Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

7-Inverse-hyperbolic-functions/7.1-Inverse-hyperbolic-sine/188-7.1.5-Inverse-hyperbolic-sine-functions

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [371]. This is test number [188].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	99.73 (370)	0.27 (1)
Rubi	99.46 (369)	0.54 (2)
Maple	67.12 (249)	32.88 (122)
Fricas	42.59 (158)	57.41 (213)
Sympy	27.22 (101)	72.78 (270)
Maxima	26.95 (100)	73.05 (271)
Giac	26.42 (98)	73.58 (273)
Mupad	21.02 (78)	78.98 (293)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

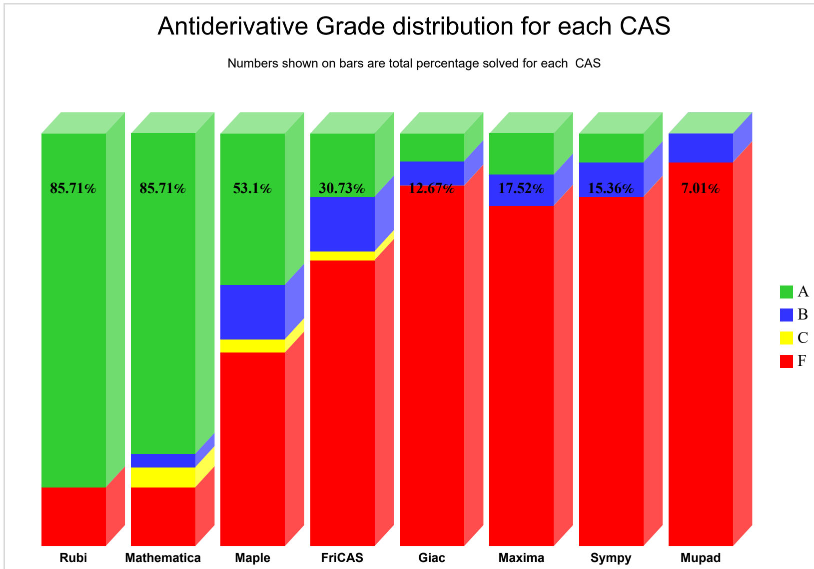
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

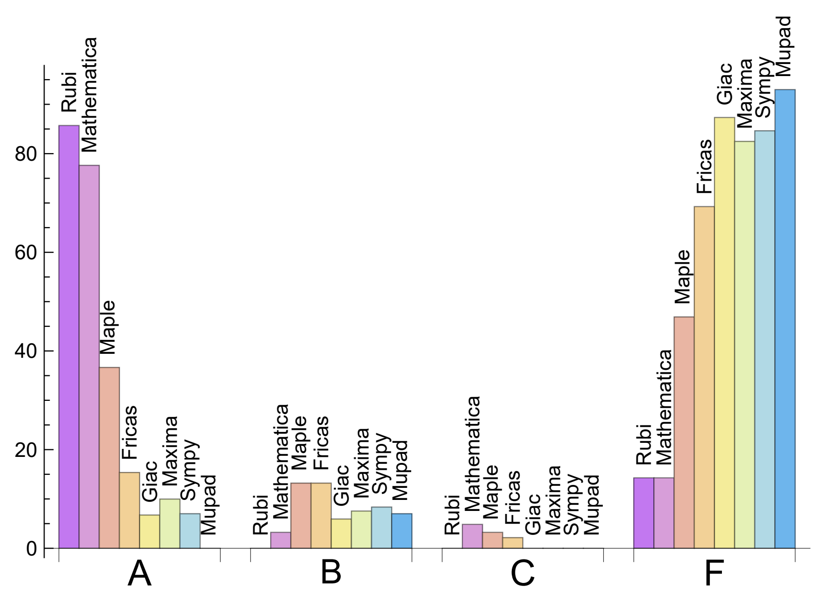
System	% A grade	% B grade	% C grade	% F grade
Mathematica	77.628	3.235	4.852	14.286
Rubi	71.698	0.000	13.747	14.555
Maple	36.658	13.208	3.235	46.900
Fricas	15.364	13.208	2.156	69.272
Maxima	9.973	7.547	0.000	82.480
Sympy	7.008	8.356	0.000	84.636
Giac	6.739	5.930	0.000	87.332
Mupad	0.000	7.008	0.000	92.992

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	1	100.00	0.00	0.00
Rubi	2	100.00	0.00	0.00
Maple	122	100.00	0.00	0.00
Fricas	213	59.62	0.00	40.38
Maxima	271	80.07	1.85	18.08
Sympy	270	83.33	6.30	10.37
Giac	273	82.42	0.37	17.22
Mupad	293	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.26
Maple	0.47
Rubi	0.68
Maxima	1.34
Mupad	2.72
Mathematica	3.05
Giac	3.50
Sympy	7.12

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	33.54	1.08	25.00	1.00
Giac	105.78	1.62	34.50	1.11
Fricas	172.00	2.23	110.00	1.72
Rubi	172.64	1.00	134.00	1.00
Mathematica	201.22	1.07	115.00	0.99
Sympy	261.81	2.25	61.00	1.44
Maple	271.97	1.45	114.00	1.10
Maxima	294.87	10.92	92.00	1.35

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

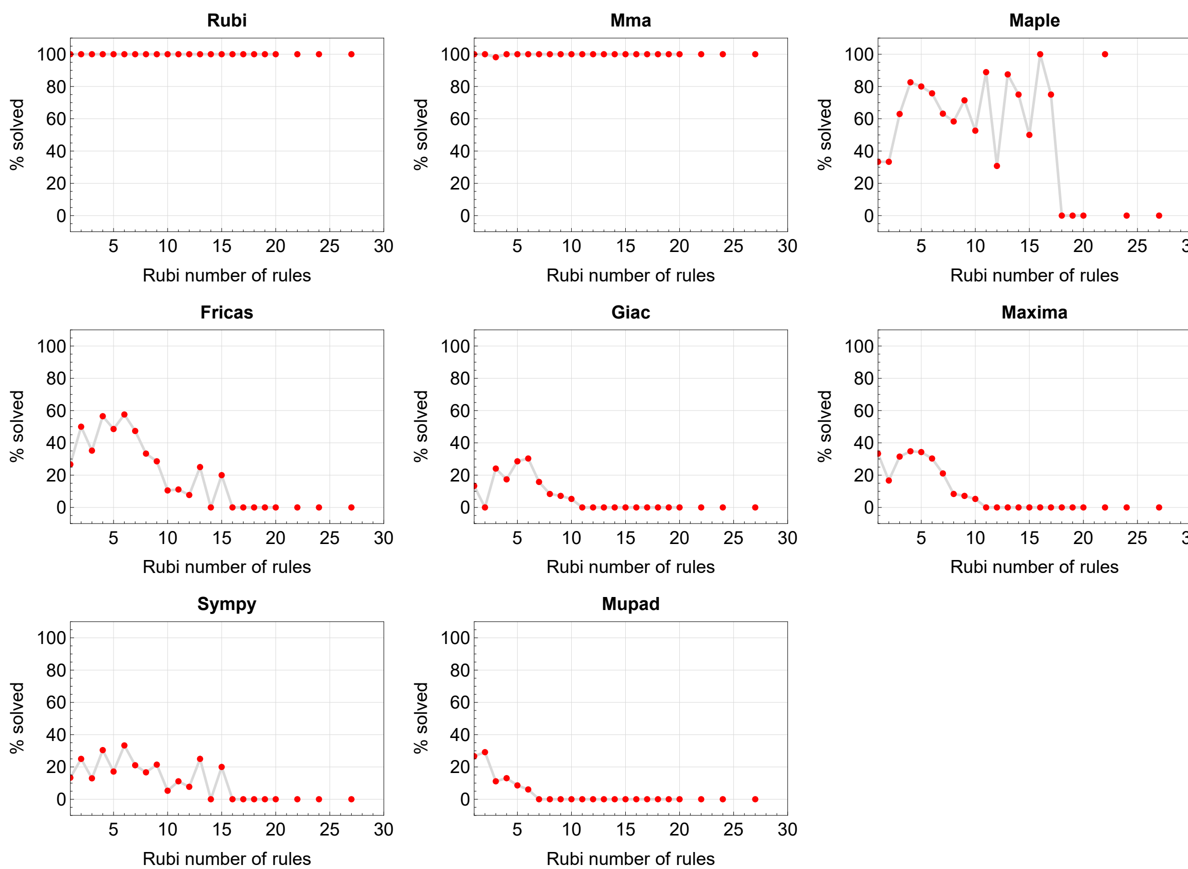


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

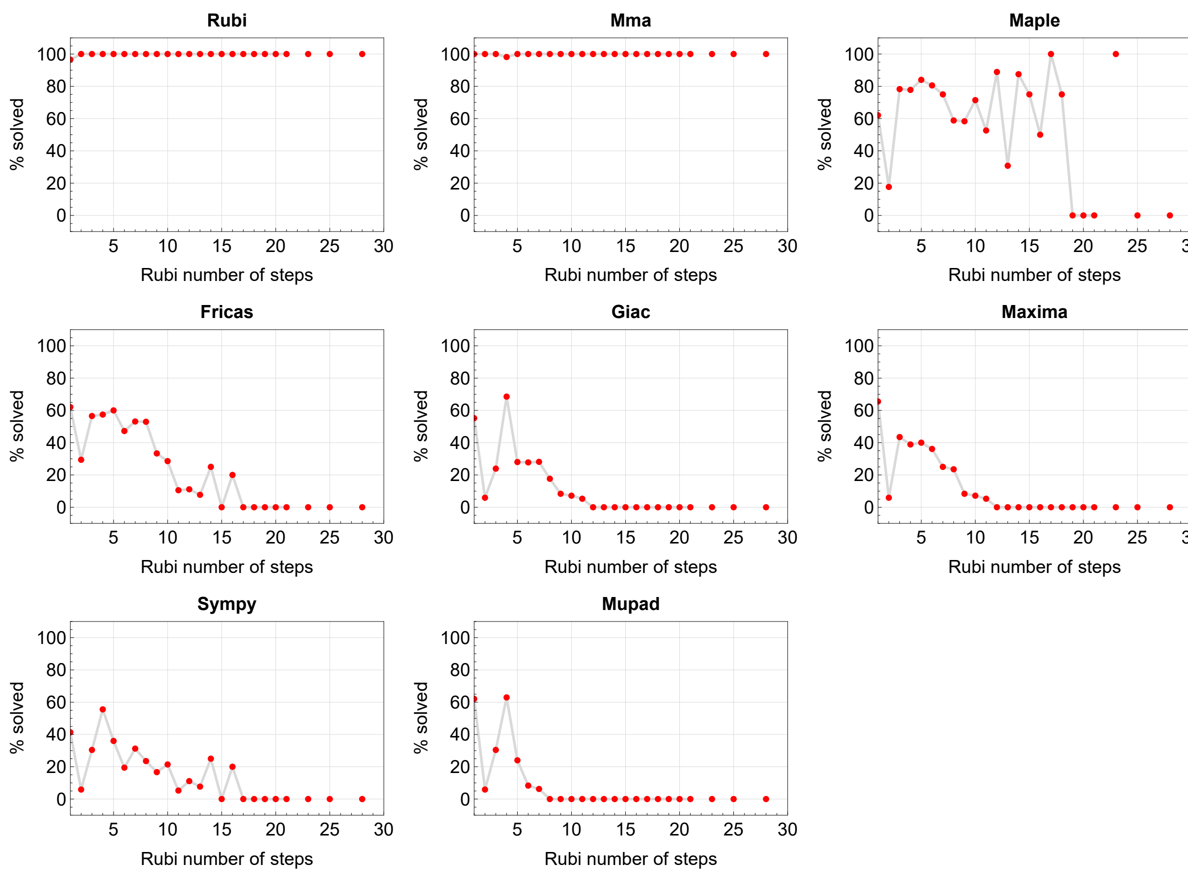


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

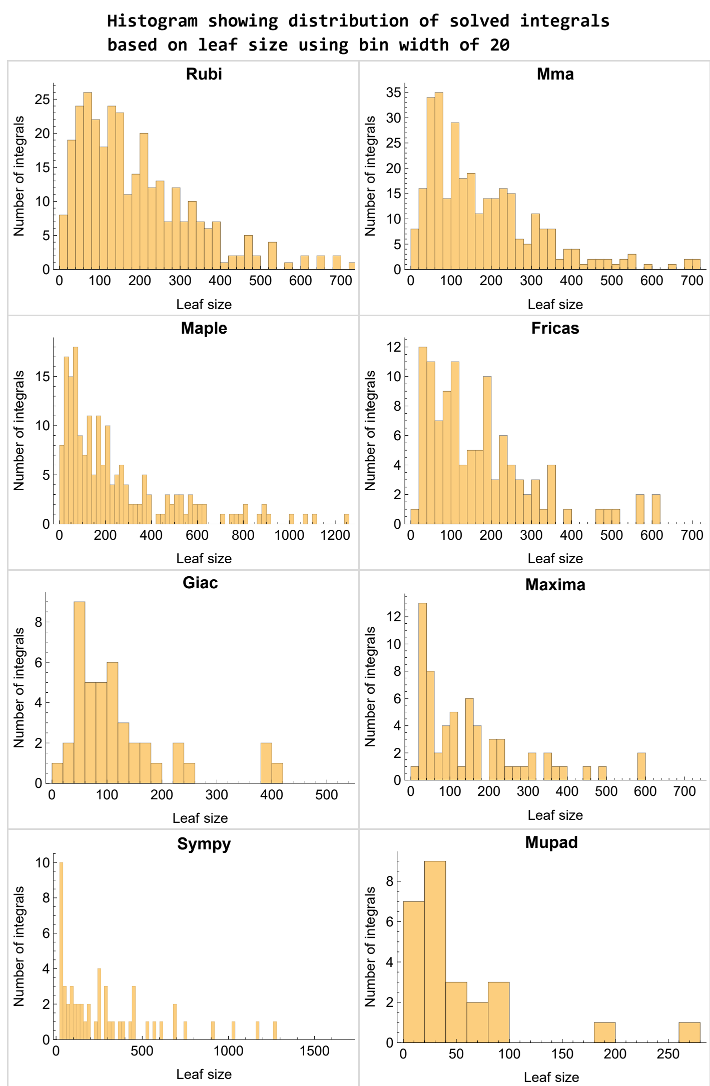


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

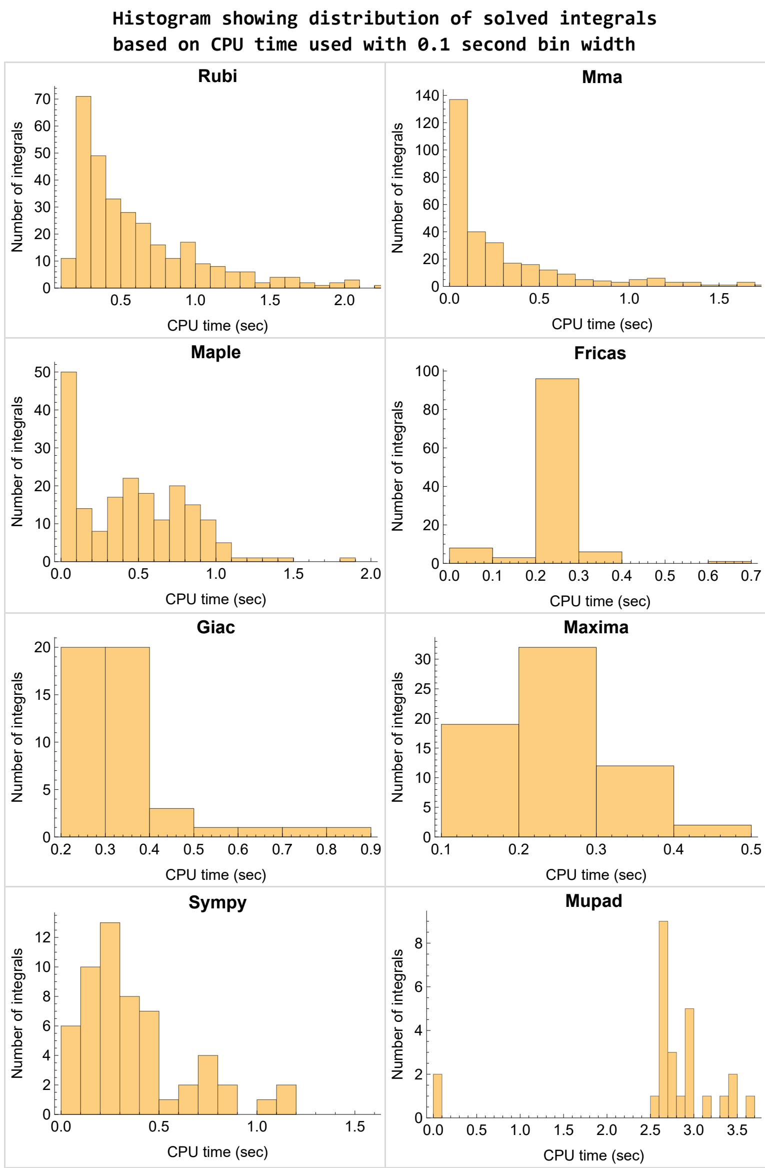


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

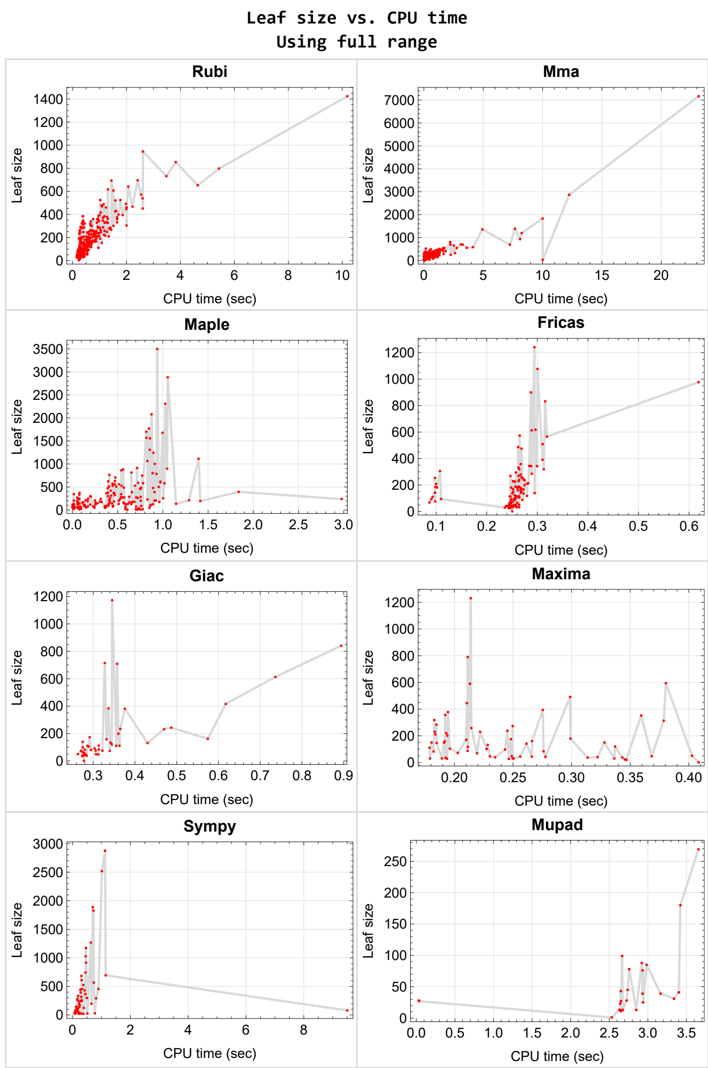


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{23, 24, 28, 29, 30, 32, 33, 53, 57, 84, 88, 92, 93, 97, 136, 146, 155, 161, 167, 173, 179, 185, 191, 197, 203, 209, 215, 221, 227, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 281, 282, 342, 346, 347, 362, 363}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {73, 74, 76, 120, 132, 133, 135, 142, 143, 144, 145, 151, 152, 153, 154, 278, 279, 343, 344, 345, 348, 366, 367}

Mathematica {37, 38, 42, 46, 145, 154, 369}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

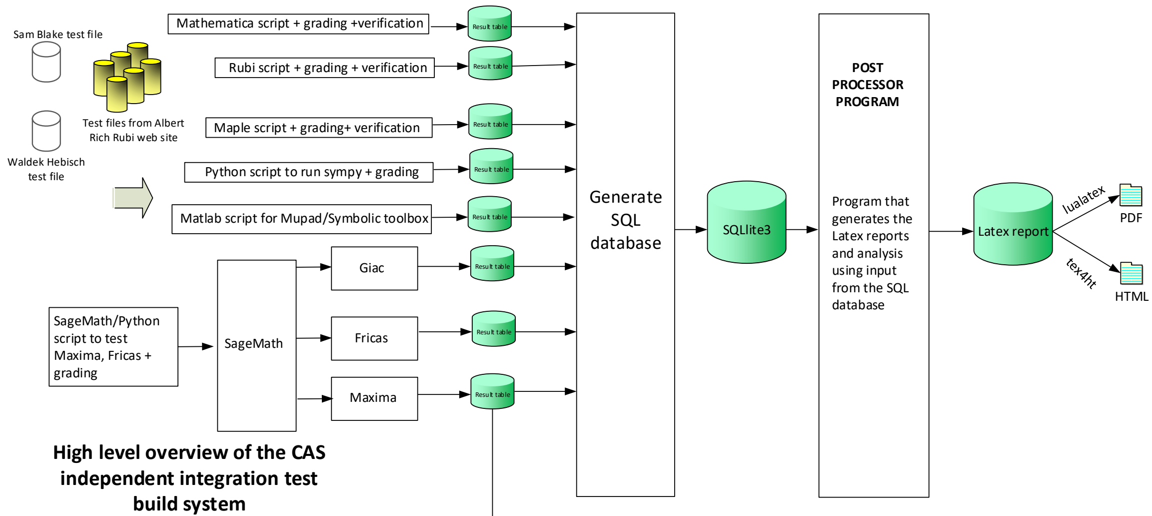
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 101, 102, 103, 105, 106, 107, 108, 110, 111, 112, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 137, 138, 139, 140, 141, 147, 148, 149, 150, 156, 157, 158, 160, 162, 163, 164, 165, 168, 170, 172, 174, 175, 177, 181, 183, 186, 188, 190, 193, 195, 200, 202, 204, 205, 206, 208, 210, 211, 212, 213, 216, 218, 220, 222, 223, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 280, 283, 284, 285, 286, 288, 289, 290, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 365, 366, 367, 368, 370, 371 }

B grade { }

C grade { 27, 100, 104, 109, 113, 120, 132, 133, 135, 142, 143, 144, 145, 151, 152, 153, 154, 159, 166, 169, 171, 176, 178, 180, 182, 184, 187, 189, 192, 194, 196, 199, 201, 207, 214, 217, 219, 224, 226, 278, 279, 287, 291, 295, 303, 311, 343, 344, 345, 348, 364 }

F normal fail { 198, 369 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 34, 35, 36, 39, 40, 41, 43, 44, 45, 47, 48, 49, 50, 51, 52, 54, 55, 56, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 153, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 198, 199, 200, 201, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 236, 237, 238, 239, 240, 241, 242, 243, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 283, 285, 287, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 364, 365, 366, 367, 370, 371 }

B grade { 61, 103, 104, 119, 145, 152, 154, 196, 202, 302, 368, 369 }

C grade { 37, 38, 42, 46, 74, 125, 228, 229, 230, 231, 232, 233, 234, 235, 284, 286, 288, 290 }

F normal fail { 31 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 4, 5, 6, 7, 8, 12, 13, 14, 15, 17, 19, 20, 21, 22, 25, 26, 27, 37, 42, 50, 51, 58, 59, 60, 61, 63, 64, 65, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 81, 82, 83, 85, 86, 87, 89, 90, 91, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 133, 134, 135, 137, 138, 139, 140, 141, 143, 144, 145, 147, 148, 154, 156, 157, 158, 159, 160, 164, 165, 166, 171, 172, 177, 178, 260, 261, 262, 263, 264, 265, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 283, 285, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 311, 317, 324, 345, 348, 351, 353, 364, 365, 366, 368 }

B grade { 9, 10, 11, 18, 34, 35, 36, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 52, 62, 66, 132, 142, 149, 150, 151, 152, 153, 162, 163, 168, 169, 170, 174, 175, 176, 266, 267, 268, 280, 343, 344, 349, 350, 352, 354, 355, 356, 357 }

C grade { 228, 229, 230, 231, 232, 233, 234, 235, 284, 286, 288, 290 }

F normal fail { 2, 3, 16, 31, 54, 55, 56, 71, 78, 79, 80, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 126, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 236, 237, 238, 239, 240, 241, 242, 243, 287, 291, 307, 308, 309, 310, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 358, 359, 360, 361, 367, 369, 370, 371 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 4, 5, 6, 7, 12, 13, 14, 58, 59, 60, 61, 67, 68, 69, 75, 76, 77, 118, 119, 260, 261, 262, 266, 267, 268, 283, 284, 285, 286, 288, 292, 293, 294, 296, 297, 298, 299, 301, 304, 305, 306, 315, 316, 317, 322, 323, 324, 349, 350, 351, 353, 355, 356, 357, 366, 369, 371 }

B grade { 9, 10, 11, 15, 63, 64, 65, 66, 70, 115, 116, 117, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 134, 137, 138, 139, 140, 141, 147, 148, 149, 150, 272, 273, 274, 275, 276, 277, 280, 289, 300, 302, 314, 321, 352, 354, 367, 368, 370 }

C grade { 228, 229, 230, 231, 232, 233, 234, 235 }

F normal fail { 1, 2, 3, 8, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 62, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 114, 120, 126, 132, 133, 135, 142, 143, 144, 145, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 236, 237, 238, 239, 240, 241, 242, 243, 263, 264, 265, 269, 270, 271, 278, 279, 287, 290, 291, 295, 303, 318, 319, 320, 325, 326, 327, 343, 344, 345, 358, 359, 360, 361, 364, 365 }

F(-1) timedout fail { }

F(-2) exception fail { 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 307, 308, 309, 310, 311, 312, 313, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 348 }

2.1.5 Maxima

A grade { 4, 5, 6, 7, 9, 10, 12, 13, 14, 15, 49, 50, 61, 64, 119, 285, 289, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 304, 306, 317, 324, 350, 353, 354, 366, 367, 369 }

B grade { 58, 59, 60, 63, 65, 66, 115, 116, 117, 118, 122, 124, 134, 262, 268, 272, 273, 274, 276, 280, 283, 305, 349, 351, 352, 355, 356, 357 }

C grade { }

F normal fail { 1, 2, 3, 8, 11, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 37, 38, 51, 52, 54, 55, 56, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 120, 123, 125, 126, 127, 128, 129, 130, 131, 132, 135, 137, 138, 139, 140, 141, 142, 144, 145, 147, 148, 149, 150, 151, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 260, 261, 263, 264, 265, 266, 267, 269, 270, 271, 275, 277, 278, 279, 284, 286, 287, 288, 290, 291, 295, 303, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 358, 359, 360, 361, 364, 365, 368, 370, 371 }

F(-1) timedout fail { 174, 175, 176, 177, 179 }

F(-2) exception fail { 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 121, 133, 143, 152, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259 }

2.1.6 Giac

A grade { 6, 7, 58, 59, 60, 280, 283, 285, 292, 293, 294, 296, 297, 298, 299, 300, 301, 304, 305, 306, 349, 350, 351, 353, 369 }

B grade { 9, 15, 61, 63, 64, 65, 66, 115, 116, 117, 118, 119, 121, 289, 302, 352, 354, 355, 356, 357, 366, 367 }

C grade { }

F normal fail { 1, 2, 3, 8, 10, 11, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 47, 48, 49, 50, 51, 52, 55, 56, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 284, 286, 287, 288, 290, 291, 295, 303, 307, 308, 309, 310, 311, 312, 313, 343, 344, 345, 348, 358, 359, 360, 361, 364, 365, 368, 370, 371 }

F(-1) timedout fail { 93 }

F(-2) exception fail { 4, 5, 12, 13, 14, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 54, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341 }

2.1.7 Mupad

A grade { }

B grade { 6, 7, 61, 119, 272, 273, 274, 275, 276, 277, 283, 285, 294, 301, 302, 304, 305, 317, 324, 353, 354, 366, 367, 368, 369, 371 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 278, 279, 280, 284, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 303, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 360, 361, 364, 365, 370 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 4, 5, 6, 7, 13, 14, 15, 60, 61, 67, 68, 69, 70, 75, 76, 77, 119, 283, 285, 294, 304, 349, 350, 351, 352, 366 }

B grade { 12, 58, 59, 115, 116, 117, 118, 127, 128, 129, 130, 131, 137, 138, 139, 140, 141, 147, 148, 149, 150, 266, 267, 268, 272, 273, 274, 275, 276, 277, 367 }

C grade { }

F normal fail { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 62, 63, 64, 65, 66, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 120, 121, 122, 123, 124, 125, 126, 132, 133, 134, 135, 142, 143, 144, 145, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 193, 194, 195, 196, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 260, 261, 262, 263, 264, 265, 269, 270, 271, 278, 279, 280, 284, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 306, 307, 308, 309, 310, 311, 312, 313, 343, 344, 345, 348, 353, 354, 355, 356, 357, 358, 359, 360, 361, 364, 365, 368, 369, 370, 371 }

F(-1) timedout fail { 43, 53, 192, 198, 199, 200, 201, 202, 203, 228, 236, 244, 245, 252, 253, 342, 347 }

F(-2) exception fail { 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	168	272	0	0	0	0	0
N.S.	1	1.00	0.99	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.618	0.010	0.706	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	260	256	240	0	0	0	0	0	0
N.S.	1	0.98	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.935	0.102	0.000	0.000	0.000	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	348	346	322	0	0	0	0	0	0
N.S.	1	0.99	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.216	0.039	0.000	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	202	166	242	230	214	316	0	0
N.S.	1	1.15	0.94	1.38	1.31	1.22	1.80	0.00	0.00
time (sec)	N/A	0.392	0.111	0.312	0.183	0.282	0.304	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	147	121	174	150	147	190	0	0
N.S.	1	1.19	0.98	1.40	1.21	1.19	1.53	0.00	0.00
time (sec)	N/A	0.301	0.079	0.323	0.180	0.252	0.229	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	106	91	84	82	87	99	122	78
N.S.	1	1.09	0.94	0.87	0.85	0.90	1.02	1.26	0.80
time (sec)	N/A	0.256	0.032	0.016	0.182	0.255	0.173	0.345	2.758

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	30	43	26	41	28
N.S.	1	1.00	1.00	1.03	1.00	1.43	0.87	1.37	0.93
time (sec)	N/A	0.157	0.007	0.013	0.189	0.250	0.070	0.275	0.032

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	175	279	0	0	0	0	0
N.S.	1	1.00	0.94	1.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.641	0.043	0.538	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	79	174	103	253	0	232	0
N.S.	1	1.00	0.96	2.12	1.26	3.09	0.00	2.83	0.00
time (sec)	N/A	0.246	0.071	0.961	0.196	0.270	0.000	0.470	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	129	166	273	158	566	0	0	0
N.S.	1	1.01	1.30	2.13	1.23	4.42	0.00	0.00	0.00
time (sec)	N/A	0.287	0.247	0.573	0.192	0.319	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	199	205	510	0	977	0	0	0
N.S.	1	1.09	1.12	2.79	0.00	5.34	0.00	0.00	0.00
time (sec)	N/A	0.353	0.300	0.406	0.000	0.619	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	387	354	526	590	475	743	0	0
N.S.	1	1.05	0.96	1.43	1.60	1.29	2.02	0.00	0.00
time (sec)	N/A	1.050	0.357	0.444	0.213	0.269	0.450	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	262	248	364	378	319	454	0	0
N.S.	1	1.10	1.04	1.52	1.58	1.33	1.90	0.00	0.00
time (sec)	N/A	0.782	0.225	0.455	0.194	0.264	0.326	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	162	142	183	219	183	233	0	0
N.S.	1	1.16	1.01	1.31	1.56	1.31	1.66	0.00	0.00
time (sec)	N/A	0.577	1.148	0.306	0.193	0.262	0.234	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	50	74	72	72	96	82	111	0
N.S.	1	1.09	1.61	1.57	1.57	2.09	1.78	2.41	0.00
time (sec)	N/A	0.285	0.033	0.000	0.203	0.256	0.099	0.363	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	285	273	0	0	0	0	0	0
N.S.	1	0.98	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.044	0.178	0.000	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	239	191	525	0	0	0	0	0
N.S.	1	0.91	0.73	2.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.996	0.145	0.543	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	331	270	815	0	0	0	0	0
N.S.	1	0.95	0.77	2.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.350	0.443	0.653	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	378	305	394	0	0	0	0	0
N.S.	1	0.96	0.77	1.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.423	0.535	1.838	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	233	188	254	0	0	0	0	0
N.S.	1	0.95	0.77	1.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.929	0.345	1.003	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	110	98	120	0	0	0	0	0
N.S.	1	0.95	0.84	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.566	0.148	0.589	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	49	45	56	0	0	0	0	0
N.S.	1	0.91	0.83	1.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.406	0.043	0.000	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	25	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.39	0.83	1.11	1.11
time (sec)	N/A	0.212	0.162	0.366	0.257	0.256	0.893	0.287	2.669

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	49	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	2.72	0.94	1.11	1.11
time (sec)	N/A	0.209	0.317	0.277	0.265	0.241	1.707	0.861	2.630

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	288	616	0	0	0	0	0
N.S.	1	1.00	0.80	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.847	1.661	0.973	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	150	272	0	0	0	0	0
N.S.	1	1.00	0.83	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.534	0.890	0.556	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	89	71	118	0	0	0	0	0
N.S.	1	1.05	0.84	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.598	0.161	0.001	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	746	51	17	20	20
N.S.	1	1.00	1.11	1.00	41.44	2.83	0.94	1.11	1.11
time (sec)	N/A	0.202	5.834	0.308	0.829	0.262	1.609	0.318	2.911

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	1050	92	19	20	20
N.S.	1	1.00	1.11	1.00	58.33	5.11	1.06	1.11	1.11
time (sec)	N/A	0.211	4.551	0.353	1.221	0.265	5.726	1.299	2.675

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	271	32	17	20	20
N.S.	1	1.00	1.11	1.00	15.06	1.78	0.94	1.11	1.11
time (sec)	N/A	0.545	5.324	1.562	0.852	0.281	5.170	0.383	2.810

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11
time (sec)	N/A	0.197	0.302	0.948	0.239	0.251	0.901	0.281	2.899

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	607	34	17	20	20
N.S.	1	1.00	1.11	1.00	33.72	1.89	0.94	1.11	1.11
time (sec)	N/A	0.205	0.669	0.928	0.961	0.277	14.414	0.297	2.650

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	640	363	410	1309	0	0	0	0	0
N.S.	1	0.57	0.64	2.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.038	0.843	0.856	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	257	301	913	0	0	0	0	0
N.S.	1	0.60	0.70	2.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.835	0.769	0.715	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	136	208	582	0	0	0	0	0
N.S.	1	0.60	0.92	2.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.528	0.959	0.781	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	664	461	1358	747	0	0	0	0	0
N.S.	1	0.69	2.05	1.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.163	4.933	0.771	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	781	572	1384	1701	0	0	0	0	0
N.S.	1	0.73	1.77	2.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.737	7.674	0.817	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	918	488	536	2079	0	0	0	0	0
N.S.	1	0.53	0.58	2.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.267	1.604	0.875	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	651	359	416	1568	0	0	0	0	0
N.S.	1	0.55	0.64	2.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.005	1.163	0.818	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	188	289	1065	0	0	0	0	0
N.S.	1	0.53	0.82	3.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.597	0.633	0.828	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	984	641	2869	1557	0	0	0	0	0
N.S.	1	0.65	2.92	1.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.241	12.244	0.858	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1228	617	810	2884	0	0	0	0	0
N.S.	1	0.50	0.66	2.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.421	2.221	1.054	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	901	470	555	2309	0	0	0	0	0
N.S.	1	0.52	0.62	2.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.194	1.726	1.026	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	494	249	390	1679	0	0	0	0	0
N.S.	1	0.50	0.79	3.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.691	0.887	0.997	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1536	945	7168	3499	0	0	0	0	0
N.S.	1	0.62	4.67	2.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.731	23.148	0.937	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	430	256	304	785	0	0	0	0	0
N.S.	1	0.60	0.71	1.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.860	1.015	0.930	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	158	233	484	0	0	0	0	0
N.S.	1	0.61	0.90	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.686	0.710	0.758	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	84	158	227	87	0	0	0	0
N.S.	1	0.70	1.32	1.89	0.72	0.00	0.00	0.00	0.00
time (sec)	N/A	0.471	0.348	0.830	0.211	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	76	77	28	0	0	0	0
N.S.	1	1.00	1.62	1.64	0.60	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	0.034	0.375	0.194	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	238	240	529	0	0	0	0	0
N.S.	1	0.73	0.74	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.009	0.352	0.691	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	327	251	1770	0	0	0	0	0
N.S.	1	0.74	0.57	3.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.350	0.407	0.846	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	32	34	34	0	34	34
N.S.	1	1.00	1.06	0.94	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.351	0.103	2.496	0.923	0.263	0.000	73.328	2.651

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	438	426	397	0	0	0	0	0	0
N.S.	1	0.97	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.712	0.141	0.000	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	332	328	304	0	0	0	0	0	0
N.S.	1	0.99	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.281	0.154	0.000	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	197	196	206	0	0	0	0	0	0
N.S.	1	0.99	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.700	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	32	34	51	31	34	34
N.S.	1	1.00	1.06	0.94	1.00	1.50	0.91	1.00	1.00
time (sec)	N/A	0.377	0.294	1.941	0.806	0.266	7.693	0.910	2.547

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	158	95	200	318	110	255	162	0
N.S.	1	1.21	0.73	1.53	2.43	0.84	1.95	1.24	0.00
time (sec)	N/A	0.383	0.063	0.053	0.183	0.257	0.323	0.574	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	117	74	130	210	91	170	131	0
N.S.	1	1.30	0.82	1.44	2.33	1.01	1.89	1.46	0.00
time (sec)	N/A	0.325	0.043	0.015	0.184	0.248	0.248	0.430	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	82	60	74	149	75	104	111	0
N.S.	1	1.08	0.79	0.97	1.96	0.99	1.37	1.46	0.00
time (sec)	N/A	0.292	0.031	0.012	0.191	0.251	0.181	0.286	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	32	78	31	30	57	46	92	76
N.S.	1	0.94	2.29	0.91	0.88	1.68	1.35	2.71	2.24
time (sec)	N/A	0.213	0.158	0.013	0.179	0.246	0.078	0.303	2.933

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	153	388	0	0	0	0	0
N.S.	1	1.00	1.17	2.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.596	0.011	0.658	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	61	57	68	111	167	0	110	0
N.S.	1	1.07	1.00	1.19	1.95	2.93	0.00	1.93	0.00
time (sec)	N/A	0.291	0.030	0.242	0.179	0.273	0.000	0.355	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	99	110	103	146	236	0	199	0
N.S.	1	1.08	1.20	1.12	1.59	2.57	0.00	2.16	0.00
time (sec)	N/A	0.316	0.163	0.017	0.185	0.275	0.000	0.360	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	148	149	203	284	285	0	381	0
N.S.	1	1.15	1.16	1.57	2.20	2.21	0.00	2.95	0.00
time (sec)	N/A	0.356	0.189	0.015	0.185	0.289	0.000	0.376	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	200	179	345	357	343	0	709	0
N.S.	1	1.20	1.07	2.07	2.14	2.05	0.00	4.25	0.00
time (sec)	N/A	0.408	0.181	0.016	0.192	0.300	0.000	0.357	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	305	145	293	0	182	366	0	0
N.S.	1	0.92	0.44	0.89	0.00	0.55	1.11	0.00	0.00
time (sec)	N/A	0.728	0.501	0.204	0.000	0.265	0.441	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	204	107	190	0	146	243	0	0
N.S.	1	0.97	0.51	0.90	0.00	0.69	1.15	0.00	0.00
time (sec)	N/A	0.604	0.411	0.167	0.000	0.252	0.300	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	120	79	113	0	114	138	0	0
N.S.	1	0.95	0.63	0.90	0.00	0.90	1.10	0.00	0.00
time (sec)	N/A	0.497	0.226	0.127	0.000	0.246	0.226	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	49	47	46	0	88	63	0	0
N.S.	1	1.09	1.04	1.02	0.00	1.96	1.40	0.00	0.00
time (sec)	N/A	0.300	0.019	0.081	0.000	0.270	0.100	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	205	207	251	0	0	0	0	0	0
N.S.	1	1.01	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.826	0.026	0.000	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	174	178	206	0	0	0	0	0
N.S.	1	0.98	1.00	1.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.814	0.089	0.250	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	237	279	374	0	0	0	0	0
N.S.	1	1.01	1.19	1.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.067	0.089	0.461	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	478	696	1830	764	0	0	0	0	0
N.S.	1	1.46	3.83	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.633	10.001	0.408	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	317	175	297	0	225	432	0	0
N.S.	1	0.89	0.49	0.84	0.00	0.63	1.22	0.00	0.00
time (sec)	N/A	0.681	0.510	0.199	0.000	0.256	0.420	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	191	129	169	0	180	248	0	0
N.S.	1	0.94	0.64	0.83	0.00	0.89	1.22	0.00	0.00
time (sec)	N/A	0.581	0.281	0.120	0.000	0.261	0.292	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	74	70	67	0	139	109	0	0
N.S.	1	0.95	0.90	0.86	0.00	1.78	1.40	0.00	0.00
time (sec)	N/A	0.372	0.022	0.093	0.000	0.295	0.142	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	275	285	346	0	0	0	0	0	0
N.S.	1	1.04	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.057	0.031	0.000	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	246	259	0	0	0	0	0	0
N.S.	1	0.92	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.129	0.097	0.000	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	514	446	524	0	0	0	0	0	0
N.S.	1	0.87	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.127	0.160	0.000	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	52	44	49	0	0	0	0	0
N.S.	1	0.87	0.73	0.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.581	0.233	0.244	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	28	30	27	0	0	0	0	0
N.S.	1	0.93	1.00	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.410	0.127	0.096	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	0	0	0	0
N.S.	1	1.00	1.00	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	0.011	0.080	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	10	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	0.83	1.17	1.17
time (sec)	N/A	0.255	0.233	0.983	0.246	0.242	0.316	0.345	2.726

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	137	83	146	0	0	0	0	0
N.S.	1	0.89	0.54	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.421	0.770	0.279	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	77	62	73	0	0	0	0	0
N.S.	1	0.92	0.74	0.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	0.186	0.171	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	35	35	34	0	0	0	0	0
N.S.	1	0.92	0.92	0.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.367	0.069	0.083	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	527	14	12	14	14
N.S.	1	1.00	1.17	1.00	43.92	1.17	1.00	1.17	1.17
time (sec)	N/A	0.258	2.129	0.846	0.948	0.244	0.390	0.340	2.707

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	225	110	215	0	0	0	0	0
N.S.	1	0.88	0.43	0.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.680	0.330	0.146	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	131	117	107	0	0	0	0	0
N.S.	1	0.89	0.80	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.464	0.135	0.131	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	57	53	51	0	0	0	0	0
N.S.	1	0.90	0.84	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	0.033	0.091	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	3273	14	12	14	14
N.S.	1	1.00	1.17	1.00	272.75	1.17	1.00	1.17	1.17
time (sec)	N/A	0.255	1.587	0.835	15.191	0.246	0.472	0.330	2.835

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	0	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	0.00	1.12
time (sec)	N/A	0.283	0.344	0.322	0.575	0.272	12.848	0.000	2.820

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	545	525	345	0	0	0	0	0	0
N.S.	1	0.96	0.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.140	0.664	0.000	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	267	262	228	0	0	0	0	0	0
N.S.	1	0.98	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.695	0.135	0.000	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	131	109	0	0	0	0	0	0
N.S.	1	1.02	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.411	0.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.12
time (sec)	N/A	0.281	0.212	0.310	0.595	0.257	0.573	1.398	2.662

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	496	525	656	0	0	0	0	0	0
N.S.	1	1.06	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.860	2.525	0.000	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	259	284	250	0	0	0	0	0	0
N.S.	1	1.10	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.048	2.267	0.000	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	121	111	0	0	0	0	0	0
N.S.	1	1.05	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.623	0.133	0.000	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	326	372	582	0	0	0	0	0	0
N.S.	1	1.14	1.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.246	4.136	0.000	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	149	272	0	0	0	0	0	0
N.S.	1	0.99	1.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.702	0.400	0.000	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	389	460	939	0	0	0	0	0	0
N.S.	1	1.18	2.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.362	8.103	0.000	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	181	458	0	0	0	0	0	0
N.S.	1	1.01	2.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.913	1.447	0.000	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	411	395	498	0	0	0	0	0	0
N.S.	1	0.96	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.086	1.599	0.000	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	204	200	217	0	0	0	0	0	0
N.S.	1	0.98	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.757	1.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	93	111	0	0	0	0	0	0
N.S.	1	1.01	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.467	0.089	0.000	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	269	258	322	0	0	0	0	0	0
N.S.	1	0.96	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.776	2.613	0.000	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	134	155	0	0	0	0	0	0
N.S.	1	1.10	1.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.658	0.115	0.000	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	365	343	555	0	0	0	0	0	0
N.S.	1	0.94	1.52	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.043	3.614	0.000	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	162	207	0	0	0	0	0	0
N.S.	1	1.03	1.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.764	0.233	0.000	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	445	417	703	0	0	0	0	0	0
N.S.	1	0.94	1.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.140	3.117	0.000	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	210	238	0	0	0	0	0	0
N.S.	1	1.08	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.966	0.192	0.000	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	89	79	0	0	0	0	0	0
N.S.	1	0.98	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.299	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	84	71	93	1231	279	527	841	0
N.S.	1	0.84	0.71	0.93	12.31	2.79	5.27	8.41	0.00
time (sec)	N/A	0.324	0.065	0.385	0.214	0.267	0.371	0.894	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	96	83	86	790	228	394	613	0
N.S.	1	0.91	0.79	0.82	7.52	2.17	3.75	5.84	0.00
time (sec)	N/A	0.305	0.050	0.360	0.211	0.254	0.282	0.736	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	67	64	73	445	168	258	416	0
N.S.	1	0.88	0.84	0.96	5.86	2.21	3.39	5.47	0.00
time (sec)	N/A	0.310	0.033	0.359	0.211	0.252	0.170	0.617	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	65	57	62	201	108	148	243	0
N.S.	1	0.96	0.84	0.91	2.96	1.59	2.18	3.57	0.00
time (sec)	N/A	0.263	0.044	0.013	0.194	0.258	0.132	0.487	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	83	36	35	65	51	99	85
N.S.	1	1.00	2.13	0.92	0.90	1.67	1.31	2.54	2.18
time (sec)	N/A	0.174	0.158	0.061	0.192	0.247	0.082	0.275	2.984

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	101	70	129	0	0	0	0	0
N.S.	1	1.25	0.86	1.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.579	0.019	0.650	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	44	43	54	0	175	0	134	0
N.S.	1	0.90	0.88	1.10	0.00	3.57	0.00	2.73	0.00
time (sec)	N/A	0.284	0.030	0.365	0.000	0.268	0.000	0.342	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	54	57	60	117	118	0	0	0
N.S.	1	0.92	0.97	1.02	1.98	2.00	0.00	0.00	0.00
time (sec)	N/A	0.270	0.039	0.368	0.212	0.280	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	71	69	74	0	343	0	0	0
N.S.	1	0.85	0.82	0.88	0.00	4.08	0.00	0.00	0.00
time (sec)	N/A	0.293	0.083	0.446	0.000	0.288	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	83	61	80	258	210	0	0	0
N.S.	1	0.92	0.68	0.89	2.87	2.33	0.00	0.00	0.00
time (sec)	N/A	0.285	0.049	0.362	0.214	0.280	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	100	61	94	0	509	0	0	0
N.S.	1	0.87	0.53	0.82	0.00	4.43	0.00	0.00	0.00
time (sec)	N/A	0.306	0.057	0.398	0.000	0.310	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	180	155	0	0	0	0	0	0
N.S.	1	0.96	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.486	0.075	0.000	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	171	192	218	0	618	1268	0	0
N.S.	1	0.87	0.97	1.11	0.00	3.14	6.44	0.00	0.00
time (sec)	N/A	0.694	0.266	0.407	0.000	0.296	0.635	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	151	170	194	0	486	916	0	0
N.S.	1	0.88	0.99	1.13	0.00	2.83	5.33	0.00	0.00
time (sec)	N/A	0.646	0.230	0.412	0.000	0.262	0.469	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	120	147	163	0	358	610	0	0
N.S.	1	0.88	1.08	1.20	0.00	2.63	4.49	0.00	0.00
time (sec)	N/A	0.529	0.213	0.411	0.000	0.267	0.316	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	96	120	135	0	230	335	0	0
N.S.	1	0.93	1.17	1.31	0.00	2.23	3.25	0.00	0.00
time (sec)	N/A	0.477	0.219	0.078	0.000	0.263	0.189	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	59	87	88	0	141	143	0	0
N.S.	1	1.04	1.53	1.54	0.00	2.47	2.51	0.00	0.00
time (sec)	N/A	0.330	0.093	0.131	0.000	0.267	0.125	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	154	100	312	0	0	0	0	0
N.S.	1	1.33	0.86	2.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.768	0.034	0.429	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	88	164	185	0	0	0	0	0
N.S.	1	0.88	1.64	1.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.573	0.547	0.459	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	74	120	150	230	319	0	0	0
N.S.	1	0.87	1.41	1.76	2.71	3.75	0.00	0.00	0.00
time (sec)	N/A	0.403	0.289	0.566	0.222	0.313	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	143	229	236	0	0	0	0	0
N.S.	1	0.85	1.36	1.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.732	1.313	0.528	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	716	55	20	25	25
N.S.	1	1.00	1.09	1.00	31.13	2.39	0.87	1.09	1.09
time (sec)	N/A	0.466	1.499	1.065	3.980	0.280	17.299	0.820	2.767

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	332	355	420	0	1077	2518	0	0
N.S.	1	1.02	1.09	1.29	0.00	3.30	7.72	0.00	0.00
time (sec)	N/A	1.348	0.431	0.423	0.000	0.300	1.017	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	288	303	365	0	832	1828	0	0
N.S.	1	1.03	1.09	1.31	0.00	2.98	6.55	0.00	0.00
time (sec)	N/A	1.161	0.364	0.376	0.000	0.316	0.722	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	207	258	302	0	613	1173	0	0
N.S.	1	0.91	1.14	1.33	0.00	2.70	5.17	0.00	0.00
time (sec)	N/A	0.830	0.323	0.393	0.000	0.289	0.465	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	151	200	243	0	391	685	0	0
N.S.	1	0.94	1.24	1.51	0.00	2.43	4.25	0.00	0.00
time (sec)	N/A	0.644	0.267	0.076	0.000	0.310	0.307	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	93	147	160	0	239	282	0	0
N.S.	1	0.93	1.47	1.60	0.00	2.39	2.82	0.00	0.00
time (sec)	N/A	0.369	0.145	0.109	0.000	0.264	0.167	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	211	128	556	0	0	0	0	0
N.S.	1	1.36	0.83	3.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.922	0.032	0.467	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	142	315	377	0	0	0	0	0
N.S.	1	0.86	1.90	2.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.761	0.651	0.392	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	163	229	325	0	0	0	0	0
N.S.	1	1.04	1.46	2.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.870	0.718	0.469	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	261	223	694	486	0	0	0	0	0
N.S.	1	0.85	2.66	1.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.228	7.245	0.569	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	940	71	20	25	25
N.S.	1	1.00	1.09	1.00	40.87	3.09	0.87	1.09	1.09
time (sec)	N/A	0.456	0.902	1.092	5.240	0.273	35.373	1.189	2.759

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	374	475	573	0	1241	2876	0	0
N.S.	1	1.07	1.36	1.64	0.00	3.56	8.24	0.00	0.00
time (sec)	N/A	1.765	0.538	0.443	0.000	0.294	1.124	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	278	412	473	0	900	1889	0	0
N.S.	1	0.99	1.47	1.68	0.00	3.20	6.72	0.00	0.00
time (sec)	N/A	1.430	0.454	0.391	0.000	0.287	0.700	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	182	300	371	0	574	1027	0	0
N.S.	1	0.93	1.54	1.90	0.00	2.94	5.27	0.00	0.00
time (sec)	N/A	0.932	0.354	0.079	0.000	0.265	0.454	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	111	226	245	0	344	444	0	0
N.S.	1	0.97	1.97	2.13	0.00	2.99	3.86	0.00	0.00
time (sec)	N/A	0.516	0.227	0.173	0.000	0.284	0.268	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	264	157	861	0	0	0	0	0
N.S.	1	1.42	0.84	4.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.100	0.044	0.541	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	B	F(-2)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	200	510	630	0	0	0	0	0
N.S.	1	0.85	2.18	2.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.951	1.324	0.468	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	214	349	561	0	0	0	0	0
N.S.	1	1.15	1.88	3.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.080	1.347	0.515	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	A	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	385	331	1198	883	0	0	0	0	0
N.S.	1	0.86	3.11	2.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.723	8.235	0.561	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	19	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.83	1.09	1.09
time (sec)	N/A	0.283	0.453	0.782	0.281	0.245	0.901	0.333	2.789

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	169	151	194	0	0	0	0	0
N.S.	1	0.79	0.71	0.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.619	0.389	1.414	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	119	109	134	0	0	0	0	0
N.S.	1	0.82	0.75	0.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.551	0.312	1.146	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	115	102	130	0	0	0	0	0
N.S.	1	0.82	0.72	0.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.514	0.301	0.511	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	68	61	66	0	0	0	0	0
N.S.	1	0.99	0.88	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.599	0.099	0.055	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	53	49	60	0	0	0	0	0
N.S.	1	0.91	0.84	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.464	0.051	0.055	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	31	34	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.35	1.48	1.09	1.09
time (sec)	N/A	0.289	0.723	0.205	0.300	0.243	1.009	0.346	2.627

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	208	281	602	0	0	0	0	0
N.S.	1	0.81	1.10	2.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.551	1.251	0.973	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	158	193	388	0	0	0	0	0
N.S.	1	0.84	1.03	2.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.467	1.099	0.908	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	154	138	342	0	0	0	0	0
N.S.	1	0.84	0.75	1.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.457	0.911	0.538	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	95	97	160	0	0	0	0	0
N.S.	1	0.92	0.94	1.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.549	0.439	0.062	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	93	77	128	0	0	0	0	0
N.S.	1	1.02	0.85	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.624	0.182	0.056	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	1086	61	73	25	25
N.S.	1	1.00	1.09	1.00	47.22	2.65	3.17	1.09	1.09
time (sec)	N/A	0.285	1.366	0.173	1.634	0.269	1.887	0.348	2.681

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	383	316	896	0	0	0	0	0
N.S.	1	1.20	0.99	2.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.234	1.021	1.047	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	285	179	579	0	0	0	0	0
N.S.	1	1.15	0.72	2.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.497	0.625	1.018	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	266	216	507	0	0	0	0	0
N.S.	1	1.08	0.88	2.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.542	0.672	0.484	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	153	120	239	0	0	0	0	0
N.S.	1	0.98	0.77	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.168	0.322	0.069	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	117	100	190	0	0	0	0	0
N.S.	1	0.94	0.80	1.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.733	0.238	0.053	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	6716	91	112	25	25
N.S.	1	1.00	1.09	1.00	292.00	3.96	4.87	1.09	1.09
time (sec)	N/A	0.281	0.904	0.152	68.046	0.281	3.600	0.366	2.701

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	477	410	1244	0	0	0	0	0
N.S.	1	1.16	1.00	3.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.120	1.835	0.896	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	362	318	800	0	0	0	0	0
N.S.	1	1.06	0.94	2.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.387	1.172	0.886	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	360	258	709	0	0	0	0	0
N.S.	1	1.09	0.78	2.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.777	0.927	0.474	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	190	181	333	0	0	0	0	0
N.S.	1	0.93	0.89	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.110	0.780	0.076	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	165	130	272	0	0	0	0	0
N.S.	1	1.03	0.81	1.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.931	0.410	0.054	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	0	121	151	25	25
N.S.	1	1.00	1.09	1.00	0.00	5.26	6.57	1.09	1.09
time (sec)	N/A	0.285	4.671	0.161	0.000	0.333	7.476	0.386	2.650

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	361	345	342	0	0	0	0	0	0
N.S.	1	0.96	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.078	0.516	0.000	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	272	248	223	0	0	0	0	0	0
N.S.	1	0.91	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.865	0.237	0.000	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	245	237	238	0	0	0	0	0	0
N.S.	1	0.97	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.870	0.318	0.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	164	154	122	0	0	0	0	0	0
N.S.	1	0.94	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.681	0.073	0.000	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	121	111	0	0	0	0	0	0
N.S.	1	1.05	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.614	0.177	0.000	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	20	25	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.80	1.00	1.00
time (sec)	N/A	0.307	2.561	0.210	0.659	0.000	0.397	1.508	2.944

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	601	731	343	0	0	0	0	0	0
N.S.	1	1.22	0.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.668	0.414	0.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	360	451	225	0	0	0	0	0	0
N.S.	1	1.25	0.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.792	0.235	0.000	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	328	394	238	0	0	0	0	0	0
N.S.	1	1.20	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.918	0.231	0.000	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	205	209	125	0	0	0	0	0	0
N.S.	1	1.02	0.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.396	0.076	0.000	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	149	272	0	0	0	0	0	0
N.S.	1	0.99	1.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.713	0.412	0.000	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	51	25	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	2.04	1.00	1.00
time (sec)	N/A	0.312	0.226	0.203	0.781	0.000	2.379	2.312	2.629

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	701	853	324	0	0	0	0	0	0
N.S.	1	1.22	0.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.059	0.367	0.000	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	455	539	209	0	0	0	0	0	0
N.S.	1	1.18	0.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.720	0.211	0.000	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	394	467	219	0	0	0	0	0	0
N.S.	1	1.19	0.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.405	0.234	0.000	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	248	126	0	0	0	0	0	0
N.S.	1	0.95	0.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.350	0.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	181	458	0	0	0	0	0	0
N.S.	1	1.01	2.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.903	1.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	88	25	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	3.52	1.00	1.00
time (sec)	N/A	0.313	0.257	0.189	1.079	0.000	22.455	3.763	2.539

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	835	0	324	0	0	0	0	0	0
N.S.	1	0.00	0.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.497	0.000	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	547	797	208	0	0	0	0	0	0
N.S.	1	1.46	0.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.755	0.220	0.000	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	481	652	220	0	0	0	0	0	0
N.S.	1	1.36	0.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.881	0.280	0.000	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	305	303	125	0	0	0	0	0	0
N.S.	1	0.99	0.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.125	0.071	0.000	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	209	698	0	0	0	0	0	0
N.S.	1	0.97	3.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.025	3.269	0.000	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	0	25	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	0.00	1.00	1.00
time (sec)	N/A	0.297	0.272	0.206	2.533	0.000	0.000	5.196	2.559

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	326	300	320	0	0	0	0	0	0
N.S.	1	0.92	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.757	0.297	0.000	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	203	205	0	0	0	0	0	0
N.S.	1	0.94	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.640	0.184	0.000	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	214	200	217	0	0	0	0	0	0
N.S.	1	0.93	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.635	0.179	0.000	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	122	119	0	0	0	0	0	0
N.S.	1	1.08	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.595	0.056	0.000	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	93	111	0	0	0	0	0	0
N.S.	1	1.01	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.454	0.071	0.000	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	36	25	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	1.44	1.00	1.00
time (sec)	N/A	0.306	0.123	0.214	0.798	0.000	0.695	1.846	2.601

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	367	338	490	0	0	0	0	0	0
N.S.	1	0.92	1.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.724	0.589	0.000	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	245	253	0	0	0	0	0	0
N.S.	1	0.94	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.577	0.338	0.000	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	255	238	327	0	0	0	0	0	0
N.S.	1	0.93	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.584	0.252	0.000	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	153	147	0	0	0	0	0	0
N.S.	1	1.03	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.565	0.100	0.000	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	134	155	0	0	0	0	0	0
N.S.	1	1.10	1.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.656	0.189	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	88	25	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	3.52	1.00	1.00
time (sec)	N/A	0.313	0.143	0.219	0.750	0.000	1.899	0.510	2.633

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	437	607	551	0	0	0	0	0	0
N.S.	1	1.39	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.593	2.772	0.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	326	429	390	0	0	0	0	0	0
N.S.	1	1.32	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.734	1.280	0.000	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	321	399	389	0	0	0	0	0	0
N.S.	1	1.24	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.851	1.147	0.000	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	209	216	227	0	0	0	0	0	0
N.S.	1	1.03	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.279	0.522	0.000	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	162	207	0	0	0	0	0	0
N.S.	1	1.03	1.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.758	0.434	0.000	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	155	25	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	6.20	1.00	1.00
time (sec)	N/A	0.318	0.139	0.208	0.709	0.000	8.739	0.587	2.705

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	531	693	701	0	0	0	0	0	0
N.S.	1	1.31	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.541	2.245	0.000	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	420	521	429	0	0	0	0	0	0
N.S.	1	1.24	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.689	1.622	0.000	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	410	492	474	0	0	0	0	0	0
N.S.	1	1.20	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.123	1.201	0.000	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	252	257	235	0	0	0	0	0	0
N.S.	1	1.02	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.251	0.647	0.000	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	210	238	0	0	0	0	0	0
N.S.	1	1.08	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.995	0.139	0.000	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	0	221	25	25
N.S.	1	1.00	1.08	0.92	1.00	0.00	8.84	1.00	1.00
time (sec)	N/A	0.323	0.138	0.246	0.798	0.000	105.850	0.675	2.593

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	323	113	238	0	306	0	0	0
N.S.	1	1.08	0.38	0.80	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.473	0.159	2.978	0.000	0.108	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	194	113	212	0	254	0	0	0
N.S.	1	1.10	0.64	1.20	0.00	1.44	0.00	0.00	0.00
time (sec)	N/A	0.352	0.127	1.289	0.000	0.098	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	286	87	205	0	185	0	0	0
N.S.	1	1.10	0.33	0.79	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.434	0.036	0.908	0.000	0.098	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	157	87	179	0	142	0	0	0
N.S.	1	1.11	0.61	1.26	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.318	0.026	0.685	0.000	0.098	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	247	61	161	0	93	0	0	0
N.S.	1	1.11	0.27	0.72	0.00	0.42	0.00	0.00	0.00
time (sec)	N/A	0.410	0.031	0.565	0.000	0.091	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	116	56	140	0	110	0	0	0
N.S.	1	1.09	0.53	1.32	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.318	0.020	0.437	0.000	0.093	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	286	58	202	0	181	0	0	0
N.S.	1	1.08	0.22	0.76	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.438	0.028	0.513	0.000	0.102	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	159	61	176	0	208	0	0	0
N.S.	1	1.10	0.42	1.21	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.339	0.031	0.661	0.000	0.100	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	135	110	0	0	0	0	0	0
N.S.	1	1.01	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.488	0.096	0.000	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	135	110	0	0	0	0	0	0
N.S.	1	1.01	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.469	0.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	135	110	0	0	0	0	0	0
N.S.	1	1.01	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.466	0.083	0.000	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	135	110	0	0	0	0	0	0
N.S.	1	1.01	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.450	0.072	0.000	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	131	110	0	0	0	0	0	0
N.S.	1	0.99	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.452	0.050	0.000	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	129	109	0	0	0	0	0	0
N.S.	1	0.99	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.457	0.073	0.000	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	131	106	0	0	0	0	0	0
N.S.	1	0.98	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.470	0.060	0.000	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	135	110	0	0	0	0	0	0
N.S.	1	1.01	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.471	0.060	0.000	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	256	0	25	25
N.S.	1	1.00	1.08	0.92	0.00	10.24	0.00	1.00	1.00
time (sec)	N/A	0.459	74.280	0.910	0.000	0.288	0.000	2.121	2.706

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	186	0	25	25
N.S.	1	1.00	1.08	0.92	0.00	7.44	0.00	1.00	1.00
time (sec)	N/A	0.461	120.369	0.219	0.000	0.283	0.000	1.823	2.659

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	100	22	25	25
N.S.	1	1.00	1.08	0.92	0.00	4.00	0.88	1.00	1.00
time (sec)	N/A	0.463	66.169	0.239	0.000	0.312	20.338	1.612	2.680

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	55	22	25	25
N.S.	1	1.00	1.08	0.92	0.00	2.20	0.88	1.00	1.00
time (sec)	N/A	0.448	104.771	0.237	0.000	0.268	3.880	1.149	2.973

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	55	22	25	25
N.S.	1	1.00	1.08	0.92	0.00	2.20	0.88	1.00	1.00
time (sec)	N/A	0.451	61.732	0.087	0.000	0.266	3.403	0.599	2.799

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	83	22	25	25
N.S.	1	1.00	1.08	0.92	0.00	3.32	0.88	1.00	1.00
time (sec)	N/A	0.452	43.205	0.329	0.000	0.265	6.559	0.751	2.798

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	97	22	25	25
N.S.	1	1.00	1.08	0.92	0.00	3.88	0.88	1.00	1.00
time (sec)	N/A	0.458	39.195	0.253	0.000	0.279	14.788	2.983	2.660

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	111	22	25	25
N.S.	1	1.00	1.08	0.92	0.00	4.44	0.88	1.00	1.00
time (sec)	N/A	0.463	73.145	0.250	0.000	0.280	61.702	0.759	2.703

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	330	0	25	25
N.S.	1	1.00	1.08	0.92	0.00	13.20	0.00	1.00	1.00
time (sec)	N/A	0.457	37.754	0.899	0.000	0.301	0.000	172.173	2.757

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	240	0	25	25
N.S.	1	1.00	1.08	0.92	0.00	9.60	0.00	1.00	1.00
time (sec)	N/A	0.453	74.088	0.225	0.000	0.282	0.000	2.364	2.810

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	130	22	25	25
N.S.	1	1.00	1.08	0.92	0.00	5.20	0.88	1.00	1.00
time (sec)	N/A	0.452	34.309	0.216	0.000	0.268	39.495	2.244	2.762

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	71	22	25	25
N.S.	1	1.00	1.08	0.92	0.00	2.84	0.88	1.00	1.00
time (sec)	N/A	0.446	57.210	0.232	0.000	0.278	6.792	1.465	2.815

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	71	22	25	25
N.S.	1	1.00	1.08	0.92	0.00	2.84	0.88	1.00	1.00
time (sec)	N/A	0.427	13.866	0.074	0.000	0.282	5.190	0.698	2.931

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	99	22	25	25
N.S.	1	1.00	1.08	0.92	0.00	3.96	0.88	1.00	1.00
time (sec)	N/A	0.445	16.330	0.263	0.000	0.279	9.407	0.906	2.737

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	113	22	25	25
N.S.	1	1.00	1.08	0.92	0.00	4.52	0.88	1.00	1.00
time (sec)	N/A	0.453	10.218	0.244	0.000	0.271	18.432	3.848	2.817

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	0	127	22	25	25
N.S.	1	1.00	1.08	0.92	0.00	5.08	0.88	1.00	1.00
time (sec)	N/A	0.458	53.079	0.259	0.000	0.280	68.421	0.921	2.809

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	122	127	204	0	199	0	0	0
N.S.	1	0.93	0.97	1.56	0.00	1.52	0.00	0.00	0.00
time (sec)	N/A	0.691	0.082	0.924	0.000	0.275	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	101	110	167	0	161	0	0	0
N.S.	1	0.94	1.03	1.56	0.00	1.50	0.00	0.00	0.00
time (sec)	N/A	0.463	0.065	0.661	0.000	0.261	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	56	61	91	238	98	0	0	0
N.S.	1	0.92	1.00	1.49	3.90	1.61	0.00	0.00	0.00
time (sec)	N/A	0.327	0.039	0.860	0.245	0.257	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	29	24	23	0	0	0	0	0
N.S.	1	0.94	0.77	0.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.389	0.052	0.585	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	33	47	44	0	0	0	0	0
N.S.	1	0.92	1.31	1.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.475	0.049	0.852	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	63	85	63	0	0	0	0	0
N.S.	1	0.89	1.20	0.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.430	0.171	0.587	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	301	266	592	0	332	694	0	0
N.S.	1	1.28	1.13	2.52	0.00	1.41	2.95	0.00	0.00
time (sec)	N/A	1.619	0.162	0.727	0.000	0.263	1.145	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	226	211	479	0	259	568	0	0
N.S.	1	1.20	1.12	2.53	0.00	1.37	3.01	0.00	0.00
time (sec)	N/A	0.765	0.110	0.892	0.000	0.269	0.736	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	116	124	262	394	160	298	0	0
N.S.	1	1.09	1.17	2.47	3.72	1.51	2.81	0.00	0.00
time (sec)	N/A	0.470	0.065	0.722	0.275	0.267	0.495	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	42	37	36	0	0	0	0	0
N.S.	1	0.89	0.79	0.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.414	0.478	0.736	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	55	70	72	0	0	0	0	0
N.S.	1	1.02	1.30	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.487	0.222	0.704	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	110	108	110	0	0	0	0	0
N.S.	1	1.31	1.29	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.035	0.211	0.869	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	179	32	26	0	13
N.S.	1	1.00	1.00	0.93	11.93	2.13	1.73	0.00	0.87
time (sec)	N/A	0.285	0.016	0.778	0.299	0.264	0.289	0.000	2.641

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	132	32	26	0	13
N.S.	1	1.00	1.00	0.93	8.80	2.13	1.73	0.00	0.87
time (sec)	N/A	0.284	0.015	0.717	0.228	0.245	0.231	0.000	2.676

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	84	32	24	0	13
N.S.	1	1.00	1.00	0.93	5.60	2.13	1.60	0.00	0.87
time (sec)	N/A	0.255	0.013	0.701	0.276	0.259	0.207	0.000	2.850

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	30	22	0	11
N.S.	1	1.00	1.00	1.09	0.00	2.73	2.00	0.00	1.00
time (sec)	N/A	0.286	0.024	0.717	0.000	0.236	0.240	0.000	2.650

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	150	32	26	0	13
N.S.	1	1.00	1.00	1.08	11.54	2.46	2.00	0.00	1.00
time (sec)	N/A	0.281	0.012	0.724	0.328	0.247	0.511	0.000	2.631

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	0	32	29	0	13
N.S.	1	1.00	1.00	0.93	0.00	2.13	1.93	0.00	0.87
time (sec)	N/A	0.278	0.011	0.750	0.000	0.246	0.772	0.000	2.634

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	119	128	203	0	0	0	0	0
N.S.	1	1.03	1.11	1.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.706	0.426	0.755	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	93	98	168	0	0	0	0	0
N.S.	1	1.08	1.14	1.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.591	0.300	0.630	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	44	62	131	119	115	0	76	0
N.S.	1	0.96	1.35	2.85	2.59	2.50	0.00	1.65	0.00
time (sec)	N/A	0.291	0.066	0.612	0.337	0.264	0.000	0.306	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	30	82	31	30	30
N.S.	1	1.00	1.07	0.93	1.00	2.73	1.03	1.00	1.00
time (sec)	N/A	0.317	0.512	0.372	0.332	0.237	1.613	0.303	2.648

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	610	82	32	30	30
N.S.	1	1.00	1.07	0.93	20.33	2.73	1.07	1.00	1.00
time (sec)	N/A	0.407	2.209	0.322	2.535	0.256	1.769	0.310	2.630

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	56	44	71	102	52	42	74	45
N.S.	1	1.12	0.88	1.42	2.04	1.04	0.84	1.48	0.90
time (sec)	N/A	0.242	0.014	0.073	0.227	0.264	0.229	0.271	2.738

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	107	75	89	0	68	0	0	0
N.S.	1	1.06	0.74	0.88	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.267	0.086	0.451	0.000	0.087	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	33	34	29	30	42	27	40	28
N.S.	1	0.97	1.00	0.85	0.88	1.24	0.79	1.18	0.82
time (sec)	N/A	0.259	0.009	0.017	0.251	0.239	0.086	0.284	2.725

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	173	35	77	0	94	0	0	0
N.S.	1	1.07	0.22	0.48	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.305	10.014	0.135	0.000	0.110	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	67	54	0	0	0	0	0	0
N.S.	1	1.24	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.356	0.007	0.000	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	42	66	0	76	0	0	0
N.S.	1	1.00	0.56	0.88	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.222	0.028	0.079	0.000	0.098	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	46	106	0	58	0
N.S.	1	1.00	1.00	0.85	1.39	3.21	0.00	1.76	0.00
time (sec)	N/A	0.224	0.006	0.014	0.230	0.265	0.000	0.278	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	203	88	101	0	0	0	0	0
N.S.	1	1.03	0.45	0.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.361	0.105	0.139	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	67	54	0	0	0	0	0	0
N.S.	1	1.24	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.344	0.008	0.000	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	82	43	47	46	40	0	50	0
N.S.	1	1.14	0.60	0.65	0.64	0.56	0.00	0.69	0.00
time (sec)	N/A	0.213	0.017	0.058	0.368	0.257	0.000	0.263	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	61	37	37	36	35	0	48	0
N.S.	1	1.09	0.66	0.66	0.64	0.62	0.00	0.86	0.00
time (sec)	N/A	0.207	0.014	0.012	0.314	0.250	0.000	0.280	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	34	46	24	23	28	29	40	31
N.S.	1	0.97	1.31	0.69	0.66	0.80	0.83	1.14	0.89
time (sec)	N/A	0.204	0.075	0.010	0.345	0.243	0.107	0.277	3.339

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	65	46	78	0	0	0	0	0
N.S.	1	1.41	1.00	1.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	0.008	0.229	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	25	0	35	0
N.S.	1	1.00	1.00	0.81	0.77	0.96	0.00	1.35	0.00
time (sec)	N/A	0.180	0.009	0.010	0.347	0.246	0.000	0.274	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	51	34	31	30	32	0	52	0
N.S.	1	1.11	0.74	0.67	0.65	0.70	0.00	1.13	0.00
time (sec)	N/A	0.196	0.011	0.011	0.336	0.252	0.000	0.280	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	72	39	41	40	37	0	67	0
N.S.	1	1.16	0.63	0.66	0.65	0.60	0.00	1.08	0.00
time (sec)	N/A	0.211	0.013	0.012	0.322	0.259	0.000	0.274	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	93	44	51	50	42	0	82	0
N.S.	1	1.19	0.56	0.65	0.64	0.54	0.00	1.05	0.00
time (sec)	N/A	0.217	0.015	0.012	0.403	0.242	0.000	0.280	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	54	69	122	0	74	0
N.S.	1	1.00	1.02	0.96	1.23	2.18	0.00	1.32	0.00
time (sec)	N/A	0.289	0.030	0.151	0.219	0.255	0.000	0.340	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	38	27	45	0	47	27
N.S.	1	1.00	0.88	1.15	0.82	1.36	0.00	1.42	0.82
time (sec)	N/A	0.237	0.018	0.015	0.246	0.255	0.000	0.312	0.036

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	77	31	43	96	0	49	25
N.S.	1	1.00	3.08	1.24	1.72	3.84	0.00	1.96	1.00
time (sec)	N/A	0.236	0.061	0.014	0.278	0.255	0.000	0.307	2.938

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	65	52	114	0	0	0	0	0
N.S.	1	1.25	1.00	2.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.354	0.008	0.197	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	31	30	49	20	39	27
N.S.	1	1.00	1.00	1.07	1.03	1.69	0.69	1.34	0.93
time (sec)	N/A	0.234	0.014	0.011	0.250	0.257	0.369	0.284	2.652

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	56	44	46	97	58	0	84	43
N.S.	1	1.12	0.88	0.92	1.94	1.16	0.00	1.68	0.86
time (sec)	N/A	0.258	0.018	0.013	0.243	0.244	0.000	0.313	2.649

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	58	48	50	47	62	0	75	0
N.S.	1	1.07	0.89	0.93	0.87	1.15	0.00	1.39	0.00
time (sec)	N/A	0.284	0.022	0.013	0.249	0.248	0.000	0.322	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	74	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	0.053	0.000	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	66	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	0.034	0.000	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	58	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.035	0.000	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.218	0.040	0.000	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	68	52	120	0	0	0	0	0
N.S.	1	1.13	0.87	2.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.376	0.014	0.256	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.239	0.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	69	62	0	0	0	0	0	0
N.S.	1	1.01	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	149	0	0	269	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	0.316	0.088	0.000	0.000	0.263	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	127	180	0	0	188	0	0	0
N.S.	1	0.98	1.40	0.00	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.289	0.098	0.000	0.000	0.257	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	114	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.50	0.00	0.00	0.00
time (sec)	N/A	0.213	0.015	0.000	0.000	0.247	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	47	44	52	0	0	39
N.S.	1	1.00	0.96	0.94	0.88	1.04	0.00	0.00	0.78
time (sec)	N/A	0.201	0.020	0.024	0.266	0.245	0.000	0.000	3.165

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	150	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.285	0.617	0.000	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	245	245	197	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	1.104	0.000	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	275	283	229	0	0	0	0	0	0
N.S.	1	1.03	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.369	0.402	0.000	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	149	0	0	269	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	0.303	0.088	0.000	0.000	0.247	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	127	180	0	0	188	0	0	0
N.S.	1	0.98	1.40	0.00	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.265	0.095	0.000	0.000	0.263	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	114	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.50	0.00	0.00	0.00
time (sec)	N/A	0.195	0.015	0.000	0.000	0.263	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	48	44	52	0	0	39
N.S.	1	1.00	0.96	0.96	0.88	1.04	0.00	0.00	0.78
time (sec)	N/A	0.173	0.020	0.025	0.256	0.254	0.000	0.000	2.931

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	191	191	146	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.249	0.574	0.000	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	244	244	196	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.286	1.153	0.000	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	272	280	227	0	0	0	0	0	0
N.S.	1	1.03	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.373	0.485	0.000	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	348	344	337	0	0	0	0	0	0
N.S.	1	0.99	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.419	0.205	0.000	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	312	312	258	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.394	0.165	0.000	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	263	263	259	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	0.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	231	180	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.275	0.005	0.000	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	291	291	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	0.284	0.000	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	326	324	308	0	0	0	0	0	0
N.S.	1	0.99	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.383	0.587	0.000	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	389	384	365	0	0	0	0	0	0
N.S.	1	0.99	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.400	0.690	0.000	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	348	343	337	0	0	0	0	0	0
N.S.	1	0.99	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.402	0.209	0.000	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	310	312	255	0	0	0	0	0	0
N.S.	1	1.01	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.395	0.184	0.000	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	262	259	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.288	0.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	231	180	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.276	0.005	0.000	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	291	291	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.326	0.254	0.000	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	326	324	308	0	0	0	0	0	0
N.S.	1	0.99	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.385	0.591	0.000	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	389	384	370	0	0	0	0	0	0
N.S.	1	0.99	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.396	0.692	0.000	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	38	0	38	39
N.S.	1	1.00	1.05	0.90	0.98	0.95	0.00	0.95	0.98
time (sec)	N/A	0.252	0.169	0.510	0.971	0.268	0.000	1.984	3.070

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	293	244	1113	0	0	0	0	0
N.S.	1	1.12	0.93	4.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.987	0.045	1.395	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	218	187	624	0	0	0	0	0
N.S.	1	1.12	0.96	3.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.836	0.040	0.395	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	143	127	258	0	0	0	0	0
N.S.	1	1.08	0.95	1.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.634	0.020	0.415	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	48	61	38	39
N.S.	1	1.00	1.05	0.90	0.98	1.20	1.52	0.95	0.98
time (sec)	N/A	0.263	0.280	0.340	0.486	0.250	163.878	5.648	2.856

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	474	91	0	38	39
N.S.	1	1.00	1.05	0.90	11.85	2.28	0.00	0.95	0.98
time (sec)	N/A	0.259	1.132	0.312	0.594	0.270	0.000	20.277	3.629

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	82	68	153	0	0	0	0	0
N.S.	1	1.08	0.89	2.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.421	0.447	0.257	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	143	119	464	491	138	456	173	0
N.S.	1	0.87	0.72	2.81	2.98	0.84	2.76	1.05	0.00
time (sec)	N/A	0.437	0.061	0.753	0.299	0.251	0.897	0.291	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	103	102	288	273	117	292	140	0
N.S.	1	0.90	0.89	2.50	2.37	1.02	2.54	1.22	0.00
time (sec)	N/A	0.386	0.075	0.628	0.250	0.258	0.814	0.275	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	58	73	145	175	93	197	106	0
N.S.	1	0.87	1.09	2.16	2.61	1.39	2.94	1.58	0.00
time (sec)	N/A	0.328	0.068	0.576	0.248	0.249	0.656	0.289	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	46	89	141	73	124	80	0
N.S.	1	1.00	1.48	2.87	4.55	2.35	4.00	2.58	0.00
time (sec)	N/A	0.226	0.027	0.761	0.261	0.247	0.394	0.295	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	99	126	160	136	0	158	180
N.S.	1	1.00	1.11	1.42	1.80	1.53	0.00	1.78	2.02
time (sec)	N/A	0.346	0.051	0.892	0.266	0.273	0.000	0.332	3.422

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	110	284	170	183	0	234	269
N.S.	1	1.00	1.11	2.87	1.72	1.85	0.00	2.36	2.72
time (sec)	N/A	0.342	0.111	0.755	0.210	0.259	0.000	0.365	3.656

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	129	459	313	181	0	384	0
N.S.	1	1.00	1.11	3.96	2.70	1.56	0.00	3.31	0.00
time (sec)	N/A	0.336	0.143	0.750	0.379	0.248	0.000	0.337	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	162	502	352	230	0	715	0
N.S.	1	1.00	1.04	3.22	2.26	1.47	0.00	4.58	0.00
time (sec)	N/A	0.362	0.077	0.954	0.359	0.252	0.000	0.328	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	192	1003	594	295	0	1173	0
N.S.	1	1.00	0.93	4.85	2.87	1.43	0.00	5.67	0.00
time (sec)	N/A	0.444	0.591	0.911	0.380	0.258	0.000	0.345	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	359	328	198	0	0	0	0	0	0
N.S.	1	0.91	0.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.953	0.249	0.000	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	251	231	138	0	0	0	0	0	0
N.S.	1	0.92	0.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.748	0.125	0.000	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	110	76	0	0	0	0	0	0
N.S.	1	0.94	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.559	0.102	0.000	0.000	0.000	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	63	44	0	0	0	0	0	0
N.S.	1	0.97	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.292	0.031	0.000	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	15	15	12	15	15
N.S.	1	1.00	1.14	0.93	1.07	1.07	0.86	1.07	1.07
time (sec)	N/A	0.210	0.147	0.007	0.624	0.257	0.360	0.317	2.566

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	15	15	14	15	15
N.S.	1	1.00	1.14	0.93	1.07	1.07	1.00	1.07	1.07
time (sec)	N/A	0.211	0.342	0.006	0.450	0.258	0.386	0.333	2.567

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	68	52	134	0	0	0	0	0
N.S.	1	1.13	0.87	2.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.461	0.015	0.404	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	0	0	0	0	0
N.S.	1	1.00	1.00	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	0.077	0.608	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	35	41	38	37	66	61	105	88
N.S.	1	0.78	0.91	0.84	0.82	1.47	1.36	2.33	1.96
time (sec)	N/A	0.288	0.019	0.020	0.343	0.260	0.179	0.289	2.921

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	B	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	37	41	0	39	152	80	113	99
N.S.	1	0.80	0.89	0.00	0.85	3.30	1.74	2.46	2.15
time (sec)	N/A	0.310	0.030	0.000	0.235	0.269	9.504	0.313	2.666

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	357	46	0	242	0	0	41
N.S.	1	1.00	7.29	0.94	0.00	4.94	0.00	0.00	0.84
time (sec)	N/A	0.299	0.895	0.360	0.000	0.273	0.000	0.000	3.401

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	A	A	F	A	B
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	148	0	1	1	0	1	1
N.S.	1	0.00	5.48	0.00	0.04	0.04	0.00	0.04	0.04
time (sec)	N/A	0.000	0.216	0.000	0.408	0.250	0.000	0.278	2.532

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	108	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	2.92	0.00	0.00	0.00
time (sec)	N/A	0.313	0.036	0.000	0.000	0.263	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	24	0	0	33	0	0	23
N.S.	1	1.00	0.83	0.00	0.00	1.14	0.00	0.00	0.79
time (sec)	N/A	0.295	0.018	0.000	0.000	0.245	0.000	0.000	2.640

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [74] had the largest ratio of [1.83332999999999990]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.00	12	0.417
2	A	7	6	0.98	14	0.429
3	A	8	7	0.99	14	0.500
4	A	6	6	1.15	16	0.375
5	A	4	4	1.19	16	0.250
6	A	4	4	1.09	14	0.286
7	A	1	1	1.00	8	0.125
8	A	6	5	1.00	16	0.312
9	A	4	3	1.00	16	0.188
10	A	5	4	1.01	16	0.250
11	A	7	6	1.09	16	0.375
12	A	3	3	1.05	18	0.167
13	A	3	3	1.10	18	0.167
14	A	3	3	1.16	16	0.188
15	A	3	3	1.09	10	0.300
16	A	7	6	0.98	18	0.333
17	A	11	10	0.91	18	0.556
18	A	15	14	0.95	18	0.778
19	A	4	3	0.96	18	0.167
20	A	4	3	0.95	18	0.167
21	A	4	3	0.95	16	0.188
22	A	9	8	0.91	10	0.800

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	N/A	1	0	1.00	18	0.000
24	N/A	1	0	1.00	18	0.000
25	A	2	2	1.00	18	0.111
26	A	2	2	1.00	16	0.125
27	C	12	11	1.05	10	1.100
28	N/A	1	0	1.00	18	0.000
29	N/A	1	0	1.00	18	0.000
30	N/A	2	0	1.00	18	0.000
31	A	4	3	1.00	16	0.188
32	N/A	1	0	1.00	18	0.000
33	N/A	1	0	1.00	18	0.000
34	A	3	3	0.57	30	0.100
35	A	3	3	0.60	30	0.100
36	A	3	3	0.60	28	0.107
37	A	7	7	0.69	30	0.233
38	A	7	7	0.73	30	0.233
39	A	3	3	0.53	30	0.100
40	A	3	3	0.55	30	0.100
41	A	3	3	0.53	28	0.107
42	A	3	3	0.65	30	0.100
43	A	3	3	0.50	30	0.100
44	A	3	3	0.52	30	0.100
45	A	3	3	0.50	28	0.107
46	A	3	3	0.62	30	0.100
47	A	3	3	0.60	30	0.100
48	A	3	3	0.61	30	0.100
49	A	3	3	0.70	28	0.107
50	A	1	1	1.00	23	0.043
51	A	10	9	0.73	30	0.300
52	A	14	13	0.74	30	0.433
53	N/A	1	0	1.00	34	0.000
54	A	9	8	0.97	34	0.235
55	A	8	7	0.99	32	0.219

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	8	7	0.99	24	0.292
57	N/A	1	0	1.00	34	0.000
58	A	10	9	1.21	10	0.900
59	A	8	7	1.30	10	0.700
60	A	9	8	1.08	8	1.000
61	A	4	3	0.94	6	0.500
62	A	9	8	1.00	10	0.800
63	A	6	5	1.07	10	0.500
64	A	8	7	1.08	10	0.700
65	A	9	8	1.15	10	0.800
66	A	11	10	1.20	10	1.000
67	A	7	6	0.92	12	0.500
68	A	6	5	0.97	12	0.417
69	A	7	6	0.95	10	0.600
70	A	5	4	1.09	8	0.500
71	A	10	9	1.01	12	0.750
72	A	12	11	0.98	12	0.917
73	A	17	16	1.01	12	1.333
74	A	23	22	1.46	12	1.833
75	A	8	7	0.89	12	0.583
76	A	9	8	0.94	10	0.800
77	A	6	5	0.95	8	0.625
78	A	11	10	1.04	12	0.833
79	A	13	12	0.92	12	1.000
80	A	20	19	0.87	12	1.583
81	A	6	5	0.87	12	0.417
82	A	7	6	0.93	10	0.600
83	A	5	4	1.00	8	0.500
84	N/A	5	0	1.00	12	0.000
85	A	5	4	0.89	12	0.333
86	A	6	5	0.92	10	0.500
87	A	7	6	0.92	8	0.750
88	N/A	5	0	1.00	12	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
89	A	5	4	0.88	12	0.333
90	A	6	5	0.89	10	0.500
91	A	7	6	0.90	8	0.750
92	N/A	5	0	1.00	12	0.000
93	N/A	3	0	1.00	16	0.000
94	A	6	5	0.96	16	0.312
95	A	7	6	0.98	14	0.429
96	A	7	6	1.02	12	0.500
97	N/A	5	0	1.00	16	0.000
98	A	8	7	1.06	18	0.389
99	A	9	8	1.10	16	0.500
100	C	11	10	1.05	14	0.714
101	A	9	8	1.14	16	0.500
102	A	11	10	0.99	14	0.714
103	A	9	8	1.18	16	0.500
104	C	13	12	1.01	14	0.857
105	A	8	7	0.96	18	0.389
106	A	9	8	0.98	16	0.500
107	A	9	8	1.01	14	0.571
108	A	6	5	0.96	16	0.312
109	C	11	10	1.10	14	0.714
110	A	6	5	0.94	16	0.312
111	A	11	10	1.03	14	0.714
112	A	6	5	0.94	16	0.312
113	C	13	12	1.08	14	0.857
114	A	4	3	0.98	21	0.143
115	A	7	6	0.84	21	0.286
116	A	7	6	0.91	21	0.286
117	A	7	6	0.88	21	0.286
118	A	6	5	0.96	19	0.263
119	A	1	1	1.00	10	0.100
120	C	11	10	1.25	21	0.476
121	A	7	6	0.90	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
122	A	5	4	0.92	21	0.190
123	A	8	7	0.85	21	0.333
124	A	6	5	0.92	21	0.238
125	A	9	8	0.87	21	0.381
126	A	4	3	0.96	23	0.130
127	A	10	9	0.87	23	0.391
128	A	9	8	0.88	23	0.348
129	A	8	7	0.88	23	0.304
130	A	7	6	0.93	21	0.286
131	A	5	4	1.04	12	0.333
132	C	12	11	1.33	23	0.478
133	C	10	9	0.88	23	0.391
134	A	6	5	0.87	23	0.217
135	C	12	11	0.85	23	0.478
136	N/A	4	0	1.00	23	0.000
137	A	16	15	1.02	23	0.652
138	A	14	13	1.03	23	0.565
139	A	11	10	0.91	23	0.435
140	A	9	8	0.94	21	0.381
141	A	5	4	0.93	12	0.333
142	C	13	12	1.36	23	0.522
143	C	11	10	0.86	23	0.435
144	C	13	12	1.04	23	0.522
145	C	16	15	0.85	23	0.652
146	N/A	4	0	1.00	23	0.000
147	A	13	12	1.07	23	0.522
148	A	14	13	0.99	23	0.565
149	A	10	9	0.93	21	0.429
150	A	7	6	0.97	12	0.500
151	C	14	13	1.42	23	0.565
152	C	12	11	0.85	23	0.478
153	C	14	13	1.15	23	0.565
154	C	16	15	0.86	23	0.652

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
155	N/A	3	0	1.00	23	0.000
156	A	6	5	0.79	23	0.217
157	A	7	6	0.82	23	0.261
158	A	6	5	0.82	23	0.217
159	C	15	14	0.99	21	0.667
160	A	10	9	0.91	12	0.750
161	N/A	4	0	1.00	23	0.000
162	A	5	4	0.81	23	0.174
163	A	5	4	0.84	23	0.174
164	A	5	4	0.84	23	0.174
165	A	11	10	0.92	21	0.476
166	C	13	12	1.02	12	1.000
167	N/A	4	0	1.00	23	0.000
168	A	8	7	1.20	23	0.304
169	C	18	17	1.15	23	0.739
170	A	16	15	1.08	23	0.652
171	C	18	17	0.98	21	0.810
172	A	12	11	0.94	12	0.917
173	N/A	4	0	1.00	23	0.000
174	A	7	6	1.16	23	0.261
175	A	14	13	1.06	23	0.565
176	C	18	17	1.09	23	0.739
177	A	14	13	0.93	21	0.619
178	C	15	14	1.03	12	1.167
179	N/A	4	0	1.00	23	0.000
180	C	10	9	0.96	25	0.360
181	A	8	7	0.91	25	0.280
182	C	10	9	0.97	25	0.360
183	A	9	8	0.94	23	0.348
184	C	11	10	1.05	14	0.714
185	N/A	4	0	1.00	25	0.000
186	A	20	19	1.22	25	0.760
187	C	21	20	1.25	25	0.800

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
188	A	16	15	1.20	25	0.600
189	C	16	15	1.02	23	0.652
190	A	11	10	0.99	14	0.714
191	N/A	4	0	1.00	25	0.000
192	C	20	19	1.22	25	0.760
193	A	14	13	1.18	25	0.520
194	C	18	17	1.19	25	0.680
195	A	12	11	0.95	23	0.478
196	C	13	12	1.01	14	0.857
197	N/A	4	0	1.00	25	0.000
198	F	0	0	N/A	0.000	N/A
199	C	25	24	1.46	25	0.960
200	A	28	27	1.36	25	1.080
201	C	19	18	0.99	23	0.783
202	A	13	12	0.97	14	0.857
203	N/A	4	0	1.00	25	0.000
204	A	6	5	0.92	25	0.200
205	A	7	6	0.94	25	0.240
206	A	6	5	0.93	25	0.200
207	C	13	12	1.08	23	0.522
208	A	9	8	1.01	14	0.571
209	N/A	4	0	1.00	25	0.000
210	A	5	4	0.92	25	0.160
211	A	5	4	0.94	25	0.160
212	A	5	4	0.93	25	0.160
213	A	10	9	1.03	23	0.391
214	C	11	10	1.10	14	0.714
215	N/A	4	0	1.00	25	0.000
216	A	8	7	1.39	25	0.280
217	C	16	15	1.32	25	0.600
218	A	15	14	1.24	25	0.560
219	C	16	15	1.03	23	0.652
220	A	11	10	1.03	14	0.714

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
221	N/A	4	0	1.00	25	0.000
222	A	7	6	1.31	25	0.240
223	A	13	12	1.24	25	0.480
224	C	16	15	1.20	25	0.600
225	A	13	12	1.02	23	0.522
226	C	13	12	1.08	14	0.857
227	N/A	4	0	1.00	25	0.000
228	A	10	9	1.08	23	0.391
229	A	7	6	1.10	23	0.261
230	A	9	8	1.10	23	0.348
231	A	6	5	1.11	23	0.217
232	A	8	7	1.11	23	0.304
233	A	5	4	1.09	23	0.174
234	A	9	8	1.08	23	0.348
235	A	6	5	1.10	23	0.217
236	A	4	3	1.01	25	0.120
237	A	4	3	1.01	25	0.120
238	A	4	3	1.01	25	0.120
239	A	4	3	1.01	25	0.120
240	A	4	3	0.99	25	0.120
241	A	4	3	0.99	25	0.120
242	A	4	3	0.98	25	0.120
243	A	4	3	1.01	25	0.120
244	N/A	4	0	1.00	25	0.000
245	N/A	4	0	1.00	25	0.000
246	N/A	4	0	1.00	25	0.000
247	N/A	4	0	1.00	25	0.000
248	N/A	4	0	1.00	25	0.000
249	N/A	4	0	1.00	25	0.000
250	N/A	4	0	1.00	25	0.000
251	N/A	4	0	1.00	25	0.000
252	N/A	4	0	1.00	25	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
253	N/A	4	0	1.00	25	0.000
254	N/A	4	0	1.00	25	0.000
255	N/A	4	0	1.00	25	0.000
256	N/A	4	0	1.00	25	0.000
257	N/A	4	0	1.00	25	0.000
258	N/A	4	0	1.00	25	0.000
259	N/A	4	0	1.00	25	0.000
260	A	8	7	0.93	30	0.233
261	A	7	6	0.94	30	0.200
262	A	5	4	0.92	28	0.143
263	A	6	5	0.94	30	0.167
264	A	9	8	0.92	30	0.267
265	A	6	5	0.89	30	0.167
266	A	16	15	1.28	30	0.500
267	A	12	11	1.20	30	0.367
268	A	8	7	1.09	28	0.250
269	A	6	5	0.89	30	0.167
270	A	6	5	1.02	30	0.167
271	A	11	10	1.31	30	0.333
272	A	3	2	1.00	30	0.067
273	A	3	2	1.00	30	0.067
274	A	3	2	1.00	28	0.071
275	A	3	2	1.00	30	0.067
276	A	3	2	1.00	30	0.067
277	A	3	2	1.00	30	0.067
278	C	11	10	1.03	30	0.333
279	C	10	9	1.08	30	0.300
280	A	4	3	0.96	28	0.107
281	N/A	3	0	1.00	30	0.000
282	N/A	4	0	1.00	30	0.000
283	A	6	5	1.12	10	0.500
284	A	4	4	1.06	10	0.400
285	A	4	3	0.97	8	0.375

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
286	A	5	5	1.07	6	0.833
287	C	9	8	1.24	10	0.800
288	A	3	3	1.00	10	0.300
289	A	6	5	1.00	10	0.500
290	A	6	6	1.03	10	0.600
291	C	9	8	1.24	10	0.800
292	A	8	7	1.14	10	0.700
293	A	7	6	1.09	8	0.750
294	A	7	6	0.97	6	1.000
295	C	9	8	1.41	10	0.800
296	A	3	3	1.00	10	0.300
297	A	4	4	1.11	10	0.400
298	A	5	5	1.16	10	0.500
299	A	6	6	1.19	10	0.600
300	A	7	6	1.00	10	0.600
301	A	3	3	1.00	8	0.375
302	A	6	5	1.00	6	0.833
303	C	9	8	1.25	10	0.800
304	A	3	3	1.00	10	0.300
305	A	6	5	1.12	10	0.500
306	A	6	5	1.07	10	0.500
307	A	3	3	1.00	10	0.300
308	A	3	3	1.00	10	0.300
309	A	3	3	1.00	8	0.375
310	A	3	3	1.00	6	0.500
311	C	9	8	1.13	10	0.800
312	A	3	3	1.00	10	0.300
313	A	3	3	1.01	10	0.300
314	A	3	3	1.00	20	0.150
315	A	2	2	0.98	20	0.100
316	A	2	2	1.00	20	0.100
317	A	1	1	1.00	18	0.056
318	A	1	1	1.00	20	0.050
319	A	1	1	1.00	20	0.050

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
320	A	2	2	1.03	20	0.100
321	A	3	3	1.00	20	0.150
322	A	2	2	0.98	20	0.100
323	A	2	2	1.00	20	0.100
324	A	1	1	1.00	18	0.056
325	A	1	1	1.00	20	0.050
326	A	1	1	1.00	20	0.050
327	A	2	2	1.03	20	0.100
328	A	2	2	0.99	22	0.091
329	A	2	2	1.00	22	0.091
330	A	1	1	1.00	22	0.045
331	A	1	1	1.00	22	0.045
332	A	1	1	1.00	22	0.045
333	A	2	2	0.99	22	0.091
334	A	2	2	0.99	22	0.091
335	A	2	2	0.99	22	0.091
336	A	2	2	1.01	22	0.091
337	A	1	1	1.00	22	0.045
338	A	1	1	1.00	22	0.045
339	A	1	1	1.00	22	0.045
340	A	2	2	0.99	22	0.091
341	A	2	2	0.99	22	0.091
342	N/A	1	0	1.00	40	0.000
343	C	12	11	1.12	40	0.275
344	C	11	10	1.12	40	0.250
345	C	10	9	1.08	38	0.237
346	N/A	1	0	1.00	40	0.000
347	N/A	1	0	1.00	40	0.000
348	C	10	9	1.08	10	0.900
349	A	7	6	0.87	12	0.500
350	A	6	5	0.90	12	0.417
351	A	7	6	0.87	10	0.600
352	A	6	5	1.00	8	0.625

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
353	A	3	3	1.00	12	0.250
354	A	3	3	1.00	12	0.250
355	A	3	3	1.00	12	0.250
356	A	3	3	1.00	12	0.250
357	A	3	3	1.00	12	0.250
358	A	7	6	0.91	14	0.429
359	A	6	5	0.92	14	0.357
360	A	7	6	0.94	12	0.500
361	A	4	3	0.97	10	0.300
362	N/A	1	0	1.00	14	0.000
363	N/A	1	0	1.00	14	0.000
364	C	11	10	1.13	19	0.526
365	A	5	4	1.00	15	0.267
366	A	5	4	0.78	12	0.333
367	A	5	4	0.80	14	0.286
368	A	7	6	1.00	10	0.600
369	F	0	0	N/A	0.000	N/A
370	A	3	2	1.00	26	0.077
371	A	3	2	1.00	26	0.077

CHAPTER 3

Listing of integrals

3.1 $\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx$

3.1.1 Optimal result

Integrand size = 12, antiderivative size = 170

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = -\frac{\operatorname{arcsinh}(cx)^2}{2e}+\frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

output
```
-1/2*arcsinh(c*x)^2/e+arcsinh(c*x)*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^
2*d^2+e^2)^(1/2)))/e+arcsinh(c*x)*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2
*d^2+e^2)^(1/2)))/e+polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2
)^(1/2)))/e+polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2))
)/e
```

3.1.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.99

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = -\frac{\operatorname{arcsinh}(cx)^2}{2e} + \frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{arcsinh}(cx)\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{PolyLog}\left(2,\frac{ee^{\operatorname{arcsinh}(cx)}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

input
```
Integrate[ArcSinh[c*x]/(d + e*x),x]
```

output
```
-1/2*ArcSinh[c*x]^2/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sq
rt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d + S
qrt[c^2*d^2 + e^2])])/e + PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2
*d^2 + e^2])]/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^
2]))]/e
```

3.1.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6242, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx$$

$$\Big\downarrow \ 6242$$

$$\int \frac{\sqrt{c^2x^2+1}\operatorname{arcsinh}(cx)}{cd+cex}\,d\operatorname{arcsinh}(cx)$$

$$\Big\downarrow \ 6095$$

$$\int \frac{e^{\operatorname{arcsinh}(cx)}\operatorname{arcsinh}(cx)}{cd+ee^{\operatorname{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}}\,d\operatorname{arcsinh}(cx) + \int \frac{e^{\operatorname{arcsinh}(cx)}\operatorname{arcsinh}(cx)}{cd+ee^{\operatorname{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}}\,d\operatorname{arcsinh}(cx) - \frac{\operatorname{arcsinh}(cx)^2}{2e}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{\int \log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd-\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}-\frac{\int \log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}+\frac{\text{arcsinh}(cx)\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^2}{2e}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd-\sqrt{c^2d^2+e^2}}+1\right)de^{\text{arcsinh}(cx)}}{e}-\frac{\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)de^{\text{arcsinh}(cx)}}{e}+\frac{\text{arcsinh}(cx)\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^2}{2e}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\text{arcsinh}(cx)\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^2}{2e}$$

input
```
Int[ArcSinh[c*x]/(d + e*x),x]
```

output
```
-1/2*ArcSinh[c*x]^2/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sq
rt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d + S
qrt[c^2*d^2 + e^2])])/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*
d^2 + e^2]))]/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^
2]))]/e
```

3.1.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

3.1.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{-\frac{c\,\mathrm{arcsinh}(cx)^2}{2e}+\frac{c\,\mathrm{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{c\,\mathrm{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}}{c}$
default	$\frac{-\frac{c\,\mathrm{arcsinh}(cx)^2}{2e}+\frac{c\,\mathrm{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{c\,\mathrm{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}}{c}$

input
```
int(arcsinh(c*x)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2*c*arcsinh(c*x)^2/e+c/e*arcsinh(c*x)*ln((-c*d-e*(c*x+(c^2*x^2+1)^
(1/2))+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))+c/e*arcsinh(c*x)*l
n((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(
1/2)))+c/e*dilog((-c*d-e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(-c*
d+(c^2*d^2+e^2)^(1/2)))+c/e*dilog((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+
e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2))))
```

3.1.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)}{ex+d}\,dx$$

input
```
integrate(arcsinh(c*x)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(arcsinh(c*x)/(e*x + d), x)
```

3.1.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = \int \frac{\operatorname{asinh}(cx)}{d+ex}\,dx$$

input
```
integrate(asinh(c*x)/(e*x+d),x)
```

output
```
Integral(asinh(c*x)/(d + e*x), x)
```

3.1.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)}{ex+d}\,dx$$

input
```
integrate(arcsinh(c*x)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(arcsinh(c*x)/(e*x + d), x)
```

3.1.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)}{ex+d}\,dx$$

input
```
integrate(arcsinh(c*x)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(arcsinh(c*x)/(e*x + d), x)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(cx)}{d+ex}\,dx = \int \frac{\operatorname{asinh}(c\,x)}{d+e\,x}\,dx$$

input
```
int(asinh(c*x)/(d + e*x),x)
```

output
```
int(asinh(c*x)/(d + e*x), x)
```

3.2 $\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex} dx$

3.2.1 Optimal result

Integrand size = 14, antiderivative size = 260

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex} dx = -\frac{\operatorname{arcsinh}(cx)^3}{3e} + \frac{\operatorname{arcsinh}(cx)^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\operatorname{arcsinh}(cx)^2 \log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{2\operatorname{arcsinh}(cx) \operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{2\operatorname{arcsinh}(cx) \operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} - \frac{2 \operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} - \frac{2 \operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

output
```
-1/3*arcsinh(c*x)^3/e+arcsinh(c*x)^2*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(
c^2*d^2+e^2)^(1/2)))/e+arcsinh(c*x)^2*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+
(c^2*d^2+e^2)^(1/2)))/e+2*arcsinh(c*x)*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2)
)/(c*d-(c^2*d^2+e^2)^(1/2)))/e+2*arcsinh(c*x)*polylog(2,-e*(c*x+(c^2*x^2+1
)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e-2*polylog(3,-e*(c*x+(c^2*x^2+1)^(1/2
))/(c*d-(c^2*d^2+e^2)^(1/2)))/e-2*polylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*
d+(c^2*d^2+e^2)^(1/2)))/e
```

3.2.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx = -\frac{\operatorname{arcsinh}(cx)^3 - 3\operatorname{arcsinh}(cx)^2\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right) - 3\operatorname{arcsinh}(cx)^2\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right) - 6\operatorname{arcsinh}(c}{}$$

input
```
Integrate[ArcSinh[c*x]^2/(d + e*x),x]
```

output
```
-1/3*(ArcSinh[c*x]^3 - 3*ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d -
Sqrt[c^2*d^2 + e^2])] - 3*ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d +
 Sqrt[c^2*d^2 + e^2])] - 6*ArcSinh[c*x]*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c
*d) + Sqrt[c^2*d^2 + e^2])] - 6*ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSinh[c*x
])/(c*d + Sqrt[c^2*d^2 + e^2]))] + 6*PolyLog[3, (e*E^ArcSinh[c*x])/(-(c*d)
 + Sqrt[c^2*d^2 + e^2])] + 6*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c
^2*d^2 + e^2]))])/e
```

3.2.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6242, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx$$

$$\Big\downarrow \ 6242$$

$$\int \frac{\sqrt{c^2x^2+1}\operatorname{arcsinh}(cx)^2}{cd+cex}d\operatorname{arcsinh}(cx)$$

$$\Big\downarrow \ 6095$$

$$\int \frac{e^{\operatorname{arcsinh}(cx)}\operatorname{arcsinh}(cx)^2}{cd+ee^{\operatorname{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}}d\operatorname{arcsinh}(cx)+\int \frac{e^{\operatorname{arcsinh}(cx)}\operatorname{arcsinh}(cx)^2}{cd+ee^{\operatorname{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}}d\operatorname{arcsinh}(cx)-\frac{\operatorname{arcsinh}(cx)^3}{3e}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{2\int \text{arcsinh}(cx)\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd-\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}-\frac{2\int \text{arcsinh}(cx)\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}+\frac{\text{arcsinh}(cx)^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^3}{3e}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{2\left(\int \text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)d\text{arcsinh}(cx)-\text{arcsinh}(cx)\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)\right)}{e}-\frac{2\left(\int \text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)d\text{arcsinh}(cx)-\text{arcsinh}(cx)\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)}{e}+\frac{\text{arcsinh}(cx)^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^3}{3e}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{2\left(\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)de^{\text{arcsinh}(cx)}-\text{arcsinh}(cx)\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)\right)}{e}-\frac{2\left(\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)de^{\text{arcsinh}(cx)}-\text{arcsinh}(cx)\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)}{e}+\frac{\text{arcsinh}(cx)^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^3}{3e}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{2\left(\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-\text{arcsinh}(cx)\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)\right)}{e}-\frac{2\left(\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)-\text{arcsinh}(cx)\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)}{e}+\frac{\text{arcsinh}(cx)^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^3}{3e}$$

input
```
Int[ArcSinh[c*x]^2/(d + e*x),x]
```

3.2. $\int \frac{\text{arcsinh}(cx)^2}{d+ex}\,dx$

output
```
-1/3*ArcSinh[c*x]^3/e + (ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d -
Sqrt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d
 + Sqrt[c^2*d^2 + e^2])])/e - (2*(-(ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSinh
[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))]) + PolyLog[3, -((e*E^ArcSinh[c*x])/(c
*d - Sqrt[c^2*d^2 + e^2]))]))/e - (2*(-(ArcSinh[c*x]*PolyLog[2, -((e*E^Arc
Sinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))]) + PolyLog[3, -((e*E^ArcSinh[c*x]
)/(c*d + Sqrt[c^2*d^2 + e^2]))]))/e
```

3.2.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.2.4 Maple [F]

$$\int \frac{\operatorname{arcsinh}(cx)^2}{ex + d}\, dx$$

input
```
int(arcsinh(c*x)^2/(e*x+d),x)
```

output
```
int(arcsinh(c*x)^2/(e*x+d),x)
```

3.2.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d + ex}\, dx = \int \frac{\operatorname{arsinh}(cx)^2}{ex + d}\, dx$$

input
```
integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral(arcsinh(c*x)^2/(e*x + d), x)
```

3.2.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d + ex}\, dx = \int \frac{\operatorname{asinh}^2(cx)}{d + ex}\, dx$$

input
```
integrate(asinh(c*x)**2/(e*x+d),x)
```

output
```
Integral(asinh(c*x)**2/(d + e*x), x)
```

3.2.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)^2}{ex+d}\,dx$$

input
```
integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(arcsinh(c*x)^2/(e*x + d), x)
```

3.2.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)^2}{ex+d}\,dx$$

input
```
integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate(arcsinh(c*x)^2/(e*x + d), x)
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(cx)^2}{d+ex}\,dx = \int \frac{\operatorname{asinh}(c\,x)^2}{d+e\,x}\,dx$$

input
```
int(asinh(c*x)^2/(d + e*x),x)
```

output
```
int(asinh(c*x)^2/(d + e*x), x)
```

3.3 $\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx$

3.3.1 Optimal result

Integrand size = 14, antiderivative size = 348

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = &-\frac{\operatorname{arcsinh}(cx)^4}{4e}+\frac{\operatorname{arcsinh}(cx)^3\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\ &+\frac{\operatorname{arcsinh}(cx)^3\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\ &+\frac{3\operatorname{arcsinh}(cx)^2\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\ &+\frac{3\operatorname{arcsinh}(cx)^2\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\ &-\frac{6\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}\\ &-\frac{6\operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(3,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\\ &+\frac{6\operatorname{PolyLog}\left(4,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{6\operatorname{PolyLog}\left(4,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\end{aligned}$$

output
```
-1/4*arcsinh(c*x)^4/e+arcsinh(c*x)^3*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(
c^2*d^2+e^2)^(1/2)))/e+arcsinh(c*x)^3*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+
(c^2*d^2+e^2)^(1/2)))/e+3*arcsinh(c*x)^2*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/
2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+3*arcsinh(c*x)^2*polylog(2,-e*(c*x+(c^2*x
^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e-6*arcsinh(c*x)*polylog(3,-e*(c*x
+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e-6*arcsinh(c*x)*polylog(3,
-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e+6*polylog(4,-e*(c*
x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+6*polylog(4,-e*(c*x+(c^2
*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e
```

3.3.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = \frac{-\operatorname{arcsinh}(cx)^4+4\operatorname{arcsinh}(cx)^3\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)+4\operatorname{arcsinh}(cx)^3\log\left(1+\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)+12\operatorname{arcsinh}(}{}$$

input
```
Integrate[ArcSinh[c*x]^3/(d + e*x),x]
```

output
```
(-ArcSinh[c*x]^4 + 4*ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqrt
[c^2*d^2 + e^2])] + 4*ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqr
t[c^2*d^2 + e^2])] + 12*ArcSinh[c*x]^2*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*
d) + Sqrt[c^2*d^2 + e^2])] + 12*ArcSinh[c*x]^2*PolyLog[2, -((e*E^ArcSinh[c
*x])/(c*d + Sqrt[c^2*d^2 + e^2]))] - 24*ArcSinh[c*x]*PolyLog[3, (e*E^ArcSi
nh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] - 24*ArcSinh[c*x]*PolyLog[3, -((e
*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))] + 24*PolyLog[4, (e*E^ArcSin
h[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] + 24*PolyLog[4, -((e*E^ArcSinh[c*x
])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(4*e)
```

3.3.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6242, 6095, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx$$

$$\Big\downarrow \text{ 6242}$$

$$\int \frac{\sqrt{c^2x^2+1}\operatorname{arcsinh}(cx)^3}{cd+cex}d\operatorname{arcsinh}(cx)$$

$$\Big\downarrow \text{ 6095}$$

$$\int \frac{e^{\operatorname{arcsinh}(cx)}\operatorname{arcsinh}(cx)^3}{cd+ee^{\operatorname{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}}d\operatorname{arcsinh}(cx)+\int \frac{e^{\operatorname{arcsinh}(cx)}\operatorname{arcsinh}(cx)^3}{cd+ee^{\operatorname{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}}d\operatorname{arcsinh}(cx)-\frac{\operatorname{arcsinh}(cx)^4}{4e}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{3\int \operatorname{arcsinh}(cx)^2\log\left(\frac{e^{\operatorname{arcsinh}(cx)}e}{cd-\sqrt{c^2d^2+e^2}}+1\right)d\operatorname{arcsinh}(cx)}{e}-\frac{3\int \operatorname{arcsinh}(cx)^2\log\left(\frac{e^{\operatorname{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)d\operatorname{arcsinh}(cx)}{e}+\frac{\operatorname{arcsinh}(cx)^3\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\operatorname{arcsinh}(cx)^3\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\operatorname{arcsinh}(cx)^4}{4e}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{3\left(2\int \operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)d\operatorname{arcsinh}(cx)-\operatorname{arcsinh}(cx)^2\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)\right)}{e}-\frac{3\left(2\int \operatorname{arcsinh}(cx)\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)d\operatorname{arcsinh}(cx)-\operatorname{arcsinh}(cx)^2\operatorname{PolyLog}\left(2,-\frac{ee^{\operatorname{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)}{e}+\frac{\operatorname{arcsinh}(cx)^3\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\operatorname{arcsinh}(cx)^3\log\left(\frac{ee^{\operatorname{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\operatorname{arcsinh}(cx)^4}{4e}$$

$$\Big\downarrow \text{ 7163}$$

$$-\frac{3\left(2\left(\text{arcsinh}(cx)\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-\int \text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)d\text{arcsinh}(cx)\right)-\text{arcsinh}(cx)^2\,\text{Poly}\right.}{e}$$

$$\frac{3\left(2\left(\text{arcsinh}(cx)\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)-\int \text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)d\text{arcsinh}(cx)\right)-\text{arcsinh}(cx)^2\,\text{PolyL}\right.}{e}$$

$$\frac{\text{arcsinh}(cx)^3\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)^3\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^4}{4e}$$

$\downarrow$ 2720

$$-\frac{3\left(2\left(\text{arcsinh}(cx)\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)de^{\text{arcsinh}(cx)}\right)-\text{arcsin}\right.}{e}$$

$$\frac{3\left(2\left(\text{arcsinh}(cx)\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)-\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)de^{\text{arcsinh}(cx)}\right)-\text{arcsinh}(\right.}{e}$$

$$\frac{\text{arcsinh}(cx)^3\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)^3\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^4}{4e}$$

$\downarrow$ 7143

$$-\frac{3\left(2\left(\text{arcsinh}(cx)\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-\text{PolyLog}\left(4,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)\right)-\text{arcsinh}(cx)^2\,\text{PolyLog}\left(2,-\frac{ee^{\text{arc}}}{cd-\sqrt{}}\right.\right.}{e}$$

$$\frac{3\left(2\left(\text{arcsinh}(cx)\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)-\text{PolyLog}\left(4,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)-\text{arcsinh}(cx)^2\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsin}}}{cd+\sqrt{c^2}}\right.\right.}{e}$$

$$\frac{\text{arcsinh}(cx)^3\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\text{arcsinh}(cx)^3\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\text{arcsinh}(cx)^4}{4e}$$

input
```
Int[ArcSinh[c*x]^3/(d + e*x),x]
```

output
```
-1/4*ArcSinh[c*x]^4/e + (ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d -
Sqrt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d
 + Sqrt[c^2*d^2 + e^2])])/e - (3*(-(ArcSinh[c*x]^2*PolyLog[2, -((e*E^ArcSi
nh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))]) + 2*(ArcSinh[c*x]*PolyLog[3, -((e*
E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))] - PolyLog[4, -((e*E^ArcSinh[
c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])))/e - (3*(-(ArcSinh[c*x]^2*PolyLog[2,
 -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))]) + 2*(ArcSinh[c*x]*Pol
yLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))] - PolyLog[4, -(
(e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])))/e
```

3.3. $\int \frac{\text{arcsinh}(cx)^3}{d+ex}\,dx$

3.3.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.3.4 Maple [F]

$$\int \frac{\operatorname{arcsinh}(cx)^3}{ex+d}dx$$

input
```
int(arcsinh(c*x)^3/(e*x+d),x)
```

output
```
int(arcsinh(c*x)^3/(e*x+d),x)
```

3.3.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)^3}{ex+d}\,dx$$

input
```
integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="fricas")
```

output
```
integral(arcsinh(c*x)^3/(e*x + d), x)
```

3.3.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = \int \frac{\operatorname{asinh}^3(cx)}{d+ex}\,dx$$

input
```
integrate(asinh(c*x)**3/(e*x+d),x)
```

output
```
Integral(asinh(c*x)**3/(d + e*x), x)
```

3.3.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)^3}{ex+d}\,dx$$

input
```
integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(arcsinh(c*x)^3/(e*x + d), x)
```

3.3.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = \int \frac{\operatorname{arsinh}(cx)^3}{ex+d}\,dx$$

input
```
integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="giac")
```

output
```
integrate(arcsinh(c*x)^3/(e*x + d), x)
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(cx)^3}{d+ex}\,dx = \int \frac{\operatorname{asinh}(c\,x)^3}{d+e\,x}\,dx$$

input
```
int(asinh(c*x)^3/(d + e*x),x)
```

output
```
int(asinh(c*x)^3/(d + e*x), x)
```

3.4 $\int (d+ex)^3(a+b\text{arcsinh}(cx))\,dx$

3.4.1 Optimal result

Integrand size = 16, antiderivative size = 176

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))\,dx = -\frac{7bd(d+ex)^2\sqrt{1+c^2x^2}}{48c} - \frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} - \frac{b(4d(19c^2d^2-16e^2)+e(26c^2d^2-9e^2)\,x)\sqrt{1+c^2x^2}}{96c^3} - \frac{b(8c^4d^4-24c^2d^2e^2+3e^4)\,\text{arcsinh}(cx)}{32c^4e} + \frac{(d+ex)^4(a+b\text{arcsinh}(cx))}{4e}$$

output
```
-1/32*b*(8*c^4*d^4-24*c^2*d^2*e^2+3*e^4)*arcsinh(c*x)/c^4/e+1/4*(e*x+d)^4*
(a+b*arcsinh(c*x))/e-7/48*b*d*(e*x+d)^2*(c^2*x^2+1)^(1/2)/c-1/16*b*(e*x+d)
^3*(c^2*x^2+1)^(1/2)/c-1/96*b*(4*d*(19*c^2*d^2-16*e^2)+e*(26*c^2*d^2-9*e^2
)*x)*(c^2*x^2+1)^(1/2)/c^3
```

3.4.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.94

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))\,dx$$

$$= \frac{24ac^4x(4d^3+6d^2ex+4de^2x^2+e^3x^3) - bc\sqrt{1+c^2x^2}(-e^2(64d+9ex)+c^2(96d^3+72d^2ex+32de^2x^2+6}{96c^4}$$

input
```
Integrate[(d + e*x)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
(24*a*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - b*c*Sqrt[1 + c^2
*x^2]*(-(e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6
*e^3*x^3)) + 3*b*(24*c^2*d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*
e^2*x^2 + e^3*x^3))*ArcSinh[c*x])/(96*c^4)
```

3.4.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6243, 497, 687, 27, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6243}$$

$$\frac{(d+ex)^4(a+b\text{arcsinh}(cx))}{4e}-\frac{bc\int \frac{(d+ex)^4}{\sqrt{c^2x^2+1}}dx}{4e}$$

$$\Big\downarrow \text{ 497}$$

$$\frac{(d+ex)^4(a+b\text{arcsinh}(cx))}{4e}-\frac{bc\left(\frac{\int \frac{(d+ex)^2\left(4d^2c^2+7dexc^2-3e^2\right)}{\sqrt{c^2x^2+1}}dx}{4c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)^3}{4c^2}\right)}{4e}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{(d+ex)^4(a+b\text{arcsinh}(cx))}{4e}-\frac{bc\left(\frac{\frac{\int \frac{c^2(d+ex)\left(d\left(12c^2d^2-23e^2\right)+e\left(26c^2d^2-9e^2\right)x\right)}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{7}{3}de\sqrt{c^2x^2+1}(d+ex)^2}{4c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)^3}{4c^2}\right)}{4e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(d+ex)^4(a+b\text{arcsinh}(cx))}{4e}-\frac{bc\left(\frac{\frac{1}{3}\int\frac{(d+ex)\left(d\left(12c^2d^2-23e^2\right)+e\left(26c^2d^2-9e^2\right)x\right)}{\sqrt{c^2x^2+1}}dx+\frac{7}{3}de\sqrt{c^2x^2+1}(d+ex)^2}{4c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)^3}{4c^2}\right)}{4e}$$

$$\Big\downarrow \text{676}$$

$$\frac{(d+ex)^4(a+b\text{arcsinh}(cx))}{4e}-\frac{bc\left(\frac{\frac{1}{3}\left(\frac{3\left(8c^4d^4-24c^2d^2e^2+3e^4\right)\int\frac{1}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{1}{2}e^2x\sqrt{c^2x^2+1}\left(26d^2-\frac{9e^2}{c^2}\right)+2de\sqrt{c^2x^2+1}\left(19d^2-\frac{16e^2}{c^2}\right)\right)+\frac{7}{3}de\sqrt{c^2x^2+1}(d+ex)^2}{4c^2}+\frac{e\sqrt{c^2x^2+}}{4c}\right.}{4e}$$

$$\Big\downarrow \text{222}$$

$$\frac{(d+ex)^4(a+b\text{arcsinh}(cx))}{4e}-\frac{bc\left(\frac{\frac{1}{3}\left(\frac{3\text{arcsinh}(cx)\left(8c^4d^4-24c^2d^2e^2+3e^4\right)}{2c^3}+\frac{1}{2}e^2x\sqrt{c^2x^2+1}\left(26d^2-\frac{9e^2}{c^2}\right)+2de\sqrt{c^2x^2+1}\left(19d^2-\frac{16e^2}{c^2}\right)\right)+\frac{7}{3}de\sqrt{c^2x^2+1}(d+ex)^2}{4c^2}+\frac{e\sqrt{c^2x^2+1}}{4c^2}\right.}{4e}$$

input
```
Int[(d + e*x)^3*(a + b*ArcSinh[c*x]),x]
```

output
```
((d + e*x)^4*(a + b*ArcSinh[c*x]))/(4*e) - (b*c*((e*(d + e*x)^3*Sqrt[1 + c
^2*x^2])/(4*c^2) + ((7*d*e*(d + e*x)^2*Sqrt[1 + c^2*x^2])/3 + (2*d*e*(19*d
^2 - (16*e^2)/c^2)*Sqrt[1 + c^2*x^2] + (e^2*(26*d^2 - (9*e^2)/c^2)*x*Sqrt[
1 + c^2*x^2])/2 + (3*(8*c^4*d^4 - 24*c^2*d^2*e^2 + 3*e^4)*ArcSinh[c*x])/(2
*c^3))/3)/(4*c^2)))/(4*e)
```

3.4.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

3.4.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.38

method	result
parts	$\frac{a(ex+d)^4}{4e} + \frac{b\left(\frac{c\,e^3 \operatorname{arcsinh}(cx)x^4}{4} + c\,e^2 \operatorname{arcsinh}(cx)x^3 d + \frac{3c \operatorname{arcsinh}(cx)d^2 e\,x^2}{2} + \operatorname{arcsinh}(cx)cx\,d^3 + \frac{c \operatorname{arcsinh}(cx)d^4}{4e} - \frac{c^4 d^4 \mathrm{a}}{}\right.}{}$
derivativedivides	$\frac{\frac{a(ecx+cd)^4}{4c^3 e} + \frac{b\left(\frac{\operatorname{arcsinh}(cx)c^4 d^4}{4e} + \operatorname{arcsinh}(cx)c^4 d^3 x + \frac{3e \operatorname{arcsinh}(cx)c^4 d^2 x^2}{2} + e^2 \operatorname{arcsinh}(cx)c^4 d\,x^3 + \frac{e^3 \operatorname{arcsinh}(cx)c^4 x^4}{4} - \frac{c^4 d^4 \operatorname{arcs}}{}\right.}{}}{}$
default	$\frac{\frac{a(ecx+cd)^4}{4c^3 e} + \frac{b\left(\frac{\operatorname{arcsinh}(cx)c^4 d^4}{4e} + \operatorname{arcsinh}(cx)c^4 d^3 x + \frac{3e \operatorname{arcsinh}(cx)c^4 d^2 x^2}{2} + e^2 \operatorname{arcsinh}(cx)c^4 d\,x^3 + \frac{e^3 \operatorname{arcsinh}(cx)c^4 x^4}{4} - \frac{c^4 d^4 \operatorname{arcs}}{}\right.}{}}{}$

input
```
int((e*x+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(e*x+d)^4/e+b/c*(1/4*c*e^3*arcsinh(c*x)*x^4+c*e^2*arcsinh(c*x)*x^3*d
+3/2*c*arcsinh(c*x)*d^2*e*x^2+arcsinh(c*x)*c*x*d^3+1/4*c/e*arcsinh(c*x)*d^
4-1/4/c^3/e*(c^4*d^4*arcsinh(c*x)+e^4*(1/4*c^3*x^3*(c^2*x^2+1)^(1/2)-3/8*c
*x*(c^2*x^2+1)^(1/2)+3/8*arcsinh(c*x))+4*d^3*c^3*e*(c^2*x^2+1)^(1/2)+6*d^2
*c^2*e^2*(1/2*c*x*(c^2*x^2+1)^(1/2)-1/2*arcsinh(c*x))+4*d*c*e^3*(1/3*c^2*x
^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))))
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.22

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))\,dx$$

$$= \frac{24\,ac^4e^3x^4 + 96\,ac^4de^2x^3 + 144\,ac^4d^2ex^2 + 96\,ac^4d^3x + 3\,(8\,bc^4e^3x^4 + 32\,bc^4de^2x^3 + 48\,bc^4d^2ex^2 + 32\,bc^4d}{}$$

input
```
integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/96*(24*a*c^4*e^3*x^4 + 96*a*c^4*d*e^2*x^3 + 144*a*c^4*d^2*e*x^2 + 96*a*c
^4*d^3*x + 3*(8*b*c^4*e^3*x^4 + 32*b*c^4*d*e^2*x^3 + 48*b*c^4*d^2*e*x^2 +
32*b*c^4*d^3*x + 24*b*c^2*d^2*e - 3*b*e^3)*log(c*x + sqrt(c^2*x^2 + 1)) -
(6*b*c^3*e^3*x^3 + 32*b*c^3*d*e^2*x^2 + 96*b*c^3*d^3 - 64*b*c*d*e^2 + 9*(8
*b*c^3*d^2*e - b*c*e^3)*x)*sqrt(c^2*x^2 + 1))/c^4
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.80

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))\,dx$$
$$=\begin{cases} ad^3x+\frac{3ad^2ex^2}{2}+ade^2x^3+\frac{ae^3x^4}{4}+bd^3x\operatorname{asinh}(cx)+\frac{3bd^2ex^2\operatorname{asinh}(cx)}{2}+bde^2x^3\operatorname{asinh}(cx)+\frac{be^3x^4\operatorname{asinh}(cx)}{4}- \\ a\left(d^3x+\frac{3d^2ex^2}{2}+de^2x^3+\frac{e^3x^4}{4}\right) \end{cases}$$

input
```
integrate((e*x+d)**3*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 +
b*d**3*x*asinh(c*x) + 3*b*d**2*e*x**2*asinh(c*x)/2 + b*d*e**2*x**3*asinh(c
*x) + b*e**3*x**4*asinh(c*x)/4 - b*d**3*sqrt(c**2*x**2 + 1)/c - 3*b*d**2*e
*x*sqrt(c**2*x**2 + 1)/(4*c) - b*d*e**2*x**2*sqrt(c**2*x**2 + 1)/(3*c) - b
*e**3*x**3*sqrt(c**2*x**2 + 1)/(16*c) + 3*b*d**2*e*asinh(c*x)/(4*c**2) + 2
*b*d*e**2*sqrt(c**2*x**2 + 1)/(3*c**3) + 3*b*e**3*x*sqrt(c**2*x**2 + 1)/(3
2*c**3) - 3*b*e**3*asinh(c*x)/(32*c**4), Ne(c, 0)), (a*(d**3*x + 3*d**2*e*
x**2/2 + d*e**2*x**3 + e**3*x**4/4), True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.31

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))\,dx$$
$$\begin{aligned}=&\ \frac{1}{4}ae^3x^4+ade^2x^3+\frac{3}{2}ad^2ex^2\\&+\frac{3}{4}\left(2x^2\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}x}{c^2}-\frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)bd^2e\\&+\frac{1}{3}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bde^2\\&+\frac{1}{32}\left(8x^4\operatorname{arsinh}(cx)-\left(\frac{2\sqrt{c^2x^2+1}x^3}{c^2}-\frac{3\sqrt{c^2x^2+1}x}{c^4}+\frac{3\operatorname{arsinh}(cx)}{c^5}\right)c\right)be^3\\&+ad^3x+\frac{\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)bd^3}{c}\end{aligned}$$

input
```
integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/4*(2*x^2*arcsinh(c*x) -
c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*b*d^2*e + 1/3*(3*x^3*arcsi
nh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d*e^2
 + 1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^
2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*b*e^3 + a*d^3*x + (c*x*arcsinh(c*x)
- sqrt(c^2*x^2 + 1))*b*d^3/c
```

3.4.8 Giac [F(-2)]

Exception generated.

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))\,dx = \int (a+b\,\text{asinh}(c\,x))\,\left(d+e\,x\right)^3\,dx$$

input
```
int((a + b*asinh(c*x))*(d + e*x)^3,x)
```

output
```
int((a + b*asinh(c*x))*(d + e*x)^3, x)
```

3.5 $\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx$

3.5.1 Optimal result

Integrand size = 16, antiderivative size = 124

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx = -\frac{b(d+ex)^2\sqrt{1+c^2x^2}}{9c} - \frac{b(4(4c^2d^2-e^2)+5c^2dex)\sqrt{1+c^2x^2}}{18c^3} - \frac{bd\left(2d^2-\frac{3e^2}{c^2}\right)\operatorname{arcsinh}(cx)}{6e} + \frac{(d+ex)^3(a+b\operatorname{arcsinh}(cx))}{3e}$$

output
```
-1/6*b*d*(2*d^2-3*e^2/c^2)*arcsinh(c*x)/e+1/3*(e*x+d)^3*(a+b*arcsinh(c*x))
/e-1/9*b*(e*x+d)^2*(c^2*x^2+1)^(1/2)/c-1/18*b*(5*c^2*d*e*x+16*c^2*d^2-4*e^
2)*(c^2*x^2+1)^(1/2)/c^3
```

3.5.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx = \frac{6ac^3x(3d^2+3dex+e^2x^2)-b\sqrt{1+c^2x^2}(-4e^2+c^2(18d^2+9dex+2e^2x^2))+3bc(6c^2d^2x+2c^2e^2x^3+3d(}{18c^3}$$

input
```
Integrate[(d + e*x)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
(6*a*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) - b*Sqrt[1 + c^2*x^2]*(-4*e^2 + c^2
*(18*d^2 + 9*d*e*x + 2*e^2*x^2)) + 3*b*c*(6*c^2*d^2*x + 2*c^2*e^2*x^3 + 3*
d*(e + 2*c^2*e*x^2))*ArcSinh[c*x])/(18*c^3)
```

3.5.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6243, 497, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6243}$$

$$\frac{(d+ex)^3(a+b\text{arcsinh}(cx))}{3e}-\frac{bc\int \frac{(d+ex)^3}{\sqrt{c^2x^2+1}}dx}{3e}$$

$$\Big\downarrow \text{497}$$

$$\frac{(d+ex)^3(a+b\text{arcsinh}(cx))}{3e}-\frac{bc\left(\frac{\int \frac{(d+ex)\left(3d^2c^2+5dexc^2-2e^2\right)}{\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)^2}{3c^2}\right)}{3e}$$

$$\Big\downarrow \text{676}$$

$$\frac{(d+ex)^3(a+b\text{arcsinh}(cx))}{3e}-\frac{bc\left(\frac{\frac{3}{2}d\left(2c^2d^2-3e^2\right)\int \frac{1}{\sqrt{c^2x^2+1}}dx+2e\sqrt{c^2x^2+1}\left(4d^2-\frac{e^2}{c^2}\right)+\frac{5}{2}de^2x\sqrt{c^2x^2+1}}{3c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)^2}{3c^2}\right)}{3e}$$

$$\Big\downarrow \text{222}$$

$$\frac{(d+ex)^3(a+b\text{arcsinh}(cx))}{3e}-\frac{bc\left(\frac{\frac{3d\text{arcsinh}(cx)\left(2c^2d^2-3e^2\right)}{2c}+2e\sqrt{c^2x^2+1}\left(4d^2-\frac{e^2}{c^2}\right)+\frac{5}{2}de^2x\sqrt{c^2x^2+1}}{3c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)^2}{3c^2}\right)}{3e}$$

input
```
Int[(d + e*x)^2*(a + b*ArcSinh[c*x]),x]
```

output
```
((d + e*x)^3*(a + b*ArcSinh[c*x]))/(3*e) - (b*c*((e*(d + e*x)^2*Sqrt[1 + c
^2*x^2])/(3*c^2) + (2*e*(4*d^2 - e^2/c^2)*Sqrt[1 + c^2*x^2] + (5*d*e^2*x*S
qrt[1 + c^2*x^2])/2 + (3*d*(2*c^2*d^2 - 3*e^2)*ArcSinh[c*x])/(2*c))/(3*c^2
)))/(3*e)
```

3.5.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

3.5.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.40

method	result
parts	$\frac{a(ex+d)^3}{3e}+\frac{b\left(\frac{c\,e^2 \operatorname{arcsinh}(cx)x^3}{3}+c \operatorname{arcsinh}(cx)de\,x^2+\operatorname{arcsinh}(cx)cx\,d^2+\frac{c \operatorname{arcsinh}(cx)d^3}{3e}-\frac{c^3d^3 \operatorname{arcsinh}(cx)+e^3\left(\frac{c^2x^2\sqrt{c}}{3}\right.}{}\right.}{c}$
derivativedivides	$\frac{\frac{a(ecx+cd)^3}{3c^2e}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)c^3d^3}{3e}+\operatorname{arcsinh}(cx)c^3d^2x+e \operatorname{arcsinh}(cx)c^3d\,x^2+\frac{e^2 \operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^3d^3 \operatorname{arcsinh}(cx)+e^3\left(\frac{c^2x^2\sqrt{c^2x^2}}{3}\right.}{}\right.}{c^2}}{c}$
default	$\frac{\frac{a(ecx+cd)^3}{3c^2e}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)c^3d^3}{3e}+\operatorname{arcsinh}(cx)c^3d^2x+e \operatorname{arcsinh}(cx)c^3d\,x^2+\frac{e^2 \operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^3d^3 \operatorname{arcsinh}(cx)+e^3\left(\frac{c^2x^2\sqrt{c^2x^2}}{3}\right.}{}\right.}{c^2}}{c}$

input
```
int((e*x+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*(e*x+d)^3/e+b/c*(1/3*c*e^2*arcsinh(c*x)*x^3+c*arcsinh(c*x)*d*e*x^2+a
rcsinh(c*x)*c*x*d^2+1/3*c/e*arcsinh(c*x)*d^3-1/3/c^2/e*(c^3*d^3*arcsinh(c*
x)+e^3*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))+3*d^2*c^2*e*(
c^2*x^2+1)^(1/2)+3*d*c*e^2*(1/2*c*x*(c^2*x^2+1)^(1/2)-1/2*arcsinh(c*x))))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.19

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{6\,ac^3e^2x^3+18\,ac^3dex^2+18\,ac^3d^2x+3\left(2\,bc^3e^2x^3+6\,bc^3dex^2+6\,bc^3d^2x+3\,bcde\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-}{18\,c^3}$$

input
```
integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/18*(6*a*c^3*e^2*x^3 + 18*a*c^3*d*e*x^2 + 18*a*c^3*d^2*x + 3*(2*b*c^3*e^2
*x^3 + 6*b*c^3*d*e*x^2 + 6*b*c^3*d^2*x + 3*b*c*d*e)*log(c*x + sqrt(c^2*x^2
 + 1)) - (2*b*c^2*e^2*x^2 + 9*b*c^2*d*e*x + 18*b*c^2*d^2 - 4*b*e^2)*sqrt(c
^2*x^2 + 1))/c^3
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.53

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx$$
$$= \begin{cases} ad^2x + adex^2 + \frac{ae^2x^3}{3} + bd^2x\operatorname{asinh}(cx) + bdex^2\operatorname{asinh}(cx) + \frac{be^2x^3\operatorname{asinh}(cx)}{3} - \frac{bd^2\sqrt{c^2x^2+1}}{c} - \frac{bdex\sqrt{c^2x^2+1}}{2c} - \\ a\left(d^2x + dex^2 + \frac{e^2x^3}{3}\right) \end{cases}$$

input
```
integrate((e*x+d)**2*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*asinh(c*x) + b
*d*e*x**2*asinh(c*x) + b*e**2*x**3*asinh(c*x)/3 - b*d**2*sqrt(c**2*x**2 +
1)/c - b*d*e*x*sqrt(c**2*x**2 + 1)/(2*c) - b*e**2*x**2*sqrt(c**2*x**2 + 1)
/(9*c) + b*d*e*asinh(c*x)/(2*c**2) + 2*b*e**2*sqrt(c**2*x**2 + 1)/(9*c**3)
, Ne(c, 0)), (a*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx$$
$$= \frac{1}{3}ae^2x^3 + adex^2 + \frac{1}{2}\left(2x^2\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)bde$$
$$+ \frac{1}{9}\left(3x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)be^2$$
$$+ ad^2x + \frac{\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bd^2}{c}$$

input
```
integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e^2*x^3 + a*d*e*x^2 + 1/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)
*x/c^2 - arcsinh(c*x)/c^3))*b*d*e + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*
x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*e^2 + a*d^2*x + (c*x*arcsin
h(c*x) - sqrt(c^2*x^2 + 1))*b*d^2/c
```

3.5.8 Giac [F(-2)]

Exception generated.

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))\,dx = \int (a+b\operatorname{asinh}(c\,x))\,(d+e\,x)^2\,dx$$

input
```
int((a + b*asinh(c*x))*(d + e*x)^2,x)
```

output
```
int((a + b*asinh(c*x))*(d + e*x)^2, x)
```

3.6 $\int (d+ex)(a+b\text{arcsinh}(cx))\,dx$

3.6.1 Optimal result

Integrand size = 14, antiderivative size = 97

$$\int (d+ex)(a+b\text{arcsinh}(cx))\,dx = -\frac{3bd\sqrt{1+c^2x^2}}{4c} - \frac{b(d+ex)\sqrt{1+c^2x^2}}{4c} - \frac{b\left(2d^2-\frac{e^2}{c^2}\right)\text{arcsinh}(cx)}{4e} + \frac{(d+ex)^2(a+b\text{arcsinh}(cx))}{2e}$$

output
```
-1/4*b*(2*d^2-e^2/c^2)*arcsinh(c*x)/e+1/2*(e*x+d)^2*(a+b*arcsinh(c*x))/e-3
/4*b*d*(c^2*x^2+1)^(1/2)/c-1/4*b*(e*x+d)*(c^2*x^2+1)^(1/2)/c
```

3.6.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int (d+ex)(a+b\text{arcsinh}(cx))\,dx = adx + \frac{1}{2}aex^2 - \frac{bd\sqrt{1+c^2x^2}}{c} - \frac{bex\sqrt{1+c^2x^2}}{4c} + \frac{be\text{arcsinh}(cx)}{4c^2} + bdx\text{arcsinh}(cx) + \frac{1}{2}bex^2\text{arcsinh}(cx)$$

input
```
Integrate[(d + e*x)*(a + b*ArcSinh[c*x]),x]
```

output
```
a*d*x + (a*e*x^2)/2 - (b*d*Sqrt[1 + c^2*x^2])/c - (b*e*x*Sqrt[1 + c^2*x^2]
)/(4*c) + (b*e*ArcSinh[c*x])/(4*c^2) + b*d*x*ArcSinh[c*x] + (b*e*x^2*ArcSi
nh[c*x])/2
```

3.6.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6243, 497, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6243}$$

$$\frac{(d+ex)^2(a+b\text{arcsinh}(cx))}{2e}-\frac{bc\int \frac{(d+ex)^2}{\sqrt{c^2x^2+1}}dx}{2e}$$

$$\Big\downarrow \text{497}$$

$$\frac{(d+ex)^2(a+b\text{arcsinh}(cx))}{2e}-\frac{bc\left(\frac{\int \frac{2d^2c^2+3dexc^2-e^2}{\sqrt{c^2x^2+1}}dx}{2c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)}{2c^2}\right)}{2e}$$

$$\Big\downarrow \text{455}$$

$$\frac{(d+ex)^2(a+b\text{arcsinh}(cx))}{2e}-\frac{bc\left(\frac{(2c^2d^2-e^2)\int \frac{1}{\sqrt{c^2x^2+1}}dx+3de\sqrt{c^2x^2+1}}{2c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)}{2c^2}\right)}{2e}$$

$$\Big\downarrow \text{222}$$

$$\frac{(d+ex)^2(a+b\text{arcsinh}(cx))}{2e}-\frac{bc\left(\frac{\frac{\text{arcsinh}(cx)(2c^2d^2-e^2)}{c}+3de\sqrt{c^2x^2+1}}{2c^2}+\frac{e\sqrt{c^2x^2+1}(d+ex)}{2c^2}\right)}{2e}$$

input
```
Int[(d + e*x)*(a + b*ArcSinh[c*x]),x]
```

output
```
((d + e*x)^2*(a + b*ArcSinh[c*x]))/(2*e) - (b*c*((e*(d + e*x)*Sqrt[1 + c^2
*x^2])/(2*c^2) + (3*d*e*Sqrt[1 + c^2*x^2] + ((2*c^2*d^2 - e^2)*ArcSinh[c*x
])/c)/(2*c^2)))/(2*e)
```

3.6.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

3.6.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

method	result	size
parts	$a\left(\frac{1}{2}e\,x^{2}+dx\right)+\frac{b\left(\frac{c\,\operatorname{arcsinh}(cx)x^{2}e}{2}+\operatorname{arcsinh}(cx)dcx-\frac{e\left(\frac{cx\sqrt{c^{2}x^{2}+1}}{2}-\frac{\operatorname{arcsinh}(cx)}{2}\right)+2dc\sqrt{c^{2}x^{2}+1}}{2c}\right)}{c}$	84
derivativedivides	$\frac{\frac{a\left(d\,c^{2}x+\frac{1}{2}c^{2}e\,x^{2}\right)}{c}+\frac{b\left(\operatorname{arcsinh}(cx)d\,c^{2}x+\frac{\operatorname{arcsinh}(cx)e\,c^{2}x^{2}}{2}-\frac{e\left(\frac{cx\sqrt{c^{2}x^{2}+1}}{2}-\frac{\operatorname{arcsinh}(cx)}{2}\right)}{2}-dc\sqrt{c^{2}x^{2}+1}\right)}{c}}{c}$	96
default	$\frac{\frac{a\left(d\,c^{2}x+\frac{1}{2}c^{2}e\,x^{2}\right)}{c}+\frac{b\left(\operatorname{arcsinh}(cx)d\,c^{2}x+\frac{\operatorname{arcsinh}(cx)e\,c^{2}x^{2}}{2}-\frac{e\left(\frac{cx\sqrt{c^{2}x^{2}+1}}{2}-\frac{\operatorname{arcsinh}(cx)}{2}\right)}{2}-dc\sqrt{c^{2}x^{2}+1}\right)}{c}}{c}$	96

input
```
int((e*x+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e*x^2+d*x)+b/c*(1/2*c*arcsinh(c*x)*x^2*e+arcsinh(c*x)*d*c*x-1/2/c*(
e*(1/2*c*x*(c^2*x^2+1)^(1/2)-1/2*arcsinh(c*x))+2*d*c*(c^2*x^2+1)^(1/2)))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))\,dx = \frac{2\,ac^2ex^2+4\,ac^2dx+(2\,bc^2ex^2+4\,bc^2dx+be)\log\left(cx+\sqrt{c^2x^2+1}\right)-(bcex+4\,bcd)\sqrt{c^2x^2+1}}{4\,c^2}$$

input
```
integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
1/4*(2*a*c^2*e*x^2 + 4*a*c^2*d*x + (2*b*c^2*e*x^2 + 4*b*c^2*d*x + b*e)*log
(c*x + sqrt(c^2*x^2 + 1)) - (b*c*e*x + 4*b*c*d)*sqrt(c^2*x^2 + 1))/c^2
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))\,dx = \begin{cases} adx+\frac{aex^2}{2}+bdx\operatorname{asinh}(cx)+\frac{bex^2\operatorname{asinh}(cx)}{2}-\frac{bd\sqrt{c^2x^2+1}}{c}-\frac{bex\sqrt{c^2x^2+1}}{4c}+\frac{be\operatorname{asinh}(cx)}{4c^2} & \text{for } c\neq 0 \\ a\left(dx+\frac{ex^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((e*x+d)*(a+b*asinh(c*x)),x)
```

output
```
Piecewise((a*d*x + a*e*x**2/2 + b*d*x*asinh(c*x) + b*e*x**2*asinh(c*x)/2 -
 b*d*sqrt(c**2*x**2 + 1)/c - b*e*x*sqrt(c**2*x**2 + 1)/(4*c) + b*e*asinh(c
*x)/(4*c**2), Ne(c, 0)), (a*(d*x + e*x**2/2), True))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int (d+ex)(a+b\text{arcsinh}(cx))\,dx = \frac{1}{2}\,aex^2 + \frac{1}{4}\left(2\,x^2\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\text{arsinh}\,(cx)}{c^3}\right)\right)be + adx + \frac{\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)bd}{c}$$

input
```
integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
1/2*a*e*x^2 + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsi
nh(c*x)/c^3))*b*e + a*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d/c
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.26

$$\int (d+ex)(a+b\text{arcsinh}(cx))\,dx = \frac{1}{2}\,aex^2 + \left(x\log\left(cx+\sqrt{c^2x^2+1}\right) - \frac{\sqrt{c^2x^2+1}}{c}\right)bd + \frac{1}{4}\left(2\,x^2\log\left(cx+\sqrt{c^2x^2+1}\right) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} + \frac{\log\left(-x|c|+\sqrt{c^2x^2+1}\right)}{c^2|c|}\right)\right)be + adx$$

input
```
integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
1/2*a*e*x^2 + (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b*d +
 1/4*(2*x^2*log(c*x + sqrt(c^2*x^2 + 1)) - c*(sqrt(c^2*x^2 + 1)*x/c^2 + lo
g(-x*abs(c) + sqrt(c^2*x^2 + 1))/(c^2*abs(c))))*b*e + a*d*x
```

3.6.9 Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))\,dx = \frac{a\,x\,(2\,d+e\,x)}{2} - \frac{b\,d\,\left(\sqrt{c^2\,x^2+1}-c\,x\,\operatorname{asinh}(c\,x)\right)}{c} - \frac{b\,e\,x\,\sqrt{c^2\,x^2+1}}{4\,c} + b\,e\,x\,\operatorname{asinh}(c\,x)\,\left(\frac{x}{2}+\frac{1}{4\,c^2\,x}\right)$$

input
```
int((a + b*asinh(c*x))*(d + e*x),x)
```

output
```
(a*x*(2*d + e*x))/2 - (b*d*((c^2*x^2 + 1)^(1/2) - c*x*asinh(c*x)))/c - (b*
e*x*(c^2*x^2 + 1)^(1/2))/(4*c) + b*e*x*asinh(c*x)*(x/2 + 1/(4*c^2*x))
```

3.7 $\int (a + b\text{arcsinh}(cx))\,dx$

3.7.1 Optimal result

Integrand size = 8, antiderivative size = 30

$$\int (a + b\text{arcsinh}(cx))\,dx = ax - \frac{b\sqrt{1+c^2x^2}}{c} + bx\text{arcsinh}(cx)$$

output
```
a*x+b*x*arcsinh(c*x)-b*(c^2*x^2+1)^(1/2)/c
```

3.7.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a + b\text{arcsinh}(cx))\,dx = ax - \frac{b\sqrt{1+c^2x^2}}{c} + bx\text{arcsinh}(cx)$$

input
```
Integrate[a + b*ArcSinh[c*x],x]
```

output
```
a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

3.7.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+bx\text{arcsinh}(cx)-\frac{b\sqrt{c^2x^2+1}}{c}$$

input
```
Int[a + b*ArcSinh[c*x],x]
```

output
```
a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

3.7.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.7.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
default	$ax+\frac{b\left(\text{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	31
parts	$ax+\frac{b\left(\text{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	31
derivativedivides	$\frac{cxa+b\left(\text{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	33

input
```
int(a+b*arcsinh(c*x),x,method=_RETURNVERBOSE)
```

output
```
a*x+b/c*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2))
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int (a + b\text{arcsinh}(cx))\,dx = \frac{bcx\log\left(cx + \sqrt{c^2x^2+1}\right) + acx - \sqrt{c^2x^2+1}b}{c}$$

input
```
integrate(a+b*arcsinh(c*x),x, algorithm="fricas")
```

output
```
(b*c*x*log(c*x + sqrt(c^2*x^2 + 1)) + a*c*x - sqrt(c^2*x^2 + 1)*b)/c
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int (a + b\text{arcsinh}(cx))\,dx = ax + b\left(\begin{cases} x\,\text{asinh}\,(cx) - \frac{\sqrt{c^2x^2+1}}{c} & \text{for } c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*asinh(c*x),x)
```

output
```
a*x + b*Piecewise((x*asinh(c*x) - sqrt(c**2*x**2 + 1)/c, Ne(c, 0)), (0, Tr
ue))
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a + b\text{arcsinh}(cx))\,dx = ax + \frac{\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)b}{c}$$

input
```
integrate(a+b*arcsinh(c*x),x, algorithm="maxima")
```

output
```
a*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b/c
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int (a + b\operatorname{arcsinh}(cx))\,dx = \left(x\log\left(cx + \sqrt{c^2x^2 + 1}\right) - \frac{\sqrt{c^2x^2 + 1}}{c}\right)b + ax$$

input
```
integrate(a+b*arcsinh(c*x),x, algorithm="giac")
```

output
```
(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b + a*x
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int (a + b\operatorname{arcsinh}(cx))\,dx = a\,x - \frac{b\,\sqrt{c^2\,x^2 + 1}}{c} + b\,x\,\operatorname{asinh}(c\,x)$$

input
```
int(a + b*asinh(c*x),x)
```

output
```
a*x - (b*(c^2*x^2 + 1)^(1/2))/c + b*x*asinh(c*x)
```

3.8 $\int \frac{a+b\text{arcsinh}(cx)}{d+ex} dx$

3.8.1 Optimal result

Integrand size = 16, antiderivative size = 187

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex} dx = -\frac{(a+b\text{arcsinh}(cx))^2}{2be}+\frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{(a+b\text{arcsinh}(cx))\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{b\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{b\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

output
```
-1/2*(a+b*arcsinh(c*x))^2/b/e+(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1)^(
1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2
+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e+b*polylog(2,-e*(c*x+(c^2*x^2+1)^(1
/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+b*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(
c*d+(c^2*d^2+e^2)^(1/2)))/e
```

3.8.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.94

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex}\,dx$$
$$= \frac{-\left((a+b\text{arcsinh}(cx))\left(a+b\text{arcsinh}(cx)-2b\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-2b\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)\right)+2b^2\text{ P}}{2be}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x),x]
```

output
```
(-((a + b*ArcSinh[c*x])*(a + b*ArcSinh[c*x] - 2*b*Log[1 + (e*E^ArcSinh[c*x
])/(c*d - Sqrt[c^2*d^2 + e^2])] - 2*b*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sq
rt[c^2*d^2 + e^2])])) + 2*b^2*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt
[c^2*d^2 + e^2])] + 2*b^2*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*
d^2 + e^2]))])/(2*b*e)
```

3.8.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6242, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex}\,dx$$
$$\Big\downarrow \text{ 6242}$$
$$\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{cd+cex}d\text{arcsinh}(cx)$$
$$\Big\downarrow \text{ 6095}$$
$$\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cd+ee^{\text{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)+\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cd+ee^{\text{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)-\frac{(a+b\text{arcsinh}(cx))^2}{2be}$$
$$\Big\downarrow \text{ 2620}$$

$$-\frac{b\int \log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd-\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}-\frac{b\int \log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}+\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{2be}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{b\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd-\sqrt{c^2d^2+e^2}}+1\right)de^{\text{arcsinh}(cx)}}{e}-\frac{b\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)de^{\text{arcsinh}(cx)}}{e}+\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{2be}$$

$$\Big\downarrow \text{2838}$$

$$\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{2be}+\frac{b\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{b\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x),x]
```

output
```
-1/2*(a + b*ArcSinh[c*x])^2/(b*e) + ((a + b*ArcSinh[c*x])*Log[1 + (e*E^Arc
Sinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + ((a + b*ArcSinh[c*x])*Log[1 +
 (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])])/e + (b*PolyLog[2, -((e*E
^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (b*PolyLog[2, -((e*E^Arc
Sinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e
```

3.8.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

3.8.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.49

method	result
parts	$\frac{a\ln(ex+d)}{e}-\frac{b\,\mathrm{arcsinh}(cx)^2}{2e}+\frac{b\,\mathrm{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{b\,\mathrm{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+ \right.}{c}\right.}{e}$
derivativedivides	$\frac{\frac{ac\ln(ecx+cd)}{e}+bc\left(-\frac{\mathrm{arcsinh}(cx)^2}{2e}+\frac{\mathrm{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\mathrm{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+\sqrt{c^2x^2+1}\right)}{cd+\sqrt{c^2d^2-}}\right.}{e}\right.}{c}$
default	$\frac{\frac{ac\ln(ecx+cd)}{e}+bc\left(-\frac{\mathrm{arcsinh}(cx)^2}{2e}+\frac{\mathrm{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\mathrm{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+\sqrt{c^2x^2+1}\right)}{cd+\sqrt{c^2d^2-}}\right.}{e}\right.}{c}$

input
```
int((a+b*arcsinh(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*ln(e*x+d)/e-1/2*b*arcsinh(c*x)^2/e+b/e*arcsinh(c*x)*ln((-c*d-e*(c*x+(c^2
*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))+b/e*arcsin
h(c*x)*ln((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^
2+e^2)^(1/2)))+b/e*dilog((-c*d-e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/
2))/(-c*d+(c^2*d^2+e^2)^(1/2)))+b/e*dilog((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(
c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))
```

3.8.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/(e*x + d), x)
```

3.8.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex}\,dx=\int \frac{a+b\,\text{asinh}\,(cx)}{d+ex}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x+d),x)
```

output
```
Integral((a + b*asinh(c*x))/(d + e*x), x)
```

3.8.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(e*x + d), x) + a*log(e*x + d)/e
```

3.8.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(e*x + d), x)
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{d+ex}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{d+e\,x}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x),x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x), x)
```

3.9 $\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^2} dx$

3.9.1 Optimal result

Integrand size = 16, antiderivative size = 82

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^2} dx = -\frac{a+b\operatorname{arcsinh}(cx)}{e(d+ex)} - \frac{b c\operatorname{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$

output
```
(-a-b*arcsinh(c*x))/e/(e*x+d)-b*c*arctanh((-c^2*d*x+e)/(c^2*d^2+e^2)^(1/2)
/(c^2*x^2+1)^(1/2))/e/(c^2*d^2+e^2)^(1/2)
```

3.9.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^2} dx = -\frac{\frac{a+b\operatorname{arcsinh}(cx)}{d+ex} + \frac{b c\operatorname{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{\sqrt{c^2d^2+e^2}}}{e}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^2,x]
```

output
```
-(((a + b*ArcSinh[c*x])/(d + e*x) + (b*c*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d
^2 + e^2]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*d^2 + e^2])/e)
```

3.9.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6243, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^2}dx$$

$$\Big\downarrow \text{ 6243}$$

$$\frac{bc\int \frac{1}{(d+ex)\sqrt{c^2x^2+1}}dx}{e}-\frac{a+b\text{arcsinh}(cx)}{e(d+ex)}$$

$$\Big\downarrow \text{ 488}$$

$$-\frac{bc\int \frac{1}{c^2d^2+e^2-\frac{\left(e-c^2dx\right)^2}{c^2x^2+1}}d\frac{e-c^2dx}{\sqrt{c^2x^2+1}}}{e}-\frac{a+b\text{arcsinh}(cx)}{e(d+ex)}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{a+b\text{arcsinh}(cx)}{e(d+ex)}-\frac{b\text{carctanh}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x)^2,x]
```

output
```
-((a + b*ArcSinh[c*x])/(e*(d + e*x))) - (b*c*ArcTanh[(e - c^2*d*x)/(Sqrt[c
^2*d^2 + e^2]*Sqrt[1 + c^2*x^2])])/(e*Sqrt[c^2*d^2 + e^2])
```

3.9.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

3.9.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(78) = 156$.

Time = 0.96 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.12

method	result
parts	$-\frac{a}{(ex+d)e}-\frac{bc\,\mathrm{arcsinh}(cx)}{(ecx+cd)e}-\frac{bc\ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+2\sqrt{\frac{c^2d^2+e^2}{e^2}}\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{cx+\frac{dc}{e}}\right)}{e^2\sqrt{\frac{c^2d^2+e^2}{e^2}}}$
derivativedivides	$\frac{-\frac{a\,c^2}{(ecx+cd)e}+b\,c^2\left(-\frac{\mathrm{arcsinh}(cx)}{(ecx+cd)e}-\frac{\ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+2\sqrt{\frac{c^2d^2+e^2}{e^2}}\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{cx+\frac{dc}{e}}\right)}{e^2\sqrt{\frac{c^2d^2+e^2}{e^2}}}\right)}{c}$
default	$\frac{-\frac{a\,c^2}{(ecx+cd)e}+b\,c^2\left(-\frac{\mathrm{arcsinh}(cx)}{(ecx+cd)e}-\frac{\ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+2\sqrt{\frac{c^2d^2+e^2}{e^2}}\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{cx+\frac{dc}{e}}\right)}{e^2\sqrt{\frac{c^2d^2+e^2}{e^2}}}\right)}{c}$

input
```
int((a+b*arcsinh(c*x))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/(e*x+d)/e-b*c/(c*e*x+c*d)/e*arcsinh(c*x)-b*c/e^2/((c^2*d^2+e^2)/e^2)^(1
/2)*ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(c*x+d*c/e)+2*((c^2*d^2+e^2)/e^2)^(1/2
)*((c*x+d*c/e)^2-2*d*c/e*(c*x+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+d*c/e)
)
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(78) = 156.

Time = 0.27 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.09

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^2}\,dx =$$
$$-\frac{ac^2d^3+ade^2-(bc^2d^2e+be^3)x\log\left(cx+\sqrt{c^2x^2+1}\right)-(bcdex+bcd^2)\sqrt{c^2d^2+e^2}\log\left(-\frac{c^3d^2x-cde+\sqrt{c^2d}}{}\right.}{c^2d^4e+d^2e^3+(c}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

output
```
-(a*c^2*d^3 + a*d*e^2 - (b*c^2*d^2*e + b*e^3)*x*log(c*x + sqrt(c^2*x^2 + 1
)) - (b*c*d*e*x + b*c*d^2)*sqrt(c^2*d^2 + e^2)*log(-(c^3*d^2*x - c*d*e + s
qrt(c^2*d^2 + e^2)*(c^2*d*x - e) + (c^2*d^2 + sqrt(c^2*d^2 + e^2)*c*d + e^
2)*sqrt(c^2*x^2 + 1))/(e*x + d)) - (b*c^2*d^3 + b*d*e^2 + (b*c^2*d^2*e + b
*e^3)*x)*log(-c*x + sqrt(c^2*x^2 + 1)))/(c^2*d^4*e + d^2*e^3 + (c^2*d^3*e^
2 + d*e^4)*x)
```

3.9.6 Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^2}\,dx = \int \frac{a+b\operatorname{asinh}(cx)}{(d+ex)^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x+d)**2,x)
```

output
```
Integral((a + b*asinh(c*x))/(d + e*x)**2, x)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.26

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^2}\,dx = -b\left(\frac{\operatorname{arsinh}(cx)}{e^2x+de} - \frac{c\operatorname{arsinh}\left(\frac{cd\sqrt{e^4}x}{e|e^2x+de|} - \frac{\sqrt{e^4}}{c|e^2x+de|}\right)}{\sqrt{\frac{c^2d^2}{e^2}+1}e^2}\right) - \frac{a}{e^2x+de}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-b*(arcsinh(c*x)/(e^2*x + d*e) - c*arcsinh(c*d*sqrt(e^4)*x/(e*abs(e^2*x +
d*e)) - sqrt(e^4)/(c*abs(e^2*x + d*e)))/(sqrt(c^2*d^2/e^2 + 1)*e^2)) - a/(
e^2*x + d*e)
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(78) = 156.

Time = 0.47 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.83

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^2}\,dx = \left(\frac{c\log\left(-c^2de+\sqrt{c^2d^2+e^2}|c||e|\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}{\sqrt{c^2d^2+e^2}|e|} - \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{(ex+d)e} - \frac{c\log\left(-c^2de+\sqrt{c^2d^2+e\right.}{\sqrt{c}} - \frac{a}{(ex+d)e}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="giac")
```

output
```
(c*log(-c^2*d*e + sqrt(c^2*d^2 + e^2)*abs(c)*abs(e))*sgn(1/(e*x + d))*sgn(
e)/(sqrt(c^2*d^2 + e^2)*abs(e)) - log(c*x + sqrt(c^2*x^2 + 1))/((e*x + d)*
e) - c*log(-c^2*d*e + sqrt(c^2*d^2 + e^2)*(sqrt(c^2 - 2*c^2*d/(e*x + d) +
c^2*d^2/(e*x + d)^2 + e^2/(e*x + d)^2) + sqrt(c^2*d^2*e^2 + e^4)/((e*x + d
)*e))*abs(e))/(sqrt(c^2*d^2 + e^2)*abs(e)*sgn(1/(e*x + d))*sgn(e)))*b - a/
((e*x + d)*e)
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^2}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{{(d+e\,x)}^2}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x)^2,x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x)^2, x)
```

3.10 $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^3}\,dx$

3.10.1 Optimal result

Integrand size = 16, antiderivative size = 128

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^3}\,dx = -\frac{bc\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)(d+ex)} - \frac{a+b\text{arcsinh}(cx)}{2e(d+ex)^2} - \frac{bc^3d\text{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{2e\left(c^2d^2+e^2\right)^{3/2}}$$

output
```
1/2*(-a-b*arcsinh(c*x))/e/(e*x+d)^2-1/2*b*c^3*d*arctanh((-c^2*d*x+e)/(c^2*
d^2+e^2)^(1/2)/(c^2*x^2+1)^(1/2))/e/(c^2*d^2+e^2)^(3/2)-1/2*b*c*(c^2*x^2+1
)^(1/2)/(c^2*d^2+e^2)/(e*x+d)
```

3.10.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.30

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^3}\,dx = \frac{1}{2}\left(-\frac{a}{e(d+ex)^2} - \frac{bc\sqrt{1+c^2x^2}}{(c^2d^2+e^2)(d+ex)} - \frac{b\text{arcsinh}(cx)}{e(d+ex)^2} + \frac{bc^3d\log(d+ex)}{e\left(c^2d^2+e^2\right)^{3/2}} - \frac{bc^3d\log\left(e-c^2dx+\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}\right)}{e\left(c^2d^2+e^2\right)^{3/2}}\right)$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^3,x]
```

output
```
(-(a/(e*(d + e*x)^2)) - (b*c*Sqrt[1 + c^2*x^2])/((c^2*d^2 + e^2)*(d + e*x)
) - (b*ArcSinh[c*x])/(e*(d + e*x)^2) + (b*c^3*d*Log[d + e*x])/(e*(c^2*d^2
+ e^2)^(3/2)) - (b*c^3*d*Log[e - c^2*d*x + Sqrt[c^2*d^2 + e^2]*Sqrt[1 + c^
2*x^2]])/(e*(c^2*d^2 + e^2)^(3/2)))/2
```

3.10.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6243, 491, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^3}\,dx$$

$$\Big\downarrow \text{6243}$$

$$\frac{bc\int \frac{1}{(d+ex)^2\sqrt{c^2x^2+1}}dx}{2e}-\frac{a+b\text{arcsinh}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{491}$$

$$\frac{bc\left(\frac{c^2d\int \frac{1}{(d+ex)\sqrt{c^2x^2+1}}dx}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)(d+ex)}\right)}{2e}-\frac{a+b\text{arcsinh}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{488}$$

$$\frac{bc\left(-\frac{c^2d\int \frac{1}{c^2d^2+e^2-\frac{\left(e-c^2dx\right)^2}{c^2x^2+1}}d\frac{e-c^2dx}{\sqrt{c^2x^2+1}}}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)(d+ex)}\right)}{2e}-\frac{a+b\text{arcsinh}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{bc\left(-\frac{c^2d\text{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{(c^2d^2+e^2)^{3/2}}-\frac{e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)(d+ex)}\right)}{2e}-\frac{a+b\text{arcsinh}(cx)}{2e(d+ex)^2}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x)^3,x]
```

output
```
-1/2*(a + b*ArcSinh[c*x])/(e*(d + e*x)^2) + (b*c*(-((e*Sqrt[1 + c^2*x^2])/
((c^2*d^2 + e^2)*(d + e*x))) - (c^2*d*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d^2
+ e^2]*Sqrt[1 + c^2*x^2])])/(c^2*d^2 + e^2)^(3/2)))/(2*e)
```

3.10.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 491
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b*(c/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

3.10.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(117) = 234$.

Time = 0.57 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.13

method	result
parts	$-\frac{a}{2(ex+d)^2 e}-\frac{b\,c^2 \operatorname{arcsinh}(cx)}{2(ecx+cd)^2 e}-\frac{b\,c^2\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{2e(c^2d^2+e^2)\left(cx+\frac{dc}{e}\right)}-\frac{b\,c^3 d\ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}\cdots}{\cdots}\right.}{2e^2(\cdots}$
derivativedivides	$\frac{-\frac{a\,c^3}{2(ecx+cd)^2 e}+b\,c^3\left(-\frac{\operatorname{arcsinh}(cx)}{2(ecx+cd)^2 e}+\frac{-\frac{e^2\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{(c^2d^2+e^2)\left(cx+\frac{dc}{e}\right)}-\frac{dce\ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+2\sqrt{\frac{c^2d\cdots}{\cdots}}}{\cdots}\right.}{(c^2d^2+e^2\cdots}}{2e^3}\cdots\right.}{c}$
default	$\frac{-\frac{a\,c^3}{2(ecx+cd)^2 e}+b\,c^3\left(-\frac{\operatorname{arcsinh}(cx)}{2(ecx+cd)^2 e}+\frac{-\frac{e^2\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{(c^2d^2+e^2)\left(cx+\frac{dc}{e}\right)}-\frac{dce\ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+2\sqrt{\frac{c^2d\cdots}{\cdots}}}{\cdots}\right.}{(c^2d^2+e^2\cdots}}{2e^3}\cdots\right.}{c}$

input
```
int((a+b*arcsinh(c*x))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/(e*x+d)^2/e-1/2*b*c^2/(c*e*x+c*d)^2/e*arcsinh(c*x)-1/2*b*c^2/e/(c^2
*d^2+e^2)/(c*x+d*c/e)*((c*x+d*c/e)^2-2*d*c/e*(c*x+d*c/e)+(c^2*d^2+e^2)/e^2
)^(1/2)-1/2*b*c^3/e^2*d/(c^2*d^2+e^2)/((c^2*d^2+e^2)/e^2)^(1/2)*ln((2*(c^2
*d^2+e^2)/e^2-2*d*c/e*(c*x+d*c/e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*((c*x+d*c/e)
^2-2*d*c/e*(c*x+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+d*c/e))
```

3.10.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(116) = 232$.

Time = 0.32 (sec) , antiderivative size = 566, normalized size of antiderivative = 4.42

$$\int \frac{a + b\text{arcsinh}(cx)}{(d+ex)^3}\,dx =$$

$$-\frac{(a+b)c^4d^6 + (2\,a+b)c^2d^4e^2 + ad^2e^4 + (bc^4d^4e^2 + bc^2d^2e^4)x^2 - (bc^3d^3e^2x^2 + 2\,bc^3d^4ex + bc^3d^5)\sqrt{c^2d^2}}{}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/2*((a + b)*c^4*d^6 + (2*a + b)*c^2*d^4*e^2 + a*d^2*e^4 + (b*c^4*d^4*e^2
 + b*c^2*d^2*e^4)*x^2 - (b*c^3*d^3*e^2*x^2 + 2*b*c^3*d^4*e*x + b*c^3*d^5)*
sqrt(c^2*d^2 + e^2)*log(-(c^3*d^2*x - c*d*e + sqrt(c^2*d^2 + e^2)*(c^2*d*x
 - e) + (c^2*d^2 + sqrt(c^2*d^2 + e^2)*c*d + e^2)*sqrt(c^2*x^2 + 1))/(e*x
+ d)) + 2*(b*c^4*d^5*e + b*c^2*d^3*e^3)*x - ((b*c^4*d^4*e^2 + 2*b*c^2*d^2*
e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e + 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*log(c*x
+ sqrt(c^2*x^2 + 1)) - (b*c^4*d^6 + 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d
^4*e^2 + 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e + 2*b*c^2*d^3*e^3 +
 b*d*e^5)*x)*log(-c*x + sqrt(c^2*x^2 + 1)) + (b*c^3*d^5*e + b*c*d^3*e^3 +
(b*c^3*d^4*e^2 + b*c*d^2*e^4)*x)*sqrt(c^2*x^2 + 1))/(c^4*d^8*e + 2*c^2*d^6
*e^3 + d^4*e^5 + (c^4*d^6*e^3 + 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*
e^2 + 2*c^2*d^5*e^4 + d^3*e^6)*x)
```

3.10.6 Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d+ex)^3}\,dx = \int \frac{a + b\,\text{asinh}\,(cx)}{(d+ex)^3}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x+d)**3,x)
```

output
```
Integral((a + b*asinh(c*x))/(d + e*x)**3, x)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.23

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^3}\,dx =$$
$$-\frac{1}{2}\left(c\left(\frac{\sqrt{c^2x^2+1}}{c^2d^2ex+c^2d^3+e^3x+de^2}-\frac{c^2d\,\text{arsinh}\left(\frac{cdx}{e\left|x+\frac{d}{e}\right|}-\frac{1}{c\left|x+\frac{d}{e}\right|}\right)}{\left(\frac{c^2d^2}{e^2}+1\right)^{\frac{3}{2}}e^4}\right)+\frac{\text{arsinh}\,(cx)}{e^3x^2+2\,de^2x+d^2e}\right)b$$
$$-\frac{a}{2\,(e^3x^2+2\,de^2x+d^2e)}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

output
```
-1/2*(c*(sqrt(c^2*x^2 + 1)/(c^2*d^2*e*x + c^2*d^3 + e^3*x + d*e^2) - c^2*d
*arcsinh(c*d*x/(e*abs(x + d/e)) - 1/(c*abs(x + d/e)))/((c^2*d^2/e^2 + 1)^(
3/2)*e^4)) + arcsinh(c*x)/(e^3*x^2 + 2*d*e^2*x + d^2*e))*b - 1/2*a/(e^3*x^
2 + 2*d*e^2*x + d^2*e)
```

3.10.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^3}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{(ex+d)^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(e*x + d)^3, x)
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^3}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{(d+e\,x)^3}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x)^3,x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x)^3, x)
```

3.11 $\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^4} dx$

3.11.1 Optimal result

Integrand size = 16, antiderivative size = 183

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^4} dx = -\frac{bc\sqrt{1+c^2x^2}}{6(c^2d^2+e^2)(d+ex)^2} - \frac{bc^3d\sqrt{1+c^2x^2}}{2(c^2d^2+e^2)^2(d+ex)} - \frac{a+b\text{arcsinh}(cx)}{3e(d+ex)^3} - \frac{bc^3(2c^2d^2-e^2)\,\text{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{6e(c^2d^2+e^2)^{5/2}}$$

output
```
1/3*(-a-b*arcsinh(c*x))/e/(e*x+d)^3-1/6*b*c^3*(2*c^2*d^2-e^2)*arctanh((-c^
2*d*x+e)/(c^2*d^2+e^2)^(1/2)/(c^2*x^2+1)^(1/2))/e/(c^2*d^2+e^2)^(5/2)-1/6*
b*c*(c^2*x^2+1)^(1/2)/(c^2*d^2+e^2)/(e*x+d)^2-1/2*b*c^3*d*(c^2*x^2+1)^(1/2
)/(c^2*d^2+e^2)^2/(e*x+d)
```

3.11.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^4} dx = \frac{1}{6}\left(-\frac{2a}{e(d+ex)^3} - \frac{bc\sqrt{1+c^2x^2}(e^2+c^2d(4d+3ex))}{(c^2d^2+e^2)^2(d+ex)^2} - \frac{2b\text{arcsinh}(cx)}{e(d+ex)^3} - \frac{bc^3(-2c^2d^2+e^2)\log(d+ex)}{e(c^2d^2+e^2)^{5/2}} + \frac{bc^3(-2c^2d^2+e^2)\log\left(e-c^2dx+\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}\right)}{e(c^2d^2+e^2)^{5/2}}\right)$$

input
```
Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^4,x]
```

output
```
((-2*a)/(e*(d + e*x)^3) - (b*c*Sqrt[1 + c^2*x^2]*(e^2 + c^2*d*(4*d + 3*e*x
)))/((c^2*d^2 + e^2)^2*(d + e*x)^2) - (2*b*ArcSinh[c*x])/(e*(d + e*x)^3) -
 (b*c^3*(-2*c^2*d^2 + e^2)*Log[d + e*x])/(e*(c^2*d^2 + e^2)^(5/2)) + (b*c^
3*(-2*c^2*d^2 + e^2)*Log[e - c^2*d*x + Sqrt[c^2*d^2 + e^2]*Sqrt[1 + c^2*x^
2]])/(e*(c^2*d^2 + e^2)^(5/2)))/6
```

3.11.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6243, 498, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^4}dx$$

$$\Big\downarrow \text{6243}$$

$$\frac{bc\int \frac{1}{(d+ex)^3\sqrt{c^2x^2+1}}dx}{3e}-\frac{a+b\text{arcsinh}(cx)}{3e(d+ex)^3}$$

$$\Big\downarrow \text{498}$$

$$\frac{bc\left(-\frac{c^2\int -\frac{2d-ex}{(d+ex)^2\sqrt{c^2x^2+1}}dx}{2(c^2d^2+e^2)}-\frac{e\sqrt{c^2x^2+1}}{2(c^2d^2+e^2)(d+ex)^2}\right)}{3e}-\frac{a+b\text{arcsinh}(cx)}{3e(d+ex)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{bc\left(\frac{c^2\int \frac{2d-ex}{(d+ex)^2\sqrt{c^2x^2+1}}dx}{2(c^2d^2+e^2)}-\frac{e\sqrt{c^2x^2+1}}{2(c^2d^2+e^2)(d+ex)^2}\right)}{3e}-\frac{a+b\text{arcsinh}(cx)}{3e(d+ex)^3}$$

$$\Big\downarrow \text{679}$$

$$\frac{bc\left(\frac{c^2\left(\frac{(2c^2d^2-e^2)\int \frac{1}{(d+ex)\sqrt{c^2x^2+1}}dx}{c^2d^2+e^2}-\frac{3de\sqrt{c^2x^2+1}}{(c^2d^2+e^2)(d+ex)}\right)}{2(c^2d^2+e^2)}-\frac{e\sqrt{c^2x^2+1}}{2(c^2d^2+e^2)(d+ex)^2}\right)}{3e}-\frac{a+b\text{arcsinh}(cx)}{3e(d+ex)^3}$$

$$\Big\downarrow \; 488$$

$$\frac{bc\left(\frac{c^2\left(-\frac{\left(2c^2d^2-e^2\right)\int \frac{1}{c^2d^2+e^2-\frac{\left(e-c^2dx\right)^2}{c^2x^2+1}}d\frac{e-c^2dx}{\sqrt{c^2x^2+1}}}{c^2d^2+e^2}-\frac{3de\sqrt{c^2x^2+1}}{\left(c^2d^2+e^2\right)(d+ex)}\right)}{2(c^2d^2+e^2)}-\frac{e\sqrt{c^2x^2+1}}{2(c^2d^2+e^2)(d+ex)^2}\right)}{3e}-\frac{a+b\text{arcsinh}(cx)}{3e(d+ex)^3}$$

$$\Big\downarrow \; 219$$

$$\frac{bc\left(\frac{c^2\left(-\frac{\left(2c^2d^2-e^2\right)\text{arctanh}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{\left(c^2d^2+e^2\right)^{3/2}}-\frac{3de\sqrt{c^2x^2+1}}{\left(c^2d^2+e^2\right)(d+ex)}\right)}{2(c^2d^2+e^2)}-\frac{e\sqrt{c^2x^2+1}}{2(c^2d^2+e^2)(d+ex)^2}\right)}{3e}-\frac{a+b\text{arcsinh}(cx)}{3e(d+ex)^3}$$

input
```
Int[(a + b*ArcSinh[c*x])/(d + e*x)^4,x]
```

output
```
-1/3*(a + b*ArcSinh[c*x])/(e*(d + e*x)^3) + (b*c*(-1/2*(e*Sqrt[1 + c^2*x^2
])/((c^2*d^2 + e^2)*(d + e*x)^2) + (c^2*((-3*d*e*Sqrt[1 + c^2*x^2])/((c^2*
d^2 + e^2)*(d + e*x)) - ((2*c^2*d^2 - e^2)*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2
*d^2 + e^2]*Sqrt[1 + c^2*x^2])])/(c^2*d^2 + e^2)^(3/2)))/(2*(c^2*d^2 + e^2
))))/(3*e)
```

3.11.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

3.11.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(168) = 336$.

Time = 0.41 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.79

method	result
parts	$-\frac{a}{3(ex+d)^3 e}-\frac{b\,c^3\operatorname{arcsinh}(cx)}{3(ecx+cd)^3 e}-\frac{b\,c^3\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{6e^2(c^2d^2+e^2)\left(cx+\frac{dc}{e}\right)^2}-\frac{b\,c^4 d\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+}}{2e(c^2d^2+e^2)^2\left(cx+\frac{dc}{e}\right)}$
derivativedivides	$-\frac{a\,c^4}{3(ecx+cd)^3 e}-\frac{b\,c^4\operatorname{arcsinh}(cx)}{3(ecx+cd)^3 e}-\frac{b\,c^4\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{6e^2\left(c^2d^2+e^2\right)\left(cx+\frac{dc}{e}\right)^2}-\frac{b\,c^5 d\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{2e\left(c^2d^2+e^2\right)^2\left(cx+\frac{dc}{e}\right)}-$
default	$-\frac{a\,c^4}{3(ecx+cd)^3 e}-\frac{b\,c^4\operatorname{arcsinh}(cx)}{3(ecx+cd)^3 e}-\frac{b\,c^4\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{6e^2\left(c^2d^2+e^2\right)\left(cx+\frac{dc}{e}\right)^2}-\frac{b\,c^5 d\sqrt{\left(cx+\frac{dc}{e}\right)^2-\frac{2dc\left(cx+\frac{dc}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{2e\left(c^2d^2+e^2\right)^2\left(cx+\frac{dc}{e}\right)}-$

input
```
int((a+b*arcsinh(c*x))/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/(e*x+d)^3/e-1/3*b*c^3/(c*e*x+c*d)^3/e*arcsinh(c*x)-1/6*b*c^3/e^2/(c
^2*d^2+e^2)/(c*x+d*c/e)^2*((c*x+d*c/e)^2-2*d*c/e*(c*x+d*c/e)+(c^2*d^2+e^2)
/e^2)^(1/2)-1/2*b*c^4/e*d/(c^2*d^2+e^2)^2/(c*x+d*c/e)*((c*x+d*c/e)^2-2*d*c
/e*(c*x+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2)-1/2*b*c^5/e^2*d^2/(c^2*d^2+e^2)^2/
((c^2*d^2+e^2)/e^2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(c*x+d*c/e)+2*((
c^2*d^2+e^2)/e^2)^(1/2)*((c*x+d*c/e)^2-2*d*c/e*(c*x+d*c/e)+(c^2*d^2+e^2)/e
^2)^(1/2))/(c*x+d*c/e))+1/6*b*c^3/e^2/(c^2*d^2+e^2)/((c^2*d^2+e^2)/e^2)^(1
/2)*ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(c*x+d*c/e)+2*((c^2*d^2+e^2)/e^2)^(1/2
)*((c*x+d*c/e)^2-2*d*c/e*(c*x+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+d*c/e)
)
```

3.11.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(167) = 334$.

Time = 0.62 (sec) , antiderivative size = 977, normalized size of antiderivative = 5.34

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^4}\,dx =$$

$$-\frac{(2a+3b)c^6d^9+3(2a+b)c^4d^7e^2+6ac^2d^5e^4+2ad^3e^6+3(bc^6d^6e^3+bc^4d^4e^5)x^3+9(bc^6d^7e^2+bc^4d^5e}{}$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="fricas")
```

output
```
-1/6*((2*a + 3*b)*c^6*d^9 + 3*(2*a + b)*c^4*d^7*e^2 + 6*a*c^2*d^5*e^4 + 2*
a*d^3*e^6 + 3*(b*c^6*d^6*e^3 + b*c^4*d^4*e^5)*x^3 + 9*(b*c^6*d^7*e^2 + b*c
^4*d^5*e^4)*x^2 + (2*b*c^5*d^8 - b*c^3*d^6*e^2 + (2*b*c^5*d^5*e^3 - b*c^3*
d^3*e^5)*x^3 + 3*(2*b*c^5*d^6*e^2 - b*c^3*d^4*e^4)*x^2 + 3*(2*b*c^5*d^7*e
- b*c^3*d^5*e^3)*x)*sqrt(c^2*d^2 + e^2)*log(-(c^3*d^2*x - c*d*e - sqrt(c^2
*d^2 + e^2)*(c^2*d*x - e) + (c^2*d^2 - sqrt(c^2*d^2 + e^2)*c*d + e^2)*sqrt
(c^2*x^2 + 1))/(e*x + d)) + 9*(b*c^6*d^8*e + b*c^4*d^6*e^3)*x - 2*((b*c^6*
d^6*e^3 + 3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 + b*e^9)*x^3 + 3*(b*c^6*d^7*e^
2 + 3*b*c^4*d^5*e^4 + 3*b*c^2*d^3*e^6 + b*d*e^8)*x^2 + 3*(b*c^6*d^8*e + 3*
b*c^4*d^6*e^3 + 3*b*c^2*d^4*e^5 + b*d^2*e^7)*x)*log(c*x + sqrt(c^2*x^2 + 1
)) - 2*(b*c^6*d^9 + 3*b*c^4*d^7*e^2 + 3*b*c^2*d^5*e^4 + b*d^3*e^6 + (b*c^6
*d^6*e^3 + 3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 + b*e^9)*x^3 + 3*(b*c^6*d^7*e
^2 + 3*b*c^4*d^5*e^4 + 3*b*c^2*d^3*e^6 + b*d*e^8)*x^2 + 3*(b*c^6*d^8*e + 3
*b*c^4*d^6*e^3 + 3*b*c^2*d^4*e^5 + b*d^2*e^7)*x)*log(-c*x + sqrt(c^2*x^2 +
 1)) + (4*b*c^5*d^8*e + 5*b*c^3*d^6*e^3 + b*c*d^4*e^5 + 3*(b*c^5*d^6*e^3 +
 b*c^3*d^4*e^5)*x^2 + (7*b*c^5*d^7*e^2 + 8*b*c^3*d^5*e^4 + b*c*d^3*e^6)*x)
*sqrt(c^2*x^2 + 1))/(c^6*d^12*e + 3*c^4*d^10*e^3 + 3*c^2*d^8*e^5 + d^6*e^7
 + (c^6*d^9*e^4 + 3*c^4*d^7*e^6 + 3*c^2*d^5*e^8 + d^3*e^10)*x^3 + 3*(c^6*d
^10*e^3 + 3*c^4*d^8*e^5 + 3*c^2*d^6*e^7 + d^4*e^9)*x^2 + 3*(c^6*d^11*e^2 +
 3*c^4*d^9*e^4 + 3*c^2*d^7*e^6 + d^5*e^8)*x)
```

3.11.6 Sympy [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(d + ex)^4} \, dx = \int \frac{a + b\,\text{asinh}\,(cx)}{(d + ex)^4} \, dx$$

input
```
integrate((a+b*asinh(c*x))/(e*x+d)**4,x)
```

output
```
Integral((a + b*asinh(c*x))/(d + e*x)**4, x)
```

3.11.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^4}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(ex+d)^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="maxima")
```

output
```
1/6*(6*c*integrate(1/3/(c^3*e^4*x^6 + 3*c^3*d*e^3*x^5 + 3*c*d^2*e^2*x^2 +
c*d^3*e*x + (3*c^3*d^2*e^2 + c*e^4)*x^4 + (c^3*d^3*e + 3*c*d*e^3)*x^3 + (c
^2*e^4*x^5 + 3*c^2*d*e^3*x^4 + 3*d^2*e^2*x + d^3*e + (3*c^2*d^2*e^2 + e^4)
*x^3 + (c^2*d^3*e + 3*d*e^3)*x^2)*sqrt(c^2*x^2 + 1)), x) - 2*(c^6*d^3 - 3*
c^4*d*e^2)*log(e*x + d)/(c^6*d^6*e + 3*c^4*d^4*e^3 + 3*c^2*d^2*e^5 + e^7)
+ (3*c^6*d^6 + 2*c^4*d^4*e^2 - c^2*d^2*e^4 + 2*(c^6*d^4*e^2 - c^2*e^6)*x^2
 + (5*c^6*d^5*e + 2*c^4*d^3*e^3 - 3*c^2*d*e^5)*x + (c^6*d^6 - 3*c^4*d^4*e^
2 + (c^6*d^3*e^3 - 3*c^4*d*e^5)*x^3 + 3*(c^6*d^4*e^2 - 3*c^4*d^2*e^4)*x^2
+ 3*(c^6*d^5*e - 3*c^4*d^3*e^3)*x)*log(c^2*x^2 + 1) - 2*(c^6*d^6 + 3*c^4*d
^4*e^2 + 3*c^2*d^2*e^4 + e^6)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^6*d^9*e + 3
*c^4*d^7*e^3 + 3*c^2*d^5*e^5 + d^3*e^7 + (c^6*d^6*e^4 + 3*c^4*d^4*e^6 + 3*
c^2*d^2*e^8 + e^10)*x^3 + 3*(c^6*d^7*e^3 + 3*c^4*d^5*e^5 + 3*c^2*d^3*e^7 +
 d*e^9)*x^2 + 3*(c^6*d^8*e^2 + 3*c^4*d^6*e^4 + 3*c^2*d^4*e^6 + d^2*e^8)*x)
 - I*(3*c^6*d^2 - c^4*e^2)*(log(I*c*x + 1) - log(-I*c*x + 1))/((c^6*d^6 +
3*c^4*d^4*e^2 + 3*c^2*d^2*e^4 + e^6)*c))*b - 1/3*a/(e^4*x^3 + 3*d*e^3*x^2
+ 3*d^2*e^2*x + d^3*e)
```

3.11.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^4}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{(ex+d)^4}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(e*x + d)^4, x)
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(d+ex)^4}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{{(d+e\,x)}^4}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + e*x)^4,x)
```

output
```
int((a + b*asinh(c*x))/(d + e*x)^4, x)
```

3.12 $\int (d+ex)^3(a+b\text{arcsinh}(cx))^2\,dx$

3.12.1 Optimal result

Integrand size = 18, antiderivative size = 368

$$\begin{aligned}\int (d+ex)^3(a+b\text{arcsinh}(cx))^2\,dx &= 2b^2d^3x-\frac{4b^2de^2x}{3c^2}+\frac{3}{4}b^2d^2ex^2-\frac{3b^2e^3x^2}{32c^2}+\frac{2}{9}b^2de^2x^3 \\ &+\frac{1}{32}b^2e^3x^4-\frac{2bd^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c} \\ &+\frac{4bde^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3c^3} \\ &-\frac{3bd^2ex\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{2c} \\ &+\frac{3be^3x\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{16c^3} \\ &-\frac{2bde^2x^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{3c} \\ &-\frac{be^3x^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{8c} \\ &-\frac{d^4(a+b\text{arcsinh}(cx))^2}{4e}+\frac{3d^2e(a+b\text{arcsinh}(cx))^2}{4c^2} \\ &-\frac{3e^3(a+b\text{arcsinh}(cx))^2}{32c^4} \\ &+\frac{(d+ex)^4(a+b\text{arcsinh}(cx))^2}{4e}\end{aligned}$$

output
```
2*b^2*d^3*x-4/3*b^2*d*e^2*x/c^2+3/4*b^2*d^2*e*x^2-3/32*b^2*e^3*x^2/c^2+2/9
*b^2*d*e^2*x^3+1/32*b^2*e^3*x^4-1/4*d^4*(a+b*arcsinh(c*x))^2/e+3/4*d^2*e*(
a+b*arcsinh(c*x))^2/c^2-3/32*e^3*(a+b*arcsinh(c*x))^2/c^4+1/4*(e*x+d)^4*(a
+b*arcsinh(c*x))^2/e-2*b*d^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+4/3*b*
d*e^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-3/2*b*d^2*e*x*(a+b*arcsinh(
c*x))*(c^2*x^2+1)^(1/2)/c+3/16*b*e^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2
)/c^3-2/3*b*d*e^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c-1/8*b*e^3*x^3
*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

3.12.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.96

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{c(72a^2c^3x(4d^3+6d^2ex+4de^2x^2+e^3x^3)-6ab\sqrt{1+c^2x^2}(-e^2(64d+9ex)+c^2(96d^3+72d^2ex+32de^2x^2}{}$$

input
```
Integrate[(d + e*x)^3*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(c*(72*a^2*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - 6*a*b*Sqrt[
1 + c^2*x^2]*(-(e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*
x^2 + 6*e^3*x^3)) + b^2*c*x*(-3*e^2*(128*d + 9*e*x) + c^2*(576*d^3 + 216*d
^2*e*x + 64*d*e^2*x^2 + 9*e^3*x^3))) - 6*b*(-3*a*(24*c^2*d^2*e - 3*e^3 + 8
*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)) + b*c*Sqrt[1 + c^2*x^2
]*(-(e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6*e^3
*x^3)))*ArcSinh[c*x] + 9*b^2*(24*c^2*d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^
2*e*x + 4*d*e^2*x^2 + e^3*x^3))*ArcSinh[c*x]^2)/(288*c^4)
```

3.12.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6243, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6243}$$

$$\frac{(d+ex)^4(a+b\text{arcsinh}(cx))^2}{4e}-\frac{bc\int \frac{(d+ex)^4(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{2e}$$

$$\Big\downarrow \text{ 6253}$$

$$\frac{(d+ex)^4(a+b\text{arcsinh}(cx))^2}{4e}-$$

$$\frac{bc\int \left(\frac{(a+b\text{arcsinh}(cx))d^4}{\sqrt{c^2x^2+1}}+\frac{4ex(a+b\text{arcsinh}(cx))d^3}{\sqrt{c^2x^2+1}}+\frac{6e^2x^2(a+b\text{arcsinh}(cx))d^2}{\sqrt{c^2x^2+1}}+\frac{4e^3x^3(a+b\text{arcsinh}(cx))d}{\sqrt{c^2x^2+1}}+\frac{e^4x^4(a+b\text{arcsinh}}{\sqrt{c^2x^2+1}}\right.}{2e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(d+ex)^4(a+b\text{arcsinh}(cx))^2}{4e}-$$

$$bc\left(\frac{3e^4(a+b\text{arcsinh}(cx))^2}{16bc^5}-\frac{3d^2e^2(a+b\text{arcsinh}(cx))^2}{2bc^3}+\frac{4d^3e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}+\frac{3d^2e^2x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}+\frac{4de}{}\right.$$

input
```
Int[(d + e*x)^3*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((d + e*x)^4*(a + b*ArcSinh[c*x])^2)/(4*e) - (b*c*((-4*b*d^3*e*x)/c + (8*b
*d*e^3*x)/(3*c^3) - (3*b*d^2*e^2*x^2)/(2*c) + (3*b*e^4*x^2)/(16*c^3) - (4*
b*d*e^3*x^3)/(9*c) - (b*e^4*x^4)/(16*c) + (4*d^3*e*Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x]))/c^2 - (8*d*e^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3
*c^4) + (3*d^2*e^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c^2 - (3*e^4*
x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c^4) + (4*d*e^3*x^2*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2) + (e^4*x^3*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x]))/(4*c^2) + (d^4*(a + b*ArcSinh[c*x])^2)/(2*b*c) - (3*d^2*e^
2*(a + b*ArcSinh[c*x])^2)/(2*b*c^3) + (3*e^4*(a + b*ArcSinh[c*x])^2)/(16*b
*c^5)))/(2*e)
```

3.12. $\int (d+ex)^3(a+b\text{arcsinh}(cx))^2\,dx$

3.12.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

3.12.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{\frac{a^2(ecx+cd)^4}{4c^3e}+\frac{b^2\left(\frac{e^3\left(8\operatorname{arcsinh}(cx)^2x^4c^4-4\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+c^4x^4+6\operatorname{arcsinh}(cx)cx\sqrt{c^2x^2+1}-3\operatorname{arcsinh}(cx)^2-3c^2x^2-\cdots\right.}{32}\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{a^2(ecx+cd)^4}{4c^3e}+\frac{b^2\left(\frac{e^3\left(8\operatorname{arcsinh}(cx)^2x^4c^4-4\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^3c^3+c^4x^4+6\operatorname{arcsinh}(cx)cx\sqrt{c^2x^2+1}-3\operatorname{arcsinh}(cx)^2-3c^2x^2-\cdots\right.}{32}\cdots\right.}{\cdots}}{\cdots}$
parts	$\frac{a^2(ex+d)^4}{4e}+\frac{b^2\left(72\operatorname{arcsinh}(cx)^2c^4x^4e^3+288\operatorname{arcsinh}(cx)^2c^4x^3d\,e^2+432\operatorname{arcsinh}(cx)^2c^4x^2d^2e+288\operatorname{arcsinh}(cx)^2c^4x\,d^3\cdots\right.}{\cdots}$

input
```
int((e*x+d)^3*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/4*a^2/c^3*(c*e*x+c*d)^4/e+b^2/c^3*(1/32*e^3*(8*arcsinh(c*x)^2*x^4*c
^4-4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3*c^3+c^4*x^4+6*arcsinh(c*x)*c*x*(c^
2*x^2+1)^(1/2)-3*arcsinh(c*x)^2-3*c^2*x^2-3)+1/9*d*c*e^2*(9*arcsinh(c*x)^2
*x^3*c^3-6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+12*arcsinh(c*x
)*(c^2*x^2+1)^(1/2)-12*c*x)+3/4*d^2*c^2*e*(2*arcsinh(c*x)^2*x^2*c^2-2*arcs
inh(c*x)*c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x)^2+c^2*x^2+1)+d^3*c^3*(arcsinh(
c*x)^2*x*c-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x))+2*a*b/c^3*(1/4/e*arcsi
nh(c*x)*c^4*d^4+arcsinh(c*x)*c^4*d^3*x+3/2*e*arcsinh(c*x)*c^4*d^2*x^2+e^2*
arcsinh(c*x)*c^4*d*x^3+1/4*e^3*arcsinh(c*x)*c^4*x^4-1/4/e*(c^4*d^4*arcsinh
(c*x)+e^4*(1/4*c^3*x^3*(c^2*x^2+1)^(1/2)-3/8*c*x*(c^2*x^2+1)^(1/2)+3/8*arc
sinh(c*x))+4*d^3*c^3*e*(c^2*x^2+1)^(1/2)+6*d^2*c^2*e^2*(1/2*c*x*(c^2*x^2+1
)^(1/2)-1/2*arcsinh(c*x))+4*d*c*e^3*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^
2*x^2+1)^(1/2)))))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.29

$$\int (d+ex)^3(a+b\text{arcsinh}(cx))^2\,dx$$
$$= \frac{9\,(8\,a^2+b^2)c^4e^3x^4+32\,(9\,a^2+2\,b^2)c^4de^2x^3+27\,(8\,(2\,a^2+b^2)c^4d^2e-b^2c^2e^3)x^2+9\,(8\,b^2c^4e^3x^4+32\,b^2c^4}{}$$

input
```
integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/288*(9*(8*a^2 + b^2)*c^4*e^3*x^4 + 32*(9*a^2 + 2*b^2)*c^4*d*e^2*x^3 + 27
*(8*(2*a^2 + b^2)*c^4*d^2*e - b^2*c^2*e^3)*x^2 + 9*(8*b^2*c^4*e^3*x^4 + 32
*b^2*c^4*d*e^2*x^3 + 48*b^2*c^4*d^2*e*x^2 + 32*b^2*c^4*d^3*x + 24*b^2*c^2*
d^2*e - 3*b^2*e^3)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 96*(3*(a^2 + 2*b^2)*c^
4*d^3 - 4*b^2*c^2*d*e^2)*x + 6*(24*a*b*c^4*e^3*x^4 + 96*a*b*c^4*d*e^2*x^3
+ 144*a*b*c^4*d^2*e*x^2 + 96*a*b*c^4*d^3*x + 72*a*b*c^2*d^2*e - 9*a*b*e^3
- (6*b^2*c^3*e^3*x^3 + 32*b^2*c^3*d*e^2*x^2 + 96*b^2*c^3*d^3 - 64*b^2*c*d*
e^2 + 9*(8*b^2*c^3*d^2*e - b^2*c*e^3)*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt
(c^2*x^2 + 1)) - 6*(6*a*b*c^3*e^3*x^3 + 32*a*b*c^3*d*e^2*x^2 + 96*a*b*c^3*
d^3 - 64*a*b*c*d*e^2 + 9*(8*a*b*c^3*d^2*e - a*b*c*e^3)*x)*sqrt(c^2*x^2 + 1
))/c^4
```

3.12.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(364) = 728$.

Time = 0.45 (sec) , antiderivative size = 743, normalized size of antiderivative = 2.02

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$= \begin{cases} a^2d^3x + \frac{3a^2d^2ex^2}{2} + a^2de^2x^3 + \frac{a^2e^3x^4}{4} + 2abd^3x\operatorname{asinh}(cx) + 3abd^2ex^2\operatorname{asinh}(cx) + 2abde^2x^3\operatorname{asinh}(cx) + \\ a^2\left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right) \end{cases}$$

input
```
integrate((e*x+d)**3*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*d**3*x + 3*a**2*d**2*e*x**2/2 + a**2*d*e**2*x**3 + a**2*e*
*3*x**4/4 + 2*a*b*d**3*x*asinh(c*x) + 3*a*b*d**2*e*x**2*asinh(c*x) + 2*a*b
*d*e**2*x**3*asinh(c*x) + a*b*e**3*x**4*asinh(c*x)/2 - 2*a*b*d**3*sqrt(c**
2*x**2 + 1)/c - 3*a*b*d**2*e*x*sqrt(c**2*x**2 + 1)/(2*c) - 2*a*b*d*e**2*x*
*2*sqrt(c**2*x**2 + 1)/(3*c) - a*b*e**3*x**3*sqrt(c**2*x**2 + 1)/(8*c) + 3
*a*b*d**2*e*asinh(c*x)/(2*c**2) + 4*a*b*d*e**2*sqrt(c**2*x**2 + 1)/(3*c**3
) + 3*a*b*e**3*x*sqrt(c**2*x**2 + 1)/(16*c**3) - 3*a*b*e**3*asinh(c*x)/(16
*c**4) + b**2*d**3*x*asinh(c*x)**2 + 2*b**2*d**3*x + 3*b**2*d**2*e*x**2*as
inh(c*x)**2/2 + 3*b**2*d**2*e*x**2/4 + b**2*d*e**2*x**3*asinh(c*x)**2 + 2*
b**2*d*e**2*x**3/9 + b**2*e**3*x**4*asinh(c*x)**2/4 + b**2*e**3*x**4/32 -
2*b**2*d**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - 3*b**2*d**2*e*x*sqrt(c**2*x
**2 + 1)*asinh(c*x)/(2*c) - 2*b**2*d*e**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c
*x)/(3*c) - b**2*e**3*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(8*c) + 3*b**2*d
**2*e*asinh(c*x)**2/(4*c**2) - 4*b**2*d*e**2*x/(3*c**2) - 3*b**2*e**3*x**2
/(32*c**2) + 4*b**2*d*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c**3) + 3*b**
2*e**3*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(16*c**3) - 3*b**2*e**3*asinh(c*x)
**2/(32*c**4), Ne(c, 0)), (a**2*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 +
e**3*x**4/4), True))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.60

$$
\begin{aligned}
\int (d+ex)^3(a+b\text{arcsinh}(cx))^2\,dx &= \frac{1}{4}\,b^2e^3x^4\,\text{arsinh}\left(cx\right)^2 + b^2de^2x^3\,\text{arsinh}\left(cx\right)^2 \\
&+ \frac{1}{4}\,a^2e^3x^4 + \frac{3}{2}\,b^2d^2ex^2\,\text{arsinh}\left(cx\right)^2 + a^2de^2x^3 + b^2d^3x\,\text{arsinh}\left(cx\right)^2 \\
&+ \frac{3}{2}\,a^2d^2ex^2 + \frac{3}{2}\left(2\,x^2\,\text{arsinh}\left(cx\right) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\text{arsinh}\left(cx\right)}{c^3}\right)\right)abd^2e \\
&+ \frac{3}{4}\left(c^2\left(\frac{x^2}{c^2} - \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4}\right) - 2\,c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\text{arsinh}\left(cx\right)}{c^3}\right)\text{arsinh}\left(cx\right)\right)b^2d^2e \\
&+ \frac{2}{3}\left(3\,x^3\,\text{arsinh}\left(cx\right) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)abde^2 \\
&- \frac{2}{9}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\text{arsinh}\left(cx\right) - \frac{c^2x^3-6\,x}{c^2}\right)b^2de^2 \\
&+ \frac{1}{16}\left(8\,x^4\,\text{arsinh}\left(cx\right) - \left(\frac{2\,\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\,\sqrt{c^2x^2+1}x}{c^4} + \frac{3\,\text{arsinh}\left(cx\right)}{c^5}\right)c\right)abe^3 \\
&+ \frac{1}{32}\left(\left(\frac{x^4}{c^2} - \frac{3\,x^2}{c^4} + \frac{3\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6}\right)c^2 - 2\left(\frac{2\,\sqrt{c^2x^2+1}x^3}{c^2} - \frac{3\,\sqrt{c^2x^2+1}x}{c^4} + \frac{3\,\text{arsinh}\left(cx\right)}{c^5}\right.\right. \\
&+ 2\,b^2d^3\left(x - \frac{\sqrt{c^2x^2+1}\,\text{arsinh}\left(cx\right)}{c}\right) + a^2d^3x + \frac{2\left(cx\,\text{arsinh}\left(cx\right) - \sqrt{c^2x^2+1}\right)abd^3}{c}
\end{aligned}
$$

input
```
integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*e^3*x^4*arcsinh(c*x)^2 + b^2*d*e^2*x^3*arcsinh(c*x)^2 + 1/4*a^2*e^
3*x^4 + 3/2*b^2*d^2*e*x^2*arcsinh(c*x)^2 + a^2*d*e^2*x^3 + b^2*d^3*x*arcsi
nh(c*x)^2 + 3/2*a^2*d^2*e*x^2 + 3/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2
+ 1)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d^2*e + 3/4*(c^2*(x^2/c^2 - log(c*x +
sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^
3)*arcsinh(c*x))*b^2*d^2*e + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1
)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d*e^2 - 2/9*(3*c*(sqrt(c^2*x^2 +
 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)
*b^2*d*e^2 + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*s
qrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c)*a*b*e^3 + 1/32*((x^4/c^2 -
 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c^2*x^2
 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5)*c*arcsinh(
c*x))*b^2*e^3 + 2*b^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^3
*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^3/c
```

3.12.8 Giac [F(-2)]

Exception generated.

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3(a+b\operatorname{arcsinh}(cx))^2\,dx = \int (a+b\operatorname{asinh}(c\,x))^2\,(d+e\,x)^3\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + e*x)^3,x)
```

output
```
int((a + b*asinh(c*x))^2*(d + e*x)^3, x)
```

3.13 $\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))^2\,dx$

3.13.1 Optimal result

Integrand size = 18, antiderivative size = 239

$$\begin{aligned}\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))^2\,dx =\ & 2b^2d^2x-\frac{4b^2e^2x}{9c^2}+\frac{1}{2}b^2dex^2+\frac{2}{27}b^2e^2x^3 \\ & -\frac{2bd^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} \\ & +\frac{4be^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{9c^3} \\ & -\frac{bdex\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} \\ & -\frac{2be^2x^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{9c} \\ & -\frac{d^3(a+b\operatorname{arcsinh}(cx))^2}{3e}+\frac{de(a+b\operatorname{arcsinh}(cx))^2}{2c^2} \\ & +\frac{(d+ex)^3(a+b\operatorname{arcsinh}(cx))^2}{3e}\end{aligned}$$

output
```
2*b^2*d^2*x-4/9*b^2*e^2*x/c^2+1/2*b^2*d*e*x^2+2/27*b^2*e^2*x^3-1/3*d^3*(a+
b*arcsinh(c*x))^2/e+1/2*d*e*(a+b*arcsinh(c*x))^2/c^2+1/3*(e*x+d)^3*(a+b*ar
csinh(c*x))^2/e-2*b*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+4/9*b*e^2*(
a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-b*d*e*x*(a+b*arcsinh(c*x))*(c^2*x^
2+1)^(1/2)/c-2/9*b*e^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

3.13.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.04

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{18a^2c^3x(3d^2+3dex+e^2x^2)-6ab\sqrt{1+c^2x^2}(-4e^2+c^2(18d^2+9dex+2e^2x^2))+b^2cx(-24e^2+c^2(108d^2$$

input
```
Integrate[(d + e*x)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(18*a^2*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(-4*e^
2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2)) + b^2*c*x*(-24*e^2 + c^2*(108*d^2
+ 27*d*e*x + 4*e^2*x^2)) - 6*b*(-3*a*(3*c*d*e + 2*c^3*x*(3*d^2 + 3*d*e*x +
 e^2*x^2)) + b*Sqrt[1 + c^2*x^2]*(-4*e^2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x
^2)))*ArcSinh[c*x] + 9*b^2*c*(6*c^2*d^2*x + 2*c^2*e^2*x^3 + 3*d*(e + 2*c^2
*e*x^2))*ArcSinh[c*x]^2)/(54*c^3)
```

3.13.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6243, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx$$
$$\Big\downarrow \text{ 6243}$$
$$\frac{(d+ex)^3(a+b\text{arcsinh}(cx))^2}{3e}-\frac{2bc\int \frac{(d+ex)^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{3e}$$
$$\Big\downarrow \text{ 6253}$$
$$\frac{(d+ex)^3(a+b\text{arcsinh}(cx))^2}{3e}-\frac{2bc\int \left(\frac{(a+b\text{arcsinh}(cx))d^3}{\sqrt{c^2x^2+1}}+\frac{3ex(a+b\text{arcsinh}(cx))d^2}{\sqrt{c^2x^2+1}}+\frac{3e^2x^2(a+b\text{arcsinh}(cx))d}{\sqrt{c^2x^2+1}}+\frac{e^3x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}\right)dx}{3e}$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{(d+ex)^3(a+b\text{arcsinh}(cx))^2}{3e} - \frac{2bc\left(-\frac{3de^2(a+b\text{arcsinh}(cx))^2}{4bc^3} + \frac{3d^2e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2} + \frac{3de^2x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2} + \frac{e^3x^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(\right.}{3c^2}}{3e}$$

input
```
Int[(d + e*x)^2*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((d + e*x)^3*(a + b*ArcSinh[c*x])^2)/(3*e) - (2*b*c*((-3*b*d^2*e*x)/c + (2
*b*e^3*x)/(3*c^3) - (3*b*d*e^2*x^2)/(4*c) - (b*e^3*x^3)/(9*c) + (3*d^2*e*S
qrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c^2 - (2*e^3*Sqrt[1 + c^2*x^2]*(a +
 b*ArcSinh[c*x]))/(3*c^4) + (3*d*e^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*
x]))/(2*c^2) + (e^3*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2) +
(d^3*(a + b*ArcSinh[c*x])^2)/(2*b*c) - (3*d*e^2*(a + b*ArcSinh[c*x])^2)/(4
*b*c^3)))/(3*e)
```

3.13.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

3.13.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.52

method	result
derivativedivides	$\frac{\frac{a^2(ecx+cd)^3}{3c^2e}+\frac{b^2\left(\frac{e^2\left(9\,\text{arcsinh}(cx)^2x^3c^3-6\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^2c^2+2c^3x^3+12\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}-12cx\right)}{27}+\frac{dce\left(2\,\text{arcsinh}(c\right.}{}\right.}{c^{}}}{}$
default	$\frac{\frac{a^2(ecx+cd)^3}{3c^2e}+\frac{b^2\left(\frac{e^2\left(9\,\text{arcsinh}(cx)^2x^3c^3-6\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,x^2c^2+2c^3x^3+12\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}-12cx\right)}{27}+\frac{dce\left(2\,\text{arcsinh}(c\right.}{}\right.}{c^{}}}{}$
parts	$\frac{a^2(ex+d)^3}{3e}+\frac{b^2\left(18\,\text{arcsinh}(cx)^2c^3x^3e^2+54\,\text{arcsinh}(cx)^2c^3x^2de+54\,\text{arcsinh}(cx)^2c^3x\,d^2-12\,\text{arcsinh}(cx)\sqrt{c^2x^2+1}\,c^2x\right.}{}$

input
```
int((e*x+d)^2*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/3*a^2/c^2*(c*e*x+c*d)^3/e+b^2/c^2*(1/27*e^2*(9*arcsinh(c*x)^2*x^3*c
^3-6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+12*arcsinh(c*x)*(c^2
*x^2+1)^(1/2)-12*c*x)+1/2*d*c*e*(2*arcsinh(c*x)^2*x^2*c^2-2*arcsinh(c*x)*c
*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x)^2+c^2*x^2+1)+d^2*c^2*(arcsinh(c*x)^2*x*c
-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x))+2*a*b/c^2*(1/3/e*arcsinh(c*x)*c^
3*d^3+arcsinh(c*x)*c^3*d^2*x+e*arcsinh(c*x)*c^3*d*x^2+1/3*e^2*arcsinh(c*x)
*c^3*x^3-1/3/e*(c^3*d^3*arcsinh(c*x)+e^3*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/
3*(c^2*x^2+1)^(1/2))+3*d^2*c^2*e*(c^2*x^2+1)^(1/2)+3*d*c*e^2*(1/2*c*x*(c^2
*x^2+1)^(1/2)-1/2*arcsinh(c*x)))))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.33

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{2\,(9\,a^2+2\,b^2)c^3e^2x^3+27\,(2\,a^2+b^2)c^3dex^2+9\,(2\,b^2c^3e^2x^3+6\,b^2c^3dex^2+6\,b^2c^3d^2x+3\,b^2cde)\log\left(cx+\sqrt{\right.}}{}$$

input
```
integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/54*(2*(9*a^2 + 2*b^2)*c^3*e^2*x^3 + 27*(2*a^2 + b^2)*c^3*d*e*x^2 + 9*(2*
b^2*c^3*e^2*x^3 + 6*b^2*c^3*d*e*x^2 + 6*b^2*c^3*d^2*x + 3*b^2*c*d*e)*log(c
*x + sqrt(c^2*x^2 + 1))^2 + 6*(9*(a^2 + 2*b^2)*c^3*d^2 - 4*b^2*c*e^2)*x +
6*(6*a*b*c^3*e^2*x^3 + 18*a*b*c^3*d*e*x^2 + 18*a*b*c^3*d^2*x + 9*a*b*c*d*e
 - (2*b^2*c^2*e^2*x^2 + 9*b^2*c^2*d*e*x + 18*b^2*c^2*d^2 - 4*b^2*e^2)*sqrt
(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 6*(2*a*b*c^2*e^2*x^2 + 9*a*b
*c^2*d*e*x + 18*a*b*c^2*d^2 - 4*a*b*e^2)*sqrt(c^2*x^2 + 1))/c^3
```

3.13.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.90

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx$$

$$= \begin{cases} a^2d^2x + a^2dex^2 + \frac{a^2e^2x^3}{3} + 2abd^2x\operatorname{asinh}(cx) + 2abdex^2\operatorname{asinh}(cx) + \frac{2abe^2x^3\operatorname{asinh}(cx)}{3} - \frac{2abd^2\sqrt{c^2x^2+1}}{c} - abde \\ a^2\left(d^2x + dex^2 + \frac{e^2x^3}{3}\right) \end{cases}$$

input
```
integrate((e*x+d)**2*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*d**2*x + a**2*d*e*x**2 + a**2*e**2*x**3/3 + 2*a*b*d**2*x*a
sinh(c*x) + 2*a*b*d*e*x**2*asinh(c*x) + 2*a*b*e**2*x**3*asinh(c*x)/3 - 2*a
*b*d**2*sqrt(c**2*x**2 + 1)/c - a*b*d*e*x*sqrt(c**2*x**2 + 1)/c - 2*a*b*e*
*2*x**2*sqrt(c**2*x**2 + 1)/(9*c) + a*b*d*e*asinh(c*x)/c**2 + 4*a*b*e**2*s
qrt(c**2*x**2 + 1)/(9*c**3) + b**2*d**2*x*asinh(c*x)**2 + 2*b**2*d**2*x +
b**2*d*e*x**2*asinh(c*x)**2 + b**2*d*e*x**2/2 + b**2*e**2*x**3*asinh(c*x)*
*2/3 + 2*b**2*e**2*x**3/27 - 2*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c
- b**2*d*e*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*e**2*x**2*sqrt(c**2
*x**2 + 1)*asinh(c*x)/(9*c) + b**2*d*e*asinh(c*x)**2/(2*c**2) - 4*b**2*e**
2*x/(9*c**2) + 4*b**2*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c**3), Ne(c,
0)), (a**2*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.58

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx = \frac{1}{3}\,b^2e^2x^3\,\text{arsinh}\,(cx)^2+b^2dex^2\,\text{arsinh}\,(cx)^2+\frac{1}{3}\,a^2e^2x^3 + b^2d^2x\,\text{arsinh}\,(cx)^2+a^2dex^2+\left(2\,x^2\,\text{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}x}{c^2}-\frac{\text{arsinh}\,(cx)}{c^3}\right)\right)abde + \frac{1}{2}\left(c^2\left(\frac{x^2}{c^2}-\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4}\right)-2\,c\left(\frac{\sqrt{c^2x^2+1}x}{c^2}-\frac{\text{arsinh}\,(cx)}{c^3}\right)\text{arsinh}\,(cx)\right)b^2de + \frac{2}{9}\left(3\,x^3\,\text{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)abe^2 - \frac{2}{27}\left(3\,c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\text{arsinh}\,(cx)-\frac{c^2x^3-6\,x}{c^2}\right)b^2e^2 + 2\,b^2d^2\left(x-\frac{\sqrt{c^2x^2+1}\,\text{arsinh}\,(cx)}{c}\right)+a^2d^2x+\frac{2\,\left(cx\,\text{arsinh}\,(cx)-\sqrt{c^2x^2+1}\right)abd^2}{c}$$

input
```
integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*e^2*x^3*arcsinh(c*x)^2 + b^2*d*e*x^2*arcsinh(c*x)^2 + 1/3*a^2*e^2*
x^3 + b^2*d^2*x*arcsinh(c*x)^2 + a^2*d*e*x^2 + (2*x^2*arcsinh(c*x) - c*(sq
rt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d*e + 1/2*(c^2*(x^2/c^2 - l
og(c*x + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsin
h(c*x)/c^3)*arcsinh(c*x))*b^2*d*e + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*
x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*e^2 - 2/27*(3*c*(sqrt(c^2
*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x
)/c^2)*b^2*e^2 + 2*b^2*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^
2*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^2/c
```

3.13.8 Giac [F(-2)]

Exception generated.

$$\int (d+ex)^2(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2(a+b\operatorname{arcsinh}(cx))^2\,dx=\int \left(a+b\,\mathrm{asinh}(c\,x)\right)^2\,{\left(d+e\,x\right)}^2\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + e*x)^2,x)
```

output
```
int((a + b*asinh(c*x))^2*(d + e*x)^2, x)
```

3.14 $\int (d+ex)(a+b\operatorname{arcsinh}(cx))^2\,dx$

3.14.1 Optimal result

Integrand size = 16, antiderivative size = 140

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))^2\,dx = 2b^2dx+\frac{1}{4}b^2ex^2-\frac{2bd\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{c} - \frac{bex\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))}{2c} - \frac{d^2(a+b\operatorname{arcsinh}(cx))^2}{2e}+\frac{e(a+b\operatorname{arcsinh}(cx))^2}{4c^2} + \frac{(d+ex)^2(a+b\operatorname{arcsinh}(cx))^2}{2e}$$

output
```
2*b^2*d*x+1/4*b^2*e*x^2-1/2*d^2*(a+b*arcsinh(c*x))^2/e+1/4*e*(a+b*arcsinh(
c*x))^2/c^2+1/2*(e*x+d)^2*(a+b*arcsinh(c*x))^2/e-2*b*d*(a+b*arcsinh(c*x))*
(c^2*x^2+1)^(1/2)/c-1/2*b*e*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

3.14.2 Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))^2\,dx = \frac{c\left(2a^2cx(2d+ex)+b^2cx(8d+ex)-2ab(4d+ex)\sqrt{1+c^2x^2}\right)+2b\left(-bc(4d+ex)\sqrt{1+c^2x^2}+a(e+4c^2d\right.}{4c^2}$$

input
```
Integrate[(d + e*x)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
(c*(2*a^2*c*x*(2*d + e*x) + b^2*c*x*(8*d + e*x) - 2*a*b*(4*d + e*x)*Sqrt[1
 + c^2*x^2]) + 2*b*(-(b*c*(4*d + e*x)*Sqrt[1 + c^2*x^2]) + a*(e + 4*c^2*d*
x + 2*c^2*e*x^2))*ArcSinh[c*x] + b^2*(e + 4*c^2*d*x + 2*c^2*e*x^2)*ArcSinh
[c*x]^2)/(4*c^2)
```

3.14.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6243, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6243}$$

$$\frac{(d+ex)^2(a+b\text{arcsinh}(cx))^2}{2e}-\frac{bc\int \frac{(d+ex)^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{e}$$

$$\Big\downarrow \text{ 6253}$$

$$\frac{(d+ex)^2(a+b\text{arcsinh}(cx))^2}{2e}-\frac{bc\int \left(\frac{(a+b\text{arcsinh}(cx))d^2}{\sqrt{c^2x^2+1}}+\frac{2ex(a+b\text{arcsinh}(cx))d}{\sqrt{c^2x^2+1}}+\frac{e^2x^2(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}\right)dx}{e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(d+ex)^2(a+b\text{arcsinh}(cx))^2}{2e}-\frac{bc\left(-\frac{e^2(a+b\text{arcsinh}(cx))^2}{4bc^3}+\frac{2de\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}+\frac{e^2x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}+\frac{d^2(a+b\text{arcsinh}(cx))^2}{2bc}-\frac{2bdex}{c}-\right.}{e}$$

input
```
Int[(d + e*x)*(a + b*ArcSinh[c*x])^2,x]
```

output
```
((d + e*x)^2*(a + b*ArcSinh[c*x])^2)/(2*e) - (b*c*((-2*b*d*e*x)/c - (b*e^2
*x^2)/(4*c) + (2*d*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c^2 + (e^2*x*
Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2) + (d^2*(a + b*ArcSinh[c*x]
)^2)/(2*b*c) - (e^2*(a + b*ArcSinh[c*x])^2)/(4*b*c^3)))/e
```

3.14.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

3.14.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.31

method	result
parts	$a^2\left(\frac{1}{2}e\,x^2+dx\right)+\frac{b^2\left(\frac{e\left(2\,\mathrm{arcsinh}(cx)^2x^2c^2-2\,\mathrm{arcsinh}(cx)cx\sqrt{c^2x^2+1}+\mathrm{arcsinh}(cx)^2+c^2x^2+1\right)}{4c}+d\left(\mathrm{arcsinh}(cx)^2xc-2\,\cdots\right.\right.}{c}$
derivativedivides	$\frac{\frac{a^2\left(d\,c^2x+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b^2\left(\frac{e\left(2\,\mathrm{arcsinh}(cx)^2x^2c^2-2\,\mathrm{arcsinh}(cx)cx\sqrt{c^2x^2+1}+\mathrm{arcsinh}(cx)^2+c^2x^2+1\right)}{4}+dc\left(\mathrm{arcsinh}(cx)^2xc-2\,\mathrm{arcsinh}\cdots\right.\right.}{c}}{c}$
default	$\frac{\frac{a^2\left(d\,c^2x+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b^2\left(\frac{e\left(2\,\mathrm{arcsinh}(cx)^2x^2c^2-2\,\mathrm{arcsinh}(cx)cx\sqrt{c^2x^2+1}+\mathrm{arcsinh}(cx)^2+c^2x^2+1\right)}{4}+dc\left(\mathrm{arcsinh}(cx)^2xc-2\,\mathrm{arcsinh}\cdots\right.\right.}{c}}{c}$

input
```
int((e*x+d)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/2*e*x^2+d*x)+b^2/c*(1/4*e*(2*arcsinh(c*x)^2*x^2*c^2-2*arcsinh(c*x)*
c*x*(c^2*x^2+1)^(1/2)+arcsinh(c*x)^2+c^2*x^2+1)/c+d*(arcsinh(c*x)^2*x*c-2*
arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x))+2*a*b/c*(1/2*c*arcsinh(c*x)*x^2*e+a
rcsinh(c*x)*d*c*x-1/2/c*(e*(1/2*c*x*(c^2*x^2+1)^(1/2)-1/2*arcsinh(c*x))+2*
d*c*(c^2*x^2+1)^(1/2)))
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.31

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\frac{(2a^2+b^2)c^2ex^2+4(a^2+2b^2)c^2dx+(2b^2c^2ex^2+4b^2c^2dx+b^2e)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\left(2abc^2ex^2+\right.}{4c^2}$$

input
```
integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
1/4*((2*a^2 + b^2)*c^2*e*x^2 + 4*(a^2 + 2*b^2)*c^2*d*x + (2*b^2*c^2*e*x^2
+ 4*b^2*c^2*d*x + b^2*e)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(2*a*b*c^2*e*x
^2 + 4*a*b*c^2*d*x + a*b*e - (b^2*c*e*x + 4*b^2*c*d)*sqrt(c^2*x^2 + 1))*lo
g(c*x + sqrt(c^2*x^2 + 1)) - 2*(a*b*c*e*x + 4*a*b*c*d)*sqrt(c^2*x^2 + 1))/
c^2
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.66

$$\int (d+ex)(a+b\operatorname{arcsinh}(cx))^2\,dx$$
$$=\begin{cases} a^2dx+\frac{a^2ex^2}{2}+2abdx\operatorname{asinh}(cx)+abex^2\operatorname{asinh}(cx)-\frac{2abd\sqrt{c^2x^2+1}}{c}-\frac{abex\sqrt{c^2x^2+1}}{2c}+\frac{abe\operatorname{asinh}(cx)}{2c^2}+b^2dx\operatorname{asin} \\ a^2\left(dx+\frac{ex^2}{2}\right) \end{cases}$$

input
```
integrate((e*x+d)*(a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*d*x + a**2*e*x**2/2 + 2*a*b*d*x*asinh(c*x) + a*b*e*x**2*as
inh(c*x) - 2*a*b*d*sqrt(c**2*x**2 + 1)/c - a*b*e*x*sqrt(c**2*x**2 + 1)/(2*
c) + a*b*e*asinh(c*x)/(2*c**2) + b**2*d*x*asinh(c*x)**2 + 2*b**2*d*x + b**
2*e*x**2*asinh(c*x)**2/2 + b**2*e*x**2/4 - 2*b**2*d*sqrt(c**2*x**2 + 1)*as
inh(c*x)/c - b**2*e*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(2*c) + b**2*e*asinh(
c*x)**2/(4*c**2), Ne(c, 0)), (a**2*(d*x + e*x**2/2), True))
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.56

$$\begin{aligned}\int (d+ex)(a+b\text{arcsinh}(cx))^2\,dx &= \frac{1}{2}\,b^2ex^2\,\text{arsinh}\,(cx)^2 + b^2dx\,\text{arsinh}\,(cx)^2 \\ &+ \frac{1}{2}\,a^2ex^2 + \frac{1}{2}\left(2\,x^2\,\text{arsinh}\,(cx) - c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\text{arsinh}\,(cx)}{c^3}\right)\right)abe \\ &+ \frac{1}{4}\left(c^2\left(\frac{x^2}{c^2} - \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4}\right) - 2\,c\left(\frac{\sqrt{c^2x^2+1}x}{c^2} - \frac{\text{arsinh}\,(cx)}{c^3}\right)\text{arsinh}\,(cx)\right)b^2e \\ &+ 2\,b^2d\left(x - \frac{\sqrt{c^2x^2+1}\,\text{arsinh}\,(cx)}{c}\right) + a^2dx + \frac{2\left(cx\,\text{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)abd}{c}\end{aligned}$$

input
```
integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*e*x^2*arcsinh(c*x)^2 + b^2*d*x*arcsinh(c*x)^2 + 1/2*a^2*e*x^2 + 1/
2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*a*
b*e + 1/4*(c^2*(x^2/c^2 - log(c*x + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(
c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3)*arcsinh(c*x))*b^2*e + 2*b^2*d*(x -
sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c
^2*x^2 + 1))*a*b*d/c
```

3.14.8 Giac [F(-2)]

Exception generated.

$$\int (d+ex)(a+b\text{arcsinh}(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)(a+b\text{arcsinh}(cx))^2\,dx = \int (a+b\,\text{asinh}(c\,x))^2\,(d+e\,x)\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + e*x),x)
```

output
```
int((a + b*asinh(c*x))^2*(d + e*x), x)
```

3.15 $\int (a+b\text{arcsinh}(cx))^2\,dx$

3.15.1 Optimal result

Integrand size = 10, antiderivative size = 46

$$\int (a+b\text{arcsinh}(cx))^2\,dx=2b^2x-\frac{2b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{c}+x(a+b\text{arcsinh}(cx))^2$$

output
```
2*b^2*x+x*(a+b*arcsinh(c*x))^2-2*b*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

3.15.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int (a+b\text{arcsinh}(cx))^2\,dx=\left(a^2+2b^2\right)x-\frac{2ab\sqrt{1+c^2x^2}}{c}+\frac{2b\left(acx-b\sqrt{1+c^2x^2}\right)\text{arcsinh}(cx)}{c}+b^2x\text{arcsinh}(cx)^2$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2,x]
```

output
```
(a^2 + 2*b^2)*x - (2*a*b*Sqrt[1 + c^2*x^2])/c + (2*b*(a*c*x - b*Sqrt[1 + c
^2*x^2])*ArcSinh[c*x])/c + b^2*x*ArcSinh[c*x]^2
```

3.15.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6187, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6187}$$

$$x(a+b\text{arcsinh}(cx))^2-2bc\int \frac{x(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6213}$$

$$x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{b\int 1dx}{c}\right)$$

$$\Big\downarrow \text{ 24}$$

$$x(a+b\text{arcsinh}(cx))^2-2bc\left(\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}-\frac{bx}{c}\right)$$

input
```
Int[(a + b*ArcSinh[c*x])^2,x]
```

output
```
x*(a + b*ArcSinh[c*x])^2 - 2*b*c*(-((b*x)/c) + (Sqrt[1 + c^2*x^2]*(a + b*A
rcSinh[c*x]))/c^2)
```

3.15.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

3.15.4 Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

method	result	size
derivativedivides	$\frac{cx\,a^2+b^2\left(\operatorname{arcsinh}(cx)^2xc-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+2ab\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	72
default	$\frac{cx\,a^2+b^2\left(\operatorname{arcsinh}(cx)^2xc-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+2ab\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	72
parts	$a^2x+\frac{b^2\left(\operatorname{arcsinh}(cx)^2xc-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)}{c}+\frac{2ab\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	73

input
```
int((a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(c*x*a^2+b^2*(arcsinh(c*x)^2*x*c-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*
x)+2*a*b*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2)))
```

3.15.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(44) = 88.

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.09

$$\int (a+b\text{arcsinh}(cx))^2\,dx$$
$$=\frac{b^2cx\log\left(cx+\sqrt{c^2x^2+1}\right)^2+(a^2+2\,b^2)cx-2\,\sqrt{c^2x^2+1}ab+2\left(abcx-\sqrt{c^2x^2+1}b^2\right)\log\left(cx+\sqrt{c^2x^2+\cdots}\right.}{c}$$

input
```
integrate((a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
(b^2*c*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + (a^2 + 2*b^2)*c*x - 2*sqrt(c^2*x
^2 + 1)*a*b + 2*(a*b*c*x - sqrt(c^2*x^2 + 1)*b^2)*log(c*x + sqrt(c^2*x^2 +
 1)))/c
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int (a + b\text{arcsinh}(cx))^2 \, dx = \begin{cases} a^2x + 2abx \operatorname{asinh}(cx) - \frac{2ab\sqrt{c^2x^2+1}}{c} + b^2x \operatorname{asinh}^2(cx) + 2b^2x - \frac{2b^2\sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{c} & \text{for } c \neq 0 \\ a^2x & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*asinh(c*x))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*x*asinh(c*x) - 2*a*b*sqrt(c**2*x**2 + 1)/c + b**
2*x*asinh(c*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c, Ne
(c, 0)), (a**2*x, True))
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

$$\int (a + b\text{arcsinh}(cx))^2 \, dx = b^2x \operatorname{arsinh}(cx)^2 + 2b^2\left(x - \frac{\sqrt{c^2x^2+1} \operatorname{arsinh}(cx)}{c}\right) + a^2x + \frac{2\left(cx \operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)ab}{c}$$

input
```
integrate((a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
b^2*x*arcsinh(c*x)^2 + 2*b^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*
x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b/c
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(44) = 88.

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.41

$$\int (a + b\text{arcsinh}(cx))^2 \, dx$$
$$= 2\left(x\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{\sqrt{c^2x^2+1}}{c}\right)ab$$
$$+ \left(x\log\left(cx + \sqrt{c^2x^2+1}\right)^2 + 2\,c\left(\frac{x}{c} - \frac{\sqrt{c^2x^2+1}\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2}\right)\right)b^2 + a^2x$$

input
```
integrate((a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
2*(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b + (x*log(c*x
+ sqrt(c^2*x^2 + 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x
^2 + 1))/c^2))*b^2 + a^2*x
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int (a + b\text{arcsinh}(cx))^2 \, dx = \int (a + b\,\text{asinh}(c\,x))^2 \, dx$$

input
```
int((a + b*asinh(c*x))^2,x)
```

output
```
int((a + b*asinh(c*x))^2, x)
```

3.16 $\int \frac{(a+b\,\text{arcsinh}(cx))^2}{d+ex}\,dx$

3.16.1 Optimal result

Integrand size = 18, antiderivative size = 291

$$\begin{aligned}\int \frac{(a+b\,\text{arcsinh}(cx))^2}{d+ex}\,dx = &-\frac{(a+b\,\text{arcsinh}(cx))^3}{3be} \\ &+\frac{(a+b\,\text{arcsinh}(cx))^2\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\ &+\frac{(a+b\,\text{arcsinh}(cx))^2\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\ &+\frac{2b(a+b\,\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\ &+\frac{2b(a+b\,\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e} \\ &-\frac{2b^2\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} \\ &-\frac{2b^2\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\end{aligned}$$

output
```
-1/3*(a+b*arcsinh(c*x))^3/b/e+(a+b*arcsinh(c*x))^2*ln(1+e*(c*x+(c^2*x^2+1)
^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+(a+b*arcsinh(c*x))^2*ln(1+e*(c*x+(c^2
*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e+2*b*(a+b*arcsinh(c*x))*polylog
(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+2*b*(a+b*arcsin
h(c*x))*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e-
2*b^2*polylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e-2*
b^2*polylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e
```

3.16.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex}\,dx = \frac{-\frac{(a+b\text{arcsinh}(cx))^3}{b}+3(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)+3(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x),x]
```

output
```
(-((a + b*ArcSinh[c*x])^3/b) + 3*(a + b*ArcSinh[c*x])^2*Log[1 + (e*E^ArcSi
nh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])] + 3*(a + b*ArcSinh[c*x])^2*Log[1 + (
e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])] + 6*b*(a + b*ArcSinh[c*x])*
PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] + 6*b*(a + b
*ArcSinh[c*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])
)] - 6*b^2*PolyLog[3, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] -
 6*b^2*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(3*e
)
```

3.16.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6242, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex}\,dx$$

$$\Big\downarrow \text{ 6242}$$

$$\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{cd+cex}d\text{arcsinh}(cx)$$

$$\Big\downarrow \text{ 6095}$$

$$\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))^2}{cd+ee^{\text{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)+\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))^2}{cd+ee^{\text{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)-\frac{(a+b\text{arcsinh}(cx))^3}{3be}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{2b\int (a+b\text{arcsinh}(cx))\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd-\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}-\frac{2b\int (a+b\text{arcsinh}(cx))\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^3}{3be}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{2b\left(b\int \text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)d\text{arcsinh}(cx)-(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)\right)}{e}-\frac{2b\left(b\int \text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)d\text{arcsinh}(cx)-(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)}{e}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^3}{3be}$$

$$\Big\downarrow \text{ 2720}$$

3.16. $\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex}\,dx$

$$-\frac{2b\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)de^{\text{arcsinh}(cx)}-(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)\right)}{e}-\frac{2b\left(b\int e^{-\text{arcsinh}(cx)}\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)de^{\text{arcsinh}(cx)}-(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)}{e}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^3}{3be}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{2b\left(b\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)\right)}{e}-\frac{2b\left(b\,\text{PolyLog}\left(3,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)-(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)}{e}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{(a+b\text{arcsinh}(cx))^2\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^3}{3be}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + e*x),x]
```

output
```
-1/3*(a + b*ArcSinh[c*x])^3/(b*e) + ((a + b*ArcSinh[c*x])^2*Log[1 + (e*E^A
rcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + ((a + b*ArcSinh[c*x])^2*Log
[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])])/e - (2*b*(-((a + b*A
rcSinh[c*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))]
) + b*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))]))/e -
(2*b*(-((a + b*ArcSinh[c*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c
^2*d^2 + e^2]))]) + b*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
+ e^2]))]))/e
```

3.16.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.16.4 Maple [F]

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^2}{ex+d}dx$$

input
```
int((a+b*arcsinh(c*x))^2/(e*x+d),x)
```

output
```
int((a+b*arcsinh(c*x))^2/(e*x+d),x)
```

3.16.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{ex+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e*x + d), x)
```

3.16.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{d+ex}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(e*x+d),x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d + e*x), x)
```

3.16.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{d+ex}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{ex+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

output
```
a^2*log(e*x + d)/e + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(e*x + d
) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(e*x + d), x)
```

3.16.8 Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{d+ex}\,dx = \int \frac{(b\operatorname{arsinh}(cx)+a)^2}{ex+d}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(e*x + d), x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{d+ex}\,dx = \int \frac{(a+b\operatorname{asinh}(c\,x))^2}{d+e\,x}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + e*x),x)
```

output
```
int((a + b*asinh(c*x))^2/(d + e*x), x)
```

3.17 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx$

3.17.1 Optimal result

Integrand size = 18, antiderivative size = 263

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx = &-\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}\\ &+\frac{2bc(a+b\text{arcsinh}(cx))\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}\\ &-\frac{2bc(a+b\text{arcsinh}(cx))\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}\\ &+\frac{2b^2c\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}\\ &-\frac{2b^2c\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}\end{aligned}$$

output
```
-(a+b*arcsinh(c*x))^2/e/(e*x+d)+2*b*c*(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*
x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(1/2)-2*b*c*(a+b*
arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e/
(c^2*d^2+e^2)^(1/2)+2*b^2*c*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2
*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(1/2)-2*b^2*c*polylog(2,-e*(c*x+(c^2*x^2
+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(1/2)
```

3.17.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.73

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx$$
$$= \frac{-\frac{(a+b\text{arcsinh}(cx))^2}{d+ex}+\frac{2bc\left((a+b\text{arcsinh}(cx))\left(\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)+b\,\text{PolyLog}\left(2,\frac{ee^{\text{arcsinh}(cx)}}{-cd+\sqrt{c^2d^2+e^2}}\right)-b\,\text{Po}}{\sqrt{c^2d^2+e^2}}}{e}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x)^2,x]
```

output
```
(-((a + b*ArcSinh[c*x])^2/(d + e*x)) + (2*b*c*((a + b*ArcSinh[c*x])*(Log[1
 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])] - Log[1 + (e*E^ArcSinh[
c*x])/(c*d + Sqrt[c^2*d^2 + e^2])]) + b*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c
*d) + Sqrt[c^2*d^2 + e^2])] - b*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqr
t[c^2*d^2 + e^2]))]))/Sqrt[c^2*d^2 + e^2])/e
```

3.17.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6243, 6258, 3042, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx$$

$$\Big\downarrow \; 6243$$

$$\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)\sqrt{c^2x^2+1}}dx}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$

$$\Big\downarrow \; 6258$$

$$\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{cd+cex}d\text{arcsinh}(cx)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}+\frac{2bc\int \frac{a+b\text{arcsinh}(cx)}{cd-ie\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{e}$$

$\Big\downarrow$ 3803

$$\frac{4bc\int -\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{-2ce^{\text{arcsinh}(cx)}d-ee^{2\text{arcsinh}(cx)}+e}d\text{arcsinh}(cx)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$

$\Big\downarrow$ 25

$$-\frac{4bc\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{-2ce^{\text{arcsinh}(cx)}d-ee^{2\text{arcsinh}(cx)}+e}d\text{arcsinh}(cx)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$

$\Big\downarrow$ 2694

$$-\frac{4bc\left(\frac{e\int -\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2\left(cd+ee^{\text{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}\right)}d\text{arcsinh}(cx)}{\sqrt{c^2d^2+e^2}}-\frac{e\int -\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2\left(cd+ee^{\text{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}\right)}d\text{arcsinh}(cx)}{\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$

$\Big\downarrow$ 27

$$-\frac{4bc\left(\frac{e\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cd+ee^{\text{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)}{2\sqrt{c^2d^2+e^2}}-\frac{e\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cd+ee^{\text{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)}{2\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$

$\Big\downarrow$ 2620

$$-\frac{4bc\left(\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{b\int \log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}\right)}{2\sqrt{c^2d^2+e^2}}-\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}\right)}{2\sqrt{c^2}}\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$

$\Big\downarrow$ 2715

3.17. $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx$

$$-\frac{4bc\left(\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{b\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)de^{\text{arcsinh}(cx)}}{e}\right)}{2\sqrt{c^2d^2+e^2}}-\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{ar}}}{cd-}\right.}{e}\right.}{}\right.}{e}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$

$\Big\downarrow$ 2838

$$-\frac{4bc\left(\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}+\frac{b\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\right)}{2\sqrt{c^2d^2+e^2}}-\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{b\,\text{PolyLo}}{}\right.}{2\sqrt{c^2d^2+e^2}}\right.}{e}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{e(d+ex)}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + e*x)^2,x]
```

output
```
-((a + b*ArcSinh[c*x])^2/(e*(d + e*x))) - (4*b*c*(-1/2*(e*(((a + b*ArcSinh
[c*x])*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + (b*Pol
yLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e))/Sqrt[c^2*d
^2 + e^2] + (e*(((a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sq
rt[c^2*d^2 + e^2])])/e + (b*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^
2*d^2 + e^2]))])/e))/(2*Sqrt[c^2*d^2 + e^2])))/e
```

3.17.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

3.17. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^2} dx$

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

3.17.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.00

method	result
derivativedivides	$-\frac{a^2c^2}{(ecx+cd)e}+b^2c^2\left(-\frac{\operatorname{arcsinh}(cx)^2}{e(ecx+cd)}+\frac{2\operatorname{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2\operatorname{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+\sqrt{c^2x\right.}}{cd+\sqrt{c}}\right.}{e\sqrt{c^2d^2+e^2}}\right.$
default	$-\frac{a^2c^2}{(ecx+cd)e}+b^2c^2\left(-\frac{\operatorname{arcsinh}(cx)^2}{e(ecx+cd)}+\frac{2\operatorname{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2\operatorname{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+\sqrt{c^2x\right.}}{cd+\sqrt{c}}\right.}{e\sqrt{c^2d^2+e^2}}\right.$
parts	$-\frac{a^2}{(ex+d)e}+\frac{b^2\left(-\frac{c^2\operatorname{arcsinh}(cx)^2}{e(ecx+cd)}+\frac{2c^2\operatorname{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2c^2\operatorname{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+\right.}{}\right.}{e\sqrt{c^2d^2}}\right.}{}$

input
```
int((a+b*arcsinh(c*x))^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-a^2*c^2/(c*e*x+c*d)/e+b^2*c^2*(-arcsinh(c*x)^2/e/(c*e*x+c*d)+2/e*arc
sinh(c*x)/(c^2*d^2+e^2)^(1/2)*ln((-c*d-e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+
e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-2/e*arcsinh(c*x)/(c^2*d^2+e^2)^(1/
2)*ln((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^
2)^(1/2)))+2/e/(c^2*d^2+e^2)^(1/2)*dilog((-c*d-e*(c*x+(c^2*x^2+1)^(1/2))+(
c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-2/e/(c^2*d^2+e^2)^(1/2)*di
log((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)
^(1/2))))+2*a*b*c^2*(-1/(c*e*x+c*d)/e*arcsinh(c*x)-1/e^2/((c^2*d^2+e^2)/e^
2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(c*x+d*c/e)+2*((c^2*d^2+e^2)/e^2)
^(1/2)*((c*x+d*c/e)^2-2*d*c/e*(c*x+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+d
*c/e))))
```

3.17.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^2}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(ex+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e^2*x^2 + 2*d*e*
x + d^2), x)
```

3.17.6 Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^2}\,dx=\int \frac{(a+b\operatorname{asinh}(cx))^2}{(d+ex)^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(e*x+d)**2,x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d + e*x)**2, x)
```

3.17.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-b^2*(log(c*x + sqrt(c^2*x^2 + 1))^2/(e^2*x + d*e) - integrate(2*(c^3*x^2
+ sqrt(c^2*x^2 + 1)*c^2*x + c)*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*e^2*x^4 +
 c^3*d*e*x^3 + c*e^2*x^2 + c*d*e*x + (c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x +
d*e)*sqrt(c^2*x^2 + 1)), x)) - 2*a*b*(arcsinh(c*x)/(e^2*x + d*e) - c*arcsi
nh(c*d*sqrt(e^4)*x/(e*abs(e^2*x + d*e)) - sqrt(e^4)/(c*abs(e^2*x + d*e)))/
(sqrt(c^2*d^2/e^2 + 1)*e^2)) - a^2/(e^2*x + d*e)
```

3.17.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex+d)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(e*x + d)^2, x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{(d+e\,x)^2}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + e*x)^2,x)
```

output
```
int((a + b*asinh(c*x))^2/(d + e*x)^2, x)
```

3.18 $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3} dx$

3.18.1 Optimal result

Integrand size = 18, antiderivative size = 349

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3} dx = -\frac{bc\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{(c^2d^2+e^2)(d+ex)} - \frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2} + \frac{bc^3d(a+b\text{arcsinh}(cx))\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e(c^2d^2+e^2)^{3/2}} - \frac{bc^3d(a+b\text{arcsinh}(cx))\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e(c^2d^2+e^2)^{3/2}} + \frac{b^2c^2\log(d+ex)}{e(c^2d^2+e^2)} + \frac{b^2c^3d\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e(c^2d^2+e^2)^{3/2}} - \frac{b^2c^3d\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e(c^2d^2+e^2)^{3/2}}$$

output
```
-1/2*(a+b*arcsinh(c*x))^2/e/(e*x+d)^2+b^2*c^2*ln(e*x+d)/e/(c^2*d^2+e^2)+b*
c^3*d*(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)
^(1/2)))/e/(c^2*d^2+e^2)^(3/2)-b*c^3*d*(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2
*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(3/2)+b^2*c^3*d*
polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2
+e^2)^(3/2)-b^2*c^3*d*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e
^2)^(1/2)))/e/(c^2*d^2+e^2)^(3/2)-b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)
/(c^2*d^2+e^2)/(e*x+d)
```

3.18.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.77

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx$$
$$= \frac{-\frac{2bce\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{(c^2d^2+e^2)(d+ex)} - \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^2} + \frac{2b^2c^2\log(d+ex)}{c^2d^2+e^2} + \frac{2bc^3d\left((a+b\text{arcsinh}(cx))\left(\log\left(1+\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-\text{lo}\right.\right.}{}}{2e}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x)^3,x]
```

output
```
((-2*b*c*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/((c^2*d^2 + e^2)*(d + e
*x)) - (a + b*ArcSinh[c*x])^2/(d + e*x)^2 + (2*b^2*c^2*Log[d + e*x])/(c^2*
d^2 + e^2) + (2*b*c^3*d*((a + b*ArcSinh[c*x])*(Log[1 + (e*E^ArcSinh[c*x])/
(c*d - Sqrt[c^2*d^2 + e^2])] - Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*
d^2 + e^2])]) + b*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e
^2])] - b*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))]))/
(c^2*d^2 + e^2)^(3/2))/(2*e)
```

3.18.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6243, 6258, 3042, 3805, 3042, 3147, 16, 3803, 25, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx$$
$$\Big\downarrow \; 6243$$
$$\frac{bc\int \frac{a+b\text{arcsinh}(cx)}{(d+ex)^2\sqrt{c^2x^2+1}}dx}{e} - \frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$
$$\Big\downarrow \; 6258$$
$$\frac{bc^2\int \frac{a+b\text{arcsinh}(cx)}{(cd+cex)^2}d\text{arcsinh}(cx)}{e} - \frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}+\frac{bc^2\int \frac{a+b\text{arcsinh}(cx)}{(cd-ie\sin(i\text{arcsinh}(cx)))^2}d\text{arcsinh}(cx)}{e}$$

$$\Big\downarrow \text{3805}$$

$$\frac{bc^2\left(\frac{cd\int \frac{a+b\text{arcsinh}(cx)}{cd+cex}d\text{arcsinh}(cx)}{c^2d^2+e^2}+\frac{be\int \frac{\sqrt{c^2x^2+1}}{cd+cex}d\text{arcsinh}(cx)}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2d^2+e^2)(cd+cex)}\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}+\frac{bc^2\left(\frac{cd\int \frac{a+b\text{arcsinh}(cx)}{cd-ie\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{c^2d^2+e^2}+\frac{be\int \frac{\cos(i\text{arcsinh}(cx))}{cd-ie\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2d^2+e^2)(cd+cex)}\right)}{e}$$

$$\Big\downarrow \text{3147}$$

$$-\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}+\frac{bc^2\left(\frac{cd\int \frac{a+b\text{arcsinh}(cx)}{cd-ie\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{c^2d^2+e^2}+\frac{b\int \frac{1}{cd+cex}d(cex)}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2d^2+e^2)(cd+cex)}\right)}{e}$$

$$\Big\downarrow \text{16}$$

$$-\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}+\frac{bc^2\left(\frac{cd\int \frac{a+b\text{arcsinh}(cx)}{cd-ie\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2d^2+e^2)(cd+cex)}+\frac{b\log(cd+cex)}{c^2d^2+e^2}\right)}{e}$$

$$\Big\downarrow \text{3803}$$

$$\frac{bc^2\left(\frac{2cd\int -\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{-2ce^{\text{arcsinh}(cx)}d-ee^{2\text{arcsinh}(cx)}+e}d\text{arcsinh}(cx)}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2d^2+e^2)(cd+cex)}+\frac{b\log(cd+cex)}{c^2d^2+e^2}\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

$$\Big\downarrow \text{25}$$

3.18. $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx$

$$\frac{bc^2\left(-\frac{2cd\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{-2ce^{\text{arcsinh}(cx)}d-ee^{2\text{arcsinh}(cx)}+e}d\text{arcsinh}(cx)}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2d^2+e^2)(cd+cex)}+\frac{b\log(cd+cex)}{c^2d^2+e^2}\right)}{e}-\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

$\big\downarrow$ 2694

$$\frac{bc^2\left(-\frac{2cd\left(\frac{e\int -\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2\left(cd+ee^{\text{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}\right)}d\text{arcsinh}(cx)}{\sqrt{c^2d^2+e^2}}-\frac{e\int -\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2\left(cd+ee^{\text{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}\right)}d\text{arcsinh}(cx)}{\sqrt{c^2d^2+e^2}}\right)}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}(a+b\text{arc}}{(c^2d^2+e^2)(cd}\right.}{e}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

$\big\downarrow$ 27

$$\frac{bc^2\left(-\frac{2cd\left(\frac{e\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cd+ee^{\text{arcsinh}(cx)}+\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)}{2\sqrt{c^2d^2+e^2}}-\frac{e\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cd+ee^{\text{arcsinh}(cx)}-\sqrt{c^2d^2+e^2}}d\text{arcsinh}(cx)}{2\sqrt{c^2d^2+e^2}}\right)}{c^2d^2+e^2}-\frac{e\sqrt{c^2x^2+1}(a+b\text{arcsinh}}{(c^2d^2+e^2)(cd+cex)}\right.}{e}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

$\big\downarrow$ 2620

3.18. $\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx$

$$bc^2\left(-\frac{2cd\left(\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{b\int\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)d\text{arcsinh}(cx)}{e}\right)}{2\sqrt{c^2d^2+e^2}}-\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+\right.}{e}\right.}{2\sqrt{}}\right)}{c^2d^2+e^2}\right.}{e}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

$\downarrow$ 2715

$$bc^2\left(-\frac{2cd\left(\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{b\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}e}{cd+\sqrt{c^2d^2+e^2}}+1\right)de^{\text{arcsinh}(cx)}}{e}\right)}{2\sqrt{c^2d^2+e^2}}-\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee}{cd}\right.}{e}\right.}{}\right)}{c^2d^2+e^2}\right.}{e}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

$\downarrow$ 2838

3.18. $\int\frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx$

$$\frac{bc^2\left(-\frac{2cd\left(\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}+\frac{b\,\text{PolyLog}\left(2,-\frac{ee^{\text{arcsinh}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}\right)}{2\sqrt{c^2d^2+e^2}}-\frac{e\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ee^{\text{arcsinh}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{b\,\text{Poly}\cdots}{\cdots}\right)}{2\sqrt{c^2d^2+e^2}}\cdots\right)}{c^2d^2+e^2}\cdots\right)}{e}$$

$$\frac{(a+b\text{arcsinh}(cx))^2}{2e(d+ex)^2}$$

input
```
Int[(a + b*ArcSinh[c*x])^2/(d + e*x)^3,x]
```

output
```
-1/2*(a + b*ArcSinh[c*x])^2/(e*(d + e*x)^2) + (b*c^2*(-((e*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/((c^2*d^2 + e^2)*(c*d + c*e*x))) + (b*Log[c*d + c
*e*x])/(c^2*d^2 + e^2) - (2*c*d*(-1/2*(e*(((a + b*ArcSinh[c*x])*Log[1 + (e
*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + (b*PolyLog[2, -((e*E^Ar
cSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e))/Sqrt[c^2*d^2 + e^2] + (e*((
(a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]
)])/e + (b*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/
e))/(2*Sqrt[c^2*d^2 + e^2])))/(c^2*d^2 + e^2)))/e
```

3.18.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.18. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^3}\,dx$

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

3.18.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(365) = 730$.

Time = 0.65 (sec) , antiderivative size = 815, normalized size of antiderivative = 2.34

method	result
derivativedivides	$\frac{-\frac{a^2 c^3}{2(ecx+cd)^2 e}+b^2 c^3 \left(-\frac{\operatorname{arcsinh}(cx)\left(2dce\sqrt{c^2x^2+1}-4d\,c^2ex+e^2\operatorname{arcsinh}(cx)+2\sqrt{c^2x^2+1}\,e^2cx-2c^2d^2-2e^2c^2x^2+c^2d^2\operatorname{arcsinh}(cx)\right.}{2e(ecx+cd)^2\left(c^2d^2+e^2\right)}\right.}{}$
default	$\frac{-\frac{a^2 c^3}{2(ecx+cd)^2 e}+b^2 c^3 \left(-\frac{\operatorname{arcsinh}(cx)\left(2dce\sqrt{c^2x^2+1}-4d\,c^2ex+e^2\operatorname{arcsinh}(cx)+2\sqrt{c^2x^2+1}\,e^2cx-2c^2d^2-2e^2c^2x^2+c^2d^2\operatorname{arcsinh}(cx)\right.}{2e(ecx+cd)^2\left(c^2d^2+e^2\right)}\right.}{}$
parts	$-\frac{a^2}{2(ex+d)^2 e}+\frac{b^2\left(-\frac{c^3\operatorname{arcsinh}(cx)\left(2dce\sqrt{c^2x^2+1}-4d\,c^2ex+e^2\operatorname{arcsinh}(cx)+2\sqrt{c^2x^2+1}\,e^2cx-2c^2d^2-2e^2c^2x^2+c^2d^2\operatorname{arcsinh}(\right.}{2e\left(c^2d^2+e^2\right)(ecx+cd)^2}\right.}{}$

input
```
int((a+b*arcsinh(c*x))^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2*a^2*c^3/(c*e*x+c*d)^2/e+b^2*c^3*(-1/2*arcsinh(c*x)*(2*d*c*e*(c^2
*x^2+1)^(1/2)-4*d*c^2*e*x+e^2*arcsinh(c*x)+2*(c^2*x^2+1)^(1/2)*e^2*c*x-2*c
^2*d^2-2*e^2*c^2*x^2+c^2*d^2*arcsinh(c*x))/e/(c*e*x+c*d)^2/(c^2*d^2+e^2)-2
/e/(c^2*d^2+e^2)*ln(c*x+(c^2*x^2+1)^(1/2))+1/e/(c^2*d^2+e^2)*ln(2*d*(c*x+(
c^2*x^2+1)^(1/2))*c+e*(c*x+(c^2*x^2+1)^(1/2))^2-e)+1/e/(c^2*d^2+e^2)^(3/2)
*d*c*arcsinh(c*x)*ln((-c*d-e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/
(-c*d+(c^2*d^2+e^2)^(1/2)))-1/e/(c^2*d^2+e^2)^(3/2)*d*c*arcsinh(c*x)*ln((c
*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)
))+1/e/(c^2*d^2+e^2)^(3/2)*d*c*dilog((-c*d-e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*
d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-1/e/(c^2*d^2+e^2)^(3/2)*d*c*di
log((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)
^(1/2))))-a*b*c^3/(c*e*x+c*d)^2/e*arcsinh(c*x)-a*b*c^3/e/(c^2*d^2+e^2)/(c*
x+d*c/e)*((c*x+d*c/e)^2-2*d*c/e*(c*x+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2)-a*b*c
^4/e^2*d/(c^2*d^2+e^2)/((c^2*d^2+e^2)/e^2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2
*d*c/e*(c*x+d*c/e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*((c*x+d*c/e)^2-2*d*c/e*(c*x
+d*c/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+d*c/e)))
```

3.18. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2}{(d+ex)^3}\,dx$

3.18.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex+d)^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e^3*x^3 + 3*d*e^
2*x^2 + 3*d^2*e*x + d^3), x)
```

3.18.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2}{(d+ex)^3}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2/(e*x+d)**3,x)
```

output
```
Integral((a + b*asinh(c*x))**2/(d + e*x)**3, x)
```

3.18.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex+d)^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="maxima")
```

output
```
-(c*(sqrt(c^2*x^2 + 1)/(c^2*d^2*e*x + c^2*d^3 + e^3*x + d*e^2) - c^2*d*arc
sinh(c*d*x/(e*abs(x + d/e)) - 1/(c*abs(x + d/e)))/((c^2*d^2/e^2 + 1)^(3/2)
*e^4)) + arcsinh(c*x)/(e^3*x^2 + 2*d*e^2*x + d^2*e))*a*b - 1/2*b^2*(log(c*
x + sqrt(c^2*x^2 + 1))^2/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 2*integrate((c^3*
x^2 + sqrt(c^2*x^2 + 1)*c^2*x + c)*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*e^3*x
^5 + 2*c^3*d*e^2*x^4 + 2*c*d*e^2*x^2 + c*d^2*e*x + (c^3*d^2*e + c*e^3)*x^3
 + (c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*x + d^2*e + (c^2*d^2*e + e^3)*
x^2)*sqrt(c^2*x^2 + 1)), x)) - 1/2*a^2/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

3.18.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{(ex+d)^3}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2/(e*x + d)^3, x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))^2}{(d+ex)^3}\,dx=\int \frac{(a+b\,\text{asinh}(c\,x))^2}{(d+e\,x)^3}\,dx$$

input
```
int((a + b*asinh(c*x))^2/(d + e*x)^3,x)
```

output
```
int((a + b*asinh(c*x))^2/(d + e*x)^3, x)
```

3.19 $\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx$

3.19.1 Optimal result

Integrand size = 18, antiderivative size = 394

$$\begin{aligned}
\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx = {} & \frac{d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc} \\
& - \frac{3de^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{4bc^3} \\
& + \frac{3de^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\text{arcsinh}(cx)\right)}{4bc^3} \\
& - \frac{3d^2e\text{Chi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2bc^2} \\
& + \frac{e^3\text{Chi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{4bc^4} \\
& - \frac{e^3\text{Chi}\left(\frac{4a}{b}+4\text{arcsinh}(cx)\right)\sinh\left(\frac{4a}{b}\right)}{8bc^4} \\
& - \frac{d^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc} \\
& + \frac{3de^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{4bc^3} \\
& + \frac{3d^2e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)}{2bc^2} \\
& - \frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)}{4bc^4} \\
& - \frac{3de^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3a}{b}+3\text{arcsinh}(cx)\right)}{4bc^3} \\
& + \frac{e^3\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4a}{b}+4\text{arcsinh}(cx)\right)}{8bc^4}
\end{aligned}$$

output
```
d^3*Chi(a/b+arcsinh(c*x))*cosh(a/b)/b/c-3/4*d*e^2*Chi(a/b+arcsinh(c*x))*co
sh(a/b)/b/c^3+3/4*d*e^2*Chi(3*a/b+3*arcsinh(c*x))*cosh(3*a/b)/b/c^3+3/2*d^
2*e*cosh(2*a/b)*Shi(2*a/b+2*arcsinh(c*x))/b/c^2-1/4*e^3*cosh(2*a/b)*Shi(2*
a/b+2*arcsinh(c*x))/b/c^4+1/8*e^3*cosh(4*a/b)*Shi(4*a/b+4*arcsinh(c*x))/b/
c^4-d^3*Shi(a/b+arcsinh(c*x))*sinh(a/b)/b/c+3/4*d*e^2*Shi(a/b+arcsinh(c*x)
)*sinh(a/b)/b/c^3-3/2*d^2*e*Chi(2*a/b+2*arcsinh(c*x))*sinh(2*a/b)/b/c^2+1/
4*e^3*Chi(2*a/b+2*arcsinh(c*x))*sinh(2*a/b)/b/c^4-3/4*d*e^2*Shi(3*a/b+3*ar
csinh(c*x))*sinh(3*a/b)/b/c^3-1/8*e^3*Chi(4*a/b+4*arcsinh(c*x))*sinh(4*a/b
)/b/c^4
```

3.19.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx$$
$$= \frac{d^3\left(\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)}{bc}$$
$$+\frac{3de^2\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsin}\right.\right.}{4bc^3}$$
$$+\frac{e^3\left(2\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)-\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right)-2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{a}\right.\right.\right.}{8bc^4}$$
$$-\frac{3d^2e\left(\text{Chi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)\sinh\left(\frac{2a}{b}\right)-\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)\right)}{2bc^2}$$

input
```
Integrate[(d + e*x)^3/(a + b*ArcSinh[c*x]),x]
```

output
```
(d^3*(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - Sinh[a/b]*SinhIntegral[
a/b + ArcSinh[c*x]]))/(b*c) + (3*d*e^2*(-(Cosh[a/b]*CoshIntegral[a/b + Arc
Sinh[c*x]]) + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + Sinh[a/
b]*SinhIntegral[a/b + ArcSinh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b +
ArcSinh[c*x])]))/(4*b*c^3) + (e^3*(2*CoshIntegral[2*(a/b + ArcSinh[c*x])]*
Sinh[(2*a)/b] - CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] - 2*Cos
h[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*SinhIntegr
al[4*(a/b + ArcSinh[c*x])]))/(8*b*c^4) - (3*d^2*e*(CoshIntegral[(2*a)/b +
2*ArcSinh[c*x]]*Sinh[(2*a)/b] - Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*Arc
Sinh[c*x]]))/(2*b*c^2)
```

3.19.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6245, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{ 6245}$$

$$\frac{\int \frac{(cd+cex)^3\sqrt{c^2x^2+1}}{a+b\text{arcsinh}(cx)}d\text{arcsinh}(cx)}{c^4}$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{\int \left(\frac{d^3\sqrt{c^2x^2+1}c^3}{a+b\text{arcsinh}(cx)}+\frac{e^3x^3\sqrt{c^2x^2+1}c^3}{a+b\text{arcsinh}(cx)}+\frac{3de^2x^2\sqrt{c^2x^2+1}c^3}{a+b\text{arcsinh}(cx)}+\frac{3d^2ex\sqrt{c^2x^2+1}c^3}{a+b\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)}{c^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^3d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{b}-\frac{c^3d^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{b}-\frac{3c^2d^2e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)}{2b}+\frac{3c^2d^2e\cosh\left(\frac{2}{\cdot}\right.}{}$$

input
```
Int[(d + e*x)^3/(a + b*ArcSinh[c*x]),x]
```

output
```
((c^3*d^3*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/b - (3*c*d*e^2*Cosh[
a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(4*b) + (3*c*d*e^2*Cosh[(3*a)/b]*Co
shIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b) - (3*c^2*d^2*e*CoshIntegral[(2
*a)/b + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(2*b) + (e^3*CoshIntegral[(2*a)/b +
 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(4*b) - (e^3*CoshIntegral[(4*a)/b + 4*ArcS
inh[c*x]]*Sinh[(4*a)/b])/(8*b) - (c^3*d^3*Sinh[a/b]*SinhIntegral[a/b + Arc
Sinh[c*x]])/b + (3*c*d*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(4*
b) + (3*c^2*d^2*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(2
*b) - (e^3*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(4*b) - (
3*c*d*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b) + (e
^3*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b))/c^4
```

3.19.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6245
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[1/c^(m + 1)   Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[
x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m,
0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.19.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{e^3 e^{\frac{4a}{b}}\,\mathrm{Ei}_1\left(4\,\mathrm{arcsinh}(cx)+\frac{4a}{b}\right)}{16c^3b}-\frac{e^3 e^{-\frac{4a}{b}}\,\mathrm{Ei}_1\left(-4\,\mathrm{arcsinh}(cx)-\frac{4a}{b}\right)}{16c^3b}+\frac{3e\,e^{\frac{2a}{b}}\,\mathrm{Ei}_1\left(2\,\mathrm{arcsinh}(cx)+\frac{2a}{b}\right)d^2}{4cb}-\frac{e^3 e^{\frac{2a}{b}}\,\mathrm{Ei}_1\left(2\,\mathrm{arcsinh}(cx\right.}{8c^3b}$
default	$\frac{e^3 e^{\frac{4a}{b}}\,\mathrm{Ei}_1\left(4\,\mathrm{arcsinh}(cx)+\frac{4a}{b}\right)}{16c^3b}-\frac{e^3 e^{-\frac{4a}{b}}\,\mathrm{Ei}_1\left(-4\,\mathrm{arcsinh}(cx)-\frac{4a}{b}\right)}{16c^3b}+\frac{3e\,e^{\frac{2a}{b}}\,\mathrm{Ei}_1\left(2\,\mathrm{arcsinh}(cx)+\frac{2a}{b}\right)d^2}{4cb}-\frac{e^3 e^{\frac{2a}{b}}\,\mathrm{Ei}_1\left(2\,\mathrm{arcsinh}(cx\right.}{8c^3b}$

input
```
int((e*x+d)^3/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/16/c^3*e^3/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)-1/16/c^3*e^3/b*e
xp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)+3/4/c*e/b*exp(2*a/b)*Ei(1,2*arcsinh
(c*x)+2*a/b)*d^2-1/8/c^3*e^3/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-3/4/c
*e/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*d^2+1/8/c^3*e^3/b*exp(-2*a/b)
*Ei(1,-2*arcsinh(c*x)-2*a/b)-3/8/c^2*d*e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c
*x)-3*a/b)-3/8/c^2*d*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/2/b*exp
(a/b)*Ei(1,arcsinh(c*x)+a/b)*d^3+3/8/c^2*d/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/
b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d^3+3/8/c^2*d/b*exp(-a/b)*E
i(1,-arcsinh(c*x)-a/b)*e^2)
```

3.19.5 Fricas [F]

$$\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex+d)^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)/(b*arcsinh(c*x) + a), x
)
```

3.19.6 Sympy [F]

$$\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(d+ex)^3}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((e*x+d)**3/(a+b*asinh(c*x)),x)
```

output
```
Integral((d + e*x)**3/(a + b*asinh(c*x)), x)
```

3.19.7 Maxima [F]

$$\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex+d)^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3/(b*arcsinh(c*x) + a), x)
```

3.19.8 Giac [F]

$$\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(ex+d)^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3/(b*arcsinh(c*x) + a), x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(d+e\,x)^3}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((d + e*x)^3/(a + b*asinh(c*x)),x)
```

output
```
int((d + e*x)^3/(a + b*asinh(c*x)), x)
```

3.20 $\int \frac{(d+ex)^2}{a+b\text{arcsinh}(cx)}\,dx$

3.20.1 Optimal result

Integrand size = 18, antiderivative size = 245

$$\begin{aligned}\int \frac{(d+ex)^2}{a+b\text{arcsinh}(cx)}\,dx =& \frac{d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc} \\ &- \frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{4bc^3} \\ &+ \frac{e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\text{arcsinh}(cx)\right)}{4bc^3} \\ &- \frac{de\text{Chi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{bc^2} \\ &- \frac{d^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc} \\ &+ \frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{4bc^3} \\ &+ \frac{de\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)}{bc^2} \\ &- \frac{e^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3a}{b}+3\text{arcsinh}(cx)\right)}{4bc^3}\end{aligned}$$

output
```
d^2*Chi(a/b+arcsinh(c*x))*cosh(a/b)/b/c-1/4*e^2*Chi(a/b+arcsinh(c*x))*cosh
(a/b)/b/c^3+1/4*e^2*Chi(3*a/b+3*arcsinh(c*x))*cosh(3*a/b)/b/c^3+d*e*cosh(2
*a/b)*Shi(2*a/b+2*arcsinh(c*x))/b/c^2-d^2*Shi(a/b+arcsinh(c*x))*sinh(a/b)/
b/c+1/4*e^2*Shi(a/b+arcsinh(c*x))*sinh(a/b)/b/c^3-d*e*Chi(2*a/b+2*arcsinh(
c*x))*sinh(2*a/b)/b/c^2-1/4*e^2*Shi(3*a/b+3*arcsinh(c*x))*sinh(3*a/b)/b/c^
3
```

3.20.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^2}{a+b\text{arcsinh}(cx)}\,dx$$
$$=\frac{(4c^2d^2-e^2)\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)+e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)-4cde\text{Chi}\left(2\left(\frac{a}{b}+\text{arc}\right.\right.}{}$$

input
```
Integrate[(d + e*x)^2/(a + b*ArcSinh[c*x]),x]
```

output
```
((4*c^2*d^2 - e^2)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + e^2*Cosh[(
3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 4*c*d*e*CoshIntegral[2*(a/b
 + ArcSinh[c*x])]*Sinh[(2*a)/b] - 4*c^2*d^2*Sinh[a/b]*SinhIntegral[a/b + A
rcSinh[c*x]] + e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 4*c*d*e*Co
sh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - e^2*Sinh[(3*a)/b]*SinhI
ntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^3)
```

3.20.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6245, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{ 6245}$$

$$\frac{\int \frac{(cd+cex)^2\sqrt{c^2x^2+1}}{a+b\text{arcsinh}(cx)}d\text{arcsinh}(cx)}{c^3}$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{\int \left(\frac{c^2\sqrt{c^2x^2+1}d^2}{a+b\text{arcsinh}(cx)}+\frac{ce\sinh(2\text{arcsinh}(cx))d}{a+b\text{arcsinh}(cx)}+\frac{c^2e^2x^2\sqrt{c^2x^2+1}}{a+b\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)}{c^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^2d^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\mathrm{arcsinh}(cx)\right)}{b}-\frac{c^2d^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{arcsinh}(cx)\right)}{b}-\frac{cde\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\mathrm{arcsinh}(cx)\right)}{b}-\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\right.}{4}$$

input
```
Int[(d + e*x)^2/(a + b*ArcSinh[c*x]),x]
```

output
```
((c^2*d^2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/b - (e^2*Cosh[a/b]*C
oshIntegral[a/b + ArcSinh[c*x]])/(4*b) + (e^2*Cosh[(3*a)/b]*CoshIntegral[(
3*a)/b + 3*ArcSinh[c*x]])/(4*b) - (c*d*e*CoshIntegral[(2*a)/b + 2*ArcSinh[
c*x]]*Sinh[(2*a)/b])/b - (c^2*d^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x
]])/b + (e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(4*b) + (c*d*e*Co
sh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/b - (e^2*Sinh[(3*a)/b]
*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b))/c^3
```

3.20.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6245
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[1/c^(m + 1)   Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[
x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m,
0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.20.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.04

method	result
derivativedivides	$-\frac{e^2 e^{\frac{3a}{b}}\,\mathrm{Ei}_1\left(3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b}-\frac{e^2 e^{-\frac{3a}{b}}\,\mathrm{Ei}_1\left(-3\,\mathrm{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^2b}-\frac{e^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)d^2}{2b}+\frac{e^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)e^2}{8c^2b}$
default	$-\frac{e^2 e^{\frac{3a}{b}}\,\mathrm{Ei}_1\left(3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b}-\frac{e^2 e^{-\frac{3a}{b}}\,\mathrm{Ei}_1\left(-3\,\mathrm{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^2b}-\frac{e^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)d^2}{2b}+\frac{e^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)e^2}{8c^2b}$

input
```
int((e*x+d)^2/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/8/c^2*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/8/c^2*e^2/b*ex
p(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b
)*d^2+1/8/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-
arcsinh(c*x)-a/b)*d^2+1/8/c^2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e^2+1/2/
c*d*e/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-1/2/c*d*e/b*exp(-2*a/b)*Ei(1
,-2*arcsinh(c*x)-2*a/b))
```

3.20.5 Fricas [F]

$$\int \frac{(d+ex)^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex+d)^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)/(b*arcsinh(c*x) + a), x)
```

3.20.6 Sympy [F]

$$\int \frac{(d+ex)^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(d+ex)^2}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((e*x+d)**2/(a+b*asinh(c*x)),x)
```

output
```
Integral((d + e*x)**2/(a + b*asinh(c*x)), x)
```

3.20.7 Maxima [F]

$$\int \frac{(d+ex)^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex+d)^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2/(b*arcsinh(c*x) + a), x)
```

3.20.8 Giac [F]

$$\int \frac{(d+ex)^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex+d)^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/(b*arcsinh(c*x) + a), x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(d+e\,x)^2}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((d + e*x)^2/(a + b*asinh(c*x)),x)
```

output
```
int((d + e*x)^2/(a + b*asinh(c*x)), x)
```

3.21 $\int \frac{d+ex}{a+b\,\mathrm{arcsinh}(cx)}\,dx$

3.21.1 Optimal result

Integrand size = 16, antiderivative size = 116

$$\int \frac{d+ex}{a+b\,\mathrm{arcsinh}(cx)}\,dx = \frac{d\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\mathrm{arcsinh}(cx)\right)}{bc} - \frac{e\,\mathrm{Chi}\left(\frac{2a}{b}+2\mathrm{arcsinh}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2bc^2} - \frac{d\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{arcsinh}(cx)\right)}{bc} + \frac{e\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\mathrm{arcsinh}(cx)\right)}{2bc^2}$$

output
```
d*Chi(a/b+arcsinh(c*x))*cosh(a/b)/b/c+1/2*e*cosh(2*a/b)*Shi(2*a/b+2*arcsin
h(c*x))/b/c^2-d*Shi(a/b+arcsinh(c*x))*sinh(a/b)/b/c-1/2*e*Chi(2*a/b+2*arcs
inh(c*x))*sinh(2*a/b)/b/c^2
```

3.21.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int \frac{d+ex}{a+b\,\mathrm{arcsinh}(cx)}\,dx = \frac{2cd\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\mathrm{arcsinh}(cx)\right) - e\,\mathrm{Chi}\left(2\left(\frac{a}{b}+\mathrm{arcsinh}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right) - 2cd\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{arcsinh}(\right.}{2bc^2}$$

input
```
Integrate[(d + e*x)/(a + b*ArcSinh[c*x]),x]
```

output
```
(2*c*d*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - e*CoshIntegral[2*(a/b
+ ArcSinh[c*x])]*Sinh[(2*a)/b] - 2*c*d*Sinh[a/b]*SinhIntegral[a/b + ArcSin
h[c*x]] + e*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(2*b*c^2)
```

3.21.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {6245, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{a+b\text{arcsinh}(cx)}dx$$

$$\Big\downarrow \text{ 6245}$$

$$\frac{\int \frac{(cd+cex)\sqrt{c^2x^2+1}}{a+b\text{arcsinh}(cx)}d\text{arcsinh}(cx)}{c^2}$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{\int \left(\frac{c\sqrt{c^2x^2+1}d}{a+b\text{arcsinh}(cx)}+\frac{cex\sqrt{c^2x^2+1}}{a+b\text{arcsinh}(cx)}\right)d\text{arcsinh}(cx)}{c^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{cd\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{b}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\text{arcsinh}(cx)\right)}{2b}-\frac{cd\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{b}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{ar}\right.}{2b}}{c^2}$$

input
```
Int[(d + e*x)/(a + b*ArcSinh[c*x]),x]
```

output
```
((c*d*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/b - (e*CoshIntegral[(2*a
)/b + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(2*b) - (c*d*Sinh[a/b]*SinhIntegral[a
/b + ArcSinh[c*x]])/b + (e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[
c*x]])/(2*b))/c^2
```

3.21.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6245
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[1/c^(m + 1)   Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[
x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m,
0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.21.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{-\frac{e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right) d}{2b}-\frac{e^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right) d}{2b}+\frac{e\, e^{\frac{2a}{b}} \operatorname{Ei}_1\left(2 \operatorname{arcsinh}(cx)+\frac{2a}{b}\right)}{4cb}-\frac{e\, e^{-\frac{2a}{b}} \operatorname{Ei}_1\left(-2 \operatorname{arcsinh}(cx)-\frac{2a}{b}\right)}{4cb}}{c}$
default	$\frac{-\frac{e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right) d}{2b}-\frac{e^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right) d}{2b}+\frac{e\, e^{\frac{2a}{b}} \operatorname{Ei}_1\left(2 \operatorname{arcsinh}(cx)+\frac{2a}{b}\right)}{4cb}-\frac{e\, e^{-\frac{2a}{b}} \operatorname{Ei}_1\left(-2 \operatorname{arcsinh}(cx)-\frac{2a}{b}\right)}{4cb}}{c}$

input
```
int((e*x+d)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d-1/2/b*exp(-a/b)*Ei(1,-arcsin
h(c*x)-a/b)*d+1/4*e/c/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-1/4*e/c/b*ex
p(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b))
```

3.21.5 Fricas [F]

$$\int \frac{d+ex}{a+b\operatorname{arcsinh}(cx)}\,dx=\int \frac{ex+d}{b\operatorname{arsinh}\left(cx\right)+a}\,dx$$

input
```
integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((e*x + d)/(b*arcsinh(c*x) + a), x)
```

3.21.6 Sympy [F]

$$\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{d+ex}{a+b\operatorname{asinh}(cx)}\,dx$$

input
```
integrate((e*x+d)/(a+b*asinh(c*x)),x)
```

output
```
Integral((d + e*x)/(a + b*asinh(c*x)), x)
```

3.21.7 Maxima [F]

$$\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{ex+d}{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x + d)/(b*arcsinh(c*x) + a), x)
```

3.21.8 Giac [F]

$$\int \frac{d+ex}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{ex+d}{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)/(b*arcsinh(c*x) + a), x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{a+b\operatorname{arcsinh}(cx)}\,dx=\int \frac{d+e\,x}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

input
```
int((d + e*x)/(a + b*asinh(c*x)),x)
```

output
```
int((d + e*x)/(a + b*asinh(c*x)), x)
```

3.22 $\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx$

3.22.1 Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx=\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}$$

output
```
Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-Shi((a+b*arcsinh(c*x))/b)*sinh(a/b
)/b/c
```

3.22.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{1}{a+b\text{arcsinh}(cx)}\,dx=\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{bc}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(-1),x]
```

output
```
(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - Sinh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]])/(b*c)
```

3.22.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6189, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\text{arcsinh}(cx)}dx$$

$$\Big\downarrow \text{ 6189}$$

$$\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3784}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\sinh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\sinh\left(\frac{a}{b}\right)\int \frac{\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\sinh\left(\frac{a}{b}\right)\int -\frac{i\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\sinh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))+\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3779}$$

$$\frac{-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{bc}$$

$$\Big\downarrow \text{3782}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{bc}$$

input
```
Int[(a + b*ArcSinh[c*x])^(-1),x]
```

output
```
(Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b] - Sinh[a/b]*SinhIntegral[(
a + b*ArcSinh[c*x])/b])/(b*c)
```

3.22.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

3.22.4 Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)}{2b}}{c}$	56
default	$\frac{-\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)}{2b}}{c}$	56

input
```
int(1/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arcsinh(
c*x)-a/b))
```

3.22.5 Fricas [F]

$$\int \frac{1}{a+b\operatorname{arcsinh}(cx)}\,dx=\int \frac{1}{b\operatorname{arsinh}(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*arcsinh(c*x) + a), x)
```

3.22.6 Sympy [F]

$$\int \frac{1}{a + b\text{arcsinh}(cx)}\,dx = \int \frac{1}{a + b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate(1/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/(a + b*asinh(c*x)), x)
```

3.22.7 Maxima [F]

$$\int \frac{1}{a + b\text{arcsinh}(cx)}\,dx = \int \frac{1}{b\,\text{arsinh}\,(cx) + a}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arcsinh(c*x) + a), x)
```

3.22.8 Giac [F]

$$\int \frac{1}{a + b\text{arcsinh}(cx)}\,dx = \int \frac{1}{b\,\text{arsinh}\,(cx) + a}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(b*arcsinh(c*x) + a), x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a + b\text{arcsinh}(cx)}\,dx = \int \frac{1}{a + b\,\text{asinh}\left(c\,x\right)}\,dx$$

input
```
int(1/(a + b*asinh(c*x)),x)
```

output
```
int(1/(a + b*asinh(c*x)), x)
```

3.23 $\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))} dx$

3.23.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))} dx = \text{Int}\left(\frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}, x\right)$$

output
```
Unintegrable(1/(e*x+d)/(a+b*arcsinh(c*x)),x)
```

3.23.2 Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))} dx = \int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))} dx$$

input
```
Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])), x]
```

3.23.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6272}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))} \, dx$$

$$\Big\downarrow \; 6272$$

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/((d + e*x)*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

3.23.3.1 Defintions of rubi rules used

rule 6272
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.23.4 Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ex+d)\,(a+b\,\text{arcsinh}\,(cx))}dx$$

input
```
int(1/(e*x+d)/(a+b*arcsinh(c*x)),x)
```

output
```
int(1/(e*x+d)/(a+b*arcsinh(c*x)),x)
```

3.23.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex+d)(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*e*x + a*d + (b*e*x + b*d)*arcsinh(c*x)), x)
```

3.23.6 Sympy [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/((a + b*asinh(c*x))*(d + e*x)), x)
```

3.23.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex+d)(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x + d)*(b*arcsinh(c*x) + a)), x)
```

3.23.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(ex+d)(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x + d)*(b*arcsinh(c*x) + a)), x)
```

3.23.9 Mupad [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))\,\,(d+e\,x)}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + e*x)),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + e*x)), x)
```

3.24 $\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx$

3.24.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x)
```

3.24.2 Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx = \int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])), x]
```

3.24.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6272}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}dx$$
$$\Big\downarrow \text{6272}$$
$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[1/((d + e*x)^2*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

3.24.3.1 Defintions of rubi rules used

rule 6272
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.24.4 Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ex+d)^2\,(a+b\,\text{arcsinh}\,(cx))}dx$$

input
```
int(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x)
```

output
```
int(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x)
```

3.24.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.72

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))} dx = \int \frac{1}{(ex+d)^2(b \operatorname{arsinh}(cx)+a)} dx$$

input
```
integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*e^2*x^2 + 2*a*d*e*x + a*d^2 + (b*e^2*x^2 + 2*b*d*e*x + b*d^2
)*arcsinh(c*x)), x)
```

3.24.6 Sympy [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))} dx = \int \frac{1}{(a+b \operatorname{asinh}(cx))(d+ex)^2} dx$$

input
```
integrate(1/(e*x+d)**2/(a+b*asinh(c*x)),x)
```

output
```
Integral(1/((a + b*asinh(c*x))*(d + e*x)**2), x)
```

3.24.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))} dx = \int \frac{1}{(ex+d)^2(b \operatorname{arsinh}(cx)+a)} dx$$

input
```
integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x + d)^2*(b*arcsinh(c*x) + a)), x)
```

3.24.8 Giac [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{1}{(ex+d)^2(b\operatorname{arsinh}(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x + d)^2*(b*arcsinh(c*x) + a)), x)
```

3.24.9 Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\operatorname{arcsinh}(cx))}\,dx = \int \frac{1}{(a+b\operatorname{asinh}(c\,x))\,(d+e\,x)^2}\,dx$$

input
```
int(1/((a + b*asinh(c*x))*(d + e*x)^2),x)
```

output
```
int(1/((a + b*asinh(c*x))*(d + e*x)^2), x)
```

3.25 $\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2} dx$

3.25.1 Optimal result

Integrand size = 18, antiderivative size = 359

$$\begin{aligned}\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}dx = &-\frac{d^2\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))}-\frac{2dex\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))}\\
&-\frac{e^2x^2\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))}+\frac{2de\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^2}\\
&-\frac{d^2\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c}\\
&+\frac{e^2\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2c^3}\\
&-\frac{3e^2\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b^2c^3}\\
&+\frac{d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}\\
&-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^3}\\
&-\frac{2de\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^2}\\
&+\frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3}\end{aligned}$$

output
```
2*d*e*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b^2/c^2+d^2*cosh(a/b)*Shi((a
+b*arcsinh(c*x))/b)/b^2/c-1/4*e^2*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/
c^3+3/4*e^2*cosh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^3-d^2*Chi((a+b*a
rcsinh(c*x))/b)*sinh(a/b)/b^2/c+1/4*e^2*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b
)/b^2/c^3-2*d*e*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c^2-3/4*e^2*Ch
i(3*(a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^3-d^2*(c^2*x^2+1)^(1/2)/b/c/(a
+b*arcsinh(c*x))-2*d*e*x*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-e^2*x^2*
(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

3.25.2 Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}\,dx =$$
$$-\frac{\frac{4bc^2d^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}+\frac{8bc^2dex\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}+\frac{4bc^2e^2x^2\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)}-8cde\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\right)+(4c^2d^2-\cdots}{\cdots}$$

input
```
Integrate[(d + e*x)^2/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-1/4*((4*b*c^2*d^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (8*b*c^2*d*e*
x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*b*c^2*e^2*x^2*Sqrt[1 + c^2*
x^2])/(a + b*ArcSinh[c*x]) - 8*c*d*e*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + A
rcSinh[c*x])] + (4*c^2*d^2 - e^2)*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/
b] + 3*e^2*CoshIntegral[3*(a/b + ArcSinh[c*x])]*Sinh[(3*a)/b] - 4*c^2*d^2*
Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + e^2*Cosh[a/b]*SinhIntegral[a/
b + ArcSinh[c*x]] + 8*c*d*e*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*
x])] - 3*e^2*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(b^2*c^3)
```

3.25.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \text{ 6244}$$

$$\int \left(\frac{d^2}{(a+b\text{arcsinh}(cx))^2}+\frac{2dex}{(a+b\text{arcsinh}(cx))^2}+\frac{e^2x^2}{(a+b\text{arcsinh}(cx))^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^3}-\frac{3e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3}-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{4b^2c^3}+\frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(cx))}{b}\right)}{4b^2c^3}+\frac{2de\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^2}-\frac{2de\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^2}-\frac{d^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}+\frac{d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}-\frac{d^2\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}-\frac{2dex\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}-\frac{e^2x^2\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

input
```
Int[(d + e*x)^2/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((d^2*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (2*d*e*x*Sqrt[1 +
c^2*x^2])/(b*c*(a + b*ArcSinh[c*x])) - (e^2*x^2*Sqrt[1 + c^2*x^2])/(b*c*(a
 + b*ArcSinh[c*x])) + (2*d*e*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[
c*x]))/b])/(b^2*c^2) - (d^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b]
)/(b^2*c) + (e^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(4*b^2*c^
3) - (3*e^2*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(4*b^2
*c^3) + (d^2*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) - (e^
2*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(4*b^2*c^3) - (2*d*e*Sin
h[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b^2*c^2) + (3*e^2*Co
sh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(4*b^2*c^3)
```

3.25. $\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}\,dx$

3.25.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6244
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

3.25.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.72

method	result
derivativedivides	$\frac{\left(4c^3x^3-4c^2x^2\sqrt{c^2x^2+1}+3cx-\sqrt{c^2x^2+1}\right)e^2}{8c^2b(a+b\,\text{arcsinh}(cx))}+\frac{3e^2\mathrm{e}^{\frac{3a}{b}}\,\text{Ei}_1\left(3\,\text{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e^2\left(4c^3x^3+3cx+4c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}\right)}{8b\,c^2(a+b\,\text{arcsinh}(cx))}-$
default	$\frac{\left(4c^3x^3-4c^2x^2\sqrt{c^2x^2+1}+3cx-\sqrt{c^2x^2+1}\right)e^2}{8c^2b(a+b\,\text{arcsinh}(cx))}+\frac{3e^2\mathrm{e}^{\frac{3a}{b}}\,\text{Ei}_1\left(3\,\text{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e^2\left(4c^3x^3+3cx+4c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}\right)}{8b\,c^2(a+b\,\text{arcsinh}(cx))}-$

input
```
int((e*x+d)^2/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/8*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))*e
^2/c^2/b/(a+b*arcsinh(c*x))+3/8*e^2/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)
+3*a/b)-1/8/b*e^2/c^2*(4*c^3*x^3+3*c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^
2+1)^(1/2))/(a+b*arcsinh(c*x))-3/8/b^2*e^2/c^2*exp(-3*a/b)*Ei(1,-3*arcsinh
(c*x)-3*a/b)+1/2*(-(c^2*x^2+1)^(1/2)+c*x)*d^2/b/(a+b*arcsinh(c*x))+1/2*d^2
/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/8*(-(c^2*x^2+1)^(1/2)+c*x)*e^2/c^2/
b/(a+b*arcsinh(c*x))-1/8/c^2*e^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b
*d^2*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/2/b^2*d^2*exp(-a/b)*Ei(1
,-arcsinh(c*x)-a/b)+1/8/c^2/b*e^2*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x
))+1/8/c^2/b^2*e^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/2*(-2*c*x*(c^2*x^2+
1)^(1/2)+2*c^2*x^2+1)*d*e/c/b/(a+b*arcsinh(c*x))-e*d/c/b^2*exp(2*a/b)*Ei(1
,2*arcsinh(c*x)+2*a/b)-1/2/b*e*d/c*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^(1/2))/(
a+b*arcsinh(c*x))-1/b^2*e*d/c*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b))
```

3.25.5 Fricas [F]

$$\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex+d)^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x
) + a^2), x)
```

3.25.6 Sympy [F]

$$\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(d+ex)^2}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate((e*x+d)**2/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((d + e*x)**2/(a + b*asinh(c*x))**2, x)
```

3.25.7 Maxima [F]

$$\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex+d)^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*e^2*x^5 + 2*c^3*d*e*x^4 + 2*c*d*e*x^2 + c*d^2*x + (c^3*d^2 + c*e^2)*
x^3 + (c^2*e^2*x^4 + 2*c^2*d*e*x^3 + 2*d*e*x + (c^2*d^2 + e^2)*x^2 + d^2)*
sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b
^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 +
 1))) + integrate((3*c^5*e^2*x^6 + 4*c^5*d*e*x^5 + 8*c^3*d*e*x^3 + (c^5*d^
2 + 6*c^3*e^2)*x^4 + 4*c*d*e*x + c*d^2 + (2*c^3*d^2 + 3*c*e^2)*x^2 + (3*c^
3*e^2*x^4 + 4*c^3*d*e*x^3 - c*d^2 + (c^3*d^2 + c*e^2)*x^2)*(c^2*x^2 + 1) +
 (6*c^4*e^2*x^5 + 8*c^4*d*e*x^4 + 8*c^2*d*e*x^2 + (2*c^4*d^2 + 7*c^2*e^2)*
x^3 + 2*d*e + (c^2*d^2 + 2*e^2)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*
x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1
)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c
^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sq
rt(c^2*x^2 + 1)), x)
```

3.25.8 Giac [F]

$$\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex+d)^2}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/(b*arcsinh(c*x) + a)^2, x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(d+e\,x)^2}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((d + e*x)^2/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + e*x)^2/(a + b*asinh(c*x))^2, x)
```

3.26 $\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx$

3.26.1 Optimal result

Integrand size = 16, antiderivative size = 180

$$\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx = -\frac{d\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))}-\frac{ex\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^2}-\frac{d\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(cx))}{b}\right)}{b^2c^2}$$

output
```
e*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b^2/c^2+d*cosh(a/b)*Shi((a+b*arc
sinh(c*x))/b)/b^2/c-d*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c-e*Shi(2*(a
+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c^2-d*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsi
nh(c*x))-e*x*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

3.26.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.83

$$\int \frac{d+ex}{(a+b\operatorname{arcsinh}(cx))^2}\,dx = -\frac{\frac{bcd\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}+\frac{bcex\sqrt{1+c^2x^2}}{a+b\operatorname{arcsinh}(cx)}-e\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\right)+cd\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(cx)\right)\sinh\left(\right.}{b^2c^2}$$

input
```
Integrate[(d + e*x)/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-(((b*c*d*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (b*c*e*x*Sqrt[1 + c^2*
x^2])/(a + b*ArcSinh[c*x]) - e*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh
[c*x])] + c*d*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - c*d*Cosh[a/b]*S
inhIntegral[a/b + ArcSinh[c*x]] + e*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + Ar
cSinh[c*x]])])/(b^2*c^2))
```

3.26.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(a+b\operatorname{arcsinh}(cx))^2}\,dx$$

$$\Big\downarrow \; 6244$$

$$\int \left(\frac{d}{(a+b\operatorname{arcsinh}(cx))^2}+\frac{ex}{(a+b\operatorname{arcsinh}(cx))^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{e\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2}-\frac{e\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)}{b^2c^2}-\frac{d\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b^2c}+\frac{d\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(cx)}{b}\right)}{b^2c}-\frac{d\sqrt{c^2x^2+1}}{bc(a+b\operatorname{arcsinh}(cx))}-\frac{ex\sqrt{c^2x^2+1}}{bc(a+b\operatorname{arcsinh}(cx))}$$

input
```
Int[(d + e*x)/(a + b*ArcSinh[c*x])^2,x]
```

output
```
-((d*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (e*x*Sqrt[1 + c^2*x^
2])/(b*c*(a + b*ArcSinh[c*x])) + (e*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*A
rcSinh[c*x]))/b])/(b^2*c^2) - (d*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh
[a/b])/(b^2*c) + (d*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c
) - (e*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b^2*c^2)
```

3.26.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6244
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

3.26.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.51

method	result
derivativedivides	$\frac{\frac{\left(-\sqrt{c^2x^2+1}+cx\right)d}{2b(a+b\,\mathrm{arcsinh}(cx))}+\frac{e^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)d}{2b^2}-\frac{\left(cx+\sqrt{c^2x^2+1}\right)d}{2b(a+b\,\mathrm{arcsinh}(cx))}-\frac{e^{-\frac{a}{b}}\,\mathrm{Ei}_1\left(-\mathrm{arcsinh}(cx)-\frac{a}{b}\right)d}{2b^2}+\frac{\left(-2cx\sqrt{c^2x^2+1}+2c^2x^2+\cdots\right.}{4cb(a+b\,\mathrm{arcsinh}(cx))}\cdots}{c}$
default	$\frac{\frac{\left(-\sqrt{c^2x^2+1}+cx\right)d}{2b(a+b\,\mathrm{arcsinh}(cx))}+\frac{e^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(cx)+\frac{a}{b}\right)d}{2b^2}-\frac{\left(cx+\sqrt{c^2x^2+1}\right)d}{2b(a+b\,\mathrm{arcsinh}(cx))}-\frac{e^{-\frac{a}{b}}\,\mathrm{Ei}_1\left(-\mathrm{arcsinh}(cx)-\frac{a}{b}\right)d}{2b^2}+\frac{\left(-2cx\sqrt{c^2x^2+1}+2c^2x^2+\cdots\right.}{4cb(a+b\,\mathrm{arcsinh}(cx))}\cdots}{c}$

input
```
int((e*x+d)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/2*(-(c^2*x^2+1)^(1/2)+c*x)*d/b/(a+b*arcsinh(c*x))+1/2/b^2*exp(a/b)*
Ei(1,arcsinh(c*x)+a/b)*d-1/2/b*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))*
d-1/2/b^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d+1/4*(-2*c*x*(c^2*x^2+1)^(1/2
)+2*c^2*x^2+1)*e/c/b/(a+b*arcsinh(c*x))-1/2*e/c/b^2*exp(2*a/b)*Ei(1,2*arcs
inh(c*x)+2*a/b)-1/4*e/c/b*(2*c^2*x^2+1+2*c*x*(c^2*x^2+1)^(1/2))/(a+b*arcsi
nh(c*x))-1/2*e/c/b^2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b))
```

3.26.5 Fricas [F]

$$\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{ex+d}{\left(b\,\text{arsinh}\,(cx)+a\right)^2}\,dx$$

input
```
integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((e*x + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

3.26.6 Sympy [F]

$$\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{d+ex}{\left(a+b\,\text{asinh}\,(cx)\right)^2}\,dx$$

input
```
integrate((e*x+d)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((d + e*x)/(a + b*asinh(c*x))**2, x)
```

3.26.7 Maxima [F]

$$\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{ex+d}{\left(b\,\text{arsinh}\,(cx)+a\right)^2}\,dx$$

input
```
integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*e*x^4 + c^3*d*x^3 + c*e*x^2 + c*d*x + (c^2*e*x^3 + c^2*d*x^2 + e*x +
 d)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c
+ (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x
^2 + 1))) + integrate((2*c^5*e*x^5 + c^5*d*x^4 + 4*c^3*e*x^3 + 2*c^3*d*x^2
 + 2*c*e*x + (2*c^3*e*x^3 + c^3*d*x^2 - c*d)*(c^2*x^2 + 1) + c*d + (4*c^4*
e*x^4 + 2*c^4*d*x^3 + 4*c^2*e*x^2 + c^2*d*x + e)*sqrt(c^2*x^2 + 1))/(a*b*c
^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4
+ (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2
*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 +
 a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

3.26.8 Giac [F]

$$\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{ex+d}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)/(b*arcsinh(c*x) + a)^2, x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{d+e\,x}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((d + e*x)/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + e*x)/(a + b*asinh(c*x))^2, x)
```

3.27 $\int \frac{1}{(a+b\text{arcsinh}(cx))^2} dx$

3.27.1 Optimal result

Integrand size = 10, antiderivative size = 85

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2} dx = -\frac{\sqrt{1+c^2x^2}}{bc(a+b\text{arcsinh}(cx))} - \frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{b^2c}$$

output
```
cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-Chi((a+b*arcsinh(c*x))/b)*sinh(a
/b)/b^2/c-(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

3.27.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2} dx = \frac{-\frac{b\sqrt{1+c^2x^2}}{a+b\text{arcsinh}(cx)} - \text{Chi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)\sinh\left(\frac{a}{b}\right) + \cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}{b^2c}$$

input
```
Integrate[(a + b*ArcSinh[c*x])^(-2),x]
```

output
```
(-((b*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])) - CoshIntegral[a/b + ArcSin
h[c*x]]*Sinh[a/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c)
```

3.27.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6188, 6234, 25, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}dx$$

$$\Big\downarrow \text{6188}$$

$$\frac{c\int \frac{x}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx}{b}-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{6234}$$

$$\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}-\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))}{b^2c}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))+\cosh\left(\frac{a}{b}\right)\int-\frac{i\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$$\Big\downarrow \; 26$$

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{a}{b}\right)\int\frac{\sinh\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{a}{b}\right)\int-\frac{i\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$$\Big\downarrow \; 26$$

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))\right)}{b^2c}$$

$$\Big\downarrow \; 3779$$

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(cx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(cx)}d(a+b\text{arcsinh}(cx))-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)}{b^2c}$$

$$\Big\downarrow \; 3782$$

$$-\frac{\sqrt{c^2x^2+1}}{bc(a+b\text{arcsinh}(cx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)}{b^2c}$$

3.27. $\int\frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx$

input
```
Int[(a + b*ArcSinh[c*x])^(-2),x]
```

output
```
-(Sqrt[1 + c^2*x^2]/(b*c*(a + b*ArcSinh[c*x]))) + (I*(I*CoshIntegral[(a +
b*ArcSinh[c*x])/b]*Sinh[a/b] - I*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x
])/b]))/(b^2*c)
```

3.27.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

3.27.4 Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

method	result	size
derivativedivides	$\frac{\frac{-\sqrt{c^2x^2+1}+cx}{2b(a+b\operatorname{arcsinh}(cx))}+\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{2b^2}-\frac{cx+\sqrt{c^2x^2+1}}{2b(a+b\operatorname{arcsinh}(cx))}-\frac{e^{-\frac{a}{b}}\operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)}{2b^2}}{c}$	118
default	$\frac{\frac{-\sqrt{c^2x^2+1}+cx}{2b(a+b\operatorname{arcsinh}(cx))}+\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(cx)+\frac{a}{b}\right)}{2b^2}-\frac{cx+\sqrt{c^2x^2+1}}{2b(a+b\operatorname{arcsinh}(cx))}-\frac{e^{-\frac{a}{b}}\operatorname{Ei}_1\left(-\operatorname{arcsinh}(cx)-\frac{a}{b}\right)}{2b^2}}{c}$	118

input
```
int(1/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/2*(-(c^2*x^2+1)^(1/2)+c*x)/b/(a+b*arcsinh(c*x))+1/2/b^2*exp(a/b)*Ei
(1,arcsinh(c*x)+a/b)-1/2/b*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/2/
b^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b))
```

3.27.5 Fricas [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

3.27.6 Sympy [F]

$$\int \frac{1}{(a + b\text{arcsinh}(cx))^2} dx = \int \frac{1}{(a + b\,\text{asinh}\,(cx))^2} dx$$

input
```
integrate(1/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((a + b*asinh(c*x))**(-2), x)
```

3.27.7 Maxima [F]

$$\int \frac{1}{(a + b\text{arcsinh}(cx))^2} dx = \int \frac{1}{(b\,\text{arsinh}\,(cx) + a)^2} dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(
c*x + sqrt(c^2*x^2 + 1))) + integrate((c^4*x^4 + 2*c^2*x^2 + (c^2*x^2 + 1)
*(c^2*x^2 - 1) + (2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) + 1)/(a*b*c^4*x^4 + (
c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 +
 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2
*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c
^2*x^2 + 1)), x)
```

3.27.8 Giac [F]

$$\int \frac{1}{(a + b\text{arcsinh}(cx))^2} dx = \int \frac{1}{(b\,\text{arsinh}\,(cx) + a)^2} dx$$

input
```
integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^(-2), x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

input
```
int(1/(a + b*asinh(c*x))^2,x)
```

output
```
int(1/(a + b*asinh(c*x))^2, x)
```

3.28 $\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx$

3.28.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x)
```

3.28.2 Mathematica [N/A]

Not integrable

Time = 5.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])^2), x]
```

3.28.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6272}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2} \, dx$$

$$\Big\downarrow \text{ 6272}$$

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[1/((d + e*x)*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

3.28.3.1 Defintions of rubi rules used

rule 6272
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.28.4 Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ex+d)\left(a+b\,\text{arcsinh}\left(cx\right)\right)^{2}}dx$$

input
```
int(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x)
```

3.28.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex+d)(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*e*x + a^2*d + (b^2*e*x + b^2*d)*arcsinh(c*x)^2 + 2*(a*b*e*
x + a*b*d)*arcsinh(c*x)), x)
```

3.28.6 Sympy [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))^2\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/((a + b*asinh(c*x))**2*(d + e*x)), x)
```

3.28.7 Maxima [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 746, normalized size of antiderivative = 41.44

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex+d)(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e*x^3 + a*b*c^3*d*x^2 + a*
b*c*e*x + a*b*c*d + (b^2*c^3*e*x^3 + b^2*c^3*d*x^2 + b^2*c*e*x + b^2*c*d +
 (b^2*c^2*e*x^2 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 +
 1)) + (a*b*c^2*e*x^2 + a*b*c^2*d*x)*sqrt(c^2*x^2 + 1)) + integrate((c^5*d
*x^4 + 2*c^3*d*x^2 + (c^3*d*x^2 - 2*c*e*x - c*d)*(c^2*x^2 + 1) + c*d + (2*
c^4*d*x^3 - 2*c^2*e*x^2 + c^2*d*x - e)*sqrt(c^2*x^2 + 1))/(a*b*c^5*e^2*x^6
 + 2*a*b*c^5*d*e*x^5 + 4*a*b*c^3*d*e*x^3 + (c^5*d^2 + 2*c^3*e^2)*a*b*x^4 +
 2*a*b*c*d*e*x + a*b*c*d^2 + (2*c^3*d^2 + c*e^2)*a*b*x^2 + (a*b*c^3*e^2*x^
4 + 2*a*b*c^3*d*e*x^3 + a*b*c^3*d^2*x^2)*(c^2*x^2 + 1) + (b^2*c^5*e^2*x^6
+ 2*b^2*c^5*d*e*x^5 + 4*b^2*c^3*d*e*x^3 + (c^5*d^2 + 2*c^3*e^2)*b^2*x^4 +
2*b^2*c*d*e*x + b^2*c*d^2 + (2*c^3*d^2 + c*e^2)*b^2*x^2 + (b^2*c^3*e^2*x^4
 + 2*b^2*c^3*d*e*x^3 + b^2*c^3*d^2*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e^2*x^5
 + 2*b^2*c^4*d*e*x^4 + 2*b^2*c^2*d*e*x^2 + b^2*c^2*d^2*x + (c^4*d^2 + c^2*
e^2)*b^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4
*e^2*x^5 + 2*a*b*c^4*d*e*x^4 + 2*a*b*c^2*d*e*x^2 + a*b*c^2*d^2*x + (c^4*d^
2 + c^2*e^2)*a*b*x^3)*sqrt(c^2*x^2 + 1)), x)
```

3.28.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex+d)(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x + d)*(b*arcsinh(c*x) + a)^2), x)
```

3.28.9 Mupad [N/A]

Not integrable

Time = 2.91 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^2\,(d+e\,x)}\,dx$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + e*x)),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + e*x)), x)
```

3.28. $\int \frac{1}{(d+ex)(a+b\text{arcsinh}(cx))^2}\,dx$

3.29 $\int \frac{1}{(d+ex)^2(a+b\mathbf{arcsinh}(cx))^2}\,dx$

3.29.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx = \text{Int}\left(\frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x)
```

3.29.2 Mathematica [N/A]

Not integrable

Time = 4.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2), x]
```

3.29.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6272}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx$$
$$\Big\downarrow \; 6272$$
$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2),x]
```

output
```
$Aborted
```

3.29.3.1 Defintions of rubi rules used

rule 6272
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.29.4 Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(ex+d\right)^{2}\left(a+b\,\text{arcsinh}\left(cx\right)\right)^{2}}dx$$

input
```
int(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x)
```

output
```
int(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x)
```

3.29.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 5.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex+d)^2(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*e^2*x^2 + 2*a^2*d*e*x + a^2*d^2 + (b^2*e^2*x^2 + 2*b^2*d*e
*x + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*e^2*x^2 + 2*a*b*d*e*x + a*b*d^2)*arc
sinh(c*x)), x)
```

3.29.6 Sympy [N/A]

Not integrable

Time = 5.73 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(cx))^2\,(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(a+b*asinh(c*x))**2,x)
```

output
```
Integral(1/((a + b*asinh(c*x))**2*(d + e*x)**2), x)
```

3.29.7 Maxima [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 1050, normalized size of antiderivative = 58.33

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex+d)^2(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e^2*x^4 + 2*a*b*c^3*d*e*x^
3 + 2*a*b*c*d*e*x + a*b*c*d^2 + (c^3*d^2 + c*e^2)*a*b*x^2 + (b^2*c^3*e^2*x
^4 + 2*b^2*c^3*d*e*x^3 + 2*b^2*c*d*e*x + b^2*c*d^2 + (c^3*d^2 + c*e^2)*b^2
*x^2 + (b^2*c^2*e^2*x^3 + 2*b^2*c^2*d*e*x^2 + b^2*c^2*d^2*x)*sqrt(c^2*x^2
+ 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^2*e^2*x^3 + 2*a*b*c^2*d*e*x^2
+ a*b*c^2*d^2*x)*sqrt(c^2*x^2 + 1)) - integrate((c^5*e*x^5 - c^5*d*x^4 + 2
*c^3*e*x^3 - 2*c^3*d*x^2 + c*e*x + (c^3*e*x^3 - c^3*d*x^2 + 3*c*e*x + c*d)
*(c^2*x^2 + 1) - c*d + (2*c^4*e*x^4 - 2*c^4*d*x^3 + 5*c^2*e*x^2 - c^2*d*x
+ 2*e)*sqrt(c^2*x^2 + 1))/(a*b*c^5*e^3*x^7 + 3*a*b*c^5*d*e^2*x^6 + (3*c^5*
d^2*e + 2*c^3*e^3)*a*b*x^5 + 3*a*b*c*d^2*e*x + (c^5*d^3 + 6*c^3*d*e^2)*a*b
*x^4 + a*b*c*d^3 + (6*c^3*d^2*e + c*e^3)*a*b*x^3 + (2*c^3*d^3 + 3*c*d*e^2)
*a*b*x^2 + (a*b*c^3*e^3*x^5 + 3*a*b*c^3*d*e^2*x^4 + 3*a*b*c^3*d^2*e*x^3 +
a*b*c^3*d^3*x^2)*(c^2*x^2 + 1) + (b^2*c^5*e^3*x^7 + 3*b^2*c^5*d*e^2*x^6 +
(3*c^5*d^2*e + 2*c^3*e^3)*b^2*x^5 + 3*b^2*c*d^2*e*x + (c^5*d^3 + 6*c^3*d*e
^2)*b^2*x^4 + b^2*c*d^3 + (6*c^3*d^2*e + c*e^3)*b^2*x^3 + (2*c^3*d^3 + 3*c
*d*e^2)*b^2*x^2 + (b^2*c^3*e^3*x^5 + 3*b^2*c^3*d*e^2*x^4 + 3*b^2*c^3*d^2*e
*x^3 + b^2*c^3*d^3*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e^3*x^6 + 3*b^2*c^4*d*e
^2*x^5 + 3*b^2*c^2*d^2*e*x^2 + b^2*c^2*d^3*x + (3*c^4*d^2*e + c^2*e^3)*b^2
*x^4 + (c^4*d^3 + 3*c^2*d*e^2)*b^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(
c^2*x^2 + 1)) + 2*(a*b*c^4*e^3*x^6 + 3*a*b*c^4*d*e^2*x^5 + 3*a*b*c^2*d^...
```

3.29.8 Giac [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{1}{(ex+d)^2(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x + d)^2*(b*arcsinh(c*x) + a)^2), x)
```

3.29.9 Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2(a+b\text{arcsinh}(cx))^2}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d+e\,x\right)}^2} \,d x$$

input
```
int(1/((a + b*asinh(c*x))^2*(d + e*x)^2),x)
```

output
```
int(1/((a + b*asinh(c*x))^2*(d + e*x)^2), x)
```

3.30 $\int (d+ex)^m(a+b\text{arcsinh}(cx))^2\,dx$

3.30.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))^2\,dx = \frac{(d+ex)^{1+m}(a+b\text{arcsinh}(cx))^2}{e(1+m)} - \frac{2bc\text{Int}\left(\frac{(d+ex)^{1+m}(a+b\text{arcsinh}(cx))}{\sqrt{1+c^2x^2}},x\right)}{e(1+m)}$$

output
```
(e*x+d)^(1+m)*(a+b*arcsinh(c*x))^2/e/(1+m)-2*b*c*Unintegrable((e*x+d)^(1+m
)*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)/e/(1+m)
```

3.30.2 Mathematica [N/A]

Not integrable

Time = 5.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))^2\,dx = \int (d+ex)^m(a+b\text{arcsinh}(cx))^2\,dx$$

input
```
Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x])^2,x]
```

output
```
Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x])^2, x]
```

3.30.3 Rubi [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6243, 6272}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))^2\,dx$$

$$\Big\downarrow \; 6243$$

$$\frac{(d+ex)^{m+1}(a+b\text{arcsinh}(cx))^2}{e(m+1)}-\frac{2bc\int \frac{(d+ex)^{m+1}(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{e(m+1)}$$

$$\Big\downarrow \; 6272$$

$$\frac{(d+ex)^{m+1}(a+b\text{arcsinh}(cx))^2}{e(m+1)}-\frac{2bc\int \frac{(d+ex)^{m+1}(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{e(m+1)}$$

input
```
Int[(d + e*x)^m*(a + b*ArcSinh[c*x])^2,x]
```

output
```
$Aborted
```

3.30.3.1 Defintions of rubi rules used

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6272
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.30.4 Maple [N/A] (verified)

Not integrable

Time = 1.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (ex+d)^m (a+b\,\text{arcsinh}\,(cx))^2 \, dx$$

input
```
int((e*x+d)^m*(a+b*arcsinh(c*x))^2,x)
```

output
```
int((e*x+d)^m*(a+b*arcsinh(c*x))^2,x)
```

3.30.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))^2 \, dx = \int (b\,\text{arsinh}\,(cx)+a)^2(ex+d)^m \, dx$$

input
```
integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*(e*x + d)^m, x)
```

3.30.6 Sympy [N/A]

Not integrable

Time = 5.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))^2 \, dx = \int (a+b\,\text{asinh}\,(cx))^2\,(d+ex)^m \, dx$$

input
```
integrate((e*x+d)**m*(a+b*asinh(c*x))**2,x)
```

output
```
Integral((a + b*asinh(c*x))**2*(d + e*x)**m, x)
```

3.30.7 Maxima [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 271, normalized size of antiderivative = 15.06

$$\int (d+ex)^m(a+b\operatorname{arcsinh}(cx))^2\,dx = \int (b\operatorname{arsinh}(cx)+a)^2(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
(b^2*e*x + b^2*d)*(e*x + d)^m*log(c*x + sqrt(c^2*x^2 + 1))^2/(e*(m + 1)) +
 (e*x + d)^(m + 1)*a^2/(e*(m + 1)) + integrate(-2*((b^2*c^2*d*x - a*b*e*(m
 + 1) - (a*b*c^2*e*(m + 1) - b^2*c^2*e)*x^2)*sqrt(c^2*x^2 + 1)*(e*x + d)^m
 + (b^2*c^3*d*x^2 + b^2*c*d - (a*b*c^3*e*(m + 1) - b^2*c^3*e)*x^3 - (a*b*c
*e*(m + 1) - b^2*c*e)*x)*(e*x + d)^m)*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*e*
(m + 1)*x^3 + c*e*(m + 1)*x + (c^2*e*(m + 1)*x^2 + e*(m + 1))*sqrt(c^2*x^2
 + 1)), x)
```

3.30.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (d+ex)^m(a+b\operatorname{arcsinh}(cx))^2\,dx = \int (b\operatorname{arsinh}(cx)+a)^2(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^2*(e*x + d)^m, x)
```

3.30.9 Mupad [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (d+ex)^m(a+b\operatorname{arcsinh}(cx))^2\,dx = \int {\left(a+b\,\mathrm{asinh}(c\,x)\right)}^2\,{\left(d+e\,x\right)}^m\,dx$$

input
```
int((a + b*asinh(c*x))^2*(d + e*x)^m,x)
```

output
```
int((a + b*asinh(c*x))^2*(d + e*x)^m, x)
```

3.30. $\int (d+ex)^m(a+b\text{arcsinh}(cx))^2\,dx$

3.31 $\int (d+ex)^m (a+b\text{arcsinh}(cx))\,dx$

3.31.1 Optimal result

Integrand size = 16, antiderivative size = 179

$$\int (d+ex)^m (a+b\text{arcsinh}(cx))\,dx = -\frac{bc(d+ex)^{2+m}\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\,\text{AppellF1}\left(2+m,\frac{1}{2},\frac{1}{2},3+m,\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}},\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}\right)}{e^2(1+m)(2+m)\sqrt{1+c^2x^2}} + \frac{(d+ex)^{1+m}(a+b\text{arcsinh}(cx))}{e(1+m)}$$

output
```
(e*x+d)^(1+m)*(a+b*arcsinh(c*x))/e/(1+m)-b*c*(e*x+d)^(2+m)*AppellF1(2+m,1/
2,1/2,3+m,(e*x+d)/(d-e/(-c^2)^(1/2)),(e*x+d)/(d+e/(-c^2)^(1/2)))*(1+(-e*x-
d)/(d-e/(-c^2)^(1/2)))^(1/2)*(1+(-e*x-d)/(d+e/(-c^2)^(1/2)))^(1/2)/e^2/(1+
m)/(2+m)/(c^2*x^2+1)^(1/2)
```

3.31.2 Mathematica [F]

$$\int (d+ex)^m (a+b\text{arcsinh}(cx))\,dx = \int (d+ex)^m (a+b\text{arcsinh}(cx))\,dx$$

input
```
Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x]),x]
```

output
```
Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x]), x]
```

3.31.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6243, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6243}$$

$$\frac{(d+ex)^{m+1}(a+b\text{arcsinh}(cx))}{e(m+1)}-\frac{bc\int \frac{(d+ex)^{m+1}}{\sqrt{c^2x^2+1}}dx}{e(m+1)}$$

$$\Big\downarrow \text{514}$$

$$\frac{(d+ex)^{m+1}(a+b\text{arcsinh}(cx))}{e(m+1)}-\frac{bc\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}\int \frac{(d+ex)^{m+1}}{\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}}d(d+ex)}{e^2(m+1)\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{150}$$

$$\frac{(d+ex)^{m+1}(a+b\text{arcsinh}(cx))}{e(m+1)}-\frac{bc\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}(d+ex)^{m+2}\text{AppellF1}\left(m+2,\frac{1}{2},\frac{1}{2},m+3,\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}},\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}\right)}{e^2(m+1)(m+2)\sqrt{c^2x^2+1}}$$

input
```
Int[(d + e*x)^m*(a + b*ArcSinh[c*x]),x]
```

output
```
-((b*c*(d + e*x)^(2 + m)*Sqrt[1 - (d + e*x)/(d - e/Sqrt[-c^2])]*Sqrt[1 - (
d + e*x)/(d + e/Sqrt[-c^2])]*AppellF1[2 + m, 1/2, 1/2, 3 + m, (d + e*x)/(d
 - e/Sqrt[-c^2]), (d + e*x)/(d + e/Sqrt[-c^2])])/(e^2*(1 + m)*(2 + m)*Sqrt
[1 + c^2*x^2])) + ((d + e*x)^(1 + m)*(a + b*ArcSinh[c*x]))/(e*(1 + m))
```

3.31.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

3.31.4 Maple [F]

$$\int (ex+d)^m (a+b\,\operatorname{arcsinh}(cx))\,dx$$

input
```
int((e*x+d)^m*(a+b*arcsinh(c*x)),x)
```

output
```
int((e*x+d)^m*(a+b*arcsinh(c*x)),x)
```

3.31.5 Fricas [F]

$$\int (d+ex)^m(a+b\text{arcsinh}(cx))\,dx = \int (b\,\text{arsinh}\,(cx)+a)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)*(e*x + d)^m, x)
```

3.31.6 Sympy [F]

$$\int (d+ex)^m(a+b\operatorname{arcsinh}(cx))\,dx=\int (a+b\operatorname{asinh}(cx))\,(d+ex)^m\,dx$$

input
```
integrate((e*x+d)**m*(a+b*asinh(c*x)),x)
```

output
```
Integral((a + b*asinh(c*x))*(d + e*x)**m, x)
```

3.31.7 Maxima [F]

$$\int (d+ex)^m(a+b\operatorname{arcsinh}(cx))\,dx=\int (b\operatorname{arsinh}(cx)+a)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
b*((e*x + d)*(e*x + d)^m*log(c*x + sqrt(c^2*x^2 + 1))/(e*(m + 1)) - integr
ate((c^2*e*x^2 + c^2*d*x)*(e*x + d)^m/(c^2*e*(m + 1)*x^2 + e*(m + 1)), x)
- integrate((c*e*x + c*d)*(e*x + d)^m/(c^3*e*(m + 1)*x^3 + c*e*(m + 1)*x +
 (c^2*e*(m + 1)*x^2 + e*(m + 1))*sqrt(c^2*x^2 + 1)), x)) + (e*x + d)^(m +
1)*a/(e*(m + 1))
```

3.31.8 Giac [F]

$$\int (d+ex)^m(a+b\operatorname{arcsinh}(cx))\,dx=\int (b\operatorname{arsinh}(cx)+a)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*(e*x + d)^m, x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(a+b\operatorname{arcsinh}(cx))\,dx=\int (a+b\,\mathrm{asinh}(c\,x))\ (d+e\,x)^m\,dx$$

input
```
int((a + b*asinh(c*x))*(d + e*x)^m,x)
```

output
```
int((a + b*asinh(c*x))*(d + e*x)^m, x)
```

3.32 $\int \frac{(d+ex)^m}{a+b\mathbf{arcsinh}(cx)}\,dx$

3.32.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx=\text{Int}\left(\frac{(d+ex)^m}{a+b\text{arcsinh}(cx)},x\right)$$

output
```
Unintegrable((e*x+d)^m/(a+b*arcsinh(c*x)),x)
```

3.32.2 Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx$$

input
```
Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x]),x]
```

output
```
Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x]), x]
```

3.32.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6272}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx$$

$$\Big\downarrow \text{6272}$$

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}dx$$

input
```
Int[(d + e*x)^m/(a + b*ArcSinh[c*x]),x]
```

output
```
$Aborted
```

3.32.3.1 Defintions of rubi rules used

rule 6272
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.32.4 Maple [N/A] (verified)

Not integrable

Time = 0.95 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(ex+d)^m}{a+b\,\text{arcsinh}\,(cx)}dx$$

input
```
int((e*x+d)^m/(a+b*arcsinh(c*x)),x)
```

output
```
int((e*x+d)^m/(a+b*arcsinh(c*x)),x)
```

3.32.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex+d)^m}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m/(b*arcsinh(c*x) + a), x)
```

3.32.6 Sympy [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(d+ex)^m}{a+b\,\text{asinh}\,(cx)}\,dx$$

input
```
integrate((e*x+d)**m/(a+b*asinh(c*x)),x)
```

output
```
Integral((d + e*x)**m/(a + b*asinh(c*x)), x)
```

3.32.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx = \int \frac{(ex+d)^m}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m/(b*arcsinh(c*x) + a), x)
```

3.32.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(ex+d)^m}{b\,\text{arsinh}\,(cx)+a}\,dx$$

input
```
integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/(b*arcsinh(c*x) + a), x)
```

3.32.9 Mupad [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{a+b\text{arcsinh}(cx)}\,dx=\int \frac{(d+e\,x)^m}{a+b\,\text{asinh}\,(c\,x)}\,dx$$

input
```
int((d + e*x)^m/(a + b*asinh(c*x)),x)
```

output
```
int((d + e*x)^m/(a + b*asinh(c*x)), x)
```

3.33 $\int \frac{(d+ex)^m}{(a+b\,\mathrm{arcsinh}(cx))^2}\,dx$

3.33.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(d+ex)^m}{(a+b\,\mathrm{arcsinh}(cx))^2}\,dx = \mathrm{Int}\left(\frac{(d+ex)^m}{(a+b\,\mathrm{arcsinh}(cx))^2},x\right)$$

output
```
Unintegrable((e*x+d)^m/(a+b*arcsinh(c*x))^2,x)
```

3.33.2 Mathematica [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{(a+b\,\mathrm{arcsinh}(cx))^2}\,dx = \int \frac{(d+ex)^m}{(a+b\,\mathrm{arcsinh}(cx))^2}\,dx$$

input
```
Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x])^2,x]
```

output
```
Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x])^2, x]
```

3.33.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6272}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2} \, dx$$
$$\Big\downarrow \; 6272$$
$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}dx$$

input
```
Int[(d + e*x)^m/(a + b*ArcSinh[c*x])^2,x]
```

output
```
$Aborted
```

3.33.3.1 Defintions of rubi rules used

rule 6272
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.33.4 Maple [N/A] (verified)

Not integrable

Time = 0.93 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(ex+d)^m}{(a+b\,\operatorname{arcsinh}(cx))^2}dx$$

input
```
int((e*x+d)^m/(a+b*arcsinh(c*x))^2,x)
```

output
```
int((e*x+d)^m/(a+b*arcsinh(c*x))^2,x)
```

3.33.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex+d)^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((e*x + d)^m/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

3.33.6 Sympy [N/A]

Not integrable

Time = 14.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(d+ex)^m}{(a+b\,\text{asinh}\,(cx))^2}\,dx$$

input
```
integrate((e*x+d)**m/(a+b*asinh(c*x))**2,x)
```

output
```
Integral((d + e*x)**m/(a + b*asinh(c*x))**2, x)
```

3.33.7 Maxima [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 607, normalized size of antiderivative = 33.72

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex+d)^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^2*x^2 + 1)^(3/2)*(e*x + d)^m + (c^3*x^3 + c*x)*(e*x + d)^m)/(a*b*c^3*
x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 +
1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^3*e*(m
 + 1)*x^3 + c^3*d*x^2 + c*e*(m - 1)*x - c*d)*(c^2*x^2 + 1)*(e*x + d)^m + (
2*c^4*e*(m + 1)*x^4 + 2*c^4*d*x^3 + c^2*e*(3*m + 1)*x^2 + c^2*d*x + e*m)*s
qrt(c^2*x^2 + 1)*(e*x + d)^m + (c^5*e*(m + 1)*x^5 + c^5*d*x^4 + 2*c^3*e*(m
 + 1)*x^3 + 2*c^3*d*x^2 + c*e*(m + 1)*x + c*d)*(e*x + d)^m)/(a*b*c^5*e*x^5
 + a*b*c^5*d*x^4 + 2*a*b*c^3*e*x^3 + 2*a*b*c^3*d*x^2 + a*b*c*e*x + a*b*c*d
 + (a*b*c^3*e*x^3 + a*b*c^3*d*x^2)*(c^2*x^2 + 1) + (b^2*c^5*e*x^5 + b^2*c^
5*d*x^4 + 2*b^2*c^3*e*x^3 + 2*b^2*c^3*d*x^2 + b^2*c*e*x + b^2*c*d + (b^2*c
^3*e*x^3 + b^2*c^3*d*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e*x^4 + b^2*c^4*d*x^3
 + b^2*c^2*e*x^2 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
+ 1)) + 2*(a*b*c^4*e*x^4 + a*b*c^4*d*x^3 + a*b*c^2*e*x^2 + a*b*c^2*d*x)*sq
rt(c^2*x^2 + 1)), x)
```

3.33.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(ex+d)^m}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/(b*arcsinh(c*x) + a)^2, x)
```

3.33.9 Mupad [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^m}{(a+b\text{arcsinh}(cx))^2}\,dx = \int \frac{(d+e\,x)^m}{(a+b\,\text{asinh}\,(c\,x))^2}\,dx$$

input
```
int((d + e*x)^m/(a + b*asinh(c*x))^2,x)
```

output
```
int((d + e*x)^m/(a + b*asinh(c*x))^2, x)
```

3.34 $\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$

3.34.1 Optimal result

Integrand size = 30, antiderivative size = 640

$$\begin{aligned}
&\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx \\
&= -\frac{bf^2gx\sqrt{d+c^2dx^2}}{c\sqrt{1+c^2x^2}}+\frac{2bg^3x\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{bcf^3x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}-\frac{3bfg^2x^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}} \\
&\quad-\frac{bcf^2gx^3\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bg^3x^3\sqrt{d+c^2dx^2}}{45c\sqrt{1+c^2x^2}}-\frac{3bcfg^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{bcg^3x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} \\
&\quad+\frac{1}{2}f^3x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{3fg^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c^2} \\
&\quad+\frac{3}{4}fg^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{f^2g(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{c^2} \\
&\quad-\frac{g^3(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c^4} \\
&\quad+\frac{g^3(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{5c^4} \\
&\quad+\frac{f^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}-\frac{3fg^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16bc^3\sqrt{1+c^2x^2}}
\end{aligned}$$

output
```
1/2*f^3*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+3/8*f*g^2*x*(a+b*arcsinh(
c*x))*(c^2*d*x^2+d)^(1/2)/c^2+3/4*f*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+
d)^(1/2)+f^2*g*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-1/3*
g^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4+1/5*g^3*(c^2*x^
2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4-b*f^2*g*x*(c^2*d*x^2+d)^
(1/2)/c/(c^2*x^2+1)^(1/2)+2/15*b*g^3*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)
^(1/2)-1/4*b*c*f^3*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/16*b*f*g^2*
x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/3*b*c*f^2*g*x^3*(c^2*d*x^2+d
)^(1/2)/(c^2*x^2+1)^(1/2)-1/45*b*g^3*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)
^(1/2)-3/16*b*c*f*g^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/25*b*c*g
^3*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*f^3*(a+b*arcsinh(c*x))^2*
(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-3/16*f*g^2*(a+b*arcsinh(c*x))^2*
(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

3.34.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.64

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx$$

$$= \frac{240ad(1+c^2x^2)^{3/2}\left(-16g^3+c^2g(120f^2+45fgx+8g^2x^2)+6c^4x(10f^3+20f^2gx+15fg^2x^2+4g^3x^3)\right)-}{}$$

input
```
Integrate[(f + g*x)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(240*a*d*(1 + c^2*x^2)^(3/2)*(-16*g^3 + c^2*g*(120*f^2 + 45*f*g*x + 8*g^2*
x^2) + 6*c^4*x*(10*f^3 + 20*f^2*g*x + 15*f*g^2*x^2 + 4*g^3*x^3)) - 9600*b*
c^2*d*f^2*g*(3*c*x + 4*c^3*x^3 + c^5*x^5 - 3*(1 + c^2*x^2)^(5/2)*ArcSinh[c
*x]) + 128*b*d*g^3*(1 + c^2*x^2)*(30*c*x - 5*c^3*x^3 - 9*c^5*x^5 + 15*Sqrt
[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x]) + 3600*a*c*Sqrt[d]*
f*(4*c^2*f^2 - 3*g^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sq
rt[d]*Sqrt[d + c^2*d*x^2]] - 3600*b*c^3*d*f^3*(1 + c^2*x^2)*(Cosh[2*ArcSin
h[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])) - 675*b*c*
d*f*g^2*(1 + c^2*x^2)*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh
[c*x]*Sinh[4*ArcSinh[c*x]]))/(28800*c^4*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x
^2])
```

3.34.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.57, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c^2dx^2+d}(f+gx)^3(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{\sqrt{c^2dx^2+d}\int (f+gx)^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{\sqrt{c^2dx^2+d}\int \left(\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))f^3+3gx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))f^2+3g^2x^2\sqrt{c^2x^2+1}(a+b\text{arcs}\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{3fg^2(a+b\text{arcsinh}(cx))^2}{16bc^3}+\frac{1}{2}f^3x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{f^2g(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{c^2}+\frac{3fg^2x\sqrt{}}{}\right.}{}$$

input
```
Int[(f + g*x)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(-((b*f^2*g*x)/c) + (2*b*g^3*x)/(15*c^3) - (b*c*f^3*x
^2)/4 - (3*b*f*g^2*x^2)/(16*c) - (b*c*f^2*g*x^3)/3 - (b*g^3*x^3)/(45*c) -
(3*b*c*f*g^2*x^4)/16 - (b*c*g^3*x^5)/25 + (f^3*x*Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x]))/2 + (3*f*g^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c
^2) + (3*f*g^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/4 + (f^2*g*(1 +
 c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/c^2 - (g^3*(1 + c^2*x^2)^(3/2)*(a +
b*ArcSinh[c*x]))/(3*c^4) + (g^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/
(5*c^4) + (f^3*(a + b*ArcSinh[c*x])^2)/(4*b*c) - (3*f*g^2*(a + b*ArcSinh[c
*x])^2)/(16*b*c^3)))/Sqrt[1 + c^2*x^2]
```

3.34.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

3.34.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1308 vs. $2(560) = 1120$.

Time = 0.86 (sec) , antiderivative size = 1309, normalized size of antiderivative = 2.05

method	result	size
default	Expression too large to display	1309
parts	Expression too large to display	1309

input
```
int((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^3*(1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^
2+d)^(1/2))/(c^2*d)^(1/2))+g^3*(1/5*x^2*(c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c
^4*(c^2*d*x^2+d)^(3/2))+3*f*g^2*(1/4*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/4/c^2*(
1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/
2))/(c^2*d)^(1/2)))+f^2*g/c^2/d*(c^2*d*x^2+d)^(3/2))+b*(1/16*(d*(c^2*x^2+1
))^(1/2)*f*arcsinh(c*x)^2*(4*c^2*f^2-3*g^2)/(c^2*x^2+1)^(1/2)/c^3+1/800*(d
*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20
*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*g^3*(-1+5
*arcsinh(c*x))/c^4/(c^2*x^2+1)+3/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^
4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*
x^2+1)^(1/2))*f*g^2*(-1+4*arcsinh(c*x))/c^3/(c^2*x^2+1)+1/288*(d*(c^2*x^2+
1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+
1)^(1/2)+1)*g*(36*arcsinh(c*x)*c^2*f^2-12*c^2*f^2-3*arcsinh(c*x)*g^2+g^2)/
c^4/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1
)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*f^3*(-1+2*arcsinh(c*x))/c/(c^2*x^2+1)+1/1
6*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*g*(6*arcsinh(c*x
)*c^2*f^2-6*c^2*f^2-arcsinh(c*x)*g^2+g^2)/c^4/(c^2*x^2+1)+1/16*(d*(c^2*x^2
+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*g*(6*arcsinh(c*x)*c^2*f^2+6*c
^2*f^2-arcsinh(c*x)*g^2-g^2)/c^4/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2
*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*f^3*(1+2*...
```

3.34.5 Fricas [F]

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int \sqrt{c^2dx^2+d}(gx+f)^3(b\operatorname{arsinh}(cx)+a)\,dx$$

input
```
integrate((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

output
```
integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3
*b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

3.34.6 Sympy [F]

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int \sqrt{d\left(c^2x^2+1\right)}(a+b\,\text{asinh}\,(cx))\,(f+gx)^3\,dx$$

input
```
integrate((g*x+f)**3*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x)**3, x)
```

3.34.7 Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.34.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx=\int (f+g\,x)^3\,(a+b\operatorname{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

3.35 $\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$

3.35.1 Optimal result

Integrand size = 30, antiderivative size = 431

$$\begin{aligned}
&\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx\\
&= -\frac{2bfgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}}-\frac{bcf^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}-\frac{bg^2x^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}}\\
&\quad-\frac{2bcfgx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}-\frac{bcg^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+\frac{1}{2}f^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\\
&\quad+\frac{g^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{8c^2}+\frac{1}{4}g^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\\
&\quad+\frac{2fg(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c^2}\\
&\quad+\frac{f^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}-\frac{g^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16bc^3\sqrt{1+c^2x^2}}
\end{aligned}$$

output
```
1/2*f^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/8*g^2*x*(a+b*arcsinh(c*
x))*(c^2*d*x^2+d)^(1/2)/c^2+1/4*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(
1/2)+2/3*f*g*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-2/3*b*
f*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/4*b*c*f^2*x^2*(c^2*d*x^2+d
)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)
^(1/2)-2/9*b*c*f*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*c*g^2*
x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*f^2*(a+b*arcsinh(c*x))^2*(c^
2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-1/16*g^2*(a+b*arcsinh(c*x))^2*(c^2*
d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

3.35.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.70

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx$$
$$=\frac{48ac\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(12c^2f^2x+3g^2x(1+2c^2x^2)+16f(g+c^2gx^2))-256bcfg\sqrt{d+c^2dx^2}\Big(3cx+c}{}$$

input
```
Integrate[(f + g*x)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(48*a*c*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(12*c^2*f^2*x + 3*g^2*x*(1 +
 2*c^2*x^2) + 16*f*(g + c^2*g*x^2)) - 256*b*c*f*g*Sqrt[d + c^2*d*x^2]*(3*c
*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]) + 144*a*Sqrt[d]*(2*c*f
- g)*(2*c*f + g)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]
] - 144*b*c^2*f^2*Sqrt[d + c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*
x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])) - 9*b*g^2*Sqrt[d + c^2*d*x^2]*(8
*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x
]]))/(1152*c^3*Sqrt[1 + c^2*x^2])
```

3.35.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.60, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c^2dx^2+d}(f+gx)^2(a+b\operatorname{arcsinh}(cx))\,dx$$
$$\Big\downarrow \text{6260}$$
$$\frac{\sqrt{c^2dx^2+d}\int (f+gx)^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$
$$\Big\downarrow \text{6253}$$
$$\frac{\sqrt{c^2dx^2+d}\int \Big(\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))f^2+2gx\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))f+g^2x^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsin}}{\sqrt{c^2x^2+1}}$$

$$\downarrow \text{2009}$$

$$\frac{\sqrt{c^2dx^2+d}\left(-\frac{g^2(a+b\text{arcsinh}(cx))^2}{16bc^3}+\frac{1}{2}f^2x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{2fg(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}+\frac{g^2x\sqrt{c^2x^2}}{}\right.}{}$$

input
```
Int[(f + g*x)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*((-2*b*f*g*x)/(3*c) - (b*c*f^2*x^2)/4 - (b*g^2*x^2)/(
16*c) - (2*b*c*f*g*x^3)/9 - (b*c*g^2*x^4)/16 + (f^2*x*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/2 + (g^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*
c^2) + (g^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/4 + (2*f*g*(1 + c^
2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^2) + (f^2*(a + b*ArcSinh[c*x])^2)/
(4*b*c) - (g^2*(a + b*ArcSinh[c*x])^2)/(16*b*c^3)))/Sqrt[1 + c^2*x^2]
```

3.35.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

3.35.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(373) = 746$.

Time = 0.72 (sec) , antiderivative size = 913, normalized size of antiderivative = 2.12

method	result
default	$a\left(f^2\left(\frac{x\sqrt{c^2dx^2+d}}{2}+\frac{d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}\right)+g^2\left(\frac{x(c^2dx^2+d)^{\frac{3}{2}}}{4c^2d}-\frac{\frac{x\sqrt{c^2dx^2+d}}{2}+\frac{d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}}{4c^2}\right)+\right.$
parts	$a\left(f^2\left(\frac{x\sqrt{c^2dx^2+d}}{2}+\frac{d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}\right)+g^2\left(\frac{x(c^2dx^2+d)^{\frac{3}{2}}}{4c^2d}-\frac{\frac{x\sqrt{c^2dx^2+d}}{2}+\frac{d\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}}{4c^2}\right)+\right.$

input
```
int((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^2*(1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^
2+d)^(1/2))/(c^2*d)^(1/2))+g^2*(1/4*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/4/c^2*(1
/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2
))/(c^2*d)^(1/2)))+2/3*f*g/c^2/d*(c^2*d*x^2+d)^(3/2))+b*(1/16*(d*(c^2*x^2+
1))^(1/2)*arcsinh(c*x)^2*(4*c^2*f^2-g^2)/(c^2*x^2+1)^(1/2)/c^3+1/256*(d*(c
^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x
^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*g^2*(-1+4*arcsinh(c*x))/c^3/
(c^2*x^2+1)+1/36*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1
/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(-1+3*arcsinh(c*x))/c^2/(c^2*
x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2
*c*x+(c^2*x^2+1)^(1/2))*f^2*(-1+2*arcsinh(c*x))/c/(c^2*x^2+1)+1/4*(d*(c^2*
x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(-1+arcsinh(c*x))/c^2/
(c^2*x^2+1)+1/4*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*f*
g*(arcsinh(c*x)+1)/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2
*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*f^2*(1+2*arcsinh(c*x))
/c/(c^2*x^2+1)+1/36*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)
^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(3*arcsinh(c*x)+1)/c^2/(c^
2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2
)+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*g^2*(1+4
*arcsinh(c*x))/c^3/(c^2*x^2+1))
```

3.35.5 Fricas [F]

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int \sqrt{c^2dx^2+d}(gx+f)^2(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 +
 2*b*f*g*x + b*f^2)*arcsinh(c*x)), x)
```

3.35.6 Sympy [F]

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int \sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))\,(f+gx)^2\,dx$$

input
```
integrate((g*x+f)**2*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x)**2, x)
```

3.35.7 Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.35.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\,dx = \int (f+g\,x)^2\,(a+b\,\operatorname{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

3.36 $\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx$

3.36.1 Optimal result

Integrand size = 28, antiderivative size = 227

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = -\frac{bgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}}-\frac{bcfx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}-\frac{bcgx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{1}{2}fx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))+\frac{g(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{3c^2}+\frac{f\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{4bc\sqrt{1+c^2x^2}}$$

output
```
1/2*f*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/3*g*(c^2*x^2+1)*(a+b*arcs
inh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-1/3*b*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2
+1)^(1/2)-1/4*b*c*f*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*b*c*g*x^
3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*f*(a+b*arcsinh(c*x))^2*(c^2*d*
x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

3.36.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.92

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\frac{1}{6}a\sqrt{d+c^2dx^2}\left(\frac{2g}{c^2}+x(3f+2gx)\right)-\frac{bg\sqrt{d+c^2dx^2}\left(3cx+c^3x^3-3(1+c^2x^2)^{3/2}\text{arcsinh}(cx)\right)}{9c^2\sqrt{1+c^2x^2}}+\frac{a\sqrt{d}f\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{2c}+\frac{bf\sqrt{d+c^2dx^2}(-\cosh(2\text{arcsinh}(cx))+2\text{arcsinh}(cx)(\text{arcsinh}(cx)+\sinh(2\text{arcsinh}(cx))))}{8c\sqrt{1+c^2x^2}}$$

input
```
Integrate[(f + g*x)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(a*Sqrt[d + c^2*d*x^2]*((2*g)/c^2 + x*(3*f + 2*g*x)))/6 - (b*g*Sqrt[d + c^
2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]))/(9*c^2*Sq
rt[1 + c^2*x^2]) + (a*Sqrt[d]*f*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/
(2*c) + (b*f*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*(
ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(8*c*Sqrt[1 + c^2*x^2])
```

3.36.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.60, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c^2dx^2+d}(f+gx)(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{\sqrt{c^2dx^2+d}\int (f+gx)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{\sqrt{c^2dx^2+d}\int\left(f\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+gx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right)dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{1}{2}fx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{g(c^2x^2+1)^{3/2}(a+b\text{arcsinh}(cx))}{3c^2}+\frac{f(a+b\text{arcsinh}(cx))^2}{4bc}-\frac{1}{4}bcfx^2-\frac{1}{9}bcg\right.}{\sqrt{c^2x^2+1}}$$

input
```
Int[(f + g*x)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(-1/3*(b*g*x)/c - (b*c*f*x^2)/4 - (b*c*g*x^3)/9 + (f*
x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 + (g*(1 + c^2*x^2)^(3/2)*(a +
b*ArcSinh[c*x]))/(3*c^2) + (f*(a + b*ArcSinh[c*x])^2)/(4*b*c)))/Sqrt[1 + c
^2*x^2]
```

3.36.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

3.36.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(195) = 390$.

Time = 0.78 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.56

method	result
default	$\frac{afx\sqrt{c^2dx^2+d}}{2}+\frac{afd\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}+\frac{ag(c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,f\operatorname{arcsinh}(cx)^2}{4\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\,(4c^4x^4+\right.$
parts	$\frac{afx\sqrt{c^2dx^2+d}}{2}+\frac{afd\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}+\frac{ag(c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,f\operatorname{arcsinh}(cx)^2}{4\sqrt{c^2x^2+1}\,c}+\frac{\sqrt{d(c^2x^2+1)}\,(4c^4x^4+\right.$

input
```
int((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/2*a*f*x*(c^2*d*x^2+d)^(1/2)+1/2*a*f*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^
2+d)^(1/2))/(c^2*d)^(1/2)+1/3*a*g/c^2/d*(c^2*d*x^2+d)^(3/2)+b*(1/4*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*f*arcsinh(c*x)^2+1/72*(d*(c^2*x^2+1))^(
1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1
/2)+1)*g*(-1+3*arcsinh(c*x))/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2
*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*f*(-1+2*arcs
inh(c*x))/c/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)
^(1/2)+1)*g*(-1+arcsinh(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c
^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*g*(arcsinh(c*x)+1)/c^2/(c^2*x^2+1)+1/16*(d
*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+
1)^(1/2))*f*(1+2*arcsinh(c*x))/c/(c^2*x^2+1)+1/72*(d*(c^2*x^2+1))^(1/2)*(4
*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*
g*(3*arcsinh(c*x)+1)/c^2/(c^2*x^2+1))
```

3.36.5 Fricas [F]

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int\sqrt{c^2dx^2+d}(gx+f)(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arcsinh(c*x)), x
)
```

3.36.6 Sympy [F]

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\int\sqrt{d\left(c^2x^2+1\right)}(a+b\,\text{asinh}\,(cx))\,(f+gx)\,\,dx$$

input
```
integrate((g*x+f)*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x), x)
```

3.36.7 Maxima [F(-2)]

Exception generated.

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.36.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\,dx = \int (f+g\,x)\,\left(a+b\,\text{asinh}(c\,x)\right)\,\sqrt{d\,c^2\,x^2+d}\,dx$$

input
```
int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

3.37 $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx$

3.37.1 Optimal result

Integrand size = 30, antiderivative size = 664

$$
\begin{aligned}
&\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx \\
&= \frac{a\sqrt{d+c^2dx^2}}{g}-\frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}}+\frac{b\sqrt{d+c^2dx^2}\text{arcsinh}(cx)}{g} \\
&\quad -\frac{cx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2bg\sqrt{1+c^2x^2}}-\frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)\sqrt{1+c^2x^2}} \\
&\quad +\frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)} \\
&\quad -\frac{a\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\text{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad +\frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\text{arcsinh}(cx)\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad -\frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\text{arcsinh}(cx)\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad +\frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}} \\
&\quad -\frac{b\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{1+c^2x^2}}
\end{aligned}
$$

output
```
a*(c^2*d*x^2+d)^(1/2)/g+b*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/g-b*c*x*(c^2*d*
x^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)-1/2*c*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d
)^(1/2)/b/g/(c^2*x^2+1)^(1/2)-1/2*(1+c^2*f^2/g^2)*(a+b*arcsinh(c*x))^2*(c^
2*d*x^2+d)^(1/2)/b/c/(g*x+f)/(c^2*x^2+1)^(1/2)-a*arctanh((-c^2*f*x+g)/(c^2
*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2))*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)
/g^2/(c^2*x^2+1)^(1/2)+b*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-
(c^2*f^2+g^2)^(1/2)))*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2
+1)^(1/2)-b*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)
^(1/2)))*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/2)+b*p
olylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*f^2+g^
2)^(1/2)*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/2)-b*polylog(2,-(c*x+(c^2*
x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+
d)^(1/2)/g^2/(c^2*x^2+1)^(1/2)+1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)*
(c^2*d*x^2+d)^(1/2)/b/c/(g*x+f)
```

3.37.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.93 (sec) , antiderivative size = 1358, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

output
```
(2*a*g*Sqrt[d + c^2*d*x^2] + 2*a*Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Log[f + g*x]
- 2*a*c*Sqrt[d]*f*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 2*a*Sqrt[d]*S
qrt[c^2*f^2 + g^2]*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Sqrt[
d + c^2*d*x^2]] + b*Sqrt[d + c^2*d*x^2]*((-2*c*g*x)/Sqrt[1 + c^2*x^2] + 2*
g*ArcSinh[c*x] - (c*f*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (2*((-I)*c*f + g
)*(I*c*f + g)*(((-I)*Pi*ArcTanh[(-g + c*f*Tanh[ArcSinh[c*x]/2])/Sqrt[c^2*f
^2 + g^2]])/Sqrt[c^2*f^2 + g^2] - (2*ArcCos[((-I)*c*f)/g]*ArcTanh[((c*f +
I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (Pi - (2*
I)*ArcSinh[c*x])*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sq
rt[-(c^2*f^2) - g^2]] + (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)
*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] - (2*I)*ArcTanh
[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*L
og[((1/2 - I/2)*Sqrt[-(c^2*f^2) - g^2])/(E^(ArcSinh[c*x]/2)*Sqrt[(-I)*g]*S
qrt[c*(f + g*x)])] + (ArcCos[((-I)*c*f)/g] + (2*I)*(ArcTanh[((c*f + I*g)*C
ot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + ArcTanh[((c*f -
 I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]]))*Log[((1/
2 + I/2)*E^(ArcSinh[c*x]/2)*Sqrt[-(c^2*f^2) - g^2])/(Sqrt[(-I)*g]*Sqrt[c*(
f + g*x)])] - (ArcCos[((-I)*c*f)/g] + (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi +
 (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*((-I)*c
*f + g + Sqrt[-(c^2*f^2) - g^2])*(1 + I*Cot[(Pi + (2*I)*ArcSinh[c*x])/4...
```

3.37.3 Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.69, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {6260, 6254, 25, 6250, 25, 6271, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{f+gx}dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{f+gx}dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6254}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)}-\frac{\int -\frac{(-gx^2c^2-2fxc^2+g)(a+b\text{arcsinh}(cx))^2}{(f+gx)^2}dx}{2bc}\right)}{\sqrt{c^2x^2+1}}$$

$$\downarrow 25$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{\int \frac{\left(-gx^2c^2-2fxc^2+g\right)(a+b\text{arcsinh}(cx))^2}{(f+gx)^2}dx}{2bc}+\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)}\right)}{\sqrt{c^2x^2+1}}$$

$$\downarrow 6250$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{-2bc\int -\frac{\left(\frac{xc^2}{g}+\frac{\frac{c^2f^2}{g^2}+1}{f+gx}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx-\frac{\left(\frac{c^2f^2}{g^2}+1\right)(a+b\text{arcsinh}(cx))^2}{f+gx}-\frac{c^2x(a+b\text{arcsinh}(cx))^2}{g}}{2bc}+\frac{(c^2x^2+1)(a+b\text{arc}}{2bc(f+g}\right.}{\sqrt{c^2x^2+1}}$$

$$\downarrow 25$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{2bc\int \frac{\left(\frac{xc^2}{g}+\frac{\frac{c^2f^2}{g^2}+1}{f+gx}\right)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx-\frac{\left(\frac{c^2f^2}{g^2}+1\right)(a+b\text{arcsinh}(cx))^2}{f+gx}-\frac{c^2x(a+b\text{arcsinh}(cx))^2}{g}}{2bc}+\frac{(c^2x^2+1)(a+b\text{arcsin}}{2bc(f+gx)}\right.}{\sqrt{c^2x^2+1}}$$

$$\downarrow 6271$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{2bc\int \left(\frac{b\text{arcsinh}(cx)\left(f^2c^2+g^2x^2c^2+fgxc^2+g^2\right)}{g^2(f+gx)\sqrt{c^2x^2+1}}+\frac{a\left(f^2c^2+g^2x^2c^2+fgxc^2+g^2\right)}{g^2(f+gx)\sqrt{c^2x^2+1}}\right)dx-\frac{\left(\frac{c^2f^2}{g^2}+1\right)(a+b\text{arcsinh}(cx))^2}{f+gx}-\frac{c^2x(a+b\text{arcs}}{g}}{2bc}\right.}{\sqrt{c^2x^2+1}}$$

$$\downarrow 2009$$

$$\sqrt{c^2dx^2+d}\left(\frac{2bc\left(-\frac{a\sqrt{c^2f^2+g^2}\text{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}}\right)}{g^2}+\frac{a\sqrt{c^2x^2+1}}{g}+\frac{b\sqrt{c^2f^2+g^2}\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2}-\frac{b\sqrt{c^2f^2+g^2}\text{PolyLog}\left(\right.}{g}\right.}{}\right.$$

3.37. $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx$

input
```
Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(((1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(2*b*c*(f + g
*x)) + (-((c^2*x*(a + b*ArcSinh[c*x])^2)/g) - ((1 + (c^2*f^2)/g^2)*(a + b*
ArcSinh[c*x])^2)/(f + g*x) + 2*b*c*(-((b*c*x)/g) + (a*Sqrt[1 + c^2*x^2])/g
 + (b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/g - (a*Sqrt[c^2*f^2 + g^2]*ArcTanh[(
g - c^2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])])/g^2 + (b*Sqrt[c^2*f
^2 + g^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^
2])])/g^2 - (b*Sqrt[c^2*f^2 + g^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)
/(c*f + Sqrt[c^2*f^2 + g^2])])/g^2 + (b*Sqrt[c^2*f^2 + g^2]*PolyLog[2, -((
E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/g^2 - (b*Sqrt[c^2*f^2 + g
^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/g^2))/(
2*b*c)))/Sqrt[1 + c^2*x^2]
```

3.37.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6250
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Simp[(a + b*ArcSinh[c*x])^n   u, x] - Simp[b
*c*n   Int[SimplifyIntegrand[u*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x
^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IG
tQ[p, 0] && EqQ[e*g - 2*d*h, 0]
```

rule 6254
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(f + g*x)^m*(d + e*x^2)*((a + b*A
rcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Simp[1/(b*c*Sqrt[d]*(n +
1))   Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcS
inh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*
d] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

rule 6271
```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

3.37.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 747, normalized size of antiderivative = 1.12

method	result
default	$\frac{a\left(\sqrt{\left(x+\frac{f}{g}\right)^2 c^2 d-\frac{2c^2 df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^2f^2+g^2\right)}{g^2}}-\frac{c^2 df \ln\left(\frac{-\frac{c^2 df}{g}+c^2 d\left(x+\frac{f}{g}\right)}{\sqrt{c^2 d}}+\sqrt{\left(x+\frac{f}{g}\right)^2 c^2 d-\frac{2c^2 df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^2f^2+g^2\right)}{g^2}}\right)}{g\sqrt{c^2 d}}-\frac{d\left(c^2f^2+g^2\right.}{}\right.}{g}$
parts	$\frac{a\left(\sqrt{\left(x+\frac{f}{g}\right)^2 c^2 d-\frac{2c^2 df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^2f^2+g^2\right)}{g^2}}-\frac{c^2 df \ln\left(\frac{-\frac{c^2 df}{g}+c^2 d\left(x+\frac{f}{g}\right)}{\sqrt{c^2 d}}+\sqrt{\left(x+\frac{f}{g}\right)^2 c^2 d-\frac{2c^2 df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^2f^2+g^2\right)}{g^2}}\right)}{g\sqrt{c^2 d}}-\frac{d\left(c^2f^2+g^2\right.}{}\right.}{g}$

input
```
int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x,method=_RETURNVERBOSE
)
```

output
```
a/g*(((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)-c^2*d
*f/g*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*
f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)-d*(c^2*f^2+g^2)/g^2/
(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+
2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*
f^2+g^2)/g^2)^(1/2))/(x+f/g)))+b*(-1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(
1/2)*f*arcsinh(c*x)^2*c/g^2+1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^
2+1)^(1/2)+1)*(-1+arcsinh(c*x))/(c^2*x^2+1)/g+1/2*(d*(c^2*x^2+1))^(1/2)*(c
^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)+1)/(c^2*x^2+1)/g+(d*(c^2*x^2
+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)*(arcsinh(c*x)*ln((-(c*x+(
c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-a
rcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c
^2*f^2+g^2)^(1/2)))+dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1
/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2
*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2))))/g^2)
```

3.37.5 Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{gx+f}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="fri
cas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(g*x + f), x)
```

3.37.6 Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \int \frac{\sqrt{d\,(c^2x^2+1)}(a+b\,\text{asinh}\,(cx))}{f+gx}\,dx$$

input
```
integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/(g*x+f),x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/(f + g*x), x)
```

3.37.7 Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{gx+f}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="max
ima")
```

output
```
-(c*sqrt(d)*f*arcsinh(c*x)/g^2 - sqrt(c^2*d*f^2/g^2 + d)*arcsinh(c*f*x/abs
(g*x + f) - g/(c*abs(g*x + f)))/g - sqrt(c^2*d*x^2 + d)/g)*a + b*integrate
(sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))/(g*x + f), x)
```

3.37.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}}{f+g\,x}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x), x)
```

3.38 $\int \frac{\sqrt{d+c^2dx^2}(a+b\,\mathrm{arcsinh}(cx))}{(f+gx)^2}\,dx$

3.38.1 Optimal result

Integrand size = 30, antiderivative size = 781

$$
\begin{aligned}
\int \frac{\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{(f+gx)^2}\,dx = & -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)}-\frac{b\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g(f+gx)} \\
& +\frac{ac^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} \\
& +\frac{bc^3f^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)^2}{2g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} \\
& -\frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} \\
& +\frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bc(f+gx)^2} \\
& +\frac{ac^2f\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
& -\frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
& +\frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
& +\frac{bc\sqrt{d+c^2dx^2}\log(f+gx)}{g^2\sqrt{1+c^2x^2}} \\
& -\frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}} \\
& +\frac{bc^2f\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}
\end{aligned}
$$

output
```
-a*(c^2*d*x^2+d)^(1/2)/g/(g*x+f)-b*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/g/(g*x
+f)+a*c^3*f^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)/(c^2*x^2+
1)^(1/2)+1/2*b*c^3*f^2*arcsinh(c*x)^2*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2
)/(c^2*x^2+1)^(1/2)-1/2*(-c^2*f*x+g)^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^
(1/2)/b/c/(c^2*f^2+g^2)/(g*x+f)^2/(c^2*x^2+1)^(1/2)+b*c*ln(g*x+f)*(c^2*d*x
^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/2)+a*c^2*f*arctanh((-c^2*f*x+g)/(c^2*f^2+g^
2)^(1/2)/(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)/(c
^2*x^2+1)^(1/2)-b*c^2*f*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(
c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+
1)^(1/2)+b*c^2*f*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2
+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2
)-b*c^2*f*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*
(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)+b*c^2*f*poly
log(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^
(1/2)/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*(a+b*arcsinh(c*x))^2*(
c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)/b/c/(g*x+f)^2
```

3.38.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.67 (sec) , antiderivative size = 1384, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x)^2,x]
```

output
```
((-2*a*g*Sqrt[d + c^2*d*x^2])/(f + g*x) - (2*a*c^2*Sqrt[d]*f*Log[f + g*x])
/Sqrt[c^2*f^2 + g^2] + 2*a*c*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^
2]] + (2*a*c^2*Sqrt[d]*f*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 + g^2]
*Sqrt[d + c^2*d*x^2]])/Sqrt[c^2*f^2 + g^2] + b*c*Sqrt[d + c^2*d*x^2]*((-2*
g*ArcSinh[c*x])/(c*f + c*g*x) + ArcSinh[c*x]^2/Sqrt[1 + c^2*x^2] + ((2*I)*
c*f*Pi*ArcTanh[(-g + c*f*Tanh[ArcSinh[c*x]/2])/Sqrt[c^2*f^2 + g^2]])/(Sqrt
[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2]) + (2*Log[1 + (g*x)/f])/Sqrt[1 + c^2*x^2
] + (2*c*f*(2*ArcCos[((-I)*c*f)/g]*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*Ar
cSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (Pi - (2*I)*ArcSinh[c*x])*ArcTan
h[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] +
 (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSin
h[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] - (2*I)*ArcTanh[((c*f - I*g)*Tan[(Pi +
 (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((1/2 - I/2)*Sqrt[-(
c^2*f^2) - g^2])/(E^(ArcSinh[c*x]/2)*Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] + (A
rcCos[((-I)*c*f)/g] + (2*I)*(ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[
c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*A
rcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]]))*Log[((1/2 + I/2)*E^(ArcSinh[c*x
]/2)*Sqrt[-(c^2*f^2) - g^2])/(Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] - (ArcCos[(
(-I)*c*f)/g] + (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]
)/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*((-I)*c*f + g + Sqrt[-(c^2*...
```

3.38.3 Rubi [A] (verified)

Time = 2.74 (sec) , antiderivative size = 572, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {6260, 6254, 27, 6249, 27, 6271, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2dx^2+d}(a+b\text{arcsinh}(cx))}{(f+gx)^2}dx$$

$$\Big\downarrow \text{ 6260}$$

$$\frac{\sqrt{c^2dx^2+d}\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(f+gx)^2}dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6254}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)^2}-\frac{\int -\frac{2\left(g-c^2fx\right)(a+b\text{arcsinh}(cx))^2}{(f+gx)^3}dx}{2bc}\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{\int \frac{\left(g-c^2fx\right)(a+b\text{arcsinh}(cx))^2}{(f+gx)^3}dx}{bc}+\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)^2}\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6249}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{-2bc\int -\frac{\left(g-c^2fx\right)^2(a+b\text{arcsinh}(cx))}{2\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{c^2x^2+1}}dx-\frac{\left(g-c^2fx\right)^2(a+b\text{arcsinh}(cx))^2}{2\left(c^2f^2+g^2\right)(f+gx)^2}}{bc}+\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)^2}\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{\frac{bc\int \frac{\left(g-c^2fx\right)^2(a+b\text{arcsinh}(cx))}{(f+gx)^2\sqrt{c^2x^2+1}}dx}{c^2f^2+g^2}-\frac{\left(g-c^2fx\right)^2(a+b\text{arcsinh}(cx))^2}{2\left(c^2f^2+g^2\right)(f+gx)^2}}{bc}+\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)^2}\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6271}$$

$$\frac{\sqrt{c^2dx^2+d}\left(\frac{\frac{bc\int \left(\frac{b\text{arcsinh}(cx)\left(c^2fx-g\right)^2}{(f+gx)^2\sqrt{c^2x^2+1}}+\frac{a\left(c^2fx-g\right)^2}{(f+gx)^2\sqrt{c^2x^2+1}}\right)dx}{c^2f^2+g^2}-\frac{\left(g-c^2fx\right)^2(a+b\text{arcsinh}(cx))^2}{2\left(c^2f^2+g^2\right)(f+gx)^2}}{bc}+\frac{(c^2x^2+1)(a+b\text{arcsinh}(cx))^2}{2bc(f+gx)^2}\right)}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

3.38. $\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx$

$$\sqrt{c^2dx^2+d}\left(\frac{\frac{bc\left(\frac{ac^3f^2\text{arcsinh}(cx)}{g^2}+\frac{ac^2f\sqrt{c^2f^2+g^2}\text{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2x^2+1}\sqrt{c^2f^2+g^2}}\right)}{g^2}-\frac{a\sqrt{c^2x^2+1}\left(c^2f^2+g^2\right)}{g(f+gx)}+\frac{bc^3f^2\text{arcsinh}(cx)^2}{2g^2}-\frac{bc^2f\sqrt{c^2 \cdots}}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

input
```
Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x)^2,x]
```

output
```
(Sqrt[d + c^2*d*x^2]*(((1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(2*b*c*(f + g
*x)^2) + (-1/2*((g - c^2*f*x)^2*(a + b*ArcSinh[c*x])^2)/((c^2*f^2 + g^2)*(
f + g*x)^2) + (b*c*(-((a*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2])/(g*(f + g*x)))
 + (a*c^3*f^2*ArcSinh[c*x])/g^2 - (b*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2]*Arc
Sinh[c*x])/(g*(f + g*x)) + (b*c^3*f^2*ArcSinh[c*x]^2)/(2*g^2) + (a*c^2*f*S
qrt[c^2*f^2 + g^2]*ArcTanh[(g - c^2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2
*x^2])])/g^2 - (b*c^2*f*Sqrt[c^2*f^2 + g^2]*ArcSinh[c*x]*Log[1 + (E^ArcSin
h[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/g^2 + (b*c^2*f*Sqrt[c^2*f^2 + g^2]
*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/g^2
 + (b*c*(c^2*f^2 + g^2)*Log[f + g*x])/g^2 - (b*c^2*f*Sqrt[c^2*f^2 + g^2]*P
olyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/g^2 + (b*c^2
*f*Sqrt[c^2*f^2 + g^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2
 + g^2]))])/g^2))/(c^2*f^2 + g^2))/(b*c)))/Sqrt[1 + c^2*x^2]
```

3.38.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6249
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_)*((f_.
) + (g_.)*(x_))^(p_.), x_Symbol] :> With[{u = IntHide[(f + g*x)^p*(d + e*x)
^m, x]}, Simp[(a + b*ArcSinh[c*x])^n   u, x] - Simp[b*c*n   Int[SimplifyInt
egrand[u*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x], x]] /; F
reeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[m, 0] &&
 LtQ[m + p + 1, 0]
```

rule 6254
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(f + g*x)^m*(d + e*x^2)*((a + b*A
rcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Simp[1/(b*c*Sqrt[d]*(n +
1))   Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcS
inh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*
d] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

rule 6271
```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

3.38.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1700 vs. $2(745) = 1490$.

Time = 0.82 (sec) , antiderivative size = 1701, normalized size of antiderivative = 2.18

method	result	size
default	Expression too large to display	1701
parts	Expression too large to display	1701

input
```
int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x,method=_RETURNVERBO
SE)
```

output
```
a/g^2*(-1/d/(c^2*f^2+g^2)*g^2/(x+f/g)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)
+d*(c^2*f^2+g^2)/g^2)^(3/2)-c^2*f*g/(c^2*f^2+g^2)*(((x+f/g)^2*c^2*d-2*c^2*
d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)-c^2*d*f/g*ln((-c^2*d*f/g+c^2*d*(x
+f/g))/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/
g^2)^(1/2))/(c^2*d)^(1/2)-d*(c^2*f^2+g^2)/g^2/(d*(c^2*f^2+g^2)/g^2)^(1/2)*
ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2
)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g)
))+2*c^2/(c^2*f^2+g^2)*g^2*(1/4*(2*c^2*d*(x+f/g)-2*c^2*d*f/g)/c^2/d*((x+f/
g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)+1/8*(4*c^2*d^2*(
c^2*f^2+g^2)/g^2-4*c^4*d^2*f^2/g^2)/c^2/d*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c
^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2
))/(c^2*d)^(1/2)))+1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c
*x)^2*c/g^2+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/g^2/(g*x+f)*x
^3*c^4*f-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/g^2/(g*x+f)*x*c^2*f-b*(d*(c^
2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/g/(g*x+f)*x^2*c^2+b*(d*(c^2*x^2+1
))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)^(1/2)/g/(g*x+f)*x*c+b*(d*(c^2*x^2+1))^(1
/2)*arcsinh(c*x)/(c^2*x^2+1)/g^2/(g*x+f)*x*c^2*f+b*(d*(c^2*x^2+1))^(1/2)*a
rcsinh(c*x)/(c^2*x^2+1)^(1/2)/g^2/(g*x+f)*c*f-b*(d*(c^2*x^2+1))^(1/2)*arcs
inh(c*x)/(c^2*x^2+1)/g/(g*x+f)-b*(d*(c^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)^(1/
2)/g^2/(c^2*f^2+g^2)^(1/2)*arcsinh(c*x)*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-...
```

3.38.5 Fricas [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx = \int \frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{(gx+f)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="f
ricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(g^2*x^2 + 2*f*g*x + f^2
), x)
```

3.38.6 Sympy [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx=\int \frac{\sqrt{d\left(c^2x^2+1\right)}(a+b\operatorname{asinh}\left(cx\right))}{\left(f+gx\right)^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/(g*x+f)**2,x)
```

output
```
Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/(f + g*x)**2, x)
```

3.38.7 Maxima [F]

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx=\int \frac{\sqrt{c^2dx^2+d}(b\operatorname{arsinh}\left(cx\right)+a)}{\left(gx+f\right)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="m
axima")
```

output
```
-(c^2*d*f*arcsinh(c*f*x/(g*abs(x + f/g)) - 1/(c*abs(x + f/g)))/(sqrt(c^2*d
*f^2/g^2 + d)*g^3) - c*sqrt(d)*arcsinh(c*x)/g^2 + sqrt(c^2*d*x^2 + d)/(g^2
*x + f*g))*a + b*integrate(sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)
)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

3.38.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="g
iac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{(f+gx)^2}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}}{(f+g\,x)^2}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x)^2,x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x)^2, x)
```

3.39 $\int (f+gx)^3 \left(d+c^2dx^2\right)^{3/2} (a+b\,\mathbf{arcsinh}(cx))\,dx$

3.39.1 Optimal result

Integrand size = 30, antiderivative size = 918

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{3/2} (a+b\operatorname{arcsinh}(cx))\,dx = -\frac{3bdf^2gx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}} + \frac{2bdg^3x\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}} - \frac{5bcdf^3x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{3bdfg^2x^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}} - \frac{2bcdf^2gx^3\sqrt{d+c^2dx^2}}{5\sqrt{1+c^2x^2}} - \frac{bdg^3x^3\sqrt{d+c^2dx^2}}{105c\sqrt{1+c^2x^2}} - \frac{bc^3df^3x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{7bcdfg^2x^4\sqrt{d+c^2dx^2}}{32\sqrt{1+c^2x^2}} - \frac{3bc^3df^2gx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} - \frac{8bcdg^3x^5\sqrt{d+c^2dx^2}}{175\sqrt{1+c^2x^2}} - \frac{bc^3dfg^2x^6\sqrt{d+c^2dx^2}}{12\sqrt{1+c^2x^2}} - \frac{bc^3dg^3x^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} + \frac{3}{8}df^3x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{3dfg^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{16c^2} + \frac{3}{8}dfg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{4}df^3x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{1}{2}dfg^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) + \frac{3df^2g(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^2} - \frac{dg^3(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5c^4} + \frac{dg^3(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^4} + \frac{3df^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}} - \frac{3dfg^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{32bc^3\sqrt{1+c^2x^2}}$$

output
```
3/8*d*f^3*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+3/16*d*f*g^2*x*(a+b*arc
sinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2+3/8*d*f*g^2*x^3*(a+b*arcsinh(c*x))*(c^2
*d*x^2+d)^(1/2)+1/4*d*f^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(
1/2)+1/2*d*f*g^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+3/
5*d*f^2*g*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-1/5*d*g
^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4+1/7*d*g^3*(c^2
*x^2+1)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4-3/5*b*d*f^2*g*x*(c^2*
d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)+2/35*b*d*g^3*x*(c^2*d*x^2+d)^(1/2)/c^3/
(c^2*x^2+1)^(1/2)-5/16*b*c*d*f^3*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
-3/32*b*d*f*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/5*b*c*d*f^2*
g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/105*b*d*g^3*x^3*(c^2*d*x^2+d
)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/16*b*c^3*d*f^3*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*
x^2+1)^(1/2)-7/32*b*c*d*f*g^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/
25*b*c^3*d*f^2*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-8/175*b*c*d*g^3
*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/12*b*c^3*d*f*g^2*x^6*(c^2*d*x
^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/49*b*c^3*d*g^3*x^7*(c^2*d*x^2+d)^(1/2)/(c^
2*x^2+1)^(1/2)+3/16*d*f^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^
2*x^2+1)^(1/2)-3/32*d*f*g^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3
/(c^2*x^2+1)^(1/2)
```

3.39.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.58

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{3/2} (a+b\operatorname{arcsinh}(cx))\,dx = \frac{-d^2(1+c^2x^2)\left(-1680a\sqrt{1+c^2x^2}(-32g^3+c^2g(336f^2+105fgx+16g^2x^2)+4c^6x^3(35\right.}{}$$

input
```
Integrate[(f + g*x)^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(-(d^2*(1 + c^2*x^2)*(-1680*a*Sqrt[1 + c^2*x^2]*(-32*g^3 + c^2*g*(336*f^2
+ 105*f*g*x + 16*g^2*x^2) + 4*c^6*x^3*(35*f^3 + 84*f^2*g*x + 70*f*g^2*x^2
+ 20*g^3*x^3) + 2*c^4*x*(175*f^3 + 336*f^2*g*x + 245*f*g^2*x^2 + 64*g^3*x^
3)) + b*(-35*c*g^2*(245*f + 1536*g*x) + 70*c^3*(1785*f^3 + 8064*f^2*g*x +
1260*f*g^2*x^2 + 128*g^3*x^3) + 168*c^5*x^2*(1750*f^3 + 2240*f^2*g*x + 122
5*f*g^2*x^2 + 256*g^3*x^3) + 16*c^7*x^4*(3675*f^3 + 7056*f^2*g*x + 4900*f*
g^2*x^2 + 1200*g^3*x^3)))) + 88200*b*c*d^2*f*(2*c^2*f^2 - g^2)*(1 + c^2*x^
2)*ArcSinh[c*x]^2 + 176400*a*c*d^(3/2)*f*(2*c^2*f^2 - g^2)*Sqrt[1 + c^2*x^
2]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 420*b*d^
2*(1 + c^2*x^2)*ArcSinh[c*x]*(35*c*f*(16*c^2*f^2 - 3*g^2)*Sinh[2*ArcSinh[c
*x]] + 35*c*f*(2*c^2*f^2 + 3*g^2)*Sinh[4*ArcSinh[c*x]] + g*(64*(1 + c^2*x^
2)^(5/2)*(-2*g^2 + c^2*(21*f^2 + 5*g^2*x^2)) + 35*c*f*g*Sinh[6*ArcSinh[c*x
]]))))/(940800*c^4*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2])
```

3.39.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 488, normalized size of antiderivative = 0.53, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{3/2}(f+gx)^3(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \; 6260$$

$$\frac{d\sqrt{c^2dx^2+d}\int (f+gx)^3\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 6253$$

$$\frac{d\sqrt{c^2dx^2+d}\int \Big(\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))f^3+3gx\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))f^2+3g^2x^2\left(c^2x^2+1\right)^{3/}}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \; 2009$$

$$\frac{d\sqrt{c^2dx^2+d}\Big(-\frac{3fg^2(a+b\text{arcsinh}(cx))^2}{32bc^3}+\frac{1}{4}f^3x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{8}f^3x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx)}{}$$

input
```
Int[(f + g*x)^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*((-3*b*f^2*g*x)/(5*c) + (2*b*g^3*x)/(35*c^3) - (5*b
*c*f^3*x^2)/16 - (3*b*f*g^2*x^2)/(32*c) - (2*b*c*f^2*g*x^3)/5 - (b*g^3*x^3
)/(105*c) - (b*c^3*f^3*x^4)/16 - (7*b*c*f*g^2*x^4)/32 - (3*b*c^3*f^2*g*x^5
)/25 - (8*b*c*g^3*x^5)/175 - (b*c^3*f*g^2*x^6)/12 - (b*c^3*g^3*x^7)/49 + (
3*f^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/8 + (3*f*g^2*x*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/(16*c^2) + (3*f*g^2*x^3*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x]))/8 + (f^3*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4
+ (f*g^2*x^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/2 + (3*f^2*g*(1 + c
^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^2) - (g^3*(1 + c^2*x^2)^(5/2)*(a
+ b*ArcSinh[c*x]))/(5*c^4) + (g^3*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x])
)/(7*c^4) + (3*f^3*(a + b*ArcSinh[c*x])^2)/(16*b*c) - (3*f*g^2*(a + b*ArcS
inh[c*x])^2)/(32*b*c^3)))/Sqrt[1 + c^2*x^2]
```

3.39.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

3.39.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2078 vs. 2(802) = 1604.

Time = 0.88 (sec) , antiderivative size = 2079, normalized size of antiderivative = 2.26

method	result	size
default	Expression too large to display	2079
parts	Expression too large to display	2079

input
```
int((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^3*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*l
n(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)))+g^3*(1/7*x^2*
(c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(c^2*d*x^2+d)^(5/2))+3*f*g^2*(1/6*x*(
c^2*d*x^2+d)^(5/2)/c^2/d-1/6/c^2*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(
c^2*d*x^2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^
2*d)^(1/2))))+3/5*f^2*g/c^2/d*(c^2*d*x^2+d)^(5/2))+b*(3/32*(d*(c^2*x^2+1))
^(1/2)*f*arcsinh(c*x)^2*(2*c^2*f^2-g^2)*d/(c^2*x^2+1)^(1/2)/c^3+1/6272*(d*
(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+11
2*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^
2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*g^3*(-1+7*arcsinh(c*x))*d/c^4/(c^2*x^2+1)
+1/768*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c
^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*x^2*(c^2*x^2+1)^(1/2
)+6*c*x+(c^2*x^2+1)^(1/2))*f*g^2*(-1+6*arcsinh(c*x))*d/c^3/(c^2*x^2+1)+1/3
200*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*
x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*g*(
60*arcsinh(c*x)*c^2*f^2-12*c^2*f^2+5*arcsinh(c*x)*g^2-g^2)*d/c^4/(c^2*x^2+
1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c
^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*f*(8*arcsinh(c
*x)*c^2*f^2-2*c^2*f^2+12*arcsinh(c*x)*g^2-3*g^2)*d/c^3/(c^2*x^2+1)+1/384*(
d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3...
```

3.39.5 Fricas [F]

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{3/2} (a+b\operatorname{arcsinh}(cx))\,dx = \int \left(c^2dx^2+d\right)^{\frac{3}{2}}(gx+f)^3(b\operatorname{arsinh}(cx)+a)\,dx$$

input
```
integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="f
ricas")
```

output
```
integral((a*c^2*d*g^3*x^5 + 3*a*c^2*d*f*g^2*x^4 + 3*a*d*f^2*g*x + a*d*f^3
+ (3*a*c^2*d*f^2*g + a*d*g^3)*x^3 + (a*c^2*d*f^3 + 3*a*d*f*g^2)*x^2 + (b*c
^2*d*g^3*x^5 + 3*b*c^2*d*f*g^2*x^4 + 3*b*d*f^2*g*x + b*d*f^3 + (3*b*c^2*d*
f^2*g + b*d*g^3)*x^3 + (b*c^2*d*f^3 + 3*b*d*f*g^2)*x^2)*arcsinh(c*x))*sqrt
(c^2*d*x^2 + d), x)
```

3.39.6 Sympy [F]

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{3/2} (a+b\operatorname{arcsinh}(cx))\,dx = \int \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}} (a+b\operatorname{asinh}(cx))\,(f+gx)^3\,dx$$

input
```
integrate((g*x+f)**3*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))*(f + g*x)**3, x)
```

3.39.7 Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{3/2} (a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.39.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3 (d + c^2dx^2)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int (f+g\,x)^3\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

3.40 $\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx$

3.40.1 Optimal result

Integrand size = 30, antiderivative size = 651

$$\begin{aligned}\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx &= -\frac{2bdfgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}} \\ &- \frac{5bcdf^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{bdg^2x^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}} - \frac{4bcdfgx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}} \\ &- \frac{bc^3df^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{7bcdg^2x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{2bc^3dfgx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} \\ &- \frac{bc^3dg^2x^6\sqrt{d+c^2dx^2}}{36\sqrt{1+c^2x^2}} + \frac{3}{8}df^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\ &+ \frac{dg^2x\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{16c^2} + \frac{1}{8}dg^2x^3\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\ &+ \frac{1}{4}df^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\ &+ \frac{1}{6}dg^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx)) \\ &+ \frac{2dfg\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{5c^2} \\ &+ \frac{3df^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}} - \frac{dg^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{32bc^3\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
3/8*d*f^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/16*d*g^2*x*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2+1/8*d*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x
^2+d)^(1/2)+1/4*d*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)
+1/6*d*g^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+2/5*d*f*
g*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-2/5*b*d*f*g*x*(
c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-5/16*b*c*d*f^2*x^2*(c^2*d*x^2+d)^(1
/2)/(c^2*x^2+1)^(1/2)-1/32*b*d*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(
1/2)-4/15*b*c*d*f*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*c^3*d
*f^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-7/96*b*c*d*g^2*x^4*(c^2*d*x
^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/25*b*c^3*d*f*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^
2*x^2+1)^(1/2)-1/36*b*c^3*d*g^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+
3/16*d*f^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-
1/32*d*g^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2
)
```

3.40.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 416, normalized size of antiderivative = 0.64

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \frac{-d^2(1+c^2x^2)\left(b(-175g^2+90c^2(85f^2+256fgx+20g^2x^2)+120c^4x^2(150f^2+128fg\right.}{}$$

input
```
Integrate[(f + g*x)^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(-(d^2*(1 + c^2*x^2)*(b*(-175*g^2 + 90*c^2*(85*f^2 + 256*f*g*x + 20*g^2*x^
2) + 120*c^4*x^2*(150*f^2 + 128*f*g*x + 35*g^2*x^2) + 16*c^6*x^4*(225*f^2
+ 288*f*g*x + 100*g^2*x^2)) - 240*a*c*Sqrt[1 + c^2*x^2]*(96*f*g*(1 + c^2*x
^2)^2 + 30*c^2*f^2*x*(5 + 2*c^2*x^2) + 5*g^2*x*(3 + 14*c^2*x^2 + 8*c^4*x^4
)))) + 1800*b*d^2*(6*c^2*f^2 - g^2)*(1 + c^2*x^2)*ArcSinh[c*x]^2 + 3600*a*
d^(3/2)*(6*c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*Log[c*d*x
+ Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 60*b*d^2*(1 + c^2*x^2)*ArcSinh[c*x]*(15*(
16*c^2*f^2 - g^2)*Sinh[2*ArcSinh[c*x]] + 15*(2*c^2*f^2 + g^2)*Sinh[4*ArcSi
nh[c*x]] + g*(384*c*f*(1 + c^2*x^2)^(5/2) + 5*g*Sinh[6*ArcSinh[c*x]])))/(5
7600*c^3*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2])
```

3.40.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.55, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{3/2}(f+gx)^2(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6260}$$

$$\frac{d\sqrt{c^2dx^2+d}\int (f+gx)^2\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6253}$$

$$\frac{d\sqrt{c^2dx^2+d}\int \left(\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))f^2+2gx\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))f+g^2x^2\left(c^2x^2+1\right)^{3/2}\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\sqrt{c^2dx^2+d}\left(-\frac{g^2(a+b\text{arcsinh}(cx))^2}{32bc^3}+\frac{1}{4}f^2x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{8}f^2x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))\right.}{}$$

input
```
Int[(f + g*x)^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*((-2*b*f*g*x)/(5*c) - (5*b*c*f^2*x^2)/16 - (b*g^2*x
^2)/(32*c) - (4*b*c*f*g*x^3)/15 - (b*c^3*f^2*x^4)/16 - (7*b*c*g^2*x^4)/96
- (2*b*c^3*f*g*x^5)/25 - (b*c^3*g^2*x^6)/36 + (3*f^2*x*Sqrt[1 + c^2*x^2]*(
a + b*ArcSinh[c*x]))/8 + (g^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(1
6*c^2) + (g^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/8 + (f^2*x*(1 +
c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (g^2*x^3*(1 + c^2*x^2)^(3/2)*(a +
 b*ArcSinh[c*x]))/6 + (2*f*g*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*
c^2) + (3*f^2*(a + b*ArcSinh[c*x])^2)/(16*b*c) - (g^2*(a + b*ArcSinh[c*x])
^2)/(32*b*c^3)))/Sqrt[1 + c^2*x^2]
```

3.40.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

3.40.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1567 vs. $2(567) = 1134$.

Time = 0.82 (sec) , antiderivative size = 1568, normalized size of antiderivative = 2.41

method	result	size
default	Expression too large to display	1568
parts	Expression too large to display	1568

input
```
int((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^2*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*l
n(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)))+g^2*(1/6*x*(c
^2*d*x^2+d)^(5/2)/c^2/d-1/6/c^2*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(c
^2*d*x^2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2
*d)^(1/2))))+2/5*f*g/c^2/d*(c^2*d*x^2+d)^(5/2))+b*(1/32*(d*(c^2*x^2+1))^(1
/2)*arcsinh(c*x)^2*(6*c^2*f^2-g^2)*d/(c^2*x^2+1)^(1/2)/c^3+1/2304*(d*(c^2*
x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c^4*x
^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^
2+1)^(1/2))*g^2*(-1+6*arcsinh(c*x))*d/c^3/(c^2*x^2+1)+1/400*(d*(c^2*x^2+1)
)^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^
2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(-1+5*arcsinh(c*x
))*d/c^2/(c^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2
*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/
2))*(8*arcsinh(c*x)*c^2*f^2-2*c^2*f^2+4*arcsinh(c*x)*g^2-g^2)*d/c^3/(c^2*x
^2+1)+1/48*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*
c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(-1+3*arcsinh(c*x))*d/c^2/(c^2*x^2+
1)+1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*
x+(c^2*x^2+1)^(1/2))*(32*arcsinh(c*x)*c^2*f^2-16*c^2*f^2-2*arcsinh(c*x)*g^
2+g^2)*d/c^3/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1
)^(1/2)+1)*f*g*(-1+arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^...
```

3.40.5 Fricas [F]

$$\int (f+gx)^2 \left(d + c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int \left(c^2dx^2+d\right)^{\frac{3}{2}}(gx+f)^2(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="f
ricas")
```

output
```
integral((a*c^2*d*g^2*x^4 + 2*a*c^2*d*f*g*x^3 + 2*a*d*f*g*x + a*d*f^2 + (a
*c^2*d*f^2 + a*d*g^2)*x^2 + (b*c^2*d*g^2*x^4 + 2*b*c^2*d*f*g*x^3 + 2*b*d*f
*g*x + b*d*f^2 + (b*c^2*d*f^2 + b*d*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2
 + d), x)
```

3.40.6 Sympy [F]

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}} (a+b\,\text{asinh}\,(cx))\,(f+gx)^2\,dx$$

input
```
integrate((g*x+f)**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))*(f + g*x)**2, x)
```

3.40.7 Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.40.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} (a+b\text{arcsinh}(cx))\,dx = \int (f+g\,x)^2\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

3.41 $\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx$

3.41.1 Optimal result

Integrand size = 28, antiderivative size = 353

$$\begin{aligned}\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx &= -\frac{bdgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}\\ &-\frac{5bcdfx^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{2bcdgx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{bc^3dfx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}\\ &-\frac{bc^3dgx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}+\frac{3}{8}dfx\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\\ &+\frac{1}{4}dfx(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))\\ &+\frac{dg(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))}{5c^2}+\frac{3df\sqrt{d+c^2dx^2}(a+b\text{arcsinh}(cx))^2}{16bc\sqrt{1+c^2x^2}}\end{aligned}$$

output
```
3/8*d*f*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/4*d*f*x*(c^2*x^2+1)*(a+
b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/5*d*g*(c^2*x^2+1)^2*(a+b*arcsinh(c*x
))*(c^2*d*x^2+d)^(1/2)/c^2-1/5*b*d*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(
1/2)-5/16*b*c*d*f*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/15*b*c*d*g*x
^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*c^3*d*f*x^4*(c^2*d*x^2+d)^
(1/2)/(c^2*x^2+1)^(1/2)-1/25*b*c^3*d*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)
^(1/2)+3/16*d*f*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(
1/2)
```

3.41.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.82

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx = \frac{-d^2(1+c^2x^2)\left(-240a\sqrt{1+c^2x^2}\left(8g(1+c^2x^2)^2+5c^2fx(5+2c^2x^2)\right)+bc(128gx(15+\right.}{}$$

input
```
Integrate[(f + g*x)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(-(d^2*(1 + c^2*x^2)*(-240*a*Sqrt[1 + c^2*x^2]*(8*g*(1 + c^2*x^2)^2 + 5*c^
2*f*x*(5 + 2*c^2*x^2)) + b*c*(128*g*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) + 75*f
*(17 + 40*c^2*x^2 + 8*c^4*x^4)))) + 1800*b*c*d^2*f*(1 + c^2*x^2)*ArcSinh[c
*x]^2 + 3600*a*c*d^(3/2)*f*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*Log[c*d*x
 + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 60*b*d^2*(1 + c^2*x^2)*ArcSinh[c*x]*(32*
g*(1 + c^2*x^2)^(5/2) + 40*c*f*Sinh[2*ArcSinh[c*x]] + 5*c*f*Sinh[4*ArcSinh
[c*x]]))/(9600*c^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2])
```

3.41.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.53, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{3/2}(f+gx)(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{d\sqrt{c^2dx^2+d}\int (f+gx)\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d\sqrt{c^2dx^2+d}\int \left(f(a+b\text{arcsinh}(cx))\left(c^2x^2+1\right)^{3/2}+gx(a+b\text{arcsinh}(cx))\left(c^2x^2+1\right)^{3/2}\right)dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\sqrt{c^2dx^2+d}\left(\frac{1}{4}fx\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{3}{8}fx\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))+\frac{g\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(\ldots}{5c^2}\right.}{\sqrt{c^2x^2+1}}$$

input
```
Int[(f + g*x)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*(-1/5*(b*g*x)/c - (5*b*c*f*x^2)/16 - (2*b*c*g*x^3)/
15 - (b*c^3*f*x^4)/16 - (b*c^3*g*x^5)/25 + (3*f*x*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x]))/8 + (f*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (g*
(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^2) + (3*f*(a + b*ArcSinh[c*
x])^2)/(16*b*c)))/Sqrt[1 + c^2*x^2]
```

3.41.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

3.41.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. $2(305) = 610$.

Time = 0.83 (sec) , antiderivative size = 1065, normalized size of antiderivative = 3.02

method	result	size
default	Expression too large to display	1065
parts	Expression too large to display	1065

input
```
int((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE
)
```

output
```
1/4*a*f*x*(c^2*d*x^2+d)^(3/2)+3/8*a*f*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a*f*d^2*
ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/5*a*g/c^2/d*
(c^2*d*x^2+d)^(5/2)+b*(3/16*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*f*ar
csinh(c*x)^2*d+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2
+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^
2+1)^(1/2)+1)*g*(-1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1)
)^(1/2)*(8*c^5*x^5+8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x
^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*f*(-1+4*arcsinh(c*x))*d/c/(c^2*x^2+1)
+1/96*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x
^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*g*(-1+3*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/16
*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x
^2+1)^(1/2))*f*(-1+2*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1
/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*g*(-1+arcsinh(c*x))*d/c^2/(c^2*x^2+1
)+1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*g*(arcsinh(
c*x)+1)*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*
(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*f*(1+2*arcsinh(c*x))*d/c/(c^2*x
^2+1)+1/96*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*
c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*g*(3*arcsinh(c*x)+1)*d/c^2/(c^2*x^2+1)+
1/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*c^4*x^4*(c^2*x^2+1)^(1/2)+12*c^3*
x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*f*(1+4*arcsinh...
```

3.41.5 Fricas [F]

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(gx+f)(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fri
cas")
```

output
```
integral((a*c^2*d*g*x^3 + a*c^2*d*f*x^2 + a*d*g*x + a*d*f + (b*c^2*d*g*x^3
 + b*c^2*d*f*x^2 + b*d*g*x + b*d*f)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

3.41.6 Sympy [F]

$$\int (f+gx)\,(d+c^2dx^2)^{3/2}\,(a+b\text{arcsinh}(cx))\,dx=\int \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}(a+b\,\text{asinh}\,(cx))\,(f+gx)\,dx$$

input
```
integrate((g*x+f)*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))*(f + g*x), x)
```

3.41.7 Maxima [F(-2)]

Exception generated.

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="max
ima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.41.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="gia
c")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(d+c^2dx^2\right)^{3/2}(a+b\operatorname{arcsinh}(cx))\,dx = \int (f+g\,x)\,(a+b\,\operatorname{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

input
```
int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

output
```
int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

3.42 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx$

3.42.1 Optimal result

Integrand size = 30, antiderivative size = 984

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{arcsinh}(cx))}{f+gx}\,dx = \frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} - \frac{bcd(c^2f^2+g^2)x\sqrt{d+c^2dx^2}}{g^3\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}} + \frac{bd(c^2f^2+g^2)\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^3} - \frac{c^2dfx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^2} + \frac{d(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g} - \frac{cdf\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^2\sqrt{1+c^2x^2}} - \frac{cd(c^2f^2+g^2)x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^3\sqrt{1+c^2x^2}} - \frac{d(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^4(f+gx)\sqrt{1+c^2x^2}} + \frac{d(c^2f^2+g^2)\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^2(f+gx)} - \frac{ad(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^4\sqrt{1+c^2x^2}} + \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}} - \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}} + \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}} - \frac{bd(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^4\sqrt{1+c^2x^2}}$$

output
```
a*d*(c^2*f^2+g^2)*(c^2*d*x^2+d)^(1/2)/g^3+b*d*(c^2*f^2+g^2)*arcsinh(c*x)*(
c^2*d*x^2+d)^(1/2)/g^3-1/2*c^2*d*f*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2
)/g^2+1/3*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g-1/3*b*c*d
*x*(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)-b*c*d*(c^2*f^2+g^2)*x*(c^2*d*x^
2+d)^(1/2)/g^3/(c^2*x^2+1)^(1/2)+1/4*b*c^3*d*f*x^2*(c^2*d*x^2+d)^(1/2)/g^2
/(c^2*x^2+1)^(1/2)-1/9*b*c^3*d*x^3*(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)
-1/4*c*d*f*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/g^2/(c^2*x^2+1)^(1/2
)-1/2*c*d*(c^2*f^2+g^2)*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/g^3/(
c^2*x^2+1)^(1/2)-1/2*d*(c^2*f^2+g^2)^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^
(1/2)/b/c/g^4/(g*x+f)/(c^2*x^2+1)^(1/2)-a*d*(c^2*f^2+g^2)^(3/2)*arctanh((-
c^2*f*x+g)/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/g^4/
(c^2*x^2+1)^(1/2)+b*d*(c^2*f^2+g^2)^(3/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+
1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^4/(c^2*x^2+1)
^(1/2)-b*d*(c^2*f^2+g^2)^(3/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g
/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^4/(c^2*x^2+1)^(1/2)+b*d*
(c^2*f^2+g^2)^(3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2
)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^4/(c^2*x^2+1)^(1/2)-b*d*(c^2*f^2+g^2)^(3/2
)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*d*x
^2+d)^(1/2)/g^4/(c^2*x^2+1)^(1/2)+1/2*d*(c^2*f^2+g^2)*(a+b*arcsinh(c*x))^2
*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(1/2)/b/c/g^2/(g*x+f)
```

3.42.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.24 (sec) , antiderivative size = 2869, normalized size of antiderivative = 2.92

$$\int \frac{(d + c^2dx^2)^{3/2} (a + b\text{arcsinh}(cx))}{f + gx} \, dx = \text{Result too large to show}$$

input
```
Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

output
```
(a*d*Sqrt[d + c^2*d*x^2]*(8*g^2 + c^2*(6*f^2 - 3*f*g*x + 2*g^2*x^2)))/(6*g
^3) + (a*d^(3/2)*(c^2*f^2 + g^2)^(3/2)*Log[f + g*x])/g^4 - (a*c*d^(3/2)*f*
(2*c^2*f^2 + 3*g^2)*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(2*g^4) - (a
*d^(3/2)*(c^2*f^2 + g^2)^(3/2)*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2
+ g^2]*Sqrt[d + c^2*d*x^2]])/g^4 + (b*d*Sqrt[d + c^2*d*x^2]*((-2*c*g*x)/Sq
rt[1 + c^2*x^2] + 2*g*ArcSinh[c*x] - (c*f*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2
] + (2*((-I)*c*f + g)*(I*c*f + g)*(((-I)*Pi*ArcTanh[(-g + c*f*Tanh[ArcSinh
[c*x]/2])/Sqrt[c^2*f^2 + g^2]])/Sqrt[c^2*f^2 + g^2] - (2*ArcCos[((-I)*c*f)
/g]*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2)
 - g^2]] + (Pi - (2*I)*ArcSinh[c*x])*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*
ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (ArcCos[((-I)*c*f)/g] - (2*I)*
ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g
^2]] - (2*I)*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-
(c^2*f^2) - g^2]])*Log[((1/2 - I/2)*Sqrt[-(c^2*f^2) - g^2])/(E^(ArcSinh[c*
x]/2)*Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] + (ArcCos[((-I)*c*f)/g] + (2*I)*(Ar
cTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2
]] + ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2
) - g^2]]))*Log[((1/2 + I/2)*E^(ArcSinh[c*x]/2)*Sqrt[-(c^2*f^2) - g^2])/(S
qrt[(-I)*g]*Sqrt[c*(f + g*x)])] - (ArcCos[((-I)*c*f)/g] + (2*I)*ArcTanh[((
c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*L...
```

3.42.3 Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 641, normalized size of antiderivative = 0.65, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6260, 6255, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^{3/2}(a+b\text{arcsinh}(cx))}{f+gx}dx$$

$$\Big\downarrow \text{ 6260}$$

$$\frac{d\sqrt{c^2dx^2+d}\int \frac{\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))}{f+gx}dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6255}$$

$$\frac{d\sqrt{c^2dx^2+d}\int\left(\frac{x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))c^2}{g}-\frac{f\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))c^2}{g^2}+\frac{(c^2f^2+g^2)\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{g^2(f+gx)}\right)dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$d\sqrt{c^2dx^2+d}\left(\frac{(c^2x^2+1)(c^2f^2+g^2)(a+b\text{arcsinh}(cx))^2}{2bcg^2(f+gx)}-\frac{(c^2f^2+g^2)^2(a+b\text{arcsinh}(cx))^2}{2bcg^4(f+gx)}-\frac{cx(c^2f^2+g^2)(a+b\text{arcsinh}(cx))^2}{2bg^3}-\frac{c^2fx}{}\right.$$

input
```
Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

output
```
(d*Sqrt[d + c^2*d*x^2]*(-1/3*(b*c*x)/g - (b*c*(c^2*f^2 + g^2)*x)/g^3 + (b*
c^3*f*x^2)/(4*g^2) - (b*c^3*x^3)/(9*g) + (a*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x
^2])/g^3 + (b*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/g^3 - (c^2*f
*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*g^2) + ((1 + c^2*x^2)^(3/2)*
(a + b*ArcSinh[c*x]))/(3*g) - (c*f*(a + b*ArcSinh[c*x])^2)/(4*b*g^2) - (c*
(c^2*f^2 + g^2)*x*(a + b*ArcSinh[c*x])^2)/(2*b*g^3) - ((c^2*f^2 + g^2)^2*(
a + b*ArcSinh[c*x])^2)/(2*b*c*g^4*(f + g*x)) + ((c^2*f^2 + g^2)*(1 + c^2*x
^2)*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^2*(f + g*x)) - (a*(c^2*f^2 + g^2)^(3/
2)*ArcTanh[(g - c^2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])])/g^4 + (
b*(c^2*f^2 + g^2)^(3/2)*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqr
t[c^2*f^2 + g^2])])/g^4 - (b*(c^2*f^2 + g^2)^(3/2)*ArcSinh[c*x]*Log[1 + (E
^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/g^4 + (b*(c^2*f^2 + g^2)^(3
/2)*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/g^4 - (
b*(c^2*f^2 + g^2)^(3/2)*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^
2 + g^2]))])/g^4))/Sqrt[1 + c^2*x^2]
```

3.42.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6255
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(
a + b*ArcSinh[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IGtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

3.42.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 1557, normalized size of antiderivative = 1.58

method	result	size
default	Expression too large to display	1557
parts	Expression too large to display	1557

input
```
int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x,method=_RETURNVERBOSE
)
```

output
```
a/g*(1/3*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(3/2)-c
^2*d*f/g*(1/4*(2*c^2*d*(x+f/g)-2*c^2*d*f/g)/c^2/d*((x+f/g)^2*c^2*d-2*c^2*d
*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)+1/8*(4*c^2*d^2*(c^2*f^2+g^2)/g^2-4
*c^4*d^2*f^2/g^2)/c^2/d*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1/2)+((x+f/
g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2))+
d*(c^2*f^2+g^2)/g^2*(((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/
g^2)^(1/2)-c^2*d*f/g*ln((-c^2*d*f/g+c^2*d*(x+f/g))/(c^2*d)^(1/2)+((x+f/g)^
2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)-d*(c
^2*f^2+g^2)/g^2/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^
2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g
*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))))+b*(c^2*f^2+g^2)^(3/2)*d*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^4*dilog((-(c*x+(c^2*x^2+1)^(1/2))*
g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-b*(c^2*f^2+g^2)^(3/
2)*d*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^4*dilog(((c*x+(c^2*x^2+1)^(
1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))-1/2*b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*f^3*arcsinh(c*x)^2*c^3*d/g^4-3/4*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*f*arcsinh(c*x)^2*c*d/g^2+1/3*b*(d*(c^2*x^
2+1))^(1/2)*d/(c^2*x^2+1)/g*arcsinh(c*x)*x^4*c^4-1/9*b*(d*(c^2*x^2+1))^(1/
2)*d/(c^2*x^2+1)^(1/2)/g*x^3*c^3+5/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)
/g*arcsinh(c*x)*x^2*c^2-4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)...
```

3.42.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx=\int \frac{(c^2dx^2+d)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)}{gx+f}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="fri
cas")
```

output
```
integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d
*x^2 + d)/(g*x + f), x)
```

3.42.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{3}{2}}(a+b\operatorname{asinh}(cx))}{f+gx}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/(g*x+f),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/(f + g*x), x)
```

3.42.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{3/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="max
ima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.42.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d + c^2dx^2)^{3/2}\,(a + b\text{arcsinh}(cx))}{f + gx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d + c^2dx^2)^{3/2}\,(a + b\text{arcsinh}(cx))}{f + gx}\,dx = \int \frac{(a + b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2 + d\right)^{3/2}}{f + g\,x}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/(f + g*x),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/(f + g*x), x)
```

3.43 $\int (f+gx)^3 (d+c^2dx^2)^{5/2} (a+b\text{arcsinh}(cx)) dx$

3.43.1 Optimal result

Integrand size = 30, antiderivative size = 1228

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\operatorname{arcsinh}(cx))\,dx = -\frac{3bd^2f^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}$$
$$+\frac{2bd^2g^3x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{25bcd^2f^3x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{15bd^2fg^2x^2\sqrt{d+c^2dx^2}}{256c\sqrt{1+c^2x^2}}$$
$$-\frac{3bcd^2f^2gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}}-\frac{bd^2g^3x^3\sqrt{d+c^2dx^2}}{189c\sqrt{1+c^2x^2}}-\frac{5bc^3d^2f^3x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}$$
$$-\frac{59bcd^2fg^2x^4\sqrt{d+c^2dx^2}}{256\sqrt{1+c^2x^2}}-\frac{9bc^3d^2f^2gx^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}}-\frac{bcd^2g^3x^5\sqrt{d+c^2dx^2}}{21\sqrt{1+c^2x^2}}$$
$$-\frac{17bc^3d^2fg^2x^6\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{3bc^5d^2f^2gx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}}-\frac{19bc^3d^2g^3x^7\sqrt{d+c^2dx^2}}{441\sqrt{1+c^2x^2}}$$
$$-\frac{3bc^5d^2fg^2x^8\sqrt{d+c^2dx^2}}{64\sqrt{1+c^2x^2}}-\frac{bc^5d^2g^3x^9\sqrt{d+c^2dx^2}}{81\sqrt{1+c^2x^2}}-\frac{bd^2f^3(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c}$$
$$+\frac{5}{16}d^2f^3x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))+\frac{15d^2fg^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{128c^2}$$
$$+\frac{15}{64}d^2fg^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))+\frac{5}{24}d^2f^3x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))$$
$$+\frac{5}{16}d^2fg^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))$$
$$+\frac{1}{6}d^2f^3x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))$$
$$+\frac{3}{8}d^2fg^2x^3\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))$$
$$+\frac{3d^2f^2g(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^2}$$
$$-\frac{d^2g^3(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^4}$$
$$+\frac{d^2g^3(1+c^2x^2)^4\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{9c^4}$$
$$+\frac{5d^2f^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{32bc\sqrt{1+c^2x^2}}-\frac{15d^2fg^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{256bc^3\sqrt{1+c^2x^2}}$$

output
```
15/128*d^2*f*g^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2+5/16*d^2*f*g
^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+3/8*d^2*f*g^2*x^
3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+3/7*d^2*f^2*g*(c^2*
x^2+1)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2+2/63*b*d^2*g^3*x*(c^2*
d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)-25/96*b*c*d^2*f^3*x^2*(c^2*d*x^2+d)^(
1/2)/(c^2*x^2+1)^(1/2)-1/189*b*d^2*g^3*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+
1)^(1/2)-5/96*b*c^3*d^2*f^3*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/21
*b*c*d^2*g^3*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-19/441*b*c^3*d^2*g^
3*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/81*b*c^5*d^2*g^3*x^9*(c^2*d*
x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/32*d^2*f^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^
2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)+5/16*d^2*f^3*x*(a+b*arcsinh(c*x))*(c^2*d*
x^2+d)^(1/2)+5/24*d^2*f^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(
1/2)+1/6*d^2*f^3*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-1/
7*d^2*g^3*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4+1/9*d^2
*g^3*(c^2*x^2+1)^4*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4-1/36*b*d^2*f
^3*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/c+15/64*d^2*f*g^2*x^3*(a+b*arcsin
h(c*x))*(c^2*d*x^2+d)^(1/2)-3/7*b*d^2*f^2*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x
^2+1)^(1/2)-15/256*b*d^2*f*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)
-3/7*b*c*d^2*f^2*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-59/256*b*c*d^
2*f*g^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-9/35*b*c^3*d^2*f^2*g*...
```

3.43.2 Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 810, normalized size of antiderivative = 0.66

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \frac{-d^3(1+c^2x^2)\left(-20160a\sqrt{1+c^2x^2}(-256g^3+c^2g(3456f^2+945fgx+128g^2x^2)+16c^{8}\right.}{}$$

input
```
Integrate[(f + g*x)^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(-(d^3*(1 + c^2*x^2)*(-20160*a*Sqrt[1 + c^2*x^2]*(-256*g^3 + c^2*g*(3456*f
^2 + 945*f*g*x + 128*g^2*x^2) + 16*c^8*x^5*(84*f^3 + 216*f^2*g*x + 189*f*g
^2*x^2 + 56*g^3*x^3) + 8*c^6*x^3*(546*f^3 + 1296*f^2*g*x + 1071*f*g^2*x^2
+ 304*g^3*x^3) + 6*c^4*x*(924*f^3 + 1728*f^2*g*x + 1239*f*g^2*x^2 + 320*g^
3*x^3)) + b*(-315*c*g^2*(7539*f + 16384*g*x) + 30240*c^5*x^2*(1848*f^3 + 2
304*f^2*g*x + 1239*f*g^2*x^2 + 256*g^3*x^3) + 3360*c^3*(6279*f^3 + 20736*f
^2*g*x + 2835*f*g^2*x^2 + 256*g^3*x^3) + 2304*c^7*x^4*(9555*f^3 + 18144*f^
2*g*x + 12495*f*g^2*x^2 + 3040*g^3*x^3) + 640*c^9*x^6*(7056*f^3 + 15552*f^
2*g*x + 11907*f*g^2*x^2 + 3136*g^3*x^3)))) + 3175200*b*c*d^3*f*(8*c^2*f^2
- 3*g^2)*(1 + c^2*x^2)*ArcSinh[c*x]^2 + 6350400*a*c*d^(5/2)*f*(8*c^2*f^2 -
 3*g^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d +
 c^2*d*x^2]] + 2520*b*d^3*(1 + c^2*x^2)*ArcSinh[c*x]*(27648*c^2*f^2*g*Sqrt
[1 + c^2*x^2] - 2048*g^3*Sqrt[1 + c^2*x^2] + 82944*c^4*f^2*g*x^2*Sqrt[1 +
c^2*x^2] + 1024*c^2*g^3*x^2*Sqrt[1 + c^2*x^2] + 82944*c^6*f^2*g*x^4*Sqrt[1
 + c^2*x^2] + 15360*c^4*g^3*x^4*Sqrt[1 + c^2*x^2] + 27648*c^8*f^2*g*x^6*Sq
rt[1 + c^2*x^2] + 19456*c^6*g^3*x^6*Sqrt[1 + c^2*x^2] + 7168*c^8*g^3*x^8*S
qrt[1 + c^2*x^2] + 3024*c*f*(5*c^2*f^2 - g^2)*Sinh[2*ArcSinh[c*x]] + 1512*
c*f*(2*c^2*f^2 + g^2)*Sinh[4*ArcSinh[c*x]] + 336*c^3*f^3*Sinh[6*ArcSinh[c*
x]] + 1008*c*f*g^2*Sinh[6*ArcSinh[c*x]] + 189*c*f*g^2*Sinh[8*ArcSinh[c*x]]
))/(162570240*c^4*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2])
```

3.43.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 617, normalized size of antiderivative = 0.50, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{5/2}(f+gx)^3(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int (f+gx)^3\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int \left(\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))f^3+3gx\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))f^2+3g^2x^2\left(c^2x^2+1\right)^{5/2}\right.}{\sqrt{c^2x^2+1}}$$

$\downarrow$ 2009

$$d^2\sqrt{c^2dx^2+d}\left(-\frac{15fg^2(a+b\text{arcsinh}(cx))^2}{256bc^3}+\frac{1}{6}f^3x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{24}f^3x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsin}\right.$$

input
```
Int[(f + g*x)^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*((-3*b*f^2*g*x)/(7*c) + (2*b*g^3*x)/(63*c^3) - (2
5*b*c*f^3*x^2)/96 - (15*b*f*g^2*x^2)/(256*c) - (3*b*c*f^2*g*x^3)/7 - (b*g^
3*x^3)/(189*c) - (5*b*c^3*f^3*x^4)/96 - (59*b*c*f*g^2*x^4)/256 - (9*b*c^3*
f^2*g*x^5)/35 - (b*c*g^3*x^5)/21 - (17*b*c^3*f*g^2*x^6)/96 - (3*b*c^5*f^2*
g*x^7)/49 - (19*b*c^3*g^3*x^7)/441 - (3*b*c^5*f*g^2*x^8)/64 - (b*c^5*g^3*x
^9)/81 - (b*f^3*(1 + c^2*x^2)^3)/(36*c) + (5*f^3*x*Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x]))/16 + (15*f*g^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
(128*c^2) + (15*f*g^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/64 + (5*
f^3*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/24 + (5*f*g^2*x^3*(1 + c^2
*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/16 + (f^3*x*(1 + c^2*x^2)^(5/2)*(a + b*A
rcSinh[c*x]))/6 + (3*f*g^2*x^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/8
 + (3*f^2*g*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^2) - (g^3*(1 +
c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4) + (g^3*(1 + c^2*x^2)^(9/2)*(a
 + b*ArcSinh[c*x]))/(9*c^4) + (5*f^3*(a + b*ArcSinh[c*x])^2)/(32*b*c) - (1
5*f*g^2*(a + b*ArcSinh[c*x])^2)/(256*b*c^3)))/Sqrt[1 + c^2*x^2]
```

3.43.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

3.43.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2883 vs. $2(1080) = 2160$.

Time = 1.05 (sec) , antiderivative size = 2884, normalized size of antiderivative = 2.35

method	result	size
default	Expression too large to display	2884
parts	Expression too large to display	2884

input
```
int((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^3*(1/6*x*(c^2*d*x^2+d)^(5/2)+5/6*d*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(
1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/
2))/(c^2*d)^(1/2))))+g^3*(1/9*x^2*(c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(c^
2*d*x^2+d)^(7/2))+3*f*g^2*(1/8*x*(c^2*d*x^2+d)^(7/2)/c^2/d-1/8/c^2*(1/6*x*
(c^2*d*x^2+d)^(5/2)+5/6*d*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(c^2*d*x
^2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1
/2)))))+3/7*f^2*g*(c^2*d*x^2+d)^(7/2)/c^2/d)+b*(5/256*(d*(c^2*x^2+1))^(1/2
)*f*arcsinh(c*x)^2*(8*c^2*f^2-3*g^2)*d^2/(c^2*x^2+1)^(1/2)/c^3+1/41472*(d*
(c^2*x^2+1))^(1/2)*(256*c^10*x^10+256*c^9*x^9*(c^2*x^2+1)^(1/2)+704*c^8*x^
8+576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6+432*c^5*x^5*(c^2*x^2+1)^(1/2)+
280*c^4*x^4+120*c^3*x^3*(c^2*x^2+1)^(1/2)+41*c^2*x^2+9*c*x*(c^2*x^2+1)^(1/
2)+1)*g^3*(-1+9*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+3/16384*(d*(c^2*x^2+1))^
(1/2)*(128*c^9*x^9+128*c^8*x^8*(c^2*x^2+1)^(1/2)+320*c^7*x^7+256*c^6*x^6*(
c^2*x^2+1)^(1/2)+272*c^5*x^5+160*c^4*x^4*(c^2*x^2+1)^(1/2)+88*c^3*x^3+32*c
^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x+(c^2*x^2+1)^(1/2))*f*g^2*(-1+8*arcsinh(c*x)
)*d^2/c^3/(c^2*x^2+1)+3/25088*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7
*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+5
6*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*g*(28*ar
csinh(c*x)*c^2*f^2-4*c^2*f^2+7*arcsinh(c*x)*g^2-g^2)*d^2/c^4/(c^2*x^2+1)+1
/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(c^2*x^2+1)^(1/2)+64...
```

3.43.5 Fricas [F]

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\operatorname{arcsinh}(cx))\,dx = \int \left(c^2dx^2+d\right)^{\frac{5}{2}}(gx+f)^3(b\operatorname{arsinh}(cx)+a)\,dx$$

input
```
integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="f
ricas")
```

output
```
integral((a*c^4*d^2*g^3*x^7 + 3*a*c^4*d^2*f*g^2*x^6 + 3*a*d^2*f^2*g*x + a*
d^2*f^3 + (3*a*c^4*d^2*f^2*g + 2*a*c^2*d^2*g^3)*x^5 + (a*c^4*d^2*f^3 + 6*a
*c^2*d^2*f*g^2)*x^4 + (6*a*c^2*d^2*f^2*g + a*d^2*g^3)*x^3 + (2*a*c^2*d^2*f
^3 + 3*a*d^2*f*g^2)*x^2 + (b*c^4*d^2*g^3*x^7 + 3*b*c^4*d^2*f*g^2*x^6 + 3*b
*d^2*f^2*g*x + b*d^2*f^3 + (3*b*c^4*d^2*f^2*g + 2*b*c^2*d^2*g^3)*x^5 + (b*
c^4*d^2*f^3 + 6*b*c^2*d^2*f*g^2)*x^4 + (6*b*c^2*d^2*f^2*g + b*d^2*g^3)*x^3
 + (2*b*c^2*d^2*f^3 + 3*b*d^2*f*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d
), x)
```

3.43.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\operatorname{arcsinh}(cx))\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**3*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Timed out
```

3.43.7 Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.43.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \int (f+g\,x)^3\,(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

input
```
int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

3.44 $\int (f+gx)^2 \left(d+c^2dx^2\right)^{5/2} (a+b\,\mathbf{arcsinh}(cx))\, dx$

3.44.1 Optimal result

Integrand size = 30, antiderivative size = 901

$$
\begin{aligned}
&\int (f+gx)^2\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx=-\frac{2bd^2fgx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}\\
&-\frac{25bcd^2f^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{5bd^2g^2x^2\sqrt{d+c^2dx^2}}{256c\sqrt{1+c^2x^2}}-\frac{2bcd^2fgx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}}\\
&-\frac{5bc^3d^2f^2x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{59bcd^2g^2x^4\sqrt{d+c^2dx^2}}{768\sqrt{1+c^2x^2}}\\
&-\frac{6bc^3d^2fgx^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}}-\frac{17bc^3d^2g^2x^6\sqrt{d+c^2dx^2}}{288\sqrt{1+c^2x^2}}\\
&-\frac{2bc^5d^2fgx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}}-\frac{bc^5d^2g^2x^8\sqrt{d+c^2dx^2}}{64\sqrt{1+c^2x^2}}\\
&-\frac{bd^2f^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c}+\frac{5}{16}d^2f^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\\
&+\frac{5d^2g^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{128c^2}\\
&+\frac{5}{64}d^2g^2x^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\\
&+\frac{5}{24}d^2f^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\\
&+\frac{5}{48}d^2g^2x^3\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\\
&+\frac{1}{6}d^2f^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\\
&+\frac{1}{8}d^2g^2x^3\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))\\
&+\frac{2d^2fg(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^2}\\
&+\frac{5d^2f^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{32bc\sqrt{1+c^2x^2}}\\
&-\frac{5d^2g^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{256bc^3\sqrt{1+c^2x^2}}
\end{aligned}
$$

output
```
-1/36*b*d^2*f^2*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/c+5/16*d^2*f^2*x*(a+
b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+5/128*d^2*g^2*x*(a+b*arcsinh(c*x))*(c^
2*d*x^2+d)^(1/2)/c^2+5/64*d^2*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/
2)+5/24*d^2*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+5/48*
d^2*g^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/6*d^2*f^2
*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/8*d^2*g^2*x^3*(c
^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+2/7*d^2*f*g*(c^2*x^2+1)
^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-2/7*b*d^2*f*g*x*(c^2*d*x^2+d
)^(1/2)/c/(c^2*x^2+1)^(1/2)-25/96*b*c*d^2*f^2*x^2*(c^2*d*x^2+d)^(1/2)/(c^2
*x^2+1)^(1/2)-5/256*b*d^2*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-
2/7*b*c*d^2*f*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5/96*b*c^3*d^2*f
^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-59/768*b*c*d^2*g^2*x^4*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-6/35*b*c^3*d^2*f*g*x^5*(c^2*d*x^2+d)^(1/2)
/(c^2*x^2+1)^(1/2)-17/288*b*c^3*d^2*g^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1
)^(1/2)-2/49*b*c^5*d^2*f*g*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/64*
b*c^5*d^2*g^2*x^8*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/32*d^2*f^2*(a+b*
arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-5/256*d^2*g^2*(a
+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

3.44.2 Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 555, normalized size of antiderivative = 0.62

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \frac{-d^3(1+c^2x^2)\left(b(-87955g^2+1120c^2(2093f^2+4608fgx+315g^2x^2)+3360c^4x^2(1848\right.}{}$$

input
```
Integrate[(f + g*x)^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(-(d^3*(1 + c^2*x^2)*(b*(-87955*g^2 + 1120*c^2*(2093*f^2 + 4608*f*g*x + 31
5*g^2*x^2) + 3360*c^4*x^2*(1848*f^2 + 1536*f*g*x + 413*g^2*x^2) + 640*c^8*
x^6*(784*f^2 + 1152*f*g*x + 441*g^2*x^2) + 1792*c^6*x^4*(1365*f^2 + 1728*f
*g*x + 595*g^2*x^2)) - 6720*a*c*Sqrt[1 + c^2*x^2]*(768*f*g*(1 + c^2*x^2)^3
 + 56*c^2*f^2*x*(33 + 26*c^2*x^2 + 8*c^4*x^4) + 7*g^2*x*(15 + 118*c^2*x^2
+ 136*c^4*x^4 + 48*c^6*x^6))) + 352800*b*d^3*(8*c^2*f^2 - g^2)*(1 + c^2*x
^2)*ArcSinh[c*x]^2 + 705600*a*d^(5/2)*(8*c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*
Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 840*b*d^3*(
1 + c^2*x^2)*ArcSinh[c*x]*(6144*c*f*g*Sqrt[1 + c^2*x^2] + 18432*c^3*f*g*x^
2*Sqrt[1 + c^2*x^2] + 18432*c^5*f*g*x^4*Sqrt[1 + c^2*x^2] + 6144*c^7*f*g*x
^6*Sqrt[1 + c^2*x^2] + 336*(15*c^2*f^2 - g^2)*Sinh[2*ArcSinh[c*x]] + 168*(
6*c^2*f^2 + g^2)*Sinh[4*ArcSinh[c*x]] + 112*c^2*f^2*Sinh[6*ArcSinh[c*x]] +
 112*g^2*Sinh[6*ArcSinh[c*x]] + 21*g^2*Sinh[8*ArcSinh[c*x]]))/(18063360*c^
3*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2])
```

3.44.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.52, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{5/2}(f+gx)^2(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int (f+gx)^2\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int \Big(f^2(a+b\text{arcsinh}(cx))\left(c^2x^2+1\right)^{5/2}+g^2x^2(a+b\text{arcsinh}(cx))\left(c^2x^2+1\right)^{5/2}+2fgx(a+b\text{arcsinh}}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\Big(-\frac{5g^2(a+b\text{arcsinh}(cx))^2}{256bc^3}+\frac{1}{6}f^2x\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{24}f^2x\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}}{}$$

input
```
Int[(f + g*x)^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*((-2*b*f*g*x)/(7*c) - (25*b*c*f^2*x^2)/96 - (5*b*
g^2*x^2)/(256*c) - (2*b*c*f*g*x^3)/7 - (5*b*c^3*f^2*x^4)/96 - (59*b*c*g^2*
x^4)/768 - (6*b*c^3*f*g*x^5)/35 - (17*b*c^3*g^2*x^6)/288 - (2*b*c^5*f*g*x^
7)/49 - (b*c^5*g^2*x^8)/64 - (b*f^2*(1 + c^2*x^2)^3)/(36*c) + (5*f^2*x*Sqr
t[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/16 + (5*g^2*x*Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x]))/(128*c^2) + (5*g^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x]))/64 + (5*f^2*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/24 + (5*g^2*
x^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/48 + (f^2*x*(1 + c^2*x^2)^(5
/2)*(a + b*ArcSinh[c*x]))/6 + (g^2*x^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[
c*x]))/8 + (2*f*g*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^2) + (5*f
^2*(a + b*ArcSinh[c*x])^2)/(32*b*c) - (5*g^2*(a + b*ArcSinh[c*x])^2)/(256*
b*c^3)))/Sqrt[1 + c^2*x^2]
```

3.44.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

3.44.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2308 vs. 2(791) = 1582.

Time = 1.03 (sec) , antiderivative size = 2309, normalized size of antiderivative = 2.56

method	result	size
default	Expression too large to display	2309
parts	Expression too large to display	2309

input
```
int((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^2*(1/6*x*(c^2*d*x^2+d)^(5/2)+5/6*d*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(
1/2*x*(c^2*d*x^2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/
2))/(c^2*d)^(1/2))))+g^2*(1/8*x*(c^2*d*x^2+d)^(7/2)/c^2/d-1/8/c^2*(1/6*x*(
c^2*d*x^2+d)^(5/2)+5/6*d*(1/4*x*(c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(c^2*d*x^
2+d)^(1/2)+1/2*d*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/
2)))))+2/7*f*g*(c^2*d*x^2+d)^(7/2)/c^2/d)+b*(5/256*(d*(c^2*x^2+1))^(1/2)*a
rcsinh(c*x)^2*(8*c^2*f^2-g^2)*d^2/(c^2*x^2+1)^(1/2)/c^3+1/16384*(d*(c^2*x^
2+1))^(1/2)*(128*c^9*x^9+128*c^8*x^8*(c^2*x^2+1)^(1/2)+320*c^7*x^7+256*c^6
*x^6*(c^2*x^2+1)^(1/2)+272*c^5*x^5+160*c^4*x^4*(c^2*x^2+1)^(1/2)+88*c^3*x^
3+32*c^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x+(c^2*x^2+1)^(1/2))*g^2*(-1+8*arcsinh(
c*x))*d^2/c^3/(c^2*x^2+1)+1/3136*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*
x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^
4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(
-1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^
7*x^7+32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5+48*c^4*x^4*(c^2*x^2+1)^(1/2)
+38*c^3*x^3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*(6*arcsi
nh(c*x)*c^2*f^2-c^2*f^2+6*arcsinh(c*x)*g^2-g^2)*d^2/c^3/(c^2*x^2+1)+1/320*
(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+
20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(-1
+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/1024*(d*(c^2*x^2+1))^(1/2)*(8*c^...
```

3.44.5 Fricas [F]

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \int \left(c^2dx^2+d\right)^{\frac{5}{2}}(gx+f)^2(b\,\text{arsinh}\,(cx)+a)\,dx$$

input
```
integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="f
ricas")
```

output
```
integral((a*c^4*d^2*g^2*x^6 + 2*a*c^4*d^2*f*g*x^5 + 4*a*c^2*d^2*f*g*x^3 +
2*a*d^2*f*g*x + a*d^2*f^2 + (a*c^4*d^2*f^2 + 2*a*c^2*d^2*g^2)*x^4 + (2*a*c
^2*d^2*f^2 + a*d^2*g^2)*x^2 + (b*c^4*d^2*g^2*x^6 + 2*b*c^4*d^2*f*g*x^5 + 4
*b*c^2*d^2*f*g*x^3 + 2*b*d^2*f*g*x + b*d^2*f^2 + (b*c^4*d^2*f^2 + 2*b*c^2*
d^2*g^2)*x^4 + (2*b*c^2*d^2*f^2 + b*d^2*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d
*x^2 + d), x)
```

3.44.6 Sympy [F]

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \int \left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}} (a+b\,\text{asinh}\,(cx))\,(f+gx)^2\,dx$$

input
```
integrate((g*x+f)**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))*(f + g*x)**2, x)
```

3.44.7 Maxima [F(-2)]

Exception generated.

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{5/2} (a+b\text{arcsinh}(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.44.8 Giac [F(-2)]

Exception generated.

$$\int (f + gx)^2 \left(d + c^2 dx^2\right)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int (f + gx)^2 (d + c^2 dx^2)^{5/2} (a + b\text{arcsinh}(cx))\, dx = \int (f + g\,x)^2 \,(a + b\,\text{asinh}(c\,x))\, \left(d\,c^2\,x^2 + d\right)^{5/2} dx$$

input
```
int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

3.45 $\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx$

3.45.1 Optimal result

Integrand size = 28, antiderivative size = 494

$$\begin{aligned}
&\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \\
&-\frac{bd^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}-\frac{25bcd^2fx^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{bcd^2gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} \\
&-\frac{5bc^3d^2fx^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{3bc^3d^2gx^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}}-\frac{bc^5d^2gx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} \\
&-\frac{bd^2f(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c}+\frac{5}{16}d^2fx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+\frac{5}{24}d^2fx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+\frac{1}{6}d^2fx\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx)) \\
&+\frac{d^2g(1+c^2x^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{7c^2} \\
&+\frac{5d^2f\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{32bc\sqrt{1+c^2x^2}}
\end{aligned}$$

output
```
-1/36*b*d^2*f*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/c+5/16*d^2*f*x*(a+b*ar
csinh(c*x))*(c^2*d*x^2+d)^(1/2)+5/24*d^2*f*x*(c^2*x^2+1)*(a+b*arcsinh(c*x)
)*(c^2*d*x^2+d)^(1/2)+1/6*d^2*f*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*
x^2+d)^(1/2)+1/7*d^2*g*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2
)/c^2-1/7*b*d^2*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-25/96*b*c*d^2*
f*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/7*b*c*d^2*g*x^3*(c^2*d*x^2+d
)^(1/2)/(c^2*x^2+1)^(1/2)-5/96*b*c^3*d^2*f*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^
2+1)^(1/2)-3/35*b*c^3*d^2*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/49
*b*c^5*d^2*g*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/32*d^2*f*(a+b*arc
sinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

3.45.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.79

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))\,dx = \frac{-d^3(1+c^2x^2)\left(-1680a\sqrt{1+c^2x^2}\left(48g(1+c^2x^2)^3+7c^2fx(33+26c^2x^2+8c^4x^4)\right)+b\right.}{}$$

input
```
Integrate[(f + g*x)*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(-(d^3*(1 + c^2*x^2)*(-1680*a*Sqrt[1 + c^2*x^2]*(48*g*(1 + c^2*x^2)^3 + 7*
c^2*f*x*(33 + 26*c^2*x^2 + 8*c^4*x^4)) + b*c*(2304*g*x*(35 + 35*c^2*x^2 +
21*c^4*x^4 + 5*c^6*x^6) + 245*f*(299 + 792*c^2*x^2 + 312*c^4*x^4 + 64*c^6*
x^6)))) + 88200*b*c*d^3*f*(1 + c^2*x^2)*ArcSinh[c*x]^2 + 176400*a*c*d^(5/2
)*f*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2
*d*x^2]] + 420*b*d^3*(1 + c^2*x^2)*ArcSinh[c*x]*(192*g*Sqrt[1 + c^2*x^2] +
 576*c^2*g*x^2*Sqrt[1 + c^2*x^2] + 576*c^4*g*x^4*Sqrt[1 + c^2*x^2] + 192*c
^6*g*x^6*Sqrt[1 + c^2*x^2] + 315*c*f*Sinh[2*ArcSinh[c*x]] + 63*c*f*Sinh[4*
ArcSinh[c*x]] + 7*c*f*Sinh[6*ArcSinh[c*x]]))/(564480*c^2*Sqrt[1 + c^2*x^2]
*Sqrt[d + c^2*d*x^2])
```

3.45.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.50, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2dx^2+d\right)^{5/2}(f+gx)(a+b\text{arcsinh}(cx))\,dx$$

$$\Big\downarrow \text{ 6260}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int (f+gx)\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 6253}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int \left(f(a+b\text{arcsinh}(cx))\left(c^2x^2+1\right)^{5/2}+gx(a+b\text{arcsinh}(cx))\left(c^2x^2+1\right)^{5/2}\right)dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$d^2\sqrt{c^2dx^2+d}\left(\frac{1}{6}fx\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))+\frac{5}{24}fx\left(c^2x^2+1\right)^{3/2}(a+b\text{arcsinh}(cx))+\frac{5}{16}fx\sqrt{c^2x^2+1}(a\right.$$

input
```
Int[(f + g*x)*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*(-1/7*(b*g*x)/c - (25*b*c*f*x^2)/96 - (b*c*g*x^3)
/7 - (5*b*c^3*f*x^4)/96 - (3*b*c^3*g*x^5)/35 - (b*c^5*g*x^7)/49 - (b*f*(1
+ c^2*x^2)^3)/(36*c) + (5*f*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/16 +
 (5*f*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/24 + (f*x*(1 + c^2*x^2)^
(5/2)*(a + b*ArcSinh[c*x]))/6 + (g*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]
))/(7*c^2) + (5*f*(a + b*ArcSinh[c*x])^2)/(32*b*c)))/Sqrt[1 + c^2*x^2]
```

3.45.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

3.45.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1678 vs. $2(430) = 860$.

Time = 1.00 (sec) , antiderivative size = 1679, normalized size of antiderivative = 3.40

method	result	size
default	Expression too large to display	1679
parts	Expression too large to display	1679

input
```
int((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE
)
```

output
```
1/6*a*f*x*(c^2*d*x^2+d)^(5/2)+5/24*a*f*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a*f*d^
2*x*(c^2*d*x^2+d)^(1/2)+5/16*a*f*d^3*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d
)^(1/2))/(c^2*d)^(1/2)+1/7*a*g*(c^2*d*x^2+d)^(7/2)/c^2/d+b*(5/32*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*f*arcsinh(c*x)^2*d^2+1/6272*(d*(c^2*x^2+1
))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*
(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*
x*(c^2*x^2+1)^(1/2)+1)*g*(-1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/2304*(d
*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*c^6*x^6*(c^2*x^2+1)^(1/2)+64*c^5*x^5+48
*c^4*x^4*(c^2*x^2+1)^(1/2)+38*c^3*x^3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(
c^2*x^2+1)^(1/2))*f*(-1+6*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+1/640*(d*(c^2*x^
2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3
*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*g*(-1+5*arcsinh(c
*x))*d^2/c^2/(c^2*x^2+1)+3/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*c^4*x^4*
(c^2*x^2+1)^(1/2)+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)
^(1/2))*f*(-1+4*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+1/128*(d*(c^2*x^2+1))^(1/2
)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)
+1)*g*(-1+3*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+15/256*(d*(c^2*x^2+1))^(1/2)
*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*f*(-1+2*a
rcsinh(c*x))*d^2/c/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c
^2*x^2+1)^(1/2)+1)*g*(-1+arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^...
```

3.45.5 Fricas [F]

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\int \left(c^2dx^2+d\right)^{\frac{5}{2}}(gx+f)(b\operatorname{arsinh}(cx)+a)\,dx$$

input
```
integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fri
cas")
```

output
```
integral((a*c^4*d^2*g*x^5 + a*c^4*d^2*f*x^4 + 2*a*c^2*d^2*g*x^3 + 2*a*c^2*
d^2*f*x^2 + a*d^2*g*x + a*d^2*f + (b*c^4*d^2*g*x^5 + b*c^4*d^2*f*x^4 + 2*b
*c^2*d^2*g*x^3 + 2*b*c^2*d^2*f*x^2 + b*d^2*g*x + b*d^2*f)*arcsinh(c*x))*sq
rt(c^2*d*x^2 + d), x)
```

3.45.6 Sympy [F]

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\int \left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}(a+b\,\text{asinh}\,(cx))\,(f+gx)\;dx$$

input
```
integrate((g*x+f)*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))*(f + g*x), x)
```

3.45.7 Maxima [F(-2)]

Exception generated.

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="max
ima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.45.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="gia
c")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))\,dx=\int (f+g\,x)\,\left(a+b\,\text{asinh}(c\,x)\right)\,{\left(d\,c^2\,x^2+d\right)}^{5/2}\,dx$$

input
```
int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

output
```
int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

3.46 $\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\operatorname{arcsinh}(cx))}{f+gx}\,dx$

3.46.1 Optimal result

Integrand size = 30, antiderivative size = 1536

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{arcsinh}(cx))}{f+gx}\,dx = \frac{ad^2(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}}{g^5}+\frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}}$$
$$-\frac{bcd^2(c^2f^2+g^2)^2x\sqrt{d+c^2dx^2}}{g^5\sqrt{1+c^2x^2}}-\frac{bcd^2(c^2f^2+2g^2)x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}}+\frac{bc^3d^2fx^2\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}}$$
$$+\frac{bc^3d^2f(c^2f^2+2g^2)x^2\sqrt{d+c^2dx^2}}{4g^4\sqrt{1+c^2x^2}}-\frac{bc^3d^2x^3\sqrt{d+c^2dx^2}}{45g\sqrt{1+c^2x^2}}-\frac{bc^3d^2(c^2f^2+2g^2)x^3\sqrt{d+c^2dx^2}}{9g^3\sqrt{1+c^2x^2}}$$
$$+\frac{bc^5d^2fx^4\sqrt{d+c^2dx^2}}{16g^2\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25g\sqrt{1+c^2x^2}}+\frac{bd^2(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)}{g^5}$$
$$-\frac{c^2d^2fx\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{8g^2}-\frac{c^2d^2f(c^2f^2+2g^2)x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{2g^4}$$
$$-\frac{c^4d^2fx^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{4g^2}-\frac{d^2(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g}$$
$$+\frac{d^2(c^2f^2+2g^2)(1+c^2x^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{3g^3}$$
$$+\frac{d^2(1+c^2x^2)^2\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))}{5g}+\frac{cd^2f\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{16bg^2\sqrt{1+c^2x^2}}$$
$$-\frac{cd^2f(c^2f^2+2g^2)\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{4bg^4\sqrt{1+c^2x^2}}$$
$$-\frac{cd^2(c^2f^2+g^2)^2x\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bg^5\sqrt{1+c^2x^2}}$$
$$-\frac{d^2(c^2f^2+g^2)^3\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^6(f+gx)\sqrt{1+c^2x^2}}$$
$$+\frac{d^2(c^2f^2+g^2)^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}(a+b\operatorname{arcsinh}(cx))^2}{2bcg^4(f+gx)}$$
$$-\frac{ad^2(c^2f^2+g^2)^{5/2}\sqrt{d+c^2dx^2}\operatorname{arctanh}\left(\frac{g-c^2fx}{\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}}\right)}{g^6\sqrt{1+c^2x^2}}$$
$$+\frac{bd^2(c^2f^2+g^2)^{5/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{1+c^2x^2}}$$
$$-\frac{bd^2(c^2f^2+g^2)^{5/2}\sqrt{d+c^2dx^2}\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{1+c^2x^2}}$$
$$+\frac{bd^2(c^2f^2+g^2)^{5/2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{1+c^2x^2}}$$
$$-\frac{bd^2(c^2f^2+g^2)^{5/2}\sqrt{d+c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^6\sqrt{1+c^2x^2}}$$

output
```
-1/8*c^2*d^2*f*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g^2-1/4*c^4*d^2*f*
x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g^2+2/15*b*c*d^2*x*(c^2*d*x^2+d
)^(1/2)/g/(c^2*x^2+1)^(1/2)-1/45*b*c^3*d^2*x^3*(c^2*d*x^2+d)^(1/2)/g/(c^2*
x^2+1)^(1/2)-1/25*b*c^5*d^2*x^5*(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)+1/
3*d^2*(c^2*f^2+2*g^2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g
^3+1/4*b*c^3*d^2*f*(c^2*f^2+2*g^2)*x^2*(c^2*d*x^2+d)^(1/2)/g^4/(c^2*x^2+1)
^(1/2)-1/4*c*d^2*f*(c^2*f^2+2*g^2)*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2
)/b/g^4/(c^2*x^2+1)^(1/2)-1/2*c*d^2*(c^2*f^2+g^2)^2*x*(a+b*arcsinh(c*x))^2
*(c^2*d*x^2+d)^(1/2)/b/g^5/(c^2*x^2+1)^(1/2)-1/2*d^2*(c^2*f^2+g^2)^3*(a+b*
arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/g^6/(g*x+f)/(c^2*x^2+1)^(1/2)+1/2*
d^2*(c^2*f^2+g^2)^2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)^(
1/2)/b/c/g^4/(g*x+f)+a*d^2*(c^2*f^2+g^2)^2*(c^2*d*x^2+d)^(1/2)/g^5-a*d^2*(
c^2*f^2+g^2)^(5/2)*arctanh((-c^2*f*x+g)/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1
/2))*(c^2*d*x^2+d)^(1/2)/g^6/(c^2*x^2+1)^(1/2)+b*d^2*(c^2*f^2+g^2)^2*arcsi
nh(c*x)*(c^2*d*x^2+d)^(1/2)/g^5+b*d^2*(c^2*f^2+g^2)^(5/2)*polylog(2,-(c*x+
(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^6/(c
^2*x^2+1)^(1/2)-b*d^2*(c^2*f^2+g^2)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2
))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^6/(c^2*x^2+1)^(1/2)-
b*c*d^2*(c^2*f^2+g^2)^2*x*(c^2*d*x^2+d)^(1/2)/g^5/(c^2*x^2+1)^(1/2)+b*d^2*
(c^2*f^2+g^2)^(5/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c...
```

3.46.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 23.15 (sec) , antiderivative size = 7168, normalized size of antiderivative = 4.67

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

output
```
Result too large to show
```

3.46.3 Rubi [A] (verified)

Time = 2.73 (sec) , antiderivative size = 945, normalized size of antiderivative = 0.62, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6260, 6255, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2dx^2+d\right)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx}dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int \frac{\left(c^2x^2+1\right)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx}dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{6255}$$

$$\frac{d^2\sqrt{c^2dx^2+d}\int \left(\frac{x^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))c^4}{g}-\frac{fx^2\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))c^4}{g^2}+\frac{(c^2f^2+2g^2)x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))c^2}{g^3}\right.}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{2009}$$

$$d^2\sqrt{c^2dx^2+d}\left(-\frac{bx^5c^5}{25g}+\frac{bfx^4c^5}{16g^2}-\frac{fx^3\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))c^4}{4g^2}-\frac{b(c^2f^2+2g^2)x^3c^3}{9g^3}-\frac{bx^3c^3}{45g}+\frac{bf(c^2f^2+2g^2)x^2c^3}{4g^4}+\frac{bfx^2c}{16g^2}\right.$$

input
```
Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

output
```
(d^2*Sqrt[d + c^2*d*x^2]*((2*b*c*x)/(15*g) - (b*c*(c^2*f^2 + g^2)^2*x)/g^5
 - (b*c*(c^2*f^2 + 2*g^2)*x)/(3*g^3) + (b*c^3*f*x^2)/(16*g^2) + (b*c^3*f*(
c^2*f^2 + 2*g^2)*x^2)/(4*g^4) - (b*c^3*x^3)/(45*g) - (b*c^3*(c^2*f^2 + 2*g
^2)*x^3)/(9*g^3) + (b*c^5*f*x^4)/(16*g^2) - (b*c^5*x^5)/(25*g) + (a*(c^2*f
^2 + g^2)^2*Sqrt[1 + c^2*x^2])/g^5 + (b*(c^2*f^2 + g^2)^2*Sqrt[1 + c^2*x^2
]*ArcSinh[c*x])/g^5 - (c^2*f*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*
g^2) - (c^2*f*(c^2*f^2 + 2*g^2)*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
(2*g^4) - (c^4*f*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(4*g^2) - ((1
 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*g) + ((c^2*f^2 + 2*g^2)*(1 + c^
2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*g^3) + ((1 + c^2*x^2)^(5/2)*(a + b*A
rcSinh[c*x]))/(5*g) + (c*f*(a + b*ArcSinh[c*x])^2)/(16*b*g^2) - (c*f*(c^2*
f^2 + 2*g^2)*(a + b*ArcSinh[c*x])^2)/(4*b*g^4) - (c*(c^2*f^2 + g^2)^2*x*(a
 + b*ArcSinh[c*x])^2)/(2*b*g^5) - ((c^2*f^2 + g^2)^3*(a + b*ArcSinh[c*x])^
2)/(2*b*c*g^6*(f + g*x)) + ((c^2*f^2 + g^2)^2*(1 + c^2*x^2)*(a + b*ArcSinh
[c*x])^2)/(2*b*c*g^4*(f + g*x)) - (a*(c^2*f^2 + g^2)^(5/2)*ArcTanh[(g - c^
2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])])/g^6 + (b*(c^2*f^2 + g^2)^
(5/2)*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])]
)/g^6 - (b*(c^2*f^2 + g^2)^(5/2)*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(
c*f + Sqrt[c^2*f^2 + g^2])])/g^6 + (b*(c^2*f^2 + g^2)^(5/2)*PolyLog[2, -((
E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/g^6 - (b*(c^2*f^2 + g^...
```

3.46.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6255
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(
a + b*ArcSinh[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IGtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

3.46.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3498 vs. $2(1420) = 2840$.

Time = 0.94 (sec) , antiderivative size = 3499, normalized size of antiderivative = 2.28

method	result	size
default	Expression too large to display	3499
parts	Expression too large to display	3499

input
```
int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x,method=_RETURNVERBOSE
)
```

output
```
b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*dilo
g((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)
^(1/2)))-1/25*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)/g*x^5*c^5+33/1
28*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c/(c^2*x^2+1)^(1/2)/g^2+1/8*b*(d*(c^2*x^2
+1))^(1/2)*f^3*d^2*c^3/(c^2*x^2+1)^(1/2)/g^4-11/45*b*(d*(c^2*x^2+1))^(1/2)
*d^2/(c^2*x^2+1)^(1/2)/g*x^3*c^3-23/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^
2+1)^(1/2)/g*c*x-b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+
1)^(1/2)/g^2*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*
f+(c^2*f^2+g^2)^(1/2)))-1/2*b*(d*(c^2*x^2+1))^(1/2)*f^3*d^2*c^6/(c^2*x^2+1
)/g^4*arcsinh(c*x)*x^3-1/4*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^6/(c^2*x^2+1)/g
^2*arcsinh(c*x)*x^5-2*b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2
*x^2+1)^(1/2)/g^4*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2)
)/(c*f+(c^2*f^2+g^2)^(1/2)))*c^2*f^2-11/8*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^
4/(c^2*x^2+1)/g^2*arcsinh(c*x)*x^3-9/8*b*(d*(c^2*x^2+1))^(1/2)*f*d^2*c^2/(
c^2*x^2+1)/g^2*arcsinh(c*x)*x-1/2*b*(d*(c^2*x^2+1))^(1/2)*f^3*d^2*c^4/(c^2
*x^2+1)/g^4*arcsinh(c*x)*x+1/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g^3
*arcsinh(c*x)*x^4*c^6*f^2+8/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g^3*
arcsinh(c*x)*x^2*c^4*f^2+b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(
c^2*x^2+1)^(1/2)/g^6*dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(
1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))*c^4*f^4-b*d^2*(d*(c^2*x^2+1))^(1/2)*(...
```

3.46.5 Fricas [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx=\int \frac{(c^2dx^2+d)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)}{gx+f}\,dx$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="fri
cas")
```

output
```
integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c
^2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/(g*x + f), x)
```

3.46.6 Sympy [F]

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx=\int \frac{(d(c^2x^2+1))^{\frac{5}{2}}(a+b\,\text{asinh}\,(cx))}{f+gx}\,dx$$

input
```
integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/(g*x+f),x)
```

output
```
Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/(f + g*x), x)
```

3.46.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="max
ima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.46.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+c^2dx^2)^{5/2}(a+b\text{arcsinh}(cx))}{f+gx}\,dx = \int \frac{(a+b\,\text{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}}{f+g\,x}\,dx$$

input
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/(f + g*x),x)
```

output
```
int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/(f + g*x), x)
```

3.47 $\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

3.47.1 Optimal result

Integrand size = 30, antiderivative size = 430

$$\begin{aligned}\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = & -\frac{3bf^2gx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}+\frac{2bg^3x\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} \\ & -\frac{3bfg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}}-\frac{bg^3x^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}} \\ & +\frac{3f^2g(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c^2\sqrt{d+c^2dx^2}} \\ & -\frac{2g^3(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3c^4\sqrt{d+c^2dx^2}} \\ & +\frac{3fg^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{2c^2\sqrt{d+c^2dx^2}} \\ & +\frac{g^3x^2(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{3c^2\sqrt{d+c^2dx^2}} \\ & +\frac{f^3\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}} \\ & -\frac{3fg^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc^3\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
3*f^2*g*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)-2/3*g^3*(c^
2*x^2+1)*(a+b*arcsinh(c*x))/c^4/(c^2*d*x^2+d)^(1/2)+3/2*f*g^2*x*(c^2*x^2+1
)*(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)+1/3*g^3*x^2*(c^2*x^2+1)*(a+b*
arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)-3*b*f^2*g*x*(c^2*x^2+1)^(1/2)/c/(c^2
*d*x^2+d)^(1/2)+2/3*b*g^3*x*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-3/4*
b*f*g^2*x^2*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)-1/9*b*g^3*x^3*(c^2*x^2
+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)+1/2*f^3*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(
1/2)/b/c/(c^2*d*x^2+d)^(1/2)-3/4*f*g^2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1
/2)/b/c^3/(c^2*d*x^2+d)^(1/2)
```

3.47.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.71

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$$
$$=\frac{4\sqrt{d}g\left(-2bcx\sqrt{1+c^2x^2}(-6g^2+c^2(27f^2+g^2x^2))+3a(1+c^2x^2)\left(-4g^2+c^2(18f^2+9fgx+2g^2x^2)\right)\right)+1}{}$$

input
```
Integrate[((f + g*x)^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
(4*Sqrt[d]*g*(-2*b*c*x*Sqrt[1 + c^2*x^2]*(-6*g^2 + c^2*(27*f^2 + g^2*x^2))
 + 3*a*(1 + c^2*x^2)*(-4*g^2 + c^2*(18*f^2 + 9*f*g*x + 2*g^2*x^2))) + 12*b
*Sqrt[d]*g*(1 + c^2*x^2)*(-4*g^2 + c^2*(18*f^2 + 9*f*g*x + 2*g^2*x^2))*Arc
Sinh[c*x] + 18*b*c*Sqrt[d]*f*(2*c^2*f^2 - 3*g^2)*Sqrt[1 + c^2*x^2]*ArcSinh
[c*x]^2 - 27*b*c*Sqrt[d]*f*g^2*Sqrt[1 + c^2*x^2]*Cosh[2*ArcSinh[c*x]] + 36
*a*c*f*(2*c^2*f^2 - 3*g^2)*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d
+ c^2*d*x^2]])/(72*c^4*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

3.47.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.60, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{\sqrt{c^2x^2+1}\int \left(\frac{(a+b\text{arcsinh}(cx))f^3}{\sqrt{c^2x^2+1}}+\frac{3gx(a+b\text{arcsinh}(cx))f^2}{\sqrt{c^2x^2+1}}+\frac{3g^2x^2(a+b\text{arcsinh}(cx))f}{\sqrt{c^2x^2+1}}+\frac{g^3x^3(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c^2x^2+1}\left(-\frac{3fg^2(a+b\text{arcsinh}(cx))^2}{4bc^3}+\frac{3f^2g\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}+\frac{3fg^2x\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{2c^2}+\frac{g^3x^2\sqrt{c^2x^2+1}(a+b\right.}{3c^2}}{\sqrt{c^2dx^2+d}}$$

input
```
Int[((f + g*x)^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
(Sqrt[1 + c^2*x^2]*((-3*b*f^2*g*x)/c + (2*b*g^3*x)/(3*c^3) - (3*b*f*g^2*x^
2)/(4*c) - (b*g^3*x^3)/(9*c) + (3*f^2*g*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x]))/c^2 - (2*g^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^4) + (3*f*
g^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2) + (g^3*x^2*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2) + (f^3*(a + b*ArcSinh[c*x])^2)/(2*b
*c) - (3*f*g^2*(a + b*ArcSinh[c*x])^2)/(4*b*c^3)))/Sqrt[d + c^2*d*x^2]
```

3.47.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
 + b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

3.47.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. 2(382) = 764.

Time = 0.93 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.83

method	result
default	$a\left(\frac{f^3\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+g^3\left(\frac{x^2\sqrt{c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{c^2d\,x^2+d}}{3d\,c^4}\right)+3f\,g^2\left(\frac{x\sqrt{c^2d\,x^2+d}}{2c^2d}-\frac{\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^2\sqrt{c^2d}}\right.\right.$
parts	$a\left(\frac{f^3\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+g^3\left(\frac{x^2\sqrt{c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{c^2d\,x^2+d}}{3d\,c^4}\right)+3f\,g^2\left(\frac{x\sqrt{c^2d\,x^2+d}}{2c^2d}-\frac{\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^2\sqrt{c^2d}}\right.\right.$

input
```
int((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^3*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+g^3*(1/
3*x^2/c^2/d*(c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2))+3*f*g^2*(1/
2*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2/c^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+
d)^(1/2))/(c^2*d)^(1/2))+3*f^2*g/c^2/d*(c^2*d*x^2+d)^(1/2))+b*(1/4*(d*(c^2
*x^2+1))^(1/2)*f*arcsinh(c*x)^2*(2*c^2*f^2-3*g^2)/(c^2*x^2+1)^(1/2)/c^3/d+
1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^
2+3*c*x*(c^2*x^2+1)^(1/2)+1)*g^3*(-1+3*arcsinh(c*x))/c^4/d/(c^2*x^2+1)+3/1
6*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*
x^2+1)^(1/2))*f*g^2*(-1+2*arcsinh(c*x))/c^3/d/(c^2*x^2+1)+3/8*(d*(c^2*x^2+
1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*g*(4*arcsinh(c*x)*c^2*f^2-4*c^
2*f^2-arcsinh(c*x)*g^2+g^2)/c^4/d/(c^2*x^2+1)+3/8*(d*(c^2*x^2+1))^(1/2)*(c
^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*g*(4*arcsinh(c*x)*c^2*f^2+4*c^2*f^2-arcsin
h(c*x)*g^2-g^2)/c^4/d/(c^2*x^2+1)+3/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*
c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*f*g^2*(1+2*arcsinh(c*x)
)/c^3/d/(c^2*x^2+1)+1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x
^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*g^3*(3*arcsinh(c*x)+1)/c^
4/d/(c^2*x^2+1))
```

3.47.5 Fricas [F]

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(gx+f)^3(b\,\text{arsinh}\,(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

output
```
integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3
*b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arcsinh(c*x))/sqrt(c^2*d*x^2 + d), x)
```

3.47.6 Sympy [F]

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(a+b\,\text{asinh}\,(cx))\,(f+gx)^3}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate((g*x+f)**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))*(f + g*x)**3/sqrt(d*(c**2*x**2 + 1)), x)
```

3.47.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.47.8 Giac [F]

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(gx+f)^3(b\,\text{arsinh}\,(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((g*x + f)^3*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(f+g\,x)^3\,(a+b\,\text{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int(((f + g*x)^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int(((f + g*x)^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.48 $\int \frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

3.48.1 Optimal result

Integrand size = 30, antiderivative size = 258

$$\begin{aligned}\int \frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = &-\frac{2bfgx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}-\frac{bg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}}\\&+\frac{2fg(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c^2\sqrt{d+c^2dx^2}}\\&+\frac{g^2x(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{2c^2\sqrt{d+c^2dx^2}}\\&+\frac{f^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}}\\&-\frac{g^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{4bc^3\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
2*f*g*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)+1/2*g^2*x*(c^
2*x^2+1)*(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)-2*b*f*g*x*(c^2*x^2+1)^
(1/2)/c/(c^2*d*x^2+d)^(1/2)-1/4*b*g^2*x^2*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d
)^(1/2)+1/2*f^2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(
1/2)-1/4*g^2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c^3/(c^2*d*x^2+d)^(1
/2)
```

3.48.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.90

$$\int \frac{(f+gx)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{4c\sqrt{d}g\left(-4bcfx\sqrt{1+c^2x^2}+a(4f+gx)\left(1+c^2x^2\right)\right)+4bc\sqrt{d}g(4f+gx)\left(1+c^2x^2\right)\operatorname{arcsinh}(cx)+2b\sqrt{d}(2}{}$$

input
```
Integrate[((f + g*x)^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
(4*c*Sqrt[d]*g*(-4*b*c*f*x*Sqrt[1 + c^2*x^2] + a*(4*f + g*x)*(1 + c^2*x^2)
) + 4*b*c*Sqrt[d]*g*(4*f + g*x)*(1 + c^2*x^2)*ArcSinh[c*x] + 2*b*Sqrt[d]*(
2*c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 - b*Sqrt[d]*g^2*Sqrt[1 +
 c^2*x^2]*Cosh[2*ArcSinh[c*x]] + 4*a*(2*c^2*f^2 - g^2)*Sqrt[d + c^2*d*x^2]
*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(8*c^3*Sqrt[d]*Sqrt[d + c^2*d*x
^2])
```

3.48.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.61, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}\,dx$$
$$\Big\downarrow \; 6260$$
$$\frac{\sqrt{c^2x^2+1}\int \frac{(f+gx)^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2dx^2+d}}$$
$$\Big\downarrow \; 6253$$
$$\frac{\sqrt{c^2x^2+1}\int \left(\frac{(a+b\operatorname{arcsinh}(cx))f^2}{\sqrt{c^2x^2+1}}+\frac{2gx(a+b\operatorname{arcsinh}(cx))f}{\sqrt{c^2x^2+1}}+\frac{g^2x^2(a+b\operatorname{arcsinh}(cx))}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{c^2dx^2+d}}$$
$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{c^2x^2+1}\left(-\frac{g^2(a+b\,\mathrm{arcsinh}(cx))^2}{4bc^3}+\frac{2fg\sqrt{c^2x^2+1}(a+b\,\mathrm{arcsinh}(cx))}{c^2}+\frac{g^2x\sqrt{c^2x^2+1}(a+b\,\mathrm{arcsinh}(cx))}{2c^2}+\frac{f^2(a+b\,\mathrm{arcsinh}(cx))^2}{2bc}-\right.}{\sqrt{c^2dx^2+d}}$$

input
```
Int[((f + g*x)^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
(Sqrt[1 + c^2*x^2]*((-2*b*f*g*x)/c - (b*g^2*x^2)/(4*c) + (2*f*g*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x]))/c^2 + (g^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]))/(2*c^2) + (f^2*(a + b*ArcSinh[c*x])^2)/(2*b*c) - (g^2*(a + b*ArcS
inh[c*x])^2)/(4*b*c^3)))/Sqrt[d + c^2*d*x^2]
```

3.48.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

3.48.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(230) = 460$.

Time = 0.76 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.88

method	result
default	$a\left(\frac{f^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+g^2\left(\frac{x\sqrt{c^2d\,x^2+d}}{2c^2d}-\frac{\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^2\sqrt{c^2d}}\right)+\frac{2fg\sqrt{c^2d\,x^2+d}}{c^2d}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\ \mathrm{a}}{4\sqrt{}}\right.$
parts	$a\left(\frac{f^2\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+g^2\left(\frac{x\sqrt{c^2d\,x^2+d}}{2c^2d}-\frac{\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^2\sqrt{c^2d}}\right)+\frac{2fg\sqrt{c^2d\,x^2+d}}{c^2d}\right)+b\left(\frac{\sqrt{d(c^2x^2+1)}\ \mathrm{a}}{4\sqrt{}}\right.$

input
```
int((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+g^2*(1/
2*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2/c^2*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+
d)^(1/2))/(c^2*d)^(1/2))+2*f*g/c^2/d*(c^2*d*x^2+d)^(1/2))+b*(1/4*(d*(c^2*x
^2+1))^(1/2)*arcsinh(c*x)^2*(2*c^2*f^2-g^2)/(c^2*x^2+1)^(1/2)/c^3/d+1/16*(
d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2
+1)^(1/2))*g^2*(-1+2*arcsinh(c*x))/c^3/d/(c^2*x^2+1)+(d*(c^2*x^2+1))^(1/2)
*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(-1+arcsinh(c*x))/c^2/d/(c^2*x^2+1)
+(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*f*g*(arcsinh(c*x)
+1)/c^2/d/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2
*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*g^2*(1+2*arcsinh(c*x))/c^3/d/(c^2*x
^2+1))
```

3.48.5 Fricas [F]

$$\int\frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int\frac{(gx+f)^2(b\,\text{arsinh}\,(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

output
```
integral((a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 + 2*b*f*g*x + b*f^2)*
arcsinh(c*x))/sqrt(c^2*d*x^2 + d), x)
```

3.48.6 Sympy [F]

$$\int \frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(a+b\operatorname{asinh}(cx))\,(f+gx)^2}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate((g*x+f)**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))*(f + g*x)**2/sqrt(d*(c**2*x**2 + 1)), x)
```

3.48.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.48.8 Giac [F]

$$\int \frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{(gx+f)^2(b\operatorname{arsinh}(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((g*x + f)^2*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{{\left(f+g\,x\right)}^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int(((f + g*x)^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int(((f + g*x)^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.49 $\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

3.49.1 Optimal result

Integrand size = 28, antiderivative size = 120

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = -\frac{bgx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{g(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{c^2\sqrt{d+c^2dx^2}} + \frac{f\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}}$$

output
```
g*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)-b*g*x*(c^2*x^2+1)
^(1/2)/c/(c^2*d*x^2+d)^(1/2)+1/2*f*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/
b/c/(c^2*d*x^2+d)^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.32

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx = \frac{2\sqrt{d}g\left(a+ac^2x^2-bcx\sqrt{1+c^2x^2}\right)+2b\sqrt{d}g(1+c^2x^2)\,\text{arcsinh}(cx)+bc\sqrt{d}f\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2+2ac}{2c^2\sqrt{d}\sqrt{d+c^2dx^2}}$$

input
```
Integrate[((f + g*x)*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
(2*Sqrt[d]*g*(a + a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2]) + 2*b*Sqrt[d]*g*(1
+ c^2*x^2)*ArcSinh[c*x] + b*c*Sqrt[d]*f*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 +
 2*a*c*f*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(2*
c^2*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

3.49.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {6260, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{c^2dx^2+d}}dx$$

$$\Big\downarrow \text{6260}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}dx}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{6253}$$

$$\frac{\sqrt{c^2x^2+1}\int \left(\frac{f(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}+\frac{gx(a+b\text{arcsinh}(cx))}{\sqrt{c^2x^2+1}}\right)dx}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c^2x^2+1}\left(\frac{g\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{c^2}+\frac{f(a+b\text{arcsinh}(cx))^2}{2bc}-\frac{bgx}{c}\right)}{\sqrt{c^2dx^2+d}}$$

input
```
Int[((f + g*x)*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

output
```
(Sqrt[1 + c^2*x^2]*(-((b*g*x)/c) + (g*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
]))/c^2 + (f*(a + b*ArcSinh[c*x])^2)/(2*b*c)))/Sqrt[d + c^2*d*x^2]
```

3.49.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

3.49.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(108) = 216.

Time = 0.83 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.89

method	result
default	$\frac{af\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{ag\sqrt{c^2d\,x^2+d}}{c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,f\,\mathrm{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,cd}+\frac{\sqrt{d(c^2x^2+1)}\,\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)g(-1+a}{2c^2d(c^2x^2+1)}\right.$
parts	$\frac{af\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{ag\sqrt{c^2d\,x^2+d}}{c^2d}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,f\,\mathrm{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,cd}+\frac{\sqrt{d(c^2x^2+1)}\,\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)g(-1+a}{2c^2d(c^2x^2+1)}\right.$

input
```
int((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
a*f*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+a*g/c^2/d*
(c^2*d*x^2+d)^(1/2)+b*(1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*f*a
rcsinh(c*x)^2+1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*
g*(-1+arcsinh(c*x))/c^2/d/(c^2*x^2+1)+1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c
*x*(c^2*x^2+1)^(1/2)+1)*g*(arcsinh(c*x)+1)/c^2/d/(c^2*x^2+1))
```

3.49.5 Fricas [F]

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(gx+f)(b\,\text{arsinh}\,(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

output
```
integral((a*g*x + a*f + (b*g*x + b*f)*arcsinh(c*x))/sqrt(c^2*d*x^2 + d), x
)
```

3.49.6 Sympy [F]

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(a+b\,\text{asinh}\,(cx))\,(f+gx)}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate((g*x+f)*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))*(f + g*x)/sqrt(d*(c**2*x**2 + 1)), x)
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int \frac{(f+gx)(a+b\text{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\frac{bf\,\text{arsinh}\,(cx)^2}{2\,c\sqrt{d}}-\frac{bgx}{c\sqrt{d}}+\frac{af\,\text{arsinh}\,(cx)}{c\sqrt{d}}+\frac{\sqrt{c^2dx^2+d}bg\,\text{arsinh}\,(cx)}{c^2d}+\frac{\sqrt{c^2dx^2+d}ag}{c^2d}$$

input
```
integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

output
```
1/2*b*f*arcsinh(c*x)^2/(c*sqrt(d)) - b*g*x/(c*sqrt(d)) + a*f*arcsinh(c*x)/
(c*sqrt(d)) + sqrt(c^2*d*x^2 + d)*b*g*arcsinh(c*x)/(c^2*d) + sqrt(c^2*d*x^
2 + d)*a*g/(c^2*d)
```

3.49.8 Giac [F]

$$\int \frac{(f+gx)(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(gx+f)(b\operatorname{arsinh}(cx)+a)}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((g*x + f)*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)(a+b\operatorname{arcsinh}(cx))}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{(f+g\,x)\,\left(a+b\operatorname{asinh}(c\,x)\right)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int(((f + g*x)*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int(((f + g*x)*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.50 $\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx$

3.50.1 Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{d+c^2dx^2}}$$

output
```
1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)
```

3.50.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\frac{b\sqrt{1+c^2x^2}\text{arcsinh}(cx)^2}{2c\sqrt{d(1+c^2x^2)}}+\frac{a\text{arctanh}\left(\frac{c\sqrt{d}x}{\sqrt{d+c^2dx^2}}\right)}{c\sqrt{d}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

output
```
(b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(2*c*Sqrt[d*(1 + c^2*x^2)]) + (a*ArcT
anh[(c*Sqrt[d]*x)/Sqrt[d + c^2*d*x^2]])/(c*Sqrt[d])
```

3.50.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}\,dx$$

$$\Big\downarrow \; 6198$$

$$\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{2bc\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

output
```
(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

3.50.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

3.50.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

method	result	size
default	$\frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,dc}$	77
parts	$\frac{a\ln\left(\frac{c^2dx}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{\sqrt{c^2d}}+\frac{b\sqrt{d(c^2x^2+1)}\,\text{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,dc}$	77

input
```
int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*ln(c^2*d*x/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d/c*arcsinh(c*x)^2
```

3.50.5 Fricas [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

3.50.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx = \int \frac{a+b\,\text{asinh}\,(cx)}{\sqrt{d\,(c^2x^2+1)}}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx = \frac{b\,\text{arsinh}\,(cx)^2}{2\,c\sqrt{d}} + \frac{a\,\text{arsinh}\,(cx)}{c\sqrt{d}}$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
1/2*b*arcsinh(c*x)^2/(c*sqrt(d)) + a*arcsinh(c*x)/(c*sqrt(d))
```

3.50.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{a+b\,\text{asinh}(c\,x)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2), x)
```

3.51 $\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx$

3.51.1 Optimal result

Integrand size = 30, antiderivative size = 325

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx &= \frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}}\\
&\quad-\frac{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}}\\
&\quad+\frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}}\\
&\quad-\frac{b\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)
))*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(1/2)/(c^2*d*x^2+d)^(1/2)-(a+b*arcsinh(
c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+1
)^(1/2)/(c^2*f^2+g^2)^(1/2)/(c^2*d*x^2+d)^(1/2)+b*polylog(2,-(c*x+(c^2*x^2
+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(1
/2)/(c^2*d*x^2+d)^(1/2)-b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f
^2+g^2)^(1/2)))*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(1/2)/(c^2*d*x^2+d)^(1/2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.74

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx$$
$$= \frac{-\frac{a\text{arctanh}\left(\frac{\sqrt{d}(g-c^2fx)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}}\right)}{\sqrt{d}} + \frac{b\sqrt{1+c^2x^2}\left(\text{arcsinh}(cx)\left(\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)-\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)\right)+\text{PolyLog}\left(2,\frac{e^{\text{arcs}\cdots}}{-cf+\cdots}\right.\right.}{\sqrt{d+c^2dx^2}}}{\sqrt{c^2f^2+g^2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/((f + g*x)*Sqrt[d + c^2*d*x^2]),x]
```

output
```
(-((a*ArcTanh[(Sqrt[d]*(g - c^2*f*x))/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*
x^2])])/Sqrt[d]) + (b*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(Log[1 + (E^ArcSinh[
c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] - Log[1 + (E^ArcSinh[c*x]*g)/(c*f + S
qrt[c^2*f^2 + g^2])]) + PolyLog[2, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f
^2 + g^2])] - PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))
]))/Sqrt[d + c^2*d*x^2])/Sqrt[c^2*f^2 + g^2]
```

3.51.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.73, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6260, 6258, 3042, 3803, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}(f+gx)}dx$$
$$\Big\downarrow \text{ 6260}$$
$$\frac{\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{c^2x^2+1}}dx}{\sqrt{c^2dx^2+d}}$$
$$\Big\downarrow \text{ 6258}$$
$$\frac{\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{cf+cgx}d\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}$$
$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{cf-ig\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 3803

$$\frac{2\sqrt{c^2x^2+1}\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2ce^{\text{arcsinh}(cx)}f+e^{2\text{arcsinh}(cx)}g-g}d\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 2694

$$\frac{2\sqrt{c^2x^2+1}\left(\frac{g\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2\left(cf+e^{\text{arcsinh}(cx)}g-\sqrt{c^2f^2+g^2}\right)}d\text{arcsinh}(cx)}{\sqrt{c^2f^2+g^2}}-\frac{g\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2\left(cf+e^{\text{arcsinh}(cx)}g+\sqrt{c^2f^2+g^2}\right)}d\text{arcsinh}(cx)}{\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 27

$$\frac{2\sqrt{c^2x^2+1}\left(\frac{g\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cf+e^{\text{arcsinh}(cx)}g-\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)}{2\sqrt{c^2f^2+g^2}}-\frac{g\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cf+e^{\text{arcsinh}(cx)}g+\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)}{2\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}}$$

$\Big\downarrow$ 2620

$$\frac{2\sqrt{c^2x^2+1}\left(\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\text{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g}-\frac{b\int \log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)d\text{arcsinh}(cx)}{g}\right)}{2\sqrt{c^2f^2+g^2}}-\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\text{arcsi}\cdots}}{\sqrt{c^2f^2\cdots}}\right.\cdots}{g}\cdots\right.}{\cdots}\cdots\right.}{\sqrt{c^2dx^2+d}\cdots}$$

$\Big\downarrow$ 2715

$$\frac{2\sqrt{c^2x^2+1}\left(\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\text{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g}-\frac{b\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)de^{\text{arcsinh}(cx)}}{g}\right)}{2\sqrt{c^2f^2+g^2}}-\frac{g\left(\frac{(a+b\text{arcsinh}(cx)\cdots}{\cdots}\right.\cdots}{\cdots}\cdots\right.}{\sqrt{c^2dx^2+d}\cdots}$$

$\Big\downarrow$ 2838

3.51. $\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx$

$$\frac{2\sqrt{c^2x^2+1}\left(\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\text{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g}+\frac{b\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g}\right)}{2\sqrt{c^2f^2+g^2}}-\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\text{arcsinh}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+1\right)}{g}\cdots\right)}{2\sqrt{c^2f^2+g^2}}\cdots\right)}{\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/((f + g*x)*Sqrt[d + c^2*d*x^2]),x]
```

output
```
(2*Sqrt[1 + c^2*x^2]*((g*(((a + b*ArcSinh[c*x])*Log[1 + (E^ArcSinh[c*x]*g)
/(c*f - Sqrt[c^2*f^2 + g^2])])/g + (b*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f
 - Sqrt[c^2*f^2 + g^2]))])/g))/(2*Sqrt[c^2*f^2 + g^2]) - (g*(((a + b*ArcSi
nh[c*x])*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/g + (b*P
olyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/g))/(2*Sqrt[
c^2*f^2 + g^2])))/Sqrt[d + c^2*d*x^2]
```

3.51.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

3.51. $\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx$

3.51.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.63

method	result
default	$-\frac{a\ln\left(\frac{\frac{2d\left(c^{2}f^{2}+g^{2}\right)}{g^{2}}-\frac{2c^{2}df\left(x+\frac{f}{g}\right)}{g}+2\sqrt{\frac{d\left(c^{2}f^{2}+g^{2}\right)}{g^{2}}}\sqrt{\left(x+\frac{f}{g}\right)^{2}c^{2}d-\frac{2c^{2}df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^{2}f^{2}+g^{2}\right)}{g^{2}}}}{x+\frac{f}{g}}\right)}{g\sqrt{\frac{d\left(c^{2}f^{2}+g^{2}\right)}{g^{2}}}}+b\left(\frac{\sqrt{d(c^{2}x^{2}+1)}\,\sqrt{c^{2}f^{2}+g^{2}}\,\sqrt{}}{}\right.$
parts	$-\frac{a\ln\left(\frac{\frac{2d\left(c^{2}f^{2}+g^{2}\right)}{g^{2}}-\frac{2c^{2}df\left(x+\frac{f}{g}\right)}{g}+2\sqrt{\frac{d\left(c^{2}f^{2}+g^{2}\right)}{g^{2}}}\sqrt{\left(x+\frac{f}{g}\right)^{2}c^{2}d-\frac{2c^{2}df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^{2}f^{2}+g^{2}\right)}{g^{2}}}}{x+\frac{f}{g}}\right)}{g\sqrt{\frac{d\left(c^{2}f^{2}+g^{2}\right)}{g^{2}}}}+b\left(\frac{\sqrt{d(c^{2}x^{2}+1)}\,\sqrt{c^{2}f^{2}+g^{2}}\,\sqrt{}}{}\right.$

input
```
int((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-a/g/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+
f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*
(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))+b*((d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^
(1/2)*(c^2*x^2+1)^(1/2)/d/(c^4*f^2*x^2+c^2*g^2*x^2+c^2*f^2+g^2)*arcsinh(c*
x)*(ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2
+g^2)^(1/2)))-ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+
(c^2*f^2+g^2)^(1/2))))+(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)*(c^2*x^2+
1)^(1/2)/d/(c^4*f^2*x^2+c^2*g^2*x^2+c^2*f^2+g^2)*(dilog((-(c*x+(c^2*x^2+1)
^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-dilog(((c*x
+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2))))
)
```

3.51.5 Fricas [F]

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx=\int \frac{b\operatorname{arsinh}(cx)+a}{\sqrt{c^2dx^2+d}(gx+f)}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*g*x^3 + c^2*d*f*x
^2 + d*g*x + d*f), x)
```

3.51.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx=\int \frac{a+b\,\text{asinh}\,(cx)}{\sqrt{d\,(c^2x^2+1)}\,(f+gx)}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(g*x+f)/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(sqrt(d*(c**2*x**2 + 1))*(f + g*x)), x)
```

3.51.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}(gx+f)}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)), x)
```

3.51.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx=\int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}(gx+f)}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)), x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c\,x)}{(f+g\,x)\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))/((f + g*x)*(d + c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/((f + g*x)*(d + c^2*d*x^2)^(1/2)), x)
```

3.52 $\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx$

3.52.1 Optimal result

Integrand size = 30, antiderivative size = 444

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx =& -\frac{g(1+c^2x^2)\,(a+b\text{arcsinh}(cx))}{(c^2f^2+g^2)\,(f+gx)\sqrt{d+c^2dx^2}}\\
&+\frac{c^2f\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}}\\
&-\frac{c^2f\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}}\\
&+\frac{bc\sqrt{1+c^2x^2}\log(f+gx)}{(c^2f^2+g^2)\sqrt{d+c^2dx^2}}\\
&+\frac{bc^2f\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}}\\
&-\frac{bc^2f\sqrt{1+c^2x^2}\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{(c^2f^2+g^2)^{3/2}\sqrt{d+c^2dx^2}}\end{aligned}$$

output
```
-g*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(c^2*f^2+g^2)/(g*x+f)/(c^2*d*x^2+d)^(1/2
)+b*c*ln(g*x+f)*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)/(c^2*d*x^2+d)^(1/2)+c^2*f*
(a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)
))*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(3/2)/(c^2*d*x^2+d)^(1/2)-c^2*f*(a+b*ar
csinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2
*x^2+1)^(1/2)/(c^2*f^2+g^2)^(3/2)/(c^2*d*x^2+d)^(1/2)+b*c^2*f*polylog(2,-(
c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+1)^(1/2)/(c^2
*f^2+g^2)^(3/2)/(c^2*d*x^2+d)^(1/2)-b*c^2*f*polylog(2,-(c*x+(c^2*x^2+1)^(1
/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(3/2)/(c
^2*d*x^2+d)^(1/2)
```

3.52.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.57

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx$$

$$= \frac{c\sqrt{1+c^2x^2}\left(-\frac{g\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}{cf+cgx}+b\log(f+gx)+\frac{cf\left((a+b\text{arcsinh}(cx))\left(\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)-\log\left(1+\frac{e^{\text{arcs}}}{cf+\sqrt{}}\right.\right.\right.}{}\right.}{(c^2f^2+g^2)\sqrt{d+c^2dx^2}}$$

input
```
Integrate[(a + b*ArcSinh[c*x])/((f + g*x)^2*Sqrt[d + c^2*d*x^2]),x]
```

output
```
(c*Sqrt[1 + c^2*x^2]*(-((g*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c*f +
c*g*x)) + b*Log[f + g*x] + (c*f*((a + b*ArcSinh[c*x])*(Log[1 + (E^ArcSinh[
c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] - Log[1 + (E^ArcSinh[c*x]*g)/(c*f + S
qrt[c^2*f^2 + g^2])]) + b*PolyLog[2, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2
*f^2 + g^2])] - b*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^
2]))]))/Sqrt[c^2*f^2 + g^2]))/((c^2*f^2 + g^2)*Sqrt[d + c^2*d*x^2])
```

3.52.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.74, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {6260, 6258, 3042, 3805, 3042, 3147, 16, 3803, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}(f+gx)^2}dx$$

$$\Big\downarrow \ 6260$$

$$\frac{\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{c^2x^2+1}}dx}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 6258$$

$$\frac{c\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{(cf+cgx)^2}d\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 3042$$

$$\frac{c\sqrt{c^2x^2+1}\int \frac{a+b\text{arcsinh}(cx)}{(cf-ig\sin(i\text{arcsinh}(cx)))^2}d\text{arcsinh}(cx)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 3805$$

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{cf\int \frac{a+b\text{arcsinh}(cx)}{cf+cgx}d\text{arcsinh}(cx)}{c^2f^2+g^2}+\frac{bg\int \frac{\sqrt{c^2x^2+1}}{cf+cgx}d\text{arcsinh}(cx)}{c^2f^2+g^2}-\frac{g\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2f^2+g^2)(cf+cgx)}\right)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 3042$$

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{cf\int \frac{a+b\text{arcsinh}(cx)}{cf-ig\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{c^2f^2+g^2}+\frac{bg\int \frac{\cos(i\text{arcsinh}(cx))}{cf-ig\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{c^2f^2+g^2}-\frac{g\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2f^2+g^2)(cf+cgx)}\right)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 3147$$

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{cf\int \frac{a+b\text{arcsinh}(cx)}{cf-ig\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{c^2f^2+g^2}+\frac{b\int \frac{1}{cf+cgx}d(cgx)}{c^2f^2+g^2}-\frac{g\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2f^2+g^2)(cf+cgx)}\right)}{\sqrt{c^2dx^2+d}}$$

$$\Big\downarrow \ 16$$

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{cf\int \frac{a+b\text{arcsinh}(cx)}{cf-ig\sin(i\text{arcsinh}(cx))}d\text{arcsinh}(cx)}{c^2f^2+g^2}-\frac{g\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2f^2+g^2)(cf+cgx)}+\frac{b\log(cf+cgx)}{c^2f^2+g^2}\right)}{\sqrt{c^2dx^2+d}}$$

$\downarrow$ 3803

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{2cf\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2ce^{\text{arcsinh}(cx)}f+e^{2\text{arcsinh}(cx)}g-g}d\text{arcsinh}(cx)}{c^2f^2+g^2}-\frac{g\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}{(c^2f^2+g^2)(cf+cgx)}+\frac{b\log(cf+cgx)}{c^2f^2+g^2}\right)}{\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2694

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{2cf\left(\frac{g\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2\left(cf+e^{\text{arcsinh}(cx)}g-\sqrt{c^2f^2+g^2}\right)}d\text{arcsinh}(cx)}{\sqrt{c^2f^2+g^2}}-\frac{g\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{2\left(cf+e^{\text{arcsinh}(cx)}g+\sqrt{c^2f^2+g^2}\right)}d\text{arcsinh}(cx)}{\sqrt{c^2f^2+g^2}}\right)}{c^2f^2+g^2}-\frac{g\sqrt{c^2x^2+1}(a}{(c^2f^2+}\right.}{\sqrt{c^2dx^2+d}}$$

$\downarrow$ 27

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{2cf\left(\frac{g\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cf+e^{\text{arcsinh}(cx)}g-\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)}{2\sqrt{c^2f^2+g^2}}-\frac{g\int \frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))}{cf+e^{\text{arcsinh}(cx)}g+\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)}{2\sqrt{c^2f^2+g^2}}\right)}{c^2f^2+g^2}-\frac{g\sqrt{c^2x^2+1}(a+b}{(c^2f^2+g^2)(}\right.}{\sqrt{c^2dx^2+d}}$$

$\downarrow$ 2620

3.52. $\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx$

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{2cf\left(\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\text{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g}-\frac{b\int\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)d\text{arcsinh}(cx)}{g}\right)}{2\sqrt{c^2f^2+g^2}}-\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\text{ar}\cdots}}{\sqrt{c^2\cdots}}\right)}{g}\cdots\right.}{\cdots}\right.}{c^2f^2+g^2}\cdots\right.}{\sqrt{c^2dx^2+d}\cdots}$$

$\downarrow$ 2715

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{2cf\left(\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\text{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g}-\frac{b\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)de^{\text{arcsinh}(cx)}}{g}\right)}{2\sqrt{c^2f^2+g^2}}-\frac{g\left(\frac{(a+b\text{arcsinh}(\cdots}{\cdots}\right.}{\cdots}\right.}{c^2f^2+g^2}\cdots\right.}{\sqrt{c^2dx^2\cdots}}$$

$\downarrow$ 2838

3.52. $\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx$

$$\frac{c\sqrt{c^2x^2+1}\left(\frac{2cf\left(\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\text{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g}+\frac{b\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g}\right)}{2\sqrt{c^2f^2+g^2}}-\frac{g\left(\frac{(a+b\text{arcsinh}(cx))\log\left(\frac{ge^{\text{arcsinh}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+\ldots\right)}{g}\ldots\right)}{2\sqrt{c^2f^2\ldots}}\right)}{c^2f^2+g^2}\ldots\right)}{\sqrt{c^2dx^2+d}}$$

input
```
Int[(a + b*ArcSinh[c*x])/((f + g*x)^2*Sqrt[d + c^2*d*x^2]),x]
```

output
```
(c*Sqrt[1 + c^2*x^2]*(-((g*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/((c^2*f
^2 + g^2)*(c*f + c*g*x))) + (b*Log[c*f + c*g*x])/(c^2*f^2 + g^2) + (2*c*f*
((g*(((a + b*ArcSinh[c*x])*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2
+ g^2])])/g + (b*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2
]))])/g))/(2*Sqrt[c^2*f^2 + g^2]) - (g*(((a + b*ArcSinh[c*x])*Log[1 + (E^A
rcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/g + (b*PolyLog[2, -((E^ArcSin
h[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/g))/(2*Sqrt[c^2*f^2 + g^2])))/(c^
2*f^2 + g^2)))/Sqrt[d + c^2*d*x^2]
```

3.52.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.52. $\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx$

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

rule 6260
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p]   Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ
[p - 1/2] &&  !GtQ[d, 0]
```

3.52.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1769 vs. $2(442) = 884$.

Time = 0.85 (sec) , antiderivative size = 1770, normalized size of antiderivative = 3.99

method	result	size
default	Expression too large to display	1770
parts	Expression too large to display	1770

input
```
int((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-a/d/(c^2*f^2+g^2)/(x+f/g)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2
+g^2)/g^2)^(1/2)-a/g*c^2*f/(c^2*f^2+g^2)/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2
*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x
+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))+b*(
d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)/(c^2*f^2+g^2)/(g*x+f)*x^3*
c^4*f-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*f^2+g^2)/(g*x+f)*x*c^2*f
-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)/(c^2*f^2+g^2)/(g*x+f)*
x^2*c^2*g+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)^(1/2)/(c^2*f^
2+g^2)/(g*x+f)*x*c*g+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)/(c
^2*f^2+g^2)/(g*x+f)*x*c^2*f+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^
2+1)^(1/2)/(c^2*f^2+g^2)/(g*x+f)*c*f-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/
d/(c^2*x^2+1)/(c^2*f^2+g^2)/(g*x+f)*g+b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^
(1/2)/d/(c^6*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g
^4)*c^3*ln((c*x+(c^2*x^2+1)^(1/2))^2*g+2*c*f*(c*x+(c^2*x^2+1)^(1/2))-g)*f^
2-2*b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^2*g^2
*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c^3*ln(c*x+(c^2*x^2+1)^(1/2))*
f^2+b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^2*g^2
*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c^2*ln((-(c*x+(c^2*x^2+1)^(1/2
))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))*arcsinh(c*x)*(c^
2*f^2+g^2)^(1/2)*f-b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4...
```

3.52.5 Fricas [F]

$$\int \frac{a + b\text{arcsinh}(cx)}{(f + gx)^2\sqrt{d + c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx) + a}{\sqrt{c^2dx^2 + d}(gx + f)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*g^2*x^4 + 2*c^2*d
*f*g*x^3 + 2*d*f*g*x + d*f^2 + (c^2*d*f^2 + d*g^2)*x^2), x)
```

3.52.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx = \int \frac{a+b\,\text{asinh}\,(cx)}{\sqrt{d\,(c^2x^2+1)}\,(f+gx)^2}\,dx$$

input
```
integrate((a+b*asinh(c*x))/(g*x+f)**2/(c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))/(sqrt(d*(c**2*x**2 + 1))*(f + g*x)**2), x)
```

3.52.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}(gx+f)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)^2), x)
```

3.52.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx = \int \frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}(gx+f)^2}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)^2), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx = \int \frac{a+b\,\text{asinh}(c\,x)}{(f+g\,x)^2\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(c*x))/((f + g*x)^2*(d + c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(c*x))/((f + g*x)^2*(d + c^2*d*x^2)^(1/2)), x)
```

3.53 $\int \frac{(a+b\,\mathrm{arcsinh}(cx))^n \log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$

3.53.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(a+b\,\mathrm{arcsinh}(cx))^n \log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx = \mathrm{Int}\left(\frac{(a+b\,\mathrm{arcsinh}(cx))^n \log(h(f+gx)^m)}{\sqrt{1+c^2x^2}},x\right)$$

output
```
Unintegrable((a+b*arcsinh(c*x))^n*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

3.53.2 Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\,\mathrm{arcsinh}(cx))^n \log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(a+b\,\mathrm{arcsinh}(cx))^n \log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$$

input
```
Integrate[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

output
```
Integrate[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2], x
]
```

3.53.3 Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6272}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^n\log\left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}}dx$$
$$\Big\downarrow \text{ 6272}$$
$$\int \frac{(a+b\text{arcsinh}(cx))^n\log\left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}}dx$$

input
```
Int[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

output
```
$Aborted
```

3.53.3.1 Defintions of rubi rules used

rule 6272
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.53.4 Maple [N/A] (verified)

Not integrable

Time = 2.50 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\,\text{arcsinh}\,(cx))^n\ln\left(h(gx+f)^m\right)}{\sqrt{c^2x^2+1}}dx$$

input
```
int((a+b*arcsinh(c*x))^n*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))^n*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

3.53.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^n\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^n\log\left((gx+f)^mh\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algor
ithm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

3.53.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^n\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asinh(c*x))**n*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
```

output
```
Timed out
```

3.53.7 Maxima [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(cx))^n\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx=\int \frac{(b\operatorname{arsinh}(cx)+a)^n\log\left((gx+f)^mh\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

3.53.8 Giac [N/A]

Not integrable

Time = 73.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^n \log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^n \log((gx+f)^m h)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

3.53.9 Mupad [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(cx))^n \log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\ln\left(h\,(f+g\,x)^m\right)\,(a+b\,\text{asinh}(c\,x))^n}{\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^n)/(c^2*x^2 + 1)^(1/2),x)
```

output
```
int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^n)/(c^2*x^2 + 1)^(1/2), x)
```

3.54 $\int \frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$

3.54.1 Optimal result

Integrand size = 34, antiderivative size = 438

$$\begin{aligned}
&\int \frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx \\
&= \frac{m(a+b\text{arcsinh}(cx))^4}{12b^2c} - \frac{m(a+b\text{arcsinh}(cx))^3\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{3bc} \\
&\quad - \frac{m(a+b\text{arcsinh}(cx))^3\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{3bc} + \frac{(a+b\text{arcsinh}(cx))^3\log(h(f+gx)^m)}{3bc} \\
&\quad - \frac{m(a+b\text{arcsinh}(cx))^2\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m(a+b\text{arcsinh}(cx))^2\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{2bm(a+b\text{arcsinh}(cx))\text{PolyLog}\left(3,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{2bm(a+b\text{arcsinh}(cx))\text{PolyLog}\left(3,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{2b^2m\,\text{PolyLog}\left(4,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{2b^2m\,\text{PolyLog}\left(4,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}
\end{aligned}$$

output
```
1/12*m*(a+b*arcsinh(c*x))^4/b^2/c+1/3*(a+b*arcsinh(c*x))^3*ln(h*(g*x+f)^m)
/b/c-1/3*m*(a+b*arcsinh(c*x))^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f
^2+g^2)^(1/2)))/b/c-1/3*m*(a+b*arcsinh(c*x))^3*ln(1+(c*x+(c^2*x^2+1)^(1/2)
)*g/(c*f+(c^2*f^2+g^2)^(1/2)))/b/c-m*(a+b*arcsinh(c*x))^2*polylog(2,-(c*x+
(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*(a+b*arcsinh(c*x))^2*p
olylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c+2*b*m*(a+
b*arcsinh(c*x))*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1
/2)))/c+2*b*m*(a+b*arcsinh(c*x))*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f
+(c^2*f^2+g^2)^(1/2)))/c-2*b^2*m*polylog(4,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f
-(c^2*f^2+g^2)^(1/2)))/c-2*b^2*m*polylog(4,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f
+(c^2*f^2+g^2)^(1/2)))/c
```

3.54.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\text{arcsinh}(cx))^2\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx =$$

$$-\frac{-\frac{m(a+b\text{arcsinh}(cx))^4}{4b}+m(a+b\text{arcsinh}(cx))^3\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)+m(a+b\text{arcsinh}(cx))^3\log\left(1+\cdots\right.}{\cdots}$$

input
```
Integrate[((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

output
```
-1/3*(-1/4*(m*(a + b*ArcSinh[c*x])^4)/b + m*(a + b*ArcSinh[c*x])^3*Log[1 +
 (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] + m*(a + b*ArcSinh[c*x])^
3*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])] - (a + b*ArcSinh
[c*x])^3*Log[h*(f + g*x)^m] + 3*b*m*((a + b*ArcSinh[c*x])^2*PolyLog[2, (E^
ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])] - 2*b*(a + b*ArcSinh[c*x])
*PolyLog[3, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])] + 2*b^2*Pol
yLog[4, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])]) + 3*b*m*((a +
b*ArcSinh[c*x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^
2]))] - 2*b*(a + b*ArcSinh[c*x])*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f + Sq
rt[c^2*f^2 + g^2]))] + 2*b^2*PolyLog[4, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c
^2*f^2 + g^2]))]))/(b*c)
```

3.54.3 Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6261, 6242, 6095, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \text{6261}$$

$$\frac{(a+b\text{arcsinh}(cx))^3\log(h(f+gx)^m)}{3bc}-\frac{gm\int\frac{(a+b\text{arcsinh}(cx))^3}{f+gx}dx}{3bc}$$

$$\Big\downarrow \text{6242}$$

$$\frac{(a+b\text{arcsinh}(cx))^3\log(h(f+gx)^m)}{3bc}-\frac{gm\int\frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^3}{cf+cgx}d\text{arcsinh}(cx)}{3bc}$$

$$\Big\downarrow \text{6095}$$

$$\frac{(a+b\text{arcsinh}(cx))^3\log(h(f+gx)^m)}{3bc}-\frac{gm\left(\int\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))^3}{cf+e^{\text{arcsinh}(cx)}g-\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)+\int\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))^3}{cf+e^{\text{arcsinh}(cx)}g+\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)-\frac{(a+b\text{arcsinh}(cx))^4}{4bg}\right)}{3bc}$$

$$\Big\downarrow \text{2620}$$

$$\frac{(a+b\text{arcsinh}(cx))^3\log(h(f+gx)^m)}{3bc}-\frac{gm\left(-\frac{3b\int(a+b\text{arcsinh}(cx))^2\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)d\text{arcsinh}(cx)}{g}-\frac{3b\int(a+b\text{arcsinh}(cx))^2\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}+1\right)d\text{arcsinh}(cx)}{g}+\right.}{3bc}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(a+b\text{arcsinh}(cx))^3\log(h(f+gx)^m)}{3bc}-\frac{gm\left(-\frac{3b\left(2b\int(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)d\text{arcsinh}(cx)-(a+b\text{arcsinh}(cx))^2\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)\right)}{g}-\frac{3b\left(\right.}{}\right.}{}$$

$$\Big\downarrow \text{7163}$$

$$\frac{(a+b\operatorname{arcsinh}(cx))^3\log\left(h(f+gx)^m\right)}{3bc}-$$
$$gm\left(-\frac{3b\left(2b\left((a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)-b\int\operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)d\operatorname{arcsinh}(cx)\right)-(a+b\operatorname{arcsinh}(cx))^2\operatorname{Pol}\right.}{g}\right.$$

$\big\downarrow$ 2720

$$\frac{(a+b\operatorname{arcsinh}(cx))^3\log\left(h(f+gx)^m\right)}{3bc}-$$
$$gm\left(-\frac{3b\left(2b\left((a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)-b\int e^{-\operatorname{arcsinh}(cx)}\operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)de^{\operatorname{arcsinh}(cx)}\right)-(a+b\operatorname{arcsi}\right.}{g}\right.$$

$\big\downarrow$ 7143

$$\frac{(a+b\operatorname{arcsinh}(cx))^3\log\left(h(f+gx)^m\right)}{3bc}-$$
$$gm\left(-\frac{3b\left(2b\left((a+b\operatorname{arcsinh}(cx))\operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)-b\operatorname{PolyLog}\left(4,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)\right)-(a+b\operatorname{arcsinh}(cx))^2\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsi}}}{cf-\sqrt{}}\right.\right.}{g}\right.$$

input
```
Int[((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

output
```
((a + b*ArcSinh[c*x])^3*Log[h*(f + g*x)^m])/(3*b*c) - (g*m*(-1/4*(a + b*Ar
cSinh[c*x])^4/(b*g) + ((a + b*ArcSinh[c*x])^3*Log[1 + (E^ArcSinh[c*x]*g)/(
c*f - Sqrt[c^2*f^2 + g^2])])/g + ((a + b*ArcSinh[c*x])^3*Log[1 + (E^ArcSin
h[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/g - (3*b*(-((a + b*ArcSinh[c*x])^2
*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))]) + 2*b*((a
+ b*ArcSinh[c*x])*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^
2]))] - b*PolyLog[4, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])))
/g - (3*b*(-((a + b*ArcSinh[c*x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f +
 Sqrt[c^2*f^2 + g^2]))]) + 2*b*((a + b*ArcSinh[c*x])*PolyLog[3, -((E^ArcSi
nh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))] - b*PolyLog[4, -((E^ArcSinh[c*x]*
g)/(c*f + Sqrt[c^2*f^2 + g^2]))])))/g))/(3*b*c)
```

3.54. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$

3.54.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

rule 6261
```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcSinh[(c_.)*(x_)]*(b_
.))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[Log[h*(f + g*x)^m]*
((a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Simp[g*(m/(b*c*S
qrt[d]*(n + 1)))   Int[(a + b*ArcSinh[c*x])^(n + 1)/(f + g*x), x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && IGtQ[n
, 0]
```

3.54. $\int \frac{(a+b\operatorname{arcsinh}(cx))^2 \log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.54.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2 \ln\left(h(gx+f)^m\right)}{\sqrt{c^2x^2+1}}dx$$

input
```
int((a+b*arcsinh(c*x))^2*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))^2*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

3.54.5 Fricas [F]

$$\int \frac{(a+b\mathrm{arcsinh}(cx))^2\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\left(b\,\mathrm{arsinh}\,(cx)+a\right)^2\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algor
ithm="fricas")
```

output
```
integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*log((g*x + f)^m*h
)/sqrt(c^2*x^2 + 1), x)
```

3.54.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))^2\log\left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*asinh(c*x))**2*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))**2*log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

3.54.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))^2\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)^2\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)^2*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

3.54.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(cx))^2\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algor
ithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{720,[0,4,1,1,1,1,4,0]%%%}+%%%{-1260,[0,4,1,1,1,1,3,1]%%%}+
%%%{360,[
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arcsinh}(cx))^2 \log\left(h(f + gx)^m\right)}{\sqrt{1 + c^2x^2}} \, dx = \int \frac{\ln\left(h\,(f + g\,x)^m\right)\,\left(a + b\,\text{asinh}(c\,x)\right)^2}{\sqrt{c^2\,x^2 + 1}} \, dx$$

input
```
int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^2)/(c^2*x^2 + 1)^(1/2),x)
```

output
```
int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^2)/(c^2*x^2 + 1)^(1/2), x)
```

3.55 $\int \frac{(a+b\text{arcsinh}(cx))\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$

3.55.1 Optimal result

Integrand size = 32, antiderivative size = 332

$$\begin{aligned}
&\int \frac{(a+b\text{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx \\
&= \frac{m(a+b\text{arcsinh}(cx))^3}{6b^2c} - \frac{m(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{2bc} \\
&\quad - \frac{m(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{2bc} + \frac{(a+b\text{arcsinh}(cx))^2\log\left(h(f+gx)^m\right)}{2bc} \\
&\quad - \frac{m(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad - \frac{m(a+b\text{arcsinh}(cx))\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} \\
&\quad + \frac{bm\,\text{PolyLog}\left(3,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} + \frac{bm\,\text{PolyLog}\left(3,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}
\end{aligned}$$

output
```
1/6*m*(a+b*arcsinh(c*x))^3/b^2/c+1/2*(a+b*arcsinh(c*x))^2*ln(h*(g*x+f)^m)/
b/c-1/2*m*(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^
2+g^2)^(1/2)))/b/c-1/2*m*(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))
*g/(c*f+(c^2*f^2+g^2)^(1/2)))/b/c-m*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^
2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*(a+b*arcsinh(c*x))*polylo
g(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c+b*m*polylog(3,
-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c+b*m*polylog(3,-(c*
x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c
```

3.55.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\text{arcsinh}(cx))\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx = \frac{\frac{m(a+b\text{arcsinh}(cx))^3}{3b} - m(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right) - m(a+b\text{arcsinh}(cx))^2\log\left(1+\frac{e^{\text{arcsinh}}}{cf+\sqrt{c^2}}\right.}{}$$

input
```
Integrate[((a + b*ArcSinh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

output
```
((m*(a + b*ArcSinh[c*x])^3)/(3*b) - m*(a + b*ArcSinh[c*x])^2*Log[1 + (E^Ar
cSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] - m*(a + b*ArcSinh[c*x])^2*Log[
1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])] + (a + b*ArcSinh[c*x])
^2*Log[h*(f + g*x)^m] + 2*b*m*(-((a + b*ArcSinh[c*x])*PolyLog[2, (E^ArcSin
h[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])]) + b*PolyLog[3, (E^ArcSinh[c*x]*
g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])]) + 2*b*m*(-((a + b*ArcSinh[c*x])*PolyLo
g[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]) + b*PolyLog[3, -(
(E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]))/(2*b*c)
```

3.55.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {6261, 6242, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(cx))\log(h(f+gx)^m)}{\sqrt{c^2x^2+1}}\,dx$$

$$\Big\downarrow \text{6261}$$

$$\frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{2bc} - \frac{gm\int \frac{(a+b\text{arcsinh}(cx))^2}{f+gx}dx}{2bc}$$

$$\Big\downarrow \text{6242}$$

$$\frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{2bc} - \frac{gm\int \frac{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))^2}{cf+cgx}d\text{arcsinh}(cx)}{2bc}$$

$$\Big\downarrow \text{6095}$$

$$\frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{2bc}-\frac{gm\left(\int\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))^2}{cf+e^{\text{arcsinh}(cx)}g-\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)+\int\frac{e^{\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))^2}{cf+e^{\text{arcsinh}(cx)}g+\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)-\frac{(a+b\text{arcsinh}(cx))^3}{3bg}\right)}{2bc}$$

$\downarrow$ 2620

$$\frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{2bc}-\frac{gm\left(-\frac{2b\int(a+b\text{arcsinh}(cx))\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)d\text{arcsinh}(cx)}{g}-\frac{2b\int(a+b\text{arcsinh}(cx))\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}+1\right)d\text{arcsinh}(cx)}{g}+\frac{(a\ldots}{\ldots}\right)}{2bc}$$

$\downarrow$ 3011

$$\frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{2bc}-\frac{gm\left(-\frac{2b\left(b\int\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)d\text{arcsinh}(cx)-(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)\right)}{g}-\frac{2b\left(b\int\text{PolyLog}\left(2,-\frac{e^{\ldots}}{cf+\ldots}\right)\ldots\right)}{\ldots}\right)}{\ldots}$$

$\downarrow$ 2720

$$\frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{2bc}-\frac{gm\left(-\frac{2b\left(b\int e^{-\text{arcsinh}(cx)}\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)de^{\text{arcsinh}(cx)}-(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)\right)}{g}-\frac{2b\left(b\int e^{-\text{arc}\ldots}\ldots\right)}{\ldots}\right)}{\ldots}$$

$\downarrow$ 7143

$$\frac{(a+b\text{arcsinh}(cx))^2\log(h(f+gx)^m)}{2bc}-\frac{gm\left(-\frac{2b\left(b\text{PolyLog}\left(3,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)-(a+b\text{arcsinh}(cx))\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)\right)}{g}-\frac{2b\left(b\text{PolyLog}\left(3,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)-(a+b\ldots\right)}{\ldots}\right)}{\ldots}$$

input
```
Int[((a + b*ArcSinh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

3.55. $\int\frac{(a+b\text{arcsinh}(cx))\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}dx$

output
```
((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/(2*b*c) - (g*m*(-1/3*(a + b*Ar
cSinh[c*x])^3/(b*g) + ((a + b*ArcSinh[c*x])^2*Log[1 + (E^ArcSinh[c*x]*g)/(
c*f - Sqrt[c^2*f^2 + g^2])])/g + ((a + b*ArcSinh[c*x])^2*Log[1 + (E^ArcSin
h[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/g - (2*b*(-((a + b*ArcSinh[c*x])*P
olyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))]) + b*PolyLog[
3, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))]))/g - (2*b*(-((a + b
*ArcSinh[c*x])*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])
)]) + b*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]))/g)
)/(2*b*c)
```

3.55.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

rule 6261
```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcSinh[(c_.)*(x_)]*(b_
.))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[Log[h*(f + g*x)^m]*
((a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Simp[g*(m/(b*c*S
qrt[d]*(n + 1)))   Int[(a + b*ArcSinh[c*x])^(n + 1)/(f + g*x), x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && IGtQ[n
, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.55.4 Maple [F]

$$\int \frac{(a + b \operatorname{arcsinh}(cx)) \ln (h(gx + f)^m)}{\sqrt{c^2x^2 + 1}} dx$$

input
```
int((a+b*arcsinh(c*x))*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

output
```
int((a+b*arcsinh(c*x))*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

3.55.5 Fricas [F]

$$\int \frac{(a + b\text{arcsinh}(cx)) \log (h(f + gx)^m)}{\sqrt{1 + c^2x^2}} dx = \int \frac{(b \operatorname{arsinh}(cx) + a) \log ((gx + f)^m h)}{\sqrt{c^2x^2 + 1}} dx$$

input
```
integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorit
hm="fricas")
```

output
```
integral((b*arcsinh(c*x) + a)*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

3.55.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(cx))\log\left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*asinh(c*x))*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral((a + b*asinh(c*x))*log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

3.55.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate((b*arcsinh(c*x) + a)*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

3.55.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \int \frac{(b\,\text{arsinh}\,(cx)+a)\log\left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorit
hm="giac")
```

output
```
integrate((b*arcsinh(c*x) + a)*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx=\int \frac{\ln\left(h\,(f+g\,x)^m\right)\,\left(a+b\,\mathrm{asinh}(c\,x)\right)}{\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int((log(h*(f + g*x)^m)*(a + b*asinh(c*x)))/(c^2*x^2 + 1)^(1/2),x)
```

output
```
int((log(h*(f + g*x)^m)*(a + b*asinh(c*x)))/(c^2*x^2 + 1)^(1/2), x)
```

3.56 $\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$

3.56.1 Optimal result

Integrand size = 24, antiderivative size = 197

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx = \frac{m\operatorname{arcsinh}(cx)^2}{2c} - \frac{m\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\operatorname{arcsinh}(cx)\log\left(1+\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c} + \frac{\operatorname{arcsinh}(cx)\log(h(f+gx)^m)}{c} - \frac{m\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}$$

output
```
1/2*m*arcsinh(c*x)^2/c+arcsinh(c*x)*ln(h*(g*x+f)^m)/c-m*arcsinh(c*x)*ln(1+
(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*arcsinh(c*x)*ln(1
+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c-m*polylog(2,-(c*x+
(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*polylog(2,-(c*x+(c^2*x
^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c
```

3.56.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \frac{m\,\text{arcsinh}(cx)^2}{2c} - \frac{m\,\text{arcsinh}(cx)\log\left(1+\frac{ce^{\text{arcsinh}(cx)}g}{c^2f-c\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\,\text{arcsinh}(cx)\log\left(1+\frac{ce^{\text{arcsinh}(cx)}g}{c^2f+c\sqrt{c^2f^2+g^2}}\right)}{c} + \frac{\text{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c} - \frac{m\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\,\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}$$

input
```
Integrate[Log[h*(f + g*x)^m]/Sqrt[1 + c^2*x^2],x]
```

output
```
(m*ArcSinh[c*x]^2)/(2*c) - (m*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x]*g)/(c
^2*f - c*Sqrt[c^2*f^2 + g^2])])/c - (m*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c
*x]*g)/(c^2*f + c*Sqrt[c^2*f^2 + g^2])])/c + (ArcSinh[c*x]*Log[h*(f + g*x)
^m])/c - (m*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])
/c - (m*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c
```

3.56.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2851, 27, 6242, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}}\,dx$$

$$\Big\downarrow \; 2851$$

$$\frac{\text{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c} - gm\int \frac{\text{arcsinh}(cx)}{c(f+gx)}dx$$

$$\Big\downarrow \; 27$$

$$\frac{\text{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\int\frac{\text{arcsinh}(cx)}{f+gx}dx}{c}$$

$\big\downarrow$ 6242

$$\frac{\text{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\int\frac{\sqrt{c^2x^2+1}\text{arcsinh}(cx)}{cf+cgx}d\text{arcsinh}(cx)}{c}$$

$\big\downarrow$ 6095

$$\frac{\text{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\left(\int\frac{e^{\text{arcsinh}(cx)}\text{arcsinh}(cx)}{cf+e^{\text{arcsinh}(cx)}g-\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)+\int\frac{e^{\text{arcsinh}(cx)}\text{arcsinh}(cx)}{cf+e^{\text{arcsinh}(cx)}g+\sqrt{c^2f^2+g^2}}d\text{arcsinh}(cx)-\frac{\text{arcsinh}(cx)^2}{2g}\right)}{c}$$

$\big\downarrow$ 2620

$$\frac{\text{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\left(-\frac{\int\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)d\text{arcsinh}(cx)}{g}-\frac{\int\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}+1\right)d\text{arcsinh}(cx)}{g}+\frac{\text{arcsinh}(cx)\log\left(\frac{ge^{\text{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g}+\frac{\text{arc}\ldots}{}\right.}{c}$$

$\big\downarrow$ 2715

$$\frac{\text{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\left(-\frac{\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}+1\right)de^{\text{arcsinh}(cx)}}{g}-\frac{\int e^{-\text{arcsinh}(cx)}\log\left(\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}+1\right)de^{\text{arcsinh}(cx)}}{g}+\frac{\text{arcsinh}(cx)\log\left(\ldots\right.}{g}\right.}{c}$$

$\big\downarrow$ 2838

$$\frac{\text{arcsinh}(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\left(\frac{\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g}+\frac{\text{PolyLog}\left(2,-\frac{e^{\text{arcsinh}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g}+\frac{\text{arcsinh}(cx)\log\left(\frac{ge^{\text{arcsinh}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g}+\frac{\text{arcsinh}(cx)\log\left(\frac{ge^{\text{arcsin}\ldots}}{\sqrt{c^2f^2+\ldots}}\right.}{g}\right.}{c}$$

input
```
Int[Log[h*(f + g*x)^m]/Sqrt[1 + c^2*x^2],x]
```

3.56. $\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$

output
```
(ArcSinh[c*x]*Log[h*(f + g*x)^m])/c - (g*m*(-1/2*ArcSinh[c*x]^2/g + (ArcSi
nh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/g + (ArcS
inh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/g + Poly
Log[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))]/g + PolyLog[2, -
((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]/g))/c
```

3.56.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2851
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Simp[b*e*n   Int[SimplifyIntegrand[u/(d + e*x)
, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

3.56.4 Maple [F]

$$\int \frac{\ln \left(h(gx+f)^{m}\right)}{\sqrt{c^{2}x^{2}+1}}dx$$

input
```
int(ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

output
```
int(ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

3.56.5 Fricas [F]

$$\int \frac{\log \left(h(f+gx)^{m}\right)}{\sqrt{1+c^{2}x^{2}}}\,dx=\int \frac{\log \left((gx+f)^{m}h\right)}{\sqrt{c^{2}x^{2}+1}}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

3.56.6 Sympy [F]

$$\int \frac{\log \left(h(f+gx)^{m}\right)}{\sqrt{1+c^{2}x^{2}}}\,dx=\int \frac{\log \left(h(f+gx)^{m}\right)}{\sqrt{c^{2}x^{2}+1}}\,dx$$

input
```
integrate(ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

3.56.7 Maxima [F]

$$\int \frac{\log (h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx=\int \frac{\log \left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

3.56.8 Giac [F]

$$\int \frac{\log (h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx=\int \frac{\log \left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log (h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx=\int \frac{\ln \left(h\,(f+g\,x)^m\right)}{\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int(log(h*(f + g*x)^m)/(c^2*x^2 + 1)^(1/2),x)
```

output
```
int(log(h*(f + g*x)^m)/(c^2*x^2 + 1)^(1/2), x)
```

3.57 $\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\,\mathbf{arcsinh}(cx))}\,dx$

3.57.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \text{Int}\left(\frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))},x\right)$$

output
```
Unintegrable(ln(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

3.57.2 Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx = \int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx$$

input
```
Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

3.57.3 Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6272}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))} \, dx$$
$$\Big\downarrow \text{ 6272}$$
$$\int \frac{\log \left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}(a+b\text{arcsinh}(cx))}dx$$

input
```
Int[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

output
```
$Aborted
```

3.57.3.1 Defintions of rubi rules used

rule 6272
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.57.4 Maple [N/A] (verified)

Not integrable

Time = 1.94 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\ln \left(h(gx+f)^m\right)}{(a+b\,\text{arcsinh}\,(cx))\sqrt{c^2x^2+1}}dx$$

input
```
int(ln(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

output
```
int(ln(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

3.57.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50

$$\int \frac{\log (h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\log \left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorit
hm="fricas")
```

output
```
integral(sqrt(c^2*x^2 + 1)*log((g*x + f)^m*h)/(a*c^2*x^2 + (b*c^2*x^2 + b)
*arcsinh(c*x) + a), x)
```

3.57.6 Sympy [N/A]

Not integrable

Time = 7.69 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\log (h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\log \left(h(f+gx)^m\right)}{(a+b\,\text{asinh}\,(cx))\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate(ln(h*(g*x+f)**m)/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(log(h*(f + g*x)**m)/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

3.57.7 Maxima [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\log (h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))}\,dx=\int \frac{\log \left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(log((g*x + f)^m*h)/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

3.57.8 Giac [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\log (h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))} \, dx = \int \frac{\log \left((gx+f)^m h\right)}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)} \, dx$$

input
```
integrate(log(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(log((g*x + f)^m*h)/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

3.57.9 Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\log (h(f+gx)^m)}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))} \, dx = \int \frac{\ln \left(h\,{\left(f+g\,x\right)}^m\right)}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{c^2\,x^2+1}} \, dx$$

input
```
int(log(h*(f + g*x)^m)/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

output
```
int(log(h*(f + g*x)^m)/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.58 $\int x^3 \text{arcsinh}(a+bx)\,dx$

3.58.1 Optimal result

Integrand size = 10, antiderivative size = 131

$$\int x^3 \text{arcsinh}(a+bx)\,dx = \frac{7ax^2\sqrt{1+(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1+(a+bx)^2}}{16b} - \frac{(4a(16-19a^2)-(9-26a^2)(a+bx))\sqrt{1+(a+bx)^2}}{96b^4} - \frac{(3-24a^2+8a^4)\,\text{arcsinh}(a+bx)}{32b^4} + \frac{1}{4}x^4\text{arcsinh}(a+bx)$$

output
```
-1/32*(8*a^4-24*a^2+3)*arcsinh(b*x+a)/b^4+1/4*x^4*arcsinh(b*x+a)+7/48*a*x^
2*(1+(b*x+a)^2)^(1/2)/b^2-1/16*x^3*(1+(b*x+a)^2)^(1/2)/b-1/96*(4*a*(-19*a^
2+16)-(-26*a^2+9)*(b*x+a))*(1+(b*x+a)^2)^(1/2)/b^4
```

3.58.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int x^3 \text{arcsinh}(a+bx)\,dx = \frac{\sqrt{1+a^2+2abx+b^2x^2}(50a^3+9bx-26a^2bx-6b^3x^3+a(-55+14b^2x^2))-3(3-24a^2+8a^4-8b^4x^4)\,\text{a}}{96b^4}$$

input
```
Integrate[x^3*ArcSinh[a + b*x],x]
```

output
```
(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(50*a^3 + 9*b*x - 26*a^2*b*x - 6*b^3*x^
3 + a*(-55 + 14*b^2*x^2)) - 3*(3 - 24*a^2 + 8*a^4 - 8*b^4*x^4)*ArcSinh[a +
 b*x])/(96*b^4)
```

3.58.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6274, 25, 27, 6243, 497, 25, 687, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{arcsinh}(a+bx)\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int x^3\text{arcsinh}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -x^3\text{arcsinh}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -b^3x^3\text{arcsinh}(a+bx)d(a+bx)}{b^4}$$

$$\Big\downarrow \text{6243}$$

$$-\frac{\frac{1}{4}\int \frac{b^4x^4}{\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{1}{4}b^4x^4\text{arcsinh}(a+bx)}{b^4}$$

$$\Big\downarrow \text{497}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{4}\int -\frac{b^2x^2\left(-4a^2+7(a+bx)a+3\right)}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{4}b^3x^3\sqrt{(a+bx)^2+1}\right)-\frac{1}{4}b^4x^4\text{arcsinh}(a+bx)}{b^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{4}b^3x^3\sqrt{(a+bx)^2+1}-\frac{1}{4}\int \frac{b^2x^2\left(-4a^2+7(a+bx)a+3\right)}{\sqrt{(a+bx)^2+1}}d(a+bx)\right)-\frac{1}{4}b^4x^4\text{arcsinh}(a+bx)}{b^4}$$

$$\Big\downarrow \text{687}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{4}\left(-\frac{1}{3}\int -\frac{bx(a(23-12a^2)-(9-26a^2)(a+bx))}{\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{7}{3}ab^2x^2\sqrt{(a+bx)^2+1}\right)+\frac{1}{4}b^3x^3\sqrt{(a+bx)^2+1}\right)-\frac{1}{4}b^4x}{b^4}$$

$\downarrow$ 676

$$-\frac{\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}(8a^4-24a^2+3)\int\frac{1}{\sqrt{(a+bx)^2+1}}d(a+bx)+2a(16-19a^2)\sqrt{(a+bx)^2+1}-\frac{1}{2}(9-26a^2)(a+bx)\sqrt{(a}\right.\right.\right.}{b^4}$$

$\downarrow$ 222

$$-\frac{\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{3}\left(2a(16-19a^2)\sqrt{(a+bx)^2+1}-\frac{1}{2}(9-26a^2)(a+bx)\sqrt{(a+bx)^2+1}+\frac{3}{2}(8a^4-24a^2+3)\,\text{arcsinh}(a+\right.\right.\right.}{b^4}$$

input
```
Int[x^3*ArcSinh[a + b*x],x]
```

output
```
-((-1/4*(b^4*x^4*ArcSinh[a + b*x]) + ((b^3*x^3*Sqrt[1 + (a + b*x)^2])/4 +
((-7*a*b^2*x^2*Sqrt[1 + (a + b*x)^2])/3 + (2*a*(16 - 19*a^2)*Sqrt[1 + (a +
 b*x)^2] - ((9 - 26*a^2)*(a + b*x)*Sqrt[1 + (a + b*x)^2])/2 + (3*(3 - 24*a
^2 + 8*a^4)*ArcSinh[a + b*x])/2)/3)/4)/4)/b^4)
```

3.58.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.58.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.53

method	result
derivativedivides	$\frac{-\operatorname{arcsinh}(bx+a)a^3(bx+a)+\frac{3\operatorname{arcsinh}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arcsinh}(bx+a)a(bx+a)^3+\frac{\operatorname{arcsinh}(bx+a)(bx+a)^4}{4}-\frac{(bx+a)^3\sqrt{1+(bx}{16}}{}$
default	$\frac{-\operatorname{arcsinh}(bx+a)a^3(bx+a)+\frac{3\operatorname{arcsinh}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arcsinh}(bx+a)a(bx+a)^3+\frac{\operatorname{arcsinh}(bx+a)(bx+a)^4}{4}-\frac{(bx+a)^3\sqrt{1+(bx}{16}}{}$
parts	$\frac{x^4\operatorname{arcsinh}(bx+a)}{4}-\frac{b\left(\frac{x^3\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}-\frac{7a\left(\frac{x^2\sqrt{b^2x^2+2abx+a^2+1}}{3b^2}-\frac{5a\left(\frac{x\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{3a\left(\frac{\sqrt{b^2x^2+2abx+}}{b^2}\right.}{}\right.}{}\right.}{}\right.}{}$

input

```
int(x^3*arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
1/b^4*(-arcsinh(b*x+a)*a^3*(b*x+a)+3/2*arcsinh(b*x+a)*a^2*(b*x+a)^2-arcsin
h(b*x+a)*a*(b*x+a)^3+1/4*arcsinh(b*x+a)*(b*x+a)^4-1/16*(b*x+a)^3*(1+(b*x+a
)^2)^(1/2)+3/32*(b*x+a)*(1+(b*x+a)^2)^(1/2)-3/32*arcsinh(b*x+a)+a^3*(1+(b*
x+a)^2)^(1/2)-3/2*a^2*(1/2*(b*x+a)*(1+(b*x+a)^2)^(1/2)-1/2*arcsinh(b*x+a))
+a*(1/3*(b*x+a)^2*(1+(b*x+a)^2)^(1/2)-2/3*(1+(b*x+a)^2)^(1/2)))
```

3.58. $\int x^3 \operatorname{arcsinh}(a+bx)\,dx$

3.58.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.84

$$\int x^3\text{arcsinh}(a+bx)\,dx$$
$$=\frac{3\,(8\,b^4x^4-8\,a^4+24\,a^2-3)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-(6\,b^3x^3-14\,ab^2x^2-50\,a^3+(26\,a^2}{96\,b^4}$$

input
```
integrate(x^3*arcsinh(b*x+a),x, algorithm="fricas")
```

output
```
1/96*(3*(8*b^4*x^4 - 8*a^4 + 24*a^2 - 3)*log(b*x + a + sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)) - (6*b^3*x^3 - 14*a*b^2*x^2 - 50*a^3 + (26*a^2 - 9)*b*x +
55*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^4
```

3.58.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(117) = 234$.

Time = 0.32 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.95

$$\int x^3\text{arcsinh}(a+bx)\,dx$$
$$=\begin{cases}-\frac{a^4\operatorname{asinh}(a+bx)}{4b^4}+\frac{25a^3\sqrt{a^2+2abx+b^2x^2+1}}{48b^4}-\frac{13a^2x\sqrt{a^2+2abx+b^2x^2+1}}{48b^3}+\frac{3a^2\operatorname{asinh}(a+bx)}{4b^4}+\frac{7ax^2\sqrt{a^2+2abx+b^2x^2+1}}{48b^2}-\frac{55a\sqrt{}}{} \\ \frac{x^4\operatorname{asinh}(a)}{4}\end{cases}$$

input
```
integrate(x**3*asinh(b*x+a),x)
```

output
```
Piecewise((-a**4*asinh(a + b*x)/(4*b**4) + 25*a**3*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)/(48*b**4) - 13*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(
48*b**3) + 3*a**2*asinh(a + b*x)/(4*b**4) + 7*a*x**2*sqrt(a**2 + 2*a*b*x +
 b**2*x**2 + 1)/(48*b**2) - 55*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(96*
b**4) + x**4*asinh(a + b*x)/4 - x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/
(16*b) + 3*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(32*b**3) - 3*asinh(a +
b*x)/(32*b**4), Ne(b, 0)), (x**4*asinh(a)/4, True))
```

3.58.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(114) = 228$.

Time = 0.18 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.43

$$\int x^3 \text{arcsinh}(a+bx)\,dx = \frac{1}{4}x^4 \operatorname{arsinh}(bx+a) - \frac{1}{96}\left(\frac{6\sqrt{b^2x^2+2abx+a^2+1}x^3}{b^2} - \frac{14\sqrt{b^2x^2+2abx+a^2+1}ax^2}{b^3} + \frac{105a^4 \operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b^5}\right.$$

input

```
integrate(x^3*arcsinh(b*x+a),x, algorithm="maxima")
```

output

```
1/4*x^4*arcsinh(b*x + a) - 1/96*(6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x^3/b
^2 - 14*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x^2/b^3 + 105*a^4*arcsinh(2*(b
^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 35*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*a^2*x/b^4 - 90*(a^2 + 1)*a^2*arcsinh(2*(b^2*x + a*b)/sqrt
(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 - 105*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)*a^3/b^5 - 9*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*x/b^4 + 9*(a^2 +
 1)^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 55
*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*a/b^5)*b
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.24

$$\int x^3 \text{arcsinh}(a+bx)\,dx = \frac{1}{4}x^4 \log\left(bx+a+\sqrt{(bx+a)^2+1}\right) - \frac{1}{96}\left(\sqrt{b^2x^2+2abx+a^2+1}\left(\left(2x\left(\frac{3x}{b^2}-\frac{7a}{b^3}\right)+\frac{26a^2b^3-9b^3}{b^7}\right)x - \frac{5(10a^3b^2-11ab^2)}{b^7}\right) - \frac{3(8a^4}{}\right.$$

input

```
integrate(x^3*arcsinh(b*x+a),x, algorithm="giac")
```

output

```
1/4*x^4*log(b*x + a + sqrt((b*x + a)^2 + 1)) - 1/96*(sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)*((2*x*(3*x/b^2 - 7*a/b^3) + (26*a^2*b^3 - 9*b^3)/b^7)*x - 5*(
10*a^3*b^2 - 11*a*b^2)/b^7) - 3*(8*a^4 - 24*a^2 + 3)*log(-a*b - (x*abs(b)
- sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^4*abs(b)))*b
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{arcsinh}(a+bx)\,dx=\int x^3\,\operatorname{asinh}(a+b\,x)\;dx$$

input
```
int(x^3*asinh(a + b*x),x)
```

output
```
int(x^3*asinh(a + b*x), x)
```

3.59 $\int x^2\text{arcsinh}(a+bx)\,dx$

3.59.1 Optimal result

Integrand size = 10, antiderivative size = 90

$$\int x^2\text{arcsinh}(a+bx)\,dx=-\frac{x^2\sqrt{1+(a+bx)^2}}{9b}+\frac{(4-11a^2+5abx)\sqrt{1+(a+bx)^2}}{18b^3}-\frac{a(3-2a^2)\,\text{arcsinh}(a+bx)}{6b^3}+\frac{1}{3}x^3\text{arcsinh}(a+bx)$$

output
```
-1/6*a*(-2*a^2+3)*arcsinh(b*x+a)/b^3+1/3*x^3*arcsinh(b*x+a)-1/9*x^2*(1+(b*
x+a)^2)^(1/2)/b+1/18*(5*a*b*x-11*a^2+4)*(1+(b*x+a)^2)^(1/2)/b^3
```

3.59.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.82

$$\int x^2\text{arcsinh}(a+bx)\,dx=\frac{(4-11a^2+5abx-2b^2x^2)\sqrt{1+a^2+2abx+b^2x^2}+(-9a+6a^3+6b^3x^3)\,\text{arcsinh}(a+bx)}{18b^3}$$

input
```
Integrate[x^2*ArcSinh[a + b*x],x]
```

output
```
((4 - 11*a^2 + 5*a*b*x - 2*b^2*x^2)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (-
9*a + 6*a^3 + 6*b^3*x^3)*ArcSinh[a + b*x])/(18*b^3)
```

3.59.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6274, 27, 6243, 497, 25, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arcsinh}(a+bx)\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int x^2\text{arcsinh}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int b^2x^2\text{arcsinh}(a+bx)d(a+bx)}{b^3}$$

$$\Big\downarrow \text{6243}$$

$$\frac{\frac{1}{3}\int -\frac{b^3x^3}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{3}b^3x^3\text{arcsinh}(a+bx)}{b^3}$$

$$\Big\downarrow \text{497}$$

$$\frac{\frac{1}{3}\left(\frac{1}{3}\int \frac{bx\left(-3a^2+5(a+bx)a+2\right)}{\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{1}{3}b^2x^2\sqrt{(a+bx)^2+1}\right)+\frac{1}{3}b^3x^3\text{arcsinh}(a+bx)}{b^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{3}\left(-\frac{1}{3}\int -\frac{bx\left(-3a^2+5(a+bx)a+2\right)}{\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{1}{3}b^2x^2\sqrt{(a+bx)^2+1}\right)+\frac{1}{3}b^3x^3\text{arcsinh}(a+bx)}{b^3}$$

$$\Big\downarrow \text{676}$$

$$\frac{\frac{1}{3}\left(\frac{1}{3}\left(-\frac{3}{2}a\left(3-2a^2\right)\int \frac{1}{\sqrt{(a+bx)^2+1}}d(a+bx)+2\left(1-4a^2\right)\sqrt{(a+bx)^2+1}+\frac{5}{2}a(a+bx)\sqrt{(a+bx)^2+1}\right)-\frac{1}{3}b^2x^2\sqrt{\ldots}\right.}{b^3}$$

$$\Big\downarrow \text{222}$$

$$\frac{\frac{1}{3}\left(\frac{1}{3}\left(-\frac{3}{2}a\left(3-2a^2\right)\text{arcsinh}(a+bx)+2\left(1-4a^2\right)\sqrt{(a+bx)^2+1}+\frac{5}{2}a(a+bx)\sqrt{(a+bx)^2+1}\right)-\frac{1}{3}b^2x^2\sqrt{(a+b\ldots}\right.}{b^3}$$

input
```
Int[x^2*ArcSinh[a + b*x],x]
```

output
```
((b^3*x^3*ArcSinh[a + b*x])/3 + (-1/3*(b^2*x^2*Sqrt[1 + (a + b*x)^2]) + (2
*(1 - 4*a^2)*Sqrt[1 + (a + b*x)^2] + (5*a*(a + b*x)*Sqrt[1 + (a + b*x)^2])
/2 - (3*a*(3 - 2*a^2)*ArcSinh[a + b*x])/2)/3)/3)/b^3
```

3.59.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.59.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{\operatorname{arcsinh}(bx+a)a^2(bx+a)-\operatorname{arcsinh}(bx+a)a(bx+a)^2+\frac{\operatorname{arcsinh}(bx+a)(bx+a)^3}{3}-\frac{(bx+a)^2\sqrt{1+(bx+a)^2}}{9}+\frac{2\sqrt{1+(bx+a)^2}}{9}-a^2\sqrt{1-}}{b^3}$
default	$\frac{\operatorname{arcsinh}(bx+a)a^2(bx+a)-\operatorname{arcsinh}(bx+a)a(bx+a)^2+\frac{\operatorname{arcsinh}(bx+a)(bx+a)^3}{3}-\frac{(bx+a)^2\sqrt{1+(bx+a)^2}}{9}+\frac{2\sqrt{1+(bx+a)^2}}{9}-a^2\sqrt{1-}}{b^3}$
parts	$\frac{x^3\operatorname{arcsinh}(bx+a)}{3}-\frac{b\left(\frac{x^2\sqrt{b^2x^2+2abx+a^2+1}}{3b^2}-\frac{5a\left(\frac{x\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{3a\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^2}-\frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2}\right.}{b\sqrt{b^2}}\right.}{2b}\right.}{3b}\right.}{}$

input
```
int(x^2*arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(arcsinh(b*x+a)*a^2*(b*x+a)-arcsinh(b*x+a)*a*(b*x+a)^2+1/3*arcsinh(b
*x+a)*(b*x+a)^3-1/9*(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+2/9*(1+(b*x+a)^2)^(1/2)-
a^2*(1+(b*x+a)^2)^(1/2)+a*(1/2*(b*x+a)*(1+(b*x+a)^2)^(1/2)-1/2*arcsinh(b*x
+a)))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int x^2 \text{arcsinh}(a + bx) \, dx$$
$$= \frac{3\,(2\,b^3x^3 + 2\,a^3 - 3\,a)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right) - (2\,b^2x^2 - 5\,abx + 11\,a^2 - 4)\sqrt{b^2x^2 + 2\,abx}}{18\,b^3}$$

input
```
integrate(x^2*arcsinh(b*x+a),x, algorithm="fricas")
```

output
```
1/18*(3*(2*b^3*x^3 + 2*a^3 - 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)) - (2*b^2*x^2 - 5*a*b*x + 11*a^2 - 4)*sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1))/b^3
```

3.59.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(83) = 166.

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.89

$$\int x^2 \text{arcsinh}(a + bx) \, dx$$
$$= \begin{cases} \frac{a^3 \operatorname{asinh}(a+bx)}{3b^3} - \frac{11a^2\sqrt{a^2+2abx+b^2x^2+1}}{18b^3} + \frac{5ax\sqrt{a^2+2abx+b^2x^2+1}}{18b^2} - \frac{a \operatorname{asinh}(a+bx)}{2b^3} + \frac{x^3 \operatorname{asinh}(a+bx)}{3} - \frac{x^2\sqrt{a^2+2abx+b^2x^2+1}}{9b} \\ \frac{x^3 \operatorname{asinh}(a)}{3} \end{cases}$$

input
```
integrate(x**2*asinh(b*x+a),x)
```

output
```
Piecewise((a**3*asinh(a + b*x)/(3*b**3) - 11*a**2*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1)/(18*b**3) + 5*a*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(18*b*
*2) - a*asinh(a + b*x)/(2*b**3) + x**3*asinh(a + b*x)/3 - x**2*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)/(9*b) + 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(
9*b**3), Ne(b, 0)), (x**3*asinh(a)/3, True))
```

3.59.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(78) = 156.

Time = 0.18 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.33

$$\int x^2\text{arcsinh}(a+bx)\,dx = \frac{1}{3}\,x^3\,\text{arsinh}\,(bx+a) - \frac{1}{18}\,b\left(\frac{2\,\sqrt{b^2x^2+2\,abx+a^2+1}x^2}{b^2} - \frac{15\,a^3\,\text{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b^4} - \frac{5\,\sqrt{b^2x^2+2\,abx+a^2+1}ax}{b^3} +$$

input
```
integrate(x^2*arcsinh(b*x+a),x, algorithm="maxima")
```

output
```
1/3*x^3*arcsinh(b*x + a) - 1/18*b*(2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x^2
/b^2 - 15*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/
b^4 - 5*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b^3 + 9*(a^2 + 1)*a*arcsinh(
2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 + 15*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*a^2/b^4 - 4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 +
1)/b^4)
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46

$$\int x^2\text{arcsinh}(a+bx)\,dx = \frac{1}{3}\,x^3\log\left(bx+a+\sqrt{(bx+a)^2+1}\right) - \frac{1}{18}\left(\sqrt{b^2x^2+2\,abx+a^2+1}\left(x\left(\frac{2\,x}{b^2}-\frac{5\,a}{b^3}\right)+\frac{11\,a^2b-4\,b}{b^5}\right)+\frac{3\,(2\,a^3-3\,a)\log\left(-ab-\left(x|b|-\sqrt{b^2x\right.\right.}{b^3|b|}\right.$$

input
```
integrate(x^2*arcsinh(b*x+a),x, algorithm="giac")
```

output
```
1/3*x^3*log(b*x + a + sqrt((b*x + a)^2 + 1)) - 1/18*(sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)*(x*(2*x/b^2 - 5*a/b^3) + (11*a^2*b - 4*b)/b^5) + 3*(2*a^3 - 3
*a)*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^3
*abs(b)))*b
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arcsinh}(a+bx)\,dx = \int x^2\,\text{asinh}(a+b\,x)\,dx$$

input
```
int(x^2*asinh(a + b*x),x)
```

output
```
int(x^2*asinh(a + b*x), x)
```

3.60 $\int x\text{arcsinh}(a+bx)\,dx$

3.60.1 Optimal result

Integrand size = 8, antiderivative size = 76

$$\int x\text{arcsinh}(a+bx)\,dx=\frac{3a\sqrt{1+(a+bx)^2}}{4b^2}-\frac{x\sqrt{1+(a+bx)^2}}{4b}+\frac{(1-2a^2)\,\text{arcsinh}(a+bx)}{4b^2}+\frac{1}{2}x^2\text{arcsinh}(a+bx)$$

output
```
1/4*(-2*a^2+1)*arcsinh(b*x+a)/b^2+1/2*x^2*arcsinh(b*x+a)+3/4*a*(1+(b*x+a)^
2)^(1/2)/b^2-1/4*x*(1+(b*x+a)^2)^(1/2)/b
```

3.60.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int x\text{arcsinh}(a+bx)\,dx=\frac{(3a-bx)\sqrt{1+a^2+2abx+b^2x^2}+(1-2a^2+2b^2x^2)\,\text{arcsinh}(a+bx)}{4b^2}$$

input
```
Integrate[x*ArcSinh[a + b*x],x]
```

output
```
((3*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (1 - 2*a^2 + 2*b^2*x^2)*A
rcSinh[a + b*x])/(4*b^2)
```

3.60.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6274, 25, 27, 6243, 497, 25, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arcsinh}(a+bx)\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int x\text{arcsinh}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -x\text{arcsinh}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -bx\text{arcsinh}(a+bx)d(a+bx)}{b^2}$$

$$\Big\downarrow \text{6243}$$

$$-\frac{\frac{1}{2}\int \frac{b^2x^2}{\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{1}{2}b^2x^2\text{arcsinh}(a+bx)}{b^2}$$

$$\Big\downarrow \text{497}$$

$$-\frac{\frac{1}{2}\left(\frac{1}{2}\int -\frac{-2a^2+3(a+bx)a+1}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}bx\sqrt{(a+bx)^2+1}\right)-\frac{1}{2}b^2x^2\text{arcsinh}(a+bx)}{b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{2}\left(\frac{1}{2}bx\sqrt{(a+bx)^2+1}-\frac{1}{2}\int \frac{-2a^2+3(a+bx)a+1}{\sqrt{(a+bx)^2+1}}d(a+bx)\right)-\frac{1}{2}b^2x^2\text{arcsinh}(a+bx)}{b^2}$$

$$\Big\downarrow \text{455}$$

$$-\frac{\frac{1}{2}\left(\frac{1}{2}\left(-(1-2a^2)\int \frac{1}{\sqrt{(a+bx)^2+1}}d(a+bx)-3a\sqrt{(a+bx)^2+1}\right)+\frac{1}{2}bx\sqrt{(a+bx)^2+1}\right)-\frac{1}{2}b^2x^2\text{arcsinh}(a+bx)}{b^2}$$

$$\Big\downarrow \text{222}$$

$$-\frac{\frac{1}{2}\left(\frac{1}{2}\left(-(1-2a^2)\operatorname{arcsinh}(a+bx)-3a\sqrt{(a+bx)^2+1}\right)+\frac{1}{2}bx\sqrt{(a+bx)^2+1}\right)-\frac{1}{2}b^2x^2\operatorname{arcsinh}(a+bx)}{b^2}$$

input
```
Int[x*ArcSinh[a + b*x],x]
```

output
```
-((-1/2*(b^2*x^2*ArcSinh[a + b*x]) + ((b*x*Sqrt[1 + (a + b*x)^2])/2 + (-3*
a*Sqrt[1 + (a + b*x)^2] - (1 - 2*a^2)*ArcSinh[a + b*x])/2)/2)/b^2)
```

3.60.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.60.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsinh}(bx+a)(bx+a)^2}{2}-\operatorname{arcsinh}(bx+a)a(bx+a)-\frac{(bx+a)\sqrt{1+(bx+a)^2}}{4}+\frac{\operatorname{arcsinh}(bx+a)}{4}+a\sqrt{1+(bx+a)^2}}{b^2}$
default	$\frac{\frac{\operatorname{arcsinh}(bx+a)(bx+a)^2}{2}-\operatorname{arcsinh}(bx+a)a(bx+a)-\frac{(bx+a)\sqrt{1+(bx+a)^2}}{4}+\frac{\operatorname{arcsinh}(bx+a)}{4}+a\sqrt{1+(bx+a)^2}}{b^2}$
parts	$\frac{x^2\operatorname{arcsinh}(bx+a)}{2}-\frac{b\left(\frac{x\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{3a\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^2}-\frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{b\sqrt{b^2}}\right)}{2b}-\frac{\left(a^2+1\right)\ln}{}\right.}{2}$

input
```
int(x*arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/2*arcsinh(b*x+a)*(b*x+a)^2-arcsinh(b*x+a)*a*(b*x+a)-1/4*(b*x+a)*(
1+(b*x+a)^2)^(1/2)+1/4*arcsinh(b*x+a)+a*(1+(b*x+a)^2)^(1/2))
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int x\operatorname{arcsinh}(a+bx)\,dx$$
$$=\frac{(2\,b^2x^2-2\,a^2+1)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\sqrt{b^2x^2+2\,abx+a^2+1}(bx-3\,a)}{4\,b^2}$$

input
```
integrate(x*arcsinh(b*x+a),x, algorithm="fricas")
```

output
```
1/4*((2*b^2*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a))/b^2
```

3.60.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.37

$$\int x\text{arcsinh}(a+bx)\,dx$$
$$= \begin{cases} -\frac{a^2 \operatorname{asinh}(a+bx)}{2b^2} + \frac{3a\sqrt{a^2+2abx+b^2x^2+1}}{4b^2} + \frac{x^2 \operatorname{asinh}(a+bx)}{2} - \frac{x\sqrt{a^2+2abx+b^2x^2+1}}{4b} + \frac{\operatorname{asinh}(a+bx)}{4b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{asinh}(a)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*asinh(b*x+a),x)
```

output
```
Piecewise((-a**2*asinh(a + b*x)/(2*b**2) + 3*a*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1)/(4*b**2) + x**2*asinh(a + b*x)/2 - x*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1)/(4*b) + asinh(a + b*x)/(4*b**2), Ne(b, 0)), (x**2*asinh(a)/2, Tr
ue))
```

3.60.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(64) = 128.

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.96

$$\int x\text{arcsinh}(a+bx)\,dx = \frac{1}{2}x^2 \operatorname{arsinh}(bx+a) - \frac{1}{4}b\left(\frac{3a^2 \operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b^3} + \frac{\sqrt{b^2x^2+2abx+a^2+1}x}{b^2} - \frac{(a^2+1)\operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b^3}\right.$$

input
```
integrate(x*arcsinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x^2*arcsinh(b*x + a) - 1/4*b*(3*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^
2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 -
(a^2 + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3
- 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^3)
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.46

$$\int x\text{arcsinh}(a+bx)\,dx = \frac{1}{2}x^2\log\left(bx+a+\sqrt{(bx+a)^2+1}\right) - \frac{1}{4}\left(\sqrt{b^2x^2+2\,abx+a^2+1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right)-\frac{(2\,a^2-1)\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{b^2|b|}\right)$$

input
```
integrate(x*arcsinh(b*x+a),x, algorithm="giac")
```

output
```
1/2*x^2*log(b*x + a + sqrt((b*x + a)^2 + 1)) - 1/4*(sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*(x/b^2 - 3*a/b^3) - (2*a^2 - 1)*log(-a*b - (x*abs(b) - sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^2*abs(b)))*b
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arcsinh}(a+bx)\,dx = \int x\,\text{asinh}(a+b\,x)\;dx$$

input
```
int(x*asinh(a + b*x),x)
```

output
```
int(x*asinh(a + b*x), x)
```

3.61 $\int \operatorname{arcsinh}(a+bx)\,dx$

3.61.1 Optimal result

Integrand size = 6, antiderivative size = 34

$$\int \operatorname{arcsinh}(a+bx)\,dx=-\frac{\sqrt{1+(a+bx)^2}}{b}+\frac{(a+bx)\operatorname{arcsinh}(a+bx)}{b}$$

output
```
(b*x+a)*arcsinh(b*x+a)/b-(1+(b*x+a)^2)^(1/2)/b
```

3.61.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(34) = 68.

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.29

$$\int \operatorname{arcsinh}(a+bx)\,dx=x\operatorname{arcsinh}(a+bx)-\frac{\sqrt{1+a^2+2abx+b^2x^2}+2a\operatorname{arctanh}\left(\frac{bx}{\sqrt{1+a^2}-\sqrt{1+a^2+2abx+b^2x^2}}\right)}{b}$$

input
```
Integrate[ArcSinh[a + b*x],x]
```

output
```
x*ArcSinh[a + b*x] - (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + 2*a*ArcTanh[(b*x
)/(Sqrt[1 + a^2] - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])])/b
```

3.61.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6273, 6187, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arcsinh}(a+bx)\,dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int \operatorname{arcsinh}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{6187}$$

$$\frac{(a+bx)\operatorname{arcsinh}(a+bx)-\int \frac{a+bx}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b}$$

$$\Big\downarrow \text{241}$$

$$\frac{(a+bx)\operatorname{arcsinh}(a+bx)-\sqrt{(a+bx)^2+1}}{b}$$

input
```
Int[ArcSinh[a + b*x],x]
```

output
```
(-Sqrt[1 + (a + b*x)^2] + (a + b*x)*ArcSinh[a + b*x])/b
```

3.61.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.61.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{(bx+a)\,\mathrm{arcsinh}(bx+a)-\sqrt{1+(bx+a)^2}}{b}$	31
default	$\frac{(bx+a)\,\mathrm{arcsinh}(bx+a)-\sqrt{1+(bx+a)^2}}{b}$	31
parts	$x\,\mathrm{arcsinh}\,(bx+a)-b\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^2}-\frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{b\sqrt{b^2}}\right)$	84

input
```
int(arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*((b*x+a)*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int \mathrm{arcsinh}(a+bx)\,dx = \frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\sqrt{b^2x^2+2\,abx+a^2+1}}{b}$$

input
```
integrate(arcsinh(b*x+a),x, algorithm="fricas")
```

output
```
((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))/b
```

3.61.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \operatorname{arcsinh}(a+bx)\,dx = \begin{cases} \frac{a \operatorname{asinh}(a+bx)}{b} + x \operatorname{asinh}(a+bx) - \frac{\sqrt{a^2+2abx+b^2x^2+1}}{b} & \text{for } b \neq 0 \\ x \operatorname{asinh}(a) & \text{otherwise} \end{cases}$$

input
```
integrate(asinh(b*x+a),x)
```

output
```
Piecewise((a*asinh(a + b*x)/b + x*asinh(a + b*x) - sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)/b, Ne(b, 0)), (x*asinh(a), True))
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \operatorname{arcsinh}(a+bx)\,dx = \frac{(bx+a)\operatorname{arsinh}(bx+a) - \sqrt{(bx+a)^2+1}}{b}$$

input
```
integrate(arcsinh(b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*arcsinh(b*x + a) - sqrt((b*x + a)^2 + 1))/b
```

3.61.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(32) = 64.

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.71

$$\int \operatorname{arcsinh}(a+bx)\,dx = -b\left(\frac{a\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{b|b|} + \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{b^2}\right) + x\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)$$

input
```
integrate(arcsinh(b*x+a),x, algorithm="giac")
```

output
```
-b*(a*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b
*abs(b)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^2) + x*log(b*x + a + sqrt((
b*x + a)^2 + 1))
```

3.61.9 Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int \operatorname{arcsinh}(a+bx)\,dx = x\,\operatorname{asinh}(a+b\,x)-\frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{b}+\frac{a\,\ln\left(\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}+\frac{x\,b^2+a\,b}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

input
```
int(asinh(a + b*x),x)
```

output
```
x*asinh(a + b*x) - (a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/b + (a*log((a^2 + b
^2*x^2 + 2*a*b*x + 1)^(1/2) + (a*b + b^2*x)/(b^2)^(1/2)))/(b^2)^(1/2)
```

3.62 $\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx$

3.62.1 Optimal result

Integrand size = 10, antiderivative size = 131

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx = -\frac{1}{2}\operatorname{arcsinh}(a+bx)^2 + \operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) + \operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)$$

output
```
-1/2*arcsinh(b*x+a)^2+arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(
a^2+1)^(1/2)))+arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^
(1/2)))+polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))+polylog(2
,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))
```

3.62.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx = -\frac{1}{2}\operatorname{arcsinh}(a+bx)^2 + \operatorname{arcsinh}(a+bx)\log\left(1+\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right) + \operatorname{arcsinh}(a+bx)\log\left(1+\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right) + \operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(a+bx)}}{-a+\sqrt{1+a^2}}\right) + \operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)$$

input
```
Integrate[ArcSinh[a + b*x]/x,x]
```

output
```
-1/2*ArcSinh[a + b*x]^2 + ArcSinh[a + b*x]*Log[1 + E^ArcSinh[a + b*x]/((-(
a/b) - Sqrt[1 + a^2]/b)*b)] + ArcSinh[a + b*x]*Log[1 + E^ArcSinh[a + b*x]/
((-(a/b) + Sqrt[1 + a^2]/b)*b)] + PolyLog[2, -(E^ArcSinh[a + b*x]/(-a + Sq
rt[1 + a^2]))] + PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

3.62.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6274, 25, 27, 6242, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(a+bx)}{x} \, dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{\text{arcsinh}(a+bx)}{x}d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{\text{arcsinh}(a+bx)}{x}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\int -\frac{\text{arcsinh}(a+bx)}{bx}d(a+bx)$$

$$\Big\downarrow \text{6242}$$

$$-\int -\frac{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)}{bx}d\text{arcsinh}(a+bx)$$

$$\Big\downarrow \text{6095}$$

$$-\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{a-e^{\text{arcsinh}(a+bx)}-\sqrt{a^2+1}}d\text{arcsinh}(a+bx)-\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{a-e^{\text{arcsinh}(a+bx)}+\sqrt{a^2+1}}d\text{arcsinh}(a+bx)-\frac{1}{2}\text{arcsinh}(a+bx)^2$$

$$\Big\downarrow \text{2620}$$

$$-\int \log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)d\text{arcsinh}(a+bx)-\int \log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)d\text{arcsinh}(a+bx)+\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)-\frac{1}{2}\text{arcsinh}(a+bx)^2$$

$$\Big\downarrow \text{2715}$$

$$-\int e^{-\text{arcsinh}(a+bx)}\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)de^{\text{arcsinh}(a+bx)}-\int e^{-\text{arcsinh}(a+bx)}\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)de^{\text{arcsinh}(a+bx)}+\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)-\frac{1}{2}\text{arcsinh}(a+bx)^2$$

$$\Big\downarrow \text{2838}$$

$$\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)+\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)-\frac{1}{2}\text{arcsinh}(a+bx)^2$$

input
```
Int[ArcSinh[a + b*x]/x,x]
```

output
```
-1/2*ArcSinh[a + b*x]^2 + ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a -
 Sqrt[1 + a^2])] + ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1
 + a^2])] + PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + PolyLog[2
, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

3.62.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.62. $\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx$

3.62.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(153) = 306$.

Time = 0.66 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.96

method	result
derivativedivides	$-\frac{\operatorname{arcsinh}(bx+a)^2}{2}+\frac{\left(a^2+1+\sqrt{a^2+1}\,a\right)\operatorname{arcsinh}(bx+a)\left(2\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx+a)^2}}{a+\sqrt{a^2+1}}\right)a^2+\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx-}}{a+\sqrt{a^2+1}}\right.\right.}{a^2+1}$
default	$-\frac{\operatorname{arcsinh}(bx+a)^2}{2}+\frac{\left(a^2+1+\sqrt{a^2+1}\,a\right)\operatorname{arcsinh}(bx+a)\left(2\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx+a)^2}}{a+\sqrt{a^2+1}}\right)a^2+\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx-}}{a+\sqrt{a^2+1}}\right.\right.}{a^2+1}$

input
```
int(arcsinh(b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arcsinh(b*x+a)^2+(a^2+1+(a^2+1)^(1/2)*a)/(a^2+1)*arcsinh(b*x+a)*(2*ln
(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))*a^2+ln(((a^2+1
)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2*ln(((a^2+1)^(1/2)-b*
x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))*(a^2+1)^(1/2)*a+ln(((a^2+1)^(1/2
)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2))))+dilog(((a^2+1)^(1/2)+b*x+(
1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))+dilog(((a^2+1)^(1/2)-b*x-(1+(b*x+a
)^2)^(1/2))/(a+(a^2+1)^(1/2)))+a*arcsinh(b*x+a)/(a^2+1)^(1/2)*ln(((a^2+1)^
(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-a*arcsinh(b*x+a)/(a^2+1)
^(1/2)*ln(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))
```

3.62.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx = \int \frac{\operatorname{arsinh}(bx+a)}{x}\,dx$$

input
```
integrate(arcsinh(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(arcsinh(b*x + a)/x, x)
```

3.62.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx = \int \frac{\operatorname{asinh}{(a+bx)}}{x}\,dx$$

input
```
integrate(asinh(b*x+a)/x,x)
```

output
```
Integral(asinh(a + b*x)/x, x)
```

3.62.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx = \int \frac{\operatorname{arsinh}(bx+a)}{x}\,dx$$

input
```
integrate(arcsinh(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(arcsinh(b*x + a)/x, x)
```

3.62.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx = \int \frac{\operatorname{arsinh}(bx+a)}{x}\,dx$$

input
```
integrate(arcsinh(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(arcsinh(b*x + a)/x, x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)}{x}\,dx$$

input
```
int(asinh(a + b*x)/x,x)
```

output
```
int(asinh(a + b*x)/x, x)
```

3.63 $\int \frac{\text{arcsinh}(a+bx)}{x^2} dx$

3.63.1 Optimal result

Integrand size = 10, antiderivative size = 57

$$\int \frac{\text{arcsinh}(a+bx)}{x^2} dx = -\frac{\text{arcsinh}(a+bx)}{x} - \frac{b\text{arctanh}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{\sqrt{1+a^2}}$$

output
```
-arcsinh(b*x+a)/x-b*arctanh((1+a*(b*x+a))/(a^2+1)^(1/2)/(1+(b*x+a)^2)^(1/2
))/(a^2+1)^(1/2)
```

3.63.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(a+bx)}{x^2} dx = -\frac{\text{arcsinh}(a+bx)}{x} - \frac{b\text{arctanh}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{\sqrt{1+a^2}}$$

input
```
Integrate[ArcSinh[a + b*x]/x^2,x]
```

output
```
-(ArcSinh[a + b*x]/x) - (b*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1
 + (a + b*x)^2])])/Sqrt[1 + a^2]
```

3.63.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6274, 27, 6243, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(a+bx)}{x^2} \, dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{\text{arcsinh}(a+bx)}{x^2}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{\text{arcsinh}(a+bx)}{b^2x^2}d(a+bx)$$

$$\Big\downarrow \text{6243}$$

$$b\left(-\int -\frac{1}{bx\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{\text{arcsinh}(a+bx)}{bx}\right)$$

$$\Big\downarrow \text{488}$$

$$b\left(\int \frac{1}{a^2-\frac{(-a(a+bx)-1)^2}{(a+bx)^2+1}+1}d\frac{-a(a+bx)-1}{\sqrt{(a+bx)^2+1}}-\frac{\text{arcsinh}(a+bx)}{bx}\right)$$

$$\Big\downarrow \text{219}$$

$$b\left(\frac{\text{arctanh}\left(\frac{-a(a+bx)-1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{\sqrt{a^2+1}}-\frac{\text{arcsinh}(a+bx)}{bx}\right)$$

input
```
Int[ArcSinh[a + b*x]/x^2,x]
```

output
```
b*(-(ArcSinh[a + b*x]/(b*x)) + ArcTanh[(-1 - a*(a + b*x))/(Sqrt[1 + a^2]*S
qrt[1 + (a + b*x)^2])]/Sqrt[1 + a^2])
```

3.63.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.63.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

method	result	size
parts	$-\frac{\operatorname{arcsinh}(bx+a)}{x}-\frac{b\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{\sqrt{a^2+1}}$	68
derivativedivides	$b\left(-\frac{\operatorname{arcsinh}(bx+a)}{bx}-\frac{\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{bx}\right)}{\sqrt{a^2+1}}\right)$	75
default	$b\left(-\frac{\operatorname{arcsinh}(bx+a)}{bx}-\frac{\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{bx}\right)}{\sqrt{a^2+1}}\right)$	75

input
```
int(arcsinh(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arcsinh(b*x+a)/x-b/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2
*x^2+2*a*b*x+a^2+1)^(1/2))/x)
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(51) = 102$.

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.93

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^2} dx = \frac{\sqrt{a^2+1}bx \log \left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2+1}a+1\right)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)+(a^2+1)x\log\left(-bx-a+\sqrt{\right.}}{(a^2+1)x}$$

input
```
integrate(arcsinh(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
(sqrt(a^2 + 1)*b*x*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/x) + (
a^2 + 1)*x*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - (a^2 - (a^2
 + 1)*x + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/((a^2 + 1)*
x)
```

3.63.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^2} dx = \int \frac{\operatorname{asinh}(a+bx)}{x^2} dx$$

input
```
integrate(asinh(b*x+a)/x**2,x)
```

output
```
Integral(asinh(a + b*x)/x**2, x)
```

3.63.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(51) = 102$.

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.95

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^2}\,dx = -\frac{b\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{\sqrt{a^2+1}} - \frac{\operatorname{arsinh}\left(bx+a\right)}{x}$$

input
```
integrate(arcsinh(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
-b*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sq
rt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1
)*b^2)*abs(x)))/sqrt(a^2 + 1) - arcsinh(b*x + a)/x
```

3.63.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(51) = 102$.

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^2}\,dx = \frac{b\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}} - \frac{\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)}{x}$$

input
```
integrate(arcsinh(b*x+a)/x^2,x, algorithm="giac")
```

output
```
b*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sqrt(a^2 +
 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt(a^2 +
1)))/sqrt(a^2 + 1) - log(b*x + a + sqrt((b*x + a)^2 + 1))/x
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)}{x^2}\,dx$$

input
```
int(asinh(a + b*x)/x^2,x)
```

output
```
int(asinh(a + b*x)/x^2, x)
```

3.64 $\int \frac{\operatorname{arcsinh}(a+bx)}{x^3} dx$

3.64.1 Optimal result

Integrand size = 10, antiderivative size = 92

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^3} dx = -\frac{b\sqrt{1+(a+bx)^2}}{2(1+a^2)x} - \frac{\operatorname{arcsinh}(a+bx)}{2x^2} + \frac{ab^2\operatorname{arctanh}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{2(1+a^2)^{3/2}}$$

output
```
-1/2*arcsinh(b*x+a)/x^2+1/2*a*b^2*arctanh((1+a*(b*x+a))/(a^2+1)^(1/2)/(1+(
b*x+a)^2)^(1/2))/(a^2+1)^(3/2)-1/2*b*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x
```

3.64.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^3} dx = -\frac{\operatorname{arcsinh}(a+bx) + \frac{bx\left(\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}+abx\log(x)-abx\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)\right)}{(1+a^2)^{3/2}}}{2x^2}$$

input
```
Integrate[ArcSinh[a + b*x]/x^3,x]
```

output
```
-1/2*(ArcSinh[a + b*x] + (b*x*(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*
x^2] + a*b*x*Log[x] - a*b*x*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a
^2 + 2*a*b*x + b^2*x^2]]))/(1 + a^2)^(3/2))/x^2
```

3.64.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6274, 25, 27, 6243, 491, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(a+bx)}{x^3} \, dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{\text{arcsinh}(a+bx)}{x^3}d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{\text{arcsinh}(a+bx)}{x^3}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-b^2 \int -\frac{\text{arcsinh}(a+bx)}{b^3x^3}d(a+bx)$$

$$\Big\downarrow \text{6243}$$

$$-b^2 \left(\frac{\text{arcsinh}(a+bx)}{2b^2x^2} - \frac{1}{2}\int \frac{1}{b^2x^2\sqrt{(a+bx)^2+1}}d(a+bx)\right)$$

$$\Big\downarrow \text{491}$$

$$-b^2 \left(\frac{1}{2}\left(\frac{\sqrt{(a+bx)^2+1}}{(a^2+1)\,bx} - \frac{a\int -\frac{1}{bx\sqrt{(a+bx)^2+1}}d(a+bx)}{a^2+1}\right) + \frac{\text{arcsinh}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \text{488}$$

$$-b^2 \left(\frac{1}{2}\left(\frac{a\int \frac{1}{a^2-\frac{(-a(a+bx)-1)^2}{(a+bx)^2+1}+1}d\frac{-a(a+bx)-1}{\sqrt{(a+bx)^2+1}}}{a^2+1} + \frac{\sqrt{(a+bx)^2+1}}{(a^2+1)\,bx}\right) + \frac{\text{arcsinh}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow 219$$

$$-b^2\left(\frac{1}{2}\left(\frac{a\text{arctanh}\left(\frac{-a(a+bx)-1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{(a^2+1)^{3/2}}+\frac{\sqrt{(a+bx)^2+1}}{(a^2+1)\,bx}\right)+\frac{\text{arcsinh}(a+bx)}{2b^2x^2}\right)$$

input
```
Int[ArcSinh[a + b*x]/x^3,x]
```

output
```
-(b^2*(ArcSinh[a + b*x]/(2*b^2*x^2) + (Sqrt[1 + (a + b*x)^2]/((1 + a^2)*b*
x) + (a*ArcTanh[(-1 - a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])
/(1 + a^2)^(3/2))/2))
```

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 491
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b*(c/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.64.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.12

method	result	size
parts	$-\frac{\operatorname{arcsinh}(bx+a)}{2x^2}+\frac{b\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{(a^2+1)x}+\frac{ab\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{(a^2+1)^{\frac{3}{2}}}\right)}{2}$	103
derivativedivides	$b^2\left(-\frac{\operatorname{arcsinh}(bx+a)}{2b^2x^2}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2(a^2+1)bx}+\frac{a\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{bx}\right)}{2(a^2+1)^{\frac{3}{2}}}\right)$	112
default	$b^2\left(-\frac{\operatorname{arcsinh}(bx+a)}{2b^2x^2}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2(a^2+1)bx}+\frac{a\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{bx}\right)}{2(a^2+1)^{\frac{3}{2}}}\right)$	112

input
```
int(arcsinh(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arcsinh(b*x+a)/x^2+1/2*b*(-1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+
a*b/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/x))
```

3.64.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(78) = 156$.

Time = 0.28 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.57

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^3}\,dx = \frac{\sqrt{a^2+1}ab^2x^2\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2+\sqrt{a^2+1}a+1\right)+(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)-(a^2+1)b^2x^2+(a^4+2\,a$$

input
```
integrate(arcsinh(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
1/2*(sqrt(a^2 + 1)*a*b^2*x^2*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)*(a^2 + sqrt(a^2 + 1)*a + 1) + (a*b*x + a^2 + 1)*sqrt(a^2 + 1) +
 a)/x) - (a^2 + 1)*b^2*x^2 + (a^4 + 2*a^2 + 1)*x^2*log(-b*x - a + sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*b
*x - (a^4 - (a^4 + 2*a^2 + 1)*x^2 + 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)))/((a^4 + 2*a^2 + 1)*x^2)
```

3.64.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{asinh}(a+bx)}{x^3}\,dx$$

input
```
integrate(asinh(b*x+a)/x**3,x)
```

output
```
Integral(asinh(a + b*x)/x**3, x)
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.59

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^3}\,dx$$
$$=\frac{1}{2}\left(\frac{ab\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{(a^2+1)^{\frac{3}{2}}}-\frac{\sqrt{b^2x^2+2\,abx+a^2+}}{(a^2+1)x}\right.$$
$$-\frac{\operatorname{arsinh}\left(bx+a\right)}{2\,x^2}$$

input
```
integrate(arcsinh(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
1/2*(a*b*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a
^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a
^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/
((a^2 + 1)*x))*b - 1/2*arcsinh(b*x + a)/x^2
```

3.64.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(78) = 156.

Time = 0.36 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.16

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^3}\,dx =$$
$$-\frac{1}{2}\left(\frac{ab\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{(a^2+1)^{\frac{3}{2}}}-\frac{2\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)ab+a^2|b|+|b|\right)}{\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-a^2-1\right)(a^2+1)}\right.$$
$$-\frac{\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)}{2\,x^2}$$

input
```
integrate(arcsinh(b*x+a)/x^3,x, algorithm="giac")
```

output
```
-1/2*(a*b*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sq
rt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqr
t(a^2 + 1)))/(a^2 + 1)^(3/2) - 2*((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1))*a*b + a^2*abs(b) + abs(b))/(((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1))^2 - a^2 - 1)*(a^2 + 1)))*b - 1/2*log(b*x + a + sqrt((b*x + a)^2 +
 1))/x^2
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)}{x^3}\,dx$$

input
```
int(asinh(a + b*x)/x^3,x)
```

output
```
int(asinh(a + b*x)/x^3, x)
```

3.65 $\int \frac{\text{arcsinh}(a+bx)}{x^4} dx$

3.65.1 Optimal result

Integrand size = 10, antiderivative size = 129

$$\int \frac{\text{arcsinh}(a+bx)}{x^4} dx = -\frac{b\sqrt{1+(a+bx)^2}}{6(1+a^2)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}}{2(1+a^2)^2 x} - \frac{\text{arcsinh}(a+bx)}{3x^3} + \frac{(1-2a^2)b^3\text{arctanh}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{6(1+a^2)^{5/2}}$$

output
```
-1/3*arcsinh(b*x+a)/x^3+1/6*(-2*a^2+1)*b^3*arctanh((1+a*(b*x+a))/(a^2+1)^(
1/2)/(1+(b*x+a)^2)^(1/2))/(a^2+1)^(5/2)-1/6*b*(1+(b*x+a)^2)^(1/2)/(a^2+1)/
x^2+1/2*a*b^2*(1+(b*x+a)^2)^(1/2)/(a^2+1)^2/x
```

3.65.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.16

$$\int \frac{\text{arcsinh}(a+bx)}{x^4} dx = \frac{-\sqrt{1+a^2}bx(1+a^2-3abx)\sqrt{1+a^2+2abx+b^2x^2} - 2(1+a^2)^{5/2}\text{arcsinh}(a+bx) + (-1+2a^2)b^3x^3\log}{6(1+a^2)^{5/2}x^3}$$

input
```
Integrate[ArcSinh[a + b*x]/x^4,x]
```

output
```
(-(Sqrt[1 + a^2]*b*x*(1 + a^2 - 3*a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]
) - 2*(1 + a^2)^(5/2)*ArcSinh[a + b*x] + (-1 + 2*a^2)*b^3*x^3*Log[x] + (1
- 2*a^2)*b^3*x^3*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*
x + b^2*x^2]])/(6*(1 + a^2)^(5/2)*x^3)
```

3.65.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6274, 27, 6243, 498, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(a+bx)}{x^4} \, dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \frac{\text{arcsinh}(a+bx)}{x^4}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$b^3 \int \frac{\text{arcsinh}(a+bx)}{b^4x^4}d(a+bx)$$

$$\Big\downarrow \text{ 6243}$$

$$b^3 \left(-\frac{1}{3} \int -\frac{1}{b^3x^3\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{\text{arcsinh}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \text{ 498}$$

$$b^3 \left(\frac{1}{3} \left(\frac{\int -\frac{3a+bx}{b^2x^2\sqrt{(a+bx)^2+1}}d(a+bx)}{2(a^2+1)}-\frac{\sqrt{(a+bx)^2+1}}{2(a^2+1)b^2x^2}\right)-\frac{\text{arcsinh}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \text{ 25}$$

$$b^3 \left(\frac{1}{3} \left(-\frac{\int \frac{3a+bx}{b^2x^2\sqrt{(a+bx)^2+1}}d(a+bx)}{2(a^2+1)}-\frac{\sqrt{(a+bx)^2+1}}{2(a^2+1)b^2x^2}\right)-\frac{\text{arcsinh}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \text{ 679}$$

$$b^3\left(\frac{1}{3}\left(-\frac{-\frac{(1-2a^2)\int -\frac{1}{bx\sqrt{(a+bx)^2+1}}d(a+bx)}{a^2+1}-\frac{3a\sqrt{(a+bx)^2+1}}{(a^2+1)bx}}{2(a^2+1)}-\frac{\sqrt{(a+bx)^2+1}}{2(a^2+1)b^2x^2}\right)-\frac{\operatorname{arcsinh}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \; 488$$

$$b^3\left(\frac{1}{3}\left(-\frac{\frac{(1-2a^2)\int \frac{1}{a^2-\frac{(-a(a+bx)-1)^2}{(a+bx)^2+1}+1}d\frac{-a(a+bx)-1}{\sqrt{(a+bx)^2+1}}}{a^2+1}-\frac{3a\sqrt{(a+bx)^2+1}}{(a^2+1)bx}}{2(a^2+1)}-\frac{\sqrt{(a+bx)^2+1}}{2(a^2+1)b^2x^2}\right)-\frac{\operatorname{arcsinh}(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \; 219$$

$$b^3\left(\frac{1}{3}\left(-\frac{\frac{(1-2a^2)\operatorname{arctanh}\left(\frac{-a(a+bx)-1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{(a^2+1)^{3/2}}-\frac{3a\sqrt{(a+bx)^2+1}}{(a^2+1)bx}}{2(a^2+1)}-\frac{\sqrt{(a+bx)^2+1}}{2(a^2+1)b^2x^2}\right)-\frac{\operatorname{arcsinh}(a+bx)}{3b^3x^3}\right)$$

input
```
Int[ArcSinh[a + b*x]/x^4,x]
```

output
```
b^3*(-1/3*ArcSinh[a + b*x]/(b^3*x^3) + (-1/2*Sqrt[1 + (a + b*x)^2]/((1 + a
^2)*b^2*x^2) - ((-3*a*Sqrt[1 + (a + b*x)^2])/((1 + a^2)*b*x) + ((1 - 2*a^2
)*ArcTanh[(-1 - a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])/(1 +
a^2)^(3/2))/(2*(1 + a^2)))/3)
```

3.65.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.65. $\int \frac{\text{arcsinh}(a+bx)}{x^4} \, dx$

3.65.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.57

method	result
parts	$-\frac{\operatorname{arcsinh}(bx+a)}{3x^3}+\frac{b\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2(a^2+1)x^2}-\frac{3ab\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{(a^2+1)x}+\frac{ab\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{(a^2+1)^{\frac{3}{2}}}\right)}{2(a^2+1)}$
derivativedivides	$b^3\left(-\frac{\operatorname{arcsinh}(bx+a)}{3b^3x^3}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{6(a^2+1)b^2x^2}-\frac{a\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{(a^2+1)bx}+\frac{a\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\sqrt{b^2x^2+2abx+}}{bx}\right)}{(a^2+1)^{\frac{3}{2}}}\right)}{2(a^2+1)}\right.$
default	$b^3\left(-\frac{\operatorname{arcsinh}(bx+a)}{3b^3x^3}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{6(a^2+1)b^2x^2}-\frac{a\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{(a^2+1)bx}+\frac{a\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\sqrt{b^2x^2+2abx+}}{bx}\right)}{(a^2+1)^{\frac{3}{2}}}\right)}{2(a^2+1)}\right.$

input
```
int(arcsinh(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arcsinh(b*x+a)/x^3+1/3*b*(-1/2/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)-3/2*a*b/(a^2+1)*(-1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b/(a^2+1
)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))
/x))+1/2*b^2/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/x))
```

3.65.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(111) = 222$.

Time = 0.29 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.21

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^4}\,dx = \frac{(2a^2-1)\sqrt{a^2+1}b^3x^3\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2abx+a^2+1}\left(a^2-\sqrt{a^2+1}a+1\right)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)+3(a^3+a)b^3x^3}{}$$

input
```
integrate(arcsinh(b*x+a)/x^4,x, algorithm="fricas")
```

output
```
1/6*((2*a^2 - 1)*sqrt(a^2 + 1)*b^3*x^3*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(
a^2 + 1) + a)/x) + 3*(a^3 + a)*b^3*x^3 + 2*(a^6 + 3*a^4 + 3*a^2 + 1)*x^3*l
og(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 2*(a^6 + 3*a^4 - (a^6 +
 3*a^4 + 3*a^2 + 1)*x^3 + 3*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)) + (3*(a^3 + a)*b^2*x^2 - (a^4 + 2*a^2 + 1)*b*x)*sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1))/((a^6 + 3*a^4 + 3*a^2 + 1)*x^3)
```

3.65.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^4}\,dx = \int \frac{\operatorname{asinh}(a+bx)}{x^4}\,dx$$

input
```
integrate(asinh(b*x+a)/x**4,x)
```

output
```
Integral(asinh(a + b*x)/x**4, x)
```

3.65.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(111) = 222.

Time = 0.18 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.20

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^4}\,dx = -\frac{1}{6}\left(\frac{3\,a^2b^2\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{(a^2+1)^{\frac{5}{2}}} - \frac{b^2\operatorname{arsinh}\Bigg(\frac{}{\sqrt{-4\,a\cdots}}\right. - \frac{\operatorname{arsinh}(bx+a)}{3\,x^3}$$

input
```
integrate(arcsinh(b*x+a)/x^4,x, algorithm="maxima")
```

output
```
-1/6*(3*a^2*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)
) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2
 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) - b^2*arcsinh(2*a*b*x/(sqrt(-
4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1
)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^
(3/2) - 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*b/((a^2 + 1)^2*x) + sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)/((a^2 + 1)*x^2))*b - 1/3*arcsinh(b*x + a)/x^3
```

3.65.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(111) = 222$.

Time = 0.38 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.95

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^4}\,dx$$
$$= \frac{1}{6}b\left(\frac{(2\,a^2b^2-b^2)\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{(a^4+2\,a^2+1)\sqrt{a^2+1}} - \frac{2\left(2\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3a^2b^2-\right.}{}\right.$$
$$-\frac{\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)}{3\,x^3}$$

input
```
integrate(arcsinh(b*x+a)/x^4,x, algorithm="giac")
```

output
```
1/6*b*((2*a^2*b^2 - b^2)*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1) + 2*sqrt(a^2 + 1)))/((a^4 + 2*a^2 + 1)*sqrt(a^2 + 1)) - 2*(2*(x*ab
s(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^2*b^2 - 6*(x*abs(b) - sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1))*a^4*b^2 - 4*a^5*b*abs(b) - (x*abs(b) - sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1))^3*b^2 - 7*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1))*a^2*b^2 - 8*a^3*b*abs(b) - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1))*b^2 - 4*a*b*abs(b))/((a^4 + 2*a^2 + 1)*((x*abs(b) - sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1))^2 - a^2 - 1)^2)) - 1/3*log(b*x + a + sqrt((b*x +
a)^2 + 1))/x^3
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^4} \, dx = \int \frac{\operatorname{asinh}(a+b\,x)}{x^4} \,dx$$

input
```
int(asinh(a + b*x)/x^4,x)
```

output
```
int(asinh(a + b*x)/x^4, x)
```

3.66 $\int \frac{\operatorname{arcsinh}(a+bx)}{x^5}\,dx$

3.66.1 Optimal result

Integrand size = 10, antiderivative size = 167

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^5}\,dx = -\frac{b\sqrt{1+(a+bx)^2}}{12(1+a^2)x^3}+\frac{5ab^2\sqrt{1+(a+bx)^2}}{24(1+a^2)^2x^2}+\frac{(4-11a^2)b^3\sqrt{1+(a+bx)^2}}{24(1+a^2)^3x}-\frac{\operatorname{arcsinh}(a+bx)}{4x^4}-\frac{a(3-2a^2)b^4\operatorname{arctanh}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{8(1+a^2)^{7/2}}$$

output
```
-1/4*arcsinh(b*x+a)/x^4-1/8*a*(-2*a^2+3)*b^4*arctanh((1+a*(b*x+a))/(a^2+1)
^(1/2)/(1+(b*x+a)^2)^(1/2))/(a^2+1)^(7/2)-1/12*b*(1+(b*x+a)^2)^(1/2)/(a^2+
1)/x^3+5/24*a*b^2*(1+(b*x+a)^2)^(1/2)/(a^2+1)^2/x^2+1/24*(-11*a^2+4)*b^3*(
1+(b*x+a)^2)^(1/2)/(a^2+1)^3/x
```

3.66.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.07

$$\int \frac{\text{arcsinh}(a+bx)}{x^5}\,dx = \frac{1}{8}\left(-\frac{b\sqrt{1+a^2+2abx+b^2x^2}(2+2a^4-5abx-5a^3bx-4b^2x^2+a^2(4+11b^2x^2))}{3\left(1+a^2\right)^3x^3} - \frac{2\text{arcsinh}(a+bx)}{x^4} - \frac{a(-3+2a^2)\,b^4\log(x)}{(1+a^2)^{7/2}} + \frac{a(-3+2a^2)\,b^4\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)}{(1+a^2)^{7/2}}\right)$$

input
```
Integrate[ArcSinh[a + b*x]/x^5,x]
```

output
```
(-1/3*(b*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2 + 2*a^4 - 5*a*b*x - 5*a^3*b*
x - 4*b^2*x^2 + a^2*(4 + 11*b^2*x^2)))/((1 + a^2)^3*x^3) - (2*ArcSinh[a +
b*x])/x^4 - (a*(-3 + 2*a^2)*b^4*Log[x])/(1 + a^2)^(7/2) + (a*(-3 + 2*a^2)*
b^4*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]]
)/(1 + a^2)^(7/2))/8
```

3.66.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6274, 25, 27, 6243, 498, 25, 688, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(a+bx)}{x^5}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{\text{arcsinh}(a+bx)}{x^5}d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{\text{arcsinh}(a+bx)}{x^5}d(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-b^4 \int -\frac{\text{arcsinh}(a+bx)}{b^5x^5}d(a+bx)$$

$$\Big\downarrow \; 6243$$

$$-b^4 \left(\frac{\text{arcsinh}(a+bx)}{4b^4x^4}-\frac{1}{4}\int \frac{1}{b^4x^4\sqrt{(a+bx)^2+1}}d(a+bx)\right)$$

$$\Big\downarrow \; 498$$

$$-b^4 \left(\frac{1}{4}\left(\frac{\int \frac{3a+2(a+bx)}{b^3x^3\sqrt{(a+bx)^2+1}}d(a+bx)}{3\left(a^2+1\right)}+\frac{\sqrt{(a+bx)^2+1}}{3\left(a^2+1\right)b^3x^3}\right)+\frac{\text{arcsinh}(a+bx)}{4b^4x^4}\right)$$

$$\Big\downarrow \; 25$$

$$-b^4 \left(\frac{1}{4}\left(\frac{\sqrt{(a+bx)^2+1}}{3\left(a^2+1\right)b^3x^3}-\frac{\int -\frac{3a+2(a+bx)}{b^3x^3\sqrt{(a+bx)^2+1}}d(a+bx)}{3\left(a^2+1\right)}\right)+\frac{\text{arcsinh}(a+bx)}{4b^4x^4}\right)$$

$$\Big\downarrow \; 688$$

$$-b^4 \left(\frac{1}{4}\left(\frac{\sqrt{(a+bx)^2+1}}{3\left(a^2+1\right)b^3x^3}-\frac{\frac{5a\sqrt{(a+bx)^2+1}}{2(a^2+1)b^2x^2}-\frac{\int \frac{2\left(2-3a^2\right)-5a(a+bx)}{b^2x^2\sqrt{(a+bx)^2+1}}d(a+bx)}{2(a^2+1)}}{3\left(a^2+1\right)}\right)+\frac{\text{arcsinh}(a+bx)}{4b^4x^4}\right)$$

$$\Big\downarrow \; 679$$

$$-b^4 \left(\frac{1}{4}\left(\frac{\sqrt{(a+bx)^2+1}}{3\left(a^2+1\right)b^3x^3}-\frac{\frac{5a\sqrt{(a+bx)^2+1}}{2(a^2+1)b^2x^2}-\frac{\frac{3a\left(3-2a^2\right)\int -\frac{1}{bx\sqrt{(a+bx)^2+1}}d(a+bx)}{a^2+1}-\frac{\left(4-11a^2\right)\sqrt{(a+bx)^2+1}}{\left(a^2+1\right)bx}}{2(a^2+1)}}{3\left(a^2+1\right)}\right)+\frac{\text{arcsinh}(a+bx)}{4b^4x^4}\right.$$

$$\Big\downarrow \; 488$$

$$-b^4 \left(\frac{1}{4}\left(\frac{\sqrt{(a+bx)^2+1}}{3\left(a^2+1\right)b^3x^3}-\frac{\frac{5a\sqrt{(a+bx)^2+1}}{2(a^2+1)b^2x^2}-\frac{-\frac{3a\left(3-2a^2\right)\int \frac{1}{a^2-\frac{(-a(a+bx)-1)^2}{(a+bx)^2+1}+1}d\frac{-a(a+bx)-1}{\sqrt{(a+bx)^2+1}}}{a^2+1}-\frac{\left(4-11a^2\right)\sqrt{(a+bx)^2+1}}{\left(a^2+1\right)bx}}{2(a^2+1)}}{3\left(a^2+1\right)}\right)+\text{arcsi}\right.$$

$$\Big\downarrow \; 219$$

3.66. $\int \frac{\text{arcsinh}(a+bx)}{x^5}dx$

$$-b^4\left(\frac{1}{4}\left(\frac{\sqrt{(a+bx)^2+1}}{3(a^2+1)b^3x^3}-\frac{\frac{5a\sqrt{(a+bx)^2+1}}{2(a^2+1)b^2x^2}-\frac{-\frac{3a(3-2a^2)\text{arctanh}\left(\frac{-a(a+bx)-1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{(a^2+1)^{3/2}}-\frac{(4-11a^2)\sqrt{(a+bx)^2+1}}{(a^2+1)bx}}{2(a^2+1)}}{3(a^2+1)}\right)+\frac{\text{arcsinh}(a}{4b^4x}\right.$$

input
```
Int[ArcSinh[a + b*x]/x^5,x]
```

output
```
-(b^4*(ArcSinh[a + b*x]/(4*b^4*x^4) + (Sqrt[1 + (a + b*x)^2]/(3*(1 + a^2)*
b^3*x^3) - ((5*a*Sqrt[1 + (a + b*x)^2])/(2*(1 + a^2)*b^2*x^2) - (-(((4 - 1
1*a^2)*Sqrt[1 + (a + b*x)^2])/((1 + a^2)*b*x)) - (3*a*(3 - 2*a^2)*ArcTanh[
(-1 - a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])/(1 + a^2)^(3/2)
)/(2*(1 + a^2)))/(3*(1 + a^2)))/4))
```

3.66.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.66. $\int \frac{\operatorname{arcsinh}(a+bx)}{x^5} \, dx$

3.66.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(145) = 290$.

Time = 0.02 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.07

method	result
parts	$-\frac{\operatorname{arcsinh}(bx+a)}{4x^4}+\frac{b\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{3\left(a^2+1\right)x^3}-\frac{5ab\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2\left(a^2+1\right)x^2}-\frac{3ab\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{\left(a^2+1\right)x}+\frac{ab\ln\left(\frac{2a^2+2+2ab\cdots}{\cdots}\right)}{\cdots}\right)}{2\left(a^2+1\right)\cdots}\right)}{\cdots}\right)}{\cdots}$
derivativedivides	$b^4\left(-\frac{\operatorname{arcsinh}(bx+a)}{4b^4x^4}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{12(a^2+1)b^3x^3}-\frac{5a\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2\left(a^2+1\right)b^2x^2}-\frac{3a\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{\left(a^2+1\right)bx}+\frac{a\ln\left(\frac{2a^2+2+\cdots}{\cdots}\right)}{\cdots}\right)}{2\left(a^2+\cdots\right)}\right)}{\cdots}\right.$
default	$b^4\left(-\frac{\operatorname{arcsinh}(bx+a)}{4b^4x^4}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{12(a^2+1)b^3x^3}-\frac{5a\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2\left(a^2+1\right)b^2x^2}-\frac{3a\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{\left(a^2+1\right)bx}+\frac{a\ln\left(\frac{2a^2+2+\cdots}{\cdots}\right)}{\cdots}\right)}{2\left(a^2+\cdots\right)}\right)}{\cdots}\right.$

input
```
int(arcsinh(b*x+a)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arcsinh(b*x+a)/x^4+1/4*b*(-1/3/(a^2+1)/x^3*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)-5/3*a*b/(a^2+1)*(-1/2/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*a*
b/(a^2+1)*(-1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b/(a^2+1)^(3/2)*ln
((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x))+1/2*b
^2/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+
1)^(1/2))/x))-2/3*b^2/(a^2+1)*(-1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+
a*b/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/x)))
```

3.66.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(145) = 290$.

Time = 0.30 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.05

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^5}\,dx$$
$$= \frac{3\,(2\,a^3-3\,a)\sqrt{a^2+1}b^4x^4\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2+\sqrt{a^2+1}a+1\right)+(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)-(11\,a^4+7\,a^2}{}$$

input
```
integrate(arcsinh(b*x+a)/x^5,x, algorithm="fricas")
```

output
```
1/24*(3*(2*a^3 - 3*a)*sqrt(a^2 + 1)*b^4*x^4*log(-(a^2*b*x + a^3 + sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + sqrt(a^2 + 1)*a + 1) + (a*b*x + a^2 + 1)*
sqrt(a^2 + 1) + a)/x) - (11*a^4 + 7*a^2 - 4)*b^4*x^4 + 6*(a^8 + 4*a^6 + 6*
a^4 + 4*a^2 + 1)*x^4*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 6
*(a^8 + 4*a^6 - (a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*x^4 + 6*a^4 + 4*a^2 + 1)
*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - ((11*a^4 + 7*a^2 - 4)*
b^3*x^3 - 5*(a^5 + 2*a^3 + a)*b^2*x^2 + 2*(a^6 + 3*a^4 + 3*a^2 + 1)*b*x)*s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*x^4)
```

3.66.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^5}\,dx = \int \frac{\operatorname{asinh}(a+bx)}{x^5}\,dx$$

input
```
integrate(asinh(b*x+a)/x**5,x)
```

output
```
Integral(asinh(a + b*x)/x**5, x)
```

3.66.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(145) = 290$.

Time = 0.19 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.14

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^5}\,dx = \frac{1}{24}\left(\frac{15\,a^3b^3\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{(a^2+1)^{\frac{7}{2}}}-\frac{9\,ab^3\operatorname{arsinh}\left(\frac{}{\sqrt{-}}\right.}{}\right. - \frac{\operatorname{arsinh}(bx+a)}{4\,x^4}$$

input
```
integrate(arcsinh(b*x+a)/x^5,x, algorithm="maxima")
```

output
```
1/24*(15*a^3*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x
)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^
2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(7/2) - 9*a*b^3*arcsinh(2*a*b*x/(s
qrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^
2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2
+ 1)^(5/2) - 15*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*b^2/((a^2 + 1)^3*x)
+ 4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2/((a^2 + 1)^2*x) + 5*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*a*b/((a^2 + 1)^2*x^2) - 2*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)/((a^2 + 1)*x^3))*b - 1/4*arcsinh(b*x + a)/x^4
```

3.66.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(145) = 290$.

Time = 0.36 (sec) , antiderivative size = 709, normalized size of antiderivative = 4.25

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^5}\,dx = -\frac{1}{24}\,b\left(\frac{3\,(2\,a^3b^3-3\,ab^3)\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{(a^6+3\,a^4+3\,a^2+1)\sqrt{a^2+1}} - \frac{2\left(6\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)\right.}{} \right. - \frac{\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)}{4\,x^4}$$

input
```
integrate(arcsinh(b*x+a)/x^5,x, algorithm="giac")
```

output
```
-1/24*b*(3*(2*a^3*b^3 - 3*a*b^3)*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1) + 2*sqrt(a^2 + 1)))/((a^6 + 3*a^4 + 3*a^2 + 1)*sqrt(a^2 +
1)) - 2*(6*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*a^3*b^3 - 16*(
x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^5*b^3 + 42*(x*abs(b) - s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^7*b^3 + 12*(x*abs(b) - sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1))^2*a^6*b^2*abs(b) + 20*a^8*b^2*abs(b) - 9*(x*abs(b) - s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*a*b^3 + 8*(x*abs(b) - sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1))^3*a^3*b^3 + 93*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1))*a^5*b^3 + 36*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^4*b
^2*abs(b) + 56*a^6*b^2*abs(b) + 24*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1))^3*a*b^3 + 60*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^3*b^
3 + 36*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^2*b^2*abs(b) + 4
8*a^4*b^2*abs(b) + 9*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a*b^3
+ 12*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*b^2*abs(b) + 8*a^2*b
^2*abs(b) - 4*b^2*abs(b))/((a^6 + 3*a^4 + 3*a^2 + 1)*((x*abs(b) - sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1))^2 - a^2 - 1)^3)) - 1/4*log(b*x + a + sqrt((b*x
+ a)^2 + 1))/x^4
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)}{x^5}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)}{x^5}\,dx$$

input
```
int(asinh(a + b*x)/x^5,x)
```

output
```
int(asinh(a + b*x)/x^5, x)
```

3.67 $\int x^3 \operatorname{arcsinh}(a+bx)^2 \, dx$

3.67.1 Optimal result

Integrand size = 12, antiderivative size = 331

$$\begin{aligned}\int x^3 \operatorname{arcsinh}(a+bx)^2 \, dx = {} & \frac{4ax}{3b^3} - \frac{2a^3x}{b^3} - \frac{3(a+bx)^2}{32b^4} + \frac{3a^2(a+bx)^2}{4b^4} - \frac{2a(a+bx)^3}{9b^4} \\ & + \frac{(a+bx)^4}{32b^4} - \frac{4a\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3b^4} \\ & + \frac{2a^3\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{b^4} \\ & + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{16b^4} \\ & - \frac{3a^2(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{2b^4} \\ & + \frac{2a(a+bx)^2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{3b^4} \\ & - \frac{(a+bx)^3\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{8b^4} \\ & - \frac{3\operatorname{arcsinh}(a+bx)^2}{32b^4} + \frac{3a^2\operatorname{arcsinh}(a+bx)^2}{4b^4} \\ & - \frac{a^4\operatorname{arcsinh}(a+bx)^2}{4b^4} + \frac{1}{4}x^4\operatorname{arcsinh}(a+bx)^2\end{aligned}$$

output
```
4/3*a*x/b^3-2*a^3*x/b^3-3/32*(b*x+a)^2/b^4+3/4*a^2*(b*x+a)^2/b^4-2/9*a*(b*
x+a)^3/b^4+1/32*(b*x+a)^4/b^4-3/32*arcsinh(b*x+a)^2/b^4+3/4*a^2*arcsinh(b*
x+a)^2/b^4-1/4*a^4*arcsinh(b*x+a)^2/b^4+1/4*x^4*arcsinh(b*x+a)^2-4/3*a*arc
sinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^4+2*a^3*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/
2)/b^4+3/16*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^4-3/2*a^2*(b*x+a)
*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^4+2/3*a*(b*x+a)^2*arcsinh(b*x+a)*(1+
(b*x+a)^2)^(1/2)/b^4-1/8*(b*x+a)^3*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^4
```

3.67.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.44

$$\int x^3 \text{arcsinh}(a+bx)^2\,dx$$
$$= \frac{bx(-300a^3+78a^2bx+a(330-28b^2x^2)+9bx(-3+b^2x^2))+6\sqrt{1+a^2+2abx+b^2x^2}(50a^3+9bx-26a}{288b^4}$$

input
```
Integrate[x^3*ArcSinh[a + b*x]^2,x]
```

output
```
(b*x*(-300*a^3 + 78*a^2*b*x + a*(330 - 28*b^2*x^2) + 9*b*x*(-3 + b^2*x^2))
 + 6*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(50*a^3 + 9*b*x - 26*a^2*b*x - 6*b^
3*x^3 + a*(-55 + 14*b^2*x^2))*ArcSinh[a + b*x] - 9*(3 - 24*a^2 + 8*a^4 - 8
*b^4*x^4)*ArcSinh[a + b*x]^2)/(288*b^4)
```

3.67.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6274, 25, 27, 6243, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{arcsinh}(a+bx)^2\,dx$$
$$\Big\downarrow \; 6274$$
$$\frac{\int x^3 \text{arcsinh}(a+bx)^2 d(a+bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -x^3 \text{arcsinh}(a+bx)^2 d(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int -b^3x^3 \text{arcsinh}(a+bx)^2 d(a+bx)}{b^4}$$

$$\Big\downarrow \; 6243$$

$$-\frac{\frac{1}{2}\int \frac{b^4x^4\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx) - \frac{1}{4}b^4x^4\text{arcsinh}(a+bx)^2}{b^4}$$

$$\Big\downarrow \; 6253$$

$$-\frac{\frac{1}{2}\int \left(\frac{\text{arcsinh}(a+bx)a^4}{\sqrt{(a+bx)^2+1}} - \frac{4(a+bx)\text{arcsinh}(a+bx)a^3}{\sqrt{(a+bx)^2+1}} + \frac{6(a+bx)^2\text{arcsinh}(a+bx)a^2}{\sqrt{(a+bx)^2+1}} - \frac{4(a+bx)^3\text{arcsinh}(a+bx)a}{\sqrt{(a+bx)^2+1}} + \frac{(a+bx)^4\text{arcsin}}{\sqrt{(a+bx)^2}}\right.}{b^4}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{1}{2}\left(\frac{1}{2}a^4\text{arcsinh}(a+bx)^2 - 4a^3\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx) + 4a^3(a+bx) - \frac{3}{2}a^2\text{arcsinh}(a+bx)^2 + 3a^2(a+b\right.}{}$$

input

```
Int[x^3*ArcSinh[a + b*x]^2,x]
```

output

```
-((-1/4*(b^4*x^4*ArcSinh[a + b*x]^2) + ((-8*a*(a + b*x))/3 + 4*a^3*(a + b*
x) + (3*(a + b*x)^2)/16 - (3*a^2*(a + b*x)^2)/2 + (4*a*(a + b*x)^3)/9 - (a
 + b*x)^4/16 + (8*a*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/3 - 4*a^3*Sqrt
[1 + (a + b*x)^2]*ArcSinh[a + b*x] - (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*Ar
cSinh[a + b*x])/8 + 3*a^2*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]
 - (4*a*(a + b*x)^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/3 + ((a + b*x)
^3*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/4 + (3*ArcSinh[a + b*x]^2)/16 -
 (3*a^2*ArcSinh[a + b*x]^2)/2 + (a^4*ArcSinh[a + b*x]^2)/2)/2)/b^4)
```

3.67. $\int x^3 \text{arcsinh}(a+bx)^2 \, dx$

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.67.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsinh}(bx+a)^2(bx+a)^4}{4}-\frac{\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)^3}{8}+\frac{3\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)}{16}-\frac{3\operatorname{arcsinh}(bx+a)^2}{32}+\frac{(bx+a)}{32}}{}$
default	$\frac{\frac{\operatorname{arcsinh}(bx+a)^2(bx+a)^4}{4}-\frac{\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)^3}{8}+\frac{3\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)}{16}-\frac{3\operatorname{arcsinh}(bx+a)^2}{32}+\frac{(bx+a)}{32}}{}$

input
```
int(x^3*arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/4*arcsinh(b*x+a)^2*(b*x+a)^4-1/8*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/
2)*(b*x+a)^3+3/16*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*(b*x+a)-3/32*arcsinh(
b*x+a)^2+1/32*(b*x+a)^4-3/32*(b*x+a)^2-3/32-1/9*a*(9*(b*x+a)^3*arcsinh(b*x
+a)^2-6*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*(b*x+a)^2+2*(b*x+a)^3+12*arcsin
h(b*x+a)*(1+(b*x+a)^2)^(1/2)-12*b*x-12*a)+3/4*a^2*(2*arcsinh(b*x+a)^2*(b*x
+a)^2-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*(b*x+a)+arcsinh(b*x+a)^2+(b*x+a
)^2+1)-a^3*(arcsinh(b*x+a)^2*(b*x+a)-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+
2*b*x+2*a))
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.55

$$\int x^3 \text{arcsinh}(a+bx)^2\,dx$$
$$= \frac{9\,b^4x^4 - 28\,ab^3x^3 + 3\,(26\,a^2-9)b^2x^2 - 30\,(10\,a^3-11\,a)bx + 9\,(8\,b^4x^4 - 8\,a^4 + 24\,a^2 - 3)\log\left(bx + a + \sqrt{\right.}}{}$$

input
```
integrate(x^3*arcsinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/288*(9*b^4*x^4 - 28*a*b^3*x^3 + 3*(26*a^2 - 9)*b^2*x^2 - 30*(10*a^3 - 11
*a)*b*x + 9*(8*b^4*x^4 - 8*a^4 + 24*a^2 - 3)*log(b*x + a + sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1))^2 - 6*(6*b^3*x^3 - 14*a*b^2*x^2 - 50*a^3 + (26*a^2 - 9
)*b*x + 55*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)))/b^4
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.11

$$\int x^3 \text{arcsinh}(a+bx)^2\,dx$$
$$= \begin{cases} -\frac{a^4 \operatorname{asinh}^2(a+bx)}{4b^4} - \frac{25a^3x}{24b^3} + \frac{25a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{24b^4} + \frac{13a^2x^2}{48b^2} - \frac{13a^2x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{24b^3} + \frac{3a^2 \operatorname{asi}}{} \\ \frac{x^4\operatorname{asinh}^2(a)}{4} \end{cases}$$

input
```
integrate(x**3*asinh(b*x+a)**2,x)
```

output
```
Piecewise((-a**4*asinh(a + b*x)**2/(4*b**4) - 25*a**3*x/(24*b**3) + 25*a**
3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(24*b**4) + 13*a**2*
x**2/(48*b**2) - 13*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a +
b*x)/(24*b**3) + 3*a**2*asinh(a + b*x)**2/(4*b**4) - 7*a*x**3/(72*b) + 7*a
*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(24*b**2) + 55*a
*x/(48*b**3) - 55*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(4
8*b**4) + x**4*asinh(a + b*x)**2/4 + x**4/32 - x**3*sqrt(a**2 + 2*a*b*x +
b**2*x**2 + 1)*asinh(a + b*x)/(8*b) - 3*x**2/(32*b**2) + 3*x*sqrt(a**2 + 2
*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(16*b**3) - 3*asinh(a + b*x)**2/(32
*b**4), Ne(b, 0)), (x**4*asinh(a)**2/4, True))
```

3.67.7 Maxima [F]

$$\int x^3 \text{arcsinh}(a+bx)^2\,dx = \int x^3 \operatorname{arsinh}(bx+a)^2\,dx$$

input
```
integrate(x^3*arcsinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*x^4*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - integrate(1/2
*(b^3*x^6 + 2*a*b^2*x^5 + (a^2*b + b)*x^4 + (b^2*x^5 + a*b*x^4)*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/
(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2
+ 1)^(3/2) + a), x)
```

3.67.8 Giac [F]

$$\int x^3 \text{arcsinh}(a+bx)^2\,dx = \int x^3 \operatorname{arsinh}(bx+a)^2\,dx$$

input
```
integrate(x^3*arcsinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*arcsinh(b*x + a)^2, x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \text{arcsinh}(a+bx)^2\,dx = \int x^3\,\text{asinh}(a+b\,x)^2\,dx$$

input
```
int(x^3*asinh(a + b*x)^2,x)
```

output
```
int(x^3*asinh(a + b*x)^2, x)
```

3.68 $\int x^2 \text{arcsinh}(a+bx)^2\,dx$

3.68.1 Optimal result

Integrand size = 12, antiderivative size = 211

$$\begin{aligned}\int x^2 \text{arcsinh}(a+bx)^2\,dx = &-\frac{4x}{9b^2}+\frac{2a^2x}{b^2}-\frac{a(a+bx)^2}{2b^3}+\frac{2(a+bx)^3}{27b^3}\\ &+\frac{4\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)}{9b^3}\\ &-\frac{2a^2\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)}{b^3}\\ &+\frac{a(a+bx)\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)}{b^3}\\ &-\frac{2(a+bx)^2\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)}{9b^3}\\ &-\frac{a\text{arcsinh}(a+bx)^2}{2b^3}+\frac{a^3\text{arcsinh}(a+bx)^2}{3b^3}+\frac{1}{3}x^3\text{arcsinh}(a+bx)^2\end{aligned}$$

output
```
-4/9*x/b^2+2*a^2*x/b^2-1/2*a*(b*x+a)^2/b^3+2/27*(b*x+a)^3/b^3-1/2*a*arcsin
h(b*x+a)^2/b^3+1/3*a^3*arcsinh(b*x+a)^2/b^3+1/3*x^3*arcsinh(b*x+a)^2+4/9*a
rcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3-2*a^2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(
1/2)/b^3+a*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3-2/9*(b*x+a)^2*ar
csinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3
```

3.68.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.51

$$\int x^2 \text{arcsinh}(a+bx)^2\,dx$$
$$= \frac{bx(-24+66a^2-15abx+4b^2x^2)-6\sqrt{1+a^2+2abx+b^2x^2}(-4+11a^2-5abx+2b^2x^2)\,\text{arcsinh}(a+bx)}{54b^3}$$

input
```
Integrate[x^2*ArcSinh[a + b*x]^2,x]
```

output
```
(b*x*(-24 + 66*a^2 - 15*a*b*x + 4*b^2*x^2) - 6*Sqrt[1 + a^2 + 2*a*b*x + b^
2*x^2]*(-4 + 11*a^2 - 5*a*b*x + 2*b^2*x^2)*ArcSinh[a + b*x] + 9*(-3*a + 2*
a^3 + 2*b^3*x^3)*ArcSinh[a + b*x]^2)/(54*b^3)
```

3.68.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6274, 27, 6243, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arcsinh}(a+bx)^2\,dx$$
$$\Big\downarrow \text{6274}$$
$$\frac{\int x^2 \text{arcsinh}(a+bx)^2 d(a+bx)}{b}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int b^2x^2 \text{arcsinh}(a+bx)^2 d(a+bx)}{b^3}$$
$$\Big\downarrow \text{6243}$$
$$\frac{\frac{2}{3}\int -\frac{b^3x^3\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{3}b^3x^3\text{arcsinh}(a+bx)^2}{b^3}$$
$$\Big\downarrow \text{6253}$$

$$\frac{\frac{2}{3}\int \left(\frac{\text{arcsinh}(a+bx)a^3}{\sqrt{(a+bx)^2+1}} - \frac{3(a+bx)\text{arcsinh}(a+bx)a^2}{\sqrt{(a+bx)^2+1}} + \frac{3(a+bx)^2\text{arcsinh}(a+bx)a}{\sqrt{(a+bx)^2+1}} - \frac{(a+bx)^3\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}\right) d(a+bx) + \frac{1}{3}b^3x^3a}{b^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}\left(\frac{1}{2}a^3\text{arcsinh}(a+bx)^2 - 3a^2\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx) + 3a^2(a+bx) - \frac{3}{4}a\text{arcsinh}(a+bx)^2 + \frac{3}{2}a(a+bx)\sqrt{}\right.$$

input
```
Int[x^2*ArcSinh[a + b*x]^2,x]
```

output
```
((b^3*x^3*ArcSinh[a + b*x]^2)/3 + (2*((-2*(a + b*x))/3 + 3*a^2*(a + b*x) -
 (3*a*(a + b*x)^2)/4 + (a + b*x)^3/9 + (2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a
+ b*x])/3 - 3*a^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x] + (3*a*(a + b*x)*
Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/2 - ((a + b*x)^2*Sqrt[1 + (a + b*x
)^2]*ArcSinh[a + b*x])/3 - (3*a*ArcSinh[a + b*x]^2)/4 + (a^3*ArcSinh[a + b
*x]^2)/2))/3)/b^3
```

3.68.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.68.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\frac{(bx+a)^3 \operatorname{arcsinh}(bx+a)^2}{3}+\frac{4 \operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}}{9}-\frac{2 \operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)^2}{9}-\frac{4bx}{9}-\frac{4a}{9}+\frac{2(bx+a)^3}{27}-\frac{a\left(2 \operatorname{arcs}\right.}{}}{}$
default	$\frac{\frac{(bx+a)^3 \operatorname{arcsinh}(bx+a)^2}{3}+\frac{4 \operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}}{9}-\frac{2 \operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)^2}{9}-\frac{4bx}{9}-\frac{4a}{9}+\frac{2(bx+a)^3}{27}-\frac{a\left(2 \operatorname{arcs}\right.}{}}{}$

input
```
int(x^2*arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*(b*x+a)^3*arcsinh(b*x+a)^2+4/9*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/
2)-2/9*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*(b*x+a)^2-4/9*b*x-4/9*a+2/27*(b*
x+a)^3-1/2*a*(2*arcsinh(b*x+a)^2*(b*x+a)^2-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^
(1/2)*(b*x+a)+arcsinh(b*x+a)^2+(b*x+a)^2+1)+a^2*(arcsinh(b*x+a)^2*(b*x+a)-
2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+2*b*x+2*a))
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.69

$$\int x^2 \operatorname{arcsinh}(a+bx)^2\,dx$$
$$= \frac{4\,b^3x^3-15\,ab^2x^2+6\,(11\,a^2-4)bx+9\,(2\,b^3x^3+2\,a^3-3\,a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-6\,(\cdots}{54\,b^3}$$

input
```
integrate(x^2*arcsinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/54*(4*b^3*x^3 - 15*a*b^2*x^2 + 6*(11*a^2 - 4)*b*x + 9*(2*b^3*x^3 + 2*a^3
 - 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - 6*(2*b^2*x^2
- 5*a*b*x + 11*a^2 - 4)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b^3
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.15

$$\int x^2 \text{arcsinh}(a+bx)^2\,dx$$
$$= \begin{cases} \frac{a^3 \operatorname{asinh}^2(a+bx)}{3b^3} + \frac{11a^2x}{9b^2} - \frac{11a^2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{9b^3} - \frac{5ax^2}{18b} + \frac{5ax\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{9b^2} - \frac{a\operatorname{asinh}^2(a+bx)}{2b^3} \\ \frac{x^3\operatorname{asinh}^2(a)}{3} \end{cases}$$

input
```
integrate(x**2*asinh(b*x+a)**2,x)
```

output
```
Piecewise((a**3*asinh(a + b*x)**2/(3*b**3) + 11*a**2*x/(9*b**2) - 11*a**2*
sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b**3) - 5*a*x**2/(1
8*b) + 5*a*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b**2)
- a*asinh(a + b*x)**2/(2*b**3) + x**3*asinh(a + b*x)**2/3 + 2*x**3/27 - 2*
x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b) - 4*x/(9*b*
*2) + 4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b**3), Ne(b
, 0)), (x**3*asinh(a)**2/3, True))
```

3.68.7 Maxima [F]

$$\int x^2 \text{arcsinh}(a+bx)^2\,dx = \int x^2 \operatorname{arsinh}(bx+a)^2\,dx$$

input
```
integrate(x^2*arcsinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*x^3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - integrate(2/3
*(b^3*x^5 + 2*a*b^2*x^4 + (a^2*b + b)*x^3 + (b^2*x^4 + a*b*x^3)*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/
(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2
+ 1)^(3/2) + a), x)
```

3.68.8 Giac [F]

$$\int x^2 \text{arcsinh}(a+bx)^2\,dx = \int x^2 \operatorname{arsinh}(bx+a)^2\,dx$$

input
```
integrate(x^2*arcsinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arcsinh(b*x + a)^2, x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arcsinh}(a+bx)^2\,dx = \int x^2 \operatorname{asinh}(a+b\,x)^2\,dx$$

input
```
int(x^2*asinh(a + b*x)^2,x)
```

output
```
int(x^2*asinh(a + b*x)^2, x)
```

3.69 $\int x\text{arcsinh}(a+bx)^2\,dx$

3.69.1 Optimal result

Integrand size = 10, antiderivative size = 126

$$\int x\text{arcsinh}(a+bx)^2\,dx=-\frac{2ax}{b}+\frac{(a+bx)^2}{4b^2}+\frac{2a\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)}{b^2}-\frac{(a+bx)\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)}{2b^2}+\frac{\text{arcsinh}(a+bx)^2}{4b^2}-\frac{a^2\text{arcsinh}(a+bx)^2}{2b^2}+\frac{1}{2}x^2\text{arcsinh}(a+bx)^2$$

output
```
-2*a*x/b+1/4*(b*x+a)^2/b^2+1/4*arcsinh(b*x+a)^2/b^2-1/2*a^2*arcsinh(b*x+a)
^2/b^2+1/2*x^2*arcsinh(b*x+a)^2+2*a*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^2
-1/2*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^2
```

3.69.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.63

$$\int x\text{arcsinh}(a+bx)^2\,dx=\frac{bx(-6a+bx)+2(3a-bx)\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)+(1-2a^2+2b^2x^2)\,\text{arcsinh}(a+bx)^2}{4b^2}$$

input
```
Integrate[x*ArcSinh[a + b*x]^2,x]
```

output
```
(b*x*(-6*a + b*x) + 2*(3*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSin
h[a + b*x] + (1 - 2*a^2 + 2*b^2*x^2)*ArcSinh[a + b*x]^2)/(4*b^2)
```

3.69.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6274, 25, 27, 6243, 6253, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arcsinh}(a+bx)^2 \, dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int x\text{arcsinh}(a+bx)^2d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -x\text{arcsinh}(a+bx)^2d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -bx\text{arcsinh}(a+bx)^2d(a+bx)}{b^2}$$

$$\Big\downarrow \text{6243}$$

$$-\frac{\int \frac{b^2x^2\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{1}{2}b^2x^2\text{arcsinh}(a+bx)^2}{b^2}$$

$$\Big\downarrow \text{6253}$$

$$-\frac{\int \left(\frac{\text{arcsinh}(a+bx)a^2}{\sqrt{(a+bx)^2+1}}-\frac{2(a+bx)\text{arcsinh}(a+bx)a}{\sqrt{(a+bx)^2+1}}+\frac{(a+bx)^2\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}\right)d(a+bx)-\frac{1}{2}b^2x^2\text{arcsinh}(a+bx)^2}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}a^2\text{arcsinh}(a+bx)^2-\frac{1}{2}b^2x^2\text{arcsinh}(a+bx)^2+\frac{1}{2}\sqrt{(a+bx)^2+1}(a+bx)\text{arcsinh}(a+bx)-\frac{1}{4}\text{arcsinh}(a+bx)^2}{b^2}$$

input
```
Int[x*ArcSinh[a + b*x]^2,x]
```

output
```
-((2*a*(a + b*x) - (a + b*x)^2/4 - 2*a*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b
*x] + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/2 - ArcSinh[a + b
*x]^2/4 + (a^2*ArcSinh[a + b*x]^2)/2 - (b^2*x^2*ArcSinh[a + b*x]^2)/2)/b^2
)
```

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6253
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.69.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsinh}(bx+a)^2\left(1+(bx+a)^2\right)}{2}-\frac{\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)}{2}-\frac{\operatorname{arcsinh}(bx+a)^2}{4}+\frac{(bx+a)^2}{4}+\frac{1}{4}-a\left(\operatorname{arcsinh}(bx+a)^2(bx+a)\cdot\right.}{b^2}$
default	$\frac{\frac{\operatorname{arcsinh}(bx+a)^2\left(1+(bx+a)^2\right)}{2}-\frac{\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)}{2}-\frac{\operatorname{arcsinh}(bx+a)^2}{4}+\frac{(bx+a)^2}{4}+\frac{1}{4}-a\left(\operatorname{arcsinh}(bx+a)^2(bx+a)\cdot\right.}{b^2}$

input
```
int(x*arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/2*arcsinh(b*x+a)^2*(1+(b*x+a)^2)-1/2*arcsinh(b*x+a)*(1+(b*x+a)^2)
^(1/2)*(b*x+a)-1/4*arcsinh(b*x+a)^2+1/4*(b*x+a)^2+1/4-a*(arcsinh(b*x+a)^2*
(b*x+a)-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+2*b*x+2*a))
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int x\operatorname{arcsinh}(a+bx)^2\,dx$$
$$=\frac{b^2x^2-6\,abx+(2\,b^2x^2-2\,a^2+1)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-2\,\sqrt{b^2x^2+2\,abx+a^2+1}(bx}{4\,b^2}$$

input
```
integrate(x*arcsinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(b^2*x^2 - 6*a*b*x + (2*b^2*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))^2 - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a
)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b^2
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.10

$$\int x\text{arcsinh}(a+bx)^2\,dx$$
$$=\begin{cases} -\frac{a^2\,\text{asinh}^2\,(a+bx)}{2b^2}-\frac{3ax}{2b}+\frac{3a\sqrt{a^2+2abx+b^2x^2+1}\,\text{asinh}\,(a+bx)}{2b^2}+\frac{x^2\,\text{asinh}^2\,(a+bx)}{2}+\frac{x^2}{4}-\frac{x\sqrt{a^2+2abx+b^2x^2+1}\,\text{asinh}\,(a+bx)}{2b}+ \\ \frac{x^2\,\text{asinh}^2\,(a)}{2} \end{cases}$$

input
```
integrate(x*asinh(b*x+a)**2,x)
```

output
```
Piecewise((-a**2*asinh(a + b*x)**2/(2*b**2) - 3*a*x/(2*b) + 3*a*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(2*b**2) + x**2*asinh(a + b*x)**
2/2 + x**2/4 - x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(2*b)
 + asinh(a + b*x)**2/(4*b**2), Ne(b, 0)), (x**2*asinh(a)**2/2, True))
```

3.69.7 Maxima [F]

$$\int x\text{arcsinh}(a+bx)^2\,dx=\int x\,\text{arsinh}\,(bx+a)^2\,dx$$

input
```
integrate(x*arcsinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*x^2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - integrate((b^
3*x^4 + 2*a*b^2*x^3 + (a^2*b + b)*x^2 + (b^2*x^3 + a*b*x^2)*sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3
*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)
^(3/2) + a), x)
```

3.69.8 Giac [F]

$$\int x\text{arcsinh}(a+bx)^2\,dx=\int x\,\text{arsinh}\left(bx+a\right)^2\,dx$$

input
```
integrate(x*arcsinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x*arcsinh(b*x + a)^2, x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arcsinh}(a+bx)^2\,dx=\int x\,\text{asinh}(a+b\,x)^2\,dx$$

input
```
int(x*asinh(a + b*x)^2,x)
```

output
```
int(x*asinh(a + b*x)^2, x)
```

3.70 $\int \operatorname{arcsinh}(a+bx)^2\,dx$

3.70.1 Optimal result

Integrand size = 8, antiderivative size = 45

$$\int \operatorname{arcsinh}(a+bx)^2\,dx=2x-\frac{2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{b}+\frac{(a+bx)\operatorname{arcsinh}(a+bx)^2}{b}$$

output
```
2*x+(b*x+a)*arcsinh(b*x+a)^2/b-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b
```

3.70.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \operatorname{arcsinh}(a+bx)^2\,dx =\frac{2(a+bx)-2\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)+(a+bx)\operatorname{arcsinh}(a+bx)^2}{b}$$

input
```
Integrate[ArcSinh[a + b*x]^2,x]
```

output
```
(2*(a + b*x) - 2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x] + (a + b*x)*ArcSin
h[a + b*x]^2)/b
```

3.70.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6273, 6187, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}(a+bx)^2\,dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int \text{arcsinh}(a+bx)^2 d(a+bx)}{b}$$

$$\Big\downarrow \text{6187}$$

$$\frac{(a+bx)\text{arcsinh}(a+bx)^2-2\int \frac{(a+bx)\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b}$$

$$\Big\downarrow \text{6213}$$

$$\frac{(a+bx)\text{arcsinh}(a+bx)^2-2\left(\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)-\int 1d(a+bx)\right)}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{(a+bx)\text{arcsinh}(a+bx)^2-2\left(\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)-a-bx\right)}{b}$$

input
```
Int[ArcSinh[a + b*x]^2,x]
```

output
```
((a + b*x)*ArcSinh[a + b*x]^2 - 2*(-a - b*x + Sqrt[1 + (a + b*x)^2]*ArcSin
h[a + b*x]))/b
```

3.70.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.70.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{\operatorname{arcsinh}(bx+a)^2(bx+a)-2\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}+2bx+2a}{b}$	46
default	$\frac{\operatorname{arcsinh}(bx+a)^2(bx+a)-2\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}+2bx+2a}{b}$	46

input
```
int(arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(arcsinh(b*x+a)^2*(b*x+a)-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+2*b*x+2
*a)
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(43) = 86.

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.96

$$\int \operatorname{arcsinh}(a+bx)^2\,dx$$
$$=\frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2+2\,bx-2\,\sqrt{b^2x^2+2\,abx+a^2+1}\log\left(bx+a+\sqrt{b^2x^2+\right.}{b}$$

input
```
integrate(arcsinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 2*b*x - 2*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)))/b
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int \operatorname{arcsinh}(a+bx)^2\,dx$$
$$=\begin{cases}\frac{a\operatorname{asinh}^2(a+bx)}{b}+x\operatorname{asinh}^2(a+bx)+2x-\frac{2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{b} & \text{for } b\neq 0\\ x\operatorname{asinh}^2(a) & \text{otherwise}\end{cases}$$

input
```
integrate(asinh(b*x+a)**2,x)
```

output
```
Piecewise((a*asinh(a + b*x)**2/b + x*asinh(a + b*x)**2 + 2*x - 2*sqrt(a**2
 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/b, Ne(b, 0)), (x*asinh(a)**2, T
rue))
```

3.70.7 Maxima [F]

$$\int \operatorname{arcsinh}(a+bx)^2\,dx = \int \operatorname{arsinh}\left(bx+a\right)^2\,dx$$

input
```
integrate(arcsinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
x*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - integrate(2*(b^3*x^
3 + 2*a*b^2*x^2 + (a^2*b + b)*x + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x
^2 + a*b*x))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^3 + 3
*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) +
 a), x)
```

3.70.8 Giac [F]

$$\int \operatorname{arcsinh}(a+bx)^2\,dx = \int \operatorname{arsinh}\left(bx+a\right)^2\,dx$$

input
```
integrate(arcsinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(arcsinh(b*x + a)^2, x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(a+bx)^2\,dx = \int \operatorname{asinh}(a+b\,x)^2\,dx$$

input
```
int(asinh(a + b*x)^2,x)
```

output
```
int(asinh(a + b*x)^2, x)
```

3.71 $\int \frac{\text{arcsinh}(a+bx)^2}{x} dx$

3.71.1 Optimal result

Integrand size = 12, antiderivative size = 205

$$\begin{aligned}\int \frac{\text{arcsinh}(a+bx)^2}{x} dx = & -\frac{1}{3}\text{arcsinh}(a+bx)^3 + \text{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\ & + \text{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\ & + 2\text{arcsinh}(a+bx)\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) \\ & + 2\text{arcsinh}(a+bx)\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right) \\ & - 2\,\text{PolyLog}\left(3,\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) - 2\,\text{PolyLog}\left(3,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)\end{aligned}$$

output
```
-1/3*arcsinh(b*x+a)^3+arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a
-(a^2+1)^(1/2)))+arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2
+1)^(1/2)))+2*arcsinh(b*x+a)*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2
+1)^(1/2)))+2*arcsinh(b*x+a)*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2
+1)^(1/2)))-2*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))-2*p
olylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))
```

3.71.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x}\,dx = -\frac{1}{3}\operatorname{arcsinh}(a+bx)^3+\operatorname{arcsinh}(a+bx)^2\log\left(1+\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right)+\operatorname{arcsinh}(a+bx)^2\log\left(1+\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right)+2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right)+2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right)-2\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)-2\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)$$

input
```
Integrate[ArcSinh[a + b*x]^2/x,x]
```

output
```
-1/3*ArcSinh[a + b*x]^3 + ArcSinh[a + b*x]^2*Log[1 + E^ArcSinh[a + b*x]/((
-(a/b) - Sqrt[1 + a^2]/b)*b)] + ArcSinh[a + b*x]^2*Log[1 + E^ArcSinh[a + b
*x]/((-(a/b) + Sqrt[1 + a^2]/b)*b)] + 2*ArcSinh[a + b*x]*PolyLog[2, -(E^Ar
cSinh[a + b*x]/((-(a/b) - Sqrt[1 + a^2]/b)*b))] + 2*ArcSinh[a + b*x]*PolyL
og[2, -(E^ArcSinh[a + b*x]/((-(a/b) + Sqrt[1 + a^2]/b)*b))] - 2*PolyLog[3,
 E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] - 2*PolyLog[3, E^ArcSinh[a + b*x]
/(a + Sqrt[1 + a^2])]
```

3.71.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6274, 25, 27, 6242, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{\text{arcsinh}(a+bx)^2}{x}d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{\text{arcsinh}(a+bx)^2}{x}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\int -\frac{\text{arcsinh}(a+bx)^2}{bx}d(a+bx)$$

$$\Big\downarrow \text{6242}$$

$$-\int -\frac{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)^2}{bx}d\text{arcsinh}(a+bx)$$

$$\Big\downarrow \text{6095}$$

$$-\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)^2}{a-e^{\text{arcsinh}(a+bx)}-\sqrt{a^2+1}}d\text{arcsinh}(a+bx)-\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)^2}{a-e^{\text{arcsinh}(a+bx)}+\sqrt{a^2+1}}d\text{arcsinh}(a+bx)-\frac{1}{3}\text{arcsinh}(a+bx)^3$$

$$\Big\downarrow \text{2620}$$

$$-2\int \text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)d\text{arcsinh}(a+bx)-2\int \text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)d\text{arcsinh}(a+bx)+\text{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\text{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)-\frac{1}{3}\text{arcsinh}(a+bx)^3$$

$$\Big\downarrow \text{3011}$$

$$-2\left(\int \text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)d\text{arcsinh}(a+bx)-\text{arcsinh}(a+bx)\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)\right)-2\left(\int \text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)d\text{arcsinh}(a+bx)-\text{arcsinh}(a+bx)\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)\right)+\text{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\text{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)-\frac{1}{3}\text{arcsinh}(a+bx)^3$$

$$\Big\downarrow \text{2720}$$

3.71. $\int \frac{\text{arcsinh}(a+bx)^2}{x}\,dx$

$$-2\left(\int e^{-\text{arcsinh}(a+bx)}\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)de^{\text{arcsinh}(a+bx)}-\text{arcsinh}(a+bx)\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)\right)-2\left(\int e^{-\text{arcsinh}(a+bx)}\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)de^{\text{arcsinh}(a+bx)}-\text{arcsinh}(a+bx)\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)\right)+\text{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\text{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)-\frac{1}{3}\text{arcsinh}(a+bx)^3$$

$$\Big\downarrow \text{7143}$$

$$-2\left(\text{PolyLog}\left(3,\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)-\text{arcsinh}(a+bx)\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)\right)-2\left(\text{PolyLog}\left(3,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)-\text{arcsinh}(a+bx)\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)\right)+\text{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\text{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)-\frac{1}{3}\text{arcsinh}(a+bx)^3$$

input
```
Int[ArcSinh[a + b*x]^2/x,x]
```

output
```
-1/3*ArcSinh[a + b*x]^3 + ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(a
 - Sqrt[1 + a^2])] + ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(a + Sq
rt[1 + a^2])] - 2*(-(ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a - S
qrt[1 + a^2])]) + PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])]) - 2*
(-(ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]) +
PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])
```

3.71.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.71. $\int \frac{\operatorname{arcsinh}(a+bx)^2}{x}\,dx$

3.71.4 Maple [F]

$$\int \frac{\operatorname{arcsinh}(bx+a)^2}{x}dx$$

input
```
int(arcsinh(b*x+a)^2/x,x)
```

output
```
int(arcsinh(b*x+a)^2/x,x)
```

3.71.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{arsinh}(bx+a)^2}{x}\,dx$$

input
```
integrate(arcsinh(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(arcsinh(b*x + a)^2/x, x)
```

3.71.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{asinh}^2(a+bx)}{x}\,dx$$

input
```
integrate(asinh(b*x+a)**2/x,x)
```

output
```
Integral(asinh(a + b*x)**2/x, x)
```

3.71.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(arcsinh(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
integrate(arcsinh(b*x + a)^2/x, x)
```

3.71.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(arcsinh(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(arcsinh(b*x + a)^2/x, x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)^2}{x}\,dx$$

input
```
int(asinh(a + b*x)^2/x,x)
```

output
```
int(asinh(a + b*x)^2/x, x)
```

3.72 $\int \frac{\text{arcsinh}(a+bx)^2}{x^2} dx$

3.72.1 Optimal result

Integrand size = 12, antiderivative size = 178

$$\int \frac{\text{arcsinh}(a+bx)^2}{x^2} dx = -\frac{\text{arcsinh}(a+bx)^2}{x} - \frac{2b\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{2b\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} - \frac{2b\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{2b\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}$$

output
```
-arcsinh(b*x+a)^2/x-2*b*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a
-(a^2+1)^(1/2)))/(a^2+1)^(1/2)+2*b*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2
)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)-2*b*polylog(2,(b*x+a+(1+(b*x+a)^
2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(1/2)+2*b*polylog(2,(b*x+a+(1+(b*x+a)
^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)
```

3.72.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^2}\,dx$$
$$= \frac{-\operatorname{arcsinh}(a+bx)\left(\sqrt{1+a^2}\operatorname{arcsinh}(a+bx)+2bx\left(-\log\left(\frac{a+\sqrt{1+a^2}-e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)+\log\left(\frac{-a+\sqrt{1+a^2}+e^{\operatorname{arcsinh}(}}{-a+\sqrt{1+a^2}}\right.\right.\right.}{\sqrt{1+a^2}x}$$

input
```
Integrate[ArcSinh[a + b*x]^2/x^2,x]
```

output
```
(-(ArcSinh[a + b*x]*(Sqrt[1 + a^2]*ArcSinh[a + b*x] + 2*b*x*(-Log[(a + Sqr
t[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[1 + a^2])] + Log[(-a + Sqrt[1 +
 a^2] + E^ArcSinh[a + b*x])/(-a + Sqrt[1 + a^2])]))) - 2*b*x*PolyLog[2, E^
ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 2*b*x*PolyLog[2, E^ArcSinh[a + b*x
]/(a + Sqrt[1 + a^2])])/(Sqrt[1 + a^2]*x)
```

3.72.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {6274, 27, 6243, 6258, 3042, 3803, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^2}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^2}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{\operatorname{arcsinh}(a+bx)^2}{b^2x^2}d(a+bx)$$

$$\Big\downarrow \text{6243}$$

$$b\left(-2\int -\frac{\operatorname{arcsinh}(a+bx)}{bx\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{\operatorname{arcsinh}(a+bx)^2}{bx}\right)$$

$$\Big\downarrow \text{6258}$$

$$b\left(-2\int -\frac{\text{arcsinh}(a+bx)}{bx}d\text{arcsinh}(a+bx)-\frac{\text{arcsinh}(a+bx)^2}{bx}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(-\frac{\text{arcsinh}(a+bx)^2}{bx}-2\int \frac{\text{arcsinh}(a+bx)}{a+i\sin(i\text{arcsinh}(a+bx))}d\text{arcsinh}(a+bx)\right)$$

$$\Big\downarrow \text{3803}$$

$$b\left(-4\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{2e^{\text{arcsinh}(a+bx)}a-e^{2\text{arcsinh}(a+bx)}+1}d\text{arcsinh}(a+bx)-\frac{\text{arcsinh}(a+bx)^2}{bx}\right)$$

$$\Big\downarrow \text{2694}$$

$$b\left(-4\left(\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{2\left(a-e^{\text{arcsinh}(a+bx)}+\sqrt{a^2+1}\right)}d\text{arcsinh}(a+bx)}{\sqrt{a^2+1}}-\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{2\left(a-e^{\text{arcsinh}(a+bx)}-\sqrt{a^2+1}\right)}d\text{arcsinh}(a+bx)}{\sqrt{a^2+1}}\right)-\frac{\text{arcsinh}(a}{bx}\right.$$

$$\Big\downarrow \text{27}$$

$$b\left(-4\left(\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{a-e^{\text{arcsinh}(a+bx)}+\sqrt{a^2+1}}d\text{arcsinh}(a+bx)}{2\sqrt{a^2+1}}-\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{a-e^{\text{arcsinh}(a+bx)}-\sqrt{a^2+1}}d\text{arcsinh}(a+bx)}{2\sqrt{a^2+1}}\right)-\frac{\text{arcsinh}(a}{bx}\right.$$

$$\Big\downarrow \text{2620}$$

$$b\left(-4\left(\frac{\int \log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)d\text{arcsinh}(a+bx)-\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{2\sqrt{a^2+1}}-\frac{\int \log\left(1-\frac{e^{\text{arcsinh}(a}}{a-\sqrt{a^2}}\right.}{}\right.\right.$$

$$\Big\downarrow \text{2715}$$

$$b\left(-4\left(\frac{\int e^{-\text{arcsinh}(a+bx)}\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)de^{\text{arcsinh}(a+bx)}-\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{2\sqrt{a^2+1}}-\frac{\int e^{-\text{arcs}}}{}\right.\right.$$

$$\Big\downarrow \text{2838}$$

$$b\left(-4\left(\frac{-\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)-\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{2\sqrt{a^2+1}}-\frac{-\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)-\text{a}}{2\sqrt{}}\right.\right.$$

3.72. $\int \frac{\text{arcsinh}(a+bx)^2}{x^2}\,dx$

input
```
Int[ArcSinh[a + b*x]^2/x^2,x]
```

output
```
b*(-(ArcSinh[a + b*x]^2/(b*x)) - 4*(-1/2*(-(ArcSinh[a + b*x]*Log[1 - E^Arc
Sinh[a + b*x]/(a - Sqrt[1 + a^2])]) - PolyLog[2, E^ArcSinh[a + b*x]/(a - S
qrt[1 + a^2])])/Sqrt[1 + a^2] + (-(ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a +
b*x]/(a + Sqrt[1 + a^2])]) - PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a
^2])])/(2*Sqrt[1 + a^2])))
```

3.72.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.72.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.16

method	result
derivativedivides	$b\left(-\frac{\operatorname{arcsinh}(bx+a)^2}{bx}+\frac{2\operatorname{arcsinh}(bx+a)\left(\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx+a)^2}}{a+\sqrt{a^2+1}}\right)-\ln\left(\frac{\sqrt{a^2+1}+bx+\sqrt{1+(bx+a)^2}}{-a+\sqrt{a^2+1}}\right)\right)}{\sqrt{a^2+1}}+\frac{2\,\mathrm{dil}}{}\right.$
default	$b\left(-\frac{\operatorname{arcsinh}(bx+a)^2}{bx}+\frac{2\operatorname{arcsinh}(bx+a)\left(\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx+a)^2}}{a+\sqrt{a^2+1}}\right)-\ln\left(\frac{\sqrt{a^2+1}+bx+\sqrt{1+(bx+a)^2}}{-a+\sqrt{a^2+1}}\right)\right)}{\sqrt{a^2+1}}+\frac{2\,\mathrm{dil}}{}\right.$

input
```
int(arcsinh(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
b*(-arcsinh(b*x+a)^2/b/x+2*arcsinh(b*x+a)*(ln(((a^2+1)^(1/2)-b*x-(1+(b*x+a
)^2)^(1/2))/(a+(a^2+1)^(1/2)))-ln(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/
(-a+(a^2+1)^(1/2))))/(a^2+1)^(1/2)+2/(a^2+1)^(1/2)*dilog(((a^2+1)^(1/2)-b*
x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2/(a^2+1)^(1/2)*dilog(((a^2+1)^(
1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2))))
```

3.72.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(arcsinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(arcsinh(b*x + a)^2/x^2, x)
```

3.72.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{asinh}^2\left(a+bx\right)}{x^2}\,dx$$

input
```
integrate(asinh(b*x+a)**2/x**2,x)
```

output
```
Integral(asinh(a + b*x)**2/x**2, x)
```

3.72.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(arcsinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
-log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/x + integrate(2*(b^3*x
^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) +
 b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^4 + 3*a*b^2*x^
3 + (3*a^2*b + b)*x^2 + (a^3 + a)*x + (b^2*x^3 + 2*a*b*x^2 + (a^2 + 1)*x)*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

3.72.8 Giac [F]

$$\int \frac{\text{arcsinh}(a+bx)^2}{x^2}\,dx = \int \frac{\text{arsinh}\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(arcsinh(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(arcsinh(b*x + a)^2/x^2, x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arcsinh}(a+bx)^2}{x^2}\,dx = \int \frac{\text{asinh}(a+b\,x)^2}{x^2}\,dx$$

input
```
int(asinh(a + b*x)^2/x^2,x)
```

output
```
int(asinh(a + b*x)^2/x^2, x)
```

3.73 $\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}\,dx$

3.73.1 Optimal result

Integrand size = 12, antiderivative size = 235

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}\,dx =& -\frac{b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{(1+a^2)\,x}-\frac{\operatorname{arcsinh}(a+bx)^2}{2x^2}\\&+\frac{ab^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}}\\&-\frac{ab^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}}+\frac{b^2\log(x)}{1+a^2}\\&+\frac{ab^2\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}}-\frac{ab^2\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}}\end{aligned}$$

output
```
-1/2*arcsinh(b*x+a)^2/x^2+b^2*ln(x)/(a^2+1)+a*b^2*arcsinh(b*x+a)*ln(1-(b*x
+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)-a*b^2*arcsinh(b*x
+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)+a*b^
2*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)-a
*b^2*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2
)-b*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x
```

3.73.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.19

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}\,dx =$$

$$-\frac{2\sqrt{1+a^2}bx\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)+\sqrt{1+a^2}\operatorname{arcsinh}(a+bx)^2+a^2\sqrt{1+a^2}\operatorname{arcsinh}(a+bx)^2 -}{}$$

input
```
Integrate[ArcSinh[a + b*x]^2/x^3,x]
```

output
```
-1/2*(2*Sqrt[1 + a^2]*b*x*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x] + Sqrt[1
+ a^2]*ArcSinh[a + b*x]^2 + a^2*Sqrt[1 + a^2]*ArcSinh[a + b*x]^2 + 2*a*b^2
*x^2*ArcSinh[a + b*x]*Log[(a + Sqrt[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sq
rt[1 + a^2])] - 2*a*b^2*x^2*ArcSinh[a + b*x]*Log[(-a + Sqrt[1 + a^2] + E^A
rcSinh[a + b*x])/(-a + Sqrt[1 + a^2])] - 2*Sqrt[1 + a^2]*b^2*x^2*Log[x] -
2*a*b^2*x^2*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 2*a*b^2*x
^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/((1 + a^2)^(3/2)*x^
2)
```

3.73.3 Rubi [A] (warning: unable to verify)

Time = 1.07 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.01, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {6274, 25, 27, 6243, 6258, 3042, 3805, 3042, 3147, 16, 3803, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}d(a+bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{\operatorname{arcsinh}(a+bx)^2}{x^3}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-b^2\int -\frac{\text{arcsinh}(a+bx)^2}{b^3x^3}d(a+bx)$$

$$\Big\downarrow \text{6243}$$

$$-b^2\left(\frac{\text{arcsinh}(a+bx)^2}{2b^2x^2}-\int \frac{\text{arcsinh}(a+bx)}{b^2x^2\sqrt{(a+bx)^2+1}}d(a+bx)\right)$$

$$\Big\downarrow \text{6258}$$

$$-b^2\left(\frac{\text{arcsinh}(a+bx)^2}{2b^2x^2}-\int \frac{\text{arcsinh}(a+bx)}{b^2x^2}d\text{arcsinh}(a+bx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-b^2\left(\frac{\text{arcsinh}(a+bx)^2}{2b^2x^2}-\int \frac{\text{arcsinh}(a+bx)}{(a+i\sin(i\text{arcsinh}(a+bx)))^2}d\text{arcsinh}(a+bx)\right)$$

$$\Big\downarrow \text{3805}$$

$$-b^2\left(\frac{\int -\frac{\sqrt{(a+bx)^2+1}}{bx}d\text{arcsinh}(a+bx)}{a^2+1}-\frac{a\int -\frac{\text{arcsinh}(a+bx)}{bx}d\text{arcsinh}(a+bx)}{a^2+1}+\frac{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)}{(a^2+1)\,bx}+\right.$$

$$\Big\downarrow \text{3042}$$

$$-b^2\left(-\frac{a\int \frac{\text{arcsinh}(a+bx)}{a+i\sin(i\text{arcsinh}(a+bx))}d\text{arcsinh}(a+bx)}{a^2+1}+\frac{\int \frac{\cos(i\text{arcsinh}(a+bx))}{a+i\sin(i\text{arcsinh}(a+bx))}d\text{arcsinh}(a+bx)}{a^2+1}+\frac{\sqrt{(a+bx)^2+1}\text{ar}}{(a^2+1}\right.$$

$$\Big\downarrow \text{3147}$$

$$-b^2\left(-\frac{a\int \frac{\text{arcsinh}(a+bx)}{a+i\sin(i\text{arcsinh}(a+bx))}d\text{arcsinh}(a+bx)}{a^2+1}-\frac{\int \frac{1}{2a+bx}d(-a-bx)}{a^2+1}+\frac{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)}{(a^2+1)\,bx}+\frac{\text{arcs}}{}\right.$$

$$\Big\downarrow \text{16}$$

$$-b^2\left(-\frac{a\int \frac{\text{arcsinh}(a+bx)}{a+i\sin(i\text{arcsinh}(a+bx))}d\text{arcsinh}(a+bx)}{a^2+1}+\frac{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)}{(a^2+1)\,bx}-\frac{\log(2a+bx)}{a^2+1}+\frac{\text{arcsinh}(a}{2b^2x}\right.$$

$$\Big\downarrow \text{3803}$$

3.73. $\int \frac{\text{arcsinh}(a+bx)^2}{x^3}\,dx$

$$-b^2\left(-\frac{2a\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{2e^{\text{arcsinh}(a+bx)}a-e^{2\text{arcsinh}(a+bx)}+1}d\text{arcsinh}(a+bx)}{a^2+1}+\frac{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)}{(a^2+1)\,bx}-\frac{\log(2a+bx)}{a^2+1}+\right.$$

$\downarrow$ 2694

$$-b^2\left(-\frac{2a\left(\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{2\left(a-e^{\text{arcsinh}(a+bx)}+\sqrt{a^2+1}\right)}d\text{arcsinh}(a+bx)}{\sqrt{a^2+1}}-\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{2\left(a-e^{\text{arcsinh}(a+bx)}-\sqrt{a^2+1}\right)}d\text{arcsinh}(a+bx)}{\sqrt{a^2+1}}\right)}{a^2+1}+\frac{\sqrt{(a+bx)^2}}{(a}\right.$$

$\downarrow$ 27

$$-b^2\left(-\frac{2a\left(\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{a-e^{\text{arcsinh}(a+bx)}+\sqrt{a^2+1}}d\text{arcsinh}(a+bx)}{2\sqrt{a^2+1}}-\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{a-e^{\text{arcsinh}(a+bx)}-\sqrt{a^2+1}}d\text{arcsinh}(a+bx)}{2\sqrt{a^2+1}}\right)}{a^2+1}+\frac{\sqrt{(a+bx)^2}}{(a}\right.$$

$\downarrow$ 2620

$$-b^2\left(-\frac{2a\left(\frac{\int \log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)d\text{arcsinh}(a+bx)-\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{2\sqrt{a^2+1}}-\frac{\int \log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)d\text{arcsinh}}{}\right.}{a^2+1}\right.$$

$\downarrow$ 2715

$$-b^2\left(-\frac{2a\left(\frac{\int e^{-\text{arcsinh}(a+bx)}\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)de^{\text{arcsinh}(a+bx)}-\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{2\sqrt{a^2+1}}-\frac{\int e^{-\text{arcsinh}(a+bx)}\log}{}\right.}{a^2+1}\right.$$

$\downarrow$ 2838

3.73. $\displaystyle\int \frac{\text{arcsinh}(a+bx)^2}{x^3}\,dx$

$$-b^2\left(-\frac{2a\left(\frac{-\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)-\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{2\sqrt{a^2+1}}-\frac{-\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)-\operatorname{arcsinh}(a+bx)}{2\sqrt{a^2+1}}\right.}{a^2+1}\right.$$

input
```
Int[ArcSinh[a + b*x]^2/x^3,x]
```

output
```
-(b^2*((Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/((1 + a^2)*b*x) + ArcSinh[
a + b*x]^2/(2*b^2*x^2) - Log[2*a + b*x]/(1 + a^2) - (2*a*(-1/2*(-(ArcSinh[
a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])]) - PolyLog[2, E^A
rcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (-(ArcSinh[a + b*x]*
Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]) - PolyLog[2, E^ArcSinh[a
+ b*x]/(a + Sqrt[1 + a^2])])/(2*Sqrt[1 + a^2])))/(1 + a^2)))
```

3.73.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

3.73. $\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3} dx$

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.73.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.59

method	result
derivativedivides	$b^2\left(-\frac{\operatorname{arcsinh}(bx+a)\left(-2(bx+a)^2+4a(bx+a)+\operatorname{arcsinh}(bx+a)+a^2\operatorname{arcsinh}(bx+a)-2a\sqrt{1+(bx+a)^2}+2(bx+a)\sqrt{1+(b\right.}{2b^2x^2(a^2+1)}\right.$
default	$b^2\left(-\frac{\operatorname{arcsinh}(bx+a)\left(-2(bx+a)^2+4a(bx+a)+\operatorname{arcsinh}(bx+a)+a^2\operatorname{arcsinh}(bx+a)-2a\sqrt{1+(bx+a)^2}+2(bx+a)\sqrt{1+(b\right.}{2b^2x^2(a^2+1)}\right.$

input
```
int(arcsinh(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
b^2*(-1/2*arcsinh(b*x+a)*(-2*(b*x+a)^2+4*a*(b*x+a)+arcsinh(b*x+a)+a^2*arcs
inh(b*x+a)-2*a*(1+(b*x+a)^2)^(1/2)+2*(b*x+a)*(1+(b*x+a)^2)^(1/2)-2*a^2)/b^
2/x^2/(a^2+1)-1/(a^2+1)^(3/2)*a*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)-b*x-(1+(b
*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))+1/(a^2+1)^(3/2)*a*arcsinh(b*x+a)*ln(((a
^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))-1/(a^2+1)^(3/2)*a
*dilog(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))+1/(a^2+1
)^(3/2)*a*dilog(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2))
)-2/(a^2+1)*ln(b*x+a+(1+(b*x+a)^2)^(1/2))+1/(a^2+1)*ln(2*a*(b*x+a+(1+(b*x+
a)^2)^(1/2))-(b*x+a+(1+(b*x+a)^2)^(1/2))^2+1))
```

3.73.5 Fricas [F]

$$\int \frac{\text{arcsinh}(a+bx)^2}{x^3}\,dx = \int \frac{\text{arsinh}\,(bx+a)^2}{x^3}\,dx$$

input
```
integrate(arcsinh(b*x+a)^2/x^3,x, algorithm="fricas")
```

output
```
integral(arcsinh(b*x + a)^2/x^3, x)
```

3.73.6 Sympy [F]

$$\int \frac{\text{arcsinh}(a+bx)^2}{x^3}\,dx = \int \frac{\text{asinh}^2\,(a+bx)}{x^3}\,dx$$

input
```
integrate(asinh(b*x+a)**2/x**3,x)
```

output
```
Integral(asinh(a + b*x)**2/x**3, x)
```

3.73.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{x^3}\,dx$$

input
```
integrate(arcsinh(b*x+a)^2/x^3,x, algorithm="maxima")
```

output
```
-1/2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/x^2 + integrate((b
^3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*
b) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^5 + 3*a*b^
2*x^4 + (3*a^2*b + b)*x^3 + (a^3 + a)*x^2 + (b^2*x^4 + 2*a*b*x^3 + (a^2 +
1)*x^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

3.73.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{x^3}\,dx$$

input
```
integrate(arcsinh(b*x+a)^2/x^3,x, algorithm="giac")
```

output
```
integrate(arcsinh(b*x + a)^2/x^3, x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)^2}{x^3}\,dx$$

input
```
int(asinh(a + b*x)^2/x^3,x)
```

output
```
int(asinh(a + b*x)^2/x^3, x)
```

3.74 $\int \frac{\text{arcsinh}(a+bx)^2}{x^4} dx$

3.74.1 Optimal result

Integrand size = 12, antiderivative size = 478

$$\begin{aligned}\int \frac{\text{arcsinh}(a+bx)^2}{x^4} dx = &-\frac{b^2}{3(1+a^2)x} - \frac{b\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)}{3(1+a^2)x^2} \\ &+ \frac{ab^2\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)}{(1+a^2)^2 x} - \frac{\text{arcsinh}(a+bx)^2}{3x^3} \\ &- \frac{a^2b^3\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{(1+a^2)^{5/2}} \\ &+ \frac{b^3\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3(1+a^2)^{3/2}} \\ &+ \frac{a^2b^3\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{(1+a^2)^{5/2}} \\ &- \frac{b^3\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3(1+a^2)^{3/2}} - \frac{ab^3\log(x)}{(1+a^2)^2} \\ &- \frac{a^2b^3\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{(1+a^2)^{5/2}} + \frac{b^3\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{3(1+a^2)^{3/2}} \\ &+ \frac{a^2b^3\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{(1+a^2)^{5/2}} - \frac{b^3\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{3(1+a^2)^{3/2}}\end{aligned}$$

output
```
-1/3*b^2/(a^2+1)/x-1/3*arcsinh(b*x+a)^2/x^3-a*b^3*ln(x)/(a^2+1)^2-a^2*b^3*
arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)
^(5/2)+1/3*b^3*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^
(1/2)))/(a^2+1)^(3/2)+a^2*b^3*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/
2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(5/2)-1/3*b^3*arcsinh(b*x+a)*ln(1-(b*x+a+(1
+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-a^2*b^3*polylog(2,(b*x
+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(5/2)+1/3*b^3*polylog(2
,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)+a^2*b^3*poly
log(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(5/2)-1/3*b^3
*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-1/
3*b*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x^2+a*b^2*arcsinh(b*x+a)*(1
+(b*x+a)^2)^(1/2)/(a^2+1)^2/x
```

3.74.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.00 (sec) , antiderivative size = 1830, normalized size of antiderivative = 3.83

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4} \, dx = \text{Too large to display}$$

input
```
Integrate[ArcSinh[a + b*x]^2/x^4,x]
```

output
```
(-((b*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/((1 + a^2)*x^2)) - ArcSinh[a
 + b*x]^2/x^3 - (b^2*(1 + a^2 - 3*a*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]
))/((1 + a^2)^2*x) + (I*b^3*Pi*ArcTanh[(-1 - a*Tanh[ArcSinh[a + b*x]/2])/S
qrt[1 + a^2]])/(1 + a^2)^(5/2) - ((2*I)*a^2*b^3*Pi*ArcTanh[(-1 - a*Tanh[Ar
cSinh[a + b*x]/2])/Sqrt[1 + a^2]])/(1 + a^2)^(5/2) - (3*a*b^3*Log[-((b*x)/
a)])/(1 + a^2)^2 + (b^3*(-2*ArcCos[I*a]*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*
ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]] - (Pi - (2*I)*ArcSinh[a + b*x])*ArcT
anh[((I + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]] + (ArcC
os[I*a] + (2*I)*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sq
rt[-1 - a^2]] + (2*I)*ArcTanh[((I + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4
])/Sqrt[-1 - a^2]])*Log[Sqrt[-1 - a^2]/(Sqrt[2]*E^(ArcSinh[a + b*x]/2)*Sqr
t[b*x])] + (ArcCos[I*a] - (2*I)*(ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh
[a + b*x])/4])/Sqrt[-1 - a^2]] + ArcTanh[((I + a)*Tan[(Pi + (2*I)*ArcSinh[
a + b*x])/4])/Sqrt[-1 - a^2]]))*Log[(I*Sqrt[-1 - a^2]*E^(ArcSinh[a + b*x]/
2))/(Sqrt[2]*Sqrt[b*x])] - (ArcCos[I*a] + (2*I)*ArcTanh[((-I + a)*Cot[(Pi
+ (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]])*Log[((I + a)*(a + I*(-1 + S
qrt[-1 - a^2]))*(I + Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(I + a - Sqrt[
-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])] - (ArcCos[I*a] - (2*I)*Ar
cTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]])*Log
[((I + a)*(a - I*(1 + Sqrt[-1 - a^2]))*(-I + Cot[(Pi + (2*I)*ArcSinh[a ...
```

3.74.3 Rubi [A] (warning: unable to verify)

Time = 2.63 (sec) , antiderivative size = 696, normalized size of antiderivative = 1.46, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 1.833$, Rules used = {6274, 27, 6243, 6258, 3042, 3806, 26, 3042, 3147, 17, 3805, 3042, 3147, 16, 3803, 2694, 27, 2620, 2715, 2838, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$b^3\int \frac{\operatorname{arcsinh}(a+bx)^2}{b^4x^4}d(a+bx)$$

$$\Big\downarrow \ 6243$$

$$b^3\left(-\frac{2}{3}\int -\frac{\text{arcsinh}(a+bx)}{b^3x^3\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{\text{arcsinh}(a+bx)^2}{3b^3x^3}\right)$$

$$\Big\downarrow \ 6258$$

$$b^3\left(-\frac{2}{3}\int -\frac{\text{arcsinh}(a+bx)}{b^3x^3}d\text{arcsinh}(a+bx)-\frac{\text{arcsinh}(a+bx)^2}{3b^3x^3}\right)$$

$$\Big\downarrow \ 3042$$

$$b^3\left(-\frac{\text{arcsinh}(a+bx)^2}{3b^3x^3}-\frac{2}{3}\int \frac{\text{arcsinh}(a+bx)}{(a+i\sin(i\text{arcsinh}(a+bx)))^3}d\text{arcsinh}(a+bx)\right)$$

$$\Big\downarrow \ 3806$$

$$b^3\left(-\frac{\text{arcsinh}(a+bx)^2}{3b^3x^3}-\frac{2}{3}\left(-\frac{\int \frac{\sqrt{(a+bx)^2+1}}{b^2x^2}d\text{arcsinh}(a+bx)}{2(a^2+1)}+\frac{a\int \frac{\text{arcsinh}(a+bx)}{b^2x^2}d\text{arcsinh}(a+bx)}{a^2+1}-\frac{i\int \frac{i(a+bx)\text{a}}{}}{}\right.\right.$$

$$\Big\downarrow \ 26$$

$$b^3\left(-\frac{2}{3}\left(-\frac{\int \frac{\sqrt{(a+bx)^2+1}}{b^2x^2}d\text{arcsinh}(a+bx)}{2(a^2+1)}+\frac{a\int \frac{\text{arcsinh}(a+bx)}{b^2x^2}d\text{arcsinh}(a+bx)}{a^2+1}+\frac{\int \frac{(a+bx)\text{arcsinh}(a+bx)}{b^2x^2}d\text{arcsinh}(}{2(a^2+1)}\right.\right.$$

$$\Big\downarrow \ 3042$$

$$b^3\left(-\frac{\text{arcsinh}(a+bx)^2}{3b^3x^3}-\frac{2}{3}\left(\frac{\int \frac{(a+bx)\text{arcsinh}(a+bx)}{b^2x^2}d\text{arcsinh}(a+bx)}{2(a^2+1)}+\frac{a\int \frac{\text{arcsinh}(a+bx)}{(a+i\sin(i\text{arcsinh}(a+bx)))^2}d\text{arcsinh}(a+bx}{a^2+1}\right.\right.$$

$$\Big\downarrow \ 3147$$

$$b^3\left(-\frac{\text{arcsinh}(a+bx)^2}{3b^3x^3}-\frac{2}{3}\left(\frac{\int \frac{(a+bx)\text{arcsinh}(a+bx)}{b^2x^2}d\text{arcsinh}(a+bx)}{2(a^2+1)}+\frac{a\int \frac{\text{arcsinh}(a+bx)}{(a+i\sin(i\text{arcsinh}(a+bx)))^2}d\text{arcsinh}(a+bx}{a^2+1}\right.\right.$$

$$\Big\downarrow \ 17$$

$$b^3\left(-\frac{\text{arcsinh}(a+bx)^2}{3b^3x^3}-\frac{2}{3}\left(\frac{\int \frac{(a+bx)\text{arcsinh}(a+bx)}{b^2x^2}d\text{arcsinh}(a+bx)}{2(a^2+1)}+\frac{a\int \frac{\text{arcsinh}(a+bx)}{(a+i\sin(i\text{arcsinh}(a+bx)))^2}d\text{arcsinh}(a+bx}{a^2+1}\right.\right.$$

$$\Big\downarrow \ 3805$$

3.74. $\int \frac{\text{arcsinh}(a+bx)^2}{x^4}\,dx$

$$b^3\left(-\frac{2}{3}\left(\frac{\int \frac{(a+bx)\text{arcsinh}(a+bx)}{b^2x^2}d\text{arcsinh}(a+bx)}{2\left(a^2+1\right)}+\frac{a\left(-\frac{\int -\frac{\sqrt{(a+bx)^2+1}}{bx}d\text{arcsinh}(a+bx)}{a^2+1}+\frac{a\int -\frac{\text{arcsinh}(a+bx)}{bx}d\text{arcsinh}(a\cdots}{a^2+1}\right.}{a^2+1}\right.\right.$$

$\big\downarrow$ 3042

$$b^3\left(-\frac{\text{arcsinh}(a+bx)^2}{3b^3x^3}-\frac{2}{3}\left(\frac{\int \frac{(a+bx)\text{arcsinh}(a+bx)}{b^2x^2}d\text{arcsinh}(a+bx)}{2\left(a^2+1\right)}+\frac{a\left(\frac{a\int \frac{\text{arcsinh}(a+bx)}{a+i\sin(i\text{arcsinh}(a+bx))}d\text{arcsinh}(a+bx)}{a^2+1}-\cdots\right.}{\cdots}\right.\right.$$

$\big\downarrow$ 3147

$$b^3\left(-\frac{\text{arcsinh}(a+bx)^2}{3b^3x^3}-\frac{2}{3}\left(\frac{\int \frac{(a+bx)\text{arcsinh}(a+bx)}{b^2x^2}d\text{arcsinh}(a+bx)}{2\left(a^2+1\right)}+\frac{a\left(\frac{a\int \frac{\text{arcsinh}(a+bx)}{a+i\sin(i\text{arcsinh}(a+bx))}d\text{arcsinh}(a+bx)}{a^2+1}+\cdots\right.}{a^2\cdots}\right.\right.$$

$\big\downarrow$ 16

$$b^3\left(-\frac{\text{arcsinh}(a+bx)^2}{3b^3x^3}-\frac{2}{3}\left(\frac{\int \frac{(a+bx)\text{arcsinh}(a+bx)}{b^2x^2}d\text{arcsinh}(a+bx)}{2\left(a^2+1\right)}+\frac{a\left(\frac{a\int \frac{\text{arcsinh}(a+bx)}{a+i\sin(i\text{arcsinh}(a+bx))}d\text{arcsinh}(a+bx)}{a^2+1}-\cdots\right.}{a^2+\cdots}\right.\right.$$

$\big\downarrow$ 3803

$$b^3\left(-\frac{2}{3}\left(\frac{\int \frac{(a+bx)\text{arcsinh}(a+bx)}{b^2x^2}d\text{arcsinh}(a+bx)}{2\left(a^2+1\right)}+\frac{a\left(\frac{2a\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{2e^{\text{arcsinh}(a+bx)}a-e^{2\text{arcsinh}(a+bx)}+1}d\text{arcsinh}(a+bx)}{a^2+1}-\frac{\sqrt{(a+bx)^2\cdots}}{(\cdots}\right.}{a^2+1}\right.\right.$$

$\big\downarrow$ 2694

3.74. $\displaystyle\int \frac{\text{arcsinh}(a+bx)^2}{x^4}\,dx$

$$b^3\left(-\frac{2}{3}\left(\frac{\int \frac{(a+bx)\text{arcsinh}(a+bx)}{b^2x^2}d\text{arcsinh}(a+bx)}{2\left(a^2+1\right)}+\frac{a\left(\frac{2a\left(\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{2\left(a-e^{\text{arcsinh}(a+bx)}+\sqrt{a^2+1}\right)}d\text{arcsinh}(a+bx)}{\sqrt{a^2+1}}-\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{ar}}{2\left(a-e^{\text{arcsinh}(a+}\right.}}{}\right.}{a^2+1}\right.}{a}\right.\right.$$

$\downarrow$ 27

$$b^3\left(-\frac{2}{3}\left(\frac{\int \frac{(a+bx)\text{arcsinh}(a+bx)}{b^2x^2}d\text{arcsinh}(a+bx)}{2\left(a^2+1\right)}+\frac{a\left(\frac{2a\left(\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{a-e^{\text{arcsinh}(a+bx)}+\sqrt{a^2+1}}d\text{arcsinh}(a+bx)}{2\sqrt{a^2+1}}-\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{ar}}{a-e^{\text{arcsinh}(a+}}}{}\right.}{a^2+1}\right.}{a}\right.\right.$$

$\downarrow$ 2620

$$b^3\left(-\frac{2}{3}\left(\frac{\int \frac{(a+bx)\text{arcsinh}(a+bx)}{b^2x^2}d\text{arcsinh}(a+bx)}{2\left(a^2+1\right)}+\frac{a\left(\frac{2a\left(\frac{\int \log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)d\text{arcsinh}(a+bx)-\text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arc}}}{\sqrt{}}\right.}{2\sqrt{a^2+1}}\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 2715

3.74. $\int \frac{\text{arcsinh}(a+bx)^2}{x^4}\,dx$

$$b^3\left(-\frac{2}{3}\left(\frac{\int \frac{(a+bx)\text{arcsinh}(a+bx)}{b^2x^2}d\text{arcsinh}(a+bx)}{2(a^2+1)}+\frac{a\left(2a\left(\frac{\int e^{-\text{arcsinh}(a+bx)}\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)de^{\text{arcsinh}(a+bx)}-\text{arcsinh}(a}{2\sqrt{a^2+1}}\right.\right.}{}\right.\right.$$

$\downarrow$ 2838

$$b^3\left(-\frac{2}{3}\left(\frac{\int \frac{(a+bx)\text{arcsinh}(a+bx)}{b^2x^2}d\text{arcsinh}(a+bx)}{2(a^2+1)}+\frac{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)}{2(a^2+1)b^2x^2}+\frac{a\left(2a\left(\frac{-\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right.}{}\right.\right.}{}\right.\right.$$

$\downarrow$ 7293

$$b^3\left(-\frac{2}{3}\left(\frac{\int\left(\frac{\text{arcsinh}(a+bx)}{bx}+\frac{a\text{arcsinh}(a+bx)}{b^2x^2}\right)d\text{arcsinh}(a+bx)}{2(a^2+1)}+\frac{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)}{2(a^2+1)b^2x^2}+\frac{a\left(2a\left(\frac{-\text{PolyL}}{}\right.\right.}{}\right.\right.$$

$\downarrow$ 2009

3.74. $\int \frac{\text{arcsinh}(a+bx)^2}{x^4}\,dx$

$$b^3\left(-\frac{2}{3}\left(\frac{\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)}{2(a^2+1)b^2x^2}+\frac{a\left(\frac{2a\left(\frac{-\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)-\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{2\sqrt{a^2+1}}-\frac{-\operatorname{Poly}}{}\right.}{a^2+1}\right.}{}\right.\right.$$

input
```
Int[ArcSinh[a + b*x]^2/x^4,x]
```

output
```
b^3*(-1/3*ArcSinh[a + b*x]^2/(b^3*x^3) - (2*(-1/2*1/((1 + a^2)*(2*a + b*x)
) + (Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(2*(1 + a^2)*b^2*x^2) + (a*(-
((Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/((1 + a^2)*b*x)) + Log[2*a + b*x
]/(1 + a^2) + (2*a*(-1/2*(-(ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a
 - Sqrt[1 + a^2])]) - PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/
Sqrt[1 + a^2] + (-(ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1
 + a^2])]) - PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(2*Sqrt[1
 + a^2])))/(1 + a^2)))/(1 + a^2) + (-((a*Sqrt[1 + (a + b*x)^2]*ArcSinh[a +
 b*x])/((1 + a^2)*b*x)) + (a^2*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]
/(a - Sqrt[1 + a^2])])/(1 + a^2)^(3/2) - (ArcSinh[a + b*x]*Log[1 - E^ArcSi
nh[a + b*x]/(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] - (a^2*ArcSinh[a + b*x]*Lo
g[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)^(3/2) + (ArcSinh[
a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^2] +
(a*Log[-(b*x)])/(1 + a^2) + (a^2*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1
 + a^2])])/(1 + a^2)^(3/2) - PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a
^2])]/Sqrt[1 + a^2] - (a^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2
])])/(1 + a^2)^(3/2) + PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]/
Sqrt[1 + a^2])/(2*(1 + a^2))))/3)
```

3.74. $\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}\,dx$

3.74.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 3806
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(-b)*(c + d*x)^m*Cos[e + f*x]*((a + b*Sin[e + f*x])^(n +
 1)/(f*(n + 1)*(a^2 - b^2))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m*(a
 + b*Sin[e + f*x])^(n + 1), x], x] - Simp[b*((n + 2)/((n + 1)*(a^2 - b^2)))
   Int[(c + d*x)^m*Sin[e + f*x]*(a + b*Sin[e + f*x])^(n + 1), x], x] + Simp
[b*d*(m/(f*(n + 1)*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*Cos[e + f*x]*(a +
b*Sin[e + f*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2
 - b^2, 0] && ILtQ[n, -2] && IGtQ[m, 0]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

3.74. $\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}\,dx$

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.74.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.60

method	result
derivativedivides	$b^3\left(-\frac{a^4\operatorname{arcsinh}(bx+a)^2-4\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,a^3+7\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,a^2(bx+a)-3\operatorname{arcsinh}(bx\right.$
default	$b^3\left(-\frac{a^4\operatorname{arcsinh}(bx+a)^2-4\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,a^3+7\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,a^2(bx+a)-3\operatorname{arcsinh}(bx\right.$

input
```
int(arcsinh(b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
b^3*(-1/3*(a^4*arcsinh(b*x+a)^2-4*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*a^3+7
*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*a^2*(b*x+a)-3*arcsinh(b*x+a)*(1+(b*x+a
)^2)^(1/2)*a*(b*x+a)^2-3*arcsinh(b*x+a)*a^4+9*arcsinh(b*x+a)*a^3*(b*x+a)-9
*arcsinh(b*x+a)*a^2*(b*x+a)^2+3*arcsinh(b*x+a)*a*(b*x+a)^3+2*a^2*arcsinh(b
*x+a)^2+a^4-2*a^3*(b*x+a)+a^2*(b*x+a)^2-arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)
*a+arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*(b*x+a)+arcsinh(b*x+a)^2+a^2-2*a*(b*
x+a)+(b*x+a)^2)/(a^2+1)^2/b^3/x^3+2/(a^2+1)^2*a*ln(b*x+a+(1+(b*x+a)^2)^(1/
2))-1/(a^2+1)^2*a*ln(2*a*(b*x+a+(1+(b*x+a)^2)^(1/2))-(b*x+a+(1+(b*x+a)^2)^
(1/2))^2+1)-1/3/(a^2+1)^(5/2)*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)-b*x-(1+(b*x
+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))+1/3/(a^2+1)^(5/2)*arcsinh(b*x+a)*ln(((a^2
+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))-1/3/(a^2+1)^(5/2)*d
ilog(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))+1/3/(a^2+1
)^(5/2)*dilog(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))+
2/3/(a^2+1)^(5/2)*a^2*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(
1/2))/(a+(a^2+1)^(1/2)))-2/3/(a^2+1)^(5/2)*a^2*arcsinh(b*x+a)*ln(((a^2+1)^
(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))+2/3/(a^2+1)^(5/2)*a^2*d
ilog(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2/3/(a^2+1
)^(5/2)*a^2*dilog(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2
))))
```

3.74.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{x^4}\,dx$$

input
```
integrate(arcsinh(b*x+a)^2/x^4,x, algorithm="fricas")
```

output
```
integral(arcsinh(b*x + a)^2/x^4, x)
```

3.74.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}\,dx = \int \frac{\operatorname{asinh}^2\left(a+bx\right)}{x^4}\,dx$$

input
```
integrate(asinh(b*x+a)**2/x**4,x)
```

output
```
Integral(asinh(a + b*x)**2/x**4, x)
```

3.74.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{x^4}\,dx$$

input
```
integrate(arcsinh(b*x+a)^2/x^4,x, algorithm="maxima")
```

output
```
-1/3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/x^3 + integrate(2/
3*(b^3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x
+ a*b) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^6 + 3*
a*b^2*x^5 + (3*a^2*b + b)*x^4 + (a^3 + a)*x^3 + (b^2*x^5 + 2*a*b*x^4 + (a^
2 + 1)*x^3)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

3.74.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{x^4}\,dx$$

input
```
integrate(arcsinh(b*x+a)^2/x^4,x, algorithm="giac")
```

output
```
integrate(arcsinh(b*x + a)^2/x^4, x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{x^4}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)^2}{x^4}\,dx$$

input
```
int(asinh(a + b*x)^2/x^4,x)
```

output
```
int(asinh(a + b*x)^2/x^4, x)
```

3.75 $\int x^2 \text{arcsinh}(a+bx)^3\,dx$

3.75.1 Optimal result

Integrand size = 12, antiderivative size = 355

$$\begin{aligned}\int x^2\text{arcsinh}(a+bx)^3\,dx=&\frac{14\sqrt{1+(a+bx)^2}}{9b^3}-\frac{6a^2\sqrt{1+(a+bx)^2}}{b^3}\\&+\frac{3a(a+bx)\sqrt{1+(a+bx)^2}}{4b^3}-\frac{2(1+(a+bx)^2)^{3/2}}{27b^3}\\&-\frac{3a\text{arcsinh}(a+bx)}{4b^3}-\frac{4(a+bx)\text{arcsinh}(a+bx)}{3b^3}\\&+\frac{6a^2(a+bx)\text{arcsinh}(a+bx)}{b^3}-\frac{3a(a+bx)^2\text{arcsinh}(a+bx)}{2b^3}\\&+\frac{2(a+bx)^3\text{arcsinh}(a+bx)}{9b^3}+\frac{2\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)^2}{3b^3}\\&-\frac{3a^2\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)^2}{b^3}\\&+\frac{3a(a+bx)\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)^2}{2b^3}\\&-\frac{(a+bx)^2\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)^2}{3b^3}\\&-\frac{a\text{arcsinh}(a+bx)^3}{2b^3}+\frac{a^3\text{arcsinh}(a+bx)^3}{3b^3}+\frac{1}{3}x^3\text{arcsinh}(a+bx)^3\end{aligned}$$

output
```
-2/27*(1+(b*x+a)^2)^(3/2)/b^3-3/4*a*arcsinh(b*x+a)/b^3-4/3*(b*x+a)*arcsinh
(b*x+a)/b^3+6*a^2*(b*x+a)*arcsinh(b*x+a)/b^3-3/2*a*(b*x+a)^2*arcsinh(b*x+a
)/b^3+2/9*(b*x+a)^3*arcsinh(b*x+a)/b^3-1/2*a*arcsinh(b*x+a)^3/b^3+1/3*a^3*
arcsinh(b*x+a)^3/b^3+1/3*x^3*arcsinh(b*x+a)^3+14/9*(1+(b*x+a)^2)^(1/2)/b^3
-6*a^2*(1+(b*x+a)^2)^(1/2)/b^3+3/4*a*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3+2/3*a
rcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^3-3*a^2*arcsinh(b*x+a)^2*(1+(b*x+a)^
2)^(1/2)/b^3+3/2*a*(b*x+a)*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^3-1/3*(b
*x+a)^2*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^3
```

3.75.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.49

$$\int x^2 \text{arcsinh}(a+bx)^3\,dx$$
$$= \frac{(160-575a^2+65abx-8b^2x^2)\sqrt{1+a^2+2abx+b^2x^2}+3(170a^3+132a^2bx+8bx(-6+b^2x^2)-15a(5+\cdots}{\cdots}$$

input
```
Integrate[x^2*ArcSinh[a + b*x]^3,x]
```

output
```
((160 - 575*a^2 + 65*a*b*x - 8*b^2*x^2)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]
+ 3*(170*a^3 + 132*a^2*b*x + 8*b*x*(-6 + b^2*x^2) - 15*a*(5 + 2*b^2*x^2))*
ArcSinh[a + b*x] - 18*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-4 + 11*a^2 - 5*a
*b*x + 2*b^2*x^2)*ArcSinh[a + b*x]^2 + 18*(-3*a + 2*a^3 + 2*b^3*x^3)*ArcSi
nh[a + b*x]^3)/(108*b^3)
```

3.75.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6274, 27, 6243, 6258, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arcsinh}(a+bx)^3\,dx$$
$$\Big\downarrow \; 6274$$

$$\frac{\int x^2 \text{arcsinh}(a+bx)^3 d(a+bx)}{b}$$

$\Big\downarrow$ 27

$$\frac{\int b^2 x^2 \text{arcsinh}(a+bx)^3 d(a+bx)}{b^3}$$

$\Big\downarrow$ 6243

$$\frac{\int -\frac{b^3 x^3 \text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}} d(a+bx) + \frac{1}{3} b^3 x^3 \text{arcsinh}(a+bx)^3}{b^3}$$

$\Big\downarrow$ 6258

$$\frac{\int -b^3 x^3 \text{arcsinh}(a+bx)^2 d\text{arcsinh}(a+bx) + \frac{1}{3} b^3 x^3 \text{arcsinh}(a+bx)^3}{b^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{3} b^3 x^3 \text{arcsinh}(a+bx)^3 + \int \text{arcsinh}(a+bx)^2 (a + i \sin(i \text{arcsinh}(a+bx)))^3 d\text{arcsinh}(a+bx)}{b^3}$$

$\Big\downarrow$ 3798

$$\frac{\int \left(\text{arcsinh}(a+bx)^2 a^3 - 3(a+bx)\text{arcsinh}(a+bx)^2 a^2 + 3(a+bx)^2 \text{arcsinh}(a+bx)^2 a - (a+bx)^3 \text{arcsinh}(a+bx)^2\right.}{b^3}$$

$\Big\downarrow$ 2009

$$\frac{1}{3} a^3 \text{arcsinh}(a+bx)^3 + 6a^2(a+bx)\text{arcsinh}(a+bx) - 3a^2 \sqrt{(a+bx)^2+1} \text{arcsinh}(a+bx)^2 - 6a^2 \sqrt{(a+bx)^2+1} +$$

input
```
Int[x^2*ArcSinh[a + b*x]^3,x]
```

output
```
((14*Sqrt[1 + (a + b*x)^2])/9 - 6*a^2*Sqrt[1 + (a + b*x)^2] + (3*a*(a + b*
x)*Sqrt[1 + (a + b*x)^2])/4 - (2*(1 + (a + b*x)^2)^(3/2))/27 - (3*a*ArcSin
h[a + b*x])/4 - (4*(a + b*x)*ArcSinh[a + b*x])/3 + 6*a^2*(a + b*x)*ArcSinh
[a + b*x] - (3*a*(a + b*x)^2*ArcSinh[a + b*x])/2 + (2*(a + b*x)^3*ArcSinh[
a + b*x])/9 + (2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^2)/3 - 3*a^2*Sqrt[
1 + (a + b*x)^2]*ArcSinh[a + b*x]^2 + (3*a*(a + b*x)*Sqrt[1 + (a + b*x)^2]
*ArcSinh[a + b*x]^2)/2 - ((a + b*x)^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*
x]^2)/3 - (a*ArcSinh[a + b*x]^3)/2 + (a^3*ArcSinh[a + b*x]^3)/3 + (b^3*x^3
*ArcSinh[a + b*x]^3)/3)/b^3
```

3.75. $\int x^2 \text{arcsinh}(a+bx)^3 \, dx$

3.75.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.75.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsinh}(bx+a)^3(bx+a)^3}{3}+\frac{2\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}}{3}-\frac{\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}\,(bx+a)^2}{3}-\frac{4(bx+a)\operatorname{arcsinh}(bx+a)}{3}+\frac{40\sqrt{1+}}{}}{}$
default	$\frac{\frac{\operatorname{arcsinh}(bx+a)^3(bx+a)^3}{3}+\frac{2\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}}{3}-\frac{\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}\,(bx+a)^2}{3}-\frac{4(bx+a)\operatorname{arcsinh}(bx+a)}{3}+\frac{40\sqrt{1+}}{}}{}$

input
```
int(x^2*arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*arcsinh(b*x+a)^3*(b*x+a)^3+2/3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(
1/2)-1/3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)*(b*x+a)^2-4/3*(b*x+a)*arcsin
h(b*x+a)+40/27*(1+(b*x+a)^2)^(1/2)+2/9*arcsinh(b*x+a)*(b*x+a)^3-2/27*(b*x+
a)^2*(1+(b*x+a)^2)^(1/2)-1/4*a*(4*arcsinh(b*x+a)^3*(b*x+a)^2-6*arcsinh(b*x
+a)^2*(1+(b*x+a)^2)^(1/2)*(b*x+a)+2*arcsinh(b*x+a)^3+6*arcsinh(b*x+a)*(b*x
+a)^2-3*(b*x+a)*(1+(b*x+a)^2)^(1/2)+3*arcsinh(b*x+a))+a^2*(arcsinh(b*x+a)^
3*(b*x+a)-3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+6*(b*x+a)*arcsinh(b*x+a)-
6*(1+(b*x+a)^2)^(1/2)))
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.63

$$\int x^2\operatorname{arcsinh}(a+bx)^3\,dx$$
$$=\frac{18\,(2\,b^3x^3+2\,a^3-3\,a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-18\,(2\,b^2x^2-5\,abx+11\,a^2-4)\sqrt{b^2x^2+}}{}$$

input
```
integrate(x^2*arcsinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/108*(18*(2*b^3*x^3 + 2*a^3 - 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1))^3 - 18*(2*b^2*x^2 - 5*a*b*x + 11*a^2 - 4)*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 3*(8*b^3
*x^3 - 30*a*b^2*x^2 + 170*a^3 + 12*(11*a^2 - 4)*b*x - 75*a)*log(b*x + a +
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - (8*b^2*x^2 - 65*a*b*x + 575*a^2 - 160
)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.22

$$\int x^2\text{arcsinh}(a+bx)^3\,dx$$
$$=\begin{cases}\frac{a^3\operatorname{asinh}^3(a+bx)}{3b^3}+\frac{85a^3\operatorname{asinh}(a+bx)}{18b^3}+\frac{11a^2x\operatorname{asinh}(a+bx)}{3b^2}-\frac{11a^2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{6b^3}-\frac{575a^2\sqrt{a^2+2abx+b^2x^2+1}}{108b^3} \\ \frac{x^3\operatorname{asinh}^3(a)}{3}\end{cases}$$

input
```
integrate(x**2*asinh(b*x+a)**3,x)
```

output
```
Piecewise((a**3*asinh(a + b*x)**3/(3*b**3) + 85*a**3*asinh(a + b*x)/(18*b*
*3) + 11*a**2*x*asinh(a + b*x)/(3*b**2) - 11*a**2*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1)*asinh(a + b*x)**2/(6*b**3) - 575*a**2*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)/(108*b**3) - 5*a*x**2*asinh(a + b*x)/(6*b) + 5*a*x*sqrt(a**2
 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(6*b**2) + 65*a*x*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1)/(108*b**2) - a*asinh(a + b*x)**3/(2*b**3) - 25*
a*asinh(a + b*x)/(12*b**3) + x**3*asinh(a + b*x)**3/3 + 2*x**3*asinh(a + b
*x)/9 - x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(3*b)
- 2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(27*b) - 4*x*asinh(a + b*x)/
(3*b**2) + 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(3*b**
3) + 40*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(27*b**3), Ne(b, 0)), (x**3*a
sinh(a)**3/3, True))
```

3.75.7 Maxima [F]

$$\int x^2\text{arcsinh}(a+bx)^3\,dx=\int x^2\operatorname{arsinh}(bx+a)^3\,dx$$

input
```
integrate(x^2*arcsinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/3*x^3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - integrate((b^
3*x^5 + 2*a*b^2*x^4 + (a^2*b + b)*x^3 + (b^2*x^4 + a*b*x^3)*sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(b
^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 +
1)^(3/2) + a), x)
```

3.75.8 Giac [F]

$$\int x^2 \operatorname{arcsinh}(a+bx)^3\,dx = \int x^2 \operatorname{arsinh}\left(bx+a\right)^3\,dx$$

input
```
integrate(x^2*arcsinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^2*arcsinh(b*x + a)^3, x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{arcsinh}(a+bx)^3\,dx = \int x^2 \operatorname{asinh}(a+b\,x)^3\,dx$$

input
```
int(x^2*asinh(a + b*x)^3,x)
```

output
```
int(x^2*asinh(a + b*x)^3, x)
```

3.76 $\int x \text{arcsinh}(a+bx)^3 \, dx$

3.76.1 Optimal result

Integrand size = 10, antiderivative size = 203

$$\begin{aligned}\int x \text{arcsinh}(a+bx)^3 \, dx = {} & \frac{6a\sqrt{1+(a+bx)^2}}{b^2} - \frac{3(a+bx)\sqrt{1+(a+bx)^2}}{8b^2} + \frac{3\text{arcsinh}(a+bx)}{8b^2} \\ & - \frac{6a(a+bx)\text{arcsinh}(a+bx)}{b^2} + \frac{3(a+bx)^2\text{arcsinh}(a+bx)}{4b^2} \\ & + \frac{3a\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)^2}{b^2} \\ & - \frac{3(a+bx)\sqrt{1+(a+bx)^2}\text{arcsinh}(a+bx)^2}{4b^2} + \frac{\text{arcsinh}(a+bx)^3}{4b^2} \\ & - \frac{a^2\text{arcsinh}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\text{arcsinh}(a+bx)^3\end{aligned}$$

output

```
3/8*arcsinh(b*x+a)/b^2-6*a*(b*x+a)*arcsinh(b*x+a)/b^2+3/4*(b*x+a)^2*arcsin
h(b*x+a)/b^2+1/4*arcsinh(b*x+a)^3/b^2-1/2*a^2*arcsinh(b*x+a)^3/b^2+1/2*x^2
*arcsinh(b*x+a)^3+6*a*(1+(b*x+a)^2)^(1/2)/b^2-3/8*(b*x+a)*(1+(b*x+a)^2)^(1
/2)/b^2+3*a*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^2-3/4*(b*x+a)*arcsinh(b
*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^2
```

3.76.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.64

$$\int x\text{arcsinh}(a+bx)^3\,dx$$
$$=\frac{3(15a-bx)\sqrt{1+a^2+2abx+b^2x^2}+(3-42a^2-36abx+6b^2x^2)\,\text{arcsinh}(a+bx)+6(3a-bx)\sqrt{1+a^2}}{8b^2}$$

input
```
Integrate[x*ArcSinh[a + b*x]^3,x]
```

output
```
(3*(15*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (3 - 42*a^2 - 36*a*b*x
 + 6*b^2*x^2)*ArcSinh[a + b*x] + 6*(3*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^
2*x^2]*ArcSinh[a + b*x]^2 + (2 - 4*a^2 + 4*b^2*x^2)*ArcSinh[a + b*x]^3)/(8
*b^2)
```

3.76.3 Rubi [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6274, 25, 27, 6243, 6258, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arcsinh}(a+bx)^3\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int x\text{arcsinh}(a+bx)^3d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -x\text{arcsinh}(a+bx)^3d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -bx\text{arcsinh}(a+bx)^3d(a+bx)}{b^2}$$

$$\Big\downarrow \text{6243}$$

$$-\frac{\frac{3}{2}\int \frac{b^2x^2\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{1}{2}b^2x^2\text{arcsinh}(a+bx)^3}{b^2}$$

$$\Big\downarrow \text{6258}$$

$$-\frac{\frac{3}{2}\int b^2x^2\text{arcsinh}(a+bx)^2d\text{arcsinh}(a+bx)-\frac{1}{2}b^2x^2\text{arcsinh}(a+bx)^3}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{2}b^2x^2\text{arcsinh}(a+bx)^3+\frac{3}{2}\int \text{arcsinh}(a+bx)^2(a+i\sin(i\text{arcsinh}(a+bx)))^2d\text{arcsinh}(a+bx)}{b^2}$$

$$\Big\downarrow \text{3798}$$

$$-\frac{\frac{3}{2}\int \left(a^2\text{arcsinh}(a+bx)^2+(a+bx)^2\text{arcsinh}(a+bx)^2-2a(a+bx)\text{arcsinh}(a+bx)^2\right)d\text{arcsinh}(a+bx)-\frac{1}{2}b^2x^2\text{ar}}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{3}{2}\Big(\frac{1}{3}a^2\text{arcsinh}(a+bx)^3-\frac{1}{6}\text{arcsinh}(a+bx)^3-2a\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)^2+\frac{1}{2}(a+bx)\sqrt{(a+bx)^2+1}\text{ar}}{}$$

input
```
Int[x*ArcSinh[a + b*x]^3,x]
```

output
```
-((-1/2*(b^2*x^2*ArcSinh[a + b*x]^3) + (3*((-a - b*x)/4 - 4*a*Sqrt[1 + (a
+ b*x)^2] + ((a + b*x)*Sqrt[1 + (a + b*x)^2])/4 + 4*a*(a + b*x)*ArcSinh[a
+ b*x] - ((a + b*x)^2*ArcSinh[a + b*x])/2 - 2*a*Sqrt[1 + (a + b*x)^2]*ArcS
inh[a + b*x]^2 + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^2)/2 -
ArcSinh[a + b*x]^3/6 + (a^2*ArcSinh[a + b*x]^3)/3))/2)/b^2)
```

3.76.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.76.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsinh}(bx+a)^3\left(1+(bx+a)^2\right)}{2}-\frac{3\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}\,(bx+a)}{4}-\frac{\operatorname{arcsinh}(bx+a)^3}{4}+\frac{3\operatorname{arcsinh}(bx+a)\left(1+(bx+a)^2\right)}{4}-\frac{3(bx+a)}{}}{}$
default	$\frac{\frac{\operatorname{arcsinh}(bx+a)^3\left(1+(bx+a)^2\right)}{2}-\frac{3\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}\,(bx+a)}{4}-\frac{\operatorname{arcsinh}(bx+a)^3}{4}+\frac{3\operatorname{arcsinh}(bx+a)\left(1+(bx+a)^2\right)}{4}-\frac{3(bx+a)}{}}{}$

input
```
int(x*arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/2*arcsinh(b*x+a)^3*(1+(b*x+a)^2)-3/4*arcsinh(b*x+a)^2*(1+(b*x+a)^
2)^(1/2)*(b*x+a)-1/4*arcsinh(b*x+a)^3+3/4*arcsinh(b*x+a)*(1+(b*x+a)^2)-3/8
*(b*x+a)*(1+(b*x+a)^2)^(1/2)-3/8*arcsinh(b*x+a)-a*(arcsinh(b*x+a)^3*(b*x+a
)-3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+6*(b*x+a)*arcsinh(b*x+a)-6*(1+(b*
x+a)^2)^(1/2)))
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.89

$$\int x\text{arcsinh}(a+bx)^3\,dx$$
$$= \frac{2\left(2\,b^2x^2-2\,a^2+1\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-6\,\sqrt{b^2x^2+2\,abx+a^2+1}(bx-3\,a)\log\left(bx\right.}{}$$

input
```
integrate(x*arcsinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8*(2*(2*b^2*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))^3 - 6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a)*log(b*x + a + sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 3*(2*b^2*x^2 - 12*a*b*x - 14*a^2 + 1)
*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)*(b*x - 15*a))/b^2
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.22

$$\int x\text{arcsinh}(a+bx)^3\,dx$$
$$= \begin{cases} -\frac{a^2\operatorname{asinh}^3(a+bx)}{2b^2}-\frac{21a^2\operatorname{asinh}(a+bx)}{4b^2}-\frac{9ax\operatorname{asinh}(a+bx)}{2b}+\frac{9a\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{4b^2}+\frac{45a\sqrt{a^2+2abx+b^2x^2+1}}{8b^2}+\frac{x^2}{} \\ \frac{x^2\operatorname{asinh}^3(a)}{2} \end{cases}$$

input
```
integrate(x*asinh(b*x+a)**3,x)
```

output
```
Piecewise((-a**2*asinh(a + b*x)**3/(2*b**2) - 21*a**2*asinh(a + b*x)/(4*b*
*2) - 9*a*x*asinh(a + b*x)/(2*b) + 9*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1
)*asinh(a + b*x)**2/(4*b**2) + 45*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(
8*b**2) + x**2*asinh(a + b*x)**3/2 + 3*x**2*asinh(a + b*x)/4 - 3*x*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(4*b) - 3*x*sqrt(a**2 + 2*
a*b*x + b**2*x**2 + 1)/(8*b) + asinh(a + b*x)**3/(4*b**2) + 3*asinh(a + b*
x)/(8*b**2), Ne(b, 0)), (x**2*asinh(a)**3/2, True))
```

3.76.7 Maxima [F]

$$\int x\text{arcsinh}(a+bx)^3\,dx = \int x\,\text{arsinh}\,(bx+a)^3\,dx$$

input
```
integrate(x*arcsinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*x^2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - integrate(3/2
*(b^3*x^4 + 2*a*b^2*x^3 + (a^2*b + b)*x^2 + (b^2*x^3 + a*b*x^2)*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^
2/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^
2 + 1)^(3/2) + a), x)
```

3.76.8 Giac [F]

$$\int x\text{arcsinh}(a+bx)^3\,dx = \int x\,\text{arsinh}\,(bx+a)^3\,dx$$

input
```
integrate(x*arcsinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x*arcsinh(b*x + a)^3, x)
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arcsinh}(a+bx)^3\,dx=\int x\,\text{asinh}(a+b\,x)^3\,dx$$

input
```
int(x*asinh(a + b*x)^3,x)
```

output
```
int(x*asinh(a + b*x)^3, x)
```

3.77 $\int \operatorname{arcsinh}(a+bx)^3 \, dx$

3.77.1 Optimal result

Integrand size = 8, antiderivative size = 78

$$\int \operatorname{arcsinh}(a+bx)^3 \, dx = -\frac{6\sqrt{1+(a+bx)^2}}{b} + \frac{6(a+bx)\operatorname{arcsinh}(a+bx)}{b} - \frac{3\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{b} + \frac{(a+bx)\operatorname{arcsinh}(a+bx)^3}{b}$$

output
```
6*(b*x+a)*arcsinh(b*x+a)/b+(b*x+a)*arcsinh(b*x+a)^3/b-6*(1+(b*x+a)^2)^(1/2
)/b-3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b
```

3.77.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \operatorname{arcsinh}(a+bx)^3 \, dx = \frac{-6\sqrt{1+(a+bx)^2} + 6(a+bx)\operatorname{arcsinh}(a+bx) - 3\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2 + (a+bx)\operatorname{arcsinh}(a}{b}$$

input
```
Integrate[ArcSinh[a + b*x]^3,x]
```

output
```
(-6*Sqrt[1 + (a + b*x)^2] + 6*(a + b*x)*ArcSinh[a + b*x] - 3*Sqrt[1 + (a +
 b*x)^2]*ArcSinh[a + b*x]^2 + (a + b*x)*ArcSinh[a + b*x]^3)/b
```

3.77.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6273, 6187, 6213, 6187, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}(a+bx)^3\,dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int \text{arcsinh}(a+bx)^3d(a+bx)}{b}$$

$$\Big\downarrow \text{6187}$$

$$\frac{(a+bx)\text{arcsinh}(a+bx)^3-3\int \frac{(a+bx)\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b}$$

$$\Big\downarrow \text{6213}$$

$$\frac{(a+bx)\text{arcsinh}(a+bx)^3-3\left(\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)^2-2\int \text{arcsinh}(a+bx)d(a+bx)\right)}{b}$$

$$\Big\downarrow \text{6187}$$

$$\frac{(a+bx)\text{arcsinh}(a+bx)^3-3\left(\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)^2-2\left((a+bx)\text{arcsinh}(a+bx)-\int \frac{a+bx}{\sqrt{(a+bx)^2+1}}d(a+bx)\right)\right)}{b}$$

$$\Big\downarrow \text{241}$$

$$\frac{(a+bx)\text{arcsinh}(a+bx)^3-3\left(\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)^2-2\left((a+bx)\text{arcsinh}(a+bx)-\sqrt{(a+bx)^2+1}\right)\right)}{b}$$

input
```
Int[ArcSinh[a + b*x]^3,x]
```

output
```
((a + b*x)*ArcSinh[a + b*x]^3 - 3*(Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^
2 - 2*(-Sqrt[1 + (a + b*x)^2] + (a + b*x)*ArcSinh[a + b*x])))/b
```

3.77.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.77.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\operatorname{arcsinh}(bx+a)^3(bx+a)-3\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}+6(bx+a)\operatorname{arcsinh}(bx+a)-6\sqrt{1+(bx+a)^2}}{b}$	67
default	$\frac{\operatorname{arcsinh}(bx+a)^3(bx+a)-3\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}+6(bx+a)\operatorname{arcsinh}(bx+a)-6\sqrt{1+(bx+a)^2}}{b}$	67

input
```
int(arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(arcsinh(b*x+a)^3*(b*x+a)-3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+6*(b*
x+a)*arcsinh(b*x+a)-6*(1+(b*x+a)^2)^(1/2))
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.78

$$\int \operatorname{arcsinh}(a+bx)^3\,dx$$
$$=\frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-3\,\sqrt{b^2x^2+2\,abx+a^2+1}\log\left(bx+a+\sqrt{b^2x^2+2\,abx\right.}}{b}$$

input
```
integrate(arcsinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - 3*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
^2 + 6*(b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 6*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1))/b
```

3.77.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.40

$$\int \operatorname{arcsinh}(a+bx)^3\,dx$$
$$=\begin{cases}\frac{a\,\operatorname{asinh}^3(a+bx)}{b}+\frac{6a\,\operatorname{asinh}(a+bx)}{b}+x\,\operatorname{asinh}^3(a+bx)+6x\,\operatorname{asinh}(a+bx)-\frac{3\sqrt{a^2+2abx+b^2x^2+1}\,\operatorname{asinh}^2(a+bx)}{b}-\frac{6\sqrt{a^2}}{} \\ x\,\operatorname{asinh}^3(a)\end{cases}$$

input
```
integrate(asinh(b*x+a)**3,x)
```

output
```
Piecewise((a*asinh(a + b*x)**3/b + 6*a*asinh(a + b*x)/b + x*asinh(a + b*x)
**3 + 6*x*asinh(a + b*x) - 3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a
+ b*x)**2/b - 6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/b, Ne(b, 0)), (x*asin
h(a)**3, True))
```

3.77.7 Maxima [F]

$$\int \operatorname{arcsinh}(a+bx)^3\,dx = \int \operatorname{arsinh}\left(bx+a\right)^3\,dx$$

input
```
integrate(arcsinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
x*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - integrate(3*(b^3*x^
3 + 2*a*b^2*x^2 + (a^2*b + b)*x + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x
^2 + a*b*x))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(b^3*x^3 +
 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)
 + a), x)
```

3.77.8 Giac [F]

$$\int \operatorname{arcsinh}(a+bx)^3\,dx = \int \operatorname{arsinh}\left(bx+a\right)^3\,dx$$

input
```
integrate(arcsinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(arcsinh(b*x + a)^3, x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(a+bx)^3\,dx = \int \operatorname{asinh}(a+b\,x)^3\,dx$$

input
```
int(asinh(a + b*x)^3,x)
```

output
```
int(asinh(a + b*x)^3, x)
```

3.78 $\int \frac{\text{arcsinh}(a+bx)^3}{x}\,dx$

3.78.1 Optimal result

Integrand size = 12, antiderivative size = 275

$$\begin{aligned}\int \frac{\text{arcsinh}(a+bx)^3}{x}\,dx = & -\frac{1}{4}\text{arcsinh}(a+bx)^4+\text{arcsinh}(a+bx)^3\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)\\ & +\text{arcsinh}(a+bx)^3\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)\\ & +3\text{arcsinh}(a+bx)^2\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)\\ & +3\text{arcsinh}(a+bx)^2\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)\\ & -6\text{arcsinh}(a+bx)\,\text{PolyLog}\left(3,\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)\\ & -6\text{arcsinh}(a+bx)\,\text{PolyLog}\left(3,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)\\ & +6\,\text{PolyLog}\left(4,\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)+6\,\text{PolyLog}\left(4,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)\end{aligned}$$

output
```
-1/4*arcsinh(b*x+a)^4+arcsinh(b*x+a)^3*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a
-(a^2+1)^(1/2)))+arcsinh(b*x+a)^3*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2
+1)^(1/2)))+3*arcsinh(b*x+a)^2*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a
^2+1)^(1/2)))+3*arcsinh(b*x+a)^2*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+
(a^2+1)^(1/2)))-6*arcsinh(b*x+a)*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-
(a^2+1)^(1/2)))-6*arcsinh(b*x+a)*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+
(a^2+1)^(1/2)))+6*polylog(4,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))
+6*polylog(4,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))
```

3.78.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x}\,dx = -\frac{1}{4}\operatorname{arcsinh}(a+bx)^4 + \operatorname{arcsinh}(a+bx)^3\log\left(1+\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right) + \operatorname{arcsinh}(a+bx)^3\log\left(1+\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right) + 3\operatorname{arcsinh}(a+bx)^2\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right) + 3\operatorname{arcsinh}(a+bx)^2\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right) - 6\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right) - 6\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(3,-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right) + 6\operatorname{PolyLog}\left(4,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right) + 6\operatorname{PolyLog}\left(4,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)$$

input
```
Integrate[ArcSinh[a + b*x]^3/x,x]
```

output
```
-1/4*ArcSinh[a + b*x]^4 + ArcSinh[a + b*x]^3*Log[1 + E^ArcSinh[a + b*x]/((
-(a/b) - Sqrt[1 + a^2]/b)*b)] + ArcSinh[a + b*x]^3*Log[1 + E^ArcSinh[a + b
*x]/((-(a/b) + Sqrt[1 + a^2]/b)*b)] + 3*ArcSinh[a + b*x]^2*PolyLog[2, -(E^
ArcSinh[a + b*x]/((-(a/b) - Sqrt[1 + a^2]/b)*b))] + 3*ArcSinh[a + b*x]^2*P
olyLog[2, -(E^ArcSinh[a + b*x]/((-(a/b) + Sqrt[1 + a^2]/b)*b))] - 6*ArcSin
h[a + b*x]*PolyLog[3, -(E^ArcSinh[a + b*x]/((-(a/b) - Sqrt[1 + a^2]/b)*b))
] - 6*ArcSinh[a + b*x]*PolyLog[3, -(E^ArcSinh[a + b*x]/((-(a/b) + Sqrt[1 +
 a^2]/b)*b))] + 6*PolyLog[4, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*P
olyLog[4, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

3.78.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6274, 25, 27, 6242, 6095, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x} \, dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{\operatorname{arcsinh}(a+bx)^3}{x} d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{\operatorname{arcsinh}(a+bx)^3}{x} d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\int -\frac{\operatorname{arcsinh}(a+bx)^3}{bx} d(a+bx)$$

$$\Big\downarrow \text{6242}$$

$$-\int -\frac{\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)^3}{bx} d\operatorname{arcsinh}(a+bx)$$

$$\Big\downarrow \text{6095}$$

$$-\int \frac{e^{\operatorname{arcsinh}(a+bx)}\operatorname{arcsinh}(a+bx)^3}{a-e^{\operatorname{arcsinh}(a+bx)}-\sqrt{a^2+1}} d\operatorname{arcsinh}(a+bx)-\int \frac{e^{\operatorname{arcsinh}(a+bx)}\operatorname{arcsinh}(a+bx)^3}{a-e^{\operatorname{arcsinh}(a+bx)}+\sqrt{a^2+1}} d\operatorname{arcsinh}(a+bx)-\frac{1}{4}\operatorname{arcsinh}(a+bx)^4$$

$$\Big\downarrow \text{2620}$$

$$-3\int \operatorname{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right) d\operatorname{arcsinh}(a+bx)-3\int \operatorname{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right) d\operatorname{arcsinh}(a+bx)+\operatorname{arcsinh}(a+bx)^3 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\operatorname{arcsinh}(a+bx)^3 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)-\frac{1}{4}\operatorname{arcsinh}(a+bx)^4$$

$$\Big\downarrow \text{3011}$$

$$-3\left(2\int \operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)d\operatorname{arcsinh}(a+bx)-\operatorname{arcsinh}(a+bx)^2\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+b}}{a-\sqrt{a^2+}}\right.\right.$$
$$3\left(2\int \operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)d\operatorname{arcsinh}(a+bx)-\operatorname{arcsinh}(a+bx)^2\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right.\right.$$
$$\operatorname{arcsinh}(a+bx)^3\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\operatorname{arcsinh}(a+bx)^3\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)-$$
$$\frac{1}{4}\operatorname{arcsinh}(a+bx)^4$$

$$\Big\downarrow \text{7163}$$

$$-3\left(2\left(\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)-\int \operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)d\operatorname{arcsinh}(a+bx)\right)-\operatorname{arcsin}\right.$$
$$3\left(2\left(\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)-\int \operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)d\operatorname{arcsinh}(a+bx)\right)-\operatorname{arcsinh}(\right.$$
$$\operatorname{arcsinh}(a+bx)^3\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\operatorname{arcsinh}(a+bx)^3\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)-$$
$$\frac{1}{4}\operatorname{arcsinh}(a+bx)^4$$

$$\Big\downarrow \text{2720}$$

$$-3\left(2\left(\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)-\int e^{-\operatorname{arcsinh}(a+bx)}\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)de^{\operatorname{arcsinh}(a+bx)}\right.\right.$$
$$3\left(2\left(\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)-\int e^{-\operatorname{arcsinh}(a+bx)}\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)de^{\operatorname{arcsinh}(a+bx)}\right)\right.$$
$$\operatorname{arcsinh}(a+bx)^3\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\operatorname{arcsinh}(a+bx)^3\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)-$$
$$\frac{1}{4}\operatorname{arcsinh}(a+bx)^4$$

$$\Big\downarrow \text{7143}$$

$$-3\left(2\left(\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)-\operatorname{PolyLog}\left(4,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)\right)-\operatorname{arcsinh}(a+bx)^2\operatorname{PolyLog}\right.$$
$$3\left(2\left(\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)-\operatorname{PolyLog}\left(4,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)\right)-\operatorname{arcsinh}(a+bx)^2\operatorname{PolyLog}\left(2\right.\right.$$
$$\operatorname{arcsinh}(a+bx)^3\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\operatorname{arcsinh}(a+bx)^3\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)-$$
$$\frac{1}{4}\operatorname{arcsinh}(a+bx)^4$$

3.78. $\int \frac{\operatorname{arcsinh}(a+bx)^3}{x}\,dx$

input
```
Int[ArcSinh[a + b*x]^3/x,x]
```

output
```
-1/4*ArcSinh[a + b*x]^4 + ArcSinh[a + b*x]^3*Log[1 - E^ArcSinh[a + b*x]/(a
 - Sqrt[1 + a^2])] + ArcSinh[a + b*x]^3*Log[1 - E^ArcSinh[a + b*x]/(a + Sq
rt[1 + a^2])] - 3*(-(ArcSinh[a + b*x]^2*PolyLog[2, E^ArcSinh[a + b*x]/(a -
 Sqrt[1 + a^2])]) + 2*(ArcSinh[a + b*x]*PolyLog[3, E^ArcSinh[a + b*x]/(a -
 Sqrt[1 + a^2])] - PolyLog[4, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])) -
3*(-(ArcSinh[a + b*x]^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
) + 2*(ArcSinh[a + b*x]*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
 - PolyLog[4, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]))
```

3.78.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6242
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.78.4 Maple [F]

$$\int \frac{\operatorname{arcsinh}(bx+a)^3}{x}\,dx$$

input
```
int(arcsinh(b*x+a)^3/x,x)
```

output
```
int(arcsinh(b*x+a)^3/x,x)
```

3.78.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{x}\,dx$$

input
```
integrate(arcsinh(b*x+a)^3/x,x, algorithm="fricas")
```

output
```
integral(arcsinh(b*x + a)^3/x, x)
```

3.78.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{asinh}^3\left(a+bx\right)}{x}\,dx$$

input
```
integrate(asinh(b*x+a)**3/x,x)
```

output
```
Integral(asinh(a + b*x)**3/x, x)
```

3.78.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{x}\,dx$$

input
```
integrate(arcsinh(b*x+a)^3/x,x, algorithm="maxima")
```

output
```
integrate(arcsinh(b*x + a)^3/x, x)
```

3.78.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{x}\,dx$$

input
```
integrate(arcsinh(b*x+a)^3/x,x, algorithm="giac")
```

output
```
integrate(arcsinh(b*x + a)^3/x, x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)^3}{x}\,dx$$

input
```
int(asinh(a + b*x)^3/x,x)
```

output
```
int(asinh(a + b*x)^3/x, x)
```

3.79 $\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2} dx$

3.79.1 Optimal result

Integrand size = 12, antiderivative size = 268

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2} dx = & -\frac{\operatorname{arcsinh}(a+bx)^3}{x} - \frac{3b\operatorname{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} \\ & + \frac{3b\operatorname{arcsinh}(a+bx)^2 \log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} \\ & - \frac{6b\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} \\ & + \frac{6b\operatorname{arcsinh}(a+bx) \operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} \\ & + \frac{6b \operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} - \frac{6b \operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}\end{aligned}$$

output
```
-arcsinh(b*x+a)^3/x-3*b*arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/
(a-(a^2+1)^(1/2)))/(a^2+1)^(1/2)+3*b*arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+
a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)-6*b*arcsinh(b*x+a)*polylog(2
,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(1/2)+6*b*arcsinh(
b*x+a)*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1
/2)+6*b*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(
1/2)-6*b*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^
(1/2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}\,dx =$$
$$-\frac{\sqrt{1+a^2}\operatorname{arcsinh}(a+bx)^3-3bx\operatorname{arcsinh}(a+bx)^2\log\left(\frac{a+\sqrt{1+a^2}-e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)+3bx\operatorname{arcsinh}(a+bx)^2\log\left(\right.}{}$$

input
```
Integrate[ArcSinh[a + b*x]^3/x^2,x]
```

output
```
-((Sqrt[1 + a^2]*ArcSinh[a + b*x]^3 - 3*b*x*ArcSinh[a + b*x]^2*Log[(a + Sq
rt[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[1 + a^2])] + 3*b*x*ArcSinh[a +
 b*x]^2*Log[(-a + Sqrt[1 + a^2] + E^ArcSinh[a + b*x])/(-a + Sqrt[1 + a^2])
] + 6*b*x*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2
])] - 6*b*x*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a
^2])] - 6*b*x*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*b*x*P
olyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(Sqrt[1 + a^2]*x))
```

3.79.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6274, 27, 6243, 6258, 3042, 3803, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}dx$$
$$\Big\downarrow \text{ 6274}$$
$$\frac{\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}d(a+bx)}{b}$$
$$\Big\downarrow \text{ 27}$$
$$b\int \frac{\operatorname{arcsinh}(a+bx)^3}{b^2x^2}d(a+bx)$$
$$\Big\downarrow \text{ 6243}$$

$$b\left(-3\int -\frac{\text{arcsinh}(a+bx)^2}{bx\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{\text{arcsinh}(a+bx)^3}{bx}\right)$$

$\Big\downarrow$ 6258

$$b\left(-3\int -\frac{\text{arcsinh}(a+bx)^2}{bx}d\text{arcsinh}(a+bx)-\frac{\text{arcsinh}(a+bx)^3}{bx}\right)$$

$\Big\downarrow$ 3042

$$b\left(-\frac{\text{arcsinh}(a+bx)^3}{bx}-3\int \frac{\text{arcsinh}(a+bx)^2}{a+i\sin(i\text{arcsinh}(a+bx))}d\text{arcsinh}(a+bx)\right)$$

$\Big\downarrow$ 3803

$$b\left(-6\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)^2}{2e^{\text{arcsinh}(a+bx)}a-e^{2\text{arcsinh}(a+bx)}+1}d\text{arcsinh}(a+bx)-\frac{\text{arcsinh}(a+bx)^3}{bx}\right)$$

$\Big\downarrow$ 2694

$$b\left(-6\left(\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)^2}{2\left(a-e^{\text{arcsinh}(a+bx)}+\sqrt{a^2+1}\right)}d\text{arcsinh}(a+bx)}{\sqrt{a^2+1}}-\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)^2}{2\left(a-e^{\text{arcsinh}(a+bx)}-\sqrt{a^2+1}\right)}d\text{arcsinh}(a+bx)}{\sqrt{a^2+1}}\right)-\text{arcsinh}$$

$\Big\downarrow$ 27

$$b\left(-6\left(\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)^2}{a-e^{\text{arcsinh}(a+bx)}+\sqrt{a^2+1}}d\text{arcsinh}(a+bx)}{2\sqrt{a^2+1}}-\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)^2}{a-e^{\text{arcsinh}(a+bx)}-\sqrt{a^2+1}}d\text{arcsinh}(a+bx)}{2\sqrt{a^2+1}}\right)-\text{arcsinh}$$

$\Big\downarrow$ 2620

$$b\left(-6\left(\frac{2\int \text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)d\text{arcsinh}(a+bx)-\text{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\text{arcsinh}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{2\sqrt{a^2+1}}-\right.\right.$$

$\Big\downarrow$ 3011

$$b\left(-6\left(\frac{2\left(\int \text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)d\text{arcsinh}(a+bx)-\text{arcsinh}(a+bx)\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)\right)-\text{arcsinh}(}{2\sqrt{a^2+1}}\right.\right.$$

$\Big\downarrow$ 2720

3.79. $\int \frac{\text{arcsinh}(a+bx)^3}{x^2}\,dx$

$$b\left(-6\left(\frac{2\left(\int e^{-\text{arcsinh}(a+bx)}\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)de^{\text{arcsinh}(a+bx)}-\text{arcsinh}(a+bx)\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)\right)}{2\sqrt{a^2+1}}\right.\right.$$

$\downarrow$ 7143

$$b\left(-6\left(\frac{2\left(\text{PolyLog}\left(3,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)-\text{arcsinh}(a+bx)\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)\right)-\text{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\text{a}}}{}\right.}{2\sqrt{a^2+1}}\right.\right.$$

input
```
Int[ArcSinh[a + b*x]^3/x^2,x]
```

output
```
b*(-(ArcSinh[a + b*x]^3/(b*x)) - 6*(-1/2*(-(ArcSinh[a + b*x]^2*Log[1 - E^A
rcSinh[a + b*x]/(a - Sqrt[1 + a^2])]) + 2*(-(ArcSinh[a + b*x]*PolyLog[2, E
^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])]) + PolyLog[3, E^ArcSinh[a + b*x]/(a
 - Sqrt[1 + a^2])]))/Sqrt[1 + a^2] + (-(ArcSinh[a + b*x]^2*Log[1 - E^ArcSi
nh[a + b*x]/(a + Sqrt[1 + a^2])]) + 2*(-(ArcSinh[a + b*x]*PolyLog[2, E^Arc
Sinh[a + b*x]/(a + Sqrt[1 + a^2])]) + PolyLog[3, E^ArcSinh[a + b*x]/(a + S
qrt[1 + a^2])]))/(2*Sqrt[1 + a^2])))
```

3.79.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.79. $\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.79.4 Maple [F]

$$\int \frac{\operatorname{arcsinh}(bx+a)^3}{x^2}dx$$

input
```
int(arcsinh(b*x+a)^3/x^2,x)
```

output
```
int(arcsinh(b*x+a)^3/x^2,x)
```

3.79.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{arsinh}(bx+a)^3}{x^2}\,dx$$

input
```
integrate(arcsinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
integral(arcsinh(b*x + a)^3/x^2, x)
```

3.79.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{asinh}^3(a+bx)}{x^2}\,dx$$

input
```
integrate(asinh(b*x+a)**3/x**2,x)
```

output
```
Integral(asinh(a + b*x)**3/x**2, x)
```

3.79.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{x^2}\,dx$$

input
```
integrate(arcsinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
-log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3/x + integrate(3*(b^3*x
^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) +
 b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(b^3*x^4 + 3*a*b^2*
x^3 + (3*a^2*b + b)*x^2 + (a^3 + a)*x + (b^2*x^3 + 2*a*b*x^2 + (a^2 + 1)*x
)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

3.79.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{x^2}\,dx$$

input
```
integrate(arcsinh(b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
integrate(arcsinh(b*x + a)^3/x^2, x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^2}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)^3}{x^2}\,dx$$

input
```
int(asinh(a + b*x)^3/x^2,x)
```

output
```
int(asinh(a + b*x)^3/x^2, x)
```

3.80 $\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^3} dx$

3.80.1 Optimal result

Integrand size = 12, antiderivative size = 514

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^3}\,dx = -\frac{3b^2\operatorname{arcsinh}(a+bx)^2}{2(1+a^2)} - \frac{3b\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{2(1+a^2)x} - \frac{\operatorname{arcsinh}(a+bx)^3}{2x^2} + \frac{3b^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{1+a^2} + \frac{3ab^2\operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{2(1+a^2)^{3/2}} + \frac{3b^2\operatorname{arcsinh}(a+bx)\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{1+a^2} - \frac{3ab^2\operatorname{arcsinh}(a+bx)^2\log\left(1-\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{2(1+a^2)^{3/2}} + \frac{3b^2\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{1+a^2} + \frac{3ab^2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}} + \frac{3b^2\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{1+a^2} - \frac{3ab^2\operatorname{arcsinh}(a+bx)\operatorname{PolyLog}\left(2,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}} - \frac{3ab^2\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}} + \frac{3ab^2\operatorname{PolyLog}\left(3,\frac{e^{\operatorname{arcsinh}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}}$$

output
```
-3/2*b^2*arcsinh(b*x+a)^2/(a^2+1)-1/2*arcsinh(b*x+a)^3/x^2+3*b^2*arcsinh(b
*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)+3/2*a*b^
2*arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^
2+1)^(3/2)+3*b^2*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1
)^(1/2)))/(a^2+1)-3/2*a*b^2*arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/
2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)+3*b^2*polylog(2,(b*x+a+(1+(b*x+a)^2)^
(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)+3*a*b^2*arcsinh(b*x+a)*polylog(2,(b*x+a+
(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)+3*b^2*polylog(2,(b*x
+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)-3*a*b^2*arcsinh(b*x+a)*
polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-3*a
*b^2*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2
)+3*a*b^2*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)
^(3/2)-3/2*b*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x
```

3.80.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^3}\,dx$$
$$= \frac{-3\sqrt{1+a^2}b^2x^2\operatorname{arcsinh}(a+bx)^2-3\sqrt{1+a^2}bx\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2-\sqrt{1+a^2}\operatorname{arcsinh}}{}$$

input
```
Integrate[ArcSinh[a + b*x]^3/x^3,x]
```

output
```
(-3*Sqrt[1 + a^2]*b^2*x^2*ArcSinh[a + b*x]^2 - 3*Sqrt[1 + a^2]*b*x*Sqrt[1
+ a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2 - Sqrt[1 + a^2]*ArcSinh[a +
b*x]^3 - a^2*Sqrt[1 + a^2]*ArcSinh[a + b*x]^3 + 6*Sqrt[1 + a^2]*b^2*x^2*Ar
cSinh[a + b*x]*Log[(a + Sqrt[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[1 +
a^2])] - 3*a*b^2*x^2*ArcSinh[a + b*x]^2*Log[(a + Sqrt[1 + a^2] - E^ArcSinh
[a + b*x])/(a + Sqrt[1 + a^2])] + 6*Sqrt[1 + a^2]*b^2*x^2*ArcSinh[a + b*x]
*Log[(-a + Sqrt[1 + a^2] + E^ArcSinh[a + b*x])/(-a + Sqrt[1 + a^2])] + 3*a
*b^2*x^2*ArcSinh[a + b*x]^2*Log[(-a + Sqrt[1 + a^2] + E^ArcSinh[a + b*x])/
(-a + Sqrt[1 + a^2])] + 6*b^2*x^2*(Sqrt[1 + a^2] + a*ArcSinh[a + b*x])*Pol
yLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*b^2*x^2*(Sqrt[1 + a^2]
 - a*ArcSinh[a + b*x])*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
- 6*a*b^2*x^2*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*a*b^2
*x^2*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(2*(1 + a^2)^(3/2
)*x^2)
```

3.80.3 Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.87, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.583, Rules used = {6274, 25, 27, 6243, 6258, 3042, 3805, 3042, 3803, 2694, 27, 2620, 3011, 2720, 6095, 2620, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(a+bx)^3}{x^3} \, dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \frac{\text{arcsinh}(a+bx)^3}{x^3}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -\frac{\text{arcsinh}(a+bx)^3}{x^3}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-b^2 \int -\frac{\text{arcsinh}(a+bx)^3}{b^3x^3}d(a+bx)$$

$$\Big\downarrow \text{ 6243}$$

$$-b^2 \left(\frac{\text{arcsinh}(a+bx)^3}{2b^2x^2} - \frac{3}{2}\int \frac{\text{arcsinh}(a+bx)^2}{b^2x^2\sqrt{(a+bx)^2+1}}d(a+bx)\right)$$

$$\Big\downarrow \text{ 6258}$$

$$-b^2 \left(\frac{\text{arcsinh}(a+bx)^3}{2b^2x^2} - \frac{3}{2}\int \frac{\text{arcsinh}(a+bx)^2}{b^2x^2}d\text{arcsinh}(a+bx)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-b^2 \left(\frac{\text{arcsinh}(a+bx)^3}{2b^2x^2} - \frac{3}{2}\int \frac{\text{arcsinh}(a+bx)^2}{(a+i\sin(i\text{arcsinh}(a+bx)))^2}d\text{arcsinh}(a+bx)\right)$$

$$\Big\downarrow \text{ 3805}$$

$$-b^2 \left(\frac{\text{arcsinh}(a+bx)^3}{2b^2x^2} - \frac{3}{2}\left(-\frac{2\int -\frac{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)}{bx}d\text{arcsinh}(a+bx)}{a^2+1} + \frac{a\int -\frac{\text{arcsinh}(a+bx)^2}{bx}d\text{arcsinh}(a+\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$-b^2\left(\frac{\text{arcsinh}(a+bx)^3}{2b^2x^2}-\frac{3}{2}\left(-\frac{2\int -\frac{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)}{bx}d\text{arcsinh}(a+bx)}{a^2+1}+\frac{a\int \frac{\text{arcsinh}(a+bx)^2}{a+i\sin(i\text{arcsinh}(a+bx))}d\text{arcsin}}{a^2+1}\right.\right.$$

$\downarrow$ 3803

$$-b^2\left(\frac{\text{arcsinh}(a+bx)^3}{2b^2x^2}-\frac{3}{2}\left(-\frac{2\int -\frac{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)}{bx}d\text{arcsinh}(a+bx)}{a^2+1}+\frac{2a\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)^2}{2e^{\text{arcsinh}(a+bx)}a-e^{2\text{arcsinh}(a+bx)}}}{a^2+1}\right.\right.$$

$\downarrow$ 2694

$$-b^2\left(\frac{\text{arcsinh}(a+bx)^3}{2b^2x^2}-\frac{3}{2}\left(-\frac{2\int -\frac{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)}{bx}d\text{arcsinh}(a+bx)}{a^2+1}+\frac{2a\left(\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)^2}{2\left(a-e^{\text{arcsinh}(a+bx)}+\sqrt{a^2+1}\right)}d}{\sqrt{a^2+1}}\right.}{}\right.\right.$$

$\downarrow$ 27

$$-b^2\left(\frac{\text{arcsinh}(a+bx)^3}{2b^2x^2}-\frac{3}{2}\left(-\frac{2\int -\frac{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)}{bx}d\text{arcsinh}(a+bx)}{a^2+1}+\frac{2a\left(\frac{\int \frac{e^{\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)^2}{a-e^{\text{arcsinh}(a+bx)}+\sqrt{a^2+1}}d}{2\sqrt{a^2+1}}\right.}{}\right.\right.$$

$\downarrow$ 2620

$$-b^2\left(\frac{\text{arcsinh}(a+bx)^3}{2b^2x^2}-\frac{3}{2}\left(-\frac{2\int -\frac{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)}{bx}d\text{arcsinh}(a+bx)}{a^2+1}+\frac{2a\left(\frac{2\int \text{arcsinh}(a+bx)\log\left(1-\frac{e^{\text{arcsin}}}{a+\sqrt{}}\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 3011

3.80. $\int \frac{\text{arcsinh}(a+bx)^3}{x^3}\,dx$

$$-b^2\left(\frac{\text{arcsinh}(a+bx)^3}{2b^2x^2}-\frac{3}{2}\left(\frac{2a\left(\frac{2\left(\int \text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)d\text{arcsinh}(a+bx)-\text{arcsinh}(a+bx)\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)\right)}{2\sqrt{a^2+1}}\cdots\right.}{\cdots}\cdots\right.\right.$$

$\downarrow$ 2720

$$-b^2\left(\frac{\text{arcsinh}(a+bx)^3}{2b^2x^2}-\frac{3}{2}\left(\frac{2a\left(\frac{2\left(\int e^{-\text{arcsinh}(a+bx)}\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)de^{\text{arcsinh}(a+bx)}-\text{arcsinh}(a+bx)\,\text{PolyLog}\left(2,\frac{e^{\text{arc}\cdots}}{\cdots}\right.\right.}{2\sqrt{a^2+1}}\cdots\right.}{\cdots}\cdots\right.\right.$$

$\downarrow$ 6095

$$-b^2\left(\frac{\text{arcsinh}(a+bx)^3}{2b^2x^2}-\frac{3}{2}\left(\frac{2a\left(\frac{2\left(\int e^{-\text{arcsinh}(a+bx)}\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)de^{\text{arcsinh}(a+bx)}-\text{arcsinh}(a+bx)\,\text{PolyLog}\left(2,\frac{e^{\text{arc}\cdots}}{\cdots}\right.\right.}{2\sqrt{a^2+1}}\cdots\right.}{\cdots}\cdots\right.\right.$$

$\downarrow$ 2620

$$-b^2\left(\frac{\text{arcsinh}(a+bx)^3}{2b^2x^2}-\frac{3}{2}\left(\frac{2a\left(\frac{2\left(\int e^{-\text{arcsinh}(a+bx)}\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)de^{\text{arcsinh}(a+bx)}-\text{arcsinh}(a+bx)\,\text{PolyLog}\left(2,\frac{e^{\text{arc}\cdots}}{\cdots}\right.\right.}{2\sqrt{a^2+1}}\cdots\right.}{\cdots}\cdots\right.\right.$$

$\downarrow$ 2715

$$-b^2\left(\frac{\text{arcsinh}(a+bx)^3}{2b^2x^2}-\frac{3}{2}\left(\frac{2a\left(\frac{2\left(\int e^{-\text{arcsinh}(a+bx)}\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)de^{\text{arcsinh}(a+bx)}-\text{arcsinh}(a+bx)\,\text{PolyLog}\left(2,\frac{e^{\text{arc}\cdots}}{\cdots}\right.\right.}{2\sqrt{a^2+1}}\cdots\right.}{\cdots}\cdots\right.\right.$$

$\downarrow$ 2838

3.80. $\int \frac{\text{arcsinh}(a+bx)^3}{x^3}\,dx$

$$-b^2\left(\frac{\text{arcsinh}(a+bx)^3}{2b^2x^2}-\frac{3}{2}\left(\frac{2a\left(\frac{2\left(\int e^{-\text{arcsinh}(a+bx)}\,\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)de^{\text{arcsinh}(a+bx)}-\text{arcsinh}(a+bx)\,\text{PolyLog}\left(2,\frac{e^{\text{arc}}}{a}\right.\right.}{2\sqrt{a^2+1}}\right.}{}\right.\right.$$

$\downarrow$ 7143

$$-b^2\left(\frac{\text{arcsinh}(a+bx)^3}{2b^2x^2}-\frac{3}{2}\left(-\frac{2\left(-\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a-\sqrt{a^2+1}}\right)-\text{PolyLog}\left(2,\frac{e^{\text{arcsinh}(a+bx)}}{a+\sqrt{a^2+1}}\right)-\text{arcsinh}(a+bx)\log\right.}{a^2}\right.\right.$$

input
```
Int[ArcSinh[a + b*x]^3/x^3,x]
```

output
```
-(b^2*(ArcSinh[a + b*x]^3/(2*b^2*x^2) - (3*(-((Sqrt[1 + (a + b*x)^2]*ArcSi
nh[a + b*x]^2)/((1 + a^2)*b*x)) - (2*(ArcSinh[a + b*x]^2/2 - ArcSinh[a + b
*x]*Log[1 - E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] - ArcSinh[a + b*x]*Log
[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])] - PolyLog[2, E^ArcSinh[a + b*
x]/(a - Sqrt[1 + a^2])] - PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2]
)]))/(1 + a^2) + (2*a*(-1/2*(-(ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*
x]/(a - Sqrt[1 + a^2])]) + 2*(-(ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a +
b*x]/(a - Sqrt[1 + a^2])]) + PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a
^2])]))/Sqrt[1 + a^2] + (-(ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(
a + Sqrt[1 + a^2])]) + 2*(-(ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]
/(a + Sqrt[1 + a^2])]) + PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])
]))/(2*Sqrt[1 + a^2])))/(1 + a^2)))/2))
```

3.80.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.80. $\int \frac{\text{arcsinh}(a+bx)^3}{x^3} dx$

rule 3803
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((
-I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /;
FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6243
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(
n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n
, 0] && NeQ[m, -1]
```

rule 6258
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[1/(c^(m + 1)*Sqrt[d])   Subst[I
nt[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (Gt
Q[m, 0] || IGtQ[n, 0])
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.80. $\int \frac{\text{arcsinh}(a+bx)^3}{x^3}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.80.4 Maple [F]

$$\int \frac{\operatorname{arcsinh}\left(bx+a\right)^{3}}{x^{3}}dx$$

input
```
int(arcsinh(b*x+a)^3/x^3,x)
```

output
```
int(arcsinh(b*x+a)^3/x^3,x)
```

3.80.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^3}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^{3}}{x^{3}}\,dx$$

input
```
integrate(arcsinh(b*x+a)^3/x^3,x, algorithm="fricas")
```

output
```
integral(arcsinh(b*x + a)^3/x^3, x)
```

3.80.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^3}\,dx = \int \frac{\operatorname{asinh}^{3}\left(a+bx\right)}{x^{3}}\,dx$$

input
```
integrate(asinh(b*x+a)**3/x**3,x)
```

output
```
Integral(asinh(a + b*x)**3/x**3, x)
```

3.80.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^3}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{x^3}\,dx$$

input
```
integrate(arcsinh(b*x+a)^3/x^3,x, algorithm="maxima")
```

output
```
-1/2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3/x^2 + integrate(3/
2*(b^3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x
+ a*b) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(b^3*x^5 +
3*a*b^2*x^4 + (3*a^2*b + b)*x^3 + (a^3 + a)*x^2 + (b^2*x^4 + 2*a*b*x^3 + (
a^2 + 1)*x^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

3.80.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^3}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{x^3}\,dx$$

input
```
integrate(arcsinh(b*x+a)^3/x^3,x, algorithm="giac")
```

output
```
integrate(arcsinh(b*x + a)^3/x^3, x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{x^3}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)^3}{x^3}\,dx$$

input
```
int(asinh(a + b*x)^3/x^3,x)
```

output
```
int(asinh(a + b*x)^3/x^3, x)
```

3.81 $\int \frac{x^2}{\operatorname{arcsinh}(a+bx)}\,dx$

3.81.1 Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)}\,dx=-\frac{\operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{4b^3}+\frac{a^2\operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{b^3}+\frac{\operatorname{Chi}(3\operatorname{arcsinh}(a+bx))}{4b^3}-\frac{a\operatorname{Shi}(2\operatorname{arcsinh}(a+bx))}{b^3}$$

output
```
-1/4*Chi(arcsinh(b*x+a))/b^3+a^2*Chi(arcsinh(b*x+a))/b^3+1/4*Chi(3*arcsinh
(b*x+a))/b^3-a*Shi(2*arcsinh(b*x+a))/b^3
```

3.81.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)}\,dx=\frac{(-1+4a^2)\operatorname{Chi}(\operatorname{arcsinh}(a+bx))+\operatorname{Chi}(3\operatorname{arcsinh}(a+bx))-4a\operatorname{Shi}(2\operatorname{arcsinh}(a+bx))}{4b^3}$$

input
```
Integrate[x^2/ArcSinh[a + b*x],x]
```

output
```
((-1 + 4*a^2)*CoshIntegral[ArcSinh[a + b*x]] + CoshIntegral[3*ArcSinh[a +
b*x]] - 4*a*SinhIntegral[2*ArcSinh[a + b*x]])/(4*b^3)
```

3.81.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6274, 27, 6245, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\text{arcsinh}(a+bx)}dx$$

$$\Big\downarrow \ 6274$$

$$\frac{\int \frac{x^2}{\text{arcsinh}(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{b^2x^2}{\text{arcsinh}(a+bx)}d(a+bx)}{b^3}$$

$$\Big\downarrow \ 6245$$

$$\frac{\int \frac{b^2x^2\sqrt{(a+bx)^2+1}}{\text{arcsinh}(a+bx)}d\text{arcsinh}(a+bx)}{b^3}$$

$$\Big\downarrow \ 7293$$

$$\frac{\int \left(\frac{\sqrt{(a+bx)^2+1}a^2}{\text{arcsinh}(a+bx)}-\frac{2(a+bx)\sqrt{(a+bx)^2+1}a}{\text{arcsinh}(a+bx)}+\frac{(a+bx)^2\sqrt{(a+bx)^2+1}}{\text{arcsinh}(a+bx)}\right)d\text{arcsinh}(a+bx)}{b^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{a^2\text{Chi}(\text{arcsinh}(a+bx))-\frac{1}{4}\text{Chi}(\text{arcsinh}(a+bx))+\frac{1}{4}\text{Chi}(3\text{arcsinh}(a+bx))-a\text{Shi}(2\text{arcsinh}(a+bx))}{b^3}$$

input
```
Int[x^2/ArcSinh[a + b*x],x]
```

output
```
(-1/4*CoshIntegral[ArcSinh[a + b*x]] + a^2*CoshIntegral[ArcSinh[a + b*x]]
+ CoshIntegral[3*ArcSinh[a + b*x]]/4 - a*SinhIntegral[2*ArcSinh[a + b*x]])
/b^3
```

3.81.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6245
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[1/c^(m + 1)   Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[
x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m,
0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.81.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{-\frac{\operatorname{Chi}(\operatorname{arcsinh}(bx+a))}{4}+\frac{\operatorname{Chi}(3\operatorname{arcsinh}(bx+a))}{4}-a\operatorname{Shi}(2\operatorname{arcsinh}(bx+a))+a^2\operatorname{Chi}(\operatorname{arcsinh}(bx+a))}{b^3}$	49
default	$\frac{-\frac{\operatorname{Chi}(\operatorname{arcsinh}(bx+a))}{4}+\frac{\operatorname{Chi}(3\operatorname{arcsinh}(bx+a))}{4}-a\operatorname{Shi}(2\operatorname{arcsinh}(bx+a))+a^2\operatorname{Chi}(\operatorname{arcsinh}(bx+a))}{b^3}$	49

input
```
int(x^2/arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(-1/4*Chi(arcsinh(b*x+a))+1/4*Chi(3*arcsinh(b*x+a))-a*Shi(2*arcsinh(
b*x+a))+a^2*Chi(arcsinh(b*x+a)))
```

3.81.5 Fricas [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)}\,dx=\int \frac{x^2}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

input
```
integrate(x^2/arcsinh(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^2/arcsinh(b*x + a), x)
```

3.81.6 Sympy [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)}\,dx=\int \frac{x^2}{\operatorname{asinh}\left(a+bx\right)}\,dx$$

input
```
integrate(x**2/asinh(b*x+a),x)
```

output
```
Integral(x**2/asinh(a + b*x), x)
```

3.81.7 Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)}\,dx=\int \frac{x^2}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

input
```
integrate(x^2/arcsinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^2/arcsinh(b*x + a), x)
```

3.81.8 Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

input
```
integrate(x^2/arcsinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2/arcsinh(b*x + a), x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{x^2}{\operatorname{asinh}\left(a+b\,x\right)}\,dx$$

input
```
int(x^2/asinh(a + b*x),x)
```

output
```
int(x^2/asinh(a + b*x), x)
```

3.82 $\int \frac{x}{\text{arcsinh}(a+bx)} dx$

3.82.1 Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{x}{\text{arcsinh}(a+bx)} dx = -\frac{a\text{Chi}(\text{arcsinh}(a+bx))}{b^2} + \frac{\text{Shi}(2\text{arcsinh}(a+bx))}{2b^2}$$

output
```
-a*Chi(arcsinh(b*x+a))/b^2+1/2*Shi(2*arcsinh(b*x+a))/b^2
```

3.82.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{x}{\text{arcsinh}(a+bx)} dx = -\frac{a\text{Chi}(\text{arcsinh}(a+bx))}{b^2} + \frac{\text{Shi}(2\text{arcsinh}(a+bx))}{2b^2}$$

input
```
Integrate[x/ArcSinh[a + b*x],x]
```

output
```
-((a*CoshIntegral[ArcSinh[a + b*x]])/b^2) + SinhIntegral[2*ArcSinh[a + b*x
]]/(2*b^2)
```

3.82.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6274, 25, 27, 6245, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{x}{\operatorname{arcsinh}(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{x}{\operatorname{arcsinh}(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -\frac{bx}{\operatorname{arcsinh}(a+bx)}d(a+bx)}{b^2}$$

$$\Big\downarrow \text{6245}$$

$$-\frac{\int -\frac{bx\sqrt{(a+bx)^2+1}}{\operatorname{arcsinh}(a+bx)}d\operatorname{arcsinh}(a+bx)}{b^2}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{\int \left(\frac{a\sqrt{(a+bx)^2+1}}{\operatorname{arcsinh}(a+bx)}-\frac{(a+bx)\sqrt{(a+bx)^2+1}}{\operatorname{arcsinh}(a+bx)}\right)d\operatorname{arcsinh}(a+bx)}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\operatorname{Chi}(\operatorname{arcsinh}(a+bx))-\frac{1}{2}\operatorname{Shi}(2\operatorname{arcsinh}(a+bx))}{b^2}$$

input
```
Int[x/ArcSinh[a + b*x],x]
```

output
```
-((a*CoshIntegral[ArcSinh[a + b*x]] - SinhIntegral[2*ArcSinh[a + b*x]]/2)/
b^2)
```

3.82.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6245
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[1/c^(m + 1)   Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[
x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m,
0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.82.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\frac{\mathrm{Shi}(2\,\mathrm{arcsinh}(bx+a))}{2}-a\,\mathrm{Chi}(\mathrm{arcsinh}(bx+a))}{b^2}$	27
default	$\frac{\frac{\mathrm{Shi}(2\,\mathrm{arcsinh}(bx+a))}{2}-a\,\mathrm{Chi}(\mathrm{arcsinh}(bx+a))}{b^2}$	27

input
```
int(x/arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/2*Shi(2*arcsinh(b*x+a))-a*Chi(arcsinh(b*x+a)))
```

3.82.5 Fricas [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)} \, dx = \int \frac{x}{\operatorname{arsinh}(bx+a)} \, dx$$

input
```
integrate(x/arcsinh(b*x+a),x, algorithm="fricas")
```

output
```
integral(x/arcsinh(b*x + a), x)
```

3.82.6 Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)} \, dx = \int \frac{x}{\operatorname{asinh}(a+bx)} \, dx$$

input
```
integrate(x/asinh(b*x+a),x)
```

output
```
Integral(x/asinh(a + b*x), x)
```

3.82.7 Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)} \, dx = \int \frac{x}{\operatorname{arsinh}(bx+a)} \, dx$$

input
```
integrate(x/arcsinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x/arcsinh(b*x + a), x)
```

3.82.8 Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)} \, dx = \int \frac{x}{\operatorname{arsinh}\left(bx+a\right)} \, dx$$

input
```
integrate(x/arcsinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x/arcsinh(b*x + a), x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)} \, dx = \int \frac{x}{\operatorname{asinh}\left(a+b\,x\right)} \, dx$$

input
```
int(x/asinh(a + b*x),x)
```

output
```
int(x/asinh(a + b*x), x)
```

3.83 $\int \frac{1}{\text{arcsinh}(a+bx)}\,dx$

3.83.1 Optimal result

Integrand size = 8, antiderivative size = 11

$$\int \frac{1}{\text{arcsinh}(a+bx)}\,dx=\frac{\text{Chi}(\text{arcsinh}(a+bx))}{b}$$

output
```
Chi(arcsinh(b*x+a))/b
```

3.83.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\text{arcsinh}(a+bx)}\,dx=\frac{\text{Chi}(\text{arcsinh}(a+bx))}{b}$$

input
```
Integrate[ArcSinh[a + b*x]^(-1),x]
```

output
```
CoshIntegral[ArcSinh[a + b*x]]/b
```

3.83.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6273, 6189, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(a+bx)}\,dx$$
$$\Big\downarrow \text{ 6273}$$
$$\frac{\int \frac{1}{\text{arcsinh}(a+bx)}d(a+bx)}{b}$$
$$\Big\downarrow \text{ 6189}$$
$$\frac{\int \frac{\sqrt{(a+bx)^2+1}}{\text{arcsinh}(a+bx)}d\text{arcsinh}(a+bx)}{b}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \frac{\sin\left(i\text{arcsinh}(a+bx)+\frac{\pi}{2}\right)}{\text{arcsinh}(a+bx)}d\text{arcsinh}(a+bx)}{b}$$
$$\Big\downarrow \text{ 3782}$$
$$\frac{\text{Chi}(\text{arcsinh}(a+bx))}{b}$$

input
```
Int[ArcSinh[a + b*x]^(-1),x]
```

output
```
CoshIntegral[ArcSinh[a + b*x]]/b
```

3.83.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.83.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\mathrm{Chi}(\operatorname{arcsinh}(bx+a))}{b}$	12
default	$\frac{\mathrm{Chi}(\operatorname{arcsinh}(bx+a))}{b}$	12

input
```
int(1/arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
Chi(arcsinh(b*x+a))/b
```

3.83.5 Fricas [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

input
```
integrate(1/arcsinh(b*x+a),x, algorithm="fricas")
```

output
```
integral(1/arcsinh(b*x + a), x)
```

3.83.6 Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{\operatorname{asinh}(a+bx)}\,dx$$

input
```
integrate(1/asinh(b*x+a),x)
```

output
```
Integral(1/asinh(a + b*x), x)
```

3.83.7 Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{\operatorname{arsinh}(bx+a)}\,dx$$

input
```
integrate(1/arcsinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(1/arcsinh(b*x + a), x)
```

3.83.8 Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{\operatorname{arsinh}(bx+a)}\,dx$$

input
```
integrate(1/arcsinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(1/arcsinh(b*x + a), x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{\operatorname{asinh}\left(a+b\,x\right)}\,dx$$

input
```
int(1/asinh(a + b*x),x)
```

output
```
int(1/asinh(a + b*x), x)
```

3.84 $\int \frac{1}{x\text{arcsinh}(a+bx)} dx$

3.84.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\text{arcsinh}(a+bx)} dx = \text{Int}\left(\frac{1}{x\text{arcsinh}(a+bx)}, x\right)$$

output
```
Unintegrable(1/x/arcsinh(b*x+a),x)
```

3.84.2 Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)} dx = \int \frac{1}{x\text{arcsinh}(a+bx)} dx$$

input
```
Integrate[1/(x*ArcSinh[a + b*x]),x]
```

output
```
Integrate[1/(x*ArcSinh[a + b*x]), x]
```

3.84.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 25, 27, 6272}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{arcsinh}(a+bx)}dx$$
$$\Big\downarrow \text{6274}$$
$$\frac{\int \frac{1}{x\text{arcsinh}(a+bx)}d(a+bx)}{b}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\int -\frac{1}{x\text{arcsinh}(a+bx)}d(a+bx)}{b}$$
$$\Big\downarrow \text{27}$$
$$-\int -\frac{1}{bx\text{arcsinh}(a+bx)}d(a+bx)$$
$$\Big\downarrow \text{6272}$$
$$-\int -\frac{1}{bx\text{arcsinh}(a+bx)}d(a+bx)$$

input
```
Int[1/(x*ArcSinh[a + b*x]),x]
```

output
```
$Aborted
```

3.84.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6272
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.84.4 Maple [N/A] (verified)

Not integrable

Time = 0.98 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \operatorname{arcsinh}(bx+a)} dx$$

input
```
int(1/x/arcsinh(b*x+a),x)
```

output
```
int(1/x/arcsinh(b*x+a),x)
```

3.84.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)}\, dx = \int \frac{1}{x \operatorname{arsinh}(bx+a)}\, dx$$

input
```
integrate(1/x/arcsinh(b*x+a),x, algorithm="fricas")
```

output
```
integral(1/(x*arcsinh(b*x + a)), x)
```

3.84.6 Sympy [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\text{arcsinh}(a+bx)}\,dx=\int \frac{1}{x\,\text{asinh}\,(a+bx)}\,dx$$

input
```
integrate(1/x/asinh(b*x+a),x)
```

output
```
Integral(1/(x*asinh(a + b*x)), x)
```

3.84.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)}\,dx=\int \frac{1}{x\,\text{arsinh}\,(bx+a)}\,dx$$

input
```
integrate(1/x/arcsinh(b*x+a),x, algorithm="maxima")
```

output
```
integrate(1/(x*arcsinh(b*x + a)), x)
```

3.84.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)}\,dx=\int \frac{1}{x\,\text{arsinh}\,(bx+a)}\,dx$$

input
```
integrate(1/x/arcsinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(1/(x*arcsinh(b*x + a)), x)
```

3.84.9 Mupad [N/A]

Not integrable

Time = 2.73 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)}\,dx = \int \frac{1}{x\,\mathrm{asinh}\left(a+b\,x\right)}\,dx$$

input
```
int(1/(x*asinh(a + b*x)),x)
```

output
```
int(1/(x*asinh(a + b*x)), x)
```

3.85 $\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^2}\,dx$

3.85.1 Optimal result

Integrand size = 12, antiderivative size = 154

$$\begin{aligned}\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^2}\,dx = &-\frac{a^2\sqrt{1+(a+bx)^2}}{b^3\operatorname{arcsinh}(a+bx)}+\frac{2a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\operatorname{arcsinh}(a+bx)}\\&-\frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{b^3\operatorname{arcsinh}(a+bx)}\\&-\frac{2a\operatorname{Chi}(2\operatorname{arcsinh}(a+bx))}{b^3}-\frac{\operatorname{Shi}(\operatorname{arcsinh}(a+bx))}{4b^3}\\&+\frac{a^2\operatorname{Shi}(\operatorname{arcsinh}(a+bx))}{b^3}+\frac{3\operatorname{Shi}(3\operatorname{arcsinh}(a+bx))}{4b^3}\end{aligned}$$

output
```
-2*a*Chi(2*arcsinh(b*x+a))/b^3-1/4*Shi(arcsinh(b*x+a))/b^3+a^2*Shi(arcsinh
(b*x+a))/b^3+3/4*Shi(3*arcsinh(b*x+a))/b^3-a^2*(1+(b*x+a)^2)^(1/2)/b^3/arc
sinh(b*x+a)+2*a*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3/arcsinh(b*x+a)-(b*x+a)^2*(
1+(b*x+a)^2)^(1/2)/b^3/arcsinh(b*x+a)
```

3.85.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.54

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^2}\,dx$$
$$= \frac{-\frac{4b^2x^2\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)} - 8a\mathrm{Chi}(2\operatorname{arcsinh}(a+bx)) + (-1+4a^2)\,\mathrm{Shi}(\operatorname{arcsinh}(a+bx)) + 3\mathrm{Shi}(3\operatorname{arcsinh}(a+}{4b^3}$$

input
```
Integrate[x^2/ArcSinh[a + b*x]^2,x]
```

output
```
((-4*b^2*x^2*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/ArcSinh[a + b*x] - 8*a*Cos
hIntegral[2*ArcSinh[a + b*x]] + (-1 + 4*a^2)*SinhIntegral[ArcSinh[a + b*x]
] + 3*SinhIntegral[3*ArcSinh[a + b*x]])/(4*b^3)
```

3.85.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6274, 27, 6244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^2}\,dx$$
$$\Big\downarrow \; 6274$$
$$\frac{\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^2}d(a+bx)}{b}$$
$$\Big\downarrow \; 27$$
$$\frac{\int \frac{b^2x^2}{\operatorname{arcsinh}(a+bx)^2}d(a+bx)}{b^3}$$
$$\Big\downarrow \; 6244$$
$$\frac{\int \left(\frac{a^2}{\operatorname{arcsinh}(a+bx)^2} - \frac{2(a+bx)a}{\operatorname{arcsinh}(a+bx)^2} + \frac{(a+bx)^2}{\operatorname{arcsinh}(a+bx)^2}\right)d(a+bx)}{b^3}$$
$$\Big\downarrow \; 2009$$

$$\frac{a^2\text{Shi}(\text{arcsinh}(a+bx)) - \frac{a^2\sqrt{(a+bx)^2+1}}{\text{arcsinh}(a+bx)} - 2a\text{Chi}(2\text{arcsinh}(a+bx)) - \frac{1}{4}\text{Shi}(\text{arcsinh}(a+bx)) + \frac{3}{4}\text{Shi}(3\text{arcsinh}(a+}{b^3}$$

input
```
Int[x^2/ArcSinh[a + b*x]^2,x]
```

output
```
(-((a^2*Sqrt[1 + (a + b*x)^2])/ArcSinh[a + b*x]) + (2*a*(a + b*x)*Sqrt[1 +
 (a + b*x)^2])/ArcSinh[a + b*x] - ((a + b*x)^2*Sqrt[1 + (a + b*x)^2])/ArcS
inh[a + b*x] - 2*a*CoshIntegral[2*ArcSinh[a + b*x]] - SinhIntegral[ArcSinh
[a + b*x]]/4 + a^2*SinhIntegral[ArcSinh[a + b*x]] + (3*SinhIntegral[3*ArcS
inh[a + b*x]])/4)/b^3
```

3.85.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6244
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.85.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{\sqrt{1+(bx+a)^2}}{4\,\mathrm{arcsinh}(bx+a)} - \frac{\mathrm{Shi}(\mathrm{arcsinh}(bx+a))}{4} - \frac{\cosh(3\,\mathrm{arcsinh}(bx+a))}{4\,\mathrm{arcsinh}(bx+a)} + \frac{3\,\mathrm{Shi}(3\,\mathrm{arcsinh}(bx+a))}{4} - \frac{a(2\,\mathrm{Chi}(2\,\mathrm{arcsinh}(bx+a))\,\mathrm{arcsinh}(bx+a)-}{\mathrm{arcsinh}(bx+a)}}{b^3}$
default	$\frac{\frac{\sqrt{1+(bx+a)^2}}{4\,\mathrm{arcsinh}(bx+a)} - \frac{\mathrm{Shi}(\mathrm{arcsinh}(bx+a))}{4} - \frac{\cosh(3\,\mathrm{arcsinh}(bx+a))}{4\,\mathrm{arcsinh}(bx+a)} + \frac{3\,\mathrm{Shi}(3\,\mathrm{arcsinh}(bx+a))}{4} - \frac{a(2\,\mathrm{Chi}(2\,\mathrm{arcsinh}(bx+a))\,\mathrm{arcsinh}(bx+a)-}{\mathrm{arcsinh}(bx+a)}}{b^3}$

input
```
int(x^2/arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/4/arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)-1/4*Shi(arcsinh(b*x+a))-1/4/
arcsinh(b*x+a)*cosh(3*arcsinh(b*x+a))+3/4*Shi(3*arcsinh(b*x+a))-a*(2*Chi(2
*arcsinh(b*x+a))*arcsinh(b*x+a)-sinh(2*arcsinh(b*x+a)))/arcsinh(b*x+a)+a^2
*(Shi(arcsinh(b*x+a))*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a))
```

3.85.5 Fricas [F]

$$\int \frac{x^2}{\mathrm{arcsinh}(a+bx)^2}\,dx = \int \frac{x^2}{\mathrm{arsinh}\,(bx+a)^2}\,dx$$

input
```
integrate(x^2/arcsinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^2/arcsinh(b*x + a)^2, x)
```

3.85.6 Sympy [F]

$$\int \frac{x^2}{\mathrm{arcsinh}(a+bx)^2}\,dx = \int \frac{x^2}{\mathrm{asinh}^2\,(a+bx)}\,dx$$

input
```
integrate(x**2/asinh(b*x+a)**2,x)
```

output
```
Integral(x**2/asinh(a + b*x)**2, x)
```

3.85.7 Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

input
```
integrate(x^2/arcsinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b + b)*x^3 + (a^3 + a)*x^2 + (b^2*x^4 + 2
*a*b*x^3 + (a^2 + 1)*x^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2
*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*lo
g(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate((3*b^5*x^6 + 1
4*a*b^4*x^5 + 2*(13*a^2*b^3 + 3*b^3)*x^4 + 8*(3*a^3*b^2 + 2*a*b^2)*x^3 + (
11*a^4*b + 14*a^2*b + 3*b)*x^2 + (3*b^3*x^4 + 8*a*b^2*x^3 + (7*a^2*b + b)*
x^2 + 2*(a^3 + a)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*(a^5 + 2*a^3 + a)*x
 + (6*b^4*x^5 + 22*a*b^3*x^4 + (30*a^2*b^2 + 7*b^2)*x^3 + (18*a^3*b + 13*a
*b)*x^2 + 2*(2*a^4 + 3*a^2 + 1)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^
5*x^4 + 4*a*b^4*x^3 + a^4*b + 2*a^2*b + 2*(3*a^2*b^3 + b^3)*x^2 + (b^3*x^2
 + 2*a*b^2*x + a^2*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*(a^3*b^2 + a*b^2)*
x + 2*(b^4*x^3 + 3*a*b^3*x^2 + a^3*b + a*b + (3*a^2*b^2 + b^2)*x)*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))), x)
```

3.85.8 Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

input
```
integrate(x^2/arcsinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2/arcsinh(b*x + a)^2, x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x^2}{\operatorname{asinh}\left(a+b\,x\right)^2}\,dx$$

input
```
int(x^2/asinh(a + b*x)^2,x)
```

output
```
int(x^2/asinh(a + b*x)^2, x)
```

3.86 $\int \frac{x}{\text{arcsinh}(a+bx)^2} dx$

3.86.1 Optimal result

Integrand size = 10, antiderivative size = 84

$$\int \frac{x}{\text{arcsinh}(a+bx)^2} dx = \frac{a\sqrt{1+(a+bx)^2}}{b^2\text{arcsinh}(a+bx)} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b^2\text{arcsinh}(a+bx)} + \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{b^2} - \frac{a\text{Shi}(\text{arcsinh}(a+bx))}{b^2}$$

output
```
Chi(2*arcsinh(b*x+a))/b^2-a*Shi(arcsinh(b*x+a))/b^2+a*(1+(b*x+a)^2)^(1/2)/
b^2/arcsinh(b*x+a)-(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^2/arcsinh(b*x+a)
```

3.86.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{x}{\text{arcsinh}(a+bx)^2} dx = -\frac{bx\sqrt{1+(a+bx)^2} - \text{arcsinh}(a+bx)\text{Chi}(2\text{arcsinh}(a+bx)) + a\text{arcsinh}(a+bx)\text{Shi}(\text{arcsinh}(a+bx))}{b^2\text{arcsinh}(a+bx)}$$

input
```
Integrate[x/ArcSinh[a + b*x]^2,x]
```

output
```
-((b*x*Sqrt[1 + (a + b*x)^2] - ArcSinh[a + b*x]*CoshIntegral[2*ArcSinh[a +
 b*x]] + a*ArcSinh[a + b*x]*SinhIntegral[ArcSinh[a + b*x]])/(b^2*ArcSinh[a
 + b*x]))
```

3.86.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6274, 25, 27, 6244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\text{arcsinh}(a+bx)^2} \, dx$$

$$\Big\downarrow \ 6274$$

$$\frac{\int \frac{x}{\text{arcsinh}(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -\frac{x}{\text{arcsinh}(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int -\frac{bx}{\text{arcsinh}(a+bx)^2}d(a+bx)}{b^2}$$

$$\Big\downarrow \ 6244$$

$$-\frac{\int \left(\frac{a}{\text{arcsinh}(a+bx)^2} - \frac{a+bx}{\text{arcsinh}(a+bx)^2}\right)d(a+bx)}{b^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\text{Chi}(2\text{arcsinh}(a+bx)) + a\text{Shi}(\text{arcsinh}(a+bx)) - \frac{a\sqrt{(a+bx)^2+1}}{\text{arcsinh}(a+bx)} + \frac{(a+bx)\sqrt{(a+bx)^2+1}}{\text{arcsinh}(a+bx)}}{b^2}$$

input
```
Int[x/ArcSinh[a + b*x]^2,x]
```

output
```
-((-((a*Sqrt[1 + (a + b*x)^2])/ArcSinh[a + b*x]) + ((a + b*x)*Sqrt[1 + (a
+ b*x)^2])/ArcSinh[a + b*x] - CoshIntegral[2*ArcSinh[a + b*x]] + a*SinhInt
egral[ArcSinh[a + b*x]])/b^2)
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6244
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.86.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{-\frac{\sinh(2\operatorname{arcsinh}(bx+a))}{2\operatorname{arcsinh}(bx+a)}+\operatorname{Chi}(2\operatorname{arcsinh}(bx+a))-\frac{a\left(\operatorname{Shi}(\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)-\sqrt{1+(bx+a)^2}\right)}{\operatorname{arcsinh}(bx+a)}}{b^2}$	73
default	$\frac{-\frac{\sinh(2\operatorname{arcsinh}(bx+a))}{2\operatorname{arcsinh}(bx+a)}+\operatorname{Chi}(2\operatorname{arcsinh}(bx+a))-\frac{a\left(\operatorname{Shi}(\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)-\sqrt{1+(bx+a)^2}\right)}{\operatorname{arcsinh}(bx+a)}}{b^2}$	73

input
```
int(x/arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(-1/2/arcsinh(b*x+a)*sinh(2*arcsinh(b*x+a))+Chi(2*arcsinh(b*x+a))-a*
(Shi(arcsinh(b*x+a))*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a))
```

3.86.5 Fricas [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

input
```
integrate(x/arcsinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x/arcsinh(b*x + a)^2, x)
```

3.86.6 Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x}{\operatorname{asinh}^2\left(a+bx\right)}\,dx$$

input
```
integrate(x/asinh(b*x+a)**2,x)
```

output
```
Integral(x/asinh(a + b*x)**2, x)
```

3.86.7 Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

input
```
integrate(x/arcsinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(b^3*x^4 + 3*a*b^2*x^3 + (3*a^2*b + b)*x^2 + (a^3 + a)*x + (b^2*x^3 + 2*a
*b*x^2 + (a^2 + 1)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a*b
^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b*
x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate((2*b^5*x^5 + 9*a*b
^4*x^4 + a^5 + 4*(4*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(7*a^3*b^2 + 5*a*b^2)*x
^2 + (2*b^3*x^3 + 5*a*b^2*x^2 + 4*a^2*b*x + a^3 + a)*(b^2*x^2 + 2*a*b*x +
a^2 + 1) + 2*(3*a^4*b + 4*a^2*b + b)*x + (4*b^4*x^4 + 14*a*b^3*x^3 + 2*a^4
 + 2*(9*a^2*b^2 + 2*b^2)*x^2 + 3*a^2 + (10*a^3*b + 7*a*b)*x + 1)*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1) + a)/((b^5*x^4 + 4*a*b^4*x^3 + a^4*b + 2*a^2*b +
2*(3*a^2*b^3 + b^3)*x^2 + (b^3*x^2 + 2*a*b^2*x + a^2*b)*(b^2*x^2 + 2*a*b*x
 + a^2 + 1) + 4*(a^3*b^2 + a*b^2)*x + 2*(b^4*x^3 + 3*a*b^3*x^2 + a^3*b + a
*b + (3*a^2*b^2 + b^2)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x +
 a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

3.86.8 Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

input
```
integrate(x/arcsinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x/arcsinh(b*x + a)^2, x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{x}{\operatorname{asinh}\left(a+b\,x\right)^2}\,dx$$

input
```
int(x/asinh(a + b*x)^2,x)
```

output
```
int(x/asinh(a + b*x)^2, x)
```

3.87 $\int \frac{1}{\operatorname{arcsinh}(a+bx)^2}\,dx$

3.87.1 Optimal result

Integrand size = 8, antiderivative size = 38

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^2}\,dx = -\frac{\sqrt{1+(a+bx)^2}}{b\operatorname{arcsinh}(a+bx)} + \frac{\operatorname{Shi}(\operatorname{arcsinh}(a+bx))}{b}$$

output
```
Shi(arcsinh(b*x+a))/b-(1+(b*x+a)^2)^(1/2)/b/arcsinh(b*x+a)
```

3.87.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^2}\,dx = \frac{-\frac{\sqrt{1+(a+bx)^2}}{\operatorname{arcsinh}(a+bx)} + \operatorname{Shi}(\operatorname{arcsinh}(a+bx))}{b}$$

input
```
Integrate[ArcSinh[a + b*x]^(-2),x]
```

output
```
(-(Sqrt[1 + (a + b*x)^2]/ArcSinh[a + b*x]) + SinhIntegral[ArcSinh[a + b*x]
])/b
```

3.87.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6273, 6188, 6234, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^2} \, dx$$

$$\Big\downarrow \; 6273$$

$$\frac{\int \frac{1}{\operatorname{arcsinh}(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \; 6188$$

$$\frac{\int \frac{a+bx}{\sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)}d(a+bx)-\frac{\sqrt{(a+bx)^2+1}}{\operatorname{arcsinh}(a+bx)}}{b}$$

$$\Big\downarrow \; 6234$$

$$\frac{\int \frac{a+bx}{\operatorname{arcsinh}(a+bx)}d\operatorname{arcsinh}(a+bx)-\frac{\sqrt{(a+bx)^2+1}}{\operatorname{arcsinh}(a+bx)}}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\sqrt{(a+bx)^2+1}}{\operatorname{arcsinh}(a+bx)}+\int -\frac{i\sin(i\operatorname{arcsinh}(a+bx))}{\operatorname{arcsinh}(a+bx)}d\operatorname{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{\sqrt{(a+bx)^2+1}}{\operatorname{arcsinh}(a+bx)}-i\int \frac{\sin(i\operatorname{arcsinh}(a+bx))}{\operatorname{arcsinh}(a+bx)}d\operatorname{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \; 3779$$

$$\frac{\operatorname{Shi}(\operatorname{arcsinh}(a+bx))-\frac{\sqrt{(a+bx)^2+1}}{\operatorname{arcsinh}(a+bx)}}{b}$$

input
```
Int[ArcSinh[a + b*x]^(-2),x]
```

output
```
(-(Sqrt[1 + (a + b*x)^2]/ArcSinh[a + b*x]) + SinhIntegral[ArcSinh[a + b*x]
])/b
```

3.87.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.87.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{\sqrt{1+(bx+a)^2}}{\operatorname{arcsinh}(bx+a)}+\operatorname{Shi}(\operatorname{arcsinh}(bx+a))}{b}$	34
default	$\frac{-\frac{\sqrt{1+(bx+a)^2}}{\operatorname{arcsinh}(bx+a)}+\operatorname{Shi}(\operatorname{arcsinh}(bx+a))}{b}$	34

input
```
int(1/arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+Shi(arcsinh(b*x+a)))
```

3.87.5 Fricas [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

input
```
integrate(1/arcsinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(arcsinh(b*x + a)^(-2), x)
```

3.87.6 Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{\operatorname{asinh}^2\left(a+bx\right)}\,dx$$

input
```
integrate(1/asinh(b*x+a)**2,x)
```

output
```
Integral(asinh(a + b*x)**(-2), x)
```

3.87.7 Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

input
```
integrate(1/arcsinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2
 + 1)^(3/2) + a)/((b^3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*(b^2*x + a*b) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
))) + integrate((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + (
b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^2 - 1) + 2*a^2 + 4*(a^
3*b + a*b)*x + (2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2*b + b)*x + a)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)/((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a
^2*b^2 + b^2)*x^2 + (b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^2
) + 2*a^2 + 4*(a^3*b + a*b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b
+ b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)*log(b*x + a + sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1))), x)
```

3.87.8 Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

input
```
integrate(1/arcsinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(arcsinh(b*x + a)^(-2), x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{\operatorname{asinh}\left(a+b\,x\right)^2}\,dx$$

input
```
int(1/asinh(a + b*x)^2,x)
```

output
```
int(1/asinh(a + b*x)^2, x)
```

3.88 $\int \frac{1}{x\textbf{arcsinh}(a+bx)^2}\,dx$

3.88.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\text{arcsinh}(a+bx)^2}\,dx = \text{Int}\left(\frac{1}{x\text{arcsinh}(a+bx)^2},x\right)$$

output
```
Unintegrable(1/x/arcsinh(b*x+a)^2,x)
```

3.88.2 Mathematica [N/A]

Not integrable

Time = 2.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{x\text{arcsinh}(a+bx)^2}\,dx$$

input
```
Integrate[1/(x*ArcSinh[a + b*x]^2),x]
```

output
```
Integrate[1/(x*ArcSinh[a + b*x]^2), x]
```

3.88.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 25, 27, 6272}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{arcsinh}(a+bx)^2} \, dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \frac{1}{x\text{arcsinh}(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{1}{x\text{arcsinh}(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\int -\frac{1}{bx\text{arcsinh}(a+bx)^2}d(a+bx)$$

$$\Big\downarrow \; 6272$$

$$-\int -\frac{1}{bx\text{arcsinh}(a+bx)^2}d(a+bx)$$

input
```
Int[1/(x*ArcSinh[a + b*x]^2),x]
```

output
```
$Aborted
```

3.88.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6272
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.88.4 Maple [N/A] (verified)

Not integrable

Time = 0.85 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \operatorname{arcsinh}\left(bx+a\right)^{2}}dx$$

input
```
int(1/x/arcsinh(b*x+a)^2,x)
```

output
```
int(1/x/arcsinh(b*x+a)^2,x)
```

3.88.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)^2}\,dx=\int \frac{1}{x \operatorname{arsinh}\left(bx+a\right)^{2}}\,dx$$

input
```
integrate(1/x/arcsinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(1/(x*arcsinh(b*x + a)^2), x)
```

3.88.6 Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{x\,\text{asinh}^2\,(a+bx)}\,dx$$

input
```
integrate(1/x/asinh(b*x+a)**2,x)
```

output
```
Integral(1/(x*asinh(a + b*x)**2), x)
```

3.88.7 Maxima [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 527, normalized size of antiderivative = 43.92

$$\int \frac{1}{x\text{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{x\,\text{arsinh}\,(bx+a)^2}\,dx$$

input
```
integrate(1/x/arcsinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2
 + 1)^(3/2) + a)/((b^3*x^3 + 2*a*b^2*x^2 + (a^2*b + b)*x + sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)*(b^2*x^2 + a*b*x))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1))) - integrate((a*b^4*x^4 + 4*a^2*b^3*x^3 + a^5 + 2*a^3 + 2*(3*
a^3*b^2 + a*b^2)*x^2 + (a*b^2*x^2 + a^3 + 2*(a^2*b + b)*x + a)*(b^2*x^2 +
2*a*b*x + a^2 + 1) + 4*(a^4*b + a^2*b)*x + (2*a*b^3*x^3 + 2*a^4 + 2*(3*a^2
*b^2 + b^2)*x^2 + 3*a^2 + (6*a^3*b + 5*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1) + a)/((b^5*x^6 + 4*a*b^4*x^5 + 2*(3*a^2*b^3 + b^3)*x^4 + 4*(a^3
*b^2 + a*b^2)*x^3 + (a^4*b + 2*a^2*b + b)*x^2 + (b^3*x^4 + 2*a*b^2*x^3 + a
^2*b*x^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*(b^4*x^5 + 3*a*b^3*x^4 + (3*a^
2*b^2 + b^2)*x^3 + (a^3*b + a*b)*x^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*l
og(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

3.88.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{x\,\text{arsinh}\left(bx+a\right)^2}\,dx$$

input
```
integrate(1/x/arcsinh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(1/(x*arcsinh(b*x + a)^2), x)
```

3.88.9 Mupad [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{x\,\text{asinh}\left(a+b\,x\right)^2}\,dx$$

input
```
int(1/(x*asinh(a + b*x)^2),x)
```

output
```
int(1/(x*asinh(a + b*x)^2), x)
```

3.89 $\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^3} dx$

3.89.1 Optimal result

Integrand size = 12, antiderivative size = 257

$$\begin{aligned}\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^3}\,dx = &-\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\operatorname{arcsinh}(a+bx)^2}+\frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\operatorname{arcsinh}(a+bx)^2}\\&-\frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\operatorname{arcsinh}(a+bx)^2}+\frac{a}{b^3\operatorname{arcsinh}(a+bx)}\\&-\frac{a+bx}{b^3\operatorname{arcsinh}(a+bx)}-\frac{a^2(a+bx)}{2b^3\operatorname{arcsinh}(a+bx)}\\&+\frac{2a(a+bx)^2}{b^3\operatorname{arcsinh}(a+bx)}-\frac{3(a+bx)^3}{2b^3\operatorname{arcsinh}(a+bx)}\\&-\frac{\operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{8b^3}+\frac{a^2\operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{2b^3}\\&+\frac{9\operatorname{Chi}(3\operatorname{arcsinh}(a+bx))}{8b^3}-\frac{2a\operatorname{Shi}(2\operatorname{arcsinh}(a+bx))}{b^3}\end{aligned}$$

output
```
a/b^3/arcsinh(b*x+a)+(-b*x-a)/b^3/arcsinh(b*x+a)-1/2*a^2*(b*x+a)/b^3/arcsi
nh(b*x+a)+2*a*(b*x+a)^2/b^3/arcsinh(b*x+a)-3/2*(b*x+a)^3/b^3/arcsinh(b*x+a
)-1/8*Chi(arcsinh(b*x+a))/b^3+1/2*a^2*Chi(arcsinh(b*x+a))/b^3+9/8*Chi(3*ar
csinh(b*x+a))/b^3-2*a*Shi(2*arcsinh(b*x+a))/b^3-1/2*a^2*(1+(b*x+a)^2)^(1/2
)/b^3/arcsinh(b*x+a)^2+a*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3/arcsinh(b*x+a)^2-
1/2*(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^3/arcsinh(b*x+a)^2
```

3.89.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.43

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^3}\,dx = \frac{-\frac{4bx\left(bx\sqrt{1+a^2+2abx+b^2x^2}+(2+2a^2+5abx+3b^2x^2)\operatorname{arcsinh}(a+bx)\right)}{\operatorname{arcsinh}(a+bx)^2}+(-1+4a^2)\operatorname{Chi}(\operatorname{arcsinh}(a+bx))+9\operatorname{Chi}(3\operatorname{arcsin}}{8b^3}$$

input
```
Integrate[x^2/ArcSinh[a + b*x]^3,x]
```

output
```
((-4*b*x*(b*x*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (2 + 2*a^2 + 5*a*b*x + 3
*b^2*x^2)*ArcSinh[a + b*x]))/ArcSinh[a + b*x]^2 + (-1 + 4*a^2)*CoshIntegra
l[ArcSinh[a + b*x]] + 9*CoshIntegral[3*ArcSinh[a + b*x]] - 16*a*SinhIntegr
al[2*ArcSinh[a + b*x]])/(8*b^3)
```

3.89.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6274, 27, 6244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^3}\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^3}d(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{b^2x^2}{\operatorname{arcsinh}(a+bx)^3}d(a+bx)}{b^3}$$

$$\Big\downarrow \; 6244$$

$$\frac{\int \left(\frac{a^2}{\operatorname{arcsinh}(a+bx)^3}-\frac{2(a+bx)a}{\operatorname{arcsinh}(a+bx)^3}+\frac{(a+bx)^2}{\operatorname{arcsinh}(a+bx)^3}\right)d(a+bx)}{b^3}$$

$\downarrow$ 2009

$$\frac{1}{2}a^2\text{Chi}(\text{arcsinh}(a+bx)) - \frac{a^2(a+bx)}{2\text{arcsinh}(a+bx)} - \frac{a^2\sqrt{(a+bx)^2+1}}{2\text{arcsinh}(a+bx)^2} - \frac{1}{8}\text{Chi}(\text{arcsinh}(a+bx)) + \frac{9}{8}\text{Chi}(3\text{arcsinh}(a+bx))$$

input
```
Int[x^2/ArcSinh[a + b*x]^3,x]
```

output
```
(-1/2*(a^2*Sqrt[1 + (a + b*x)^2])/ArcSinh[a + b*x]^2 + (a*(a + b*x)*Sqrt[1
 + (a + b*x)^2])/ArcSinh[a + b*x]^2 - ((a + b*x)^2*Sqrt[1 + (a + b*x)^2])/
(2*ArcSinh[a + b*x]^2) + a/ArcSinh[a + b*x] - (a + b*x)/ArcSinh[a + b*x] -
 (a^2*(a + b*x))/(2*ArcSinh[a + b*x]) + (2*a*(a + b*x)^2)/ArcSinh[a + b*x]
 - (3*(a + b*x)^3)/(2*ArcSinh[a + b*x]) - CoshIntegral[ArcSinh[a + b*x]]/8
 + (a^2*CoshIntegral[ArcSinh[a + b*x]])/2 + (9*CoshIntegral[3*ArcSinh[a +
b*x]])/8 - 2*a*SinhIntegral[2*ArcSinh[a + b*x]])/b^3
```

3.89.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6244
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.89.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\sqrt{1+(bx+a)^2}}{8\operatorname{arcsinh}(bx+a)^2}+\frac{bx+a}{8\operatorname{arcsinh}(bx+a)}-\frac{\mathrm{Chi}(\operatorname{arcsinh}(bx+a))}{8}-\frac{\cosh(3\operatorname{arcsinh}(bx+a))}{8\operatorname{arcsinh}(bx+a)^2}-\frac{3\sinh(3\operatorname{arcsinh}(bx+a))}{8\operatorname{arcsinh}(bx+a)}+\frac{9\,\mathrm{Chi}(3\operatorname{arcsinh}(bx+$
default	$\frac{\sqrt{1+(bx+a)^2}}{8\operatorname{arcsinh}(bx+a)^2}+\frac{bx+a}{8\operatorname{arcsinh}(bx+a)}-\frac{\mathrm{Chi}(\operatorname{arcsinh}(bx+a))}{8}-\frac{\cosh(3\operatorname{arcsinh}(bx+a))}{8\operatorname{arcsinh}(bx+a)^2}-\frac{3\sinh(3\operatorname{arcsinh}(bx+a))}{8\operatorname{arcsinh}(bx+a)}+\frac{9\,\mathrm{Chi}(3\operatorname{arcsinh}(bx+$

input
```
int(x^2/arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/8/arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+1/8/arcsinh(b*x+a)*(b*x+a)
-1/8*Chi(arcsinh(b*x+a))-1/8/arcsinh(b*x+a)^2*cosh(3*arcsinh(b*x+a))-3/8/a
rcsinh(b*x+a)*sinh(3*arcsinh(b*x+a))+9/8*Chi(3*arcsinh(b*x+a))-1/2*a*(4*Sh
i(2*arcsinh(b*x+a))*arcsinh(b*x+a)^2-2*cosh(2*arcsinh(b*x+a))*arcsinh(b*x+
a)-sinh(2*arcsinh(b*x+a)))/arcsinh(b*x+a)^2+1/2*a^2*(Chi(arcsinh(b*x+a))*a
rcsinh(b*x+a)^2-(b*x+a)*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a)
^2)
```

3.89.5 Fricas [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^3}\,dx=\int \frac{x^2}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

input
```
integrate(x^2/arcsinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(x^2/arcsinh(b*x + a)^3, x)
```

3.89.6 Sympy [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{x^2}{\operatorname{asinh}^3(a+bx)}\,dx$$

input
```
integrate(x**2/asinh(b*x+a)**3,x)
```

output
```
Integral(x**2/asinh(a + b*x)**3, x)
```

3.89.7 Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{x^2}{\operatorname{arsinh}(bx+a)^3}\,dx$$

input
```
integrate(x^2/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(b^8*x^9 + 7*a*b^7*x^8 + 3*(7*a^2*b^6 + b^6)*x^7 + 5*(7*a^3*b^5 + 3*a
*b^5)*x^6 + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^5 + 3*(7*a^5*b^3 + 10*a^3*
b^3 + 3*a*b^3)*x^4 + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^3 + (a^7
*b + 3*a^5*b + 3*a^3*b + a*b)*x^2 + (b^5*x^6 + 4*a*b^4*x^5 + (6*a^2*b^3 +
b^3)*x^4 + 2*(2*a^3*b^2 + a*b^2)*x^3 + (a^4*b + a^2*b)*x^2)*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2) + (3*b^6*x^7 + 15*a*b^5*x^6 + 5*(6*a^2*b^4 + b^4)*x^
5 + 15*(2*a^3*b^3 + a*b^3)*x^4 + (15*a^4*b^2 + 15*a^2*b^2 + 2*b^2)*x^3 + (
3*a^5*b + 5*a^3*b + 2*a*b)*x^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (3*b^8*x^9
 + 23*a*b^7*x^8 + (77*a^2*b^6 + 9*b^6)*x^7 + 3*(49*a^3*b^5 + 17*a*b^5)*x^6
 + (175*a^4*b^4 + 120*a^2*b^4 + 9*b^4)*x^5 + (133*a^5*b^3 + 150*a^3*b^3 +
33*a*b^3)*x^4 + 3*(21*a^6*b^2 + 35*a^4*b^2 + 15*a^2*b^2 + b^2)*x^3 + (17*a
^7*b + 39*a^5*b + 27*a^3*b + 5*a*b)*x^2 + (3*b^5*x^6 + 14*a*b^4*x^5 + 2*(1
3*a^2*b^3 + 2*b^3)*x^4 + 12*(2*a^3*b^2 + a*b^2)*x^3 + (11*a^4*b + 12*a^2*b
 + b)*x^2 + 2*(a^5 + 2*a^3 + a)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (
9*b^6*x^7 + 51*a*b^5*x^6 + (120*a^2*b^4 + 17*b^4)*x^5 + 5*(30*a^3*b^3 + 13
*a*b^3)*x^4 + (105*a^4*b^2 + 93*a^2*b^2 + 10*b^2)*x^3 + (39*a^5*b + 59*a^3
*b + 20*a*b)*x^2 + 2*(3*a^6 + 7*a^4 + 5*a^2 + 1)*x)*(b^2*x^2 + 2*a*b*x + a
^2 + 1) + 2*(a^8 + 3*a^6 + 3*a^4 + a^2)*x + (9*b^7*x^8 + 60*a*b^6*x^7 + (1
71*a^2*b^5 + 22*b^5)*x^6 + 2*(135*a^3*b^4 + 52*a*b^4)*x^5 + (255*a^4*b^3 +
 196*a^2*b^3 + 18*b^3)*x^4 + 2*(72*a^5*b^2 + 92*a^3*b^2 + 25*a*b^2)*x^3...
```

3.89.8 Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

input
```
integrate(x^2/arcsinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^2/arcsinh(b*x + a)^3, x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{x^2}{\operatorname{asinh}\left(a+b\,x\right)^3}\,dx$$

input
```
int(x^2/asinh(a + b*x)^3,x)
```

output
```
int(x^2/asinh(a + b*x)^3, x)
```

3.90 $\int \frac{x}{\mathbf{arcsinh}(a+bx)^3} \, dx$

3.90.1 Optimal result

Integrand size = 10, antiderivative size = 147

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^3} \, dx = \frac{a\sqrt{1+(a+bx)^2}}{2b^2\operatorname{arcsinh}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\operatorname{arcsinh}(a+bx)^2} - \frac{1}{2b^2\operatorname{arcsinh}(a+bx)} + \frac{a(a+bx)}{2b^2\operatorname{arcsinh}(a+bx)} - \frac{(a+bx)^2}{b^2\operatorname{arcsinh}(a+bx)} - \frac{a\operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{2b^2} + \frac{\operatorname{Shi}(2\operatorname{arcsinh}(a+bx))}{b^2}$$

output
```
-1/2/b^2/arcsinh(b*x+a)+1/2*a*(b*x+a)/b^2/arcsinh(b*x+a)-(b*x+a)^2/b^2/arc
sinh(b*x+a)-1/2*a*Chi(arcsinh(b*x+a))/b^2+Shi(2*arcsinh(b*x+a))/b^2+1/2*a*
(1+(b*x+a)^2)^(1/2)/b^2/arcsinh(b*x+a)^2-1/2*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b
^2/arcsinh(b*x+a)^2
```

3.90.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.80

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^3} \, dx = -\frac{bx\sqrt{1+a^2+2abx+b^2x^2} + \operatorname{arcsinh}(a+bx) + a^2\operatorname{arcsinh}(a+bx) + 3abx\operatorname{arcsinh}(a+bx) + 2b^2x^2\operatorname{arcsin}}{2b^2\operatorname{arcsinh}(a}$$

input
```
Integrate[x/ArcSinh[a + b*x]^3,x]
```

output
```
-1/2*(b*x*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + ArcSinh[a + b*x] + a^2*ArcSi
nh[a + b*x] + 3*a*b*x*ArcSinh[a + b*x] + 2*b^2*x^2*ArcSinh[a + b*x] + a*Ar
cSinh[a + b*x]^2*CoshIntegral[ArcSinh[a + b*x]] - 2*ArcSinh[a + b*x]^2*Sin
hIntegral[2*ArcSinh[a + b*x]])/(b^2*ArcSinh[a + b*x]^2)
```

3.90.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6274, 25, 27, 6244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^3}\,dx$$

$$\Big\downarrow \ 6274$$

$$\frac{\int \frac{x}{\operatorname{arcsinh}(a+bx)^3}d(a+bx)}{b}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -\frac{x}{\operatorname{arcsinh}(a+bx)^3}d(a+bx)}{b}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int -\frac{bx}{\operatorname{arcsinh}(a+bx)^3}d(a+bx)}{b^2}$$

$$\Big\downarrow \ 6244$$

$$-\frac{\int \left(\frac{a}{\operatorname{arcsinh}(a+bx)^3}-\frac{a+bx}{\operatorname{arcsinh}(a+bx)^3}\right)d(a+bx)}{b^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\frac{1}{2}a\text{Chi}(\operatorname{arcsinh}(a+bx))-\text{Shi}(2\operatorname{arcsinh}(a+bx))+\frac{(a+bx)^2}{\operatorname{arcsinh}(a+bx)}-\frac{a(a+bx)}{2\operatorname{arcsinh}(a+bx)}+\frac{\sqrt{(a+bx)^2+1}(a+bx)}{2\operatorname{arcsinh}(a+bx)^2}+\frac{}{2\text{arcsin}}}{b^2}$$

input
```
Int[x/ArcSinh[a + b*x]^3,x]
```

output
```
-((-1/2*(a*Sqrt[1 + (a + b*x)^2])/ArcSinh[a + b*x]^2 + ((a + b*x)*Sqrt[1 +
 (a + b*x)^2])/(2*ArcSinh[a + b*x]^2) + 1/(2*ArcSinh[a + b*x]) - (a*(a + b
*x))/(2*ArcSinh[a + b*x]) + (a + b*x)^2/ArcSinh[a + b*x] + (a*CoshIntegral
[ArcSinh[a + b*x]])/2 - SinhIntegral[2*ArcSinh[a + b*x]])/b^2)
```

3.90.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6244
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.90.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{-\frac{\sinh(2\ \operatorname{arcsinh}(bx+a))}{4 \operatorname{arcsinh}(bx+a)^2}-\frac{\cosh(2\ \operatorname{arcsinh}(bx+a))}{2\ \operatorname{arcsinh}(bx+a)}+\operatorname{Shi}(2\ \operatorname{arcsinh}(bx+a))-\frac{a\left(\operatorname{Chi}(\operatorname{arcsinh}(bx+a)) \operatorname{arcsinh}(bx+a)^2-(bx+a) \operatorname{arcsinh}\right.}{2 \operatorname{arcsinh}(bx+a)^2}}{b^2}$
default	$\frac{-\frac{\sinh(2\ \operatorname{arcsinh}(bx+a))}{4 \operatorname{arcsinh}(bx+a)^2}-\frac{\cosh(2\ \operatorname{arcsinh}(bx+a))}{2\ \operatorname{arcsinh}(bx+a)}+\operatorname{Shi}(2\ \operatorname{arcsinh}(bx+a))-\frac{a\left(\operatorname{Chi}(\operatorname{arcsinh}(bx+a)) \operatorname{arcsinh}(bx+a)^2-(bx+a) \operatorname{arcsinh}\right.}{2 \operatorname{arcsinh}(bx+a)^2}}{b^2}$

input
```
int(x/arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(-1/4/arcsinh(b*x+a)^2*sinh(2*arcsinh(b*x+a))-1/2/arcsinh(b*x+a)*cos
h(2*arcsinh(b*x+a))+Shi(2*arcsinh(b*x+a))-1/2*a*(Chi(arcsinh(b*x+a))*arcsi
nh(b*x+a)^2-(b*x+a)*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a)^2)
```

3.90.5 Fricas [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^3} \, dx = \int \frac{x}{\operatorname{arsinh}\left(bx+a\right)^3} \, dx$$

input
```
integrate(x/arcsinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(x/arcsinh(b*x + a)^3, x)
```

3.90.6 Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^3} \, dx = \int \frac{x}{\operatorname{asinh}^3\left(a+bx\right)} \, dx$$

input
```
integrate(x/asinh(b*x+a)**3,x)
```

output
```
Integral(x/asinh(a + b*x)**3, x)
```

3.90.7 Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^3} \, dx = \int \frac{x}{\operatorname{arsinh}\left(bx+a\right)^3} \, dx$$

input
```
integrate(x/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(b^8*x^8 + 7*a*b^7*x^7 + 3*(7*a^2*b^6 + b^6)*x^6 + 5*(7*a^3*b^5 + 3*a
*b^5)*x^5 + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 3*(7*a^5*b^3 + 10*a^3*
b^3 + 3*a*b^3)*x^3 + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + (b^5
*x^5 + 4*a*b^4*x^4 + (6*a^2*b^3 + b^3)*x^3 + 2*(2*a^3*b^2 + a*b^2)*x^2 + (
a^4*b + a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b^6*x^6 + 15*a*
b^5*x^5 + 5*(6*a^2*b^4 + b^4)*x^4 + 15*(2*a^3*b^3 + a*b^3)*x^3 + (15*a^4*b
^2 + 15*a^2*b^2 + 2*b^2)*x^2 + (3*a^5*b + 5*a^3*b + 2*a*b)*x)*(b^2*x^2 + 2
*a*b*x + a^2 + 1) + (a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + (2*b^8*x^8 + 15*
a*b^7*x^7 + a^8 + (49*a^2*b^6 + 6*b^6)*x^6 + 3*a^6 + (91*a^3*b^5 + 33*a*b^
5)*x^5 + 3*(35*a^4*b^4 + 25*a^2*b^4 + 2*b^4)*x^4 + 3*a^4 + (77*a^5*b^3 + 9
0*a^3*b^3 + 21*a*b^3)*x^3 + (35*a^6*b^2 + 60*a^4*b^2 + 27*a^2*b^2 + 2*b^2)
*x^2 + (2*b^5*x^5 + 9*a*b^4*x^4 + a^5 + 2*(8*a^2*b^3 + b^3)*x^3 + 2*a^3 +
2*(7*a^3*b^2 + 3*a*b^2)*x^2 + 6*(a^4*b + a^2*b)*x + a)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1)^(3/2) + (6*b^6*x^6 + 33*a*b^5*x^5 + 3*a^6 + 5*(15*a^2*b^4 + 2*b
^4)*x^4 + 7*a^4 + (90*a^3*b^3 + 37*a*b^3)*x^3 + (60*a^4*b^2 + 51*a^2*b^2 +
 5*b^2)*x^2 + 5*a^2 + (21*a^5*b + 31*a^3*b + 10*a*b)*x + 1)*(b^2*x^2 + 2*a
*b*x + a^2 + 1) + a^2 + 3*(3*a^7*b + 7*a^5*b + 5*a^3*b + a*b)*x + (6*b^7*x
^7 + 39*a*b^6*x^6 + 3*a^7 + 2*(54*a^2*b^5 + 7*b^5)*x^5 + 8*a^5 + (165*a^3*
b^4 + 64*a*b^4)*x^4 + (150*a^4*b^3 + 116*a^2*b^3 + 11*b^3)*x^3 + 7*a^3 + (
81*a^5*b^2 + 104*a^3*b^2 + 29*a*b^2)*x^2 + (24*a^6*b + 46*a^4*b + 25*a^...
```

3.90.8 Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{x}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

input
```
integrate(x/arcsinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x/arcsinh(b*x + a)^3, x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(a+bx)^3}\,dx=\int \frac{x}{{\mathrm{asinh}\left(a+b\,x\right)}^3} \,d x$$

input
```
int(x/asinh(a + b*x)^3,x)
```

output
```
int(x/asinh(a + b*x)^3, x)
```

3.91 $\int \frac{1}{\operatorname{arcsinh}(a+bx)^3}\,dx$

3.91.1 Optimal result

Integrand size = 8, antiderivative size = 63

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^3}\,dx=-\frac{\sqrt{1+(a+bx)^2}}{2b\operatorname{arcsinh}(a+bx)^2}-\frac{a+bx}{2b\operatorname{arcsinh}(a+bx)}+\frac{\operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{2b}$$

output
```
1/2*(-b*x-a)/b/arcsinh(b*x+a)+1/2*Chi(arcsinh(b*x+a))/b-1/2*(1+(b*x+a)^2)^
(1/2)/b/arcsinh(b*x+a)^2
```

3.91.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^3}\,dx=\frac{-\frac{\sqrt{1+(a+bx)^2}}{\operatorname{arcsinh}(a+bx)^2}-\frac{a+bx}{\operatorname{arcsinh}(a+bx)}+\operatorname{Chi}(\operatorname{arcsinh}(a+bx))}{2b}$$

input
```
Integrate[ArcSinh[a + b*x]^(-3),x]
```

output
```
(-(Sqrt[1 + (a + b*x)^2]/ArcSinh[a + b*x]^2) - (a + b*x)/ArcSinh[a + b*x]
+ CoshIntegral[ArcSinh[a + b*x]])/(2*b)
```

3.91.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6273, 6188, 6233, 6189, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arcsinh}(a+bx)^3}dx$$
$$\Big\downarrow \text{6273}$$
$$\frac{\int \frac{1}{\text{arcsinh}(a+bx)^3}d(a+bx)}{b}$$
$$\Big\downarrow \text{6188}$$
$$\frac{\frac{1}{2}\int \frac{a+bx}{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)^2}d(a+bx)-\frac{\sqrt{(a+bx)^2+1}}{2\text{arcsinh}(a+bx)^2}}{b}$$
$$\Big\downarrow \text{6233}$$
$$\frac{\frac{1}{2}\left(\int \frac{1}{\text{arcsinh}(a+bx)}d(a+bx)-\frac{a+bx}{\text{arcsinh}(a+bx)}\right)-\frac{\sqrt{(a+bx)^2+1}}{2\text{arcsinh}(a+bx)^2}}{b}$$
$$\Big\downarrow \text{6189}$$
$$\frac{\frac{1}{2}\left(\int \frac{\sqrt{(a+bx)^2+1}}{\text{arcsinh}(a+bx)}d\text{arcsinh}(a+bx)-\frac{a+bx}{\text{arcsinh}(a+bx)}\right)-\frac{\sqrt{(a+bx)^2+1}}{2\text{arcsinh}(a+bx)^2}}{b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{-\frac{\sqrt{(a+bx)^2+1}}{2\text{arcsinh}(a+bx)^2}+\frac{1}{2}\left(-\frac{a+bx}{\text{arcsinh}(a+bx)}+\int \frac{\sin\left(i\text{arcsinh}(a+bx)+\frac{\pi}{2}\right)}{\text{arcsinh}(a+bx)}d\text{arcsinh}(a+bx)\right)}{b}$$
$$\Big\downarrow \text{3782}$$
$$\frac{\frac{1}{2}\left(\text{Chi}(\text{arcsinh}(a+bx))-\frac{a+bx}{\text{arcsinh}(a+bx)}\right)-\frac{\sqrt{(a+bx)^2+1}}{2\text{arcsinh}(a+bx)^2}}{b}$$

input
```
Int[ArcSinh[a + b*x]^(-3),x]
```

output
```
(-1/2*Sqrt[1 + (a + b*x)^2]/ArcSinh[a + b*x]^2 + (-((a + b*x)/ArcSinh[a +
b*x]) + CoshIntegral[ArcSinh[a + b*x]])/2)/b
```

3.91.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.91.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{-\frac{\sqrt{1+(bx+a)^2}}{2\,\mathrm{arcsinh}(bx+a)^2}-\frac{bx+a}{2\,\mathrm{arcsinh}(bx+a)}+\frac{\mathrm{Chi}(\mathrm{arcsinh}(bx+a))}{2}}{b}$	51
default	$\frac{-\frac{\sqrt{1+(bx+a)^2}}{2\,\mathrm{arcsinh}(bx+a)^2}-\frac{bx+a}{2\,\mathrm{arcsinh}(bx+a)}+\frac{\mathrm{Chi}(\mathrm{arcsinh}(bx+a))}{2}}{b}$	51

input
```
int(1/arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)-1/2/arcsinh(b*x+a)*(b*x+a)+
1/2*Chi(arcsinh(b*x+a)))
```

3.91.5 Fricas [F]

$$\int \frac{1}{\mathrm{arcsinh}(a+bx)^3}\,dx=\int \frac{1}{\mathrm{arsinh}\,(bx+a)^3}\,dx$$

input
```
integrate(1/arcsinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(arcsinh(b*x + a)^(-3), x)
```

3.91.6 Sympy [F]

$$\int \frac{1}{\mathrm{arcsinh}(a+bx)^3}\,dx=\int \frac{1}{\mathrm{asinh}^3\,(a+bx)}\,dx$$

input
```
integrate(1/asinh(b*x+a)**3,x)
```

output
```
Integral(asinh(a + b*x)**(-3), x)
```

3.91.7 Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

input
```
integrate(1/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7
*a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 +
3*(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (b^4*x^4 + 4*a*b^3*x^3 + a^4 +
(6*a^2*b^2 + b^2)*x^2 + a^2 + 2*(2*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^
2 + 1)^(3/2) + (3*b^5*x^5 + 15*a*b^4*x^4 + 3*a^5 + 5*(6*a^2*b^3 + b^3)*x^3
 + 5*a^3 + 15*(2*a^3*b^2 + a*b^2)*x^2 + (15*a^4*b + 15*a^2*b + 2*b)*x + 2*
a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x +
(b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*
b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*
a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4 - 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^5*x^5
 + 5*a*b^4*x^4 + a^5 + (10*a^2*b^3 + b^3)*x^3 + a^3 + (10*a^3*b^2 + 3*a*b^
2)*x^2 + (5*a^4*b + 3*a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (7*a^6*b +
 15*a^4*b + 9*a^2*b + b)*x + (3*b^6*x^6 + 18*a*b^5*x^5 + 3*a^6 + 3*(15*a^2
*b^4 + 2*b^4)*x^4 + 6*a^4 + 12*(5*a^3*b^3 + 2*a*b^3)*x^3 + (45*a^4*b^2 + 3
6*a^2*b^2 + 4*b^2)*x^2 + 4*a^2 + 2*(9*a^5*b + 12*a^3*b + 4*a*b)*x + 1)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1) + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)) + (3*b^6*x^6 + 18*a*b^5*x^5 + 3*a^6 + (45*a^2*b^4 + 7*b^4)*x^4
+ 7*a^4 + 4*(15*a^3*b^3 + 7*a*b^3)*x^3 + (45*a^4*b^2 + 42*a^2*b^2 + 5*b^2)
*x^2 + 5*a^2 + 2*(9*a^5*b + 14*a^3*b + 5*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a...
```

3.91.8 Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

input
```
integrate(1/arcsinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(arcsinh(b*x + a)^(-3), x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{\operatorname{asinh}\left(a+b\,x\right)^3}\,dx$$

input
```
int(1/asinh(a + b*x)^3,x)
```

output
```
int(1/asinh(a + b*x)^3, x)
```

3.92 $\int \frac{1}{x\text{arcsinh}(a+bx)^3}\,dx$

3.92.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\text{arcsinh}(a+bx)^3}\,dx=\text{Int}\left(\frac{1}{x\text{arcsinh}(a+bx)^3},x\right)$$

output
```
Unintegrable(1/x/arcsinh(b*x+a)^3,x)
```

3.92.2 Mathematica [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)^3}\,dx=\int \frac{1}{x\text{arcsinh}(a+bx)^3}\,dx$$

input
```
Integrate[1/(x*ArcSinh[a + b*x]^3),x]
```

output
```
Integrate[1/(x*ArcSinh[a + b*x]^3), x]
```

3.92.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 25, 27, 6272}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{arcsinh}(a+bx)^3}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{1}{x\text{arcsinh}(a+bx)^3}d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{1}{x\text{arcsinh}(a+bx)^3}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\int -\frac{1}{bx\text{arcsinh}(a+bx)^3}d(a+bx)$$

$$\Big\downarrow \text{6272}$$

$$-\int -\frac{1}{bx\text{arcsinh}(a+bx)^3}d(a+bx)$$

input
```
Int[1/(x*ArcSinh[a + b*x]^3),x]
```

output
```
$Aborted
```

3.92.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6272
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.92.4 Maple [N/A] (verified)

Not integrable

Time = 0.84 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \operatorname{arcsinh}(bx+a)^3} dx$$

input
```
int(1/x/arcsinh(b*x+a)^3,x)
```

output
```
int(1/x/arcsinh(b*x+a)^3,x)
```

3.92.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)^3} dx = \int \frac{1}{x \operatorname{arsinh}(bx+a)^3} dx$$

input
```
integrate(1/x/arcsinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(1/(x*arcsinh(b*x + a)^3), x)
```

3.92.6 Sympy [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{x\,\text{asinh}^3\,(a+bx)}\,dx$$

input
```
integrate(1/x/asinh(b*x+a)**3,x)
```

output
```
Integral(1/(x*asinh(a + b*x)**3), x)
```

3.92.7 Maxima [N/A]

Not integrable

Time = 15.19 (sec) , antiderivative size = 3273, normalized size of antiderivative = 272.75

$$\int \frac{1}{x\text{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{x\,\text{arsinh}\,(bx+a)^3}\,dx$$

input
```
integrate(1/x/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(b^8*x^8 + 7*a*b^7*x^7 + 3*(7*a^2*b^6 + b^6)*x^6 + 5*(7*a^3*b^5 + 3*a
*b^5)*x^5 + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 3*(7*a^5*b^3 + 10*a^3*
b^3 + 3*a*b^3)*x^3 + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + (b^5
*x^5 + 4*a*b^4*x^4 + (6*a^2*b^3 + b^3)*x^3 + 2*(2*a^3*b^2 + a*b^2)*x^2 + (
a^4*b + a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b^6*x^6 + 15*a*
b^5*x^5 + 5*(6*a^2*b^4 + b^4)*x^4 + 15*(2*a^3*b^3 + a*b^3)*x^3 + (15*a^4*b
^2 + 15*a^2*b^2 + 2*b^2)*x^2 + (3*a^5*b + 5*a^3*b + 2*a*b)*x)*(b^2*x^2 + 2
*a*b*x + a^2 + 1) + (a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x - (a*b^7*x^7 + 7*a
^2*b^6*x^6 + a^8 + 3*a^6 + 3*(7*a^3*b^5 + a*b^5)*x^5 + 5*(7*a^4*b^4 + 3*a^
2*b^4)*x^4 + 3*a^4 + (35*a^5*b^3 + 30*a^3*b^3 + 3*a*b^3)*x^3 + 3*(7*a^6*b^
2 + 10*a^4*b^2 + 3*a^2*b^2)*x^2 + (a*b^4*x^4 + a^5 + 2*(2*a^2*b^3 + b^3)*x
^3 + 2*a^3 + 6*(a^3*b^2 + a*b^2)*x^2 + 2*(2*a^4*b + 3*a^2*b + b)*x + a)*(b
^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*a*b^5*x^5 + 3*a^6 + (15*a^2*b^4 + 4
*b^4)*x^4 + 7*a^4 + (30*a^3*b^3 + 19*a*b^3)*x^3 + (30*a^4*b^2 + 33*a^2*b^2
 + 5*b^2)*x^2 + 5*a^2 + 5*(3*a^5*b + 5*a^3*b + 2*a*b)*x + 1)*(b^2*x^2 + 2*
a*b*x + a^2 + 1) + a^2 + (7*a^7*b + 15*a^5*b + 9*a^3*b + a*b)*x + (3*a*b^6
*x^6 + 3*a^7 + 2*(9*a^2*b^5 + b^5)*x^5 + 8*a^5 + (45*a^3*b^4 + 16*a*b^4)*x
^4 + (60*a^4*b^3 + 44*a^2*b^3 + 3*b^3)*x^3 + 7*a^3 + (45*a^5*b^2 + 56*a^3*
b^2 + 13*a*b^2)*x^2 + (18*a^6*b + 34*a^4*b + 17*a^2*b + b)*x + 2*a)*sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2...
```

3.92.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{x\,\text{arsinh}\,(bx+a)^3}\,dx$$

input
```
integrate(1/x/arcsinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(1/(x*arcsinh(b*x + a)^3), x)
```

3.92.9 Mupad [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(a+bx)^3} \, dx = \int \frac{1}{x\,\mathrm{asinh}\left(a+b\,x\right)^3} \,dx$$

input
```
int(1/(x*asinh(a + b*x)^3),x)
```

output
```
int(1/(x*asinh(a + b*x)^3), x)
```

3.93 $\int x^m(a+b\text{arcsinh}(c+dx))^n\,dx$

3.93.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^m(a+b\text{arcsinh}(c+dx))^n\,dx=\text{Int}(x^m(a+b\text{arcsinh}(c+dx))^n,x)$$

output
```
Unintegrable(x^m*(a+b*arcsinh(d*x+c))^n,x)
```

3.93.2 Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m(a+b\text{arcsinh}(c+dx))^n\,dx=\int x^m(a+b\text{arcsinh}(c+dx))^n\,dx$$

input
```
Integrate[x^m*(a + b*ArcSinh[c + d*x])^n,x]
```

output
```
Integrate[x^m*(a + b*ArcSinh[c + d*x])^n, x]
```

3.93.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6274, 6272}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m(a+b\operatorname{arcsinh}(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \left(\frac{c+dx}{d}-\frac{c}{d}\right)^m (a+b\operatorname{arcsinh}(c+dx))^n d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6272}$$

$$\frac{\int \left(\frac{c+dx}{d}-\frac{c}{d}\right)^m (a+b\operatorname{arcsinh}(c+dx))^n d(c+dx)}{d}$$

input
```
Int[x^m*(a + b*ArcSinh[c + d*x])^n,x]
```

output
```
$Aborted
```

3.93.3.1 Defintions of rubi rules used

rule 6272
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.93.4 Maple [N/A] (verified)

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^m (a + b \operatorname{arcsinh}(dx + c))^n \, dx$$

input
```
int(x^m*(a+b*arcsinh(d*x+c))^n,x)
```

output
```
int(x^m*(a+b*arcsinh(d*x+c))^n,x)
```

3.93.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m (a + b\text{arcsinh}(c + dx))^n \, dx = \int (b \operatorname{arsinh}(dx + c) + a)^n x^m \, dx$$

input
```
integrate(x^m*(a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(d*x + c) + a)^n*x^m, x)
```

3.93.6 Sympy [N/A]

Not integrable

Time = 12.85 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^m (a + b\text{arcsinh}(c + dx))^n \, dx = \int x^m (a + b \operatorname{asinh}(c + dx))^n \, dx$$

input
```
integrate(x**m*(a+b*asinh(d*x+c))**n,x)
```

output
```
Integral(x**m*(a + b*asinh(c + d*x))**n, x)
```

3.93.7 Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m(a + b\text{arcsinh}(c + dx))^n\, dx = \int (b\,\text{arsinh}\,(dx + c) + a)^n x^m\, dx$$

input
```
integrate(x^m*(a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^n*x^m, x)
```

3.93.8 Giac [F(-1)]

Timed out.

$$\int x^m(a + b\text{arcsinh}(c + dx))^n\, dx = \text{Timed out}$$

input
```
integrate(x^m*(a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
```

output
```
Timed out
```

3.93.9 Mupad [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m(a + b\text{arcsinh}(c + dx))^n\, dx = \int x^m\,(a + b\,\text{asinh}(c + d\,x))^n\, dx$$

input
```
int(x^m*(a + b*asinh(c + d*x))^n,x)
```

output
```
int(x^m*(a + b*asinh(c + d*x))^n, x)
```

3.94 $\int x^2(a+b\text{arcsinh}(c+dx))^n\,dx$

3.94.1 Optimal result

Integrand size = 16, antiderivative size = 545

$$
\begin{aligned}
&\int x^2(a+b\text{arcsinh}(c+dx))^n\,dx \\
&= \frac{3^{-1-n}e^{-\frac{3a}{b}}(a+b\text{arcsinh}(c+dx))^n\left(-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{8d^3} \\
&- \frac{2^{-2-n}ce^{-\frac{2a}{b}}(a+b\text{arcsinh}(c+dx))^n\left(-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{d^3} \\
&- \frac{e^{-\frac{a}{b}}(a+b\text{arcsinh}(c+dx))^n\left(-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{8d^3} \\
&+ \frac{c^2e^{-\frac{a}{b}}(a+b\text{arcsinh}(c+dx))^n\left(-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{2d^3} \\
&+ \frac{e^{a/b}(a+b\text{arcsinh}(c+dx))^n\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{8d^3} \\
&- \frac{c^2e^{a/b}(a+b\text{arcsinh}(c+dx))^n\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{2d^3} \\
&- \frac{2^{-2-n}ce^{\frac{2a}{b}}(a+b\text{arcsinh}(c+dx))^n\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{d^3} \\
&- \frac{3^{-1-n}e^{\frac{3a}{b}}(a+b\text{arcsinh}(c+dx))^n\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{8d^3}
\end{aligned}
$$

output
```
1/8*3^(-1-n)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,-3*(a+b*arcsinh(d*x+c))/b)/d
^3/exp(3*a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)-2^(-2-n)*c*(a+b*arcsinh(d*x+c)
)^n*GAMMA(1+n,-2*(a+b*arcsinh(d*x+c))/b)/d^3/exp(2*a/b)/(((-a-b*arcsinh(d*
x+c))/b)^n)-1/8*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(-a-b*arcsinh(d*x+c))/b)/
d^3/exp(a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)+1/2*c^2*(a+b*arcsinh(d*x+c))^n*
GAMMA(1+n,(-a-b*arcsinh(d*x+c))/b)/d^3/exp(a/b)/(((-a-b*arcsinh(d*x+c))/b)
^n)+1/8*exp(a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(a+b*arcsinh(d*x+c))/b)/
d^3/(((a+b*arcsinh(d*x+c))/b)^n)-1/2*c^2*exp(a/b)*(a+b*arcsinh(d*x+c))^n*G
AMMA(1+n,(a+b*arcsinh(d*x+c))/b)/d^3/(((a+b*arcsinh(d*x+c))/b)^n)-2^(-2-n)
*c*exp(2*a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,2*(a+b*arcsinh(d*x+c))/b)/d
^3/(((a+b*arcsinh(d*x+c))/b)^n)-1/8*3^(-1-n)*exp(3*a/b)*(a+b*arcsinh(d*x+c
))^n*GAMMA(1+n,3*(a+b*arcsinh(d*x+c))/b)/d^3/(((a+b*arcsinh(d*x+c))/b)^n)
```

3.94.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.63

$$\int x^2(a+b\operatorname{arcsinh}(c+dx))^n\,dx$$

$$= \frac{2^{-3-n}3^{-1-n}e^{-\frac{3a}{b}}(a+b\operatorname{arcsinh}(c+dx))^n\left(-\frac{(a+b\operatorname{arcsinh}(c+dx))^2}{b^2}\right)^{-n}\left(-2^n3^{1+n}(-1+4c^2)\,e^{\frac{4a}{b}}\left(-\frac{a+b\operatorname{arcsinh}}{b}\right.\right.}{}$$

input
```
Integrate[x^2*(a + b*ArcSinh[c + d*x])^n,x]
```

output
```
(2^(-3 - n)*3^(-1 - n)*(a + b*ArcSinh[c + d*x])^n*(-(2^n*3^(1 + n)*(-1 + 4
*c^2)*E^((4*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n*Gamma[1 + n, a/b + Arc
Sinh[c + d*x]]) + 2^n*(a/b + ArcSinh[c + d*x])^n*Gamma[1 + n, (-3*(a + b*A
rcSinh[c + d*x]))/b] - 2*3^(1 + n)*c*E^(a/b)*(a/b + ArcSinh[c + d*x])^n*Ga
mma[1 + n, (-2*(a + b*ArcSinh[c + d*x]))/b] + 2^n*3^(1 + n)*(-1 + 4*c^2)*E
^((2*a)/b)*(a/b + ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*
x])/b)] - E^((5*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n*(2*3^(1 + n)*c*Gam
ma[1 + n, (2*(a + b*ArcSinh[c + d*x]))/b] + 2^n*E^(a/b)*Gamma[1 + n, (3*(a
 + b*ArcSinh[c + d*x]))/b])))/(d^3*E^((3*a)/b)*(-((a + b*ArcSinh[c + d*x])
^2/b^2))^n)
```

3.94.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6274, 27, 6245, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\text{arcsinh}(c+dx))^n\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int x^2(a+b\text{arcsinh}(c+dx))^n d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int d^2x^2(a+b\text{arcsinh}(c+dx))^n d(c+dx)}{d^3}$$

$$\Big\downarrow \text{6245}$$

$$\frac{\int d^2x^2\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^n d\text{arcsinh}(c+dx)}{d^3}$$

$$\Big\downarrow \text{7293}$$

$$\frac{\int \Big(c^2\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^n+(c+dx)^2\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^n-2c(c+dx)\sqrt{(}}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}c^2e^{-\frac{a}{b}}(a+b\text{arcsinh}(c+dx))^n\left(-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)-\frac{1}{2}c^2e^{a/b}(a+b\text{arcsinh}(c+$$

input
```
Int[x^2*(a + b*ArcSinh[c + d*x])^n,x]
```

output
```
((3^(-1 - n)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c
+ d*x]))/b])/(8*E^((3*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (2^(-2 -
n)*c*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c + d*x]))
/b])/(E^((2*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - ((a + b*ArcSinh[c +
 d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)])/(8*E^(a/b)*(-((a + b
*ArcSinh[c + d*x])/b))^n) + (c^2*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, -
((a + b*ArcSinh[c + d*x])/b)])/(2*E^(a/b)*(-((a + b*ArcSinh[c + d*x])/b))^
n) + (E^(a/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (a + b*ArcSinh[c + d
*x])/b])/(8*((a + b*ArcSinh[c + d*x])/b)^n) - (c^2*E^(a/b)*(a + b*ArcSinh[
c + d*x])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*((a + b*ArcSinh[c
 + d*x])/b)^n) - (2^(-2 - n)*c*E^((2*a)/b)*(a + b*ArcSinh[c + d*x])^n*Gamm
a[1 + n, (2*(a + b*ArcSinh[c + d*x]))/b])/((a + b*ArcSinh[c + d*x])/b)^n -
 (3^(-1 - n)*E^((3*a)/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (3*(a + b
*ArcSinh[c + d*x]))/b])/(8*((a + b*ArcSinh[c + d*x])/b)^n))/d^3
```

3.94.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6245
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[1/c^(m + 1)   Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[
x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m,
0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.94.4 Maple [F]

$$\int x^2(a+b\,\mathrm{arcsinh}\,(dx+c))^n\,dx$$

input
```
int(x^2*(a+b*arcsinh(d*x+c))^n,x)
```

output
```
int(x^2*(a+b*arcsinh(d*x+c))^n,x)
```

3.94.5 Fricas [F]

$$\int x^2(a+b\mathrm{arcsinh}(c+dx))^n\,dx = \int (b\,\mathrm{arsinh}\,(dx+c)+a)^n x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(d*x + c) + a)^n*x^2, x)
```

3.94.6 Sympy [F]

$$\int x^2(a+b\mathrm{arcsinh}(c+dx))^n\,dx = \int x^2(a+b\,\mathrm{asinh}\,(c+dx))^n\,dx$$

input
```
integrate(x**2*(a+b*asinh(d*x+c))**n,x)
```

output
```
Integral(x**2*(a + b*asinh(c + d*x))**n, x)
```

3.94.7 Maxima [F]

$$\int x^2(a+b\text{arcsinh}(c+dx))^n\,dx=\int(b\,\text{arsinh}\,(dx+c)+a)^nx^2\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^n*x^2, x)
```

3.94.8 Giac [F]

$$\int x^2(a+b\text{arcsinh}(c+dx))^n\,dx=\int(b\,\text{arsinh}\,(dx+c)+a)^nx^2\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^n*x^2, x)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\text{arcsinh}(c+dx))^n\,dx=\int x^2\,(a+b\,\text{asinh}(c+d\,x))^n\,dx$$

input
```
int(x^2*(a + b*asinh(c + d*x))^n,x)
```

output
```
int(x^2*(a + b*asinh(c + d*x))^n, x)
```

3.95 $\int x(a+b\text{arcsinh}(c+dx))^n\,dx$

3.95.1 Optimal result

Integrand size = 14, antiderivative size = 267

$$\begin{aligned}
&\int x(a+b\text{arcsinh}(c+dx))^n\,dx \\
&= \frac{2^{-3-n}e^{-\frac{2a}{b}}(a+b\text{arcsinh}(c+dx))^n\left(-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{d^2} \\
&\quad -\frac{ce^{-\frac{a}{b}}(a+b\text{arcsinh}(c+dx))^n\left(-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{2d^2} \\
&\quad +\frac{ce^{a/b}(a+b\text{arcsinh}(c+dx))^n\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{2d^2} \\
&\quad +\frac{2^{-3-n}e^{\frac{2a}{b}}(a+b\text{arcsinh}(c+dx))^n\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{d^2}
\end{aligned}$$

output
```
2^(-3-n)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,-2*(a+b*arcsinh(d*x+c))/b)/d^2/e
xp(2*a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)-1/2*c*(a+b*arcsinh(d*x+c))^n*GAMMA
(1+n,(-a-b*arcsinh(d*x+c))/b)/d^2/exp(a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)+1
/2*c*exp(a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(a+b*arcsinh(d*x+c))/b)/d^2
/(((a+b*arcsinh(d*x+c))/b)^n)+2^(-3-n)*exp(2*a/b)*(a+b*arcsinh(d*x+c))^n*G
AMMA(1+n,2*(a+b*arcsinh(d*x+c))/b)/d^2/(((a+b*arcsinh(d*x+c))/b)^n)
```

3.95.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.85

$$\int x(a+b\text{arcsinh}(c+dx))^n\,dx$$
$$= \frac{2^{-3-n}e^{-\frac{2a}{b}}(a+b\text{arcsinh}(c+dx))^n\left(-\frac{(a+b\text{arcsinh}(c+dx))^2}{b^2}\right)^{-n}\left(2^{2+n}ce^{\frac{3a}{b}}\left(-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^n\Gamma\left(1+n,\frac{a}{b}+\right.\right.}{}$$

input
```
Integrate[x*(a + b*ArcSinh[c + d*x])^n,x]
```

output
```
(2^(-3 - n)*(a + b*ArcSinh[c + d*x])^n*(2^(2 + n)*c*E^((3*a)/b)*(-((a + b*
ArcSinh[c + d*x])/b))^n*Gamma[1 + n, a/b + ArcSinh[c + d*x]] + (a/b + ArcS
inh[c + d*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c + d*x]))/b] - 2^(2 + n)*
c*E^(a/b)*(a/b + ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x
])/b)] + E^((4*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n*Gamma[1 + n, (2*(a
+ b*ArcSinh[c + d*x]))/b]))/(d^2*E^((2*a)/b)*(-((a + b*ArcSinh[c + d*x])^2
/b^2))^n)
```

3.95.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6274, 25, 27, 6245, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\text{arcsinh}(c+dx))^n\,dx$$
$$\Big\downarrow \text{6274}$$
$$\frac{\int x(a+b\text{arcsinh}(c+dx))^n d(c+dx)}{d}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\int -x(a+b\text{arcsinh}(c+dx))^n d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$

$$-\frac{\int -dx(a+b\text{arcsinh}(c+dx))^n d(c+dx)}{d^2}$$

$$\Big\downarrow \ 6245$$

$$-\frac{\int -dx\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^n d\text{arcsinh}(c+dx)}{d^2}$$

$$\Big\downarrow \ 7293$$

$$-\frac{\int \left(c\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^n-(c+dx)\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^n\right) d\text{arcsinh}(c+dx}{d^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-2^{-n-3}e^{-\frac{2a}{b}}(a+b\text{arcsinh}(c+dx))^n\left(-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)+\frac{1}{2}ce^{-\frac{a}{b}}(a+b\text{arcs}}{}$$

input
```
Int[x*(a + b*ArcSinh[c + d*x])^n,x]
```

output
```
-((-((2^(-3 - n)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-2*(a + b*ArcSin
h[c + d*x]))/b])/(E^((2*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n)) + (c*(a
+ b*ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)])/(2*E^
(a/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (c*E^(a/b)*(a + b*ArcSinh[c + d
*x])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*((a + b*ArcSinh[c + d*
x])/b)^n) - (2^(-3 - n)*E^((2*a)/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n
, (2*(a + b*ArcSinh[c + d*x]))/b])/((a + b*ArcSinh[c + d*x])/b)^n)/d^2)
```

3.95.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6245
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[1/c^(m + 1)   Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[
x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m,
0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.95.4 Maple [F]

$$\int x(a+b\operatorname{arcsinh}(dx+c))^n\,dx$$

input
```
int(x*(a+b*arcsinh(d*x+c))^n,x)
```

output
```
int(x*(a+b*arcsinh(d*x+c))^n,x)
```

3.95.5 Fricas [F]

$$\int x(a+b\text{arcsinh}(c+dx))^n\,dx=\int(b\operatorname{arsinh}(dx+c)+a)^nx\,dx$$

input
```
integrate(x*(a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(d*x + c) + a)^n*x, x)
```

3.95.6 Sympy [F]

$$\int x(a+b\text{arcsinh}(c+dx))^n\,dx=\int x(a+b\,\text{asinh}\,(c+dx))^n\;dx$$

input
```
integrate(x*(a+b*asinh(d*x+c))**n,x)
```

output
```
Integral(x*(a + b*asinh(c + d*x))**n, x)
```

3.95.7 Maxima [F]

$$\int x(a+b\text{arcsinh}(c+dx))^n\,dx=\int (b\,\text{arsinh}\,(dx+c)+a)^n x\,dx$$

input
```
integrate(x*(a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^n*x, x)
```

3.95.8 Giac [F]

$$\int x(a+b\text{arcsinh}(c+dx))^n\,dx=\int (b\,\text{arsinh}\,(dx+c)+a)^n x\,dx$$

input
```
integrate(x*(a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^n*x, x)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int x(a+b\text{arcsinh}(c+dx))^n\,dx=\int x\,(a+b\,\text{asinh}(c+d\,x))^n\,dx$$

input
```
int(x*(a + b*asinh(c + d*x))^n,x)
```

output
```
int(x*(a + b*asinh(c + d*x))^n, x)
```

3.96 $\int (a + b\text{arcsinh}(c + dx))^n\,dx$

3.96.1 Optimal result

Integrand size = 12, antiderivative size = 128

$$\int (a+b\text{arcsinh}(c+dx))^n\,dx = \frac{e^{-\frac{a}{b}}(a+b\text{arcsinh}(c+dx))^n \left(-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{2d} - \frac{e^{a/b}(a+b\text{arcsinh}(c+dx))^n\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{2d}$$

output
```
1/2*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(-a-b*arcsinh(d*x+c))/b)/d/exp(a/b)/(
((-a-b*arcsinh(d*x+c))/b)^n)-1/2*exp(a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n
,(a+b*arcsinh(d*x+c))/b)/d/(((a+b*arcsinh(d*x+c))/b)^n)
```

3.96.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.85

$$\int (a+b\text{arcsinh}(c+dx))^n\,dx = \frac{e^{-\frac{a}{b}}(a+b\text{arcsinh}(c+dx))^n\left(-e^{\frac{2a}{b}}\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)^{-n}\Gamma\left(1+n,\frac{a}{b}+\text{arcsinh}(c+dx)\right)+\left(-\frac{a+b\text{arcsin}}{b}\right.\right.}{2d}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^n,x]
```

output
```
((a + b*ArcSinh[c + d*x])^n*(-((E^((2*a)/b)*Gamma[1 + n, a/b + ArcSinh[c +
 d*x]])/(a/b + ArcSinh[c + d*x])^n) + Gamma[1 + n, -((a + b*ArcSinh[c + d*
x])/b)]/(-((a + b*ArcSinh[c + d*x])/b))^n))/(2*d*E^(a/b))
```

3.96.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6273, 6189, 3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(c+dx))^n\,dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int (a+b\text{arcsinh}(c+dx))^n d(c+dx)}{d}$$

$$\Big\downarrow \text{6189}$$

$$\frac{\int (a+b\text{arcsinh}(c+dx))^n \cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (a+b\text{arcsinh}(c+dx))^n \sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{3788}$$

$$\frac{\frac{1}{2}i\int -ie^{\frac{a-c-dx}{b}}(a+b\text{arcsinh}(c+dx))^n d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int ie^{-\frac{a-c-dx}{b}}(a+b\text{arcsinh}(c+dx))^n d(a+b\text{arc}}{bd}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{1}{2}\int e^{-\frac{a-c-dx}{b}}(a+b\text{arcsinh}(c+dx))^n d(a+b\text{arcsinh}(c+dx))+\frac{1}{2}\int e^{\frac{a-c-dx}{b}}(a+b\text{arcsinh}(c+dx))^n d(a+b\text{arcsinh}(}{bd}$$

$$\Big\downarrow \text{2612}$$

$$\frac{\frac{1}{2}be^{-\frac{a}{b}}(a+b\text{arcsinh}(c+dx))^n\left(-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)-\frac{1}{2}be^{a/b}(a+b\text{arcsinh}(c+dx}{bd}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^n,x]
```

output
```
((b*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)]
)/(2*E^(a/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (b*E^(a/b)*(a + b*ArcSin
h[c + d*x])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*((a + b*ArcSinh
[c + d*x])/b)^n))/(b*d)
```

3.96.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.96.4 Maple [F]

$$\int (a + b \operatorname{arcsinh}(dx + c))^n \, dx$$

input
```
int((a+b*arcsinh(d*x+c))^n,x)
```

output
```
int((a+b*arcsinh(d*x+c))^n,x)
```

3.96.5 Fricas [F]

$$\int (a + b\text{arcsinh}(c + dx))^n \, dx = \int (b \operatorname{arsinh}(dx + c) + a)^n \, dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(d*x + c) + a)^n, x)
```

3.96.6 Sympy [F]

$$\int (a + b\text{arcsinh}(c + dx))^n \, dx = \int (a + b \operatorname{asinh}(c + dx))^n \, dx$$

input
```
integrate((a+b*asinh(d*x+c))**n,x)
```

output
```
Integral((a + b*asinh(c + d*x))**n, x)
```

3.96.7 Maxima [F]

$$\int (a + b\text{arcsinh}(c + dx))^n \, dx = \int (b \operatorname{arsinh}(dx + c) + a)^n \, dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^n, x)
```

3.96.8 Giac [F]

$$\int (a + b\text{arcsinh}(c + dx))^n \, dx = \int (b \operatorname{arsinh}(dx + c) + a)^n \, dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^n, x)
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int (a + b\text{arcsinh}(c + dx))^n \, dx = \int (a + b \operatorname{asinh}(c + d\,x))^n \, dx$$

input
```
int((a + b*asinh(c + d*x))^n,x)
```

output
```
int((a + b*asinh(c + d*x))^n, x)
```

3.97 $\int \frac{(a+b\text{arcsinh}(c+dx))^n}{x} dx$

3.97.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(a+b\text{arcsinh}(c+dx))^n}{x} dx = \text{Int}\left(\frac{(a+b\text{arcsinh}(c+dx))^n}{x}, x\right)$$

output
```
Unintegrable((a+b*arcsinh(d*x+c))^n/x,x)
```

3.97.2 Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\text{arcsinh}(c+dx))^n}{x} dx = \int \frac{(a+b\text{arcsinh}(c+dx))^n}{x} dx$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^n/x,x]
```

output
```
Integrate[(a + b*ArcSinh[c + d*x])^n/x, x]
```

3.97.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 25, 27, 6272}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^n}{x}dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^n}{x}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -\frac{(a+b\text{arcsinh}(c+dx))^n}{x}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$-\int -\frac{(a+b\text{arcsinh}(c+dx))^n}{dx}d(c+dx)$$

$$\Big\downarrow \text{ 6272}$$

$$-\int -\frac{(a+b\text{arcsinh}(c+dx))^n}{dx}d(c+dx)$$

input
```
Int[(a + b*ArcSinh[c + d*x])^n/x,x]
```

output
```
$Aborted
```

3.97.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6272
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.97.4 Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \operatorname{arcsinh}(dx + c))^n}{x} dx$$

input
```
int((a+b*arcsinh(d*x+c))^n/x,x)
```

output
```
int((a+b*arcsinh(d*x+c))^n/x,x)
```

3.97.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a + b\text{arcsinh}(c + dx))^n}{x} dx = \int \frac{(b \operatorname{arsinh}(dx + c) + a)^n}{x} dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^n/x,x, algorithm="fricas")
```

output
```
integral((b*arcsinh(d*x + c) + a)^n/x, x)
```

3.97.6 Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\text{arcsinh}(c+dx))^n}{x}\,dx = \int \frac{(a+b\,\text{asinh}\,(c+dx))^n}{x}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**n/x,x)
```

output
```
Integral((a + b*asinh(c + d*x))**n/x, x)
```

3.97.7 Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\text{arcsinh}(c+dx))^n}{x}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^n}{x}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^n/x,x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^n/x, x)
```

3.97.8 Giac [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\text{arcsinh}(c+dx))^n}{x}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^n}{x}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^n/x,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^n/x, x)
```

3.97.9 Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a + b\operatorname{arcsinh}(c + dx))^n}{x} \, dx = \int \frac{(a + b\,\mathrm{asinh}(c + d\,x))^n}{x} \, dx$$

input
```
int((a + b*asinh(c + d*x))^n/x,x)
```

output
```
int((a + b*asinh(c + d*x))^n/x, x)
```

3.98 $\int x^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$

3.98.1 Optimal result

Integrand size = 18, antiderivative size = 496

$$\int x^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \frac{c^2(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d^3} + \frac{(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}}{3d^3} - \frac{c\sqrt{a+b\text{arcsinh}(c+dx)}\cosh(2\text{arcsinh}(c+dx))}{2d^3} - \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^3} + \frac{\sqrt{b}c^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^3} + \frac{\sqrt{b}ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^3} + \frac{\sqrt{b}e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{48d^3} + \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^3} - \frac{\sqrt{b}c^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^3} + \frac{\sqrt{b}ce^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^3} - \frac{\sqrt{b}e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{48d^3}$$

output
```
1/144*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*3
^(1/2)*Pi^(1/2)/d^3-1/144*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))
*b^(1/2)*3^(1/2)*Pi^(1/2)/d^3/exp(3*a/b)+1/16*c*exp(2*a/b)*erf(2^(1/2)*(a+
b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^3+1/16*c*erfi(
2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^3/e
xp(2*a/b)-1/16*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi
^(1/2)/d^3+1/4*c^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2
)*Pi^(1/2)/d^3+1/16*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1
/2)/d^3/exp(a/b)-1/4*c^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*
Pi^(1/2)/d^3/exp(a/b)+c^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d^3+1/3*(d*x+
c)^3*(a+b*arcsinh(d*x+c))^(1/2)/d^3-1/2*c*cosh(2*arcsinh(d*x+c))*(a+b*arcs
inh(d*x+c))^(1/2)/d^3
```

3.98.2 Mathematica [A] (verified)

Time = 2.53 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.32

$$\int x^2 \sqrt{a + b\text{arcsinh}(c + dx)}\,dx$$

$$= \frac{-36(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)} + 144c^2(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)} - 72c\sqrt{a+b\text{arcsinh}(c+dx)}}{}$$

input
```
Integrate[x^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(-36*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]] + 144*c^2*(c + d*x)*Sqrt[a + b
*ArcSinh[c + d*x]] - 72*c*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c +
d*x]] + Sqrt[b]*Sqrt[3*Pi]*Cosh[(3*a)/b]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c
 + d*x]])/Sqrt[b]] + 9*Sqrt[b]*Sqrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[
c + d*x]]/Sqrt[b]] - 36*Sqrt[b]*c^2*Sqrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*Arc
Sinh[c + d*x]]/Sqrt[b]] + 9*Sqrt[b]*c*Sqrt[2*Pi]*Cosh[(2*a)/b]*Erfi[(Sqrt[
2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - Sqrt[b]*Sqrt[3*Pi]*Cosh[(3*a)/
b]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - 9*Sqrt[b]*Sqrt[P
i]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 36*Sqrt[b]*c^2*S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 9*Sqrt[b]*(
-1 + 4*c^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b]
+ Sinh[a/b]) - 9*Sqrt[b]*c*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c +
 d*x]])/Sqrt[b]]*Sinh[(2*a)/b] + 9*Sqrt[b]*c*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[
a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + Sqrt[b
]*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(3*a
)/b] + Sqrt[b]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]]*Sinh[(3*a)/b] + 12*Sqrt[a + b*ArcSinh[c + d*x]]*Sinh[3*ArcSinh[c + d*
x]])/(144*d^3)
```

3.98.3 Rubi [A] (verified)

Time = 1.86 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6274, 27, 6245, 7267, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int x^2\sqrt{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int d^2x^2\sqrt{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d^3}$$

$$\Big\downarrow \text{ 6245}$$

$$\frac{\int d^2x^2\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}d\text{arcsinh}(c+dx)}{d^3}$$

$$\Big\downarrow \text{7267}$$

$$\frac{2\int d^2x^2\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))d\sqrt{a+b\text{arcsinh}(c+dx)}}{bd^3}$$

$$\Big\downarrow \text{7292}$$

$$\frac{2\int d^2x^2(a+b\text{arcsinh}(c+dx))\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)d\sqrt{a+b\text{arcsinh}(c+dx)}}{bd^3}$$

$$\Big\downarrow \text{7293}$$

$$\frac{2\int\left((a+b\text{arcsinh}(c+dx))\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)c^2+(a+b\text{arcsinh}(c+dx))\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx}{b}\right.\right.}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{1}{8}\sqrt{\pi}b^{3/2}c^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\pi}b^{3/2}c^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{\pi}b^{3/2}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{ar}}}{}\right.\right.}{}$$

input
```
Int[x^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(2*(-1/4*(b*c*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[(2*a)/b - (2*(a + b*ArcSin
h[c + d*x]))/b]) - (b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*
x]]/Sqrt[b]])/32 + (b^(3/2)*c^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c
+ d*x]]/Sqrt[b]])/8 + (b^(3/2)*c*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[
a + b*ArcSinh[c + d*x]])/Sqrt[b]])/16 + (b^(3/2)*E^((3*a)/b)*Sqrt[Pi/3]*Er
f[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/96 + (b^(3/2)*Sqrt[Pi]*
Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(32*E^(a/b)) - (b^(3/2)*c^2*Sq
rt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*E^(a/b)) + (b^(3/2)*
c*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*E^(
(2*a)/b)) - (b^(3/2)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]])/(96*E^((3*a)/b)) - (b*c^2*Sqrt[a + b*ArcSinh[c + d*x]]*Sinh[a/
b - (a + b*ArcSinh[c + d*x])/b])/2 - (b*Sqrt[a + b*ArcSinh[c + d*x]]*Sinh[
a/b - (a + b*ArcSinh[c + d*x])/b]^3)/6))/(b*d^3)
```

3.98. $\int x^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$

3.98.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6245
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[1/c^(m + 1)   Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[
x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m,
0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.98.4 Maple [F]

$$\int x^2\sqrt{a+b\,\operatorname{arcsinh}\left(dx+c\right)}dx$$

input
```
int(x^2*(a+b*arcsinh(d*x+c))^(1/2),x)
```

output
```
int(x^2*(a+b*arcsinh(d*x+c))^(1/2),x)
```

3.98.5 Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.98.6 Sympy [F]

$$\int x^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int x^2\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx$$

input
```
integrate(x**2*(a+b*asinh(d*x+c))**(1/2),x)
```

output
```
Integral(x**2*sqrt(a + b*asinh(c + d*x)), x)
```

3.98.7 Maxima [F]

$$\int x^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int \sqrt{b\,\text{arsinh}\,(dx+c)+a}x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(d*x + c) + a)*x^2, x)
```

3.98.8 Giac [F]

$$\int x^2 \sqrt{a + b\text{arcsinh}(c + dx)}\, dx = \int \sqrt{b \operatorname{arsinh}(dx + c) + a}x^2\, dx$$

input
```
integrate(x^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(d*x + c) + a)*x^2, x)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt{a + b\text{arcsinh}(c + dx)}\, dx = \int x^2 \sqrt{a + b \operatorname{asinh}(c + d\,x)}\, dx$$

input
```
int(x^2*(a + b*asinh(c + d*x))^(1/2),x)
```

output
```
int(x^2*(a + b*asinh(c + d*x))^(1/2), x)
```

3.99 $\int x\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$

3.99.1 Optimal result

Integrand size = 16, antiderivative size = 259

$$\int x\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = -\frac{c(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d^2} + \frac{\sqrt{a+b\text{arcsinh}(c+dx)}\cosh(2\text{arcsinh}(c+dx))}{4d^2} - \frac{\sqrt{b}ce^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^2} - \frac{\sqrt{b}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^2} + \frac{\sqrt{b}ce^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d^2} - \frac{\sqrt{b}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^2}$$

output
```
-1/32*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*2
^(1/2)*Pi^(1/2)/d^2-1/32*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*
b^(1/2)*2^(1/2)*Pi^(1/2)/d^2/exp(2*a/b)-1/4*c*exp(a/b)*erf((a+b*arcsinh(d*
x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d^2+1/4*c*erfi((a+b*arcsinh(d*x+c))^
(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d^2/exp(a/b)-c*(d*x+c)*(a+b*arcsinh(d*x+c)
)^(1/2)/d^2+1/4*cosh(2*arcsinh(d*x+c))*(a+b*arcsinh(d*x+c))^(1/2)/d^2
```

3.99.2 Mathematica [A] (verified)

Time = 2.27 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.97

$$\int x\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx =$$

$$-\frac{-8\sqrt{a+b\operatorname{arcsinh}(c+dx)}\cosh(2\operatorname{arcsinh}(c+dx))+16ce^{-\frac{a}{b}}\sqrt{a+b\operatorname{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\operatorname{arcsin}\right.}{\sqrt{\frac{a}{b}+\operatorname{arcsinh}(}}\right.}{}$$

input
```
Integrate[x*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
-1/32*(-8*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + (16*c*Sq
rt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c + d*
x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*x])
/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + Sqrt[b]*Sqrt[2*Pi]*Er
fi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(
2*a)/b]) + Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/S
qrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]))/d^2
```

3.99.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6274, 25, 27, 6245, 7267, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int x\sqrt{a+b\operatorname{arcsinh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -x\sqrt{a+b\operatorname{arcsinh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -dx\sqrt{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d^2}$$

$$\Big\downarrow \; 6245$$

$$-\frac{\int -dx\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}d\text{arcsinh}(c+dx)}{d^2}$$

$$\Big\downarrow \; 7267$$

$$-\frac{2\int -dx\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))d\sqrt{a+b\text{arcsinh}(c+dx)}}{bd^2}$$

$$\Big\downarrow \; 7292$$

$$-\frac{2\int -dx(a+b\text{arcsinh}(c+dx))\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)d\sqrt{a+b\text{arcsinh}(c+dx)}}{bd^2}$$

$$\Big\downarrow \; 7293$$

$$-\frac{2\int \left(c(a+b\text{arcsinh}(c+dx))\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)+\frac{1}{2}(a+b\text{arcsinh}(c+dx))\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+\right.\right.}{bd^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\left(\frac{1}{8}\sqrt{\pi}b^{3/2}ce^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\frac{\pi}{2}}b^{3/2}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\pi}b^{3/2}ce^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+\right.\right.}{}$$

input
```
Int[x*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(-2*(-1/8*(b*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[(2*a)/b - (2*(a + b*ArcSinh
[c + d*x]))/b]) + (b^(3/2)*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]])/8 + (b^(3/2)*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b
*ArcSinh[c + d*x]])/Sqrt[b]])/32 - (b^(3/2)*c*Sqrt[Pi]*Erfi[Sqrt[a + b*Arc
Sinh[c + d*x]]/Sqrt[b]])/(8*E^(a/b)) + (b^(3/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*S
qrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*E^((2*a)/b)) - (b*c*Sqrt[a + b*
ArcSinh[c + d*x]]*Sinh[a/b - (a + b*ArcSinh[c + d*x])/b])/2))/(b*d^2)
```

3.99. $\int x\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$

3.99.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6245
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[1/c^(m + 1)   Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[
x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m,
0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.99.4 Maple [F]

$$\int x\sqrt{a+b\ \mathrm{arcsinh}\left(dx+c\right)}dx$$

input
```
int(x*(a+b*arcsinh(d*x+c))^(1/2),x)
```

output
```
int(x*(a+b*arcsinh(d*x+c))^(1/2),x)
```

3.99.5 Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+b\mathrm{arcsinh}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.99.6 Sympy [F]

$$\int x\sqrt{a+b\mathrm{arcsinh}(c+dx)}\,dx = \int x\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,dx$$

input
```
integrate(x*(a+b*asinh(d*x+c))**(1/2),x)
```

output
```
Integral(x*sqrt(a + b*asinh(c + d*x)), x)
```

3.99.7 Maxima [F]

$$\int x\sqrt{a+b\text{arcsinh}(c+dx)}\,dx=\int\sqrt{b\,\text{arsinh}\,(dx+c)+a}x\,dx$$

input
```
integrate(x*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(d*x + c) + a)*x, x)
```

3.99.8 Giac [F]

$$\int x\sqrt{a+b\text{arcsinh}(c+dx)}\,dx=\int\sqrt{b\,\text{arsinh}\,(dx+c)+a}x\,dx$$

input
```
integrate(x*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(d*x + c) + a)*x, x)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+b\text{arcsinh}(c+dx)}\,dx=\int x\,\sqrt{a+b\,\text{asinh}\,(c+d\,x)}\,dx$$

input
```
int(x*(a + b*asinh(c + d*x))^(1/2),x)
```

output
```
int(x*(a + b*asinh(c + d*x))^(1/2), x)
```

3.100 $\int \sqrt{a+b\text{arcsinh}(c+dx)}\,dx$

3.100.1 Optimal result

Integrand size = 14, antiderivative size = 115

$$\int \sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d} + \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d} - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d}$$

output
```
1/4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/
4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(a/b)+(d*
x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

3.100.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \frac{e^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}} + \frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}}\right)}{2d}$$

input
```
Integrate[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c +
 d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*
x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b))
```

3.100.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6273, 6187, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\text{arcsinh}(c+dx)}\,dx$$

$$\Big\downarrow \; 6273$$

$$\frac{\int \sqrt{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \; 6187$$

$$\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}b\int \frac{c+dx}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \; 6234$$

$$\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{2}\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))+(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{2}\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{d}$$

$\downarrow$ 26

$$\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{d}$$

$\downarrow$ 3789

$$\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\left(\frac{1}{2}i\int \frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int \frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(\right.}{d}$$

$\downarrow$ 2611

$$\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-i\int e^{\frac{a+b\text{arcsinh}(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+\right.}{d}$$

$\downarrow$ 2633

$$\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcs}}}{\sqrt{}}\right.\right.}{d}$$

$\downarrow$ 2634

$$\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx}}{\sqrt{b}}\right.\right.}{d}$$

input
```
Int[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
((c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]] - (I/2)*((I/2)*Sqrt[b]*E^(a/b)*Sqr
t[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi]*
Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/E^(a/b)))/d
```

3.100. $\int \sqrt{a+b\text{arcsinh}(c+dx)}\,dx$

3.100.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.100.4 Maple [F]

$$\int \sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}dx$$

input
```
int((a+b*arcsinh(d*x+c))^(1/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^(1/2),x)
```

3.100.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\mathrm{arcsinh}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.100.6 Sympy [F]

$$\int \sqrt{a+b\mathrm{arcsinh}(c+dx)}\,dx = \int \sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*asinh(c + d*x)), x)
```

3.100.7 Maxima [F]

$$\int \sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx=\int \sqrt{b\operatorname{arsinh}(dx+c)+a}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

3.100.8 Giac [F]

$$\int \sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx=\int \sqrt{b\operatorname{arsinh}(dx+c)+a}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx=\int \sqrt{a+b\operatorname{asinh}(c+d\,x)}\,dx$$

input
```
int((a + b*asinh(c + d*x))^(1/2),x)
```

output
```
int((a + b*asinh(c + d*x))^(1/2), x)
```

3.101 $\int x(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

3.101.1 Optimal result

Integrand size = 16, antiderivative size = 326

$$\begin{aligned}\int x(a+b\text{arcsinh}(c+dx))^{3/2}\,dx &= \frac{3bc\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{2d^2} \\ &- \frac{c(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}{d^2} \\ &+ \frac{(a+b\text{arcsinh}(c+dx))^{3/2}\cosh(2\text{arcsinh}(c+dx))}{4d^2} \\ &- \frac{3b^{3/2}ce^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^2} - \frac{3b^{3/2}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d^2} \\ &- \frac{3b^{3/2}ce^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d^2} + \frac{3b^{3/2}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d^2} \\ &- \frac{3b\sqrt{a+b\text{arcsinh}(c+dx)}\sinh(2\text{arcsinh}(c+dx))}{16d^2}\end{aligned}$$

output
```
-c*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d^2+1/4*(a+b*arcsinh(d*x+c))^(3/2)*c
osh(2*arcsinh(d*x+c))/d^2-3/128*b^(3/2)*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsin
h(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^2+3/128*b^(3/2)*erfi(2^(1/2)*(
a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^2/exp(2*a/b)-3/8*b^(
3/2)*c*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^2-3/8*b
^(3/2)*c*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^2/exp(a/b)-3/
16*b*sinh(2*arcsinh(d*x+c))*(a+b*arcsinh(d*x+c))^(1/2)/d^2+3/2*b*c*(1+(d*x
+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d^2
```

3.101.2 Mathematica [A] (verified)

Time = 4.14 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.79

$$\int x(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \frac{-64ace^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}}\right)-16\cdots}{\cdots}$$

input
```
Integrate[x*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
((-64*a*c*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + Ar
cSinh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSi
nh[c + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) - 16*Sqrt[b
]*c*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*
(c + d*x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh
[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sq
rt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + 4*a*(8*Sqrt
[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + Sqrt[b]*Sqrt[2*Pi]*Erf
i[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(-Cosh[(2*a)/b] + Sinh[(
2*a)/b]) - Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/S
qrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b])) + Sqrt[b]*((4*a + 3*b)*Sqrt[2*Pi]
*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sin
h[(2*a)/b]) + (4*a - 3*b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d
*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[b]*Sqrt[a + b*ArcS
inh[c + d*x]]*(4*ArcSinh[c + d*x]*Cosh[2*ArcSinh[c + d*x]] - 3*Sinh[2*ArcS
inh[c + d*x]]))))/(128*d^2)
```

3.101.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6274, 25, 27, 6245, 7267, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$$

$\big\downarrow$ 6274

$$\frac{\int x(a+b\text{arcsinh}(c+dx))^{3/2}d(c+dx)}{d}$$

$\big\downarrow$ 25

$$-\frac{\int -x(a+b\text{arcsinh}(c+dx))^{3/2}d(c+dx)}{d}$$

$\big\downarrow$ 27

$$-\frac{\int -dx(a+b\text{arcsinh}(c+dx))^{3/2}d(c+dx)}{d^2}$$

$\big\downarrow$ 6245

$$-\frac{\int -dx\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}d\text{arcsinh}(c+dx)}{d^2}$$

$\big\downarrow$ 7267

$$-\frac{2\int -dx\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2d\sqrt{a+b\text{arcsinh}(c+dx)}}{bd^2}$$

$\big\downarrow$ 7292

$$-\frac{2\int -dx(a+b\text{arcsinh}(c+dx))^2\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)d\sqrt{a+b\text{arcsinh}(c+dx)}}{bd^2}$$

$\big\downarrow$ 7293

$$-\frac{2\int\left(c\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)(a+b\text{arcsinh}(c+dx))^2+\frac{1}{2}\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)(a+b\text{arcsinh}(c}{bd^2}$$

$\big\downarrow$ 2009

$$-\frac{2\left(\frac{3}{16}\sqrt{\pi}b^{5/2}ce^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{3}{128}\sqrt{\frac{\pi}{2}}b^{5/2}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{3}{16}\sqrt{\pi}b^{5/2}ce^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{}}{}\right.\right.}{}$$

input
```
Int[x*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

3.101. $\int x(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

output
```
(-2*(-1/8*(b*(a + b*ArcSinh[c + d*x])^(3/2)*Cosh[(2*a)/b - (2*(a + b*ArcSi
nh[c + d*x]))/b]) - (3*b^2*c*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[a/b - (a +
b*ArcSinh[c + d*x])/b])/4 + (3*b^(5/2)*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*A
rcSinh[c + d*x]]/Sqrt[b]])/16 + (3*b^(5/2)*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqr
t[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/128 + (3*b^(5/2)*c*Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*E^(a/b)) - (3*b^(5/2)*Sqrt[
Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(128*E^((2*a)/
b)) - (3*b^2*Sqrt[a + b*ArcSinh[c + d*x]]*Sinh[(2*a)/b - (2*(a + b*ArcSinh
[c + d*x]))/b])/32 - (b*c*(a + b*ArcSinh[c + d*x])^(3/2)*Sinh[a/b - (a + b
*ArcSinh[c + d*x])/b])/2))/(b*d^2)
```

3.101.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6245
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[1/c^(m + 1)   Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[
x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m,
0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.101.4 Maple [F]

$$\int x(a + b \operatorname{arcsinh}(dx + c))^{\frac{3}{2}} dx$$

input
```
int(x*(a+b*arcsinh(d*x+c))^(3/2),x)
```

output
```
int(x*(a+b*arcsinh(d*x+c))^(3/2),x)
```

3.101.5 Fricas [F(-2)]

Exception generated.

$$\int x(a + b\text{arcsinh}(c + dx))^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.101.6 Sympy [F]

$$\int x(a + b\text{arcsinh}(c + dx))^{3/2} dx = \int x(a + b \operatorname{asinh}(c + dx))^{\frac{3}{2}} dx$$

input
```
integrate(x*(a+b*asinh(d*x+c))**(3/2),x)
```

output
```
Integral(x*(a + b*asinh(c + d*x))**(3/2), x)
```

3.101.7 Maxima [F]

$$\int x(a+b\text{arcsinh}(c+dx))^{3/2}\,dx=\int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(3/2)*x, x)
```

3.101.8 Giac [F]

$$\int x(a+b\text{arcsinh}(c+dx))^{3/2}\,dx=\int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(3/2)*x, x)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int x(a+b\text{arcsinh}(c+dx))^{3/2}\,dx=\int x\,(a+b\,\text{asinh}(c+d\,x))^{3/2}\,dx$$

input
```
int(x*(a + b*asinh(c + d*x))^(3/2),x)
```

output
```
int(x*(a + b*asinh(c + d*x))^(3/2), x)
```

3.102 $\int (a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

3.102.1 Optimal result

Integrand size = 14, antiderivative size = 150

$$\int (a+b\text{arcsinh}(c+dx))^{3/2}\,dx = -\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{2d} + \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}{d} + \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d} + \frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d}$$

output
```
(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d+3/8*b^(3/2)*exp(a/b)*erf((a+b*arcsinh
(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/8*b^(3/2)*erfi((a+b*arcsinh(d*x+c))^(
1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-3/2*b*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d
*x+c))^(1/2)/d
```

3.102.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.81

$$\int (a+b\text{arcsinh}(c+dx))^{3/2}\,dx=\frac{ae^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}}\right)}{2d}$$
$$+\frac{\sqrt{b}\left(4\sqrt{b}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-3\sqrt{1+(c+dx)^2}+2(c+dx)\text{arcsinh}(c+dx)\right)+(2a+3b)\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a}}{}\right.\right.}{8d}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
(a*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c
 + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c +
d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b)) + (Sqrt[b]*(
4*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c +
d*x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(8*d)
```

3.102.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6273, 6187, 6213, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(c+dx))^{3/2}\,dx$$
$$\Big\downarrow \text{ 6273}$$
$$\frac{\int (a+b\text{arcsinh}(c+dx))^{3/2}d(c+dx)}{d}$$
$$\Big\downarrow \text{ 6187}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\int\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)}{d}$$

$\downarrow$ 6213

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left(\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}b\int\frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+d\right.}{d}$$

$\downarrow$ 6189

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left(\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}\int\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(\right.}{d}$$

$\downarrow$ 3042

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left(\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right.}{\sqrt{a+b\text{arcsinh}(c+dx)}}\right.}{d}$$

$\downarrow$ 3788

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left(\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{2}\left(\frac{1}{2}i\int\frac{ie^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a\right.\right.}{d}$$

$\downarrow$ 26

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}\int\frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c\right.\right.}}{d}$$

$\downarrow$ 2611

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\left(-\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\int e^{\frac{a+b\text{arcsinh}(c+dx)}{b}-\frac{a}{b}}\right.\right.}{d}$$

$\downarrow$ 2633

3.102. $\int(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\left(-\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a}}{\right.}\right.\right.}{d}$$

$$\Big\downarrow \text{2634}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\left(-\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsin}}}{\sqrt{b}}\right.\right.\right.}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
((c + d*x)*(a + b*ArcSinh[c + d*x])^(3/2) - (3*b*(Sqrt[1 + (c + d*x)^2]*Sq
rt[a + b*ArcSinh[c + d*x]] + (-1/2*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a +
b*ArcSinh[c + d*x]]/Sqrt[b]]) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[
c + d*x]]/Sqrt[b]])/(2*E^(a/b)))/2))/2)/d
```

3.102.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.102.4 Maple [F]

$$\int (a + b \operatorname{arcsinh}(dx + c))^{\frac{3}{2}} dx$$

input
```
int((a+b*arcsinh(d*x+c))^(3/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^(3/2),x)
```

3.102.5 Fricas [F(-2)]

Exception generated.

$$\int (a + b\text{arcsinh}(c + dx))^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.102.6 Sympy [F]

$$\int (a + b\text{arcsinh}(c + dx))^{3/2} \, dx = \int (a + b \operatorname{asinh}(c + dx))^{\frac{3}{2}} \, dx$$

input
```
integrate((a+b*asinh(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**(3/2), x)
```

3.102.7 Maxima [F]

$$\int (a + b\text{arcsinh}(c + dx))^{3/2} \, dx = \int (b \operatorname{arsinh}(dx + c) + a)^{\frac{3}{2}} \, dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.102.8 Giac [F]

$$\int (a+b\text{arcsinh}(c+dx))^{3/2}\,dx=\int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arcsinh}(c+dx))^{3/2}\,dx=\int (a+b\,\text{asinh}(c+d\,x))^{3/2}\,dx$$

input
```
int((a + b*asinh(c + d*x))^(3/2),x)
```

output
```
int((a + b*asinh(c + d*x))^(3/2), x)
```

3.103 $\int x(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx$

3.103.1 Optimal result

Integrand size = 16, antiderivative size = 389

$$\begin{aligned}\int x(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx &= -\frac{15b^2c(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d^2}\\
&+\frac{5bc\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{2d^2}\\
&-\frac{c(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{d^2}\\
&+\frac{15b^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}\cosh(2\operatorname{arcsinh}(c+dx))}{64d^2}\\
&+\frac{(a+b\operatorname{arcsinh}(c+dx))^{5/2}\cosh(2\operatorname{arcsinh}(c+dx))}{4d^2}\\
&-\frac{15b^{5/2}ce^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^2}\\
&-\frac{15b^{5/2}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d^2}\\
&+\frac{15b^{5/2}ce^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d^2}\\
&-\frac{15b^{5/2}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d^2}\\
&-\frac{5b(a+b\operatorname{arcsinh}(c+dx))^{3/2}\sinh(2\operatorname{arcsinh}(c+dx))}{16d^2}\end{aligned}$$

output
```
-c*(d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)/d^2+1/4*(a+b*arcsinh(d*x+c))^(5/2)*c
osh(2*arcsinh(d*x+c))/d^2-5/16*b*(a+b*arcsinh(d*x+c))^(3/2)*sinh(2*arcsinh
(d*x+c))/d^2-15/512*b^(5/2)*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1
/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^2-15/512*b^(5/2)*erfi(2^(1/2)*(a+b*arcsinh
(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^2/exp(2*a/b)-15/16*b^(5/2)*c*ex
p(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^2+15/16*b^(5/2)*
c*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^2/exp(a/b)+5/2*b*c*(
a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d^2-15/4*b^2*c*(d*x+c)*(a+b*
arcsinh(d*x+c))^(1/2)/d^2+15/64*b^2*cosh(2*arcsinh(d*x+c))*(a+b*arcsinh(d*
x+c))^(1/2)/d^2
```

3.103.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 939 vs. $2(389) = 778$.

Time = 8.10 (sec) , antiderivative size = 939, normalized size of antiderivative = 2.41

$$\int x(a + b\text{arcsinh}(c + dx))^{5/2} \, dx = \frac{-1920b^2c^2\sqrt{a + b\text{arcsinh}(c + dx)} - 1920b^2cdx\sqrt{a + b\text{arcsinh}(c + dx)} + 1280abc\sqrt{1 + c^2 + 2}}{}$$

input
```
Integrate[x*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
(-1920*b^2*c^2*Sqrt[a + b*ArcSinh[c + d*x]] - 1920*b^2*c*d*x*Sqrt[a + b*Ar
cSinh[c + d*x]] + 1280*a*b*c*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]*Sqrt[a + b*
ArcSinh[c + d*x]] - 1024*a*b*c^2*ArcSinh[c + d*x]*Sqrt[a + b*ArcSinh[c + d
*x]] - 1024*a*b*c*d*x*ArcSinh[c + d*x]*Sqrt[a + b*ArcSinh[c + d*x]] + 1280
*b^2*c*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]*ArcSinh[c + d*x]*Sqrt[a + b*ArcSi
nh[c + d*x]] - 512*b^2*c^2*ArcSinh[c + d*x]^2*Sqrt[a + b*ArcSinh[c + d*x]]
 - 512*b^2*c*d*x*ArcSinh[c + d*x]^2*Sqrt[a + b*ArcSinh[c + d*x]] + 128*a^2
*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + 120*b^2*Sqrt[a +
b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + 256*a*b*ArcSinh[c + d*x]*Sq
rt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + 128*b^2*ArcSinh[c +
d*x]^2*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] - 128*a^2*Sqr
t[b]*c*Sqrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] + 480
*b^(5/2)*c*Sqrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] -
 15*b^(5/2)*Sqrt[2*Pi]*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]] + (256*a^2*b*c*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[
3/2, a/b + ArcSinh[c + d*x]])/Sqrt[a + b*ArcSinh[c + d*x]] + (256*a^2*b*c*
Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[3/2, -((a + b*ArcSinh[c + d*x])/
b)])/(E^(a/b)*Sqrt[a + b*ArcSinh[c + d*x]]) + 128*a^2*Sqrt[b]*c*Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] - 480*b^(5/2)*c*Sqrt[P
i]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 32*Sqrt[b]*(4...
```

3.103.3 Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6274, 25, 27, 6245, 7267, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int x(a+b\text{arcsinh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -x(a+b\text{arcsinh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int -dx(a+b\text{arcsinh}(c+dx))^{5/2}d(c+dx)}{d^2}$$

$\Big\downarrow$ 6245

$$-\frac{\int -dx\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}d\text{arcsinh}(c+dx)}{d^2}$$

$\Big\downarrow$ 7267

$$-\frac{2\int -dx\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3d\sqrt{a+b\text{arcsinh}(c+dx)}}{bd^2}$$

$\Big\downarrow$ 7292

$$-\frac{2\int -dx(a+b\text{arcsinh}(c+dx))^3\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)d\sqrt{a+b\text{arcsinh}(c+dx)}}{bd^2}$$

$\Big\downarrow$ 7293

$$-\frac{2\int \left(c\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)(a+b\text{arcsinh}(c+dx))^3+\frac{1}{2}\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)(a+b\text{arcsinh}(c}{bd^2}$$

$\Big\downarrow$ 2009

$$-\frac{2\left(\frac{15}{32}\sqrt{\pi}b^{7/2}ce^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{15}{512}\sqrt{\frac{\pi}{2}}b^{7/2}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{15}{32}\sqrt{\pi}b^{7/2}ce^{-\frac{a}{b}}\text{erfi}\left(\right.\right.}{}$$

input
```
Int[x*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
(-2*((-15*b^3*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[(2*a)/b - (2*(a + b*ArcSin
h[c + d*x]))/b])/128 - (b*(a + b*ArcSinh[c + d*x])^(5/2)*Cosh[(2*a)/b - (2
*(a + b*ArcSinh[c + d*x]))/b])/8 - (5*b^2*c*(a + b*ArcSinh[c + d*x])^(3/2)
*Cosh[a/b - (a + b*ArcSinh[c + d*x])/b])/4 + (15*b^(7/2)*c*E^(a/b)*Sqrt[Pi
]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/32 + (15*b^(7/2)*E^((2*a)/b)*
Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/512 - (15*
b^(7/2)*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(32*E^(a/b)
) + (15*b^(7/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]])/(512*E^((2*a)/b)) - (5*b^2*(a + b*ArcSinh[c + d*x])^(3/2)*Sinh[(2*a
)/b - (2*(a + b*ArcSinh[c + d*x]))/b])/32 - (15*b^3*c*Sqrt[a + b*ArcSinh[c
 + d*x]]*Sinh[a/b - (a + b*ArcSinh[c + d*x])/b])/8 - (b*c*(a + b*ArcSinh[c
 + d*x])^(5/2)*Sinh[a/b - (a + b*ArcSinh[c + d*x])/b])/2))/(b*d^2)
```

3.103. $\int x(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6245
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[1/c^(m + 1)   Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[
x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m,
0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.103.4 Maple [F]

$$\int x(a+b\,\operatorname{arcsinh}(dx+c))^{\frac{5}{2}}\,dx$$

input
```
int(x*(a+b*arcsinh(d*x+c))^(5/2),x)
```

output
```
int(x*(a+b*arcsinh(d*x+c))^(5/2),x)
```

3.103.5 Fricas [F(-2)]

Exception generated.

$$\int x(a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.103.6 Sympy [F]

$$\int x(a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\int x(a+b\,\operatorname{asinh}(c+dx))^{\frac{5}{2}}\,dx$$

input
```
integrate(x*(a+b*asinh(d*x+c))**(5/2),x)
```

output
```
Integral(x*(a + b*asinh(c + d*x))**(5/2), x)
```

3.103.7 Maxima [F]

$$\int x(a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}x\,dx$$

input
```
integrate(x*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(5/2)*x, x)
```

3.103.8 Giac [F]

$$\int x(a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}x\,dx$$

input
```
integrate(x*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(5/2)*x, x)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int x(a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\int x\,(a+b\,\text{asinh}(c+d\,x))^{5/2}\,dx$$

input
```
int(x*(a + b*asinh(c + d*x))^(5/2),x)
```

output
```
int(x*(a + b*asinh(c + d*x))^(5/2), x)
```

3.104 $\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

3.104.1 Optimal result

Integrand size = 14, antiderivative size = 179

$$\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\frac{15b^2(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{4d}-\frac{5b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{3/2}}{2d}+\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}}{d}+\frac{15b^{5/2}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15b^{5/2}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d}$$

output
```
(d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)/d+15/16*b^(5/2)*exp(a/b)*erf((a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/16*b^(5/2)*erfi((a+b*arcsinh(d*x+c
))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-5/2*b*(a+b*arcsinh(d*x+c))^(3/2)*(1+
(d*x+c)^2)^(1/2)/d+15/4*b^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

3.104.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 458 vs. 2(179) = 358.

Time = 1.45 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.56

$$\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\frac{8a^2e^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}}\right)+4a\sqrt{b}\cdots}{\cdots}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
((8*a^2*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcS
inh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh
[c + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + 4*a*Sqrt[b]
*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c
+ d*x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c
+ d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(4*Sqr
t[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(2*Sqrt[1 + (c + d*x)^2]*(a - 5*b*ArcSin
h[c + d*x]) + b*(c + d*x)*(15 + 4*ArcSinh[c + d*x]^2)) + (4*a^2 + 12*a*b +
 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] +
 Sinh[a/b]) + (4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c
+ d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(16*d)
```

3.104.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6273, 6187, 6213, 6187, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int (a+b\text{arcsinh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{6187}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\int \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}{\sqrt{(c+dx)^2+1}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6213}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\Big(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\int \sqrt{a+b\text{arcsinh}(c+dx)}d}{d}$$

$$\Big\downarrow \text{6187}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx)\sqrt{a+b\text{arcsinh}(\right.\right.}{d}$$

$$\Big\downarrow \text{6234}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx)\sqrt{a+b\text{arcsinh}(\right.\right.}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\int\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c-}{b}\right.}{\sqrt{a+b\text{arcsinh}(c+dx}}\right.\right.}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx)\sqrt{a+b\text{arcsinh}(\right.\right.}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx)\sqrt{a+b\text{arcsinh}(\right.\right.}{d}$$

$$\Big\downarrow \text{3789}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx)\sqrt{a+b\text{arcsinh}(\right.\right.}{}$$

$$\Big\downarrow \text{2611}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx)\sqrt{a+b\text{arcsinh}(\right.\right.}{d}$$

3.104. $\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

$\Big\downarrow$ 2633

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\Bigg((c+dx)\sqrt{a+b\text{arcsinh}(}}{d}$$

$\Big\downarrow$ 2634

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\Bigg((c+dx)\sqrt{a+b\text{arcsinh}(}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
((c + d*x)*(a + b*ArcSinh[c + d*x])^(5/2) - (5*b*(Sqrt[1 + (c + d*x)^2]*(a
 + b*ArcSinh[c + d*x])^(3/2) - (3*b*((c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]
] - (I/2)*((I/2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt
[b]])/E^(a/b))))/2))/2)/d
```

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.104. $\int (a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx$

3.104.4 Maple [F]

$$\int (a+b\,\text{arcsinh}\,(dx+c))^{\frac{5}{2}}\,dx$$

input
```
int((a+b*arcsinh(d*x+c))^(5/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^(5/2),x)
```

3.104.5 Fricas [F(-2)]

Exception generated.

$$\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.104.6 Sympy [F]

$$\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \int (a+b\,\text{asinh}\,(c+dx))^{\frac{5}{2}}\,\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**(5/2), x)
```

3.104.7 Maxima [F]

$$\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.104.8 Giac [F]

$$\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\int (a+b\,\text{asinh}(c+d\,x))^{5/2}\,dx$$

input
```
int((a + b*asinh(c + d*x))^(5/2),x)
```

output
```
int((a + b*asinh(c + d*x))^(5/2), x)
```

3.105 $\int \frac{x^2}{\sqrt{a+b\,\mathrm{arcsinh}(c+dx)}}\,dx$

3.105.1 Optimal result

Integrand size = 18, antiderivative size = 411

$$\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = -\frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d^3} + \frac{c^2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} + \frac{ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} + \frac{e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d^3} - \frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d^3} + \frac{c^2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} - \frac{ce^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^3} + \frac{e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d^3}$$

output
```
1/24*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi
^(1/2)/d^3/b^(1/2)+1/24*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3
^(1/2)*Pi^(1/2)/d^3/exp(3*a/b)/b^(1/2)+1/4*c*exp(2*a/b)*erf(2^(1/2)*(a+b*a
rcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^3/b^(1/2)-1/4*c*erfi(2^(1
/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^3/exp(2*a/b)/b^
(1/2)-1/8*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^3/b^
(1/2)+1/2*c^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^
3/b^(1/2)-1/8*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^3/exp(a/
b)/b^(1/2)+1/2*c^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^3/e
xp(a/b)/b^(1/2)
```

3.105.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$$
$$= \frac{\sqrt{\frac{\pi}{6}}\left(\sqrt{2}\cosh\left(\frac{3a}{b}\right)\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\sqrt{6}\cosh\left(\frac{a}{b}\right)\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+4\sqrt{6}c^2\cosh\left(\frac{a}{b}\right)\text{e}\right.}{}$$

input
```
Integrate[x^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(Sqrt[Pi/6]*(Sqrt[2]*Cosh[(3*a)/b]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x
]])/Sqrt[b]] - Sqrt[6]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]
] + 4*Sqrt[6]*c^2*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - 4
*Sqrt[3]*c*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]] + Sqrt[2]*Cosh[(3*a)/b]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sq
rt[b]] + Sqrt[6]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] - 4*
Sqrt[6]*c^2*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + Sqrt[6]
*(-1 + 4*c^2)*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[
a/b]) + 4*Sqrt[3]*c*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*S
inh[(2*a)/b] + 4*Sqrt[3]*c*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + Sqrt[2]*Erf[(Sqrt[3]*Sqrt[a + b*Arc
Sinh[c + d*x]])/Sqrt[b]]*Sinh[(3*a)/b] - Sqrt[2]*Erfi[(Sqrt[3]*Sqrt[a + b*
ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(3*a)/b]))/(8*Sqrt[b]*d^3)
```

3.105.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6274, 27, 6245, 7267, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{d}$$
$$\Big\downarrow \; 27$$
$$\frac{\int \frac{d^2x^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{d^3}$$
$$\Big\downarrow \; 6245$$
$$\frac{\int \frac{d^2x^2\sqrt{(c+dx)^2+1}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d\text{arcsinh}(c+dx)}{d^3}$$
$$\Big\downarrow \; 7267$$
$$\frac{2\int d^2x^2\sqrt{(c+dx)^2+1}d\sqrt{a+b\text{arcsinh}(c+dx)}}{bd^3}$$
$$\Big\downarrow \; 7292$$
$$\frac{2\int d^2x^2\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)d\sqrt{a+b\text{arcsinh}(c+dx)}}{bd^3}$$
$$\Big\downarrow \; 7293$$
$$\frac{2\int \left(\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)c^2+\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)c+\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh^2\left(\frac{a}{b}\right.\right.}{bd^3}$$
$$\Big\downarrow \; 2009$$
$$2\left(\frac{1}{4}\sqrt{\pi}\sqrt{b}c^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\pi}\sqrt{b}c^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}ce^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{ar}}}{}\right.\right.$$

input
```
Int[x^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

3.105. $\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}dx$

output
```
(2*(-1/16*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[
b]]) + (Sqrt[b]*c^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt
[b]])/4 + (Sqrt[b]*c*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSin
h[c + d*x]])/Sqrt[b]])/4 + (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sq
rt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/16 - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*E^(a/b)) + (Sqrt[b]*c^2*Sqrt[Pi]*Erfi[S
qrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*E^(a/b)) - (Sqrt[b]*c*Sqrt[Pi/2]*
Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(4*E^((2*a)/b)) + (S
qrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(1
6*E^((3*a)/b))))/(b*d^3)
```

3.105.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6245
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[1/c^(m + 1)   Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[
x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m,
0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.105. $\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx$

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.105.4 Maple [F]

$$\int \frac{x^2}{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}dx$$

input
```
int(x^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

output
```
int(x^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

3.105.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+b\mathrm{arcsinh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.105.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a+b\mathrm{arcsinh}(c+dx)}}\,dx = \int \frac{x^2}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx$$

input
```
integrate(x**2/(a+b*asinh(d*x+c))**(1/2),x)
```

output
```
Integral(x**2/sqrt(a + b*asinh(c + d*x)), x)
```

3.105.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{a + b\text{arcsinh}(c + dx)}}\,dx = \int \frac{x^2}{\sqrt{b\,\text{arsinh}\,(dx + c) + a}}\,dx$$

input
```
integrate(x^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.105.8 Giac [F]

$$\int \frac{x^2}{\sqrt{a + b\text{arcsinh}(c + dx)}}\,dx = \int \frac{x^2}{\sqrt{b\,\text{arsinh}\,(dx + c) + a}}\,dx$$

input
```
integrate(x^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a + b\text{arcsinh}(c + dx)}}\,dx = \int \frac{x^2}{\sqrt{a + b\,\text{asinh}\,(c + d\,x)}}\,dx$$

input
```
int(x^2/(a + b*asinh(c + d*x))^(1/2),x)
```

output
```
int(x^2/(a + b*asinh(c + d*x))^(1/2), x)
```

3.106 $\int \frac{x}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$

3.106.1 Optimal result

Integrand size = 16, antiderivative size = 204

$$\int \frac{x}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = -\frac{ce^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^2} - \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d^2} - \frac{ce^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d^2} + \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d^2}$$

output
```
-1/8*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi
^(1/2)/d^2/b^(1/2)+1/8*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^
(1/2)*Pi^(1/2)/d^2/exp(2*a/b)/b^(1/2)-1/2*c*exp(a/b)*erf((a+b*arcsinh(d*x+
c))^(1/2)/b^(1/2))*Pi^(1/2)/d^2/b^(1/2)-1/2*c*erfi((a+b*arcsinh(d*x+c))^(1
/2)/b^(1/2))*Pi^(1/2)/d^2/exp(a/b)/b^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.06

$$\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \frac{\frac{e^{-\frac{a}{b}}\left(4ce^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)-4c\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)\right)}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}} - \frac{\sqrt{2\pi}\left(\operatorname{erfi}\left(\frac{\sqrt{2}\ldots\right.\right.}{\ldots}}{8d^2}$$

input
```
Integrate[x/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
((4*c*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c
+ d*x]] - 4*c*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcS
inh[c + d*x])/b)])/(E^(a/b)*Sqrt[a + b*ArcSinh[c + d*x]]) - (Sqrt[2*Pi]*(E
rfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(-Cosh[(2*a)/b] + Sinh
[(2*a)/b]) + Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*
a)/b] + Sinh[(2*a)/b])))/Sqrt[b])/(8*d^2)
```

3.106.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6274, 25, 27, 6245, 7267, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{x}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int -\frac{dx}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{d^2}$$

$\Big\downarrow$ 6245

$$-\frac{\int -\frac{dx\sqrt{(c+dx)^2+1}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d\text{arcsinh}(c+dx)}{d^2}$$

$\Big\downarrow$ 7267

$$-\frac{2\int -dx\sqrt{(c+dx)^2+1}d\sqrt{a+b\text{arcsinh}(c+dx)}}{bd^2}$$

$\Big\downarrow$ 7292

$$-\frac{2\int -dx\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)d\sqrt{a+b\text{arcsinh}(c+dx)}}{bd^2}$$

$\Big\downarrow$ 7293

$$-\frac{2\int \left(c\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)+\frac{1}{2}\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)\right)d\sqrt{a+b\text{arcsinh}(c+dx)}}{bd^2}$$

$\Big\downarrow$ 2009

$$-\frac{2\left(\frac{1}{4}\sqrt{\pi}\sqrt{b}ce^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\pi}\sqrt{b}ce^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arc}}}{}\right.\right.}{bd^2}$$

input
```
Int[x/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(-2*((Sqrt[b]*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]
)/4 + (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/8 + (Sqrt[b]*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]])/(4*E^(a/b)) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(8*E^((2*a)/b))))/(b*d^2)
```

3.106. $\int \frac{x}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$

3.106.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6245
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[1/c^(m + 1)   Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[
x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m,
0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.106.4 Maple [F]

$$\int \frac{x}{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}dx$$

input
```
int(x/(a+b*arcsinh(d*x+c))^(1/2),x)
```

output
```
int(x/(a+b*arcsinh(d*x+c))^(1/2),x)
```

3.106.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+b\mathrm{arcsinh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.106.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+b\mathrm{arcsinh}(c+dx)}}\,dx = \int \frac{x}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx$$

input
```
integrate(x/(a+b*asinh(d*x+c))**(1/2),x)
```

output
```
Integral(x/sqrt(a + b*asinh(c + d*x)), x)
```

3.106.7 Maxima [F]

$$\int \frac{x}{\sqrt{a + b\text{arcsinh}(c + dx)}}\,dx = \int \frac{x}{\sqrt{b\,\text{arsinh}\,(dx + c) + a}}\,dx$$

input
```
integrate(x/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.106.8 Giac [F]

$$\int \frac{x}{\sqrt{a + b\text{arcsinh}(c + dx)}}\,dx = \int \frac{x}{\sqrt{b\,\text{arsinh}\,(dx + c) + a}}\,dx$$

input
```
integrate(x/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a + b\text{arcsinh}(c + dx)}}\,dx = \int \frac{x}{\sqrt{a + b\,\text{asinh}\,(c + d\,x)}}\,dx$$

input
```
int(x/(a + b*asinh(c + d*x))^(1/2),x)
```

output
```
int(x/(a + b*asinh(c + d*x))^(1/2), x)
```

3.107 $\int \frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$

3.107.1 Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx=\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

output
```
1/2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/
2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

3.107.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx=\frac{e^{-\frac{a}{b}}\left(-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)+\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\text{arcsinh}(c+dx)}{b}\right.\right.}{2d\sqrt{a+b\text{arcsinh}(c+dx)}}$$

input
```
Integrate[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d
*x]]) + Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c
+ d*x])/b)])/(2*d*E^(a/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

3.107.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6273, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}dx$$

$$\Big\downarrow \; 6273$$

$$\frac{\int \frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \; 6189$$

$$\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \; 3788$$

$$\frac{\frac{1}{2}i\int -\frac{ie^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int \frac{ie^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{1}{2}\int \frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))+\frac{1}{2}\int \frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \textcolor{blue}{2611}$$

$$\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}+\int e^{\frac{a+b\text{arcsinh}(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}}{bd}$$

$$\Big\downarrow \textcolor{blue}{2633}$$

$$\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{bd}$$

$$\Big\downarrow \textcolor{blue}{2634}$$

$$\frac{\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{bd}$$

input
```
Int[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/2 +
(Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*E^(a/b)))
/(b*d)
```

3.107.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.107.4 Maple [F]

$$\int \frac{1}{\sqrt{a + b \operatorname{arcsinh}\left(dx + c\right)}}dx$$

input
```
int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

output
```
int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

3.107.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a + b\text{arcsinh}(c + dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.107.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+b\,\text{asinh}\,(c+dx)}}\,dx$$

input
```
integrate(1/(a+b*asinh(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*asinh(c + d*x)), x)
```

3.107.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\,\text{arsinh}\,(dx+c)+a}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.107.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\,\text{arsinh}\,(dx+c)+a}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + b\text{arcsinh}(c + dx)}}\, dx = \int \frac{1}{\sqrt{a + b\,\text{asinh}\,(c + d\,x)}}\, dx$$

input
```
int(1/(a + b*asinh(c + d*x))^(1/2),x)
```

output
```
int(1/(a + b*asinh(c + d*x))^(1/2), x)
```

3.108 $\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$

3.108.1 Optimal result

Integrand size = 16, antiderivative size = 269

$$\begin{aligned}\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx &= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\text{arcsinh}(c+dx)}} \\ &- \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{ce^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2} \\ &+ \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2} - \frac{ce^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2} \\ &+ \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}\end{aligned}$$

output
```
1/2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^
(1/2)/b^(3/2)/d^2+1/2*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(
1/2)*Pi^(1/2)/b^(3/2)/d^2/exp(2*a/b)+c*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(
1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d^2-c*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*Pi^(1/2)/b^(3/2)/d^2/exp(a/b)+2*c*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsi
nh(d*x+c))^(1/2)-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsinh(d*x+c))^
(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.20

$$\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \frac{\frac{4\sqrt{b}c\sqrt{1+(c+dx)^2}}{\sqrt{a+b\text{arcsinh}(c+dx)}} - 2c\sqrt{\pi}\cosh\left(\frac{a}{b}\right)\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right) + \sqrt{2\pi}\cosh}{}$$

input
```
Integrate[x/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
((4*Sqrt[b]*c*Sqrt[1 + (c + d*x)^2])/Sqrt[a + b*ArcSinh[c + d*x]] - 2*c*Sq
rt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] + Sqrt[2*Pi]*C
osh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] + 2*c*Sq
rt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 2*c*Sqrt[Pi]
*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) - Sqrt[
2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] +
 Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a
)/b] + Sinh[(2*a)/b]) - (2*Sqrt[b]*Sinh[2*ArcSinh[c + d*x]])/Sqrt[a + b*Ar
cSinh[c + d*x]])/(2*b^(3/2)*d^2)
```

3.108.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6274, 25, 27, 6244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{x}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int -\frac{dx}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{d^2}$$

$$\Big\downarrow \ 6244$$

$$-\frac{\int \left(\frac{c}{(a+b\text{arcsinh}(c+dx))^{3/2}}-\frac{c+dx}{(a+b\text{arcsinh}(c+dx))^{3/2}}\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\frac{\sqrt{\pi}ce^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}}+\frac{\sqrt{\pi}ce^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}}-\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}e}{d^2}$$

input
```
Int[x/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
-(((-2*c*Sqrt[1 + (c + d*x)^2])/(b*Sqrt[a + b*ArcSinh[c + d*x]]) + (2*(c +
 d*x)*Sqrt[1 + (c + d*x)^2])/(b*Sqrt[a + b*ArcSinh[c + d*x]]) - (c*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/b^(3/2) - (E^((2*a)/b
)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/b^(3/2)
+ (c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*E^(a/b)
) - (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(b^(
3/2)*E^((2*a)/b)))/d^2)
```

3.108.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6244
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

3.108. $\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.108.4 Maple [F]

$$\int \frac{x}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{3}{2}}} dx$$

input
```
int(x/(a+b*arcsinh(d*x+c))^(3/2),x)
```

output
```
int(x/(a+b*arcsinh(d*x+c))^(3/2),x)
```

3.108.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a + b\text{arcsinh}(c + dx))^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.108.6 Sympy [F]

$$\int \frac{x}{(a + b\text{arcsinh}(c + dx))^{3/2}} \, dx = \int \frac{x}{(a + b \operatorname{asinh}(c + dx))^{\frac{3}{2}}} \, dx$$

input
```
integrate(x/(a+b*asinh(d*x+c))**(3/2),x)
```

output
```
Integral(x/(a + b*asinh(c + d*x))**(3/2), x)
```

3.108.7 Maxima [F]

$$\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{x}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.108.8 Giac [F]

$$\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{x}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{x}{(a+b\,\text{asinh}\,(c+d\,x))^{3/2}}\,dx$$

input
```
int(x/(a + b*asinh(c + d*x))^(3/2),x)
```

output
```
int(x/(a + b*asinh(c + d*x))^(3/2), x)
```

3.109 $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$

3.109.1 Optimal result

Integrand size = 14, antiderivative size = 122

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}$$

output
```
-exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d+erfi(
(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a/b)-2*(1+(d*x+
c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \frac{e^{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\left(-e^{a/b}\left(1+e^{2\text{arcsinh}(c+dx)}\right)+e^{\frac{2a}{b}+\text{arcsinh}(c+dx)}\sqrt{\frac{a}{b}+\text{arcsinl}\right.}{b}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

output
```
(-(E^(a/b)*(1 + E^(2*ArcSinh[c + d*x])) + E^((2*a)/b + ArcSinh[c + d*x])*
Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*x]] + E^ArcSin
h[c + d*x]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh
[c + d*x])/b)])/(b*d*E^((a + b*ArcSinh[c + d*x])/b)*Sqrt[a + b*ArcSinh[c +
 d*x]])
```

3.109.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6273, 6188, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6188}$$

$$\frac{\frac{2\int \frac{c+dx}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}}{d}$$

$$\Big\downarrow \text{6234}$$

$$\frac{\frac{2\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{2\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{2\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}}{d}$$

$$\Big\downarrow \text{3789}$$

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\int \frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int \frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{b^2}}{d}$$

$$\Big\downarrow \text{2611}$$

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-i\int e^{\frac{a+b\text{arcsinh}(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}\right)}{b^2}}{d}$$

$$\Big\downarrow \text{2633}$$

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}}{d}$$

$$\Big\downarrow \text{2634}$$

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

3.109. $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$

output
```
((-2*Sqrt[1 + (c + d*x)^2])/(b*Sqrt[a + b*ArcSinh[c + d*x]]) + ((2*I)*((I/
2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - ((
I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/E^(a/b))
)/b^2)/d
```

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.109. $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.109.4 Maple [F]

$$\int \frac{1}{\left(a+b \operatorname{arcsinh}\left(d x+c\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

output
```
int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

3.109.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.109.6 Sympy [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*asinh(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**(-3/2), x)
```

3.109.7 Maxima [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

3.109.8 Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+b\operatorname{asinh}\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(a + b*asinh(c + d*x))^(3/2),x)
```

output
```
int(1/(a + b*asinh(c + d*x))^(3/2), x)
```

3.110 $\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

3.110.1 Optimal result

Integrand size = 16, antiderivative size = 365

$$\begin{aligned}\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx &= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2(a+b\text{arcsinh}(c+dx))^{3/2}} \\ &- \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{4}{3b^2d^2\sqrt{a+b\text{arcsinh}(c+dx)}} \\ &+ \frac{4c(c+dx)}{3b^2d^2\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{8(c+dx)^2}{3b^2d^2\sqrt{a+b\text{arcsinh}(c+dx)}} \\ &- \frac{2ce^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2} - \frac{2e^{\frac{2a}{b}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2} \\ &- \frac{2ce^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2} + \frac{2e^{-\frac{2a}{b}}\sqrt{2\pi}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}\end{aligned}$$

output
```
-2/3*c*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d
^2-2/3*c*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d^2/exp
(a/b)-2/3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/
2)*Pi^(1/2)/b^(5/2)/d^2+2/3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2
))*2^(1/2)*Pi^(1/2)/b^(5/2)/d^2/exp(2*a/b)+2/3*c*(1+(d*x+c)^2)^(1/2)/b/d^2
/(a+b*arcsinh(d*x+c))^(3/2)-2/3*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arc
sinh(d*x+c))^(3/2)-4/3/b^2/d^2/(a+b*arcsinh(d*x+c))^(1/2)+4/3*c*(d*x+c)/b^
2/d^2/(a+b*arcsinh(d*x+c))^(1/2)-8/3*(d*x+c)^2/b^2/d^2/(a+b*arcsinh(d*x+c)
)^(1/2)
```

3.110.2 Mathematica [A] (verified)

Time = 3.61 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.52

$$\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx =$$

$$-\frac{-4a\sqrt{b}c(c+dx)-2b^{3/2}c\sqrt{1+(c+dx)^2}-4b^{3/2}c(c+dx)\text{arcsinh}(c+dx)+4a\sqrt{b}\cosh(2\text{arcsinh}(c+dx)}{}$$

input
```
Integrate[x/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
-1/3*(-4*a*Sqrt[b]*c*(c + d*x) - 2*b^(3/2)*c*Sqrt[1 + (c + d*x)^2] - 4*b^(
3/2)*c*(c + d*x)*ArcSinh[c + d*x] + 4*a*Sqrt[b]*Cosh[2*ArcSinh[c + d*x]] +
 4*b^(3/2)*ArcSinh[c + d*x]*Cosh[2*ArcSinh[c + d*x]] + 2*c*Sqrt[Pi]*(a + b
*ArcSinh[c + d*x])^(3/2)*Cosh[a/b]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b
]] + 2*Sqrt[2*Pi]*(a + b*ArcSinh[c + d*x])^(3/2)*Cosh[(2*a)/b]*Erf[(Sqrt[2
]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] + 2*c*Sqrt[Pi]*(a + b*ArcSinh[c +
 d*x])^(3/2)*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - 2*Sqrt
[2*Pi]*(a + b*ArcSinh[c + d*x])^(3/2)*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a +
 b*ArcSinh[c + d*x]])/Sqrt[b]] + 2*c*Sqrt[Pi]*(a + b*ArcSinh[c + d*x])^(3/
2)*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] - 2*c*Sqrt[Pi]*(a +
 b*ArcSinh[c + d*x])^(3/2)*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh
[a/b] + 2*Sqrt[2*Pi]*(a + b*ArcSinh[c + d*x])^(3/2)*Erf[(Sqrt[2]*Sqrt[a +
b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] + 2*Sqrt[2*Pi]*(a + b*ArcSinh[
c + d*x])^(3/2)*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[
(2*a)/b] + b^(3/2)*Sinh[2*ArcSinh[c + d*x]])/(b^(5/2)*d^2*(a + b*ArcSinh[c
 + d*x])^(3/2))
```

3.110.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6274, 25, 27, 6244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{x}{(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int -\frac{dx}{(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{d^2}$$

$$\Big\downarrow \; 6244$$

$$-\frac{\int \left(\frac{c}{(a+b\text{arcsinh}(c+dx))^{5/2}}-\frac{c+dx}{(a+b\text{arcsinh}(c+dx))^{5/2}}\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{2\sqrt{\pi}ce^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}}+\frac{2\sqrt{2\pi}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}}+\frac{2\sqrt{\pi}ce^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}}-\frac{2\sqrt{2\pi}e^{-\frac{2a}{b}}\cdots}{\cdots}}{\cdots}$$

input
```
Int[x/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
-(((-2*c*Sqrt[1 + (c + d*x)^2])/(3*b*(a + b*ArcSinh[c + d*x])^(3/2)) + (2*
(c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b*(a + b*ArcSinh[c + d*x])^(3/2)) + 4/
(3*b^2*Sqrt[a + b*ArcSinh[c + d*x]]) - (4*c*(c + d*x))/(3*b^2*Sqrt[a + b*A
rcSinh[c + d*x]]) + (8*(c + d*x)^2)/(3*b^2*Sqrt[a + b*ArcSinh[c + d*x]]) +
 (2*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/
2)) + (2*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])
/Sqrt[b]])/(3*b^(5/2)) + (2*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/S
qrt[b]])/(3*b^(5/2)*E^(a/b)) - (2*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*E^((2*a)/b)))/d^2)
```

3.110. $\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6244
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.110.4 Maple [F]

$$\int \frac{x}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(x/(a+b*arcsinh(d*x+c))^(5/2),x)
```

output
```
int(x/(a+b*arcsinh(d*x+c))^(5/2),x)
```

3.110.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+b\mathrm{arcsinh}(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

3.110.6 Sympy [F]

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{x}{(a+b\operatorname{asinh}(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a+b*asinh(d*x+c))**(5/2),x)
```

output
```
Integral(x/(a + b*asinh(c + d*x))**(5/2), x)
```

3.110.7 Maxima [F]

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{x}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.110.8 Giac [F]

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{x}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(x/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{x}{\left(a+b\operatorname{asinh}\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(x/(a + b*asinh(c + d*x))^(5/2),x)
```

output
```
int(x/(a + b*asinh(c + d*x))^(5/2), x)
```

3.111 $\int \frac{1}{(a+b\,\mathrm{arcsinh}(c+dx))^{5/2}}\,dx$

3.111.1 Optimal result

Integrand size = 14, antiderivative size = 158

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}(c+dx))^{5/2}}\,dx=-\frac{2\sqrt{1+(c+dx)^2}}{3bd(a+b\,\mathrm{arcsinh}(c+dx))^{3/2}}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\,\mathrm{arcsinh}(c+dx)}}+\frac{2e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}$$

output
```
2/3*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d+2/
3*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(a/b)-2/3
*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3/2)-4/3*(d*x+c)/b^2/d/(a+b
*arcsinh(d*x+c))^(1/2)
```

3.111.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.31

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \frac{e^{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\left(-e^{a/b}\left(b+2a\left(-1+e^{2\text{arcsinh}(c+dx)}\right)-2b\text{arcsinh}(c+dx)\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

output
```
(-(E^(a/b)*(b + 2*a*(-1 + E^(2*ArcSinh[c + d*x])) - 2*b*ArcSinh[c + d*x] +
 b*E^(2*ArcSinh[c + d*x])*(1 + 2*ArcSinh[c + d*x]))) - 2*E^((2*a)/b + ArcS
inh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[
1/2, a/b + ArcSinh[c + d*x]] - 2*b*E^ArcSinh[c + d*x]*(-((a + b*ArcSinh[c
+ d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)])/(3*b^2*d*E^((
a + b*ArcSinh[c + d*x])/b)*(a + b*ArcSinh[c + d*x])^(3/2))
```

3.111.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6273, 6188, 6233, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 6273}$$

$$\frac{\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6188}$$

$$\frac{\frac{2\int \frac{c+dx}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{ 6233}$$

$$\frac{\frac{2\left(\frac{2\int \frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \; 6189$$

$$\frac{\frac{2\left(\frac{2\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}\right)}{3b}}{d}$$

$$\Big\downarrow \; 3788$$

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2\left(\frac{1}{2}i\int -\frac{ie^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int \frac{ie^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d\right.}{b^2}}{3b}}{d}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{2\left(\frac{2\left(\frac{1}{2}\int \frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))+\frac{1}{2}\int \frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2\sqrt{}}{3b(a+b\text{ar}}}{d}$$

$$\Big\downarrow \; 2611$$

$$\frac{\frac{2\left(\frac{2\left(\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}+\int e^{\frac{a+b\text{arcsinh}(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}\right)}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2\sqrt{(c}}{3b(a+b\text{arcs}}}{d}$$

3.111. $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \text{2633}$$

$$\frac{\frac{2\left(\frac{2\left(\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\frac{2\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

output
```
((-2*Sqrt[1 + (c + d*x)^2])/(3*b*(a + b*ArcSinh[c + d*x])^(3/2)) + (2*((-2
*(c + d*x))/(b*Sqrt[a + b*ArcSinh[c + d*x]]) + (2*((Sqrt[b]*E^(a/b)*Sqrt[P
i]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/2 + (Sqrt[b]*Sqrt[Pi]*Erfi[S
qrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*E^(a/b))))/b^2))/(3*b))/d
```

3.111.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.111. $\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx$

3.111.4 Maple [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
```

output
```
int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
```

3.111.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.111.6 Sympy [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\operatorname{asinh}(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*asinh(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**(-5/2), x)
```

3.111.7 Maxima [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

3.111.8 Giac [F]

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\operatorname{asinh}(c+d\,x))^{5/2}}\,dx$$

input
```
int(1/(a + b*asinh(c + d*x))^(5/2),x)
```

output
```
int(1/(a + b*asinh(c + d*x))^(5/2), x)
```

3.112 $\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

3.112.1 Optimal result

Integrand size = 16, antiderivative size = 445

$$\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{4}{15b^2d^2(a+b\text{arcsinh}(c+dx))^{3/2}} + \frac{4c(c+dx)}{15b^2d^2(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{8(c+dx)^2}{15b^2d^2(a+b\text{arcsinh}(c+dx))^{3/2}} + \frac{8c\sqrt{1+(c+dx)^2}}{15b^3d^2\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{32(c+dx)\sqrt{1+(c+dx)^2}}{15b^3d^2\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{4ce^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2} + \frac{8e^{\frac{2a}{b}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2} - \frac{4ce^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2} + \frac{8e^{-\frac{2a}{b}}\sqrt{2\pi}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}$$

output
```
-4/15/b^2/d^2/(a+b*arcsinh(d*x+c))^(3/2)+4/15*c*(d*x+c)/b^2/d^2/(a+b*arcsi
nh(d*x+c))^(3/2)-8/15*(d*x+c)^2/b^2/d^2/(a+b*arcsinh(d*x+c))^(3/2)+4/15*c*
exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d^2-4/15
*c*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d^2/exp(a/b)+
8/15*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi
^(1/2)/b^(7/2)/d^2+8/15*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2
^(1/2)*Pi^(1/2)/b^(7/2)/d^2/exp(2*a/b)+2/5*c*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+
b*arcsinh(d*x+c))^(5/2)-2/5*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsinh
(d*x+c))^(5/2)+8/15*c*(1+(d*x+c)^2)^(1/2)/b^3/d^2/(a+b*arcsinh(d*x+c))^(1/
2)-32/15*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b^3/d^2/(a+b*arcsinh(d*x+c))^(1/2)
```

3.112.2 Mathematica [A] (verified)

Time = 3.12 (sec) , antiderivative size = 703, normalized size of antiderivative = 1.58

$$\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \frac{4ab^{3/2}c(c+dx)+8a^2\sqrt{b}c\sqrt{1+(c+dx)^2}+6b^{5/2}c\sqrt{1+(c+dx)^2}+4b^{5/2}}{}$$

input
```
Integrate[x/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

output
```
(4*a*b^(3/2)*c*(c + d*x) + 8*a^2*Sqrt[b]*c*Sqrt[1 + (c + d*x)^2] + 6*b^(5/
2)*c*Sqrt[1 + (c + d*x)^2] + 4*b^(5/2)*c*(c + d*x)*ArcSinh[c + d*x] + 16*a
*b^(3/2)*c*Sqrt[1 + (c + d*x)^2]*ArcSinh[c + d*x] + 8*b^(5/2)*c*Sqrt[1 + (
c + d*x)^2]*ArcSinh[c + d*x]^2 - 4*a*b^(3/2)*Cosh[2*ArcSinh[c + d*x]] - 4*
b^(5/2)*ArcSinh[c + d*x]*Cosh[2*ArcSinh[c + d*x]] + 4*c*Sqrt[Pi]*(a + b*Ar
cSinh[c + d*x])^(5/2)*Cosh[a/b]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]
+ 8*Sqrt[2*Pi]*(a + b*ArcSinh[c + d*x])^(5/2)*Cosh[(2*a)/b]*Erf[(Sqrt[2]*S
qrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - 4*c*Sqrt[Pi]*(a + b*ArcSinh[c + d*
x])^(5/2)*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] + 8*Sqrt[2*
Pi]*(a + b*ArcSinh[c + d*x])^(5/2)*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*
ArcSinh[c + d*x]])/Sqrt[b]] + 4*c*Sqrt[Pi]*(a + b*ArcSinh[c + d*x])^(5/2)*
Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 4*c*Sqrt[Pi]*(a + b*
ArcSinh[c + d*x])^(5/2)*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/
b] + 8*Sqrt[2*Pi]*(a + b*ArcSinh[c + d*x])^(5/2)*Erf[(Sqrt[2]*Sqrt[a + b*A
rcSinh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] - 8*Sqrt[2*Pi]*(a + b*ArcSinh[c +
 d*x])^(5/2)*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(2*
a)/b] - 16*a^2*Sqrt[b]*Sinh[2*ArcSinh[c + d*x]] - 3*b^(5/2)*Sinh[2*ArcSinh
[c + d*x]] - 32*a*b^(3/2)*ArcSinh[c + d*x]*Sinh[2*ArcSinh[c + d*x]] - 16*b
^(5/2)*ArcSinh[c + d*x]^2*Sinh[2*ArcSinh[c + d*x]])/(15*b^(7/2)*d^2*(a + b
*ArcSinh[c + d*x])^(5/2))
```

3.112.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6274, 25, 27, 6244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \frac{x}{(a+b\text{arcsinh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{x}{(a+b\text{arcsinh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int -\frac{dx}{(a+b\text{arcsinh}(c+dx))^{7/2}}d(c+dx)}{d^2}$$

$$\Big\downarrow \; 6244$$

$$-\frac{\int \left(\frac{c}{(a+b\text{arcsinh}(c+dx))^{7/2}}-\frac{c+dx}{(a+b\text{arcsinh}(c+dx))^{7/2}}\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{-\frac{4\sqrt{\pi}ce^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}}-\frac{8\sqrt{2\pi}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}}+\frac{4\sqrt{\pi}ce^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}}-8\sqrt{2\pi}e^{-}}{}$$

input
```
Int[x/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

output
```
-(((-2*c*Sqrt[1 + (c + d*x)^2])/(5*b*(a + b*ArcSinh[c + d*x])^(5/2)) + (2*
(c + d*x)*Sqrt[1 + (c + d*x)^2])/(5*b*(a + b*ArcSinh[c + d*x])^(5/2)) + 4/
(15*b^2*(a + b*ArcSinh[c + d*x])^(3/2)) - (4*c*(c + d*x))/(15*b^2*(a + b*A
rcSinh[c + d*x])^(3/2)) + (8*(c + d*x)^2)/(15*b^2*(a + b*ArcSinh[c + d*x])
^(3/2)) - (8*c*Sqrt[1 + (c + d*x)^2])/(15*b^3*Sqrt[a + b*ArcSinh[c + d*x]]
) + (32*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(15*b^3*Sqrt[a + b*ArcSinh[c + d*
x]]) - (4*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(1
5*b^(7/2)) - (8*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c +
 d*x]])/Sqrt[b]])/(15*b^(7/2)) + (4*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
 d*x]]/Sqrt[b]])/(15*b^(7/2)*E^(a/b)) - (8*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a
 + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*E^((2*a)/b)))/d^2)
```

3.112.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6244
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.112.4 Maple [F]

$$\int \frac{x}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}}} dx$$

input
```
int(x/(a+b*arcsinh(d*x+c))^(7/2),x)
```

output
```
int(x/(a+b*arcsinh(d*x+c))^(7/2),x)
```

3.112.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a + b\text{arcsinh}(c + dx))^{7/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.112.6 Sympy [F]

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{x}{(a+b\operatorname{asinh}(c+dx))^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/(a+b*asinh(d*x+c))**(7/2),x)
```

output
```
Integral(x/(a + b*asinh(c + d*x))**(7/2), x)
```

3.112.7 Maxima [F]

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{x}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate(x/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.112.8 Giac [F]

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{x}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(x/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{x}{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^{7/2}} \,dx$$

input
```
int(x/(a + b*asinh(c + d*x))^(7/2),x)
```

output
```
int(x/(a + b*asinh(c + d*x))^(7/2), x)
```

3.113 $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}} dx$

3.113.1 Optimal result

Integrand size = 14, antiderivative size = 195

$$\begin{aligned}\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx &= -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} \\ &\quad -\frac{4(c+dx)}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}}-\frac{8\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} \\ &\quad -\frac{4e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}\end{aligned}$$

output
```
-4/15*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-4/15*exp(a/b)*erf((a+b*arcs
inh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+4/15*erfi((a+b*arcsinh(d*x+c
))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)-2/5*(1+(d*x+c)^2)^(1/2)/b/d/
(a+b*arcsinh(d*x+c))^(5/2)-8/15*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x
+c))^(1/2)
```

3.113.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \frac{-6b^2e^{\text{arcsinh}(c+dx)} - 2e^{-\text{arcsinh}(c+dx)}(4a^2+2ab(-1+4\text{arcsinh}(c+dx))+b}{}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

output
```
(-6*b^2*E^ArcSinh[c + d*x] - (2*(4*a^2 + 2*a*b*(-1 + 4*ArcSinh[c + d*x]) +
 b^2*(3 - 2*ArcSinh[c + d*x] + 4*ArcSinh[c + d*x]^2)))/E^ArcSinh[c + d*x]
+ 8*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*Gamma[
1/2, a/b + ArcSinh[c + d*x]] - (4*(a + b*ArcSinh[c + d*x])*(E^(a/b + ArcSi
nh[c + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b*(-((a + b*ArcSinh[c +
d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b))/(30*b
^3*d*(a + b*ArcSinh[c + d*x])^(5/2))
```

3.113.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6273, 6188, 6233, 6188, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6188}$$

$$\frac{\frac{2\int \frac{c+dx}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{5b} - \frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \text{6233}$$

$$\frac{\frac{2\left(\frac{2\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}-\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}}{d}$$

$\downarrow$ 6188

$$\frac{\frac{2\left(\frac{2\left(\frac{2\int \frac{c+dx}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}-\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}}{d}$$

$\downarrow$ 6234

$$\frac{\frac{2\left(\frac{2\left(\frac{2\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}-\frac{2\sqrt{(c+dx)^2+}}{5b(a+b\text{arcsinh}(c+}}{d}$$

$\downarrow$ 25

$$\frac{\frac{2\left(\frac{2\left(-\frac{2\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}-\frac{2\sqrt{(c+dx)^2+}}{5b(a+b\text{arcsinh}(c+}}{d}$$

$\downarrow$ 3042

3.113. $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}+\frac{2\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{2\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arc}}{b^2}}{3b}}{5b}}{d}$$

$\downarrow$ 26

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}+\frac{2\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsi}}{b^2}}{3b}}{5b}}{d}$$

$\downarrow$ 3789

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}+\frac{2\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\int \frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+}{}\right.}{3b}}{5b}}{d}$$

$\downarrow$ 2611

3.113. $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}+\frac{2\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+\cdots}{\cdots}\right)}{3b}\cdots\right)}{5b}\cdots}{d}$$

$\downarrow$ 2633

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}+\frac{2\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+\cdots}{b^{\cdots}}\right)}{3b}\cdots\right)}{5b}\cdots}{d}$$

$\downarrow$ 2634

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}+\frac{2\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\cdots}{b^2}\right)}{3b}\cdots\right)}{5b}\cdots}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

output
```
((-2*Sqrt[1 + (c + d*x)^2])/(5*b*(a + b*ArcSinh[c + d*x])^(5/2)) + (2*((-2
*(c + d*x))/(3*b*(a + b*ArcSinh[c + d*x])^(3/2)) + (2*((-2*Sqrt[1 + (c + d
*x)^2])/(b*Sqrt[a + b*ArcSinh[c + d*x]]) + ((2*I)*((I/2)*Sqrt[b]*E^(a/b)*S
qrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi
]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/E^(a/b)))/b^2))/(3*b)))/(5*b
))/d
```

3.113. $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

3.113.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.113.4 Maple [F]

$$\int \frac{1}{(a+b\ \mathrm{arcsinh}\,(dx+c))^{\frac{7}{2}}}dx$$

input
```
int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

output
```
int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

3.113.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\mathrm{arcsinh}(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.113.6 Sympy [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(a+b\operatorname{asinh}(c+dx))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*asinh(d*x+c))**(7/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**(-7/2), x)
```

3.113.7 Maxima [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

3.113.8 Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^{7/2}}\,dx$$

input
```
int(1/(a + b*asinh(c + d*x))^(7/2),x)
```

output
```
int(1/(a + b*asinh(c + d*x))^(7/2), x)
```

3.114 $\int (ce + dex)^m (a + b\text{arcsinh}(c + dx))\,dx$

3.114.1 Optimal result

Integrand size = 21, antiderivative size = 91

$$\begin{aligned}\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))\,dx &= \frac{(e(c+dx))^{1+m}(a+b\text{arcsinh}(c+dx))}{de(1+m)} \\ &\quad - \frac{b(e(c+dx))^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-(c+dx)^2\right)}{de^2(1+m)(2+m)}\end{aligned}$$

output
```
(e*(d*x+c))^(1+m)*(a+b*arcsinh(d*x+c))/d/e/(1+m)-b*(e*(d*x+c))^(2+m)*hyper
geom([1/2, 1+1/2*m],[2+1/2*m],-(d*x+c)^2)/d/e^2/(1+m)/(2+m)
```

3.114.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.87

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))\,dx = -\frac{(c+dx)(e(c+dx))^m\left(-((2+m)(a+b\text{arcsinh}(c+dx)))+b(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2}\right.\right.}{d(1+m)(2+m)}$$

input
```
Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x]),x]
```

output
```
-(((c + d*x)*(e*(c + d*x))^m*(-((2 + m)*(a + b*ArcSinh[c + d*x])) + b*(c +
 d*x)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c + d*x)^2]))/(d*(1 +
 m)*(2 + m)))
```

3.114.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6274, 6191, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int (e(c+dx))^m(a+b\text{arcsinh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{(e(c+dx))^{m+1}(a+b\text{arcsinh}(c+dx))}{e(m+1)}-\frac{b\int \frac{(e(c+dx))^{m+1}}{\sqrt{(c+dx)^2+1}}d(c+dx)}{e(m+1)}}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{\frac{(e(c+dx))^{m+1}(a+b\text{arcsinh}(c+dx))}{e(m+1)}-\frac{b(e(c+dx))^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-(c+dx)^2\right)}{e^2(m+1)(m+2)}}{d}$$

input
```
Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x]),x]
```

output
```
(((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x]))/(e*(1 + m)) - (b*(e*(c +
 d*x))^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c + d*x)^2])
/(e^2*(1 + m)*(2 + m)))/d
```

3.114.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.114.4 Maple [F]

$$\int (dex + ce)^m (a + b \operatorname{arcsinh}(dx + c))\, dx$$

input
```
int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x)
```

output
```
int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x)
```

3.114.5 Fricas [F]

$$\int (ce + dex)^m (a + b\text{arcsinh}(c + dx))\, dx = \int (b \operatorname{arsinh}(dx + c) + a)(dex + ce)^m\, dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(d*x + c) + a)*(d*e*x + c*e)^m, x)
```

3.114.6 Sympy [F]

$$\int (ce+dex)^m(a+b\operatorname{arcsinh}(c+dx))\,dx = \int (e(c+dx))^m\,(a+b\operatorname{asinh}\,(c+dx))\;dx$$

input
```
integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c)),x)
```

output
```
Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x)), x)
```

3.114.7 Maxima [F]

$$\int (ce+dex)^m(a+b\operatorname{arcsinh}(c+dx))\,dx = \int (b\operatorname{arsinh}\,(dx+c)+a)(dex+ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

output
```
b*((d*e^m*x + c*e^m)*(d*x + c)^m*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))/(d*(m + 1)) - integrate((d^2*e^m*x^2 + 2*c*d*e^m*x + c^2*e^m)*(d*x
 + c)^m/(d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) + m + 1), x) - in
tegrate((d*e^m*x + c*e^m)*(d*x + c)^m/(d^3*(m + 1)*x^3 + 3*c*d^2*(m + 1)*x
^2 + c^3*(m + 1) + c*(m + 1) + (3*c^2*d*(m + 1) + d*(m + 1))*x + (d^2*(m +
 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) + m + 1)*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)), x)) + (d*e*x + c*e)^(m + 1)*a/(d*e*(m + 1))
```

3.114.8 Giac [F]

$$\int (ce+dex)^m(a+b\operatorname{arcsinh}(c+dx))\,dx = \int (b\operatorname{arsinh}\,(dx+c)+a)(dex+ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)*(d*e*x + c*e)^m, x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))\,dx=\int (c\,e+d\,e\,x)^m\,(a+b\,\text{asinh}(c+d\,x))\,dx$$

input
```
int((c*e + d*e*x)^m*(a + b*asinh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^m*(a + b*asinh(c + d*x)), x)
```

3.115 $\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))\,dx$

3.115.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))\,dx = -\frac{be^4\sqrt{1+(c+dx)^2}}{5d}+\frac{2be^4(1+(c+dx)^2)^{3/2}}{15d}-\frac{be^4(1+(c+dx)^2)^{5/2}}{25d}+\frac{e^4(c+dx)^5(a+b\text{arcsinh}(c+dx))}{5d}$$

output
```
2/15*b*e^4*(1+(d*x+c)^2)^(3/2)/d-1/25*b*e^4*(1+(d*x+c)^2)^(5/2)/d+1/5*e^4*
(d*x+c)^5*(a+b*arcsinh(d*x+c))/d-1/5*b*e^4*(1+(d*x+c)^2)^(1/2)/d
```

3.115.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.71

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))\,dx = \frac{e^4\left(-\frac{1}{75}b\sqrt{1+(c+dx)^2}\left(5-10(c+dx)^2+3(1+(c+dx)^2)^2\right)+\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))\right)}{d}$$

input
```
Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x]),x]
```

output
```
(e^4*(-1/75*(b*Sqrt[1 + (c + d*x)^2]*(5 - 10*(c + d*x)^2 + 3*(1 + (c + d*x
)^2)^2)) + ((c + d*x)^5*(a + b*ArcSinh[c + d*x]))/5))/d
```

3.115.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6274, 27, 6191, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int e^4(c+dx)^4(a+b\text{arcsinh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^4\int (c+dx)^4(a+b\text{arcsinh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))-\frac{1}{5}b\int \frac{(c+dx)^5}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{243}$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))-\frac{1}{10}b\int \frac{(c+dx)^4}{\sqrt{(c+dx)^2+1}}d(c+dx)^2\right)}{d}$$

$$\Big\downarrow \text{53}$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))-\frac{1}{10}b\int \left(\left((c+dx)^2+1\right)^{3/2}-2\sqrt{(c+dx)^2+1}+\frac{1}{\sqrt{(c+dx)^2+1}}\right)d(c+dx)^2\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))-\frac{1}{10}b\left(\frac{2}{5}\left((c+dx)^2+1\right)^{5/2}-\frac{4}{3}\left((c+dx)^2+1\right)^{3/2}+2\sqrt{(c+dx)^2+1}\right)\right)}{d}$$

input
```
Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x]),x]
```

output
```
(e^4*(-1/10*(b*(2*Sqrt[1 + (c + d*x)^2] - (4*(1 + (c + d*x)^2)^(3/2))/3 +
(2*(1 + (c + d*x)^2)^(5/2))/5)) + ((c + d*x)^5*(a + b*ArcSinh[c + d*x]))/5
))/d
```

3.115.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.115.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\frac{e^4 a(dx+c)^5}{5}+e^4 b\left(\frac{(dx+c)^5 \operatorname{arcsinh}(dx+c)}{5}-\frac{(dx+c)^4\sqrt{1+(dx+c)^2}}{25}+\frac{4(dx+c)^2\sqrt{1+(dx+c)^2}}{75}-\frac{8\sqrt{1+(dx+c)^2}}{75}\right)}{d}$	93
default	$\frac{\frac{e^4 a(dx+c)^5}{5}+e^4 b\left(\frac{(dx+c)^5 \operatorname{arcsinh}(dx+c)}{5}-\frac{(dx+c)^4\sqrt{1+(dx+c)^2}}{25}+\frac{4(dx+c)^2\sqrt{1+(dx+c)^2}}{75}-\frac{8\sqrt{1+(dx+c)^2}}{75}\right)}{d}$	93
parts	$\frac{e^4 a(dx+c)^5}{5d}+\frac{e^4 b\left(\frac{(dx+c)^5 \operatorname{arcsinh}(dx+c)}{5}-\frac{(dx+c)^4\sqrt{1+(dx+c)^2}}{25}+\frac{4(dx+c)^2\sqrt{1+(dx+c)^2}}{75}-\frac{8\sqrt{1+(dx+c)^2}}{75}\right)}{d}$	95

input
```
int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*e^4*a*(d*x+c)^5+e^4*b*(1/5*(d*x+c)^5*arcsinh(d*x+c)-1/25*(d*x+c)^
4*(1+(d*x+c)^2)^(1/2)+4/75*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-8/75*(1+(d*x+c)^2
)^(1/2)))
```

3.115.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(86) = 172.

Time = 0.27 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.79

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))\,dx$$
$$=\frac{15\,ad^5e^4x^5+75\,acd^4e^4x^4+150\,ac^2d^3e^4x^3+150\,ac^3d^2e^4x^2+75\,ac^4de^4x+15\,(bd^5e^4x^5+5\,bcd^4e^4x^4+10\,b}{}$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/75*(15*a*d^5*e^4*x^5 + 75*a*c*d^4*e^4*x^4 + 150*a*c^2*d^3*e^4*x^3 + 150*
a*c^3*d^2*e^4*x^2 + 75*a*c^4*d*e^4*x + 15*(b*d^5*e^4*x^5 + 5*b*c*d^4*e^4*x
^4 + 10*b*c^2*d^3*e^4*x^3 + 10*b*c^3*d^2*e^4*x^2 + 5*b*c^4*d*e^4*x + b*c^5
*e^4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (3*b*d^4*e^4*x^4
+ 12*b*c*d^3*e^4*x^3 + 2*(9*b*c^2 - 2*b)*d^2*e^4*x^2 + 4*(3*b*c^3 - 2*b*c)
*d*e^4*x + (3*b*c^4 - 4*b*c^2 + 8*b)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
))/d
```

3.115.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(85) = 170$.

Time = 0.37 (sec) , antiderivative size = 527, normalized size of antiderivative = 5.27

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))\,dx$$
$$= \begin{cases} ac^4e^4x + 2ac^3de^4x^2 + 2ac^2d^2e^4x^3 + acd^3e^4x^4 + \frac{ad^4e^4x^5}{5} + \frac{bc^5e^4\operatorname{asinh}(c+dx)}{5d} + bc^4e^4x\operatorname{asinh}(c+dx) - \frac{bc^4e^4\sqrt{\cdot}}{} \\ c^4e^4x(a+b\operatorname{asinh}(c)) \end{cases}$$

input
```
integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c)),x)
```

output
```
Piecewise((a*c**4*e**4*x + 2*a*c**3*d*e**4*x**2 + 2*a*c**2*d**2*e**4*x**3
+ a*c*d**3*e**4*x**4 + a*d**4*e**4*x**5/5 + b*c**5*e**4*asinh(c + d*x)/(5*
d) + b*c**4*e**4*x*asinh(c + d*x) - b*c**4*e**4*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)/(25*d) + 2*b*c**3*d*e**4*x**2*asinh(c + d*x) - 4*b*c**3*e**4*x*
sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 2*b*c**2*d**2*e**4*x**3*asinh(c
+ d*x) - 6*b*c**2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 4*
b*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(75*d) + b*c*d**3*e**4*x*
*4*asinh(c + d*x) - 4*b*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)/25 + 8*b*c*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/75 + b*d**4*e**
4*x**5*asinh(c + d*x)/5 - b*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/25 + 4*b*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/75 - 8*b*e
**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(75*d), Ne(d, 0)), (c**4*e**4*x*(
a + b*asinh(c)), True))
```

3.115.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1231 vs. $2(86) = 172$.

Time = 0.21 (sec) , antiderivative size = 1231, normalized size of antiderivative = 12.31

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/5*a*d^4*e^4*x^5 + a*c*d^3*e^4*x^4 + 2*a*c^2*d^2*e^4*x^3 + 2*a*c^3*d*e^4*
x^2 + (2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c
^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 -
 (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3
 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*b*c^3*d*e^4 + 1/3*(6*x^3*ar
csinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*a
rcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)
/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*b*c^2*
d^2*e^4 + 1/24*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*ar
csinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x +
 c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4
+ 9*(c^2 + 1)^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2)
)/d^5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*b*c*d^3*e
^4 + 1/600*(120*x^5*arcsinh(d*x + c) - (24*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*x^4/d^2 - 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^3/d^3 + 126*sqrt(...
```

3.115.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 841 vs. $2(86) = 172$.

Time = 0.89 (sec) , antiderivative size = 841, normalized size of antiderivative = 8.41

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{1}{5}ad^4e^4x^5 + acd^3e^4x^4 + 2\,ac^2d^2e^4x^3 + 2\,ac^3de^4x^2$$
$$-\left(d\left(\frac{c\log\left(-cd-\left(x|d|-\sqrt{d^2x^2+2\,cdx+c^2+1}\right)|d|\right)}{d|d|}+\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}}{d^2}\right)-x\log\left(dx+c+\right.\right.$$
$$+\left(2\,x^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(\frac{x}{d^2}-\frac{3\,c}{d^3}\right)-\frac{(2\,c^2-1)\log}{}\right.\right.$$
$$+\frac{1}{3}\left(6\,x^3\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(x\left(\frac{2\,x}{d^2}-\frac{5\,c}{d^3}\right)+\frac{11\,c^2d-}{d^5}\right.\right.\right.$$
$$+\frac{1}{24}\left(24\,x^4\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(\left(2\,x\left(\frac{3\,x}{d^2}-\frac{7\,c}{d^3}\right)+\frac{26}{}\right.\right.\right.\right.$$
$$+\frac{1}{600}\left(120\,x^5\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(\left(2\left(3\,x\left(\frac{4\,x}{d^2}-\frac{9\,c}{d^3}\right.\right.\right.\right.\right.\right.$$
$$+ac^4e^4x$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

output
```
1/5*a*d^4*e^4*x^5 + a*c*d^3*e^4*x^4 + 2*a*c^2*d^2*e^4*x^3 + 2*a*c^3*d*e^4*
x^2 - (d*(c*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(
d))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))*b*c^4*e^4 + (2*x^2*log(d*x + c + sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x/d^2
- 3*c/d^3) - (2*c^2 - 1)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1))*abs(d))/(d^2*abs(d)))*d)*b*c^3*d*e^4 + 1/3*(6*x^3*log(d*x + c + s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x*
(2*x/d^2 - 5*c/d^3) + (11*c^2*d - 4*d)/d^5) + 3*(2*c^3 - 3*c)*log(-c*d - (
x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^3*abs(d)))*d)*b*c
^2*d^2*e^4 + 1/24*(24*x^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
 - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((2*x*(3*x/d^2 - 7*c/d^3) + (26*c^2*
d^3 - 9*d^3)/d^7)*x - 5*(10*c^3*d^2 - 11*c*d^2)/d^7) - 3*(8*c^4 - 24*c^2 +
 3)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^4
*abs(d)))*d)*b*c*d^3*e^4 + 1/600*(120*x^5*log(d*x + c + sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((2*(3*x*(4*x/d^2 -
9*c/d^3) + (47*c^2*d^5 - 16*d^5)/d^9)*x - 7*(22*c^3*d^4 - 23*c*d^4)/d^9)*x
 + (274*c^4*d^3 - 607*c^2*d^3 + 64*d^3)/d^9) + 15*(8*c^5 - 40*c^3 + 15*c)*
log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^5*abs
(d)))*d)*b*d^4*e^4 + a*c^4*e^4*x
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))\,dx=\int (c\,e+d\,e\,x)^4\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)\,dx$$

input
```
int((c*e + d*e*x)^4*(a + b*asinh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^4*(a + b*asinh(c + d*x)), x)
```

3.116 $\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))\,dx$

3.116.1 Optimal result

Integrand size = 21, antiderivative size = 105

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))\,dx = \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}}{32d} - \frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}}{16d} - \frac{3be^3\text{arcsinh}(c+dx)}{32d} + \frac{e^3(c+dx)^4(a+b\text{arcsinh}(c+dx))}{4d}$$

output
```
-3/32*b*e^3*arcsinh(d*x+c)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))/d+3/32
*b*e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)/d-1/16*b*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1
/2)/d
```

3.116.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.79

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))\,dx = \frac{e^3\left(3b(c+dx)\sqrt{1+(c+dx)^2} - 2b(c+dx)^3\sqrt{1+(c+dx)^2} - 3b\text{arcsinh}(c+dx) + 8(c+dx)^4(a+b\text{arcsi}\right.}{32d}$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x]),x]
```

output
```
(e^3*(3*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^3*Sqrt[1 + (c +
d*x)^2] - 3*b*ArcSinh[c + d*x] + 8*(c + d*x)^4*(a + b*ArcSinh[c + d*x])))/
(32*d)
```

3.116.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6274, 27, 6191, 262, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int e^3(c+dx)^3(a+b\text{arcsinh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^3\int (c+dx)^3(a+b\text{arcsinh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))-\frac{1}{4}b\int \frac{(c+dx)^4}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{262}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))-\frac{1}{4}b\left(\frac{1}{4}(c+dx)^3\sqrt{(c+dx)^2+1}-\frac{3}{4}\int \frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)\right)}{d}$$

$$\Big\downarrow \text{262}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))-\frac{1}{4}b\left(\frac{1}{4}(c+dx)^3\sqrt{(c+dx)^2+1}-\frac{3}{4}\left(\frac{1}{2}(c+dx)\sqrt{(c+dx)^2+1}-\frac{1}{2}\int \frac{1}{\sqrt{(c+dx)^2+1}}\right.\right.\right.}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))-\frac{1}{4}b\left(\frac{1}{4}(c+dx)^3\sqrt{(c+dx)^2+1}-\frac{3}{4}\left(\frac{1}{2}(c+dx)\sqrt{(c+dx)^2+1}-\frac{1}{2}\text{arcsinh}(\right.\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x]),x]
```

output
```
(e^3*(-1/4*(b*(((c + d*x)^3*Sqrt[1 + (c + d*x)^2])/4 - (3*(((c + d*x)*Sqrt
[1 + (c + d*x)^2])/2 - ArcSinh[c + d*x]/2))/4)) + ((c + d*x)^4*(a + b*ArcS
inh[c + d*x]))/4))/d
```

3.116.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.116.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{e^3 a(dx+c)^4}{4}+e^3 b\left(\frac{(dx+c)^4 \operatorname{arcsinh}(dx+c)}{4}-\frac{(dx+c)^3\sqrt{1+(dx+c)^2}}{16}+\frac{3(dx+c)\sqrt{1+(dx+c)^2}}{32}-\frac{3 \operatorname{arcsinh}(dx+c)}{32}\right)}{d}$	86
default	$\frac{\frac{e^3 a(dx+c)^4}{4}+e^3 b\left(\frac{(dx+c)^4 \operatorname{arcsinh}(dx+c)}{4}-\frac{(dx+c)^3\sqrt{1+(dx+c)^2}}{16}+\frac{3(dx+c)\sqrt{1+(dx+c)^2}}{32}-\frac{3 \operatorname{arcsinh}(dx+c)}{32}\right)}{d}$	86
parts	$\frac{e^3 a(dx+c)^4}{4d}+\frac{e^3 b\left(\frac{(dx+c)^4 \operatorname{arcsinh}(dx+c)}{4}-\frac{(dx+c)^3\sqrt{1+(dx+c)^2}}{16}+\frac{3(dx+c)\sqrt{1+(dx+c)^2}}{32}-\frac{3 \operatorname{arcsinh}(dx+c)}{32}\right)}{d}$	88

input
```
int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*e^3*a*(d*x+c)^4+e^3*b*(1/4*(d*x+c)^4*arcsinh(d*x+c)-1/16*(d*x+c)^
3*(1+(d*x+c)^2)^(1/2)+3/32*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/32*arcsinh(d*x+c)
))
```

3.116.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(93) = 186.

Time = 0.25 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.17

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))\,dx$$
$$=\frac{8\,ad^4e^3x^4+32\,acd^3e^3x^3+48\,ac^2d^2e^3x^2+32\,ac^3de^3x+(8\,bd^4e^3x^4+32\,bcd^3e^3x^3+48\,bc^2d^2e^3x^2+32\,bc^3d}{}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/32*(8*a*d^4*e^3*x^4 + 32*a*c*d^3*e^3*x^3 + 48*a*c^2*d^2*e^3*x^2 + 32*a*c
^3*d*e^3*x + (8*b*d^4*e^3*x^4 + 32*b*c*d^3*e^3*x^3 + 48*b*c^2*d^2*e^3*x^2
+ 32*b*c^3*d*e^3*x + (8*b*c^4 - 3*b)*e^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1)) - (2*b*d^3*e^3*x^3 + 6*b*c*d^2*e^3*x^2 + 3*(2*b*c^2 - b)*
d*e^3*x + (2*b*c^3 - 3*b*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

3.116.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(94) = 188$.

Time = 0.28 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.75

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))\,dx$$
$$= \begin{cases} ac^3e^3x + \frac{3ac^2de^3x^2}{2} + acd^2e^3x^3 + \frac{ad^3e^3x^4}{4} + \frac{bc^4e^3\operatorname{asinh}(c+dx)}{4d} + bc^3e^3x\operatorname{asinh}(c+dx) - \frac{bc^3e^3\sqrt{c^2+2cdx+d^2x^2+1}}{16d} - \\ c^3e^3x(a+b\operatorname{asinh}(c)) \end{cases}$$

input
```
integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c)),x)
```

output
```
Piecewise((a*c**3*e**3*x + 3*a*c**2*d*e**3*x**2/2 + a*c*d**2*e**3*x**3 + a
*d**3*e**3*x**4/4 + b*c**4*e**3*asinh(c + d*x)/(4*d) + b*c**3*e**3*x*asinh
(c + d*x) - b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(16*d) + 3*b*
c**2*d*e**3*x**2*asinh(c + d*x)/2 - 3*b*c**2*e**3*x*sqrt(c**2 + 2*c*d*x +
d**2*x**2 + 1)/16 + b*c*d**2*e**3*x**3*asinh(c + d*x) - 3*b*c*d*e**3*x**2*
sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 3*b*c*e**3*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)/(32*d) + b*d**3*e**3*x**4*asinh(c + d*x)/4 - b*d**2*e**3*x
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 3*b*e**3*x*sqrt(c**2 + 2*c*d
*x + d**2*x**2 + 1)/32 - 3*b*e**3*asinh(c + d*x)/(32*d), Ne(d, 0)), (c**3*
e**3*x*(a + b*asinh(c)), True))
```

3.116.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. $2(93) = 186$.

Time = 0.21 (sec) , antiderivative size = 790, normalized size of antiderivative = 7.52

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{1}{4}ad^3e^3x^4 + acd^2e^3x^3 + \frac{3}{2}ac^2de^3x^2$$
$$+\frac{3}{4}\left(2x^2\operatorname{arsinh}(dx+c) - d\left(\frac{3c^2\operatorname{arsinh}\left(\frac{2(d^2x+cd)}{\sqrt{-4c^2d^2+4(c^2+1)d^2}}\right)}{d^3} + \frac{\sqrt{d^2x^2+2cdx+c^2+1}x}{d^2} - \frac{(c^2+1)\operatorname{ar}\cdots}{\cdots}\right.\right.$$
$$+\frac{1}{6}\left(6x^3\operatorname{arsinh}(dx+c) - d\left(\frac{2\sqrt{d^2x^2+2cdx+c^2+1}x^2}{d^2} - \frac{15c^3\operatorname{arsinh}\left(\frac{2(d^2x+cd)}{\sqrt{-4c^2d^2+4(c^2+1)d^2}}\right)}{d^4} - \frac{5\sqrt{d^2x\cdots}}{\cdots}\right.\right.$$
$$+\frac{1}{96}\left(24x^4\operatorname{arsinh}(dx+c) - \left(\frac{6\sqrt{d^2x^2+2cdx+c^2+1}x^3}{d^2} - \frac{14\sqrt{d^2x^2+2cdx+c^2+1}cx^2}{d^3} + \frac{105c^4\operatorname{ar}\cdots}{\cdots}\right.\right.$$
$$+ac^3e^3x + \frac{\left((dx+c)\operatorname{arsinh}(dx+c) - \sqrt{(dx+c)^2+1}\right)bc^3e^3}{d}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*a*d^3*e^3*x^4 + a*c*d^2*e^3*x^3 + 3/2*a*c^2*d*e^3*x^2 + 3/4*(2*x^2*arc
sinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2
+ 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsi
nh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)*c/d^3))*b*c^2*d*e^3 + 1/6*(6*x^3*arcsinh(d*x + c) -
 d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x
+ c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2
 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 -
4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*b*c*d^2*e^3 + 1/96*(24
*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^3/d^2 - 14*
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(2*(d^2*x + c
*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*
d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3/d^
5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c^2 + 1)^2*ar
csinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 55*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*b*d^3*e^3 + a*c^3*e^3*x + (
(d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*b*c^3*e^3/d
```

3.116. $\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))\,dx$

3.116.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(93) = 186$.

Time = 0.74 (sec) , antiderivative size = 613, normalized size of antiderivative = 5.84

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{1}{4}ad^3e^3x^4 + acd^2e^3x^3 + \frac{3}{2}ac^2de^3x^2$$

$$-\left(d\left(\frac{c\log\left(-cd-\left(x|d|-\sqrt{d^2x^2+2\,cdx+c^2+1}\right)|d|\right)}{d|d|}+\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}}{d^2}\right)-x\log\left(dx+c+\right.\right.$$

$$+\frac{3}{4}\left(2\,x^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(\frac{x}{d^2}-\frac{3\,c}{d^3}\right)-\frac{(2\,c^2-1)\,\mathrm{lo}}{}\right.\right.$$

$$+\frac{1}{6}\left(6\,x^3\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(x\left(\frac{2\,x}{d^2}-\frac{5\,c}{d^3}\right)+\frac{11\,c^2d-}{d^5}\right.\right.\right.$$

$$+\frac{1}{96}\left(24\,x^4\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(\left(2\,x\left(\frac{3\,x}{d^2}-\frac{7\,c}{d^3}\right)+\frac{26}{}\right.\right.\right.\right.$$

$$+ac^3e^3x$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

output
```
1/4*a*d^3*e^3*x^4 + a*c*d^2*e^3*x^3 + 3/2*a*c^2*d*e^3*x^2 - (d*(c*log(-c*d
 - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d*abs(d)) + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)))*b*c^3*e^3 + 3/4*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x/d^2 - 3*c/d^3) - (2*
c^2 - 1)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))
/(d^2*abs(d)))*d)*b*c^2*d*e^3 + 1/6*(6*x^3*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x*(2*x/d^2 - 5*c/d
^3) + (11*c^2*d - 4*d)/d^5) + 3*(2*c^3 - 3*c)*log(-c*d - (x*abs(d) - sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^3*abs(d)))*d)*b*c*d^2*e^3 + 1/96*
(24*x^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1)*((2*x*(3*x/d^2 - 7*c/d^3) + (26*c^2*d^3 - 9*d^3)/d^7)*
x - 5*(10*c^3*d^2 - 11*c*d^2)/d^7) - 3*(8*c^4 - 24*c^2 + 3)*log(-c*d - (x*
abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^4*abs(d)))*d)*b*d^3
*e^3 + a*c^3*e^3*x
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))\,dx=\int (c\,e+d\,e\,x)^3\,(a+b\,\operatorname{asinh}(c+d\,x))\,dx$$

input
```
int((c*e + d*e*x)^3*(a + b*asinh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^3*(a + b*asinh(c + d*x)), x)
```

3.117 $\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))\,dx$

3.117.1 Optimal result

Integrand size = 21, antiderivative size = 76

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))\,dx = \frac{be^2\sqrt{1+(c+dx)^2}}{3d} - \frac{be^2(1+(c+dx)^2)^{3/2}}{9d} + \frac{e^2(c+dx)^3(a+b\text{arcsinh}(c+dx))}{3d}$$

output
```
-1/9*b*e^2*(1+(d*x+c)^2)^(3/2)/d+1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))/d+
1/3*b*e^2*(1+(d*x+c)^2)^(1/2)/d
```

3.117.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))\,dx = \frac{e^2\left(-\frac{1}{9}b(-2+c^2+2cdx+d^2x^2)\sqrt{1+(c+dx)^2}+\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))\right)}{d}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x]),x]
```

output
```
(e^2*(-1/9*(b*(-2 + c^2 + 2*c*d*x + d^2*x^2)*Sqrt[1 + (c + d*x)^2]) + ((c
+ d*x)^3*(a + b*ArcSinh[c + d*x]))/3))/d
```

3.117.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6274, 27, 6191, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int e^2(c+dx)^2(a+b\text{arcsinh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\int (c+dx)^2(a+b\text{arcsinh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))-\frac{1}{3}b\int \frac{(c+dx)^3}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{243}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))-\frac{1}{6}b\int \frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}}d(c+dx)^2\right)}{d}$$

$$\Big\downarrow \text{53}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))-\frac{1}{6}b\int \left(\sqrt{(c+dx)^2+1}-\frac{1}{\sqrt{(c+dx)^2+1}}\right)d(c+dx)^2\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))-\frac{1}{6}b\left(\frac{2}{3}\left((c+dx)^2+1\right)^{3/2}-2\sqrt{(c+dx)^2+1}\right)\right)}{d}$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x]),x]
```

output
```
(e^2*(-1/6*(b*(-2*Sqrt[1 + (c + d*x)^2] + (2*(1 + (c + d*x)^2)^(3/2))/3))
+ ((c + d*x)^3*(a + b*ArcSinh[c + d*x]))/3))/d
```

3.117.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.117.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{a\,e^{2}(dx+c)^{3}}{3}+e^{2}b\left(\frac{(dx+c)^{3}\operatorname{arcsinh}(dx+c)}{3}-\frac{(dx+c)^{2}\sqrt{1+(dx+c)^{2}}}{9}+\frac{2\sqrt{1+(dx+c)^{2}}}{9}\right)}{d}$	73
default	$\frac{\frac{a\,e^{2}(dx+c)^{3}}{3}+e^{2}b\left(\frac{(dx+c)^{3}\operatorname{arcsinh}(dx+c)}{3}-\frac{(dx+c)^{2}\sqrt{1+(dx+c)^{2}}}{9}+\frac{2\sqrt{1+(dx+c)^{2}}}{9}\right)}{d}$	73
parts	$\frac{a\,e^{2}(dx+c)^{3}}{3d}+\frac{e^{2}b\left(\frac{(dx+c)^{3}\operatorname{arcsinh}(dx+c)}{3}-\frac{(dx+c)^{2}\sqrt{1+(dx+c)^{2}}}{9}+\frac{2\sqrt{1+(dx+c)^{2}}}{9}\right)}{d}$	75

input
```
int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*a*e^2*(d*x+c)^3+e^2*b*(1/3*(d*x+c)^3*arcsinh(d*x+c)-1/9*(d*x+c)^2
*(1+(d*x+c)^2)^(1/2)+2/9*(1+(d*x+c)^2)^(1/2)))
```

3.117.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(66) = 132.

Time = 0.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.21

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))\,dx$$
$$=\frac{3\,ad^3e^2x^3+9\,acd^2e^2x^2+9\,ac^2de^2x+3\left(bd^3e^2x^3+3\,bcd^2e^2x^2+3\,bc^2de^2x+bc^3e^2\right)\log\left(dx+c+\sqrt{d^2x^2+\right.}{9\,d}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/9*(3*a*d^3*e^2*x^3 + 9*a*c*d^2*e^2*x^2 + 9*a*c^2*d*e^2*x + 3*(b*d^3*e^2*
x^3 + 3*b*c*d^2*e^2*x^2 + 3*b*c^2*d*e^2*x + b*c^3*e^2)*log(d*x + c + sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)) - (b*d^2*e^2*x^2 + 2*b*c*d*e^2*x + (b*c^2 -
2*b)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

3.117.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(63) = 126$.

Time = 0.17 (sec) , antiderivative size = 258, normalized size of antiderivative = 3.39

$$\int (ce + dex)^2 (a + b\text{arcsinh}(c + dx))\, dx = \begin{cases} ac^2e^2x + acde^2x^2 + \frac{ad^2e^2x^3}{3} + \frac{bc^3e^2\operatorname{asinh}(c+dx)}{3d} + bc^2e^2x\operatorname{asinh}(c+dx) - \frac{bc^2e^2\sqrt{c^2+2cdx+d^2x^2+1}}{9d} + bcde^2x^2\operatorname{asi}\\ c^2e^2x(a + b\operatorname{asinh}(c)) \end{cases}$$

input
```
integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c)),x)
```

output
```
Piecewise((a*c**2*e**2*x + a*c*d*e**2*x**2 + a*d**2*e**2*x**3/3 + b*c**3*e
**2*asinh(c + d*x)/(3*d) + b*c**2*e**2*x*asinh(c + d*x) - b*c**2*e**2*sqrt
(c**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d) + b*c*d*e**2*x**2*asinh(c + d*x) -
2*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9 + b*d**2*e**2*x**3*asi
nh(c + d*x)/3 - b*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9 + 2*b
*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d), Ne(d, 0)), (c**2*e**2*x*
(a + b*asinh(c)), True))
```

3.117.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(66) = 132$.

Time = 0.21 (sec) , antiderivative size = 445, normalized size of antiderivative = 5.86

$$\int (ce + dex)^2 (a + b\text{arcsinh}(c + dx))\, dx = \frac{1}{3}ad^2e^2x^3 + acde^2x^2$$
$$+ \frac{1}{2}\left(2x^2\operatorname{arsinh}(dx + c) - d\left(\frac{3c^2\operatorname{arsinh}\left(\frac{2(d^2x+cd)}{\sqrt{-4c^2d^2+4(c^2+1)d^2}}\right)}{d^3} + \frac{\sqrt{d^2x^2 + 2cdx + c^2 + 1}x}{d^2} - \frac{(c^2+1)\operatorname{ar}}{}\right.\right.$$
$$+ \frac{1}{18}\left(6x^3\operatorname{arsinh}(dx + c) - d\left(\frac{2\sqrt{d^2x^2 + 2cdx + c^2 + 1}x^2}{d^2} - \frac{15c^3\operatorname{arsinh}\left(\frac{2(d^2x+cd)}{\sqrt{-4c^2d^2+4(c^2+1)d^2}}\right)}{d^4} - \frac{5\sqrt{d^2}}{}\right.\right.$$
$$+ ac^2e^2x + \frac{\left((dx + c)\operatorname{arsinh}(dx + c) - \sqrt{(dx + c)^2 + 1}\right)bc^2e^2}{d}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*a*d^2*e^2*x^3 + a*c*d*e^2*x^2 + 1/2*(2*x^2*arcsinh(d*x + c) - d*(3*c^2
*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(
-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c
/d^3))*b*c*d*e^2 + 1/18*(6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 +
4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c
^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4
+ 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)*(c^2 + 1)/d^4))*b*d^2*e^2 + a*c^2*e^2*x + ((d*x + c)*arcsinh(d*
x + c) - sqrt((d*x + c)^2 + 1))*b*c^2*e^2/d
```

3.117.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(66) = 132$.

Time = 0.62 (sec) , antiderivative size = 416, normalized size of antiderivative = 5.47

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))\,dx = \frac{1}{3}\,ad^2e^2x^3 + acde^2x^2$$
$$-\left(d\left(\frac{c\log\left(-cd-\left(x|d|-\sqrt{d^2x^2+2\,cdx+c^2+1}\right)|d|\right)}{d|d|}+\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}}{d^2}\right)-x\log\left(dx+c+\right.\right.$$
$$+\frac{1}{2}\left(2\,x^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(\frac{x}{d^2}-\frac{3\,c}{d^3}\right)-\frac{(2\,c^2-1)\log}{}\right.\right.$$
$$+\frac{1}{18}\left(6\,x^3\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(x\left(\frac{2\,x}{d^2}-\frac{5\,c}{d^3}\right)+\frac{11\,c^2d}{d^5}\right.\right.\right.$$
$$+ac^2e^2x$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

output
```
1/3*a*d^2*e^2*x^3 + a*c*d*e^2*x^2 - (d*(c*log(-c*d - (x*abs(d) - sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))*b*c^2*e^
2 + 1/2*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*(x/d^2 - 3*c/d^3) - (2*c^2 - 1)*log(-c*d - (x*a
bs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^2*abs(d)))*d)*b*c*d*
e^2 + 1/18*(6*x^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x*(2*x/d^2 - 5*c/d^3) + (11*c^2*d - 4*d)/d^
5) + 3*(2*c^3 - 3*c)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))*abs(d))/(d^3*abs(d)))*d)*b*d^2*e^2 + a*c^2*e^2*x
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))\,dx=\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{asinh}(c+d\,x))\,dx$$

input
```
int((c*e + d*e*x)^2*(a + b*asinh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^2*(a + b*asinh(c + d*x)), x)
```

3.118 $\int (ce+dex)(a+b\text{arcsinh}(c+dx))\,dx$

3.118.1 Optimal result

Integrand size = 19, antiderivative size = 68

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))\,dx = -\frac{be(c+dx)\sqrt{1+(c+dx)^2}}{4d}+\frac{be\text{arcsinh}(c+dx)}{4d}+\frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))}{2d}$$

output
```
1/4*b*e*arcsinh(d*x+c)/d+1/2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))/d-1/4*b*e*(d
*x+c)*(1+(d*x+c)^2)^(1/2)/d
```

3.118.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))\,dx = \frac{e\left(-b(c+dx)\sqrt{1+(c+dx)^2}+b\text{arcsinh}(c+dx)+2(c+dx)^2(a+b\text{arcsinh}(c+dx))\right)}{4d}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x]),x]
```

output
```
(e*(-(b*(c + d*x)*Sqrt[1 + (c + d*x)^2]) + b*ArcSinh[c + d*x] + 2*(c + d*x
)^2*(a + b*ArcSinh[c + d*x])))/(4*d)
```

3.118.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6274, 27, 6191, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int e(c+dx)(a+b\text{arcsinh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int (c+dx)(a+b\text{arcsinh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))-\frac{1}{2}b\int \frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{262}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))-\frac{1}{2}b\left(\frac{1}{2}(c+dx)\sqrt{(c+dx)^2+1}-\frac{1}{2}\int \frac{1}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)\right)}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))-\frac{1}{2}b\left(\frac{1}{2}(c+dx)\sqrt{(c+dx)^2+1}-\frac{1}{2}\text{arcsinh}(c+dx)\right)\right)}{d}$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x]),x]
```

output
```
(e*(-1/2*(b*(((c + d*x)*Sqrt[1 + (c + d*x)^2])/2 - ArcSinh[c + d*x]/2)) +
((c + d*x)^2*(a + b*ArcSinh[c + d*x]))/2))/d
```

3.118.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.118.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\frac{ea(dx+c)^2}{2}+eb\left(\frac{(dx+c)^2\operatorname{arcsinh}(dx+c)}{2}-\frac{(dx+c)\sqrt{1+(dx+c)^2}}{4}+\frac{\operatorname{arcsinh}(dx+c)}{4}\right)}{d}$	62
default	$\frac{\frac{ea(dx+c)^2}{2}+eb\left(\frac{(dx+c)^2\operatorname{arcsinh}(dx+c)}{2}-\frac{(dx+c)\sqrt{1+(dx+c)^2}}{4}+\frac{\operatorname{arcsinh}(dx+c)}{4}\right)}{d}$	62
parts	$ea\left(\frac{1}{2}d\,x^2+cx\right)+\frac{eb\left(\frac{(dx+c)^2\operatorname{arcsinh}(dx+c)}{2}-\frac{(dx+c)\sqrt{1+(dx+c)^2}}{4}+\frac{\operatorname{arcsinh}(dx+c)}{4}\right)}{d}$	63

input
```
int((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*e*a*(d*x+c)^2+e*b*(1/2*(d*x+c)^2*arcsinh(d*x+c)-1/4*(d*x+c)*(1+(d
*x+c)^2)^(1/2)+1/4*arcsinh(d*x+c)))
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.59

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))\,dx$$
$$= \frac{2\,ad^2ex^2 + 4\,acdex + (2\,bd^2ex^2 + 4\,bcdex + (2\,bc^2 + b)e)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right) - (bdex -}{4\,d}$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*a*d^2*e*x^2 + 4*a*c*d*e*x + (2*b*d^2*e*x^2 + 4*b*c*d*e*x + (2*b*c^2
 + b)*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (b*d*e*x + b*c
*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

3.118.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(58) = 116$.

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.18

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))\,dx$$
$$= \begin{cases} acex + \frac{adex^2}{2} + \frac{bc^2e\operatorname{asinh}(c+dx)}{2d} + bcex\operatorname{asinh}(c + dx) - \frac{bce\sqrt{c^2+2cdx+d^2x^2+1}}{4d} + \frac{bdex^2\operatorname{asinh}(c+dx)}{2} - \frac{bex\sqrt{c^2+2cdx+}}{4} \\ cex(a + b\operatorname{asinh}(c)) \end{cases}$$

input
```
integrate((d*e*x+c*e)*(a+b*asinh(d*x+c)),x)
```

output
```
Piecewise((a*c*e*x + a*d*e*x**2/2 + b*c**2*e*asinh(c + d*x)/(2*d) + b*c*e*
x*asinh(c + d*x) - b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(4*d) + b*d*
e*x**2*asinh(c + d*x)/2 - b*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/4 + b
*e*asinh(c + d*x)/(4*d), Ne(d, 0)), (c*e*x*(a + b*asinh(c)), True))
```

3.118.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(60) = 120$.

Time = 0.19 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.96

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))\,dx = \frac{1}{2}\,adex^2 + \frac{1}{4}\left(2\,x^2\,\text{arsinh}\,(dx+c) - d\left(\frac{3\,c^2\,\text{arsinh}\left(\frac{2\,(d^2x+cd)}{\sqrt{-4\,c^2d^2+4\,(c^2+1)d^2}}\right)}{d^3} + \frac{\sqrt{d^2x^2+2\,cdx+c^2+1}x}{d^2} - \frac{(c^2+1)\,\text{ar}}{}\right.\right.$$
$$+ acex + \frac{\left((dx+c)\,\text{arsinh}\,(dx+c) - \sqrt{(dx+c)^2+1}\right)bce}{d}$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*a*d*e*x^2 + 1/4*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x +
c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2
+ 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*b*d*e + a*c*e
*x + ((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*b*c*e/d
```

3.118.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(60) = 120$.

Time = 0.49 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.57

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))\,dx = \frac{1}{2}\,adex^2 - \left(d\left(\frac{c\log\left(-cd-\left(x|d|-\sqrt{d^2x^2+2\,cdx+c^2+1}\right)|d|\right)}{d|d|} + \frac{\sqrt{d^2x^2+2\,cdx+c^2+1}}{d^2}\right) - x\log\left(dx+c+\right.\right.$$
$$+\frac{1}{4}\left(2\,x^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right) - \left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(\frac{x}{d^2}-\frac{3\,c}{d^3}\right) - \frac{(2\,c^2-1)\,\text{lo}}{}\right.\right.$$
$$+ acex$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*a*d*e*x^2 - (d*(c*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1))*abs(d))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log(
d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))*b*c*e + 1/4*(2*x^2*log(d*x +
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*(x/d^2 - 3*c/d^3) - (2*c^2 - 1)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2
*c*d*x + c^2 + 1))*abs(d))/(d^2*abs(d)))*d)*b*d*e + a*c*e*x
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int (ce + dex)(a + b\text{arcsinh}(c + dx))\,dx = \int (c\,e + d\,e\,x)\,(a + b\,\text{asinh}(c + d\,x))\,dx$$

input
```
int((c*e + d*e*x)*(a + b*asinh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)*(a + b*asinh(c + d*x)), x)
```

3.119 $\int (a+b\text{arcsinh}(c+dx))\,dx$

3.119.1 Optimal result

Integrand size = 10, antiderivative size = 39

$$\int (a+b\text{arcsinh}(c+dx))\,dx=ax-\frac{b\sqrt{1+(c+dx)^2}}{d}+\frac{b(c+dx)\text{arcsinh}(c+dx)}{d}$$

output
```
a*x+b*(d*x+c)*arcsinh(d*x+c)/d-b*(1+(d*x+c)^2)^(1/2)/d
```

3.119.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 83 vs. 2(39) = 78.

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.13

$$\int (a+b\text{arcsinh}(c+dx))\,dx$$
$$=ax+bx\text{arcsinh}(c+dx)$$
$$-\frac{b\left(\sqrt{1+c^2+2cdx+d^2x^2}+2c\text{arctanh}\left(\frac{dx}{\sqrt{1+c^2}-\sqrt{1+c^2+2cdx+d^2x^2}}\right)\right)}{d}$$

input
```
Integrate[a + b*ArcSinh[c + d*x],x]
```

output
```
a*x + b*x*ArcSinh[c + d*x] - (b*(Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2] + 2*c*A
rcTanh[(d*x)/(Sqrt[1 + c^2] - Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])]))/d
```

3.119.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\text{arcsinh}(c + dx))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax + \frac{b(c + dx)\text{arcsinh}(c + dx)}{d} - \frac{b\sqrt{(c + dx)^2 + 1}}{d}$$

input
```
Int[a + b*ArcSinh[c + d*x],x]
```

output
```
a*x - (b*Sqrt[1 + (c + d*x)^2])/d + (b*(c + d*x)*ArcSinh[c + d*x])/d
```

3.119.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.119.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

method	result	size
default	$ax + \frac{b\left((dx+c)\operatorname{arcsinh}(dx+c)-\sqrt{1+(dx+c)^2}\right)}{d}$	36
parts	$ax + \frac{b\left((dx+c)\operatorname{arcsinh}(dx+c)-\sqrt{1+(dx+c)^2}\right)}{d}$	36
derivativedivides	$\frac{(dx+c)a+b\left((dx+c)\operatorname{arcsinh}(dx+c)-\sqrt{1+(dx+c)^2}\right)}{d}$	41

input
```
int(a+b*arcsinh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+b/d*((d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2))
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\int (a+b\text{arcsinh}(c+dx))\,dx = \frac{adx+(bdx+bc)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\sqrt{d^2x^2+2\,cdx+c^2+1}b}{d}$$

input
```
integrate(a+b*arcsinh(d*x+c),x, algorithm="fricas")
```

output
```
(a*d*x + (b*d*x + b*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) -
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*b)/d
```

3.119.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int (a+b\text{arcsinh}(c+dx))\,dx = ax+b\left(\begin{cases}\frac{c\,\text{asinh}\,(c+dx)}{d}+x\,\text{asinh}\,(c+dx)-\frac{\sqrt{c^2+2cdx+d^2x^2+1}}{d} & \text{for } d\neq 0\\ x\,\text{asinh}\,(c) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*asinh(d*x+c),x)
```

output
```
a*x + b*Piecewise((c*asinh(c + d*x)/d + x*asinh(c + d*x) - sqrt(c**2 + 2*c
*d*x + d**2*x**2 + 1)/d, Ne(d, 0)), (x*asinh(c), True))
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int (a+b\text{arcsinh}(c+dx))\,dx = ax+\frac{\left((dx+c)\,\text{arsinh}\,(dx+c)-\sqrt{(dx+c)^2+1}\right)b}{d}$$

input
```
integrate(a+b*arcsinh(d*x+c),x, algorithm="maxima")
```

output
```
a*x + ((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*b/d
```

3.119.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.54

$$\int (a + b\text{arcsinh}(c + dx))\, dx =$$
$$-\left(d\left(\frac{c\log\left(-cd - \left(x|d| - \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)|d|\right)}{d|d|} + \frac{\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}}{d^2}\right) - x\log\left(dx + c + \right.\right.$$
$$+ ax$$

input
```
integrate(a+b*arcsinh(d*x+c),x, algorithm="giac")
```

output
```
-(d*(c*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(
d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log(d*x + c + sqrt(
(d*x + c)^2 + 1)))*b + a*x
```

3.119.9 Mupad [B] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.18

$$\int (a + b\text{arcsinh}(c + dx))\, dx = a\,x + b\,x\,\text{asinh}(c + d\,x) - \frac{b\sqrt{c^2 + 2\,c\,d\,x + d^2\,x^2 + 1}}{d} + \frac{b\,c\,d^2\,\ln\left(\sqrt{c^2 + 2\,c\,d\,x + d^2\,x^2 + 1} + \frac{x\,d^2 + c\,d}{\sqrt{d^2}}\right)}{(d^2)^{3/2}}$$

input
```
int(a + b*asinh(c + d*x),x)
```

output
```
a*x + b*x*asinh(c + d*x) - (b*(c^2 + d^2*x^2 + 2*c*d*x + 1)^(1/2))/d + (b*
c*d^2*log((c^2 + d^2*x^2 + 2*c*d*x + 1)^(1/2) + (c*d + d^2*x)/(d^2)^(1/2))
)/(d^2)^(3/2)
```

3.120 $\int \frac{a+b\text{arcsinh}(c+dx)}{ce+dex}\,dx$

3.120.1 Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{a+b\text{arcsinh}(c+dx)}{ce+dex}\,dx = \frac{(a+b\text{arcsinh}(c+dx))^2}{2bde} + \frac{(a+b\text{arcsinh}(c+dx))\log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de} - \frac{b\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(c+dx)}\right)}{2de}$$

output
```
1/2*(a+b*arcsinh(d*x+c))^2/b/d/e+(a+b*arcsinh(d*x+c))*ln(1-1/(d*x+c+(1+(d*
x+c)^2)^(1/2))^2)/d/e-1/2*b*polylog(2,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e
```

3.120.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{a+b\text{arcsinh}(c+dx)}{ce+dex}\,dx = \frac{-\left((a+b\text{arcsinh}(c+dx))\left(a+b\text{arcsinh}(c+dx)-2b\log\left(1-e^{2\text{arcsinh}(c+dx)}\right)\right)\right)+b^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(c+}\right.}{2bde}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x),x]
```

output
```
(-((a + b*ArcSinh[c + d*x])*(a + b*ArcSinh[c + d*x] - 2*b*Log[1 - E^(2*Arc
Sinh[c + d*x])])) + b^2*PolyLog[2, E^(2*ArcSinh[c + d*x])])/(2*b*d*e)
```

3.120.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6274, 27, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{ce+dex}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{a+b\text{arcsinh}(c+dx)}{e(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\text{arcsinh}(c+dx)}{c+dx}d(c+dx)}{de}$$

$$\Big\downarrow \text{6190}$$

$$\frac{\int -\left((a+b\text{arcsinh}(c+dx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\right)d(a+b\text{arcsinh}(c+dx))}{bde}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int (a+b\text{arcsinh}(c+dx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)d(a+b\text{arcsinh}(c+dx))}{bde}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -i(a+b\text{arcsinh}(c+dx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(c+dx))}{bde}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int (a+b\text{arcsinh}(c+dx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)d(a+b\text{arcsinh}(c+dx))}{bde}$$

$$\Big\downarrow \text{4201}$$

$$\frac{i\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}(a+b\text{arcsinh}(c+dx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i(a+b\text{arcsinh}(c+dx))^2\right)}{bde}$$

$\downarrow$ 2620

$$\frac{i\left(2i\left(\frac{1}{2}b\int \log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}b(a+b\text{arcsinh}(c+dx))\log\left(1+\exp\left(-\right.\right.\right.\right.}{bde}$$

$\downarrow$ 2715

$$\frac{i\left(2i\left(-\frac{1}{4}b^2\int \exp\left(-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(c+dx))}{b}+i\pi\right)\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right)de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right.\right.}{bde}$$

$\downarrow$ 2838

$$\frac{i\left(2i\left(\frac{1}{4}b^2\operatorname{PolyLog}(2,-c-dx)-\frac{1}{2}b(a+b\text{arcsinh}(c+dx))\log\left(1+\exp\left(-\frac{2(a+b\text{arcsinh}(c+dx))}{b}+\frac{2a}{b}-i\pi\right)\right)\right)-\frac{1}{2}i\right.}{bde}$$

input
```
Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x),x]
```

output
```
(I*((-1/2*I)*(a + b*ArcSinh[c + d*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcSinh[c
+ d*x])*Log[1 + E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[c + d*x]))/b)]) + (b
^2*PolyLog[2, -c - d*x])/4)))/(b*d*e)
```

3.120.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.120. $\int \frac{a+b\text{arcsinh}(c+dx)}{ce+dex}\,dx$

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.120. $\int \frac{a+b\operatorname{arcsinh}(c+dx)}{ce+dex}\,dx$

3.120.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.59

method	result
derivativedivides	$\frac{\frac{a\ln(dx+c)}{e}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)^2}{2}+\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)+\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right)+\operatorname{arcsinh}(dx+c)\ln}{e}}{d}$
default	$\frac{\frac{a\ln(dx+c)}{e}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)^2}{2}+\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)+\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right)+\operatorname{arcsinh}(dx+c)\ln}{e}}{d}$
parts	$\frac{a\ln(dx+c)}{ed}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)^2}{2}+\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)+\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right)-}{ed}$

input
```
int((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a/e*ln(d*x+c)+b/e*(-1/2*arcsinh(d*x+c)^2+arcsinh(d*x+c)*ln(1+d*x+c+(1
+(d*x+c)^2)^(1/2))+polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+arcsinh(d*x+c)*ln
(1-d*x-c-(1+(d*x+c)^2)^(1/2))+polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))))
```

3.120.5 Fricas [F]

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{ce+dex}\,dx=\int \frac{b\operatorname{arsinh}(dx+c)+a}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
integral((b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

3.120.6 Sympy [F]

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{ce+dex}\,dx=\frac{\int \frac{a}{c+dx}\,dx+\int \frac{b\operatorname{asinh}(c+dx)}{c+dx}\,dx}{e}$$

input
```
integrate((a+b*asinh(d*x+c))/(d*e*x+c*e),x)
```

output
```
(Integral(a/(c + d*x), x) + Integral(b*asinh(c + d*x)/(c + d*x), x))/e
```

3.120.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{ce+dex}\,dx=\int \frac{b\,\text{arsinh}\,(dx+c)+a}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
b*integrate(log(d*x + c + sqrt((d*x + c)^2 + 1))/(d*e*x + c*e), x) + a*log
(d*e*x + c*e)/(d*e)
```

3.120.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{ce+dex}\,dx=\int \frac{b\,\text{arsinh}\,(dx+c)+a}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{ce+dex}\,dx=\int \frac{a+b\,\text{asinh}(c+d\,x)}{c\,e+d\,e\,x}\,dx$$

input
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x),x)
```

output
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x), x)
```

3.121 $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^2}\,dx$

3.121.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^2}\,dx=-\frac{a+b\text{arcsinh}(c+dx)}{de^2(c+dx)}-\frac{b\text{arctanh}\left(\sqrt{1+(c+dx)^2}\right)}{de^2}$$

output
```
(-a-b*arcsinh(d*x+c))/d/e^2/(d*x+c)-b*arctanh((1+(d*x+c)^2)^(1/2))/d/e^2
```

3.121.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^2}\,dx=-\frac{\frac{a+b\text{arcsinh}(c+dx)}{c+dx}+b\text{arctanh}\left(\sqrt{1+(c+dx)^2}\right)}{de^2}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^2,x]
```

output
```
-(((a + b*ArcSinh[c + d*x])/(c + d*x) + b*ArcTanh[Sqrt[1 + (c + d*x)^2]])/
(d*e^2))
```

3.121.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6274, 27, 6191, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^2}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{a+b\text{arcsinh}(c+dx)}{e^2(c+dx)^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\text{arcsinh}(c+dx)}{(c+dx)^2}d(c+dx)}{de^2}$$

$$\Big\downarrow \text{6191}$$

$$\frac{b\int \frac{1}{(c+dx)\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{a+b\text{arcsinh}(c+dx)}{c+dx}}{de^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}b\int \frac{1}{(c+dx)^2\sqrt{(c+dx)^2+1}}d(c+dx)^2-\frac{a+b\text{arcsinh}(c+dx)}{c+dx}}{de^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{b\int \frac{1}{(c+dx)^4-1}d\sqrt{(c+dx)^2+1}-\frac{a+b\text{arcsinh}(c+dx)}{c+dx}}{de^2}$$

$$\Big\downarrow \text{220}$$

$$\frac{-\frac{a+b\text{arcsinh}(c+dx)}{c+dx}-b\,\text{arctanh}\left(\sqrt{(c+dx)^2+1}\right)}{de^2}$$

input
```
Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^2,x]
```

output
```
(-((a + b*ArcSinh[c + d*x])/(c + d*x)) - b*ArcTanh[Sqrt[1 + (c + d*x)^2]])
/(d*e^2)
```

3.121.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.121.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{-\frac{a}{e^2(dx+c)}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{dx+c}-\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)\right)}{e^2}}{d}$	54
default	$\frac{-\frac{a}{e^2(dx+c)}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{dx+c}-\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)\right)}{e^2}}{d}$	54
parts	$-\frac{a}{e^2(dx+c)d}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{dx+c}-\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)\right)}{e^2d}$	56

input
```
int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a/e^2/(d*x+c)+b/e^2*(-1/(d*x+c)*arcsinh(d*x+c)-arctanh(1/(1+(d*x+c)^
2)^(1/2))))
```

3.121.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(47) = 94.

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.57

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^2}\,dx$$
$$=\frac{bdx\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-ac-(bcdx+bc^2)\log\left(-dx-c+\sqrt{d^2x^2+2\,cdx+c^2+1}+\right.}{c}$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="fricas")
```

output
```
(b*d*x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - a*c - (b*c*d*x +
 b*c^2)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1) + (b*d*x + b
*c)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (b*c*d*x + b*c^2)*
log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) - 1))/(c*d^2*e^2*x + c^2*
d*e^2)
```

3.121.6 Sympy [F]

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{(ce + dex)^2} dx = \frac{\int \frac{a}{c^2+2cdx+d^2x^2} dx + \int \frac{b \operatorname{asinh}(c+dx)}{c^2+2cdx+d^2x^2} dx}{e^2}$$

input
```
integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**2,x)
```

output
```
(Integral(a/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b*asinh(c + d*x)/(
c**2 + 2*c*d*x + d**2*x**2), x))/e**2
```

3.121.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{(ce + dex)^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.121.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(47) = 94.

Time = 0.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.73

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{(ce + dex)^2} dx = -b\left(\frac{\log\left(dx + c + \sqrt{(dx + c)^2 + 1}\right)}{(dex + ce)de} + \frac{d\log\left(\sqrt{\frac{e^2}{(dex+ce)^2} + 1} + \frac{\sqrt{d^2e^4}}{(dex+ce)de}\right)}{e^2|d|^2\operatorname{sgn}\left(\frac{1}{dex+ce}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}\right) - \frac{a}{(dex + ce)de}$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")
```

output
```
-b*(log(d*x + c + sqrt((d*x + c)^2 + 1))/((d*e*x + c*e)*d*e) + d*log(sqrt(
e^2/(d*e*x + c*e)^2 + 1) + sqrt(d^2*e^4)/((d*e*x + c*e)*d*e))/(e^2*abs(d)^
2*sgn(1/(d*e*x + c*e))*sgn(d)*sgn(e))) - a/((d*e*x + c*e)*d*e)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^2}\,dx = \int \frac{a+b\,\text{asinh}(c+d\,x)}{(c\,e+d\,e\,x)^2}\,dx$$

input
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^2,x)
```

output
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^2, x)
```

3.122 $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^3}\,dx$

3.122.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^3}\,dx = -\frac{b\sqrt{1+(c+dx)^2}}{2de^3(c+dx)} - \frac{a+b\text{arcsinh}(c+dx)}{2de^3(c+dx)^2}$$

output
```
1/2*(-a-b*arcsinh(d*x+c))/d/e^3/(d*x+c)^2-1/2*b*(1+(d*x+c)^2)^(1/2)/d/e^3/
(d*x+c)
```

3.122.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^3}\,dx = \frac{-\frac{b\sqrt{1+(c+dx)^2}}{2(c+dx)} + \frac{-a-b\text{arcsinh}(c+dx)}{2(c+dx)^2}}{de^3}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^3,x]
```

output
```
(-1/2*(b*Sqrt[1 + (c + d*x)^2])/(c + d*x) + (-a - b*ArcSinh[c + d*x])/(2*(
c + d*x)^2))/(d*e^3)
```

3.122.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6274, 27, 6191, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^3}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{a+b\text{arcsinh}(c+dx)}{e^3(c+dx)^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\text{arcsinh}(c+dx)}{(c+dx)^3}d(c+dx)}{de^3}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{1}{2}b\int \frac{1}{(c+dx)^2\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{a+b\text{arcsinh}(c+dx)}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{242}$$

$$\frac{-\frac{a+b\text{arcsinh}(c+dx)}{2(c+dx)^2}-\frac{b\sqrt{(c+dx)^2+1}}{2(c+dx)}}{de^3}$$

input
```
Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^3,x]
```

output
```
(-1/2*(b*Sqrt[1 + (c + d*x)^2])/(c + d*x) - (a + b*ArcSinh[c + d*x])/(2*(c
 + d*x)^2))/(d*e^3)
```

3.122.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.122.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{2(dx+c)^2}-\frac{\sqrt{1+(dx+c)^2}}{2(dx+c)}\right)}{e^3}}{d}$	60
default	$\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{2(dx+c)^2}-\frac{\sqrt{1+(dx+c)^2}}{2(dx+c)}\right)}{e^3}}{d}$	60
parts	$-\frac{a}{2e^3(dx+c)^2d}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{2(dx+c)^2}-\frac{\sqrt{1+(dx+c)^2}}{2(dx+c)}\right)}{e^3d}$	62

input
```
int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a/e^3/(d*x+c)^2+b/e^3*(-1/2/(d*x+c)^2*arcsinh(d*x+c)-1/2/(d*x+c)
*(1+(d*x+c)^2)^(1/2)))
```

3.122.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(53) = 106$.

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.00

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^3}\,dx = \frac{ad^2x^2 + 2\,acdx - bc^2\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right) - (bc^2dx + bc^3)\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}}{2\,(c^2d^3e^3x^2 + 2\,c^3d^2e^3x + c^4de^3)}$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="fricas")
```

output
```
1/2*(a*d^2*x^2 + 2*a*c*d*x - b*c^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)) - (b*c^2*d*x + b*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(c^2*d^
3*e^3*x^2 + 2*c^3*d^2*e^3*x + c^4*d*e^3)
```

3.122.6 Sympy [F]

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^3}\,dx = \frac{\int \frac{a}{c^3 + 3c^2dx + 3cd^2x^2 + d^3x^3}\,dx + \int \frac{b\,\text{asinh}\,(c + dx)}{c^3 + 3c^2dx + 3cd^2x^2 + d^3x^3}\,dx}{e^3}$$

input
```
integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**3,x)
```

output
```
(Integral(a/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral
(b*asinh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**
3
```

3.122.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(53) = 106$.

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.98

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^3}\,dx = -\frac{1}{2}\,b\left(\frac{\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}d}{d^3e^3x + cd^2e^3} + \frac{\text{arsinh}\,(dx + c)}{d^3e^3x^2 + 2\,cd^2e^3x + c^2de^3}\right) - \frac{a}{2\,(d^3e^3x^2 + 2\,cd^2e^3x + c^2de^3)}$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="maxima")
```

output
```
-1/2*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d/(d^3*e^3*x + c*d^2*e^3) + arcs
inh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)) - 1/2*a/(d^3*e^3*x
^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

3.122.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^3}\,dx=\int \frac{b\,\text{arsinh}\,(dx+c)+a}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^3, x)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^3}\,dx=\int \frac{a+b\,\text{asinh}(c+d\,x)}{(c\,e+d\,e\,x)^3}\,dx$$

input
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^3,x)
```

output
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^3, x)
```

3.123 $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^4}\,dx$

3.123.1 Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^4}\,dx=-\frac{b\sqrt{1+(c+dx)^2}}{6de^4(c+dx)^2}-\frac{a+b\text{arcsinh}(c+dx)}{3de^4(c+dx)^3}+\frac{b\text{arctanh}\left(\sqrt{1+(c+dx)^2}\right)}{6de^4}$$

output
```
1/3*(-a-b*arcsinh(d*x+c))/d/e^4/(d*x+c)^3+1/6*b*arctanh((1+(d*x+c)^2)^(1/2
))/d/e^4-1/6*b*(1+(d*x+c)^2)^(1/2)/d/e^4/(d*x+c)^2
```

3.123.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^4}\,dx=\frac{-\frac{2(a+b\text{arcsinh}(c+dx))}{(c+dx)^3}+b\left(-\frac{\sqrt{1+(c+dx)^2}}{(c+dx)^2}+\text{arctanh}\left(\sqrt{1+(c+dx)^2}\right)\right)}{6de^4}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^4,x]
```

output
```
((-2*(a + b*ArcSinh[c + d*x]))/(c + d*x)^3 + b*(-(Sqrt[1 + (c + d*x)^2]/(c
 + d*x)^2) + ArcTanh[Sqrt[1 + (c + d*x)^2]]))/(6*d*e^4)
```

3.123.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6274, 27, 6191, 243, 52, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^4} \, dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \frac{a+b\operatorname{arcsinh}(c+dx)}{e^4(c+dx)^4}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(c+dx)^4}d(c+dx)}{de^4}$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{\frac{1}{3}b\int \frac{1}{(c+dx)^3\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{a+b\operatorname{arcsinh}(c+dx)}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\frac{1}{6}b\int \frac{1}{(c+dx)^4\sqrt{(c+dx)^2+1}}d(c+dx)^2-\frac{a+b\operatorname{arcsinh}(c+dx)}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{\frac{1}{6}b\left(-\frac{1}{2}\int \frac{1}{(c+dx)^2\sqrt{(c+dx)^2+1}}d(c+dx)^2-\frac{\sqrt{(c+dx)^2+1}}{(c+dx)^2}\right)-\frac{a+b\operatorname{arcsinh}(c+dx)}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{1}{6}b\left(-\int \frac{1}{(c+dx)^4-1}d\sqrt{(c+dx)^2+1}-\frac{\sqrt{(c+dx)^2+1}}{(c+dx)^2}\right)-\frac{a+b\operatorname{arcsinh}(c+dx)}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{\frac{1}{6}b\left(\operatorname{arctanh}\left(\sqrt{(c+dx)^2+1}\right)-\frac{\sqrt{(c+dx)^2+1}}{(c+dx)^2}\right)-\frac{a+b\operatorname{arcsinh}(c+dx)}{3(c+dx)^3}}{de^4}$$

input
```
Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^4,x]
```

3.123. $\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^4} \, dx$

output
```
(-1/3*(a + b*ArcSinh[c + d*x])/(c + d*x)^3 + (b*(-(Sqrt[1 + (c + d*x)^2]/(
c + d*x)^2) + ArcTanh[Sqrt[1 + (c + d*x)^2]]))/6)/(d*e^4)
```

3.123.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.123. $\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^4}\,dx$

3.123.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{-\frac{a}{3e^4(dx+c)^3}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{3(dx+c)^3}-\frac{\sqrt{1+(dx+c)^2}}{6(dx+c)^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)}{6}\right)}{e^4}}{d}$	74
default	$\frac{-\frac{a}{3e^4(dx+c)^3}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{3(dx+c)^3}-\frac{\sqrt{1+(dx+c)^2}}{6(dx+c)^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)}{6}\right)}{e^4}}{d}$	74
parts	$-\frac{a}{3e^4(dx+c)^3d}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{3(dx+c)^3}-\frac{\sqrt{1+(dx+c)^2}}{6(dx+c)^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)}{6}\right)}{e^4d}$	76

input
```
int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a/e^4/(d*x+c)^3+b/e^4*(-1/3/(d*x+c)^3*arcsinh(d*x+c)-1/6/(d*x+c)
^2*(1+(d*x+c)^2)^(1/2)+1/6*arctanh(1/(1+(d*x+c)^2)^(1/2))))
```

3.123.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(74) = 148$.

Time = 0.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 4.08

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^4}\,dx =$$
$$-\frac{2\,ac^3-2\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(bc^3d^3x^3+3\,bc^4d^2x^2+\right.}{}$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="fricas")
```

output
```
-1/6*(2*a*c^3 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x)*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (b*c^3*d^3*x^3 + 3*b*c^4*d^2*x^2 + 3*
b*c^5*d*x + b*c^6)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1) -
 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(-d*x - c + sqrt(d
^2*x^2 + 2*c*d*x + c^2 + 1)) + (b*c^3*d^3*x^3 + 3*b*c^4*d^2*x^2 + 3*b*c^5*
d*x + b*c^6)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) - 1) + (b*c^
3*d*x + b*c^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(c^3*d^4*e^4*x^3 + 3*c^4
*d^3*e^4*x^2 + 3*c^5*d^2*e^4*x + c^6*d*e^4)
```

3.123.6 Sympy [F]

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{(ce + dex)^4}\,dx = \frac{\int \frac{a}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b\operatorname{asinh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

input
```
integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**4,x)
```

output
```
(Integral(a/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x
**4), x) + Integral(b*asinh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2
 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

3.123.7 Maxima [F]

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{(ce + dex)^4}\,dx = \int \frac{b\operatorname{arsinh}(dx + c) + a}{(dex + ce)^4}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="maxima")
```

output
```
-1/6*b*(2*(d^2*x^2 + 2*c*d*x + c^2 + log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)))/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4)
 - I*(log(I*(d^2*x + c*d)/d + 1) - log(-I*(d^2*x + c*d)/d + 1))/(d*e^4) -
6*integrate(1/3/(d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4 + (15*c
^2*d^4*e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(5*c^4*d
^2*e^4 + 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x + (d^5*e^4*x
^5 + 5*c*d^4*e^4*x^4 + c^5*e^4 + c^3*e^4 + (10*c^2*d^3*e^4 + d^3*e^4)*x^3
+ (10*c^3*d^2*e^4 + 3*c*d^2*e^4)*x^2 + (5*c^4*d*e^4 + 3*c^2*d*e^4)*x)*sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)) - 1/3*a/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2
 + 3*c^2*d^2*e^4*x + c^3*d*e^4)
```

3.123.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^4}\,dx=\int \frac{b\,\text{arsinh}\,(dx+c)+a}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^4, x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^4}\,dx=\int \frac{a+b\,\text{asinh}(c+d\,x)}{(c\,e+d\,e\,x)^4}\,dx$$

input
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^4,x)
```

output
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^4, x)
```

3.124 $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^5}\,dx$

3.124.1 Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^5}\,dx = -\frac{b\sqrt{1+(c+dx)^2}}{12de^5(c+dx)^3}+\frac{b\sqrt{1+(c+dx)^2}}{6de^5(c+dx)}-\frac{a+b\text{arcsinh}(c+dx)}{4de^5(c+dx)^4}$$

output
```
1/4*(-a-b*arcsinh(d*x+c))/d/e^5/(d*x+c)^4-1/12*b*(1+(d*x+c)^2)^(1/2)/d/e^5
/(d*x+c)^3+1/6*b*(1+(d*x+c)^2)^(1/2)/d/e^5/(d*x+c)
```

3.124.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.68

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^5}\,dx = -\frac{b(c+dx)\left(1-2(c+dx)^2\right)\sqrt{1+(c+dx)^2}+3(a+b\text{arcsinh}(c+dx))}{12de^5(c+dx)^4}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^5,x]
```

output
```
-1/12*(b*(c + d*x)*(1 - 2*(c + d*x)^2)*Sqrt[1 + (c + d*x)^2] + 3*(a + b*Ar
cSinh[c + d*x]))/(d*e^5*(c + d*x)^4)
```

3.124.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6274, 27, 6191, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^5}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{a+b\text{arcsinh}(c+dx)}{e^5(c+dx)^5}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\text{arcsinh}(c+dx)}{(c+dx)^5}d(c+dx)}{de^5}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{1}{4}b\int \frac{1}{(c+dx)^4\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{a+b\text{arcsinh}(c+dx)}{4(c+dx)^4}}{de^5}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{1}{4}b\left(-\frac{2}{3}\int \frac{1}{(c+dx)^2\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{\sqrt{(c+dx)^2+1}}{3(c+dx)^3}\right)-\frac{a+b\text{arcsinh}(c+dx)}{4(c+dx)^4}}{de^5}$$

$$\Big\downarrow \text{242}$$

$$\frac{\frac{1}{4}b\left(\frac{2\sqrt{(c+dx)^2+1}}{3(c+dx)}-\frac{\sqrt{(c+dx)^2+1}}{3(c+dx)^3}\right)-\frac{a+b\text{arcsinh}(c+dx)}{4(c+dx)^4}}{de^5}$$

input
```
Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^5,x]
```

output
```
((b*(-1/3*Sqrt[1 + (c + d*x)^2]/(c + d*x)^3 + (2*Sqrt[1 + (c + d*x)^2])/(3
*(c + d*x))))/4 - (a + b*ArcSinh[c + d*x])/(4*(c + d*x)^4))/(d*e^5)
```

3.124.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.124.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{a}{4e^5(dx+c)^4}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{4(dx+c)^4}-\frac{\sqrt{1+(dx+c)^2}}{12(dx+c)^3}+\frac{\sqrt{1+(dx+c)^2}}{6dx+6c}\right)}{e^5}}{d}$	80
default	$\frac{-\frac{a}{4e^5(dx+c)^4}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{4(dx+c)^4}-\frac{\sqrt{1+(dx+c)^2}}{12(dx+c)^3}+\frac{\sqrt{1+(dx+c)^2}}{6dx+6c}\right)}{e^5}}{d}$	80
parts	$-\frac{a}{4e^5(dx+c)^4d}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{4(dx+c)^4}-\frac{\sqrt{1+(dx+c)^2}}{12(dx+c)^3}+\frac{\sqrt{1+(dx+c)^2}}{6dx+6c}\right)}{e^5d}$	82

input
```
int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*a/e^5/(d*x+c)^4+b/e^5*(-1/4/(d*x+c)^4*arcsinh(d*x+c)-1/12/(d*x+c
)^3*(1+(d*x+c)^2)^(1/2)+1/6/(d*x+c)*(1+(d*x+c)^2)^(1/2)))
```

3.124.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(80) = 160$.

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.33

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^5}\,dx$$
$$= \frac{3\,ad^4x^4 + 12\,acd^3x^3 + 18\,ac^2d^2x^2 + 12\,ac^3dx - 3\,bc^4\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right) + (2\,bc^4d^3x^3 \cdots}{12\,(c^4d^5e^5x^4 + 4\,c^5d^4e^5x^3 + 6\,c^6d^3e^5x^2 + 4\,c^7d^2e^5 \cdots}$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="fricas")
```

output
```
1/12*(3*a*d^4*x^4 + 12*a*c*d^3*x^3 + 18*a*c^2*d^2*x^2 + 12*a*c^3*d*x - 3*b
*c^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (2*b*c^4*d^3*x^3 +
 6*b*c^5*d^2*x^2 + 2*b*c^7 - b*c^5 + (6*b*c^6 - b*c^4)*d*x)*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1))/(c^4*d^5*e^5*x^4 + 4*c^5*d^4*e^5*x^3 + 6*c^6*d^3*e^5*
x^2 + 4*c^7*d^2*e^5*x + c^8*d*e^5)
```

3.124.6 Sympy [F]

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^5}\,dx$$
$$= \frac{\int \frac{a}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx + \int \frac{b\,\text{asinh}\,(c+dx)}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx}{e^5}$$

input
```
integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**5,x)
```

output
```
(Integral(a/(c**5 + 5*c**4*d*x + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5
*c*d**4*x**4 + d**5*x**5), x) + Integral(b*asinh(c + d*x)/(c**5 + 5*c**4*d
*x + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*c*d**4*x**4 + d**5*x**5), x
))/e**5
```

3.124.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(80) = 160$.

Time = 0.21 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.87

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^5}\,dx$$
$$= \frac{1}{12}b\left(\frac{(2\,d^4x^4 + 8\,cd^3x^3 + 2\,c^4 + (12\,c^2d^2 + d^2)x^2 + c^2 + 2\,(4\,c^3d + cd)x - 1)d}{(d^5e^5x^3 + 3\,cd^4e^5x^2 + 3\,c^2d^3e^5x + c^3d^2e^5)\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}} - \frac{3\text{ arsin}}{d^5e^5x^4 + 4\,cd^4e^5x^3 + 6\,c^2}\right.$$
$$- \frac{a}{4\,(d^5e^5x^4 + 4\,cd^4e^5x^3 + 6\,c^2d^3e^5x^2 + 4\,c^3d^2e^5x + c^4de^5)}$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="maxima")
```

output
```
1/12*b*((2*d^4*x^4 + 8*c*d^3*x^3 + 2*c^4 + (12*c^2*d^2 + d^2)*x^2 + c^2 +
2*(4*c^3*d + c*d)*x - 1)*d/((d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 + 3*c^2*d^3*e^5
*x + c^3*d^2*e^5)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*arcsinh(d*x + c)/
(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4
*d*e^5)) - 1/4*a/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^
3*d^2*e^5*x + c^4*d*e^5)
```

3.124.8 Giac [F]

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^5}\,dx = \int \frac{b\,\text{arsinh}\,(dx + c) + a}{(dex + ce)^5}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^5, x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^5}\,dx=\int \frac{a+b\,\mathrm{asinh}(c+d\,x)}{{\left(c\,e+d\,e\,x\right)}^5}\,dx$$

input
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^5,x)
```

output
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^5, x)
```

3.125 $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^6}\,dx$

3.125.1 Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^6}\,dx = -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4}+\frac{3b\sqrt{1+(c+dx)^2}}{40de^6(c+dx)^2} - \frac{a+b\text{arcsinh}(c+dx)}{5de^6(c+dx)^5}-\frac{3b\text{arctanh}\left(\sqrt{1+(c+dx)^2}\right)}{40de^6}$$

output
```
1/5*(-a-b*arcsinh(d*x+c))/d/e^6/(d*x+c)^5-3/40*b*arctanh((1+(d*x+c)^2)^(1/
2))/d/e^6-1/20*b*(1+(d*x+c)^2)^(1/2)/d/e^6/(d*x+c)^4+3/40*b*(1+(d*x+c)^2)^
(1/2)/d/e^6/(d*x+c)^2
```

3.125.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^6}\,dx = -\frac{\frac{a+b\text{arcsinh}(c+dx)}{(c+dx)^5}+b\sqrt{1+(c+dx)^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1+(c+dx)^2\right)}{5de^6}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^6,x]
```

output
```
-1/5*((a + b*ArcSinh[c + d*x])/(c + d*x)^5 + b*Sqrt[1 + (c + d*x)^2]*Hyper
geometric2F1[1/2, 3, 3/2, 1 + (c + d*x)^2])/(d*e^6)
```

3.125.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6274, 27, 6191, 243, 52, 52, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^6}\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \frac{a+b\text{arcsinh}(c+dx)}{e^6(c+dx)^6}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a+b\text{arcsinh}(c+dx)}{(c+dx)^6}d(c+dx)}{de^6}$$

$$\Big\downarrow \; 6191$$

$$\frac{\frac{1}{5}b\int \frac{1}{(c+dx)^5\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{a+b\text{arcsinh}(c+dx)}{5(c+dx)^5}}{de^6}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{1}{10}b\int \frac{1}{(c+dx)^6\sqrt{(c+dx)^2+1}}d(c+dx)^2-\frac{a+b\text{arcsinh}(c+dx)}{5(c+dx)^5}}{de^6}$$

$$\Big\downarrow \; 52$$

$$\frac{\frac{1}{10}b\left(-\frac{3}{4}\int \frac{1}{(c+dx)^4\sqrt{(c+dx)^2+1}}d(c+dx)^2-\frac{\sqrt{(c+dx)^2+1}}{2(c+dx)^4}\right)-\frac{a+b\text{arcsinh}(c+dx)}{5(c+dx)^5}}{de^6}$$

$$\Big\downarrow \; 52$$

$$\frac{\frac{1}{10}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int \frac{1}{(c+dx)^2\sqrt{(c+dx)^2+1}}d(c+dx)^2-\frac{\sqrt{(c+dx)^2+1}}{(c+dx)^2}\right)-\frac{\sqrt{(c+dx)^2+1}}{2(c+dx)^4}\right)-\frac{a+b\text{arcsinh}(c+dx)}{5(c+dx)^5}}{de^6}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{1}{10}b\left(-\frac{3}{4}\left(-\int \frac{1}{(c+dx)^4-1}d\sqrt{(c+dx)^2+1}-\frac{\sqrt{(c+dx)^2+1}}{(c+dx)^2}\right)-\frac{\sqrt{(c+dx)^2+1}}{2(c+dx)^4}\right)-\frac{a+b\text{arcsinh}(c+dx)}{5(c+dx)^5}}{de^6}$$

$$\Big\downarrow \ 220$$

$$\frac{\frac{1}{10}b\left(-\frac{3}{4}\left(\text{arctanh}\left(\sqrt{(c+dx)^2+1}\right)-\frac{\sqrt{(c+dx)^2+1}}{(c+dx)^2}\right)-\frac{\sqrt{(c+dx)^2+1}}{2(c+dx)^4}\right)-\frac{a+b\text{arcsinh}(c+dx)}{5(c+dx)^5}}{de^6}$$

input
```
Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^6,x]
```

output
```
(-1/5*(a + b*ArcSinh[c + d*x])/(c + d*x)^5 + (b*(-1/2*Sqrt[1 + (c + d*x)^2
]/(c + d*x)^4 - (3*(-(Sqrt[1 + (c + d*x)^2]/(c + d*x)^2) + ArcTanh[Sqrt[1
+ (c + d*x)^2]]))/4))/10)/(d*e^6)
```

3.125.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

3.125. $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^6}\,dx$

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.125.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{-\frac{a}{5e^6(dx+c)^5}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{5(dx+c)^5}-\frac{\sqrt{1+(dx+c)^2}}{20(dx+c)^4}+\frac{3\sqrt{1+(dx+c)^2}}{40(dx+c)^2}-\frac{3\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)}{40}\right)}{e^6}}{d}$	94
default	$\frac{-\frac{a}{5e^6(dx+c)^5}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{5(dx+c)^5}-\frac{\sqrt{1+(dx+c)^2}}{20(dx+c)^4}+\frac{3\sqrt{1+(dx+c)^2}}{40(dx+c)^2}-\frac{3\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)}{40}\right)}{e^6}}{d}$	94
parts	$-\frac{a}{5e^6(dx+c)^5d}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{5(dx+c)^5}-\frac{\sqrt{1+(dx+c)^2}}{20(dx+c)^4}+\frac{3\sqrt{1+(dx+c)^2}}{40(dx+c)^2}-\frac{3\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)}{40}\right)}{e^6d}$	96

input
```
int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/5*a/e^6/(d*x+c)^5+b/e^6*(-1/5/(d*x+c)^5*arcsinh(d*x+c)-1/20/(d*x+c
)^4*(1+(d*x+c)^2)^(1/2)+3/40/(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-3/40*arctanh(1/
(1+(d*x+c)^2)^(1/2))))
```

3.125.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(101) = 202$.

Time = 0.31 (sec) , antiderivative size = 509, normalized size of antiderivative = 4.43

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{(ce + dex)^6}\,dx = -\frac{8\,ac^5 - 8\,(bd^5x^5 + 5\,bcd^4x^4 + 10\,bc^2d^3x^3 + 10\,bc^3d^2x^2 + 5\,bc^4dx)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)}{}$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="fricas")
```

output
```
-1/40*(8*a*c^5 - 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^
3*d^2*x^2 + 5*b*c^4*d*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
+ 3*(b*c^5*d^5*x^5 + 5*b*c^6*d^4*x^4 + 10*b*c^7*d^3*x^3 + 10*b*c^8*d^2*x^2
 + 5*b*c^9*d*x + b*c^10)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
+ 1) - 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2
+ 5*b*c^4*d*x + b*c^5)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) -
 3*(b*c^5*d^5*x^5 + 5*b*c^6*d^4*x^4 + 10*b*c^7*d^3*x^3 + 10*b*c^8*d^2*x^2
+ 5*b*c^9*d*x + b*c^10)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) -
 1) - (3*b*c^5*d^3*x^3 + 9*b*c^6*d^2*x^2 + 3*b*c^8 - 2*b*c^6 + (9*b*c^7 -
2*b*c^5)*d*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(c^5*d^6*e^6*x^5 + 5*c^6*
d^5*e^6*x^4 + 10*c^7*d^4*e^6*x^3 + 10*c^8*d^3*e^6*x^2 + 5*c^9*d^2*e^6*x +
c^10*d*e^6)
```

3.125.6 Sympy [F]

$$\int \frac{a + b\operatorname{arcsinh}(c + dx)}{(ce + dex)^6}\,dx = \frac{\int \frac{a}{c^6+6c^5dx+15c^4d^2x^2+20c^3d^3x^3+15c^2d^4x^4+6cd^5x^5+d^6x^6}\,dx + \int \frac{b\operatorname{asinh}(c+dx)}{c^6+6c^5dx+15c^4d^2x^2+20c^3d^3x^3+15c^2d^4x^4+6cd^5x^5+d^6x^6}\,dx}{e^6}$$

input
```
integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**6,x)
```

output
```
(Integral(a/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 1
5*c**2*d**4*x**4 + 6*c*d**5*x**5 + d**6*x**6), x) + Integral(b*asinh(c + d
*x)/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15*c**2*d
**4*x**4 + 6*c*d**5*x**5 + d**6*x**6), x))/e**6
```

3.125.7 Maxima [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^6}\,dx=\int \frac{b\,\text{arsinh}\,(dx+c)+a}{(dex+ce)^6}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="maxima")
```

output
```
1/30*b*(2*(3*d^4*x^4 + 12*c*d^3*x^3 + 3*c^4 + (18*c^2*d^2 - d^2)*x^2 - c^2
 + 2*(6*c^3*d - c*d)*x - 3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
))/(d^6*e^6*x^5 + 5*c*d^5*e^6*x^4 + 10*c^2*d^4*e^6*x^3 + 10*c^3*d^3*e^6*x^
2 + 5*c^4*d^2*e^6*x + c^5*d*e^6) - 3*I*(log(I*(d^2*x + c*d)/d + 1) - log(-
I*(d^2*x + c*d)/d + 1))/(d*e^6) + 30*integrate(1/5/(d^8*e^6*x^8 + 8*c*d^7*
e^6*x^7 + c^8*e^6 + c^6*e^6 + (28*c^2*d^6*e^6 + d^6*e^6)*x^6 + 2*(28*c^3*d
^5*e^6 + 3*c*d^5*e^6)*x^5 + 5*(14*c^4*d^4*e^6 + 3*c^2*d^4*e^6)*x^4 + 4*(14
*c^5*d^3*e^6 + 5*c^3*d^3*e^6)*x^3 + (28*c^6*d^2*e^6 + 15*c^4*d^2*e^6)*x^2
+ 2*(4*c^7*d*e^6 + 3*c^5*d*e^6)*x + (d^7*e^6*x^7 + 7*c*d^6*e^6*x^6 + c^7*e
^6 + c^5*e^6 + (21*c^2*d^5*e^6 + d^5*e^6)*x^5 + 5*(7*c^3*d^4*e^6 + c*d^4*e
^6)*x^4 + 5*(7*c^4*d^3*e^6 + 2*c^2*d^3*e^6)*x^3 + (21*c^5*d^2*e^6 + 10*c^3
*d^2*e^6)*x^2 + (7*c^6*d*e^6 + 5*c^4*d*e^6)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)), x)) - 1/5*a/(d^6*e^6*x^5 + 5*c*d^5*e^6*x^4 + 10*c^2*d^4*e^6*x^3 +
 10*c^3*d^3*e^6*x^2 + 5*c^4*d^2*e^6*x + c^5*d*e^6)
```

3.125.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^6}\,dx=\int \frac{b\,\text{arsinh}\,(dx+c)+a}{(dex+ce)^6}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^6, x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^6}\,dx=\int \frac{a+b\,\mathrm{asinh}(c+d\,x)}{(c\,e+d\,e\,x)^6}\,dx$$

input
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^6,x)
```

output
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^6, x)
```

3.126 $\int (ce + dex)^m (a + b\text{arcsinh}(c + dx))^2 \, dx$

3.126.1 Optimal result

Integrand size = 23, antiderivative size = 187

$$\int (ce + dex)^m (a + b\text{arcsinh}(c + dx))^2 \, dx = \frac{(e(c + dx))^{1+m} (a + b\text{arcsinh}(c + dx))^2}{de(1 + m)} - \frac{2b(e(c + dx))^{2+m} (a + b\text{arcsinh}(c + dx)) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -(c + dx)^2\right)}{de^2(1 + m)(2 + m)} + \frac{2b^2(e(c + dx))^{3+m} {}_3F_2\left(1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}; 2 + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}; -(c + dx)^2\right)}{de^3(1 + m)(2 + m)(3 + m)}$$

output
```
(e*(d*x+c))^(1+m)*(a+b*arcsinh(d*x+c))^2/d/e/(1+m)-2*b*(e*(d*x+c))^(2+m)*(
a+b*arcsinh(d*x+c))*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-(d*x+c)^2)/d/e^2/(
1+m)/(2+m)+2*b^2*(e*(d*x+c))^(3+m)*hypergeom([1, 3/2+1/2*m, 3/2+1/2*m],[2+
1/2*m, 5/2+1/2*m],-(d*x+c)^2)/d/e^3/(3+m)/(m^2+3*m+2)
```

3.126.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.83

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^2\,dx$$
$$=\frac{(c+dx)(e(c+dx))^m\Big((a+b\text{arcsinh}(c+dx))^2-\frac{2b(c+dx)(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-(c+dx)^2\right)}{2+m}$$
$$d(1+m)$$

input
```
Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
((c + d*x)*(e*(c + d*x))^m*((a + b*ArcSinh[c + d*x])^2 - (2*b*(c + d*x)*(a
 + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c +
d*x)^2])/(2 + m) + (2*b^2*(c + d*x)^2*HypergeometricPFQ[{1, 3/2 + m/2, 3/2
 + m/2}, {2 + m/2, 5/2 + m/2}, -(c + d*x)^2])/((2 + m)*(3 + m))))/(d*(1 +
m))
```

3.126.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6274, 6191, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^2\,dx$$
$$\Big\downarrow \text{6274}$$
$$\frac{\int (e(c+dx))^m(a+b\text{arcsinh}(c+dx))^2d(c+dx)}{d}$$
$$\Big\downarrow \text{6191}$$
$$\frac{\frac{(e(c+dx))^{m+1}(a+b\text{arcsinh}(c+dx))^2}{e(m+1)}-\frac{2b\int \frac{(e(c+dx))^{m+1}(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)}{e(m+1)}}{d}$$
$$\Big\downarrow \text{6232}$$

$$\frac{\frac{(e(c+dx))^{m+1}(a+b\operatorname{arcsinh}(c+dx))^2}{e(m+1)} - \frac{2b\left(\frac{(e(c+dx))^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-(c+dx)^2\right)(a+b\operatorname{arcsinh}(c+dx))}{e(m+2)} - \frac{b(e(c+dx))^{m+3}\,{}_3F_2(}{}\right.}{e(m+1)}}{d}$$

input
```
Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^2)/(e*(1 + m)) - (2*b*(((
e*(c + d*x))^(2 + m)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, (2 +
m)/2, (4 + m)/2, -(c + d*x)^2])/(e*(2 + m)) - (b*(e*(c + d*x))^(3 + m)*Hyp
ergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, -(c + d*x)
^2])/(e^2*(2 + m)*(3 + m))))/(e*(1 + m)))/d
```

3.126.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.126.4 Maple [F]

$$\int (dex + ce)^m (a + b \operatorname{arcsinh}(dx + c))^2 \, dx$$

input
```
int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x)
```

output
```
int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x)
```

3.126.5 Fricas [F]

$$\int (ce + dex)^m (a + b\text{arcsinh}(c + dx))^2 \, dx = \int (b \operatorname{arsinh}(dx + c) + a)^2 (dex + ce)^m \, dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*(d*e*x +
c*e)^m, x)
```

3.126.6 Sympy [F]

$$\int (ce + dex)^m (a + b\text{arcsinh}(c + dx))^2 \, dx = \int (e(c + dx))^m (a + b \operatorname{asinh}(c + dx))^2 \, dx$$

input
```
integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c))**2,x)
```

output
```
Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x))**2, x)
```

3.126.7 Maxima [F]

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^2\,dx=\int (b\,\text{arsinh}\,(dx+c)+a)^2(dex+ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
(b^2*d*e^m*x + b^2*c*e^m)*(d*x + c)^m*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))^2/(d*(m + 1)) + (d*e*x + c*e)^(m + 1)*a^2/(d*e*(m + 1)) + int
egrate(-2*((b^2*c^2*e^m - (c^2*e^m*(m + 1) + e^m*(m + 1))*a*b - (a*b*d^2*e
^m*(m + 1) - b^2*d^2*e^m)*x^2 - 2*(a*b*c*d*e^m*(m + 1) - b^2*c*d*e^m)*x)*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m - ((a*b*d^3*e^m*(m + 1) - b^2
*d^3*e^m)*x^3 + (c^3*e^m*(m + 1) + c*e^m*(m + 1))*a*b - (c^3*e^m + c*e^m)*
b^2 + 3*(a*b*c*d^2*e^m*(m + 1) - b^2*c*d^2*e^m)*x^2 + ((3*c^2*d*e^m*(m + 1
) + d*e^m*(m + 1))*a*b - (3*c^2*d*e^m + d*e^m)*b^2)*x)*(d*x + c)^m)*log(d*
x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*(m + 1)*x^3 + 3*c*d^2*(m +
 1)*x^2 + c^3*(m + 1) + c*(m + 1) + (3*c^2*d*(m + 1) + d*(m + 1))*x + (d^2
*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) + m + 1)*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)), x)
```

3.126.8 Giac [F]

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^2\,dx=\int (b\,\text{arsinh}\,(dx+c)+a)^2(dex+ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^2*(d*e*x + c*e)^m, x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^m(a+b\operatorname{arcsinh}(c+dx))^2\,dx=\int (c\,e+d\,e\,x)^m\,(a+b\,\mathrm{asinh}(c+d\,x))^2\,dx$$

input
```
int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^2, x)
```

3.127 $\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^2\,dx$

3.127.1 Optimal result

Integrand size = 23, antiderivative size = 197

$$\begin{aligned}
&\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^2\,dx \\
&= \frac{16}{75}b^2e^4x - \frac{8b^2e^4(c+dx)^3}{225d} + \frac{2b^2e^4(c+dx)^5}{125d} - \frac{16be^4\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))}{75d} \\
&\quad + \frac{8be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))}{75d} \\
&\quad - \frac{2be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))}{25d} \\
&\quad + \frac{e^4(c+dx)^5(a+b\text{arcsinh}(c+dx))^2}{5d}
\end{aligned}$$

output
```
16/75*b^2*e^4*x-8/225*b^2*e^4*(d*x+c)^3/d+2/125*b^2*e^4*(d*x+c)^5/d+1/5*e^
4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^2/d-16/75*b*e^4*(a+b*arcsinh(d*x+c))*(1+(
d*x+c)^2)^(1/2)/d+8/75*b*e^4*(d*x+c)^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^
(1/2)/d-2/25*b*e^4*(d*x+c)^4*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d
```

3.127.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.97

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^2\,dx = \frac{e^4\left(240b^2(c+dx)-40b^2(c+dx)^3+9(25a^2+2b^2)(c+dx)^5+30ab\sqrt{1+(c+dx)^2}(-8+4(c+dx)^2-3\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(e^4*(240*b^2*(c + d*x) - 40*b^2*(c + d*x)^3 + 9*(25*a^2 + 2*b^2)*(c + d*x
)^5 + 30*a*b*Sqrt[1 + (c + d*x)^2]*(-8 + 4*(c + d*x)^2 - 3*(c + d*x)^4) +
30*b*(15*a*(c + d*x)^5 - 8*b*Sqrt[1 + (c + d*x)^2] + 4*b*(c + d*x)^2*Sqrt[
1 + (c + d*x)^2] - 3*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]
 + 225*b^2*(c + d*x)^5*ArcSinh[c + d*x]^2))/(1125*d)
```

3.127.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.87, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6274, 27, 6191, 6227, 15, 6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^2\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int e^4(c+dx)^4(a+b\text{arcsinh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^4\int (c+dx)^4(a+b\text{arcsinh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^2-\frac{2}{5}b\int \frac{(c+dx)^5(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6227}$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^2-\frac{2}{5}b\left(-\frac{4}{5}\int \frac{(c+dx)^3(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{5}b\int (c+dx)^4d(c+dx)+\right.\right.}{d}$$

$$\Big\downarrow \text{15}$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^2-\frac{2}{5}b\left(-\frac{4}{5}\int \frac{(c+dx)^3(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{5}\sqrt{(c+dx)^2+1}(c+dx)^4\right.\right.}{d}$$

$$\Big\downarrow \text{6227}$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^2-\frac{2}{5}b\left(-\frac{4}{5}\left(-\frac{2}{3}\int \frac{(c+dx)(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{3}b\int (c+dx)^2d(c+dx\right.\right.\right.$$

$$\Big\downarrow \text{15}$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^2-\frac{2}{5}b\left(-\frac{4}{5}\left(-\frac{2}{3}\int \frac{(c+dx)(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{3}\sqrt{(c+dx)^2+1}(c+\right.\right.\right.$$

$$\Big\downarrow \text{6213}$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^2-\frac{2}{5}b\left(-\frac{4}{5}\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))-b\int 1d(c+dx)\right)+\right.\right.\right.$$

$$\Big\downarrow \text{24}$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^2-\frac{2}{5}b\left(\frac{1}{5}\sqrt{(c+dx)^2+1}(c+dx)^4(a+b\text{arcsinh}(c+dx))-\frac{4}{5}\left(\frac{1}{3}\sqrt{(c+dx)^2\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(e^4*(((c + d*x)^5*(a + b*ArcSinh[c + d*x])^2)/5 - (2*b*(-1/25*(b*(c + d*x
)^5) + ((c + d*x)^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/5 - (4
*(-1/9*(b*(c + d*x)^3) + ((c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh
[c + d*x]))/3 - (2*(-(b*(c + d*x)) + Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[
c + d*x])))/3))/5))/5))/d
```

3.127. $\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^2\,dx$

3.127.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.127.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{e^4a^2(dx+c)^5}{5}+e^4b^2\left(\frac{(dx+c)^5 \operatorname{arcsinh}(dx+c)^2}{5}-\frac{16 \operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{75}-\frac{2(dx+c)^4 \operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{25}+\frac{8 \operatorname{arcsinh}}{}\right.}{}$
default	$\frac{\frac{e^4a^2(dx+c)^5}{5}+e^4b^2\left(\frac{(dx+c)^5 \operatorname{arcsinh}(dx+c)^2}{5}-\frac{16 \operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{75}-\frac{2(dx+c)^4 \operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{25}+\frac{8 \operatorname{arcsinh}}{}\right.}{}$
parts	$\frac{e^4a^2(dx+c)^5}{5d}+\frac{e^4b^2\left(\frac{(dx+c)^5 \operatorname{arcsinh}(dx+c)^2}{5}-\frac{16 \operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{75}-\frac{2(dx+c)^4 \operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{25}+\frac{8 \operatorname{arcs}}{}\right.}{d}$

input
```
int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*e^4*a^2*(d*x+c)^5+e^4*b^2*(1/5*(d*x+c)^5*arcsinh(d*x+c)^2-16/75*a
rcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)-2/25*(d*x+c)^4*arcsinh(d*x+c)*(1+(d*x+c)
^2)^(1/2)+8/75*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)^2+16/75*d*x+16/7
5*c+2/125*(d*x+c)^5-8/225*(d*x+c)^3)+2*e^4*a*b*(1/5*(d*x+c)^5*arcsinh(d*x+
c)-1/25*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+4/75*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-8
/75*(1+(d*x+c)^2)^(1/2)))
```

3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(177) = 354$.

Time = 0.30 (sec) , antiderivative size = 618, normalized size of antiderivative = 3.14

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^2\,dx$$
$$= \frac{9\,(25\,a^2+2\,b^2)d^5e^4x^5+45\,(25\,a^2+2\,b^2)cd^4e^4x^4+10\,(9\,(25\,a^2+2\,b^2)c^2-4\,b^2)d^3e^4x^3+30\,(3\,(25\,a^2+2\,b^2}{}$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/1125*(9*(25*a^2 + 2*b^2)*d^5*e^4*x^5 + 45*(25*a^2 + 2*b^2)*c*d^4*e^4*x^4
 + 10*(9*(25*a^2 + 2*b^2)*c^2 - 4*b^2)*d^3*e^4*x^3 + 30*(3*(25*a^2 + 2*b^2
)*c^3 - 4*b^2*c)*d^2*e^4*x^2 + 15*(3*(25*a^2 + 2*b^2)*c^4 - 8*b^2*c^2 + 16
*b^2)*d*e^4*x + 225*(b^2*d^5*e^4*x^5 + 5*b^2*c*d^4*e^4*x^4 + 10*b^2*c^2*d^
3*e^4*x^3 + 10*b^2*c^3*d^2*e^4*x^2 + 5*b^2*c^4*d*e^4*x + b^2*c^5*e^4)*log(
d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 30*(15*a*b*d^5*e^4*x^5 +
75*a*b*c*d^4*e^4*x^4 + 150*a*b*c^2*d^3*e^4*x^3 + 150*a*b*c^3*d^2*e^4*x^2 +
 75*a*b*c^4*d*e^4*x + 15*a*b*c^5*e^4 - (3*b^2*d^4*e^4*x^4 + 12*b^2*c*d^3*e
^4*x^3 + 2*(9*b^2*c^2 - 2*b^2)*d^2*e^4*x^2 + 4*(3*b^2*c^3 - 2*b^2*c)*d*e^4
*x + (3*b^2*c^4 - 4*b^2*c^2 + 8*b^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 30*(3*a*b*d^4*e^4*x^
4 + 12*a*b*c*d^3*e^4*x^3 + 2*(9*a*b*c^2 - 2*a*b)*d^2*e^4*x^2 + 4*(3*a*b*c^
3 - 2*a*b*c)*d*e^4*x + (3*a*b*c^4 - 4*a*b*c^2 + 8*a*b)*e^4)*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1))/d
```

3.127.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. $2(184) = 368$.

Time = 0.63 (sec) , antiderivative size = 1268, normalized size of antiderivative = 6.44

$$\int (ce + dex)^4 (a + b\text{arcsinh}(c + dx))^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**2,x)
```

output
```
Piecewise((a**2*c**4*e**4*x + 2*a**2*c**3*d*e**4*x**2 + 2*a**2*c**2*d**2*e
**4*x**3 + a**2*c*d**3*e**4*x**4 + a**2*d**4*e**4*x**5/5 + 2*a*b*c**5*e**4
*asinh(c + d*x)/(5*d) + 2*a*b*c**4*e**4*x*asinh(c + d*x) - 2*a*b*c**4*e**4
*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 4*a*b*c**3*d*e**4*x**2*asin
h(c + d*x) - 8*a*b*c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 4
*a*b*c**2*d**2*e**4*x**3*asinh(c + d*x) - 12*a*b*c**2*d*e**4*x**2*sqrt(c**
2 + 2*c*d*x + d**2*x**2 + 1)/25 + 8*a*b*c**2*e**4*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)/(75*d) + 2*a*b*c*d**3*e**4*x**4*asinh(c + d*x) - 8*a*b*c*d**2
*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 16*a*b*c*e**4*x*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)/75 + 2*a*b*d**4*e**4*x**5*asinh(c + d*x)/5
 - 2*a*b*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 8*a*b*d*
e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/75 - 16*a*b*e**4*sqrt(c**2
+ 2*c*d*x + d**2*x**2 + 1)/(75*d) + b**2*c**5*e**4*asinh(c + d*x)**2/(5*d)
 + b**2*c**4*e**4*x*asinh(c + d*x)**2 + 2*b**2*c**4*e**4*x/25 - 2*b**2*c**
4*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(25*d) + 2*b**2
*c**3*d*e**4*x**2*asinh(c + d*x)**2 + 4*b**2*c**3*d*e**4*x**2/25 - 8*b**2*
c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 + 2*b**
2*c**2*d**2*e**4*x**3*asinh(c + d*x)**2 + 4*b**2*c**2*d**2*e**4*x**3/25 -
12*b**2*c**2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*
x)/25 - 8*b**2*c**2*e**4*x/75 + 8*b**2*c**2*e**4*sqrt(c**2 + 2*c*d*x + ...
```

3.127.7 Maxima [F]

$$\int (ce + dex)^4 (a + b\text{arcsinh}(c + dx))^2 \, dx = \int (dex + ce)^4 (b \operatorname{arsinh}(dx + c) + a)^2 \, dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/5*a^2*d^4*e^4*x^5 + a^2*c*d^3*e^4*x^4 + 2*a^2*c^2*d^2*e^4*x^3 + 2*a^2*c^
3*d*e^4*x^2 + 2*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)
/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)
*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a*b*c^3*d*e^4 + 2
/3*(6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^
2 - 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4
 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(
d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2
*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/
d^4))*a*b*c^2*d^2*e^4 + 1/12*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^
3 + 105*c^4*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^
5 + 35*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcs
inh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c
^2 + 1)*x/d^4 + 9*(c^2 + 1)^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*
(c^2 + 1)*d^2))/d^5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5
)*d)*a*b*c*d^3*e^4 + 1/300*(120*x^5*arcsinh(d*x + c) - (24*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1)*x^4/d^2 - 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^3...
```

3.127.8 Giac [F]

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^2\,dx=\int (dex+ce)^4(b\,\text{arsinh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^2, x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^2\,dx=\int (c\,e+d\,e\,x)^4\,(a+b\,\mathrm{asinh}(c+d\,x))^2\,dx$$

input
```
int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^2, x)
```

3.128 $\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^2\,dx$

3.128.1 Optimal result

Integrand size = 23, antiderivative size = 172

$$\begin{aligned}&\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^2\,dx\\&=-\frac{3b^2e^3(c+dx)^2}{32d}+\frac{b^2e^3(c+dx)^4}{32d}+\frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))}{16d}\\&\quad-\frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))}{8d}\\&\quad-\frac{3e^3(a+b\text{arcsinh}(c+dx))^2}{32d}+\frac{e^3(c+dx)^4(a+b\text{arcsinh}(c+dx))^2}{4d}\end{aligned}$$

output
```
-3/32*b^2*e^3*(d*x+c)^2/d+1/32*b^2*e^3*(d*x+c)^4/d-3/32*e^3*(a+b*arcsinh(d
*x+c))^2/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^2/d+3/16*b*e^3*(d*x+c)*(
a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d-1/8*b*e^3*(d*x+c)^3*(a+b*arcsinh
(d*x+c))*(1+(d*x+c)^2)^(1/2)/d
```

3.128.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.99

$$\begin{aligned}&\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^2\,dx\\&=\frac{e^3\left(-3b^2(c+dx)^2+(8a^2+b^2)(c+dx)^4+2ab(c+dx)(3-2(c+dx)^2)\sqrt{1+(c+dx)^2}-6ab\text{arcsinh}(c-\right.}{}\end{aligned}$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(e^3*(-3*b^2*(c + d*x)^2 + (8*a^2 + b^2)*(c + d*x)^4 + 2*a*b*(c + d*x)*(3
- 2*(c + d*x)^2)*Sqrt[1 + (c + d*x)^2] - 6*a*b*ArcSinh[c + d*x] + 2*b*(c +
 d*x)*(8*a*(c + d*x)^3 + 3*b*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^2*Sqrt[
1 + (c + d*x)^2])*ArcSinh[c + d*x] + b^2*(-3 + 8*(c + d*x)^4)*ArcSinh[c +
d*x]^2))/(32*d)
```

3.128.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6274, 27, 6191, 6227, 15, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int e^3(c+dx)^3(a+b\text{arcsinh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^3\int (c+dx)^3(a+b\text{arcsinh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^2-\frac{1}{2}b\int \frac{(c+dx)^4(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 6227}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^2-\frac{1}{2}b\left(-\frac{3}{4}\int \frac{(c+dx)^2(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{4}b\int (c+dx)^3d(c+dx)+\right.\right.}{d}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^2-\frac{1}{2}b\left(-\frac{3}{4}\int \frac{(c+dx)^2(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{4}\sqrt{(c+dx)^2+1}(c+dx)^3\right.\right.}{d}$$

$$\Big\downarrow \text{6227}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^2-\frac{1}{2}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{a+b\text{arcsinh}(c+dx)}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{2}b\int(c+dx)d(c+dx)+\frac{1}{2}(c\right.\right.\right.}{}$$

$$\Big\downarrow \text{15}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^2-\frac{1}{2}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{a+b\text{arcsinh}(c+dx)}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{2}\sqrt{(c+dx)^2+1}(c+dx)(a+\right.\right.\right.}{d}$$

$$\Big\downarrow \text{6198}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^2-\frac{1}{2}b\left(\frac{1}{4}\sqrt{(c+dx)^2+1}(c+dx)^3(a+b\text{arcsinh}(c+dx))-\frac{3}{4}\left(\frac{1}{2}\sqrt{(c+dx)^2}\right.\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(e^3*(((c + d*x)^4*(a + b*ArcSinh[c + d*x])^2)/4 - (b*(-1/16*(b*(c + d*x)^
4) + ((c + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/4 - (3*(
-1/4*(b*(c + d*x)^2) + ((c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c +
 d*x]))/2 - (a + b*ArcSinh[c + d*x])^2/(4*b)))/4))/2))/d
```

3.128.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.128.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{e^3a^2(dx+c)^4}{4}+e^3b^2\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^2}{4}-\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{8}+\frac{3\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)}{16}-\frac{3\operatorname{arc}}{}\right.}{}$
default	$\frac{\frac{e^3a^2(dx+c)^4}{4}+e^3b^2\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^2}{4}-\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{8}+\frac{3\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)}{16}-\frac{3\operatorname{arc}}{}\right.}{}$
parts	$\frac{e^3a^2(dx+c)^4}{4d}+\frac{e^3b^2\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^2}{4}-\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{8}+\frac{3\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)}{16}-\frac{3}{}\right.}{d}$

input
```
int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*e^3*a^2*(d*x+c)^4+e^3*b^2*(1/4*(d*x+c)^4*arcsinh(d*x+c)^2-1/8*(d*
x+c)^3*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+3/16*(1+(d*x+c)^2)^(1/2)*(d*x+c)
*arcsinh(d*x+c)-3/32*arcsinh(d*x+c)^2+1/32*(d*x+c)^4-3/32*(d*x+c)^2-3/32)+
2*e^3*a*b*(1/4*(d*x+c)^4*arcsinh(d*x+c)-1/16*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)
+3/32*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/32*arcsinh(d*x+c)))
```

3.128.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(156) = 312$.

Time = 0.26 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.83

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^2\,dx$$
$$= \frac{(8a^2+b^2)d^4e^3x^4+4(8a^2+b^2)cd^3e^3x^3+3(2(8a^2+b^2)c^2-b^2)d^2e^3x^2+2(2(8a^2+b^2)c^3-3b^2c)de^3x}{}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/32*((8*a^2 + b^2)*d^4*e^3*x^4 + 4*(8*a^2 + b^2)*c*d^3*e^3*x^3 + 3*(2*(8*
a^2 + b^2)*c^2 - b^2)*d^2*e^3*x^2 + 2*(2*(8*a^2 + b^2)*c^3 - 3*b^2*c)*d*e^
3*x + (8*b^2*d^4*e^3*x^4 + 32*b^2*c*d^3*e^3*x^3 + 48*b^2*c^2*d^2*e^3*x^2 +
 32*b^2*c^3*d*e^3*x + (8*b^2*c^4 - 3*b^2)*e^3)*log(d*x + c + sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))^2 + 2*(8*a*b*d^4*e^3*x^4 + 32*a*b*c*d^3*e^3*x^3 + 48
*a*b*c^2*d^2*e^3*x^2 + 32*a*b*c^3*d*e^3*x + (8*a*b*c^4 - 3*a*b)*e^3 - (2*b
^2*d^3*e^3*x^3 + 6*b^2*c*d^2*e^3*x^2 + 3*(2*b^2*c^2 - b^2)*d*e^3*x + (2*b^
2*c^3 - 3*b^2*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*(2*a*b*d^3*e^3*x^3 + 6*a*b*c*d^2*e^3*x
^2 + 3*(2*a*b*c^2 - a*b)*d*e^3*x + (2*a*b*c^3 - 3*a*b*c)*e^3)*sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1))/d
```

3.128.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(155) = 310$.

Time = 0.47 (sec) , antiderivative size = 916, normalized size of antiderivative = 5.33

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^2\,dx$$
$$= \begin{cases} a^2c^3e^3x + \frac{3a^2c^2de^3x^2}{2} + a^2cd^2e^3x^3 + \frac{a^2d^3e^3x^4}{4} + \frac{abc^4e^3\operatorname{asinh}(c+dx)}{2d} + 2abc^3e^3x\operatorname{asinh}(c+dx) - \frac{abc^3e^3\sqrt{c^2+2cdx+}}{8d} \\ c^3e^3x(a+b\operatorname{asinh}(c))^2 \end{cases}$$

input
```
integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**2,x)
```

output
```
Piecewise((a**2*c**3*e**3*x + 3*a**2*c**2*d*e**3*x**2/2 + a**2*c*d**2*e**3
*x**3 + a**2*d**3*e**3*x**4/4 + a*b*c**4*e**3*asinh(c + d*x)/(2*d) + 2*a*b
*c**3*e**3*x*asinh(c + d*x) - a*b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**
2 + 1)/(8*d) + 3*a*b*c**2*d*e**3*x**2*asinh(c + d*x) - 3*a*b*c**2*e**3*x*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + 2*a*b*c*d**2*e**3*x**3*asinh(c + d
*x) - 3*a*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + 3*a*b*c
*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(16*d) + a*b*d**3*e**3*x**4*asi
nh(c + d*x)/2 - a*b*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8
+ 3*a*b*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 - 3*a*b*e**3*asinh(
c + d*x)/(16*d) + b**2*c**4*e**3*asinh(c + d*x)**2/(4*d) + b**2*c**3*e**3*
x*asinh(c + d*x)**2 + b**2*c**3*e**3*x/8 - b**2*c**3*e**3*sqrt(c**2 + 2*c*
d*x + d**2*x**2 + 1)*asinh(c + d*x)/(8*d) + 3*b**2*c**2*d*e**3*x**2*asinh(
c + d*x)**2/2 + 3*b**2*c**2*d*e**3*x**2/16 - 3*b**2*c**2*e**3*x*sqrt(c**2
+ 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/8 + b**2*c*d**2*e**3*x**3*asinh(
c + d*x)**2 + b**2*c*d**2*e**3*x**3/8 - 3*b**2*c*d*e**3*x**2*sqrt(c**2 + 2
*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/8 - 3*b**2*c*e**3*x/16 + 3*b**2*c*e
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(16*d) + b**2*d**3
*e**3*x**4*asinh(c + d*x)**2/4 + b**2*d**3*e**3*x**4/32 - b**2*d**2*e**3*x
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/8 - 3*b**2*d*e**3*
x**2/32 + 3*b**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + ...
```

3.128.7 Maxima [F]

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^2\,dx=\int (dex+ce)^3(b\,\text{arsinh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*d^3*e^3*x^4 + a^2*c*d^2*e^3*x^3 + 3/2*a^2*c^2*d*e^3*x^2 + 3/2*(2*x
^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4
*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)
*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a*b*c^2*d*e^3 + 1/3*(6*x^3*arcsinh(d*
x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2
*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4
*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^
2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a*b*c*d^2*e^3
+ 1/48*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^3
/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(2*
(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/sq
rt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c^2
 + 1)^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 +
55*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a*b*d^3*e^3 + a^2
*c^3*e^3*x + 2*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a*b*c^
3*e^3/d + 1/4*(b^2*d^3*e^3*x^4 + 4*b^2*c*d^2*e^3*x^3 + 6*b^2*c^2*d*e^3*...
```

3.128.8 Giac [F]

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^2\,dx=\int (dex+ce)^3(b\,\text{arsinh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^2, x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^2\,dx=\int (c\,e+d\,e\,x)^3\,(a+b\,\mathrm{asinh}(c+d\,x))^2\,dx$$

input
```
int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^2, x)
```

3.129 $\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^2\,dx$

3.129.1 Optimal result

Integrand size = 23, antiderivative size = 136

$$\begin{aligned}\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^2\,dx &= -\frac{4}{9}b^2e^2x+\frac{2b^2e^2(c+dx)^3}{27d}+\frac{4be^2\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))}{9d}\\ &\quad-\frac{2be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))}{9d}\\ &\quad+\frac{e^2(c+dx)^3(a+b\text{arcsinh}(c+dx))^2}{3d}\end{aligned}$$

output
```
-4/9*b^2*e^2*x+2/27*b^2*e^2*(d*x+c)^3/d+1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x
+c))^2/d+4/9*b*e^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d-2/9*b*e^2*(d
*x+c)^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d
```

3.129.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.08

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^2\,dx = \frac{e^2\Big(-12b^2(c+dx)+(9a^2+2b^2)(c+dx)^3+6ab(2-(c+dx)^2)\sqrt{1+(c+dx)^2}+6b\Big(3a(c+dx)^3+2b\sqrt{}}{27d}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(e^2*(-12*b^2*(c + d*x) + (9*a^2 + 2*b^2)*(c + d*x)^3 + 6*a*b*(2 - (c + d*
x)^2)*Sqrt[1 + (c + d*x)^2] + 6*b*(3*a*(c + d*x)^3 + 2*b*Sqrt[1 + (c + d*x
)^2] - b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 9*b^2*(c +
d*x)^3*ArcSinh[c + d*x]^2))/(27*d)
```

3.129.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6274, 27, 6191, 6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^2\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int e^2(c+dx)^2(a+b\text{arcsinh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\int (c+dx)^2(a+b\text{arcsinh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^2-\frac{2}{3}b\int \frac{(c+dx)^3(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6227}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^2-\frac{2}{3}b\left(-\frac{2}{3}\int \frac{(c+dx)(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{3}b\int (c+dx)^2d(c+dx)+\frac{1}{3}\right.\right.}{d}$$

$$\Big\downarrow \text{15}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^2-\frac{2}{3}b\left(-\frac{2}{3}\int \frac{(c+dx)(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{3}\sqrt{(c+dx)^2+1}(c+dx)^2(\right.\right.}{d}$$

$$\Big\downarrow \text{6213}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^2-\frac{2}{3}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))-b\int 1d(c+dx)\right)+\frac{1}{3}\sqrt{(}\right.\right.}{d}$$

$\downarrow$ 24

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^2-\frac{2}{3}b\left(\frac{1}{3}\sqrt{(c+dx)^2+1}(c+dx)^2(a+b\text{arcsinh}(c+dx))-\frac{2}{3}\left(\sqrt{(c+dx)^2+}\right.\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(e^2*(((c + d*x)^3*(a + b*ArcSinh[c + d*x])^2)/3 - (2*b*(-1/9*(b*(c + d*x)
^3) + ((c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/3 - (2*
(-(b*(c + d*x)) + Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])))/3))/3))
/d
```

3.129.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.129.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{a^2e^2(dx+c)^3}{3}+e^2b^2\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^2}{3}+\frac{4\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{9}-\frac{2\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}\,(dx+c)^2}{9}-\frac{4dx}{9}-\frac{4c}{9}\cdots\right.}{d}$
default	$\frac{\frac{a^2e^2(dx+c)^3}{3}+e^2b^2\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^2}{3}+\frac{4\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{9}-\frac{2\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}\,(dx+c)^2}{9}-\frac{4dx}{9}-\frac{4c}{9}\cdots\right.}{d}$
parts	$\frac{a^2e^2(dx+c)^3}{3d}+\frac{e^2b^2\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^2}{3}+\frac{4\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{9}-\frac{2\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}\,(dx+c)^2}{9}-\frac{4dx}{9}\cdots\right.}{d}$

input
```
int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*a^2*e^2*(d*x+c)^3+e^2*b^2*(1/3*(d*x+c)^3*arcsinh(d*x+c)^2+4/9*arc
sinh(d*x+c)*(1+(d*x+c)^2)^(1/2)-2/9*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*
x+c)^2-4/9*d*x-4/9*c+2/27*(d*x+c)^3)+2*e^2*a*b*(1/3*(d*x+c)^3*arcsinh(d*x+
c)-1/9*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+2/9*(1+(d*x+c)^2)^(1/2)))
```

3.129.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(122) = 244$.

Time = 0.27 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.63

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^2\,dx$$
$$=\frac{(9\,a^2+2\,b^2)d^3e^2x^3+3\,(9\,a^2+2\,b^2)cd^2e^2x^2+3\,((9\,a^2+2\,b^2)c^2-4\,b^2)de^2x+9\,(b^2d^3e^2x^3+3\,b^2cd^2e^2x^2+}{}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/27*((9*a^2 + 2*b^2)*d^3*e^2*x^3 + 3*(9*a^2 + 2*b^2)*c*d^2*e^2*x^2 + 3*((
9*a^2 + 2*b^2)*c^2 - 4*b^2)*d*e^2*x + 9*(b^2*d^3*e^2*x^3 + 3*b^2*c*d^2*e^2
*x^2 + 3*b^2*c^2*d*e^2*x + b^2*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))^2 + 6*(3*a*b*d^3*e^2*x^3 + 9*a*b*c*d^2*e^2*x^2 + 9*a*b*c^2*
d*e^2*x + 3*a*b*c^3*e^2 - (b^2*d^2*e^2*x^2 + 2*b^2*c*d*e^2*x + (b^2*c^2 -
2*b^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1)) - 6*(a*b*d^2*e^2*x^2 + 2*a*b*c*d*e^2*x + (a*b*c^2 -
2*a*b)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

3.129.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(126) = 252$.

Time = 0.32 (sec) , antiderivative size = 610, normalized size of antiderivative = 4.49

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^2\,dx$$
$$=\begin{cases} a^2c^2e^2x+a^2cde^2x^2+\frac{a^2d^2e^2x^3}{3}+\frac{2abc^3e^2\operatorname{asinh}(c+dx)}{3d}+2abc^2e^2x\operatorname{asinh}(c+dx)-\frac{2abc^2e^2\sqrt{c^2+2cdx+d^2x^2+1}}{9d}+2a \\ c^2e^2x(a+b\operatorname{asinh}(c))^2 \end{cases}$$

input
```
integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**2,x)
```

output
```
Piecewise((a**2*c**2*e**2*x + a**2*c*d*e**2*x**2 + a**2*d**2*e**2*x**3/3 +
 2*a*b*c**3*e**2*asinh(c + d*x)/(3*d) + 2*a*b*c**2*e**2*x*asinh(c + d*x) -
 2*a*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d) + 2*a*b*c*d*e*
*2*x**2*asinh(c + d*x) - 4*a*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
1)/9 + 2*a*b*d**2*e**2*x**3*asinh(c + d*x)/3 - 2*a*b*d*e**2*x**2*sqrt(c**2
 + 2*c*d*x + d**2*x**2 + 1)/9 + 4*a*b*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/(9*d) + b**2*c**3*e**2*asinh(c + d*x)**2/(3*d) + b**2*c**2*e**2*x*as
inh(c + d*x)**2 + 2*b**2*c**2*e**2*x/9 - 2*b**2*c**2*e**2*sqrt(c**2 + 2*c*
d*x + d**2*x**2 + 1)*asinh(c + d*x)/(9*d) + b**2*c*d*e**2*x**2*asinh(c + d
*x)**2 + 2*b**2*c*d*e**2*x**2/9 - 4*b**2*c*e**2*x*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)*asinh(c + d*x)/9 + b**2*d**2*e**2*x**3*asinh(c + d*x)**2/3 +
2*b**2*d**2*e**2*x**3/27 - 2*b**2*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x
**2 + 1)*asinh(c + d*x)/9 - 4*b**2*e**2*x/9 + 4*b**2*e**2*sqrt(c**2 + 2*c*
d*x + d**2*x**2 + 1)*asinh(c + d*x)/(9*d), Ne(d, 0)), (c**2*e**2*x*(a + b*
asinh(c))**2, True))
```

3.129.7 Maxima [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^2\,dx=\int (dex+ce)^2(b\,\text{arsinh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*d^2*e^2*x^3 + a^2*c*d*e^2*x^2 + (2*x^2*arcsinh(d*x + c) - d*(3*c^2
*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(
-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c
/d^3))*a*b*c*d*e^2 + 1/9*(6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 +
 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(
c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4
 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1)*(c^2 + 1)/d^4))*a*b*d^2*e^2 + a^2*c^2*e^2*x + 2*((d*x + c)*arc
sinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a*b*c^2*e^2/d + 1/3*(b^2*d^2*e^2*x^
3 + 3*b^2*c*d*e^2*x^2 + 3*b^2*c^2*e^2*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1))^2 - integrate(2/3*(b^2*d^5*e^2*x^5 + 5*b^2*c*d^4*e^2*x^4 +
 (10*c^2*d^3*e^2 + d^3*e^2)*b^2*x^3 + 3*(3*c^3*d^2*e^2 + c*d^2*e^2)*b^2*x^
2 + 3*(c^4*d*e^2 + c^2*d*e^2)*b^2*x + (b^2*d^4*e^2*x^4 + 4*b^2*c*d^3*e^2*x
^3 + 6*b^2*c^2*d^2*e^2*x^2 + 3*b^2*c^3*d*e^2*x)*sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 + 3*c*d
^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c),
 x)
```

3.129.8 Giac [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^2\,dx=\int (dex+ce)^2(b\,\text{arsinh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^2, x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^2\,dx=\int (c\,e+d\,e\,x)^2\,(a+b\,\text{asinh}(c+d\,x))^2\,dx$$

input
```
int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^2, x)
```

3.130 $\int (ce + dex)(a + b\text{arcsinh}(c + dx))^2 \, dx$

3.130.1 Optimal result

Integrand size = 21, antiderivative size = 103

$$\int (ce + dex)(a + b\text{arcsinh}(c + dx))^2 \, dx = \frac{b^2 e(c + dx)^2}{4d} - \frac{be(c + dx)\sqrt{1 + (c + dx)^2}(a + b\text{arcsinh}(c + dx))}{2d} + \frac{e(a + b\text{arcsinh}(c + dx))^2}{4d} + \frac{e(c + dx)^2(a + b\text{arcsinh}(c + dx))^2}{2d}$$

output
```
1/4*b^2*e*(d*x+c)^2/d+1/4*e*(a+b*arcsinh(d*x+c))^2/d+1/2*e*(d*x+c)^2*(a+b*
arcsinh(d*x+c))^2/d-1/2*b*e*(d*x+c)*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/
2)/d
```

3.130.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.17

$$\int (ce + dex)(a + b\text{arcsinh}(c + dx))^2 \, dx = \frac{e\Big((2a^2 + b^2)(c + dx)^2 - 2ab(c + dx)\sqrt{1 + (c + dx)^2} + 2ab\text{arcsinh}(c + dx) + 2b(c + dx)\Big(2a(c + dx) - b}{4d}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(e*((2*a^2 + b^2)*(c + d*x)^2 - 2*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] + 2*
a*b*ArcSinh[c + d*x] + 2*b*(c + d*x)*(2*a*(c + d*x) - b*Sqrt[1 + (c + d*x)
^2])*ArcSinh[c + d*x] + b^2*(1 + 2*(c + d*x)^2)*ArcSinh[c + d*x]^2))/(4*d)
```

3.130.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6274, 27, 6191, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^2\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int e(c+dx)(a+b\text{arcsinh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int (c+dx)(a+b\text{arcsinh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^2-b\int \frac{(c+dx)^2(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6227}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^2-b\left(-\frac{1}{2}\int \frac{a+b\text{arcsinh}(c+dx)}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{2}b\int (c+dx)d(c+dx)+\frac{1}{2}(c+dx)\sqrt{\cdots}\right.\right.}{d}$$

$$\Big\downarrow \text{15}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^2-b\left(-\frac{1}{2}\int \frac{a+b\text{arcsinh}(c+dx)}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{2}\sqrt{(c+dx)^2+1}(c+dx)(a+\text{barcsi}\cdots\right.\right.}{d}$$

$$\Big\downarrow \text{6198}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^2-b\left(\frac{1}{2}\sqrt{(c+dx)^2+1}(c+dx)(a+b\text{arcsinh}(c+dx))-\frac{(a+b\text{arcsinh}(c+dx))^2}{4b}-\cdots\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(e*(((c + d*x)^2*(a + b*ArcSinh[c + d*x])^2)/2 - b*(-1/4*(b*(c + d*x)^2) +
 ((c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/2 - (a + b*Arc
Sinh[c + d*x])^2/(4*b))))/d
```

3.130.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.130.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{\frac{e a^2 (dx+c)^2}{2}+e b^2\left(\frac{\operatorname{arcsinh}(dx+c)^2\left(1+(dx+c)^2\right)}{2}-\frac{\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)}{2}-\frac{\operatorname{arcsinh}(dx+c)^2}{4}+\frac{(dx+c)^2}{4}+\frac{1}{4}\right)+2eab(}{d}$
default	$\frac{\frac{e a^2 (dx+c)^2}{2}+e b^2\left(\frac{\operatorname{arcsinh}(dx+c)^2\left(1+(dx+c)^2\right)}{2}-\frac{\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)}{2}-\frac{\operatorname{arcsinh}(dx+c)^2}{4}+\frac{(dx+c)^2}{4}+\frac{1}{4}\right)+2eab(}{d}$
parts	$e a^2\left(\frac{1}{2} d x^2+cx\right)+\frac{e b^2\left(\frac{\operatorname{arcsinh}(dx+c)^2\left(1+(dx+c)^2\right)}{2}-\frac{\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)}{2}-\frac{\operatorname{arcsinh}(dx+c)^2}{4}+\frac{(dx+c)^2}{4}\right.}{d}$

input
```
int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*e*a^2*(d*x+c)^2+e*b^2*(1/2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/2*(1+
(d*x+c)^2)^(1/2)*(d*x+c)*arcsinh(d*x+c)-1/4*arcsinh(d*x+c)^2+1/4*(d*x+c)^2
+1/4)+2*e*a*b*(1/2*(d*x+c)^2*arcsinh(d*x+c)-1/4*(d*x+c)*(1+(d*x+c)^2)^(1/2
)+1/4*arcsinh(d*x+c)))
```

3.130.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(93) = 186.

Time = 0.26 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.23

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^2\,dx$$

$$=\frac{(2a^2+b^2)d^2ex^2+2(2a^2+b^2)cdex+(2b^2d^2ex^2+4b^2cdex+(2b^2c^2+b^2)e)\log\left(dx+c+\sqrt{d^2x^2+2cdx}\right.}{}$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/4*((2*a^2 + b^2)*d^2*e*x^2 + 2*(2*a^2 + b^2)*c*d*e*x + (2*b^2*d^2*e*x^2
+ 4*b^2*c*d*e*x + (2*b^2*c^2 + b^2)*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1))^2 + 2*(2*a*b*d^2*e*x^2 + 4*a*b*c*d*e*x + (2*a*b*c^2 + a*b)*e
 - (b^2*d*e*x + b^2*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*(a*b*d*e*x + a*b*c*e)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))/d
```

3.130.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(88) = 176$.

Time = 0.19 (sec) , antiderivative size = 335, normalized size of antiderivative = 3.25

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))^2\,dx$$
$$= \begin{cases} a^2cex + \frac{a^2dex^2}{2} + \frac{abc^2e\operatorname{asinh}(c+dx)}{d} + 2abcex\operatorname{asinh}(c+dx) - \frac{abce\sqrt{c^2+2cdx+d^2x^2+1}}{2d} + abdex^2\operatorname{asinh}(c+dx) - \\ cex(a + b\operatorname{asinh}(c))^2 \end{cases}$$

input
```
integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**2,x)
```

output
```
Piecewise((a**2*c*e*x + a**2*d*e*x**2/2 + a*b*c**2*e*asinh(c + d*x)/d + 2*
a*b*c*e*x*asinh(c + d*x) - a*b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(2
*d) + a*b*d*e*x**2*asinh(c + d*x) - a*b*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**
2 + 1)/2 + a*b*e*asinh(c + d*x)/(2*d) + b**2*c**2*e*asinh(c + d*x)**2/(2*d
) + b**2*c*e*x*asinh(c + d*x)**2 + b**2*c*e*x/2 - b**2*c*e*sqrt(c**2 + 2*c
*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(2*d) + b**2*d*e*x**2*asinh(c + d*x)*
*2/2 + b**2*d*e*x**2/4 - b**2*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asi
nh(c + d*x)/2 + b**2*e*asinh(c + d*x)**2/(4*d), Ne(d, 0)), (c*e*x*(a + b*a
sinh(c))**2, True))
```

3.130.7 Maxima [F]

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))^2\,dx = \int (dex + ce)(b\operatorname{arsinh}(dx + c) + a)^2\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*d*e*x^2 + 1/2*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x
+ c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^
2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a*b*d*e + a
^2*c*e*x + 2*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a*b*c*e/
d + 1/2*(b^2*d*e*x^2 + 2*b^2*c*e*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1))^2 - integrate((b^2*d^4*e*x^4 + 4*b^2*c*d^3*e*x^3 + (5*c^2*d^2*e
 + d^2*e)*b^2*x^2 + 2*(c^3*d*e + c*d*e)*b^2*x + (b^2*d^3*e*x^3 + 3*b^2*c*d
^2*e*x^2 + 2*b^2*c^2*d*e*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c
 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^
2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

3.130.8 Giac [F]

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^2\,dx=\int (dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^2, x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^2\,dx=\int (c\,e+d\,e\,x)\,\left(a+b\,\text{asinh}(c+d\,x)\right)^2\,dx$$

input
```
int((c*e + d*e*x)*(a + b*asinh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)*(a + b*asinh(c + d*x))^2, x)
```

3.131 $\int (a+b\text{arcsinh}(c+dx))^2\,dx$

3.131.1 Optimal result

Integrand size = 12, antiderivative size = 57

$$\int (a+b\text{arcsinh}(c+dx))^2\,dx = 2b^2x - \frac{2b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))}{d} + \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^2}{d}$$

output
```
2*b^2*x+(d*x+c)*(a+b*arcsinh(d*x+c))^2/d-2*b*(a+b*arcsinh(d*x+c))*(1+(d*x+
c)^2)^(1/2)/d
```

3.131.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.53

$$\int (a+b\text{arcsinh}(c+dx))^2\,dx = \frac{(a^2+2b^2)(c+dx) - 2ab\sqrt{1+(c+dx)^2} + 2b\left(ac+adx-b\sqrt{1+(c+dx)^2}\right)\text{arcsinh}(c+dx) + b^2(c+d}{d}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
((a^2 + 2*b^2)*(c + d*x) - 2*a*b*Sqrt[1 + (c + d*x)^2] + 2*b*(a*c + a*d*x
- b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + b^2*(c + d*x)*ArcSinh[c + d*
x]^2)/d
```

3.131.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6273, 6187, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(c+dx))^2 \, dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int (a+b\text{arcsinh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{6187}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^2-2b\int \frac{(c+dx)(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6213}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^2-2b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))-b\int 1d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{24}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^2-2b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))-b(c+dx)\right)}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
((c + d*x)*(a + b*ArcSinh[c + d*x])^2 - 2*b*(-(b*(c + d*x)) + Sqrt[1 + (c
+ d*x)^2]*(a + b*ArcSinh[c + d*x])))/d
```

3.131.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.131.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.54

method	result
parts	$a^2x+\frac{b^2\left((dx+c)\operatorname{arcsinh}(dx+c)^2-2\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}+2dx+2c\right)}{d}+\frac{2ab\left((dx+c)\operatorname{arcsinh}(dx+c)-\sqrt{1+\right.}{d}$
derivativedivides	$\frac{(dx+c)a^2+b^2\left((dx+c)\operatorname{arcsinh}(dx+c)^2-2\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}+2dx+2c\right)+2ab\left((dx+c)\operatorname{arcsinh}(dx+c)-\sqrt{1+(\right.}{d}$
default	$\frac{(dx+c)a^2+b^2\left((dx+c)\operatorname{arcsinh}(dx+c)^2-2\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}+2dx+2c\right)+2ab\left((dx+c)\operatorname{arcsinh}(dx+c)-\sqrt{1+(\right.}{d}$

input
```
int((a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+b^2/d*((d*x+c)*arcsinh(d*x+c)^2-2*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)
+2*d*x+2*c)+2*a*b/d*((d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2))
```

3.131.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(55) = 110.

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.47

$$\int (a + b\text{arcsinh}(c + dx))^2 \, dx$$
$$= \frac{(a^2 + 2\,b^2)dx + (b^2dx + b^2c)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)^2 - 2\,\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}ab + 2\,(a}{d}$$

input
```
integrate((a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
((a^2 + 2*b^2)*d*x + (b^2*d*x + b^2*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1))^2 - 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*a*b + 2*(a*b*d*x + a
*b*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*b^2)*log(d*x + c + sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1)))/d
```

3.131.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(51) = 102.

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.51

$$\int (a + b\text{arcsinh}(c + dx))^2 \, dx$$
$$= \begin{cases} a^2x + \frac{2abc\,\text{asinh}\,(c+dx)}{d} + 2abx\,\text{asinh}\,(c + dx) - \frac{2ab\sqrt{c^2+2cdx+d^2x^2+1}}{d} + \frac{b^2c\,\text{asinh}^2\,(c+dx)}{d} + b^2x\,\text{asinh}^2\,(c + dx) + \\ x(a + b\,\text{asinh}\,(c))^2 \end{cases}$$

input
```
integrate((a+b*asinh(d*x+c))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*c*asinh(c + d*x)/d + 2*a*b*x*asinh(c + d*x) - 2*
a*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d + b**2*c*asinh(c + d*x)**2/d +
b**2*x*asinh(c + d*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)*asinh(c + d*x)/d, Ne(d, 0)), (x*(a + b*asinh(c))**2, True))
```

3.131.7 Maxima [F]

$$\int (a+b\text{arcsinh}(c+dx))^2\,dx=\int (b\operatorname{arsinh}(dx+c)+a)^2\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
(x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - integrate(2*(d^3*x
^3 + 2*c*d^2*x^2 + (c^2*d + d)*x + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d^2*
x^2 + c*d*x))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 +
3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2)
+ c), x))*b^2 + a^2*x + 2*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 +
 1))*a*b/d
```

3.131.8 Giac [F]

$$\int (a+b\text{arcsinh}(c+dx))^2\,dx=\int (b\operatorname{arsinh}(dx+c)+a)^2\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^2, x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arcsinh}(c+dx))^2\,dx=\int (a+b\operatorname{asinh}(c+d\,x))^2\,dx$$

input
```
int((a + b*asinh(c + d*x))^2,x)
```

output
```
int((a + b*asinh(c + d*x))^2, x)
```

3.132 $\int \frac{(a+b\text{arcsinh}(c+dx))^2}{ce+dex}\,dx$

3.132.1 Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{ce+dex}\,dx = \frac{(a+b\text{arcsinh}(c+dx))^3}{3bde} + \frac{(a+b\text{arcsinh}(c+dx))^2\log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de} - \frac{b(a+b\text{arcsinh}(c+dx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(c+dx)}\right)}{de} - \frac{b^2\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(c+dx)}\right)}{2de}$$

output
```
1/3*(a+b*arcsinh(d*x+c))^3/b/d/e+(a+b*arcsinh(d*x+c))^2*ln(1-1/(d*x+c+(1+(
d*x+c)^2)^(1/2))^2)/d/e-b*(a+b*arcsinh(d*x+c))*polylog(2,1/(d*x+c+(1+(d*x+
c)^2)^(1/2))^2)/d/e-1/2*b^2*polylog(3,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e
```

3.132.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{ce+dex}\,dx = \frac{-2(a+b\text{arcsinh}(c+dx))^2\left(a+b\text{arcsinh}(c+dx)-3b\log\left(1-e^{2\text{arcsinh}(c+dx)}\right)\right)+6b^2(a+b\text{arcsinh}(c+dx}{6bde}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x),x]
```

output
```
(-2*(a + b*ArcSinh[c + d*x])^2*(a + b*ArcSinh[c + d*x] - 3*b*Log[1 - E^(2*
ArcSinh[c + d*x])]) + 6*b^2*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^(2*ArcSi
nh[c + d*x])] - 3*b^3*PolyLog[3, E^(2*ArcSinh[c + d*x])])/(6*b*d*e)
```

3.132.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.33, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {6274, 27, 6190, 25, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{ce+dex}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^2}{e(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^2}{c+dx}d(c+dx)}{de}$$

$$\Big\downarrow \text{6190}$$

$$\frac{\int -(a+b\text{arcsinh}(c+dx))^2\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)d(a+b\text{arcsinh}(c+dx))}{bde}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int (a+b\text{arcsinh}(c+dx))^2\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)d(a+b\text{arcsinh}(c+dx))}{bde}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -i(a+b\text{arcsinh}(c+dx))^2\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(c+dx))}{bde}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int (a+b\text{arcsinh}(c+dx))^2\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)d(a+b\text{arcsinh}(c+dx))}{bde}$$

$\Big\downarrow$ 4201

$$\frac{i\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}(a+b\text{arcsinh}(c+dx))^2}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{3}i(a+b\text{arcsinh}(c+dx))^3\right)}{bde}$$

$\Big\downarrow$ 2620

$$\frac{i\left(2i\left(b\int (a+b\text{arcsinh}(c+dx))\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}b(a+b\text{arcsinh}(c+\right.\right.}{bde}$$

$\Big\downarrow$ 3011

$$\frac{i\left(2i\left(b\left(\frac{1}{2}b(a+b\text{arcsinh}(c+dx))\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right)-\frac{1}{2}b\int\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c\right.\right.\right.\right.}{}$$

$\Big\downarrow$ 2720

$$\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int\exp\left(-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(c+dx))}{b}+i\pi\right)\text{PolyLog}(2,-c-dx)de^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}+\frac{1}{2}b(a+b\text{arc}\right.\right.\right.}{}$$

$\Big\downarrow$ 7143

$$\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\text{PolyLog}(3,-c-dx)+\frac{1}{2}b(a+b\text{arcsinh}(c+dx))\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right)\right)-\frac{1}{2}b(a+\right.\right.}{bde}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x),x]
```

output
```
(I*((-1/3*I)*(a + b*ArcSinh[c + d*x])^3 + (2*I)*(-1/2*(b*(a + b*ArcSinh[c
+ d*x])^2*Log[1 + E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[c + d*x]))/b)]) +
b*((b*(a + b*ArcSinh[c + d*x])*PolyLog[2, -E^((2*a)/b - I*Pi - (2*(a + b*A
rcSinh[c + d*x]))/b)])/2 + (b^2*PolyLog[3, -c - d*x])/4))))/(b*d*e)
```

3.132. $\int \frac{(a+b\text{arcsinh}(c+dx))^2}{ce+dex}\,dx$

3.132.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.132.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(136) = 272$.

Time = 0.43 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.69

method	result
derivativedivides	$\frac{\frac{a^2\ln(dx+c)}{e}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(dx+c)^3}{3}+\operatorname{arcsinh}(dx+c)^2\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)+2\operatorname{arcsinh}(dx+c)\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right.\right.}{}}{}$
default	$\frac{\frac{a^2\ln(dx+c)}{e}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(dx+c)^3}{3}+\operatorname{arcsinh}(dx+c)^2\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)+2\operatorname{arcsinh}(dx+c)\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right.\right.}{}}{}$
parts	$\frac{a^2\ln(dx+c)}{ed}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(dx+c)^3}{3}+\operatorname{arcsinh}(dx+c)^2\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)+2\operatorname{arcsinh}(dx+c)\operatorname{polylog}\left(2,-dx\right.\right.}{}$

input
```
int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2/e*ln(d*x+c)+b^2/e*(-1/3*arcsinh(d*x+c)^3+arcsinh(d*x+c)^2*ln(1+d*
x+c+(1+(d*x+c)^2)^(1/2))+2*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(
1/2))-2*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+arcsinh(d*x+c)^2*ln(1-d*x-c-
(1+(d*x+c)^2)^(1/2))+2*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))
-2*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2)))+2*a*b/e*(-1/2*arcsinh(d*x+c)^2+ar
csinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+polylog(2,-d*x-c-(1+(d*x+c)^2
)^(1/2))+arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+polylog(2,d*x+c+(1
+(d*x+c)^2)^(1/2))))
```

3.132.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{ce+dex}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(dx+c\right)+a\right)^2}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)/(d*e*x +
c*e), x)
```

3.132.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{ce+dex}\,dx=\frac{\int \frac{a^2}{c+dx}\,dx+\int \frac{b^2\operatorname{asinh}^2\left(c+dx\right)}{c+dx}\,dx+\int \frac{2ab\operatorname{asinh}\left(c+dx\right)}{c+dx}\,dx}{e}$$

input
```
integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e),x)
```

output
```
(Integral(a**2/(c + d*x), x) + Integral(b**2*asinh(c + d*x)**2/(c + d*x),
x) + Integral(2*a*b*asinh(c + d*x)/(c + d*x), x))/e
```

3.132.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{ce+dex}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(dx+c\right)+a\right)^2}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
a^2*log(d*e*x + c*e)/(d*e) + integrate(b^2*log(d*x + c + sqrt((d*x + c)^2
+ 1))^2/(d*e*x + c*e) + 2*a*b*log(d*x + c + sqrt((d*x + c)^2 + 1))/(d*e*x
+ c*e), x)
```

3.132.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{ce+dex}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^2}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e), x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{ce+dex}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^2}{c\,e+d\,e\,x}\,dx$$

input
```
int((a + b*asinh(c + d*x))^2/(c*e + d*e*x),x)
```

output
```
int((a + b*asinh(c + d*x))^2/(c*e + d*e*x), x)
```

3.133 $\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx$

3.133.1 Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx = -\frac{(a+b\text{arcsinh}(c+dx))^2}{de^2(c+dx)} - \frac{4b(a+b\text{arcsinh}(c+dx))\text{arctanh}\left(e^{\text{arcsinh}(c+dx)}\right)}{de^2} - \frac{2b^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(c+dx)}\right)}{de^2} + \frac{2b^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(c+dx)}\right)}{de^2}$$

output
```
-(a+b*arcsinh(d*x+c))^2/d/e^2/(d*x+c)-4*b*(a+b*arcsinh(d*x+c))*arctanh(d*x
+c+(1+(d*x+c)^2)^(1/2))/d/e^2-2*b^2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))/
d/e^2+2*b^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2
```

3.133.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.64

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx = \frac{-\frac{a^2}{c+dx} - 2ab\left(\frac{\text{arcsinh}(c+dx)}{c+dx} + \log\left(\frac{1}{2}(c+dx)\text{csch}\left(\frac{1}{2}\text{arcsinh}(c+dx)\right)\right) - \log\left(\sinh\left(\frac{1}{2}\text{arcsinh}(c+dx)\right)\right)\right) + \cdots}{\cdots}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

output
```
(-(a^2/(c + d*x)) - 2*a*b*(ArcSinh[c + d*x]/(c + d*x) + Log[((c + d*x)*Csc
h[ArcSinh[c + d*x]/2])/2] - Log[Sinh[ArcSinh[c + d*x]/2]]) + b^2*(ArcSinh[
c + d*x]*(-(ArcSinh[c + d*x]/(c + d*x)) + 2*Log[1 - E^(-ArcSinh[c + d*x])]
 - 2*Log[1 + E^(-ArcSinh[c + d*x])]) + 2*PolyLog[2, -E^(-ArcSinh[c + d*x])
] - 2*PolyLog[2, E^(-ArcSinh[c + d*x])]))/(d*e^2)
```

3.133.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6274, 27, 6191, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^2}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^2}{e^2(c+dx)^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(c+dx)^2}d(c+dx)}{de^2}$$

$$\Big\downarrow \text{6191}$$

$$\frac{2b\int \frac{a+b\text{arcsinh}(c+dx)}{(c+dx)\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{(a+b\text{arcsinh}(c+dx))^2}{c+dx}}{de^2}$$

$$\Big\downarrow \text{6231}$$

$$\frac{2b\int \frac{a+b\text{arcsinh}(c+dx)}{c+dx}d\text{arcsinh}(c+dx)-\frac{(a+b\text{arcsinh}(c+dx))^2}{c+dx}}{de^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^2}{c+dx}+2b\int i(a+b\text{arcsinh}(c+dx))\csc(i\text{arcsinh}(c+dx))d\text{arcsinh}(c+dx)}{de^2}$$

$$\Big\downarrow \ 26$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^2}{c+dx}+2ib\int(a+b\text{arcsinh}(c+dx))\csc(i\text{arcsinh}(c+dx))d\text{arcsinh}(c+dx)}{de^2}$$

$$\Big\downarrow \ 4670$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^2}{c+dx}+2ib\left(ib\int\log\left(1-e^{\text{arcsinh}(c+dx)}\right)d\text{arcsinh}(c+dx)-ib\int\log\left(1+e^{\text{arcsinh}(c+dx)}\right)d\text{arcsinh}(c+\right.}{de^2}$$

$$\Big\downarrow \ 2715$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^2}{c+dx}+2ib\left(-ib\int e^{-\text{arcsinh}(c+dx)}\log\left(1+e^{\text{arcsinh}(c+dx)}\right)de^{\text{arcsinh}(c+dx)}+ib\int e^{-\text{arcsinh}(c+dx)}\log(-c\right.}{de^2}$$

$$\Big\downarrow \ 2838$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^2}{c+dx}+2ib\left(2i\text{arctanh}\left(e^{\text{arcsinh}(c+dx)}\right)(a+b\text{arcsinh}(c+dx))-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(c+dx)}\right)+ib\,\text{Po}\right.}{de^2}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

output
```
(-((a + b*ArcSinh[c + d*x])^2/(c + d*x)) + (2*I)*b*((2*I)*(a + b*ArcSinh[c
 + d*x])*ArcTanh[E^ArcSinh[c + d*x]] - I*b*PolyLog[2, E^ArcSinh[c + d*x]]
+ I*b*PolyLog[2, -c - d*x]))/(d*e^2)
```

3.133.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.133. $\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx$

3.133.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.85

method	result
derivativedivides	$\frac{-\frac{a^2}{e^2(dx+c)}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(dx+c)^2}{dx+c}-2\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-2\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right)+2\operatorname{arcsinh}(\cdots\right)}{e^2}\cdots}{d}$
default	$\frac{-\frac{a^2}{e^2(dx+c)}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(dx+c)^2}{dx+c}-2\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-2\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right)+2\operatorname{arcsinh}(\cdots\right)}{e^2}\cdots}{d}$
parts	$-\frac{a^2}{e^2(dx+c)d}+\frac{b^2\left(-\frac{\operatorname{arcsinh}(dx+c)^2}{dx+c}-2\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-2\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx\cdots}\right)\right)}{e^2d}$

input
```
int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2/e^2/(d*x+c)+b^2/e^2*(-1/(d*x+c)*arcsinh(d*x+c)^2-2*arcsinh(d*x+c
)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+
2*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+2*polylog(2,d*x+c+(1+(d*x
+c)^2)^(1/2)))+2*a*b/e^2*(-1/(d*x+c)*arcsinh(d*x+c)-arctanh(1/(1+(d*x+c)^2
)^(1/2))))
```

3.133.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx = \int \frac{(b\operatorname{arsinh}(dx+c)+a)^2}{(dex+ce)^2}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)/(d^2*e^2*
x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

3.133.6 Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx = \frac{\int \frac{a^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^2\operatorname{asinh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{2ab\operatorname{asinh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

input
```
integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**2,x)
```

output
```
(Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*asinh(c +
d*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*asinh(c + d*x)/(
c**2 + 2*c*d*x + d**2*x**2), x))/e**2
```

3.133.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.133.8 Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx = \int \frac{(b\operatorname{arsinh}(dx+c)+a)^2}{(dex+ce)^2}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^2, x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^2}\,dx=\int \frac{(a+b\,\mathrm{asinh}(c+d\,x))^2}{(c\,e+d\,e\,x)^2}\,dx$$

input
```
int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^2,x)
```

output
```
int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^2, x)
```

3.134 $\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^3} dx$

3.134.1 Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^3} dx = -\frac{b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))}{de^3(c+dx)} - \frac{(a+b\text{arcsinh}(c+dx))^2}{2de^3(c+dx)^2} + \frac{b^2\log(c+dx)}{de^3}$$

output
```
-1/2*(a+b*arcsinh(d*x+c))^2/d/e^3/(d*x+c)^2+b^2*ln(d*x+c)/d/e^3-b*(a+b*arc
sinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d/e^3/(d*x+c)
```

3.134.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.41

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^3} dx = -\frac{a\left(a+2b(c+dx)\sqrt{1+c^2+2cdx+d^2x^2}\right)+2b\left(a+b(c+dx)\sqrt{1+c^2+2cdx+d^2x^2}\right)\text{arcsinh}(c+dx) -}{2de^3(c+dx)^2}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

output
```
-1/2*(a*(a + 2*b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]) + 2*b*(a + b
*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])*ArcSinh[c + d*x] + b^2*ArcSi
nh[c + d*x]^2 - 2*b^2*(c + d*x)^2*Log[c + d*x])/(d*e^3*(c + d*x)^2)
```

3.134.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6274, 27, 6191, 6215, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^3}\,dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^2}{e^3(c+dx)^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(c+dx)^3}d(c+dx)}{de^3}$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{b\int \frac{a+b\text{arcsinh}(c+dx)}{(c+dx)^2\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{(a+b\text{arcsinh}(c+dx))^2}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{ 6215}$$

$$\frac{b\left(b\int \frac{1}{c+dx}d(c+dx)-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))}{c+dx}\right)-\frac{(a+b\text{arcsinh}(c+dx))^2}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{b\left(b\log(c+dx)-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))}{c+dx}\right)-\frac{(a+b\text{arcsinh}(c+dx))^2}{2(c+dx)^2}}{de^3}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

output
```
(-1/2*(a + b*ArcSinh[c + d*x])^2/(c + d*x)^2 + b*(-((Sqrt[1 + (c + d*x)^2]
*(a + b*ArcSinh[c + d*x]))/(c + d*x)) + b*Log[c + d*x]))/(d*e^3)
```

3.134.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.134.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.76

method	result
derivativedivides	$\frac{-\frac{a^2}{2e^3(dx+c)^2}+\frac{b^2\left(-2\operatorname{arcsinh}(dx+c)-\frac{\operatorname{arcsinh}(dx+c)\left(-2(dx+c)^2+2(dx+c)\sqrt{1+(dx+c)^2}+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}+\ln\left(\left(dx+c+\sqrt{1+(dx+\right.\right.}{e^3}}{d}$
default	$\frac{-\frac{a^2}{2e^3(dx+c)^2}+\frac{b^2\left(-2\operatorname{arcsinh}(dx+c)-\frac{\operatorname{arcsinh}(dx+c)\left(-2(dx+c)^2+2(dx+c)\sqrt{1+(dx+c)^2}+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}+\ln\left(\left(dx+c+\sqrt{1+(dx+\right.\right.}{e^3}}{d}$
parts	$-\frac{a^2}{2e^3(dx+c)^2d}+\frac{b^2\left(-2\operatorname{arcsinh}(dx+c)-\frac{\operatorname{arcsinh}(dx+c)\left(-2(dx+c)^2+2(dx+c)\sqrt{1+(dx+c)^2}+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}+\ln\left(\left(dx+c\right.\right.}{e^3d}$

input
```
int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a^2/e^3/(d*x+c)^2+b^2/e^3*(-2*arcsinh(d*x+c)-1/2*arcsinh(d*x+c)*
(-2*(d*x+c)^2+2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+arcsinh(d*x+c))/(d*x+c)^2+ln((
d*x+c+(1+(d*x+c)^2)^(1/2))^2-1))+2*a*b/e^3*(-1/2/(d*x+c)^2*arcsinh(d*x+c)-
1/2/(d*x+c)*(1+(d*x+c)^2)^(1/2)))
```

3.134.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(81) = 162$.

Time = 0.31 (sec) , antiderivative size = 319, normalized size of antiderivative = 3.75

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^3}\,dx =$$
$$-\frac{2\,abc^2d^2x^2+4\,abc^3dx+2\,abc^4+b^2c^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2+a^2c^2-2\left(abd^2x^2+2\,ab\right.}{}$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*c^2*d^2*x^2 + 4*a*b*c^3*d*x + 2*a*b*c^4 + b^2*c^2*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + a^2*c^2 - 2*(a*b*d^2*x^2 + 2*a*b*
c*d*x - (b^2*c^2*d*x + b^2*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*(b^2*c^2*d^2*x^2 + 2*b^2*c^3
*d*x + b^2*c^4)*log(d*x + c) - 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*log
(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*c^2*d*x + a*b*c^3)
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(c^2*d^3*e^3*x^2 + 2*c^3*d^2*e^3*x + c
^4*d*e^3)
```

3.134.6 Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^3}\,dx = \frac{\int \frac{a^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^2\operatorname{asinh}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2ab\operatorname{asinh}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

input
```
integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**3,x)
```

output
```
(Integral(a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integ
ral(b**2*asinh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)
, x) + Integral(2*a*b*asinh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
d**3*x**3), x))/e**3
```

3.134.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(81) = 162$.

Time = 0.22 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.71

$$\begin{aligned}\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^3}\,dx &= -\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}d\operatorname{arsinh}(dx+c)}{d^3e^3x+cd^2e^3} - \frac{\log(dx+c)}{de^3}\right)b^2 \\ &\quad - ab\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}d}{d^3e^3x+cd^2e^3} + \frac{\operatorname{arsinh}(dx+c)}{d^3e^3x^2+2\,cd^2e^3x+c^2de^3}\right) \\ &\quad - \frac{b^2\operatorname{arsinh}(dx+c)^2}{2\,(d^3e^3x^2+2\,cd^2e^3x+c^2de^3)} - \frac{a^2}{2\,(d^3e^3x^2+2\,cd^2e^3x+c^2de^3)}\end{aligned}$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="maxima")
```

output
```
-(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d*arcsinh(d*x + c)/(d^3*e^3*x + c*d^2*
e^3) - log(d*x + c)/(d*e^3))*b^2 - a*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*
d/(d^3*e^3*x + c*d^2*e^3) + arcsinh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x
+ c^2*d*e^3)) - 1/2*b^2*arcsinh(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x +
c^2*d*e^3) - 1/2*a^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

3.134.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^2}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^3, x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^3}\,dx=\int \frac{(a+b\,\text{asinh}(c+d\,x))^2}{(c\,e+d\,e\,x)^3}\,dx$$

input
```
int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^3,x)
```

output
```
int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^3, x)
```

3.135 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx$

3.135.1 Optimal result

Integrand size = 23, antiderivative size = 169

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx = -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{3de^4(c+dx)^2} - \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{3de^4(c+dx)^3} + \frac{2b(a+b\operatorname{arcsinh}(c+dx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4} + \frac{b^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4} - \frac{b^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4}$$

output
```
-1/3*b^2/d/e^4/(d*x+c)-1/3*(a+b*arcsinh(d*x+c))^2/d/e^4/(d*x+c)^3+2/3*b*(a
+b*arcsinh(d*x+c))*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4+1/3*b^2*polylo
g(2,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4-1/3*b^2*polylog(2,d*x+c+(1+(d*x+c)^2
)^(1/2))/d/e^4-1/3*b*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d/e^4/(d*x+c
)^2
```

3.135.2 Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.36

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx =$$
$$-\frac{4a^2+ab\left(8\text{arcsinh}(c+dx)+2\sinh(2\text{arcsinh}(c+dx))+\left(\log\left(\cosh\left(\frac{1}{2}\text{arcsinh}(c+dx)\right)\right)-\log\left(\sinh\left(\frac{1}{2}\right.\right.\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

output
```
-1/12*(4*a^2 + a*b*(8*ArcSinh[c + d*x] + 2*Sinh[2*ArcSinh[c + d*x]] + (Log
[Cosh[ArcSinh[c + d*x]/2]] - Log[Sinh[ArcSinh[c + d*x]/2]])*(3*c + 3*d*x -
 Sinh[3*ArcSinh[c + d*x]])) + b^2*(4*(c + d*x)^2 + 4*ArcSinh[c + d*x]^2 +
4*(c + d*x)^3*PolyLog[2, -E^(-ArcSinh[c + d*x])] - 4*(c + d*x)^3*PolyLog[2
, E^(-ArcSinh[c + d*x])] + ArcSinh[c + d*x]*(2*Sinh[2*ArcSinh[c + d*x]] +
(Log[1 - E^(-ArcSinh[c + d*x])] - Log[1 + E^(-ArcSinh[c + d*x])])*(-3*(c +
 d*x) + Sinh[3*ArcSinh[c + d*x]]))))/(d*e^4*(c + d*x)^3)
```

3.135.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {6274, 27, 6191, 6224, 15, 6231, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^4}dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^2}{e^4(c+dx)^4}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(c+dx)^4}d(c+dx)}{de^4}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2}{3}b\int \frac{a+b\text{arcsinh}(c+dx)}{(c+dx)^3\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{(a+b\text{arcsinh}(c+dx))^2}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \text{6224}$$

$$\frac{\frac{2}{3}b\left(-\frac{1}{2}\int \frac{a+b\text{arcsinh}(c+dx)}{(c+dx)\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{2}b\int \frac{1}{(c+dx)^2}d(c+dx)-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))}{2(c+dx)^2}\right)-\frac{(a+b\text{arcsinh}(c+dx))}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \text{15}$$

$$\frac{\frac{2}{3}b\left(-\frac{1}{2}\int \frac{a+b\text{arcsinh}(c+dx)}{(c+dx)\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))}{2(c+dx)^2}-\frac{b}{2(c+dx)}\right)-\frac{(a+b\text{arcsinh}(c+dx))^2}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \text{6231}$$

$$\frac{\frac{2}{3}b\left(-\frac{1}{2}\int \frac{a+b\text{arcsinh}(c+dx)}{c+dx}d\text{arcsinh}(c+dx)-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))}{2(c+dx)^2}-\frac{b}{2(c+dx)}\right)-\frac{(a+b\text{arcsinh}(c+dx))^2}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^2}{3(c+dx)^3}+\frac{2}{3}b\left(-\frac{1}{2}\int i(a+b\text{arcsinh}(c+dx))\csc(i\text{arcsinh}(c+dx))d\text{arcsinh}(c+dx)-\frac{\sqrt{(c+dx)^2+1}(a+b\text{a}}{2(c+d}\right.}{de^4}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^2}{3(c+dx)^3}+\frac{2}{3}b\left(-\frac{1}{2}i\int (a+b\text{arcsinh}(c+dx))\csc(i\text{arcsinh}(c+dx))d\text{arcsinh}(c+dx)-\frac{\sqrt{(c+dx)^2+1}(a+b\text{a}}{2(c+d}\right.}{de^4}$$

$$\Big\downarrow \text{4670}$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^2}{3(c+dx)^3}+\frac{2}{3}b\left(-\frac{1}{2}i\left(ib\int \log\left(1-e^{\text{arcsinh}(c+dx)}\right)d\text{arcsinh}(c+dx)-ib\int \log\left(1+e^{\text{arcsinh}(c+dx)}\right)d\text{arcsin}\right.\right.}{de^4}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{(a+b\text{arcsinh}(c+dx))^2}{3(c+dx)^3}+\frac{2}{3}b\left(-\frac{1}{2}i\left(-ib\int e^{-\text{arcsinh}(c+dx)}\log\left(1+e^{\text{arcsinh}(c+dx)}\right)de^{\text{arcsinh}(c+dx)}+ib\int e^{-\text{arcsinh}(c+dx)}\log\right.\right.$$

$$\Big\downarrow \text{2838}$$

3.135. $\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^2}{3(c+dx)^3}+\frac{2}{3}b\Big(-\frac{1}{2}i\big(2i\text{arctanh}\big(e^{\text{arcsinh}(c+dx)}\big)\,(a+b\text{arcsinh}(c+dx))-ib\,\text{PolyLog}\left(2,e^{\text{arcsinh}(c+dx)}\right)+}{de^4}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

output
```
(-1/3*(a + b*ArcSinh[c + d*x])^2/(c + d*x)^3 + (2*b*(-1/2*b/(c + d*x) - (S
qrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(2*(c + d*x)^2) - (I/2)*((2
*I)*(a + b*ArcSinh[c + d*x])*ArcTanh[E^ArcSinh[c + d*x]] - I*b*PolyLog[2,
E^ArcSinh[c + d*x]] + I*b*PolyLog[2, -c - d*x])))/3)/(d*e^4)
```

3.135.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.135. $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx$

3.135.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.40

method	result
derivativedivides	$-\frac{a^2}{3e^4(dx+c)^3}+\frac{b^2\left(-\frac{\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)+\operatorname{arcsinh}(dx+c)^2+(dx+c)^2}{3(dx+c)^3}+\frac{\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{3}+\right.}{}$
default	$-\frac{a^2}{3e^4(dx+c)^3}+\frac{b^2\left(-\frac{\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)+\operatorname{arcsinh}(dx+c)^2+(dx+c)^2}{3(dx+c)^3}+\frac{\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{3}+\right.}{}$
parts	$-\frac{a^2}{3e^4(dx+c)^3d}+\frac{b^2\left(-\frac{\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)+\operatorname{arcsinh}(dx+c)^2+(dx+c)^2}{3(dx+c)^3}+\frac{\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+\right.}{3}\right.}{}$

input
```
int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a^2/e^4/(d*x+c)^3+b^2/e^4*(-1/3*((1+(d*x+c)^2)^(1/2)*(d*x+c)*arc
sinh(d*x+c)+arcsinh(d*x+c)^2+(d*x+c)^2)/(d*x+c)^3+1/3*arcsinh(d*x+c)*ln(1+
d*x+c+(1+(d*x+c)^2)^(1/2))+1/3*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-1/3*a
rcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-1/3*polylog(2,d*x+c+(1+(d*x+
c)^2)^(1/2)))+2*a*b/e^4*(-1/3/(d*x+c)^3*arcsinh(d*x+c)-1/6/(d*x+c)^2*(1+(d
*x+c)^2)^(1/2)+1/6*arctanh(1/(1+(d*x+c)^2)^(1/2))))
```

3.135.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx=\int \frac{(b\operatorname{arsinh}(dx+c)+a)^2}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)/(d^4*e^4*
x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

3.135.6 Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(c + dx))^2}{(ce + dex)^4}\,dx = \frac{\int \frac{a^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^2\operatorname{asinh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{2ab\operatorname{asinh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

input
```
integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**4,x)
```

output
```
(Integral(a**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**
4*x**4), x) + Integral(b**2*asinh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*
d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(2*a*b*asinh(c + d*x)
/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e
**4
```

3.135.7 Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(c + dx))^2}{(ce + dex)^4}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx + c) + a)^2}{(dex + ce)^4}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="maxima")
```

output
```
-1/3*b^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^4*e^4*x^3 +
 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^2/(d^4*e^4*x^3 + 3
*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(2/3*((3*a*b*d^3
+ b^2*d^3)*x^3 + 3*(c^3 + c)*a*b + (c^3 + c)*b^2 + 3*(3*a*b*c*d^2 + b^2*c*
d^2)*x^2 + (3*(3*c^2*d + d)*a*b + (3*c^2*d + d)*b^2)*x + (b^2*c^2 + 3*(c^2
 + 1)*a*b + (3*a*b*d^2 + b^2*d^2)*x^2 + 2*(3*a*b*c*d + b^2*c*d)*x)*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
))/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 + c^5*e^4 + (21*c^2*d^5*e^4 +
d^5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 + c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 + 2*c^
2*d^3*e^4)*x^3 + (21*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*x^2 + (7*c^6*d*e^4 + 5*
c^4*d*e^4)*x + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4 + (15*c^
2*d^4*e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(5*c^4*d^
2*e^4 + 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x)*sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1)), x)
```

3.135.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^2}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^4, x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^4}\,dx=\int \frac{(a+b\,\text{asinh}(c+d\,x))^2}{(c\,e+d\,e\,x)^4}\,dx$$

input
```
int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^4,x)
```

output
```
int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^4, x)
```

3.136 $\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^3\,dx$

3.136.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^3\,dx=\frac{(e(c+dx))^{1+m}(a+b\text{arcsinh}(c+dx))^3}{de(1+m)}-\frac{3b\text{Int}\left(\frac{(e(c+dx))^{1+m}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{1+(c+dx)^2}},x\right)}{e(1+m)}$$

output
```
(e*(d*x+c))^(1+m)*(a+b*arcsinh(d*x+c))^3/d/e/(1+m)-3*b*Unintegrable((e*(d*
x+c))^(1+m)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e/(1+m)
```

3.136.2 Mathematica [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^3\,dx=\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^3\,dx$$

input
```
Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^3, x]
```

3.136.3 Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^3\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int (e(c+dx))^m(a+b\text{arcsinh}(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{(e(c+dx))^{m+1}(a+b\text{arcsinh}(c+dx))^3}{e(m+1)}-\frac{3b\int \frac{(e(c+dx))^{m+1}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)}{e(m+1)}}{d}$$

$$\Big\downarrow \text{6239}$$

$$\frac{\frac{(e(c+dx))^{m+1}(a+b\text{arcsinh}(c+dx))^3}{e(m+1)}-\frac{3b\int \frac{(e(c+dx))^{m+1}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)}{e(m+1)}}{d}$$

input
```
Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
$Aborted
```

3.136.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.136.4 Maple [N/A] (verified)

Not integrable

Time = 1.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (dex + ce)^m (a + b \operatorname{arcsinh}(dx + c))^3 \, dx$$

input
```
int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x)
```

output
```
int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x)
```

3.136.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int (ce + dex)^m (a + b\text{arcsinh}(c + dx))^3 \, dx = \int (b \operatorname{arsinh}(dx + c) + a)^3 (dex + ce)^m \, dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*ar
csinh(d*x + c) + a^3)*(d*e*x + c*e)^m, x)
```

3.136.6 Sympy [N/A]

Not integrable

Time = 17.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^3\,dx=\int (e(c+dx))^m\left(a+b\operatorname{asinh}\left(c+dx\right)\right)^3\,dx$$

input
```
integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c))**3,x)
```

output
```
Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x))**3, x)
```

3.136.7 Maxima [N/A]

Not integrable

Time = 3.98 (sec) , antiderivative size = 716, normalized size of antiderivative = 31.13

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^3\,dx=\int (b\operatorname{arsinh}\left(dx+c\right)+a)^3(dex+ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
(b^3*d*e^m*x + b^3*c*e^m)*(d*x + c)^m*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))^3/(d*(m + 1)) + (d*e*x + c*e)^(m + 1)*a^3/(d*e*(m + 1)) + int
egrate(-3*(((b^3*c^2*e^m - (c^2*e^m*(m + 1) + e^m*(m + 1))*a*b^2 - (a*b^2*
d^2*e^m*(m + 1) - b^3*d^2*e^m)*x^2 - 2*(a*b^2*c*d*e^m*(m + 1) - b^3*c*d*e^
m)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m - ((c^3*e^m*(m + 1) +
c*e^m*(m + 1))*a*b^2 - (c^3*e^m + c*e^m)*b^3 + (a*b^2*d^3*e^m*(m + 1) - b^
3*d^3*e^m)*x^3 + 3*(a*b^2*c*d^2*e^m*(m + 1) - b^3*c*d^2*e^m)*x^2 + ((3*c^2
*d*e^m*(m + 1) + d*e^m*(m + 1))*a*b^2 - (3*c^2*d*e^m + d*e^m)*b^3)*x)*(d*x
 + c)^m)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - ((a^2*b*d^2*
e^m*(m + 1)*x^2 + 2*a^2*b*c*d*e^m*(m + 1)*x + (c^2*e^m*(m + 1) + e^m*(m +
1))*a^2*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m + (a^2*b*d^3*e^m*
(m + 1)*x^3 + 3*a^2*b*c*d^2*e^m*(m + 1)*x^2 + (3*c^2*d*e^m*(m + 1) + d*e^m
*(m + 1))*a^2*b*x + (c^3*e^m*(m + 1) + c*e^m*(m + 1))*a^2*b)*(d*x + c)^m)*
log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^3*(m + 1)*x^3 + 3*c*d
^2*(m + 1)*x^2 + c^3*(m + 1) + c*(m + 1) + (3*c^2*d*(m + 1) + d*(m + 1))*x
 + (d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) + m + 1)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1)), x)
```

3.136.8 Giac [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce + dex)^m(a + b\text{arcsinh}(c + dx))^3\,dx = \int (b\,\text{arsinh}\,(dx + c) + a)^3(dex + ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^3*(d*e*x + c*e)^m, x)
```

3.136.9 Mupad [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce + dex)^m(a + b\text{arcsinh}(c + dx))^3\,dx = \int (c\,e + d\,e\,x)^m\,(a + b\,\text{asinh}(c + d\,x))^3\,dx$$

input
```
int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^3, x)
```

3.137 $\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^3\,dx$

3.137.1 Optimal result

Integrand size = 23, antiderivative size = 326

$$\begin{aligned}\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^3\,dx &= \frac{16}{25}ab^2e^4x-\frac{298b^3e^4\sqrt{1+(c+dx)^2}}{375d}+\frac{76b^3e^4(1+(c+dx)^2)^{3/2}}{1125d}-\frac{6b^3e^4(1+(c+dx)^2)^{5/2}}{625d}\\ &+\frac{16b^3e^4(c+dx)\text{arcsinh}(c+dx)}{25d}-\frac{8b^2e^4(c+dx)^3(a+b\text{arcsinh}(c+dx))}{75d}\\ &+\frac{6b^2e^4(c+dx)^5(a+b\text{arcsinh}(c+dx))}{125d}-\frac{8be^4\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^2}{25d}\\ &+\frac{4be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^2}{25d}\\ &-\frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^2}{25d}\\ &+\frac{e^4(c+dx)^5(a+b\text{arcsinh}(c+dx))^3}{5d}\end{aligned}$$

output
```
16/25*a*b^2*e^4*x+76/1125*b^3*e^4*(1+(d*x+c)^2)^(3/2)/d-6/625*b^3*e^4*(1+(
d*x+c)^2)^(5/2)/d+16/25*b^3*e^4*(d*x+c)*arcsinh(d*x+c)/d-8/75*b^2*e^4*(d*x
+c)^3*(a+b*arcsinh(d*x+c))/d+6/125*b^2*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))/
d+1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^3/d-298/375*b^3*e^4*(1+(d*x+c)^2)
^(1/2)/d-8/25*b*e^4*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d+4/25*b*e^
4*(d*x+c)^2*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d-3/25*b*e^4*(d*x+c
)^4*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d
```

3.137.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.09

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^3\,dx$$
$$=\frac{e^4\Big(240ab^2(c+dx)-40ab^2(c+dx)^3+3a(25a^2+6b^2)(c+dx)^5+\frac{1}{15}b\sqrt{1+(c+dx)^2}(-8(225a^2+518b^2)}{}$$

input
```
Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
(e^4*(240*a*b^2*(c + d*x) - 40*a*b^2*(c + d*x)^3 + 3*a*(25*a^2 + 6*b^2)*(c
 + d*x)^5 + (b*Sqrt[1 + (c + d*x)^2]*(-8*(225*a^2 + 518*b^2) + 4*(225*a^2
+ 68*b^2)*(c + d*x)^2 - 27*(25*a^2 + 2*b^2)*(c + d*x)^4))/15 - b*(-240*b^2
*(c + d*x) + 40*b^2*(c + d*x)^3 - 225*a^2*(c + d*x)^5 - 18*b^2*(c + d*x)^5
 + 240*a*b*Sqrt[1 + (c + d*x)^2] - 120*a*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^
2] + 90*a*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] - 15*b^2*(
-15*a*(c + d*x)^5 + 8*b*Sqrt[1 + (c + d*x)^2] - 4*b*(c + d*x)^2*Sqrt[1 + (
c + d*x)^2] + 3*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 +
75*b^3*(c + d*x)^5*ArcSinh[c + d*x]^3))/(375*d)
```

3.137.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {6274, 27, 6191, 6227, 6191, 243, 53, 2009, 6227, 6191, 243, 53, 2009, 6213, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^3\,dx$$
$$\Big\downarrow \ 6274$$
$$\frac{\int e^4(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^3d(c+dx)}{d}$$
$$\Big\downarrow \ 27$$
$$\frac{e^4\int (c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^3d(c+dx)}{d}$$
$$\Big\downarrow \ 6191$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^3-\frac{3}{5}b\int\frac{(c+dx)^5(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$\downarrow$ 6227

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^3-\frac{3}{5}b\left(-\frac{2}{5}b\int(c+dx)^4(a+b\text{arcsinh}(c+dx))d(c+dx)-\frac{4}{5}\int\frac{(c+dx)^3(a+b\text{arcs}}{\sqrt{(c+dx)}}\right.\right.}{d}$$

$\downarrow$ 6191

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^3-\frac{3}{5}b\left(-\frac{2}{5}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))-\frac{1}{5}b\int\frac{(c+dx)^5}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)\right.\right.}{d}$$

$\downarrow$ 243

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^3-\frac{3}{5}b\left(-\frac{2}{5}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))-\frac{1}{10}b\int\frac{(c+dx)^4}{\sqrt{(c+dx)^2+1}}d(c+dx)^2\right.\right.\right.}{d}$$

$\downarrow$ 53

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^3-\frac{3}{5}b\left(-\frac{2}{5}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))-\frac{1}{10}b\int\left(\left((c+dx)^2+1\right)^{3/2}-\right.\right.\right.\right.$$

$\downarrow$ 2009

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^3-\frac{3}{5}b\left(-\frac{4}{5}\int\frac{(c+dx)^3(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{5}\sqrt{(c+dx)^2+1}(c+dx)\right.\right.$$

$\downarrow$ 6227

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^3-\frac{3}{5}b\left(-\frac{4}{5}\left(-\frac{2}{3}b\int(c+dx)^2(a+b\text{arcsinh}(c+dx))d(c+dx)-\frac{2}{3}\int\frac{(c+dx)(a+b}{\sqrt{(}}\right.\right.\right.$$

$\downarrow$ 6191

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^3-\frac{3}{5}b\left(-\frac{4}{5}\left(-\frac{2}{3}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))-\frac{1}{3}b\int\frac{(c+dx)^3}{\sqrt{(c+dx)^2+1}}d(c+\right.\right.\right.\right.$$

$\downarrow$ 243

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^3-\frac{3}{5}b\left(-\frac{4}{5}\left(-\frac{2}{3}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))-\frac{1}{6}b\int\frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}}d(c+\right.\right.\right.\right.$$

3.137. $\int(ce+dex)^4(a+b\text{arcsinh}(c+dx))^3\,dx$

$\downarrow$ 53

$$e^4\Bigg(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^3-\frac{3}{5}b\Bigg(-\frac{4}{5}\Bigg(-\frac{2}{3}b\Bigg(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))-\frac{1}{6}b\int\Bigg(\sqrt{(c+dx)^2+1}$$

$\downarrow$ 2009

$$e^4\Bigg(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^3-\frac{3}{5}b\Bigg(-\frac{4}{5}\Bigg(-\frac{2}{3}\int\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{3}(c+dx)^2\sqrt{(c+dx)^2}$$

$\downarrow$ 6213

$$e^4\Bigg(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^3-\frac{3}{5}b\Bigg(-\frac{4}{5}\Bigg(-\frac{2}{3}\Bigg(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2-2b\int(a+b\text{arcsinh}$$

$\downarrow$ 2009

$$e^4\Bigg(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^3-\frac{3}{5}b\Bigg(\frac{1}{5}\sqrt{(c+dx)^2+1}(c+dx)^4(a+b\text{arcsinh}(c+dx))^2-\frac{2}{5}b\Bigg(\frac{1}{5}(c+dx)^5($$

input
```
Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
(e^4*(((c + d*x)^5*(a + b*ArcSinh[c + d*x])^3)/5 - (3*b*(((c + d*x)^4*Sqrt
[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/5 - (2*b*(-1/10*(b*(2*Sqrt[1
 + (c + d*x)^2] - (4*(1 + (c + d*x)^2)^(3/2))/3 + (2*(1 + (c + d*x)^2)^(5/
2))/5)) + ((c + d*x)^5*(a + b*ArcSinh[c + d*x]))/5))/5 - (4*(((c + d*x)^2*
Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/3 - (2*b*(-1/6*(b*(-2*Sq
rt[1 + (c + d*x)^2] + (2*(1 + (c + d*x)^2)^(3/2))/3)) + ((c + d*x)^3*(a +
b*ArcSinh[c + d*x]))/3))/3 - (2*(Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c +
d*x])^2 - 2*b*(a*(c + d*x) - b*Sqrt[1 + (c + d*x)^2] + b*(c + d*x)*ArcSinh
[c + d*x])))/3))/5))/5))/d
```

3.137. $\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^3\,dx$

3.137.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

3.137. $\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^3\,dx$

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.137.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{\frac{e^4a^3(dx+c)^5}{5}+e^4b^3\left(\frac{(dx+c)^5\operatorname{arcsinh}(dx+c)^3}{5}-\frac{8\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{25}-\frac{3(dx+c)^4\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{25}+\frac{4(dx+c)}{}\right.}{}$
default	$\frac{\frac{e^4a^3(dx+c)^5}{5}+e^4b^3\left(\frac{(dx+c)^5\operatorname{arcsinh}(dx+c)^3}{5}-\frac{8\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{25}-\frac{3(dx+c)^4\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{25}+\frac{4(dx+c)}{}\right.}{}$
parts	$\frac{e^4a^3(dx+c)^5}{5d}+\frac{e^4b^3\left(\frac{(dx+c)^5\operatorname{arcsinh}(dx+c)^3}{5}-\frac{8\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{25}-\frac{3(dx+c)^4\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{25}+\frac{4(dx}{}\right.}{}$

input
```
int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*e^4*a^3*(d*x+c)^5+e^4*b^3*(1/5*(d*x+c)^5*arcsinh(d*x+c)^3-8/25*ar
csinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-3/25*(d*x+c)^4*arcsinh(d*x+c)^2*(1+(d*x
+c)^2)^(1/2)+4/25*(d*x+c)^2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+16/25*(d*
x+c)*arcsinh(d*x+c)-4144/5625*(1+(d*x+c)^2)^(1/2)+6/125*(d*x+c)^5*arcsinh(
d*x+c)-6/625*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+272/5625*(d*x+c)^2*(1+(d*x+c)^2
)^(1/2)-8/75*(d*x+c)^3*arcsinh(d*x+c))+3*e^4*a*b^2*(1/5*(d*x+c)^5*arcsinh(
d*x+c)^2-16/75*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)-2/25*(d*x+c)^4*arcsinh(d
*x+c)*(1+(d*x+c)^2)^(1/2)+8/75*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)^
2+16/75*d*x+16/75*c+2/125*(d*x+c)^5-8/225*(d*x+c)^3)+3*e^4*a^2*b*(1/5*(d*x
+c)^5*arcsinh(d*x+c)-1/25*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+4/75*(d*x+c)^2*(1+
(d*x+c)^2)^(1/2)-8/75*(1+(d*x+c)^2)^(1/2)))
```

3.137.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1077 vs. $2(292) = 584$.

Time = 0.30 (sec) , antiderivative size = 1077, normalized size of antiderivative = 3.30

$$\int (ce + dex)^4 (a + b\text{arcsinh}(c + dx))^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/5625*(45*(25*a^3 + 6*a*b^2)*d^5*e^4*x^5 + 225*(25*a^3 + 6*a*b^2)*c*d^4*e
^4*x^4 - 150*(4*a*b^2 - 3*(25*a^3 + 6*a*b^2)*c^2)*d^3*e^4*x^3 - 450*(4*a*b
^2*c - (25*a^3 + 6*a*b^2)*c^3)*d^2*e^4*x^2 - 225*(8*a*b^2*c^2 - (25*a^3 +
6*a*b^2)*c^4 - 16*a*b^2)*d*e^4*x + 1125*(b^3*d^5*e^4*x^5 + 5*b^3*c*d^4*e^4
*x^4 + 10*b^3*c^2*d^3*e^4*x^3 + 10*b^3*c^3*d^2*e^4*x^2 + 5*b^3*c^4*d*e^4*x
 + b^3*c^5*e^4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 225*(
15*a*b^2*d^5*e^4*x^5 + 75*a*b^2*c*d^4*e^4*x^4 + 150*a*b^2*c^2*d^3*e^4*x^3
+ 150*a*b^2*c^3*d^2*e^4*x^2 + 75*a*b^2*c^4*d*e^4*x + 15*a*b^2*c^5*e^4 - (3
*b^3*d^4*e^4*x^4 + 12*b^3*c*d^3*e^4*x^3 + 2*(9*b^3*c^2 - 2*b^3)*d^2*e^4*x^
2 + 4*(3*b^3*c^3 - 2*b^3*c)*d*e^4*x + (3*b^3*c^4 - 4*b^3*c^2 + 8*b^3)*e^4)
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1))^2 + 15*(9*(25*a^2*b + 2*b^3)*d^5*e^4*x^5 + 45*(25*a^2*b + 2*b^3
)*c*d^4*e^4*x^4 - 10*(4*b^3 - 9*(25*a^2*b + 2*b^3)*c^2)*d^3*e^4*x^3 - 30*(
4*b^3*c - 3*(25*a^2*b + 2*b^3)*c^3)*d^2*e^4*x^2 - 15*(8*b^3*c^2 - 3*(25*a^
2*b + 2*b^3)*c^4 - 16*b^3)*d*e^4*x - (40*b^3*c^3 - 9*(25*a^2*b + 2*b^3)*c^
5 - 240*b^3*c)*e^4 - 30*(3*a*b^2*d^4*e^4*x^4 + 12*a*b^2*c*d^3*e^4*x^3 + 2*
(9*a*b^2*c^2 - 2*a*b^2)*d^2*e^4*x^2 + 4*(3*a*b^2*c^3 - 2*a*b^2*c)*d*e^4*x
+ (3*a*b^2*c^4 - 4*a*b^2*c^2 + 8*a*b^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (27*(25*a^2*b + 2
*b^3)*d^4*e^4*x^4 + 108*(25*a^2*b + 2*b^3)*c*d^3*e^4*x^3 - 2*(450*a^2*b...
```

3.137.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2518 vs. $2(306) = 612$.

Time = 1.02 (sec) , antiderivative size = 2518, normalized size of antiderivative = 7.72

$$\int (ce + dex)^4 (a + b\text{arcsinh}(c + dx))^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**3,x)
```

output
```
Piecewise((a**3*c**4*e**4*x + 2*a**3*c**3*d*e**4*x**2 + 2*a**3*c**2*d**2*e
**4*x**3 + a**3*c*d**3*e**4*x**4 + a**3*d**4*e**4*x**5/5 + 3*a**2*b*c**5*e
**4*asinh(c + d*x)/(5*d) + 3*a**2*b*c**4*e**4*x*asinh(c + d*x) - 3*a**2*b*
c**4*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 6*a**2*b*c**3*d*e*
*4*x**2*asinh(c + d*x) - 12*a**2*b*c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)/25 + 6*a**2*b*c**2*d**2*e**4*x**3*asinh(c + d*x) - 18*a**2*b*c**
2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 4*a**2*b*c**2*e**4
*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 3*a**2*b*c*d**3*e**4*x**4*a
sinh(c + d*x) - 12*a**2*b*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/25 + 8*a**2*b*c*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 3*a
**2*b*d**4*e**4*x**5*asinh(c + d*x)/5 - 3*a**2*b*d**3*e**4*x**4*sqrt(c**2
+ 2*c*d*x + d**2*x**2 + 1)/25 + 4*a**2*b*d*e**4*x**2*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)/25 - 8*a**2*b*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(2
5*d) + 3*a*b**2*c**5*e**4*asinh(c + d*x)**2/(5*d) + 3*a*b**2*c**4*e**4*x*a
sinh(c + d*x)**2 + 6*a*b**2*c**4*e**4*x/25 - 6*a*b**2*c**4*e**4*sqrt(c**2
+ 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(25*d) + 6*a*b**2*c**3*d*e**4*x*
*2*asinh(c + d*x)**2 + 12*a*b**2*c**3*d*e**4*x**2/25 - 24*a*b**2*c**3*e**4
*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 + 6*a*b**2*c**2*
d**2*e**4*x**3*asinh(c + d*x)**2 + 12*a*b**2*c**2*d**2*e**4*x**3/25 - 36*a
*b**2*c**2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d...
```

3.137.7 Maxima [F]

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^3\,dx=\int (dex+ce)^4(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/5*a^3*d^4*e^4*x^5 + a^3*c*d^3*e^4*x^4 + 2*a^3*c^2*d^2*e^4*x^3 + 2*a^3*c^
3*d*e^4*x^2 + 3*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)
/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)
*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^2*b*c^3*d*e^4 +
 (6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2
- 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 -
 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^
2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^
4))*a^2*b*c^2*d^2*e^4 + 1/8*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2
*c*d*x + c^2 + 1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3
 + 105*c^4*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5
 + 35*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsi
nh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^
2 + 1)*x/d^4 + 9*(c^2 + 1)^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(
c^2 + 1)*d^2))/d^5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)
*d)*a^2*b*c*d^3*e^4 + 1/200*(120*x^5*arcsinh(d*x + c) - (24*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1)*x^4/d^2 - 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^...
```

3.137.8 Giac [F]

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^3\,dx=\int (dex+ce)^4(b\operatorname{arsinh}(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^3, x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^3\,dx=\int (c\,e+d\,e\,x)^4\,(a+b\,\mathrm{asinh}(c+d\,x))^3\,dx$$

input
```
int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^3, x)
```

3.138 $\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^3\,dx$

3.138.1 Optimal result

Integrand size = 23, antiderivative size = 279

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^3\,dx = \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}}{256d}-\frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{128d}-\frac{45b^3e^3\text{arcsinh}(c+dx)}{256d}-\frac{9b^2e^3(c+dx)^2(a+b\text{arcsinh}(c+dx))}{32d}+\frac{3b^2e^3(c+dx)^4(a+b\text{arcsinh}(c+dx))}{32d}+\frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^2}{32d}-\frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^2}{16d}-\frac{3e^3(a+b\text{arcsinh}(c+dx))^3}{32d}+\frac{e^3(c+dx)^4(a+b\text{arcsinh}(c+dx))^3}{4d}$$

output
```
-45/256*b^3*e^3*arcsinh(d*x+c)/d-9/32*b^2*e^3*(d*x+c)^2*(a+b*arcsinh(d*x+c
))/d+3/32*b^2*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))/d-3/32*e^3*(a+b*arcsinh(d
*x+c))^3/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^3/d+45/256*b^3*e^3*(d*x+
c)*(1+(d*x+c)^2)^(1/2)/d-3/128*b^3*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/d+9/3
2*b*e^3*(d*x+c)*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d-3/16*b*e^3*(d
*x+c)^3*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d
```

3.138.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.09

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^3\,dx$$
$$= \frac{e^3\left(-72ab^2(c+dx)^2+8a(8a^2+3b^2)(c+dx)^4+3b(c+dx)\sqrt{1+(c+dx)^2}(3(8a^2+5b^2)-2(8a^2+b^2)(c\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
(e^3*(-72*a*b^2*(c + d*x)^2 + 8*a*(8*a^2 + 3*b^2)*(c + d*x)^4 + 3*b*(c + d
*x)*Sqrt[1 + (c + d*x)^2]*(3*(8*a^2 + 5*b^2) - 2*(8*a^2 + b^2)*(c + d*x)^2
) - 9*b*(8*a^2 + 5*b^2)*ArcSinh[c + d*x] - 24*b*(c + d*x)*(3*b^2*(c + d*x)
 - 8*a^2*(c + d*x)^3 - b^2*(c + d*x)^3 - 6*a*b*Sqrt[1 + (c + d*x)^2] + 4*a
*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 24*b^2*(-3*a + 8*
a*(c + d*x)^4 + 3*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^3*Sqrt
[1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 + 8*b^3*(-3 + 8*(c + d*x)^4)*ArcSinh
[c + d*x]^3))/(256*d)
```

3.138.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {6274, 27, 6191, 6227, 6191, 262, 262, 222, 6227, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^3\,dx$$
$$\Big\downarrow \text{6274}$$
$$\frac{\int e^3(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^3d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{e^3\int (c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^3d(c+dx)}{d}$$
$$\Big\downarrow \text{6191}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^3-\frac{3}{4}b\int\frac{(c+dx)^4(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$\big\downarrow$ 6227

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^3-\frac{3}{4}b\left(-\frac{1}{2}b\int(c+dx)^3(a+b\text{arcsinh}(c+dx))d(c+dx)-\frac{3}{4}\int\frac{(c+dx)^2(a+b\text{arcs}}{\sqrt{(c+dx)}}\right.\right.}{d}$$

$\big\downarrow$ 6191

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^3-\frac{3}{4}b\left(-\frac{1}{2}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))-\frac{1}{4}b\int\frac{(c+dx)^4}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)\right.\right.}{d}$$

$\big\downarrow$ 262

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^3-\frac{3}{4}b\left(-\frac{1}{2}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))-\frac{1}{4}b\left(\frac{1}{4}(c+dx)^3\sqrt{(c+dx)^2}\right.\right.\right.\right.$$

$\big\downarrow$ 262

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^3-\frac{3}{4}b\left(-\frac{1}{2}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))-\frac{1}{4}b\left(\frac{1}{4}(c+dx)^3\sqrt{(c+dx)^2}\right.\right.\right.\right.$$

$\big\downarrow$ 222

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^3-\frac{3}{4}b\left(-\frac{3}{4}\int\frac{(c+dx)^2(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{4}\sqrt{(c+dx)^2+1}(c+dx)\right.\right.$$

$\big\downarrow$ 6227

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^3-\frac{3}{4}b\left(-\frac{3}{4}\left(-b\int(c+dx)(a+b\text{arcsinh}(c+dx))d(c+dx)-\frac{1}{2}\int\frac{(a+b\text{arcsinh}(}{\sqrt{(c+dx)^2}}\right.\right.\right.$$

$\big\downarrow$ 6191

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^3-\frac{3}{4}b\left(-\frac{3}{4}\left(-b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))-\frac{1}{2}b\int\frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}}d(c+d\right.\right.\right.\right.$$

$\big\downarrow$ 262

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^3-\frac{3}{4}b\left(-\frac{3}{4}\left(-b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))-\frac{1}{2}b\left(\frac{1}{2}(c+dx)\sqrt{(c+dx}\right.\right.\right.\right.\right.$$

3.138. $\int(ce+dex)^3(a+b\text{arcsinh}(c+dx))^3\,dx$

$\downarrow$ 222

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^3-\frac{3}{4}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{2}(c+dx)\sqrt{(c+dx)^2+1}(a\right.\right.\right.$$

$\downarrow$ 6198

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^3-\frac{3}{4}b\left(\frac{1}{4}\sqrt{(c+dx)^2+1}(c+dx)^3(a+b\text{arcsinh}(c+dx))^2-\frac{1}{2}b\left(\frac{1}{4}(c+dx)^4(\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
(e^3*(((c + d*x)^4*(a + b*ArcSinh[c + d*x])^3)/4 - (3*b*(((c + d*x)^3*Sqrt
[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/4 - (b*(-1/4*(b*(((c + d*x)^
3*Sqrt[1 + (c + d*x)^2])/4 - (3*(((c + d*x)*Sqrt[1 + (c + d*x)^2])/2 - Arc
Sinh[c + d*x]/2))/4)) + ((c + d*x)^4*(a + b*ArcSinh[c + d*x]))/4))/2 - (3*
(((c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/2 - (a + b*A
rcSinh[c + d*x])^3/(6*b) - b*(-1/2*(b*(((c + d*x)*Sqrt[1 + (c + d*x)^2])/2
 - ArcSinh[c + d*x]/2)) + ((c + d*x)^2*(a + b*ArcSinh[c + d*x]))/2)))/4))/
4))/d
```

3.138.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.138.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{\frac{e^3a^3(dx+c)^4}{4}+e^3b^3\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^3}{4}-\frac{3(dx+c)^3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{16}+\frac{9\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}\,(dx+c)}{32}-\cdots\right.}{\cdots}$
default	$\frac{\frac{e^3a^3(dx+c)^4}{4}+e^3b^3\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^3}{4}-\frac{3(dx+c)^3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{16}+\frac{9\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}\,(dx+c)}{32}-\cdots\right.}{\cdots}$
parts	$\frac{e^3a^3(dx+c)^4}{4d}+\frac{e^3b^3\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^3}{4}-\frac{3(dx+c)^3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{16}+\frac{9\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}\,(dx+c)}{32}\cdots\right.}{\cdots}$

input
```
int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*e^3*a^3*(d*x+c)^4+e^3*b^3*(1/4*(d*x+c)^4*arcsinh(d*x+c)^3-3/16*(d
*x+c)^3*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+9/32*arcsinh(d*x+c)^2*(1+(d*x
+c)^2)^(1/2)*(d*x+c)-3/32*arcsinh(d*x+c)^3+3/32*(d*x+c)^4*arcsinh(d*x+c)-3
/128*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+45/256*(d*x+c)*(1+(d*x+c)^2)^(1/2)+27/2
56*arcsinh(d*x+c)-9/32*(1+(d*x+c)^2)*arcsinh(d*x+c))+3*e^3*a*b^2*(1/4*(d*x
+c)^4*arcsinh(d*x+c)^2-1/8*(d*x+c)^3*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+3/
16*(1+(d*x+c)^2)^(1/2)*(d*x+c)*arcsinh(d*x+c)-3/32*arcsinh(d*x+c)^2+1/32*(
d*x+c)^4-3/32*(d*x+c)^2-3/32)+3*e^3*a^2*b*(1/4*(d*x+c)^4*arcsinh(d*x+c)-1/
16*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+3/32*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/32*arc
sinh(d*x+c)))
```

3.138.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(253) = 506$.

Time = 0.32 (sec) , antiderivative size = 832, normalized size of antiderivative = 2.98

$$\int (ce + dex)^3 (a + b\operatorname{arcsinh}(c + dx))^3 \, dx$$

$$= \frac{8\,(8\,a^3 + 3\,ab^2)d^4e^3x^4 + 32\,(8\,a^3 + 3\,ab^2)cd^3e^3x^3 - 24\,(3\,ab^2 - 2\,(8\,a^3 + 3\,ab^2)c^2)d^2e^3x^2 - 16\,(9\,ab^2c - 2\,(8}{}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/256*(8*(8*a^3 + 3*a*b^2)*d^4*e^3*x^4 + 32*(8*a^3 + 3*a*b^2)*c*d^3*e^3*x^
3 - 24*(3*a*b^2 - 2*(8*a^3 + 3*a*b^2)*c^2)*d^2*e^3*x^2 - 16*(9*a*b^2*c - 2
*(8*a^3 + 3*a*b^2)*c^3)*d*e^3*x + 8*(8*b^3*d^4*e^3*x^4 + 32*b^3*c*d^3*e^3*
x^3 + 48*b^3*c^2*d^2*e^3*x^2 + 32*b^3*c^3*d*e^3*x + (8*b^3*c^4 - 3*b^3)*e^
3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 24*(8*a*b^2*d^4*e^
3*x^4 + 32*a*b^2*c*d^3*e^3*x^3 + 48*a*b^2*c^2*d^2*e^3*x^2 + 32*a*b^2*c^3*d
*e^3*x + (8*a*b^2*c^4 - 3*a*b^2)*e^3 - (2*b^3*d^3*e^3*x^3 + 6*b^3*c*d^2*e^
3*x^2 + 3*(2*b^3*c^2 - b^3)*d*e^3*x + (2*b^3*c^3 - 3*b^3*c)*e^3)*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
^2 + 3*(8*(8*a^2*b + b^3)*d^4*e^3*x^4 + 32*(8*a^2*b + b^3)*c*d^3*e^3*x^3 -
 24*(b^3 - 2*(8*a^2*b + b^3)*c^2)*d^2*e^3*x^2 - 16*(3*b^3*c - 2*(8*a^2*b +
 b^3)*c^3)*d*e^3*x - (24*b^3*c^2 - 8*(8*a^2*b + b^3)*c^4 + 24*a^2*b + 15*b
^3)*e^3 - 16*(2*a*b^2*d^3*e^3*x^3 + 6*a*b^2*c*d^2*e^3*x^2 + 3*(2*a*b^2*c^2
 - a*b^2)*d*e^3*x + (2*a*b^2*c^3 - 3*a*b^2*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*(2*(8*a^2
*b + b^3)*d^3*e^3*x^3 + 6*(8*a^2*b + b^3)*c*d^2*e^3*x^2 - 3*(8*a^2*b + 5*b
^3 - 2*(8*a^2*b + b^3)*c^2)*d*e^3*x + (2*(8*a^2*b + b^3)*c^3 - 3*(8*a^2*b
+ 5*b^3)*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

3.138.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1828 vs. 2(260) = 520.

Time = 0.72 (sec) , antiderivative size = 1828, normalized size of antiderivative = 6.55

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**3,x)
```

output
```
Piecewise((a**3*c**3*e**3*x + 3*a**3*c**2*d*e**3*x**2/2 + a**3*c*d**2*e**3
*x**3 + a**3*d**3*e**3*x**4/4 + 3*a**2*b*c**4*e**3*asinh(c + d*x)/(4*d) +
3*a**2*b*c**3*e**3*x*asinh(c + d*x) - 3*a**2*b*c**3*e**3*sqrt(c**2 + 2*c*d
*x + d**2*x**2 + 1)/(16*d) + 9*a**2*b*c**2*d*e**3*x**2*asinh(c + d*x)/2 -
9*a**2*b*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 3*a**2*b*c*
d**2*e**3*x**3*asinh(c + d*x) - 9*a**2*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 + 1)/16 + 9*a**2*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1
)/(32*d) + 3*a**2*b*d**3*e**3*x**4*asinh(c + d*x)/4 - 3*a**2*b*d**2*e**3*x
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 9*a**2*b*e**3*x*sqrt(c**2 +
2*c*d*x + d**2*x**2 + 1)/32 - 9*a**2*b*e**3*asinh(c + d*x)/(32*d) + 3*a*b*
*2*c**4*e**3*asinh(c + d*x)**2/(4*d) + 3*a*b**2*c**3*e**3*x*asinh(c + d*x)
**2 + 3*a*b**2*c**3*e**3*x/8 - 3*a*b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)*asinh(c + d*x)/(8*d) + 9*a*b**2*c**2*d*e**3*x**2*asinh(c + d*
x)**2/2 + 9*a*b**2*c**2*d*e**3*x**2/16 - 9*a*b**2*c**2*e**3*x*sqrt(c**2 +
2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/8 + 3*a*b**2*c*d**2*e**3*x**3*asin
h(c + d*x)**2 + 3*a*b**2*c*d**2*e**3*x**3/8 - 9*a*b**2*c*d*e**3*x**2*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/8 - 9*a*b**2*c*e**3*x/16 +
9*a*b**2*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(16*d)
 + 3*a*b**2*d**3*e**3*x**4*asinh(c + d*x)**2/4 + 3*a*b**2*d**3*e**3*x**4/3
2 - 3*a*b**2*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(...
```

3.138.7 Maxima [F]

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^3\,dx=\int (dex+ce)^3(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/4*a^3*d^3*e^3*x^4 + a^3*c*d^2*e^3*x^3 + 3/2*a^3*c^2*d*e^3*x^2 + 9/4*(2*x
^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4
*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)
*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^2*b*c^2*d*e^3 + 1/2*(6*x^3*arcsinh(
d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh
(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(
-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*
c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a^2*b*c*d^2*
e^3 + 1/32*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsin
h(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d
)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*
(c^2 + 1)^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^
5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a^2*b*d^3*e^3
 + a^3*c^3*e^3*x + 3*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*
a^2*b*c^3*e^3/d + 1/4*(b^3*d^3*e^3*x^4 + 4*b^3*c*d^2*e^3*x^3 + 6*b^3*c^...
```

3.138.8 Giac [F]

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^3\,dx=\int (dex+ce)^3(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^3, x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^3\,dx=\int (c\,e+d\,e\,x)^3\,(a+b\,\text{asinh}(c+d\,x))^3\,dx$$

input
```
int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^3, x)
```

3.139 $\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^3\,dx$

3.139.1 Optimal result

Integrand size = 23, antiderivative size = 227

$$\begin{aligned}\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^3\,dx &= -\frac{4}{3}ab^2e^2x+\frac{14b^3e^2\sqrt{1+(c+dx)^2}}{9d}-\frac{2b^3e^2(1+(c+dx)^2)^{3/2}}{27d}\\ &-\frac{4b^3e^2(c+dx)\text{arcsinh}(c+dx)}{3d}+\frac{2b^2e^2(c+dx)^3(a+b\text{arcsinh}(c+dx))}{9d}\\ &+\frac{2be^2\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^2}{3d}\\ &-\frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^2}{3d}\\ &+\frac{e^2(c+dx)^3(a+b\text{arcsinh}(c+dx))^3}{3d}\end{aligned}$$

output
```
-4/3*a*b^2*e^2*x-2/27*b^3*e^2*(1+(d*x+c)^2)^(3/2)/d-4/3*b^3*e^2*(d*x+c)*ar
csinh(d*x+c)/d+2/9*b^2*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))/d+1/3*e^2*(d*x+c
)^3*(a+b*arcsinh(d*x+c))^3/d+14/9*b^3*e^2*(1+(d*x+c)^2)^(1/2)/d+2/3*b*e^2*
(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d-1/3*b*e^2*(d*x+c)^2*(a+b*arcs
inh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d
```

3.139.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.14

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^3\,dx$$
$$=\frac{e^2\left(-12ab^2(c+dx)+a(3a^2+2b^2)(c+dx)^3+\frac{1}{3}b\sqrt{1+(c+dx)^2}(18a^2+40b^2-(9a^2+2b^2)(c+dx)^2)-\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
(e^2*(-12*a*b^2*(c + d*x) + a*(3*a^2 + 2*b^2)*(c + d*x)^3 + (b*Sqrt[1 + (c
 + d*x)^2]*(18*a^2 + 40*b^2 - (9*a^2 + 2*b^2)*(c + d*x)^2))/3 - b*(12*b^2*
(c + d*x) - 9*a^2*(c + d*x)^3 - 2*b^2*(c + d*x)^3 - 12*a*b*Sqrt[1 + (c + d
*x)^2] + 6*a*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] - 3*b^2
*(-3*a*(c + d*x)^3 - 2*b*Sqrt[1 + (c + d*x)^2] + b*(c + d*x)^2*Sqrt[1 + (c
 + d*x)^2])*ArcSinh[c + d*x]^2 + 3*b^3*(c + d*x)^3*ArcSinh[c + d*x]^3))/(9
*d)
```

3.139.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {6274, 27, 6191, 6227, 6191, 243, 53, 2009, 6213, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^3\,dx$$
$$\Big\downarrow \text{6274}$$
$$\frac{\int e^2(c+dx)^2(a+b\text{arcsinh}(c+dx))^3d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{e^2\int (c+dx)^2(a+b\text{arcsinh}(c+dx))^3d(c+dx)}{d}$$
$$\Big\downarrow \text{6191}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^3-b\int\frac{(c+dx)^3(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$\big\downarrow$ 6227

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^3-b\left(-\frac{2}{3}b\int(c+dx)^2(a+b\text{arcsinh}(c+dx))d(c+dx)-\frac{2}{3}\int\frac{(c+dx)(a+b\text{arcsin}}{\sqrt{(c+dx)^2-}}\right.\right.}{d}$$

$\big\downarrow$ 6191

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^3-b\left(-\frac{2}{3}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))-\frac{1}{3}b\int\frac{(c+dx)^3}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)-\right.\right.}{d}$$

$\big\downarrow$ 243

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^3-b\left(-\frac{2}{3}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))-\frac{1}{6}b\int\frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}}d(c+dx)^2\right)\right.\right.}{d}$$

$\big\downarrow$ 53

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^3-b\left(-\frac{2}{3}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))-\frac{1}{6}b\int\left(\sqrt{(c+dx)^2+1}-\frac{}{\sqrt{(c}}\right.\right.\right.\right.}{}$$

$\big\downarrow$ 2009

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^3-b\left(-\frac{2}{3}\int\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{3}(c+dx)^2\sqrt{(c+dx)^2+1}(\right.\right.}{}$$

$\big\downarrow$ 6213

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^3-b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2-2b\int(a+b\text{arcsinh}(c+d\right.\right.\right.}{}$$

$\big\downarrow$ 2009

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^3-b\left(\frac{1}{3}(c+dx)^2\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2-\frac{2}{3}b\left(\frac{1}{3}(c+dx)^3(a\right.\right.\right.}{}$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^3,x]
```

3.139. $\int(ce+dex)^2(a+b\text{arcsinh}(c+dx))^3\,dx$

output
```
(e^2*(((c + d*x)^3*(a + b*ArcSinh[c + d*x])^3)/3 - b*(((c + d*x)^2*Sqrt[1
+ (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/3 - (2*b*(-1/6*(b*(-2*Sqrt[1 +
(c + d*x)^2] + (2*(1 + (c + d*x)^2)^(3/2))/3)) + ((c + d*x)^3*(a + b*ArcSi
nh[c + d*x]))/3))/3 - (2*(Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2
 - 2*b*(a*(c + d*x) - b*Sqrt[1 + (c + d*x)^2] + b*(c + d*x)*ArcSinh[c + d*
x])))/3)))/d
```

3.139.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.139.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{\frac{e^2a^3(dx+c)^3}{3}+e^2b^3\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^3}{3}+\frac{2\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{3}-\frac{(dx+c)^2\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{3}-\frac{4(dx+c)\,a}{}\right.}{}$
default	$\frac{\frac{e^2a^3(dx+c)^3}{3}+e^2b^3\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^3}{3}+\frac{2\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{3}-\frac{(dx+c)^2\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{3}-\frac{4(dx+c)\,a}{}\right.}{}$
parts	$\frac{e^2a^3(dx+c)^3}{3d}+\frac{e^2b^3\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^3}{3}+\frac{2\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{3}-\frac{(dx+c)^2\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{3}-\frac{4(dx+}{}\right.}{d}$

input
```
int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*e^2*a^3*(d*x+c)^3+e^2*b^3*(1/3*(d*x+c)^3*arcsinh(d*x+c)^3+2/3*arc
sinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-1/3*(d*x+c)^2*arcsinh(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)-4/3*(d*x+c)*arcsinh(d*x+c)+40/27*(1+(d*x+c)^2)^(1/2)+2/9*(d*x+c
)^3*arcsinh(d*x+c)-2/27*(d*x+c)^2*(1+(d*x+c)^2)^(1/2))+3*e^2*a*b^2*(1/3*(d
*x+c)^3*arcsinh(d*x+c)^2+4/9*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)-2/9*arcsin
h(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)^2-4/9*d*x-4/9*c+2/27*(d*x+c)^3)+3*e^2
*a^2*b*(1/3*(d*x+c)^3*arcsinh(d*x+c)-1/9*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+2/9
*(1+(d*x+c)^2)^(1/2)))
```

3.139.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(203) = 406$.

Time = 0.29 (sec) , antiderivative size = 613, normalized size of antiderivative = 2.70

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^3\,dx$$
$$= \frac{3\,(3\,a^3+2\,ab^2)d^3e^2x^3+9\,(3\,a^3+2\,ab^2)cd^2e^2x^2-9\,(4\,ab^2-(3\,a^3+2\,ab^2)c^2)de^2x+9\,(b^3d^3e^2x^3+3\,b^3cd^2e}{}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/27*(3*(3*a^3 + 2*a*b^2)*d^3*e^2*x^3 + 9*(3*a^3 + 2*a*b^2)*c*d^2*e^2*x^2
- 9*(4*a*b^2 - (3*a^3 + 2*a*b^2)*c^2)*d*e^2*x + 9*(b^3*d^3*e^2*x^3 + 3*b^3
*c*d^2*e^2*x^2 + 3*b^3*c^2*d*e^2*x + b^3*c^3*e^2)*log(d*x + c + sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1))^3 + 9*(3*a*b^2*d^3*e^2*x^3 + 9*a*b^2*c*d^2*e^2*x^
2 + 9*a*b^2*c^2*d*e^2*x + 3*a*b^2*c^3*e^2 - (b^3*d^2*e^2*x^2 + 2*b^3*c*d*e
^2*x + (b^3*c^2 - 2*b^3)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x +
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 3*((9*a^2*b + 2*b^3)*d^3*e^2*x
^3 + 3*(9*a^2*b + 2*b^3)*c*d^2*e^2*x^2 - 3*(4*b^3 - (9*a^2*b + 2*b^3)*c^2)
*d*e^2*x - (12*b^3*c - (9*a^2*b + 2*b^3)*c^3)*e^2 - 6*(a*b^2*d^2*e^2*x^2 +
 2*a*b^2*c*d*e^2*x + (a*b^2*c^2 - 2*a*b^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - ((9*a^2*b + 2*
b^3)*d^2*e^2*x^2 + 2*(9*a^2*b + 2*b^3)*c*d*e^2*x - (18*a^2*b + 40*b^3 - (9
*a^2*b + 2*b^3)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

3.139.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1173 vs. $2(211) = 422$.

Time = 0.47 (sec) , antiderivative size = 1173, normalized size of antiderivative = 5.17

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**3,x)
```

output
```
Piecewise((a**3*c**2*e**2*x + a**3*c*d*e**2*x**2 + a**3*d**2*e**2*x**3/3 +
 a**2*b*c**3*e**2*asinh(c + d*x)/d + 3*a**2*b*c**2*e**2*x*asinh(c + d*x) -
 a**2*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(3*d) + 3*a**2*b*c*
d*e**2*x**2*asinh(c + d*x) - 2*a**2*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)/3 + a**2*b*d**2*e**2*x**3*asinh(c + d*x) - a**2*b*d*e**2*x**2*sq
rt(c**2 + 2*c*d*x + d**2*x**2 + 1)/3 + 2*a**2*b*e**2*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)/(3*d) + a*b**2*c**3*e**2*asinh(c + d*x)**2/d + 3*a*b**2*c*
*2*e**2*x*asinh(c + d*x)**2 + 2*a*b**2*c**2*e**2*x/3 - 2*a*b**2*c**2*e**2*
sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(3*d) + 3*a*b**2*c*d*e
**2*x**2*asinh(c + d*x)**2 + 2*a*b**2*c*d*e**2*x**2/3 - 4*a*b**2*c*e**2*x*
sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/3 + a*b**2*d**2*e**2*x
**3*asinh(c + d*x)**2 + 2*a*b**2*d**2*e**2*x**3/9 - 2*a*b**2*d*e**2*x**2*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/3 - 4*a*b**2*e**2*x/3 +
 4*a*b**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(3*d) +
 b**3*c**3*e**2*asinh(c + d*x)**3/(3*d) + 2*b**3*c**3*e**2*asinh(c + d*x)/
(9*d) + b**3*c**2*e**2*x*asinh(c + d*x)**3 + 2*b**3*c**2*e**2*x*asinh(c +
d*x)/3 - b**3*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x
)**2/(3*d) - 2*b**3*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(27*d)
+ b**3*c*d*e**2*x**2*asinh(c + d*x)**3 + 2*b**3*c*d*e**2*x**2*asinh(c + d*
x)/3 - 2*b**3*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d...
```

3.139.7 Maxima [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^3\,dx=\int (dex+ce)^2(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/3*a^3*d^2*e^2*x^3 + a^3*c*d*e^2*x^2 + 3/2*(2*x^2*arcsinh(d*x + c) - d*(3
*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/s
qrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*c/d^3))*a^2*b*c*d*e^2 + 1/6*(6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2
*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3
 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2
))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2
*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a^2*b*d^2*e^2 + a^3*c^2*e^2*x + 3*((d*x
+ c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^2*b*c^2*e^2/d + 1/3*(b^3*
d^2*e^2*x^3 + 3*b^3*c*d*e^2*x^2 + 3*b^3*c^2*e^2*x)*log(d*x + c + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1))^3 + integrate(((3*a*b^2*d^5*e^2 - b^3*d^5*e^2)*x
^5 + 5*(3*a*b^2*c*d^4*e^2 - b^3*c*d^4*e^2)*x^4 + 3*(c^5*e^2 + c^3*e^2)*a*b
^2 + (3*(10*c^2*d^3*e^2 + d^3*e^2)*a*b^2 - (10*c^2*d^3*e^2 + d^3*e^2)*b^3)
*x^3 + 3*((10*c^3*d^2*e^2 + 3*c*d^2*e^2)*a*b^2 - (3*c^3*d^2*e^2 + c*d^2*e^
2)*b^3)*x^2 + 3*((5*c^4*d*e^2 + 3*c^2*d*e^2)*a*b^2 - (c^4*d*e^2 + c^2*d*e^
2)*b^3)*x + ((3*a*b^2*d^4*e^2 - b^3*d^4*e^2)*x^4 + 3*(c^4*e^2 + c^2*e^2)*a
*b^2 + 4*(3*a*b^2*c*d^3*e^2 - b^3*c*d^3*e^2)*x^3 - 3*(2*b^3*c^2*d^2*e^2 -
(6*c^2*d^2*e^2 + d^2*e^2)*a*b^2)*x^2 - 3*(b^3*c^3*d*e^2 - 2*(2*c^3*d*e^...
```

3.139.8 Giac [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^3\,dx = \int (dex+ce)^2(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^3, x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int (ce + dex)^2 (a + b\operatorname{arcsinh}(c + dx))^3 \, dx = \int (c\,e + d\,e\,x)^2 \, (a + b\operatorname{asinh}(c + d\,x))^3 \, dx$$

input
```
int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^3, x)
```

3.140 $\int (ce+dex)(a+b\text{arcsinh}(c+dx))^3\,dx$

3.140.1 Optimal result

Integrand size = 21, antiderivative size = 161

$$\begin{aligned}\int (ce+dex)(a+b\text{arcsinh}(c+dx))^3\,dx &= -\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}}{8d}+\frac{3b^3e\text{arcsinh}(c+dx)}{8d}\\&\quad+\frac{3b^2e(c+dx)^2(a+b\text{arcsinh}(c+dx))}{4d}\\&\quad-\frac{3be(c+dx)\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^2}{4d}\\&\quad+\frac{e(a+b\text{arcsinh}(c+dx))^3}{4d}+\frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^3}{2d}\end{aligned}$$

output
```
3/8*b^3*e*arcsinh(d*x+c)/d+3/4*b^2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))/d+1/4*
e*(a+b*arcsinh(d*x+c))^3/d+1/2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^3/d-3/8*b^
3*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/d-3/4*b*e*(d*x+c)*(a+b*arcsinh(d*x+c))^2*(
1+(d*x+c)^2)^(1/2)/d
```

3.140.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.24

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^3\,dx$$
$$=\frac{e\Big(2a(2a^2+3b^2)\,(c+dx)^2-3b(2a^2+b^2)\,(c+dx)\sqrt{1+(c+dx)^2}+3b(2a^2+b^2)\,\text{arcsinh}(c+dx)-6b(c\cdots}{}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
(e*(2*a*(2*a^2 + 3*b^2)*(c + d*x)^2 - 3*b*(2*a^2 + b^2)*(c + d*x)*Sqrt[1 +
 (c + d*x)^2] + 3*b*(2*a^2 + b^2)*ArcSinh[c + d*x] - 6*b*(c + d*x)*(-2*a^2
*(c + d*x) - b^2*(c + d*x) + 2*a*b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]
 + 6*b^2*(a + 2*a*(c + d*x)^2 - b*(c + d*x)*Sqrt[1 + (c + d*x)^2])*ArcSinh
[c + d*x]^2 + 2*b^3*(1 + 2*(c + d*x)^2)*ArcSinh[c + d*x]^3))/(8*d)
```

3.140.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6274, 27, 6191, 6227, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^3\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int e(c+dx)(a+b\text{arcsinh}(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e\int (c+dx)(a+b\text{arcsinh}(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \; 6191$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^3-\frac{3}{2}b\int \frac{(c+dx)^2(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 6227}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^3-\frac{3}{2}b\left(-b\int(c+dx)(a+b\text{arcsinh}(c+dx))d(c+dx)-\frac{1}{2}\int\frac{(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}\right.\right.}{d}$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^3-\frac{3}{2}b\left(-b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))-\frac{1}{2}b\int\frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)-\right.\right.}{d}$$

$$\Big\downarrow \text{ 262}$$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^3-\frac{3}{2}b\left(-b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))-\frac{1}{2}b\left(\frac{1}{2}(c+dx)\sqrt{(c+dx)^2+1}\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 222}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^3-\frac{3}{2}b\left(-\frac{1}{2}\int\frac{(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{2}(c+dx)\sqrt{(c+dx)^2+1}(a+ba\right.\right.}{d}$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^3-\frac{3}{2}b\left(-\frac{(a+b\text{arcsinh}(c+dx))^3}{6b}+\frac{1}{2}(c+dx)\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
(e*(((c + d*x)^2*(a + b*ArcSinh[c + d*x])^3)/2 - (3*b*(((c + d*x)*Sqrt[1 +
 (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/2 - (a + b*ArcSinh[c + d*x])^3/(
6*b) - b*(-1/2*(b*(((c + d*x)*Sqrt[1 + (c + d*x)^2])/2 - ArcSinh[c + d*x]/
2)) + ((c + d*x)^2*(a + b*ArcSinh[c + d*x]))/2)))/2))/d
```

3.140. $\int(ce+dex)(a+b\text{arcsinh}(c+dx))^3\,dx$

3.140.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.140.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.51

method	result
derivativedivides	$\frac{\frac{e\,a^3(dx+c)^2}{2}+e\,b^3\left(\frac{\operatorname{arcsinh}(dx+c)^3\left(1+(dx+c)^2\right)}{2}-\frac{3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}\,(dx+c)}{4}-\frac{\operatorname{arcsinh}(dx+c)^3}{4}+\frac{3\left(1+(dx+c)^2\right)\operatorname{arcsi}}{4}\right.}{}$
default	$\frac{\frac{e\,a^3(dx+c)^2}{2}+e\,b^3\left(\frac{\operatorname{arcsinh}(dx+c)^3\left(1+(dx+c)^2\right)}{2}-\frac{3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}\,(dx+c)}{4}-\frac{\operatorname{arcsinh}(dx+c)^3}{4}+\frac{3\left(1+(dx+c)^2\right)\operatorname{arcsi}}{4}\right.}{}$
parts	$e\,a^3\left(\frac{1}{2}d\,x^2+cx\right)+\frac{e\,b^3\left(\frac{\operatorname{arcsinh}(dx+c)^3\left(1+(dx+c)^2\right)}{2}-\frac{3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}\,(dx+c)}{4}-\frac{\operatorname{arcsinh}(dx+c)^3}{4}+\frac{3\left(1+\right.}{}\right.}{d}$

input
```
int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*e*a^3*(d*x+c)^2+e*b^3*(1/2*arcsinh(d*x+c)^3*(1+(d*x+c)^2)-3/4*arc
sinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(d*x+c)-1/4*arcsinh(d*x+c)^3+3/4*(1+(d*x
+c)^2)*arcsinh(d*x+c)-3/8*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/8*arcsinh(d*x+c))+
3*e*a*b^2*(1/2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/2*(1+(d*x+c)^2)^(1/2)*(d*x
+c)*arcsinh(d*x+c)-1/4*arcsinh(d*x+c)^2+1/4*(d*x+c)^2+1/4)+3*e*a^2*b*(1/2*
(d*x+c)^2*arcsinh(d*x+c)-1/4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1/4*arcsinh(d*x+c
)))
```

3.140.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(145) = 290.

Time = 0.31 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.43

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^3\,dx$$

$$=\frac{2\left(2\,a^3+3\,ab^2\right)d^2ex^2+4\left(2\,a^3+3\,ab^2\right)cdex+2\left(2\,b^3d^2ex^2+4\,b^3cdex+\left(2\,b^3c^2+b^3\right)e\right)\log\left(dx+c+\sqrt{d^2x}\right.}{}$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/8*(2*(2*a^3 + 3*a*b^2)*d^2*e*x^2 + 4*(2*a^3 + 3*a*b^2)*c*d*e*x + 2*(2*b^
3*d^2*e*x^2 + 4*b^3*c*d*e*x + (2*b^3*c^2 + b^3)*e)*log(d*x + c + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1))^3 + 6*(2*a*b^2*d^2*e*x^2 + 4*a*b^2*c*d*e*x + (2*
a*b^2*c^2 + a*b^2)*e - (b^3*d*e*x + b^3*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 3*(2*(2*a^2*b +
 b^3)*d^2*e*x^2 + 4*(2*a^2*b + b^3)*c*d*e*x + (2*a^2*b + b^3 + 2*(2*a^2*b
+ b^3)*c^2)*e - 4*(a*b^2*d*e*x + a*b^2*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*((2*a^2*b + b^3)
*d*e*x + (2*a^2*b + b^3)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

3.140.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(148) = 296$.

Time = 0.31 (sec) , antiderivative size = 685, normalized size of antiderivative = 4.25

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))^3\,dx$$
$$= \begin{cases} a^3cex + \frac{a^3dex^2}{2} + \frac{3a^2bc^2e\operatorname{asinh}(c+dx)}{2d} + 3a^2bcex\operatorname{asinh}(c+dx) - \frac{3a^2bce\sqrt{c^2+2cdx+d^2x^2+1}}{4d} + \frac{3a^2bdex^2\operatorname{asinh}(c+dx)}{2} - \\ cex(a + b\operatorname{asinh}(c))^3 \end{cases}$$

input
```
integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**3,x)
```

output
```
Piecewise((a**3*c*e*x + a**3*d*e*x**2/2 + 3*a**2*b*c**2*e*asinh(c + d*x)/(
2*d) + 3*a**2*b*c*e*x*asinh(c + d*x) - 3*a**2*b*c*e*sqrt(c**2 + 2*c*d*x +
d**2*x**2 + 1)/(4*d) + 3*a**2*b*d*e*x**2*asinh(c + d*x)/2 - 3*a**2*b*e*x*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/4 + 3*a**2*b*e*asinh(c + d*x)/(4*d) +
3*a*b**2*c**2*e*asinh(c + d*x)**2/(2*d) + 3*a*b**2*c*e*x*asinh(c + d*x)**2
 + 3*a*b**2*c*e*x/2 - 3*a*b**2*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*as
inh(c + d*x)/(2*d) + 3*a*b**2*d*e*x**2*asinh(c + d*x)**2/2 + 3*a*b**2*d*e*
x**2/4 - 3*a*b**2*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/
2 + 3*a*b**2*e*asinh(c + d*x)**2/(4*d) + b**3*c**2*e*asinh(c + d*x)**3/(2*
d) + 3*b**3*c**2*e*asinh(c + d*x)/(4*d) + b**3*c*e*x*asinh(c + d*x)**3 + 3
*b**3*c*e*x*asinh(c + d*x)/2 - 3*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2
+ 1)*asinh(c + d*x)**2/(4*d) - 3*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2
+ 1)/(8*d) + b**3*d*e*x**2*asinh(c + d*x)**3/2 + 3*b**3*d*e*x**2*asinh(c +
 d*x)/4 - 3*b**3*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**
2/4 - 3*b**3*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + b**3*e*asinh(c +
 d*x)**3/(4*d) + 3*b**3*e*asinh(c + d*x)/(8*d), Ne(d, 0)), (c*e*x*(a + b*a
sinh(c))**3, True))
```

3.140.7 Maxima [F]

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^3\,dx=\int (dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*a^3*d*e*x^2 + 3/4*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x
+ c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^
2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^2*b*d*e +
 a^3*c*e*x + 3*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^2*b*
c*e/d + 1/2*(b^3*d*e*x^2 + 2*b^3*c*e*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))^3 + integrate(3/2*((2*a*b^2*d^4*e - b^3*d^4*e)*x^4 + 2*(c^4
*e + c^2*e)*a*b^2 + 4*(2*a*b^2*c*d^3*e - b^3*c*d^3*e)*x^3 + (2*(6*c^2*d^2*
e + d^2*e)*a*b^2 - (5*c^2*d^2*e + d^2*e)*b^3)*x^2 + 2*(2*(2*c^3*d*e + c*d*
e)*a*b^2 - (c^3*d*e + c*d*e)*b^3)*x + (2*(c^3*e + c*e)*a*b^2 + (2*a*b^2*d^
3*e - b^3*d^3*e)*x^3 + 3*(2*a*b^2*c*d^2*e - b^3*c*d^2*e)*x^2 - 2*(b^3*c^2*
d*e - (3*c^2*d*e + d*e)*a*b^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d
*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^3*x^3 + 3*c*d^2*x^2 + c^3
 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

3.140.8 Giac [F]

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^3\,dx=\int (dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^3, x)
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^3\,dx=\int (c\,e+d\,e\,x)\,\left(a+b\,\operatorname{asinh}(c+d\,x)\right)^3\,dx$$

input
```
int((c*e + d*e*x)*(a + b*asinh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)*(a + b*asinh(c + d*x))^3, x)
```

3.141 $\int (a+b\text{arcsinh}(c+dx))^3\,dx$

3.141.1 Optimal result

Integrand size = 12, antiderivative size = 100

$$\int (a+b\text{arcsinh}(c+dx))^3\,dx = 6ab^2x-\frac{6b^3\sqrt{1+(c+dx)^2}}{d}+\frac{6b^3(c+dx)\text{arcsinh}(c+dx)}{d}-\frac{3b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^2}{d}+\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^3}{d}$$

output
```
6*a*b^2*x+6*b^3*(d*x+c)*arcsinh(d*x+c)/d+(d*x+c)*(a+b*arcsinh(d*x+c))^3/d-
6*b^3*(1+(d*x+c)^2)^(1/2)/d-3*b*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)
/d
```

3.141.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.47

$$\int (a+b\text{arcsinh}(c+dx))^3\,dx$$

$$= \frac{a(a^2+6b^2)(c+dx)-3b(a^2+2b^2)\sqrt{1+(c+dx)^2}-3b\Big(-a^2(c+dx)-2b^2(c+dx)+2ab\sqrt{1+(c+dx}}{}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
(a*(a^2 + 6*b^2)*(c + d*x) - 3*b*(a^2 + 2*b^2)*Sqrt[1 + (c + d*x)^2] - 3*b
*(-(a^2*(c + d*x)) - 2*b^2*(c + d*x) + 2*a*b*Sqrt[1 + (c + d*x)^2])*ArcSin
h[c + d*x] - 3*b^2*(-(a*(c + d*x)) + b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c +
d*x]^2 + b^3*(c + d*x)*ArcSinh[c + d*x]^3)/d
```

3.141.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6273, 6187, 6213, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(c+dx))^3\,dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int (a+b\text{arcsinh}(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \text{6187}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^3-3b\int \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6213}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^3-3b\Big(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2-2b\int (a+b\text{arcsinh}(c+dx))d(c+}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^3-3b\Big(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2-2b\Big(a(c+dx)+b(c+dx)\text{arcsinh}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
((c + d*x)*(a + b*ArcSinh[c + d*x])^3 - 3*b*(Sqrt[1 + (c + d*x)^2]*(a + b*
ArcSinh[c + d*x])^2 - 2*b*(a*(c + d*x) - b*Sqrt[1 + (c + d*x)^2] + b*(c +
d*x)*ArcSinh[c + d*x])))/d
```

3.141.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.141.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{(dx+c)a^3+b^3\left((dx+c)\operatorname{arcsinh}(dx+c)^3-3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}+6(dx+c)\operatorname{arcsinh}(dx+c)-6\sqrt{1+(dx+c)^2}\right)+3a}{d}$
default	$\frac{(dx+c)a^3+b^3\left((dx+c)\operatorname{arcsinh}(dx+c)^3-3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}+6(dx+c)\operatorname{arcsinh}(dx+c)-6\sqrt{1+(dx+c)^2}\right)+3a}{d}$
parts	$x\,a^3+\frac{b^3\left((dx+c)\operatorname{arcsinh}(dx+c)^3-3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}+6(dx+c)\operatorname{arcsinh}(dx+c)-6\sqrt{1+(dx+c)^2}\right)}{d}+\frac{3}{}$

input
```
int((a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((d*x+c)*a^3+b^3*((d*x+c)*arcsinh(d*x+c)^3-3*arcsinh(d*x+c)^2*(1+(d*x+
c)^2)^(1/2)+6*(d*x+c)*arcsinh(d*x+c)-6*(1+(d*x+c)^2)^(1/2))+3*a*b^2*((d*x+
c)*arcsinh(d*x+c)^2-2*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*d*x+2*c)+3*a^2*
b*((d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2)))
```

3.141.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(96) = 192$.

Time = 0.26 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.39

$$\int (a + b\text{arcsinh}(c + dx))^3\,dx$$
$$= \frac{(b^3dx + b^3c)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)^3 + (a^3 + 6\,ab^2)dx + 3\left(ab^2dx + ab^2c - \sqrt{d^2x^2 + 2\,cdx}\right.}{}$$

input
```
integrate((a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
((b^3*d*x + b^3*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + (a
^3 + 6*a*b^2)*d*x + 3*(a*b^2*d*x + a*b^2*c - sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)*b^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - 3*(2*sqrt(d
^2*x^2 + 2*c*d*x + c^2 + 1)*a*b^2 - (a^2*b + 2*b^3)*d*x - (a^2*b + 2*b^3)*
c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1)*(a^2*b + 2*b^3))/d
```

3.141.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(92) = 184$.

Time = 0.17 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.82

$$\int (a + b\text{arcsinh}(c + dx))^3\,dx$$
$$= \begin{cases} a^3x + \frac{3a^2bc\operatorname{asinh}(c+dx)}{d} + 3a^2bx\operatorname{asinh}(c + dx) - \frac{3a^2b\sqrt{c^2+2cdx+d^2x^2+1}}{d} + \frac{3ab^2c\operatorname{asinh}^2(c+dx)}{d} + 3ab^2x\operatorname{asinh}^2(c + \\ x(a + b\operatorname{asinh}(c))^3 \end{cases}$$

input
```
integrate((a+b*asinh(d*x+c))**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*c*asinh(c + d*x)/d + 3*a**2*b*x*asinh(c + d*x
) - 3*a**2*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d + 3*a*b**2*c*asinh(c +
 d*x)**2/d + 3*a*b**2*x*asinh(c + d*x)**2 + 6*a*b**2*x - 6*a*b**2*sqrt(c**
2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/d + b**3*c*asinh(c + d*x)**3/d
 + 6*b**3*c*asinh(c + d*x)/d + b**3*x*asinh(c + d*x)**3 + 6*b**3*x*asinh(c
 + d*x) - 3*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/d
- 6*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d, Ne(d, 0)), (x*(a + b*asin
h(c))**3, True))
```

3.141.7 Maxima [F]

$$\int (a+b\text{arcsinh}(c+dx))^3\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
b^3*x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + a^3*x + 3*((d*x
 + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^2*b/d + integrate(3*((c^
3 + c)*a*b^2 + (a*b^2*d^3 - b^3*d^3)*x^3 + (3*a*b^2*c*d^2 - 2*b^3*c*d^2)*x
^2 + ((3*c^2*d + d)*a*b^2 - (c^2*d + d)*b^3)*x + ((c^2 + 1)*a*b^2 + (a*b^2
*d^2 - b^3*d^2)*x^2 + (2*a*b^2*c*d - b^3*c*d)*x)*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^3*x^3 + 3*
c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) +
c), x)
```

3.141.8 Giac [F]

$$\int (a+b\text{arcsinh}(c+dx))^3\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^3, x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arcsinh}(c+dx))^3\,dx = \int (a+b\,\text{asinh}(c+d\,x))^3\,dx$$

input
```
int((a + b*asinh(c + d*x))^3,x)
```

output
```
int((a + b*asinh(c + d*x))^3, x)
```

3.142 $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{ce+dex}\,dx$

3.142.1 Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{ce+dex}\,dx = \frac{(a+b\text{arcsinh}(c+dx))^4}{4bde} + \frac{(a+b\text{arcsinh}(c+dx))^3\log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de} - \frac{3b(a+b\text{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,e^{-2\text{arcsinh}(c+dx)}\right)}{2de} - \frac{3b^2(a+b\text{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,e^{-2\text{arcsinh}(c+dx)}\right)}{2de} - \frac{3b^3\operatorname{PolyLog}\left(4,e^{-2\text{arcsinh}(c+dx)}\right)}{4de}$$

output
```
1/4*(a+b*arcsinh(d*x+c))^4/b/d/e+(a+b*arcsinh(d*x+c))^3*ln(1-1/(d*x+c+(1+(
d*x+c)^2)^(1/2))^2)/d/e-3/2*b*(a+b*arcsinh(d*x+c))^2*polylog(2,1/(d*x+c+(1
+(d*x+c)^2)^(1/2))^2)/d/e-3/2*b^2*(a+b*arcsinh(d*x+c))*polylog(3,1/(d*x+c+
(1+(d*x+c)^2)^(1/2))^2)/d/e-3/4*b^3*polylog(4,1/(d*x+c+(1+(d*x+c)^2)^(1/2)
)^2)/d/e
```

3.142.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{ce+dex}\,dx$$
$$= \frac{-\frac{(a+b\text{arcsinh}(c+dx))^4}{b}+4(a+b\text{arcsinh}(c+dx))^3\log\left(1-e^{2\text{arcsinh}(c+dx)}\right)+6b(a+b\text{arcsinh}(c+dx))^2\,\text{PolyL}}{}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x),x]
```

output
```
(-((a + b*ArcSinh[c + d*x])^4/b) + 4*(a + b*ArcSinh[c + d*x])^3*Log[1 - E^
(2*ArcSinh[c + d*x])] + 6*b*(a + b*ArcSinh[c + d*x])^2*PolyLog[2, E^(2*Arc
Sinh[c + d*x])] - 6*b^2*(a + b*ArcSinh[c + d*x])*PolyLog[3, E^(2*ArcSinh[c
 + d*x])] + 3*b^3*PolyLog[4, E^(2*ArcSinh[c + d*x])])/(4*d*e)
```

3.142.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.36, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {6274, 27, 6190, 25, 3042, 26, 4201, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{ce+dex}\,dx$$
$$\Big\downarrow \text{ 6274}$$
$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^3}{e(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^3}{c+dx}d(c+dx)}{de}$$
$$\Big\downarrow \text{ 6190}$$
$$\frac{\int -(a+b\text{arcsinh}(c+dx))^3\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)d(a+b\text{arcsinh}(c+dx))}{bde}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int (a+b\text{arcsinh}(c+dx))^3 \coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)d(a+b\text{arcsinh}(c+dx))}{bde}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int -i(a+b\text{arcsinh}(c+dx))^3 \tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(c+dx))}{bde}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\int (a+b\text{arcsinh}(c+dx))^3 \tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)d(a+b\text{arcsinh}(c+dx))}{bde}$$

$$\Big\downarrow \text{ 4201}$$

$$\frac{i\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}(a+b\text{arcsinh}(c+dx))^3}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{4}i(a+b\text{arcsinh}(c+dx))^4\right)}{bde}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{i\left(2i\left(\frac{3}{2}b\int (a+b\text{arcsinh}(c+dx))^2\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}b(a+b\text{arcsinh}(\right.\right.}{bde}$$

$$\Big\downarrow \text{ 3011}$$

$$i\left(2i\left(\frac{3}{2}b\left(\frac{1}{2}b(a+b\text{arcsinh}(c+dx))^2\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right)-b\int (a+b\text{arcsinh}(c+dx))\text{PolyLo}\right.\right.\right.$$

$$\Big\downarrow \text{ 7163}$$

$$i\left(2i\left(\frac{3}{2}b\left(\frac{1}{2}b(a+b\text{arcsinh}(c+dx))^2\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right)-b\left(\frac{1}{2}b\int \text{PolyLog}\left(3,-e^{\frac{2a}{b}-\frac{2(a+b\text{arc}}{}}\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 2720}$$

$$i\left(2i\left(\frac{3}{2}b\left(\frac{1}{2}b(a+b\text{arcsinh}(c+dx))^2\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right)-b\left(-\frac{1}{4}b^2\int \exp\left(-\frac{2a}{b}+\frac{2(a+b\text{arcsin}}{b}\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 7143}$$

3.142. $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{ce+dex}\,dx$

$$i\left(2i\left(\frac{3}{2}b\left(\frac{1}{2}b(a+b\text{arcsinh}(c+dx))^2\,\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right)-b\left(-\frac{1}{4}b^2\,\text{PolyLog}(4,-c-dx)-\frac{1}{2}\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x),x]
```

output
```
(I*((-1/4*I)*(a + b*ArcSinh[c + d*x])^4 + (2*I)*(-1/2*(b*(a + b*ArcSinh[c
+ d*x])^3*Log[1 + E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[c + d*x]))/b)]) +
(3*b*((b*(a + b*ArcSinh[c + d*x])^2*PolyLog[2, -E^((2*a)/b - I*Pi - (2*(a
+ b*ArcSinh[c + d*x]))/b)])/2 - b*(-1/2*(b*(a + b*ArcSinh[c + d*x])*PolyLo
g[3, -E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[c + d*x]))/b)]) - (b^2*PolyLog
[4, -c - d*x])/4)))/2)))/(b*d*e)
```

3.142.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.142. $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{ce+dex} \, dx$

3.142.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(179) = 358$.

Time = 0.47 (sec) , antiderivative size = 556, normalized size of antiderivative = 3.59

method	result
derivativedivides	$\frac{\frac{a^3 \ln(dx+c)}{e}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)^4}{4}+\operatorname{arcsinh}(dx+c)^3 \ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)+3 \operatorname{arcsinh}(dx+c)^2 \operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right.}{}}{}$
default	$\frac{\frac{a^3 \ln(dx+c)}{e}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)^4}{4}+\operatorname{arcsinh}(dx+c)^3 \ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)+3 \operatorname{arcsinh}(dx+c)^2 \operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right.}{}}{}$
parts	$\frac{a^3 \ln(dx+c)}{ed}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)^4}{4}+\operatorname{arcsinh}(dx+c)^3 \ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)+3 \operatorname{arcsinh}(dx+c)^2 \operatorname{polylog}\left(2,-dx\right.}{}$

input
```
int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3/e*ln(d*x+c)+b^3/e*(-1/4*arcsinh(d*x+c)^4+arcsinh(d*x+c)^3*ln(1+d*
x+c+(1+(d*x+c)^2)^(1/2))+3*arcsinh(d*x+c)^2*polylog(2,-d*x-c-(1+(d*x+c)^2)
^(1/2))-6*arcsinh(d*x+c)*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+6*polylog(4
,-d*x-c-(1+(d*x+c)^2)^(1/2))+arcsinh(d*x+c)^3*ln(1-d*x-c-(1+(d*x+c)^2)^(1/
2))+3*arcsinh(d*x+c)^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-6*arcsinh(d*x+
c)*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))+6*polylog(4,d*x+c+(1+(d*x+c)^2)^(1
/2)))+3*a*b^2/e*(-1/3*arcsinh(d*x+c)^3+arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x
+c)^2)^(1/2))+2*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-2*pol
ylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^
2)^(1/2))+2*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-2*polylog(
3,d*x+c+(1+(d*x+c)^2)^(1/2)))+3*a^2*b/e*(-1/2*arcsinh(d*x+c)^2+arcsinh(d*x
+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+
arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+polylog(2,d*x+c+(1+(d*x+c)^
2)^(1/2))))
```

3.142.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{ce+dex}\,dx=\int \frac{(b\operatorname{arsinh}(dx+c)+a)^3}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*ar
csinh(d*x + c) + a^3)/(d*e*x + c*e), x)
```

3.142.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{ce+dex}\,dx = \frac{\int \frac{a^3}{c+dx}\,dx + \int \frac{b^3 \operatorname{asinh}^3(c+dx)}{c+dx}\,dx + \int \frac{3ab^2 \operatorname{asinh}^2(c+dx)}{c+dx}\,dx + \int \frac{3a^2 b \operatorname{asinh}(c+dx)}{c+dx}\,dx}{e}$$

input
```
integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e),x)
```

output
```
(Integral(a**3/(c + d*x), x) + Integral(b**3*asinh(c + d*x)**3/(c + d*x),
x) + Integral(3*a*b**2*asinh(c + d*x)**2/(c + d*x), x) + Integral(3*a**2*b
*asinh(c + d*x)/(c + d*x), x))/e
```

3.142.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{ce+dex}\,dx = \int \frac{(b\,\operatorname{arsinh}(dx+c)+a)^3}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
a^3*log(d*e*x + c*e)/(d*e) + integrate(b^3*log(d*x + c + sqrt((d*x + c)^2
+ 1))^3/(d*e*x + c*e) + 3*a*b^2*log(d*x + c + sqrt((d*x + c)^2 + 1))^2/(d*
e*x + c*e) + 3*a^2*b*log(d*x + c + sqrt((d*x + c)^2 + 1))/(d*e*x + c*e), x
)
```

3.142.8 Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{ce+dex}\,dx=\int \frac{(b\operatorname{arsinh}(dx+c)+a)^3}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e), x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{ce+dex}\,dx=\int \frac{(a+b\operatorname{asinh}(c+d\,x))^3}{c\,e+d\,e\,x}\,dx$$

input
```
int((a + b*asinh(c + d*x))^3/(c*e + d*e*x),x)
```

output
```
int((a + b*asinh(c + d*x))^3/(c*e + d*e*x), x)
```

3.143 $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx$

3.143.1 Optimal result

Integrand size = 23, antiderivative size = 166

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx = -\frac{(a+b\operatorname{arcsinh}(c+dx))^3}{de^2(c+dx)} - \frac{6b(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} - \frac{6b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} + \frac{6b^2(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} + \frac{6b^3\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2} - \frac{6b^3\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^2}$$

output
```
-(a+b*arcsinh(d*x+c))^3/d/e^2/(d*x+c)-6*b*(a+b*arcsinh(d*x+c))^2*arctanh(d
*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2-6*b^2*(a+b*arcsinh(d*x+c))*polylog(2,-d*x-
c-(1+(d*x+c)^2)^(1/2))/d/e^2+6*b^2*(a+b*arcsinh(d*x+c))*polylog(2,d*x+c+(1
+(d*x+c)^2)^(1/2))/d/e^2+6*b^3*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^2
-6*b^3*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2
```

3.143.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.90

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx$$
$$= \frac{-\frac{a^3}{c+dx}-\frac{3a^2b\text{arcsinh}(c+dx)}{c+dx}+3a^2b\log(c+dx)-3a^2b\log\left(1+\sqrt{1+c^2+2cdx+d^2x^2}\right)+3ab^2\Big(\text{arcsinh}(c+}{}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

output
```
(-(a^3/(c + d*x)) - (3*a^2*b*ArcSinh[c + d*x])/(c + d*x) + 3*a^2*b*Log[c +
 d*x] - 3*a^2*b*Log[1 + Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]] + 3*a*b^2*(ArcS
inh[c + d*x]*(-(ArcSinh[c + d*x]/(c + d*x)) + 2*Log[1 - E^(-ArcSinh[c + d*
x])] - 2*Log[1 + E^(-ArcSinh[c + d*x])]) + 2*PolyLog[2, -E^(-ArcSinh[c + d
*x])] - 2*PolyLog[2, E^(-ArcSinh[c + d*x])]) + b^3*(-(ArcSinh[c + d*x]^3/(
c + d*x)) + 3*ArcSinh[c + d*x]^2*Log[1 - E^(-ArcSinh[c + d*x])] - 3*ArcSin
h[c + d*x]^2*Log[1 + E^(-ArcSinh[c + d*x])] + 6*ArcSinh[c + d*x]*PolyLog[2
, -E^(-ArcSinh[c + d*x])] - 6*ArcSinh[c + d*x]*PolyLog[2, E^(-ArcSinh[c +
d*x])] + 6*PolyLog[3, -E^(-ArcSinh[c + d*x])] - 6*PolyLog[3, E^(-ArcSinh[c
 + d*x])]))/(d*e^2)
```

3.143.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.86, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {6274, 27, 6191, 6231, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx$$
$$\Big\downarrow \text{6274}$$
$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^3}{e^2(c+dx)^2}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(c+dx)^2}d(c+dx)}{de^2}$$

$\Big\downarrow$ 6191

$$\frac{3b\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(c+dx)\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{(a+b\text{arcsinh}(c+dx))^3}{c+dx}}{de^2}$$

$\Big\downarrow$ 6231

$$\frac{3b\int \frac{(a+b\text{arcsinh}(c+dx))^2}{c+dx}d\text{arcsinh}(c+dx)-\frac{(a+b\text{arcsinh}(c+dx))^3}{c+dx}}{de^2}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^3}{c+dx}+3b\int i(a+b\text{arcsinh}(c+dx))^2\csc(i\text{arcsinh}(c+dx))d\text{arcsinh}(c+dx)}{de^2}$$

$\Big\downarrow$ 26

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^3}{c+dx}+3ib\int (a+b\text{arcsinh}(c+dx))^2\csc(i\text{arcsinh}(c+dx))d\text{arcsinh}(c+dx)}{de^2}$$

$\Big\downarrow$ 4670

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^3}{c+dx}+3ib\left(2ib\int (a+b\text{arcsinh}(c+dx))\log\left(1-e^{\text{arcsinh}(c+dx)}\right)d\text{arcsinh}(c+dx)-2ib\int (a+b\text{arcsi}\right.}{de^2}$$

$\Big\downarrow$ 3011

$$-\frac{(a+b\text{arcsinh}(c+dx))^3}{c+dx}+3ib\left(-2ib\left(b\int \text{PolyLog}\left(2,-e^{\text{arcsinh}(c+dx)}\right)d\text{arcsinh}(c+dx)-\text{PolyLog}\left(2,-e^{\text{arcsinh}(c+dx)}\right)\right.\right.$$

$\Big\downarrow$ 2720

$$-\frac{(a+b\text{arcsinh}(c+dx))^3}{c+dx}+3ib\left(2ib\left(b\int e^{-\text{arcsinh}(c+dx)}\text{PolyLog}\left(2,e^{\text{arcsinh}(c+dx)}\right)de^{\text{arcsinh}(c+dx)}-\text{PolyLog}\left(2,e^{\text{arcsinh}(c+}\right.\right.\right.$$

$\Big\downarrow$ 7143

$$-\frac{(a+b\text{arcsinh}(c+dx))^3}{c+dx}+3ib\left(2i\text{arctanh}\left(e^{\text{arcsinh}(c+dx)}\right)(a+b\text{arcsinh}(c+dx))^2+2ib\left(b\,\text{PolyLog}\left(3,e^{\text{arcsinh}(c+dx)}\right)-\right.\right.$$

input
```
Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

3.143. $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx$

output
```
(-((a + b*ArcSinh[c + d*x])^3/(c + d*x)) + (3*I)*b*((2*I)*(a + b*ArcSinh[c
 + d*x])^2*ArcTanh[E^ArcSinh[c + d*x]] + (2*I)*b*(-((a + b*ArcSinh[c + d*x
])*PolyLog[2, E^ArcSinh[c + d*x]]) + b*PolyLog[3, E^ArcSinh[c + d*x]]) - (
2*I)*b*(-((a + b*ArcSinh[c + d*x])*PolyLog[2, -E^ArcSinh[c + d*x]]) + b*Po
lyLog[3, -c - d*x])))/(d*e^2)
```

3.143.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.143. $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx$

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.143.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.27

method	result
derivativedivides	$-\frac{a^3}{e^2(dx+c)}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)^3}{dx+c}-3\operatorname{arcsinh}(dx+c)^2\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-6\operatorname{arcsinh}(dx+c)\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c}\right.\right.}{}$
default	$-\frac{a^3}{e^2(dx+c)}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)^3}{dx+c}-3\operatorname{arcsinh}(dx+c)^2\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-6\operatorname{arcsinh}(dx+c)\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx+c}\right.\right.}{}$
parts	$-\frac{a^3}{e^2(dx+c)d}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)^3}{dx+c}-3\operatorname{arcsinh}(dx+c)^2\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-6\operatorname{arcsinh}(dx+c)\operatorname{polylog}\left(2,-\right.\right.}{}$

input
```
int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^3/e^2/(d*x+c)+b^3/e^2*(-1/(d*x+c)*arcsinh(d*x+c)^3-3*arcsinh(d*x+c
)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-6*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(
d*x+c)^2)^(1/2))+6*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+3*arcsinh(d*x+c)^
2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+6*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x
+c)^2)^(1/2))-6*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2)))+3*a*b^2/e^2*(-1/(d*x
+c)*arcsinh(d*x+c)^2-2*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-2*po
lylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+2*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)
^2)^(1/2))+2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2)))+3*a^2*b/e^2*(-1/(d*x+c)
*arcsinh(d*x+c)-arctanh(1/(1+(d*x+c)^2)^(1/2))))
```

3.143.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx=\int \frac{(b\operatorname{arsinh}(dx+c)+a)^3}{(dex+ce)^2}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="fricas")
```

output
```
integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*ar
csinh(d*x + c) + a^3)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

3.143.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx = \frac{\int \frac{a^3}{c^2+2cdx+d^2x^2}\,dx+\int \frac{b^3\operatorname{asinh}^3(c+dx)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{3ab^2\operatorname{asinh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{3a^2b\operatorname{asinh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

input
```
integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**2,x)
```

output
```
(Integral(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**3*asinh(c +
d*x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a*b**2*asinh(c + d*x
)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a**2*b*asinh(c + d*x)/(
c**2 + 2*c*d*x + d**2*x**2), x))/e**2
```

3.143.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.143.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{(dex+ce)^2}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^2, x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^2}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^3}{(c\,e+d\,e\,x)^2}\,dx$$

input
```
int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^2,x)
```

output
```
int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^2, x)
```

3.144 $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^3} dx$

3.144.1 Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^3} dx = \frac{3b(a+b\text{arcsinh}(c+dx))^2}{2de^3} - \frac{3b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^2}{2de^3(c+dx)} - \frac{(a+b\text{arcsinh}(c+dx))^3}{2de^3(c+dx)^2} + \frac{3b^2(a+b\text{arcsinh}(c+dx))\log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de^3} - \frac{3b^3\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(c+dx)}\right)}{2de^3}$$

output
```
3/2*b*(a+b*arcsinh(d*x+c))^2/d/e^3-1/2*(a+b*arcsinh(d*x+c))^3/d/e^3/(d*x+c
)^2+3*b^2*(a+b*arcsinh(d*x+c))*ln(1-1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3
-3/2*b^3*polylog(2,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3-3/2*b*(a+b*arcsi
nh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d/e^3/(d*x+c)
```

3.144.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.46

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx =$$

$$-\frac{3b^2\left(a+b(c+dx)\left(-c-dx+\sqrt{1+c^2+2cdx+d^2x^2}\right)\right)\text{arcsinh}(c+dx)^2+b^3\text{arcsinh}(c+dx)^3+3b\text{arc}}{}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

output
```
-1/2*(3*b^2*(a + b*(c + d*x)*(-c - d*x + Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]
))*ArcSinh[c + d*x]^2 + b^3*ArcSinh[c + d*x]^3 + 3*b*ArcSinh[c + d*x]*(a*(
a + 2*b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]) - 2*b^2*(c + d*x)^2*L
og[1 - E^(-2*ArcSinh[c + d*x])]) + a*(a*(a + 3*b*(c + d*x)*Sqrt[1 + c^2 +
2*c*d*x + d^2*x^2]) - 6*b^2*(c + d*x)^2*Log[c + d*x]) + 3*b^3*(c + d*x)^2*
PolyLog[2, E^(-2*ArcSinh[c + d*x])])/(d*e^3*(c + d*x)^2)
```

3.144.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {6274, 27, 6191, 6215, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^3}{e^3(c+dx)^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(c+dx)^3}d(c+dx)}{de^3}$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{\frac{3}{2}b\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(c+dx)^2\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{(a+b\text{arcsinh}(c+dx))^3}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{6215}$$

$$\frac{\frac{3}{2}b\left(2b\int \frac{a+b\text{arcsinh}(c+dx)}{c+dx}d(c+dx)-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}{c+dx}\right)-\frac{(a+b\text{arcsinh}(c+dx))^3}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{6190}$$

$$\frac{\frac{3}{2}b\left(2\int -\left((a+b\text{arcsinh}(c+dx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\right)d(a+b\text{arcsinh}(c+dx))-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(\cdots}{c+dx}}{de^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3}{2}b\left(-2\int (a+b\text{arcsinh}(c+dx))\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)d(a+b\text{arcsinh}(c+dx))-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+d\cdots}{c+dx}}{de^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^3}{2(c+dx)^2}+\frac{3}{2}b\left(-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}{c+dx}-2\int -i(a+b\text{arcsinh}(c+dx))\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}\cdots}{b}\right.\right.}{de^3}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^3}{2(c+dx)^2}+\frac{3}{2}b\left(-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}{c+dx}+2i\int (a+b\text{arcsinh}(c+dx))\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+ba\cdots}{}\right.\right.}{de^3}$$

$$\Big\downarrow \text{4201}$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^3}{2(c+dx)^2}+\frac{3}{2}b\left(-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}{c+dx}+2i\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}(a+b\text{arcsinh}(c+dx))}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}}d(\cdots\right.\right.}{de^3}$$

$$\Big\downarrow \text{2620}$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^3}{2(c+dx)^2}+\frac{3}{2}b\left(-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}{c+dx}+2i\left(2i\left(\frac{1}{2}b\int \log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right)d(a\cdots\right.\right.\right.}{}$$

$$\Big\downarrow \text{2715}$$

3.144. $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^3}{2(c+dx)^2}+\frac{3}{2}b\left(-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}{c+dx}+2i\left(2i\left(-\frac{1}{4}b^2\int\exp\left(-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(c+dx))}{b}+i\pi\right.\right.\right.\right.}{}$$

$$\Big\downarrow \; 2838$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^3}{2(c+dx)^2}+\frac{3}{2}b\left(-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}{c+dx}+2i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}(2,-c-dx)-\frac{1}{2}b(a+b\text{arcsinh}(c\right.\right.\right.}{de^3}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

output
```
(-1/2*(a + b*ArcSinh[c + d*x])^3/(c + d*x)^2 + (3*b*(-((Sqrt[1 + (c + d*x)
^2]*(a + b*ArcSinh[c + d*x])^2)/(c + d*x)) + (2*I)*((-1/2*I)*(a + b*ArcSin
h[c + d*x])^2 + (2*I)*(-1/2*(b*(a + b*ArcSinh[c + d*x])*Log[1 + E^((2*a)/b
 - I*Pi - (2*(a + b*ArcSinh[c + d*x]))/b)]) + (b^2*PolyLog[2, -c - d*x])/4
))))/2)/(d*e^3)
```

3.144.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.144. $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx$

3.144.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.07

method	result
derivativedivides	$-\frac{a^3}{2e^3(dx+c)^2}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)^2\left(3(dx+c)\sqrt{1+(dx+c)^2}-3(dx+c)^2+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}-3\operatorname{arcsinh}(dx+c)^2+3\operatorname{arcsinh}(dx+c)\ln\left(1\right.\right.}{}$
default	$-\frac{a^3}{2e^3(dx+c)^2}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)^2\left(3(dx+c)\sqrt{1+(dx+c)^2}-3(dx+c)^2+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}-3\operatorname{arcsinh}(dx+c)^2+3\operatorname{arcsinh}(dx+c)\ln\left(1\right.\right.}{}$
parts	$-\frac{a^3}{2e^3(dx+c)^2d}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)^2\left(3(dx+c)\sqrt{1+(dx+c)^2}-3(dx+c)^2+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}-3\operatorname{arcsinh}(dx+c)^2+3\operatorname{arcsinh}(\right.}{}$

input
```
int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a^3/e^3/(d*x+c)^2+b^3/e^3*(-1/2*arcsinh(d*x+c)^2*(3*(d*x+c)*(1+(
d*x+c)^2)^(1/2)-3*(d*x+c)^2+arcsinh(d*x+c))/(d*x+c)^2-3*arcsinh(d*x+c)^2+3
*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+3*polylog(2,-d*x-c-(1+(d*x
+c)^2)^(1/2))+3*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+3*polylog(2
,d*x+c+(1+(d*x+c)^2)^(1/2)))+3*a*b^2/e^3*(-2*arcsinh(d*x+c)-1/2*arcsinh(d*
x+c)*(-2*(d*x+c)^2+2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+arcsinh(d*x+c))/(d*x+c)^2
+ln((d*x+c+(1+(d*x+c)^2)^(1/2))^2-1))+3*a^2*b/e^3*(-1/2/(d*x+c)^2*arcsinh(
d*x+c)-1/2/(d*x+c)*(1+(d*x+c)^2)^(1/2)))
```

3.144.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx=\int \frac{\left(b\operatorname{arsinh}\left(dx+c\right)+a\right)^3}{\left(dex+ce\right)^3}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="fricas")
```

output
```
integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*ar
csinh(d*x + c) + a^3)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3
*e^3), x)
```

3.144.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx = \frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^3\,\text{asinh}^3\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3ab^2\,\text{asinh}^2\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3a^2b\,\text{asinh}\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

input
```
integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**3,x)
```

output
```
(Integral(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integ
ral(b**3*asinh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)
, x) + Integral(3*a*b**2*asinh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x
**2 + d**3*x**3), x) + Integral(3*a**2*b*asinh(c + d*x)/(c**3 + 3*c**2*d*x
 + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

3.144.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="maxima")
```

output
```
-3*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d*arcsinh(d*x + c)/(d^3*e^3*x + c*d^
2*e^3) - log(d*x + c)/(d*e^3))*a*b^2 - 1/2*(log(d*x + c + sqrt(d^2*x^2 + 2
*c*d*x + c^2 + 1))^3/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 2*integra
te(3/2*(d^2*x^2 + 2*c*d*x + c^2 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x +
 c) + 1)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^5*e^3*x^5 +
 5*c*d^4*e^3*x^4 + c^5*e^3 + c^3*e^3 + (10*c^2*d^3*e^3 + d^3*e^3)*x^3 + (1
0*c^3*d^2*e^3 + 3*c*d^2*e^3)*x^2 + (5*c^4*d*e^3 + 3*c^2*d*e^3)*x + (d^4*e^
3*x^4 + 4*c*d^3*e^3*x^3 + c^4*e^3 + c^2*e^3 + (6*c^2*d^2*e^3 + d^2*e^3)*x^
2 + 2*(2*c^3*d*e^3 + c*d*e^3)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x))*b
^3 - 3/2*a^2*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d/(d^3*e^3*x + c*d^2*e^3
) + arcsinh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)) - 3/2*a*b^
2*arcsinh(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 1/2*a^3/(
d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

3.144.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx=\int \frac{(b\operatorname{arsinh}(dx+c)+a)^3}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^3, x)
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^3}\,dx=\int \frac{(a+b\,\mathrm{asinh}(c+d\,x))^3}{(c\,e+d\,e\,x)^3}\,dx$$

input
```
int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^3,x)
```

output
```
int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^3, x)
```

3.145 $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx$

3.145.1 Optimal result

Integrand size = 23, antiderivative size = 261

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx = &-\frac{b^2(a+b\text{arcsinh}(c+dx))}{de^4(c+dx)} \\ &-\frac{b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^2}{2de^4(c+dx)^2} \\ &-\frac{(a+b\text{arcsinh}(c+dx))^3}{3de^4(c+dx)^3} \\ &+\frac{b(a+b\text{arcsinh}(c+dx))^2\text{arctanh}\left(e^{\text{arcsinh}(c+dx)}\right)}{de^4} \\ &-\frac{b^3\text{arctanh}\left(\sqrt{1+(c+dx)^2}\right)}{de^4} \\ &+\frac{b^2(a+b\text{arcsinh}(c+dx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(c+dx)}\right)}{de^4} \\ &-\frac{b^2(a+b\text{arcsinh}(c+dx))\,\text{PolyLog}\left(2,e^{\text{arcsinh}(c+dx)}\right)}{de^4} \\ &-\frac{b^3\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(c+dx)}\right)}{de^4} \\ &+\frac{b^3\,\text{PolyLog}\left(3,e^{\text{arcsinh}(c+dx)}\right)}{de^4}\end{aligned}$$

output
```
-b^2*(a+b*arcsinh(d*x+c))/d/e^4/(d*x+c)-1/3*(a+b*arcsinh(d*x+c))^3/d/e^4/(
d*x+c)^3+b*(a+b*arcsinh(d*x+c))^2*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4
-b^3*arctanh((1+(d*x+c)^2)^(1/2))/d/e^4+b^2*(a+b*arcsinh(d*x+c))*polylog(2
,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4-b^2*(a+b*arcsinh(d*x+c))*polylog(2,d*x+
c+(1+(d*x+c)^2)^(1/2))/d/e^4-b^3*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e
^4+b^3*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-1/2*b*(a+b*arcsinh(d*x+c
))^2*(1+(d*x+c)^2)^(1/2)/d/e^4/(d*x+c)^2
```

3.145.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 694 vs. $2(261) = 522$.

Time = 7.25 (sec) , antiderivative size = 694, normalized size of antiderivative = 2.66

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx = -\frac{a^3}{3de^4(c+dx)^3} - \frac{a^2b\sqrt{1+c^2+2cdx+d^2x^2}}{2de^4(c+dx)^2}$$
$$- \frac{a^2b\text{arcsinh}(c+dx)}{de^4(c+dx)^3} - \frac{a^2b\log(c+dx)}{2de^4} + \frac{a^2b\log\left(1+\sqrt{1+c^2+2cdx+d^2x^2}\right)}{2de^4}$$
$$+ \frac{ab^2\left(-8\,\text{PolyLog}\left(2,-e^{-\text{arcsinh}(c+dx)}\right) - \frac{2\left(-2+4\text{arcsinh}(c+dx)^2+2\cosh(2\text{arcsinh}(c+dx))-3(c+dx)\text{arcsinh}(c+dx)\log\right.}{\cdots}\right.}{\cdots}$$
$$+ \frac{b^3\left(-24\text{arcsinh}(c+dx)\coth\left(\frac{1}{2}\text{arcsinh}(c+dx)\right) + 4\text{arcsinh}(c+dx)^3\coth\left(\frac{1}{2}\text{arcsinh}(c+dx)\right) - 6\text{arcsi}\cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

output
```
-1/3*a^3/(d*e^4*(c + d*x)^3) - (a^2*b*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])/(
2*d*e^4*(c + d*x)^2) - (a^2*b*ArcSinh[c + d*x])/(d*e^4*(c + d*x)^3) - (a^2
*b*Log[c + d*x])/(2*d*e^4) + (a^2*b*Log[1 + Sqrt[1 + c^2 + 2*c*d*x + d^2*x
^2]])/(2*d*e^4) + (a*b^2*(-8*PolyLog[2, -E^(-ArcSinh[c + d*x])] - (2*(-2 +
 4*ArcSinh[c + d*x]^2 + 2*Cosh[2*ArcSinh[c + d*x]] - 3*(c + d*x)*ArcSinh[c
 + d*x]*Log[1 - E^(-ArcSinh[c + d*x])] + 3*(c + d*x)*ArcSinh[c + d*x]*Log[
1 + E^(-ArcSinh[c + d*x])] - 4*(c + d*x)^3*PolyLog[2, E^(-ArcSinh[c + d*x]
)] + 2*ArcSinh[c + d*x]*Sinh[2*ArcSinh[c + d*x]] + ArcSinh[c + d*x]*Log[1
- E^(-ArcSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]] - ArcSinh[c + d*x]*Log[1
 + E^(-ArcSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]]))/(c + d*x)^3))/(8*d*e^
4) + (b^3*(-24*ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2] + 4*ArcSinh[c + d
*x]^3*Coth[ArcSinh[c + d*x]/2] - 6*ArcSinh[c + d*x]^2*Csch[ArcSinh[c + d*x
]/2]^2 - (c + d*x)*ArcSinh[c + d*x]^3*Csch[ArcSinh[c + d*x]/2]^4 - 24*ArcS
inh[c + d*x]^2*Log[1 - E^(-ArcSinh[c + d*x])] + 24*ArcSinh[c + d*x]^2*Log[
1 + E^(-ArcSinh[c + d*x])] + 48*Log[Tanh[ArcSinh[c + d*x]/2]] - 48*ArcSinh
[c + d*x]*PolyLog[2, -E^(-ArcSinh[c + d*x])] + 48*ArcSinh[c + d*x]*PolyLog
[2, E^(-ArcSinh[c + d*x])] - 48*PolyLog[3, -E^(-ArcSinh[c + d*x])] + 48*Po
lyLog[3, E^(-ArcSinh[c + d*x])] - 6*ArcSinh[c + d*x]^2*Sech[ArcSinh[c + d*
x]/2]^2 - (16*ArcSinh[c + d*x]^3*Sinh[ArcSinh[c + d*x]/2]^4)/(c + d*x)^3 +
 24*ArcSinh[c + d*x]*Tanh[ArcSinh[c + d*x]/2] - 4*ArcSinh[c + d*x]^3*Ta...
```

3.145.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.85, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {6274, 27, 6191, 6224, 6191, 243, 73, 220, 6231, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^4}dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^3}{e^4(c+dx)^4}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(c+dx)^4}d(c+dx)}{de^4}$$

$\Big\downarrow$ 6191

$$\frac{b\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(c+dx)^3\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{(a+b\text{arcsinh}(c+dx))^3}{3(c+dx)^3}}{de^4}$$

$\Big\downarrow$ 6224

$$\frac{b\left(b\int \frac{a+b\text{arcsinh}(c+dx)}{(c+dx)^2}d(c+dx)-\frac{1}{2}\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(c+dx)\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}{2(c+dx)^2}\right)-\frac{(a+b\text{arcsin}}{3(c+}}{de^4}$$

$\Big\downarrow$ 6191

$$\frac{b\left(b\left(b\int \frac{1}{(c+dx)\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{a+b\text{arcsinh}(c+dx)}{c+dx}\right)-\frac{1}{2}\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(c+dx)\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsi}}{2(c+dx)^2}}{de^4}$$

$\Big\downarrow$ 243

$$\frac{b\left(b\left(\frac{1}{2}b\int \frac{1}{(c+dx)^2\sqrt{(c+dx)^2+1}}d(c+dx)^2-\frac{a+b\text{arcsinh}(c+dx)}{c+dx}\right)-\frac{1}{2}\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(c+dx)\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{\sqrt{(c+dx)^2+1}(a+b\text{ar}}{2(c+dx}}{de^4}$$

$\Big\downarrow$ 73

$$\frac{b\left(b\left(b\int \frac{1}{(c+dx)^4-1}d\sqrt{(c+dx)^2+1}-\frac{a+b\text{arcsinh}(c+dx)}{c+dx}\right)-\frac{1}{2}\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(c+dx)\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsi}}{2(c+dx)^2}}{de^4}$$

$\Big\downarrow$ 220

$$\frac{b\left(-\frac{1}{2}\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(c+dx)\sqrt{(c+dx)^2+1}}d(c+dx)+b\left(-\frac{a+b\text{arcsinh}(c+dx)}{c+dx}-b\text{arctanh}\left(\sqrt{(c+dx)^2+1}\right)\right)-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcs}}{2(c+dx)^2}}{de^4}$$

$\Big\downarrow$ 6231

$$\frac{b\left(-\frac{1}{2}\int \frac{(a+b\text{arcsinh}(c+dx))^2}{c+dx}d\text{arcsinh}(c+dx)+b\left(-\frac{a+b\text{arcsinh}(c+dx)}{c+dx}-b\text{arctanh}\left(\sqrt{(c+dx)^2+1}\right)\right)-\frac{\sqrt{(c+dx)^2+1}(}{}}{de^4}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^3}{3(c+dx)^3}+b\left(-\frac{1}{2}\int i(a+b\text{arcsinh}(c+dx))^2\csc(i\text{arcsinh}(c+dx))d\text{arcsinh}(c+dx)+b\left(-\frac{a+b\text{arcsinh}(}{c+dx}\right.\right.}{de^4}$$

$\Big\downarrow$ 26

3.145. $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx$

$$-\frac{(a+b\text{arcsinh}(c+dx))^3}{3(c+dx)^3}+b\Big(-\frac{1}{2}i\int(a+b\text{arcsinh}(c+dx))^2\csc(i\text{arcsinh}(c+dx))d\text{arcsinh}(c+dx)+b\Big(-\frac{a+b\text{arcsinh}(c}{c+dx}$$
$$de^4$$

$\downarrow$ 4670

$$-\frac{(a+b\text{arcsinh}(c+dx))^3}{3(c+dx)^3}+b\Big(-\frac{1}{2}i\big(2ib\int(a+b\text{arcsinh}(c+dx))\log\left(1-e^{\text{arcsinh}(c+dx)}\right)d\text{arcsinh}(c+dx)-2ib\int(a+ba$$

$\downarrow$ 3011

$$-\frac{(a+b\text{arcsinh}(c+dx))^3}{3(c+dx)^3}+b\Big(-\frac{1}{2}i\big(-2ib\big(b\int\text{PolyLog}\left(2,-e^{\text{arcsinh}(c+dx)}\right)d\text{arcsinh}(c+dx)-\text{PolyLog}\left(2,-e^{\text{arcsinh}(c+dx}$$

$\downarrow$ 2720

$$-\frac{(a+b\text{arcsinh}(c+dx))^3}{3(c+dx)^3}+b\Big(-\frac{1}{2}i\big(2ib\big(b\int e^{-\text{arcsinh}(c+dx)}\text{PolyLog}\left(2,e^{\text{arcsinh}(c+dx)}\right)de^{\text{arcsinh}(c+dx)}-\text{PolyLog}\left(2,e^{\text{arcsin}}$$

$\downarrow$ 7143

$$-\frac{(a+b\text{arcsinh}(c+dx))^3}{3(c+dx)^3}+b\Big(-\frac{1}{2}i\big(2i\text{arctanh}\left(e^{\text{arcsinh}(c+dx)}\right)(a+b\text{arcsinh}(c+dx))^2+2ib\big(b\,\text{PolyLog}\left(3,e^{\text{arcsinh}(c+dx)}\right)$$

input
```
Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

output
```
(-1/3*(a + b*ArcSinh[c + d*x])^3/(c + d*x)^3 + b*(-1/2*(Sqrt[1 + (c + d*x)
^2]*(a + b*ArcSinh[c + d*x])^2)/(c + d*x)^2 + b*(-((a + b*ArcSinh[c + d*x]
)/(c + d*x)) - b*ArcTanh[Sqrt[1 + (c + d*x)^2]]) - (I/2)*((2*I)*(a + b*Arc
Sinh[c + d*x])^2*ArcTanh[E^ArcSinh[c + d*x]] + (2*I)*b*(-((a + b*ArcSinh[c
 + d*x])*PolyLog[2, E^ArcSinh[c + d*x]]) + b*PolyLog[3, E^ArcSinh[c + d*x]
]) - (2*I)*b*(-((a + b*ArcSinh[c + d*x])*PolyLog[2, -E^ArcSinh[c + d*x]])
+ b*PolyLog[3, -c - d*x])))))/(d*e^4)
```

3.145. $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx$

3.145.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.145. $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx$

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.145. $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx$

3.145.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.86

method	result
derivativedivides	$-\frac{a^3}{3e^4(dx+c)^3}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)\left(3\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)+2\operatorname{arcsinh}(dx+c)^2+6(dx+c)^2\right)}{6(dx+c)^3}+\frac{\operatorname{arcsinh}(dx+c)^2\ln\left(1+d\right.}{2}\right.}{}$
default	$-\frac{a^3}{3e^4(dx+c)^3}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)\left(3\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)+2\operatorname{arcsinh}(dx+c)^2+6(dx+c)^2\right)}{6(dx+c)^3}+\frac{\operatorname{arcsinh}(dx+c)^2\ln\left(1+d\right.}{2}\right.}{}$
parts	$-\frac{a^3}{3e^4(dx+c)^3d}+\frac{b^3\left(-\frac{\operatorname{arcsinh}(dx+c)\left(3\sqrt{1+(dx+c)^2}\,(dx+c)\operatorname{arcsinh}(dx+c)+2\operatorname{arcsinh}(dx+c)^2+6(dx+c)^2\right)}{6(dx+c)^3}+\frac{\operatorname{arcsinh}(dx+c)^2\,l}{}\right.}{}$

input
```
int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a^3/e^4/(d*x+c)^3+b^3/e^4*(-1/6/(d*x+c)^3*arcsinh(d*x+c)*(3*(1+(
d*x+c)^2)^(1/2)*(d*x+c)*arcsinh(d*x+c)+2*arcsinh(d*x+c)^2+6*(d*x+c)^2)+1/2
*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+arcsinh(d*x+c)*polylog(2
,-d*x-c-(1+(d*x+c)^2)^(1/2))-polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))-1/2*arc
sinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-arcsinh(d*x+c)*polylog(2,d*x
+c+(1+(d*x+c)^2)^(1/2))+polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))-2*arctanh(d*x
+c+(1+(d*x+c)^2)^(1/2)))+3*a*b^2/e^4*(-1/3*((1+(d*x+c)^2)^(1/2)*(d*x+c)*ar
csinh(d*x+c)+arcsinh(d*x+c)^2+(d*x+c)^2)/(d*x+c)^3+1/3*arcsinh(d*x+c)*ln(1
+d*x+c+(1+(d*x+c)^2)^(1/2))+1/3*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-1/3*
arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-1/3*polylog(2,d*x+c+(1+(d*x
+c)^2)^(1/2)))+3*a^2*b/e^4*(-1/3/(d*x+c)^3*arcsinh(d*x+c)-1/6/(d*x+c)^2*(1
+(d*x+c)^2)^(1/2)+1/6*arctanh(1/(1+(d*x+c)^2)^(1/2))))
```

3.145.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")
```

output
```
integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*ar
csinh(d*x + c) + a^3)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 +
 4*c^3*d*e^4*x + c^4*e^4), x)
```

3.145.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx$$
$$=\frac{\int \frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{b^3\,\text{asinh}^3\,(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{3ab^2\,\text{asinh}^2\,(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int}{e^4}$$

input
```
integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**4,x)
```

output
```
(Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**
4*x**4), x) + Integral(b**3*asinh(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*
d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*asinh(c + d
*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),
 x) + Integral(3*a**2*b*asinh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x*
*2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

3.145.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")
```

output
```
-1/3*b^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/(d^4*e^4*x^3 +
 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^3/(d^4*e^4*x^3 + 3
*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(((3*(c^3 + c)*a*
b^2 + (c^3 + c)*b^3 + (3*a*b^2*d^3 + b^3*d^3)*x^3 + 3*(3*a*b^2*c*d^2 + b^3
*c*d^2)*x^2 + (3*(3*c^2*d + d)*a*b^2 + (3*c^2*d + d)*b^3)*x + (b^3*c^2 + 3
*(c^2 + 1)*a*b^2 + (3*a*b^2*d^2 + b^3*d^2)*x^2 + 2*(3*a*b^2*c*d + b^3*c*d)
*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1))^2 + 3*(a^2*b*d^3*x^3 + 3*a^2*b*c*d^2*x^2 + (3*c^2*d + d)*a^2
*b*x + (c^3 + c)*a^2*b + (a^2*b*d^2*x^2 + 2*a^2*b*c*d*x + (c^2 + 1)*a^2*b)
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)))/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 + c^5*e^4 + (21*c^2*d
^5*e^4 + d^5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 + c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e
^4 + 2*c^2*d^3*e^4)*x^3 + (21*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*x^2 + (7*c^6*d
*e^4 + 5*c^4*d*e^4)*x + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4
 + (15*c^2*d^4*e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*
(5*c^4*d^2*e^4 + 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x)*sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

3.145.8 Giac [F]

$$\int \frac{(a + b\text{arcsinh}(c + dx))^3}{(ce + dex)^4} \, dx = \int \frac{(b \operatorname{arsinh}(dx + c) + a)^3}{(dex + ce)^4} \, dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^4, x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^4}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3}{{\left(c\,e+d\,e\,x\right)}^4}\,dx$$

input
```
int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^4,x)
```

output
```
int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^4, x)
```

3.146 $\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx$

3.146.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx=\frac{(e(c+dx))^{1+m}(a+b\text{arcsinh}(c+dx))^4}{de(1+m)}-\frac{4b\text{Int}\left(\frac{(e(c+dx))^{1+m}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{1+(c+dx)^2}},x\right)}{e(1+m)}$$

output
```
(e*(d*x+c))^(1+m)*(a+b*arcsinh(d*x+c))^4/d/e/(1+m)-4*b*Unintegrable((e*(d*
x+c))^(1+m)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e/(1+m)
```

3.146.2 Mathematica [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx=\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx$$

input
```
Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^4, x]
```

3.146.3 Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int (e(c+dx))^m(a+b\text{arcsinh}(c+dx))^4d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{(e(c+dx))^{m+1}(a+b\text{arcsinh}(c+dx))^4}{e(m+1)}-\frac{4b\int \frac{(e(c+dx))^{m+1}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{(c+dx)^2+1}}d(c+dx)}{e(m+1)}}{d}$$

$$\Big\downarrow \text{6239}$$

$$\frac{\frac{(e(c+dx))^{m+1}(a+b\text{arcsinh}(c+dx))^4}{e(m+1)}-\frac{4b\int \frac{(e(c+dx))^{m+1}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{(c+dx)^2+1}}d(c+dx)}{e(m+1)}}{d}$$

input
```
Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
$Aborted
```

3.146.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.146.4 Maple [N/A] (verified)

Not integrable

Time = 1.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (dex + ce)^m (a + b \operatorname{arcsinh}(dx + c))^4 \, dx$$

input
```
int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x)
```

output
```
int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x)
```

3.146.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int (ce + dex)^m (a + b\operatorname{arcsinh}(c + dx))^4 \, dx = \int (b \operatorname{arsinh}(dx + c) + a)^4 (dex + ce)^m \, dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*
arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*(d*e*x + c*e)^m, x)
```

3.146.6 Sympy [N/A]

Not integrable

Time = 35.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx=\int (e(c+dx))^m\,(a+b\operatorname{asinh}{(c+dx)})^4\,dx$$

input
```
integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c))**4,x)
```

output
```
Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x))**4, x)
```

3.146.7 Maxima [N/A]

Not integrable

Time = 5.24 (sec) , antiderivative size = 940, normalized size of antiderivative = 40.87

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx=\int (b\,\text{arsinh}\,(dx+c)+a)^4(dex+ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
(b^4*d*e^m*x + b^4*c*e^m)*(d*x + c)^m*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))^4/(d*(m + 1)) + (d*e*x + c*e)^(m + 1)*a^4/(d*e*(m + 1)) + int
egrate(-2*(2*((b^4*c^2*e^m - (c^2*e^m*(m + 1) + e^m*(m + 1))*a*b^3 - (a*b^
3*d^2*e^m*(m + 1) - b^4*d^2*e^m)*x^2 - 2*(a*b^3*c*d*e^m*(m + 1) - b^4*c*d*
e^m)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m - ((c^3*e^m*(m + 1)
+ c*e^m*(m + 1))*a*b^3 - (c^3*e^m + c*e^m)*b^4 + (a*b^3*d^3*e^m*(m + 1) -
b^4*d^3*e^m)*x^3 + 3*(a*b^3*c*d^2*e^m*(m + 1) - b^4*c*d^2*e^m)*x^2 + ((3*c
^2*d*e^m*(m + 1) + d*e^m*(m + 1))*a*b^3 - (3*c^2*d*e^m + d*e^m)*b^4)*x)*(d
*x + c)^m)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 - 3*((a^2*b^
2*d^2*e^m*(m + 1)*x^2 + 2*a^2*b^2*c*d*e^m*(m + 1)*x + (c^2*e^m*(m + 1) + e
^m*(m + 1))*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m + (a^2*
b^2*d^3*e^m*(m + 1)*x^3 + 3*a^2*b^2*c*d^2*e^m*(m + 1)*x^2 + (3*c^2*d*e^m*(
m + 1) + d*e^m*(m + 1))*a^2*b^2*x + (c^3*e^m*(m + 1) + c*e^m*(m + 1))*a^2*
b^2)*(d*x + c)^m)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - 2*(
(a^3*b*d^2*e^m*(m + 1)*x^2 + 2*a^3*b*c*d*e^m*(m + 1)*x + (c^2*e^m*(m + 1)
+ e^m*(m + 1))*a^3*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m + (a^3
*b*d^3*e^m*(m + 1)*x^3 + 3*a^3*b*c*d^2*e^m*(m + 1)*x^2 + (3*c^2*d*e^m*(m +
 1) + d*e^m*(m + 1))*a^3*b*x + (c^3*e^m*(m + 1) + c*e^m*(m + 1))*a^3*b)*(d
*x + c)^m)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^3*(m + 1)*
x^3 + 3*c*d^2*(m + 1)*x^2 + c^3*(m + 1) + c*(m + 1) + (3*c^2*d*(m + 1) ...
```

3.146.8 Giac [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx=\int (b\,\text{arsinh}\,(dx+c)+a)^4(dex+ce)^m\,dx$$

input
```
integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^4*(d*e*x + c*e)^m, x)
```

3.146.9 Mupad [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (ce+dex)^m(a+b\text{arcsinh}(c+dx))^4\,dx=\int (c\,e+d\,e\,x)^m\,(a+b\,\text{asinh}(c+d\,x))^4\,dx$$

input
```
int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^4, x)
```

3.147 $\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^4\,dx$

3.147.1 Optimal result

Integrand size = 23, antiderivative size = 349

$$\begin{aligned}
&\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^4\,dx \\
&= -\frac{45b^4e^3(c+dx)^2}{128d}+\frac{3b^4e^3(c+dx)^4}{128d} \\
&\quad+\frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{64d} \\
&\quad-\frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))}{32d}-\frac{45b^2e^3(a+b\operatorname{arcsinh}(c+dx))^2}{128d} \\
&\quad-\frac{9b^2e^3(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^2}{16d}+\frac{3b^2e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^2}{16d} \\
&\quad+\frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{8d} \\
&\quad-\frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{4d} \\
&\quad-\frac{3e^3(a+b\operatorname{arcsinh}(c+dx))^4}{32d}+\frac{e^3(c+dx)^4(a+b\operatorname{arcsinh}(c+dx))^4}{4d}
\end{aligned}$$

output
```
-45/128*b^4*e^3*(d*x+c)^2/d+3/128*b^4*e^3*(d*x+c)^4/d-45/128*b^2*e^3*(a+b*
arcsinh(d*x+c))^2/d-9/16*b^2*e^3*(d*x+c)^2*(a+b*arcsinh(d*x+c))^2/d+3/16*b
^2*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^2/d-3/32*e^3*(a+b*arcsinh(d*x+c))^4/
d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^4/d+45/64*b^3*e^3*(d*x+c)*(a+b*ar
csinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d-3/32*b^3*e^3*(d*x+c)^3*(a+b*arcsinh(d*
x+c))*(1+(d*x+c)^2)^(1/2)/d+3/8*b*e^3*(d*x+c)*(a+b*arcsinh(d*x+c))^3*(1+(d
*x+c)^2)^(1/2)/d-1/4*b*e^3*(d*x+c)^3*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2)^
(1/2)/d
```

3.147.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.36

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^4\,dx$$
$$=\frac{e^3\left(-9b^2(8a^2+5b^2)(c+dx)^2+(32a^4+24a^2b^2+3b^4)(c+dx)^4+2ab(c+dx)\sqrt{1+(c+dx)^2}(24a^2+45\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
(e^3*(-9*b^2*(8*a^2 + 5*b^2)*(c + d*x)^2 + (32*a^4 + 24*a^2*b^2 + 3*b^4)*(
c + d*x)^4 + 2*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(24*a^2 + 45*b^2 - 2*(8
*a^2 + 3*b^2)*(c + d*x)^2) - 6*a*b*(8*a^2 + 15*b^2)*ArcSinh[c + d*x] + 2*b
*(c + d*x)*(-72*a*b^2*(c + d*x) + 64*a^3*(c + d*x)^3 + 24*a*b^2*(c + d*x)^
3 + 72*a^2*b*Sqrt[1 + (c + d*x)^2] + 45*b^3*Sqrt[1 + (c + d*x)^2] - 48*a^2
*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] - 6*b^3*(c + d*x)^2*Sqrt[1 + (c + d*x
)^2])*ArcSinh[c + d*x] + 3*b^2*(-24*a^2 - 15*b^2 - 24*b^2*(c + d*x)^2 + 64
*a^2*(c + d*x)^4 + 8*b^2*(c + d*x)^4 + 48*a*b*(c + d*x)*Sqrt[1 + (c + d*x)
^2] - 32*a*b*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 + 16*b^
3*(-3*a + 8*a*(c + d*x)^4 + 3*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 2*b*(c +
 d*x)^3*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^3 + 4*b^4*(-3 + 8*(c + d*x
)^4)*ArcSinh[c + d*x]^4))/(128*d)
```

3.147.3 Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {6274, 27, 6191, 6227, 6191, 6227, 15, 6191, 6198, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^4\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int e^3(c+dx)^3(a+b\text{arcsinh}(c+dx))^4d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^3\int (c+dx)^3(a+b\text{arcsinh}(c+dx))^4d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^4-b\int \frac{(c+dx)^4(a+b\text{arcsinh}(c+dx))^3}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6227}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^4-b\left(-\frac{3}{4}b\int (c+dx)^3(a+b\text{arcsinh}(c+dx))^2d(c+dx)-\frac{3}{4}\int \frac{(c+dx)^2(a+b\text{arcs}}{\sqrt{(c+dx)}}\right.\right.}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^4-b\left(-\frac{3}{4}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^2-\frac{1}{2}b\int \frac{(c+dx)^4(a+b\text{arcsinh}(c+dx}{\sqrt{(c+dx)^2+1}}\right.\right.\right.}{d}$$

$$\Big\downarrow \text{6227}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^4-b\left(-\frac{3}{4}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^2-\frac{1}{2}b\left(-\frac{3}{4}\int \frac{(c+dx)^2(a+b\text{arcsinh}}{\sqrt{(c+dx)^2+1}}\right.\right.\right.\right.}{}$$

$$\Big\downarrow \text{15}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^4-b\left(-\frac{3}{4}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^2-\frac{1}{2}b\left(-\frac{3}{4}\int \frac{(c+dx)^2(a+b\text{arcsinh}}{\sqrt{(c+dx)^2+1}}\right.\right.\right.\right.}{}$$

$$\Big\downarrow \text{6191}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^4-b\left(-\frac{3}{4}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^2-\frac{1}{2}b\left(-\frac{3}{4}\int\frac{(c+dx)^2(a+b\text{arcsinh}}{\sqrt{(c+dx)^2+1}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{6198}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^4-b\left(-\frac{3}{4}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^2-\frac{1}{2}b\left(-\frac{3}{4}\int\frac{(c+dx)^2(a+b\text{arcsinh}}{\sqrt{(c+dx)^2+1}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{6227}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^4-b\left(-\frac{3}{4}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^2-\frac{1}{2}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{a+b\text{arcsinh}(c+}{\sqrt{(c+dx)^2+1}}\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{15}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^4-b\left(-\frac{3}{4}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^2-\frac{1}{2}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{a+b\text{arcsinh}(c+}{\sqrt{(c+dx)^2+1}}\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{6198}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^4-b\left(\frac{1}{4}(c+dx)^3\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3-\frac{3}{4}b\left(\frac{1}{4}(c+dx)^4(a\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
(e^3*(((c + d*x)^4*(a + b*ArcSinh[c + d*x])^4)/4 - b*(((c + d*x)^3*Sqrt[1
+ (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^3)/4 - (3*b*(((c + d*x)^4*(a + b*A
rcSinh[c + d*x])^2)/4 - (b*(-1/16*(b*(c + d*x)^4) + ((c + d*x)^3*Sqrt[1 +
(c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/4 - (3*(-1/4*(b*(c + d*x)^2) + ((c
+ d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/2 - (a + b*ArcSinh[
c + d*x])^2/(4*b)))/4))/2))/4 - (3*(((c + d*x)*Sqrt[1 + (c + d*x)^2]*(a +
b*ArcSinh[c + d*x])^3)/2 - (a + b*ArcSinh[c + d*x])^4/(8*b) - (3*b*(((c +
d*x)^2*(a + b*ArcSinh[c + d*x])^2)/2 - b*(-1/4*(b*(c + d*x)^2) + ((c + d*x
)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/2 - (a + b*ArcSinh[c + d
*x])^2/(4*b))))/2))/4)))/d
```

3.147. $\int(ce+dex)^3(a+b\text{arcsinh}(c+dx))^4\,dx$

3.147.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.147.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.64

method	result
derivativedivides	$\frac{\frac{e^3a^4(dx+c)^4}{4}+e^3b^4\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^4}{4}-\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{4}+\frac{3(dx+c)\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{8}-\frac{3}{}\right.}{}$
default	$\frac{\frac{e^3a^4(dx+c)^4}{4}+e^3b^4\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^4}{4}-\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{4}+\frac{3(dx+c)\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{8}-\frac{3}{}\right.}{}$
parts	$\frac{e^3a^4(dx+c)^4}{4d}+\frac{e^3b^4\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)^4}{4}-\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{4}+\frac{3(dx+c)\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{8}\right.}{}$

input
```
int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*e^3*a^4*(d*x+c)^4+e^3*b^4*(1/4*(d*x+c)^4*arcsinh(d*x+c)^4-1/4*(d*
x+c)^3*arcsinh(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+3/8*(d*x+c)*arcsinh(d*x+c)^3*(
1+(d*x+c)^2)^(1/2)-3/32*arcsinh(d*x+c)^4+3/16*(d*x+c)^4*arcsinh(d*x+c)^2-3
/32*(d*x+c)^3*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+45/64*(1+(d*x+c)^2)^(1/2)
*(d*x+c)*arcsinh(d*x+c)+27/128*arcsinh(d*x+c)^2+3/128*(d*x+c)^4-45/128*(d*
x+c)^2-45/128-9/16*arcsinh(d*x+c)^2*(1+(d*x+c)^2))+4*e^3*a*b^3*(1/4*(d*x+c
)^4*arcsinh(d*x+c)^3-3/16*(d*x+c)^3*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+9
/32*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(d*x+c)-3/32*arcsinh(d*x+c)^3+3/3
2*(d*x+c)^4*arcsinh(d*x+c)-3/128*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+45/256*(d*x
+c)*(1+(d*x+c)^2)^(1/2)+27/256*arcsinh(d*x+c)-9/32*(1+(d*x+c)^2)*arcsinh(d
*x+c))+6*e^3*a^2*b^2*(1/4*(d*x+c)^4*arcsinh(d*x+c)^2-1/8*(d*x+c)^3*arcsinh
(d*x+c)*(1+(d*x+c)^2)^(1/2)+3/16*(1+(d*x+c)^2)^(1/2)*(d*x+c)*arcsinh(d*x+c
)-3/32*arcsinh(d*x+c)^2+1/32*(d*x+c)^4-3/32*(d*x+c)^2-3/32)+4*e^3*b*a^3*(1
/4*(d*x+c)^4*arcsinh(d*x+c)-1/16*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+3/32*(d*x+c
)*(1+(d*x+c)^2)^(1/2)-3/32*arcsinh(d*x+c)))
```

3.147.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1241 vs. 2(319) = 638.

Time = 0.29 (sec) , antiderivative size = 1241, normalized size of antiderivative = 3.56

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/128*((32*a^4 + 24*a^2*b^2 + 3*b^4)*d^4*e^3*x^4 + 4*(32*a^4 + 24*a^2*b^2
+ 3*b^4)*c*d^3*e^3*x^3 - 3*(24*a^2*b^2 + 15*b^4 - 2*(32*a^4 + 24*a^2*b^2 +
 3*b^4)*c^2)*d^2*e^3*x^2 + 2*(2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^3 - 9*(8*a
^2*b^2 + 5*b^4)*c)*d*e^3*x + 4*(8*b^4*d^4*e^3*x^4 + 32*b^4*c*d^3*e^3*x^3 +
 48*b^4*c^2*d^2*e^3*x^2 + 32*b^4*c^3*d*e^3*x + (8*b^4*c^4 - 3*b^4)*e^3)*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + 16*(8*a*b^3*d^4*e^3*x^4
 + 32*a*b^3*c*d^3*e^3*x^3 + 48*a*b^3*c^2*d^2*e^3*x^2 + 32*a*b^3*c^3*d*e^3*
x + (8*a*b^3*c^4 - 3*a*b^3)*e^3 - (2*b^4*d^3*e^3*x^3 + 6*b^4*c*d^2*e^3*x^2
 + 3*(2*b^4*c^2 - b^4)*d*e^3*x + (2*b^4*c^3 - 3*b^4*c)*e^3)*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 +
3*(8*(8*a^2*b^2 + b^4)*d^4*e^3*x^4 + 32*(8*a^2*b^2 + b^4)*c*d^3*e^3*x^3 -
24*(b^4 - 2*(8*a^2*b^2 + b^4)*c^2)*d^2*e^3*x^2 - 16*(3*b^4*c - 2*(8*a^2*b^
2 + b^4)*c^3)*d*e^3*x - (24*b^4*c^2 - 8*(8*a^2*b^2 + b^4)*c^4 + 24*a^2*b^2
 + 15*b^4)*e^3 - 16*(2*a*b^3*d^3*e^3*x^3 + 6*a*b^3*c*d^2*e^3*x^2 + 3*(2*a*
b^3*c^2 - a*b^3)*d*e^3*x + (2*a*b^3*c^3 - 3*a*b^3*c)*e^3)*sqrt(d^2*x^2 + 2
*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 2*
(8*(8*a^3*b + 3*a*b^3)*d^4*e^3*x^4 + 32*(8*a^3*b + 3*a*b^3)*c*d^3*e^3*x^3
- 24*(3*a*b^3 - 2*(8*a^3*b + 3*a*b^3)*c^2)*d^2*e^3*x^2 - 16*(9*a*b^3*c - 2
*(8*a^3*b + 3*a*b^3)*c^3)*d*e^3*x - (72*a*b^3*c^2 - 8*(8*a^3*b + 3*a*b^3)*
c^4 + 24*a^3*b + 45*a*b^3)*e^3 - 3*(2*(8*a^2*b^2 + b^4)*d^3*e^3*x^3 + 6...
```

3.147.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2876 vs. 2(325) = 650.

Time = 1.12 (sec) , antiderivative size = 2876, normalized size of antiderivative = 8.24

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**4,x)
```

output
```
Piecewise((a**4*c**3*e**3*x + 3*a**4*c**2*d*e**3*x**2/2 + a**4*c*d**2*e**3
*x**3 + a**4*d**3*e**3*x**4/4 + a**3*b*c**4*e**3*asinh(c + d*x)/d + 4*a**3
*b*c**3*e**3*x*asinh(c + d*x) - a**3*b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**
2*x**2 + 1)/(4*d) + 6*a**3*b*c**2*d*e**3*x**2*asinh(c + d*x) - 3*a**3*b*c*
*2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/4 + 4*a**3*b*c*d**2*e**3*x*
*3*asinh(c + d*x) - 3*a**3*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/4 + 3*a**3*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(8*d) + a**
3*b*d**3*e**3*x**4*asinh(c + d*x) - a**3*b*d**2*e**3*x**3*sqrt(c**2 + 2*c*
d*x + d**2*x**2 + 1)/4 + 3*a**3*b*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)/8 - 3*a**3*b*e**3*asinh(c + d*x)/(8*d) + 3*a**2*b**2*c**4*e**3*asinh(c
 + d*x)**2/(2*d) + 6*a**2*b**2*c**3*e**3*x*asinh(c + d*x)**2 + 3*a**2*b**2
*c**3*e**3*x/4 - 3*a**2*b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1
)*asinh(c + d*x)/(4*d) + 9*a**2*b**2*c**2*d*e**3*x**2*asinh(c + d*x)**2 +
9*a**2*b**2*c**2*d*e**3*x**2/8 - 9*a**2*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d
*x + d**2*x**2 + 1)*asinh(c + d*x)/4 + 6*a**2*b**2*c*d**2*e**3*x**3*asinh(
c + d*x)**2 + 3*a**2*b**2*c*d**2*e**3*x**3/4 - 9*a**2*b**2*c*d*e**3*x**2*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/4 - 9*a**2*b**2*c*e**3*
x/8 + 9*a**2*b**2*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*
x)/(8*d) + 3*a**2*b**2*d**3*e**3*x**4*asinh(c + d*x)**2/2 + 3*a**2*b**2*d*
*3*e**3*x**4/16 - 3*a**2*b**2*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2...
```

3.147.7 Maxima [F]

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^4\,dx = \int (dex+ce)^3(b\,\text{arsinh}\,(dx+c)+a)^4\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/4*a^4*d^3*e^3*x^4 + a^4*c*d^2*e^3*x^3 + 3/2*a^4*c^2*d*e^3*x^2 + 3*(2*x^2
*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(
c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*a
rcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^3*b*c^2*d*e^3 + 2/3*(6*x^3*arcsinh(d*
x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2
*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4
*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^
2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a^3*b*c*d^2*e^
3 + 1/24*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x
^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(
2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/
sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c
^2 + 1)^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5
+ 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a^3*b*d^3*e^3 +
 a^4*c^3*e^3*x + 4*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^
3*b*c^3*e^3/d + 1/4*(b^4*d^3*e^3*x^4 + 4*b^4*c*d^2*e^3*x^3 + 6*b^4*c^2*...
```

3.147.8 Giac [F]

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^4\,dx=\int (dex+ce)^3(b\operatorname{arsinh}(dx+c)+a)^4\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^4, x)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^4\,dx=\int (c\,e+d\,e\,x)^3\,(a+b\,\mathrm{asinh}(c+d\,x))^4\,dx$$

input
```
int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^4, x)
```

3.148 $\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^4\,dx$

3.148.1 Optimal result

Integrand size = 23, antiderivative size = 281

$$\begin{aligned}
&\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^4\,dx \\
&= -\frac{160}{27}b^4e^2x+\frac{8b^4e^2(c+dx)^3}{81d}+\frac{160b^3e^2\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))}{27d} \\
&\quad -\frac{8b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))}{27d} \\
&\quad -\frac{8b^2e^2(c+dx)(a+b\text{arcsinh}(c+dx))^2}{3d}+\frac{4b^2e^2(c+dx)^3(a+b\text{arcsinh}(c+dx))^2}{9d} \\
&\quad +\frac{8be^2\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^3}{9d} \\
&\quad -\frac{4be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^3}{9d} \\
&\quad +\frac{e^2(c+dx)^3(a+b\text{arcsinh}(c+dx))^4}{3d}
\end{aligned}$$

output
```
-160/27*b^4*e^2*x+8/81*b^4*e^2*(d*x+c)^3/d-8/3*b^2*e^2*(d*x+c)*(a+b*arcsin
h(d*x+c))^2/d+4/9*b^2*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))^2/d+1/3*e^2*(d*x+
c)^3*(a+b*arcsinh(d*x+c))^4/d+160/27*b^3*e^2*(a+b*arcsinh(d*x+c))*(1+(d*x+
c)^2)^(1/2)/d-8/27*b^3*e^2*(d*x+c)^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1
/2)/d+8/9*b*e^2*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d-4/9*b*e^2*(d*
x+c)^2*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d
```

3.148.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.47

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^4\,dx$$
$$= \frac{e^2\left(-24b^2(9a^2+20b^2)(c+dx)+(27a^4+36a^2b^2+8b^4)(c+dx)^3+12ab\sqrt{1+(c+dx)^2}(6a^2+40b^2-(3\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
(e^2*(-24*b^2*(9*a^2 + 20*b^2)*(c + d*x) + (27*a^4 + 36*a^2*b^2 + 8*b^4)*(
c + d*x)^3 + 12*a*b*Sqrt[1 + (c + d*x)^2]*(6*a^2 + 40*b^2 - (3*a^2 + 2*b^2
)*(c + d*x)^2) + 12*b*(-36*a*b^2*(c + d*x) + 9*a^3*(c + d*x)^3 + 6*a*b^2*(
c + d*x)^3 + 18*a^2*b*Sqrt[1 + (c + d*x)^2] + 40*b^3*Sqrt[1 + (c + d*x)^2]
 - 9*a^2*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] - 2*b^3*(c + d*x)^2*Sqrt[1 +
(c + d*x)^2])*ArcSinh[c + d*x] + 18*b^2*(-12*b^2*(c + d*x) + 9*a^2*(c + d*
x)^3 + 2*b^2*(c + d*x)^3 + 12*a*b*Sqrt[1 + (c + d*x)^2] - 6*a*b*(c + d*x)^
2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 - 36*b^3*(-3*a*(c + d*x)^3 - 2
*b*Sqrt[1 + (c + d*x)^2] + b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c
+ d*x]^3 + 27*b^4*(c + d*x)^3*ArcSinh[c + d*x]^4))/(81*d)
```

3.148.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {6274, 27, 6191, 6227, 6191, 6213, 6187, 6213, 24, 6227, 15, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^4\,dx$$
$$\Big\downarrow \text{6274}$$
$$\frac{\int e^2(c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^4d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{e^2\int (c+dx)^2(a+b\operatorname{arcsinh}(c+dx))^4d(c+dx)}{d}$$

$\Big\downarrow$ 6191

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^4-\frac{4}{3}b\int\frac{(c+dx)^3(a+b\text{arcsinh}(c+dx))^3}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$\Big\downarrow$ 6227

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^4-\frac{4}{3}b\left(-b\int(c+dx)^2(a+b\text{arcsinh}(c+dx))^2d(c+dx)-\frac{2}{3}\int\frac{(c+dx)(a+b\text{arcsi}}{\sqrt{(c+dx)^2}}\right.\right.}{d}$$

$\Big\downarrow$ 6191

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^4-\frac{4}{3}b\left(-b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^2-\frac{2}{3}b\int\frac{(c+dx)^3(a+b\text{arcsinh}(c+dx}{\sqrt{(c+dx)^2+1}}\right.\right.\right.}{d}$$

$\Big\downarrow$ 6213

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^4-\frac{4}{3}b\left(-b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^2-\frac{2}{3}b\int\frac{(c+dx)^3(a+b\text{arcsinh}(c+dx}{\sqrt{(c+dx)^2+1}}\right.\right.\right.$$

$\Big\downarrow$ 6187

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^4-\frac{4}{3}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3-3b\left((c+dx)(a+b\text{arcs}\right.\right.\right.\right.$$

$\Big\downarrow$ 6213

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^4-\frac{4}{3}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3-3b\left((c+dx)(a+b\text{arcs}\right.\right.\right.\right.$$

$\Big\downarrow$ 24

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^4-\frac{4}{3}b\left(-b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^2-\frac{2}{3}b\int\frac{(c+dx)^3(a+b\text{arcsinh}(c+dx}{\sqrt{(c+dx)^2+1}}\right.\right.\right.$$

$\Big\downarrow$ 6227

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^4-\frac{4}{3}b\left(-b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^2-\frac{2}{3}b\left(-\frac{2}{3}\int\frac{(c+dx)(a+b\text{arcsinh}(}{\sqrt{(c+dx)^2+1}}\right.\right.\right.\right.$$

$\Big\downarrow$ 15

3.148. $\int(ce+dex)^2(a+b\text{arcsinh}(c+dx))^4\,dx$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^4-\frac{4}{3}b\left(-b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^2-\frac{2}{3}b\left(-\frac{2}{3}\int\frac{(c+dx)(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}\right.\right.\right.\right.$$

$\downarrow$ 6213

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^4-\frac{4}{3}b\left(-b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^2-\frac{2}{3}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+\right.\right.\right.\right.\right.$$

$\downarrow$ 24

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^4-\frac{4}{3}b\left(\frac{1}{3}(c+dx)^2\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3-b\left(\frac{1}{3}(c+dx)^3(a\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
(e^2*(((c + d*x)^3*(a + b*ArcSinh[c + d*x])^4)/3 - (4*b*(((c + d*x)^2*Sqrt
[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^3)/3 - b*(((c + d*x)^3*(a + b*A
rcSinh[c + d*x])^2)/3 - (2*b*(-1/9*(b*(c + d*x)^3) + ((c + d*x)^2*Sqrt[1 +
 (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/3 - (2*(-(b*(c + d*x)) + Sqrt[1 +
(c + d*x)^2]*(a + b*ArcSinh[c + d*x])))/3))/3) - (2*(Sqrt[1 + (c + d*x)^2]
*(a + b*ArcSinh[c + d*x])^3 - 3*b*((c + d*x)*(a + b*ArcSinh[c + d*x])^2 -
2*b*(-(b*(c + d*x)) + Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))))/3
))/3))/d
```

3.148.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6187

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6191

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6213

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6274

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.148. $\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^4\,dx$

3.148.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.68

method	result
derivativedivides	$\frac{\frac{e^2a^4(dx+c)^3}{3}+e^2b^4\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^4}{3}+\frac{8\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{9}-\frac{4(dx+c)^2\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{9}-\frac{8(dx+c)}{}\right.}{}$
default	$\frac{\frac{e^2a^4(dx+c)^3}{3}+e^2b^4\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^4}{3}+\frac{8\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{9}-\frac{4(dx+c)^2\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{9}-\frac{8(dx+c)}{}\right.}{}$
parts	$\frac{e^2a^4(dx+c)^3}{3d}+\frac{e^2b^4\left(\frac{(dx+c)^3\operatorname{arcsinh}(dx+c)^4}{3}+\frac{8\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{9}-\frac{4(dx+c)^2\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{9}-\frac{8(dx}{}\right.}{}$

input
```
int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*e^2*a^4*(d*x+c)^3+e^2*b^4*(1/3*(d*x+c)^3*arcsinh(d*x+c)^4+8/9*arc
sinh(d*x+c)^3*(1+(d*x+c)^2)^(1/2)-4/9*(d*x+c)^2*arcsinh(d*x+c)^3*(1+(d*x+c
)^2)^(1/2)-8/3*(d*x+c)*arcsinh(d*x+c)^2+160/27*arcsinh(d*x+c)*(1+(d*x+c)^2
)^(1/2)-160/27*d*x-160/27*c+4/9*(d*x+c)^3*arcsinh(d*x+c)^2-8/27*arcsinh(d*
x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)^2+8/81*(d*x+c)^3)+4*e^2*a*b^3*(1/3*(d*x+c
)^3*arcsinh(d*x+c)^3+2/3*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-1/3*(d*x+c)^
2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-4/3*(d*x+c)*arcsinh(d*x+c)+40/27*(1
+(d*x+c)^2)^(1/2)+2/9*(d*x+c)^3*arcsinh(d*x+c)-2/27*(d*x+c)^2*(1+(d*x+c)^2
)^(1/2))+6*e^2*a^2*b^2*(1/3*(d*x+c)^3*arcsinh(d*x+c)^2+4/9*arcsinh(d*x+c)*
(1+(d*x+c)^2)^(1/2)-2/9*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)^2-4/9*d
*x-4/9*c+2/27*(d*x+c)^3)+4*e^2*b*a^3*(1/3*(d*x+c)^3*arcsinh(d*x+c)-1/9*(d*
x+c)^2*(1+(d*x+c)^2)^(1/2)+2/9*(1+(d*x+c)^2)^(1/2)))
```

3.148.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(255) = 510$.

Time = 0.29 (sec) , antiderivative size = 900, normalized size of antiderivative = 3.20

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^4\,dx$$
$$=\frac{(27\,a^4+36\,a^2b^2+8\,b^4)d^3e^2x^3+3\,(27\,a^4+36\,a^2b^2+8\,b^4)cd^2e^2x^2-3\,(72\,a^2b^2+160\,b^4-(27\,a^4+36\,a^2b^2}{}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/81*((27*a^4 + 36*a^2*b^2 + 8*b^4)*d^3*e^2*x^3 + 3*(27*a^4 + 36*a^2*b^2 +
 8*b^4)*c*d^2*e^2*x^2 - 3*(72*a^2*b^2 + 160*b^4 - (27*a^4 + 36*a^2*b^2 + 8
*b^4)*c^2)*d*e^2*x + 27*(b^4*d^3*e^2*x^3 + 3*b^4*c*d^2*e^2*x^2 + 3*b^4*c^2
*d*e^2*x + b^4*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4
 + 36*(3*a*b^3*d^3*e^2*x^3 + 9*a*b^3*c*d^2*e^2*x^2 + 9*a*b^3*c^2*d*e^2*x +
 3*a*b^3*c^3*e^2 - (b^4*d^2*e^2*x^2 + 2*b^4*c*d*e^2*x + (b^4*c^2 - 2*b^4)*
e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))^3 + 18*((9*a^2*b^2 + 2*b^4)*d^3*e^2*x^3 + 3*(9*a^2*b^2 + 2*
b^4)*c*d^2*e^2*x^2 - 3*(4*b^4 - (9*a^2*b^2 + 2*b^4)*c^2)*d*e^2*x - (12*b^4
*c - (9*a^2*b^2 + 2*b^4)*c^3)*e^2 - 6*(a*b^3*d^2*e^2*x^2 + 2*a*b^3*c*d*e^2
*x + (a*b^3*c^2 - 2*a*b^3)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 12*(3*(3*a^3*b + 2*a*b^3)*d^
3*e^2*x^3 + 9*(3*a^3*b + 2*a*b^3)*c*d^2*e^2*x^2 - 9*(4*a*b^3 - (3*a^3*b +
2*a*b^3)*c^2)*d*e^2*x - 3*(12*a*b^3*c - (3*a^3*b + 2*a*b^3)*c^3)*e^2 - ((9
*a^2*b^2 + 2*b^4)*d^2*e^2*x^2 + 2*(9*a^2*b^2 + 2*b^4)*c*d*e^2*x - (18*a^2*
b^2 + 40*b^4 - (9*a^2*b^2 + 2*b^4)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 12*((3*a^3*b + 2*
a*b^3)*d^2*e^2*x^2 + 2*(3*a^3*b + 2*a*b^3)*c*d*e^2*x - (6*a^3*b + 40*a*b^3
 - (3*a^3*b + 2*a*b^3)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

3.148.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1889 vs. $2(264) = 528$.

Time = 0.70 (sec) , antiderivative size = 1889, normalized size of antiderivative = 6.72

$$\int (ce + dex)^2(a + b\text{arcsinh}(c + dx))^4\,dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**4,x)
```

output
```
Piecewise((a**4*c**2*e**2*x + a**4*c*d*e**2*x**2 + a**4*d**2*e**2*x**3/3 +
 4*a**3*b*c**3*e**2*asinh(c + d*x)/(3*d) + 4*a**3*b*c**2*e**2*x*asinh(c +
d*x) - 4*a**3*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d) + 4*a
**3*b*c*d*e**2*x**2*asinh(c + d*x) - 8*a**3*b*c*e**2*x*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 + 1)/9 + 4*a**3*b*d**2*e**2*x**3*asinh(c + d*x)/3 - 4*a**3*b*
d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9 + 8*a**3*b*e**2*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 + 1)/(9*d) + 2*a**2*b**2*c**3*e**2*asinh(c + d*x)
**2/d + 6*a**2*b**2*c**2*e**2*x*asinh(c + d*x)**2 + 4*a**2*b**2*c**2*e**2*
x/3 - 4*a**2*b**2*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
 d*x)/(3*d) + 6*a**2*b**2*c*d*e**2*x**2*asinh(c + d*x)**2 + 4*a**2*b**2*c*
d*e**2*x**2/3 - 8*a**2*b**2*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*
asinh(c + d*x)/3 + 2*a**2*b**2*d**2*e**2*x**3*asinh(c + d*x)**2 + 4*a**2*b
**2*d**2*e**2*x**3/9 - 4*a**2*b**2*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)*asinh(c + d*x)/3 - 8*a**2*b**2*e**2*x/3 + 8*a**2*b**2*e**2*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(3*d) + 4*a*b**3*c**3*e**2*
asinh(c + d*x)**3/(3*d) + 8*a*b**3*c**3*e**2*asinh(c + d*x)/(9*d) + 4*a*b*
*3*c**2*e**2*x*asinh(c + d*x)**3 + 8*a*b**3*c**2*e**2*x*asinh(c + d*x)/3 -
 4*a*b**3*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2
/(3*d) - 8*a*b**3*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(27*d) +
4*a*b**3*c*d*e**2*x**2*asinh(c + d*x)**3 + 8*a*b**3*c*d*e**2*x**2*asinh...
```

3.148.7 Maxima [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^4\,dx=\int (dex+ce)^2(b\,\text{arsinh}\,(dx+c)+a)^4\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/3*a^4*d^2*e^2*x^3 + a^4*c*d*e^2*x^2 + 2*(2*x^2*arcsinh(d*x + c) - d*(3*c
^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqr
t(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
*c/d^3))*a^3*b*c*d*e^2 + 2/9*(6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d
^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 +
 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))
/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a^3*b*d^2*e^2 + a^4*c^2*e^2*x + 4*((d*x +
c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^3*b*c^2*e^2/d + 1/3*(b^4*d^
2*e^2*x^3 + 3*b^4*c*d*e^2*x^2 + 3*b^4*c^2*e^2*x)*log(d*x + c + sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1))^4 + integrate(2/3*(2*((3*a*b^3*d^5*e^2 - b^4*d^5*e
^2)*x^5 + 3*(c^5*e^2 + c^3*e^2)*a*b^3 + 5*(3*a*b^3*c*d^4*e^2 - b^4*c*d^4*e
^2)*x^4 + (3*(10*c^2*d^3*e^2 + d^3*e^2)*a*b^3 - (10*c^2*d^3*e^2 + d^3*e^2)
*b^4)*x^3 + 3*((10*c^3*d^2*e^2 + 3*c*d^2*e^2)*a*b^3 - (3*c^3*d^2*e^2 + c*d
^2*e^2)*b^4)*x^2 + 3*((5*c^4*d*e^2 + 3*c^2*d*e^2)*a*b^3 - (c^4*d*e^2 + c^2
*d*e^2)*b^4)*x + (3*(c^4*e^2 + c^2*e^2)*a*b^3 + (3*a*b^3*d^4*e^2 - b^4*d^4
*e^2)*x^4 + 4*(3*a*b^3*c*d^3*e^2 - b^4*c*d^3*e^2)*x^3 - 3*(2*b^4*c^2*d^2*e
^2 - (6*c^2*d^2*e^2 + d^2*e^2)*a*b^3)*x^2 - 3*(b^4*c^3*d*e^2 - 2*(2*c^3...
```

3.148.8 Giac [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^4\,dx=\int (dex+ce)^2(b\,\text{arsinh}\,(dx+c)+a)^4\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^4, x)
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^4\,dx=\int (c\,e+d\,e\,x)^2\,(a+b\,\text{asinh}(c+d\,x))^4\,dx$$

input
```
int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^4, x)
```

3.149 $\int (ce+dex)(a+b\text{arcsinh}(c+dx))^4\,dx$

3.149.1 Optimal result

Integrand size = 21, antiderivative size = 195

$$\begin{aligned}\int (ce+dex)(a+b\text{arcsinh}(c+dx))^4\,dx &= \frac{3b^4e(c+dx)^2}{4d}-\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))}{2d}\\&+\frac{3b^2e(a+b\text{arcsinh}(c+dx))^2}{4d}+\frac{3b^2e(c+dx)^2(a+b\text{arcsinh}(c+dx))^2}{2d}\\&-\frac{be(c+dx)\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^3}{d}\\&+\frac{e(a+b\text{arcsinh}(c+dx))^4}{4d}+\frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^4}{2d}\end{aligned}$$

output
```
3/4*b^4*e*(d*x+c)^2/d+3/4*b^2*e*(a+b*arcsinh(d*x+c))^2/d+3/2*b^2*e*(d*x+c)
^2*(a+b*arcsinh(d*x+c))^2/d+1/4*e*(a+b*arcsinh(d*x+c))^4/d+1/2*e*(d*x+c)^2
*(a+b*arcsinh(d*x+c))^4/d-3/2*b^3*e*(d*x+c)*(a+b*arcsinh(d*x+c))*(1+(d*x+c
)^2)^(1/2)/d-b*e*(d*x+c)*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d
```

3.149.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.54

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4\,dx$$
$$= \frac{e\Big((2a^4+6a^2b^2+3b^4)(c+dx)^2-2ab(2a^2+3b^2)(c+dx)\sqrt{1+(c+dx)^2}+2ab(2a^2+3b^2)\operatorname{arcsinh}(c+d}{}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
(e*((2*a^4 + 6*a^2*b^2 + 3*b^4)*(c + d*x)^2 - 2*a*b*(2*a^2 + 3*b^2)*(c + d
*x)*Sqrt[1 + (c + d*x)^2] + 2*a*b*(2*a^2 + 3*b^2)*ArcSinh[c + d*x] - 2*b*(
c + d*x)*(-4*a^3*(c + d*x) - 6*a*b^2*(c + d*x) + 6*a^2*b*Sqrt[1 + (c + d*x
)^2] + 3*b^3*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 3*b^2*(2*a^2 + b^2
+ 4*a^2*(c + d*x)^2 + 2*b^2*(c + d*x)^2 - 4*a*b*(c + d*x)*Sqrt[1 + (c + d*
x)^2])*ArcSinh[c + d*x]^2 + 4*b^3*(a + 2*a*(c + d*x)^2 - b*(c + d*x)*Sqrt[
1 + (c + d*x)^2])*ArcSinh[c + d*x]^3 + b^4*(1 + 2*(c + d*x)^2)*ArcSinh[c +
 d*x]^4))/(4*d)
```

3.149.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6274, 27, 6191, 6227, 6191, 6198, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4\,dx$$
$$\Big\downarrow \; 6274$$
$$\frac{\int e(c+dx)(a+b\operatorname{arcsinh}(c+dx))^4d(c+dx)}{d}$$
$$\Big\downarrow \; 27$$
$$\frac{e\int (c+dx)(a+b\operatorname{arcsinh}(c+dx))^4d(c+dx)}{d}$$
$$\Big\downarrow \; 6191$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^4-2b\int \frac{(c+dx)^2(a+b\text{arcsinh}(c+dx))^3}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$\Big\downarrow$ 6227

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^4-2b\left(-\frac{3}{2}b\int (c+dx)(a+b\text{arcsinh}(c+dx))^2d(c+dx)-\frac{1}{2}\int \frac{(a+b\text{arcsinh}(c+d}{\sqrt{(c+dx)^2+1}}\right.\right.}{d}$$

$\Big\downarrow$ 6191

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^4-2b\left(-\frac{3}{2}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^2-b\int \frac{(c+dx)^2(a+b\text{arcsinh}(c+dx)}{\sqrt{(c+dx)^2+1}}\right.\right.\right.}{d}$$

$\Big\downarrow$ 6198

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^4-2b\left(-\frac{3}{2}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^2-b\int \frac{(c+dx)^2(a+b\text{arcsinh}(c+dx)}{\sqrt{(c+dx)^2+1}}\right.\right.\right.}{d}$$

$\Big\downarrow$ 6227

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^4-2b\left(-\frac{3}{2}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^2-b\left(-\frac{1}{2}\int \frac{a+b\text{arcsinh}(c+dx)}{\sqrt{(c+dx)^2+1}}d(\right.\right.\right.\right.$$

$\Big\downarrow$ 15

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^4-2b\left(-\frac{3}{2}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^2-b\left(-\frac{1}{2}\int \frac{a+b\text{arcsinh}(c+dx)}{\sqrt{(c+dx)^2+1}}d(\right.\right.\right.\right.$$

$\Big\downarrow$ 6198

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^4-2b\left(-\frac{(a+b\text{arcsinh}(c+dx))^4}{8b}+\frac{1}{2}(c+dx)\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx\right.\right.$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
(e*(((c + d*x)^2*(a + b*ArcSinh[c + d*x])^4)/2 - 2*b*(((c + d*x)*Sqrt[1 +
(c + d*x)^2]*(a + b*ArcSinh[c + d*x])^3)/2 - (a + b*ArcSinh[c + d*x])^4/(8
*b) - (3*b*(((c + d*x)^2*(a + b*ArcSinh[c + d*x])^2)/2 - b*(-1/4*(b*(c + d
*x)^2) + ((c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/2 - (a
 + b*ArcSinh[c + d*x])^2/(4*b))))/2))/d
```

3.149. $\int (ce+dex)(a+b\text{arcsinh}(c+dx))^4\,dx$

3.149.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.149.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(179) = 358$.

Time = 0.08 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.90

method	result
derivativedivides	$\frac{\frac{e\,a^4(dx+c)^2}{2}+e\,b^4\left(\frac{\left(1+(dx+c)^2\right)\operatorname{arcsinh}(dx+c)^4}{2}-(dx+c)\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}-\frac{\operatorname{arcsinh}(dx+c)^4}{4}+\frac{3\operatorname{arcsinh}(dx+c)^2}{2}\right.}{}$
default	$\frac{\frac{e\,a^4(dx+c)^2}{2}+e\,b^4\left(\frac{\left(1+(dx+c)^2\right)\operatorname{arcsinh}(dx+c)^4}{2}-(dx+c)\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}-\frac{\operatorname{arcsinh}(dx+c)^4}{4}+\frac{3\operatorname{arcsinh}(dx+c)^2}{2}\right.}{}$
parts	$e\,a^4\left(\frac{1}{2}d\,x^2+cx\right)+\frac{e\,b^4\left(\frac{\left(1+(dx+c)^2\right)\operatorname{arcsinh}(dx+c)^4}{2}-(dx+c)\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}-\frac{\operatorname{arcsinh}(dx+c)^4}{4}+\frac{3\,a}{}\right.}{}$

input
```
int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*e*a^4*(d*x+c)^2+e*b^4*(1/2*(1+(d*x+c)^2)*arcsinh(d*x+c)^4-(d*x+c)
*arcsinh(d*x+c)^3*(1+(d*x+c)^2)^(1/2)-1/4*arcsinh(d*x+c)^4+3/2*arcsinh(d*x
+c)^2*(1+(d*x+c)^2)-3/2*(1+(d*x+c)^2)^(1/2)*(d*x+c)*arcsinh(d*x+c)-3/4*arc
sinh(d*x+c)^2+3/4*(d*x+c)^2+3/4)+4*e*a*b^3*(1/2*arcsinh(d*x+c)^3*(1+(d*x+c
)^2)-3/4*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(d*x+c)-1/4*arcsinh(d*x+c)^3
+3/4*(1+(d*x+c)^2)*arcsinh(d*x+c)-3/8*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/8*arcs
inh(d*x+c))+6*e*a^2*b^2*(1/2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/2*(1+(d*x+c)
^2)^(1/2)*(d*x+c)*arcsinh(d*x+c)-1/4*arcsinh(d*x+c)^2+1/4*(d*x+c)^2+1/4)+4
*e*b*a^3*(1/2*(d*x+c)^2*arcsinh(d*x+c)-1/4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1/4
*arcsinh(d*x+c)))
```

3.149.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(179) = 358$.

Time = 0.27 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.94

$$\int (ce + dex)(a + b\operatorname{arcsinh}(c + dx))^4 \, dx$$

$$= \frac{(2\,a^4 + 6\,a^2b^2 + 3\,b^4)d^2ex^2 + 2\,(2\,a^4 + 6\,a^2b^2 + 3\,b^4)cdex + (2\,b^4d^2ex^2 + 4\,b^4cdex + (2\,b^4c^2 + b^4)e)\log\left(dx\right.}{}$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/4*((2*a^4 + 6*a^2*b^2 + 3*b^4)*d^2*e*x^2 + 2*(2*a^4 + 6*a^2*b^2 + 3*b^4)
*c*d*e*x + (2*b^4*d^2*e*x^2 + 4*b^4*c*d*e*x + (2*b^4*c^2 + b^4)*e)*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + 4*(2*a*b^3*d^2*e*x^2 + 4*a*b
^3*c*d*e*x + (2*a*b^3*c^2 + a*b^3)*e - (b^4*d*e*x + b^4*c*e)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 +
 3*(2*(2*a^2*b^2 + b^4)*d^2*e*x^2 + 4*(2*a^2*b^2 + b^4)*c*d*e*x + (2*a^2*b
^2 + b^4 + 2*(2*a^2*b^2 + b^4)*c^2)*e - 4*(a*b^3*d*e*x + a*b^3*c*e)*sqrt(d
^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1))^2 + 2*(2*(2*a^3*b + 3*a*b^3)*d^2*e*x^2 + 4*(2*a^3*b + 3*a*b^3)*c*d*e*x
 + (2*a^3*b + 3*a*b^3 + 2*(2*a^3*b + 3*a*b^3)*c^2)*e - 3*((2*a^2*b^2 + b^4
)*d*e*x + (2*a^2*b^2 + b^4)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*
x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*((2*a^3*b + 3*a*b^3)*d*e*x
+ (2*a^3*b + 3*a*b^3)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

3.149.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(178) = 356$.

Time = 0.45 (sec) , antiderivative size = 1027, normalized size of antiderivative = 5.27

$$\int (ce + dex)(a + b\text{arcsinh}(c + dx))^4\,dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**4,x)
```

output
```
Piecewise((a**4*c*e*x + a**4*d*e*x**2/2 + 2*a**3*b*c**2*e*asinh(c + d*x)/d
 + 4*a**3*b*c*e*x*asinh(c + d*x) - a**3*b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x
**2 + 1)/d + 2*a**3*b*d*e*x**2*asinh(c + d*x) - a**3*b*e*x*sqrt(c**2 + 2*c
*d*x + d**2*x**2 + 1) + a**3*b*e*asinh(c + d*x)/d + 3*a**2*b**2*c**2*e*asi
nh(c + d*x)**2/d + 6*a**2*b**2*c*e*x*asinh(c + d*x)**2 + 3*a**2*b**2*c*e*x
 - 3*a**2*b**2*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/d +
 3*a**2*b**2*d*e*x**2*asinh(c + d*x)**2 + 3*a**2*b**2*d*e*x**2/2 - 3*a**2*
b**2*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x) + 3*a**2*b**2
*e*asinh(c + d*x)**2/(2*d) + 2*a*b**3*c**2*e*asinh(c + d*x)**3/d + 3*a*b**
3*c**2*e*asinh(c + d*x)/d + 4*a*b**3*c*e*x*asinh(c + d*x)**3 + 6*a*b**3*c*
e*x*asinh(c + d*x) - 3*a*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asi
nh(c + d*x)**2/d - 3*a*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(2*d)
 + 2*a*b**3*d*e*x**2*asinh(c + d*x)**3 + 3*a*b**3*d*e*x**2*asinh(c + d*x)
- 3*a*b**3*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2 - 3*
a*b**3*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/2 + a*b**3*e*asinh(c + d*x
)**3/d + 3*a*b**3*e*asinh(c + d*x)/(2*d) + b**4*c**2*e*asinh(c + d*x)**4/(
2*d) + 3*b**4*c**2*e*asinh(c + d*x)**2/(2*d) + b**4*c*e*x*asinh(c + d*x)**
4 + 3*b**4*c*e*x*asinh(c + d*x)**2 + 3*b**4*c*e*x/2 - b**4*c*e*sqrt(c**2 +
 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/d - 3*b**4*c*e*sqrt(c**2 + 2*c
*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(2*d) + b**4*d*e*x**2*asinh(c + d*...
```

3.149.7 Maxima [F]

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^4\,dx=\int (dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^4\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/2*a^4*d*e*x^2 + (2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*
d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 +
1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^3*b*d*e + a^4
*c*e*x + 4*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^3*b*c*e/
d + 1/2*(b^4*d*e*x^2 + 2*b^4*c*e*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1))^4 + integrate(2*((2*(c^4*e + c^2*e)*a*b^3 + (2*a*b^3*d^4*e - b^
4*d^4*e)*x^4 + 4*(2*a*b^3*c*d^3*e - b^4*c*d^3*e)*x^3 + (2*(6*c^2*d^2*e + d
^2*e)*a*b^3 - (5*c^2*d^2*e + d^2*e)*b^4)*x^2 + 2*(2*(2*c^3*d*e + c*d*e)*a*
b^3 - (c^3*d*e + c*d*e)*b^4)*x + (2*(c^3*e + c*e)*a*b^3 + (2*a*b^3*d^3*e -
 b^4*d^3*e)*x^3 + 3*(2*a*b^3*c*d^2*e - b^4*c*d^2*e)*x^2 - 2*(b^4*c^2*d*e -
 (3*c^2*d*e + d*e)*a*b^3)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x +
c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 3*(a^2*b^2*d^4*e*x^4 + 4*a^2*b^
2*c*d^3*e*x^3 + (6*c^2*d^2*e + d^2*e)*a^2*b^2*x^2 + 2*(2*c^3*d*e + c*d*e)*
a^2*b^2*x + (c^4*e + c^2*e)*a^2*b^2 + (a^2*b^2*d^3*e*x^3 + 3*a^2*b^2*c*d^2
*e*x^2 + (3*c^2*d*e + d*e)*a^2*b^2*x + (c^3*e + c*e)*a^2*b^2)*sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2)
/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2
 + 1)^(3/2) + c), x)
```

3.149.8 Giac [F]

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^4\,dx=\int (dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^4\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^4, x)
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^4\,dx=\int (c\,e+d\,e\,x)\,\left(a+b\,\text{asinh}(c+d\,x)\right)^4\,dx$$

input
```
int((c*e + d*e*x)*(a + b*asinh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)*(a + b*asinh(c + d*x))^4, x)
```

3.150 $\int (a+b\text{arcsinh}(c+dx))^4\,dx$

3.150.1 Optimal result

Integrand size = 12, antiderivative size = 115

$$\int (a+b\text{arcsinh}(c+dx))^4\,dx=24b^4x-\frac{24b^3\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))}{d}+\frac{12b^2(c+dx)(a+b\text{arcsinh}(c+dx))^2}{d}-\frac{4b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^3}{d}+\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^4}{d}$$

output
```
24*b^4*x+12*b^2*(d*x+c)*(a+b*arcsinh(d*x+c))^2/d+(d*x+c)*(a+b*arcsinh(d*x+
c))^4/d-24*b^3*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d-4*b*(a+b*arcsinh
(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d
```

3.150.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.97

$$\int (a+b\text{arcsinh}(c+dx))^4\,dx$$

$$=\frac{(a^4+12a^2b^2+24b^4)(c+dx)-4ab(a^2+6b^2)\sqrt{1+(c+dx)^2}-4b\Big(-a^3(c+dx)-6ab^2(c+dx)+3a^2b\sqrt{\ldots}}{\ldots}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
((a^4 + 12*a^2*b^2 + 24*b^4)*(c + d*x) - 4*a*b*(a^2 + 6*b^2)*Sqrt[1 + (c +
 d*x)^2] - 4*b*(-(a^3*(c + d*x)) - 6*a*b^2*(c + d*x) + 3*a^2*b*Sqrt[1 + (c
 + d*x)^2] + 6*b^3*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 6*b^2*(a^2*(c
 + d*x) + 2*b^2*(c + d*x) - 2*a*b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^
2 - 4*b^3*(-(a*(c + d*x)) + b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^3 +
b^4*(c + d*x)*ArcSinh[c + d*x]^4)/d
```

3.150.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6273, 6187, 6213, 6187, 6213, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(c+dx))^4\,dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int (a+b\text{arcsinh}(c+dx))^4d(c+dx)}{d}$$

$$\Big\downarrow \text{6187}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^4-4b\int \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^3}{\sqrt{(c+dx)^2+1}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6213}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^4-4b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3-3b\int (a+b\text{arcsinh}(c+dx))^2d(c+$$

$$\Big\downarrow \text{6187}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^4-4b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3-3b\left((c+dx)(a+b\text{arcsinh}(c+dx$$

$$\Big\downarrow \text{6213}$$

$$\frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^4-4b\Big(\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^3-3b\Big((c+dx)(a+b\operatorname{arcsinh}(c+dx)}{d}$$

$\downarrow$ 24

$$\frac{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^4-4b\Big(\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^3-3b\Big((c+dx)(a+b\operatorname{arcsinh}(c+dx}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
((c + d*x)*(a + b*ArcSinh[c + d*x])^4 - 4*b*(Sqrt[1 + (c + d*x)^2]*(a + b*
ArcSinh[c + d*x])^3 - 3*b*((c + d*x)*(a + b*ArcSinh[c + d*x])^2 - 2*b*(-(b
*(c + d*x)) + Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])))))/d
```

3.150.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.150.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(111) = 222$.

Time = 0.17 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.13

method	result
derivativedivides	$\frac{(dx+c)a^4+b^4\left((dx+c)\operatorname{arcsinh}(dx+c)^4-4\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}+12(dx+c)\operatorname{arcsinh}(dx+c)^2-24\operatorname{arcsinh}(dx+c)\right.}{}$
default	$\frac{(dx+c)a^4+b^4\left((dx+c)\operatorname{arcsinh}(dx+c)^4-4\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}+12(dx+c)\operatorname{arcsinh}(dx+c)^2-24\operatorname{arcsinh}(dx+c)\right.}{}$
parts	$a^4x+\frac{b^4\left((dx+c)\operatorname{arcsinh}(dx+c)^4-4\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}+12(dx+c)\operatorname{arcsinh}(dx+c)^2-24\operatorname{arcsinh}(dx+c)\sqrt{1\right.}{d}$

input
```
int((a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*((d*x+c)*a^4+b^4*((d*x+c)*arcsinh(d*x+c)^4-4*arcsinh(d*x+c)^3*(1+(d*x+
c)^2)^(1/2)+12*(d*x+c)*arcsinh(d*x+c)^2-24*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1
/2)+24*d*x+24*c)+4*a*b^3*((d*x+c)*arcsinh(d*x+c)^3-3*arcsinh(d*x+c)^2*(1+(
d*x+c)^2)^(1/2)+6*(d*x+c)*arcsinh(d*x+c)-6*(1+(d*x+c)^2)^(1/2))+6*a^2*b^2*
((d*x+c)*arcsinh(d*x+c)^2-2*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*d*x+2*c)+
4*b*a^3*((d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2)))
```

3.150.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(111) = 222$.

Time = 0.28 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.99

$$\int (a+b\operatorname{arcsinh}(c+dx))^4\,dx$$

$$=\frac{(b^4dx+b^4c)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^4+4\left(ab^3dx+ab^3c-\sqrt{d^2x^2+2\,cdx+c^2+1}b^4\right)\log\left(\right.}{}$$

input
```
integrate((a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
((b^4*d*x + b^4*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + 4*
(a*b^3*d*x + a*b^3*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*b^4)*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + (a^4 + 12*a^2*b^2 + 24*b^4)*d*x -
 6*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*a*b^3 - (a^2*b^2 + 2*b^4)*d*x - (a
^2*b^2 + 2*b^4)*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 4*
((a^3*b + 6*a*b^3)*d*x + (a^3*b + 6*a*b^3)*c - 3*(a^2*b^2 + 2*b^4)*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
)) - 4*(a^3*b + 6*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

3.150.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(105) = 210$.

Time = 0.27 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.86

$$\int (a + b\operatorname{arcsinh}(c + dx))^4 \, dx$$
$$= \begin{cases} a^4x + \frac{4a^3bc\operatorname{asinh}(c+dx)}{d} + 4a^3bx\operatorname{asinh}(c+dx) - \frac{4a^3b\sqrt{c^2+2cdx+d^2x^2+1}}{d} + \frac{6a^2b^2c\operatorname{asinh}^2(c+dx)}{d} + 6a^2b^2x\operatorname{asinh}^2(c \\ x(a + b\operatorname{asinh}(c))^4 \end{cases}$$

input
```
integrate((a+b*asinh(d*x+c))**4,x)
```

output
```
Piecewise((a**4*x + 4*a**3*b*c*asinh(c + d*x)/d + 4*a**3*b*x*asinh(c + d*x
) - 4*a**3*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d + 6*a**2*b**2*c*asinh(
c + d*x)**2/d + 6*a**2*b**2*x*asinh(c + d*x)**2 + 12*a**2*b**2*x - 12*a**2
*b**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/d + 4*a*b**3*c*a
sinh(c + d*x)**3/d + 24*a*b**3*c*asinh(c + d*x)/d + 4*a*b**3*x*asinh(c + d
*x)**3 + 24*a*b**3*x*asinh(c + d*x) - 12*a*b**3*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)*asinh(c + d*x)**2/d - 24*a*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/d + b**4*c*asinh(c + d*x)**4/d + 12*b**4*c*asinh(c + d*x)**2/d + b**
4*x*asinh(c + d*x)**4 + 12*b**4*x*asinh(c + d*x)**2 + 24*b**4*x - 4*b**4*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/d - 24*b**4*sqrt(c**
2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/d, Ne(d, 0)), (x*(a + b*asinh(
c))**4, True))
```

3.150.7 Maxima [F]

$$\int (a+b\operatorname{arcsinh}(c+dx))^4\,dx=\int (b\operatorname{arsinh}(dx+c)+a)^4\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
b^4*x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + a^4*x + 4*((d*x
 + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^3*b/d + integrate(2*(2*(
(c^3 + c)*a*b^3 + (a*b^3*d^3 - b^4*d^3)*x^3 + (3*a*b^3*c*d^2 - 2*b^4*c*d^2
)*x^2 + ((3*c^2*d + d)*a*b^3 - (c^2*d + d)*b^4)*x + ((c^2 + 1)*a*b^3 + (a*
b^3*d^2 - b^4*d^2)*x^2 + (2*a*b^3*c*d - b^4*c*d)*x)*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 3*(a^2*b
^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d + d)*a^2*b^2*x + (c^3 + c)*a^2
*b^2 + (a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (c^2 + 1)*a^2*b^2)*sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2
)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^
2 + 1)^(3/2) + c), x)
```

3.150.8 Giac [F]

$$\int (a+b\operatorname{arcsinh}(c+dx))^4\,dx=\int (b\operatorname{arsinh}(dx+c)+a)^4\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^4, x)
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\operatorname{arcsinh}(c+dx))^4\,dx=\int (a+b\operatorname{asinh}(c+d\,x))^4\,dx$$

input
```
int((a + b*asinh(c + d*x))^4,x)
```

output
```
int((a + b*asinh(c + d*x))^4, x)
```

3.151 $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{ce+dex}\,dx$

3.151.1 Optimal result

Integrand size = 23, antiderivative size = 186

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{ce+dex}\,dx = \frac{(a+b\text{arcsinh}(c+dx))^5}{5bde} + \frac{(a+b\text{arcsinh}(c+dx))^4\log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de} - \frac{2b(a+b\text{arcsinh}(c+dx))^3\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(c+dx)}\right)}{de} - \frac{3b^2(a+b\text{arcsinh}(c+dx))^2\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(c+dx)}\right)}{de} - \frac{3b^3(a+b\text{arcsinh}(c+dx))\,\text{PolyLog}\left(4,e^{-2\text{arcsinh}(c+dx)}\right)}{de} - \frac{3b^4\,\text{PolyLog}\left(5,e^{-2\text{arcsinh}(c+dx)}\right)}{2de}$$

output
```
1/5*(a+b*arcsinh(d*x+c))^5/b/d/e+(a+b*arcsinh(d*x+c))^4*ln(1-1/(d*x+c+(1+(
d*x+c)^2)^(1/2))^2)/d/e-2*b*(a+b*arcsinh(d*x+c))^3*polylog(2,1/(d*x+c+(1+(
d*x+c)^2)^(1/2))^2)/d/e-3*b^2*(a+b*arcsinh(d*x+c))^2*polylog(3,1/(d*x+c+(1
+(d*x+c)^2)^(1/2))^2)/d/e-3*b^3*(a+b*arcsinh(d*x+c))*polylog(4,1/(d*x+c+(1
+(d*x+c)^2)^(1/2))^2)/d/e-3/2*b^4*polylog(5,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^
2)/d/e
```

3.151.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{ce+dex}\,dx$$
$$= \frac{-\frac{(a+b\text{arcsinh}(c+dx))^5}{5b}+(a+b\text{arcsinh}(c+dx))^4\log\left(1-e^{2\text{arcsinh}(c+dx)}\right)+2b(a+b\text{arcsinh}(c+dx))^3\,\text{PolyL}}{}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x),x]
```

output
```
(-1/5*(a + b*ArcSinh[c + d*x])^5/b + (a + b*ArcSinh[c + d*x])^4*Log[1 - E^
(2*ArcSinh[c + d*x])] + 2*b*(a + b*ArcSinh[c + d*x])^3*PolyLog[2, E^(2*Arc
Sinh[c + d*x])] - 3*b^2*(a + b*ArcSinh[c + d*x])^2*PolyLog[3, E^(2*ArcSinh
[c + d*x])] + 3*b^3*(a + b*ArcSinh[c + d*x])*PolyLog[4, E^(2*ArcSinh[c + d
*x])] - (3*b^4*PolyLog[5, E^(2*ArcSinh[c + d*x])])/2)/(d*e)
```

3.151.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.42, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {6274, 27, 6190, 25, 3042, 26, 4201, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{ce+dex}\,dx$$
$$\Big\downarrow \text{6274}$$
$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^4}{e(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^4}{c+dx}d(c+dx)}{de}$$
$$\Big\downarrow \text{6190}$$

$$\frac{\int -(a+b\text{arcsinh}(c+dx))^4\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)d(a+b\text{arcsinh}(c+dx))}{bde}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int (a+b\text{arcsinh}(c+dx))^4\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)d(a+b\text{arcsinh}(c+dx))}{bde}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -i(a+b\text{arcsinh}(c+dx))^4\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)d(a+b\text{arcsinh}(c+dx))}{bde}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int (a+b\text{arcsinh}(c+dx))^4\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)d(a+b\text{arcsinh}(c+dx))}{bde}$$

$$\Big\downarrow \text{4201}$$

$$\frac{i\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}(a+b\text{arcsinh}(c+dx))^4}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{5}i(a+b\text{arcsinh}(c+dx))^5\right)}{bde}$$

$$\Big\downarrow \text{2620}$$

$$\frac{i\left(2i\left(2b\int (a+b\text{arcsinh}(c+dx))^3\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right)d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}b(a+b\text{arcsinh}(\right.\right.}{bde}$$

$$\Big\downarrow \text{3011}$$

$$i\left(2i\left(2b\left(\frac{1}{2}b(a+b\text{arcsinh}(c+dx))^3\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right)-\frac{3}{2}b\int (a+b\text{arcsinh}(c+dx))^2\text{PolyI}\right.\right.\right.$$

$$\Big\downarrow \text{7163}$$

$$i\left(2i\left(2b\left(\frac{1}{2}b(a+b\text{arcsinh}(c+dx))^3\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right)-\frac{3}{2}b\left(b\int (a+b\text{arcsinh}(c+dx))\text{Pol}\right.\right.\right.\right.$$

$$\Big\downarrow \text{7163}$$

$$i\left(2i\left(2b\left(\frac{1}{2}b(a+b\text{arcsinh}(c+dx))^3\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right)-\frac{3}{2}b\left(b\left(\frac{1}{2}b\int \text{PolyLog}\left(4,-e^{\frac{2a}{b}-\frac{2(a}{}}\right.\right.\right.\right.\right.\right.$$

3.151. $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{ce+dex}\,dx$

$$\Big\downarrow \text{2720}$$

$$i\left(2i\left(2b\left(\frac{1}{2}b(a+b\text{arcsinh}(c+dx))^3\,\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right)-\frac{3}{2}b\left(b\left(-\frac{1}{4}b^2\int\exp\left(-\frac{2a}{b}+\frac{2(a+b\text{ar}\right.\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$i\left(2i\left(2b\left(\frac{1}{2}b(a+b\text{arcsinh}(c+dx))^3\,\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}\right)-\frac{3}{2}b\left(b\left(-\frac{1}{4}b^2\,\text{PolyLog}(5,-c-dx)\right.\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x),x]
```

output
```
(I*((-1/5*I)*(a + b*ArcSinh[c + d*x])^5 + (2*I)*(-1/2*(b*(a + b*ArcSinh[c
+ d*x])^4*Log[1 + E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[c + d*x]))/b)]) +
2*b*((b*(a + b*ArcSinh[c + d*x])^3*PolyLog[2, -E^((2*a)/b - I*Pi - (2*(a +
 b*ArcSinh[c + d*x]))/b)])/2 - (3*b*(-1/2*(b*(a + b*ArcSinh[c + d*x])^2*Po
lyLog[3, -E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[c + d*x]))/b)]) + b*(-1/2*
(b*(a + b*ArcSinh[c + d*x])*PolyLog[4, -E^((2*a)/b - I*Pi - (2*(a + b*ArcS
inh[c + d*x]))/b)]) - (b^2*PolyLog[5, -c - d*x])/4)))/2))))/(b*d*e)
```

3.151.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.151. $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{ce+dex}\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.151. $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{ce+dex}\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.151.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(222) = 444$.

Time = 0.54 (sec) , antiderivative size = 861, normalized size of antiderivative = 4.63

method	result	size
derivativedivides	Expression too large to display	861
default	Expression too large to display	861
parts	Expression too large to display	872

input
```
int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4/e*ln(d*x+c)+b^4/e*(-1/5*arcsinh(d*x+c)^5+arcsinh(d*x+c)^4*ln(1+d*
x+c+(1+(d*x+c)^2)^(1/2))+4*arcsinh(d*x+c)^3*polylog(2,-d*x-c-(1+(d*x+c)^2)
^(1/2))-12*arcsinh(d*x+c)^2*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+24*arcsi
nh(d*x+c)*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))-24*polylog(5,-d*x-c-(1+(d*
x+c)^2)^(1/2))+arcsinh(d*x+c)^4*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+4*arcsinh(
d*x+c)^3*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-12*arcsinh(d*x+c)^2*polylog(
3,d*x+c+(1+(d*x+c)^2)^(1/2))+24*arcsinh(d*x+c)*polylog(4,d*x+c+(1+(d*x+c)^
2)^(1/2))-24*polylog(5,d*x+c+(1+(d*x+c)^2)^(1/2)))+4*a*b^3/e*(-1/4*arcsinh
(d*x+c)^4+arcsinh(d*x+c)^3*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+3*arcsinh(d*x+c
)^2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-6*arcsinh(d*x+c)*polylog(3,-d*x-
c-(1+(d*x+c)^2)^(1/2))+6*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))+arcsinh(d*x
+c)^3*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+3*arcsinh(d*x+c)^2*polylog(2,d*x+c+(
1+(d*x+c)^2)^(1/2))-6*arcsinh(d*x+c)*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))+
6*polylog(4,d*x+c+(1+(d*x+c)^2)^(1/2)))+6*a^2*b^2/e*(-1/3*arcsinh(d*x+c)^3
+arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+2*arcsinh(d*x+c)*polylog
(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-2*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+arc
sinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+2*arcsinh(d*x+c)*polylog(2,d
*x+c+(1+(d*x+c)^2)^(1/2))-2*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2)))+4*b*a^3/
e*(-1/2*arcsinh(d*x+c)^2+arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+po
lylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c...
```

3.151.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{ce+dex}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^4}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*
arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/(d*e*x + c*e), x)
```

3.151.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{ce+dex}\,dx = \frac{\int \frac{a^4}{c+dx}\,dx+\int \frac{b^4\,\text{asinh}^4\,(c+dx)}{c+dx}\,dx+\int \frac{4ab^3\,\text{asinh}^3\,(c+dx)}{c+dx}\,dx+\int \frac{6a^2b^2\,\text{asinh}^2\,(c+dx)}{c+dx}\,dx+\int \frac{4a^3b\,\text{asinh}\,(c+dx)}{c+dx}\,dx}{e}$$

input
```
integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e),x)
```

output
```
(Integral(a**4/(c + d*x), x) + Integral(b**4*asinh(c + d*x)**4/(c + d*x),
x) + Integral(4*a*b**3*asinh(c + d*x)**3/(c + d*x), x) + Integral(6*a**2*b
**2*asinh(c + d*x)**2/(c + d*x), x) + Integral(4*a**3*b*asinh(c + d*x)/(c
+ d*x), x))/e
```

3.151.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{ce+dex}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^4}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
a^4*log(d*e*x + c*e)/(d*e) + integrate(b^4*log(d*x + c + sqrt((d*x + c)^2
+ 1))^4/(d*e*x + c*e) + 4*a*b^3*log(d*x + c + sqrt((d*x + c)^2 + 1))^3/(d*
e*x + c*e) + 6*a^2*b^2*log(d*x + c + sqrt((d*x + c)^2 + 1))^2/(d*e*x + c*e
) + 4*a^3*b*log(d*x + c + sqrt((d*x + c)^2 + 1))/(d*e*x + c*e), x)
```

3.151.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{ce+dex}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^4}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e), x)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{ce+dex}\,dx=\int \frac{(a+b\,\text{asinh}(c+d\,x))^4}{c\,e+d\,e\,x}\,dx$$

input
```
int((a + b*asinh(c + d*x))^4/(c*e + d*e*x),x)
```

output
```
int((a + b*asinh(c + d*x))^4/(c*e + d*e*x), x)
```

3.152 $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^2}\,dx$

3.152.1 Optimal result

Integrand size = 23, antiderivative size = 234

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^2}\,dx = &-\frac{(a+b\text{arcsinh}(c+dx))^4}{de^2(c+dx)}\\
&-\frac{8b(a+b\text{arcsinh}(c+dx))^3\text{arctanh}\left(e^{\text{arcsinh}(c+dx)}\right)}{de^2}\\
&-\frac{12b^2(a+b\text{arcsinh}(c+dx))^2\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(c+dx)}\right)}{de^2}\\
&+\frac{12b^2(a+b\text{arcsinh}(c+dx))^2\,\text{PolyLog}\left(2,e^{\text{arcsinh}(c+dx)}\right)}{de^2}\\
&+\frac{24b^3(a+b\text{arcsinh}(c+dx))\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(c+dx)}\right)}{de^2}\\
&-\frac{24b^3(a+b\text{arcsinh}(c+dx))\,\text{PolyLog}\left(3,e^{\text{arcsinh}(c+dx)}\right)}{de^2}\\
&-\frac{24b^4\,\text{PolyLog}\left(4,-e^{\text{arcsinh}(c+dx)}\right)}{de^2}\\
&+\frac{24b^4\,\text{PolyLog}\left(4,e^{\text{arcsinh}(c+dx)}\right)}{de^2}\end{aligned}$$

output
```
-(a+b*arcsinh(d*x+c))^4/d/e^2/(d*x+c)-8*b*(a+b*arcsinh(d*x+c))^3*arctanh(d
*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2-12*b^2*(a+b*arcsinh(d*x+c))^2*polylog(2,-d
*x-c-(1+(d*x+c)^2)^(1/2))/d/e^2+12*b^2*(a+b*arcsinh(d*x+c))^2*polylog(2,d*
x+c+(1+(d*x+c)^2)^(1/2))/d/e^2+24*b^3*(a+b*arcsinh(d*x+c))*polylog(3,-d*x-
c-(1+(d*x+c)^2)^(1/2))/d/e^2-24*b^3*(a+b*arcsinh(d*x+c))*polylog(3,d*x+c+(
1+(d*x+c)^2)^(1/2))/d/e^2-24*b^4*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e
^2+24*b^4*polylog(4,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2
```

3.152.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 510 vs. $2(234) = 468$.

Time = 1.32 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.18

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^2}\,dx = \frac{-\frac{2a^4}{c+dx} - 8a^3b\left(\frac{\text{arcsinh}(c+dx)}{c+dx} + \log\left(\frac{1}{2}(c+dx)\text{csch}\left(\frac{1}{2}\text{arcsinh}(c+dx)\right)\right) - \log\left(\sinh\left(\frac{1}{2}\text{arcsinh}(c+dx)\right)\right)\right) -}{}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^2,x]
```

output
```
((-2*a^4)/(c + d*x) - 8*a^3*b*(ArcSinh[c + d*x]/(c + d*x) + Log[((c + d*x)
*Csch[ArcSinh[c + d*x]/2])/2] - Log[Sinh[ArcSinh[c + d*x]/2]]) + 12*a^2*b^
2*(ArcSinh[c + d*x]*(-(ArcSinh[c + d*x]/(c + d*x)) + 2*Log[1 - E^(-ArcSinh
[c + d*x])] - 2*Log[1 + E^(-ArcSinh[c + d*x])]) + 2*PolyLog[2, -E^(-ArcSin
h[c + d*x])] - 2*PolyLog[2, E^(-ArcSinh[c + d*x])]) + 8*a*b^3*(-(ArcSinh[c
 + d*x]^3/(c + d*x)) + 3*ArcSinh[c + d*x]^2*Log[1 - E^(-ArcSinh[c + d*x])]
 - 3*ArcSinh[c + d*x]^2*Log[1 + E^(-ArcSinh[c + d*x])] + 6*ArcSinh[c + d*x
]*PolyLog[2, -E^(-ArcSinh[c + d*x])] - 6*ArcSinh[c + d*x]*PolyLog[2, E^(-A
rcSinh[c + d*x])] + 6*PolyLog[3, -E^(-ArcSinh[c + d*x])] - 6*PolyLog[3, E^
(-ArcSinh[c + d*x])]) + b^4*(Pi^4 - 2*ArcSinh[c + d*x]^4 - (2*ArcSinh[c +
d*x]^4)/(c + d*x) - 8*ArcSinh[c + d*x]^3*Log[1 + E^(-ArcSinh[c + d*x])] +
8*ArcSinh[c + d*x]^3*Log[1 - E^ArcSinh[c + d*x]] + 24*ArcSinh[c + d*x]^2*P
olyLog[2, -E^(-ArcSinh[c + d*x])] + 24*ArcSinh[c + d*x]^2*PolyLog[2, E^Arc
Sinh[c + d*x]] + 48*ArcSinh[c + d*x]*PolyLog[3, -E^(-ArcSinh[c + d*x])] -
48*ArcSinh[c + d*x]*PolyLog[3, E^ArcSinh[c + d*x]] + 48*PolyLog[4, -E^(-Ar
cSinh[c + d*x])] + 48*PolyLog[4, E^ArcSinh[c + d*x]]))/(2*d*e^2)
```

3.152.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.85, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {6274, 27, 6191, 6231, 3042, 26, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^2}dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^4}{e^2(c+dx)^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(c+dx)^2}d(c+dx)}{de^2}$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{4b\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(c+dx)\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{(a+b\text{arcsinh}(c+dx))^4}{c+dx}}{de^2}$$

$$\Big\downarrow \text{ 6231}$$

$$\frac{4b\int \frac{(a+b\text{arcsinh}(c+dx))^3}{c+dx}d\text{arcsinh}(c+dx)-\frac{(a+b\text{arcsinh}(c+dx))^4}{c+dx}}{de^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^4}{c+dx}+4b\int i(a+b\text{arcsinh}(c+dx))^3\csc(i\text{arcsinh}(c+dx))d\text{arcsinh}(c+dx)}{de^2}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^4}{c+dx}+4ib\int (a+b\text{arcsinh}(c+dx))^3\csc(i\text{arcsinh}(c+dx))d\text{arcsinh}(c+dx)}{de^2}$$

$$\Big\downarrow \text{ 4670}$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^4}{c+dx}+4ib\left(3ib\int (a+b\text{arcsinh}(c+dx))^2\log\left(1-e^{\text{arcsinh}(c+dx)}\right)d\text{arcsinh}(c+dx)-3ib\int (a+b\text{arcs}\right.}{de^2}$$

$\downarrow$ 3011

$$-\frac{(a+b\text{arcsinh}(c+dx))^4}{c+dx}+4ib\left(-3ib\left(2b\int(a+b\text{arcsinh}(c+dx))\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(c+dx)}\right)d\text{arcsinh}(c+dx)-\text{PolyL}\right.\right.$$

$\downarrow$ 7163

$$-\frac{(a+b\text{arcsinh}(c+dx))^4}{c+dx}+4ib\left(-3ib\left(2b\left(\text{PolyLog}\left(3,-e^{\text{arcsinh}(c+dx)}\right)(a+b\text{arcsinh}(c+dx))-b\int\text{PolyLog}\left(3,-e^{\text{arcsin}}\right.\right.\right.\right.$$

$\downarrow$ 2720

$$-\frac{(a+b\text{arcsinh}(c+dx))^4}{c+dx}+4ib\left(3ib\left(2b\left(\text{PolyLog}\left(3,e^{\text{arcsinh}(c+dx)}\right)(a+b\text{arcsinh}(c+dx))-b\int e^{-\text{arcsinh}(c+dx)}\,\text{PolyLog}\right.\right.\right.$$

$\downarrow$ 7143

$$-\frac{(a+b\text{arcsinh}(c+dx))^4}{c+dx}+4ib\left(2i\,\text{arctanh}\left(e^{\text{arcsinh}(c+dx)}\right)(a+b\text{arcsinh}(c+dx))^3+3ib\left(2b\left(\text{PolyLog}\left(3,e^{\text{arcsinh}(c+dx)}\right)\right(\right.\right.\right.$$

input
```
Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^2,x]
```

output
```
(-((a + b*ArcSinh[c + d*x])^4/(c + d*x)) + (4*I)*b*((2*I)*(a + b*ArcSinh[c
 + d*x])^3*ArcTanh[E^ArcSinh[c + d*x]] + (3*I)*b*(-((a + b*ArcSinh[c + d*x
])^2*PolyLog[2, E^ArcSinh[c + d*x]]) + 2*b*((a + b*ArcSinh[c + d*x])*PolyL
og[3, E^ArcSinh[c + d*x]] - b*PolyLog[4, E^ArcSinh[c + d*x]])) - (3*I)*b*(
-((a + b*ArcSinh[c + d*x])^2*PolyLog[2, -E^ArcSinh[c + d*x]]) + 2*b*((a +
b*ArcSinh[c + d*x])*PolyLog[3, -E^ArcSinh[c + d*x]] - b*PolyLog[4, -c - d*
x]))))/(d*e^2)
```

3.152.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.152. $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^2}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.152.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(299) = 598$.

Time = 0.47 (sec) , antiderivative size = 630, normalized size of antiderivative = 2.69

method	result
derivativedivides	$-\frac{a^4}{e^2(dx+c)}+\frac{b^4\left(-\frac{\operatorname{arcsinh}(dx+c)^4}{dx+c}-4\operatorname{arcsinh}(dx+c)^3\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-12\operatorname{arcsinh}(dx+c)^2\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx-\right.\right.}{}$
default	$-\frac{a^4}{e^2(dx+c)}+\frac{b^4\left(-\frac{\operatorname{arcsinh}(dx+c)^4}{dx+c}-4\operatorname{arcsinh}(dx+c)^3\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-12\operatorname{arcsinh}(dx+c)^2\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(dx-\right.\right.}{}$
parts	$-\frac{a^4}{e^2(dx+c)d}+\frac{b^4\left(-\frac{\operatorname{arcsinh}(dx+c)^4}{dx+c}-4\operatorname{arcsinh}(dx+c)^3\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)-12\operatorname{arcsinh}(dx+c)^2\operatorname{polylog}\left(2,\right.\right.}{}$

input
```
int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^4/e^2/(d*x+c)+b^4/e^2*(-1/(d*x+c)*arcsinh(d*x+c)^4-4*arcsinh(d*x+c
)^3*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-12*arcsinh(d*x+c)^2*polylog(2,-d*x-c-(
1+(d*x+c)^2)^(1/2))+24*arcsinh(d*x+c)*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2)
)-24*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))+4*arcsinh(d*x+c)^3*ln(1-d*x-c-(
1+(d*x+c)^2)^(1/2))+12*arcsinh(d*x+c)^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2
))-24*arcsinh(d*x+c)*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))+24*polylog(4,d*x
+c+(1+(d*x+c)^2)^(1/2)))+4*a*b^3/e^2*(-1/(d*x+c)*arcsinh(d*x+c)^3-3*arcsin
h(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-6*arcsinh(d*x+c)*polylog(2,-d*x
-c-(1+(d*x+c)^2)^(1/2))+6*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+3*arcsinh(
d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+6*arcsinh(d*x+c)*polylog(2,d*x+c+
(1+(d*x+c)^2)^(1/2))-6*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2)))+6*a^2*b^2/e^2
*(-1/(d*x+c)*arcsinh(d*x+c)^2-2*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1
/2))-2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+2*arcsinh(d*x+c)*ln(1-d*x-c-(
1+(d*x+c)^2)^(1/2))+2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2)))+4*b*a^3/e^2*(-
1/(d*x+c)*arcsinh(d*x+c)-arctanh(1/(1+(d*x+c)^2)^(1/2))))
```

3.152.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^2}\,dx=\int \frac{(b\operatorname{arsinh}(dx+c)+a)^4}{(dex+ce)^2}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="fricas")
```

output
```
integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*
arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/(d^2*e^2*x^2 + 2*c*d*
e^2*x + c^2*e^2), x)
```

3.152.6 Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^2}\,dx$$
$$=\frac{\int \frac{a^4}{c^2+2cdx+d^2x^2}\,dx+\int \frac{b^4\operatorname{asinh}^4(c+dx)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{4ab^3\operatorname{asinh}^3(c+dx)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{6a^2b^2\operatorname{asinh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{4a^3b\operatorname{asinh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

input
```
integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**2,x)
```

output
```
(Integral(a**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**4*asinh(c +
d*x)**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a*b**3*asinh(c + d*x
)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(6*a**2*b**2*asinh(c + d*x
)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a**3*b*asinh(c + d*x)/(
c**2 + 2*c*d*x + d**2*x**2), x))/e**2
```

3.152.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(c + dx))^4}{(ce + dex)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.152.8 Giac [F]

$$\int \frac{(a + b\text{arcsinh}(c + dx))^4}{(ce + dex)^2} \, dx = \int \frac{(b \operatorname{arsinh}(dx + c) + a)^4}{(dex + ce)^2} \, dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^2, x)
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^2}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4}{{\left(c\,e+d\,e\,x\right)}^2}\,dx$$

input
```
int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^2,x)
```

output
```
int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^2, x)
```

3.153 $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx$

3.153.1 Optimal result

Integrand size = 23, antiderivative size = 186

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx &= \frac{2b(a+b\text{arcsinh}(c+dx))^3}{de^3} \\ &\quad - \frac{2b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^3}{de^3(c+dx)} \\ &\quad - \frac{(a+b\text{arcsinh}(c+dx))^4}{2de^3(c+dx)^2} \\ &\quad + \frac{6b^2(a+b\text{arcsinh}(c+dx))^2\log\left(1-e^{-2\text{arcsinh}(c+dx)}\right)}{de^3} \\ &\quad - \frac{6b^3(a+b\text{arcsinh}(c+dx))\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}(c+dx)}\right)}{de^3} \\ &\quad - \frac{3b^4\,\text{PolyLog}\left(3,e^{-2\text{arcsinh}(c+dx)}\right)}{de^3}\end{aligned}$$

output
```
2*b*(a+b*arcsinh(d*x+c))^3/d/e^3-1/2*(a+b*arcsinh(d*x+c))^4/d/e^3/(d*x+c)^
2+6*b^2*(a+b*arcsinh(d*x+c))^2*ln(1-1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3
-6*b^3*(a+b*arcsinh(d*x+c))*polylog(2,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e
^3-3*b^4*polylog(3,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3-2*b*(a+b*arcsinh
(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d/e^3/(d*x+c)
```

3.153.2 Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.88

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx =$$

$$-\frac{\frac{a^4}{(c+dx)^2}+\frac{4a^3b\sqrt{1+(c+dx)^2}}{c+dx}+\frac{4a^3b\operatorname{arcsinh}(c+dx)}{(c+dx)^2}+\frac{b^4\operatorname{arcsinh}(c+dx)^4}{(c+dx)^2}-12a^2b^2\left(-\frac{\sqrt{1+(c+dx)^2}\operatorname{arcsinh}(c+dx)}{c+dx}-\operatorname{arc}\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^3,x]
```

output
```
-1/2*(a^4/(c + d*x)^2 + (4*a^3*b*Sqrt[1 + (c + d*x)^2])/(c + d*x) + (4*a^3
*b*ArcSinh[c + d*x])/(c + d*x)^2 + (b^4*ArcSinh[c + d*x]^4)/(c + d*x)^2 -
12*a^2*b^2*(-((Sqrt[1 + (c + d*x)^2]*ArcSinh[c + d*x])/(c + d*x)) - ArcSin
h[c + d*x]^2/(2*(c + d*x)^2) + Log[c + d*x]) - 4*a*b^3*(ArcSinh[c + d*x]*(
3*ArcSinh[c + d*x] - (3*Sqrt[1 + (c + d*x)^2]*ArcSinh[c + d*x])/(c + d*x)
- ArcSinh[c + d*x]^2/(c + d*x)^2 + 6*Log[1 - E^(-2*ArcSinh[c + d*x])]) - 3
*PolyLog[2, E^(-2*ArcSinh[c + d*x])]) - 2*b^4*(2*ArcSinh[c + d*x]^2*(ArcSi
nh[c + d*x] - (Sqrt[1 + (c + d*x)^2]*ArcSinh[c + d*x])/(c + d*x) + 3*Log[1
 - E^(-2*ArcSinh[c + d*x])]) - 6*ArcSinh[c + d*x]*PolyLog[2, E^(-2*ArcSinh
[c + d*x])] - 3*PolyLog[3, E^(-2*ArcSinh[c + d*x])]))/(d*e^3)
```

3.153.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {6274, 27, 6191, 6215, 6190, 25, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{e^3(c+dx)^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(c+dx)^3}d(c+dx)}{de^3}$$

$\big\downarrow$ 6191

$$\frac{2b\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(c+dx)^2\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{(a+b\text{arcsinh}(c+dx))^4}{2(c+dx)^2}}{de^3}$$

$\big\downarrow$ 6215

$$\frac{2b\left(3b\int \frac{(a+b\text{arcsinh}(c+dx))^2}{c+dx}d(c+dx)-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3}{c+dx}\right)-\frac{(a+b\text{arcsinh}(c+dx))^4}{2(c+dx)^2}}{de^3}$$

$\big\downarrow$ 6190

$$\frac{2b\left(3\int -(a+b\text{arcsinh}(c+dx))^2\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)d(a+b\text{arcsinh}(c+dx))-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+}{c+dx}\right.}{de^3}$$

$\big\downarrow$ 25

$$\frac{2b\left(-3\int (a+b\text{arcsinh}(c+dx))^2\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)d(a+b\text{arcsinh}(c+dx))-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+}{c+dx}\right.}{de^3}$$

$\big\downarrow$ 3042

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^4}{2(c+dx)^2}+2b\left(-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3}{c+dx}-3\int -i(a+b\text{arcsinh}(c+dx))^2\tan\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}}{b}\right.\right.}{de^3}$$

$\big\downarrow$ 26

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^4}{2(c+dx)^2}+2b\left(-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3}{c+dx}+3i\int (a+b\text{arcsinh}(c+dx))^2\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i(a+b}{}\right.\right.}{de^3}$$

$\big\downarrow$ 4201

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^4}{2(c+dx)^2}+2b\left(-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3}{c+dx}+3i\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}(a+b\text{arcsinh}(c+dx))^2}{1+e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}-i\pi}}d\right.\right.}{de^3}$$

$\big\downarrow$ 2620

$$-\frac{(a+b\text{arcsinh}(c+dx))^4}{2(c+dx)^2}+2b\left(-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3}{c+dx}+3i\left(2i\left(b\int (a+b\text{arcsinh}(c+dx))\log\left(1+e^{\frac{2a}{b}-\frac{2(a+b}{}}\right.\right.\right.\right.$$

3.153. $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx$

$\downarrow$ 3011

$$-\frac{(a+b\text{arcsinh}(c+dx))^4}{2(c+dx)^2}+2b\left(-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3}{c+dx}+3i\left(2i\left(b\left(\frac{1}{2}b(a+b\text{arcsinh}(c+dx))\operatorname{PolyLog}\left(2,-e\right.\right.\right.\right.\right.$$

$\downarrow$ 2720

$$-\frac{(a+b\text{arcsinh}(c+dx))^4}{2(c+dx)^2}+2b\left(-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3}{c+dx}+3i\left(2i\left(b\left(\frac{1}{4}b^2\int\exp\left(-\frac{2a}{b}+\frac{2(a+b\text{arcsinh}(c+dx))}{b}+i\right.\right.\right.\right.\right.$$

$\downarrow$ 7143

$$-\frac{(a+b\text{arcsinh}(c+dx))^4}{2(c+dx)^2}+2b\left(-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3}{c+dx}+3i\left(2i\left(b\left(\frac{1}{4}b^2\operatorname{PolyLog}(3,-c-dx)+\frac{1}{2}b(a+b\text{arcsin}\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^3,x]
```

output
```
(-1/2*(a + b*ArcSinh[c + d*x])^4/(c + d*x)^2 + 2*b*(-((Sqrt[1 + (c + d*x)^
2]*(a + b*ArcSinh[c + d*x])^3)/(c + d*x)) + (3*I)*((-1/3*I)*(a + b*ArcSinh
[c + d*x])^3 + (2*I)*(-1/2*(b*(a + b*ArcSinh[c + d*x])^2*Log[1 + E^((2*a)/
b - I*Pi - (2*(a + b*ArcSinh[c + d*x]))/b)]) + b*((b*(a + b*ArcSinh[c + d*
x])*PolyLog[2, -E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[c + d*x]))/b)])/2 +
(b^2*PolyLog[3, -c - d*x])/4)))))/(d*e^3)
```

3.153.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.153. $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx$

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

3.153. $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx$

rule 6215
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b
*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ
[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.153.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(206) = 412$.

Time = 0.52 (sec) , antiderivative size = 561, normalized size of antiderivative = 3.02

method	result
derivativedivides	$-\frac{a^4}{2e^3(dx+c)^2}+\frac{b^4\left(-\frac{\operatorname{arcsinh}(dx+c)^3\left(-4(dx+c)^2+4(dx+c)\sqrt{1+(dx+c)^2}+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}-4\operatorname{arcsinh}(dx+c)^3+6\operatorname{arcsinh}(dx+c)^2\ln$
default	$-\frac{a^4}{2e^3(dx+c)^2}+\frac{b^4\left(-\frac{\operatorname{arcsinh}(dx+c)^3\left(-4(dx+c)^2+4(dx+c)\sqrt{1+(dx+c)^2}+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}-4\operatorname{arcsinh}(dx+c)^3+6\operatorname{arcsinh}(dx+c)^2\ln$
parts	$-\frac{a^4}{2e^3(dx+c)^2d}+\frac{b^4\left(-\frac{\operatorname{arcsinh}(dx+c)^3\left(-4(dx+c)^2+4(dx+c)\sqrt{1+(dx+c)^2}+\operatorname{arcsinh}(dx+c)\right)}{2(dx+c)^2}-4\operatorname{arcsinh}(dx+c)^3+6\operatorname{arcsinh}$

input
```
int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a^4/e^3/(d*x+c)^2+b^4/e^3*(-1/2*arcsinh(d*x+c)^3*(-4*(d*x+c)^2+4
*(d*x+c)*(1+(d*x+c)^2)^(1/2)+arcsinh(d*x+c))/(d*x+c)^2-4*arcsinh(d*x+c)^3+
6*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+12*arcsinh(d*x+c)*polyl
og(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-12*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+
6*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+12*arcsinh(d*x+c)*polyl
og(2,d*x+c+(1+(d*x+c)^2)^(1/2))-12*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2)))+4
*a*b^3/e^3*(-1/2*arcsinh(d*x+c)^2*(3*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3*(d*x+c)
^2+arcsinh(d*x+c))/(d*x+c)^2-3*arcsinh(d*x+c)^2+3*arcsinh(d*x+c)*ln(1+d*x+
c+(1+(d*x+c)^2)^(1/2))+3*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+3*arcsinh(d
*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+3*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2
)))+6*a^2*b^2/e^3*(-2*arcsinh(d*x+c)-1/2*arcsinh(d*x+c)*(-2*(d*x+c)^2+2*(d
*x+c)*(1+(d*x+c)^2)^(1/2)+arcsinh(d*x+c))/(d*x+c)^2+ln((d*x+c+(1+(d*x+c)^2
)^(1/2))^2-1))+4*b*a^3/e^3*(-1/2/(d*x+c)^2*arcsinh(d*x+c)-1/2/(d*x+c)*(1+(
d*x+c)^2)^(1/2)))
```

3.153.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx=\int \frac{(b\operatorname{arsinh}(dx+c)+a)^4}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="fricas")
```

output
```
integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*
arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/(d^3*e^3*x^3 + 3*c*d^
2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

3.153.6 Sympy [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx$$
$$=\frac{\int \frac{a^4}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{b^4\operatorname{asinh}^4(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{4ab^3\operatorname{asinh}^3(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{6a^2b^2\operatorname{asinh}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

input
```
integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**3,x)
```

output
```
(Integral(a**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integ
ral(b**4*asinh(c + d*x)**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)
, x) + Integral(4*a*b**3*asinh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x
**2 + d**3*x**3), x) + Integral(6*a**2*b**2*asinh(c + d*x)**2/(c**3 + 3*c*
*2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(4*a**3*b*asinh(c + d*x)
/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

3.153.7 Maxima [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^4}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="maxima")
```

output
```
-1/2*b^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4/(d^3*e^3*x^2 +
 2*c*d^2*e^3*x + c^2*d*e^3) - 6*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d*arcsi
nh(d*x + c)/(d^3*e^3*x + c*d^2*e^3) - log(d*x + c)/(d*e^3))*a^2*b^2 - 2*a^
3*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d/(d^3*e^3*x + c*d^2*e^3) + arcsinh
(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)) - 3*a^2*b^2*arcsinh(d
*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 1/2*a^4/(d^3*e^3*x^2
 + 2*c*d^2*e^3*x + c^2*d*e^3) + integrate(2*(2*(c^3 + c)*a*b^3 + (c^3 + c)
*b^4 + (2*a*b^3*d^3 + b^4*d^3)*x^3 + 3*(2*a*b^3*c*d^2 + b^4*c*d^2)*x^2 + (
2*(3*c^2*d + d)*a*b^3 + (3*c^2*d + d)*b^4)*x + (b^4*c^2 + 2*(c^2 + 1)*a*b^
3 + (2*a*b^3*d^2 + b^4*d^2)*x^2 + 2*(2*a*b^3*c*d + b^4*c*d)*x)*sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3
/(d^6*e^3*x^6 + 6*c*d^5*e^3*x^5 + c^6*e^3 + c^4*e^3 + (15*c^2*d^4*e^3 + d^
4*e^3)*x^4 + 4*(5*c^3*d^3*e^3 + c*d^3*e^3)*x^3 + 3*(5*c^4*d^2*e^3 + 2*c^2*
d^2*e^3)*x^2 + 2*(3*c^5*d*e^3 + 2*c^3*d*e^3)*x + (d^5*e^3*x^5 + 5*c*d^4*e^
3*x^4 + c^5*e^3 + c^3*e^3 + (10*c^2*d^3*e^3 + d^3*e^3)*x^3 + (10*c^3*d^2*e
^3 + 3*c*d^2*e^3)*x^2 + (5*c^4*d*e^3 + 3*c^2*d*e^3)*x)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)), x)
```

3.153.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^4}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^3, x)
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^3}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^4}{(c\,e+d\,e\,x)^3}\,dx$$

input
```
int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^3,x)
```

output
```
int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^3, x)
```

3.154 $\int \frac{(a+b\,\mathrm{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx$

3.154.1 Optimal result

Integrand size = 23, antiderivative size = 385

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx = -\frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^3}{3de^4(c+dx)^2} - \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{3de^4(c+dx)^3} - \frac{8b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} + \frac{4b(a+b\operatorname{arcsinh}(c+dx))^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(c+dx)}\right)}{3de^4} - \frac{4b^4\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} + \frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} + \frac{4b^4\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} - \frac{2b^2(a+b\operatorname{arcsinh}(c+dx))^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} - \frac{4b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} + \frac{4b^3(a+b\operatorname{arcsinh}(c+dx))\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} + \frac{4b^4\operatorname{PolyLog}\left(4,-e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4} - \frac{4b^4\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(c+dx)}\right)}{de^4}$$

output
```
-2*b^2*(a+b*arcsinh(d*x+c))^2/d/e^4/(d*x+c)-1/3*(a+b*arcsinh(d*x+c))^4/d/e
^4/(d*x+c)^3-8*b^3*(a+b*arcsinh(d*x+c))*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))
/d/e^4+4/3*b*(a+b*arcsinh(d*x+c))^3*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))/d/e
^4-4*b^4*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4+2*b^2*(a+b*arcsinh(d*
x+c))^2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4+4*b^4*polylog(2,d*x+c+
(1+(d*x+c)^2)^(1/2))/d/e^4-2*b^2*(a+b*arcsinh(d*x+c))^2*polylog(2,d*x+c+(1
+(d*x+c)^2)^(1/2))/d/e^4-4*b^3*(a+b*arcsinh(d*x+c))*polylog(3,-d*x-c-(1+(d
*x+c)^2)^(1/2))/d/e^4+4*b^3*(a+b*arcsinh(d*x+c))*polylog(3,d*x+c+(1+(d*x+c
)^2)^(1/2))/d/e^4+4*b^4*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4-4*b^4*
polylog(4,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-2/3*b*(a+b*arcsinh(d*x+c))^3*(1
+(d*x+c)^2)^(1/2)/d/e^4/(d*x+c)^2
```

3.154.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1198 vs. $2(385) = 770$.

Time = 8.24 (sec) , antiderivative size = 1198, normalized size of antiderivative = 3.11

$$\int \frac{(a + b\text{arcsinh}(c + dx))^4}{(ce + dex)^4} \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^4,x]
```

output
```
-1/3*a^4/(d*e^4*(c + d*x)^3) + (a^2*b^2*(-8*PolyLog[2, -E^(-ArcSinh[c + d*
x])] - (2*(-2 + 4*ArcSinh[c + d*x]^2 + 2*Cosh[2*ArcSinh[c + d*x]] - 3*(c +
 d*x)*ArcSinh[c + d*x]*Log[1 - E^(-ArcSinh[c + d*x])] + 3*(c + d*x)*ArcSin
h[c + d*x]*Log[1 + E^(-ArcSinh[c + d*x])] - 4*(c + d*x)^3*PolyLog[2, E^(-A
rcSinh[c + d*x])] + 2*ArcSinh[c + d*x]*Sinh[2*ArcSinh[c + d*x]] + ArcSinh[
c + d*x]*Log[1 - E^(-ArcSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]] - ArcSinh
[c + d*x]*Log[1 + E^(-ArcSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]]))/(c + d
*x)^3))/(4*d*e^4) + (a*b^3*(-24*ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2]
+ 4*ArcSinh[c + d*x]^3*Coth[ArcSinh[c + d*x]/2] - 6*ArcSinh[c + d*x]^2*Csc
h[ArcSinh[c + d*x]/2]^2 - (c + d*x)*ArcSinh[c + d*x]^3*Csch[ArcSinh[c + d*
x]/2]^4 - 24*ArcSinh[c + d*x]^2*Log[1 - E^(-ArcSinh[c + d*x])] + 24*ArcSin
h[c + d*x]^2*Log[1 + E^(-ArcSinh[c + d*x])] + 48*Log[Tanh[ArcSinh[c + d*x]
/2]] - 48*ArcSinh[c + d*x]*PolyLog[2, -E^(-ArcSinh[c + d*x])] + 48*ArcSinh
[c + d*x]*PolyLog[2, E^(-ArcSinh[c + d*x])] - 48*PolyLog[3, -E^(-ArcSinh[c
 + d*x])] + 48*PolyLog[3, E^(-ArcSinh[c + d*x])] - 6*ArcSinh[c + d*x]^2*Se
ch[ArcSinh[c + d*x]/2]^2 - (16*ArcSinh[c + d*x]^3*Sinh[ArcSinh[c + d*x]/2]
^4)/(c + d*x)^3 + 24*ArcSinh[c + d*x]*Tanh[ArcSinh[c + d*x]/2] - 4*ArcSinh
[c + d*x]^3*Tanh[ArcSinh[c + d*x]/2]))/(12*d*e^4) + (b^4*(-2*Pi^4 + 4*ArcS
inh[c + d*x]^4 - 24*ArcSinh[c + d*x]^2*Coth[ArcSinh[c + d*x]/2] + 2*ArcSin
h[c + d*x]^4*Coth[ArcSinh[c + d*x]/2] - 4*ArcSinh[c + d*x]^3*Csch[ArcSi...
```

3.154.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.86, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {6274, 27, 6191, 6224, 6191, 6231, 3042, 26, 4670, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^4}{e^4(c+dx)^4}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(c+dx)^4}d(c+dx)}{de^4}$$

$$\Big\downarrow \; 6191$$

$$\frac{\frac{4}{3}b\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(c+dx)^3\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{(a+b\text{arcsinh}(c+dx))^4}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \; 6224$$

$$\frac{\frac{4}{3}b\left(\frac{3}{2}b\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(c+dx)^2}d(c+dx)-\frac{1}{2}\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(c+dx)\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3}{2(c+dx)^2}\right)-\frac{(a+b}{}}{de^4}$$

$$\Big\downarrow \; 6191$$

$$\frac{\frac{4}{3}b\left(\frac{3}{2}b\left(2b\int \frac{a+b\text{arcsinh}(c+dx)}{(c+dx)\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{(a+b\text{arcsinh}(c+dx))^2}{c+dx}\right)-\frac{1}{2}\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(c+dx)\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{\sqrt{(c+dx)^2+1}(a+}{2(c}}{de^4}$$

$$\Big\downarrow \; 6231$$

$$\frac{\frac{4}{3}b\left(\frac{3}{2}b\left(2b\int \frac{a+b\text{arcsinh}(c+dx)}{c+dx}d\text{arcsinh}(c+dx)-\frac{(a+b\text{arcsinh}(c+dx))^2}{c+dx}\right)-\frac{1}{2}\int \frac{(a+b\text{arcsinh}(c+dx))^3}{c+dx}d\text{arcsinh}(c+dx)-}{de^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^4}{3(c+dx)^3}+\frac{4}{3}b\left(\frac{3}{2}b\left(-\frac{(a+b\text{arcsinh}(c+dx))^2}{c+dx}+2b\int i(a+b\text{arcsinh}(c+dx))\csc(i\text{arcsinh}(c+dx))d\text{arcsinh}}{de}$$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{(a+b\text{arcsinh}(c+dx))^4}{3(c+dx)^3}+\frac{4}{3}b\left(\frac{3}{2}b\left(-\frac{(a+b\text{arcsinh}(c+dx))^2}{c+dx}+2ib\int (a+b\text{arcsinh}(c+dx))\csc(i\text{arcsinh}(c+dx))d\text{arcsinh}(}{de}$$

$$\Big\downarrow \; 4670$$

3.154. $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx$

$$-\frac{(a+b\text{arcsinh}(c+dx))^4}{3(c+dx)^3}+\frac{4}{3}b\left(\frac{3}{2}b\left(-\frac{(a+b\text{arcsinh}(c+dx))^2}{c+dx}+2ib\left(ib\int\log\left(1-e^{\text{arcsinh}(c+dx)}\right)d\text{arcsinh}(c+dx)-ib\int\log\right.\right.\right.$$

$\downarrow$ 2715

$$-\frac{(a+b\text{arcsinh}(c+dx))^4}{3(c+dx)^3}+\frac{4}{3}b\left(-\frac{1}{2}i\left(3ib\int(a+b\text{arcsinh}(c+dx))^2\log\left(1-e^{\text{arcsinh}(c+dx)}\right)d\text{arcsinh}(c+dx)-3ib\int(a+\right.\right.$$

$\downarrow$ 2838

$$-\frac{(a+b\text{arcsinh}(c+dx))^4}{3(c+dx)^3}+\frac{4}{3}b\left(-\frac{1}{2}i\left(3ib\int(a+b\text{arcsinh}(c+dx))^2\log\left(1-e^{\text{arcsinh}(c+dx)}\right)d\text{arcsinh}(c+dx)-3ib\int(a+\right.\right.$$

$\downarrow$ 3011

$$-\frac{(a+b\text{arcsinh}(c+dx))^4}{3(c+dx)^3}+\frac{4}{3}b\left(-\frac{1}{2}i\left(-3ib\left(2b\int(a+b\text{arcsinh}(c+dx))\text{PolyLog}\left(2,-e^{\text{arcsinh}(c+dx)}\right)d\text{arcsinh}(c+dx)-\right.\right.\right.$$

$\downarrow$ 7163

$$-\frac{(a+b\text{arcsinh}(c+dx))^4}{3(c+dx)^3}+\frac{4}{3}b\left(-\frac{1}{2}i\left(-3ib\left(2b\left(\text{PolyLog}\left(3,-e^{\text{arcsinh}(c+dx)}\right)(a+b\text{arcsinh}(c+dx))-b\int\text{PolyLog}\left(3,-e\right.\right.\right.\right.\right.$$

$\downarrow$ 2720

$$-\frac{(a+b\text{arcsinh}(c+dx))^4}{3(c+dx)^3}+\frac{4}{3}b\left(-\frac{1}{2}i\left(3ib\left(2b\left(\text{PolyLog}\left(3,e^{\text{arcsinh}(c+dx)}\right)(a+b\text{arcsinh}(c+dx))-b\int e^{-\text{arcsinh}(c+dx)}\text{Poly}\right.\right.\right.\right.$$

$\downarrow$ 7143

$$-\frac{(a+b\text{arcsinh}(c+dx))^4}{3(c+dx)^3}+\frac{4}{3}b\left(\frac{3}{2}b\left(-\frac{(a+b\text{arcsinh}(c+dx))^2}{c+dx}+2ib\left(2i\text{arctanh}\left(e^{\text{arcsinh}(c+dx)}\right)(a+b\text{arcsinh}(c+dx))-ib\,\text{P}\right.\right.\right.$$

input
```
Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^4,x]
```

3.154. $\int\frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx$

output
```
(-1/3*(a + b*ArcSinh[c + d*x])^4/(c + d*x)^3 + (4*b*(-1/2*(Sqrt[1 + (c + d
*x)^2]*(a + b*ArcSinh[c + d*x])^3)/(c + d*x)^2 + (3*b*(-((a + b*ArcSinh[c
+ d*x])^2/(c + d*x)) + (2*I)*b*((2*I)*(a + b*ArcSinh[c + d*x])*ArcTanh[E^A
rcSinh[c + d*x]] - I*b*PolyLog[2, E^ArcSinh[c + d*x]] + I*b*PolyLog[2, -c
- d*x])))/2 - (I/2)*((2*I)*(a + b*ArcSinh[c + d*x])^3*ArcTanh[E^ArcSinh[c
+ d*x]] + (3*I)*b*(-((a + b*ArcSinh[c + d*x])^2*PolyLog[2, E^ArcSinh[c + d
*x]]) + 2*b*((a + b*ArcSinh[c + d*x])*PolyLog[3, E^ArcSinh[c + d*x]] - b*P
olyLog[4, E^ArcSinh[c + d*x]])) - (3*I)*b*(-((a + b*ArcSinh[c + d*x])^2*Po
lyLog[2, -E^ArcSinh[c + d*x]]) + 2*b*((a + b*ArcSinh[c + d*x])*PolyLog[3,
-E^ArcSinh[c + d*x]] - b*PolyLog[4, -c - d*x])))))/3)/(d*e^4)
```

3.154.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.154. $\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6224
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Simp[c^2*((m + 2*p + 3)/(f^2*(m +
1)))   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Sim
p[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m +
1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 6231
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]]   Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.154. $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.154.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 883, normalized size of antiderivative = 2.29

method	result	size
derivativedivides	Expression too large to display	883
default	Expression too large to display	883
parts	Expression too large to display	894

input
```
int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a^4/e^4/(d*x+c)^3+b^4/e^4*(-1/3/(d*x+c)^3*arcsinh(d*x+c)^2*(2*(1
+(d*x+c)^2)^(1/2)*(d*x+c)*arcsinh(d*x+c)+arcsinh(d*x+c)^2+6*(d*x+c)^2)+2/3
*arcsinh(d*x+c)^3*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+2*arcsinh(d*x+c)^2*polyl
og(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-4*arcsinh(d*x+c)*polylog(3,-d*x-c-(1+(d*x
+c)^2)^(1/2))+4*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))-2/3*arcsinh(d*x+c)^3
*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-2*arcsinh(d*x+c)^2*polylog(2,d*x+c+(1+(d*
x+c)^2)^(1/2))+4*arcsinh(d*x+c)*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))-4*pol
ylog(4,d*x+c+(1+(d*x+c)^2)^(1/2))-4*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2
)^(1/2))-4*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+4*arcsinh(d*x+c)*ln(1-d*x
-c-(1+(d*x+c)^2)^(1/2))+4*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2)))+4*a*b^3/e^
4*(-1/6/(d*x+c)^3*arcsinh(d*x+c)*(3*(1+(d*x+c)^2)^(1/2)*(d*x+c)*arcsinh(d*
x+c)+2*arcsinh(d*x+c)^2+6*(d*x+c)^2)+1/2*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d
*x+c)^2)^(1/2))+arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-polyl
og(3,-d*x-c-(1+(d*x+c)^2)^(1/2))-1/2*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c
)^2)^(1/2))-arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))+polylog(3,
d*x+c+(1+(d*x+c)^2)^(1/2))-2*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2)))+6*a^2*b^2
/e^4*(-1/3*((1+(d*x+c)^2)^(1/2)*(d*x+c)*arcsinh(d*x+c)+arcsinh(d*x+c)^2+(d
*x+c)^2)/(d*x+c)^3+1/3*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+1/3*
polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-1/3*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*
x+c)^2)^(1/2))-1/3*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2)))+4*b*a^3/e^4*(-...
```

3.154.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx = \int \frac{(b\operatorname{arsinh}(dx+c)+a)^4}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="fricas")
```

output
```
integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*
arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/(d^4*e^4*x^4 + 4*c*d^
3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

3.154.6 Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(c + dx))^4}{(ce + dex)^4} dx$$
$$= \frac{\int \frac{a^4}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4} dx + \int \frac{b^4 \operatorname{asinh}^4(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4} dx + \int \frac{4ab^3 \operatorname{asinh}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4} dx + \int \frac{}{c}}{e^4}$$

input
```
integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**4,x)
```

output
```
(Integral(a**4/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**
4*x**4), x) + Integral(b**4*asinh(c + d*x)**4/(c**4 + 4*c**3*d*x + 6*c**2*
d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a*b**3*asinh(c + d
*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),
 x) + Integral(6*a**2*b**2*asinh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d
**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a**3*b*asinh(c + d*
x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))
/e**4
```

3.154.7 Maxima [F]

$$\int \frac{(a + b\text{arcsinh}(c + dx))^4}{(ce + dex)^4} dx = \int \frac{(b \operatorname{arsinh}(dx + c) + a)^4}{(dex + ce)^4} dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="maxima")
```

output
```
-1/3*b^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4/(d^4*e^4*x^3 +
 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^4/(d^4*e^4*x^3 + 3
*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(2/3*(2*(3*(c^3 +
 c)*a*b^3 + (c^3 + c)*b^4 + (3*a*b^3*d^3 + b^4*d^3)*x^3 + 3*(3*a*b^3*c*d^2
 + b^4*c*d^2)*x^2 + (3*(3*c^2*d + d)*a*b^3 + (3*c^2*d + d)*b^4)*x + (b^4*c
^2 + 3*(c^2 + 1)*a*b^3 + (3*a*b^3*d^2 + b^4*d^2)*x^2 + 2*(3*a*b^3*c*d + b^
4*c*d)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1))^3 + 9*(a^2*b^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*
d + d)*a^2*b^2*x + (c^3 + c)*a^2*b^2 + (a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x
+ (c^2 + 1)*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 6*(a^3*b*d^3*x^3 + 3*a^3*b*c*d^2*x^2 +
(3*c^2*d + d)*a^3*b*x + (c^3 + c)*a^3*b + (a^3*b*d^2*x^2 + 2*a^3*b*c*d*x +
 (c^2 + 1)*a^3*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)))/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 + c^
5*e^4 + (21*c^2*d^5*e^4 + d^5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 + c*d^4*e^4)*x^4
 + 5*(7*c^4*d^3*e^4 + 2*c^2*d^3*e^4)*x^3 + (21*c^5*d^2*e^4 + 10*c^3*d^2*e^
4)*x^2 + (7*c^6*d*e^4 + 5*c^4*d*e^4)*x + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 +
c^6*e^4 + c^4*e^4 + (15*c^2*d^4*e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*
d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 + 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*
c^3*d*e^4)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

3.154.8 Giac [F]

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx=\int \frac{(b\operatorname{arsinh}(dx+c)+a)^4}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^4, x)
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^4}\,dx = \int \frac{(a+b\operatorname{asinh}(c+d\,x))^4}{(c\,e+d\,e\,x)^4}\,dx$$

input
```
int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^4,x)
```

output
```
int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^4, x)
```

3.155 $\int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx$

3.155.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx=\text{Int}\left(\frac{(e(c+dx))^m}{a+b\text{arcsinh}(c+dx)},x\right)$$

output
```
Unintegrable((e*(d*x+c))^m/(a+b*arcsinh(d*x+c)),x)
```

3.155.2 Mathematica [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx=\int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx$$

input
```
Integrate[(c*e + d*e*x)^m/(a + b*ArcSinh[c + d*x]),x]
```

output
```
Integrate[(c*e + d*e*x)^m/(a + b*ArcSinh[c + d*x]), x]
```

3.155.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)} \, dx$$
$$\Big\downarrow \text{6274}$$
$$\frac{\int \frac{(e(c+dx))^m}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \text{6196}$$
$$\frac{\int \frac{(e(c+dx))^m}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$

input
```
Int[(c*e + d*e*x)^m/(a + b*ArcSinh[c + d*x]),x]
```

output
```
$Aborted
```

3.155.3.1 Defintions of rubi rules used

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.155.4 Maple [N/A] (verified)

Not integrable

Time = 0.78 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(dex+ce)^m}{a+b\,\mathrm{arcsinh}\,(dx+c)}dx$$

input
```
int((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x)
```

output
```
int((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x)
```

3.155.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{a+b\mathrm{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^m}{b\,\mathrm{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)^m/(b*arcsinh(d*x + c) + a), x)
```

3.155.6 Sympy [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{(ce+dex)^m}{a+b\mathrm{arcsinh}(c+dx)}\,dx = \int \frac{(e(c+dx))^m}{a+b\,\mathrm{asinh}\,(c+dx)}\,dx$$

input
```
integrate((d*e*x+c*e)**m/(a+b*asinh(d*x+c)),x)
```

output
```
Integral((e*(c + d*x))**m/(a + b*asinh(c + d*x)), x)
```

3.155.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx=\int \frac{(dex+ce)^m}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^m/(b*arcsinh(d*x + c) + a), x)
```

3.155.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx=\int \frac{(dex+ce)^m}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^m/(b*arcsinh(d*x + c) + a), x)
```

3.155.9 Mupad [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(ce+dex)^m}{a+b\text{arcsinh}(c+dx)}\,dx=\int \frac{(c\,e+d\,e\,x)^m}{a+b\,\text{asinh}\,(c+d\,x)}\,dx$$

input
```
int((c*e + d*e*x)^m/(a + b*asinh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^m/(a + b*asinh(c + d*x)), x)
```

3.156 $\int \frac{(ce+dex)^4}{a+b\text{arcsinh}(c+dx)}\,dx$

3.156.1 Optimal result

Integrand size = 23, antiderivative size = 213

$$\int \frac{(ce+dex)^4}{a+b\text{arcsinh}(c+dx)}\,dx = \frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{8bd} - \frac{3e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{16bd} + \frac{e^4\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)}{16bd} - \frac{e^4\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{8bd} + \frac{3e^4\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{16bd} - \frac{e^4\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)}{16bd}$$

output
```
1/8*e^4*Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b/d-3/16*e^4*Chi(3*(a+b*arcs
inh(d*x+c))/b)*cosh(3*a/b)/b/d+1/16*e^4*Chi(5*(a+b*arcsinh(d*x+c))/b)*cosh
(5*a/b)/b/d-1/8*e^4*Shi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)/b/d+3/16*e^4*Shi
(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b/d-1/16*e^4*Shi(5*(a+b*arcsinh(d*x
+c))/b)*sinh(5*a/b)/b/d
```

3.156.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.71

$$\int \frac{(ce+dex)^4}{a+b\text{arcsinh}(c+dx)}\,dx$$
$$= \frac{e^4\left(2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)-3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}\right.\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x]),x]
```

output
```
(e^4*(2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - 3*Cosh[(3*a)/b]*C
oshIntegral[3*(a/b + ArcSinh[c + d*x])] + Cosh[(5*a)/b]*CoshIntegral[5*(a/
b + ArcSinh[c + d*x])] - 2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]]
 + 3*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])] - Sinh[(5*a)/b]
*SinhIntegral[5*(a/b + ArcSinh[c + d*x])]))/(16*b*d)
```

3.156.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6274, 27, 6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{a+b\text{arcsinh}(c+dx)}\,dx$$
$$\Big\downarrow \ 6274$$
$$\frac{\int \frac{e^4(c+dx)^4}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \ 27$$
$$\frac{e^4\int \frac{(c+dx)^4}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \ 6195$$
$$\frac{e^4\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{5971}$$

$$\frac{e^4 \int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)}{16(a+b\text{arcsinh}(c+dx))}-\frac{3\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{16(a+b\text{arcsinh}(c+dx))}+\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{8(a+b\text{arcsinh}(c+dx))}\right)d(a+b\text{arcsinh}(c}{bd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^4\left(\frac{1}{8}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)-\frac{3}{16}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)+\frac{1}{16}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(c+}{b}\right.\right.}{}$$

input
```
Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x]),x]
```

output
```
(e^4*((Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c + d*x])/b])/8 - (3*Cosh[(3*
a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/16 + (Cosh[(5*a)/b]*Co
shIntegral[(5*(a + b*ArcSinh[c + d*x]))/b])/16 - (Sinh[a/b]*SinhIntegral[(
a + b*ArcSinh[c + d*x])/b])/8 + (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*Ar
cSinh[c + d*x]))/b])/16 - (Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c
+ d*x]))/b])/16))/(b*d)
```

3.156.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

3.156. $\int \frac{(ce+dex)^4}{a+b\text{arcsinh}(c+dx)}\,dx$

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.156.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{e^4 e^{\frac{5a}{b}} \mathrm{Ei}_1\left(5 \operatorname{arcsinh}(dx+c)+\frac{5a}{b}\right)}{32b}+\frac{3e^4 e^{\frac{3a}{b}} \mathrm{Ei}_1\left(3 \operatorname{arcsinh}(dx+c)+\frac{3a}{b}\right)}{32b}-\frac{e^4 e^{\frac{a}{b}} \mathrm{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{16b}-\frac{e^4 e^{-\frac{a}{b}} \mathrm{Ei}_1\left(-\operatorname{arcsinh}(\cdots\right.}{16b}\cdots}{d}$
default	$\frac{-\frac{e^4 e^{\frac{5a}{b}} \mathrm{Ei}_1\left(5 \operatorname{arcsinh}(dx+c)+\frac{5a}{b}\right)}{32b}+\frac{3e^4 e^{\frac{3a}{b}} \mathrm{Ei}_1\left(3 \operatorname{arcsinh}(dx+c)+\frac{3a}{b}\right)}{32b}-\frac{e^4 e^{\frac{a}{b}} \mathrm{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{16b}-\frac{e^4 e^{-\frac{a}{b}} \mathrm{Ei}_1\left(-\operatorname{arcsinh}(\cdots\right.}{16b}\cdots}{d}$

input
```
int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/32*e^4/b*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)+3/32*e^4/b*exp(3*
a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)-1/16*e^4/b*exp(a/b)*Ei(1,arcsinh(d*x+c)+
a/b)-1/16*e^4/b*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)+3/32*e^4/b*exp(-3*a/b)
*Ei(1,-3*arcsinh(d*x+c)-3*a/b)-1/32*e^4/b*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+
c)-5*a/b))
```

3.156.5 Fricas [F]

$$\int \frac{(ce+dex)^4}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^4}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*
x + c^4*e^4)/(b*arcsinh(d*x + c) + a), x)
```

3.156.6 Sympy [F]

$$\int \frac{(ce+dex)^4}{a+b\text{arcsinh}(c+dx)}\,dx = e^4\left(\int \frac{c^4}{a+b\operatorname{asinh}(c+dx)}\,dx + \int \frac{d^4x^4}{a+b\operatorname{asinh}(c+dx)}\,dx + \int \frac{4cd^3x^3}{a+b\operatorname{asinh}(c+dx)}\,dx + \int \frac{6c^2d^2x^2}{a+b\operatorname{asinh}(c+dx)}\,dx + \int \frac{4c^3dx}{a+b\operatorname{asinh}(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c)),x)
```

output
```
e**4*(Integral(c**4/(a + b*asinh(c + d*x)), x) + Integral(d**4*x**4/(a + b
*asinh(c + d*x)), x) + Integral(4*c*d**3*x**3/(a + b*asinh(c + d*x)), x) +
 Integral(6*c**2*d**2*x**2/(a + b*asinh(c + d*x)), x) + Integral(4*c**3*d*
x/(a + b*asinh(c + d*x)), x))
```

3.156.7 Maxima [F]

$$\int \frac{(ce+dex)^4}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^4}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a), x)
```

3.156.8 Giac [F]

$$\int \frac{(ce+dex)^4}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^4}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a), x)
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{{\left(c\,e+d\,e\,x\right)}^4}{a+b\,\mathrm{asinh}\left(c+d\,x\right)}\,dx$$

input
```
int((c*e + d*e*x)^4/(a + b*asinh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^4/(a + b*asinh(c + d*x)), x)
```

3.157 $\int \frac{(ce+dex)^3}{a+b\text{arcsinh}(c+dx)} \, dx$

3.157.1 Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{(ce+dex)^3}{a+b\text{arcsinh}(c+dx)} \, dx = \frac{e^3\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{4bd} - \frac{e^3\text{Chi}\left(\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{8bd} - \frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{4bd} + \frac{e^3\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{8bd}$$

output
```
-1/4*e^3*cosh(2*a/b)*Shi(2*(a+b*arcsinh(d*x+c))/b)/b/d+1/8*e^3*cosh(4*a/b)
*Shi(4*(a+b*arcsinh(d*x+c))/b)/b/d+1/4*e^3*Chi(2*(a+b*arcsinh(d*x+c))/b)*s
inh(2*a/b)/b/d-1/8*e^3*Chi(4*(a+b*arcsinh(d*x+c))/b)*sinh(4*a/b)/b/d
```

3.157.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.75

$$\int \frac{(ce+dex)^3}{a+b\text{arcsinh}(c+dx)}\,dx$$
$$=\frac{e^3\left(2\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)\right)\sinh\left(\frac{2a}{b}\right)-\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)\right)\sinh\left(\frac{4a}{b}\right)-2\cosh\left(\frac{2a}{b}\right)\text{Shi}(2\cdots}{8bd}$$

input
```
Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x]),x]
```

output
```
(e^3*(2*CoshIntegral[2*(a/b + ArcSinh[c + d*x])]*Sinh[(2*a)/b] - CoshInteg
ral[4*(a/b + ArcSinh[c + d*x])]*Sinh[(4*a)/b] - 2*Cosh[(2*a)/b]*SinhIntegr
al[2*(a/b + ArcSinh[c + d*x])] + Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSi
nh[c + d*x])]))/(8*b*d)
```

3.157.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6274, 27, 6195, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{a+b\text{arcsinh}(c+dx)}\,dx$$
$$\Big\downarrow \ 6274$$
$$\frac{\int \frac{e^3(c+dx)^3}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \ 27$$
$$\frac{e^3\int \frac{(c+dx)^3}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \ 6195$$
$$\frac{e^3\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\downarrow 25$$

$$-\frac{e^3\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\downarrow 5971$$

$$-\frac{e^3\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{8(a+b\text{arcsinh}(c+dx))}-\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{4(a+b\text{arcsinh}(c+dx))}\right)d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\downarrow 2009$$

$$\frac{e^3\left(\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)-\frac{1}{8}\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)-\frac{1}{4}\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+\right.\right.}{bd}$$

input
```
Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x]),x]
```

output
```
(e^3*((CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(2*a)/b])/4 - (Co
shIntegral[(4*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(4*a)/b])/8 - (Cosh[(2*a)/
b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/4 + (Cosh[(4*a)/b]*SinhIn
tegral[(4*(a + b*ArcSinh[c + d*x]))/b])/8))/(b*d)
```

3.157.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.157. $\int \frac{(ce+dex)^3}{a+b\text{arcsinh}(c+dx)}\,dx$

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.157.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\frac{e^3 \mathrm{e}^{\frac{4a}{b}} \mathrm{Ei}_1\left(4 \operatorname{arcsinh}(dx+c)+\frac{4a}{b}\right)}{16b}-\frac{e^3 \mathrm{e}^{\frac{2a}{b}} \mathrm{Ei}_1\left(2 \operatorname{arcsinh}(dx+c)+\frac{2a}{b}\right)}{8b}+\frac{e^3 \mathrm{e}^{-\frac{2a}{b}} \mathrm{Ei}_1\left(-2 \operatorname{arcsinh}(dx+c)-\frac{2a}{b}\right)}{8b}-\frac{e^3 \mathrm{e}^{-\frac{4a}{b}} \mathrm{Ei}_1\left(-4 \mathrm{a}\right.}{1}}{d}$
default	$\frac{\frac{e^3 \mathrm{e}^{\frac{4a}{b}} \mathrm{Ei}_1\left(4 \operatorname{arcsinh}(dx+c)+\frac{4a}{b}\right)}{16b}-\frac{e^3 \mathrm{e}^{\frac{2a}{b}} \mathrm{Ei}_1\left(2 \operatorname{arcsinh}(dx+c)+\frac{2a}{b}\right)}{8b}+\frac{e^3 \mathrm{e}^{-\frac{2a}{b}} \mathrm{Ei}_1\left(-2 \operatorname{arcsinh}(dx+c)-\frac{2a}{b}\right)}{8b}-\frac{e^3 \mathrm{e}^{-\frac{4a}{b}} \mathrm{Ei}_1\left(-4 \mathrm{a}\right.}{1}}{d}$

input
```
int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/16*e^3/b*exp(4*a/b)*Ei(1,4*arcsinh(d*x+c)+4*a/b)-1/8*e^3/b*exp(2*a/
b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)+1/8*e^3/b*exp(-2*a/b)*Ei(1,-2*arcsinh(d*x+
c)-2*a/b)-1/16*e^3/b*exp(-4*a/b)*Ei(1,-4*arcsinh(d*x+c)-4*a/b))
```

3.157.5 Fricas [F]

$$\int \frac{(ce+dex)^3}{a+b\text{arcsinh}(c+dx)}\,dx=\int \frac{(dex+ce)^3}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b*arcs
inh(d*x + c) + a), x)
```

3.157.6 Sympy [F]

$$\int \frac{(ce+dex)^3}{a+b\text{arcsinh}(c+dx)}\,dx = e^3\left(\int \frac{c^3}{a+b\operatorname{asinh}(c+dx)}\,dx + \int \frac{d^3x^3}{a+b\operatorname{asinh}(c+dx)}\,dx + \int \frac{3cd^2x^2}{a+b\operatorname{asinh}(c+dx)}\,dx + \int \frac{3c^2dx}{a+b\operatorname{asinh}(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c)),x)
```

output
```
e**3*(Integral(c**3/(a + b*asinh(c + d*x)), x) + Integral(d**3*x**3/(a + b
*asinh(c + d*x)), x) + Integral(3*c*d**2*x**2/(a + b*asinh(c + d*x)), x) +
 Integral(3*c**2*d*x/(a + b*asinh(c + d*x)), x))
```

3.157.7 Maxima [F]

$$\int \frac{(ce+dex)^3}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^3}{b\operatorname{arsinh}(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a), x)
```

3.157.8 Giac [F]

$$\int \frac{(ce+dex)^3}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^3}{b\operatorname{arsinh}(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a), x)
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{a+b\operatorname{arcsinh}(c+dx)}\,dx=\int \frac{{\left(c\,e+d\,e\,x\right)}^3}{a+b\,\mathrm{asinh}\left(c+d\,x\right)}\,dx$$

input
```
int((c*e + d*e*x)^3/(a + b*asinh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^3/(a + b*asinh(c + d*x)), x)
```

3.158 $\int \frac{(ce+dex)^2}{a+b\text{arcsinh}(c+dx)}\,dx$

3.158.1 Optimal result

Integrand size = 23, antiderivative size = 141

$$\int \frac{(ce+dex)^2}{a+b\text{arcsinh}(c+dx)}\,dx=-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{4bd}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{4bd}+\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{4bd}-\frac{e^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{4bd}$$

output
```
-1/4*e^2*Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b/d+1/4*e^2*Chi(3*(a+b*arcs
inh(d*x+c))/b)*cosh(3*a/b)/b/d+1/4*e^2*Shi((a+b*arcsinh(d*x+c))/b)*sinh(a/
b)/b/d-1/4*e^2*Shi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b/d
```

3.158.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.72

$$\int \frac{(ce+dex)^2}{a+b\text{arcsinh}(c+dx)}\,dx$$
$$=\frac{e^2\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arc}\right.\right.}{4bd}$$

input
```
Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x]),x]
```

output
```
(e^2*(-(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]]) + Cosh[(3*a)/b]*Co
shIntegral[3*(a/b + ArcSinh[c + d*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcS
inh[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]))/(
4*b*d)
```

3.158.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6274, 27, 6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{a+b\text{arcsinh}(c+dx)}\,dx$$
$$\Big\downarrow \text{6274}$$
$$\frac{\int \frac{e^2(c+dx)^2}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{e^2\int \frac{(c+dx)^2}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \text{6195}$$
$$\frac{e^2\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$\Big\downarrow$ 5971

$$\frac{e^2\int\left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{4(a+b\text{arcsinh}(c+dx))}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{4(a+b\text{arcsinh}(c+dx))}\right)d(a+b\text{arcsinh}(c+dx))}{bd}$$

$\Big\downarrow$ 2009

$$\frac{e^2\left(-\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)+\frac{1}{4}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)+\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right.\right.}{bd}$$

input
```
Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x]),x]
```

output
```
(e^2*(-1/4*(Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c + d*x])/b]) + (Cosh[(3
*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/4 + (Sinh[a/b]*SinhIn
tegral[(a + b*ArcSinh[c + d*x])/b])/4 - (Sinh[(3*a)/b]*SinhIntegral[(3*(a
+ b*ArcSinh[c + d*x]))/b])/4))/(b*d)
```

3.158.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.158.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{-\frac{e^2 \mathrm{e}^{\frac{3a}{b}} \mathrm{Ei}_1\left(3\,\mathrm{arcsinh}(dx+c)+\frac{3a}{b}\right)}{8b}+\frac{e^2 \mathrm{e}^{\frac{a}{b}} \mathrm{Ei}_1\left(\mathrm{arcsinh}(dx+c)+\frac{a}{b}\right)}{8b}+\frac{e^2 \mathrm{e}^{-\frac{a}{b}} \mathrm{Ei}_1\left(-\,\mathrm{arcsinh}(dx+c)-\frac{a}{b}\right)}{8b}-\frac{e^2 \mathrm{e}^{-\frac{3a}{b}} \mathrm{Ei}_1\left(-3\,\mathrm{arcsinh}\right.}{8b}}{d}$
default	$\frac{-\frac{e^2 \mathrm{e}^{\frac{3a}{b}} \mathrm{Ei}_1\left(3\,\mathrm{arcsinh}(dx+c)+\frac{3a}{b}\right)}{8b}+\frac{e^2 \mathrm{e}^{\frac{a}{b}} \mathrm{Ei}_1\left(\mathrm{arcsinh}(dx+c)+\frac{a}{b}\right)}{8b}+\frac{e^2 \mathrm{e}^{-\frac{a}{b}} \mathrm{Ei}_1\left(-\,\mathrm{arcsinh}(dx+c)-\frac{a}{b}\right)}{8b}-\frac{e^2 \mathrm{e}^{-\frac{3a}{b}} \mathrm{Ei}_1\left(-3\,\mathrm{arcsinh}\right.}{8b}}{d}$

input
```
int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)+1/8*e^2/b*exp(a/b)
*Ei(1,arcsinh(d*x+c)+a/b)+1/8*e^2/b*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)-1/
8*e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b))
```

3.158.5 Fricas [F]

$$\int \frac{(ce+dex)^2}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^2}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b*arcsinh(d*x + c) + a), x
)
```

3.158.6 Sympy [F]

$$\int \frac{(ce+dex)^2}{a+b\text{arcsinh}(c+dx)}\,dx = e^2\left(\int \frac{c^2}{a+b\,\text{asinh}\,(c+dx)}\,dx + \int \frac{d^2x^2}{a+b\,\text{asinh}\,(c+dx)}\,dx + \int \frac{2cdx}{a+b\,\text{asinh}\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c)),x)
```

output
```
e**2*(Integral(c**2/(a + b*asinh(c + d*x)), x) + Integral(d**2*x**2/(a + b
*asinh(c + d*x)), x) + Integral(2*c*d*x/(a + b*asinh(c + d*x)), x))
```

3.158.7 Maxima [F]

$$\int \frac{(ce+dex)^2}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^2}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a), x)
```

3.158.8 Giac [F]

$$\int \frac{(ce+dex)^2}{a+b\text{arcsinh}(c+dx)}\,dx = \int \frac{(dex+ce)^2}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a), x)
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{a+b\operatorname{arcsinh}(c+dx)}\,dx=\int \frac{{\left(c\,e+d\,e\,x\right)}^2}{a+b\,\mathrm{asinh}\left(c+d\,x\right)}\,dx$$

input
```
int((c*e + d*e*x)^2/(a + b*asinh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^2/(a + b*asinh(c + d*x)), x)
```

3.159 $\int \frac{ce+dex}{a+b\text{arcsinh}(c+dx)}\,dx$

3.159.1 Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{ce+dex}{a+b\text{arcsinh}(c+dx)}\,dx = -\frac{e\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{2bd} + \frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2bd}$$

output
```
1/2*e*cosh(2*a/b)*Shi(2*(a+b*arcsinh(d*x+c))/b)/b/d-1/2*e*Chi(2*(a+b*arcsi
nh(d*x+c))/b)*sinh(2*a/b)/b/d
```

3.159.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{ce+dex}{a+b\text{arcsinh}(c+dx)}\,dx = -\frac{e\left(\text{Chi}\left(\frac{2a}{b}+2\text{arcsinh}(c+dx)\right)\sinh\left(\frac{2a}{b}\right)-\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{arcsinh}(c+dx)\right)\right)}{2bd}$$

input
```
Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x]),x]
```

output
```
-1/2*(e*(CoshIntegral[(2*a)/b + 2*ArcSinh[c + d*x]]*Sinh[(2*a)/b] - Cosh[(
2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c + d*x]]))/(b*d)
```

3.159.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6274, 27, 6195, 25, 5971, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{a+b\text{arcsinh}(c+dx)} \, dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \frac{e(c+dx)}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\int \frac{c+dx}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6195}$$

$$\frac{e\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{e\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{ 5971}$$

$$-\frac{e\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2(a+b\text{arcsinh}(c+dx))}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{e\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{2bd}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{e\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{2bd}$$

$\downarrow$ 26

$$\frac{ie\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{2bd}$$

$\downarrow$ 3784

$$\frac{ie\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))+\cosh\left(\frac{2a}{b}\right)\int -\frac{i\sinh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\right.}{2bd}$$

$\downarrow$ 26

$$\frac{ie\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\int \frac{\sinh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+ba\right.}{2bd}$$

$\downarrow$ 3042

$$\frac{ie\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\int -\frac{i\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a\right.}{2bd}$$

$\downarrow$ 26

$$\frac{ie\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-\cosh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+ba\right.}{2bd}$$

$\downarrow$ 3779

$$\frac{ie\left(i\sinh\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)\right)}{2bd}$$

$\downarrow$ 3782

3.159. $\int \frac{ce+dex}{a+b\text{arcsinh}(c+dx)}\,dx$

$$\frac{ie\left(i\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2(a+b\mathrm{arcsinh}(c+dx))}{b}\right)-i\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2(a+b\mathrm{arcsinh}(c+dx))}{b}\right)\right)}{2bd}$$

input
```
Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x]),x]
```

output
```
((I/2)*e*(I*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(2*a)/b] - I
*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b]))/(b*d)
```

3.159.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

3.159. $\int \frac{ce+dex}{a+b\mathrm{arcsinh}(c+dx)}\,dx$

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.159.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{e\,\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}_1\left(2\,\mathrm{arcsinh}(dx+c)+\frac{2a}{b}\right)}{4b}-\frac{e\,\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{Ei}_1\left(-2\,\mathrm{arcsinh}(dx+c)-\frac{2a}{b}\right)}{4b}}{d}$	66
default	$\frac{\frac{e\,\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}_1\left(2\,\mathrm{arcsinh}(dx+c)+\frac{2a}{b}\right)}{4b}-\frac{e\,\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{Ei}_1\left(-2\,\mathrm{arcsinh}(dx+c)-\frac{2a}{b}\right)}{4b}}{d}$	66

input
```
int((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*e/b*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/4*e/b*exp(-2*a/b)*E
i(1,-2*arcsinh(d*x+c)-2*a/b))
```

3.159.5 Fricas [F]

$$\int \frac{ce+dex}{a+b\text{arcsinh}(c+dx)}\,dx=\int \frac{dex+ce}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)/(b*arcsinh(d*x + c) + a), x)
```

3.159.6 Sympy [F]

$$\int \frac{ce+dex}{a+b\text{arcsinh}(c+dx)}\,dx=e\left(\int \frac{c}{a+b\,\text{asinh}\,(c+dx)}\,dx+\int \frac{dx}{a+b\,\text{asinh}\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)/(a+b*asinh(d*x+c)),x)
```

output
```
e*(Integral(c/(a + b*asinh(c + d*x)), x) + Integral(d*x/(a + b*asinh(c + d
*x)), x))
```

3.159.7 Maxima [F]

$$\int \frac{ce+dex}{a+b\text{arcsinh}(c+dx)}\,dx=\int \frac{dex+ce}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a), x)
```

3.159.8 Giac [F]

$$\int \frac{ce+dex}{a+b\text{arcsinh}(c+dx)}\,dx=\int \frac{dex+ce}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a), x)
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{a+b\text{arcsinh}(c+dx)}\,dx=\int \frac{c\,e+d\,e\,x}{a+b\,\text{asinh}\,(c+d\,x)}\,dx$$

input
```
int((c*e + d*e*x)/(a + b*asinh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)/(a + b*asinh(c + d*x)), x)
```

3.160 $\int \frac{1}{a+b\text{arcsinh}(c+dx)}\,dx$

3.160.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{a+b\text{arcsinh}(c+dx)}\,dx = \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{bd} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{bd}$$

output
```
Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b/d-Shi((a+b*arcsinh(d*x+c))/b)*sinh
(a/b)/b/d
```

3.160.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{1}{a+b\text{arcsinh}(c+dx)}\,dx = \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right) - \sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)}{bd}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(-1),x]
```

output
```
(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - Sinh[a/b]*SinhIntegral[a
/b + ArcSinh[c + d*x]])/(b*d)
```

3.160.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6273, 6189, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\text{arcsinh}(c+dx)}dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int \frac{1}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{6189}$$

$$\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{3784}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\sinh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c}{bd}$$

$$\Big\downarrow \text{26}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-\sinh\left(\frac{a}{b}\right)\int \frac{\sinh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+d}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-\sinh\left(\frac{a}{b}\right)\int -\frac{i\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}}{bd}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(}{bd}$$

$$\Big\downarrow \text{3779}$$

$$\frac{-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{3782}$$

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{bd}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(-1),x]
```

output
```
(Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c + d*x])/b] - Sinh[a/b]*SinhIntegr
al[(a + b*ArcSinh[c + d*x])/b])/(b*d)
```

3.160.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.160.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{-\frac{e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arcsinh}(dx+c)-\frac{a}{b}\right)}{2b}}{d}$	60
default	$\frac{-\frac{e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arcsinh}(dx+c)-\frac{a}{b}\right)}{2b}}{d}$	60

input
```
int(1/(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/b*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arcsin
h(d*x+c)-a/b))
```

3.160.5 Fricas [F]

$$\int \frac{1}{a+b\text{arcsinh}(c+dx)}\,dx=\int \frac{1}{b\,\operatorname{arsinh}(dx+c)+a}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(1/(b*arcsinh(d*x + c) + a), x)
```

3.160.6 Sympy [F]

$$\int \frac{1}{a+b\text{arcsinh}(c+dx)}\,dx=\int \frac{1}{a+b\,\text{asinh}\,(c+dx)}\,dx$$

input
```
integrate(1/(a+b*asinh(d*x+c)),x)
```

output
```
Integral(1/(a + b*asinh(c + d*x)), x)
```

3.160.7 Maxima [F]

$$\int \frac{1}{a+b\text{arcsinh}(c+dx)}\,dx=\int \frac{1}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arcsinh(d*x + c) + a), x)
```

3.160.8 Giac [F]

$$\int \frac{1}{a+b\text{arcsinh}(c+dx)}\,dx=\int \frac{1}{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/(b*arcsinh(d*x + c) + a), x)
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\text{arcsinh}(c+dx)}\,dx=\int \frac{1}{a+b\,\mathrm{asinh}\left(c+d\,x\right)}\,dx$$

input
```
int(1/(a + b*asinh(c + d*x)),x)
```

output
```
int(1/(a + b*asinh(c + d*x)), x)
```

3.161 $\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))}\,dx$

3.161.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))}\,dx = \frac{\text{Int}\left(\frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))},x\right)}{e}$$

output
```
Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c)),x)/e
```

3.161.2 Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))}\,dx = \int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))}\,dx$$

input
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])),x]
```

output
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])), x]
```

3.161.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 27, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))}dx$$
$$\Big\downarrow \text{6274}$$
$$\frac{\int \frac{1}{e(c+dx)(a+b\text{arcsinh}(c+dx))}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))}d(c+dx)}{de}$$
$$\Big\downarrow \text{6196}$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))}d(c+dx)}{de}$$

input
```
Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])),x]
```

output
```
$Aborted
```

3.161.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.161.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dex+ce)\,(a+b\ \mathrm{arcsinh}\,(dx+c))}dx$$

input
```
int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x)
```

output
```
int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x)
```

3.161.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1}{(ce+dex)(a+b\mathrm{arcsinh}(c+dx))}\,dx = \int \frac{1}{(dex+ce)(b\,\mathrm{arsinh}\,(dx+c)+a)}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(1/(a*d*e*x + a*c*e + (b*d*e*x + b*c*e)*arcsinh(d*x + c)), x)
```

3.161.6 Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{1}{(ce+dex)(a+b\mathrm{arcsinh}(c+dx))}\,dx = \frac{\int \frac{1}{ac+adx+bc\,\mathrm{asinh}\,(c+dx)+bdx\,\mathrm{asinh}\,(c+dx)}\,dx}{e}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c)),x)
```

output
```
Integral(1/(a*c + a*d*x + b*c*asinh(c + d*x) + b*d*x*asinh(c + d*x)), x)/e
```

3.161.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)), x)
```

3.161.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)), x)
```

3.161.9 Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))}\,dx = \int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}\,dx$$

input
```
int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))),x)
```

output
```
int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))), x)
```

3.162 $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^2}\,dx$

3.162.1 Optimal result

Integrand size = 23, antiderivative size = 256

$$\begin{aligned}\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^2}\,dx =& -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd(a+b\text{arcsinh}(c+dx))} \\ & - \frac{e^4\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b^2d} \\ & + \frac{9e^4\text{Chi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16b^2d} \\ & - \frac{5e^4\text{Chi}\left(\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{16b^2d} \\ & + \frac{e^4\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{8b^2d} \\ & - \frac{9e^4\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{16b^2d} \\ & + \frac{5e^4\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)}{16b^2d}\end{aligned}$$

output
```
1/8*e^4*cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^2/d-9/16*e^4*cosh(3*a/b)*S
hi(3*(a+b*arcsinh(d*x+c))/b)/b^2/d+5/16*e^4*cosh(5*a/b)*Shi(5*(a+b*arcsinh
(d*x+c))/b)/b^2/d-1/8*e^4*Chi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)/b^2/d+9/16
*e^4*Chi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^2/d-5/16*e^4*Chi(5*(a+b*a
rcsinh(d*x+c))/b)*sinh(5*a/b)/b^2/d-e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)/b/d/
(a+b*arcsinh(d*x+c))
```

3.162.2 Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.10

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^2}\,dx = \frac{e^4\left(-\frac{16b(c+dx)^4\sqrt{1+(c+dx)^2}}{a+b\text{arcsinh}(c+dx)}+16\left(3\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)\right)\sinh\left(\right.\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(e^4*((-16*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x]) +
 16*(3*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - CoshIntegral[3*(a/
b + ArcSinh[c + d*x])]*Sinh[(3*a)/b] - 3*Cosh[a/b]*SinhIntegral[a/b + ArcS
inh[c + d*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]) -
5*(10*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - 5*CoshIntegral[3*(a
/b + ArcSinh[c + d*x])]*Sinh[(3*a)/b] + CoshIntegral[5*(a/b + ArcSinh[c +
d*x])]*Sinh[(5*a)/b] - 10*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] +
 5*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])] - Cosh[(5*a)/b]*
SinhIntegral[5*(a/b + ArcSinh[c + d*x])])))/(16*b^2*d)
```

3.162.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6274, 27, 6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^2}\,dx$$

$$\Big\downarrow \ 6274$$

$$\frac{\int \frac{e^4(c+dx)^4}{(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{e^4\int \frac{(c+dx)^4}{(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{d}$$

$$\Big\downarrow \ 6193$$

$$\frac{e^4\left(\frac{\int \left(-\frac{5\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)}{16(a+b\text{arcsinh}(c+dx))}+\frac{9\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{16(a+b\text{arcsinh}(c+dx))}-\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{8(a+b\text{arcsinh}(c+dx))}\right)d(a+b\text{arcsinh}(c+dx))}{b^2}-\right.}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{e^4\left(\frac{-\frac{1}{8}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)+\frac{9}{16}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)-\frac{5}{16}\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)+\frac{1}{8}\cosh\left(\frac{a}{b}\right.}{b^2}\right.}{}$$

input
```
Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(e^4*(-(((c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*(a + b*ArcSinh[c + d*x])))
+ (-1/8*(CoshIntegral[(a + b*ArcSinh[c + d*x])/b]*Sinh[a/b]) + (9*CoshInte
gral[(3*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(3*a)/b])/16 - (5*CoshIntegral[(
5*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(5*a)/b])/16 + (Cosh[a/b]*SinhIntegral
[(a + b*ArcSinh[c + d*x])/b])/8 - (9*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*
ArcSinh[c + d*x]))/b])/16 + (5*Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSin
h[c + d*x]))/b])/16)/b^2))/d
```

3.162. $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^2}\,dx$

3.162.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.162.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(242) = 484$.

Time = 0.97 (sec) , antiderivative size = 602, normalized size of antiderivative = 2.35

method	result
derivativedivides	$\frac{\left(-16(dx+c)^4\sqrt{1+(dx+c)^2}+16(dx+c)^5-12(dx+c)^2\sqrt{1+(dx+c)^2}+20(dx+c)^3-\sqrt{1+(dx+c)^2}+5dx+5c\right)e^4}{32b(a+b\operatorname{arcsinh}(dx+c))}+\frac{5e^4 e^{\frac{5a}{b}}\operatorname{Ei}_1\left(5\operatorname{arcsinh}(\right.}{32b^2}$
default	$\frac{\left(-16(dx+c)^4\sqrt{1+(dx+c)^2}+16(dx+c)^5-12(dx+c)^2\sqrt{1+(dx+c)^2}+20(dx+c)^3-\sqrt{1+(dx+c)^2}+5dx+5c\right)e^4}{32b(a+b\operatorname{arcsinh}(dx+c))}+\frac{5e^4 e^{\frac{5a}{b}}\operatorname{Ei}_1\left(5\operatorname{arcsinh}(\right.}{32b^2}$

input
```
int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/32*(-16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+16*(d*x+c)^5-12*(d*x+c)^2*(1+
(d*x+c)^2)^(1/2)+20*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+5*d*x+5*c)*e^4/b/(a+b*ar
csinh(d*x+c))+5/32*e^4/b^2*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)-3/32*(-
4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+3*d*x+3*c)
*e^4/b/(a+b*arcsinh(d*x+c))-9/32*e^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+
3*a/b)+1/16*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^4/b/(a+b*arcsinh(d*x+c))+1/16*e
^4/b^2*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)-1/16/b*e^4*(d*x+c+(1+(d*x+c)^2)^(
1/2))/(a+b*arcsinh(d*x+c))-1/16/b^2*e^4*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b
)+3/32/b*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*
x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+9/32/b^2*e^4*exp(-3*a/b)*Ei(1,-3*arcsi
nh(d*x+c)-3*a/b)-1/32/b*e^4*(16*(d*x+c)^5+20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*
x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/
2))/(a+b*arcsinh(d*x+c))-5/32/b^2*e^4*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5
*a/b))
```

3.162.5 Fricas [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^4}{(b\operatorname{arsinh}(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*
x + c^4*e^4)/(b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2), x)
```

3.162.6 Sympy [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^2}\,dx = e^4\left(\int \frac{c^4}{a^2+2ab\operatorname{asinh}(c+dx)+b^2\operatorname{asinh}^2(c+dx)}\,dx + \int \frac{d^4x^4}{a^2+2ab\operatorname{asinh}(c+dx)+b^2\operatorname{asinh}^2(c+dx)}\,dx + \int \frac{4cd^3x^3}{a^2+2ab\operatorname{asinh}(c+dx)+b^2\operatorname{asinh}^2(c+dx)}\,dx + \int \frac{6c^2d^2x^2}{a^2+2ab\operatorname{asinh}(c+dx)+b^2\operatorname{asinh}^2(c+dx)}\,dx + \int \frac{4c^3dx}{a^2+2ab\operatorname{asinh}(c+dx)+b^2\operatorname{asinh}^2(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**2,x)
```

output
```
e**4*(Integral(c**4/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2)
, x) + Integral(d**4*x**4/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*
x)**2), x) + Integral(4*c*d**3*x**3/(a**2 + 2*a*b*asinh(c + d*x) + b**2*as
inh(c + d*x)**2), x) + Integral(6*c**2*d**2*x**2/(a**2 + 2*a*b*asinh(c + d
*x) + b**2*asinh(c + d*x)**2), x) + Integral(4*c**3*d*x/(a**2 + 2*a*b*asin
h(c + d*x) + b**2*asinh(c + d*x)**2), x))
```

3.162.7 Maxima [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^2}\,dx=\int \frac{(dex+ce)^4}{(b\,\text{arsinh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 + c^5*e^4 + (21*c^2*d^5*e^4 + d^
5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 + c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 + 2*c^2*
d^3*e^4)*x^3 + (21*c^5*d^2*e^4 + 10*c^3*d^2*e^4)*x^2 + (7*c^6*d*e^4 + 5*c^
4*d*e^4)*x + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 + c^4*e^4 + (15*c^2*
d^4*e^4 + d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*
e^4 + 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 + 2*c^3*d*e^4)*x)*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1))/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d + d)*a*b + (b^2
*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
)) + (a*b*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate(
(5*d^8*e^4*x^8 + 40*c*d^7*e^4*x^7 + 5*c^8*e^4 + 10*c^6*e^4 + 5*c^4*e^4 + 1
0*(14*c^2*d^6*e^4 + d^6*e^4)*x^6 + 20*(14*c^3*d^5*e^4 + 3*c*d^5*e^4)*x^5 +
 5*(70*c^4*d^4*e^4 + 30*c^2*d^4*e^4 + d^4*e^4)*x^4 + 20*(14*c^5*d^3*e^4 +
10*c^3*d^3*e^4 + c*d^3*e^4)*x^3 + 10*(14*c^6*d^2*e^4 + 15*c^4*d^2*e^4 + 3*
c^2*d^2*e^4)*x^2 + (5*d^6*e^4*x^6 + 30*c*d^5*e^4*x^5 + 5*c^6*e^4 + 3*c^4*e
^4 + 3*(25*c^2*d^4*e^4 + d^4*e^4)*x^4 + 4*(25*c^3*d^3*e^4 + 3*c*d^3*e^4)*x
^3 + 3*(25*c^4*d^2*e^4 + 6*c^2*d^2*e^4)*x^2 + 6*(5*c^5*d*e^4 + 2*c^3*d*e^4
)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 20*(2*c^7*d*e^4 + 3*c^5*d*e^4 + c^3*d
*e^4)*x + (10*d^7*e^4*x^7 + 70*c*d^6*e^4*x^6 + 10*c^7*e^4 + 13*c^5*e^4 + 4
*c^3*e^4 + (210*c^2*d^5*e^4 + 13*d^5*e^4)*x^5 + 5*(70*c^3*d^4*e^4 + 13*...
```

3.162.8 Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^2}\,dx=\int \frac{(dex+ce)^4}{(b\,\text{arsinh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^2, x)
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^2}\,dx=\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\text{asinh}\,(c+d\,x))^2}\,dx$$

input
```
int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^2, x)
```

3.163 $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^2}\,dx$

3.163.1 Optimal result

Integrand size = 23, antiderivative size = 188

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^2}\,dx = -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd(a+b\text{arcsinh}(c+dx))} - \frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2b^2d} + \frac{e^3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{2b^2d} + \frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2b^2d} - \frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{2b^2d}$$

output
```
-1/2*e^3*Chi(2*(a+b*arcsinh(d*x+c))/b)*cosh(2*a/b)/b^2/d+1/2*e^3*Chi(4*(a+
b*arcsinh(d*x+c))/b)*cosh(4*a/b)/b^2/d+1/2*e^3*Shi(2*(a+b*arcsinh(d*x+c))/
b)*sinh(2*a/b)/b^2/d-1/2*e^3*Shi(4*(a+b*arcsinh(d*x+c))/b)*sinh(4*a/b)/b^2
/d-e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))
```

3.163.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.03

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^2}\,dx =$$
$$-\frac{e^3\left(\frac{2b(c+dx)^3\sqrt{1+(c+dx)^2}}{a+b\text{arcsinh}(c+dx)}+\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)\right)-\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\text{arcsinh}(c+dx\right.\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
-1/2*(e^3*((2*b*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x]
) + Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c + d*x])] - Cosh[(4*a)/b]
*CoshIntegral[4*(a/b + ArcSinh[c + d*x])] - 3*Log[a + b*ArcSinh[c + d*x]]
- 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])] + 3*(Log[a + b*
ArcSinh[c + d*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])]
) + Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c + d*x])]))/(b^2*d)
```

3.163.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6274, 27, 6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^2}dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \frac{e^3(c+dx)^3}{(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e^3\int \frac{(c+dx)^3}{(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{d}$$

$$\Big\downarrow \; 6193$$

$$\frac{e^3\left(\frac{\int \left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{2(a+b\text{arcsinh}(c+dx))}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2(a+b\text{arcsinh}(c+dx))}\right)d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)^3\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}\right)}{d}$$

$\downarrow$ 2009

$$\frac{e^3\left(\frac{-\frac{1}{2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)+\frac{1}{2}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)+\frac{1}{2}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)-\frac{1}{2}\sinh(}{b^2}\right.}{d}$$

input
```
Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(e^3*(-(((c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*(a + b*ArcSinh[c + d*x])))
+ (-1/2*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b]) + (Co
sh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c + d*x]))/b])/2 + (Sinh[(2*a)/
b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/2 - (Sinh[(4*a)/b]*SinhIn
tegral[(4*(a + b*ArcSinh[c + d*x]))/b])/2)/b^2))/d
```

3.163.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

3.163. $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^2}\,dx$

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.163.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(178) = 356$.

Time = 0.91 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.06

method	result
derivativedivides	$\frac{\left(-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^4-4(dx+c)\sqrt{1+(dx+c)^2}+8(dx+c)^2+1\right)e^3}{16b(a+b\,\mathrm{arcsinh}(dx+c))} - \frac{e^3 e^{\frac{4a}{b}}\,\mathrm{Ei}_1\left(4\,\mathrm{arcsinh}(dx+c)+\frac{4a}{b}\right)}{4b^2} - \frac{\left(-2(dx+c)\sqrt{1 \ldots}\right.}{8b(a \ldots}$
default	$\frac{\left(-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^4-4(dx+c)\sqrt{1+(dx+c)^2}+8(dx+c)^2+1\right)e^3}{16b(a+b\,\mathrm{arcsinh}(dx+c))} - \frac{e^3 e^{\frac{4a}{b}}\,\mathrm{Ei}_1\left(4\,\mathrm{arcsinh}(dx+c)+\frac{4a}{b}\right)}{4b^2} - \frac{\left(-2(dx+c)\sqrt{1 \ldots}\right.}{8b(a \ldots}$

input
```
int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/16*(-8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+8*(d*x+c)^4-4*(d*x+c)*(1+(d*x+
c)^2)^(1/2)+8*(d*x+c)^2+1)*e^3/b/(a+b*arcsinh(d*x+c))-1/4*e^3/b^2*exp(4*a/
b)*Ei(1,4*arcsinh(d*x+c)+4*a/b)-1/8*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x
+c)^2+1)*e^3/b/(a+b*arcsinh(d*x+c))+1/4*e^3/b^2*exp(2*a/b)*Ei(1,2*arcsinh(
d*x+c)+2*a/b)+1/8/b*e^3*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b
*arcsinh(d*x+c))+1/4/b^2*e^3*exp(-2*a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b)-1/1
6/b*e^3*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)
*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c))-1/4/b^2*e^3*exp(-4*a/b)*Ei(1,
-4*arcsinh(d*x+c)-4*a/b))
```

3.163.5 Fricas [F]

$$\int \frac{(ce+dex)^3}{(a+b\mathrm{arcsinh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^3}{(b\,\mathrm{arsinh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^2*ar
csinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2), x)
```

3.163.6 Sympy [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^2}\,dx = e^3\left(\int \frac{c^3}{a^2+2ab\,\text{asinh}\,(c+dx)+b^2\,\text{asinh}^2\,(c+dx)}\,dx + \int \frac{d^3x^3}{a^2+2ab\,\text{asinh}\,(c+dx)+b^2\,\text{asinh}^2\,(c+dx)}\,dx + \int \frac{3cd^2x^2}{a^2+2ab\,\text{asinh}\,(c+dx)+b^2\,\text{asinh}^2\,(c+dx)}\,dx + \int \frac{3c^2dx}{a^2+2ab\,\text{asinh}\,(c+dx)+b^2\,\text{asinh}^2\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**2,x)
```

output
```
e**3*(Integral(c**3/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2)
, x) + Integral(d**3*x**3/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*
x)**2), x) + Integral(3*c*d**2*x**2/(a**2 + 2*a*b*asinh(c + d*x) + b**2*as
inh(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2 + 2*a*b*asinh(c + d*x) +
b**2*asinh(c + d*x)**2), x))
```

3.163.7 Maxima [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(d^6*e^3*x^6 + 6*c*d^5*e^3*x^5 + c^6*e^3 + c^4*e^3 + (15*c^2*d^4*e^3 + d^
4*e^3)*x^4 + 4*(5*c^3*d^3*e^3 + c*d^3*e^3)*x^3 + 3*(5*c^4*d^2*e^3 + 2*c^2*
d^2*e^3)*x^2 + 2*(3*c^5*d*e^3 + 2*c^3*d*e^3)*x + (d^5*e^3*x^5 + 5*c*d^4*e^
3*x^4 + c^5*e^3 + c^3*e^3 + (10*c^2*d^3*e^3 + d^3*e^3)*x^3 + (10*c^3*d^2*e
^3 + 3*c*d^2*e^3)*x^2 + (5*c^4*d*e^3 + 3*c^2*d*e^3)*x)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1))/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3*
x^2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) +
(a*b*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate((4*d^
7*e^3*x^7 + 28*c*d^6*e^3*x^6 + 4*c^7*e^3 + 8*c^5*e^3 + 4*c^3*e^3 + 4*(21*c
^2*d^5*e^3 + 2*d^5*e^3)*x^5 + 20*(7*c^3*d^4*e^3 + 2*c*d^4*e^3)*x^4 + 4*(35
*c^4*d^3*e^3 + 20*c^2*d^3*e^3 + d^3*e^3)*x^3 + 4*(21*c^5*d^2*e^3 + 20*c^3*
d^2*e^3 + 3*c*d^2*e^3)*x^2 + 2*(2*d^5*e^3*x^5 + 10*c*d^4*e^3*x^4 + 2*c^5*e
^3 + c^3*e^3 + (20*c^2*d^3*e^3 + d^3*e^3)*x^3 + (20*c^3*d^2*e^3 + 3*c*d^2*
e^3)*x^2 + (10*c^4*d*e^3 + 3*c^2*d*e^3)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) +
 4*(7*c^6*d*e^3 + 10*c^4*d*e^3 + 3*c^2*d*e^3)*x + (8*d^6*e^3*x^6 + 48*c*d^
5*e^3*x^5 + 8*c^6*e^3 + 10*c^4*e^3 + 3*c^2*e^3 + 10*(12*c^2*d^4*e^3 + d^4*
e^3)*x^4 + 40*(4*c^3*d^3*e^3 + c*d^3*e^3)*x^3 + 3*(40*c^4*d^2*e^3 + 20*c^2
*d^2*e^3 + d^2*e^3)*x^2 + 2*(24*c^5*d*e^3 + 20*c^3*d*e^3 + 3*c*d*e^3)*x)*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3...
```

3.163.8 Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^2}\,dx=\int \frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^2, x)
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^2}\,dx=\int \frac{{\left(c\,e+d\,e\,x\right)}^3}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^2}\,dx$$

input
```
int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^2, x)
```

3.164 $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^2}\,dx$

3.164.1 Optimal result

Integrand size = 23, antiderivative size = 184

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^2}\,dx = -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd(a+b\text{arcsinh}(c+dx))} + \frac{e^2\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2d} - \frac{3e^2\text{Chi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b^2d} - \frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{4b^2d} + \frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{4b^2d}$$

output
```
-1/4*e^2*cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^2/d+3/4*e^2*cosh(3*a/b)*S
hi(3*(a+b*arcsinh(d*x+c))/b)/b^2/d+1/4*e^2*Chi((a+b*arcsinh(d*x+c))/b)*sin
h(a/b)/b^2/d-3/4*e^2*Chi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^2/d-e^2*(
d*x+c)^2*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))
```

3.164.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.75

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^2}\,dx$$
$$= \frac{e^2\left(-\frac{4b(c+dx)^2\sqrt{1+(c+dx)^2}}{a+b\text{arcsinh}(c+dx)}+\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-3\text{Chi}\left(3\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)\right)\sinh\left(\frac{3a}{b}\right)-\right.}{4b^2d}$$

input
```
Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(e^2*((-4*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x]) +
CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - 3*CoshIntegral[3*(a/b + A
rcSinh[c + d*x])]*Sinh[(3*a)/b] - Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c +
 d*x]] + 3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]))/(4*b^2
*d)
```

3.164.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6274, 27, 6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^2}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{e^2(c+dx)^2}{(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\int \frac{(c+dx)^2}{(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{6193}$$

$$\frac{e^2\left(\frac{\int \left(\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{4(a+b\text{arcsinh}(c+dx))}-\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{4(a+b\text{arcsinh}(c+dx))}\right)d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)^2\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2\left(\frac{\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)-\frac{3}{4}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)-\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)+\frac{3}{4}\cosh\left(\frac{3a}{b}\right)\text{Shi}(}{b^2}\right.}{d}$$

input
```
Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(e^2*(-(((c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*(a + b*ArcSinh[c + d*x])))
+ ((CoshIntegral[(a + b*ArcSinh[c + d*x])/b]*Sinh[a/b])/4 - (3*CoshIntegra
l[(3*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(3*a)/b])/4 - (Cosh[a/b]*SinhIntegr
al[(a + b*ArcSinh[c + d*x])/b])/4 + (3*Cosh[(3*a)/b]*SinhIntegral[(3*(a +
b*ArcSinh[c + d*x]))/b])/4)/b^2))/d
```

3.164.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

3.164. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^2}\,dx$

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.164.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.86

method	result
derivativedivides	$\frac{\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2}{8b(a+b\ \mathrm{arcsinh}(dx+c))}+\frac{3e^2 e^{\frac{3a}{b}}\ \mathrm{Ei}_1\left(3\ \mathrm{arcsinh}(dx+c)+\frac{3a}{b}\right)}{8b^2}-\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)e}{8b(a+b\ \mathrm{arcsinh}(dx+c))}}{}$
default	$\frac{\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2}{8b(a+b\ \mathrm{arcsinh}(dx+c))}+\frac{3e^2 e^{\frac{3a}{b}}\ \mathrm{Ei}_1\left(3\ \mathrm{arcsinh}(dx+c)+\frac{3a}{b}\right)}{8b^2}-\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)e}{8b(a+b\ \mathrm{arcsinh}(dx+c))}}{}$

input
```
int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*(-4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)
+3*d*x+3*c)*e^2/b/(a+b*arcsinh(d*x+c))+3/8*e^2/b^2*exp(3*a/b)*Ei(1,3*arcsi
nh(d*x+c)+3*a/b)-1/8*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^2/b/(a+b*arcsinh(d*x+c
))-1/8*e^2/b^2*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)+1/8/b*e^2*(d*x+c+(1+(d*x+
c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+1/8/b^2*e^2*exp(-a/b)*Ei(1,-arcsinh(d*x+
c)-a/b)-1/8/b*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(
1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-3/8/b^2*e^2*exp(-3*a/b)*Ei(1,-3*a
rcsinh(d*x+c)-3*a/b))
```

3.164.5 Fricas [F]

$$\int \frac{(ce+dex)^2}{(a+b\mathrm{arcsinh}(c+dx))^2}\,dx=\int \frac{(dex+ce)^2}{(b\,\mathrm{arsinh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^2*arcsinh(d*x + c)^2 + 2
*a*b*arcsinh(d*x + c) + a^2), x)
```

3.164.6 Sympy [F]

$$\int \frac{(ce+dex)^2}{(a+b\mathrm{arcsinh}(c+dx))^2}\,dx = e^2\left(\int \frac{c^2}{a^2+2ab\,\mathrm{asinh}\,(c+dx)+b^2\,\mathrm{asinh}^2\,(c+dx)}\,dx + \int \frac{d^2x^2}{a^2+2ab\,\mathrm{asinh}\,(c+dx)+b^2\,\mathrm{asinh}^2\,(c+dx)}\,dx + \int \frac{2cdx}{a^2+2ab\,\mathrm{asinh}\,(c+dx)+b^2\,\mathrm{asinh}^2\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**2,x)
```

output
```
e**2*(Integral(c**2/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2)
, x) + Integral(d**2*x**2/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*
x)**2), x) + Integral(2*c*d*x/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c
+ d*x)**2), x))
```

3.164.7 Maxima [F]

$$\int \frac{(ce+dex)^2}{(a+b\mathrm{arcsinh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^2}{(b\,\mathrm{arsinh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(d^5*e^2*x^5 + 5*c*d^4*e^2*x^4 + c^5*e^2 + c^3*e^2 + (10*c^2*d^3*e^2 + d^
3*e^2)*x^3 + (10*c^3*d^2*e^2 + 3*c*d^2*e^2)*x^2 + (5*c^4*d*e^2 + 3*c^2*d*e
^2)*x + (d^4*e^2*x^4 + 4*c*d^3*e^2*x^3 + c^4*e^2 + c^2*e^2 + (6*c^2*d^2*e^
2 + d^2*e^2)*x^2 + 2*(2*c^3*d*e^2 + c*d*e^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1))/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3*x^2 + 2
*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^
2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate((3*d^6*e^2*x
^6 + 18*c*d^5*e^2*x^5 + 3*c^6*e^2 + 6*c^4*e^2 + 3*(15*c^2*d^4*e^2 + 2*d^4*
e^2)*x^4 + 3*c^2*e^2 + 12*(5*c^3*d^3*e^2 + 2*c*d^3*e^2)*x^3 + 3*(15*c^4*d^
2*e^2 + 12*c^2*d^2*e^2 + d^2*e^2)*x^2 + (3*d^4*e^2*x^4 + 12*c*d^3*e^2*x^3
+ 3*c^4*e^2 + c^2*e^2 + (18*c^2*d^2*e^2 + d^2*e^2)*x^2 + 2*(6*c^3*d*e^2 +
c*d*e^2)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 6*(3*c^5*d*e^2 + 4*c^3*d*e^2 +
 c*d*e^2)*x + (6*d^5*e^2*x^5 + 30*c*d^4*e^2*x^4 + 6*c^5*e^2 + 7*c^3*e^2 +
(60*c^2*d^3*e^2 + 7*d^3*e^2)*x^3 + 2*c*e^2 + 3*(20*c^3*d^2*e^2 + 7*c*d^2*e
^2)*x^2 + (30*c^4*d*e^2 + 21*c^2*d*e^2 + 2*d*e^2)*x)*sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1))/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*a*b*x^2
 + 4*(c^3*d + c*d)*a*b*x + (c^4 + 2*c^2 + 1)*a*b + (a*b*d^2*x^2 + 2*a*b*c*
d*x + a*b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*
x^3 + 2*(3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b^2*x + (c^4 + 2*c^...
```

3.164.8 Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{(dex+ce)^2}{(b\,\text{arsinh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^2, x)
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx=\int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\mathrm{asinh}\,(c+d\,x))^2}\,dx$$

input
```
int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^2, x)
```

3.165 $\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx$

3.165.1 Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{bd(a+b\operatorname{arcsinh}(c+dx))} + \frac{e\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{b^2d} - \frac{e\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{b^2d}$$

output
```
e*Chi(2*(a+b*arcsinh(d*x+c))/b)*cosh(2*a/b)/b^2/d-e*Shi(2*(a+b*arcsinh(d*x
+c))/b)*sinh(2*a/b)/b^2/d-e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d
*x+c))
```

3.165.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = \frac{e\left(-\frac{b(c+dx)\sqrt{1+c^2+2cdx+d^2x^2}}{a+b\operatorname{arcsinh}(c+dx)} + \cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)\right) - \sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(2\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right.\right.\right.}{b^2d}$$

input
```
Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(e*(-((b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])/(a + b*ArcSinh[c + d
*x])) + Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c + d*x])] - Sinh[(2*a
)/b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])]))/(b^2*d)
```

3.165.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6274, 27, 6193, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^2}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{e(c+dx)}{(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int \frac{c+dx}{(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{6193}$$

$$\frac{e\left(\frac{\int \frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e\left(-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}\right)}{d}$$

$$\Big\downarrow \text{3784}$$

$$e\left(-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{\cosh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\sinh\left(\frac{2a}{b}\right)\int-\frac{i\sinh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}}{b^2}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$\downarrow$ 26

$$e\left(\frac{\cosh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-\sinh\left(\frac{2a}{b}\right)\int\frac{\sinh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+}{b(a+}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$\downarrow$ 3042

$$e\left(-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-\sinh\left(\frac{2a}{b}\right)\int-\frac{i\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx)}{b}\right.}{a+b\text{arcsinh}(c+dx)}}{b^2}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$\downarrow$ 26

$$e\left(-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))+\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}}{b^2}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$\downarrow$ 3779

$$e\left(-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)+\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx)}{b^2}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$\downarrow$ 3782

$$\frac{e\left(\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{b^2}-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}\right)}{d}$$

3.165. $\int\frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^2}\,dx$

input
```
Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(e*(-(((c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*(a + b*ArcSinh[c + d*x]))) + (C
osh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b] - Sinh[(2*a)/b]*
SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/b^2))/d
```

3.165.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.165.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.55

method	result
derivativedivides	$\frac{\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e}{4b(a+b\,\text{arcsinh}(dx+c))}-\frac{e\,e^{\frac{2a}{b}}\,\text{Ei}_1\left(2\,\text{arcsinh}(dx+c)+\frac{2a}{b}\right)}{2b^2}-\frac{e\left(2(dx+c)^2+1+2(dx+c)\sqrt{1+(dx+c)^2}\right)}{4b(a+b\,\text{arcsinh}(dx+c))}-\frac{e\,e^{-\frac{2a}{b}}\,\text{E}}{}}{d}$
default	$\frac{\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e}{4b(a+b\,\text{arcsinh}(dx+c))}-\frac{e\,e^{\frac{2a}{b}}\,\text{Ei}_1\left(2\,\text{arcsinh}(dx+c)+\frac{2a}{b}\right)}{2b^2}-\frac{e\left(2(dx+c)^2+1+2(dx+c)\sqrt{1+(dx+c)^2}\right)}{4b(a+b\,\text{arcsinh}(dx+c))}-\frac{e\,e^{-\frac{2a}{b}}\,\text{E}}{}}{d}$

input
```
int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x+c)^2+1)*e/b/(a+b*arcsinh(d
*x+c))-1/2*e/b^2*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/4/b*e*(2*(d*x+c
)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/2/b^2*e*exp(-2
*a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b))
```

3.165.5 Fricas [F]

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{dex+ce}{\left(b\,\text{arsinh}\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)/(b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) +
a^2), x)
```

3.165.6 Sympy [F]

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = e\left(\int \frac{c}{a^2+2ab\operatorname{asinh}(c+dx)+b^2\operatorname{asinh}^2(c+dx)}\,dx + \int \frac{dx}{a^2+2ab\operatorname{asinh}(c+dx)+b^2\operatorname{asinh}^2(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**2,x)
```

output
```
e*(Integral(c/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2), x) +
 Integral(d*x/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2), x))
```

3.165.7 Maxima [F]

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^2}\,dx = \int \frac{dex+ce}{(b\operatorname{arsinh}(dx+c)+a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(d^4*e*x^4 + 4*c*d^3*e*x^3 + c^4*e + c^2*e + (6*c^2*d^2*e + d^2*e)*x^2 +
2*(2*c^3*d*e + c*d*e)*x + (d^3*e*x^3 + 3*c*d^2*e*x^2 + c^3*e + c*e + (3*c^
2*d*e + d*e)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^3*x^2 + 2*a*b*c*
d^2*x + (c^2*d + d)*a*b + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^2 +
 (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1)) + integrate((2*d^5*e*x^5 + 10*c*d^4*e*x^4 + 2*c^5*e + 4*
c^3*e + 4*(5*c^2*d^3*e + d^3*e)*x^3 + 4*(5*c^3*d^2*e + 3*c*d^2*e)*x^2 + 2*
(d^3*e*x^3 + 3*c*d^2*e*x^2 + 3*c^2*d*e*x + c^3*e)*(d^2*x^2 + 2*c*d*x + c^2
 + 1) + 2*c*e + 2*(5*c^4*d*e + 6*c^2*d*e + d*e)*x + (4*d^4*e*x^4 + 16*c*d^
3*e*x^3 + 4*c^4*e + 4*c^2*e + 4*(6*c^2*d^2*e + d^2*e)*x^2 + 8*(2*c^3*d*e +
 c*d*e)*x + e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^4*x^4 + 4*a*b*c*d
^3*x^3 + 2*(3*c^2*d^2 + d^2)*a*b*x^2 + 4*(c^3*d + c*d)*a*b*x + (c^4 + 2*c^
2 + 1)*a*b + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d^2*x^2 + 2*c*d*x + c^
2 + 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*b^2*x^2 + 4*
(c^3*d + c*d)*b^2*x + (c^4 + 2*c^2 + 1)*b^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + (3*c^2*d + d)*b^2*x + (c^3 + c)*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*d^3*x^3 + 3*a*b
*c*d^2*x^2 + (3*c^2*d + d)*a*b*x + (c^3 + c)*a*b)*sqrt(d^2*x^2 + 2*c*d*...
```

3.165.8 Giac [F]

$$\int \frac{ce + dex}{(a + b\text{arcsinh}(c + dx))^2}\,dx = \int \frac{dex + ce}{(b\,\text{arsinh}\,(dx + c) + a)^2}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^2, x)
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int \frac{ce + dex}{(a + b\text{arcsinh}(c + dx))^2} dx = \int \frac{c\,e + d\,e\,x}{(a + b\,\text{asinh}\,(c + d\,x))^2} dx$$

input
```
int((c*e + d*e*x)/(a + b*asinh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)/(a + b*asinh(c + d*x))^2, x)
```

3.166 $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^2} dx$

3.166.1 Optimal result

Integrand size = 12, antiderivative size = 91

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^2} dx = -\frac{\sqrt{1+(c+dx)^2}}{bd(a+b\text{arcsinh}(c+dx))} - \frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2d} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{b^2d}$$

output
```
cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^2/d-Chi((a+b*arcsinh(d*x+c))/b)*si
nh(a/b)/b^2/d-(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))
```

3.166.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^2} dx = \frac{-\frac{b\sqrt{1+(c+dx)^2}}{a+b\text{arcsinh}(c+dx)} - \text{Chi}\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)\sinh\left(\frac{a}{b}\right) + \cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)}{b^2d}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(-2),x]
```

output
```
(-((b*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])) - CoshIntegral[a/b
+ ArcSinh[c + d*x]]*Sinh[a/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d
*x]])/(b^2*d)
```

3.166.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6273, 6188, 6234, 25, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^2}dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int \frac{1}{(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{6188}$$

$$\frac{\frac{\int \frac{c+dx}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))}d(c+dx)}{b}-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}}{d}$$

$$\Big\downarrow \text{6234}$$

$$\frac{\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}-\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}}{d}$$

$\Big\downarrow$ 26

$$\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}}{d}$$

$\Big\downarrow$ 3784

$$\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))+\cosh\left(\frac{a}{b}\right)\int-\frac{i\sinh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+ba$$

$\Big\downarrow$ 26

$$\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\cosh\left(\frac{a}{b}\right)\int\frac{\sinh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+bar$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\cosh\left(\frac{a}{b}\right)\int-\frac{i\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}$$

$\Big\downarrow$ 26

$$\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a-$$

$\Big\downarrow$ 3779

$$\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\right)}{b^2}}{d}$$

$\Big\downarrow$ 3782

3.166. $\int\frac{1}{(a+b\text{arcsinh}(c+dx))^2}\,dx$

$$\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\right)}{b^2}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(-2),x]
```

output
```
(-(Sqrt[1 + (c + d*x)^2]/(b*(a + b*ArcSinh[c + d*x]))) + (I*(I*CoshIntegra
l[(a + b*ArcSinh[c + d*x])/b]*Sinh[a/b] - I*Cosh[a/b]*SinhIntegral[(a + b*
ArcSinh[c + d*x])/b]))/b^2)/d
```

3.166.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.166.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$\frac{\frac{-\sqrt{1+(dx+c)^2}+dx+c}{2b(a+b\,\mathrm{arcsinh}(dx+c))}+\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(dx+c)+\frac{a}{b}\right)}{2b^2}-\frac{dx+c+\sqrt{1+(dx+c)^2}}{2b(a+b\,\mathrm{arcsinh}(dx+c))}-\frac{\mathrm{e}^{-\frac{a}{b}}\,\mathrm{Ei}_1\left(-\,\mathrm{arcsinh}(dx+c)-\frac{a}{b}\right)}{2b^2}}{d}$	128
default	$\frac{\frac{-\sqrt{1+(dx+c)^2}+dx+c}{2b(a+b\,\mathrm{arcsinh}(dx+c))}+\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\mathrm{arcsinh}(dx+c)+\frac{a}{b}\right)}{2b^2}-\frac{dx+c+\sqrt{1+(dx+c)^2}}{2b(a+b\,\mathrm{arcsinh}(dx+c))}-\frac{\mathrm{e}^{-\frac{a}{b}}\,\mathrm{Ei}_1\left(-\,\mathrm{arcsinh}(dx+c)-\frac{a}{b}\right)}{2b^2}}{d}$	128

input
```
int(1/(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*(-(1+(d*x+c)^2)^(1/2)+d*x+c)/b/(a+b*arcsinh(d*x+c))+1/2/b^2*exp(a
/b)*Ei(1,arcsinh(d*x+c)+a/b)-1/2/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsin
h(d*x+c))-1/2/b^2*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b))
```

3.166.5 Fricas [F]

$$\int \frac{1}{(a + b\text{arcsinh}(c + dx))^2} dx = \int \frac{1}{(b \operatorname{arsinh}(dx + c) + a)^2} dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2), x)
```

3.166.6 Sympy [F]

$$\int \frac{1}{(a + b\text{arcsinh}(c + dx))^2} dx = \int \frac{1}{(a + b \operatorname{asinh}(c + dx))^2} dx$$

input
```
integrate(1/(a+b*asinh(d*x+c))**2,x)
```

output
```
Integral((a + b*asinh(c + d*x))**(-2), x)
```

3.166.7 Maxima [F]

$$\int \frac{1}{(a + b\text{arcsinh}(c + dx))^2} dx = \int \frac{1}{(b \operatorname{arsinh}(dx + c) + a)^2} dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2
 + 1)^(3/2) + c)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3
*x^2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) +
 (a*b*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate((d^4
*x^4 + 4*c*d^3*x^3 + c^4 + 2*(3*c^2*d^2 + d^2)*x^2 + (d^2*x^2 + 2*c*d*x +
c^2 + 1)*(d^2*x^2 + 2*c*d*x + c^2 - 1) + 2*c^2 + 4*(c^3*d + c*d)*x + (2*d^
3*x^3 + 6*c*d^2*x^2 + 2*c^3 + (6*c^2*d + d)*x + c)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1) + 1)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*a*b*x
^2 + 4*(c^3*d + c*d)*a*b*x + (c^4 + 2*c^2 + 1)*a*b + (a*b*d^2*x^2 + 2*a*b*
c*d*x + a*b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^4*x^4 + 4*b^2*c*d^
3*x^3 + 2*(3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b^2*x + (c^4 + 2*c^2
 + 1)*b^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x^2 + 2*c*d*x + c^2
 + 1) + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*b^2*x + (c^3 + c)
*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)) + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d + d)*a*b*x +
 (c^3 + c)*a*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

3.166.8 Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^2}\,dx=\int \frac{1}{(b\,\text{arsinh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(-2), x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^2}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(c+d\,x))^2}\,dx$$

input
```
int(1/(a + b*asinh(c + d*x))^2,x)
```

output
```
int(1/(a + b*asinh(c + d*x))^2, x)
```

3.167 $\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^2}\,dx$

3.167.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^2}\,dx = \frac{\text{Int}\left(\frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))^2},x\right)}{e}$$

output
```
Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^2,x)/e
```

3.167.2 Mathematica [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^2}\,dx$$

input
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2),x]
```

output
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2), x]
```

3.167.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 27, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^2}dx$$
$$\Big\downarrow \text{6274}$$
$$\frac{\int \frac{1}{e(c+dx)(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{de}$$
$$\Big\downarrow \text{6196}$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{de}$$

input
```
Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2),x]
```

output
```
$Aborted
```

3.167.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.167.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dex+ce)\,(a+b\,\operatorname{arcsinh}\,(dx+c))^2}dx$$

input
```
int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x)
```

output
```
int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x)
```

3.167.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d*e*x + a^2*c*e + (b^2*d*e*x + b^2*c*e)*arcsinh(d*x + c)^2
 + 2*(a*b*d*e*x + a*b*c*e)*arcsinh(d*x + c)), x)
```

3.167.6 Sympy [N/A]

Not integrable

Time = 1.89 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.17

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^2}\,dx = \frac{\int \frac{1}{a^2c+a^2dx+2abc\operatorname{asinh}(c+dx)+2abdx\operatorname{asinh}(c+dx)+b^2c\operatorname{asinh}^2(c+dx)+b^2dx\operatorname{asinh}^2(c+dx)}\,dx}{e}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**2,x)
```

output
```
Integral(1/(a**2*c + a**2*d*x + 2*a*b*c*asinh(c + d*x) + 2*a*b*d*x*asinh(c
 + d*x) + b**2*c*asinh(c + d*x)**2 + b**2*d*x*asinh(c + d*x)**2), x)/e
```

3.167.7 Maxima [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 1086, normalized size of antiderivative = 47.22

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{1}{(dex+ce)(b\operatorname{arsinh}(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2
 + 1)^(3/2) + c)/(a*b*d^4*e*x^3 + 3*a*b*c*d^3*e*x^2 + (3*c^2*d^2*e + d^2*e
)*a*b*x + (c^3*d*e + c*d*e)*a*b + (b^2*d^4*e*x^3 + 3*b^2*c*d^3*e*x^2 + (3*
c^2*d^2*e + d^2*e)*b^2*x + (c^3*d*e + c*d*e)*b^2 + (b^2*d^3*e*x^2 + 2*b^2*
c*d^2*e*x + b^2*c^2*d*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^3*e*x^2 + 2*a*b*c*d^2*e*x + a*
b*c^2*d*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - integrate((2*(d^2*x^2 + 2*
c*d*x + c^2 + 1)*(d*x + c) + (2*d^2*x^2 + 4*c*d*x + 2*c^2 + 1)*sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1))/(a*b*d^6*e*x^6 + 6*a*b*c*d^5*e*x^5 + (15*c^2*d^4*e
 + 2*d^4*e)*a*b*x^4 + 4*(5*c^3*d^3*e + 2*c*d^3*e)*a*b*x^3 + (15*c^4*d^2*e
+ 12*c^2*d^2*e + d^2*e)*a*b*x^2 + 2*(3*c^5*d*e + 4*c^3*d*e + c*d*e)*a*b*x
+ (c^6*e + 2*c^4*e + c^2*e)*a*b + (a*b*d^4*e*x^4 + 4*a*b*c*d^3*e*x^3 + 6*a
*b*c^2*d^2*e*x^2 + 4*a*b*c^3*d*e*x + a*b*c^4*e)*(d^2*x^2 + 2*c*d*x + c^2 +
 1) + (b^2*d^6*e*x^6 + 6*b^2*c*d^5*e*x^5 + (15*c^2*d^4*e + 2*d^4*e)*b^2*x^
4 + 4*(5*c^3*d^3*e + 2*c*d^3*e)*b^2*x^3 + (15*c^4*d^2*e + 12*c^2*d^2*e + d
^2*e)*b^2*x^2 + 2*(3*c^5*d*e + 4*c^3*d*e + c*d*e)*b^2*x + (c^6*e + 2*c^4*e
 + c^2*e)*b^2 + (b^2*d^4*e*x^4 + 4*b^2*c*d^3*e*x^3 + 6*b^2*c^2*d^2*e*x^2 +
 4*b^2*c^3*d*e*x + b^2*c^4*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b^2*d^5*e
*x^5 + 5*b^2*c*d^4*e*x^4 + (10*c^2*d^3*e + d^3*e)*b^2*x^3 + (10*c^3*d^2*e
+ 3*c*d^2*e)*b^2*x^2 + (5*c^4*d*e + 3*c^2*d*e)*b^2*x + (c^5*e + c^3*e)*...
```

3.167.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^2), x)
```

3.167.9 Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^2}\,dx = \int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^2),x)
```

output
```
int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^2), x)
```

3.168 $\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx$

3.168.1 Optimal result

Integrand size = 23, antiderivative size = 320

$$\begin{aligned}\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = & -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2}-\frac{2e^4(c+dx)^3}{b^2d(a+b\operatorname{arcsinh}(c+dx))}\\ & -\frac{5e^4(c+dx)^5}{2b^2d(a+b\operatorname{arcsinh}(c+dx))}\\ & +\frac{e^4\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{16b^3d}\\ & -\frac{27e^4\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^3d}\\ & +\frac{25e^4\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^3d}\\ & -\frac{e^4\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)}{16b^3d}\\ & +\frac{27e^4\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^3d}\\ & -\frac{25e^4\sinh\left(\frac{5a}{b}\right)\operatorname{Shi}\left(\frac{5(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{32b^3d}\end{aligned}$$

output
```
-2*e^4*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))-5/2*e^4*(d*x+c)^5/b^2/d/(a+b*a
rcsinh(d*x+c))+1/16*e^4*Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b^3/d-27/32*
e^4*Chi(3*(a+b*arcsinh(d*x+c))/b)*cosh(3*a/b)/b^3/d+25/32*e^4*Chi(5*(a+b*a
rcsinh(d*x+c))/b)*cosh(5*a/b)/b^3/d-1/16*e^4*Shi((a+b*arcsinh(d*x+c))/b)*s
inh(a/b)/b^3/d+27/32*e^4*Shi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^3/d-2
5/32*e^4*Shi(5*(a+b*arcsinh(d*x+c))/b)*sinh(5*a/b)/b^3/d-1/2*e^4*(d*x+c)^4
*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^2
```

3.168.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.99

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^3}\,dx = \frac{e^4\left(-\frac{16b^2(c+dx)^4\sqrt{1+(c+dx)^2}}{(a+b\text{arcsinh}(c+dx))^2}+\frac{16b(-4(c+dx)^3-5(c+dx)^5)}{a+b\text{arcsinh}(c+dx)}+48\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
(e^4*((-16*b^2*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])
^2 + (16*b*(-4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcSinh[c + d*x]) + 48
*(-(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]]) + Cosh[(3*a)/b]*CoshIn
tegral[3*(a/b + ArcSinh[c + d*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcSinh[
c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]) + 25*(
2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - 3*Cosh[(3*a)/b]*CoshInt
egral[3*(a/b + ArcSinh[c + d*x])] + Cosh[(5*a)/b]*CoshIntegral[5*(a/b + Ar
cSinh[c + d*x])] - 2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 3*Si
nh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])] - Sinh[(5*a)/b]*SinhI
ntegral[5*(a/b + ArcSinh[c + d*x])])))/(32*b^3*d)
```

3.168.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6274, 27, 6194, 6233, 6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.168. $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^3}\,dx$$

$\Big\downarrow$ 6274

$$\frac{\int \frac{e^4(c+dx)^4}{(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{d}$$

$\Big\downarrow$ 27

$$\frac{e^4\int \frac{(c+dx)^4}{(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{d}$$

$\Big\downarrow$ 6194

$$\frac{e^4\left(\frac{2\int \frac{(c+dx)^3}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{b}+\frac{5\int \frac{(c+dx)^5}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{2b}-\frac{\sqrt{(c+dx)^2+1}(c+dx)^4}{2b(a+b\text{arcsinh}(c+dx))^2}\right)}{d}$$

$\Big\downarrow$ 6233

$$\frac{e^4\left(\frac{2\left(\frac{3\int \frac{(c+dx)^2}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{b}-\frac{(c+dx)^3}{b(a+b\text{arcsinh}(c+dx))}\right)}{b}+\frac{5\left(\frac{5\int \frac{(c+dx)^4}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{b}-\frac{(c+dx)^5}{b(a+b\text{arcsinh}(c+dx))}\right)}{2b}-\frac{\sqrt{(c+dx}}{2b(a+b\text{arc}}\right.}{d}$$

$\Big\downarrow$ 6195

$$\frac{e^4\left(\frac{5\left(\frac{5\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)^5}{b(a+b\text{arcsinh}(c+dx))}\right)}{2b}+\frac{2\left(\frac{3\int \frac{\cosh\left(\frac{a}{b}-\frac{a+}{}\right.}{}}{}\right.}{}\right.}{d}$$

$\Big\downarrow$ 5971

3.168. $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

$$e^4\left(\frac{2\left(\frac{3\int\left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{4(a+b\text{arcsinh}(c+dx))}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{4(a+b\text{arcsinh}(c+dx))}\right)d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)^3}{b(a+b\text{arcsinh}(c+dx))}\right)}{b}+\frac{5\left(\frac{5\int\left(\frac{\cos\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 2009

$$e^4\left(\frac{2\left(\frac{3\left(-\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)+\frac{1}{4}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)+\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)-\frac{1}{4}\sinh\left(\frac{3a}{b}\right)\text{S}\cdots\right.}{b^2}\right.}{b}\right.$$

input
```
Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
(e^4*(-1/2*((c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*(a + b*ArcSinh[c + d*x])
^2) + (2*(-((c + d*x)^3/(b*(a + b*ArcSinh[c + d*x]))) + (3*(-1/4*(Cosh[a/b
]*CoshIntegral[(a + b*ArcSinh[c + d*x])/b]) + (Cosh[(3*a)/b]*CoshIntegral[
(3*(a + b*ArcSinh[c + d*x]))/b])/4 + (Sinh[a/b]*SinhIntegral[(a + b*ArcSin
h[c + d*x])/b])/4 - (Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c + d*x]
))/b])/4))/b^2))/b + (5*(-((c + d*x)^5/(b*(a + b*ArcSinh[c + d*x]))) + (5*
((Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c + d*x])/b])/8 - (3*Cosh[(3*a)/b]
*CoshIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/16 + (Cosh[(5*a)/b]*CoshInt
egral[(5*(a + b*ArcSinh[c + d*x]))/b])/16 - (Sinh[a/b]*SinhIntegral[(a + b
*ArcSinh[c + d*x])/b])/8 + (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh
[c + d*x]))/b])/16 - (Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c + d*x
]))/b])/16))/b^2))/(2*b)))/d
```

3.168. $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

3.168.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.168. $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

3.168.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(302) = 604$.

Time = 1.05 (sec) , antiderivative size = 896, normalized size of antiderivative = 2.80

method	result	size
derivativedivides	Expression too large to display	896
default	Expression too large to display	896

input
```
int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/64*(-16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+16*(d*x+c)^5-12*(d*x+c)^2*(1
+(d*x+c)^2)^(1/2)+20*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+5*d*x+5*c)*e^4*(5*b*arc
sinh(d*x+c)+5*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)-25/
64*e^4/b^3*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)+3/64*(-4*(d*x+c)^2*(1+(
d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+3*d*x+3*c)*e^4*(3*b*arcsin
h(d*x+c)+3*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+27/64*
e^4/b^3*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)-1/32*(-(1+(d*x+c)^2)^(1/2)
+d*x+c)*e^4*(b*arcsinh(d*x+c)+a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh
(d*x+c)+a^2)-1/32*e^4/b^3*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)-1/32/b*e^4*(d*
x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/32/b^2*e^4*(d*x+c+(1+(d*
x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/32/b^3*e^4*exp(-a/b)*Ei(1,-arcsinh(d
*x+c)-a/b)+3/64/b*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/
2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+9/64/b^2*e^4*(4*(d*x+c)^3+3
*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh
(d*x+c))+27/64/b^3*e^4*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b)-1/64/b*e^
4*(16*(d*x+c)^5+20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12
*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2
-5/64/b^2*e^4*(16*(d*x+c)^5+20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+
5*d*x+5*c+12*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsi
nh(d*x+c))-25/64/b^3*e^4*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5*a/b))
```

3.168.5 Fricas [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{(dex+ce)^4}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*
x + c^4*e^4)/(b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*
b*arcsinh(d*x + c) + a^3), x)
```

3.168.6 Sympy [F]

$$\begin{aligned}
&\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^3}\,dx \\
&= e^4\left(\int \frac{c^4}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx\right. \\
&\quad + \int \frac{d^4x^4}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx \\
&\quad + \int \frac{4cd^3x^3}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx \\
&\quad + \int \frac{6c^2d^2x^2}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx \\
&\quad \left. + \int \frac{4c^3dx}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx\right)
\end{aligned}$$

input
```
integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**3,x)
```

output
```
e**4*(Integral(c**4/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d
*x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(d**4*x**4/(a**3 + 3*a**2*b
*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x)
 + Integral(4*c*d**3*x**3/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh
(c + d*x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(6*c**2*d**2*x**2/(a*
*3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c +
 d*x)**3), x) + Integral(4*c**3*d*x/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*
b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x))
```

3.168.7 Maxima [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^3}\,dx=\int \frac{(dex+ce)^4}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*((5*a*d^11*e^4 + b*d^11*e^4)*x^11 + 11*(5*a*c*d^10*e^4 + b*c*d^10*e^4
)*x^10 + (5*(55*c^2*d^9*e^4 + 3*d^9*e^4)*a + (55*c^2*d^9*e^4 + 3*d^9*e^4)*
b)*x^9 + 3*(5*(55*c^3*d^8*e^4 + 9*c*d^8*e^4)*a + (55*c^3*d^8*e^4 + 9*c*d^8
*e^4)*b)*x^8 + 3*(5*(110*c^4*d^7*e^4 + 36*c^2*d^7*e^4 + d^7*e^4)*a + (110*
c^4*d^7*e^4 + 36*c^2*d^7*e^4 + d^7*e^4)*b)*x^7 + 21*(5*(22*c^5*d^6*e^4 + 1
2*c^3*d^6*e^4 + c*d^6*e^4)*a + (22*c^5*d^6*e^4 + 12*c^3*d^6*e^4 + c*d^6*e^
4)*b)*x^6 + (5*(462*c^6*d^5*e^4 + 378*c^4*d^5*e^4 + 63*c^2*d^5*e^4 + d^5*e
^4)*a + (462*c^6*d^5*e^4 + 378*c^4*d^5*e^4 + 63*c^2*d^5*e^4 + d^5*e^4)*b)*
x^5 + (5*(330*c^7*d^4*e^4 + 378*c^5*d^4*e^4 + 105*c^3*d^4*e^4 + 5*c*d^4*e^
4)*a + (330*c^7*d^4*e^4 + 378*c^5*d^4*e^4 + 105*c^3*d^4*e^4 + 5*c*d^4*e^4)
*b)*x^4 + (5*(165*c^8*d^3*e^4 + 252*c^6*d^3*e^4 + 105*c^4*d^3*e^4 + 10*c^2
*d^3*e^4)*a + (165*c^8*d^3*e^4 + 252*c^6*d^3*e^4 + 105*c^4*d^3*e^4 + 10*c^
2*d^3*e^4)*b)*x^3 + (5*(55*c^9*d^2*e^4 + 108*c^7*d^2*e^4 + 63*c^5*d^2*e^4
+ 10*c^3*d^2*e^4)*a + (55*c^9*d^2*e^4 + 108*c^7*d^2*e^4 + 63*c^5*d^2*e^4 +
 10*c^3*d^2*e^4)*b)*x^2 + ((5*a*d^8*e^4 + b*d^8*e^4)*x^8 + 8*(5*a*c*d^7*e^
4 + b*c*d^7*e^4)*x^7 + (4*(35*c^2*d^6*e^4 + 2*d^6*e^4)*a + (28*c^2*d^6*e^4
 + d^6*e^4)*b)*x^6 + 2*(4*(35*c^3*d^5*e^4 + 6*c*d^5*e^4)*a + (28*c^3*d^5*e
^4 + 3*c*d^5*e^4)*b)*x^5 + ((350*c^4*d^4*e^4 + 120*c^2*d^4*e^4 + 3*d^4*e^4
)*a + 5*(14*c^4*d^4*e^4 + 3*c^2*d^4*e^4)*b)*x^4 + 4*((70*c^5*d^3*e^4 + 40*
c^3*d^3*e^4 + 3*c*d^3*e^4)*a + (14*c^5*d^3*e^4 + 5*c^3*d^3*e^4)*b)*x^3 ...
```

3.168.8 Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^3}\,dx=\int \frac{(dex+ce)^4}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^3, x)
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^3}\,dx=\int \frac{{\left(c\,e+d\,e\,x\right)}^4}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^3, x)
```

3.169 $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

3.169.1 Optimal result

Integrand size = 23, antiderivative size = 247

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx = -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} - \frac{3e^3(c+dx)^2}{2b^2d(a+b\text{arcsinh}(c+dx))} - \frac{2e^3(c+dx)^4}{b^2d(a+b\text{arcsinh}(c+dx))} + \frac{e^3\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{2b^3d} - \frac{e^3\text{Chi}\left(\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{b^3d} - \frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2b^3d} + \frac{e^3\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{b^3d}$$

output
```
-3/2*e^3*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+c))-2*e^3*(d*x+c)^4/b^2/d/(a+b*a
rcsinh(d*x+c))-1/2*e^3*cosh(2*a/b)*Shi(2*(a+b*arcsinh(d*x+c))/b)/b^3/d+e^3
*cosh(4*a/b)*Shi(4*(a+b*arcsinh(d*x+c))/b)/b^3/d+1/2*e^3*Chi(2*(a+b*arcsin
h(d*x+c))/b)*sinh(2*a/b)/b^3/d-e^3*Chi(4*(a+b*arcsinh(d*x+c))/b)*sinh(4*a/
b)/b^3/d-1/2*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^2
```

3.169.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.72

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx = \frac{e^3\left(-\frac{b^2(c+dx)^3\sqrt{1+(c+dx)^2}}{(a+b\text{arcsinh}(c+dx))^2}+\frac{b(-3(c+dx)^2-4(c+dx)^4)}{a+b\text{arcsinh}(c+dx)}+\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)\right)\sinh\left(\frac{2a}{b}\right)-2\text{Chi}\left(4\left(\frac{a}{b}+\text{ar}\right.\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
(e^3*(-((b^2*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2
) + (b*(-3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcSinh[c + d*x]) + CoshIn
tegral[2*(a/b + ArcSinh[c + d*x])]*Sinh[(2*a)/b] - 2*CoshIntegral[4*(a/b +
 ArcSinh[c + d*x])]*Sinh[(4*a)/b] - Cosh[(2*a)/b]*SinhIntegral[2*(a/b + Ar
cSinh[c + d*x])] + 2*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c + d*x])
]))/(2*b^3*d)
```

3.169.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.15, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {6274, 27, 6194, 6233, 6195, 25, 5971, 27, 2009, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{e^3(c+dx)^3}{(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^3\int \frac{(c+dx)^3}{(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{d}$$

$$\Big\downarrow \ 6194$$

$$\frac{e^3\left(\frac{3\int \frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{2b}+\frac{2\int \frac{(c+dx)^4}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{b}-\frac{\sqrt{(c+dx)^2+1}(c+dx)^3}{2b(a+b\text{arcsinh}(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \ 6233$$

$$\frac{e^3\left(\frac{3\left(\frac{2\int \frac{c+dx}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{b}-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}\right)}{2b}+\frac{2\left(\frac{4\int \frac{(c+dx)^3}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{b}-\frac{(c+dx)^4}{b(a+b\text{arcsinh}(c+dx))}\right)}{b}-\frac{\sqrt{(c+dx\cdots}}{2b(a+b\text{ar}\cdots}\cdots\right.}{d}$$

$$\Big\downarrow \ 6195$$

$$\frac{e^3\left(\frac{2\left(\frac{4\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)^4}{b(a+b\text{arcsinh}(c+dx))}\right)}{b}+\frac{3\left(\frac{2\int -\frac{\cosh\left(\frac{a}{b}-\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{e^3\left(\frac{2\left(-\frac{4\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)^4}{b(a+b\text{arcsinh}(c+dx))}\right)}{b}+\frac{3\left(-\frac{2\int \frac{\cosh\left(\frac{a}{b}-\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{d}$$

$$\Big\downarrow \ 5971$$

3.169. $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

$$\frac{e^3\left(\frac{3\left(-\frac{2\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2(a+b\text{arcsinh}(c+dx))}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}\right)}{2b}+\frac{2\left(-\frac{4\int\left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{8(a+b\text{arcsinh}(c+dx))}-\cdots\right.}{\cdots}\right.}{\cdots}\right.}{d\cdots}$$

$\downarrow$ 27

$$\frac{e^3\left(\frac{3\left(-\frac{\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}\right)}{2b}+\frac{2\left(-\frac{4\int\left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{8(a+b\text{arcsinh}(c+dx))}-\cdots\right.}{\cdots}\right.}{\cdots}\right.}{d\cdots}$$

$\downarrow$ 2009

$$\frac{e^3\left(\frac{3\left(-\frac{\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}\right)}{2b}+\frac{2\left(\frac{4\left(\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$\downarrow$ 3042

3.169. $\displaystyle\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

$$e^3\left(\frac{3\left(-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}-\frac{\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}\right)}{2b}+\frac{2\left(\frac{4\left(\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+d\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 26

$$e^3\left(\frac{3\left(-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}\right)}{2b}+\frac{2\left(\frac{4\left(\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx)\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 3784

$$e^3\left(\frac{3\left(-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))+\cosh\left(\frac{2a}{b}\right)\int-\frac{i\sinh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}\right.}{b^2}\right.}{2b}\right.$$

$\downarrow$ 26

3.169. $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

$$e^3\left(\frac{3\left(-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\int\frac{\sinh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}\right.}{b^2}\right.}{2b}\right.$$

$\downarrow$ 3042

$$e^3\left(\frac{3\left(-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\int-\frac{i\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right.}{a+b\text{arcsinh}(c+dx)}\right.}{b^2}\right.}{2b}\right.$$

$\downarrow$ 26

$$e^3\left(\frac{3\left(-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}\right.}{b^2}\right.}{2b}\right.$$

$\downarrow$ 3779

3.169. $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

$$e^3\left(\frac{3\left(-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)\right.}{b^2}\right.}{2b}\right.$$

$\big\downarrow$ 3782

$$e^3\left(\frac{3\left(-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)\right)}{b^2}\right)}{2b}+\frac{2\left(\frac{4\left(\frac{1}{4}\sinh\left(\frac{2a}{b}\right)\text{C}\right.}{}\right.}{}\right.$$

input
```
Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
(e^3*(-1/2*((c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*(a + b*ArcSinh[c + d*x])
^2) + (3*(-((c + d*x)^2/(b*(a + b*ArcSinh[c + d*x]))) + (I*(I*CoshIntegral
[(2*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(2*a)/b] - I*Cosh[(2*a)/b]*SinhInteg
ral[(2*(a + b*ArcSinh[c + d*x]))/b]))/b^2))/(2*b) + (2*(-((c + d*x)^4/(b*(
a + b*ArcSinh[c + d*x]))) + (4*((CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))
/b]*Sinh[(2*a)/b])/4 - (CoshIntegral[(4*(a + b*ArcSinh[c + d*x]))/b]*Sinh[
(4*a)/b])/8 - (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])
/4 + (Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c + d*x]))/b])/8))/b^2)
)/b))/d
```

3.169. $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

3.169.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.169. $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3} \, dx$

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.169.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(237) = 474$.

Time = 1.02 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.34

method	result
derivativedivides	$\frac{-\frac{\left(-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^4-4(dx+c)\sqrt{1+(dx+c)^2}+8(dx+c)^2+1\right)e^3(4b\ \operatorname{arcsinh}(dx+c)+4a-b)}{32b^2\left(b^2 \operatorname{arcsinh}(dx+c)^2+2ab\ \operatorname{arcsinh}(dx+c)+a^2\right)}+\frac{e^3 e^{\frac{4a}{b}}\ \mathrm{Ei}_1\left(4\ \operatorname{arcsinh}(d\cdots\right.}{2b^3}}{\cdots}$
default	$\frac{-\frac{\left(-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^4-4(dx+c)\sqrt{1+(dx+c)^2}+8(dx+c)^2+1\right)e^3(4b\ \operatorname{arcsinh}(dx+c)+4a-b)}{32b^2\left(b^2 \operatorname{arcsinh}(dx+c)^2+2ab\ \operatorname{arcsinh}(dx+c)+a^2\right)}+\frac{e^3 e^{\frac{4a}{b}}\ \mathrm{Ei}_1\left(4\ \operatorname{arcsinh}(d\cdots\right.}{2b^3}}{\cdots}$

input
```
int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

3.169. $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

output
```
1/d*(-1/32*(-8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+8*(d*x+c)^4-4*(d*x+c)*(1+(d*x
+c)^2)^(1/2)+8*(d*x+c)^2+1)*e^3*(4*b*arcsinh(d*x+c)+4*a-b)/b^2/(b^2*arcsin
h(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+1/2*e^3/b^3*exp(4*a/b)*Ei(1,4*arcsinh
(d*x+c)+4*a/b)+1/16*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x+c)^2+1)*e^3*(2*
b*arcsinh(d*x+c)+2*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2
)-1/4*e^3/b^3*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)+1/16/b*e^3*(2*(d*x+c
)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+1/8/b^2*e^3*(2
*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+1/4/b^3*e
^3*exp(-2*a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b)-1/32/b*e^3*(8*(d*x+c)^4+8*(d*
x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a
+b*arcsinh(d*x+c))^2-1/8/b^2*e^3*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(
d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c))-1/2/
b^3*e^3*exp(-4*a/b)*Ei(1,-4*arcsinh(d*x+c)-4*a/b))
```

3.169.5 Fricas [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx=\int \frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^3*ar
csinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arcsinh(d*x + c) +
 a^3), x)
```

3.169.6 Sympy [F]

$$\begin{aligned}&\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx\\&=e^3\left(\int \frac{c^3}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx\right.\\&\quad+\int \frac{d^3x^3}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx\\&\quad+\int \frac{3cd^2x^2}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx\\&\quad\left.+\int \frac{3c^2dx}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx\right)\end{aligned}$$

input
```
integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**3,x)
```

output
```
e**3*(Integral(c**3/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d
*x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(d**3*x**3/(a**3 + 3*a**2*b
*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x)
 + Integral(3*c*d**2*x**2/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh
(c + d*x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(3*c**2*d*x/(a**3 + 3
*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)*
*3), x))
```

3.169.7 Maxima [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*((4*a*d^10*e^3 + b*d^10*e^3)*x^10 + 10*(4*a*c*d^9*e^3 + b*c*d^9*e^3)*
x^9 + 3*(4*(15*c^2*d^8*e^3 + d^8*e^3)*a + (15*c^2*d^8*e^3 + d^8*e^3)*b)*x^
8 + 24*(4*(5*c^3*d^7*e^3 + c*d^7*e^3)*a + (5*c^3*d^7*e^3 + c*d^7*e^3)*b)*x
^7 + 3*(4*(70*c^4*d^6*e^3 + 28*c^2*d^6*e^3 + d^6*e^3)*a + (70*c^4*d^6*e^3
+ 28*c^2*d^6*e^3 + d^6*e^3)*b)*x^6 + 6*(4*(42*c^5*d^5*e^3 + 28*c^3*d^5*e^3
 + 3*c*d^5*e^3)*a + (42*c^5*d^5*e^3 + 28*c^3*d^5*e^3 + 3*c*d^5*e^3)*b)*x^5
 + (4*(210*c^6*d^4*e^3 + 210*c^4*d^4*e^3 + 45*c^2*d^4*e^3 + d^4*e^3)*a + (
210*c^6*d^4*e^3 + 210*c^4*d^4*e^3 + 45*c^2*d^4*e^3 + d^4*e^3)*b)*x^4 + 4*(
4*(30*c^7*d^3*e^3 + 42*c^5*d^3*e^3 + 15*c^3*d^3*e^3 + c*d^3*e^3)*a + (30*c
^7*d^3*e^3 + 42*c^5*d^3*e^3 + 15*c^3*d^3*e^3 + c*d^3*e^3)*b)*x^3 + 3*(4*(1
5*c^8*d^2*e^3 + 28*c^6*d^2*e^3 + 15*c^4*d^2*e^3 + 2*c^2*d^2*e^3)*a + (15*c
^8*d^2*e^3 + 28*c^6*d^2*e^3 + 15*c^4*d^2*e^3 + 2*c^2*d^2*e^3)*b)*x^2 + ((4
*a*d^7*e^3 + b*d^7*e^3)*x^7 + 7*(4*a*c*d^6*e^3 + b*c*d^6*e^3)*x^6 + (6*(14
*c^2*d^5*e^3 + d^5*e^3)*a + (21*c^2*d^5*e^3 + d^5*e^3)*b)*x^5 + 5*(2*(14*c
^3*d^4*e^3 + 3*c*d^4*e^3)*a + (7*c^3*d^4*e^3 + c*d^4*e^3)*b)*x^4 + (2*(70*
c^4*d^3*e^3 + 30*c^2*d^3*e^3 + d^3*e^3)*a + 5*(7*c^4*d^3*e^3 + 2*c^2*d^3*e
^3)*b)*x^3 + (6*(14*c^5*d^2*e^3 + 10*c^3*d^2*e^3 + c*d^2*e^3)*a + (21*c^5*
d^2*e^3 + 10*c^3*d^2*e^3)*b)*x^2 + 2*(2*c^7*e^3 + 3*c^5*e^3 + c^3*e^3)*a +
 (c^7*e^3 + c^5*e^3)*b + (2*(14*c^6*d*e^3 + 15*c^4*d*e^3 + 3*c^2*d*e^3)*a
+ (7*c^6*d*e^3 + 5*c^4*d*e^3)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2)...
```

3.169.8 Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^3, x)
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\text{asinh}\,(c+d\,x))^3}\,dx$$

input
```
int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^3, x)
```

3.170 $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

3.170.1 Optimal result

Integrand size = 23, antiderivative size = 246

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^3}\,dx = -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd(a+b\text{arcsinh}(c+dx))^2} - \frac{e^2(c+dx)}{b^2d(a+b\text{arcsinh}(c+dx))} - \frac{3e^2(c+dx)^3}{2b^2d(a+b\text{arcsinh}(c+dx))} - \frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{8b^3d} + \frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{8b^3d} + \frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{8b^3d} - \frac{9e^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{8b^3d}$$

output
```
-e^2*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))-3/2*e^2*(d*x+c)^3/b^2/d/(a+b*arcsi
nh(d*x+c))-1/8*e^2*Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b^3/d+9/8*e^2*Chi
(3*(a+b*arcsinh(d*x+c))/b)*cosh(3*a/b)/b^3/d+1/8*e^2*Shi((a+b*arcsinh(d*x+
c))/b)*sinh(a/b)/b^3/d-9/8*e^2*Shi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b
^3/d-1/2*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^2
```

3.170.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.88

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^3}\,dx = \frac{e^2\left(-\frac{4b^2(c+dx)^2\sqrt{1+(c+dx)^2}}{(a+b\text{arcsinh}(c+dx))^2}+\frac{4b(-2(c+dx)-3(c+dx)^3)}{a+b\text{arcsinh}(c+dx)}+8\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)-8\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
(e^2*((-4*b^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^
2 + (4*b*(-2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcSinh[c + d*x]) + 8*Cosh
[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - 8*Sinh[a/b]*SinhIntegral[a/b
+ ArcSinh[c + d*x]] + 9*(-(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]])
 + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c + d*x])] + Sinh[a/b]*Sinh
Integral[a/b + ArcSinh[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + Arc
Sinh[c + d*x])])))/(8*b^3*d)
```

3.170.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {6274, 27, 6194, 6233, 6189, 3042, 3784, 26, 3042, 26, 3779, 3782, 6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^3}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{e^2(c+dx)^2}{(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\int \frac{(c+dx)^2}{(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{6194}$$

$$\frac{e^2\left(\frac{\int \frac{c+dx}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{b}+\frac{3\int \frac{(c+dx)^3}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{2b}-\frac{\sqrt{(c+dx)^2+1}(c+dx)^2}{2b(a+b\text{arcsinh}(c+dx))^2}\right)}{d}$$

$\big\downarrow$ 6233

$$\frac{e^2\left(\frac{\frac{\int \frac{1}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{b}-\frac{c+dx}{b(a+b\text{arcsinh}(c+dx))}}{b}+\frac{3\left(\frac{3\int \frac{(c+dx)^2}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{b}-\frac{(c+dx)^3}{b(a+b\text{arcsinh}(c+dx))}\right)}{2b}-\frac{\sqrt{(c+dx)^2+1}(\cdots}{2b(a+b\text{arcsinh}\cdots}\right.}{d}$$

$\big\downarrow$ 6189

$$\frac{e^2\left(\frac{\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{c+dx}{b(a+b\text{arcsinh}(c+dx))}}{b}+\frac{3\left(\frac{3\int \frac{(c+dx)^2}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{b}-\frac{(c+dx)^3}{b(a+b\text{arcsinh}(c+d\cdots}\right.}{2b}\right.}{d}$$

$\big\downarrow$ 3042

$$\frac{e^2\left(\frac{-\frac{c+dx}{b(a+b\text{arcsinh}(c+dx))}+\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}}{b}+\frac{3\left(\frac{3\int \frac{(c+dx)^2}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{b}-\frac{(c+dx\cdots}{b(a+b\text{arcsin}\cdots}\right.}{2b}\right.}{d}$$

$\big\downarrow$ 3784

$$\frac{e^2\left(\frac{-\frac{c+dx}{b(a+b\text{arcsinh}(c+dx))}+\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\sinh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsi}\cdots}{b^2}}{b}\right.}{d}$$

$\big\downarrow$ 26

3.170. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

$$e^2\left(\frac{\frac{\frac{\cosh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-\sinh\left(\frac{a}{b}\right)\int\frac{\sinh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{c+dx}{b(a+b\text{arcsinh}(c+}}{b}}{d}\right.$$

$$\downarrow \text{3042}$$

$$e^2\left(\frac{-\frac{c+dx}{b(a+b\text{arcsinh}(c+dx))}+\frac{\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-\sinh\left(\frac{a}{b}\right)\int-\frac{i\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{ar}}{b^2}}{b}\right.$$

$$\downarrow \text{26}$$

$$e^2\left(\frac{-\frac{c+dx}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arc}}{b^2}}{b}\right.$$

$$\downarrow \text{3779}$$

$$e^2\left(\frac{-\frac{c+dx}{b(a+b\text{arcsinh}(c+dx))}+\frac{-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}}{b}+\frac{3\left(\frac{3\int}{}\right.}{}\right.$$

$$\downarrow \text{3782}$$

$$e^2\left(\frac{3\left(\frac{3\int\frac{(c+dx)^2}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{b}-\frac{(c+dx)^3}{b(a+b\text{arcsinh}(c+dx))}\right)}{2b}+\frac{\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{b^2}-}{b}\right.$$

3.170. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

$\Big\downarrow$ 6195

$$e^2\left(\frac{3\left(\frac{3\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)^3}{b(a+b\text{arcsinh}(c+dx))}\right)}{2b}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\dots}{}\right)}{}\right.$$

$$\overline{\qquad\qquad d\qquad\qquad}$$

$\Big\downarrow$ 5971

$$e^2\left(\frac{3\left(\frac{3\int\left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{4(a+b\text{arcsinh}(c+dx))}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{4(a+b\text{arcsinh}(c+dx))}\right)d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)^3}{b(a+b\text{arcsinh}(c+dx))}\right)}{2b}+\frac{\cosh\left(\frac{a}{b}\right)\text{C}\dots}{}\right.$$

$$\overline{\qquad\qquad d\qquad\qquad}$$

$\Big\downarrow$ 2009

$$e^2\left(\frac{\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{b^2}-\frac{c+dx}{b(a+b\text{arcsinh}(c+dx))}}{b}+\frac{3\left(\frac{3\left(-\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c\dots}{b}\right.\right.}{}\right.}{}\right.$$

input
```
Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^3,x]
```

3.170. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

output
```
(e^2*(-1/2*((c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*(a + b*ArcSinh[c + d*x])
^2) + (-((c + d*x)/(b*(a + b*ArcSinh[c + d*x]))) + (Cosh[a/b]*CoshIntegral
[(a + b*ArcSinh[c + d*x])/b] - Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d
*x])/b])/b^2)/b + (3*(-((c + d*x)^3/(b*(a + b*ArcSinh[c + d*x]))) + (3*(-1
/4*(Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c + d*x])/b]) + (Cosh[(3*a)/b]*C
oshIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/4 + (Sinh[a/b]*SinhIntegral[(
a + b*ArcSinh[c + d*x])/b])/4 - (Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcS
inh[c + d*x]))/b])/4))/b^2))/(2*b)))/d
```

3.170.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

3.170. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.170. $\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx$

3.170.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(232) = 464$.

Time = 0.48 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.06

method	result
derivativedivides	$\frac{-\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2(3b\operatorname{arcsinh}(dx+c)+3a-b)}{16b^2\left(b^2\operatorname{arcsinh}(dx+c)^2+2ab\operatorname{arcsinh}(dx+c)+a^2\right)}-\frac{9e^2e^{\frac{3a}{b}}\operatorname{Ei}_1\left(3\operatorname{arcsinh}(dx+c)+\frac{3a}{b}\right)}{16b^3}}{}$
default	$\frac{-\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2(3b\operatorname{arcsinh}(dx+c)+3a-b)}{16b^2\left(b^2\operatorname{arcsinh}(dx+c)^2+2ab\operatorname{arcsinh}(dx+c)+a^2\right)}-\frac{9e^2e^{\frac{3a}{b}}\operatorname{Ei}_1\left(3\operatorname{arcsinh}(dx+c)+\frac{3a}{b}\right)}{16b^3}}{}$

input
```
int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/16*(-4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/
2)+3*d*x+3*c)*e^2*(3*b*arcsinh(d*x+c)+3*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a
*b*arcsinh(d*x+c)+a^2)-9/16*e^2/b^3*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b
)+1/16*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^2*(b*arcsinh(d*x+c)+a-b)/b^2/(b^2*ar
csinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+1/16*e^2/b^3*exp(a/b)*Ei(1,arcsin
h(d*x+c)+a/b)+1/16/b*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^
2+1/16/b^2*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+1/16/b^3*e
^2*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)-1/16/b*e^2*(4*(d*x+c)^3+3*d*x+3*c+4
*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2
-3/16/b^2*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d
*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-9/16/b^3*e^2*exp(-3*a/b)*Ei(1,-3*arcs
inh(d*x+c)-3*a/b))
```

3.170.5 Fricas [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx=\int \frac{(dex+ce)^2}{(b\operatorname{arsinh}(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^3*arcsinh(d*x + c)^3 + 3
*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arcsinh(d*x + c) + a^3), x)
```

3.170.6 Sympy [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^3}\,dx = e^2\left(\int \frac{c^2}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx + \int \frac{d^2x^2}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx + \int \frac{2cdx}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**3,x)
```

output
```
e**2*(Integral(c**2/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d
*x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(d**2*x**2/(a**3 + 3*a**2*b
*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x)
 + Integral(2*c*d*x/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d
*x)**2 + b**3*asinh(c + d*x)**3), x))
```

3.170.7 Maxima [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{(dex+ce)^2}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*((3*a*d^9*e^2 + b*d^9*e^2)*x^9 + 9*(3*a*c*d^8*e^2 + b*c*d^8*e^2)*x^8
+ 3*(3*(12*c^2*d^7*e^2 + d^7*e^2)*a + (12*c^2*d^7*e^2 + d^7*e^2)*b)*x^7 +
21*(3*(4*c^3*d^6*e^2 + c*d^6*e^2)*a + (4*c^3*d^6*e^2 + c*d^6*e^2)*b)*x^6 +
 3*(3*(42*c^4*d^5*e^2 + 21*c^2*d^5*e^2 + d^5*e^2)*a + (42*c^4*d^5*e^2 + 21
*c^2*d^5*e^2 + d^5*e^2)*b)*x^5 + 3*(3*(42*c^5*d^4*e^2 + 35*c^3*d^4*e^2 + 5
*c*d^4*e^2)*a + (42*c^5*d^4*e^2 + 35*c^3*d^4*e^2 + 5*c*d^4*e^2)*b)*x^4 + (
3*(84*c^6*d^3*e^2 + 105*c^4*d^3*e^2 + 30*c^2*d^3*e^2 + d^3*e^2)*a + (84*c^
6*d^3*e^2 + 105*c^4*d^3*e^2 + 30*c^2*d^3*e^2 + d^3*e^2)*b)*x^3 + 3*(3*(12*
c^7*d^2*e^2 + 21*c^5*d^2*e^2 + 10*c^3*d^2*e^2 + c*d^2*e^2)*a + (12*c^7*d^2
*e^2 + 21*c^5*d^2*e^2 + 10*c^3*d^2*e^2 + c*d^2*e^2)*b)*x^2 + ((3*a*d^6*e^2
 + b*d^6*e^2)*x^6 + 6*(3*a*c*d^5*e^2 + b*c*d^5*e^2)*x^5 + ((45*c^2*d^4*e^2
 + 4*d^4*e^2)*a + (15*c^2*d^4*e^2 + d^4*e^2)*b)*x^4 + 4*((15*c^3*d^3*e^2 +
 4*c*d^3*e^2)*a + (5*c^3*d^3*e^2 + c*d^3*e^2)*b)*x^3 + ((45*c^4*d^2*e^2 +
24*c^2*d^2*e^2 + d^2*e^2)*a + 3*(5*c^4*d^2*e^2 + 2*c^2*d^2*e^2)*b)*x^2 + (
3*c^6*e^2 + 4*c^4*e^2 + c^2*e^2)*a + (c^6*e^2 + c^4*e^2)*b + 2*((9*c^5*d*e
^2 + 8*c^3*d*e^2 + c*d*e^2)*a + (3*c^5*d*e^2 + 2*c^3*d*e^2)*b)*x)*(d^2*x^2
 + 2*c*d*x + c^2 + 1)^(3/2) + (3*(3*a*d^7*e^2 + b*d^7*e^2)*x^7 + 21*(3*a*c
*d^6*e^2 + b*c*d^6*e^2)*x^6 + ((189*c^2*d^5*e^2 + 17*d^5*e^2)*a + (63*c^2*
d^5*e^2 + 5*d^5*e^2)*b)*x^5 + 5*((63*c^3*d^4*e^2 + 17*c*d^4*e^2)*a + (21*c
^3*d^4*e^2 + 5*c*d^4*e^2)*b)*x^4 + (5*(63*c^4*d^3*e^2 + 34*c^2*d^3*e^2 ...
```

3.170.8 Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^3}\,dx=\int \frac{(dex+ce)^2}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^3, x)
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^3}\,dx=\int \frac{{\left(c\,e+d\,e\,x\right)}^2}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^3, x)
```

3.171 $\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx$

3.171.1 Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx = -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd(a+b\operatorname{arcsinh}(c+dx))^2} - \frac{e}{2b^2d(a+b\operatorname{arcsinh}(c+dx))} - \frac{e(c+dx)^2}{b^2d(a+b\operatorname{arcsinh}(c+dx))} - \frac{e\operatorname{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{b^3d} + \frac{e\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{b^3d}$$

output
```
-1/2*e/b^2/d/(a+b*arcsinh(d*x+c))-e*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+c))+e
*cosh(2*a/b)*Shi(2*(a+b*arcsinh(d*x+c))/b)/b^3/d-e*Chi(2*(a+b*arcsinh(d*x+
c))/b)*sinh(2*a/b)/b^3/d-1/2*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsin
h(d*x+c))^2
```

3.171.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.77

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^3}\,dx = \frac{e\left(-\frac{b^2(c+dx)\sqrt{1+(c+dx)^2}}{(a+b\text{arcsinh}(c+dx))^2}+\frac{b(-1-2(c+dx)^2)}{a+b\text{arcsinh}(c+dx)}-2\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)\right)\sinh\left(\frac{2a}{b}\right)+2\cosh\left(\frac{2a}{b}\right)\text{Shi}(2(\right.}{2b^3d}$$

input
```
Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
(e*(-((b^2*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2) +
(b*(-1 - 2*(c + d*x)^2))/(a + b*ArcSinh[c + d*x]) - 2*CoshIntegral[2*(a/b
+ ArcSinh[c + d*x])]*Sinh[(2*a)/b] + 2*Cosh[(2*a)/b]*SinhIntegral[2*(a/b +
 ArcSinh[c + d*x])]))/(2*b^3*d)
```

3.171.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.810, Rules used = {6274, 27, 6194, 6198, 6233, 6195, 25, 5971, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^3}dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \frac{e(c+dx)}{(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\int \frac{c+dx}{(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6194}$$

$$\frac{e\left(\frac{\int \frac{1}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{2b}+\frac{\int \frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{b}-\frac{\sqrt{(c+dx)^2+1}(c+dx)}{2b(a+b\text{arcsinh}(c+dx))^2}\right)}{d}$$

$$\downarrow 6198$$

$$\frac{e\left(\frac{\int \frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{b}-\frac{1}{2b^2(a+b\text{arcsinh}(c+dx))}-\frac{\sqrt{(c+dx)^2+1}(c+dx)}{2b(a+b\text{arcsinh}(c+dx))^2}\right)}{d}$$

$$\downarrow 6233$$

$$\frac{e\left(\frac{\frac{2\int \frac{c+dx}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{b}-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}}{b}-\frac{1}{2b^2(a+b\text{arcsinh}(c+dx))}-\frac{\sqrt{(c+dx)^2+1}(c+dx)}{2b(a+b\text{arcsinh}(c+dx))^2}\right)}{d}$$

$$\downarrow 6195$$

$$\frac{e\left(\frac{\frac{2\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}}{b}-\frac{1}{2b^2(a+b\text{arcsinh}(c+dx)}\right.}{d}$$

$$\downarrow 25$$

$$\frac{e\left(\frac{-\frac{2\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}}{b}-\frac{1}{2b^2(a+b\text{arcsinh}(c+dx}\right.}{d}$$

$$\downarrow 5971$$

3.171. $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

$$\frac{e\left(\frac{-\frac{2\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2(a+b\text{arcsinh}(c+dx))}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}}{b}-\frac{1}{2b^2(a+b\text{arcsinh}(c+dx))}-\frac{\sqrt{(c+dx)^2+1}(c+\cdots}{2b(a+b\text{arcsinh}(c+\cdots}\right.}{d}$$

$\downarrow$ 27

$$\frac{e\left(\frac{-\frac{\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}}{b}-\frac{1}{2b^2(a+b\text{arcsinh}(c+dx))}-\frac{\sqrt{(c+dx)^2+1}(c+dx\cdots}{2b(a+b\text{arcsinh}(c+dx\cdots}\right.}{d}$$

$\downarrow$ 3042

$$\frac{e\left(\frac{-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}-\frac{\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}}{b}-\frac{1}{2b^2(a+b\text{arcsinh}(c+dx))}-\frac{\sqrt{(c+dx)^2+1}(c\cdots}{2b(a+b\text{arcsinh}(c\cdots}\right.}{d}$$

$\downarrow$ 26

$$\frac{e\left(\frac{-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}}{b}-\frac{1}{2b^2(a+b\text{arcsinh}(c+dx))}-\frac{\sqrt{(c+dx)^2+1}(c+\cdots}{2b(a+b\text{arcsinh}(c+\cdots}\right.}{d}$$

$\downarrow$ 3784

3.171. $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

$$e\left(\frac{-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))+\cosh\left(\frac{2a}{b}\right)\int-\frac{i\sinh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a-\right.}{b^2}}{b}\right.}{d}$$

$\downarrow$ 26

$$e\left(\frac{-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\int\frac{\sinh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\right.}{b^2}}{b}\right.}{d}$$

$\downarrow$ 3042

$$e\left(\frac{-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\int-\frac{i\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}\right.}{b^2}}{b}\right.}{d}$$

$\downarrow$ 26

$$e\left(\frac{-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+\right.}{b^2}}{b}\right.}{d}$$

$\downarrow$ 3779

3.171. $\int\frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

$$\frac{e\left(\frac{-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)\right)}{b^2}}{b} -\right.}{d}$$

$$\Big\downarrow \text{3782}$$

$$\frac{e\left(\frac{-\frac{(c+dx)^2}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)-i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)\right)}{b^2}}{b}-\frac{1}{2b^2(a+b\text{arcsinh}(c+dx)}\right.}{d}$$

input
```
Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
(e*(-1/2*((c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*(a + b*ArcSinh[c + d*x])^2)
- 1/(2*b^2*(a + b*ArcSinh[c + d*x])) + (-((c + d*x)^2/(b*(a + b*ArcSinh[c
+ d*x]))) + (I*(I*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(2*a)/
b] - I*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b]))/b^2)/b
))/d
```

3.171.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

3.171. $\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx$

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.171.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.53

method	result
derivativedivides	$\frac{-\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e(2b\operatorname{arcsinh}(dx+c)+2a-b)}{8b^2\left(b^2\operatorname{arcsinh}(dx+c)^2+2ab\operatorname{arcsinh}(dx+c)+a^2\right)}+\frac{e\,e^{\frac{2a}{b}}\operatorname{Ei}_1\left(2\operatorname{arcsinh}(dx+c)+\frac{2a}{b}\right)}{2b^3}-\frac{e\left(2(dx+c)^2+1+2(dx+c\right.}{8b(a+b\operatorname{arcsinh}(}}{d}$
default	$\frac{-\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e(2b\operatorname{arcsinh}(dx+c)+2a-b)}{8b^2\left(b^2\operatorname{arcsinh}(dx+c)^2+2ab\operatorname{arcsinh}(dx+c)+a^2\right)}+\frac{e\,e^{\frac{2a}{b}}\operatorname{Ei}_1\left(2\operatorname{arcsinh}(dx+c)+\frac{2a}{b}\right)}{2b^3}-\frac{e\left(2(dx+c)^2+1+2(dx+c\right.}{8b(a+b\operatorname{arcsinh}(}}{d}$

input
```
int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x+c)^2+1)*e*(2*b*arcsinh(d*
x+c)+2*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+1/2*e/b^3*
exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/8/b*e*(2*(d*x+c)^2+1+2*(d*x+c)*(
1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/4/b^2*e*(2*(d*x+c)^2+1+2*(d*x
+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/2/b^3*e*exp(-2*a/b)*Ei(1,-
2*arcsinh(d*x+c)-2*a/b))
```

3.171.5 Fricas [F]

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^3}\,dx=\int \frac{dex+ce}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)/(b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^
2 + 3*a^2*b*arcsinh(d*x + c) + a^3), x)
```

3.171.6 Sympy [F]

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^3}\,dx = e\left(\int \frac{c}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx + \int \frac{dx}{a^3+3a^2b\,\text{asinh}\,(c+dx)+3ab^2\,\text{asinh}^2\,(c+dx)+b^3\,\text{asinh}^3\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**3,x)
```

output
```
e*(Integral(c/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2
 + b**3*asinh(c + d*x)**3), x) + Integral(d*x/(a**3 + 3*a**2*b*asinh(c + d
*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x))
```

3.171.7 Maxima [F]

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^3}\,dx=\int \frac{dex+ce}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*((2*a*d^8*e + b*d^8*e)*x^8 + 8*(2*a*c*d^7*e + b*c*d^7*e)*x^7 + (2*(28
*c^2*d^6*e + 3*d^6*e)*a + (28*c^2*d^6*e + 3*d^6*e)*b)*x^6 + 2*(2*(28*c^3*d
^5*e + 9*c*d^5*e)*a + (28*c^3*d^5*e + 9*c*d^5*e)*b)*x^5 + (2*(70*c^4*d^4*e
 + 45*c^2*d^4*e + 3*d^4*e)*a + (70*c^4*d^4*e + 45*c^2*d^4*e + 3*d^4*e)*b)*
x^4 + 4*(2*(14*c^5*d^3*e + 15*c^3*d^3*e + 3*c*d^3*e)*a + (14*c^5*d^3*e + 1
5*c^3*d^3*e + 3*c*d^3*e)*b)*x^3 + (2*(28*c^6*d^2*e + 45*c^4*d^2*e + 18*c^2
*d^2*e + d^2*e)*a + (28*c^6*d^2*e + 45*c^4*d^2*e + 18*c^2*d^2*e + d^2*e)*b
)*x^2 + ((2*a*d^5*e + b*d^5*e)*x^5 + 5*(2*a*c*d^4*e + b*c*d^4*e)*x^4 + (2*
(10*c^2*d^3*e + d^3*e)*a + (10*c^2*d^3*e + d^3*e)*b)*x^3 + (2*(10*c^3*d^2*
e + 3*c*d^2*e)*a + (10*c^3*d^2*e + 3*c*d^2*e)*b)*x^2 + 2*(c^5*e + c^3*e)*a
 + (c^5*e + c^3*e)*b + (2*(5*c^4*d*e + 3*c^2*d*e)*a + (5*c^4*d*e + 3*c^2*d
*e)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (3*(2*a*d^6*e + b*d^6*e)*x
^6 + 18*(2*a*c*d^5*e + b*c*d^5*e)*x^5 + 5*(2*(9*c^2*d^4*e + d^4*e)*a + (9*
c^2*d^4*e + d^4*e)*b)*x^4 + 20*(2*(3*c^3*d^3*e + c*d^3*e)*a + (3*c^3*d^3*e
 + c*d^3*e)*b)*x^3 + (5*(18*c^4*d^2*e + 12*c^2*d^2*e + d^2*e)*a + (45*c^4*
d^2*e + 30*c^2*d^2*e + 2*d^2*e)*b)*x^2 + (6*c^6*e + 10*c^4*e + 5*c^2*e + e
)*a + (3*c^6*e + 5*c^4*e + 2*c^2*e)*b + 2*((18*c^5*d*e + 20*c^3*d*e + 5*c*
d*e)*a + (9*c^5*d*e + 10*c^3*d*e + 2*c*d*e)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2
 + 1) + 2*(c^8*e + 3*c^6*e + 3*c^4*e + c^2*e)*a + (c^8*e + 3*c^6*e + 3*c^4
*e + c^2*e)*b + 2*(2*(4*c^7*d*e + 9*c^5*d*e + 6*c^3*d*e + c*d*e)*a + (4...
```

3.171.8 Giac [F]

$$\int \frac{ce + dex}{(a + b\text{arcsinh}(c + dx))^3}\,dx = \int \frac{dex + ce}{(b\,\text{arsinh}\,(dx + c) + a)^3}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^3, x)
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx=\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int((c*e + d*e*x)/(a + b*asinh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)/(a + b*asinh(c + d*x))^3, x)
```

3.172 $\int \frac{1}{(a+b\,\mathrm{arcsinh}(c+dx))^3}\,dx$

3.172.1 Optimal result

Integrand size = 12, antiderivative size = 125

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}(c+dx))^3}\,dx=-\frac{\sqrt{1+(c+dx)^2}}{2bd(a+b\,\mathrm{arcsinh}(c+dx))^2}-\frac{c+dx}{2b^2d(a+b\,\mathrm{arcsinh}(c+dx))}+\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\,\mathrm{arcsinh}(c+dx)}{b}\right)}{2b^3d}-\frac{\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\,\mathrm{arcsinh}(c+dx)}{b}\right)}{2b^3d}$$

output
```
1/2*(-d*x-c)/b^2/d/(a+b*arcsinh(d*x+c))+1/2*Chi((a+b*arcsinh(d*x+c))/b)*co
sh(a/b)/b^3/d-1/2*Shi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)/b^3/d-1/2*(1+(d*x+
c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^2
```

3.172.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}(c+dx))^3}\,dx=-\frac{\frac{b^2\sqrt{1+(c+dx)^2}}{(a+b\,\mathrm{arcsinh}(c+dx))^2}+\frac{b(c+dx)}{a+b\,\mathrm{arcsinh}(c+dx)}-\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\mathrm{arcsinh}(c+dx)\right)+\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{arcsinh}\right.}{2b^3d}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(-3),x]
```

output
```
-1/2*((b^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2 + (b*(c + d*x
))/(a + b*ArcSinh[c + d*x]) - Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x
]] + Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]])/(b^3*d)
```

3.172.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6273, 6188, 6233, 6189, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^3}dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int \frac{1}{(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{6188}$$

$$\frac{\frac{\int \frac{c+dx}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{2b}-\frac{\sqrt{(c+dx)^2+1}}{2b(a+b\text{arcsinh}(c+dx))^2}}{d}$$

$$\Big\downarrow \text{6233}$$

$$\frac{\frac{\frac{\int \frac{1}{a+b\text{arcsinh}(c+dx)}d(c+dx)}{b}-\frac{c+dx}{b(a+b\text{arcsinh}(c+dx))}}{2b}-\frac{\sqrt{(c+dx)^2+1}}{2b(a+b\text{arcsinh}(c+dx))^2}}{d}$$

$$\Big\downarrow \text{6189}$$

$$\frac{\frac{\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{c+dx}{b(a+b\text{arcsinh}(c+dx))}}{2b}-\frac{\sqrt{(c+dx)^2+1}}{2b(a+b\text{arcsinh}(c+dx))^2}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\sqrt{(c+dx)^2+1}}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{c+dx}{b(a+b\text{arcsinh}(c+dx))}+\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}}{2b}}{d}$$

$\downarrow$ 3784

$$\frac{-\frac{\sqrt{(c+dx)^2+1}}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{c+dx}{b(a+b\text{arcsinh}(c+dx))}+\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\sinh\left(\frac{a}{b}\right)\int -\frac{i\sinh\left(\frac{a+b\text{ar}}{}\right.}{a+b\text{arcs}}}{b^2}}{2b}}{d}$$

$\downarrow$ 26

$$\frac{\frac{\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-\sinh\left(\frac{a}{b}\right)\int \frac{\sinh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{c+dx}{b(a+b\text{arcsinh}(c+dx))}}{2b}}{d}$$

$\downarrow$ 3042

$$\frac{-\frac{\sqrt{(c+dx)^2+1}}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{c+dx}{b(a+b\text{arcsinh}(c+dx))}+\frac{\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-\sinh\left(\frac{a}{b}\right)\int -\frac{i\sin\left(\frac{i(a+}{}\right.}{a+b\text{a}}}{b^2}}{2b}}{d}$$

$\downarrow$ 26

$$\frac{-\frac{\sqrt{(c+dx)^2+1}}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{c+dx}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\sinh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))+\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arc}}{}\right.}{a+b\text{arc}}}{b^2}}{2b}}{d}$$

$\downarrow$ 3779

$$\frac{-\frac{\sqrt{(c+dx)^2+1}}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{c+dx}{b(a+b\text{arcsinh}(c+dx))}+\frac{-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)+\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{a}}{b^2}}{2b}}{d}$$

$\downarrow$ 3782

3.172. $\quad \displaystyle\int \frac{1}{(a+b\text{arcsinh}(c+dx))^3}\,dx$

$$\frac{\frac{\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{b^2}-\frac{c+dx}{b(a+b\text{arcsinh}(c+dx))}}{2b}-\frac{\sqrt{(c+dx)^2+1}}{2b(a+b\text{arcsinh}(c+dx))^2}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(-3),x]
```

output
```
(-1/2*Sqrt[1 + (c + d*x)^2]/(b*(a + b*ArcSinh[c + d*x])^2) + (-((c + d*x)/
(b*(a + b*ArcSinh[c + d*x]))) + (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c +
 d*x])/b] - Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/b^2)/(2*b)
)/d
```

3.172.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.172.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.52

method	result
derivativedivides	$\frac{-\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)(b\operatorname{arcsinh}(dx+c)+a-b)}{4b^2\left(b^2\operatorname{arcsinh}(dx+c)^2+2ab\operatorname{arcsinh}(dx+c)+a^2\right)}-\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{4b^3}-\frac{dx+c+\sqrt{1+(dx+c)^2}}{4b(a+b\operatorname{arcsinh}(dx+c))^2}-\frac{dx+c+\sqrt{1+(dx+c)^2}}{4b^2(a+b\operatorname{arcsinh}(dx+\cdots}}{d}$
default	$\frac{-\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)(b\operatorname{arcsinh}(dx+c)+a-b)}{4b^2\left(b^2\operatorname{arcsinh}(dx+c)^2+2ab\operatorname{arcsinh}(dx+c)+a^2\right)}-\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{4b^3}-\frac{dx+c+\sqrt{1+(dx+c)^2}}{4b(a+b\operatorname{arcsinh}(dx+c))^2}-\frac{dx+c+\sqrt{1+(dx+c)^2}}{4b^2(a+b\operatorname{arcsinh}(dx+\cdots}}{d}$

input
```
int(1/(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*(b*arcsinh(d*x+c)+a-b)/b^2/(b^2*arc
sinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)-1/4/b^3*exp(a/b)*Ei(1,arcsinh(d*x+
c)+a/b)-1/4/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/4/b^2*(
d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/4/b^3*exp(-a/b)*Ei(1,-ar
csinh(d*x+c)-a/b))
```

3.172.5 Fricas [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^3}\,dx=\int \frac{1}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*
arcsinh(d*x + c) + a^3), x)
```

3.172.6 Sympy [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^3}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(c+dx))^3}\,dx$$

input
```
integrate(1/(a+b*asinh(d*x+c))**3,x)
```

output
```
Integral((a + b*asinh(c + d*x))**(-3), x)
```

3.172.7 Maxima [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^3}\,dx=\int \frac{1}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*((a*d^7 + b*d^7)*x^7 + 7*(a*c*d^6 + b*c*d^6)*x^6 + 3*((7*c^2*d^5 + d^
5)*a + (7*c^2*d^5 + d^5)*b)*x^5 + 5*((7*c^3*d^4 + 3*c*d^4)*a + (7*c^3*d^4
+ 3*c*d^4)*b)*x^4 + ((35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*a + (35*c^4*d^3 + 3
0*c^2*d^3 + 3*d^3)*b)*x^3 + 3*((7*c^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*a + (7*c
^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*b)*x^2 + ((a*d^4 + b*d^4)*x^4 + 4*(a*c*d^3
+ b*c*d^3)*x^3 + (6*a*c^2*d^2 + (6*c^2*d^2 + d^2)*b)*x^2 + (c^4 - 1)*a + (
c^4 + c^2)*b + 2*(2*a*c^3*d + (2*c^3*d + c*d)*b)*x)*(d^2*x^2 + 2*c*d*x + c
^2 + 1)^(3/2) + (3*(a*d^5 + b*d^5)*x^5 + 15*(a*c*d^4 + b*c*d^4)*x^4 + (3*(
10*c^2*d^3 + d^3)*a + 5*(6*c^2*d^3 + d^3)*b)*x^3 + 3*((10*c^3*d^2 + 3*c*d^
2)*a + 5*(2*c^3*d^2 + c*d^2)*b)*x^2 + 3*(c^5 + c^3)*a + (3*c^5 + 5*c^3 + 2
*c)*b + (3*(5*c^4*d + 3*c^2*d)*a + (15*c^4*d + 15*c^2*d + 2*d)*b)*x)*(d^2*
x^2 + 2*c*d*x + c^2 + 1) + (c^7 + 3*c^5 + 3*c^3 + c)*a + (c^7 + 3*c^5 + 3*
c^3 + c)*b + ((7*c^6*d + 15*c^4*d + 9*c^2*d + d)*a + (7*c^6*d + 15*c^4*d +
 9*c^2*d + d)*b)*x + (b*d^7*x^7 + 7*b*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*b*x^
5 + 5*(7*c^3*d^4 + 3*c*d^4)*b*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*b*x^
3 + 3*(7*c^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*b*x^2 + (7*c^6*d + 15*c^4*d + 9*c
^2*d + d)*b*x + (b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
 + (c^4 - 1)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(b*d^5*x^5 + 5*b*c
*d^4*x^4 + (10*c^2*d^3 + d^3)*b*x^3 + (10*c^3*d^2 + 3*c*d^2)*b*x^2 + (5*c^
4*d + 3*c^2*d)*b*x + (c^5 + c^3)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (c^...
```

3.172.8 Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^3}\,dx=\int \frac{1}{(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(-3), x)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^3}\,dx=\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(1/(a + b*asinh(c + d*x))^3,x)
```

output
```
int(1/(a + b*asinh(c + d*x))^3, x)
```

3.173 $\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^3}\,dx$

3.173.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^3}\,dx = \frac{\text{Int}\left(\frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))^3},x\right)}{e}$$

output
```
Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^3,x)/e
```

3.173.2 Mathematica [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^3}\,dx$$

input
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3),x]
```

output
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3), x]
```

3.173.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6274, 27, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^3}\,dx$$
$$\Big\downarrow \; 6274$$
$$\frac{\int \frac{1}{e(c+dx)(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{d}$$
$$\Big\downarrow \; 27$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{de}$$
$$\Big\downarrow \; 6196$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{de}$$

input
```
Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3),x]
```

output
```
$Aborted
```

3.173.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.173.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dex + ce)(a + b \operatorname{arcsinh}(dx + c))^3} dx$$

input
```
int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x)
```

output
```
int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x)
```

3.173.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.96

$$\int \frac{1}{(ce + dex)(a + b\text{arcsinh}(c + dx))^3} dx = \int \frac{1}{(dex + ce)(b \operatorname{arsinh}(dx + c) + a)^3} dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(1/(a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arcsinh(d*x + c)^3
 + 3*(a*b^2*d*e*x + a*b^2*c*e)*arcsinh(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b
*c*e)*arcsinh(d*x + c)), x)
```

3.173.6 Sympy [N/A]

Not integrable

Time = 3.60 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.87

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^3}\,dx = \frac{\int \frac{1}{a^3c+a^3dx+3a^2bc\,\text{asinh}\,(c+dx)+3a^2bdx\,\text{asinh}\,(c+dx)+3ab^2c\,\text{asinh}^2\,(c+dx)+3ab^2dx\,\text{asinh}^2\,(c+dx)+b^3c\,\text{asinh}^3\,(c+dx)+b^3dx\,\text{asinh}^3\,(c+dx)}\,dx}{e}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**3,x)
```

output
```
Integral(1/(a**3*c + a**3*d*x + 3*a**2*b*c*asinh(c + d*x) + 3*a**2*b*d*x*a
sinh(c + d*x) + 3*a*b**2*c*asinh(c + d*x)**2 + 3*a*b**2*d*x*asinh(c + d*x)
**2 + b**3*c*asinh(c + d*x)**3 + b**3*d*x*asinh(c + d*x)**3), x)/e
```

3.173.7 Maxima [N/A]

Not integrable

Time = 68.05 (sec) , antiderivative size = 6716, normalized size of antiderivative = 292.00

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(b*d^8*x^8 + 8*b*c*d^7*x^7 + (28*c^2*d^6 + 3*d^6)*b*x^6 + 2*(28*c^3*d
^5 + 9*c*d^5)*b*x^5 + (70*c^4*d^4 + 45*c^2*d^4 + 3*d^4)*b*x^4 + 4*(14*c^5*
d^3 + 15*c^3*d^3 + 3*c*d^3)*b*x^3 + (28*c^6*d^2 + 45*c^4*d^2 + 18*c^2*d^2
+ d^2)*b*x^2 + 2*(4*c^7*d + 9*c^5*d + 6*c^3*d + c*d)*b*x + (b*d^5*x^5 + 5*
b*c*d^4*x^4 - (2*a*d^3 - (10*c^2*d^3 + d^3)*b)*x^3 - (6*a*c*d^2 - (10*c^3*
d^2 + 3*c*d^2)*b)*x^2 - 2*(c^3 + c)*a + (c^5 + c^3)*b - (2*(3*c^2*d + d)*a
 - (5*c^4*d + 3*c^2*d)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (3*b*d^
6*x^6 + 18*b*c*d^5*x^5 - (4*a*d^4 - 5*(9*c^2*d^4 + d^4)*b)*x^4 - 4*(4*a*c*
d^3 - 5*(3*c^3*d^3 + c*d^3)*b)*x^3 - ((24*c^2*d^2 + 5*d^2)*a - (45*c^4*d^2
 + 30*c^2*d^2 + 2*d^2)*b)*x^2 - (4*c^4 + 5*c^2 + 1)*a + (3*c^6 + 5*c^4 + 2
*c^2)*b - 2*((8*c^3*d + 5*c*d)*a - (9*c^5*d + 10*c^3*d + 2*c*d)*b)*x)*(d^2
*x^2 + 2*c*d*x + c^2 + 1) + (c^8 + 3*c^6 + 3*c^4 + c^2)*b - (2*(b*d^3*x^3
+ 3*b*c*d^2*x^2 + (3*c^2*d + d)*b*x + (c^3 + c)*b)*(d^2*x^2 + 2*c*d*x + c^
2 + 1)^(3/2) + (4*b*d^4*x^4 + 16*b*c*d^3*x^3 + (24*c^2*d^2 + 5*d^2)*b*x^2
+ 2*(8*c^3*d + 5*c*d)*b*x + (4*c^4 + 5*c^2 + 1)*b)*(d^2*x^2 + 2*c*d*x + c^
2 + 1) + (2*b*d^5*x^5 + 10*b*c*d^4*x^4 + (20*c^2*d^3 + 3*d^3)*b*x^3 + (20*
c^3*d^2 + 9*c*d^2)*b*x^2 + (10*c^4*d + 9*c^2*d + d)*b*x + (2*c^5 + 3*c^3 +
 c)*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1)) + (3*b*d^7*x^7 + 21*b*c*d^6*x^6 - (2*a*d^5 - 7*(9*c^2*d^5
 + d^5)*b)*x^5 - 5*(2*a*c*d^4 - 7*(3*c^3*d^4 + c*d^4)*b)*x^4 - ((20*c^2...
```

3.173.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^3), x)
```

3.173.9 Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^3}\,dx = \int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^3),x)
```

output
```
int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^3), x)
```

3.174 $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

3.174.1 Optimal result

Integrand size = 23, antiderivative size = 410

$$\begin{aligned}\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^4}\,dx = & -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3}-\frac{2e^4(c+dx)^3}{3b^2d(a+b\text{arcsinh}(c+dx))^2}\\
&-\frac{5e^4(c+dx)^5}{6b^2d(a+b\text{arcsinh}(c+dx))^2}\\
&-\frac{2e^4(c+dx)^2\sqrt{1+(c+dx)^2}}{b^3d(a+b\text{arcsinh}(c+dx))}\\
&-\frac{25e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{6b^3d(a+b\text{arcsinh}(c+dx))}\\
&-\frac{e^4\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{48b^4d}\\
&+\frac{27e^4\text{Chi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{32b^4d}\\
&-\frac{125e^4\text{Chi}\left(\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{5a}{b}\right)}{96b^4d}\\
&+\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{48b^4d}\\
&-\frac{27e^4\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{32b^4d}\\
&+\frac{125e^4\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)}{96b^4d}\end{aligned}$$

output
```
-2/3*e^4*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))^2-5/6*e^4*(d*x+c)^5/b^2/d/(a
+b*arcsinh(d*x+c))^2+1/48*e^4*cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^4/d-
27/32*e^4*cosh(3*a/b)*Shi(3*(a+b*arcsinh(d*x+c))/b)/b^4/d+125/96*e^4*cosh(
5*a/b)*Shi(5*(a+b*arcsinh(d*x+c))/b)/b^4/d-1/48*e^4*Chi((a+b*arcsinh(d*x+c
))/b)*sinh(a/b)/b^4/d+27/32*e^4*Chi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/
b^4/d-125/96*e^4*Chi(5*(a+b*arcsinh(d*x+c))/b)*sinh(5*a/b)/b^4/d-1/3*e^4*(
d*x+c)^4*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^3-2*e^4*(d*x+c)^2*(1
+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))-25/6*e^4*(d*x+c)^4*(1+(d*x+c)
^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))
```

3.174.2 Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^4}\,dx =$$

$$-\frac{e^4\left(\frac{32b^3(c+dx)^4\sqrt{1+(c+dx)^2}}{(a+b\text{arcsinh}(c+dx))^3}-\frac{16b^2(-4(c+dx)^3-5(c+dx)^5)}{(a+b\text{arcsinh}(c+dx))^2}+\frac{16b\sqrt{1+(c+dx)^2}(12(c+dx)^2+25(c+dx)^4)}{a+b\text{arcsinh}(c+dx)}+384\left(\text{Chi}\left(\frac{a}{b}+\text{arcs}\right.\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
-1/96*(e^4*((32*b^3*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c +
d*x])^3 - (16*b^2*(-4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcSinh[c + d*x
])^2 + (16*b*Sqrt[1 + (c + d*x)^2]*(12*(c + d*x)^2 + 25*(c + d*x)^4))/(a +
 b*ArcSinh[c + d*x]) + 384*(CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b]
 - Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]]) + 544*(-3*CoshIntegral[
a/b + ArcSinh[c + d*x]]*Sinh[a/b] + CoshIntegral[3*(a/b + ArcSinh[c + d*x]
)]*Sinh[(3*a)/b] + 3*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] - Cosh
[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]) + 125*(10*CoshIntegral
[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - 5*CoshIntegral[3*(a/b + ArcSinh[c + d
*x])]*Sinh[(3*a)/b] + CoshIntegral[5*(a/b + ArcSinh[c + d*x])]*Sinh[(5*a)/
b] - 10*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 5*Cosh[(3*a)/b]*S
inhIntegral[3*(a/b + ArcSinh[c + d*x])] - Cosh[(5*a)/b]*SinhIntegral[5*(a/
b + ArcSinh[c + d*x])])))/(b^4*d)
```

3.174.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6274, 27, 6194, 6233, 6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^4}dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \frac{e^4(c+dx)^4}{(a+b\text{arcsinh}(c+dx))^4}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^4\int \frac{(c+dx)^4}{(a+b\text{arcsinh}(c+dx))^4}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6194}$$

$$\frac{e^4\left(\frac{4\int \frac{(c+dx)^3}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{3b}+\frac{5\int \frac{(c+dx)^5}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{3b}-\frac{\sqrt{(c+dx)^2+1}(c+dx)^4}{3b(a+b\text{arcsinh}(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \text{ 6233}$$

$$\frac{e^4\left(\frac{4\left(\frac{3\int \frac{(c+dx)^2}{(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{2b}-\frac{(c+dx)^3}{2b(a+b\text{arcsinh}(c+dx))^2}\right)}{3b}+\frac{5\left(\frac{5\int \frac{(c+dx)^4}{(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{2b}-\frac{(c+dx)^5}{2b(a+b\text{arcsinh}(c+dx))^2}\right)}{3b}-\cdots\right)}{d}$$

$$\Big\downarrow \text{ 6193}$$

$$e^4\left(\frac{5\left(\frac{5\left(\frac{\int\left(-\frac{5\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)}{16(a+b\text{arcsinh}(c+dx))}+\frac{9\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{16(a+b\text{arcsinh}(c+dx))}-\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{8(a+b\text{arcsinh}(c+dx))}\right)d(a+b\text{arcsinh}(c+dx))}{b^2}\right.}{2b}\right.}{3b}\right.$$

$\downarrow$ 2009

$$e^4\left(\frac{4\left(\frac{3\left(\frac{\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)-\frac{3}{4}\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)-\frac{1}{4}\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)+\frac{3}{4}\cosh\left(\frac{3a}{b}\right)\text{Sl}}{b^2}\right.}{2b}\right.}{3b}\right.$$

input
```
Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^4,x]
```

3.174. $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

output
```
(e^4*(-1/3*((c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*(a + b*ArcSinh[c + d*x])
^3) + (4*(-1/2*(c + d*x)^3/(b*(a + b*ArcSinh[c + d*x])^2) + (3*(-(((c + d*
x)^2*Sqrt[1 + (c + d*x)^2])/(b*(a + b*ArcSinh[c + d*x]))) + ((CoshIntegral
[(a + b*ArcSinh[c + d*x])/b]*Sinh[a/b])/4 - (3*CoshIntegral[(3*(a + b*ArcS
inh[c + d*x]))/b]*Sinh[(3*a)/b])/4 - (Cosh[a/b]*SinhIntegral[(a + b*ArcSin
h[c + d*x])/b])/4 + (3*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c + d*
x]))/b])/4)/b^2))/(2*b)))/(3*b) + (5*(-1/2*(c + d*x)^5/(b*(a + b*ArcSinh[c
 + d*x])^2) + (5*(-(((c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*(a + b*ArcSinh[
c + d*x]))) + (-1/8*(CoshIntegral[(a + b*ArcSinh[c + d*x])/b]*Sinh[a/b]) +
 (9*CoshIntegral[(3*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(3*a)/b])/16 - (5*Co
shIntegral[(5*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(5*a)/b])/16 + (Cosh[a/b]*
SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/8 - (9*Cosh[(3*a)/b]*SinhIntegra
l[(3*(a + b*ArcSinh[c + d*x]))/b])/16 + (5*Cosh[(5*a)/b]*SinhIntegral[(5*(
a + b*ArcSinh[c + d*x]))/b])/16)/b^2))/(2*b)))/(3*b)))/d
```

3.174.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

3.174. $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.174.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1243 vs. $2(384) = 768$.

Time = 0.90 (sec) , antiderivative size = 1244, normalized size of antiderivative = 3.03

method	result	size
derivativedivides	Expression too large to display	1244
default	Expression too large to display	1244

input
```
int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/192*(-16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+16*(d*x+c)^5-12*(d*x+c)^2*(1
+(d*x+c)^2)^(1/2)+20*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+5*d*x+5*c)*e^4*(25*b^2*
arcsinh(d*x+c)^2+50*a*b*arcsinh(d*x+c)-5*b^2*arcsinh(d*x+c)+25*a^2-5*a*b+2
*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d
*x+c)+a^3)+125/192*e^4/b^4*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)-1/64*(-
4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+3*d*x+3*c)
*e^4*(9*b^2*arcsinh(d*x+c)^2+18*a*b*arcsinh(d*x+c)-3*b^2*arcsinh(d*x+c)+9*
a^2-3*a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)^2+3*a^2*
b*arcsinh(d*x+c)+a^3)-27/64*e^4/b^4*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b
)+1/96*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^4*(b^2*arcsinh(d*x+c)^2+2*a*b*arcsin
h(d*x+c)-b^2*arcsinh(d*x+c)+a^2-a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b
^2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)+1/96*e^4/b^4*exp(a/b)*Ei(1
,arcsinh(d*x+c)+a/b)-1/48/b*e^4*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d
*x+c))^3-1/96/b^2*e^4*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1
/96/b^3*e^4*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/96/b^4*e^4*
exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)+1/32/b*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d
*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3+3/
64/b^2*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+
c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+9/64/b^3*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*
(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))...
```

3.174.5 Fricas [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^4}\,dx = \int \frac{(dex+ce)^4}{(b\,\text{arsinh}\,(dx+c)+a)^4}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*
x + c^4*e^4)/(b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*
b^2*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

3.174.6 Sympy [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^4}\,dx$$
$$= e^4\left(\int \frac{c^4}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx\right.$$
$$+\int \frac{d^4x^4}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx$$
$$+\int \frac{4cd^3x^3}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx$$
$$+\int \frac{6c^2d^2x^2}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx$$
$$\left.+\int \frac{4c^3dx}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**4,x)
```

output
```
e**4*(Integral(c**4/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c
+ d*x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Int
egral(d**4*x**4/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*
x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integra
l(4*c*d**3*x**3/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*
x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integra
l(6*c**2*d**2*x**2/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c +
 d*x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Inte
gral(4*c**3*d*x/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*
x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x))
```

3.174.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
Timed out
```

3.174.8 Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^4}\,dx=\int \frac{(dex+ce)^4}{(b\,\text{arsinh}\,(dx+c)+a)^4}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^4, x)
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^4}\,dx=\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\text{asinh}\,(c+d\,x))^4}\,dx$$

input
```
int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^4, x)
```

3.175 $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

3.175.1 Optimal result

Integrand size = 23, antiderivative size = 340

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^4}\,dx = -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3}-\frac{e^3(c+dx)^2}{2b^2d(a+b\text{arcsinh}(c+dx))^2}-\frac{2e^3(c+dx)^4}{3b^2d(a+b\text{arcsinh}(c+dx))^2}-\frac{e^3(c+dx)\sqrt{1+(c+dx)^2}}{b^3d(a+b\text{arcsinh}(c+dx))}-\frac{8e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3b^3d(a+b\text{arcsinh}(c+dx))}-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{3b^4d}+\frac{4e^3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{3b^4d}+\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{3b^4d}-\frac{4e^3\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{3b^4d}$$

output
```
-1/2*e^3*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+c))^2-2/3*e^3*(d*x+c)^4/b^2/d/(a
+b*arcsinh(d*x+c))^2-1/3*e^3*Chi(2*(a+b*arcsinh(d*x+c))/b)*cosh(2*a/b)/b^4
/d+4/3*e^3*Chi(4*(a+b*arcsinh(d*x+c))/b)*cosh(4*a/b)/b^4/d+1/3*e^3*Shi(2*(
a+b*arcsinh(d*x+c))/b)*sinh(2*a/b)/b^4/d-4/3*e^3*Shi(4*(a+b*arcsinh(d*x+c)
)/b)*sinh(4*a/b)/b^4/d-1/3*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcs
inh(d*x+c))^3-e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))-8
/3*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))
```

3.175.2 Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.94

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$$
$$= \frac{e^3\left(-\frac{2b^3(c+dx)^3\sqrt{1+(c+dx)^2}}{(a+b\operatorname{arcsinh}(c+dx))^3}+\frac{b^2(-3(c+dx)^2-4(c+dx)^4)}{(a+b\operatorname{arcsinh}(c+dx))^2}-\frac{2b\sqrt{1+(c+dx)^2}(3(c+dx)+8(c+dx)^3)}{a+b\operatorname{arcsinh}(c+dx)}+6\log(a+b\operatorname{arcsinh}(c+d\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
(e^3*((-2*b^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^
3 + (b^2*(-3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcSinh[c + d*x])^2 - (2
*b*Sqrt[1 + (c + d*x)^2]*(3*(c + d*x) + 8*(c + d*x)^3))/(a + b*ArcSinh[c +
 d*x]) + 6*Log[a + b*ArcSinh[c + d*x]] + 30*(Cosh[(2*a)/b]*CoshIntegral[2*
(a/b + ArcSinh[c + d*x])] - Log[a + b*ArcSinh[c + d*x]] - Sinh[(2*a)/b]*Si
nhIntegral[2*(a/b + ArcSinh[c + d*x])]) + 8*(-4*Cosh[(2*a)/b]*CoshIntegral
[2*(a/b + ArcSinh[c + d*x])] + Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcSinh
[c + d*x])] + 3*Log[a + b*ArcSinh[c + d*x]] + 4*Sinh[(2*a)/b]*SinhIntegral
[2*(a/b + ArcSinh[c + d*x])] - Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh
[c + d*x])])))/(6*b^4*d)
```

3.175.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {6274, 27, 6194, 6233, 6193, 2009, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \frac{e^3(c+dx)^3}{(a+b\operatorname{arcsinh}(c+dx))^4}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e^3\int \frac{(c+dx)^3}{(a+b\operatorname{arcsinh}(c+dx))^4}d(c+dx)}{d}$$

$$\Big\downarrow \; 6194$$

$$\frac{e^3\left(\frac{\int \frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^3}d(c+dx)}{b}+\frac{4\int \frac{(c+dx)^4}{\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^3}d(c+dx)}{3b}-\frac{\sqrt{(c+dx)^2+1}(c+dx)^3}{3b(a+b\operatorname{arcsinh}(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \; 6233$$

$$\frac{e^3\left(\frac{\frac{\int \frac{c+dx}{(a+b\operatorname{arcsinh}(c+dx))^2}d(c+dx)}{b}-\frac{(c+dx)^2}{2b(a+b\operatorname{arcsinh}(c+dx))^2}}{b}+\frac{4\left(\frac{2\int \frac{(c+dx)^3}{(a+b\operatorname{arcsinh}(c+dx))^2}d(c+dx)}{b}-\frac{(c+dx)^4}{2b(a+b\operatorname{arcsinh}(c+dx))^2}\right)}{3b}-\frac{\sqrt{(c-}}{3b(a+b}\right.}{d}$$

$$\Big\downarrow \; 6193$$

3.175. $\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$

$$e^3\left(\frac{\frac{\frac{\int \frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}}{b}}{b}-\frac{(c+dx)^2}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{4\left(\frac{2\left(\frac{\int\left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\ldots}{}\right)}{2(a+b\text{arcs}\ldots}\right.}{}\right.}{}\right.}{}\right.$$

$\downarrow$ 2009

$$e^3\left(\frac{\frac{\frac{\int \frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}}{b}}{b}-\frac{(c+dx)^2}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{4\left(\frac{2\left(\frac{-\frac{1}{2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\ldots\right.}{}\right.}{}\right.}{}\right.$$

$\downarrow$ 3042

3.175. $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

$$e^3\left(\frac{-\frac{(c+dx)^2}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{\int\frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}}{b}}{b}+\frac{4\left(2\left(\frac{-\frac{1}{2}\cosh\left(\frac{2a}{b}\right)\ldots}{\ldots}\right.\right.}{\ldots}\right.$$

$\downarrow$ 3784

$$e^3\left(\frac{-\frac{(c+dx)^2}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{\cosh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\sinh\left(\frac{2a}{b}\right)\int-\frac{i\sinh\left(\frac{2\ldots}{\ldots}\right)}{a\ldots}\ldots}{b^2}}{b}}{b}\ldots\right.$$

$\downarrow$ 26

$$e^3\left(\frac{\frac{\frac{\cosh\left(\frac{2a}{b}\right)\int\frac{\cosh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-\sinh\left(\frac{2a}{b}\right)\int\frac{\sinh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)\sqrt{\ldots}}{b(a+b\text{arcs}\ldots}}{b}\ldots}{b}\ldots\right.$$

$\downarrow$ 3042

3.175. $\displaystyle\int\frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

$$e^3\left(\frac{-\frac{(c+dx)^2}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-\sinh\left(\frac{2a}{b}\right)\int-\frac{i\sin\left(\right.}{}}{b^2}}{b}}{b}\right.$$

$\downarrow$ 26

$$e^3\left(\frac{-\frac{(c+dx)^2}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))+\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+}{}\right.}{a+}}{b^2}}{b}}{b}\right.$$

$\downarrow$ 3779

$$e^3\left(\frac{-\frac{(c+dx)^2}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)+\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}}{b^2}}{b}}{b}\right.$$

$\downarrow$ 3782

3.175. $\displaystyle\int\frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

$$e^3\left(\frac{\frac{\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\operatorname{arcsinh}(c+dx))}{b}\right)}{b^2}}{b}-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\operatorname{arcsinh}(c+dx))}}{b}-\frac{(c+dx)^2}{2b(a+b\operatorname{arcsinh}(c+dx))^2}+\frac{4\left(2\cdots\right.}{\cdots}\right.$$

input
```
Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
(e^3*(-1/3*((c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*(a + b*ArcSinh[c + d*x])
^3) + (-1/2*(c + d*x)^2/(b*(a + b*ArcSinh[c + d*x])^2) + (-(((c + d*x)*Sqr
t[1 + (c + d*x)^2])/(b*(a + b*ArcSinh[c + d*x]))) + (Cosh[(2*a)/b]*CoshInt
egral[(2*(a + b*ArcSinh[c + d*x]))/b] - Sinh[(2*a)/b]*SinhIntegral[(2*(a +
 b*ArcSinh[c + d*x]))/b])/b^2)/b)/b + (4*(-1/2*(c + d*x)^4/(b*(a + b*ArcSi
nh[c + d*x])^2) + (2*(-(((c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*(a + b*ArcS
inh[c + d*x]))) + (-1/2*(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c +
d*x]))/b]) + (Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c + d*x]))/b])/
2 + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/2 - (Sinh
[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c + d*x]))/b])/2)/b^2))/b))/(3*b)
))/d
```

3.175.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.175. $\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

3.175. $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.175.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(318) = 636$.

Time = 0.89 (sec) , antiderivative size = 800, normalized size of antiderivative = 2.35

method	result
derivativedivides	$\frac{\left(-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^4-4(dx+c)\sqrt{1+(dx+c)^2}+8(dx+c)^2+1\right)e^3\left(8b^2 \operatorname{arcsinh}(dx+c)^2+16ab \operatorname{arcsinh}(dx+c)-2b^2 \operatorname{arcsinh}\right.}{48b^3\left(b^3 \operatorname{arcsinh}(dx+c)^3+3a\,b^2 \operatorname{arcsinh}(dx+c)^2+3a^2b \operatorname{arcsinh}(dx+c)+a^3\right)}$
default	$\frac{\left(-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^4-4(dx+c)\sqrt{1+(dx+c)^2}+8(dx+c)^2+1\right)e^3\left(8b^2 \operatorname{arcsinh}(dx+c)^2+16ab \operatorname{arcsinh}(dx+c)-2b^2 \operatorname{arcsinh}\right.}{48b^3\left(b^3 \operatorname{arcsinh}(dx+c)^3+3a\,b^2 \operatorname{arcsinh}(dx+c)^2+3a^2b \operatorname{arcsinh}(dx+c)+a^3\right)}$

input
```
int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/48*(-8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+8*(d*x+c)^4-4*(d*x+c)*(1+(d*x+
c)^2)^(1/2)+8*(d*x+c)^2+1)*e^3*(8*b^2*arcsinh(d*x+c)^2+16*a*b*arcsinh(d*x+
c)-2*b^2*arcsinh(d*x+c)+8*a^2-2*a*b+b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2
*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)-2/3*e^3/b^4*exp(4*a/b)*Ei(1,
4*arcsinh(d*x+c)+4*a/b)-1/24*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x+c)^2+1
)*e^3*(2*b^2*arcsinh(d*x+c)^2+4*a*b*arcsinh(d*x+c)-b^2*arcsinh(d*x+c)+2*a^
2-a*b+b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcs
inh(d*x+c)+a^3)+1/6*e^3/b^4*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)+1/24/b
*e^3*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3+
1/24/b^2*e^3*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*
x+c))^2+1/12/b^3*e^3*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*ar
csinh(d*x+c))+1/6/b^4*e^3*exp(-2*a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b)-1/48/b
*e^3*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1
+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c))^3-1/24/b^2*e^3*(8*(d*x+c)^4+8*(d
*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(
a+b*arcsinh(d*x+c))^2-1/6/b^3*e^3*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+
(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c))-2/3
/b^4*e^3*exp(-4*a/b)*Ei(1,-4*arcsinh(d*x+c)-4*a/b))
```

3.175.5 Fricas [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^4}\,dx=\int \frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^4}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^4*ar
csinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*arcsinh(d*x + c)
^2 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

3.175.6 Sympy [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^4}\,dx$$
$$=e^3\left(\int \frac{c^3}{a^4+4a^3b\,\text{asinh}\,(c+dx)+6a^2b^2\,\text{asinh}^2\,(c+dx)+4ab^3\,\text{asinh}^3\,(c+dx)+b^4\,\text{asinh}^4\,(c+dx)}\,dx\right.$$
$$+\int \frac{d^3x^3}{a^4+4a^3b\,\text{asinh}\,(c+dx)+6a^2b^2\,\text{asinh}^2\,(c+dx)+4ab^3\,\text{asinh}^3\,(c+dx)+b^4\,\text{asinh}^4\,(c+dx)}\,dx$$
$$+\int \frac{3cd^2x^2}{a^4+4a^3b\,\text{asinh}\,(c+dx)+6a^2b^2\,\text{asinh}^2\,(c+dx)+4ab^3\,\text{asinh}^3\,(c+dx)+b^4\,\text{asinh}^4\,(c+dx)}\,dx$$
$$\left.+\int \frac{3c^2dx}{a^4+4a^3b\,\text{asinh}\,(c+dx)+6a^2b^2\,\text{asinh}^2\,(c+dx)+4ab^3\,\text{asinh}^3\,(c+dx)+b^4\,\text{asinh}^4\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**4,x)
```

output
```
e**3*(Integral(c**3/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c
+ d*x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Int
egral(d**3*x**3/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*
x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integra
l(3*c*d**2*x**2/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*
x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integra
l(3*c**2*d*x/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)*
*2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x))
```

3.175.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
Timed out
```

3.175.8 Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \int \frac{(dex+ce)^3}{(b\operatorname{arsinh}(dx+c)+a)^4}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^4, x)
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \int \frac{(c\,e+d\,e\,x)^3}{(a+b\operatorname{asinh}(c+d\,x))^4}\,dx$$

input
```
int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^4, x)
```

3.176 $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

3.176.1 Optimal result

Integrand size = 23, antiderivative size = 331

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx = -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} - \frac{e^2(c+dx)}{3b^2d(a+b\text{arcsinh}(c+dx))^2} - \frac{e^2(c+dx)^3}{2b^2d(a+b\text{arcsinh}(c+dx))^2} - \frac{e^2\sqrt{1+(c+dx)^2}}{3b^3d(a+b\text{arcsinh}(c+dx))} - \frac{3e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2b^3d(a+b\text{arcsinh}(c+dx))} + \frac{e^2\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{24b^4d} - \frac{9e^2\text{Chi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{8b^4d} - \frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{24b^4d} + \frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{8b^4d}$$

output
```
-1/3*e^2*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))^2-1/2*e^2*(d*x+c)^3/b^2/d/(a+b
*arcsinh(d*x+c))^2-1/24*e^2*cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^4/d+9/
8*e^2*cosh(3*a/b)*Shi(3*(a+b*arcsinh(d*x+c))/b)/b^4/d+1/24*e^2*Chi((a+b*ar
csinh(d*x+c))/b)*sinh(a/b)/b^4/d-9/8*e^2*Chi(3*(a+b*arcsinh(d*x+c))/b)*sin
h(3*a/b)/b^4/d-1/3*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+
c))^3-1/3*e^2*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))-3/2*e^2*(d*x+
c)^2*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))
```

3.176.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.78

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx$$
$$= \frac{e^2\left(-\frac{8b^3(c+dx)^2\sqrt{1+(c+dx)^2}}{(a+b\text{arcsinh}(c+dx))^3}+\frac{4b^2(-2(c+dx)-3(c+dx)^3)}{(a+b\text{arcsinh}(c+dx))^2}-\frac{4b\sqrt{1+(c+dx)^2}(2+9(c+dx)^2)}{a+b\text{arcsinh}(c+dx)}-80\text{Chi}\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)\text{s}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
(e^2*((-8*b^3*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^
3 + (4*b^2*(-2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcSinh[c + d*x])^2 - (4
*b*Sqrt[1 + (c + d*x)^2]*(2 + 9*(c + d*x)^2))/(a + b*ArcSinh[c + d*x]) - 8
0*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] + 80*Cosh[a/b]*SinhIntegr
al[a/b + ArcSinh[c + d*x]] + 27*(3*CoshIntegral[a/b + ArcSinh[c + d*x]]*Si
nh[a/b] - CoshIntegral[3*(a/b + ArcSinh[c + d*x])]*Sinh[(3*a)/b] - 3*Cosh[
a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(
a/b + ArcSinh[c + d*x])])))/(24*b^4*d)
```

3.176.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.78 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {6274, 27, 6194, 6233, 6188, 6193, 2009, 6234, 25, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.176. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \frac{e^2(c+dx)^2}{(a+b\text{arcsinh}(c+dx))^4}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e^2\int \frac{(c+dx)^2}{(a+b\text{arcsinh}(c+dx))^4}d(c+dx)}{d}$$

$$\Big\downarrow \; 6194$$

$$\frac{e^2\left(\frac{2\int \frac{c+dx}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{3b}+\frac{\int \frac{(c+dx)^3}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{b}-\frac{\sqrt{(c+dx)^2+1}(c+dx)^2}{3b(a+b\text{arcsinh}(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \; 6233$$

$$\frac{e^2\left(\frac{2\left(\frac{\int \frac{1}{(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{2b}-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}\right)}{3b}+\frac{\frac{3\int \frac{(c+dx)^2}{(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{2b}-\frac{(c+dx)^3}{2b(a+b\text{arcsinh}(c+dx))^2}}{b}-\frac{\sqrt{(c-}{3b(a+b}\right.}{d}$$

$$\Big\downarrow \; 6188$$

$$\frac{e^2\left(\frac{\frac{3\int \frac{(c+dx)^2}{(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{2b}-\frac{(c+dx)^3}{2b(a+b\text{arcsinh}(c+dx))^2}}{b}+\frac{2\left(\frac{\frac{\int \frac{c+dx}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))}d(c+dx)}{b}-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}}{2b}-\frac{}{2b(}\right.}{3b}\right.}{d}$$

$$\Big\downarrow \; 6193$$

3.176. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

$$e^2\left(\frac{\frac{3\left(\frac{\int \left(\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{4(a+b\text{arcsinh}(c+dx))}-\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{4(a+b\text{arcsinh}(c+dx))}\right)d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)^2\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}\right)}{2b}-\frac{(c+dx}{2b(a+b\text{arcsin}}}{b}\right.$$

$$\Big\downarrow \textbf{2009}$$

$$e^2\left(\frac{2\left(\frac{\frac{\int \frac{c+dx}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))}d(c+dx)}{b}-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}}{2b}-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}\right)}{3b}+\frac{3\left(\frac{\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+}{b}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \textbf{6234}$$

$$e^2\left(\frac{2\left(\frac{\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}}{2b}-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}\right)}{3b}+\frac{3\left(\frac{\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b}{}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \textbf{25}$$

3.176. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

$$e^2\left(\frac{2\left(\frac{-\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}}{2b}-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}\right)}{3b}+\frac{3\left(\frac{\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 3042

$$e^2\left(\frac{2\left(-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}-\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}}{2b}\right)}{3b}+\frac{3\left(\frac{\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Ch}\cdots}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 26

$$e^2\left(\frac{2\left(-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}}{2b}\right)}{3b}+\frac{3\left(\frac{\frac{1}{4}\sinh\left(\frac{a}{b}\right)\text{Chi}\cdots}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 3784

3.176. $\displaystyle\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

$$e^2\left(\frac{2\left(-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))+\cosh\left(\frac{a}{b}\right)\int-\frac{i\sinh\left(\cdots\right.}{a+\cdots}\right.}{b^2}}{2b}\right.}{3b}\right.$$

$\downarrow$ 26

$$e^2\left(\frac{2\left(-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\cosh\left(\frac{a}{b}\right)\int\frac{\sinh\left(\frac{a}{\cdots}\right.}{a+\cdots}\right.}{b^2}}{2b}\right.}{3b}\right.$$

$\downarrow$ 3042

3.176. $\int\frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

$$e^2\left(\frac{2\left(-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\cosh\left(\frac{a}{b}\right)\int-\cdots\right.}{b^2}}{2b}\right.}{3b}\right.$$

$$\downarrow 26$$

$$e^2\left(\frac{2\left(-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\cdots\right.}{\cdots}\right.}{b^2}}{2b}\right.}{3b}\right.$$

$$\downarrow 3779$$

3.176. $\int\frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

$$e^2\left(\frac{2\left(-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\cosh\left(\frac{a}{b}\right)\text{Shi}\right.}{b^2}}{2b}\right)}{3b}\right.$$

$\big\downarrow$ 3782

$$e^2\left(\frac{2\left(-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\right)}{b^2}}{2b}\right)}{3b}\right.$$

input
```
Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
(e^2*(-1/3*((c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*(a + b*ArcSinh[c + d*x])
^3) + (2*(-1/2*(c + d*x)/(b*(a + b*ArcSinh[c + d*x])^2) + (-(Sqrt[1 + (c +
 d*x)^2]/(b*(a + b*ArcSinh[c + d*x]))) + (I*(I*CoshIntegral[(a + b*ArcSinh
[c + d*x])/b]*Sinh[a/b] - I*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x]
)/b]))/b^2)/(2*b)))/(3*b) + (-1/2*(c + d*x)^3/(b*(a + b*ArcSinh[c + d*x])^
2) + (3*(-(((c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*(a + b*ArcSinh[c + d*x])
)) + ((CoshIntegral[(a + b*ArcSinh[c + d*x])/b]*Sinh[a/b])/4 - (3*CoshInte
gral[(3*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(3*a)/b])/4 - (Cosh[a/b]*SinhInt
egral[(a + b*ArcSinh[c + d*x])/b])/4 + (3*Cosh[(3*a)/b]*SinhIntegral[(3*(a
 + b*ArcSinh[c + d*x]))/b])/4)/b^2))/(2*b))/b))/d
```

3.176. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

3.176.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

3.176. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.176. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

3.176.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 708 vs. $2(307) = 614$.

Time = 0.47 (sec) , antiderivative size = 709, normalized size of antiderivative = 2.14

method	result
derivativedivides	$\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2\left(9b^2\operatorname{arcsinh}(dx+c)^2+18ab\operatorname{arcsinh}(dx+c)-3b^2\operatorname{arcsinh}(dx+c)+9a^2\right.}{48b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}$
default	$\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2\left(9b^2\operatorname{arcsinh}(dx+c)^2+18ab\operatorname{arcsinh}(dx+c)-3b^2\operatorname{arcsinh}(dx+c)+9a^2\right.}{48b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}$

input
```
int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/48*(-4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2
)+3*d*x+3*c)*e^2*(9*b^2*arcsinh(d*x+c)^2+18*a*b*arcsinh(d*x+c)-3*b^2*arcsi
nh(d*x+c)+9*a^2-3*a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(d*x
+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)+9/16*e^2/b^4*exp(3*a/b)*Ei(1,3*arcsinh(d
*x+c)+3*a/b)-1/48*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^2*(b^2*arcsinh(d*x+c)^2+2
*a*b*arcsinh(d*x+c)-b^2*arcsinh(d*x+c)+a^2-a*b+2*b^2)/b^3/(b^3*arcsinh(d*x
+c)^3+3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)-1/48*e^2/b^4*ex
p(a/b)*Ei(1,arcsinh(d*x+c)+a/b)+1/24/b*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+
b*arcsinh(d*x+c))^3+1/48/b^2*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(
d*x+c))^2+1/48/b^3*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+1/
48/b^4*e^2*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)-1/24/b*e^2*(4*(d*x+c)^3+3*d
*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d
*x+c))^3-1/16/b^2*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/
2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-3/16/b^3*e^2*(4*(d*x+c)^3+3
*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh
(d*x+c))-9/16/b^4*e^2*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b))
```

3.176.5 Fricas [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx = \int \frac{(dex+ce)^2}{(b\,\text{arsinh}\,(dx+c)+a)^4}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^4*arcsinh(d*x + c)^4 + 4
*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh
(d*x + c) + a^4), x)
```

3.176.6 Sympy [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx$$
$$= e^2\left(\int \frac{c^2}{a^4+4a^3b\,\text{asinh}\,(c+dx)+6a^2b^2\,\text{asinh}^2\,(c+dx)+4ab^3\,\text{asinh}^3\,(c+dx)+b^4\,\text{asinh}^4\,(c+dx)}\,dx\right.$$
$$+\int \frac{d^2x^2}{a^4+4a^3b\,\text{asinh}\,(c+dx)+6a^2b^2\,\text{asinh}^2\,(c+dx)+4ab^3\,\text{asinh}^3\,(c+dx)+b^4\,\text{asinh}^4\,(c+dx)}\,dx$$
$$\left.+\int \frac{2cdx}{a^4+4a^3b\,\text{asinh}\,(c+dx)+6a^2b^2\,\text{asinh}^2\,(c+dx)+4ab^3\,\text{asinh}^3\,(c+dx)+b^4\,\text{asinh}^4\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**4,x)
```

output
```
e**2*(Integral(c**2/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c
+ d*x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Int
egral(d**2*x**2/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*
x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integra
l(2*c*d*x/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2
+ 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x))
```

3.176.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
Timed out
```

3.176.8 Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx = \int \frac{(dex+ce)^2}{(b\,\text{arsinh}\,(dx+c)+a)^4}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^4, x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^4}\,dx = \int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\text{asinh}\,(c+d\,x))^4}\,dx$$

input
```
int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^4, x)
```

3.177 $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

3.177.1 Optimal result

Integrand size = 21, antiderivative size = 204

$$\begin{aligned}\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^4}\,dx &= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3}-\frac{e}{6b^2d(a+b\text{arcsinh}(c+dx))^2}\\ &\quad-\frac{e(c+dx)^2}{3b^2d(a+b\text{arcsinh}(c+dx))^2}\\ &\quad-\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3b^3d(a+b\text{arcsinh}(c+dx))}\\ &\quad+\frac{2e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{3b^4d}\\ &\quad-\frac{2e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{3b^4d}\end{aligned}$$

output
```
-1/6*e/b^2/d/(a+b*arcsinh(d*x+c))^2-1/3*e*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x
+c))^2+2/3*e*Chi(2*(a+b*arcsinh(d*x+c))/b)*cosh(2*a/b)/b^4/d-2/3*e*Shi(2*(
a+b*arcsinh(d*x+c))/b)*sinh(2*a/b)/b^4/d-1/3*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)
/b/d/(a+b*arcsinh(d*x+c))^3-2/3*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*a
rcsinh(d*x+c))
```

3.177.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.89

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^4}\,dx$$
$$= \frac{e\left(-\frac{2b^3(c+dx)\sqrt{1+(c+dx)^2}}{(a+b\text{arcsinh}(c+dx))^3}+\frac{b^2(-1-2(c+dx)^2)}{(a+b\text{arcsinh}(c+dx))^2}-\frac{4b(c+dx)\sqrt{1+(c+dx)^2}}{a+b\text{arcsinh}(c+dx)}+4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)\right)\right.}{6}$$

input
```
Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
(e*((-2*b^3*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3 +
(b^2*(-1 - 2*(c + d*x)^2))/(a + b*ArcSinh[c + d*x])^2 - (4*b*(c + d*x)*Sqr
t[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x]) + 4*Cosh[(2*a)/b]*CoshIntegra
l[2*(a/b + ArcSinh[c + d*x])] + 4*Log[a + b*ArcSinh[c + d*x]] - 4*(Log[a +
 b*ArcSinh[c + d*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x
])])))/(6*b^4*d)
```

3.177.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {6274, 27, 6194, 6198, 6233, 6193, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^4}\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \frac{e(c+dx)}{(a+b\text{arcsinh}(c+dx))^4}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e\int \frac{c+dx}{(a+b\text{arcsinh}(c+dx))^4}d(c+dx)}{d}$$

$$\Big\downarrow \; 6194$$

$$\frac{e\left(\frac{\int \frac{1}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{3b}+\frac{2\int \frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{3b}-\frac{\sqrt{(c+dx)^2+1}(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \text{6198}$$

$$\frac{e\left(\frac{2\int \frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^3}d(c+dx)}{3b}-\frac{1}{6b^2(a+b\text{arcsinh}(c+dx))^2}-\frac{\sqrt{(c+dx)^2+1}(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \text{6233}$$

$$\frac{e\left(\frac{2\left(\frac{\int \frac{c+dx}{(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{b}-\frac{(c+dx)^2}{2b(a+b\text{arcsinh}(c+dx))^2}\right)}{3b}-\frac{1}{6b^2(a+b\text{arcsinh}(c+dx))^2}-\frac{\sqrt{(c+dx)^2+1}(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \text{6193}$$

$$\frac{e\left(\frac{2\left(\frac{\frac{\int \frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}}{b}-\frac{(c+dx)^2}{2b(a+b\text{arcsinh}(c+dx))^2}\right)}{3b}-\frac{1}{6b^2(a+b\text{arcsinh}(c+\cdots}\right.}{d}$$

$$\Big\downarrow \text{3042}$$

3.177. $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

$$\frac{e\left(\frac{2\left(-\frac{(c+dx)^2}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}}{b}\right)}{3b}-\frac{1}{6b^2(a+b\text{arcsi}\cdots}\cdots\right)}{d}\cdots$$

$\downarrow$ 3784

$$\frac{e\left(\frac{2\left(-\frac{(c+dx)^2}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{\cosh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-i\sinh\left(\frac{2a}{b}\right)\int -\frac{i\sinh\cdots}{\cdots}}{b^2}}{b}\cdots\right)}{3b}\cdots\right)}{d}\cdots$$

$\downarrow$ 26

$$\frac{e\left(\frac{2\left(\frac{\frac{\cosh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-\sinh\left(\frac{2a}{b}\right)\int \frac{\sinh\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{(c+dx)\cdots}{b(a+b\text{ar}\cdots}}{b}\cdots\right)}{3b}\cdots\right)}{d}\cdots$$

$\downarrow$ 3042

3.177. $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

$$e\left(\frac{2\left(-\frac{(c+dx)^2}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))-\sinh\left(\frac{2a}{b}\right)\int-\frac{i\sin\cdots}{\cdots}}{b^2}}{b}\cdots\right)}{3b}\cdots\right)$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$\downarrow$ 26

$$e\left(\frac{2\left(-\frac{(c+dx)^2}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))+\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(\cdots}{\cdots}\right)}{a\cdots}}{b^2}}{b}\cdots\right)}{3b}\cdots\right)$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$\downarrow$ 3779

$$e\left(\frac{2\left(-\frac{(c+dx)^2}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)+\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}\cdots}{b^2}}{b}\cdots\right)}{3b}\cdots\right)$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$\downarrow$ 3782

3.177. $\int\frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

$$\frac{e\left(\frac{2\left(\frac{\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{b^2}-\frac{(c+dx)\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}}{b}-\frac{(c+dx)^2}{2b(a+b\text{arcsinh}(c+dx))^2}\right)}{3b}-\frac{}{6}\right.}{d}$$

input
```
Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
(e*(-1/3*((c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*(a + b*ArcSinh[c + d*x])^3)
- 1/(6*b^2*(a + b*ArcSinh[c + d*x])^2) + (2*(-1/2*(c + d*x)^2/(b*(a + b*Ar
cSinh[c + d*x])^2) + (-(((c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*(a + b*ArcSin
h[c + d*x]))) + (Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b
] - Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/b^2)/b))/(
3*b)))/d
```

3.177.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

3.177. $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.177.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.63

method	result
derivativedivides	$\frac{\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e\left(2b^2\operatorname{arcsinh}(dx+c)^2+4ab\operatorname{arcsinh}(dx+c)-b^2\operatorname{arcsinh}(dx+c)+2a^2-ab+b^2\right)}{12b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}-\frac{e\,\mathrm{e}^{\frac{2a}{b}}\operatorname{Ei}_1\left(2\,a\right.}{}}{}$
default	$\frac{\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e\left(2b^2\operatorname{arcsinh}(dx+c)^2+4ab\operatorname{arcsinh}(dx+c)-b^2\operatorname{arcsinh}(dx+c)+2a^2-ab+b^2\right)}{12b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}-\frac{e\,\mathrm{e}^{\frac{2a}{b}}\operatorname{Ei}_1\left(2\,a\right.}{}}{}$

input
```
int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/12*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x+c)^2+1)*e*(2*b^2*arcsinh(
d*x+c)^2+4*a*b*arcsinh(d*x+c)-b^2*arcsinh(d*x+c)+2*a^2-a*b+b^2)/b^3/(b^3*a
rcsinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)-1/3*e
/b^4*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/12/b*e*(2*(d*x+c)^2+1+2*(d*
x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1/12/b^2*e*(2*(d*x+c)^2+1
+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/6/b^3*e*(2*(d*x+c
)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/3/b^4*e*exp(-2
*a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b))
```

3.177.5 Fricas [F]

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^4}\,dx=\int \frac{dex+ce}{\left(b\,\text{arsinh}\left(dx+c\right)+a\right)^4}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral((d*e*x + c*e)/(b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^
3 + 6*a^2*b^2*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

3.177.6 Sympy [F]

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$$
$$=e\left(\int \frac{c}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx\right.$$
$$\left.+\int \frac{dx}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**4,x)
```

output
```
e*(Integral(c/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)
**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(
d*x/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2 + 4*a*
b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x))
```

3.177.7 Maxima [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
Timed out
```

3.177.8 Giac [F]

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \int \frac{dex+ce}{(b\operatorname{arsinh}(dx+c)+a)^4}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^4, x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx=\int \frac{c\,e+d\,e\,x}{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^4}\,dx$$

input
```
int((c*e + d*e*x)/(a + b*asinh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)/(a + b*asinh(c + d*x))^4, x)
```

3.178 $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

3.178.1 Optimal result

Integrand size = 12, antiderivative size = 160

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^4}\,dx = -\frac{\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^3} - \frac{c+dx}{6b^2d(a+b\text{arcsinh}(c+dx))^2} - \frac{\sqrt{1+(c+dx)^2}}{6b^3d(a+b\text{arcsinh}(c+dx))} - \frac{\text{Chi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{6b^4d} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{6b^4d}$$

output
```
1/6*(-d*x-c)/b^2/d/(a+b*arcsinh(d*x+c))^2+1/6*cosh(a/b)*Shi((a+b*arcsinh(d
*x+c))/b)/b^4/d-1/6*Chi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)/b^4/d-1/3*(1+(d*
x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^3-1/6*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+
b*arcsinh(d*x+c))
```

3.178.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = -\frac{\frac{2b^3\sqrt{1+(c+dx)^2}}{(a+b\operatorname{arcsinh}(c+dx))^3}+\frac{b^2(c+dx)}{(a+b\operatorname{arcsinh}(c+dx))^2}+\frac{b\sqrt{1+(c+dx)^2}}{a+b\operatorname{arcsinh}(c+dx)}+\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-\operatorname{cosh}}{6b^4d}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(-4),x]
```

output
```
-1/6*((2*b^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3 + (b^2*(c +
 d*x))/(a + b*ArcSinh[c + d*x])^2 + (b*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSi
nh[c + d*x]) + CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - Cosh[a/b]*
SinhIntegral[a/b + ArcSinh[c + d*x]])/(b^4*d)
```

3.178.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {6273, 6188, 6233, 6188, 6234, 25, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^4}dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^4}d(c+dx)}{d}$$

$$\Big\downarrow \text{6188}$$

$$\frac{\frac{\int \frac{c+dx}{\sqrt{(c+dx)^2+1}(a+b\operatorname{arcsinh}(c+dx))^3}d(c+dx)}{3b}-\frac{\sqrt{(c+dx)^2+1}}{3b(a+b\operatorname{arcsinh}(c+dx))^3}}{d}$$

$$\Big\downarrow \text{6233}$$

$$\frac{\frac{\frac{\int \frac{1}{(a+b\text{arcsinh}(c+dx))^2}d(c+dx)}{2b}-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}}{3b}-\frac{\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^3}}{d}$$

$$\Big\downarrow \; 6188$$

$$\frac{\frac{\frac{\frac{\int \frac{c+dx}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))}d(c+dx)}{b}-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}}{2b}-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}}{3b}-\frac{\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^3}}{d}$$

$$\Big\downarrow \; 6234$$

$$\frac{\frac{\frac{\frac{\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}}{2b}-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}}{3b}-\frac{\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^3}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{-\frac{\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}}{2b}-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}}{3b}-\frac{\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^3}}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^3}+\frac{-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}-\frac{\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}}{2b}}{3b}}{d}$$

$$\Big\downarrow \; 26$$

$$\frac{-\frac{\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^3}+\frac{-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+dx))}{b^2}}{2b}}{3b}}{d}$$

$$\Big\downarrow \; 3784$$

3.178. $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

$$-\frac{\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^3}+\frac{-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+\ldots\right.}{\ldots}}{2b}}{3b}}{d}$$

$\Big\downarrow$ 26

$$-\frac{\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^3}+\frac{-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\cosh\left(\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsinh}(c+\ldots\right.}{\ldots}}{2b}}{3b}}{d}$$

$\Big\downarrow$ 3042

$$-\frac{\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^3}+\frac{-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsin}\ldots\right.}{\ldots}}{2b}}{3b}}{d}$$

$\Big\downarrow$ 26

$$-\frac{\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^3}+\frac{-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsin}\ldots\right.}{\ldots}}{2b}}{3b}}{d}$$

$\Big\downarrow$ 3779

$$-\frac{\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^3}+\frac{-\frac{c+dx}{2b(a+b\text{arcsinh}(c+dx))^2}+\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\text{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{a+b\text{arcsinh}(c+dx)}d(a+b\text{arcsin}\ldots\right.}{b^2}}{2b}}{3b}}{d}$$

$\Big\downarrow$ 3782

3.178. $\displaystyle\int\frac{1}{(a+b\text{arcsinh}(c+dx))^4}\,dx$

$$\frac{-\frac{\sqrt{(c+dx)^2+1}}{3b(a+b\operatorname{arcsinh}(c+dx))^3}+\frac{-\frac{c+dx}{2b(a+b\operatorname{arcsinh}(c+dx))^2}+\frac{-\frac{\sqrt{(c+dx)^2+1}}{b(a+b\operatorname{arcsinh}(c+dx))}+\frac{i\left(i\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right)-i\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+}{}\right.\right.}{b^2}}{2b}}{3b}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(-4),x]
```

output
```
(-1/3*Sqrt[1 + (c + d*x)^2]/(b*(a + b*ArcSinh[c + d*x])^3) + (-1/2*(c + d*
x)/(b*(a + b*ArcSinh[c + d*x])^2) + (-(Sqrt[1 + (c + d*x)^2]/(b*(a + b*Arc
Sinh[c + d*x]))) + (I*(I*CoshIntegral[(a + b*ArcSinh[c + d*x])/b]*Sinh[a/b
] - I*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b]))/b^2)/(2*b))/(3*
b))/d
```

3.178.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.178.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.70

method	result
derivativedivides	$\frac{\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)\left(b^2 \operatorname{arcsinh}(dx+c)^2+2ab \operatorname{arcsinh}(dx+c)-b^2 \operatorname{arcsinh}(dx+c)+a^2-ab+2b^2\right)}{12b^3\left(b^3 \operatorname{arcsinh}(dx+c)^3+3a b^2 \operatorname{arcsinh}(dx+c)^2+3a^2 b \operatorname{arcsinh}(dx+c)+a^3\right)}+\frac{e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{12b^4}-\frac{}{6}}{d}$
default	$\frac{\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)\left(b^2 \operatorname{arcsinh}(dx+c)^2+2ab \operatorname{arcsinh}(dx+c)-b^2 \operatorname{arcsinh}(dx+c)+a^2-ab+2b^2\right)}{12b^3\left(b^3 \operatorname{arcsinh}(dx+c)^3+3a b^2 \operatorname{arcsinh}(dx+c)^2+3a^2 b \operatorname{arcsinh}(dx+c)+a^3\right)}+\frac{e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{12b^4}-\frac{}{6}}{d}$

input
```
int(1/(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/12*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh
(d*x+c)-b^2*arcsinh(d*x+c)+a^2-a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^
2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)+1/12/b^4*exp(a/b)*Ei(1,arcs
inh(d*x+c)+a/b)-1/6/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1
/12/b^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/12/b^3*(d*x+c
+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/12/b^4*exp(-a/b)*Ei(1,-arcsin
h(d*x+c)-a/b))
```

3.178.5 Fricas [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^4}\,dx=\int \frac{1}{(b\,\text{arsinh}\,(dx+c)+a)^4}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral(1/(b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^
2*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

3.178.6 Sympy [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^4}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(c+dx))^4}\,dx$$

input
```
integrate(1/(a+b*asinh(d*x+c))**4,x)
```

output
```
Integral((a + b*asinh(c + d*x))**(-4), x)
```

3.178.7 Maxima [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^4} dx = \int \frac{1}{(b \operatorname{arsinh}(dx+c)+a)^4} dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/6*((a^2*d^11 + a*b*d^11 + 2*b^2*d^11)*x^11 + 11*(a^2*c*d^10 + a*b*c*d^1
0 + 2*b^2*c*d^10)*x^10 + 5*((11*c^2*d^9 + d^9)*a^2 + (11*c^2*d^9 + d^9)*a*
b + 2*(11*c^2*d^9 + d^9)*b^2)*x^9 + 15*((11*c^3*d^8 + 3*c*d^8)*a^2 + (11*c
^3*d^8 + 3*c*d^8)*a*b + 2*(11*c^3*d^8 + 3*c*d^8)*b^2)*x^8 + 10*((33*c^4*d^
7 + 18*c^2*d^7 + d^7)*a^2 + (33*c^4*d^7 + 18*c^2*d^7 + d^7)*a*b + 2*(33*c^
4*d^7 + 18*c^2*d^7 + d^7)*b^2)*x^7 + 14*((33*c^5*d^6 + 30*c^3*d^6 + 5*c*d^
6)*a^2 + (33*c^5*d^6 + 30*c^3*d^6 + 5*c*d^6)*a*b + 2*(33*c^5*d^6 + 30*c^3*
d^6 + 5*c*d^6)*b^2)*x^6 + 2*((231*c^6*d^5 + 315*c^4*d^5 + 105*c^2*d^5 + 5*
d^5)*a^2 + (231*c^6*d^5 + 315*c^4*d^5 + 105*c^2*d^5 + 5*d^5)*a*b + 2*(231*
c^6*d^5 + 315*c^4*d^5 + 105*c^2*d^5 + 5*d^5)*b^2)*x^5 + 10*((33*c^7*d^4 +
63*c^5*d^4 + 35*c^3*d^4 + 5*c*d^4)*a^2 + (33*c^7*d^4 + 63*c^5*d^4 + 35*c^3
*d^4 + 5*c*d^4)*a*b + 2*(33*c^7*d^4 + 63*c^5*d^4 + 35*c^3*d^4 + 5*c*d^4)*b
^2)*x^4 + 5*((33*c^8*d^3 + 84*c^6*d^3 + 70*c^4*d^3 + 20*c^2*d^3 + d^3)*a^2
 + (33*c^8*d^3 + 84*c^6*d^3 + 70*c^4*d^3 + 20*c^2*d^3 + d^3)*a*b + 2*(33*c
^8*d^3 + 84*c^6*d^3 + 70*c^4*d^3 + 20*c^2*d^3 + d^3)*b^2)*x^3 + ((a^2*d^6
+ a*b*d^6 + 2*b^2*d^6)*x^6 + 6*(a^2*c*d^5 + a*b*c*d^5 + 2*b^2*c*d^5)*x^5 +
 (15*a*b*c^2*d^4 + (15*c^2*d^4 + d^4)*a^2 + 2*(15*c^2*d^4 + d^4)*b^2)*x^4
+ 4*(5*a*b*c^3*d^3 + (5*c^3*d^3 + c*d^3)*a^2 + 2*(5*c^3*d^3 + c*d^3)*b^2)*
x^3 + (c^6 + c^4 + 3*c^2 + 3)*a^2 + (c^6 - c^2)*a*b + 2*(c^6 + c^4)*b^2 +
(3*(5*c^4*d^2 + 2*c^2*d^2 + d^2)*a^2 + (15*c^4*d^2 - d^2)*a*b + 6*(5*c^...
```

3.178.8 Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^4} dx = \int \frac{1}{(b \operatorname{arsinh}(dx+c)+a)^4} dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(-4), x)
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^4}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4}\,dx$$

input
```
int(1/(a + b*asinh(c + d*x))^4,x)
```

output
```
int(1/(a + b*asinh(c + d*x))^4, x)
```

3.179 $\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^4} dx$

3.179.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^4} dx = \frac{\text{Int}\left(\frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))^4},x\right)}{e}$$

output
```
Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^4,x)/e
```

3.179.2 Mathematica [N/A]

Not integrable

Time = 4.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^4} dx = \int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^4} dx$$

input
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4),x]
```

output
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4), x]
```

3.179.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 27, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^4}dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \frac{1}{e(c+dx)(a+b\text{arcsinh}(c+dx))^4}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))^4}d(c+dx)}{de}$$

$$\Big\downarrow \text{ 6196}$$

$$\frac{\int \frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))^4}d(c+dx)}{de}$$

input
```
Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4),x]
```

output
```
$Aborted
```

3.179.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.179.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dex + ce)\left(a + b\ \text{arcsinh}\left(dx + c\right)\right)^4} dx$$

input
```
int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x)
```

output
```
int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x)
```

3.179.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 5.26

$$\int \frac{1}{(ce + dex)(a + b\text{arcsinh}(c + dx))^4} \, dx = \int \frac{1}{(dex + ce)(b\,\text{arsinh}\left(dx + c\right) + a)^4} \, dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral(1/(a^4*d*e*x + a^4*c*e + (b^4*d*e*x + b^4*c*e)*arcsinh(d*x + c)^4
 + 4*(a*b^3*d*e*x + a*b^3*c*e)*arcsinh(d*x + c)^3 + 6*(a^2*b^2*d*e*x + a^2
*b^2*c*e)*arcsinh(d*x + c)^2 + 4*(a^3*b*d*e*x + a^3*b*c*e)*arcsinh(d*x + c
)), x)
```

3.179.6 Sympy [N/A]

Not integrable

Time = 7.48 (sec) , antiderivative size = 151, normalized size of antiderivative = 6.57

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4}\,dx$$
$$= \frac{\int \frac{1}{a^4c+a^4dx+4a^3bc\operatorname{asinh}(c+dx)+4a^3bdx\operatorname{asinh}(c+dx)+6a^2b^2c\operatorname{asinh}^2(c+dx)+6a^2b^2dx\operatorname{asinh}^2(c+dx)+4ab^3c\operatorname{asinh}^3(c+dx)+4ab^3dx\operatorname{asinh}^3(c}}{e}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**4,x)
```

output
```
Integral(1/(a**4*c + a**4*d*x + 4*a**3*b*c*asinh(c + d*x) + 4*a**3*b*d*x*a
sinh(c + d*x) + 6*a**2*b**2*c*asinh(c + d*x)**2 + 6*a**2*b**2*d*x*asinh(c
+ d*x)**2 + 4*a*b**3*c*asinh(c + d*x)**3 + 4*a*b**3*d*x*asinh(c + d*x)**3
+ b**4*c*asinh(c + d*x)**4 + b**4*d*x*asinh(c + d*x)**4), x)/e
```

3.179.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
Timed out
```

3.179.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^4}\,dx = \int \frac{1}{(dex+ce)(b\operatorname{arsinh}(dx+c)+a)^4}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^4), x)
```

3.179.9 Mupad [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^4}\,dx=\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^4}\,dx$$

input
```
int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^4),x)
```

output
```
int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^4), x)
```

3.180 $\int (ce+dex)^4\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$

3.180.1 Optimal result

Integrand size = 25, antiderivative size = 361

$$\begin{aligned}\int (ce+dex)^4\sqrt{a+b\text{arcsinh}(c+dx)}\,dx &= \frac{e^4(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}}{5d}\\
&+\frac{\sqrt{b}e^4e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d}\\
&-\frac{\sqrt{b}e^4e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d}\\
&+\frac{\sqrt{b}e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{320d}\\
&-\frac{\sqrt{b}e^4e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d}\\
&+\frac{\sqrt{b}e^4e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d}\\
&-\frac{\sqrt{b}e^4e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{320d}\end{aligned}$$

output
```
1/1600*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1
/2)*5^(1/2)*Pi^(1/2)/d-1/1600*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/
b^(1/2))*b^(1/2)*5^(1/2)*Pi^(1/2)/d/exp(5*a/b)-1/192*e^4*exp(3*a/b)*erf(3^
(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/d+1/192
*e^4*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(
1/2)/d/exp(3*a/b)+1/32*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2)
)*b^(1/2)*Pi^(1/2)/d-1/32*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(
1/2)*Pi^(1/2)/d/exp(a/b)+1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(1/2)/d
```

3.180.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.95

$$\int (ce+dex)^4\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$$
$$= \frac{e^4e^{-\frac{5a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-150e^{\frac{6a}{b}}\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)+3\sqrt{5}\sqrt{\frac{a}{b}+\text{arcsi}}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(e^4*Sqrt[a + b*ArcSinh[c + d*x]]*(-150*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[
c + d*x])/b)]*Gamma[3/2, a/b + ArcSinh[c + d*x]] + 3*Sqrt[5]*Sqrt[a/b + Ar
cSinh[c + d*x]]*Gamma[3/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 25*Sqrt[3]*E
^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c +
d*x]))/b] + 150*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, -((a +
 b*ArcSinh[c + d*x])/b)] + 25*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*ArcSinh[c
+ d*x])/b)]*Gamma[3/2, (3*(a + b*ArcSinh[c + d*x]))/b] - 3*Sqrt[5]*E^((10*
a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[3/2, (5*(a + b*ArcSinh[c +
 d*x]))/b]))/(2400*d*E^((5*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

3.180.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {6274, 27, 6192, 6234, 25, 3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^4\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int e^4(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^4\int (c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{6192}$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{10}b\int \frac{(c+dx)^5}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6234}$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{10}\int -\frac{\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^4\left(\frac{1}{10}\int \frac{\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))+\frac{1}{5}(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{10}\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)^5}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{d}$$

$\downarrow$ 26

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{10}i\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)^5}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{d}$$

$\downarrow$ 3793

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{10}i\int\left(\frac{i\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{5i\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{5}{}\right.\right.}{d}$$

$\downarrow$ 2009

$$e^4\left(\frac{1}{5}(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{10}i\left(\frac{5}{16}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{5}{32}i\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+ba}}{}\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^4*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(e^4*(((c + d*x)^5*Sqrt[a + b*ArcSinh[c + d*x]])/5 - (I/10)*(((5*I)/16)*Sq
rt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - ((5*I)/
32)*Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x
]])/Sqrt[b]] + (I/32)*Sqrt[b]*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a +
 b*ArcSinh[c + d*x]])/Sqrt[b]] - (((5*I)/16)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a
+ b*ArcSinh[c + d*x]]/Sqrt[b]])/E^(a/b) + (((5*I)/32)*Sqrt[b]*Sqrt[Pi/3]*E
rfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/E^((3*a)/b) - ((I/32)
*Sqrt[b]*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/
E^((5*a)/b))))/d
```

3.180.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.180.4 Maple [F]

$$\int (dex+ce)^4 \sqrt{a+b\,\operatorname{arcsinh}(dx+c)}dx$$

input
```
int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x)
```

output
```
int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x)
```

3.180.5 Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)^4\sqrt{a+b\text{arcsinh}(c+dx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.180.6 Sympy [F]

$$\begin{aligned}\int (ce+dex)^4\sqrt{a+b\text{arcsinh}(c+dx)}\,dx=e^4\bigg(&\int c^4\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx\\&+\int d^4x^4\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx\\&+\int 4cd^3x^3\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx\\&+\int 6c^2d^2x^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx\\&+\int 4c^3dx\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx\bigg)\end{aligned}$$

input
```
integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(1/2),x)
```

output
```
e**4*(Integral(c**4*sqrt(a + b*asinh(c + d*x)), x) + Integral(d**4*x**4*sq
rt(a + b*asinh(c + d*x)), x) + Integral(4*c*d**3*x**3*sqrt(a + b*asinh(c +
 d*x)), x) + Integral(6*c**2*d**2*x**2*sqrt(a + b*asinh(c + d*x)), x) + In
tegral(4*c**3*d*x*sqrt(a + b*asinh(c + d*x)), x))
```

3.180.7 Maxima [F]

$$\int (ce+dex)^4\sqrt{a+b\text{arcsinh}(c+dx)}\,dx=\int (dex+ce)^4\sqrt{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^4*sqrt(b*arcsinh(d*x + c) + a), x)
```

3.180.8 Giac [F]

$$\int (ce+dex)^4\sqrt{a+b\text{arcsinh}(c+dx)}\,dx=\int (dex+ce)^4\sqrt{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4*sqrt(b*arcsinh(d*x + c) + a), x)
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^4\sqrt{a+b\text{arcsinh}(c+dx)}\,dx=\int (c\,e+d\,e\,x)^4\,\sqrt{a+b\,\text{asinh}\,(c+d\,x)}\,dx$$

input
```
int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(1/2),x)
```

output
```
int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(1/2), x)
```

3.181 $\int (ce+dex)^3 \sqrt{a+b\text{arcsinh}(c+dx)}\,dx$

3.181.1 Optimal result

Integrand size = 25, antiderivative size = 272

$$\int (ce+dex)^3 \sqrt{a+b\text{arcsinh}(c+dx)}\,dx = -\frac{3e^3\sqrt{a+b\text{arcsinh}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}}{4d} - \frac{\sqrt{b}e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d} + \frac{\sqrt{b}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d} - \frac{\sqrt{b}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d} + \frac{\sqrt{b}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d}$$

output
```
1/64*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2
)*2^(1/2)*Pi^(1/2)/d+1/64*e^3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1
/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-1/256*e^3*exp(4*a/b)*erf(2*(a+b
*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/256*e^3*erfi(2*(a+b*a
rcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(4*a/b)-3/32*e^3*(a+b*
arcsinh(d*x+c))^(1/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^(1/2)/d
```

3.181.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.82

$$\int (ce+dex)^3\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$$
$$=\frac{e^3e^{-\frac{4a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}\Gamma\left(\frac{3}{2},-\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)-4\sqrt{2}e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}\ldots}\right.}{12\ldots}$$

input
```
Integrate[(c*e + d*e*x)^3*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(e^3*Sqrt[a + b*ArcSinh[c + d*x]]*(Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2,
 (-4*(a + b*ArcSinh[c + d*x]))/b] - 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcSi
nh[c + d*x]]*Gamma[3/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*Sqr
t[-((a + b*ArcSinh[c + d*x])/b)]*(-4*Sqrt[2]*Gamma[3/2, (2*(a + b*ArcSinh[
c + d*x]))/b] + E^((2*a)/b)*Gamma[3/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/
(128*d*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

3.181.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6274, 27, 6192, 6234, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int e^3(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^3\int (c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6192}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{8}b\int\frac{(c+dx)^4}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6234}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{8}\int\frac{\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{8}\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)^4}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{d}$$

$$\Big\downarrow \text{3793}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{8}\int\left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{8\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{}{8\sqrt{a+}}\right.\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$e^3\left(\frac{1}{8}\left(-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{}}{}\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^3*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(e^3*(((c + d*x)^4*Sqrt[a + b*ArcSinh[c + d*x]])/4 + ((-3*Sqrt[a + b*ArcSi
nh[c + d*x]])/4 - (Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[
c + d*x]])/Sqrt[b]])/32 + (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqr
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/4 - (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a
+ b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*E^((4*a)/b)) + (Sqrt[b]*Sqrt[Pi/2]*Er
fi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(4*E^((2*a)/b)))/8))/d
```

3.181. $\int (ce+dex)^3\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$

3.181.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.181.4 Maple [F]

$$\int (dex+ce)^3 \sqrt{a+b \operatorname{arcsinh}(dx+c)}dx$$

input
```
int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x)
```

output
```
int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x)
```

3.181.5 Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)^3 \sqrt{a+b\text{arcsinh}(c+dx)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.181.6 Sympy [F]

$$\int (ce+dex)^3 \sqrt{a+b\text{arcsinh}(c+dx)}\, dx = e^3\left(\int c^3\sqrt{a+b\operatorname{asinh}(c+dx)}\, dx + \int d^3x^3\sqrt{a+b\operatorname{asinh}(c+dx)}\, dx + \int 3cd^2x^2\sqrt{a+b\operatorname{asinh}(c+dx)}\, dx + \int 3c^2dx\sqrt{a+b\operatorname{asinh}(c+dx)}\, dx\right)$$

input
```
integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(1/2),x)
```

output
```
e**3*(Integral(c**3*sqrt(a + b*asinh(c + d*x)), x) + Integral(d**3*x**3*sq
rt(a + b*asinh(c + d*x)), x) + Integral(3*c*d**2*x**2*sqrt(a + b*asinh(c +
 d*x)), x) + Integral(3*c**2*d*x*sqrt(a + b*asinh(c + d*x)), x))
```

3.181.7 Maxima [F]

$$\int (ce+dex)^3\sqrt{a+b\text{arcsinh}(c+dx)}\,dx=\int (dex+ce)^3\sqrt{b\operatorname{arsinh}(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^3*sqrt(b*arcsinh(d*x + c) + a), x)
```

3.181.8 Giac [F]

$$\int (ce+dex)^3\sqrt{a+b\text{arcsinh}(c+dx)}\,dx=\int (dex+ce)^3\sqrt{b\operatorname{arsinh}(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3*sqrt(b*arcsinh(d*x + c) + a), x)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3\sqrt{a+b\text{arcsinh}(c+dx)}\,dx=\int (c\,e+d\,e\,x)^3\,\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}\,dx$$

input
```
int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(1/2),x)
```

output
```
int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(1/2), x)
```

3.182 $\int (ce+dex)^2 \sqrt{a+b\text{arcsinh}(c+dx)}\,dx$

3.182.1 Optimal result

Integrand size = 25, antiderivative size = 245

$$\int (ce+dex)^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx=\frac{e^2(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}}{3d}-\frac{\sqrt{b}e^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{\sqrt{b}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{48d}+\frac{\sqrt{b}e^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{b}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{48d}$$

output
```
1/144*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/
2)*3^(1/2)*Pi^(1/2)/d-1/144*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^
(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)-1/16*e^2*exp(a/b)*erf((a+b*ar
csinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d+1/16*e^2*erfi((a+b*arcsinh
(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(a/b)+1/3*e^2*(d*x+c)^3*(a+b
*arcsinh(d*x+c))^(1/2)/d
```

3.182.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.97

$$\int (ce+dex)^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$$
$$= \frac{e^2e^{-\frac{3a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\text{arcsinh}(c\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(e^2*Sqrt[a + b*ArcSinh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c +
 d*x])/b)]*Gamma[3/2, a/b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh
[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 9*E^((2*a)/b)*Sqr
t[a/b + ArcSinh[c + d*x]]*Gamma[3/2, -((a + b*ArcSinh[c + d*x])/b)] - Sqrt
[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*A
rcSinh[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2
/b^2)])
```

3.182.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {6274, 27, 6192, 6234, 25, 3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$$
$$\Big\downarrow \ 6274$$
$$\frac{\int e^2(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \ 27$$
$$\frac{e^2\int (c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{6192}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{6}b\int\frac{(c+dx)^3}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6234}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{6}\int-\frac{\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^2\left(\frac{1}{6}\int\frac{\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))+\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{6}\int\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)^3}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{6}i\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)^3}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{d}$$

$$\Big\downarrow \text{3793}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{6}i\int\left(\frac{3i\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{i\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(c+dx)}}\right)d(a+\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{6}i\left(\frac{3}{8}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}i\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcs}}}{\sqrt{b}}\right.\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

3.182. $\int (ce+dex)^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$

output
```
(e^2*(((c + d*x)^3*Sqrt[a + b*ArcSinh[c + d*x]])/3 + (I/6)*(((3*I)/8)*Sqrt
[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - (I/8)*Sqr
t[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]] - (((3*I)/8)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt
[b]])/E^(a/b) + ((I/8)*Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh
[c + d*x]])/Sqrt[b]])/E^((3*a)/b))))/d
```

3.182.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.182.4 Maple [F]

$$\int (dex+ce)^2 \sqrt{a+b\operatorname{arcsinh}(dx+c)}dx$$

input
```
int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x)
```

output
```
int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x)
```

3.182.5 Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.182.6 Sympy [F]

$$\int (ce+dex)^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = e^2\left(\int c^2\sqrt{a+b\operatorname{asinh}{(c+dx)}}\,dx + \int d^2x^2\sqrt{a+b\operatorname{asinh}{(c+dx)}}\,dx + \int 2cdx\sqrt{a+b\operatorname{asinh}{(c+dx)}}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(1/2),x)
```

output
```
e**2*(Integral(c**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(d**2*x**2*sq
rt(a + b*asinh(c + d*x)), x) + Integral(2*c*d*x*sqrt(a + b*asinh(c + d*x))
, x))
```

3.182.7 Maxima [F]

$$\int (ce+dex)^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int (dex+ce)^2\sqrt{b\operatorname{arsinh}(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2*sqrt(b*arcsinh(d*x + c) + a), x)
```

3.182.8 Giac [F]

$$\int (ce+dex)^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int (dex+ce)^2\sqrt{b\operatorname{arsinh}(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2*sqrt(b*arcsinh(d*x + c) + a), x)
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2\sqrt{a+b\text{arcsinh}(c+dx)}\,dx=\int (c\,e+d\,e\,x)^2\,\sqrt{a+b\,\mathrm{asinh}\,(c+d\,x)}\,dx$$

input
```
int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(1/2),x)
```

output
```
int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(1/2), x)
```

3.183 $\int (ce+dex)\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx$

3.183.1 Optimal result

Integrand size = 23, antiderivative size = 164

$$\int (ce+dex)\sqrt{a+b\operatorname{arcsinh}(c+dx)}\,dx = \frac{e\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{2d} - \frac{\sqrt{b}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d} - \frac{\sqrt{b}ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d}$$

output
```
-1/32*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)
*2^(1/2)*Pi^(1/2)/d-1/32*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2
)*b^(1/2)*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)+1/4*e*(a+b*arcsinh(d*x+c))^(1/2)/d
+1/2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(1/2)/d
```

3.183.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.74

$$\int (ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$$
$$=\frac{ee^{-\frac{2a}{b}}\left(-b\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{3}{2},-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)+be^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}\Gamma\left(\frac{3}{2},\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)\right)}{8\sqrt{2}d\sqrt{a+b\text{arcsinh}(c+dx)}}$$

input
```
Integrate[(c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(e*(-(b*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[3/2, (-2*(a + b*ArcSinh[
c + d*x]))/b]) + b*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, (2*
(a + b*ArcSinh[c + d*x]))/b]))/(8*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSinh
[c + d*x]])
```

3.183.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6274, 27, 6192, 6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$$
$$\Big\downarrow \text{ 6274}$$
$$\frac{\int e(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{e\int (c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \text{ 6192}$$
$$\frac{e\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{4}b\int \frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6234}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{4}\int\frac{\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{4}\int-\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{4}\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{d}$$

$$\Big\downarrow \text{3793}$$

$$\frac{e\left(\frac{1}{4}\int\left(\frac{1}{2\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(c+dx)}}\right)d(a+b\text{arcsinh}(c+dx))+\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arcsinh}}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\left(\frac{1}{4}\left(-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\sqrt{a+b\text{arcsinh}(c+\right.\right.}{d}$$

input

```
Int[(c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output

```
(e*(((c + d*x)^2*Sqrt[a + b*ArcSinh[c + d*x]])/2 + (Sqrt[a + b*ArcSinh[c +
 d*x]] - (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c
 + d*x]])/Sqrt[b]])/4 - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(4*E^((2*a)/b)))/4))/d
```

3.183. $\int (ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}\,dx$

3.183.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.183. $\int (ce + dex)\sqrt{a + b\operatorname{arcsinh}(c + dx)}\,dx$

3.183.4 Maple [F]

$$\int (dex + ce)\sqrt{a + b\,\mathrm{arcsinh}\,(dx + c)}dx$$

input
```
int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x)
```

output
```
int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x)
```

3.183.5 Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)\sqrt{a + b\mathrm{arcsinh}(c + dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.183.6 Sympy [F]

$$\int (ce + dex)\sqrt{a + b\mathrm{arcsinh}(c + dx)}\,dx = e\left(\int c\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,dx + \int dx\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(1/2),x)
```

output
```
e*(Integral(c*sqrt(a + b*asinh(c + d*x)), x) + Integral(d*x*sqrt(a + b*asi
nh(c + d*x)), x))
```

3.183.7 Maxima [F]

$$\int (ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int (dex+ce)\sqrt{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a), x)
```

3.183.8 Giac [F]

$$\int (ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int (dex+ce)\sqrt{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a), x)
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \int (c\,e+d\,e\,x)\,\sqrt{a+b\,\text{asinh}\,(c+d\,x)}\,dx$$

input
```
int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(1/2),x)
```

output
```
int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(1/2), x)
```

3.184 $\int \sqrt{a+b\text{arcsinh}(c+dx)}\,dx$

3.184.1 Optimal result

Integrand size = 14, antiderivative size = 115

$$\int \sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d} + \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d} - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4d}$$

output
```
1/4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/
4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(a/b)+(d*
x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

3.184.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \sqrt{a+b\text{arcsinh}(c+dx)}\,dx = \frac{e^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}}\right)}{2d}$$

input
```
Integrate[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c +
 d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*
x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b))
```

3.184.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6273, 6187, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\text{arcsinh}(c+dx)}\,dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int \sqrt{a+b\text{arcsinh}(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{6187}$$

$$\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}b\int \frac{c+dx}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6234}$$

$$\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))+(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{2}\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{d}$$

$$\Big\downarrow \text{3789}$$

$$\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\left(\frac{1}{2}i\int \frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int \frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(\right.}{d}$$

$$\Big\downarrow \text{2611}$$

$$\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-i\int e^{\frac{a+b\text{arcsinh}(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+\right.}{d}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcs}}}{\sqrt{}}\right.\right.}{d}$$

$$\Big\downarrow \text{2634}$$

$$\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx}}{\sqrt{b}}\right.\right.}{d}$$

input
```
Int[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
((c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]] - (I/2)*((I/2)*Sqrt[b]*E^(a/b)*Sqr
t[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi]*
Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/E^(a/b)))/d
```

3.184. $\int \sqrt{a+b\text{arcsinh}(c+dx)}\,dx$

3.184.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.184.4 Maple [F]

$$\int \sqrt{a + b \operatorname{arcsinh}(dx + c)}dx$$

input
```
int((a+b*arcsinh(d*x+c))^(1/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^(1/2),x)
```

3.184.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a + b\text{arcsinh}(c + dx)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.184.6 Sympy [F]

$$\int \sqrt{a + b\text{arcsinh}(c + dx)}\, dx = \int \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx$$

input
```
integrate((a+b*asinh(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*asinh(c + d*x)), x)
```

3.184.7 Maxima [F]

$$\int \sqrt{a+b\text{arcsinh}(c+dx)}\,dx=\int \sqrt{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

3.184.8 Giac [F]

$$\int \sqrt{a+b\text{arcsinh}(c+dx)}\,dx=\int \sqrt{b\,\text{arsinh}\,(dx+c)+a}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\text{arcsinh}(c+dx)}\,dx=\int \sqrt{a+b\,\text{asinh}\,(c+d\,x)}\,dx$$

input
```
int((a + b*asinh(c + d*x))^(1/2),x)
```

output
```
int((a + b*asinh(c + d*x))^(1/2), x)
```

3.185 $\int \frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{ce+dex}\,dx$

3.185.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{ce+dex}\,dx=\frac{\text{Int}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{c+dx},x\right)}{e}$$

output
```
Unintegrable((a+b*arcsinh(d*x+c))^(1/2)/(d*x+c),x)/e
```

3.185.2 Mathematica [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{ce+dex}\,dx=\int \frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{ce+dex}\,dx$$

input
```
Integrate[Sqrt[a + b*ArcSinh[c + d*x]]/(c*e + d*e*x),x]
```

output
```
Integrate[Sqrt[a + b*ArcSinh[c + d*x]]/(c*e + d*e*x), x]
```

3.185.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6274, 27, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{ce+dex} \, dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{e(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{c+dx}d(c+dx)}{de}$$

$$\Big\downarrow \text{6196}$$

$$\frac{\int \frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{c+dx}d(c+dx)}{de}$$

input
```
Int[Sqrt[a + b*ArcSinh[c + d*x]]/(c*e + d*e*x),x]
```

output
```
$Aborted
```

3.185.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.185.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}{dex+ce}dx$$

input
```
int((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x)
```

output
```
int((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x)
```

3.185.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{ce+dex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.185.6 Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{ce+dex}\,dx = \frac{\int \frac{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}{c+dx}\,dx}{e}$$

input
```
integrate((a+b*asinh(d*x+c))**(1/2)/(d*e*x+c*e),x)
```

output
```
Integral(sqrt(a + b*asinh(c + d*x))/(c + d*x), x)/e
```

3.185.7 Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{ce+dex}\,dx=\int \frac{\sqrt{b\,\text{arsinh}\left(dx+c\right)+a}}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

3.185.8 Giac [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{ce+dex}\,dx=\int \frac{\sqrt{b\,\text{arsinh}\left(dx+c\right)+a}}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

3.185.9 Mupad [N/A]

Not integrable

Time = 2.94 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{ce+dex}\,dx=\int \frac{\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}}{c\,e+d\,e\,x}\,dx$$

input
```
int((a + b*asinh(c + d*x))^(1/2)/(c*e + d*e*x),x)
```

output
```
int((a + b*asinh(c + d*x))^(1/2)/(c*e + d*e*x), x)
```

3.186 $\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

3.186.1 Optimal result

Integrand size = 25, antiderivative size = 601

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{25d} + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{25d} - \frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{50d} + \frac{e^4(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}}{5d} + \frac{3b^{3/2}e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} - \frac{b^{3/2}e^4e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{200d} - \frac{3b^{3/2}e^4e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d} + \frac{3b^{3/2}e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\,\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d} + \frac{3b^{3/2}e^4e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d} - \frac{b^{3/2}e^4e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{200d} - \frac{3b^{3/2}e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d} + \frac{3b^{3/2}e^4e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\,\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3200d}$$

output
```
1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(3/2)/d+3/16000*b^(3/2)*e^4*exp(5*a
/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/d+3/1
6000*b^(3/2)*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*
Pi^(1/2)/d/exp(5*a/b)-1/384*b^(3/2)*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsin
h(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d-1/384*b^(3/2)*e^4*erfi(3^(1/2)
*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)+3/64*b^
(3/2)*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/64
*b^(3/2)*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-
4/25*b*e^4*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d+2/25*b*e^4*(d*
x+c)^2*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d-3/50*b*e^4*(d*x+c)
^4*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

3.186.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.57

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{3/2}\,dx =$$

$$-\frac{be^4e^{-\frac{5a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(2250e^{\frac{6a}{b}}\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{5}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)+9\sqrt{5}\sqrt{\frac{a}{b}+\text{arcsi}}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
-1/36000*(b*e^4*Sqrt[a + b*ArcSinh[c + d*x]]*(2250*E^((6*a)/b)*Sqrt[-((a +
 b*ArcSinh[c + d*x])/b)]*Gamma[5/2, a/b + ArcSinh[c + d*x]] + 9*Sqrt[5]*Sq
rt[a/b + ArcSinh[c + d*x]]*Gamma[5/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 1
25*Sqrt[3]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2, (-3*(a + b*
ArcSinh[c + d*x]))/b] + 2250*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamm
a[5/2, -((a + b*ArcSinh[c + d*x])/b)] - 125*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a
+ b*ArcSinh[c + d*x])/b)]*Gamma[5/2, (3*(a + b*ArcSinh[c + d*x]))/b] + 9*S
qrt[5]*E^((10*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[5/2, (5*(a +
 b*ArcSinh[c + d*x]))/b]))/(d*E^((5*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^
2/b^2)])
```

3.186.3 Rubi [A] (verified)

Time = 3.67 (sec) , antiderivative size = 731, normalized size of antiderivative = 1.22, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.760, Rules used = {6274, 27, 6192, 6227, 6195, 5971, 2009, 6227, 6195, 5971, 2009, 6213, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int e^4(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e^4\int (c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}d(c+dx)}{d}$$

$$\Big\downarrow \; 6192$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\int \frac{(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \; 6227$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{1}{10}b\int \frac{(c+dx)^4}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)-\frac{4}{5}\int \frac{(c+dx)^3\sqrt{a+b\text{arcsinh}(c+d}}{\sqrt{(c+dx)^2+1}}\right.\right.}{d}$$

$$\Big\downarrow \; 6195$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{4}{5}\int \frac{(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{10}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsin}}{b}\right.}{\sqrt{}}\right.\right.}{}$$

$$\Big\downarrow \; 5971$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{4}{5}\int \frac{(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{10}\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{ar}}{}\right.}{16\sqrt{a+b\text{arcs}}}\right.\right.\right.}{}$$

$$\Big\downarrow \; 2009$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{4}{5}\int\frac{(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{10}\left(-\frac{1}{16}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\right.\right.\right.$$

$$\Big\downarrow \; 6227$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{4}{5}\left(-\frac{1}{6}b\int\frac{(c+dx)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)-\frac{2}{3}\int\frac{(c+dx)\sqrt{a+b\text{arcsinh}(}}{\sqrt{(c+dx)^2+1}}\right.\right.\right.$$

$$\Big\downarrow \; 6195$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{4}{5}\left(-\frac{2}{3}\int\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{6}\int\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arc}}{}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \; 5971$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{4}{5}\left(-\frac{2}{3}\int\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{6}\int\left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a}{}\right.}{4\sqrt{a+ba}}\right.\right.\right.\right.$$

$$\Big\downarrow \; 2009$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{4}{5}\left(-\frac{2}{3}\int\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{6}\left(\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\right.\right.\right.\right.$$

$$\Big\downarrow \; 6213$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{4}{5}\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}b\int\frac{}{\sqrt{a+b\text{arc}}}\right.\right.\right.\right.$$

$$\Big\downarrow \; 6189$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{4}{5}\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}\int\frac{\cosh\left(\frac{a}{b}-\right.}{\sqrt{a+b}}\right.\right.\right.\right.$$

3.186. $\int(ce+dex)^4(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

$\downarrow$ 3042

$$e^4\Bigg(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\Bigg(-\frac{4}{5}\Bigg(-\frac{2}{3}\Bigg(\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}\int\frac{\sin\Big(\frac{ia}{b}-\frac{i(}{}}{\sqrt{a}}$$

$\downarrow$ 3788

$$e^4\Bigg(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\Bigg(-\frac{4}{5}\Bigg(-\frac{2}{3}\Bigg(\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{2}\Bigg(\frac{1}{2}i\int\frac{i}{\sqrt{a+b}}$$

$\downarrow$ 26

$$e^4\Bigg(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\Bigg(-\frac{4}{5}\Bigg(-\frac{2}{3}\Bigg(\frac{1}{2}\Bigg(-\frac{1}{2}\int\frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}$$

$\downarrow$ 2611

$$e^4\Bigg(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\Bigg(-\frac{4}{5}\Bigg(-\frac{2}{3}\Bigg(\frac{1}{2}\Bigg(-\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\int e$$

$\downarrow$ 2633

$$e^4\Bigg(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\Bigg(-\frac{4}{5}\Bigg(-\frac{2}{3}\Bigg(\frac{1}{2}\Bigg(-\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}\sqrt{}$$

$\downarrow$ 2634

$$e^4\Bigg(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\Bigg(-\frac{4}{5}\Bigg(-\frac{2}{3}\Bigg(\frac{1}{2}\Bigg(-\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\Bigg(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\Bigg)-\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-}$$

input
```
Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

3.186. $\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

output
```
(e^4*(((c + d*x)^5*(a + b*ArcSinh[c + d*x])^(3/2))/5 - (3*b*(((c + d*x)^4*
Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/5 - (4*(((c + d*x)^2*S
qrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/3 - (2*(Sqrt[1 + (c + d
*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]] + (-1/2*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b
*ArcSinh[c + d*x]]/Sqrt[b]])/(2*E^(a/b)))/2))/3 + ((Sqrt[b]*E^(a/b)*Sqrt[P
i]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/8 - (Sqrt[b]*E^((3*a)/b)*Sqr
t[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/8 + (Sqrt[b]*
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*E^(a/b)) - (Sqrt[b
]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*E^((
3*a)/b)))/6))/5 + (-1/16*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[
c + d*x]]/Sqrt[b]]) + (Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a
+ b*ArcSinh[c + d*x]])/Sqrt[b]])/32 - (Sqrt[b]*E^((5*a)/b)*Sqrt[Pi/5]*Erf[
(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/32 - (Sqrt[b]*Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*E^(a/b)) + (Sqrt[b]*Sqrt[3*P
i]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*E^((3*a)/b))
- (Sqrt[b]*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(32*E^((5*a)/b)))/10))/10))/d
```

3.186.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

3.186. $\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.186.4 Maple [F]

$$\int (dex + ce)^4 (a + b \operatorname{arcsinh}(dx + c))^{\frac{3}{2}} dx$$

input
```
int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x)
```

output
```
int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x)
```

3.186.5 Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)^4 (a + b\text{arcsinh}(c + dx))^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.186.6 Sympy [F]

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = e^4\left(\int ac^4\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx + \int ad^4x^4\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx + \int bc^4\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx + \int 4acd^3x^3\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx + \int 6ac^2d^2x^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx + \int 4ac^3dx\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx + \int bd^4x^4\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx + \int 4bcd^3x^3\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx + \int 6bc^2d^2x^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx + \int 4bc^3dx\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(3/2),x)
```

output
```
e**4*(Integral(a*c**4*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d**4*x**
4*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*c**4*sqrt(a + b*asinh(c + d*
x))*asinh(c + d*x), x) + Integral(4*a*c*d**3*x**3*sqrt(a + b*asinh(c + d*x
)), x) + Integral(6*a*c**2*d**2*x**2*sqrt(a + b*asinh(c + d*x)), x) + Inte
gral(4*a*c**3*d*x*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*d**4*x**4*sq
rt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(4*b*c*d**3*x**3*sqr
t(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(6*b*c**2*d**2*x**2*s
qrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(4*b*c**3*d*x*sqrt(
a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

3.186.7 Maxima [F]

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (dex+ce)^4(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.186.8 Giac [F]

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{3/2}\,dx=\int (dex+ce)^4(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{3/2}\,dx=\int (c\,e+d\,e\,x)^4\,(a+b\,\text{asinh}(c+d\,x))^{3/2}\,dx$$

input
```
int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(3/2),x)
```

output
```
int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(3/2), x)
```

3.187 $\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

3.187.1 Optimal result

Integrand size = 25, antiderivative size = 360

$$\begin{aligned}\int (ce+dex)^3(a&+b\text{arcsinh}(c+dx))^{3/2}\,dx = \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{64d}\\ &-\frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{32d}\\ &-\frac{3e^3(a+b\text{arcsinh}(c+dx))^{3/2}}{32d}+\frac{e^3(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}}{4d}\\ &-\frac{3b^{3/2}e^3e^{\frac{4a}{b}}\sqrt{\pi}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d}\\ &+\frac{3b^{3/2}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d}\\ &+\frac{3b^{3/2}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d}\\ &-\frac{3b^{3/2}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d}\end{aligned}$$

output
```
-3/32*e^3*(a+b*arcsinh(d*x+c))^(3/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+
c))^(3/2)/d+3/256*b^(3/2)*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^
(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d-3/256*b^(3/2)*e^3*erfi(2^(1/2)*(a+b*arcs
inh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-3/2048*b^(3/2)*e^
3*exp(4*a/b)*erf(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/2048*b
^(3/2)*e^3*erfi(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(4*a/b
)+9/64*b*e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d-3/32
*b*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

3.187.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.62

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \frac{be^3e^{-\frac{4a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}\Gamma\left(\frac{5}{2},-\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)+8\sqrt{2}e^{\frac{2}{b}}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
(b*e^3*Sqrt[a + b*ArcSinh[c + d*x]]*(-(Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[
5/2, (-4*(a + b*ArcSinh[c + d*x]))/b]) + 8*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b +
ArcSinh[c + d*x]]*Gamma[5/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b
)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*(-8*Sqrt[2]*Gamma[5/2, (2*(a + b*Arc
Sinh[c + d*x]))/b] + E^((2*a)/b)*Gamma[5/2, (4*(a + b*ArcSinh[c + d*x]))/b
])))/(512*d*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

3.187.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.79 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.25, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6274, 27, 6192, 6227, 6195, 25, 5971, 2009, 6227, 6195, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \ 6274$$

$$\frac{\int e^3(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}d(c+dx)}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{e^3\int (c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}d(c+dx)}{d}$$

$$\Big\downarrow \ 6192$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\int \frac{(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \ 6227$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{1}{8}b\int \frac{(c+dx)^3}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)-\frac{3}{4}\int \frac{(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}\right.\right.}{d}$$

$$\Big\downarrow \ 6195$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\int \frac{(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{8}\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}}{b}\right.}{\sqrt{}}\right.\right.$$

$$\Big\downarrow \ 25$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\int \frac{(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{8}\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(}{b}\right.}{\sqrt{a}}\right.\right.$$

$$\Big\downarrow \ 5971$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\int \frac{(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{8}\int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcs}}{}\right.}{8\sqrt{a+b\text{arcsinh}}}\right.\right.\right.$$

$$\Big\downarrow \ 2009$$

3.187. $\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\int\frac{(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{8}\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{}}{}\right.\right.\right.\right.$$

$\downarrow$ 6227

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{4}b\int\frac{c+dx}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d\right.\right.\right.$$

$\downarrow$ 6195

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{4}\int-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}}{b}\right.}{\sqrt{a}}\right.\right.\right.$$

$\downarrow$ 25

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{4}\int\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c}{b}\right.}{\sqrt{a+}}\right.\right.\right.$$

$\downarrow$ 5971

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{4}\int\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}}{b}\right.}{2\sqrt{a+b\text{arcsinh}(c+}}\right.\right.\right.$$

$\downarrow$ 27

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{8}\int\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}}{b}\right.}{\sqrt{a+b\text{arcsinh}(c+}}\right.\right.\right.$$

$\downarrow$ 3042

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{8}\int-\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcs}}{b}\right.}{\sqrt{a+b\text{arcsinh}(}}\right.\right.\right.$$

$\downarrow$ 26

3.187. $\int(ce+dex)^3(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{8}i\int\frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}}{b}\right.}{\sqrt{a+b\text{arcsinh}(c}}\right.\right.\right.$$

$\downarrow$ 3789

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{8}i\left(\frac{1}{2}i\int\frac{e^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int\right.\right.\right.\right.$$

$\downarrow$ 2611

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{8}i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-i\int e\right.\right.\right.\right.$$

$\downarrow$ 2633

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{8}i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\sqrt{}\right.\right.\right.\right.$$

$\downarrow$ 2634

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{8}i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}}{}\right.\right.\right.\right.\right.$$

$\downarrow$ 6198

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(\frac{1}{8}\left(\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a}}{}\right.\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

3.187. $\int(ce+dex)^3(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

output
```
(e^3*(((c + d*x)^4*(a + b*ArcSinh[c + d*x])^(3/2))/4 - (3*b*(((c + d*x)^3*
Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/4 + ((Sqrt[b]*E^((4*a)
/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/32 - (Sqrt[b]*
E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/8 - (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3
2*E^((4*a)/b)) + (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/(8*E^((2*a)/b)))/8 - (3*(((c + d*x)*Sqrt[1 + (c + d*x)^2]
*Sqrt[a + b*ArcSinh[c + d*x]])/2 - (a + b*ArcSinh[c + d*x])^(3/2)/(3*b) -
(I/8)*((I/2)*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSin
h[c + d*x]])/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b
*ArcSinh[c + d*x]])/Sqrt[b]])/E^((2*a)/b))))/4))/8))/d
```

3.187.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

3.187. $\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.187.4 Maple [F]

$$\int (dex+ce)^3 \left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^{\frac{3}{2}} dx$$

input
```
int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x)
```

output
```
int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x)
```

3.187.5 Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)^3(a+b\mathrm{arcsinh}(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.187.6 Sympy [F]

$$\begin{aligned}\int (ce+dex)^3(a+b\mathrm{arcsinh}(c+dx))^{3/2}\,dx &= e^3\left(\int ac^3\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,dx\right.\\&+\int ad^3x^3\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,dx+\int bc^3\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)\,dx\\&+\int 3acd^2x^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,dx+\int 3ac^2dx\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,dx\\&+\int bd^3x^3\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)\,dx\\&+\int 3bcd^2x^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)\,dx\\&\left.+\int 3bc^2dx\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)\,dx\right)\end{aligned}$$

input
```
integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(3/2),x)
```

output
```
e**3*(Integral(a*c**3*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d**3*x**
3*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*c**3*sqrt(a + b*asinh(c + d*
x))*asinh(c + d*x), x) + Integral(3*a*c*d**2*x**2*sqrt(a + b*asinh(c + d*x
)), x) + Integral(3*a*c**2*d*x*sqrt(a + b*asinh(c + d*x)), x) + Integral(b
*d**3*x**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(3*b*c*
d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(3*b*c**
2*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

3.187.7 Maxima [F]

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{3/2}\,dx=\int (dex+ce)^3(b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.187.8 Giac [F]

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{3/2}\,dx=\int (dex+ce)^3(b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{3/2}\,dx=\int (c\,e+d\,e\,x)^3\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^{3/2}\,dx$$

input
```
int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(3/2),x)
```

output
```
int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(3/2), x)
```

3.188 $\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

3.188.1 Optimal result

Integrand size = 25, antiderivative size = 328

$$\begin{aligned}\int (ce+dex)^2(a&+b\text{arcsinh}(c+dx))^{3/2}\,dx = \frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{3d}\\&-\frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{6d}\\&+\frac{e^2(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}}{3d}-\frac{3b^{3/2}e^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d}\\&+\frac{b^{3/2}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{96d}-\frac{3b^{3/2}e^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d}\\&+\frac{b^{3/2}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{96d}\end{aligned}$$

output

```
1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(3/2)/d+1/288*b^(3/2)*e^2*exp(3*a/b
)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d+1/288
*b^(3/2)*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(
1/2)/d/exp(3*a/b)-3/32*b^(3/2)*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)
/b^(1/2))*Pi^(1/2)/d-3/32*b^(3/2)*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1
/2))*Pi^(1/2)/d/exp(a/b)+1/3*b*e^2*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c)
)^(1/2)/d-1/6*b*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/
2)/d
```

3.188.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.73

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{3/2}\,dx =$$

$$-\frac{be^2e^{-\frac{3a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-27e^{\frac{4a}{b}}\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{5}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\text{arcsin}}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
-1/216*(b*e^2*Sqrt[a + b*ArcSinh[c + d*x]]*(-27*E^((4*a)/b)*Sqrt[-((a + b*
ArcSinh[c + d*x])/b)]*Gamma[5/2, a/b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[a/
b + ArcSinh[c + d*x]]*Gamma[5/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 27*E^(
(2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2, -((a + b*ArcSinh[c + d*x]
)/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[5/2,
 (3*(a + b*ArcSinh[c + d*x]))/b]))/(d*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c
+ d*x])^2/b^2)])
```

3.188.3 Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.20, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6274, 27, 6192, 6227, 6195, 5971, 2009, 6213, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int e^2(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e^2\int (c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}d(c+dx)}{d}$$

$$\Big\downarrow \; 6192$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\int\frac{(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$\downarrow$ 6227

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{1}{6}b\int\frac{(c+dx)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)-\frac{2}{3}\int\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d\right.\right.}{d}$$

$\downarrow$ 6195

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\int\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{6}\int\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c}{b}\right.}{\sqrt{a+}}\right.\right.}{d}$$

$\downarrow$ 5971

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\int\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{6}\int\left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsi}}{b}\right.}{4\sqrt{a+b\text{arcsinh}}}\right.\right.\right.}{}$$

$\downarrow$ 2009

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\int\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{6}\left(\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+}}{}\right.\right.\right.\right.}{}$$

$\downarrow$ 6213

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}b\int\frac{1}{\sqrt{a+b\text{arcsinh}(c}}\right.\right.\right.}{}$$

$\downarrow$ 6189

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}\int\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcs}}{}\right.}{\sqrt{a+b\text{arcsin}}}\right.\right.\right.}{}$$

$\downarrow$ 3042

3.188. $\int(ce+dex)^2(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcs}}{}\right.}{\sqrt{a+b\text{arcsi}}}\right.\right.\right.$$

↓ 3788

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{2}\left(\frac{1}{2}i\int\frac{ie^{-\frac{a-c-}{b}}}{\sqrt{a+b\text{arcsin}}}\right.\right.\right.\right.$$

↓ 26

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}\int\frac{}{\sqrt{a}}\right.\right.\right.\right.$$

↓ 2611

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\left(\frac{1}{2}\left(-\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\int e^{\frac{a+b\text{arc}}{}}\right.\right.\right.\right.$$

↓ 2633

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\left(\frac{1}{2}\left(-\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}\sqrt{\pi}\sqrt{b}e\right.\right.\right.\right.$$

↓ 2634

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\left(\frac{1}{2}\left(-\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\right.\right.\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

3.188. $\int(ce+dex)^2(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

output
```
(e^2*(((c + d*x)^3*(a + b*ArcSinh[c + d*x])^(3/2))/3 - (b*(((c + d*x)^2*Sq
rt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/3 - (2*(Sqrt[1 + (c + d*
x)^2]*Sqrt[a + b*ArcSinh[c + d*x]] + (-1/2*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]])/(2*E^(a/b)))/2))/3 + ((Sqrt[b]*E^(a/b)*Sqrt[Pi
]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/8 - (Sqrt[b]*E^((3*a)/b)*Sqrt
[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/8 + (Sqrt[b]*S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*E^(a/b)) - (Sqrt[b]
*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*E^((3
*a)/b)))/6))/2))/d
```

3.188.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

3.188. $\int (ce + dex)^2 (a + b\text{arcsinh}(c + dx))^{3/2}\, dx$

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.188.4 Maple [F]

$$\int (dex+ce)^2 \left(a+b \operatorname{arcsinh}(dx+c)\right)^{\frac{3}{2}} dx$$

input
```
int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x)
```

output
```
int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x)
```

3.188.5 Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.188.6 Sympy [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = e^2\left(\int ac^2\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx + \int ad^2x^2\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx + \int bc^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx + \int 2acdx\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx + \int bd^2x^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx + \int 2bcdx\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(3/2),x)
```

output
```
e**2*(Integral(a*c**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d**2*x**
2*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*c**2*sqrt(a + b*asinh(c + d*
x))*asinh(c + d*x), x) + Integral(2*a*c*d*x*sqrt(a + b*asinh(c + d*x)), x)
 + Integral(b*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + In
tegral(2*b*c*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

3.188.7 Maxima [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (dex+ce)^2(b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.188.8 Giac [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{3/2}\,dx=\int (dex+ce)^2(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{3/2}\,dx=\int (c\,e+d\,e\,x)^2\,(a+b\,\text{asinh}(c+d\,x))^{3/2}\,dx$$

input
```
int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(3/2),x)
```

output
```
int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(3/2), x)
```

3.189 $\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

3.189.1 Optimal result

Integrand size = 23, antiderivative size = 205

$$\begin{aligned}\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = &-\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{8d}\\ &+\frac{e(a+b\text{arcsinh}(c+dx))^{3/2}}{4d}+\frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}}{2d}\\ &-\frac{3b^{3/2}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{3b^{3/2}ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d}\end{aligned}$$

output
```
1/4*e*(a+b*arcsinh(d*x+c))^(3/2)/d+1/2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(3
/2)/d-3/128*b^(3/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^
(1/2))*2^(1/2)*Pi^(1/2)/d+3/128*b^(3/2)*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c)
)^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-3/8*b*e*(d*x+c)*(1+(d*x+c)^
2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

3.189.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.61

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \frac{ee^{-\frac{2a}{b}}\left(b^2\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{5}{2},-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)+b^2e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}\Gamma\left(\frac{5}{2},\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)\right)}{16\sqrt{2}d\sqrt{a+b\text{arcsinh}(c+dx)}}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
(e*(b^2*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[5/2, (-2*(a + b*ArcSinh[
c + d*x]))/b] + b^2*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2, (2
*(a + b*ArcSinh[c + d*x]))/b]))/(16*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSi
nh[c + d*x]])
```

3.189.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {6274, 27, 6192, 6227, 6195, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int e(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\int (c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6192}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\int\frac{(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$\big\downarrow$ 6227

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{4}b\int\frac{c+dx}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+d\right.\right.}{d}$$

$\big\downarrow$ 6195

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{4}\int-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcs}}}\right.\right.}{d}$$

$\big\downarrow$ 25

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{4}\int\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\text{si}}{\sqrt{a+b\text{arcsin}}}\right.\right.}{d}$$

$\big\downarrow$ 5971

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{4}\int\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(c+dx)}}\right.\right.}{d}$$

$\big\downarrow$ 27

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{8}\int\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}\right.\right.}{d}$$

$\big\downarrow$ 3042

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)+\frac{1}{8}\int-\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+d}{b}\right.}{\sqrt{a+b\text{arcsinh}(c+dx)}}\right.\right.}{d}$$

$\big\downarrow$ 26

3.189. $\int(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{8}i\int\frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right.}{\sqrt{a+b\text{arcsinh}(c+dx)}}\right.\right.}{d}$$

$\downarrow$ 3789

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{8}i\left(\frac{1}{2}i\int\frac{e^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int\frac{e^{-\frac{2}{}}}{\sqrt{a+b\text{ar}}}\right.\right.\right.$$

$\downarrow$ 2611

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{8}i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-i\int e^{\frac{2(a+b\text{ar}}{}}\right.\right.\right.$$

$\downarrow$ 2633

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{8}i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e\right.\right.\right.$$

$\downarrow$ 2634

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)-\frac{1}{8}i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{ar}}}{}\right.\right.\right.\right.$$

$\downarrow$ 6198

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{8}i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\right.\right.\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

3.189. $\int(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

output
```
(e*(((c + d*x)^2*(a + b*ArcSinh[c + d*x])^(3/2))/2 - (3*b*(((c + d*x)*Sqrt
[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/2 - (a + b*ArcSinh[c + d*x
])^(3/2)/(3*b) - (I/8)*((I/2)*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*
Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi/2]*Erfi[(S
qrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/E^((2*a)/b))))/4))/d
```

3.189.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.189. $\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

3.189.4 Maple [F]

$$\int (dex+ce)\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^{\frac{3}{2}}dx$$

input
```
int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x)
```

output
```
int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x)
```

3.189.5 Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)(a+b\mathrm{arcsinh}(c+dx))^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.189.6 Sympy [F]

$$\int (ce+dex)(a+b\mathrm{arcsinh}(c+dx))^{3/2}\,dx=e\bigg(\int ac\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,dx+\int adx\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,dx+\int bc\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)\,dx+\int bdx\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)\,dx\bigg)$$

input
```
integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(3/2),x)
```

output
```
e*(Integral(a*c*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d*x*sqrt(a + b
*asinh(c + d*x)), x) + Integral(b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x), x) + Integral(b*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

3.189.7 Maxima [F]

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}\,dx=\int (dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.189.8 Giac [F]

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}\,dx=\int (dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}\,dx=\int (c\,e+d\,e\,x)\,\left(a+b\,\text{asinh}(c+d\,x)\right)^{3/2}\,dx$$

input
```
int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(3/2),x)
```

output
```
int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(3/2), x)
```

3.190 $\int (a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

3.190.1 Optimal result

Integrand size = 14, antiderivative size = 150

$$\int (a+b\text{arcsinh}(c+dx))^{3/2}\,dx = -\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{2d} + \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}{d} + \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d} + \frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8d}$$

output
```
(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d+3/8*b^(3/2)*exp(a/b)*erf((a+b*arcsinh
(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/8*b^(3/2)*erfi((a+b*arcsinh(d*x+c))^(
1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-3/2*b*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d
*x+c))^(1/2)/d
```

3.190.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.81

$$\int (a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \frac{ae^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}}\right)}{2d}$$
$$+\frac{\sqrt{b}\left(4\sqrt{b}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-3\sqrt{1+(c+dx)^2}+2(c+dx)\text{arcsinh}(c+dx)\right)+(2a+3b)\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a}\cdots}{\cdots}\right.\cdots}{8d\cdots}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
(a*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c
 + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c +
d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b)) + (Sqrt[b]*(
4*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c +
d*x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(8*d)
```

3.190.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6273, 6187, 6213, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 6273}$$

$$\frac{\int (a+b\text{arcsinh}(c+dx))^{3/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6187}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\int\frac{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)}{d}$$

$\downarrow$ 6213

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left(\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}b\int\frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx\right.}{d}$$

$\downarrow$ 6189

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left(\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}\int\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(\right.}{d}$$

$\downarrow$ 3042

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left(\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right.}{\sqrt{a+b\text{arcsinh}(c+dx)}}\right.}{d}$$

$\downarrow$ 3788

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left(\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{2}\left(\frac{1}{2}i\int\frac{ie^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a\right.\right.}{d}$$

$\downarrow$ 26

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}\int\frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c\right.\right.}}{d}$$

$\downarrow$ 2611

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\left(-\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\int e^{\frac{a+b\text{arcsinh}(c+dx)}{b}-\frac{a}{b}}\right.\right.}{d}$$

$\downarrow$ 2633

3.190. $\int(a+b\text{arcsinh}(c+dx))^{3/2}\,dx$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\left(-\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a}\right.}{}\right.\right.}{d}$$

$\downarrow$ 2634

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left(\frac{1}{2}\left(-\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsin}}}{\sqrt{b}}\right.\right.\right.}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
((c + d*x)*(a + b*ArcSinh[c + d*x])^(3/2) - (3*b*(Sqrt[1 + (c + d*x)^2]*Sq
rt[a + b*ArcSinh[c + d*x]] + (-1/2*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a +
b*ArcSinh[c + d*x]]/Sqrt[b]]) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[
c + d*x]]/Sqrt[b]])/(2*E^(a/b)))/2))/2)/d
```

3.190.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.190.4 Maple [F]

$$\int (a + b \operatorname{arcsinh}(dx + c))^{\frac{3}{2}} dx$$

input
```
int((a+b*arcsinh(d*x+c))^(3/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^(3/2),x)
```

3.190.5 Fricas [F(-2)]

Exception generated.

$$\int (a + b\text{arcsinh}(c + dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.190.6 Sympy [F]

$$\int (a + b\text{arcsinh}(c + dx))^{3/2}\,dx = \int (a + b\,\text{asinh}\,(c + dx))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**(3/2), x)
```

3.190.7 Maxima [F]

$$\int (a + b\text{arcsinh}(c + dx))^{3/2}\,dx = \int (b\,\text{arsinh}\,(dx + c) + a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.190.8 Giac [F]

$$\int (a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arcsinh}(c+dx))^{3/2}\,dx = \int (a+b\,\text{asinh}(c+d\,x))^{3/2}\,dx$$

input
```
int((a + b*asinh(c + d*x))^(3/2),x)
```

output
```
int((a + b*asinh(c + d*x))^(3/2), x)
```

3.191 $\int \frac{(a+b\text{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx$

3.191.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx = \frac{\text{Int}\left(\frac{(a+b\text{arcsinh}(c+dx))^{3/2}}{c+dx},x\right)}{e}$$

output
```
Unintegrable((a+b*arcsinh(d*x+c))^(3/2)/(d*x+c),x)/e
```

3.191.2 Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(3/2)/(c*e + d*e*x),x]
```

output
```
Integrate[(a + b*ArcSinh[c + d*x])^(3/2)/(c*e + d*e*x), x]
```

3.191.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 27, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^{3/2}}{e(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^{3/2}}{c+dx}d(c+dx)}{de}$$

$$\Big\downarrow \text{6196}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^{3/2}}{c+dx}d(c+dx)}{de}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(3/2)/(c*e + d*e*x),x]
```

output
```
$Aborted
```

3.191.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.191.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{3}{2}}}{dex+ce}dx$$

input
```
int((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x)
```

output
```
int((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x)
```

3.191.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\mathrm{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.191.6 Sympy [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{(a+b\mathrm{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx = \frac{\int \frac{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}{c+dx}\,dx + \int \frac{b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}{c+dx}\,dx}{e}$$

input
```
integrate((a+b*asinh(d*x+c))**(3/2)/(d*e*x+c*e),x)
```

output
```
(Integral(a*sqrt(a + b*asinh(c + d*x))/(c + d*x), x) + Integral(b*sqrt(a +
 b*asinh(c + d*x))*asinh(c + d*x)/(c + d*x), x))/e
```

3.191.7 Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(3/2)/(d*e*x + c*e), x)
```

3.191.8 Giac [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(3/2)/(d*e*x + c*e), x)
```

3.191.9 Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{3/2}}{ce+dex}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{3/2}}{c\,e+d\,e\,x}\,dx$$

input
```
int((a + b*asinh(c + d*x))^(3/2)/(c*e + d*e*x),x)
```

output
```
int((a + b*asinh(c + d*x))^(3/2)/(c*e + d*e*x), x)
```

3.192 $\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

3.192.1 Optimal result

Integrand size = 25, antiderivative size = 701

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \frac{2b^2e^4(c+dx)\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{15d} + \frac{3b^2e^4(c+dx)^5\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{100d} - \frac{4be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} + \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} - \frac{be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{10d} + \frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{5d} + \frac{15b^{5/2}e^4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d} - \frac{b^{5/2}e^4e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{240d} - \frac{b^{5/2}e^4e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1280d} + \frac{3b^{5/2}e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6400d} - \frac{15b^{5/2}e^4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d} + \frac{b^{5/2}e^4e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{240d} + \frac{b^{5/2}e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1280d} - \frac{3b^{5/2}e^4e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6400d}$$

output
```
1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(5/2)/d+3/32000*b^(5/2)*e^4*exp(5*a
/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/d-3/3
2000*b^(5/2)*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*
Pi^(1/2)/d/exp(5*a/b)-5/2304*b^(5/2)*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d+5/2304*b^(5/2)*e^4*erfi(3^(1/
2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)+15/12
8*b^(5/2)*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-
15/128*b^(5/2)*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp
(a/b)-4/15*b*e^4*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d+2/15*b*e
^4*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d-1/10*b*e^4*(
d*x+c)^4*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d+2/5*b^2*e^4*(d*x
+c)*(a+b*arcsinh(d*x+c))^(1/2)/d-1/15*b^2*e^4*(d*x+c)^3*(a+b*arcsinh(d*x+c
))^(1/2)/d+3/100*b^2*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(1/2)/d
```

3.192.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.46

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx =$$

$$-\frac{b^3e^4e^{-\frac{5a}{b}}\left(33750e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}\,\Gamma\left(\frac{7}{2},\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)+27\sqrt{5}\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\,\Gamma\left(\frac{7}{2},-\frac{5(\cdots}{}\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
-1/540000*(b^3*e^4*(33750*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7
/2, a/b + ArcSinh[c + d*x]] + 27*Sqrt[5]*Sqrt[-((a + b*ArcSinh[c + d*x])/b
)]*Gamma[7/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 625*Sqrt[3]*E^((2*a)/b)*S
qrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, (-3*(a + b*ArcSinh[c + d*x])
)/b] + 33750*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, -(
(a + b*ArcSinh[c + d*x])/b)] - 625*Sqrt[3]*E^((8*a)/b)*Sqrt[a/b + ArcSinh[
c + d*x]]*Gamma[7/2, (3*(a + b*ArcSinh[c + d*x]))/b] + 27*Sqrt[5]*E^((10*a
)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, (5*(a + b*ArcSinh[c + d*x]))/
b]))/(d*E^((5*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

3.192.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 4.06 (sec) , antiderivative size = 853, normalized size of antiderivative = 1.22, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.760, Rules used = {6274, 27, 6192, 6227, 6192, 6227, 6192, 6213, 6187, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int e^4(c+dx)^4(a+b\text{arcsinh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^4\int (c+dx)^4(a+b\text{arcsinh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6192}$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{1}{2}b\int \frac{(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 6227}$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{1}{2}b\left(-\frac{3}{10}b\int (c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}d(c+dx)-\frac{4}{5}\int \frac{(c+dx)^3(a+b\ldots}{\sqrt{(\ldots}}\right.\right.}{d}$$

$$\Big\downarrow \text{ 6192}$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{1}{2}b\left(-\frac{3}{10}b\left(\frac{1}{5}(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{10}b\int \frac{(c+dx)\ldots}{\sqrt{(c+dx)^2+1}\sqrt{a+b\ldots}}\right.\right.\right.$$

$$\Big\downarrow \text{ 6227}$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{1}{2}b\left(-\frac{3}{10}b\left(\frac{1}{5}(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{10}b\int \frac{(c+dx)\ldots}{\sqrt{(c+dx)^2+1}\sqrt{a+b\ldots}}\right.\right.\right.$$

$$\Big\downarrow \text{ 6192}$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{1}{2}b\left(-\frac{3}{10}b\left(\frac{1}{5}(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{10}b\int\frac{(c+dx)}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{a}}}\right.\right.\right.$$

$\downarrow$ 6213

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{1}{2}b\left(-\frac{3}{10}b\left(\frac{1}{5}(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{10}b\int\frac{(c+dx)}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{a}}}\right.\right.\right.$$

$\downarrow$ 6187

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{1}{2}b\left(-\frac{4}{5}\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx\right.\right.\right.\right.\right.$$

$\downarrow$ 6234

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{1}{2}b\left(-\frac{3}{10}b\left(\frac{1}{5}(c+dx)^5\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{10}\int-\frac{\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arcs}}{}\right.}{\sqrt{a+b\text{arcsin}}}\right.\right.\right.$$

$\downarrow$ 25

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{1}{2}b\left(-\frac{3}{10}b\left(\frac{1}{10}\int\frac{\sinh^5\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))+\frac{1}{5}\right.\right.\right.$$

$\downarrow$ 3042

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{1}{2}b\left(-\frac{4}{5}\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+d\right.\right.\right.\right.\right.$$

$\downarrow$ 26

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{1}{2}b\left(-\frac{4}{5}\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+d\right.\right.\right.\right.\right.$$

$\downarrow$ 3789

3.192. $\int(ce+dex)^4(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{1}{2}b\left(-\frac{4}{5}\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx\right.\right.\right.\right.\right.$$

$\downarrow$ 2611

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{1}{2}b\left(-\frac{4}{5}\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx\right.\right.\right.\right.\right.$$

$\downarrow$ 2633

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{1}{2}b\left(-\frac{4}{5}\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx\right.\right.\right.\right.\right.$$

$\downarrow$ 2634

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{1}{2}b\left(-\frac{4}{5}\left(-\frac{1}{2}b\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{6}i\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{a}}{}\right.}{\sqrt{a+b\text{arc}}}\right.\right.\right.\right.$$

$\downarrow$ 3793

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{1}{2}b\left(-\frac{4}{5}\left(-\frac{1}{2}b\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{6}i\int\left(\frac{3i\sinh\left(\frac{a}{b}-\frac{a}{}\right.}{4\sqrt{a+b\text{a}}}\right.\right.\right.\right.\right.$$

$\downarrow$ 2009

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{1}{2}b\left(\frac{1}{5}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}(c+dx)^4-\frac{4}{5}\left(\frac{1}{3}\sqrt{(c+\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

3.192. $\int(ce+dex)^4(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

output
```
(e^4*(((c + d*x)^5*(a + b*ArcSinh[c + d*x])^(5/2))/5 - (b*(((c + d*x)^4*Sq
rt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/5 - (4*(((c + d*x)^2*S
qrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/3 - (2*(Sqrt[1 + (c +
 d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2) - (3*b*((c + d*x)*Sqrt[a + b*ArcSi
nh[c + d*x]] - (I/2)*((I/2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSin
h[c + d*x]]/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
 d*x]]/Sqrt[b]])/E^(a/b))))/2))/3 - (b*(((c + d*x)^3*Sqrt[a + b*ArcSinh[c
+ d*x]])/3 + (I/6)*(((3*I)/8)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcS
inh[c + d*x]]/Sqrt[b]] - (I/8)*Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]
*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - (((3*I)/8)*Sqrt[b]*Sqrt[Pi]*Erfi
[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/E^(a/b) + ((I/8)*Sqrt[b]*Sqrt[Pi/3
]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/E^((3*a)/b))))/2))
/5 - (3*b*(((c + d*x)^5*Sqrt[a + b*ArcSinh[c + d*x]])/5 - (I/10)*(((5*I)/1
6)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - ((
5*I)/32)*Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c
+ d*x]])/Sqrt[b]] + (I/32)*Sqrt[b]*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqr
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - (((5*I)/16)*Sqrt[b]*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/E^(a/b) + (((5*I)/32)*Sqrt[b]*Sqrt[Pi
/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/E^((3*a)/b) - ((
I/32)*Sqrt[b]*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sq...
```

3.192.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

3.192. $\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.192.4 Maple [F]

$$\int (dex + ce)^4 (a + b\operatorname{arcsinh}(dx + c))^{\frac{5}{2}} dx$$

input
```
int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x)
```

output
```
int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x)
```

3.192.5 Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)^4 (a + b\operatorname{arcsinh}(c + dx))^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

3.192.6 Sympy [F(-1)]

Timed out.

$$\int (ce + dex)^4 (a + b\operatorname{arcsinh}(c + dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.192.7 Maxima [F]

$$\int (ce + dex)^4 (a + b\operatorname{arcsinh}(c + dx))^{5/2}\,dx = \int (dex + ce)^4 (b\operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.192.8 Giac [F]

$$\int (ce + dex)^4 (a + b\operatorname{arcsinh}(c + dx))^{5/2}\,dx = \int (dex + ce)^4 (b\operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\int (c\,e+d\,e\,x)^4\,(a+b\,\text{asinh}(c+d\,x))^{5/2}\,dx$$

input
```
int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(5/2),x)
```

output
```
int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(5/2), x)
```

3.193 $\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

3.193.1 Optimal result

Integrand size = 25, antiderivative size = 455

$$\begin{aligned}
&\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \\
&-\frac{225b^2e^3\sqrt{a+b\text{arcsinh}(c+dx)}}{2048d}-\frac{45b^2e^3(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}}{256d} \\
&+\frac{15b^2e^3(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}}{256d} \\
&+\frac{15be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{3/2}}{64d} \\
&-\frac{5be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{3/2}}{32d} \\
&-\frac{3e^3(a+b\text{arcsinh}(c+dx))^{5/2}}{32d}+\frac{e^3(c+dx)^4(a+b\text{arcsinh}(c+dx))^{5/2}}{4d} \\
&-\frac{15b^{5/2}e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16384d} \\
&+\frac{15b^{5/2}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{512d} \\
&-\frac{15b^{5/2}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16384d} \\
&+\frac{15b^{5/2}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{512d}
\end{aligned}$$

output
```
-3/32*e^3*(a+b*arcsinh(d*x+c))^(5/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+
c))^(5/2)/d+15/1024*b^(5/2)*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c)
)^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d+15/1024*b^(5/2)*e^3*erfi(2^(1/2)*(a+b*
arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-15/16384*b^(5
/2)*e^3*exp(4*a/b)*erf(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15
/16384*b^(5/2)*e^3*erfi(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/e
xp(4*a/b)+15/64*b*e^3*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/
2)/d-5/32*b*e^3*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d
-225/2048*b^2*e^3*(a+b*arcsinh(d*x+c))^(1/2)/d-45/256*b^2*e^3*(d*x+c)^2*(a
+b*arcsinh(d*x+c))^(1/2)/d+15/256*b^2*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^(
1/2)/d
```

3.193.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.46

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \frac{b^3e^3e^{-\frac{4a}{b}}\left(-\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{7}{2},-\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)+16\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma \cdots\right.}{\cdots}$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
(b^3*e^3*(-(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, (-4*(a + b*ArcS
inh[c + d*x]))/b]) + 16*Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x]
)/b)]*Gamma[7/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[a/b +
 ArcSinh[c + d*x]]*(-16*Sqrt[2]*Gamma[7/2, (2*(a + b*ArcSinh[c + d*x]))/b]
 + E^((2*a)/b)*Gamma[7/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/(2048*d*E^((4
*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

3.193.3 Rubi [A] (verified)

Time = 2.72 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.18, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {6274, 27, 6192, 6227, 6192, 6227, 6192, 6198, 6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int e^3(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^3\int (c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6192}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{8}b\int \frac{(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 6227}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{8}b\left(-\frac{3}{8}b\int (c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}d(c+dx)-\frac{3}{4}\int \frac{(c+dx)^2(a+b\text{ar}}{\sqrt{(c\text{-}}}\right.\right.}{d}$$

$$\Big\downarrow \text{ 6192}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{8}b\left(-\frac{3}{8}b\left(\frac{1}{4}(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{8}b\int \frac{(c+dx)^4}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arc}}}\right.\right.\right.}{}$$

$$\Big\downarrow \text{ 6227}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{8}b\left(-\frac{3}{8}b\left(\frac{1}{4}(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{8}b\int \frac{(c+dx)^4}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arc}}}\right.\right.\right.}{}$$

$$\Big\downarrow \text{ 6192}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{8}b\left(-\frac{3}{8}b\left(\frac{1}{4}(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{8}b\int\frac{(c+dx)^4}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arc}}}\right.\right.\right.$$

$\downarrow$ 6198

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{8}b\left(-\frac{3}{4}\left(-\frac{3}{4}b\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{4}b\int\frac{(c-}{\sqrt{(c+dx)^2+1}\sqrt{a}}\right.\right.\right.\right.$$

$\downarrow$ 6234

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{8}b\left(-\frac{3}{8}b\left(\frac{1}{4}(c+dx)^4\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{8}\int\frac{\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}}{b}\right.}{\sqrt{a+b\text{arcsinh}(c+}}\right.\right.\right.$$

$\downarrow$ 3042

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{8}b\left(-\frac{3}{4}\left(-\frac{3}{4}b\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{4}\int-\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b}{}\right.}{\sqrt{a+b\text{ar}}}\right.\right.\right.\right.$$

$\downarrow$ 25

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{8}b\left(-\frac{3}{4}\left(-\frac{3}{4}b\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{4}\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{ar}}{}\right.}{\sqrt{a+b\text{arcs}}}\right.\right.\right.\right.$$

$\downarrow$ 3793

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{8}b\left(-\frac{3}{4}\left(-\frac{3}{4}b\left(\frac{1}{4}\int\left(\frac{1}{2\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right.}{2\sqrt{a+b\text{arcsinh}(c+dx)}}\right.\right.\right.\right.\right.$$

$\downarrow$ 2009

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{8}b\left(-\frac{3}{8}b\left(\frac{1}{8}\left(-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\right.\right.\right.\right.$$

3.193. $\int(ce+dex)^3(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

input
```
Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
(e^3*(((c + d*x)^4*(a + b*ArcSinh[c + d*x])^(5/2))/4 - (5*b*(((c + d*x)^3*
Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/4 - (3*b*(((c + d*x)
^4*Sqrt[a + b*ArcSinh[c + d*x]])/4 + ((-3*Sqrt[a + b*ArcSinh[c + d*x]])/4
- (Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/32 + (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[
c + d*x]])/Sqrt[b]])/4 - (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/(32*E^((4*a)/b)) + (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt
[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(4*E^((2*a)/b)))/8))/8 - (3*(((c + d*x
)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/2 - (a + b*ArcSinh
[c + d*x])^(5/2)/(5*b) - (3*b*(((c + d*x)^2*Sqrt[a + b*ArcSinh[c + d*x]])/
2 + (Sqrt[a + b*ArcSinh[c + d*x]] - (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(S
qrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/4 - (Sqrt[b]*Sqrt[Pi/2]*Erf
i[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(4*E^((2*a)/b)))/4))/4)
)/4))/8))/d
```

3.193.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.193.4 Maple [F]

$$\int (dex + ce)^3 (a + b \operatorname{arcsinh}(dx + c))^{\frac{5}{2}} dx$$

input
```
int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x)
```

output
```
int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x)
```

3.193.5 Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)^3 (a + b\text{arcsinh}(c + dx))^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.193.6 Sympy [F]

$$\int (ce+dex)^3 (a+b\text{arcsinh}(c+dx))^{5/2} \, dx = e^3 \left(\int a^2c^3\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx \right.$$
$$+ \int a^2d^3x^3\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx$$
$$+ \int b^2c^3\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}^2\,(c+dx)\,dx$$
$$+ \int 2abc^3\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx$$
$$+ \int 3a^2cd^2x^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx + \int 3a^2c^2dx\sqrt{a+b\,\text{asinh}\,(c+dx)}\,dx$$
$$+ \int b^2d^3x^3\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}^2\,(c+dx)\,dx$$
$$+ \int 2abd^3x^3\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx$$
$$+ \int 3b^2cd^2x^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}^2\,(c+dx)\,dx$$
$$+ \int 3b^2c^2dx\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}^2\,(c+dx)\,dx$$
$$+ \int 6abcd^2x^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx$$
$$\left. + \int 6abc^2dx\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)\,dx \right)$$

input
```
integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(5/2),x)
```

output
```
e**3*(Integral(a**2*c**3*sqrt(a + b*asinh(c + d*x)), x) + Integral(a**2*d*
*3*x**3*sqrt(a + b*asinh(c + d*x)), x) + Integral(b**2*c**3*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x)**2, x) + Integral(2*a*b*c**3*sqrt(a + b*asinh(
c + d*x))*asinh(c + d*x), x) + Integral(3*a**2*c*d**2*x**2*sqrt(a + b*asin
h(c + d*x)), x) + Integral(3*a**2*c**2*d*x*sqrt(a + b*asinh(c + d*x)), x)
+ Integral(b**2*d**3*x**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2, x)
 + Integral(2*a*b*d**3*x**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x)
+ Integral(3*b**2*c*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2
, x) + Integral(3*b**2*c**2*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)*
*2, x) + Integral(6*a*b*c*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x), x) + Integral(6*a*b*c**2*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
), x))
```

3.193.7 Maxima [F]

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\int (dex+ce)^3(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.193.8 Giac [F]

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\int (dex+ce)^3(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx=\int (c\,e+d\,e\,x)^3\,(a+b\,\mathrm{asinh}(c+d\,x))^{5/2}\,dx$$

input
```
int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(5/2),x)
```

output
```
int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(5/2), x)
```

3.194 $\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

3.194.1 Optimal result

Integrand size = 25, antiderivative size = 394

$$\begin{aligned}\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{5/2}\,dx =& -\frac{5b^2e^2(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{6d}+\frac{5b^2e^2(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}}{36d}\\ &+\frac{5be^2\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{3/2}}{9d}\\ &-\frac{5be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{3/2}}{18d}\\ &+\frac{e^2(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}}{3d}-\frac{15b^{5/2}e^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d}\\ &+\frac{5b^{5/2}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{576d}\\ &+\frac{15b^{5/2}e^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64d}\\ &-\frac{5b^{5/2}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{576d}\end{aligned}$$

output
```
1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(5/2)/d+5/1728*b^(5/2)*e^2*exp(3*a/
b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d-5/17
28*b^(5/2)*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi
^(1/2)/d/exp(3*a/b)-15/64*b^(5/2)*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1
/2)/b^(1/2))*Pi^(1/2)/d+15/64*b^(5/2)*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/
b^(1/2))*Pi^(1/2)/d/exp(a/b)+5/9*b*e^2*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+
c)^2)^(1/2)/d-5/18*b*e^2*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2
)^(1/2)/d-5/6*b^2*e^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d+5/36*b^2*e^2*(d
*x+c)^3*(a+b*arcsinh(d*x+c))^(1/2)/d
```

3.194.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.56

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{5/2}\,dx =$$

$$-\frac{b^3e^2e^{-\frac{3a}{b}}\left(-81e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}\Gamma\left(\frac{7}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)+\sqrt{3}\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{7}{2},-\frac{3(a+b\text{a}}{}\right.\right.}{648}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
-1/648*(b^3*e^2*(-81*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, a
/b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma
[7/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 81*E^((2*a)/b)*Sqrt[-((a + b*ArcS
inh[c + d*x])/b)]*Gamma[7/2, -((a + b*ArcSinh[c + d*x])/b)] + Sqrt[3]*E^((
6*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, (3*(a + b*ArcSinh[c + d*x]
))/b]))/(d*E^((3*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

3.194.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.41 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.19, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {6274, 27, 6192, 6227, 6192, 6213, 6187, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.194. $\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \textbf{6274}$$

$$\frac{\int e^2(c+dx)^2(a+b\text{arcsinh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{e^2\int (c+dx)^2(a+b\text{arcsinh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\Big\downarrow \textbf{6192}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{6}b\int \frac{(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \textbf{6227}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{1}{2}b\int (c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}d(c+dx)-\frac{2}{3}\int \frac{(c+dx)(a+b\text{arc}}{\sqrt{(c+}}\right.\right.}{d}$$

$$\Big\downarrow \textbf{6192}$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{1}{2}b\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{6}b\int \frac{(c+dx)^3}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arc}}}\right.\right.\right.$$

$$\Big\downarrow \textbf{6213}$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{1}{2}b\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{6}b\int \frac{(c+dx)^3}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arc}}}\right.\right.\right.$$

$$\Big\downarrow \textbf{6187}$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx)\sqrt{a}\right.\right.\right.\right.$$

$$\Big\downarrow \textbf{6234}$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{1}{2}b\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{6}\int -\frac{\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsin}}{b}\right.}{\sqrt{a+b\text{arcsinh}(}}\right.\right.\right.$$

3.194. $\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

$\downarrow$ 25

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{1}{2}b\left(\frac{1}{6}\int\frac{\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))+\frac{1}{3}(c\right.\right.\right.$$

$\downarrow$ 3042

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx)\sqrt{a}\right.\right.\right.\right.$$

$\downarrow$ 26

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx)\sqrt{a}\right.\right.\right.\right.$$

$\downarrow$ 3789

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx)\sqrt{a}\right.\right.\right.\right.$$

$\downarrow$ 2611

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx)\sqrt{a}\right.\right.\right.\right.$$

$\downarrow$ 2633

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx)\sqrt{a}\right.\right.\right.\right.$$

$\downarrow$ 2634

3.194. $\int(ce+dex)^2(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{1}{2}b\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{6}i\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}}{b}\right.}{\sqrt{a+b\text{arcsinh}(}}\right.\right.\right.$$

$\downarrow$ 3793

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{1}{2}b\left(\frac{1}{3}(c+dx)^3\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{6}i\int\left(\frac{3i\sinh\left(\frac{a}{b}-\frac{a+b\text{arcs}}{}\right.}{4\sqrt{a+b\text{arcsin}}}\right.\right.\right.\right.$$

$\downarrow$ 2009

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{6}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx)\sqrt{a}\right.\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
(e^2*(((c + d*x)^3*(a + b*ArcSinh[c + d*x])^(5/2))/3 - (5*b*(((c + d*x)^2*
Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/3 - (2*(Sqrt[1 + (c
+ d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2) - (3*b*((c + d*x)*Sqrt[a + b*ArcS
inh[c + d*x]] - (I/2)*((I/2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSi
nh[c + d*x]]/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c
+ d*x]]/Sqrt[b]])/E^(a/b))))/2))/3 - (b*(((c + d*x)^3*Sqrt[a + b*ArcSinh[c
 + d*x]])/3 + (I/6)*(((3*I)/8)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Arc
Sinh[c + d*x]]/Sqrt[b]] - (I/8)*Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3
]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - (((3*I)/8)*Sqrt[b]*Sqrt[Pi]*Erf
i[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/E^(a/b) + ((I/8)*Sqrt[b]*Sqrt[Pi/
3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/E^((3*a)/b))))/2)
)/6))/d
```

3.194. $\int(ce+dex)^2(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

3.194.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.194. $\int (ce+dex)^2(a+b\,\text{arcsinh}(c+dx))^{5/2}\,dx$

3.194.4 Maple [F]

$$\int (dex+ce)^2 \left(a+b \operatorname{arcsinh}\left(dx+c\right)\right)^{\frac{5}{2}} dx$$

input
```
int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x)
```

output
```
int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x)
```

3.194.5 Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.194.6 Sympy [F]

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = e^2\left(\int a^2c^2\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx + \int a^2d^2x^2\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx + \int b^2c^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c+dx)\,dx + \int 2abc^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx + \int 2a^2cdx\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx + \int b^2d^2x^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c+dx)\,dx + \int 2abd^2x^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx + \int 2b^2cdx\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c+dx)\,dx + \int 4abcdx\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(5/2),x)
```

output
```
e**2*(Integral(a**2*c**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(a**2*d*
*2*x**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(b**2*c**2*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x)**2, x) + Integral(2*a*b*c**2*sqrt(a + b*asinh(
c + d*x))*asinh(c + d*x), x) + Integral(2*a**2*c*d*x*sqrt(a + b*asinh(c +
d*x)), x) + Integral(b**2*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)**2, x) + Integral(2*a*b*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c +
 d*x), x) + Integral(2*b**2*c*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
)**2, x) + Integral(4*a*b*c*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x),
 x))
```

3.194.7 Maxima [F]

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \int (dex+ce)^2(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.194.8 Giac [F]

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \int (dex+ce)^2(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx = \int (c\,e+d\,e\,x)^2\,(a+b\operatorname{asinh}(c+d\,x))^{5/2}\,dx$$

input
```
int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(5/2),x)
```

output
```
int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(5/2), x)
```

3.195 $\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

3.195.1 Optimal result

Integrand size = 23, antiderivative size = 262

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \frac{15b^2e\sqrt{a+b\text{arcsinh}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}}{32d} - \frac{5be(c+dx)\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{3/2}}{8d} + \frac{e(a+b\text{arcsinh}(c+dx))^{5/2}}{4d} + \frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^{5/2}}{2d} - \frac{15b^{5/2}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d} - \frac{15b^{5/2}ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d}$$

output
```
1/4*e*(a+b*arcsinh(d*x+c))^(5/2)/d+1/2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(5
/2)/d-15/512*b^(5/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b
^(1/2))*2^(1/2)*Pi^(1/2)/d-15/512*b^(5/2)*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+
c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-5/8*b*e*(d*x+c)*(a+b*arcs
inh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d+15/64*b^2*e*(a+b*arcsinh(d*x+c))^(
1/2)/d+15/32*b^2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(1/2)/d
```

3.195.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.48

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \frac{ee^{-\frac{2a}{b}}\left(-b^3\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{7}{2},-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)+b^3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}\Gamma\left(\frac{7}{2}\right.}{32\sqrt{2}d\sqrt{a+b\text{arcsinh}(c+dx)}}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
(e*(-(b^3*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, (-2*(a + b*ArcSin
h[c + d*x]))/b]) + b^3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2,
 (2*(a + b*ArcSinh[c + d*x]))/b]))/(32*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*Ar
cSinh[c + d*x]])
```

3.195.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {6274, 27, 6192, 6227, 6192, 6198, 6234, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int e(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\int (c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6192}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{4}b\int \frac{(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6227}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{4}b\left(-\frac{3}{4}b\int(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}d(c+dx)-\frac{1}{2}\int\frac{(a+b\text{arcsinh}(c+}{\sqrt{(c+dx)^2+}}\right.\right.}{d}$$

$$\Big\downarrow \text{6192}$$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{4}b\left(-\frac{3}{4}b\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{4}b\int\frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcs}}}\right.\right.\right.$$

$$\Big\downarrow \text{6198}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{4}b\left(-\frac{3}{4}b\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{4}b\int\frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcs}}}\right.\right.\right.}{d}$$

$$\Big\downarrow \text{6234}$$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{4}b\left(-\frac{3}{4}b\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{4}\int\frac{\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c}{b}\right.}{\sqrt{a+b\text{arcsinh}(c+d}}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{4}b\left(-\frac{3}{4}b\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{4}\int-\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}}{b}\right.}{\sqrt{a+b\text{arcsinh}(c}}\right.\right.\right.$$

$$\Big\downarrow \text{25}$$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{4}b\left(-\frac{3}{4}b\left(\frac{1}{2}(c+dx)^2\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{4}\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c}{b}\right.}{\sqrt{a+b\text{arcsinh}(c+}}\right.\right.\right.$$

$$\Big\downarrow \text{3793}$$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{4}b\left(-\frac{3}{4}b\left(\frac{1}{4}\int\left(\frac{1}{2\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(c+dx)}}\right)d(\right.\right.\right.$$

3.195. $\int(ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

$$\Big\downarrow \text{2009}$$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{4}b\left(-\frac{3}{4}b\left(\frac{1}{4}\left(-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{e}\right.\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
(e*(((c + d*x)^2*(a + b*ArcSinh[c + d*x])^(5/2))/2 - (5*b*(((c + d*x)*Sqrt
[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/2 - (a + b*ArcSinh[c + d
*x])^(5/2)/(5*b) - (3*b*(((c + d*x)^2*Sqrt[a + b*ArcSinh[c + d*x]])/2 + (S
qrt[a + b*ArcSinh[c + d*x]] - (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]
*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/4 - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqr
t[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(4*E^((2*a)/b)))/4))/4))/4))/
d
```

3.195.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.195.4 Maple [F]

$$\int (dex + ce) (a + b \operatorname{arcsinh}(dx + c))^{\frac{5}{2}} dx$$

input
```
int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x)
```

output
```
int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x)
```

3.195.5 Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.195.6 Sympy [F]

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}\,dx = e\left(\int a^2c\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx + \int a^2dx\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx + \int b^2c\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c+dx)\,dx + \int 2abc\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx + \int b^2dx\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c+dx)\,dx + \int 2abdx\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(5/2),x)
```

output
```
e*(Integral(a**2*c*sqrt(a + b*asinh(c + d*x)), x) + Integral(a**2*d*x*sqrt
(a + b*asinh(c + d*x)), x) + Integral(b**2*c*sqrt(a + b*asinh(c + d*x))*as
inh(c + d*x)**2, x) + Integral(2*a*b*c*sqrt(a + b*asinh(c + d*x))*asinh(c
+ d*x), x) + Integral(b**2*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**
2, x) + Integral(2*a*b*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

3.195.7 Maxima [F]

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\int (dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.195.8 Giac [F]

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\int (dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\int (c\,e+d\,e\,x)\,\left(a+b\,\text{asinh}(c+d\,x)\right)^{5/2}\,dx$$

input
```
int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(5/2),x)
```

output
```
int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(5/2), x)
```

3.196 $\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

3.196.1 Optimal result

Integrand size = 14, antiderivative size = 179

$$\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\frac{15b^2(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}{4d}-\frac{5b\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{3/2}}{2d}+\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}}{d}+\frac{15b^{5/2}e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15b^{5/2}e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16d}$$

output
```
(d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)/d+15/16*b^(5/2)*exp(a/b)*erf((a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/16*b^(5/2)*erfi((a+b*arcsinh(d*x+c
))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-5/2*b*(a+b*arcsinh(d*x+c))^(3/2)*(1+
(d*x+c)^2)^(1/2)/d+15/4*b^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

3.196.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 458 vs. 2(179) = 358.

Time = 1.04 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.56

$$\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx=\frac{8a^2e^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}}\right)+4a\sqrt{b}\cdots}{\cdots}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
((8*a^2*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcS
inh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh
[c + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + 4*a*Sqrt[b]
*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c
+ d*x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c
+ d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(4*Sqr
t[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(2*Sqrt[1 + (c + d*x)^2]*(a - 5*b*ArcSin
h[c + d*x]) + b*(c + d*x)*(15 + 4*ArcSinh[c + d*x]^2)) + (4*a^2 + 12*a*b +
 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] +
 Sinh[a/b]) + (4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c
+ d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(16*d)
```

3.196.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6273, 6187, 6213, 6187, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 6273}$$

$$\frac{\int (a+b\text{arcsinh}(c+dx))^{5/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6187}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\int \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}{\sqrt{(c+dx)^2+1}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\Big(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\int \sqrt{a+b\text{arcsinh}(c+dx)}d}{d}$$

$$\Big\downarrow \; 6187$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\Bigg((c+dx)\sqrt{a+b\text{arcsinh}(}{d}$$

$$\Big\downarrow \; 6234$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\Bigg((c+dx)\sqrt{a+b\text{arcsinh}(}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\Bigg(\frac{1}{2}\int\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c}{b}\right.}{\sqrt{a+b\text{arcsinh}(c+dx}}}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\Bigg((c+dx)\sqrt{a+b\text{arcsinh}(}{d}$$

$$\Big\downarrow \; 26$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\Bigg((c+dx)\sqrt{a+b\text{arcsinh}(}{d}$$

$$\Big\downarrow \; 3789$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\Bigg(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\Bigg((c+dx)\sqrt{a+b\text{arcsinh}(}{}$$

$$\Big\downarrow \; 2611$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\Big(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\Big((c+dx)\sqrt{a+b\text{arcsinh}(}{d}$$

3.196. $\int (a+b\text{arcsinh}(c+dx))^{5/2}\,dx$

$$\Big\downarrow \text{2633}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx)\sqrt{a+b\text{arcsinh}(\right.\right.}{d}$$

$$\Big\downarrow \text{2634}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{2}b\left((c+dx)\sqrt{a+b\text{arcsinh}(\right.\right.}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
((c + d*x)*(a + b*ArcSinh[c + d*x])^(5/2) - (5*b*(Sqrt[1 + (c + d*x)^2]*(a
 + b*ArcSinh[c + d*x])^(3/2) - (3*b*((c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]
] - (I/2)*((I/2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt
[b]])/E^(a/b))))/2))/2)/d
```

3.196.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.196. $\int (a+b\operatorname{arcsinh}(c+dx))^{5/2}\,dx$

3.196.4 Maple [F]

$$\int (a + b \operatorname{arcsinh}(dx + c))^{\frac{5}{2}}\, dx$$

input
```
int((a+b*arcsinh(d*x+c))^(5/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^(5/2),x)
```

3.196.5 Fricas [F(-2)]

Exception generated.

$$\int (a + b\text{arcsinh}(c + dx))^{5/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.196.6 Sympy [F]

$$\int (a + b\text{arcsinh}(c + dx))^{5/2}\, dx = \int (a + b \operatorname{asinh}(c + dx))^{\frac{5}{2}}\, dx$$

input
```
integrate((a+b*asinh(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**(5/2), x)
```

3.196.7 Maxima [F]

$$\int (a + b\text{arcsinh}(c + dx))^{5/2} \, dx = \int (b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}} \, dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.196.8 Giac [F]

$$\int (a + b\text{arcsinh}(c + dx))^{5/2} \, dx = \int (b \operatorname{arsinh}(dx + c) + a)^{\frac{5}{2}} \, dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int (a + b\text{arcsinh}(c + dx))^{5/2} \, dx = \int (a + b \operatorname{asinh}(c + d\,x))^{5/2} \, dx$$

input
```
int((a + b*asinh(c + d*x))^(5/2),x)
```

output
```
int((a + b*asinh(c + d*x))^(5/2), x)
```

3.197 $\int \frac{(a+b\text{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx$

3.197.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx=\frac{\text{Int}\left(\frac{(a+b\text{arcsinh}(c+dx))^{5/2}}{c+dx},x\right)}{e}$$

output
```
Unintegrable((a+b*arcsinh(d*x+c))^(5/2)/(d*x+c),x)/e
```

3.197.2 Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx=\int \frac{(a+b\text{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(5/2)/(c*e + d*e*x),x]
```

output
```
Integrate[(a + b*ArcSinh[c + d*x])^(5/2)/(c*e + d*e*x), x]
```

3.197.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 27, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{5/2}}{ce+dex} \, dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^{5/2}}{e(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^{5/2}}{c+dx}d(c+dx)}{de}$$

$$\Big\downarrow \text{6196}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^{5/2}}{c+dx}d(c+dx)}{de}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(5/2)/(c*e + d*e*x),x]
```

output
```
$Aborted
```

3.197.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.197.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{5}{2}}}{dex+ce}dx$$

input
```
int((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x)
```

output
```
int((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x)
```

3.197.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\mathrm{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.197.6 Sympy [N/A]

Not integrable

Time = 22.46 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.52

$$\int \frac{(a+b\mathrm{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx = \frac{\int \frac{a^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}{c+dx}\,dx + \int \frac{b^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}^2\,(c+dx)}{c+dx}\,dx + \int \frac{2ab\sqrt{a+b\,\mathrm{asinh}}}{}}{e}$$

input
```
integrate((a+b*asinh(d*x+c))**(5/2)/(d*e*x+c*e),x)
```

output
```
(Integral(a**2*sqrt(a + b*asinh(c + d*x))/(c + d*x), x) + Integral(b**2*sq
rt(a + b*asinh(c + d*x))*asinh(c + d*x)**2/(c + d*x), x) + Integral(2*a*b*
sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)/(c + d*x), x))/e
```

3.197.7 Maxima [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(5/2)/(d*e*x + c*e), x)
```

3.197.8 Giac [N/A]

Not integrable

Time = 3.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{5/2}}{ce+dex}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(5/2)/(d*e*x + c*e), x)
```

3.197.9 Mupad [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a + b\text{arcsinh}(c + dx))^{5/2}}{ce + dex}\,dx = \int \frac{(a + b\,\text{asinh}(c + d\,x))^{5/2}}{c\,e + d\,e\,x}\,dx$$

input
```
int((a + b*asinh(c + d*x))^(5/2)/(c*e + d*e*x),x)
```

output
```
int((a + b*asinh(c + d*x))^(5/2)/(c*e + d*e*x), x)
```

3.198 $\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$

3.198.1 Optimal result

Integrand size = 25, antiderivative size = 835

$$
\begin{aligned}
\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx =
&-\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{1125d} \\
&+\frac{119b^3e^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{1125d} \\
&-\frac{21b^3e^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{1000d} \\
&+\frac{14b^2e^4(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{15d} \\
&-\frac{7b^2e^4(c+dx)^3(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{45d} \\
&+\frac{7b^2e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{3/2}}{100d} \\
&-\frac{28be^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{75d} \\
&+\frac{14be^4(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{75d} \\
&-\frac{7be^4(c+dx)^4\sqrt{1+(c+dx)^2}(a+b\operatorname{arcsinh}(c+dx))^{5/2}}{50d} \\
&+\frac{e^4(c+dx)^5(a+b\operatorname{arcsinh}(c+dx))^{7/2}}{5d} \\
&+\frac{105b^{7/2}e^4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d} \\
&-\frac{119b^{7/2}e^4e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{18000d} \\
&-\frac{21b^{7/2}e^4e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64000d} \\
&+\frac{21b^{7/2}e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64000d} \\
&+\frac{105b^{7/2}e^4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{256d} \\
&-\frac{119b^{7/2}e^4e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{18000d} \\
&-\frac{21b^{7/2}e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{64000d}
\end{aligned}
$$

output
```
14/15*b^2*e^4*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d-7/45*b^2*e^4*(d*x+c)^3*
(a+b*arcsinh(d*x+c))^(3/2)/d+7/100*b^2*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^
(3/2)/d+1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(7/2)/d+21/320000*b^(7/2)*e
^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(
1/2)/d+21/320000*b^(7/2)*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*5^(1/2)*Pi^(1/2)/d/exp(5*a/b)-35/13824*b^(7/2)*e^4*exp(3*a/b)*erf(3^(1
/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d-35/13824*b^(7/2
)*e^4*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/
exp(3*a/b)+105/256*b^(7/2)*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(
1/2))*Pi^(1/2)/d+105/256*b^(7/2)*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*Pi^(1/2)/d/exp(a/b)-28/75*b*e^4*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^
2)^(1/2)/d+14/75*b*e^4*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^
(1/2)/d-7/50*b*e^4*(d*x+c)^4*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2
)/d-1813/1125*b^3*e^4*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d+119
/1125*b^3*e^4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d-2
1/1000*b^3*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

3.198.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.39

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{7/2}\,dx = \frac{b^4e^4e^{-\frac{5a}{b}}\left(-506250e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}\Gamma\left(\frac{9}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)+81\sqrt{5}\sqrt{-\frac{a+b\text{arcsin}}{b}}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

output
```
(b^4*e^4*(-506250*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, a/b
+ ArcSinh[c + d*x]] + 81*Sqrt[5]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma
[9/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 3125*Sqrt[3]*E^((2*a)/b)*Sqrt[-((
a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, (-3*(a + b*ArcSinh[c + d*x]))/b] +
506250*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, -((a + b
*ArcSinh[c + d*x])/b)] + 3125*Sqrt[3]*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c + d
*x]]*Gamma[9/2, (3*(a + b*ArcSinh[c + d*x]))/b] - 81*Sqrt[5]*E^((10*a)/b)*
Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, (5*(a + b*ArcSinh[c + d*x]))/b]))/
(8100000*d*E^((5*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

3.198.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int e^4(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^4\int (c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{6192}$$

$$\frac{e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\int \frac{(c+dx)^5(a+b\text{arcsinh}(c+dx))^{5/2}}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6227}$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(-\frac{1}{2}b\int (c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}d(c+dx)-\frac{4}{5}\int \frac{(c+dx)^3(a-\right.\right.$$

$$d$$

$$\Big\downarrow \text{6192}$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(-\frac{1}{2}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\int \frac{(c+dx)^5\sqrt{a+b\text{arc}}}{\sqrt{(c+dx)}}\right.\right.\right.$$

$$\Big\downarrow \text{6227}$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(-\frac{1}{2}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{1}{10}b\int \frac{(c}{\sqrt{a+b\text{ar}}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{6192}$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(-\frac{1}{2}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{1}{10}b\int \frac{(c}{\sqrt{a+b\text{ar}}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{6195}$$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(-\frac{4}{5}\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\int\frac{(c+dx)^3\sqrt{a}}{\sqrt{(}}\right.\right.\right.\right.$$

$\downarrow$ 5971

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(-\frac{4}{5}\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\int\frac{(c+dx)^3\sqrt{a}}{\sqrt{(}}\right.\right.\right.\right.$$

$\downarrow$ 2009

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(-\frac{1}{2}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{4}{5}\int\frac{(c+dx)^3\sqrt{c}}{\sqrt{}}\right.\right.\right.\right.$$

$\downarrow$ 6213

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(-\frac{1}{2}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{4}{5}\int\frac{(c+dx)^3\sqrt{c}}{\sqrt{}}\right.\right.\right.\right.$$

$\downarrow$ 6187

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(-\frac{1}{2}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{4}{5}\int\frac{(c+dx)^3\sqrt{c}}{\sqrt{}}\right.\right.\right.\right.$$

$\downarrow$ 6213

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(-\frac{1}{2}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{4}{5}\int\frac{(c+dx)^3\sqrt{c}}{\sqrt{}}\right.\right.\right.\right.$$

$\downarrow$ 6189

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(-\frac{1}{2}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{4}{5}\int\frac{(c+dx)^3\sqrt{}}{\sqrt{}}\right.\right.\right.\right.$$

$\downarrow$ 3042

3.198. $\int(ce+dex)^4(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(-\frac{1}{2}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{4}{5}\int\frac{(c+dx)^3\sqrt{}}{\sqrt{}}\right.\right.\right.\right.$$

$\downarrow$ 3788

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(\frac{1}{5}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^4-\frac{1}{2}b\left(\frac{1}{5}(c+\right.\right.\right.$$

$\downarrow$ 26

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(\frac{1}{5}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^4-\frac{1}{2}b\left(\frac{1}{5}(c+\right.\right.\right.$$

$\downarrow$ 2611

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(-\frac{1}{2}b\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{10}b\left(-\frac{4}{5}\int\frac{(c+dx)^3\sqrt{}}{\sqrt{}}\right.\right.\right.\right.$$

$\downarrow$ 2633

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(\frac{1}{5}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^4-\frac{1}{2}b\left(\frac{1}{5}(c+\right.\right.\right.$$

$\downarrow$ 2634

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(\frac{1}{5}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^4-\frac{1}{2}b\left(\frac{1}{5}(c+\right.\right.\right.$$

$\downarrow$ 6227

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(\frac{1}{5}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^4-\frac{1}{2}b\left(\frac{1}{5}(c+\right.\right.\right.$$

$\downarrow$ 6195

3.198. $\int(ce+dex)^4(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(\frac{1}{5}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^4-\frac{1}{2}b\left(\frac{1}{5}(c+\right.\right.\right.$$

$\downarrow$ 5971

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(\frac{1}{5}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^4-\frac{1}{2}b\left(\frac{1}{5}(c+\right.\right.\right.$$

$\downarrow$ 2009

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(\frac{1}{5}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^4-\frac{1}{2}b\left(\frac{1}{5}(c+\right.\right.\right.$$

$\downarrow$ 6213

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(\frac{1}{5}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^4-\frac{1}{2}b\left(\frac{1}{5}(c+\right.\right.\right.$$

$\downarrow$ 6189

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(\frac{1}{5}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^4-\frac{1}{2}b\left(\frac{1}{5}(c+\right.\right.\right.$$

$\downarrow$ 3042

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(\frac{1}{5}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^4-\frac{1}{2}b\left(\frac{1}{5}(c+\right.\right.\right.$$

$\downarrow$ 3788

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(\frac{1}{5}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^4-\frac{1}{2}b\left(\frac{1}{5}(c+\right.\right.\right.$$

$\downarrow$ 26

3.198. $\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(\frac{1}{5}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^4-\frac{1}{2}b\left(\frac{1}{5}(c+\right.\right.\right.$$

$\downarrow$ 2611

$$e^4\left(\frac{1}{5}(c+dx)^5(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{10}b\left(\frac{1}{5}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^4-\frac{1}{2}b\left(\frac{1}{5}(c+\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

output
```
$Aborted
```

3.198.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

3.198. $\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx$

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.198.4 Maple [F]

$$\int (dex+ce)^4 (a+b\operatorname{arcsinh}(dx+c))^{\frac{7}{2}}\,dx$$

input
```
int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x)
```

output
```
int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x)
```

3.198.5 Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)^4(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.198.6 Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.198.7 Maxima [F]

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{7/2}\,dx=\int (dex+ce)^4(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.198.8 Giac [F]

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{7/2}\,dx=\int (dex+ce)^4(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^4(a+b\text{arcsinh}(c+dx))^{7/2}\,dx=\int (c\,e+d\,e\,x)^4\,(a+b\,\text{asinh}(c+d\,x))^{7/2}\,dx$$

input
```
int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(7/2),x)
```

output
```
int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(7/2), x)
```

3.199 $\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$

3.199.1 Optimal result

Integrand size = 25, antiderivative size = 547

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{7/2}\,dx = \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{4096d} - \frac{105b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{2048d} - \frac{525b^2e^3(a+b\text{arcsinh}(c+dx))^{3/2}}{2048d} - \frac{105b^2e^3(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}}{256d} + \frac{35b^2e^3(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}}{256d} + \frac{21be^3(c+dx)\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{5/2}}{64d} - \frac{7be^3(c+dx)^3\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{5/2}}{32d} - \frac{3e^3(a+b\text{arcsinh}(c+dx))^{7/2}}{32d} + \frac{e^3(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}}{4d} - \frac{105b^{7/2}e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{131072d} + \frac{105b^{7/2}e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d} + \frac{105b^{7/2}e^3e^{-\frac{4a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{131072d} - \frac{105b^{7/2}e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2048d}$$

output
```
-525/2048*b^2*e^3*(a+b*arcsinh(d*x+c))^(3/2)/d-105/256*b^2*e^3*(d*x+c)^2*(
a+b*arcsinh(d*x+c))^(3/2)/d+35/256*b^2*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^
(3/2)/d-3/32*e^3*(a+b*arcsinh(d*x+c))^(7/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsi
nh(d*x+c))^(7/2)/d+105/4096*b^(7/2)*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsin
h(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d-105/4096*b^(7/2)*e^3*erfi(2^(1
/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-105/
131072*b^(7/2)*e^3*exp(4*a/b)*erf(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi
^(1/2)/d+105/131072*b^(7/2)*e^3*erfi(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))
*Pi^(1/2)/d/exp(4*a/b)+21/64*b*e^3*(d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)*(1+(
d*x+c)^2)^(1/2)/d-7/32*b*e^3*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+
c)^2)^(1/2)/d+1575/4096*b^3*e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d
*x+c))^(1/2)/d-105/2048*b^3*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh
(d*x+c))^(1/2)/d
```

3.199.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.38

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{7/2}\,dx = \frac{b^4e^3e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{9}{2},-\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)-32\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{9}{2}\right.\right.}{8}$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

output
```
(b^4*e^3*(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, (-4*(a + b*ArcSin
h[c + d*x]))/b] - 32*Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b
)]*Gamma[9/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[a/b + Ar
cSinh[c + d*x]]*(-32*Sqrt[2]*Gamma[9/2, (2*(a + b*ArcSinh[c + d*x]))/b] +
E^((2*a)/b)*Gamma[9/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/(8192*d*E^((4*a)
/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

3.199.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 5.75 (sec) , antiderivative size = 797, normalized size of antiderivative = 1.46, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {6274, 27, 6192, 6227, 6192, 6227, 6192, 6195, 25, 5971, 2009, 6198, 6227, 6195, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int e^3(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^3\int (c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6192}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\int \frac{(c+dx)^4(a+b\text{arcsinh}(c+dx))^{5/2}}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 6227}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(-\frac{5}{8}b\int (c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}d(c+dx)-\frac{3}{4}\int \frac{(c+dx)^2(a+}{\sqrt{}}\right.\right.}{d}$$

$$\Big\downarrow \text{ 6192}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(-\frac{5}{8}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\int \frac{(c+dx)^4\sqrt{a+b\text{arcsi}}}{\sqrt{(c+dx)^2}}\right.\right.\right.$$

$$\Big\downarrow \text{ 6227}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(-\frac{5}{8}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{1}{8}b\int \frac{(c+d}{\sqrt{a+b\text{arcsi}}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 6192}$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(-\frac{5}{8}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{1}{8}b\int\frac{(c+d}{\sqrt{a+b\text{arcsi}}}\right.\right.\right.\right.$$

$\downarrow$ 6195

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(-\frac{3}{4}\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\int\frac{(c+dx)^2\sqrt{a+}}{\sqrt{(c}}\right.\right.\right.\right.$$

$\downarrow$ 25

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(-\frac{3}{4}\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\int\frac{(c+dx)^2\sqrt{a+}}{\sqrt{(c}}\right.\right.\right.\right.$$

$\downarrow$ 5971

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(-\frac{3}{4}\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\int\frac{(c+dx)^2\sqrt{a+}}{\sqrt{(c}}\right.\right.\right.\right.$$

$\downarrow$ 2009

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(-\frac{5}{8}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\int\frac{(c+dx)^2\sqrt{a+}}{\sqrt{(c-}}\right.\right.\right.\right.$$

$\downarrow$ 6198

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(-\frac{5}{8}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\int\frac{(c+dx)^2\sqrt{a+}}{\sqrt{(c-}}\right.\right.\right.\right.$$

$\downarrow$ 6227

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(-\frac{5}{8}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{4}b\int\frac{}{\sqrt{a+b}}\right.\right.\right.\right.\right.$$

$\downarrow$ 6195

3.199. $\int(ce+dex)^3(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(-\frac{5}{8}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{\sqrt{a+b}}{\sqrt{}}\right.\right.\right.\right.\right.$$

$\downarrow$ 25

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(-\frac{5}{8}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{\sqrt{a+b}}{\sqrt{}}\right.\right.\right.\right.\right.$$

$\downarrow$ 5971

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(-\frac{5}{8}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{\sqrt{a+b}}{\sqrt{}}\right.\right.\right.\right.\right.$$

$\downarrow$ 27

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(-\frac{5}{8}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{\sqrt{a+b}}{\sqrt{}}\right.\right.\right.\right.\right.$$

$\downarrow$ 3042

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(-\frac{5}{8}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{\sqrt{a+b}}{\sqrt{}}\right.\right.\right.\right.\right.$$

$\downarrow$ 26

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(-\frac{5}{8}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{\sqrt{a+b}}{\sqrt{}}\right.\right.\right.\right.\right.$$

$\downarrow$ 3789

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(\frac{1}{4}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^3-\frac{5}{8}b\left(\frac{1}{4}(c+\right.\right.\right.$$

3.199. $\int(ce+dex)^3(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$

$$\Big\downarrow \ 2611$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(-\frac{5}{8}b\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{8}b\left(-\frac{3}{4}\left(-\frac{1}{8}i\left(i\int e^{\frac{2a}{b}}\right.\right.\right.\right.\right.\right.$$

$$\Big\downarrow \ 2633$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(\frac{1}{4}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^3-\frac{5}{8}b\left(\frac{1}{4}(c+d\right.\right.\right.$$

$$\Big\downarrow \ 2634$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(\frac{1}{4}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^3-\frac{5}{8}b\left(\frac{1}{4}(c+d\right.\right.\right.$$

$$\Big\downarrow \ 6198$$

$$e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{8}b\left(\frac{1}{4}\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}(c+dx)^3-\frac{5}{8}b\left(\frac{1}{4}(c+d\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

3.199. $\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$

output
```
(e^3*(((c + d*x)^4*(a + b*ArcSinh[c + d*x])^(7/2))/4 - (7*b*(((c + d*x)^3*
Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/4 - (5*b*(((c + d*x)
^4*(a + b*ArcSinh[c + d*x])^(3/2))/4 - (3*b*(((c + d*x)^3*Sqrt[1 + (c + d*
x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/4 + ((Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf
[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/32 - (Sqrt[b]*E^((2*a)/b)*Sqrt
[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/8 - (Sqrt[b]*S
qrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*E^((4*a)/b)) +
 (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(8*E^((2*a)/b)))/8 - (3*(((c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcS
inh[c + d*x]])/2 - (a + b*ArcSinh[c + d*x])^(3/2)/(3*b) - (I/8)*((I/2)*Sqr
t[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x
]])/Sqrt[b]])/E^((2*a)/b))))/4))/8))/8 - (3*(((c + d*x)*Sqrt[1 + (c + d*x)
^2]*(a + b*ArcSinh[c + d*x])^(5/2))/2 - (a + b*ArcSinh[c + d*x])^(7/2)/(7*
b) - (5*b*(((c + d*x)^2*(a + b*ArcSinh[c + d*x])^(3/2))/2 - (3*b*(((c + d*
x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/2 - (a + b*ArcSinh[
c + d*x])^(3/2)/(3*b) - (I/8)*((I/2)*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(S
qrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi/2]*
Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/E^((2*a)/b))))/4))/4
))/4))/8))/d
```

3.199.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

3.199. $\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx$

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.199.4 Maple [F]

$$\int (dex + ce)^3 (a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}} dx$$

input
```
int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x)
```

output
```
int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x)
```

3.199.5 Fricas [F(-2)]

Exception generated.

$$\int (ce + dex)^3 (a + b\operatorname{arcsinh}(c + dx))^{7/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.199.6 Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.199.7 Maxima [F]

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx=\int (dex+ce)^3(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.199.8 Giac [F]

$$\int (ce+dex)^3(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx=\int (dex+ce)^3(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^3(a+b\text{arcsinh}(c+dx))^{7/2}\,dx=\int (c\,e+d\,e\,x)^3\,(a+b\,\text{asinh}(c+d\,x))^{7/2}\,dx$$

input
```
int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(7/2),x)
```

output
```
int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(7/2), x)
```

3.200 $\int (ce+dex)^2(a+b\,\text{arcsinh}(c+dx))^{7/2}\,dx$

3.200.1 Optimal result

Integrand size = 25, antiderivative size = 481

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{7/2}\,dx = \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{54d} - \frac{35b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{216d} - \frac{35b^2e^2(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}{18d} + \frac{35b^2e^2(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}}{108d} + \frac{7be^2\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{5/2}}{9d} - \frac{7be^2(c+dx)^2\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{5/2}}{18d} + \frac{e^2(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}}{3d} - \frac{105b^{7/2}e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d} + \frac{35b^{7/2}e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3456d} - \frac{105b^{7/2}e^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{128d} + \frac{35b^{7/2}e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3456d}$$

output
```
-35/18*b^2*e^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d+35/108*b^2*e^2*(d*x+c)
^3*(a+b*arcsinh(d*x+c))^(3/2)/d+1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(7/
2)/d+35/10368*b^(7/2)*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2
)/b^(1/2))*3^(1/2)*Pi^(1/2)/d+35/10368*b^(7/2)*e^2*erfi(3^(1/2)*(a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)-105/128*b^(7/2)*e^
2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-105/128*b^(7
/2)*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)+7/9*b
*e^2*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-7/18*b*e^2*(d*x+c)^2
*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d+175/54*b^3*e^2*(1+(d*x+c
)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d-35/216*b^3*e^2*(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

3.200.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.46

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \frac{b^4e^2e^{-\frac{3a}{b}}\left(243e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}\Gamma\left(\frac{9}{2},\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)+\sqrt{3}\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\Gamma$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

output
```
(b^4*e^2*(243*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, a/b + Ar
cSinh[c + d*x]] + Sqrt[3]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, (
-3*(a + b*ArcSinh[c + d*x]))/b] - 243*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c
+ d*x])/b)]*Gamma[9/2, -((a + b*ArcSinh[c + d*x])/b)] - Sqrt[3]*E^((6*a)/b
)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, (3*(a + b*ArcSinh[c + d*x]))/b])
)/(1944*d*E^((3*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

3.200.3 Rubi [A] (verified)

Time = 4.88 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.36, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 1.080$, Rules used = {6274, 27, 6192, 6227, 6192, 6213, 6187, 6213, 6189, 3042, 3788, 26, 2611, 2633, 2634, 6227, 6195, 5971, 2009, 6213, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int e^2(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^2\int (c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6192}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\int \frac{(c+dx)^3(a+b\text{arcsinh}(c+dx))^{5/2}}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 6227}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\int (c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}d(c+dx)-\frac{2}{3}\int \frac{(c+dx)(a+b}{\sqrt{(}}\right.\right.}{d}$$

$$\Big\downarrow \text{ 6192}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\int \frac{(c+dx)^3\sqrt{a+b\text{arcs}}}{\sqrt{(c+dx)^2-}}\right.\right.\right.}{}$$

$$\Big\downarrow \text{ 6213}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\int \frac{(c+dx)^3\sqrt{a+b\text{arcs}}}{\sqrt{(c+dx)^2-}}\right.\right.\right.}{}$$

$$\Big\downarrow \text{ 6187}$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+\right.\right.\right.\right.$$

$\downarrow$ 6213

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+\right.\right.\right.\right.$$

$\downarrow$ 6189

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\int\frac{(c+dx)^3\sqrt{a+b\text{arcs}}}{\sqrt{(c+dx)^2}}\right.\right.\right.$$

$\downarrow$ 3042

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\int\frac{(c+dx)^3\sqrt{a+b\text{arcs}}}{\sqrt{(c+dx)^2}}\right.\right.\right.$$

$\downarrow$ 3788

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+\right.\right.\right.\right.$$

$\downarrow$ 26

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+\right.\right.\right.\right.$$

$\downarrow$ 2611

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+\right.\right.\right.\right.$$

$\downarrow$ 2633

3.200. $\int(ce+dex)^2(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+\right.\right.\right.\right.$$

$\downarrow$ 2634

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\int\frac{(c+dx)^3\sqrt{a+b\text{arcsi}}}{\sqrt{(c+dx)^2-}}\right.\right.\right.$$

$\downarrow$ 6227

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{1}{6}b\int\frac{(c+d}{\sqrt{a+b\text{arcsi}}}\right.\right.\right.\right.$$

$\downarrow$ 6195

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\int\frac{(c+dx)\sqrt{a+}}{\sqrt{(c}}\right.\right.\right.\right.$$

$\downarrow$ 5971

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\int\frac{(c+dx)\sqrt{a+}}{\sqrt{(c}}\right.\right.\right.\right.$$

$\downarrow$ 2009

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\int\frac{(c+dx)\sqrt{a+b}}{\sqrt{(c+}}\right.\right.\right.\right.$$

$\downarrow$ 6213

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2}\right.\right.\right.\right.\right.$$

$\downarrow$ 6189

3.200. $\int(ce+dex)^2(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)\right.\right.\right.\right.\right.$$

$\downarrow$ 3042

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)\right.\right.\right.\right.\right.$$

$\downarrow$ 3788

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2\right.\right.\right.\right.\right.$$

$\downarrow$ 26

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{}{\sqrt{a}}\right.\right.\right.\right.\right.\right.$$

$\downarrow$ 2611

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\left(\frac{1}{2}\left(-\int e^{\frac{a}{b}-}\right.\right.\right.\right.\right.\right.$$

$\downarrow$ 2633

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{5}{6}b\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{1}{2}b\left(-\frac{2}{3}\left(\frac{1}{2}\left(-\int e^{\frac{a}{b}-}\right.\right.\right.\right.\right.\right.$$

$\downarrow$ 2634

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{6}b\left(-\frac{2}{3}\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+\right.\right.\right.\right.$$

3.200. $\int(ce+dex)^2(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

output
```
(e^2*(((c + d*x)^3*(a + b*ArcSinh[c + d*x])^(7/2))/3 - (7*b*(((c + d*x)^2*
Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/3 - (2*(Sqrt[1 + (c
+ d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2) - (5*b*((c + d*x)*(a + b*ArcSinh[
c + d*x])^(3/2) - (3*b*(Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]]
 + (-1/2*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b
]]) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*E^(
a/b)))/2))/2))/2))/3 - (5*b*(((c + d*x)^3*(a + b*ArcSinh[c + d*x])^(3/2))/
3 - (b*(((c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/3
 - (2*(Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]] + (-1/2*(Sqrt[b]
*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]) - (Sqrt[b]*Sq
rt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*E^(a/b)))/2))/3 + ((
Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/8 - (S
qrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/S
qrt[b]])/8 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])
/(8*E^(a/b)) - (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*
x]])/Sqrt[b]])/(8*E^((3*a)/b)))/6))/2))/6))/6))/d
```

3.200.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

3.200. $\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.200.4 Maple [F]

$$\int (dex+ce)^2 (a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{7}{2}}\,dx$$

input
```
int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x)
```

output
```
int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x)
```

3.200.5 Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)^2(a+b\mathrm{arcsinh}(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.200.6 Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.200.7 Maxima [F]

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \int (dex+ce)^2(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.200.8 Giac [F]

$$\int (ce+dex)^2(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx = \int (dex+ce)^2(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arcsinh}(c+dx))^{7/2}\,dx=\int (c\,e+d\,e\,x)^2\,(a+b\,\text{asinh}(c+d\,x))^{7/2}\,dx$$

input
```
int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(7/2),x)
```

output
```
int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(7/2), x)
```

3.201 $\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$

3.201.1 Optimal result

Integrand size = 23, antiderivative size = 305

$$\begin{aligned}\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{7/2}\,dx =&-\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\text{arcsinh}(c+dx)}}{128d}\\&+\frac{35b^2e(a+b\text{arcsinh}(c+dx))^{3/2}}{64d}+\frac{35b^2e(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}}{32d}\\&-\frac{7be(c+dx)\sqrt{1+(c+dx)^2}(a+b\text{arcsinh}(c+dx))^{5/2}}{8d}\\&+\frac{e(a+b\text{arcsinh}(c+dx))^{7/2}}{4d}+\frac{e(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}}{2d}\\&-\frac{105b^{7/2}ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1024d}\\&+\frac{105b^{7/2}ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{1024d}\end{aligned}$$

output
```
35/64*b^2*e*(a+b*arcsinh(d*x+c))^(3/2)/d+35/32*b^2*e*(d*x+c)^2*(a+b*arcsin
h(d*x+c))^(3/2)/d+1/4*e*(a+b*arcsinh(d*x+c))^(7/2)/d+1/2*e*(d*x+c)^2*(a+b*
arcsinh(d*x+c))^(7/2)/d-105/2048*b^(7/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arc
sinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d+105/2048*b^(7/2)*e*erfi(2^(
1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-7/8
*b*e*(d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-105/128*b^3*
e*(d*x+c)*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

3.201.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.41

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{7/2}\,dx = \frac{ee^{-\frac{2a}{b}}\left(b^4\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{9}{2},-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)+b^4e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}\Gamma\left(\frac{9}{2},\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)\right)}{64\sqrt{2}d\sqrt{a+b\text{arcsinh}(c+dx)}}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

output
```
(e*(b^4*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, (-2*(a + b*ArcSinh[
c + d*x]))/b] + b^4*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, (2
*(a + b*ArcSinh[c + d*x]))/b]))/(64*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSi
nh[c + d*x]])
```

3.201.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.783, Rules used = {6274, 27, 6192, 6227, 6192, 6198, 6227, 6195, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int e(c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int (c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{6192}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{4}b\int\frac{(c+dx)^2(a+b\text{arcsinh}(c+dx))^{5/2}}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6227}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\int(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}d(c+dx)-\frac{1}{2}\int\frac{(a+b\text{arcsinh}(\cdots}{\sqrt{(c+dx\cdots}}\right.\right.}{d}$$

$$\Big\downarrow \text{6192}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\int\frac{(c+dx)^2\sqrt{a+b\text{arcsin}\cdots}}{\sqrt{(c+dx)^2+\cdots}}\right.\right.\right.}{d}$$

$$\Big\downarrow \text{6198}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\int\frac{(c+dx)^2\sqrt{a+b\text{arcsin}\cdots}}{\sqrt{(c+dx)^2+\cdots}}\right.\right.\right.}{d}$$

$$\Big\downarrow \text{6227}$$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{4}b\int\frac{c+dx\cdots}{\sqrt{a+b\text{arcsin}\cdots}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{6195}$$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsin}\cdots}}{\sqrt{(c+dx)\cdots}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{25}$$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsin}\cdots}}{\sqrt{(c+dx)\cdots}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{5971}$$

3.201. $\int(ce+dex)(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsin}}}{\sqrt{(c+dx)}}\right.\right.\right.\right.$$

$\big\downarrow$ 27

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsin}}}{\sqrt{(c+dx)}}\right.\right.\right.\right.$$

$\big\downarrow$ 3042

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsin}}}{\sqrt{(c+dx)}}\right.\right.\right.\right.$$

$\big\downarrow$ 26

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsin}}}{\sqrt{(c+dx)}}\right.\right.\right.\right.$$

$\big\downarrow$ 3789

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{8}i\left(\frac{1}{2}i\int\frac{e^{2(}}{\sqrt{a+b\text{ar}}}\right.\right.\right.\right.\right.$$

$\big\downarrow$ 2611

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{8}i\left(i\int e^{\frac{2a}{b}-\frac{2(a+}{}}\right.\right.\right.\right.\right.$$

$\big\downarrow$ 2633

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{8}i\left(i\int e^{\frac{2a}{b}-\frac{2(a+}{}}\right.\right.\right.\right.\right.$$

$\big\downarrow$ 2634

3.201. $\int (ce+dex)(a+b\text{arcsinh}(c+dx))^{7/2}\,dx$

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{2}\int\frac{\sqrt{a+b\text{arcsinh}}}{\sqrt{(c+dx)^2}}\right.\right.\right.\right.$$

$\downarrow$ 6198

$$e\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{4}b\left(-\frac{5}{4}b\left(\frac{1}{2}(c+dx)^2(a+b\text{arcsinh}(c+dx))^{3/2}-\frac{3}{4}b\left(-\frac{1}{8}i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\right.\right.\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

output
```
(e*(((c + d*x)^2*(a + b*ArcSinh[c + d*x])^(7/2))/2 - (7*b*(((c + d*x)*Sqrt
[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/2 - (a + b*ArcSinh[c + d
*x])^(7/2)/(7*b) - (5*b*(((c + d*x)^2*(a + b*ArcSinh[c + d*x])^(3/2))/2 -
(3*b*(((c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/2 - (
a + b*ArcSinh[c + d*x])^(3/2)/(3*b) - (I/8)*((I/2)*Sqrt[b]*E^((2*a)/b)*Sqr
t[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - ((I/2)*Sqrt[
b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/E^((2*
a)/b))))/4))/4))/4))/d
```

3.201.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6192
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Simp[b*c*(n/(m + 1))   Int
[x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; Free
Q[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

3.201. $\int (ce+dex)(a+b\,\mathrm{arcsinh}(c+dx))^{7/2}\,dx$

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.201.4 Maple [F]

$$\int (dex+ce)\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^{\frac{7}{2}}dx$$

input
```
int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x)
```

output
```
int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x)
```

3.201.5 Fricas [F(-2)]

Exception generated.

$$\int (ce+dex)(a+b\mathrm{arcsinh}(c+dx))^{7/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.201.6 Sympy [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.201.7 Maxima [F]

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx=\int (dex+ce)(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.201.8 Giac [F]

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx=\int (dex+ce)(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}\,dx=\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^{7/2}\,dx$$

input
```
int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(7/2),x)
```

output
```
int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(7/2), x)
```

3.202 $\int (a + b\text{arcsinh}(c + dx))^{7/2}\, dx$

3.202.1 Optimal result

Integrand size = 14, antiderivative size = 216

$$\begin{aligned}\int (a + b\text{arcsinh}(c + dx))^{7/2}\, dx = &-\frac{105b^3\sqrt{1 + (c + dx)^2}\sqrt{a + b\text{arcsinh}(c + dx)}}{8d} \\ &+ \frac{35b^2(c + dx)(a + b\text{arcsinh}(c + dx))^{3/2}}{4d} \\ &- \frac{7b\sqrt{1 + (c + dx)^2}(a + b\text{arcsinh}(c + dx))^{5/2}}{2d} + \frac{(c + dx)(a + b\text{arcsinh}(c + dx))^{7/2}}{d} \\ &+ \frac{105b^{7/2}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d} + \frac{105b^{7/2}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32d}\end{aligned}$$

output
```
35/4*b^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d+(d*x+c)*(a+b*arcsinh(d*x+c))
^(7/2)/d+105/32*b^(7/2)*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*P
i^(1/2)/d+105/32*b^(7/2)*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)
/d/exp(a/b)-7/2*b*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-105/8*b
^3*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

3.202.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 698 vs. $2(216) = 432$.

Time = 3.27 (sec) , antiderivative size = 698, normalized size of antiderivative = 3.23

$$\int (a+b\text{arcsinh}(c+dx))^{7/2}\,dx = \frac{16a^3e^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}}\right)+12a^2 \cdots}{\cdots}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(7/2),x]
```

output
```
((16*a^3*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + Arc
Sinh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSin
h[c + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + 12*a^2*Sqr
t[b]*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2
*(c + d*x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSin
h[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + 6*a*Sqrt[b
]*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(2*Sqrt[1 + (c + d*x)^2]*(a - 5*
b*ArcSinh[c + d*x]) + b*(c + d*x)*(15 + 4*ArcSinh[c + d*x]^2)) + (4*a^2 +
12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(-Cos
h[a/b] + Sinh[a/b]) + (4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*Ar
cSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(4*Sqrt[b]*Sqr
t[a + b*ArcSinh[c + d*x]]*(2*b*(c + d*x)*(-10*a + 35*b*ArcSinh[c + d*x] +
4*b*ArcSinh[c + d*x]^3) + Sqrt[1 + (c + d*x)^2]*(-4*a^2 + 4*a*b*ArcSinh[c
+ d*x] - 7*b^2*(15 + 4*ArcSinh[c + d*x]^2))) + (8*a^3 + 36*a^2*b + 90*a*b^
2 + 105*b^3)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b
] - Sinh[a/b]) + (-8*a^3 + 36*a^2*b - 90*a*b^2 + 105*b^3)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(32*d)
```

3.202.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6273, 6187, 6213, 6187, 6213, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arcsinh}(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int (a+b\text{arcsinh}(c+dx))^{7/2}d(c+dx)}{d}$$

$$\Big\downarrow \text{6187}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{2}b\int \frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}}{\sqrt{(c+dx)^2+1}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6213}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\int (a+b\text{arcsinh}(c+dx))^{3/}\right.}{d}$$

$$\Big\downarrow \text{6187}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+b\text{arcsinh}(c\right.\right.}{d}$$

$$\Big\downarrow \text{6213}$$

$$\frac{(c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+b\text{arcsinh}(c\right.\right.}{d}$$

$$\Big\downarrow \text{6189}$$

$$(c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+b\text{arcsinh}(c\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$(c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+b\text{arcsinh}(c\right.\right.$$

$\downarrow$ 3788

$$(c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+b\text{arcsinh}(c\right.\right.$$

$\downarrow$ 26

$$(c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+b\text{arcsinh}(c\right.\right.$$

$\downarrow$ 2611

$$(c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+b\text{arcsinh}(c\right.\right.$$

$\downarrow$ 2633

$$(c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+b\text{arcsinh}(c\right.\right.$$

$\downarrow$ 2634

$$(c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}-\frac{7}{2}b\left(\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}-\frac{5}{2}b\left((c+dx)(a+b\text{arcsinh}(c\right.\right.$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(7/2),x]
```

3.202. $\int (a+b\text{arcsinh}(c+dx))^{7/2}\,dx$

output
```
((c + d*x)*(a + b*ArcSinh[c + d*x])^(7/2) - (7*b*(Sqrt[1 + (c + d*x)^2]*(a
 + b*ArcSinh[c + d*x])^(5/2) - (5*b*((c + d*x)*(a + b*ArcSinh[c + d*x])^(3
/2) - (3*b*(Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]] + (-1/2*(Sq
rt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]) - (Sqrt[
b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*E^(a/b)))/2))/2
))/2))/2)/d
```

3.202.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.202.4 Maple [F]

$$\int (a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{7}{2}}\,dx$$

input
```
int((a+b*arcsinh(d*x+c))^(7/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^(7/2),x)
```

3.202.5 Fricas [F(-2)]

Exception generated.

$$\int (a+b\mathrm{arcsinh}(c+dx))^{7/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.202.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\text{arcsinh}(c+dx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asinh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.202.7 Maxima [F]

$$\int (a+b\text{arcsinh}(c+dx))^{7/2}\,dx=\int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.202.8 Giac [F]

$$\int (a+b\text{arcsinh}(c+dx))^{7/2}\,dx=\int (b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arcsinh}(c+dx))^{7/2}\,dx=\int (a+b\,\text{asinh}(c+d\,x))^{7/2}\,dx$$

input
```
int((a + b*asinh(c + d*x))^(7/2),x)
```

output
```
int((a + b*asinh(c + d*x))^(7/2), x)
```

3.203 $\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx$

3.203.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx = \frac{\text{Int}\left(\frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{c+dx},x\right)}{e}$$

output
```
Unintegrable((a+b*arcsinh(d*x+c))^(7/2)/(d*x+c),x)/e
```

3.203.2 Mathematica [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(7/2)/(c*e + d*e*x),x]
```

output
```
Integrate[(a + b*ArcSinh[c + d*x])^(7/2)/(c*e + d*e*x), x]
```

3.203.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 27, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx$$
$$\Big\downarrow \text{6274}$$
$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{e(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{c+dx}d(c+dx)}{de}$$
$$\Big\downarrow \text{6196}$$
$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{c+dx}d(c+dx)}{de}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(7/2)/(c*e + d*e*x),x]
```

output
```
$Aborted
```

3.203.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.203.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{7}{2}}}{dex+ce}dx$$

input
```
int((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x)
```

output
```
int((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x)
```

3.203.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\mathrm{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.203.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\mathrm{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asinh(d*x+c))**(7/2)/(d*e*x+c*e),x)
```

output
```
Timed out
```

3.203.7 Maxima [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(7/2)/(d*e*x + c*e), x)
```

3.203.8 Giac [N/A]

Not integrable

Time = 5.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}{dex+ce}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(7/2)/(d*e*x + c*e), x)
```

3.203.9 Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^{7/2}}{ce+dex}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{7/2}}{c\,e+d\,e\,x}\,dx$$

input
```
int((a + b*asinh(c + d*x))^(7/2)/(c*e + d*e*x),x)
```

output
```
int((a + b*asinh(c + d*x))^(7/2)/(c*e + d*e*x), x)
```

3.204 $\int \frac{(ce+dex)^4}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$

3.204.1 Optimal result

Integrand size = 25, antiderivative size = 326

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \frac{e^4e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d} - \frac{e^4e^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} + \frac{e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} + \frac{e^4e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d} - \frac{e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} + \frac{e^4e^{-\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}$$

output
```
1/160*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/
2)*Pi^(1/2)/d/b^(1/2)+1/160*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^
(1/2))*5^(1/2)*Pi^(1/2)/d/exp(5*a/b)/b^(1/2)+1/16*e^4*exp(a/b)*erf((a+b*ar
csinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/16*e^4*erfi((a+b*arcsinh
(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)-1/32*e^4*exp(3*a/b)*er
f(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/b^(1/2)-1
/32*e^4*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/
d/exp(3*a/b)/b^(1/2)
```

3.204.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.98

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$$
$$= \frac{e^4e^{-\frac{5a}{b}}\left(-10e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)+\sqrt{5}\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{1}{2},-\frac{5(a+b\text{arc}}{}\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(e^4*(-10*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSin
h[c + d*x]] + Sqrt[5]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-5*(
a + b*ArcSinh[c + d*x]))/b] - 5*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[
c + d*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c + d*x]))/b] + 10*E^((4*a)/b)
*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c + d*x])
/b)] + 5*Sqrt[3]*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (3*(a
 + b*ArcSinh[c + d*x]))/b] - Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcSinh[c + d
*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c + d*x]))/b]))/(160*d*E^((5*a)/b)*Sqrt[
a + b*ArcSinh[c + d*x]])
```

3.204.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6274, 27, 6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\text{arcsinh}(c+dx)}}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{e^4(c+dx)^4}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^4\int \frac{(c+dx)^4}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6195}$$

$$\frac{e^4\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{5971}$$

$$\frac{e^4\int \left(\frac{\cosh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{3\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{8\sqrt{a+b\text{arcsinh}(c+dx)}}\right)d(a+b\text{arcsinh}(c}{bd}$$

$$\Big\downarrow \text{2009}$$

$$e^4\left(\frac{1}{16}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\frac{\pi}{5}}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b}\right.\right.$$

input
```
Int[(c*e + d*e*x)^4/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(e^4*((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])
/16 - (Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/32 + (Sqrt[b]*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a
+ b*ArcSinh[c + d*x]])/Sqrt[b]])/32 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*Ar
cSinh[c + d*x]]/Sqrt[b]])/(16*E^(a/b)) - (Sqrt[b]*Sqrt[3*Pi]*Erfi[(Sqrt[3]
*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*E^((3*a)/b)) + (Sqrt[b]*Sqrt[
Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*E^((5*a)/b
))))/(b*d)
```

3.204.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.204.4 Maple [F]

$$\int \frac{(dex+ce)^4}{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}dx$$

input
```
int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x)
```

output
```
int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x)
```

3.204.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\mathrm{arcsinh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.204.6 Sympy [F]

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\mathrm{arcsinh}(c+dx)}}\,dx = e^4\left(\int \frac{c^4}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx + \int \frac{d^4x^4}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx + \int \frac{4cd^3x^3}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx + \int \frac{6c^2d^2x^2}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx + \int \frac{4c^3dx}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(1/2),x)
```

output
```
e**4*(Integral(c**4/sqrt(a + b*asinh(c + d*x)), x) + Integral(d**4*x**4/sq
rt(a + b*asinh(c + d*x)), x) + Integral(4*c*d**3*x**3/sqrt(a + b*asinh(c +
 d*x)), x) + Integral(6*c**2*d**2*x**2/sqrt(a + b*asinh(c + d*x)), x) + In
tegral(4*c**3*d*x/sqrt(a + b*asinh(c + d*x)), x))
```

3.204.7 Maxima [F]

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx=\int \frac{(dex+ce)^4}{\sqrt{b\,\text{arsinh}\,(dx+c)+a}}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^4/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.204.8 Giac [F]

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx=\int \frac{(dex+ce)^4}{\sqrt{b\,\text{arsinh}\,(dx+c)+a}}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx=\int \frac{(c\,e+d\,e\,x)^4}{\sqrt{a+b\,\text{asinh}\,(c+d\,x)}}\,dx$$

input
```
int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(1/2),x)
```

output
```
int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(1/2), x)
```

3.205 $\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$

3.205.1 Optimal result

Integrand size = 25, antiderivative size = 217

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = -\frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} + \frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} + \frac{e^3e^{-\frac{4a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d} - \frac{e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}$$

output
```
1/16*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2
)*Pi^(1/2)/d/b^(1/2)-1/16*e^3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1
/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)/b^(1/2)-1/32*e^3*exp(4*a/b)*erf(2*(a+b*
arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/32*e^3*erfi(2*(a+b*arc
sinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(4*a/b)/b^(1/2)
```

3.205.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.94

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \frac{e^3e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{1}{2},-\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)-2\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{1}{2},-\frac{2(a+b\text{arcsinh}}{b}\right.\right.}{32d\sqrt{a+b\text{arc}}}$$

input
```
Integrate[(c*e + d*e*x)^3/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(e^3*(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c
+ d*x]))/b] - 2*Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Ga
mma[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[a/b + ArcSinh
[c + d*x]]*(-2*Sqrt[2]*Gamma[1/2, (2*(a + b*ArcSinh[c + d*x]))/b] + E^((2*
a)/b)*Gamma[1/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/(32*d*E^((4*a)/b)*Sqrt
[a + b*ArcSinh[c + d*x]])
```

3.205.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6274, 27, 6195, 25, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$$
$$\Big\downarrow \; 6274$$
$$\frac{\int \frac{e^3(c+dx)^3}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{d}$$
$$\Big\downarrow \; 27$$
$$\frac{e^3\int \frac{(c+dx)^3}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{d}$$

$$\downarrow \text{6195}$$

$$\frac{e^3 \int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\downarrow \text{25}$$

$$-\frac{e^3 \int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\downarrow \text{5971}$$

$$-\frac{e^3 \int \left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{8\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(c+dx)}}\right)d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\downarrow \text{2009}$$

$$\frac{e^3\left(-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b}\right.\right.}{bd}$$

input
```
Int[(c*e + d*e*x)^3/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(e^3*(-1/32*(Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*
x]])/Sqrt[b]]) + (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*A
rcSinh[c + d*x]])/Sqrt[b]])/8 + (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(32*E^((4*a)/b)) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[
2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*E^((2*a)/b))))/(b*d)
```

3.205.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.205. $\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.205.4 Maple [F]

$$\int \frac{(dex+ce)^3}{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}dx$$

input
```
int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x)
```

output
```
int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x)
```

3.205.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\mathrm{arcsinh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.205.6 Sympy [F]

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = e^3\left(\int \frac{c^3}{\sqrt{a+b\,\text{asinh}\,(c+dx)}}\,dx + \int \frac{d^3x^3}{\sqrt{a+b\,\text{asinh}\,(c+dx)}}\,dx + \int \frac{3cd^2x^2}{\sqrt{a+b\,\text{asinh}\,(c+dx)}}\,dx + \int \frac{3c^2dx}{\sqrt{a+b\,\text{asinh}\,(c+dx)}}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(1/2),x)
```

output
```
e**3*(Integral(c**3/sqrt(a + b*asinh(c + d*x)), x) + Integral(d**3*x**3/sq
rt(a + b*asinh(c + d*x)), x) + Integral(3*c*d**2*x**2/sqrt(a + b*asinh(c +
 d*x)), x) + Integral(3*c**2*d*x/sqrt(a + b*asinh(c + d*x)), x))
```

3.205.7 Maxima [F]

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \int \frac{(dex+ce)^3}{\sqrt{b\,\text{arsinh}\,(dx+c)+a}}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^3/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.205.8 Giac [F]

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \int \frac{(dex+ce)^3}{\sqrt{b\,\text{arsinh}\,(dx+c)+a}}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx=\int \frac{{\left(c\,e+d\,e\,x\right)}^3}{\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}}\,dx$$

input
```
int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(1/2),x)
```

output
```
int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(1/2), x)
```

3.206 $\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$

3.206.1 Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = -\frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} + \frac{e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} - \frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d} + \frac{e^2e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}$$

output
```
1/24*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2
)*Pi^(1/2)/d/b^(1/2)+1/24*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1
/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)/b^(1/2)-1/8*e^2*exp(a/b)*erf((a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)-1/8*e^2*erfi((a+b*arcsinh(d*x
+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

3.206.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.01

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$$
$$= \frac{e^2e^{-\frac{3a}{b}}\left(3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arcsinh}(c+dx)\right)+\sqrt{3}\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{1}{2},-\frac{3(a+b\text{arcsin}}{b}\right.\right.}{24d\sqrt{}}$$

input
```
Integrate[(c*e + d*e*x)^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(e^2*(3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[
c + d*x]] + Sqrt[3]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-3*(a
+ b*ArcSinh[c + d*x]))/b] - 3*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/
b)]*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[a
/b + ArcSinh[c + d*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b]))/(24*d*
E^((3*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

3.206.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6274, 27, 6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$$
$$\Big\downarrow \; 6274$$
$$\frac{\int \frac{e^2(c+dx)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{d}$$
$$\Big\downarrow \; 27$$
$$\frac{e^2\int \frac{(c+dx)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6195}$$

$$\frac{e^2 \int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{e^2 \int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(c+dx)}}\right)d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^2\left(-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcs}}}{\sqrt{}}\right.\right.}{bd}$$

input

```
Int[(c*e + d*e*x)^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output

```
(e^2*(-1/8*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt
[b]]) + (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c
+ d*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]])/(8*E^(a/b)) + (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(8*E^((3*a)/b))))/(b*d)
```

3.206.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

3.206. $\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.206.4 Maple [F]

$$\int \frac{(dex + ce)^2}{\sqrt{a + b \operatorname{arcsinh}(dx + c)}} dx$$

input
```
int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

output
```
int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

3.206.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce + dex)^2}{\sqrt{a + b\text{arcsinh}(c + dx)}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.206.6 Sympy [F]

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = e^2\left(\int \frac{c^2}{\sqrt{a+b\operatorname{asinh}(c+dx)}}\,dx + \int \frac{d^2x^2}{\sqrt{a+b\operatorname{asinh}(c+dx)}}\,dx + \int \frac{2cdx}{\sqrt{a+b\operatorname{asinh}(c+dx)}}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(1/2),x)
```

output
```
e**2*(Integral(c**2/sqrt(a + b*asinh(c + d*x)), x) + Integral(d**2*x**2/sq
rt(a + b*asinh(c + d*x)), x) + Integral(2*c*d*x/sqrt(a + b*asinh(c + d*x))
, x))
```

3.206.7 Maxima [F]

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \int \frac{(dex+ce)^2}{\sqrt{b\operatorname{arsinh}(dx+c)+a}}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.206.8 Giac [F]

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \int \frac{(dex+ce)^2}{\sqrt{b\operatorname{arsinh}(dx+c)+a}}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx=\int \frac{{\left(c\,e+d\,e\,x\right)}^2}{\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}}\,dx$$

input
```
int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(1/2),x)
```

output
```
int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(1/2), x)
```

3.207 $\int \frac{ce+dex}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$

3.207.1 Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \frac{ce+dex}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx=-\frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}+\frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}$$

output
```
-1/8*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*
Pi^(1/2)/d/b^(1/2)+1/8*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*
2^(1/2)*Pi^(1/2)/d/exp(2*a/b)/b^(1/2)
```

3.207.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

$$\int \frac{ce+dex}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \frac{ee^{-\frac{2a}{b}}\left(\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{1}{2},-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)+e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(c+dx)}\Gamma\left(\frac{1}{2},\frac{2(a+b\text{arcsinh}(c+dx}{b}\right.\right.}{4\sqrt{2}d\sqrt{a+b\text{arcsinh}(c+dx)}}$$

input
```
Integrate[(c*e + d*e*x)/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(e*(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c +
d*x]))/b] + E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (2*(a + b*
ArcSinh[c + d*x]))/b]))/(4*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c + d*
x]])
```

3.207.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {6274, 27, 6195, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{\sqrt{a+b\text{arcsinh}(c+dx)}}dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \frac{e(c+dx)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\int \frac{c+dx}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6195}$$

$$\frac{e\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{e\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{ 5971}$$

$$-\frac{e\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{e\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{2bd}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{e\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{2bd}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{ie\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{2bd}$$

$$\Big\downarrow \text{ 3789}$$

$$\frac{ie\left(\frac{1}{2}i\int \frac{e^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int \frac{e^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{2bd}$$

$$\Big\downarrow \text{ 2611}$$

$$\frac{ie\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-i\int e^{\frac{2(a+b\text{arcsinh}(c+dx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}\right)}{2bd}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{ie\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{2bd}$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{ie\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{2bd}$$

3.207. $\int \frac{ce+dex}{\sqrt{a+b\text{arcsinh}(c+dx)}}dx$

input
```
Int[(c*e + d*e*x)/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
((I/2)*e*((I/2)*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*Arc
Sinh[c + d*x]])/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a
+ b*ArcSinh[c + d*x]])/Sqrt[b]])/E^((2*a)/b)))/(b*d)
```

3.207.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.207.4 Maple [F]

$$\int \frac{dex + ce}{\sqrt{a + b \operatorname{arcsinh}(dx + c)}} dx$$

input
```
int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

output
```
int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

3.207.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{ce + dex}{\sqrt{a + b\text{arcsinh}(c + dx)}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.207.6 Sympy [F]

$$\int \frac{ce+dex}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = e\left(\int \frac{c}{\sqrt{a+b\,\text{asinh}\,(c+dx)}}\,dx + \int \frac{dx}{\sqrt{a+b\,\text{asinh}\,(c+dx)}}\,dx\right)$$

input
```
integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(1/2),x)
```

output
```
e*(Integral(c/sqrt(a + b*asinh(c + d*x)), x) + Integral(d*x/sqrt(a + b*asi
nh(c + d*x)), x))
```

3.207.7 Maxima [F]

$$\int \frac{ce+dex}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \int \frac{dex+ce}{\sqrt{b\,\text{arsinh}\,(dx+c)+a}}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.207.8 Giac [F]

$$\int \frac{ce+dex}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \int \frac{dex+ce}{\sqrt{b\,\text{arsinh}\,(dx+c)+a}}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \frac{ce + dex}{\sqrt{a + b\text{arcsinh}(c + dx)}}\, dx = \int \frac{c\,e + d\,e\,x}{\sqrt{a + b\,\mathrm{asinh}\,(c + d\,x)}}\, dx$$

input
```
int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(1/2),x)
```

output
```
int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(1/2), x)
```

3.208 $\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx$

3.208.1 Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}d}$$

output

```
1/2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/
2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

3.208.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(c+dx)}}\,dx = \frac{e^{-\frac{a}{b}}\left(-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)+\sqrt{-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\operatorname{arcsinh}(c+dx)}{b}\right.\right.}{2d\sqrt{a+b\operatorname{arcsinh}(c+dx)}}$$

input
```
Integrate[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
(-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d
*x]]) + Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c
+ d*x])/b)])/(2*d*E^(a/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

3.208.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6273, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int \frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6189}$$

$$\frac{\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{3788}$$

$$\frac{\frac{1}{2}i\int -\frac{ie^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int \frac{ie^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{1}{2}\int \frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))+\frac{1}{2}\int \frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{bd}$$

$$\Big\downarrow \text{2611}$$

$$\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}+\int e^{\frac{a+b\text{arcsinh}(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}}{bd}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{bd}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{bd}$$

input
```
Int[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

output
```
((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/2 +
(Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*E^(a/b)))
/(b*d)
```

3.208.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.208.4 Maple [F]

$$\int \frac{1}{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}dx$$

input
```
int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

output
```
int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

3.208.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\mathrm{arcsinh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.208.6 Sympy [F]

$$\int \frac{1}{\sqrt{a + b\text{arcsinh}(c + dx)}}\,dx = \int \frac{1}{\sqrt{a + b\,\text{asinh}\,(c + dx)}}\,dx$$

input
```
integrate(1/(a+b*asinh(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*asinh(c + d*x)), x)
```

3.208.7 Maxima [F]

$$\int \frac{1}{\sqrt{a + b\text{arcsinh}(c + dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{arsinh}\,(dx + c) + a}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.208.8 Giac [F]

$$\int \frac{1}{\sqrt{a + b\text{arcsinh}(c + dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{arsinh}\,(dx + c) + a}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(a + b*asinh(c + d*x))^(1/2),x)
```

output
```
int(1/(a + b*asinh(c + d*x))^(1/2), x)
```

3.209 $\int \frac{1}{(ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$

3.209.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \frac{\text{Int}\left(\frac{1}{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}},x\right)}{e}$$

output
```
Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^(1/2),x)/e
```

3.209.2 Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx = \int \frac{1}{(ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx$$

input
```
Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]]),x]
```

output
```
Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]]), x]
```

3.209.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 27, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{1}{e(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{de}$$

$$\Big\downarrow \text{6196}$$

$$\frac{\int \frac{1}{(c+dx)\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{de}$$

input
```
Int[1/((c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]]),x]
```

output
```
$Aborted
```

3.209.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.209.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(dex+ce)\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}dx$$

input
```
int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

output
```
int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

3.209.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ce+dex)\sqrt{a+b\mathrm{arcsinh}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.209.6 Sympy [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{1}{(ce+dex)\sqrt{a+b\mathrm{arcsinh}(c+dx)}}\,dx = \frac{\int \frac{1}{c\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+dx\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx}{e}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(1/2),x)
```

output
```
Integral(1/(c*sqrt(a + b*asinh(c + d*x)) + d*x*sqrt(a + b*asinh(c + d*x)))
, x)/e
```

3.209.7 Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx=\int \frac{1}{(dex+ce)\sqrt{b\,\text{arsinh}\,(dx+c)+a}}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a)), x)
```

3.209.8 Giac [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx=\int \frac{1}{(dex+ce)\sqrt{b\,\text{arsinh}\,(dx+c)+a}}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a)), x)
```

3.209.9 Mupad [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)\sqrt{a+b\text{arcsinh}(c+dx)}}\,dx=\int \frac{1}{(c\,e+d\,e\,x)\,\sqrt{a+b\,\mathrm{asinh}\,(c+d\,x)}}\,dx$$

input
```
int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(1/2)),x)
```

output
```
int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(1/2)), x)
```

3.210 $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$

3.210.1 Optimal result

Integrand size = 25, antiderivative size = 367

$$\begin{aligned}\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = &-\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}}\\
&-\frac{e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d}+\frac{3e^4e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}\\
&-\frac{e^4e^{\frac{5a}{b}}\sqrt{5\pi}\,\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+\frac{e^4e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d}\\
&-\frac{3e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+\frac{e^4e^{-\frac{5a}{b}}\sqrt{5\pi}\,\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}\end{aligned}$$

output
```
-1/8*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)
/d+1/8*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp
(a/b)+3/16*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*
3^(1/2)*Pi^(1/2)/b^(3/2)/d-3/16*e^4*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2
)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(3*a/b)-1/16*e^4*exp(5*a/b)*erf(5
^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/d+1/16
*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(
3/2)/d/exp(5*a/b)-2*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x
+c))^(1/2)
```

3.210.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.34

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \frac{e^4 e^{-5\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)}\left(-e^{\frac{5a}{b}}+3e^{\frac{5a}{b}+2\text{arcsinh}(c+dx)}-2e^{\frac{5a}{b}+4\text{arcsinh}(c+dx)}-2e^{\frac{5}{}}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
(e^4*(-E^((5*a)/b) + 3*E^((5*a)/b + 2*ArcSinh[c + d*x]) - 2*E^((5*a)/b + 4
*ArcSinh[c + d*x]) - 2*E^((5*a)/b + 6*ArcSinh[c + d*x]) + 3*E^((5*a)/b + 8
*ArcSinh[c + d*x]) - E^((5*a)/b + 10*ArcSinh[c + d*x]) + 2*E^((6*a)/b + 5*
ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c
+ d*x]] + Sqrt[5]*E^(5*ArcSinh[c + d*x])*Sqrt[-((a + b*ArcSinh[c + d*x])/b
)]*Gamma[1/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 3*Sqrt[3]*E^((2*a)/b + 5*
ArcSinh[c + d*x])*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-3*(a +
b*ArcSinh[c + d*x]))/b] + 2*E^((4*a)/b + 5*ArcSinh[c + d*x])*Sqrt[-((a + b
*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] - 3*Sqrt[
3]*E^((8*a)/b + 5*ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2
, (3*(a + b*ArcSinh[c + d*x]))/b] + Sqrt[5]*E^(5*((2*a)/b + ArcSinh[c + d*
x]))*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c + d*x]))/
b]))/(16*b*d*E^(5*(a/b + ArcSinh[c + d*x]))*Sqrt[a + b*ArcSinh[c + d*x]])
```

3.210.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6274, 27, 6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{e^4(c+dx)^4}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^4\int \frac{(c+dx)^4}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$\downarrow$ 6193

$$\frac{e^4\left(\frac{2\int \left(-\frac{5\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{9\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{8\sqrt{a+b\text{arcsinh}(c+dx)}}\right)d(a+b\text{arcsinh}(c+dx))}{b^2}\right.}{d}$$

$\downarrow$ 2009

$$\frac{e^4\left(\frac{2\left(-\frac{1}{16}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{3}{32}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{5\pi}\sqrt{b}e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right.\right.}{}\right.}{}$$

input
```
Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
(e^4*((-2*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*Sqrt[a + b*ArcSinh[c + d*x
]]) + (2*(-1/16*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]]) + (3*Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*Arc
Sinh[c + d*x]])/Sqrt[b]])/32 - (Sqrt[b]*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5
]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/32 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt
[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*E^(a/b)) - (3*Sqrt[b]*Sqrt[3*Pi]*Er
fi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*E^((3*a)/b)) + (Sq
rt[b]*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32
*E^((5*a)/b))))/b^2))/d
```

3.210.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.210. $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.210.4 Maple [F]

$$\int \frac{(dex+ce)^4}{(a+b\operatorname{arcsinh}(dx+c))^{\frac{3}{2}}}dx$$

input
```
int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x)
```

output
```
int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x)
```

3.210.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.210.6 Sympy [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = e^4\left(\int \frac{c^4}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\,dx\right.$$
$$+\int \frac{d^4x^4}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\,dx$$
$$+\int \frac{4cd^3x^3}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\,dx$$
$$+\int \frac{6c^2d^2x^2}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\,dx$$
$$\left.+\int \frac{4c^3dx}{a\sqrt{a+b\operatorname{asinh}(c+dx)}+b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(3/2),x)
```

output
```
e**4*(Integral(c**4/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c +
 d*x))*asinh(c + d*x)), x) + Integral(d**4*x**4/(a*sqrt(a + b*asinh(c + d*
x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(4*c*d**3
*x**3/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c
 + d*x)), x) + Integral(6*c**2*d**2*x**2/(a*sqrt(a + b*asinh(c + d*x)) + b
*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(4*c**3*d*x/(a*s
qrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)),
x))
```

3.210.7 Maxima [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{(dex+ce)^4}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.210.8 Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx=\int \frac{(dex+ce)^4}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx=\int \frac{(c\,e+d\,e\,x)^4}{(a+b\operatorname{asinh}(c+d\,x))^{3/2}}\,dx$$

input
```
int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(3/2),x)
```

output
```
int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(3/2), x)
```

3.211 $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$

3.211.1 Optimal result

Integrand size = 25, antiderivative size = 262

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d} + \frac{e^3e^{-\frac{4a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{e^3e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d}$$

output
```
-1/4*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2
)*Pi^(1/2)/b^(3/2)/d-1/4*e^3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*2^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(2*a/b)+1/4*e^3*exp(4*a/b)*erf(2*(a+b*ar
csinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d+1/4*e^3*erfi(2*(a+b*arcsin
h(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(4*a/b)-2*e^3*(d*x+c)^3*(1+
(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(1/2)
```

3.211.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.97

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \frac{e^3e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{1}{2},-\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)-\sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\ldots}{}}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
(e^3*(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c
+ d*x]))/b] - Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamm
a[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b] - E^((4*a)/b)*(-(Sqrt[2]*E^((2*a)/
b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c + d*x]))/b]
) + E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (4*(a + b*ArcSinh[
c + d*x]))/b] - 2*Sinh[2*ArcSinh[c + d*x]] + Sinh[4*ArcSinh[c + d*x]])))/(
4*b*d*E^((4*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

3.211.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6274, 27, 6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{e^3(c+dx)^3}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^3\int \frac{(c+dx)^3}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6193}$$

$$\frac{e^3\left(\frac{2\int\left(\frac{\cosh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(c+dx)}}\right)d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)^3\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^3\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\right.}{b^2}}{d}$$

input
```
Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
(e^3*((-2*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*Sqrt[a + b*ArcSinh[c + d*x
]]) + (2*((Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]
])/Sqrt[b]])/8 - (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*A
rcSinh[c + d*x]])/Sqrt[b]])/4 + (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(8*E^((4*a)/b)) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2
]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(4*E^((2*a)/b))))/b^2))/d
```

3.211.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.211.4 Maple [F]

$$\int \frac{(dex + ce)^3}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{3}{2}}} dx$$

input
```
int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x)
```

output
```
int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x)
```

3.211.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce + dex)^3}{(a + b\operatorname{arcsinh}(c + dx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.211.6 Sympy [F]

$$\int \frac{(ce + dex)^3}{(a + b\operatorname{arcsinh}(c + dx))^{3/2}} dx = e^3 \left(\int \frac{c^3}{a\sqrt{a + b \operatorname{asinh}(c + dx)} + b\sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx)} dx + \int \frac{d^3x^3}{a\sqrt{a + b \operatorname{asinh}(c + dx)} + b\sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx)} dx + \int \frac{3cd^2x^2}{a\sqrt{a + b \operatorname{asinh}(c + dx)} + b\sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx)} dx + \int \frac{3c^2dx}{a\sqrt{a + b \operatorname{asinh}(c + dx)} + b\sqrt{a + b \operatorname{asinh}(c + dx)} \operatorname{asinh}(c + dx)} dx \right)$$

input
```
integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(3/2),x)
```

output
```
e**3*(Integral(c**3/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c +
 d*x))*asinh(c + d*x)), x) + Integral(d**3*x**3/(a*sqrt(a + b*asinh(c + d*
x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(3*c*d**2
*x**2/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c
 + d*x)), x) + Integral(3*c**2*d*x/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(
a + b*asinh(c + d*x))*asinh(c + d*x)), x))
```

3.211.7 Maxima [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.211.8 Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx=\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\mathrm{asinh}\,(c+d\,x))^{3/2}}\,dx$$

input
```
int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(3/2),x)
```

output
```
int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(3/2), x)
```

3.212 $\int \frac{(ce+dex)^2}{(a+b\,\text{arcsinh}(c+dx))^{3/2}}\,dx$

3.212.1 Optimal result

Integrand size = 25, antiderivative size = 255

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{e^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{e^2e^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} - \frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} + \frac{e^2e^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}$$

output
```
1/4*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/
d-1/4*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(
a/b)-1/4*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^
(1/2)*Pi^(1/2)/b^(3/2)/d+1/4*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b
^(1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(3*a/b)-2*e^2*(d*x+c)^2*(1+(d*x+c)^2
)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(1/2)
```

3.212.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.28

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \frac{e^2e^{-3\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)}\left(-e^{\frac{3a}{b}}+e^{\frac{3a}{b}+2\text{arcsinh}(c+dx)}+e^{\frac{3a}{b}+4\text{arcsinh}(c+dx)}-e^{\frac{3a}{b}+6}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
(e^2*(-E^((3*a)/b) + E^((3*a)/b + 2*ArcSinh[c + d*x]) + E^((3*a)/b + 4*Arc
Sinh[c + d*x]) - E^((3*a)/b + 6*ArcSinh[c + d*x]) - E^((4*a)/b + 3*ArcSinh
[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*x]]
 + Sqrt[3]*E^(3*ArcSinh[c + d*x])*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamm
a[1/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - E^((2*a)/b + 3*ArcSinh[c + d*x])
*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c + d*x])
/b)] + Sqrt[3]*E^((6*a)/b + 3*ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x
]]*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b]))/(4*b*d*E^(3*(a/b + ArcSinh
[c + d*x]))*Sqrt[a + b*ArcSinh[c + d*x]])
```

3.212.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6274, 27, 6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \frac{e^2(c+dx)^2}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^2\int \frac{(c+dx)^2}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6193}$$

$$\frac{e^2\left(\frac{2\int\left(\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(c+dx)}}\right)d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)^2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{d}$$

$\downarrow$ 2009

$$\frac{e^2\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{}\right.}{b^2}\right.}{d}$$

input
```
Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
(e^2*((-2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*Sqrt[a + b*ArcSinh[c + d*x
]]) + (2*((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[
b]])/8 - (Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c
 + d*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]
]/Sqrt[b]])/(8*E^(a/b)) + (Sqrt[b]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*Arc
Sinh[c + d*x]])/Sqrt[b]])/(8*E^((3*a)/b))))/b^2))/d
```

3.212.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.212.4 Maple [F]

$$\int \frac{(dex+ce)^2}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{3}{2}}}dx$$

input
```
int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x)
```

output
```
int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x)
```

3.212.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^2}{(a+b\mathrm{arcsinh}(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.212.6 Sympy [F]

$$\int \frac{(ce+dex)^2}{(a+b\mathrm{arcsinh}(c+dx))^{3/2}}\,dx = e^2\left(\int \frac{c^2}{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}\,dx + \int \frac{d^2x^2}{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}\,dx + \int \frac{2cdx}{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(3/2),x)
```

output
```
e**2*(Integral(c**2/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c +
 d*x))*asinh(c + d*x)), x) + Integral(d**2*x**2/(a*sqrt(a + b*asinh(c + d*
x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(2*c*d*x/
(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
)), x))
```

3.212.7 Maxima [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx=\int \frac{(dex+ce)^2}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.212.8 Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx=\int \frac{(dex+ce)^2}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx=\int \frac{{\left(c\,e+d\,e\,x\right)}^2}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(3/2),x)
```

output
```
int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(3/2), x)
```

3.213 $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$

3.213.1 Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}$$

output
```
1/2*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(3/2)/d+1/2*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2
^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(2*a/b)-2*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d/(
a+b*arcsinh(d*x+c))^(1/2)
```

3.213.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.99

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \frac{ee^{-\frac{2a}{b}}\left(\sqrt{2}\sqrt{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\Gamma\left(\frac{1}{2},-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right) - \sqrt{2}e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\cdots}\right.}{2bd\sqrt{a+b\text{ar}\cdots}}$$

input
```
Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
(e*(Sqrt[2]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-2*(a + b*ArcS
inh[c + d*x]))/b] - Sqrt[2]*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma
[1/2, (2*(a + b*ArcSinh[c + d*x]))/b] - 2*E^((2*a)/b)*Sinh[2*ArcSinh[c + d
*x]]))/(2*b*d*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

3.213.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6274, 27, 6193, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{e(c+dx)}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int \frac{c+dx}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6193}$$

$$\frac{e\left(\frac{2\int \frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e\left(-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}\right)}{d}$$

$$\Big\downarrow \text{3788}$$

$$\frac{e\left(-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2\left(\frac{1}{2}i\int -\frac{ie^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int \frac{ie^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{b^2}\right)}{d}$$

$$\Big\downarrow \; 26$$

$$\frac{e\left(\frac{2\left(\frac{1}{2}\int \frac{e^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))+\frac{1}{2}\int \frac{e^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{b^2}-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{d}$$

$$\Big\downarrow \; 2611$$

$$\frac{e\left(\frac{2\left(\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}+\int e^{\frac{2(a+b\text{arcsinh}(c+dx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}\right)}{b^2}-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{d}$$

$$\Big\downarrow \; 2633$$

$$\frac{e\left(\frac{2\left(\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{d}$$

$$\Big\downarrow \; 2634$$

$$\frac{e\left(\frac{2\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{d}$$

input
```
Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

output
```
(e*((-2*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*Sqrt[a + b*ArcSinh[c + d*x]])
+ (2*((Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/2 + (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[
c + d*x]])/Sqrt[b]])/(2*E^((2*a)/b))))/b^2))/d
```

3.213. $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$

3.213.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

3.213. $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.213.4 Maple [F]

$$\int \frac{dex + ce}{\left(a + b \operatorname{arcsinh}\left(dx + c\right)\right)^{\frac{3}{2}}} dx$$

input
```
int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
```

output
```
int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
```

3.213.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{ce + dex}{(a + b\text{arcsinh}(c + dx))^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.213.6 Sympy [F]

$$\int \frac{ce + dex}{(a + b\text{arcsinh}(c + dx))^{3/2}} \, dx = e\left(\int \frac{c}{a\sqrt{a + b \operatorname{asinh}\left(c + dx\right)} + b\sqrt{a + b \operatorname{asinh}\left(c + dx\right)} \operatorname{asinh}\left(c + dx\right)} \, dx + \int \frac{dx}{a\sqrt{a + b \operatorname{asinh}\left(c + dx\right)} + b\sqrt{a + b \operatorname{asinh}\left(c + dx\right)} \operatorname{asinh}\left(c + dx\right)} \, dx\right)$$

input
```
integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(3/2),x)
```

output
```
e*(Integral(c/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))
*asinh(c + d*x)), x) + Integral(d*x/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt
(a + b*asinh(c + d*x))*asinh(c + d*x)), x))
```

3.213.7 Maxima [F]

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx=\int \frac{dex+ce}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.213.8 Giac [F]

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx=\int \frac{dex+ce}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx=\int \frac{c\,e+d\,e\,x}{(a+b\,\text{asinh}\,(c+d\,x))^{3/2}}\,dx$$

input
```
int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(3/2),x)
```

output
```
int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(3/2), x)
```

3.214 $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$

3.214.1 Optimal result

Integrand size = 14, antiderivative size = 122

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}$$

output
```
-exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d+erfi(
(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a/b)-2*(1+(d*x+
c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(1/2)
```

3.214.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \frac{e^{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\left(-e^{a/b}\left(1+e^{2\text{arcsinh}(c+dx)}\right)+e^{\frac{2a}{b}+\text{arcsinh}(c+dx)}\sqrt{\frac{a}{b}+\text{arcsinl}\cdots\right.}{b\cdots}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

output
```
(-(E^(a/b)*(1 + E^(2*ArcSinh[c + d*x]))) + E^((2*a)/b + ArcSinh[c + d*x])*
Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*x]] + E^ArcSin
h[c + d*x]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh
[c + d*x])/b)])/(b*d*E^((a + b*ArcSinh[c + d*x])/b)*Sqrt[a + b*ArcSinh[c +
 d*x]])
```

3.214.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6273, 6188, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6188}$$

$$\frac{\frac{2\int \frac{c+dx}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}}{d}$$

$$\Big\downarrow \text{6234}$$

$$\frac{\frac{2\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{2\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{2\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}}{d}$$

$\Big\downarrow$ 26

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}}{d}$$

$\Big\downarrow$ 3789

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\int \frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int \frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{b^2}}{d}$$

$\Big\downarrow$ 2611

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-i\int e^{\frac{a+b\text{arcsinh}(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}\right)}{b^2}}{d}$$

$\Big\downarrow$ 2633

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}}{d}$$

$\Big\downarrow$ 2634

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

3.214. $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$

output
```
((-2*Sqrt[1 + (c + d*x)^2])/(b*Sqrt[a + b*ArcSinh[c + d*x]]) + ((2*I)*((I/
2)*Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - ((
I/2)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/E^(a/b))
)/b^2)/d
```

3.214.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.214. $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.214.4 Maple [F]

$$\int \frac{1}{\left(a+b \,\operatorname{arcsinh}\left(d x +c \right)\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

output
```
int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

3.214.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.214.6 Sympy [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\operatorname{asinh}(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*asinh(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**(-3/2), x)
```

3.214.7 Maxima [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

3.214.8 Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{arsinh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(a + b*asinh(c + d*x))^(3/2),x)
```

output
```
int(1/(a + b*asinh(c + d*x))^(3/2), x)
```

3.215 $\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$

3.215.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx=\frac{\text{Int}\left(\frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}},x\right)}{e}$$

output
```
Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^(3/2),x)/e
```

3.215.2 Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx=\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx$$

input
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2)),x]
```

output
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2)), x]
```

3.215.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 27, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{ 6274}$$
$$\frac{\int \frac{1}{e(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{d}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{de}$$
$$\Big\downarrow \text{ 6196}$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{de}$$

input
```
Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2)),x]
```

output
```
$Aborted
```

3.215.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.215.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(dex + ce)\,(a + b\ \mathrm{arcsinh}\,(dx + c))^{\frac{3}{2}}}dx$$

input
```
int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
```

output
```
int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
```

3.215.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ce + dex)(a + b\mathrm{arcsinh}(c + dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.215.6 Sympy [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.52

$$\int \frac{1}{(ce + dex)(a + b\mathrm{arcsinh}(c + dx))^{3/2}}\,dx = \frac{\int \frac{1}{ac\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+adx\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+bc\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx}}{e}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(3/2),x)
```

output
```
Integral(1/(a*c*sqrt(a + b*asinh(c + d*x)) + a*d*x*sqrt(a + b*asinh(c + d*
x)) + b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b*d*x*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x)), x)/e
```

3.215.7 Maxima [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2)), x)
```

3.215.8 Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx = \int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2)), x)
```

3.215.9 Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{3/2}}\,dx=\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(3/2)),x)
```

output
```
int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(3/2)), x)
```

3.216 $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

3.216.1 Optimal result

Integrand size = 25, antiderivative size = 437

$$\begin{aligned}\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx &= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} \\ &\quad -\frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} \\ &\quad +\frac{e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d}-\frac{3e^4e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d} \\ &\quad +\frac{5e^4e^{\frac{5a}{b}}\sqrt{5\pi}\,\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}+\frac{e^4e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d} \\ &\quad -\frac{3e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d} \\ &\quad +\frac{5e^4e^{-\frac{5a}{b}}\sqrt{5\pi}\,\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}\end{aligned}$$

output
```
1/12*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)
/d+1/12*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/ex
p(a/b)-3/8*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*
3^(1/2)*Pi^(1/2)/b^(5/2)/d-3/8*e^4*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)
/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(3*a/b)+5/24*e^4*exp(5*a/b)*erf(5^
(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(5/2)/d+5/24*
e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(5
/2)/d/exp(5*a/b)-2/3*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*
x+c))^(3/2)-16/3*e^4*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)-20/3*e^4*(
d*x+c)^5/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)
```

3.216.2 Mathematica [A] (verified)

Time = 2.77 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.26

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \frac{e^4\left(-2e^{\text{arcsinh}(c+dx)}(2a+b+2b\text{arcsinh}(c+dx))+e^{-\text{arcsinh}(c+dx)}\left(4a-2b\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
(e^4*(-2*E^ArcSinh[c + d*x]*(2*a + b + 2*b*ArcSinh[c + d*x]) + (4*a - 2*b
+ 4*b*ArcSinh[c + d*x] - 4*E^(a/b + ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c
 + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, a/b + ArcSinh[c + d*x]])/E^Ar
cSinh[c + d*x] + (-(E^(5*(a/b + ArcSinh[c + d*x]))*(10*a + b + 10*b*ArcSin
h[c + d*x])) - 10*Sqrt[5]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/
2, (-5*(a + b*ArcSinh[c + d*x]))/b])/E^((5*a)/b) + (3*E^(3*(a/b + ArcSinh[
c + d*x]))*(6*a + b + 6*b*ArcSinh[c + d*x]) + 18*Sqrt[3]*b*(-((a + b*ArcSi
nh[c + d*x])/b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcSinh[c + d*x]))/b])/E^((3*
a)/b) - (4*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*Arc
Sinh[c + d*x])/b)])/E^(a/b) + (3*(-6*a + b - 6*b*ArcSinh[c + d*x] + 6*Sqrt
[3]*E^(3*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*Arc
Sinh[c + d*x])*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b]))/E^(3*ArcSinh[c
 + d*x]) + (10*a - b + 10*b*ArcSinh[c + d*x] - 10*Sqrt[5]*E^(5*(a/b + ArcS
inh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma
[1/2, (5*(a + b*ArcSinh[c + d*x]))/b])/E^(5*ArcSinh[c + d*x])))/(48*b^2*d*
(a + b*ArcSinh[c + d*x])^(3/2))
```

3.216.3 Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.39, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6274, 27, 6194, 6233, 6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{e^4(c+dx)^4}{(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^4\int \frac{(c+dx)^4}{(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6194}$$

$$\frac{e^4\left(\frac{8\int \frac{(c+dx)^3}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{3b}+\frac{10\int \frac{(c+dx)^5}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2\sqrt{(c+dx)^2+1}(c+dx)^4}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \text{6233}$$

$$\frac{e^4\left(\frac{8\left(\frac{6\int \frac{(c+dx)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)^3}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}+\frac{10\left(\frac{10\int \frac{(c+dx)^4}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)^5}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2}{3b(a}\right.}{d}$$

$$\Big\downarrow \text{6195}$$

$$e^4\left(\frac{10\left(\frac{10\int\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh^4\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)^5}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}+\frac{8\left(\frac{6\int\frac{\cosh\left(\frac{a}{b}-\right.}{}}{}\right.}{}\right.$$

$$\overline{\qquad\qquad d}$$

$\downarrow$ 5971

$$e^4\left(\frac{8\left(\frac{6\int\left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(c+dx)}}\right)d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)^3}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}+\frac{10\left(\frac{10\int\left(\right.}{}\right.}{}\right.$$

$\downarrow$ 2009

$$e^4\left(\frac{8\left(\frac{6\left(-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\right.}{b^2}\right.}{3b}\right.$$

input
```
Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

3.216. $\int\frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

output
```
(e^4*((-2*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(3*b*(a + b*ArcSinh[c + d*x])
^(3/2)) + (8*((-2*(c + d*x)^3)/(b*Sqrt[a + b*ArcSinh[c + d*x]]) + (6*(-1/8
*(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]) + (S
qrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/S
qrt[b]])/8 - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])
/(8*E^(a/b)) + (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*
x]])/Sqrt[b]])/(8*E^((3*a)/b))))/b^2))/(3*b) + (10*((-2*(c + d*x)^5)/(b*Sq
rt[a + b*ArcSinh[c + d*x]]) + (10*((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a +
b*ArcSinh[c + d*x]]/Sqrt[b]])/16 - (Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sq
rt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/32 + (Sqrt[b]*E^((5*a)/b)*Sq
rt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/32 + (Sqrt[b
]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*E^(a/b)) - (Sqr
t[b]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*
E^((3*a)/b)) + (Sqrt[b]*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*
x]])/Sqrt[b]])/(32*E^((5*a)/b))))/b^2))/(3*b)))/d
```

3.216.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

3.216. $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.216.4 Maple [F]

$$\int \frac{(dex + ce)^4}{(a + b\,\mathrm{arcsinh}\,(dx + c))^{\frac{5}{2}}}dx$$

input
```
int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x)
```

output
```
int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x)
```

3.216.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce + dex)^4}{(a + b\mathrm{arcsinh}(c + dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.216.6 Sympy [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = e^4\Bigg(\int \frac{c^4}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx}$$

$$+\int \frac{d^4x^4}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c}$$

$$+\int \frac{4cd^3x^3}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c}$$

$$+\int \frac{6c^2d^2x^2}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c}$$

$$+\int \frac{4c^3dx}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c}$$

input
```
integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(5/2),x)
```

output
```
e**4*(Integral(c**4/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*as
inh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)**2), x) + Integral(d**4*x**4/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*
sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x)
)*asinh(c + d*x)**2), x) + Integral(4*c*d**3*x**3/(a**2*sqrt(a + b*asinh(c
 + d*x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a +
 b*asinh(c + d*x))*asinh(c + d*x)**2), x) + Integral(6*c**2*d**2*x**2/(a**
2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c +
d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2), x) + Integral(4
*c**3*d*x/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c + d*
x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2), x
))
```

3.216.7 Maxima [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{(dex+ce)^4}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.216.8 Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx=\int \frac{(dex+ce)^4}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx=\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\text{asinh}\,(c+d\,x))^{5/2}}\,dx$$

input
```
int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(5/2),x)
```

output
```
int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(5/2), x)
```

3.217 $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

3.217.1 Optimal result

Integrand size = 25, antiderivative size = 326

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{2e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{e^3e^{\frac{2a}{b}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{2e^3e^{-\frac{4a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} - \frac{e^3e^{-\frac{2a}{b}}\sqrt{2\pi}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}$$

output
```
-2/3*e^3*exp(4*a/b)*erf(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(
5/2)/d+2/3*e^3*erfi(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)
/d/exp(4*a/b)+1/3*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^
(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)/d-1/3*e^3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c)
)^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(2*a/b)-2/3*e^3*(d*x+c)^3*(
1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3/2)-4*e^3*(d*x+c)^2/b^2/d/(a
+b*arcsinh(d*x+c))^(1/2)-16/3*e^3*(d*x+c)^4/b^2/d/(a+b*arcsinh(d*x+c))^(1/
2)
```

3.217.2 Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.20

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \frac{e^3e^{-4\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)}\left(-8be^{4\text{arcsinh}(c+dx)}\left(-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{4(a+}{}\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
(e^3*(-8*b*E^(4*ArcSinh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Ga
mma[1/2, (-4*(a + b*ArcSinh[c + d*x]))/b] + 4*Sqrt[2]*b*E^((2*a)/b + 4*Arc
Sinh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-2*(a + b
*ArcSinh[c + d*x]))/b] + (E^((4*a)/b)*(-((-1 + E^(2*ArcSinh[c + d*x]))^2*(
b*(-1 + E^(4*ArcSinh[c + d*x])) + 8*a*(1 + E^(2*ArcSinh[c + d*x]) + E^(4*A
rcSinh[c + d*x])) + 8*b*(1 + E^(2*ArcSinh[c + d*x]) + E^(4*ArcSinh[c + d*x
]))*ArcSinh[c + d*x])) - 8*Sqrt[2]*E^((2*a)/b + 4*ArcSinh[c + d*x])*Sqrt[a
/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (2*(a + b*ArcSi
nh[c + d*x]))/b] + 16*E^(4*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c
+ d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (4*(a + b*ArcSinh[c + d*x]))/b
]))/2))/(12*b^2*d*E^(4*(a/b + ArcSinh[c + d*x]))*(a + b*ArcSinh[c + d*x])^
(3/2))
```

3.217.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.32, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6274, 27, 6194, 6233, 6195, 25, 5971, 27, 2009, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{e^3(c+dx)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e^3\int \frac{(c+dx)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \; 6194$$

$$\frac{e^3\left(\frac{2\int \frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{b}+\frac{8\int \frac{(c+dx)^4}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2\sqrt{(c+dx)^2+1}(c+dx)^3}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \; 6233$$

$$\frac{e^3\left(\frac{2\left(\frac{4\int \frac{c+dx}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{b}+\frac{8\left(\frac{8\int \frac{(c+dx)^3}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)^4}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2\sqrt{}}{3b(a+}\right.}{d}$$

$$\Big\downarrow \; 6195$$

$$\frac{e^3\left(\frac{8\left(\frac{8\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)^4}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}+\frac{2\left(\frac{4\int -\frac{\cosh\left(\frac{a}{b}-\right.}{}}{}\right.}{}\right.}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{e^3\left(\frac{8\left(-\frac{8\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh^3\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)^4}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}+\frac{2\left(-\frac{4\int \frac{\cosh\left(\frac{a}{b}\right.}{}}{}\right.}{}\right.}{d}$$

3.217. $\displaystyle\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \text{5971}$$

$$\frac{e^3\left(\frac{2\left(-\frac{4\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{b}+\frac{8\left(-\frac{8\int\left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{8\sqrt{a+b\text{arcsinh}(c+dx)}}\right.}{\ldots}\right.}{\ldots}\right.}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^3\left(\frac{2\left(-\frac{2\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{b}+\frac{8\left(-\frac{8\int\left(\frac{\sinh\left(\frac{4a}{b}-\frac{4(a+b\text{arcsinh}(c+dx))}{b}\right)}{8\sqrt{a+b\text{arcsinh}(c+dx)}}\right.}{\ldots}\right.}{\ldots}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$e^3\left(\frac{2\left(-\frac{2\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{b}+\frac{8\left(\frac{8\left(-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+\ldots}}{\sqrt{b}}\right.\right.}{\ldots}\right.}{\ldots}\right.$$

$$\Big\downarrow \text{3042}$$

3.217. $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

$$e^3\left(\frac{2\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{2\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}\right)}{b}+\frac{8\left(\frac{8\left(-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}\cdots}}{\sqrt{b}}\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow \ 26$$

$$e^3\left(\frac{2\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}\right)}{b}+\frac{8\left(\frac{8\left(-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c\cdots}}{\sqrt{b}}\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow \ 3789$$

$$e^3\left(\frac{2\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\int \frac{e^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int \frac{e^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{b^2}\right)}{b}+\cdots\right.$$

$$\Big\downarrow \ 2611$$

$$e^3\left(\frac{2\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-i\int e^{\frac{2(a+b\text{arcsinh}(c+dx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}\right)}{b^2}\right)}{b}\cdots\right.$$

$$\Big\downarrow \ 2633$$

3.217. $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

$$e^3\left(\frac{2\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}\right)}{b}+\right.$$

$$\downarrow \text{2634}$$

$$e^3\left(\frac{8\left(\frac{8\left(-\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right.}{b^2}\right.}{3b}\right.$$

input
```
Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
(e^3*((-2*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(3*b*(a + b*ArcSinh[c + d*x])
^(3/2)) + (8*((-2*(c + d*x)^4)/(b*Sqrt[a + b*ArcSinh[c + d*x]]) + (8*(-1/3
2*(Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]]) + (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c +
 d*x]])/Sqrt[b]])/8 + (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x
]])/Sqrt[b]])/(32*E^((4*a)/b)) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a
+ b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*E^((2*a)/b))))/b^2))/(3*b) + (2*((-2*(
c + d*x)^2)/(b*Sqrt[a + b*ArcSinh[c + d*x]]) + ((2*I)*((I/2)*Sqrt[b]*E^((2
*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - ((
I/2)*Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b
]])/E^((2*a)/b)))/b^2))/b))/d
```

3.217. $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

3.217.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.217. $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.217.4 Maple [F]

$$\int \frac{(dex + ce)^3}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{5}{2}}} dx$$

input
```
int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x)
```

output
```
int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x)
```

3.217.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.217.6 Sympy [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = e^3\Bigg(\int \frac{c^3}{a^2\sqrt{a+b\,\text{asinh}\,(c+dx)}+2ab\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx}$$
$$+\int \frac{d^3x^3}{a^2\sqrt{a+b\,\text{asinh}\,(c+dx)}+2ab\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)+b^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}^2\,(c}$$
$$+\int \frac{3cd^2x^2}{a^2\sqrt{a+b\,\text{asinh}\,(c+dx)}+2ab\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)+b^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}^2\,(c}$$
$$+\int \frac{3c^2dx}{a^2\sqrt{a+b\,\text{asinh}\,(c+dx)}+2ab\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)+b^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}^2\,(c}$$

input
```
integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(5/2),x)
```

output
```
e**3*(Integral(c**3/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*as
inh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)**2), x) + Integral(d**3*x**3/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*
sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x)
)*asinh(c + d*x)**2), x) + Integral(3*c*d**2*x**2/(a**2*sqrt(a + b*asinh(c
 + d*x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a +
 b*asinh(c + d*x))*asinh(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2*sqrt
(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) +
 b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2), x))
```

3.217.7 Maxima [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.217.8 Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\text{asinh}\,(c+d\,x))^{5/2}}\,dx$$

input
```
int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(5/2),x)
```

output
```
int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(5/2), x)
```

3.218 $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

3.218.1 Optimal result

Integrand size = 25, antiderivative size = 321

$$\begin{aligned}\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx &= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}}\\ &\quad-\frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\text{arcsinh}(c+dx)}}\\ &\quad-\frac{e^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}+\frac{e^2e^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d}\\ &\quad-\frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}+\frac{e^2e^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d}\end{aligned}$$

output
```
-1/6*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)
/d-1/6*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp
(a/b)+1/2*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3
^(1/2)*Pi^(1/2)/b^(5/2)/d+1/2*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/
b^(1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(3*a/b)-2/3*e^2*(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3/2)-8/3*e^2*(d*x+c)/b^2/d/(a+b*arcsi
nh(d*x+c))^(1/2)-4*e^2*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.21

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \frac{e^2e^{-3\left(\frac{a}{b}+\operatorname{arcsinh}(c+dx)\right)}\left(2e^{\frac{4a}{b}+3\operatorname{arcsinh}(c+dx)}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(c+dx)}(a+b\operatorname{arcsin}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
(e^2*(2*E^((4*a)/b + 3*ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a +
 b*ArcSinh[c + d*x])*Gamma[1/2, a/b + ArcSinh[c + d*x]] - 6*Sqrt[3]*b*E^(3
*ArcSinh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-3*(a
 + b*ArcSinh[c + d*x]))/b] + 2*b*E^((2*a)/b + 3*ArcSinh[c + d*x])*(-((a +
b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] -
E^((3*a)/b)*((-1 + E^(2*ArcSinh[c + d*x]))*(b*(-1 + E^(4*ArcSinh[c + d*x])
) + a*(6 + 4*E^(2*ArcSinh[c + d*x]) + 6*E^(4*ArcSinh[c + d*x])) + 2*b*(3 +
 2*E^(2*ArcSinh[c + d*x]) + 3*E^(4*ArcSinh[c + d*x]))*ArcSinh[c + d*x]) +
6*Sqrt[3]*E^(3*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]*(a +
 b*ArcSinh[c + d*x])*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b])))/(12*b^2
*d*E^(3*(a/b + ArcSinh[c + d*x]))*(a + b*ArcSinh[c + d*x])^(3/2))
```

3.218.3 Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.24, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {6274, 27, 6194, 6233, 6189, 3042, 3788, 26, 2611, 2633, 2634, 6195, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \frac{e^2(c+dx)^2}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e^2 \int \frac{(c+dx)^2}{(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$\downarrow$ 6194

$$\frac{e^2\left(\frac{4\int \frac{c+dx}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{3b}+\frac{2\int \frac{(c+dx)^3}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{b}-\frac{2\sqrt{(c+dx)^2+1}(c+dx)^2}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{d}$$

$\downarrow$ 6233

$$\frac{e^2\left(\frac{4\left(\frac{2\int\frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}+\frac{2\left(\frac{6\int\frac{(c+dx)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)^3}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{b}-\frac{2\sqrt{\cdots}}{3b(a+\cdots}\cdots\right.}{d}$$

$\downarrow$ 6189

$$\frac{e^2\left(\frac{4\left(\frac{2\int\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}+\frac{2\left(\frac{6\int\frac{(c+dx)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+\cdots}{b\sqrt{a+b\text{arcs}\cdots}}\cdots\right.}{b}\cdots\right.}{d}$$

$\downarrow$ 3042

$$\frac{e^2\left(\frac{4\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2\int\frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}\right)}{3b}+\frac{2\left(\frac{6\int\frac{(c+dx)^2}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{\cdots}{b\sqrt{a\cdots}}\cdots\right.}{b}\cdots\right.}{d}$$

$\downarrow$ 3788

3.218. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

$$e^2\left(\frac{4\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2\left(\frac{1}{2}i\int -\frac{ie^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int \frac{ie^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{b^2}\right)}{3b}+\right.$$

$$d$$

$\downarrow$ 26

$$e^2\left(\frac{4\left(\frac{2\left(\frac{1}{2}\int \frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))+\frac{1}{2}\int \frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}+\frac{2\left(6\right.}{}\right.$$

$$d$$

$\downarrow$ 2611

$$e^2\left(\frac{4\left(\frac{2\left(\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}+\int e^{\frac{a+b\text{arcsinh}(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}\right)}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}+\frac{2\left(6\int\right.}{}\right.$$

$$d$$

$\downarrow$ 2633

$$e^2\left(\frac{4\left(\frac{2\left(\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}+\frac{2\left(6\int\frac{}{\sqrt{a+b}}\right.}{}\right.$$

$$d$$

$\downarrow$ 2634

3.218. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

$$\frac{e^2\left(\frac{2\left(\frac{6\int \frac{(c+dx)^2}{\sqrt{a+b\,\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)^3}{b\sqrt{a+b\,\text{arcsinh}(c+dx)}}\right)}{b}+\frac{4\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\,\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\,\text{ar}\cdots}}{\cdots}\right)\cdots\right)}{b^2}\cdots\right)}{3b}\cdots\right)}{d}$$

$\downarrow$ 6195

$$\frac{e^2\left(\frac{2\left(\frac{6\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\,\text{arcsinh}(c+dx)}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{a+b\,\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\,\text{arcsinh}(c+dx)}}d(a+b\,\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)^3}{b\sqrt{a+b\,\text{arcsinh}(c+dx)}}\right)}{b}+\frac{4\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{d}$$

$\downarrow$ 5971

$$\frac{e^2\left(\frac{2\left(\frac{6\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3(a+b\,\text{arcsinh}(c+dx))}{b}\right)}{4\sqrt{a+b\,\text{arcsinh}(c+dx)}}-\frac{\cosh\left(\frac{a}{b}-\frac{a+b\,\text{arcsinh}(c+dx)}{b}\right)}{4\sqrt{a+b\,\text{arcsinh}(c+dx)}}\right)d(a+b\,\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)^3}{b\sqrt{a+b\,\text{arcsinh}(c+dx)}}\right)}{b}+\frac{4\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{d}$$

$\downarrow$ 2009

$$e^2\left(\frac{4\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\,\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\,\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\,\text{arcsinh}(c+dx)}}\right)}{3b}+\frac{2\left(\frac{6\left(-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)\cdots$$

3.218. $\int \frac{(ce+dex)^2}{(a+b\,\text{arcsinh}(c+dx))^{5/2}}\,dx$

input
```
Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
(e^2*((-2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(3*b*(a + b*ArcSinh[c + d*x])
^(3/2)) + (4*((-2*(c + d*x))/(b*Sqrt[a + b*ArcSinh[c + d*x]]) + (2*((Sqrt[
b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/2 + (Sqrt[b
]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*E^(a/b))))/b^2))
/(3*b) + (2*((-2*(c + d*x)^3)/(b*Sqrt[a + b*ArcSinh[c + d*x]]) + (6*(-1/8*
(Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]) + (Sq
rt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sq
rt[b]])/8 - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/
(8*E^(a/b)) + (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x
]])/Sqrt[b]])/(8*E^((3*a)/b))))/b^2))/b))/d
```

3.218.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

3.218. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

3.218. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.218.4 Maple [F]

$$\int \frac{(dex+ce)^2}{(a+b\operatorname{arcsinh}(dx+c))^{\frac{5}{2}}}dx$$

input
```
int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x)
```

output
```
int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x)
```

3.218.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.218.6 Sympy [F]

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx = e^2\left(\int \frac{c^2}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx\right.$$
$$+\int \frac{d^2x^2}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c}$$
$$+\int \frac{2cdx}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c}$$

input
```
integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(5/2),x)
```

output
```
e**2*(Integral(c**2/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*as
inh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)**2), x) + Integral(d**2*x**2/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*
sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x)
)*asinh(c + d*x)**2), x) + Integral(2*c*d*x/(a**2*sqrt(a + b*asinh(c + d*x
)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x)**2), x))
```

3.218.7 Maxima [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{(dex+ce)^2}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.218.8 Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{(dex+ce)^2}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx=\int \frac{{\left(c\,e+d\,e\,x\right)}^2}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(5/2),x)
```

output
```
int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(5/2), x)
```

3.219 $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

3.219.1 Optimal result

Integrand size = 23, antiderivative size = 209

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{2ee^{\frac{2a}{b}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{2ee^{-\frac{2a}{b}}\sqrt{2\pi}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}$$

output
```
-2/3*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*
Pi^(1/2)/b^(5/2)/d+2/3*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*
2^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(2*a/b)-2/3*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/
d/(a+b*arcsinh(d*x+c))^(3/2)-4/3*e/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)-8/3*e*
(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)
```

3.219.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.09

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \frac{ee^{-2\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)}\left(-4\sqrt{2}be^{2\text{arcsinh}(c+dx)}\left(-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-2\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
(e*(-4*Sqrt[2]*b*E^(2*ArcSinh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3
/2)*Gamma[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((2*a)/b)*(-4*a + b -
4*a*E^(4*ArcSinh[c + d*x]) - b*E^(4*ArcSinh[c + d*x]) - 4*b*(1 + E^(4*ArcS
inh[c + d*x]))*ArcSinh[c + d*x] + 4*Sqrt[2]*E^(2*(a/b + ArcSinh[c + d*x]))
*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (2*(a +
b*ArcSinh[c + d*x]))/b])))/(6*b^2*d*E^(2*(a/b + ArcSinh[c + d*x]))*(a + b*
ArcSinh[c + d*x])^(3/2))
```

3.219.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.03, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {6274, 27, 6194, 6198, 6233, 6195, 25, 5971, 27, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{e(c+dx)}{(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int \frac{c+dx}{(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6194}$$

$$\frac{e\left(\frac{2\int \frac{1}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{3b}+\frac{4\int \frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2\sqrt{(c+dx)^2+1}(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \text{6198}$$

$$\frac{e\left(\frac{4\int \frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{4}{3b^2\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{2\sqrt{(c+dx)^2+1}(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \text{6233}$$

$$\frac{e\left(\frac{4\left(\frac{4\int \frac{c+dx}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{4}{3b^2\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{2\sqrt{(c+dx)^2+1}(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \text{6195}$$

$$\frac{e\left(\frac{4\left(\frac{4\int -\frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{4}{3b^2\sqrt{a+b\text{arcsinh}}\dots}\right.}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{e\left(\frac{4\left(-\frac{4\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{4}{3b^2\sqrt{a+b\text{arcsinh}}\dots}\right.}{d}$$

3.219. $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \text{5971}$$

$$\frac{e\left(\frac{4\left(-\frac{4\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{2\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{4}{3b^2\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{2\sqrt{(c+dx)^2}}{3b(a+b\text{arcsin}}\right.}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\left(\frac{4\left(-\frac{2\int \frac{\sinh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{4}{3b^2\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{2\sqrt{(c+dx)^2}}{3b(a+b\text{arcsin}}\right.}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e\left(\frac{4\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{2\int -\frac{i\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}\right)}{3b}-\frac{4}{3b^2\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{2\sqrt{(c+d}}{3b(a+b\text{arcs}}\right.}{d}$$

$$\Big\downarrow \text{26}$$

3.219. $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

$$\frac{e\left(\frac{4\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\int\frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}\right)}{3b}-\frac{4}{3b^2\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{2\sqrt{(c+dx)}}{3b(a+b\text{arcsi}}\right.}{d}$$

$\downarrow$ 3789

$$\frac{e\left(\frac{4\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\int\frac{e^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int\frac{e^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{b^2}\right)}{3b}-\frac{}{3b}\right.}{d}$$

$\downarrow$ 2611

$$\frac{e\left(\frac{4\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-i\int e^{\frac{2(a+b\text{arcsinh}(c+dx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}\right)}{b^2}\right)}{3b}\right.}{d}$$

$\downarrow$ 2633

$$\frac{e\left(\frac{4\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}\right)}{3b}-\frac{}{3}\right.}{d}$$

$\downarrow$ 2634

3.219. $\int\frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

$$e\left(\frac{4\left(-\frac{2(c+dx)^2}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}\right)}{3b}-\frac{}{3b^2\sqrt{a-}}\right.$$
$$\overline{\qquad d \qquad}$$

input
```
Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

output
```
(e*((-2*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b*(a + b*ArcSinh[c + d*x])^(3/
2)) - 4/(3*b^2*Sqrt[a + b*ArcSinh[c + d*x]]) + (4*((-2*(c + d*x)^2)/(b*Sqr
t[a + b*ArcSinh[c + d*x]]) + ((2*I)*((I/2)*Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*
Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[
Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/E^((2*a)/b)))/
b^2))/(3*b)))/d
```

3.219.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

3.219. $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.219.4 Maple [F]

$$\int \frac{dex + ce}{(a + b\,\text{arcsinh}\,(dx + c))^{\frac{5}{2}}}dx$$

input
```
int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

output
```
int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

3.219.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{ce + dex}{(a + b\text{arcsinh}(c + dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.219.6 Sympy [F]

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = e\left(\int \frac{c}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\right.$$
$$+\int \frac{dx}{a^2\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c}$$

input
```
integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(5/2),x)
```

output
```
e*(Integral(c/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c
+ d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2
), x) + Integral(d*x/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*a
sinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c +
d*x)**2), x))
```

3.219.7 Maxima [F]

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{dex+ce}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.219.8 Giac [F]

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{dex+ce}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx=\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

input
```
int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(5/2),x)
```

output
```
int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(5/2), x)
```

3.220 $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

3.220.1 Optimal result

Integrand size = 14, antiderivative size = 158

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{1+(c+dx)^2}}{3bd(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d} + \frac{2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}$$

output
```
2/3*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d+2/
3*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(a/b)-2/3
*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3/2)-4/3*(d*x+c)/b^2/d/(a+b
*arcsinh(d*x+c))^(1/2)
```

3.220.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.31

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \frac{e^{-\frac{a+b\text{arcsinh}(c+dx)}{b}}\left(-e^{a/b}\left(b+2a\left(-1+e^{2\text{arcsinh}(c+dx)}\right)-2b\text{arcsinh}(c+dx)\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

output
```
(-(E^(a/b)*(b + 2*a*(-1 + E^(2*ArcSinh[c + d*x])) - 2*b*ArcSinh[c + d*x] +
 b*E^(2*ArcSinh[c + d*x])*(1 + 2*ArcSinh[c + d*x]))) - 2*E^((2*a)/b + ArcS
inh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[
1/2, a/b + ArcSinh[c + d*x]] - 2*b*E^ArcSinh[c + d*x]*(-((a + b*ArcSinh[c
+ d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)])/(3*b^2*d*E^((
a + b*ArcSinh[c + d*x])/b)*(a + b*ArcSinh[c + d*x])^(3/2))
```

3.220.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6273, 6188, 6233, 6189, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6188}$$

$$\frac{\frac{2\int \frac{c+dx}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{6233}$$

$$\frac{\frac{2\left(\frac{2\int \frac{1}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}}{d}$$

$\Big\downarrow$ 6189

$$\frac{\frac{2\left(\frac{2\int \frac{\cosh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}}{d}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}\right)}{3b}}{d}$$

$\Big\downarrow$ 3788

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2\left(\frac{1}{2}i\int -\frac{ie^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int \frac{ie^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d \cdots\right.}{b^2}\right.}{3b}}{d}$$

$\Big\downarrow$ 26

$$\frac{\frac{2\left(\frac{2\left(\frac{1}{2}\int \frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))+\frac{1}{2}\int \frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2\sqrt{\cdots}}{3b(a+b\text{ar}\cdots}}{d}$$

$\Big\downarrow$ 2611

$$\frac{\frac{2\left(\frac{2\left(\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}+\int e^{\frac{a+b\text{arcsinh}(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}\right)}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2\sqrt{(c\cdots}}{3b(a+b\text{arcs}\cdots}}{d}$$

3.220. $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \ 2633$$

$$\frac{\frac{2\left(\frac{2\left(\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \ 2634$$

$$\frac{\frac{2\left(\frac{2\left(\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2(c+dx)}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2\sqrt{(c+dx)^2+1}}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

output
```
((-2*Sqrt[1 + (c + d*x)^2])/(3*b*(a + b*ArcSinh[c + d*x])^(3/2)) + (2*((-2
*(c + d*x))/(b*Sqrt[a + b*ArcSinh[c + d*x]]) + (2*((Sqrt[b]*E^(a/b)*Sqrt[P
i]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/2 + (Sqrt[b]*Sqrt[Pi]*Erfi[S
qrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*E^(a/b))))/b^2))/(3*b))/d
```

3.220.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

3.220. $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6189
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/(b*c)   S
ubst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.220. $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}} dx$

3.220.4 Maple [F]

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
```

output
```
int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
```

3.220.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\mathrm{arcsinh}(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.220.6 Sympy [F]

$$\int \frac{1}{(a+b\mathrm{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*asinh(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**(-5/2), x)
```

3.220.7 Maxima [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

3.220.8 Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c+d\,x))^{5/2}}\,dx$$

input
```
int(1/(a + b*asinh(c + d*x))^(5/2),x)
```

output
```
int(1/(a + b*asinh(c + d*x))^(5/2), x)
```

3.221 $\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$

3.221.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx=\frac{\text{Int}\left(\frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}},x\right)}{e}$$

output
```
Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^(5/2),x)/e
```

3.221.2 Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx=\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx$$

input
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2)),x]
```

output
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2)), x]
```

3.221.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 27, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{1}{e(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{de}$$

$$\Big\downarrow \text{6196}$$

$$\frac{\int \frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{de}$$

input
```
Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2)),x]
```

output
```
$Aborted
```

3.221.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.221.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(dex + ce)\left(a + b\operatorname{arcsinh}\left(dx + c\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

output
```
int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

3.221.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ce + dex)(a + b\text{arcsinh}(c + dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.221.6 Sympy [N/A]

Not integrable

Time = 8.74 (sec) , antiderivative size = 155, normalized size of antiderivative = 6.20

$$\int \frac{1}{(ce + dex)(a + b\text{arcsinh}(c + dx))^{5/2}}\,dx = \int \frac{}{a^2c\sqrt{a+b\operatorname{asinh}(c+dx)}+a^2dx\sqrt{a+b\operatorname{asinh}(c+dx)}+2abc\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(5/2),x)
```

output
```
Integral(1/(a**2*c*sqrt(a + b*asinh(c + d*x)) + a**2*d*x*sqrt(a + b*asinh(
c + d*x)) + 2*a*b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + 2*a*b*d*x*
sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*c*sqrt(a + b*asinh(c + d*
x))*asinh(c + d*x)**2 + b**2*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)
**2), x)/e
```

3.221.7 Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx=\int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2)), x)
```

3.221.8 Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{5/2}}\,dx=\int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2)), x)
```

3.221.9 Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{5/2}}\,dx=\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(5/2)),x)
```

output
```
int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(5/2)), x)
```

3.222 $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

3.222.1 Optimal result

Integrand size = 25, antiderivative size = 531

$$\begin{aligned}
\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx &= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} \\
&- \frac{16e^4(c+dx)^3}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{4e^4(c+dx)^5}{3b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} \\
&- \frac{32e^4(c+dx)^2\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{40e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} \\
&- \frac{e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d} + \frac{9e^4e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d} \\
&- \frac{5e^4e^{\frac{5a}{b}}\sqrt{5\pi}\,\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d} + \frac{e^4e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d} \\
&- \frac{9e^4e^{-\frac{3a}{b}}\sqrt{3\pi}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d} \\
&+ \frac{5e^4e^{-\frac{5a}{b}}\sqrt{5\pi}\,\text{erfi}\left(\frac{\sqrt{5}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d}
\end{aligned}$$

output
```
-16/15*e^4*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-4/3*e^4*(d*x+c)^5/b^
2/d/(a+b*arcsinh(d*x+c))^(3/2)-1/30*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^
(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+1/30*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)
/b^(1/2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)+9/20*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b
*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d-9/20*e^4*erfi(3
^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(
3*a/b)-5/12*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))
*5^(1/2)*Pi^(1/2)/b^(7/2)/d+5/12*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/
2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(5*a/b)-2/5*e^4*(d*x+c)^4*(1+(d*
x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(5/2)-32/5*e^4*(d*x+c)^2*(1+(d*x+c)
^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)-40/3*e^4*(d*x+c)^4*(1+(d*x+c)^2
)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)
```

3.222.2 Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.32

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \frac{e^4\left(-6b^2e^{\text{arcsinh}(c+dx)}-3b^2e^{5\text{arcsinh}(c+dx)}+e^{-\text{arcsinh}(c+dx)}\left(-8a^2+4ab-6b\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

output
```
(e^4*(-6*b^2*E^ArcSinh[c + d*x] - 3*b^2*E^(5*ArcSinh[c + d*x]) + (-8*a^2 +
 4*a*b - 6*b^2 - 4*(4*a - b)*b*ArcSinh[c + d*x] - 8*b^2*ArcSinh[c + d*x]^2
 + 8*E^(a/b + ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSin
h[c + d*x])^2*Gamma[1/2, a/b + ArcSinh[c + d*x]])/E^ArcSinh[c + d*x] - (10
*(a + b*ArcSinh[c + d*x])*(E^(5*(a/b + ArcSinh[c + d*x]))*(10*a + b + 10*b
*ArcSinh[c + d*x]) + 10*Sqrt[5]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Ga
mma[1/2, (-5*(a + b*ArcSinh[c + d*x]))/b]))/E^((5*a)/b) + 9*(b^2*E^(3*ArcS
inh[c + d*x]) + (2*(a + b*ArcSinh[c + d*x])*(E^(3*(a/b + ArcSinh[c + d*x])
)*(6*a + b + 6*b*ArcSinh[c + d*x]) + 6*Sqrt[3]*b*(-((a + b*ArcSinh[c + d*x
])/b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcSinh[c + d*x]))/b]))/E^((3*a)/b)) -
(4*(a + b*ArcSinh[c + d*x])*(E^(a/b + ArcSinh[c + d*x])*(2*a + b + 2*b*Arc
Sinh[c + d*x]) + 2*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a
 + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + (9*(b^2 + 2*(a + b*ArcSinh[c + d*x]
)*(6*a - b + 6*b*ArcSinh[c + d*x] - 6*Sqrt[3]*E^(3*(a/b + ArcSinh[c + d*x]
))*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (3*(a
+ b*ArcSinh[c + d*x]))/b])))/E^(3*ArcSinh[c + d*x]) - (3*b^2 + 10*(a + b*A
rcSinh[c + d*x])*(10*a - b + 10*b*ArcSinh[c + d*x] - 10*Sqrt[5]*E^(5*(a/b
+ ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])
*Gamma[1/2, (5*(a + b*ArcSinh[c + d*x]))/b]))/E^(5*ArcSinh[c + d*x])))/(24
0*b^3*d*(a + b*ArcSinh[c + d*x])^(5/2))
```

3.222.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6274, 27, 6194, 6233, 6193, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \frac{e^4(c+dx)^4}{(a+b\text{arcsinh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^4\int \frac{(c+dx)^4}{(a+b\text{arcsinh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \ 6194$$

$$\frac{e^4\left(\frac{8\int \frac{(c+dx)^3}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{5b}+\frac{2\int \frac{(c+dx)^5}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{b}-\frac{2\sqrt{(c+dx)^2+1}(c+dx)^4}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}\right)}{d}$$

$$\Big\downarrow \ 6233$$

$$\frac{e^4\left(\frac{8\left(\frac{2\int \frac{(c+dx)^2}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{b}-\frac{2(c+dx)^3}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}+\frac{2\left(\frac{10\int \frac{(c+dx)^4}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2(c+dx)^5}{3b(a+b\text{arcsinh}(c+dx))^3}\right.}{b}}{d}$$

$$\Big\downarrow \ 6193$$

$$e^4\left(\frac{2\left(\frac{10\left(\frac{2\int\left(-\frac{5\sinh\left(\frac{5a}{b}-\frac{5(a+b\text{arcsinh}(c+dx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{9\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{16\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{8\sqrt{a+b\text{arcsinh}(c+dx)}}\right)d(a+b\text{arcsinh}(c+dx}{b^2}\right.}{3b}\right.}{b}\right.$$

$$\Big\downarrow \ 2009$$

3.222. $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

$$e^4\left(\frac{8\left(\frac{2\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{3\pi}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{8}\right.}{b^2}\right.}{b}\right.}{5b}\right.$$

input
```
Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

output
```
(e^4*((-2*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(5*b*(a + b*ArcSinh[c + d*x])
^(5/2)) + (8*((-2*(c + d*x)^3)/(3*b*(a + b*ArcSinh[c + d*x])^(3/2)) + (2*(
(-2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*Sqrt[a + b*ArcSinh[c + d*x]]) +
(2*((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/8
 - (Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x
]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt
[b]])/(8*E^(a/b)) + (Sqrt[b]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c
 + d*x]])/Sqrt[b]])/(8*E^((3*a)/b))))/b^2))/b))/(5*b) + (2*((-2*(c + d*x)^
5)/(3*b*(a + b*ArcSinh[c + d*x])^(3/2)) + (10*((-2*(c + d*x)^4*Sqrt[1 + (c
 + d*x)^2])/(b*Sqrt[a + b*ArcSinh[c + d*x]]) + (2*(-1/16*(Sqrt[b]*E^(a/b)*
Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]) + (3*Sqrt[b]*E^((3*a)/
b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/32 - (S
qrt[b]*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/S
qrt[b]])/32 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]
)/(16*E^(a/b)) - (3*Sqrt[b]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c
+ d*x]])/Sqrt[b]])/(32*E^((3*a)/b)) + (Sqrt[b]*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sq
rt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*E^((5*a)/b))))/b^2))/(3*b)))/b))
/d
```

3.222. $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

3.222.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.222. $\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

3.222.4 Maple [F]

$$\int \frac{(dex+ce)^4}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{7}{2}}}dx$$

input
```
int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x)
```

output
```
int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x)
```

3.222.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^4}{(a+b\mathrm{arcsinh}(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.222.6 Sympy [F]

$$\int \frac{(ce+dex)^4}{(a+b\mathrm{arcsinh}(c+dx))^{7/2}}\,dx = e^4\left(\int \frac{}{a^3\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+d} \right.$$
$$+\int \frac{d^4x^4}{a^3\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)+3ab^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}}$$
$$+\int \frac{4cd^3x^3}{a^3\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)+3ab^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}}$$
$$+\int \frac{6c^2d^2x^2}{a^3\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)+3ab^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}}$$
$$+\int \frac{4c^3dx}{a^3\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)+3ab^2\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}}$$

input
```
integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(7/2),x)
```

output
```
e**4*(Integral(c**4/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b
*asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asin
h(c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + I
ntegral(d**4*x**4/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*a
sinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(
c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Int
egral(4*c*d**3*x**3/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b
*asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asin
h(c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + I
ntegral(6*c**2*d**2*x**2/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(
a + b*asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))
*asinh(c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x
) + Integral(4*c**3*d*x/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a
 + b*asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*
asinh(c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x)
)
```

3.222.7 Maxima [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{(dex+ce)^4}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.222.8 Giac [F]

$$\int \frac{(ce+dex)^4}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{(dex+ce)^4}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce + dex)^4}{(a + b\text{arcsinh}(c + dx))^{7/2}}\,dx = \int \frac{(c\,e + d\,e\,x)^4}{(a + b\,\text{asinh}\,(c + d\,x))^{7/2}}\,dx$$

input
```
int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(7/2),x)
```

output
```
int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(7/2), x)
```

3.223 $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

3.223.1 Optimal result

Integrand size = 25, antiderivative size = 420

$$\begin{aligned}\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx =& -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} \\ &-\frac{4e^3(c+dx)^2}{5b^2d(a+b\text{arcsinh}(c+dx))^{3/2}}-\frac{16e^3(c+dx)^4}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} \\ &-\frac{16e^3(c+dx)\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{128e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} \\ &+\frac{16e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4e^3e^{\frac{2a}{b}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} \\ &+\frac{16e^3e^{-\frac{4a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{2\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4e^3e^{-\frac{2a}{b}}\sqrt{2\pi}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}\end{aligned}$$

output
```
-4/5*e^3*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-16/15*e^3*(d*x+c)^4/b^
2/d/(a+b*arcsinh(d*x+c))^(3/2)+16/15*e^3*exp(4*a/b)*erf(2*(a+b*arcsinh(d*x
+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+16/15*e^3*erfi(2*(a+b*arcsinh(d*x+c
))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d/exp(4*a/b)-4/15*e^3*exp(2*a/b)*erf(2^
(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d-4/15*
e^3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7
/2)/d/exp(2*a/b)-2/5*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*
x+c))^(5/2)-16/5*e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c)
)^(1/2)-128/15*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c)
)^(1/2)
```

3.223.2 Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.02

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \frac{e^3\left(4(a+b\text{arcsinh}(c+dx))\left(-4ae^{-2\text{arcsinh}(c+dx)}+be^{-2\text{arcsinh}(c+dx)}-4be^{-}\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

output
```
(e^3*(4*(a + b*ArcSinh[c + d*x])*((-4*a)/E^(2*ArcSinh[c + d*x]) + b/E^(2*A
rcSinh[c + d*x]) - (4*b*ArcSinh[c + d*x])/E^(2*ArcSinh[c + d*x]) + E^(2*Ar
cSinh[c + d*x])*(4*a + b + 4*b*ArcSinh[c + d*x]) + (4*Sqrt[2]*b*(-((a + b*
ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b])/E
^((2*a)/b) + 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*Arc
Sinh[c + d*x])*Gamma[1/2, (2*(a + b*ArcSinh[c + d*x]))/b]) - 4*(a + b*ArcS
inh[c + d*x])*((-8*a)/E^(4*ArcSinh[c + d*x]) + (b*(1 - 8*ArcSinh[c + d*x])
)/E^(4*ArcSinh[c + d*x]) + E^(4*ArcSinh[c + d*x])*(8*a + b + 8*b*ArcSinh[c
 + d*x]) + (16*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-4*(a +
 b*ArcSinh[c + d*x]))/b])/E^((4*a)/b) + 16*E^((4*a)/b)*Sqrt[a/b + ArcSinh[
c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (4*(a + b*ArcSinh[c + d*x]))
/b]) + 6*b^2*Sinh[2*ArcSinh[c + d*x]] - 3*b^2*Sinh[4*ArcSinh[c + d*x]]))/(
60*b^3*d*(a + b*ArcSinh[c + d*x])^(5/2))
```

3.223.3 Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {6274, 27, 6194, 6233, 6193, 2009, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{e^3(c+dx)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^3\int \frac{(c+dx)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$\Big\downarrow$ 6194

$$\frac{e^3\left(\frac{6\int \frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{5b}+\frac{8\int \frac{(c+dx)^4}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{5b}-\frac{2\sqrt{(c+dx)^2+1}(c+dx)^3}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}\right)}{d}$$

$\Big\downarrow$ 6233

$$\frac{e^3\left(\frac{6\left(\frac{4\int \frac{c+dx}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2(c+dx)^2}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}+\frac{8\left(\frac{8\int \frac{(c+dx)^3}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2(c+dx)^4}{3b(a+b\text{arcsinh}(c+dx))^{3/}}\right.}{5b}\right.}{d}$$

$\Big\downarrow$ 6193

$$\frac{e^3\left(\frac{6\left(\frac{4\left(\frac{2\int \frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)^2}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}+\frac{8\left(\frac{8\left(\frac{2\int\left(\ldots\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}$$

$\Big\downarrow$ 2009

3.223. $\displaystyle\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

$$e^3\left(\frac{6\left(\frac{4\left(\frac{2\int \frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\,\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\,\text{arcsinh}(c+dx)}}d(a+b\,\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\,\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)^2}{3b(a+b\,\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}+\frac{8\left(\frac{8\left(2\left(\frac{1}{8}\sqrt{\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 3042

$$e^3\left(\frac{6\left(-\frac{2(c+dx)^2}{3b(a+b\,\text{arcsinh}(c+dx))^{3/2}}+\frac{4\left(-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\,\text{arcsinh}(c+dx)}}+\frac{2\int \frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\,\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\,\text{arcsinh}(c+dx)}}d(a+b\,\text{arcsinh}(c+dx))}{b^2}\right)}{3b}\right)}{5b}+\frac{8\left(\frac{8\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 3788

3.223. $\displaystyle\int \frac{(ce+dex)^3}{(a+b\,\text{arcsinh}(c+dx))^{7/2}}\,dx$

$$e^3\left(\frac{6\left(-\frac{2(c+dx)^2}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{4\left(-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2\left(\frac{1}{2}i\int -\frac{ie^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int \frac{ie^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+\cdots}}\cdots\right)}{b^2}\cdots\right)}{3b}\cdots\right)}{5b}\cdots\right.$$

$\downarrow$ 26

$$e^3\left(\frac{6\left(\frac{4\left(\frac{2\left(\frac{1}{2}\int \frac{e^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))+\frac{1}{2}\int \frac{e^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{b^2}-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{\cdots}{3b(a\cdots}\cdots\right)}{5b}\cdots\right.$$

$\downarrow$ 2611

3.223. $\displaystyle\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

$$e^3\left(\frac{6\left(\frac{4\left(\frac{2\left(\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}+\int e^{\frac{2(a+b\text{arcsinh}(c+dx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}\right)}{b^2}-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\cdots\right)}{5b}\cdots\right.$$

$$\Big\downarrow \text{2633}$$

$$e^3\left(\frac{6\left(\frac{4\left(\frac{2\left(\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{\cdots}{3b(\cdots}\right)}{5b}\cdots\right.$$

$$\Big\downarrow \text{2634}$$

3.223. $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

$$e^3\left(\frac{8\left(\frac{8\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\operatorname{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\right.}{b^2}}{3b}}{5b}\right.$$

input
```
Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

output
```
(e^3*((-2*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(5*b*(a + b*ArcSinh[c + d*x])
^(5/2)) + (8*((-2*(c + d*x)^4)/(3*b*(a + b*ArcSinh[c + d*x])^(3/2)) + (8*(
(-2*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*Sqrt[a + b*ArcSinh[c + d*x]]) +
(2*((Sqrt[b]*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]])/8 - (Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh
[c + d*x]])/Sqrt[b]])/4 + (Sqrt[b]*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c +
 d*x]])/Sqrt[b]])/(8*E^((4*a)/b)) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt
[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(4*E^((2*a)/b))))/b^2))/(3*b)))/(5*b)
+ (6*((-2*(c + d*x)^2)/(3*b*(a + b*ArcSinh[c + d*x])^(3/2)) + (4*((-2*(c +
 d*x)*Sqrt[1 + (c + d*x)^2])/(b*Sqrt[a + b*ArcSinh[c + d*x]]) + (2*((Sqrt[
b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/2 + (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/S
qrt[b]])/(2*E^((2*a)/b))))/b^2))/(3*b)))/(5*b)))/d
```

3.223.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

3.223. $\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.223.4 Maple [F]

$$\int \frac{(dex + ce)^3}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}}} dx$$

input
```
int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x)
```

output
```
int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x)
```

3.223.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce + dex)^3}{(a + b\text{arcsinh}(c + dx))^{7/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.223.6 Sympy [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = e^3\left(\int \frac{}{a^3\sqrt{a+b\,\text{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+d\right.$$

$$+\int \frac{d^3x^3}{a^3\sqrt{a+b\,\text{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)+3ab^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}}$$

$$+\int \frac{3cd^2x^2}{a^3\sqrt{a+b\,\text{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)+3ab^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}}$$

$$+\int \frac{3c^2dx}{a^3\sqrt{a+b\,\text{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)+3ab^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}}$$

input
```
integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(7/2),x)
```

output
```
e**3*(Integral(c**3/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b
*asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asin
h(c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + I
ntegral(d**3*x**3/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*a
sinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(
c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Int
egral(3*c*d**2*x**2/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b
*asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asin
h(c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + I
ntegral(3*c**2*d*x/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*
asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh
(c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x))
```

3.223.7 Maxima [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.223.8 Giac [F]

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx=\int \frac{(dex+ce)^3}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^3}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx=\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\text{asinh}\,(c+d\,x))^{7/2}}\,dx$$

input
```
int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(7/2),x)
```

output
```
int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(7/2), x)
```

3.224 $\int \frac{(ce+dex)^2}{(a+b\,\mathrm{arcsinh}(c+dx))^{7/2}}\,dx$

3.224.1 Optimal result

Integrand size = 25, antiderivative size = 410

$$\begin{aligned}\int \frac{(ce+dex)^2}{(a+b\,\mathrm{arcsinh}(c+dx))^{7/2}}\,dx = &-\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd(a+b\,\mathrm{arcsinh}(c+dx))^{5/2}} \\ &-\frac{8e^2(c+dx)}{15b^2d(a+b\,\mathrm{arcsinh}(c+dx))^{3/2}}-\frac{4e^2(c+dx)^3}{5b^2d(a+b\,\mathrm{arcsinh}(c+dx))^{3/2}} \\ &-\frac{16e^2\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\,\mathrm{arcsinh}(c+dx)}}-\frac{24e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5b^3d\sqrt{a+b\,\mathrm{arcsinh}(c+dx)}} \\ &+\frac{e^2e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{3e^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\,\mathrm{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d} \\ &-\frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{3e^2e^{-\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\,\mathrm{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d}\end{aligned}$$

output
```
-8/15*e^2*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-4/5*e^2*(d*x+c)^3/b^2/d
/(a+b*arcsinh(d*x+c))^(3/2)+1/15*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/
2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d-1/15*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^
(1/2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)-3/5*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arc
sinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d+3/5*e^2*erfi(3^(1/2
)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(3*a/b
)-2/5*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(5/2)-16/
15*e^2*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)-24/5*e^2*(d*x+
c)^2*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)
```

3.224.2 Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.16

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \frac{e^2\left(3b^2e^{\operatorname{arcsinh}(c+dx)}+e^{-\operatorname{arcsinh}(c+dx)}\left(4a^2-2ab+3b^2+2(4a-b)b\operatorname{arcsinh}(\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

output
```
(e^2*(3*b^2*E^ArcSinh[c + d*x] + (4*a^2 - 2*a*b + 3*b^2 + 2*(4*a - b)*b*Ar
cSinh[c + d*x] + 4*b^2*ArcSinh[c + d*x]^2 - 4*E^(a/b + ArcSinh[c + d*x])*S
qrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*Gamma[1/2, a/b + Ar
cSinh[c + d*x]])/E^ArcSinh[c + d*x] - 3*(b^2*E^(3*ArcSinh[c + d*x]) + (2*(
a + b*ArcSinh[c + d*x])*(E^(3*(a/b + ArcSinh[c + d*x]))*(6*a + b + 6*b*Arc
Sinh[c + d*x]) + 6*Sqrt[3]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1
/2, (-3*(a + b*ArcSinh[c + d*x]))/b]))/E^((3*a)/b)) + (2*(a + b*ArcSinh[c
+ d*x])*(E^(a/b + ArcSinh[c + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b
*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x
])/b)]))/E^(a/b) - (3*(b^2 + 2*(a + b*ArcSinh[c + d*x])*(6*a - b + 6*b*Arc
Sinh[c + d*x] - 6*Sqrt[3]*E^(3*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSin
h[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]
))/b])))/E^(3*ArcSinh[c + d*x])))/(60*b^3*d*(a + b*ArcSinh[c + d*x])^(5/2)
)
```

3.224.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.20, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6274, 27, 6194, 6233, 6188, 6193, 2009, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \frac{e^2(c+dx)^2}{(a+b\text{arcsinh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^2\int \frac{(c+dx)^2}{(a+b\text{arcsinh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6194}$$

$$\frac{e^2\left(\frac{4\int \frac{c+dx}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{5b}+\frac{6\int \frac{(c+dx)^3}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{5b}-\frac{2\sqrt{(c+dx)^2+1}(c+dx)^2}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}\right)}{d}$$

$$\Big\downarrow \text{ 6233}$$

$$\frac{e^2\left(\frac{4\left(\frac{2\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}+\frac{6\left(\frac{2\int \frac{(c+dx)^2}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{b}-\frac{2(c+dx)^3}{3b(a+b\text{arcsinh}(c+dx))^{3/}\cdots}\right.}{5b}\cdots\right.}{d}$$

$$\Big\downarrow \text{ 6188}$$

$$\frac{e^2\left(\frac{6\left(\frac{2\int \frac{(c+dx)^2}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{b}-\frac{2(c+dx)^3}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}+\frac{4\left(\frac{2\left(\frac{2\int \frac{c+dx}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{2\sqrt{(c+dx}\cdots}{b\sqrt{a+b\text{arcsi}\cdots}}\right.}{3b}\cdots\right.}{5b}\cdots\right.}{d}$$

$$\Big\downarrow \text{ 6193}$$

3.224. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

$$e^2\left(\frac{6\left(\frac{2\left(\frac{2\int\left(\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{4\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{3\sinh\left(\frac{3a}{b}-\frac{3(a+b\text{arcsinh}(c+dx))}{b}\right)}{4\sqrt{a+b\text{arcsinh}(c+dx)}}\right)d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)^2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{b}-\frac{\cdots}{3b(a+b\text{ar}\cdots}\right)}{5b}\cdots\right.$$

$$\downarrow \; 2009$$

$$e^2\left(\frac{4\left(\frac{2\left(\frac{2\int\frac{c+dx}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}+\frac{6\left(\frac{2\left(\frac{2\left(\frac{1}{8}\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{\cdots}}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\cdots\right.$$

$$\downarrow \; 6234$$

3.224. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

$$e^2\left(\frac{4\left(\frac{2\left(\frac{2\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}+\frac{6\left(\frac{2\left(2\left(\frac{1}{8}\sqrt{\pi}\sqrt{\right.\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 25

$$e^2\left(\frac{4\left(\frac{2\left(-\frac{2\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}+\frac{6\left(\frac{2\left(2\left(\frac{1}{8}\sqrt{\pi}\sqrt{\right.\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 3042

3.224. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

$$e^2\left(\frac{4\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{2\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}\right)}{3b}\right)}{5b}+\frac{6\left(2\left(2\left(\cdots\right.\right.\right.}{\cdots}\right.$$

$\downarrow$ 26

$$e^2\left(\frac{4\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}\right)}{3b}\right)}{5b}+\frac{6\left(2\left(2\left(\frac{1}{8}\cdots\right.\right.\right.}{\cdots}\right.$$

$\downarrow$ 3789

3.224. $\displaystyle\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

$$e^2\left(\frac{4\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\int\frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int\frac{e^{-\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+d}}\right.}{b^2}\right.}{3b}\right.}{5b}\right.$$

$\downarrow$ 2611

$$e^2\left(\frac{4\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-i\int e^{\frac{a+b\text{arcsinh}(c+dx)}{b}-\frac{a}{b}}\right.}{b^2}\right.}{3b}\right.}{5b}\right.$$

$\downarrow$ 2633

3.224. $\displaystyle\int\frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

$$e^2\left(\frac{4\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{a}}}{}\right.\right.}{b^2}\right.}{3b}\right.}{5b}\right.$$

$$\downarrow 2634$$

$$e^2\left(\frac{4\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsin}}}{\sqrt{b}}\right.\right.}{b^2}\right.}{3b}\right.}{5b}\right.$$

input

```
Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

3.224. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

output
```
(e^2*((-2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(5*b*(a + b*ArcSinh[c + d*x])
^(5/2)) + (4*((-2*(c + d*x))/(3*b*(a + b*ArcSinh[c + d*x])^(3/2)) + (2*((-
2*Sqrt[1 + (c + d*x)^2])/(b*Sqrt[a + b*ArcSinh[c + d*x]]) + ((2*I)*((I/2)*
Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - ((I/2
)*Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/E^(a/b)))/b
^2))/(3*b)))/(5*b) + (6*((-2*(c + d*x)^3)/(3*b*(a + b*ArcSinh[c + d*x])^(3
/2)) + (2*((-2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*Sqrt[a + b*ArcSinh[c
+ d*x]]) + (2*((Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/
Sqrt[b]])/8 - (Sqrt[b]*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
 d*x]]/Sqrt[b]])/(8*E^(a/b)) + (Sqrt[b]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a +
b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*E^((3*a)/b))))/b^2))/b))/(5*b)))/d
```

3.224.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

3.224. $\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.224.4 Maple [F]

$$\int \frac{(dex+ce)^2}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{7}{2}}}dx$$

input
```
int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x)
```

output
```
int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x)
```

3.224.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(ce+dex)^2}{(a+b\mathrm{arcsinh}(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.224.6 Sympy [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = e^2\left(\int \frac{}{a^3\sqrt{a+b\,\text{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+d}\right.$$
$$+\int \frac{d^2x^2}{a^3\sqrt{a+b\,\text{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)+3ab^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}}$$
$$+\int \frac{2cdx}{a^3\sqrt{a+b\,\text{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)+3ab^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}}$$

input
```
integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(7/2),x)
```

output
```
e**2*(Integral(c**2/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b
*asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asin
h(c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + I
ntegral(d**2*x**2/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*a
sinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(
c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Int
egral(2*c*d*x/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c +
d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x))
```

3.224.7 Maxima [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{(dex+ce)^2}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.224.8 Giac [F]

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx=\int \frac{(dex+ce)^2}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)^2}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx=\int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\text{asinh}\,(c+d\,x))^{7/2}}\,dx$$

input
```
int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(7/2),x)
```

output
```
int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(7/2), x)
```

3.225 $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

3.225.1 Optimal result

Integrand size = 23, antiderivative size = 252

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{4e}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{8e(c+dx)^2}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{32e(c+dx)\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} + \frac{8ee^{\frac{2a}{b}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} + \frac{8ee^{-\frac{2a}{b}}\sqrt{2\pi}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}$$

output
```
-4/15*e/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-8/15*e*(d*x+c)^2/b^2/d/(a+b*arcsi
nh(d*x+c))^(3/2)+8/15*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/
b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d+8/15*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c
))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(2*a/b)-2/5*e*(d*x+c)*(1+(
d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(5/2)-32/15*e*(d*x+c)*(1+(d*x+c)^
2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)
```

3.225.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.93

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx =$$
$$-\frac{e\left((a+b\text{arcsinh}(c+dx))\left(e^{-\frac{2a}{b}}\left(2e^{2\left(\frac{a}{b}+\text{arcsinh}(c+dx)\right)}(4a+b+4b\text{arcsinh}(c+dx))+8\sqrt{2}b\left(-\frac{a+b\text{arcsinh}(c+\cdots}{b}\right.\right.\right.\right.}{\cdots}$$

input
```
Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

output
```
-1/15*(e*((a + b*ArcSinh[c + d*x])*((2*E^(2*(a/b + ArcSinh[c + d*x]))*(4*a
 + b + 4*b*ArcSinh[c + d*x]) + 8*Sqrt[2]*b*(-((a + b*ArcSinh[c + d*x])/b))
^(3/2)*Gamma[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b])/E^((2*a)/b) + (-8*a +
2*b - 8*b*ArcSinh[c + d*x] + 8*Sqrt[2]*E^(2*(a/b + ArcSinh[c + d*x]))*Sqrt
[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (2*(a + b*Arc
Sinh[c + d*x]))/b])/E^(2*ArcSinh[c + d*x])) + 3*b^2*Sinh[2*ArcSinh[c + d*x
]]))/(b^3*d*(a + b*ArcSinh[c + d*x])^(5/2))
```

3.225.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {6274, 27, 6194, 6198, 6233, 6193, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$$
$$\Big\downarrow \text{6274}$$
$$\frac{\int \frac{e(c+dx)}{(a+b\text{arcsinh}(c+dx))^{7/2}}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{e\int \frac{c+dx}{(a+b\text{arcsinh}(c+dx))^{7/2}}d(c+dx)}{d}$$
$$\Big\downarrow \text{6194}$$

$$\frac{e\left(\frac{2\int \frac{1}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{5b}+\frac{4\int \frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{5b}-\frac{2\sqrt{(c+dx)^2+1}(c+dx)}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}\right)}{d}$$

$\downarrow$ 6198

$$\frac{e\left(\frac{4\int \frac{(c+dx)^2}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{5b}-\frac{4}{15b^2(a+b\text{arcsinh}(c+dx))^{3/2}}-\frac{2\sqrt{(c+dx)^2+1}(c+dx)}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}\right)}{d}$$

$\downarrow$ 6233

$$\frac{e\left(\frac{4\left(\frac{4\int \frac{c+dx}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2(c+dx)^2}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}-\frac{4}{15b^2(a+b\text{arcsinh}(c+dx))^{3/2}}-\frac{2\sqrt{(c+dx)^2+1}(c+dx)}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}\right)}{d}$$

$\downarrow$ 6193

$$\frac{e\left(\frac{4\left(\frac{4\left(\frac{2\int \frac{\cosh\left(\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)^2}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}-\frac{}{15b^2(a+b\text{arcs}}\right.}{d}$$

$\downarrow$ 3042

3.225. $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

$$\frac{e\left(\frac{4\left(-\frac{2(c+dx)^2}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{4\left(-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2\int\frac{\sin\left(\frac{2ia}{b}-\frac{2i(a+b\text{arcsinh}(c+dx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}\right)}{3b}\right)}{5b}-\frac{}{15b^2(a}\right.}{d}$$

$\downarrow$ 3788

$$\frac{e\left(\frac{4\left(-\frac{2(c+dx)^2}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{4\left(-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2\left(\frac{1}{2}i\int-\frac{ie^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))-\frac{1}{2}i\int\frac{ie^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx}}\right.}{b^2}\right.}{3b}\right.}{5b}\right.}{d}$$

$\downarrow$ 26

3.225. $\int\frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

$$e\left(\frac{4\left(\frac{4\left(\frac{2\left(\frac{1}{2}\int\frac{e^{-\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))+\frac{1}{2}\int\frac{e^{\frac{2(a-c-dx)}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))\right)}{b^2}-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{}{3b(a+}}{5b}\right.$$

$$\frac{}{d}$$

$\downarrow$ 2611

$$e\left(\frac{4\left(\frac{4\left(\frac{2\left(\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}+\int e^{\frac{2(a+b\text{arcsinh}(c+dx))}{b}-\frac{2a}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}\right)}{b^2}-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{}{3}}{5b}\right.$$

$$\frac{}{d}$$

$\downarrow$ 2633

3.225. $\int\frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

$$\frac{e\left(\frac{4\left(\frac{4\left(\frac{2\left(\int e^{\frac{2a}{b}-\frac{2(a+b\text{arcsinh}(c+dx))}{b}}d\sqrt{a+b\text{arcsinh}(c+dx)}+\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{}{3b(a+}}{5b}\right.}{d}$$

$\downarrow$ 2634

$$\frac{e\left(\frac{4\left(\frac{4\left(\frac{2\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)\right)}{b^2}-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2(c}{3b(a+b\text{arcs}}}{5b}\right.}{d}$$

input
```
Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

output
```
(e*((-2*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(5*b*(a + b*ArcSinh[c + d*x])^(5/
2)) - 4/(15*b^2*(a + b*ArcSinh[c + d*x])^(3/2)) + (4*((-2*(c + d*x)^2)/(3*
b*(a + b*ArcSinh[c + d*x])^(3/2)) + (4*((-2*(c + d*x)*Sqrt[1 + (c + d*x)^2
])/(b*Sqrt[a + b*ArcSinh[c + d*x]]) + (2*((Sqrt[b]*E^((2*a)/b)*Sqrt[Pi/2]*
Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/2 + (Sqrt[b]*Sqrt[Pi/
2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2*E^((2*a)/b))))
/b^2))/(3*b)))/(5*b)))/d
```

3.225. $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

3.225.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

3.225. $\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

rule 6194
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/
Sqrt[1 + c^2*x^2]), x], x] - Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] &&
IGtQ[m, 0] && LtQ[n, -2]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.225.4 Maple [F]

$$\int \frac{dex + ce}{(a + b\,\mathrm{arcsinh}\,(dx + c))^{\frac{7}{2}}}dx$$

input
```
int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

output
```
int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

3.225.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.225.6 Sympy [F]

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = e\left(\int \frac{}{a^3\sqrt{a+b\,\text{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx}\right.$$

$$+\int \frac{dx}{a^3\sqrt{a+b\,\text{asinh}\,(c+dx)}+3a^2b\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}\,(c+dx)+3ab^2\sqrt{a+b\,\text{asinh}\,(c+dx)}\,\text{asinh}}$$

input
```
integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(7/2),x)
```

output
```
e*(Integral(c/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c +
d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integra
l(d*x/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(c + d*x
))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2
+ b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x))
```

3.225.7 Maxima [F]

$$\int \frac{ce+dex}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{dex+ce}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.225.8 Giac [F]

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{dex+ce}{(b\operatorname{arsinh}(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{ce+dex}{(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{c\,e+d\,e\,x}{(a+b\operatorname{asinh}(c+d\,x))^{7/2}}\,dx$$

input
```
int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(7/2),x)
```

output
```
int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(7/2), x)
```

3.226 $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}} dx$

3.226.1 Optimal result

Integrand size = 14, antiderivative size = 195

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}} dx = -\frac{2\sqrt{1+(c+dx)^2}}{5bd(a+b\text{arcsinh}(c+dx))^{5/2}} - \frac{4(c+dx)}{15b^2d(a+b\text{arcsinh}(c+dx))^{3/2}} - \frac{8\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\text{arcsinh}(c+dx)}} - \frac{4e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d} + \frac{4e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}$$

output
```
-4/15*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-4/15*exp(a/b)*erf((a+b*arcs
inh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+4/15*erfi((a+b*arcsinh(d*x+c
))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)-2/5*(1+(d*x+c)^2)^(1/2)/b/d/
(a+b*arcsinh(d*x+c))^(5/2)-8/15*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x
+c))^(1/2)
```

3.226.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \frac{-6b^2e^{\text{arcsinh}(c+dx)}-2e^{-\text{arcsinh}(c+dx)}(4a^2+2ab(-1+4\text{arcsinh}(c+dx))+b}{}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

output
```
(-6*b^2*E^ArcSinh[c + d*x] - (2*(4*a^2 + 2*a*b*(-1 + 4*ArcSinh[c + d*x]) +
 b^2*(3 - 2*ArcSinh[c + d*x] + 4*ArcSinh[c + d*x]^2)))/E^ArcSinh[c + d*x]
+ 8*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*Gamma[
1/2, a/b + ArcSinh[c + d*x]] - (4*(a + b*ArcSinh[c + d*x])*(E^(a/b + ArcSi
nh[c + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b*(-((a + b*ArcSinh[c +
d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b))/(30*b
^3*d*(a + b*ArcSinh[c + d*x])^(5/2))
```

3.226.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6273, 6188, 6233, 6188, 6234, 25, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6188}$$

$$\frac{\frac{2\int \frac{c+dx}{\sqrt{(c+dx)^2+1}(a+b\text{arcsinh}(c+dx))^{5/2}}d(c+dx)}{5b}-\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \text{6233}$$

$$\frac{\frac{2\left(\frac{2\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{3/2}}d(c+dx)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}-\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \; 6188$$

$$\frac{\frac{2\left(\frac{2\left(\frac{2\int \frac{c+dx}{\sqrt{(c+dx)^2+1}\sqrt{a+b\text{arcsinh}(c+dx)}}d(c+dx)}{b}-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}-\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \; 6234$$

$$\frac{\frac{2\left(\frac{2\left(\frac{2\int -\frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}-\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2\left(\frac{2\left(-\frac{2\int \frac{\sinh\left(\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+dx))}{b^2}-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}\right)}{3b}-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}\right)}{5b}-\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \; 3042$$

3.226. $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}+\frac{2\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}-\frac{2\int -\frac{i\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arc}\cdots}{b^2}\right)}{3b}\right)}{5b}}{d}$$

$\downarrow$ 26

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}+\frac{2\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\int \frac{\sin\left(\frac{ia}{b}-\frac{i(a+b\text{arcsinh}(c+dx))}{b}\right)}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsi}\cdots}{b^2}\right)}{3b}\right)}{5b}}{d}$$

$\downarrow$ 3789

$$\frac{-\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}+\frac{2\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\int \frac{e^{\frac{a-c-dx}{b}}}{\sqrt{a+b\text{arcsinh}(c+dx)}}d(a+b\text{arcsinh}(c+\cdots\right)}{}\right)}{3b}\right)}{5b}}{d}$$

$\downarrow$ 2611

3.226. $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

$$-\frac{\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}+\frac{2\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+\cdots}{\cdots}\right)}{3b}\right)}{5b}}{d}$$

$\downarrow$ 2633

$$-\frac{\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}+\frac{2\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(i\int e^{\frac{a}{b}-\frac{a+b\text{arcsinh}(c+dx)}{b}}d\sqrt{a+b\text{arcsinh}(c+\cdots}{b^{\cdots}}\right)}{3b}\right)}{5b}}{d}$$

$\downarrow$ 2634

$$-\frac{\frac{2\sqrt{(c+dx)^2+1}}{5b(a+b\text{arcsinh}(c+dx))^{5/2}}+\frac{2\left(-\frac{2(c+dx)}{3b(a+b\text{arcsinh}(c+dx))^{3/2}}+\frac{2\left(-\frac{2\sqrt{(c+dx)^2+1}}{b\sqrt{a+b\text{arcsinh}(c+dx)}}+\frac{2i\left(\frac{1}{2}i\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(c+dx)}}{\sqrt{b}}\right)-\frac{1}{2}i\cdots\right)}{b^2}\right)}{3b}\right)}{5b}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

output
```
((-2*Sqrt[1 + (c + d*x)^2])/(5*b*(a + b*ArcSinh[c + d*x])^(5/2)) + (2*((-2
*(c + d*x))/(3*b*(a + b*ArcSinh[c + d*x])^(3/2)) + (2*((-2*Sqrt[1 + (c + d
*x)^2])/(b*Sqrt[a + b*ArcSinh[c + d*x]]) + ((2*I)*((I/2)*Sqrt[b]*E^(a/b)*S
qrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - ((I/2)*Sqrt[b]*Sqrt[Pi
]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/E^(a/b)))/b^2))/(3*b)))/(5*b
))/d
```

3.226. $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

3.226.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 6188
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1)
)   Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

3.226. $\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$

rule 6233
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Simp[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a +
b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e
, c^2*d] && LtQ[n, -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

3.226.4 Maple [F]

$$\int \frac{1}{\left(a+b \,\operatorname{arcsinh}\left(d x +c \right)\right)^{\frac{7}{2}}}d x$$

input
```
int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

output
```
int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

3.226.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.226.6 Sympy [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx=\int \frac{1}{(a+b\,\text{asinh}\,(c+dx))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*asinh(d*x+c))**(7/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**(-7/2), x)
```

3.226.7 Maxima [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx=\int \frac{1}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

3.226.8 Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx=\int \frac{1}{(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\,(c+d\,x))^{7/2}}\,dx$$

input
```
int(1/(a + b*asinh(c + d*x))^(7/2),x)
```

output
```
int(1/(a + b*asinh(c + d*x))^(7/2), x)
```

3.227 $\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx$

3.227.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx=\frac{\operatorname{Int}\left(\frac{1}{(c+dx)(a+b\operatorname{arcsinh}(c+dx))^{7/2}},x\right)}{e}$$

output
```
Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^(7/2),x)/e
```

3.227.2 Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx=\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx$$

input
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2)),x]
```

output
```
Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2)), x]
```

3.227.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 27, 6196}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx$$
$$\Big\downarrow \text{ 6274}$$
$$\frac{\int \frac{1}{e(c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}}d(c+dx)}{d}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}}d(c+dx)}{de}$$
$$\Big\downarrow \text{ 6196}$$
$$\frac{\int \frac{1}{(c+dx)(a+b\text{arcsinh}(c+dx))^{7/2}}d(c+dx)}{de}$$

input
```
Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2)),x]
```

output
```
$Aborted
```

3.227.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6196
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Unintegrable[(d*x)^m*(a + b*ArcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, m
, n}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.227.4 Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(dex+ce)\left(a+b\operatorname{arcsinh}\left(dx+c\right)\right)^{\frac{7}{2}}}dx$$

input
```
int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

output
```
int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

3.227.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.227.6 Sympy [N/A]

Not integrable

Time = 105.85 (sec) , antiderivative size = 221, normalized size of antiderivative = 8.84

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx = \int \frac{}{a^3c\sqrt{a+b\operatorname{asinh}(c+dx)}+a^3dx\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^2bc\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(7/2),x)
```

output
```
Integral(1/(a**3*c*sqrt(a + b*asinh(c + d*x)) + a**3*d*x*sqrt(a + b*asinh(
c + d*x)) + 3*a**2*b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + 3*a**2*
b*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*c*sqrt(a + b*as
inh(c + d*x))*asinh(c + d*x)**2 + 3*a*b**2*d*x*sqrt(a + b*asinh(c + d*x))*
asinh(c + d*x)**2 + b**3*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3 +
b**3*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x)/e
```

3.227.7 Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx=\int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2)), x)
```

3.227.8 Giac [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\text{arcsinh}(c+dx))^{7/2}}\,dx=\int \frac{1}{(dex+ce)(b\,\text{arsinh}\,(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2)), x)
```

3.227.9 Mupad [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)(a+b\operatorname{arcsinh}(c+dx))^{7/2}}\,dx=\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^{7/2}}\,dx$$

input
```
int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(7/2)),x)
```

output
```
int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(7/2)), x)
```

3.228 $\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))\,dx$

3.228.1 Optimal result

Integrand size = 23, antiderivative size = 298

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))\,dx=\frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d}-\frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx)^2}}{81d}-\frac{28be^3\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{135d(1+c+dx)}+\frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))}{9de}+\frac{28be^{7/2}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}E\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{135d\sqrt{1+(c+dx)^2}}-\frac{14be^{7/2}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{135d\sqrt{1+(c+dx)^2}}$$

output
```
2/9*(e*(d*x+c))^(9/2)*(a+b*arcsinh(d*x+c))/d/e+28/405*b*e^2*(e*(d*x+c))^(3
/2)*(1+(d*x+c)^2)^(1/2)/d-4/81*b*(e*(d*x+c))^(7/2)*(1+(d*x+c)^2)^(1/2)/d-2
8/135*b*e^3*(e*(d*x+c))^(1/2)*(1+(d*x+c)^2)^(1/2)/d/(d*x+c+1)+28/135*b*e^(
7/2)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/2)/cos(2*ar
ctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticE(sin(2*arctan((e*(d*x+c))^(1/2)/
e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/(1+(d*x+c)^2)^(
1/2)-14/135*b*e^(7/2)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^
2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*arctan((
e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/
d/(1+(d*x+c)^2)^(1/2)
```

3.228.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.38

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))\,dx=\frac{2(e(c+dx))^{7/2}\left(45a(c+dx)^3+14b\sqrt{1+(c+dx)^2}-10b(c+dx)^2\sqrt{1+(c+dx)^2}\right.}{405d(c+}$$

input
```
Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x]),x]
```

output
```
(2*(e*(c + d*x))^(7/2)*(45*a*(c + d*x)^3 + 14*b*Sqrt[1 + (c + d*x)^2] - 10
*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] + 45*b*(c + d*x)^3*ArcSinh[c + d*x] -
 14*b*Hypergeometric2F1[1/2, 3/4, 7/4, -(c + d*x)^2]))/(405*d*(c + d*x)^2)
```

3.228.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6274, 6191, 262, 262, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int (e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))}{9e}-\frac{2b\int \frac{(e(c+dx))^{9/2}}{\sqrt{(c+dx)^2+1}}d(c+dx)}{9e}}{d}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))}{9e}-\frac{2b\left(\frac{2}{9}e\sqrt{(c+dx)^2+1}(e(c+dx))^{7/2}-\frac{7}{9}e^2\int \frac{(e(c+dx))^{5/2}}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{9e}}{d}$$

$\Big\downarrow$ 262

$$\frac{\frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))}{9e}-\frac{2b\left(\frac{2}{9}e\sqrt{(c+dx)^2+1}(e(c+dx))^{7/2}-\frac{7}{9}e^2\left(\frac{2}{5}e\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}-\frac{3}{5}e^2\int\frac{\sqrt{e(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)\right)}{9e}}{d}$$

$\Big\downarrow$ 266

$$\frac{\frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))}{9e}-\frac{2b\left(\frac{2}{9}e\sqrt{(c+dx)^2+1}(e(c+dx))^{7/2}-\frac{7}{9}e^2\left(\frac{2}{5}e\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}-\frac{6}{5}e\int\frac{e(c+dx)}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}\right)\right.}{9e}}{d}$$

$\Big\downarrow$ 834

$$\frac{\frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))}{9e}-\frac{2b\left(\frac{2}{9}e\sqrt{(c+dx)^2+1}(e(c+dx))^{7/2}-\frac{7}{9}e^2\left(\frac{2}{5}e\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}-\frac{6}{5}e\left(e\int\frac{1}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx}\right.\right.\right.}{9e}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))}{9e}-\frac{2b\left(\frac{2}{9}e\sqrt{(c+dx)^2+1}(e(c+dx))^{7/2}-\frac{7}{9}e^2\left(\frac{2}{5}e\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}-\frac{6}{5}e\left(e\int\frac{1}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx}\right.\right.\right.}{9e}}{d}$$

$\Big\downarrow$ 761

$$\frac{\frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))}{9e}-\frac{2b\left(\frac{2}{9}e\sqrt{(c+dx)^2+1}(e(c+dx))^{7/2}-\frac{7}{9}e^2\left(\frac{2}{5}e\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}-\frac{6}{5}e\left(\frac{\sqrt{e}(e(c+dx)+e)\sqrt{\frac{e^2(c+dx)^2}{(e(c+dx)+}}}{2}\right.\right.\right.}{9e}}{d}$$

$\Big\downarrow$ 1510

$$\frac{\frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))}{9e}-\frac{2b\left(\frac{2}{9}e\sqrt{(c+dx)^2+1}(e(c+dx))^{7/2}-\frac{7}{9}e^2\left(\frac{2}{5}e\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}-\frac{6}{5}e\left(\frac{\sqrt{e}(e(c+dx)+e)\sqrt{\frac{e^2(c+dx)^2}{(e(c+dx)+}}}{2}\right.\right.\right.}{}}{d}$$

input
```
Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x]),x]
```

3.228. $\int(ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))\,dx$

output
```
((2*(e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x]))/(9*e) - (2*b*((2*e*(e*(c
 + d*x))^(7/2)*Sqrt[1 + (c + d*x)^2])/9 - (7*e^2*((2*e*(e*(c + d*x))^(3/2)
*Sqrt[1 + (c + d*x)^2])/5 - (6*e*((e^2*Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x
)^2])/(e + e*(c + d*x)) - (Sqrt[e]*(e + e*(c + d*x))*Sqrt[(e^2 + e^2*(c +
d*x)^2)/(e + e*(c + d*x))^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]]
, 1/2])/Sqrt[1 + (c + d*x)^2] + (Sqrt[e]*(e + e*(c + d*x))*Sqrt[(e^2 + e^2
*(c + d*x)^2)/(e + e*(c + d*x))^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sq
rt[e]], 1/2])/(2*Sqrt[1 + (c + d*x)^2])))/5))/9))/(9*e))/d
```

3.228.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.228.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.98 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{2a(dex+ce)^{\frac{9}{2}}}{9}+2b\left(\frac{(dex+ce)^{\frac{9}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{9}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{7}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{9}-\frac{7e^4(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{45}+\frac{7ie^5\sqrt{1-}}{}\right.}{}\right.}{de}$
default	$\frac{\frac{2a(dex+ce)^{\frac{9}{2}}}{9}+2b\left(\frac{(dex+ce)^{\frac{9}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{9}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{7}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{9}-\frac{7e^4(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{45}+\frac{7ie^5\sqrt{1-}}{}\right.}{}\right.}{de}$
parts	$\frac{2a(dex+ce)^{\frac{9}{2}}}{9de}+\frac{2b\left(\frac{(dex+ce)^{\frac{9}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{9}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{7}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{9}-\frac{7e^4(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{45}+\frac{7ie^5\sqrt{}}{}\right.}{}\right.}{de}$

input
```
int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d/e*(1/9*a*(d*e*x+c*e)^(9/2)+b*(1/9*(d*e*x+c*e)^(9/2)*arcsinh(1/e*(d*e*x
+c*e))-2/9/e*(1/9*e^2*(d*e*x+c*e)^(7/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)-7/45
*e^4*(d*e*x+c*e)^(3/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)+7/15*I*e^5/(I/e)^(1/2
)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*(d*e*x+c*e)^2
+1)^(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-EllipticE((d*e*x+c*e
)^(1/2)*(I/e)^(1/2),I)))))
```

3.228.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.03

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))\,dx = \frac{2\left(42\sqrt{d^3e}be^3\text{weierstrassZeta}\left(-\frac{4}{d^2},0,\text{weierstrassPInverse}\left(-\frac{4}{d^2},0,\frac{dx+c}{d}\right)\right)+45\,(bd^5e\right.}{}$$

input
```
integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

output
```
2/405*(42*sqrt(d^3*e)*b*e^3*weierstrassZeta(-4/d^2, 0, weierstrassPInverse
(-4/d^2, 0, (d*x + c)/d)) + 45*(b*d^5*e^3*x^4 + 4*b*c*d^4*e^3*x^3 + 6*b*c^
2*d^3*e^3*x^2 + 4*b*c^3*d^2*e^3*x + b*c^4*d*e^3)*sqrt(d*e*x + c*e)*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*(5*b*d^4*e^3*x^3 + 15*b*c*d^
3*e^3*x^2 + (15*b*c^2 - 7*b)*d^2*e^3*x + (5*b*c^3 - 7*b*c)*d*e^3)*sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*e*x + c*e) + 45*(a*d^5*e^3*x^4 + 4*a*c*d^
4*e^3*x^3 + 6*a*c^2*d^3*e^3*x^2 + 4*a*c^3*d^2*e^3*x + a*c^4*d*e^3)*sqrt(d*
e*x + c*e))/d^2
```

3.228.6 Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))\,dx=\text{Timed out}$$

input
```
integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c)),x)
```

output
```
Timed out
```

3.228.7 Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.228.8 Giac [F]

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))\,dx=\int (dex+ce)^{\frac{7}{2}}(b\operatorname{arsinh}(dx+c)+a)\,dx$$

input
```
integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a), x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))\,dx=\int {\left(c\,e+d\,e\,x\right)}^{7/2}\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)\,dx$$

input
```
int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x)), x)
```

3.229 $\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))\,dx$

3.229.1 Optimal result

Integrand size = 23, antiderivative size = 177

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))\,dx = \frac{20be^2\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{147d} - \frac{4b(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}}{49d} + \frac{2(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))}{7de} - \frac{10be^{5/2}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{147d\sqrt{1+(c+dx)^2}}$$

output
```
2/7*(e*(d*x+c))^(7/2)*(a+b*arcsinh(d*x+c))/d/e-4/49*b*(e*(d*x+c))^(5/2)*(1
+(d*x+c)^2)^(1/2)/d+20/147*b*e^2*(e*(d*x+c))^(1/2)*(1+(d*x+c)^2)^(1/2)/d-1
0/147*b*e^(5/2)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/
2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*arctan((e*(d*x
+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/(1+(
d*x+c)^2)^(1/2)
```

3.229.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.64

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))\,dx=\frac{2(e(c+dx))^{5/2}\left(21a(c+dx)^3+10b\sqrt{1+(c+dx)^2}-6b(c+dx)^2\sqrt{1+(c+dx)^2}-\right.}{147d(c+}$$

input
```
Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x]),x]
```

output
```
(2*(e*(c + d*x))^(5/2)*(21*a*(c + d*x)^3 + 10*b*Sqrt[1 + (c + d*x)^2] - 6*
b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] + 21*b*(c + d*x)^3*ArcSinh[c + d*x] -
10*b*Hypergeometric2F1[1/4, 1/2, 5/4, -(c + d*x)^2]))/(147*d*(c + d*x)^2)
```

3.229.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6274, 6191, 262, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int (e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))}{7e}-\frac{2b\int \frac{(e(c+dx))^{7/2}}{\sqrt{(c+dx)^2+1}}d(c+dx)}{7e}}{d}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))}{7e}-\frac{2b\left(\frac{2}{7}e\sqrt{(c+dx)^2+1}(e(c+dx))^{5/2}-\frac{5}{7}e^2\int \frac{(e(c+dx))^{3/2}}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{7e}}{d}$$

$$\Big\downarrow \; 262$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))}{7e}-\frac{2b\left(\frac{2}{7}e\sqrt{(c+dx)^2+1}(e(c+dx))^{5/2}-\frac{5}{7}e^2\left(\frac{2}{3}e\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}-\frac{1}{3}e^2\int\frac{1}{\sqrt{e(c+dx)}\sqrt{(c+dx)^2+1}}d(c+d\right.\right.}{7e}}{d}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))}{7e}-\frac{2b\left(\frac{2}{7}e\sqrt{(c+dx)^2+1}(e(c+dx))^{5/2}-\frac{5}{7}e^2\left(\frac{2}{3}e\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}-\frac{2}{3}e\int\frac{1}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}\right)\right)}{7e}}{d}$$

$$\Big\downarrow \; 761$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))}{7e}-\frac{2b\left(\frac{2}{7}e\sqrt{(c+dx)^2+1}(e(c+dx))^{5/2}-\frac{5}{7}e^2\left(\frac{2}{3}e\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}-\frac{\sqrt{e}(e(c+dx)+e)\sqrt{\frac{e^2(c+dx)^2+e^2}{(e(c+dx)+e)^2}}\text{Ell}}{3\sqrt{(c+dx}}\right.\right.}{7e}}{d}$$

input
```
Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x]),x]
```

output
```
((2*(e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x]))/(7*e) - (2*b*((2*e*(e*(c
 + d*x))^(5/2)*Sqrt[1 + (c + d*x)^2])/7 - (5*e^2*((2*e*Sqrt[e*(c + d*x)]*S
qrt[1 + (c + d*x)^2])/3 - (Sqrt[e]*(e + e*(c + d*x))*Sqrt[(e^2 + e^2*(c +
d*x)^2)/(e + e*(c + d*x))^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]]
, 1/2])/(3*Sqrt[1 + (c + d*x)^2])))/7))/(7*e))/d
```

3.229.3.1 Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.229.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{2(dex+ce)^{\frac{7}{2}} a}{7}+2b\left(\frac{(dex+ce)^{\frac{7}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{7}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{5}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{7}-\frac{5e^4\sqrt{dex+ce}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{21}+\frac{5e^4\sqrt{1-\frac{i(\cdots}{\cdots}}\cdots}{\cdots}\right)}{7e}\cdots\right)}{de}$
default	$\frac{\frac{2(dex+ce)^{\frac{7}{2}} a}{7}+2b\left(\frac{(dex+ce)^{\frac{7}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{7}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{5}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{7}-\frac{5e^4\sqrt{dex+ce}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{21}+\frac{5e^4\sqrt{1-\frac{i(\cdots}{\cdots}}\cdots}{\cdots}\right)}{7e}\cdots\right)}{de}$
parts	$\frac{2a(dex+ce)^{\frac{7}{2}}}{7de}+\frac{2b\left(\frac{(dex+ce)^{\frac{7}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{7}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{5}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{7}-\frac{5e^4\sqrt{dex+ce}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{21}+\frac{5e^4\sqrt{1-\cdots}}{\cdots}\right)}{7e}\cdots\right)}{de}$

input
```
int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d/e*(1/7*(d*e*x+c*e)^(7/2)*a+b*(1/7*(d*e*x+c*e)^(7/2)*arcsinh(1/e*(d*e*x
+c*e))-2/7/e*(1/7*e^2*(d*e*x+c*e)^(5/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)-5/21
*e^4*(d*e*x+c*e)^(1/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)+5/21*e^4/(I/e)^(1/2)*
(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*(d*e*x+c*e)^2+1
)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

3.229.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.44

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))\,dx =$$

$$-\frac{2\left(10\sqrt{d^3e}be^2\text{weierstrassPInverse}\left(-\frac{4}{d^2},0,\frac{dx+c}{d}\right)-21\left(bd^5e^2x^3+3\,bcd^4e^2x^2+3\,bc^2d^3e^2x+bc^3d^2e^2\right)\sqrt{dex}\right.}{}$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

output
```
-2/147*(10*sqrt(d^3*e)*b*e^2*weierstrassPInverse(-4/d^2, 0, (d*x + c)/d) -
 21*(b*d^5*e^2*x^3 + 3*b*c*d^4*e^2*x^2 + 3*b*c^2*d^3*e^2*x + b*c^3*d^2*e^2
)*sqrt(d*e*x + c*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(
3*b*d^4*e^2*x^2 + 6*b*c*d^3*e^2*x + (3*b*c^2 - 5*b)*d^2*e^2)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1)*sqrt(d*e*x + c*e) - 21*(a*d^5*e^2*x^3 + 3*a*c*d^4*e^2
*x^2 + 3*a*c^2*d^3*e^2*x + a*c^3*d^2*e^2)*sqrt(d*e*x + c*e))/d^3
```

3.229.6 Sympy [F]

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))\,dx = \int \left(e(c+dx)\right)^{\frac{5}{2}}\left(a+b\operatorname{asinh}\left(c+dx\right)\right)\,dx$$

input
```
integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c)),x)
```

output
```
Integral((e*(c + d*x))**(5/2)*(a + b*asinh(c + d*x)), x)
```

3.229.7 Maxima [F(-2)]

Exception generated.

$$\int (ce + dex)^{5/2}(a + b\operatorname{arcsinh}(c + dx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.229.8 Giac [F]

$$\int (ce + dex)^{5/2}(a + b\operatorname{arcsinh}(c + dx))\,dx = \int (dex + ce)^{\frac{5}{2}}(b\operatorname{arsinh}(dx + c) + a)\,dx$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a), x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int (ce + dex)^{5/2}(a + b\operatorname{arcsinh}(c + dx))\,dx = \int (c\,e + d\,e\,x)^{5/2}\,(a + b\,\operatorname{asinh}(c + d\,x))\,dx$$

input
```
int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x)), x)
```

3.230 $\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))\,dx$

3.230.1 Optimal result

Integrand size = 23, antiderivative size = 261

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))\,dx=-\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d}+\frac{12be\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{25d(1+c+dx)}+\frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))}{5de}-\frac{12be^{3/2}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}E\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{25d\sqrt{1+(c+dx)^2}}+\frac{6be^{3/2}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{25d\sqrt{1+(c+dx)^2}}$$

output
```
2/5*(e*(d*x+c))^(5/2)*(a+b*arcsinh(d*x+c))/d/e-4/25*b*(e*(d*x+c))^(3/2)*(1
+(d*x+c)^2)^(1/2)/d+12/25*b*e*(e*(d*x+c))^(1/2)*(1+(d*x+c)^2)^(1/2)/d/(d*x
+c+1)-12/25*b*e^(3/2)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^
2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticE(sin(2*arctan((
e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/
d/(1+(d*x+c)^2)^(1/2)+6/25*b*e^(3/2)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(
1/2)/e^(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF
(sin(2*arctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x
+c+1)^2)^(1/2)/d/(1+(d*x+c)^2)^(1/2)
```

3.230.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.33

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))\,dx = \frac{2(e(c+dx))^{3/2}\left(5ac+5adx-2b\sqrt{1+(c+dx)^2}+5bc\text{arcsinh}(c+dx)+5bdx\text{arcsi}\right.}{25d}$$

input
```
Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x]),x]
```

output
```
(2*(e*(c + d*x))^(3/2)*(5*a*c + 5*a*d*x - 2*b*Sqrt[1 + (c + d*x)^2] + 5*b*
c*ArcSinh[c + d*x] + 5*b*d*x*ArcSinh[c + d*x] + 2*b*Hypergeometric2F1[1/2,
 3/4, 7/4, -(c + d*x)^2]))/(25*d)
```

3.230.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6274, 6191, 262, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int (e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))}{5e}-\frac{2b\int \frac{(e(c+dx))^{5/2}}{\sqrt{(c+dx)^2+1}}d(c+dx)}{5e}}{d}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))}{5e}-\frac{2b\left(\frac{2}{5}e\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}-\frac{3}{5}e^2\int \frac{\sqrt{e(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{5e}}{d}$$

$$\Big\downarrow \ 266$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))}{5e}-\frac{2b\left(\frac{2}{5}e\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}-\frac{6}{5}e\int\frac{e(c+dx)}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}\right)}{5e}}{d}$$

$$\Big\downarrow \ 834$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))}{5e}-\frac{2b\left(\frac{2}{5}e\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}-\frac{6}{5}e\left(e\int\frac{1}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}-e\int\frac{e-e(c+dx)}{e\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}\right)\right)}{5e}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))}{5e}-\frac{2b\left(\frac{2}{5}e\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}-\frac{6}{5}e\left(e\int\frac{1}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}-\int\frac{e-e(c+dx)}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}\right)\right)}{5e}}{d}$$

$$\Big\downarrow \ 761$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))}{5e}-\frac{2b\left(\frac{2}{5}e\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}-\frac{6}{5}e\left(\frac{\sqrt{e}(e(c+dx)+e)\sqrt{\frac{e^2(c+dx)^2+e^2}{(e(c+dx)+e)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{2\sqrt{(c+dx)^2+1}}\right.\right.}{5e}}{d}$$

$$\Big\downarrow \ 1510$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))}{5e}-\frac{2b\left(\frac{2}{5}e\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}-\frac{6}{5}e\left(\frac{\sqrt{e}(e(c+dx)+e)\sqrt{\frac{e^2(c+dx)^2+e^2}{(e(c+dx)+e)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{2\sqrt{(c+dx)^2+1}}\right.\right.}{5e}}{d}$$

input
```
Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x]),x]
```

output
```
((2*(e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x]))/(5*e) - (2*b*((2*e*(e*(c
 + d*x))^(3/2)*Sqrt[1 + (c + d*x)^2])/5 - (6*e*((e^2*Sqrt[e*(c + d*x)]*Sqr
t[1 + (c + d*x)^2])/(e + e*(c + d*x)) - (Sqrt[e]*(e + e*(c + d*x))*Sqrt[(e
^2 + e^2*(c + d*x)^2)/(e + e*(c + d*x))^2]*EllipticE[2*ArcTan[Sqrt[e*(c +
d*x)]/Sqrt[e]], 1/2])/Sqrt[1 + (c + d*x)^2] + (Sqrt[e]*(e + e*(c + d*x))*S
qrt[(e^2 + e^2*(c + d*x)^2)/(e + e*(c + d*x))^2]*EllipticF[2*ArcTan[Sqrt[e
*(c + d*x)]/Sqrt[e]], 1/2])/(2*Sqrt[1 + (c + d*x)^2])))/5))/(5*e))/d
```

3.230. $\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))\,dx$

3.230.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.230.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\frac{2(dex+ce)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dex+ce)^{\frac{5}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{5}-\frac{3ie^3\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{Ellip}\ldots\right.}{5\sqrt{\frac{i}{e}}\ldots}\right.}{5e}\right.}{de}$
default	$\frac{\frac{2(dex+ce)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dex+ce)^{\frac{5}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{5}-\frac{3ie^3\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{Ellip}\ldots\right.}{5\sqrt{\frac{i}{e}}\ldots}\right.}{5e}\right.}{de}$
parts	$\frac{2a(dex+ce)^{\frac{5}{2}}}{5de}+\frac{2b\left(\frac{(dex+ce)^{\frac{5}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{5}-\frac{3ie^3\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{E}\ldots\right.}{5\ldots}\right.}{5e}\right.}{de}$

input
```
int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d/e*(1/5*(d*e*x+c*e)^(5/2)*a+b*(1/5*(d*e*x+c*e)^(5/2)*arcsinh(1/e*(d*e*x
+c*e))-2/5/e*(1/5*e^2*(d*e*x+c*e)^(3/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)-3/5*
I*e^3/(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e
^2*(d*e*x+c*e)^2+1)^(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-Elli
pticE((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)))))
```

3.230.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.71

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))\,dx =$$

$$-\frac{2\left(6\sqrt{d^3e}\,be\,\text{weierstrassZeta}\left(-\frac{4}{d^2},0,\text{weierstrassPInverse}\left(-\frac{4}{d^2},0,\frac{dx+c}{d}\right)\right)-5\left(bd^3ex^2+2\,bcd^2ex+bc^2de\right)\sqrt{}\right.}{}$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

output
```
-2/25*(6*sqrt(d^3*e)*b*e*weierstrassZeta(-4/d^2, 0, weierstrassPInverse(-4
/d^2, 0, (d*x + c)/d)) - 5*(b*d^3*e*x^2 + 2*b*c*d^2*e*x + b*c^2*d*e)*sqrt(
d*e*x + c*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(b*d^2*e
*x + b*c*d*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*e*x + c*e) - 5*(a*d
^3*e*x^2 + 2*a*c*d^2*e*x + a*c^2*d*e)*sqrt(d*e*x + c*e))/d^2
```

3.230.6 Sympy [F]

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))\,dx = \int (e(c+dx))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(c+dx))\;dx$$

input
```
integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c)),x)
```

output
```
Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x)), x)
```

3.230.7 Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.230.8 Giac [F]

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))\,dx=\int (dex+ce)^{\frac{3}{2}}(b\,\text{arsinh}\,(dx+c)+a)\,dx$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a), x)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))\,dx=\int (c\,e+d\,e\,x)^{3/2}\,(a+b\,\text{asinh}(c+d\,x))\,dx$$

input
```
int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x)), x)
```

3.231 $\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))\,dx$

3.231.1 Optimal result

Integrand size = 23, antiderivative size = 142

$$\begin{aligned}\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))\,dx &= -\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{9d}+\frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))}{3de}\\ &\quad+\frac{2b\sqrt{e}(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{9d\sqrt{1+(c+dx)^2}}\end{aligned}$$

output
```
2/3*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))/d/e-4/9*b*(e*(d*x+c))^(1/2)*(1+
(d*x+c)^2)^(1/2)/d+2/9*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)
))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*arcta
n((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*e^(1/2)*((1+(d*x+c)^2)/(d*x+c+1
)^2)^(1/2)/d/(1+(d*x+c)^2)^(1/2)
```

3.231.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.61

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))\,dx = \frac{2\sqrt{e(c+dx)}\left(3ac+3adx-2b\sqrt{1+(c+dx)^2}+3bc\text{arcsinh}(c+dx)+3bdx\text{arcsinh}(c+dx)+2b\,\text{Hyperg}\right.}{9d}$$

input
```
Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x]),x]
```

output
```
(2*Sqrt[e*(c + d*x)]*(3*a*c + 3*a*d*x - 2*b*Sqrt[1 + (c + d*x)^2] + 3*b*c*
ArcSinh[c + d*x] + 3*b*d*x*ArcSinh[c + d*x] + 2*b*Hypergeometric2F1[1/4, 1
/2, 5/4, -(c + d*x)^2]))/(9*d)
```

3.231.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6274, 6191, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))\,dx$$

$$\Big\downarrow \ 6274$$

$$\frac{\int \sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \ 6191$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))}{3e}-\frac{2b\int \frac{(e(c+dx))^{3/2}}{\sqrt{(c+dx)^2+1}}d(c+dx)}{3e}}{d}$$

$$\Big\downarrow \ 262$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))}{3e}-\frac{2b\left(\frac{2}{3}e\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}-\frac{1}{3}e^2\int \frac{1}{\sqrt{e(c+dx)}\sqrt{(c+dx)^2+1}}d(c+dx)\right)}{3e}}{d}$$

$$\Big\downarrow \ 266$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))}{3e}-\frac{2b\left(\frac{2}{3}e\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}-\frac{2}{3}e\int \frac{1}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}\right)}{3e}}{d}$$

$$\Big\downarrow \ 761$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))}{3e}-\frac{2b\left(\frac{2}{3}e\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}-\frac{\sqrt{e}(e(c+dx)+e)\sqrt{\frac{e^2(c+dx)^2+e^2}{(e(c+dx)+e)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{3\sqrt{(c+dx)^2+1}}\right)}{3e}}{d}$$

input
```
Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x]),x]
```

output
```
((2*(e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x]))/(3*e) - (2*b*((2*e*Sqrt[
e*(c + d*x)]*Sqrt[1 + (c + d*x)^2])/3 - (Sqrt[e]*(e + e*(c + d*x))*Sqrt[(e
^2 + e^2*(c + d*x)^2)/(e + e*(c + d*x))^2]*EllipticF[2*ArcTan[Sqrt[e*(c +
d*x)]/Sqrt[e]], 1/2])/(3*Sqrt[1 + (c + d*x)^2])))/(3*e))/d
```

3.231.3.1 Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.231.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{\frac{2(dex+ce)^{\frac{3}{2}}a}{3}+2b\Bigg(\frac{(dex+ce)^{\frac{3}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3}-\frac{2\Bigg(\frac{e^2\sqrt{dex+ce}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3}-\frac{e^2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\operatorname{EllipticF}(}{3\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{3e}}{de}$
default	$\frac{\frac{2(dex+ce)^{\frac{3}{2}}a}{3}+2b\Bigg(\frac{(dex+ce)^{\frac{3}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3}-\frac{2\Bigg(\frac{e^2\sqrt{dex+ce}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3}-\frac{e^2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\operatorname{EllipticF}(}{3\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{3e}}{de}$
parts	$\frac{2a(dex+ce)^{\frac{3}{2}}}{3de}+\frac{2b\Bigg(\frac{(dex+ce)^{\frac{3}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3}-\frac{2\Bigg(\frac{e^2\sqrt{dex+ce}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3}-\frac{e^2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\operatorname{Ellipt}}{3\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+}}}{3e}}{de}$

input
```
int((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/e*(1/3*(d*e*x+c*e)^(3/2)*a+b*(1/3*(d*e*x+c*e)^(3/2)*arcsinh(1/e*(d*e*x
+c*e))-2/3/e*(1/3*e^2*(d*e*x+c*e)^(1/2)*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)-1/3*
e^2/(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2
*(d*e*x+c*e)^2+1)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)))
```

3.231.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))\,dx =$$
$$-\frac{2\left(2\sqrt{d^2x^2+2\,cdx+c^2+1}\sqrt{dex+ce}bd^2-3\,(bd^3x+bcd^2)\sqrt{dex+ce}\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+\right.\right.}{9\,d^3}$$

input
```
integrate((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

output
```
-2/9*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*e*x + c*e)*b*d^2 - 3*(b*d
^3*x + b*c*d^2)*sqrt(d*e*x + c*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)) - 2*sqrt(d^3*e)*b*weierstrassPInverse(-4/d^2, 0, (d*x + c)/d) - 3
*(a*d^3*x + a*c*d^2)*sqrt(d*e*x + c*e))/d^3
```

3.231.6 Sympy [F]

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))\,dx = \int \sqrt{e\,(c+dx)}(a+b\,\text{asinh}\,(c+dx))\;dx$$

input
```
integrate((a+b*asinh(d*x+c))*(d*e*x+c*e)**(1/2),x)
```

output
```
Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x)), x)
```

3.231.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.231.8 Giac [F]

$$\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))\,dx=\int \sqrt{dex+ce}(b\operatorname{arsinh}(dx+c)+a)\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a), x)
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))\,dx=\int \sqrt{c\,e+d\,e\,x}\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)\,dx$$

input
```
int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x)),x)
```

output
```
int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x)), x)
```

3.232 $\int \frac{a+b\text{arcsinh}(c+dx)}{\sqrt{ce+dex}}\,dx$

3.232.1 Optimal result

Integrand size = 23, antiderivative size = 223

$$\int \frac{a+b\text{arcsinh}(c+dx)}{\sqrt{ce+dex}}\,dx = -\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{de(1+c+dx)} + \frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))}{de} + \frac{4b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}E\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{d\sqrt{e}\sqrt{1+(c+dx)^2}} - \frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{d\sqrt{e}\sqrt{1+(c+dx)^2}}$$

output
```
2*(a+b*arcsinh(d*x+c))*(e*(d*x+c))^(1/2)/d/e-4*b*(e*(d*x+c))^(1/2)*(1+(d*x
+c)^2)^(1/2)/d/e/(d*x+c+1)+4*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e
^(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticE(sin(2
*arctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^
2)^(1/2)/d/e^(1/2)/(1+(d*x+c)^2)^(1/2)-2*b*(d*x+c+1)*(cos(2*arctan((e*(d*x
+c))^(1/2)/e^(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*Ell
ipticF(sin(2*arctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2
)/(d*x+c+1)^2)^(1/2)/d/e^(1/2)/(1+(d*x+c)^2)^(1/2)
```

3.232.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.27

$$\int \frac{a+b\text{arcsinh}(c+dx)}{\sqrt{ce+dex}}\,dx = -\frac{2\sqrt{e(c+dx)}\left(-3(a+b\text{arcsinh}(c+dx))+2b(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-(c+dx)^2\right)\right)}{3de}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])/Sqrt[c*e + d*e*x],x]
```

output
```
(-2*Sqrt[e*(c + d*x)]*(-3*(a + b*ArcSinh[c + d*x]) + 2*b*(c + d*x)*Hyperge
ometric2F1[1/2, 3/4, 7/4, -(c + d*x)^2]))/(3*d*e)
```

3.232.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6274, 6191, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{\sqrt{ce+dex}}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{a+b\text{arcsinh}(c+dx)}{\sqrt{e(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))}{e}-\frac{2b\int \frac{\sqrt{e(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)}{e}}{d}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))}{e}-\frac{4b\int \frac{e(c+dx)}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}}{e^2}}{d}$$

$$\Big\downarrow \; 834$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))}{e}-\frac{4b\left(e\int \frac{1}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}-e\int \frac{e-e(c+dx)}{e\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}\right)}{e^2}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))}{e}-\frac{4b\left(e\int \frac{1}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}-\int \frac{e-e(c+dx)}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}\right)}{e^2}}{d}$$

$$\Big\downarrow \; 761$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))}{e}-\frac{4b\left(\frac{\sqrt{e}(e(c+dx)+e)\sqrt{\frac{e^2(c+dx)^2+e^2}{(e(c+dx)+e)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{2\sqrt{(c+dx)^2+1}}-\int \frac{e-e(c+dx)}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}\right)}{e^2}}{d}$$

$$\Big\downarrow \; 1510$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))}{e}-\frac{4b\left(\frac{\sqrt{e}(e(c+dx)+e)\sqrt{\frac{e^2(c+dx)^2+e^2}{(e(c+dx)+e)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{2\sqrt{(c+dx)^2+1}}-\frac{\sqrt{e}(e(c+dx)+e)\sqrt{\frac{e^2(c+dx)^2+e^2}{(e(c+dx)+e)^2}}E\left(2\text{arc}\right.}{\sqrt{(c+dx)^2+1}}\right.}{e^2}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])/Sqrt[c*e + d*e*x],x]
```

output
```
((2*Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x]))/e - (4*b*((e^2*Sqrt[e*(c +
 d*x)]*Sqrt[1 + (c + d*x)^2])/(e + e*(c + d*x)) - (Sqrt[e]*(e + e*(c + d*x
))*Sqrt[(e^2 + e^2*(c + d*x)^2)/(e + e*(c + d*x))^2]*EllipticE[2*ArcTan[Sq
rt[e*(c + d*x)]/Sqrt[e]], 1/2])/Sqrt[1 + (c + d*x)^2] + (Sqrt[e]*(e + e*(c
 + d*x))*Sqrt[(e^2 + e^2*(c + d*x)^2)/(e + e*(c + d*x))^2]*EllipticF[2*Arc
Tan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2])/(2*Sqrt[1 + (c + d*x)^2])))/e^2)/d
```

3.232. $\int \frac{a+b\text{arcsinh}(c+dx)}{\sqrt{ce+dex}}\,dx$

3.232.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.232. $\int \frac{a+b\,\text{arcsinh}(c+dx)}{\sqrt{ce+dex}}\,dx$

3.232.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{2a\sqrt{dex+ce}+2b\left(\sqrt{dex+ce}\,\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)-\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)-\operatorname{EllipticE}\left(\sqrt{de}\right.\right.}{\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right.}{de}$
default	$\frac{2a\sqrt{dex+ce}+2b\left(\sqrt{dex+ce}\,\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)-\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)-\operatorname{EllipticE}\left(\sqrt{de}\right.\right.}{\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right.}{de}$
parts	$\frac{2a\sqrt{dex+ce}}{de}+\frac{2b\left(\sqrt{dex+ce}\,\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)-\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)-\operatorname{EllipticE}\left(\sqrt{\right.\right.}{\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right.}{de}$

input
```
int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/e*(a*(d*e*x+c*e)^(1/2)+b*((d*e*x+c*e)^(1/2)*arcsinh(1/e*(d*e*x+c*e))-2
*I/(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*
(d*e*x+c*e)^2+1)^(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-Ellipti
cE((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

3.232.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.42

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{\sqrt{ce+dex}}\,dx$$
$$=\frac{2\left(\sqrt{dex+ce}bd\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)+\sqrt{dex+ce}ad+2\sqrt{d^3e}b\,\text{weierstrassZeta}\left(-\frac{4}{d^2},0\right.\right.}{d^2e}$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(d*e*x + c*e)*b*d*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
+ sqrt(d*e*x + c*e)*a*d + 2*sqrt(d^3*e)*b*weierstrassZeta(-4/d^2, 0, weier
strassPInverse(-4/d^2, 0, (d*x + c)/d)))/(d^2*e)
```

3.232.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{\sqrt{ce+dex}}\,dx=\int \frac{a+b\,\text{asinh}\,(c+dx)}{\sqrt{e\,(c+dx)}}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(1/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))/sqrt(e*(c + d*x)), x)
```

3.232.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{\sqrt{ce+dex}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.232.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{\sqrt{ce+dex}}\,dx=\int \frac{b\,\text{arsinh}\,(dx+c)+a}{\sqrt{dex+ce}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)/sqrt(d*e*x + c*e), x)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(c + dx)}{\sqrt{ce + dex}}\, dx = \int \frac{a + b\,\mathrm{asinh}(c + d\,x)}{\sqrt{c\,e + d\,e\,x}}\, dx$$

input
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(1/2),x)
```

output
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(1/2), x)
```

3.233 $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx$

3.233.1 Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx=-\frac{2(a+b\text{arcsinh}(c+dx))}{de\sqrt{e(c+dx)}}+\frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{de^{3/2}\sqrt{1+(c+dx)^2}}$$

output
```
-2*(a+b*arcsinh(d*x+c))/d/e/(e*(d*x+c))^(1/2)+2*b*(d*x+c+1)*(cos(2*arctan(
(e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2
)))*EllipticF(sin(2*arctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d
*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/e^(3/2)/(1+(d*x+c)^2)^(1/2)
```

3.233.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.53

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx=-\frac{2\left(a+b\text{arcsinh}(c+dx)-2b(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-(c+dx)^2\right)\right)}{de\sqrt{e(c+dx)}}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

output
```
(-2*(a + b*ArcSinh[c + d*x] - 2*b*(c + d*x)*Hypergeometric2F1[1/4, 1/2, 5/
4, -(c + d*x)^2]))/(d*e*Sqrt[e*(c + d*x)])
```

3.233.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6274, 6191, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{3/2}}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{a+b\text{arcsinh}(c+dx)}{(e(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2b\int \frac{1}{\sqrt{e(c+dx)}\sqrt{(c+dx)^2+1}}d(c+dx)}{e}-\frac{2(a+b\text{arcsinh}(c+dx))}{e\sqrt{e(c+dx)}}}{d}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{4b\int \frac{1}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}}{e^2}-\frac{2(a+b\text{arcsinh}(c+dx))}{e\sqrt{e(c+dx)}}}{d}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{2b(e(c+dx)+e)\sqrt{\frac{e^2(c+dx)^2+e^2}{(e(c+dx)+e)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{e^{5/2}\sqrt{(c+dx)^2+1}}-\frac{2(a+b\text{arcsinh}(c+dx))}{e\sqrt{e(c+dx)}}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

output
```
((-2*(a + b*ArcSinh[c + d*x]))/(e*Sqrt[e*(c + d*x)]) + (2*b*(e + e*(c + d*
x))*Sqrt[(e^2 + e^2*(c + d*x)^2)/(e + e*(c + d*x))^2]*EllipticF[2*ArcTan[S
qrt[e*(c + d*x)]/Sqrt[e]], 1/2])/(e^(5/2)*Sqrt[1 + (c + d*x)^2]))/d
```

3.233.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.233. $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx$

3.233.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$\frac{-\frac{2a}{\sqrt{dex+ce}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{\sqrt{dex+ce}}+\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\,\sqrt{1+\frac{i(dex+ce)}{e}}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{\frac{i}{e}},i\right)}{e\sqrt{\frac{i}{e}}\,\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right)}{de}$	140
default	$\frac{-\frac{2a}{\sqrt{dex+ce}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{\sqrt{dex+ce}}+\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\,\sqrt{1+\frac{i(dex+ce)}{e}}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{\frac{i}{e}},i\right)}{e\sqrt{\frac{i}{e}}\,\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right)}{de}$	140
parts	$-\frac{2a}{\sqrt{dex+ce}\,de}+\frac{2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{\sqrt{dex+ce}}+\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\,\sqrt{1+\frac{i(dex+ce)}{e}}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{\frac{i}{e}},i\right)}{e\sqrt{\frac{i}{e}}\,\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right)}{de}$	145

input
```
int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/e*(-a/(d*e*x+c*e)^(1/2)+b*(-1/(d*e*x+c*e)^(1/2)*arcsinh(1/e*(d*e*x+c*e
))+2/e/(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/
e^2*(d*e*x+c*e)^2+1)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)))
```

3.233.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx =$$
$$-\frac{2\left(\sqrt{dex+ce}bd^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)+\sqrt{dex+ce}ad^2-2\sqrt{d^3e}(bdx+bc)\text{weierstrassP}\right.}{d^4e^2x+cd^3e^2}$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(d*e*x + c*e)*b*d^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
)) + sqrt(d*e*x + c*e)*a*d^2 - 2*sqrt(d^3*e)*(b*d*x + b*c)*weierstrassPInv
erse(-4/d^2, 0, (d*x + c)/d))/(d^4*e^2*x + c*d^3*e^2)
```

3.233.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx = \int \frac{a+b\,\text{asinh}\,(c+dx)}{(e\,(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(3/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))/(e*(c + d*x))**(3/2), x)
```

3.233.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.233.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx = \int \frac{b\,\text{arsinh}\,(dx+c)+a}{(dex+ce)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^(3/2), x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asinh}(c+d\,x)}{(c\,e+d\,e\,x)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(3/2),x)
```

output
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(3/2), x)
```

3.234 $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{5/2}}\,dx$

3.234.1 Optimal result

Integrand size = 23, antiderivative size = 266

$$\begin{aligned}\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{5/2}}\,dx &= -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}}+\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{3de^3(1+c+dx)} \\ &\quad -\frac{2(a+b\text{arcsinh}(c+dx))}{3de(e(c+dx))^{3/2}}-\frac{4b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}E\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{3de^{5/2}\sqrt{1+(c+dx)^2}} \\ &\quad +\frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{3de^{5/2}\sqrt{1+(c+dx)^2}}\end{aligned}$$

output
```
-2/3*(a+b*arcsinh(d*x+c))/d/e/(e*(d*x+c))^(3/2)-4/3*b*(1+(d*x+c)^2)^(1/2)/
d/e^2/(e*(d*x+c))^(1/2)+4/3*b*(e*(d*x+c))^(1/2)*(1+(d*x+c)^2)^(1/2)/d/e^3/
(d*x+c+1)-4/3*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/
2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticE(sin(2*arctan((e*(d*x
+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/e^(5
/2)/(1+(d*x+c)^2)^(1/2)+2/3*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^
(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*
arctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2
)^(1/2)/d/e^(5/2)/(1+(d*x+c)^2)^(1/2)
```

3.234.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.22

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{5/2}}\,dx = -\frac{2\left(a+b\text{arcsinh}(c+dx)+2b(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-(c+dx)^2\right)\right)}{3de(e(c+dx))^{3/2}}$$

input

```
Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

output

```
(-2*(a + b*ArcSinh[c + d*x] + 2*b*(c + d*x)*Hypergeometric2F1[-1/4, 1/2, 3
/4, -(c + d*x)^2]))/(3*d*e*(e*(c + d*x))^(3/2))
```

3.234.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6274, 6191, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{5/2}}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{a+b\text{arcsinh}(c+dx)}{(e(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2b\int \frac{1}{(e(c+dx))^{3/2}\sqrt{(c+dx)^2+1}}d(c+dx)}{3e}-\frac{2(a+b\text{arcsinh}(c+dx))}{3e(e(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{2b\left(\frac{\int \frac{\sqrt{e(c+dx)}}{\sqrt{(c+dx)^2+1}}d(c+dx)}{e^2}-\frac{2\sqrt{(c+dx)^2+1}}{e\sqrt{e(c+dx)}}\right)}{3e}-\frac{2(a+b\text{arcsinh}(c+dx))}{3e(e(c+dx))^{3/2}}}{d}$$

$\Big\downarrow$ 266

$$\frac{\frac{2b\left(\frac{2\int \frac{e(c+dx)}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}}{e^3}-\frac{2\sqrt{(c+dx)^2+1}}{e\sqrt{e(c+dx)}}\right)}{3e}-\frac{2(a+b\text{arcsinh}(c+dx))}{3e(e(c+dx))^{3/2}}}{d}$$

$\Big\downarrow$ 834

$$\frac{\frac{2b\left(\frac{2\left(e\int \frac{1}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}-e\int \frac{e-e(c+dx)}{e\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}\right)}{e^3}-\frac{2\sqrt{(c+dx)^2+1}}{e\sqrt{e(c+dx)}}\right)}{3e}-\frac{2(a+b\text{arcsinh}(c+dx))}{3e(e(c+dx))^{3/2}}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{2b\left(\frac{2\left(e\int \frac{1}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}-\int \frac{e-e(c+dx)}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}\right)}{e^3}-\frac{2\sqrt{(c+dx)^2+1}}{e\sqrt{e(c+dx)}}\right)}{3e}-\frac{2(a+b\text{arcsinh}(c+dx))}{3e(e(c+dx))^{3/2}}}{d}$$

$\Big\downarrow$ 761

$$\frac{\frac{2b\left(\frac{2\left(\frac{\sqrt{e}(e(c+dx)+e)\sqrt{\frac{e^2(c+dx)^2+e^2}{(e(c+dx)+e)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{2\sqrt{(c+dx)^2+1}}-\int \frac{e-e(c+dx)}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}\right)}{e^3}-\frac{2\sqrt{(c+dx)^2+1}}{e\sqrt{e(c+dx)}}\right)}{3e}-\frac{2(a+b\text{arcsinh}(c+dx))}{3e(e(c+dx))^{3/2}}}{d}$$

$\Big\downarrow$ 1510

$$\frac{\frac{2b\left(\frac{2\left(\frac{\sqrt{e}(e(c+dx)+e)\sqrt{\frac{e^2(c+dx)^2+e^2}{(e(c+dx)+e)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{2\sqrt{(c+dx)^2+1}}-\frac{\sqrt{e}(e(c+dx)+e)\sqrt{\frac{e^2(c+dx)^2+e^2}{(e(c+dx)+e)^2}}E\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{\sqrt{(c+dx)^2+1}}+\frac{e^2\sqrt{(c+dx)^2+1}\cdots}{e(c+dx)\cdots}\cdots}{e^3}\cdots}{3e}\cdots}{d}$$

3.234. $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{5/2}}dx$

input
```
Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

output
```
((-2*(a + b*ArcSinh[c + d*x]))/(3*e*(e*(c + d*x))^(3/2)) + (2*b*((-2*Sqrt[
1 + (c + d*x)^2])/(e*Sqrt[e*(c + d*x)]) + (2*((e^2*Sqrt[e*(c + d*x)]*Sqrt[
1 + (c + d*x)^2])/(e + e*(c + d*x)) - (Sqrt[e]*(e + e*(c + d*x))*Sqrt[(e^2
 + e^2*(c + d*x)^2)/(e + e*(c + d*x))^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*
x)]/Sqrt[e]], 1/2])/Sqrt[1 + (c + d*x)^2] + (Sqrt[e]*(e + e*(c + d*x))*Sqr
t[(e^2 + e^2*(c + d*x)^2)/(e + e*(c + d*x))^2]*EllipticF[2*ArcTan[Sqrt[e*(
c + d*x)]/Sqrt[e]], 1/2])/(2*Sqrt[1 + (c + d*x)^2])))/e^3))/(3*e))/d
```

3.234.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.234. $\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{5/2}}\,dx$

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.234.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{-\frac{2a}{3(dex+ce)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3(dex+ce)^{\frac{3}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3\sqrt{dex+ce}}+\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)-\text{E}\ldots\right.}{3e\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{e}\ldots\right.}{de}$
default	$\frac{-\frac{2a}{3(dex+ce)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3(dex+ce)^{\frac{3}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3\sqrt{dex+ce}}+\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)-\text{E}\ldots\right.}{3e\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{e}\ldots\right.}{de}$
parts	$-\frac{2a}{3(dex+ce)^{\frac{3}{2}}de}+\frac{2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3(dex+ce)^{\frac{3}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3\sqrt{dex+ce}}+\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{EllipticF}\left(\sqrt{dex+ce}\sqrt{\ldots}\right.\right.}{3e\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{e}\ldots\right.}{de}$

input
```
int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/e*(-1/3*a/(d*e*x+c*e)^(3/2)+b*(-1/3/(d*e*x+c*e)^(3/2)*arcsinh(1/e*(d*e
*x+c*e))+2/3/e*(-(1/e^2*(d*e*x+c*e)^2+1)^(1/2)/(d*e*x+c*e)^(1/2)+I/e/(I/e)
^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*(d*e*x+c
*e)^2+1)^(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-EllipticE((d*e*
x+c*e)^(1/2)*(I/e)^(1/2),I)))))
```

3.234.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.68

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{5/2}}\,dx = -\frac{2\left(\sqrt{dex+ce}bd\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)+\sqrt{dex+ce}ad+2\left(bd^2x^2+2\,bcdx+bc^2\right)\sqrt{d^3e}\text{we}\cdots}{3\left(d^4e^3x^2+2\cdots\right.}$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(d*e*x + c*e)*b*d*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
)) + sqrt(d*e*x + c*e)*a*d + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(d^3*e)
*weierstrassZeta(-4/d^2, 0, weierstrassPInverse(-4/d^2, 0, (d*x + c)/d)) +
 2*(b*d^2*x + b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*e*x + c*e))/
(d^4*e^3*x^2 + 2*c*d^3*e^3*x + c^2*d^2*e^3)
```

3.234.6 Sympy [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{5/2}}\,dx = \int \frac{a+b\,\text{asinh}\,(c+dx)}{(e\,(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(5/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))/(e*(c + d*x))**(5/2), x)
```

3.234.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^{5/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.234.8 Giac [F]

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^{5/2}} \, dx = \int \frac{b \operatorname{arsinh}(dx + c) + a}{(dex + ce)^{\frac{5}{2}}} \, dx$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^(5/2), x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^{5/2}} \, dx = \int \frac{a + b \operatorname{asinh}(c + d\,x)}{(c\,e + d\,e\,x)^{5/2}} \, dx$$

input
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(5/2),x)
```

output
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(5/2), x)
```

3.235 $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{7/2}}\,dx$

3.235.1 Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{7/2}}\,dx = -\frac{4b\sqrt{1+(c+dx)^2}}{15de^2(e(c+dx))^{3/2}} - \frac{2(a+b\text{arcsinh}(c+dx))}{5de(e(c+dx))^{5/2}} - \frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{15de^{7/2}\sqrt{1+(c+dx)^2}}$$

output
```
-2/5*(a+b*arcsinh(d*x+c))/d/e/(e*(d*x+c))^(5/2)-4/15*b*(1+(d*x+c)^2)^(1/2)
/d/e^2/(e*(d*x+c))^(3/2)-2/15*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/
e^(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(
2*arctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)
^2)^(1/2)/d/e^(7/2)/(1+(d*x+c)^2)^(1/2)
```

3.235.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.42

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{7/2}}\,dx = \frac{-6(a+b\text{arcsinh}(c+dx)) - 4b(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-(c+}{15de(e(c+dx))^{5/2}}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(7/2),x]
```

output
```
(-6*(a + b*ArcSinh[c + d*x]) - 4*b*(c + d*x)*Hypergeometric2F1[-3/4, 1/2,
1/4, -(c + d*x)^2])/(15*d*e*(e*(c + d*x))^(5/2))
```

3.235.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6274, 6191, 264, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{7/2}}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{a+b\text{arcsinh}(c+dx)}{(e(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2b\int \frac{1}{(e(c+dx))^{5/2}\sqrt{(c+dx)^2+1}}d(c+dx)}{5e}-\frac{2(a+b\text{arcsinh}(c+dx))}{5e(e(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{2b\left(-\frac{\int \frac{1}{\sqrt{e(c+dx)}\sqrt{(c+dx)^2+1}}d(c+dx)}{3e^2}-\frac{2\sqrt{(c+dx)^2+1}}{3e(e(c+dx))^{3/2}}\right)}{5e}-\frac{2(a+b\text{arcsinh}(c+dx))}{5e(e(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{2b\left(-\frac{2\int \frac{1}{\sqrt{(c+dx)^2+1}}d\sqrt{e(c+dx)}}{3e^3}-\frac{2\sqrt{(c+dx)^2+1}}{3e(e(c+dx))^{3/2}}\right)}{5e}-\frac{2(a+b\text{arcsinh}(c+dx))}{5e(e(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{2b\left(-\frac{(e(c+dx)+e)\sqrt{\frac{e^2(c+dx)^2+e^2}{(e(c+dx)+e)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right),\frac{1}{2}\right)}{3e^{7/2}\sqrt{(c+dx)^2+1}}-\frac{2\sqrt{(c+dx)^2+1}}{3e(e(c+dx))^{3/2}}\right)}{5e}-\frac{2(a+b\text{arcsinh}(c+dx))}{5e(e(c+dx))^{5/2}}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(7/2),x]
```

output
```
((-2*(a + b*ArcSinh[c + d*x]))/(5*e*(e*(c + d*x))^(5/2)) + (2*b*((-2*Sqrt[
1 + (c + d*x)^2])/(3*e*(e*(c + d*x))^(3/2)) - ((e + e*(c + d*x))*Sqrt[(e^2
 + e^2*(c + d*x)^2)/(e + e*(c + d*x))^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*
x)]/Sqrt[e]], 1/2])/(3*e^(7/2)*Sqrt[1 + (c + d*x)^2])))/(5*e))/d
```

3.235.3.1 Defintions of rubi rules used

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.235. $\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{7/2}}\,dx$

3.235.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{2a}{5(dex+ce)^{\frac{5}{2}}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5(dex+ce)^{\frac{5}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{15(dex+ce)^{\frac{3}{2}}}-\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)}{15e^2\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{e}\right)}{de}$
default	$\frac{-\frac{2a}{5(dex+ce)^{\frac{5}{2}}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5(dex+ce)^{\frac{5}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{15(dex+ce)^{\frac{3}{2}}}-\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},i\right)}{15e^2\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{e}\right)}{de}$
parts	$-\frac{2a}{5(dex+ce)^{\frac{5}{2}}de}+\frac{2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5(dex+ce)^{\frac{5}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{15(dex+ce)^{\frac{3}{2}}}-\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{\frac{i}{e}},\right.}{15e^2\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{e}\right.}{de}$

input
```
int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/e*(-1/5*a/(d*e*x+c*e)^(5/2)+b*(-1/5/(d*e*x+c*e)^(5/2)*arcsinh(1/e*(d*e
*x+c*e))+2/5/e*(-1/3*(1/e^2*(d*e*x+c*e)^2+1)^(1/2)/(d*e*x+c*e)^(3/2)-1/3/e
^2/(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/(1/e^2*
(d*e*x+c*e)^2+1)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

3.235.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.43

$$\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(ce+dex)^{7/2}}\,dx =$$

$$-\frac{2\left(3\sqrt{dex+ce}bd^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)+3\sqrt{dex+ce}ad^2+2\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2\right.\right.}{15\left(d^6e^4x^3+3\,cd^5e^4x^2+\right.}$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(3*sqrt(d*e*x + c*e)*b*d^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)) + 3*sqrt(d*e*x + c*e)*a*d^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c
^2*d*x + b*c^3)*sqrt(d^3*e)*weierstrassPInverse(-4/d^2, 0, (d*x + c)/d) +
2*(b*d^3*x + b*c*d^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*e*x + c*e))
/(d^6*e^4*x^3 + 3*c*d^5*e^4*x^2 + 3*c^2*d^4*e^4*x + c^3*d^3*e^4)
```

3.235.6 Sympy [F]

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^{7/2}}\,dx = \int \frac{a + b\,\text{asinh}\,(c + dx)}{(e\,(c + dx))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(7/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))/(e*(c + d*x))**(7/2), x)
```

3.235.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{arcsinh}(c + dx)}{(ce + dex)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.235.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{7/2}}\,dx=\int \frac{b\,\text{arsinh}\,(dx+c)+a}{(dex+ce)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^(7/2), x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}(c+dx)}{(ce+dex)^{7/2}}\,dx=\int \frac{a+b\,\text{asinh}(c+d\,x)}{(c\,e+d\,e\,x)^{7/2}}\,dx$$

input
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(7/2),x)
```

output
```
int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(7/2), x)
```

3.236 $\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^2\,dx$

3.236.1 Optimal result

Integrand size = 25, antiderivative size = 134

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^2\,dx = \frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))^2}{9de} - \frac{8b(e(c+dx))^{11/2}(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{4},\frac{15}{4},-(c+dx)^2\right)}{99de^2} + \frac{16b^2(e(c+dx))^{13/2}\,{}_3F_2\left(1,\frac{13}{4},\frac{13}{4};\frac{15}{4},\frac{17}{4};-(c+dx)^2\right)}{1287de^3}$$

output
```
2/9*(e*(d*x+c))^(9/2)*(a+b*arcsinh(d*x+c))^2/d/e-8/99*b*(e*(d*x+c))^(11/2)
*(a+b*arcsinh(d*x+c))*hypergeom([1/2, 11/4],[15/4],-(d*x+c)^2)/d/e^2+16/12
87*b^2*(e*(d*x+c))^(13/2)*hypergeom([1, 13/4, 13/4],[15/4, 17/4],-(d*x+c)^
2)/d/e^3
```

3.236.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.82

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^2\,dx = \frac{2(e(c+dx))^{9/2}\left(143(a+b\text{arcsinh}(c+dx))^2-52b(c+dx)(a+b\text{arcsinh}(c+dx))\,\text{H}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(2*(e*(c + d*x))^(9/2)*(143*(a + b*ArcSinh[c + d*x])^2 - 52*b*(c + d*x)*(a
 + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 11/4, 15/4, -(c + d*x)^2] +
8*b^2*(c + d*x)^2*HypergeometricPFQ[{1, 13/4, 13/4}, {15/4, 17/4}, -(c + d
*x)^2]))/(1287*d*e)
```

3.236.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6274, 6191, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^2\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int (e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))^2}{9e}-\frac{4b\int \frac{(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)}{9e}}{d}$$

$$\Big\downarrow \text{6232}$$

$$\frac{\frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))^2}{9e}-\frac{4b\left(\frac{2(e(c+dx))^{11/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{4},\frac{15}{4},-(c+dx)^2\right)(a+b\text{arcsinh}(c+dx))}{11e}-\frac{4b(e(c+dx))^{13/2}{}_3F_2\left(1,\frac{1}{\cdots}\right.}{143\cdots}\right.}{9e}}{d}$$

input
```
Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
((2*(e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^2)/(9*e) - (4*b*((2*(e*(c
 + d*x))^(11/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 11/4, 15/4
, -(c + d*x)^2])/(11*e) - (4*b*(e*(c + d*x))^(13/2)*HypergeometricPFQ[{1,
13/4, 13/4}, {15/4, 17/4}, -(c + d*x)^2])/(143*e^2)))/(9*e))/d
```

3.236.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.236.4 Maple [F]

$$\int (dex+ce)^{\frac{7}{2}} (a+b\,\mathrm{arcsinh}\,(dx+c))^2\,dx$$

input
```
int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x)
```

output
```
int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x)
```

3.236.5 Fricas [F]

$$\int (ce+dex)^{7/2}(a+b\mathrm{arcsinh}(c+dx))^2\,dx = \int (dex+ce)^{\frac{7}{2}}(b\,\mathrm{arsinh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((a^2*d^3*e^3*x^3 + 3*a^2*c*d^2*e^3*x^2 + 3*a^2*c^2*d*e^3*x + a^2*
c^3*e^3 + (b^2*d^3*e^3*x^3 + 3*b^2*c*d^2*e^3*x^2 + 3*b^2*c^2*d*e^3*x + b^2
*c^3*e^3)*arcsinh(d*x + c)^2 + 2*(a*b*d^3*e^3*x^3 + 3*a*b*c*d^2*e^3*x^2 +
3*a*b*c^2*d*e^3*x + a*b*c^3*e^3)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

3.236.6 Sympy [F(-1)]

Timed out.

$$\int (ce + dex)^{7/2}(a + b\operatorname{arcsinh}(c + dx))^2\,dx = \text{Timed out}$$

input
```
integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c))**2,x)
```

output
```
Timed out
```

3.236.7 Maxima [F(-2)]

Exception generated.

$$\int (ce + dex)^{7/2}(a + b\operatorname{arcsinh}(c + dx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.236.8 Giac [F]

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^2\,dx=\int (dex+ce)^{\frac{7}{2}}(b\,\text{arsinh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a)^2, x)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^2\,dx=\int (c\,e+d\,e\,x)^{7/2}\,(a+b\,\text{asinh}(c+d\,x))^2\,dx$$

input
```
int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^2, x)
```

3.237 $\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^2\,dx$

3.237.1 Optimal result

Integrand size = 25, antiderivative size = 134

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^2\,dx = \frac{2(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^2}{7de} - \frac{8b(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{4},\frac{13}{4},-(c+dx)^2\right)}{63de^2} + \frac{16b^2(e(c+dx))^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};-(c+dx)^2\right)}{693de^3}$$

output
```
2/7*(e*(d*x+c))^(7/2)*(a+b*arcsinh(d*x+c))^2/d/e-8/63*b*(e*(d*x+c))^(9/2)*
(a+b*arcsinh(d*x+c))*hypergeom([1/2, 9/4],[13/4],-(d*x+c)^2)/d/e^2+16/693*
b^2*(e*(d*x+c))^(11/2)*hypergeom([1, 11/4, 11/4],[13/4, 15/4],-(d*x+c)^2)/
d/e^3
```

3.237.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.82

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^2\,dx = \frac{2(e(c+dx))^{7/2}\left(99(a+b\text{arcsinh}(c+dx))^2-44b(c+dx)(a+b\text{arcsinh}(c+dx))\,\text{Hy}\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(2*(e*(c + d*x))^(7/2)*(99*(a + b*ArcSinh[c + d*x])^2 - 44*b*(c + d*x)*(a
+ b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 9/4, 13/4, -(c + d*x)^2] + 8*
b^2*(c + d*x)^2*HypergeometricPFQ[{1, 11/4, 11/4}, {13/4, 15/4}, -(c + d*x
)^2]))/(693*d*e)
```

3.237.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6274, 6191, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^2\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int (e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \; 6191$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^2}{7e}-\frac{4b\int \frac{(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)}{7e}}{d}$$

$$\Big\downarrow \; 6232$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^2}{7e}-\frac{4b\left(\frac{2(e(c+dx))^{9/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{4},\frac{13}{4},-(c+dx)^2\right)(a+b\text{arcsinh}(c+dx))}{9e}-\frac{4b(e(c+dx))^{11/2}{}_3F_2\left(1,\frac{11}{4},\right.}{99e^2}\right.}{7e}}{d}$$

input
```
Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
((2*(e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^2)/(7*e) - (4*b*((2*(e*(c
 + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 9/4, 13/4,
-(c + d*x)^2])/(9*e) - (4*b*(e*(c + d*x))^(11/2)*HypergeometricPFQ[{1, 11/
4, 11/4}, {13/4, 15/4}, -(c + d*x)^2])/(99*e^2)))/(7*e))/d
```

3.237.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.237.4 Maple [F]

$$\int (dex+ce)^{\frac{5}{2}} (a+b \operatorname{arcsinh}(dx+c))^2 \, dx$$

input
```
int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x)
```

output
```
int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x)
```

3.237.5 Fricas [F]

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^2 \, dx = \int (dex+ce)^{\frac{5}{2}} (b \operatorname{arsinh}(dx+c)+a)^2 \, dx$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((a^2*d^2*e^2*x^2 + 2*a^2*c*d*e^2*x + a^2*c^2*e^2 + (b^2*d^2*e^2*x
^2 + 2*b^2*c*d*e^2*x + b^2*c^2*e^2)*arcsinh(d*x + c)^2 + 2*(a*b*d^2*e^2*x^
2 + 2*a*b*c*d*e^2*x + a*b*c^2*e^2)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

3.237.6 Sympy [F]

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^2\,dx=\int (e(c+dx))^{\frac{5}{2}}\,(a+b\,\text{asinh}\,(c+dx))^2\,dx$$

input
```
integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c))**2,x)
```

output
```
Integral((e*(c + d*x))**(5/2)*(a + b*asinh(c + d*x))**2, x)
```

3.237.7 Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^2\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.237.8 Giac [F]

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^2\,dx=\int (dex+ce)^{\frac{5}{2}}(b\,\text{arsinh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a)^2, x)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int (ce + dex)^{5/2}(a + b\text{arcsinh}(c + dx))^2 \, dx = \int (c\,e + d\,e\,x)^{5/2}\,(a + b\,\text{asinh}(c + d\,x))^2 \, dx$$

input
```
int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^2, x)
```

3.238 $\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^2\,dx$

3.238.1 Optimal result

Integrand size = 25, antiderivative size = 134

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^2\,dx = \frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^2}{5de} - \frac{8b(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{4},\frac{11}{4},-(c+dx)^2\right)}{35de^2} + \frac{16b^2(e(c+dx))^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};-(c+dx)^2\right)}{315de^3}$$

output
```
2/5*(e*(d*x+c))^(5/2)*(a+b*arcsinh(d*x+c))^2/d/e-8/35*b*(e*(d*x+c))^(7/2)*
(a+b*arcsinh(d*x+c))*hypergeom([1/2, 7/4],[11/4],-(d*x+c)^2)/d/e^2+16/315*
b^2*(e*(d*x+c))^(9/2)*hypergeom([1, 9/4, 9/4],[11/4, 13/4],-(d*x+c)^2)/d/e
^3
```

3.238.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.82

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^2\,dx = \frac{2(e(c+dx))^{5/2}\left(63(a+b\text{arcsinh}(c+dx))^2-36b(c+dx)(a+b\text{arcsinh}(c+dx))\,\text{Hy}\right.}{3}$$

input
```
Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(2*(e*(c + d*x))^(5/2)*(63*(a + b*ArcSinh[c + d*x])^2 - 36*b*(c + d*x)*(a
+ b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 7/4, 11/4, -(c + d*x)^2] + 8*
b^2*(c + d*x)^2*HypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, -(c + d*x)^
2]))/(315*d*e)
```

3.238.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6274, 6191, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce + dex)^{3/2}(a + b\text{arcsinh}(c + dx))^2 \, dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int (e(c + dx))^{3/2}(a + b\text{arcsinh}(c + dx))^2 d(c + dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^2}{5e} - \frac{4b\int \frac{(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)}{5e}}{d}$$

$$\Big\downarrow \text{6232}$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^2}{5e} - \frac{4b\left(\frac{2(e(c+dx))^{7/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{4},\frac{11}{4},-(c+dx)^2\right)(a+b\text{arcsinh}(c+dx))}{7e} - \frac{4b(e(c+dx))^{9/2}{}_3F_2\left(1,\frac{9}{4},\frac{9}{4}\right.}{63e^2}}{5e}}{d}$$

input
```
Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
((2*(e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^2)/(5*e) - (4*b*((2*(e*(c
 + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 7/4, 11/4,
-(c + d*x)^2])/(7*e) - (4*b*(e*(c + d*x))^(9/2)*HypergeometricPFQ[{1, 9/4,
 9/4}, {11/4, 13/4}, -(c + d*x)^2])/(63*e^2)))/(5*e))/d
```

3.238.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.238.4 Maple [F]

$$\int (dex+ce)^{\frac{3}{2}} (a+b\operatorname{arcsinh}(dx+c))^2 dx$$

input
```
int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x)
```

output
```
int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x)
```

3.238.5 Fricas [F]

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^2 dx = \int (dex+ce)^{\frac{3}{2}}(b\operatorname{arsinh}(dx+c)+a)^2 dx$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((a^2*d*e*x + a^2*c*e + (b^2*d*e*x + b^2*c*e)*arcsinh(d*x + c)^2 +
 2*(a*b*d*e*x + a*b*c*e)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

3.238.6 Sympy [F]

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^2\,dx=\int \left(e(c+dx)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(c+dx\right)\right)^2\,dx$$

input
```
integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c))**2,x)
```

output
```
Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x))**2, x)
```

3.238.7 Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^2\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.238.8 Giac [F]

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^2\,dx=\int (dex+ce)^{\frac{3}{2}}\left(b\operatorname{arsinh}\left(dx+c\right)+a\right)^2\,dx$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a)^2, x)
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^2\,dx=\int (c\,e+d\,e\,x)^{3/2}\,(a+b\,\mathrm{asinh}(c+d\,x))^2\,dx$$

input
```
int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^2, x)
```

3.239 $\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^2\,dx$

3.239.1 Optimal result

Integrand size = 25, antiderivative size = 134

$$\begin{aligned}\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^2\,dx &= \frac{2(e(c+dx))^{3/2}(a+b\operatorname{arcsinh}(c+dx))^2}{3de} \\ &\quad -\frac{8b(e(c+dx))^{5/2}(a+b\operatorname{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},-(c+dx)^2\right)}{15de^2} \\ &\quad +\frac{16b^2(e(c+dx))^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};-(c+dx)^2\right)}{105de^3}\end{aligned}$$

output
```
2/3*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))^2/d/e-8/15*b*(e*(d*x+c))^(5/2)*
(a+b*arcsinh(d*x+c))*hypergeom([1/2, 5/4],[9/4],-(d*x+c)^2)/d/e^2+16/105*b
^2*(e*(d*x+c))^(7/2)*hypergeom([1, 7/4, 7/4],[9/4, 11/4],-(d*x+c)^2)/d/e^3
```

3.239.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.82

$$\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^2\,dx = \frac{2(e(c+dx))^{3/2}\left(35(a+b\operatorname{arcsinh}(c+dx))^2-28b(c+dx)(a+b\operatorname{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4}\right.\right.}{105de}$$

input
```
Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
(2*(e*(c + d*x))^(3/2)*(35*(a + b*ArcSinh[c + d*x])^2 - 28*b*(c + d*x)*(a
+ b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 5/4, 9/4, -(c + d*x)^2] + 8*b
^2*(c + d*x)^2*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, -(c + d*x)^2]
))/(105*d*e)
```

3.239.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6274, 6191, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^2\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \; 6191$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^2}{3e}-\frac{4b\int \frac{(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)}{3e}}{d}$$

$$\Big\downarrow \; 6232$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^2}{3e}-\frac{4b\left(\frac{2(e(c+dx))^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},-(c+dx)^2\right)(a+b\text{arcsinh}(c+dx))}{5e}-\frac{4b(e(c+dx))^{7/2}{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4}\ldots\right)}{35e^2}\right)}{3e}}{d}$$

input
```
Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^2,x]
```

output
```
((2*(e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^2)/(3*e) - (4*b*((2*(e*(c
 + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 5/4, 9/4, -
(c + d*x)^2])/(5*e) - (4*b*(e*(c + d*x))^(7/2)*HypergeometricPFQ[{1, 7/4,
7/4}, {9/4, 11/4}, -(c + d*x)^2])/(35*e^2)))/(3*e))/d
```

3.239.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.239.4 Maple [F]

$$\int (a + b \operatorname{arcsinh}(dx + c))^2 \sqrt{dex + ce}dx$$

input
```
int((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x)
```

3.239.5 Fricas [F]

$$\int \sqrt{ce + dex}(a + b\mathrm{arcsinh}(c + dx))^2 \, dx = \int \sqrt{dex + ce}(b \operatorname{arsinh}(dx + c) + a)^2 \, dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*e*
x + c*e), x)
```

3.239.6 Sympy [F]

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^2\,dx=\int \sqrt{e\,(c+dx)}(a+b\operatorname{asinh}\,(c+dx))^2\;dx$$

input
```
integrate((a+b*asinh(d*x+c))**2*(d*e*x+c*e)**(1/2),x)
```

output
```
Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x))**2, x)
```

3.239.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^2\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.239.8 Giac [F]

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^2\,dx=\int \sqrt{dex+ce}(b\,\text{arsinh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^2, x)
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^2\,dx=\int \sqrt{c\,e+d\,e\,x}\,\left(a+b\,\text{asinh}(c+d\,x)\right)^2\,dx$$

input
```
int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^2, x)
```

3.240 $\int \frac{(a+b\text{arcsinh}(c+dx))^2}{\sqrt{ce+dex}}\,dx$

3.240.1 Optimal result

Integrand size = 25, antiderivative size = 132

$$\begin{aligned}\int \frac{(a+b\text{arcsinh}(c+dx))^2}{\sqrt{ce+dex}}\,dx &= \frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^2}{de}\\ &\quad-\frac{8b(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-(c+dx)^2\right)}{3de^2}\\ &\quad+\frac{16b^2(e(c+dx))^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};-(c+dx)^2\right)}{15de^3}\end{aligned}$$

output
```
-8/3*b*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))*hypergeom([1/2, 3/4],[7/4],-
(d*x+c)^2)/d/e^2+16/15*b^2*(e*(d*x+c))^(5/2)*hypergeom([1, 5/4, 5/4],[7/4,
 9/4],-(d*x+c)^2)/d/e^3+2*(a+b*arcsinh(d*x+c))^2*(e*(d*x+c))^(1/2)/d/e
```

3.240.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{\sqrt{ce+dex}}\,dx = \frac{2\sqrt{e(c+dx)}\left(15(a+b\text{arcsinh}(c+dx))^2-20b(c+dx)(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4}\right.\right.}{15de}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
```

output
```
(2*Sqrt[e*(c + d*x)]*(15*(a + b*ArcSinh[c + d*x])^2 - 20*b*(c + d*x)*(a +
b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4, -(c + d*x)^2] + 8*b^2
*(c + d*x)^2*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, -(c + d*x)^2]))/
(15*d*e)
```

3.240.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6274, 6191, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{\sqrt{ce+dex}}\,dx$$

$$\Big\downarrow \text{ 6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^2}{\sqrt{e(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6191}$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^2}{e}-\frac{4b\int \frac{\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))}{\sqrt{(c+dx)^2+1}}d(c+dx)}{e}}{d}$$

$$\Big\downarrow \text{ 6232}$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^2}{e}-\frac{4b\left(\frac{2(e(c+dx))^{3/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-(c+dx)^2\right)(a+b\text{arcsinh}(c+dx))}{3e}-\frac{4b(e(c+dx))^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4}\right.}{15e^2}\right.}{e}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
```

output
```
((2*Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^2)/e - (4*b*((2*(e*(c + d*x
))^(3/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4, -(c + d
*x)^2])/(3*e) - (4*b*(e*(c + d*x))^(5/2)*HypergeometricPFQ[{1, 5/4, 5/4},
{7/4, 9/4}, -(c + d*x)^2])/(15*e^2)))/e)/d
```

3.240. $\int \frac{(a+b\text{arcsinh}(c+dx))^2}{\sqrt{ce+dex}}\,dx$

3.240.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.240.4 Maple [F]

$$\int \frac{(a + b\ \mathrm{arcsinh}\,(dx + c))^2}{\sqrt{dex + ce}}dx$$

input
```
int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
```

3.240.5 Fricas [F]

$$\int \frac{(a + b\mathrm{arcsinh}(c + dx))^2}{\sqrt{ce + dex}}\,dx = \int \frac{(b\,\mathrm{arsinh}\,(dx + c) + a)^2}{\sqrt{dex + ce}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)/sqrt(d*e*
x + c*e), x)
```

3.240.6 Sympy [F]

$$\int \frac{(a + b\text{arcsinh}(c + dx))^2}{\sqrt{ce + dex}}\,dx = \int \frac{(a + b\operatorname{asinh}(c + dx))^2}{\sqrt{e(c + dx)}}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(1/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**2/sqrt(e*(c + d*x)), x)
```

3.240.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\text{arcsinh}(c + dx))^2}{\sqrt{ce + dex}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.240.8 Giac [F]

$$\int \frac{(a + b\text{arcsinh}(c + dx))^2}{\sqrt{ce + dex}}\,dx = \int \frac{(b\,\text{arsinh}(dx + c) + a)^2}{\sqrt{dex + ce}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^2/sqrt(d*e*x + c*e), x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{\sqrt{ce+dex}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^2}{\sqrt{c\,e+d\,e\,x}}\,dx$$

input
```
int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(1/2),x)
```

output
```
int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(1/2), x)
```

3.241 $\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx$

3.241.1 Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx=-\frac{2(a+b\text{arcsinh}(c+dx))^2}{de\sqrt{e(c+dx)}}+\frac{8b\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-(c+dx)^2\right)}{de^2}-\frac{16b^2(e(c+dx))^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};-(c+dx)^2\right)}{3de^3}$$

output
```
-16/3*b^2*(e*(d*x+c))^(3/2)*hypergeom([3/4, 3/4, 1],[5/4, 7/4],-(d*x+c)^2)
/d/e^3-2*(a+b*arcsinh(d*x+c))^2/d/e/(e*(d*x+c))^(1/2)+8*b*(a+b*arcsinh(d*x
+c))*hypergeom([1/4, 1/2],[5/4],-(d*x+c)^2)*(e*(d*x+c))^(1/2)/d/e^2
```

3.241.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx=\frac{2\left(-3(a+b\text{arcsinh}(c+dx))^2-4b(c+dx)\left(-3(a+b\text{arcsinh}(c+dx))\,\text{Hyp}\right.\right.}{3de\sqrt{}}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
```

output
```
(2*(-3*(a + b*ArcSinh[c + d*x])^2 - 4*b*(c + d*x)*(-3*(a + b*ArcSinh[c + d
*x])*Hypergeometric2F1[1/4, 1/2, 5/4, -(c + d*x)^2] + 2*b*(c + d*x)*Hyperg
eometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, -(c + d*x)^2])))/(3*d*e*Sqrt[e*(c +
 d*x)])
```

3.241.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6274, 6191, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(e(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{4b\int \frac{a+b\text{arcsinh}(c+dx)}{\sqrt{e(c+dx)}\sqrt{(c+dx)^2+1}}d(c+dx)}{e}-\frac{2(a+b\text{arcsinh}(c+dx))^2}{e\sqrt{e(c+dx)}}}{d}$$

$$\Big\downarrow \text{6232}$$

$$\frac{\frac{4b\left(\frac{2\sqrt{e(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-(c+dx)^2\right)(a+b\text{arcsinh}(c+dx))}{e}-\frac{4b(e(c+dx))^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};-(c+dx)^2\right)}{3e^2}\right)}{e}-\frac{2(a+b\text{arcsinh}(c+dx))^2}{e\sqrt{e(c+dx)}}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
```

output
```
((-2*(a + b*ArcSinh[c + d*x])^2)/(e*Sqrt[e*(c + d*x)]) + (4*b*((2*Sqrt[e*(
c + d*x)]*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/4, 1/2, 5/4, -(c +
d*x)^2])/e - (4*b*(e*(c + d*x))^(3/2)*HypergeometricPFQ[{3/4, 3/4, 1}, {5/
4, 7/4}, -(c + d*x)^2])/(3*e^2)))/e)/d
```

3.241.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.241.4 Maple [F]

$$\int \frac{(a + b \operatorname{arcsinh}(dx + c))^2}{(dex + ce)^{\frac{3}{2}}} dx$$

input
```
int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

3.241.5 Fricas [F]

$$\int \frac{(a + b\operatorname{arcsinh}(c + dx))^2}{(ce + dex)^{3/2}} dx = \int \frac{(b \operatorname{arsinh}(dx + c) + a)^2}{(dex + ce)^{\frac{3}{2}}} dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*e*
x + c*e)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

3.241.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx=\int \frac{(a+b\operatorname{asinh}{(c+dx)})^2}{(e\,(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(3/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**2/(e*(c + d*x))**(3/2), x)
```

3.241.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.241.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx=\int \frac{(b\operatorname{arsinh}(dx+c)+a)^2}{(dex+ce)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^(3/2), x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^2}{(c\,e+d\,e\,x)^{3/2}}\,dx$$

input
```
int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(3/2),x)
```

output
```
int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(3/2), x)
```

3.242 $\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx$

3.242.1 Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx = -\frac{2(a+b\text{arcsinh}(c+dx))^2}{3de(e(c+dx))^{3/2}} - \frac{8b(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-(c+dx)^2\right)}{3de^2\sqrt{e(c+dx)}} + \frac{16b^2\sqrt{e(c+dx)}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};-(c+dx)^2\right)}{3de^3}$$

output
```
-2/3*(a+b*arcsinh(d*x+c))^2/d/e/(e*(d*x+c))^(3/2)-8/3*b*(a+b*arcsinh(d*x+c
))*hypergeom([-1/4, 1/2],[3/4],-(d*x+c)^2)/d/e^2/(e*(d*x+c))^(1/2)+16/3*b^
2*hypergeom([1/4, 1/4, 1],[3/4, 5/4],-(d*x+c)^2)*(e*(d*x+c))^(1/2)/d/e^3
```

3.242.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx = -\frac{2\left((a+b\text{arcsinh}(c+dx))^2+4b(c+dx)\left((a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-(c+dx)^2\right.\right.\right.}{3de(e(c+dx))^{3/2}}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]
```

output
```
(-2*((a + b*ArcSinh[c + d*x])^2 + 4*b*(c + d*x)*((a + b*ArcSinh[c + d*x])*
Hypergeometric2F1[-1/4, 1/2, 3/4, -(c + d*x)^2] - 2*b*(c + d*x)*Hypergeome
tricPFQ[{1/4, 1/4, 1}, {3/4, 5/4}, -(c + d*x)^2])))/(3*d*e*(e*(c + d*x))^(
3/2))
```

3.242.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6274, 6191, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}} \, dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(e(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{4b\int \frac{a+b\text{arcsinh}(c+dx)}{(e(c+dx))^{3/2}\sqrt{(c+dx)^2+1}}d(c+dx)}{3e}-\frac{2(a+b\text{arcsinh}(c+dx))^2}{3e(e(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{6232}$$

$$\frac{\frac{4b\left(\frac{4b\sqrt{e(c+dx)}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};-(c+dx)^2\right)}{e^2}-\frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-(c+dx)^2\right)(a+b\text{arcsinh}(c+dx))}{e\sqrt{e(c+dx)}}\right)}{3e}-\frac{2(a+b\text{arcsinh}(c+dx))^2}{3e(e(c+dx))^{3/2}}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]
```

output
```
((-2*(a + b*ArcSinh[c + d*x])^2)/(3*e*(e*(c + d*x))^(3/2)) + (4*b*((-2*(a
+ b*ArcSinh[c + d*x])*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c + d*x)^2])/(e*
Sqrt[e*(c + d*x)]) + (4*b*Sqrt[e*(c + d*x)]*HypergeometricPFQ[{1/4, 1/4, 1
}, {3/4, 5/4}, -(c + d*x)^2])/e^2))/(3*e))/d
```

3.242.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.242.4 Maple [F]

$$\int \frac{(a + b \operatorname{arcsinh}(dx + c))^2}{(dex + ce)^{\frac{5}{2}}} dx$$

input
```
int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

3.242.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^2}{(dex+ce)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*e*
x + c*e)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

3.242.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx=\int \frac{(a+b\,\text{asinh}\,(c+dx))^2}{(e\,(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(5/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**2/(e*(c + d*x))**(5/2), x)
```

3.242.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.242.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^2}{(dex+ce)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^(5/2), x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{5/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^2}{(c\,e+d\,e\,x)^{5/2}}\,dx$$

input
```
int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(5/2),x)
```

output
```
int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(5/2), x)
```

3.243 $\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx$

3.243.1 Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx = -\frac{2(a+b\text{arcsinh}(c+dx))^2}{5de(e(c+dx))^{5/2}} - \frac{8b(a+b\text{arcsinh}(c+dx))\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-(c+dx)^2\right)}{15de^2(e(c+dx))^{3/2}} - \frac{16b^2\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},1;\frac{1}{4},\frac{3}{4};-(c+dx)^2\right)}{15de^3\sqrt{e(c+dx)}}$$

output
```
-2/5*(a+b*arcsinh(d*x+c))^2/d/e/(e*(d*x+c))^(5/2)-8/15*b*(a+b*arcsinh(d*x+
c))*hypergeom([-3/4, 1/2],[1/4],-(d*x+c)^2)/d/e^2/(e*(d*x+c))^(3/2)-16/15*
b^2*hypergeom([-1/4, -1/4, 1],[1/4, 3/4],-(d*x+c)^2)/d/e^3/(e*(d*x+c))^(1/
2)
```

3.243.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.82

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx = -\frac{2\left((a+b\text{arcsinh}(c+dx))\left(3(a+b\text{arcsinh}(c+dx))+4b(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-(c+dx)^2\right)\right.\right.}{15de(e(c+dx))^{5/2}}$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(7/2),x]
```

output
```
(-2*((a + b*ArcSinh[c + d*x])*(3*(a + b*ArcSinh[c + d*x]) + 4*b*(c + d*x)*
Hypergeometric2F1[-3/4, 1/2, 1/4, -(c + d*x)^2]) + 8*b^2*(c + d*x)^2*Hyper
geometricPFQ[{-1/4, -1/4, 1}, {1/4, 3/4}, -(c + d*x)^2]))/(15*d*e*(e*(c +
d*x))^(5/2))
```

3.243.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6274, 6191, 6232}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{(a+b\operatorname{arcsinh}(c+dx))^2}{(e(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{4b\int \frac{a+b\operatorname{arcsinh}(c+dx)}{(e(c+dx))^{5/2}\sqrt{(c+dx)^2+1}}d(c+dx)}{5e}-\frac{2(a+b\operatorname{arcsinh}(c+dx))^2}{5e(e(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \text{6232}$$

$$\frac{\frac{4b\left(-\frac{4b\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},1;\frac{1}{4},\frac{3}{4};-(c+dx)^2\right)}{3e^2\sqrt{e(c+dx)}}-\frac{2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-(c+dx)^2\right)(a+b\operatorname{arcsinh}(c+dx))}{3e(e(c+dx))^{3/2}}\right)}{5e}-\frac{2(a+b\operatorname{arcsinh}(c+dx))^2}{5e(e(c+dx))^{5/2}}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(7/2),x]
```

output
```
((-2*(a + b*ArcSinh[c + d*x])^2)/(5*e*(e*(c + d*x))^(5/2)) + (4*b*((-2*(a
+ b*ArcSinh[c + d*x])*Hypergeometric2F1[-3/4, 1/2, 1/4, -(c + d*x)^2])/(3*
e*(e*(c + d*x))^(3/2)) - (4*b*HypergeometricPFQ[{-1/4, -1/4, 1}, {1/4, 3/4
}, -(c + d*x)^2])/(3*e^2*Sqrt[e*(c + d*x)])))/(5*e))/d
```

3.243.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6232
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.243.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(dx+c))^2}{(dex+ce)^{\frac{7}{2}}}dx$$

input
```
int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x)
```

3.243.5 Fricas [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^2}{(dex+ce)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

output
```
integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*e*
x + c*e)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*
x + c^4*e^4), x)
```

3.243.6 Sympy [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(c+dx))^2}{(e\,(c+dx))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(7/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**2/(e*(c + d*x))**(7/2), x)
```

3.243.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.243.8 Giac [F]

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^2}{(dex+ce)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^(7/2), x)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(ce+dex)^{7/2}}\,dx = \int \frac{(a+b\,\text{asinh}(c+d\,x))^2}{(c\,e+d\,e\,x)^{7/2}}\,dx$$

input
```
int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(7/2),x)
```

output
```
int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(7/2), x)
```

3.244 $\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^3\,dx$

3.244.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^3\,dx=\frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))^3}{9de}-\frac{2b\text{Int}\left(\frac{(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{1+(c+dx)^2}},x\right)}{3e}$$

output
```
2/9*(e*(d*x+c))^(9/2)*(a+b*arcsinh(d*x+c))^3/d/e-2/3*b*Unintegrable((e*(d*
x+c))^(9/2)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e
```

3.244.2 Mathematica [N/A]

Not integrable

Time = 74.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^3\,dx=\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^3\,dx$$

input
```
Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^3, x]
```

3.244.3 Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^3\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int (e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))^3}{9e}-\frac{2b\int \frac{(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)}{3e}}{d}$$

$$\Big\downarrow \text{6239}$$

$$\frac{\frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))^3}{9e}-\frac{2b\int \frac{(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)}{3e}}{d}$$

input
```
Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
$Aborted
```

3.244.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.244.4 Maple [N/A] (verified)

Not integrable

Time = 0.91 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (dex + ce)^{\frac{7}{2}} (a + b \operatorname{arcsinh}(dx + c))^3 \, dx$$

input
```
int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x)
```

output
```
int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x)
```

3.244.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 256, normalized size of antiderivative = 10.24

$$\int (ce + dex)^{7/2} (a + b\operatorname{arcsinh}(c + dx))^3 \, dx = \int (dex + ce)^{\frac{7}{2}} (b \operatorname{arsinh}(dx + c) + a)^3 \, dx$$

input
```
integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((a^3*d^3*e^3*x^3 + 3*a^3*c*d^2*e^3*x^2 + 3*a^3*c^2*d*e^3*x + a^3*
c^3*e^3 + (b^3*d^3*e^3*x^3 + 3*b^3*c*d^2*e^3*x^2 + 3*b^3*c^2*d*e^3*x + b^3
*c^3*e^3)*arcsinh(d*x + c)^3 + 3*(a*b^2*d^3*e^3*x^3 + 3*a*b^2*c*d^2*e^3*x^
2 + 3*a*b^2*c^2*d*e^3*x + a*b^2*c^3*e^3)*arcsinh(d*x + c)^2 + 3*(a^2*b*d^3
*e^3*x^3 + 3*a^2*b*c*d^2*e^3*x^2 + 3*a^2*b*c^2*d*e^3*x + a^2*b*c^3*e^3)*ar
csinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

3.244.6 Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^3\,dx=\text{Timed out}$$

input
```
integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c))**3,x)
```

output
```
Timed out
```

3.244.7 Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^3\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.244.8 Giac [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^3\,dx=\int (dex+ce)^{\frac{7}{2}}(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a)^3, x)
```

3.244.9 Mupad [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^3\,dx=\int {\left(c\,e+d\,e\,x\right)}^{7/2}\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3 \,d x$$

input
```
int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^3, x)
```

3.245 $\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^3\,dx$

3.245.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^3\,dx=\frac{2(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^3}{7de}-\frac{6b\text{Int}\left(\frac{(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{1+(c+dx)^2}},x\right)}{7e}$$

output
```
2/7*(e*(d*x+c))^(7/2)*(a+b*arcsinh(d*x+c))^3/d/e-6/7*b*Unintegrable((e*(d*
x+c))^(7/2)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e
```

3.245.2 Mathematica [N/A]

Not integrable

Time = 120.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^3\,dx=\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^3\,dx$$

input
```
Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^3, x]
```

3.245.3 Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^3\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int (e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^3}{7e}-\frac{6b\int \frac{(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)}{7e}}{d}$$

$$\Big\downarrow \text{6239}$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^3}{7e}-\frac{6b\int \frac{(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)}{7e}}{d}$$

input
```
Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
$Aborted
```

3.245.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.245.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (dex + ce)^{\frac{5}{2}} (a + b \operatorname{arcsinh}(dx + c))^3 \, dx$$

input
```
int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x)
```

output
```
int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x)
```

3.245.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 7.44

$$\int (ce + dex)^{5/2}(a + b\operatorname{arcsinh}(c + dx))^3 \, dx = \int (dex + ce)^{\frac{5}{2}} (b \operatorname{arsinh}(dx + c) + a)^3 \, dx$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((a^3*d^2*e^2*x^2 + 2*a^3*c*d*e^2*x + a^3*c^2*e^2 + (b^3*d^2*e^2*x
^2 + 2*b^3*c*d*e^2*x + b^3*c^2*e^2)*arcsinh(d*x + c)^3 + 3*(a*b^2*d^2*e^2*
x^2 + 2*a*b^2*c*d*e^2*x + a*b^2*c^2*e^2)*arcsinh(d*x + c)^2 + 3*(a^2*b*d^2
*e^2*x^2 + 2*a^2*b*c*d*e^2*x + a^2*b*c^2*e^2)*arcsinh(d*x + c))*sqrt(d*e*x
 + c*e), x)
```

3.245.6 Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^3\,dx=\text{Timed out}$$

input
```
integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c))**3,x)
```

output
```
Timed out
```

3.245.7 Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^3\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.245.8 Giac [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^3\,dx=\int (dex+ce)^{\frac{5}{2}}(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a)^3, x)
```

3.245.9 Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^3\,dx=\int (c\,e+d\,e\,x)^{5/2}\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^3\,dx$$

input
```
int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^3, x)
```

3.246 $\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^3\,dx$

3.246.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^3\,dx=\frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^3}{5de}-\frac{6b\text{Int}\left(\frac{(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{1+(c+dx)^2}},x\right)}{5e}$$

output
```
2/5*(e*(d*x+c))^(5/2)*(a+b*arcsinh(d*x+c))^3/d/e-6/5*b*Unintegrable((e*(d*
x+c))^(5/2)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e
```

3.246.2 Mathematica [N/A]

Not integrable

Time = 66.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^3\,dx=\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^3\,dx$$

input
```
Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^3, x]
```

3.246.3 Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^3\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int (e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^3}{5e}-\frac{6b\int \frac{(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)}{5e}}{d}$$

$$\Big\downarrow \text{6239}$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^3}{5e}-\frac{6b\int \frac{(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)}{5e}}{d}$$

input
```
Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
$Aborted
```

3.246.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.246.4 Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (dex + ce)^{\frac{3}{2}} (a + b \operatorname{arcsinh}(dx + c))^3 dx$$

input
```
int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x)
```

output
```
int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x)
```

3.246.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.00

$$\int (ce + dex)^{3/2} (a + b\text{arcsinh}(c + dx))^3 dx = \int (dex + ce)^{\frac{3}{2}} (b \operatorname{arsinh}(dx + c) + a)^3 dx$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arcsinh(d*x + c)^3 +
 3*(a*b^2*d*e*x + a*b^2*c*e)*arcsinh(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c
*e)*arcsinh(d*x + c))*sqrt(d*e*x + c*e), x)
```

3.246.6 Sympy [N/A]

Not integrable

Time = 20.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^3\,dx=\int (e(c+dx))^{\frac{3}{2}}\left(a+b\,\text{asinh}\,(c+dx)\right)^3\,dx$$

input
```
integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c))**3,x)
```

output
```
Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x))**3, x)
```

3.246.7 Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^3\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.246.8 Giac [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^3\,dx=\int (dex+ce)^{\frac{3}{2}}(b\,\text{arsinh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a)^3, x)
```

3.246.9 Mupad [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^3\,dx=\int {\left(c\,e+d\,e\,x\right)}^{3/2}\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3 \,d x$$

input
```
int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^3, x)
```

3.247 $\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^3\,dx$

3.247.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^3\,dx=\frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^3}{3de}-\frac{2b\text{Int}\left(\frac{(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{1+(c+dx)^2}},x\right)}{e}$$

output
```
2/3*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))^3/d/e-2*b*Unintegrable((e*(d*x+
c))^(3/2)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e
```

3.247.2 Mathematica [N/A]

Not integrable

Time = 104.77 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^3\,dx=\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^3\,dx$$

input
```
Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^3, x]
```

3.247.3 Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^3\,dx$$

$$\Big\downarrow \ 6274$$

$$\frac{\int \sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \ 6191$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^3}{3e}-\frac{2b\int \frac{(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)}{e}}{d}$$

$$\Big\downarrow \ 6239$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^3}{3e}-\frac{2b\int \frac{(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)}{e}}{d}$$

input
```
Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^3,x]
```

output
```
$Aborted
```

3.247.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.247.4 Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (a+b\,\mathrm{arcsinh}\,(dx+c))^3\,\sqrt{dex+ce}dx$$

input
```
int((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x)
```

3.247.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int \sqrt{ce+dex}(a+b\mathrm{arcsinh}(c+dx))^3\,dx = \int \sqrt{dex+ce}(b\,\mathrm{arsinh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*ar
csinh(d*x + c) + a^3)*sqrt(d*e*x + c*e), x)
```

3.247.6 Sympy [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^3\,dx=\int \sqrt{e\left(c+dx\right)}(a+b\operatorname{asinh}\left(c+dx\right))^3\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**3*(d*e*x+c*e)**(1/2),x)
```

output
```
Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x))**3, x)
```

3.247.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^3\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.247.8 Giac [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^3\,dx=\int \sqrt{dex+ce}(b\operatorname{arsinh}\left(dx+c\right)+a)^3\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^3, x)
```

3.247.9 Mupad [N/A]

Not integrable

Time = 2.97 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{ce+dex}(a+b\operatorname{arcsinh}(c+dx))^3\,dx=\int \sqrt{c\,e+d\,e\,x}\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^3\,dx$$

input
```
int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^3, x)
```

3.248 $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx$

3.248.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx = \frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^3}{de} - \frac{6b\text{Int}\left(\frac{\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{1+(c+dx)^2}},x\right)}{e}$$

output
```
2*(a+b*arcsinh(d*x+c))^3*(e*(d*x+c))^(1/2)/d/e-6*b*Unintegrable((a+b*arcsi
nh(d*x+c))^2*(e*(d*x+c))^(1/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

3.248.2 Mathematica [N/A]

Not integrable

Time = 61.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^3/Sqrt[c*e + d*e*x],x]
```

output
```
Integrate[(a + b*ArcSinh[c + d*x])^3/Sqrt[c*e + d*e*x], x]
```

3.248.3 Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^3}{\sqrt{e(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^3}{e}-\frac{6b\int \frac{\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)}{e}}{d}$$

$$\Big\downarrow \text{6239}$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^3}{e}-\frac{6b\int \frac{\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^2}{\sqrt{(c+dx)^2+1}}d(c+dx)}{e}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^3/Sqrt[c*e + d*e*x],x]
```

output
```
$Aborted
```

3.248.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.248.4 Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a + b \operatorname{arcsinh}(dx + c))^3}{\sqrt{dex + ce}} dx$$

input
```
int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)
```

3.248.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int \frac{(a + b\operatorname{arcsinh}(c + dx))^3}{\sqrt{ce + dex}} dx = \int \frac{(b \operatorname{arsinh}(dx + c) + a)^3}{\sqrt{dex + ce}} dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*ar
csinh(d*x + c) + a^3)/sqrt(d*e*x + c*e), x)
```

3.248.6 Sympy [N/A]

Not integrable

Time = 3.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx=\int \frac{(a+b\operatorname{asinh}(c+dx))^3}{\sqrt{e(c+dx)}}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(1/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**3/sqrt(e*(c + d*x)), x)
```

3.248.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.248.8 Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx=\int \frac{(b\operatorname{arsinh}(dx+c)+a)^3}{\sqrt{dex+ce}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^3/sqrt(d*e*x + c*e), x)
```

3.248.9 Mupad [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{\sqrt{ce+dex}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3}{\sqrt{c\,e+d\,e\,x}}\,dx$$

input
```
int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(1/2),x)
```

output
```
int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(1/2), x)
```

3.249 $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx$

3.249.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx=-\frac{2(a+b\text{arcsinh}(c+dx))^3}{de\sqrt{e(c+dx)}}+\frac{6b\text{Int}\left(\frac{(a+b\text{arcsinh}(c+dx))^2}{\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}},x\right)}{e}$$

output
```
-2*(a+b*arcsinh(d*x+c))^3/d/e/(e*(d*x+c))^(1/2)+6*b*Unintegrable((a+b*arcs
inh(d*x+c))^2/(e*(d*x+c))^(1/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

3.249.2 Mathematica [N/A]

Not integrable

Time = 43.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx=\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
```

output
```
Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(3/2), x]
```

3.249.3 Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(e(c+dx))^{3/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{6b\int \frac{(a+b\text{arcsinh}(c+dx))^2}{\sqrt{e(c+dx)}\sqrt{(c+dx)^2+1}}d(c+dx)}{e}-\frac{2(a+b\text{arcsinh}(c+dx))^3}{e\sqrt{e(c+dx)}}}{d}$$

$$\Big\downarrow \text{6239}$$

$$\frac{\frac{6b\int \frac{(a+b\text{arcsinh}(c+dx))^2}{\sqrt{e(c+dx)}\sqrt{(c+dx)^2+1}}d(c+dx)}{e}-\frac{2(a+b\text{arcsinh}(c+dx))^3}{e\sqrt{e(c+dx)}}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
```

output
```
$Aborted
```

3.249.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.249.4 Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\,\operatorname{arcsinh}(dx+c))^3}{(dex+ce)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
```

3.249.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.32

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx = \int \frac{(b\operatorname{arsinh}(dx+c)+a)^3}{(dex+ce)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*ar
csinh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e
^2), x)
```

3.249.6 Sympy [N/A]

Not integrable

Time = 6.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx=\int \frac{(a+b\,\text{asinh}\,(c+dx))^3}{(e\,(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(3/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**3/(e*(c + d*x))**(3/2), x)
```

3.249.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.249.8 Giac [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{(dex+ce)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^(3/2), x)
```

3.249.9 Mupad [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{3/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3}{{\left(c\,e+d\,e\,x\right)}^{3/2}}\,dx$$

input
```
int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(3/2),x)
```

output
```
int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(3/2), x)
```

3.250 $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx$

3.250.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx = -\frac{2(a+b\text{arcsinh}(c+dx))^3}{3de(e(c+dx))^{3/2}}+\frac{2b\text{Int}\left(\frac{(a+b\text{arcsinh}(c+dx))^2}{(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}},x\right)}{e}$$

output
```
-2/3*(a+b*arcsinh(d*x+c))^3/d/e/(e*(d*x+c))^(3/2)+2*b*Unintegrable((a+b*ar
csinh(d*x+c))^2/(e*(d*x+c))^(3/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

3.250.2 Mathematica [N/A]

Not integrable

Time = 39.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
```

output
```
Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(5/2), x]
```

3.250.3 Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(e(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2b\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(e(c+dx))^{3/2}\sqrt{(c+dx)^2+1}}d(c+dx)}{e}-\frac{2(a+b\text{arcsinh}(c+dx))^3}{3e(e(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{6239}$$

$$\frac{\frac{2b\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(e(c+dx))^{3/2}\sqrt{(c+dx)^2+1}}d(c+dx)}{e}-\frac{2(a+b\text{arcsinh}(c+dx))^3}{3e(e(c+dx))^{3/2}}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
```

output
```
$Aborted
```

3.250.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.250.4 Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\operatorname{arcsinh}(dx+c))^3}{(dex+ce)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

3.250.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.88

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx=\int \frac{(b\operatorname{arsinh}(dx+c)+a)^3}{(dex+ce)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

output
```
integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*ar
csinh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3
*c^2*d*e^3*x + c^3*e^3), x)
```

3.250.6 Sympy [N/A]

Not integrable

Time = 14.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx=\int \frac{(a+b\,\text{asinh}\,(c+dx))^3}{(e\,(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(5/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**3/(e*(c + d*x))**(5/2), x)
```

3.250.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.250.8 Giac [N/A]

Not integrable

Time = 2.98 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^3}{(dex+ce)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^(5/2), x)
```

3.250.9 Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{5/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3}{{\left(c\,e+d\,e\,x\right)}^{5/2}} \,d x$$

input
```
int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(5/2),x)
```

output
```
int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(5/2), x)
```

3.251 $\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx$

3.251.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx = -\frac{2(a+b\text{arcsinh}(c+dx))^3}{5de(e(c+dx))^{5/2}} + \frac{6b\text{Int}\left(\frac{(a+b\text{arcsinh}(c+dx))^2}{(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}},x\right)}{5e}$$

output
```
-2/5*(a+b*arcsinh(d*x+c))^3/d/e/(e*(d*x+c))^(5/2)+6/5*b*Unintegrable((a+b*
arcsinh(d*x+c))^2/(e*(d*x+c))^(5/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

3.251.2 Mathematica [N/A]

Not integrable

Time = 73.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(7/2),x]
```

output
```
Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(7/2), x]
```

3.251.3 Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(e(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \; 6191$$

$$\frac{\frac{6b\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(e(c+dx))^{5/2}\sqrt{(c+dx)^2+1}}d(c+dx)}{5e}-\frac{2(a+b\text{arcsinh}(c+dx))^3}{5e(e(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \; 6239$$

$$\frac{\frac{6b\int \frac{(a+b\text{arcsinh}(c+dx))^2}{(e(c+dx))^{5/2}\sqrt{(c+dx)^2+1}}d(c+dx)}{5e}-\frac{2(a+b\text{arcsinh}(c+dx))^3}{5e(e(c+dx))^{5/2}}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(7/2),x]
```

output
```
$Aborted
```

3.251.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.251.4 Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a + b \operatorname{arcsinh}(dx + c))^3}{(dex + ce)^{\frac{7}{2}}} dx$$

input
```
int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x)
```

3.251.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.44

$$\int \frac{(a + b\text{arcsinh}(c + dx))^3}{(ce + dex)^{7/2}} dx = \int \frac{(b \operatorname{arsinh}(dx + c) + a)^3}{(dex + ce)^{\frac{7}{2}}} dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

output
```
integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*ar
csinh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6
*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

3.251.6 Sympy [N/A]

Not integrable

Time = 61.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx=\int \frac{(a+b\operatorname{asinh}(c+dx))^3}{(e(c+dx))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(7/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**3/(e*(c + d*x))**(7/2), x)
```

3.251.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.251.8 Giac [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx=\int \frac{(b\operatorname{arsinh}(dx+c)+a)^3}{(dex+ce)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^(7/2), x)
```

3.251.9 Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(ce+dex)^{7/2}}\,dx = \int \frac{(a+b\,\mathrm{asinh}(c+d\,x))^3}{(c\,e+d\,e\,x)^{7/2}}\,dx$$

input
```
int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(7/2),x)
```

output
```
int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(7/2), x)
```

3.252 $\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^4\,dx$

3.252.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^4\,dx=\frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))^4}{9de}-\frac{8b\text{Int}\left(\frac{(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{1+(c+dx)^2}},x\right)}{9e}$$

output
```
2/9*(e*(d*x+c))^(9/2)*(a+b*arcsinh(d*x+c))^4/d/e-8/9*b*Unintegrable((e*(d*
x+c))^(9/2)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e
```

3.252.2 Mathematica [N/A]

Not integrable

Time = 37.75 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^4\,dx=\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^4\,dx$$

input
```
Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^4, x]
```

3.252.3 Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{7/2}(a+b\text{arcsinh}(c+dx))^4\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int (e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^4d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))^4}{9e}-\frac{8b\int \frac{(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{(c+dx)^2+1}}d(c+dx)}{9e}}{d}$$

$$\Big\downarrow \text{6239}$$

$$\frac{\frac{2(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))^4}{9e}-\frac{8b\int \frac{(e(c+dx))^{9/2}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{(c+dx)^2+1}}d(c+dx)}{9e}}{d}$$

input
```
Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
$Aborted
```

3.252.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.252.4 Maple [N/A] (verified)

Not integrable

Time = 0.90 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (dex+ce)^{\frac{7}{2}} (a+b\,\mathrm{arcsinh}\,(dx+c))^4\,dx$$

input
```
int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x)
```

output
```
int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x)
```

3.252.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 330, normalized size of antiderivative = 13.20

$$\int (ce+dex)^{7/2}(a+b\mathrm{arcsinh}(c+dx))^4\,dx=\int (dex+ce)^{\frac{7}{2}}(b\,\mathrm{arsinh}\,(dx+c)+a)^4\,dx$$

input
```
integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral((a^4*d^3*e^3*x^3 + 3*a^4*c*d^2*e^3*x^2 + 3*a^4*c^2*d*e^3*x + a^4*
c^3*e^3 + (b^4*d^3*e^3*x^3 + 3*b^4*c*d^2*e^3*x^2 + 3*b^4*c^2*d*e^3*x + b^4
*c^3*e^3)*arcsinh(d*x + c)^4 + 4*(a*b^3*d^3*e^3*x^3 + 3*a*b^3*c*d^2*e^3*x^
2 + 3*a*b^3*c^2*d*e^3*x + a*b^3*c^3*e^3)*arcsinh(d*x + c)^3 + 6*(a^2*b^2*d
^3*e^3*x^3 + 3*a^2*b^2*c*d^2*e^3*x^2 + 3*a^2*b^2*c^2*d*e^3*x + a^2*b^2*c^3
*e^3)*arcsinh(d*x + c)^2 + 4*(a^3*b*d^3*e^3*x^3 + 3*a^3*b*c*d^2*e^3*x^2 +
3*a^3*b*c^2*d*e^3*x + a^3*b*c^3*e^3)*arcsinh(d*x + c))*sqrt(d*e*x + c*e),
x)
```

3.252.6 Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx=\text{Timed out}$$

input
```
integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c))**4,x)
```

output
```
Timed out
```

3.252.7 Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.252.8 Giac [N/A]

Not integrable

Time = 172.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{7/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx=\int (dex+ce)^{\frac{7}{2}}(b\operatorname{arsinh}(dx+c)+a)^4\,dx$$

input
```
integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a)^4, x)
```

3.252.9 Mupad [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce + dex)^{7/2}(a + b\text{arcsinh}(c + dx))^4\, dx = \int (c\,e + d\,e\,x)^{7/2}\,(a + b\,\text{asinh}(c + d\,x))^4\, dx$$

input
```
int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^4, x)
```

3.253 $\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^4\,dx$

3.253.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^4\,dx = \frac{2(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^4}{7de} - \frac{8b\text{Int}\left(\frac{(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{1+(c+dx)^2}},x\right)}{7e}$$

output
```
2/7*(e*(d*x+c))^(7/2)*(a+b*arcsinh(d*x+c))^4/d/e-8/7*b*Unintegrable((e*(d*
x+c))^(7/2)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e
```

3.253.2 Mathematica [N/A]

Not integrable

Time = 74.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^4\,dx = \int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^4\,dx$$

input
```
Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^4, x]
```

3.253.3 Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^4\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int (e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^4d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^4}{7e}-\frac{8b\int \frac{(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{(c+dx)^2+1}}d(c+dx)}{7e}}{d}$$

$$\Big\downarrow \text{6239}$$

$$\frac{\frac{2(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^4}{7e}-\frac{8b\int \frac{(e(c+dx))^{7/2}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{(c+dx)^2+1}}d(c+dx)}{7e}}{d}$$

input
```
Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
$Aborted
```

3.253.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.253.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (dex + ce)^{\frac{5}{2}} (a + b \operatorname{arcsinh}(dx + c))^4 \, dx$$

input
```
int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x)
```

output
```
int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x)
```

3.253.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 240, normalized size of antiderivative = 9.60

$$\int (ce + dex)^{5/2} (a + b\text{arcsinh}(c + dx))^4 \, dx = \int (dex + ce)^{\frac{5}{2}} (b \operatorname{arsinh}(dx + c) + a)^4 \, dx$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral((a^4*d^2*e^2*x^2 + 2*a^4*c*d*e^2*x + a^4*c^2*e^2 + (b^4*d^2*e^2*x
^2 + 2*b^4*c*d*e^2*x + b^4*c^2*e^2)*arcsinh(d*x + c)^4 + 4*(a*b^3*d^2*e^2*
x^2 + 2*a*b^3*c*d*e^2*x + a*b^3*c^2*e^2)*arcsinh(d*x + c)^3 + 6*(a^2*b^2*d
^2*e^2*x^2 + 2*a^2*b^2*c*d*e^2*x + a^2*b^2*c^2*e^2)*arcsinh(d*x + c)^2 + 4
*(a^3*b*d^2*e^2*x^2 + 2*a^3*b*c*d*e^2*x + a^3*b*c^2*e^2)*arcsinh(d*x + c))
*sqrt(d*e*x + c*e), x)
```

3.253.6 Sympy [F(-1)]

Timed out.

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx=\text{Timed out}$$

input
```
integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c))**4,x)
```

output
```
Timed out
```

3.253.7 Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.253.8 Giac [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{5/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx=\int (dex+ce)^{\frac{5}{2}}(b\operatorname{arsinh}(dx+c)+a)^4\,dx$$

input
```
integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a)^4, x)
```

3.253.9 Mupad [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{5/2}(a+b\text{arcsinh}(c+dx))^4\,dx=\int (c\,e+d\,e\,x)^{5/2}\,(a+b\,\text{asinh}(c+d\,x))^4\,dx$$

input
```
int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^4, x)
```

3.254 $\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx$

3.254.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx=\frac{2(e(c+dx))^{5/2}(a+b\operatorname{arcsinh}(c+dx))^4}{5de}-\frac{8b\operatorname{Int}\left(\frac{(e(c+dx))^{5/2}(a+b\operatorname{arcsinh}(c+dx))^3}{\sqrt{1+(c+dx)^2}},x\right)}{5e}$$

output
```
2/5*(e*(d*x+c))^(5/2)*(a+b*arcsinh(d*x+c))^4/d/e-8/5*b*Unintegrable((e*(d*
x+c))^(5/2)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e
```

3.254.2 Mathematica [N/A]

Not integrable

Time = 34.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx=\int (ce+dex)^{3/2}(a+b\operatorname{arcsinh}(c+dx))^4\,dx$$

input
```
Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^4, x]
```

3.254.3 Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^4\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int (e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^4d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^4}{5e}-\frac{8b\int \frac{(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{(c+dx)^2+1}}d(c+dx)}{5e}}{d}$$

$$\Big\downarrow \text{6239}$$

$$\frac{\frac{2(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^4}{5e}-\frac{8b\int \frac{(e(c+dx))^{5/2}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{(c+dx)^2+1}}d(c+dx)}{5e}}{d}$$

input
```
Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
$Aborted
```

3.254.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.254.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (dex + ce)^{\frac{3}{2}} (a + b \operatorname{arcsinh}(dx + c))^4 \, dx$$

input
```
int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x)
```

output
```
int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x)
```

3.254.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 5.20

$$\int (ce + dex)^{3/2}(a + b\text{arcsinh}(c + dx))^4 \, dx = \int (dex + ce)^{\frac{3}{2}} (b \operatorname{arsinh}(dx + c) + a)^4 \, dx$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
integral((a^4*d*e*x + a^4*c*e + (b^4*d*e*x + b^4*c*e)*arcsinh(d*x + c)^4 +
 4*(a*b^3*d*e*x + a*b^3*c*e)*arcsinh(d*x + c)^3 + 6*(a^2*b^2*d*e*x + a^2*b
^2*c*e)*arcsinh(d*x + c)^2 + 4*(a^3*b*d*e*x + a^3*b*c*e)*arcsinh(d*x + c))
*sqrt(d*e*x + c*e), x)
```

3.254.6 Sympy [N/A]

Not integrable

Time = 39.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^4\,dx=\int (e(c+dx))^{\frac{3}{2}}\,(a+b\,\text{asinh}\,(c+dx))^4\,\,dx$$

input
```
integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c))**4,x)
```

output
```
Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x))**4, x)
```

3.254.7 Maxima [F(-2)]

Exception generated.

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^4\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.254.8 Giac [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^4\,dx=\int (dex+ce)^{\frac{3}{2}}(b\,\text{arsinh}\,(dx+c)+a)^4\,dx$$

input
```
integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a)^4, x)
```

3.254.9 Mupad [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ce+dex)^{3/2}(a+b\text{arcsinh}(c+dx))^4\,dx=\int (c\,e+d\,e\,x)^{3/2}\,(a+b\,\text{asinh}(c+d\,x))^4\,dx$$

input
```
int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^4, x)
```

3.255 $\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^4\,dx$

3.255.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^4\,dx = \frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^4}{3de} - \frac{8b\text{Int}\left(\frac{(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{1+(c+dx)^2}},x\right)}{3e}$$

output
```
2/3*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))^4/d/e-8/3*b*Unintegrable((e*(d*
x+c))^(3/2)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e
```

3.255.2 Mathematica [N/A]

Not integrable

Time = 57.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^4\,dx = \int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^4\,dx$$

input
```
Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^4, x]
```

3.255.3 Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^4\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^4d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^4}{3e}-\frac{8b\int \frac{(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{(c+dx)^2+1}}d(c+dx)}{3e}}{d}$$

$$\Big\downarrow \text{6239}$$

$$\frac{\frac{2(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^4}{3e}-\frac{8b\int \frac{(e(c+dx))^{3/2}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{(c+dx)^2+1}}d(c+dx)}{3e}}{d}$$

input
```
Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^4,x]
```

output
```
$Aborted
```

3.255.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.255.4 Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (a + b \operatorname{arcsinh}(dx + c))^4 \sqrt{dex + ce}dx$$

input
```
int((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x)
```

3.255.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.84

$$\int \sqrt{ce + dex}(a + b\text{arcsinh}(c + dx))^4 \, dx = \int \sqrt{dex + ce}(b \operatorname{arsinh}(dx + c) + a)^4 \, dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*
arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*e*x + c*e), x)
```

3.255.6 Sympy [N/A]

Not integrable

Time = 6.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^4\,dx = \int \sqrt{e\,(c+dx)}(a+b\operatorname{asinh}{(c+dx)})^4\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**4*(d*e*x+c*e)**(1/2),x)
```

output
```
Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x))**4, x)
```

3.255.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^4\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.255.8 Giac [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^4\,dx = \int \sqrt{dex+ce}(b\,\text{arsinh}\,(dx+c)+a)^4\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^4, x)
```

3.255.9 Mupad [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{ce+dex}(a+b\text{arcsinh}(c+dx))^4\,dx=\int \sqrt{c\,e+d\,e\,x}\,\left(a+b\,\text{asinh}(c+d\,x)\right)^4\,dx$$

input
```
int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^4,x)
```

output
```
int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^4, x)
```

3.256 $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx$

3.256.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx=\frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^4}{de}-\frac{8b\text{Int}\left(\frac{\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{1+(c+dx)^2}},x\right)}{e}$$

output
```
2*(a+b*arcsinh(d*x+c))^4*(e*(d*x+c))^(1/2)/d/e-8*b*Unintegrable((a+b*arcsi
nh(d*x+c))^3*(e*(d*x+c))^(1/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

3.256.2 Mathematica [N/A]

Not integrable

Time = 13.87 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx=\int \frac{(a+b\text{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^4/Sqrt[c*e + d*e*x],x]
```

output
```
Integrate[(a + b*ArcSinh[c + d*x])^4/Sqrt[c*e + d*e*x], x]
```

3.256.3 Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^4}{\sqrt{e(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^4}{e}-\frac{8b\int \frac{\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{(c+dx)^2+1}}d(c+dx)}{e}}{d}$$

$$\Big\downarrow \text{6239}$$

$$\frac{\frac{2\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^4}{e}-\frac{8b\int \frac{\sqrt{e(c+dx)}(a+b\text{arcsinh}(c+dx))^3}{\sqrt{(c+dx)^2+1}}d(c+dx)}{e}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^4/Sqrt[c*e + d*e*x],x]
```

output
```
$Aborted
```

3.256.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.256.4 Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\operatorname{arcsinh}(dx+c))^4}{\sqrt{dex+ce}}dx$$

input
```
int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x)
```

3.256.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.84

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx=\int \frac{(b\,\text{arsinh}\,(dx+c)+a)^4}{\sqrt{dex+ce}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*
arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)/sqrt(d*e*x + c*e), x)
```

3.256.6 Sympy [N/A]

Not integrable

Time = 5.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx=\int \frac{(a+b\operatorname{asinh}(c+dx))^4}{\sqrt{e\,(c+dx)}}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(1/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**4/sqrt(e*(c + d*x)), x)
```

3.256.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.256.8 Giac [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx=\int \frac{(b\operatorname{arsinh}\,(dx+c)+a)^4}{\sqrt{dex+ce}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^4/sqrt(d*e*x + c*e), x)
```

3.256.9 Mupad [N/A]

Not integrable

Time = 2.93 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{\sqrt{ce+dex}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4}{\sqrt{c\,e+d\,e\,x}}\,dx$$

input
```
int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(1/2),x)
```

output
```
int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(1/2), x)
```

3.257 $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx$

3.257.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx = -\frac{2(a+b\text{arcsinh}(c+dx))^4}{de\sqrt{e(c+dx)}}+\frac{8b\text{Int}\left(\frac{(a+b\text{arcsinh}(c+dx))^3}{\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}},x\right)}{e}$$

output
```
-2*(a+b*arcsinh(d*x+c))^4/d/e/(e*(d*x+c))^(1/2)+8*b*Unintegrable((a+b*arcs
inh(d*x+c))^3/(e*(d*x+c))^(1/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

3.257.2 Mathematica [N/A]

Not integrable

Time = 16.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx = \int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(3/2),x]
```

output
```
Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(3/2), x]
```

3.257.3 Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}dx$$
$$\Big\downarrow \text{ 6274}$$
$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(e(c+dx))^{3/2}}d(c+dx)}{d}$$
$$\Big\downarrow \text{ 6191}$$
$$\frac{\frac{8b\int \frac{(a+b\text{arcsinh}(c+dx))^3}{\sqrt{e(c+dx)}\sqrt{(c+dx)^2+1}}d(c+dx)}{e}-\frac{2(a+b\text{arcsinh}(c+dx))^4}{e\sqrt{e(c+dx)}}}{d}$$
$$\Big\downarrow \text{ 6239}$$
$$\frac{\frac{8b\int \frac{(a+b\text{arcsinh}(c+dx))^3}{\sqrt{e(c+dx)}\sqrt{(c+dx)^2+1}}d(c+dx)}{e}-\frac{2(a+b\text{arcsinh}(c+dx))^4}{e\sqrt{e(c+dx)}}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(3/2),x]
```

output
```
$Aborted
```

3.257.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.257.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a + b \operatorname{arcsinh}(dx + c))^4}{(dex + ce)^{\frac{3}{2}}} dx$$

input
```
int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x)
```

3.257.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.96

$$\int \frac{(a + b\text{arcsinh}(c + dx))^4}{(ce + dex)^{3/2}} dx = \int \frac{(b \operatorname{arsinh}(dx + c) + a)^4}{(dex + ce)^{\frac{3}{2}}} dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*
arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^
2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

3.257.6 Sympy [N/A]

Not integrable

Time = 9.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\,\text{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx = \int \frac{(a+b\,\text{asinh}\,(c+dx))^4}{(e\,(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(3/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**4/(e*(c + d*x))**(3/2), x)
```

3.257.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\,\text{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.257.8 Giac [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\,\text{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx = \int \frac{(b\,\text{arsinh}\,(dx+c)+a)^4}{(dex+ce)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^(3/2), x)
```

3.257.9 Mupad [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{3/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4}{{\left(c\,e+d\,e\,x\right)}^{3/2}} \,d x$$

input
```
int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(3/2),x)
```

output
```
int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(3/2), x)
```

3.258 $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx$

3.258.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx=-\frac{2(a+b\text{arcsinh}(c+dx))^4}{3de(e(c+dx))^{3/2}}+\frac{8b\text{Int}\left(\frac{(a+b\text{arcsinh}(c+dx))^3}{(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}},x\right)}{3e}$$

output
```
-2/3*(a+b*arcsinh(d*x+c))^4/d/e/(e*(d*x+c))^(3/2)+8/3*b*Unintegrable((a+b*
arcsinh(d*x+c))^3/(e*(d*x+c))^(3/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

3.258.2 Mathematica [N/A]

Not integrable

Time = 10.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx=\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(5/2),x]
```

output
```
Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(5/2), x]
```

3.258.3 Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(e(c+dx))^{5/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{8b\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(e(c+dx))^{3/2}\sqrt{(c+dx)^2+1}}d(c+dx)}{3e}-\frac{2(a+b\text{arcsinh}(c+dx))^4}{3e(e(c+dx))^{3/2}}}{d}$$

$$\Big\downarrow \text{6239}$$

$$\frac{\frac{8b\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(e(c+dx))^{3/2}\sqrt{(c+dx)^2+1}}d(c+dx)}{3e}-\frac{2(a+b\text{arcsinh}(c+dx))^4}{3e(e(c+dx))^{3/2}}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(5/2),x]
```

output
```
$Aborted
```

3.258.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.258.4 Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a + b \operatorname{arcsinh}(dx + c))^4}{(dex + ce)^{\frac{5}{2}}} dx$$

input
```
int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x)
```

3.258.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.52

$$\int \frac{(a + b\text{arcsinh}(c + dx))^4}{(ce + dex)^{5/2}} dx = \int \frac{(b \operatorname{arsinh}(dx + c) + a)^4}{(dex + ce)^{\frac{5}{2}}} dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

output
```
integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*
arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^
3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

3.258.6 Sympy [N/A]

Not integrable

Time = 18.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx=\int \frac{(a+b\operatorname{asinh}(c+dx))^4}{(e(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(5/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**4/(e*(c + d*x))**(5/2), x)
```

3.258.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.258.8 Giac [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx=\int \frac{(b\operatorname{arsinh}(dx+c)+a)^4}{(dex+ce)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^(5/2), x)
```

3.258.9 Mupad [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{5/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4}{{\left(c\,e+d\,e\,x\right)}^{5/2}}\,dx$$

input
```
int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(5/2),x)
```

output
```
int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(5/2), x)
```

3.259 $\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx$

3.259.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx=-\frac{2(a+b\text{arcsinh}(c+dx))^4}{5de(e(c+dx))^{5/2}}+\frac{8b\text{Int}\left(\frac{(a+b\text{arcsinh}(c+dx))^3}{(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}},x\right)}{5e}$$

output
```
-2/5*(a+b*arcsinh(d*x+c))^4/d/e/(e*(d*x+c))^(5/2)+8/5*b*Unintegrable((a+b*
arcsinh(d*x+c))^3/(e*(d*x+c))^(5/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

3.259.2 Mathematica [N/A]

Not integrable

Time = 53.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx=\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx$$

input
```
Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(7/2),x]
```

output
```
Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(7/2), x]
```

3.259.3 Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6274, 6191, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}} \, dx$$

$$\Big\downarrow \text{6274}$$

$$\frac{\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(e(c+dx))^{7/2}}d(c+dx)}{d}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{8b\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(e(c+dx))^{5/2}\sqrt{(c+dx)^2+1}}d(c+dx)}{5e}-\frac{2(a+b\text{arcsinh}(c+dx))^4}{5e(e(c+dx))^{5/2}}}{d}$$

$$\Big\downarrow \text{6239}$$

$$\frac{\frac{8b\int \frac{(a+b\text{arcsinh}(c+dx))^3}{(e(c+dx))^{5/2}\sqrt{(c+dx)^2+1}}d(c+dx)}{5e}-\frac{2(a+b\text{arcsinh}(c+dx))^4}{5e(e(c+dx))^{5/2}}}{d}$$

input
```
Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(7/2),x]
```

output
```
$Aborted
```

3.259.3.1 Defintions of rubi rules used

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.259.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b \operatorname{arcsinh}(dx+c))^4}{(dex+ce)^{\frac{7}{2}}} dx$$

input
```
int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x)
```

output
```
int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x)
```

3.259.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 5.08

$$\int \frac{(a+b\operatorname{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\, dx = \int \frac{(b \operatorname{arsinh}(dx+c)+a)^4}{(dex+ce)^{\frac{7}{2}}}\, dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

output
```
integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*
arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^
4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4)
, x)
```

3.259.6 Sympy [N/A]

Not integrable

Time = 68.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx=\int \frac{(a+b\operatorname{asinh}{(c+dx)})^4}{(e(c+dx))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(7/2),x)
```

output
```
Integral((a + b*asinh(c + d*x))**4/(e*(c + d*x))**(7/2), x)
```

3.259.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.259.8 Giac [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx=\int \frac{(b\operatorname{arsinh}(dx+c)+a)^4}{(dex+ce)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^(7/2), x)
```

3.259.9 Mupad [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{arcsinh}(c+dx))^4}{(ce+dex)^{7/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4}{{\left(c\,e+d\,e\,x\right)}^{7/2}}\,dx$$

input
```
int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(7/2),x)
```

output
```
int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(7/2), x)
```

3.260 $\int \sqrt{1+a^2+2abx+b^2x^2}\,\mathbf{arcsinh}(a+bx)^3\,dx$

3.260.1 Optimal result

Integrand size = 30, antiderivative size = 131

$$\begin{aligned}\int \sqrt{1+a^2+2abx+b^2x^2}\,\text{arcsinh}(a+bx)^3\,dx &= -\frac{3(a+bx)^2}{8b}+\frac{3(a+bx)\sqrt{1+(a+bx)^2}\,\text{arcsinh}(a+bx)}{4b}\\ &\quad-\frac{3\,\text{arcsinh}(a+bx)^2}{8b}-\frac{3(a+bx)^2\text{arcsinh}(a+bx)^2}{4b}\\ &\quad+\frac{(a+bx)\sqrt{1+(a+bx)^2}\,\text{arcsinh}(a+bx)^3}{2b}+\frac{\text{arcsinh}(a+bx)^4}{8b}\end{aligned}$$

output
```
-3/8*(b*x+a)^2/b-3/8*arcsinh(b*x+a)^2/b-3/4*(b*x+a)^2*arcsinh(b*x+a)^2/b+1
/8*arcsinh(b*x+a)^4/b+3/4*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b+1/2
*(b*x+a)*arcsinh(b*x+a)^3*(1+(b*x+a)^2)^(1/2)/b
```

3.260.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97

$$\int \sqrt{1+a^2+2abx+b^2x^2}\,\text{arcsinh}(a+bx)^3\,dx = \frac{-3bx(2a+bx)+6(a+bx)\sqrt{1+a^2+2abx+b^2x^2}\,\text{arcsinh}(a+bx)-3(1+2a^2+4abx+2b^2x^2)\,\text{arcsinh}(a}{8b}$$

input
```
Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3,x]
```

output
```
(-3*b*x*(2*a + b*x) + 6*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSin
h[a + b*x] - 3*(1 + 2*a^2 + 4*a*b*x + 2*b^2*x^2)*ArcSinh[a + b*x]^2 + 4*(a
 + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3 + ArcSinh[a +
 b*x]^4)/(8*b)
```

3.260.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {6275, 6200, 6191, 6198, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx+b^2x^2+1}\text{arcsinh}(a+bx)^3\,dx$$

$$\Big\downarrow \text{6275}$$

$$\frac{\int \sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)^3d(a+bx)}{b}$$

$$\Big\downarrow \text{6200}$$

$$\frac{-\frac{3}{2}\int (a+bx)\text{arcsinh}(a+bx)^2d(a+bx)+\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)^3}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}(a+bx)\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx}{b}$$

$$\Big\downarrow \text{6191}$$

$$\frac{-\frac{3}{2}\left(\frac{1}{2}(a+bx)^2\text{arcsinh}(a+bx)^2-\int \frac{(a+bx)^2\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)\right)+\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)^3}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}(a+bx)\sqrt{}}{b}$$

$$\Big\downarrow \text{6198}$$

$$\frac{-\frac{3}{2}\left(\frac{1}{2}(a+bx)^2\text{arcsinh}(a+bx)^2-\int \frac{(a+bx)^2\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)\right)+\frac{1}{8}\text{arcsinh}(a+bx)^4+\frac{1}{2}(a+bx)\sqrt{(a+bx)^2}}{b}$$

$$\Big\downarrow \text{6227}$$

$$\frac{-\frac{3}{2}\left(\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}\int (a+bx)d(a+bx)+\frac{1}{2}(a+bx)^2\text{arcsinh}(a+bx)^2-\frac{1}{2}(a+bx)\sqrt{(a+bx)^2+1}\right.}{b}$$

$$\Big\downarrow \ 15$$

$$\frac{-\frac{3}{2}\left(\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}(a+bx)^2\text{arcsinh}(a+bx)^2-\frac{1}{2}\sqrt{(a+bx)^2+1}(a+bx)\text{arcsinh}(a+bx)+\frac{1}{4}(a\right.}{b}$$

$$\Big\downarrow \ 6198$$

$$\frac{\frac{1}{8}\text{arcsinh}(a+bx)^4+\frac{1}{2}(a+bx)\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)^3-\frac{3}{2}\left(\frac{1}{2}(a+bx)^2\text{arcsinh}(a+bx)^2-\frac{1}{2}\sqrt{(a+bx)^2}\right.}{b}$$

input
```
Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3,x]
```

output
```
(((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^3)/2 + ArcSinh[a + b*x]
^4/8 - (3*((a + b*x)^2/4 - ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*
x])/2 + ArcSinh[a + b*x]^2/4 + ((a + b*x)^2*ArcSinh[a + b*x]^2)/2))/2)/b
```

3.260.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.260.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.56

method	result
default	$\frac{4\,\mathrm{arcsinh}(bx+a)^3\sqrt{b^2x^2+2abx+a^2+1}\,bx-6\,\mathrm{arcsinh}(bx+a)^2b^2x^2+4\,\mathrm{arcsinh}(bx+a)^3\sqrt{b^2x^2+2abx+a^2+1}\,a-12\,\mathrm{arcsinh}(bx+a)^2abx+a}{}$

input
```
int(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/8*(4*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x-6*arcsinh(b*x+a)
^2*b^2*x^2+4*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-12*arcsinh(b
*x+a)^2*a*b*x+arcsinh(b*x+a)^4-6*a^2*arcsinh(b*x+a)^2+6*arcsinh(b*x+a)*(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x-3*b^2*x^2+6*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x
+a^2+1)^(1/2)*a-6*a*b*x-3*arcsinh(b*x+a)^2-3*a^2-3)/b
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.52

$$\int \sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^3\,dx$$
$$= \frac{4\sqrt{b^2x^2+2\,abx+a^2+1}(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3 - 3\,b^2x^2 + \log\left(bx+a+\sqrt{b^2x^2\cdots}\right.}{}$$

input
```
integrate(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fri
cas")
```

output
```
1/8*(4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log(b*x + a + sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1))^3 - 3*b^2*x^2 + log(b*x + a + sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1))^4 - 6*a*b*x - 3*(2*b^2*x^2 + 4*a*b*x + 2*a^2 + 1)*log(b*x
 + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 6*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*(b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

3.260.6 Sympy [F]

$$\int \sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^3\,dx = \int \sqrt{a^2+2abx+b^2x^2+1}\,\text{asinh}^3\,(a+bx)\,dx$$

input
```
integrate(asinh(b*x+a)**3*(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

output
```
Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3, x)
```

3.260.7 Maxima [F]

$$\int \sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^3\,dx = \int \sqrt{b^2x^2+2\,abx+a^2+1}\,\text{arsinh}\,(bx+a)^3\,\,dx$$

input
```
integrate(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3, x)
```

3.260.8 Giac [F]

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3\,dx=\int \sqrt{b^2x^2+2\,abx+a^2+1}\operatorname{arsinh}\left(bx+a\right)^3\,dx$$

input
```
integrate(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3, x)
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3\,dx=\int \operatorname{asinh}(a+b\,x)^3\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}\,dx$$

input
```
int(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

output
```
int(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.261 $\int \sqrt{1+a^2+2abx+b^2x^2}\,\text{arcsinh}(a+bx)^2\,dx$

3.261.1 Optimal result

Integrand size = 30, antiderivative size = 107

$$\int \sqrt{1+a^2+2abx+b^2x^2}\,\text{arcsinh}(a+bx)^2\,dx = \frac{(a+bx)\sqrt{1+(a+bx)^2}}{4b} - \frac{\text{arcsinh}(a+bx)}{4b} - \frac{(a+bx)^2\text{arcsinh}(a+bx)}{2b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\text{arcsinh}(a+bx)^2}{2b} + \frac{\text{arcsinh}(a+bx)^3}{6b}$$

output
```
-1/4*arcsinh(b*x+a)/b-1/2*(b*x+a)^2*arcsinh(b*x+a)/b+1/6*arcsinh(b*x+a)^3/
b+1/4*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b+1/2*(b*x+a)*arcsinh(b*x+a)^2*(1+(b*x+a
)^2)^(1/2)/b
```

3.261.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03

$$\int \sqrt{1+a^2+2abx+b^2x^2}\,\text{arcsinh}(a+bx)^2\,dx = \frac{3(a+bx)\sqrt{1+a^2+2abx+b^2x^2} - 3(1+2a^2+4abx+2b^2x^2)\,\text{arcsinh}(a+bx) + 6(a+bx)\sqrt{1+a^2+2ab}}{12b}$$

input
```
Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2,x]
```

output
```
(3*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] - 3*(1 + 2*a^2 + 4*a*b*x +
2*b^2*x^2)*ArcSinh[a + b*x] + 6*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2
]*ArcSinh[a + b*x]^2 + 2*ArcSinh[a + b*x]^3)/(12*b)
```

3.261.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6275, 6200, 6191, 262, 222, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx+b^2x^2+1}\text{arcsinh}(a+bx)^2\,dx$$

$$\Big\downarrow \text{6275}$$

$$\frac{\int \sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)^2d(a+bx)}{b}$$

$$\Big\downarrow \text{6200}$$

$$\frac{-\int (a+bx)\text{arcsinh}(a+bx)d(a+bx)+\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}(a+bx)\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)^2}{b}$$

$$\Big\downarrow \text{6191}$$

$$\frac{\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}\int \frac{(a+bx)^2}{\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{1}{2}(a+bx)^2\text{arcsinh}(a+bx)+\frac{1}{2}\sqrt{(a+bx)^2+1}(a+bx)a}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}\left(\frac{1}{2}(a+bx)\sqrt{(a+bx)^2+1}-\frac{1}{2}\int \frac{1}{\sqrt{(a+bx)^2+1}}d(a+bx)\right)-\frac{1}{2}(a+bx)^2\text{arcsinh}(a+}{b}$$

$$\Big\downarrow \text{222}$$

$$\frac{\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{1}{2}(a+bx)^2\text{arcsinh}(a+bx)+\frac{1}{2}\sqrt{(a+bx)^2+1}(a+bx)\text{arcsinh}(a+bx)^2+\frac{1}{2}\left(\frac{1}{2}(a+\right.}{b}$$

$$\Big\downarrow \text{6198}$$

$$\frac{\frac{1}{6}\text{arcsinh}(a+bx)^3+\frac{1}{2}(a+bx)\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)^2-\frac{1}{2}(a+bx)^2\text{arcsinh}(a+bx)+\frac{1}{2}\left(\frac{1}{2}(a+bx)\sqrt{(a-}{b}$$

input
```
Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2,x]
```

output
```
((((a + b*x)*Sqrt[1 + (a + b*x)^2])/2 - ArcSinh[a + b*x]/2)/2 - ((a + b*x)
^2*ArcSinh[a + b*x])/2 + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]
^2)/2 + ArcSinh[a + b*x]^3/6)/b
```

3.261.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

3.261. $\int \sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^2\,dx$

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.261.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.56

method	result
default	$\frac{6\,\mathrm{arcsinh}(bx+a)^2\sqrt{b^2x^2+2abx+a^2+1}\,bx-6\ \mathrm{arcsinh}(bx+a)b^2x^2+6\,\mathrm{arcsinh}(bx+a)^2\sqrt{b^2x^2+2abx+a^2+1}\,a-12\ \mathrm{arcsinh}(bx+a)abx+2\,a}{12b}$

input
```
int(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/12*(6*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x-6*arcsinh(b*x+a
)*b^2*x^2+6*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-12*arcsinh(b*
x+a)*a*b*x+2*arcsinh(b*x+a)^3-6*a^2*arcsinh(b*x+a)+3*(b^2*x^2+2*a*b*x+a^2+
1)^(1/2)*b*x+3*a*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*arcsinh(b*x+a))/b
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.50

$$\int\sqrt{1+a^2+2abx+b^2x^2}\mathrm{arcsinh}(a+bx)^2\,dx$$
$$=\frac{6\sqrt{b^2x^2+2\,abx+a^2+1}(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2+2\,\log\left(bx+a+\sqrt{b^2x^2+2\,ab\right.}{}$$

input
```
integrate(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fri
cas")
```

output
```
1/12*(6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log(b*x + a + sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1))^2 + 2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1))^3 - 3*(2*b^2*x^2 + 4*a*b*x + 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)) + 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a))/b
```

3.261.6 Sympy [F]

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2\,dx=\int \sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2\left(a+bx\right)dx$$

input
```
integrate(asinh(b*x+a)**2*(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

output
```
Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2, x)
```

3.261.7 Maxima [F]

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2\,dx=\int \sqrt{b^2x^2+2\,abx+a^2+1}\operatorname{arsinh}\left(bx+a\right)^2\,dx$$

input
```
integrate(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2, x)
```

3.261.8 Giac [F]

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2\,dx = \int \sqrt{b^2x^2+2\,abx+a^2+1}\operatorname{arsinh}\left(bx+a\right)^2\,dx$$

input
```
integrate(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2, x)
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^2\,dx = \int \operatorname{asinh}(a+b\,x)^2\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}\,dx$$

input
```
int(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

output
```
int(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.262 $\int \sqrt{1+a^2+2abx+b^2x^2}\,\mathrm{arcsinh}(a+bx)\,dx$

3.262.1 Optimal result

Integrand size = 28, antiderivative size = 61

$$\int \sqrt{1+a^2+2abx+b^2x^2}\,\mathrm{arcsinh}(a+bx)\,dx = -\frac{(a+bx)^2}{4b}+\frac{(a+bx)\sqrt{1+(a+bx)^2}\,\mathrm{arcsinh}(a+bx)}{2b}+\frac{\mathrm{arcsinh}(a+bx)^2}{4b}$$

output
```
-1/4*(b*x+a)^2/b+1/4*arcsinh(b*x+a)^2/b+1/2*(b*x+a)*arcsinh(b*x+a)*(1+(b*x
+a)^2)^(1/2)/b
```

3.262.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \sqrt{1+a^2+2abx+b^2x^2}\,\mathrm{arcsinh}(a+bx)\,dx = \frac{-bx(2a+bx)+2(a+bx)\sqrt{1+a^2+2abx+b^2x^2}\,\mathrm{arcsinh}(a+bx)+\mathrm{arcsinh}(a+bx)^2}{4b}$$

input
```
Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x],x]
```

output
```
(-(b*x*(2*a + b*x)) + 2*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSin
h[a + b*x] + ArcSinh[a + b*x]^2)/(4*b)
```

3.262.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6275, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx+b^2x^2+1}\text{arcsinh}(a+bx)\,dx$$

$$\Big\downarrow \text{6275}$$

$$\frac{\int \sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{6200}$$

$$\frac{\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{1}{2}\int (a+bx)d(a+bx)+\frac{1}{2}(a+bx)\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}\sqrt{(a+bx)^2+1}(a+bx)\text{arcsinh}(a+bx)-\frac{1}{4}(a+bx)^2}{b}$$

$$\Big\downarrow \text{6198}$$

$$\frac{\frac{1}{2}\sqrt{(a+bx)^2+1}(a+bx)\text{arcsinh}(a+bx)+\frac{1}{4}\text{arcsinh}(a+bx)^2-\frac{1}{4}(a+bx)^2}{b}$$

input
```
Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x],x]
```

output
```
(-1/4*(a + b*x)^2 + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/2 +
 ArcSinh[a + b*x]^2/4)/b
```

3.262.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.262.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{2\,\mathrm{arcsinh}(bx+a)\sqrt{b^2x^2+2abx+a^2+1}\,bx-b^2x^2+2\,\mathrm{arcsinh}(bx+a)\sqrt{b^2x^2+2abx+a^2+1}\,a-2abx+\mathrm{arcsinh}(bx+a)^2-a^2-1}{4b}$	91

input
```
int(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x-b^2*x^2+2*arcsinh(
b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-2*a*b*x+arcsinh(b*x+a)^2-a^2-1)/b
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.61

$$\int \sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)\,dx =$$
$$-\frac{b^2x^2+2\,abx-2\,\sqrt{b^2x^2+2\,abx+a^2+1}(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\log\left(bx+a\right.}{4\,b}$$

input
```
integrate(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="frica
s")
```

output
```
-1/4*(b^2*x^2 + 2*a*b*x - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*lo
g(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - log(b*x + a + sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))^2)/b
```

3.262.6 Sympy [F]

$$\int \sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)\,dx = \int \sqrt{a^2+2abx+b^2x^2+1}\,\text{asinh}\,(a+bx)\,dx$$

input
```
integrate(asinh(b*x+a)*(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

output
```
Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x), x)
```

3.262.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(53) = 106.

Time = 0.25 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.90

$$\int \sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)\,dx =$$
$$-\frac{1}{4}\left(x^2+\frac{2\,ax}{b}+\frac{2\,\text{arsinh}\,(bx+a)\,\text{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b^2}-\frac{\text{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)^2}{b^2}\right)b$$
$$-\frac{1}{2}\left(\frac{a^2\,\text{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b}-\sqrt{b^2x^2+2\,abx+a^2+1}x-\frac{(a^2+1)\,\text{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b}-\right.$$
$$+a)$$

input
```
integrate(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/4*(x^2 + 2*a*x/b + 2*arcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a
^2*b^2 + 4*(a^2 + 1)*b^2))/b^2 - arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 +
 4*(a^2 + 1)*b^2))^2/b^2)*b - 1/2*(a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2
*b^2 + 4*(a^2 + 1)*b^2))/b - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x - (a^2 +
1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b - sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*a/b)*arcsinh(b*x + a)
```

3.262.8 Giac [F]

$$\int \sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)\,dx = \int \sqrt{b^2x^2+2\,abx+a^2+1}\,\text{arsinh}\left(bx+a\right)\,dx$$

input
```
integrate(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a), x)
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)\,dx = \int \text{asinh}(a+b\,x)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}\,dx$$

input
```
int(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

output
```
int(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.263 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)}\,dx$

3.263.1 Optimal result

Integrand size = 30, antiderivative size = 31

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)}\,dx=\frac{\operatorname{Chi}(2\operatorname{arcsinh}(a+bx))}{2b}+\frac{\log(\operatorname{arcsinh}(a+bx))}{2b}$$

output
```
1/2*Chi(2*arcsinh(b*x+a))/b+1/2*ln(arcsinh(b*x+a))/b
```

3.263.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)}\,dx=\frac{\operatorname{Chi}(2\operatorname{arcsinh}(a+bx))+\log(\operatorname{arcsinh}(a+bx))}{2b}$$

input
```
Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x],x]
```

output
```
(CoshIntegral[2*ArcSinh[a + b*x]] + Log[ArcSinh[a + b*x]])/(2*b)
```

3.263.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6275, 6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{\text{arcsinh}(a+bx)}dx$$

$$\Big\downarrow \text{6275}$$

$$\frac{\int \frac{\sqrt{(a+bx)^2+1}}{\text{arcsinh}(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{6206}$$

$$\frac{\int \frac{(a+bx)^2+1}{\text{arcsinh}(a+bx)}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(i\text{arcsinh}(a+bx)+\frac{\pi}{2}\right)^2}{\text{arcsinh}(a+bx)}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{\cosh(2\text{arcsinh}(a+bx))}{2\text{arcsinh}(a+bx)}+\frac{1}{2\text{arcsinh}(a+bx)}\right)d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\text{Chi}(2\text{arcsinh}(a+bx))+\frac{1}{2}\log(\text{arcsinh}(a+bx))}{b}$$

input
```
Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x],x]
```

output
```
(CoshIntegral[2*ArcSinh[a + b*x]]/2 + Log[ArcSinh[a + b*x]]/2)/b
```

3.263.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.263.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\ln(\operatorname{arcsinh}(bx+a))+\operatorname{Chi}(2\operatorname{arcsinh}(bx+a))}{2b}$	23

input
```
int((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(ln(arcsinh(b*x+a))+Chi(2*arcsinh(b*x+a)))/b
```

3.263.5 Fricas [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x, algorithm="frica
s")
```

output
```
integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a), x)
```

3.263.6 Sympy [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{\operatorname{asinh}\left(a+bx\right)}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2+1)**(1/2)/asinh(b*x+a),x)
```

output
```
Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/asinh(a + b*x), x)
```

3.263.7 Maxima [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a), x)
```

3.263.8 Giac [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)}\,dx=\int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x, algorithm="giac"
)
```

output
```
integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a), x)
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)}\,dx=\int \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{\operatorname{asinh}\left(a+b\,x\right)}\,dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x),x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x), x)
```

3.264 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\text{arcsinh}(a+bx)^2}\,dx$

3.264.1 Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\text{arcsinh}(a+bx)^2}\,dx = -\frac{1+(a+bx)^2}{b\text{arcsinh}(a+bx)} + \frac{\text{Shi}(2\text{arcsinh}(a+bx))}{b}$$

output
```
(-1-(b*x+a)^2)/b/arcsinh(b*x+a)+Shi(2*arcsinh(b*x+a))/b
```

3.264.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\text{arcsinh}(a+bx)^2}\,dx = -\frac{1+a^2+2abx+b^2x^2-\text{arcsinh}(a+bx)\text{Shi}(2\text{arcsinh}(a+bx))}{b\text{arcsinh}(a+bx)}$$

input
```
Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^2,x]
```

output
```
-((1 + a^2 + 2*a*b*x + b^2*x^2 - ArcSinh[a + b*x]*SinhIntegral[2*ArcSinh[a
 + b*x]])/(b*ArcSinh[a + b*x]))
```

3.264.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6275, 6205, 6195, 5971, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{\operatorname{arcsinh}(a+bx)^2} dx$$

$$\Big\downarrow \text{6275}$$

$$\frac{\int \frac{\sqrt{(a+bx)^2+1}}{\operatorname{arcsinh}(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \text{6205}$$

$$\frac{2\int \frac{a+bx}{\operatorname{arcsinh}(a+bx)}d(a+bx)-\frac{(a+bx)^2+1}{\operatorname{arcsinh}(a+bx)}}{b}$$

$$\Big\downarrow \text{6195}$$

$$\frac{2\int \frac{(a+bx)\sqrt{(a+bx)^2+1}}{\operatorname{arcsinh}(a+bx)}d\operatorname{arcsinh}(a+bx)-\frac{(a+bx)^2+1}{\operatorname{arcsinh}(a+bx)}}{b}$$

$$\Big\downarrow \text{5971}$$

$$\frac{2\int \frac{\sinh(2\operatorname{arcsinh}(a+bx))}{2\operatorname{arcsinh}(a+bx)}d\operatorname{arcsinh}(a+bx)-\frac{(a+bx)^2+1}{\operatorname{arcsinh}(a+bx)}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sinh(2\operatorname{arcsinh}(a+bx))}{\operatorname{arcsinh}(a+bx)}d\operatorname{arcsinh}(a+bx)-\frac{(a+bx)^2+1}{\operatorname{arcsinh}(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{(a+bx)^2+1}{\operatorname{arcsinh}(a+bx)}+\int -\frac{i\sin(2i\operatorname{arcsinh}(a+bx))}{\operatorname{arcsinh}(a+bx)}d\operatorname{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{-\frac{(a+bx)^2+1}{\operatorname{arcsinh}(a+bx)}-i\int \frac{\sin(2i\operatorname{arcsinh}(a+bx))}{\operatorname{arcsinh}(a+bx)}d\operatorname{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{3779}$$

$$\frac{\text{Shi}(2\text{arcsinh}(a+bx)) - \frac{(a+bx)^2+1}{\text{arcsinh}(a+bx)}}{b}$$

input
```
Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^2,x]
```

output
```
(-((1 + (a + b*x)^2)/ArcSinh[a + b*x]) + SinhIntegral[2*ArcSinh[a + b*x]])
/b
```

3.264.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6195
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
1/(b*c^(m + 1))   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
 a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.264.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{2\,\mathrm{Shi}(2\,\mathrm{arcsinh}(bx+a))\,\mathrm{arcsinh}(bx+a)-\cosh(2\,\mathrm{arcsinh}(bx+a))-1}{2b\,\mathrm{arcsinh}(bx+a)}$	44

input
```
int((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE
)
```

output
```
1/2/b*(2*Shi(2*arcsinh(b*x+a))*arcsinh(b*x+a)-cosh(2*arcsinh(b*x+a))-1)/ar
csinh(b*x+a)
```

3.264.5 Fricas [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\mathrm{arcsinh}(a+bx)^2}\,dx = \int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\mathrm{arsinh}\,(bx+a)^2}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x, algorithm="fri
cas")
```

output
```
integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^2, x)
```

3.264.6 Sympy [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{\operatorname{asinh}^2(a+bx)}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2+1)**(1/2)/asinh(b*x+a)**2,x)
```

output
```
Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/asinh(a + b*x)**2, x)
```

3.264.7 Maxima [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}(bx+a)^2}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x, algorithm="max
ima")
```

output
```
-((b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*
b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a*b^2*x + a
^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b*x + a +
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate(((2*b^2*x^2 + 4*a*b*x + 2*
a^2 - 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 2*(2*b^3*x^3 + 6*a*b^2*x^2
+ 2*a^3 + (6*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (2*b^4*x^4
+ 8*a*b^3*x^3 + 2*a^4 + 3*(4*a^2*b^2 + b^2)*x^2 + 3*a^2 + 2*(4*a^3*b + 3*a
*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^4*x^4 + 4*a*b^3*x^3 + a^
4 + 2*(3*a^2*b^2 + b^2)*x^2 + (b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2 + 2*a
*b*x + a^2) + 2*a^2 + 4*(a^3*b + a*b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 +
 (3*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)*log(b*x + a +
 sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

3.264.8 Giac [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x, algorithm="gia
c")
```

output
```
integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^2, x)
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{\operatorname{asinh}\left(a+b\,x\right)^2}\,dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x)^2,x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x)^2, x)
```

3.265 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\text{arcsinh}(a+bx)^3}\,dx$

3.265.1 Optimal result

Integrand size = 30, antiderivative size = 71

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\text{arcsinh}(a+bx)^3}\,dx=\frac{-1-(a+bx)^2}{2b\text{arcsinh}(a+bx)^2}-\frac{(a+bx)\sqrt{1+(a+bx)^2}}{b\text{arcsinh}(a+bx)}+\frac{\text{Chi}(2\text{arcsinh}(a+bx))}{b}$$

output
```
1/2*(-1-(b*x+a)^2)/b/arcsinh(b*x+a)^2+Chi(2*arcsinh(b*x+a))/b-(b*x+a)*(1+(
b*x+a)^2)^(1/2)/b/arcsinh(b*x+a)
```

3.265.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\text{arcsinh}(a+bx)^3}\,dx=-\frac{1+a^2+2abx+b^2x^2+2(a+bx)\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)-2\text{arcsinh}(a+bx)^2\text{Chi}(2\text{arcs}}{2b\text{arcsinh}(a+bx)^2}$$

input
```
Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^3,x]
```

output
```
-1/2*(1 + a^2 + 2*a*b*x + b^2*x^2 + 2*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b
^2*x^2]*ArcSinh[a + b*x] - 2*ArcSinh[a + b*x]^2*CoshIntegral[2*ArcSinh[a +
 b*x]])/(b*ArcSinh[a + b*x]^2)
```

3.265.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6275, 6205, 6193, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{\text{arcsinh}(a+bx)^3}dx$$

$$\Big\downarrow \text{6275}$$

$$\frac{\int \frac{\sqrt{(a+bx)^2+1}}{\text{arcsinh}(a+bx)^3}d(a+bx)}{b}$$

$$\Big\downarrow \text{6205}$$

$$\frac{\int \frac{a+bx}{\text{arcsinh}(a+bx)^2}d(a+bx)-\frac{(a+bx)^2+1}{2\text{arcsinh}(a+bx)^2}}{b}$$

$$\Big\downarrow \text{6193}$$

$$\frac{\int \frac{\cosh(2\text{arcsinh}(a+bx))}{\text{arcsinh}(a+bx)}d\text{arcsinh}(a+bx)-\frac{\sqrt{(a+bx)^2+1}(a+bx)}{\text{arcsinh}(a+bx)}-\frac{(a+bx)^2+1}{2\text{arcsinh}(a+bx)^2}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(2i\text{arcsinh}(a+bx)+\frac{\pi}{2}\right)}{\text{arcsinh}(a+bx)}d\text{arcsinh}(a+bx)-\frac{\sqrt{(a+bx)^2+1}(a+bx)}{\text{arcsinh}(a+bx)}-\frac{(a+bx)^2+1}{2\text{arcsinh}(a+bx)^2}}{b}$$

$$\Big\downarrow \text{3782}$$

$$\frac{\text{Chi}(2\text{arcsinh}(a+bx))-\frac{\sqrt{(a+bx)^2+1}(a+bx)}{\text{arcsinh}(a+bx)}-\frac{(a+bx)^2+1}{2\text{arcsinh}(a+bx)^2}}{b}$$

input
```
Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^3,x]
```

output
```
(-1/2*(1 + (a + b*x)^2)/ArcSinh[a + b*x]^2 - ((a + b*x)*Sqrt[1 + (a + b*x)
^2])/ArcSinh[a + b*x] + CoshIntegral[2*ArcSinh[a + b*x]])/b
```

3.265.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 6193
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Si
mp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-
a/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSi
nh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -
1]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.265.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{4\,\mathrm{Chi}\left(2\,\mathrm{arcsinh}\left(bx+a\right)\right)\mathrm{arcsinh}\left(bx+a\right)^{2}-2\sinh\left(2\,\mathrm{arcsinh}\left(bx+a\right)\right)\mathrm{arcsinh}\left(bx+a\right)-\cosh\left(2\,\mathrm{arcsinh}\left(bx+a\right)\right)-1}{4b\,\mathrm{arcsinh}\left(bx+a\right)^{2}}$	63

input
```
int((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE
)
```

output
```
1/4/b*(4*Chi(2*arcsinh(b*x+a))*arcsinh(b*x+a)^2-2*sinh(2*arcsinh(b*x+a))*a
rcsinh(b*x+a)-cosh(2*arcsinh(b*x+a))-1)/arcsinh(b*x+a)^2
```

3.265.5 Fricas [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x, algorithm="fri
cas")
```

output
```
integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^3, x)
```

3.265.6 Sympy [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{\operatorname{asinh}^3\left(a+bx\right)}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2+1)**(1/2)/asinh(b*x+a)**3,x)
```

output
```
Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/asinh(a + b*x)**3, x)
```

3.265.7 Maxima [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x, algorithm="max
ima")
```

output
```
-1/2*((b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*a^2*b^2 + b^2)*x^2 + a^2 + 2*(2*a^
3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (3*b^5*x^5 + 15*a*b^4*x^4
+ 3*a^5 + 5*(6*a^2*b^3 + b^3)*x^3 + 5*a^3 + 15*(2*a^3*b^2 + a*b^2)*x^2 + (
15*a^4*b + 15*a^2*b + 2*b)*x + 2*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) +
(3*b^6*x^6 + 18*a*b^5*x^5 + 3*a^6 + (45*a^2*b^4 + 7*b^4)*x^4 + 7*a^4 + 4*(
15*a^3*b^3 + 7*a*b^3)*x^3 + (45*a^4*b^2 + 42*a^2*b^2 + 5*b^2)*x^2 + 5*a^2
+ 2*(9*a^5*b + 14*a^3*b + 5*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + ((
2*b^4*x^4 + 8*a*b^3*x^3 + 2*a^4 + (12*a^2*b^2 + b^2)*x^2 + a^2 + 2*(4*a^3*
b + a*b)*x - 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (6*b^5*x^5 + 30*a*b^4*x^
4 + 6*a^5 + (60*a^2*b^3 + 7*b^3)*x^3 + 7*a^3 + 3*(20*a^3*b^2 + 7*a*b^2)*x^
2 + (30*a^4*b + 21*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) +
 (6*b^6*x^6 + 36*a*b^5*x^5 + 6*a^6 + (90*a^2*b^4 + 11*b^4)*x^4 + 11*a^4 +
4*(30*a^3*b^3 + 11*a*b^3)*x^3 + 6*(15*a^4*b^2 + 11*a^2*b^2 + b^2)*x^2 + 6*
a^2 + 4*(9*a^5*b + 11*a^3*b + 3*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)
+ (2*b^7*x^7 + 14*a*b^6*x^6 + 2*a^7 + (42*a^2*b^5 + 5*b^5)*x^5 + 5*a^5 + 5
*(14*a^3*b^4 + 5*a*b^4)*x^4 + 2*(35*a^4*b^3 + 25*a^2*b^3 + 2*b^3)*x^3 + 4*
a^3 + 2*(21*a^5*b^2 + 25*a^3*b^2 + 6*a*b^2)*x^2 + (14*a^6*b + 25*a^4*b + 1
2*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)) + (b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^
5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a...
```

3.265.8 Giac [F]

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x, algorithm="gia
c")
```

output
```
integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^3, x)
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{\operatorname{asinh}\left(a+b\,x\right)^3}\,dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x)^3,x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x)^3, x)
```

3.266 $\int (1+a^2+2abx+b^2x^2)^{3/2} \operatorname{arcsinh}(a+bx)^3\,dx$

3.266.1 Optimal result

Integrand size = 30, antiderivative size = 235

$$\begin{aligned}
\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \operatorname{arcsinh}(a+bx)^3\,dx &= -\frac{51(a+bx)^2}{128b} \\
&- \frac{3(a+bx)^4}{128b} + \frac{45(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{64b} \\
&+ \frac{3(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)}{32b} - \frac{27\operatorname{arcsinh}(a+bx)^2}{128b} \\
&- \frac{9(a+bx)^2\operatorname{arcsinh}(a+bx)^2}{16b} - \frac{3\left(1+(a+bx)^2\right)^2\operatorname{arcsinh}(a+bx)^2}{16b} \\
&+ \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^3}{8b} \\
&+ \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^3}{4b} + \frac{3\operatorname{arcsinh}(a+bx)^4}{32b}
\end{aligned}$$

output
```
-51/128*(b*x+a)^2/b-3/128*(b*x+a)^4/b+3/32*(b*x+a)*(1+(b*x+a)^2)^(3/2)*arc
sinh(b*x+a)/b-27/128*arcsinh(b*x+a)^2/b-9/16*(b*x+a)^2*arcsinh(b*x+a)^2/b-
3/16*(1+(b*x+a)^2)^2*arcsinh(b*x+a)^2/b+1/4*(b*x+a)*(1+(b*x+a)^2)^(3/2)*ar
csinh(b*x+a)^3/b+3/32*arcsinh(b*x+a)^4/b+45/64*(b*x+a)*arcsinh(b*x+a)*(1+(
b*x+a)^2)^(1/2)/b+3/8*(b*x+a)*arcsinh(b*x+a)^3*(1+(b*x+a)^2)^(1/2)/b
```

3.266.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.13

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)^3\,dx =$$
$$-\frac{6a(17+2a^2)\,bx+3(17+6a^2)\,b^2x^2+12ab^3x^3+3b^4x^4-6\sqrt{1+a^2+2abx+b^2x^2}(17a+2a^3+17bx+6a^2}{}$$

input
```
Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^3,x]
```

output
```
-1/128*(6*a*(17 + 2*a^2)*b*x + 3*(17 + 6*a^2)*b^2*x^2 + 12*a*b^3*x^3 + 3*b
^4*x^4 - 6*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(17*a + 2*a^3 + 17*b*x + 6*a^
2*b*x + 6*a*b^2*x^2 + 2*b^3*x^3)*ArcSinh[a + b*x] + 3*(17 + 8*a^4 + 32*a^3
*b*x + 40*b^2*x^2 + 8*b^4*x^4 + 16*a*b*x*(5 + 2*b^2*x^2) + 8*a^2*(5 + 6*b^
2*x^2))*ArcSinh[a + b*x]^2 - 16*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(5*a + 2
*a^3 + 5*b*x + 6*a^2*b*x + 6*a*b^2*x^2 + 2*b^3*x^3)*ArcSinh[a + b*x]^3 - 1
2*ArcSinh[a + b*x]^4)/b
```

3.266.3 Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.28, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6275, 6201, 6200, 6191, 6198, 6213, 6201, 244, 2009, 6200, 15, 6198, 6227, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2+1\right)^{3/2}\operatorname{arcsinh}(a+bx)^3\,dx$$

$$\Big\downarrow \text{6275}$$

$$\frac{\int \left((a+bx)^2+1\right)^{3/2}\operatorname{arcsinh}(a+bx)^3d(a+bx)}{b}$$

$$\Big\downarrow \text{6201}$$

$$\frac{-\frac{3}{4}\int (a+bx)\left((a+bx)^2+1\right)\operatorname{arcsinh}(a+bx)^2d(a+bx)+\frac{3}{4}\int \sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)^3d(a+bx)+\frac{1}{4}(a+}{b}$$

$$\Big\downarrow \text{6200}$$

$$-\frac{3}{4}\int(a+bx)\left((a+bx)^2+1\right)\text{arcsinh}(a+bx)^2d(a+bx)+\frac{3}{4}\left(-\frac{3}{2}\int(a+bx)\text{arcsinh}(a+bx)^2d(a+bx)+\frac{1}{2}\int\frac{\text{arcsi}}{\sqrt{(a}}\right.$$

$\downarrow$ 6191

$$-\frac{3}{4}\int(a+bx)\left((a+bx)^2+1\right)\text{arcsinh}(a+bx)^2d(a+bx)+\frac{3}{4}\left(-\frac{3}{2}\left(\frac{1}{2}(a+bx)^2\text{arcsinh}(a+bx)^2-\int\frac{(a+bx)^2\text{arcsinh}(}{\sqrt{(a+bx)^2+}}\right.\right.$$

$\downarrow$ 6198

$$\frac{3}{4}\left(-\frac{3}{2}\left(\frac{1}{2}(a+bx)^2\text{arcsinh}(a+bx)^2-\int\frac{(a+bx)^2\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)\right)+\frac{1}{8}\text{arcsinh}(a+bx)^4+\frac{1}{2}(a+bx)\sqrt{(a+bx}\right.$$

$\downarrow$ 6213

$$\frac{3}{4}\left(-\frac{3}{2}\left(\frac{1}{2}(a+bx)^2\text{arcsinh}(a+bx)^2-\int\frac{(a+bx)^2\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)\right)+\frac{1}{8}\text{arcsinh}(a+bx)^4+\frac{1}{2}(a+bx)\sqrt{(a+bx}\right.$$

$\downarrow$ 6201

$$\frac{3}{4}\left(-\frac{3}{2}\left(\frac{1}{2}(a+bx)^2\text{arcsinh}(a+bx)^2-\int\frac{(a+bx)^2\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)\right)+\frac{1}{8}\text{arcsinh}(a+bx)^4+\frac{1}{2}(a+bx)\sqrt{(a+bx}\right.$$

$\downarrow$ 244

$$\frac{3}{4}\left(-\frac{3}{2}\left(\frac{1}{2}(a+bx)^2\text{arcsinh}(a+bx)^2-\int\frac{(a+bx)^2\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)\right)+\frac{1}{8}\text{arcsinh}(a+bx)^4+\frac{1}{2}(a+bx)\sqrt{(a+bx}\right.$$

$\downarrow$ 2009

$$\frac{3}{4}\left(-\frac{3}{2}\left(\frac{1}{2}(a+bx)^2\text{arcsinh}(a+bx)^2-\int\frac{(a+bx)^2\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)\right)+\frac{1}{8}\text{arcsinh}(a+bx)^4+\frac{1}{2}(a+bx)\sqrt{(a+bx}\right.$$

$\downarrow$ 6200

$$-\frac{3}{4}\left(\frac{1}{2}\left(-\frac{3}{4}\left(\frac{1}{2}\int\frac{\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{1}{2}\int(a+bx)d(a+bx)+\frac{1}{2}(a+bx)\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)\right)-\frac{1}{4}(\right.\right.$$

$\downarrow$ 15

3.266. $\int\left(1+a^2+2abx+b^2x^2\right)^{3/2}\text{arcsinh}(a+bx)^3\,dx$

$$-\frac{3}{4}\left(\frac{1}{2}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}\sqrt{(a+bx)^2+1}(a+bx)\text{arcsinh}(a+bx)-\frac{1}{4}(a+bx)^2\right)-\frac{1}{4}(a+bx)\left((\right.\right.\right.$$

$\downarrow$ 6198

$$\frac{3}{4}\left(-\frac{3}{2}\left(\frac{1}{2}(a+bx)^2\text{arcsinh}(a+bx)^2-\int \frac{(a+bx)^2\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)\right)+\frac{1}{8}\text{arcsinh}(a+bx)^4+\frac{1}{2}(a+bx)\sqrt{(a+bx\right.$$

$\downarrow$ 6227

$$\frac{3}{4}\left(-\frac{3}{2}\left(\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}\int (a+bx)d(a+bx)+\frac{1}{2}(a+bx)^2\text{arcsinh}(a+bx)^2-\frac{1}{2}(a+bx)\sqrt{(a+bx)^2\right.\right.$$

$\downarrow$ 15

$$\frac{3}{4}\left(-\frac{3}{2}\left(\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}(a+bx)^2\text{arcsinh}(a+bx)^2-\frac{1}{2}\sqrt{(a+bx)^2+1}(a+bx)\text{arcsinh}(a+bx)+\frac{1}{4}(\right.\right.$$

$\downarrow$ 6198

$$\frac{1}{4}(a+bx)\left((a+bx)^2+1\right)^{3/2}\text{arcsinh}(a+bx)^3+\frac{3}{4}\left(\frac{1}{8}\text{arcsinh}(a+bx)^4+\frac{1}{2}(a+bx)\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)\right.$$

input
```
Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^3,x]
```

output
```
(((a + b*x)*(1 + (a + b*x)^2)^(3/2)*ArcSinh[a + b*x]^3)/4 + (3*(((a + b*x)
*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^3)/2 + ArcSinh[a + b*x]^4/8 - (3*(
(a + b*x)^2/4 - ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/2 + Arc
Sinh[a + b*x]^2/4 + ((a + b*x)^2*ArcSinh[a + b*x]^2)/2))/2))/4 - (3*(((1 +
 (a + b*x)^2)^2*ArcSinh[a + b*x]^2)/4 + (((a + b*x)^2/2 + (a + b*x)^4/4)/4
 - ((a + b*x)*(1 + (a + b*x)^2)^(3/2)*ArcSinh[a + b*x])/4 - (3*(-1/4*(a +
b*x)^2 + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/2 + ArcSinh[a
+ b*x]^2/4))/4)/2))/4)/b
```

3.266. $\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\text{arcsinh}(a+bx)^3\,dx$

3.266.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

3.266. $\int (1+a^2+2abx+b^2x^2)^{3/2} \operatorname{arcsinh}(a+bx)^3 \, dx$

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6227
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Simp[f^2*((m - 1)/(c^2*(m +
2*p + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
 - Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int
[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[
m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.266.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(207) = 414$.

Time = 0.73 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.52

method	result
default	$\frac{-48-102abx-51a^2-51b^2x^2+12\operatorname{arcsinh}(bx+a)^4-51\operatorname{arcsinh}(bx+a)^2-12a\,b^3x^3-3b^4x^4+96\operatorname{arcsinh}(bx+a)^3\sqrt{b^2x^2+2abx+a^2+1}\,a\,b^2}{}$

input
```
int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE
)
```

output
```
1/128*(-48-102*a*b*x-51*a^2-51*b^2*x^2+12*arcsinh(b*x+a)^4-51*arcsinh(b*x+
a)^2-12*a*b^3*x^3-3*b^4*x^4+96*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)*a*b^2*x^2+96*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^2*b*x+36
*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a*b^2*x^2+36*arcsinh(b*x+a)*
(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^2*b*x-3*a^4+12*arcsinh(b*x+a)*(b^2*x^2+2*a
*b*x+a^2+1)^(1/2)*b^3*x^3+32*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)*b^3*x^3-96*arcsinh(b*x+a)^2*a*b^3*x^3-144*arcsinh(b*x+a)^2*a^2*b^2*x^2-9
6*arcsinh(b*x+a)^2*a^3*b*x+80*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)*b*x-240*arcsinh(b*x+a)^2*a*b*x+102*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+
1)^(1/2)*b*x-24*a^4*arcsinh(b*x+a)^2-120*a^2*arcsinh(b*x+a)^2-120*arcsinh(
b*x+a)^2*b^2*x^2+80*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+102*a
rcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a+12*arcsinh(b*x+a)*(b^2*x^2+2
*a*b*x+a^2+1)^(1/2)*a^3+32*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*
a^3-24*arcsinh(b*x+a)^2*b^4*x^4-18*a^2*b^2*x^2-12*a^3*b*x)/b
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.41

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \operatorname{arcsinh}(a+bx)^3\,dx =$$

$$-\frac{3\,b^4x^4+12\,ab^3x^3+3\,(6\,a^2+17)b^2x^2-16\,(2\,b^3x^3+6\,ab^2x^2+2\,a^3+(6\,a^2+5)bx+5\,a)\sqrt{b^2x^2+2\,abx+a}}{}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x, algorithm="fri
cas")
```

output
```
-1/128*(3*b^4*x^4 + 12*a*b^3*x^3 + 3*(6*a^2 + 17)*b^2*x^2 - 16*(2*b^3*x^3
+ 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 5)*b*x + 5*a)*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - 12*log(b*x + a
 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4 + 6*(2*a^3 + 17*a)*b*x + 3*(8*b^4*
x^4 + 32*a*b^3*x^3 + 8*(6*a^2 + 5)*b^2*x^2 + 8*a^4 + 16*(2*a^3 + 5*a)*b*x
+ 40*a^2 + 17)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - 6*(2*b
^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 17)*b*x + 17*a)*sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

3.266.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(223) = 446$.

Time = 1.15 (sec) , antiderivative size = 694, normalized size of antiderivative = 2.95

$$\int (1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)^3\,dx = \begin{cases} -\frac{3a^4\operatorname{asinh}^2(a+bx)}{16b} - \frac{3a^3x\operatorname{asinh}^2(a+bx)}{4} - \frac{3a^3x}{32} + \frac{a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^3(a+bx)}{4b} + \frac{3a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asin}}{32b} \\ x(a^2+1)^{\frac{3}{2}}\operatorname{asinh}^3(a) \end{cases}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)*asinh(b*x+a)**3,x)
```

output
```
Piecewise((-3*a**4*asinh(a + b*x)**2/(16*b) - 3*a**3*x*asinh(a + b*x)**2/4
 - 3*a**3*x/32 + a**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)*
*3/(4*b) + 3*a**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(32*
b) - 9*a**2*b*x**2*asinh(a + b*x)**2/8 - 9*a**2*b*x**2/64 + 3*a**2*x*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/4 + 9*a**2*x*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/32 - 15*a**2*asinh(a + b*x)**2/(1
6*b) - 3*a*b**2*x**3*asinh(a + b*x)**2/4 - 3*a*b**2*x**3/32 + 3*a*b*x**2*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/4 + 9*a*b*x**2*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/32 - 15*a*x*asinh(a + b*x)*
*2/8 - 51*a*x/64 + 5*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)
**3/(8*b) + 51*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(64*b
) - 3*b**3*x**4*asinh(a + b*x)**2/16 - 3*b**3*x**4/128 + b**2*x**3*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/4 + 3*b**2*x**3*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/32 - 15*b*x**2*asinh(a + b*x)**2
/16 - 51*b*x**2/128 + 5*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b
*x)**3/8 + 51*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/64 + 3
*asinh(a + b*x)**4/(32*b) - 51*asinh(a + b*x)**2/(128*b), Ne(b, 0)), (x*(a
**2 + 1)**(3/2)*asinh(a)**3, True))
```

3.266.7 Maxima [F]

$$\int (1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)^3\,dx = \int (b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}\operatorname{arsinh}(bx+a)^3\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x, algorithm="max
ima")
```

output
```
integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^3, x)
```

3.266.8 Giac [F]

$$\int (1 + a^2 + 2abx + b^2x^2)^{3/2} \operatorname{arcsinh}(a + bx)^3 \, dx = \int \left(b^2x^2 + 2\,abx + a^2 + 1\right)^{\frac{3}{2}} \operatorname{arsinh}\left(bx + a\right)^3 \, dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x, algorithm="gia
c")
```

output
```
integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^3, x)
```

3.266.9 Mupad [F(-1)]

Timed out.

$$\int (1 + a^2 + 2abx + b^2x^2)^{3/2} \operatorname{arcsinh}(a + bx)^3 \, dx = \int \operatorname{asinh}(a + b\,x)^3 \left(a^2 + 2\,a\,b\,x + b^2\,x^2 + 1\right)^{3/2} dx$$

input
```
int(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

output
```
int(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

3.267 $\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \operatorname{arcsinh}(a+bx)^2 \, dx$

3.267.1 Optimal result

Integrand size = 30, antiderivative size = 189

$$\begin{aligned}\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \operatorname{arcsinh}(a+bx)^2 \, dx &= \frac{15(a+bx)\sqrt{1+(a+bx)^2}}{64b} \\ &+ \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{32b} - \frac{9\operatorname{arcsinh}(a+bx)}{64b} - \frac{3(a+bx)^2\operatorname{arcsinh}(a+bx)}{8b} \\ &- \frac{\left(1+(a+bx)^2\right)^2 \operatorname{arcsinh}(a+bx)}{8b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)^2}{8b} \\ &+ \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2} \operatorname{arcsinh}(a+bx)^2}{4b} + \frac{\operatorname{arcsinh}(a+bx)^3}{8b}\end{aligned}$$

output
```
1/32*(b*x+a)*(1+(b*x+a)^2)^(3/2)/b-9/64*arcsinh(b*x+a)/b-3/8*(b*x+a)^2*arc
sinh(b*x+a)/b-1/8*(1+(b*x+a)^2)^2*arcsinh(b*x+a)/b+1/4*(b*x+a)*(1+(b*x+a)^
2)^(3/2)*arcsinh(b*x+a)^2/b+1/8*arcsinh(b*x+a)^3/b+15/64*(b*x+a)*(1+(b*x+a
)^2)^(1/2)/b+3/8*(b*x+a)*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b
```

3.267.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.12

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \operatorname{arcsinh}(a+bx)^2\,dx = \frac{\sqrt{1+a^2+2abx+b^2x^2}\left(17a+2a^3+17bx+6a^2bx+6ab^2x^2+2b^3x^3\right)-\left(17+40a^2+8a^4\right)\operatorname{arcsin}$$

input
```
Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2,x]
```

output
```
(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(17*a + 2*a^3 + 17*b*x + 6*a^2*b*x + 6*
a*b^2*x^2 + 2*b^3*x^3) - (17 + 40*a^2 + 8*a^4)*ArcSinh[a + b*x] - 8*b*x*(1
0*a + 4*a^3 + 5*b*x + 6*a^2*b*x + 4*a*b^2*x^2 + b^3*x^3)*ArcSinh[a + b*x]
+ 8*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(5*a + 2*a^3 + 5*b*x + 6*a^2*b*x + 6
*a*b^2*x^2 + 2*b^3*x^3)*ArcSinh[a + b*x]^2 + 8*ArcSinh[a + b*x]^3)/(64*b)
```

3.267.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {6275, 6201, 6200, 6191, 262, 222, 6198, 6213, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2+1\right)^{3/2} \operatorname{arcsinh}(a+bx)^2\,dx$$

$$\Big\downarrow \text{ 6275}$$

$$\frac{\int \left((a+bx)^2+1\right)^{3/2} \operatorname{arcsinh}(a+bx)^2 d(a+bx)}{b}$$

$$\Big\downarrow \text{ 6201}$$

$$\frac{-\frac{1}{2}\int (a+bx)\left((a+bx)^2+1\right) \operatorname{arcsinh}(a+bx)d(a+bx)+\frac{3}{4}\int \sqrt{(a+bx)^2+1}\operatorname{arcsinh}(a+bx)^2 d(a+bx)+\frac{1}{4}(a+b}{b}$$

$$\Big\downarrow \text{ 6200}$$

$$\frac{-\frac{1}{2}\int (a+bx)\left((a+bx)^2+1\right)\text{arcsinh}(a+bx)d(a+bx)+\frac{3}{4}\left(-\int (a+bx)\text{arcsinh}(a+bx)d(a+bx)+\frac{1}{2}\int \frac{\text{arcsinh}(a}{\sqrt{(a+bx}}\right.}{b}$$

$$\Big\downarrow \; 6191$$

$$-\frac{1}{2}\int (a+bx)\left((a+bx)^2+1\right)\text{arcsinh}(a+bx)d(a+bx)+\frac{3}{4}\left(\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}\int \frac{(a+bx)^2}{\sqrt{(a+bx)^2+1}}d(a+b\right.$$

$$\Big\downarrow \; 262$$

$$-\frac{1}{2}\int (a+bx)\left((a+bx)^2+1\right)\text{arcsinh}(a+bx)d(a+bx)+\frac{3}{4}\left(\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}\left(\frac{1}{2}(a+bx)\sqrt{(a+bx}\right.\right.$$

$$\Big\downarrow \; 222$$

$$-\frac{1}{2}\int (a+bx)\left((a+bx)^2+1\right)\text{arcsinh}(a+bx)d(a+bx)+\frac{3}{4}\left(\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{1}{2}(a+bx)^2\text{arcsinh}(a+\right.$$

$$\Big\downarrow \; 6198$$

$$-\frac{1}{2}\int (a+bx)\left((a+bx)^2+1\right)\text{arcsinh}(a+bx)d(a+bx)+\frac{1}{4}(a+bx)\left((a+bx)^2+1\right)^{3/2}\text{arcsinh}(a+bx)^2+\frac{3}{4}\left(\frac{1}{6}\text{arc}\right.$$

$$\Big\downarrow \; 6213$$

$$\frac{1}{2}\left(\frac{1}{4}\int \left((a+bx)^2+1\right)^{3/2}d(a+bx)-\frac{1}{4}\left((a+bx)^2+1\right)^2\text{arcsinh}(a+bx)\right)+\frac{1}{4}(a+bx)\left((a+bx)^2+1\right)^{3/2}\text{arcsinh}($$

$$\Big\downarrow \; 211$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{3}{4}\int \sqrt{(a+bx)^2+1}d(a+bx)+\frac{1}{4}(a+bx)\left((a+bx)^2+1\right)^{3/2}\right)-\frac{1}{4}\left((a+bx)^2+1\right)^2\text{arcsinh}(a+bx)\right)+\frac{1}{4}(a$$

$$\Big\downarrow \; 211$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}\sqrt{(a+bx)^2+1}(a+bx)\right)+\frac{1}{4}(a+bx)\left((a+bx)^2+1\right)^{3/2}\right)-\frac{1}{4}\left((a+bx)^2+\right.\right.$$

$$\Big\downarrow \; 222$$

3.267. $\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\text{arcsinh}(a+bx)^2\,dx$

$$\frac{1}{4}(a+bx)\left((a+bx)^2+1\right)^{3/2}\text{arcsinh}(a+bx)^2+\frac{1}{2}\left(\frac{1}{4}\left(\frac{3}{4}\left(\frac{1}{2}\text{arcsinh}(a+bx)+\frac{1}{2}\sqrt{(a+bx)^2+1}(a+bx)\right)+\frac{1}{4}(a+\right.\right.$$

input
```
Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2,x]
```

output
```
(((a + b*x)*(1 + (a + b*x)^2)^(3/2)*ArcSinh[a + b*x]^2)/4 + ((((a + b*x)*(
1 + (a + b*x)^2)^(3/2))/4 + (3*(((a + b*x)*Sqrt[1 + (a + b*x)^2])/2 + ArcS
inh[a + b*x]/2))/4)/4 - ((1 + (a + b*x)^2)^2*ArcSinh[a + b*x])/4)/2 + (3*(
(((a + b*x)*Sqrt[1 + (a + b*x)^2])/2 - ArcSinh[a + b*x]/2)/2 - ((a + b*x)^
2*ArcSinh[a + b*x])/2 + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^
2)/2 + ArcSinh[a + b*x]^3/6))/4)/b
```

3.267.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6191
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Simp[b*c*
(n/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 +
c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6213
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]
 Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.267. $\int (1+a^2+2abx+b^2x^2)^{3/2} \operatorname{arcsinh}(a+bx)^2\,dx$

3.267.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(165) = 330$.

Time = 0.89 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.53

method	result
default	$\frac{16 \operatorname{arcsinh}(bx+a)^2 \sqrt{b^2x^2+2abx+a^2+1}\, b^3x^3 - 8 \operatorname{arcsinh}(bx+a) b^4x^4 + 48 \operatorname{arcsinh}(bx+a)^2 \sqrt{b^2x^2+2abx+a^2+1}\, a\, b^2x^2 - 32 \operatorname{arcsinh}(bx+}{}$

input
```
int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE
)
```

output
```
1/64*(16*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b^3*x^3-8*arcsinh(
b*x+a)*b^4*x^4+48*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a*b^2*x^2
-32*arcsinh(b*x+a)*a*b^3*x^3+48*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)*a^2*b*x+2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b^3*x^3-48*arcsinh(b*x+a)*a^2
*b^2*x^2+16*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3+6*(b^2*x^2+
2*a*b*x+a^2+1)^(1/2)*a*b^2*x^2-32*arcsinh(b*x+a)*a^3*b*x+40*arcsinh(b*x+a)
^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x+6*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^2*b
*x-8*arcsinh(b*x+a)*a^4-40*arcsinh(b*x+a)*b^2*x^2+40*arcsinh(b*x+a)^2*(b^2
*x^2+2*a*b*x+a^2+1)^(1/2)*a+2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3-80*arcsinh
(b*x+a)*a*b*x+17*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x+8*arcsinh(b*x+a)^3-40*a
^2*arcsinh(b*x+a)+17*a*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-17*arcsinh(b*x+a))/b
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.37

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \operatorname{arcsinh}(a+bx)^2\, dx = \frac{8\,(2\,b^3x^3+6\,ab^2x^2+2\,a^3+(6\,a^2+5)bx+5\,a)\sqrt{b^2x^2+2\,abx+a^2+1}\,\log\left(bx+a+\sqrt{b^2x^2+2\,a}\right.}{}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x, algorithm="fri
cas")
```

output
```
1/64*(8*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 5)*b*x + 5*a)*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
^2 + 8*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - (8*b^4*x^4 + 3
2*a*b^3*x^3 + 8*(6*a^2 + 5)*b^2*x^2 + 8*a^4 + 16*(2*a^3 + 5*a)*b*x + 40*a^
2 + 17)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (2*b^3*x^3 + 6*
a*b^2*x^2 + 2*a^3 + (6*a^2 + 17)*b*x + 17*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))/b
```

3.267.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(173) = 346$.

Time = 0.74 (sec) , antiderivative size = 568, normalized size of antiderivative = 3.01

$$\int (1 + a^2 + 2abx + b^2x^2)^{3/2} \operatorname{arcsinh}(a+bx)^2\,dx = \begin{cases} -\frac{a^4 \operatorname{asinh}(a+bx)}{8b} - \frac{a^3 x \operatorname{asinh}(a+bx)}{2} + \frac{a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{4b} + \frac{a^3\sqrt{a^2+2abx+b^2x^2+1}}{32b} - \frac{3a^2bx^2\operatorname{asinh}(a}{4} \\ x(a^2+1)^{\frac{3}{2}}\operatorname{asinh}^2(a) \end{cases}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)*asinh(b*x+a)**2,x)
```

output
```
Piecewise((-a**4*asinh(a + b*x)/(8*b) - a**3*x*asinh(a + b*x)/2 + a**3*sqr
t(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(4*b) + a**3*sqrt(a**2
 + 2*a*b*x + b**2*x**2 + 1)/(32*b) - 3*a**2*b*x**2*asinh(a + b*x)/4 + 3*a*
*2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/4 + 3*a**2*x*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/32 - 5*a**2*asinh(a + b*x)/(8*b) - a*b
**2*x**3*asinh(a + b*x)/2 + 3*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1
)*asinh(a + b*x)**2/4 + 3*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/32
 - 5*a*x*asinh(a + b*x)/4 + 5*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh
(a + b*x)**2/(8*b) + 17*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(64*b) - b*
*3*x**4*asinh(a + b*x)/8 + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*
asinh(a + b*x)**2/4 + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/32 -
5*b*x**2*asinh(a + b*x)/8 + 5*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh
(a + b*x)**2/8 + 17*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/64 + asinh(a +
b*x)**3/(8*b) - 17*asinh(a + b*x)/(64*b), Ne(b, 0)), (x*(a**2 + 1)**(3/2)*
asinh(a)**2, True))
```

3.267.7 Maxima [F]

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \operatorname{arcsinh}(a+bx)^2\,dx = \int \left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}} \operatorname{arsinh}\left(bx+a\right)^2\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x, algorithm="max
ima")
```

output
```
integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^2, x)
```

3.267.8 Giac [F]

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \operatorname{arcsinh}(a+bx)^2\,dx = \int \left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}} \operatorname{arsinh}\left(bx+a\right)^2\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x, algorithm="gia
c")
```

output
```
integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^2, x)
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \operatorname{arcsinh}(a+bx)^2\,dx = \int \operatorname{asinh}(a+b\,x)^2\,\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2}\,dx$$

input
```
int(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

output
```
int(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

3.268 $\int (1+a^2+2abx+b^2x^2)^{3/2} \operatorname{arcsinh}(a+bx)\,dx$

3.268.1 Optimal result

Integrand size = 28, antiderivative size = 106

$$\int (1+a^2+2abx+b^2x^2)^{3/2} \operatorname{arcsinh}(a+bx)\,dx = -\frac{5(a+bx)^2}{16b} - \frac{(a+bx)^4}{16b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\operatorname{arcsinh}(a+bx)}{8b} + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\operatorname{arcsinh}(a+bx)}{4b} + \frac{3\operatorname{arcsinh}(a+bx)^2}{16b}$$

output

```
-5/16*(b*x+a)^2/b-1/16*(b*x+a)^4/b+1/4*(b*x+a)*(1+(b*x+a)^2)^(3/2)*arcsinh
(b*x+a)/b+3/16*arcsinh(b*x+a)^2/b+3/8*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)
^(1/2)/b
```

3.268.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.17

$$\int (1+a^2+2abx+b^2x^2)^{3/2} \operatorname{arcsinh}(a+bx)\,dx = \frac{-bx(10a+4a^3+5bx+6a^2bx+4ab^2x^2+b^3x^3)+2\sqrt{1+a^2+2abx+b^2x^2}(5a+2a^3+5bx+6a^2}{16b}$$

input

```
Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x],x]
```

output
```
(-(b*x*(10*a + 4*a^3 + 5*b*x + 6*a^2*b*x + 4*a*b^2*x^2 + b^3*x^3)) + 2*Sqr
t[1 + a^2 + 2*a*b*x + b^2*x^2]*(5*a + 2*a^3 + 5*b*x + 6*a^2*b*x + 6*a*b^2*
x^2 + 2*b^3*x^3)*ArcSinh[a + b*x] + 3*ArcSinh[a + b*x]^2)/(16*b)
```

3.268.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6275, 6201, 244, 2009, 6200, 15, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx+b^2x^2+1\right)^{3/2}\text{arcsinh}(a+bx)\,dx$$

$$\Big\downarrow \text{6275}$$

$$\frac{\int \left((a+bx)^2+1\right)^{3/2}\text{arcsinh}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{6201}$$

$$\frac{\frac{3}{4}\int \sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)d(a+bx)-\frac{1}{4}\int (a+bx)\left((a+bx)^2+1\right)d(a+bx)+\frac{1}{4}(a+bx)\left((a+bx)^2+1\right)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\frac{3}{4}\int \sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)d(a+bx)-\frac{1}{4}\int \left((a+bx)^3+a+bx\right)d(a+bx)+\frac{1}{4}(a+bx)\left((a+bx)^2+1\right)^{3/2}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3}{4}\int \sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)d(a+bx)+\frac{1}{4}(a+bx)\left((a+bx)^2+1\right)^{3/2}\text{arcsinh}(a+bx)+\frac{1}{4}\left(-\frac{1}{4}(a+bx)^4-\right.}{b}$$

$$\Big\downarrow \text{6200}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{1}{2}\int (a+bx)d(a+bx)+\frac{1}{2}(a+bx)\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)\right)+\frac{1}{4}(a+bx)\,\left(\right.}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int \frac{\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}\sqrt{(a+bx)^2+1}(a+bx)\text{arcsinh}(a+bx)-\frac{1}{4}(a+bx)^2\right)+\frac{1}{4}(a+bx)\left((a+bx)^2\right. -}{b}$$

$\downarrow$ 6198

$$\frac{\frac{1}{4}(a+bx)\left((a+bx)^2+1\right)^{3/2}\text{arcsinh}(a+bx)+\frac{3}{4}\left(\frac{1}{2}\sqrt{(a+bx)^2+1}(a+bx)\text{arcsinh}(a+bx)+\frac{1}{4}\text{arcsinh}(a+bx)^2\right. -}{b}$$

input
```
Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x],x]
```

output
```
((-1/2*(a + b*x)^2 - (a + b*x)^4/4)/4 + ((a + b*x)*(1 + (a + b*x)^2)^(3/2)
*ArcSinh[a + b*x])/4 + (3*(-1/4*(a + b*x)^2 + ((a + b*x)*Sqrt[1 + (a + b*x
)^2]*ArcSinh[a + b*x])/2 + ArcSinh[a + b*x]^2/4))/4)/b
```

3.268.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6200
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Simp[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]]   Int[(a + b*ArcSinh[c*x])^n/Sq
rt[1 + c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*
x^2]]   Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e
}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

rule 6201
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Simp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x
], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[x*
(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.268.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(92) = 184$.

Time = 0.72 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.47

method	result
default	$\frac{4 \operatorname{arcsinh}(bx+a)\sqrt{b^2x^2+2abx+a^2+1}\, b^3x^3-b^4x^4+12 \operatorname{arcsinh}(bx+a)\sqrt{b^2x^2+2abx+a^2+1}\, a\, b^2x^2-4a\, b^3x^3+12 \operatorname{arcsinh}(bx+a)\sqrt{b^2x^2+}}{}$

input
```
int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b^3*x^3-b^4*x^4+12*ar
csinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a*b^2*x^2-4*a*b^3*x^3+12*arcsin
h(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^2*b*x-6*a^2*b^2*x^2+4*arcsinh(b*x
+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a^3-4*a^3*b*x+10*arcsinh(b*x+a)*(b^2*x^2
+2*a*b*x+a^2+1)^(1/2)*b*x-a^4-5*b^2*x^2+10*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x
+a^2+1)^(1/2)*a-10*a*b*x+3*arcsinh(b*x+a)^2-5*a^2-4)/b
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.51

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \operatorname{arcsinh}(a+bx)\, dx = -\frac{b^4x^4+4\,ab^3x^3+(6\,a^2+5)b^2x^2+2\,(2\,a^3+5\,a)bx-2\,(2\,b^3x^3+6\,ab^2x^2+2\,a^3+(6\,a^2+5)bx+5\,a)\sqrt{b^2x^2}}{16\,b}$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x, algorithm="frica
s")
```

output
```
-1/16*(b^4*x^4 + 4*a*b^3*x^3 + (6*a^2 + 5)*b^2*x^2 + 2*(2*a^3 + 5*a)*b*x -
 2*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 5)*b*x + 5*a)*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3*
log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)/b
```

3.268.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(95) = 190$.

Time = 0.49 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.81

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \operatorname{arcsinh}(a+bx)\, dx = \begin{cases} -\frac{a^3x}{4}+\frac{a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{4b}-\frac{3a^2bx^2}{8}+\frac{3a^2x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{4}-\frac{ab^2x^3}{4}+\frac{3abx^2\sqrt{a^2+2}}{} \\ x(a^2+1)^{\frac{3}{2}}\operatorname{asinh}(a) \end{cases}$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)*asinh(b*x+a),x)
```

output
```
Piecewise((-a**3*x/4 + a**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a +
 b*x)/(4*b) - 3*a**2*b*x**2/8 + 3*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1)*asinh(a + b*x)/4 - a*b**2*x**3/4 + 3*a*b*x**2*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1)*asinh(a + b*x)/4 - 5*a*x/8 + 5*a*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1)*asinh(a + b*x)/(8*b) - b**3*x**4/16 + b**2*x**3*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1)*asinh(a + b*x)/4 - 5*b*x**2/16 + 5*x*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1)*asinh(a + b*x)/8 + 3*asinh(a + b*x)**2/(16*b), Ne(b, 0
)), (x*(a**2 + 1)**(3/2)*asinh(a), True))
```

3.268.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(92) = 184.

Time = 0.28 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.72

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2} \operatorname{arcsinh}(a+bx)\,dx =$$

$$-\frac{1}{16}\left(b^2x^4+4\,abx^3+6\,a^2x^2+\frac{4\,a^3x}{b}+5\,x^2+\frac{10\,ax}{b}+\frac{6\,\operatorname{arsinh}\left(bx+a\right)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b^2}-\frac{3\,a$$

$$+\frac{1}{8}\left(2\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}x+\frac{2\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a}{b}+\frac{3\,(a^2b^2-(a^2+1)b^2)a^2\,\operatorname{arsinh}\left(\frac{2\,(b^2x}{\sqrt{-4\,a^2b^2+}}\right.}{b^3}\right.$$

$$+a)$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x, algorithm="maxim
a")
```

output
```
-1/16*(b^2*x^4 + 4*a*b*x^3 + 6*a^2*x^2 + 4*a^3*x/b + 5*x^2 + 10*a*x/b + 6*
arcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2
))/b^2 - 3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))^2/b
^2)*b + 1/8*(2*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x + 2*(b^2*x^2 + 2*a*b*
x + a^2 + 1)^(3/2)*a/b + 3*(a^2*b^2 - (a^2 + 1)*b^2)*a^2*arcsinh(2*(b^2*x
+ a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 3*(a^2*b^2 - (a^2 + 1)*b^
2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 - 3*(a^2*b^2 - (a^2 + 1)*b^2)*(
a^2 + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 -
 3*(a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^3)*arcs
inh(b*x + a)
```

3.268.8 Giac [F]

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\operatorname{arcsinh}(a+bx)\,dx = \int \left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}\operatorname{arsinh}\left(bx+a\right)\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x, algorithm="giac"
)
```

output
```
integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a), x)
```

3.268.9 Mupad [F(-1)]

Timed out.

$$\int (1+a^2+2abx+b^2x^2)^{3/2} \operatorname{arcsinh}(a+bx)\,dx = \int \operatorname{asinh}(a+b\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2}\,dx$$

input
```
int(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

output
```
int(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

3.269 $\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\text{arcsinh}(a+bx)} \, dx$

3.269.1 Optimal result

Integrand size = 30, antiderivative size = 47

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\text{arcsinh}(a+bx)} \, dx = \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{2b} + \frac{\text{Chi}(4\text{arcsinh}(a+bx))}{8b} + \frac{3\log(\text{arcsinh}(a+bx))}{8b}$$

output
```
1/2*Chi(2*arcsinh(b*x+a))/b+1/8*Chi(4*arcsinh(b*x+a))/b+3/8*ln(arcsinh(b*x
+a))/b
```

3.269.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\text{arcsinh}(a+bx)} \, dx = \frac{4\text{Chi}(2\text{arcsinh}(a+bx)) + \text{Chi}(4\text{arcsinh}(a+bx)) + 3\log(\text{arcsinh}(a+bx))}{8b}$$

input
```
Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x],x]
```

output
```
(4*CoshIntegral[2*ArcSinh[a + b*x]] + CoshIntegral[4*ArcSinh[a + b*x]] + 3
*Log[ArcSinh[a + b*x]])/(8*b)
```

3.269.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6275, 6206, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx+b^2x^2+1)^{3/2}}{\text{arcsinh}(a+bx)}dx$$

$$\Big\downarrow \ 6275$$

$$\frac{\int \frac{((a+bx)^2+1)^{3/2}}{\text{arcsinh}(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \ 6206$$

$$\frac{\int \frac{((a+bx)^2+1)^2}{\text{arcsinh}(a+bx)}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\sin\left(i\text{arcsinh}(a+bx)+\frac{\pi}{2}\right)^4}{\text{arcsinh}(a+bx)}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \ 3793$$

$$\frac{\int \left(\frac{\cosh(2\text{arcsinh}(a+bx))}{2\text{arcsinh}(a+bx)}+\frac{\cosh(4\text{arcsinh}(a+bx))}{8\text{arcsinh}(a+bx)}+\frac{3}{8\text{arcsinh}(a+bx)}\right)d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{2}\text{Chi}(2\text{arcsinh}(a+bx))+\frac{1}{8}\text{Chi}(4\text{arcsinh}(a+bx))+\frac{3}{8}\log(\text{arcsinh}(a+bx))}{b}$$

input
```
Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x],x]
```

output
```
(CoshIntegral[2*ArcSinh[a + b*x]]/2 + CoshIntegral[4*ArcSinh[a + b*x]]/8 +
 (3*Log[ArcSinh[a + b*x]])/8)/b
```

3.269.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.269.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{3\ln(\operatorname{arcsinh}(bx+a))+4\ \mathrm{Chi}(2\ \operatorname{arcsinh}(bx+a))+\mathrm{Chi}(4\ \operatorname{arcsinh}(bx+a))}{8b}$	36

input
```
int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*(3*ln(arcsinh(b*x+a))+4*Chi(2*arcsinh(b*x+a))+Chi(4*arcsinh(b*x+a)))/b
```

3.269.5 Fricas [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="frica
s")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a), x)
```

3.269.6 Sympy [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{(a^2+2abx+b^2x^2+1)^{\frac{3}{2}}}{\operatorname{asinh}\left(a+bx\right)}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a),x)
```

output
```
Integral((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)/asinh(a + b*x), x)
```

3.269.7 Maxima [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx+a\right)}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="maxim
a")
```

output
```
integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a), x)
```

3.269.8 Giac [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{\operatorname{arsinh}\,(bx+a)}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="giac"
)
```

output
```
integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a), x)
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{(a^2+2\,a\,b\,x+b^2\,x^2+1)^{3/2}}{\operatorname{asinh}\,(a+b\,x)}\,dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x),x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x), x)
```

3.270 $\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^2}\,dx$

3.270.1 Optimal result

Integrand size = 30, antiderivative size = 54

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = -\frac{(1+(a+bx)^2)^2}{b\operatorname{arcsinh}(a+bx)} + \frac{\operatorname{Shi}(2\operatorname{arcsinh}(a+bx))}{b} + \frac{\operatorname{Shi}(4\operatorname{arcsinh}(a+bx))}{2b}$$

output
```
-(1+(b*x+a)^2)^2/b/arcsinh(b*x+a)+Shi(2*arcsinh(b*x+a))/b+1/2*Shi(4*arcsin
h(b*x+a))/b
```

3.270.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^2}\,dx = \frac{-2(1+a^2+2abx+b^2x^2)^2 + 2\operatorname{arcsinh}(a+bx)\operatorname{Shi}(2\operatorname{arcsinh}(a+bx)) + a}{2b\operatorname{arcsinh}(a+bx)}$$

input
```
Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^2,x]
```

output
```
(-2*(1 + a^2 + 2*a*b*x + b^2*x^2)^2 + 2*ArcSinh[a + b*x]*SinhIntegral[2*Ar
cSinh[a + b*x]] + ArcSinh[a + b*x]*SinhIntegral[4*ArcSinh[a + b*x]])/(2*b*
ArcSinh[a + b*x])
```

3.270.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6275, 6205, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2+1\right)^{3/2}}{\text{arcsinh}(a+bx)^2}dx$$

$$\Big\downarrow \text{ 6275}$$

$$\frac{\int \frac{\left((a+bx)^2+1\right)^{3/2}}{\text{arcsinh}(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 6205}$$

$$\frac{4\int \frac{(a+bx)\left((a+bx)^2+1\right)}{\text{arcsinh}(a+bx)}d(a+bx)-\frac{\left((a+bx)^2+1\right)^2}{\text{arcsinh}(a+bx)}}{b}$$

$$\Big\downarrow \text{ 6234}$$

$$\frac{4\int \frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}}{\text{arcsinh}(a+bx)}d\text{arcsinh}(a+bx)-\frac{\left((a+bx)^2+1\right)^2}{\text{arcsinh}(a+bx)}}{b}$$

$$\Big\downarrow \text{ 5971}$$

$$\frac{4\int \left(\frac{\sinh(2\text{arcsinh}(a+bx))}{4\text{arcsinh}(a+bx)}+\frac{\sinh(4\text{arcsinh}(a+bx))}{8\text{arcsinh}(a+bx)}\right)d\text{arcsinh}(a+bx)-\frac{\left((a+bx)^2+1\right)^2}{\text{arcsinh}(a+bx)}}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4\left(\frac{1}{4}\text{Shi}(2\text{arcsinh}(a+bx))+\frac{1}{8}\text{Shi}(4\text{arcsinh}(a+bx))\right)-\frac{\left((a+bx)^2+1\right)^2}{\text{arcsinh}(a+bx)}}{b}$$

input
```
Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^2,x]
```

output
```
(-((1 + (a + b*x)^2)^2/ArcSinh[a + b*x]) + 4*(SinhIntegral[2*ArcSinh[a + b
*x]]/4 + SinhIntegral[4*ArcSinh[a + b*x]]/8))/b
```

3.270.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.270.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

method	result
default	$\frac{8\,\mathrm{Shi}(2\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)+4\,\mathrm{Shi}(4\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)-4\cosh(2\operatorname{arcsinh}(bx+a))-\cosh(4\operatorname{arcsinh}(bx+a))-}{8b\operatorname{arcsinh}(bx+a)}$

input
```
int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE
)
```

output
```
1/8/b*(8*Shi(2*arcsinh(b*x+a))*arcsinh(b*x+a)+4*Shi(4*arcsinh(b*x+a))*arcs
inh(b*x+a)-4*cosh(2*arcsinh(b*x+a))-cosh(4*arcsinh(b*x+a))-3)/arcsinh(b*x+
a)
```

3.270.5 Fricas [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^2}\,dx=\int \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="fri
cas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^2, x)
```

3.270.6 Sympy [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^2}\,dx=\int \frac{(a^2+2abx+b^2x^2+1)^{\frac{3}{2}}}{\operatorname{asinh}^2\left(a+bx\right)}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a)**2,x)
```

output
```
Integral((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)/asinh(a + b*x)**2, x)
```

3.270.7 Maxima [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^2}\,dx=\int \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="max
ima")
```

output
```
-((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + 2*a^2 + 4*(a^3*
b + a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (b^5*x^5 + 5*a*b^4*x^4 + a
^5 + 2*(5*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(5*a^3*b^2 + 3*a*b^2)*x^2 + (5*a^
4*b + 6*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2
*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*lo
g(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate(((4*b^4*x^4 +
16*a*b^3*x^3 + 4*a^4 + 3*(8*a^2*b^2 + b^2)*x^2 + 3*a^2 + 2*(8*a^3*b + 3*a*
b)*x - 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 4*(2*b^5*x^5 + 10*a*b^4*x^
4 + 2*a^5 + (20*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + (20*a^3*b^2 + 9*a*b^2)*x^2
+ (10*a^4*b + 9*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (4*b^6*x
^6 + 24*a*b^5*x^5 + 4*a^6 + 3*(20*a^2*b^4 + 3*b^4)*x^4 + 9*a^4 + 4*(20*a^3
*b^3 + 9*a*b^3)*x^3 + 6*(10*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + 6*a^2 + 12*(2
*a^5*b + 3*a^3*b + a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^4*x^
4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + (b^2*x^2 + 2*a*b*x + a^2
 + 1)*(b^2*x^2 + 2*a*b*x + a^2) + 2*a^2 + 4*(a^3*b + a*b)*x + 2*(b^3*x^3 +
 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
) + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

3.270.8 Giac [F]

$$\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\text{arcsinh}(a+bx)^2}\,dx=\int \frac{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{\text{arsinh}\left(bx+a\right)^2}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="gia
c")
```

output
```
integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^2, x)
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\text{arcsinh}(a+bx)^2}\,dx=\int \frac{{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{3/2}}{{\mathrm{asinh}\left(a+b\,x\right)}^2}\,dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x)^2,x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x)^2, x)
```

3.271 $\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\text{arcsinh}(a+bx)^3}\,dx$

3.271.1 Optimal result

Integrand size = 30, antiderivative size = 84

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\text{arcsinh}(a+bx)^3}\,dx = -\frac{(1+(a+bx)^2)^2}{2b\text{arcsinh}(a+bx)^2} - \frac{2(a+bx)(1+(a+bx)^2)^{3/2}}{b\text{arcsinh}(a+bx)} + \frac{\text{Chi}(2\text{arcsinh}(a+bx))}{b} + \frac{\text{Chi}(4\text{arcsinh}(a+bx))}{b}$$

output
```
-1/2*(1+(b*x+a)^2)^2/b/arcsinh(b*x+a)^2-2*(b*x+a)*(1+(b*x+a)^2)^(3/2)/b/ar
csinh(b*x+a)+Chi(2*arcsinh(b*x+a))/b+Chi(4*arcsinh(b*x+a))/b
```

3.271.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.29

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\text{arcsinh}(a+bx)^3}\,dx = \frac{-\frac{(1+a^2+2abx+b^2x^2)\left(1+a^2+2abx+b^2x^2+4(a+bx)\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)\right)}{\text{arcsinh}(a+bx)^2} + 2C}{2b}$$

input
```
Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^3,x]
```

output
```
(-(((1 + a^2 + 2*a*b*x + b^2*x^2)*(1 + a^2 + 2*a*b*x + b^2*x^2 + 4*(a + b*
x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]))/ArcSinh[a + b*x]^2
) + 2*CoshIntegral[2*ArcSinh[a + b*x]] + 2*CoshIntegral[4*ArcSinh[a + b*x]
])/(2*b)
```

3.271.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6275, 6205, 6229, 6206, 3042, 3793, 2009, 6234, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx+b^2x^2+1\right)^{3/2}}{\text{arcsinh}(a+bx)^3}\,dx$$

$$\Big\downarrow \text{ 6275}$$

$$\frac{\int \frac{\left((a+bx)^2+1\right)^{3/2}}{\text{arcsinh}(a+bx)^3}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 6205}$$

$$\frac{2\int \frac{(a+bx)\left((a+bx)^2+1\right)}{\text{arcsinh}(a+bx)^2}d(a+bx)-\frac{\left((a+bx)^2+1\right)^2}{2\text{arcsinh}(a+bx)^2}}{b}$$

$$\Big\downarrow \text{ 6229}$$

$$\frac{2\left(\int \frac{\sqrt{(a+bx)^2+1}}{\text{arcsinh}(a+bx)}d(a+bx)+4\int \frac{(a+bx)^2\sqrt{(a+bx)^2+1}}{\text{arcsinh}(a+bx)}d(a+bx)-\frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}}{\text{arcsinh}(a+bx)}\right)-\frac{\left((a+bx)^2+1\right)^2}{2\text{arcsinh}(a+bx)^2}}{b}$$

$$\Big\downarrow \text{ 6206}$$

$$\frac{2\left(4\int \frac{(a+bx)^2\sqrt{(a+bx)^2+1}}{\text{arcsinh}(a+bx)}d(a+bx)+\int \frac{(a+bx)^2+1}{\text{arcsinh}(a+bx)}d\text{arcsinh}(a+bx)-\frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}}{\text{arcsinh}(a+bx)}\right)-\frac{\left((a+bx)^2+1\right)^2}{2\text{arcsinh}(a+bx)^2}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{\left((a+bx)^2+1\right)^2}{2\text{arcsinh}(a+bx)^2}+2\left(4\int \frac{(a+bx)^2\sqrt{(a+bx)^2+1}}{\text{arcsinh}(a+bx)}d(a+bx)+\int \frac{\sin\left(i\text{arcsinh}(a+bx)+\frac{\pi}{2}\right)^2}{\text{arcsinh}(a+bx)}d\text{arcsinh}(a+bx)-\frac{(a+bx)\left((a+bx)^2\right.}{\text{arcsinh}(a}\right.}{b}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{2\left(4\int \frac{(a+bx)^2\sqrt{(a+bx)^2+1}}{\text{arcsinh}(a+bx)}d(a+bx)+\int \left(\frac{\cosh(2\text{arcsinh}(a+bx))}{2\text{arcsinh}(a+bx)}+\frac{1}{2\text{arcsinh}(a+bx)}\right)d\text{arcsinh}(a+bx)-\frac{(a+bx)\left((a+bx)^2+1\right.}{\text{arcsinh}(a+bx}\right.}{b}$$

$$\downarrow \text{2009}$$

$$\frac{2\left(4\int \frac{(a+bx)^2\sqrt{(a+bx)^2+1}}{\text{arcsinh}(a+bx)}d(a+bx)+\frac{1}{2}\text{Chi}(2\text{arcsinh}(a+bx))-\frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}}{\text{arcsinh}(a+bx)}+\frac{1}{2}\log(\text{arcsinh}(a+bx))\right)-\frac{}{2}}{b}$$

$$\downarrow \text{6234}$$

$$\frac{2\left(4\int \frac{(a+bx)^2\left((a+bx)^2+1\right)}{\text{arcsinh}(a+bx)}d\text{arcsinh}(a+bx)+\frac{1}{2}\text{Chi}(2\text{arcsinh}(a+bx))-\frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}}{\text{arcsinh}(a+bx)}+\frac{1}{2}\log(\text{arcsinh}(a+bx$$

$$b$$

$$\downarrow \text{5971}$$

$$\frac{2\left(4\int \left(\frac{\cosh(4\text{arcsinh}(a+bx))}{8\text{arcsinh}(a+bx)}-\frac{1}{8\text{arcsinh}(a+bx)}\right)d\text{arcsinh}(a+bx)+\frac{1}{2}\text{Chi}(2\text{arcsinh}(a+bx))-\frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}}{\text{arcsinh}(a+bx)}+\right.}{b}$$

$$\downarrow \text{2009}$$

$$\frac{2\left(\frac{1}{2}\text{Chi}(2\text{arcsinh}(a+bx))+4\left(\frac{1}{8}\text{Chi}(4\text{arcsinh}(a+bx))-\frac{1}{8}\log(\text{arcsinh}(a+bx))\right)-\frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}}{\text{arcsinh}(a+bx)}+\frac{1}{2}\log(a\right.}{b}$$

input
```
Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^3,x]
```

output
```
(-1/2*(1 + (a + b*x)^2)^2/ArcSinh[a + b*x]^2 + 2*(-(((a + b*x)*(1 + (a + b
*x)^2)^(3/2))/ArcSinh[a + b*x]) + CoshIntegral[2*ArcSinh[a + b*x]]/2 + 4*(
CoshIntegral[4*ArcSinh[a + b*x]]/8 - Log[ArcSinh[a + b*x]]/8) + Log[ArcSin
h[a + b*x]]/2))/b
```

3.271.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.271. $\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\text{arcsinh}(a+bx)^3}\,dx$

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6206
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Subst[Int
[x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

rule 6229
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(
p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*
(n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 + c^2*
x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

3.271. $\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\text{arcsinh}(a+bx)^3}\,dx$

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.271.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.31

method	result
default	$\frac{16 \operatorname{Chi}(2 \operatorname{arcsinh}(bx+a)) \operatorname{arcsinh}(bx+a)^2+16 \operatorname{Chi}(4 \operatorname{arcsinh}(bx+a)) \operatorname{arcsinh}(bx+a)^2-8 \sinh(2 \operatorname{arcsinh}(bx+a)) \operatorname{arcsinh}(bx+a)-4 \sin}{16b \operatorname{arcsinh}(bx+a)^2}$

input
```
int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE
)
```

output
```
1/16/b*(16*Chi(2*arcsinh(b*x+a))*arcsinh(b*x+a)^2+16*Chi(4*arcsinh(b*x+a))
*arcsinh(b*x+a)^2-8*sinh(2*arcsinh(b*x+a))*arcsinh(b*x+a)-4*sinh(4*arcsinh
(b*x+a))*arcsinh(b*x+a)-4*cosh(2*arcsinh(b*x+a))-cosh(4*arcsinh(b*x+a))-3)
/arcsinh(b*x+a)^2
```

3.271.5 Fricas [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x, algorithm="fri
cas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^3, x)
```

3.271.6 Sympy [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{(a^2+2abx+b^2x^2+1)^{\frac{3}{2}}}{\operatorname{asinh}^3(a+bx)}\,dx$$

input
```
integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a)**3,x)
```

output
```
Integral((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)/asinh(a + b*x)**3, x)
```

3.271.7 Maxima [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{\operatorname{arsinh}(bx+a)^3}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x, algorithm="max
ima")
```

output
```
-1/2*((b^6*x^6 + 6*a*b^5*x^5 + a^6 + (15*a^2*b^4 + 2*b^4)*x^4 + 2*a^4 + 4*
(5*a^3*b^3 + 2*a*b^3)*x^3 + (15*a^4*b^2 + 12*a^2*b^2 + b^2)*x^2 + a^2 + 2*
(3*a^5*b + 4*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (3*b^7*x^7
+ 21*a*b^6*x^6 + 3*a^7 + (63*a^2*b^5 + 8*b^5)*x^5 + 8*a^5 + 5*(21*a^3*b^4
+ 8*a*b^4)*x^4 + (105*a^4*b^3 + 80*a^2*b^3 + 7*b^3)*x^3 + 7*a^3 + (63*a^5*
b^2 + 80*a^3*b^2 + 21*a*b^2)*x^2 + (21*a^6*b + 40*a^4*b + 21*a^2*b + 2*b)*
x + 2*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b^8*x^8 + 24*a*b^7*x^7 +
 3*a^8 + 2*(42*a^2*b^6 + 5*b^6)*x^6 + 10*a^6 + 12*(14*a^3*b^5 + 5*a*b^5)*x
^5 + 6*(35*a^4*b^4 + 25*a^2*b^4 + 2*b^4)*x^4 + 12*a^4 + 8*(21*a^5*b^3 + 25
*a^3*b^3 + 6*a*b^3)*x^3 + 6*(14*a^6*b^2 + 25*a^4*b^2 + 12*a^2*b^2 + b^2)*x
^2 + 6*a^2 + 12*(2*a^7*b + 5*a^5*b + 4*a^3*b + a*b)*x + 1)*(b^2*x^2 + 2*a*
b*x + a^2 + 1) + ((4*b^6*x^6 + 24*a*b^5*x^5 + 4*a^6 + (60*a^2*b^4 + 7*b^4)
*x^4 + 7*a^4 + 4*(20*a^3*b^3 + 7*a*b^3)*x^3 + 2*(30*a^4*b^2 + 21*a^2*b^2 +
 b^2)*x^2 + 2*a^2 + 4*(6*a^5*b + 7*a^3*b + a*b)*x - 1)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1)^2 + 3*(4*b^7*x^7 + 28*a*b^6*x^6 + 4*a^7 + 3*(28*a^2*b^5 + 3*b^5
)*x^5 + 9*a^5 + 5*(28*a^3*b^4 + 9*a*b^4)*x^4 + 2*(70*a^4*b^3 + 45*a^2*b^3
+ 3*b^3)*x^3 + 6*a^3 + 6*(14*a^5*b^2 + 15*a^3*b^2 + 3*a*b^2)*x^2 + (28*a^6
*b + 45*a^4*b + 18*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) +
 (12*b^8*x^8 + 96*a*b^7*x^7 + 12*a^8 + 3*(112*a^2*b^6 + 11*b^6)*x^6 + 33*a
^6 + 6*(112*a^3*b^5 + 33*a*b^5)*x^5 + (840*a^4*b^4 + 495*a^2*b^4 + 31*b...
```

3.271.8 Giac [F]

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{\operatorname{arsinh}\left(bx+a\right)^3}\,dx$$

input
```
integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x, algorithm="gia
c")
```

output
```
integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^3, x)
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\operatorname{arcsinh}(a+bx)^3}\,dx = \int \frac{(a^2+2\,a\,b\,x+b^2\,x^2+1)^{3/2}}{\operatorname{asinh}\left(a+b\,x\right)^3}\,dx$$

input
```
int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x)^3,x)
```

output
```
int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x)^3, x)
```

3.272 $\int \frac{\text{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

3.272.1 Optimal result

Integrand size = 30, antiderivative size = 15

$$\int \frac{\text{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx=\frac{\text{arcsinh}(a+bx)^4}{4b}$$

output
```
1/4*arcsinh(b*x+a)^4/b
```

3.272.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx=\frac{\text{arcsinh}(a+bx)^4}{4b}$$

input
```
Integrate[ArcSinh[a + b*x]^3/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
ArcSinh[a + b*x]^4/(4*b)
```

3.272.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {6275, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(a+bx)^3}{\sqrt{a^2+2abx+b^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6275}$$

$$\frac{\int \frac{\text{arcsinh}(a+bx)^3}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{\text{arcsinh}(a+bx)^4}{4b}$$

input
```
Int[ArcSinh[a + b*x]^3/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
ArcSinh[a + b*x]^4/(4*b)
```

3.272.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.272.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\operatorname{arcsinh}(bx+a)^{4}}{4b}$	14

input
```
int(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/4*arcsinh(b*x+a)^4/b
```

3.272.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^4}{4\,b}$$

input
```
integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fri
cas")
```

output
```
1/4*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4/b
```

3.272.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \begin{cases} \frac{\operatorname{asinh}^{4}(a+bx)}{4b} & \text{for}\ b \neq 0 \\ \frac{x\operatorname{asinh}^{3}(a)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

input
```
integrate(asinh(b*x+a)**3/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

output
```
Piecewise((asinh(a + b*x)**4/(4*b), Ne(b, 0)), (x*asinh(a)**3/sqrt(a**2 +
1), True))
```

3.272.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(13) = 26$.

Time = 0.30 (sec) , antiderivative size = 179, normalized size of antiderivative = 11.93

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{arsinh}(bx+a)^3 \operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b} - \frac{3\operatorname{arsinh}(bx+a)^2 \operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)^2}{2\,b} + \frac{\operatorname{arsinh}(bx+a) \operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)^3}{b} - \frac{\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)^4}{4\,b}$$

input
```
integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="max
ima")
```

output
```
arcsinh(b*x + a)^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b
^2))/b - 3/2*arcsinh(b*x + a)^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 +
4*(a^2 + 1)*b^2))^2/b + arcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a
^2*b^2 + 4*(a^2 + 1)*b^2))^3/b - 1/4*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b
^2 + 4*(a^2 + 1)*b^2))^4/b
```

3.272.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(bx+a)^3}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(arcsinh(b*x + a)^3/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

3.272.9 Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{asinh}(a+b\,x)^4}{4\,b}$$

input
```
int(asinh(a + b*x)^3/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

output
```
asinh(a + b*x)^4/(4*b)
```

3.273 $\int \frac{\operatorname{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

3.273.1 Optimal result

Integrand size = 30, antiderivative size = 15

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{arcsinh}(a+bx)^3}{3b}$$

output
```
1/3*arcsinh(b*x+a)^3/b
```

3.273.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{arcsinh}(a+bx)^3}{3b}$$

input
```
Integrate[ArcSinh[a + b*x]^2/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
ArcSinh[a + b*x]^3/(3*b)
```

3.273.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {6275, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\sqrt{a^2+2abx+b^2x^2+1}}dx$$

$$\Big\downarrow \text{6275}$$

$$\frac{\int \frac{\operatorname{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b}$$

$$\Big\downarrow \text{6198}$$

$$\frac{\operatorname{arcsinh}(a+bx)^3}{3b}$$

input
```
Int[ArcSinh[a + b*x]^2/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
ArcSinh[a + b*x]^3/(3*b)
```

3.273.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.273.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\operatorname{arcsinh}(bx+a)^3}{3b}$	14

input
```
int(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/3*arcsinh(b*x+a)^3/b
```

3.273.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3}{3\,b}$$

input
```
integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fri
cas")
```

output
```
1/3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3/b
```

3.273.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \begin{cases} \frac{\operatorname{asinh}^3(a+bx)}{3b} & \text{for } b\neq 0 \\ \frac{x\operatorname{asinh}^2(a)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

input
```
integrate(asinh(b*x+a)**2/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

output
```
Piecewise((asinh(a + b*x)**3/(3*b), Ne(b, 0)), (x*asinh(a)**2/sqrt(a**2 +
1), True))
```

3.273.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(13) = 26$.

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 8.80

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{arsinh}\left(bx+a\right)^2 \operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b} - \frac{\operatorname{arsinh}\left(bx+a\right) \operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)^2}{b} + \frac{\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)^3}{3\,b}$$

input
```
integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="max
ima")
```

output
```
arcsinh(b*x + a)^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b
^2))/b - arcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2
 + 1)*b^2))^2/b + 1/3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1
)*b^2))^3/b
```

3.273.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(arcsinh(b*x + a)^2/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

3.273.9 Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{asinh}(a+b\,x)^3}{3\,b}$$

input
```
int(asinh(a + b*x)^2/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

output
```
asinh(a + b*x)^3/(3*b)
```

3.274 $\int \frac{\text{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

3.274.1 Optimal result

Integrand size = 28, antiderivative size = 15

$$\int \frac{\text{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\text{arcsinh}(a+bx)^2}{2b}$$

output
```
1/2*arcsinh(b*x+a)^2/b
```

3.274.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\text{arcsinh}(a+bx)^2}{2b}$$

input
```
Integrate[ArcSinh[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
ArcSinh[a + b*x]^2/(2*b)
```

3.274.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {6275, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(a+bx)}{\sqrt{a^2+2abx+b^2x^2+1}}dx$$

$$\Big\downarrow \text{ 6275}$$

$$\frac{\int \frac{\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 6198}$$

$$\frac{\text{arcsinh}(a+bx)^2}{2b}$$

input
```
Int[ArcSinh[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
ArcSinh[a + b*x]^2/(2*b)
```

3.274.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.274.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\operatorname{arcsinh}(bx+a)^2}{2b}$	14

input
```
int(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(b*x+a)^2/b
```

3.274.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2}{2\,b}$$

input
```
integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="frica
s")
```

output
```
1/2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/b
```

3.274.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(10) = 20.

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \begin{cases} \frac{\operatorname{asinh}^2(a+bx)}{2b} & \text{for } b \neq 0 \\ \frac{x \operatorname{asinh}(a)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

input
```
integrate(asinh(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

output
```
Piecewise((asinh(a + b*x)**2/(2*b), Ne(b, 0)), (x*asinh(a)/sqrt(a**2 + 1),
 True))
```

3.274.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(13) = 26$.

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 5.60

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{arsinh}(bx+a)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b} - \frac{\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)^2}{2\,b}$$

input
```
integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxim
a")
```

output
```
arcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2
))/b - 1/2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))^2/b
```

3.274.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{arsinh}(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(arcsinh(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

3.274.9 Mupad [B] (verification not implemented)

Time = 2.85 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{asinh}(a+b\,x)^2}{2\,b}$$

input
```
int(asinh(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

output
```
asinh(a + b*x)^2/(2*b)
```

3.275 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\,\text{arcsinh}(a+bx)}\,dx$

3.275.1 Optimal result

Integrand size = 30, antiderivative size = 11

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\,\text{arcsinh}(a+bx)}\,dx = \frac{\log(\text{arcsinh}(a+bx))}{b}$$

output
```
ln(arcsinh(b*x+a))/b
```

3.275.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\,\text{arcsinh}(a+bx)}\,dx = \frac{\log(\text{arcsinh}(a+bx))}{b}$$

input
```
Integrate[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]),x]
```

output
```
Log[ArcSinh[a + b*x]]/b
```

3.275.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {6275, 6197}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+2abx+b^2x^2+1}\text{arcsinh}(a+bx)}dx$$

$$\Big\downarrow \text{ 6275}$$

$$\frac{\int \frac{1}{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 6197}$$

$$\frac{\log(\text{arcsinh}(a+bx))}{b}$$

input
```
Int[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]),x]
```

output
```
Log[ArcSinh[a + b*x]]/b
```

3.275.3.1 Defintions of rubi rules used

rule 6197
```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*Log[a + b*A
rcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.275.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\ln(\operatorname{arcsinh}(bx+a))}{b}$	12

input
```
int(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
ln(arcsinh(b*x+a))/b
```

3.275.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(11) = 22.

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)}\,dx = \frac{\log\left(\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)\right)}{b}$$

input
```
integrate(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fri
cas")
```

output
```
log(log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

3.275.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(8) = 16.

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)}\,dx = \begin{cases} \frac{\log(\operatorname{asinh}(a+bx))}{b} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{a^2+1}\operatorname{asinh}(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/asinh(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

output
```
Piecewise((log(asinh(a + b*x))/b, Ne(b, 0)), (x/(sqrt(a**2 + 1)*asinh(a)),
 True))
```

3.275.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)}\,dx=\int \frac{1}{\sqrt{b^2x^2+2\,abx+a^2+1}\,\text{arsinh}\,(bx+a)}\,dx$$

input
```
integrate(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(1/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)), x)
```

3.275.8 Giac [F]

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)}\,dx=\int \frac{1}{\sqrt{b^2x^2+2\,abx+a^2+1}\,\text{arsinh}\,(bx+a)}\,dx$$

input
```
integrate(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(1/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)), x)
```

3.275.9 Mupad [B] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)}\,dx=\frac{\ln\left(\text{asinh}(a+b\,x)\right)}{b}$$

input
```
int(1/(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)),x)
```

output
```
log(asinh(a + b*x))/b
```

3.276 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^2}\,dx$

3.276.1 Optimal result

Integrand size = 30, antiderivative size = 13

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^2}\,dx = -\frac{1}{b\text{arcsinh}(a+bx)}$$

output
```
-1/b/arcsinh(b*x+a)
```

3.276.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^2}\,dx = -\frac{1}{b\text{arcsinh}(a+bx)}$$

input
```
Integrate[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2),x]
```

output
```
-(1/(b*ArcSinh[a + b*x]))
```

3.276.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {6275, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+2abx+b^2x^2+1}\text{arcsinh}(a+bx)^2}dx$$

$$\Big\downarrow \text{6275}$$

$$\frac{\int \frac{1}{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{1}{b\text{arcsinh}(a+bx)}$$

input
```
Int[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2),x]
```

output
```
-(1/(b*ArcSinh[a + b*x]))
```

3.276.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.276.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{1}{b\,\mathrm{arcsinh}(bx+a)}$	14

input
```
int(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/b/arcsinh(b*x+a)
```

3.276.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(13) = 26$.

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.46

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\mathrm{arcsinh}(a+bx)^2}\,dx = -\frac{1}{b\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}$$

input
```
integrate(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/(b*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))
```

3.276.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\mathrm{arcsinh}(a+bx)^2}\,dx = \begin{cases} -\frac{1}{b\,\mathrm{asinh}\,(a+bx)} & \text{for } b\neq 0 \\ \frac{x}{\sqrt{a^2+1}\,\mathrm{asinh}^2\,(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/asinh(b*x+a)**2/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

output
```
Piecewise((-1/(b*asinh(a + b*x)), Ne(b, 0)), (x/(sqrt(a**2 + 1)*asinh(a)**
2), True))
```

3.276.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(13) = 26$.

Time = 0.33 (sec) , antiderivative size = 150, normalized size of antiderivative = 11.54

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^2}\,dx = -\frac{b^3x^3+3\,ab^2x^2+a^3+(3\,a^2b+b)x+(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}+a}{\left((b^2x^2+2\,abx+a^2+1)(b^2x+ab)+(b^3x^2+2\,ab^2x+a^2b+b)\sqrt{b^2x^2+2\,abx+a^2+1}\right)\log\left(bx+a+\sqrt{\ldots}\right)}$$

input
```
integrate(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="m
axima")
```

output
```
-(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2
 + 1)^(3/2) + a)/(((b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + (b^3*x^2
+ 2*a*b^2*x + a^2*b + b)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a +
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))
```

3.276.8 Giac [F]

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{\sqrt{b^2x^2+2\,abx+a^2+1}\,\text{arsinh}\left(bx+a\right)^2}\,dx$$

input
```
integrate(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="g
iac")
```

output
```
integrate(1/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2), x)
```

3.276.9 Mupad [B] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^2}\,dx=-\frac{1}{b\,\text{asinh}\left(a+b\,x\right)}$$

input
```
int(1/(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)),x)
```

output
```
-1/(b*asinh(a + b*x))
```

3.277 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^3}\,dx$

3.277.1 Optimal result

Integrand size = 30, antiderivative size = 15

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^3}\,dx = -\frac{1}{2b\text{arcsinh}(a+bx)^2}$$

output
```
-1/2/b/arcsinh(b*x+a)^2
```

3.277.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^3}\,dx = -\frac{1}{2b\text{arcsinh}(a+bx)^2}$$

input
```
Integrate[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3),x]
```

output
```
-1/2*1/(b*ArcSinh[a + b*x]^2)
```

3.277.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {6275, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+2abx+b^2x^2+1}\text{arcsinh}(a+bx)^3}dx$$

$$\Big\downarrow \text{6275}$$

$$\frac{\int \frac{1}{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)^3}d(a+bx)}{b}$$

$$\Big\downarrow \text{6198}$$

$$-\frac{1}{2b\text{arcsinh}(a+bx)^2}$$

input
```
Int[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3),x]
```

output
```
-1/2*1/(b*ArcSinh[a + b*x]^2)
```

3.277.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.277.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{2b \operatorname{arcsinh}(bx+a)^{2}}$	14

input
```
int(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/2/b/arcsinh(b*x+a)^2
```

3.277.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3}\,dx = -\frac{1}{2\,b\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2}$$

input
```
integrate(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/2/(b*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)
```

3.277.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(14) = 28.

Time = 0.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\operatorname{arcsinh}(a+bx)^3}\,dx = \begin{cases} -\frac{1}{2b\operatorname{asinh}^2(a+bx)} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{a^2+1}\operatorname{asinh}^3(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/asinh(b*x+a)**3/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

output
```
Piecewise((-1/(2*b*asinh(a + b*x)**2), Ne(b, 0)), (x/(sqrt(a**2 + 1)*asinh
(a)**3), True))
```

3.277.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{\sqrt{b^2x^2+2\,abx+a^2+1}\,\text{arsinh}\left(bx+a\right)^3}\,dx$$

input
```
integrate(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="m
axima")
```

output
```
-1/2*(b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7
*a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 +
3*(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (b^4*x^4 + 4*a*b^3*x^3 + a^4 +
(6*a^2*b^2 + b^2)*x^2 + a^2 + 2*(2*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^
2 + 1)^(3/2) + (3*b^5*x^5 + 15*a*b^4*x^4 + 3*a^5 + 5*(6*a^2*b^3 + b^3)*x^3
 + 5*a^3 + 15*(2*a^3*b^2 + a*b^2)*x^2 + (15*a^4*b + 15*a^2*b + 2*b)*x + 2*
a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x +
(b^5*x^5 + 5*a*b^4*x^4 + a^5 + 2*(5*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(5*a^3*
b^2 + 3*a*b^2)*x^2 - (b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2) - (b^3*x^3 + 3*a*
b^2*x^2 + a^3 + (3*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (5*a^
4*b + 6*a^2*b + b)*x + (b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2 + b^2)*
x^2 + 2*a^2 + 4*(a^3*b + a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + a
)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (3*b^6*x^6 + 18*a*b^5
*x^5 + 3*a^6 + (45*a^2*b^4 + 7*b^4)*x^4 + 7*a^4 + 4*(15*a^3*b^3 + 7*a*b^3)
*x^3 + (45*a^4*b^2 + 42*a^2*b^2 + 5*b^2)*x^2 + 5*a^2 + 2*(9*a^5*b + 14*a^3
*b + 5*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + a)/(((b^4*x^3 + 3*a
*b^3*x^2 + 3*a^2*b^2*x + a^3*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 3*(b^5*x
^4 + 4*a*b^4*x^3 + a^4*b + a^2*b + (6*a^2*b^3 + b^3)*x^2 + 2*(2*a^3*b^2 +
a*b^2)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^6*x^5 + 5*a*b^5*x^4 +
 a^5*b + 2*a^3*b + 2*(5*a^2*b^4 + b^4)*x^3 + 2*(5*a^3*b^3 + 3*a*b^3)*x^...
```

3.277.8 Giac [F]

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^3}\,dx = \int \frac{1}{\sqrt{b^2x^2+2\,abx+a^2+1}\,\text{arsinh}\,(bx+a)^3}\,dx$$

input
```
integrate(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="g
iac")
```

output
```
integrate(1/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3), x)
```

3.277.9 Mupad [B] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\text{arcsinh}(a+bx)^3}\,dx = -\frac{1}{2\,b\,\text{asinh}\,(a+b\,x)^2}$$

input
```
int(1/(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)),x)
```

output
```
-1/(2*b*asinh(a + b*x)^2)
```

3.278 $\int \frac{\text{arcsinh}(a+bx)^3}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx$

3.278.1 Optimal result

Integrand size = 30, antiderivative size = 115

$$\int \frac{\text{arcsinh}(a+bx)^3}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{\text{arcsinh}(a+bx)^3}{b} + \frac{(a+bx)\text{arcsinh}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\text{arcsinh}(a+bx)^2\log\left(1+e^{2\text{arcsinh}(a+bx)}\right)}{b} - \frac{3\text{arcsinh}(a+bx)\,\text{PolyLog}\left(2,-e^{2\text{arcsinh}(a+bx)}\right)}{b} + \frac{3\,\text{PolyLog}\left(3,-e^{2\text{arcsinh}(a+bx)}\right)}{2b}$$

output
```
arcsinh(b*x+a)^3/b-3*arcsinh(b*x+a)^2*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/
b-3*arcsinh(b*x+a)*polylog(2,-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b+3/2*polylog
(3,-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b+(b*x+a)*arcsinh(b*x+a)^3/b/(1+(b*x+a)
^2)^(1/2)
```

3.278.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{\text{arcsinh}(a+bx)^3}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{2\text{arcsinh}(a+bx)^2\left(\frac{\left(a+bx-\sqrt{1+a^2+2abx+b^2x^2}\right)\text{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} - 3\log\left(1+e^{-2\text{ar}\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[ArcSinh[a + b*x]^3/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(2*ArcSinh[a + b*x]^2*(((a + b*x - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])*ArcS
inh[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] - 3*Log[1 + E^(-2*ArcSinh[
a + b*x])]) + 6*ArcSinh[a + b*x]*PolyLog[2, -E^(-2*ArcSinh[a + b*x])] + 3*
PolyLog[3, -E^(-2*ArcSinh[a + b*x])])/(2*b)
```

3.278.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6275, 6202, 6212, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(a+bx)^3}{\left(a^2+2abx+b^2x^2+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{6275}$$

$$\frac{\int \frac{\text{arcsinh}(a+bx)^3}{\left((a+bx)^2+1\right)^{3/2}}d(a+bx)}{b}$$

$$\Big\downarrow \text{6202}$$

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)^3}{\sqrt{(a+bx)^2+1}}-3\int \frac{(a+bx)\text{arcsinh}(a+bx)^2}{(a+bx)^2+1}d(a+bx)}{b}$$

$$\Big\downarrow \text{6212}$$

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)^3}{\sqrt{(a+bx)^2+1}}-3\int \frac{(a+bx)\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)^3}{\sqrt{(a+bx)^2+1}}-3\int -i\text{arcsinh}(a+bx)^2\tan(i\text{arcsinh}(a+bx))d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)^3}{\sqrt{(a+bx)^2+1}}+3i\int \text{arcsinh}(a+bx)^2\tan(i\text{arcsinh}(a+bx))d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{4201}$$

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)^3}{\sqrt{(a+bx)^2+1}}+3i\left(2i\int \frac{e^{2\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)^2}{1+e^{2\text{arcsinh}(a+bx)}}d\text{arcsinh}(a+bx)-\frac{1}{3}i\text{arcsinh}(a+bx)^3\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)^3}{\sqrt{(a+bx)^2+1}}+3i(2i(\frac{1}{2}\text{arcsinh}(a+bx)^2\log\left(e^{2\text{arcsinh}(a+bx)}+1\right)-\int \text{arcsinh}(a+bx)\log\left(1+e^{2\text{arcsinh}(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)^3}{\sqrt{(a+bx)^2+1}}+3i(2i(-\frac{1}{2}\int \text{PolyLog}\left(2,-e^{2\text{arcsinh}(a+bx)}\right)d\text{arcsinh}(a+bx)+\frac{1}{2}\text{arcsinh}(a+bx)\text{PolyLog}\left(2,-\right.}{b}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)^3}{\sqrt{(a+bx)^2+1}}+3i(2i(-\frac{1}{4}\int e^{-2\text{arcsinh}(a+bx)}\text{PolyLog}(2,-a-bx)de^{2\text{arcsinh}(a+bx)}+\frac{1}{2}\text{arcsinh}(a+bx)\text{PolyLog}}{b}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)^3}{\sqrt{(a+bx)^2+1}}+3i(2i(\frac{1}{2}\text{arcsinh}(a+bx)\text{PolyLog}\left(2,-e^{2\text{arcsinh}(a+bx)}\right)+\frac{1}{2}\text{arcsinh}(a+bx)^2\log\left(e^{2\text{arcsinh}(a+bx}\right.}{b}$$

input
```
Int[ArcSinh[a + b*x]^3/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(((a + b*x)*ArcSinh[a + b*x]^3)/Sqrt[1 + (a + b*x)^2] + (3*I)*((-1/3*I)*Ar
cSinh[a + b*x]^3 + (2*I)*((ArcSinh[a + b*x]^2*Log[1 + E^(2*ArcSinh[a + b*x
])])/2 + (ArcSinh[a + b*x]*PolyLog[2, -E^(2*ArcSinh[a + b*x])])/2 - PolyLo
g[3, -a - b*x]/4)))/b
```

3.278. $\int \frac{\text{arcsinh}(a+bx)^3}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx$

3.278.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

3.278. $\int \frac{\operatorname{arcsinh}(a+bx)^3}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx$

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.278.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.77

method	result
default	$-\frac{\left(b^2x^2-\sqrt{b^2x^2+2abx+a^2+1}\,bx+2abx-a\sqrt{b^2x^2+2abx+a^2+1}+a^2+1\right)\operatorname{arcsinh}(bx+a)^3}{b(b^2x^2+2abx+a^2+1)}+\frac{2\operatorname{arcsinh}(bx+a)^3}{b}-\frac{3\operatorname{arcsinh}(bx+a)^2}{}$

input
```
int(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
-(b^2*x^2-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x+2*a*b*x-a*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2)+a^2+1)/b/(b^2*x^2+2*a*b*x+a^2+1)*arcsinh(b*x+a)^3+2*arcsinh(b*x+
a)^3/b-3*arcsinh(b*x+a)^2*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b-3*arcsinh(
b*x+a)*polylog(2,-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b+3/2*polylog(3,-(b*x+a+(
1+(b*x+a)^2)^(1/2))^2)/b
```

3.278.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3/(b^4*x^4 + 4
*a*b^3*x^3 + 2*(3*a^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1), x
)
```

3.278.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{asinh}^3\left(a+bx\right)}{(a^2+2abx+b^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(asinh(b*x+a)**3/(b**2*x**2+2*a*b*x+a**2+1)**(3/2),x)
```

output
```
Integral(asinh(a + b*x)**3/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2), x)
```

3.278.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="max
ima")
```

output
```
integrate(arcsinh(b*x + a)^3/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

3.278.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{arsinh}\left(bx+a\right)^3}{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="gia
c")
```

output
```
integrate(arcsinh(b*x + a)^3/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)^3}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)^3}{(a^2+2\,a\,b\,x+b^2\,x^2+1)^{3/2}}\,dx$$

input
```
int(asinh(a + b*x)^3/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

output
```
int(asinh(a + b*x)^3/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

3.279 $\int \frac{\operatorname{arcsinh}(a+bx)^2}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx$

3.279.1 Optimal result

Integrand size = 30, antiderivative size = 86

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{\operatorname{arcsinh}(a+bx)^2}{b} + \frac{(a+bx)\operatorname{arcsinh}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\operatorname{arcsinh}(a+bx)\log\left(1+e^{2\operatorname{arcsinh}(a+bx)}\right)}{b} - \frac{\operatorname{PolyLog}\left(2,-e^{2\operatorname{arcsinh}(a+bx)}\right)}{b}$$

output
```
arcsinh(b*x+a)^2/b-2*arcsinh(b*x+a)*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b-
polylog(2,-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b+(b*x+a)*arcsinh(b*x+a)^2/b/(1+
(b*x+a)^2)^(1/2)
```

3.279.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{\operatorname{arcsinh}(a+bx)\left(\frac{\left(a+bx-\sqrt{1+a^2+2abx+b^2x^2}\right)\operatorname{arcsinh}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} - 2\log\left(1+e^{-2\operatorname{arcsi}}\right.\right.}{b}$$

input
```
Integrate[ArcSinh[a + b*x]^2/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(ArcSinh[a + b*x]*(((a + b*x - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])*ArcSinh[
a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] - 2*Log[1 + E^(-2*ArcSinh[a +
b*x])]) + PolyLog[2, -E^(-2*ArcSinh[a + b*x])])/b
```

3.279.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6275, 6202, 6212, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(a+bx)^2}{(a^2+2abx+b^2x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{6275}$$

$$\frac{\int \frac{\text{arcsinh}(a+bx)^2}{((a+bx)^2+1)^{3/2}}d(a+bx)}{b}$$

$$\Big\downarrow \text{6202}$$

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}-2\int \frac{(a+bx)\text{arcsinh}(a+bx)}{(a+bx)^2+1}d(a+bx)}{b}$$

$$\Big\downarrow \text{6212}$$

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}-2\int \frac{(a+bx)\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}-2\int -i\text{arcsinh}(a+bx)\tan(i\text{arcsinh}(a+bx))d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}+2i\int \text{arcsinh}(a+bx)\tan(i\text{arcsinh}(a+bx))d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{4201}$$

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}+2i\left(2i\int \frac{e^{2\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{1+e^{2\text{arcsinh}(a+bx)}}d\text{arcsinh}(a+bx)-\frac{1}{2}i\text{arcsinh}(a+bx)^2\right)}{b}$$

$$\Big\downarrow \text{2620}$$

3.279. $\int \frac{\text{arcsinh}(a+bx)^2}{(1+a^2+2abx+b^2x^2)^{3/2}}dx$

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}+2i\left(2i\left(\frac{1}{2}\text{arcsinh}(a+bx)\log\left(e^{2\text{arcsinh}(a+bx)}+1\right)-\frac{1}{2}\int\log\left(1+e^{2\text{arcsinh}(a+bx)}\right)d\text{arcsinh}(a+\right.\right.}{b}$$

$\Big\downarrow$ 2715

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}+2i\left(2i\left(\frac{1}{2}\text{arcsinh}(a+bx)\log\left(e^{2\text{arcsinh}(a+bx)}+1\right)-\frac{1}{4}\int e^{-2\text{arcsinh}(a+bx)}\log\left(1+e^{2\text{arcsinh}(a+bx)}\right.\right.\right.}{b}$$

$\Big\downarrow$ 2838

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)^2}{\sqrt{(a+bx)^2+1}}+2i\left(2i\left(\frac{1}{2}\text{arcsinh}(a+bx)\log\left(e^{2\text{arcsinh}(a+bx)}+1\right)+\frac{1}{4}\text{PolyLog}(2,-a-bx)\right)-\frac{1}{2}i\text{arcsinh}(a+\right.}{b}$$

input
```
Int[ArcSinh[a + b*x]^2/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(((a + b*x)*ArcSinh[a + b*x]^2)/Sqrt[1 + (a + b*x)^2] + (2*I)*((-1/2*I)*Ar
cSinh[a + b*x]^2 + (2*I)*((ArcSinh[a + b*x]*Log[1 + E^(2*ArcSinh[a + b*x])
])/2 + PolyLog[2, -a - b*x]/4)))/b
```

3.279.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.279. $\int \frac{\text{arcsinh}(a+bx)^2}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6212
```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.279.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.95

method	result
default	$-\frac{\left(b^2x^2-\sqrt{b^2x^2+2abx+a^2+1}\,bx+2abx-a\sqrt{b^2x^2+2abx+a^2+1}+a^2+1\right)\operatorname{arcsinh}(bx+a)^2}{b(b^2x^2+2abx+a^2+1)}+\frac{2\operatorname{arcsinh}(bx+a)^2}{b}-\frac{2\operatorname{arcsinh}(bx+a)}{}$

input
```
int(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
-(b^2*x^2-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x+2*a*b*x-a*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2)+a^2+1)/b/(b^2*x^2+2*a*b*x+a^2+1)*arcsinh(b*x+a)^2+2*arcsinh(b*x+
a)^2/b-2*arcsinh(b*x+a)*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b-polylog(2,-(
b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b
```

3.279.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\int \frac{\operatorname{arsinh}\left(bx+a\right)^2}{\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2/(b^4*x^4 + 4
*a*b^3*x^3 + 2*(3*a^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1), x
)
```

3.279.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)^2}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx=\int \frac{\operatorname{asinh}^2\left(a+bx\right)}{\left(a^2+2abx+b^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(asinh(b*x+a)**2/(b**2*x**2+2*a*b*x+a**2+1)**(3/2),x)
```

output
```
Integral(asinh(a + b*x)**2/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2), x)
```

3.279.7 Maxima [F]

$$\int \frac{\text{arcsinh}(a+bx)^2}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{\text{arsinh}\,(bx+a)^2}{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="max
ima")
```

output
```
integrate(arcsinh(b*x + a)^2/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

3.279.8 Giac [F]

$$\int \frac{\text{arcsinh}(a+bx)^2}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{\text{arsinh}\,(bx+a)^2}{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="gia
c")
```

output
```
integrate(arcsinh(b*x + a)^2/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arcsinh}(a+bx)^2}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{\text{asinh}(a+b\,x)^2}{(a^2+2\,a\,b\,x+b^2\,x^2+1)^{3/2}}\,dx$$

input
```
int(asinh(a + b*x)^2/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

output
```
int(asinh(a + b*x)^2/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

3.280 $\int \frac{\text{arcsinh}(a+bx)}{(1+a^2+2abx+b^2x^2)^{3/2}} dx$

3.280.1 Optimal result

Integrand size = 28, antiderivative size = 46

$$\int \frac{\text{arcsinh}(a+bx)}{(1+a^2+2abx+b^2x^2)^{3/2}} dx = \frac{(a+bx)\text{arcsinh}(a+bx)}{b\sqrt{1+(a+bx)^2}} - \frac{\log(1+(a+bx)^2)}{2b}$$

output
```
-1/2*ln(1+(b*x+a)^2)/b+(b*x+a)*arcsinh(b*x+a)/b/(1+(b*x+a)^2)^(1/2)
```

3.280.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \frac{\text{arcsinh}(a+bx)}{(1+a^2+2abx+b^2x^2)^{3/2}} dx = \frac{(a+bx)\text{arcsinh}(a+bx)}{b\sqrt{1+a^2+2abx+b^2x^2}} - \frac{\log(1+a^2+2abx+b^2x^2)}{2b}$$

input
```
Integrate[ArcSinh[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
((a + b*x)*ArcSinh[a + b*x])/(b*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) - Log[1
 + a^2 + 2*a*b*x + b^2*x^2]/(2*b)
```

3.280.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {6275, 6202, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(a+bx)}{(a^2+2abx+b^2x^2+1)^{3/2}}\,dx$$

$$\Big\downarrow \text{6275}$$

$$\frac{\int \frac{\text{arcsinh}(a+bx)}{((a+bx)^2+1)^{3/2}}d(a+bx)}{b}$$

$$\Big\downarrow \text{6202}$$

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}} - \int \frac{a+bx}{(a+bx)^2+1}d(a+bx)}{b}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{(a+bx)\text{arcsinh}(a+bx)}{\sqrt{(a+bx)^2+1}} - \frac{1}{2}\log\left((a+bx)^2+1\right)}{b}$$

input
```
Int[ArcSinh[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

output
```
(((a + b*x)*ArcSinh[a + b*x])/Sqrt[1 + (a + b*x)^2] - Log[1 + (a + b*x)^2]
/2)/b
```

3.280.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6202
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSinh[
c*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
 c^2*d] && GtQ[n, 0]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.280.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(42) = 84$.

Time = 0.61 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.85

method	result
default	$\frac{2\,\operatorname{arcsinh}(bx+a)}{b} - \frac{\left(b^2x^2-\sqrt{b^2x^2+2abx+a^2+1}\,bx+2abx-a\sqrt{b^2x^2+2abx+a^2+1}+a^2+1\right)\operatorname{arcsinh}(bx+a)}{b(b^2x^2+2abx+a^2+1)} - \frac{\ln\left(1+\left(bx+a+\sqrt{1+\right.\right.}{b}$

input
```
int(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsinh(b*x+a)/b-(b^2*x^2-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x+2*a*b*x-a*(b
^2*x^2+2*a*b*x+a^2+1)^(1/2)+a^2+1)/b/(b^2*x^2+2*a*b*x+a^2+1)*arcsinh(b*x+a
)-1/b*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)
```

3.280.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(42) = 84$.

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.50

$$\int \frac{\operatorname{arcsinh}(a+bx)}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{2\sqrt{b^2x^2+2\,abx+a^2+1}(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{2\left(b^3x^2+2\,ab^2x+(a^2+\right.}$$

input
```
integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="frica
s")
```

output
```
1/2*(2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log(b*x + a + sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)) - (b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b^2*x^2 + 2*
a*b*x + a^2 + 1))/(b^3*x^2 + 2*a*b^2*x + (a^2 + 1)*b)
```

3.280.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{asinh}(a+bx)}{(a^2+2abx+b^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(asinh(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(3/2),x)
```

output
```
Integral(asinh(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2), x)
```

3.280.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(42) = 84$.

Time = 0.34 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.59

$$\int \frac{\operatorname{arcsinh}(a+bx)}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = -\left(\frac{b^2x}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}} + \frac{ab}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}}\right)\operatorname{arsinh}(bx+a) - \frac{\log(b^2x^2+2\,abx+a^2+1)}{2\,b}$$

input
```
integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="maxim
a")
```

output
```
-(b^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + a*
b/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))*arcsinh(b
*x + a) - 1/2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b
```

3.280.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.65

$$\int \frac{\operatorname{arcsinh}(a+bx)}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{\left(x+\frac{a}{b}\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{\sqrt{b^2x^2+2\,abx+a^2+1}} - \frac{\log(b^2x^2+2\,abx+a^2+1)}{2\,b}$$

input
```
integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="giac"
)
```

output
```
(x + a/b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1) - 1/2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b
```

3.280.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(a+bx)}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \int \frac{\operatorname{asinh}(a+b\,x)}{(a^2+2\,a\,b\,x+b^2\,x^2+1)^{3/2}}\,dx$$

input
```
int(asinh(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

output
```
int(asinh(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

3.281 $\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)}\,dx$

3.281.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)}\,dx = \operatorname{Int}\left(\frac{1}{(1+(a+bx)^2)^{3/2}\operatorname{arcsinh}(a+bx)},x\right)$$

output
```
Unintegrable(1/(1+(b*x+a)^2)^(3/2)/arcsinh(b*x+a),x)
```

3.281.2 Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)}\,dx$$

input
```
Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]),x]
```

output
```
Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]), x]
```

3.281.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6275, 6209}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx+b^2x^2+1)^{3/2}\operatorname{arcsinh}(a+bx)}dx$$

$$\Big\downarrow \text{ 6275}$$

$$\frac{\int \frac{1}{((a+bx)^2+1)^{3/2}\operatorname{arcsinh}(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 6209}$$

$$\frac{\int \frac{1}{((a+bx)^2+1)^{3/2}\operatorname{arcsinh}(a+bx)}d(a+bx)}{b}$$

input
```
Int[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]),x]
```

output
```
$Aborted
```

3.281.3.1 Defintions of rubi rules used

rule 6209
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Unintegrable[(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x] /; FreeQ
[{a, b, c, d, e, n, p}, x]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.281.4 Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}} \operatorname{arcsinh}(bx+a)}dx$$

input
```
int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x)
```

output
```
int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x)
```

3.281.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.73

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2} \operatorname{arcsinh}(a+bx)} dx = \int \frac{1}{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}} \operatorname{arsinh}(bx+a)} dx$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="fri
cas")
```

output
```
integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/((b^4*x^4 + 4*a*b^3*x^3 + 2*(3*
a^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1)*arcsinh(b*x + a)), x
)
```

3.281.6 Sympy [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2} \operatorname{arcsinh}(a+bx)} dx = \int \frac{1}{(a^2+2abx+b^2x^2+1)^{\frac{3}{2}} \operatorname{asinh}(a+bx)} dx$$

input
```
integrate(1/(b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a),x)
```

output
```
Integral(1/((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)*asinh(a + b*x)), x)
```

3.281.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}\operatorname{arsinh}\left(bx+a\right)}\,dx$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="max
ima")
```

output
```
integrate(1/((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)), x)
```

3.281.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}\operatorname{arsinh}\left(bx+a\right)}\,dx$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="gia
c")
```

output
```
integrate(1/((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)), x)
```

3.281.9 Mupad [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)}\,dx = \int \frac{1}{\operatorname{asinh}\left(a+b\,x\right)\,{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{3/2}}\,dx$$

input
```
int(1/(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)),x)
```

output
```
int(1/(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)), x)
```

3.282 $\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\text{arcsinh}(a+bx)^2}\,dx$

3.282.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\,\text{arcsinh}(a+bx)^2}\,dx = -\frac{1}{b\,(1+(a+bx)^2)\,\text{arcsinh}(a+bx)} - 2\text{Int}\left(\frac{a+bx}{(1+(a+bx)^2)^2\,\text{arcsinh}(a+bx)},x\right)$$

output
```
-1/b/(1+(b*x+a)^2)/arcsinh(b*x+a)-2*Unintegrable((b*x+a)/(1+(b*x+a)^2)^2/a
rcsinh(b*x+a),x)
```

3.282.2 Mathematica [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\,\text{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\,\text{arcsinh}(a+bx)^2}\,dx$$

input
```
Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2),x]
```

output
```
Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2), x]
```

3.282.3 Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6275, 6205, 6239}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx+b^2x^2+1)^{3/2}\operatorname{arcsinh}(a+bx)^2}dx$$

$$\Big\downarrow \text{6275}$$

$$\frac{\int \frac{1}{((a+bx)^2+1)^{3/2}\operatorname{arcsinh}(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \text{6205}$$

$$\frac{-2\int \frac{a+bx}{((a+bx)^2+1)^2\operatorname{arcsinh}(a+bx)}d(a+bx)-\frac{1}{((a+bx)^2+1)\operatorname{arcsinh}(a+bx)}}{b}$$

$$\Big\downarrow \text{6239}$$

$$\frac{-2\int \frac{a+bx}{((a+bx)^2+1)^2\operatorname{arcsinh}(a+bx)}d(a+bx)-\frac{1}{((a+bx)^2+1)\operatorname{arcsinh}(a+bx)}}{b}$$

input
```
Int[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2),x]
```

output
```
$Aborted
```

3.282.3.1 Defintions of rubi rules used

rule 6205
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p]   Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])
^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
 -1]
```

rule 6239
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^2)^p*(a + b*A
rcSinh[c*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 6275
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2
)^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C
, n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.282.4 Maple [N/A] (verified)

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(b^{2} x^{2}+2 a b x+a^{2}+1\right)^{\frac{3}{2}} \operatorname{arcsinh}\left(b x+a\right)^{2}}dx$$

input
```
int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x)
```

output
```
int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x)
```

3.282.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.73

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2} \operatorname{arcsinh}(a+bx)^2} \, dx=\int \frac{1}{{\left(b^{2} x^{2} + 2 \, a b x + a^{2} + 1\right)}^{\frac{3}{2}} \operatorname{arsinh}\left(b x + a\right)^{2}} \,d x$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="f
ricas")
```

output
```
integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/((b^4*x^4 + 4*a*b^3*x^3 + 2*(3*
a^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1)*arcsinh(b*x + a)^2),
 x)
```

3.282.6 Sympy [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{(a^2+2abx+b^2x^2+1)^{\frac{3}{2}}\operatorname{asinh}^2(a+bx)}\,dx$$

input
```
integrate(1/(b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a)**2,x)
```

output
```
Integral(1/((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)*asinh(a + b*x)**2), x)
```

3.282.7 Maxima [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 610, normalized size of antiderivative = 20.33

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}\operatorname{arsinh}(bx+a)^2}\,dx$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="m
axima")
```

output
```
-(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((((b^2*x^2 + 2*a*b*x + a^2
+ 1)*(b^2*x + a*b) + (b^3*x^2 + 2*a*b^2*x + a^2*b + b)*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) - integr
ate((2*b^4*x^4 + 8*a*b^3*x^3 + 2*a^4 + (12*a^2*b^2 + b^2)*x^2 + (2*b^2*x^2
 + 4*a*b*x + 2*a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + a^2 + 2*(4*a^3*b +
 a*b)*x + 2*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2*b + b)*x + a)*sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1) - 1)/((((b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*a^2*b
^2 + b^2)*x^2 + a^2 + 2*(2*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(
3/2) + 2*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + 2*(5*a^2*b^3 + b^3)*x^3 + 2*a^3 +
2*(5*a^3*b^2 + 3*a*b^2)*x^2 + (5*a^4*b + 6*a^2*b + b)*x + a)*(b^2*x^2 + 2*
a*b*x + a^2 + 1) + (b^6*x^6 + 6*a*b^5*x^5 + a^6 + 3*(5*a^2*b^4 + b^4)*x^4
+ 3*a^4 + 4*(5*a^3*b^3 + 3*a*b^3)*x^3 + 3*(5*a^4*b^2 + 6*a^2*b^2 + b^2)*x^
2 + 3*a^2 + 6*(a^5*b + 2*a^3*b + a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

3.282.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}\operatorname{arsinh}\left(bx+a\right)^2}\,dx$$

input
```
integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="g
iac")
```

output
```
integrate(1/((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^2), x)
```

3.282.9 Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\operatorname{arcsinh}(a+bx)^2}\,dx = \int \frac{1}{\operatorname{asinh}\left(a+b\,x\right)^2\,\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2}}\,dx$$

input
```
int(1/(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)),x)
```

output
```
int(1/(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)), x)
```

3.283 $\int x^3 \text{arcsinh}(ax^2)\, dx$

3.283.1 Optimal result

Integrand size = 10, antiderivative size = 50

$$\int x^3 \text{arcsinh}(ax^2)\, dx = -\frac{x^2\sqrt{1+a^2x^4}}{8a} + \frac{\text{arcsinh}(ax^2)}{8a^2} + \frac{1}{4}x^4 \text{arcsinh}(ax^2)$$

output
```
1/8*arcsinh(a*x^2)/a^2+1/4*x^4*arcsinh(a*x^2)-1/8*x^2*(a^2*x^4+1)^(1/2)/a
```

3.283.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int x^3 \text{arcsinh}(ax^2)\, dx = \frac{-ax^2\sqrt{1+a^2x^4} + (1+2a^2x^4)\,\text{arcsinh}(ax^2)}{8a^2}$$

input
```
Integrate[x^3*ArcSinh[a*x^2],x]
```

output
```
(-(a*x^2*Sqrt[1 + a^2*x^4]) + (1 + 2*a^2*x^4)*ArcSinh[a*x^2])/(8*a^2)
```

3.283.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6290, 27, 807, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{arcsinh}(ax^2) \, dx$$

$$\Big\downarrow \ 6290$$

$$\frac{1}{4}x^4\text{arcsinh}(ax^2) - \frac{1}{4}\int \frac{2ax^5}{\sqrt{a^2x^4+1}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}x^4\text{arcsinh}(ax^2) - \frac{1}{2}a\int \frac{x^5}{\sqrt{a^2x^4+1}}dx$$

$$\Big\downarrow \ 807$$

$$\frac{1}{4}x^4\text{arcsinh}(ax^2) - \frac{1}{4}a\int \frac{x^4}{\sqrt{a^2x^4+1}}dx^2$$

$$\Big\downarrow \ 262$$

$$\frac{1}{4}x^4\text{arcsinh}(ax^2) - \frac{1}{4}a\left(\frac{x^2\sqrt{a^2x^4+1}}{2a^2} - \frac{\int \frac{1}{\sqrt{a^2x^4+1}}dx^2}{2a^2}\right)$$

$$\Big\downarrow \ 222$$

$$\frac{1}{4}x^4\text{arcsinh}(ax^2) - \frac{1}{4}a\left(\frac{x^2\sqrt{a^2x^4+1}}{2a^2} - \frac{\text{arcsinh}(ax^2)}{2a^3}\right)$$

input
```
Int[x^3*ArcSinh[a*x^2],x]
```

output
```
(x^4*ArcSinh[a*x^2])/4 - (a*((x^2*Sqrt[1 + a^2*x^4])/(2*a^2) - ArcSinh[a*x
^2]/(2*a^3)))/4
```

3.283.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6290
```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x],
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u
, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[
u, x]
```

3.283.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

method	result	size
default	$\frac{x^{4} \operatorname{arcsinh}\left(a\,x^{2}\right)}{4}-\frac{a\left(\frac{x^{2} \sqrt{a^{2} x^{4}+1}}{4 a^{2}}-\frac{\ln\left(\frac{a^{2} x^{2}}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{4}+1}\right)}{4 a^{2} \sqrt{a^{2}}}\right)}{2}$	71
parts	$\frac{x^{4} \operatorname{arcsinh}\left(a\,x^{2}\right)}{4}-\frac{a\left(\frac{x^{2} \sqrt{a^{2} x^{4}+1}}{4 a^{2}}-\frac{\ln\left(\frac{a^{2} x^{2}}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{4}+1}\right)}{4 a^{2} \sqrt{a^{2}}}\right)}{2}$	71

input
```
int(x^3*arcsinh(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*arcsinh(a*x^2)-1/2*a*(1/4*x^2/a^2*(a^2*x^4+1)^(1/2)-1/4/a^2*ln(a^2
*x^2/(a^2)^(1/2)+(a^2*x^4+1)^(1/2))/(a^2)^(1/2))
```

3.283.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int x^3 \operatorname{arcsinh}(ax^2)\, dx = -\frac{\sqrt{a^2x^4+1}ax^2 - (2\,a^2x^4+1)\log\left(ax^2+\sqrt{a^2x^4+1}\right)}{8\,a^2}$$

input
```
integrate(x^3*arcsinh(a*x^2),x, algorithm="fricas")
```

output
```
-1/8*(sqrt(a^2*x^4 + 1)*a*x^2 - (2*a^2*x^4 + 1)*log(a*x^2 + sqrt(a^2*x^4 +
 1)))/a^2
```

3.283.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int x^3 \operatorname{arcsinh}(ax^2)\, dx = \begin{cases} \frac{x^4 \operatorname{asinh}(ax^2)}{4} - \frac{x^2\sqrt{a^2x^4+1}}{8a} + \frac{\operatorname{asinh}(ax^2)}{8a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*asinh(a*x**2),x)
```

output
```
Piecewise((x**4*asinh(a*x**2)/4 - x**2*sqrt(a**2*x**4 + 1)/(8*a) + asinh(a
*x**2)/(8*a**2), Ne(a, 0)), (0, True))
```

3.283.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(42) = 84$.

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.04

$$\int x^3 \text{arcsinh}(ax^2)\, dx = \frac{1}{4}x^4 \operatorname{arsinh}(ax^2) + \frac{1}{16}a\left(\frac{\log\left(a + \frac{\sqrt{a^2x^4+1}}{x^2}\right)}{a^3} - \frac{\log\left(-a + \frac{\sqrt{a^2x^4+1}}{x^2}\right)}{a^3} + \frac{2\sqrt{a^2x^4+1}}{\left(a^4 - \frac{(a^2x^4+1)a^2}{x^4}\right)x^2}\right)$$

input
```
integrate(x^3*arcsinh(a*x^2),x, algorithm="maxima")
```

output
```
1/4*x^4*arcsinh(a*x^2) + 1/16*a*(log(a + sqrt(a^2*x^4 + 1)/x^2)/a^3 - log(
-a + sqrt(a^2*x^4 + 1)/x^2)/a^3 + 2*sqrt(a^2*x^4 + 1)/((a^4 - (a^2*x^4 + 1
)*a^2/x^4)*x^2))
```

3.283.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.48

$$\int x^3 \text{arcsinh}(ax^2)\, dx = \frac{1}{4}x^4 \log\left(ax^2 + \sqrt{a^2x^4+1}\right) - \frac{1}{8}a\left(\frac{\sqrt{a^2x^4+1}x^2}{a^2} + \frac{\log\left(-x^2|a| + \sqrt{a^2x^4+1}\right)}{a^2|a|}\right)$$

input
```
integrate(x^3*arcsinh(a*x^2),x, algorithm="giac")
```

output
```
1/4*x^4*log(a*x^2 + sqrt(a^2*x^4 + 1)) - 1/8*a*(sqrt(a^2*x^4 + 1)*x^2/a^2
+ log(-x^2*abs(a) + sqrt(a^2*x^4 + 1))/(a^2*abs(a)))
```

3.283.9 Mupad [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int x^3 \operatorname{arcsinh}(ax^2)\, dx = \frac{x^2\, \operatorname{asinh}(a\, x^2)\, \left(\frac{x^2}{2}+\frac{1}{4\, a^2\, x^2}\right)}{2} - \frac{x^2\, \sqrt{a^2\, x^4+1}}{8\, a}$$

input
```
int(x^3*asinh(a*x^2),x)
```

output
```
(x^2*asinh(a*x^2)*(x^2/2 + 1/(4*a^2*x^2)))/2 - (x^2*(a^2*x^4 + 1)^(1/2))/(
8*a)
```

3.284 $\int x^2 \text{arcsinh}(ax^2)\,dx$

3.284.1 Optimal result

Integrand size = 10, antiderivative size = 101

$$\int x^2 \text{arcsinh}\left(ax^2\right)\,dx = -\frac{2x\sqrt{1+a^2x^4}}{9a} + \frac{1}{3}x^3\text{arcsinh}\left(ax^2\right) + \frac{(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{9a^{3/2}\sqrt{1+a^2x^4}}$$

output
```
1/3*x^3*arcsinh(a*x^2)-2/9*x*(a^2*x^4+1)^(1/2)/a+1/9*(a*x^2+1)*(cos(2*arct
an(x*a^(1/2)))^2)^(1/2)/cos(2*arctan(x*a^(1/2)))*EllipticF(sin(2*arctan(x*
a^(1/2))),1/2*2^(1/2))*((a^2*x^4+1)/(a*x^2+1)^2)^(1/2)/a^(3/2)/(a^2*x^4+1)
^(1/2)
```

3.284.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int x^2 \text{arcsinh}\left(ax^2\right)\,dx = \frac{1}{9}\left(-\frac{2(x+a^2x^5)}{a\sqrt{1+a^2x^4}} + 3x^3\text{arcsinh}\left(ax^2\right) - \frac{2\sqrt{ia}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{ia}x\right),-1\right)}{a^2}\right)$$

input
```
Integrate[x^2*ArcSinh[a*x^2],x]
```

output
```
((-2*(x + a^2*x^5))/(a*Sqrt[1 + a^2*x^4]) + 3*x^3*ArcSinh[a*x^2] - (2*Sqrt
[I*a]*EllipticF[I*ArcSinh[Sqrt[I*a]*x], -1])/a^2)/9
```

3.284.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6290, 27, 843, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arcsinh}\left(ax^2\right)\,dx$$

$$\Big\downarrow \text{6290}$$

$$\frac{1}{3}x^3\text{arcsinh}\left(ax^2\right)-\frac{1}{3}\int \frac{2ax^4}{\sqrt{a^2x^4+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\text{arcsinh}\left(ax^2\right)-\frac{2}{3}a\int \frac{x^4}{\sqrt{a^2x^4+1}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{3}x^3\text{arcsinh}\left(ax^2\right)-\frac{2}{3}a\left(\frac{x\sqrt{a^2x^4+1}}{3a^2}-\frac{\int \frac{1}{\sqrt{a^2x^4+1}}dx}{3a^2}\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{3}x^3\text{arcsinh}\left(ax^2\right)-\frac{2}{3}a\left(\frac{x\sqrt{a^2x^4+1}}{3a^2}-\frac{\left(ax^2+1\right)\sqrt{\frac{a^2x^4+1}{\left(ax^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{6a^{5/2}\sqrt{a^2x^4+1}}\right)$$

input
```
Int[x^2*ArcSinh[a*x^2],x]
```

output
```
(x^3*ArcSinh[a*x^2])/3 - (2*a*((x*Sqrt[1 + a^2*x^4])/(3*a^2) - ((1 + a*x^2
)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticF[2*ArcTan[Sqrt[a]*x], 1/2])/(
6*a^(5/2)*Sqrt[1 + a^2*x^4])))/3
```

3.284.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 6290
```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x],
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u
, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[
u, x]
```

3.284.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{x^{3} \operatorname{arcsinh}\left(a \,x^{2}\right)}{3}-\frac{2 a \left(\frac{x \sqrt{a^{2} x^{4}+1}}{3 a^{2}}-\frac{\sqrt{-i a \,x^{2}+1}\, \sqrt{i a \,x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{i a}, i\right)}{3 a^{2} \sqrt{i a}\, \sqrt{a^{2} x^{4}+1}}\right)}{3}$	89
parts	$\frac{x^{3} \operatorname{arcsinh}\left(a \,x^{2}\right)}{3}-\frac{2 a \left(\frac{x \sqrt{a^{2} x^{4}+1}}{3 a^{2}}-\frac{\sqrt{-i a \,x^{2}+1}\, \sqrt{i a \,x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{i a}, i\right)}{3 a^{2} \sqrt{i a}\, \sqrt{a^{2} x^{4}+1}}\right)}{3}$	89

input
```
int(x^2*arcsinh(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arcsinh(a*x^2)-2/3*a*(1/3*x/a^2*(a^2*x^4+1)^(1/2)-1/3/a^2/(I*a)^(1
/2)*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*x^4+1)^(1/2)*EllipticF(x*(I*a
)^(1/2),I))
```

3.284.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int x^2 \operatorname{arcsinh}(ax^2)\,dx = \frac{3\,ax^3\log\left(ax^2+\sqrt{a^2x^4+1}\right)+2\,a\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{1}{a^2}\right)^{\frac{1}{4}}}{x}\right)\mid -1)-2\sqrt{a^2x^4+1}x}{9\,a}$$

input
```
integrate(x^2*arcsinh(a*x^2),x, algorithm="fricas")
```

output
```
1/9*(3*a*x^3*log(a*x^2 + sqrt(a^2*x^4 + 1)) + 2*a*(-1/a^2)^(3/4)*elliptic_
f(arcsin((-1/a^2)^(1/4)/x), -1) - 2*sqrt(a^2*x^4 + 1)*x)/a
```

3.284.6 Sympy [F]

$$\int x^2 \operatorname{arcsinh}(ax^2)\,dx = \int x^2 \operatorname{asinh}(ax^2)\,dx$$

input
```
integrate(x**2*asinh(a*x**2),x)
```

output
```
Integral(x**2*asinh(a*x**2), x)
```

3.284.7 Maxima [F]

$$\int x^2\text{arcsinh}\left(ax^2\right)\,dx=\int x^2\,\text{arsinh}\left(ax^2\right)\,dx$$

input
```
integrate(x^2*arcsinh(a*x^2),x, algorithm="maxima")
```

output
```
1/3*x^3*log(a*x^2 + sqrt(a^2*x^4 + 1)) - 2/9*x^3 - 2*a*integrate(1/3*x^4/(
a^3*x^6 + a*x^2 + (a^2*x^4 + 1)^(3/2)), x) - 1/12*I*sqrt(2)*(log(1/2*I*sqr
t(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a) + 1) - log(-1/2*I*sqrt(2)*(2*a*x +
sqrt(2)*sqrt(a))/sqrt(a) + 1))/a^(3/2) - 1/12*I*sqrt(2)*(log(1/2*I*sqrt(2)
*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1) - log(-1/2*I*sqrt(2)*(2*a*x - sqrt
(2)*sqrt(a))/sqrt(a) + 1))/a^(3/2) - 1/12*sqrt(2)*log(a*x^2 + sqrt(2)*sqrt
(a)*x + 1)/a^(3/2) + 1/12*sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/a^(3/
2)
```

3.284.8 Giac [F]

$$\int x^2\text{arcsinh}\left(ax^2\right)\,dx=\int x^2\,\text{arsinh}\left(ax^2\right)\,dx$$

input
```
integrate(x^2*arcsinh(a*x^2),x, algorithm="giac")
```

output
```
integrate(x^2*arcsinh(a*x^2), x)
```

3.284.9 Mupad [F(-1)]

Timed out.

$$\int x^2\text{arcsinh}\left(ax^2\right)\,dx=\int x^2\,\text{asinh}\left(a\,x^2\right)\,dx$$

input
```
int(x^2*asinh(a*x^2),x)
```

output
```
int(x^2*asinh(a*x^2), x)
```

3.285 $\int x\text{arcsinh}(ax^2)\,dx$

3.285.1 Optimal result

Integrand size = 8, antiderivative size = 34

$$\int x\text{arcsinh}(ax^2)\,dx=-\frac{\sqrt{1+a^2x^4}}{2a}+\frac{1}{2}x^2\text{arcsinh}(ax^2)$$

output
```
1/2*x^2*arcsinh(a*x^2)-1/2*(a^2*x^4+1)^(1/2)/a
```

3.285.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int x\text{arcsinh}(ax^2)\,dx=-\frac{\sqrt{1+a^2x^4}}{2a}+\frac{1}{2}x^2\text{arcsinh}(ax^2)$$

input
```
Integrate[x*ArcSinh[a*x^2],x]
```

output
```
-1/2*Sqrt[1 + a^2*x^4]/a + (x^2*ArcSinh[a*x^2])/2
```

3.285.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {7266, 6187, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arcsinh}(ax^2)\ dx$$

$$\Big\downarrow \text{ 7266}$$

$$\frac{1}{2}\int \text{arcsinh}(ax^2)\,dx^2$$

$$\Big\downarrow \text{ 6187}$$

$$\frac{1}{2}\left(x^2\text{arcsinh}(ax^2)-a\int \frac{x^2}{\sqrt{a^2x^4+1}}dx^2\right)$$

$$\Big\downarrow \text{ 241}$$

$$\frac{1}{2}\left(x^2\text{arcsinh}(ax^2)-\frac{\sqrt{a^2x^4+1}}{a}\right)$$

input
```
Int[x*ArcSinh[a*x^2],x]
```

output
```
(-(Sqrt[1 + a^2*x^4]/a) + x^2*ArcSinh[a*x^2])/2
```

3.285.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.285.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

method	result	size
parts	$\frac{x^2 \operatorname{arcsinh}(a\,x^2)}{2}-\frac{\sqrt{a^2x^4+1}}{2a}$	29
derivativedivides	$\frac{a\,x^2 \operatorname{arcsinh}(a\,x^2)-\sqrt{a^2x^4+1}}{2a}$	31
default	$\frac{a\,x^2 \operatorname{arcsinh}(a\,x^2)-\sqrt{a^2x^4+1}}{2a}$	31

input
```
int(x*arcsinh(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arcsinh(a*x^2)-1/2*(a^2*x^4+1)^(1/2)/a
```

3.285.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int x\text{arcsinh}\left(ax^2\right)\,dx = \frac{ax^2\log\left(ax^2+\sqrt{a^2x^4+1}\right)-\sqrt{a^2x^4+1}}{2\,a}$$

input
```
integrate(x*arcsinh(a*x^2),x, algorithm="fricas")
```

output
```
1/2*(a*x^2*log(a*x^2 + sqrt(a^2*x^4 + 1)) - sqrt(a^2*x^4 + 1))/a
```

3.285.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int x\operatorname{arcsinh}(ax^2)\,dx = \begin{cases} \frac{x^2\operatorname{asinh}(ax^2)}{2} - \frac{\sqrt{a^2x^4+1}}{2a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*asinh(a*x**2),x)
```

output
```
Piecewise((x**2*asinh(a*x**2)/2 - sqrt(a**2*x**4 + 1)/(2*a), Ne(a, 0)), (0
, True))
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int x\operatorname{arcsinh}(ax^2)\,dx = \frac{ax^2\operatorname{arsinh}(ax^2) - \sqrt{a^2x^4+1}}{2a}$$

input
```
integrate(x*arcsinh(a*x^2),x, algorithm="maxima")
```

output
```
1/2*(a*x^2*arcsinh(a*x^2) - sqrt(a^2*x^4 + 1))/a
```

3.285.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

$$\int x\operatorname{arcsinh}(ax^2)\,dx = \frac{1}{2}x^2\log\left(ax^2+\sqrt{a^2x^4+1}\right) - \frac{\sqrt{a^2x^4+1}}{2a}$$

input
```
integrate(x*arcsinh(a*x^2),x, algorithm="giac")
```

output
```
1/2*x^2*log(a*x^2 + sqrt(a^2*x^4 + 1)) - 1/2*sqrt(a^2*x^4 + 1)/a
```

3.285.9 Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int x\text{arcsinh}(ax^2)\ dx = \frac{x^2\,\text{asinh}(a\,x^2)}{2} - \frac{\sqrt{a^2\,x^4+1}}{2\,a}$$

input
```
int(x*asinh(a*x^2),x)
```

output
```
(x^2*asinh(a*x^2))/2 - (a^2*x^4 + 1)^(1/2)/(2*a)
```

3.286 $\int \operatorname{arcsinh}(ax^2)\,dx$

3.286.1 Optimal result

Integrand size = 6, antiderivative size = 162

$$\int \operatorname{arcsinh}(ax^2)\,dx = -\frac{2x\sqrt{1+a^2x^4}}{1+ax^2}+x\operatorname{arcsinh}(ax^2)+\frac{2(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}E\left(2\arctan\left(\sqrt{a}x\right)\middle|\frac{1}{2}\right)}{\sqrt{a}\sqrt{1+a^2x^4}}-\frac{(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{\sqrt{a}\sqrt{1+a^2x^4}}$$

output
```
x*arcsinh(a*x^2)-2*x*(a^2*x^4+1)^(1/2)/(a*x^2+1)+2*(a*x^2+1)*(cos(2*arctan
(x*a^(1/2)))^2)^(1/2)/cos(2*arctan(x*a^(1/2)))*EllipticE(sin(2*arctan(x*a^
(1/2))),1/2*2^(1/2))*((a^2*x^4+1)/(a*x^2+1)^2)^(1/2)/a^(1/2)/(a^2*x^4+1)^(
1/2)-(a*x^2+1)*(cos(2*arctan(x*a^(1/2)))^2)^(1/2)/cos(2*arctan(x*a^(1/2)))
*EllipticF(sin(2*arctan(x*a^(1/2))),1/2*2^(1/2))*((a^2*x^4+1)/(a*x^2+1)^2)
^(1/2)/a^(1/2)/(a^2*x^4+1)^(1/2)
```

3.286.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.22

$$\int \operatorname{arcsinh}\left(ax^2\right) dx = x\operatorname{arcsinh}\left(ax^2\right) - \frac{2}{3}ax^3 \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-a^2x^4\right)$$

input
```
Integrate[ArcSinh[a*x^2],x]
```

output
```
x*ArcSinh[a*x^2] - (2*a*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -(a^2*x^4)])/
3
```

3.286.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6289, 27, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arcsinh}\left(ax^2\right) dx$$

$$\Big\downarrow \text{ 6289}$$

$$x\operatorname{arcsinh}\left(ax^2\right) - \int \frac{2ax^2}{\sqrt{a^2x^4+1}}dx$$

$$\Big\downarrow \text{ 27}$$

$$x\operatorname{arcsinh}\left(ax^2\right) - 2a\int \frac{x^2}{\sqrt{a^2x^4+1}}dx$$

$$\Big\downarrow \text{ 834}$$

$$x\operatorname{arcsinh}\left(ax^2\right) - 2a\left(\frac{\int \frac{1}{\sqrt{a^2x^4+1}}dx}{a} - \frac{\int \frac{1-ax^2}{\sqrt{a^2x^4+1}}dx}{a}\right)$$

$$\Big\downarrow \text{ 761}$$

$$x\text{arcsinh}(ax^2) - 2a\left(\frac{(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{2a^{3/2}\sqrt{a^2x^4+1}} - \frac{\int \frac{1-ax^2}{\sqrt{a^2x^4+1}}dx}{a}\right)$$

$$\Big\downarrow \text{1510}$$

$$x\text{arcsinh}(ax^2) - 2a\left(\frac{(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{2a^{3/2}\sqrt{a^2x^4+1}} - \frac{\frac{(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}E\left(2\arctan(\sqrt{a}x)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{a^2x^4+1}} - \frac{x\sqrt{a^2x^4+1}}{ax^2+1}}{a}\right)$$

input
```
Int[ArcSinh[a*x^2],x]
```

output
```
x*ArcSinh[a*x^2] - 2*a*(-((-((x*Sqrt[1 + a^2*x^4])/(1 + a*x^2)) + ((1 + a*
x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticE[2*ArcTan[Sqrt[a]*x], 1/2]
)/(Sqrt[a]*Sqrt[1 + a^2*x^4]))/a) + ((1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a
*x^2)^2]*EllipticF[2*ArcTan[Sqrt[a]*x], 1/2])/(2*a^(3/2)*Sqrt[1 + a^2*x^4]
))
```

3.286.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6289
```
Int[ArcSinh[u_], x_Symbol] :> Simp[x*ArcSinh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/Sqrt[1 + u^2]), x], x] /; InverseFunctionFreeQ[u, x] && !Funct
ionOfExponentialQ[u, x]
```

3.286.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.48

method	result	size
default	$x \operatorname{arcsinh}\left(a\,x^{2}\right)-\frac{2 i \sqrt{-i a\,x^{2}+1}\,\sqrt{i a\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(x \sqrt{i a}, i\right)-\operatorname{EllipticE}\left(x \sqrt{i a}, i\right)\right)}{\sqrt{i a}\,\sqrt{a^{2} x^{4}+1}}$	77
parts	$x \operatorname{arcsinh}\left(a\,x^{2}\right)-\frac{2 i \sqrt{-i a\,x^{2}+1}\,\sqrt{i a\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(x \sqrt{i a}, i\right)-\operatorname{EllipticE}\left(x \sqrt{i a}, i\right)\right)}{\sqrt{i a}\,\sqrt{a^{2} x^{4}+1}}$	77

input
```
int(arcsinh(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
x*arcsinh(a*x^2)-2*I/(I*a)^(1/2)*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*
x^4+1)^(1/2)*(EllipticF(x*(I*a)^(1/2),I)-EllipticE(x*(I*a)^(1/2),I))
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.58

$$\int \operatorname{arcsinh}(ax^2)\,dx = \frac{ax^2\log\left(ax^2+\sqrt{a^2x^4+1}\right)-2\,ax\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{1}{a^2}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+2\,ax\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{1}{a^2}\right)^{\frac{1}{4}}}{x}\right)\,|}{ax}$$

input
```
integrate(arcsinh(a*x^2),x, algorithm="fricas")
```

output
```
(a*x^2*log(a*x^2 + sqrt(a^2*x^4 + 1)) - 2*a*x*(-1/a^2)^(3/4)*elliptic_e(ar
csin((-1/a^2)^(1/4)/x), -1) + 2*a*x*(-1/a^2)^(3/4)*elliptic_f(arcsin((-1/a
^2)^(1/4)/x), -1) - 2*sqrt(a^2*x^4 + 1))/(a*x)
```

3.286.6 Sympy [F]

$$\int \operatorname{arcsinh}(ax^2) \, dx = \int \operatorname{asinh}(ax^2) \, dx$$

input
```
integrate(asinh(a*x**2),x)
```

output
```
Integral(asinh(a*x**2), x)
```

3.286.7 Maxima [F]

$$\int \operatorname{arcsinh}(ax^2) \, dx = \int \operatorname{arsinh}(ax^2) \, dx$$

input
```
integrate(arcsinh(a*x^2),x, algorithm="maxima")
```

output
```
-2*a*integrate(x^2/(a^3*x^6 + a*x^2 + (a^2*x^4 + 1)^(3/2)), x) + x*log(a*x
^2 + sqrt(a^2*x^4 + 1)) - 2*x - 1/4*I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*a*x +
sqrt(2)*sqrt(a))/sqrt(a) + 1) - log(-1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a
))/sqrt(a) + 1))/sqrt(a) - 1/4*I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*a*x - sqrt(
2)*sqrt(a))/sqrt(a) + 1) - log(-1/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sq
rt(a) + 1))/sqrt(a) + 1/4*sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/sqrt(
a) - 1/4*sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/sqrt(a)
```

3.286.8 Giac [F]

$$\int \operatorname{arcsinh}(ax^2)\,dx = \int \operatorname{arsinh}\left(ax^2\right)\,dx$$

input
```
integrate(arcsinh(a*x^2),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x^2), x)
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(ax^2)\,dx = \int \operatorname{asinh}\left(a\,x^2\right)\,dx$$

input
```
int(asinh(a*x^2),x)
```

output
```
int(asinh(a*x^2), x)
```

3.287 $\int \frac{\text{arcsinh}(ax^2)}{x}\,dx$

3.287.1 Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{\text{arcsinh}(ax^2)}{x}\,dx = -\frac{1}{4}\text{arcsinh}(ax^2)^2 + \frac{1}{2}\text{arcsinh}(ax^2)\log\left(1 - e^{2\text{arcsinh}(ax^2)}\right) + \frac{1}{4}\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax^2)}\right)$$

output
```
-1/4*arcsinh(a*x^2)^2+1/2*arcsinh(a*x^2)*ln(1-(a*x^2+(a^2*x^4+1)^(1/2))^2)
+1/4*polylog(2,(a*x^2+(a^2*x^4+1)^(1/2))^2)
```

3.287.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(ax^2)}{x}\,dx = -\frac{1}{4}\text{arcsinh}(ax^2)^2 + \frac{1}{2}\text{arcsinh}(ax^2)\log\left(1 - e^{2\text{arcsinh}(ax^2)}\right) + \frac{1}{4}\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax^2)}\right)$$

input
```
Integrate[ArcSinh[a*x^2]/x,x]
```

output
```
-1/4*ArcSinh[a*x^2]^2 + (ArcSinh[a*x^2]*Log[1 - E^(2*ArcSinh[a*x^2])])/2 +
 PolyLog[2, E^(2*ArcSinh[a*x^2])]/4
```

3.287.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6284, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x}dx$$

$$\Big\downarrow \text{6284}$$

$$\frac{1}{2}\int \frac{\sqrt{a^2x^4+1}\operatorname{arcsinh}(ax^2)}{ax^2}d\operatorname{arcsinh}(ax^2)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int -i\operatorname{arcsinh}(ax^2)\tan\left(i\operatorname{arcsinh}(ax^2)+\frac{\pi}{2}\right)d\operatorname{arcsinh}(ax^2)$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{2}i\int \operatorname{arcsinh}(ax^2)\tan\left(i\operatorname{arcsinh}(ax^2)+\frac{\pi}{2}\right)d\operatorname{arcsinh}(ax^2)$$

$$\Big\downarrow \text{4199}$$

$$-\frac{1}{2}i\left(2i\int -\frac{e^{2\operatorname{arcsinh}(ax^2)}\operatorname{arcsinh}(ax^2)}{1-e^{2\operatorname{arcsinh}(ax^2)}}d\operatorname{arcsinh}(ax^2)-\frac{1}{2}i\operatorname{arcsinh}(ax^2)^2\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}i\left(-2i\int \frac{e^{2\operatorname{arcsinh}(ax^2)}\operatorname{arcsinh}(ax^2)}{1-e^{2\operatorname{arcsinh}(ax^2)}}d\operatorname{arcsinh}(ax^2)-\frac{1}{2}i\operatorname{arcsinh}(ax^2)^2\right)$$

$$\Big\downarrow \text{2620}$$

$$-\frac{1}{2}i\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2\operatorname{arcsinh}(ax^2)}\right)d\operatorname{arcsinh}(ax^2)-\frac{1}{2}\operatorname{arcsinh}(ax^2)\log\left(1-e^{2\operatorname{arcsinh}(ax^2)}\right)\right)-\frac{1}{2}i\operatorname{arcsinh}(ax^2$$

$$\Big\downarrow \text{2715}$$

$$-\frac{1}{2}i\left(-2i\left(\frac{1}{4}\int e^{-2\operatorname{arcsinh}(ax^2)}\log\left(1-e^{2\operatorname{arcsinh}(ax^2)}\right)de^{2\operatorname{arcsinh}(ax^2)}-\frac{1}{2}\operatorname{arcsinh}(ax^2)\log\left(1-e^{2\operatorname{arcsinh}(ax^2)}\right)\right)-\frac{1}{2}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{1}{2}i\left(-2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax^2)}\right)-\frac{1}{2}\operatorname{arcsinh}(ax^2)\log\left(1-e^{2\operatorname{arcsinh}(ax^2)}\right)\right)-\frac{1}{2}i\operatorname{arcsinh}(ax^2)^2\right)$$

input
```
Int[ArcSinh[a*x^2]/x,x]
```

output
```
(-1/2*I)*((-1/2*I)*ArcSinh[a*x^2]^2 - (2*I)*(-1/2*(ArcSinh[a*x^2]*Log[1 -
E^(2*ArcSinh[a*x^2])]) - PolyLog[2, E^(2*ArcSinh[a*x^2])]/4))
```

3.287.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6284
```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Simp[1/p   Subst[Int[
x^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

3.287.4 Maple [F]

$$\int \frac{\operatorname{arcsinh}\left(a\,x^{2}\right)}{x}dx$$

input
```
int(arcsinh(a*x^2)/x,x)
```

output
```
int(arcsinh(a*x^2)/x,x)
```

3.287.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax^2\right)}{x}\,dx$$

input
```
integrate(arcsinh(a*x^2)/x,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x^2)/x, x)
```

3.287.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x}\,dx = \int \frac{\operatorname{asinh}\left(ax^2\right)}{x}\,dx$$

input
```
integrate(asinh(a*x**2)/x,x)
```

output
```
Integral(asinh(a*x**2)/x, x)
```

3.287.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax^2\right)}{x}\,dx$$

input
```
integrate(arcsinh(a*x^2)/x,x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x^2)/x, x)
```

3.287.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax^2\right)}{x}\,dx$$

input
```
integrate(arcsinh(a*x^2)/x,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x^2)/x, x)
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x}\,dx = \int \frac{\operatorname{asinh}(a\,x^2)}{x}\,dx$$

input
```
int(asinh(a*x^2)/x,x)
```

output
```
int(asinh(a*x^2)/x, x)
```

3.288 $\int \frac{\text{arcsinh}(ax^2)}{x^2} dx$

3.288.1 Optimal result

Integrand size = 10, antiderivative size = 75

$$\int \frac{\text{arcsinh}(ax^2)}{x^2}\,dx = -\frac{\text{arcsinh}(ax^2)}{x}+\frac{\sqrt{a}(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{\sqrt{1+a^2x^4}}$$

output
```
-arcsinh(a*x^2)/x+(a*x^2+1)*(cos(2*arctan(x*a^(1/2)))^2)^(1/2)/cos(2*arcta
n(x*a^(1/2)))*EllipticF(sin(2*arctan(x*a^(1/2))),1/2*2^(1/2))*a^(1/2)*((a^
2*x^4+1)/(a*x^2+1)^2)^(1/2)/(a^2*x^4+1)^(1/2)
```

3.288.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.56

$$\int \frac{\text{arcsinh}(ax^2)}{x^2}\,dx = -\frac{\text{arcsinh}(ax^2)+2\sqrt{ia}x\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{ia}x\right),-1\right)}{x}$$

input
```
Integrate[ArcSinh[a*x^2]/x^2,x]
```

output
```
-((ArcSinh[a*x^2] + 2*Sqrt[I*a]*x*EllipticF[I*ArcSinh[Sqrt[I*a]*x], -1])/x
)
```

3.288.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6290, 27, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax^2)}{x^2} \, dx$$

$$\Big\downarrow \; 6290$$

$$\int \frac{2a}{\sqrt{a^2x^4+1}}dx-\frac{\text{arcsinh}(ax^2)}{x}$$

$$\Big\downarrow \; 27$$

$$2a\int \frac{1}{\sqrt{a^2x^4+1}}dx-\frac{\text{arcsinh}(ax^2)}{x}$$

$$\Big\downarrow \; 761$$

$$\frac{\sqrt{a}(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{\sqrt{a^2x^4+1}}-\frac{\text{arcsinh}(ax^2)}{x}$$

input
```
Int[ArcSinh[a*x^2]/x^2,x]
```

output
```
-(ArcSinh[a*x^2]/x) + (Sqrt[a]*(1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^
2]*EllipticF[2*ArcTan[Sqrt[a]*x], 1/2])/Sqrt[1 + a^2*x^4]
```

3.288.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 6290
```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x],
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u
, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[
u, x]
```

3.288.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\operatorname{arcsinh}\left(a\,x^{2}\right)}{x}+\frac{2 a \sqrt{-i a\,x^{2}+1}\,\sqrt{i a\,x^{2}+1}\;\operatorname{EllipticF}\left(x \sqrt{i a}, i\right)}{\sqrt{i a}\,\sqrt{a^{2} x^{4}+1}}$	66
parts	$-\frac{\operatorname{arcsinh}\left(a\,x^{2}\right)}{x}+\frac{2 a \sqrt{-i a\,x^{2}+1}\,\sqrt{i a\,x^{2}+1}\;\operatorname{EllipticF}\left(x \sqrt{i a}, i\right)}{\sqrt{i a}\,\sqrt{a^{2} x^{4}+1}}$	66

input
```
int(arcsinh(a*x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arcsinh(a*x^2)/x+2*a/(I*a)^(1/2)*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2
*x^4+1)^(1/2)*EllipticF(x*(I*a)^(1/2),I)
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^2}\,dx$$
$$=\frac{2\,a^2x\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{1}{a^2}\right)^{\frac{1}{4}}}{x}\right)\mid -1)+(x-1)\log\left(ax^2+\sqrt{a^2x^4+1}\right)+x\log\left(ax^2-\sqrt{a^2x^4+1}\right)}{x}$$

input
```
integrate(arcsinh(a*x^2)/x^2,x, algorithm="fricas")
```

output
```
(2*a^2*x*(-1/a^2)^(3/4)*elliptic_f(arcsin((-1/a^2)^(1/4)/x), -1) + (x - 1)
*log(a*x^2 + sqrt(a^2*x^4 + 1)) + x*log(a*x^2 - sqrt(a^2*x^4 + 1)))/x
```

3.288.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^2}\,dx = \int \frac{\operatorname{asinh}(ax^2)}{x^2}\,dx$$

input
```
integrate(asinh(a*x**2)/x**2,x)
```

output
```
Integral(asinh(a*x**2)/x**2, x)
```

3.288.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^2}\,dx = \int \frac{\operatorname{arsinh}(ax^2)}{x^2}\,dx$$

input
```
integrate(arcsinh(a*x^2)/x^2,x, algorithm="maxima")
```

output
```
-1/4*a^2*(I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a) +
 1) - log(-1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a) + 1))/a^(3/2) +
 I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1) - log
(-1/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1))/a^(3/2) + sqrt(2)*
log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/a^(3/2) - sqrt(2)*log(a*x^2 - sqrt(2)*s
qrt(a)*x + 1)/a^(3/2)) + 2*a*integrate(1/(a^3*x^6 + a*x^2 + (a^2*x^4 + 1)^
(3/2)), x) - log(a*x^2 + sqrt(a^2*x^4 + 1))/x
```

3.288.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^2}\,dx = \int \frac{\operatorname{arsinh}(ax^2)}{x^2}\,dx$$

input
```
integrate(arcsinh(a*x^2)/x^2,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x^2)/x^2, x)
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^2}\,dx = \int \frac{\operatorname{asinh}(a\,x^2)}{x^2}\,dx$$

input
```
int(asinh(a*x^2)/x^2,x)
```

output
```
int(asinh(a*x^2)/x^2, x)
```

3.289 $\int \frac{\operatorname{arcsinh}(ax^2)}{x^3} dx$

3.289.1 Optimal result

Integrand size = 10, antiderivative size = 33

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^3} dx = -\frac{\operatorname{arcsinh}(ax^2)}{2x^2} - \frac{1}{2}a\operatorname{arctanh}\left(\sqrt{1+a^2x^4}\right)$$

output
```
-1/2*arcsinh(a*x^2)/x^2-1/2*a*arctanh((a^2*x^4+1)^(1/2))
```

3.289.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^3} dx = -\frac{\operatorname{arcsinh}(ax^2)}{2x^2} - \frac{1}{2}a\operatorname{arctanh}\left(\sqrt{1+a^2x^4}\right)$$

input
```
Integrate[ArcSinh[a*x^2]/x^3,x]
```

output
```
-1/2*ArcSinh[a*x^2]/x^2 - (a*ArcTanh[Sqrt[1 + a^2*x^4]])/2
```

3.289.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6290, 27, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax^2)}{x^3} \, dx$$

$$\Big\downarrow \text{6290}$$

$$\frac{1}{2}\int \frac{2a}{x\sqrt{a^2x^4+1}}dx-\frac{\text{arcsinh}(ax^2)}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$a\int \frac{1}{x\sqrt{a^2x^4+1}}dx-\frac{\text{arcsinh}(ax^2)}{2x^2}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}a\int \frac{1}{x^4\sqrt{a^2x^4+1}}dx^4-\frac{\text{arcsinh}(ax^2)}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{x^8}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^4+1}}{2a}-\frac{\text{arcsinh}(ax^2)}{2x^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{1}{2}a\text{arctanh}\left(\sqrt{a^2x^4+1}\right)-\frac{\text{arcsinh}(ax^2)}{2x^2}$$

input
```
Int[ArcSinh[a*x^2]/x^3,x]
```

output
```
-1/2*ArcSinh[a*x^2]/x^2 - (a*ArcTanh[Sqrt[1 + a^2*x^4]])/2
```

3.289.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6290
```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x],
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u
, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[
u, x]
```

3.289.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{\operatorname{arcsinh}\left(a\,x^{2}\right)}{2x^{2}}-\frac{a\,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{4}+1}}\right)}{2}$	28
parts	$-\frac{\operatorname{arcsinh}\left(a\,x^{2}\right)}{2x^{2}}-\frac{a\,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{4}+1}}\right)}{2}$	28

input
```
int(arcsinh(a*x^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arcsinh(a*x^2)/x^2-1/2*a*arctanh(1/(a^2*x^4+1)^(1/2))
```

3.289.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(27) = 54$.

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.21

$$\int \frac{\text{arcsinh}(ax^2)}{x^3}\,dx = -\frac{ax^2\log\left(-ax^2+\sqrt{a^2x^4+1}+1\right)-ax^2\log\left(-ax^2+\sqrt{a^2x^4+1}-1\right)-x^2\log\left(-ax^2+\sqrt{a^2x^4+1}\right)-(x^2-1)\log\left(ax^2+\sqrt{a^2x^4+1}\right)}{2\,x^2}$$

input
```
integrate(arcsinh(a*x^2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a*x^2*log(-a*x^2 + sqrt(a^2*x^4 + 1) + 1) - a*x^2*log(-a*x^2 + sqrt(
a^2*x^4 + 1) - 1) - x^2*log(-a*x^2 + sqrt(a^2*x^4 + 1)) - (x^2 - 1)*log(a*
x^2 + sqrt(a^2*x^4 + 1)))/x^2
```

3.289.6 Sympy [F]

$$\int \frac{\text{arcsinh}(ax^2)}{x^3}\,dx = \int \frac{\text{asinh}\,(ax^2)}{x^3}\,dx$$

input
```
integrate(asinh(a*x**2)/x**3,x)
```

output
```
Integral(asinh(a*x**2)/x**3, x)
```

3.289.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^3}\,dx = -\frac{1}{4}\,a\left(\log\left(\sqrt{a^2x^4+1}+1\right)-\log\left(\sqrt{a^2x^4+1}-1\right)\right)-\frac{\operatorname{arsinh}\left(ax^2\right)}{2\,x^2}$$

input
```
integrate(arcsinh(a*x^2)/x^3,x, algorithm="maxima")
```

output
```
-1/4*a*(log(sqrt(a^2*x^4 + 1) + 1) - log(sqrt(a^2*x^4 + 1) - 1)) - 1/2*arc
sinh(a*x^2)/x^2
```

3.289.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(27) = 54.

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^3}\,dx = -\frac{1}{4}\,a\left(\log\left(\sqrt{a^2x^4+1}+1\right)-\log\left(\sqrt{a^2x^4+1}-1\right)\right) - \frac{\log\left(ax^2+\sqrt{a^2x^4+1}\right)}{2\,x^2}$$

input
```
integrate(arcsinh(a*x^2)/x^3,x, algorithm="giac")
```

output
```
-1/4*a*(log(sqrt(a^2*x^4 + 1) + 1) - log(sqrt(a^2*x^4 + 1) - 1)) - 1/2*log
(a*x^2 + sqrt(a^2*x^4 + 1))/x^2
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^3}\,dx = \int \frac{\operatorname{asinh}(a\,x^2)}{x^3}\,dx$$

input
```
int(asinh(a*x^2)/x^3,x)
```

output
```
int(asinh(a*x^2)/x^3, x)
```

3.290 $\int \frac{\operatorname{arcsinh}(ax^2)}{x^4} dx$

3.290.1 Optimal result

Integrand size = 10, antiderivative size = 197

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^4} dx = -\frac{2a\sqrt{1+a^2x^4}}{3x} + \frac{2a^2x\sqrt{1+a^2x^4}}{3(1+ax^2)} - \frac{\operatorname{arcsinh}(ax^2)}{3x^3} - \frac{2a^{3/2}(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}E\left(2\arctan\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{3\sqrt{1+a^2x^4}} + \frac{a^{3/2}(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{3\sqrt{1+a^2x^4}}$$

output
```
-1/3*arcsinh(a*x^2)/x^3-2/3*a*(a^2*x^4+1)^(1/2)/x+2/3*a^2*x*(a^2*x^4+1)^(1
/2)/(a*x^2+1)-2/3*a^(3/2)*(a*x^2+1)*(cos(2*arctan(x*a^(1/2)))^2)^(1/2)/cos
(2*arctan(x*a^(1/2)))*EllipticE(sin(2*arctan(x*a^(1/2))),1/2*2^(1/2))*((a^
2*x^4+1)/(a*x^2+1)^2)^(1/2)/(a^2*x^4+1)^(1/2)+1/3*a^(3/2)*(a*x^2+1)*(cos(2
*arctan(x*a^(1/2)))^2)^(1/2)/cos(2*arctan(x*a^(1/2)))*EllipticF(sin(2*arct
an(x*a^(1/2))),1/2*2^(1/2))*((a^2*x^4+1)/(a*x^2+1)^2)^(1/2)/(a^2*x^4+1)^(1
/2)
```

3.290.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.45

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^4}\,dx = \frac{1}{3}\left(-\frac{2a\sqrt{1+a^2x^4}}{x}-\frac{\operatorname{arcsinh}(ax^2)}{x^3} + \frac{2a^2\left(E\left(i\operatorname{arcsinh}\left(\sqrt{ia}x\right)\middle|-1\right)-\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{ia}x\right),-1\right)\right)}{\sqrt{ia}}\right)$$

input
```
Integrate[ArcSinh[a*x^2]/x^4,x]
```

output
```
((-2*a*Sqrt[1 + a^2*x^4])/x - ArcSinh[a*x^2]/x^3 + (2*a^2*(EllipticE[I*Arc
Sinh[Sqrt[I*a]*x], -1] - EllipticF[I*ArcSinh[Sqrt[I*a]*x], -1]))/Sqrt[I*a]
)/3
```

3.290.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6290, 27, 847, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^4}\,dx$$

$$\Big\downarrow \text{6290}$$

$$\frac{1}{3}\int \frac{2a}{x^2\sqrt{a^2x^4+1}}dx-\frac{\operatorname{arcsinh}(ax^2)}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}a\int \frac{1}{x^2\sqrt{a^2x^4+1}}dx-\frac{\operatorname{arcsinh}(ax^2)}{3x^3}$$

$$\Big\downarrow \text{847}$$

$$\frac{2}{3}a\left(a^2\int \frac{x^2}{\sqrt{a^2x^4+1}}dx-\frac{\sqrt{a^2x^4+1}}{x}\right)-\frac{\operatorname{arcsinh}(ax^2)}{3x^3}$$

$\Big\downarrow$ 834

$$\frac{2}{3}a\left(a^2\left(\frac{\int \frac{1}{\sqrt{a^2x^4+1}}dx}{a}-\frac{\int \frac{1-ax^2}{\sqrt{a^2x^4+1}}dx}{a}\right)-\frac{\sqrt{a^2x^4+1}}{x}\right)-\frac{\operatorname{arcsinh}(ax^2)}{3x^3}$$

$\Big\downarrow$ 761

$$\frac{2}{3}a\left(a^2\left(\frac{(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{2a^{3/2}\sqrt{a^2x^4+1}}-\frac{\int \frac{1-ax^2}{\sqrt{a^2x^4+1}}dx}{a}\right)-\frac{\sqrt{a^2x^4+1}}{x}\right)-\frac{\operatorname{arcsinh}(ax^2)}{3x^3}$$

$\Big\downarrow$ 1510

$$\frac{2}{3}a\left(a^2\left(\frac{(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{a}x\right),\frac{1}{2}\right)}{2a^{3/2}\sqrt{a^2x^4+1}}-\frac{\frac{(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}E\left(2\arctan(\sqrt{a}x)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{a^2x^4+1}}-\frac{x\sqrt{a^2x^4+1}}{ax^2+1}}{a}\right)-\right.$$

$$\frac{\operatorname{arcsinh}(ax^2)}{3x^3}$$

input
```
Int[ArcSinh[a*x^2]/x^4,x]
```

output
```
-1/3*ArcSinh[a*x^2]/x^3 + (2*a*(-(Sqrt[1 + a^2*x^4]/x) + a^2*(-((-((x*Sqrt
[1 + a^2*x^4])/(1 + a*x^2)) + ((1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^
2]*EllipticE[2*ArcTan[Sqrt[a]*x], 1/2])/(Sqrt[a]*Sqrt[1 + a^2*x^4]))/a) +
((1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticF[2*ArcTan[Sqrt[a]*
x], 1/2])/(2*a^(3/2)*Sqrt[1 + a^2*x^4]))))/3
```

3.290. $\int \frac{\operatorname{arcsinh}(ax^2)}{x^4}\,dx$

3.290.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6290
```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x],
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u
, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[
u, x]
```

3.290. $\int \frac{\text{arcsinh}(ax^2)}{x^4} \, dx$

3.290.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.51

method	result	size
default	$-\frac{\operatorname{arcsinh}\left(a\,x^{2}\right)}{3x^{3}}+\frac{2a\left(-\frac{\sqrt{a^{2}x^{4}+1}}{x}+\frac{ia\sqrt{-ia\,x^{2}+1}\,\sqrt{ia\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{ia},i\right)-\operatorname{EllipticE}\left(x\sqrt{ia},i\right)\right)}{\sqrt{ia}\,\sqrt{a^{2}x^{4}+1}}\right)}{3}$	101
parts	$-\frac{\operatorname{arcsinh}\left(a\,x^{2}\right)}{3x^{3}}+\frac{2a\left(-\frac{\sqrt{a^{2}x^{4}+1}}{x}+\frac{ia\sqrt{-ia\,x^{2}+1}\,\sqrt{ia\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{ia},i\right)-\operatorname{EllipticE}\left(x\sqrt{ia},i\right)\right)}{\sqrt{ia}\,\sqrt{a^{2}x^{4}+1}}\right)}{3}$	101

input
```
int(arcsinh(a*x^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arcsinh(a*x^2)/x^3+2/3*a*(-(a^2*x^4+1)^(1/2)/x+I*a/(I*a)^(1/2)*(1-I*a
*x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*x^4+1)^(1/2)*(EllipticF(x*(I*a)^(1/2),I
)-EllipticE(x*(I*a)^(1/2),I)))
```

3.290.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^4}\,dx=\int \frac{\operatorname{arsinh}\left(ax^2\right)}{x^4}\,dx$$

input
```
integrate(arcsinh(a*x^2)/x^4,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x^2)/x^4, x)
```

3.290.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^4}\,dx=\int \frac{\operatorname{asinh}\left(ax^2\right)}{x^4}\,dx$$

input
```
integrate(asinh(a*x**2)/x**4,x)
```

output
```
Integral(asinh(a*x**2)/x**4, x)
```

3.290.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^4}\,dx = \int \frac{\operatorname{arsinh}(ax^2)}{x^4}\,dx$$

input
```
integrate(arcsinh(a*x^2)/x^4,x, algorithm="maxima")
```

output
```
-1/12*I*sqrt(2)*a^(3/2)*(log(1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(
a) + 1) - log(-1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a) + 1)) - 1/1
2*I*sqrt(2)*a^(3/2)*(log(1/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a) +
 1) - log(-1/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1)) + 1/12*sq
rt(2)*a^(3/2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1) - 1/12*sqrt(2)*a^(3/2)*lo
g(a*x^2 - sqrt(2)*sqrt(a)*x + 1) + 2*a*integrate(1/3/(a^3*x^8 + a*x^4 + (a
^2*x^6 + x^2)*sqrt(a^2*x^4 + 1)), x) - 1/3*log(a*x^2 + sqrt(a^2*x^4 + 1))/
x^3
```

3.290.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^4}\,dx = \int \frac{\operatorname{arsinh}(ax^2)}{x^4}\,dx$$

input
```
integrate(arcsinh(a*x^2)/x^4,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x^2)/x^4, x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax^2)}{x^4}\,dx = \int \frac{\operatorname{asinh}(a\,x^2)}{x^4}\,dx$$

input
```
int(asinh(a*x^2)/x^4,x)
```

output
```
int(asinh(a*x^2)/x^4, x)
```

3.291 $\int \frac{\operatorname{arcsinh}(ax^5)}{x} dx$

3.291.1 Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{\operatorname{arcsinh}(ax^5)}{x} dx = -\frac{1}{10}\operatorname{arcsinh}(ax^5)^2 + \frac{1}{5}\operatorname{arcsinh}(ax^5)\log\left(1 - e^{2\operatorname{arcsinh}(ax^5)}\right) + \frac{1}{10}\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}(ax^5)}\right)$$

output
```
-1/10*arcsinh(a*x^5)^2+1/5*arcsinh(a*x^5)*ln(1-(a*x^5+(a^2*x^10+1)^(1/2))^
2)+1/10*polylog(2,(a*x^5+(a^2*x^10+1)^(1/2))^2)
```

3.291.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax^5)}{x} dx = -\frac{1}{10}\operatorname{arcsinh}(ax^5)^2 + \frac{1}{5}\operatorname{arcsinh}(ax^5)\log\left(1 - e^{2\operatorname{arcsinh}(ax^5)}\right) + \frac{1}{10}\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}(ax^5)}\right)$$

input
```
Integrate[ArcSinh[a*x^5]/x,x]
```

output
```
-1/10*ArcSinh[a*x^5]^2 + (ArcSinh[a*x^5]*Log[1 - E^(2*ArcSinh[a*x^5])])/5
+ PolyLog[2, E^(2*ArcSinh[a*x^5])]/10
```

3.291.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6284, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax^5)}{x}\,dx$$

$$\Big\downarrow \text{ 6284}$$

$$\frac{1}{5}\int \frac{\sqrt{a^2x^{10}+1}\text{arcsinh}(ax^5)}{ax^5}d\text{arcsinh}(ax^5)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\int -i\text{arcsinh}(ax^5)\tan\left(i\text{arcsinh}(ax^5)+\frac{\pi}{2}\right)d\text{arcsinh}(ax^5)$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{1}{5}i\int \text{arcsinh}(ax^5)\tan\left(i\text{arcsinh}(ax^5)+\frac{\pi}{2}\right)d\text{arcsinh}(ax^5)$$

$$\Big\downarrow \text{ 4199}$$

$$-\frac{1}{5}i\left(2i\int -\frac{e^{2\text{arcsinh}(ax^5)}\text{arcsinh}(ax^5)}{1-e^{2\text{arcsinh}(ax^5)}}d\text{arcsinh}(ax^5)-\frac{1}{2}i\text{arcsinh}(ax^5)^2\right)$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{5}i\left(-2i\int \frac{e^{2\text{arcsinh}(ax^5)}\text{arcsinh}(ax^5)}{1-e^{2\text{arcsinh}(ax^5)}}d\text{arcsinh}(ax^5)-\frac{1}{2}i\text{arcsinh}(ax^5)^2\right)$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{1}{5}i\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2\text{arcsinh}(ax^5)}\right)d\text{arcsinh}(ax^5)-\frac{1}{2}\text{arcsinh}(ax^5)\log\left(1-e^{2\text{arcsinh}(ax^5)}\right)\right)-\frac{1}{2}i\text{arcsinh}(ax^5\right.$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{1}{5}i\left(-2i\left(\frac{1}{4}\int e^{-2\text{arcsinh}(ax^5)}\log\left(1-e^{2\text{arcsinh}(ax^5)}\right)de^{2\text{arcsinh}(ax^5)}-\frac{1}{2}\text{arcsinh}(ax^5)\log\left(1-e^{2\text{arcsinh}(ax^5)}\right)\right)-\frac{1}{2}\right.$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{1}{5}i\left(-2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax^5)}\right)-\frac{1}{2}\operatorname{arcsinh}(ax^5)\log\left(1-e^{2\operatorname{arcsinh}(ax^5)}\right)\right)-\frac{1}{2}i\operatorname{arcsinh}(ax^5)^2\right)$$

input
```
Int[ArcSinh[a*x^5]/x,x]
```

output
```
(-1/5*I)*((-1/2*I)*ArcSinh[a*x^5]^2 - (2*I)*(-1/2*(ArcSinh[a*x^5]*Log[1 -
E^(2*ArcSinh[a*x^5])]) - PolyLog[2, E^(2*ArcSinh[a*x^5])]/4))
```

3.291.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6284
```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Simp[1/p   Subst[Int[
x^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

3.291.4 Maple [F]

$$\int \frac{\operatorname{arcsinh}\left(a\,x^{5}\right)}{x}dx$$

input
```
int(arcsinh(a*x^5)/x,x)
```

output
```
int(arcsinh(a*x^5)/x,x)
```

3.291.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax^5)}{x}\,dx=\int \frac{\operatorname{arsinh}\left(ax^{5}\right)}{x}\,dx$$

input
```
integrate(arcsinh(a*x^5)/x,x, algorithm="fricas")
```

output
```
integral(arcsinh(a*x^5)/x, x)
```

3.291.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax^5)}{x}\,dx = \int \frac{\operatorname{asinh}(ax^5)}{x}\,dx$$

input
```
integrate(asinh(a*x**5)/x,x)
```

output
```
Integral(asinh(a*x**5)/x, x)
```

3.291.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax^5)}{x}\,dx = \int \frac{\operatorname{arsinh}(ax^5)}{x}\,dx$$

input
```
integrate(arcsinh(a*x^5)/x,x, algorithm="maxima")
```

output
```
integrate(arcsinh(a*x^5)/x, x)
```

3.291.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax^5)}{x}\,dx = \int \frac{\operatorname{arsinh}(ax^5)}{x}\,dx$$

input
```
integrate(arcsinh(a*x^5)/x,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x^5)/x, x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax^5)}{x}\,dx = \int \frac{\operatorname{asinh}(a\,x^5)}{x}\,dx$$

input
```
int(asinh(a*x^5)/x,x)
```

output
```
int(asinh(a*x^5)/x, x)
```

3.292 $\int x^2 \text{arcsinh}(\sqrt{x})\,dx$

3.292.1 Optimal result

Integrand size = 10, antiderivative size = 72

$$\int x^2 \text{arcsinh}(\sqrt{x})\,dx = -\frac{5}{48}\sqrt{x}\sqrt{1+x}+\frac{5}{72}x^{3/2}\sqrt{1+x}-\frac{1}{18}x^{5/2}\sqrt{1+x}+\frac{5\text{arcsinh}(\sqrt{x})}{48}+\frac{1}{3}x^3\text{arcsinh}(\sqrt{x})$$

output
```
5/48*arcsinh(x^(1/2))+1/3*x^3*arcsinh(x^(1/2))+5/72*x^(3/2)*(1+x)^(1/2)-1/
18*x^(5/2)*(1+x)^(1/2)-5/48*x^(1/2)*(1+x)^(1/2)
```

3.292.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

$$\int x^2 \text{arcsinh}(\sqrt{x})\,dx = \frac{1}{144}\left(\sqrt{x}\sqrt{1+x}(-15+10x-8x^2)+3(5+16x^3)\,\text{arcsinh}(\sqrt{x})\right)$$

input
```
Integrate[x^2*ArcSinh[Sqrt[x]],x]
```

output
```
(Sqrt[x]*Sqrt[1 + x]*(-15 + 10*x - 8*x^2) + 3*(5 + 16*x^3)*ArcSinh[Sqrt[x]
])/144
```

3.292.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6290, 27, 60, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arcsinh}\left(\sqrt{x}\right) \, dx$$

$$\Big\downarrow \ 6290$$

$$\frac{1}{3}x^3\text{arcsinh}\left(\sqrt{x}\right)-\frac{1}{3}\int \frac{x^{5/2}}{2\sqrt{x+1}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}x^3\text{arcsinh}\left(\sqrt{x}\right)-\frac{1}{6}\int \frac{x^{5/2}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \ 60$$

$$\frac{1}{6}\left(\frac{5}{6}\int \frac{x^{3/2}}{\sqrt{x+1}}dx-\frac{1}{3}x^{5/2}\sqrt{x+1}\right)+\frac{1}{3}x^3\text{arcsinh}\left(\sqrt{x}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{6}\left(\frac{5}{6}\left(\frac{1}{2}x^{3/2}\sqrt{x+1}-\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{x+1}}dx\right)-\frac{1}{3}x^{5/2}\sqrt{x+1}\right)+\frac{1}{3}x^3\text{arcsinh}\left(\sqrt{x}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{6}\left(\frac{5}{6}\left(\frac{1}{2}x^{3/2}\sqrt{x+1}-\frac{3}{4}\left(\sqrt{x}\sqrt{x+1}-\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx\right)\right)-\frac{1}{3}x^{5/2}\sqrt{x+1}\right)+\frac{1}{3}x^3\text{arcsinh}\left(\sqrt{x}\right)$$

$$\Big\downarrow \ 63$$

$$\frac{1}{6}\left(\frac{5}{6}\left(\frac{1}{2}x^{3/2}\sqrt{x+1}-\frac{3}{4}\left(\sqrt{x}\sqrt{x+1}-\int \frac{1}{\sqrt{x+1}}d\sqrt{x}\right)\right)-\frac{1}{3}x^{5/2}\sqrt{x+1}\right)+\frac{1}{3}x^3\text{arcsinh}\left(\sqrt{x}\right)$$

$$\Big\downarrow \ 222$$

$$\frac{1}{6}\left(\frac{5}{6}\left(\frac{1}{2}x^{3/2}\sqrt{x+1}-\frac{3}{4}\left(\sqrt{x}\sqrt{x+1}-\text{arcsinh}\left(\sqrt{x}\right)\right)\right)-\frac{1}{3}x^{5/2}\sqrt{x+1}\right)+\frac{1}{3}x^3\text{arcsinh}\left(\sqrt{x}\right)$$

input
```
Int[x^2*ArcSinh[Sqrt[x]],x]
```

output
```
(-1/3*(x^(5/2)*Sqrt[1 + x]) + (5*((x^(3/2)*Sqrt[1 + x])/2 - (3*(Sqrt[x]*Sq
rt[1 + x] - ArcSinh[Sqrt[x]]))/4))/6)/6 + (x^3*ArcSinh[Sqrt[x]])/3
```

3.292.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6290
```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x],
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u
, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[
u, x]
```

3.292.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{5\,\text{arcsinh}\left(\sqrt{x}\right)}{48}+\frac{x^{3}\,\text{arcsinh}\left(\sqrt{x}\right)}{3}+\frac{5x^{\frac{3}{2}}\sqrt{1+x}}{72}-\frac{x^{\frac{5}{2}}\sqrt{1+x}}{18}-\frac{5\sqrt{x}\,\sqrt{1+x}}{48}$	47
default	$\frac{5\,\text{arcsinh}\left(\sqrt{x}\right)}{48}+\frac{x^{3}\,\text{arcsinh}\left(\sqrt{x}\right)}{3}+\frac{5x^{\frac{3}{2}}\sqrt{1+x}}{72}-\frac{x^{\frac{5}{2}}\sqrt{1+x}}{18}-\frac{5\sqrt{x}\,\sqrt{1+x}}{48}$	47
parts	$\frac{x^{3}\,\text{arcsinh}\left(\sqrt{x}\right)}{3}-\frac{x^{\frac{5}{2}}\sqrt{1+x}}{18}+\frac{5x^{\frac{3}{2}}\sqrt{1+x}}{72}-\frac{5\sqrt{x}\,\sqrt{1+x}}{48}+\frac{5\sqrt{x(1+x)}\,\ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right)}{96\sqrt{x}\,\sqrt{1+x}}$	69

input
```
int(x^2*arcsinh(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
5/48*arcsinh(x^(1/2))+1/3*x^3*arcsinh(x^(1/2))+5/72*x^(3/2)*(1+x)^(1/2)-1/
18*x^(5/2)*(1+x)^(1/2)-5/48*x^(1/2)*(1+x)^(1/2)
```

3.292.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int x^2\text{arcsinh}\left(\sqrt{x}\right)\,dx=-\frac{1}{144}\left(8\,x^{2}-10\,x+15\right)\sqrt{x+1}\sqrt{x}+\frac{1}{48}\left(16\,x^{3}+5\right)\log\left(\sqrt{x+1}+\sqrt{x}\right)$$

input
```
integrate(x^2*arcsinh(x^(1/2)),x, algorithm="fricas")
```

output
```
-1/144*(8*x^2 - 10*x + 15)*sqrt(x + 1)*sqrt(x) + 1/48*(16*x^3 + 5)*log(sqr
t(x + 1) + sqrt(x))
```

3.292.6 Sympy [F]

$$\int x^2\text{arcsinh}\left(\sqrt{x}\right)\,dx=\int x^{2}\operatorname{asinh}\left(\sqrt{x}\right)\,dx$$

input
```
integrate(x**2*asinh(x**(1/2)),x)
```

output
```
Integral(x**2*asinh(sqrt(x)), x)
```

3.292.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

$$\int x^2 \operatorname{arcsinh}(\sqrt{x})\, dx = \frac{1}{3}\, x^3 \operatorname{arsinh}\left(\sqrt{x}\right) - \frac{1}{18}\, \sqrt{x+1} x^{\frac{5}{2}} + \frac{5}{72}\, \sqrt{x+1} x^{\frac{3}{2}} - \frac{5}{48}\, \sqrt{x+1}\sqrt{x} + \frac{5}{48}\, \operatorname{arsinh}\left(\sqrt{x}\right)$$

input
```
integrate(x^2*arcsinh(x^(1/2)),x, algorithm="maxima")
```

output
```
1/3*x^3*arcsinh(sqrt(x)) - 1/18*sqrt(x + 1)*x^(5/2) + 5/72*sqrt(x + 1)*x^(
3/2) - 5/48*sqrt(x + 1)*sqrt(x) + 5/48*arcsinh(sqrt(x))
```

3.292.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int x^2 \operatorname{arcsinh}(\sqrt{x})\, dx = \frac{1}{3}\, x^3 \log\left(\sqrt{x+1} + \sqrt{x}\right) - \frac{1}{144}\, (2\,(4\,x - 5)x + 15)\sqrt{x+1}\sqrt{x} - \frac{5}{48}\, \log\left(\sqrt{x+1} - \sqrt{x}\right)$$

input
```
integrate(x^2*arcsinh(x^(1/2)),x, algorithm="giac")
```

output
```
1/3*x^3*log(sqrt(x + 1) + sqrt(x)) - 1/144*(2*(4*x - 5)*x + 15)*sqrt(x + 1
)*sqrt(x) - 5/48*log(sqrt(x + 1) - sqrt(x))
```

3.292.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{arcsinh}(\sqrt{x})\, dx = \int x^2 \operatorname{asinh}(\sqrt{x})\, dx$$

input
```
int(x^2*asinh(x^(1/2)),x)
```

output
```
int(x^2*asinh(x^(1/2)), x)
```

3.293 $\int x\text{arcsinh}(\sqrt{x})\,dx$

3.293.1 Optimal result

Integrand size = 8, antiderivative size = 56

$$\int x\text{arcsinh}(\sqrt{x})\,dx=\frac{3}{16}\sqrt{x}\sqrt{1+x}-\frac{1}{8}x^{3/2}\sqrt{1+x}-\frac{3\text{arcsinh}(\sqrt{x})}{16}+\frac{1}{2}x^2\text{arcsinh}(\sqrt{x})$$

output
```
-3/16*arcsinh(x^(1/2))+1/2*x^2*arcsinh(x^(1/2))-1/8*x^(3/2)*(1+x)^(1/2)+3/
16*x^(1/2)*(1+x)^(1/2)
```

3.293.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int x\text{arcsinh}(\sqrt{x})\,dx=\frac{1}{16}\left((3-2x)\sqrt{x}\sqrt{1+x}+(-3+8x^2)\,\text{arcsinh}(\sqrt{x})\right)$$

input
```
Integrate[x*ArcSinh[Sqrt[x]],x]
```

output
```
((3 - 2*x)*Sqrt[x]*Sqrt[1 + x] + (-3 + 8*x^2)*ArcSinh[Sqrt[x]])/16
```

3.293.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6290, 27, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arcsinh}\left(\sqrt{x}\right) \, dx$$

$$\Big\downarrow \ 6290$$

$$\frac{1}{2}x^2\text{arcsinh}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{x^{3/2}}{2\sqrt{x+1}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}x^2\text{arcsinh}\left(\sqrt{x}\right)-\frac{1}{4}\int \frac{x^{3/2}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \ 60$$

$$\frac{1}{4}\left(\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{x+1}}dx-\frac{1}{2}x^{3/2}\sqrt{x+1}\right)+\frac{1}{2}x^2\text{arcsinh}\left(\sqrt{x}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\sqrt{x}\sqrt{x+1}-\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx\right)-\frac{1}{2}x^{3/2}\sqrt{x+1}\right)+\frac{1}{2}x^2\text{arcsinh}\left(\sqrt{x}\right)$$

$$\Big\downarrow \ 63$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\sqrt{x}\sqrt{x+1}-\int \frac{1}{\sqrt{x+1}}d\sqrt{x}\right)-\frac{1}{2}x^{3/2}\sqrt{x+1}\right)+\frac{1}{2}x^2\text{arcsinh}\left(\sqrt{x}\right)$$

$$\Big\downarrow \ 222$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\sqrt{x}\sqrt{x+1}-\text{arcsinh}\left(\sqrt{x}\right)\right)-\frac{1}{2}x^{3/2}\sqrt{x+1}\right)+\frac{1}{2}x^2\text{arcsinh}\left(\sqrt{x}\right)$$

input
```
Int[x*ArcSinh[Sqrt[x]],x]
```

output
```
(-1/2*(x^(3/2)*Sqrt[1 + x]) + (3*(Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]))
/4)/4 + (x^2*ArcSinh[Sqrt[x]])/2
```

3.293.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6290
```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x],
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u
, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[
u, x]
```

3.293.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$-\frac{3 \operatorname{arcsinh}\left(\sqrt{x}\right)}{16}+\frac{x^{2} \operatorname{arcsinh}\left(\sqrt{x}\right)}{2}-\frac{x^{\frac{3}{2}} \sqrt{1+x}}{8}+\frac{3 \sqrt{x}\, \sqrt{1+x}}{16}$	37
default	$-\frac{3 \operatorname{arcsinh}\left(\sqrt{x}\right)}{16}+\frac{x^{2} \operatorname{arcsinh}\left(\sqrt{x}\right)}{2}-\frac{x^{\frac{3}{2}} \sqrt{1+x}}{8}+\frac{3 \sqrt{x}\, \sqrt{1+x}}{16}$	37
parts	$\frac{x^{2} \operatorname{arcsinh}\left(\sqrt{x}\right)}{2}-\frac{x^{\frac{3}{2}} \sqrt{1+x}}{8}+\frac{3 \sqrt{x}\, \sqrt{1+x}}{16}-\frac{3 \sqrt{x(1+x)}\, \ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right)}{32 \sqrt{x}\, \sqrt{1+x}}$	59

input
```
int(x*arcsinh(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-3/16*arcsinh(x^(1/2))+1/2*x^2*arcsinh(x^(1/2))-1/8*x^(3/2)*(1+x)^(1/2)+3/
16*x^(1/2)*(1+x)^(1/2)
```

3.293.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

$$\int x\text{arcsinh}\left(\sqrt{x}\right)\,dx = -\frac{1}{16}\,(2\,x-3)\sqrt{x+1}\sqrt{x}+\frac{1}{16}\,\left(8\,x^2-3\right)\log\left(\sqrt{x+1}+\sqrt{x}\right)$$

input
```
integrate(x*arcsinh(x^(1/2)),x, algorithm="fricas")
```

output
```
-1/16*(2*x - 3)*sqrt(x + 1)*sqrt(x) + 1/16*(8*x^2 - 3)*log(sqrt(x + 1) + s
qrt(x))
```

3.293.6 Sympy [F]

$$\int x\text{arcsinh}\left(\sqrt{x}\right)\,dx = \int x\,\text{asinh}\left(\sqrt{x}\right)dx$$

input
```
integrate(x*asinh(x**(1/2)),x)
```

output
```
Integral(x*asinh(sqrt(x)), x)
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int x\text{arcsinh}\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x^2\,\text{arsinh}\left(\sqrt{x}\right)-\frac{1}{8}\,\sqrt{x+1}x^{\frac{3}{2}}+\frac{3}{16}\,\sqrt{x+1}\sqrt{x}-\frac{3}{16}\,\text{arsinh}\left(\sqrt{x}\right)$$

input
```
integrate(x*arcsinh(x^(1/2)),x, algorithm="maxima")
```

output
```
1/2*x^2*arcsinh(sqrt(x)) - 1/8*sqrt(x + 1)*x^(3/2) + 3/16*sqrt(x + 1)*sqrt
(x) - 3/16*arcsinh(sqrt(x))
```

3.293.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int x\text{arcsinh}\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x^2\log\left(\sqrt{x+1}+\sqrt{x}\right) - \frac{1}{16}\,\sqrt{x^2+x}(2\,x-3) + \frac{3}{32}\,\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)$$

input
```
integrate(x*arcsinh(x^(1/2)),x, algorithm="giac")
```

output
```
1/2*x^2*log(sqrt(x + 1) + sqrt(x)) - 1/16*sqrt(x^2 + x)*(2*x - 3) + 3/32*l
og(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

3.293.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arcsinh}\left(\sqrt{x}\right)\,dx = \int x\,\text{asinh}\left(\sqrt{x}\right)\,dx$$

input
```
int(x*asinh(x^(1/2)),x)
```

output
```
int(x*asinh(x^(1/2)), x)
```

3.294 $\int \text{arcsinh}(\sqrt{x})\,dx$

3.294.1 Optimal result

Integrand size = 6, antiderivative size = 35

$$\int \text{arcsinh}(\sqrt{x})\,dx = -\frac{1}{2}\sqrt{x}\sqrt{1+x} + \frac{\text{arcsinh}(\sqrt{x})}{2} + x\text{arcsinh}(\sqrt{x})$$

output
```
1/2*arcsinh(x^(1/2))+x*arcsinh(x^(1/2))-1/2*x^(1/2)*(1+x)^(1/2)
```

3.294.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \text{arcsinh}(\sqrt{x})\,dx = \frac{1}{2}\left(-\frac{x}{\sqrt{\frac{x}{1+x}}} + 2x\text{arcsinh}(\sqrt{x}) - \log\left(-\sqrt{x} + \sqrt{1+x}\right)\right)$$

input
```
Integrate[ArcSinh[Sqrt[x]],x]
```

output
```
(-(x/Sqrt[x/(1 + x)]) + 2*x*ArcSinh[Sqrt[x]] - Log[-Sqrt[x] + Sqrt[1 + x]]
)/2
```

3.294.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6289, 27, 2050, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{6289}$$

$$x\text{arcsinh}\left(\sqrt{x}\right)-\int \frac{1}{2}\sqrt{\frac{x}{x+1}}dx$$

$$\Big\downarrow \text{27}$$

$$x\text{arcsinh}\left(\sqrt{x}\right)-\frac{1}{2}\int \sqrt{\frac{x}{x+1}}dx$$

$$\Big\downarrow \text{2050}$$

$$x\text{arcsinh}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx-\sqrt{x}\sqrt{x+1}\right)+x\text{arcsinh}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{63}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt{x+1}}d\sqrt{x}-\sqrt{x}\sqrt{x+1}\right)+x\text{arcsinh}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{222}$$

$$x\text{arcsinh}\left(\sqrt{x}\right)+\frac{1}{2}\left(\text{arcsinh}\left(\sqrt{x}\right)-\sqrt{x}\sqrt{x+1}\right)$$

input
```
Int[ArcSinh[Sqrt[x]],x]
```

output
```
x*ArcSinh[Sqrt[x]] + (-(Sqrt[x]*Sqrt[1 + x]) + ArcSinh[Sqrt[x]])/2
```

3.294.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

rule 6289
```
Int[ArcSinh[u_], x_Symbol] :> Simp[x*ArcSinh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/Sqrt[1 + u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Funct
ionOfExponentialQ[u, x]
```

3.294.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{\operatorname{arcsinh}(\sqrt{x})}{2}+x\operatorname{arcsinh}\left(\sqrt{x}\right)-\frac{\sqrt{x}\,\sqrt{1+x}}{2}$	24
default	$\frac{\operatorname{arcsinh}(\sqrt{x})}{2}+x\operatorname{arcsinh}\left(\sqrt{x}\right)-\frac{\sqrt{x}\,\sqrt{1+x}}{2}$	24
parts	$x\operatorname{arcsinh}\left(\sqrt{x}\right)-\frac{\sqrt{x}\,\sqrt{1+x}}{2}+\frac{\sqrt{x(1+x)}\,\ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right)}{4\sqrt{x}\,\sqrt{1+x}}$	46

input
```
int(arcsinh(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(x^(1/2))+x*arcsinh(x^(1/2))-1/2*x^(1/2)*(1+x)^(1/2)
```

3.294.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \operatorname{arcsinh}(\sqrt{x})\,dx=\frac{1}{2}\,(2\,x+1)\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x+1}\sqrt{x}$$

input
```
integrate(arcsinh(x^(1/2)),x, algorithm="fricas")
```

output
```
1/2*(2*x + 1)*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x + 1)*sqrt(x)
```

3.294.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \operatorname{arcsinh}(\sqrt{x})\,dx=-\frac{\sqrt{x}\sqrt{x+1}}{2}+x\operatorname{asinh}\left(\sqrt{x}\right)+\frac{\operatorname{asinh}\left(\sqrt{x}\right)}{2}$$

input
```
integrate(asinh(x**(1/2)),x)
```

output
```
-sqrt(x)*sqrt(x + 1)/2 + x*asinh(sqrt(x)) + asinh(sqrt(x))/2
```

3.294.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \operatorname{arcsinh}\left(\sqrt{x}\right)\,dx = x\operatorname{arsinh}\left(\sqrt{x}\right) - \frac{1}{2}\sqrt{x+1}\sqrt{x} + \frac{1}{2}\operatorname{arsinh}\left(\sqrt{x}\right)$$

input
```
integrate(arcsinh(x^(1/2)),x, algorithm="maxima")
```

output
```
x*arcsinh(sqrt(x)) - 1/2*sqrt(x + 1)*sqrt(x) + 1/2*arcsinh(sqrt(x))
```

3.294.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \operatorname{arcsinh}\left(\sqrt{x}\right)\,dx = x\log\left(\sqrt{x+1}+\sqrt{x}\right) - \frac{1}{2}\sqrt{x^2+x} - \frac{1}{4}\log\left(\left|-2\,x+2\sqrt{x^2+x}-1\right|\right)$$

input
```
integrate(arcsinh(x^(1/2)),x, algorithm="giac")
```

output
```
x*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x^2 + x) - 1/4*log(abs(-2*x + 2*sq
rt(x^2 + x) - 1))
```

3.294.9 Mupad [B] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \operatorname{arcsinh}\left(\sqrt{x}\right)\,dx = \operatorname{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right) + x\operatorname{asinh}\left(\sqrt{x}\right) - \frac{\sqrt{x}\sqrt{x+1}}{2}$$

input
```
int(asinh(x^(1/2)),x)
```

output
```
atanh(x^(1/2)/((x + 1)^(1/2) - 1)) + x*asinh(x^(1/2)) - (x^(1/2)*(x + 1)^(
1/2))/2
```

3.295 $\int \frac{\text{arcsinh}(\sqrt{x})}{x} \, dx$

3.295.1 Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{\text{arcsinh}(\sqrt{x})}{x} \, dx = -\text{arcsinh}(\sqrt{x})^2 + 2\text{arcsinh}(\sqrt{x}) \log\left(1 - e^{2\text{arcsinh}(\sqrt{x})}\right) + \text{PolyLog}\left(2, e^{2\text{arcsinh}(\sqrt{x})}\right)$$

output
```
-arcsinh(x^(1/2))^2+2*arcsinh(x^(1/2))*ln(1-(x^(1/2)+(1+x)^(1/2))^2)+polyl
og(2,(x^(1/2)+(1+x)^(1/2))^2)
```

3.295.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(\sqrt{x})}{x} \, dx = -\text{arcsinh}(\sqrt{x})^2 + 2\text{arcsinh}(\sqrt{x}) \log\left(1 - e^{2\text{arcsinh}(\sqrt{x})}\right) + \text{PolyLog}\left(2, e^{2\text{arcsinh}(\sqrt{x})}\right)$$

input
```
Integrate[ArcSinh[Sqrt[x]]/x,x]
```

output
```
-ArcSinh[Sqrt[x]]^2 + 2*ArcSinh[Sqrt[x]]*Log[1 - E^(2*ArcSinh[Sqrt[x]])] +
 PolyLog[2, E^(2*ArcSinh[Sqrt[x]])]
```

3.295.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6284, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(\sqrt{x})}{x} \, dx$$

$$\Big\downarrow \text{ 6284}$$

$$2\int \frac{\sqrt{x+1}\text{arcsinh}(\sqrt{x})}{\sqrt{x}}d\text{arcsinh}(\sqrt{x})$$

$$\Big\downarrow \text{ 3042}$$

$$2\int -i\text{arcsinh}(\sqrt{x})\tan\left(i\text{arcsinh}(\sqrt{x})+\frac{\pi}{2}\right)d\text{arcsinh}(\sqrt{x})$$

$$\Big\downarrow \text{ 26}$$

$$-2i\int \text{arcsinh}(\sqrt{x})\tan\left(i\text{arcsinh}(\sqrt{x})+\frac{\pi}{2}\right)d\text{arcsinh}(\sqrt{x})$$

$$\Big\downarrow \text{ 4199}$$

$$-2i\left(2i\int -\frac{e^{2\text{arcsinh}(\sqrt{x})}\text{arcsinh}(\sqrt{x})}{1-e^{2\text{arcsinh}(\sqrt{x})}}d\text{arcsinh}(\sqrt{x})-\frac{1}{2}i\text{arcsinh}(\sqrt{x})^2\right)$$

$$\Big\downarrow \text{ 25}$$

$$-2i\left(-2i\int \frac{e^{2\text{arcsinh}(\sqrt{x})}\text{arcsinh}(\sqrt{x})}{1-e^{2\text{arcsinh}(\sqrt{x})}}d\text{arcsinh}(\sqrt{x})-\frac{1}{2}i\text{arcsinh}(\sqrt{x})^2\right)$$

$$\Big\downarrow \text{ 2620}$$

$$-2i\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2\text{arcsinh}(\sqrt{x})}\right)d\text{arcsinh}(\sqrt{x})-\frac{1}{2}\text{arcsinh}(\sqrt{x})\log\left(1-e^{2\text{arcsinh}(\sqrt{x})}\right)\right)-\frac{1}{2}i\text{arcsinh}(\sqrt{x})^2\right.$$

$$\Big\downarrow \text{ 2715}$$

$$-2i\left(-2i\left(\frac{1}{4}\int e^{-2\text{arcsinh}(\sqrt{x})}\log\left(1-e^{2\text{arcsinh}(\sqrt{x})}\right)de^{2\text{arcsinh}(\sqrt{x})}-\frac{1}{2}\text{arcsinh}(\sqrt{x})\log\left(1-e^{2\text{arcsinh}(\sqrt{x})}\right)\right)-\frac{1}{2}ia\right.$$

$$\Big\downarrow \text{ 2838}$$

$$-2i\left(-2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(\sqrt{x})}\right)-\frac{1}{2}\operatorname{arcsinh}(\sqrt{x})\log\left(1-e^{2\operatorname{arcsinh}(\sqrt{x})}\right)\right)-\frac{1}{2}i\operatorname{arcsinh}(\sqrt{x})^2\right)$$

input
```
Int[ArcSinh[Sqrt[x]]/x,x]
```

output
```
(-2*I)*((-1/2*I)*ArcSinh[Sqrt[x]]^2 - (2*I)*(-1/2*(ArcSinh[Sqrt[x]]*Log[1
- E^(2*ArcSinh[Sqrt[x]])]) - PolyLog[2, E^(2*ArcSinh[Sqrt[x]])]/4))
```

3.295.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.295. $\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x}\,dx$

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6284
```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Simp[1/p   Subst[Int[
x^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

3.295.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.70

method	result
derivativedivides	$-\operatorname{arcsinh}\left(\sqrt{x}\right)^{2}+2\operatorname{arcsinh}\left(\sqrt{x}\right)\ln\left(1+\sqrt{x}+\sqrt{1+x}\right)+2\operatorname{polylog}\left(2,-\sqrt{x}-\sqrt{1+\right.$
default	$-\operatorname{arcsinh}\left(\sqrt{x}\right)^{2}+2\operatorname{arcsinh}\left(\sqrt{x}\right)\ln\left(1+\sqrt{x}+\sqrt{1+x}\right)+2\operatorname{polylog}\left(2,-\sqrt{x}-\sqrt{1+\right.$

input
```
int(arcsinh(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
-arcsinh(x^(1/2))^2+2*arcsinh(x^(1/2))*ln(1+x^(1/2)+(1+x)^(1/2))+2*polylog
(2,-x^(1/2)-(1+x)^(1/2))+2*arcsinh(x^(1/2))*ln(1-x^(1/2)-(1+x)^(1/2))+2*po
lylog(2,x^(1/2)+(1+x)^(1/2))
```

3.295.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x}\,dx=\int \frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(arcsinh(x^(1/2))/x,x, algorithm="fricas")
```

output
```
integral(arcsinh(sqrt(x))/x, x)
```

3.295.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x}\,dx = \int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(asinh(x**(1/2))/x,x)
```

output
```
Integral(asinh(sqrt(x))/x, x)
```

3.295.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x}\,dx = \int \frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(arcsinh(x^(1/2))/x,x, algorithm="maxima")
```

output
```
integrate(arcsinh(sqrt(x))/x, x)
```

3.295.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x}\,dx = \int \frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(arcsinh(x^(1/2))/x,x, algorithm="giac")
```

output
```
integrate(arcsinh(sqrt(x))/x, x)
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
int(asinh(x^(1/2))/x,x)
```

output
```
int(asinh(x^(1/2))/x, x)
```

3.296 $\int \frac{\text{arcsinh}(\sqrt{x})}{x^2} \, dx$

3.296.1 Optimal result

Integrand size = 10, antiderivative size = 26

$$\int \frac{\text{arcsinh}(\sqrt{x})}{x^2} \, dx = -\frac{\sqrt{1+x}}{\sqrt{x}} - \frac{\text{arcsinh}(\sqrt{x})}{x}$$

output
```
-arcsinh(x^(1/2))/x-(1+x)^(1/2)/x^(1/2)
```

3.296.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}(\sqrt{x})}{x^2} \, dx = -\frac{\sqrt{1+x}}{\sqrt{x}} - \frac{\text{arcsinh}(\sqrt{x})}{x}$$

input
```
Integrate[ArcSinh[Sqrt[x]]/x^2,x]
```

output
```
-(Sqrt[1 + x]/Sqrt[x]) - ArcSinh[Sqrt[x]]/x
```

3.296.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6290, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^2}dx$$

$$\Big\downarrow \text{ 6290}$$

$$\int \frac{1}{2x^{3/2}\sqrt{x+1}}dx-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{1}{x^{3/2}\sqrt{x+1}}dx-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x}-\frac{\sqrt{x+1}}{\sqrt{x}}$$

input
```
Int[ArcSinh[Sqrt[x]]/x^2,x]
```

output
```
-(Sqrt[1 + x]/Sqrt[x]) - ArcSinh[Sqrt[x]]/x
```

3.296.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 6290
```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x],
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u
, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[
u, x]
```

3.296.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x}-\frac{\sqrt{1+x}}{\sqrt{x}}$	21
default	$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x}-\frac{\sqrt{1+x}}{\sqrt{x}}$	21
parts	$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x}-\frac{\sqrt{1+x}}{\sqrt{x}}$	21

input
```
int(arcsinh(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arcsinh(x^(1/2))/x-(1+x)^(1/2)/x^(1/2)
```

3.296.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^2}\,dx=-\frac{\sqrt{x+1}\sqrt{x}+\log\left(\sqrt{x+1}+\sqrt{x}\right)}{x}$$

input
```
integrate(arcsinh(x^(1/2))/x^2,x, algorithm="fricas")
```

output
```
-(sqrt(x + 1)*sqrt(x) + log(sqrt(x + 1) + sqrt(x)))/x
```

3.296.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^2}\,dx = \int \frac{\operatorname{asinh}(\sqrt{x})}{x^2}\,dx$$

input
```
integrate(asinh(x**(1/2))/x**2,x)
```

output
```
Integral(asinh(sqrt(x))/x**2, x)
```

3.296.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^2}\,dx = -\frac{\sqrt{x+1}}{\sqrt{x}} - \frac{\operatorname{arsinh}(\sqrt{x})}{x}$$

input
```
integrate(arcsinh(x^(1/2))/x^2,x, algorithm="maxima")
```

output
```
-sqrt(x + 1)/sqrt(x) - arcsinh(sqrt(x))/x
```

3.296.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^2}\,dx = -\frac{\log(\sqrt{x+1}+\sqrt{x})}{x} + \frac{2}{(\sqrt{x+1}-\sqrt{x})^2-1}$$

input
```
integrate(arcsinh(x^(1/2))/x^2,x, algorithm="giac")
```

output
```
-log(sqrt(x + 1) + sqrt(x))/x + 2/((sqrt(x + 1) - sqrt(x))^2 - 1)
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^2}\,dx = \int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^2}\,dx$$

input
```
int(asinh(x^(1/2))/x^2,x)
```

output
```
int(asinh(x^(1/2))/x^2, x)
```

3.297 $\int \frac{\text{arcsinh}(\sqrt{x})}{x^3} dx$

3.297.1 Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{\text{arcsinh}(\sqrt{x})}{x^3} dx = -\frac{\sqrt{1+x}}{6x^{3/2}} + \frac{\sqrt{1+x}}{3\sqrt{x}} - \frac{\text{arcsinh}(\sqrt{x})}{2x^2}$$

output
```
-1/2*arcsinh(x^(1/2))/x^2-1/6*(1+x)^(1/2)/x^(3/2)+1/3*(1+x)^(1/2)/x^(1/2)
```

3.297.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{\text{arcsinh}(\sqrt{x})}{x^3} dx = \frac{\sqrt{x}\sqrt{1+x}(-1+2x) - 3\text{arcsinh}(\sqrt{x})}{6x^2}$$

input
```
Integrate[ArcSinh[Sqrt[x]]/x^3,x]
```

output
```
(Sqrt[x]*Sqrt[1 + x]*(-1 + 2*x) - 3*ArcSinh[Sqrt[x]])/(6*x^2)
```

3.297.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6290, 27, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^3}\,dx$$

$$\Big\downarrow \text{ 6290}$$

$$\frac{1}{2}\int \frac{1}{2x^{5/2}\sqrt{x+1}}dx-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\int \frac{1}{x^{5/2}\sqrt{x+1}}dx-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{1}{4}\left(-\frac{2}{3}\int \frac{1}{x^{3/2}\sqrt{x+1}}dx-\frac{2\sqrt{x+1}}{3x^{3/2}}\right)-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{1}{4}\left(\frac{4\sqrt{x+1}}{3\sqrt{x}}-\frac{2\sqrt{x+1}}{3x^{3/2}}\right)-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2x^2}$$

input
```
Int[ArcSinh[Sqrt[x]]/x^3,x]
```

output
```
((-2*Sqrt[1 + x])/(3*x^(3/2)) + (4*Sqrt[1 + x])/(3*Sqrt[x]))/4 - ArcSinh[S
qrt[x]]/(2*x^2)
```

3.297.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 6290
```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x],
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u
, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[
u, x]
```

3.297.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2x^{2}}-\frac{\sqrt{1+x}}{6x^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{3\sqrt{x}}$	31
default	$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2x^{2}}-\frac{\sqrt{1+x}}{6x^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{3\sqrt{x}}$	31
parts	$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2x^{2}}-\frac{\sqrt{1+x}}{6x^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{3\sqrt{x}}$	31

input
```
int(arcsinh(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arcsinh(x^(1/2))/x^2-1/6*(1+x)^(1/2)/x^(3/2)+1/3*(1+x)^(1/2)/x^(1/2)
```

3.297.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{\text{arcsinh}(\sqrt{x})}{x^3}\,dx = \frac{(2\,x-1)\sqrt{x+1}\sqrt{x} - 3\,\log\left(\sqrt{x+1}+\sqrt{x}\right)}{6\,x^2}$$

input
```
integrate(arcsinh(x^(1/2))/x^3,x, algorithm="fricas")
```

output
```
1/6*((2*x - 1)*sqrt(x + 1)*sqrt(x) - 3*log(sqrt(x + 1) + sqrt(x)))/x^2
```

3.297.6 Sympy [F]

$$\int \frac{\text{arcsinh}(\sqrt{x})}{x^3}\,dx = \int \frac{\text{asinh}\left(\sqrt{x}\right)}{x^3}\,dx$$

input
```
integrate(asinh(x**(1/2))/x**3,x)
```

output
```
Integral(asinh(sqrt(x))/x**3, x)
```

3.297.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

$$\int \frac{\text{arcsinh}(\sqrt{x})}{x^3}\,dx = \frac{\sqrt{x+1}}{3\,\sqrt{x}} - \frac{\sqrt{x+1}}{6\,x^{\frac{3}{2}}} - \frac{\text{arsinh}\left(\sqrt{x}\right)}{2\,x^2}$$

input
```
integrate(arcsinh(x^(1/2))/x^3,x, algorithm="maxima")
```

output
```
1/3*sqrt(x + 1)/sqrt(x) - 1/6*sqrt(x + 1)/x^(3/2) - 1/2*arcsinh(sqrt(x))/x
^2
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^3}\,dx = -\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{2\,x^2} + \frac{2\left(3\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right)}{3\left(\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right)^3}$$

input
```
integrate(arcsinh(x^(1/2))/x^3,x, algorithm="giac")
```

output
```
-1/2*log(sqrt(x + 1) + sqrt(x))/x^2 + 2/3*(3*(sqrt(x + 1) - sqrt(x))^2 - 1
)/((sqrt(x + 1) - sqrt(x))^2 - 1)^3
```

3.297.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^3}\,dx = \int \frac{\operatorname{asinh}(\sqrt{x})}{x^3}\,dx$$

input
```
int(asinh(x^(1/2))/x^3,x)
```

output
```
int(asinh(x^(1/2))/x^3, x)
```

3.298 $\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^4} dx$

3.298.1 Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^4} dx = -\frac{\sqrt{1+x}}{15x^{5/2}} + \frac{4\sqrt{1+x}}{45x^{3/2}} - \frac{8\sqrt{1+x}}{45\sqrt{x}} - \frac{\operatorname{arcsinh}(\sqrt{x})}{3x^3}$$

output
```
-1/3*arcsinh(x^(1/2))/x^3-1/15*(1+x)^(1/2)/x^(5/2)+4/45*(1+x)^(1/2)/x^(3/2
)-8/45*(1+x)^(1/2)/x^(1/2)
```

3.298.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.63

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^4} dx = \frac{\sqrt{x}\sqrt{1+x}(-3+4x-8x^2) - 15\operatorname{arcsinh}(\sqrt{x})}{45x^3}$$

input
```
Integrate[ArcSinh[Sqrt[x]]/x^4,x]
```

output
```
(Sqrt[x]*Sqrt[1 + x]*(-3 + 4*x - 8*x^2) - 15*ArcSinh[Sqrt[x]])/(45*x^3)
```

3.298.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6290, 27, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{x^4}\,dx$$

$$\Big\downarrow \ 6290$$

$$\frac{1}{3}\int \frac{1}{2x^{7/2}\sqrt{x+1}}dx-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{6}\int \frac{1}{x^{7/2}\sqrt{x+1}}dx-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \ 55$$

$$\frac{1}{6}\left(-\frac{4}{5}\int \frac{1}{x^{5/2}\sqrt{x+1}}dx-\frac{2\sqrt{x+1}}{5x^{5/2}}\right)-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \ 55$$

$$\frac{1}{6}\left(-\frac{4}{5}\left(-\frac{2}{3}\int \frac{1}{x^{3/2}\sqrt{x+1}}dx-\frac{2\sqrt{x+1}}{3x^{3/2}}\right)-\frac{2\sqrt{x+1}}{5x^{5/2}}\right)-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \ 48$$

$$\frac{1}{6}\left(-\frac{4}{5}\left(\frac{4\sqrt{x+1}}{3\sqrt{x}}-\frac{2\sqrt{x+1}}{3x^{3/2}}\right)-\frac{2\sqrt{x+1}}{5x^{5/2}}\right)-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{3x^3}$$

input
```
Int[ArcSinh[Sqrt[x]]/x^4,x]
```

output
```
((-2*Sqrt[1 + x])/(5*x^(5/2)) - (4*((-2*Sqrt[1 + x])/(3*x^(3/2)) + (4*Sqrt
[1 + x])/(3*Sqrt[x])))/5)/6 - ArcSinh[Sqrt[x]]/(3*x^3)
```

3.298.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 6290
```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x],
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u
, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[
u, x]
```

3.298.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{3x^{3}}-\frac{\sqrt{1+x}}{15x^{\frac{5}{2}}}+\frac{4\sqrt{1+x}}{45x^{\frac{3}{2}}}-\frac{8\sqrt{1+x}}{45\sqrt{x}}$	41
default	$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{3x^{3}}-\frac{\sqrt{1+x}}{15x^{\frac{5}{2}}}+\frac{4\sqrt{1+x}}{45x^{\frac{3}{2}}}-\frac{8\sqrt{1+x}}{45\sqrt{x}}$	41
parts	$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{3x^{3}}-\frac{\sqrt{1+x}}{15x^{\frac{5}{2}}}+\frac{4\sqrt{1+x}}{45x^{\frac{3}{2}}}-\frac{8\sqrt{1+x}}{45\sqrt{x}}$	41

input
```
int(arcsinh(x^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arcsinh(x^(1/2))/x^3-1/15*(1+x)^(1/2)/x^(5/2)+4/45*(1+x)^(1/2)/x^(3/2
)-8/45*(1+x)^(1/2)/x^(1/2)
```

3.298.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^4}\,dx = -\frac{(8\,x^2-4\,x+3)\sqrt{x+1}\sqrt{x}+15\,\log\left(\sqrt{x+1}+\sqrt{x}\right)}{45\,x^3}$$

input
```
integrate(arcsinh(x^(1/2))/x^4,x, algorithm="fricas")
```

output
```
-1/45*((8*x^2 - 4*x + 3)*sqrt(x + 1)*sqrt(x) + 15*log(sqrt(x + 1) + sqrt(x
)))/x^3
```

3.298.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^4}\,dx = \int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^4}\,dx$$

input
```
integrate(asinh(x**(1/2))/x**4,x)
```

output
```
Integral(asinh(sqrt(x))/x**4, x)
```

3.298.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^4}\,dx = -\frac{8\,\sqrt{x+1}}{45\,\sqrt{x}}+\frac{4\,\sqrt{x+1}}{45\,x^{\frac{3}{2}}}-\frac{\sqrt{x+1}}{15\,x^{\frac{5}{2}}}-\frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{3\,x^3}$$

input
```
integrate(arcsinh(x^(1/2))/x^4,x, algorithm="maxima")
```

output
```
-8/45*sqrt(x + 1)/sqrt(x) + 4/45*sqrt(x + 1)/x^(3/2) - 1/15*sqrt(x + 1)/x^
(5/2) - 1/3*arcsinh(sqrt(x))/x^3
```

3.298.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^4}\,dx = -\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{3\,x^3} + \frac{16\left(10\left(\sqrt{x+1}-\sqrt{x}\right)^4 - 5\left(\sqrt{x+1}-\sqrt{x}\right)^2 + 1\right)}{45\left(\left(\sqrt{x+1}-\sqrt{x}\right)^2 - 1\right)^5}$$

input
```
integrate(arcsinh(x^(1/2))/x^4,x, algorithm="giac")
```

output
```
-1/3*log(sqrt(x + 1) + sqrt(x))/x^3 + 16/45*(10*(sqrt(x + 1) - sqrt(x))^4
- 5*(sqrt(x + 1) - sqrt(x))^2 + 1)/((sqrt(x + 1) - sqrt(x))^2 - 1)^5
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^4}\,dx = \int \frac{\operatorname{asinh}(\sqrt{x})}{x^4}\,dx$$

input
```
int(asinh(x^(1/2))/x^4,x)
```

output
```
int(asinh(x^(1/2))/x^4, x)
```

3.299 $\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^5} dx$

3.299.1 Optimal result

Integrand size = 10, antiderivative size = 78

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^5} dx = -\frac{\sqrt{1+x}}{28x^{7/2}} + \frac{3\sqrt{1+x}}{70x^{5/2}} - \frac{2\sqrt{1+x}}{35x^{3/2}} + \frac{4\sqrt{1+x}}{35\sqrt{x}} - \frac{\operatorname{arcsinh}(\sqrt{x})}{4x^4}$$

output
```
-1/4*arcsinh(x^(1/2))/x^4-1/28*(1+x)^(1/2)/x^(7/2)+3/70*(1+x)^(1/2)/x^(5/2
)-2/35*(1+x)^(1/2)/x^(3/2)+4/35*(1+x)^(1/2)/x^(1/2)
```

3.299.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.56

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^5} dx = \frac{\sqrt{x}\sqrt{1+x}(-5+6x-8x^2+16x^3) - 35\operatorname{arcsinh}(\sqrt{x})}{140x^4}$$

input
```
Integrate[ArcSinh[Sqrt[x]]/x^5,x]
```

output
```
(Sqrt[x]*Sqrt[1 + x]*(-5 + 6*x - 8*x^2 + 16*x^3) - 35*ArcSinh[Sqrt[x]])/(1
40*x^4)
```

3.299.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6290, 27, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}\left(\sqrt{x}\right)}{x^5} \, dx$$

$$\Big\downarrow \text{ 6290}$$

$$\frac{1}{4} \int \frac{1}{2x^{9/2}\sqrt{x+1}}dx - \frac{\text{arcsinh}\left(\sqrt{x}\right)}{4x^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8} \int \frac{1}{x^{9/2}\sqrt{x+1}}dx - \frac{\text{arcsinh}\left(\sqrt{x}\right)}{4x^4}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{1}{8}\left(-\frac{6}{7} \int \frac{1}{x^{7/2}\sqrt{x+1}}dx - \frac{2\sqrt{x+1}}{7x^{7/2}}\right) - \frac{\text{arcsinh}\left(\sqrt{x}\right)}{4x^4}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{1}{8}\left(-\frac{6}{7}\left(-\frac{4}{5} \int \frac{1}{x^{5/2}\sqrt{x+1}}dx - \frac{2\sqrt{x+1}}{5x^{5/2}}\right) - \frac{2\sqrt{x+1}}{7x^{7/2}}\right) - \frac{\text{arcsinh}\left(\sqrt{x}\right)}{4x^4}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{1}{8}\left(-\frac{6}{7}\left(-\frac{4}{5}\left(-\frac{2}{3} \int \frac{1}{x^{3/2}\sqrt{x+1}}dx - \frac{2\sqrt{x+1}}{3x^{3/2}}\right) - \frac{2\sqrt{x+1}}{5x^{5/2}}\right) - \frac{2\sqrt{x+1}}{7x^{7/2}}\right) - \frac{\text{arcsinh}\left(\sqrt{x}\right)}{4x^4}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{1}{8}\left(-\frac{6}{7}\left(-\frac{4}{5}\left(\frac{4\sqrt{x+1}}{3\sqrt{x}} - \frac{2\sqrt{x+1}}{3x^{3/2}}\right) - \frac{2\sqrt{x+1}}{5x^{5/2}}\right) - \frac{2\sqrt{x+1}}{7x^{7/2}}\right) - \frac{\text{arcsinh}\left(\sqrt{x}\right)}{4x^4}$$

input
```
Int[ArcSinh[Sqrt[x]]/x^5,x]
```

output
```
((-2*Sqrt[1 + x])/(7*x^(7/2)) - (6*((-2*Sqrt[1 + x])/(5*x^(5/2)) - (4*((-2
*Sqrt[1 + x])/(3*x^(3/2)) + (4*Sqrt[1 + x])/(3*Sqrt[x])))/5))/7)/8 - ArcSi
nh[Sqrt[x]]/(4*x^4)
```

3.299.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 6290
```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x],
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u
, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[
u, x]
```

3.299.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{4x^4}-\frac{\sqrt{1+x}}{28x^{\frac{7}{2}}}+\frac{3\sqrt{1+x}}{70x^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{35x^{\frac{3}{2}}}+\frac{4\sqrt{1+x}}{35\sqrt{x}}$	51
default	$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{4x^4}-\frac{\sqrt{1+x}}{28x^{\frac{7}{2}}}+\frac{3\sqrt{1+x}}{70x^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{35x^{\frac{3}{2}}}+\frac{4\sqrt{1+x}}{35\sqrt{x}}$	51
parts	$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{4x^4}-\frac{\sqrt{1+x}}{28x^{\frac{7}{2}}}+\frac{3\sqrt{1+x}}{70x^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{35x^{\frac{3}{2}}}+\frac{4\sqrt{1+x}}{35\sqrt{x}}$	51

input
```
int(arcsinh(x^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arcsinh(x^(1/2))/x^4-1/28*(1+x)^(1/2)/x^(7/2)+3/70*(1+x)^(1/2)/x^(5/2
)-2/35*(1+x)^(1/2)/x^(3/2)+4/35*(1+x)^(1/2)/x^(1/2)
```

3.299.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.54

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^5}\,dx = \frac{(16\,x^3-8\,x^2+6\,x-5)\sqrt{x+1}\sqrt{x}-35\,\log\left(\sqrt{x+1}+\sqrt{x}\right)}{140\,x^4}$$

input
```
integrate(arcsinh(x^(1/2))/x^5,x, algorithm="fricas")
```

output
```
1/140*((16*x^3 - 8*x^2 + 6*x - 5)*sqrt(x + 1)*sqrt(x) - 35*log(sqrt(x + 1)
 + sqrt(x)))/x^4
```

3.299.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^5}\,dx = \int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^5}\,dx$$

input
```
integrate(asinh(x**(1/2))/x**5,x)
```

output
```
Integral(asinh(sqrt(x))/x**5, x)
```

3.299.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^5}\,dx = \frac{4\sqrt{x+1}}{35\sqrt{x}} - \frac{2\sqrt{x+1}}{35\,x^{\frac{3}{2}}} + \frac{3\sqrt{x+1}}{70\,x^{\frac{5}{2}}} - \frac{\sqrt{x+1}}{28\,x^{\frac{7}{2}}} - \frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{4\,x^4}$$

input
```
integrate(arcsinh(x^(1/2))/x^5,x, algorithm="maxima")
```

output
```
4/35*sqrt(x + 1)/sqrt(x) - 2/35*sqrt(x + 1)/x^(3/2) + 3/70*sqrt(x + 1)/x^(
5/2) - 1/28*sqrt(x + 1)/x^(7/2) - 1/4*arcsinh(sqrt(x))/x^4
```

3.299.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^5}\,dx = -\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{4\,x^4} + \frac{8\left(35\left(\sqrt{x+1}-\sqrt{x}\right)^6-21\left(\sqrt{x+1}-\sqrt{x}\right)^4+7\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right)}{35\left(\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right)^7}$$

input
```
integrate(arcsinh(x^(1/2))/x^5,x, algorithm="giac")
```

output
```
-1/4*log(sqrt(x + 1) + sqrt(x))/x^4 + 8/35*(35*(sqrt(x + 1) - sqrt(x))^6 -
 21*(sqrt(x + 1) - sqrt(x))^4 + 7*(sqrt(x + 1) - sqrt(x))^2 - 1)/((sqrt(x
+ 1) - sqrt(x))^2 - 1)^7
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(\sqrt{x})}{x^5}\,dx = \int \frac{\operatorname{asinh}(\sqrt{x})}{x^5}\,dx$$

input
```
int(asinh(x^(1/2))/x^5,x)
```

output
```
int(asinh(x^(1/2))/x^5, x)
```

3.300 $\int x^2 \text{arcsinh}\left(\frac{a}{x}\right) dx$

3.300.1 Optimal result

Integrand size = 10, antiderivative size = 56

$$\int x^2 \text{arcsinh}\left(\frac{a}{x}\right) dx = \frac{1}{6}a\sqrt{1+\frac{a^2}{x^2}}x^2 + \frac{1}{3}x^3\text{csch}^{-1}\left(\frac{x}{a}\right) - \frac{1}{6}a^3\text{arctanh}\left(\sqrt{1+\frac{a^2}{x^2}}\right)$$

output
```
1/3*x^3*arccsch(x/a)-1/6*a^3*arctanh((a^2/x^2+1)^(1/2))+1/6*a*x^2*(a^2/x^2
+1)^(1/2)
```

3.300.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int x^2 \text{arcsinh}\left(\frac{a}{x}\right) dx = \frac{1}{6}\left(a\sqrt{1+\frac{a^2}{x^2}}x^2 + 2x^3\text{arcsinh}\left(\frac{a}{x}\right) - a^3\log\left(\left(1+\sqrt{1+\frac{a^2}{x^2}}\right)x\right)\right)$$

input
```
Integrate[x^2*ArcSinh[a/x],x]
```

output
```
(a*Sqrt[1 + a^2/x^2]*x^2 + 2*x^3*ArcSinh[a/x] - a^3*Log[(1 + Sqrt[1 + a^2/
x^2])*x])/6
```

3.300.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6285, 6838, 798, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arcsinh}\left(\frac{a}{x}\right) \, dx$$

$$\Big\downarrow \text{6285}$$

$$\int x^2 \text{csch}^{-1}\left(\frac{x}{a}\right) dx$$

$$\Big\downarrow \text{6838}$$

$$\frac{1}{3} a \int \frac{x}{\sqrt{\frac{a^2}{x^2}+1}} dx + \frac{1}{3} x^3 \text{csch}^{-1}\left(\frac{x}{a}\right)$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} x^3 \text{csch}^{-1}\left(\frac{x}{a}\right) - \frac{1}{6} a \int \frac{x^4}{\sqrt{\frac{a^2}{x^2}+1}} d\frac{1}{x^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3} x^3 \text{csch}^{-1}\left(\frac{x}{a}\right) - \frac{1}{6} a \left(-\frac{1}{2} a^2 \int \frac{x^2}{\sqrt{\frac{a^2}{x^2}+1}} d\frac{1}{x^2} - x^2 \sqrt{\frac{a^2}{x^2}+1}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3} x^3 \text{csch}^{-1}\left(\frac{x}{a}\right) - \frac{1}{6} a \left(x^2 \left(-\sqrt{\frac{a^2}{x^2}+1}\right) - \int \frac{1}{\frac{1}{a^2 x^4} - \frac{1}{a^2}} d\sqrt{\frac{a^2}{x^2}+1}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3} x^3 \text{csch}^{-1}\left(\frac{x}{a}\right) - \frac{1}{6} a \left(a^2 \text{arctanh}\left(\sqrt{\frac{a^2}{x^2}+1}\right) - x^2 \sqrt{\frac{a^2}{x^2}+1}\right)$$

input
```
Int[x^2*ArcSinh[a/x],x]
```

output
```
(x^3*ArcCsch[x/a])/3 - (a*(-(Sqrt[1 + a^2/x^2]*x^2) + a^2*ArcTanh[Sqrt[1 +
 a^2/x^2]]))/6
```

3.300.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6285
```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.300.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-a^{3}\left(-\frac{x^{3}\operatorname{arcsinh}\left(\frac{a}{x}\right)}{3a^{3}}-\frac{x^{2}\sqrt{\frac{a^{2}}{x^{2}}+1}}{6a^{2}}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{\frac{a^{2}}{x^{2}}+1}}\right)}{6}\right)$	54
default	$-a^{3}\left(-\frac{x^{3}\operatorname{arcsinh}\left(\frac{a}{x}\right)}{3a^{3}}-\frac{x^{2}\sqrt{\frac{a^{2}}{x^{2}}+1}}{6a^{2}}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{\frac{a^{2}}{x^{2}}+1}}\right)}{6}\right)$	54
parts	$\frac{x^{3}\operatorname{arcsinh}\left(\frac{a}{x}\right)}{3}+\frac{a\sqrt{a^{2}+x^{2}}\,\left(-a^{2}\ln\left(x+\sqrt{a^{2}+x^{2}}\right)+x\sqrt{a^{2}+x^{2}}\right)}{6\sqrt{\frac{a^{2}+x^{2}}{x^{2}}}\,x}$	70

input
```
int(x^2*arcsinh(a/x),x,method=_RETURNVERBOSE)
```

output
```
-a^3*(-1/3/a^3*x^3*arcsinh(a/x)-1/6/a^2*x^2*(a^2/x^2+1)^(1/2)+1/6*arctanh(
1/(a^2/x^2+1)^(1/2)))
```

3.300.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(46) = 92$.

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.18

$$\int x^2\operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx=\frac{1}{6}\,a^3\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}-x\right)+\frac{1}{6}\,ax^2\sqrt{\frac{a^2+x^2}{x^2}}+\frac{1}{3}\left(x^3-1\right)\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)+\frac{1}{3}\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}+a-x\right)-\frac{1}{3}\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}-a-x\right)$$

input
```
integrate(x^2*arcsinh(a/x),x, algorithm="fricas")
```

output
```
1/6*a^3*log(x*sqrt((a^2 + x^2)/x^2) - x) + 1/6*a*x^2*sqrt((a^2 + x^2)/x^2)
 + 1/3*(x^3 - 1)*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) + 1/3*log(x*sqrt((a^
2 + x^2)/x^2) + a - x) - 1/3*log(x*sqrt((a^2 + x^2)/x^2) - a - x)
```

3.300.6 Sympy [F]

$$\int x^2\text{arcsinh}\left(\frac{a}{x}\right)\,dx = \int x^2\operatorname{asinh}\left(\frac{a}{x}\right)dx$$

input
```
integrate(x**2*asinh(a/x),x)
```

output
```
Integral(x**2*asinh(a/x), x)
```

3.300.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

$$\begin{aligned}\int x^2\text{arcsinh}\left(\frac{a}{x}\right)\,dx &= \frac{1}{3}\,x^3\,\text{arsinh}\left(\frac{a}{x}\right) \\ &\quad - \frac{1}{12}\left(a^2\log\left(\sqrt{\frac{a^2}{x^2}+1}+1\right) - a^2\log\left(\sqrt{\frac{a^2}{x^2}+1}-1\right) - 2\,x^2\sqrt{\frac{a^2}{x^2}+1}\right)a\end{aligned}$$

input
```
integrate(x^2*arcsinh(a/x),x, algorithm="maxima")
```

output
```
1/3*x^3*arcsinh(a/x) - 1/12*(a^2*log(sqrt(a^2/x^2 + 1) + 1) - a^2*log(sqrt
(a^2/x^2 + 1) - 1) - 2*x^2*sqrt(a^2/x^2 + 1))*a
```

3.300.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.32

$$\int x^2 \text{arcsinh}\left(\frac{a}{x}\right) dx = -\frac{1}{6} a^3 \log(|a|) \operatorname{sgn}(x) + \frac{1}{3} x^3 \log\left(\sqrt{\frac{a^2}{x^2}+1} + \frac{a}{x}\right) + \frac{a^3 \log\left(-x+\sqrt{a^2+x^2}\right)}{6 \operatorname{sgn}(x)} + \frac{\sqrt{a^2+x^2} a x}{6 \operatorname{sgn}(x)}$$

input
```
integrate(x^2*arcsinh(a/x),x, algorithm="giac")
```

output
```
-1/6*a^3*log(abs(a))*sgn(x) + 1/3*x^3*log(sqrt(a^2/x^2 + 1) + a/x) + 1/6*a
^3*log(-x + sqrt(a^2 + x^2))/sgn(x) + 1/6*sqrt(a^2 + x^2)*a*x/sgn(x)
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arcsinh}\left(\frac{a}{x}\right) dx = \int x^2 \operatorname{asinh}\left(\frac{a}{x}\right) dx$$

input
```
int(x^2*asinh(a/x),x)
```

output
```
int(x^2*asinh(a/x), x)
```

3.301 $\int x\text{arcsinh}\left(\frac{a}{x}\right)\,dx$

3.301.1 Optimal result

Integrand size = 8, antiderivative size = 33

$$\int x\text{arcsinh}\left(\frac{a}{x}\right)\,dx=\frac{1}{2}a\sqrt{1+\frac{a^2}{x^2}}x+\frac{1}{2}x^2\text{csch}^{-1}\left(\frac{x}{a}\right)$$

output
```
1/2*x^2*arccsch(x/a)+1/2*a*x*(a^2/x^2+1)^(1/2)
```

3.301.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x\text{arcsinh}\left(\frac{a}{x}\right)\,dx=\frac{1}{2}x\left(a\sqrt{1+\frac{a^2}{x^2}}+x\text{arcsinh}\left(\frac{a}{x}\right)\right)$$

input
```
Integrate[x*ArcSinh[a/x],x]
```

output
```
(x*(a*Sqrt[1 + a^2/x^2] + x*ArcSinh[a/x]))/2
```

3.301.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6285, 6838, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arcsinh}\left(\frac{a}{x}\right)dx$$

$$\Big\downarrow \text{6285}$$

$$\int x\text{csch}^{-1}\left(\frac{x}{a}\right)dx$$

$$\Big\downarrow \text{6838}$$

$$\frac{1}{2}a\int \frac{1}{\sqrt{\frac{a^2}{x^2}+1}}dx+\frac{1}{2}x^2\text{csch}^{-1}\left(\frac{x}{a}\right)$$

$$\Big\downarrow \text{746}$$

$$\frac{1}{2}ax\sqrt{\frac{a^2}{x^2}+1}+\frac{1}{2}x^2\text{csch}^{-1}\left(\frac{x}{a}\right)$$

input
```
Int[x*ArcSinh[a/x],x]
```

output
```
(a*Sqrt[1 + a^2/x^2]*x)/2 + (x^2*ArcCsch[x/a])/2
```

3.301.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 6285
```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.301.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

method	result	size
derivativedivides	$-a^{2}\left(-\frac{x^{2} \operatorname{arcsinh}\left(\frac{a}{x}\right)}{2 a^{2}}-\frac{x \sqrt{\frac{a^{2}}{x^{2}}+1}}{2 a}\right)$	38
default	$-a^{2}\left(-\frac{x^{2} \operatorname{arcsinh}\left(\frac{a}{x}\right)}{2 a^{2}}-\frac{x \sqrt{\frac{a^{2}}{x^{2}}+1}}{2 a}\right)$	38
parts	$\frac{x^{2} \operatorname{arcsinh}\left(\frac{a}{x}\right)}{2}+\frac{a\left(a^{2}+x^{2}\right)}{2 \sqrt{\frac{a^{2}+x^{2}}{x^{2}}}\, x}$	39

input
```
int(x*arcsinh(a/x),x,method=_RETURNVERBOSE)
```

output
```
-a^2*(-1/2/a^2*x^2*arcsinh(a/x)-1/2/a*x*(a^2/x^2+1)^(1/2))
```

3.301.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int x\text{arcsinh}\left(\frac{a}{x}\right)\,dx=\frac{1}{2}\,x^{2}\log\left(\frac{x\sqrt{\frac{a^{2}+x^{2}}{x^{2}}}+a}{x}\right)+\frac{1}{2}\,ax\sqrt{\frac{a^{2}+x^{2}}{x^{2}}}$$

input
```
integrate(x*arcsinh(a/x),x, algorithm="fricas")
```

output
```
1/2*x^2*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) + 1/2*a*x*sqrt((a^2 + x^2)/x^
2)
```

3.301.6 Sympy [F]

$$\int x \operatorname{arcsinh}\left(\frac{a}{x}\right) dx = \int x \operatorname{asinh}\left(\frac{a}{x}\right) dx$$

input
```
integrate(x*asinh(a/x),x)
```

output
```
Integral(x*asinh(a/x), x)
```

3.301.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x \operatorname{arcsinh}\left(\frac{a}{x}\right) dx = \frac{1}{2} x^2 \operatorname{arsinh}\left(\frac{a}{x}\right) + \frac{1}{2} a x \sqrt{\frac{a^2}{x^2} + 1}$$

input
```
integrate(x*arcsinh(a/x),x, algorithm="maxima")
```

output
```
1/2*x^2*arcsinh(a/x) + 1/2*a*x*sqrt(a^2/x^2 + 1)
```

3.301.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int x \operatorname{arcsinh}\left(\frac{a}{x}\right) dx = \frac{1}{2} x^2 \log\left(\sqrt{\frac{a^2}{x^2} + 1} + \frac{a}{x}\right) - \frac{1}{2} a|a|\operatorname{sgn}(x) + \frac{\sqrt{a^2 + x^2} a}{2 \operatorname{sgn}(x)}$$

input
```
integrate(x*arcsinh(a/x),x, algorithm="giac")
```

output
```
1/2*x^2*log(sqrt(a^2/x^2 + 1) + a/x) - 1/2*a*abs(a)*sgn(x) + 1/2*sqrt(a^2
+ x^2)*a/sgn(x)
```

3.301.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x\operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx=\frac{x^2\,\operatorname{asinh}\left(\frac{a}{x}\right)}{2}+\frac{a\,x\,\sqrt{\frac{a^2}{x^2}+1}}{2}$$

input
```
int(x*asinh(a/x),x)
```

output
```
(x^2*asinh(a/x))/2 + (a*x*(a^2/x^2 + 1)^(1/2))/2
```

3.302 $\int \operatorname{arcsinh}\left(\frac{a}{x}\right) dx$

3.302.1 Optimal result

Integrand size = 6, antiderivative size = 25

$$\int \operatorname{arcsinh}\left(\frac{a}{x}\right) dx = x\operatorname{csch}^{-1}\left(\frac{x}{a}\right) + a\operatorname{arctanh}\left(\sqrt{1+\frac{a^2}{x^2}}\right)$$

output
```
x*arccsch(x/a)+a*arctanh((a^2/x^2+1)^(1/2))
```

3.302.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 77 vs. 2(25) = 50.

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.08

$$\int \operatorname{arcsinh}\left(\frac{a}{x}\right) dx = x\operatorname{arcsinh}\left(\frac{a}{x}\right) + \frac{a\sqrt{a^2+x^2}\left(-\log\left(1-\frac{x}{\sqrt{a^2+x^2}}\right)+\log\left(1+\frac{x}{\sqrt{a^2+x^2}}\right)\right)}{2\sqrt{1+\frac{a^2}{x^2}}x}$$

input
```
Integrate[ArcSinh[a/x],x]
```

output
```
x*ArcSinh[a/x] + (a*Sqrt[a^2 + x^2]*(-Log[1 - x/Sqrt[a^2 + x^2]] + Log[1 +
 x/Sqrt[a^2 + x^2]]))/(2*Sqrt[1 + a^2/x^2]*x)
```

3.302.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6285, 6832, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arcsinh}\left(\frac{a}{x}\right) dx$$

$$\Big\downarrow \text{ 6285}$$

$$\int \operatorname{csch}^{-1}\left(\frac{x}{a}\right) dx$$

$$\Big\downarrow \text{ 6832}$$

$$a \int \frac{1}{\sqrt{\frac{a^2}{x^2}+1x}}dx + x\operatorname{csch}^{-1}\left(\frac{x}{a}\right)$$

$$\Big\downarrow \text{ 798}$$

$$x\operatorname{csch}^{-1}\left(\frac{x}{a}\right) - \frac{1}{2}a\int \frac{x^2}{\sqrt{\frac{a^2}{x^2}+1}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 73}$$

$$x\operatorname{csch}^{-1}\left(\frac{x}{a}\right) - \frac{\int \frac{1}{\frac{1}{a^2x^4}-\frac{1}{a^2}}d\sqrt{\frac{a^2}{x^2}+1}}{a}$$

$$\Big\downarrow \text{ 221}$$

$$a\operatorname{arctanh}\left(\sqrt{\frac{a^2}{x^2}+1}\right) + x\operatorname{csch}^{-1}\left(\frac{x}{a}\right)$$

input
```
Int[ArcSinh[a/x],x]
```

output
```
x*ArcCsch[x/a] + a*ArcTanh[Sqrt[1 + a^2/x^2]]
```

3.302.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6285
```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

rule 6832
```
Int[ArcCsch[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsch[c*x], x] + Simp[1/c
Int[1/(x*Sqrt[1 + 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

3.302.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

method	result	size
derivativedivides	$-a\left(-\frac{x\,\mathrm{arcsinh}\left(\frac{a}{x}\right)}{a}-\mathrm{arctanh}\left(\frac{1}{\sqrt{\frac{a^2}{x^2}+1}}\right)\right)$	31
default	$-a\left(-\frac{x\,\mathrm{arcsinh}\left(\frac{a}{x}\right)}{a}-\mathrm{arctanh}\left(\frac{1}{\sqrt{\frac{a^2}{x^2}+1}}\right)\right)$	31
parts	$x\,\mathrm{arcsinh}\left(\frac{a}{x}\right)+\frac{a\sqrt{a^2+x^2}\,\ln\left(x+\sqrt{a^2+x^2}\right)}{\sqrt{\frac{a^2+x^2}{x^2}}\,x}$	49

input
```
int(arcsinh(a/x),x,method=_RETURNVERBOSE)
```

output
```
-a*(-1/a*x*arcsinh(a/x)-arctanh(1/(a^2/x^2+1)^(1/2)))
```

3.302.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 3.84

$$\int \operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = -a\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}-x\right)+(x-1)\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right) + \log\left(x\sqrt{\frac{a^2+x^2}{x^2}}+a-x\right)-\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}-a-x\right)$$

input
```
integrate(arcsinh(a/x),x, algorithm="fricas")
```

output
```
-a*log(x*sqrt((a^2 + x^2)/x^2) - x) + (x - 1)*log((x*sqrt((a^2 + x^2)/x^2)
 + a)/x) + log(x*sqrt((a^2 + x^2)/x^2) + a - x) - log(x*sqrt((a^2 + x^2)/x
^2) - a - x)
```

3.302.6 Sympy [F]

$$\int \operatorname{arcsinh}\left(\frac{a}{x}\right)\,dx = \int \operatorname{asinh}\left(\frac{a}{x}\right)\,dx$$

input
```
integrate(asinh(a/x),x)
```

output
```
Integral(asinh(a/x), x)
```

3.302.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \operatorname{arcsinh}\left(\frac{a}{x}\right) dx = \frac{1}{2}a\left(\log\left(\sqrt{\frac{a^2}{x^2}+1}+1\right)-\log\left(\sqrt{\frac{a^2}{x^2}+1}-1\right)\right)+x\operatorname{arsinh}\left(\frac{a}{x}\right)$$

input
```
integrate(arcsinh(a/x),x, algorithm="maxima")
```

output
```
1/2*a*(log(sqrt(a^2/x^2 + 1) + 1) - log(sqrt(a^2/x^2 + 1) - 1)) + x*arcsin
h(a/x)
```

3.302.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(23) = 46.

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \operatorname{arcsinh}\left(\frac{a}{x}\right) dx = a\log(|a|)\operatorname{sgn}(x)+x\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)-\frac{a\log\left(-x+\sqrt{a^2+x^2}\right)}{\operatorname{sgn}(x)}$$

input
```
integrate(arcsinh(a/x),x, algorithm="giac")
```

output
```
a*log(abs(a))*sgn(x) + x*log(sqrt(a^2/x^2 + 1) + a/x) - a*log(-x + sqrt(a^
2 + x^2))/sgn(x)
```

3.302.9 Mupad [B] (verification not implemented)

Time = 2.94 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \operatorname{arcsinh}\left(\frac{a}{x}\right) dx = x\operatorname{asinh}\left(\frac{a}{x}\right)+a\ \ln\left(x+\sqrt{a^2+x^2}\right)\ \operatorname{sign}(x)$$

input
```
int(asinh(a/x),x)
```

output
```
x*asinh(a/x) + a*log(x + (a^2 + x^2)^(1/2))*sign(x)
```

3.303 $\int \frac{\text{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx$

3.303.1 Optimal result

Integrand size = 10, antiderivative size = 52

$$\int \frac{\text{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx = \frac{1}{2}\text{arcsinh}\left(\frac{a}{x}\right)^2 - \text{arcsinh}\left(\frac{a}{x}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{a}{x}\right)}\right) - \frac{1}{2}\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{a}{x}\right)}\right)$$

output
```
1/2*arcsinh(a/x)^2-arcsinh(a/x)*ln(1-(a/x+(a^2/x^2+1)^(1/2))^2)-1/2*polylo
g(2,(a/x+(a^2/x^2+1)^(1/2))^2)
```

3.303.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\text{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx = \frac{1}{2}\text{arcsinh}\left(\frac{a}{x}\right)^2 - \text{arcsinh}\left(\frac{a}{x}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{a}{x}\right)}\right) - \frac{1}{2}\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{a}{x}\right)}\right)$$

input
```
Integrate[ArcSinh[a/x]/x,x]
```

output
```
ArcSinh[a/x]^2/2 - ArcSinh[a/x]*Log[1 - E^(2*ArcSinh[a/x])] - PolyLog[2, E
^(2*ArcSinh[a/x])]/2
```

3.303.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6284, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}\left(\frac{a}{x}\right)}{x}dx$$

$$\Big\downarrow \text{6284}$$

$$-\int \frac{\sqrt{\frac{a^2}{x^2}+1}x\text{arcsinh}\left(\frac{a}{x}\right)}{a}d\text{arcsinh}\left(\frac{a}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int -i\text{arcsinh}\left(\frac{a}{x}\right)\tan\left(i\text{arcsinh}\left(\frac{a}{x}\right)+\frac{\pi}{2}\right)d\text{arcsinh}\left(\frac{a}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$i\int \text{arcsinh}\left(\frac{a}{x}\right)\tan\left(i\text{arcsinh}\left(\frac{a}{x}\right)+\frac{\pi}{2}\right)d\text{arcsinh}\left(\frac{a}{x}\right)$$

$$\Big\downarrow \text{4199}$$

$$i\left(2i\int -\frac{e^{2\text{arcsinh}\left(\frac{a}{x}\right)}\text{arcsinh}\left(\frac{a}{x}\right)}{1-e^{2\text{arcsinh}\left(\frac{a}{x}\right)}}d\text{arcsinh}\left(\frac{a}{x}\right)-\frac{1}{2}i\text{arcsinh}\left(\frac{a}{x}\right)^2\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-2i\int \frac{e^{2\text{arcsinh}\left(\frac{a}{x}\right)}\text{arcsinh}\left(\frac{a}{x}\right)}{1-e^{2\text{arcsinh}\left(\frac{a}{x}\right)}}d\text{arcsinh}\left(\frac{a}{x}\right)-\frac{1}{2}i\text{arcsinh}\left(\frac{a}{x}\right)^2\right)$$

$$\Big\downarrow \text{2620}$$

$$i\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2\text{arcsinh}\left(\frac{a}{x}\right)}\right)d\text{arcsinh}\left(\frac{a}{x}\right)-\frac{1}{2}\text{arcsinh}\left(\frac{a}{x}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{a}{x}\right)}\right)\right)-\frac{1}{2}i\text{arcsinh}\left(\frac{a}{x}\right)^2\right)$$

$$\Big\downarrow \text{2715}$$

$$i\left(-2i\left(\frac{1}{4}\int e^{-2\text{arcsinh}\left(\frac{a}{x}\right)}\log\left(1-e^{2\text{arcsinh}\left(\frac{a}{x}\right)}\right)de^{2\text{arcsinh}\left(\frac{a}{x}\right)}-\frac{1}{2}\text{arcsinh}\left(\frac{a}{x}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{a}{x}\right)}\right)\right)-\frac{1}{2}i\text{arcsinh}\left(\right.\right.$$

$$\Big\downarrow 2838$$

$$i\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{a}{x}\right)}\right)-\frac{1}{2}\text{arcsinh}\left(\frac{a}{x}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{a}{x}\right)}\right)\right)-\frac{1}{2}i\text{arcsinh}\left(\frac{a}{x}\right)^2\right)$$

input
```
Int[ArcSinh[a/x]/x,x]
```

output
```
I*((-1/2*I)*ArcSinh[a/x]^2 - (2*I)*(-1/2*(ArcSinh[a/x]*Log[1 - E^(2*ArcSin
h[a/x])]) - PolyLog[2, E^(2*ArcSinh[a/x])]/4))
```

3.303.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.303. $\int \frac{\text{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx$

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6284
```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Simp[1/p   Subst[Int[
x^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

3.303.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.19

method	result
derivativedivides	$\frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)^{2}}{2}-\operatorname{arcsinh}\left(\frac{a}{x}\right)\ln\left(1+\frac{a}{x}+\sqrt{\frac{a^{2}}{x^{2}}+1}\right)-\operatorname{polylog}\left(2,-\frac{a}{x}-\sqrt{\frac{a^{2}}{x^{2}}+1}\right)-\text{arcsi}$
default	$\frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)^{2}}{2}-\operatorname{arcsinh}\left(\frac{a}{x}\right)\ln\left(1+\frac{a}{x}+\sqrt{\frac{a^{2}}{x^{2}}+1}\right)-\operatorname{polylog}\left(2,-\frac{a}{x}-\sqrt{\frac{a^{2}}{x^{2}}+1}\right)-\text{arcsi}$

input
```
int(arcsinh(a/x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(a/x)^2-arcsinh(a/x)*ln(1+a/x+(a^2/x^2+1)^(1/2))-polylog(2,-a/x
-(a^2/x^2+1)^(1/2))-arcsinh(a/x)*ln(1-a/x-(a^2/x^2+1)^(1/2))-polylog(2,a/x
+(a^2/x^2+1)^(1/2))
```

3.303.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx=\int \frac{\operatorname{arsinh}\left(\frac{a}{x}\right)}{x}\,dx$$

input
```
integrate(arcsinh(a/x)/x,x, algorithm="fricas")
```

output
```
integral(arcsinh(a/x)/x, x)
```

3.303.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x}\,dx$$

input
```
integrate(asinh(a/x)/x,x)
```

output
```
Integral(asinh(a/x)/x, x)
```

3.303.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(\frac{a}{x}\right)}{x}\,dx$$

input
```
integrate(arcsinh(a/x)/x,x, algorithm="maxima")
```

output
```
a*integrate(x*log(x)/(a^3 + a*x^2 + (a^2 + x^2)^(3/2)), x) + log(a + sqrt(
a^2 + x^2))*log(x) - 1/2*log(x)^2 - 1/2*log(x)*log(x^2/a^2 + 1) - 1/4*dilo
g(-x^2/a^2)
```

3.303.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(\frac{a}{x}\right)}{x}\,dx$$

input
```
integrate(arcsinh(a/x)/x,x, algorithm="giac")
```

output
```
integrate(arcsinh(a/x)/x, x)
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x}\,dx$$

input

```
int(asinh(a/x)/x,x)
```

output

```
int(asinh(a/x)/x, x)
```

3.304 $\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^2} dx$

3.304.1 Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^2} dx = \frac{\sqrt{1+\frac{a^2}{x^2}}}{a} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x}$$

output
```
-arccsch(x/a)/x+(a^2/x^2+1)^(1/2)/a
```

3.304.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^2} dx = \frac{\sqrt{1+\frac{a^2}{x^2}}}{a} - \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}$$

input
```
Integrate[ArcSinh[a/x]/x^2,x]
```

output
```
Sqrt[1 + a^2/x^2]/a - ArcSinh[a/x]/x
```

3.304.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6285, 6838, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}\left(\frac{a}{x}\right)}{x^2} \, dx$$
$$\Big\downarrow \; 6285$$
$$\int \frac{\text{csch}^{-1}\left(\frac{x}{a}\right)}{x^2}dx$$
$$\Big\downarrow \; 6838$$
$$-a\int \frac{1}{\sqrt{\frac{a^2}{x^2}+1}x^3}dx-\frac{\text{csch}^{-1}\left(\frac{x}{a}\right)}{x}$$
$$\Big\downarrow \; 793$$
$$\frac{\sqrt{\frac{a^2}{x^2}+1}}{a}-\frac{\text{csch}^{-1}\left(\frac{x}{a}\right)}{x}$$

input
```
Int[ArcSinh[a/x]/x^2,x]
```

output
```
Sqrt[1 + a^2/x^2]/a - ArcCsch[x/a]/x
```

3.304.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 6285
```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.304.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$-\frac{\frac{a\,\mathrm{arcsinh}\left(\frac{a}{x}\right)}{x}-\sqrt{\frac{a^{2}}{x^{2}}+1}}{a}$	31
default	$-\frac{\frac{a\,\mathrm{arcsinh}\left(\frac{a}{x}\right)}{x}-\sqrt{\frac{a^{2}}{x^{2}}+1}}{a}$	31
parts	$-\frac{\mathrm{arcsinh}\left(\frac{a}{x}\right)}{x}+\frac{a^{2}+x^{2}}{a\sqrt{\frac{a^{2}+x^{2}}{x^{2}}}\,x^{2}}$	40

input
```
int(arcsinh(a/x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a*(a/x*arcsinh(a/x)-(a^2/x^2+1)^(1/2))
```

3.304.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{\mathrm{arcsinh}\left(\frac{a}{x}\right)}{x^2}\,dx=-\frac{a\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)-x\sqrt{\frac{a^2+x^2}{x^2}}}{ax}$$

input
```
integrate(arcsinh(a/x)/x^2,x, algorithm="fricas")
```

output
```
-(a*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) - x*sqrt((a^2 + x^2)/x^2))/(a*x)
```

3.304.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^2}\,dx = \begin{cases} -\frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x} + \frac{\sqrt{\frac{a^2}{x^2}+1}}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(asinh(a/x)/x**2,x)
```

output
```
Piecewise((-asinh(a/x)/x + sqrt(a**2/x**2 + 1)/a, Ne(a, 0)), (0, True))
```

3.304.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{\frac{a \operatorname{arsinh}\left(\frac{a}{x}\right)}{x} - \sqrt{\frac{a^2}{x^2}+1}}{a}$$

input
```
integrate(arcsinh(a/x)/x^2,x, algorithm="maxima")
```

output
```
-(a*arcsinh(a/x)/x - sqrt(a^2/x^2 + 1))/a
```

3.304.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)}{x} + \frac{\sqrt{\frac{a^2}{x^2}+1}}{a}$$

input
```
integrate(arcsinh(a/x)/x^2,x, algorithm="giac")
```

output
```
-log(sqrt(a^2/x^2 + 1) + a/x)/x + sqrt(a^2/x^2 + 1)/a
```

3.304.9 Mupad [B] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^2}\,dx=\frac{\sqrt{\frac{a^2}{x^2}+1}}{a}-\frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x}$$

input
```
int(asinh(a/x)/x^2,x)
```

output
```
(a^2/x^2 + 1)^(1/2)/a - asinh(a/x)/x
```

3.305 $\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^3} dx$

3.305.1 Optimal result

Integrand size = 10, antiderivative size = 50

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^3} dx = \frac{\sqrt{1+\frac{a^2}{x^2}}}{4ax} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{4a^2} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

output
```
-1/4*arccsch(x/a)/a^2-1/2*arccsch(x/a)/x^2+1/4*(a^2/x^2+1)^(1/2)/a/x
```

3.305.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^3} dx = \frac{a\sqrt{1+\frac{a^2}{x^2}}x - (2a^2+x^2)\operatorname{arcsinh}\left(\frac{a}{x}\right)}{4a^2x^2}$$

input
```
Integrate[ArcSinh[a/x]/x^3,x]
```

output
```
(a*Sqrt[1 + a^2/x^2]*x - (2*a^2 + x^2)*ArcSinh[a/x])/(4*a^2*x^2)
```

3.305.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6285, 6838, 858, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}\left(\frac{a}{x}\right)}{x^3} \, dx$$

$$\Big\downarrow \text{ 6285}$$

$$\int \frac{\text{csch}^{-1}\left(\frac{x}{a}\right)}{x^3}dx$$

$$\Big\downarrow \text{ 6838}$$

$$-\frac{1}{2}a\int \frac{1}{\sqrt{\frac{a^2}{x^2}+1}x^4}dx-\frac{\text{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{1}{2}a\int \frac{1}{\sqrt{\frac{a^2}{x^2}+1}x^2}d\frac{1}{x}-\frac{\text{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{2}a\left(\frac{\sqrt{\frac{a^2}{x^2}+1}}{2a^2x}-\frac{\int \frac{1}{\sqrt{\frac{a^2}{x^2}+1}}d\frac{1}{x}}{2a^2}\right)-\frac{\text{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}a\left(\frac{\sqrt{\frac{a^2}{x^2}+1}}{2a^2x}-\frac{\text{arcsinh}\left(\frac{a}{x}\right)}{2a^3}\right)-\frac{\text{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

input
```
Int[ArcSinh[a/x]/x^3,x]
```

output
```
-1/2*ArcCsch[x/a]/x^2 + (a*(Sqrt[1 + a^2/x^2]/(2*a^2*x) - ArcSinh[a/x]/(2*
a^3)))/2
```

3.305.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6285
```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.305.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{\frac{a^2 \operatorname{arcsinh}\left(\frac{a}{x}\right)}{2x^2}-\frac{a\sqrt{\frac{a^2}{x^2}+1}}{4x}+\frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{4}}{a^2}$	46
default	$-\frac{\frac{a^2 \operatorname{arcsinh}\left(\frac{a}{x}\right)}{2x^2}-\frac{a\sqrt{\frac{a^2}{x^2}+1}}{4x}+\frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{4}}{a^2}$	46
parts	$-\frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{2x^2}+\frac{\sqrt{a^2+x^2}\left(-\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2+x^2}}{x}\right)x^2+\sqrt{a^2}\,\sqrt{a^2+x^2}\right)}{4a\sqrt{\frac{a^2+x^2}{x^2}}\,x^3\sqrt{a^2}}$	94

input
```
int(arcsinh(a/x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(1/2*a^2/x^2*arcsinh(a/x)-1/4*a/x*(a^2/x^2+1)^(1/2)+1/4*arcsinh(a/x
))
```

3.305.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^3}\,dx = \frac{ax\sqrt{\frac{a^2+x^2}{x^2}} - (2\,a^2+x^2)\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)}{4\,a^2x^2}$$

input
```
integrate(arcsinh(a/x)/x^3,x, algorithm="fricas")
```

output
```
1/4*(a*x*sqrt((a^2 + x^2)/x^2) - (2*a^2 + x^2)*log((x*sqrt((a^2 + x^2)/x^2
) + a)/x))/(a^2*x^2)
```

3.305.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^3}\,dx = \int \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x^3}\,dx$$

input
```
integrate(asinh(a/x)/x**3,x)
```

output
```
Integral(asinh(a/x)/x**3, x)
```

3.305.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(42) = 84$.

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.94

$$\int \frac{\text{arcsinh}\left(\frac{a}{x}\right)}{x^3}\,dx$$
$$= \frac{1}{8}\,a\left(\frac{2\,x\sqrt{\frac{a^2}{x^2}+1}}{a^2x^2\left(\frac{a^2}{x^2}+1\right)-a^4} - \frac{\log\left(x\sqrt{\frac{a^2}{x^2}+1}+a\right)}{a^3} + \frac{\log\left(x\sqrt{\frac{a^2}{x^2}+1}-a\right)}{a^3}\right) - \frac{\text{arsinh}\left(\frac{a}{x}\right)}{2\,x^2}$$

input
```
integrate(arcsinh(a/x)/x^3,x, algorithm="maxima")
```

output
```
1/8*a*(2*x*sqrt(a^2/x^2 + 1)/(a^2*x^2*(a^2/x^2 + 1) - a^4) - log(x*sqrt(a^
2/x^2 + 1) + a)/a^3 + log(x*sqrt(a^2/x^2 + 1) - a)/a^3) - 1/2*arcsinh(a/x)
/x^2
```

3.305.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.68

$$\int \frac{\text{arcsinh}\left(\frac{a}{x}\right)}{x^3}\,dx = -\frac{a\left(\frac{\log\left(a+\sqrt{a^2+x^2}\right)}{a^3} - \frac{\log\left(-a+\sqrt{a^2+x^2}\right)}{a^3} - \frac{2\sqrt{a^2+x^2}}{a^2x^2}\right)}{8\,\text{sgn}\left(x\right)} - \frac{\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)}{2\,x^2}$$

input
```
integrate(arcsinh(a/x)/x^3,x, algorithm="giac")
```

output
```
-1/8*a*(log(a + sqrt(a^2 + x^2))/a^3 - log(-a + sqrt(a^2 + x^2))/a^3 - 2*s
qrt(a^2 + x^2)/(a^2*x^2))/sgn(x) - 1/2*log(sqrt(a^2/x^2 + 1) + a/x)/x^2
```

3.305.9 Mupad [B] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^3}\,dx = \frac{\sqrt{\frac{a^2}{x^2}+1}}{4\,a\,x} - \frac{\operatorname{asinh}\left(\frac{a}{x}\right)\,\left(\frac{x}{4\,a^2}+\frac{1}{2\,x}\right)}{x}$$

input
```
int(asinh(a/x)/x^3,x)
```

output
```
(a^2/x^2 + 1)^(1/2)/(4*a*x) - (asinh(a/x)*(x/(4*a^2) + 1/(2*x)))/x
```

3.306 $\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^4} dx$

3.306.1 Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^4} dx = -\frac{\sqrt{1+\frac{a^2}{x^2}}}{3a^3} + \frac{\left(1+\frac{a^2}{x^2}\right)^{3/2}}{9a^3} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

output
```
1/9*(a^2/x^2+1)^(3/2)/a^3-1/3*arccsch(x/a)/x^3-1/3*(a^2/x^2+1)^(1/2)/a^3
```

3.306.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^4} dx = \left(-\frac{2}{9a^3} + \frac{1}{9ax^2}\right)\sqrt{\frac{a^2+x^2}{x^2}} - \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{3x^3}$$

input
```
Integrate[ArcSinh[a/x]/x^4,x]
```

output
```
(-2/(9*a^3) + 1/(9*a*x^2))*Sqrt[(a^2 + x^2)/x^2] - ArcSinh[a/x]/(3*x^3)
```

3.306.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6285, 6838, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}\left(\frac{a}{x}\right)}{x^4} \, dx$$

$$\Big\downarrow \text{ 6285}$$

$$\int \frac{\text{csch}^{-1}\left(\frac{x}{a}\right)}{x^4}dx$$

$$\Big\downarrow \text{ 6838}$$

$$-\frac{1}{3}a\int \frac{1}{\sqrt{\frac{a^2}{x^2}+1}x^5}dx-\frac{\text{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{6}a\int \frac{1}{\sqrt{\frac{a^2}{x^2}+1}x^2}d\frac{1}{x^2}-\frac{\text{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{6}a\int \left(\frac{\sqrt{\frac{a^2}{x^2}+1}}{a^2}-\frac{1}{a^2\sqrt{\frac{a^2}{x^2}+1}}\right)d\frac{1}{x^2}-\frac{\text{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6}a\left(\frac{2\left(\frac{a^2}{x^2}+1\right)^{3/2}}{3a^4}-\frac{2\sqrt{\frac{a^2}{x^2}+1}}{a^4}\right)-\frac{\text{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

input
```
Int[ArcSinh[a/x]/x^4,x]
```

output
```
(a*((-2*Sqrt[1 + a^2/x^2])/a^4 + (2*(1 + a^2/x^2)^(3/2))/(3*a^4)))/6 - Arc
Csch[x/a]/(3*x^3)
```

3.306.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6285
```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.306.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

method	result	size
parts	$-\frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{3x^3}+\frac{(a^2+x^2)(a^2-2x^2)}{9a^3\sqrt{\frac{a^2+x^2}{x^2}}\,x^4}$	50
derivativedivides	$-\frac{\frac{a^3\operatorname{arcsinh}\left(\frac{a}{x}\right)}{3x^3}-\frac{a^2\sqrt{\frac{a^2}{x^2}+1}}{9x^2}+\frac{2\sqrt{\frac{a^2}{x^2}+1}}{9}}{a^3}$	53
default	$-\frac{\frac{a^3\operatorname{arcsinh}\left(\frac{a}{x}\right)}{3x^3}-\frac{a^2\sqrt{\frac{a^2}{x^2}+1}}{9x^2}+\frac{2\sqrt{\frac{a^2}{x^2}+1}}{9}}{a^3}$	53

input
```
int(arcsinh(a/x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arcsinh(a/x)/x^3+1/9/a^3*(a^2+x^2)*(a^2-2*x^2)/((a^2+x^2)/x^2)^(1/2)/
x^4
```

3.306.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^4}\,dx = -\frac{3\,a^3\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)-(a^2x-2\,x^3)\sqrt{\frac{a^2+x^2}{x^2}}}{9\,a^3x^3}$$

input
```
integrate(arcsinh(a/x)/x^4,x, algorithm="fricas")
```

output
```
-1/9*(3*a^3*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) - (a^2*x - 2*x^3)*sqrt((a
^2 + x^2)/x^2))/(a^3*x^3)
```

3.306.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^4}\,dx = \int \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x^4}\,dx$$

input
```
integrate(asinh(a/x)/x**4,x)
```

output
```
Integral(asinh(a/x)/x**4, x)
```

3.306.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^4}\,dx = \frac{1}{9}\,a\left(\frac{\left(\frac{a^2}{x^2}+1\right)^{\frac{3}{2}}}{a^4}-\frac{3\,\sqrt{\frac{a^2}{x^2}+1}}{a^4}\right)-\frac{\operatorname{arsinh}\left(\frac{a}{x}\right)}{3\,x^3}$$

input
```
integrate(arcsinh(a/x)/x^4,x, algorithm="maxima")
```

output
```
1/9*a*((a^2/x^2 + 1)^(3/2)/a^4 - 3*sqrt(a^2/x^2 + 1)/a^4) - 1/3*arcsinh(a/
x)/x^3
```

3.306.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.39

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^4}\,dx = -\frac{\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)}{3\,x^3} - \frac{4\left(a^2-3\left(x-\sqrt{a^2+x^2}\right)^2\right)a}{9\left(a^2-\left(x-\sqrt{a^2+x^2}\right)^2\right)^3\operatorname{sgn}\left(x\right)}$$

input
```
integrate(arcsinh(a/x)/x^4,x, algorithm="giac")
```

output
```
-1/3*log(sqrt(a^2/x^2 + 1) + a/x)/x^3 - 4/9*(a^2 - 3*(x - sqrt(a^2 + x^2))
^2)*a/((a^2 - (x - sqrt(a^2 + x^2))^2)^3*sgn(x))
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{x^4}\,dx = \int \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x^4}\,dx$$

input
```
int(asinh(a/x)/x^4,x)
```

output
```
int(asinh(a/x)/x^4, x)
```

3.307 $\int x^m \text{arcsinh}(ax^n)\,dx$

3.307.1 Optimal result

Integrand size = 10, antiderivative size = 77

$$\int x^m \text{arcsinh}(ax^n)\,dx = \frac{x^{1+m}\text{arcsinh}(ax^n)}{1+m} - \frac{anx^{1+m+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m+n}{2n},\frac{1+m+3n}{2n},-a^2x^{2n}\right)}{(1+m)(1+m+n)}$$

output
```
x^(1+m)*arcsinh(a*x^n)/(1+m)-a*n*x^(1+m+n)*hypergeom([1/2, 1/2*(1+m+n)/n],
[1/2*(1+m+3*n)/n],-a^2*x^(2*n))/(1+m)/(1+m+n)
```

3.307.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int x^m \text{arcsinh}(ax^n)\,dx = \frac{x^{1+m}\left((1+m+n)\text{arcsinh}(ax^n) - anx^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m+n}{2n},\frac{1+m+3n}{2n},-a^2x^{2n}\right)\right)}{(1+m)(1+m+n)}$$

input
```
Integrate[x^m*ArcSinh[a*x^n],x]
```

output
```
(x^(1 + m)*((1 + m + n)*ArcSinh[a*x^n] - a*n*x^n*Hypergeometric2F1[1/2, (1
 + m + n)/(2*n), (1 + m + 3*n)/(2*n), -(a^2*x^(2*n))]))/((1 + m)*(1 + m +
n))
```

3.307.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6290, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \operatorname{arcsinh}(ax^n)\,dx$$

$$\Big\downarrow \text{ 6290}$$

$$\frac{x^{m+1}\operatorname{arcsinh}(ax^n)}{m+1}-\frac{\int \frac{anx^{m+n}}{\sqrt{a^2x^{2n}+1}}dx}{m+1}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^{m+1}\operatorname{arcsinh}(ax^n)}{m+1}-\frac{an\int \frac{x^{m+n}}{\sqrt{a^2x^{2n}+1}}dx}{m+1}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^{m+1}\operatorname{arcsinh}(ax^n)}{m+1}-\frac{anx^{m+n+1}\text{ Hypergeometric2F1}\left(\frac{1}{2},\frac{m+n+1}{2n},\frac{m+3n+1}{2n},-a^2x^{2n}\right)}{(m+1)(m+n+1)}$$

input
```
Int[x^m*ArcSinh[a*x^n],x]
```

output
```
(x^(1 + m)*ArcSinh[a*x^n])/(1 + m) - (a*n*x^(1 + m + n)*Hypergeometric2F1[
1/2, (1 + m + n)/(2*n), (1 + m + 3*n)/(2*n), -(a^2*x^(2*n))])/((1 + m)*(1
+ m + n))
```

3.307.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6290
```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x],
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u
, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[
u, x]
```

3.307.4 Maple [F]

$$\int x^m \operatorname{arcsinh}(a x^n) dx$$

input
```
int(x^m*arcsinh(a*x^n),x)
```

output
```
int(x^m*arcsinh(a*x^n),x)
```

3.307.5 Fricas [F(-2)]

Exception generated.

$$\int x^m \operatorname{arcsinh}(ax^n) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*arcsinh(a*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.307.6 Sympy [F]

$$\int x^m \operatorname{arcsinh}(ax^n) \, dx = \int x^m \operatorname{asinh}(ax^n) \, dx$$

input
```
integrate(x**m*asinh(a*x**n),x)
```

output
```
Integral(x**m*asinh(a*x**n), x)
```

3.307.7 Maxima [F]

$$\int x^m \text{arcsinh}(ax^n)\,dx = \int x^m \operatorname{arsinh}(ax^n)\,dx$$

input
```
integrate(x^m*arcsinh(a*x^n),x, algorithm="maxima")
```

output
```
-a*n*integrate(e^(m*log(x) + n*log(x))/(a^3*(m + 1)*x^(3*n) + a*(m + 1)*x^
n + (a^2*(m + 1)*x^(2*n) + m + 1)*sqrt(a^2*x^(2*n) + 1)), x) + n*integrate
(x^m/(a^2*(m + 1)*x^(2*n) + m + 1), x) + ((m + 1)*x*x^m*log(a*x^n + sqrt(a
^2*x^(2*n) + 1)) - n*x*x^m)/(m^2 + 2*m + 1)
```

3.307.8 Giac [F]

$$\int x^m \text{arcsinh}(ax^n)\,dx = \int x^m \operatorname{arsinh}(ax^n)\,dx$$

input
```
integrate(x^m*arcsinh(a*x^n),x, algorithm="giac")
```

output
```
integrate(x^m*arcsinh(a*x^n), x)
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int x^m \text{arcsinh}(ax^n)\,dx = \int x^m \operatorname{asinh}(a\,x^n)\,dx$$

input
```
int(x^m*asinh(a*x^n),x)
```

output
```
int(x^m*asinh(a*x^n), x)
```

3.308 $\int x^2 \text{arcsinh}(ax^n)\,dx$

3.308.1 Optimal result

Integrand size = 10, antiderivative size = 64

$$\int x^2 \text{arcsinh}(ax^n)\,dx = \frac{1}{3}x^3\text{arcsinh}(ax^n) - \frac{anx^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2n},\frac{3(1+n)}{2n},-a^2x^{2n}\right)}{3(3+n)}$$

output
```
1/3*x^3*arcsinh(a*x^n)-1/3*a*n*x^(3+n)*hypergeom([1/2, 1/2*(3+n)/n],[3/2*(
1+n)/n],-a^2*x^(2*n))/(3+n)
```

3.308.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int x^2 \text{arcsinh}(ax^n)\,dx = \frac{1}{3}x^3\text{arcsinh}(ax^n) - \frac{anx^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2n},1+\frac{3+n}{2n},-a^2x^{2n}\right)}{3(3+n)}$$

input
```
Integrate[x^2*ArcSinh[a*x^n],x]
```

output
```
(x^3*ArcSinh[a*x^n])/3 - (a*n*x^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/(2*
n), 1 + (3 + n)/(2*n), -(a^2*x^(2*n))])/(3*(3 + n))
```

3.308.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6290, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arcsinh}(ax^n)\, dx$$

$$\Big\downarrow \text{ 6290}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax^n)-\frac{1}{3}\int \frac{anx^{n+2}}{\sqrt{a^2x^{2n}+1}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax^n)-\frac{1}{3}an\int \frac{x^{n+2}}{\sqrt{a^2x^{2n}+1}}dx$$

$$\Big\downarrow \text{ 888}$$

$$\frac{1}{3}x^3\text{arcsinh}(ax^n)-\frac{anx^{n+3}\text{ Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2n},\frac{3(n+1)}{2n},-a^2x^{2n}\right)}{3(n+3)}$$

input
```
Int[x^2*ArcSinh[a*x^n],x]
```

output
```
(x^3*ArcSinh[a*x^n])/3 - (a*n*x^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/(2*
n), (3*(1 + n))/(2*n), -(a^2*x^(2*n))])/(3*(3 + n))
```

3.308.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6290
```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x],
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u
, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[
u, x]
```

3.308.4 Maple [F]

$$\int x^2 \operatorname{arcsinh}(a\,x^n)\,dx$$

input
```
int(x^2*arcsinh(a*x^n),x)
```

output
```
int(x^2*arcsinh(a*x^n),x)
```

3.308.5 Fricas [F(-2)]

Exception generated.

$$\int x^2 \operatorname{arcsinh}(ax^n)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arcsinh(a*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.308.6 Sympy [F]

$$\int x^2 \operatorname{arcsinh}(ax^n)\,dx = \int x^2 \operatorname{asinh}(ax^n)\,dx$$

input
```
integrate(x**2*asinh(a*x**n),x)
```

output
```
Integral(x**2*asinh(a*x**n), x)
```

3.308.7 Maxima [F]

$$\int x^2 \text{arcsinh}(ax^n)\,dx = \int x^2 \operatorname{arsinh}(ax^n)\,dx$$

input
```
integrate(x^2*arcsinh(a*x^n),x, algorithm="maxima")
```

output
```
-1/9*n*x^3 + 1/3*x^3*log(a*x^n + sqrt(a^2*x^(2*n) + 1)) - a*n*integrate(1/
3*x^2*x^n/(a^3*x^(3*n) + a*x^n + (a^2*x^(2*n) + 1)^(3/2)), x) + n*integrat
e(1/3*x^2/(a^2*x^(2*n) + 1), x)
```

3.308.8 Giac [F]

$$\int x^2 \text{arcsinh}(ax^n)\,dx = \int x^2 \operatorname{arsinh}(ax^n)\,dx$$

input
```
integrate(x^2*arcsinh(a*x^n),x, algorithm="giac")
```

output
```
integrate(x^2*arcsinh(a*x^n), x)
```

3.308.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arcsinh}(ax^n)\,dx = \int x^2 \operatorname{asinh}(a\,x^n)\,dx$$

input
```
int(x^2*asinh(a*x^n),x)
```

output
```
int(x^2*asinh(a*x^n), x)
```

3.309 $\int x\text{arcsinh}(ax^n)\,dx$

3.309.1 Optimal result

Integrand size = 8, antiderivative size = 65

$$\int x\text{arcsinh}(ax^n)\,dx = \frac{1}{2}x^2\text{arcsinh}(ax^n) - \frac{anx^{2+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2n},\frac{1}{2}\left(3+\frac{2}{n}\right),-a^2x^{2n}\right)}{2(2+n)}$$

output
```
1/2*x^2*arcsinh(a*x^n)-1/2*a*n*x^(2+n)*hypergeom([1/2, 1/2*(2+n)/n],[3/2+1
/n],-a^2*x^(2*n))/(2+n)
```

3.309.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int x\text{arcsinh}(ax^n)\,dx = \frac{x^2\left((2+n)\text{arcsinh}(ax^n) - anx^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n},\frac{3}{2}+\frac{1}{n},-a^2x^{2n}\right)\right)}{2(2+n)}$$

input
```
Integrate[x*ArcSinh[a*x^n],x]
```

output
```
(x^2*((2 + n)*ArcSinh[a*x^n] - a*n*x^n*Hypergeometric2F1[1/2, 1/2 + n^(-1)
, 3/2 + n^(-1), -(a^2*x^(2*n))]))/(2*(2 + n))
```

3.309.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6290, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arcsinh}(ax^n)\,dx$$

$$\Big\downarrow \text{ 6290}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax^n)-\frac{1}{2}\int \frac{anx^{n+1}}{\sqrt{a^2x^{2n}+1}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax^n)-\frac{1}{2}an\int \frac{x^{n+1}}{\sqrt{a^2x^{2n}+1}}dx$$

$$\Big\downarrow \text{ 888}$$

$$\frac{1}{2}x^2\text{arcsinh}(ax^n)-\frac{anx^{n+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2n},\frac{1}{2}\left(3+\frac{2}{n}\right),-a^2x^{2n}\right)}{2(n+2)}$$

input
```
Int[x*ArcSinh[a*x^n],x]
```

output
```
(x^2*ArcSinh[a*x^n])/2 - (a*n*x^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/(2*
n), (3 + 2/n)/2, -(a^2*x^(2*n))])/(2*(2 + n))
```

3.309.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6290
```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x],
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u
, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[
u, x]
```

3.309.4 Maple [F]

$$\int x \operatorname{arcsinh}\left(a\,x^{n}\right) dx$$

input
```
int(x*arcsinh(a*x^n),x)
```

output
```
int(x*arcsinh(a*x^n),x)
```

3.309.5 Fricas [F(-2)]

Exception generated.

$$\int x\text{arcsinh}(ax^n)\;dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arcsinh(a*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.309.6 Sympy [F]

$$\int x\text{arcsinh}(ax^n)\;dx = \int x \operatorname{asinh}\left(ax^{n}\right) dx$$

input
```
integrate(x*asinh(a*x**n),x)
```

output
```
Integral(x*asinh(a*x**n), x)
```

3.309.7 Maxima [F]

$$\int x\text{arcsinh}(ax^n)\,dx = \int x\,\text{arsinh}\,(ax^n)\,dx$$

input
```
integrate(x*arcsinh(a*x^n),x, algorithm="maxima")
```

output
```
-1/4*n*x^2 - a*n*integrate(1/2*x*x^n/(a^3*x^(3*n) + a*x^n + (a^2*x^(2*n) +
 1)^(3/2)), x) + 1/2*x^2*log(a*x^n + sqrt(a^2*x^(2*n) + 1)) + n*integrate(
1/2*x/(a^2*x^(2*n) + 1), x)
```

3.309.8 Giac [F]

$$\int x\text{arcsinh}(ax^n)\,dx = \int x\,\text{arsinh}\,(ax^n)\,dx$$

input
```
integrate(x*arcsinh(a*x^n),x, algorithm="giac")
```

output
```
integrate(x*arcsinh(a*x^n), x)
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arcsinh}(ax^n)\,dx = \int x\,\text{asinh}(a\,x^n)\,dx$$

input
```
int(x*asinh(a*x^n),x)
```

output
```
int(x*asinh(a*x^n), x)
```

3.310 $\int \operatorname{arcsinh}(ax^n)\,dx$

3.310.1 Optimal result

Integrand size = 6, antiderivative size = 56

$$\int \operatorname{arcsinh}(ax^n)\,dx = x\operatorname{arcsinh}(ax^n) - \frac{anx^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-a^2x^{2n}\right)}{1+n}$$

output
```
x*arcsinh(a*x^n)-a*n*x^(1+n)*hypergeom([1/2, 1/2*(1+n)/n],[3/2+1/2/n],-a^2
*x^(2*n))/(1+n)
```

3.310.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \operatorname{arcsinh}(ax^n)\,dx = x\operatorname{arcsinh}(ax^n) - \frac{anx^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-a^2x^{2n}\right)}{1+n}$$

input
```
Integrate[ArcSinh[a*x^n],x]
```

output
```
x*ArcSinh[a*x^n] - (a*n*x^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/(2*n), (3
 + n^(-1))/2, -(a^2*x^(2*n))])/(1 + n)
```

3.310.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6289, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}(ax^n)\,dx$$

$$\Big\downarrow \text{6289}$$

$$x\text{arcsinh}(ax^n)-\int \frac{anx^n}{\sqrt{a^2x^{2n}+1}}dx$$

$$\Big\downarrow \text{27}$$

$$x\text{arcsinh}(ax^n)-an\int \frac{x^n}{\sqrt{a^2x^{2n}+1}}dx$$

$$\Big\downarrow \text{888}$$

$$x\text{arcsinh}(ax^n)-\frac{anx^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),-a^2x^{2n}\right)}{n+1}$$

input
```
Int[ArcSinh[a*x^n],x]
```

output
```
x*ArcSinh[a*x^n] - (a*n*x^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/(2*n), (3
 + n^(-1))/2, -(a^2*x^(2*n))])/(1 + n)
```

3.310.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6289
```
Int[ArcSinh[u_], x_Symbol] :> Simp[x*ArcSinh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/Sqrt[1 + u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Funct
ionOfExponentialQ[u, x]
```

3.310.4 Maple [F]

$$\int \operatorname{arcsinh}\left(a\,x^{n}\right)dx$$

input
```
int(arcsinh(a*x^n),x)
```

output
```
int(arcsinh(a*x^n),x)
```

3.310.5 Fricas [F(-2)]

Exception generated.

$$\int \operatorname{arcsinh}(ax^n)\;dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.310.6 Sympy [F]

$$\int \operatorname{arcsinh}(ax^n)\;dx = \int \operatorname{asinh}\left(ax^{n}\right)dx$$

input
```
integrate(asinh(a*x**n),x)
```

output
```
Integral(asinh(a*x**n), x)
```

3.310.7 Maxima [F]

$$\int \operatorname{arcsinh}(ax^n)\,dx = \int \operatorname{arsinh}\left(ax^n\right)\,dx$$

input
```
integrate(arcsinh(a*x^n),x, algorithm="maxima")
```

output
```
-a*n*integrate(x^n/(a^3*x^(3*n) + a*x^n + (a^2*x^(2*n) + 1)^(3/2)), x) - n
*x + n*integrate(1/(a^2*x^(2*n) + 1), x) + x*log(a*x^n + sqrt(a^2*x^(2*n)
+ 1))
```

3.310.8 Giac [F]

$$\int \operatorname{arcsinh}(ax^n)\,dx = \int \operatorname{arsinh}\left(ax^n\right)\,dx$$

input
```
integrate(arcsinh(a*x^n),x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x^n), x)
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(ax^n)\,dx = \int \operatorname{asinh}(a\,x^n)\,dx$$

input
```
int(asinh(a*x^n),x)
```

output
```
int(asinh(a*x^n), x)
```

3.311 $\int \frac{\operatorname{arcsinh}(ax^n)}{x} dx$

3.311.1 Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x} dx = -\frac{\operatorname{arcsinh}(ax^n)^2}{2n} + \frac{\operatorname{arcsinh}(ax^n)\log\left(1-e^{2\operatorname{arcsinh}(ax^n)}\right)}{n} + \frac{\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax^n)}\right)}{2n}$$

output
```
-1/2*arcsinh(a*x^n)^2/n+arcsinh(a*x^n)*ln(1-(a*x^n+(1+a^2*(x^n)^2)^(1/2))^
2)/n+1/2*polylog(2,(a*x^n+(1+a^2*(x^n)^2)^(1/2))^2)/n
```

3.311.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x} dx = \frac{-\operatorname{arcsinh}(ax^n)\left(\operatorname{arcsinh}(ax^n)-2\log\left(1-e^{2\operatorname{arcsinh}(ax^n)}\right)\right)+\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax^n)}\right)}{2n}$$

input
```
Integrate[ArcSinh[a*x^n]/x,x]
```

output
```
(-(ArcSinh[a*x^n]*(ArcSinh[a*x^n] - 2*Log[1 - E^(2*ArcSinh[a*x^n])])) + Po
lyLog[2, E^(2*ArcSinh[a*x^n])])/(2*n)
```

3.311.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6284, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax^n)}{x} dx$$

$$\Big\downarrow \text{6284}$$

$$\frac{\int \frac{x^{-n}\sqrt{a^2x^{2n}+1}\text{arcsinh}(ax^n)}{a}d\text{arcsinh}(ax^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -i\text{arcsinh}(ax^n)\tan\left(i\text{arcsinh}(ax^n)+\frac{\pi}{2}\right)d\text{arcsinh}(ax^n)}{n}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \text{arcsinh}(ax^n)\tan\left(i\text{arcsinh}(ax^n)+\frac{\pi}{2}\right)d\text{arcsinh}(ax^n)}{n}$$

$$\Big\downarrow \text{4199}$$

$$-\frac{i\left(2i\int -\frac{e^{2\text{arcsinh}(ax^n)}\text{arcsinh}(ax^n)}{1-e^{2\text{arcsinh}(ax^n)}}d\text{arcsinh}(ax^n)-\frac{1}{2}i\text{arcsinh}(ax^n)^2\right)}{n}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i\left(-2i\int \frac{e^{2\text{arcsinh}(ax^n)}\text{arcsinh}(ax^n)}{1-e^{2\text{arcsinh}(ax^n)}}d\text{arcsinh}(ax^n)-\frac{1}{2}i\text{arcsinh}(ax^n)^2\right)}{n}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{i\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2\text{arcsinh}(ax^n)}\right)d\text{arcsinh}(ax^n)-\frac{1}{2}\text{arcsinh}(ax^n)\log\left(1-e^{2\text{arcsinh}(ax^n)}\right)\right)-\frac{1}{2}i\text{arcsinh}(ax^n)^2\right)}{n}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{i\left(-2i\left(\frac{1}{4}\int e^{-2\text{arcsinh}(ax^n)}\log\left(1-e^{2\text{arcsinh}(ax^n)}\right)de^{2\text{arcsinh}(ax^n)}-\frac{1}{2}\text{arcsinh}(ax^n)\log\left(1-e^{2\text{arcsinh}(ax^n)}\right)\right)-\frac{1}{2}i\text{arcsi}}{n}$$

$\downarrow$ 2838

$$-\frac{i\left(-2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax^n)}\right)-\frac{1}{2}\operatorname{arcsinh}(ax^n)\log\left(1-e^{2\operatorname{arcsinh}(ax^n)}\right)\right)-\frac{1}{2}i\operatorname{arcsinh}(ax^n)^2\right)}{n}$$

input
```
Int[ArcSinh[a*x^n]/x,x]
```

output
```
((-I)*((-1/2*I)*ArcSinh[a*x^n]^2 - (2*I)*(-1/2*(ArcSinh[a*x^n]*Log[1 - E^(
2*ArcSinh[a*x^n])]) - PolyLog[2, E^(2*ArcSinh[a*x^n])]/4)))/n
```

3.311.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.311. $\int \frac{\operatorname{arcsinh}(ax^n)}{x}\,dx$

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6284
```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Simp[1/p   Subst[Int[
x^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

3.311.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.00

method	result
derivativedivides	$\frac{-\frac{\operatorname{arcsinh}(a\,x^{n})^{2}}{2}+\operatorname{arcsinh}(a\,x^{n})\ln\left(1+a\,x^{n}+\sqrt{1+a^{2}x^{2n}}\right)+\operatorname{polylog}\left(2,-a\,x^{n}-\sqrt{1+a^{2}x^{2n}}\right)+\operatorname{arcsinh}(a\,x^{n})\ln\left(1-a\,x^{n}-\right.}{n}$
default	$\frac{-\frac{\operatorname{arcsinh}(a\,x^{n})^{2}}{2}+\operatorname{arcsinh}(a\,x^{n})\ln\left(1+a\,x^{n}+\sqrt{1+a^{2}x^{2n}}\right)+\operatorname{polylog}\left(2,-a\,x^{n}-\sqrt{1+a^{2}x^{2n}}\right)+\operatorname{arcsinh}(a\,x^{n})\ln\left(1-a\,x^{n}-\right.}{n}$

input
```
int(arcsinh(a*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(-1/2*arcsinh(a*x^n)^2+arcsinh(a*x^n)*ln(1+a*x^n+(1+a^2*(x^n)^2)^(1/2)
)+polylog(2,-a*x^n-(1+a^2*(x^n)^2)^(1/2))+arcsinh(a*x^n)*ln(1-a*x^n-(1+a^2
*(x^n)^2)^(1/2))+polylog(2,a*x^n+(1+a^2*(x^n)^2)^(1/2)))
```

3.311.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x^n)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.311.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x}\,dx = \int \frac{\operatorname{asinh}(ax^n)}{x}\,dx$$

input
```
integrate(asinh(a*x**n)/x,x)
```

output
```
Integral(asinh(a*x**n)/x, x)
```

3.311.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x}\,dx = \int \frac{\operatorname{arsinh}(ax^n)}{x}\,dx$$

input
```
integrate(arcsinh(a*x^n)/x,x, algorithm="maxima")
```

output
```
-a*n*integrate(x^n*log(x)/(a^3*x*x^(3*n) + a*x*x^n + (a^2*x*x^(2*n) + x)*s
qrt(a^2*x^(2*n) + 1)), x) - 1/2*n*log(x)^2 + n*integrate(log(x)/(a^2*x*x^(
2*n) + x), x) + log(a*x^n + sqrt(a^2*x^(2*n) + 1))*log(x)
```

3.311.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x}\,dx = \int \frac{\operatorname{arsinh}(ax^n)}{x}\,dx$$

input
```
integrate(arcsinh(a*x^n)/x,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x^n)/x, x)
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x}\,dx = \int \frac{\operatorname{asinh}(a\,x^n)}{x}\,dx$$

input
```
int(asinh(a*x^n)/x,x)
```

output
```
int(asinh(a*x^n)/x, x)
```

3.312 $\int \frac{\text{arcsinh}(ax^n)}{x^2}\,dx$

3.312.1 Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \frac{\text{arcsinh}(ax^n)}{x^2}\,dx = -\frac{\text{arcsinh}(ax^n)}{x} - \frac{anx^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1-n}{2n},\frac{1}{2}\left(3-\frac{1}{n}\right),-a^2x^{2n}\right)}{1-n}$$

output
```
-arcsinh(a*x^n)/x-a*n*x^(-1+n)*hypergeom([1/2, 1/2*(-1+n)/n],[3/2-1/2/n],-
a^2*x^(2*n))/(1-n)
```

3.312.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{\text{arcsinh}(ax^n)}{x^2}\,dx = -\frac{\text{arcsinh}(ax^n)}{x} + \frac{anx^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{-1+n}{2n},1+\frac{-1+n}{2n},-a^2x^{2n}\right)}{-1+n}$$

input
```
Integrate[ArcSinh[a*x^n]/x^2,x]
```

output
```
-(ArcSinh[a*x^n]/x) + (a*n*x^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/(2*n
), 1 + (-1 + n)/(2*n), -(a^2*x^(2*n))])/(-1 + n)
```

3.312.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6290, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(ax^n)}{x^2} \, dx$$

$$\Big\downarrow \text{6290}$$

$$\int \frac{anx^{n-2}}{\sqrt{a^2x^{2n}+1}}dx - \frac{\text{arcsinh}(ax^n)}{x}$$

$$\Big\downarrow \text{27}$$

$$an\int \frac{x^{n-2}}{\sqrt{a^2x^{2n}+1}}dx - \frac{\text{arcsinh}(ax^n)}{x}$$

$$\Big\downarrow \text{888}$$

$$-\frac{anx^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1-n}{2n},\frac{1}{2}\left(3-\frac{1}{n}\right),-a^2x^{2n}\right)}{1-n} - \frac{\text{arcsinh}(ax^n)}{x}$$

input
```
Int[ArcSinh[a*x^n]/x^2,x]
```

output
```
-(ArcSinh[a*x^n]/x) - (a*n*x^(-1 + n)*Hypergeometric2F1[1/2, -1/2*(1 - n)/
n, (3 - n^(-1))/2, -(a^2*x^(2*n))])/(1 - n)
```

3.312.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6290
```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x],
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u
, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[
u, x]
```

3.312.4 Maple [F]

$$\int \frac{\operatorname{arcsinh}(a\,x^n)}{x^2}dx$$

input
```
int(arcsinh(a*x^n)/x^2,x)
```

output
```
int(arcsinh(a*x^n)/x^2,x)
```

3.312.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x^n)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.312.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^2}\,dx = \int \frac{\operatorname{asinh}(ax^n)}{x^2}\,dx$$

input
```
integrate(asinh(a*x**n)/x**2,x)
```

output
```
Integral(asinh(a*x**n)/x**2, x)
```

3.312.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^2}\,dx = \int \frac{\operatorname{arsinh}(ax^n)}{x^2}\,dx$$

input
```
integrate(arcsinh(a*x^n)/x^2,x, algorithm="maxima")
```

output
```
a*n*integrate(x^n/(a^3*x^2*x^(3*n) + a*x^2*x^n + (a^2*x^2*x^(2*n) + x^2)*s
qrt(a^2*x^(2*n) + 1)), x) - n*integrate(1/(a^2*x^2*x^(2*n) + x^2), x) - (n
 + log(a*x^n + sqrt(a^2*x^(2*n) + 1)))/x
```

3.312.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^2}\,dx = \int \frac{\operatorname{arsinh}(ax^n)}{x^2}\,dx$$

input
```
integrate(arcsinh(a*x^n)/x^2,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x^n)/x^2, x)
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^2}\,dx = \int \frac{\operatorname{asinh}(a\,x^n)}{x^2}\,dx$$

input
```
int(asinh(a*x^n)/x^2,x)
```

output
```
int(asinh(a*x^n)/x^2, x)
```

3.313 $\int \frac{\text{arcsinh}(ax^n)}{x^3}\,dx$

3.313.1 Optimal result

Integrand size = 10, antiderivative size = 68

$$\int \frac{\text{arcsinh}(ax^n)}{x^3}\,dx = -\frac{\text{arcsinh}(ax^n)}{2x^2} - \frac{anx^{-2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(1-\frac{2}{n}\right),\frac{1}{2}\left(3-\frac{2}{n}\right),-a^2x^{2n}\right)}{2(2-n)}$$

output
```
-1/2*arcsinh(a*x^n)/x^2-1/2*a*n*x^(-2+n)*hypergeom([1/2, 1/2-1/n],[3/2-1/n
],-a^2*x^(2*n))/(2-n)
```

3.313.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{\text{arcsinh}(ax^n)}{x^3}\,dx = \frac{-((-2+n)\text{arcsinh}(ax^n)) + anx^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-\frac{1}{n},\frac{3}{2}-\frac{1}{n},-a^2x^{2n}\right)}{2(-2+n)x^2}$$

input
```
Integrate[ArcSinh[a*x^n]/x^3,x]
```

output
```
(-((-2 + n)*ArcSinh[a*x^n]) + a*n*x^n*Hypergeometric2F1[1/2, 1/2 - n^(-1),
 3/2 - n^(-1), -(a^2*x^(2*n))])/(2*(-2 + n)*x^2)
```

3.313.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6290, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^3} \, dx$$

$$\Big\downarrow \; 6290$$

$$\frac{1}{2} \int \frac{anx^{n-3}}{\sqrt{a^2x^{2n}+1}} dx - \frac{\operatorname{arcsinh}(ax^n)}{2x^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2} an \int \frac{x^{n-3}}{\sqrt{a^2x^{2n}+1}} dx - \frac{\operatorname{arcsinh}(ax^n)}{2x^2}$$

$$\Big\downarrow \; 888$$

$$-\frac{anx^{n-2} \text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{2-n}{2n}, \frac{1}{2}\left(3-\frac{2}{n}\right), -a^2x^{2n}\right)}{2(2-n)} - \frac{\operatorname{arcsinh}(ax^n)}{2x^2}$$

input
```
Int[ArcSinh[a*x^n]/x^3,x]
```

output
```
-1/2*ArcSinh[a*x^n]/x^2 - (a*n*x^(-2 + n)*Hypergeometric2F1[1/2, -1/2*(2 -
 n)/n, (3 - 2/n)/2, -(a^2*x^(2*n))])/(2*(2 - n))
```

3.313.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6290
```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x],
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u
, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[
u, x]
```

3.313.4 Maple [F]

$$\int \frac{\operatorname{arcsinh}\left(a\,x^{n}\right)}{x^{3}}dx$$

input
```
int(arcsinh(a*x^n)/x^3,x)
```

output
```
int(arcsinh(a*x^n)/x^3,x)
```

3.313.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(a*x^n)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.313.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax^n)}{x^3}\,dx = \int \frac{\operatorname{asinh}\left(ax^n\right)}{x^3}\,dx$$

input
```
integrate(asinh(a*x**n)/x**3,x)
```

output
```
Integral(asinh(a*x**n)/x**3, x)
```

3.313.7 Maxima [F]

$$\int \frac{\text{arcsinh}(ax^n)}{x^3}\,dx = \int \frac{\text{arsinh}\,(ax^n)}{x^3}\,dx$$

input
```
integrate(arcsinh(a*x^n)/x^3,x, algorithm="maxima")
```

output
```
a*n*integrate(1/2*x^n/(a^3*x^3*x^(3*n) + a*x^3*x^n + (a^2*x^3*x^(2*n) + x^
3)*sqrt(a^2*x^(2*n) + 1)), x) - n*integrate(1/2/(a^2*x^3*x^(2*n) + x^3), x
) - 1/4*(n + 2*log(a*x^n + sqrt(a^2*x^(2*n) + 1)))/x^2
```

3.313.8 Giac [F]

$$\int \frac{\text{arcsinh}(ax^n)}{x^3}\,dx = \int \frac{\text{arsinh}\,(ax^n)}{x^3}\,dx$$

input
```
integrate(arcsinh(a*x^n)/x^3,x, algorithm="giac")
```

output
```
integrate(arcsinh(a*x^n)/x^3, x)
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arcsinh}(ax^n)}{x^3}\,dx = \int \frac{\text{asinh}(a\,x^n)}{x^3}\,dx$$

input
```
int(asinh(a*x^n)/x^3,x)
```

output
```
int(asinh(a*x^n)/x^3, x)
```

3.314 $\int (a+ib\arcsin(1-idx^2))^4\,dx$

3.314.1 Optimal result

Integrand size = 20, antiderivative size = 153

$$\int (a+ib\arcsin(1-idx^2))^4\,dx = 384b^4x - \frac{192b^3\sqrt{2idx^2+d^2x^4}(a+ib\arcsin(1-idx^2))}{dx} + 48b^2x(a+ib\arcsin(1-idx^2))^2 - \frac{8b\sqrt{2idx^2+d^2x^4}(a+ib\arcsin(1-idx^2))^3}{dx} + x(a+ib\arcsin(1-idx^2))^4$$

output
```
384*b^4*x+48*b^2*x*(a-I*b*arcsin(-1+I*d*x^2))^2+x*(a-I*b*arcsin(-1+I*d*x^2
))^4-192*b^3*(a-I*b*arcsin(-1+I*d*x^2))*(2*I*d*x^2+d^2*x^4)^(1/2)/d/x-8*b*
(a-I*b*arcsin(-1+I*d*x^2))^3*(2*I*d*x^2+d^2*x^4)^(1/2)/d/x
```

3.314.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

$$\int (a+ib\arcsin(1-idx^2))^4\,dx = -\frac{8b\sqrt{dx^2(2i+dx^2)}(a+ib\arcsin(1-idx^2))^3}{dx} + x(a+ib\arcsin(1-idx^2))^4 + 48b^2\left(8b^2x - \frac{4b\sqrt{dx^2(2i+dx^2)}(a+ib\arcsin(1-idx^2))}{dx} + x(a+ib\arcsin(1-idx^2))^2\right)$$

input
```
Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^4,x]
```

output
```
(-8*b*Sqrt[d*x^2*(2*I + d*x^2)]*(a + I*b*ArcSin[1 - I*d*x^2])^3)/(d*x) + x
*(a + I*b*ArcSin[1 - I*d*x^2])^4 + 48*b^2*(8*b^2*x - (4*b*Sqrt[d*x^2*(2*I
+ d*x^2)]*(a + I*b*ArcSin[1 - I*d*x^2]))/(d*x) + x*(a + I*b*ArcSin[1 - I*d
*x^2])^2)
```

3.314.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5313, 5313, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^4dx$$

$$\Big\downarrow \text{5313}$$

$$48b^2\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^2dx-\frac{8b\sqrt{d^2x^4+2idx^2}\left(a+ib\arcsin\left(1-idx^2\right)\right)^3}{dx}+x\left(a+ib\arcsin\left(1-idx^2\right)\right)^4$$

$$\Big\downarrow \text{5313}$$

$$48b^2\left(8b^2\int 1dx-\frac{4b\sqrt{d^2x^4+2idx^2}\left(a+ib\arcsin\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\arcsin\left(1-idx^2\right)\right)^2\right)-\frac{8b\sqrt{d^2x^4+2idx^2}\left(a+ib\arcsin\left(1-idx^2\right)\right)^3}{dx}+x\left(a+ib\arcsin\left(1-idx^2\right)\right)^4$$

$$\Big\downarrow \text{24}$$

$$48b^2\left(-\frac{4b\sqrt{d^2x^4+2idx^2}\left(a+ib\arcsin\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\arcsin\left(1-idx^2\right)\right)^2+8b^2x\right)-\frac{8b\sqrt{d^2x^4+2idx^2}\left(a+ib\arcsin\left(1-idx^2\right)\right)^3}{dx}+x\left(a+ib\arcsin\left(1-idx^2\right)\right)^4$$

input
```
Int[(a + I*b*ArcSin[1 - I*d*x^2])^4,x]
```

output
```
(-8*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2])^3)/(d*x) +
 x*(a + I*b*ArcSin[1 - I*d*x^2])^4 + 48*b^2*(8*b^2*x - (4*b*Sqrt[(2*I)*d*x
^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2]))/(d*x) + x*(a + I*b*ArcSin[1 -
 I*d*x^2])^2)
```

3.314.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5313
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a
 + b*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
+ b*ArcSin[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

3.314.4 Maple [F]

$$\int \left(a+b\,\mathrm{arcsinh}\left(d\,x^{2}+i\right)\right)^{4}dx$$

input
```
int((a+b*arcsinh(I+d*x^2))^4,x)
```

output
```
int((a+b*arcsinh(I+d*x^2))^4,x)
```

3.314.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(129) = 258.

Time = 0.26 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.76

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^4\,dx$$
$$=\frac{b^4dx\log\left(dx^2+\sqrt{d^2x^2+2i\,d}x+i\right)^4+4\left(ab^3dx-2\sqrt{d^2x^2+2i\,d}b^4\right)\log\left(dx^2+\sqrt{d^2x^2+2i\,d}x+i\right)^3+(a^4$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^4,x, algorithm="fricas")
```

output
```
(b^4*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^4 + 4*(a*b^3*d*x - 2*sqr
t(d^2*x^2 + 2*I*d)*b^4)*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^3 + (a^4
+ 48*a^2*b^2 + 384*b^4)*d*x - 6*(4*sqrt(d^2*x^2 + 2*I*d)*a*b^3 - (a^2*b^2
+ 8*b^4)*d*x)*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^2 + 4*((a^3*b + 24*
a*b^3)*d*x - 6*(a^2*b^2 + 8*b^4)*sqrt(d^2*x^2 + 2*I*d))*log(d*x^2 + sqrt(d
^2*x^2 + 2*I*d)*x + I) - 8*(a^3*b + 24*a*b^3)*sqrt(d^2*x^2 + 2*I*d))/d
```

3.314.6 Sympy [F(-2)]

Exception generated.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^4\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*asinh(I+d*x**2))**4,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real I
```

3.314.7 Maxima [F]

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^4\,dx=\int \left(b\,\mathrm{arsinh}\left(dx^2+i\right)+a\right)^4\,dx$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^4,x, algorithm="maxima")
```

output
```
b^4*x*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^4 + 4*(x*arcsinh(d*x^2
+ I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*d))*a^3*b + a^4*x
+ integrate(2*(2*((a*b^3*d^2 - 2*b^4*d^2)*x^4 - 2*a*b^3 - (-3*I*a*b^3*d +
4*I*b^4*d)*x^2 + ((a*b^3*d^(3/2) - 2*b^4*d^(3/2))*x^3 - 2*(-I*a*b^3*sqrt(d
) + I*b^4*sqrt(d))*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqr
t(d)*x + I)^3 + 3*(a^2*b^2*d^2*x^4 + 3*I*a^2*b^2*d*x^2 - 2*a^2*b^2 + (a^2*
b^2*d^(3/2)*x^3 + 2*I*a^2*b^2*sqrt(d)*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sq
rt(d*x^2 + 2*I)*sqrt(d)*x + I)^2)/(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*x^3 + 2*
I*sqrt(d)*x)*sqrt(d*x^2 + 2*I) - 2), x)
```

3.314.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^4\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^4\,dx=\int \left(a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)\right)^4\,dx$$

input
```
int((a + b*asinh(d*x^2 + 1i))^4,x)
```

output
```
int((a + b*asinh(d*x^2 + 1i))^4, x)
```

3.315 $\int (a+ib\arcsin(1-idx^2))^3\,dx$

3.315.1 Optimal result

Integrand size = 20, antiderivative size = 129

$$\int (a+ib\arcsin(1-idx^2))^3\,dx = 24ab^2x-\frac{48b^3\sqrt{2idx^2+d^2x^4}}{dx}+24ib^3x\arcsin(1-idx^2)-\frac{6b\sqrt{2idx^2+d^2x^4}(a+ib\arcsin(1-idx^2))^2}{dx}+x(a+ib\arcsin(1-idx^2))^3$$

output
```
24*a*b^2*x-24*I*b^3*x*arcsin(-1+I*d*x^2)+x*(a-I*b*arcsin(-1+I*d*x^2))^3-48
*b^3*(2*I*d*x^2+d^2*x^4)^(1/2)/d/x-6*b*(a-I*b*arcsin(-1+I*d*x^2))^2*(2*I*d
*x^2+d^2*x^4)^(1/2)/d/x
```

3.315.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.40

$$\int (a+ib\arcsin(1-idx^2))^3\,dx = \frac{a(a^2+24b^2)\,dx^2-6b(a^2+8b^2)\sqrt{dx^2(2i+dx^2)}+3ib\left(a^2dx^2+8b^2dx^2-4ab\sqrt{dx^2(2i+dx^2)}\right)\arcsin(1}{dx}$$

input
```
Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^3,x]
```

output
```
(a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2*(2*I + d*x^2)] + (3
*I)*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2*(2*I + d*x^2)])*ArcSin[1
 - I*d*x^2] + 3*b^2*(-(a*d*x^2) + 2*b*Sqrt[d*x^2*(2*I + d*x^2)])*ArcSin[1
- I*d*x^2]^2 - I*b^3*d*x^2*ArcSin[1 - I*d*x^2]^3)/(d*x)
```

3.315.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5313, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^3dx$$

$$\Big\downarrow \text{5313}$$

$$24b^2\int \left(a+ib\arcsin\left(1-idx^2\right)\right)dx-\frac{6b\sqrt{d^2x^4+2idx^2}\left(a+ib\arcsin\left(1-idx^2\right)\right)^2}{dx}+x\left(a+ib\arcsin\left(1-idx^2\right)\right)^3$$

$$\Big\downarrow \text{2009}$$

$$24b^2\left(ax+ibx\arcsin\left(1-idx^2\right)-\frac{2b\sqrt{d^2x^4+2idx^2}}{dx}\right)-\frac{6b\sqrt{d^2x^4+2idx^2}\left(a+ib\arcsin\left(1-idx^2\right)\right)^2}{dx}+x\left(a+ib\arcsin\left(1-idx^2\right)\right)^3$$

input
```
Int[(a + I*b*ArcSin[1 - I*d*x^2])^3,x]
```

output
```
(-6*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2])^2)/(d*x) +
 x*(a + I*b*ArcSin[1 - I*d*x^2])^3 + 24*b^2*(a*x - (2*b*Sqrt[(2*I)*d*x^2 +
 d^2*x^4])/(d*x) + I*b*x*ArcSin[1 - I*d*x^2])
```

3.315.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5313
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a
 + b*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
+ b*ArcSin[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

3.315.4 Maple [F]

$$\int \left(a+b\,\operatorname{arcsinh}\left(d\,x^{2}+i\right)\right)^{3}dx$$

input
```
int((a+b*arcsinh(I+d*x^2))^3,x)
```

output
```
int((a+b*arcsinh(I+d*x^2))^3,x)
```

3.315.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.46

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^3\,dx$$
$$= \frac{b^3dx\log\left(dx^2+\sqrt{d^2x^2+2i\,d}x+i\right)^3+(a^3+24\,ab^2)dx+3\left(ab^2dx-2\sqrt{d^2x^2+2i\,d}b^3\right)\log\left(dx^2+\sqrt{d^2x^2\cdots}\right.}{}$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^3,x, algorithm="fricas")
```

output
```
(b^3*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^3 + (a^3 + 24*a*b^2)*d*x
 + 3*(a*b^2*d*x - 2*sqrt(d^2*x^2 + 2*I*d)*b^3)*log(d*x^2 + sqrt(d^2*x^2 +
2*I*d)*x + I)^2 - 3*(4*sqrt(d^2*x^2 + 2*I*d)*a*b^2 - (a^2*b + 8*b^3)*d*x)*
log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) - 6*sqrt(d^2*x^2 + 2*I*d)*(a^2*b
+ 8*b^3))/d
```

3.315.6 Sympy [F(-2)]

Exception generated.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^3 dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*asinh(I+d*x**2))**3,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real I
```

3.315.7 Maxima [F]

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^3 dx = \int \left(b\,\mathrm{arsinh}\left(dx^2+i\right)+a\right)^3 dx$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^3,x, algorithm="maxima")
```

output
```
b^3*x*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^3 + 3*(x*arcsinh(d*x^2
+ I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*d))*a^2*b + a^3*x
+ integrate(3*((a*b^2*d^2 - 2*b^3*d^2)*x^4 - 2*a*b^2 - (-3*I*a*b^2*d + 4*I
*b^3*d)*x^2 + ((a*b^2*d^(3/2) - 2*b^3*d^(3/2))*x^3 - 2*(-I*a*b^2*sqrt(d) +
 I*b^3*sqrt(d))*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d
)*x + I)^2/(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*x^3 + 2*I*sqrt(d)*x)*sqrt(d*x^2
 + 2*I) - 2), x)
```

3.315.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^3 dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+ib \arcsin \left(1-idx^2\right)\right)^3 \, dx=\int {\left(a+b\,\mathrm{asinh}\left(d\,x^2+1{}\mathrm{i}\right)\right)}^3 \,d x$$

input
```
int((a + b*asinh(d*x^2 + 1i))^3,x)
```

output
```
int((a + b*asinh(d*x^2 + 1i))^3, x)
```

3.316 $\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^2\,dx$

3.316.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^2\,dx=8b^2x-\frac{4b\sqrt{2idx^2+d^2x^4}\left(a+ib\arcsin\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\arcsin\left(1-idx^2\right)\right)^2$$

output
```
8*b^2*x+x*(a-I*b*arcsin(-1+I*d*x^2))^2-4*b*(a-I*b*arcsin(-1+I*d*x^2))*(2*I
*d*x^2+d^2*x^4)^(1/2)/d/x
```

3.316.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^2\,dx=8b^2x-\frac{4b\sqrt{2idx^2+d^2x^4}\left(a+ib\arcsin\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\arcsin\left(1-idx^2\right)\right)^2$$

input
```
Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^2,x]
```

output
```
8*b^2*x - (4*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2]))/
(d*x) + x*(a + I*b*ArcSin[1 - I*d*x^2])^2
```

3.316.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5313, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^2 dx$$

$$\Big\downarrow \; 5313$$

$$8b^2\int 1dx-\frac{4b\sqrt{d^2x^4+2idx^2}\left(a+ib\arcsin\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\arcsin\left(1-idx^2\right)\right)^2$$

$$\Big\downarrow \; 24$$

$$-\frac{4b\sqrt{d^2x^4+2idx^2}\left(a+ib\arcsin\left(1-idx^2\right)\right)}{dx}+x\left(a+ib\arcsin\left(1-idx^2\right)\right)^2+8b^2x$$

input
```
Int[(a + I*b*ArcSin[1 - I*d*x^2])^2,x]
```

output
```
8*b^2*x - (4*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2]))/
(d*x) + x*(a + I*b*ArcSin[1 - I*d*x^2])^2
```

3.316.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5313
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a
 + b*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
+ b*ArcSin[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

3.316.4 Maple [F]

$$\int \left(a+b\,\mathrm{arcsinh}\left(d\,x^{2}+i\right)\right)^{2}dx$$

input
```
int((a+b*arcsinh(I+d*x^2))^2,x)
```

output
```
int((a+b*arcsinh(I+d*x^2))^2,x)
```

3.316.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.50

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^2\,dx$$
$$=\frac{b^2dx\log\left(dx^2+\sqrt{d^2x^2+2i\,d}x+i\right)^2+(a^2+8\,b^2)dx-4\,\sqrt{d^2x^2+2i\,d}ab+2\left(abdx-2\,\sqrt{d^2x^2+2i\,d}b^2\right)\log\left(dx^2+\sqrt{d^2x^2+2i\,d}x+i\right)}{d}$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^2,x, algorithm="fricas")
```

output
```
(b^2*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^2 + (a^2 + 8*b^2)*d*x -
4*sqrt(d^2*x^2 + 2*I*d)*a*b + 2*(a*b*d*x - 2*sqrt(d^2*x^2 + 2*I*d)*b^2)*lo
g(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I))/d
```

3.316.6 Sympy [F(-2)]

Exception generated.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*asinh(I+d*x**2))**2,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real I
```

3.316.7 Maxima [F]

$$\int \left(a+ib\arcsin \left(1-idx^2\right)\right)^2\,dx=\int \left(b\operatorname{arsinh}\left(dx^2+i\right)+a\right)^2\,dx$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^2,x, algorithm="maxima")
```

output
```
2*(x*arcsinh(d*x^2 + I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)
*d))*a*b + (x*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^2 - integrate(4
*(d^2*x^4 + 2*I*d*x^2 + (d^(3/2)*x^3 + I*sqrt(d)*x)*sqrt(d*x^2 + 2*I))*log
(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)/(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*
x^3 + 2*I*sqrt(d)*x)*sqrt(d*x^2 + 2*I) - 2), x))*b^2 + a^2*x
```

3.316.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+ib\arcsin \left(1-idx^2\right)\right)^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+ib\arcsin \left(1-idx^2\right)\right)^2\,dx=\int \left(a+b\operatorname{asinh}\left(d\,x^2+1\mathrm{i}\right)\right)^2\,dx$$

input
```
int((a + b*asinh(d*x^2 + 1i))^2,x)
```

output
```
int((a + b*asinh(d*x^2 + 1i))^2, x)
```

3.317 $\int (a+ib\arcsin(1-idx^2))\,dx$

3.317.1 Optimal result

Integrand size = 18, antiderivative size = 50

$$\int (a+ib\arcsin(1-idx^2))\,dx = ax-\frac{2b\sqrt{2idx^2+d^2x^4}}{dx}+ibx\arcsin(1-idx^2)$$

output
```
a*x-I*b*x*arcsin(-1+I*d*x^2)-2*b*(2*I*d*x^2+d^2*x^4)^(1/2)/d/x
```

3.317.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+ib\arcsin(1-idx^2))\,dx = ax-\frac{2b\sqrt{dx^2(2i+dx^2)}}{dx}+ibx\arcsin(1-idx^2)$$

input
```
Integrate[a + I*b*ArcSin[1 - I*d*x^2],x]
```

output
```
a*x - (2*b*Sqrt[d*x^2*(2*I + d*x^2)])/(d*x) + I*b*x*ArcSin[1 - I*d*x^2]
```

3.317.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$ax+ibx\arcsin\left(1-idx^2\right)-\frac{2b\sqrt{d^2x^4+2idx^2}}{dx}$$

input
```
Int[a + I*b*ArcSin[1 - I*d*x^2],x]
```

output
```
a*x - (2*b*Sqrt[(2*I)*d*x^2 + d^2*x^4])/(d*x) + I*b*x*ArcSin[1 - I*d*x^2]
```

3.317.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.317.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

method	result	size
default	$ax+b\left(x\,\operatorname{arcsinh}\left(d\,x^2+i\right)-\frac{2x\left(d\,x^2+2i\right)}{\sqrt{d^2x^4+2id\,x^2}}\right)$	47
parts	$ax+b\left(x\,\operatorname{arcsinh}\left(d\,x^2+i\right)-\frac{2x\left(d\,x^2+2i\right)}{\sqrt{d^2x^4+2id\,x^2}}\right)$	47

input
```
int(a+b*arcsinh(I+d*x^2),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*(x*arcsinh(I+d*x^2)-2/(2*I*d*x^2+d^2*x^4)^(1/2)*x*(d*x^2+2*I))
```

3.317.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)\,dx=\frac{bdx\log\left(dx^2+\sqrt{d^2x^2+2i\,d}x+i\right)+adx-2\sqrt{d^2x^2+2i\,d}b}{d}$$

input
```
integrate(a+b*arcsinh(I+d*x^2),x, algorithm="fricas")
```

output
```
(b*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a*d*x - 2*sqrt(d^2*x^2 +
 2*I*d)*b)/d
```

3.317.6 Sympy [F(-2)]

Exception generated.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(a+b*asinh(I+d*x**2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real I
```

3.317.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)\,dx=\left(x\,\text{arsinh}\left(dx^2+i\right)-\frac{2\left(d^{\frac{3}{2}}x^2+2i\sqrt{d}\right)}{\sqrt{dx^2+2id}}\right)b+ax$$

input
```
integrate(a+b*arcsinh(I+d*x^2),x, algorithm="maxima")
```

output
```
(x*arcsinh(d*x^2 + I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*d
))*b + a*x
```

3.317.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(a+b*arcsinh(I+d*x^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.317.9 Mupad [B] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)\,dx=a\,x+b\,x\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)-\frac{2\,b\,\sqrt{{\left(d\,x^2+1\mathrm{i}\right)}^2+1}}{d\,x}$$

input
```
int(a + b*asinh(d*x^2 + 1i),x)
```

output
```
a*x + b*x*asinh(d*x^2 + 1i) - (2*b*((d*x^2 + 1i)^2 + 1)^(1/2))/(d*x)
```

3.318 $\int \frac{1}{a+ib \arcsin(1-idx^2)} \, dx$

3.318.1 Optimal result

Integrand size = 20, antiderivative size = 194

$$\int \frac{1}{a+ib \arcsin(1-idx^2)} \, dx = \frac{x \operatorname{CosIntegral}\left(-\frac{i(a+ib\arcsin(1-idx^2))}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)} - \frac{x\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\operatorname{Si}\left(\frac{ia}{2b}-\frac{1}{2}\arcsin(1-idx^2)\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}$$

output
```
1/2*x*Ci(-1/2*I*(a-I*b*arcsin(-1+I*d*x^2))/b)*(I*cosh(1/2*a/b)-sinh(1/2*a/
b))/b/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))-1/2*x*Si(1
/2*I*a/b+1/2*arcsin(-1+I*d*x^2))*(I*cosh(1/2*a/b)+sinh(1/2*a/b))/b/(cos(1/
2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))
```

3.318.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

$$\int \frac{1}{a+ib \arcsin(1-idx^2)} \, dx = \frac{x\left(\operatorname{CosIntegral}\left(\frac{1}{2}\left(-\frac{ia}{b}+\arcsin(1-idx^2)\right)\right)\left(i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)+\left(-i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\operatorname{Si}\left(\frac{ia}{2b}-\right.}{2b\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}$$

input
```
Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-1),x]
```

output
```
(x*(CosIntegral[(((-I)*a)/b + ArcSin[1 - I*d*x^2])/2]*(I*Cosh[a/(2*b)] - S
inh[a/(2*b)]) + ((-I)*Cosh[a/(2*b)] - Sinh[a/(2*b)])*SinIntegral[((I/2)*a)
/b - ArcSin[1 - I*d*x^2]/2]))/(2*b*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSi
n[1 - I*d*x^2]/2]))
```

3.318.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5315}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+ib\arcsin\left(1-idx^2\right)}dx$$

$$\Big\downarrow \text{5315}$$

$$\frac{x\left(-\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(-\frac{i\left(a+ib\arcsin\left(1-idx^2\right)\right)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$

input
```
Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-1),x]
```

output
```
(x*CosIntegral[((-1/2*I)*(a + I*b*ArcSin[1 - I*d*x^2]))/b]*(I*Cosh[a/(2*b)
] - Sinh[a/(2*b)]))/(2*b*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*
x^2]/2])) - (x*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b -
 ArcSin[1 - I*d*x^2]/2])/(2*b*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 -
 I*d*x^2]/2]))
```

3.318.3.1 Defintions of rubi rules used

rule 5315
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(-x
)*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c +
d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x]
 - Simp[x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Arc
Sin[c + d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2
]))), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

3.318.4 Maple [F]

$$\int \frac{1}{a+b\,\mathrm{arcsinh}\left(d\,x^{2}+i\right)}dx$$

input
```
int(1/(a+b*arcsinh(I+d*x^2)),x)
```

output
```
int(1/(a+b*arcsinh(I+d*x^2)),x)
```

3.318.5 Fricas [F]

$$\int \frac{1}{a+ib\arcsin(1-idx^2)}\,dx = \int \frac{1}{b\,\mathrm{arsinh}\left(dx^2+i\right)+a}\,dx$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2)),x, algorithm="fricas")
```

output
```
integral(1/(b*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a), x)
```

3.318.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{a+ib\arcsin(1-idx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*asinh(I+d*x**2)),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real I
```

3.318.7 Maxima [F]

$$\int \frac{1}{a+ib\arcsin\left(1-idx^2\right)}\,dx=\int \frac{1}{b\,\text{arsinh}\left(dx^2+i\right)+a}\,dx$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arcsinh(d*x^2 + I) + a), x)
```

3.318.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{a+ib\arcsin\left(1-idx^2\right)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+ib\arcsin\left(1-idx^2\right)}\,dx=\int \frac{1}{a+b\,\text{asinh}\left(d\,x^2+1\mathrm{i}\right)}\,dx$$

input
```
int(1/(a + b*asinh(d*x^2 + 1i)),x)
```

output
```
int(1/(a + b*asinh(d*x^2 + 1i)), x)
```

3.319 $\int \frac{1}{(a+ib\arcsin(1-idx^2))^2}\,dx$

3.319.1 Optimal result

Integrand size = 20, antiderivative size = 245

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^2}\,dx = -\frac{\sqrt{2idx^2+d^2x^4}}{2bdx\,(a+ib\arcsin(1-idx^2))} + \frac{x\,\text{CosIntegral}\left(-\frac{i(a+ib\arcsin(1-idx^2))}{2b}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)} + \frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\arcsin(1-idx^2)\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}$$

output
```
1/4*x*Ci(-1/2*I*(a-I*b*arcsin(-1+I*d*x^2))/b)*(cosh(1/2*a/b)-I*sinh(1/2*a/
b))/b^2/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))+1/4*x*Si
(1/2*I*a/b+1/2*arcsin(-1+I*d*x^2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))/b^2/(co
s(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))-1/2*(2*I*d*x^2+d^2*
x^4)^(1/2)/b/d/x/(a-I*b*arcsin(-1+I*d*x^2))
```

3.319.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^2}\,dx$$
$$= \frac{-\frac{2b\sqrt{dx^2(2i+dx^2)}}{d(a+ib\arcsin(1-idx^2))} + \frac{x^2 (\operatorname{CosIntegral}(\frac{1}{2}(-\frac{ia}{b}+\arcsin(1-idx^2)))(\cosh(\frac{a}{2b})-i\sinh(\frac{a}{2b}))+(\cosh(\frac{a}{2b})+i\sinh(\frac{a}{2b}))\operatorname{Si}(\frac{ia}{2b}-\frac{1}{2}\arcsin(1-i}{\cos(\frac{1}{2}\arcsin(1-idx^2))-\sin(\frac{1}{2}\arcsin(1-idx^2))}}{4b^2x}$$

input
```
Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-2),x]
```

output
```
((-2*b*Sqrt[d*x^2*(2*I + d*x^2)])/(d*(a + I*b*ArcSin[1 - I*d*x^2])) + (x^2
*(CosIntegral[(((-I)*a)/b + ArcSin[1 - I*d*x^2])/2]*(Cosh[a/(2*b)] - I*Sin
h[a/(2*b)]) + (Cosh[a/(2*b)] + I*Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b -
ArcSin[1 - I*d*x^2]/2]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*
x^2]/2]))/(4*b^2*x)
```

3.319.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5324}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^2}\,dx$$
$$\Big\downarrow \; 5324$$
$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\operatorname{CosIntegral}\left(-\frac{i(a+ib\arcsin(1-idx^2))}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}+\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\operatorname{Si}\left(\frac{ia}{2b}-\frac{1}{2}\arcsin(1-idx^2)\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}-\frac{\sqrt{d^2x^4+2idx^2}}{2bdx\,(a+ib\arcsin(1-idx^2))}$$

input
```
Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-2),x]
```

output
```
-1/2*Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])) + (
x*CosIntegral[((-1/2*I)*(a + I*b*ArcSin[1 - I*d*x^2]))/b]*(Cosh[a/(2*b)] -
 I*Sinh[a/(2*b)]))/(4*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d
*x^2]/2])) + (x*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b
- ArcSin[1 - I*d*x^2]/2])/(4*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[
1 - I*d*x^2]/2]))
```

3.319.3.1 Defintions of rubi rules used

rule 5324
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> Simp[-Sq
rt[-2*c*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcSin[c + d*x^2])), x] + (-Simp[x
*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d
*x^2])]/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x
] + Simp[x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Ar
cSin[c + d*x^2])]/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2
]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

3.319.4 Maple [F]

$$\int \frac{1}{\left(a+b\ \mathrm{arcsinh}\left(d\,x^2+i\right)\right)^2}dx$$

input
```
int(1/(a+b*arcsinh(I+d*x^2))^2,x)
```

output
```
int(1/(a+b*arcsinh(I+d*x^2))^2,x)
```

3.319.5 Fricas [F]

$$\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\,\mathrm{arsinh}\left(dx^2+i\right)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^2,x, algorithm="fricas")
```

output
```
1/2*(2*(b^2*d*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a*b*d)*integral(1
/2*sqrt(d^2*x^2 + 2*I*d)*x/(a*b*d*x^2 + 2*I*a*b + (b^2*d*x^2 + 2*I*b^2)*lo
g(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)), x) - sqrt(d^2*x^2 + 2*I*d))/(b^2*
d*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a*b*d)
```

3.319.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*asinh(I+d*x**2))**2,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real I
```

3.319.7 Maxima [F]

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^2}\,dx = \int \frac{1}{(b\,\text{arsinh}(dx^2+i)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^2,x, algorithm="maxima")
```

output
```
-1/2*(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*x^3 + 2*I*sqrt(d)*x)*sqrt(d*x^2 + 2*I
) - 2)/(a*b*d^2*x^3 + 2*I*a*b*d*x + (b^2*d^2*x^3 + 2*I*b^2*d*x + (b^2*d^(3
/2)*x^2 + I*b^2*sqrt(d))*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)*
sqrt(d)*x + I) + (a*b*d^(3/2)*x^2 + I*a*b*sqrt(d))*sqrt(d*x^2 + 2*I)) + in
tegrate(1/2*(d^3*x^6 + 3*I*d^2*x^4 + (d^2*x^4 + I*d*x^2 - 2)*(d*x^2 + 2*I)
 + (2*d^(5/2)*x^5 + 4*I*d^(3/2)*x^3 - sqrt(d)*x)*sqrt(d*x^2 + 2*I) + 4*I)/
(a*b*d^3*x^6 + 4*I*a*b*d^2*x^4 - 4*a*b*d*x^2 + (a*b*d^2*x^4 + 2*I*a*b*d*x^
2 - a*b)*(d*x^2 + 2*I) + (b^2*d^3*x^6 + 4*I*b^2*d^2*x^4 - 4*b^2*d*x^2 + (b
^2*d^2*x^4 + 2*I*b^2*d*x^2 - b^2)*(d*x^2 + 2*I) + 2*(b^2*d^(5/2)*x^5 + 3*I
*b^2*d^(3/2)*x^3 - 2*b^2*sqrt(d)*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*
x^2 + 2*I)*sqrt(d)*x + I) + 2*(a*b*d^(5/2)*x^5 + 3*I*a*b*d^(3/2)*x^3 - 2*a
*b*sqrt(d)*x)*sqrt(d*x^2 + 2*I)), x)
```

3.319.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\,(d\,x^2+1\mathrm{i}))^2}\,dx$$

input
```
int(1/(a + b*asinh(d*x^2 + 1i))^2,x)
```

output
```
int(1/(a + b*asinh(d*x^2 + 1i))^2, x)
```

3.320 $\int \frac{1}{(a+ib\arcsin(1-idx^2))^3} dx$

3.320.1 Optimal result

Integrand size = 20, antiderivative size = 275

$$\begin{aligned}\int \frac{1}{(a+ib\arcsin(1-idx^2))^3}\,dx &= -\frac{\sqrt{2idx^2+d^2x^4}}{4bdx(a+ib\arcsin(1-idx^2))^2}-\frac{x}{8b^2(a+ib\arcsin(1-idx^2))}\\ &\quad+\frac{x\,\text{CosIntegral}\left(-\frac{i(a+ib\arcsin(1-idx^2))}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{16b^3\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}\\ &\quad-\frac{x\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\arcsin(1-idx^2)\right)}{16b^3\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}\end{aligned}$$

output
```
-1/8*x/b^2/(a-I*b*arcsin(-1+I*d*x^2))+1/16*x*Ci(-1/2*I*(a-I*b*arcsin(-1+I*
d*x^2))/b)*(I*cosh(1/2*a/b)-sinh(1/2*a/b))/b^3/(cos(1/2*arcsin(-1+I*d*x^2)
)+sin(1/2*arcsin(-1+I*d*x^2)))-1/16*x*Si(1/2*I*a/b+1/2*arcsin(-1+I*d*x^2))
*(I*cosh(1/2*a/b)+sinh(1/2*a/b))/b^3/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*
arcsin(-1+I*d*x^2)))-1/4*(2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-I*b*arcsin(-1+
I*d*x^2))^2
```

3.320.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^3}\,dx = \frac{-\frac{8b^2\sqrt{dx^2(2i+dx^2)}}{d(a+ib\arcsin(1-idx^2))^2}-\frac{4bx^2}{a+ib\arcsin(1-idx^2)}+\frac{2ix^2\left(\text{CosIntegral}\left(\frac{1}{2}\left(-\frac{ia}{b}+\arcsin(1-idx^2)\right)\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)-i\sin\right.\right.}{\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)}}{32b^3x}$$

input
```
Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-3),x]
```

output
```
((-8*b^2*Sqrt[d*x^2*(2*I + d*x^2)])/(d*(a + I*b*ArcSin[1 - I*d*x^2])^2) -
(4*b*x^2)/(a + I*b*ArcSin[1 - I*d*x^2]) + ((2*I)*x^2*(CosIntegral[(((-I)*a
)/b + ArcSin[1 - I*d*x^2])/2]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]) - (Cosh[a/
(2*b)] - I*Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - ArcSin[1 - I*d*x^2]/2]
))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))/(32*b^3*x)
```

3.320.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5327, 5315}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^3}\,dx$$

$$\Big\downarrow \text{5327}$$

$$\frac{\int \frac{1}{a+ib\arcsin(1-idx^2)}dx}{8b^2}-\frac{x}{8b^2(a+ib\arcsin(1-idx^2))}-\frac{\sqrt{d^2x^4+2idx^2}}{4bdx(a+ib\arcsin(1-idx^2))^2}$$

$$\Big\downarrow \text{5315}$$

$$\frac{\frac{x\left(-\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(-\frac{i\left(a+ib\arcsin\left(1-idx^2\right)\right)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\arcsin(1-idx^2)\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}}{8b^2}-\frac{x}{8b^2(a+ib\arcsin(1-idx^2))}-\frac{\sqrt{d^2x^4+2idx^2}}{4bdx(a+ib\arcsin(1-idx^2))^2}$$

input
```
Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-3),x]
```

output
```
-1/4*Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^2) -
 x/(8*b^2*(a + I*b*ArcSin[1 - I*d*x^2])) + ((x*CosIntegral[((-1/2*I)*(a +
I*b*ArcSin[1 - I*d*x^2]))/b]*(I*Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(2*b*(Cos[
ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])) - (x*(I*Cosh[a/(2*b)
] + Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - ArcSin[1 - I*d*x^2]/2])/(2*b*
(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])))/(8*b^2)
```

3.320.3.1 Defintions of rubi rules used

rule 5315
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(-x
)*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c +
d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x]
 - Simp[x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Arc
Sin[c + d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2
]))), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

rule 5327
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Simp[Sqrt
[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n + 1)*x)
), x] - Simp[1/(4*b^2*(n + 1)*(n + 2))   Int[(a + b*ArcSin[c + d*x^2])^(n +
 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

3.320.4 Maple [F]

$$\int \frac{1}{\left(a+b \,\operatorname{arcsinh}\left(d \,x^{2}+i\right)\right)^{3}}d x$$

input
```
int(1/(a+b*arcsinh(I+d*x^2))^3,x)
```

output
```
int(1/(a+b*arcsinh(I+d*x^2))^3,x)
```

3.320.5 Fricas [F]

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^3}\,dx = \int \frac{1}{(b\,\text{arsinh}(dx^2+i)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^3,x, algorithm="fricas")
```

output
```
-1/8*(b*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a*d*x - 8*(b^4*d*lo
g(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^2 + 2*a*b^3*d*log(d*x^2 + sqrt(d^2*
x^2 + 2*I*d)*x + I) + a^2*b^2*d)*integral(1/8/(b^3*log(d*x^2 + sqrt(d^2*x^
2 + 2*I*d)*x + I) + a*b^2), x) + 2*sqrt(d^2*x^2 + 2*I*d)*b)/(b^4*d*log(d*x
^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^2 + 2*a*b^3*d*log(d*x^2 + sqrt(d^2*x^2 +
 2*I*d)*x + I) + a^2*b^2*d)
```

3.320.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*asinh(I+d*x**2))**3,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real I
```

3.320.7 Maxima [F]

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^3}\,dx = \int \frac{1}{(b\,\text{arsinh}(dx^2+i)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^3,x, algorithm="maxima")
```

output
```
-1/8*((a*d^(11/2) + 2*b*d^(11/2))*x^10 - 2*(-3*I*a*d^(9/2) - 7*I*b*d^(9/2)
)*x^8 - (11*a*d^(7/2) + 36*b*d^(7/2))*x^6 - 2*(I*a*d^(5/2) + 20*I*b*d^(5/2
))*x^4 - 4*(3*a*d^(3/2) - 4*b*d^(3/2))*x^2 + ((a*d^4 + 2*b*d^4)*x^7 - (-3*
I*a*d^3 - 8*I*b*d^3)*x^5 - 2*(2*a*d^2 + 5*b*d^2)*x^3 - 4*(I*a*d + I*b*d)*x
)*(d*x^2 + 2*I)^(3/2) + (3*(a*d^(9/2) + 2*b*d^(9/2))*x^8 - 6*(-2*I*a*d^(7/
2) - 5*I*b*d^(7/2))*x^6 - 2*(8*a*d^(5/2) + 25*b*d^(5/2))*x^4 - 10*(I*a*d^(
3/2) + 3*I*b*d^(3/2))*x^2 + 4*a*sqrt(d) + 4*b*sqrt(d))*(d*x^2 + 2*I) + (b*
d^(11/2)*x^10 + 6*I*b*d^(9/2)*x^8 - 11*b*d^(7/2)*x^6 - 2*I*b*d^(5/2)*x^4 -
 12*b*d^(3/2)*x^2 + (b*d^4*x^7 + 3*I*b*d^3*x^5 - 4*b*d^2*x^3 - 4*I*b*d*x)*
(d*x^2 + 2*I)^(3/2) + (3*b*d^(9/2)*x^8 + 12*I*b*d^(7/2)*x^6 - 16*b*d^(5/2)
*x^4 - 10*I*b*d^(3/2)*x^2 + 4*b*sqrt(d))*(d*x^2 + 2*I) + (3*b*d^5*x^9 + 15
*I*b*d^4*x^7 - 23*b*d^3*x^5 - 7*I*b*d^2*x^3 - 6*b*d*x)*sqrt(d*x^2 + 2*I) -
 8*I*b*sqrt(d))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I) + (3*(a*d^5 +
 2*b*d^5)*x^9 - 3*(-5*I*a*d^4 - 12*I*b*d^4)*x^7 - (23*a*d^3 + 76*b*d^3)*x^
5 - (7*I*a*d^2 + 64*I*b*d^2)*x^3 - 2*(3*a*d - 8*b*d)*x)*sqrt(d*x^2 + 2*I)
- 8*I*a*sqrt(d))/(a^2*b^2*d^(11/2)*x^9 + 6*I*a^2*b^2*d^(9/2)*x^7 - 12*a^2*
b^2*d^(7/2)*x^5 - 8*I*a^2*b^2*d^(5/2)*x^3 + (b^4*d^(11/2)*x^9 + 6*I*b^4*d^
(9/2)*x^7 - 12*b^4*d^(7/2)*x^5 - 8*I*b^4*d^(5/2)*x^3 + (b^4*d^4*x^6 + 3*I*
b^4*d^3*x^4 - 3*b^4*d^2*x^2 - I*b^4*d)*(d*x^2 + 2*I)^(3/2) + 3*(b^4*d^(9/2
)*x^7 + 4*I*b^4*d^(7/2)*x^5 - 5*b^4*d^(5/2)*x^3 - 2*I*b^4*d^(3/2)*x)*(d...
```

3.320.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a + ib\arcsin(1 - idx^2))^3}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.320.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^3}\,dx=\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)\right)^3}\,dx$$

input
```
int(1/(a + b*asinh(d*x^2 + 1i))^3,x)
```

output
```
int(1/(a + b*asinh(d*x^2 + 1i))^3, x)
```

3.321 $\int (a - ib \arcsin(1 + idx^2))^4 \, dx$

3.321.1 Optimal result

Integrand size = 20, antiderivative size = 153

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^4 dx = 384b^4x - \frac{192b^3\sqrt{-2idx^2 + d^2x^4}(a - ib\arcsin(1 + idx^2))}{dx} + 48b^2x\left(a - ib\arcsin\left(1 + idx^2\right)\right)^2 - \frac{8b\sqrt{-2idx^2 + d^2x^4}(a - ib\arcsin(1 + idx^2))^3}{dx} + x\left(a - ib\arcsin\left(1 + idx^2\right)\right)^4$$

output
```
384*b^4*x+48*b^2*x*(a-I*b*arcsin(1+I*d*x^2))^2+x*(a-I*b*arcsin(1+I*d*x^2))
^4-192*b^3*(a-I*b*arcsin(1+I*d*x^2))*(-2*I*d*x^2+d^2*x^4)^(1/2)/d/x-8*b*(a
-I*b*arcsin(1+I*d*x^2))^3*(-2*I*d*x^2+d^2*x^4)^(1/2)/d/x
```

3.321.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^4 dx = -\frac{8b\sqrt{dx^2(-2i + dx^2)}(a - ib\arcsin(1 + idx^2))^3}{dx} + x\left(a - ib\arcsin\left(1 + idx^2\right)\right)^4 + 48b^2\left(8b^2x - \frac{4b\sqrt{dx^2(-2i + dx^2)}(a - ib\arcsin(1 + idx^2))}{dx} + x\left(a - ib\arcsin\left(1 + idx^2\right)\right)^2\right)$$

input
```
Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^4,x]
```

output
```
(-8*b*Sqrt[d*x^2*(-2*I + d*x^2)]*(a - I*b*ArcSin[1 + I*d*x^2])^3)/(d*x) +
x*(a - I*b*ArcSin[1 + I*d*x^2])^4 + 48*b^2*(8*b^2*x - (4*b*Sqrt[d*x^2*(-2*
I + d*x^2)]*(a - I*b*ArcSin[1 + I*d*x^2]))/(d*x) + x*(a - I*b*ArcSin[1 + I
*d*x^2])^2)
```

3.321.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5313, 5313, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^4 dx$$

$$\Big\downarrow \text{ 5313}$$

$$48b^2\int \left(a-ib\arcsin\left(idx^2+1\right)\right)^2dx-\frac{8b\sqrt{d^2x^4-2idx^2}\left(a-ib\arcsin\left(1+idx^2\right)\right)^3}{dx}+x\left(a-ib\arcsin\left(1+idx^2\right)\right)^4$$

$$\Big\downarrow \text{ 5313}$$

$$48b^2\left(8b^2\int 1dx-\frac{4b\sqrt{d^2x^4-2idx^2}\left(a-ib\arcsin\left(1+idx^2\right)\right)}{dx}+x\left(a-ib\arcsin\left(1+idx^2\right)\right)^2\right)-\frac{8b\sqrt{d^2x^4-2idx^2}\left(a-ib\arcsin\left(1+idx^2\right)\right)^3}{dx}+x\left(a-ib\arcsin\left(1+idx^2\right)\right)^4$$

$$\Big\downarrow \text{ 24}$$

$$48b^2\left(-\frac{4b\sqrt{d^2x^4-2idx^2}\left(a-ib\arcsin\left(1+idx^2\right)\right)}{dx}+x\left(a-ib\arcsin\left(1+idx^2\right)\right)^2+8b^2x\right)-\frac{8b\sqrt{d^2x^4-2idx^2}\left(a-ib\arcsin\left(1+idx^2\right)\right)^3}{dx}+x\left(a-ib\arcsin\left(1+idx^2\right)\right)^4$$

input
```
Int[(a - I*b*ArcSin[1 + I*d*x^2])^4,x]
```

output
```
(-8*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2])^3)/(d*x)
+ x*(a - I*b*ArcSin[1 + I*d*x^2])^4 + 48*b^2*(8*b^2*x - (4*b*Sqrt[(-2*I)*d
*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2]))/(d*x) + x*(a - I*b*ArcSin[1
 + I*d*x^2])^2)
```

3.321.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5313
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a
 + b*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
+ b*ArcSin[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

3.321.4 Maple [F]

$$\int \left(a+b\,\mathrm{arcsinh}\left(d\,x^{2}-i\right)\right)^{4}dx$$

input
```
int((a+b*arcsinh(-I+d*x^2))^4,x)
```

output
```
int((a+b*arcsinh(-I+d*x^2))^4,x)
```

3.321.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(129) = 258.

Time = 0.25 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.76

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^4\,dx$$
$$=\frac{b^4dx\log\left(dx^2+\sqrt{d^2x^2-2i\,d}\,x-i\right)^4+4\left(ab^3dx-2\sqrt{d^2x^2-2i\,d}b^4\right)\log\left(dx^2+\sqrt{d^2x^2-2i\,d}\,x-i\right)^3+(a$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^4,x, algorithm="fricas")
```

output
```
(b^4*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^4 + 4*(a*b^3*d*x - 2*sqr
t(d^2*x^2 - 2*I*d)*b^4)*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^3 + (a^4
+ 48*a^2*b^2 + 384*b^4)*d*x - 6*(4*sqrt(d^2*x^2 - 2*I*d)*a*b^3 - (a^2*b^2
+ 8*b^4)*d*x)*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^2 + 4*((a^3*b + 24*
a*b^3)*d*x - 6*(a^2*b^2 + 8*b^4)*sqrt(d^2*x^2 - 2*I*d))*log(d*x^2 + sqrt(d
^2*x^2 - 2*I*d)*x - I) - 8*(a^3*b + 24*a*b^3)*sqrt(d^2*x^2 - 2*I*d))/d
```

3.321.6 Sympy [F(-2)]

Exception generated.

$$\int \left(a - ib\arcsin\left(1 + idx^2\right)\right)^4\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*asinh(-I+d*x**2))**4,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

3.321.7 Maxima [F]

$$\int \left(a - ib\arcsin\left(1 + idx^2\right)\right)^4\,dx = \int \left(b\,\text{arsinh}\left(dx^2 - i\right) + a\right)^4\,dx$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^4,x, algorithm="maxima")
```

output
```
b^4*x*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^4 + 4*(x*arcsinh(d*x^2
- I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*d))*a^3*b + a^4*x
+ integrate(2*(2*((a*b^3*d^2 - 2*b^4*d^2)*x^4 - 2*a*b^3 - (3*I*a*b^3*d - 4
*I*b^4*d)*x^2 + ((a*b^3*d^(3/2) - 2*b^4*d^(3/2))*x^3 - 2*(I*a*b^3*sqrt(d)
- I*b^4*sqrt(d))*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(
d)*x - I)^3 + 3*(a^2*b^2*d^2*x^4 - 3*I*a^2*b^2*d*x^2 - 2*a^2*b^2 + (a^2*b^
2*d^(3/2)*x^3 - 2*I*a^2*b^2*sqrt(d)*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt
(d*x^2 - 2*I)*sqrt(d)*x - I)^2)/(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*x^3 - 2*I*
sqrt(d)*x)*sqrt(d*x^2 - 2*I) - 2), x)
```

3.321.8 Giac [F(-2)]

Exception generated.

$$\int \left(a - ib\arcsin\left(1 + idx^2\right)\right)^4 dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int \left(a - ib\arcsin\left(1 + idx^2\right)\right)^4 dx = \int \left(a + b\,\mathrm{asinh}\left(d\,x^2 - \mathrm{i}\right)\right)^4 dx$$

input
```
int((a + b*asinh(d*x^2 - 1i))^4,x)
```

output
```
int((a + b*asinh(d*x^2 - 1i))^4, x)
```

3.322 $\int (a - ib\arcsin(1 + idx^2))^3 \, dx$

3.322.1 Optimal result

Integrand size = 20, antiderivative size = 129

$$\int (a - ib\arcsin(1 + idx^2))^3 \, dx = 24ab^2x - \frac{48b^3\sqrt{-2idx^2 + d^2x^4}}{dx} - 24ib^3x\arcsin(1 + idx^2) - \frac{6b\sqrt{-2idx^2 + d^2x^4}(a - ib\arcsin(1 + idx^2))^2}{dx} + x(a - ib\arcsin(1 + idx^2))^3$$

output
```
24*a*b^2*x-24*I*b^3*x*arcsin(1+I*d*x^2)+x*(a-I*b*arcsin(1+I*d*x^2))^3-48*b
^3*(-2*I*d*x^2+d^2*x^4)^(1/2)/d/x-6*b*(a-I*b*arcsin(1+I*d*x^2))^2*(-2*I*d*
x^2+d^2*x^4)^(1/2)/d/x
```

3.322.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.40

$$\int (a - ib\arcsin(1 + idx^2))^3 \, dx = \frac{a(a^2 + 24b^2)\,dx^2 - 6b(a^2 + 8b^2)\sqrt{dx^2(-2i + dx^2)} - 3ib\left(a^2dx^2 + 8b^2dx^2 - 4ab\sqrt{dx^2(-2i + dx^2)}\right)\text{arcsi}}{dx}$$

input
```
Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^3,x]
```

output
```
(a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2*(-2*I + d*x^2)] - (
3*I)*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2*(-2*I + d*x^2)])*ArcSin
[1 + I*d*x^2] + 3*b^2*(-(a*d*x^2) + 2*b*Sqrt[d*x^2*(-2*I + d*x^2)])*ArcSin
[1 + I*d*x^2]^2 + I*b^3*d*x^2*ArcSin[1 + I*d*x^2]^3)/(d*x)
```

3.322.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5313, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a - ib\arcsin\left(1 + idx^2\right)\right)^3 dx$$

$$\Big\downarrow \text{5313}$$

$$24b^2\int \left(a - ib\arcsin\left(idx^2+1\right)\right)dx - \frac{6b\sqrt{d^2x^4-2idx^2}\left(a - ib\arcsin\left(1+idx^2\right)\right)^2}{dx} + x\left(a - ib\arcsin\left(1+idx^2\right)\right)^3$$

$$\Big\downarrow \text{2009}$$

$$24b^2\left(ax - ibx\arcsin\left(1+idx^2\right) - \frac{2b\sqrt{d^2x^4-2idx^2}}{dx}\right) - \frac{6b\sqrt{d^2x^4-2idx^2}\left(a - ib\arcsin\left(1+idx^2\right)\right)^2}{dx} + x\left(a - ib\arcsin\left(1+idx^2\right)\right)^3$$

input
```
Int[(a - I*b*ArcSin[1 + I*d*x^2])^3,x]
```

output
```
(-6*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2])^2)/(d*x)
+ x*(a - I*b*ArcSin[1 + I*d*x^2])^3 + 24*b^2*(a*x - (2*b*Sqrt[(-2*I)*d*x^2
 + d^2*x^4])/(d*x) - I*b*x*ArcSin[1 + I*d*x^2])
```

3.322.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5313
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a
 + b*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
+ b*ArcSin[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

3.322.4 Maple [F]

$$\int \left(a+b\,\mathrm{arcsinh}\left(d\,x^{2}-i\right)\right)^{3}dx$$

input
```
int((a+b*arcsinh(-I+d*x^2))^3,x)
```

output
```
int((a+b*arcsinh(-I+d*x^2))^3,x)
```

3.322.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.46

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^3\,dx$$
$$=\frac{b^3dx\log\left(dx^2+\sqrt{d^2x^2-2i\,d}x-i\right)^3+(a^3+24\,ab^2)dx+3\left(ab^2dx-2\sqrt{d^2x^2-2i\,d}b^3\right)\log\left(dx^2+\sqrt{d^2x^2}\right.}{}$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^3,x, algorithm="fricas")
```

output
```
(b^3*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^3 + (a^3 + 24*a*b^2)*d*x
 + 3*(a*b^2*d*x - 2*sqrt(d^2*x^2 - 2*I*d)*b^3)*log(d*x^2 + sqrt(d^2*x^2 -
2*I*d)*x - I)^2 - 3*(4*sqrt(d^2*x^2 - 2*I*d)*a*b^2 - (a^2*b + 8*b^3)*d*x)*
log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) - 6*sqrt(d^2*x^2 - 2*I*d)*(a^2*b
+ 8*b^3))/d
```

3.322.6 Sympy [F(-2)]

Exception generated.

$$\int \left(a - ib\arcsin\left(1 + idx^2\right)\right)^3 dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*asinh(-I+d*x**2))**3,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

3.322.7 Maxima [F]

$$\int \left(a - ib\arcsin\left(1 + idx^2\right)\right)^3 dx = \int \left(b\,\text{arsinh}\left(dx^2 - i\right) + a\right)^3 dx$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^3,x, algorithm="maxima")
```

output
```
b^3*x*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^3 + 3*(x*arcsinh(d*x^2
- I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*d))*a^2*b + a^3*x
+ integrate(3*((a*b^2*d^2 - 2*b^3*d^2)*x^4 - 2*a*b^2 - (3*I*a*b^2*d - 4*I*
b^3*d)*x^2 + ((a*b^2*d^(3/2) - 2*b^3*d^(3/2))*x^3 - 2*(I*a*b^2*sqrt(d) - I
*b^3*sqrt(d))*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*
x - I)^2/(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*x^3 - 2*I*sqrt(d)*x)*sqrt(d*x^2 -
 2*I) - 2), x)
```

3.322.8 Giac [F(-2)]

Exception generated.

$$\int \left(a - ib\arcsin\left(1 + idx^2\right)\right)^3 dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^3 \, dx = \int \left(a + b \operatorname{asinh}\left(d\,x^2 - \mathrm{i}\right)\right)^3 dx$$

input
```
int((a + b*asinh(d*x^2 - 1i))^3,x)
```

output
```
int((a + b*asinh(d*x^2 - 1i))^3, x)
```

3.323 $\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^2\,dx$

3.323.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^2\,dx=8b^2x-\frac{4b\sqrt{-2idx^2+d^2x^4}\left(a-ib\arcsin\left(1+idx^2\right)\right)}{dx}+x\left(a-ib\arcsin\left(1+idx^2\right)\right)^2$$

output
```
8*b^2*x+x*(a-I*b*arcsin(1+I*d*x^2))^2-4*b*(a-I*b*arcsin(1+I*d*x^2))*(-2*I*
d*x^2+d^2*x^4)^(1/2)/d/x
```

3.323.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^2\,dx=8b^2x-\frac{4b\sqrt{-2idx^2+d^2x^4}\left(a-ib\arcsin\left(1+idx^2\right)\right)}{dx}+x\left(a-ib\arcsin\left(1+idx^2\right)\right)^2$$

input
```
Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^2,x]
```

output
```
8*b^2*x - (4*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2]))
/(d*x) + x*(a - I*b*ArcSin[1 + I*d*x^2])^2
```

3.323.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5313, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^2 dx$$

$$\Big\downarrow \text{5313}$$

$$8b^2\int 1dx-\frac{4b\sqrt{d^2x^4-2idx^2}\left(a-ib\arcsin\left(1+idx^2\right)\right)}{dx}+x\left(a-ib\arcsin\left(1+idx^2\right)\right)^2$$

$$\Big\downarrow \text{24}$$

$$-\frac{4b\sqrt{d^2x^4-2idx^2}\left(a-ib\arcsin\left(1+idx^2\right)\right)}{dx}+x\left(a-ib\arcsin\left(1+idx^2\right)\right)^2+8b^2x$$

input
```
Int[(a - I*b*ArcSin[1 + I*d*x^2])^2,x]
```

output
```
8*b^2*x - (4*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2]))
/(d*x) + x*(a - I*b*ArcSin[1 + I*d*x^2])^2
```

3.323.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5313
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a
 + b*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
+ b*ArcSin[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

3.323.4 Maple [F]

$$\int \left(a+b\ \mathrm{arcsinh}\left(d\,x^{2}-i\right)\right)^{2}dx$$

input
```
int((a+b*arcsinh(-I+d*x^2))^2,x)
```

output
```
int((a+b*arcsinh(-I+d*x^2))^2,x)
```

3.323.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.50

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^2\,dx$$
$$=\frac{b^{2}dx\log\left(dx^{2}+\sqrt{d^{2}x^{2}-2i\,d}x-i\right)^{2}+(a^{2}+8\,b^{2})dx-4\,\sqrt{d^{2}x^{2}-2i\,d}ab+2\left(abdx-2\,\sqrt{d^{2}x^{2}-2i\,d}b^{2}\right)\mathrm{lo}}{d}$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^2,x, algorithm="fricas")
```

output
```
(b^2*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^2 + (a^2 + 8*b^2)*d*x -
4*sqrt(d^2*x^2 - 2*I*d)*a*b + 2*(a*b*d*x - 2*sqrt(d^2*x^2 - 2*I*d)*b^2)*lo
g(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I))/d
```

3.323.6 Sympy [F(-2)]

Exception generated.

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*asinh(-I+d*x**2))**2,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

3.323.7 Maxima [F]

$$\int \left(a - ib\arcsin\left(1 + idx^2\right)\right)^2 dx = \int \left(b\,\mathrm{arsinh}\left(dx^2 - i\right) + a\right)^2 dx$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^2,x, algorithm="maxima")
```

output
```
2*(x*arcsinh(d*x^2 - I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)
*d))*a*b + (x*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^2 - integrate(4
*(d^2*x^4 - 2*I*d*x^2 + (d^(3/2)*x^3 - I*sqrt(d)*x)*sqrt(d*x^2 - 2*I))*log
(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)/(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*
x^3 - 2*I*sqrt(d)*x)*sqrt(d*x^2 - 2*I) - 2), x))*b^2 + a^2*x
```

3.323.8 Giac [F(-2)]

Exception generated.

$$\int \left(a - ib\arcsin\left(1 + idx^2\right)\right)^2 dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \left(a - ib\arcsin\left(1 + idx^2\right)\right)^2 dx = \int \left(a + b\,\mathrm{asinh}\left(d\,x^2 - \mathrm{i}\right)\right)^2 dx$$

input
```
int((a + b*asinh(d*x^2 - 1i))^2,x)
```

output
```
int((a + b*asinh(d*x^2 - 1i))^2, x)
```

3.324 $\int \left(a - ib \arcsin\left(1 + idx^2\right)\right) dx$

3.324.1 Optimal result

Integrand size = 18, antiderivative size = 50

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right) dx = ax - \frac{2b\sqrt{-2idx^2 + d^2x^4}}{dx} - ibx \arcsin\left(1 + idx^2\right)$$

output
```
a*x-I*b*x*arcsin(1+I*d*x^2)-2*b*(-2*I*d*x^2+d^2*x^4)^(1/2)/d/x
```

3.324.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right) dx = ax - \frac{2b\sqrt{dx^2\left(-2i + dx^2\right)}}{dx} - ibx \arcsin\left(1 + idx^2\right)$$

input
```
Integrate[a - I*b*ArcSin[1 + I*d*x^2],x]
```

output
```
a*x - (2*b*Sqrt[d*x^2*(-2*I + d*x^2)])/(d*x) - I*b*x*ArcSin[1 + I*d*x^2]
```

3.324.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right) dx$$

$$\Big\downarrow \ 2009$$

$$ax - ibx \arcsin\left(1 + idx^2\right) - \frac{2b\sqrt{d^2x^4 - 2idx^2}}{dx}$$

input
```
Int[a - I*b*ArcSin[1 + I*d*x^2],x]
```

output
```
a*x - (2*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4])/(d*x) - I*b*x*ArcSin[1 + I*d*x^2]
```

3.324.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.324.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

method	result	size
default	$ax + b\left(x \operatorname{arcsinh}\left(d\,x^2 - i\right) + \frac{2x\left(-d\,x^2+2i\right)}{\sqrt{d^2x^4-2id\,x^2}}\right)$	48
parts	$ax + b\left(x \operatorname{arcsinh}\left(d\,x^2 - i\right) + \frac{2x\left(-d\,x^2+2i\right)}{\sqrt{d^2x^4-2id\,x^2}}\right)$	48

input
```
int(a+b*arcsinh(-I+d*x^2),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*(x*arcsinh(-I+d*x^2)+2/(-2*I*d*x^2+d^2*x^4)^(1/2)*x*(-d*x^2+2*I))
```

3.324.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int (a-ib\arcsin(1+idx^2))\,dx = \frac{bdx\log\left(dx^2+\sqrt{d^2x^2-2i\,d}x-i\right)+adx-2\sqrt{d^2x^2-2i\,d}b}{d}$$

input
```
integrate(a+b*arcsinh(-I+d*x^2),x, algorithm="fricas")
```

output
```
(b*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a*d*x - 2*sqrt(d^2*x^2 -
 2*I*d)*b)/d
```

3.324.6 Sympy [F(-2)]

Exception generated.

$$\int (a-ib\arcsin(1+idx^2))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(a+b*asinh(-I+d*x**2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

3.324.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int (a-ib\arcsin(1+idx^2))\,dx = \left(x\,\mathrm{arsinh}\left(dx^2-i\right)-\frac{2\left(d^{\frac{3}{2}}x^2-2i\sqrt{d}\right)}{\sqrt{dx^2-2id}}\right)b+ax$$

input
```
integrate(a+b*arcsinh(-I+d*x^2),x, algorithm="maxima")
```

output
```
(x*arcsinh(d*x^2 - I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*d
))*b + a*x
```

3.324.8 Giac [F(-2)]

Exception generated.

$$\int \left(a - ib \arcsin \left(1 + idx^2\right)\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(a+b*arcsinh(-I+d*x^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.324.9 Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \left(a - ib \arcsin \left(1 + idx^2\right)\right) dx = a\,x + b\,x\,\mathrm{asinh}\left(d\,x^2 - \mathrm{i}\right) - \frac{2\,b\,\sqrt{{\left(d\,x^2-\mathrm{i}\right)}^2+1}}{d\,x}$$

input
```
int(a + b*asinh(d*x^2 - 1i),x)
```

output
```
a*x + b*x*asinh(d*x^2 - 1i) - (2*b*((d*x^2 - 1i)^2 + 1)^(1/2))/(d*x)
```

3.325 $\int \frac{1}{a-ib\arcsin(1+idx^2)}\,dx$

3.325.1 Optimal result

Integrand size = 20, antiderivative size = 191

$$\int \frac{1}{a-ib\arcsin(1+idx^2)}\,dx = -\frac{x\,\text{CosIntegral}\left(\frac{i(a-ib\arcsin(1+idx^2))}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)} + \frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\arcsin(1+idx^2)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}$$

output

```
1/2*x*Shi(1/2*(a-I*b*arcsin(1+I*d*x^2))/b)*(cosh(1/2*a/b)+I*sinh(1/2*a/b))
/b/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/2*x*Ci(1/2*I*
(a-I*b*arcsin(1+I*d*x^2))/b)*(I*cosh(1/2*a/b)+sinh(1/2*a/b))/b/(cos(1/2*ar
csin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))
```

3.325.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.76

$$\int \frac{1}{a-ib\arcsin(1+idx^2)}\,dx = \frac{x\left(\text{CosIntegral}\left(\frac{1}{2}\left(\frac{ia}{b}+\arcsin(1+idx^2)\right)\right)\left(-i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)+\left(-i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{1}{2}\left(\frac{i}{b}\right.\right.\right.}{2b\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}$$

input

```
Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-1),x]
```

output
```
(x*(CosIntegral[((I*a)/b + ArcSin[1 + I*d*x^2])/2]*((-I)*Cosh[a/(2*b)] - S
inh[a/(2*b)]) + ((-I)*Cosh[a/(2*b)] + Sinh[a/(2*b)])*SinIntegral[((I*a)/b
+ ArcSin[1 + I*d*x^2])/2]))/(2*b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[
1 + I*d*x^2]/2]))
```

3.325.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5315}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-ib\arcsin\left(1+idx^2\right)}dx$$

$$\Big\downarrow \text{ 5315}$$

$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\arcsin(idx^2+1)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{i(a-ib\arcsin(idx^2+1))}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}$$

input
```
Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-1),x]
```

output
```
-1/2*(x*CosIntegral[((I/2)*(a - I*b*ArcSin[1 + I*d*x^2]))/b]*(I*Cosh[a/(2*
b)] + Sinh[a/(2*b)]))/(b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*
x^2]/2])) + (x*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])*SinhIntegral[(a - I*b*Arc
Sin[1 + I*d*x^2])/(2*b)])/(2*b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1
+ I*d*x^2]/2]))
```

3.325.3.1 Defintions of rubi rules used

rule 5315
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(-x
)*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c +
d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x]
 - Simp[x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Arc
Sin[c + d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2
]))), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

3.325.4 Maple [F]

$$\int \frac{1}{a + b \operatorname{arcsinh}\left(d\,x^{2} - i\right)}dx$$

input
```
int(1/(a+b*arcsinh(-I+d*x^2)),x)
```

output
```
int(1/(a+b*arcsinh(-I+d*x^2)),x)
```

3.325.5 Fricas [F]

$$\int \frac{1}{a - ib \arcsin\left(1 + idx^2\right)}\,dx = \int \frac{1}{b \operatorname{arsinh}\left(dx^2 - i\right) + a}\,dx$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2)),x, algorithm="fricas")
```

output
```
integral(1/(b*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a), x)
```

3.325.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{a - ib \arcsin\left(1 + idx^2\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*asinh(-I+d*x**2)),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

3.325.7 Maxima [F]

$$\int \frac{1}{a - ib \arcsin(1 + idx^2)} dx = \int \frac{1}{b \operatorname{arsinh}(dx^2 - i) + a} dx$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arcsinh(d*x^2 - I) + a), x)
```

3.325.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{a - ib \arcsin(1 + idx^2)} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a - ib \arcsin(1 + idx^2)} dx = \int \frac{1}{a + b \operatorname{asinh}(d x^2 - \mathrm{i})} dx$$

input
```
int(1/(a + b*asinh(d*x^2 - 1i)),x)
```

output
```
int(1/(a + b*asinh(d*x^2 - 1i)), x)
```

3.326 $\int \frac{1}{(a-ib\arcsin(1+idx^2))^2}\,dx$

3.326.1 Optimal result

Integrand size = 20, antiderivative size = 244

$$\begin{aligned}\int \frac{1}{(a-ib\arcsin(1+idx^2))^2}\,dx &= -\frac{\sqrt{-2idx^2+d^2x^4}}{2bdx(a-ib\arcsin(1+idx^2))}\\ &\quad+\frac{x\,\mathrm{CosIntegral}\left(\frac{i(a-ib\arcsin(1+idx^2))}{2b}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}\\ &\quad-\frac{x\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Shi}\left(\frac{a-ib\arcsin(1+idx^2)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}\end{aligned}$$

output
```
1/4*x*Ci(1/2*I*(a-I*b*arcsin(1+I*d*x^2))/b)*(cosh(1/2*a/b)+I*sinh(1/2*a/b)
)/b^2/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/4*x*Shi(1/
2*(a-I*b*arcsin(1+I*d*x^2))/b)*(I*cosh(1/2*a/b)+sinh(1/2*a/b))/b^2/(cos(1/
2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/2*(-2*I*d*x^2+d^2*x^4)^
(1/2)/b/d/x/(a-I*b*arcsin(1+I*d*x^2))
```

3.326.2 Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a-ib\arcsin(1+idx^2))^2}\,dx$$
$$= \frac{-\frac{2b\sqrt{dx^2(-2i+dx^2)}}{d(a-ib\arcsin(1+idx^2))} + \frac{x^2\left(\text{CosIntegral}\left(\frac{1}{2}\left(\frac{ia}{b}+\arcsin(1+idx^2)\right)\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{1}{2}\left(\frac{ia}{b}+\arcsin(1+id\right.\right.\right.}{\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)}}{4b^2x}$$

input
```
Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-2),x]
```

output
```
((-2*b*Sqrt[d*x^2*(-2*I + d*x^2)])/(d*(a - I*b*ArcSin[1 + I*d*x^2])) + (x^
2*(CosIntegral[((I*a)/b + ArcSin[1 + I*d*x^2])/2]*(Cosh[a/(2*b)] + I*Sinh[
a/(2*b)]) - (Cosh[a/(2*b)] - I*Sinh[a/(2*b)])*SinIntegral[((I*a)/b + ArcSi
n[1 + I*d*x^2])/2]))/(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]
/2]))/(4*b^2*x)
```

3.326.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5324}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-ib\arcsin(1+idx^2))^2}\,dx$$
$$\Big\downarrow \ 5324$$
$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{i(a-ib\arcsin(idx^2+1))}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)} - \frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\arcsin(idx^2+1)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)} - \frac{\sqrt{d^2x^4-2idx^2}}{2bdx(a-ib\arcsin(1+idx^2))}$$

input
```
Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-2),x]
```

output
```
-1/2*Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])) +
(x*CosIntegral[((I/2)*(a - I*b*ArcSin[1 + I*d*x^2]))/b]*(Cosh[a/(2*b)] + I
*Sinh[a/(2*b)]))/(4*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x
^2]/2])) - (x*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)])*SinhIntegral[(a - I*b*ArcS
in[1 + I*d*x^2])/(2*b)])/(4*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1
 + I*d*x^2]/2]))
```

3.326.3.1 Defintions of rubi rules used

rule 5324
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> Simp[-Sq
rt[-2*c*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcSin[c + d*x^2])), x] + (-Simp[x
*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d
*x^2])]/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x
] + Simp[x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Ar
cSin[c + d*x^2])]/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2
]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

3.326.4 Maple [F]

$$\int \frac{1}{\left(a+b\ \mathrm{arcsinh}\left(d\,x^{2}-i\right)\right)^{2}}dx$$

input
```
int(1/(a+b*arcsinh(-I+d*x^2))^2,x)
```

output
```
int(1/(a+b*arcsinh(-I+d*x^2))^2,x)
```

3.326.5 Fricas [F]

$$\int \frac{1}{(a-ib\arcsin(1+idx^2))^2}\,dx=\int \frac{1}{(b\,\mathrm{arsinh}\,(dx^2-i)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^2,x, algorithm="fricas")
```

output
```
1/2*(2*(b^2*d*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a*b*d)*integral(1
/2*sqrt(d^2*x^2 - 2*I*d)*x/(a*b*d*x^2 - 2*I*a*b + (b^2*d*x^2 - 2*I*b^2)*lo
g(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)), x) - sqrt(d^2*x^2 - 2*I*d))/(b^2*
d*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a*b*d)
```

3.326.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*asinh(-I+d*x**2))**2,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

3.326.7 Maxima [F]

$$\int \frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^2}\,dx = \int \frac{1}{\left(b\,\mathrm{arsinh}\left(dx^2-i\right)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^2,x, algorithm="maxima")
```

output
```
-1/2*(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*x^3 - 2*I*sqrt(d)*x)*sqrt(d*x^2 - 2*I
) - 2)/(a*b*d^2*x^3 - 2*I*a*b*d*x + (b^2*d^2*x^3 - 2*I*b^2*d*x + (b^2*d^(3
/2)*x^2 - I*b^2*sqrt(d))*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)*
sqrt(d)*x - I) + (a*b*d^(3/2)*x^2 - I*a*b*sqrt(d))*sqrt(d*x^2 - 2*I)) + in
tegrate(1/2*(d^3*x^6 - 3*I*d^2*x^4 + (d^2*x^4 - I*d*x^2 - 2)*(d*x^2 - 2*I)
 + (2*d^(5/2)*x^5 - 4*I*d^(3/2)*x^3 - sqrt(d)*x)*sqrt(d*x^2 - 2*I) - 4*I)/
(a*b*d^3*x^6 - 4*I*a*b*d^2*x^4 - 4*a*b*d*x^2 + (a*b*d^2*x^4 - 2*I*a*b*d*x^
2 - a*b)*(d*x^2 - 2*I) + (b^2*d^3*x^6 - 4*I*b^2*d^2*x^4 - 4*b^2*d*x^2 + (b
^2*d^2*x^4 - 2*I*b^2*d*x^2 - b^2)*(d*x^2 - 2*I) + 2*(b^2*d^(5/2)*x^5 - 3*I
*b^2*d^(3/2)*x^3 - 2*b^2*sqrt(d)*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*
x^2 - 2*I)*sqrt(d)*x - I) + 2*(a*b*d^(5/2)*x^5 - 3*I*a*b*d^(3/2)*x^3 - 2*a
*b*sqrt(d)*x)*sqrt(d*x^2 - 2*I)), x)
```

3.326.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a - ib\arcsin(1 + idx^2))^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a - ib\arcsin(1 + idx^2))^2} \, dx = \int \frac{1}{(a + b\,\mathrm{asinh}\,(d\,x^2 - \mathrm{i}))^2} \, dx$$

input
```
int(1/(a + b*asinh(d*x^2 - 1i))^2,x)
```

output
```
int(1/(a + b*asinh(d*x^2 - 1i))^2, x)
```

3.327 $\int \frac{1}{(a-ib\arcsin(1+idx^2))^3}\,dx$

3.327.1 Optimal result

Integrand size = 20, antiderivative size = 272

$$\begin{aligned}\int \frac{1}{(a-ib\arcsin(1+idx^2))^3}\,dx &= -\frac{\sqrt{-2idx^2+d^2x^4}}{4bdx\,(a-ib\arcsin(1+idx^2))^2}-\frac{x}{8b^2\,(a-ib\arcsin(1+idx^2))}\\&\quad-\frac{x\,\text{CosIntegral}\left(\frac{i(a-ib\arcsin(1+idx^2))}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{16b^3\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}\\&\quad+\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\arcsin(1+idx^2)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}\end{aligned}$$

output
```
-1/8*x/b^2/(a-I*b*arcsin(1+I*d*x^2))+1/16*x*Shi(1/2*(a-I*b*arcsin(1+I*d*x^
2))/b)*(cosh(1/2*a/b)+I*sinh(1/2*a/b))/b^3/(cos(1/2*arcsin(1+I*d*x^2))-sin
(1/2*arcsin(1+I*d*x^2)))-1/16*x*Ci(1/2*I*(a-I*b*arcsin(1+I*d*x^2))/b)*(I*c
osh(1/2*a/b)+sinh(1/2*a/b))/b^3/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin
(1+I*d*x^2)))-1/4*(-2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-I*b*arcsin(1+I*d*x^2
))^2
```

3.327.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a-ib\arcsin(1+idx^2))^3}\,dx$$
$$= \frac{-\frac{4bx^2}{a-ib\arcsin(1+idx^2)}+\frac{8b^2\sqrt{dx^2(-2i+dx^2)}}{d(ia+b\arcsin(1+idx^2))^2}-\frac{2ix^2\left(\text{CosIntegral}\left(\frac{1}{2}\left(\frac{ia}{b}+\arcsin(1+idx^2)\right)\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)+\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\right.\right.}{\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)}}{32b^3x}$$

input
```
Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-3),x]
```

output
```
((-4*b*x^2)/(a - I*b*ArcSin[1 + I*d*x^2]) + (8*b^2*Sqrt[d*x^2*(-2*I + d*x^
2)])/(d*(I*a + b*ArcSin[1 + I*d*x^2])^2) - ((2*I)*x^2*(CosIntegral[((I*a)/
b + ArcSin[1 + I*d*x^2])/2]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) + (Cosh[a/(2
*b)] + I*Sinh[a/(2*b)])*SinIntegral[((I*a)/b + ArcSin[1 + I*d*x^2])/2]))/(
Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))/(32*b^3*x)
```

3.327.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5327, 5315}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-ib\arcsin(1+idx^2))^3}\,dx$$
$$\Big\downarrow \ 5327$$
$$\frac{\int \frac{1}{a-ib\arcsin(idx^2+1)}dx}{8b^2}-\frac{x}{8b^2(a-ib\arcsin(1+idx^2))}-\frac{\sqrt{d^2x^4-2idx^2}}{4bdx(a-ib\arcsin(1+idx^2))^2}$$
$$\Big\downarrow \ 5315$$
$$\frac{\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\arcsin\left(idx^2+1\right)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{i\left(a-ib\arcsin\left(idx^2+1\right)\right)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}}{8b^2}-\frac{x}{8b^2(a-ib\arcsin(1+idx^2))}-\frac{\sqrt{d^2x^4-2idx^2}}{4bdx(a-ib\arcsin(1+idx^2))^2}$$

input
```
Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-3),x]
```

output
```
-1/4*Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^2)
- x/(8*b^2*(a - I*b*ArcSin[1 + I*d*x^2])) + (-1/2*(x*CosIntegral[((I/2)*(a
 - I*b*ArcSin[1 + I*d*x^2]))/b]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(b*(Cos
[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) + (x*(Cosh[a/(2*b)]
 + I*Sinh[a/(2*b)])*SinhIntegral[(a - I*b*ArcSin[1 + I*d*x^2])/(2*b)])/(2*
b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])))/(8*b^2)
```

3.327.3.1 Defintions of rubi rules used

rule 5315
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(-x
)*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c +
d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x]
 - Simp[x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Arc
Sin[c + d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2
]))), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

rule 5327
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Simp[Sqrt
[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n + 1)*x)
), x] - Simp[1/(4*b^2*(n + 1)*(n + 2))   Int[(a + b*ArcSin[c + d*x^2])^(n +
 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

3.327.4 Maple [F]

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^{2}-i\right)\right)^{3}}dx$$

input
```
int(1/(a+b*arcsinh(-I+d*x^2))^3,x)
```

output
```
int(1/(a+b*arcsinh(-I+d*x^2))^3,x)
```

3.327.5 Fricas [F]

$$\int \frac{1}{(a - ib\arcsin(1 + idx^2))^3}\,dx = \int \frac{1}{(b\,\text{arsinh}(dx^2 - i) + a)^3}\,dx$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^3,x, algorithm="fricas")
```

output
```
-1/8*(b*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a*d*x - 8*(b^4*d*lo
g(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^2 + 2*a*b^3*d*log(d*x^2 + sqrt(d^2*
x^2 - 2*I*d)*x - I) + a^2*b^2*d)*integral(1/8/(b^3*log(d*x^2 + sqrt(d^2*x^
2 - 2*I*d)*x - I) + a*b^2), x) + 2*sqrt(d^2*x^2 - 2*I*d)*b)/(b^4*d*log(d*x
^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^2 + 2*a*b^3*d*log(d*x^2 + sqrt(d^2*x^2 -
 2*I*d)*x - I) + a^2*b^2*d)
```

3.327.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a - ib\arcsin(1 + idx^2))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*asinh(-I+d*x**2))**3,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

3.327.7 Maxima [F]

$$\int \frac{1}{(a - ib\arcsin(1 + idx^2))^3}\,dx = \int \frac{1}{(b\,\text{arsinh}(dx^2 - i) + a)^3}\,dx$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^3,x, algorithm="maxima")
```

output
```
-1/8*((a*d^(11/2) + 2*b*d^(11/2))*x^10 - 2*(3*I*a*d^(9/2) + 7*I*b*d^(9/2))
*x^8 - (11*a*d^(7/2) + 36*b*d^(7/2))*x^6 - 2*(-I*a*d^(5/2) - 20*I*b*d^(5/2
))*x^4 - 4*(3*a*d^(3/2) - 4*b*d^(3/2))*x^2 + ((a*d^4 + 2*b*d^4)*x^7 - (3*I
*a*d^3 + 8*I*b*d^3)*x^5 - 2*(2*a*d^2 + 5*b*d^2)*x^3 - 4*(-I*a*d - I*b*d)*x
)*(d*x^2 - 2*I)^(3/2) + (3*(a*d^(9/2) + 2*b*d^(9/2))*x^8 - 6*(2*I*a*d^(7/2
) + 5*I*b*d^(7/2))*x^6 - 2*(8*a*d^(5/2) + 25*b*d^(5/2))*x^4 - 10*(-I*a*d^(
3/2) - 3*I*b*d^(3/2))*x^2 + 4*a*sqrt(d) + 4*b*sqrt(d))*(d*x^2 - 2*I) + (b*
d^(11/2)*x^10 - 6*I*b*d^(9/2)*x^8 - 11*b*d^(7/2)*x^6 + 2*I*b*d^(5/2)*x^4 -
 12*b*d^(3/2)*x^2 + (b*d^4*x^7 - 3*I*b*d^3*x^5 - 4*b*d^2*x^3 + 4*I*b*d*x)*
(d*x^2 - 2*I)^(3/2) + (3*b*d^(9/2)*x^8 - 12*I*b*d^(7/2)*x^6 - 16*b*d^(5/2)
*x^4 + 10*I*b*d^(3/2)*x^2 + 4*b*sqrt(d))*(d*x^2 - 2*I) + (3*b*d^5*x^9 - 15
*I*b*d^4*x^7 - 23*b*d^3*x^5 + 7*I*b*d^2*x^3 - 6*b*d*x)*sqrt(d*x^2 - 2*I) +
 8*I*b*sqrt(d))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I) + (3*(a*d^5 +
 2*b*d^5)*x^9 - 3*(5*I*a*d^4 + 12*I*b*d^4)*x^7 - (23*a*d^3 + 76*b*d^3)*x^5
 - (-7*I*a*d^2 - 64*I*b*d^2)*x^3 - 2*(3*a*d - 8*b*d)*x)*sqrt(d*x^2 - 2*I)
+ 8*I*a*sqrt(d))/(a^2*b^2*d^(11/2)*x^9 - 6*I*a^2*b^2*d^(9/2)*x^7 - 12*a^2*
b^2*d^(7/2)*x^5 + 8*I*a^2*b^2*d^(5/2)*x^3 + (b^4*d^(11/2)*x^9 - 6*I*b^4*d^
(9/2)*x^7 - 12*b^4*d^(7/2)*x^5 + 8*I*b^4*d^(5/2)*x^3 + (b^4*d^4*x^6 - 3*I*
b^4*d^3*x^4 - 3*b^4*d^2*x^2 + I*b^4*d)*(d*x^2 - 2*I)^(3/2) + 3*(b^4*d^(9/2
)*x^7 - 4*I*b^4*d^(7/2)*x^5 - 5*b^4*d^(5/2)*x^3 + 2*I*b^4*d^(3/2)*x)*(d...
```

3.327.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a - ib\arcsin(1 + idx^2))^3}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a - ib\arcsin(1 + idx^2))^3}\,dx = \int \frac{1}{(a + b\,\mathrm{asinh}(d\,x^2 - \mathrm{i}))^3}\,dx$$

input
```
int(1/(a + b*asinh(d*x^2 - 1i))^3,x)
```

output
```
int(1/(a + b*asinh(d*x^2 - 1i))^3, x)
```

3.328 $\int (a+ib\arcsin(1-idx^2))^{5/2}\,dx$

3.328.1 Optimal result

Integrand size = 22, antiderivative size = 348

$$\begin{aligned}\int (a+ib\arcsin(1-idx^2))^{5/2}\,dx &= 15b^2x\sqrt{a+ib\arcsin(1-idx^2)} \\ &\quad -\frac{5b\sqrt{2idx^2+d^2x^4}(a+ib\arcsin(1-idx^2))^{3/2}}{dx}+x(a+ib\arcsin(1-idx^2))^{5/2} \\ &\quad +\frac{15b^2\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)} \\ &\quad -\frac{15\sqrt{-\frac{i}{b}}b^3\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)}\end{aligned}$$

output
```
x*(a-I*b*arcsin(-1+I*d*x^2))^(5/2)+15*b^2*x*FresnelS((-I/b)^(1/2)*(a-I*b*a
rcsin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2
)/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))/(-I/b)^(1/2)-1
5*b^3*x*FresnelC((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(
I*cosh(1/2*a/b)+sinh(1/2*a/b))*(-I/b)^(1/2)*Pi^(1/2)/(cos(1/2*arcsin(-1+I*
d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))-5*b*(a-I*b*arcsin(-1+I*d*x^2))^(3/2)*
(2*I*d*x^2+d^2*x^4)^(1/2)/d/x+15*b^2*x*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)
```

3.328.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.97

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}\,dx =$$
$$-\frac{5b\sqrt{dx^2\left(2i+dx^2\right)}\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}{dx}+x\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}$$
$$+\frac{15b^2x\left(\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)-\sqrt{\pi}\,\text{Fresnel}\right.}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1\right.\right.\right.}$$

input
```
Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(5/2),x]
```

output
```
(-5*b*Sqrt[d*x^2*(2*I + d*x^2)]*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2))/(d*x)
 + x*(a + I*b*ArcSin[1 - I*d*x^2])^(5/2) + (15*b^2*x*(Sqrt[(-I)/b]*Sqrt[a
+ I*b*ArcSin[1 - I*d*x^2]]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*
d*x^2]/2]) - Sqrt[Pi]*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x
^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) + Sqrt[Pi]*FresnelS[(Sqr
t[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*
Sinh[a/(2*b)])))/(Sqrt[(-I)/b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1
- I*d*x^2]/2]))
```

3.328.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5313, 5310}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}\,dx$$
$$\Big\downarrow \; 5313$$
$$15b^2\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}dx-\frac{5b\sqrt{d^2x^4+2idx^2}\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}{dx}+$$
$$x\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}$$

$\big\downarrow$ 5310

$$15b^2\left(-\frac{\sqrt{\pi}\sqrt{-\frac{i}{b}}bx\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}+\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{I}}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1-id\right.\right.\right.}\right.$$

$$\frac{5b\sqrt{d^2x^4+2idx^2}\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}{dx}+x\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}$$

input
```
Int[(a + I*b*ArcSin[1 - I*d*x^2])^(5/2),x]
```

output
```
(-5*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2))/(d*
x) + x*(a + I*b*ArcSin[1 - I*d*x^2])^(5/2) + 15*b^2*(x*Sqrt[a + I*b*ArcSin
[1 - I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1
- I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[(-I)/b]*(C
os[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])) - (Sqrt[(-I)/b]*b
*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt
[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin
[ArcSin[1 - I*d*x^2]/2]))
```

3.328.3.1 Defintions of rubi rules used

rule 5310
```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a
/(2*b)])*(FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[c/b]
*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] + Simp[Sqrt[
Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]]/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
d*x^2]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

rule 5313
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a
 + b*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
+ b*ArcSin[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

3.328.4 Maple [F]

$$\int \left(a + b \operatorname{arcsinh}\left(d\,x^{2} + i\right)\right)^{\frac{5}{2}} dx$$

input
```
int((a+b*arcsinh(I+d*x^2))^(5/2),x)
```

output
```
int((a+b*arcsinh(I+d*x^2))^(5/2),x)
```

3.328.5 Fricas [F(-2)]

Exception generated.

$$\int \left(a + ib \arcsin\left(1 - idx^{2}\right)\right)^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.328.6 Sympy [F(-2)]

Exception generated.

$$\int \left(a + ib \arcsin\left(1 - idx^{2}\right)\right)^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*asinh(I+d*x**2))**(5/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real I
```

3.328.7 Maxima [F]

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}\,dx=\int \left(b\,\mathrm{arsinh}\left(dx^2+i\right)+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x^2 + I) + a)^(5/2), x)
```

3.328.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}\,dx=\int \left(a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)\right)^{5/2}\,dx$$

input
```
int((a + b*asinh(d*x^2 + 1i))^(5/2),x)
```

output
```
int((a + b*asinh(d*x^2 + 1i))^(5/2), x)
```

3.329 $\int (a+ib\arcsin(1-idx^2))^{3/2}\,dx$

3.329.1 Optimal result

Integrand size = 22, antiderivative size = 312

$$\int (a+ib\arcsin(1-idx^2))^{3/2}\,dx = -\frac{3b\sqrt{2idx^2+d^2x^4}\sqrt{a+ib\arcsin(1-idx^2)}}{dx}+x(a+ib\arcsin(1-idx^2))^{3/2}+\frac{3\sqrt{ib}b\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)}-\frac{3b^2\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}$$

output
```
x*(a-I*b*arcsin(-1+I*d*x^2))^(3/2)-3*b^2*x*FresnelS((a-I*b*arcsin(-1+I*d*x
^2))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/
(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))/(I*b)^(1/2)+3*b*
x*FresnelC((a-I*b*arcsin(-1+I*d*x^2))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(I*cosh(
1/2*a/b)-sinh(1/2*a/b))*(I*b)^(1/2)*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2))+
sin(1/2*arcsin(-1+I*d*x^2)))-3*b*(2*I*d*x^2+d^2*x^4)^(1/2)*(a-I*b*arcsin(-
1+I*d*x^2))^(1/2)/d/x
```

3.329.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.83

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}\,dx =$$
$$-\frac{3b\sqrt{dx^2\left(2i+dx^2\right)}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{dx}+x\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}$$
$$+\frac{3b^2\sqrt{\pi}x\left(-\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)-\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\right.\right.\right.}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}$$

input
```
Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(3/2),x]
```

output
```
(-3*b*Sqrt[d*x^2*(2*I + d*x^2)]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/(d*x) +
 x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2) + (3*b^2*Sqrt[Pi]*x*(-(FresnelS[Sqr
t[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Si
nh[a/(2*b)])) - FresnelC[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt
[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*
x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

3.329.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5313, 5318}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}dx$$
$$\Big\downarrow \text{ 5313}$$
$$3b^2\int \frac{1}{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}dx-\frac{3b\sqrt{d^2x^4+2idx^2}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{dx}+ x\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}$$
$$\Big\downarrow \text{ 5318}$$

$$3b^2\left(-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{a}}{}\right.}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{arc}\right.\right.}\right.$$

$$\frac{3b\sqrt{d^2x^4+2idx^2}\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{dx}+x\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}$$

input
```
Int[(a + I*b*ArcSin[1 - I*d*x^2])^(3/2),x]
```

output
```
(-3*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/(d*x)
 + x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2) + 3*b^2*(-((Sqrt[Pi]*x*FresnelS[S
qrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*
Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*
d*x^2]/2]))) - (Sqrt[Pi]*x*FresnelC[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqr
t[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSi
n[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])))
```

3.329.3.1 Defintions of rubi rules used

rule 5313
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a
 + b*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
+ b*ArcSin[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

rule 5318
```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-
Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi
]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c
*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/
(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(
Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /;
FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

3.329.4 Maple [F]

$$\int \left(a+b \,\mathrm{arcsinh}\left(d\,x^{2}+i\right)\right)^{\frac{3}{2}}dx$$

input
```
int((a+b*arcsinh(I+d*x^2))^(3/2),x)
```

output
```
int((a+b*arcsinh(I+d*x^2))^(3/2),x)
```

3.329.5 Fricas [F(-2)]

Exception generated.

$$\int \left(a+ib \arcsin \left(1-idx^2\right)\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.329.6 Sympy [F(-2)]

Exception generated.

$$\int \left(a+ib \arcsin \left(1-idx^2\right)\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*asinh(I+d*x**2))**(3/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real I
```

3.329.7 Maxima [F]

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}\,dx=\int \left(b\,\mathrm{arsinh}\left(dx^2+i\right)+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x^2 + I) + a)^(3/2), x)
```

3.329.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}\,dx=\int \left(a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)\right)^{3/2}\,dx$$

input
```
int((a + b*asinh(d*x^2 + 1i))^(3/2),x)
```

output
```
int((a + b*asinh(d*x^2 + 1i))^(3/2), x)
```

3.330 $\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx$

3.330.1 Optimal result

Integrand size = 22, antiderivative size = 263

$$\begin{aligned}&\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx\\&=x\sqrt{a+ib\arcsin\left(1-idx^2\right)}\\&+\frac{\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}\\&-\frac{\sqrt{-\frac{i}{b}}b\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}\end{aligned}$$

output
```
x*FresnelS((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1
/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arc
sin(-1+I*d*x^2)))/(-I/b)^(1/2)-b*x*FresnelC((-I/b)^(1/2)*(a-I*b*arcsin(-1+
I*d*x^2))^(1/2)/Pi^(1/2))*(I*cosh(1/2*a/b)+sinh(1/2*a/b))*(-I/b)^(1/2)*Pi^
(1/2)/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))+x*(a-I*b*a
rcsin(-1+I*d*x^2))^(1/2)
```

3.330.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.98

$$\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx$$
$$=\frac{x\left(\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin\left(1-idx^2\right)}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)-\sqrt{\pi}\,\text{FresnelC}\left(\right.\right.}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1-\right.\right.\right.}$$

input
```
Integrate[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

output
```
(x*(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]*(Cos[ArcSin[1 - I*d*x^2
]/2] - Sin[ArcSin[1 - I*d*x^2]/2]) - Sqrt[Pi]*FresnelC[(Sqrt[(-I)/b]*Sqrt[
a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)])
+ Sqrt[Pi]*FresnelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[
Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[(-I)/b]*(Cos[ArcSin[1 - I*d
*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

3.330.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {5310}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx$$
$$\Big\downarrow \text{5310}$$
$$-\frac{\sqrt{\pi}\sqrt{-\frac{i}{b}}bx\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}+\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}+x\sqrt{a+ib\arcsin\left(1-idx^2\right)}$$

input
```
Int[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

output
```
x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*S
qrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b
)]))/(Sqrt[(-I)/b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2
])) - (Sqrt[(-I)/b]*b*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSi
n[1 - I*d*x^2]])/Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Cos[ArcSin[
1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])
```

3.330.3.1 Defintions of rubi rules used

rule 5310
```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a
/(2*b)])*(FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[c/b]
*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] + Simp[Sqrt[
Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]]/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
d*x^2]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

3.330.4 Maple [F]

$$\int \sqrt{a+b\,\mathrm{arcsinh}\left(d\,x^{2}+i\right)}dx$$

input
```
int((a+b*arcsinh(I+d*x^2))^(1/2),x)
```

output
```
int((a+b*arcsinh(I+d*x^2))^(1/2),x)
```

3.330.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.330.6 Sympy [F(-2)]

Exception generated.

$$\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*asinh(I+d*x**2))**(1/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real I
```

3.330.7 Maxima [F]

$$\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx = \int \sqrt{b\,\text{arsinh}\left(dx^2+i\right)+a}\,dx$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(d*x^2 + I) + a), x)
```

3.330.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+ib\arcsin\left(1-idx^2\right)}\,dx=\int \sqrt{a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)}\,dx$$

input
```
int((a + b*asinh(d*x^2 + 1i))^(1/2),x)
```

output
```
int((a + b*asinh(d*x^2 + 1i))^(1/2), x)
```

3.331 $\int \frac{1}{\sqrt{a+ib\arcsin(1-idx^2)}}\,dx$

3.331.1 Optimal result

Integrand size = 22, antiderivative size = 231

$$\int \frac{1}{\sqrt{a+ib\arcsin(1-idx^2)}}\,dx = -\frac{\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)} - \frac{\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}$$

output
```
-x*FresnelS((a-I*b*arcsin(-1+I*d*x^2))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(cosh(1
/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arc
sin(-1+I*d*x^2)))/(I*b)^(1/2)-x*FresnelC((a-I*b*arcsin(-1+I*d*x^2))^(1/2)/
(I*b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*ar
csin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))/(I*b)^(1/2)
```

3.331.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a+ib\arcsin(1-idx^2)}}\,dx$$
$$= \frac{\sqrt{\pi}x\left(-\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)-\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)\right.\right.}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}$$

input
```
Integrate[1/Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

output
```
(Sqrt[Pi]*x*(-(FresnelS[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[
Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)])) - FresnelC[Sqrt[a + I*b*ArcSin[1
- I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Sqr
t[I*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

3.331.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {5318}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ib\arcsin(1-idx^2)}}\,dx$$
$$\Big\downarrow \ 5318$$
$$-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}$$

input
```
Int[1/Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

output
```
-((Sqrt[Pi]*x*FresnelS[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[P
i])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2
]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))) - (Sqrt[Pi]*x*FresnelC[Sqrt[a + I*b*A
rcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]
))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

3.331.3.1 Defintions of rubi rules used

rule 5318
```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-
Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi
]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c
*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/
(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(
Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /;
FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

3.331.4 Maple [F]

$$\int \frac{1}{\sqrt{a + b \,\mathrm{arcsinh}\left(d\,x^{2}+i\right)}}dx$$

input
```
int(1/(a+b*arcsinh(I+d*x^2))^(1/2),x)
```

output
```
int(1/(a+b*arcsinh(I+d*x^2))^(1/2),x)
```

3.331.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+ib\arcsin(1-idx^2)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.331.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+ib\arcsin(1-idx^2)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*asinh(I+d*x**2))**(1/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real I
```

3.331.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+ib\arcsin(1-idx^2)}}\,dx = \int \frac{1}{\sqrt{b\,\text{arsinh}(dx^2+i)+a}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*arcsinh(d*x^2 + I) + a), x)
```

3.331.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+ib\arcsin(1-idx^2)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}\,dx=\int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)}}\,dx$$

input
```
int(1/(a + b*asinh(d*x^2 + 1i))^(1/2),x)
```

output
```
int(1/(a + b*asinh(d*x^2 + 1i))^(1/2), x)
```

3.332 $\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}\,dx$

3.332.1 Optimal result

Integrand size = 22, antiderivative size = 291

$$\begin{aligned}\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}\,dx &= -\frac{\sqrt{2idx^2+d^2x^4}}{bdx\sqrt{a+ib\arcsin\left(1-idx^2\right)}}\\
&-\frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}\\
&+\frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)}\end{aligned}$$

output
```
-(-I/b)^(3/2)*x*FresnelC((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)/Pi^
(1/2))*(cosh(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2
))+sin(1/2*arcsin(-1+I*d*x^2)))+(-I/b)^(3/2)*x*FresnelS((-I/b)^(1/2)*(a-I*
b*arcsin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(
1/2)/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))-(2*I*d*x^2+
d^2*x^4)^(1/2)/b/d/x/(a-I*b*arcsin(-1+I*d*x^2))^(1/2)
```

3.332.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{3/2}}\,dx = -\frac{\sqrt{2idx^2+d^2x^4}}{bdx\sqrt{a+ib\arcsin(1-idx^2)}} - \frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)} + \frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)}$$

input
```
Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-3/2),x]
```

output
```
-(Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])) -
 (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 -
 I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I
*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*Fres
nelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2
*b)] + I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*
x^2]/2])
```

3.332.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {5321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{3/2}}\,dx$$

$$\Big\downarrow \; 5321$$

$$-\frac{\sqrt{d^2x^4+2idx^2}}{bdx\sqrt{a+ib\arcsin(1-idx^2)}}-\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)}+\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)}$$

input
```
Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-3/2),x]
```

output
```
-(Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])) -
 (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 -
 I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I
*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*Fres
nelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2
*b)] + I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*
x^2]/2])
```

3.332.3.1 Defintions of rubi rules used

rule 5321
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[-
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*Si
n[ArcSin[c + d*x^2]/2])), x] + Simp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Co
s[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2])), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

3.332.4 Maple [F]

$$\int \frac{1}{(a+b\,\text{arcsinh}\,(d\,x^2+i))^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*arcsinh(I+d*x^2))^(3/2),x)
```

output
```
int(1/(a+b*arcsinh(I+d*x^2))^(3/2),x)
```

3.332.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.332.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*asinh(I+d*x**2))**(3/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real I
```

3.332.7 Maxima [F]

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{3/2}}\,dx = \int \frac{1}{(b\operatorname{arsinh}(dx^2+i)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x^2 + I) + a)^(-3/2), x)
```

3.332.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{3/2}}\,dx = \int \frac{1}{(a+b\operatorname{asinh}(d\,x^2+1\mathrm{i}))^{3/2}}\,dx$$

input
```
int(1/(a + b*asinh(d*x^2 + 1i))^(3/2),x)
```

output
```
int(1/(a + b*asinh(d*x^2 + 1i))^(3/2), x)
```

3.333 $\int \frac{1}{(a+ib\arcsin(1-idx^2))^{5/2}}\,dx$

3.333.1 Optimal result

Integrand size = 22, antiderivative size = 326

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{5/2}}\,dx = -\frac{\sqrt{2idx^2+d^2x^4}}{3bdx(a+ib\arcsin(1-idx^2))^{3/2}} - \frac{x}{3b^2\sqrt{a+ib\arcsin(1-idx^2)}} - \frac{\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{3\sqrt{ib}b^2\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)} - \frac{\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{3\sqrt{ib}b^2\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}$$

output
```
-1/3*x*FresnelS((a-I*b*arcsin(-1+I*d*x^2))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(co
sh(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arcsin(-1+I*d*x^2))+sin
(1/2*arcsin(-1+I*d*x^2)))/(I*b)^(1/2)-1/3*x*FresnelC((a-I*b*arcsin(-1+I*d*
x^2))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)
/b^2/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))/(I*b)^(1/2)
-1/3*(2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-I*b*arcsin(-1+I*d*x^2))^(3/2)-1/3*
x/b^2/(a-I*b*arcsin(-1+I*d*x^2))^(1/2)
```

3.333.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{5/2}}\,dx =$$

$$-\frac{\frac{b\sqrt{dx^2(2i+dx^2)}}{dx(a+ib\arcsin(1-idx^2))^{3/2}}+\frac{x}{\sqrt{a+ib\arcsin(1-idx^2)}}+\frac{\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}+\frac{\sqrt{\pi}x\,\text{FresnelC}\left(\cdots\right.}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\,\mathrm{a}\cdots\right.\right.}}{3b^2}$$

input
```
Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-5/2),x]
```

output
```
-1/3*((b*Sqrt[d*x^2*(2*I + d*x^2)])/(d*x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/
2)) + x/Sqrt[a + I*b*ArcSin[1 - I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[Sqrt[a +
I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(
2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2
])) + (Sqrt[Pi]*x*FresnelC[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sq
rt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d
*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])))/b^2
```

3.333.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5327, 5318}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{5/2}}\,dx$$

$$\Big\downarrow \; 5327$$

$$\frac{\int \frac{1}{\sqrt{a+ib\arcsin(1-idx^2)}}dx}{3b^2}-\frac{x}{3b^2\sqrt{a+ib\arcsin(1-idx^2)}}-\frac{\sqrt{d^2x^4+2idx^2}}{3bdx\,(a+ib\arcsin(1-idx^2))^{3/2}}$$

$$\Big\downarrow \; 5318$$

$$\frac{-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{a+ib\arcsin\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)\right)}}{3b^2}-\frac{x}{3b^2\sqrt{a+ib\arcsin\left(1-idx^2\right)}}-\frac{\sqrt{d^2x^4+2idx^2}}{3bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}$$

input
```
Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-5/2),x]
```

output
```
-1/3*Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2
)) - x/(3*b^2*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]) + (-((Sqrt[Pi]*x*FresnelS
[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] -
I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 -
I*d*x^2]/2]))) - (Sqrt[Pi]*x*FresnelC[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(S
qrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[Arc
Sin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])))/(3*b^2)
```

3.333.3.1 Defintions of rubi rules used

rule 5318
```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-
Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi
]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c
*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/
(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(
Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /;
FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

rule 5327
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Simp[Sqrt
[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n + 1)*x)
), x] - Simp[1/(4*b^2*(n + 1)*(n + 2))   Int[(a + b*ArcSin[c + d*x^2])^(n +
 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

3.333. $\int \frac{1}{(a+ib\arcsin(1-idx^2))^{5/2}}\,dx$

3.333.4 Maple [F]

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}\,(d\,x^2+i))^{\frac{5}{2}}}dx$$

input
```
int(1/(a+b*arcsinh(I+d*x^2))^(5/2),x)
```

output
```
int(1/(a+b*arcsinh(I+d*x^2))^(5/2),x)
```

3.333.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.333.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*asinh(I+d*x**2))**(5/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real I
```

3.333.7 Maxima [F]

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{5/2}}\,dx = \int \frac{1}{(b\,\mathrm{arsinh}(dx^2+i)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x^2 + I) + a)^(-5/2), x)
```

3.333.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{5/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}(d\,x^2+1\mathrm{i}))^{5/2}}\,dx$$

input
```
int(1/(a + b*asinh(d*x^2 + 1i))^(5/2),x)
```

output
```
int(1/(a + b*asinh(d*x^2 + 1i))^(5/2), x)
```

3.334 $\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^{7/2}}\,dx$

3.334.1 Optimal result

Integrand size = 22, antiderivative size = 389

$$\begin{aligned}\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^{7/2}}\,dx &= -\frac{\sqrt{2idx^2+d^2x^4}}{5bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}}\\
&-\frac{x}{15b^2\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}-\frac{\sqrt{2idx^2+d^2x^4}}{15b^3dx\sqrt{a+ib\arcsin\left(1-idx^2\right)}}\\
&-\frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\mathrm{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{15b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}\\
&+\frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\mathrm{FresnelS}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{15b^2\left(\cos\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1-idx^2\right)\right)\right)}\end{aligned}$$

output
```
-1/15*x/b^2/(a-I*b*arcsin(-1+I*d*x^2))^(3/2)-1/15*(-I/b)^(3/2)*x*FresnelC(
(-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)-I*s
inh(1/2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+
I*d*x^2)))+1/15*(-I/b)^(3/2)*x*FresnelS((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*
x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/b^2/(cos(1/
2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))-1/5*(2*I*d*x^2+d^2*x^4)
^(1/2)/b/d/x/(a-I*b*arcsin(-1+I*d*x^2))^(5/2)-1/15*(2*I*d*x^2+d^2*x^4)^(1/
2)/b^3/d/x/(a-I*b*arcsin(-1+I*d*x^2))^(1/2)
```

3.334.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{7/2}}\,dx = \frac{-\frac{3b\sqrt{dx^2(2i+dx^2)}}{d}-x^2(a+ib\arcsin(1-idx^2))+\frac{\sqrt{dx^2(2i+dx^2)}(-ia+b\arcsin(1-idx^2))^2}{bd}}{x(a+ib\arcsin(1-idx^2))^{5/2}} - \frac{(-\frac{i}{b})}{}$$

input
```
Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-7/2),x]
```

output
```
(((-3*b*Sqrt[d*x^2*(2*I + d*x^2)])/d - x^2*(a + I*b*ArcSin[1 - I*d*x^2]) +
 (Sqrt[d*x^2*(2*I + d*x^2)]*((-I)*a + b*ArcSin[1 - I*d*x^2])^2)/(b*d))/(x*
(a + I*b*ArcSin[1 - I*d*x^2])^(5/2)) - (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelC
[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)]
 - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]
/2]) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcS
in[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Cos[ArcSin
[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))/(15*b^2)
```

3.334.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5327, 5321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{7/2}}dx$$

$$\Big\downarrow \text{5327}$$

$$\frac{\int \frac{1}{(a+ib\arcsin(1-idx^2))^{3/2}}dx}{15b^2} - \frac{x}{15b^2(a+ib\arcsin(1-idx^2))^{3/2}} - \frac{\sqrt{d^2x^4+2idx^2}}{5bdx(a+ib\arcsin(1-idx^2))^{5/2}}$$

$$\Big\downarrow \text{5321}$$

$$\frac{-\frac{\sqrt{d^2x^4+2idx^2}}{bdx\sqrt{a+ib\arcsin(1-idx^2)}}-\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\arcsin(1-idx^2)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\arcsin(1-idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1-idx^2)\right)}+\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\ldots\right.}{\cos\left(\frac{1}{2}\arcsin(1-\ldots\right.}}{15b^2}$$

$$\frac{x}{15b^2\left(a+ib\arcsin\left(1-idx^2\right)\right)^{3/2}}-\frac{\sqrt{d^2x^4+2idx^2}}{5bdx\left(a+ib\arcsin\left(1-idx^2\right)\right)^{5/2}}$$

input
```
Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-7/2),x]
```

output
```
-1/5*Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^(5/2
)) - x/(15*b^2*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2)) + (-(Sqrt[(2*I)*d*x^2
+ d^2*x^4]/(b*d*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])) - (((-I)/b)^(3/2)*Sq
rt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi
]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[Ar
cSin[1 - I*d*x^2]/2]) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*
Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*
b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))/(15*b^2)
```

3.334.3.1 Defintions of rubi rules used

rule 5321
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[-
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*Si
n[ArcSin[c + d*x^2]/2])), x] + Simp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Co
s[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2])), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

rule 5327
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Simp[Sqrt
[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n + 1)*x)
), x] - Simp[1/(4*b^2*(n + 1)*(n + 2))   Int[(a + b*ArcSin[c + d*x^2])^(n +
 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

3.334. $\int \frac{1}{\left(a+ib\arcsin\left(1-idx^2\right)\right)^{7/2}}\,dx$

3.334.4 Maple [F]

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^{2}+i\right)\right)^{\frac{7}{2}}}dx$$

input
```
int(1/(a+b*arcsinh(I+d*x^2))^(7/2),x)
```

output
```
int(1/(a+b*arcsinh(I+d*x^2))^(7/2),x)
```

3.334.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.334.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*asinh(I+d*x**2))**(7/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real I
```

3.334.7 Maxima [F]

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{7/2}}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(dx^2+i)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x^2 + I) + a)^(-7/2), x)
```

3.334.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(I+d*x^2))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+ib\arcsin(1-idx^2))^{7/2}}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(d\,x^2+1\text{i}))^{7/2}}\,dx$$

input
```
int(1/(a + b*asinh(d*x^2 + 1i))^(7/2),x)
```

output
```
int(1/(a + b*asinh(d*x^2 + 1i))^(7/2), x)
```

3.335 $\int (a - ib \arcsin(1 + idx^2))^{5/2} \, dx$

3.335.1 Optimal result

Integrand size = 22, antiderivative size = 348

$$\int (a - ib \arcsin(1 + idx^2))^{5/2} \, dx = 15b^2 x \sqrt{a - ib \arcsin(1 + idx^2)} - \frac{5b\sqrt{-2idx^2 + d^2x^4}(a - ib \arcsin(1 + idx^2))^{3/2}}{dx} + x(a - ib \arcsin(1 + idx^2))^{5/2} + \frac{15b^2\sqrt{\pi}x \operatorname{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right) - i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin(1 + idx^2)\right) - \sin\left(\frac{1}{2}\arcsin(1 + idx^2)\right)\right)} - \frac{15b^2\sqrt{\pi}x \operatorname{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin(1 + idx^2)\right) - \sin\left(\frac{1}{2}\arcsin(1 + idx^2)\right)\right)}$$

output
```
x*(a-I*b*arcsin(1+I*d*x^2))^(5/2)+15*b^2*x*FresnelS((I/b)^(1/2)*(a-I*b*arc
sin(1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(
cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))/(I/b)^(1/2)-15*b^2*
x*FresnelC((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2
*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin
(1+I*d*x^2)))/(I/b)^(1/2)-5*b*(a-I*b*arcsin(1+I*d*x^2))^(3/2)*(-2*I*d*x^2+
d^2*x^4)^(1/2)/d/x+15*b^2*x*(a-I*b*arcsin(1+I*d*x^2))^(1/2)
```

3.335.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.97

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2}\,dx =$$
$$-\frac{5b\sqrt{dx^2\left(-2i+dx^2\right)}\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}}{dx}+x\left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2}$$
$$+\frac{15b^2x\left(\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin\left(1+idx^2\right)}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)+\sqrt{\pi}\,\mathrm{FresnelS}\right.}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1+ic\right.\right.\right.}$$

input
```
Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(5/2),x]
```

output
```
(-5*b*Sqrt[d*x^2*(-2*I + d*x^2)]*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2))/(d*x
) + x*(a - I*b*ArcSin[1 + I*d*x^2])^(5/2) + (15*b^2*x*(Sqrt[I/b]*Sqrt[a -
I*b*ArcSin[1 + I*d*x^2]]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*
x^2]/2]) + Sqrt[Pi]*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])
/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) - Sqrt[Pi]*FresnelC[(Sqrt[I/b
]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(
2*b)])))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/
2]))
```

3.335.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5313, 5310}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2}\,dx$$
$$\Big\downarrow \; 5313$$
$$15b^2\int \sqrt{a-ib\arcsin\left(idx^2+1\right)}dx-\frac{5b\sqrt{d^2x^4-2idx^2}\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}}{dx}+ x\left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2}$$

$$\Big\downarrow \text{5310}$$

$$15b^2\left(-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(idx^2+1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}+\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\right.}\right.$$

$$\frac{5b\sqrt{d^2x^4-2idx^2}\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}}{dx}+x\left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2}$$

input
```
Int[(a - I*b*ArcSin[1 + I*d*x^2])^(5/2),x]
```

output
```
(-5*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2))/(d
*x) + x*(a - I*b*ArcSin[1 + I*d*x^2])^(5/2) + 15*b^2*(x*Sqrt[a - I*b*ArcSi
n[1 + I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 +
I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[Ar
cSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) - (Sqrt[Pi]*x*FresnelC
[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] +
I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 +
I*d*x^2]/2])))
```

3.335.3.1 Defintions of rubi rules used

rule 5310
```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a
/(2*b)])*(FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[c/b]
*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] + Simp[Sqrt[
Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]]/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
d*x^2]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

rule 5313
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a
 + b*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
+ b*ArcSin[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

3.335.4 Maple [F]

$$\int \left(a + b \operatorname{arcsinh}\left(d\,x^{2} - i\right)\right)^{\frac{5}{2}} dx$$

input
```
int((a+b*arcsinh(-I+d*x^2))^(5/2),x)
```

output
```
int((a+b*arcsinh(-I+d*x^2))^(5/2),x)
```

3.335.5 Fricas [F(-2)]

Exception generated.

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.335.6 Sympy [F(-2)]

Exception generated.

$$\int \left(a - ib \arcsin\left(1 + idx^2\right)\right)^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*asinh(-I+d*x**2))**(5/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

3.335.7 Maxima [F]

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2} dx = \int \left(b\operatorname{arsinh}\left(dx^2-i\right)+a\right)^{\frac{5}{2}} dx$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x^2 - I) + a)^(5/2), x)
```

3.335.8 Giac [F(-2)]

Exception generated.

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2} dx = \int \left(a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)\right)^{5/2} dx$$

input
```
int((a + b*asinh(d*x^2 - 1i))^(5/2),x)
```

output
```
int((a + b*asinh(d*x^2 - 1i))^(5/2), x)
```

3.336 $\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}\,dx$

3.336.1 Optimal result

Integrand size = 22, antiderivative size = 310

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}\,dx = -\frac{3b\sqrt{-2idx^2+d^2x^4}\sqrt{a-ib\arcsin\left(1+idx^2\right)}}{dx}+x\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2} - \frac{3b^2\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)} - \frac{3\sqrt{-ib}b\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)}$$

output
```
x*(a-I*b*arcsin(1+I*d*x^2))^(3/2)-3*b^2*x*FresnelS((a-I*b*arcsin(1+I*d*x^2
))^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(
cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))/(-I*b)^(1/2)-3*b*x*
FresnelC((a-I*b*arcsin(1+I*d*x^2))^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(I*cosh(1/
2*a/b)+sinh(1/2*a/b))*(-I*b)^(1/2)*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-si
n(1/2*arcsin(1+I*d*x^2)))-3*b*(-2*I*d*x^2+d^2*x^4)^(1/2)*(a-I*b*arcsin(1+I
*d*x^2))^(1/2)/d/x
```

3.336.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.82

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}\,dx =$$
$$-\frac{3b\sqrt{dx^2\left(-2i+dx^2\right)}\sqrt{a-ib\arcsin\left(1+idx^2\right)}}{dx}+x\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}$$
$$-\frac{3(-ib)^{3/2}\sqrt{\pi}x\left(-\operatorname{FresnelC}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)-\operatorname{FresnelS}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\right.\right.}{\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)}$$

input
```
Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(3/2),x]
```

output
```
(-3*b*Sqrt[d*x^2*(-2*I + d*x^2)]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/(d*x)
+ x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2) - (3*((-I)*b)^(3/2)*Sqrt[Pi]*x*(-(
FresnelC[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[
a/(2*b)] - I*Sinh[a/(2*b)])) - FresnelS[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/
(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Cos[ArcSin[1
 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

3.336.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5313, 5318}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{5313}$$

$$3b^2\int \frac{1}{\sqrt{a-ib\arcsin\left(idx^2+1\right)}}dx-\frac{3b\sqrt{d^2x^4-2idx^2}\sqrt{a-ib\arcsin\left(1+idx^2\right)}}{dx}+$$
$$x\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}$$

$$\Big\downarrow \text{5318}$$

$$3b^2\left(-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a-ib\arcsin(idx^2+1)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{a}}{}\right.}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\right.\right.}\right.$$
$$\frac{3b\sqrt{d^2x^4-2idx^2}\sqrt{a-ib\arcsin\left(1+idx^2\right)}}{dx}+x\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}$$

input
```
Int[(a - I*b*ArcSin[1 + I*d*x^2])^(3/2),x]
```

output
```
(-3*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/(d*x
) + x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2) + 3*b^2*(-((Sqrt[Pi]*x*FresnelC[
Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)]
- I*Sinh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin
[1 + I*d*x^2]/2]))) - (Sqrt[Pi]*x*FresnelS[Sqrt[a - I*b*ArcSin[1 + I*d*x^2
]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[(-I)*
b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])))
```

3.336.3.1 Defintions of rubi rules used

rule 5313
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a
 + b*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
+ b*ArcSin[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

rule 5318
```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-
Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi
]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c
*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/
(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(
Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /;
FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

3.336. $\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}\,dx$

3.336.4 Maple [F]

$$\int \left(a+b\,\mathrm{arcsinh}\left(d\,x^{2}-i\right)\right)^{\frac{3}{2}}dx$$

input
```
int((a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

output
```
int((a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

3.336.5 Fricas [F(-2)]

Exception generated.

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.336.6 Sympy [F(-2)]

Exception generated.

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*asinh(-I+d*x**2))**(3/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

3.336.7 Maxima [F]

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2} dx = \int \left(b\,\mathrm{arsinh}\left(dx^2-i\right)+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x^2 - I) + a)^(3/2), x)
```

3.336.8 Giac [F(-2)]

Exception generated.

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2} dx = \int \left(a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)\right)^{3/2} dx$$

input
```
int((a + b*asinh(d*x^2 - 1i))^(3/2),x)
```

output
```
int((a + b*asinh(d*x^2 - 1i))^(3/2), x)
```

3.337 $\int \sqrt{a-ib\arcsin\left(1+idx^2\right)}\,dx$

3.337.1 Optimal result

Integrand size = 22, antiderivative size = 262

$$\int \sqrt{a-ib\arcsin\left(1+idx^2\right)}\,dx = x\sqrt{a-ib\arcsin\left(1+idx^2\right)} + \frac{\sqrt{\pi}x\operatorname{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)} - \frac{\sqrt{\pi}x\operatorname{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)\right)}$$

output
```
x*FresnelS((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2
*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin
(1+I*d*x^2)))/(I/b)^(1/2)-x*FresnelC((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))
^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(
1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))/(I/b)^(1/2)+x*(a-I*b*arcsin(1+I*d*
x^2))^(1/2)
```

3.337.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.99

$$\int \sqrt{a-ib\arcsin(1+idx^2)}\,dx$$
$$= \frac{x\left(\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)+\sqrt{\pi}\,\text{FresnelS}\left(\sqrt{\right.\right.}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin(1+idx\right.\right.}$$

input
```
Integrate[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

output
```
(x*(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]*(Cos[ArcSin[1 + I*d*x^2]/2
] - Sin[ArcSin[1 + I*d*x^2]/2]) + Sqrt[Pi]*FresnelS[(Sqrt[I/b]*Sqrt[a - I*
b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) - Sqrt
[Pi]*FresnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cos
h[a/(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - S
in[ArcSin[1 + I*d*x^2]/2]))
```

3.337.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {5310}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-ib\arcsin(1+idx^2)}\,dx$$
$$\Big\downarrow \text{5310}$$
$$-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(idx^2+1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}+\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(idx^2+1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}+x\sqrt{a-ib\arcsin(1+idx^2)}$$

input
```
Int[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

output
```
x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt
[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)])
)/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) -
(Sqrt[Pi]*x*FresnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi
]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/
2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

3.337.3.1 Defintions of rubi rules used

rule 5310
```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a
/(2*b)])*(FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[c/b]
*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] + Simp[Sqrt[
Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]]/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
d*x^2]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

3.337.4 Maple [F]

$$\int \sqrt{a + b \operatorname{arcsinh}(d x^2 - i)} dx$$

input
```
int((a+b*arcsinh(-I+d*x^2))^(1/2),x)
```

output
```
int((a+b*arcsinh(-I+d*x^2))^(1/2),x)
```

3.337.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a - ib \arcsin(1 + idx^2)} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.337.6 Sympy [F(-2)]

Exception generated.

$$\int \sqrt{a - ib \arcsin\left(1 + idx^2\right)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*asinh(-I+d*x**2))**(1/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

3.337.7 Maxima [F]

$$\int \sqrt{a - ib \arcsin\left(1 + idx^2\right)}\, dx = \int \sqrt{b\, \operatorname{arsinh}\left(dx^2 - i\right) + a}\, dx$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arcsinh(d*x^2 - I) + a), x)
```

3.337.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{a - ib \arcsin\left(1 + idx^2\right)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a - ib \arcsin(1 + idx^2)}\, dx = \int \sqrt{a + b \operatorname{asinh}(d x^2 - \mathrm{i})}\, dx$$

input
```
int((a + b*asinh(d*x^2 - 1i))^(1/2),x)
```

output
```
int((a + b*asinh(d*x^2 - 1i))^(1/2), x)
```

3.338 $\int \frac{1}{\sqrt{a-ib\arcsin(1+idx^2)}}\,dx$

3.338.1 Optimal result

Integrand size = 22, antiderivative size = 231

$$\begin{aligned}\int \frac{1}{\sqrt{a-ib\arcsin(1+idx^2)}}\,dx &= -\frac{\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}\\ &\quad-\frac{\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}\end{aligned}$$

output
```
-x*FresnelC((a-I*b*arcsin(1+I*d*x^2))^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(cosh(1
/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcs
in(1+I*d*x^2)))/(-I*b)^(1/2)-x*FresnelS((a-I*b*arcsin(1+I*d*x^2))^(1/2)/(-
I*b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arc
sin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))/(-I*b)^(1/2)
```

3.338.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a-ib\arcsin(1+idx^2)}}\,dx$$
$$= \frac{\sqrt{\pi}x\left(-\text{FresnelC}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)-\text{FresnelS}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)\right.\right.}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}$$

input
```
Integrate[1/Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

output
```
(Sqrt[Pi]*x*(-(FresnelC[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sq
rt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)])) - FresnelS[Sqrt[a - I*b*ArcSin
[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))
)/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

3.338.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {5318}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-ib\arcsin(1+idx^2)}}\,dx$$
$$\Big\downarrow \; 5318$$
$$-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a-ib\arcsin(idx^2+1)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{a-ib\arcsin(idx^2+1)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}$$

input
```
Int[1/Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

output
```
-((Sqrt[Pi]*x*FresnelC[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqr
t[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I
*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))) - (Sqrt[Pi]*x*FresnelS[Sqrt[a -
 I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh
[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d
*x^2]/2]))
```

3.338.3.1 Defintions of rubi rules used

rule 5318
```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-
Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi
]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c
*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/
(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(
Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /;
FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

3.338.4 Maple [F]

$$\int \frac{1}{\sqrt{a+b\,\mathrm{arcsinh}\left(d\,x^{2}-i\right)}}dx$$

input
```
int(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x)
```

output
```
int(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x)
```

3.338.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a-ib\arcsin\left(1+idx^2\right)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.338.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a - ib\arcsin\left(1 + idx^2\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*asinh(-I+d*x**2))**(1/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

3.338.7 Maxima [F]

$$\int \frac{1}{\sqrt{a - ib\arcsin\left(1 + idx^2\right)}}\,dx = \int \frac{1}{\sqrt{b\,\text{arsinh}\left(dx^2 - i\right) + a}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*arcsinh(d*x^2 - I) + a), x)
```

3.338.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a - ib\arcsin\left(1 + idx^2\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a - ib\arcsin(1 + idx^2)}}\,dx = \int \frac{1}{\sqrt{a + b\,\mathrm{asinh}\left(d\,x^2 - \mathrm{i}\right)}}\,dx$$

input
```
int(1/(a + b*asinh(d*x^2 - 1i))^(1/2),x)
```

output
```
int(1/(a + b*asinh(d*x^2 - 1i))^(1/2), x)
```

3.339 $\int \frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}}\,dx$

3.339.1 Optimal result

Integrand size = 22, antiderivative size = 291

$$\int \frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}}\,dx = -\frac{\sqrt{-2idx^2+d^2x^4}}{bdx\sqrt{a-ib\arcsin\left(1+idx^2\right)}} + \frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)} - \frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\arcsin\left(1+idx^2\right)\right)}$$

output
```
(I/b)^(3/2)*x*FresnelS((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2
))*(cosh(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-si
n(1/2*arcsin(1+I*d*x^2)))-(I/b)^(3/2)*x*FresnelC((I/b)^(1/2)*(a-I*b*arcsin
(1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos
(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-(-2*I*d*x^2+d^2*x^4)^(
1/2)/b/d/x/(a-I*b*arcsin(1+I*d*x^2))^(1/2)
```

3.339.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a-ib\arcsin(1+idx^2))^{3/2}}\,dx = -\frac{\sqrt{-2idx^2+d^2x^4}}{bdx\sqrt{a-ib\arcsin(1+idx^2)}} + \frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)} - \frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)}$$

input
```
Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-3/2),x]
```

output
```
-(Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]))
+ ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*
x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 + I*d*x^
2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]) - ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sq
rt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Si
nh[a/(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

3.339.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {5321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-ib\arcsin(1+idx^2))^{3/2}}\,dx$$
$$\Big\downarrow \text{5321}$$

$$-\frac{\sqrt{d^2x^4-2idx^2}}{bdx\sqrt{a-ib\arcsin(1+idx^2)}}-\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(idx^2+1)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)}+\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(idx^2+1)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)}$$

input
```
Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-3/2),x]
```

output
```
-(Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]))
+ ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*
x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 + I*d*x^
2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]) - ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sq
rt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Si
nh[a/(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

3.339.3.1 Defintions of rubi rules used

rule 5321
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[-
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*Si
n[ArcSin[c + d*x^2]/2])), x] + Simp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Co
s[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2])), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

3.339.4 Maple [F]

$$\int \frac{1}{\left(a+b\,\operatorname{arcsinh}\left(d\,x^2-i\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

output
```
int(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

3.339.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a - ib \arcsin(1 + idx^2))^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.339.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a - ib \arcsin(1 + idx^2))^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*asinh(-I+d*x**2))**(3/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

3.339.7 Maxima [F]

$$\int \frac{1}{(a - ib \arcsin(1 + idx^2))^{3/2}} \, dx = \int \frac{1}{(b \operatorname{arsinh}(dx^2 - i) + a)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x^2 - I) + a)^(-3/2), x)
```

3.339.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a - ib \arcsin(1 + idx^2))^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a - ib \arcsin(1 + idx^2))^{3/2}} dx = \int \frac{1}{(a + b \operatorname{asinh}(d x^2 - \mathrm{i}))^{3/2}} dx$$

input
```
int(1/(a + b*asinh(d*x^2 - 1i))^(3/2),x)
```

output
```
int(1/(a + b*asinh(d*x^2 - 1i))^(3/2), x)
```

3.340 $\int \frac{1}{(a-ib\arcsin(1+idx^2))^{5/2}}\,dx$

3.340.1 Optimal result

Integrand size = 22, antiderivative size = 326

$$\int \frac{1}{(a-ib\arcsin(1+idx^2))^{5/2}}\,dx = -\frac{\sqrt{-2idx^2+d^2x^4}}{3bdx\,(a-ib\arcsin(1+idx^2))^{3/2}} - \frac{x}{3b^2\sqrt{a-ib\arcsin(1+idx^2)}} - \frac{\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{3\sqrt{-ib}b^2\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)} - \frac{\sqrt{-ib}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{3b^3\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}$$

output
```
-1/3*x*FresnelS((a-I*b*arcsin(1+I*d*x^2))^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(co
sh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arcsin(1+I*d*x^2))-sin(
1/2*arcsin(1+I*d*x^2)))/(-I*b)^(1/2)-1/3*x*FresnelC((a-I*b*arcsin(1+I*d*x^
2))^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(I*cosh(1/2*a/b)+sinh(1/2*a/b))*(-I*b)^(1
/2)*Pi^(1/2)/b^3/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1
/3*(-2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-I*b*arcsin(1+I*d*x^2))^(3/2)-1/3*x/
b^2/(a-I*b*arcsin(1+I*d*x^2))^(1/2)
```

3.340.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a - ib\arcsin(1 + idx^2))^{5/2}}\, dx =$$

$$-\frac{\frac{b\sqrt{dx^2(-2i+dx^2)}}{dx(a-ib\arcsin(1+idx^2))^{3/2}} + \frac{x}{\sqrt{a-ib\arcsin(1+idx^2)}} + \frac{\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)} + \frac{\sqrt{\pi}x\,\text{FresnelS}\left(\right.}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\right.\right.}}{3b^2}$$

input
```
Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-5/2),x]
```

output
```
-1/3*((b*Sqrt[d*x^2*(-2*I + d*x^2)])/(d*x*(a - I*b*ArcSin[1 + I*d*x^2])^(3
/2)) + x/Sqrt[a - I*b*ArcSin[1 + I*d*x^2]] + (Sqrt[Pi]*x*FresnelC[Sqrt[a -
 I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh
[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d
*x^2]/2])) + (Sqrt[Pi]*x*FresnelS[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[
(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[A
rcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])))/b^2
```

3.340.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5327, 5318}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a - ib\arcsin(1 + idx^2))^{5/2}}\, dx$$

$$\Big\downarrow \text{5327}$$

$$\frac{\int \frac{1}{\sqrt{a-ib\arcsin(idx^2+1)}}dx}{3b^2} - \frac{x}{3b^2\sqrt{a - ib\arcsin(1 + idx^2)}} - \frac{\sqrt{d^2x^4 - 2idx^2}}{3bdx\,(a - ib\arcsin(1 + idx^2))^{3/2}}$$

$$\Big\downarrow \text{5318}$$

$$-\frac{-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a-ib\arcsin\left(idx^2+1\right)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}-\frac{\sqrt{\pi}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelS}\left(\frac{\sqrt{a-ib\arcsin\left(idx^2+1\right)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}}{3b^2} - \frac{x}{3b^2\sqrt{a-ib\arcsin\left(1+idx^2\right)}}-\frac{\sqrt{d^2x^4-2idx^2}}{3bdx\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}}$$

input
```
Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-5/2),x]
```

output
```
-1/3*Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/
2)) - x/(3*b^2*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]) + (-((Sqrt[Pi]*x*Fresnel
C[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)
] - I*Sinh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcS
in[1 + I*d*x^2]/2]))) - (Sqrt[Pi]*x*FresnelS[Sqrt[a - I*b*ArcSin[1 + I*d*x
^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[(-I
)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])))/(3*b^2)
```

3.340.3.1 Defintions of rubi rules used

rule 5318
```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-
Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi
]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c
*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/
(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(
Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /;
FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

rule 5327
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Simp[Sqrt
[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n + 1)*x)
), x] - Simp[1/(4*b^2*(n + 1)*(n + 2))   Int[(a + b*ArcSin[c + d*x^2])^(n +
 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

3.340. $\int \frac{1}{(a-ib\arcsin(1+idx^2))^{5/2}}\,dx$

3.340.4 Maple [F]

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^{2}-i\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x)
```

output
```
int(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x)
```

3.340.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a-ib\arcsin(1+idx^2))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.340.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a-ib\arcsin(1+idx^2))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*asinh(-I+d*x**2))**(5/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

3.340.7 Maxima [F]

$$\int \frac{1}{(a-ib\arcsin(1+idx^2))^{5/2}}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(dx^2-i)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x^2 - I) + a)^(-5/2), x)
```

3.340.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-ib\arcsin(1+idx^2))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-ib\arcsin(1+idx^2))^{5/2}}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(d\,x^2-\mathrm{i}))^{5/2}}\,dx$$

input
```
int(1/(a + b*asinh(d*x^2 - 1i))^(5/2),x)
```

output
```
int(1/(a + b*asinh(d*x^2 - 1i))^(5/2), x)
```

3.341 $\int \frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^{7/2}}\,dx$

3.341.1 Optimal result

Integrand size = 22, antiderivative size = 389

$$\begin{aligned}\int \frac{1}{(a-ib\arcsin(1+idx^2))^{7/2}}\,dx &= -\frac{\sqrt{-2idx^2+d^2x^4}}{5bdx\,(a-ib\arcsin(1+idx^2))^{5/2}} \\ &- \frac{x}{15b^2\,(a-ib\arcsin(1+idx^2))^{3/2}} - \frac{\sqrt{-2idx^2+d^2x^4}}{15b^3dx\sqrt{a-ib\arcsin(1+idx^2)}} \\ &- \frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{15b^2\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)} \\ &+ \frac{\sqrt{\frac{i}{b}}\sqrt{\pi}x\,\text{FresnelS}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin(1+idx^2)}}{\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{15b^3\left(\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)\right)}\end{aligned}$$

output
```
-1/15*x/b^2/(a-I*b*arcsin(1+I*d*x^2))^(3/2)-1/15*(I/b)^(3/2)*x*FresnelC((I
/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(
1/2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^
2)))+1/15*x*FresnelS((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2))
*(I*cosh(1/2*a/b)+sinh(1/2*a/b))*(I/b)^(1/2)*Pi^(1/2)/b^3/(cos(1/2*arcsin(
1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/5*(-2*I*d*x^2+d^2*x^4)^(1/2)/b/d
/x/(a-I*b*arcsin(1+I*d*x^2))^(5/2)-1/15*(-2*I*d*x^2+d^2*x^4)^(1/2)/b^3/d/x
/(a-I*b*arcsin(1+I*d*x^2))^(1/2)
```

3.341.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a - ib\arcsin(1 + idx^2))^{7/2}}\,dx = \frac{\frac{-\frac{3b\sqrt{dx^2(-2i+dx^2)}}{d} - x^2(a-ib\arcsin(1+idx^2)) + \frac{\sqrt{dx^2(-2i+dx^2)}(ia+b\arcsin(1+idx^2))^2}{bd}}{x(a-ib\arcsin(1+idx^2))^{5/2}} + \frac{\sqrt{\frac{i}{b}}\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-7/2),x]
```

output
```
(((-3*b*Sqrt[d*x^2*(-2*I + d*x^2)])/d - x^2*(a - I*b*ArcSin[1 + I*d*x^2])
+ (Sqrt[d*x^2*(-2*I + d*x^2)]*(I*a + b*ArcSin[1 + I*d*x^2])^2)/(b*d))/(x*(
a - I*b*ArcSin[1 + I*d*x^2])^(5/2)) + (Sqrt[I/b]*Sqrt[Pi]*x*FresnelC[(Sqrt
[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*((-I)*Cosh[a/(2*b)] + S
inh[a/(2*b)]))/(b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]
)) + (Sqrt[I/b]*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d
*x^2]])/Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(b*(Cos[ArcSin[1 + I*
d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])))/(15*b^2)
```

3.341.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5327, 5321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a - ib\arcsin(1 + idx^2))^{7/2}}\,dx$$

$$\Big\downarrow \text{5327}$$

$$\frac{\int \frac{1}{(a-ib\arcsin(idx^2+1))^{3/2}}dx}{15b^2} - \frac{x}{15b^2(a - ib\arcsin(1 + idx^2))^{3/2}} - \frac{\sqrt{d^2x^4 - 2idx^2}}{5bdx(a - ib\arcsin(1 + idx^2))^{5/2}}$$

$$\Big\downarrow \text{5321}$$

$$\frac{-\frac{\sqrt{d^2x^4-2idx^2}}{bdx\sqrt{a-ib\arcsin(1+idx^2)}}-\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\arcsin\left(idx^2+1\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)-\sin\left(\frac{1}{2}\arcsin(1+idx^2)\right)}+\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\arcsin(1+idx^2)\right)}}{15b^2}$$

$$\frac{x}{15b^2\left(a-ib\arcsin\left(1+idx^2\right)\right)^{3/2}}-\frac{\sqrt{d^2x^4-2idx^2}}{5bdx\left(a-ib\arcsin\left(1+idx^2\right)\right)^{5/2}}$$

input
```
Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-7/2),x]
```

output
```
-1/5*Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^(5/
2)) - x/(15*b^2*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2)) + (-(Sqrt[(-2*I)*d*x^
2 + d^2*x^4]/(b*d*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])) + ((I/b)^(3/2)*Sqr
t[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(
Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin
[1 + I*d*x^2]/2]) - ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[I/b]*Sqrt[a - I
*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Cos
[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))/(15*b^2)
```

3.341.3.1 Defintions of rubi rules used

rule 5321
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[-
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*Si
n[ArcSin[c + d*x^2]/2])), x] + Simp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Co
s[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2])), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

rule 5327
```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Simp[Sqrt
[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n + 1)*x)
), x] - Simp[1/(4*b^2*(n + 1)*(n + 2))   Int[(a + b*ArcSin[c + d*x^2])^(n +
 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

3.341. $\int \frac{1}{\left(a-ib\arcsin\left(1+idx^2\right)\right)^{7/2}}\,dx$

3.341.4 Maple [F]

$$\int \frac{1}{\left(a + b\,\mathrm{arcsinh}\left(d\,x^{2} - i\right)\right)^{\frac{7}{2}}}dx$$

input
```
int(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x)
```

output
```
int(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x)
```

3.341.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a - ib\arcsin(1 + idx^2))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.341.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a - ib\arcsin(1 + idx^2))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*asinh(-I+d*x**2))**(7/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

3.341.7 Maxima [F]

$$\int \frac{1}{(a - ib \arcsin (1 + idx^2))^{7/2}} \, dx = \int \frac{1}{(b \operatorname{arsinh} (dx^2 - i) + a)^{\frac{7}{2}}} \, dx$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsinh(d*x^2 - I) + a)^(-7/2), x)
```

3.341.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a - ib \arcsin (1 + idx^2))^{7/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.341.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a - ib \arcsin (1 + idx^2))^{7/2}} \, dx = \int \frac{1}{(a + b \operatorname{asinh} (d\, x^2 - \mathrm{i}))^{7/2}} \, dx$$

input
```
int(1/(a + b*asinh(d*x^2 - 1i))^(7/2),x)
```

output
```
int(1/(a + b*asinh(d*x^2 - 1i))^(7/2), x)
```

3.342 $\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

3.342.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\operatorname{Int}\left(\frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2},x\right)$$

output
```
Unintegrable((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

3.342.2 Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

3.342.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2} \, dx$$

$$\Big\downarrow \; 7234$$

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2}dx$$

input
```
Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
$Aborted
```

3.342.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !IGt
Q[n, 0]
```

3.342.4 Maple [N/A] (verified)

Not integrable

Time = 0.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\,\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n}}{-c^{2}x^{2}+1}dx$$

input
```
int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

output
```
int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

3.342.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, al
gorithm="fricas")
```

output
```
integral(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

3.342.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

output
```
Timed out
```

3.342.7 Maxima [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, al
gorithm="maxima")
```

output
```
-integrate((b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1),
x)
```

3.342.8 Giac [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, al
gorithm="giac")
```

output
```
integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1),
x)
```

3.342.9 Mupad [N/A]

Not integrable

Time = 3.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\int \frac{\left(a+b\,\text{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

output
```
-int((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.343 $\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

3.343.1 Optimal result

Integrand size = 40, antiderivative size = 261

$$\begin{aligned}
&\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx \\
&= -\frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1-e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} \\
&\quad + \frac{3b\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}\left(2,e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} \\
&\quad + \frac{3b^2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(3,e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} \\
&\quad + \frac{3b^3\text{PolyLog}\left(4,e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{4c}
\end{aligned}$$

output
```
-1/4*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^4/b/c-(a+b*arcsinh((-c*x+
1)^(1/2)/(c*x+1)^(1/2)))^3*ln(1-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1
)/(c*x+1))^(1/2))^2)/c+3/2*b*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2
*polylog(2,1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/
c+3/2*b^2*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,1/((-c*x+1
)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c+3/4*b^3*polylog(4,1
/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c
```

3.343.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \frac{\frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{b}-4\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)-6b\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{}$$

input
```
Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

output
```
((a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/b - 4*(a + b*ArcSinh[Sqrt[
1 - c*x]/Sqrt[1 + c*x]])^3*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x
]])] - 6*b*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, E^(2*
ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])] + 6*b^2*(a + b*ArcSinh[Sqrt[1 - c*x
]/Sqrt[1 + c*x]])*PolyLog[3, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])] -
 3*b^3*PolyLog[4, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(4*c)
```

3.343.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {7232, 6190, 25, 3042, 26, 4201, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{1-c^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{6190}$$

$$-\frac{\int -\frac{(1-cx)^{3/2}\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{b}\right)}{(cx+1)^{3/2}}d\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(1-cx)^{3/2}\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{b}\right)}{(cx+1)^{3/2}}d\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\frac{i(1-cx)^{3/2}\tan\left(\frac{ia}{b}-\frac{i\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}+\frac{\pi}{2}\right)}{(cx+1)^{3/2}}d\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \frac{(1-cx)^{3/2}\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}\right)}{(cx+1)^{3/2}}d\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{i\left(2i\int \frac{\exp\left(\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}-i\pi\right)(1-cx)^{3/2}}{\left(1+\exp\left(\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}-i\pi\right)\right)(cx+1)^{3/2}}d\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{i(1-cx)^2}{4(cx+1)^2}\right)}{bc}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{i\left(2i\left(\frac{3}{2}b\int \frac{(1-cx)\log\left(1+\exp\left(\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}-i\pi\right)\right)}{cx+1}d\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{b(1-cx)^{3/2}\log\left(1+\exp\left(-\frac{2(}{}\right.\right.}{2(}\right.\right.}{bc}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{i\left(2i\left(\frac{3}{2}b\left(\frac{b(1-cx)\text{PolyLog}\left(2,-\exp\left(\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}-i\pi\right)\right)}{2(cx+1)}-b\int \left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\text{PolyLog}\left(2,-e\right.\right.\right.\right.}{}$$

3.343. $\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

$\Big\downarrow$ 7163

$$-\frac{i\left(2i\left(\frac{3}{2}b\left(\frac{b(1-cx)\,\text{PolyLog}\left(2,-\exp\left(\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}-i\pi\right)\right)}{2(cx+1)}-b\left(\frac{1}{2}b\int \text{PolyLog}\left(3,-\exp\left(\frac{2a}{b}-\frac{2\left(a+b\text{arcs}\right.}{}\right.\right.\right.\right.\right.\right.}{}$$

$\Big\downarrow$ 2720

$$-\frac{i\left(2i\left(\frac{3}{2}b\left(\frac{b(1-cx)\,\text{PolyLog}\left(2,-\exp\left(\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}-i\pi\right)\right)}{2(cx+1)}-b\left(-\frac{1}{4}b^2\int\frac{\sqrt{cx+1}\,\text{PolyLog}\left(3,-a-b\text{arcsinh}\left(\frac{\sqrt{1-c}}{\sqrt{cx+}}\right.\right.}{\sqrt{1-cx}}\right.\right.\right.\right.}{}$$

$\Big\downarrow$ 7143

$$-\frac{i\left(2i\left(\frac{3}{2}b\left(\frac{b(1-cx)\,\text{PolyLog}\left(2,-\exp\left(\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}-i\pi\right)\right)}{2(cx+1)}-b\left(-\frac{1}{4}b^2\,\text{PolyLog}\left(4,-a-b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right.\right.\right.\right.\right.\right.}{}$$

input
```
Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

output
```
((-I)*(((-1/4*I)*(1 - c*x)^2)/(1 + c*x)^2 + (2*I)*(-1/2*(b*(1 - c*x)^(3/2)
*Log[1 + E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]
]))/b)])/(1 + c*x)^(3/2) + (3*b*((b*(1 - c*x)*PolyLog[2, -E^((2*a)/b - I*P
i - (2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))/b)])/(2*(1 + c*x)) -
b*(-1/2*(b*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -E^((2*
a)/b - I*Pi - (2*Sqrt[1 - c*x])/(b*Sqrt[1 + c*x]))]) - (b^2*PolyLog[4, -a
- b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]]])/4)))/2)))/(b*c)
```

3.343. $\displaystyle\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

3.343.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

3.343. $\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.343.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. $2(285) = 570$.

Time = 1.40 (sec) , antiderivative size = 1113, normalized size of antiderivative = 4.26

method	result	size
default	Expression too large to display	1113
parts	Expression too large to display	1113

input
```
int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_R
ETURNVERBOSE)
```

output
```
1/2*a^3/c*ln(c*x+1)-1/2*a^3/c*ln(c*x-1)-b^3*(-1/4/c*arcsinh((-c*x+1)^(1/2)
/(c*x+1)^(1/2))^4+1/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln((-c*x+1)^
(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2)+1)+3/c*arcsinh((-c*x+1)^(1/
2)/(c*x+1)^(1/2))^2*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c
*x+1))^(1/2))-6/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-(-c*x+1
)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+6/c*polylog(4,-(-c*x+1)^
(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+1/c*arcsinh((-c*x+1)^(1/2)
/(c*x+1)^(1/2))^3*ln(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(
1/2))+3/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,(-c*x+1)^(1/2)
/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))-6/c*arcsinh((-c*x+1)^(1/2)/(c*x
+1)^(1/2))*polylog(3,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/
2))+6/c*polylog(4,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))
)-3*a*b^2*(-1/3/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3+1/c*arcsinh((-c*
x+1)^(1/2)/(c*x+1)^(1/2))^2*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c
*x+1))^(1/2)+1)+2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(-c*x
+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))-2/c*polylog(3,-(-c*x+1
)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+1/c*arcsinh((-c*x+1)^(1/
2)/(c*x+1)^(1/2))^2*ln(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))
^(1/2))+2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,(-c*x+1)^(1/2)
/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))-2/c*polylog(3,(-c*x+1)^(1/2)...
```

3.343.5 Fricas [F]

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, al
gorithm="fricas")
```

output
```
integral(-(b^3*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arcsinh(s
qrt(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x +
 1)) + a^3)/(c^2*x^2 - 1), x)
```

3.343.6 Sympy [F]

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\int \frac{a^3}{c^2x^2-1}\,dx - \int \frac{b^3\operatorname{asinh}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3ab^2\operatorname{asinh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3a^2b\operatorname{asinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

output
```
-Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*asinh(sqrt(-c*x + 1)/sq
rt(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*asinh(sqrt(-c*x +
1)/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*asinh(sqrt(-c
*x + 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

3.343.7 Maxima [F]

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(b\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, al
gorithm="maxima")
```

output
```
1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^3*log(c*x + 1) - b^3*lo
g(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1))^3/c + integrate(1/8*((sqrt(2)*b
^3 + sqrt(-c*x + 1)*b^3)*log(c*x + 1)^3 - 6*(sqrt(2)*a*b^2 + sqrt(-c*x + 1
)*a*b^2)*log(c*x + 1)^2 - 6*(4*sqrt(2)*a*b^2 - 2*(sqrt(2)*b^3 + sqrt(-c*x
+ 1)*b^3)*log(c*x + 1) + (4*a*b^2 + (b^3*c*x + b^3)*log(c*x + 1) - (b^3*c*
x + b^3)*log(-c*x + 1))*sqrt(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1))^2 +
12*(sqrt(2)*a^2*b + sqrt(-c*x + 1)*a^2*b)*log(c*x + 1) - 6*(4*sqrt(2)*a^2*
b + 4*sqrt(-c*x + 1)*a^2*b + (sqrt(2)*b^3 + sqrt(-c*x + 1)*b^3)*log(c*x +
1)^2 - 4*(sqrt(2)*a*b^2 + sqrt(-c*x + 1)*a*b^2)*log(c*x + 1))*log(sqrt(2)
+ sqrt(-c*x + 1)))/(sqrt(2)*c^2*x^2 + (c^2*x^2 - 1)*sqrt(-c*x + 1) - sqrt(
2)), x)
```

3.343.8 Giac [F]

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, al
gorithm="giac")
```

output
```
integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1),
x)
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(a+b\,\text{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.344 $\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

3.344.1 Optimal result

Integrand size = 40, antiderivative size = 194

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1-e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b^2\operatorname{PolyLog}\left(3,e^{-2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}$$

output
```
-1/3*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/b/c-(a+b*arcsinh((-c*x+
1)^(1/2)/(c*x+1)^(1/2)))^2*ln(1-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1
)/(c*x+1))^(1/2))^2)/c+b*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polyl
og(2,1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c+1/2*
b^2*polylog(3,1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^
2)/c
```

3.344.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$
$$=\frac{2\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)-3b\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)\right)-6b^2\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{6bc}$$

input
```
Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
(2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*(a + b*ArcSinh[Sqrt[1 -
c*x]/Sqrt[1 + c*x]] - 3*b*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]
])]) - 6*b^2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, E^(2*
ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])] + 3*b^3*PolyLog[3, E^(2*ArcSinh[Sqr
t[1 - c*x]/Sqrt[1 + c*x]])])/(6*b*c)
```

3.344.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7232, 6190, 25, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{1-c^2x^2}\,dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{6190}$$

$$-\frac{\int -\frac{(1-cx)\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{b}\right)}{cx+1}d\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(1-cx)\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{b}\right)}{cx+1}d\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int -\frac{i(1-cx)\tan\left(\frac{ia}{b}-\frac{i\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}+\frac{\pi}{2}\right)}{cx+1}d\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \ 26$$

$$-\frac{i\int \frac{(1-cx)\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}\right)}{cx+1}d\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \ 4201$$

$$-\frac{i\left(2i\int \frac{\exp\left(\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}-i\pi\right)(1-cx)}{\left(1+\exp\left(\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}-i\pi\right)\right)(cx+1)}d\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{i(1-cx)^{3/2}}{3(cx+1)^{3/2}}\right)}{bc}$$

$$\Big\downarrow \ 2620$$

$$-\frac{i\left(2i\left(b\int \left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\log\left(1+e^{\frac{2a}{b}-\frac{2\sqrt{1-cx}}{b\sqrt{cx+1}}-i\pi}\right)d\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{b(1-cx)\log\left(1+\exp\left(-\frac{2\left(a+\cdots\right.}{\cdots}\right.\right.}{2\cdots}\right.\right.}{bc}$$

$$\Big\downarrow \ 3011$$

$$-\frac{i\left(2i\left(b\left(\frac{1}{2}b\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2\sqrt{1-cx}}{b\sqrt{cx+1}}-i\pi}\right)-\frac{1}{2}b\int \text{PolyLog}\left(2,-\exp\left(\frac{2a}{b}-\frac{2\left(a+b\text{arcsi}\cdots\right.}{\cdots}\right.\right.\right.\right.\right.\right.}{\cdots}$$

3.344. $\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}dx$

$$\Big\downarrow \text{2720}$$

$$-\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\int\frac{\sqrt{cx+1}\,\text{PolyLog}\left(2,-a-b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{\sqrt{1-cx}}d\exp\left(\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}-i\pi\right)+\frac{1}{2}b\left(a+b\text{arcsin}\right.\right.\right.\right.}{}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{i\left(2i\left(b\left(\frac{1}{4}b^2\,\text{PolyLog}\left(3,-a-b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)+\frac{1}{2}b\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\text{PolyLog}\left(2,-e^{\frac{2a}{b}-\frac{2\sqrt{1-cx}}{b\sqrt{cx+1}}-i\pi}\right)\right.\right.\right.}{bc}$$

input
```
Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
((-I)*(((-1/3*I)*(1 - c*x)^(3/2))/(1 + c*x)^(3/2) + (2*I)*(-1/2*(b*(1 - c*
x)*Log[1 + E^((2*a)/b - I*Pi - (2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]]))/b)])/(1 + c*x) + b*((b*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*
PolyLog[2, -E^((2*a)/b - I*Pi - (2*Sqrt[1 - c*x])/(b*Sqrt[1 + c*x]))])/2 +
 (b^2*PolyLog[3, -a - b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]]])/4))))/(b*c)
```

3.344.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.344. $\int\frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.344. $\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

3.344.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(214) = 428$.

Time = 0.40 (sec) , antiderivative size = 624, normalized size of antiderivative = 3.22

method	result
default	$\frac{a^2 \ln(cx+1)}{2c} - \frac{a^2 \ln(cx-1)}{2c} - b^2 \Bigg(-\frac{\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3}{3c} + \frac{\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 \ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}} + \sqrt{1+\frac{-cx+1}{cx+1}}+1\right)}{c} + \frac{2 \operatorname{arcsinh}(}{}$
parts	$\frac{a^2 \ln(cx+1)}{2c} - \frac{a^2 \ln(cx-1)}{2c} - b^2 \Bigg(-\frac{\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3}{3c} + \frac{\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 \ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}} + \sqrt{1+\frac{-cx+1}{cx+1}}+1\right)}{c} + \frac{2 \operatorname{arcsinh}(}{}$

input
```
int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_R
ETURNVERBOSE)
```

output
```
1/2*a^2/c*ln(c*x+1)-1/2*a^2/c*ln(c*x-1)-b^2*(-1/3/c*arcsinh((-c*x+1)^(1/2)
/(c*x+1)^(1/2))^3+1/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln((-c*x+1)^
(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2)+1)+2/c*arcsinh((-c*x+1)^(1/
2)/(c*x+1)^(1/2))*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x
+1))^(1/2))-2/c*polylog(3,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1
))^(1/2))+1/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-(-c*x+1)^(1/2)/
(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+2/c*arcsinh((-c*x+1)^(1/2)/(c*x+
1)^(1/2))*polylog(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2
))-2/c*polylog(3,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2)))
-2*a*b*(-1/2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2+1/c*arcsinh((-c*x+1
)^(1/2)/(c*x+1)^(1/2))*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1)
)^(1/2)+1)+1/c*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1)
)^(1/2))+1/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(-c*x+1)^(1/2)/(c*
x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+1/c*polylog(2,(-c*x+1)^(1/2)/(c*x+1
)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2)))
```

3.344.5 Fricas [F]

$$\int \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(b\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, al
gorithm="fricas")
```

output
```
integral(-(b^2*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arcsinh(sqr
t(-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

3.344.6 Sympy [F]

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\int \frac{a^2}{c^2x^2-1}\,dx - \int \frac{b^2\operatorname{asinh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{2ab\operatorname{asinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

output
```
-Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*asinh(sqrt(-c*x + 1)/sq
rt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*asinh(sqrt(-c*x + 1)/
sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

3.344.7 Maxima [F]

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(b\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, al
gorithm="maxima")
```

output
```
1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^2*log(c*x + 1) - b^2*lo
g(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1))^2/c + integrate(-1/4*((sqrt(2)*
b^2 + sqrt(-c*x + 1)*b^2)*log(c*x + 1)^2 - 4*(sqrt(2)*a*b + sqrt(-c*x + 1)
*a*b)*log(c*x + 1) + 2*(4*sqrt(2)*a*b - 2*(sqrt(2)*b^2 + sqrt(-c*x + 1)*b^
2)*log(c*x + 1) + (4*a*b + (b^2*c*x + b^2)*log(c*x + 1) - (b^2*c*x + b^2)*
log(-c*x + 1))*sqrt(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1)))/(sqrt(2)*c^2
*x^2 + (c^2*x^2 - 1)*sqrt(-c*x + 1) - sqrt(2)), x)
```

3.344.8 Giac [F]

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, al
gorithm="giac")
```

output
```
integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1),
x)
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(a+b\,\text{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.345 $\int \frac{a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

3.345.1 Optimal result

Integrand size = 38, antiderivative size = 133

$$\int \frac{a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1-e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}$$

output
```
-1/2*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/b/c-(a+b*arcsinh((-c*x+
1)^(1/2)/(c*x+1)^(1/2)))*ln(1-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/
(c*x+1))^(1/2))^2)/c+1/2*b*polylog(2,1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-
c*x+1)/(c*x+1))^(1/2))^2)/c
```

3.345.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

$$\int \frac{a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \frac{\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)-2b\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)\right)-b^2\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsin}}\right.}{2bc}$$

input
```
Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

output
```
((a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*(a + b*ArcSinh[Sqrt[1 - c*x]
/Sqrt[1 + c*x]] - 2*b*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])
 - b^2*PolyLog[2, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*b*c)
```

3.345.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {7232, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{1-c^2x^2}\,dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{6190}$$

$$-\frac{\int -\left(\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\coth\left(\frac{a}{b}-\frac{\sqrt{1-cx}}{b\sqrt{cx+1}}\right)\right)d\left(a+b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\coth\left(\frac{a}{b}-\frac{\sqrt{1-cx}}{b\sqrt{cx+1}}\right)d\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -i\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\tan\left(\frac{ia}{b}-\frac{i\sqrt{1-cx}}{b\sqrt{cx+1}}+\frac{\pi}{2}\right)d\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i\sqrt{1-cx}}{b\sqrt{cx+1}}\right)d\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{i\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2\sqrt{1-cx}}{b\sqrt{cx+1}}-i\pi}\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{1+e^{\frac{2a}{b}-\frac{2\sqrt{1-cx}}{b\sqrt{cx+1}}-i\pi}}d\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{i(1-cx)}{2(cx+1)}\right)}{bc}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{i\left(2i\left(\frac{1}{2}b\int \log\left(1+\exp\left(\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}-i\pi\right)\right)d\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{1}{2}b\left(a+b\text{arcsinh}\left(\frac{\sqrt{1}}{\sqrt{c}}\right.\right.\right.\right.}{bc}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{i\left(2i\left(-\frac{1}{4}b^2\int \frac{\sqrt{cx+1}\log\left(1+\exp\left(\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}-i\pi\right)\right)}{\sqrt{1-cx}}d\exp\left(\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{b}-i\pi\right)-\frac{1}{2}b\left(a\right.\right.\right.}{bc}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{i\left(2i\left(\frac{1}{4}b^2\,\text{PolyLog}\left(2,-a-b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{1}{2}b\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\log\left(1+e^{\frac{2a}{b}-\frac{2\sqrt{1-cx}}{b\sqrt{cx+1}}-i\pi}\right)\right)-\frac{i(1-c}{2(cx+}\right.}{bc}$$

input
```
Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

3.345. $\int \frac{a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

output
```
((-I)*(((-1/2*I)*(1 - c*x))/(1 + c*x) + (2*I)*(-1/2*(b*(a + b*ArcSinh[Sqrt
[1 - c*x]/Sqrt[1 + c*x]])*Log[1 + E^((2*a)/b - I*Pi - (2*Sqrt[1 - c*x])/(b
*Sqrt[1 + c*x]))]) + (b^2*PolyLog[2, -a - b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 +
 c*x]]])/4)))/(b*c)
```

3.345.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

3.345. $\int \frac{a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.345.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.94

method	result
default	$\frac{a\ln(cx+1)}{2c}-\frac{a\ln(cx-1)}{2c}-b\left(-\frac{\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{2c}+\frac{\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{1+\frac{-cx+1}{cx+1}}+1\right)}{c}+\frac{\operatorname{polylog}\left(2,-\frac{\sqrt{-}}{\sqrt{}}\right.}{}\right.$
parts	$\frac{a\ln(cx+1)}{2c}-\frac{a\ln(cx-1)}{2c}-b\left(-\frac{\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{2c}+\frac{\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{1+\frac{-cx+1}{cx+1}}+1\right)}{c}+\frac{\operatorname{polylog}\left(2,-\frac{\sqrt{-}}{\sqrt{}}\right.}{}\right.$

input
```
int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
```

output
```
1/2*a/c*ln(c*x+1)-1/2*a/c*ln(c*x-1)-b*(-1/2/c*arcsinh((-c*x+1)^(1/2)/(c*x+
1)^(1/2))^2+1/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln((-c*x+1)^(1/2)/(c
*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2)+1)+1/c*polylog(2,-(-c*x+1)^(1/2)/(c
*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+1/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)
^(1/2))*ln(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-(1+(-c*x+1)/(c*x+1))^(1/2))+1/c*
polylog(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2)))
```

3.345.5 Fricas [F]

$$\int \frac{a + b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1 - c^2x^2}\,dx = \int -\frac{b\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a}{c^2x^2 - 1}\,dx$$

input
```
integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algo
rithm="fricas")
```

output
```
integral(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

3.345.6 Sympy [F]

$$\int \frac{a + b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1 - c^2x^2}\,dx = -\int \frac{a}{c^2x^2 - 1}\,dx - \int \frac{b\operatorname{asinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2 - 1}\,dx$$

input
```
integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

output
```
-Integral(a/(c**2*x**2 - 1), x) - Integral(b*asinh(sqrt(-c*x + 1)/sqrt(c*x
 + 1))/(c**2*x**2 - 1), x)
```

3.345.7 Maxima [F]

$$\int \frac{a + b\operatorname{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1 - c^2x^2}\,dx = \int -\frac{b\operatorname{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a}{c^2x^2 - 1}\,dx$$

input
```
integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algo
rithm="maxima")
```

output
```
-1/8*b*((2*(log(c*x + 1) - log(-c*x + 1))*log(c*x + 1) - log(c*x + 1)^2 +
2*log(c*x + 1)*log(-c*x + 1) - log(-c*x + 1)^2 - 4*(log(c*x + 1) - log(-c*
x + 1))*log(sqrt(2) + sqrt(-c*x + 1)))/c + 8*integrate(-1/4*(sqrt(2)*log(c
*x + 1) - sqrt(2)*log(-c*x + 1))/(sqrt(2)*c*x + (c*x - 1)*sqrt(-c*x + 1) -
 sqrt(2)), x)) + 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c)
```

3.345.8 Giac [F]

$$\int \frac{a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algo
rithm="giac")
```

output
```
integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

3.345.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{a+b\,\text{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

3.346 $\int \frac{1}{(1-c^2x^2)\left(a+b\,\mathbf{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

3.346.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)},x\right)$$

output
```
Unintegrable(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

3.346.2 Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x
]
```

output
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),
x]
```

3.346.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$
$$\Big\downarrow \text{ 7234}$$
$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

output
```
$Aborted
```

3.346.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !IGt
Q[n, 0]
```

3.346.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\,\text{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}dx$$

input
```
int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

3.346.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, al
gorithm="fricas")
```

output
```
integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arcsinh(sqrt(-c*x + 1)/sqrt(c*x +
 1)) - a), x)
```

3.346.6 Sympy [N/A]

Not integrable

Time = 163.88 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$
$$= -\int \frac{1}{ac^2x^2-a+bc^2x^2\,\text{asinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b\,\text{asinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

output
```
-Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*asinh(sqrt(-c*x + 1)/sqrt(c*x +
 1)) - b*asinh(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

3.346.7 Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, al
gorithm="maxima")
```

output
```
-integrate(1/((c^2*x^2 - 1)*(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a))
, x)
```

3.346.8 Giac [N/A]

Not integrable

Time = 5.65 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, al
gorithm="giac")
```

output
```
integrate(-1/((c^2*x^2 - 1)*(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a))
, x)
```

3.346.9 Mupad [N/A]

Not integrable

Time = 2.86 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\int \frac{1}{\left(a+b\,\text{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,(c^2\,x^2-1)}\,dx$$

input
```
int(-1/((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.346. $\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

3.347 $\int \frac{1}{(1-c^2x^2)\left(a+b\,\mathrm{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

3.347.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\,\mathrm{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\mathrm{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\,\mathrm{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,
x)
```

3.347.2 Mathematica [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\,\mathrm{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\,\mathrm{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
,x]
```

output
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
, x]
```

3.347.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}\,dx$$

$$\Big\downarrow \text{7234}$$

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}dx$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

output
```
$Aborted
```

3.347.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !IGt
Q[n, 0]
```

3.347.4 Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\ \text{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}dx$$

input
```
int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

3.347.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.28

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x,
algorithm="fricas")
```

output
```
integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arcsinh(sqrt(-c*x + 1)/sqrt
(c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arcsinh(sqrt(-c*x + 1)/sqrt(c*x
 + 1))), x)
```

3.347.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,
x)
```

output
```
Timed out
```

3.347.7 Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 474, normalized size of antiderivative = 11.85

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x,
algorithm="maxima")
```

output
```
-4*(sqrt(2) + sqrt(-c*x + 1))/(2*sqrt(2)*a*b*c^2*x - 2*sqrt(2)*a*b*c - 4*s
qrt(-c*x + 1)*a*b*c - (sqrt(2)*b^2*c^2*x - sqrt(2)*b^2*c - 2*sqrt(-c*x + 1
)*b^2*c)*log(c*x + 1) + 2*(sqrt(2)*b^2*c^2*x - sqrt(2)*b^2*c - 2*sqrt(-c*x
 + 1)*b^2*c)*log(sqrt(2) + sqrt(-c*x + 1))) - integrate((4*c*x + (sqrt(2)*
c*x - 3*sqrt(2))*sqrt(-c*x + 1) - 4)/(2*a*b*c^3*x^3 - 6*a*b*c^2*x^2 + 6*a*
b*c*x - 4*(a*b*c*x - a*b)*(c*x - 1) - 2*a*b - (b^2*c^3*x^3 - 3*b^2*c^2*x^2
 + 3*b^2*c*x - 2*(b^2*c*x - b^2)*(c*x - 1) - b^2 - 2*(sqrt(2)*b^2*c^2*x^2
- 2*sqrt(2)*b^2*c*x + sqrt(2)*b^2)*sqrt(-c*x + 1))*log(c*x + 1) + 2*(b^2*c
^3*x^3 - 3*b^2*c^2*x^2 + 3*b^2*c*x - 2*(b^2*c*x - b^2)*(c*x - 1) - b^2 - 2
*(sqrt(2)*b^2*c^2*x^2 - 2*sqrt(2)*b^2*c*x + sqrt(2)*b^2)*sqrt(-c*x + 1))*l
og(sqrt(2) + sqrt(-c*x + 1)) - 4*(sqrt(2)*a*b*c^2*x^2 - 2*sqrt(2)*a*b*c*x
+ sqrt(2)*a*b)*sqrt(-c*x + 1)), x)
```

3.347.8 Giac [N/A]

Not integrable

Time = 20.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\,\text{arsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x,
algorithm="giac")
```

output
```
integrate(-1/((c^2*x^2 - 1)*(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^
2), x)
```

3.347.9 Mupad [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arcsinh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=-\int \frac{1}{\left(a+b\,\text{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,\left(c^2\,x^2-1\right)}\,dx$$

input
```
int(-1/((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x
)
```

3.348 $\int \operatorname{arcsinh}\left(ce^{a+bx}\right)\,dx$

3.348.1 Optimal result

Integrand size = 10, antiderivative size = 76

$$\int \operatorname{arcsinh}\left(ce^{a+bx}\right)\,dx = -\frac{\operatorname{arcsinh}\left(ce^{a+bx}\right)^2}{2b} + \frac{\operatorname{arcsinh}\left(ce^{a+bx}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(ce^{a+bx}\right)}\right)}{b} + \frac{\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}\left(ce^{a+bx}\right)}\right)}{2b}$$

output
```
-1/2*arcsinh(c*exp(b*x+a))^2/b+arcsinh(c*exp(b*x+a))*ln(1-(c*exp(b*x+a)+(1
+c^2*exp(b*x+a)^2)^(1/2))^2)/b+1/2*polylog(2,(c*exp(b*x+a)+(1+c^2*exp(b*x+
a)^2)^(1/2))^2)/b
```

3.348.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \operatorname{arcsinh}\left(ce^{a+bx}\right)\,dx = \frac{-\operatorname{arcsinh}\left(ce^{a+bx}\right)\left(\operatorname{arcsinh}\left(ce^{a+bx}\right)-2\log\left(1-e^{2\operatorname{arcsinh}\left(ce^{a+bx}\right)}\right)\right)+\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}\left(ce^{a+bx}\right)}\right)}{2b}$$

input
```
Integrate[ArcSinh[c*E^(a + b*x)],x]
```

output
```
(-(ArcSinh[c*E^(a + b*x)]*(ArcSinh[c*E^(a + b*x)] - 2*Log[1 - E^(2*ArcSinh
[c*E^(a + b*x)])])) + PolyLog[2, E^(2*ArcSinh[c*E^(a + b*x)])])/(2*b)
```

3.348.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {2720, 6190, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}\left(ce^{a+bx}\right) dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int e^{-a-bx}\text{arcsinh}\left(ce^{a+bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \text{6190}$$

$$\frac{\int \frac{e^{-a-bx}\sqrt{e^{2a+2bx}c^2+1}\text{arcsinh}\left(ce^{a+bx}\right)}{c}d\text{arcsinh}\left(ce^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -i\text{arcsinh}\left(ce^{a+bx}\right)\tan\left(i\text{arcsinh}\left(ce^{a+bx}\right)+\frac{\pi}{2}\right)d\text{arcsinh}\left(ce^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \text{arcsinh}\left(ce^{a+bx}\right)\tan\left(i\text{arcsinh}\left(ce^{a+bx}\right)+\frac{\pi}{2}\right)d\text{arcsinh}\left(ce^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{4199}$$

$$-\frac{i\left(2i\int -\frac{e^{a+bx+2\text{arcsinh}\left(ce^{a+bx}\right)}}{1-e^{2\text{arcsinh}\left(ce^{a+bx}\right)}}d\text{arcsinh}\left(ce^{a+bx}\right)-\frac{1}{2}ie^{2a+2bx}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i\left(-2i\int \frac{e^{a+bx+2\text{arcsinh}\left(ce^{a+bx}\right)}}{1-e^{2\text{arcsinh}\left(ce^{a+bx}\right)}}d\text{arcsinh}\left(ce^{a+bx}\right)-\frac{1}{2}ie^{2a+2bx}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{i\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2\text{arcsinh}\left(ce^{a+bx}\right)}\right)d\text{arcsinh}\left(ce^{a+bx}\right)-\frac{1}{2}\text{arcsinh}\left(ce^{a+bx}\right)\log\left(1-e^{2\text{arcsinh}\left(ce^{a+bx}\right)}\right)\right)-\frac{1}{2}ie^{2a+2}\right.}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{i\left(-2i\left(\frac{1}{4}\int e^{-a-bx}\log\left(1-e^{2\text{arcsinh}\left(ce^{a+bx}\right)}\right)de^{2\text{arcsinh}\left(ce^{a+bx}\right)}-\frac{1}{2}\text{arcsinh}\left(ce^{a+bx}\right)\log\left(1-e^{2\text{arcsinh}\left(ce^{a+bx}\right)}\right)\right)-\frac{1}{2}\right.}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{i\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(ce^{a+bx}\right)}\right)-\frac{1}{2}\text{arcsinh}\left(ce^{a+bx}\right)\log\left(1-e^{2\text{arcsinh}\left(ce^{a+bx}\right)}\right)\right)-\frac{1}{2}ie^{2a+2bx}\right)}{b}$$

input
```
Int[ArcSinh[c*E^(a + b*x)],x]
```

output
```
((-I)*((-1/2*I)*E^(2*a + 2*b*x) - (2*I)*(-1/2*(ArcSinh[c*E^(a + b*x)]*Log[
1 - E^(2*ArcSinh[c*E^(a + b*x)])]) - PolyLog[2, E^(2*ArcSinh[c*E^(a + b*x)
])]/4)))/b
```

3.348.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

3.348.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.01

method	result
derivativedivides	$\frac{-\frac{\operatorname{arcsinh}\left(c\,\mathrm{e}^{bx+a}\right)^2}{2}+\operatorname{arcsinh}\left(c\,\mathrm{e}^{bx+a}\right)\ln\left(1+c\,\mathrm{e}^{bx+a}+\sqrt{1+c^2\mathrm{e}^{2bx+2a}}\right)+\operatorname{polylog}\left(2,-c\,\mathrm{e}^{bx+a}-\sqrt{1+c^2\mathrm{e}^{2bx+2a}}\right)+\operatorname{arcsin}}{b}$
default	$\frac{-\frac{\operatorname{arcsinh}\left(c\,\mathrm{e}^{bx+a}\right)^2}{2}+\operatorname{arcsinh}\left(c\,\mathrm{e}^{bx+a}\right)\ln\left(1+c\,\mathrm{e}^{bx+a}+\sqrt{1+c^2\mathrm{e}^{2bx+2a}}\right)+\operatorname{polylog}\left(2,-c\,\mathrm{e}^{bx+a}-\sqrt{1+c^2\mathrm{e}^{2bx+2a}}\right)+\operatorname{arcsin}}{b}$

input
```
int(arcsinh(c*exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*arcsinh(c*exp(b*x+a))^2+arcsinh(c*exp(b*x+a))*ln(1+c*exp(b*x+a)+
(1+c^2*exp(b*x+a)^2)^(1/2))+polylog(2,-c*exp(b*x+a)-(1+c^2*exp(b*x+a)^2)^(
1/2))+arcsinh(c*exp(b*x+a))*ln(1-c*exp(b*x+a)-(1+c^2*exp(b*x+a)^2)^(1/2))+
polylog(2,c*exp(b*x+a)+(1+c^2*exp(b*x+a)^2)^(1/2)))
```

3.348.5 Fricas [F(-2)]

Exception generated.

$$\int \operatorname{arcsinh}\left(ce^{a+bx}\right)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arcsinh(c*exp(b*x+a)),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.348.6 Sympy [F]

$$\int \operatorname{arcsinh}\left(ce^{a+bx}\right)\,dx = \int \operatorname{asinh}\left(ce^{a+bx}\right)\,dx$$

input
```
integrate(asinh(c*exp(b*x+a)),x)
```

output
```
Integral(asinh(c*exp(a + b*x)), x)
```

3.348.7 Maxima [F]

$$\int \operatorname{arcsinh}\left(ce^{a+bx}\right)\,dx = \int \operatorname{arsinh}\left(ce^{(bx+a)}\right)\,dx$$

input
```
integrate(arcsinh(c*exp(b*x+a)),x, algorithm="maxima")
```

output
```
-b*c*integrate(x*e^(b*x + a)/(c^3*e^(3*b*x + 3*a) + c*e^(b*x + a) + (c^2*e
^(2*b*x + 2*a) + 1)^(3/2)), x) + x*log(c*e^(b*x + a) + sqrt(c^2*e^(2*b*x +
 2*a) + 1)) - 1/4*(2*b*x*log(c^2*e^(2*b*x + 2*a) + 1) + dilog(-c^2*e^(2*b*
x + 2*a)))/b
```

3.348.8 Giac [F]

$$\int \operatorname{arcsinh}\left(ce^{a+bx}\right)\,dx = \int \operatorname{arsinh}\left(ce^{(bx+a)}\right)\,dx$$

input
```
integrate(arcsinh(c*exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arcsinh(c*e^(b*x + a)), x)
```

3.348.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}\left(ce^{a+bx}\right)\,dx = \int \operatorname{asinh}\left(c\,e^{b\,x}\,e^{a}\right)\,dx$$

input
```
int(asinh(c*exp(a + b*x)),x)
```

output
```
int(asinh(c*exp(b*x)*exp(a)), x)
```

3.349 $\int e^{\text{arcsinh}(a+bx)}x^3\,dx$

3.349.1 Optimal result

Integrand size = 12, antiderivative size = 165

$$\int e^{\text{arcsinh}(a+bx)}x^3\,dx=\frac{e^{-3\text{arcsinh}(a+bx)}}{48b^4}+\frac{3ae^{-2\text{arcsinh}(a+bx)}}{16b^4}-\frac{(1-6a^2)\,e^{-\text{arcsinh}(a+bx)}}{8b^4}+\frac{a(3-4a^2)\,e^{2\text{arcsinh}(a+bx)}}{16b^4}-\frac{(1-6a^2)\,e^{3\text{arcsinh}(a+bx)}}{24b^4}-\frac{3ae^{4\text{arcsinh}(a+bx)}}{32b^4}+\frac{e^{5\text{arcsinh}(a+bx)}}{80b^4}+\frac{a(3-4a^2)\,\text{arcsinh}(a+bx)}{8b^4}$$

output
```
1/48/b^4/(b*x+a+(1+(b*x+a)^2)^(1/2))^3+3/16*a/b^4/(b*x+a+(1+(b*x+a)^2)^(1/
2))^2+1/8*(6*a^2-1)/b^4/(b*x+a+(1+(b*x+a)^2)^(1/2))+1/16*a*(-4*a^2+3)*(b*x
+a+(1+(b*x+a)^2)^(1/2))^2/b^4-1/24*(-6*a^2+1)*(b*x+a+(1+(b*x+a)^2)^(1/2))^
3/b^4-3/32*a*(b*x+a+(1+(b*x+a)^2)^(1/2))^4/b^4+1/80*(b*x+a+(1+(b*x+a)^2)^(
1/2))^5/b^4+1/8*a*(-4*a^2+3)*arcsinh(b*x+a)/b^4
```

3.349.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.72

$$\int e^{\text{arcsinh}(a+bx)}x^3\,dx=\frac{30ab^4x^4+24b^5x^5-\sqrt{1+a^2+2abx+b^2x^2}(16-83a^2+6a^4+a(29-6a^2)\,bx+2(-4+3a^2)\,b^2x^2-6ab^3}{120b^4}$$

input
```
Integrate[E^ArcSinh[a + b*x]*x^3,x]
```

output
```
(30*a*b^4*x^4 + 24*b^5*x^5 - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(16 - 83*a^
2 + 6*a^4 + a*(29 - 6*a^2)*b*x + 2*(-4 + 3*a^2)*b^2*x^2 - 6*a*b^3*x^3 - 24
*b^4*x^4) + 15*a*(3 - 4*a^2)*ArcSinh[a + b*x])/(120*b^4)
```

3.349.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6288, 25, 2720, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{\text{arcsinh}(a+bx)} dx$$

$$\Big\downarrow \text{6288}$$

$$\frac{\int -e^{\text{arcsinh}(a+bx)}\left(\frac{a}{b}-\frac{a+bx}{b}\right)^3\sqrt{(a+bx)^2+1}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int e^{\text{arcsinh}(a+bx)}\left(\frac{a}{b}-\frac{a+bx}{b}\right)^3\sqrt{(a+bx)^2+1}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\int \frac{e^{-4\text{arcsinh}(a+bx)}\left(2e^{\text{arcsinh}(a+bx)}a-e^{2\text{arcsinh}(a+bx)}+1\right)^3\left(1+e^{2\text{arcsinh}(a+bx)}\right)}{16b^3}de^{\text{arcsinh}(a+bx)}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int e^{-4\text{arcsinh}(a+bx)}\left(2e^{\text{arcsinh}(a+bx)}a-e^{2\text{arcsinh}(a+bx)}+1\right)^3\left(1+e^{2\text{arcsinh}(a+bx)}\right)de^{\text{arcsinh}(a+bx)}}{16b^4}$$

$$\Big\downarrow \text{2159}$$

$$-\frac{\int \left(6e^{-3\text{arcsinh}(a+bx)}a+2\left(4a^2-3\right)e^{-\text{arcsinh}(a+bx)}a+2\left(4a^2-3\right)e^{\text{arcsinh}(a+bx)}a+6e^{3\text{arcsinh}(a+bx)}a+e^{-4\text{arcsinh}(a+}}{16b^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-(3-4a^2)\,ae^{2\text{arcsinh}(a+bx)}+2(1-6a^2)\,e^{-\text{arcsinh}(a+bx)}+\frac{2}{3}(1-6a^2)\,e^{3\text{arcsinh}(a+bx)}-2(3-4a^2)\,a\log\left(e^{\text{arcsinh}(a-\right.}{16b^4}$$

input
```
Int[E^ArcSinh[a + b*x]*x^3,x]
```

output
```
-1/16*(-1/3*1/E^(3*ArcSinh[a + b*x]) - (3*a)/E^(2*ArcSinh[a + b*x]) + (2*(
1 - 6*a^2))/E^ArcSinh[a + b*x] - a*(3 - 4*a^2)*E^(2*ArcSinh[a + b*x]) + (2
*(1 - 6*a^2)*E^(3*ArcSinh[a + b*x]))/3 + (3*a*E^(4*ArcSinh[a + b*x]))/2 -
E^(5*ArcSinh[a + b*x])/5 - 2*a*(3 - 4*a^2)*Log[E^ArcSinh[a + b*x]])/b^4
```

3.349.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6288
```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Simp[1/b   Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSin
h[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.349.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(205) = 410$.

Time = 0.75 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.81

method	result
default	$\frac{x^2(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{5b^2} - \frac{7a\left(\frac{x(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{4b^2} - \frac{5a\left(\frac{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{3b^2} - \frac{a\left(\frac{(2b^2x+2ab)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2} + \frac{(4b^2(a^2+1)}{}\right.}{b}\right.}{4b}\right.}{}$

input
```
int((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-7/5*a/b*(1/4*x*(b^2*x^2+2*a*b*x+
a^2+1)^(3/2)/b^2-5/4*a/b*(1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-a/b*(1/4*(
2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*
b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(
1/2)))-1/4*(a^2+1)/b^2*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+
2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)))-2/5*(a^2+1)/b^2*(1/3*(b^2*x^2+2*a*b*x+
a^2+1)^(3/2)/b^2-a/b*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)))+1/5*b*x^5+1/4*a*x^4
```

3.349.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.84

$$\int e^{\operatorname{arcsinh}(a+bx)}x^3\,dx$$
$$= \frac{24\,b^5x^5 + 30\,ab^4x^4 + 15\,(4\,a^3-3\,a)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right) + (24\,b^4x^4 + 6\,ab^3x^3 - 2\,(3\,a}{120\,b^4}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x, algorithm="fricas")
```

output
```
1/120*(24*b^5*x^5 + 30*a*b^4*x^4 + 15*(4*a^3 - 3*a)*log(-b*x - a + sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)) + (24*b^4*x^4 + 6*a*b^3*x^3 - 2*(3*a^2 - 4)*b^
2*x^2 - 6*a^4 + (6*a^3 - 29*a)*b*x + 83*a^2 - 16)*sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1))/b^4
```

3.349.6 Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.76

$$\int e^{\operatorname{arcsinh}(a+bx)}x^3\,dx = \frac{ax^4}{4} + \frac{bx^5}{5} + \begin{cases} \frac{\left(-\frac{a\left(-\frac{3a\left(-\frac{5a\left(\frac{1}{5}-\frac{3a^2}{20}\right)}{3b}-\frac{a(3a^2+3)}{20b}\right)}{2b}-\frac{\left(\frac{1}{5}-\frac{3a^2}{20}\right)(2a^2+2)}{3b^2}\right)}{b}-\frac{(a^2+1)\left(-\frac{5a\left(\frac{1}{5}-\frac{3a^2}{20}\right)}{3b}-\frac{a(3a^2+3)}{20b}\right)}{2b^2}\right)\log\left(2ab+2b^2x+2\sqrt{a^2+2abx+b^2x\cdots}\right.}{\sqrt{b^2}} \\ \frac{\frac{(-3a^2-3)(a^2+2abx+1)^{\frac{7}{2}}}{7}+\frac{(a^2+2abx+1)^{\frac{9}{2}}}{9}+\frac{(a^2+2abx+1)^{\frac{5}{2}}\cdot(3a^4+6a^2+3)}{5}+\frac{(a^2+2abx+1)^{\frac{3}{2}}(-a^6-3a^4-3a^2-1)}{3}}{8a^4b^4} \\ \frac{x^4\sqrt{a^2+1}}{4} \end{cases}$$

input
```
integrate((b*x+a+(1+(b*x+a)**2)**(1/2))*x**3,x)
```

output
```
a*x**4/4 + b*x**5/5 + Piecewise(((-a*(-3*a*(-5*a*(1/5 - 3*a**2/20)/(3*b) -
 a*(3*a**2 + 3)/(20*b))/(2*b) - (1/5 - 3*a**2/20)*(2*a**2 + 2)/(3*b**2))/b
 - (a**2 + 1)*(-5*a*(1/5 - 3*a**2/20)/(3*b) - a*(3*a**2 + 3)/(20*b))/(2*b*
*2))*log(2*a*b + 2*b**2*x + 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*sqrt(b*
*2))/sqrt(b**2) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*(a*x**3/(20*b) + x*
*4/5 + x**2*(1/5 - 3*a**2/20)/(3*b**2) + x*(-5*a*(1/5 - 3*a**2/20)/(3*b) -
 a*(3*a**2 + 3)/(20*b))/(2*b**2) + (-3*a*(-5*a*(1/5 - 3*a**2/20)/(3*b) - a
*(3*a**2 + 3)/(20*b))/(2*b) - (1/5 - 3*a**2/20)*(2*a**2 + 2)/(3*b**2))/b**
2), Ne(b**2, 0)), (((-3*a**2 - 3)*(a**2 + 2*a*b*x + 1)**(7/2)/7 + (a**2 +
2*a*b*x + 1)**(9/2)/9 + (a**2 + 2*a*b*x + 1)**(5/2)*(3*a**4 + 6*a**2 + 3)/
5 + (a**2 + 2*a*b*x + 1)**(3/2)*(-a**6 - 3*a**4 - 3*a**2 - 1)/3)/(8*a**4*b
**4), Ne(a*b, 0)), (x**4*sqrt(a**2 + 1)/4, True))
```

3.349.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(205) = 410$.

Time = 0.30 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.98

$$\int e^{\operatorname{arcsinh}(a+bx)}x^3\,dx = \frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4 + \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}x^2}{5\,b^2} - \frac{(a^2+1)a^3\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{5\,b^4} - \frac{7\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}ax}{20\,b^3} + \frac{\sqrt{b^2x^2+2\,abx+a^2+1}(a^2+1)ax}{5\,b^3} + \frac{(a^2+1)^2a\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{5\,b^4} + \frac{7\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a^2}{12\,b^4} + \frac{\sqrt{b^2x^2+2\,abx+a^2+1}(a^2+1)a^2}{5\,b^4} + \frac{7\,(5\,a^2b^2-(a^2+1)b^2)a^3\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{40\,b^6} - \frac{2\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}(a^2+1)}{15\,b^4} - \frac{7\,(5\,a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}ax}{40\,b^5} - \frac{7\,(5\,a^2b^2-(a^2+1)b^2)(a^2+1)a\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{40\,b^6} - \frac{7\,(5\,a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}a^2}{40\,b^6}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x, algorithm="maxima")
```

output
```
1/5*b*x^5 + 1/4*a*x^4 + 1/5*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x^2/b^2 -
1/5*(a^2 + 1)*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^
2))/b^4 - 7/20*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a*x/b^3 + 1/5*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*a*x/b^3 + 1/5*(a^2 + 1)^2*a*arcsinh(2*(
b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 + 7/12*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2)*a^2/b^4 + 1/5*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2
 + 1)*a^2/b^4 + 7/40*(5*a^2*b^2 - (a^2 + 1)*b^2)*a^3*arcsinh(2*(b^2*x + a*
b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^6 - 2/15*(b^2*x^2 + 2*a*b*x + a^2
 + 1)^(3/2)*(a^2 + 1)/b^4 - 7/40*(5*a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*a*x/b^5 - 7/40*(5*a^2*b^2 - (a^2 + 1)*b^2)*(a^2 + 1)*
a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^6 - 7/40*(
5*a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/b^6
```

3.349.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.05

$$\int e^{\operatorname{arcsinh}(a+bx)}x^3\,dx = \frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4 + \frac{1}{120}\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\left(\left(2\left(3\left(4\,x + \frac{a}{b}\right)x - \frac{3\,a^2b^5 - 4\,b^5}{b^7}\right)x + \frac{6\,a^3b^4 - 29\,ab^4}{b^7}\right)x - \frac{6\,a^4b^3 - 83 \ldots}{b \ldots}\right. + \frac{(4\,a^3 - 3\,a)\log\left(-ab - \left(x|b| - \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)|b|\right)}{8\,b^3|b|}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x, algorithm="giac")
```

output
```
1/5*b*x^5 + 1/4*a*x^4 + 1/120*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((2*(3*(4*
x + a/b)*x - (3*a^2*b^5 - 4*b^5)/b^7)*x + (6*a^3*b^4 - 29*a*b^4)/b^7)*x -
(6*a^4*b^3 - 83*a^2*b^3 + 16*b^3)/b^7) + 1/8*(4*a^3 - 3*a)*log(-a*b - (x*a
bs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^3*abs(b))
```

3.349.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{arcsinh}(a+bx)}x^3\,dx = \int x^3\left(a+\sqrt{(a+b\,x)^2+1}+b\,x\right)\,dx$$

input
```
int(x^3*(a + ((a + b*x)^2 + 1)^(1/2) + b*x),x)
```

output
```
int(x^3*(a + ((a + b*x)^2 + 1)^(1/2) + b*x), x)
```

3.350 $\int e^{\text{arcsinh}(a+bx)}x^2\,dx$

3.350.1 Optimal result

Integrand size = 12, antiderivative size = 115

$$\int e^{\text{arcsinh}(a+bx)}x^2\,dx=-\frac{e^{-2\text{arcsinh}(a+bx)}}{16b^3}-\frac{ae^{-\text{arcsinh}(a+bx)}}{2b^3}-\frac{(1-4a^2)\,e^{2\text{arcsinh}(a+bx)}}{16b^3}-\frac{ae^{3\text{arcsinh}(a+bx)}}{6b^3}+\frac{e^{4\text{arcsinh}(a+bx)}}{32b^3}-\frac{(1-4a^2)\,\text{arcsinh}(a+bx)}{8b^3}$$

output
```
-1/16/b^3/(b*x+a+(1+(b*x+a)^2)^(1/2))^2-1/2*a/b^3/(b*x+a+(1+(b*x+a)^2)^(1/
2))-1/16*(-4*a^2+1)*(b*x+a+(1+(b*x+a)^2)^(1/2))^2/b^3-1/6*a*(b*x+a+(1+(b*x
+a)^2)^(1/2))^3/b^3+1/32*(b*x+a+(1+(b*x+a)^2)^(1/2))^4/b^3-1/8*(-4*a^2+1)*
arcsinh(b*x+a)/b^3
```

3.350.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int e^{\text{arcsinh}(a+bx)}x^2\,dx$$
$$=\frac{8ab^3x^3+6b^4x^4+\sqrt{1+a^2+2abx+b^2x^2}(2a^3+3bx-2a^2bx+6b^3x^3+a(-13+2b^2x^2))+3(-1+2a)(1}{24b^3}$$

input
```
Integrate[E^ArcSinh[a + b*x]*x^2,x]
```

output
```
(8*a*b^3*x^3 + 6*b^4*x^4 + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2*a^3 + 3*b*
x - 2*a^2*b*x + 6*b^3*x^3 + a*(-13 + 2*b^2*x^2)) + 3*(-1 + 2*a)*(1 + 2*a)*
ArcSinh[a + b*x])/(24*b^3)
```

3.350.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6288, 2720, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\text{arcsinh}(a+bx)} dx$$

$$\Big\downarrow \text{6288}$$

$$\frac{\int e^{\text{arcsinh}(a+bx)}\left(\frac{a}{b}-\frac{a+bx}{b}\right)^2\sqrt{(a+bx)^2+1}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^{-3\text{arcsinh}(a+bx)}\left(2e^{\text{arcsinh}(a+bx)}a-e^{2\text{arcsinh}(a+bx)}+1\right)^2\left(1+e^{2\text{arcsinh}(a+bx)}\right)}{8b^2}de^{\text{arcsinh}(a+bx)}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int e^{-3\text{arcsinh}(a+bx)}\left(2e^{\text{arcsinh}(a+bx)}a-e^{2\text{arcsinh}(a+bx)}+1\right)^2\left(1+e^{2\text{arcsinh}(a+bx)}\right)de^{\text{arcsinh}(a+bx)}}{8b^3}$$

$$\Big\downarrow \text{2159}$$

$$\frac{\int \left(4e^{-2\text{arcsinh}(a+bx)}a-4e^{2\text{arcsinh}(a+bx)}a+e^{-3\text{arcsinh}(a+bx)}+\left(4a^2-1\right)e^{-\text{arcsinh}(a+bx)}+\left(4a^2-1\right)e^{\text{arcsinh}(a+bx)}+e^{3\text{arcsinh}(a+bx)}\right)de^{\text{arcsinh}(a+bx)}}{8b^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\left(1-4a^2\right)e^{2\text{arcsinh}(a+bx)}-\left(1-4a^2\right)\log\left(e^{\text{arcsinh}(a+bx)}\right)-4ae^{-\text{arcsinh}(a+bx)}-\frac{4}{3}ae^{3\text{arcsinh}(a+bx)}-\frac{1}{2}e^{-2\text{arcsinh}(a+bx)}+\frac{1}{4}e^{4\text{arcsinh}(a+bx)}}{8b^3}$$

input
```
Int[E^ArcSinh[a + b*x]*x^2,x]
```

output
```
(-1/2*1/E^(2*ArcSinh[a + b*x]) - (4*a)/E^ArcSinh[a + b*x] - ((1 - 4*a^2)*E
^(2*ArcSinh[a + b*x]))/2 - (4*a*E^(3*ArcSinh[a + b*x]))/3 + E^(4*ArcSinh[a
 + b*x])/4 - (1 - 4*a^2)*Log[E^ArcSinh[a + b*x]])/(8*b^3)
```

3.350.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6288
```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Simp[1/b   Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSin
h[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.350.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(143) = 286$.

Time = 0.63 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.50

method	result
default	$\frac{x(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{4b^2} - \frac{5a\left(\frac{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{3b^2} - \frac{a\left(\frac{(2b^2x+2ab)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2} + \frac{(4b^2(a^2+1)-4a^2b^2)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2}\right.}{8b^2\sqrt{b^2}}\right.}{b}\right.}{4b}$

input
```
int((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-5/4*a/b*(1/3*(b^2*x^2+2*a*b*x+a^2+
1)^(3/2)/b^2-a/b*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/
8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/(b^2)^(1/2)))-1/4*(a^2+1)/b^2*(1/4*(2*b^2*x+2*a*b)/b^2*(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b
)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2))+1/4*b*x^4+1/3*a*
x^3
```

3.350.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02

$$\int e^{\operatorname{arcsinh}(a+bx)}x^2\,dx$$
$$= \frac{6\,b^4x^4 + 8\,ab^3x^3 - 3\,(4\,a^2-1)\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right) + (6\,b^3x^3 + 2\,ab^2x^2 + 2\,a^3 - (2\,a^2}{24\,b^3}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x, algorithm="fricas")
```

output
```
1/24*(6*b^4*x^4 + 8*a*b^3*x^3 - 3*(4*a^2 - 1)*log(-b*x - a + sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)) + (6*b^3*x^3 + 2*a*b^2*x^2 + 2*a^3 - (2*a^2 - 3)*b*x
 - 13*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

3.350.6 Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.54

$$\int e^{\operatorname{arcsinh}(a+bx)}x^2\,dx = \frac{ax^3}{3} + \frac{bx^4}{4} + \begin{cases} \frac{\left(-\frac{a\left(-\frac{3a\left(\frac{1}{4}-\frac{a^2}{6}\right)}{2b}-\frac{a(2a^2+2)}{12b}\right)}{b}-\frac{\left(\frac{1}{4}-\frac{a^2}{6}\right)(a^2+1)}{2b^2}\right)\log\left(2ab+2b^2x+2\sqrt{a^2+2abx+b^2x^2+1}\sqrt{b^2}\right)}{\sqrt{b^2}} + \sqrt{a^2+2abx+b^2x^2+1}\Bigg(\\ \frac{\frac{(-2a^2-2)(a^2+2abx+1)^{\frac{5}{2}}}{5}+\frac{(a^2+2abx+1)^{\frac{7}{2}}}{7}+\frac{(a^2+2abx+1)^{\frac{3}{2}}(a^4+2a^2+1)}{3}}{4a^3b^3} \\ \frac{x^3\sqrt{a^2+1}}{3} \end{cases}$$

input
```
integrate((b*x+a+(1+(b*x+a)**2)**(1/2))*x**2,x)
```

output
```
a*x**3/3 + b*x**4/4 + Piecewise(((-a*(-3*a*(1/4 - a**2/6)/(2*b) - a*(2*a**
2 + 2)/(12*b))/b - (1/4 - a**2/6)*(a**2 + 1)/(2*b**2))*log(2*a*b + 2*b**2*
x + 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*sqrt(b**2))/sqrt(b**2) + sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1)*(a*x**2/(12*b) + x**3/4 + x*(1/4 - a**2/6)/
(2*b**2) + (-3*a*(1/4 - a**2/6)/(2*b) - a*(2*a**2 + 2)/(12*b))/b**2), Ne(b
**2, 0)), (((-2*a**2 - 2)*(a**2 + 2*a*b*x + 1)**(5/2)/5 + (a**2 + 2*a*b*x
+ 1)**(7/2)/7 + (a**2 + 2*a*b*x + 1)**(3/2)*(a**4 + 2*a**2 + 1)/3)/(4*a**3
*b**3), Ne(a*b, 0)), (x**3*sqrt(a**2 + 1)/3, True))
```

3.350.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.37

$$\int e^{\operatorname{arcsinh}(a+bx)}x^2\,dx = \frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3 + \frac{(b^2x^2 + 2\,abx + a^2 + 1)^{\frac{3}{2}}x}{4\,b^2} - \frac{5\,(b^2x^2 + 2\,abx + a^2 + 1)^{\frac{3}{2}}a}{12\,b^3} - \frac{(5\,a^2b^2 - (a^2 + 1)b^2)a^2\operatorname{arsinh}\left(\frac{2\,(b^2x + ab)}{\sqrt{-4\,a^2b^2 + 4\,(a^2 + 1)b^2}}\right)}{8\,b^5} + \frac{(5\,a^2b^2 - (a^2 + 1)b^2)\sqrt{b^2x^2 + 2\,abx + a^2 + 1}x}{8\,b^4} + \frac{(5\,a^2b^2 - (a^2 + 1)b^2)(a^2 + 1)\operatorname{arsinh}\left(\frac{2\,(b^2x + ab)}{\sqrt{-4\,a^2b^2 + 4\,(a^2 + 1)b^2}}\right)}{8\,b^5} + \frac{(5\,a^2b^2 - (a^2 + 1)b^2)\sqrt{b^2x^2 + 2\,abx + a^2 + 1}a}{8\,b^5}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x, algorithm="maxima")
```

output
```
1/4*b*x^4 + 1/3*a*x^3 + 1/4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x/b^2 - 5/
12*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/b^3 - 1/8*(5*a^2*b^2 - (a^2 + 1)*
b^2)*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 +
 1/8*(5*a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^4 +
 1/8*(5*a^2*b^2 - (a^2 + 1)*b^2)*(a^2 + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4
*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 1/8*(5*a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*a/b^5
```

3.350.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.22

$$\int e^{\operatorname{arcsinh}(a+bx)}x^2\,dx = \frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3 + \frac{1}{24}\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\left(\left(2\left(3\,x + \frac{a}{b}\right)x - \frac{2\,a^2b^3 - 3\,b^3}{b^5}\right)x + \frac{2\,a^3b^2 - 13\,ab^2}{b^5}\right) - \frac{(4\,a^2 - 1)\log\left(-ab - \left(x|b| - \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)|b|\right)}{8\,b^2|b|}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x, algorithm="giac")
```

output
```
1/4*b*x^4 + 1/3*a*x^3 + 1/24*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((2*(3*x +
a/b)*x - (2*a^2*b^3 - 3*b^3)/b^5)*x + (2*a^3*b^2 - 13*a*b^2)/b^5) - 1/8*(4
*a^2 - 1)*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b)
)/(b^2*abs(b))
```

3.350.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{arcsinh}(a+bx)} x^2 \, dx = \int x^2 \left(a + \sqrt{(a+b\,x)^2+1} + b\,x\right) \, dx$$

input
```
int(x^2*(a + ((a + b*x)^2 + 1)^(1/2) + b*x),x)
```

output
```
int(x^2*(a + ((a + b*x)^2 + 1)^(1/2) + b*x), x)
```

3.351 $\int e^{\text{arcsinh}(a+bx)}x\,dx$

3.351.1 Optimal result

Integrand size = 10, antiderivative size = 67

$$\int e^{\text{arcsinh}(a+bx)}x\,dx=\frac{e^{-\text{arcsinh}(a+bx)}}{4b^2}-\frac{ae^{2\text{arcsinh}(a+bx)}}{4b^2}+\frac{e^{3\text{arcsinh}(a+bx)}}{12b^2}-\frac{a\text{arcsinh}(a+bx)}{2b^2}$$

output
```
1/4/b^2/(b*x+a+(1+(b*x+a)^2)^(1/2))-1/4*a*(b*x+a+(1+(b*x+a)^2)^(1/2))^2/b^
2+1/12*(b*x+a+(1+(b*x+a)^2)^(1/2))^3/b^2-1/2*a*arcsinh(b*x+a)/b^2
```

3.351.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int e^{\text{arcsinh}(a+bx)}x\,dx=\frac{1}{6}\left(3ax^2+2bx^3+\frac{\sqrt{1+a^2+2abx+b^2x^2}(2-a^2+abx+2b^2x^2)}{b^2}-\frac{3a\text{arcsinh}(a+bx)}{b^2}\right)$$

input
```
Integrate[E^ArcSinh[a + b*x]*x,x]
```

output
```
(3*a*x^2 + 2*b*x^3 + (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2 - a^2 + a*b*x +
 2*b^2*x^2))/b^2 - (3*a*ArcSinh[a + b*x])/b^2)/6
```

3.351.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6288, 25, 2720, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\text{arcsinh}(a+bx)}\,dx$$

$$\Big\downarrow \text{6288}$$

$$\frac{\int -e^{\text{arcsinh}(a+bx)}\left(\frac{a}{b}-\frac{a+bx}{b}\right)\sqrt{(a+bx)^2+1}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int e^{\text{arcsinh}(a+bx)}\left(\frac{a}{b}-\frac{a+bx}{b}\right)\sqrt{(a+bx)^2+1}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\int \frac{e^{-2\text{arcsinh}(a+bx)}\left(2e^{\text{arcsinh}(a+bx)}a-e^{2\text{arcsinh}(a+bx)}+1\right)\left(1+e^{2\text{arcsinh}(a+bx)}\right)}{4b}de^{\text{arcsinh}(a+bx)}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int e^{-2\text{arcsinh}(a+bx)}\left(2e^{\text{arcsinh}(a+bx)}a-e^{2\text{arcsinh}(a+bx)}+1\right)\left(1+e^{2\text{arcsinh}(a+bx)}\right)de^{\text{arcsinh}(a+bx)}}{4b^2}$$

$$\Big\downarrow \text{2159}$$

$$-\frac{\int \left(2e^{-\text{arcsinh}(a+bx)}a+2e^{\text{arcsinh}(a+bx)}a+e^{-2\text{arcsinh}(a+bx)}-e^{2\text{arcsinh}(a+bx)}\right)de^{\text{arcsinh}(a+bx)}}{4b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ae^{2\text{arcsinh}(a+bx)}-e^{-\text{arcsinh}(a+bx)}-\frac{1}{3}e^{3\text{arcsinh}(a+bx)}+2a\log\left(e^{\text{arcsinh}(a+bx)}\right)}{4b^2}$$

input
```
Int[E^ArcSinh[a + b*x]*x,x]
```

output
```
-1/4*(-E^(-ArcSinh[a + b*x]) + a*E^(2*ArcSinh[a + b*x]) - E^(3*ArcSinh[a +
 b*x])/3 + 2*a*Log[E^ArcSinh[a + b*x]])/b^2
```

3.351.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6288
```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Simp[1/b   Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSin
h[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

3.351.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.16

method	result
default	$\frac{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{3b^2} - \frac{a\left(\frac{(2b^2x+2ab)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2} + \frac{(4b^2(a^2+1)-4a^2b^2)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{8b^2\sqrt{b^2}}\right)}{b} + \frac{b\,x^3}{3} + a$

input
```
int((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x,method=_RETURNVERBOSE)
```

output
```
1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-a/b*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^
2+2*a*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b
^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2))+1/3*b*x^3+1/2*a*x^2
```

3.351.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.39

$$\int e^{\operatorname{arcsinh}(a+bx)}x\,dx = \frac{2\,b^3x^3+3\,ab^2x^2+3\,a\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+\left(2\,b^2x^2+abx-a^2+2\right)\sqrt{b^2x^2+2\,abx+}}{6\,b^2}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3 + 3*a*b^2*x^2 + 3*a*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)) + (2*b^2*x^2 + a*b*x - a^2 + 2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1))/b^2
```

3.351.6 Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.94

$$\int e^{\operatorname{arcsinh}(a+bx)}x\,dx = \frac{ax^2}{2}+\frac{bx^3}{3} + \begin{cases} \frac{\left(-\frac{a\left(\frac{1}{3}-\frac{a^2}{6}\right)}{b}-\frac{a(a^2+1)}{6b}\right)\log\left(2ab+2b^2x+2\sqrt{a^2+2abx+b^2x^2+1}\sqrt{b^2}\right)}{\sqrt{b^2}}+\left(\frac{ax}{6b}+\frac{x^2}{3}+\frac{\frac{1}{3}-\frac{a^2}{6}}{b^2}\right)\sqrt{a^2+2abx+b^2x^2+1} & \text{f} \\ \frac{\frac{(-a^2-1)(a^2+2abx+1)^{\frac{3}{2}}}{3}+\frac{(a^2+2abx+1)^{\frac{5}{2}}}{5}}{2a^2b^2} & \text{f} \\ \frac{x^2\sqrt{a^2+1}}{2} & \text{o} \end{cases}$$

input
```
integrate((b*x+a+(1+(b*x+a)**2)**(1/2))*x,x)
```

output
```
a*x**2/2 + b*x**3/3 + Piecewise(((-a*(1/3 - a**2/6)/b - a*(a**2 + 1)/(6*b)
)*log(2*a*b + 2*b**2*x + 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*sqrt(b**2)
)/sqrt(b**2) + (a*x/(6*b) + x**2/3 + (1/3 - a**2/6)/b**2)*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1), Ne(b**2, 0)), (((-a**2 - 1)*(a**2 + 2*a*b*x + 1)**(3
/2)/3 + (a**2 + 2*a*b*x + 1)**(5/2)/5)/(2*a**2*b**2), Ne(a*b, 0)), (x**2*s
qrt(a**2 + 1)/2, True))
```

3.351.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(83) = 166$.

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.61

$$\int e^{\operatorname{arcsinh}(a+bx)}x\,dx = \frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2 + \frac{a^3\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^2} - \frac{\sqrt{b^2x^2+2\,abx+a^2+1}ax}{2\,b} - \frac{(a^2+1)a\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^2} - \frac{\sqrt{b^2x^2+2\,abx+a^2+1}a^2}{2\,b^2} + \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{3\,b^2}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x, algorithm="maxima")
```

output
```
1/3*b*x^3 + 1/2*a*x^2 + 1/2*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 +
4*(a^2 + 1)*b^2))/b^2 - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b - 1/2*
(a^2 + 1)*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^
2 - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/b^2 + 1/3*(b^2*x^2 + 2*a*b*x
 + a^2 + 1)^(3/2)/b^2
```

3.351.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.58

$$\int e^{\operatorname{arcsinh}(a+bx)}x\,dx = \frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2 + \frac{1}{6}\,\sqrt{b^2x^2+2\,abx+a^2+1}\left(\left(2\,x+\frac{a}{b}\right)x - \frac{a^2b-2\,b}{b^3}\right) + \frac{a\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{2\,b|b|}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x, algorithm="giac")
```

output
```
1/3*b*x^3 + 1/2*a*x^2 + 1/6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((2*x + a/b)
*x - (a^2*b - 2*b)/b^3) + 1/2*a*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1))*abs(b))/(b*abs(b))
```

3.351.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{arcsinh}(a+bx)}x\,dx = \int x\left(a+\sqrt{(a+b\,x)^2+1}+b\,x\right)\,dx$$

input
```
int(x*(a + ((a + b*x)^2 + 1)^(1/2) + b*x),x)
```

output
```
int(x*(a + ((a + b*x)^2 + 1)^(1/2) + b*x), x)
```

3.352 $\int e^{\text{arcsinh}(a+bx)} dx$

3.352.1 Optimal result

Integrand size = 8, antiderivative size = 31

$$\int e^{\text{arcsinh}(a+bx)} dx = \frac{e^{2\text{arcsinh}(a+bx)}}{4b} + \frac{\text{arcsinh}(a+bx)}{2b}$$

output
```
1/4*(b*x+a+(1+(b*x+a)^2)^(1/2))^2/b+1/2*arcsinh(b*x+a)/b
```

3.352.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int e^{\text{arcsinh}(a+bx)} dx = \frac{(a+bx)\left(a+bx+\sqrt{1+a^2+2abx+b^2x^2}\right)+\text{arcsinh}(a+bx)}{2b}$$

input
```
Integrate[E^ArcSinh[a + b*x],x]
```

output
```
((a + b*x)*(a + b*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) + ArcSinh[a + b*x
])/(2*b)
```

3.352.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6287, 2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\text{arcsinh}(a+bx)}dx$$

$$\Big\downarrow \text{6287}$$

$$\frac{\int e^{\text{arcsinh}(a+bx)}\sqrt{(a+bx)^2+1}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{2}e^{-\text{arcsinh}(a+bx)}\left(1+e^{2\text{arcsinh}(a+bx)}\right)de^{\text{arcsinh}(a+bx)}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int e^{-\text{arcsinh}(a+bx)}\left(1+e^{2\text{arcsinh}(a+bx)}\right)de^{\text{arcsinh}(a+bx)}}{2b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(e^{-\text{arcsinh}(a+bx)}+e^{\text{arcsinh}(a+bx)}\right)de^{\text{arcsinh}(a+bx)}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}e^{2\text{arcsinh}(a+bx)}+\log\left(e^{\text{arcsinh}(a+bx)}\right)}{2b}$$

input
```
Int[E^ArcSinh[a + b*x],x]
```

output
```
(E^(2*ArcSinh[a + b*x])/2 + Log[E^ArcSinh[a + b*x]])/(2*b)
```

3.352.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6287
```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Simp[1/b
 Subst[Int[f^(c*x^n)*Cosh[x], x], x, ArcSinh[a + b*x]], x] /; FreeQ[{a, b,
c, f}, x] && IGtQ[n, 0]
```

3.352.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(35) = 70$.

Time = 0.76 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.87

method	result	size
default	$ax+\frac{(2b^2x+2ab)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}+\frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2\sqrt{b^2}}+\frac{b\,x^2}{2}$	89
parts	$ax+\frac{(2b^2x+2ab)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}+\frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2\sqrt{b^2}}+\frac{b\,x^2}{2}$	89

input
```
int(b*x+a+(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*x+1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*ln((b^2*x+a*
b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*b*x^2
```

3.352.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.35

$$\int e^{\operatorname{arcsinh}(a+bx)}\,dx = \frac{b^2x^2 + 2\,abx + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}(bx + a) - \log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)}{2\,b}$$

input
```
integrate(b*x+a+(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 + 2*a*b*x + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a) - log
(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

3.352.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 124, normalized size of antiderivative = 4.00

$$\int e^{\operatorname{arcsinh}(a+bx)}\,dx = ax + \frac{bx^2}{2} + \begin{cases} \left(\frac{a}{2b} + \frac{x}{2}\right)\sqrt{a^2 + 2abx + b^2x^2 + 1} + \frac{\log\left(2ab + 2b^2x + 2\sqrt{a^2 + 2abx + b^2x^2 + 1}\sqrt{b^2}\right)}{2\sqrt{b^2}} & \text{for } b^2 \neq 0 \\ \frac{(a^2 + 2abx + 1)^{\frac{3}{2}}}{3ab} & \text{for } ab \neq 0 \\ x\sqrt{a^2 + 1} & \text{otherwise} \end{cases}$$

input
```
integrate(b*x+a+(1+(b*x+a)**2)**(1/2),x)
```

output
```
a*x + b*x**2/2 + Piecewise(((a/(2*b) + x/2)*sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1) + log(2*a*b + 2*b**2*x + 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*sqr
t(b**2))/(2*sqrt(b**2)), Ne(b**2, 0)), ((a**2 + 2*a*b*x + 1)**(3/2)/(3*a*b
), Ne(a*b, 0)), (x*sqrt(a**2 + 1), True))
```

3.352.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(35) = 70.

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 4.55

$$\int e^{\text{arcsinh}(a+bx)}\,dx = \frac{1}{2}\,bx^2 + ax - \frac{a^2\,\text{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b} + \frac{1}{2}\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1}x + \frac{(a^2+1)\,\text{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b} + \frac{\sqrt{b^2x^2 + 2\,abx + a^2 + 1}a}{2\,b}$$

input
```
integrate(b*x+a+(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*b*x^2 + a*x - 1/2*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2
 + 1)*b^2))/b + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x + 1/2*(a^2 + 1)*ar
csinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b + 1/2*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*a/b
```

3.352.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(35) = 70.

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.58

$$\int e^{\text{arcsinh}(a+bx)}\,dx = \frac{1}{2}\,bx^2 + ax + \frac{1}{2}\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\left(x + \frac{a}{b}\right) - \frac{\log\left(-ab - \left(x|b| - \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)|b|\right)}{2\,|b|}$$

input
```
integrate(b*x+a+(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*b*x^2 + a*x + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(x + a/b) - 1/2*lo
g(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/abs(b)
```

3.352.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{arcsinh}(a+bx)}\,dx = \int a + \sqrt{(a+b\,x)^2+1} + b\,x\,dx$$

input
```
int(a + ((a + b*x)^2 + 1)^(1/2) + b*x,x)
```

output
```
int(a + ((a + b*x)^2 + 1)^(1/2) + b*x, x)
```

3.353 $\int \frac{e^{\text{arcsinh}(a+bx)}}{x} \, dx$

3.353.1 Optimal result

Integrand size = 12, antiderivative size = 89

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x} \, dx = bx + \sqrt{1+a^2+2abx+b^2x^2} + a\text{arcsinh}(a+bx) - \sqrt{1+a^2}\text{arctanh}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right) + a\log(x)$$

output
```
b*x+a*arcsinh(b*x+a)+a*ln(x)-arctanh((a*b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*x^2+
2*a*b*x+a^2+1)^(1/2))*(a^2+1)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
```

3.353.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x} \, dx = bx + \sqrt{1+a^2+2abx+b^2x^2} + a\text{arcsinh}(a+bx) + \left(a+\sqrt{1+a^2}\right)\log(x) - \sqrt{1+a^2}\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)$$

input
```
Integrate[E^ArcSinh[a + b*x]/x,x]
```

output
```
b*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + a*ArcSinh[a + b*x] + (a + Sqrt[1
 + a^2])*Log[x] - Sqrt[1 + a^2]*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1
 + a^2 + 2*a*b*x + b^2*x^2]]
```

3.353.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6293, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x}\,dx$$

$$\Big\downarrow \text{ 6293}$$

$$\int \frac{\sqrt{(a+bx)^2+1}+a+bx}{x}dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(\frac{\sqrt{a^2+2abx+b^2x^2+1}}{x}+\frac{a}{x}+b\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\sqrt{a^2+1}\text{arctanh}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)+\sqrt{a^2+2abx+b^2x^2+1}+a\text{arcsinh}(a+bx)+a\log(x)+bx$$

input
```
Int[E^ArcSinh[a + b*x]/x,x]
```

output
```
b*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + a*ArcSinh[a + b*x] - Sqrt[1 + a^
2]*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x
^2])] + a*Log[x]
```

3.353.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 6293
```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

3.353.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.42

method	result
default	$\sqrt{b^2x^2+2abx+a^2+1}+\frac{ab\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}-\sqrt{a^2+1}\,\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2ab}}{x}\right.$

input
```
int((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*
b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^
(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+b*x+a*ln(x)
```

3.353.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.53

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x}\,dx = bx - a\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+a\log(x)$$
$$+\sqrt{a^2+1}\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}(a^2-\sqrt{a^2+1}a+1)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right.$$
$$+\sqrt{b^2x^2+2\,abx+a^2+1}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x, algorithm="fricas")
```

output
```
b*x - a*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + a*log(x) + sqr
t(a^2 + 1)*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 -
sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/x) + sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)
```

3.353.6 Sympy [F]

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x}\,dx = \int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x}\,dx$$

input
```
integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x,x)
```

output
```
Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x, x)
```

3.353.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.80

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x}\,dx = bx + a\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right) + a\log\left(x\right) - \sqrt{a^2+1}\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|} + \frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|} + \frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right) + \sqrt{b^2x^2+2\,abx+a^2+1}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x, algorithm="maxima")
```

output
```
b*x + a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)) + a*lo
g(x) - sqrt(a^2 + 1)*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*a
bs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^
2*b^2 + 4*(a^2 + 1)*b^2)*abs(x))) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
```

3.353.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.78

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x}\,dx = bx - \frac{ab\log\left(-ab - \left(x|b| - \sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{|b|} + a\log\left(|x|\right) + \sqrt{a^2+1}\log\left(\frac{\left|-2\,x|b| + 2\,\sqrt{b^2x^2+2\,abx+a^2+1} - 2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b| + 2\,\sqrt{b^2x^2+2\,abx+a^2+1} + 2\,\sqrt{a^2+1}\right|}\right) + \sqrt{b^2x^2+2\,abx+a^2+1}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x, algorithm="giac")
```

output
```
b*x - a*b*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b)
)/abs(b) + a*log(abs(x)) + sqrt(a^2 + 1)*log(abs(-2*x*abs(b) + 2*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1) + 2*sqrt(a^2 + 1))) + sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)
```

3.353.9 Mupad [B] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.02

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x}\,dx = \sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1} + b\,x + a\,\ln(x) - \frac{\ln\left(a\,b + \frac{a^2+1}{x} + \frac{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)}{\sqrt{a^2+1}} - \frac{a^2\,\ln\left(a\,b + \frac{a^2+1}{x} + \frac{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)}{\sqrt{a^2+1}} + \frac{a\,b\,\ln\left(\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1} + \frac{x\,b^2+a\,b}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

input
```
int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x,x)
```

output
```
(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2) + b*x + a*log(x) - log(a*b + (a^2 + 1)
/x + ((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2))/x)/(a^2 + 1)^(1
/2) - (a^2*log(a*b + (a^2 + 1)/x + ((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b
*x + 1)^(1/2))/x))/(a^2 + 1)^(1/2) + (a*b*log((a^2 + b^2*x^2 + 2*a*b*x + 1
)^(1/2) + (a*b + b^2*x)/(b^2)^(1/2)))/(b^2)^(1/2)
```

3.354 $\int \frac{e^{\text{arcsinh}(a+bx)}}{x^2}\,dx$

3.354.1 Optimal result

Integrand size = 12, antiderivative size = 99

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x^2}\,dx = -\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\text{arcsinh}(a+bx)-\frac{ab\text{arctanh}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{\sqrt{1+a^2}}+b\log(x)$$

output
```
-a/x+b*arcsinh(b*x+a)+b*ln(x)-a*b*arctanh((a*b*x+a^2+1)/(a^2+1)^(1/2)/(b^2
*x^2+2*a*b*x+a^2+1)^(1/2))/(a^2+1)^(1/2)-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/x
```

3.354.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.11

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x^2}\,dx = b\text{arcsinh}(a+bx)-\frac{a+\sqrt{1+a^2+2abx+b^2x^2}+\left(-1-\frac{a}{\sqrt{1+a^2}}\right)bx\log(x)+\frac{abx\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)}{\sqrt{1+a^2}}}{x}$$

input
```
Integrate[E^ArcSinh[a + b*x]/x^2,x]
```

output
```
b*ArcSinh[a + b*x] - (a + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (-1 - a/Sqrt
[1 + a^2])*b*x*Log[x] + (a*b*x*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1
+ a^2 + 2*a*b*x + b^2*x^2]])/Sqrt[1 + a^2])/x
```

3.354.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6293, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x^2}dx$$
$$\Big\downarrow \text{ 6293}$$
$$\int \frac{\sqrt{(a+bx)^2+1}+a+bx}{x^2}dx$$
$$\Big\downarrow \text{ 2010}$$
$$\int \left(\frac{\sqrt{a^2+2abx+b^2x^2+1}}{x^2}+\frac{a}{x^2}+\frac{b}{x}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{ab\text{arctanh}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{\sqrt{a^2+1}}-\frac{\sqrt{a^2+2abx+b^2x^2+1}}{x}+b\text{arcsinh}(a+bx)-\frac{a}{x}+b\log(x)$$

input
```
Int[E^ArcSinh[a + b*x]/x^2,x]
```

output
```
-(a/x) - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/x + b*ArcSinh[a + b*x] - (a*b*A
rcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])
])/Sqrt[1 + a^2] + b*Log[x]
```

3.354.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 6293
```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

3.354.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(91) = 182$.

Time = 0.76 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.87

method	result
default	$-\frac{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{(a^2+1)x}+\frac{ab\left(\sqrt{b^2x^2+2abx+a^2+1}+\frac{ab\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}-\sqrt{a^2+1}\,\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2}}{x}\right.\right.}{a^2+1}$

input
```
int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+a*b/(a^2+1)*((b^2*x^2+2*a*b*x+a
^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/
(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a
*b*x+a^2+1)^(1/2))/x))+2*b^2/(a^2+1)*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a
*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(
1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2))+b*ln(x)-a/x
```

3.354.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(91) = 182$.

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.85

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^2}\,dx$$
$$= \frac{\sqrt{a^2+1}abx\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2+1}a+1\right)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)-(a^2+1)bx\log\left(-bx-a+\right.}{(a^2+}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x, algorithm="fricas")
```

output
```
(sqrt(a^2 + 1)*a*b*x*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/x) -
 (a^2 + 1)*b*x*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (a^2 +
1)*b*x*log(x) - a^3 - (a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a
^2 + 1) - a)/((a^2 + 1)*x)
```

3.354.6 Sympy [F]

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^2}\,dx = \int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x^2}\,dx$$

input
```
integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**2,x)
```

output
```
Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**2, x)
```

3.354.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.72

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^2}\,dx$$
$$= -\frac{ab\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{\sqrt{a^2+1}}$$
$$+b\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)+b\log(x)-\frac{a}{x}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{x}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x, algorithm="maxima")
```

output
```
-a*b*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(
sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 +
 1)*b^2)*abs(x)))/sqrt(a^2 + 1) + b*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^
2 + 4*(a^2 + 1)*b^2)) + b*log(x) - a/x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
/x
```

3.354.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(91) = 182$.

Time = 0.36 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.36

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^2}\,dx = \frac{ab\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}} - \frac{b^2\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{|b|} + b\log\left(|x|\right) - \frac{a}{x} + \frac{2\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)ab^5+a^2b^4|b|+b^4|b|\right)}{\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-a^2-1\right)b^4}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x, algorithm="giac")
```

output
```
a*b*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sqrt(a^2
 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt(a^2
+ 1)))/sqrt(a^2 + 1) - b^2*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1))*abs(b))/abs(b) + b*log(abs(x)) - a/x + 2*((x*abs(b) - sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1))*a*b^5 + a^2*b^4*abs(b) + b^4*abs(b))/(((x*abs(b)
 - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - a^2 - 1)*b^4)
```

3.354.9 Mupad [B] (verification not implemented)

Time = 3.66 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.72

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^2}\,dx = b\,\ln(x) - \frac{a}{x} + \ln\left(\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1} + \frac{x\,b^2+a\,b}{\sqrt{b^2}}\right)\sqrt{b^2} - \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x\,(a^2+1)} + \frac{a^3\,b\,\mathrm{atanh}\left(\frac{a^2+b\,x\,a+1}{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\right)}{(a^2+1)^{3/2}} - \frac{a^2\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x\,(a^2+1)} + \frac{a\,b\,\mathrm{atanh}\left(\frac{a^2+b\,x\,a+1}{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\right)}{(a^2+1)^{3/2}} - \frac{2\,a\,b\,\ln\left(a\,b+\frac{a^2+1}{x}+\frac{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)}{\sqrt{a^2+1}}$$

input

```
int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^2,x)
```

output

```
b*log(x) - a/x + log((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2) + (a*b + b^2*x)/(
b^2)^(1/2))*(b^2)^(1/2) - (a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/(x*(a^2 + 1)
) + (a^3*b*atanh((a^2 + a*b*x + 1)/((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b
*x + 1)^(1/2))))/(a^2 + 1)^(3/2) - (a^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2
))/(x*(a^2 + 1)) + (a*b*atanh((a^2 + a*b*x + 1)/((a^2 + 1)^(1/2)*(a^2 + b^
2*x^2 + 2*a*b*x + 1)^(1/2))))/(a^2 + 1)^(3/2) - (2*a*b*log(a*b + (a^2 + 1)
/x + ((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2))/x))/(a^2 + 1)^(
1/2)
```

3.355 $\int \frac{e^{\text{arcsinh}(a+bx)}}{x^3}\,dx$

3.355.1 Optimal result

Integrand size = 12, antiderivative size = 116

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x^3}\,dx = -\frac{a}{2x^2}-\frac{b}{x}-\frac{(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2(1+a^2)x^2} - \frac{b^2\text{arctanh}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{2(1+a^2)^{3/2}}$$

output
```
-1/2*a/x^2-b/x-1/2*b^2*arctanh((a*b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/(a^2+1)^(3/2)-1/2*(a*b*x+a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)/(a^2+1)/x^2
```

3.355.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.11

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x^3}\,dx = \frac{1}{2}\left(-\frac{a}{x^2}-\frac{2b}{x}-\frac{(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{(1+a^2)x^2}+\frac{b^2\log(x)}{(1+a^2)^{3/2}} - \frac{b^2\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)}{(1+a^2)^{3/2}}\right)$$

input
```
Integrate[E^ArcSinh[a + b*x]/x^3,x]
```

output
```
(-(a/x^2) - (2*b)/x - ((1 + a^2 + a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]
)/((1 + a^2)*x^2) + (b^2*Log[x])/(1 + a^2)^(3/2) - (b^2*Log[1 + a^2 + a*b*
x + Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(1 + a^2)^(3/2))/2
```

3.355.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6293, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x^3} \, dx$$

$$\Big\downarrow \text{ 6293}$$

$$\int \frac{\sqrt{(a+bx)^2+1}+a+bx}{x^3}dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(\frac{\sqrt{a^2+2abx+b^2x^2+1}}{x^3}+\frac{a}{x^3}+\frac{b}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^2\text{arctanh}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{2\left(a^2+1\right)^{3/2}}-\frac{\left(a^2+abx+1\right)\sqrt{a^2+2abx+b^2x^2+1}}{2\left(a^2+1\right)x^2}-\frac{a}{2x^2}-\frac{b}{x}$$

input
```
Int[E^ArcSinh[a + b*x]/x^3,x]
```

output
```
-1/2*a/x^2 - b/x - ((1 + a^2 + a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/(
2*(1 + a^2)*x^2) - (b^2*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 +
a^2 + 2*a*b*x + b^2*x^2])])/(2*(1 + a^2)^(3/2))
```

3.355.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 6293
```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

3.355.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(102) = 204$.

Time = 0.75 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.96

method	result
default	$-\frac{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{2(a^2+1)x^2} - \frac{ab\left(-\frac{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{(a^2+1)x} + \frac{ab\left(\sqrt{b^2x^2+2abx+a^2+1}+\frac{ab\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}} - \sqrt{a^2+1}\,\ln\left(2\cdots\right)\right)}{a^2+1}\cdots\right)}{\cdots}$

input
```
int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/2*a*b/(a^2+1)*(-1/(a^2+1)
/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+a*b/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1)^(1/2
)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2
)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1
)^(1/2))/x))+2*b^2/(a^2+1)*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1
)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*
x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)))+1/2*b^2/(a^2+1)*((b^2*x^2+2*a*b*x+
a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))
/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/x))-b/x-1/2*a/x^2
```

3.355.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.56

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^3}\,dx$$
$$= \frac{\sqrt{a^2+1}b^2x^2\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2+1}a+1\right)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)-a^5-(a^3+a)b^2x^2-2\,a^3}{2\,(a^4+2\,a^2+1)x^2}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x, algorithm="fricas")
```

output
```
1/2*(sqrt(a^2 + 1)*b^2*x^2*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a
)/x) - a^5 - (a^3 + a)*b^2*x^2 - 2*a^3 - 2*(a^4 + 2*a^2 + 1)*b*x - (a^4 +
(a^3 + a)*b*x + 2*a^2 + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - a)/((a^4 +
2*a^2 + 1)*x^2)
```

3.355.6 Sympy [F]

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^3}\,dx = \int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x^3}\,dx$$

input
```
integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**3,x)
```

output
```
Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**3, x)
```

3.355.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(102) = 204$.

Time = 0.38 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.70

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^3}\,dx = \frac{a^2b^2\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{2\,(a^2+1)^{\frac{3}{2}}} - \frac{b^2\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{2\sqrt{a^2+1}} + \frac{\sqrt{b^2x^2+2\,abx+a^2+1}b^2}{2\,(a^2+1)} + \frac{\sqrt{b^2x^2+2\,abx+a^2+1}ab}{2\,(a^2+1)x} - \frac{b}{x} - \frac{a}{2\,x^2} - \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{2\,(a^2+1)x^2}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x, algorithm="maxima")
```

output
```
1/2*a^2*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) +
2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4
*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) - 1/2*b^2*arcsinh(2*a*b*x/(sqrt(-
4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1
)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/sqrt(a^2 +
 1) + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2/(a^2 + 1) + 1/2*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)*a*b/((a^2 + 1)*x) - b/x - 1/2*a/x^2 - 1/2*(b^2*x^2
 + 2*a*b*x + a^2 + 1)^(3/2)/((a^2 + 1)*x^2)
```

3.355.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(102) = 204.

Time = 0.34 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.31

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^3}\,dx = \frac{b^2\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{2\,(a^2+1)^{\frac{3}{2}}} - \frac{2\,bx+a}{2\,x^2} + \frac{2\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3a^2b^2+2\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)a^4b^2+4\left(x|b|-\sqrt{b^2x^2+2\right.}{}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x, algorithm="giac")
```

output
```
1/2*b^2*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sqrt
(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt(
a^2 + 1)))/(a^2 + 1)^(3/2) - 1/2*(2*b*x + a)/x^2 + (2*(x*abs(b) - sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1))^3*a^2*b^2 + 2*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))*a^4*b^2 + 4*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2
*a^3*b*abs(b) + (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*b^2 + 3*(
x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^2*b^2 + 4*(x*abs(b) - sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a*b*abs(b) + (x*abs(b) - sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1))*b^2)/(((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2
 - a^2 - 1)^2*(a^2 + 1))
```

3.355.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x^3}\,dx = \int \frac{a+\sqrt{(a+b\,x)^2+1}+b\,x}{x^3}\,dx$$

input
```
int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^3,x)
```

output
```
int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^3, x)
```

3.356 $\int \frac{e^{\text{arcsinh}(a+bx)}}{x^4} dx$

3.356.1 Optimal result

Integrand size = 12, antiderivative size = 156

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x^4} dx = -\frac{a}{3x^3} - \frac{b}{2x^2} + \frac{ab(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2(1+a^2)^2x^2} - \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{3(1+a^2)x^3} + \frac{ab^3\text{arctanh}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{2(1+a^2)^{5/2}}$$

output
```
-1/3*a/x^3-1/2*b/x^2-1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^3+1/2*a*b
^3*arctanh((a*b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(a^2
+1)^(5/2)+1/2*a*b*(a*b*x+a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/(a^2+1)^2/x^
2
```

3.356.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.04

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x^4} dx = \frac{1}{6}\left(-\frac{2a}{x^3} - \frac{3b}{x^2} - \frac{\sqrt{1+a^2+2abx+b^2x^2}(2+2a^4+abx+a^3bx+2b^2x^2+a^2(4-b^2x^2))}{(1+a^2)^2x^3} - \frac{3ab^3\log(x)}{(1+a^2)^{5/2}} + \frac{3ab^3\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)}{(1+a^2)^{5/2}}\right)$$

input
```
Integrate[E^ArcSinh[a + b*x]/x^4,x]
```

output
```
((-2*a)/x^3 - (3*b)/x^2 - (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2 + 2*a^4 +
a*b*x + a^3*b*x + 2*b^2*x^2 + a^2*(4 - b^2*x^2)))/((1 + a^2)^2*x^3) - (3*a
*b^3*Log[x])/(1 + a^2)^(5/2) + (3*a*b^3*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2
]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(1 + a^2)^(5/2))/6
```

3.356.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6293, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x^4} \, dx$$

$$\Big\downarrow \text{ 6293}$$

$$\int \frac{\sqrt{(a+bx)^2+1}+a+bx}{x^4}dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(\frac{\sqrt{a^2+2abx+b^2x^2+1}}{x^4}+\frac{a}{x^4}+\frac{b}{x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ab^3\text{arctanh}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{2\left(a^2+1\right)^{5/2}}+\frac{ab\left(a^2+abx+1\right)\sqrt{a^2+2abx+b^2x^2+1}}{2\left(a^2+1\right)^2x^2}-\frac{\left(a^2+2abx+b^2x^2+1\right)^{3/2}}{3\left(a^2+1\right)x^3}-\frac{a}{3x^3}-\frac{b}{2x^2}$$

input
```
Int[E^ArcSinh[a + b*x]/x^4,x]
```

output
```
-1/3*a/x^3 - b/(2*x^2) + (a*b*(1 + a^2 + a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b
^2*x^2])/(2*(1 + a^2)^2*x^2) - (1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(3*(1 +
 a^2)*x^3) + (a*b^3*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2
+ 2*a*b*x + b^2*x^2])])/(2*(1 + a^2)^(5/2))
```

3.356.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 6293
```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

3.356.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(136) = 272$.

Time = 0.95 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.22

method	result
default	$-\frac{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{3(a^2+1)x^3} - \frac{ab\left(-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{2\left(a^2+1\right)x^2} - \frac{ab\left(-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{\left(a^2+1\right)x} + \frac{ab\left(\sqrt{b^2x^2+2abx+a^2+1}+\frac{ab\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2}\right.}{\sqrt{b^2}}\right.}{}\right.}{}\right.}{}$

input
```
int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^3-a*b/(a^2+1)*(-1/2/(a^2+1)/x
^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/2*a*b/(a^2+1)*(-1/(a^2+1)/x*(b^2*x^2+2*
a*b*x+a^2+1)^(3/2)+a*b/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*
x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2
)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x))+2
*b^2/(a^2+1)*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/8*(4
*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^
2+1)^(1/2))/(b^2)^(1/2)))+1/2*b^2/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a
*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(
a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(
1/2))/x)))-1/3*a/x^3-1/2*b/x^2
```

3.356.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.47

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^4}\,dx$$

$$= \frac{3\sqrt{a^2+1}ab^3x^3\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2+\sqrt{a^2+1}a+1\right)+(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)-2\,a^7+(a^4-a^2-2)b^3}{}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x, algorithm="fricas")
```

output
```
1/6*(3*sqrt(a^2 + 1)*a*b^3*x^3*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)*(a^2 + sqrt(a^2 + 1)*a + 1) + (a*b*x + a^2 + 1)*sqrt(a^2 + 1)
 + a)/x) - 2*a^7 + (a^4 - a^2 - 2)*b^3*x^3 - 6*a^5 - 6*a^3 - 3*(a^6 + 3*a^
4 + 3*a^2 + 1)*b*x - (2*a^6 - (a^4 - a^2 - 2)*b^2*x^2 + 6*a^4 + (a^5 + 2*a
^3 + a)*b*x + 6*a^2 + 2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*a)/((a^6 +
3*a^4 + 3*a^2 + 1)*x^3)
```

3.356.6 Sympy [F]

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^4}\,dx = \int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x^4}\,dx$$

input
```
integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**4,x)
```

output
```
Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**4, x)
```

3.356.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(136) = 272$.

Time = 0.36 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.26

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^4}\,dx = -\frac{a^3b^3\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{2\,(a^2+1)^{\frac{5}{2}}} + \frac{ab^3\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{2\,(a^2+1)^{\frac{3}{2}}} - \frac{\sqrt{b^2x^2+2\,abx+a^2+1}ab^3}{2\,(a^2+1)^2} - \frac{\sqrt{b^2x^2+2\,abx+a^2+1}a^2b^2}{2\,(a^2+1)^2x} + \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}ab}{2\,(a^2+1)^2x^2} - \frac{b}{2\,x^2} - \frac{a}{3\,x^3} - \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{3\,(a^2+1)x^3}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x, algorithm="maxima")
```

output
```
-1/2*a^3*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) +
 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 +
4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) + 1/2*a*b^3*arcsinh(2*a*b*x/(sqr
t(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2
+ 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 +
1)^(3/2) - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*b^3/(a^2 + 1)^2 - 1/2*s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*b^2/((a^2 + 1)^2*x) + 1/2*(b^2*x^2 +
2*a*b*x + a^2 + 1)^(3/2)*a*b/((a^2 + 1)^2*x^2) - 1/2*b/x^2 - 1/3*a/x^3 - 1
/3*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/((a^2 + 1)*x^3)
```

3.356.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(136) = 272$.

Time = 0.33 (sec) , antiderivative size = 715, normalized size of antiderivative = 4.58

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x^4}\,dx = -\frac{ab^3\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{2\,(a^4+2\,a^2+1)\sqrt{a^2+1}} - \frac{3\,bx+2\,a}{6\,x^3} + \frac{20\,\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3 a^5b^3 + 12\,\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)a^7b^3 + 6\,\left(x|b|-\sqrt{b^2x^2+\right.}{}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x, algorithm="giac")
```

output
```
-1/2*a*b^3*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*s
qrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sq
rt(a^2 + 1)))/((a^4 + 2*a^2 + 1)*sqrt(a^2 + 1)) - 1/6*(3*b*x + 2*a)/x^3 +
1/3*(20*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^5*b^3 + 12*(x*a
bs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^7*b^3 + 6*(x*abs(b) - sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1))^4*a^4*b^2*abs(b) + 24*(x*abs(b) - sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))^2*a^6*b^2*abs(b) + 2*a^8*b^2*abs(b) + 3*(x*abs(b) -
 sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*a*b^3 + 32*(x*abs(b) - sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1))^3*a^3*b^3 + 33*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1))*a^5*b^3 + 12*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*a^
2*b^2*abs(b) + 48*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^4*b^2
*abs(b) + 8*a^6*b^2*abs(b) + 12*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))^3*a*b^3 + 30*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^3*b^3 +
 6*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*b^2*abs(b) + 24*(x*abs
(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^2*b^2*abs(b) + 12*a^4*b^2*abs
(b) + 9*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a*b^3 + 8*a^2*b^2*a
bs(b) + 2*b^2*abs(b))/((a^4 + 2*a^2 + 1)*((x*abs(b) - sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1))^2 - a^2 - 1)^3)
```

3.356.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^4}\,dx = \int \frac{a+\sqrt{(a+b\,x)^2+1}+b\,x}{x^4}\,dx$$

input
```
int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^4,x)
```

output
```
int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^4, x)
```

3.357 $\int \frac{e^{\text{arcsinh}(a+bx)}}{x^5}\,dx$

3.357.1 Optimal result

Integrand size = 12, antiderivative size = 207

$$\begin{aligned}\int \frac{e^{\text{arcsinh}(a+bx)}}{x^5}\,dx =& -\frac{a}{4x^4}-\frac{b}{3x^3}+\frac{(1-4a^2)\,b^2(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{8\,(1+a^2)^3\,x^2}\\ &-\frac{(1+a^2+2abx+b^2x^2)^{3/2}}{4\,(1+a^2)\,x^4}+\frac{5ab(1+a^2+2abx+b^2x^2)^{3/2}}{12\,(1+a^2)^2\,x^3}\\ &+\frac{(1-4a^2)\,b^4\text{arctanh}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{8\,(1+a^2)^{7/2}}\end{aligned}$$

output
```
-1/4*a/x^4-1/3*b/x^3-1/4*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^4+5/12*a*
b*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)^2/x^3+1/8*(-4*a^2+1)*b^4*arctanh((
a*b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(a^2+1)^(7/2)+1/
8*(-4*a^2+1)*b^2*(a*b*x+a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/(a^2+1)^3/x^2
```

3.357.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.93

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x^5} dx = \frac{1}{24}\left(-\frac{6a}{x^4} - \frac{8b}{x^3} - \frac{\sqrt{1+a^2+2abx+b^2x^2}\left(6+\frac{2abx}{1+a^2} - \frac{(-3+2a^2)b^2x^2}{(1+a^2)^2} + \frac{a(-13+2a^2)b^3x^3}{(1+a^2)^3}\right)}{x^4} + \frac{3(-1+2a)(1+2a)b^4\log(x)}{(1+a^2)^{7/2}} - \frac{3(-1+2a)(1+2a)b^4\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)}{(1+a^2)^{7/2}}\right)$$

input
```
Integrate[E^ArcSinh[a + b*x]/x^5,x]
```

output
```
((-6*a)/x^4 - (8*b)/x^3 - (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(6 + (2*a*b*x
)/(1 + a^2) - ((-3 + 2*a^2)*b^2*x^2)/(1 + a^2)^2 + (a*(-13 + 2*a^2)*b^3*x^
3)/(1 + a^2)^3))/x^4 + (3*(-1 + 2*a)*(1 + 2*a)*b^4*Log[x])/(1 + a^2)^(7/2)
 - (3*(-1 + 2*a)*(1 + 2*a)*b^4*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1
+ a^2 + 2*a*b*x + b^2*x^2]])/(1 + a^2)^(7/2))/24
```

3.357.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6293, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x^5} dx$$

$$\Big\downarrow \; 6293$$

$$\int \frac{\sqrt{(a+bx)^2+1}+a+bx}{x^5}dx$$

$$\Big\downarrow \; 2010$$

$$\int \left(\frac{\sqrt{a^2+2abx+b^2x^2+1}}{x^5}+\frac{a}{x^5}+\frac{b}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(1-4a^2)\,b^4\text{arctanh}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{8\,(a^2+1)^{7/2}}+\frac{(1-4a^2)\,b^2(a^2+abx+1)\sqrt{a^2+2abx+b^2x^2+1}}{8\,(a^2+1)^3\,x^2}-\frac{(a^2+2abx+b^2x^2+1)^{3/2}}{4\,(a^2+1)\,x^4}+\frac{5ab(a^2+2abx+b^2x^2+1)^{3/2}}{12\,(a^2+1)^2\,x^3}-\frac{a}{4x^4}-\frac{b}{3x^3}$$

input
```
Int[E^ArcSinh[a + b*x]/x^5,x]
```

output
```
-1/4*a/x^4 - b/(3*x^3) + ((1 - 4*a^2)*b^2*(1 + a^2 + a*b*x)*Sqrt[1 + a^2 +
 2*a*b*x + b^2*x^2])/(8*(1 + a^2)^3*x^2) - (1 + a^2 + 2*a*b*x + b^2*x^2)^(
3/2)/(4*(1 + a^2)*x^4) + (5*a*b*(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(12*(
1 + a^2)^2*x^3) + ((1 - 4*a^2)*b^4*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2
]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])])/(8*(1 + a^2)^(7/2))
```

3.357.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 6293
```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

3.357. $\int \frac{e^{\text{arcsinh}(a+bx)}}{x^5}\,dx$

3.357.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1002 vs. 2(183) = 366.

Time = 0.91 (sec) , antiderivative size = 1003, normalized size of antiderivative = 4.85

method	result	size
default	Expression too large to display	1003

input
```
int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^4-5/4*a*b/(a^2+1)*(-1/3*(b^2*
x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^3-a*b/(a^2+1)*(-1/2/(a^2+1)/x^2*(b^2*x^
2+2*a*b*x+a^2+1)^(3/2)-1/2*a*b/(a^2+1)*(-1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+
1)^(3/2)+a*b/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^
2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^
2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x))+2*b^2/(a^2+
1)*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+
1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
)/(b^2)^(1/2)))+1/2*b^2/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2
*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/
2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)))
)-1/4*b^2/(a^2+1)*(-1/2/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/2*a*b/
(a^2+1)*(-1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+a*b/(a^2+1)*((b^2*x^2+
2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1
)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^
2*x^2+2*a*b*x+a^2+1)^(1/2))/x))+2*b^2/(a^2+1)*(1/4*(2*b^2*x+2*a*b)/b^2*(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b
)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)))+1/2*b^2/(a^2+1)
*((b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^...
```

3.357.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.43

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^5}\,dx$$
$$= \frac{3\,(4\,a^2-1)\sqrt{a^2+1}b^4x^4\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2+1}a+1\right)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)-6\,a^9-(2\,a^5-}{}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x, algorithm="fricas")
```

output
```
1/24*(3*(4*a^2 - 1)*sqrt(a^2 + 1)*b^4*x^4*log(-(a^2*b*x + a^3 + sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sq
rt(a^2 + 1) + a)/x) - 6*a^9 - (2*a^5 - 11*a^3 - 13*a)*b^4*x^4 - 24*a^7 - 3
6*a^5 - 24*a^3 - 8*(a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*b*x - (6*a^8 + (2*a^5
 - 11*a^3 - 13*a)*b^3*x^3 + 24*a^6 - (2*a^6 + a^4 - 4*a^2 - 3)*b^2*x^2 + 3
6*a^4 + 2*(a^7 + 3*a^5 + 3*a^3 + a)*b*x + 24*a^2 + 6)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1) - 6*a)/((a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*x^4)
```

3.357.6 Sympy [F]

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^5}\,dx = \int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x^5}\,dx$$

input
```
integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**5,x)
```

output
```
Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**5, x)
```

3.357.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(183) = 366$.

Time = 0.38 (sec) , antiderivative size = 594, normalized size of antiderivative = 2.87

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^5}\,dx$$
$$= \frac{5\,a^4b^4\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{8\,(a^2+1)^{\frac{7}{2}}}$$
$$-\frac{3\,a^2b^4\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{4\,(a^2+1)^{\frac{5}{2}}}$$
$$+\frac{5\,\sqrt{b^2x^2+2\,abx+a^2+1}a^2b^4}{8\,(a^2+1)^3}$$
$$+\frac{b^4\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{8\,(a^2+1)^{\frac{3}{2}}}$$
$$-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}b^4}{8\,(a^2+1)^2}+\frac{5\,\sqrt{b^2x^2+2\,abx+a^2+1}a^3b^3}{8\,(a^2+1)^3x}$$
$$-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}ab^3}{8\,(a^2+1)^2x}-\frac{5\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a^2b^2}{8\,(a^2+1)^3x^2}$$
$$+\frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}b^2}{8\,(a^2+1)^2x^2}+\frac{5\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}ab}{12\,(a^2+1)^2x^3}$$
$$-\frac{b}{3\,x^3}-\frac{a}{4\,x^4}-\frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{4\,(a^2+1)x^4}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x, algorithm="maxima")
```

3.357. $\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^5}\,dx$

output
```
5/8*a^4*b^4*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) +
2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4
*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(7/2) - 3/4*a^2*b^4*arcsinh(2*a*b*x/(sq
rt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2
 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 +
 1)^(5/2) + 5/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*b^4/(a^2 + 1)^3 + 1/
8*b^4*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/
(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2
+ 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) - 1/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*b^4/(a^2 + 1)^2 + 5/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3*b^3/((a^2 + 1
)^3*x) - 1/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*b^3/((a^2 + 1)^2*x) - 5/8
*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2*b^2/((a^2 + 1)^3*x^2) + 1/8*(b^2*
x^2 + 2*a*b*x + a^2 + 1)^(3/2)*b^2/((a^2 + 1)^2*x^2) + 5/12*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2)*a*b/((a^2 + 1)^2*x^3) - 1/3*b/x^3 - 1/4*a/x^4 - 1/4*
(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/((a^2 + 1)*x^4)
```

3.357.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1173 vs. $2(183) = 366$.

Time = 0.35 (sec) , antiderivative size = 1173, normalized size of antiderivative = 5.67

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)}}{x^5} \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x, algorithm="giac")
```

output
```
1/8*(4*a^2*b^4 - b^4)*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1) + 2*sqrt(a^2 + 1)))/((a^6 + 3*a^4 + 3*a^2 + 1)*sqrt(a^2 + 1)) - 1/12*
(4*b*x + 3*a)/x^4 + 1/12*(32*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
)^5*a^6*b^4 + 256*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^8*b^4
 + 96*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^10*b^4 + 144*(x*abs
(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*a^7*b^3*abs(b) + 224*(x*abs(b)
- sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^9*b^3*abs(b) + 16*a^11*b^3*abs(b)
 - 12*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^7*a^2*b^4 + 140*(x*ab
s(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*a^4*b^4 + 716*(x*abs(b) - sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^6*b^4 + 372*(x*abs(b) - sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1))*a^8*b^4 + 432*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1))^4*a^5*b^3*abs(b) + 704*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
))^2*a^7*b^3*abs(b) + 80*a^9*b^3*abs(b) + 3*(x*abs(b) - sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1))^7*b^4 + 129*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
)^5*a^2*b^4 + 685*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^4*b^4
 + 543*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^6*b^4 + 432*(x*abs
(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*a^3*b^3*abs(b) + 768*(x*abs(b)
- sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^5*b^3*abs(b) + 160*a^7*b^3*abs(b)
 + 21*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*b^4 + 246*(x*abs...
```

3.357.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\text{arcsinh}(a+bx)}}{x^5} dx = \int \frac{a+\sqrt{(a+b\,x)^2+1}+b\,x}{x^5} dx$$

input
```
int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^5,x)
```

output
```
int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^5, x)
```

3.358 $\int e^{\operatorname{arcsinh}(a+bx)^2} x^3 \, dx$

3.358.1 Optimal result

Integrand size = 14, antiderivative size = 359

$$\begin{aligned}\int e^{\operatorname{arcsinh}(a+bx)^2} x^3 \, dx = &-\frac{\sqrt{\pi}\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))}{16b^4e} + \frac{3a^2\sqrt{\pi}\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))}{8b^4e} \\ &+ \frac{\sqrt{\pi}\operatorname{erfi}(2-\operatorname{arcsinh}(a+bx))}{32b^4e^4} - \frac{\sqrt{\pi}\operatorname{erfi}(1+\operatorname{arcsinh}(a+bx))}{16b^4e} \\ &+ \frac{3a^2\sqrt{\pi}\operatorname{erfi}(1+\operatorname{arcsinh}(a+bx))}{8b^4e} + \frac{\sqrt{\pi}\operatorname{erfi}(2+\operatorname{arcsinh}(a+bx))}{32b^4e^4} \\ &- \frac{3a\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-3+2\operatorname{arcsinh}(a+bx))\right)}{16b^4e^{9/4}} \\ &+ \frac{3a\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-1+2\operatorname{arcsinh}(a+bx))\right)}{16b^4\sqrt[4]{e}} \\ &- \frac{a^3\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-1+2\operatorname{arcsinh}(a+bx))\right)}{4b^4\sqrt[4]{e}} \\ &+ \frac{3a\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(1+2\operatorname{arcsinh}(a+bx))\right)}{16b^4\sqrt[4]{e}} \\ &- \frac{a^3\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(1+2\operatorname{arcsinh}(a+bx))\right)}{4b^4\sqrt[4]{e}} \\ &- \frac{3a\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(3+2\operatorname{arcsinh}(a+bx))\right)}{16b^4e^{9/4}}\end{aligned}$$

output
```
-1/32*erfi(-2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(4)+1/16*erfi(-1+arcsinh(b*x
+a))*Pi^(1/2)/b^4/exp(1)-3/8*a^2*erfi(-1+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(
1)-1/16*erfi(1+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(1)+3/8*a^2*erfi(1+arcsinh(
b*x+a))*Pi^(1/2)/b^4/exp(1)+1/32*erfi(2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(4
)-3/16*a*erfi(-3/2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(9/4)+3/16*a*erfi(-1/2+
arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(1/4)-1/4*a^3*erfi(-1/2+arcsinh(b*x+a))*Pi
^(1/2)/b^4/exp(1/4)+3/16*a*erfi(1/2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(1/4)-
1/4*a^3*erfi(1/2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(1/4)-3/16*a*erfi(3/2+arc
sinh(b*x+a))*Pi^(1/2)/b^4/exp(9/4)
```

3.358.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.55

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x^3 \, dx = \frac{\sqrt{\pi}\left(2a(-3+4a^2)\, e^{15/4}\operatorname{erfi}\left(\frac{1}{2}-\operatorname{arcsinh}(a+bx)\right)+2(-1+6a^2)\, e^3\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))+6ae^{7/4}\operatorname{erfi}\left(\right.\right.}{}$$

input
```
Integrate[E^ArcSinh[a + b*x]^2*x^3,x]
```

output
```
(Sqrt[Pi]*(2*a*(-3 + 4*a^2)*E^(15/4)*Erfi[1/2 - ArcSinh[a + b*x]] + 2*(-1
+ 6*a^2)*E^3*Erfi[1 - ArcSinh[a + b*x]] + 6*a*E^(7/4)*Erfi[3/2 - ArcSinh[a
 + b*x]] + Erfi[2 - ArcSinh[a + b*x]] + 6*a*E^(15/4)*Erfi[1/2 + ArcSinh[a
+ b*x]] - 8*a^3*E^(15/4)*Erfi[1/2 + ArcSinh[a + b*x]] - 2*E^3*Erfi[1 + Arc
Sinh[a + b*x]] + 12*a^2*E^3*Erfi[1 + ArcSinh[a + b*x]] - 6*a*E^(7/4)*Erfi[
3/2 + ArcSinh[a + b*x]] + Erfi[2 + ArcSinh[a + b*x]]))/(32*b^4*E^4)
```

3.358.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6288, 25, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{\operatorname{arcsinh}(a+bx)^2} \, dx$$

$$\Big\downarrow \; 6288$$

$$\frac{\int -e^{\text{arcsinh}(a+bx)^2}\left(\frac{a}{b}-\frac{a+bx}{b}\right)^3\sqrt{(a+bx)^2+1}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int e^{\text{arcsinh}(a+bx)^2}\left(\frac{a}{b}-\frac{a+bx}{b}\right)^3\sqrt{(a+bx)^2+1}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \; 7292$$

$$-\frac{\int -e^{\text{arcsinh}(a+bx)^2}x^3\sqrt{(a+bx)^2+1}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int -b^3e^{\text{arcsinh}(a+bx)^2}x^3\sqrt{(a+bx)^2+1}d\text{arcsinh}(a+bx)}{b^4}$$

$$\Big\downarrow \; 7293$$

$$-\frac{\int \left(e^{\text{arcsinh}(a+bx)^2}\sqrt{(a+bx)^2+1}a^3-3e^{\text{arcsinh}(a+bx)^2}(a+bx)\sqrt{(a+bx)^2+1}a^2+3e^{\text{arcsinh}(a+bx)^2}(a+bx)^2\sqrt{(a+\right.}{b^4}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{\sqrt{\pi}a^3\text{erfi}\left(\frac{1}{2}(2\text{arcsinh}(a+bx)-1)\right)}{4\sqrt[4]{e}}+\frac{\sqrt{\pi}a^3\text{erfi}\left(\frac{1}{2}(2\text{arcsinh}(a+bx)+1)\right)}{4\sqrt[4]{e}}-\frac{3\sqrt{\pi}a^2\text{erfi}(1-\text{arcsinh}(a+bx))}{8e}-\frac{3\sqrt{\pi}a^2\text{erfi}(\text{arcsinh}(a}{8e}}{}$$

input
```
Int[E^ArcSinh[a + b*x]^2*x^3,x]
```

output
```
-(((Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/(16*E) - (3*a^2*Sqrt[Pi]*Erfi[1 -
 ArcSinh[a + b*x]])/(8*E) - (Sqrt[Pi]*Erfi[2 - ArcSinh[a + b*x]])/(32*E^4)
 + (Sqrt[Pi]*Erfi[1 + ArcSinh[a + b*x]])/(16*E) - (3*a^2*Sqrt[Pi]*Erfi[1 +
 ArcSinh[a + b*x]])/(8*E) - (Sqrt[Pi]*Erfi[2 + ArcSinh[a + b*x]])/(32*E^4)
 + (3*a*Sqrt[Pi]*Erfi[(-3 + 2*ArcSinh[a + b*x])/2])/(16*E^(9/4)) - (3*a*Sq
rt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(16*E^(1/4)) + (a^3*Sqrt[Pi]*Erf
i[(-1 + 2*ArcSinh[a + b*x])/2])/(4*E^(1/4)) - (3*a*Sqrt[Pi]*Erfi[(1 + 2*Ar
cSinh[a + b*x])/2])/(16*E^(1/4)) + (a^3*Sqrt[Pi]*Erfi[(1 + 2*ArcSinh[a + b
*x])/2])/(4*E^(1/4)) + (3*a*Sqrt[Pi]*Erfi[(3 + 2*ArcSinh[a + b*x])/2])/(16
*E^(9/4)))/b^4)
```

3.358. $\int e^{\text{arcsinh}(a+bx)^2}x^3\,dx$

3.358.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6288
```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Simp[1/b   Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSin
h[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.358.4 Maple [F]

$$\int \mathrm{e}^{\operatorname{arcsinh}(bx+a)^2} x^3 dx$$

input
```
int(exp(arcsinh(b*x+a)^2)*x^3,x)
```

output
```
int(exp(arcsinh(b*x+a)^2)*x^3,x)
```

3.358.5 Fricas [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2}x^3\,dx=\int x^3e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2)*x^3,x, algorithm="fricas")
```

output
```
integral(x^3*e^(arcsinh(b*x + a)^2), x)
```

3.358.6 Sympy [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2}x^3\,dx=\int x^3e^{\operatorname{asinh}^2(a+bx)}\,dx$$

input
```
integrate(exp(asinh(b*x+a)**2)*x**3,x)
```

output
```
Integral(x**3*exp(asinh(a + b*x)**2), x)
```

3.358.7 Maxima [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2}x^3\,dx=\int x^3e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2)*x^3,x, algorithm="maxima")
```

output
```
integrate(x^3*e^(arcsinh(b*x + a)^2), x)
```

3.358.8 Giac [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2}x^3\,dx = \int x^3 e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2)*x^3,x, algorithm="giac")
```

output
```
integrate(x^3*e^(arcsinh(b*x + a)^2), x)
```

3.358.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{arcsinh}(a+bx)^2}x^3\,dx = \int x^3\,\mathrm{e}^{\operatorname{asinh}(a+b\,x)^2}\,dx$$

input
```
int(x^3*exp(asinh(a + b*x)^2),x)
```

output
```
int(x^3*exp(asinh(a + b*x)^2), x)
```

3.359 $\int e^{\operatorname{arcsinh}(a+bx)^2} x^2 \, dx$

3.359.1 Optimal result

Integrand size = 14, antiderivative size = 251

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x^2 \, dx = -\frac{a\sqrt{\pi}\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))}{4b^3e} - \frac{a\sqrt{\pi}\operatorname{erfi}(1+\operatorname{arcsinh}(a+bx))}{4b^3e} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-3+2\operatorname{arcsinh}(a+bx))\right)}{16b^3e^{9/4}} - \frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-1+2\operatorname{arcsinh}(a+bx))\right)}{16b^3\sqrt[4]{e}} + \frac{a^2\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-1+2\operatorname{arcsinh}(a+bx))\right)}{4b^3\sqrt[4]{e}} - \frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(1+2\operatorname{arcsinh}(a+bx))\right)}{16b^3\sqrt[4]{e}} + \frac{a^2\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(1+2\operatorname{arcsinh}(a+bx))\right)}{4b^3\sqrt[4]{e}} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(3+2\operatorname{arcsinh}(a+bx))\right)}{16b^3e^{9/4}}$$

output
```
1/4*a*erfi(-1+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(1)-1/4*a*erfi(1+arcsinh(b*x
+a))*Pi^(1/2)/b^3/exp(1)+1/16*erfi(-3/2+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(9
/4)-1/16*erfi(-1/2+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)+1/4*a^2*erfi(-1/2
+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)-1/16*erfi(1/2+arcsinh(b*x+a))*Pi^(1
/2)/b^3/exp(1/4)+1/4*a^2*erfi(1/2+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)+1/
16*erfi(3/2+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(9/4)
```

3.359.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.55

$$\int e^{\operatorname{arcsinh}(a+bx)^2}x^2\,dx =$$
$$-\frac{\sqrt{\pi}\left((-1+4a^2)\,e^2\operatorname{erfi}\left(\frac{1}{2}-\operatorname{arcsinh}(a+bx)\right)+4ae^{5/4}\operatorname{erfi}(1-\operatorname{arcsinh}(a+bx))+\operatorname{erfi}\left(\frac{3}{2}-\operatorname{arcsinh}(a+b\right.\right.}{}$$

input
```
Integrate[E^ArcSinh[a + b*x]^2*x^2,x]
```

output
```
-1/16*(Sqrt[Pi]*((-1 + 4*a^2)*E^2*Erfi[1/2 - ArcSinh[a + b*x]] + 4*a*E^(5/
4)*Erfi[1 - ArcSinh[a + b*x]] + Erfi[3/2 - ArcSinh[a + b*x]] + E^2*Erfi[1/
2 + ArcSinh[a + b*x]] - 4*a^2*E^2*Erfi[1/2 + ArcSinh[a + b*x]] + 4*a*E^(5/
4)*Erfi[1 + ArcSinh[a + b*x]] - Erfi[3/2 + ArcSinh[a + b*x]]))/(b^3*E^(9/4
))
```

3.359.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6288, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\operatorname{arcsinh}(a+bx)^2}\,dx$$
$$\Big\downarrow \; 6288$$
$$\frac{\int e^{\operatorname{arcsinh}(a+bx)^2}\left(\frac{a}{b}-\frac{a+bx}{b}\right)^2\sqrt{(a+bx)^2+1}\,d\operatorname{arcsinh}(a+bx)}{b}$$
$$\Big\downarrow \; 7292$$
$$\frac{\int e^{\operatorname{arcsinh}(a+bx)^2}x^2\sqrt{(a+bx)^2+1}\,d\operatorname{arcsinh}(a+bx)}{b}$$
$$\Big\downarrow \; 27$$
$$\frac{\int b^2e^{\operatorname{arcsinh}(a+bx)^2}x^2\sqrt{(a+bx)^2+1}\,d\operatorname{arcsinh}(a+bx)}{b^3}$$
$$\Big\downarrow \; 7293$$

$$\frac{\int \left(e^{\text{arcsinh}(a+bx)^2}\sqrt{(a+bx)^2+1}a^2 - 2e^{\text{arcsinh}(a+bx)^2}(a+bx)\sqrt{(a+bx)^2+1}a + e^{\text{arcsinh}(a+bx)^2}(a+bx)^2\sqrt{(a+bx)}\right.}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt{\pi}a^2\text{erfi}\left(\frac{1}{2}(2\text{arcsinh}(a+bx)-1)\right)}{4\sqrt[4]{e}} + \frac{\sqrt{\pi}a^2\text{erfi}\left(\frac{1}{2}(2\text{arcsinh}(a+bx)+1)\right)}{4\sqrt[4]{e}} - \frac{\sqrt{\pi}a\text{erfi}(1-\text{arcsinh}(a+bx))}{4e} - \frac{\sqrt{\pi}a\text{erfi}(\text{arcsinh}(a+bx)+}{4e}}{}$$

input
```
Int[E^ArcSinh[a + b*x]^2*x^2,x]
```

output
```
(-1/4*(a*Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/E - (a*Sqrt[Pi]*Erfi[1 + Arc
Sinh[a + b*x]])/(4*E) + (Sqrt[Pi]*Erfi[(-3 + 2*ArcSinh[a + b*x])/2])/(16*E
^(9/4)) - (Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(16*E^(1/4)) + (a^2
*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*E^(1/4)) - (Sqrt[Pi]*Erfi[
(1 + 2*ArcSinh[a + b*x])/2])/(16*E^(1/4)) + (a^2*Sqrt[Pi]*Erfi[(1 + 2*ArcS
inh[a + b*x])/2])/(4*E^(1/4)) + (Sqrt[Pi]*Erfi[(3 + 2*ArcSinh[a + b*x])/2]
)/(16*E^(9/4)))/b^3
```

3.359.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6288
```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Simp[1/b   Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSin
h[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.359.4 Maple [F]

$$\int \mathrm{e}^{\operatorname{arcsinh}(bx+a)^2} x^2 dx$$

input
```
int(exp(arcsinh(b*x+a)^2)*x^2,x)
```

output
```
int(exp(arcsinh(b*x+a)^2)*x^2,x)
```

3.359.5 Fricas [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x^2 \, dx = \int x^2 e^{\left(\operatorname{arsinh}(bx+a)^2\right)} \, dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2)*x^2,x, algorithm="fricas")
```

output
```
integral(x^2*e^(arcsinh(b*x + a)^2), x)
```

3.359.6 Sympy [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2} x^2 \, dx = \int x^2 e^{\operatorname{asinh}^2(a+bx)} \, dx$$

input
```
integrate(exp(asinh(b*x+a)**2)*x**2,x)
```

output
```
Integral(x**2*exp(asinh(a + b*x)**2), x)
```

3.359.7 Maxima [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2}x^2\,dx = \int x^2 e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2)*x^2,x, algorithm="maxima")
```

output
```
integrate(x^2*e^(arcsinh(b*x + a)^2), x)
```

3.359.8 Giac [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2}x^2\,dx = \int x^2 e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2)*x^2,x, algorithm="giac")
```

output
```
integrate(x^2*e^(arcsinh(b*x + a)^2), x)
```

3.359.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{arcsinh}(a+bx)^2}x^2\,dx = \int x^2\,\mathrm{e}^{\operatorname{asinh}(a+b\,x)^2}\,dx$$

input
```
int(x^2*exp(asinh(a + b*x)^2),x)
```

output
```
int(x^2*exp(asinh(a + b*x)^2), x)
```

3.360 $\int e^{\text{arcsinh}(a+bx)^2} x\,dx$

3.360.1 Optimal result

Integrand size = 12, antiderivative size = 117

$$\int e^{\text{arcsinh}(a+bx)^2} x\,dx = \frac{\sqrt{\pi}\text{erfi}(1-\text{arcsinh}(a+bx))}{8b^2e} + \frac{\sqrt{\pi}\text{erfi}(1+\text{arcsinh}(a+bx))}{8b^2e} - \frac{a\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(-1+2\text{arcsinh}(a+bx))\right)}{4b^2\sqrt[4]{e}} - \frac{a\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(1+2\text{arcsinh}(a+bx))\right)}{4b^2\sqrt[4]{e}}$$

output
```
-1/8*erfi(-1+arcsinh(b*x+a))*Pi^(1/2)/b^2/exp(1)+1/8*erfi(1+arcsinh(b*x+a)
)*Pi^(1/2)/b^2/exp(1)-1/4*a*erfi(-1/2+arcsinh(b*x+a))*Pi^(1/2)/b^2/exp(1/4
)-1/4*a*erfi(1/2+arcsinh(b*x+a))*Pi^(1/2)/b^2/exp(1/4)
```

3.360.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int e^{\text{arcsinh}(a+bx)^2} x\,dx = \frac{\sqrt{\pi}\left(2ae^{3/4}\text{erfi}\left(\frac{1}{2}-\text{arcsinh}(a+bx)\right)+\text{erfi}(1-\text{arcsinh}(a+bx))-2ae^{3/4}\text{erfi}\left(\frac{1}{2}+\text{arcsinh}(a+bx)\right)+\text{erfi}\cdots}{8b^2e}$$

input
```
Integrate[E^ArcSinh[a + b*x]^2*x,x]
```

output
```
(Sqrt[Pi]*(2*a*E^(3/4)*Erfi[1/2 - ArcSinh[a + b*x]] + Erfi[1 - ArcSinh[a +
 b*x]] - 2*a*E^(3/4)*Erfi[1/2 + ArcSinh[a + b*x]] + Erfi[1 + ArcSinh[a + b
*x]]))/(8*b^2*E)
```

3.360.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6288, 25, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\text{arcsinh}(a+bx)^2}dx$$

$$\Big\downarrow \text{6288}$$

$$\frac{\int -e^{\text{arcsinh}(a+bx)^2}\left(\frac{a}{b}-\frac{a+bx}{b}\right)\sqrt{(a+bx)^2+1}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int e^{\text{arcsinh}(a+bx)^2}\left(\frac{a}{b}-\frac{a+bx}{b}\right)\sqrt{(a+bx)^2+1}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{7292}$$

$$-\frac{\int -e^{\text{arcsinh}(a+bx)^2}x\sqrt{(a+bx)^2+1}d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -be^{\text{arcsinh}(a+bx)^2}x\sqrt{(a+bx)^2+1}d\text{arcsinh}(a+bx)}{b^2}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{\int \left(ae^{\text{arcsinh}(a+bx)^2}\sqrt{(a+bx)^2+1}-e^{\text{arcsinh}(a+bx)^2}(a+bx)\sqrt{(a+bx)^2+1}\right)d\text{arcsinh}(a+bx)}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{\sqrt{\pi}\text{erfi}(1-\text{arcsinh}(a+bx))}{8e}-\frac{\sqrt{\pi}\text{erfi}(\text{arcsinh}(a+bx)+1)}{8e}+\frac{\sqrt{\pi}a\text{erfi}\left(\frac{1}{2}(2\text{arcsinh}(a+bx)-1)\right)}{4\sqrt[4]{e}}+\frac{\sqrt{\pi}a\text{erfi}\left(\frac{1}{2}(2\text{arcsinh}(a+bx)+1\right.}{4\sqrt[4]{e}}}{b^2}$$

input
```
Int[E^ArcSinh[a + b*x]^2*x,x]
```

output
```
-((-1/8*(Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/E - (Sqrt[Pi]*Erfi[1 + ArcSi
nh[a + b*x]])/(8*E) + (a*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*E^
(1/4)) + (a*Sqrt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(4*E^(1/4)))/b^2)
```

3.360.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6288
```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Simp[1/b   Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSin
h[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.360.4 Maple [F]

$$\int e^{\operatorname{arcsinh}(bx+a)^2} x dx$$

input
```
int(exp(arcsinh(b*x+a)^2)*x,x)
```

output
```
int(exp(arcsinh(b*x+a)^2)*x,x)
```

3.360.5 Fricas [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2}x\,dx=\int xe^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2)*x,x, algorithm="fricas")
```

output
```
integral(x*e^(arcsinh(b*x + a)^2), x)
```

3.360.6 Sympy [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2}x\,dx=\int xe^{\operatorname{asinh}^2(a+bx)}\,dx$$

input
```
integrate(exp(asinh(b*x+a)**2)*x,x)
```

output
```
Integral(x*exp(asinh(a + b*x)**2), x)
```

3.360.7 Maxima [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2}x\,dx=\int xe^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2)*x,x, algorithm="maxima")
```

output
```
integrate(x*e^(arcsinh(b*x + a)^2), x)
```

3.360.8 Giac [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2}x\,dx = \int xe^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2)*x,x, algorithm="giac")
```

output
```
integrate(x*e^(arcsinh(b*x + a)^2), x)
```

3.360.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{arcsinh}(a+bx)^2}x\,dx = \int x\,\mathrm{e}^{\operatorname{asinh}(a+b\,x)^2}\,dx$$

input
```
int(x*exp(asinh(a + b*x)^2),x)
```

output
```
int(x*exp(asinh(a + b*x)^2), x)
```

3.361 $\int e^{\text{arcsinh}(a+bx)^2}\,dx$

3.361.1 Optimal result

Integrand size = 10, antiderivative size = 65

$$\int e^{\text{arcsinh}(a+bx)^2}\,dx=\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(-1+2\text{arcsinh}(a+bx))\right)}{4b\sqrt[4]{e}}+\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(1+2\text{arcsinh}(a+bx))\right)}{4b\sqrt[4]{e}}$$

output
```
1/4*erfi(-1/2+arcsinh(b*x+a))*Pi^(1/2)/b/exp(1/4)+1/4*erfi(1/2+arcsinh(b*x
+a))*Pi^(1/2)/b/exp(1/4)
```

3.361.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int e^{\text{arcsinh}(a+bx)^2}\,dx=\frac{\sqrt{\pi}\left(\text{erfi}\left(\frac{1}{2}+\text{arcsinh}(a+bx)\right)+\text{erfi}\left(\frac{1}{2}(-1+2\text{arcsinh}(a+bx))\right)\right)}{4b\sqrt[4]{e}}$$

input
```
Integrate[E^ArcSinh[a + b*x]^2,x]
```

output
```
(Sqrt[Pi]*(Erfi[1/2 + ArcSinh[a + b*x]] + Erfi[(-1 + 2*ArcSinh[a + b*x])/2
]))/(4*b*E^(1/4))
```

3.361.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6287, 6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\text{arcsinh}(a+bx)^2} \, dx$$

$$\Big\downarrow \ 6287$$

$$\frac{\int e^{\text{arcsinh}(a+bx)^2} \sqrt{(a+bx)^2+1} d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \ 6039$$

$$\frac{\int \left(\frac{1}{2} e^{\text{arcsinh}(a+bx)^2-\text{arcsinh}(a+bx)}+\frac{1}{2} e^{\text{arcsinh}(a+bx)^2+\text{arcsinh}(a+bx)}\right) d\text{arcsinh}(a+bx)}{b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(2\text{arcsinh}(a+bx)-1)\right)}{4\sqrt[4]{e}}+\frac{\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(2\text{arcsinh}(a+bx)+1)\right)}{4\sqrt[4]{e}}}{b}$$

input
```
Int[E^ArcSinh[a + b*x]^2,x]
```

output
```
((Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*E^(1/4)) + (Sqrt[Pi]*Erfi
[(1 + 2*ArcSinh[a + b*x])/2])/(4*E^(1/4)))/b
```

3.361.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

rule 6287
```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Simp[1/b
 Subst[Int[f^(c*x^n)*Cosh[x], x], x, ArcSinh[a + b*x]], x] /; FreeQ[{a, b,
c, f}, x] && IGtQ[n, 0]
```

3.361.4 Maple [F]

$$\int \mathrm{e}^{\operatorname{arcsinh}(bx+a)^2} dx$$

input
```
int(exp(arcsinh(b*x+a)^2),x)
```

output
```
int(exp(arcsinh(b*x+a)^2),x)
```

3.361.5 Fricas [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2} dx = \int e^{\left(\operatorname{arsinh}(bx+a)^2\right)} dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2),x, algorithm="fricas")
```

output
```
integral(e^(arcsinh(b*x + a)^2), x)
```

3.361.6 Sympy [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2} dx = \int e^{\operatorname{asinh}^2(a+bx)} dx$$

input
```
integrate(exp(asinh(b*x+a)**2),x)
```

output
```
Integral(exp(asinh(a + b*x)**2), x)
```

3.361.7 Maxima [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2}\,dx = \int e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2),x, algorithm="maxima")
```

output
```
integrate(e^(arcsinh(b*x + a)^2), x)
```

3.361.8 Giac [F]

$$\int e^{\operatorname{arcsinh}(a+bx)^2}\,dx = \int e^{\left(\operatorname{arsinh}(bx+a)^2\right)}\,dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2),x, algorithm="giac")
```

output
```
integrate(e^(arcsinh(b*x + a)^2), x)
```

3.361.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{arcsinh}(a+bx)^2}\,dx = \int \mathrm{e}^{\operatorname{asinh}(a+b\,x)^2}\,dx$$

input
```
int(exp(asinh(a + b*x)^2),x)
```

output
```
int(exp(asinh(a + b*x)^2), x)
```

3.362 $\int \frac{e^{\text{arcsinh}(a+bx)^2}}{x} \, dx$

3.362.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{e^{\text{arcsinh}(a+bx)^2}}{x} \, dx = \text{Int}\left(\frac{e^{\text{arcsinh}(a+bx)^2}}{x}, x\right)$$

output
```
CannotIntegrate(exp(arcsinh(b*x+a)^2)/x,x)
```

3.362.2 Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{e^{\text{arcsinh}(a+bx)^2}}{x} \, dx = \int \frac{e^{\text{arcsinh}(a+bx)^2}}{x} \, dx$$

input
```
Integrate[E^ArcSinh[a + b*x]^2/x,x]
```

output
```
Integrate[E^ArcSinh[a + b*x]^2/x, x]
```

3.362.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x}dx$$

input
```
Int[E^ArcSinh[a + b*x]^2/x,x]
```

output
```
$Aborted
```

3.362.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.362.4 Maple [N/A] (verified)

Not integrable

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{\mathrm{e}^{\operatorname{arcsinh}(bx+a)^2}}{x}dx$$

input
```
int(exp(arcsinh(b*x+a)^2)/x,x)
```

output
```
int(exp(arcsinh(b*x+a)^2)/x,x)
```

3.362.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{\text{arcsinh}(a+bx)^2}}{x}\,dx = \int \frac{e^{\left(\text{arsinh}(bx+a)^2\right)}}{x}\,dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2)/x,x, algorithm="fricas")
```

output
```
integral(e^(arcsinh(b*x + a)^2)/x, x)
```

3.362.6 Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^{\text{arcsinh}(a+bx)^2}}{x}\,dx = \int \frac{e^{\text{asinh}^2(a+bx)}}{x}\,dx$$

input
```
integrate(exp(asinh(b*x+a)**2)/x,x)
```

output
```
Integral(exp(asinh(a + b*x)**2)/x, x)
```

3.362.7 Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{\text{arcsinh}(a+bx)^2}}{x}\,dx = \int \frac{e^{\left(\text{arsinh}(bx+a)^2\right)}}{x}\,dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2)/x,x, algorithm="maxima")
```

output
```
integrate(e^(arcsinh(b*x + a)^2)/x, x)
```

3.362.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{\text{arcsinh}(a+bx)^2}}{x}\,dx = \int \frac{e^{\left(\text{arsinh}(bx+a)^2\right)}}{x}\,dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2)/x,x, algorithm="giac")
```

output
```
integrate(e^(arcsinh(b*x + a)^2)/x, x)
```

3.362.9 Mupad [N/A]

Not integrable

Time = 2.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{\text{arcsinh}(a+bx)^2}}{x}\,dx = \int \frac{\mathrm{e}^{\mathrm{asinh}(a+b\,x)^2}}{x}\,dx$$

input
```
int(exp(asinh(a + b*x)^2)/x,x)
```

output
```
int(exp(asinh(a + b*x)^2)/x, x)
```

3.363 $\int \frac{e^{\mathbf{arcsinh}(a+bx)^2}}{x^2} dx$

3.363.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{e^{\text{arcsinh}(a+bx)^2}}{x^2} dx = \text{Int}\left(\frac{e^{\text{arcsinh}(a+bx)^2}}{x^2}, x\right)$$

output
```
CannotIntegrate(exp(arcsinh(b*x+a)^2)/x^2,x)
```

3.363.2 Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{e^{\text{arcsinh}(a+bx)^2}}{x^2} dx = \int \frac{e^{\text{arcsinh}(a+bx)^2}}{x^2} dx$$

input
```
Integrate[E^ArcSinh[a + b*x]^2/x^2,x]
```

output
```
Integrate[E^ArcSinh[a + b*x]^2/x^2, x]
```

3.363.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{arcsinh}(a+bx)^2}}{x^2} \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{e^{\text{arcsinh}(a+bx)^2}}{x^2}dx$$

input
```
Int[E^ArcSinh[a + b*x]^2/x^2,x]
```

output
```
$Aborted
```

3.363.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.363.4 Maple [N/A] (verified)

Not integrable

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{\mathrm{e}^{\text{arcsinh}(bx+a)^2}}{x^2}dx$$

input
```
int(exp(arcsinh(b*x+a)^2)/x^2,x)
```

output
```
int(exp(arcsinh(b*x+a)^2)/x^2,x)
```

3.363.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x^2}\,dx = \int \frac{e^{\left(\operatorname{arsinh}(bx+a)^2\right)}}{x^2}\,dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2)/x^2,x, algorithm="fricas")
```

output
```
integral(e^(arcsinh(b*x + a)^2)/x^2, x)
```

3.363.6 Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x^2}\,dx = \int \frac{e^{\operatorname{asinh}^2(a+bx)}}{x^2}\,dx$$

input
```
integrate(exp(asinh(b*x+a)**2)/x**2,x)
```

output
```
Integral(exp(asinh(a + b*x)**2)/x**2, x)
```

3.363.7 Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x^2}\,dx = \int \frac{e^{\left(\operatorname{arsinh}(bx+a)^2\right)}}{x^2}\,dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2)/x^2,x, algorithm="maxima")
```

output
```
integrate(e^(arcsinh(b*x + a)^2)/x^2, x)
```

3.363.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x^2} \, dx = \int \frac{e^{\left(\operatorname{arsinh}(bx+a)^2\right)}}{x^2} \, dx$$

input
```
integrate(exp(arcsinh(b*x+a)^2)/x^2,x, algorithm="giac")
```

output
```
integrate(e^(arcsinh(b*x + a)^2)/x^2, x)
```

3.363.9 Mupad [N/A]

Not integrable

Time = 2.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{\operatorname{arcsinh}(a+bx)^2}}{x^2} \, dx = \int \frac{e^{\operatorname{asinh}(a+b\,x)^2}}{x^2} \, dx$$

input
```
int(exp(asinh(a + b*x)^2)/x^2,x)
```

output
```
int(exp(asinh(a + b*x)^2)/x^2, x)
```

3.364 $\int \frac{\text{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx$

3.364.1 Optimal result

Integrand size = 19, antiderivative size = 60

$$\int \frac{\text{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx=-\frac{\text{arcsinh}(a+bx)^2}{2d}+\frac{\text{arcsinh}(a+bx)\log\left(1-e^{2\text{arcsinh}(a+bx)}\right)}{d}+\frac{\text{PolyLog}\left(2,e^{2\text{arcsinh}(a+bx)}\right)}{2d}$$

output
```
-1/2*arcsinh(b*x+a)^2/d+arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)
/d+1/2*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/d
```

3.364.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{\text{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{-\text{arcsinh}(a+bx)\left(\text{arcsinh}(a+bx)-2\log\left(1-e^{2\text{arcsinh}(a+bx)}\right)\right)+\text{PolyLog}\left(2,e^{2\text{arcsinh}(a+bx)}\right)}{2d}$$

input
```
Integrate[ArcSinh[a + b*x]/((a*d)/b + d*x),x]
```

output
```
(-(ArcSinh[a + b*x]*(ArcSinh[a + b*x] - 2*Log[1 - E^(2*ArcSinh[a + b*x])])
) + PolyLog[2, E^(2*ArcSinh[a + b*x])])/(2*d)
```

3.364.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {6274, 27, 6190, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}(a+bx)}{\frac{ad}{b}+dx}dx$$

$$\Big\downarrow \; 6274$$

$$\frac{\int \frac{b\text{arcsinh}(a+bx)}{d(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\text{arcsinh}(a+bx)}{a+bx}d(a+bx)}{d}$$

$$\Big\downarrow \; 6190$$

$$\frac{\int \frac{\sqrt{(a+bx)^2+1}\text{arcsinh}(a+bx)}{a+bx}d\text{arcsinh}(a+bx)}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int -i\text{arcsinh}(a+bx)\tan\left(i\text{arcsinh}(a+bx)+\frac{\pi}{2}\right)d\text{arcsinh}(a+bx)}{d}$$

$$\Big\downarrow \; 26$$

$$-\frac{i\int \text{arcsinh}(a+bx)\tan\left(i\text{arcsinh}(a+bx)+\frac{\pi}{2}\right)d\text{arcsinh}(a+bx)}{d}$$

$$\Big\downarrow \; 4199$$

$$-\frac{i\left(2i\int -\frac{e^{2\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{1-e^{2\text{arcsinh}(a+bx)}}d\text{arcsinh}(a+bx)-\frac{1}{2}i\text{arcsinh}(a+bx)^2\right)}{d}$$

$$\Big\downarrow \; 25$$

$$-\frac{i\left(-2i\int \frac{e^{2\text{arcsinh}(a+bx)}\text{arcsinh}(a+bx)}{1-e^{2\text{arcsinh}(a+bx)}}d\text{arcsinh}(a+bx)-\frac{1}{2}i\text{arcsinh}(a+bx)^2\right)}{d}$$

$$\Big\downarrow \; 2620$$

$$-\frac{i\left(-2i\left(\frac{1}{2}\int\log\left(1-e^{2\text{arcsinh}(a+bx)}\right)d\text{arcsinh}(a+bx)-\frac{1}{2}\text{arcsinh}(a+bx)\log\left(1-e^{2\text{arcsinh}(a+bx)}\right)\right)-\frac{1}{2}i\text{arcsinh}(a+\right.}{d}$$

$\big\downarrow$ 2715

$$-\frac{i\left(-2i\left(\frac{1}{4}\int e^{-2\text{arcsinh}(a+bx)}\log(-a-bx+1)de^{2\text{arcsinh}(a+bx)}-\frac{1}{2}\text{arcsinh}(a+bx)\log\left(1-e^{2\text{arcsinh}(a+bx)}\right)\right)-\frac{1}{2}i\text{arcs}\right.}{d}$$

$\big\downarrow$ 2838

$$-\frac{i\left(-2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2\text{arcsinh}(a+bx)}\right)-\frac{1}{2}\text{arcsinh}(a+bx)\log\left(1-e^{2\text{arcsinh}(a+bx)}\right)\right)-\frac{1}{2}i\text{arcsinh}(a+bx)^2\right)}{d}$$

input
```
Int[ArcSinh[a + b*x]/((a*d)/b + d*x),x]
```

output
```
((-I)*((-1/2*I)*ArcSinh[a + b*x]^2 - (2*I)*(-1/2*(ArcSinh[a + b*x]*Log[1 -
 E^(2*ArcSinh[a + b*x])]) - PolyLog[2, E^(2*ArcSinh[a + b*x])]/4)))/d
```

3.364.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6274
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.364.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.23

method	result
derivativedivides	$\frac{-\frac{b \operatorname{arcsinh}(bx+a)^2}{2d}+\frac{b \operatorname{arcsinh}(bx+a)\ln\left(1+bx+a+\sqrt{1+(bx+a)^2}\right)}{d}+\frac{b\operatorname{polylog}\left(2,-bx-a-\sqrt{1+(bx+a)^2}\right)}{d}+\frac{b \operatorname{arcsinh}(bx+a)\ln(1-ba}{d}}{b}$
default	$\frac{-\frac{b \operatorname{arcsinh}(bx+a)^2}{2d}+\frac{b \operatorname{arcsinh}(bx+a)\ln\left(1+bx+a+\sqrt{1+(bx+a)^2}\right)}{d}+\frac{b\operatorname{polylog}\left(2,-bx-a-\sqrt{1+(bx+a)^2}\right)}{d}+\frac{b \operatorname{arcsinh}(bx+a)\ln(1-ba}{d}}{b}$

input
```
int(arcsinh(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*b/d*arcsinh(b*x+a)^2+b/d*arcsinh(b*x+a)*ln(1+b*x+a+(1+(b*x+a)^2)
^(1/2))+b/d*polylog(2,-b*x-a-(1+(b*x+a)^2)^(1/2))+b/d*arcsinh(b*x+a)*ln(1-
b*x-a-(1+(b*x+a)^2)^(1/2))+b/d*polylog(2,b*x+a+(1+(b*x+a)^2)^(1/2)))
```

3.364.5 Fricas [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\operatorname{arsinh}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arcsinh(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

output
```
integral(b*arcsinh(b*x + a)/(b*d*x + a*d), x)
```

3.364.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{b\int \frac{\operatorname{asinh}(a+bx)}{a+bx}\,dx}{d}$$

input
```
integrate(asinh(b*x+a)/(a*d/b+d*x),x)
```

output
```
b*Integral(asinh(a + b*x)/(a + b*x), x)/d
```

3.364.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\operatorname{arsinh}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arcsinh(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

output
```
integrate(arcsinh(b*x + a)/(d*x + a*d/b), x)
```

3.364.8 Giac [F]

$$\int \frac{\text{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\text{arsinh}\,(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arcsinh(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

output
```
integrate(arcsinh(b*x + a)/(d*x + a*d/b), x)
```

3.364.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arcsinh}(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\text{asinh}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

input
```
int(asinh(a + b*x)/(d*x + (a*d)/b),x)
```

output
```
int(asinh(a + b*x)/(d*x + (a*d)/b), x)
```

3.365 $\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)} dx$

3.365.1 Optimal result

Integrand size = 15, antiderivative size = 3

$$\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)} dx = \text{Shi}(\text{arcsinh}(x))$$

output
```
Shi(arcsinh(x))
```

3.365.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)} dx = \text{Shi}(\text{arcsinh}(x))$$

input
```
Integrate[x/(Sqrt[1 + x^2]*ArcSinh[x]),x]
```

output
```
SinhIntegral[ArcSinh[x]]
```

3.365.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6234, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^2+1}\text{arcsinh}(x)}\,dx$$
$$\Big\downarrow \text{6234}$$
$$\int \frac{x}{\text{arcsinh}(x)}d\text{arcsinh}(x)$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{i\sin(i\text{arcsinh}(x))}{\text{arcsinh}(x)}d\text{arcsinh}(x)$$
$$\Big\downarrow \text{26}$$
$$-i\int \frac{\sin(i\text{arcsinh}(x))}{\text{arcsinh}(x)}d\text{arcsinh}(x)$$
$$\Big\downarrow \text{3779}$$
$$\text{Shi}(\text{arcsinh}(x))$$

input
```
Int[x/(Sqrt[1 + x^2]*ArcSinh[x]),x]
```

output
```
SinhIntegral[ArcSinh[x]]
```

3.365.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 6234
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p]   Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
 && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

3.365.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
default	$\operatorname{Shi}(\operatorname{arcsinh}(x))$	4

input
```
int(x/arcsinh(x)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
Shi(arcsinh(x))
```

3.365.5 Fricas [F]

$$\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)}\,dx = \int \frac{x}{\sqrt{x^2+1}\,\text{arsinh}\,(x)}\,dx$$

input
```
integrate(x/arcsinh(x)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(x/(sqrt(x^2 + 1)*arcsinh(x)), x)
```

3.365.6 Sympy [F]

$$\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)}\,dx = \int \frac{x}{\sqrt{x^2+1}\,\text{asinh}\,(x)}\,dx$$

input
```
integrate(x/asinh(x)/(x**2+1)**(1/2),x)
```

output
```
Integral(x/(sqrt(x**2 + 1)*asinh(x)), x)
```

3.365.7 Maxima [F]

$$\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)}\,dx = \int \frac{x}{\sqrt{x^2+1}\,\text{arsinh}\,(x)}\,dx$$

input
```
integrate(x/arcsinh(x)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^2 + 1)*arcsinh(x)), x)
```

3.365.8 Giac [F]

$$\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)}\,dx = \int \frac{x}{\sqrt{x^2+1}\,\text{arsinh}\,(x)}\,dx$$

input
```
integrate(x/arcsinh(x)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(x^2 + 1)*arcsinh(x)), x)
```

3.365.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1+x^2}\text{arcsinh}(x)}\,dx = \int \frac{x}{\text{asinh}\,(x)\,\sqrt{x^2+1}}\,dx$$

input
```
int(x/(asinh(x)*(x^2 + 1)^(1/2)),x)
```

output
```
int(x/(asinh(x)*(x^2 + 1)^(1/2)), x)
```

3.366 $\int x^3 \textbf{arcsinh}(a+bx^4)\,dx$

3.366.1 Optimal result

Integrand size = 12, antiderivative size = 45

$$\int x^3 \text{arcsinh}(a+bx^4)\,dx = -\frac{\sqrt{1+(a+bx^4)^2}}{4b} + \frac{(a+bx^4)\,\text{arcsinh}(a+bx^4)}{4b}$$

output
```
1/4*(b*x^4+a)*arcsinh(b*x^4+a)/b-1/4*(1+(b*x^4+a)^2)^(1/2)/b
```

3.366.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int x^3 \text{arcsinh}(a+bx^4)\,dx = \frac{-\sqrt{1+(a+bx^4)^2} + (a+bx^4)\,\text{arcsinh}(a+bx^4)}{4b}$$

input
```
Integrate[x^3*ArcSinh[a + b*x^4],x]
```

output
```
(-Sqrt[1 + (a + b*x^4)^2] + (a + b*x^4)*ArcSinh[a + b*x^4])/(4*b)
```

3.366.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7266, 6273, 6187, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{arcsinh}\left(a+bx^4\right) \, dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{4}\int \text{arcsinh}\left(bx^4+a\right)dx^4$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int \text{arcsinh}\left(bx^4+a\right)d\left(bx^4+a\right)}{4b}$$

$$\Big\downarrow \text{6187}$$

$$\frac{\left(a+bx^4\right)\text{arcsinh}\left(a+bx^4\right)-\int \frac{bx^4+a}{\sqrt{x^8+1}}d\left(bx^4+a\right)}{4b}$$

$$\Big\downarrow \text{241}$$

$$\frac{\left(a+bx^4\right)\text{arcsinh}\left(a+bx^4\right)-\sqrt{x^8+1}}{4b}$$

input
```
Int[x^3*ArcSinh[a + b*x^4],x]
```

output
```
(-Sqrt[1 + x^8] + (a + b*x^4)*ArcSinh[a + b*x^4])/(4*b)
```

3.366.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.366.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{(b\,x^4+a)\,\mathrm{arcsinh}(b\,x^4+a)-\sqrt{1+(b\,x^4+a)^2}}{4b}$	38
default	$\frac{(b\,x^4+a)\,\mathrm{arcsinh}(b\,x^4+a)-\sqrt{1+(b\,x^4+a)^2}}{4b}$	38

input
```
int(x^3*arcsinh(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*((b*x^4+a)*arcsinh(b*x^4+a)-(1+(b*x^4+a)^2)^(1/2))
```

3.366.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int x^3\mathrm{arcsinh}(a+bx^4)\,dx = \frac{(bx^4+a)\log\left(bx^4+a+\sqrt{b^2x^8+2\,abx^4+a^2+1}\right)-\sqrt{b^2x^8+2\,abx^4+a^2+1}}{4\,b}$$

input
```
integrate(x^3*arcsinh(b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*((b*x^4 + a)*log(b*x^4 + a + sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 + 1)) - sq
rt(b^2*x^8 + 2*a*b*x^4 + a^2 + 1))/b
```

3.366.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int x^3 \text{arcsinh}\left(a+bx^4\right) dx = \begin{cases} \frac{a \operatorname{asinh}\left(a+bx^4\right)}{4b} + \frac{x^4 \operatorname{asinh}\left(a+bx^4\right)}{4} - \frac{\sqrt{a^2+2abx^4+b^2x^8+1}}{4b} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{asinh}(a)}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*asinh(b*x**4+a),x)
```

output
```
Piecewise((a*asinh(a + b*x**4)/(4*b) + x**4*asinh(a + b*x**4)/4 - sqrt(a**
2 + 2*a*b*x**4 + b**2*x**8 + 1)/(4*b), Ne(b, 0)), (x**4*asinh(a)/4, True))
```

3.366.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int x^3 \text{arcsinh}\left(a+bx^4\right) dx = \frac{(bx^4+a) \operatorname{arsinh}\left(bx^4+a\right) - \sqrt{(bx^4+a)^2+1}}{4\,b}$$

input
```
integrate(x^3*arcsinh(b*x^4+a),x, algorithm="maxima")
```

output
```
1/4*((b*x^4 + a)*arcsinh(b*x^4 + a) - sqrt((b*x^4 + a)^2 + 1))/b
```

3.366.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(39) = 78.

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.33

$$\begin{aligned} &\int x^3 \text{arcsinh}\left(a+bx^4\right) dx \\ &= \frac{1}{4}\,x^4 \log\left(bx^4+a+\sqrt{(bx^4+a)^2+1}\right) \\ &\quad - \frac{1}{4}\,b\left(\frac{a \log\left(-ab-\left(x^4|b|-\sqrt{b^2x^8+2\,abx^4+a^2+1}\right)|b|\right)}{b|b|} + \frac{\sqrt{b^2x^8+2\,abx^4+a^2+1}}{b^2}\right) \end{aligned}$$

input
```
integrate(x^3*arcsinh(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*x^4*log(b*x^4 + a + sqrt((b*x^4 + a)^2 + 1)) - 1/4*b*(a*log(-a*b - (x^
4*abs(b) - sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 + 1))*abs(b))/(b*abs(b)) + sqrt(
b^2*x^8 + 2*a*b*x^4 + a^2 + 1)/b^2)
```

3.366.9 Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.96

$$\int x^3 \text{arcsinh}\left(a+bx^4\right)\,dx = \frac{x^4\,\text{asinh}(b\,x^4+a)}{4} - \frac{\sqrt{a^2+2\,a\,b\,x^4+b^2\,x^8+1}}{4\,b} + \frac{a\,\ln\left(\sqrt{a^2+2\,a\,b\,x^4+b^2\,x^8+1}+\frac{b^2\,x^4+a\,b}{\sqrt{b^2}}\right)}{4\,\sqrt{b^2}}$$

input
```
int(x^3*asinh(a + b*x^4),x)
```

output
```
(x^4*asinh(a + b*x^4))/4 - (a^2 + b^2*x^8 + 2*a*b*x^4 + 1)^(1/2)/(4*b) + (
a*log((a^2 + b^2*x^8 + 2*a*b*x^4 + 1)^(1/2) + (a*b + b^2*x^4)/(b^2)^(1/2))
)/(4*(b^2)^(1/2))
```

3.367 $\int x^{-1+n}\text{arcsinh}(a+bx^n)\,dx$

3.367.1 Optimal result

Integrand size = 14, antiderivative size = 46

$$\int x^{-1+n}\text{arcsinh}(a+bx^n)\,dx=-\frac{\sqrt{1+(a+bx^n)^2}}{bn}+\frac{(a+bx^n)\,\text{arcsinh}(a+bx^n)}{bn}$$

output
```
(a+b*x^n)*arcsinh(a+b*x^n)/b/n-(1+(a+b*x^n)^2)^(1/2)/b/n
```

3.367.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int x^{-1+n}\text{arcsinh}(a+bx^n)\,dx=\frac{-\sqrt{1+(a+bx^n)^2}+(a+bx^n)\,\text{arcsinh}(a+bx^n)}{bn}$$

input
```
Integrate[x^(-1 + n)*ArcSinh[a + b*x^n],x]
```

output
```
(-Sqrt[1 + (a + b*x^n)^2] + (a + b*x^n)*ArcSinh[a + b*x^n])/(b*n)
```

3.367.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7266, 6273, 6187, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}\text{arcsinh}(a+bx^n)\,dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{\int \text{arcsinh}(bx^n+a)\,dx^n}{n}$$

$$\Big\downarrow \text{6273}$$

$$\frac{\int \text{arcsinh}(bx^n+a)\,d(bx^n+a)}{bn}$$

$$\Big\downarrow \text{6187}$$

$$\frac{(a+bx^n)\,\text{arcsinh}(a+bx^n)-\int \frac{bx^n+a}{\sqrt{x^{2n}+1}}d(bx^n+a)}{bn}$$

$$\Big\downarrow \text{241}$$

$$\frac{(a+bx^n)\,\text{arcsinh}(a+bx^n)-\sqrt{x^{2n}+1}}{bn}$$

input
```
Int[x^(-1 + n)*ArcSinh[a + b*x^n],x]
```

output
```
(-Sqrt[1 + x^(2*n)] + (a + b*x^n)*ArcSinh[a + b*x^n])/(b*n)
```

3.367.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6273
```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
, n}, x]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.367.4 Maple [F]

$$\int x^{n-1} \operatorname{arcsinh}\left(a+b\,x^{n}\right)dx$$

input
```
int(x^(n-1)*arcsinh(a+b*x^n),x)
```

output
```
int(x^(n-1)*arcsinh(a+b*x^n),x)
```

3.367.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(44) = 88.

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 3.30

$$\int x^{-1+n}\operatorname{arcsinh}(a+bx^n)\,dx$$
$$= \frac{(b\cosh(n\log(x)) + b\sinh(n\log(x)) + a)\log\left(b\cosh(n\log(x)) + b\sinh(n\log(x)) + a + \sqrt{\frac{2ab+(a^2+b^2+1)}{\cos}}\right.}{bn}$$

input
```
integrate(x^(-1+n)*arcsinh(a+b*x^n),x, algorithm="fricas")
```

output
```
((b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)*log(b*cosh(n*log(x)) + b*sinh(n
*log(x)) + a + sqrt((2*a*b + (a^2 + b^2 + 1)*cosh(n*log(x)) - (a^2 - b^2 +
 1)*sinh(n*log(x)))/(cosh(n*log(x)) - sinh(n*log(x))))) - sqrt((2*a*b + (a
^2 + b^2 + 1)*cosh(n*log(x)) - (a^2 - b^2 + 1)*sinh(n*log(x)))/(cosh(n*log
(x)) - sinh(n*log(x)))))/(b*n)
```

3.367.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(34) = 68$.

Time = 9.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int x^{-1+n}\text{arcsinh}(a+bx^n)\,dx = \begin{cases} \log{(x)}\operatorname{asinh}{(a)} & \text{for}\: b=0 \wedge n=0 \\ \frac{xx^{n-1}\operatorname{asinh}{(a)}}{n} & \text{for}\: b=0 \\ \log{(x)}\operatorname{asinh}{(a+b)} & \text{for}\: n=0 \\ \frac{a\operatorname{asinh}{(a+bx^n)}}{bn} + \frac{x^n\operatorname{asinh}{(a+bx^n)}}{n} - \frac{\sqrt{a^2+2abx^n+b^2x^{2n}+1}}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)*asinh(a+b*x**n),x)
```

output
```
Piecewise((log(x)*asinh(a), Eq(b, 0) & Eq(n, 0)), (x*x**(n - 1)*asinh(a)/n
, Eq(b, 0)), (log(x)*asinh(a + b), Eq(n, 0)), (a*asinh(a + b*x**n)/(b*n) +
 x**n*asinh(a + b*x**n)/n - sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n) + 1)/(b
*n), True))
```

3.367.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int x^{-1+n}\text{arcsinh}(a+bx^n)\,dx = \frac{(bx^n+a)\,\text{arsinh}\,(bx^n+a)-\sqrt{(bx^n+a)^2+1}}{bn}$$

input
```
integrate(x^(-1+n)*arcsinh(a+b*x^n),x, algorithm="maxima")
```

output
```
((b*x^n + a)*arcsinh(b*x^n + a) - sqrt((b*x^n + a)^2 + 1))/(b*n)
```

3.367.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(44) = 88.

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.46

$$\int x^{-1+n}\text{arcsinh}(a+bx^n)\,dx = -\frac{b\left(\frac{a\log\left(-ab-\left(x^n|b|-\sqrt{b^2x^{2n}+2abx^n+a^2+1}\right)|b|\right)}{b|b|}+\frac{\sqrt{b^2x^{2n}+2abx^n+a^2+1}}{b^2}\right)-x^n\log\left(bx^n+a+\sqrt{(bx^n+a)^2+1}\right)}{n}$$

input
```
integrate(x^(-1+n)*arcsinh(a+b*x^n),x, algorithm="giac")
```

output
```
-(b*(a*log(-a*b - (x^n*abs(b) - sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 + 1))*a
bs(b))/(b*abs(b)) + sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 + 1)/b^2) - x^n*log
(b*x^n + a + sqrt((b*x^n + a)^2 + 1)))/n
```

3.367.9 Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.15

$$\int x^{-1+n}\text{arcsinh}(a+bx^n)\,dx = \frac{x^n\,\text{asinh}(a+b\,x^n)}{n}-\frac{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n+1}}{b\,n}+\frac{a\,\ln\left(\frac{a\,b+b^2\,x^n}{\sqrt{b^2}}+\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n+1}\right)}{n\,\sqrt{b^2}}$$

input
```
int(x^(n - 1)*asinh(a + b*x^n),x)
```

output
```
(x^n*asinh(a + b*x^n))/n - (a^2 + b^2*x^(2*n) + 2*a*b*x^n + 1)^(1/2)/(b*n)
 + (a*log((a*b + b^2*x^n)/(b^2)^(1/2) + (a^2 + b^2*x^(2*n) + 2*a*b*x^n + 1
)^(1/2)))/(n*(b^2)^(1/2))
```

3.368 $\int \operatorname{arcsinh}\left(\frac{c}{a+bx}\right) dx$

3.368.1 Optimal result

Integrand size = 10, antiderivative size = 49

$$\int \operatorname{arcsinh}\left(\frac{c}{a+bx}\right) dx = \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} + \frac{c\operatorname{arctanh}\left(\sqrt{1+\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}}\right)}{b}$$

output

```
(b*x+a)*arccsch(a/c+b*x/c)/b+c*arctanh((1+1/(a/c+b*x/c)^2)^(1/2))/b
```

3.368.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 357 vs. $2(49) = 98$.

Time = 0.89 (sec) , antiderivative size = 357, normalized size of antiderivative = 7.29

$$\int \operatorname{arcsinh}\left(\frac{c}{a+bx}\right) dx = x\operatorname{arcsinh}\left(\frac{c}{a+bx}\right) + \frac{(a+bx)\sqrt{\frac{a^2+c^2+2abx+b^2x^2}{(a+bx)^2}}\left(a\operatorname{arctanh}\left(\frac{bcx\left(\sqrt{a^2+c^2}-\sqrt{a^2+c^2+2abx+b^2x^2}\right)}{a\left(a^2+c^2+abx-\sqrt{a^2+c^2}\sqrt{a^2+c^2+2abx+b^2x^2}\right)}\right)+a\operatorname{arctanh}\left(\frac{-a^4-a^3bx+b^2c^2x^2}{\cdots}\right.\right.}{\cdots}$$

input

```
Integrate[ArcSinh[c/(a + b*x)],x]
```

output
```
x*ArcSinh[c/(a + b*x)] + ((a + b*x)*Sqrt[(a^2 + c^2 + 2*a*b*x + b^2*x^2)/(
a + b*x)^2]*(a*ArcTanh[(b*c*x*(Sqrt[a^2 + c^2] - Sqrt[a^2 + c^2 + 2*a*b*x
+ b^2*x^2]))/(a*(a^2 + c^2 + a*b*x - Sqrt[a^2 + c^2]*Sqrt[a^2 + c^2 + 2*a*
b*x + b^2*x^2]))] + a*ArcTanh[(-a^4 - a^3*b*x + b^2*c^2*x^2 + a^2*(-c^2 +
Sqrt[a^2 + c^2]*Sqrt[a^2 + c^2 + 2*a*b*x + b^2*x^2]))/(b^2*c*Sqrt[a^2 + c^
2]*x^2)] + c*(Log[Sqrt[a^2 + c^2] - b*x - Sqrt[a^2 + c^2 + 2*a*b*x + b^2*x
^2]] - Log[b^2*(Sqrt[a^2 + c^2] + b*x - Sqrt[a^2 + c^2 + 2*a*b*x + b^2*x^2
])])))/(b*Sqrt[a^2 + c^2 + 2*a*b*x + b^2*x^2])
```

3.368.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6285, 6868, 895, 798, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}\left(\frac{c}{a+bx}\right) dx$$

$$\Big\downarrow \text{ 6285}$$

$$\int \text{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right) dx$$

$$\Big\downarrow \text{ 6868}$$

$$\int \frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)\sqrt{1+\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}}}dx+\frac{(a+bx)\text{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}$$

$$\Big\downarrow \text{ 895}$$

$$\frac{c\int \frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)\sqrt{1+\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}}}d\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}+\frac{(a+bx)\text{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{(a+bx)\text{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{c\int \frac{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}{\sqrt{1+\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}}}d\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}}{2b}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(a+bx)\text{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{c\int \frac{1}{\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^4}-1}d\sqrt{1+\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}}}{b}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{c\,\text{arctanh}\left(\sqrt{\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}+1}\right)}{b}+\frac{(a+bx)\text{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}$$

input
```
Int[ArcSinh[c/(a + b*x)],x]
```

output
```
((a + b*x)*ArcCsch[a/c + (b*x)/c])/b + (c*ArcTanh[Sqrt[1 + (a/c + (b*x)/c)
^(-2)]])/b
```

3.368.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 6285
```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

rule 6868
```
Int[ArcCsch[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcCsch[c + d*
x]/d), x] + Int[1/((c + d*x)*Sqrt[1 + 1/(c + d*x)^2]), x] /; FreeQ[{c, d},
x]
```

3.368.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{c\left(-\frac{(bx+a)\,\mathrm{arcsinh}\left(\frac{c}{bx+a}\right)}{c}-\mathrm{arctanh}\left(\frac{1}{\sqrt{\frac{c^2}{(bx+a)^2}+1}}\right)\right)}{b}$
default	$-\frac{c\left(-\frac{(bx+a)\,\mathrm{arcsinh}\left(\frac{c}{bx+a}\right)}{c}-\mathrm{arctanh}\left(\frac{1}{\sqrt{\frac{c^2}{(bx+a)^2}+1}}\right)\right)}{b}$
parts	$x\ \mathrm{arcsinh}\left(\frac{c}{bx+a}\right)+\frac{c\sqrt{b^2x^2+2abx+a^2+c^2}\left(\ln\left(\frac{b^2x+\sqrt{b^2x^2+2abx+a^2+c^2}\,\sqrt{b^2}+ab}{\sqrt{b^2}}\right)b\sqrt{c^2}+a\ln\left(\frac{2\left(\sqrt{c^2}\,\sqrt{b^2x^2+2a\cdots}\right.}{bx+\cdots}\right.\right.}{b\sqrt{\frac{b^2x^2+2abx+a^2+c^2}{(bx+a)^2}}\,(bx+a)\sqrt{b^2}\,\sqrt{c^2}}$

input
```
int(arcsinh(c/(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/b*c*(-1/c*(b*x+a)*arcsinh(c/(b*x+a))-arctanh(1/(c^2/(b*x+a)^2+1)^(1/2))
)
```

3.368.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(47) = 94.

Time = 0.27 (sec) , antiderivative size = 242, normalized size of antiderivative = 4.94

$$\int \mathrm{arcsinh}\left(\frac{c}{a+bx}\right)\,dx$$
$$=\frac{bx\log\left(\frac{(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+c^2}{b^2x^2+2\,abx+a^2}}+c}{bx+a}\right)+a\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+c^2}{b^2x^2+2\,abx+a^2}}-a+c\right)-a\log\left(-bx+(bx\cdots\right.}{b}$$

input
```
integrate(arcsinh(c/(b*x+a)),x, algorithm="fricas")
```

output
```
(b*x*log(((b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + c^2)/(b^2*x^2 + 2*a*b*
x + a^2)) + c)/(b*x + a)) + a*log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x
 + a^2 + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - a + c) - a*log(-b*x + (b*x + a)
*sqrt((b^2*x^2 + 2*a*b*x + a^2 + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - a - c)
- c*log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + c^2)/(b^2*x^2 + 2
*a*b*x + a^2)) - a))/b
```

3.368.6 Sympy [F]

$$\int \operatorname{arcsinh}\left(\frac{c}{a+bx}\right) dx = \int \operatorname{asinh}\left(\frac{c}{a+bx}\right) dx$$

input
```
integrate(asinh(c/(b*x+a)),x)
```

output
```
Integral(asinh(c/(a + b*x)), x)
```

3.368.7 Maxima [F]

$$\int \operatorname{arcsinh}\left(\frac{c}{a+bx}\right) dx = \int \operatorname{arsinh}\left(\frac{c}{bx+a}\right) dx$$

input
```
integrate(arcsinh(c/(b*x+a)),x, algorithm="maxima")
```

output
```
-1/2*I*c*(log(I*(b^2*x + a*b)/(b*c) + 1) - log(-I*(b^2*x + a*b)/(b*c) + 1)
)/b + 1/2*(2*b*x*log(c + sqrt(b^2*x^2 + 2*a*b*x + a^2 + c^2)) + a*log(b^2*
x^2 + 2*a*b*x + a^2 + c^2) - 2*(b*x + a)*log(b*x + a))/b + integrate((b^2*
c*x^2 + a*b*c*x)/(b^2*c*x^2 + 2*a*b*c*x + a^2*c + c^3 + (b^2*x^2 + 2*a*b*x
 + a^2 + c^2)^(3/2)), x)
```

3.368.8 Giac [F]

$$\int \operatorname{arcsinh}\left(\frac{c}{a+bx}\right)\,dx = \int \operatorname{arsinh}\left(\frac{c}{bx+a}\right)\,dx$$

input
```
integrate(arcsinh(c/(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arcsinh(c/(b*x + a)), x)
```

3.368.9 Mupad [B] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \operatorname{arcsinh}\left(\frac{c}{a+bx}\right)\,dx = \frac{c\,\operatorname{atanh}\left(\sqrt{\frac{c^2}{(a+b\,x)^2}+1}\right)}{b} + \frac{\operatorname{asinh}\left(\frac{c}{a+b\,x}\right)\,(a+b\,x)}{b}$$

input
```
int(asinh(c/(a + b*x)),x)
```

output
```
(c*atanh((c^2/(a + b*x)^2 + 1)^(1/2)))/b + (asinh(c/(a + b*x))*(a + b*x))/
b
```

3.369 $\int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx$

3.369.1 Optimal result

Integrand size = 7, antiderivative size = 27

$$\int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx = \operatorname{arcsinh}(\sinh(x)) + \log(\operatorname{arcsinh}(\sinh(x)))\left(-\operatorname{arcsinh}(\sinh(x)) + x\sqrt{\cosh^2(x)}\operatorname{sech}(x)\right)$$

output
```
arcsinh(sinh(x))+ln(arcsinh(sinh(x)))*(-arcsinh(sinh(x))+x*sech(x)*(cosh(x
)^2)^(1/2))
```

3.369.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 148 vs. 2(27) = 54.

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 5.48

$$\int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx = -\log\left(\frac{1}{2}e^{-x}\left(-1+e^{2x}+e^x\sqrt{2+e^{-2x}+e^{2x}}\right)\right)\left(-1 + \log\left(\log\left(\frac{1}{2}e^{-x}\left(-1+e^{2x}+e^x\sqrt{2+e^{-2x}+e^{2x}}\right)\right)\right)\right) + \frac{e^x\sqrt{2+e^{-2x}+e^{2x}}\,x\log\left(\log\left(\frac{1}{2}e^{-x}\left(-1+e^{2x}+e^x\sqrt{2+e^{-2x}+e^{2x}}\right)\right)\right)}{1+e^{2x}}$$

input
```
Integrate[x/ArcSinh[Sinh[x]],x]
```

output
```
-(Log[(-1 + E^(2*x) + E^x*Sqrt[2 + E^(-2*x) + E^(2*x)])/(2*E^x)]*(-1 + Log
[Log[(-1 + E^(2*x) + E^x*Sqrt[2 + E^(-2*x) + E^(2*x)])/(2*E^x)]])) + (E^x*
Sqrt[2 + E^(-2*x) + E^(2*x)]*x*Log[Log[(-1 + E^(2*x) + E^x*Sqrt[2 + E^(-2*
x) + E^(2*x)])/(2*E^x)]])/(1 + E^(2*x))
```

3.369.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\text{arcsinh}(\sinh(x))} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{x}{\text{arcsinh}(\sinh(x))}dx$$

input
```
Int[x/ArcSinh[Sinh[x]],x]
```

output
```
$Aborted
```

3.369.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.369.4 Maple [F]

$$\int \frac{x}{\text{arcsinh}\left(\sinh\left(x\right)\right)}dx$$

input
```
int(x/arcsinh(sinh(x)),x)
```

output
```
int(x/arcsinh(sinh(x)),x)
```

3.369.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.04

$$\int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx = x$$

input
```
integrate(x/arcsinh(sinh(x)),x, algorithm="fricas")
```

output
```
x
```

3.369.6 Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx = \int \frac{x}{\operatorname{asinh}\left(\sinh\left(x\right)\right)}\,dx$$

input
```
integrate(x/asinh(sinh(x)),x)
```

output
```
Integral(x/asinh(sinh(x)), x)
```

3.369.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.04

$$\int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx = x$$

input
```
integrate(x/arcsinh(sinh(x)),x, algorithm="maxima")
```

output
```
x
```

3.369.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.04

$$\int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx = x$$

input
```
integrate(x/arcsinh(sinh(x)),x, algorithm="giac")
```

output
```
x
```

3.369.9 Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.04

$$\int \frac{x}{\operatorname{arcsinh}(\sinh(x))}\,dx = x$$

input
```
int(x/asinh(sinh(x)),x)
```

output
```
x
```

3.370 $\int \frac{\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx$

3.370.1 Optimal result

Integrand size = 26, antiderivative size = 37

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx=\frac{\sqrt{bx^2}\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^{1+n}}{b(1+n)x}$$

output
```
arcsinh((b*x^2-1)^(1/2))^(1+n)*(b*x^2)^(1/2)/b/(1+n)/x
```

3.370.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx=\frac{\sqrt{bx^2}\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^{1+n}}{b(1+n)x}$$

input
```
Integrate[ArcSinh[Sqrt[-1 + b*x^2]]^n/Sqrt[-1 + b*x^2],x]
```

output
```
(Sqrt[b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]^(1 + n))/(b*(1 + n)*x)
```

3.370.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6286, 6198}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arcsinh}\left(\sqrt{bx^2-1}\right)^n}{\sqrt{bx^2-1}}dx$$

$$\Big\downarrow \text{6286}$$

$$\frac{\sqrt{bx^2}\int \frac{\text{arcsinh}\left(\sqrt{bx^2-1}\right)^n}{\sqrt{bx^2}}d\sqrt{bx^2-1}}{bx}$$

$$\Big\downarrow \text{6198}$$

$$\frac{\sqrt{bx^2}\text{arcsinh}\left(\sqrt{bx^2-1}\right)^{n+1}}{b(n+1)x}$$

input
```
Int[ArcSinh[Sqrt[-1 + b*x^2]]^n/Sqrt[-1 + b*x^2],x]
```

output
```
(Sqrt[b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]^(1 + n))/(b*(1 + n)*x)
```

3.370.3.1 Defintions of rubi rules used

rule 6198
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

rule 6286
```
Int[ArcSinh[Sqrt[-1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[-1 + (b_.)*(x_)^2], x_Symbo
l] :> Simp[Sqrt[b*x^2]/(b*x)   Subst[Int[ArcSinh[x]^n/Sqrt[1 + x^2], x], x,
 Sqrt[-1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

3.370.4 Maple [F]

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{b\,x^{2}-1}\right)^{n}}{\sqrt{b\,x^{2}-1}}dx$$

input
```
int(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x)
```

output
```
int(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x)
```

3.370.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(33) = 66$.

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.92

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx = \frac{\sqrt{bx^2}\cosh\left(n\log\left(\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\right)\right)\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)+\sqrt{bx^2}\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\sin}{(bn+b)x}$$

input
```
integrate(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x, algorithm="fricas"
)
```

output
```
(sqrt(b*x^2)*cosh(n*log(log(sqrt(b*x^2 - 1) + sqrt(b*x^2))))*log(sqrt(b*x^
2 - 1) + sqrt(b*x^2)) + sqrt(b*x^2)*log(sqrt(b*x^2 - 1) + sqrt(b*x^2))*sin
h(n*log(log(sqrt(b*x^2 - 1) + sqrt(b*x^2)))))/((b*n + b)*x)
```

3.370.6 Sympy [F]

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx = \begin{cases} -\frac{2x}{\pi} & \text{for}\: b = 0 \wedge n = -1 \\ -ix\left(\frac{i\pi}{2}\right)^n & \text{for}\: b = 0 \\ \int \frac{1}{\sqrt{bx^2-1}\operatorname{asinh}\left(\sqrt{bx^2-1}\right)}\,dx & \text{for}\: n = -1 \\ \frac{\sqrt{bx^2}\operatorname{asinh}\left(\sqrt{bx^2-1}\right)\operatorname{asinh}^{n}\left(\sqrt{bx^2-1}\right)}{bnx+bx} & \text{otherwise} \end{cases}$$

input
```
integrate(asinh((b*x**2-1)**(1/2))**n/(b*x**2-1)**(1/2),x)
```

output
```
Piecewise((-2*x/pi, Eq(b, 0) & Eq(n, -1)), (-I*x*(I*pi/2)**n, Eq(b, 0)), (
Integral(1/(sqrt(b*x**2 - 1)*asinh(sqrt(b*x**2 - 1))), x), Eq(n, -1)), (sq
rt(b*x**2)*asinh(sqrt(b*x**2 - 1))*asinh(sqrt(b*x**2 - 1))**n/(b*n*x + b*x
), True))
```

3.370.7 Maxima [F]

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx = \int \frac{\operatorname{arsinh}\left(\sqrt{bx^2-1}\right)^n}{\sqrt{bx^2-1}}\,dx$$

input
```
integrate(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(arcsinh(sqrt(b*x^2 - 1))^n/sqrt(b*x^2 - 1), x)
```

3.370.8 Giac [F]

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx = \int \frac{\operatorname{arsinh}\left(\sqrt{bx^2-1}\right)^n}{\sqrt{bx^2-1}}\,dx$$

input
```
integrate(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.370.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx = \int \frac{\operatorname{asinh}\left(\sqrt{b\,x^2-1}\right)^n}{\sqrt{b\,x^2-1}}\,dx$$

input
```
int(asinh((b*x^2 - 1)^(1/2))^n/(b*x^2 - 1)^(1/2),x)
```

output
```
int(asinh((b*x^2 - 1)^(1/2))^n/(b*x^2 - 1)^(1/2), x)
```

3.371 $\int \frac{1}{\sqrt{-1+bx^2}\text{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx$

3.371.1 Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{1}{\sqrt{-1+bx^2}\text{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx=\frac{\sqrt{bx^2}\log\left(\text{arcsinh}\left(\sqrt{-1+bx^2}\right)\right)}{bx}$$

output
```
ln(arcsinh((b*x^2-1)^(1/2)))*(b*x^2)^(1/2)/b/x
```

3.371.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-1+bx^2}\text{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx=\frac{x\log\left(\text{arcsinh}\left(\sqrt{-1+bx^2}\right)\right)}{\sqrt{bx^2}}$$

input
```
Integrate[1/(Sqrt[-1 + b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]),x]
```

output
```
(x*Log[ArcSinh[Sqrt[-1 + b*x^2]]])/Sqrt[b*x^2]
```

3.371.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {6286, 6197}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx^2-1}\text{arcsinh}\left(\sqrt{bx^2-1}\right)}dx$$

$$\Big\downarrow \text{6286}$$

$$\frac{\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}\text{arcsinh}\left(\sqrt{bx^2-1}\right)}d\sqrt{bx^2-1}}{bx}$$

$$\Big\downarrow \text{6197}$$

$$\frac{\sqrt{bx^2}\log\left(\text{arcsinh}\left(\sqrt{bx^2-1}\right)\right)}{bx}$$

input
```
Int[1/(Sqrt[-1 + b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]),x]
```

output
```
(Sqrt[b*x^2]*Log[ArcSinh[Sqrt[-1 + b*x^2]]])/(b*x)
```

3.371.3.1 Defintions of rubi rules used

rule 6197
```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*Log[a + b*A
rcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 6286
```
Int[ArcSinh[Sqrt[-1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[-1 + (b_.)*(x_)^2], x_Symbo
l] :> Simp[Sqrt[b*x^2]/(b*x)   Subst[Int[ArcSinh[x]^n/Sqrt[1 + x^2], x], x,
 Sqrt[-1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

3.371.4 Maple [F]

$$\int \frac{1}{\operatorname{arcsinh}\left(\sqrt{b\,x^2-1}\right)\sqrt{b\,x^2-1}}dx$$

input
```
int(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x)
```

output
```
int(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x)
```

3.371.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{-1+bx^2}\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx = \frac{\sqrt{bx^2}\log\left(\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\right)}{bx}$$

input
```
integrate(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x, algorithm="fricas"
)
```

output
```
sqrt(b*x^2)*log(log(sqrt(b*x^2 - 1) + sqrt(b*x^2)))/(b*x)
```

3.371.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+bx^2}\operatorname{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx = \int \frac{1}{\sqrt{bx^2-1}\operatorname{asinh}\left(\sqrt{bx^2-1}\right)}\,dx$$

input
```
integrate(1/asinh((b*x**2-1)**(1/2))/(b*x**2-1)**(1/2),x)
```

output
```
Integral(1/(sqrt(b*x**2 - 1)*asinh(sqrt(b*x**2 - 1))), x)
```

3.371.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1+bx^2}\text{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx = \int \frac{1}{\sqrt{bx^2-1}\,\text{arsinh}\left(\sqrt{bx^2-1}\right)}\,dx$$

input
```
integrate(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(b*x^2 - 1)*arcsinh(sqrt(b*x^2 - 1))), x)
```

3.371.8 Giac [F]

$$\int \frac{1}{\sqrt{-1+bx^2}\text{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx = \int \frac{1}{\sqrt{bx^2-1}\,\text{arsinh}\left(\sqrt{bx^2-1}\right)}\,dx$$

input
```
integrate(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.371.9 Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{-1+bx^2}\text{arcsinh}\left(\sqrt{-1+bx^2}\right)}\,dx = \frac{\ln\left(\text{asinh}\left(\sqrt{b\,x^2-1}\right)\right)\,\sqrt{x^2}}{\sqrt{b}\,x}$$

input
```
int(1/(asinh((b*x^2 - 1)^(1/2))*(b*x^2 - 1)^(1/2)),x)
```

output
```
(log(asinh((b*x^2 - 1)^(1/2)))*(x^2)^(1/2))/(b^(1/2)*x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```