Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/197-7.3.7-Inverse-hyperbolic-tangent-functions

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [361]. This is test number [197].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (361)	0.00 (0)
Mathematica	99.45 (359)	0.55 (2)
Fricas	96.95 (350)	3.05 (11)
Maple	94.74 (342)	5.26 (19)
Maxima	74.24 (268)	25.76 (93)
Giac	72.30 (261)	27.70 (100)
Mupad	66.20 (239)	33.80 (122)
Sympy	27.42 (99)	72.58 (262)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

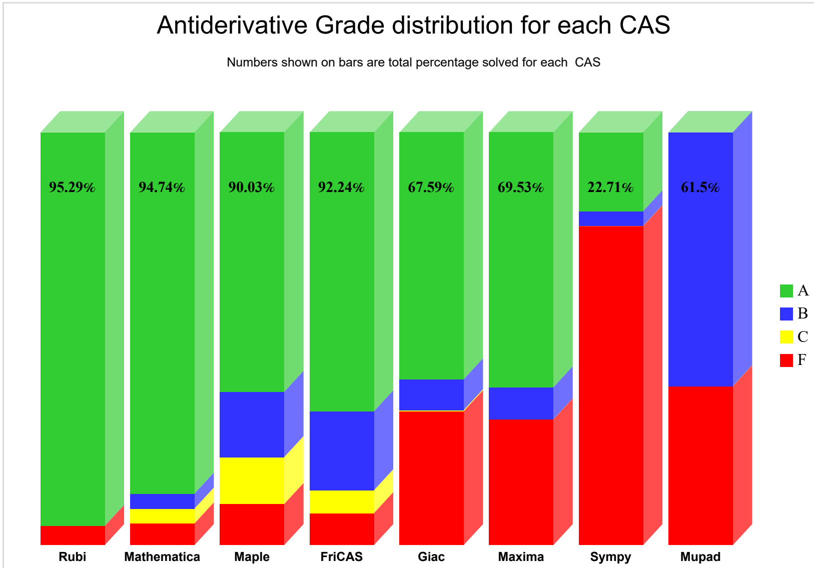
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

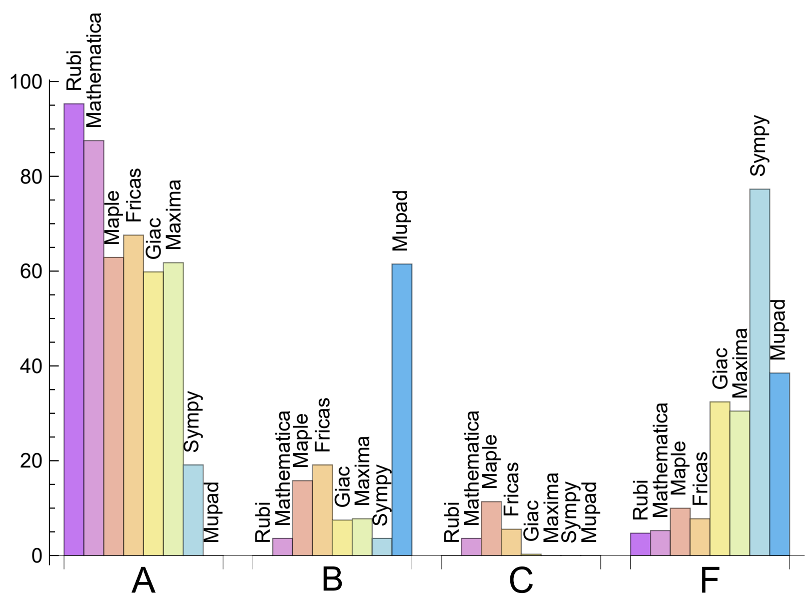
System	% A grade	% B grade	% C grade	% F grade
Rubi	93.906	0.277	1.108	4.709
Mathematica	87.535	3.601	3.601	5.263
Fricas	67.590	19.114	5.540	7.756
Maple	62.881	15.789	11.357	9.972
Maxima	61.773	7.756	0.000	30.471
Giac	59.834	7.479	0.277	32.410
Sympy	19.114	3.601	0.000	77.285
Mupad	0.000	61.496	0.000	38.504

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	2	100.00	0.00	0.00
Fricas	11	100.00	0.00	0.00
Maple	19	100.00	0.00	0.00
Maxima	93	100.00	0.00	0.00
Giac	100	99.00	0.00	1.00
Mupad	122	0.00	100.00	0.00
Sympy	262	88.55	11.07	0.38

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.25
Rubi	0.36
Mathematica	0.41
Maxima	0.46
Giac	2.31
Maple	2.65
Mupad	4.26
Sympy	6.23

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	50.79	1.43	48.00	1.01
Giac	61.26	0.96	46.00	0.64
Maxima	75.48	1.08	54.00	0.86
Mathematica	93.79	1.04	71.00	0.89
Rubi	109.37	1.09	88.00	1.04
Fricas	159.67	1.38	73.00	1.02
Mupad	367.64	4.36	211.00	3.17
Maple	411.07	2.81	95.00	1.03

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

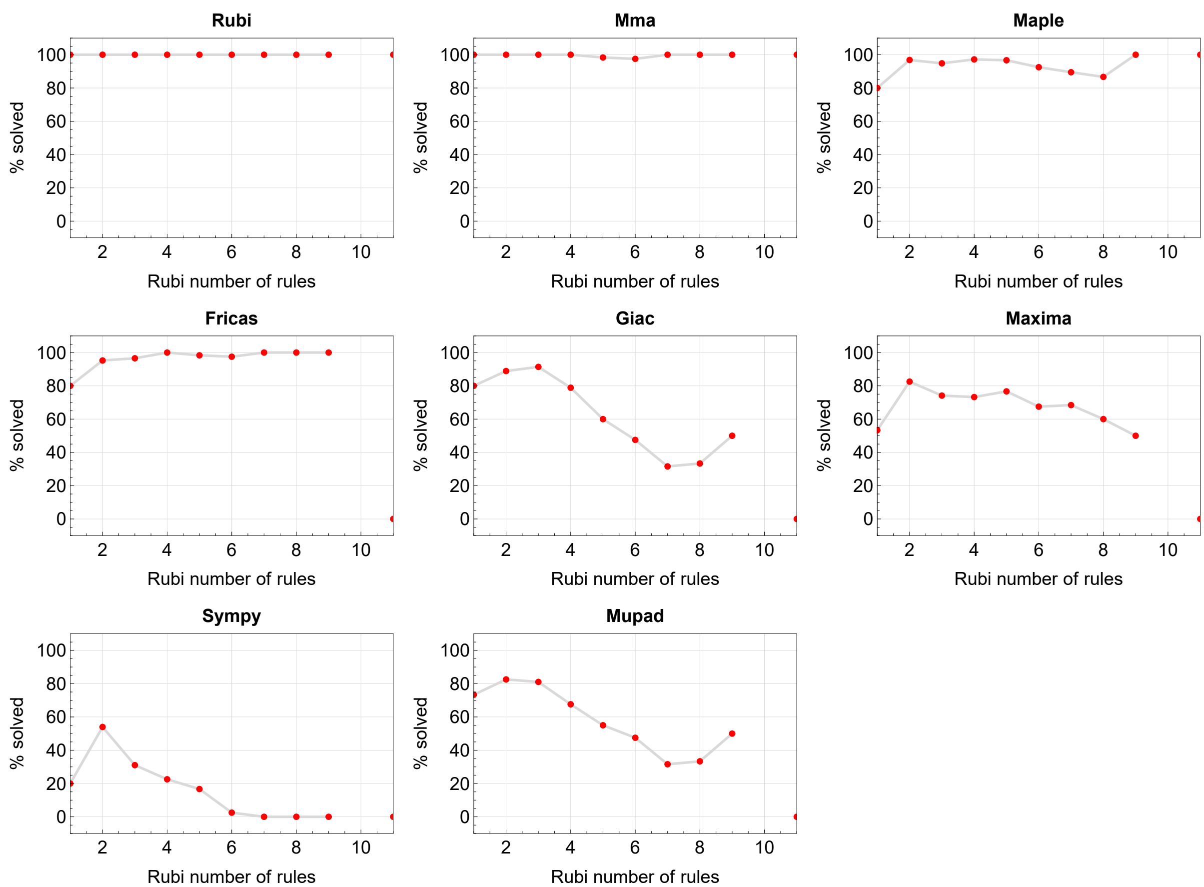


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

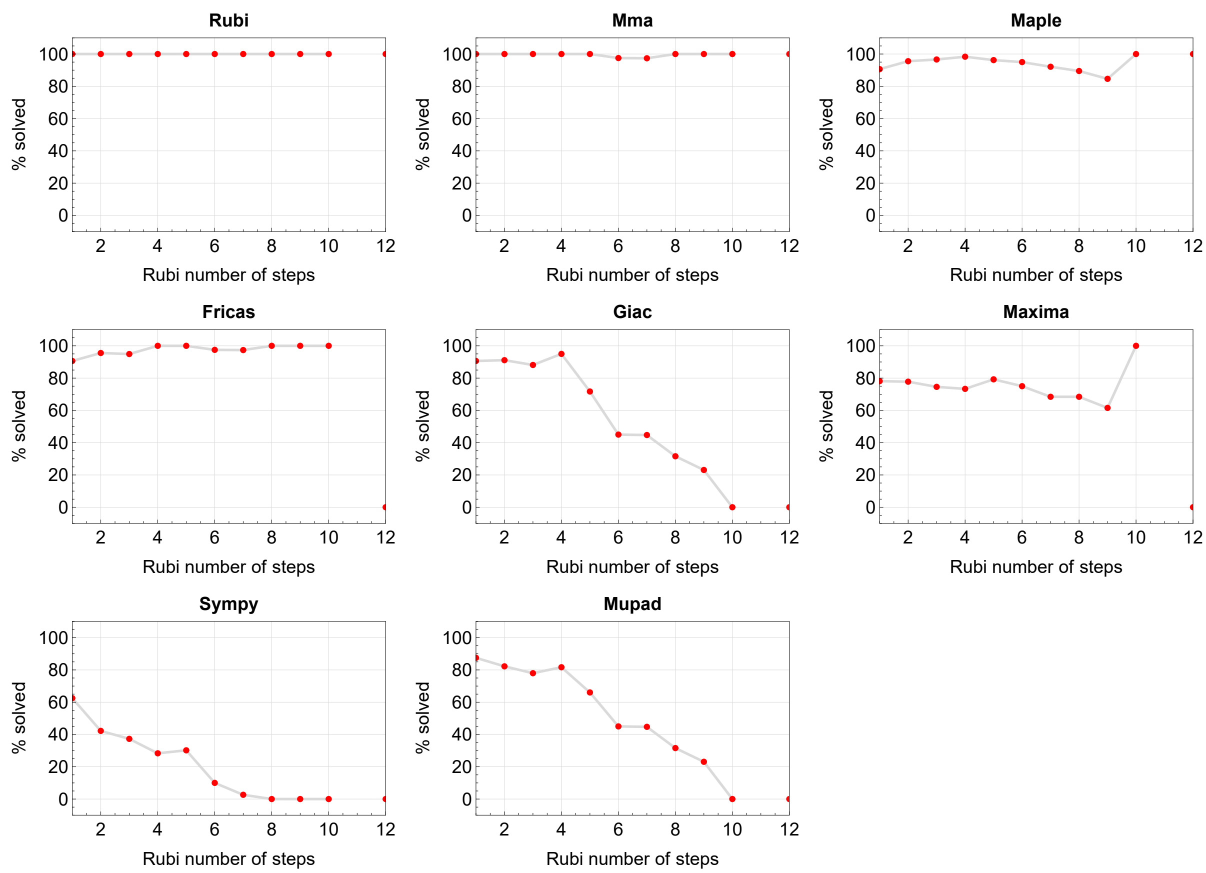


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

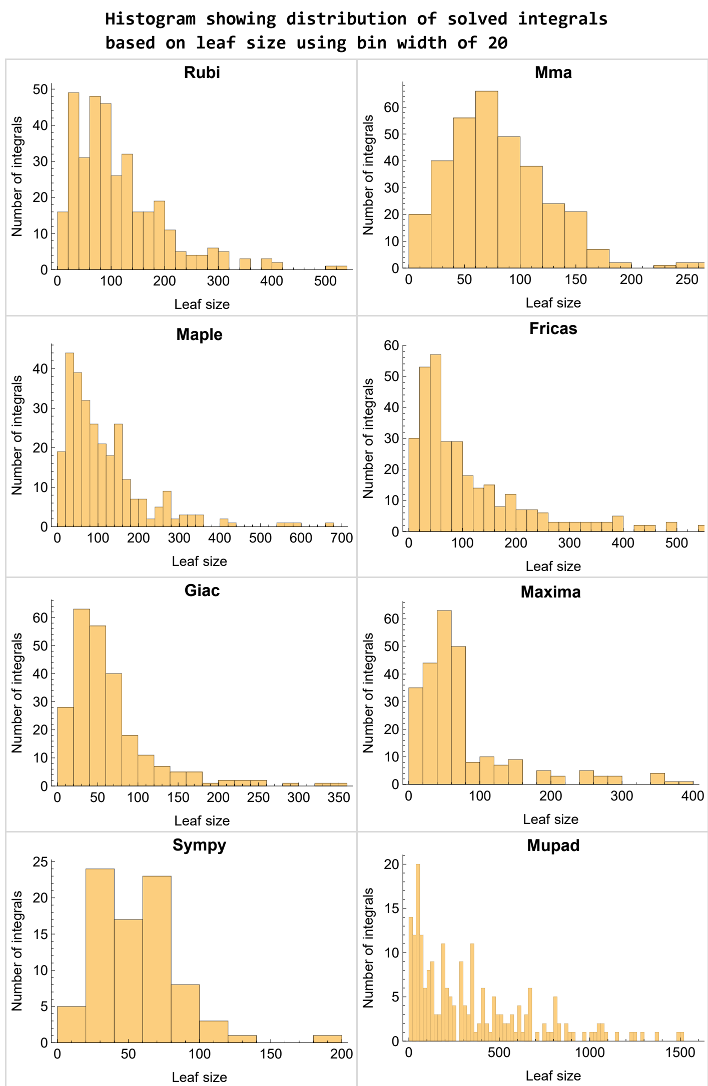


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

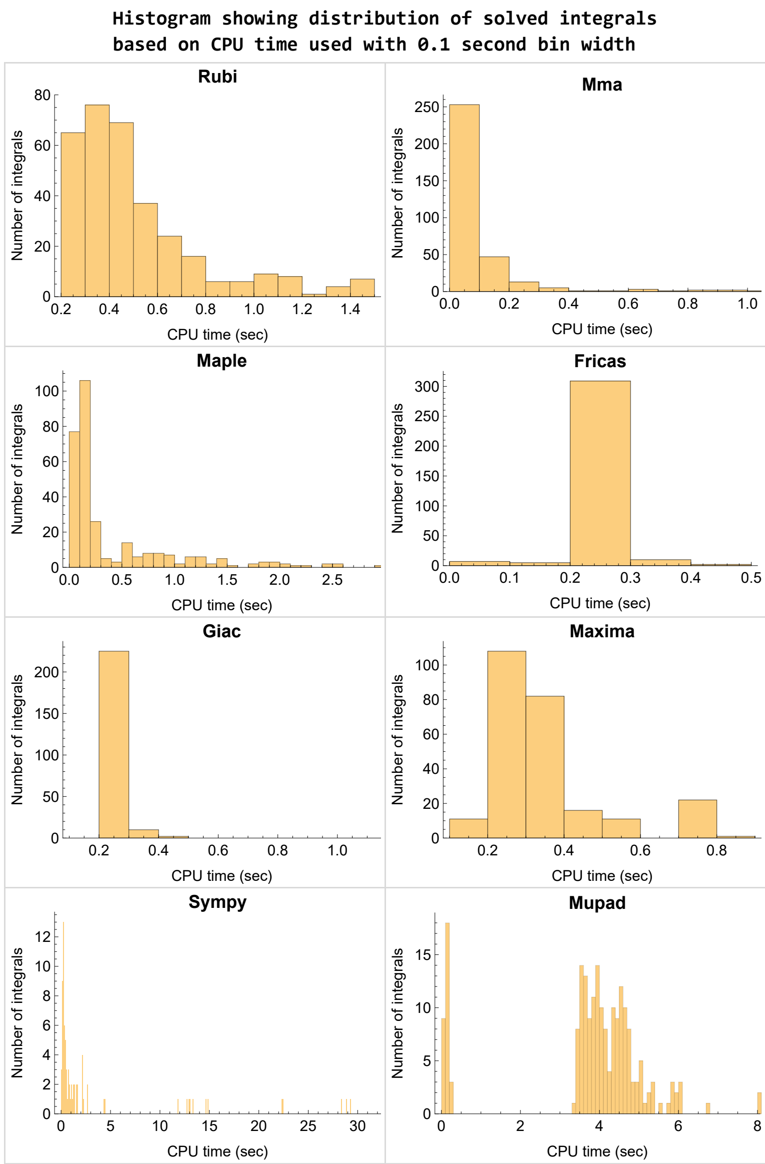


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

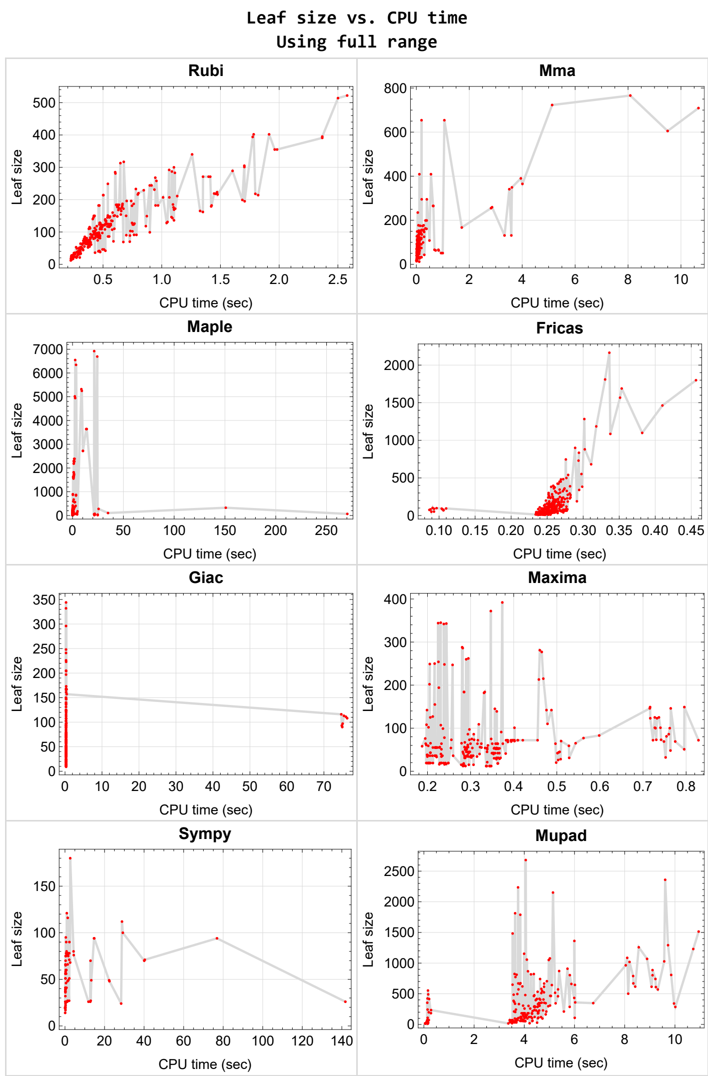


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{30, 34, 35, 287, 292, 297, 301, 306, 311, 316, 320, 324, 328, 333, 337, 341, 345\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{28, 29, 355, 360\}$

Mathematica $\{319, 323, 327, 336, 340, 344\}$

Maple $\{282, 283, 284, 285, 288, 289, 290, 293, 294, 295, 298, 299, 302, 303, 304, 307, 308, 309, 312, 313, 314, 317, 318, 321, 322, 325, 326, 329, 330, 331, 334, 335, 338, 339, 342, 343, 355, 356, 359, 360, 361\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the **Timofeev** test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

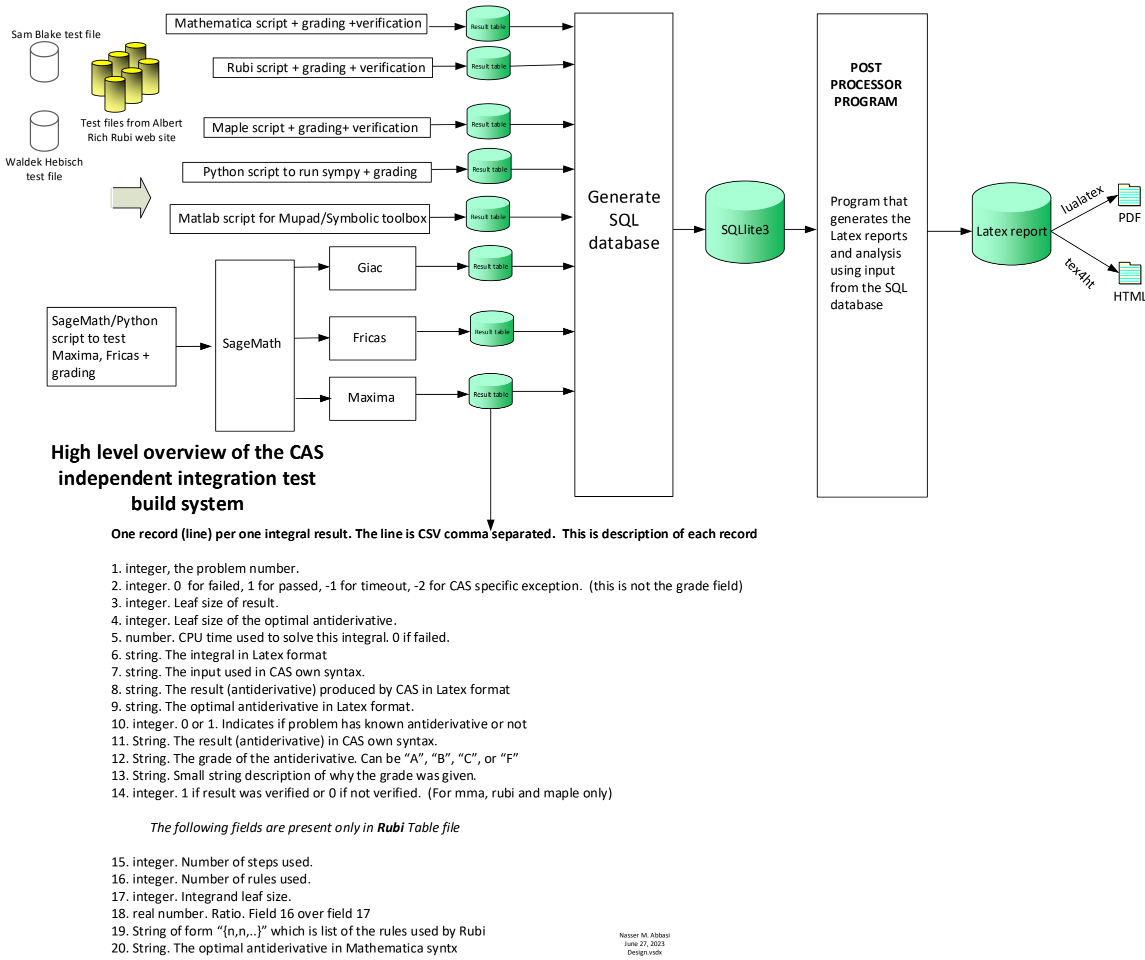
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 298, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 338, 339, 340, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361 }

B grade { 81 }

C grade { 4, 281, 282, 283 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 28, 29, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 298, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 313, 314, 315, 317, 318, 319, 321, 322, 325, 326, 330, 331, 332, 334, 335, 338, 339, 342, 343, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360 }

B grade { 47, 57, 71, 78, 84, 312, 323, 327, 329, 336, 340, 344, 346 }

C grade { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 361 }

F normal fail { 31, 32 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 4, 6, 7, 8, 13, 14, 28, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 74, 75, 76, 77, 79, 80, 81, 82, 83, 87, 88, 89, 90, 91, 92, 96, 97, 98, 99, 100, 101, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 268, 269, 270, 274, 275, 276, 277, 278, 279, 280, 281, 346, 347, 348, 349, 352, 357, 358 }

B grade { 1, 2, 3, 5, 9, 10, 11, 12, 15, 29, 31, 32, 33, 71, 73, 78, 84, 86, 94, 95, 103, 104, 134, 149, 150, 151, 158, 159, 191, 192, 199, 207, 218, 227, 235, 236, 260, 266, 267, 286, 291, 296, 300, 305, 310, 315, 319, 323, 327, 332, 336, 340, 344, 350, 351, 353, 354 }

C grade { 282, 283, 284, 285, 288, 289, 290, 293, 294, 295, 298, 299, 302, 303, 304, 307, 308, 309, 312, 313, 314, 317, 318, 321, 322, 325, 326, 329, 330, 331, 334, 335, 338, 339, 342, 343, 355, 356, 359, 360, 361 }

F normal fail { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 85, 93, 102, 265, 271, 272, 273 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 28, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 269, 270, 283, 338, 339, 340, 342, 343, 344, 352, 354, 356, 357, 359 }

B grade { 13, 14, 15, 29, 44, 54, 58, 65, 72, 84, 133, 162, 266, 267, 268, 281, 282, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 298, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 346, 347, 348, 349, 350, 351, 353, 355, 360, 361 }

C grade { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 274, 275, 276, 277, 278, 279, 280, 358 }

F normal fail { 4, 31, 32, 33, 85, 93, 102, 265, 271, 272, 273 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 5, 6, 7, 8, 9, 10, 11, 28, 29, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 123, 124, 129, 130, 131, 132, 133, 140, 141, 142, 143, 144, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 218, 219, 220, 221, 222, 228, 229, 230, 231, 237, 238, 239, 240, 245, 246, 247, 248, 254, 255, 256, 262, 263, 264, 266, 267, 268, 269, 270, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 298, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 348, 350, 351, 352, 353, 354, 356, 357, 358, 359 }

B grade { 12, 48, 58, 71, 72, 78, 84, 315, 319, 321, 322, 323, 325, 326, 327, 332, 336, 338, 339, 340, 342, 343, 344, 346, 347, 349, 355, 360 }

C grade { }

F normal fail { 1, 2, 3, 4, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 85, 93, 102, 116, 117, 118, 119, 125, 126, 127, 128, 134, 135, 136, 137, 138, 139, 145, 146, 147, 148, 154, 155, 156, 157, 163, 164, 165, 166, 215, 216, 217, 223, 224, 225, 226, 227, 232, 233, 234, 235, 236, 241, 242, 243, 244, 249, 250, 251, 252, 253, 257, 258, 259, 260, 261, 265, 271, 272, 273, 312, 313, 314, 317, 318, 329, 330, 331, 334, 335, 361 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { 12, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 125, 126, 127, 128, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 267, 268, 269, 270, 355, 357, 358, 360 }

B grade { 28, 29, 58, 72, 84, 113, 114, 120, 121, 122, 123, 124, 129, 130, 131, 132, 133, 219, 266, 274, 275, 276, 277, 279, 280, 356, 359 }

C grade { 278 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 44, 54, 65, 85, 93, 102, 265, 271, 272, 273, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 298, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 338, 339, 340, 342, 343, 344, 346, 347, 348, 349, 350, 351, 353, 354, 361 }

F(-1) timedout fail { }

F(-2) exception fail { 352 }

2.1.7 Mupad

A grade { }

B grade { 12, 28, 29, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 219, 220, 221, 222, 228, 229, 230, 231, 237, 238, 239, 240, 245, 246, 247, 248, 253, 254, 255, 256, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 274, 275, 276, 277, 278, 279, 280, 355, 356, 357, 358, 359, 360 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 85, 93, 102, 215, 216, 217, 218, 223, 224, 225, 226, 227, 232, 233, 234, 235, 236, 241, 242, 243, 244, 249, 250, 251, 252, 257, 258, 259, 265, 271, 272, 273, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 298, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 338, 339, 340, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 361 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 9, 10, 11, 12, 28, 37, 39, 41, 42, 43, 45, 46, 47, 48, 51, 52, 53, 55, 56, 57, 58, 62, 64, 66, 67, 68, 69, 70, 71, 72, 77, 79, 80, 81, 82, 83, 84, 89, 107, 115, 123, 124, 130, 131, 132, 133, 144, 152, 153, 159, 160, 161, 162, 167, 168, 169, 172, 173, 174, 176, 181, 182, 184, 189, 190, 357 }

B grade { 38, 63, 78, 97, 98, 105, 106, 270, 275, 276, 277, 279, 280 }

C grade { }

F normal fail { 4, 5, 6, 7, 8, 13, 14, 15, 17, 18, 19, 20, 24, 25, 26, 31, 32, 33, 36, 40, 44, 49, 50, 54, 59, 60, 61, 65, 73, 74, 75, 76, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 125, 126, 127, 128, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 154, 155, 156, 157, 158, 163, 164, 165, 166, 170, 171, 177, 178, 179, 180, 185, 186, 187, 188, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 233, 234, 235, 241, 242, 243, 244, 245, 246, 250, 251, 252, 253, 254, 255, 258, 259, 260, 261, 262, 265, 266, 267, 268, 269, 271, 272, 273, 274, 278, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 298, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 338, 339, 340, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 360, 361 }

F(-1) timedout fail { 16, 21, 22, 23, 27, 129, 175, 183, 220, 221, 222, 228, 229, 230, 231, 232, 236, 237, 238, 239, 240, 247, 248, 249, 256, 257, 263, 264, 359 }

F(-2) exception fail { 29 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	145	99	253	0	86	121	0	0
N.S.	1	1.14	0.78	1.99	0.00	0.68	0.95	0.00	0.00
time (sec)	N/A	0.275	0.061	0.019	0.000	0.263	0.915	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	115	88	205	0	75	95	0	0
N.S.	1	1.14	0.87	2.03	0.00	0.74	0.94	0.00	0.00
time (sec)	N/A	0.247	0.038	0.015	0.000	0.257	0.420	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	85	76	157	0	59	66	0	0
N.S.	1	1.13	1.01	2.09	0.00	0.79	0.88	0.00	0.00
time (sec)	N/A	0.224	0.029	0.013	0.000	0.265	0.219	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	185	167	209	0	0	0	0	0
N.S.	1	0.78	0.70	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.680	1.717	0.091	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	50	111	51	54	0	0	0
N.S.	1	1.00	0.94	2.09	0.96	1.02	0.00	0.00	0.00
time (sec)	N/A	0.209	0.027	0.014	0.232	0.270	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	80	63	62	61	67	0	0	0
N.S.	1	1.01	0.80	0.78	0.77	0.85	0.00	0.00	0.00
time (sec)	N/A	0.224	0.032	0.013	0.197	0.269	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	110	74	110	102	78	0	0	0
N.S.	1	1.05	0.70	1.05	0.97	0.74	0.00	0.00	0.00
time (sec)	N/A	0.249	0.040	0.014	0.202	0.261	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	140	85	158	125	89	0	0	0
N.S.	1	1.07	0.65	1.21	0.95	0.68	0.00	0.00	0.00
time (sec)	N/A	0.263	0.043	0.017	0.206	0.277	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	112	79	224	155	88	116	0	0
N.S.	1	0.98	0.69	1.96	1.36	0.77	1.02	0.00	0.00
time (sec)	N/A	0.265	0.042	0.020	0.217	0.261	1.389	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	93	68	176	127	77	90	0	0
N.S.	1	1.02	0.75	1.93	1.40	0.85	0.99	0.00	0.00
time (sec)	N/A	0.260	0.063	0.014	0.212	0.274	0.609	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	72	56	128	99	65	65	0	0
N.S.	1	1.06	0.82	1.88	1.46	0.96	0.96	0.00	0.00
time (sec)	N/A	0.240	0.037	0.016	0.236	0.260	0.310	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	76	65	51	36	59	32
N.S.	1	1.00	1.00	1.90	1.62	1.28	0.90	1.48	0.80
time (sec)	N/A	0.187	0.010	0.015	0.207	0.253	0.217	0.327	3.623

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	61	84	0	273	0	0	0
N.S.	1	1.00	1.11	1.53	0.00	4.96	0.00	0.00	0.00
time (sec)	N/A	0.216	0.038	0.014	0.000	0.282	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	81	92	123	0	340	0	0	0
N.S.	1	0.95	1.08	1.45	0.00	4.00	0.00	0.00	0.00
time (sec)	N/A	0.230	0.066	0.014	0.000	0.294	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	107	171	0	383	0	0	0
N.S.	1	1.00	0.96	1.54	0.00	3.45	0.00	0.00	0.00
time (sec)	N/A	0.253	0.078	0.015	0.000	0.299	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	196	214	161	0	0	94	0	0	0
N.S.	1	1.09	0.82	0.00	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	0.314	0.384	0.000	0.000	0.110	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	168	182	147	0	0	83	0	0	0
N.S.	1	1.08	0.88	0.00	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.301	0.293	0.000	0.000	0.105	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	150	135	0	0	69	0	0	0
N.S.	1	1.06	0.95	0.00	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.272	0.206	0.000	0.000	0.106	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	113	111	0	0	50	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.255	0.093	0.000	0.000	0.089	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	150	142	0	0	75	0	0	0
N.S.	1	1.03	0.98	0.00	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.275	0.159	0.000	0.000	0.087	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	182	154	0	0	90	0	0	0
N.S.	1	1.05	0.89	0.00	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.299	0.242	0.000	0.000	0.093	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	201	214	163	0	0	101	0	0	0
N.S.	1	1.06	0.81	0.00	0.00	0.50	0.00	0.00	0.00
time (sec)	N/A	0.328	0.341	0.000	0.000	0.091	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	317	124	0	0	92	0	0	0
N.S.	1	1.07	0.42	0.00	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.429	0.082	0.000	0.000	0.104	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	269	285	109	0	0	77	0	0	0
N.S.	1	1.06	0.41	0.00	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.380	0.071	0.000	0.000	0.106	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	232	249	85	0	0	52	0	0	0
N.S.	1	1.07	0.37	0.00	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.353	0.081	0.000	0.000	0.093	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	272	281	118	0	0	82	0	0	0
N.S.	1	1.03	0.43	0.00	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.398	0.080	0.000	0.000	0.088	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	302	313	131	0	0	98	0	0	0
N.S.	1	1.04	0.43	0.00	0.00	0.32	0.00	0.00	0.00
time (sec)	N/A	0.403	0.078	0.000	0.000	0.097	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	36	39	39	37	59	60	223	90
N.S.	1	0.82	0.89	0.89	0.84	1.34	1.36	5.07	2.05
time (sec)	N/A	0.291	0.017	0.260	0.201	0.264	0.711	0.301	3.715

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	38	42	121	40	109	0	124	56
N.S.	1	0.81	0.89	2.57	0.85	2.32	0.00	2.64	1.19
time (sec)	N/A	0.302	0.032	4.658	0.231	0.263	0.000	0.318	3.928

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	38	34	38	39
N.S.	1	1.00	1.05	0.90	0.98	0.95	0.85	0.95	0.98
time (sec)	N/A	0.248	0.159	0.154	0.789	0.281	9.639	0.729	3.959

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	409	402	0	1623	0	0	0	0	0
N.S.	1	0.98	0.00	3.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.212	0.000	0.553	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	268	271	0	893	0	0	0	0	0
N.S.	1	1.01	0.00	3.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.857	0.000	0.262	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	84	43	357	0	0	0	0	0
N.S.	1	0.94	0.48	4.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.299	0.299	0.276	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	48	61	38	39
N.S.	1	1.00	1.05	0.90	0.98	1.20	1.52	0.95	0.98
time (sec)	N/A	0.246	0.245	0.150	0.348	0.250	3.787	0.443	3.676

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	246	91	126	38	39
N.S.	1	1.00	1.05	0.90	6.15	2.28	3.15	0.95	0.98
time (sec)	N/A	0.246	0.980	0.140	0.408	0.269	10.756	0.812	4.465

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	41	38	62	0	43	96
N.S.	1	1.00	0.92	1.11	1.03	1.68	0.00	1.16	2.59
time (sec)	N/A	0.184	0.050	0.189	0.199	0.268	0.000	0.293	4.122

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	13	19	13	19
N.S.	1	1.00	0.87	0.87	0.83	0.57	0.83	0.57	0.83
time (sec)	N/A	0.160	0.015	0.058	0.229	0.236	0.116	0.276	0.141

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	13	39	13	19
N.S.	1	1.00	0.87	0.87	0.83	0.57	1.70	0.57	0.83
time (sec)	N/A	0.179	0.014	0.056	0.237	0.239	0.173	0.271	3.513

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	16	10	19	10	16
N.S.	1	1.00	1.12	0.94	1.00	0.62	1.19	0.62	1.00
time (sec)	N/A	0.153	0.008	0.063	0.240	0.238	0.069	0.272	0.051

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	23	34	8	0	9	58
N.S.	1	1.00	0.90	1.10	1.62	0.38	0.00	0.43	2.76
time (sec)	N/A	0.183	0.014	0.063	0.232	0.247	0.000	0.294	3.954

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	18	17	13	14	12	17
N.S.	1	1.00	1.06	1.06	1.00	0.76	0.82	0.71	1.00
time (sec)	N/A	0.156	0.014	0.065	0.241	0.236	0.089	0.300	0.092

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	17	19	11	19	11	16
N.S.	1	1.00	0.78	0.74	0.83	0.48	0.83	0.48	0.70
time (sec)	N/A	0.157	0.015	0.062	0.248	0.246	0.186	0.280	3.364

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	19	19	13	20	13	19
N.S.	1	1.00	0.87	0.83	0.83	0.57	0.87	0.57	0.83
time (sec)	N/A	0.158	0.015	0.094	0.248	0.242	0.222	0.276	0.072

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	66	62	98	73	161	0	0	203
N.S.	1	0.93	0.87	1.38	1.03	2.27	0.00	0.00	2.86
time (sec)	N/A	0.227	0.125	0.463	0.255	0.250	0.000	0.000	3.640

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	45	37	37	36	24	37	24	36
N.S.	1	1.07	0.88	0.88	0.86	0.57	0.88	0.57	0.86
time (sec)	N/A	0.197	0.025	24.744	0.286	0.240	0.209	0.284	0.139

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	45	37	37	36	24	37	24	36
N.S.	1	1.07	0.88	0.88	0.86	0.57	0.88	0.57	0.86
time (sec)	N/A	0.200	0.039	22.155	0.292	0.235	0.145	0.273	3.448

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	74	37	36	24	41	24	36
N.S.	1	1.00	2.18	1.09	1.06	0.71	1.21	0.71	1.06
time (sec)	N/A	0.200	0.191	21.416	0.301	0.237	0.196	0.274	3.445

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	33	20	20	20	33
N.S.	1	1.00	1.00	0.94	2.06	1.25	1.25	1.25	2.06
time (sec)	N/A	0.158	0.006	0.163	0.299	0.244	0.093	0.269	3.496

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	50	53	67	20	20	0	21	183
N.S.	1	1.02	1.08	1.37	0.41	0.41	0.00	0.43	3.73
time (sec)	N/A	0.206	0.095	0.098	0.499	0.239	0.000	0.280	0.282

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	37	41	54	24	0	21	198
N.S.	1	1.00	0.95	1.05	1.38	0.62	0.00	0.54	5.08
time (sec)	N/A	0.198	0.034	0.069	0.254	0.235	0.000	0.282	0.194

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	42	35	34	26	32	22	34
N.S.	1	1.00	1.17	0.97	0.94	0.72	0.89	0.61	0.94
time (sec)	N/A	0.201	0.030	0.159	0.315	0.243	0.191	0.278	3.571

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	34	33	36	22	37	22	32
N.S.	1	1.00	1.10	1.06	1.16	0.71	1.19	0.71	1.03
time (sec)	N/A	0.168	0.035	0.146	0.306	0.238	0.240	0.279	3.686

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	64	37	36	36	24	39	24	36
N.S.	1	1.52	0.88	0.86	0.86	0.57	0.93	0.57	0.86
time (sec)	N/A	0.228	0.028	0.182	0.303	0.236	0.298	0.273	3.462

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	95	97	177	109	300	0	0	332
N.S.	1	0.86	0.88	1.61	0.99	2.73	0.00	0.00	3.02
time (sec)	N/A	0.282	0.112	4.926	0.319	0.252	0.000	0.000	3.786

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	67	54	56	54	35	58	35	53
N.S.	1	1.10	0.89	0.92	0.89	0.57	0.95	0.57	0.87
time (sec)	N/A	0.243	0.024	0.039	0.334	0.245	0.304	0.267	3.560

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	61	54	56	54	35	56	35	53
N.S.	1	1.15	1.02	1.06	1.02	0.66	1.06	0.66	1.00
time (sec)	N/A	0.243	0.019	0.048	0.339	0.244	0.211	0.295	0.149

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	99	54	54	34	41	34	53
N.S.	1	1.00	2.91	1.59	1.59	1.00	1.21	1.00	1.56
time (sec)	N/A	0.186	0.255	21.152	0.360	0.253	0.270	0.291	3.558

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	51	31	20	31	47
N.S.	1	1.00	1.00	0.94	3.19	1.94	1.25	1.94	2.94
time (sec)	N/A	0.155	0.009	21.432	0.339	0.242	0.121	0.279	0.112

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	79	104	147	31	31	0	32	306
N.S.	1	1.03	1.35	1.91	0.40	0.40	0.00	0.42	3.97
time (sec)	N/A	0.266	0.109	0.171	0.529	0.236	0.000	0.278	3.952

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	85	65	36	0	33	415
N.S.	1	1.00	0.91	1.25	0.96	0.53	0.00	0.49	6.10
time (sec)	N/A	0.257	0.033	0.150	0.544	0.258	0.000	0.274	0.211

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	61	66	59	72	37	0	31	365
N.S.	1	1.02	1.10	0.98	1.20	0.62	0.00	0.52	6.08
time (sec)	N/A	0.254	0.038	0.175	0.317	0.236	0.000	0.286	3.601

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	60	52	52	37	51	35	51
N.S.	1	1.00	1.09	0.95	0.95	0.67	0.93	0.64	0.93
time (sec)	N/A	0.250	0.020	0.598	0.358	0.243	0.233	0.279	3.475

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	50	49	53	33	56	33	48
N.S.	1	1.00	1.61	1.58	1.71	1.06	1.81	1.06	1.55
time (sec)	N/A	0.168	0.024	0.609	0.361	0.247	0.303	0.280	3.440

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	54	53	54	35	60	35	53
N.S.	1	1.00	0.84	0.83	0.84	0.55	0.94	0.55	0.83
time (sec)	N/A	0.228	0.027	0.654	0.355	0.242	0.402	0.282	3.516

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	124	137	278	145	483	0	0	479
N.S.	1	0.81	0.89	1.81	0.94	3.14	0.00	0.00	3.11
time (sec)	N/A	0.353	0.130	25.642	0.356	0.272	0.000	0.000	4.040

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	89	71	74	72	46	75	46	242
N.S.	1	1.11	0.89	0.92	0.90	0.58	0.94	0.58	3.02
time (sec)	N/A	0.294	0.046	0.046	0.398	0.243	1.649	0.273	3.666

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	89	71	74	72	46	76	46	242
N.S.	1	1.11	0.89	0.92	0.90	0.58	0.95	0.58	3.02
time (sec)	N/A	0.296	0.026	0.048	0.385	0.239	1.017	0.282	0.149

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	89	71	74	72	46	78	46	77
N.S.	1	1.11	0.89	0.92	0.90	0.58	0.98	0.58	0.96
time (sec)	N/A	0.300	0.028	0.055	0.398	0.239	0.729	0.279	3.947

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	88	71	74	72	45	75	45	70
N.S.	1	1.22	0.99	1.03	1.00	0.62	1.04	0.62	0.97
time (sec)	N/A	0.301	0.021	0.046	0.386	0.249	0.448	0.280	0.197

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	61	71	74	72	45	75	45	70
N.S.	1	1.15	1.34	1.40	1.36	0.85	1.42	0.85	1.32
time (sec)	N/A	0.253	0.038	0.035	0.382	0.248	0.310	0.281	0.167

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	125	71	72	46	41	46	70
N.S.	1	1.00	3.68	2.09	2.12	1.35	1.21	1.35	2.06
time (sec)	N/A	0.198	0.293	21.867	0.394	0.238	0.354	0.293	0.134

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	69	42	20	42	67
N.S.	1	1.00	1.00	0.94	4.31	2.62	1.25	2.62	4.19
time (sec)	N/A	0.157	0.007	24.750	0.394	0.237	0.181	0.283	3.463

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	108	175	265	42	42	0	43	423
N.S.	1	1.03	1.67	2.52	0.40	0.40	0.00	0.41	4.03
time (sec)	N/A	0.326	0.145	0.224	0.501	0.251	0.000	0.291	0.145

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	97	85	165	77	47	0	44	553
N.S.	1	1.02	0.89	1.74	0.81	0.49	0.00	0.46	5.82
time (sec)	N/A	0.313	0.060	0.231	0.562	0.260	0.000	0.277	0.164

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	88	81	103	83	47	0	43	672
N.S.	1	1.01	0.93	1.18	0.95	0.54	0.00	0.49	7.72
time (sec)	N/A	0.306	0.034	0.254	0.598	0.240	0.000	0.297	3.848

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	83	82	77	91	48	0	42	571
N.S.	1	1.08	1.06	1.00	1.18	0.62	0.00	0.55	7.42
time (sec)	N/A	0.300	0.035	0.458	0.371	0.241	0.000	0.284	3.768

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	78	69	72	48	70	46	68
N.S.	1	1.00	1.05	0.93	0.97	0.65	0.95	0.62	0.92
time (sec)	N/A	0.295	0.028	2.033	0.409	0.239	0.306	0.279	3.506

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	66	65	70	44	75	44	64
N.S.	1	1.00	2.13	2.10	2.26	1.42	2.42	1.42	2.06
time (sec)	N/A	0.176	0.043	1.849	0.402	0.245	0.420	0.279	3.531

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	71	70	72	46	78	46	70
N.S.	1	1.00	1.11	1.09	1.12	0.72	1.22	0.72	1.09
time (sec)	N/A	0.228	0.028	1.871	0.402	0.242	0.563	0.278	3.733

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	116	71	70	72	46	80	46	70
N.S.	1	1.18	0.72	0.71	0.73	0.47	0.82	0.47	0.71
time (sec)	N/A	0.310	0.039	1.973	0.455	0.250	0.801	0.276	3.669

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	168	71	70	72	46	76	46	70
N.S.	1	2.10	0.89	0.88	0.90	0.58	0.95	0.58	0.88
time (sec)	N/A	0.399	0.030	2.158	0.421	0.241	1.048	0.290	3.528

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	89	71	70	72	46	76	46	70
N.S.	1	1.11	0.89	0.88	0.90	0.58	0.95	0.58	0.88
time (sec)	N/A	0.297	0.050	2.424	0.410	0.248	1.520	0.267	3.418

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	89	71	70	72	46	78	46	70
N.S.	1	1.11	0.89	0.88	0.90	0.58	0.98	0.58	0.88
time (sec)	N/A	0.289	0.032	2.904	0.398	0.241	2.177	0.263	3.703

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	177	105	110	68	41	68	104
N.S.	1	1.00	5.21	3.09	3.24	2.00	1.21	2.00	3.06
time (sec)	N/A	0.194	0.276	34.920	0.478	0.245	0.768	0.282	4.013

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	0.076	0.000	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	87	79	163	42	41	0	43	354
N.S.	1	1.07	0.98	2.01	0.52	0.51	0.00	0.53	4.37
time (sec)	N/A	0.304	0.039	0.954	0.349	0.246	0.000	0.268	0.136

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	59	55	86	29	29	0	30	234
N.S.	1	1.05	0.98	1.54	0.52	0.52	0.00	0.54	4.18
time (sec)	N/A	0.248	0.030	0.228	0.367	0.248	0.000	0.270	0.286

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	32	18	17	0	19	108
N.S.	1	1.00	1.00	1.03	0.58	0.55	0.00	0.61	3.48
time (sec)	N/A	0.194	0.023	0.129	0.363	0.249	0.000	0.263	0.151

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	13	10	17	11	12
N.S.	1	1.00	1.00	1.08	1.08	0.83	1.42	0.92	1.00
time (sec)	N/A	0.154	0.109	0.060	0.286	0.244	0.250	0.259	0.154

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	29	43	18	16	0	20	107
N.S.	1	1.00	0.66	0.98	0.41	0.36	0.00	0.45	2.43
time (sec)	N/A	0.217	0.018	4.529	0.363	0.252	0.000	0.252	6.009

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	81	45	64	28	26	0	30	210
N.S.	1	1.25	0.69	0.98	0.43	0.40	0.00	0.46	3.23
time (sec)	N/A	0.274	0.022	0.038	0.360	0.245	0.000	0.261	5.576

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	121	66	87	40	41	0	45	286
N.S.	1	1.32	0.72	0.95	0.43	0.45	0.00	0.49	3.11
time (sec)	N/A	0.359	0.024	0.031	0.353	0.272	0.000	0.274	5.808

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	51	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.239	0.937	0.000	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	110	106	262	70	73	0	62	669
N.S.	1	1.12	1.08	2.67	0.71	0.74	0.00	0.63	6.83
time (sec)	N/A	0.373	0.075	0.979	0.510	0.252	0.000	0.255	3.759

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	82	83	169	59	62	0	48	490
N.S.	1	1.09	1.11	2.25	0.79	0.83	0.00	0.64	6.53
time (sec)	N/A	0.289	0.042	0.255	0.528	0.251	0.000	0.268	0.173

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	54	56	86	44	47	0	34	302
N.S.	1	1.08	1.12	1.72	0.88	0.94	0.00	0.68	6.04
time (sec)	N/A	0.241	0.058	0.143	0.505	0.242	0.000	0.266	4.167

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	35	26	28	94	24	28
N.S.	1	1.00	0.96	1.25	0.93	1.00	3.36	0.86	1.00
time (sec)	N/A	0.187	0.120	0.084	0.503	0.242	14.804	0.278	0.146

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	12	13	94	12	14
N.S.	1	1.00	1.00	1.07	0.86	0.93	6.71	0.86	1.00
time (sec)	N/A	0.153	0.008	0.057	0.283	0.239	14.685	0.257	0.158

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	88	53	67	28	39	0	31	359
N.S.	1	1.26	0.76	0.96	0.40	0.56	0.00	0.44	5.13
time (sec)	N/A	0.286	0.111	270.431	0.509	0.259	0.000	0.265	6.026

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	135	70	91	45	63	0	45	432
N.S.	1	1.32	0.69	0.89	0.44	0.62	0.00	0.44	4.24
time (sec)	N/A	0.361	0.046	0.038	0.507	0.256	0.000	0.280	5.988

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	187	92	116	64	86	0	64	660
N.S.	1	1.31	0.64	0.81	0.45	0.60	0.00	0.45	4.62
time (sec)	N/A	0.475	0.043	0.043	0.497	0.242	0.000	0.258	5.875

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	97	51	0	0	0	0	0	0
N.S.	1	1.03	0.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.296	0.996	0.000	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	107	114	266	81	95	0	61	867
N.S.	1	1.16	1.24	2.89	0.88	1.03	0.00	0.66	9.42
time (sec)	N/A	0.358	0.033	0.293	0.754	0.246	0.000	0.262	4.146

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	81	86	169	69	83	0	44	620
N.S.	1	1.14	1.21	2.38	0.97	1.17	0.00	0.62	8.73
time (sec)	N/A	0.302	0.055	0.159	0.749	0.246	0.000	0.274	3.924

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	52	49	54	48	61	112	37	46
N.S.	1	1.11	1.04	1.15	1.02	1.30	2.38	0.79	0.98
time (sec)	N/A	0.241	0.038	0.104	0.764	0.251	28.823	0.266	3.545

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	27	26	32	32	100	18	25
N.S.	1	1.00	0.79	0.76	0.94	0.94	2.94	0.53	0.74
time (sec)	N/A	0.190	0.112	0.088	0.752	0.251	29.242	0.263	0.087

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	12	24	24	12	14
N.S.	1	1.00	1.00	0.94	0.75	1.50	1.50	0.75	0.88
time (sec)	N/A	0.153	0.009	0.065	0.288	0.244	28.397	0.278	0.059

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	134	74	92	51	80	0	43	645
N.S.	1	1.38	0.76	0.95	0.53	0.82	0.00	0.44	6.65
time (sec)	N/A	0.363	0.126	0.039	0.796	0.247	0.000	0.264	6.015

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	181	93	117	69	109	0	60	804
N.S.	1	1.38	0.71	0.89	0.53	0.83	0.00	0.46	6.14
time (sec)	N/A	0.457	0.039	0.043	0.775	0.240	0.000	0.275	5.813

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	231	107	145	86	130	0	73	909
N.S.	1	1.36	0.63	0.85	0.51	0.76	0.00	0.43	5.35
time (sec)	N/A	0.578	0.036	0.042	0.760	0.263	0.000	0.285	5.720

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	125	83	154	64	64	0	150	811
N.S.	1	1.24	0.82	1.52	0.63	0.63	0.00	1.49	8.03
time (sec)	N/A	0.358	0.030	0.121	0.361	0.251	0.000	0.263	3.695

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	94	66	124	53	53	0	125	648
N.S.	1	1.18	0.82	1.55	0.66	0.66	0.00	1.56	8.10
time (sec)	N/A	0.287	0.027	0.110	0.362	0.272	0.000	0.261	3.531

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	67	49	69	42	42	0	102	485
N.S.	1	1.14	0.83	1.17	0.71	0.71	0.00	1.73	8.22
time (sec)	N/A	0.241	0.026	0.119	0.363	0.258	0.000	0.274	3.576

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	42	30	30	0	75	151
N.S.	1	1.00	0.84	1.11	0.79	0.79	0.00	1.97	3.97
time (sec)	N/A	0.191	0.169	0.109	0.360	0.254	0.000	0.266	4.054

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	12	12	26	18	95
N.S.	1	1.00	1.00	0.83	0.67	0.67	1.44	1.00	5.28
time (sec)	N/A	0.146	0.008	0.099	0.336	0.255	0.135	0.258	3.624

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	54	0	73	0	40	308
N.S.	1	1.00	0.97	0.86	0.00	1.16	0.00	0.63	4.89
time (sec)	N/A	0.244	0.116	0.097	0.000	0.253	0.000	0.259	5.025

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	65	63	0	93	0	51	341
N.S.	1	1.00	0.98	0.95	0.00	1.41	0.00	0.77	5.17
time (sec)	N/A	0.216	0.033	0.099	0.000	0.249	0.000	0.269	9.963

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	124	89	92	0	119	0	75	741
N.S.	1	0.99	0.71	0.74	0.00	0.95	0.00	0.60	5.93
time (sec)	N/A	0.319	0.077	0.095	0.000	0.277	0.000	0.272	9.222

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	196	115	185	0	145	0	93	964
N.S.	1	1.09	0.64	1.03	0.00	0.81	0.00	0.52	5.39
time (sec)	N/A	0.458	0.071	0.097	0.000	0.267	0.000	0.264	8.040

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	125	83	154	64	75	0	241	1813
N.S.	1	1.24	0.82	1.52	0.63	0.74	0.00	2.39	17.95
time (sec)	N/A	0.349	0.031	0.109	0.363	0.259	0.000	0.267	3.634

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	96	66	124	53	64	0	205	1483
N.S.	1	1.20	0.82	1.55	0.66	0.80	0.00	2.56	18.54
time (sec)	N/A	0.287	0.030	0.108	0.354	0.277	0.000	0.281	3.535

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	67	49	69	42	53	0	168	1153
N.S.	1	1.14	0.83	1.17	0.71	0.90	0.00	2.85	19.54
time (sec)	N/A	0.239	0.025	0.106	0.367	0.241	0.000	0.275	3.993

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	42	31	41	49	131	823
N.S.	1	1.00	0.84	1.11	0.82	1.08	1.29	3.45	21.66
time (sec)	N/A	0.185	0.193	0.107	0.369	0.246	1.673	0.259	3.612

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	12	28	26	84	97
N.S.	1	1.00	1.00	0.83	0.67	1.56	1.44	4.67	5.39
time (sec)	N/A	0.145	0.007	0.104	0.337	0.248	0.807	0.269	3.882

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	94	80	131	0	88	0	57	501
N.S.	1	1.03	0.88	1.44	0.00	0.97	0.00	0.63	5.51
time (sec)	N/A	0.267	0.117	0.090	0.000	0.248	0.000	0.282	8.139

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	85	79	85	0	102	0	69	459
N.S.	1	1.05	0.98	1.05	0.00	1.26	0.00	0.85	5.67
time (sec)	N/A	0.262	0.034	0.093	0.000	0.250	0.000	0.272	5.341

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	90	88	91	0	124	0	73	609
N.S.	1	0.98	0.96	0.99	0.00	1.35	0.00	0.79	6.62
time (sec)	N/A	0.262	0.052	0.102	0.000	0.263	0.000	0.283	9.239

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	148	117	116	0	145	0	93	1019
N.S.	1	1.01	0.80	0.79	0.00	0.99	0.00	0.64	6.98
time (sec)	N/A	0.369	0.073	0.104	0.000	0.269	0.000	0.288	8.191

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	125	83	154	64	86	0	344	2681
N.S.	1	1.24	0.82	1.52	0.63	0.85	0.00	3.41	26.54
time (sec)	N/A	0.341	0.028	0.395	0.362	0.238	0.000	0.274	4.058

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	96	66	124	53	75	94	296	2235
N.S.	1	1.20	0.82	1.55	0.66	0.94	1.18	3.70	27.94
time (sec)	N/A	0.285	0.035	0.206	0.367	0.252	76.866	0.271	3.751

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	67	49	69	42	64	71	248	1789
N.S.	1	1.14	0.83	1.17	0.71	1.08	1.20	4.20	30.32
time (sec)	N/A	0.228	0.022	0.137	0.357	0.240	40.355	0.266	3.841

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	42	31	52	49	197	773
N.S.	1	1.00	0.84	1.11	0.82	1.37	1.29	5.18	20.34
time (sec)	N/A	0.185	0.189	0.114	0.364	0.241	22.306	0.275	3.647

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	12	39	26	136	337
N.S.	1	1.00	1.00	0.83	0.67	2.17	1.44	7.56	18.72
time (sec)	N/A	0.146	0.008	0.107	0.348	0.242	11.870	0.283	3.632

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	125	99	222	0	114	0	73	789
N.S.	1	1.03	0.82	1.83	0.00	0.94	0.00	0.60	6.52
time (sec)	N/A	0.319	0.111	0.092	0.000	0.250	0.000	0.258	8.334

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	116	106	193	0	126	0	89	616
N.S.	1	1.05	0.96	1.75	0.00	1.15	0.00	0.81	5.60
time (sec)	N/A	0.317	0.041	0.104	0.000	0.252	0.000	0.267	8.418

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	109	108	142	0	133	0	92	614
N.S.	1	0.99	0.98	1.29	0.00	1.21	0.00	0.84	5.58
time (sec)	N/A	0.315	0.033	0.135	0.000	0.246	0.000	0.272	4.787

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	114	107	144	0	146	0	88	669
N.S.	1	1.01	0.95	1.27	0.00	1.29	0.00	0.78	5.92
time (sec)	N/A	0.300	0.052	0.236	0.000	0.260	0.000	0.270	8.342

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	172	134	169	0	167	0	108	1069
N.S.	1	1.03	0.80	1.01	0.00	1.00	0.00	0.65	6.40
time (sec)	N/A	0.419	0.081	0.531	0.000	0.260	0.000	0.277	8.890

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	244	150	262	0	189	0	123	1292
N.S.	1	1.10	0.68	1.19	0.00	0.86	0.00	0.56	5.85
time (sec)	N/A	0.565	0.087	1.227	0.000	0.258	0.000	0.273	9.725

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	119	83	153	64	53	0	61	496
N.S.	1	1.20	0.84	1.55	0.65	0.54	0.00	0.62	5.01
time (sec)	N/A	0.344	0.029	0.088	0.358	0.249	0.000	0.259	3.723

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	92	66	123	53	42	0	49	385
N.S.	1	1.21	0.87	1.62	0.70	0.55	0.00	0.64	5.07
time (sec)	N/A	0.290	0.038	0.093	0.364	0.241	0.000	0.257	3.573

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	63	49	68	42	31	0	37	211
N.S.	1	1.11	0.86	1.19	0.74	0.54	0.00	0.65	3.70
time (sec)	N/A	0.233	0.025	0.092	0.353	0.251	0.000	0.272	3.591

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	32	56	30	19	0	23	105
N.S.	1	1.00	0.89	1.56	0.83	0.53	0.00	0.64	2.92
time (sec)	N/A	0.190	0.121	0.091	0.359	0.239	0.000	0.269	3.767

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	12	12	24	12	52
N.S.	1	1.00	1.00	0.94	0.75	0.75	1.50	0.75	3.25
time (sec)	N/A	0.151	0.008	0.099	0.336	0.252	0.268	0.263	3.632

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	47	42	0	56	0	21	285
N.S.	1	1.00	0.96	0.86	0.00	1.14	0.00	0.43	5.82
time (sec)	N/A	0.182	0.107	0.104	0.000	0.270	0.000	0.276	10.015

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	100	78	95	0	93	0	47	570
N.S.	1	1.06	0.83	1.01	0.00	0.99	0.00	0.50	6.06
time (sec)	N/A	0.269	0.047	0.098	0.000	0.258	0.000	0.272	9.318

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	172	98	148	0	123	0	69	802
N.S.	1	1.09	0.62	0.94	0.00	0.78	0.00	0.44	5.08
time (sec)	N/A	0.402	0.062	0.095	0.000	0.252	0.000	0.268	9.106

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	244	117	200	0	145	0	84	1086
N.S.	1	1.15	0.55	0.94	0.00	0.68	0.00	0.40	5.12
time (sec)	N/A	0.557	0.081	0.095	0.000	0.252	0.000	0.265	8.099

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	117	83	319	64	63	0	77	1057
N.S.	1	1.23	0.87	3.36	0.67	0.66	0.00	0.81	11.13
time (sec)	N/A	0.350	0.036	0.092	0.375	0.275	0.000	0.271	4.102

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	88	66	201	52	51	0	61	660
N.S.	1	1.19	0.89	2.72	0.70	0.69	0.00	0.82	8.92
time (sec)	N/A	0.293	0.034	0.089	0.360	0.253	0.000	0.271	3.825

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	61	49	106	41	40	0	46	259
N.S.	1	1.11	0.89	1.93	0.75	0.73	0.00	0.84	4.71
time (sec)	N/A	0.238	0.029	0.087	0.381	0.248	0.000	0.268	4.236

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	29	40	30	29	46	29	152
N.S.	1	1.00	0.85	1.18	0.88	0.85	1.35	0.85	4.47
time (sec)	N/A	0.185	0.115	0.089	0.354	0.248	0.507	0.272	3.953

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	12	20	26	12	97
N.S.	1	1.00	1.00	0.94	0.75	1.25	1.62	0.75	6.06
time (sec)	N/A	0.151	0.008	0.094	0.344	0.246	0.471	0.268	3.865

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	75	68	0	110	0	37	614
N.S.	1	1.00	0.96	0.87	0.00	1.41	0.00	0.47	7.87
time (sec)	N/A	0.232	0.142	0.090	0.000	0.254	0.000	0.267	9.069

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	148	91	105	0	151	0	64	807
N.S.	1	1.19	0.73	0.85	0.00	1.22	0.00	0.52	6.51
time (sec)	N/A	0.349	0.056	0.098	0.000	0.256	0.000	0.269	9.850

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	220	115	131	0	189	0	80	1028
N.S.	1	1.15	0.60	0.69	0.00	0.99	0.00	0.42	5.38
time (sec)	N/A	0.499	0.086	0.102	0.000	0.291	0.000	0.266	9.572

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	292	133	186	0	211	0	95	1258
N.S.	1	1.19	0.54	0.76	0.00	0.86	0.00	0.39	5.13
time (sec)	N/A	0.670	0.085	0.103	0.000	0.265	0.000	0.265	8.559

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	117	83	295	64	74	0	75	817
N.S.	1	1.18	0.84	2.98	0.65	0.75	0.00	0.76	8.25
time (sec)	N/A	0.345	0.038	0.097	0.350	0.261	0.000	0.270	4.265

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	88	65	186	52	62	90	59	533
N.S.	1	1.16	0.86	2.45	0.68	0.82	1.18	0.78	7.01
time (sec)	N/A	0.291	0.035	0.095	0.364	0.253	2.234	0.268	4.530

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	63	48	91	42	52	71	39	259
N.S.	1	1.07	0.81	1.54	0.71	0.88	1.20	0.66	4.39
time (sec)	N/A	0.241	0.030	0.090	0.351	0.254	2.150	0.268	4.008

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	31	42	31	41	51	20	152
N.S.	1	1.00	0.82	1.11	0.82	1.08	1.34	0.53	4.00
time (sec)	N/A	0.191	0.123	0.097	0.360	0.240	2.193	0.273	4.101

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	12	31	27	12	103
N.S.	1	1.00	1.00	0.83	0.67	1.72	1.50	0.67	5.72
time (sec)	N/A	0.155	0.008	0.098	0.346	0.240	2.103	0.274	4.308

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	126	91	93	0	177	0	45	886
N.S.	1	1.17	0.84	0.86	0.00	1.64	0.00	0.42	8.20
time (sec)	N/A	0.306	0.190	0.088	0.000	0.260	0.000	0.274	9.107

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	196	113	130	0	221	0	65	1230
N.S.	1	1.26	0.73	0.84	0.00	1.43	0.00	0.42	7.94
time (sec)	N/A	0.441	0.096	0.108	0.000	0.266	0.000	0.271	10.730

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	268	133	157	0	255	0	93	1514
N.S.	1	1.20	0.59	0.70	0.00	1.14	0.00	0.42	6.76
time (sec)	N/A	0.606	0.078	0.105	0.000	0.257	0.000	0.282	10.943

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	340	150	211	0	277	0	115	2359
N.S.	1	1.22	0.54	0.76	0.00	1.00	0.00	0.41	8.49
time (sec)	N/A	0.774	0.137	0.099	0.000	0.252	0.000	0.275	9.611

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	19	18	26	13	57
N.S.	1	1.00	0.85	0.74	0.70	0.67	0.96	0.48	2.11
time (sec)	N/A	0.151	0.031	0.118	0.202	0.239	141.793	0.263	3.982

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	19	18	26	13	57
N.S.	1	1.00	0.85	0.74	0.70	0.67	0.96	0.48	2.11
time (sec)	N/A	0.149	0.023	0.118	0.204	0.244	12.778	0.257	3.966

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	19	18	26	13	57
N.S.	1	1.00	0.85	0.74	0.70	0.67	0.96	0.48	2.11
time (sec)	N/A	0.153	0.021	0.145	0.199	0.250	1.131	0.272	3.876

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	19	16	0	13	57
N.S.	1	1.00	0.85	0.74	0.70	0.59	0.00	0.48	2.11
time (sec)	N/A	0.153	0.020	0.122	0.204	0.244	0.000	0.263	3.836

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	20	19	12	0	13	56
N.S.	1	1.00	0.92	0.80	0.76	0.48	0.00	0.52	2.24
time (sec)	N/A	0.152	0.017	0.119	0.202	0.238	0.000	0.265	4.318

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	12	22	13	56
N.S.	1	1.00	0.87	0.87	0.83	0.52	0.96	0.57	2.43
time (sec)	N/A	0.152	0.021	0.107	0.207	0.252	0.278	0.268	3.985

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	21	20	19	11	27	11	52
N.S.	1	1.00	0.78	0.74	0.70	0.41	1.00	0.41	1.93
time (sec)	N/A	0.150	0.017	0.108	0.197	0.241	1.285	0.271	4.044

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	19	13	27	13	57
N.S.	1	1.00	0.85	0.74	0.70	0.48	1.00	0.48	2.11
time (sec)	N/A	0.156	0.018	0.125	0.212	0.244	13.009	0.257	4.026

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	51	40	38	36	29	0	24	122
N.S.	1	1.06	0.83	0.79	0.75	0.60	0.00	0.50	2.54
time (sec)	N/A	0.194	0.056	0.250	0.218	0.246	0.000	0.254	4.188

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	51	40	38	36	29	48	24	122
N.S.	1	1.06	0.83	0.79	0.75	0.60	1.00	0.50	2.54
time (sec)	N/A	0.193	0.035	0.253	0.217	0.244	22.477	0.280	4.075

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	51	40	38	36	29	0	24	122
N.S.	1	1.06	0.83	0.79	0.75	0.60	0.00	0.50	2.54
time (sec)	N/A	0.197	0.033	0.212	0.211	0.243	0.000	0.266	3.991

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	51	40	38	36	27	0	24	122
N.S.	1	1.06	0.83	0.79	0.75	0.56	0.00	0.50	2.54
time (sec)	N/A	0.196	0.040	0.217	0.235	0.251	0.000	0.257	4.268

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	47	40	47	36	24	0	24	122
N.S.	1	1.02	0.87	1.02	0.78	0.52	0.00	0.52	2.65
time (sec)	N/A	0.194	0.023	0.214	0.220	0.251	0.000	0.267	3.851

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	45	40	37	36	23	0	24	122
N.S.	1	1.02	0.91	0.84	0.82	0.52	0.00	0.55	2.77
time (sec)	N/A	0.194	0.034	0.221	0.259	0.242	0.000	0.265	3.881

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	47	40	38	36	24	48	23	122
N.S.	1	0.98	0.83	0.79	0.75	0.50	1.00	0.48	2.54
time (sec)	N/A	0.198	0.037	0.220	0.206	0.250	1.284	0.250	3.882

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	51	40	38	36	24	49	24	122
N.S.	1	1.06	0.83	0.79	0.75	0.50	1.02	0.50	2.54
time (sec)	N/A	0.192	0.034	0.229	0.219	0.251	13.306	0.266	4.042

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	75	57	56	55	40	0	35	182
N.S.	1	1.09	0.83	0.81	0.80	0.58	0.00	0.51	2.64
time (sec)	N/A	0.236	0.035	0.859	0.221	0.235	0.000	0.274	4.136

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	75	57	56	55	40	70	35	182
N.S.	1	1.09	0.83	0.81	0.80	0.58	1.01	0.51	2.64
time (sec)	N/A	0.244	0.039	0.853	0.212	0.261	40.073	0.270	3.891

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	75	57	56	55	40	0	35	182
N.S.	1	1.09	0.83	0.81	0.80	0.58	0.00	0.51	2.64
time (sec)	N/A	0.240	0.026	0.783	0.228	0.257	0.000	0.263	4.300

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	73	57	56	55	38	0	35	182
N.S.	1	1.06	0.83	0.81	0.80	0.55	0.00	0.51	2.64
time (sec)	N/A	0.238	0.027	0.767	0.219	0.239	0.000	0.261	3.942

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	71	57	69	55	35	0	35	182
N.S.	1	1.09	0.88	1.06	0.85	0.54	0.00	0.54	2.80
time (sec)	N/A	0.240	0.024	0.803	0.223	0.250	0.000	0.258	3.979

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	67	57	64	55	34	0	35	182
N.S.	1	1.06	0.90	1.02	0.87	0.54	0.00	0.56	2.89
time (sec)	N/A	0.236	0.034	0.865	0.217	0.250	0.000	0.279	4.021

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	67	55	55	55	34	66	34	182
N.S.	1	1.03	0.85	0.85	0.85	0.52	1.02	0.52	2.80
time (sec)	N/A	0.235	0.025	0.776	0.218	0.244	1.362	0.271	3.998

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	71	57	56	55	35	70	34	182
N.S.	1	1.03	0.83	0.81	0.80	0.51	1.01	0.49	2.64
time (sec)	N/A	0.241	0.026	0.740	0.221	0.256	12.907	0.271	4.168

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	154	129	262	65	153	0	70	475
N.S.	1	1.08	0.90	1.83	0.45	1.07	0.00	0.49	3.32
time (sec)	N/A	0.391	0.054	0.161	0.279	0.252	0.000	0.265	4.402

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	124	108	201	54	132	0	59	415
N.S.	1	1.07	0.93	1.73	0.47	1.14	0.00	0.51	3.58
time (sec)	N/A	0.326	0.087	0.122	0.290	0.268	0.000	0.263	4.497

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	94	86	120	42	103	0	45	354
N.S.	1	1.06	0.97	1.35	0.47	1.16	0.00	0.51	3.98
time (sec)	N/A	0.267	0.072	0.108	0.295	0.261	0.000	0.271	4.690

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	62	66	31	85	0	31	296
N.S.	1	1.00	0.97	1.03	0.48	1.33	0.00	0.48	4.62
time (sec)	N/A	0.207	0.041	0.105	0.287	0.251	0.000	0.262	4.955

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	41	18	68	0	18	347
N.S.	1	1.00	0.96	0.77	0.34	1.28	0.00	0.34	6.55
time (sec)	N/A	0.169	0.023	0.112	0.284	0.257	0.000	0.265	6.746

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	73	76	31	93	0	31	464
N.S.	1	1.00	0.96	1.00	0.41	1.22	0.00	0.41	6.11
time (sec)	N/A	0.219	0.055	0.112	0.286	0.252	0.000	0.270	5.059

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	118	89	98	41	118	0	41	642
N.S.	1	1.17	0.88	0.97	0.41	1.17	0.00	0.41	6.36
time (sec)	N/A	0.288	0.126	0.108	0.283	0.263	0.000	0.273	5.098

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	160	107	120	52	144	0	52	822
N.S.	1	1.25	0.84	0.94	0.41	1.12	0.00	0.41	6.42
time (sec)	N/A	0.359	0.122	0.112	0.289	0.247	0.000	0.270	4.379

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	151	144	280	75	188	0	76	523
N.S.	1	1.12	1.07	2.07	0.56	1.39	0.00	0.56	3.87
time (sec)	N/A	0.384	0.165	0.779	0.303	0.253	0.000	0.269	4.783

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	121	119	181	64	161	0	65	463
N.S.	1	1.12	1.10	1.68	0.59	1.49	0.00	0.60	4.29
time (sec)	N/A	0.316	0.112	0.546	0.325	0.253	0.000	0.267	4.381

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	91	81	95	50	134	0	46	403
N.S.	1	1.10	0.98	1.14	0.60	1.61	0.00	0.55	4.86
time (sec)	N/A	0.254	0.061	0.468	0.287	0.253	0.000	0.271	4.637

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	70	61	37	115	0	36	344
N.S.	1	1.00	0.96	0.84	0.51	1.58	0.00	0.49	4.71
time (sec)	N/A	0.213	0.047	0.504	0.293	0.268	0.000	0.276	5.247

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	103	80	82	35	116	0	35	516
N.S.	1	1.06	0.82	0.85	0.36	1.20	0.00	0.36	5.32
time (sec)	N/A	0.271	0.044	0.516	0.292	0.249	0.000	0.284	5.227

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	145	104	93	51	147	0	49	705
N.S.	1	1.21	0.87	0.78	0.42	1.22	0.00	0.41	5.88
time (sec)	N/A	0.342	0.088	0.557	0.291	0.253	0.000	0.267	4.900

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	187	120	115	64	184	0	58	871
N.S.	1	1.29	0.83	0.79	0.44	1.27	0.00	0.40	6.01
time (sec)	N/A	0.438	0.162	0.582	0.300	0.266	0.000	0.267	5.388

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	229	139	137	75	210	0	70	1051
N.S.	1	1.33	0.81	0.80	0.44	1.22	0.00	0.41	6.11
time (sec)	N/A	0.523	0.186	0.695	0.348	0.262	0.000	0.271	4.974

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	150	147	255	86	227	0	77	571
N.S.	1	1.11	1.09	1.89	0.64	1.68	0.00	0.57	4.23
time (sec)	N/A	0.386	0.091	0.589	0.294	0.282	0.000	0.297	5.345

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	120	104	153	73	200	0	59	511
N.S.	1	1.09	0.95	1.39	0.66	1.82	0.00	0.54	4.65
time (sec)	N/A	0.320	0.080	0.583	0.297	0.270	0.000	0.271	4.679

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	102	96	85	61	185	0	47	667
N.S.	1	1.04	0.98	0.87	0.62	1.89	0.00	0.48	6.81
time (sec)	N/A	0.269	0.060	0.501	0.293	0.258	0.000	0.274	4.789

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	132	107	98	64	186	0	52	580
N.S.	1	1.06	0.86	0.78	0.51	1.49	0.00	0.42	4.64
time (sec)	N/A	0.320	0.109	0.579	0.316	0.253	0.000	0.271	4.679

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	174	118	112	60	186	0	47	741
N.S.	1	1.14	0.78	0.74	0.39	1.22	0.00	0.31	4.88
time (sec)	N/A	0.410	0.067	0.579	0.340	0.262	0.000	0.263	4.628

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	216	141	114	73	214	0	59	1077
N.S.	1	1.23	0.80	0.65	0.41	1.22	0.00	0.34	6.12
time (sec)	N/A	0.487	0.120	0.624	0.308	0.259	0.000	0.285	5.019

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	258	156	136	86	250	0	71	1362
N.S.	1	1.28	0.78	0.68	0.43	1.24	0.00	0.35	6.78
time (sec)	N/A	0.568	0.197	0.727	0.307	0.258	0.000	0.270	5.994

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	300	174	158	97	276	0	80	2151
N.S.	1	1.32	0.76	0.69	0.43	1.21	0.00	0.35	9.43
time (sec)	N/A	0.692	0.235	0.947	0.297	0.279	0.000	0.287	5.142

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	145	104	121	0	141	0	60	0
N.S.	1	1.02	0.73	0.85	0.00	0.99	0.00	0.42	0.00
time (sec)	N/A	0.336	0.066	0.197	0.000	0.258	0.000	0.276	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	101	84	86	0	114	0	48	0
N.S.	1	0.97	0.81	0.83	0.00	1.10	0.00	0.46	0.00
time (sec)	N/A	0.273	0.072	0.190	0.000	0.255	0.000	0.276	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	62	49	0	93	0	36	0
N.S.	1	1.00	1.02	0.80	0.00	1.52	0.00	0.59	0.00
time (sec)	N/A	0.202	0.034	0.178	0.000	0.257	0.000	0.270	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	52	97	40	89	0	57	0
N.S.	1	1.00	1.06	1.98	0.82	1.82	0.00	1.16	0.00
time (sec)	N/A	0.199	0.043	0.169	0.321	0.262	0.000	0.276	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	29	15	15	0	59	210
N.S.	1	1.00	0.97	0.83	0.43	0.43	0.00	1.69	6.00
time (sec)	N/A	0.176	0.037	0.178	0.281	0.255	0.000	0.263	4.877

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	48	59	34	34	0	112	174
N.S.	1	1.00	0.67	0.82	0.47	0.47	0.00	1.56	2.42
time (sec)	N/A	0.224	0.042	0.180	0.287	0.245	0.000	0.290	4.311

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	128	66	105	45	45	0	138	234
N.S.	1	1.16	0.60	0.95	0.41	0.41	0.00	1.25	2.13
time (sec)	N/A	0.294	0.067	0.181	0.282	0.246	0.000	0.278	4.401

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	184	82	151	56	56	0	166	294
N.S.	1	1.24	0.55	1.02	0.38	0.38	0.00	1.12	1.99
time (sec)	N/A	0.390	0.056	0.186	0.286	0.264	0.000	0.275	4.397

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	183	122	150	0	163	0	147	0
N.S.	1	1.03	0.69	0.85	0.00	0.92	0.00	0.83	0.00
time (sec)	N/A	0.412	0.070	0.178	0.000	0.256	0.000	0.289	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	105	115	0	140	0	122	0
N.S.	1	1.00	0.76	0.83	0.00	1.01	0.00	0.88	0.00
time (sec)	N/A	0.324	0.056	0.173	0.000	0.273	0.000	0.290	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	99	83	80	0	119	0	97	0
N.S.	1	0.98	0.82	0.79	0.00	1.18	0.00	0.96	0.00
time (sec)	N/A	0.263	0.054	0.174	0.000	0.269	0.000	75.073	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	83	77	126	0	109	0	90	0
N.S.	1	1.02	0.95	1.56	0.00	1.35	0.00	1.11	0.00
time (sec)	N/A	0.256	0.044	0.173	0.000	0.247	0.000	74.966	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	72	74	172	0	109	0	93	0
N.S.	1	1.03	1.06	2.46	0.00	1.56	0.00	1.33	0.00
time (sec)	N/A	0.246	0.050	0.182	0.000	0.256	0.000	74.833	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	29	15	31	0	33	332
N.S.	1	1.00	0.97	0.83	0.43	0.89	0.00	0.94	9.49
time (sec)	N/A	0.172	0.040	0.178	0.278	0.248	0.000	0.289	4.545

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	48	59	34	45	0	59	228
N.S.	1	1.00	0.67	0.82	0.47	0.62	0.00	0.82	3.17
time (sec)	N/A	0.221	0.046	0.183	0.289	0.253	0.000	0.293	4.471

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	128	66	105	45	56	0	78	288
N.S.	1	1.16	0.60	0.95	0.41	0.51	0.00	0.71	2.62
time (sec)	N/A	0.293	0.040	0.186	0.290	0.249	0.000	0.313	4.529

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	184	82	151	56	67	0	97	348
N.S.	1	1.24	0.55	1.02	0.38	0.45	0.00	0.66	2.35
time (sec)	N/A	0.381	0.060	0.203	0.322	0.250	0.000	0.292	4.627

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	177	121	144	0	162	0	204	0
N.S.	1	1.02	0.70	0.83	0.00	0.93	0.00	1.17	0.00
time (sec)	N/A	0.398	0.066	0.178	0.000	0.258	0.000	0.307	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	137	101	109	0	141	0	116	0
N.S.	1	1.01	0.74	0.80	0.00	1.04	0.00	0.85	0.00
time (sec)	N/A	0.322	0.047	0.176	0.000	0.265	0.000	74.698	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	101	155	0	137	0	108	0
N.S.	1	1.00	0.83	1.28	0.00	1.13	0.00	0.89	0.00
time (sec)	N/A	0.330	0.063	0.165	0.000	0.253	0.000	76.284	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	109	97	201	0	138	0	113	0
N.S.	1	1.03	0.92	1.90	0.00	1.30	0.00	1.07	0.00
time (sec)	N/A	0.294	0.061	0.175	0.000	0.253	0.000	75.406	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	95	95	247	0	137	0	111	0
N.S.	1	1.02	1.02	2.66	0.00	1.47	0.00	1.19	0.00
time (sec)	N/A	0.301	0.042	0.183	0.000	0.256	0.000	75.947	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	29	15	42	0	33	396
N.S.	1	1.00	0.97	0.83	0.43	1.20	0.00	0.94	11.31
time (sec)	N/A	0.158	0.053	0.185	0.296	0.256	0.000	0.292	4.454

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	48	59	34	56	0	59	293
N.S.	1	1.00	0.67	0.82	0.47	0.78	0.00	0.82	4.07
time (sec)	N/A	0.222	0.041	0.188	0.294	0.244	0.000	0.286	4.424

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	128	66	105	45	67	0	78	353
N.S.	1	1.16	0.60	0.95	0.41	0.61	0.00	0.71	3.21
time (sec)	N/A	0.304	0.051	0.206	0.292	0.244	0.000	0.307	4.795

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	184	82	151	56	78	0	97	413
N.S.	1	1.24	0.55	1.02	0.38	0.53	0.00	0.66	2.79
time (sec)	N/A	0.386	0.069	0.267	0.292	0.272	0.000	0.293	4.498

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	151	105	124	0	140	0	64	0
N.S.	1	1.04	0.72	0.86	0.00	0.97	0.00	0.44	0.00
time (sec)	N/A	0.346	0.077	0.186	0.000	0.264	0.000	0.270	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	88	89	0	119	0	52	0
N.S.	1	1.00	0.82	0.83	0.00	1.11	0.00	0.49	0.00
time (sec)	N/A	0.279	0.065	0.190	0.000	0.270	0.000	0.266	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	66	53	0	91	0	38	0
N.S.	1	1.00	1.05	0.84	0.00	1.44	0.00	0.60	0.00
time (sec)	N/A	0.213	0.054	0.177	0.000	0.267	0.000	0.275	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	33	24	0	57	0	23	0
N.S.	1	1.00	1.10	0.80	0.00	1.90	0.00	0.77	0.00
time (sec)	N/A	0.160	0.027	0.176	0.000	0.271	0.000	0.267	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	29	15	15	0	30	101
N.S.	1	1.00	0.97	0.88	0.45	0.45	0.00	0.91	3.06
time (sec)	N/A	0.168	0.049	0.179	0.307	0.244	0.000	0.273	4.544

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	46	59	33	23	0	55	218
N.S.	1	1.00	0.64	0.82	0.46	0.32	0.00	0.76	3.03
time (sec)	N/A	0.222	0.054	0.184	0.302	0.251	0.000	0.291	4.331

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	128	66	105	45	34	0	77	227
N.S.	1	1.16	0.60	0.95	0.41	0.31	0.00	0.70	2.06
time (sec)	N/A	0.299	0.050	0.256	0.298	0.252	0.000	0.273	4.592

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	184	82	151	55	45	0	103	287
N.S.	1	1.24	0.55	1.02	0.37	0.30	0.00	0.70	1.94
time (sec)	N/A	0.393	0.060	0.188	0.325	0.264	0.000	0.271	4.393

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	178	122	146	0	196	0	75	0
N.S.	1	1.07	0.73	0.88	0.00	1.18	0.00	0.45	0.00
time (sec)	N/A	0.406	0.096	0.184	0.000	0.261	0.000	0.296	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	134	104	111	0	175	0	63	0
N.S.	1	1.05	0.81	0.87	0.00	1.37	0.00	0.49	0.00
time (sec)	N/A	0.329	0.086	0.184	0.000	0.265	0.000	0.277	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	90	81	75	0	145	0	48	0
N.S.	1	1.05	0.94	0.87	0.00	1.69	0.00	0.56	0.00
time (sec)	N/A	0.268	0.072	0.170	0.000	0.259	0.000	0.284	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	55	42	0	119	0	39	0
N.S.	1	1.00	1.06	0.81	0.00	2.29	0.00	0.75	0.00
time (sec)	N/A	0.200	0.048	0.169	0.000	0.269	0.000	0.272	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	29	0	22	0	15	163
N.S.	1	1.00	0.97	0.88	0.00	0.67	0.00	0.45	4.94
time (sec)	N/A	0.165	0.030	0.179	0.000	0.239	0.000	0.281	4.853

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	43	59	32	34	0	50	281
N.S.	1	1.00	0.63	0.87	0.47	0.50	0.00	0.74	4.13
time (sec)	N/A	0.221	0.051	0.199	0.302	0.243	0.000	0.268	4.426

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	124	64	105	45	49	0	107	286
N.S.	1	1.13	0.58	0.95	0.41	0.45	0.00	0.97	2.60
time (sec)	N/A	0.299	0.058	0.174	0.338	0.256	0.000	0.308	4.527

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	180	80	151	54	58	0	161	346
N.S.	1	1.22	0.54	1.02	0.36	0.39	0.00	1.09	2.34
time (sec)	N/A	0.389	0.061	0.180	0.304	0.257	0.000	0.298	4.852

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	165	121	133	0	241	0	75	0
N.S.	1	1.08	0.79	0.87	0.00	1.58	0.00	0.49	0.00
time (sec)	N/A	0.412	0.101	0.177	0.000	0.282	0.000	0.272	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	121	101	97	0	214	0	61	0
N.S.	1	1.09	0.91	0.87	0.00	1.93	0.00	0.55	0.00
time (sec)	N/A	0.319	0.092	0.175	0.000	0.264	0.000	0.281	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	80	78	59	0	186	0	49	0
N.S.	1	1.07	1.04	0.79	0.00	2.48	0.00	0.65	0.00
time (sec)	N/A	0.252	0.061	0.177	0.000	0.279	0.000	0.280	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	92	0	33	0	15	229
N.S.	1	1.00	0.97	2.63	0.00	0.94	0.00	0.43	6.54
time (sec)	N/A	0.163	0.048	0.174	0.000	0.265	0.000	0.292	4.745

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	47	58	0	43	0	25	346
N.S.	1	1.00	0.66	0.82	0.00	0.61	0.00	0.35	4.87
time (sec)	N/A	0.227	0.036	0.184	0.000	0.260	0.000	0.277	4.615

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	123	66	104	45	58	0	62	348
N.S.	1	1.16	0.62	0.98	0.42	0.55	0.00	0.58	3.28
time (sec)	N/A	0.298	0.057	0.177	0.296	0.263	0.000	0.287	4.582

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	177	79	150	56	71	0	119	406
N.S.	1	1.21	0.54	1.03	0.38	0.49	0.00	0.82	2.78
time (sec)	N/A	0.389	0.060	0.189	0.294	0.255	0.000	0.294	4.763

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	233	100	196	67	82	0	173	466
N.S.	1	1.25	0.54	1.05	0.36	0.44	0.00	0.93	2.51
time (sec)	N/A	0.490	0.063	0.183	0.287	0.278	0.000	0.314	5.032

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	71	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	150	146	406	139	374	0	332	546
N.S.	1	0.91	0.88	2.46	0.84	2.27	0.00	2.01	3.31
time (sec)	N/A	0.429	0.084	3.064	0.361	0.261	0.000	0.262	4.711

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	116	106	277	101	255	0	226	418
N.S.	1	0.96	0.88	2.29	0.83	2.11	0.00	1.87	3.45
time (sec)	N/A	0.345	0.071	1.209	0.402	0.253	0.000	0.275	4.603

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	71	163	68	168	0	140	304
N.S.	1	1.00	0.87	1.99	0.83	2.05	0.00	1.71	3.71
time (sec)	N/A	0.276	0.048	0.597	0.387	0.267	0.000	0.269	4.607

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	41	77	42	91	0	76	205
N.S.	1	1.00	0.85	1.60	0.88	1.90	0.00	1.58	4.27
time (sec)	N/A	0.217	0.040	0.365	0.353	0.259	0.000	0.267	4.545

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	21	39	51	28	121
N.S.	1	1.00	1.00	1.05	1.05	1.95	2.55	1.40	6.05
time (sec)	N/A	0.161	0.015	0.315	0.339	0.250	0.271	0.262	4.412

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	60	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.201	0.072	0.000	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	67	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	0.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	98	67	0	0	0	0	0	0
N.S.	1	0.97	0.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.301	0.036	0.000	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	49	38	81	0	90	96
N.S.	1	1.00	0.92	1.32	1.03	2.19	0.00	2.43	2.59
time (sec)	N/A	0.179	0.048	0.151	0.200	0.247	0.000	0.283	4.723

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	19	76	71	19
N.S.	1	1.00	0.87	0.87	0.83	0.83	3.30	3.09	0.83
time (sec)	N/A	0.169	0.022	0.145	0.233	0.246	4.431	0.281	0.081

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	19	180	71	19
N.S.	1	1.00	0.87	0.87	0.83	0.83	7.83	3.09	0.83
time (sec)	N/A	0.181	0.015	0.116	0.231	0.240	2.639	0.258	4.211

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	16	14	78	69	16
N.S.	1	1.00	1.12	0.94	1.00	0.88	4.88	4.31	1.00
time (sec)	N/A	0.154	0.008	0.120	0.227	0.234	1.541	0.273	0.050

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	23	34	15	0	15	59
N.S.	1	1.00	0.90	1.10	1.62	0.71	0.00	0.71	2.81
time (sec)	N/A	0.174	0.015	0.099	0.200	0.248	0.000	0.283	4.327

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	18	17	18	68	70	17
N.S.	1	1.00	1.06	1.06	1.00	1.06	4.00	4.12	1.00
time (sec)	N/A	0.160	0.016	0.115	0.243	0.243	2.619	0.275	0.081

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	20	19	16	80	71	16
N.S.	1	1.00	0.78	0.87	0.83	0.70	3.48	3.09	0.70
time (sec)	N/A	0.162	0.014	0.117	0.241	0.241	4.332	0.275	0.065

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	40	39	36	33	58	0	0	0
N.S.	1	1.48	1.44	1.33	1.22	2.15	0.00	0.00	0.00
time (sec)	N/A	0.304	0.019	0.244	0.221	0.254	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	51	69	70	400	56	88	0	0	0
N.S.	1	1.35	1.37	7.84	1.10	1.73	0.00	0.00	0.00
time (sec)	N/A	0.427	0.014	0.189	0.228	0.258	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	77	99	91	422	78	118	0	0	0
N.S.	1	1.29	1.18	5.48	1.01	1.53	0.00	0.00	0.00
time (sec)	N/A	0.572	0.023	0.210	0.239	0.261	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	307	395	265	5320	281	900	0	0	0
N.S.	1	1.29	0.86	17.33	0.92	2.93	0.00	0.00	0.00
time (sec)	N/A	1.429	0.644	8.682	0.461	0.289	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	231	305	199	5016	215	746	0	0	0
N.S.	1	1.32	0.86	21.71	0.93	3.23	0.00	0.00	0.00
time (sec)	N/A	1.046	0.346	2.250	0.469	0.276	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	207	131	348	142	552	0	0	0
N.S.	1	1.38	0.87	2.32	0.95	3.68	0.00	0.00	0.00
time (sec)	N/A	0.625	3.590	1.888	0.476	0.298	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13
time (sec)	N/A	0.293	10.638	0.086	0.885	0.260	0.808	0.783	4.826

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	155	199	148	1721	149	451	0	0	0
N.S.	1	1.28	0.95	11.10	0.96	2.91	0.00	0.00	0.00
time (sec)	N/A	1.042	0.145	1.386	0.716	0.267	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	128	165	122	1662	125	382	0	0	0
N.S.	1	1.29	0.95	12.98	0.98	2.98	0.00	0.00	0.00
time (sec)	N/A	0.823	0.085	1.061	0.727	0.256	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	101	131	91	1579	101	323	0	0	0
N.S.	1	1.30	0.90	15.63	1.00	3.20	0.00	0.00	0.00
time (sec)	N/A	0.654	0.097	0.858	0.740	0.272	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	90	63	255	72	239	0	0	0
N.S.	1	1.30	0.91	3.70	1.04	3.46	0.00	0.00	0.00
time (sec)	N/A	0.454	0.843	0.847	0.829	0.272	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12
time (sec)	N/A	0.248	3.883	0.084	0.861	0.272	0.521	0.356	4.634

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	168	218	148	1725	146	424	0	0	0
N.S.	1	1.30	0.88	10.27	0.87	2.52	0.00	0.00	0.00
time (sec)	N/A	1.086	0.154	1.325	0.764	0.264	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	139	182	123	1668	123	360	0	0	0
N.S.	1	1.31	0.88	12.00	0.88	2.59	0.00	0.00	0.00
time (sec)	N/A	0.871	0.096	1.162	0.717	0.274	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	110	146	93	1587	100	306	0	0	0
N.S.	1	1.33	0.85	14.43	0.91	2.78	0.00	0.00	0.00
time (sec)	N/A	0.684	0.094	0.893	0.729	0.263	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	103	66	272	73	228	0	0	0
N.S.	1	1.36	0.87	3.58	0.96	3.00	0.00	0.00	0.00
time (sec)	N/A	0.463	0.834	0.858	0.731	0.278	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	20	19	17	19	19
N.S.	1	1.00	1.11	0.89	1.05	1.00	0.89	1.00	1.00
time (sec)	N/A	0.235	4.688	0.085	0.902	0.247	0.524	0.391	4.266

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	303	391	265	5248	277	880	0	0	0
N.S.	1	1.29	0.87	17.32	0.91	2.90	0.00	0.00	0.00
time (sec)	N/A	1.434	0.631	9.159	0.465	0.302	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	301	199	4944	213	730	0	0	0
N.S.	1	1.31	0.87	21.59	0.93	3.19	0.00	0.00	0.00
time (sec)	N/A	1.056	0.390	2.591	0.458	0.294	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	207	131	348	142	540	0	0	0
N.S.	1	1.38	0.87	2.32	0.95	3.60	0.00	0.00	0.00
time (sec)	N/A	0.640	3.332	1.997	0.488	0.279	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13
time (sec)	N/A	0.292	10.329	0.092	0.962	0.251	1.847	0.781	4.543

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	152	195	145	1693	146	424	0	0	0
N.S.	1	1.28	0.95	11.14	0.96	2.79	0.00	0.00	0.00
time (sec)	N/A	1.013	0.141	1.405	0.716	0.266	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	126	162	120	1636	123	360	0	0	0
N.S.	1	1.29	0.95	12.98	0.98	2.86	0.00	0.00	0.00
time (sec)	N/A	0.826	0.092	1.178	0.732	0.259	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	100	128	90	1555	100	306	0	0	0
N.S.	1	1.28	0.90	15.55	1.00	3.06	0.00	0.00	0.00
time (sec)	N/A	0.655	0.087	0.925	0.762	0.268	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	89	63	255	72	227	0	0	0
N.S.	1	1.29	0.91	3.70	1.04	3.29	0.00	0.00	0.00
time (sec)	N/A	0.451	0.728	0.936	0.723	0.258	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12
time (sec)	N/A	0.242	3.788	0.094	0.916	0.255	1.603	0.359	4.404

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	165	214	151	1753	149	451	0	0	0
N.S.	1	1.30	0.92	10.62	0.90	2.73	0.00	0.00	0.00
time (sec)	N/A	1.088	0.151	1.457	0.796	0.269	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	137	179	125	1694	125	382	0	0	0
N.S.	1	1.31	0.91	12.36	0.91	2.79	0.00	0.00	0.00
time (sec)	N/A	0.869	0.093	1.201	0.737	0.260	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	109	143	94	1611	101	323	0	0	0
N.S.	1	1.31	0.86	14.78	0.93	2.96	0.00	0.00	0.00
time (sec)	N/A	0.683	0.094	0.967	0.727	0.261	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	102	66	272	73	240	0	0	0
N.S.	1	1.34	0.87	3.58	0.96	3.16	0.00	0.00	0.00
time (sec)	N/A	0.483	0.685	0.914	0.742	0.251	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	20	19	17	19	19
N.S.	1	1.00	1.11	0.89	1.05	1.00	0.89	1.00	1.00
time (sec)	N/A	0.252	3.902	0.074	0.913	0.256	1.630	0.379	4.322

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	302	355	654	3640	0	1809	0	0	0
N.S.	1	1.18	2.17	12.05	0.00	5.99	0.00	0.00	0.00
time (sec)	N/A	1.177	1.059	13.451	0.000	0.330	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	234	271	409	2719	0	1283	0	0	0
N.S.	1	1.16	1.75	11.62	0.00	5.48	0.00	0.00	0.00
time (sec)	N/A	0.886	0.552	10.254	0.000	0.302	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	162	182	295	1818	0	835	0	0	0
N.S.	1	1.12	1.82	11.22	0.00	5.15	0.00	0.00	0.00
time (sec)	N/A	0.608	0.396	1.081	0.000	0.294	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	86	127	169	182	499	0	0	0
N.S.	1	1.09	1.61	2.14	2.30	6.32	0.00	0.00	0.00
time (sec)	N/A	0.361	0.031	0.750	0.331	0.277	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13
time (sec)	N/A	0.211	7.540	0.096	1.516	0.254	0.511	0.996	4.836

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	395	522	349	6921	0	2165	0	0	0
N.S.	1	1.32	0.88	17.52	0.00	5.48	0.00	0.00	0.00
time (sec)	N/A	1.549	3.611	21.303	0.000	0.336	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	295	402	259	6547	0	1689	0	0	0
N.S.	1	1.36	0.88	22.19	0.00	5.73	0.00	0.00	0.00
time (sec)	N/A	1.095	2.861	2.595	0.000	0.354	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	194	287	365	557	372	1185	0	0	0
N.S.	1	1.48	1.88	2.87	1.92	6.11	0.00	0.00	0.00
time (sec)	N/A	0.692	4.008	1.975	0.347	0.318	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13
time (sec)	N/A	0.306	2.158	0.073	3.417	0.264	0.673	1.544	5.517

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	170	218	155	2277	343	345	0	0	0
N.S.	1	1.28	0.91	13.39	2.02	2.03	0.00	0.00	0.00
time (sec)	N/A	0.902	0.262	1.737	0.244	0.267	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	133	171	119	2187	248	293	0	0	0
N.S.	1	1.29	0.89	16.44	1.86	2.20	0.00	0.00	0.00
time (sec)	N/A	0.697	0.167	1.138	0.239	0.281	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	93	123	766	307	260	218	0	0	0
N.S.	1	1.32	8.24	3.30	2.80	2.34	0.00	0.00	0.00
time (sec)	N/A	0.487	8.086	1.246	0.290	0.271	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	144	37	17	20	21
N.S.	1	1.00	1.10	0.90	7.20	1.85	0.85	1.00	1.05
time (sec)	N/A	0.263	0.883	0.087	4.152	0.250	0.701	0.806	5.113

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	171	223	156	2387	342	345	0	0	0
N.S.	1	1.30	0.91	13.96	2.00	2.02	0.00	0.00	0.00
time (sec)	N/A	0.923	0.245	1.590	0.238	0.273	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	134	176	120	2289	247	293	0	0	0
N.S.	1	1.31	0.90	17.08	1.84	2.19	0.00	0.00	0.00
time (sec)	N/A	0.714	0.158	1.123	0.258	0.266	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	94	128	723	332	262	219	0	0	0
N.S.	1	1.36	7.69	3.53	2.79	2.33	0.00	0.00	0.00
time (sec)	N/A	0.482	5.131	1.272	0.295	0.275	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	143	37	17	21	22
N.S.	1	1.00	1.10	0.90	6.81	1.76	0.81	1.00	1.05
time (sec)	N/A	0.263	0.805	0.092	4.266	0.264	0.760	0.812	4.723

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	302	355	654	3640	0	1567	0	0	0
N.S.	1	1.18	2.17	12.05	0.00	5.19	0.00	0.00	0.00
time (sec)	N/A	1.183	0.196	13.998	0.000	0.351	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	234	271	409	2719	0	1085	0	0	0
N.S.	1	1.16	1.75	11.62	0.00	4.64	0.00	0.00	0.00
time (sec)	N/A	0.853	0.115	10.208	0.000	0.338	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	162	182	295	1819	0	681	0	0	0
N.S.	1	1.12	1.82	11.23	0.00	4.20	0.00	0.00	0.00
time (sec)	N/A	0.605	0.202	1.115	0.000	0.311	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	86	127	145	184	389	0	0	0
N.S.	1	1.09	1.61	1.84	2.33	4.92	0.00	0.00	0.00
time (sec)	N/A	0.350	0.019	0.754	0.333	0.282	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13
time (sec)	N/A	0.218	0.087	0.107	1.619	0.252	0.654	0.434	5.019

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	391	514	341	6693	0	1799	0	0	0
N.S.	1	1.31	0.87	17.12	0.00	4.60	0.00	0.00	0.00
time (sec)	N/A	1.523	3.514	24.297	0.000	0.457	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	293	394	255	6343	0	1463	0	0	0
N.S.	1	1.34	0.87	21.65	0.00	4.99	0.00	0.00	0.00
time (sec)	N/A	1.128	2.823	3.452	0.000	0.410	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	194	283	390	563	392	1099	0	0	0
N.S.	1	1.46	2.01	2.90	2.02	5.66	0.00	0.00	0.00
time (sec)	N/A	0.691	3.949	2.409	0.374	0.382	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13
time (sec)	N/A	0.298	1.990	0.165	3.387	0.249	0.959	0.696	5.562

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	168	219	155	2387	344	180	0	0	0
N.S.	1	1.30	0.92	14.21	2.05	1.07	0.00	0.00	0.00
time (sec)	N/A	0.908	0.280	2.031	0.225	0.273	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	132	172	119	2289	249	157	0	0	0
N.S.	1	1.30	0.90	17.34	1.89	1.19	0.00	0.00	0.00
time (sec)	N/A	0.739	0.183	1.421	0.205	0.250	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	93	126	709	307	288	122	0	0	0
N.S.	1	1.35	7.62	3.30	3.10	1.31	0.00	0.00	0.00
time (sec)	N/A	0.492	10.662	1.496	0.280	0.270	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	143	37	17	20	21
N.S.	1	1.00	1.10	0.90	7.15	1.85	0.85	1.00	1.05
time (sec)	N/A	0.260	0.837	0.135	4.752	0.251	1.002	0.469	4.939

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	169	216	155	2277	345	180	0	0	0
N.S.	1	1.28	0.92	13.47	2.04	1.07	0.00	0.00	0.00
time (sec)	N/A	0.917	0.252	1.796	0.231	0.270	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	133	169	119	2187	250	157	0	0	0
N.S.	1	1.27	0.89	16.44	1.88	1.18	0.00	0.00	0.00
time (sec)	N/A	0.704	0.190	1.271	0.217	0.252	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	94	123	605	332	286	122	0	0	0
N.S.	1	1.31	6.44	3.53	3.04	1.30	0.00	0.00	0.00
time (sec)	N/A	0.484	9.496	1.431	0.282	0.258	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	144	37	19	21	22
N.S.	1	1.00	1.10	0.90	6.86	1.76	0.90	1.00	1.05
time (sec)	N/A	0.263	0.842	0.131	5.077	0.254	1.025	0.451	4.554

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	51	18	58	65	0	0	0
N.S.	1	1.00	2.43	0.86	2.76	3.10	0.00	0.00	0.00
time (sec)	N/A	0.198	0.080	0.066	0.187	0.253	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	42	71	32	59	95	0	0	0
N.S.	1	0.98	1.65	0.74	1.37	2.21	0.00	0.00	0.00
time (sec)	N/A	0.341	0.018	0.061	0.202	0.243	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	70	93	49	76	120	0	0	0
N.S.	1	1.21	1.60	0.84	1.31	2.07	0.00	0.00	0.00
time (sec)	N/A	0.460	0.019	0.076	0.195	0.251	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	33	68	32	107	138	0	0	0
N.S.	1	0.94	1.94	0.91	3.06	3.94	0.00	0.00	0.00
time (sec)	N/A	0.209	0.016	0.138	0.231	0.255	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	113	155	108	199	0	0	0
N.S.	1	1.00	1.59	2.18	1.52	2.80	0.00	0.00	0.00
time (sec)	N/A	0.383	0.038	0.148	0.198	0.248	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	118	149	197	142	248	0	0	0
N.S.	1	1.17	1.48	1.95	1.41	2.46	0.00	0.00	0.00
time (sec)	N/A	0.548	0.031	0.194	0.199	0.268	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	149	108	160	202	284	0	0	0
N.S.	1	0.89	0.64	0.95	1.20	1.69	0.00	0.00	0.00
time (sec)	N/A	0.528	0.495	0.674	0.205	0.257	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	177	596	194	396	0	0	0
N.S.	1	1.00	0.84	2.82	0.92	1.88	0.00	0.00	0.00
time (sec)	N/A	0.686	0.079	0.690	0.226	0.258	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	289	235	672	254	480	0	0	0
N.S.	1	1.09	0.89	2.55	0.96	1.82	0.00	0.00	0.00
time (sec)	N/A	0.994	0.054	1.142	0.225	0.269	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	107	91	153	868	184	234	0	157	179
N.S.	1	0.85	1.43	8.11	1.72	2.19	0.00	1.47	1.67
time (sec)	N/A	0.491	0.096	3.449	0.285	0.255	0.000	0.445	4.543

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	49	46	60	887	64	93	0	147	111
N.S.	1	0.94	1.22	18.10	1.31	1.90	0.00	3.00	2.27
time (sec)	N/A	0.331	0.060	0.319	0.232	0.247	0.000	0.318	4.563

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	41	46	37	43	46	56	35	28
N.S.	1	0.91	1.02	0.82	0.96	1.02	1.24	0.78	0.62
time (sec)	N/A	0.306	0.043	0.168	0.207	0.235	0.562	0.280	0.173

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	41	46	29	43	57	0	40	28
N.S.	1	0.91	1.02	0.64	0.96	1.27	0.00	0.89	0.62
time (sec)	N/A	0.305	0.043	0.204	0.207	0.253	0.000	0.264	4.510

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	49	46	59	872	64	92	0	98	119
N.S.	1	0.94	1.20	17.80	1.31	1.88	0.00	2.00	2.43
time (sec)	N/A	0.328	0.046	0.340	0.196	0.257	0.000	0.373	4.665

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	107	91	150	842	184	233	0	167	187
N.S.	1	0.85	1.40	7.87	1.72	2.18	0.00	1.56	1.75
time (sec)	N/A	0.512	0.088	3.425	0.284	0.271	0.000	0.436	4.562

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	136	136	114	327	0	327	0	0	0
N.S.	1	1.00	0.84	2.40	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.754	0.182	150.796	0.000	0.277	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [281] had the largest ratio of [2.3333300000000013]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.14	23	0.261
2	A	6	5	1.14	23	0.217
3	A	5	4	1.13	21	0.190
4	C	12	11	0.78	23	0.478
5	A	2	2	1.00	23	0.087
6	A	3	3	1.01	23	0.130
7	A	4	4	1.05	23	0.174
8	A	5	5	1.07	23	0.217
9	A	5	4	0.98	23	0.174
10	A	5	4	1.02	23	0.174
11	A	5	4	1.06	23	0.174
12	A	2	2	1.00	19	0.105
13	A	5	4	1.00	23	0.174
14	A	6	5	0.95	23	0.217
15	A	7	6	1.00	23	0.261
16	A	7	6	1.09	25	0.240
17	A	6	5	1.08	25	0.200
18	A	5	4	1.06	25	0.160
19	A	4	3	1.00	25	0.120
20	A	5	4	1.03	25	0.160
21	A	6	5	1.05	25	0.200
22	A	7	6	1.06	25	0.240

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	9	8	1.07	25	0.320
24	A	8	7	1.06	25	0.280
25	A	7	6	1.07	25	0.240
26	A	8	7	1.03	25	0.280
27	A	9	8	1.04	25	0.320
28	A	5	4	0.82	12	0.333
29	A	5	4	0.81	14	0.286
30	N/A	1	0	1.00	40	0.000
31	A	7	6	0.98	40	0.150
32	A	6	5	1.01	40	0.125
33	A	3	2	0.94	38	0.053
34	N/A	1	0	1.00	40	0.000
35	N/A	1	0	1.00	40	0.000
36	A	2	2	1.00	11	0.182
37	A	2	2	1.00	11	0.182
38	A	2	2	1.00	9	0.222
39	A	3	2	1.00	7	0.286
40	A	2	2	1.00	11	0.182
41	A	2	2	1.00	11	0.182
42	A	2	2	1.00	11	0.182
43	A	2	2	1.00	11	0.182
44	A	3	3	0.93	13	0.231
45	A	3	3	1.07	13	0.231
46	A	3	3	1.07	13	0.231
47	A	4	3	1.00	11	0.273
48	A	3	2	1.00	9	0.222
49	A	3	3	1.02	13	0.231
50	A	3	3	1.00	13	0.231
51	A	3	3	1.00	13	0.231
52	A	1	1	1.00	13	0.077
53	A	2	2	1.52	13	0.154
54	A	4	4	0.86	13	0.308
55	A	4	4	1.10	13	0.308

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	5	4	1.15	13	0.308
57	A	4	3	1.00	11	0.273
58	A	3	2	1.00	9	0.222
59	A	4	4	1.03	13	0.308
60	A	4	4	1.00	13	0.308
61	A	4	4	1.02	13	0.308
62	A	4	4	1.00	13	0.308
63	A	1	1	1.00	13	0.077
64	A	2	2	1.00	13	0.154
65	A	5	5	0.81	13	0.385
66	A	5	5	1.11	13	0.385
67	A	5	5	1.11	13	0.385
68	A	5	5	1.11	13	0.385
69	A	6	5	1.22	13	0.385
70	A	5	4	1.15	13	0.308
71	A	4	3	1.00	11	0.273
72	A	3	2	1.00	9	0.222
73	A	5	5	1.03	13	0.385
74	A	5	5	1.02	13	0.385
75	A	5	5	1.01	13	0.385
76	A	5	5	1.08	13	0.385
77	A	5	5	1.00	13	0.385
78	A	1	1	1.00	13	0.077
79	A	2	2	1.00	13	0.154
80	A	3	3	1.18	13	0.231
81	B	4	4	2.10	13	0.308
82	A	5	5	1.11	13	0.385
83	A	5	5	1.11	13	0.385
84	A	4	3	1.00	11	0.273
85	A	1	1	1.00	13	0.077
86	A	6	5	1.07	13	0.385
87	A	5	4	1.05	13	0.308
88	A	4	3	1.00	11	0.273
89	A	3	2	1.00	9	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
90	A	5	4	1.00	13	0.308
91	A	6	5	1.25	13	0.385
92	A	7	6	1.32	13	0.462
93	A	2	2	1.00	13	0.154
94	A	7	6	1.12	13	0.462
95	A	6	5	1.09	13	0.385
96	A	5	4	1.08	13	0.308
97	A	4	3	1.00	11	0.273
98	A	3	2	1.00	9	0.222
99	A	6	5	1.26	13	0.385
100	A	7	6	1.32	13	0.462
101	A	8	7	1.31	13	0.538
102	A	3	3	1.03	13	0.231
103	A	7	6	1.16	13	0.462
104	A	6	5	1.14	13	0.385
105	A	5	4	1.11	13	0.308
106	A	4	3	1.00	11	0.273
107	A	3	2	1.00	9	0.222
108	A	7	6	1.38	13	0.462
109	A	8	7	1.38	13	0.538
110	A	9	8	1.36	13	0.615
111	A	7	6	1.24	15	0.400
112	A	6	5	1.18	15	0.333
113	A	5	4	1.14	15	0.267
114	A	4	3	1.00	13	0.231
115	A	3	2	1.00	11	0.182
116	A	2	2	1.00	15	0.133
117	A	2	2	1.00	15	0.133
118	A	4	4	0.99	15	0.267
119	A	6	6	1.09	15	0.400
120	A	7	6	1.24	15	0.400
121	A	6	5	1.20	15	0.333
122	A	5	4	1.14	15	0.267
123	A	4	3	1.00	13	0.231

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
124	A	3	2	1.00	11	0.182
125	A	3	3	1.03	15	0.200
126	A	3	3	1.05	15	0.200
127	A	3	3	0.98	15	0.200
128	A	5	5	1.01	15	0.333
129	A	7	6	1.24	15	0.400
130	A	6	5	1.20	15	0.333
131	A	5	4	1.14	15	0.267
132	A	4	3	1.00	13	0.231
133	A	3	2	1.00	11	0.182
134	A	4	4	1.03	15	0.267
135	A	4	4	1.05	15	0.267
136	A	4	4	0.99	15	0.267
137	A	4	4	1.01	15	0.267
138	A	6	6	1.03	15	0.400
139	A	8	8	1.10	15	0.533
140	A	7	6	1.20	15	0.400
141	A	6	5	1.21	15	0.333
142	A	5	4	1.11	15	0.267
143	A	4	3	1.00	13	0.231
144	A	3	2	1.00	11	0.182
145	A	1	1	1.00	15	0.067
146	A	3	3	1.06	15	0.200
147	A	5	5	1.09	15	0.333
148	A	7	7	1.15	15	0.467
149	A	7	6	1.23	15	0.400
150	A	6	5	1.19	15	0.333
151	A	5	4	1.11	15	0.267
152	A	4	3	1.00	13	0.231
153	A	3	2	1.00	11	0.182
154	A	2	2	1.00	15	0.133
155	A	4	4	1.19	15	0.267
156	A	6	6	1.15	15	0.400
157	A	8	8	1.19	15	0.533

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
158	A	7	6	1.18	15	0.400
159	A	6	5	1.16	15	0.333
160	A	5	4	1.07	15	0.267
161	A	4	3	1.00	13	0.231
162	A	3	2	1.00	11	0.182
163	A	3	3	1.17	15	0.200
164	A	5	5	1.26	15	0.333
165	A	7	7	1.20	15	0.467
166	A	9	9	1.22	15	0.600
167	A	2	2	1.00	13	0.154
168	A	2	2	1.00	13	0.154
169	A	2	2	1.00	13	0.154
170	A	2	2	1.00	13	0.154
171	A	2	2	1.00	13	0.154
172	A	2	2	1.00	13	0.154
173	A	2	2	1.00	13	0.154
174	A	2	2	1.00	13	0.154
175	A	3	3	1.06	15	0.200
176	A	3	3	1.06	15	0.200
177	A	3	3	1.06	15	0.200
178	A	3	3	1.06	15	0.200
179	A	3	3	1.02	15	0.200
180	A	3	3	1.02	15	0.200
181	A	3	3	0.98	15	0.200
182	A	3	3	1.06	15	0.200
183	A	4	4	1.09	15	0.267
184	A	4	4	1.09	15	0.267
185	A	4	4	1.09	15	0.267
186	A	4	4	1.06	15	0.267
187	A	4	4	1.09	15	0.267
188	A	4	4	1.06	15	0.267
189	A	4	4	1.03	15	0.267
190	A	4	4	1.03	15	0.267
191	A	5	5	1.08	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
192	A	4	4	1.07	15	0.267
193	A	3	3	1.06	15	0.200
194	A	2	2	1.00	15	0.133
195	A	1	1	1.00	15	0.067
196	A	2	2	1.00	15	0.133
197	A	3	3	1.17	15	0.200
198	A	4	4	1.25	15	0.267
199	A	5	5	1.12	15	0.333
200	A	4	4	1.12	15	0.267
201	A	3	3	1.10	15	0.200
202	A	2	2	1.00	15	0.133
203	A	3	3	1.06	15	0.200
204	A	4	4	1.21	15	0.267
205	A	5	5	1.29	15	0.333
206	A	6	6	1.33	15	0.400
207	A	5	5	1.11	15	0.333
208	A	4	4	1.09	15	0.267
209	A	3	3	1.04	15	0.200
210	A	4	4	1.06	15	0.267
211	A	5	5	1.14	15	0.333
212	A	6	6	1.23	15	0.400
213	A	7	7	1.28	15	0.467
214	A	8	8	1.32	15	0.533
215	A	4	4	1.02	17	0.235
216	A	3	3	0.97	17	0.176
217	A	2	2	1.00	17	0.118
218	A	2	2	1.00	17	0.118
219	A	1	1	1.00	17	0.059
220	A	2	2	1.00	17	0.118
221	A	3	3	1.16	17	0.176
222	A	4	4	1.24	17	0.235
223	A	5	5	1.03	17	0.294
224	A	4	4	1.00	17	0.235
225	A	3	3	0.98	17	0.176

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
226	A	3	3	1.02	17	0.176
227	A	3	3	1.03	17	0.176
228	A	1	1	1.00	17	0.059
229	A	2	2	1.00	17	0.118
230	A	3	3	1.16	17	0.176
231	A	4	4	1.24	17	0.235
232	A	5	5	1.02	17	0.294
233	A	4	4	1.01	17	0.235
234	A	4	4	1.00	17	0.235
235	A	4	4	1.03	17	0.235
236	A	4	4	1.02	17	0.235
237	A	1	1	1.00	17	0.059
238	A	2	2	1.00	17	0.118
239	A	3	3	1.16	17	0.176
240	A	4	4	1.24	17	0.235
241	A	4	4	1.04	17	0.235
242	A	3	3	1.00	17	0.176
243	A	2	2	1.00	17	0.118
244	A	1	1	1.00	17	0.059
245	A	1	1	1.00	17	0.059
246	A	2	2	1.00	17	0.118
247	A	3	3	1.16	17	0.176
248	A	4	4	1.24	17	0.235
249	A	5	5	1.07	17	0.294
250	A	4	4	1.05	17	0.235
251	A	3	3	1.05	17	0.176
252	A	2	2	1.00	17	0.118
253	A	1	1	1.00	17	0.059
254	A	2	2	1.00	17	0.118
255	A	3	3	1.13	17	0.176
256	A	4	4	1.22	17	0.235
257	A	5	5	1.08	17	0.294
258	A	4	4	1.09	17	0.235
259	A	3	3	1.07	17	0.176

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
260	A	1	1	1.00	17	0.059
261	A	2	2	1.00	17	0.118
262	A	3	3	1.16	17	0.176
263	A	4	4	1.21	17	0.235
264	A	5	5	1.25	17	0.294
265	A	1	1	1.00	13	0.077
266	A	7	6	0.91	13	0.462
267	A	6	5	0.96	13	0.385
268	A	5	4	1.00	13	0.308
269	A	4	3	1.00	11	0.273
270	A	3	2	1.00	9	0.222
271	A	1	1	1.00	13	0.077
272	A	2	2	1.00	13	0.154
273	A	3	3	0.97	13	0.231
274	A	2	2	1.00	11	0.182
275	A	2	2	1.00	11	0.182
276	A	2	2	1.00	9	0.222
277	A	3	2	1.00	7	0.286
278	A	2	2	1.00	11	0.182
279	A	2	2	1.00	11	0.182
280	A	2	2	1.00	11	0.182
281	C	8	7	1.48	3	2.333
282	C	9	8	1.35	5	1.600
283	C	10	9	1.29	7	1.286
284	A	7	6	1.29	15	0.400
285	A	6	5	1.32	13	0.385
286	A	5	4	1.38	11	0.364
287	N/A	1	0	1.00	15	0.000
288	A	9	8	1.28	16	0.500
289	A	8	7	1.29	16	0.438
290	A	7	6	1.30	14	0.429
291	A	6	5	1.30	12	0.417
292	N/A	1	0	1.00	16	0.000

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
293	A	9	8	1.30	19	0.421
294	A	8	7	1.31	19	0.368
295	A	7	6	1.33	17	0.353
296	A	6	5	1.36	15	0.333
297	N/A	1	0	1.00	19	0.000
298	A	7	6	1.29	15	0.400
299	A	6	5	1.31	13	0.385
300	A	5	4	1.38	11	0.364
301	N/A	1	0	1.00	15	0.000
302	A	9	8	1.28	16	0.500
303	A	8	7	1.29	16	0.438
304	A	7	6	1.28	14	0.429
305	A	6	5	1.29	12	0.417
306	N/A	1	0	1.00	16	0.000
307	A	9	8	1.30	19	0.421
308	A	8	7	1.31	19	0.368
309	A	7	6	1.31	17	0.353
310	A	6	5	1.34	15	0.333
311	N/A	1	0	1.00	19	0.000
312	A	9	8	1.18	15	0.533
313	A	8	7	1.16	15	0.467
314	A	7	6	1.12	13	0.462
315	A	6	5	1.09	7	0.714
316	N/A	1	0	1.00	15	0.000
317	A	7	6	1.32	15	0.400
318	A	6	5	1.36	13	0.385
319	A	5	4	1.48	11	0.364
320	N/A	1	0	1.00	15	0.000
321	A	8	7	1.28	20	0.350
322	A	7	6	1.29	18	0.333
323	A	6	5	1.32	16	0.312
324	N/A	1	0	1.00	20	0.000
325	A	8	7	1.30	21	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
326	A	7	6	1.31	19	0.316
327	A	6	5	1.36	17	0.294
328	N/A	1	0	1.00	21	0.000
329	A	9	8	1.18	15	0.533
330	A	8	7	1.16	15	0.467
331	A	7	6	1.12	13	0.462
332	A	6	5	1.09	7	0.714
333	N/A	1	0	1.00	15	0.000
334	A	7	6	1.31	15	0.400
335	A	6	5	1.34	13	0.385
336	A	5	4	1.46	11	0.364
337	N/A	1	0	1.00	15	0.000
338	A	8	7	1.30	20	0.350
339	A	7	6	1.30	18	0.333
340	A	6	5	1.35	16	0.312
341	N/A	1	0	1.00	20	0.000
342	A	8	7	1.28	21	0.333
343	A	7	6	1.27	19	0.316
344	A	6	5	1.31	17	0.294
345	N/A	1	0	1.00	21	0.000
346	A	3	2	1.00	4	0.500
347	A	5	4	0.98	6	0.667
348	A	6	5	1.21	8	0.625
349	A	3	2	0.94	8	0.250
350	A	5	4	1.00	10	0.400
351	A	6	5	1.17	12	0.417
352	A	9	8	0.89	12	0.667
353	A	6	5	1.00	14	0.357
354	A	7	6	1.09	16	0.375
355	A	8	7	0.85	20	0.350
356	A	7	6	0.94	20	0.300
357	A	4	3	0.91	20	0.150
358	A	4	3	0.91	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
359	A	7	6	0.94	20	0.300
360	A	9	8	0.85	20	0.400
361	A	2	2	1.00	24	0.083

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int x^5 \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

3.1.1 Optimal result

Integrand size = 23, antiderivative size = 127

$$\int x^5 \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = -\frac{5d^2x\sqrt{d+ex^2}}{96e^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}} - \frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}} + \frac{5d^3 \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{96e^3} + \frac{1}{6}x^6 \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

output
```
5/96*d^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^3+1/6*x^6*arctanh(x*e^(1/2)/
(e*x^2+d)^(1/2))-5/96*d^2*x*(e*x^2+d)^(1/2)/e^(5/2)+5/144*d*x^3*(e*x^2+d)^
(1/2)/e^(3/2)-1/36*x^5*(e*x^2+d)^(1/2)/e^(1/2)
```

3.1.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.78

$$\int x^5 \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \frac{\sqrt{e}x\sqrt{d+ex^2}(-15d^2+10dex^2-8e^2x^4)+48e^3x^6 \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+15d^3\log\left(\sqrt{e}x+\sqrt{d+ex^2}\right)}{288e^3}$$

input
```
Integrate[x^5*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(Sqrt[e]*x*Sqrt[d + e*x^2]*(-15*d^2 + 10*d*e*x^2 - 8*e^2*x^4) + 48*e^3*x^6
*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] + 15*d^3*Log[Sqrt[e]*x + Sqrt[d + e*
x^2]])/(288*e^3)
```

3.1.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6775, 262, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$$

$$\Big\downarrow \; 6775$$

$$\frac{1}{6}x^6 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{e}\int \frac{x^6}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \; 262$$

$$\frac{1}{6}x^6 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{e}\left(\frac{x^5\sqrt{d+ex^2}}{6e} - \frac{5d\int \frac{x^4}{\sqrt{ex^2+d}}dx}{6e}\right)$$

$$\Big\downarrow \; 262$$

$$\frac{1}{6}x^6 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{e}\left(\frac{x^5\sqrt{d+ex^2}}{6e} - \frac{5d\left(\frac{x^3\sqrt{d+ex^2}}{4e} - \frac{3d\int \frac{x^2}{\sqrt{ex^2+d}}dx}{4e}\right)}{6e}\right)$$

$$\Big\downarrow \; 262$$

$$\frac{1}{6}x^6 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{e}\left(\frac{x^5\sqrt{d+ex^2}}{6e} - \frac{5d\left(\frac{x^3\sqrt{d+ex^2}}{4e} - \frac{3d\left(\frac{x\sqrt{d+ex^2}}{2e} - \frac{d\int \frac{1}{\sqrt{ex^2+d}}dx}{2e}\right)}{4e}\right)}{6e}\right)$$

$$\Big\downarrow \; 224$$

$$\frac{1}{6}x^6\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{e}\left(\frac{x^5\sqrt{d+ex^2}}{6e}-\frac{5d\left(\frac{x^3\sqrt{d+ex^2}}{4e}-\frac{3d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}\right)}{4e}\right)}{6e}\right)$$

$$\Big\downarrow \textbf{219}$$

$$\frac{1}{6}x^6\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{e}\left(\frac{x^5\sqrt{d+ex^2}}{6e}-\frac{5d\left(\frac{x^3\sqrt{d+ex^2}}{4e}-\frac{3d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)}{4e}\right)}{6e}\right)$$

input
```
Int[x^5*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(x^6*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/6 - (Sqrt[e]*((x^5*Sqrt[d + e*x
^2])/(6*e) - (5*d*((x^3*Sqrt[d + e*x^2])/(4*e) - (3*d*((x*Sqrt[d + e*x^2])
/(2*e) - (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*e^(3/2))))/(4*e)))/(6
*e)))/6
```

3.1.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.1.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(97) = 194$.

Time = 0.02 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.99

method	result
default	$\frac{x^{6}\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{6}+\frac{e^{\frac{3}{2}}\left(\frac{x^{7}\sqrt{e\,x^{2}+d}}{8e}-\frac{7d\left(\frac{x^{5}\sqrt{e\,x^{2}+d}}{6e}-\frac{5d\left(\frac{x^{3}\sqrt{e\,x^{2}+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^{2}+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)}{6e}\right)}{8e}\right)}{6d}-\ \sqrt{e}$
parts	$\frac{x^{6}\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{6}+\frac{e^{\frac{3}{2}}\left(\frac{x^{7}\sqrt{e\,x^{2}+d}}{8e}-\frac{7d\left(\frac{x^{5}\sqrt{e\,x^{2}+d}}{6e}-\frac{5d\left(\frac{x^{3}\sqrt{e\,x^{2}+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^{2}+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)}{6e}\right)}{8e}\right)}{6d}-\ \sqrt{e}$

input
```
int(x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/6*e^(3/2)/d*(1/8*x^7/e*(e*x^2
+d)^(1/2)-7/8*d/e*(1/6*x^5/e*(e*x^2+d)^(1/2)-5/6*d/e*(1/4*x^3/e*(e*x^2+d)^
(1/2)-3/4*d/e*(1/2*x/e*(e*x^2+d)^(1/2)-1/2*d/e^(3/2)*ln(x*e^(1/2)+(e*x^2+d
)^(1/2))))))-1/6*e^(1/2)/d*(1/8*x^5*(e*x^2+d)^(3/2)/e-5/8*d/e*(1/6*x^3*(e*
x^2+d)^(3/2)/e-1/2*d/e*(1/4*x*(e*x^2+d)^(3/2)/e-1/4*d/e*(1/2*x*(e*x^2+d)^(
1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))))))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.68

$$\int x^5 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = -\frac{2\,(8\,e^2x^5 - 10\,dex^3 + 15\,d^2x)\sqrt{ex^2+d}\sqrt{e} - 3\,(16\,e^3x^6 + 5\,d^3)\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)}{576\,e^3}$$

input
```
integrate(x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

output
```
-1/576*(2*(8*e^2*x^5 - 10*d*e*x^3 + 15*d^2*x)*sqrt(e*x^2 + d)*sqrt(e) - 3*
(16*e^3*x^6 + 5*d^3)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d))/e
^3
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95

$$\int x^5 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \begin{cases} \frac{5d^3 \operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{96e^3} - \frac{5d^2x\sqrt{d+ex^2}}{96e^{\frac{5}{2}}} + \frac{5dx^3\sqrt{d+ex^2}}{144e^{\frac{3}{2}}} + \frac{x^6 \operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6} - \frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Piecewise((5*d**3*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(96*e**3) - 5*d**2*x*s
qrt(d + e*x**2)/(96*e**(5/2)) + 5*d*x**3*sqrt(d + e*x**2)/(144*e**(3/2)) +
 x**6*atanh(sqrt(e)*x/sqrt(d + e*x**2))/6 - x**5*sqrt(d + e*x**2)/(36*sqrt
(e)), Ne(e, 0)), (0, True))
```

3.1.7 Maxima [F]

$$\int x^5 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \int x^5 \operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right) dx$$

input
```
integrate(x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

output
```
1/12*x^6*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/12*x^6*log(-sqrt(e)*x + sqrt
(e*x^2 + d)) - 1/2*d*sqrt(e)*integrate(-1/3*sqrt(e*x^2 + d)*x^6/(e^2*x^4 +
 d*e*x^2 - (e*x^2 + d)^2), x)
```

3.1.8 Giac [F]

$$\int x^5 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \int x^5 \operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right) dx$$

input
```
integrate(x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int x^5 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \int x^5 \operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right) dx$$

input
```
int(x^5*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^5*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.2 $\int x^3\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

3.2.1 Optimal result

Integrand size = 23, antiderivative size = 101

$$\int x^3\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \frac{3dx\sqrt{d+ex^2}}{32e^{3/2}} - \frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}} - \frac{3d^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32e^2} + \frac{1}{4}x^4\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

output
```
-3/32*d^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^2+1/4*x^4*arctanh(x*e^(1/2)
/(e*x^2+d)^(1/2))+3/32*d*x*(e*x^2+d)^(1/2)/e^(3/2)-1/16*x^3*(e*x^2+d)^(1/2
)/e^(1/2)
```

3.2.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int x^3\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \frac{\sqrt{e}x(3d-2ex^2)\sqrt{d+ex^2} + 8e^2x^4\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - 3d^2\log\left(\sqrt{e}x+\sqrt{d+ex^2}\right)}{32e^2}$$

input
```
Integrate[x^3*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(Sqrt[e]*x*(3*d - 2*e*x^2)*Sqrt[d + e*x^2] + 8*e^2*x^4*ArcTanh[(Sqrt[e]*x)
/Sqrt[d + e*x^2]] - 3*d^2*Log[Sqrt[e]*x + Sqrt[d + e*x^2]])/(32*e^2)
```

3.2.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6775, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$$

$$\Big\downarrow \text{6775}$$

$$\frac{1}{4}x^4 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{4}\sqrt{e}\int \frac{x^4}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{4}x^4 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{4}\sqrt{e}\left(\frac{x^3\sqrt{d+ex^2}}{4e} - \frac{3d\int \frac{x^2}{\sqrt{ex^2+d}}dx}{4e}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{4}x^4 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{4}\sqrt{e}\left(\frac{x^3\sqrt{d+ex^2}}{4e} - \frac{3d\left(\frac{x\sqrt{d+ex^2}}{2e} - \frac{d\int \frac{1}{\sqrt{ex^2+d}}dx}{2e}\right)}{4e}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{4}x^4 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{4}\sqrt{e}\left(\frac{x^3\sqrt{d+ex^2}}{4e} - \frac{3d\left(\frac{x\sqrt{d+ex^2}}{2e} - \frac{d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}\right)}{4e}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}x^4\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{4}\sqrt{e}\left(\frac{x^3\sqrt{d+ex^2}}{4e}-\frac{3d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)}{4e}\right)$$

input
```
Int[x^3*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(x^4*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/4 - (Sqrt[e]*((x^3*Sqrt[d + e*x
^2])/(4*e) - (3*d*((x*Sqrt[d + e*x^2])/(2*e) - (d*ArcTanh[(Sqrt[e]*x)/Sqrt
[d + e*x^2]])/(2*e^(3/2))))/(4*e)))/4
```

3.2.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.2.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(77) = 154.

Time = 0.02 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.03

method	result
default	$\frac{x^4 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e x^2+d}}\right)}{4}+\frac{e^{\frac{3}{2}}\left(\frac{x^5\sqrt{e x^2+d}}{6e}-\frac{5d\left(\frac{x^3\sqrt{e x^2+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e x^2+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)}{6e}\right)}{4d}-\frac{\sqrt{e}\left(\frac{x^3\left(e x^2+d\right)^{\frac{3}{2}}}{6e}-\right.}{}$
parts	$\frac{x^4 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e x^2+d}}\right)}{4}+\frac{e^{\frac{3}{2}}\left(\frac{x^5\sqrt{e x^2+d}}{6e}-\frac{5d\left(\frac{x^3\sqrt{e x^2+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e x^2+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)}{6e}\right)}{4d}-\frac{\sqrt{e}\left(\frac{x^3\left(e x^2+d\right)^{\frac{3}{2}}}{6e}-\right.}{}$

input
```
int(x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/4*e^(3/2)/d*(1/6*x^5/e*(e*x^2
+d)^(1/2)-5/6*d/e*(1/4*x^3/e*(e*x^2+d)^(1/2)-3/4*d/e*(1/2*x/e*(e*x^2+d)^(1
/2)-1/2*d/e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2)))))-1/4*e^(1/2)/d*(1/6*x^3*
(e*x^2+d)^(3/2)/e-1/2*d/e*(1/4*x*(e*x^2+d)^(3/2)/e-1/4*d/e*(1/2*x*(e*x^2+d
)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2)))))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int x^3 \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = -\frac{2\left(2\,ex^3-3\,dx\right)\sqrt{ex^2+d}\sqrt{e}-\left(8\,e^2x^4-3\,d^2\right)\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)}{64\,e^2}$$

input
```
integrate(x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

output
```
-1/64*(2*(2*e*x^3 - 3*d*x)*sqrt(e*x^2 + d)*sqrt(e) - (8*e^2*x^4 - 3*d^2)*l
og((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d))/e^2
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int x^3\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$$
$$=\begin{cases}-\frac{3d^2\,\text{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32e^2}+\frac{3dx\sqrt{d+ex^2}}{32e^{\frac{3}{2}}}+\frac{x^4\,\text{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4}-\frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}} & \text{for } e\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate(x**3*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Piecewise((-3*d**2*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(32*e**2) + 3*d*x*sqr
t(d + e*x**2)/(32*e**(3/2)) + x**4*atanh(sqrt(e)*x/sqrt(d + e*x**2))/4 - x
**3*sqrt(d + e*x**2)/(16*sqrt(e)), Ne(e, 0)), (0, True))
```

3.2.7 Maxima [F]

$$\int x^3\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^3\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

output
```
1/8*x^4*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/8*x^4*log(-sqrt(e)*x + sqrt(e
*x^2 + d)) - 1/2*d*sqrt(e)*integrate(-1/2*sqrt(e*x^2 + d)*x^4/(e^2*x^4 + d
*e*x^2 - (e*x^2 + d)^2), x)
```

3.2.8 Giac [F]

$$\int x^3 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \int x^3 \operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right) dx$$

input
```
integrate(x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
sage0*x
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \int x^3 \operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right) dx$$

input
```
int(x^3*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^3*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.3 $\int x\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

3.3.1 Optimal result

Integrand size = 21, antiderivative size = 75

$$\int x\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = -\frac{x\sqrt{d+ex^2}}{4\sqrt{e}}+\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4e}+\frac{1}{2}x^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

output
```
1/4*d*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e+1/2*x^2*arctanh(x*e^(1/2)/(e*x^
2+d)^(1/2))-1/4*x*(e*x^2+d)^(1/2)/e^(1/2)
```

3.3.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int x\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = -\frac{x\sqrt{d+ex^2}}{4\sqrt{e}}+\frac{1}{2}x^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{d\log\left(\sqrt{e}x+\sqrt{d+ex^2}\right)}{4e}$$

input
```
Integrate[x*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
-1/4*(x*Sqrt[d + e*x^2])/Sqrt[e] + (x^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2
]])/2 + (d*Log[Sqrt[e]*x + Sqrt[d + e*x^2]])/(4*e)
```

3.3.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6775, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx$$

$$\Big\downarrow \text{6775}$$

$$\frac{1}{2}x^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{2}\sqrt{e}\int\frac{x^2}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}x^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{2}\sqrt{e}\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int\frac{1}{\sqrt{ex^2+d}}dx}{2e}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}x^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{2}\sqrt{e}\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}x^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{2}\sqrt{e}\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)$$

input
```
Int[x*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(x^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/2 - (Sqrt[e]*((x*Sqrt[d + e*x^2
])/(2*e) - (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*e^(3/2))))/2
```

3.3.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.3.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(57) = 114$.

Time = 0.01 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.09

method	result
default	$\frac{x^2 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{2}+\frac{e^{\frac{3}{2}}\left(\frac{x^3\sqrt{e\,x^2+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^2+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)}{2d}-\frac{\sqrt{e}\left(\frac{x\left(e\,x^2+d\right)^{\frac{3}{2}}}{4e}-\frac{d\left(\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}\right.}{}\right.}{4e}\right.}{2d}$
parts	$\frac{x^2 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{2}+\frac{e^{\frac{3}{2}}\left(\frac{x^3\sqrt{e\,x^2+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^2+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)}{2d}-\frac{\sqrt{e}\left(\frac{x\left(e\,x^2+d\right)^{\frac{3}{2}}}{4e}-\frac{d\left(\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}\right.}{}\right.}{4e}\right.}{2d}$

input
```
int(x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/2*e^(3/2)/d*(1/4*x^3/e*(e*x^2
+d)^(1/2)-3/4*d/e*(1/2*x/e*(e*x^2+d)^(1/2)-1/2*d/e^(3/2)*ln(x*e^(1/2)+(e*x
^2+d)^(1/2))))-1/2*e^(1/2)/d*(1/4*x*(e*x^2+d)^(3/2)/e-1/4*d/e*(1/2*x*(e*x^
2+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int x\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = -\frac{2\sqrt{ex^2+d}\sqrt{e}x - (2\,ex^2+d)\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)}{8\,e}$$

input
```
integrate(x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(e*x^2 + d)*sqrt(e)*x - (2*e*x^2 + d)*log((2*e*x^2 + 2*sqrt(e*
x^2 + d)*sqrt(e)*x + d)/d))/e
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int x\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \begin{cases} \frac{d\,\text{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4e} + \frac{x^2\,\text{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2} - \frac{x\sqrt{d+ex^2}}{4\sqrt{e}} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Piecewise((d*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(4*e) + x**2*atanh(sqrt(e)*
x/sqrt(d + e*x**2))/2 - x*sqrt(d + e*x**2)/(4*sqrt(e)), Ne(e, 0)), (0, Tru
e))
```

3.3.7 Maxima [F]

$$\int x\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

output
```
1/4*x^2*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/4*x^2*log(-sqrt(e)*x + sqrt(e
*x^2 + d)) - 1/2*d*sqrt(e)*integrate(-sqrt(e*x^2 + d)*x^2/(e^2*x^4 + d*e*x
^2 - (e*x^2 + d)^2), x)
```

3.3.8 Giac [F]

$$\int x\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
sage0*x
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x\,\text{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

input
```
int(x*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.4 $\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx$

3.4.1 Optimal result

Integrand size = 23, antiderivative size = 238

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{d+ex^2}} + \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{d+ex^2}}$$

output
```
arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*ln(x)-1/2*arcsinh(x*e^(1/2)/d^(1/2))^2*
d^(1/2)*(1+e*x^2/d)^(1/2)/(e*x^2+d)^(1/2)+arcsinh(x*e^(1/2)/d^(1/2))*ln(1-
(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(1+e*x^2/d)^(1/2)/(e*x^2+
d)^(1/2)-arcsinh(x*e^(1/2)/d^(1/2))*ln(x)*d^(1/2)*(1+e*x^2/d)^(1/2)/(e*x^2
+d)^(1/2)+1/2*polylog(2,(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(
1+e*x^2/d)^(1/2)/(e*x^2+d)^(1/2)
```

3.4.2 Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.70

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{\sqrt{e}\sqrt{1+\frac{ex^2}{d}}\left(\operatorname{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)^2 + 2\operatorname{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)\log\left(1-e^{-2\operatorname{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)}\right) - 2\log(x)\log\left(\sqrt{\frac{e}{d}}x+\sqrt{1}\right.}{2\sqrt{\frac{e}{d}}\sqrt{d+ex^2}}$$

input
```
Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x,x]
```

output
```
ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]*Log[x] + (Sqrt[e]*Sqrt[1 + (e*x^2)/d]
*(ArcSinh[Sqrt[e/d]*x]^2 + 2*ArcSinh[Sqrt[e/d]*x]*Log[1 - E^(-2*ArcSinh[Sq
rt[e/d]*x])] - 2*Log[x]*Log[Sqrt[e/d]*x + Sqrt[1 + (e*x^2)/d]] - PolyLog[2
, E^(-2*ArcSinh[Sqrt[e/d]*x])]))/(2*Sqrt[e/d]*Sqrt[d + e*x^2])
```

3.4.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.78, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {6773, 2764, 2762, 6190, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx$$

$$\Big\downarrow \; 6773$$

$$\log(x)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \sqrt{e}\int \frac{\log(x)}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \; 2764$$

$$\log(x)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{\sqrt{e}\sqrt{\frac{ex^2}{d}+1}\int \frac{\log(x)}{\sqrt{\frac{ex^2}{d}+1}}dx}{\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 2762$$

$$\log(x)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{\sqrt{d}\int\frac{\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{\sqrt{e}}\right)}{\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 6190}$$

$$\log(x)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{\sqrt{d}\int\frac{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}x}d\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}\right)}{\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\log(x)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{\sqrt{d}\int -i\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\tan\left(i\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\frac{\pi}{2}\right)d\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}\right)}{\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 26}$$

$$\log(x)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{i\sqrt{d}\int \text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\tan\left(i\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\frac{\pi}{2}\right)d\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}\right)}{\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 4199}$$

$$\log(x)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{i\sqrt{d}\left(2i\int -\frac{e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}}d\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)-\frac{1}{2}i\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2\right)}{\sqrt{e}}\right)}{\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 25}$$

3.4. $\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}dx$

$$\log(x)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{i\sqrt{d}\left(-2i\int\frac{e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}}d\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)-\frac{1}{2}i\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2\right)}{\sqrt{e}}\right)}{\sqrt{d+ex^2}}$$

$\downarrow$ 2620

$$\log(x)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{i\sqrt{d}\left(-2i\left(\frac{1}{2}\int\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)d\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)-\frac{1}{2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arcsi}\right.\right.\right.}{\sqrt{e}}\right.}{\sqrt{d+ex^2}}$$

$\downarrow$ 2715

$$\log(x)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{i\sqrt{d}\left(-2i\left(\frac{1}{4}\int e^{-2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)de^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}-\frac{1}{2}\text{arcsinh}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.\right.}{\sqrt{e}}\right.}{\sqrt{d+ex^2}}$$

$\downarrow$ 2838

$$\log(x)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{i\sqrt{d}\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)-\frac{1}{2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)\right)\right.}{\sqrt{e}}\right.}{\sqrt{d+ex^2}}$$

input
```
Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x,x]
```

3.4. $\int\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}dx$

output
```
ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]*Log[x] - (Sqrt[e]*Sqrt[1 + (e*x^2)/d]
*((Sqrt[d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[x])/Sqrt[e] + (I*Sqrt[d]*((-1/
2*I)*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2 - (2*I)*(-1/2*(ArcSinh[(Sqrt[e]*x)/Sqr
t[d]]*Log[1 - E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])]) - PolyLog[2, E^(2*ArcSi
nh[(Sqrt[e]*x)/Sqrt[d]])]/4)))/Sqrt[e]))/Sqrt[d + e*x^2]
```

3.4.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2762
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[e, 2]), x]
 - Simp[b*(n/Rt[e, 2])   Int[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]/x, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

rule 2764
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[Sqrt[1 + (e/d)*x^2]/Sqrt[d + e*x^2]   Int[(a + b*Log[c*x^n])/Sq
rt[1 + (e/d)*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] &&  !GtQ[d, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.4. $\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6773
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]/(x_), x_Symbol] :> Sim
p[ArcTanh[c*(x/Sqrt[a + b*x^2])]*Log[x], x] - Simp[c   Int[Log[x]/Sqrt[a +
b*x^2], x], x] /; FreeQ[{a, b, c}, x] && EqQ[b, c^2]
```

3.4.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.88

method	result
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)^2}{2}+\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)\ln\left(1+\frac{\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}+1}{\sqrt{-\frac{x^2 e}{e\,x^2+d}+1}}\right)+\operatorname{polylog}\left(2,-\frac{\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}+1}{\sqrt{-\frac{x^2 e}{e\,x^2+d}+1}}\right)+\operatorname{arc}$

input
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))^2+arctanh(x*e^(1/2)/(e*x^2+d)^(1/2
))*ln(1+(x*e^(1/2)/(e*x^2+d)^(1/2)+1)/(-x^2*e/(e*x^2+d)+1)^(1/2))+polylog(
2,-(x*e^(1/2)/(e*x^2+d)^(1/2)+1)/(-x^2*e/(e*x^2+d)+1)^(1/2))+arctanh(x*e^(
1/2)/(e*x^2+d)^(1/2))*ln(1-(x*e^(1/2)/(e*x^2+d)^(1/2)+1)/(-x^2*e/(e*x^2+d)
+1)^(1/2))+polylog(2,(x*e^(1/2)/(e*x^2+d)^(1/2)+1)/(-x^2*e/(e*x^2+d)+1)^(1
/2))
```

3.4.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="fricas")
```

output
```
integral(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x, x)
```

3.4.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx$$

input
```
integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x,x)
```

output
```
Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x, x)
```

3.4.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="maxima")
```

output
```
integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x, x)
```

3.4.8 Giac [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x}\,dx$$

input
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x,x)
```

output
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x, x)
```

3.5 $\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx$

3.5.1 Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = -\frac{\sqrt{e}\sqrt{d+ex^2}}{2dx} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

output
```
-1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2-1/2*e^(1/2)*(e*x^2+d)^(1/2)/d/
x
```

3.5.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = -\frac{\sqrt{e}x\sqrt{d+ex^2} + d\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2dx^2}$$

input
```
Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^3,x]
```

output
```
-1/2*(Sqrt[e]*x*Sqrt[d + e*x^2] + d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/
(d*x^2)
```

3.5.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6775, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^3} \, dx$$

$$\Big\downarrow \text{ 6775}$$

$$\frac{1}{2}\sqrt{e}\int \frac{1}{x^2\sqrt{ex^2+d}}dx - \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2x^2} - \frac{\sqrt{e}\sqrt{d+ex^2}}{2dx}$$

input
```
Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^3,x]
```

output
```
-1/2*(Sqrt[e]*Sqrt[d + e*x^2])/(d*x) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]
]/(2*x^2)
```

3.5.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.5.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(41) = 82.

Time = 0.01 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.09

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{2x^{2}}-\frac{e\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2d}+\frac{\sqrt{e}\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{dx}+\frac{2e\left(\frac{x\sqrt{e\,x^{2}+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2\sqrt{e}}\right)}{d}\right)}{2d}$	111
parts	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{2x^{2}}-\frac{e\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2d}+\frac{\sqrt{e}\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{dx}+\frac{2e\left(\frac{x\sqrt{e\,x^{2}+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2\sqrt{e}}\right)}{d}\right)}{2d}$	111

input
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2-1/2*e/d*ln(x*e^(1/2)+(e*x^2+d)
^(1/2))+1/2*e^(1/2)/d*(-1/d/x*(e*x^2+d)^(3/2)+2*e/d*(1/2*x*(e*x^2+d)^(1/2)
+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = -\frac{2\sqrt{ex^2+d}\sqrt{e}x+d\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)}{4\,dx^2}$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(e*x^2 + d)*sqrt(e)*x + d*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqr
t(e)*x + d)/d))/(d*x^2)
```

3.5.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx$$

input
```
integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**3,x)
```

output
```
Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**3, x)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = -\frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{2\,x^2} - \frac{e^{\frac{3}{2}}x^2 + d\sqrt{e}}{2\,\sqrt{ex^2+d}dx}$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="maxima")
```

output
```
-1/2*arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^2 - 1/2*(e^(3/2)*x^2 + d*sqrt(e)
)/(sqrt(e*x^2 + d)*d*x)
```

3.5.8 Giac [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^3}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^3}\,dx$$

input
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^3,x)
```

output
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^3, x)
```

3.6 $\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^5} dx$

3.6.1 Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^5} dx = -\frac{\sqrt{e}\sqrt{d+ex^2}}{12dx^3} + \frac{e^{3/2}\sqrt{d+ex^2}}{6d^2x} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

output
```
-1/4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4+1/6*e^(3/2)*(e*x^2+d)^(1/2)/d^
2/x-1/12*e^(1/2)*(e*x^2+d)^(1/2)/d/x^3
```

3.6.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^5} dx = \frac{\sqrt{e}x\sqrt{d+ex^2}(-d+2ex^2) - 3d^2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{12d^2x^4}$$

input
```
Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^5,x]
```

output
```
(Sqrt[e]*x*Sqrt[d + e*x^2]*(-d + 2*e*x^2) - 3*d^2*ArcTanh[(Sqrt[e]*x)/Sqrt
[d + e*x^2]])/(12*d^2*x^4)
```

3.6.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6775, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^5} \, dx$$

$$\Big\downarrow \text{6775}$$

$$\frac{1}{4}\sqrt{e}\int \frac{1}{x^4\sqrt{ex^2+d}}dx - \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{4}\sqrt{e}\left(-\frac{2e\int \frac{1}{x^2\sqrt{ex^2+d}}dx}{3d} - \frac{\sqrt{d+ex^2}}{3dx^3}\right) - \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

$$\Big\downarrow \text{242}$$

$$\frac{1}{4}\sqrt{e}\left(\frac{2e\sqrt{d+ex^2}}{3d^2x} - \frac{\sqrt{d+ex^2}}{3dx^3}\right) - \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

input
```
Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^5,x]
```

output
```
(Sqrt[e]*(-1/3*Sqrt[d + e*x^2]/(d*x^3) + (2*e*Sqrt[d + e*x^2])/(3*d^2*x)))
/4 - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(4*x^4)
```

3.6.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.6.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{4x^{4}}+\frac{e^{\frac{3}{2}}\sqrt{e\,x^{2}+d}}{4d^{2}x}-\frac{\sqrt{e}\,\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{12d^{2}x^{3}}$	62
parts	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{4x^{4}}+\frac{e^{\frac{3}{2}}\sqrt{e\,x^{2}+d}}{4d^{2}x}-\frac{\sqrt{e}\,\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{12d^{2}x^{3}}$	62

input
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4+1/4*e^(3/2)*(e*x^2+d)^(1/2)/d^
2/x-1/12*e^(1/2)/d^2/x^3*(e*x^2+d)^(3/2)
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx=-\frac{3\,d^2\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)-2\,(2\,ex^3-dx)\sqrt{ex^2+d}\sqrt{e}}{24\,d^2x^4}$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="fricas")
```

output
```
-1/24*(3*d^2*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*(2*e*x
^3 - d*x)*sqrt(e*x^2 + d)*sqrt(e))/(d^2*x^4)
```

3.6.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx$$

input
```
integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**5,x)
```

output
```
Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**5, x)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = \frac{\sqrt{ex^2+d}e^{\frac{3}{2}}}{4\,d^2x} - \frac{(ex^2+d)^{\frac{3}{2}}\sqrt{e}}{12\,d^2x^3} - \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{4\,x^4}$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="maxima")
```

output
```
1/4*sqrt(e*x^2 + d)*e^(3/2)/(d^2*x) - 1/12*(e*x^2 + d)^(3/2)*sqrt(e)/(d^2*
x^3) - 1/4*arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^4
```

3.6.8 Giac [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^5}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="giac")
```

output
```
sage0*x
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^5}\,dx$$

input
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^5,x)
```

output
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^5, x)
```

3.7 $\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx$

3.7.1 Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = -\frac{\sqrt{e}\sqrt{d+ex^2}}{30dx^5}+\frac{2e^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{4e^{5/2}\sqrt{d+ex^2}}{45d^3x}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

output
```
-1/6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6+2/45*e^(3/2)*(e*x^2+d)^(1/2)/d
^2/x^3-4/45*e^(5/2)*(e*x^2+d)^(1/2)/d^3/x-1/30*e^(1/2)*(e*x^2+d)^(1/2)/d/x
^5
```

3.7.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.70

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = \frac{\sqrt{e}x\sqrt{d+ex^2}(-3d^2+4dex^2-8e^2x^4)-15d^3\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{90d^3x^6}$$

input
```
Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^7,x]
```

output
```
(Sqrt[e]*x*Sqrt[d + e*x^2]*(-3*d^2 + 4*d*e*x^2 - 8*e^2*x^4) - 15*d^3*ArcTa
nh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(90*d^3*x^6)
```

3.7.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6775, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^7} \, dx$$

$$\Big\downarrow \text{6775}$$

$$\frac{1}{6}\sqrt{e}\int \frac{1}{x^6\sqrt{ex^2+d}}dx-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{6}\sqrt{e}\left(-\frac{4e\int \frac{1}{x^4\sqrt{ex^2+d}}dx}{5d}-\frac{\sqrt{d+ex^2}}{5dx^5}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{6}\sqrt{e}\left(-\frac{4e\left(-\frac{2e\int \frac{1}{x^2\sqrt{ex^2+d}}dx}{3d}-\frac{\sqrt{d+ex^2}}{3dx^3}\right)}{5d}-\frac{\sqrt{d+ex^2}}{5dx^5}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

$$\Big\downarrow \text{242}$$

$$\frac{1}{6}\sqrt{e}\left(-\frac{4e\left(\frac{2e\sqrt{d+ex^2}}{3d^2x}-\frac{\sqrt{d+ex^2}}{3dx^3}\right)}{5d}-\frac{\sqrt{d+ex^2}}{5dx^5}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

input
```
Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^7,x]
```

output
```
(Sqrt[e]*(-1/5*Sqrt[d + e*x^2]/(d*x^5) - (4*e*(-1/3*Sqrt[d + e*x^2]/(d*x^3
) + (2*e*Sqrt[d + e*x^2])/(3*d^2*x)))/(5*d)))/6 - ArcTanh[(Sqrt[e]*x)/Sqrt
[d + e*x^2]]/(6*x^6)
```

3.7.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.7.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.05

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{6x^6}-\frac{e^{\frac{3}{2}}\left(-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}\right)}{6d}+\frac{\sqrt{e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{5d\,x^5}+\frac{2e\left(e\,x^2+d\right)^{\frac{3}{2}}}{15d^2x^3}\right)}{6d}$	110
parts	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{6x^6}-\frac{e^{\frac{3}{2}}\left(-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}\right)}{6d}+\frac{\sqrt{e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{5d\,x^5}+\frac{2e\left(e\,x^2+d\right)^{\frac{3}{2}}}{15d^2x^3}\right)}{6d}$	110

input
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6-1/6*e^(3/2)/d*(-1/3/d/x^3*(e*x
^2+d)^(1/2)+2/3*e/d^2/x*(e*x^2+d)^(1/2))+1/6*e^(1/2)/d*(-1/5/d/x^5*(e*x^2+
d)^(3/2)+2/15*e/d^2/x^3*(e*x^2+d)^(3/2))
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx$$
$$= -\frac{15\,d^3\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)+2\,(8\,e^2x^5-4\,dex^3+3\,d^2x)\sqrt{ex^2+d}\sqrt{e}}{180\,d^3x^6}$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="fricas")
```

output
```
-1/180*(15*d^3*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) + 2*(8*e
^2*x^5 - 4*d*e*x^3 + 3*d^2*x)*sqrt(e*x^2 + d)*sqrt(e))/(d^3*x^6)
```

3.7.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx$$

input
```
integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**7,x)
```

output
```
Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**7, x)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = -\frac{(2\,e^2x^4+dex^2-d^2)e^{\frac{3}{2}}}{18\,\sqrt{ex^2+d}d^3x^3} - \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{6\,x^6}$$
$$+ \frac{(2\,e^2x^4-dex^2-3\,d^2)\sqrt{ex^2+d}\sqrt{e}}{90\,d^3x^5}$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="maxima")
```

output
```
-1/18*(2*e^2*x^4 + d*e*x^2 - d^2)*e^(3/2)/(sqrt(e*x^2 + d)*d^3*x^3) - 1/6*
arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^6 + 1/90*(2*e^2*x^4 - d*e*x^2 - 3*d^2
)*sqrt(e*x^2 + d)*sqrt(e)/(d^3*x^5)
```

3.7.8 Giac [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^7}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="giac")
```

output
```
sage0*x
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^7}\,dx$$

input
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^7,x)
```

output
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^7, x)
```

3.8 $\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^9} dx$

3.8.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^9} dx = -\frac{\sqrt{e}\sqrt{d+ex^2}}{56dx^7} + \frac{3e^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{e^{5/2}\sqrt{d+ex^2}}{35d^3x^3} + \frac{2e^{7/2}\sqrt{d+ex^2}}{35d^4x} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

output
```
-1/8*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^8+3/140*e^(3/2)*(e*x^2+d)^(1/2)/
d^2/x^5-1/35*e^(5/2)*(e*x^2+d)^(1/2)/d^3/x^3+2/35*e^(7/2)*(e*x^2+d)^(1/2)/
d^4/x-1/56*e^(1/2)*(e*x^2+d)^(1/2)/d/x^7
```

3.8.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.65

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^9} dx = \frac{\sqrt{e}x\sqrt{d+ex^2}(-5d^3+6d^2ex^2-8de^2x^4+16e^3x^6) - 35d^4\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{280d^4x^8}$$

input
```
Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

output
```
(Sqrt[e]*x*Sqrt[d + e*x^2]*(-5*d^3 + 6*d^2*e*x^2 - 8*d*e^2*x^4 + 16*e^3*x^
6) - 35*d^4*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(280*d^4*x^8)
```

3.8.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6775, 245, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^9} \, dx$$

$$\Big\downarrow \text{6775}$$

$$\frac{1}{8}\sqrt{e}\int \frac{1}{x^8\sqrt{ex^2+d}}dx-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{8}\sqrt{e}\left(-\frac{6e\int \frac{1}{x^6\sqrt{ex^2+d}}dx}{7d}-\frac{\sqrt{d+ex^2}}{7dx^7}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{8}\sqrt{e}\left(-\frac{6e\left(-\frac{4e\int \frac{1}{x^4\sqrt{ex^2+d}}dx}{5d}-\frac{\sqrt{d+ex^2}}{5dx^5}\right)}{7d}-\frac{\sqrt{d+ex^2}}{7dx^7}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{8}\sqrt{e}\left(-\frac{6e\left(-\frac{4e\left(-\frac{2e\int \frac{1}{x^2\sqrt{ex^2+d}}dx}{3d}-\frac{\sqrt{d+ex^2}}{3dx^3}\right)}{5d}-\frac{\sqrt{d+ex^2}}{5dx^5}\right)}{7d}-\frac{\sqrt{d+ex^2}}{7dx^7}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

$$\downarrow \text{242}$$

$$\frac{1}{8}\sqrt{e}\left(-\frac{6e\left(-\frac{4e\left(\frac{2e\sqrt{d+ex^2}}{3d^2x}-\frac{\sqrt{d+ex^2}}{3dx^3}\right)}{5d}-\frac{\sqrt{d+ex^2}}{5dx^5}\right)}{7d}-\frac{\sqrt{d+ex^2}}{7dx^7}\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

input
```
Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

output
```
(Sqrt[e]*(-1/7*Sqrt[d + e*x^2]/(d*x^7) - (6*e*(-1/5*Sqrt[d + e*x^2]/(d*x^5
) - (4*e*(-1/3*Sqrt[d + e*x^2]/(d*x^3) + (2*e*Sqrt[d + e*x^2])/(3*d^2*x)))
/(5*d)))/(7*d)))/8 - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(8*x^8)
```

3.8.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.8. $\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx$

3.8.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.21

method	result
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{8x^{8}}-\frac{e^{\frac{3}{2}}\left(-\frac{\sqrt{e\,x^{2}+d}}{5d\,x^{5}}-\frac{4e\left(-\frac{\sqrt{e\,x^{2}+d}}{3d\,x^{3}}+\frac{2e\sqrt{e\,x^{2}+d}}{3d^{2}x}\right)}{5d}\right)}{8d}+\frac{\sqrt{e}\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{7d\,x^{7}}-\frac{4e\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{5d\,x^{5}}+\frac{2e\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{15d^{2}x^{3}}\right.}{7d}\right.}{8d}$
parts	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{8x^{8}}-\frac{e^{\frac{3}{2}}\left(-\frac{\sqrt{e\,x^{2}+d}}{5d\,x^{5}}-\frac{4e\left(-\frac{\sqrt{e\,x^{2}+d}}{3d\,x^{3}}+\frac{2e\sqrt{e\,x^{2}+d}}{3d^{2}x}\right)}{5d}\right)}{8d}+\frac{\sqrt{e}\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{7d\,x^{7}}-\frac{4e\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{5d\,x^{5}}+\frac{2e\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{15d^{2}x^{3}}\right.}{7d}\right.}{8d}$

input
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^8-1/8*e^(3/2)/d*(-1/5/d/x^5*(e*x
^2+d)^(1/2)-4/5*e/d*(-1/3/d/x^3*(e*x^2+d)^(1/2)+2/3*e/d^2/x*(e*x^2+d)^(1/2
)))+1/8*e^(1/2)/d*(-1/7/d/x^7*(e*x^2+d)^(3/2)-4/7*e/d*(-1/5/d/x^5*(e*x^2+d
)^(3/2)+2/15*e/d^2/x^3*(e*x^2+d)^(3/2)))
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.68

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = -\frac{35\,d^4\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)-2\,(16\,e^3x^7-8\,de^2x^5+6\,d^2ex^3-5\,d^3x)\sqrt{ex^2+d}\sqrt{e}}{560\,d^4x^8}$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="fricas")
```

output
```
-1/560*(35*d^4*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*(16*
e^3*x^7 - 8*d*e^2*x^5 + 6*d^2*e*x^3 - 5*d^3*x)*sqrt(e*x^2 + d)*sqrt(e))/(d
^4*x^8)
```

3.8.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^9} dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^9} dx$$

input
```
integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**9,x)
```

output
```
Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**9, x)
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^9} dx = \frac{(8\,e^3x^6 + 4\,de^2x^4 - d^2ex^2 + 3\,d^3)e^{\frac{3}{2}}}{120\,\sqrt{ex^2+d}d^4x^5} - \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{8\,x^8} - \frac{(8\,e^3x^6 - 4\,de^2x^4 + 3\,d^2ex^2 + 15\,d^3)\sqrt{ex^2+d}\sqrt{e}}{840\,d^4x^7}$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="maxima")
```

output
```
1/120*(8*e^3*x^6 + 4*d*e^2*x^4 - d^2*e*x^2 + 3*d^3)*e^(3/2)/(sqrt(e*x^2 +
d)*d^4*x^5) - 1/8*arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^8 - 1/840*(8*e^3*x^
6 - 4*d*e^2*x^4 + 3*d^2*e*x^2 + 15*d^3)*sqrt(e*x^2 + d)*sqrt(e)/(d^4*x^7)
```

3.8.8 Giac [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^9} dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^9} dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="giac")
```

output
```
sage0*x
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^9}\,dx$$

input
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^9,x)
```

output
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^9, x)
```

3.9 $\int x^6 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

3.9.1 Optimal result

Integrand size = 23, antiderivative size = 114

$$\int x^6 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \frac{d^3\sqrt{d+ex^2}}{7e^{7/2}} - \frac{d^2(d+ex^2)^{3/2}}{7e^{7/2}} + \frac{3d(d+ex^2)^{5/2}}{35e^{7/2}} - \frac{(d+ex^2)^{7/2}}{49e^{7/2}} + \frac{1}{7}x^7 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

output
```
-1/7*d^2*(e*x^2+d)^(3/2)/e^(7/2)+3/35*d*(e*x^2+d)^(5/2)/e^(7/2)-1/49*(e*x^
2+d)^(7/2)/e^(7/2)+1/7*x^7*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/7*d^3*(e*x
^2+d)^(1/2)/e^(7/2)
```

3.9.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

$$\int x^6 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \frac{\sqrt{d+ex^2}(16d^3 - 8d^2ex^2 + 6de^2x^4 - 5e^3x^6)}{245e^{7/2}} + \frac{1}{7}x^7 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

input
```
Integrate[x^6*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(Sqrt[d + e*x^2]*(16*d^3 - 8*d^2*e*x^2 + 6*d*e^2*x^4 - 5*e^3*x^6))/(245*e^
(7/2)) + (x^7*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/7
```

3.9.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6775, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$$

$$\Big\downarrow \text{6775}$$

$$\frac{1}{7}x^7\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{7}\sqrt{e}\int \frac{x^7}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{7}x^7\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{14}\sqrt{e}\int \frac{x^6}{\sqrt{ex^2+d}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{7}x^7\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{14}\sqrt{e}\int \left(-\frac{d^3}{e^3\sqrt{ex^2+d}}+\frac{3\sqrt{ex^2+d}d^2}{e^3}-\frac{3\left(ex^2+d\right)^{3/2}d}{e^3}+\frac{\left(ex^2+d\right)^{5/2}}{e^3}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^7\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{14}\sqrt{e}\left(-\frac{2d^3\sqrt{d+ex^2}}{e^4}+\frac{2d^2\left(d+ex^2\right)^{3/2}}{e^4}+\frac{2\left(d+ex^2\right)^{7/2}}{7e^4}-\frac{6d\left(d+ex^2\right)^{5/2}}{5e^4}\right)$$

input
```
Int[x^6*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
-1/14*(Sqrt[e]*((-2*d^3*Sqrt[d + e*x^2])/e^4 + (2*d^2*(d + e*x^2)^(3/2))/e
^4 - (6*d*(d + e*x^2)^(5/2))/(5*e^4) + (2*(d + e*x^2)^(7/2))/(7*e^4))) + (
x^7*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/7
```

3.9.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.9.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(84) = 168$.

Time = 0.02 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.96

method	result
default	$\frac{x^7 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{7}+\frac{e^{\frac{3}{2}}\left(\frac{x^8\sqrt{e\,x^2+d}}{9e}-\frac{8d\left(\frac{x^6\sqrt{e\,x^2+d}}{7e}-\frac{6d\left(\frac{x^4\sqrt{e\,x^2+d}}{5e}-\frac{4d\left(\frac{x^2\sqrt{e\,x^2+d}}{3e}-\frac{2d\sqrt{e\,x^2+d}}{3e^2}\right)}{5e}\right)}{7e}\right)}{9e}\right)}{7d}-\sqrt{e}\left(\frac{x^6\left(e\,x^2\right.}{9e}\right.$
parts	$\frac{x^7 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{7}+\frac{e^{\frac{3}{2}}\left(\frac{x^8\sqrt{e\,x^2+d}}{9e}-\frac{8d\left(\frac{x^6\sqrt{e\,x^2+d}}{7e}-\frac{6d\left(\frac{x^4\sqrt{e\,x^2+d}}{5e}-\frac{4d\left(\frac{x^2\sqrt{e\,x^2+d}}{3e}-\frac{2d\sqrt{e\,x^2+d}}{3e^2}\right)}{5e}\right)}{7e}\right)}{9e}\right)}{7d}-\sqrt{e}\left(\frac{x^6\left(e\,x^2\right.}{9e}\right.$

input
```
int(x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/7*x^7*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/7*e^(3/2)/d*(1/9*x^8/e*(e*x^2
+d)^(1/2)-8/9*d/e*(1/7*x^6/e*(e*x^2+d)^(1/2)-6/7*d/e*(1/5*x^4/e*(e*x^2+d)^
(1/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2)))))-1/7
*e^(1/2)/d*(1/9*x^6*(e*x^2+d)^(3/2)/e-2/3*d/e*(1/7*x^4*(e*x^2+d)^(3/2)/e-4
/7*d/e*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2))))
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int x^6 \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{35\,e^4x^7\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)-2\,(5\,e^3x^6-6\,de^2x^4+8\,d^2ex^2-16\,d^3)\sqrt{ex^2+d}\sqrt{e}}{490\,e^4}$$

input
```
integrate(x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

output
```
1/490*(35*e^4*x^7*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*(
5*e^3*x^6 - 6*d*e^2*x^4 + 8*d^2*e*x^2 - 16*d^3)*sqrt(e*x^2 + d)*sqrt(e))/e
^4
```

3.9.6 Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02

$$\int x^6 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \begin{cases} \frac{16d^3\sqrt{d+ex^2}}{245e^{\frac{7}{2}}} - \frac{8d^2x^2\sqrt{d+ex^2}}{245e^{\frac{5}{2}}} + \frac{6dx^4\sqrt{d+ex^2}}{245e^{\frac{3}{2}}} + \frac{x^7\,\text{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7} - \frac{x^6\sqrt{d+ex^2}}{49\sqrt{e}} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**6*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Piecewise((16*d**3*sqrt(d + e*x**2)/(245*e**(7/2)) - 8*d**2*x**2*sqrt(d +
e*x**2)/(245*e**(5/2)) + 6*d*x**4*sqrt(d + e*x**2)/(245*e**(3/2)) + x**7*a
tanh(sqrt(e)*x/sqrt(d + e*x**2))/7 - x**6*sqrt(d + e*x**2)/(49*sqrt(e)), N
e(e, 0)), (0, True))
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.36

$$\int x^6 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{1}{7}x^7\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right) - \frac{35\,(ex^2+d)^{\frac{9}{2}} - 135\,(ex^2+d)^{\frac{7}{2}}d + 189\,(ex^2+d)^{\frac{5}{2}}d^2 - 105\,(ex^2+d)^{\frac{3}{2}}d^3}{2205\,de^{\frac{7}{2}}} + \frac{35\,(ex^2+d)^{\frac{9}{2}} - 180\,(ex^2+d)^{\frac{7}{2}}d + 378\,(ex^2+d)^{\frac{5}{2}}d^2 - 420\,(ex^2+d)^{\frac{3}{2}}d^3 + 315\,\sqrt{ex^2+d}d^4}{2205\,de^{\frac{7}{2}}}$$

input
```
integrate(x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

output
```
1/7*x^7*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)) - 1/2205*(35*(e*x^2 + d)^(9/2)
- 135*(e*x^2 + d)^(7/2)*d + 189*(e*x^2 + d)^(5/2)*d^2 - 105*(e*x^2 + d)^(3
/2)*d^3)/(d*e^(7/2)) + 1/2205*(35*(e*x^2 + d)^(9/2) - 180*(e*x^2 + d)^(7/2
)*d + 378*(e*x^2 + d)^(5/2)*d^2 - 420*(e*x^2 + d)^(3/2)*d^3 + 315*sqrt(e*x
^2 + d)*d^4)/(d*e^(7/2))
```

3.9.8 Giac [F]

$$\int x^6 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \int x^6\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
sage0*x
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int x^6 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \int x^6\,\text{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

input
```
int(x^6*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^6*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.10 $\int x^4 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

3.10.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int x^4 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = -\frac{d^2\sqrt{d+ex^2}}{5e^{5/2}} + \frac{2d(d+ex^2)^{3/2}}{15e^{5/2}} - \frac{(d+ex^2)^{5/2}}{25e^{5/2}} + \frac{1}{5}x^5 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

output
```
2/15*d*(e*x^2+d)^(3/2)/e^(5/2)-1/25*(e*x^2+d)^(5/2)/e^(5/2)+1/5*x^5*arctan
h(x*e^(1/2)/(e*x^2+d)^(1/2))-1/5*d^2*(e*x^2+d)^(1/2)/e^(5/2)
```

3.10.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int x^4 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = -\frac{\sqrt{d+ex^2}(8d^2-4dex^2+3e^2x^4)}{75e^{5/2}} + \frac{1}{5}x^5 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

input
```
Integrate[x^4*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
-1/75*(Sqrt[d + e*x^2]*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4))/e^(5/2) + (x^5*Arc
Tanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/5
```

3.10.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6775, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) \, dx$$

$$\Big\downarrow \text{6775}$$

$$\frac{1}{5}x^5\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{5}\sqrt{e}\int \frac{x^5}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{5}x^5\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{10}\sqrt{e}\int \frac{x^4}{\sqrt{ex^2+d}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{5}x^5\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{10}\sqrt{e}\int \left(\frac{d^2}{e^2\sqrt{ex^2+d}}-\frac{2\sqrt{ex^2+d}d}{e^2}+\frac{\left(ex^2+d\right)^{3/2}}{e^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{10}\sqrt{e}\left(\frac{2d^2\sqrt{d+ex^2}}{e^3}+\frac{2\left(d+ex^2\right)^{5/2}}{5e^3}-\frac{4d\left(d+ex^2\right)^{3/2}}{3e^3}\right)$$

input
```
Int[x^4*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
-1/10*(Sqrt[e]*((2*d^2*Sqrt[d + e*x^2])/e^3 - (4*d*(d + e*x^2)^(3/2))/(3*e
^3) + (2*(d + e*x^2)^(5/2))/(5*e^3))) + (x^5*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
e*x^2]])/5
```

3.10.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.10.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(67) = 134$.

Time = 0.01 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.93

method	result
default	$\frac{x^5 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e x^2+d}}\right)}{5} + \frac{e^{\frac{3}{2}}\left(\frac{x^6\sqrt{e x^2+d}}{7e} - \frac{6d\left(\frac{x^4\sqrt{e x^2+d}}{5e} - \frac{4d\left(\frac{x^2\sqrt{e x^2+d}}{3e} - \frac{2d\sqrt{e x^2+d}}{3e^2}\right)}{5e}\right)}{7e}\right)}{5d} - \frac{\sqrt{e}\left(\frac{x^4\left(e x^2+d\right)^{\frac{3}{2}}}{7e} - \frac{4d\left(\frac{x^2\left(e x^{\cdots}\right.}{5\cdots}\right.}{\cdots}\right.}{5d}$
parts	$\frac{x^5 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e x^2+d}}\right)}{5} + \frac{e^{\frac{3}{2}}\left(\frac{x^6\sqrt{e x^2+d}}{7e} - \frac{6d\left(\frac{x^4\sqrt{e x^2+d}}{5e} - \frac{4d\left(\frac{x^2\sqrt{e x^2+d}}{3e} - \frac{2d\sqrt{e x^2+d}}{3e^2}\right)}{5e}\right)}{7e}\right)}{5d} - \frac{\sqrt{e}\left(\frac{x^4\left(e x^2+d\right)^{\frac{3}{2}}}{7e} - \frac{4d\left(\frac{x^2\left(e x^{\cdots}\right.}{5\cdots}\right.}{\cdots}\right.}{5d}$

input
```
int(x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/5*e^(3/2)/d*(1/7*x^6/e*(e*x^2
+d)^(1/2)-6/7*d/e*(1/5*x^4/e*(e*x^2+d)^(1/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^
(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2))))-1/5*e^(1/2)/d*(1/7*x^4*(e*x^2+d)^(3/2)/
e-4/7*d/e*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2)))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.85

$$\int x^4 \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \frac{15\, e^3 x^5 \log\left(\frac{2\, ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right) - 2\,(3\, e^2 x^4 - 4\, dex^2 + 8\, d^2)\sqrt{ex^2+d}\sqrt{e}}{150\, e^3}$$

input
```
integrate(x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

output
```
1/150*(15*e^3*x^5*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*(
3*e^2*x^4 - 4*d*e*x^2 + 8*d^2)*sqrt(e*x^2 + d)*sqrt(e))/e^3
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int x^4 \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \begin{cases} -\frac{8d^2\sqrt{d+ex^2}}{75e^{\frac{5}{2}}} + \frac{4dx^2\sqrt{d+ex^2}}{75e^{\frac{3}{2}}} + \frac{x^5 \operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5} - \frac{x^4\sqrt{d+ex^2}}{25\sqrt{e}} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Piecewise((-8*d**2*sqrt(d + e*x**2)/(75*e**(5/2)) + 4*d*x**2*sqrt(d + e*x*
*2)/(75*e**(3/2)) + x**5*atanh(sqrt(e)*x/sqrt(d + e*x**2))/5 - x**4*sqrt(d
 + e*x**2)/(25*sqrt(e)), Ne(e, 0)), (0, True))
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.40

$$\int x^4 \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$$
$$= \frac{1}{5} x^5 \operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right) - \frac{15\,(ex^2+d)^{\frac{7}{2}} - 42\,(ex^2+d)^{\frac{5}{2}}d + 35\,(ex^2+d)^{\frac{3}{2}}d^2}{525\,de^{\frac{5}{2}}}$$
$$+ \frac{5\,(ex^2+d)^{\frac{7}{2}} - 21\,(ex^2+d)^{\frac{5}{2}}d + 35\,(ex^2+d)^{\frac{3}{2}}d^2 - 35\,\sqrt{ex^2+d}d^3}{175\,de^{\frac{5}{2}}}$$

input
```
integrate(x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

output
```
1/5*x^5*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)) - 1/525*(15*(e*x^2 + d)^(7/2) -
 42*(e*x^2 + d)^(5/2)*d + 35*(e*x^2 + d)^(3/2)*d^2)/(d*e^(5/2)) + 1/175*(5
*(e*x^2 + d)^(7/2) - 21*(e*x^2 + d)^(5/2)*d + 35*(e*x^2 + d)^(3/2)*d^2 - 3
5*sqrt(e*x^2 + d)*d^3)/(d*e^(5/2))
```

3.10.8 Giac [F]

$$\int x^4 \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \int x^4 \operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right) dx$$

input
```
integrate(x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
sage0*x
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \int x^4 \operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right) dx$$

input
```
int(x^4*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^4*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.11 $\int x^2 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

3.11.1 Optimal result

Integrand size = 23, antiderivative size = 68

$$\int x^2 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \frac{d\sqrt{d+ex^2}}{3e^{3/2}} - \frac{(d+ex^2)^{3/2}}{9e^{3/2}} + \frac{1}{3}x^3 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

output
```
-1/9*(e*x^2+d)^(3/2)/e^(3/2)+1/3*x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/
3*d*(e*x^2+d)^(1/2)/e^(3/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int x^2 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{9}\left(\frac{(2d-ex^2)\sqrt{d+ex^2}}{e^{3/2}} + 3x^3 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\right)$$

input
```
Integrate[x^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(((2*d - e*x^2)*Sqrt[d + e*x^2])/e^(3/2) + 3*x^3*ArcTanh[(Sqrt[e]*x)/Sqrt[
d + e*x^2]])/9
```

3.11.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6775, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) \, dx$$

$$\Big\downarrow \text{6775}$$

$$\frac{1}{3}x^3\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{3}\sqrt{e}\int \frac{x^3}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{3}x^3\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{e}\int \frac{x^2}{\sqrt{ex^2+d}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}x^3\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{e}\int \left(\frac{\sqrt{ex^2+d}}{e}-\frac{d}{e\sqrt{ex^2+d}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{e}\left(\frac{2\left(d+ex^2\right)^{3/2}}{3e^2}-\frac{2d\sqrt{d+ex^2}}{e^2}\right)$$

input
```
Int[x^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
-1/6*(Sqrt[e]*((-2*d*Sqrt[d + e*x^2])/e^2 + (2*(d + e*x^2)^(3/2))/(3*e^2))
) + (x^3*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/3
```

3.11.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.11.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(50) = 100$.

Time = 0.02 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.88

method	result	size
default	$\frac{x^{3} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{3}+\frac{e^{\frac{3}{2}} \left(\frac{x^{4} \sqrt{e\,x^{2}+d}}{5 e}-\frac{4 d \left(\frac{x^{2} \sqrt{e\,x^{2}+d}}{3 e}-\frac{2 d \sqrt{e\,x^{2}+d}}{3 e^{2}}\right)}{5 e}\right)}{3 d}-\frac{\sqrt{e}\, \left(\frac{x^{2} \left(e\,x^{2}+d\right)^{\frac{3}{2}}}{5 e}-\frac{2 d \left(e\,x^{2}+d\right)^{\frac{3}{2}}}{15 e^{2}}\right)}{3 d}$	128
parts	$\frac{x^{3} \operatorname{arctanh}\left(\frac{x \sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{3}+\frac{e^{\frac{3}{2}} \left(\frac{x^{4} \sqrt{e\,x^{2}+d}}{5 e}-\frac{4 d \left(\frac{x^{2} \sqrt{e\,x^{2}+d}}{3 e}-\frac{2 d \sqrt{e\,x^{2}+d}}{3 e^{2}}\right)}{5 e}\right)}{3 d}-\frac{\sqrt{e}\, \left(\frac{x^{2} \left(e\,x^{2}+d\right)^{\frac{3}{2}}}{5 e}-\frac{2 d \left(e\,x^{2}+d\right)^{\frac{3}{2}}}{15 e^{2}}\right)}{3 d}$	128

input
```
int(x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/3*e^(3/2)/d*(1/5*x^4/e*(e*x^2
+d)^(1/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2)))-1
/3*e^(1/2)/d*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int x^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{3\,e^2x^3\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)-2\,\sqrt{ex^2+d}(ex^2-2\,d)\sqrt{e}}{18\,e^2}$$

input
```
integrate(x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

output
```
1/18*(3*e^2*x^3*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*sqr
t(e*x^2 + d)*(e*x^2 - 2*d)*sqrt(e))/e^2
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int x^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \begin{cases} \frac{2d\sqrt{d+ex^2}}{9e^{\frac{3}{2}}}+\frac{x^3\,\text{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3}-\frac{x^2\sqrt{d+ex^2}}{9\sqrt{e}} & \text{for } e\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Piecewise((2*d*sqrt(d + e*x**2)/(9*e**(3/2)) + x**3*atanh(sqrt(e)*x/sqrt(d
 + e*x**2))/3 - x**2*sqrt(d + e*x**2)/(9*sqrt(e)), Ne(e, 0)), (0, True))
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.46

$$\int x^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{1}{3}\,x^3\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)-\frac{3\,(ex^2+d)^{\frac{5}{2}}-5\,(ex^2+d)^{\frac{3}{2}}d}{45\,de^{\frac{3}{2}}}+\frac{3\,(ex^2+d)^{\frac{5}{2}}-10\,(ex^2+d)^{\frac{3}{2}}d+15\,\sqrt{ex^2+d}d^2}{45\,de^{\frac{3}{2}}}$$

input
```
integrate(x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)) - 1/45*(3*(e*x^2 + d)^(5/2) - 5
*(e*x^2 + d)^(3/2)*d)/(d*e^(3/2)) + 1/45*(3*(e*x^2 + d)^(5/2) - 10*(e*x^2
+ d)^(3/2)*d + 15*sqrt(e*x^2 + d)*d^2)/(d*e^(3/2))
```

3.11.8 Giac [F]

$$\int x^2 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \int x^2 \,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right) dx$$

input
```
integrate(x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
sage0*x
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \int x^2 \,\text{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right) dx$$

input
```
int(x^2*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^2*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.12 $\int \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

3.12.1 Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = -\frac{\sqrt{d+ex^2}}{\sqrt{e}} + x\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

output
```
x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))-(e*x^2+d)^(1/2)/e^(1/2)
```

3.12.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = -\frac{\sqrt{d+ex^2}}{\sqrt{e}} + x\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

input
```
Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
-(Sqrt[d + e*x^2]/Sqrt[e]) + x*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
```

3.12.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6771, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$$

$$\Big\downarrow \text{6771}$$

$$x\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\sqrt{e}\int \frac{x}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{241}$$

$$x\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{\sqrt{e}}$$

input
```
Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
-(Sqrt[d + e*x^2]/Sqrt[e]) + x*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
```

3.12.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6771
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]], x_Symbol] :> Simp[x*A
rcTanh[(c*x)/Sqrt[a + b*x^2]], x] - Simp[c   Int[x/Sqrt[a + b*x^2], x], x]
/; FreeQ[{a, b, c}, x] && EqQ[b, c^2]
```

3.12.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(32) = 64.

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.90

method	result	size
default	$x \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)+\frac{e^{\frac{3}{2}}\left(\frac{x^2\sqrt{e\,x^2+d}}{3e}-\frac{2d\sqrt{e\,x^2+d}}{3e^2}\right)}{d}-\frac{(e\,x^2+d)^{\frac{3}{2}}}{3\sqrt{e}\,d}$	76
parts	$x \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)+\frac{e^{\frac{3}{2}}\left(\frac{x^2\sqrt{e\,x^2+d}}{3e}-\frac{2d\sqrt{e\,x^2+d}}{3e^2}\right)}{d}-\frac{(e\,x^2+d)^{\frac{3}{2}}}{3\sqrt{e}\,d}$	76

input
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+e^(3/2)/d*(1/3*x^2/e*(e*x^2+d)^(1/2)-
2/3*d/e^2*(e*x^2+d)^(1/2))-1/3/e^(1/2)/d*(e*x^2+d)^(3/2)
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{ex\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)-2\,\sqrt{ex^2+d}\sqrt{e}}{2\,e}$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

output
```
1/2*(e*x*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*sqrt(e*x^2
 + d)*sqrt(e))/e
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \begin{cases} x\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{\sqrt{e}} & \text{for } e\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Piecewise((x*atanh(sqrt(e)*x/sqrt(d + e*x**2)) - sqrt(d + e*x**2)/sqrt(e),
 Ne(e, 0)), (0, True))
```

3.12.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(32) = 64$.

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.62

$$\int \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$$
$$= x\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right) - \frac{(ex^2+d)^{\frac{3}{2}}}{3\,d\sqrt{e}} + \frac{(ex^2+d)^{\frac{3}{2}} - 3\sqrt{ex^2+d}d}{3\,d\sqrt{e}}$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

output
```
x*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)) - 1/3*(e*x^2 + d)^(3/2)/(d*sqrt(e)) +
 1/3*((e*x^2 + d)^(3/2) - 3*sqrt(e*x^2 + d)*d)/(d*sqrt(e))
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{1}{2}\,x\log\left(-\frac{\frac{\sqrt{e}x}{\sqrt{ex^2+d}}+1}{\frac{\sqrt{e}x}{\sqrt{ex^2+d}}-1}\right) - \frac{\sqrt{e^2x^2+de}}{e}$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
1/2*x*log(-(sqrt(e)*x/sqrt(e*x^2 + d) + 1)/(sqrt(e)*x/sqrt(e*x^2 + d) - 1)
) - sqrt(e^2*x^2 + d*e)/e
```

3.12.9 Mupad [B] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = x \operatorname{atanh}\left(\frac{\sqrt{e}\, x}{\sqrt{e\, x^2+d}}\right) - \frac{\sqrt{e\, x^2+d}}{\sqrt{e}}$$

input
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
x*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)) - (d + e*x^2)^(1/2)/e^(1/2)
```

3.13 $\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx$

3.13.1 Optimal result

Integrand size = 23, antiderivative size = 55

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x} - \frac{\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

output
```
-arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x-arctanh((e*x^2+d)^(1/2)/d^(1/2))*e^(
1/2)/d^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x} + \frac{\sqrt{e}\left(\log(x) - \log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)\right)}{\sqrt{d}}$$

input
```
Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^2,x]
```

output
```
-(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x) + (Sqrt[e]*(Log[x] - Log[d + Sqr
t[d]*Sqrt[d + e*x^2]]))/Sqrt[d]
```

3.13.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6775, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^2} \, dx$$

$$\Big\downarrow \text{ 6775}$$

$$\sqrt{e}\int \frac{1}{x\sqrt{ex^2+d}}dx-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\sqrt{e}\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{\sqrt{e}}-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

input
```
Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^2,x]
```

output
```
-(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x) - (Sqrt[e]*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]])/Sqrt[d]
```

3.13.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.13.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.53

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{x}-\frac{\sqrt{e}\,\sqrt{e\,x^{2}+d}}{d}+\frac{\sqrt{e}\left(\sqrt{e\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)\right)}{d}$	84
parts	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{x}-\frac{\sqrt{e}\,\sqrt{e\,x^{2}+d}}{d}+\frac{\sqrt{e}\left(\sqrt{e\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)\right)}{d}$	84

input
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x-e^(1/2)/d*(e*x^2+d)^(1/2)+e^(1/2)/d*
((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))
```

3.13.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(43) = 86.

Time = 0.28 (sec) , antiderivative size = 273, normalized size of antiderivative = 4.96

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx$$
$$= \left[\frac{x\sqrt{\frac{e}{d}}\log\left(-\frac{e^2x^2-2\sqrt{ex^2+d}d\sqrt{e}\sqrt{\frac{e}{d}}+2\,de}{x^2}\right)+(x-1)\log\left(\frac{2\,ex^2+2\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)-x\log\left(\frac{ex+\sqrt{ex^2+d}\sqrt{e}}{x}\right)+x\log}{2\,x}\right.$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="fricas")
```

output
```
[1/2*(x*sqrt(e/d)*log(-(e^2*x^2 - 2*sqrt(e*x^2 + d)*d*sqrt(e)*sqrt(e/d) +
2*d*e)/x^2) + (x - 1)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) -
 x*log((e*x + sqrt(e*x^2 + d)*sqrt(e))/x) + x*log((e*x - sqrt(e*x^2 + d)*s
qrt(e))/x))/x, 1/2*(2*x*sqrt(-e/d)*arctan(sqrt(e*x^2 + d)*d*sqrt(e)*sqrt(-
e/d)/(e^2*x^2 + d*e)) + (x - 1)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x
 + d)/d) - x*log((e*x + sqrt(e*x^2 + d)*sqrt(e))/x) + x*log((e*x - sqrt(e*
x^2 + d)*sqrt(e))/x))/x]
```

3.13.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx$$

input
```
integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**2,x)
```

output
```
Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**2, x)
```

3.13.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^2}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="maxima")
```

output
```
d*sqrt(e)*integrate(-sqrt(e*x^2 + d)/(e^2*x^5 + d*e*x^3 - (e*x^3 + d*x)*(e
*x^2 + d)), x) - 1/2*(log(sqrt(e)*x + sqrt(e*x^2 + d)) - log(-sqrt(e)*x +
sqrt(e*x^2 + d)))/x
```

3.13.8 Giac [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^2}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^2}\,dx$$

input
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^2,x)
```

output
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^2, x)
```

3.14 $\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx$

3.14.1 Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = -\frac{\sqrt{e}\sqrt{d+ex^2}}{6dx^2} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3} + \frac{e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}$$

output
```
-1/3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3+1/6*e^(3/2)*arctanh((e*x^2+d)^
(1/2)/d^(1/2))/d^(3/2)-1/6*e^(1/2)*(e*x^2+d)^(1/2)/d/x^2
```

3.14.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{\sqrt{e}x\left(\sqrt{d}\sqrt{d+ex^2}+ex^2\log(x)-ex^2\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)\right)}{d^{3/2}}}{6x^3}$$

input
```
Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

output
```
-1/6*(2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] + (Sqrt[e]*x*(Sqrt[d]*Sqrt[d
+ e*x^2] + e*x^2*Log[x] - e*x^2*Log[d + Sqrt[d]*Sqrt[d + e*x^2]]))/d^(3/2)
)/x^3
```

3.14.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6775, 243, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^4}dx$$

$$\Big\downarrow \text{6775}$$

$$\frac{1}{3}\sqrt{e}\int \frac{1}{x^3\sqrt{ex^2+d}}dx-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{6}\sqrt{e}\int \frac{1}{x^4\sqrt{ex^2+d}}dx^2-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{6}\sqrt{e}\left(-\frac{e\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2}{2d}-\frac{\sqrt{d+ex^2}}{dx^2}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\sqrt{e}\left(-\frac{\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{d}-\frac{\sqrt{d+ex^2}}{dx^2}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}\sqrt{e}\left(\frac{e\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{d+ex^2}}{dx^2}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

input
```
Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

output
```
-1/3*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^3 + (Sqrt[e]*(-(Sqrt[d + e*x^2
]/(d*x^2)) + (e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/d^(3/2)))/6
```

3.14.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.14.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{3x^{3}}+\frac{e^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)}{3d^{\frac{3}{2}}}+\frac{\sqrt{e}\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{2d\,x^{2}}+\frac{e\left(\sqrt{e\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)\right)}{2d}\right)}{3d}$	123
parts	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{3x^{3}}+\frac{e^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)}{3d^{\frac{3}{2}}}+\frac{\sqrt{e}\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{2d\,x^{2}}+\frac{e\left(\sqrt{e\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)\right)}{2d}\right)}{3d}$	123

input
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3+1/3*e^(3/2)/d^(3/2)*ln((2*d+2*
d^(1/2)*(e*x^2+d)^(1/2))/x)+1/3*e^(1/2)/d*(-1/2/d/x^2*(e*x^2+d)^(3/2)+1/2*
e/d*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)))
```

3.14.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(63) = 126$.

Time = 0.29 (sec) , antiderivative size = 340, normalized size of antiderivative = 4.00

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx$$
$$= \left[\frac{ex^3\sqrt{\frac{e}{d}}\log\left(-\frac{e^2x^2+2\sqrt{ex^2+d}d\sqrt{e}\sqrt{\frac{e}{d}}+2\,de}{x^2}\right) - 2\,dx^3\log\left(\frac{ex+\sqrt{ex^2+d}\sqrt{e}}{x}\right) + 2\,dx^3\log\left(\frac{ex-\sqrt{ex^2+d}\sqrt{e}}{x}\right) - 2\sqrt{ex^2 \cdots}}{12\,dx^3}\right.$$
$$\left. -\frac{ex^3\sqrt{-\frac{e}{d}}\arctan\left(\frac{\sqrt{ex^2+d}d\sqrt{e}\sqrt{-\frac{e}{d}}}{e^2x^2+de}\right) + dx^3\log\left(\frac{ex+\sqrt{ex^2+d}\sqrt{e}}{x}\right) - dx^3\log\left(\frac{ex-\sqrt{ex^2+d}\sqrt{e}}{x}\right) + \sqrt{ex^2+d}\sqrt{e}x \cdots}{6\,dx^3}\right]$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="fricas")
```

output
```
[1/12*(e*x^3*sqrt(e/d)*log(-(e^2*x^2 + 2*sqrt(e*x^2 + d)*d*sqrt(e)*sqrt(e/
d) + 2*d*e)/x^2) - 2*d*x^3*log((e*x + sqrt(e*x^2 + d)*sqrt(e))/x) + 2*d*x^
3*log((e*x - sqrt(e*x^2 + d)*sqrt(e))/x) - 2*sqrt(e*x^2 + d)*sqrt(e)*x + 2
*(d*x^3 - d)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d))/(d*x^3),
-1/6*(e*x^3*sqrt(-e/d)*arctan(sqrt(e*x^2 + d)*d*sqrt(e)*sqrt(-e/d)/(e^2*x^
2 + d*e)) + d*x^3*log((e*x + sqrt(e*x^2 + d)*sqrt(e))/x) - d*x^3*log((e*x
- sqrt(e*x^2 + d)*sqrt(e))/x) + sqrt(e*x^2 + d)*sqrt(e)*x - (d*x^3 - d)*lo
g((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d))/(d*x^3)]
```

3.14.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx$$

input
```
integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**4,x)
```

output
```
Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**4, x)
```

3.14.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^4}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="maxima")
```

output
```
d*sqrt(e)*integrate(-1/3*sqrt(e*x^2 + d)/(e^2*x^7 + d*e*x^5 - (e*x^5 + d*x
^3)*(e*x^2 + d)), x) - 1/6*(log(sqrt(e)*x + sqrt(e*x^2 + d)) - log(-sqrt(e
)*x + sqrt(e*x^2 + d)))/x^3
```

3.14.8 Giac [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^4}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^4}\,dx$$

input
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^4,x)
```

output
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^4, x)
```

3.15 $\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^6} dx$

3.15.1 Optimal result

Integrand size = 23, antiderivative size = 111

$$\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^6} dx = -\frac{\sqrt{e}\sqrt{d+ex^2}}{20dx^4} + \frac{3e^{3/2}\sqrt{d+ex^2}}{40d^2x^2} - \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5} - \frac{3e^{5/2}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}$$

output
```
-1/5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5-3/40*e^(5/2)*arctanh((e*x^2+d)
^(1/2)/d^(1/2))/d^(5/2)+3/40*e^(3/2)*(e*x^2+d)^(1/2)/d^2/x^2-1/20*e^(1/2)*
(e*x^2+d)^(1/2)/d/x^4
```

3.15.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^6} dx = \frac{-8\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{\sqrt{e}x\left(\sqrt{d}\sqrt{d+ex^2}(-2d+3ex^2)+3e^2x^4\log(x)-3e^2x^4\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)\right)}{d^{5/2}}}{40x^5}$$

input
```
Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

output
```
(-8*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] + (Sqrt[e]*x*(Sqrt[d]*Sqrt[d + e*
x^2]*(-2*d + 3*e*x^2) + 3*e^2*x^4*Log[x] - 3*e^2*x^4*Log[d + Sqrt[d]*Sqrt[
d + e*x^2]]))/d^(5/2))/(40*x^5)
```

3.15.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6775, 243, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^6}dx$$

$$\Big\downarrow \ 6775$$

$$\frac{1}{5}\sqrt{e}\int \frac{1}{x^5\sqrt{ex^2+d}}dx-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

$$\Big\downarrow \ 243$$

$$\frac{1}{10}\sqrt{e}\int \frac{1}{x^6\sqrt{ex^2+d}}dx^2-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

$$\Big\downarrow \ 52$$

$$\frac{1}{10}\sqrt{e}\left(-\frac{3e\int \frac{1}{x^4\sqrt{ex^2+d}}dx^2}{4d}-\frac{\sqrt{d+ex^2}}{2dx^4}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

$$\Big\downarrow \ 52$$

$$\frac{1}{10}\sqrt{e}\left(-\frac{3e\left(-\frac{e\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2}{2d}-\frac{\sqrt{d+ex^2}}{dx^2}\right)}{4d}-\frac{\sqrt{d+ex^2}}{2dx^4}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

$$\Big\downarrow \ 73$$

$$\frac{1}{10}\sqrt{e}\left(-\frac{3e\left(-\frac{\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{d}-\frac{\sqrt{d+ex^2}}{dx^2}\right)}{4d}-\frac{\sqrt{d+ex^2}}{2dx^4}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{10}\sqrt{e}\left(-\frac{3e\left(\frac{e\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{d+ex^2}}{dx^2}\right)}{4d}-\frac{\sqrt{d+ex^2}}{2dx^4}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

input
```
Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

output
```
-1/5*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^5 + (Sqrt[e]*(-1/2*Sqrt[d + e*
x^2]/(d*x^4) - (3*e*(-(Sqrt[d + e*x^2]/(d*x^2)) + (e*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]])/d^(3/2)))/(4*d)))/10
```

3.15.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.15.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(83) = 166.

Time = 0.02 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.54

method	result
default	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{5x^5}-\frac{e^{\frac{3}{2}}\left(-\frac{\sqrt{e\,x^2+d}}{2x^2 d}+\frac{e\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)}{2d^{\frac{3}{2}}}\right)}{5d}+\frac{\sqrt{e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{4d\,x^4}-\frac{e\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{2d\,x^2}+\frac{e\left(\sqrt{e\,x^2+d}-\sqrt{d}\,\ln\left(\right.\right.}{2d}\right.}{4d}\right.}{5d}$
parts	$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{5x^5}-\frac{e^{\frac{3}{2}}\left(-\frac{\sqrt{e\,x^2+d}}{2x^2 d}+\frac{e\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)}{2d^{\frac{3}{2}}}\right)}{5d}+\frac{\sqrt{e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{4d\,x^4}-\frac{e\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{2d\,x^2}+\frac{e\left(\sqrt{e\,x^2+d}-\sqrt{d}\,\ln\left(\right.\right.}{2d}\right.}{4d}\right.}{5d}$

input
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5-1/5*e^(3/2)/d*(-1/2*(e*x^2+d)^
(1/2)/x^2/d+1/2*e/d^(3/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))+1/5*e^(1/
2)/d*(-1/4/d/x^4*(e*x^2+d)^(3/2)-1/4*e/d*(-1/2/d/x^2*(e*x^2+d)^(3/2)+1/2*e
/d*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))))
```

3.15.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(83) = 166.

Time = 0.30 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.45

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = \left[\frac{3\,e^2x^5\sqrt{\frac{e}{d}}\log\left(-\frac{e^2x^2-2\,\sqrt{ex^2+d}d\sqrt{e}\sqrt{\frac{e}{d}}+2\,de}{x^2}\right)-8\,d^2x^5\log\left(\frac{ex+\sqrt{ex^2+d}\sqrt{e}}{x}\right)+8\,d^2x^5\log\left(\frac{ex-\sqrt{ex^2+d}\sqrt{e}}{x}\right)+2\,(3\cdots}{80\,d^2x^5}\right.$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="fricas")
```

output
```
[1/80*(3*e^2*x^5*sqrt(e/d)*log(-(e^2*x^2 - 2*sqrt(e*x^2 + d)*d*sqrt(e)*sqr
t(e/d) + 2*d*e)/x^2) - 8*d^2*x^5*log((e*x + sqrt(e*x^2 + d)*sqrt(e))/x) +
8*d^2*x^5*log((e*x - sqrt(e*x^2 + d)*sqrt(e))/x) + 2*(3*e*x^3 - 2*d*x)*sqr
t(e*x^2 + d)*sqrt(e) + 8*(d^2*x^5 - d^2)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*
sqrt(e)*x + d)/d))/(d^2*x^5), 1/40*(3*e^2*x^5*sqrt(-e/d)*arctan(sqrt(e*x^2
 + d)*d*sqrt(e)*sqrt(-e/d)/(e^2*x^2 + d*e)) - 4*d^2*x^5*log((e*x + sqrt(e*
x^2 + d)*sqrt(e))/x) + 4*d^2*x^5*log((e*x - sqrt(e*x^2 + d)*sqrt(e))/x) +
(3*e*x^3 - 2*d*x)*sqrt(e*x^2 + d)*sqrt(e) + 4*(d^2*x^5 - d^2)*log((2*e*x^2
 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d))/(d^2*x^5)]
```

3.15.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx$$

input
```
integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**6,x)
```

output
```
Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**6, x)
```

3.15.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^6}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="maxima")
```

output
```
d*sqrt(e)*integrate(-1/5*sqrt(e*x^2 + d)/(e^2*x^9 + d*e*x^7 - (e*x^7 + d*x
^5)*(e*x^2 + d)), x) - 1/10*(log(sqrt(e)*x + sqrt(e*x^2 + d)) - log(-sqrt(
e)*x + sqrt(e*x^2 + d)))/x^5
```

3.15.8 Giac [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^6}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="giac")
```

output
```
sage0*x
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^6}\,dx$$

input
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^6,x)
```

output
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^6, x)
```

3.16 $\int x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$

3.16.1 Optimal result

Integrand size = 25, antiderivative size = 196

$$\int x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=-\frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847e^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}}-\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}}+\frac{2}{11}x^{11/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+\frac{30d^{11/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{847e^{11/4}\sqrt{d+ex^2}}$$

output
```
2/11*x^(11/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+36/847*d*x^(5/2)*(e*x^2+d
)^(1/2)/e^(3/2)-4/121*x^(9/2)*(e*x^2+d)^(1/2)/e^(1/2)-60/847*d^2*x^(1/2)*(
e*x^2+d)^(1/2)/e^(5/2)+30/847*d^(11/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/
4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arcta
n(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d
^(1/2)+x*e^(1/2))^2)^(1/2)/e^(11/4)/(e*x^2+d)^(1/2)
```

3.16.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.82

$$\int x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\frac{2}{847}\sqrt{x}\left(-\frac{2\sqrt{d+ex^2}(15d^2-9dex^2+7e^2x^4)}{e^{5/2}}+77x^5\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\right)+\frac{60d^{5/2}\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{1+\frac{d}{ex^2}}x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{847e^2\sqrt{d+ex^2}}$$

input
```
Integrate[x^(9/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(2*Sqrt[x]*((-2*Sqrt[d + e*x^2]*(15*d^2 - 9*d*e*x^2 + 7*e^2*x^4))/e^(5/2)
+ 77*x^5*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]))/847 + (60*d^(5/2)*Sqrt[(I*
Sqrt[d])/Sqrt[e]]*Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d
])/Sqrt[e]]/Sqrt[x]], -1])/(847*e^2*Sqrt[d + e*x^2])
```

3.16.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6775, 262, 262, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$$

$$\Big\downarrow \ 6775$$

$$\frac{2}{11}x^{11/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{11}\sqrt{e}\int\frac{x^{11/2}}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \ 262$$

$$\frac{2}{11}x^{11/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{11}\sqrt{e}\left(\frac{2x^{9/2}\sqrt{d+ex^2}}{11e}-\frac{9d\int\frac{x^{7/2}}{\sqrt{ex^2+d}}dx}{11e}\right)$$

$$\Big\downarrow \ 262$$

$$\frac{2}{11}x^{11/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{11}\sqrt{e}\left(\frac{2x^{9/2}\sqrt{d+ex^2}}{11e}-\frac{9d\left(\frac{2x^{5/2}\sqrt{d+ex^2}}{7e}-\frac{5d\int\frac{x^{3/2}}{\sqrt{ex^2+d}}dx}{7e}\right)}{11e}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{11}x^{11/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{11}\sqrt{e}\left(\frac{2x^{9/2}\sqrt{d+ex^2}}{11e}-\frac{9d\left(\frac{2x^{5/2}\sqrt{d+ex^2}}{7e}-\frac{5d\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e}-\frac{d\int\frac{1}{\sqrt{x}\sqrt{ex^2+d}}dx}{3e}\right)}{7e}\right)}{11e}\right)$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{11}x^{11/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{11}\sqrt{e}\left(\frac{2x^{9/2}\sqrt{d+ex^2}}{11e}-\frac{9d\left(\frac{2x^{5/2}\sqrt{d+ex^2}}{7e}-\frac{5d\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e}-\frac{2d\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{3e}\right)}{7e}\right)}{11e}\right)$$

$$\Big\downarrow \text{761}$$

3.16. $\int x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$

$$\frac{2}{11}x^{11/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{11}\sqrt{e}\left(\frac{2x^{9/2}\sqrt{d+ex^2}}{11e}-\frac{9d\left(\frac{2x^{5/2}\sqrt{d+ex^2}}{7e}-\frac{5d\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e}-\frac{d^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3e^{5/4}\sqrt{d+ex^2}}\right)}{7e}\right)}{11e}\right)$$

input
```
Int[x^(9/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(2*x^(11/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/11 - (2*Sqrt[e]*((2*x^(9
/2)*Sqrt[d + e*x^2])/(11*e) - (9*d*((2*x^(5/2)*Sqrt[d + e*x^2])/(7*e) - (5
*d*((2*Sqrt[x]*Sqrt[d + e*x^2])/(3*e) - (d^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqr
t[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x]
)/d^(1/4)], 1/2])/(3*e^(5/4)*Sqrt[d + e*x^2])))/(7*e)))/(11*e)))/11
```

3.16.3.1 Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.16.4 Maple [F]

$$\int x^{\frac{9}{2}} \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right) dx$$

input
```
int(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

output
```
int(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

3.16.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.48

$$\int x^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{77\,e^3x^{\frac{11}{2}}\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)+60\,d^3\text{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)}{847\,e^3}$$

input
```
integrate(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas"
)
```

output
```
1/847*(77*e^3*x^(11/2)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d)
+ 60*d^3*weierstrassPInverse(-4*d/e, 0, x) - 4*(7*e^2*x^4 - 9*d*e*x^2 + 15
*d^2)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(x))/e^3
```

3.16.6 Sympy [F(-1)]

Timed out.

$$\int x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Timed out
```

3.16.7 Maxima [F]

$$\int x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \int x^{\frac{9}{2}}\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima"
)
```

output
```
1/11*x^(11/2)*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/11*x^(11/2)*log(-sqrt(e
)*x + sqrt(e*x^2 + d)) - 2*d*sqrt(e)*integrate(-1/11*x*e^(1/2*log(e*x^2 +
d) + 9/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

3.16.8 Giac [F]

$$\int x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \int x^{\frac{9}{2}}\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
integrate(x^(9/2)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{9/2}\,\text{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

input
```
int(x^(9/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^(9/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.17 $\int x^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$

3.17.1 Optimal result

Integrand size = 25, antiderivative size = 168

$$\int x^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{20d\sqrt{x}\sqrt{d+ex^2}}{147e^{3/2}} - \frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}} + \frac{2}{7}x^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{10d^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{147e^{7/4}\sqrt{d+ex^2}}$$

output
```
2/7*x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))-4/49*x^(5/2)*(e*x^2+d)^(1/2
)/e^(1/2)+20/147*d*x^(1/2)*(e*x^2+d)^(1/2)/e^(3/2)-10/147*d^(7/4)*(cos(2*a
rctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1
/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/
2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(7/4)/(e*x^2+d)^(1
/2)
```

3.17.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.88

$$\int x^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\frac{2}{147}\sqrt{x}\left(\frac{2(5d-3ex^2)\sqrt{d+ex^2}}{e^{3/2}}+21x^3\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\right)+\frac{20\sqrt{d}\left(\frac{i\sqrt{d}}{\sqrt{e}}\right)^{5/2}\sqrt{1+\frac{d}{ex^2}}\,x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{147\sqrt{d+ex^2}}$$

input
```
Integrate[x^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(2*Sqrt[x]*((2*(5*d - 3*e*x^2)*Sqrt[d + e*x^2])/e^(3/2) + 21*x^3*ArcTanh[(
Sqrt[e]*x)/Sqrt[d + e*x^2]]))/147 + (20*Sqrt[d]*((I*Sqrt[d])/Sqrt[e])^(5/2
)*Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt
[x]], -1])/(147*Sqrt[d + e*x^2])
```

3.17.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6775, 262, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$$

$$\Big\downarrow \text{6775}$$

$$\frac{2}{7}x^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{7}\sqrt{e}\int\frac{x^{7/2}}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{7}x^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{7}\sqrt{e}\left(\frac{2x^{5/2}\sqrt{d+ex^2}}{7e}-\frac{5d\int\frac{x^{3/2}}{\sqrt{ex^2+d}}dx}{7e}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{7}x^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{7}\sqrt{e}\left(\frac{2x^{5/2}\sqrt{d+ex^2}}{7e}-\frac{5d\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e}-\frac{d\int\frac{1}{\sqrt{x}\sqrt{ex^2+d}}dx}{3e}\right)}{7e}\right)$$

$\Big\downarrow$ 266

$$\frac{2}{7}x^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{7}\sqrt{e}\left(\frac{2x^{5/2}\sqrt{d+ex^2}}{7e}-\frac{5d\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e}-\frac{2d\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{3e}\right)}{7e}\right)$$

$\Big\downarrow$ 761

$$\frac{2}{7}x^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{7}\sqrt{e}\left(\frac{2x^{5/2}\sqrt{d+ex^2}}{7e}-\frac{5d\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e}-\frac{d^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3e^{5/4}\sqrt{d+ex^2}}\right)}{7e}\right)$$

input
```
Int[x^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(2*x^(7/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/7 - (2*Sqrt[e]*((2*x^(5/2
)*Sqrt[d + e*x^2])/(7*e) - (5*d*((2*Sqrt[x]*Sqrt[d + e*x^2])/(3*e) - (d^(3
/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*Ellipt
icF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(3*e^(5/4)*Sqrt[d + e*x^2])
))/(7*e)))/7
```

3.17.3.1 Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.17.4 Maple [F]

$$\int x^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)dx$$

input
```
int(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

output
```
int(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

3.17.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.49

$$\int x^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{21\,e^2x^{\frac{7}{2}}\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)-20\,d^2\text{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)-}{147\,e^2}$$

input
```
integrate(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas"
)
```

output
```
1/147*(21*e^2*x^(7/2)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) -
 20*d^2*weierstrassPInverse(-4*d/e, 0, x) - 4*(3*e*x^2 - 5*d)*sqrt(e*x^2 +
 d)*sqrt(e)*sqrt(x))/e^2
```

3.17.6 Sympy [F]

$$\int x^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{\frac{5}{2}}\,\text{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$$

input
```
integrate(x**(5/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Integral(x**(5/2)*atanh(sqrt(e)*x/sqrt(d + e*x**2)), x)
```

3.17.7 Maxima [F]

$$\int x^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{\frac{5}{2}}\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima"
)
```

output
```
1/7*x^(7/2)*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/7*x^(7/2)*log(-sqrt(e)*x
+ sqrt(e*x^2 + d)) - 2*d*sqrt(e)*integrate(-1/7*x*e^(1/2*log(e*x^2 + d) +
5/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

3.17.8 Giac [F]

$$\int x^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{\frac{5}{2}}\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
integrate(x^(5/2)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int x^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{5/2}\,\text{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

input
```
int(x^(5/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^(5/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.18 $\int \sqrt{x}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$

3.18.1 Optimal result

Integrand size = 25, antiderivative size = 142

$$\begin{aligned}&\int \sqrt{x}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx\\&=-\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{e}}+\frac{2}{3}x^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\\&\quad+\frac{2d^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{9e^{3/4}\sqrt{d+ex^2}}\end{aligned}$$

output
```
2/3*x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))-4/9*x^(1/2)*(e*x^2+d)^(1/2)
/e^(1/2)+2/9*d^(3/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(
2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/
d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^
2)^(1/2)/e^(3/4)/(e*x^2+d)^(1/2)
```

3.18.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.95

$$\int \sqrt{x}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\frac{2}{9}\sqrt{x}\left(-\frac{2\sqrt{d+ex^2}}{\sqrt{e}}+3x\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\right)+\frac{4\sqrt{d}\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{1+\frac{d}{ex^2}}x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{9\sqrt{d+ex^2}}$$

input
```
Integrate[Sqrt[x]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(2*Sqrt[x]*((-2*Sqrt[d + e*x^2])/Sqrt[e] + 3*x*ArcTanh[(Sqrt[e]*x)/Sqrt[d
+ e*x^2]]))/9 + (4*Sqrt[d]*Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[1 + d/(e*x^2)]*x
*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(9*Sqrt[d +
e*x^2])
```

3.18.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6775, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$$

$$\Big\downarrow \text{ 6775}$$

$$\frac{2}{3}x^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{3}\sqrt{e}\int \frac{x^{3/2}}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{2}{3}x^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{3}\sqrt{e}\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e}-\frac{d\int \frac{1}{\sqrt{x}\sqrt{ex^2+d}}dx}{3e}\right)$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2}{3}x^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{3}\sqrt{e}\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e}-\frac{2d\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{3e}\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{3}x^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{3}\sqrt{e}\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e}-\frac{d^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3e^{5/4}\sqrt{d+ex^2}}\right)$$

input
```
Int[Sqrt[x]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(2*x^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/3 - (2*Sqrt[e]*((2*Sqrt[x
]*Sqrt[d + e*x^2])/(3*e) - (d^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)
/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1
/2])/(3*e^(5/4)*Sqrt[d + e*x^2])))/3
```

3.18.3.1 Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.18.4 Maple [F]

$$\int \sqrt{x}\, \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right) dx$$

input
```
int(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

output
```
int(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

3.18.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.49

$$\int \sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \frac{3\,ex^{\frac{3}{2}}\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)+4\,d\text{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)-4\,\sqrt{ex^2+d}\sqrt{e}\sqrt{x}}{9\,e}$$

input
```
integrate(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas"
)
```

output
```
1/9*(3*e*x^(3/2)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) + 4*d*
weierstrassPInverse(-4*d/e, 0, x) - 4*sqrt(e*x^2 + d)*sqrt(e)*sqrt(x))/e
```

3.18.6 Sympy [F]

$$\int \sqrt{x}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\int \sqrt{x}\,\text{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$$

input
```
integrate(x**(1/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Integral(sqrt(x)*atanh(sqrt(e)*x/sqrt(d + e*x**2)), x)
```

3.18.7 Maxima [F]

$$\int \sqrt{x}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\int \sqrt{x}\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima"
)
```

output
```
-2*d*sqrt(e)*integrate(-1/3*x*e^(1/2*log(e*x^2 + d) + 1/2*log(x))/(e^2*x^4
 + d*e*x^2 - (e*x^2 + d)^2), x) + 1/3*x^(3/2)*log(sqrt(e)*x + sqrt(e*x^2 +
 d)) - 1/3*x^(3/2)*log(-sqrt(e)*x + sqrt(e*x^2 + d))
```

3.18.8 Giac [F]

$$\int \sqrt{x}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\int \sqrt{x}\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
integrate(sqrt(x)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\int \sqrt{x}\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

input
```
int(x^(1/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^(1/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.19 $\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx$

3.19.1 Optimal result

Integrand size = 25, antiderivative size = 113

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{2\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{d+ex^2}}$$

output
```
-2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2)+2*e^(1/4)*(cos(2*arctan(e^(1
/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*Elli
pticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/
2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(1/4)/(e*x^2+d)^(1/2)
```

3.19.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{4i\sqrt{e}\sqrt{1+\frac{d}{ex^2}}\,x\,\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}$$

input
```
Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
```

output
```
(-2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + ((4*I)*Sqrt[e]*Sqrt[1
+ d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1]
)/(Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])
```

3.19.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6775, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{6775}$$

$$2\sqrt{e}\int \frac{1}{\sqrt{x}\sqrt{ex^2+d}}dx - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{266}$$

$$4\sqrt{e}\int \frac{1}{\sqrt{ex^2+d}}d\sqrt{x} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}$$

input
```
Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
```

output
```
(-2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + (2*e^(1/4)*(Sqrt[d] +
Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e
^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(d^(1/4)*Sqrt[d + e*x^2])
```

3.19.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.19.4 Maple [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{x^{\frac{3}{2}}}dx$$

input
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
```

output
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
```

3.19.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.44

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = \frac{4\,x\text{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right) - \sqrt{x}\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)}{x}$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="fricas"
)
```

output
```
(4*x*weierstrassPInverse(-4*d/e, 0, x) - sqrt(x)*log((2*e*x^2 + 2*sqrt(e*x
^2 + d)*sqrt(e)*x + d)/d))/x
```

3.19.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(3/2),x)
```

output
```
Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**(3/2), x)
```

3.19.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="maxima"
)
```

output
```
2*d*sqrt(e)*integrate(-sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(3/2) - (e
*x^2 + d)*e^(log(e*x^2 + d) + 3/2*log(x))), x) - log(sqrt(e)*x + sqrt(e*x^
2 + d))/sqrt(x) + log(-sqrt(e)*x + sqrt(e*x^2 + d))/sqrt(x)
```

3.19.8 Giac [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="giac")
```

output
```
integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(3/2), x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{3/2}}\,dx$$

input
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(3/2),x)
```

output
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(3/2), x)
```

3.20 $\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx$

3.20.1 Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = -\frac{4\sqrt{e}\sqrt{d+ex^2}}{15dx^{3/2}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} - \frac{2e^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{15d^{5/4}\sqrt{d+ex^2}}$$

output
```
-2/5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2)-4/15*e^(1/2)*(e*x^2+d)^(1/
2)/d/x^(3/2)-2/15*e^(5/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)
/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(
1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1
/2))^2)^(1/2)/d^(5/4)/(e*x^2+d)^(1/2)
```

3.20.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = -\frac{2\left(2\sqrt{e}x\sqrt{d+ex^2}+3d\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\right)}{15dx^{5/2}} - \frac{4\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}e^2\sqrt{1+\frac{d}{ex^2}}x\,\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{15d^{3/2}\sqrt{d+ex^2}}$$

input
```
Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```

output
```
(-2*(2*Sqrt[e]*x*Sqrt[d + e*x^2] + 3*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]
]))/(15*d*x^(5/2)) - (4*Sqrt[(I*Sqrt[d])/Sqrt[e]]*e^2*Sqrt[1 + d/(e*x^2)]*
x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(15*d^(3/2)
*Sqrt[d + e*x^2])
```

3.20.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6775, 264, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}dx$$

$$\Big\downarrow \text{6775}$$

$$\frac{2}{5}\sqrt{e}\int \frac{1}{x^{5/2}\sqrt{ex^2+d}}dx - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{5}\sqrt{e}\left(-\frac{e\int \frac{1}{\sqrt{x}\sqrt{ex^2+d}}dx}{3d} - \frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right) - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{5}\sqrt{e}\left(-\frac{2e\int \frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{3d}-\frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{5}\sqrt{e}\left(-\frac{e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3d^{5/4}\sqrt{d+ex^2}}-\frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}$$

input
```
Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```

output
```
(-2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(5*x^(5/2)) + (2*Sqrt[e]*((-2*Sq
rt[d + e*x^2])/(3*d*x^(3/2)) - (e^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*
x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)
], 1/2])/(3*d^(5/4)*Sqrt[d + e*x^2])))/5
```

3.20.3.1 Defintions of rubi rules used

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.20. $\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx$

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.20.4 Maple [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{7}{2}}}dx$$

input
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)
```

output
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)
```

3.20.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.52

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = -\frac{4\,ex^3 \text{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)+4\,\sqrt{ex^2+d}\sqrt{e}x^{\frac{3}{2}}+3\,d\sqrt{x}\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)}{15\,dx^3}$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="fricas"
)
```

output
```
-1/15*(4*e*x^3*weierstrassPInverse(-4*d/e, 0, x) + 4*sqrt(e*x^2 + d)*sqrt(
e)*x^(3/2) + 3*d*sqrt(x)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d
))/(d*x^3)
```

3.20.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(7/2),x)
```

output
```
Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**(7/2), x)
```

3.20.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="maxima"
)
```

output
```
2*d*sqrt(e)*integrate(-1/5*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(7/2)
- (e*x^2 + d)*e^(log(e*x^2 + d) + 7/2*log(x))), x) - 1/5*log(sqrt(e)*x + s
qrt(e*x^2 + d))/x^(5/2) + 1/5*log(-sqrt(e)*x + sqrt(e*x^2 + d))/x^(5/2)
```

3.20.8 Giac [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="giac")
```

output
```
integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(7/2), x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{7/2}}\,dx$$

input
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(7/2),x)
```

output
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(7/2), x)
```

3.21 $\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx$

3.21.1 Optimal result

Integrand size = 25, antiderivative size = 173

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = -\frac{4\sqrt{e}\sqrt{d+ex^2}}{63dx^{7/2}}+\frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}$$
$$+\frac{10e^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{189d^{9/4}\sqrt{d+ex^2}}$$

output
```
-2/9*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2)+20/189*e^(3/2)*(e*x^2+d)^(
1/2)/d^2/x^(3/2)-4/63*e^(1/2)*(e*x^2+d)^(1/2)/d/x^(7/2)+10/189*e^(9/4)*(co
s(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)
/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(
d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(9/4)/(e*x^2+
d)^(1/2)
```

3.21.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.89

$$\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \frac{4\sqrt{e}x\sqrt{d+ex^2}(-3d+5ex^2)-42d^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{189d^2x^{9/2}} + \frac{20\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}e^3\sqrt{1+\frac{d}{ex^2}}x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{189d^{5/2}\sqrt{d+ex^2}}$$

input
```
Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

output
```
(4*Sqrt[e]*x*Sqrt[d + e*x^2]*(-3*d + 5*e*x^2) - 42*d^2*ArcTanh[(Sqrt[e]*x)
/Sqrt[d + e*x^2]])/(189*d^2*x^(9/2)) + (20*Sqrt[(I*Sqrt[d])/Sqrt[e]]*e^3*S
qrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]
], -1])/(189*d^(5/2)*Sqrt[d + e*x^2])
```

3.21.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6775, 264, 264, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx$$

$$\Big\downarrow \text{ 6775}$$

$$\frac{2}{9}\sqrt{e}\int \frac{1}{x^{9/2}\sqrt{ex^2+d}}dx-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{2}{9}\sqrt{e}\left(-\frac{5e\int \frac{1}{x^{5/2}\sqrt{ex^2+d}}dx}{7d}-\frac{2\sqrt{d+ex^2}}{7dx^{7/2}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{9}\sqrt{e}\left(-\frac{5e\left(-\frac{e\int \frac{1}{\sqrt{x}\sqrt{ex^2+d}}dx}{3d}-\frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right)}{7d}-\frac{2\sqrt{d+ex^2}}{7dx^{7/2}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{9}\sqrt{e}\left(-\frac{5e\left(-\frac{2e\int \frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{3d}-\frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right)}{7d}-\frac{2\sqrt{d+ex^2}}{7dx^{7/2}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{9}\sqrt{e}\left(-\frac{5e\left(-\frac{e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3d^{5/4}\sqrt{d+ex^2}}-\frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right)}{7d}-\frac{2\sqrt{d+ex^2}}{7dx^{7/2}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}$$

input
```
Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

output
```
(-2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(9*x^(9/2)) + (2*Sqrt[e]*((-2*Sq
rt[d + e*x^2])/(7*d*x^(7/2)) - (5*e*((-2*Sqrt[d + e*x^2])/(3*d*x^(3/2)) -
(e^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*E
llipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(3*d^(5/4)*Sqrt[d + e*
x^2])))/(7*d)))/9
```

3.21. $\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}dx$

3.21.3.1 Defintions of rubi rules used

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.21.4 Maple [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{x^{\frac{11}{2}}}dx$$

input
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

output
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

3.21.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.52

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \frac{20\,e^2x^5\text{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)-21\,d^2\sqrt{x}\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)+4\,(5}{189\,d^2x^5}$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="fricas
")
```

output
```
1/189*(20*e^2*x^5*weierstrassPInverse(-4*d/e, 0, x) - 21*d^2*sqrt(x)*log((
2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) + 4*(5*e*x^3 - 3*d*x)*sqrt(e
*x^2 + d)*sqrt(e)*sqrt(x))/(d^2*x^5)
```

3.21.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(11/2),x)
```

output
```
Timed out
```

3.21.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="maxima
")
```

output
```
2*d*sqrt(e)*integrate(-1/9*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(11/2)
 - (e*x^2 + d)*e^(log(e*x^2 + d) + 11/2*log(x))), x) - 1/9*log(sqrt(e)*x +
 sqrt(e*x^2 + d))/x^(9/2) + 1/9*log(-sqrt(e)*x + sqrt(e*x^2 + d))/x^(9/2)
```

3.21.8 Giac [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="giac")
```

output
```
integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(11/2), x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{11/2}}\,dx$$

input
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(11/2),x)
```

output
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(11/2), x)
```

3.22 $\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx$

3.22.1 Optimal result

Integrand size = 25, antiderivative size = 201

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = -\frac{4\sqrt{e}\sqrt{d+ex^2}}{143dx^{11/2}} + \frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} - \frac{30e^{13/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{1001d^{13/4}\sqrt{d+ex^2}}$$

output
```
-2/13*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(13/2)+36/1001*e^(3/2)*(e*x^2+d
)^(1/2)/d^2/x^(7/2)-60/1001*e^(5/2)*(e*x^2+d)^(1/2)/d^3/x^(3/2)-4/143*e^(1
/2)*(e*x^2+d)^(1/2)/d/x^(11/2)-30/1001*e^(13/4)*(cos(2*arctan(e^(1/4)*x^(1
/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(si
n(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*
x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(13/4)/(e*x^2+d)^(1/2)
```

3.22.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.81

$$\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = \frac{2\left(-\frac{2\sqrt{e}x\sqrt{d+ex^2}(7d^2-9dex^2+15e^2x^4)}{d^3}-77\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{30\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}e^4\sqrt{1+\frac{d}{ex^2}}x^{15/2}\,\text{Ellip}}{d^{7/2}\sqrt{}}\right.}{1001x^{13/2}}$$

input
```
Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```

output
```
(2*((-2*Sqrt[e]*x*Sqrt[d + e*x^2]*(7*d^2 - 9*d*e*x^2 + 15*e^2*x^4))/d^3 -
77*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] - (30*Sqrt[(I*Sqrt[d])/Sqrt[e]]*e^
4*Sqrt[1 + d/(e*x^2)]*x^(15/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e
]]/Sqrt[x]], -1])/(d^(7/2)*Sqrt[d + e*x^2])))/(1001*x^(13/2))
```

3.22.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6775, 264, 264, 264, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}dx$$

$$\Big\downarrow \text{ 6775}$$

$$\frac{2}{13}\sqrt{e}\int \frac{1}{x^{13/2}\sqrt{ex^2+d}}dx-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{2}{13}\sqrt{e}\left(-\frac{9e\int \frac{1}{x^{9/2}\sqrt{ex^2+d}}dx}{11d}-\frac{2\sqrt{d+ex^2}}{11dx^{11/2}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{2}{13}\sqrt{e}\left(-\frac{9e\left(-\frac{5e\int \frac{1}{x^{5/2}\sqrt{ex^2+d}}dx}{7d}-\frac{2\sqrt{d+ex^2}}{7dx^{7/2}}\right)}{11d}-\frac{2\sqrt{d+ex^2}}{11dx^{11/2}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{13}\sqrt{e}\left(-\frac{9e\left(-\frac{5e\left(-\frac{e\int \frac{1}{\sqrt{x}\sqrt{ex^2+d}}dx}{3d}-\frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right)}{7d}-\frac{2\sqrt{d+ex^2}}{7dx^{7/2}}\right)}{11d}-\frac{2\sqrt{d+ex^2}}{11dx^{11/2}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{13}\sqrt{e}\left(-\frac{9e\left(-\frac{5e\left(-\frac{2e\int \frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{3d}-\frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right)}{7d}-\frac{2\sqrt{d+ex^2}}{7dx^{7/2}}\right)}{11d}-\frac{2\sqrt{d+ex^2}}{11dx^{11/2}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{13}\sqrt{e}\left(-\frac{9e\left(-\frac{5e\left(-\frac{e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3d^{5/4}\sqrt{d+ex^2}}-\frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right)}{7d}-\frac{2\sqrt{d+ex^2}}{7dx^{7/2}}\right)}{11d}-\frac{2\sqrt{d+ex^2}}{11dx^{11/2}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}$$

3.22. $\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}dx$

input
```
Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```

output
```
(-2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(13*x^(13/2)) + (2*Sqrt[e]*((-2*
Sqrt[d + e*x^2])/(11*d*x^(11/2)) - (9*e*((-2*Sqrt[d + e*x^2])/(7*d*x^(7/2)
) - (5*e*((-2*Sqrt[d + e*x^2])/(3*d*x^(3/2)) - (e^(3/4)*(Sqrt[d] + Sqrt[e]
*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*
Sqrt[x])/d^(1/4)], 1/2])/(3*d^(5/4)*Sqrt[d + e*x^2])))/(7*d)))/(11*d)))/13
```

3.22.3.1 Defintions of rubi rules used

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.22.4 Maple [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{15}{2}}}dx$$

input
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
```

output
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
```

3.22.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.50

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = -\frac{60\,e^3x^7\mathrm{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)+77\,d^3\sqrt{x}\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)+4\,(15\,e^2x^5-9\,dex^3+7\,d^2x)}{1001\,d^3x^7}$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="fricas
")
```

output
```
-1/1001*(60*e^3*x^7*weierstrassPInverse(-4*d/e, 0, x) + 77*d^3*sqrt(x)*log
((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) + 4*(15*e^2*x^5 - 9*d*e*x^
3 + 7*d^2*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(x))/(d^3*x^7)
```

3.22.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(15/2),x)
```

output
```
Timed out
```

3.22.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{15}{2}}}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="maxima
")
```

output
```
2*d*sqrt(e)*integrate(-1/13*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(15/2
) - (e*x^2 + d)*e^(log(e*x^2 + d) + 15/2*log(x))), x) - 1/13*log(sqrt(e)*x
 + sqrt(e*x^2 + d))/x^(13/2) + 1/13*log(-sqrt(e)*x + sqrt(e*x^2 + d))/x^(1
3/2)
```

3.22.8 Giac [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{15}{2}}}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="giac")
```

output
```
integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(15/2), x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{15/2}}\,dx$$

input
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(15/2),x)
```

output
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(15/2), x)
```

3.23 $\int x^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$

3.23.1 Optimal result

Integrand size = 25, antiderivative size = 297

$$\int x^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{28dx^{3/2}\sqrt{d+ex^2}}{405e^{3/2}} - \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}} - \frac{28d^2\sqrt{x}\sqrt{d+ex^2}}{135e^2\left(\sqrt{d}+\sqrt{e}x\right)}$$
$$+\frac{2}{9}x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{28d^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{135e^{9/4}\sqrt{d+ex^2}} - \frac{14d^{9/4}\left(\sqrt{d}+\sqrt{}\right.}{}$$

output
```
2/9*x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+28/405*d*x^(3/2)*(e*x^2+d)^
(1/2)/e^(3/2)-4/81*x^(7/2)*(e*x^2+d)^(1/2)/e^(1/2)-28/135*d^2*x^(1/2)*(e*x
^2+d)^(1/2)/e^2/(d^(1/2)+x*e^(1/2))+28/135*d^(9/4)*(cos(2*arctan(e^(1/4)*x
^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE
(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*(
(e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(9/4)/(e*x^2+d)^(1/2)-14/135*d^(9
/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*
x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(
1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(9/4)/
(e*x^2+d)^(1/2)
```

3.23.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.42

$$\int x^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{2x^{3/2}\left(14d^2+4dex^2-10e^2x^4+45e^{3/2}x^3\sqrt{d+ex^2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-14\right.}{405e^{3/2}\sqrt{d+ex^2}}$$

input
```
Integrate[x^(7/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(2*x^(3/2)*(14*d^2 + 4*d*e*x^2 - 10*e^2*x^4 + 45*e^(3/2)*x^3*Sqrt[d + e*x^
2]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] - 14*d^2*Sqrt[1 + (e*x^2)/d]*Hyper
geometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)]))/(405*e^(3/2)*Sqrt[d + e*x^2])
```

3.23.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {6775, 262, 262, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$$

$$\Big\downarrow \text{6775}$$

$$\frac{2}{9}x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{9}\sqrt{e}\int\frac{x^{9/2}}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{9}x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{9}\sqrt{e}\left(\frac{2x^{7/2}\sqrt{d+ex^2}}{9e}-\frac{7d\int\frac{x^{5/2}}{\sqrt{ex^2+d}}dx}{9e}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{9}x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{9}\sqrt{e}\left(\frac{2x^{7/2}\sqrt{d+ex^2}}{9e}-\frac{7d\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{3d\int\frac{\sqrt{x}}{\sqrt{ex^2+d}}dx}{5e}\right)}{9e}\right)$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2}{9}x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{9}\sqrt{e}\left(\frac{2x^{7/2}\sqrt{d+ex^2}}{9e}-\frac{7d\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\int\frac{x}{\sqrt{ex^2+d}}d\sqrt{x}}{5e}\right)}{9e}\right)$$

$$\Big\downarrow \text{ 834}$$

$$\frac{2}{9}x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{9}\sqrt{e}\left(\frac{2x^{7/2}\sqrt{d+ex^2}}{9e}-\frac{7d\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\left(\frac{\sqrt{d}\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\sqrt{d}\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{d}\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{5e}\right)}{9e}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{9}x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{9}\sqrt{e}\left(\frac{2x^{7/2}\sqrt{d+ex^2}}{9e}-\frac{7d\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\left(\frac{\sqrt{d}\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{5e}\right)}{9e}\right)$$

$$\Big\downarrow \text{ 761}$$

3.23. $\int x^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$

$$\frac{2}{9}x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{2}{9}\sqrt{e}\left(\frac{2x^{7/2}\sqrt{d+ex^2}}{9e} - \frac{7d\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e} - \frac{6d\left(\frac{\sqrt[4]{d}(\sqrt{d}+\sqrt{e}x)\sqrt{\frac{d+ex^2}{(\sqrt{d}+\sqrt{e}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}} - \frac{\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{5e}\right)}{9e}\right)$$

$\downarrow$ 1510

$$\frac{2}{9}x^{9/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{2}{9}\sqrt{e}\left(\frac{2x^{7/2}\sqrt{d+ex^2}}{9e} - \frac{7d\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e} - \frac{6d\left(\frac{\sqrt[4]{d}(\sqrt{d}+\sqrt{e}x)\sqrt{\frac{d+ex^2}{(\sqrt{d}+\sqrt{e}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}} - \frac{\sqrt[4]{d}(\sqrt{d}+\sqrt{e}x)\sqrt{\cdots}}{\cdots}\right)}{5e}\right)}{9e}\right)$$

3.23. $\int x^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$

input
```
Int[x^(7/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(2*x^(9/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/9 - (2*Sqrt[e]*((2*x^(7/2
)*Sqrt[d + e*x^2])/(9*e) - (7*d*((2*x^(3/2)*Sqrt[d + e*x^2])/(5*e) - (6*d*
(-((-((Sqrt[x]*Sqrt[d + e*x^2])/(Sqrt[d] + Sqrt[e]*x)) + (d^(1/4)*(Sqrt[d]
 + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan
[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(1/4)*Sqrt[d + e*x^2]))/Sqrt[e]) + (
d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*El
lipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(2*e^(3/4)*Sqrt[d + e*x
^2])))/(5*e)))/(9*e)))/9
```

3.23.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.23.4 Maple [F]

$$\int x^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right) dx$$

input
```
int(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

output
```
int(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

3.23.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.31

$$\int x^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{45\,e^2x^{\frac{9}{2}}\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)+84\,d^2\text{weierstrassZeta}\left(-\frac{4\,d}{e},0,\text{weierstr}\right.}{405\,e^2}$$

input
```
integrate(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas"
)
```

output
```
1/405*(45*e^2*x^(9/2)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) +
 84*d^2*weierstrassZeta(-4*d/e, 0, weierstrassPInverse(-4*d/e, 0, x)) - 4*
(5*e*x^3 - 7*d*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(x))/e^2
```

3.23.6 Sympy [F(-1)]

Timed out.

$$\int x^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\text{Timed out}$$

input
```
integrate(x**(7/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Timed out
```

3.23.7 Maxima [F]

$$\int x^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{\frac{7}{2}}\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima"
)
```

output
```
1/9*x^(9/2)*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/9*x^(9/2)*log(-sqrt(e)*x
+ sqrt(e*x^2 + d)) - 2*d*sqrt(e)*integrate(-1/9*x*e^(1/2*log(e*x^2 + d) +
7/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

3.23.8 Giac [F]

$$\int x^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{\frac{7}{2}}\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
integrate(x^(7/2)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int x^{7/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{7/2}\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

input
```
int(x^(7/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^(7/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.24 $\int x^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$

3.24.1 Optimal result

Integrand size = 25, antiderivative size = 269

$$\int x^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=-\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{e}}+\frac{12d\sqrt{x}\sqrt{d+ex^2}}{25e\left(\sqrt{d}+\sqrt{e}x\right)}$$
$$+\frac{2}{5}x^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{12d^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{5/4}\sqrt{d+ex^2}}+\frac{6d^{5/4}\left(\sqrt{d}+\sqrt{e}\right.}{}$$

output
```
2/5*x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))-4/25*x^(3/2)*(e*x^2+d)^(1/2
)/e^(1/2)+12/25*d*x^(1/2)*(e*x^2+d)^(1/2)/e/(d^(1/2)+x*e^(1/2))-12/25*d^(5
/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*
x^(1/2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(
1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(5/4)/
(e*x^2+d)^(1/2)+6/25*d^(5/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1
/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*
x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e
^(1/2))^2)^(1/2)/e^(5/4)/(e*x^2+d)^(1/2)
```

3.24.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.41

$$\int x^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{2x^{3/2}\left(-2(d+ex^2)+5\sqrt{e}x\sqrt{d+ex^2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+2d\sqrt{1+\frac{ex^2}{d}}\,\text{Hy}\right.}{25\sqrt{e}\sqrt{d+ex^2}}$$

input
```
Integrate[x^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(2*x^(3/2)*(-2*(d + e*x^2) + 5*Sqrt[e]*x*Sqrt[d + e*x^2]*ArcTanh[(Sqrt[e]*
x)/Sqrt[d + e*x^2]] + 2*d*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[1/2, 3/4,
7/4, -((e*x^2)/d)]))/(25*Sqrt[e]*Sqrt[d + e*x^2])
```

3.24.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6775, 262, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$$

$$\Big\downarrow \text{6775}$$

$$\frac{2}{5}x^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{5}\sqrt{e}\int\frac{x^{5/2}}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{5}x^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{5}\sqrt{e}\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{3d\int\frac{\sqrt{x}}{\sqrt{ex^2+d}}dx}{5e}\right)$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{5}x^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{5}\sqrt{e}\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\int\frac{x}{\sqrt{ex^2+d}}d\sqrt{x}}{5e}\right)$$

$$\Big\downarrow \text{834}$$

$$\frac{2}{5}x^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{5}\sqrt{e}\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\left(\frac{\sqrt{d}\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\sqrt{d}\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{d}\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{5e}\right)$$

$\downarrow$ 27

$$\frac{2}{5}x^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{5}\sqrt{e}\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\left(\frac{\sqrt{d}\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{5e}\right)$$

$\downarrow$ 761

$$\frac{2}{5}x^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{5}\sqrt{e}\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{5e}\right)$$

$\downarrow$ 1510

$$\frac{2}{5}x^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{5}\sqrt{e}\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\,\text{arc}\ldots\right.}{\sqrt[4]{e}\sqrt{d+ex^2}}}{\sqrt{e}}\ldots}{5e}\ldots\right.$$

input
```
Int[x^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

3.24. $\int x^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$

output
```
(2*x^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/5 - (2*Sqrt[e]*((2*x^(3/2
)*Sqrt[d + e*x^2])/(5*e) - (6*d*(-((-((Sqrt[x]*Sqrt[d + e*x^2])/(Sqrt[d] +
 Sqrt[e]*x)) + (d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] +
Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(1/4
)*Sqrt[d + e*x^2]))/Sqrt[e]) + (d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*
x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)
], 1/2])/(2*e^(3/4)*Sqrt[d + e*x^2])))/(5*e)))/5
```

3.24.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.24.4 Maple [F]

$$\int x^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right) dx$$

input
```
int(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

output
```
int(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

3.24.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.29

$$\int x^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \frac{5\,ex^{\frac{5}{2}}\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right) - 4\,\sqrt{ex^2+d}\sqrt{e}x^{\frac{3}{2}} - 12\,d\text{weierstrassZeta}}{25\,e}$$

input
```
integrate(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas"
)
```

output
```
1/25*(5*e*x^(5/2)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 4*s
qrt(e*x^2 + d)*sqrt(e)*x^(3/2) - 12*d*weierstrassZeta(-4*d/e, 0, weierstra
ssPInverse(-4*d/e, 0, x)))/e
```

3.24.6 Sympy [F]

$$\int x^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \int x^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx$$

input
```
integrate(x**(3/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Integral(x**(3/2)*atanh(sqrt(e)*x/sqrt(d + e*x**2)), x)
```

3.24.7 Maxima [F]

$$\int x^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \int x^{\frac{3}{2}}\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima"
)
```

output
```
1/5*x^(5/2)*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/5*x^(5/2)*log(-sqrt(e)*x
+ sqrt(e*x^2 + d)) - 2*d*sqrt(e)*integrate(-1/5*x*e^(1/2*log(e*x^2 + d) +
3/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

3.24.8 Giac [F]

$$\int x^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx = \int x^{\frac{3}{2}}\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
integrate(x^(3/2)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

input
```
int(x^(3/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^(3/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.25 $\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx$

3.25.1 Optimal result

Integrand size = 25, antiderivative size = 232

$$\begin{aligned}&\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx\\
&= -\frac{4\sqrt{x}\sqrt{d+ex^2}}{\sqrt{d}+\sqrt{e}x}+2\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\\
&\quad+\frac{4\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{e}\sqrt{d+ex^2}}\\
&\quad-\frac{2\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{\sqrt[4]{e}\sqrt{d+ex^2}}\end{aligned}$$

output
```
2*x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))-4*x^(1/2)*(e*x^2+d)^(1/2)/(d^
(1/2)+x*e^(1/2))+4*d^(1/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2
)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^
(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(
1/2))^2)^(1/2)/e^(1/4)/(e*x^2+d)^(1/2)-2*d^(1/4)*(cos(2*arctan(e^(1/4)*x^(
1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(s
in(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e
*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(1/4)/(e*x^2+d)^(1/2)
```

3.25.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.37

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{4\sqrt{e}x^{3/2}\sqrt{1+\frac{ex^2}{d}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{ex^2}{d}\right)}{3\sqrt{d+ex^2}}$$

input
```
Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] - (4*Sqrt[e]*x^(3/2)*Sqrt[1
 + (e*x^2)/d]*Hypergeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(3*Sqrt[d +
e*x^2])
```

3.25.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6775, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}dx$$

$$\Big\downarrow \ 6775$$

$$2\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - 2\sqrt{e}\int \frac{\sqrt{x}}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \ 266$$

$$2\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - 4\sqrt{e}\int \frac{x}{\sqrt{ex^2+d}}d\sqrt{x}$$

$$\Big\downarrow \ 834$$

$$2\sqrt{x}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-4\sqrt{e}\left(\frac{\sqrt{d}\int \frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\sqrt{d}\int \frac{\sqrt{d}-\sqrt{e}x}{\sqrt{d}\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)$$

$$\Big\downarrow \; 27$$

$$2\sqrt{x}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-4\sqrt{e}\left(\frac{\sqrt{d}\int \frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\int \frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)$$

$$\Big\downarrow \; 761$$

$$2\sqrt{x}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-$$
$$4\sqrt{e}\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\int \frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)$$

$$\Big\downarrow \; 1510$$

$$2\sqrt{x}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-$$
$$4\sqrt{e}\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\ldots}{\sqrt[4]{\ldots}}\right.\right.}{\sqrt[4]{e}\sqrt{d+ex^2}}}{\sqrt{e}}\right.$$

input
```
Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] - 4*Sqrt[e]*(-((-((Sqrt[x]*
Sqrt[d + e*x^2])/(Sqrt[d] + Sqrt[e]*x)) + (d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*S
qrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[
x])/d^(1/4)], 1/2])/(e^(1/4)*Sqrt[d + e*x^2]))/Sqrt[e]) + (d^(1/4)*(Sqrt[d
] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTa
n[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(2*e^(3/4)*Sqrt[d + e*x^2]))
```

3.25. $\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx$

3.25.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.25. $\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx$

3.25.4 Maple [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{\sqrt{x}}dx$$

input
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
```

output
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
```

3.25.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.22

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = \sqrt{x}\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right) + 4\,\mathrm{weierstrassZeta}\left(-\frac{4\,d}{e},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)\right)$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="fricas"
)
```

output
```
sqrt(x)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) + 4*weierstrass
Zeta(-4*d/e, 0, weierstrassPInverse(-4*d/e, 0, x))
```

3.25.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx$$

input
```
integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(1/2),x)
```

output
```
Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/sqrt(x), x)
```

3.25.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{\sqrt{x}}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="maxima"
)
```

output
```
-2*d*sqrt(e)*integrate(sqrt(e*x^2 + d)*x/((e*x^2 + d)*e^(log(e*x^2 + d) +
1/2*log(x)) - (e^2*x^4 + d*e*x^2)*sqrt(x)), x) + sqrt(x)*log(sqrt(e)*x + s
qrt(e*x^2 + d)) - sqrt(x)*log(-sqrt(e)*x + sqrt(e*x^2 + d))
```

3.25.8 Giac [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{\sqrt{x}}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/sqrt(x), x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{\sqrt{x}}\,dx$$

input
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(1/2),x)
```

output
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(1/2), x)
```

3.26 $\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx$

3.26.1 Optimal result

Integrand size = 25, antiderivative size = 272

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = -\frac{4\sqrt{e}\sqrt{d+ex^2}}{3d\sqrt{x}}+\frac{4e\sqrt{x}\sqrt{d+ex^2}}{3d\left(\sqrt{d}+\sqrt{e}x\right)} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} - \frac{4e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}} + \frac{2e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}$$

output
```
-2/3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2)-4/3*e^(1/2)*(e*x^2+d)^(1/2
)/d/x^(1/2)+4/3*e*x^(1/2)*(e*x^2+d)^(1/2)/d/(d^(1/2)+x*e^(1/2))-4/3*e^(3/4
)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^
(1/2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/
2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(3/4)/(e
*x^2+d)^(1/2)+2/3*e^(3/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)
/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(
1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1
/2))^2)^(1/2)/d^(3/4)/(e*x^2+d)^(1/2)
```

3.26.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.43

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = -\frac{4\sqrt{e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{4e^{3/2}x^{3/2}\sqrt{1+\frac{ex^2}{d}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{ex^2}{d}\right)}{9d\sqrt{d+ex^2}}$$

input
```
Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

output
```
(-4*Sqrt[e]*Sqrt[d + e*x^2])/(3*d*Sqrt[x]) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d
 + e*x^2]])/(3*x^(3/2)) + (4*e^(3/2)*x^(3/2)*Sqrt[1 + (e*x^2)/d]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(9*d*Sqrt[d + e*x^2])
```

3.26.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6775, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{6775}$$

$$\frac{2}{3}\sqrt{e}\int \frac{1}{x^{3/2}\sqrt{ex^2+d}}dx - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{3}\sqrt{e}\left(\frac{e\int \frac{\sqrt{x}}{\sqrt{ex^2+d}}dx}{d} - \frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right) - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{3}\sqrt{e}\left(\frac{2e\int \frac{x}{\sqrt{ex^2+d}}d\sqrt{x}}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}$$

$$\Big\downarrow \; 834$$

$$\frac{2}{3}\sqrt{e}\left(\frac{2e\left(\frac{\sqrt{d}\int \frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\sqrt{d}\int \frac{\sqrt{d}-\sqrt{e}x}{\sqrt{d}\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{3}\sqrt{e}\left(\frac{2e\left(\frac{\sqrt{d}\int \frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\int \frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}$$

$$\Big\downarrow \; 761$$

$$\frac{2}{3}\sqrt{e}\left(\frac{2e\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\int \frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}$$

$$\Big\downarrow \; 1510$$

$$\frac{2}{3}\sqrt{e}\left(\frac{2e\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{e}\sqrt{d+ex^2}}-\frac{\sqrt{x}\cdots}{\sqrt{\cdots}}}{\sqrt{e}}\cdots\right)}{d}\cdots\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}$$

3.26. $\displaystyle\int \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx$

input
```
Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

output
```
(-2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(3*x^(3/2)) + (2*Sqrt[e]*((-2*Sq
rt[d + e*x^2])/(d*Sqrt[x]) + (2*e*(-((-((Sqrt[x]*Sqrt[d + e*x^2])/(Sqrt[d]
 + Sqrt[e]*x)) + (d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d]
+ Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(1
/4)*Sqrt[d + e*x^2]))/Sqrt[e]) + (d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d +
e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/
4)], 1/2])/(2*e^(3/4)*Sqrt[d + e*x^2])))/d))/3
```

3.26.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.26.4 Maple [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{x^{\frac{5}{2}}}dx$$

input
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
```

output
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
```

3.26.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.30

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = -\frac{4\,ex^{2}\text{weierstrassZeta}\left(-\frac{4\,d}{e},0,\text{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)\right)+4\,\sqrt{ex^{2}+d}\sqrt{e}x^{\frac{3}{2}}+d\sqrt{x}\log\left(\frac{2\,ex^{2}+2\,\sqrt{ex^{2}-}}{d}\right.}{3\,dx^{2}}$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="fricas"
)
```

output
```
-1/3*(4*e*x^2*weierstrassZeta(-4*d/e, 0, weierstrassPInverse(-4*d/e, 0, x)
) + 4*sqrt(e*x^2 + d)*sqrt(e)*x^(3/2) + d*sqrt(x)*log((2*e*x^2 + 2*sqrt(e*
x^2 + d)*sqrt(e)*x + d)/d))/(d*x^2)
```

3.26.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(5/2),x)
```

output
```
Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**(5/2), x)
```

3.26.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="maxima"
)
```

output
```
2*d*sqrt(e)*integrate(-1/3*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(5/2)
- (e*x^2 + d)*e^(log(e*x^2 + d) + 5/2*log(x))), x) - 1/3*log(sqrt(e)*x + s
qrt(e*x^2 + d))/x^(3/2) + 1/3*log(-sqrt(e)*x + sqrt(e*x^2 + d))/x^(3/2)
```

3.26.8 Giac [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="giac")
```

output
```
integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(5/2), x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{5/2}}\,dx$$

input
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(5/2),x)
```

output
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(5/2), x)
```

3.27 $\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx$

3.27.1 Optimal result

Integrand size = 25, antiderivative size = 302

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = -\frac{4\sqrt{e}\sqrt{d+ex^2}}{35dx^{5/2}}+\frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}}-\frac{12e^2\sqrt{x}\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}x\right)} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}+\frac{12e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}} - \frac{6e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}}$$

output
```
-2/7*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2)-4/35*e^(1/2)*(e*x^2+d)^(1/
2)/d/x^(5/2)+12/35*e^(3/2)*(e*x^2+d)^(1/2)/d^2/x^(1/2)-12/35*e^2*x^(1/2)*(
e*x^2+d)^(1/2)/d^2/(d^(1/2)+x*e^(1/2))+12/35*e^(7/4)*(cos(2*arctan(e^(1/4)
*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*Ellipti
cE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))
*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(7/4)/(e*x^2+d)^(1/2)-6/35*e^(7
/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*
x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(
1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(7/4)/
(e*x^2+d)^(1/2)
```

3.27.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.43

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \frac{4\sqrt{e}x(-d^2+2dex^2+3e^2x^4)-10d^2\sqrt{d+ex^2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-4e^{5/2}x^5\sqrt{1+\frac{e}{}}}{35d^2x^{7/2}\sqrt{d+ex^2}}$$

input
```
Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
```

output
```
(4*Sqrt[e]*x*(-d^2 + 2*d*e*x^2 + 3*e^2*x^4) - 10*d^2*Sqrt[d + e*x^2]*ArcTa
nh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] - 4*e^(5/2)*x^5*Sqrt[1 + (e*x^2)/d]*Hyperg
eometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(35*d^2*x^(7/2)*Sqrt[d + e*x^2])
```

3.27.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {6775, 264, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}dx$$

$$\Big\downarrow \text{ 6775}$$

$$\frac{2}{7}\sqrt{e}\int \frac{1}{x^{7/2}\sqrt{ex^2+d}}dx-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{2}{7}\sqrt{e}\left(-\frac{3e\int \frac{1}{x^{3/2}\sqrt{ex^2+d}}dx}{5d}-\frac{2\sqrt{d+ex^2}}{5dx^{5/2}}\right)-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{2}{7}\sqrt{e}\left(-\frac{3e\left(\frac{e\int\frac{\sqrt{x}}{\sqrt{ex^2+d}}dx}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)}{5d}-\frac{2\sqrt{d+ex^2}}{5dx^{5/2}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{7}\sqrt{e}\left(-\frac{3e\left(\frac{2e\int\frac{x}{\sqrt{ex^2+d}}d\sqrt{x}}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)}{5d}-\frac{2\sqrt{d+ex^2}}{5dx^{5/2}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

$$\Big\downarrow \text{834}$$

$$\frac{2}{7}\sqrt{e}\left(-\frac{3e\left(\frac{2e\left(\frac{\sqrt{d}\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\sqrt{d}\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{d}\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)}{5d}-\frac{2\sqrt{d+ex^2}}{5dx^{5/2}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{7}\sqrt{e}\left(-\frac{3e\left(\frac{2e\left(\frac{\sqrt{d}\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)}{5d}-\frac{2\sqrt{d+ex^2}}{5dx^{5/2}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

$$\Big\downarrow \text{761}$$

3.27. $\displaystyle\int\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}dx$

$$\frac{2}{7}\sqrt{e}\left(-\frac{3e\left(\frac{2e\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)}{5d}-\frac{2\sqrt{d+ex^2}}{5dx^{5/2}}\right)-$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

$\downarrow$ 1510

$$\frac{2}{7}\sqrt{e}\left(-\frac{3e\left(\frac{2e\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{e}\sqrt{d+ex^2}}-\frac{\sqrt{x}\sqrt{\ldots}}{\sqrt{d\ldots}}}{\sqrt{e}}\ldots}{d}\ldots}{5d}\ldots\right.$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

3.27. $\int\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}dx$

input
```
Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
```

output
```
(-2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(7*x^(7/2)) + (2*Sqrt[e]*((-2*Sq
rt[d + e*x^2])/(5*d*x^(5/2)) - (3*e*((-2*Sqrt[d + e*x^2])/(d*Sqrt[x]) + (2
*e*(-((-((Sqrt[x]*Sqrt[d + e*x^2])/(Sqrt[d] + Sqrt[e]*x)) + (d^(1/4)*(Sqrt
[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*Arc
Tan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(1/4)*Sqrt[d + e*x^2]))/Sqrt[e])
+ (d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]
*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(2*e^(3/4)*Sqrt[d +
e*x^2])))/d))/(5*d)))/7
```

3.27.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6775
```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[(d*x)^(m + 1)*(ArcTanh[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))),
 x] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Fre
eQ[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

3.27.4 Maple [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{x^{\frac{9}{2}}}dx$$

input
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)
```

output
```
int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)
```

3.27.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.32

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \frac{12\,e^{2}x^{4}\text{weierstrassZeta}\left(-\frac{4\,d}{e},0,\text{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)\right)-5\,d^{2}\sqrt{x}\log\left(\frac{2\,e}{}\right.}{35\,d^{2}x^{4}}$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="fricas"
)
```

output
```
1/35*(12*e^2*x^4*weierstrassZeta(-4*d/e, 0, weierstrassPInverse(-4*d/e, 0,
 x)) - 5*d^2*sqrt(x)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) +
4*(3*e*x^3 - d*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(x))/(d^2*x^4)
```

3.27.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(9/2),x)
```

output
```
Timed out
```

3.27.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="maxima"
)
```

output
```
2*d*sqrt(e)*integrate(-1/7*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(9/2)
- (e*x^2 + d)*e^(log(e*x^2 + d) + 9/2*log(x))), x) - 1/7*log(sqrt(e)*x + s
qrt(e*x^2 + d))/x^(7/2) + 1/7*log(-sqrt(e)*x + sqrt(e*x^2 + d))/x^(7/2)
```

3.27.8 Giac [F]

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="giac")
```

output
```
integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(9/2), x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{9/2}}\,dx$$

input
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(9/2),x)
```

output
```
int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(9/2), x)
```

3.28 $\int x^3 \operatorname{arctanh}(a + bx^4)\, dx$

3.28.1 Optimal result

Integrand size = 12, antiderivative size = 44

$$\int x^3 \operatorname{arctanh}\left(a+bx^4\right)\, dx = \frac{(a+bx^4)\operatorname{arctanh}(a+bx^4)}{4b} + \frac{\log\left(1-(a+bx^4)^2\right)}{8b}$$

output
```
1/4*(b*x^4+a)*arctanh(b*x^4+a)/b+1/8*ln(1-(b*x^4+a)^2)/b
```

3.28.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int x^3 \operatorname{arctanh}\left(a+bx^4\right)\, dx = \frac{2(a+bx^4)\operatorname{arctanh}(a+bx^4) + \log\left(1-(a+bx^4)^2\right)}{8b}$$

input
```
Integrate[x^3*ArcTanh[a + b*x^4],x]
```

output
```
(2*(a + b*x^4)*ArcTanh[a + b*x^4] + Log[1 - (a + b*x^4)^2])/(8*b)
```

3.28.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7266, 6653, 6436, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{arctanh}\left(a+bx^4\right)\, dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{4}\int \text{arctanh}\left(bx^4+a\right)dx^4$$

$$\Big\downarrow \text{6653}$$

$$\frac{\int \text{arctanh}\left(bx^4+a\right)d\left(bx^4+a\right)}{4b}$$

$$\Big\downarrow \text{6436}$$

$$\frac{\left(a+bx^4\right)\text{arctanh}\left(a+bx^4\right)-\int \frac{bx^4+a}{1-x^8}d\left(bx^4+a\right)}{4b}$$

$$\Big\downarrow \text{240}$$

$$\frac{\left(a+bx^4\right)\text{arctanh}\left(a+bx^4\right)+\frac{1}{2}\log\left(1-x^8\right)}{4b}$$

input
```
Int[x^3*ArcTanh[a + b*x^4],x]
```

output
```
((a + b*x^4)*ArcTanh[a + b*x^4] + Log[1 - x^8]/2)/(4*b)
```

3.28.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6653
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
}, x] && IGtQ[p, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.28.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\left(b\,x^{4}+a\right) \operatorname{arctanh}\left(b\,x^{4}+a\right)+\frac{\ln\left(1-\left(b\,x^{4}+a\right)^{2}\right)}{2}}{4b}$
default	$\frac{\left(b\,x^{4}+a\right) \operatorname{arctanh}\left(b\,x^{4}+a\right)+\frac{\ln\left(1-\left(b\,x^{4}+a\right)^{2}\right)}{2}}{4b}$
parts	$\frac{x^{4} \operatorname{arctanh}\left(b\,x^{4}+a\right)}{4}-b\left(\frac{\left(-1+a\right) \ln\left(b\,x^{4}+a-1\right)}{8b^{2}}+\frac{\left(-1-a\right) \ln\left(b\,x^{4}+a+1\right)}{8b^{2}}\right)$
parallelrisch	$-\frac{-\operatorname{arctanh}\left(b\,x^{4}+a\right) x^{4} b^{2}-\operatorname{arctanh}\left(b\,x^{4}+a\right) a b-\ln\left(b\,x^{4}+a-1\right) b-\operatorname{arctanh}\left(b\,x^{4}+a\right) b}{4b^{2}}$
risch	$\frac{x^{4} \ln\left(b\,x^{4}+a+1\right)}{8}-\frac{x^{4} \ln\left(-b\,x^{4}-a+1\right)}{8}-\frac{\ln\left(-b\,x^{4}-a+1\right) a}{8b}+\frac{\ln\left(b\,x^{4}+a+1\right) a}{8b}+\frac{\ln\left(-b\,x^{4}-a+1\right)}{8b}+\frac{\ln\left(b\,x^{4}+a+\right.}{8b}$

input
```
int(x^3*arctanh(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*((b*x^4+a)*arctanh(b*x^4+a)+1/2*ln(1-(b*x^4+a)^2))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

$$\int x^3 \operatorname{arctanh}\left(a+bx^4\right)\,dx = \frac{bx^4\log\left(-\frac{bx^4+a+1}{bx^4+a-1}\right)+(a+1)\log\left(bx^4+a+1\right)-(a-1)\log\left(bx^4+a-1\right)}{8\,b}$$

input
```
integrate(x^3*arctanh(b*x^4+a),x, algorithm="fricas")
```

output
```
1/8*(b*x^4*log(-(b*x^4 + a + 1)/(b*x^4 + a - 1)) + (a + 1)*log(b*x^4 + a +
 1) - (a - 1)*log(b*x^4 + a - 1))/b
```

3.28.6 Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

$$\int x^3 \operatorname{arctanh}(a+bx^4)\, dx = \begin{cases} \frac{a \operatorname{atanh}(a+bx^4)}{4b} + \frac{x^4 \operatorname{atanh}(a+bx^4)}{4} + \frac{\log(a+bx^4+1)}{4b} - \frac{\operatorname{atanh}(a+bx^4)}{4b} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{atanh}(a)}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*atanh(b*x**4+a),x)
```

output
```
Piecewise((a*atanh(a + b*x**4)/(4*b) + x**4*atanh(a + b*x**4)/4 + log(a +
b*x**4 + 1)/(4*b) - atanh(a + b*x**4)/(4*b), Ne(b, 0)), (x**4*atanh(a)/4,
True))
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int x^3 \operatorname{arctanh}(a+bx^4)\, dx = \frac{2\,(bx^4+a)\operatorname{artanh}(bx^4+a) + \log\left(-(bx^4+a)^2+1\right)}{8\,b}$$

input
```
integrate(x^3*arctanh(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*(2*(b*x^4 + a)*arctanh(b*x^4 + a) + log(-(b*x^4 + a)^2 + 1))/b
```

3.28.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(40) = 80$.

Time = 0.30 (sec) , antiderivative size = 223, normalized size of antiderivative = 5.07

$$\int x^3 \operatorname{arctanh}(a+bx^4)\,dx = \frac{1}{8}\left((a+1)b-(a-1)b\right)\left(\frac{\log\left(\frac{|-bx^4-a-1|}{|bx^4+a-1|}\right)}{b^2}-\frac{\log\left(\left|-\frac{bx^4+a+1}{bx^4+a-1}+1\right|\right)}{b^2}+\frac{\log\left(-\frac{a-\frac{\left(\frac{(bx^4+a+1)(a-1)}{bx^4+a-1}-a-1\right)b}{\frac{(bx^4+a+1)b}{bx^4+a-1}-b}+1}{a-\frac{\left(\frac{(bx^4+a+1)(a-1)}{bx^4+a-1}-a-1\right)b}{\frac{(bx^4+a+1)b}{bx^4+a-1}-b}-1}\right)}{b^2\left(\frac{bx^4+a+1}{bx^4+a-1}-1\right)}\right)$$

input
```
integrate(x^3*arctanh(b*x^4+a),x, algorithm="giac")
```

output
```
1/8*((a + 1)*b - (a - 1)*b)*(log(abs(-b*x^4 - a - 1)/abs(b*x^4 + a - 1))/b
^2 - log(abs(-(b*x^4 + a + 1)/(b*x^4 + a - 1) + 1))/b^2 + log(-(a - ((b*x^
4 + a + 1)*(a - 1)/(b*x^4 + a - 1) - a - 1)*b/((b*x^4 + a + 1)*b/(b*x^4 +
a - 1) - b) + 1)/(a - ((b*x^4 + a + 1)*(a - 1)/(b*x^4 + a - 1) - a - 1)*b/
((b*x^4 + a + 1)*b/(b*x^4 + a - 1) - b) - 1))/(b^2*((b*x^4 + a + 1)/(b*x^4
 + a - 1) - 1)))
```

3.28.9 Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.05

$$\int x^3 \operatorname{arctanh}(a+bx^4)\,dx = \frac{\ln(b\,x^4+a-1)}{8\,b}-\frac{x^4\,\ln(-b\,x^4-a+1)}{8}+\frac{\ln(b\,x^4+a+1)}{8\,b}+\frac{x^4\,\ln(b\,x^4+a+1)}{8}-\frac{a\,\ln(b\,x^4+a-1)}{8\,b}+\frac{a\,\ln(b\,x^4+a+1)}{8\,b}$$

input
```
int(x^3*atanh(a + b*x^4),x)
```

output
```
log(a + b*x^4 - 1)/(8*b) - (x^4*log(1 - b*x^4 - a))/8 + log(a + b*x^4 + 1)
/(8*b) + (x^4*log(a + b*x^4 + 1))/8 - (a*log(a + b*x^4 - 1))/(8*b) + (a*lo
g(a + b*x^4 + 1))/(8*b)
```

3.28. $\int x^3 \text{arctanh}(a + bx^4)\, dx$

3.29 $\int x^{-1+n} \text{arctanh}(a + bx^n) \, dx$

3.29.1 Optimal result

Integrand size = 14, antiderivative size = 47

$$\int x^{-1+n} \text{arctanh}(a + bx^n) \, dx = \frac{(a + bx^n) \, \text{arctanh}(a + bx^n)}{bn} + \frac{\log \left(1 - (a + bx^n)^2\right)}{2bn}$$

output
```
(a+b*x^n)*arctanh(a+b*x^n)/b/n+1/2*ln(1-(a+b*x^n)^2)/b/n
```

3.29.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int x^{-1+n} \text{arctanh}(a + bx^n) \, dx = \frac{2(a + bx^n) \, \text{arctanh}(a + bx^n) + \log \left(1 - (a + bx^n)^2\right)}{2bn}$$

input
```
Integrate[x^(-1 + n)*ArcTanh[a + b*x^n],x]
```

output
```
(2*(a + b*x^n)*ArcTanh[a + b*x^n] + Log[1 - (a + b*x^n)^2])/(2*b*n)
```

3.29.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7266, 6653, 6436, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}\text{arctanh}(a+bx^n)\,dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{\int \text{arctanh}(bx^n+a)\,dx^n}{n}$$

$$\Big\downarrow \text{6653}$$

$$\frac{\int \text{arctanh}(bx^n+a)\,d(bx^n+a)}{bn}$$

$$\Big\downarrow \text{6436}$$

$$\frac{(a+bx^n)\,\text{arctanh}(a+bx^n)-\int \frac{bx^n+a}{1-x^{2n}}d(bx^n+a)}{bn}$$

$$\Big\downarrow \text{240}$$

$$\frac{(a+bx^n)\,\text{arctanh}(a+bx^n)+\frac{1}{2}\log\left(1-x^{2n}\right)}{bn}$$

input
```
Int[x^(-1 + n)*ArcTanh[a + b*x^n],x]
```

output
```
((a + b*x^n)*ArcTanh[a + b*x^n] + Log[1 - x^(2*n)]/2)/(b*n)
```

3.29.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6653
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
}, x] && IGtQ[p, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.29.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(45) = 90$.

Time = 4.66 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.57

method	result	size
risch	$\frac{x^n \ln(a+b\,x^n+1)}{2n} - \frac{x^n \ln(1-a-b\,x^n)}{2n} - \frac{\ln\left(x^n+\frac{-1+a}{b}\right)a}{2bn} + \frac{\ln\left(x^n+\frac{1+a}{b}\right)a}{2bn} + \frac{\ln\left(x^n+\frac{-1+a}{b}\right)}{2bn} + \frac{\ln\left(x^n+\frac{1+a}{b}\right)}{2bn}$	121

input
```
int(x^(-1+n)*arctanh(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/2/n*x^n*ln(a+b*x^n+1)-1/2/n*x^n*ln(1-a-b*x^n)-1/2/b/n*ln(x^n+(-1+a)/b)*a
+1/2/b/n*ln(x^n+(1+a)/b)*a+1/2/b/n*ln(x^n+(-1+a)/b)+1/2/b/n*ln(x^n+(1+a)/b
)
```

3.29.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(45) = 90$.

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.32

$$\int x^{-1+n}\text{arctanh}(a+bx^n)\,dx$$
$$= \frac{(a+1)\log\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a+1\right)-(a-1)\log\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right.\right.\right.}{2\,bn}$$

input
```
integrate(x^(-1+n)*arctanh(a+b*x^n),x, algorithm="fricas")
```

output
```
1/2*((a + 1)*log(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a + 1) - (a - 1)*lo
g(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a - 1) + (b*cosh(n*log(x)) + b*sin
h(n*log(x)))*log(-(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a + 1)/(b*cosh(n*
log(x)) + b*sinh(n*log(x)) + a - 1)))/(b*n)
```

3.29.6 Sympy [F(-2)]

Exception generated.

$$\int x^{-1+n} \operatorname{arctanh}(a+bx^n)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1+n)*atanh(a+b*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int x^{-1+n} \operatorname{arctanh}(a+bx^n)\,dx = \frac{2\,(bx^n+a)\operatorname{artanh}\,(bx^n+a)+\log\left(-(bx^n+a)^2+1\right)}{2\,bn}$$

input
```
integrate(x^(-1+n)*arctanh(a+b*x^n),x, algorithm="maxima")
```

output
```
1/2*(2*(b*x^n + a)*arctanh(b*x^n + a) + log(-(b*x^n + a)^2 + 1))/(b*n)
```

3.29.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(45) = 90.

Time = 0.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.64

$$\int x^{-1+n} \operatorname{arctanh}(a+bx^n)\,dx$$
$$= \frac{((a+1)b-(a-1)b)\left(\frac{\log\left(\frac{|-bx^n-a-1|}{|bx^n+a-1|}\right)}{b^2}-\frac{\log\left(\left|-\frac{bx^n+a+1}{bx^n+a-1}+1\right|\right)}{b^2}+\frac{\log\left(-\frac{bx^n+a+1}{bx^n+a-1}\right)}{b^2\left(\frac{bx^n+a+1}{bx^n+a-1}-1\right)}\right)}{2\,n}$$

input
```
integrate(x^(-1+n)*arctanh(a+b*x^n),x, algorithm="giac")
```

output
```
1/2*((a + 1)*b - (a - 1)*b)*(log(abs(-b*x^n - a - 1)/abs(b*x^n + a - 1))/b
^2 - log(abs(-(b*x^n + a + 1)/(b*x^n + a - 1) + 1))/b^2 + log(-(b*x^n + a
+ 1)/(b*x^n + a - 1))/(b^2*((b*x^n + a + 1)/(b*x^n + a - 1) - 1)))/n
```

3.29.9 Mupad [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.19

$$\int x^{-1+n}\text{arctanh}(a+bx^n)\,dx=\frac{x^n\,\text{atanh}(a+b\,x^n)}{n}-\frac{\ln\left(a+b\,x^n-1\right)\,\left(a-1\right)}{2\,b\,n}+\frac{\ln\left(a+b\,x^n+1\right)\,\left(a+1\right)}{2\,b\,n}$$

input
```
int(x^(n - 1)*atanh(a + b*x^n),x)
```

output
```
(x^n*atanh(a + b*x^n))/n - (log(a + b*x^n - 1)*(a - 1))/(2*b*n) + (log(a +
 b*x^n + 1)*(a + 1))/(2*b*n)
```

3.30 $\int \frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

3.30.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \text{Int}\left(\frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2},x\right)$$

output
```
Unintegrable((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

3.30.2 Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \int \frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

3.30.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2}dx$$

$$\Big\downarrow \ 7234$$

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2}dx$$

input
```
Int[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
$Aborted
```

3.30.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !IGt
Q[n, 0]
```

3.30.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\,\operatorname{arctanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{-c^2x^2+1}dx$$

input
```
int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

output
```
int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

3.30.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, al
gorithm="fricas")
```

output
```
integral(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

3.30.6 Sympy [N/A]

Not integrable

Time = 9.64 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\int \frac{\left(a+b\operatorname{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

output
```
-Integral((a + b*atanh(sqrt(-c*x + 1)/sqrt(c*x + 1)))**n/(c**2*x**2 - 1),
x)
```

3.30.7 Maxima [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\mathrm{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, al
gorithm="maxima")
```

output
```
-integrate((b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1),
x)
```

3.30.8 Giac [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\mathrm{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, al
gorithm="giac")
```

output
```
integrate(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1),
x)
```

3.30.9 Mupad [N/A]

Not integrable

Time = 3.96 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

output
```
-int((a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.30. $\int \frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

3.31 $\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

3.31.1 Optimal result

Integrand size = 40, antiderivative size = 409

$$\begin{aligned}
&\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx \\
&= -\frac{2\left(a+b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\mathrm{arctanh}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} \\
&+\frac{3b\left(a+b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\mathrm{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&-\frac{3b\left(a+b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\mathrm{PolyLog}\left(2,-1+\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&-\frac{3b^2\left(a+b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\mathrm{PolyLog}\left(3,1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&+\frac{3b^2\left(a+b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\mathrm{PolyLog}\left(3,-1+\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&+\frac{3b^3\,\mathrm{PolyLog}\left(4,1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{4c}-\frac{3b^3\,\mathrm{PolyLog}\left(4,-1+\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{4c}
\end{aligned}$$

output
```
2*arctanh(-1+2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))*(a+b*arctanh((-c*x+1)^(1/
2)/(c*x+1)^(1/2)))^3/c+3/2*b*(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2
*polylog(2,1-2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/2*b*(a+b*arctanh((-c*
x+1)^(1/2)/(c*x+1)^(1/2)))^2*polylog(2,-1+2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2
)))/c-3/2*b^2*(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,1-2/(1
-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c+3/2*b^2*(a+b*arctanh((-c*x+1)^(1/2)/(c*x
+1)^(1/2)))*polylog(3,-1+2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c+3/4*b^3*pol
ylog(4,1-2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/4*b^3*polylog(4,-1+2/(1-(
-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c
```

3.31.2 Mathematica [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int \frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

3.31.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7232, 6448, 6614, 6620, 6624, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{1-c^2x^2}\,dx$$

$$\Big\downarrow \text{ 7232}$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{ 6448}$$

$$-\frac{2\text{arctanh}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\int\frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2\text{arctanh}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$\big\downarrow$ 6614

$$-\frac{2\text{arctanh}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\left(\frac{1}{2}\int\frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2\log\left(2-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}-\frac{1}{2}\right.}{c}$$

$\big\downarrow$ 6620

$$-\frac{2\text{arctanh}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\left(\frac{1}{2}\left(\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-c}}{\sqrt{cx+}}\right.\right.\right.\right.}{}$$

$\big\downarrow$ 6624

$$-\frac{2\text{arctanh}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\left(\frac{1}{2}\left(\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-c}}{\sqrt{cx+}}\right.\right.\right.\right.}{}$$

$\big\downarrow$ 7164

$$-\frac{2\text{arctanh}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\left(\frac{1}{2}\left(\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right.\right.\right.\right.}{}$$

input
```
Int[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

3.31. $\int\frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}dx$

output
```
-((2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcTanh[1 - 2/(1 - Sqr
t[1 - c*x]/Sqrt[1 + c*x])] - 6*b*((((a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 +
c*x]])^2*PolyLog[2, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/2 - b*(((a +
 b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1 - 2/(1 - Sqrt[1 - c*
x]/Sqrt[1 + c*x])])/2 - (b*PolyLog[4, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*
x])])/4))/2 + (-1/2*((a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLo
g[2, -1 + 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])]) + b*(((a + b*ArcTanh[Sqrt[
1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -1 + 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x
])])/2 - (b*PolyLog[4, -1 + 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/4))/2))/
c)
```

3.31.3.1 Defintions of rubi rules used

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6624
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.31.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1622 vs. $2(349) = 698$.

Time = 0.55 (sec) , antiderivative size = 1623, normalized size of antiderivative = 3.97

method	result	size
default	Expression too large to display	1623
parts	Expression too large to display	1623

input
```
int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_R
ETURNVERBOSE)
```

output
```
-1/2*a^3/c*ln(c*x-1)+1/2*a^3/c*ln(c*x+1)-b^3*(1/c*arctanh((-c*x+1)^(1/2)/(
c*x+1)^(1/2))^3*ln(1-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1
)^(1/2))+3/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,((-c*x+1)^(
1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-6/c*arctanh((-c*x+1)^(1
/2)/(c*x+1)^(1/2))*polylog(3,((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(
c*x+1)+1)^(1/2))+6/c*polylog(4,((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)
/(c*x+1)+1)^(1/2))+1/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+((-c*x
+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+3/c*arctanh((-c*x+
1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-
c*x+1)/(c*x+1)+1)^(1/2))-6/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog
(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+6/c*poly
log(4,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-1/c*a
rctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+
1)^2/(-(-c*x+1)/(c*x+1)+1))-3/2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*
polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))+3/2/c
*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^
(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))-3/4/c*polylog(4,-((-c*x+1)^(1/2)/(c*x+1)
^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1)))-3*a*b^2*(1/c*arctanh((-c*x+1)^(1/2)/(c
*x+1)^(1/2))^2*ln(1-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)
^(1/2))+2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,((-c*x+1)^(...
```

3.31.5 Fricas [F]

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(b\operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, al
gorithm="fricas")
```

output
```
integral(-(b^3*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arctanh(s
qrt(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arctanh(sqrt(-c*x + 1)/sqrt(c*x +
 1)) + a^3)/(c^2*x^2 - 1), x)
```

3.31.6 Sympy [F]

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\int \frac{a^3}{c^2x^2-1}\,dx - \int \frac{b^3\operatorname{atanh}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3ab^2\operatorname{atanh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3a^2b\operatorname{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

output
```
-Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*atanh(sqrt(-c*x + 1)/sq
rt(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*atanh(sqrt(-c*x +
1)/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*atanh(sqrt(-c
*x + 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

3.31.7 Maxima [F]

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(b\operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, al
gorithm="maxima")
```

output
```
1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) - 1/16*(b^3*log(c*x + 1) - b^3*l
og(-c*x + 1))*log(sqrt(c*x + 1) - sqrt(-c*x + 1))^3/c - integrate(1/32*(4*
(sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x + 1) + sqrt(-c*x + 1
))^3 + 24*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*log(sqrt(c*x + 1) +
 sqrt(-c*x + 1))^2 + 3*(4*(sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqr
t(c*x + 1) + sqrt(-c*x + 1)) + (8*a*b^2 - (b^3*c*x - b^3)*log(c*x + 1) + (
b^3*c*x - b^3)*log(-c*x + 1))*sqrt(c*x + 1) - (8*a*b^2 - (b^3*c*x + b^3)*l
og(c*x + 1) + (b^3*c*x + b^3)*log(-c*x + 1))*sqrt(-c*x + 1))*log(sqrt(c*x
+ 1) - sqrt(-c*x + 1))^2 + 48*(sqrt(c*x + 1)*a^2*b - sqrt(-c*x + 1)*a^2*b)
*log(sqrt(c*x + 1) + sqrt(-c*x + 1)) - 12*(4*sqrt(c*x + 1)*a^2*b - 4*sqrt(
-c*x + 1)*a^2*b + (sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x +
1) + sqrt(-c*x + 1))^2 + 4*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*lo
g(sqrt(c*x + 1) + sqrt(-c*x + 1)))*log(sqrt(c*x + 1) - sqrt(-c*x + 1)))/((
c^2*x^2 - 1)*sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)
```

3.31.8 Giac [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, al
gorithm="giac")
```

output
```
integrate(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1),
x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(a+b\,\text{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.32 $\int \frac{\left(a+b\textbf{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

3.32.1 Optimal result

Integrand size = 40, antiderivative size = 268

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{2\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{arctanh}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{b\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{b\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2,-1+\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{b^2\text{PolyLog}\left(3,1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} + \frac{b^2\text{PolyLog}\left(3,-1+\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c}$$

output
```
2*arctanh(-1+2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))*(a+b*arctanh((-c*x+1)^(1/
2)/(c*x+1)^(1/2)))^2/c+b*(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polyl
og(2,1-2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-b*(a+b*arctanh((-c*x+1)^(1/2)
/(c*x+1)^(1/2)))*polylog(2,-1+2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-1/2*b^
2*polylog(3,1-2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c+1/2*b^2*polylog(3,-1+2
/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c
```

3.32.2 Mathematica [F]

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]
```

3.32.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7232, 6448, 6614, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{1-c^2x^2}\,dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{6448}$$

$$-\frac{2\operatorname{arctanh}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-4b\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\operatorname{arctanh}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{6614}$$

$$-\frac{2\operatorname{arctanh}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-4b\left(\frac{1}{2}\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\log\left(2-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}-\frac{1}{2}\right.}{c}$$

$\downarrow$ 6620

$$-\frac{2\text{arctanh}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-4b\left(\frac{1}{2}\left(\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-c}}{\sqrt{cx+}}\right.\right.\right.\right.}{}$$

$\downarrow$ 7164

$$-\frac{2\text{arctanh}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-4b\left(\frac{1}{2}\left(\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right.\right.\right.\right.}{}$$

input
```
Int[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
-((2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcTanh[1 - 2/(1 - Sqr
t[1 - c*x]/Sqrt[1 + c*x])] - 4*b*((((a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 +
c*x]])*PolyLog[2, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/2 - (b*PolyLog
[3, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/4)/2 + (-1/2*((a + b*ArcTanh
[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, -1 + 2/(1 - Sqrt[1 - c*x]/Sqrt[1
 + c*x])]) + (b*PolyLog[3, -1 + 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/4)/2
))/c)
```

3.32.3.1 Defintions of rubi rules used

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

3.32. $\int \frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.32.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(232) = 464$.

Time = 0.26 (sec) , antiderivative size = 893, normalized size of antiderivative = 3.33

method	result
default	$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}-b^2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(1-\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}{\sqrt{-\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\operatorname{polylog}\left(2,\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{-\frac{-cx+}{cx+1}}}\right.}{c}\right.$
parts	$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}-b^2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(1-\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}{\sqrt{-\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\operatorname{polylog}\left(2,\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{-\frac{-cx+}{cx+1}}}\right.}{c}\right.$

input
```
int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_R
ETURNVERBOSE)
```

output
```
-1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)-b^2*(1/c*arctanh((-c*x+1)^(1/2)/(
c*x+1)^(1/2))^2*ln(1-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1
)^(1/2))+2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,((-c*x+1)^(1/
2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-2/c*polylog(3,((-c*x+1)^(
1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+1/c*arctanh((-c*x+1)^(1
/2)/(c*x+1)^(1/2))^2*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x
+1)+1)^(1/2))+2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-((-c*x+
1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-2/c*polylog(3,-((-c
*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-1/c*arctanh((-c*
x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*
x+1)/(c*x+1)+1))-1/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-((-c
*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))+1/2/c*polylog(3,-((-
c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1)))-2*a*b*(1/c*arctanh
((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c
*x+1)/(c*x+1)+1)^(1/2))+1/c*polylog(2,((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(
-c*x+1)/(c*x+1)+1)^(1/2))+1/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(
(-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+1/c*polylog(2
,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-1/c*arctan
h((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-
(-c*x+1)/(c*x+1)+1))-1/2/c*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^...
```

3.32.5 Fricas [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, al
gorithm="fricas")
```

output
```
integral(-(b^2*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arctanh(sqr
t(-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

3.32.6 Sympy [F]

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\int \frac{a^2}{c^2x^2-1}\,dx - \int \frac{b^2\operatorname{atanh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{2ab\operatorname{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

output
```
-Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*atanh(sqrt(-c*x + 1)/sq
rt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*atanh(sqrt(-c*x + 1)/
sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

3.32.7 Maxima [F]

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(b\operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, al
gorithm="maxima")
```

output
```
1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/8*(b^2*log(c*x + 1) - b^2*lo
g(-c*x + 1))*log(sqrt(c*x + 1) - sqrt(-c*x + 1))^2/c + integrate(-1/8*(2*(
sqrt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1) + sqrt(-c*x + 1)
)^2 + 8*(sqrt(c*x + 1)*a*b - sqrt(-c*x + 1)*a*b)*log(sqrt(c*x + 1) + sqrt(
-c*x + 1)) - (4*(sqrt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1)
 + sqrt(-c*x + 1)) + (8*a*b - (b^2*c*x - b^2)*log(c*x + 1) + (b^2*c*x - b^
2)*log(-c*x + 1))*sqrt(c*x + 1) - (8*a*b - (b^2*c*x + b^2)*log(c*x + 1) +
(b^2*c*x + b^2)*log(-c*x + 1))*sqrt(-c*x + 1))*log(sqrt(c*x + 1) - sqrt(-c
*x + 1)))/((c^2*x^2 - 1)*sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)
```

3.32.8 Giac [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, al
gorithm="giac")
```

output
```
integrate(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1),
x)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(a+b\,\text{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.33 $\int \frac{a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

3.33.1 Optimal result

Integrand size = 38, antiderivative size = 89

$$\int \frac{a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}+\frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c} - \frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}$$

output
```
-a*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))/c+1/2*b*polylog(2,-(-c*x+1)^(1/2)/(c*x
+1)^(1/2))/c-1/2*b*polylog(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2))/c
```

3.33.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.48

$$\int \frac{a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \frac{a\text{arctanh}(cx)}{c} + \frac{b\left(\text{PolyLog}\left(2,-e^{-\text{arctanh}(cx)}\right)-\text{PolyLog}\left(2,e^{-\text{arctanh}(cx)}\right)\right)}{2c}$$

input
```
Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

output
```
(a*ArcTanh[c*x])/c + (b*(PolyLog[2, -E^(-ArcTanh[c*x])] - PolyLog[2, E^(-A
rcTanh[c*x])]))/(2*c)
```

3.33.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {7232, 6446}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{1-c^2x^2} \, dx$$
$$\Big\downarrow \text{7232}$$
$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$
$$\Big\downarrow \text{6446}$$
$$-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)+\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

input
```
Int[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

output
```
-((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]] - (b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt
[1 + c*x])])/2 + (b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/2)/c)
```

3.33.3.1 Defintions of rubi rules used

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.33.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(73) = 146.

Time = 0.28 (sec) , antiderivative size = 357, normalized size of antiderivative = 4.01

method	result
default	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}-b\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1-\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}{\sqrt{-\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{\operatorname{polylog}\left(2,\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}{\sqrt{-\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{-cx}}{\sqrt{cx+}}\right.}{}\right.$
parts	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}-b\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1-\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}{\sqrt{-\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{\operatorname{polylog}\left(2,\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}{\sqrt{-\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{-cx}}{\sqrt{cx+}}\right.}{}\right.$

input
```
int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
```

output
```
-1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)-b*(1/c*arctanh((-c*x+1)^(1/2)/(c*x+1)
^(1/2))*ln(1-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))
+1/c*polylog(2,((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2
))+1/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+((-c*x+1)^(1/2)/(c*x+1)^
(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+1/c*polylog(2,-((-c*x+1)^(1/2)/(c*x+
1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-1/c*arctanh((-c*x+1)^(1/2)/(c*x+1
)^(1/2))*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))-1/
2/c*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1)))
```

3.33.5 Fricas [F]

$$\int \frac{a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algo
rithm="fricas")
```

output
```
integral(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

3.33.6 Sympy [F]

$$\int \frac{a + b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\int \frac{a}{c^2x^2-1}\,dx - \int \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

output
```
-Integral(a/(c**2*x**2 - 1), x) - Integral(b*atanh(sqrt(-c*x + 1)/sqrt(c*x
 + 1))/(c**2*x**2 - 1), x)
```

3.33.7 Maxima [F]

$$\int \frac{a + b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \int -\frac{b\,\mathrm{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algo
rithm="maxima")
```

output
```
1/4*b*(((log(c*x + 1) - log(-c*x + 1))*log(sqrt(c*x + 1) + sqrt(-c*x + 1))
 - (log(c*x + 1) - log(-c*x + 1))*log(sqrt(c*x + 1) - sqrt(-c*x + 1)))/c -
 2*integrate(-1/2*sqrt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 -
 1)*sqrt(c*x + 1) + (c^2*x^2 - 1)*sqrt(-c*x + 1)), x) - 2*integrate(1/2*sq
rt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)*sqrt(c*x + 1) -
(c^2*x^2 - 1)*sqrt(-c*x + 1)), x)) + 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/
c)
```

3.33.8 Giac [F]

$$\int \frac{a + b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \int -\frac{b\,\mathrm{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algo
rithm="giac")
```

output
```
integrate(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1 - c^2x^2}\,dx = \int -\frac{a + b\,\text{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2 - 1}\,dx$$

input
```
int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

3.34 $\int \frac{1}{(1-c^2x^2)\left(a+b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

3.34.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \mathrm{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)},x\right)$$

output
```
Unintegrable(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

3.34.2 Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\,\mathrm{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x
]
```

output
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),
x]
```

3.34.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$

$$\Big\downarrow \text{7234}$$

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

output
```
$Aborted
```

3.34.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !IGt
Q[n, 0]
```

3.34.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\,\text{arctanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}dx$$

input
```
int(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

3.34.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, al
gorithm="fricas")
```

output
```
integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arctanh(sqrt(-c*x + 1)/sqrt(c*x +
 1)) - a), x)
```

3.34.6 Sympy [N/A]

Not integrable

Time = 3.79 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$
$$= -\int \frac{1}{ac^2x^2-a+bc^2x^2\,\text{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b\,\text{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

output
```
-Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*atanh(sqrt(-c*x + 1)/sqrt(c*x +
 1)) - b*atanh(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

3.34.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, al
gorithm="maxima")
```

output
```
-integrate(1/((c^2*x^2 - 1)*(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a))
, x)
```

3.34.8 Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, al
gorithm="giac")
```

output
```
integrate(-1/((c^2*x^2 - 1)*(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a))
, x)
```

3.34.9 Mupad [N/A]

Not integrable

Time = 3.68 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=-\int \frac{1}{\left(a+b\,\text{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,(c^2\,x^2-1)}\,dx$$

input
```
int(-1/((a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.34. $\int \frac{1}{(1-c^2x^2)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

3.35 $\int \frac{1}{(1-c^2x^2)\left(a+b\mathbf{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2} dx$

3.35.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2} dx = \text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}, x\right)$$

output
```
Unintegrable(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,
x)
```

3.35.2 Mathematica [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2} dx = \int \frac{1}{(1-c^2x^2)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2} dx$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
,x]
```

output
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
, x]
```

3.35.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}dx$$

$$\Big\downarrow \text{7234}$$

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}dx$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

output
```
$Aborted
```

3.35.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !IGt
Q[n, 0]
```

3.35.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\,\text{arctanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}dx$$

input
```
int(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

3.35.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.28

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x,
algorithm="fricas")
```

output
```
integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arctanh(sqrt(-c*x + 1)/sqrt
(c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arctanh(sqrt(-c*x + 1)/sqrt(c*x
 + 1))), x)
```

3.35.6 Sympy [N/A]

Not integrable

Time = 10.76 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.15

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\int \frac{1}{a^2c^2x^2-a^2+2abc^2x^2\,\text{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-2ab\,\text{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+b^2c^2x^2\,\text{atanh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b^2\,\text{atanh}$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,
x)
```

output
```
-Integral(1/(a**2*c**2*x**2 - a**2 + 2*a*b*c**2*x**2*atanh(sqrt(-c*x + 1)/
sqrt(c*x + 1)) - 2*a*b*atanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + b**2*c**2*x**
2*atanh(sqrt(-c*x + 1)/sqrt(c*x + 1))**2 - b**2*atanh(sqrt(-c*x + 1)/sqrt(
c*x + 1))**2), x)
```

3.35.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 246, normalized size of antiderivative = 6.15

$$\int \frac{1}{(1-c^2x^2)\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x,
algorithm="maxima")
```

output
```
4*c*x/(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*c*log(sqrt(c*x + 1) + sqrt(-c*x +
1)) - sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*c*log(sqrt(c*x + 1) - sqrt(-c*x + 1
)) + 2*sqrt(c*x + 1)*sqrt(-c*x + 1)*a*b*c) - integrate(-4/((b^2*c^2*x^2 -
b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(sqrt(c*x + 1) + sqrt(-c*x + 1)) - (b
^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(sqrt(c*x + 1) - sqrt(-c
*x + 1)) + 2*(a*b*c^2*x^2 - a*b)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)
```

3.35.8 Giac [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x,
algorithm="giac")
```

output
```
integrate(-1/((c^2*x^2 - 1)*(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^
2), x)
```

3.35.9 Mupad [N/A]

Not integrable

Time = 4.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=-\int \frac{1}{\left(a+b\,\text{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,\left(c^2\,x^2-1\right)}\,dx$$

input
```
int(-1/((a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x
)
```

3.36 $\int x^m \textbf{arctanh}(\tanh(a+bx))\,dx$

3.36.1 Optimal result

Integrand size = 11, antiderivative size = 37

$$\int x^m \operatorname{arctanh}(\tanh(a+bx))\,dx = -\frac{bx^{2+m}}{2+3m+m^2} + \frac{x^{1+m}\operatorname{arctanh}(\tanh(a+bx))}{1+m}$$

output
```
-b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*arctanh(tanh(b*x+a))/(1+m)
```

3.36.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x^m \operatorname{arctanh}(\tanh(a+bx))\,dx = x^m\left(\frac{bx^2}{2+m} + \frac{x(-bx+\operatorname{arctanh}(\tanh(a+bx)))}{1+m}\right)$$

input
```
Integrate[x^m*ArcTanh[Tanh[a + b*x]],x]
```

output
```
x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/(1 + m))
```

3.36.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{arctanh}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\text{arctanh}(\tanh(a+bx))}{m+1}-\frac{b\int x^{m+1}dx}{m+1}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x^{m+1}\text{arctanh}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{(m+1)(m+2)}$$

input
```
Int[x^m*ArcTanh[Tanh[a + b*x]],x]
```

output
```
-((b*x^(2 + m))/((1 + m)*(2 + m))) + (x^(1 + m)*ArcTanh[Tanh[a + b*x]])/(1
 + m)
```

3.36.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.36.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

method	result
default	$\frac{b\,x^{2} {\mathrm e}^{m \ln \left(x \right)}}{2+m}+\frac{\left(\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)-b x \right) x \,{\mathrm e}^{m \ln \left(x \right)}}{1+m}$
parallelrisch	$-\frac{-2 \,\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right) x^{m} x +b \,x^{2} x^{m}-x \,x^{m} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right) m}{\left(1+m \right) \left(2+m \right)}$
risch	$\frac{x \,x^{m} \ln \left({\mathrm e}^{b x +a}\right)}{1+m}-\frac{x \left(4 b x +2 i \pi \,\operatorname{csgn}\left(\frac{i {\mathrm e}^{2 b x +2 a}}{{\mathrm e}^{2 b x +2 a}+1}\right)^{3}-i \pi \,\operatorname{csgn}\left(i {\mathrm e}^{2 b x +2 a}\right) \operatorname{csgn}\left(\frac{i {\mathrm e}^{2 b x +2 a}}{{\mathrm e}^{2 b x +2 a}+1}\right)^{2} m +i \pi \,\operatorname{csgn}\left(i {\mathrm e}^{2 b x +2 a}\right)^{3} m -2 i \pi \,\operatorname{cs}$

input
```
int(x^m*arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
b/(2+m)*x^2*exp(m*ln(x))+(arctanh(tanh(b*x+a))-b*x)/(1+m)*x*exp(m*ln(x))
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.68

$$\int x^m \operatorname{arctanh}(\tanh(a+bx))\,dx = \frac{((bm+b)x^2+(am+2\,a)x)\cosh\left(m\log\left(x\right)\right)+((bm+b)x^2+(am+2\,a)x)\sinh\left(m\log\left(x\right)\right)}{m^2+3\,m+2}$$

input
```
integrate(x^m*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
(((b*m + b)*x^2 + (a*m + 2*a)*x)*cosh(m*log(x)) + ((b*m + b)*x^2 + (a*m +
2*a)*x)*sinh(m*log(x)))/(m^2 + 3*m + 2)
```

3.36.6 Sympy [F]

$$\int x^m \operatorname{arctanh}(\tanh(a+bx))\,dx = \begin{cases} b\log\left(x\right)-\frac{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{x} & \text{for}\: m=-2 \\ \int \frac{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{x}\,dx & \text{for}\: m=-1 \\ -\frac{bx^2x^m}{m^2+3m+2}+\frac{mxx^m\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{m^2+3m+2}+\frac{2xx^m\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{m^2+3m+2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*atanh(tanh(b*x+a)),x)
```

output
```
Piecewise((b*log(x) - atanh(tanh(a + b*x))/x, Eq(m, -2)), (Integral(atanh(
tanh(a + b*x))/x, x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x*x**
m*atanh(tanh(a + b*x))/(m**2 + 3*m + 2) + 2*x*x**m*atanh(tanh(a + b*x))/(m
**2 + 3*m + 2), True))
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int x^m \text{arctanh}(\tanh(a+bx))\,dx = -\frac{bx^2x^m}{(m+2)(m+1)} + \frac{x^{m+1}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)}{m+1}$$

input
```
integrate(x^m*arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-b*x^2*x^m/((m + 2)*(m + 1)) + x^(m + 1)*arctanh(tanh(b*x + a))/(m + 1)
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

$$\int x^m \text{arctanh}(\tanh(a+bx))\,dx = \frac{bmx^2x^m + amxx^m + bx^2x^m + 2\,axx^m}{m^2+3\,m+2}$$

input
```
integrate(x^m*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
(b*m*x^2*x^m + a*m*x*x^m + b*x^2*x^m + 2*a*x*x^m)/(m^2 + 3*m + 2)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 4.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.59

$$\int x^m \operatorname{arctanh}(\tanh(a+bx))\,dx = \frac{2\,b\,x^m\,x^2\,(m+1)}{2\,m^2+6\,m+4} - \frac{x\,x^m\,(m+2)\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,m^2+6\,m+4}$$

input
```
int(x^m*atanh(tanh(a + b*x)),x)
```

output
```
(2*b*x^m*x^2*(m + 1))/(6*m + 2*m^2 + 4) - (x*x^m*(m + 2)*(log(2/(exp(2*a)*
exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x))/(6*m + 2*m^2 + 4)
```

3.37 $\int x^2 \mathbf{arctanh}(\tanh(a+bx))\,dx$

3.37.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int x^2 \text{arctanh}(\tanh(a+bx))\,dx = -\frac{bx^4}{12} + \frac{1}{3}x^3 \text{arctanh}(\tanh(a+bx))$$

output
```
-1/12*b*x^4+1/3*x^3*arctanh(tanh(b*x+a))
```

3.37.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x^2 \text{arctanh}(\tanh(a+bx))\,dx = -\frac{1}{12}x^3(bx - 4\text{arctanh}(\tanh(a+bx)))$$

input
```
Integrate[x^2*ArcTanh[Tanh[a + b*x]],x]
```

output
```
-1/12*(x^3*(b*x - 4*ArcTanh[Tanh[a + b*x]]))
```

3.37.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arctanh}(\tanh(a+bx))\,dx$$

$$\downarrow \text{2599}$$

$$\frac{1}{3}x^3\text{arctanh}(\tanh(a+bx)) - \frac{b\int x^3 dx}{3}$$

$$\downarrow \text{15}$$

$$\frac{1}{3}x^3\text{arctanh}(\tanh(a+bx)) - \frac{bx^4}{12}$$

input
```
Int[x^2*ArcTanh[Tanh[a + b*x]],x]
```

output
```
-1/12*(b*x^4) + (x^3*ArcTanh[Tanh[a + b*x]])/3
```

3.37.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.37.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{b x^4}{12}+\frac{x^3 \operatorname{arctanh}(\tanh(bx+a))}{3}$
parallelrisch	$-\frac{b x^4}{12}+\frac{x^3 \operatorname{arctanh}(\tanh(bx+a))}{3}$
parts	$-\frac{b x^4}{12}+\frac{x^3 \operatorname{arctanh}(\tanh(bx+a))}{3}$
risch	$\frac{x^3 \ln\left(e^{bx+a}\right)}{3}-\frac{b x^4}{12}-\frac{i\pi x^3 \operatorname{csgn}\left(i e^{2bx+2a}\right)^3}{12}+\frac{i\pi x^3 \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right) \operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2}{12}-\frac{i\pi x^3 \operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^3}{12}$

input
```
int(x^2*arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*b*x^4+1/3*x^3*arctanh(tanh(b*x+a))
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x^2 \operatorname{arctanh}(\tanh(a+bx))\, dx = \frac{1}{4}\, bx^4 + \frac{1}{3}\, ax^3$$

input
```
integrate(x^2*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

3.37.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \operatorname{arctanh}(\tanh(a+bx))\, dx = -\frac{bx^4}{12} + \frac{x^3 \operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{3}$$

input
```
integrate(x**2*atanh(tanh(b*x+a)),x)
```

output
```
-b*x**4/12 + x**3*atanh(tanh(a + b*x))/3
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \text{arctanh}(\tanh(a+bx))\,dx = -\frac{1}{12}\,bx^4 + \frac{1}{3}\,x^3\,\text{artanh}\left(\tanh\left(bx+a\right)\right)$$

input
```
integrate(x^2*arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/12*b*x^4 + 1/3*x^3*arctanh(tanh(b*x + a))
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x^2 \text{arctanh}(\tanh(a+bx))\,dx = \frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

input
```
integrate(x^2*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

3.37.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \text{arctanh}(\tanh(a+bx))\,dx = \frac{x^3\,\text{atanh}(\tanh(a+b\,x))}{3} - \frac{b\,x^4}{12}$$

input
```
int(x^2*atanh(tanh(a + b*x)),x)
```

output
```
(x^3*atanh(tanh(a + b*x)))/3 - (b*x^4)/12
```

3.38 $\int x\text{arctanh}(\tanh(a+bx))\,dx$

3.38.1 Optimal result

Integrand size = 9, antiderivative size = 23

$$\int x\text{arctanh}(\tanh(a+bx))\,dx=-\frac{bx^3}{6}+\frac{1}{2}x^2\text{arctanh}(\tanh(a+bx))$$

output
```
-1/6*b*x^3+1/2*x^2*arctanh(tanh(b*x+a))
```

3.38.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x\text{arctanh}(\tanh(a+bx))\,dx=-\frac{1}{6}x^2(bx-3\text{arctanh}(\tanh(a+bx)))$$

input
```
Integrate[x*ArcTanh[Tanh[a + b*x]],x]
```

output
```
-1/6*(x^2*(b*x - 3*ArcTanh[Tanh[a + b*x]]))
```

3.38.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6793, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{6793}$$

$$\frac{1}{2}b\int -x^2dx+\frac{1}{2}x^2\text{arctanh}(\tanh(a+bx))$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}x^2\text{arctanh}(\tanh(a+bx))-\frac{bx^3}{6}$$

input
```
Int[x*ArcTanh[Tanh[a + b*x]],x]
```

output
```
-1/6*(b*x^3) + (x^2*ArcTanh[Tanh[a + b*x]])/2
```

3.38.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6793
```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

3.38.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{b\,x^{3}}{6}+\frac{x^{2} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)}{2}$
parallelrisch	$-\frac{b\,x^{3}}{6}+\frac{x^{2} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)}{2}$
parts	$-\frac{b\,x^{3}}{6}+\frac{x^{2} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)}{2}$
risch	$\frac{x^{2} \ln \left({\mathrm e}^{b x +a}\right)}{2}-\frac{b\,x^{3}}{6}-\frac{i \pi \,x^{2} \operatorname{csgn}\left(i {\mathrm e}^{2 b x +2 a}\right)^{3}}{8}+\frac{i \pi \,x^{2} \operatorname{csgn}\left(i {\mathrm e}^{b x +a}\right) \operatorname{csgn}\left(i {\mathrm e}^{2 b x +2 a}\right)^{2}}{4}-\frac{i \pi \,x^{2} \operatorname{csgn}\left(\frac{i {\mathrm e}^{2 b x +2 a}}{{\mathrm e}^{2 b x +2 a}+1}\right)^{3}}{8}-i\pi$

input
```
int(x*arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*b*x^3+1/2*x^2*arctanh(tanh(b*x+a))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x\text{arctanh}(\tanh(a+bx))\,dx = \frac{1}{3}\,bx^{3} + \frac{1}{2}\,ax^{2}$$

input
```
integrate(x*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/3*b*x^3 + 1/2*a*x^2
```

3.38.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(19) = 38$.

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x\text{arctanh}(\tanh(a+bx))\,dx = \begin{cases} \frac{x\,\text{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}{2b} - \frac{\text{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{6b^{2}} & \text{for}\ b \neq 0 \\ \frac{x^{2}\,\text{atanh}\left(\tanh\left(a\right)\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*atanh(tanh(b*x+a)),x)
```

output
```
Piecewise((x*atanh(tanh(a + b*x))**2/(2*b) - atanh(tanh(a + b*x))**3/(6*b*
*2), Ne(b, 0)), (x**2*atanh(tanh(a))/2, True))
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x\mathrm{arctanh}(\tanh(a+bx))\,dx = -\frac{1}{6}\,bx^3 + \frac{1}{2}\,x^2\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)$$

input
```
integrate(x*arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/6*b*x^3 + 1/2*x^2*arctanh(tanh(b*x + a))
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x\mathrm{arctanh}(\tanh(a+bx))\,dx = \frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2$$

input
```
integrate(x*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
1/3*b*x^3 + 1/2*a*x^2
```

3.38.9 Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x\mathrm{arctanh}(\tanh(a+bx))\,dx = \frac{x^2\,\mathrm{atanh}(\tanh(a+b\,x))}{2} - \frac{b\,x^3}{6}$$

input
```
int(x*atanh(tanh(a + b*x)),x)
```

output
```
(x^2*atanh(tanh(a + b*x)))/2 - (b*x^3)/6
```

3.39 $\int \textbf{arctanh}(\tanh(a+bx))\,dx$

3.39.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \operatorname{arctanh}(\tanh(a+bx))\,dx=\frac{\operatorname{arctanh}(\tanh(a+bx))^2}{2b}$$

output
```
1/2*arctanh(tanh(b*x+a))^2/b
```

3.39.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \operatorname{arctanh}(\tanh(a+bx))\,dx=-\frac{bx^2}{2}+x\operatorname{arctanh}(\tanh(a+bx))$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]],x]
```

output
```
-1/2*(b*x^2) + x*ArcTanh[Tanh[a + b*x]]
```

3.39.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \operatorname{arctanh}(\tanh(a+bx))d\operatorname{arctanh}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\operatorname{arctanh}(\tanh(a+bx))^2}{2b}$$

input
```
Int[ArcTanh[Tanh[a + b*x]],x]
```

output
```
ArcTanh[Tanh[a + b*x]]^2/(2*b)
```

3.39.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.39.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2b}$
default	$\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2b}$
parallelrisch	$-\frac{b\,x^2}{2}+x\ \operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)$
parts	$-\frac{b\,x^2}{2}+x\ \operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)$
risch	$x\ln\left(\mathrm{e}^{bx+a}\right)-\frac{i\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)\pi x}{4}+\frac{i\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)}{4}$

input
```
int(arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(tanh(b*x+a))^2/b
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \operatorname{arctanh}(\tanh(a+bx))\,dx=\frac{1}{2}\,bx^2+ax$$

input
```
integrate(arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*b*x^2 + a*x
```

3.39.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \operatorname{arctanh}(\tanh(a+bx))\,dx=\begin{cases}\frac{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{2b} & \text{for } b\neq 0\\ x\operatorname{atanh}\left(\tanh\left(a\right)\right) & \text{otherwise}\end{cases}$$

input
```
integrate(atanh(tanh(b*x+a)),x)
```

output
```
Piecewise((atanh(tanh(a + b*x))**2/(2*b), Ne(b, 0)), (x*atanh(tanh(a)), Tr
ue))
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \operatorname{arctanh}(\tanh(a+bx))\,dx = -\frac{1}{2}\,bx^2 + x\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)$$

input
```
integrate(arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/2*b*x^2 + x*arctanh(tanh(b*x + a))
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \operatorname{arctanh}(\tanh(a+bx))\,dx = \frac{1}{2}\,bx^2 + ax$$

input
```
integrate(arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
1/2*b*x^2 + a*x
```

3.39.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \operatorname{arctanh}(\tanh(a+bx))\,dx = x\,\operatorname{atanh}(\tanh(a+b\,x)) - \frac{b\,x^2}{2}$$

input
```
int(atanh(tanh(a + b*x)),x)
```

output
```
x*atanh(tanh(a + b*x)) - (b*x^2)/2
```

3.40 $\int \frac{\text{arctanh}(\tanh(a+bx))}{x} dx$

3.40.1 Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \frac{\text{arctanh}(\tanh(a+bx))}{x} dx = bx - (bx - \text{arctanh}(\tanh(a+bx))) \log(x)$$

output
```
b*x-(b*x-arctanh(tanh(b*x+a)))*ln(x)
```

3.40.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\text{arctanh}(\tanh(a+bx))}{x} dx = bx + (-bx + \text{arctanh}(\tanh(a+bx))) \log(x)$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]/x,x]
```

output
```
b*x + (-(b*x) + ArcTanh[Tanh[a + b*x]])*Log[x]
```

3.40.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x} \, dx$$

$$\Big\downarrow \ 2589$$

$$bx-(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{1}{x}dx$$

$$\Big\downarrow \ 14$$

$$bx-\log(x)(bx-\operatorname{arctanh}(\tanh(a+bx)))$$

input
```
Int[ArcTanh[Tanh[a + b*x]]/x,x]
```

output
```
b*x - (b*x - ArcTanh[Tanh[a + b*x]])*Log[x]
```

3.40.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

3.40.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

method	result
default	$\ln(x)\operatorname{arctanh}(\tanh(bx+a)) - b(x\ln(x) - x)$
parts	$\ln(x)\operatorname{arctanh}(\tanh(bx+a)) - b(x\ln(x) - x)$
risch	$\ln(x)\ln\left(\mathrm{e}^{bx+a}\right) - b\ln(x)\,x + bx - \frac{i\pi\left(\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(i\,\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\,\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right) - \operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\right.}{}$

input
```
int(arctanh(tanh(b*x+a))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arctanh(tanh(b*x+a))-b*(x*ln(x)-x)
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x}\,dx = bx + a\log(x)$$

input
```
integrate(arctanh(tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
b*x + a*log(x)
```

3.40.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{atanh}(\tanh(a+bx))}{x}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))/x,x)
```

output
```
Integral(atanh(tanh(a + b*x))/x, x)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x}\,dx = -b\left(x+\frac{a}{b}\right)\log(x)+b\left(x+\frac{a\log(x)}{b}\right) + \operatorname{artanh}(\tanh(bx+a))\log(x)$$

input
```
integrate(arctanh(tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
-b*(x + a/b)*log(x) + b*(x + a*log(x)/b) + arctanh(tanh(b*x + a))*log(x)
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.43

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x}\,dx = bx + a\log(|x|)$$

input
```
integrate(arctanh(tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
b*x + a*log(abs(x))
```

3.40.9 Mupad [B] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.76

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x}\,dx = b\,x-\frac{\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)\,\ln(x)}{2}+\frac{\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)\,\ln(x)}{2}-b\,x\,\ln(x)$$

input
```
int(atanh(tanh(a + b*x))/x,x)
```

output
```
b*x - (log(1/(exp(2*a)*exp(2*b*x) + 1))*log(x))/2 + (log((exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))*log(x))/2 - b*x*log(x)
```

3.41 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^2} \, dx$

3.41.1 Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^2} \, dx = -\frac{\operatorname{arctanh}(\tanh(a+bx))}{x} + b\log(x)$$

output
```
-arctanh(tanh(b*x+a))/x+b*ln(x)
```

3.41.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^2} \, dx = b - \frac{\operatorname{arctanh}(\tanh(a+bx))}{x} + b\log(x)$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]/x^2,x]
```

output
```
b - ArcTanh[Tanh[a + b*x]]/x + b*Log[x]
```

3.41.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))}{x^2}dx$$
$$\Big\downarrow \ 2599$$
$$b\int \frac{1}{x}dx-\frac{\text{arctanh}(\tanh(a+bx))}{x}$$
$$\Big\downarrow \ 14$$
$$b\log(x)-\frac{\text{arctanh}(\tanh(a+bx))}{x}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]/x^2,x]
```

output
```
-(ArcTanh[Tanh[a + b*x]]/x) + b*Log[x]
```

3.41.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.41.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))}{x}+b\ln(x)$
parts	$-\frac{\operatorname{arctanh}(\tanh(bx+a))}{x}+b\ln(x)$
parallelrisch	$\frac{b\ln(x)x-\operatorname{arctanh}(\tanh(bx+a))}{x}$
risch	$-\frac{\ln(e^{bx+a})}{x}+\frac{-i\pi\,\mathrm{csgn}(ie^{2bx+2a})\,\mathrm{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2+i\pi\,\mathrm{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^3-2i\pi\,\mathrm{csgn}(ie^{bx+a})\,\mathrm{csgn}(ie^{2bx+2a})^2-i\pi\,\mathrm{csg}$

input
```
int(arctanh(tanh(b*x+a))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arctanh(tanh(b*x+a))/x+b*ln(x)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^2}\,dx=\frac{bx\log(x)-a}{x}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^2,x, algorithm="fricas")
```

output
```
(b*x*log(x) - a)/x
```

3.41.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^2}\,dx=b\log(x)-\frac{\operatorname{atanh}(\tanh(a+bx))}{x}$$

input
```
integrate(atanh(tanh(b*x+a))/x**2,x)
```

output
```
b*log(x) - atanh(tanh(a + b*x))/x
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^2}\,dx = b\log(x) - \frac{\operatorname{artanh}(\tanh(bx+a))}{x}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^2,x, algorithm="maxima")
```

output
```
b*log(x) - arctanh(tanh(b*x + a))/x
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^2}\,dx = b\log(|x|) - \frac{a}{x}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^2,x, algorithm="giac")
```

output
```
b*log(abs(x)) - a/x
```

3.41.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^2}\,dx = b\,\ln(x) - \frac{\operatorname{atanh}(\tanh(a+b\,x))}{x}$$

input
```
int(atanh(tanh(a + b*x))/x^2,x)
```

output
```
b*log(x) - atanh(tanh(a + b*x))/x
```

3.42 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^3} dx$

3.42.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^3} dx = -\frac{b}{2x} - \frac{\operatorname{arctanh}(\tanh(a+bx))}{2x^2}$$

output
```
-1/2*b/x-1/2*arctanh(tanh(b*x+a))/x^2
```

3.42.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^3} dx = -\frac{bx + \operatorname{arctanh}(\tanh(a+bx))}{2x^2}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]/x^3,x]
```

output
```
-1/2*(b*x + ArcTanh[Tanh[a + b*x]])/x^2
```

3.42.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))}{x^3} \, dx$$

$$\Big\downarrow \; 2599$$

$$\frac{1}{2} b \int \frac{1}{x^2} dx - \frac{\text{arctanh}(\tanh(a+bx))}{2x^2}$$

$$\Big\downarrow \; 15$$

$$-\frac{\text{arctanh}(\tanh(a+bx))}{2x^2} - \frac{b}{2x}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]/x^3,x]
```

output
```
-1/2*b/x - ArcTanh[Tanh[a + b*x]]/(2*x^2)
```

3.42.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.42.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

method	result
parallelrisch	$-\frac{bx+\operatorname{arctanh}(\tanh(bx+a))}{2x^2}$
default	$-\frac{b}{2x}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{2x^2}$
parts	$-\frac{b}{2x}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{2x^2}$
risch	$-\frac{\ln(\mathrm{e}^{bx+a})}{2x^2}-\frac{4bx-i\pi\,\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)+i\pi\,\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^2-i\pi}{}$

input
```
int(arctanh(tanh(b*x+a))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x+arctanh(tanh(b*x+a)))/x^2
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^3}\,dx=-\frac{2\,bx+a}{2\,x^2}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b*x + a)/x^2
```

3.42.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^3}\,dx=-\frac{b}{2x}-\frac{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{2x^2}$$

input
```
integrate(atanh(tanh(b*x+a))/x**3,x)
```

output
```
-b/(2*x) - atanh(tanh(a + b*x))/(2*x**2)
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\text{arctanh}(\tanh(a+bx))}{x^3}\,dx = -\frac{b}{2\,x} - \frac{\text{artanh}\left(\tanh\left(bx+a\right)\right)}{2\,x^2}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^3,x, algorithm="maxima")
```

output
```
-1/2*b/x - 1/2*arctanh(tanh(b*x + a))/x^2
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int \frac{\text{arctanh}(\tanh(a+bx))}{x^3}\,dx = -\frac{2\,bx+a}{2\,x^2}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^3,x, algorithm="giac")
```

output
```
-1/2*(2*b*x + a)/x^2
```

3.42.9 Mupad [B] (verification not implemented)

Time = 3.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\text{arctanh}(\tanh(a+bx))}{x^3}\,dx = -\frac{\text{atanh}(\tanh(a+b\,x))+b\,x}{2\,x^2}$$

input
```
int(atanh(tanh(a + b*x))/x^3,x)
```

output
```
-(atanh(tanh(a + b*x)) + b*x)/(2*x^2)
```

3.43 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^4}\,dx$

3.43.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^4}\,dx = -\frac{b}{6x^2} - \frac{\operatorname{arctanh}(\tanh(a+bx))}{3x^3}$$

output
```
-1/6*b/x^2-1/3*arctanh(tanh(b*x+a))/x^3
```

3.43.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^4}\,dx = -\frac{bx + 2\operatorname{arctanh}(\tanh(a+bx))}{6x^3}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]/x^4,x]
```

output
```
-1/6*(b*x + 2*ArcTanh[Tanh[a + b*x]])/x^3
```

3.43.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))}{x^4} dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{3}b\int \frac{1}{x^3}dx-\frac{\text{arctanh}(\tanh(a+bx))}{3x^3}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\text{arctanh}(\tanh(a+bx))}{3x^3}-\frac{b}{6x^2}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]/x^4,x]
```

output
```
-1/6*b/x^2 - ArcTanh[Tanh[a + b*x]]/(3*x^3)
```

3.43.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.43.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result
parallelrisch	$-\frac{bx+2\,\operatorname{arctanh}(\tanh(bx+a))}{6x^{3}}$
default	$-\frac{b}{6x^{2}}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{3x^{3}}$
parts	$-\frac{b}{6x^{2}}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{3x^{3}}$
risch	$-\frac{\ln\left(\mathrm{e}^{bx+a}\right)}{3x^{3}}-\frac{2bx+2i\pi\,\operatorname{csgn}\left(i\mathrm{e}^{bx+a}\right)\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{2}+i\pi\,\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}-i\pi\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{3}-i\pi\,\operatorname{csg}}{}$

input
```
int(arctanh(tanh(b*x+a))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x+2*arctanh(tanh(b*x+a)))/x^3
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^4}\,dx=-\frac{3\,bx+2\,a}{6\,x^3}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^4,x, algorithm="fricas")
```

output
```
-1/6*(3*b*x + 2*a)/x^3
```

3.43.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^4}\,dx=-\frac{b}{6x^2}-\frac{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{3x^3}$$

input
```
integrate(atanh(tanh(b*x+a))/x**4,x)
```

output
```
-b/(6*x**2) - atanh(tanh(a + b*x))/(3*x**3)
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^4}\,dx = -\frac{b}{6\,x^2} - \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{3\,x^3}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^4,x, algorithm="maxima")
```

output
```
-1/6*b/x^2 - 1/3*arctanh(tanh(b*x + a))/x^3
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^4}\,dx = -\frac{3\,bx+2\,a}{6\,x^3}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^4,x, algorithm="giac")
```

output
```
-1/6*(3*b*x + 2*a)/x^3
```

3.43.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^4}\,dx = -\frac{\operatorname{atanh}(\tanh(a+b\,x))}{3\,x^3} - \frac{b}{6\,x^2}$$

input
```
int(atanh(tanh(a + b*x))/x^4,x)
```

output
```
- atanh(tanh(a + b*x))/(3*x^3) - b/(6*x^2)
```

3.44 $\int x^m \textbf{arctanh}(\tanh(a+bx))^2\,dx$

3.44.1 Optimal result

Integrand size = 13, antiderivative size = 71

$$\int x^m \text{arctanh}(\tanh(a+bx))^2\,dx = \frac{2b^2x^{3+m}}{6+11m+6m^2+m^3} - \frac{2bx^{2+m}\text{arctanh}(\tanh(a+bx))}{2+3m+m^2} + \frac{x^{1+m}\text{arctanh}(\tanh(a+bx))^2}{1+m}$$

output
```
2*b^2*x^(3+m)/(m^3+6*m^2+11*m+6)-2*b*x^(2+m)*arctanh(tanh(b*x+a))/(m^2+3*m
+2)+x^(1+m)*arctanh(tanh(b*x+a))^2/(1+m)
```

3.44.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int x^m \text{arctanh}(\tanh(a+bx))^2\,dx = \frac{x^{1+m}(2b^2x^2 - 2b(3+m)x\text{arctanh}(\tanh(a+bx)) + (6+5m+m^2)\,\text{arctanh}(\tanh(a+bx))^2)}{(1+m)(2+m)(3+m)}$$

input
```
Integrate[x^m*ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(x^(1 + m)*(2*b^2*x^2 - 2*b*(3 + m)*x*ArcTanh[Tanh[a + b*x]] + (6 + 5*m +
m^2)*ArcTanh[Tanh[a + b*x]]^2))/((1 + m)*(2 + m)*(3 + m))
```

3.44.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{arctanh}(\tanh(a+bx))^2 \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^{m+1}\text{arctanh}(\tanh(a+bx))^2}{m+1}-\frac{2b\int x^{m+1}\text{arctanh}(\tanh(a+bx))dx}{m+1}$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^{m+1}\text{arctanh}(\tanh(a+bx))^2}{m+1}-\frac{2b\left(\frac{x^{m+2}\text{arctanh}(\tanh(a+bx))}{m+2}-\frac{b\int x^{m+2}dx}{m+2}\right)}{m+1}$$

$$\Big\downarrow \text{15}$$

$$\frac{x^{m+1}\text{arctanh}(\tanh(a+bx))^2}{m+1}-\frac{2b\left(\frac{x^{m+2}\text{arctanh}(\tanh(a+bx))}{m+2}-\frac{bx^{m+3}}{(m+2)(m+3)}\right)}{m+1}$$

input
```
Int[x^m*ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(x^(1 + m)*ArcTanh[Tanh[a + b*x]]^2)/(1 + m) - (2*b*(-((b*x^(3 + m))/((2 +
 m)*(3 + m))) + (x^(2 + m)*ArcTanh[Tanh[a + b*x]])/(2 + m)))/(1 + m)
```

3.44.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.44.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.38

method	result
default	$\frac{b^2 x^3 e^{m \ln(x)}}{3+m} + \frac{\left(a^2+2a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2\right) x\, e^{m \ln(x)}}{1+m} + \frac{2b(\operatorname{arctanh}(\tanh(bx\cdots}{2+\cdots}$
parallelrisch	$-\frac{-6 \operatorname{arctanh}(\tanh(bx+a))^2 x^m x + 2x^2 x^m \operatorname{arctanh}(\tanh(bx+a)) b m - 2b^2 x^m x^3 + 6b \operatorname{arctanh}(\tanh(bx+a)) x^2 x^m - x\, x^m \operatorname{arctanh}\cdots}{(1+m)(2+m)(3+m)}$
risch	Expression too large to display

input
```
int(x^m*arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
b^2/(3+m)*x^3*exp(m*ln(x))+(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(
tanh(b*x+a))-b*x-a)^2)/(1+m)*x*exp(m*ln(x))+2*b*(arctanh(tanh(b*x+a))-b*x)
/(2+m)*x^2*exp(m*ln(x))
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(71) = 142.

Time = 0.25 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.27

$$\int x^m \operatorname{arctanh}(\tanh(a+bx))^2 \, dx$$
$$= \frac{((b^2m^2 + 3\,b^2m + 2\,b^2)x^3 + 2\,(abm^2 + 4\,abm + 3\,ab)x^2 + (a^2m^2 + 5\,a^2m + 6\,a^2)x)\cosh(m\log(x)) + ((b^2\cdots}{m^3 + 6\,m^2 + 11\,\cdots}$$

input
```
integrate(x^m*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
(((b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a
^2*m^2 + 5*a^2*m + 6*a^2)*x)*cosh(m*log(x)) + ((b^2*m^2 + 3*b^2*m + 2*b^2)
*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^2*m^2 + 5*a^2*m + 6*a^2)*x)*
sinh(m*log(x)))/(m^3 + 6*m^2 + 11*m + 6)
```

3.44.6 Sympy [F]

$$\int x^m \text{arctanh}(\tanh(a+bx))^2\,dx$$

$$= \begin{cases} b^2\log(x) - \frac{b\,\text{atanh}(\tanh(a+bx))}{x} - \frac{\text{atanh}^2(\tanh(a+bx))}{2x^2} \\ \int \frac{\text{atanh}^2(\tanh(a+bx))}{x^2}\,dx \\ \int \frac{\text{atanh}^2(\tanh(a+bx))}{x}\,dx \\ \frac{2b^2x^3x^m}{m^3+6m^2+11m+6} - \frac{2bmx^2x^m\,\text{atanh}(\tanh(a+bx))}{m^3+6m^2+11m+6} - \frac{6bx^2x^m\,\text{atanh}(\tanh(a+bx))}{m^3+6m^2+11m+6} + \frac{m^2xx^m\,\text{atanh}^2(\tanh(a+bx))}{m^3+6m^2+11m+6} + \frac{5mxx^m\,\text{atanh}^2}{m^3+6m^2} \end{cases}$$

input
```
integrate(x**m*atanh(tanh(b*x+a))**2,x)
```

output
```
Piecewise((b**2*log(x) - b*atanh(tanh(a + b*x))/x - atanh(tanh(a + b*x))**
2/(2*x**2), Eq(m, -3)), (Integral(atanh(tanh(a + b*x))**2/x**2, x), Eq(m,
-2)), (Integral(atanh(tanh(a + b*x))**2/x, x), Eq(m, -1)), (2*b**2*x**3*x*
*m/(m**3 + 6*m**2 + 11*m + 6) - 2*b*m*x**2*x**m*atanh(tanh(a + b*x))/(m**3
 + 6*m**2 + 11*m + 6) - 6*b*x**2*x**m*atanh(tanh(a + b*x))/(m**3 + 6*m**2
+ 11*m + 6) + m**2*x*x**m*atanh(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m +
6) + 5*m*x*x**m*atanh(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6) + 6*x*x
**m*atanh(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6), True))
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int x^m \text{arctanh}(\tanh(a+bx))^2\,dx = \frac{2\,b^2x^3x^m}{(m+3)(m+2)(m+1)} - \frac{2\,bx^2x^m\,\text{artanh}\left(\tanh\left(bx+a\right)\right)}{(m+2)(m+1)} + \frac{x^{m+1}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2}{m+1}$$

input
```
integrate(x^m*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
2*b^2*x^3*x^m/((m + 3)*(m + 2)*(m + 1)) - 2*b*x^2*x^m*arctanh(tanh(b*x + a
))/((m + 2)*(m + 1)) + x^(m + 1)*arctanh(tanh(b*x + a))^2/(m + 1)
```

3.44.8 Giac [F]

$$\int x^m \text{arctanh}(\tanh(a+bx))^2\,dx = \int x^m\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2\,dx$$

input
```
integrate(x^m*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
integrate(x^m*arctanh(tanh(b*x + a))^2, x)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.86

$$\int x^m \text{arctanh}(\tanh(a+bx))^2\,dx = \frac{4\,b^2\,x^m\,x^3\,\left(m^2+3\,m+2\right)}{4\,m^3+24\,m^2+44\,m+24} + \frac{x\,x^m\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2\,\left(m^2+5\,m+6\right)}{4\,m^3+24\,m^2+44\,m+24} - \frac{4\,b\,x^m\,x^2\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)\,\left(m^2+4\,m+3\right)}{4\,m^3+24\,m^2+44\,m+24}$$

input
```
int(x^m*atanh(tanh(a + b*x))^2,x)
```

output
```
(4*b^2*x^m*x^3*(3*m + m^2 + 2))/(44*m + 24*m^2 + 4*m^3 + 24) + (x*x^m*(log
(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^2*(5*m + m^2 + 6))/(44*m + 24*m^2 + 4*m^3 + 24) - (4
*b*x^m*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)*(4*m + m^2 + 3))/(44*m + 24*m^2 + 4*m
^3 + 24)
```

3.44. $\int x^m \text{arctanh}(\tanh(a+bx))^2\,dx$

3.45 $\int x^3 \mathbf{arctanh}(\tanh(a+bx))^2\,dx$

3.45.1 Optimal result

Integrand size = 13, antiderivative size = 42

$$\int x^3 \operatorname{arctanh}(\tanh(a+bx))^2\,dx = \frac{b^2x^6}{60} - \frac{1}{10}bx^5 \operatorname{arctanh}(\tanh(a+bx)) + \frac{1}{4}x^4 \operatorname{arctanh}(\tanh(a+bx))^2$$

output
```
1/60*b^2*x^6-1/10*b*x^5*arctanh(tanh(b*x+a))+1/4*x^4*arctanh(tanh(b*x+a))^
2
```

3.45.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int x^3 \operatorname{arctanh}(\tanh(a+bx))^2\,dx = \frac{1}{60}x^4\left(b^2x^2 - 6bx\operatorname{arctanh}(\tanh(a+bx)) + 15\operatorname{arctanh}(\tanh(a+bx))^2\right)$$

input
```
Integrate[x^3*ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(x^4*(b^2*x^2 - 6*b*x*ArcTanh[Tanh[a + b*x]] + 15*ArcTanh[Tanh[a + b*x]]^2
))/60
```

3.45.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{arctanh}(\tanh(a+bx))^2\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{4}x^4\text{arctanh}(\tanh(a+bx))^2-\frac{1}{2}b\int x^4\text{arctanh}(\tanh(a+bx))dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{4}x^4\text{arctanh}(\tanh(a+bx))^2-\frac{1}{2}b\left(\frac{1}{5}x^5\text{arctanh}(\tanh(a+bx))-\frac{b\int x^5dx}{5}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{4}x^4\text{arctanh}(\tanh(a+bx))^2-\frac{1}{2}b\left(\frac{1}{5}x^5\text{arctanh}(\tanh(a+bx))-\frac{bx^6}{30}\right)$$

input
```
Int[x^3*ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(x^4*ArcTanh[Tanh[a + b*x]]^2)/4 - (b*(-1/30*(b*x^6) + (x^5*ArcTanh[Tanh[a
 + b*x]])/5))/2
```

3.45.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.45.4 Maple [A] (verified)

Time = 24.74 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
parallelrisch	$\frac{b^{2} x^{6}}{60}-\frac{b\,x^{5} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)}{10}+\frac{x^{4} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{2}}{4}$	37
default	$\frac{x^{4} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{2}}{4}-\frac{b \left(\frac{x^{5} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)}{5}-\frac{b\,x^{6}}{30}\right)}{2}$	38
parts	$\frac{x^{4} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{2}}{4}-\frac{b \left(\frac{x^{5} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)}{5}-\frac{b\,x^{6}}{30}\right)}{2}$	38
risch	Expression too large to display	2083

input
```
int(x^3*arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
1/60*b^2*x^6-1/10*b*x^5*arctanh(tanh(b*x+a))+1/4*x^4*arctanh(tanh(b*x+a))^
2
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int x^3 \text{arctanh}(\tanh(a+bx))^2\,dx = \frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

3.45.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int x^3 \text{arctanh}(\tanh(a+bx))^2\,dx = \frac{b^2x^6}{60} - \frac{bx^5\,\text{atanh}\,(\tanh\,(a+bx))}{10} + \frac{x^4\,\text{atanh}^2\,(\tanh\,(a+bx))}{4}$$

input
```
integrate(x**3*atanh(tanh(b*x+a))**2,x)
```

output
```
b**2*x**6/60 - b*x**5*atanh(tanh(a + b*x))/10 + x**4*atanh(tanh(a + b*x))*
*2/4
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int x^3 \text{arctanh}(\tanh(a+bx))^2\,dx = \frac{1}{60}\,b^2x^6 - \frac{1}{10}\,bx^5\,\text{artanh}\,(\tanh\,(bx+a)) + \frac{1}{4}\,x^4\,\text{artanh}\,(\tanh\,(bx+a))^2$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/60*b^2*x^6 - 1/10*b*x^5*arctanh(tanh(b*x + a)) + 1/4*x^4*arctanh(tanh(b*
x + a))^2
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int x^3 \text{arctanh}(\tanh(a+bx))^2 \, dx = \frac{1}{6} b^2 x^6 + \frac{2}{5} abx^5 + \frac{1}{4} a^2 x^4$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int x^3 \text{arctanh}(\tanh(a+bx))^2 \, dx = \frac{b^2 x^6}{60} - \frac{b x^5 \operatorname{atanh}(\tanh(a+bx))}{10} + \frac{x^4 \operatorname{atanh}(\tanh(a+bx))^2}{4}$$

input
```
int(x^3*atanh(tanh(a + b*x))^2,x)
```

output
```
(x^4*atanh(tanh(a + b*x))^2)/4 + (b^2*x^6)/60 - (b*x^5*atanh(tanh(a + b*x)
))/10
```

3.46 $\int x^2 \mathbf{arctanh}(\tanh(a+bx))^2\,dx$

3.46.1 Optimal result

Integrand size = 13, antiderivative size = 42

$$\int x^2 \text{arctanh}(\tanh(a+bx))^2\,dx = \frac{b^2x^5}{30} - \frac{1}{6}bx^4\text{arctanh}(\tanh(a+bx)) + \frac{1}{3}x^3\text{arctanh}(\tanh(a+bx))^2$$

output
```
1/30*b^2*x^5-1/6*b*x^4*arctanh(tanh(b*x+a))+1/3*x^3*arctanh(tanh(b*x+a))^2
```

3.46.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int x^2 \text{arctanh}(\tanh(a+bx))^2\,dx = \frac{1}{30}x^3\left(b^2x^2 - 5bx\text{arctanh}(\tanh(a+bx)) + 10\text{arctanh}(\tanh(a+bx))^2\right)$$

input
```
Integrate[x^2*ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(x^3*(b^2*x^2 - 5*b*x*ArcTanh[Tanh[a + b*x]] + 10*ArcTanh[Tanh[a + b*x]]^2
))/30
```

3.46.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arctanh}(\tanh(a+bx))^2 \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{3}x^3\text{arctanh}(\tanh(a+bx))^2-\frac{2}{3}b\int x^3\text{arctanh}(\tanh(a+bx))dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{3}x^3\text{arctanh}(\tanh(a+bx))^2-\frac{2}{3}b\left(\frac{1}{4}x^4\text{arctanh}(\tanh(a+bx))-\frac{b\int x^4dx}{4}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{3}x^3\text{arctanh}(\tanh(a+bx))^2-\frac{2}{3}b\left(\frac{1}{4}x^4\text{arctanh}(\tanh(a+bx))-\frac{bx^5}{20}\right)$$

input
```
Int[x^2*ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(x^3*ArcTanh[Tanh[a + b*x]]^2)/3 - (2*b*(-1/20*(b*x^5) + (x^4*ArcTanh[Tanh
[a + b*x]])/4))/3
```

3.46.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.46.4 Maple [A] (verified)

Time = 22.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
parallelrisch	$\frac{b^2x^5}{30}-\frac{b\,x^4\operatorname{arctanh}(\tanh(bx+a))}{6}+\frac{x^3\operatorname{arctanh}(\tanh(bx+a))^2}{3}$	37
default	$\frac{x^3\operatorname{arctanh}(\tanh(bx+a))^2}{3}-\frac{2b\left(\frac{x^4\operatorname{arctanh}(\tanh(bx+a))}{4}-\frac{b\,x^5}{20}\right)}{3}$	38
parts	$\frac{x^3\operatorname{arctanh}(\tanh(bx+a))^2}{3}-\frac{2b\left(\frac{x^4\operatorname{arctanh}(\tanh(bx+a))}{4}-\frac{b\,x^5}{20}\right)}{3}$	38
risch	Expression too large to display	2083

input
```
int(x^2*arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
1/30*b^2*x^5-1/6*b*x^4*arctanh(tanh(b*x+a))+1/3*x^3*arctanh(tanh(b*x+a))^2
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int x^2 \text{arctanh}(\tanh(a+bx))^2 \, dx = \frac{1}{5} b^2 x^5 + \frac{1}{2} a b x^4 + \frac{1}{3} a^2 x^3$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

3.46.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int x^2 \text{arctanh}(\tanh(a+bx))^2 \, dx = \frac{b^2 x^5}{30} - \frac{b x^4 \operatorname{atanh}(\tanh(a+bx))}{6} + \frac{x^3 \operatorname{atanh}^2(\tanh(a+bx))}{3}$$

input
```
integrate(x**2*atanh(tanh(b*x+a))**2,x)
```

output
```
b**2*x**5/30 - b*x**4*atanh(tanh(a + b*x))/6 + x**3*atanh(tanh(a + b*x))**
2/3
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int x^2 \text{arctanh}(\tanh(a+bx))^2 \, dx = \frac{1}{30} b^2 x^5 - \frac{1}{6} b x^4 \operatorname{artanh}(\tanh(bx+a)) + \frac{1}{3} x^3 \operatorname{artanh}(\tanh(bx+a))^2$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/30*b^2*x^5 - 1/6*b*x^4*arctanh(tanh(b*x + a)) + 1/3*x^3*arctanh(tanh(b*x
 + a))^2
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int x^2 \text{arctanh}(\tanh(a+bx))^2 \, dx = \frac{1}{5} b^2 x^5 + \frac{1}{2} abx^4 + \frac{1}{3} a^2 x^3$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

3.46.9 Mupad [B] (verification not implemented)

Time = 3.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int x^2 \text{arctanh}(\tanh(a+bx))^2 \, dx = \frac{b^2 x^5}{30} - \frac{b x^4 \operatorname{atanh}(\tanh(a+b x))}{6} + \frac{x^3 \operatorname{atanh}(\tanh(a+b x))^2}{3}$$

input
```
int(x^2*atanh(tanh(a + b*x))^2,x)
```

output
```
(x^3*atanh(tanh(a + b*x))^2)/3 + (b^2*x^5)/30 - (b*x^4*atanh(tanh(a + b*x)
))/6
```

3.47 $\int x\mathbf{arctanh}(\tanh(a+bx))^2\,dx$

3.47.1 Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x\operatorname{arctanh}(\tanh(a+bx))^2\,dx=\frac{x\operatorname{arctanh}(\tanh(a+bx))^3}{3b}-\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{12b^2}$$

output
```
1/3*x*arctanh(tanh(b*x+a))^3/b-1/12*arctanh(tanh(b*x+a))^4/b^2
```

3.47.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 74 vs. 2(34) = 68.

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.18

$$\int x\operatorname{arctanh}(\tanh(a+bx))^2\,dx$$
$$=\frac{(a+bx)\left(-((3a-bx)(a+bx)^2)+4(2a^2+abx-b^2x^2)\operatorname{arctanh}(\tanh(a+bx))-6(a-bx)\operatorname{arctanh}(\tanh(\right.}{12b^2}$$

input
```
Integrate[x*ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
((a + b*x)*(-((3*a - b*x)*(a + b*x)^2) + 4*(2*a^2 + a*b*x - b^2*x^2)*ArcTa
nh[Tanh[a + b*x]] - 6*(a - b*x)*ArcTanh[Tanh[a + b*x]]^2))/(12*b^2)
```

3.47.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(\tanh(a+bx))^2\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x\text{arctanh}(\tanh(a+bx))^3}{3b}-\frac{\int \text{arctanh}(\tanh(a+bx))^3dx}{3b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{x\text{arctanh}(\tanh(a+bx))^3}{3b}-\frac{\int \text{arctanh}(\tanh(a+bx))^3d\text{arctanh}(\tanh(a+bx))}{3b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{x\text{arctanh}(\tanh(a+bx))^3}{3b}-\frac{\text{arctanh}(\tanh(a+bx))^4}{12b^2}$$

input
```
Int[x*ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(x*ArcTanh[Tanh[a + b*x]]^3)/(3*b) - ArcTanh[Tanh[a + b*x]]^4/(12*b^2)
```

3.47.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.47.4 Maple [A] (verified)

Time = 21.42 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$-\frac{x^3 \operatorname{arctanh}(\tanh(bx+a))b}{3}+\frac{x^2 \operatorname{arctanh}(\tanh(bx+a))^2}{2}+\frac{b^2x^4}{12}$	37
default	$\frac{x^2 \operatorname{arctanh}(\tanh(bx+a))^2}{2}-b\left(-\frac{b\,x^4}{12}+\frac{x^3 \operatorname{arctanh}(\tanh(bx+a))}{3}\right)$	38
parts	$\frac{x^2 \operatorname{arctanh}(\tanh(bx+a))^2}{2}-b\left(-\frac{b\,x^4}{12}+\frac{x^3 \operatorname{arctanh}(\tanh(bx+a))}{3}\right)$	38
risch	Expression too large to display	2083

input
```
int(x*arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3*arctanh(tanh(b*x+a))*b+1/2*x^2*arctanh(tanh(b*x+a))^2+1/12*b^2*x^
4
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int x\operatorname{arctanh}(\tanh(a+bx))^2\,dx=\frac{1}{4}\,b^2x^4+\frac{2}{3}\,abx^3+\frac{1}{2}\,a^2x^2$$

input
```
integrate(x*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

3.47.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int x\mathrm{arctanh}(\tanh(a+bx))^2\,dx = \begin{cases} \frac{x\,\mathrm{atanh}^3\left(\tanh\left(a+bx\right)\right)}{3b} - \frac{\mathrm{atanh}^4\left(\tanh\left(a+bx\right)\right)}{12b^2} & \text{for } b\neq 0 \\ \frac{x^2\,\mathrm{atanh}^2\left(\tanh\left(a\right)\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*atanh(tanh(b*x+a))**2,x)
```

output
```
Piecewise((x*atanh(tanh(a + b*x))**3/(3*b) - atanh(tanh(a + b*x))**4/(12*b
**2), Ne(b, 0)), (x**2*atanh(tanh(a))**2/2, True))
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int x\mathrm{arctanh}(\tanh(a+bx))^2\,dx = \frac{1}{12}\,b^2x^4 - \frac{1}{3}\,bx^3\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right) + \frac{1}{2}\,x^2\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

input
```
integrate(x*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/12*b^2*x^4 - 1/3*b*x^3*arctanh(tanh(b*x + a)) + 1/2*x^2*arctanh(tanh(b*x
 + a))^2
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int x\mathrm{arctanh}(\tanh(a+bx))^2\,dx = \frac{1}{4}\,b^2x^4 + \frac{2}{3}\,abx^3 + \frac{1}{2}\,a^2x^2$$

input
```
integrate(x*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

3.47.9 Mupad [B] (verification not implemented)

Time = 3.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int x\text{arctanh}(\tanh(a+bx))^2\,dx=\frac{b^2\,x^4}{12}-\frac{b\,x^3\,\text{atanh}(\tanh(a+b\,x))}{3}+\frac{x^2\,\text{atanh}(\tanh(a+b\,x))^2}{2}$$

input
```
int(x*atanh(tanh(a + b*x))^2,x)
```

output
```
(x^2*atanh(tanh(a + b*x))^2)/2 + (b^2*x^4)/12 - (b*x^3*atanh(tanh(a + b*x)
))/3
```

3.48 $\int \operatorname{arctanh}(\tanh(a+bx))^2\,dx$

3.48.1 Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \operatorname{arctanh}(\tanh(a+bx))^2\,dx = \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{3b}$$

output
```
1/3*arctanh(tanh(b*x+a))^3/b
```

3.48.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \operatorname{arctanh}(\tanh(a+bx))^2\,dx = \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{3b}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
ArcTanh[Tanh[a + b*x]]^3/(3*b)
```

3.48.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(\tanh(a+bx))^2\,dx$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \operatorname{arctanh}(\tanh(a+bx))^2 d\operatorname{arctanh}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\operatorname{arctanh}(\tanh(a+bx))^3}{3b}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
ArcTanh[Tanh[a + b*x]]^3/(3*b)
```

3.48.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.48.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{3b}$	15
default	$\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{3b}$	15
parallelrisch	$\frac{b^2x^3}{3}-x^2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)b+x\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2$	34
risch	Expression too large to display	6270

input
```
int(arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*arctanh(tanh(b*x+a))^3/b
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \operatorname{arctanh}(\tanh(a+bx))^2\,dx = \frac{1}{3}\,b^2x^3 + abx^2 + a^2x$$

input
```
integrate(arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

3.48.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \operatorname{arctanh}(\tanh(a+bx))^2\,dx = \begin{cases} \frac{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{3b} & \text{for } b\neq 0 \\ x\operatorname{atanh}^2\left(\tanh\left(a\right)\right) & \text{otherwise} \end{cases}$$

input
```
integrate(atanh(tanh(b*x+a))**2,x)
```

output
```
Piecewise((atanh(tanh(a + b*x))**3/(3*b), Ne(b, 0)), (x*atanh(tanh(a))**2,
 True))
```

3.48.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \operatorname{arctanh}(\tanh(a+bx))^2\,dx = \frac{1}{3}\,b^2x^3 - bx^2\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right) + x\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

input
```
integrate(arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3 - b*x^2*arctanh(tanh(b*x + a)) + x*arctanh(tanh(b*x + a))^2
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \operatorname{arctanh}(\tanh(a+bx))^2\,dx = \frac{1}{3}\,b^2x^3 + abx^2 + a^2x$$

input
```
integrate(arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

3.48.9 Mupad [B] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \operatorname{arctanh}(\tanh(a+bx))^2\,dx = \frac{b^2\,x^3}{3} - b\,x^2\,\operatorname{atanh}(\tanh(a+b\,x)) + x\,\operatorname{atanh}(\tanh(a+b\,x))^2$$

input
```
int(atanh(tanh(a + b*x))^2,x)
```

output
```
x*atanh(tanh(a + b*x))^2 + (b^2*x^3)/3 - b*x^2*atanh(tanh(a + b*x))
```

3.49 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x} dx$

3.49.1 Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x} dx = -bx(bx-\operatorname{arctanh}(\tanh(a+bx)))+\frac{1}{2}\operatorname{arctanh}(\tanh(a+bx))^2 + (bx-\operatorname{arctanh}(\tanh(a+bx)))^2\log(x)$$

output
```
-b*x*(b*x-arctanh(tanh(b*x+a)))+1/2*arctanh(tanh(b*x+a))^2+(b*x-arctanh(ta
nh(b*x+a)))^2*ln(x)
```

3.49.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x} dx = \frac{1}{2}(a+bx)^2-(a+bx)(a+2bx-2\operatorname{arctanh}(\tanh(a+bx))) + (-bx+\operatorname{arctanh}(\tanh(a+bx)))^2\log(bx)$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^2/x,x]
```

output
```
(a + b*x)^2/2 - (a + b*x)*(a + 2*b*x - 2*ArcTanh[Tanh[a + b*x]]) + (-(b*x)
 + ArcTanh[Tanh[a + b*x]])^2*Log[b*x]
```

3.49.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2590, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x} \, dx$$

$$\Big\downarrow \text{ 2590}$$

$$\frac{1}{2}\text{arctanh}(\tanh(a+bx))^2-(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{\text{arctanh}(\tanh(a+bx))}{x}dx$$

$$\Big\downarrow \text{ 2589}$$

$$\frac{1}{2}\text{arctanh}(\tanh(a+bx))^2-(bx-\text{arctanh}(\tanh(a+bx)))\left(bx-(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{x}dx\right)$$

$$\Big\downarrow \text{ 14}$$

$$\frac{1}{2}\text{arctanh}(\tanh(a+bx))^2-(bx-\text{arctanh}(\tanh(a+bx)))(bx-\log(x)(bx-\text{arctanh}(\tanh(a+bx))))$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^2/x,x]
```

output
```
ArcTanh[Tanh[a + b*x]]^2/2 - (b*x - ArcTanh[Tanh[a + b*x]])*(b*x - (b*x -
ArcTanh[Tanh[a + b*x]])*Log[x])
```

3.49.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

3.49.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

method	result
default	$\ln(x)\operatorname{arctanh}(\tanh(bx+a))^2 - 2b\left(b\left(\frac{x^2\ln(x)}{2} - \frac{x^2}{4}\right) + a(x\ln(x) - x) + (\operatorname{arctanh}(\tanh(bx+a\right.\right.$
parts	$\ln(x)\operatorname{arctanh}(\tanh(bx+a))^2 - 2b\left(b\left(\frac{x^2\ln(x)}{2} - \frac{x^2}{4}\right) + a(x\ln(x) - x) + (\operatorname{arctanh}(\tanh(bx+a\right.\right.$
risch	$\ln(x)\ln\left(e^{bx+a}\right)^2 + b^2\ln(x)x^2 - \frac{3b^2x^2}{2} - 2b\ln\left(e^{bx+a}\right)\ln(x)x + 2b\ln\left(e^{bx+a}\right)x - \frac{\pi^2\left(\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right.\right.}{}$

input
```
int(arctanh(tanh(b*x+a))^2/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arctanh(tanh(b*x+a))^2-2*b*(b*(1/2*x^2*ln(x)-1/4*x^2)+a*(x*ln(x)-x)+
(arctanh(tanh(b*x+a))-b*x-a)*(x*ln(x)-x))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.41

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x}\,dx = \frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log(x)$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x,x, algorithm="fricas")
```

output
```
1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

3.49.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x}\,dx = \int \frac{\operatorname{atanh}^2(\tanh(a+bx))}{x}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**2/x,x)
```

output
```
Integral(atanh(tanh(a + b*x))**2/x, x)
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.41

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x}\,dx = \frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log(x)$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.43

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x}\,dx = \frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log(|x|)$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x,x, algorithm="giac")
```

output
```
1/2*b^2*x^2 + 2*a*b*x + a^2*log(abs(x))
```

3.49.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.73

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x}\,dx = \ln(x)\left(\frac{\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{4} - a\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right) + a^2\right) + \frac{b^2\,x^2}{2} - b\,x\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)$$

input

```
int(atanh(tanh(a + b*x))^2/x,x)
```

output

```
log(x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/4 - a*(2*a - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x) + a^2) + (b^2*x^2)/2 - b*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
```

3.50 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^2}\,dx$

3.50.1 Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^2}\,dx = 2b^2x-\frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x} - 2b(bx-\operatorname{arctanh}(\tanh(a+bx)))\log(x)$$

output
```
2*b^2*x-arctanh(tanh(b*x+a))^2/x-2*b*(b*x-arctanh(tanh(b*x+a)))*ln(x)
```

3.50.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^2}\,dx = -\frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x} - 2b^2x\log(x) + 2b\operatorname{arctanh}(\tanh(a+bx))(1+\log(x))$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^2/x^2,x]
```

output
```
-(ArcTanh[Tanh[a + b*x]]^2/x) - 2*b^2*x*Log[x] + 2*b*ArcTanh[Tanh[a + b*x]
]*(1 + Log[x])
```

3.50.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2599, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^2} \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$2b \int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x} dx - \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x}$$

$$\Big\downarrow \text{ 2589}$$

$$2b\left(bx - (bx - \operatorname{arctanh}(\tanh(a+bx))) \int \frac{1}{x} dx\right) - \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x}$$

$$\Big\downarrow \text{ 14}$$

$$2b(bx - \log(x)(bx - \operatorname{arctanh}(\tanh(a+bx)))) - \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^2/x^2,x]
```

output
```
-(ArcTanh[Tanh[a + b*x]]^2/x) + 2*b*(b*x - (b*x - ArcTanh[Tanh[a + b*x]])*
Log[x])
```

3.50.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.50.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

method	result
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{x}+2b(\ln(x)\operatorname{arctanh}(\tanh(bx+a))-b(x\ln(x)-x))$
parts	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{x}+2b(\ln(x)\operatorname{arctanh}(\tanh(bx+a))-b(x\ln(x)-x))$
risch	$-\frac{\ln(\mathrm{e}^{bx+a})^2}{x}-2\ln(x)\,x\,b^2+2\ln(x)\ln(\mathrm{e}^{bx+a})\,b+2b^2x+\frac{\pi^2\left(\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+\cdots}\right.\right.}{\cdots}$

input
```
int(arctanh(tanh(b*x+a))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arctanh(tanh(b*x+a))^2/x+2*b*(ln(x)*arctanh(tanh(b*x+a))-b*(x*ln(x)-x))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.62

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^2}\,dx=\frac{b^2x^2+2\,abx\log(x)-a^2}{x}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^2,x, algorithm="fricas")
```

output
```
(b^2*x^2 + 2*a*b*x*log(x) - a^2)/x
```

3.50.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^2}\,dx = \int \frac{\operatorname{atanh}^2(\tanh(a+bx))}{x^2}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**2/x**2,x)
```

output
```
Integral(atanh(tanh(a + b*x))**2/x**2, x)
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^2}\,dx = 2\,b\,\operatorname{artanh}(\tanh(bx+a))\log(x) - 2\left(b\left(x+\frac{a}{b}\right)\log(x) - b\left(x+\frac{a\log(x)}{b}\right)\right)b - \frac{\operatorname{artanh}(\tanh(bx+a))^2}{x}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^2,x, algorithm="maxima")
```

output
```
2*b*arctanh(tanh(b*x + a))*log(x) - 2*(b*(x + a/b)*log(x) - b*(x + a*log(x
)/b))*b - arctanh(tanh(b*x + a))^2/x
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.54

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^2}\,dx = b^2x + 2\,ab\log(|x|) - \frac{a^2}{x}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^2,x, algorithm="giac")
```

output
```
b^2*x + 2*a*b*log(abs(x)) - a^2/x
```

3.50.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 198, normalized size of antiderivative = 5.08

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^2}\,dx = b\,\ln\left(\frac{e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{4\,x} - b\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{4\,x} + b\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln(x) - 2\,b^2\,x\,\ln(x) - b\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln(x) + \frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2\,x}$$

input
```
int(atanh(tanh(a + b*x))^2/x^2,x)
```

output
```
b*log(exp(2*b*x)/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1))^2/(4*x) - b*log(1/(exp(2*a)*exp(2*b*x) + 1)) - lo
g(1/(exp(2*a)*exp(2*b*x) + 1))^2/(4*x) + b*log((exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1))*log(x) - 2*b^2*x*log(x) - b*log(1/(exp(2*a)*exp(2*b*
x) + 1))*log(x) + (log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)))/(2*x)
```

3.51 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^3}\,dx$

3.51.1 Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^3}\,dx = -\frac{b\operatorname{arctanh}(\tanh(a+bx))}{x} - \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{2x^2} + b^2\log(x)$$

output
```
-b*arctanh(tanh(b*x+a))/x-1/2*arctanh(tanh(b*x+a))^2/x^2+b^2*ln(x)
```

3.51.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^3}\,dx = -\frac{2bx\operatorname{arctanh}(\tanh(a+bx)) + \operatorname{arctanh}(\tanh(a+bx))^2 - b^2x^2(3+2\log(x))}{2x^2}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^2/x^3,x]
```

output
```
-1/2*(2*b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2 - b^2*x^2*(3
 + 2*Log[x]))/x^2
```

3.51.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2599, 2599, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^3} \, dx$$

$$\Big\downarrow \text{2599}$$

$$b \int \frac{\text{arctanh}(\tanh(a+bx))}{x^2}dx - \frac{\text{arctanh}(\tanh(a+bx))^2}{2x^2}$$

$$\Big\downarrow \text{2599}$$

$$b\left(b\int \frac{1}{x}dx - \frac{\text{arctanh}(\tanh(a+bx))}{x}\right) - \frac{\text{arctanh}(\tanh(a+bx))^2}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$b\left(b\log(x) - \frac{\text{arctanh}(\tanh(a+bx))}{x}\right) - \frac{\text{arctanh}(\tanh(a+bx))^2}{2x^2}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^2/x^3,x]
```

output
```
-1/2*ArcTanh[Tanh[a + b*x]]^2/x^2 + b*(-(ArcTanh[Tanh[a + b*x]]/x) + b*Log
[x])
```

3.51.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.51.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2x^2}+b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{x}+b\ln(x)\right)$	35
parts	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2x^2}+b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{x}+b\ln(x)\right)$	35
parallelrisch	$\frac{2b^2\ln(x)x^2-2bx\operatorname{arctanh}(\tanh(bx+a))-\operatorname{arctanh}(\tanh(bx+a))^2}{2x^2}$	39
risch	Expression too large to display	1974

input
```
int(arctanh(tanh(b*x+a))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(tanh(b*x+a))^2/x^2+b*(-arctanh(tanh(b*x+a))/x+b*ln(x))
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^3}\,dx=\frac{2\,b^2x^2\log(x)-4\,abx-a^2}{2\,x^2}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)/x^2
```

3.51.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^3}\,dx=b^2\log(x)-\frac{b\operatorname{atanh}(\tanh(a+bx))}{x}-\frac{\operatorname{atanh}^2(\tanh(a+bx))}{2x^2}$$

input
```
integrate(atanh(tanh(b*x+a))**2/x**3,x)
```

output
```
b**2*log(x) - b*atanh(tanh(a + b*x))/x - atanh(tanh(a + b*x))**2/(2*x**2)
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^3}\,dx$$
$$= b^2\log(x) - \frac{b\operatorname{artanh}(\tanh(bx+a))}{x} - \frac{\operatorname{artanh}(\tanh(bx+a))^2}{2x^2}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^3,x, algorithm="maxima")
```

output
```
b^2*log(x) - b*arctanh(tanh(b*x + a))/x - 1/2*arctanh(tanh(b*x + a))^2/x^2
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^3}\,dx = b^2\log(|x|) - \frac{4abx+a^2}{2x^2}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^3,x, algorithm="giac")
```

output
```
b^2*log(abs(x)) - 1/2*(4*a*b*x + a^2)/x^2
```

3.51.9 Mupad [B] (verification not implemented)

Time = 3.57 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^3}\,dx = b^2\ln(x) - \frac{\frac{\operatorname{atanh}(\tanh(a+b\,x))^2}{2} + b\,x\operatorname{atanh}(\tanh(a+b\,x))}{x^2}$$

input
```
int(atanh(tanh(a + b*x))^2/x^3,x)
```

output
```
b^2*log(x) - (atanh(tanh(a + b*x))^2/2 + b*x*atanh(tanh(a + b*x)))/x^2
```

3.52 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^4} dx$

3.52.1 Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^4} dx = \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{3x^3(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

output
```
1/3*arctanh(tanh(b*x+a))^3/x^3/(b*x-arctanh(tanh(b*x+a)))
```

3.52.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^4} dx = -\frac{b^2x^2+bx\operatorname{arctanh}(\tanh(a+bx))+\operatorname{arctanh}(\tanh(a+bx))^2}{3x^3}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^2/x^4,x]
```

output
```
-1/3*(b^2*x^2 + b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2)/x^3
```

3.52.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^4} \, dx$$

$$\Big\downarrow \text{ 2598}$$

$$\frac{\text{arctanh}(\tanh(a+bx))^3}{3x^3(bx-\text{arctanh}(\tanh(a+bx)))}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^2/x^4,x]
```

output
```
ArcTanh[Tanh[a + b*x]]^3/(3*x^3*(b*x - ArcTanh[Tanh[a + b*x]]))
```

3.52.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

3.52.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

method	result	size
parallelrisch	$-\frac{b^2x^2+bx\,\text{arctanh}(\tanh(bx+a))+\text{arctanh}(\tanh(bx+a))^2}{3x^3}$	33
default	$-\frac{\text{arctanh}(\tanh(bx+a))^2}{3x^3}+\frac{2b\left(-\frac{b}{2x}-\frac{\text{arctanh}(\tanh(bx+a))}{2x^2}\right)}{3}$	38
parts	$-\frac{\text{arctanh}(\tanh(bx+a))^2}{3x^3}+\frac{2b\left(-\frac{b}{2x}-\frac{\text{arctanh}(\tanh(bx+a))}{2x^2}\right)}{3}$	38
risch	Expression too large to display	1978

input
```
int(arctanh(tanh(b*x+a))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b^2*x^2+b*x*arctanh(tanh(b*x+a))+arctanh(tanh(b*x+a))^2)/x^3
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^4}\,dx = -\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^4,x, algorithm="fricas")
```

output
```
-1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

3.52.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^4}\,dx = -\frac{b^2}{3x} - \frac{b\,\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{3x^2} - \frac{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{3x^3}$$

input
```
integrate(atanh(tanh(b*x+a))**2/x**4,x)
```

output
```
-b**2/(3*x) - b*atanh(tanh(a + b*x))/(3*x**2) - atanh(tanh(a + b*x))**2/(3
*x**3)
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^4}\,dx = -\frac{b^2}{3\,x} - \frac{b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{3\,x^2} - \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{3\,x^3}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^4,x, algorithm="maxima")
```

output
```
-1/3*b^2/x - 1/3*b*arctanh(tanh(b*x + a))/x^2 - 1/3*arctanh(tanh(b*x + a))
^2/x^3
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^4}\,dx = -\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^4,x, algorithm="giac")
```

output
```
-1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

3.52.9 Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^4}\,dx = -\frac{b^2\,x^2+b\,x\,\mathrm{atanh}(\tanh(a+b\,x))+\mathrm{atanh}(\tanh(a+b\,x))^2}{3\,x^3}$$

input
```
int(atanh(tanh(a + b*x))^2/x^4,x)
```

output
```
-(atanh(tanh(a + b*x))^2 + b^2*x^2 + b*x*atanh(tanh(a + b*x)))/(3*x^3)
```

3.53 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^5}\,dx$

3.53.1 Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{b^2}{12x^2} - \frac{b\operatorname{arctanh}(\tanh(a+bx))}{6x^3} - \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{4x^4}$$

output
```
-1/12*b^2/x^2-1/6*b*arctanh(tanh(b*x+a))/x^3-1/4*arctanh(tanh(b*x+a))^2/x^
4
```

3.53.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{b^2x^2 + 2bx\operatorname{arctanh}(\tanh(a+bx)) + 3\operatorname{arctanh}(\tanh(a+bx))^2}{12x^4}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^2/x^5,x]
```

output
```
-1/12*(b^2*x^2 + 2*b*x*ArcTanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2
)/x^4
```

3.53.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.52, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^5} \, dx$$

$$\Big\downarrow \text{ 2602}$$

$$\frac{b \int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^4}dx}{4(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{\text{arctanh}(\tanh(a+bx))^3}{4x^4(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{ 2598}$$

$$\frac{\text{arctanh}(\tanh(a+bx))^3}{4x^4(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{b\text{arctanh}(\tanh(a+bx))^3}{12x^3(bx-\text{arctanh}(\tanh(a+bx)))^2}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^2/x^5,x]
```

output
```
(b*ArcTanh[Tanh[a + b*x]]^3)/(12*x^3*(b*x - ArcTanh[Tanh[a + b*x]])^2) + A
rcTanh[Tanh[a + b*x]]^3/(4*x^4*(b*x - ArcTanh[Tanh[a + b*x]]))
```

3.53.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.53.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
parallelrisch	$-\frac{b^{2} x^{2}+2 b x \,\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)+3 \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{2}}{12 x^{4}}$	36
default	$-\frac{\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{2}}{4 x^{4}}+\frac{b \left(-\frac{b}{6 x^{2}}-\frac{\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)}{3 x^{3}}\right)}{2}$	38
parts	$-\frac{\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{2}}{4 x^{4}}+\frac{b \left(-\frac{b}{6 x^{2}}-\frac{\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)}{3 x^{3}}\right)}{2}$	38
risch	Expression too large to display	1978

input
```
int(arctanh(tanh(b*x+a))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b^2*x^2+2*b*x*arctanh(tanh(b*x+a))+3*arctanh(tanh(b*x+a))^2)/x^4
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^5,x, algorithm="fricas")
```

output
```
-1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

3.53.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{b^2}{12x^2} - \frac{b\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{6x^3} - \frac{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{4x^4}$$

input
```
integrate(atanh(tanh(b*x+a))**2/x**5,x)
```

output
```
-b**2/(12*x**2) - b*atanh(tanh(a + b*x))/(6*x**3) - atanh(tanh(a + b*x))**
2/(4*x**4)
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{b^2}{12\,x^2} - \frac{b\operatorname{artanh}(\tanh(bx+a))}{6\,x^3} - \frac{\operatorname{artanh}(\tanh(bx+a))^2}{4\,x^4}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^5,x, algorithm="maxima")
```

output
```
-1/12*b^2/x^2 - 1/6*b*arctanh(tanh(b*x + a))/x^3 - 1/4*arctanh(tanh(b*x +
a))^2/x^4
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{6\,b^2x^2 + 8\,abx + 3\,a^2}{12\,x^4}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^5,x, algorithm="giac")
```

output
```
-1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

3.53.9 Mupad [B] (verification not implemented)

Time = 3.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{\operatorname{atanh}(\tanh(a+b\,x))^2}{4\,x^4} - \frac{b^2}{12\,x^2} - \frac{b\operatorname{atanh}(\tanh(a+b\,x))}{6\,x^3}$$

input
```
int(atanh(tanh(a + b*x))^2/x^5,x)
```

output
```
- atanh(tanh(a + b*x))^2/(4*x^4) - b^2/(12*x^2) - (b*atanh(tanh(a + b*x)))
/(6*x^3)
```

3.54 $\int x^m \mathbf{arctanh}(\tanh(a+bx))^3\,dx$

3.54.1 Optimal result

Integrand size = 13, antiderivative size = 110

$$\int x^m \operatorname{arctanh}(\tanh(a+bx))^3\,dx = -\frac{6b^3x^{4+m}}{(1+m)\,(24+26m+9m^2+m^3)} + \frac{6b^2x^{3+m}\operatorname{arctanh}(\tanh(a+bx))}{6+11m+6m^2+m^3} - \frac{3bx^{2+m}\operatorname{arctanh}(\tanh(a+bx))^2}{2+3m+m^2} + \frac{x^{1+m}\operatorname{arctanh}(\tanh(a+bx))^3}{1+m}$$

output
```
-6*b^3*x^(4+m)/(1+m)/(m^3+9*m^2+26*m+24)+6*b^2*x^(3+m)*arctanh(tanh(b*x+a)
)/(m^3+6*m^2+11*m+6)-3*b*x^(2+m)*arctanh(tanh(b*x+a))^2/(m^2+3*m+2)+x^(1+m
)*arctanh(tanh(b*x+a))^3/(1+m)
```

3.54.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int x^m \operatorname{arctanh}(\tanh(a+bx))^3\,dx$$
$$= \frac{x^{1+m}\left(-6b^3x^3+6b^2(4+m)x^2\operatorname{arctanh}(\tanh(a+bx))-3b(12+7m+m^2)\,x\operatorname{arctanh}(\tanh(a+bx))^2+(24\right.}{(1+m)(2+m)(3+m)(4+m)}$$

input
```
Integrate[x^m*ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(x^(1 + m)*(-6*b^3*x^3 + 6*b^2*(4 + m)*x^2*ArcTanh[Tanh[a + b*x]] - 3*b*(1
2 + 7*m + m^2)*x*ArcTanh[Tanh[a + b*x]]^2 + (24 + 26*m + 9*m^2 + m^3)*ArcT
anh[Tanh[a + b*x]]^3))/((1 + m)*(2 + m)*(3 + m)*(4 + m))
```

3.54.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{arctanh}(\tanh(a+bx))^3 \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\text{arctanh}(\tanh(a+bx))^3}{m+1}-\frac{3b\int x^{m+1}\text{arctanh}(\tanh(a+bx))^2dx}{m+1}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\text{arctanh}(\tanh(a+bx))^3}{m+1}-\frac{3b\left(\frac{x^{m+2}\text{arctanh}(\tanh(a+bx))^2}{m+2}-\frac{2b\int x^{m+2}\text{arctanh}(\tanh(a+bx))dx}{m+2}\right)}{m+1}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\text{arctanh}(\tanh(a+bx))^3}{m+1}-\frac{3b\left(\frac{x^{m+2}\text{arctanh}(\tanh(a+bx))^2}{m+2}-\frac{2b\left(\frac{x^{m+3}\text{arctanh}(\tanh(a+bx))}{m+3}-\frac{b\int x^{m+3}dx}{m+3}\right)}{m+2}\right)}{m+1}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x^{m+1}\text{arctanh}(\tanh(a+bx))^3}{m+1}-\frac{3b\left(\frac{x^{m+2}\text{arctanh}(\tanh(a+bx))^2}{m+2}-\frac{2b\left(\frac{x^{m+3}\text{arctanh}(\tanh(a+bx))}{m+3}-\frac{bx^{m+4}}{(m+3)(m+4)}\right)}{m+2}\right)}{m+1}$$

input
```
Int[x^m*ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(x^(1 + m)*ArcTanh[Tanh[a + b*x]]^3)/(1 + m) - (3*b*((x^(2 + m)*ArcTanh[Ta
nh[a + b*x]]^2)/(2 + m) - (2*b*(-((b*x^(4 + m))/((3 + m)*(4 + m))) + (x^(3
 + m)*ArcTanh[Tanh[a + b*x]])/(3 + m)))/(2 + m)))/(1 + m)
```

3.54.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.54.4 Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.61

method	result
default	$\frac{b^3x^4e^{m\ln(x)}}{4+m}+\frac{\left(a^3+3a^2(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+3a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2+(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\right.}{1+m}$
parallelrisch	$-\frac{-26x\,x^m\operatorname{arctanh}(\tanh(bx+a))^3m-x\,x^m\operatorname{arctanh}(\tanh(bx+a))^3m^3-9x\,x^m\operatorname{arctanh}(\tanh(bx+a))^3m^2-6x^3x^m\operatorname{arctanh}(\tan}{}$
risch	Expression too large to display

input
```
int(x^m*arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
b^3/(4+m)*x^4*exp(m*ln(x))+(a^3+3*a^2*(arctanh(tanh(b*x+a))-b*x-a)+3*a*(ar
ctanh(tanh(b*x+a))-b*x-a)^2+(arctanh(tanh(b*x+a))-b*x-a)^3)/(1+m)*x*exp(m*
ln(x))+3*b*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x
-a)^2)/(2+m)*x^2*exp(m*ln(x))+3*b^2*(arctanh(tanh(b*x+a))-b*x)/(3+m)*x^3*e
xp(m*ln(x))
```

3.54.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(110) = 220$.

Time = 0.25 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.73

$$\int x^m \text{arctanh}(\tanh(a+bx))^3\,dx$$
$$= \frac{((b^3m^3 + 6\,b^3m^2 + 11\,b^3m + 6\,b^3)x^4 + 3\,(ab^2m^3 + 7\,ab^2m^2 + 14\,ab^2m + 8\,ab^2)x^3 + 3\,(a^2bm^3 + 8\,a^2bm^2 +}{}$$

input
```
integrate(x^m*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
(((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^
2 + 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m +
12*a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*cosh(m*log(x)
) + ((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2
*m^2 + 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m
 + 12*a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*sinh(m*log
(x)))/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

3.54.6 Sympy [F]

$$\int x^m \text{arctanh}(\tanh(a+bx))^3\,dx$$
$$= \begin{cases} b^3\log(x) - \frac{b^2\,\text{atanh}(\tanh(a+bx))}{x} - \frac{b\,\text{atanh}^2(\tanh(a+bx))}{2x^2} - \frac{\text{atanh}^3(\tanh(a+bx))}{3x^3} \\ \int \frac{\text{atanh}^3(\tanh(a+bx))}{x^3}\,dx \\ \int \frac{\text{atanh}^3(\tanh(a+bx))}{x^2}\,dx \\ \int \frac{\text{atanh}^3(\tanh(a+bx))}{x}\,dx \\ -\frac{6b^3x^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{6b^2mx^3x^m\,\text{atanh}(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} + \frac{24b^2x^3x^m\,\text{atanh}(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} - \frac{3bm^2x^2x^m\,\text{atanh}^2(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} \end{cases}$$

input
```
integrate(x**m*atanh(tanh(b*x+a))**3,x)
```

output
```
Piecewise((b**3*log(x) - b**2*atanh(tanh(a + b*x))/x - b*atanh(tanh(a + b*
x))**2/(2*x**2) - atanh(tanh(a + b*x))**3/(3*x**3), Eq(m, -4)), (Integral(
atanh(tanh(a + b*x))**3/x**3, x), Eq(m, -3)), (Integral(atanh(tanh(a + b*x
))**3/x**2, x), Eq(m, -2)), (Integral(atanh(tanh(a + b*x))**3/x, x), Eq(m,
 -1)), (-6*b**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**2*
m*x**3*x**m*atanh(tanh(a + b*x))/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
24*b**2*x**3*x**m*atanh(tanh(a + b*x))/(m**4 + 10*m**3 + 35*m**2 + 50*m +
24) - 3*b*m**2*x**2*x**m*atanh(tanh(a + b*x))**2/(m**4 + 10*m**3 + 35*m**2
 + 50*m + 24) - 21*b*m*x**2*x**m*atanh(tanh(a + b*x))**2/(m**4 + 10*m**3 +
 35*m**2 + 50*m + 24) - 36*b*x**2*x**m*atanh(tanh(a + b*x))**2/(m**4 + 10*
m**3 + 35*m**2 + 50*m + 24) + m**3*x*x**m*atanh(tanh(a + b*x))**3/(m**4 +
10*m**3 + 35*m**2 + 50*m + 24) + 9*m**2*x*x**m*atanh(tanh(a + b*x))**3/(m*
*4 + 10*m**3 + 35*m**2 + 50*m + 24) + 26*m*x*x**m*atanh(tanh(a + b*x))**3/
(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*x*x**m*atanh(tanh(a + b*x))**3
/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24), True))
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.99

$$\int x^m \text{arctanh}(\tanh(a+bx))^3\,dx = -\frac{3\,bx^2x^m\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2}{(m+2)(m+1)} + \frac{x^{m+1}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^3}{m+1} - \frac{6\left(\frac{b^2x^4x^m}{(m+4)(m+3)(m+2)} - \frac{bx^3x^m\,\text{artanh}(\tanh(bx+a))}{(m+3)(m+2)}\right)b}{m+1}$$

input
```
integrate(x^m*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-3*b*x^2*x^m*arctanh(tanh(b*x + a))^2/((m + 2)*(m + 1)) + x^(m + 1)*arctan
h(tanh(b*x + a))^3/(m + 1) - 6*(b^2*x^4*x^m/((m + 4)*(m + 3)*(m + 2)) - b*
x^3*x^m*arctanh(tanh(b*x + a))/((m + 3)*(m + 2)))*b/(m + 1)
```

3.54.8 Giac [F]

$$\int x^m \text{arctanh}(\tanh(a+bx))^3\,dx = \int x^m\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^3\,dx$$

input
```
integrate(x^m*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
integrate(x^m*arctanh(tanh(b*x + a))^3, x)
```

3.54.9 Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.02

$$\begin{aligned}\int x^m \text{arctanh}(\tanh(a+bx))^3\,dx &= \frac{8\,b^3\,x^m\,x^4\,(m^3+6\,m^2+11\,m+6)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192} \\ &- \frac{x\,x^m\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3(m^3+9\,m^2+26\,m+24)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192} \\ &- \frac{12\,b^2\,x^m\,x^3\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)(m^3+7\,m^2+14\,m+8)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192} \\ &+ \frac{6\,b\,x^m\,x^2\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2(m^3+8\,m^2+19\,m+12)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192}\end{aligned}$$

input
```
int(x^m*atanh(tanh(a + b*x))^3,x)
```

output
```
(8*b^3*x^m*x^4*(11*m + 6*m^2 + m^3 + 6))/(400*m + 280*m^2 + 80*m^3 + 8*m^4
 + 192) - (x*x^m*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3*(26*m + 9*m^2 + m^3 + 24))/(4
00*m + 280*m^2 + 80*m^3 + 8*m^4 + 192) - (12*b^2*x^m*x^3*(log(2/(exp(2*a)*
exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x)*(14*m + 7*m^2 + m^3 + 8))/(400*m + 280*m^2 + 80*m^3 + 8*m^4 + 192
) + (6*b*x^m*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2*(19*m + 8*m^2 + m^3 + 12))/(4
00*m + 280*m^2 + 80*m^3 + 8*m^4 + 192)
```

3.55 $\int x^3\mathbf{arctanh}(\tanh(a+bx))^3\,dx$

3.55.1 Optimal result

Integrand size = 13, antiderivative size = 61

$$\int x^3\text{arctanh}(\tanh(a+bx))^3\,dx = -\frac{1}{140}b^3x^7+\frac{1}{20}b^2x^6\text{arctanh}(\tanh(a+bx)) - \frac{3}{20}bx^5\text{arctanh}(\tanh(a+bx))^2 + \frac{1}{4}x^4\text{arctanh}(\tanh(a+bx))^3$$

output
```
-1/140*b^3*x^7+1/20*b^2*x^6*arctanh(tanh(b*x+a))-3/20*b*x^5*arctanh(tanh(b
*x+a))^2+1/4*x^4*arctanh(tanh(b*x+a))^3
```

3.55.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int x^3\text{arctanh}(\tanh(a+bx))^3\,dx = -\frac{1}{140}x^4\left(b^3x^3-7b^2x^2\text{arctanh}(\tanh(a+bx)) + 21bx\text{arctanh}(\tanh(a+bx))^2 - 35\text{arctanh}(\tanh(a+bx))^3\right)$$

input
```
Integrate[x^3*ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
-1/140*(x^4*(b^3*x^3 - 7*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 21*b*x*ArcTanh[T
anh[a + b*x]]^2 - 35*ArcTanh[Tanh[a + b*x]]^3))
```

3.55.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{arctanh}(\tanh(a+bx))^3 \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{4}x^4\text{arctanh}(\tanh(a+bx))^3-\frac{3}{4}b\int x^4\text{arctanh}(\tanh(a+bx))^2dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{4}x^4\text{arctanh}(\tanh(a+bx))^3-\frac{3}{4}b\left(\frac{1}{5}x^5\text{arctanh}(\tanh(a+bx))^2-\frac{2}{5}b\int x^5\text{arctanh}(\tanh(a+bx))dx\right)$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{4}x^4\text{arctanh}(\tanh(a+bx))^3-\frac{3}{4}b\left(\frac{1}{5}x^5\text{arctanh}(\tanh(a+bx))^2-\frac{2}{5}b\left(\frac{1}{6}x^6\text{arctanh}(\tanh(a+bx))-\frac{b\int x^6dx}{6}\right)\right)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{4}x^4\text{arctanh}(\tanh(a+bx))^3-\frac{3}{4}b\left(\frac{1}{5}x^5\text{arctanh}(\tanh(a+bx))^2-\frac{2}{5}b\left(\frac{1}{6}x^6\text{arctanh}(\tanh(a+bx))-\frac{bx^7}{42}\right)\right)$$

input
```
Int[x^3*ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(x^4*ArcTanh[Tanh[a + b*x]]^3)/4 - (3*b*((x^5*ArcTanh[Tanh[a + b*x]]^2)/5
- (2*b*(-1/42*(b*x^7) + (x^6*ArcTanh[Tanh[a + b*x]])/6))/5))/4
```

3.55.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.55.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\frac{x^4 \operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^3}{4} - \frac{3b\left(\frac{x^5 \operatorname{arctanh}(\tanh(bx+a))^2}{5} - \frac{2b\left(\frac{x^6 \operatorname{arctanh}(\tanh(bx+a))}{6} - \frac{b\,x^7}{42}\right)}{5}\right)}{4}$$

input
```
int(x^3*arctanh(tanh(b*x+a))^3,x)
```

output
```
1/4*x^4*arctanh(tanh(b*x+a))^3-3/4*b*(1/5*x^5*arctanh(tanh(b*x+a))^2-2/5*b
*(1/6*x^6*arctanh(tanh(b*x+a))-1/42*b*x^7))
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.57

$$\int x^3 \operatorname{arctanh}(\tanh(a+bx))^3\,dx = \frac{1}{7}\,b^3x^7 + \frac{1}{2}\,ab^2x^6 + \frac{3}{5}\,a^2bx^5 + \frac{1}{4}\,a^3x^4$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

3.55.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int x^3\text{arctanh}(\tanh(a+bx))^3\,dx = -\frac{b^3x^7}{140} + \frac{b^2x^6\operatorname{atanh}(\tanh(a+bx))}{20} - \frac{3bx^5\operatorname{atanh}^2(\tanh(a+bx))}{20} + \frac{x^4\operatorname{atanh}^3(\tanh(a+bx))}{4}$$

input
```
integrate(x**3*atanh(tanh(b*x+a))**3,x)
```

output
```
-b**3*x**7/140 + b**2*x**6*atanh(tanh(a + b*x))/20 - 3*b*x**5*atanh(tanh(a
 + b*x))**2/20 + x**4*atanh(tanh(a + b*x))**3/4
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int x^3\text{arctanh}(\tanh(a+bx))^3\,dx = -\frac{3}{20}\,bx^5\operatorname{artanh}(\tanh(bx+a))^2 + \frac{1}{4}\,x^4\operatorname{artanh}(\tanh(bx+a))^3 - \frac{1}{140}\left(b^2x^7 - 7\,bx^6\operatorname{artanh}(\tanh(bx+a))\right)b$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-3/20*b*x^5*arctanh(tanh(b*x + a))^2 + 1/4*x^4*arctanh(tanh(b*x + a))^3 -
1/140*(b^2*x^7 - 7*b*x^6*arctanh(tanh(b*x + a)))*b
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.57

$$\int x^3\text{arctanh}(\tanh(a+bx))^3\,dx = \frac{1}{7}\,b^3x^7 + \frac{1}{2}\,ab^2x^6 + \frac{3}{5}\,a^2bx^5 + \frac{1}{4}\,a^3x^4$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

3.55.9 Mupad [B] (verification not implemented)

Time = 3.56 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int x^3 \operatorname{arctanh}(\tanh(a+bx))^3\,dx = -\frac{b^3\,x^7}{140} + \frac{b^2\,x^6\,\mathrm{atanh}(\tanh(a+b\,x))}{20} - \frac{3\,b\,x^5\,\mathrm{atanh}(\tanh(a+b\,x))^2}{20} + \frac{x^4\,\mathrm{atanh}(\tanh(a+b\,x))^3}{4}$$

input
```
int(x^3*atanh(tanh(a + b*x))^3,x)
```

output
```
(x^4*atanh(tanh(a + b*x))^3)/4 - (b^3*x^7)/140 - (3*b*x^5*atanh(tanh(a + b
*x))^2)/20 + (b^2*x^6*atanh(tanh(a + b*x)))/20
```

3.56 $\int x^2 \textbf{arctanh}(\tanh(a+bx))^3\,dx$

3.56.1 Optimal result

Integrand size = 13, antiderivative size = 53

$$\int x^2\text{arctanh}(\tanh(a+bx))^3\,dx = \frac{x^2\text{arctanh}(\tanh(a+bx))^4}{4b} - \frac{x\text{arctanh}(\tanh(a+bx))^5}{10b^2} + \frac{\text{arctanh}(\tanh(a+bx))^6}{60b^3}$$

output
```
1/4*x^2*arctanh(tanh(b*x+a))^4/b-1/10*x*arctanh(tanh(b*x+a))^5/b^2+1/60*ar
ctanh(tanh(b*x+a))^6/b^3
```

3.56.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int x^2\text{arctanh}(\tanh(a+bx))^3\,dx = -\frac{1}{60}x^3\big(b^3x^3 - 6b^2x^2\text{arctanh}(\tanh(a+bx)) + 15bx\text{arctanh}(\tanh(a+bx))^2 - 20\text{arctanh}(\tanh(a+bx))^3\big)$$

input
```
Integrate[x^2*ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
-1/60*(x^3*(b^3*x^3 - 6*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 15*b*x*ArcTanh[Ta
nh[a + b*x]]^2 - 20*ArcTanh[Tanh[a + b*x]]^3))
```

3.56.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arctanh}(\tanh(a+bx))^3 \, dx$$

$$\Big\downarrow \ 2599$$

$$\frac{x^2 \text{arctanh}(\tanh(a+bx))^4}{4b} - \frac{\int x \text{arctanh}(\tanh(a+bx))^4 dx}{2b}$$

$$\Big\downarrow \ 2599$$

$$\frac{x^2 \text{arctanh}(\tanh(a+bx))^4}{4b} - \frac{\frac{x \text{arctanh}(\tanh(a+bx))^5}{5b} - \frac{\int \text{arctanh}(\tanh(a+bx))^5 dx}{5b}}{2b}$$

$$\Big\downarrow \ 2588$$

$$\frac{x^2 \text{arctanh}(\tanh(a+bx))^4}{4b} - \frac{\frac{x \text{arctanh}(\tanh(a+bx))^5}{5b} - \frac{\int \text{arctanh}(\tanh(a+bx))^5 d\text{arctanh}(\tanh(a+bx))}{5b^2}}{2b}$$

$$\Big\downarrow \ 15$$

$$\frac{x^2 \text{arctanh}(\tanh(a+bx))^4}{4b} - \frac{\frac{x \text{arctanh}(\tanh(a+bx))^5}{5b} - \frac{\text{arctanh}(\tanh(a+bx))^6}{30b^2}}{2b}$$

input
```
Int[x^2*ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(x^2*ArcTanh[Tanh[a + b*x]]^4)/(4*b) - ((x*ArcTanh[Tanh[a + b*x]]^5)/(5*b)
 - ArcTanh[Tanh[a + b*x]]^6/(30*b^2))/(2*b)
```

3.56.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.56.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\frac{x^{3} \operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{3}}{3}-b \left(\frac{x^{4} \operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{2}}{4}-\frac{b \left(\frac{x^{5} \operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)}{5}-\frac{b \,x^{6}}{30}\right)}{2}\right)$$

input
```
int(x^2*arctanh(tanh(b*x+a))^3,x)
```

output
```
1/3*x^3*arctanh(tanh(b*x+a))^3-b*(1/4*x^4*arctanh(tanh(b*x+a))^2-1/2*b*(1/
5*x^5*arctanh(tanh(b*x+a))-1/30*b*x^6))
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int x^2 \operatorname{arctanh}(\tanh(a+bx))^3 \, dx = \frac{1}{6} \, b^{3} x^{6} + \frac{3}{5} \, a b^{2} x^{5} + \frac{3}{4} \, a^{2} b x^{4} + \frac{1}{3} \, a^{3} x^{3}$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

3.56.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int x^2 \operatorname{arctanh}(\tanh(a+bx))^3\,dx = -\frac{b^3x^6}{60} + \frac{b^2x^5\operatorname{atanh}(\tanh(a+bx))}{10} - \frac{bx^4\operatorname{atanh}^2(\tanh(a+bx))}{4} + \frac{x^3\operatorname{atanh}^3(\tanh(a+bx))}{3}$$

input
```
integrate(x**2*atanh(tanh(b*x+a))**3,x)
```

output
```
-b**3*x**6/60 + b**2*x**5*atanh(tanh(a + b*x))/10 - b*x**4*atanh(tanh(a +
b*x))**2/4 + x**3*atanh(tanh(a + b*x))**3/3
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int x^2 \operatorname{arctanh}(\tanh(a+bx))^3\,dx = -\frac{1}{4}\,bx^4\operatorname{artanh}(\tanh(bx+a))^2 + \frac{1}{3}\,x^3\operatorname{artanh}(\tanh(bx+a))^3 - \frac{1}{60}\left(b^2x^6 - 6\,bx^5\operatorname{artanh}(\tanh(bx+a))\right)b$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-1/4*b*x^4*arctanh(tanh(b*x + a))^2 + 1/3*x^3*arctanh(tanh(b*x + a))^3 - 1
/60*(b^2*x^6 - 6*b*x^5*arctanh(tanh(b*x + a)))*b
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int x^2 \text{arctanh}(\tanh(a+bx))^3 \, dx = \frac{1}{6} b^3 x^6 + \frac{3}{5} ab^2 x^5 + \frac{3}{4} a^2 bx^4 + \frac{1}{3} a^3 x^3$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

3.56.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int x^2 \text{arctanh}(\tanh(a+bx))^3 \, dx = -\frac{b^3 x^6}{60} + \frac{b^2 x^5 \operatorname{atanh}(\tanh(a+b x))}{10} - \frac{b x^4 \operatorname{atanh}(\tanh(a+b x))^2}{4} + \frac{x^3 \operatorname{atanh}(\tanh(a+b x))^3}{3}$$

input
```
int(x^2*atanh(tanh(a + b*x))^3,x)
```

output
```
(x^3*atanh(tanh(a + b*x))^3)/3 - (b^3*x^6)/60 - (b*x^4*atanh(tanh(a + b*x)
)^2)/4 + (b^2*x^5*atanh(tanh(a + b*x)))/10
```

3.57 $\int x\textbf{arctanh}(\tanh(a+bx))^3\,dx$

3.57.1 Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x\text{arctanh}(\tanh(a+bx))^3\,dx=\frac{x\text{arctanh}(\tanh(a+bx))^4}{4b}-\frac{\text{arctanh}(\tanh(a+bx))^5}{20b^2}$$

output
```
1/4*x*arctanh(tanh(b*x+a))^4/b-1/20*arctanh(tanh(b*x+a))^5/b^2
```

3.57.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 99 vs. $2(34)=68$.

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.91

$$\int x\text{arctanh}(\tanh(a+bx))^3\,dx$$
$$=\frac{(a+bx)\left((4a-bx)(a+bx)^3-5(3a-bx)(a+bx)^2\text{arctanh}(\tanh(a+bx))+10(2a^2+abx-b^2x^2)\,\text{arctan}\right.}{20b^2}$$

input
```
Integrate[x*ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
((a + b*x)*((4*a - b*x)*(a + b*x)^3 - 5*(3*a - b*x)*(a + b*x)^2*ArcTanh[Ta
nh[a + b*x]] + 10*(2*a^2 + a*b*x - b^2*x^2)*ArcTanh[Tanh[a + b*x]]^2 - 10*
(a - b*x)*ArcTanh[Tanh[a + b*x]]^3))/(20*b^2)
```

3.57.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(\tanh(a+bx))^3\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x\text{arctanh}(\tanh(a+bx))^4}{4b}-\frac{\int \text{arctanh}(\tanh(a+bx))^4dx}{4b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{x\text{arctanh}(\tanh(a+bx))^4}{4b}-\frac{\int \text{arctanh}(\tanh(a+bx))^4d\text{arctanh}(\tanh(a+bx))}{4b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{x\text{arctanh}(\tanh(a+bx))^4}{4b}-\frac{\text{arctanh}(\tanh(a+bx))^5}{20b^2}$$

input
```
Int[x*ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(x*ArcTanh[Tanh[a + b*x]]^4)/(4*b) - ArcTanh[Tanh[a + b*x]]^5/(20*b^2)
```

3.57.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.57.4 Maple [A] (verified)

Time = 21.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

method	result	size
parallelrisch	$-\frac{b^{3} x^{5}}{20}+\frac{b^{2} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right) x^{4}}{4}-\frac{b \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{2} x^{3}}{2}+\frac{x^{2} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{3}}{2}$	54
default	$\frac{x^{2} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{3}}{2}-\frac{3 b \left(\frac{x^{3} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{2}}{3}-\frac{2 b \left(\frac{x^{4} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)}{4}-\frac{b \,x^{5}}{20}\right)}{3}\right)}{2}$	56
parts	$\frac{x^{2} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{3}}{2}-\frac{3 b \left(\frac{x^{3} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{2}}{3}-\frac{2 b \left(\frac{x^{4} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)}{4}-\frac{b \,x^{5}}{20}\right)}{3}\right)}{2}$	56
risch	Expression too large to display	8165

input
```
int(x*arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/20*b^3*x^5+1/4*b^2*arctanh(tanh(b*x+a))*x^4-1/2*b*arctanh(tanh(b*x+a))^
2*x^3+1/2*x^2*arctanh(tanh(b*x+a))^3
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int x \operatorname{arctanh}(\tanh(a+bx))^3\,dx = \frac{1}{5}\,b^3x^5 + \frac{3}{4}\,ab^2x^4 + a^2bx^3 + \frac{1}{2}\,a^3x^2$$

input
```
integrate(x*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int x\text{arctanh}(\tanh(a+bx))^3\,dx = \begin{cases} \frac{x\,\text{atanh}^4\left(\tanh\left(a+bx\right)\right)}{4b} - \frac{\text{atanh}^5\left(\tanh\left(a+bx\right)\right)}{20b^2} & \text{for } b\neq 0 \\ \frac{x^2\,\text{atanh}^3\left(\tanh\left(a\right)\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*atanh(tanh(b*x+a))**3,x)
```

output
```
Piecewise(((x*atanh(tanh(a + b*x))**4/(4*b) - atanh(tanh(a + b*x))**5/(20*b
**2), Ne(b, 0)), (x**2*atanh(tanh(a))**3/2, True))
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\begin{aligned}\int x\text{arctanh}(\tanh(a+bx))^3\,dx = & -\frac{1}{2}\,bx^3\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2 \\ & +\frac{1}{2}\,x^2\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^3 \\ & -\frac{1}{20}\left(b^2x^5-5\,bx^4\,\text{artanh}\left(\tanh\left(bx+a\right)\right)\right)b\end{aligned}$$

input
```
integrate(x*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-1/2*b*x^3*arctanh(tanh(b*x + a))^2 + 1/2*x^2*arctanh(tanh(b*x + a))^3 - 1
/20*(b^2*x^5 - 5*b*x^4*arctanh(tanh(b*x + a)))*b
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int x\text{arctanh}(\tanh(a+bx))^3\,dx = \frac{1}{5}\,b^3x^5 + \frac{3}{4}\,ab^2x^4 + a^2bx^3 + \frac{1}{2}\,a^3x^2$$

input
```
integrate(x*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

3.57.9 Mupad [B] (verification not implemented)

Time = 3.56 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int x\text{arctanh}(\tanh(a+bx))^3\,dx = -\frac{b^3\,x^5}{20}+\frac{b^2\,x^4\,\mathrm{atanh}(\tanh(a+b\,x))}{4} - \frac{b\,x^3\,\mathrm{atanh}(\tanh(a+b\,x))^2}{2}+\frac{x^2\,\mathrm{atanh}(\tanh(a+b\,x))^3}{2}$$

input
```
int(x*atanh(tanh(a + b*x))^3,x)
```

output
```
(x^2*atanh(tanh(a + b*x))^3)/2 - (b^3*x^5)/20 - (b*x^3*atanh(tanh(a + b*x)
)^2)/2 + (b^2*x^4*atanh(tanh(a + b*x)))/4
```

3.58 $\int \operatorname{arctanh}(\tanh(a+bx))^3\,dx$

3.58.1 Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \operatorname{arctanh}(\tanh(a+bx))^3\,dx=\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{4b}$$

output
```
1/4*arctanh(tanh(b*x+a))^4/b
```

3.58.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \operatorname{arctanh}(\tanh(a+bx))^3\,dx=\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{4b}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
ArcTanh[Tanh[a + b*x]]^4/(4*b)
```

3.58.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(\tanh(a+bx))^3\,dx$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \operatorname{arctanh}(\tanh(a+bx))^3 d\operatorname{arctanh}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{4b}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
ArcTanh[Tanh[a + b*x]]^4/(4*b)
```

3.58.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.58.4 Maple [A] (verified)

Time = 21.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{4b}$
default	$\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{4b}$
parallelrisch	$b^2x^3 \operatorname{arctanh}(\tanh(bx+a)) - \frac{3b \operatorname{arctanh}(\tanh(bx+a))^2 x^2}{2} + x \operatorname{arctanh}(\tanh(bx+a))^3 - \frac{b^3 x}{4}$
risch	Expression too large to display

input
```
int(arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*arctanh(tanh(b*x+a))^4/b
```

3.58.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(14) = 28.

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \operatorname{arctanh}(\tanh(a+bx))^3 \, dx = \frac{1}{4} b^3x^4 + ab^2x^3 + \frac{3}{2} a^2bx^2 + a^3x$$

input
```
integrate(arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

3.58.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \operatorname{arctanh}(\tanh(a+bx))^3 \, dx = \begin{cases} \frac{\operatorname{atanh}^4(\tanh(a+bx))}{4b} & \text{for } b \neq 0 \\ x \operatorname{atanh}^3(\tanh(a)) & \text{otherwise} \end{cases}$$

input
```
integrate(atanh(tanh(b*x+a))**3,x)
```

output
```
Piecewise((atanh(tanh(a + b*x))**4/(4*b), Ne(b, 0)), (x*atanh(tanh(a))**3,
 True))
```

3.58.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(14) = 28$.

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int \operatorname{arctanh}(\tanh(a+bx))^3\,dx = -\frac{3}{2}\,bx^2\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2 + x\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3 - \frac{1}{4}\left(b^2x^4 - 4\,bx^3\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

input
```
integrate(arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-3/2*b*x^2*arctanh(tanh(b*x + a))^2 + x*arctanh(tanh(b*x + a))^3 - 1/4*(b^
2*x^4 - 4*b*x^3*arctanh(tanh(b*x + a)))*b
```

3.58.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \operatorname{arctanh}(\tanh(a+bx))^3\,dx = \frac{1}{2}\left(bx^2 + 2\,ax\right)a^2 + \frac{1}{4}\left(bx^2 + 2\,ax\right)^2 b$$

input
```
integrate(arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
1/2*(b*x^2 + 2*a*x)*a^2 + 1/4*(b*x^2 + 2*a*x)^2*b
```

3.58.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.94

$$\int \operatorname{arctanh}(\tanh(a+bx))^3\,dx$$
$$=\frac{x\,(2\,\mathrm{atanh}(\tanh(a+b\,x))-b\,x)\,\left(b^2\,x^2-2\,b\,x\,\mathrm{atanh}(\tanh(a+b\,x))+2\,\mathrm{atanh}(\tanh(a+b\,x))^2\right)}{4}$$

input
```
int(atanh(tanh(a + b*x))^3,x)
```

output
```
(x*(2*atanh(tanh(a + b*x)) - b*x)*(2*atanh(tanh(a + b*x))^2 + b^2*x^2 - 2*
b*x*atanh(tanh(a + b*x))))/4
```

3.59 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x}\,dx$

3.59.1 Optimal result

Integrand size = 13, antiderivative size = 77

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x}\,dx = bx(bx-\operatorname{arctanh}(\tanh(a+bx)))^2 - \frac{1}{2}(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))^2 + \frac{1}{3}\operatorname{arctanh}(\tanh(a+bx))^3 - (bx-\operatorname{arctanh}(\tanh(a+bx)))^3\log(x)$$

output
```
b*x*(b*x-arctanh(tanh(b*x+a)))^2-1/2*(b*x-arctanh(tanh(b*x+a)))*arctanh(ta
nh(b*x+a))^2+1/3*arctanh(tanh(b*x+a))^3-(b*x-arctanh(tanh(b*x+a)))^3*ln(x)
```

3.59.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x}\,dx = \frac{1}{3}(a+bx)^3+(a+bx)\left(a^2-3a(a+bx-\operatorname{arctanh}(\tanh(a+bx))) + 3(a+bx-\operatorname{arctanh}(\tanh(a+bx)))^2\right) - \frac{1}{2}(a+bx)^2(2a+3bx-3\operatorname{arctanh}(\tanh(a+bx))) + (-bx+\operatorname{arctanh}(\tanh(a+bx)))^3\log(bx)$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^3/x,x]
```

output
```
(a + b*x)^3/3 + (a + b*x)*(a^2 - 3*a*(a + b*x - ArcTanh[Tanh[a + b*x]]) +
3*(a + b*x - ArcTanh[Tanh[a + b*x]])^2) - ((a + b*x)^2*(2*a + 3*b*x - 3*Ar
cTanh[Tanh[a + b*x]]))/2 + (-(b*x) + ArcTanh[Tanh[a + b*x]])^3*Log[b*x]
```

3.59.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2590, 2590, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^3}{x} \, dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{1}{3}\text{arctanh}(\tanh(a+bx))^3-(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x}dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{1}{3}\text{arctanh}(\tanh(a+bx))^3-(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{2}\text{arctanh}(\tanh(a+bx))^2-(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{\text{arctanh}(\tanh(a+bx))}{x}dx\right)$$

$$\Big\downarrow \text{2589}$$

$$\frac{1}{3}\text{arctanh}(\tanh(a+bx))^3-(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{2}\text{arctanh}(\tanh(a+bx))^2-(bx-\text{arctanh}(\tanh(a+bx)))\left(bx-(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{x}dx\right)\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{3}\text{arctanh}(\tanh(a+bx))^3-(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{2}\text{arctanh}(\tanh(a+bx))^2-(bx-\text{arctanh}(\tanh(a+bx)))(bx-\log(x)(bx-\text{arctanh}(\tanh(a+bx))))\right)$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^3/x,x]
```

output
```
ArcTanh[Tanh[a + b*x]]^3/3 - (b*x - ArcTanh[Tanh[a + b*x]])*(ArcTanh[Tanh[
a + b*x]]^2/2 - (b*x - ArcTanh[Tanh[a + b*x]])*(b*x - (b*x - ArcTanh[Tanh[
a + b*x]])*Log[x]))
```

3.59.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]]
```

3.59.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.91

method	result
default	$\ln(x)\operatorname{arctanh}(\tanh(bx+a))^3 - 3b\left(b^2\left(\frac{x^3\ln(x)}{3} - \frac{x^3}{9}\right) + 2ab\left(\frac{x^2\ln(x)}{2} - \frac{x^2}{4}\right) + 2b(\operatorname{arctanh}(\tanh($
parts	$\ln(x)\operatorname{arctanh}(\tanh(bx+a))^3 - 3b\left(b^2\left(\frac{x^3\ln(x)}{3} - \frac{x^3}{9}\right) + 2ab\left(\frac{x^2\ln(x)}{2} - \frac{x^2}{4}\right) + 2b(\operatorname{arctanh}(\tanh($
risch	Expression too large to display

input
```
int(arctanh(tanh(b*x+a))^3/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arctanh(tanh(b*x+a))^3-3*b*(b^2*(1/3*x^3*ln(x)-1/9*x^3)+2*a*b*(1/2*x
^2*ln(x)-1/4*x^2)+2*b*(arctanh(tanh(b*x+a))-b*x-a)*(1/2*x^2*ln(x)-1/4*x^2)
+a^2*(x*ln(x)-x)+2*a*(arctanh(tanh(b*x+a))-b*x-a)*(x*ln(x)-x)+(arctanh(tan
h(b*x+a))-b*x-a)^2*(x*ln(x)-x))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.40

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x}\,dx = \frac{1}{3}\,b^3x^3 + \frac{3}{2}\,ab^2x^2 + 3\,a^2bx + a^3\log\left(x\right)$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x,x, algorithm="fricas")
```

output
```
1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

3.59.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x}\,dx = \int \frac{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{x}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**3/x,x)
```

output
```
Integral(atanh(tanh(a + b*x))**3/x, x)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.40

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x}\,dx = \frac{1}{3}\,b^3x^3 + \frac{3}{2}\,ab^2x^2 + 3\,a^2bx + a^3\log\left(x\right)$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x,x, algorithm="maxima")
```

output
```
1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.42

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x}\,dx = \frac{1}{3}\,b^3x^3 + \frac{3}{2}\,ab^2x^2 + 3\,a^2bx + a^3\log\left(|x|\right)$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x,x, algorithm="giac")
```

output
```
1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(abs(x))
```

3.59.9 Mupad [B] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.97

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x}\,dx = \frac{b^3\,x^3}{3} - \ln(x)\left(\frac{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{8} - a^3 - \frac{3\,a\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{4} + \frac{3\,a^2\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2}\right) - \frac{3\,b^2\,x^2\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{4} + \frac{3\,b\,x\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{4}$$

input
```
int(atanh(tanh(a + b*x))^3/x,x)
```

output
```
(b^3*x^3)/3 - log(x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/8 - a^3 - (3*a*(2*
a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^2)/4 + (3*a^2*(2*a - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
)/2) - (3*b^2*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/4 + (3*b*x*(log(2/(exp(2*a)*e
xp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^2)/4
```

3.59. $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x}\,dx$

3.60 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^2} dx$

3.60.1 Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^2} dx = -3b^2x(bx - \operatorname{arctanh}(\tanh(a+bx))) + \frac{3}{2}b\operatorname{arctanh}(\tanh(a+bx))^2 - \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x} + 3b(bx - \operatorname{arctanh}(\tanh(a+bx)))^2 \log(x)$$

output
```
-3*b^2*x*(b*x-arctanh(tanh(b*x+a)))+3/2*b*arctanh(tanh(b*x+a))^2-arctanh(t
anh(b*x+a))^3/x+3*b*(b*x-arctanh(tanh(b*x+a)))^2*ln(x)
```

3.60.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^2} dx = -\frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x} - 6b^2x\operatorname{arctanh}(\tanh(a+bx)) \log(x) + 3b\operatorname{arctanh}(\tanh(a+bx))^2(1+\log(x)) + \frac{3}{2}b^3x^2(-1+2\log(x))$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^3/x^2,x]
```

output
```
-(ArcTanh[Tanh[a + b*x]]^3/x) - 6*b^2*x*ArcTanh[Tanh[a + b*x]]*Log[x] + 3*
b*ArcTanh[Tanh[a + b*x]]^2*(1 + Log[x]) + (3*b^3*x^2*(-1 + 2*Log[x]))/2
```

3.60.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2590, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^2}dx$$

$$\Big\downarrow \text{2599}$$

$$3b\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x}dx-\frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x}$$

$$\Big\downarrow \text{2590}$$

$$3b\left(\frac{1}{2}\operatorname{arctanh}(\tanh(a+bx))^2-(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x}dx\right)-\frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x}$$

$$\Big\downarrow \text{2589}$$

$$3b\left(\frac{1}{2}\operatorname{arctanh}(\tanh(a+bx))^2-(bx-\operatorname{arctanh}(\tanh(a+bx)))\left(bx-(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{1}{x}dx\right)\right)-\frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x}$$

$$\Big\downarrow \text{14}$$

$$3b\left(\frac{1}{2}\operatorname{arctanh}(\tanh(a+bx))^2-(bx-\operatorname{arctanh}(\tanh(a+bx)))(bx-\log(x)(bx-\operatorname{arctanh}(\tanh(a+bx))))\right)-\frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^3/x^2,x]
```

output
```
-(ArcTanh[Tanh[a + b*x]]^3/x) + 3*b*(ArcTanh[Tanh[a + b*x]]^2/2 - (b*x - A
rcTanh[Tanh[a + b*x]])*(b*x - (b*x - ArcTanh[Tanh[a + b*x]])*Log[x]))
```

3.60.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.60.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25

method	result
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{x}+3b\Big(\ln(x)\operatorname{arctanh}(\tanh(bx+a))^2-2b\Big(b\Big(\frac{x^2\ln(x)}{2}-\frac{x^2}{4}\Big)+a(x\ln(x)-x)$
parts	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{x}+3b\Big(\ln(x)\operatorname{arctanh}(\tanh(bx+a))^2-2b\Big(b\Big(\frac{x^2\ln(x)}{2}-\frac{x^2}{4}\Big)+a(x\ln(x)-x)$
risch	Expression too large to display

input
```
int(arctanh(tanh(b*x+a))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arctanh(tanh(b*x+a))^3/x+3*b*(ln(x)*arctanh(tanh(b*x+a))^2-2*b*(b*(1/2*x^
2*ln(x)-1/4*x^2)+a*(x*ln(x)-x)+(arctanh(tanh(b*x+a))-b*x-a)*(x*ln(x)-x)))
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.53

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^2}\,dx = \frac{b^3x^3 + 6\,ab^2x^2 + 6\,a^2bx\log(x) - 2\,a^3}{2\,x}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^2,x, algorithm="fricas")
```

output
```
1/2*(b^3*x^3 + 6*a*b^2*x^2 + 6*a^2*b*x*log(x) - 2*a^3)/x
```

3.60.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^2}\,dx = \int \frac{\operatorname{atanh}^3(\tanh(a+bx))}{x^2}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**3/x**2,x)
```

output
```
Integral(atanh(tanh(a + b*x))**3/x**2, x)
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^2}\,dx \\ &= 3\,b\,\operatorname{artanh}(\tanh(bx+a))^2\log(x) \\ &\quad + \frac{3}{2}\left(b^2x^2 + 4\,abx + 2\,a^2\log(x) - 2\,\operatorname{artanh}(\tanh(bx+a))^2\log(x)\right)b \\ &\quad - \frac{\operatorname{artanh}(\tanh(bx+a))^3}{x}\end{aligned}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^2,x, algorithm="maxima")
```

output
```
3*b*arctanh(tanh(b*x + a))^2*log(x) + 3/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x
) - 2*arctanh(tanh(b*x + a))^2*log(x))*b - arctanh(tanh(b*x + a))^3/x
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.49

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^2}\,dx = \frac{1}{2}\,b^3x^2 + 3\,ab^2x + 3\,a^2b\log\left(|x|\right) - \frac{a^3}{x}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^2,x, algorithm="giac")
```

output
```
1/2*b^3*x^2 + 3*a*b^2*x + 3*a^2*b*log(abs(x)) - a^3/x
```

3.60.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 415, normalized size of antiderivative = 6.10

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^2}\,dx = \frac{3\,b\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{4} - \frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{8\,x} + \frac{3\,b\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{4} + \frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{8\,x} - \frac{3\,b^3\,x^2}{2} + \frac{3\,b\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2\,\ln(x)}{4} + \frac{3\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{8\,x} - \frac{3\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{8\,x} + \frac{3\,b\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2\,\ln(x)}{4} - \frac{3\,b\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} + 3\,b^3\,x^2\,\ln(x) - \frac{3\,b\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln(x)}{2} + 3\,b^2\,x\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln(x) - 3\,b^2\,x\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln(x)$$

input
```
int(atanh(tanh(a + b*x))^3/x^2,x)
```

output
```
(3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))^2)/4 - log((exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))^3/(8*x) + (3*b*log((exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1))^2)/4 + log(1/(exp(2*a)*exp(2*b*x) + 1))^3/(8*x) - (3*b^3
*x^2)/2 + (3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log(x))/4 + (3*log(1/(ex
p(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
))^2)/(8*x) - (3*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)))/(8*x) + (3*b*log((exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))^2*log(x))/4 - (3*b*log(1/(exp(2*a)*exp(2*b*x) + 1)
)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/2 + 3*b^3*x^2*log(
x) - (3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1))*log(x))/2 + 3*b^2*x*log(1/(exp(2*a)*exp(2*b*x) + 1))
*log(x) - 3*b^2*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log
(x)
```

3.60. $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^2}\,dx$

3.61 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^3}\,dx$

3.61.1 Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^3}\,dx = 3b^3x-\frac{3b\operatorname{arctanh}(\tanh(a+bx))^2}{2x}-\frac{\operatorname{arctanh}(\tanh(a+bx))^3}{2x^2} - 3b^2(bx-\operatorname{arctanh}(\tanh(a+bx)))\log(x)$$

output
```
3*b^3*x-3/2*b*arctanh(tanh(b*x+a))^2/x-1/2*arctanh(tanh(b*x+a))^3/x^2-3*b^
2*(b*x-arctanh(tanh(b*x+a)))*ln(x)
```

3.61.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^3}\,dx = b^3x-\frac{3b(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2}{x} - \frac{(-bx+\operatorname{arctanh}(\tanh(a+bx)))^3}{2x^2} + 3b^2(-bx+\operatorname{arctanh}(\tanh(a+bx)))\log(x)$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^3/x^3,x]
```

output
```
b^3*x - (3*b*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)/x - (-(b*x) + ArcTanh[Ta
nh[a + b*x]])^3/(2*x^2) + 3*b^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])*Log[x]
```

3.61.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2599, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^3} \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{3}{2} b \int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^2} dx - \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{2x^2}$$

$$\Big\downarrow \text{2599}$$

$$\frac{3}{2} b \left(2b \int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x} dx - \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x}\right) - \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{2x^2}$$

$$\Big\downarrow \text{2589}$$

$$\frac{3}{2} b \left(2b \left(bx - (bx - \operatorname{arctanh}(\tanh(a+bx))) \int \frac{1}{x} dx\right) - \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x}\right) - \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{3}{2} b \left(2b (bx - \log(x)(bx - \operatorname{arctanh}(\tanh(a+bx)))) - \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x}\right) - \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{2x^2}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^3/x^3,x]
```

output
```
-1/2*ArcTanh[Tanh[a + b*x]]^3/x^2 + (3*b*(-(ArcTanh[Tanh[a + b*x]]^2/x) +
2*b*(b*x - (b*x - ArcTanh[Tanh[a + b*x]])*Log[x])))/2
```

3.61.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.61.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{2x^2}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{x}+2b(\ln(x)\operatorname{arctanh}(\tanh(bx+a))-b(x\ln(x)-x))\right)}{2}$	59
parts	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{2x^2}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{x}+2b(\ln(x)\operatorname{arctanh}(\tanh(bx+a))-b(x\ln(x)-x))\right)}{2}$	59
risch	Expression too large to display	2008

input
```
int(arctanh(tanh(b*x+a))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(tanh(b*x+a))^3/x^2+3/2*b*(-arctanh(tanh(b*x+a))^2/x+2*b*(ln(x
)*arctanh(tanh(b*x+a))-b*(x*ln(x)-x)))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.62

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^3}\,dx = \frac{2\,b^3x^3+6\,ab^2x^2\log\left(x\right)-6\,a^2bx-a^3}{2\,x^2}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*b^3*x^3 + 6*a*b^2*x^2*log(x) - 6*a^2*b*x - a^3)/x^2
```

3.61.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^3}\,dx = \int \frac{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{x^3}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**3/x**3,x)
```

output
```
Integral(atanh(tanh(a + b*x))**3/x**3, x)
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^3}\,dx &= 3\left(b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\log\left(x\right)-\left(b\left(x+\frac{a}{b}\right)\log\left(x\right)-b\left(x+\frac{a\log\left(x\right)}{b}\right)\right)b\right)b\\&\quad-\frac{3\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{2\,x}-\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}{2\,x^2}\end{aligned}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^3,x, algorithm="maxima")
```

output
```
3*(b*arctanh(tanh(b*x + a))*log(x) - (b*(x + a/b)*log(x) - b*(x + a*log(x)
/b))*b)*b - 3/2*b*arctanh(tanh(b*x + a))^2/x - 1/2*arctanh(tanh(b*x + a))^
3/x^2
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.52

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^3}\,dx = b^3x + 3\,ab^2\log\left(|x|\right) - \frac{6\,a^2bx + a^3}{2\,x^2}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^3,x, algorithm="giac")
```

output
```
b^3*x + 3*a*b^2*log(abs(x)) - 1/2*(6*a^2*b*x + a^3)/x^2
```

3.61.9 Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 365, normalized size of antiderivative = 6.08

$$\begin{aligned}\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^3}\,dx = {} & \frac{9\,b^2\,\ln\left(\frac{e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{4} - \frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{16\,x^2} \\ & - \frac{9\,b^2\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{4} - \frac{3\,b^3\,x}{2} + \frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{16\,x^2} \\ & - \frac{3\,b\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{8\,x} - \frac{3\,b^2\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(x\right)}{2} \\ & + \frac{3\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{16\,x^2} \\ & - \frac{3\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{16\,x^2} \\ & - \frac{3\,b\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{8\,x} + \frac{3\,b^2\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(x\right)}{2} \\ & - 3\,b^3\,x\,\ln\left(x\right) + \frac{3\,b\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{4\,x}\end{aligned}$$

input
```
int(atanh(tanh(a + b*x))^3/x^3,x)
```

output
```
(9*b^2*log(exp(2*b*x)/(exp(2*a)*exp(2*b*x) + 1)))/4 - log((exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))^3/(16*x^2) - (9*b^2*log(1/(exp(2*a)*exp(2
*b*x) + 1)))/4 - (3*b^3*x)/2 + log(1/(exp(2*a)*exp(2*b*x) + 1))^3/(16*x^2)
 - (3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))^2)/(8*x) - (3*b^2*log(1/(exp(2*a)
*exp(2*b*x) + 1))*log(x))/2 + (3*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(16*x^2) - (3*log(1/(exp(2
*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
))/(16*x^2) - (3*b*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)
/(8*x) + (3*b^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(x
))/2 - 3*b^3*x*log(x) + (3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/(4*x)
```

3.61. $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^3}\,dx$

3.62 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^4} \, dx$

3.62.1 Optimal result

Integrand size = 13, antiderivative size = 55

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^4} \, dx = -\frac{b^2 \operatorname{arctanh}(\tanh(a+bx))}{x} - \frac{b \operatorname{arctanh}(\tanh(a+bx))^2}{2x^2} - \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{3x^3} + b^3 \log(x)$$

output
```
-b^2*arctanh(tanh(b*x+a))/x-1/2*b*arctanh(tanh(b*x+a))^2/x^2-1/3*arctanh(t
anh(b*x+a))^3/x^3+b^3*ln(x)
```

3.62.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^4} \, dx = \frac{-6b^2x^2 \operatorname{arctanh}(\tanh(a+bx)) - 3bx \operatorname{arctanh}(\tanh(a+bx))^2 - 2 \operatorname{arctanh}(\tanh(a+bx))^3 + b^3x^3(11 + 6 \,\mathrm{lo}}{6x^3}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^3/x^4,x]
```

output
```
(-6*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 3*b*x*ArcTanh[Tanh[a + b*x]]^2 - 2*Ar
cTanh[Tanh[a + b*x]]^3 + b^3*x^3*(11 + 6*Log[x]))/(6*x^3)
```

3.62.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2599, 2599, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^3}{x^4}dx$$

$$\Big\downarrow \text{2599}$$

$$b\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^3}dx-\frac{\text{arctanh}(\tanh(a+bx))^3}{3x^3}$$

$$\Big\downarrow \text{2599}$$

$$b\left(b\int \frac{\text{arctanh}(\tanh(a+bx))}{x^2}dx-\frac{\text{arctanh}(\tanh(a+bx))^2}{2x^2}\right)-\frac{\text{arctanh}(\tanh(a+bx))^3}{3x^3}$$

$$\Big\downarrow \text{2599}$$

$$b\left(b\left(b\int \frac{1}{x}dx-\frac{\text{arctanh}(\tanh(a+bx))}{x}\right)-\frac{\text{arctanh}(\tanh(a+bx))^2}{2x^2}\right)-\frac{\text{arctanh}(\tanh(a+bx))^3}{3x^3}$$

$$\Big\downarrow \text{14}$$

$$b\left(b\left(b\log(x)-\frac{\text{arctanh}(\tanh(a+bx))}{x}\right)-\frac{\text{arctanh}(\tanh(a+bx))^2}{2x^2}\right)-\frac{\text{arctanh}(\tanh(a+bx))^3}{3x^3}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^3/x^4,x]
```

output
```
-1/3*ArcTanh[Tanh[a + b*x]]^3/x^3 + b*(-1/2*ArcTanh[Tanh[a + b*x]]^2/x^2 +
 b*(-(ArcTanh[Tanh[a + b*x]]/x) + b*Log[x]))
```

3.62.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.62.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{3x^3}+b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2x^2}+b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{x}+b\ln(x)\right)\right)$	52
parts	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{3x^3}+b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2x^2}+b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{x}+b\ln(x)\right)\right)$	52
parallelrisch	$\frac{6b^3\ln(x)x^3-6b^2\operatorname{arctanh}(\tanh(bx+a))x^2-3b\operatorname{arctanh}(\tanh(bx+a))^2x-2\operatorname{arctanh}(\tanh(bx+a))^3}{6x^3}$	56
risch	Expression too large to display	7816

input
```
int(arctanh(tanh(b*x+a))^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arctanh(tanh(b*x+a))^3/x^3+b*(-1/2*arctanh(tanh(b*x+a))^2/x^2+b*(-arc
tanh(tanh(b*x+a))/x+b*ln(x)))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^4}\,dx=\frac{6\,b^3x^3\log(x)-18\,ab^2x^2-9\,a^2bx-2\,a^3}{6\,x^3}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^4,x, algorithm="fricas")
```

output
```
1/6*(6*b^3*x^3*log(x) - 18*a*b^2*x^2 - 9*a^2*b*x - 2*a^3)/x^3
```

3.62.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^4}\,dx = b^3\log(x) - \frac{b^2\operatorname{atanh}(\tanh(a+bx))}{x} - \frac{b\operatorname{atanh}^2(\tanh(a+bx))}{2x^2} - \frac{\operatorname{atanh}^3(\tanh(a+bx))}{3x^3}$$

input
```
integrate(atanh(tanh(b*x+a))**3/x**4,x)
```

output
```
b**3*log(x) - b**2*atanh(tanh(a + b*x))/x - b*atanh(tanh(a + b*x))**2/(2*x
**2) - atanh(tanh(a + b*x))**3/(3*x**3)
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^4}\,dx = \left(b^2\log(x) - \frac{b\operatorname{artanh}(\tanh(bx+a))}{x}\right)b - \frac{b\operatorname{artanh}(\tanh(bx+a))^2}{2x^2} - \frac{\operatorname{artanh}(\tanh(bx+a))^3}{3x^3}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^4,x, algorithm="maxima")
```

output
```
(b^2*log(x) - b*arctanh(tanh(b*x + a))/x)*b - 1/2*b*arctanh(tanh(b*x + a))
^2/x^2 - 1/3*arctanh(tanh(b*x + a))^3/x^3
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^4}\,dx = b^3\log(|x|) - \frac{18\,ab^2x^2 + 9\,a^2bx + 2\,a^3}{6\,x^3}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^4,x, algorithm="giac")
```

output
```
b^3*log(abs(x)) - 1/6*(18*a*b^2*x^2 + 9*a^2*b*x + 2*a^3)/x^3
```

3.62.9 Mupad [B] (verification not implemented)

Time = 3.47 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^4}\,dx$$
$$= b^3 \ln(x) - \frac{b^2\,x^2\,\operatorname{atanh}(\tanh(a+b\,x)) + \frac{b\,x\,\operatorname{atanh}(\tanh(a+b\,x))^2}{2} + \frac{\operatorname{atanh}(\tanh(a+b\,x))^3}{3}}{x^3}$$

input
```
int(atanh(tanh(a + b*x))^3/x^4,x)
```

output
```
b^3*log(x) - (atanh(tanh(a + b*x))^3/3 + (b*x*atanh(tanh(a + b*x))^2)/2 +
b^2*x^2*atanh(tanh(a + b*x)))/x^3
```

3.63 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^5}\,dx$

3.63.1 Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^5}\,dx = \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{4x^4(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

output
```
1/4*arctanh(tanh(b*x+a))^4/x^4/(b*x-arctanh(tanh(b*x+a)))
```

3.63.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^5}\,dx = -\frac{b^3x^3+b^2x^2\operatorname{arctanh}(\tanh(a+bx))+bx\operatorname{arctanh}(\tanh(a+bx))^2+\operatorname{arctanh}(\tanh(a+bx))^3}{4x^4}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^3/x^5,x]
```

output
```
-1/4*(b^3*x^3 + b^2*x^2*ArcTanh[Tanh[a + b*x]] + b*x*ArcTanh[Tanh[a + b*x]
]^2 + ArcTanh[Tanh[a + b*x]]^3)/x^4
```

3.63.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^5} \, dx$$

$$\Big\downarrow \ 2598$$

$$\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{4x^4(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^3/x^5,x]
```

output
```
ArcTanh[Tanh[a + b*x]]^4/(4*x^4*(b*x - ArcTanh[Tanh[a + b*x]]))
```

3.63.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

3.63.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

method	result	size
parallelrisch	$-\frac{b^{3} x^{3}+b^{2} \operatorname{arctanh}(\tanh(bx+a)) x^{2}+b \operatorname{arctanh}(\tanh(bx+a))^{2} x+\operatorname{arctanh}(\tanh(bx+a))^{3}}{4x^{4}}$	49
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^{3}}{4x^{4}}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{2}}{3x^{3}}+\frac{2b\left(-\frac{b}{2x}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{2x^{2}}\right)}{3}\right)}{4}$	56
parts	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^{3}}{4x^{4}}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{2}}{3x^{3}}+\frac{2b\left(-\frac{b}{2x}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{2x^{2}}\right)}{3}\right)}{4}$	56
risch	Expression too large to display	7814

input
```
int(arctanh(tanh(b*x+a))^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b^3*x^3+b^2*arctanh(tanh(b*x+a))*x^2+b*arctanh(tanh(b*x+a))^2*x+arct
anh(tanh(b*x+a))^3)/x^4
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^5}\,dx = -\frac{4\,b^3x^3+6\,ab^2x^2+4\,a^2bx+a^3}{4\,x^4}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^5,x, algorithm="fricas")
```

output
```
-1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

3.63.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(26) = 52.

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^5}\,dx = -\frac{b^3}{4x} - \frac{b^2\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{4x^2} - \frac{b\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{4x^3} - \frac{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{4x^4}$$

input
```
integrate(atanh(tanh(b*x+a))**3/x**5,x)
```

output
```
-b**3/(4*x) - b**2*atanh(tanh(a + b*x))/(4*x**2) - b*atanh(tanh(a + b*x))*
*2/(4*x**3) - atanh(tanh(a + b*x))**3/(4*x**4)
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^5}\,dx = -\frac{1}{4}\,b\left(\frac{b^2}{x} + \frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{x^2}\right) - \frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{4\,x^3} - \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}{4\,x^4}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^5,x, algorithm="maxima")
```

output
```
-1/4*b*(b^2/x + b*arctanh(tanh(b*x + a))/x^2) - 1/4*b*arctanh(tanh(b*x + a
))^2/x^3 - 1/4*arctanh(tanh(b*x + a))^3/x^4
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^5}\,dx = -\frac{4\,b^3x^3 + 6\,ab^2x^2 + 4\,a^2bx + a^3}{4\,x^4}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^5,x, algorithm="giac")
```

output
```
-1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

3.63.9 Mupad [B] (verification not implemented)

Time = 3.44 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^5}\,dx = -\frac{b^3\,x^3 + b^2\,x^2\operatorname{atanh}(\tanh(a+b\,x)) + b\,x\operatorname{atanh}(\tanh(a+b\,x))^2 + \operatorname{atanh}(\tanh(a+b\,x))^3}{4\,x^4}$$

input
```
int(atanh(tanh(a + b*x))^3/x^5,x)
```

output
```
-(atanh(tanh(a + b*x))^3 + b^3*x^3 + b*x*atanh(tanh(a + b*x))^2 + b^2*x^2*
atanh(tanh(a + b*x)))/(4*x^4)
```

3.64 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^6} dx$

3.64.1 Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^6} dx = \frac{b\operatorname{arctanh}(\tanh(a+bx))^4}{20x^4(bx-\operatorname{arctanh}(\tanh(a+bx)))^2} + \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{5x^5(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

output
```
1/20*b*arctanh(tanh(b*x+a))^4/x^4/(b*x-arctanh(tanh(b*x+a)))^2+1/5*arctanh
(tanh(b*x+a))^4/x^5/(b*x-arctanh(tanh(b*x+a)))
```

3.64.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^6} dx = -\frac{b^3x^3 + 2b^2x^2\operatorname{arctanh}(\tanh(a+bx)) + 3bx\operatorname{arctanh}(\tanh(a+bx))^2 + 4\operatorname{arctanh}(\tanh(a+bx))^3}{20x^5}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^3/x^6,x]
```

output
```
-1/20*(b^3*x^3 + 2*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 3*b*x*ArcTanh[Tanh[a +
 b*x]]^2 + 4*ArcTanh[Tanh[a + b*x]]^3)/x^5
```

3.64.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^6}\,dx$$

$$\Big\downarrow \; 2602$$

$$\frac{b\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^5}dx}{5(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{5x^5(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \; 2598$$

$$\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{5x^5(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{b\operatorname{arctanh}(\tanh(a+bx))^4}{20x^4(bx-\operatorname{arctanh}(\tanh(a+bx)))^2}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^3/x^6,x]
```

output
```
(b*ArcTanh[Tanh[a + b*x]]^4)/(20*x^4*(b*x - ArcTanh[Tanh[a + b*x]])^2) + A
rcTanh[Tanh[a + b*x]]^4/(5*x^5*(b*x - ArcTanh[Tanh[a + b*x]]))
```

3.64.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.64.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

method	result	size
parallelrisch	$-\frac{b^{3}x^{3}+2b^{2}\operatorname{arctanh}(\tanh(bx+a))x^{2}+3b\operatorname{arctanh}(\tanh(bx+a))^{2}x+4\operatorname{arctanh}(\tanh(bx+a))^{3}}{20x^{5}}$	53
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^{3}}{5x^{5}}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{2}}{4x^{4}}+\frac{b\left(-\frac{b}{6x^{2}}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{3x^{3}}\right)}{2}\right)}{5}$	56
parts	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^{3}}{5x^{5}}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{2}}{4x^{4}}+\frac{b\left(-\frac{b}{6x^{2}}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{3x^{3}}\right)}{2}\right)}{5}$	56
risch	Expression too large to display	7813

input
```
int(arctanh(tanh(b*x+a))^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/20*(b^3*x^3+2*b^2*arctanh(tanh(b*x+a))*x^2+3*b*arctanh(tanh(b*x+a))^2*x
+4*arctanh(tanh(b*x+a))^3)/x^5
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.55

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^6}\,dx = -\frac{10\,b^{3}x^{3}+20\,ab^{2}x^{2}+15\,a^{2}bx+4\,a^{3}}{20\,x^{5}}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^6,x, algorithm="fricas")
```

output
```
-1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

3.64.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^6}\,dx = -\frac{b^{3}}{20x^{2}}-\frac{b^{2}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{10x^{3}} - \frac{3b\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}{20x^{4}}-\frac{\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{5x^{5}}$$

input
```
integrate(atanh(tanh(b*x+a))**3/x**6,x)
```

output
```
-b**3/(20*x**2) - b**2*atanh(tanh(a + b*x))/(10*x**3) - 3*b*atanh(tanh(a +
 b*x))**2/(20*x**4) - atanh(tanh(a + b*x))**3/(5*x**5)
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^6}\,dx = -\frac{1}{20}\,b\left(\frac{b^2}{x^2}+\frac{2\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{x^3}\right) - \frac{3\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{20\,x^4} - \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}{5\,x^5}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^6,x, algorithm="maxima")
```

output
```
-1/20*b*(b^2/x^2 + 2*b*arctanh(tanh(b*x + a))/x^3) - 3/20*b*arctanh(tanh(b
*x + a))^2/x^4 - 1/5*arctanh(tanh(b*x + a))^3/x^5
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.55

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^6}\,dx = -\frac{10\,b^3x^3+20\,ab^2x^2+15\,a^2bx+4\,a^3}{20\,x^5}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^6,x, algorithm="giac")
```

output
```
-1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

3.64.9 Mupad [B] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^6}\,dx = -\frac{\operatorname{atanh}(\tanh(a+b\,x))^3}{5\,x^5} - \frac{b^3}{20\,x^2} - \frac{b^2\operatorname{atanh}(\tanh(a+b\,x))}{10\,x^3} - \frac{3\,b\operatorname{atanh}(\tanh(a+b\,x))^2}{20\,x^4}$$

input
```
int(atanh(tanh(a + b*x))^3/x^6,x)
```

output
```
- atanh(tanh(a + b*x))^3/(5*x^5) - b^3/(20*x^2) - (b^2*atanh(tanh(a + b*x)
))/(10*x^3) - (3*b*atanh(tanh(a + b*x))^2)/(20*x^4)
```

3.65 $\int x^m \textbf{arctanh}(\tanh(a+bx))^4\,dx$

3.65.1 Optimal result

Integrand size = 13, antiderivative size = 154

$$\int x^m \text{arctanh}(\tanh(a+bx))^4\,dx = \frac{24b^4x^{5+m}}{(1+m)(2+m)(3+m)\,(20+9m+m^2)} - \frac{24b^3x^{4+m}\text{arctanh}(\tanh(a+bx))}{(1+m)\,(24+26m+9m^2+m^3)} + \frac{12b^2x^{3+m}\text{arctanh}(\tanh(a+bx))^2}{6+11m+6m^2+m^3} - \frac{4bx^{2+m}\text{arctanh}(\tanh(a+bx))^3}{2+3m+m^2} + \frac{x^{1+m}\text{arctanh}(\tanh(a+bx))^4}{1+m}$$

output
```
24*b^4*x^(5+m)/(1+m)/(2+m)/(3+m)/(m^2+9*m+20)-24*b^3*x^(4+m)*arctanh(tanh(
b*x+a))/(1+m)/(m^3+9*m^2+26*m+24)+12*b^2*x^(3+m)*arctanh(tanh(b*x+a))^2/(m
^3+6*m^2+11*m+6)-4*b*x^(2+m)*arctanh(tanh(b*x+a))^3/(m^2+3*m+2)+x^(1+m)*ar
ctanh(tanh(b*x+a))^4/(1+m)
```

3.65.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.89

$$\int x^m \text{arctanh}(\tanh(a+bx))^4\,dx$$
$$= \frac{x^{1+m}(24b^4x^4 - 24b^3(5+m)x^3\text{arctanh}(\tanh(a+bx)) + 12b^2(20+9m+m^2)\,x^2\text{arctanh}(\tanh(a+bx))^2 -}{(1+m)(2+}$$

input
```
Integrate[x^m*ArcTanh[Tanh[a + b*x]]^4,x]
```

output
```
(x^(1 + m)*(24*b^4*x^4 - 24*b^3*(5 + m)*x^3*ArcTanh[Tanh[a + b*x]] + 12*b^
2*(20 + 9*m + m^2)*x^2*ArcTanh[Tanh[a + b*x]]^2 - 4*b*(60 + 47*m + 12*m^2
+ m^3)*x*ArcTanh[Tanh[a + b*x]]^3 + (120 + 154*m + 71*m^2 + 14*m^3 + m^4)*
ArcTanh[Tanh[a + b*x]]^4))/((1 + m)*(2 + m)*(3 + m)*(4 + m)*(5 + m))
```

3.65.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2599, 2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{arctanh}(\tanh(a+bx))^4\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\text{arctanh}(\tanh(a+bx))^4}{m+1} - \frac{4b\int x^{m+1}\text{arctanh}(\tanh(a+bx))^3dx}{m+1}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\text{arctanh}(\tanh(a+bx))^4}{m+1} - \frac{4b\left(\frac{x^{m+2}\text{arctanh}(\tanh(a+bx))^3}{m+2} - \frac{3b\int x^{m+2}\text{arctanh}(\tanh(a+bx))^2dx}{m+2}\right)}{m+1}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\text{arctanh}(\tanh(a+bx))^4}{m+1} - \frac{4b\left(\frac{x^{m+2}\text{arctanh}(\tanh(a+bx))^3}{m+2} - \frac{3b\left(\frac{x^{m+3}\text{arctanh}(\tanh(a+bx))^2}{m+3} - \frac{2b\int x^{m+3}\text{arctanh}(\tanh(a+bx))dx}{m+3}\right)}{m+2}\right)}{m+1}$$

$$\Big\downarrow \; 2599$$

$$\frac{x^{m+1}\text{arctanh}(\tanh(a+bx))^4}{m+1} - \frac{4b\left(\frac{x^{m+2}\text{arctanh}(\tanh(a+bx))^3}{m+2} - \frac{3b\left(\frac{x^{m+3}\text{arctanh}(\tanh(a+bx))^2}{m+3} - \frac{2b\left(\frac{x^{m+4}\text{arctanh}(\tanh(a+bx))}{m+4} - \frac{b\int x^{m+4}dx}{m+4}\right)}{m+3}\right)}{m+2}\right)}{m+1}$$

$$\Big\downarrow \; 15$$

$$\frac{x^{m+1}\text{arctanh}(\tanh(a+bx))^4}{m+1} - \frac{4b\left(\frac{x^{m+2}\text{arctanh}(\tanh(a+bx))^3}{m+2} - \frac{3b\left(\frac{x^{m+3}\text{arctanh}(\tanh(a+bx))^2}{m+3} - \frac{2b\left(\frac{x^{m+4}\text{arctanh}(\tanh(a+bx))}{m+4} - \frac{bx^{m+5}}{(m+4)(m+5)}\right)}{m+3}\right)}{m+2}\right)}{m+1}$$

input
```
Int[x^m*ArcTanh[Tanh[a + b*x]]^4,x]
```

output
```
(x^(1 + m)*ArcTanh[Tanh[a + b*x]]^4)/(1 + m) - (4*b*((x^(2 + m)*ArcTanh[Ta
nh[a + b*x]]^3)/(2 + m) - (3*b*((x^(3 + m)*ArcTanh[Tanh[a + b*x]]^2)/(3 +
m) - (2*b*(-((b*x^(5 + m))/((4 + m)*(5 + m))) + (x^(4 + m)*ArcTanh[Tanh[a
+ b*x]])/(4 + m)))/(3 + m)))/(2 + m)))/(1 + m)
```

3.65. $\int x^m \text{arctanh}(\tanh(a+bx))^4 \, dx$

3.65.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.65.4 Maple [A] (verified)

Time = 25.64 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.81

method	result
default	$\frac{b^4x^5e^{m\ln(x)}}{5+m}+\frac{\left(a^4+4a^3(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+6a^2(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2+4a(\operatorname{arctanh}(\tanh(bx+a))-bx\right.}{1+m}$
parallelrisch	$-\frac{-108x^3x^m\operatorname{arctanh}(\tanh(bx+a))^2b^2m+48x^2x^m\operatorname{arctanh}(\tanh(bx+a))^3bm^2+188x^2x^m\operatorname{arctanh}(\tanh(bx+a))^3bm+24x^4x^m}{}$
risch	Expression too large to display

input
```
int(x^m*arctanh(tanh(b*x+a))^4,x,method=_RETURNVERBOSE)
```

output
```
b^4/(5+m)*x^5*exp(m*ln(x))+(a^4+4*a^3*(arctanh(tanh(b*x+a))-b*x-a)+6*a^2*(
arctanh(tanh(b*x+a))-b*x-a)^2+4*a*(arctanh(tanh(b*x+a))-b*x-a)^3+(arctanh(
tanh(b*x+a))-b*x-a)^4)/(1+m)*x*exp(m*ln(x))+4*b*(a^3+3*a^2*(arctanh(tanh(b
*x+a))-b*x-a)+3*a*(arctanh(tanh(b*x+a))-b*x-a)^2+(arctanh(tanh(b*x+a))-b*x
-a)^3)/(2+m)*x^2*exp(m*ln(x))+6*b^2*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+
(arctanh(tanh(b*x+a))-b*x-a)^2)/(3+m)*x^3*exp(m*ln(x))+4*b^3*(arctanh(tanh
(b*x+a))-b*x)/(4+m)*x^4*exp(m*ln(x))
```

3.65.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(154) = 308$.

Time = 0.27 (sec) , antiderivative size = 483, normalized size of antiderivative = 3.14

$$\int x^m \operatorname{arctanh}(\tanh(a+bx))^4\,dx$$
$$= \frac{((b^4m^4 + 10\,b^4m^3 + 35\,b^4m^2 + 50\,b^4m + 24\,b^4)x^5 + 4\,(ab^3m^4 + 11\,ab^3m^3 + 41\,ab^3m^2 + 61\,ab^3m + 30\,ab^3)}{}$$

input
```
integrate(x^m*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

output
```
(((b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)*x^5 + 4*(a*b^3*m
^4 + 11*a*b^3*m^3 + 41*a*b^3*m^2 + 61*a*b^3*m + 30*a*b^3)*x^4 + 6*(a^2*b^2
*m^4 + 12*a^2*b^2*m^3 + 49*a^2*b^2*m^2 + 78*a^2*b^2*m + 40*a^2*b^2)*x^3 +
4*(a^3*b*m^4 + 13*a^3*b*m^3 + 59*a^3*b*m^2 + 107*a^3*b*m + 60*a^3*b)*x^2 +
 (a^4*m^4 + 14*a^4*m^3 + 71*a^4*m^2 + 154*a^4*m + 120*a^4)*x)*cosh(m*log(x
)) + ((b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)*x^5 + 4*(a*b
^3*m^4 + 11*a*b^3*m^3 + 41*a*b^3*m^2 + 61*a*b^3*m + 30*a*b^3)*x^4 + 6*(a^2
*b^2*m^4 + 12*a^2*b^2*m^3 + 49*a^2*b^2*m^2 + 78*a^2*b^2*m + 40*a^2*b^2)*x^
3 + 4*(a^3*b*m^4 + 13*a^3*b*m^3 + 59*a^3*b*m^2 + 107*a^3*b*m + 60*a^3*b)*x
^2 + (a^4*m^4 + 14*a^4*m^3 + 71*a^4*m^2 + 154*a^4*m + 120*a^4)*x)*sinh(m*l
og(x)))/(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)
```

3.65.6 Sympy [F]

$$\int x^m \operatorname{arctanh}(\tanh(a+bx))^4\,dx = \text{Too large to display}$$

input
```
integrate(x**m*atanh(tanh(b*x+a))**4,x)
```

output
```
Piecewise((b**4*log(x) - b**3*atanh(tanh(a + b*x))/x - b**2*atanh(tanh(a +
 b*x))**2/(2*x**2) - b*atanh(tanh(a + b*x))**3/(3*x**3) - atanh(tanh(a + b
*x))**4/(4*x**4), Eq(m, -5)), (Integral(atanh(tanh(a + b*x))**4/x**4, x),
Eq(m, -4)), (Integral(atanh(tanh(a + b*x))**4/x**3, x), Eq(m, -3)), (Integ
ral(atanh(tanh(a + b*x))**4/x**2, x), Eq(m, -2)), (Integral(atanh(tanh(a +
 b*x))**4/x, x), Eq(m, -1)), (24*b**4*x**5*x**m/(m**5 + 15*m**4 + 85*m**3
+ 225*m**2 + 274*m + 120) - 24*b**3*m*x**4*x**m*atanh(tanh(a + b*x))/(m**5
 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) - 120*b**3*x**4*x**m*atanh(
tanh(a + b*x))/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 12*b*
*2*m**2*x**3*x**m*atanh(tanh(a + b*x))**2/(m**5 + 15*m**4 + 85*m**3 + 225*
m**2 + 274*m + 120) + 108*b**2*m*x**3*x**m*atanh(tanh(a + b*x))**2/(m**5 +
 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 240*b**2*x**3*x**m*atanh(ta
nh(a + b*x))**2/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) - 4*b*
m**3*x**2*x**m*atanh(tanh(a + b*x))**3/(m**5 + 15*m**4 + 85*m**3 + 225*m**
2 + 274*m + 120) - 48*b*m**2*x**2*x**m*atanh(tanh(a + b*x))**3/(m**5 + 15*
m**4 + 85*m**3 + 225*m**2 + 274*m + 120) - 188*b*m*x**2*x**m*atanh(tanh(a
+ b*x))**3/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) - 240*b*x**
2*x**m*atanh(tanh(a + b*x))**3/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*
m + 120) + m**4*x*x**m*atanh(tanh(a + b*x))**4/(m**5 + 15*m**4 + 85*m**3 +
 225*m**2 + 274*m + 120) + 14*m**3*x*x**m*atanh(tanh(a + b*x))**4/(m**5...
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int x^m \text{arctanh}(\tanh(a+bx))^4\,dx = -\frac{4\,bx^2x^m\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^3}{(m+2)(m+1)} + \frac{x^{m+1}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^4}{m+1} + \frac{12\left(\frac{bx^3x^m\,\text{artanh}(\tanh(bx+a))^2}{(m+3)(m+2)} + \frac{2\left(\frac{b^2x^5x^m}{(m+5)(m+4)(m+3)} - \frac{bx^4x^m\,\text{artanh}(\tanh(bx+a))}{(m+4)(m+3)}\right)b}{m+2}\right)b}{m+1}$$

input
```
integrate(x^m*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

output
```
-4*b*x^2*x^m*arctanh(tanh(b*x + a))^3/((m + 2)*(m + 1)) + x^(m + 1)*arctan
h(tanh(b*x + a))^4/(m + 1) + 12*(b*x^3*x^m*arctanh(tanh(b*x + a))^2/((m +
3)*(m + 2)) + 2*(b^2*x^5*x^m/((m + 5)*(m + 4)*(m + 3)) - b*x^4*x^m*arctanh
(tanh(b*x + a))/((m + 4)*(m + 3)))*b/(m + 2))*b/(m + 1)
```

3.65.8 Giac [F]

$$\int x^m \text{arctanh}(\tanh(a+bx))^4\,dx = \int x^m\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^4\,dx$$

input
```
integrate(x^m*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

output
```
integrate(x^m*arctanh(tanh(b*x + a))^4, x)
```

3.65.9 Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 479, normalized size of antiderivative = 3.11

$$\begin{aligned}&\int x^m \text{arctanh}(\tanh(a+bx))^4\,dx\\&=\frac{x\,x^m\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^4\left(m^4+14\,m^3+71\,m^2+154\,m+120\right)}{16\,m^5+240\,m^4+1360\,m^3+3600\,m^2+4384\,m+1920}\\&\quad+\frac{16\,b^4\,x^m\,x^5\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}{16\,m^5+240\,m^4+1360\,m^3+3600\,m^2+4384\,m+1920}\\&\quad+\frac{24\,b^2\,x^m\,x^3\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2\left(m^4+12\,m^3+49\,m^2+78\,m+40\right)}{16\,m^5+240\,m^4+1360\,m^3+3600\,m^2+4384\,m+1920}\\&\quad-\frac{32\,b^3\,x^m\,x^4\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)\left(m^4+11\,m^3+41\,m^2+61\,m+30\right)}{16\,m^5+240\,m^4+1360\,m^3+3600\,m^2+4384\,m+1920}\\&\quad-\frac{8\,b\,x^m\,x^2\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3\left(m^4+13\,m^3+59\,m^2+107\,m+60\right)}{16\,m^5+240\,m^4+1360\,m^3+3600\,m^2+4384\,m+1920}\end{aligned}$$

input
```
int(x^m*atanh(tanh(a + b*x))^4,x)
```

output
```
(x*x^m*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4*(154*m + 71*m^2 + 14*m^3 + m^4 + 120))/
(4384*m + 3600*m^2 + 1360*m^3 + 240*m^4 + 16*m^5 + 1920) + (16*b^4*x^m*x^5
*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))/(4384*m + 3600*m^2 + 1360*m^3 + 240*
m^4 + 16*m^5 + 1920) + (24*b^2*x^m*x^3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2*(78*m +
 49*m^2 + 12*m^3 + m^4 + 40))/(4384*m + 3600*m^2 + 1360*m^3 + 240*m^4 + 16
*m^5 + 1920) - (32*b^3*x^m*x^4*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)*(61*m + 41*m^2 +
11*m^3 + m^4 + 30))/(4384*m + 3600*m^2 + 1360*m^3 + 240*m^4 + 16*m^5 + 192
0) - (8*b*x^m*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3*(107*m + 59*m^2 + 13*m^3 + m
^4 + 60))/(4384*m + 3600*m^2 + 1360*m^3 + 240*m^4 + 16*m^5 + 1920)
```

3.65. $\int x^m \text{arctanh}(\tanh(a+bx))^4 \, dx$

3.66 $\int x^6 \mathbf{arctanh}(\tanh(a+bx))^4 \, dx$

3.66.1 Optimal result

Integrand size = 13, antiderivative size = 80

$$\begin{aligned}\int x^6 \text{arctanh}(\tanh(a+bx))^4 \, dx = & \frac{b^4 x^{11}}{2310} - \frac{1}{210} b^3 x^{10} \text{arctanh}(\tanh(a+bx)) \\ & + \frac{1}{42} b^2 x^9 \text{arctanh}(\tanh(a+bx))^2 \\ & - \frac{1}{14} b x^8 \text{arctanh}(\tanh(a+bx))^3 \\ & + \frac{1}{7} x^7 \text{arctanh}(\tanh(a+bx))^4\end{aligned}$$

output
```
1/2310*b^4*x^11-1/210*b^3*x^10*arctanh(tanh(b*x+a))+1/42*b^2*x^9*arctanh(t
anh(b*x+a))^2-1/14*b*x^8*arctanh(tanh(b*x+a))^3+1/7*x^7*arctanh(tanh(b*x+a
))^4
```

3.66.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\begin{aligned}&\int x^6 \text{arctanh}(\tanh(a+bx))^4 \, dx \\ &= \frac{x^7 (b^4 x^4 - 11 b^3 x^3 \text{arctanh}(\tanh(a+bx)) + 55 b^2 x^2 \text{arctanh}(\tanh(a+bx))^2 - 165 b x \text{arctanh}(\tanh(a+bx))^3}{2310}\end{aligned}$$

input
```
Integrate[x^6*ArcTanh[Tanh[a + b*x]]^4,x]
```

output
```
(x^7*(b^4*x^4 - 11*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 55*b^2*x^2*ArcTanh[Tan
h[a + b*x]]^2 - 165*b*x*ArcTanh[Tanh[a + b*x]]^3 + 330*ArcTanh[Tanh[a + b*
x]]^4))/2310
```

3.66.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2599, 2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6\text{arctanh}(\tanh(a+bx))^4\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{7}x^7\text{arctanh}(\tanh(a+bx))^4-\frac{4}{7}b\int x^7\text{arctanh}(\tanh(a+bx))^3dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{7}x^7\text{arctanh}(\tanh(a+bx))^4- \frac{4}{7}b\left(\frac{1}{8}x^8\text{arctanh}(\tanh(a+bx))^3-\frac{3}{8}b\int x^8\text{arctanh}(\tanh(a+bx))^2dx\right)$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{7}x^7\text{arctanh}(\tanh(a+bx))^4- \frac{4}{7}b\left(\frac{1}{8}x^8\text{arctanh}(\tanh(a+bx))^3-\frac{3}{8}b\left(\frac{1}{9}x^9\text{arctanh}(\tanh(a+bx))^2-\frac{2}{9}b\int x^9\text{arctanh}(\tanh(a+bx))dx\right)\right)$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{7}x^7\text{arctanh}(\tanh(a+bx))^4- \frac{4}{7}b\left(\frac{1}{8}x^8\text{arctanh}(\tanh(a+bx))^3-\frac{3}{8}b\left(\frac{1}{9}x^9\text{arctanh}(\tanh(a+bx))^2-\frac{2}{9}b\left(\frac{1}{10}x^{10}\text{arctanh}(\tanh(a+bx))-\frac{b\int x^{10}

$$\frac{1}{7}x^7\text{arctanh}(\tanh(a+bx))^4 - \frac{4}{7}b\left(\frac{1}{8}x^8\text{arctanh}(\tanh(a+bx))^3 - \frac{3}{8}b\left(\frac{1}{9}x^9\text{arctanh}(\tanh(a+bx))^2 - \frac{2}{9}b\left(\frac{1}{10}x^{10}\text{arctanh}(\tanh(a+bx)) - \frac{bx^{11}}{110}\right)\right.\right.$$

input
```
Int[x^6*ArcTanh[Tanh[a + b*x]]^4,x]
```

output
```
(x^7*ArcTanh[Tanh[a + b*x]]^4)/7 - (4*b*((x^8*ArcTanh[Tanh[a + b*x]]^3)/8
- (3*b*((x^9*ArcTanh[Tanh[a + b*x]]^2)/9 - (2*b*(-1/110*(b*x^11) + (x^10*A
rcTanh[Tanh[a + b*x]])/10))/9))/8))/7
```

3.66.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.66.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\frac{x^7 \operatorname{arctanh}(\tanh(bx+a))^4}{7} - \frac{4b\left(\frac{x^8 \operatorname{arctanh}(\tanh(bx+a))^3}{8} - \frac{3b\left(\frac{x^9 \operatorname{arctanh}(\tanh(bx+a))^2}{9} - \frac{2b\left(\frac{x^{10}\operatorname{arctanh}(\tanh(bx+a))}{10} - \frac{x^{11}b}{110}\right)}{9}\right)}{8}\right)}{7}$$

input
```
int(x^6*arctanh(tanh(b*x+a))^4,x)
```

output
```
1/7*x^7*arctanh(tanh(b*x+a))^4-4/7*b*(1/8*x^8*arctanh(tanh(b*x+a))^3-3/8*b
*(1/9*x^9*arctanh(tanh(b*x+a))^2-2/9*b*(1/10*x^10*arctanh(tanh(b*x+a))-1/1
10*x^11*b)))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int x^6 \text{arctanh}(\tanh(a+bx))^4\,dx = \frac{1}{11}\,b^4x^{11} + \frac{2}{5}\,ab^3x^{10} + \frac{2}{3}\,a^2b^2x^9 + \frac{1}{2}\,a^3bx^8 + \frac{1}{7}\,a^4x^7$$

input
```
integrate(x^6*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

output
```
1/11*b^4*x^11 + 2/5*a*b^3*x^10 + 2/3*a^2*b^2*x^9 + 1/2*a^3*b*x^8 + 1/7*a^4
*x^7
```

3.66.6 Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int x^6 \text{arctanh}(\tanh(a+bx))^4\,dx = \frac{b^4x^{11}}{2310} - \frac{b^3x^{10}\,\text{atanh}\,(\tanh\,(a+bx))}{210} + \frac{b^2x^9\,\text{atanh}^2\,(\tanh\,(a+bx))}{42} - \frac{bx^8\,\text{atanh}^3\,(\tanh\,(a+bx))}{14} + \frac{x^7\,\text{atanh}^4\,(\tanh\,(a+bx))}{7}$$

input
```
integrate(x**6*atanh(tanh(b*x+a))**4,x)
```

output
```
b**4*x**11/2310 - b**3*x**10*atanh(tanh(a + b*x))/210 + b**2*x**9*atanh(ta
nh(a + b*x))**2/42 - b*x**8*atanh(tanh(a + b*x))**3/14 + x**7*atanh(tanh(a
 + b*x))**4/7
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int x^6 \text{arctanh}(\tanh(a+bx))^4 \, dx$$
$$= -\frac{1}{14} bx^8 \operatorname{artanh}(\tanh(bx+a))^3 + \frac{1}{7} x^7 \operatorname{artanh}(\tanh(bx+a))^4$$
$$+ \frac{1}{2310} \left(55 bx^9 \operatorname{artanh}(\tanh(bx+a))^2 + \left(b^2 x^{11} - 11 bx^{10} \operatorname{artanh}(\tanh(bx+a))\right) b\right) b$$

input
```
integrate(x^6*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

output
```
-1/14*b*x^8*arctanh(tanh(b*x + a))^3 + 1/7*x^7*arctanh(tanh(b*x + a))^4 +
1/2310*(55*b*x^9*arctanh(tanh(b*x + a))^2 + (b^2*x^11 - 11*b*x^10*arctanh(
tanh(b*x + a)))*b)*b
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int x^6 \text{arctanh}(\tanh(a+bx))^4 \, dx = \frac{1}{11} b^4 x^{11} + \frac{2}{5} ab^3 x^{10} + \frac{2}{3} a^2 b^2 x^9 + \frac{1}{2} a^3 bx^8 + \frac{1}{7} a^4 x^7$$

input
```
integrate(x^6*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

output
```
1/11*b^4*x^11 + 2/5*a*b^3*x^10 + 2/3*a^2*b^2*x^9 + 1/2*a^3*b*x^8 + 1/7*a^4
*x^7
```

3.66.9 Mupad [B] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.02

$$\int x^6 \operatorname{arctanh}(\tanh(a+bx))^4\,dx = \frac{x^7\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^4}{112}+\frac{b^4\,x^{11}}{11} - \frac{b\,x^8\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^3}{16} - \frac{b^3\,x^{10}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}{5} + \frac{b^2\,x^9\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^2}{6}$$

input
```
int(x^6*atanh(tanh(a + b*x))^4,x)
```

output
```
(x^7*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)/112 + (b^4*x^11)/11 - (b*x^8*(log(2/(exp
(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^3)/16 - (b^3*x^10*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/5 + (b^2*x^9*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^2)/6
```

3.67 $\int x^5 \textbf{arctanh}(\tanh(a+bx))^4\,dx$

3.67.1 Optimal result

Integrand size = 13, antiderivative size = 80

$$\int x^5\operatorname{arctanh}(\tanh(a+bx))^4\,dx=\frac{b^4x^{10}}{1260}-\frac{1}{126}b^3x^9\operatorname{arctanh}(\tanh(a+bx))+\frac{1}{28}b^2x^8\operatorname{arctanh}(\tanh(a+bx))^2-\frac{2}{21}bx^7\operatorname{arctanh}(\tanh(a+bx))^3+\frac{1}{6}x^6\operatorname{arctanh}(\tanh(a+bx))^4$$

output
```
1/1260*b^4*x^10-1/126*b^3*x^9*arctanh(tanh(b*x+a))+1/28*b^2*x^8*arctanh(ta
nh(b*x+a))^2-2/21*b*x^7*arctanh(tanh(b*x+a))^3+1/6*x^6*arctanh(tanh(b*x+a)
)^4
```

3.67.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int x^5\operatorname{arctanh}(\tanh(a+bx))^4\,dx=\frac{x^6(b^4x^4-10b^3x^3\operatorname{arctanh}(\tanh(a+bx))+45b^2x^2\operatorname{arctanh}(\tanh(a+bx))^2-120bx\operatorname{arctanh}(\tanh(a+bx))^3}{1260}$$

input
```
Integrate[x^5*ArcTanh[Tanh[a + b*x]]^4,x]
```

output
```
(x^6*(b^4*x^4 - 10*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 45*b^2*x^2*ArcTanh[Tan
h[a + b*x]]^2 - 120*b*x*ArcTanh[Tanh[a + b*x]]^3 + 210*ArcTanh[Tanh[a + b*
x]]^4))/1260
```

3.67.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2599, 2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \text{arctanh}(\tanh(a+bx))^4 \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{6}x^6\text{arctanh}(\tanh(a+bx))^4-\frac{2}{3}b\int x^6\text{arctanh}(\tanh(a+bx))^3dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{6}x^6\text{arctanh}(\tanh(a+bx))^4- \frac{2}{3}b\left(\frac{1}{7}x^7\text{arctanh}(\tanh(a+bx))^3-\frac{3}{7}b\int x^7\text{arctanh}(\tanh(a+bx))^2dx\right)$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{6}x^6\text{arctanh}(\tanh(a+bx))^4- \frac{2}{3}b\left(\frac{1}{7}x^7\text{arctanh}(\tanh(a+bx))^3-\frac{3}{7}b\left(\frac{1}{8}x^8\text{arctanh}(\tanh(a+bx))^2-\frac{1}{4}b\int x^8\text{arctanh}(\tanh(a+bx))dx\right)\right)$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{6}x^6\text{arctanh}(\tanh(a+bx))^4- \frac{2}{3}b\left(\frac{1}{7}x^7\text{arctanh}(\tanh(a+bx))^3-\frac{3}{7}b\left(\frac{1}{8}x^8\text{arctanh}(\tanh(a+bx))^2-\frac{1}{4}b\left(\frac{1}{9}x^9\text{arctanh}(\tanh(a+bx))-\frac{b\int x^9dx}{9}\right)\right)\right)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{6}x^6\text{arctanh}(\tanh(a+bx))^4 - \frac{2}{3}b\left(\frac{1}{7}x^7\text{arctanh}(\tanh(a+bx))^3 - \frac{3}{7}b\left(\frac{1}{8}x^8\text{arctanh}(\tanh(a+bx))^2 - \frac{1}{4}b\left(\frac{1}{9}x^9\text{arctanh}(\tanh(a+bx)) - \frac{bx^{10}}{90}\right)\right)\right)$$

input
```
Int[x^5*ArcTanh[Tanh[a + b*x]]^4,x]
```

output
```
(x^6*ArcTanh[Tanh[a + b*x]]^4)/6 - (2*b*((x^7*ArcTanh[Tanh[a + b*x]]^3)/7
- (3*b*((x^8*ArcTanh[Tanh[a + b*x]]^2)/8 - (b*(-1/90*(b*x^10) + (x^9*ArcTa
nh[Tanh[a + b*x]])/9))/4))/7))/3
```

3.67.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.67.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\frac{x^6\,\text{arctanh}\left(\tanh\left(bx+a\right)\right)^4}{6} - \frac{2b\left(\frac{x^7\,\text{arctanh}(\tanh(bx+a))^3}{7} - \frac{3b\left(\frac{x^8\,\text{arctanh}(\tanh(bx+a))^2}{8} - \frac{b\left(\frac{x^9\,\text{arctanh}(\tanh(bx+a))}{9} - \frac{x^{10}b}{90}\right)}{4}\right)}{7}\right)}{3}$$

input
```
int(x^5*arctanh(tanh(b*x+a))^4,x)
```

output
```
1/6*x^6*arctanh(tanh(b*x+a))^4-2/3*b*(1/7*x^7*arctanh(tanh(b*x+a))^3-3/7*b
*(1/8*x^8*arctanh(tanh(b*x+a))^2-1/4*b*(1/9*x^9*arctanh(tanh(b*x+a))-1/90*
x^10*b)))
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int x^5 \operatorname{arctanh}(\tanh(a+bx))^4\,dx = \frac{1}{10}\,b^4x^{10} + \frac{4}{9}\,ab^3x^9 + \frac{3}{4}\,a^2b^2x^8 + \frac{4}{7}\,a^3bx^7 + \frac{1}{6}\,a^4x^6$$

input
```
integrate(x^5*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

output
```
1/10*b^4*x^10 + 4/9*a*b^3*x^9 + 3/4*a^2*b^2*x^8 + 4/7*a^3*b*x^7 + 1/6*a^4*
x^6
```

3.67.6 Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int x^5 \operatorname{arctanh}(\tanh(a+bx))^4\,dx = \frac{b^4x^{10}}{1260} - \frac{b^3x^9 \operatorname{atanh}(\tanh(a+bx))}{126} + \frac{b^2x^8 \operatorname{atanh}^2(\tanh(a+bx))}{28} - \frac{2bx^7 \operatorname{atanh}^3(\tanh(a+bx))}{21} + \frac{x^6 \operatorname{atanh}^4(\tanh(a+bx))}{6}$$

input
```
integrate(x**5*atanh(tanh(b*x+a))**4,x)
```

output
```
b**4*x**10/1260 - b**3*x**9*atanh(tanh(a + b*x))/126 + b**2*x**8*atanh(tan
h(a + b*x))**2/28 - 2*b*x**7*atanh(tanh(a + b*x))**3/21 + x**6*atanh(tanh(
a + b*x))**4/6
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int x^5\text{arctanh}(\tanh(a+bx))^4\,dx$$
$$= -\frac{2}{21}\,bx^7\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^3+\frac{1}{6}\,x^6\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^4$$
$$+\frac{1}{1260}\left(45\,bx^8\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2+\left(b^2x^{10}-10\,bx^9\,\text{artanh}\left(\tanh\left(bx+a\right)\right)\right)b\right)b$$

input
```
integrate(x^5*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

output
```
-2/21*b*x^7*arctanh(tanh(b*x + a))^3 + 1/6*x^6*arctanh(tanh(b*x + a))^4 +
1/1260*(45*b*x^8*arctanh(tanh(b*x + a))^2 + (b^2*x^10 - 10*b*x^9*arctanh(t
anh(b*x + a)))*b)*b
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int x^5\text{arctanh}(\tanh(a+bx))^4\,dx = \frac{1}{10}\,b^4x^{10}+\frac{4}{9}\,ab^3x^9+\frac{3}{4}\,a^2b^2x^8+\frac{4}{7}\,a^3bx^7+\frac{1}{6}\,a^4x^6$$

input
```
integrate(x^5*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

output
```
1/10*b^4*x^10 + 4/9*a*b^3*x^9 + 3/4*a^2*b^2*x^8 + 4/7*a^3*b*x^7 + 1/6*a^4*
x^6
```

3.67.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.02

$$\int x^5 \operatorname{arctanh}(\tanh(a+bx))^4\,dx = \frac{x^6\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^4}{96}+\frac{b^4\,x^{10}}{10} - \frac{b\,x^7\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^3}{14} - \frac{2\,b^3\,x^9\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}{9} + \frac{3\,b^2\,x^8\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^2}{16}$$

input
```
int(x^5*atanh(tanh(a + b*x))^4,x)
```

output
```
(x^6*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)/96 + (b^4*x^10)/10 - (b*x^7*(log(2/(exp(
2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x)^3)/14 - (2*b^3*x^9*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/9 + (3*b^2*x^8
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^2)/16
```

3.68 $\int x^4 \textbf{arctanh}(\tanh(a+bx))^4\,dx$

3.68.1 Optimal result

Integrand size = 13, antiderivative size = 80

$$\int x^4\text{arctanh}(\tanh(a+bx))^4\,dx = \frac{b^4x^9}{630} - \frac{1}{70}b^3x^8\text{arctanh}(\tanh(a+bx)) + \frac{2}{35}b^2x^7\text{arctanh}(\tanh(a+bx))^2 - \frac{2}{15}bx^6\text{arctanh}(\tanh(a+bx))^3 + \frac{1}{5}x^5\text{arctanh}(\tanh(a+bx))^4$$

output
```
1/630*b^4*x^9-1/70*b^3*x^8*arctanh(tanh(b*x+a))+2/35*b^2*x^7*arctanh(tanh(
b*x+a))^2-2/15*b*x^6*arctanh(tanh(b*x+a))^3+1/5*x^5*arctanh(tanh(b*x+a))^4
```

3.68.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int x^4\text{arctanh}(\tanh(a+bx))^4\,dx = \frac{1}{630}x^5\big(b^4x^4 - 9b^3x^3\text{arctanh}(\tanh(a+bx)) + 36b^2x^2\text{arctanh}(\tanh(a+bx))^2 - 84bx\text{arctanh}(\tanh(a+bx))^3 + 126\text{arctanh}(\tanh(a+bx))^4\big)$$

input
```
Integrate[x^4*ArcTanh[Tanh[a + b*x]]^4,x]
```

output
```
(x^5*(b^4*x^4 - 9*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 36*b^2*x^2*ArcTanh[Tanh
[a + b*x]]^2 - 84*b*x*ArcTanh[Tanh[a + b*x]]^3 + 126*ArcTanh[Tanh[a + b*x]
]^4))/630
```

3.68.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2599, 2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{arctanh}(\tanh(a+bx))^4 \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{5}x^5\text{arctanh}(\tanh(a+bx))^4-\frac{4}{5}b\int x^5\text{arctanh}(\tanh(a+bx))^3dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{5}x^5\text{arctanh}(\tanh(a+bx))^4-\frac{4}{5}b\left(\frac{1}{6}x^6\text{arctanh}(\tanh(a+bx))^3-\frac{1}{2}b\int x^6\text{arctanh}(\tanh(a+bx))^2dx\right)$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{5}x^5\text{arctanh}(\tanh(a+bx))^4-\frac{4}{5}b\left(\frac{1}{6}x^6\text{arctanh}(\tanh(a+bx))^3-\frac{1}{2}b\left(\frac{1}{7}x^7\text{arctanh}(\tanh(a+bx))^2-\frac{2}{7}b\int x^7\text{arctanh}(\tanh(a+bx))dx\right)\right)$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{5}x^5\text{arctanh}(\tanh(a+bx))^4-\frac{4}{5}b\left(\frac{1}{6}x^6\text{arctanh}(\tanh(a+bx))^3-\frac{1}{2}b\left(\frac{1}{7}x^7\text{arctanh}(\tanh(a+bx))^2-\frac{2}{7}b\left(\frac{1}{8}x^8\text{arctanh}(\tanh(a+bx))-\frac{b\int x^8dx}{8}\right.\right.\right.$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{5}x^5\text{arctanh}(\tanh(a+bx))^4-\frac{4}{5}b\left(\frac{1}{6}x^6\text{arctanh}(\tanh(a+bx))^3-\frac{1}{2}b\left(\frac{1}{7}x^7\text{arctanh}(\tanh(a+bx))^2-\frac{2}{7}b\left(\frac{1}{8}x^8\text{arctanh}(\tanh(a+bx))-\frac{bx^9}{72}\right)\right)\right)$$

input
```
Int[x^4*ArcTanh[Tanh[a + b*x]]^4,x]
```

output
```
(x^5*ArcTanh[Tanh[a + b*x]]^4)/5 - (4*b*((x^6*ArcTanh[Tanh[a + b*x]]^3)/6
- (b*((x^7*ArcTanh[Tanh[a + b*x]]^2)/7 - (2*b*(-1/72*(b*x^9) + (x^8*ArcTan
h[Tanh[a + b*x]])/8))/7))/2))/5
```

3.68.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.68.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\frac{x^5\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^4}{5}-\frac{4b\left(\frac{x^6\operatorname{arctanh}(\tanh(bx+a))^3}{6}-\frac{b\left(\frac{x^7\operatorname{arctanh}(\tanh(bx+a))^2}{7}-\frac{2b\left(\frac{x^8\operatorname{arctanh}(\tanh(bx+a))}{8}-\frac{b\,x^9}{72}\right)}{7}\right)}{2}\right)}{5}$$

input
```
int(x^4*arctanh(tanh(b*x+a))^4,x)
```

output
```
1/5*x^5*arctanh(tanh(b*x+a))^4-4/5*b*(1/6*x^6*arctanh(tanh(b*x+a))^3-1/2*b
*(1/7*x^7*arctanh(tanh(b*x+a))^2-2/7*b*(1/8*x^8*arctanh(tanh(b*x+a))-1/72*
b*x^9)))
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int x^4 \operatorname{arctanh}(\tanh(a+bx))^4\,dx = \frac{1}{9}\,b^4x^9 + \frac{1}{2}\,ab^3x^8 + \frac{6}{7}\,a^2b^2x^7 + \frac{2}{3}\,a^3bx^6 + \frac{1}{5}\,a^4x^5$$

input
```
integrate(x^4*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

output
```
1/9*b^4*x^9 + 1/2*a*b^3*x^8 + 6/7*a^2*b^2*x^7 + 2/3*a^3*b*x^6 + 1/5*a^4*x^
5
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int x^4 \operatorname{arctanh}(\tanh(a+bx))^4\,dx = \frac{b^4x^9}{630} - \frac{b^3x^8\operatorname{atanh}(\tanh(a+bx))}{70} + \frac{2b^2x^7\operatorname{atanh}^2(\tanh(a+bx))}{35} - \frac{2bx^6\operatorname{atanh}^3(\tanh(a+bx))}{15} + \frac{x^5\operatorname{atanh}^4(\tanh(a+bx))}{5}$$

input
```
integrate(x**4*atanh(tanh(b*x+a))**4,x)
```

output
```
b**4*x**9/630 - b**3*x**8*atanh(tanh(a + b*x))/70 + 2*b**2*x**7*atanh(tanh
(a + b*x))**2/35 - 2*b*x**6*atanh(tanh(a + b*x))**3/15 + x**5*atanh(tanh(a
 + b*x))**4/5
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int x^4 \text{arctanh}(\tanh(a+bx))^4\,dx = -\frac{2}{15}\,bx^6 \,\text{artanh}\left(\tanh\left(bx+a\right)\right)^3 + \frac{1}{5}\,x^5 \,\text{artanh}\left(\tanh\left(bx+a\right)\right)^4 + \frac{1}{630}\left(36\,bx^7 \,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2 + \left(b^2x^9 - 9\,bx^8 \,\text{artanh}\left(\tanh\left(bx+a\right)\right)\right)b\right)b$$

input
```
integrate(x^4*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

output
```
-2/15*b*x^6*arctanh(tanh(b*x + a))^3 + 1/5*x^5*arctanh(tanh(b*x + a))^4 +
1/630*(36*b*x^7*arctanh(tanh(b*x + a))^2 + (b^2*x^9 - 9*b*x^8*arctanh(tanh
(b*x + a)))*b)*b
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int x^4 \text{arctanh}(\tanh(a+bx))^4\,dx = \frac{1}{9}\,b^4x^9 + \frac{1}{2}\,ab^3x^8 + \frac{6}{7}\,a^2b^2x^7 + \frac{2}{3}\,a^3bx^6 + \frac{1}{5}\,a^4x^5$$

input
```
integrate(x^4*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

output
```
1/9*b^4*x^9 + 1/2*a*b^3*x^8 + 6/7*a^2*b^2*x^7 + 2/3*a^3*b*x^6 + 1/5*a^4*x^
5
```

3.68.9 Mupad [B] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int x^4 \text{arctanh}(\tanh(a+bx))^4\,dx = \frac{\text{atanh}(\tanh(a+b\,x))^5\,\left(126\,b^4\,x^4 - 84\,b^3\,x^3\,\text{atanh}(\tanh(a+b\,x)) + 36\,b^2\,x^2\,\text{atanh}(\tanh(a+b\,x))^2 - 9\,b\,x\right.}{630\,b^5}$$

input
```
int(x^4*atanh(tanh(a + b*x))^4,x)
```

output
```
(atanh(tanh(a + b*x))^5*(atanh(tanh(a + b*x))^4 + 126*b^4*x^4 + 36*b^2*x^2
*atanh(tanh(a + b*x))^2 - 9*b*x*atanh(tanh(a + b*x))^3 - 84*b^3*x^3*atanh(
tanh(a + b*x))))/(630*b^5)
```

3.68. $\int x^4 \text{arctanh}(\tanh(a+bx))^4 \, dx$

3.69 $\int x^3 \textbf{arctanh}(\tanh(a+bx))^4\,dx$

3.69.1 Optimal result

Integrand size = 13, antiderivative size = 72

$$\int x^3\text{arctanh}(\tanh(a+bx))^4\,dx=\frac{x^3\text{arctanh}(\tanh(a+bx))^5}{5b}-\frac{x^2\text{arctanh}(\tanh(a+bx))^6}{10b^2}+\frac{x\text{arctanh}(\tanh(a+bx))^7}{35b^3}-\frac{\text{arctanh}(\tanh(a+bx))^8}{280b^4}$$

output
```
1/5*x^3*arctanh(tanh(b*x+a))^5/b-1/10*x^2*arctanh(tanh(b*x+a))^6/b^2+1/35*
x*arctanh(tanh(b*x+a))^7/b^3-1/280*arctanh(tanh(b*x+a))^8/b^4
```

3.69.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int x^3\text{arctanh}(\tanh(a+bx))^4\,dx=\frac{1}{280}x^4\left(b^4x^4-8b^3x^3\text{arctanh}(\tanh(a+bx))+28b^2x^2\text{arctanh}(\tanh(a+bx))^2-56bx\text{arctanh}(\tanh(a+bx))^3+70\text{arctanh}(\tanh(a+bx))^4\right)$$

input
```
Integrate[x^3*ArcTanh[Tanh[a + b*x]]^4,x]
```

output
```
(x^4*(b^4*x^4 - 8*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 28*b^2*x^2*ArcTanh[Tanh
[a + b*x]]^2 - 56*b*x*ArcTanh[Tanh[a + b*x]]^3 + 70*ArcTanh[Tanh[a + b*x]]
^4))/280
```

3.69.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{arctanh}(\tanh(a+bx))^4 \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^3 \text{arctanh}(\tanh(a+bx))^5}{5b} - \frac{3 \int x^2 \text{arctanh}(\tanh(a+bx))^5 dx}{5b}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^3 \text{arctanh}(\tanh(a+bx))^5}{5b} - \frac{3 \left(\frac{x^2 \text{arctanh}(\tanh(a+bx))^6}{6b} - \frac{\int x \text{arctanh}(\tanh(a+bx))^6 dx}{3b}\right)}{5b}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^3 \text{arctanh}(\tanh(a+bx))^5}{5b} - \frac{3 \left(\frac{x^2 \text{arctanh}(\tanh(a+bx))^6}{6b} - \frac{\frac{x \text{arctanh}(\tanh(a+bx))^7}{7b} - \frac{\int \text{arctanh}(\tanh(a+bx))^7 dx}{7b}}{3b}\right)}{5b}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{x^3 \text{arctanh}(\tanh(a+bx))^5}{5b} - \frac{3 \left(\frac{x^2 \text{arctanh}(\tanh(a+bx))^6}{6b} - \frac{\frac{x \text{arctanh}(\tanh(a+bx))^7}{7b} - \frac{\int \text{arctanh}(\tanh(a+bx))^7 d\text{arctanh}(\tanh(a+bx))}{7b^2}}{3b}\right)}{5b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x^3 \text{arctanh}(\tanh(a+bx))^5}{5b} - \frac{3 \left(\frac{x^2 \text{arctanh}(\tanh(a+bx))^6}{6b} - \frac{\frac{x \text{arctanh}(\tanh(a+bx))^7}{7b} - \frac{\text{arctanh}(\tanh(a+bx))^8}{56b^2}}{3b}\right)}{5b}$$

input
```
Int[x^3*ArcTanh[Tanh[a + b*x]]^4,x]
```

output
```
(x^3*ArcTanh[Tanh[a + b*x]]^5)/(5*b) - (3*((x^2*ArcTanh[Tanh[a + b*x]]^6)/
(6*b) - ((x*ArcTanh[Tanh[a + b*x]]^7)/(7*b) - ArcTanh[Tanh[a + b*x]]^8/(56
*b^2))/(3*b)))/(5*b)
```

3.69.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.69.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\frac{x^4 \operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^4}{4} - b\left(\frac{x^5 \operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^3}{5} - \frac{3b\left(\frac{x^6 \operatorname{arctanh}(\tanh(bx+a))^2}{6} - \frac{b\left(\frac{x^7 \operatorname{arctanh}(\tanh(bx+a))}{7}\right.}{3}\right.}{5}\right.$$

input
```
int(x^3*arctanh(tanh(b*x+a))^4,x)
```

output
```
1/4*x^4*arctanh(tanh(b*x+a))^4-b*(1/5*x^5*arctanh(tanh(b*x+a))^3-3/5*b*(1/
6*x^6*arctanh(tanh(b*x+a))^2-1/3*b*(1/7*x^7*arctanh(tanh(b*x+a))-1/56*b*x^
8)))
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int x^3 \text{arctanh}(\tanh(a+bx))^4\,dx = \frac{1}{8}\,b^4x^8 + \frac{4}{7}\,ab^3x^7 + a^2b^2x^6 + \frac{4}{5}\,a^3bx^5 + \frac{1}{4}\,a^4x^4$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

output
```
1/8*b^4*x^8 + 4/7*a*b^3*x^7 + a^2*b^2*x^6 + 4/5*a^3*b*x^5 + 1/4*a^4*x^4
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int x^3 \text{arctanh}(\tanh(a+bx))^4\,dx = \frac{b^4x^8}{280} - \frac{b^3x^7\,\text{atanh}\,(\tanh\,(a+bx))}{35} + \frac{b^2x^6\,\text{atanh}^2\,(\tanh\,(a+bx))}{10} - \frac{bx^5\,\text{atanh}^3\,(\tanh\,(a+bx))}{5} + \frac{x^4\,\text{atanh}^4\,(\tanh\,(a+bx))}{4}$$

input
```
integrate(x**3*atanh(tanh(b*x+a))**4,x)
```

output
```
b**4*x**8/280 - b**3*x**7*atanh(tanh(a + b*x))/35 + b**2*x**6*atanh(tanh(a
 + b*x))**2/10 - b*x**5*atanh(tanh(a + b*x))**3/5 + x**4*atanh(tanh(a + b*
x))**4/4
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int x^3 \text{arctanh}(\tanh(a+bx))^4\,dx = -\frac{1}{5}\,bx^5\,\text{artanh}\,(\tanh\,(bx+a))^3 + \frac{1}{4}\,x^4\,\text{artanh}\,(\tanh\,(bx+a))^4 + \frac{1}{280}\,\left(28\,bx^6\,\text{artanh}\,(\tanh\,(bx+a))^2 + \left(b^2x^8 - 8\,bx^7\,\text{artanh}\,(\tanh\,(bx+a))\right)b\right)b$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

output
```
-1/5*b*x^5*arctanh(tanh(b*x + a))^3 + 1/4*x^4*arctanh(tanh(b*x + a))^4 + 1
/280*(28*b*x^6*arctanh(tanh(b*x + a))^2 + (b^2*x^8 - 8*b*x^7*arctanh(tanh(
b*x + a)))*b)*b
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int x^3\text{arctanh}(\tanh(a+bx))^4\,dx = \frac{1}{8}\,b^4x^8+\frac{4}{7}\,ab^3x^7+a^2b^2x^6+\frac{4}{5}\,a^3bx^5+\frac{1}{4}\,a^4x^4$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

output
```
1/8*b^4*x^8 + 4/7*a*b^3*x^7 + a^2*b^2*x^6 + 4/5*a^3*b*x^5 + 1/4*a^4*x^4
```

3.69.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int x^3\text{arctanh}(\tanh(a+bx))^4\,dx = \frac{b^4\,x^8}{280}-\frac{b^3\,x^7\,\text{atanh}(\tanh(a+b\,x))}{35}+\frac{b^2\,x^6\,\text{atanh}(\tanh(a+b\,x))^2}{10}-\frac{b\,x^5\,\text{atanh}(\tanh(a+b\,x))^3}{5}+\frac{x^4\,\text{atanh}(\tanh(a+b\,x))^4}{4}$$

input
```
int(x^3*atanh(tanh(a + b*x))^4,x)
```

output
```
(x^4*atanh(tanh(a + b*x))^4)/4 + (b^4*x^8)/280 + (b^2*x^6*atanh(tanh(a + b
*x))^2)/10 - (b*x^5*atanh(tanh(a + b*x))^3)/5 - (b^3*x^7*atanh(tanh(a + b*
x)))/35
```

3.70 $\int x^2 \textbf{arctanh}(\tanh(a+bx))^4 \, dx$

3.70.1 Optimal result

Integrand size = 13, antiderivative size = 53

$$\int x^2 \text{arctanh}(\tanh(a+bx))^4 \, dx = \frac{x^2 \text{arctanh}(\tanh(a+bx))^5}{5b} - \frac{x \text{arctanh}(\tanh(a+bx))^6}{15b^2} + \frac{\text{arctanh}(\tanh(a+bx))^7}{105b^3}$$

output
```
1/5*x^2*arctanh(tanh(b*x+a))^5/b-1/15*x*arctanh(tanh(b*x+a))^6/b^2+1/105*a
rctanh(tanh(b*x+a))^7/b^3
```

3.70.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

$$\int x^2 \text{arctanh}(\tanh(a+bx))^4 \, dx = \frac{1}{105} x^3 \left(b^4 x^4 - 7b^3 x^3 \text{arctanh}(\tanh(a+bx)) + 21b^2 x^2 \text{arctanh}(\tanh(a+bx))^2 - 35bx \text{arctanh}(\tanh(a+bx))^3 + 35 \text{arctanh}(\tanh(a+bx))^4\right)$$

input
```
Integrate[x^2*ArcTanh[Tanh[a + b*x]]^4,x]
```

output
```
(x^3*(b^4*x^4 - 7*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 21*b^2*x^2*ArcTanh[Tanh
[a + b*x]]^2 - 35*b*x*ArcTanh[Tanh[a + b*x]]^3 + 35*ArcTanh[Tanh[a + b*x]]
^4))/105
```

3.70.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arctanh}(\tanh(a+bx))^4 \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^2 \text{arctanh}(\tanh(a+bx))^5}{5b} - \frac{2\int x \text{arctanh}(\tanh(a+bx))^5 dx}{5b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^2 \text{arctanh}(\tanh(a+bx))^5}{5b} - \frac{2\left(\frac{x \text{arctanh}(\tanh(a+bx))^6}{6b} - \frac{\int \text{arctanh}(\tanh(a+bx))^6 dx}{6b}\right)}{5b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{x^2 \text{arctanh}(\tanh(a+bx))^5}{5b} - \frac{2\left(\frac{x \text{arctanh}(\tanh(a+bx))^6}{6b} - \frac{\int \text{arctanh}(\tanh(a+bx))^6 d\text{arctanh}(\tanh(a+bx))}{6b^2}\right)}{5b}$$

$$\Big\downarrow \text{15}$$

$$\frac{x^2 \text{arctanh}(\tanh(a+bx))^5}{5b} - \frac{2\left(\frac{x \text{arctanh}(\tanh(a+bx))^6}{6b} - \frac{\text{arctanh}(\tanh(a+bx))^7}{42b^2}\right)}{5b}$$

input
```
Int[x^2*ArcTanh[Tanh[a + b*x]]^4,x]
```

output
```
(x^2*ArcTanh[Tanh[a + b*x]]^5)/(5*b) - (2*((x*ArcTanh[Tanh[a + b*x]]^6)/(6
*b) - ArcTanh[Tanh[a + b*x]]^7/(42*b^2)))/(5*b)
```

3.70.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.70.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\frac{x^3 \operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^4}{3}-\frac{4b\left(\frac{x^4 \operatorname{arctanh}(\tanh(bx+a))^3}{4}-\frac{3b\left(\frac{x^5 \operatorname{arctanh}(\tanh(bx+a))^2}{5}-\frac{2b\left(\frac{x^6 \operatorname{arctanh}(\tanh(bx+a))}{6}-\frac{b\,x^7}{42}\right)}{5}\right)}{4}\right)}{3}$$

input
```
int(x^2*arctanh(tanh(b*x+a))^4,x)
```

output
```
1/3*x^3*arctanh(tanh(b*x+a))^4-4/3*b*(1/4*x^4*arctanh(tanh(b*x+a))^3-3/4*b
*(1/5*x^5*arctanh(tanh(b*x+a))^2-2/5*b*(1/6*x^6*arctanh(tanh(b*x+a))-1/42*
b*x^7)))
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int x^2 \text{arctanh}(\tanh(a+bx))^4\,dx = \frac{1}{7}\,b^4x^7 + \frac{2}{3}\,ab^3x^6 + \frac{6}{5}\,a^2b^2x^5 + a^3bx^4 + \frac{1}{3}\,a^4x^3$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

output
```
1/7*b^4*x^7 + 2/3*a*b^3*x^6 + 6/5*a^2*b^2*x^5 + a^3*b*x^4 + 1/3*a^4*x^3
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.42

$$\int x^2 \text{arctanh}(\tanh(a+bx))^4\,dx = \frac{b^4x^7}{105} - \frac{b^3x^6\,\text{atanh}\left(\tanh\left(a+bx\right)\right)}{15} + \frac{b^2x^5\,\text{atanh}^2\left(\tanh\left(a+bx\right)\right)}{5} - \frac{bx^4\,\text{atanh}^3\left(\tanh\left(a+bx\right)\right)}{3} + \frac{x^3\,\text{atanh}^4\left(\tanh\left(a+bx\right)\right)}{3}$$

input
```
integrate(x**2*atanh(tanh(b*x+a))**4,x)
```

output
```
b**4*x**7/105 - b**3*x**6*atanh(tanh(a + b*x))/15 + b**2*x**5*atanh(tanh(a
 + b*x))**2/5 - b*x**4*atanh(tanh(a + b*x))**3/3 + x**3*atanh(tanh(a + b*x
))**4/3
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.36

$$\int x^2 \text{arctanh}(\tanh(a+bx))^4\,dx = -\frac{1}{3}\,bx^4\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^3 + \frac{1}{3}\,x^3\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^4 + \frac{1}{105}\,\left(21\,bx^5\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2 + \left(b^2x^7 - 7\,bx^6\,\text{artanh}\left(\tanh\left(bx+a\right)\right)\right)b\right)b$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

output
```
-1/3*b*x^4*arctanh(tanh(b*x + a))^3 + 1/3*x^3*arctanh(tanh(b*x + a))^4 + 1
/105*(21*b*x^5*arctanh(tanh(b*x + a))^2 + (b^2*x^7 - 7*b*x^6*arctanh(tanh(
b*x + a)))*b)*b
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int x^2\text{arctanh}(\tanh(a+bx))^4\,dx = \frac{1}{7}\,b^4x^7 + \frac{2}{3}\,ab^3x^6 + \frac{6}{5}\,a^2b^2x^5 + a^3bx^4 + \frac{1}{3}\,a^4x^3$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

output
```
1/7*b^4*x^7 + 2/3*a*b^3*x^6 + 6/5*a^2*b^2*x^5 + a^3*b*x^4 + 1/3*a^4*x^3
```

3.70.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int x^2\text{arctanh}(\tanh(a+bx))^4\,dx = \frac{b^4\,x^7}{105} - \frac{b^3\,x^6\,\text{atanh}(\tanh(a+b\,x))}{15} + \frac{b^2\,x^5\,\text{atanh}(\tanh(a+b\,x))^2}{5} - \frac{b\,x^4\,\text{atanh}(\tanh(a+b\,x))^3}{3} + \frac{x^3\,\text{atanh}(\tanh(a+b\,x))^4}{3}$$

input
```
int(x^2*atanh(tanh(a + b*x))^4,x)
```

output
```
(x^3*atanh(tanh(a + b*x))^4)/3 + (b^4*x^7)/105 + (b^2*x^5*atanh(tanh(a + b
*x))^2)/5 - (b*x^4*atanh(tanh(a + b*x))^3)/3 - (b^3*x^6*atanh(tanh(a + b*x
)))/15
```

3.71 $\int x\textbf{arctanh}(\tanh(a+bx))^4\,dx$

3.71.1 Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x\text{arctanh}(\tanh(a+bx))^4\,dx=\frac{x\text{arctanh}(\tanh(a+bx))^5}{5b}-\frac{\text{arctanh}(\tanh(a+bx))^6}{30b^2}$$

output
```
1/5*x*arctanh(tanh(b*x+a))^5/b-1/30*arctanh(tanh(b*x+a))^6/b^2
```

3.71.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 125 vs. 2(34) = 68.

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.68

$$\int x\text{arctanh}(\tanh(a+bx))^4\,dx=$$
$$-\frac{(a+bx)\left((5a-bx)(a+bx)^4-6(4a-bx)(a+bx)^3\text{arctanh}(\tanh(a+bx))+15(3a-bx)(a+bx)^2\text{arct}\ldots\right)}{3\ldots}$$

input
```
Integrate[x*ArcTanh[Tanh[a + b*x]]^4,x]
```

output
```
-1/30*((a + b*x)*((5*a - b*x)*(a + b*x)^4 - 6*(4*a - b*x)*(a + b*x)^3*ArcT
anh[Tanh[a + b*x]] + 15*(3*a - b*x)*(a + b*x)^2*ArcTanh[Tanh[a + b*x]]^2 -
 20*(2*a^2 + a*b*x - b^2*x^2)*ArcTanh[Tanh[a + b*x]]^3 + 15*(a - b*x)*ArcT
anh[Tanh[a + b*x]]^4))/b^2
```

3.71.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(\tanh(a+bx))^4\,dx$$

$$\Big\downarrow 2599$$

$$\frac{x\text{arctanh}(\tanh(a+bx))^5}{5b}-\frac{\int \text{arctanh}(\tanh(a+bx))^5dx}{5b}$$

$$\Big\downarrow 2588$$

$$\frac{x\text{arctanh}(\tanh(a+bx))^5}{5b}-\frac{\int \text{arctanh}(\tanh(a+bx))^5d\text{arctanh}(\tanh(a+bx))}{5b^2}$$

$$\Big\downarrow 15$$

$$\frac{x\text{arctanh}(\tanh(a+bx))^5}{5b}-\frac{\text{arctanh}(\tanh(a+bx))^6}{30b^2}$$

input
```
Int[x*ArcTanh[Tanh[a + b*x]]^4,x]
```

output
```
(x*ArcTanh[Tanh[a + b*x]]^5)/(5*b) - ArcTanh[Tanh[a + b*x]]^6/(30*b^2)
```

3.71.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.71.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(30) = 60$.

Time = 21.87 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.09

method	result
parallelrisch	$\frac{b^4x^6}{30}+\frac{x^2\operatorname{arctanh}(\tanh(bx+a))^4}{2}-\frac{b^3\operatorname{arctanh}(\tanh(bx+a))x^5}{5}+\frac{b^2\operatorname{arctanh}(\tanh(bx+a))^2x^4}{2}-\frac{2b\operatorname{arctanh}(\tanh(bx+a}{3}$
default	$\frac{x^2\operatorname{arctanh}(\tanh(bx+a))^4}{2}-2b\left(\frac{x^3\operatorname{arctanh}(\tanh(bx+a))^3}{3}-b\left(\frac{x^4\operatorname{arctanh}(\tanh(bx+a))^2}{4}-\frac{b\left(\frac{x^5\operatorname{arctanh}(\tanh(bx+a))}{5}\right.}{2}\right.\right.$
parts	$\frac{x^2\operatorname{arctanh}(\tanh(bx+a))^4}{2}-2b\left(\frac{x^3\operatorname{arctanh}(\tanh(bx+a))^3}{3}-b\left(\frac{x^4\operatorname{arctanh}(\tanh(bx+a))^2}{4}-\frac{b\left(\frac{x^5\operatorname{arctanh}(\tanh(bx+a))}{5}\right.}{2}\right.\right.$
risch	Expression too large to display

input
```
int(x*arctanh(tanh(b*x+a))^4,x,method=_RETURNVERBOSE)
```

output
```
1/30*b^4*x^6+1/2*x^2*arctanh(tanh(b*x+a))^4-1/5*b^3*arctanh(tanh(b*x+a))*x
^5+1/2*b^2*arctanh(tanh(b*x+a))^2*x^4-2/3*b*arctanh(tanh(b*x+a))^3*x^3
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int x\operatorname{arctanh}(\tanh(a+bx))^4\,dx = \frac{1}{6}\,b^4x^6+\frac{4}{5}\,ab^3x^5+\frac{3}{2}\,a^2b^2x^4+\frac{4}{3}\,a^3bx^3+\frac{1}{2}\,a^4x^2$$

input
```
integrate(x*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

output
```
1/6*b^4*x^6 + 4/5*a*b^3*x^5 + 3/2*a^2*b^2*x^4 + 4/3*a^3*b*x^3 + 1/2*a^4*x^
2
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int x\text{arctanh}(\tanh(a+bx))^4\,dx = \begin{cases} \frac{x\,\text{atanh}^5\left(\tanh\left(a+bx\right)\right)}{5b} - \frac{\text{atanh}^6\left(\tanh\left(a+bx\right)\right)}{30b^2} & \text{for } b \neq 0 \\ \frac{x^2\,\text{atanh}^4\left(\tanh\left(a\right)\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*atanh(tanh(b*x+a))**4,x)
```

output
```
Piecewise((x*atanh(tanh(a + b*x))**5/(5*b) - atanh(tanh(a + b*x))**6/(30*b
**2), Ne(b, 0)), (x**2*atanh(tanh(a))**4/2, True))
```

3.71.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(30) = 60$.

Time = 0.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.12

$$\begin{aligned}&\int x\text{arctanh}(\tanh(a+bx))^4\,dx \\ &= -\frac{2}{3}\,bx^3\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^3 + \frac{1}{2}\,x^2\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^4 \\ &\quad + \frac{1}{30}\left(15\,bx^4\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2 + \left(b^2x^6 - 6\,bx^5\,\text{artanh}\left(\tanh\left(bx+a\right)\right)\right)b\right)b\end{aligned}$$

input
```
integrate(x*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

output
```
-2/3*b*x^3*arctanh(tanh(b*x + a))^3 + 1/2*x^2*arctanh(tanh(b*x + a))^4 + 1
/30*(15*b*x^4*arctanh(tanh(b*x + a))^2 + (b^2*x^6 - 6*b*x^5*arctanh(tanh(b
*x + a)))*b)*b
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int x\text{arctanh}(\tanh(a+bx))^4\,dx = \frac{1}{6}\,b^4x^6+\frac{4}{5}\,ab^3x^5+\frac{3}{2}\,a^2b^2x^4+\frac{4}{3}\,a^3bx^3+\frac{1}{2}\,a^4x^2$$

input
```
integrate(x*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

output
```
1/6*b^4*x^6 + 4/5*a*b^3*x^5 + 3/2*a^2*b^2*x^4 + 4/3*a^3*b*x^3 + 1/2*a^4*x^
2
```

3.71.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.06

$$\int x\text{arctanh}(\tanh(a+bx))^4\,dx = \frac{b^4\,x^6}{30}-\frac{b^3\,x^5\,\text{atanh}(\tanh(a+b\,x))}{5}+\frac{b^2\,x^4\,\text{atanh}(\tanh(a+b\,x))^2}{2}-\frac{2\,b\,x^3\,\text{atanh}(\tanh(a+b\,x))^3}{3}+\frac{x^2\,\text{atanh}(\tanh(a+b\,x))^4}{2}$$

input
```
int(x*atanh(tanh(a + b*x))^4,x)
```

output
```
(x^2*atanh(tanh(a + b*x))^4)/2 + (b^4*x^6)/30 + (b^2*x^4*atanh(tanh(a + b*
x))^2)/2 - (2*b*x^3*atanh(tanh(a + b*x))^3)/3 - (b^3*x^5*atanh(tanh(a + b*
x)))/5
```

3.72 $\int \operatorname{arctanh}(\tanh(a+bx))^4\,dx$

3.72.1 Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \operatorname{arctanh}(\tanh(a+bx))^4\,dx=\frac{\operatorname{arctanh}(\tanh(a+bx))^5}{5b}$$

output
```
1/5*arctanh(tanh(b*x+a))^5/b
```

3.72.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \operatorname{arctanh}(\tanh(a+bx))^4\,dx=\frac{\operatorname{arctanh}(\tanh(a+bx))^5}{5b}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^4,x]
```

output
```
ArcTanh[Tanh[a + b*x]]^5/(5*b)
```

3.72.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(\tanh(a+bx))^4\,dx$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \operatorname{arctanh}(\tanh(a+bx))^4 d\operatorname{arctanh}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\operatorname{arctanh}(\tanh(a+bx))^5}{5b}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^4,x]
```

output
```
ArcTanh[Tanh[a + b*x]]^5/(5*b)
```

3.72.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.72.4 Maple [A] (verified)

Time = 24.75 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\operatorname{arctanh}(\tanh(bx+a))^5}{5b}$
default	$\frac{\operatorname{arctanh}(\tanh(bx+a))^5}{5b}$
parallelrisch	$\frac{b^4x^5}{5} + x \operatorname{arctanh}(\tanh(bx+a))^4 - b^3 \operatorname{arctanh}(\tanh(bx+a))\,x^4 + 2b^2 \operatorname{arctanh}(\tanh($
risch	Expression too large to display

input
```
int(arctanh(tanh(b*x+a))^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*arctanh(tanh(b*x+a))^5/b
```

3.72.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(14) = 28.

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.62

$$\int \operatorname{arctanh}(\tanh(a+bx))^4\,dx = \frac{1}{5}b^4x^5 + ab^3x^4 + 2\,a^2b^2x^3 + 2\,a^3bx^2 + a^4x$$

input
```
integrate(arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

output
```
1/5*b^4*x^5 + a*b^3*x^4 + 2*a^2*b^2*x^3 + 2*a^3*b*x^2 + a^4*x
```

3.72.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \operatorname{arctanh}(\tanh(a+bx))^4\,dx = \begin{cases} \frac{\operatorname{atanh}^5(\tanh(a+bx))}{5b} & \text{for } b \neq 0 \\ x \operatorname{atanh}^4(\tanh(a)) & \text{otherwise} \end{cases}$$

input
```
integrate(atanh(tanh(b*x+a))**4,x)
```

output
```
Piecewise((atanh(tanh(a + b*x))**5/(5*b), Ne(b, 0)), (x*atanh(tanh(a))**4,
 True))
```

3.72.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(14) = 28$.

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 4.31

$$\begin{aligned}\int \operatorname{arctanh}(\tanh(a+bx))^4\,dx &= -2\,bx^2\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3 + x\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4\\ &\quad+\frac{1}{5}\left(10\,bx^3\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2 + \left(b^2x^5 - 5\,bx^4\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\right)b\right)b\end{aligned}$$

input
```
integrate(arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

output
```
-2*b*x^2*arctanh(tanh(b*x + a))^3 + x*arctanh(tanh(b*x + a))^4 + 1/5*(10*b
*x^3*arctanh(tanh(b*x + a))^2 + (b^2*x^5 - 5*b*x^4*arctanh(tanh(b*x + a)))
*b)*b
```

3.72.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.62

$$\int \operatorname{arctanh}(\tanh(a+bx))^4\,dx = \frac{1}{5}\,b^4x^5 + ab^3x^4 + 2\,a^2b^2x^3 + 2\,a^3bx^2 + a^4x$$

input
```
integrate(arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

output
```
1/5*b^4*x^5 + a*b^3*x^4 + 2*a^2*b^2*x^3 + 2*a^3*b*x^2 + a^4*x
```

3.72.9 Mupad [B] (verification not implemented)

Time = 3.46 (sec) , antiderivative size = 67, normalized size of antiderivative = 4.19

$$\int \operatorname{arctanh}(\tanh(a+bx))^4\,dx = \frac{b^4\,x^5}{5} - b^3\,x^4\,\mathrm{atanh}(\tanh(a+b\,x)) + 2\,b^2\,x^3\,\mathrm{atanh}(\tanh(a+b\,x))^2 - 2\,b\,x^2\,\mathrm{atanh}(\tanh(a+b\,x))^3 + x\,\mathrm{atanh}(\tanh(a+b\,x))^4$$

input
```
int(atanh(tanh(a + b*x))^4,x)
```

output
```
x*atanh(tanh(a + b*x))^4 + (b^4*x^5)/5 + 2*b^2*x^3*atanh(tanh(a + b*x))^2
- 2*b*x^2*atanh(tanh(a + b*x))^3 - b^3*x^4*atanh(tanh(a + b*x))
```

3.73 $\int \frac{\textbf{arctanh}(\tanh(a+bx))^4}{x} dx$

3.73.1 Optimal result

Integrand size = 13, antiderivative size = 105

$$\begin{aligned}\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x} dx = &-bx(bx-\operatorname{arctanh}(\tanh(a+bx)))^3\\&+\frac{1}{2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2\operatorname{arctanh}(\tanh(a+bx))^2\\&-\frac{1}{3}(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))^3\\&+\frac{1}{4}\operatorname{arctanh}(\tanh(a+bx))^4\\&+(bx-\operatorname{arctanh}(\tanh(a+bx)))^4\log(x)\end{aligned}$$

output
```
-b*x*(b*x-arctanh(tanh(b*x+a)))^3+1/2*(b*x-arctanh(tanh(b*x+a)))^2*arctanh
(tanh(b*x+a))^2-1/3*(b*x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^3+1/4*
arctanh(tanh(b*x+a))^4+(b*x-arctanh(tanh(b*x+a)))^4*ln(x)
```

3.73.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x}\,dx = \frac{1}{4}(a+bx)^4 + \frac{1}{2}(a+bx)^2\left(a^2-4a(a+bx-\operatorname{arctanh}(\tanh(a+bx))) + 6(a+bx-\operatorname{arctanh}(\tanh(a+bx)))^2\right) + (a+bx)\left(a^3-4a^2(a+bx-\operatorname{arctanh}(\tanh(a+bx))) + 6a(a+bx-\operatorname{arctanh}(\tanh(a+bx)))^2 - 4(a+bx-\operatorname{arctanh}(\tanh(a+bx)))^3\right) - \frac{1}{3}(a+bx)^3(3a+4bx-4\operatorname{arctanh}(\tanh(a+bx))) + (-bx+\operatorname{arctanh}(\tanh(a+bx)))^4\log(bx)$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^4/x,x]
```

output
```
(a + b*x)^4/4 + ((a + b*x)^2*(a^2 - 4*a*(a + b*x - ArcTanh[Tanh[a + b*x]])
 + 6*(a + b*x - ArcTanh[Tanh[a + b*x]])^2))/2 + (a + b*x)*(a^3 - 4*a^2*(a
+ b*x - ArcTanh[Tanh[a + b*x]]) + 6*a*(a + b*x - ArcTanh[Tanh[a + b*x]])^2
 - 4*(a + b*x - ArcTanh[Tanh[a + b*x]])^3) - ((a + b*x)^3*(3*a + 4*b*x - 4
*ArcTanh[Tanh[a + b*x]]))/3 + (-(b*x) + ArcTanh[Tanh[a + b*x]])^4*Log[b*x]
```

3.73.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2590, 2590, 2590, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x}\,dx$$

$$\Big\downarrow \text{ 2590}$$

$$\frac{1}{4}\operatorname{arctanh}(\tanh(a+bx))^4-(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x}dx$$

$$\Big\downarrow \text{ 2590}$$

$$\frac{1}{4}\text{arctanh}(\tanh(a+bx))^4-(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{3}\text{arctanh}(\tanh(a+bx))^3-(bx-\text{arctanh}(\tanh(a+bx)))\int\frac{\text{arctanh}(\tanh(a+bx))^2}{x}dx\right)$$

$\downarrow$ 2590

$$\frac{1}{4}\text{arctanh}(\tanh(a+bx))^4-(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{3}\text{arctanh}(\tanh(a+bx))^3-(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{2}\text{arctanh}(\tanh(a+bx))^2-(bx-\text{arctanh}(\tan\right.\right.$$

$\downarrow$ 2589

$$\frac{1}{4}\text{arctanh}(\tanh(a+bx))^4-(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{3}\text{arctanh}(\tanh(a+bx))^3-(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{2}\text{arctanh}(\tanh(a+bx))^2-(bx-\text{arctanh}(\tan\right.\right.$$

$\downarrow$ 14

$$\frac{1}{4}\text{arctanh}(\tanh(a+bx))^4-(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{3}\text{arctanh}(\tanh(a+bx))^3-(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{2}\text{arctanh}(\tanh(a+bx))^2-(bx-\text{arctanh}(\tan\right.\right.$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^4/x,x]
```

output
```
ArcTanh[Tanh[a + b*x]]^4/4 - (b*x - ArcTanh[Tanh[a + b*x]])*(ArcTanh[Tanh[
a + b*x]]^3/3 - (b*x - ArcTanh[Tanh[a + b*x]])*(ArcTanh[Tanh[a + b*x]]^2/2
 - (b*x - ArcTanh[Tanh[a + b*x]])*(b*x - (b*x - ArcTanh[Tanh[a + b*x]])*Lo
g[x])))
```

3.73.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

3.73.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(99) = 198$.

Time = 0.22 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.52

method	result
default	$\ln(x)\operatorname{arctanh}(\tanh(bx+a))^4 - 4b\left(b^3\left(\frac{x^4\ln(x)}{4} - \frac{x^4}{16}\right) + 3a\,b^2\left(\frac{x^3\ln(x)}{3} - \frac{x^3}{9}\right) + 3b^2(\operatorname{arctanh}(\tanh$
parts	$\ln(x)\operatorname{arctanh}(\tanh(bx+a))^4 - 4b\left(b^3\left(\frac{x^4\ln(x)}{4} - \frac{x^4}{16}\right) + 3a\,b^2\left(\frac{x^3\ln(x)}{3} - \frac{x^3}{9}\right) + 3b^2(\operatorname{arctanh}(\tanh$
risch	Expression too large to display

input
```
int(arctanh(tanh(b*x+a))^4/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arctanh(tanh(b*x+a))^4-4*b*(b^3*(1/4*x^4*ln(x)-1/16*x^4)+3*a*b^2*(1/
3*x^3*ln(x)-1/9*x^3)+3*b^2*(arctanh(tanh(b*x+a))-b*x-a)*(1/3*x^3*ln(x)-1/9
*x^3)+3*b*a^2*(1/2*x^2*ln(x)-1/4*x^2)+6*a*b*(arctanh(tanh(b*x+a))-b*x-a)*(
1/2*x^2*ln(x)-1/4*x^2)+3*b*(arctanh(tanh(b*x+a))-b*x-a)^2*(1/2*x^2*ln(x)-1
/4*x^2)+a^3*(x*ln(x)-x)+3*a^2*(arctanh(tanh(b*x+a))-b*x-a)*(x*ln(x)-x)+3*a
*(arctanh(tanh(b*x+a))-b*x-a)^2*(x*ln(x)-x)+(arctanh(tanh(b*x+a))-b*x-a)^3
*(x*ln(x)-x))
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.40

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x}\,dx = \frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + 3\,a^2b^2x^2 + 4\,a^3bx + a^4\log(x)$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x,x, algorithm="fricas")
```

output
```
1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + a^4*log(x)
```

3.73.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x} \, dx = \int \frac{\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{x} \, dx$$

input
```
integrate(atanh(tanh(b*x+a))**4/x,x)
```

output
```
Integral(atanh(tanh(a + b*x))**4/x, x)
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.40

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x} \, dx = \frac{1}{4} b^4x^4 + \frac{4}{3} ab^3x^3 + 3 a^2b^2x^2 + 4 a^3bx + a^4 \log\left(x\right)$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x,x, algorithm="maxima")
```

output
```
1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + a^4*log(x)
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.41

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x} \, dx = \frac{1}{4} b^4x^4 + \frac{4}{3} ab^3x^3 + 3 a^2b^2x^2 + 4 a^3bx + a^4 \log\left(|x|\right)$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x,x, algorithm="giac")
```

output
```
1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + a^4*log(abs(x))
```

3.73.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 423, normalized size of antiderivative = 4.03

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x}\,dx = \ln(x)\left(\frac{\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^4}{16}+\frac{3a^2\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}{2}+a^4-\frac{a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3}{2}-2a^3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)\right)+\frac{b^4x^4}{4}-\frac{2b^3x^3\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}{3}+\frac{3b^2x^2\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}{4}-\frac{bx\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3}{2}$$

input
```
int(atanh(tanh(a + b*x))^4/x,x)
```

output
```
log(x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4/16 + (3*a^2*(2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x)^2)/2 + a^4 - (a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/2 - 2*a^3*(
2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (b^4*x^4)/4 - (2*b^3*x^3*(log(2/(exp(2*a
)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x))/3 + (3*b^2*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/4 - (b*x*(log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x)^3)/2
```

3.74 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^2}\,dx$

3.74.1 Optimal result

Integrand size = 13, antiderivative size = 95

$$\begin{aligned}\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^2}\,dx &= 4b^2x(bx-\operatorname{arctanh}(\tanh(a+bx)))^2\\ &\quad-2b(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))^2\\ &\quad+\frac{4}{3}b\operatorname{arctanh}(\tanh(a+bx))^3-\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x}\\ &\quad-4b(bx-\operatorname{arctanh}(\tanh(a+bx)))^3\log(x)\end{aligned}$$

output
```
4*b^2*x*(b*x-arctanh(tanh(b*x+a)))^2-2*b*(b*x-arctanh(tanh(b*x+a)))*arctan
h(tanh(b*x+a))^2+4/3*b*arctanh(tanh(b*x+a))^3-arctanh(tanh(b*x+a))^4/x-4*b
*(b*x-arctanh(tanh(b*x+a)))^3*ln(x)
```

3.74.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^2}\,dx &= -\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x}+\frac{2}{3}b^4x^3(5-6\log(x))\\ &\quad-12b^2x\operatorname{arctanh}(\tanh(a+bx))^2\log(x)\\ &\quad+4b\operatorname{arctanh}(\tanh(a+bx))^3(1+\log(x))\\ &\quad+6b^3x^2\operatorname{arctanh}(\tanh(a+bx))(-1+2\log(x))\end{aligned}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^4/x^2,x]
```

output
```
-(ArcTanh[Tanh[a + b*x]]^4/x) + (2*b^4*x^3*(5 - 6*Log[x]))/3 - 12*b^2*x*Ar
cTanh[Tanh[a + b*x]]^2*Log[x] + 4*b*ArcTanh[Tanh[a + b*x]]^3*(1 + Log[x])
+ 6*b^3*x^2*ArcTanh[Tanh[a + b*x]]*(-1 + 2*Log[x])
```

3.74.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2599, 2590, 2590, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^2}dx$$

$$\Big\downarrow \text{ 2599}$$

$$4b\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x}dx-\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x}$$

$$\Big\downarrow \text{ 2590}$$

$$4b\left(\frac{1}{3}\operatorname{arctanh}(\tanh(a+bx))^3-(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x}dx\right)-\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x}$$

$$\Big\downarrow \text{ 2590}$$

$$4b\left(\frac{1}{3}\operatorname{arctanh}(\tanh(a+bx))^3-(bx-\operatorname{arctanh}(\tanh(a+bx)))\left(\frac{1}{2}\operatorname{arctanh}(\tanh(a+bx))^2-(bx-\operatorname{arctanh}(\tanh(a\right.\right.$$
$$\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x}$$

$$\Big\downarrow \text{ 2589}$$

$$4b\left(\frac{1}{3}\operatorname{arctanh}(\tanh(a+bx))^3-(bx-\operatorname{arctanh}(\tanh(a+bx)))\left(\frac{1}{2}\operatorname{arctanh}(\tanh(a+bx))^2-(bx-\operatorname{arctanh}(\tanh(a\right.\right.$$
$$\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x}$$

$$\Big\downarrow \text{ 14}$$

$$4b\left(\frac{1}{3}\text{arctanh}(\tanh(a+bx))^3-(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{2}\text{arctanh}(\tanh(a+bx))^2-(bx-\text{arctanh}(\tanh(a\right.\right.$$
$$\frac{\text{arctanh}(\tanh(a+bx))^4}{x}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^4/x^2,x]
```

output
```
-(ArcTanh[Tanh[a + b*x]]^4/x) + 4*b*(ArcTanh[Tanh[a + b*x]]^3/3 - (b*x - A
rcTanh[Tanh[a + b*x]])*(ArcTanh[Tanh[a + b*x]]^2/2 - (b*x - ArcTanh[Tanh[a
 + b*x]])*(b*x - (b*x - ArcTanh[Tanh[a + b*x]])*Log[x])))
```

3.74.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.74.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{x}+4b\Big(\ln(x)\operatorname{arctanh}(\tanh(bx+a))^3-3b\Big(b^2\Big(\frac{x^3\ln(x)}{3}-\frac{x^3}{9}\Big)+2ab\Big(\frac{x^2\ln(x)}{2}-$
parts	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{x}+4b\Big(\ln(x)\operatorname{arctanh}(\tanh(bx+a))^3-3b\Big(b^2\Big(\frac{x^3\ln(x)}{3}-\frac{x^3}{9}\Big)+2ab\Big(\frac{x^2\ln(x)}{2}-$
risch	Expression too large to display

input
```
int(arctanh(tanh(b*x+a))^4/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arctanh(tanh(b*x+a))^4/x+4*b*(ln(x)*arctanh(tanh(b*x+a))^3-3*b*(b^2*(1/3*
x^3*ln(x)-1/9*x^3)+2*a*b*(1/2*x^2*ln(x)-1/4*x^2)+2*b*(arctanh(tanh(b*x+a))
-b*x-a)*(1/2*x^2*ln(x)-1/4*x^2)+a^2*(x*ln(x)-x)+2*a*(arctanh(tanh(b*x+a))-
b*x-a)*(x*ln(x)-x)+(arctanh(tanh(b*x+a))-b*x-a)^2*(x*ln(x)-x)))
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.49

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^2}\,dx = \frac{b^4x^4+6\,ab^3x^3+18\,a^2b^2x^2+12\,a^3bx\log(x)-3\,a^4}{3\,x}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^2,x, algorithm="fricas")
```

output
```
1/3*(b^4*x^4 + 6*a*b^3*x^3 + 18*a^2*b^2*x^2 + 12*a^3*b*x*log(x) - 3*a^4)/x
```

3.74.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^2}\,dx = \int \frac{\operatorname{atanh}^4(\tanh(a+bx))}{x^2}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**4/x**2,x)
```

output
```
Integral(atanh(tanh(a + b*x))**4/x**2, x)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^2}\,dx$$
$$= 4\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3\log\left(x\right) - \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4}{x}$$
$$+ \frac{2}{3}\left(2\,b^3x^3 + 9\,ab^2x^2 + 18\,a^2bx + 6\,a^3\log\left(x\right) - 6\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3\log\left(x\right)\right)b$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^2,x, algorithm="maxima")
```

output
```
4*b*arctanh(tanh(b*x + a))^3*log(x) - arctanh(tanh(b*x + a))^4/x + 2/3*(2*
b^3*x^3 + 9*a*b^2*x^2 + 18*a^2*b*x + 6*a^3*log(x) - 6*arctanh(tanh(b*x + a
))^3*log(x))*b
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.46

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^2}\,dx = \frac{1}{3}\,b^4x^3 + 2\,ab^3x^2 + 6\,a^2b^2x + 4\,a^3b\log\left(|x|\right) - \frac{a^4}{x}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^2,x, algorithm="giac")
```

output
```
1/3*b^4*x^3 + 2*a*b^3*x^2 + 6*a^2*b^2*x + 4*a^3*b*log(abs(x)) - a^4/x
```

3.74.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 553, normalized size of antiderivative = 5.82

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^2}\,dx$$

$$= \ln(x)\left(4\,a^3\,b - \frac{b\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^3}{2} + 3\,a\,b\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2 - 6\,a^2\,b\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)\right)$$

$$- \frac{\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^4 + 24\,a^2\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right.}{}$$

$$+ \frac{b^4\,x^3}{3} + \frac{3\,b^2\,x\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2}{2} - b^3\,x^2\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)$$

input
```
int(atanh(tanh(a + b*x))^4/x^2,x)
```

output
```
log(x)*(4*a^3*b - (b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/2 + 3*a*b*(2*a - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^2 - 6*a^2*b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - ((2
*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2
 + 16*a^4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 32*a^3*(2*a - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x))/(16*x) + (b^4*x^3)/3 + (3*b^2*x*(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)^2)/2 - b^3*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
```

3.75 $\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^3} dx$

3.75.1 Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^3} dx = -6b^3x(bx - \text{arctanh}(\tanh(a+bx))) + 3b^2\text{arctanh}(\tanh(a+bx))^2 - \frac{2b\text{arctanh}(\tanh(a+bx))^3}{x} - \frac{\text{arctanh}(\tanh(a+bx))^4}{2x^2} + 6b^2(bx - \text{arctanh}(\tanh(a+bx)))^2 \log(x)$$

output
```
-6*b^3*x*(b*x-arctanh(tanh(b*x+a)))+3*b^2*arctanh(tanh(b*x+a))^2-2*b*arcta
nh(tanh(b*x+a))^3/x-1/2*arctanh(tanh(b*x+a))^4/x^2+6*b^2*(b*x-arctanh(tanh
(b*x+a)))^2*ln(x)
```

3.75.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^3} dx = -\frac{2b\text{arctanh}(\tanh(a+bx))^3}{x} - \frac{\text{arctanh}(\tanh(a+bx))^4}{2x^2} + 6b^4x^2\log(x) - 6b^3x\text{arctanh}(\tanh(a+bx))(1+2\log(x)) + 3b^2\text{arctanh}(\tanh(a+bx))^2(3+2\log(x))$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^4/x^3,x]
```

output
```
(-2*b*ArcTanh[Tanh[a + b*x]]^3)/x - ArcTanh[Tanh[a + b*x]]^4/(2*x^2) + 6*b
^4*x^2*Log[x] - 6*b^3*x*ArcTanh[Tanh[a + b*x]]*(1 + 2*Log[x]) + 3*b^2*ArcT
anh[Tanh[a + b*x]]^2*(3 + 2*Log[x])
```

3.75.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2599, 2599, 2590, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^3}dx$$

$$\Big\downarrow \text{2599}$$

$$2b\int \frac{\text{arctanh}(\tanh(a+bx))^3}{x^2}dx-\frac{\text{arctanh}(\tanh(a+bx))^4}{2x^2}$$

$$\Big\downarrow \text{2599}$$

$$2b\left(3b\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x}dx-\frac{\text{arctanh}(\tanh(a+bx))^3}{x}\right)-\frac{\text{arctanh}(\tanh(a+bx))^4}{2x^2}$$

$$\Big\downarrow \text{2590}$$

$$2b\left(3b\left(\frac{1}{2}\text{arctanh}(\tanh(a+bx))^2-(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{\text{arctanh}(\tanh(a+bx))}{x}dx\right)-\frac{\text{arctanh}(\tanh(}{x}\right.$$
$$\frac{\text{arctanh}(\tanh(a+bx))^4}{2x^2}$$

$$\Big\downarrow \text{2589}$$

$$2b\left(3b\left(\frac{1}{2}\text{arctanh}(\tanh(a+bx))^2-(bx-\text{arctanh}(\tanh(a+bx)))\left(bx-(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{x}dx\right)\right)\right.$$
$$\frac{\text{arctanh}(\tanh(a+bx))^4}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$2b\left(3b\left(\frac{1}{2}\text{arctanh}(\tanh(a+bx))^2-(bx-\text{arctanh}(\tanh(a+bx)))(bx-\log(x)(bx-\text{arctanh}(\tanh(a+bx))))\right)-\right.$$
$$\frac{\text{arctanh}(\tanh(a+bx))^4}{2x^2}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^4/x^3,x]
```

output
```
-1/2*ArcTanh[Tanh[a + b*x]]^4/x^2 + 2*b*(-(ArcTanh[Tanh[a + b*x]]^3/x) + 3
*b*(ArcTanh[Tanh[a + b*x]]^2/2 - (b*x - ArcTanh[Tanh[a + b*x]])*(b*x - (b*
x - ArcTanh[Tanh[a + b*x]])*Log[x])))
```

3.75.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.75.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.18

method	result
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{2x^2}+2b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{x}+3b\left(\ln(x)\operatorname{arctanh}(\tanh(bx+a))^2-2b\left(b\left(\frac{x^2\ln(x)}{2}\right.\right.\right.\right.$
parts	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{2x^2}+2b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{x}+3b\left(\ln(x)\operatorname{arctanh}(\tanh(bx+a))^2-2b\left(b\left(\frac{x^2\ln(x)}{2}\right.\right.\right.\right.$
risch	Expression too large to display

input
```
int(arctanh(tanh(b*x+a))^4/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(tanh(b*x+a))^4/x^2+2*b*(-arctanh(tanh(b*x+a))^3/x+3*b*(ln(x)*
arctanh(tanh(b*x+a))^2-2*b*(b*(1/2*x^2*ln(x)-1/4*x^2)+a*(x*ln(x)-x)+(arcta
nh(tanh(b*x+a))-b*x-a)*(x*ln(x)-x))))
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.54

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^3}\,dx=\frac{b^4x^4+8\,ab^3x^3+12\,a^2b^2x^2\log(x)-8\,a^3bx-a^4}{2\,x^2}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^3,x, algorithm="fricas")
```

output
```
1/2*(b^4*x^4 + 8*a*b^3*x^3 + 12*a^2*b^2*x^2*log(x) - 8*a^3*b*x - a^4)/x^2
```

3.75.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^3}\,dx=\int \frac{\operatorname{atanh}^4(\tanh(a+bx))}{x^3}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**4/x**3,x)
```

output
```
Integral(atanh(tanh(a + b*x))**4/x**3, x)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^3}\,dx = -\frac{2\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}{x}$$
$$+3\left(2\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2\log\left(x\right)+\left(b^2x^2+4\,abx+2\,a^2\log\left(x\right)-2\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2\log\left(x\right)\right)$$
$$-\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4}{2\,x^2}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^3,x, algorithm="maxima")
```

output
```
-2*b*arctanh(tanh(b*x + a))^3/x + 3*(2*b*arctanh(tanh(b*x + a))^2*log(x) +
 (b^2*x^2 + 4*a*b*x + 2*a^2*log(x) - 2*arctanh(tanh(b*x + a))^2*log(x))*b)
*b - 1/2*arctanh(tanh(b*x + a))^4/x^2
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.49

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^3}\,dx = \frac{1}{2}\,b^4x^2+4\,ab^3x+6\,a^2b^2\log\left(|x|\right)-\frac{8\,a^3bx+a^4}{2\,x^2}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^3,x, algorithm="giac")
```

output
```
1/2*b^4*x^2 + 4*a*b^3*x + 6*a^2*b^2*log(abs(x)) - 1/2*(8*a^3*b*x + a^4)/x^
2
```

3.75.9 Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 672, normalized size of antiderivative = 7.72

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^3}\,dx = \frac{9\,b^2\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{4} - \frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^4}{32\,x^2} - \frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^4}{32\,x^2} + \frac{9\,b^2\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{4} - 3\,b^3\,x\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + \frac{b\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{4\,x} + \frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{8\,x^2} + \frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{8\,x^2} - \frac{b\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{4\,x} + \frac{3\,b^2\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2\,\ln\left(x\right)}{2} - \frac{3\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{16\,x^2} + \frac{3\,b^2\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2\,\ln\left(x\right)}{2} + 6\,b^4\,x^2\,\ln\left(x\right) - \frac{9\,b^2\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} + 3\,b^3\,x\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + \frac{3\,b\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{4\,x} - \frac{3\,b\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{4\,x} - 3\,b^2\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(x\right) + 6\,b^3\,x\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(x\right) - 6\,b^3\,x\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(x\right)$$

input
```
int(atanh(tanh(a + b*x))^4/x^3,x)
```

output
```
(9*b^2*log(1/(exp(2*a)*exp(2*b*x) + 1))^2)/4 - log((exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1))^4/(32*x^2) - log(1/(exp(2*a)*exp(2*b*x) + 1))^4/
(32*x^2) + (9*b^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/
4 - 3*b^3*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + (b*log(
1/(exp(2*a)*exp(2*b*x) + 1))^3)/(4*x) + (log(1/(exp(2*a)*exp(2*b*x) + 1))*
log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3)/(8*x^2) + (log(1/(
exp(2*a)*exp(2*b*x) + 1))^3*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)))/(8*x^2) - (b*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^
3)/(4*x) + (3*b^2*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log(x))/2 - (3*log(1/
(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1))^2)/(16*x^2) + (3*b^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1))^2*log(x))/2 + 6*b^4*x^2*log(x) - (9*b^2*log(1/(exp(2*a)*exp(2*b*x)
 + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/2 + 3*b^3*x*l
og(1/(exp(2*a)*exp(2*b*x) + 1)) + (3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))*lo
g((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(4*x) - (3*b*log(1/(
exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)))/(4*x) - 3*b^2*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(x) + 6*b^3*x*log(1/(exp(2*a)*exp(2*b
*x) + 1))*log(x) - 6*b^3*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1))*log(x)
```

3.75. $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^3}\,dx$

3.76 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^4}\,dx$

3.76.1 Optimal result

Integrand size = 13, antiderivative size = 77

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^4}\,dx = 4b^4x - \frac{2b^2\operatorname{arctanh}(\tanh(a+bx))^2}{x} - \frac{2b\operatorname{arctanh}(\tanh(a+bx))^3}{3x^2} - \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{3x^3} - 4b^3(bx-\operatorname{arctanh}(\tanh(a+bx)))\log(x)$$

output
```
4*b^4*x-2*b^2*arctanh(tanh(b*x+a))^2/x-2/3*b*arctanh(tanh(b*x+a))^3/x^2-1/
3*arctanh(tanh(b*x+a))^4/x^3-4*b^3*(b*x-arctanh(tanh(b*x+a)))*ln(x)
```

3.76.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^4}\,dx = -\frac{6b^2x^2\operatorname{arctanh}(\tanh(a+bx))^2 + 2bx\operatorname{arctanh}(\tanh(a+bx))^3 + \operatorname{arctanh}(\tanh(a+bx))^4 + 2b^4x^4(5+6\,\mathrm{lo}}{3x^3}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^4/x^4,x]
```

output
```
-1/3*(6*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 + 2*b*x*ArcTanh[Tanh[a + b*x]]^3
+ ArcTanh[Tanh[a + b*x]]^4 + 2*b^4*x^4*(5 + 6*Log[x]) - 2*b^3*x^3*ArcTanh[
Tanh[a + b*x]]*(11 + 6*Log[x]))/x^3
```

3.76.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2599, 2599, 2599, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^4}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{4}{3}b\int \frac{\text{arctanh}(\tanh(a+bx))^3}{x^3}dx-\frac{\text{arctanh}(\tanh(a+bx))^4}{3x^3}$$

$$\Big\downarrow \text{2599}$$

$$\frac{4}{3}b\left(\frac{3}{2}b\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^2}dx-\frac{\text{arctanh}(\tanh(a+bx))^3}{2x^2}\right)-\frac{\text{arctanh}(\tanh(a+bx))^4}{3x^3}$$

$$\Big\downarrow \text{2599}$$

$$\frac{4}{3}b\left(\frac{3}{2}b\left(2b\int \frac{\text{arctanh}(\tanh(a+bx))}{x}dx-\frac{\text{arctanh}(\tanh(a+bx))^2}{x}\right)-\frac{\text{arctanh}(\tanh(a+bx))^3}{2x^2}\right)-\frac{\text{arctanh}(\tanh(a+bx))^4}{3x^3}$$

$$\Big\downarrow \text{2589}$$

$$\frac{4}{3}b\left(\frac{3}{2}b\left(2b\left(bx-(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{x}dx\right)-\frac{\text{arctanh}(\tanh(a+bx))^2}{x}\right)-\frac{\text{arctanh}(\tanh(a+bx))^3}{2x^2}\right)-\frac{\text{arctanh}(\tanh(a+bx))^4}{3x^3}$$

$$\Big\downarrow \text{14}$$

$$\frac{4}{3}b\left(\frac{3}{2}b\left(2b(bx-\log(x)(bx-\text{arctanh}(\tanh(a+bx))))-\frac{\text{arctanh}(\tanh(a+bx))^2}{x}\right)-\frac{\text{arctanh}(\tanh(a+bx))^3}{2x^2}\right)-\frac{\text{arctanh}(\tanh(a+bx))^4}{3x^3}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^4/x^4,x]
```

output
```
-1/3*ArcTanh[Tanh[a + b*x]]^4/x^3 + (4*b*(-1/2*ArcTanh[Tanh[a + b*x]]^3/x^
2 + (3*b*(-(ArcTanh[Tanh[a + b*x]]^2/x) + 2*b*(b*x - (b*x - ArcTanh[Tanh[a
 + b*x]])*Log[x])))/2))/3
```

3.76.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.76.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

method	result
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{3x^3}+\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{2x^2}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{x}+2b(\ln(x)\operatorname{arctanh}(\tanh(bx+a))-b(x\ln(x)-x))\right)}{2}\right)}{3}$
parts	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{3x^3}+\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{2x^2}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{x}+2b(\ln(x)\operatorname{arctanh}(\tanh(bx+a))-b(x\ln(x)-x))\right)}{2}\right)}{3}$
risch	Expression too large to display

input
```
int(arctanh(tanh(b*x+a))^4/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arctanh(tanh(b*x+a))^4/x^3+4/3*b*(-1/2*arctanh(tanh(b*x+a))^3/x^2+3/2
*b*(-arctanh(tanh(b*x+a))^2/x+2*b*(ln(x)*arctanh(tanh(b*x+a))-b*(x*ln(x)-x
))))
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^4}\,dx = \frac{3\,b^4x^4 + 12\,ab^3x^3\log(x) - 18\,a^2b^2x^2 - 6\,a^3bx - a^4}{3\,x^3}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^4,x, algorithm="fricas")
```

output
```
1/3*(3*b^4*x^4 + 12*a*b^3*x^3*log(x) - 18*a^2*b^2*x^2 - 6*a^3*b*x - a^4)/x
^3
```

3.76.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^4}\,dx = \int \frac{\operatorname{atanh}^4(\tanh(a+bx))}{x^4}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**4/x**4,x)
```

output
```
Integral(atanh(tanh(a + b*x))**4/x**4, x)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^4}\,dx$$
$$= 2\left(2\left(b\operatorname{artanh}(\tanh(bx+a))\log(x) - \left(b\left(x+\frac{a}{b}\right)\log(x) - b\left(x+\frac{a\log(x)}{b}\right)\right)b\right)b - \frac{b\operatorname{artanh}(\tanh(b\ldots}{x}\right.$$
$$- \frac{2\,b\operatorname{artanh}(\tanh(bx+a))^3}{3\,x^2} - \frac{\operatorname{artanh}(\tanh(bx+a))^4}{3\,x^3}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^4,x, algorithm="maxima")
```

output
```
2*(2*(b*arctanh(tanh(b*x + a))*log(x) - (b*(x + a/b)*log(x) - b*(x + a*log
(x)/b))*b)*b - b*arctanh(tanh(b*x + a))^2/x)*b - 2/3*b*arctanh(tanh(b*x +
a))^3/x^2 - 1/3*arctanh(tanh(b*x + a))^4/x^3
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^4}\,dx = b^4x + 4\,ab^3\log\left(|x|\right) - \frac{18\,a^2b^2x^2 + 6\,a^3bx + a^4}{3\,x^3}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^4,x, algorithm="giac")
```

output
```
b^4*x + 4*a*b^3*log(abs(x)) - 1/3*(18*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/x^3
```

3.76.9 Mupad [B] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 571, normalized size of antiderivative = 7.42

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^4}\,dx = \frac{11\,b^3\,\ln\left(\frac{e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{3} - \frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^4}{48\,x^3} - \frac{11\,b^3\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{3} - \frac{10\,b^4\,x}{3} - \frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^4}{48\,x^3} + \frac{b\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{12\,x^2} - 2\,b^3\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln(x) + \frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{12\,x^3} + \frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{12\,x^3} - \frac{b\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{12\,x^2} + 2\,b^3\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln(x) - 4\,b^4\,x\,\ln(x) - \frac{b^2\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{2\,x} - \frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{8\,x^3} - \frac{b^2\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{2\,x} + \frac{b^2\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{x} + \frac{b\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{4\,x^2} - \frac{b\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{4\,x^2}$$

input
```
int(atanh(tanh(a + b*x))^4/x^4,x)
```

output
```
(11*b^3*log(exp(2*b*x)/(exp(2*a)*exp(2*b*x) + 1)))/3 - log((exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1))^4/(48*x^3) - (11*b^3*log(1/(exp(2*a)*exp
(2*b*x) + 1)))/3 - (10*b^4*x)/3 - log(1/(exp(2*a)*exp(2*b*x) + 1))^4/(48*x
^3) + (b*log(1/(exp(2*a)*exp(2*b*x) + 1))^3)/(12*x^2) - 2*b^3*log(1/(exp(2
*a)*exp(2*b*x) + 1))*log(x) + (log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3)/(12*x^3) + (log(1/(exp(2*a)*
exp(2*b*x) + 1))^3*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/(
12*x^3) - (b*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3)/(12*x
^2) + 2*b^3*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(x) -
4*b^4*x*log(x) - (b^2*log(1/(exp(2*a)*exp(2*b*x) + 1))^2)/(2*x) - (log(1/(
exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1))^2)/(8*x^3) - (b^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1))^2)/(2*x) + (b^2*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)))/x + (b*log(1/(exp(2*a)*exp(2*b*x) + 1))*l
og((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(4*x^2) - (b*log(1/
(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)))/(4*x^2)
```

3.76. $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^4}\,dx$

3.77 $\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^5} \, dx$

3.77.1 Optimal result

Integrand size = 13, antiderivative size = 74

$$\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^5} \, dx = -\frac{b^3\text{arctanh}(\tanh(a+bx))}{x} - \frac{b^2\text{arctanh}(\tanh(a+bx))^2}{2x^2} - \frac{b\text{arctanh}(\tanh(a+bx))^3}{3x^3} - \frac{\text{arctanh}(\tanh(a+bx))^4}{4x^4} + b^4\log(x)$$

output
```
-b^3*arctanh(tanh(b*x+a))/x-1/2*b^2*arctanh(tanh(b*x+a))^2/x^2-1/3*b*arcta
nh(tanh(b*x+a))^3/x^3-1/4*arctanh(tanh(b*x+a))^4/x^4+b^4*ln(x)
```

3.77.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^5} \, dx = -\frac{12b^3x^3\text{arctanh}(\tanh(a+bx)) + 6b^2x^2\text{arctanh}(\tanh(a+bx))^2 + 4bx\text{arctanh}(\tanh(a+bx))^3 + 3\text{arctan}}{12x^4}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^4/x^5,x]
```

output
```
-1/12*(12*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 6*b^2*x^2*ArcTanh[Tanh[a + b*x]
]^2 + 4*b*x*ArcTanh[Tanh[a + b*x]]^3 + 3*ArcTanh[Tanh[a + b*x]]^4 - b^4*x^
4*(25 + 12*Log[x]))/x^4
```

3.77.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2599, 2599, 2599, 2599, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^5} \, dx$$

$$\Big\downarrow \text{2599}$$

$$b \int \frac{\text{arctanh}(\tanh(a+bx))^3}{x^4} dx - \frac{\text{arctanh}(\tanh(a+bx))^4}{4x^4}$$

$$\Big\downarrow \text{2599}$$

$$b \left(b \int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^3} dx - \frac{\text{arctanh}(\tanh(a+bx))^3}{3x^3} \right) - \frac{\text{arctanh}(\tanh(a+bx))^4}{4x^4}$$

$$\Big\downarrow \text{2599}$$

$$b \left(b \left(b \int \frac{\text{arctanh}(\tanh(a+bx))}{x^2} dx - \frac{\text{arctanh}(\tanh(a+bx))^2}{2x^2} \right) - \frac{\text{arctanh}(\tanh(a+bx))^3}{3x^3} \right) - \frac{\text{arctanh}(\tanh(a+bx))^4}{4x^4}$$

$$\Big\downarrow \text{2599}$$

$$b \left(b \left(b \left(b \int \frac{1}{x} dx - \frac{\text{arctanh}(\tanh(a+bx))}{x} \right) - \frac{\text{arctanh}(\tanh(a+bx))^2}{2x^2} \right) - \frac{\text{arctanh}(\tanh(a+bx))^3}{3x^3} \right) - \frac{\text{arctanh}(\tanh(a+bx))^4}{4x^4}$$

$$\Big\downarrow \text{14}$$

$$b \left(b \left(b \left(b \log(x) - \frac{\text{arctanh}(\tanh(a+bx))}{x} \right) - \frac{\text{arctanh}(\tanh(a+bx))^2}{2x^2} \right) - \frac{\text{arctanh}(\tanh(a+bx))^3}{3x^3} \right) - \frac{\text{arctanh}(\tanh(a+bx))^4}{4x^4}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^4/x^5,x]
```

output
```
-1/4*ArcTanh[Tanh[a + b*x]]^4/x^4 + b*(-1/3*ArcTanh[Tanh[a + b*x]]^3/x^3 +
 b*(-1/2*ArcTanh[Tanh[a + b*x]]^2/x^2 + b*(-(ArcTanh[Tanh[a + b*x]]/x) + b
*Log[x])))
```

3.77.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.77.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{4x^4}+b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{3x^3}+b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2x^2}+b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{x}\right.\right.\right.$
parts	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{4x^4}+b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{3x^3}+b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2x^2}+b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{x}\right.\right.\right.$
parallelrisch	$\frac{12b^4x^4\ln(x)-12b^3\operatorname{arctanh}(\tanh(bx+a))x^3-6b^2x^2\operatorname{arctanh}(\tanh(bx+a))^2-4bx\operatorname{arctanh}(\tanh(bx+a))^3-3\operatorname{arctanh}(\tanh(bx+a}{12x^4}$
risch	Expression too large to display

input
```
int(arctanh(tanh(b*x+a))^4/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arctanh(tanh(b*x+a))^4/x^4+b*(-1/3*arctanh(tanh(b*x+a))^3/x^3+b*(-1/2
*arctanh(tanh(b*x+a))^2/x^2+b*(-arctanh(tanh(b*x+a))/x+b*ln(x))))
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^5}\,dx = \frac{12\,b^4x^4\log{(x)} - 48\,ab^3x^3 - 36\,a^2b^2x^2 - 16\,a^3bx - 3\,a^4}{12\,x^4}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^5,x, algorithm="fricas")
```

output
```
1/12*(12*b^4*x^4*log(x) - 48*a*b^3*x^3 - 36*a^2*b^2*x^2 - 16*a^3*b*x - 3*a
^4)/x^4
```

3.77.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^5}\,dx = b^4\log{(x)} - \frac{b^3\operatorname{atanh}{(\tanh{(a+bx)})}}{x} - \frac{b^2\operatorname{atanh}^2{(\tanh{(a+bx)})}}{2x^2} - \frac{b\operatorname{atanh}^3{(\tanh{(a+bx)})}}{3x^3} - \frac{\operatorname{atanh}^4{(\tanh{(a+bx)})}}{4x^4}$$

input
```
integrate(atanh(tanh(b*x+a))**4/x**5,x)
```

output
```
b**4*log(x) - b**3*atanh(tanh(a + b*x))/x - b**2*atanh(tanh(a + b*x))**2/(
2*x**2) - b*atanh(tanh(a + b*x))**3/(3*x**3) - atanh(tanh(a + b*x))**4/(4*
x**4)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^5}\,dx = \frac{1}{2}\left(2\left(b^2\log{(x)} - \frac{b\,\operatorname{artanh}{(\tanh{(bx+a)})}}{x}\right)b - \frac{b\,\operatorname{artanh}{(\tanh{(bx+a)})}^2}{x^2}\right)b - \frac{b\,\operatorname{artanh}{(\tanh{(bx+a)})}^3}{3\,x^3} - \frac{\operatorname{artanh}{(\tanh{(bx+a)})}^4}{4\,x^4}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^5,x, algorithm="maxima")
```

output
```
1/2*(2*(b^2*log(x) - b*arctanh(tanh(b*x + a))/x)*b - b*arctanh(tanh(b*x +
a))^2/x^2)*b - 1/3*b*arctanh(tanh(b*x + a))^3/x^3 - 1/4*arctanh(tanh(b*x +
 a))^4/x^4
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.62

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^5}\,dx = b^4\log(|x|) - \frac{48\,ab^3x^3 + 36\,a^2b^2x^2 + 16\,a^3bx + 3\,a^4}{12\,x^4}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^5,x, algorithm="giac")
```

output
```
b^4*log(abs(x)) - 1/12*(48*a*b^3*x^3 + 36*a^2*b^2*x^2 + 16*a^3*b*x + 3*a^4
)/x^4
```

3.77.9 Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^5}\,dx = b^4\ln(x) - \frac{\operatorname{atanh}(\tanh(a+b\,x))^4}{4\,x^4} - \frac{b^2\operatorname{atanh}(\tanh(a+b\,x))^2}{2\,x^2} - \frac{b^3\operatorname{atanh}(\tanh(a+b\,x))}{x} - \frac{b\operatorname{atanh}(\tanh(a+b\,x))^3}{3\,x^3}$$

input
```
int(atanh(tanh(a + b*x))^4/x^5,x)
```

output
```
b^4*log(x) - atanh(tanh(a + b*x))^4/(4*x^4) - (b^2*atanh(tanh(a + b*x))^2)
/(2*x^2) - (b^3*atanh(tanh(a + b*x)))/x - (b*atanh(tanh(a + b*x))^3)/(3*x^
3)
```

3.78 $\int \frac{\textbf{arctanh}(\tanh(a+bx))^4}{x^6}\,dx$

3.78.1 Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^6}\,dx = \frac{\text{arctanh}(\tanh(a+bx))^5}{5x^5(bx-\text{arctanh}(\tanh(a+bx)))}$$

output
```
1/5*arctanh(tanh(b*x+a))^5/x^5/(b*x-arctanh(tanh(b*x+a)))
```

3.78.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 66 vs. 2(31) = 62.

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.13

$$\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^6}\,dx = -\frac{b^4x^4+b^3x^3\text{arctanh}(\tanh(a+bx))+b^2x^2\text{arctanh}(\tanh(a+bx))^2+bx\text{arctanh}(\tanh(a+bx))^3+\text{arctan}}{5x^5}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^4/x^6,x]
```

output
```
-1/5*(b^4*x^4 + b^3*x^3*ArcTanh[Tanh[a + b*x]] + b^2*x^2*ArcTanh[Tanh[a +
b*x]]^2 + b*x*ArcTanh[Tanh[a + b*x]]^3 + ArcTanh[Tanh[a + b*x]]^4)/x^5
```

3.78.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^6} \, dx$$

$$\Big\downarrow \text{ 2598}$$

$$\frac{\operatorname{arctanh}(\tanh(a+bx))^5}{5x^5(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^4/x^6,x]
```

output
```
ArcTanh[Tanh[a + b*x]]^5/(5*x^5*(b*x - ArcTanh[Tanh[a + b*x]]))
```

3.78.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

3.78.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(29) = 58$.

Time = 1.85 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.10

method	result	s
parallelrisch	$-\frac{b^{4} x^{4}+b^{3} \operatorname{arctanh}\left(\tanh \left(b x+a\right)\right) x^{3}+b^{2} x^{2} \operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{2}+b x \operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{3}+\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{4}}{5 x^{5}}$	6
default	$-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{4}}{5 x^{5}}+\frac{4 b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{3}}{4 x^{4}}+\frac{3 b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{2}}{3 x^{3}}+\frac{2 b \left(-\frac{b}{2 x}-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)}{2 x^{2}}\right)}{3}\right)}{4}\right)}{5}$	7
parts	$-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{4}}{5 x^{5}}+\frac{4 b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{3}}{4 x^{4}}+\frac{3 b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{2}}{3 x^{3}}+\frac{2 b \left(-\frac{b}{2 x}-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)}{2 x^{2}}\right)}{3}\right)}{4}\right)}{5}$	7
risch	Expression too large to display	2

input
```
int(arctanh(tanh(b*x+a))^4/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(b^4*x^4+b^3*arctanh(tanh(b*x+a))*x^3+b^2*x^2*arctanh(tanh(b*x+a))^2+
b*x*arctanh(tanh(b*x+a))^3+arctanh(tanh(b*x+a))^4)/x^5
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^6}\,dx = -\frac{5\,b^4x^4 + 10\,ab^3x^3 + 10\,a^2b^2x^2 + 5\,a^3bx + a^4}{5\,x^5}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^6,x, algorithm="fricas")
```

output
```
-1/5*(5*b^4*x^4 + 10*a*b^3*x^3 + 10*a^2*b^2*x^2 + 5*a^3*b*x + a^4)/x^5
```

3.78.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(26) = 52.

Time = 0.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.42

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^6}\,dx = -\frac{b^4}{5x} - \frac{b^3\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{5x^2} - \frac{b^2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{5x^3} - \frac{b\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{5x^4} - \frac{\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{5x^5}$$

input
```
integrate(atanh(tanh(b*x+a))**4/x**6,x)
```

output
```
-b**4/(5*x) - b**3*atanh(tanh(a + b*x))/(5*x**2) - b**2*atanh(tanh(a + b*x
))**2/(5*x**3) - b*atanh(tanh(a + b*x))**3/(5*x**4) - atanh(tanh(a + b*x))
**4/(5*x**5)
```

3.78.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(29) = 58.

Time = 0.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^6}\,dx = -\frac{1}{5}\left(b\left(\frac{b^2}{x} + \frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{x^2}\right) + \frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{x^3}\right)b - \frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}{5\,x^4} - \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4}{5\,x^5}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^6,x, algorithm="maxima")
```

output
```
-1/5*(b*(b^2/x + b*arctanh(tanh(b*x + a))/x^2) + b*arctanh(tanh(b*x + a))^
2/x^3)*b - 1/5*b*arctanh(tanh(b*x + a))^3/x^4 - 1/5*arctanh(tanh(b*x + a))
^4/x^5
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^6}\,dx = -\frac{5\,b^4x^4+10\,ab^3x^3+10\,a^2b^2x^2+5\,a^3bx+a^4}{5\,x^5}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^6,x, algorithm="giac")
```

output
```
-1/5*(5*b^4*x^4 + 10*a*b^3*x^3 + 10*a^2*b^2*x^2 + 5*a^3*b*x + a^4)/x^5
```

3.78.9 Mupad [B] (verification not implemented)

Time = 3.53 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.06

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^6}\,dx = -\frac{b^4\,x^4+b^3\,x^3\,\operatorname{atanh}(\tanh(a+b\,x))+b^2\,x^2\,\operatorname{atanh}(\tanh(a+b\,x))^2+b\,x\,\operatorname{atanh}(\tanh(a+b\,x))^3+\operatorname{atanh}(\mathrm{t}}{5\,x^5}$$

input
```
int(atanh(tanh(a + b*x))^4/x^6,x)
```

output
```
-(atanh(tanh(a + b*x))^4 + b^4*x^4 + b^2*x^2*atanh(tanh(a + b*x))^2 + b*x*
atanh(tanh(a + b*x))^3 + b^3*x^3*atanh(tanh(a + b*x)))/(5*x^5)
```

3.79 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^7}\,dx$

3.79.1 Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^7}\,dx = \frac{b\operatorname{arctanh}(\tanh(a+bx))^5}{30x^5(bx-\operatorname{arctanh}(\tanh(a+bx)))^2} + \frac{\operatorname{arctanh}(\tanh(a+bx))^5}{6x^6(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

output
```
1/30*b*arctanh(tanh(b*x+a))^5/x^5/(b*x-arctanh(tanh(b*x+a)))^2+1/6*arctanh
(tanh(b*x+a))^5/x^6/(b*x-arctanh(tanh(b*x+a)))
```

3.79.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^7}\,dx = -\frac{b^4x^4+2b^3x^3\operatorname{arctanh}(\tanh(a+bx))+3b^2x^2\operatorname{arctanh}(\tanh(a+bx))^2+4bx\operatorname{arctanh}(\tanh(a+bx))^3+5a}{30x^6}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^4/x^7,x]
```

output
```
-1/30*(b^4*x^4 + 2*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 3*b^2*x^2*ArcTanh[Tanh
[a + b*x]]^2 + 4*b*x*ArcTanh[Tanh[a + b*x]]^3 + 5*ArcTanh[Tanh[a + b*x]]^4
)/x^6
```

3.79.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^7}\,dx$$

$$\Big\downarrow \text{ 2602}$$

$$\frac{b\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^6}dx}{6(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{\text{arctanh}(\tanh(a+bx))^5}{6x^6(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{ 2598}$$

$$\frac{\text{arctanh}(\tanh(a+bx))^5}{6x^6(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{b\text{arctanh}(\tanh(a+bx))^5}{30x^5(bx-\text{arctanh}(\tanh(a+bx)))^2}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^4/x^7,x]
```

output
```
(b*ArcTanh[Tanh[a + b*x]]^5)/(30*x^5*(b*x - ArcTanh[Tanh[a + b*x]])^2) + A
rcTanh[Tanh[a + b*x]]^5/(6*x^6*(b*x - ArcTanh[Tanh[a + b*x]]))
```

3.79.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.79.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

method	result
parallelrisch	$-\frac{b^{4} x^{4}+2 b^{3} \operatorname{arctanh}\left(\tanh \left(b x+a\right)\right) x^{3}+3 b^{2} x^{2} \operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{2}+4 b x \operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{3}+5 \operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{4}}{30 x^{6}}$
default	$-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{4}}{6 x^{6}}+\frac{2 b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{3}}{5 x^{5}}+\frac{3 b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{2}}{4 x^{4}}+\frac{b \left(-\frac{b}{6 x^{2}}-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)}{3 x^{3}}\right)}{2}\right)}{5}\right)}{3}$
parts	$-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{4}}{6 x^{6}}+\frac{2 b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{3}}{5 x^{5}}+\frac{3 b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{2}}{4 x^{4}}+\frac{b \left(-\frac{b}{6 x^{2}}-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)}{3 x^{3}}\right)}{2}\right)}{5}\right)}{3}$
risch	Expression too large to display

input
```
int(arctanh(tanh(b*x+a))^4/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/30*(b^4*x^4+2*b^3*arctanh(tanh(b*x+a))*x^3+3*b^2*x^2*arctanh(tanh(b*x+a
))^2+4*b*x*arctanh(tanh(b*x+a))^3+5*arctanh(tanh(b*x+a))^4)/x^6
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^7}\,dx = -\frac{15\,b^4x^4+40\,ab^3x^3+45\,a^2b^2x^2+24\,a^3bx+5\,a^4}{30\,x^6}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^7,x, algorithm="fricas")
```

output
```
-1/30*(15*b^4*x^4 + 40*a*b^3*x^3 + 45*a^2*b^2*x^2 + 24*a^3*b*x + 5*a^4)/x^
6
```

3.79.6 Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^7}\,dx = -\frac{b^4}{30x^2} - \frac{b^3\operatorname{atanh}(\tanh(a+bx))}{15x^3} - \frac{b^2\operatorname{atanh}^2(\tanh(a+bx))}{10x^4} - \frac{2b\operatorname{atanh}^3(\tanh(a+bx))}{15x^5} - \frac{\operatorname{atanh}^4(\tanh(a+bx))}{6x^6}$$

input
```
integrate(atanh(tanh(b*x+a))**4/x**7,x)
```

output
```
-b**4/(30*x**2) - b**3*atanh(tanh(a + b*x))/(15*x**3) - b**2*atanh(tanh(a
+ b*x))**2/(10*x**4) - 2*b*atanh(tanh(a + b*x))**3/(15*x**5) - atanh(tanh(
a + b*x))**4/(6*x**6)
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^7}\,dx = -\frac{1}{30}\left(b\left(\frac{b^2}{x^2} + \frac{2\,b\,\operatorname{artanh}(\tanh(bx+a))}{x^3}\right) + \frac{3\,b\,\operatorname{artanh}(\tanh(bx+a))^2}{x^4}\right)b - \frac{2\,b\,\operatorname{artanh}(\tanh(bx+a))^3}{15\,x^5} - \frac{\operatorname{artanh}(\tanh(bx+a))^4}{6\,x^6}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^7,x, algorithm="maxima")
```

output
```
-1/30*(b*(b^2/x^2 + 2*b*arctanh(tanh(b*x + a))/x^3) + 3*b*arctanh(tanh(b*x
 + a))^2/x^4)*b - 2/15*b*arctanh(tanh(b*x + a))^3/x^5 - 1/6*arctanh(tanh(b
*x + a))^4/x^6
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^7}\,dx = -\frac{15\,b^4x^4 + 40\,ab^3x^3 + 45\,a^2b^2x^2 + 24\,a^3bx + 5\,a^4}{30\,x^6}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^7,x, algorithm="giac")
```

output
```
-1/30*(15*b^4*x^4 + 40*a*b^3*x^3 + 45*a^2*b^2*x^2 + 24*a^3*b*x + 5*a^4)/x^
6
```

3.79.9 Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^7}\,dx = -\frac{\operatorname{atanh}(\tanh(a+b\,x))^4}{6\,x^6} - \frac{b^4}{30\,x^2} - \frac{b^2\operatorname{atanh}(\tanh(a+b\,x))^2}{10\,x^4} - \frac{b^3\operatorname{atanh}(\tanh(a+b\,x))}{15\,x^3} - \frac{2\,b\operatorname{atanh}(\tanh(a+b\,x))^3}{15\,x^5}$$

input
```
int(atanh(tanh(a + b*x))^4/x^7,x)
```

output
```
- atanh(tanh(a + b*x))^4/(6*x^6) - b^4/(30*x^2) - (b^2*atanh(tanh(a + b*x)
)^2)/(10*x^4) - (b^3*atanh(tanh(a + b*x)))/(15*x^3) - (2*b*atanh(tanh(a +
b*x))^3)/(15*x^5)
```

3.80 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^8} dx$

3.80.1 Optimal result

Integrand size = 13, antiderivative size = 98

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^8} dx = \frac{b^2 \operatorname{arctanh}(\tanh(a+bx))^5}{105x^5(bx-\operatorname{arctanh}(\tanh(a+bx)))^3} + \frac{b \operatorname{arctanh}(\tanh(a+bx))^5}{21x^6(bx-\operatorname{arctanh}(\tanh(a+bx)))^2} + \frac{\operatorname{arctanh}(\tanh(a+bx))^5}{7x^7(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

output
```
1/105*b^2*arctanh(tanh(b*x+a))^5/x^5/(b*x-arctanh(tanh(b*x+a)))^3+1/21*b*a
rctanh(tanh(b*x+a))^5/x^6/(b*x-arctanh(tanh(b*x+a)))^2+1/7*arctanh(tanh(b*
x+a))^5/x^7/(b*x-arctanh(tanh(b*x+a)))
```

3.80.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^8} dx = -\frac{b^4x^4 + 3b^3x^3 \operatorname{arctanh}(\tanh(a+bx)) + 6b^2x^2 \operatorname{arctanh}(\tanh(a+bx))^2 + 10bx \operatorname{arctanh}(\tanh(a+bx))^3 + 1}{105x^7}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^4/x^8,x]
```

output
```
-1/105*(b^4*x^4 + 3*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 6*b^2*x^2*ArcTanh[Tan
h[a + b*x]]^2 + 10*b*x*ArcTanh[Tanh[a + b*x]]^3 + 15*ArcTanh[Tanh[a + b*x]
]^4)/x^7
```

3.80.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2602, 2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^8}\,dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{2b\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^7}dx}{7(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{\text{arctanh}(\tanh(a+bx))^5}{7x^7(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2602}$$

$$\frac{2b\left(\frac{b\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^6}dx}{6(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{\text{arctanh}(\tanh(a+bx))^5}{6x^6(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{7(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{\text{arctanh}(\tanh(a+bx))^5}{7x^7(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2598}$$

$$\frac{\text{arctanh}(\tanh(a+bx))^5}{7x^7(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2b\left(\frac{\text{arctanh}(\tanh(a+bx))^5}{6x^6(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{b\,\text{arctanh}(\tanh(a+bx))^5}{30x^5(bx-\text{arctanh}(\tanh(a+bx)))^2}\right)}{7(bx-\text{arctanh}(\tanh(a+bx)))}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^4/x^8,x]
```

output
```
ArcTanh[Tanh[a + b*x]]^5/(7*x^7*(b*x - ArcTanh[Tanh[a + b*x]])) + (2*b*((b
*ArcTanh[Tanh[a + b*x]]^5)/(30*x^5*(b*x - ArcTanh[Tanh[a + b*x]])^2) + Arc
Tanh[Tanh[a + b*x]]^5/(6*x^6*(b*x - ArcTanh[Tanh[a + b*x]]))))/(7*(b*x - A
rcTanh[Tanh[a + b*x]]))
```

3.80.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.80.4 Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

method	result
parallelrisch	$-\frac{b^{4} x^{4}+3 b^{3} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right) x^{3}+6 b^{2} x^{2} \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{2}+10 b x \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{3}+15 \operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{4}}{105 x^{7}}$
default	$-\frac{\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{4}}{7 x^{7}}+\frac{4 b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{3}}{6 x^{6}}+\frac{b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{2}}{5 x^{5}}+\frac{2 b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)}{4 x^{4}}-\frac{b}{12 x^{3}}\right)}{5}\right)}{2}\right)}{7}$
parts	$-\frac{\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{4}}{7 x^{7}}+\frac{4 b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{3}}{6 x^{6}}+\frac{b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{2}}{5 x^{5}}+\frac{2 b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)}{4 x^{4}}-\frac{b}{12 x^{3}}\right)}{5}\right)}{2}\right)}{7}$
risch	Expression too large to display

input
```
int(arctanh(tanh(b*x+a))^4/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/105*(b^4*x^4+3*b^3*arctanh(tanh(b*x+a))*x^3+6*b^2*x^2*arctanh(tanh(b*x+
a))^2+10*b*x*arctanh(tanh(b*x+a))^3+15*arctanh(tanh(b*x+a))^4)/x^7
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.47

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^8}\,dx = -\frac{35\,b^4x^4 + 105\,ab^3x^3 + 126\,a^2b^2x^2 + 70\,a^3bx + 15\,a^4}{105\,x^7}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^8,x, algorithm="fricas")
```

output
```
-1/105*(35*b^4*x^4 + 105*a*b^3*x^3 + 126*a^2*b^2*x^2 + 70*a^3*b*x + 15*a^4
)/x^7
```

3.80.6 Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^8}\,dx = -\frac{b^4}{105x^3} - \frac{b^3\operatorname{atanh}(\tanh(a+bx))}{35x^4} - \frac{2b^2\operatorname{atanh}^2(\tanh(a+bx))}{35x^5} - \frac{2b\operatorname{atanh}^3(\tanh(a+bx))}{21x^6} - \frac{\operatorname{atanh}^4(\tanh(a+bx))}{7x^7}$$

input
```
integrate(atanh(tanh(b*x+a))**4/x**8,x)
```

output
```
-b**4/(105*x**3) - b**3*atanh(tanh(a + b*x))/(35*x**4) - 2*b**2*atanh(tanh
(a + b*x))**2/(35*x**5) - 2*b*atanh(tanh(a + b*x))**3/(21*x**6) - atanh(ta
nh(a + b*x))**4/(7*x**7)
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.73

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^8}\,dx = -\frac{1}{105}\left(b\left(\frac{b^2}{x^3} + \frac{3\,b\operatorname{artanh}(\tanh(bx+a))}{x^4}\right) + \frac{6\,b\operatorname{artanh}(\tanh(bx+a))^2}{x^5}\right)b - \frac{2\,b\operatorname{artanh}(\tanh(bx+a))^3}{21\,x^6} - \frac{\operatorname{artanh}(\tanh(bx+a))^4}{7\,x^7}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^8,x, algorithm="maxima")
```

output
```
-1/105*(b*(b^2/x^3 + 3*b*arctanh(tanh(b*x + a))/x^4) + 6*b*arctanh(tanh(b*
x + a))^2/x^5)*b - 2/21*b*arctanh(tanh(b*x + a))^3/x^6 - 1/7*arctanh(tanh(
b*x + a))^4/x^7
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.47

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^8}\,dx = -\frac{35\,b^4x^4 + 105\,ab^3x^3 + 126\,a^2b^2x^2 + 70\,a^3bx + 15\,a^4}{105\,x^7}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^8,x, algorithm="giac")
```

output
```
-1/105*(35*b^4*x^4 + 105*a*b^3*x^3 + 126*a^2*b^2*x^2 + 70*a^3*b*x + 15*a^4
)/x^7
```

3.80.9 Mupad [B] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^8}\,dx = -\frac{\operatorname{atanh}(\tanh(a+b\,x))^4}{7\,x^7} - \frac{b^4}{105\,x^3} - \frac{2\,b^2\operatorname{atanh}(\tanh(a+b\,x))^2}{35\,x^5} - \frac{b^3\operatorname{atanh}(\tanh(a+b\,x))}{35\,x^4} - \frac{2\,b\operatorname{atanh}(\tanh(a+b\,x))^3}{21\,x^6}$$

input
```
int(atanh(tanh(a + b*x))^4/x^8,x)
```

output
```
- atanh(tanh(a + b*x))^4/(7*x^7) - b^4/(105*x^3) - (2*b^2*atanh(tanh(a + b
*x))^2)/(35*x^5) - (b^3*atanh(tanh(a + b*x)))/(35*x^4) - (2*b*atanh(tanh(a
 + b*x))^3)/(21*x^6)
```

3.81 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^9}\,dx$

3.81.1 Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^9}\,dx = -\frac{b^4}{280x^4} - \frac{b^3\operatorname{arctanh}(\tanh(a+bx))}{70x^5} - \frac{b^2\operatorname{arctanh}(\tanh(a+bx))^2}{28x^6} - \frac{b\operatorname{arctanh}(\tanh(a+bx))^3}{14x^7} - \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{8x^8}$$

output
```
-1/280*b^4/x^4-1/70*b^3*arctanh(tanh(b*x+a))/x^5-1/28*b^2*arctanh(tanh(b*x
+a))^2/x^6-1/14*b*arctanh(tanh(b*x+a))^3/x^7-1/8*arctanh(tanh(b*x+a))^4/x^
8
```

3.81.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^9}\,dx = -\frac{b^4x^4 + 4b^3x^3\operatorname{arctanh}(\tanh(a+bx)) + 10b^2x^2\operatorname{arctanh}(\tanh(a+bx))^2 + 20bx\operatorname{arctanh}(\tanh(a+bx))^3 +}{280x^8}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^4/x^9,x]
```

output
```
-1/280*(b^4*x^4 + 4*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 10*b^2*x^2*ArcTanh[Ta
nh[a + b*x]]^2 + 20*b*x*ArcTanh[Tanh[a + b*x]]^3 + 35*ArcTanh[Tanh[a + b*x
]]^4)/x^8
```

3.81.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 168 vs. $2(80) = 160$.

Time = 0.40 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2602, 2602, 2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^9}\,dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{3b\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^8}dx}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{\operatorname{arctanh}(\tanh(a+bx))^5}{8x^8(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2602}$$

$$\frac{3b\left(\frac{2b\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^7}dx}{7(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{\operatorname{arctanh}(\tanh(a+bx))^5}{7x^7(bx-\operatorname{arctanh}(\tanh(a+bx)))}\right)}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{\operatorname{arctanh}(\tanh(a+bx))^5}{8x^8(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2602}$$

$$\frac{3b\left(\frac{2b\left(\frac{b\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^6}dx}{6(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{\operatorname{arctanh}(\tanh(a+bx))^5}{6x^6(bx-\operatorname{arctanh}(\tanh(a+bx)))}\right)}{7(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{\operatorname{arctanh}(\tanh(a+bx))^5}{7x^7(bx-\operatorname{arctanh}(\tanh(a+bx)))}\right)}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{\operatorname{arctanh}(\tanh(a+bx))^5}{8x^8(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2598}$$

$$\frac{\operatorname{arctanh}(\tanh(a+bx))^5}{8x^8(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{3b\left(\frac{\operatorname{arctanh}(\tanh(a+bx))^5}{7x^7(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{2b\left(\frac{\operatorname{arctanh}(\tanh(a+bx))^5}{6x^6(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{b\operatorname{arctanh}(\tanh(a+bx))^5}{30x^5(bx-\operatorname{arctanh}(\tanh(a+bx)))^2}\right)}{7(bx-\operatorname{arctanh}(\tanh(a+bx)))}\right)}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^4/x^9,x]
```

output
```
ArcTanh[Tanh[a + b*x]]^5/(8*x^8*(b*x - ArcTanh[Tanh[a + b*x]])) + (3*b*(Ar
cTanh[Tanh[a + b*x]]^5/(7*x^7*(b*x - ArcTanh[Tanh[a + b*x]])) + (2*b*((b*A
rcTanh[Tanh[a + b*x]]^5)/(30*x^5*(b*x - ArcTanh[Tanh[a + b*x]])^2) + ArcTa
nh[Tanh[a + b*x]]^5/(6*x^6*(b*x - ArcTanh[Tanh[a + b*x]]))))/(7*(b*x - Arc
Tanh[Tanh[a + b*x]]))))/(8*(b*x - ArcTanh[Tanh[a + b*x]]))
```

3.81.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.81.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

method	result
parallelrisch	$-\frac{b^{4} x^{4}+4 b^{3} \operatorname{arctanh}\left(\tanh \left(b x+a\right)\right) x^{3}+10 b^{2} x^{2} \operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{2}+20 b x \operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{3}+35 \operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{4}}{280 x^{8}}$
default	$-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{4}}{8 x^{8}}+\frac{b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{3}}{7 x^{7}}+\frac{3 b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{2}}{6 x^{6}}+\frac{b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)}{5 x^{5}}-\frac{b}{20 x^{4}}\right)}{3}\right)}{7}\right)}{2}$
parts	$-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{4}}{8 x^{8}}+\frac{b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{3}}{7 x^{7}}+\frac{3 b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)^{2}}{6 x^{6}}+\frac{b \left(-\frac{\operatorname{arctanh}\left(\tanh \left(b x+a\right)\right)}{5 x^{5}}-\frac{b}{20 x^{4}}\right)}{3}\right)}{7}\right)}{2}$
risch	Expression too large to display

input
```
int(arctanh(tanh(b*x+a))^4/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/280*(b^4*x^4+4*b^3*arctanh(tanh(b*x+a))*x^3+10*b^2*x^2*arctanh(tanh(b*x
+a))^2+20*b*x*arctanh(tanh(b*x+a))^3+35*arctanh(tanh(b*x+a))^4)/x^8
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^9}\,dx = -\frac{70\,b^4x^4 + 224\,ab^3x^3 + 280\,a^2b^2x^2 + 160\,a^3bx + 35\,a^4}{280\,x^8}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^9,x, algorithm="fricas")
```

output
```
-1/280*(70*b^4*x^4 + 224*a*b^3*x^3 + 280*a^2*b^2*x^2 + 160*a^3*b*x + 35*a^
4)/x^8
```

3.81.6 Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^9}\,dx = -\frac{b^4}{280x^4} - \frac{b^3\operatorname{atanh}(\tanh(a+bx))}{70x^5} - \frac{b^2\operatorname{atanh}^2(\tanh(a+bx))}{28x^6} - \frac{b\operatorname{atanh}^3(\tanh(a+bx))}{14x^7} - \frac{\operatorname{atanh}^4(\tanh(a+bx))}{8x^8}$$

input
```
integrate(atanh(tanh(b*x+a))**4/x**9,x)
```

output
```
-b**4/(280*x**4) - b**3*atanh(tanh(a + b*x))/(70*x**5) - b**2*atanh(tanh(a
 + b*x))**2/(28*x**6) - b*atanh(tanh(a + b*x))**3/(14*x**7) - atanh(tanh(a
 + b*x))**4/(8*x**8)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^9}\,dx = -\frac{1}{280}\left(b\left(\frac{b^2}{x^4} + \frac{4\,b\operatorname{artanh}(\tanh(bx+a))}{x^5}\right) + \frac{10\,b\operatorname{artanh}(\tanh(bx+a))^2}{x^6}\right)b - \frac{b\operatorname{artanh}(\tanh(bx+a))^3}{14\,x^7} - \frac{\operatorname{artanh}(\tanh(bx+a))^4}{8\,x^8}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^9,x, algorithm="maxima")
```

output
```
-1/280*(b*(b^2/x^4 + 4*b*arctanh(tanh(b*x + a))/x^5) + 10*b*arctanh(tanh(b
*x + a))^2/x^6)*b - 1/14*b*arctanh(tanh(b*x + a))^3/x^7 - 1/8*arctanh(tanh
(b*x + a))^4/x^8
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^9}\,dx = -\frac{70\,b^4x^4+224\,ab^3x^3+280\,a^2b^2x^2+160\,a^3bx+35\,a^4}{280\,x^8}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^9,x, algorithm="giac")
```

output
```
-1/280*(70*b^4*x^4 + 224*a*b^3*x^3 + 280*a^2*b^2*x^2 + 160*a^3*b*x + 35*a^
4)/x^8
```

3.81.9 Mupad [B] (verification not implemented)

Time = 3.53 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^9}\,dx = -\frac{\operatorname{atanh}(\tanh(a+b\,x))^4}{8\,x^8}-\frac{b^4}{280\,x^4}-\frac{b^2\operatorname{atanh}(\tanh(a+b\,x))^2}{28\,x^6}-\frac{b^3\operatorname{atanh}(\tanh(a+b\,x))}{70\,x^5}-\frac{b\operatorname{atanh}(\tanh(a+b\,x))^3}{14\,x^7}$$

input
```
int(atanh(tanh(a + b*x))^4/x^9,x)
```

output
```
- atanh(tanh(a + b*x))^4/(8*x^8) - b^4/(280*x^4) - (b^2*atanh(tanh(a + b*x
))^2)/(28*x^6) - (b^3*atanh(tanh(a + b*x)))/(70*x^5) - (b*atanh(tanh(a + b
*x))^3)/(14*x^7)
```

3.82 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^{10}}\,dx$

3.82.1 Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^{10}}\,dx = -\frac{b^4}{630x^5} - \frac{b^3\operatorname{arctanh}(\tanh(a+bx))}{126x^6} - \frac{b^2\operatorname{arctanh}(\tanh(a+bx))^2}{42x^7} - \frac{b\operatorname{arctanh}(\tanh(a+bx))^3}{18x^8} - \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{9x^9}$$

output
```
-1/630*b^4/x^5-1/126*b^3*arctanh(tanh(b*x+a))/x^6-1/42*b^2*arctanh(tanh(b*
x+a))^2/x^7-1/18*b*arctanh(tanh(b*x+a))^3/x^8-1/9*arctanh(tanh(b*x+a))^4/x
^9
```

3.82.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^{10}}\,dx = -\frac{b^4x^4 + 5b^3x^3\operatorname{arctanh}(\tanh(a+bx)) + 15b^2x^2\operatorname{arctanh}(\tanh(a+bx))^2 + 35bx\operatorname{arctanh}(\tanh(a+bx))^3 +}{630x^9}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^4/x^10,x]
```

output
```
-1/630*(b^4*x^4 + 5*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 15*b^2*x^2*ArcTanh[Ta
nh[a + b*x]]^2 + 35*b*x*ArcTanh[Tanh[a + b*x]]^3 + 70*ArcTanh[Tanh[a + b*x
]]^4)/x^9
```

3.82.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2599, 2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^{10}}dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{4}{9}b\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^9}dx-\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{9x^9}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{4}{9}b\left(\frac{3}{8}b\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^8}dx-\frac{\operatorname{arctanh}(\tanh(a+bx))^3}{8x^8}\right)-\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{9x^9}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{4}{9}b\left(\frac{3}{8}b\left(\frac{2}{7}b\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^7}dx-\frac{\operatorname{arctanh}(\tanh(a+bx))^2}{7x^7}\right)-\frac{\operatorname{arctanh}(\tanh(a+bx))^3}{8x^8}\right)-\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{9x^9}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{4}{9}b\left(\frac{3}{8}b\left(\frac{2}{7}b\left(\frac{1}{6}b\int \frac{1}{x^6}dx-\frac{\operatorname{arctanh}(\tanh(a+bx))}{6x^6}\right)-\frac{\operatorname{arctanh}(\tanh(a+bx))^2}{7x^7}\right)-\frac{\operatorname{arctanh}(\tanh(a+bx))^3}{8x^8}\right)-\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{9x^9}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{4}{9}b\left(\frac{3}{8}b\left(\frac{2}{7}b\left(-\frac{\operatorname{arctanh}(\tanh(a+bx))}{6x^6}-\frac{b}{30x^5}\right)-\frac{\operatorname{arctanh}(\tanh(a+bx))^2}{7x^7}\right)-\frac{\operatorname{arctanh}(\tanh(a+bx))^3}{8x^8}\right)-\frac{\operatorname{arctanh}(\tanh(a+bx))^4}{9x^9}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^4/x^10,x]
```

output
```
-1/9*ArcTanh[Tanh[a + b*x]]^4/x^9 + (4*b*(-1/8*ArcTanh[Tanh[a + b*x]]^3/x^
8 + (3*b*(-1/7*ArcTanh[Tanh[a + b*x]]^2/x^7 + (2*b*(-1/30*b/x^5 - ArcTanh[
Tanh[a + b*x]]/(6*x^6)))/7))/8))/9
```

3.82.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.82.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

method	result
parallelrisch	$-\frac{b^4x^4+5b^3\operatorname{arctanh}(\tanh(bx+a))x^3+15b^2x^2\operatorname{arctanh}(\tanh(bx+a))^2+35bx\operatorname{arctanh}(\tanh(bx+a))^3+70\operatorname{arctanh}(\tanh(bx+a))^4}{630x^9}$
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{9x^9}+\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{8x^8}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{7x^7}+\frac{2b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{6x^6}-\frac{b}{30x^5}\right)}{7}\right)}{8}\right)}{9}$
parts	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{9x^9}+\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{8x^8}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{7x^7}+\frac{2b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{6x^6}-\frac{b}{30x^5}\right)}{7}\right)}{8}\right)}{9}$
risch	Expression too large to display

input
```
int(arctanh(tanh(b*x+a))^4/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/630*(b^4*x^4+5*b^3*arctanh(tanh(b*x+a))*x^3+15*b^2*x^2*arctanh(tanh(b*x
+a))^2+35*b*x*arctanh(tanh(b*x+a))^3+70*arctanh(tanh(b*x+a))^4)/x^9
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^{10}}\,dx = -\frac{126\,b^4x^4+420\,ab^3x^3+540\,a^2b^2x^2+315\,a^3bx+70\,a^4}{630\,x^9}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^10,x, algorithm="fricas")
```

output
```
-1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a
^4)/x^9
```

3.82.6 Sympy [A] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^{10}}\,dx = -\frac{b^4}{630x^5} - \frac{b^3\operatorname{atanh}(\tanh(a+bx))}{126x^6} - \frac{b^2\operatorname{atanh}^2(\tanh(a+bx))}{42x^7} - \frac{b\operatorname{atanh}^3(\tanh(a+bx))}{18x^8} - \frac{\operatorname{atanh}^4(\tanh(a+bx))}{9x^9}$$

input
```
integrate(atanh(tanh(b*x+a))**4/x**10,x)
```

output
```
-b**4/(630*x**5) - b**3*atanh(tanh(a + b*x))/(126*x**6) - b**2*atanh(tanh(
a + b*x))**2/(42*x**7) - b*atanh(tanh(a + b*x))**3/(18*x**8) - atanh(tanh(
a + b*x))**4/(9*x**9)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^{10}}\,dx$$
$$= -\frac{1}{630}\left(b\left(\frac{b^2}{x^5} + \frac{5\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{x^6}\right) + \frac{15\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{x^7}\right)b$$
$$- \frac{b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}{18\,x^8} - \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4}{9\,x^9}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^10,x, algorithm="maxima")
```

output
```
-1/630*(b*(b^2/x^5 + 5*b*arctanh(tanh(b*x + a))/x^6) + 15*b*arctanh(tanh(b
*x + a))^2/x^7)*b - 1/18*b*arctanh(tanh(b*x + a))^3/x^8 - 1/9*arctanh(tanh
(b*x + a))^4/x^9
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^{10}}\,dx = -\frac{126\,b^4x^4 + 420\,ab^3x^3 + 540\,a^2b^2x^2 + 315\,a^3bx + 70\,a^4}{630\,x^9}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^10,x, algorithm="giac")
```

output
```
-1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a
^4)/x^9
```

3.82.9 Mupad [B] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^{10}}\,dx = -\frac{\operatorname{atanh}(\tanh(a+b\,x))^4}{9\,x^9} - \frac{b^4}{630\,x^5}$$
$$- \frac{b^2\operatorname{atanh}(\tanh(a+b\,x))^2}{42\,x^7} - \frac{b^3\operatorname{atanh}(\tanh(a+b\,x))}{126\,x^6}$$
$$- \frac{b\operatorname{atanh}(\tanh(a+b\,x))^3}{18\,x^8}$$

input
```
int(atanh(tanh(a + b*x))^4/x^10,x)
```

output
```
- atanh(tanh(a + b*x))^4/(9*x^9) - b^4/(630*x^5) - (b^2*atanh(tanh(a + b*x
))^2)/(42*x^7) - (b^3*atanh(tanh(a + b*x)))/(126*x^6) - (b*atanh(tanh(a +
b*x))^3)/(18*x^8)
```

3.82. $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^{10}}\,dx$

3.83 $\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^{11}} \, dx$

3.83.1 Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^{11}} \, dx = -\frac{b^4}{1260x^6} - \frac{b^3\text{arctanh}(\tanh(a+bx))}{210x^7} - \frac{b^2\text{arctanh}(\tanh(a+bx))^2}{60x^8} - \frac{2b\text{arctanh}(\tanh(a+bx))^3}{45x^9} - \frac{\text{arctanh}(\tanh(a+bx))^4}{10x^{10}}$$

output
```
-1/1260*b^4/x^6-1/210*b^3*arctanh(tanh(b*x+a))/x^7-1/60*b^2*arctanh(tanh(b
*x+a))^2/x^8-2/45*b*arctanh(tanh(b*x+a))^3/x^9-1/10*arctanh(tanh(b*x+a))^4
/x^10
```

3.83.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^{11}} \, dx = -\frac{b^4x^4 + 6b^3x^3\text{arctanh}(\tanh(a+bx)) + 21b^2x^2\text{arctanh}(\tanh(a+bx))^2 + 56bx\text{arctanh}(\tanh(a+bx))^3 +}{1260x^{10}}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^4/x^11,x]
```

output
```
-1/1260*(b^4*x^4 + 6*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 21*b^2*x^2*ArcTanh[T
anh[a + b*x]]^2 + 56*b*x*ArcTanh[Tanh[a + b*x]]^3 + 126*ArcTanh[Tanh[a + b
*x]]^4)/x^10
```

3.83.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2599, 2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^4}{x^{11}}dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{5}b\int \frac{\text{arctanh}(\tanh(a+bx))^3}{x^{10}}dx-\frac{\text{arctanh}(\tanh(a+bx))^4}{10x^{10}}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{5}b\left(\frac{1}{3}b\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^9}dx-\frac{\text{arctanh}(\tanh(a+bx))^3}{9x^9}\right)-\frac{\text{arctanh}(\tanh(a+bx))^4}{10x^{10}}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{5}b\left(\frac{1}{3}b\left(\frac{1}{4}b\int \frac{\text{arctanh}(\tanh(a+bx))}{x^8}dx-\frac{\text{arctanh}(\tanh(a+bx))^2}{8x^8}\right)-\frac{\text{arctanh}(\tanh(a+bx))^3}{9x^9}\right)-\frac{\text{arctanh}(\tanh(a+bx))^4}{10x^{10}}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{5}b\left(\frac{1}{3}b\left(\frac{1}{4}b\left(\frac{1}{7}b\int \frac{1}{x^7}dx-\frac{\text{arctanh}(\tanh(a+bx))}{7x^7}\right)-\frac{\text{arctanh}(\tanh(a+bx))^2}{8x^8}\right)-\frac{\text{arctanh}(\tanh(a+bx))^3}{9x^9}\right)-\frac{\text{arctanh}(\tanh(a+bx))^4}{10x^{10}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2}{5}b\left(\frac{1}{3}b\left(\frac{1}{4}b\left(-\frac{\text{arctanh}(\tanh(a+bx))}{7x^7}-\frac{b}{42x^6}\right)-\frac{\text{arctanh}(\tanh(a+bx))^2}{8x^8}\right)-\frac{\text{arctanh}(\tanh(a+bx))^3}{9x^9}\right)-\frac{\text{arctanh}(\tanh(a+bx))^4}{10x^{10}}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^4/x^11,x]
```

output
```
-1/10*ArcTanh[Tanh[a + b*x]]^4/x^10 + (2*b*(-1/9*ArcTanh[Tanh[a + b*x]]^3/
x^9 + (b*(-1/8*ArcTanh[Tanh[a + b*x]]^2/x^8 + (b*(-1/42*b/x^6 - ArcTanh[Ta
nh[a + b*x]]/(7*x^7)))/4))/3))/5
```

3.83.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.83.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

method	result
parallelrisch	$-\frac{b^4x^4+6b^3\operatorname{arctanh}(\tanh(bx+a))x^3+21b^2x^2\operatorname{arctanh}(\tanh(bx+a))^2+56bx\operatorname{arctanh}(\tanh(bx+a))^3+126\operatorname{arctanh}(\tanh(bx+a))}{1260x^{10}}$
default	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{10x^{10}}+\frac{2b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{9x^9}+\frac{b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{8x^8}+\frac{b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{7x^7}-\frac{b}{42x^6}\right)}{4}\right)}{3}\right)}{5}$
parts	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{10x^{10}}+\frac{2b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{9x^9}+\frac{b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{8x^8}+\frac{b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{7x^7}-\frac{b}{42x^6}\right)}{4}\right)}{3}\right)}{5}$
risch	Expression too large to display

input
```
int(arctanh(tanh(b*x+a))^4/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/1260*(b^4*x^4+6*b^3*arctanh(tanh(b*x+a))*x^3+21*b^2*x^2*arctanh(tanh(b*
x+a))^2+56*b*x*arctanh(tanh(b*x+a))^3+126*arctanh(tanh(b*x+a))^4)/x^10
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^{11}}\,dx = -\frac{210\,b^4x^4 + 720\,ab^3x^3 + 945\,a^2b^2x^2 + 560\,a^3bx + 126\,a^4}{1260\,x^{10}}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^11,x, algorithm="fricas")
```

output
```
-1/1260*(210*b^4*x^4 + 720*a*b^3*x^3 + 945*a^2*b^2*x^2 + 560*a^3*b*x + 126
*a^4)/x^10
```

3.83.6 Sympy [A] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^{11}}\,dx = -\frac{b^4}{1260x^6} - \frac{b^3\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{210x^7} - \frac{b^2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{60x^8} - \frac{2b\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{45x^9} - \frac{\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{10x^{10}}$$

input
```
integrate(atanh(tanh(b*x+a))**4/x**11,x)
```

output
```
-b**4/(1260*x**6) - b**3*atanh(tanh(a + b*x))/(210*x**7) - b**2*atanh(tanh
(a + b*x))**2/(60*x**8) - 2*b*atanh(tanh(a + b*x))**3/(45*x**9) - atanh(ta
nh(a + b*x))**4/(10*x**10)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^{11}}\,dx = -\frac{1}{1260}\left(b\left(b\left(\frac{b^2}{x^6}+\frac{6\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{x^7}\right)+\frac{21\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{x^8}\right)b\right. \\ -\frac{2\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}{45\,x^9}-\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4}{10\,x^{10}}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^11,x, algorithm="maxima")
```

output
```
-1/1260*(b*(b^2/x^6 + 6*b*arctanh(tanh(b*x + a))/x^7) + 21*b*arctanh(tanh(
b*x + a))^2/x^8)*b - 2/45*b*arctanh(tanh(b*x + a))^3/x^9 - 1/10*arctanh(ta
nh(b*x + a))^4/x^10
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^{11}}\,dx = -\frac{210\,b^4x^4+720\,ab^3x^3+945\,a^2b^2x^2+560\,a^3bx+126\,a^4}{1260\,x^{10}}$$

input
```
integrate(arctanh(tanh(b*x+a))^4/x^11,x, algorithm="giac")
```

output
```
-1/1260*(210*b^4*x^4 + 720*a*b^3*x^3 + 945*a^2*b^2*x^2 + 560*a^3*b*x + 126
*a^4)/x^10
```

3.83.9 Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^{11}}\,dx = -\frac{\operatorname{atanh}(\tanh(a+b\,x))^4}{10\,x^{10}}-\frac{b^4}{1260\,x^6} \\ -\frac{b^2\operatorname{atanh}(\tanh(a+b\,x))^2}{60\,x^8}-\frac{b^3\operatorname{atanh}(\tanh(a+b\,x))}{210\,x^7} \\ -\frac{2\,b\operatorname{atanh}(\tanh(a+b\,x))^3}{45\,x^9}$$

input
```
int(atanh(tanh(a + b*x))^4/x^11,x)
```

output
```
- atanh(tanh(a + b*x))^4/(10*x^10) - b^4/(1260*x^6) - (b^2*atanh(tanh(a +
b*x))^2)/(60*x^8) - (b^3*atanh(tanh(a + b*x)))/(210*x^7) - (2*b*atanh(tanh
(a + b*x))^3)/(45*x^9)
```

3.83. $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^4}{x^{11}}\,dx$

3.84 $\int x\mathbf{arctanh}(\tanh(a+bx))^6\,dx$

3.84.1 Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x\text{arctanh}(\tanh(a+bx))^6\,dx=\frac{x\text{arctanh}(\tanh(a+bx))^7}{7b}-\frac{\text{arctanh}(\tanh(a+bx))^8}{56b^2}$$

output
```
1/7*x*arctanh(tanh(b*x+a))^7/b-1/56*arctanh(tanh(b*x+a))^8/b^2
```

3.84.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. $2(34)=68$.

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 5.21

$$\int x\text{arctanh}(\tanh(a+bx))^6\,dx=-\frac{(a+bx)\left((7a-bx)(a+bx)^6-8(6a-bx)(a+bx)^5\text{arctanh}(\tanh(a+bx))+28(5a-bx)(a+bx)^4\text{arct}\right.}{}$$

input
```
Integrate[x*ArcTanh[Tanh[a + b*x]]^6,x]
```

output
```
-1/56*((a + b*x)*((7*a - b*x)*(a + b*x)^6 - 8*(6*a - b*x)*(a + b*x)^5*ArcT
anh[Tanh[a + b*x]] + 28*(5*a - b*x)*(a + b*x)^4*ArcTanh[Tanh[a + b*x]]^2 -
 56*(4*a - b*x)*(a + b*x)^3*ArcTanh[Tanh[a + b*x]]^3 + 70*(3*a - b*x)*(a +
 b*x)^2*ArcTanh[Tanh[a + b*x]]^4 - 56*(2*a^2 + a*b*x - b^2*x^2)*ArcTanh[Ta
nh[a + b*x]]^5 + 28*(a - b*x)*ArcTanh[Tanh[a + b*x]]^6))/b^2
```

3.84.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(\tanh(a+bx))^6\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x\text{arctanh}(\tanh(a+bx))^7}{7b}-\frac{\int \text{arctanh}(\tanh(a+bx))^7dx}{7b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{x\text{arctanh}(\tanh(a+bx))^7}{7b}-\frac{\int \text{arctanh}(\tanh(a+bx))^7d\text{arctanh}(\tanh(a+bx))}{7b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{x\text{arctanh}(\tanh(a+bx))^7}{7b}-\frac{\text{arctanh}(\tanh(a+bx))^8}{56b^2}$$

input
```
Int[x*ArcTanh[Tanh[a + b*x]]^6,x]
```

output
```
(x*ArcTanh[Tanh[a + b*x]]^7)/(7*b) - ArcTanh[Tanh[a + b*x]]^8/(56*b^2)
```

3.84.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.84.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(30) = 60$.

Time = 34.92 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.09

method	result
parallelrisch	$\frac{x^2 \operatorname{arctanh}(\tanh(bx+a))^6}{2} - b^3 \operatorname{arctanh}(\tanh(bx+a))^3 x^5 + \frac{5b^2 \operatorname{arctanh}(\tanh(bx+a))^4 x^4}{4} - b \operatorname{arctanh}(\text{ta}$
default	$\frac{x^2 \operatorname{arctanh}(\tanh(bx+a))^6}{2} - 3b \left(\frac{x^3 \operatorname{arctanh}(\tanh(bx+a))^5}{3} - \frac{5b \left(\frac{x^4 \operatorname{arctanh}(\tanh(bx+a))^4}{4} - b \left(\frac{x^5 \operatorname{arctanh}(\tanh(bx+a))^3}{5} - \right.\right.}{}\right.$
parts	$\frac{x^2 \operatorname{arctanh}(\tanh(bx+a))^6}{2} - 3b \left(\frac{x^3 \operatorname{arctanh}(\tanh(bx+a))^5}{3} - \frac{5b \left(\frac{x^4 \operatorname{arctanh}(\tanh(bx+a))^4}{4} - b \left(\frac{x^5 \operatorname{arctanh}(\tanh(bx+a))^3}{5} - \right.\right.}{}\right.$
risch	Expression too large to display

input
```
int(x*arctanh(tanh(b*x+a))^6,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arctanh(tanh(b*x+a))^6-b^3*arctanh(tanh(b*x+a))^3*x^5+5/4*b^2*arct
anh(tanh(b*x+a))^4*x^4-b*arctanh(tanh(b*x+a))^5*x^3-1/7*b^5*arctanh(tanh(b
*x+a))*x^7+1/2*b^4*arctanh(tanh(b*x+a))^2*x^6+1/56*b^6*x^8
```

3.84.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(30) = 60$.

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int x\text{arctanh}(\tanh(a+bx))^6\,dx = \frac{1}{8}\,b^6x^8 + \frac{6}{7}\,ab^5x^7 + \frac{5}{2}\,a^2b^4x^6 + 4\,a^3b^3x^5 + \frac{15}{4}\,a^4b^2x^4 + 2\,a^5bx^3 + \frac{1}{2}\,a^6x^2$$

input
```
integrate(x*arctanh(tanh(b*x+a))^6,x, algorithm="fricas")
```

output
```
1/8*b^6*x^8 + 6/7*a*b^5*x^7 + 5/2*a^2*b^4*x^6 + 4*a^3*b^3*x^5 + 15/4*a^4*b
^2*x^4 + 2*a^5*b*x^3 + 1/2*a^6*x^2
```

3.84.6 Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int x\text{arctanh}(\tanh(a+bx))^6\,dx = \begin{cases} \frac{x\,\text{atanh}^7\,(\tanh\,(a+bx))}{7b} - \frac{\text{atanh}^8\,(\tanh\,(a+bx))}{56b^2} & \text{for } b \neq 0 \\ \frac{x^2\,\text{atanh}^6\,(\tanh\,(a))}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*atanh(tanh(b*x+a))**6,x)
```

output
```
Piecewise((x*atanh(tanh(a + b*x))**7/(7*b) - atanh(tanh(a + b*x))**8/(56*b
**2), Ne(b, 0)), (x**2*atanh(tanh(a))**6/2, True))
```

3.84.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(30) = 60.

Time = 0.48 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.24

$$\int x\text{arctanh}(\tanh(a+bx))^6\,dx = -bx^3\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^5 + \frac{1}{2}\,x^2\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^6 + \frac{1}{56}\left(70\,bx^4\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^4 - \left(56\,bx^5\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^3 - \left(28\,bx^6\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2\right.\right.\right.$$

input
```
integrate(x*arctanh(tanh(b*x+a))^6,x, algorithm="maxima")
```

output
```
-b*x^3*arctanh(tanh(b*x + a))^5 + 1/2*x^2*arctanh(tanh(b*x + a))^6 + 1/56*
(70*b*x^4*arctanh(tanh(b*x + a))^4 - (56*b*x^5*arctanh(tanh(b*x + a))^3 -
(28*b*x^6*arctanh(tanh(b*x + a))^2 + (b^2*x^8 - 8*b*x^7*arctanh(tanh(b*x +
 a)))*b)*b)*b)*b
```

3.84.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(30) = 60.

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int x\text{arctanh}(\tanh(a+bx))^6\,dx = \frac{1}{8}\,b^6x^8 + \frac{6}{7}\,ab^5x^7 + \frac{5}{2}\,a^2b^4x^6 + 4\,a^3b^3x^5 + \frac{15}{4}\,a^4b^2x^4 + 2\,a^5bx^3 + \frac{1}{2}\,a^6x^2$$

input
```
integrate(x*arctanh(tanh(b*x+a))^6,x, algorithm="giac")
```

output
```
1/8*b^6*x^8 + 6/7*a*b^5*x^7 + 5/2*a^2*b^4*x^6 + 4*a^3*b^3*x^5 + 15/4*a^4*b
^2*x^4 + 2*a^5*b*x^3 + 1/2*a^6*x^2
```

3.84.9 Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.06

$$\int x\text{arctanh}(\tanh(a+bx))^6\,dx = \frac{b^6\,x^8}{56} - \frac{b^5\,x^7\,\text{atanh}(\tanh(a+b\,x))}{7} + \frac{b^4\,x^6\,\text{atanh}(\tanh(a+b\,x))^2}{2} - b^3\,x^5\,\text{atanh}(\tanh(a+b\,x))^3 + \frac{5\,b^2\,x^4\,\text{atanh}(\tanh(a+b\,x))^4}{4} - b\,x^3\,\text{atanh}(\tanh(a+b\,x))^5 + \frac{x^2\,\text{atanh}(\tanh(a+b\,x))^6}{2}$$

input
```
int(x*atanh(tanh(a + b*x))^6,x)
```

output
```
(x^2*atanh(tanh(a + b*x))^6)/2 + (b^6*x^8)/56 + (5*b^2*x^4*atanh(tanh(a +
b*x))^4)/4 - b^3*x^5*atanh(tanh(a + b*x))^3 + (b^4*x^6*atanh(tanh(a + b*x)
)^2)/2 - b*x^3*atanh(tanh(a + b*x))^5 - (b^5*x^7*atanh(tanh(a + b*x)))/7
```

3.85 $\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))}\,dx$

3.85.1 Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = -\frac{x^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{bx}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)}{(1+m)(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

output
```
-x^(1+m)*hypergeom([1, 1+m],[2+m],b*x/(b*x-arctanh(tanh(b*x+a))))/(1+m)/(b
*x-arctanh(tanh(b*x+a)))
```

3.85.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{bx}{-bx+\operatorname{arctanh}(\tanh(a+bx))}\right)}{(1+m)(-bx+\operatorname{arctanh}(\tanh(a+bx)))}$$

input
```
Integrate[x^m/ArcTanh[Tanh[a + b*x]],x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcTanh[Ta
nh[a + b*x]]))])/((1 + m)*(-(b*x) + ArcTanh[Tanh[a + b*x]]))
```

3.85.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2595}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\text{arctanh}(\tanh(a+bx))} \, dx$$

$$\Big\downarrow \text{2595}$$

$$-\frac{x^{m+1} \text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{bx}{bx-\text{arctanh}(\tanh(a+bx))}\right)}{(m+1)(bx-\text{arctanh}(\tanh(a+bx)))}$$

input
```
Int[x^m/ArcTanh[Tanh[a + b*x]],x]
```

output
```
-((x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (b*x)/(b*x - ArcTanh[Tanh[
a + b*x]])])/((1 + m)*(b*x - ArcTanh[Tanh[a + b*x]])))
```

3.85.3.1 Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

3.85.4 Maple [F]

$$\int \frac{x^m}{\text{arctanh}\left(\tanh\left(bx+a\right)\right)}dx$$

input
```
int(x^m/arctanh(tanh(b*x+a)),x)
```

output
```
int(x^m/arctanh(tanh(b*x+a)),x)
```

3.85.5 Fricas [F]

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int \frac{x^m}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}\,dx$$

input
```
integrate(x^m/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
integral(x^m/arctanh(tanh(b*x + a)), x)
```

3.85.6 Sympy [F]

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int \frac{x^m}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**m/atanh(tanh(b*x+a)),x)
```

output
```
Integral(x**m/atanh(tanh(a + b*x)), x)
```

3.85.7 Maxima [F]

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int \frac{x^m}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}\,dx$$

input
```
integrate(x^m/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(x^m/arctanh(tanh(b*x + a)), x)
```

3.85.8 Giac [F]

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int \frac{x^m}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}\,dx$$

input
```
integrate(x^m/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^m/arctanh(tanh(b*x + a)), x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int \frac{x^m}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}\,dx$$

input
```
int(x^m/atanh(tanh(a + b*x)),x)
```

output
```
int(x^m/atanh(tanh(a + b*x)), x)
```

3.86 $\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))}\,dx$

3.86.1 Optimal result

Integrand size = 13, antiderivative size = 81

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{x^3}{3b} + \frac{x^2(bx-\operatorname{arctanh}(\tanh(a+bx)))}{2b^2} + \frac{x(bx-\operatorname{arctanh}(\tanh(a+bx)))^2}{b^3} + \frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))^3\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^4}$$

output
```
1/3*x^3/b+1/2*x^2*(b*x-arctanh(tanh(b*x+a)))/b^2+x*(b*x-arctanh(tanh(b*x+a
)))^2/b^3+(b*x-arctanh(tanh(b*x+a)))^3*ln(arctanh(tanh(b*x+a)))/b^4
```

3.86.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{x^3}{3b} - \frac{x^2(-bx+\operatorname{arctanh}(\tanh(a+bx)))}{2b^2} + \frac{x(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2}{b^3} - \frac{(-bx+\operatorname{arctanh}(\tanh(a+bx)))^3\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^4}$$

input
```
Integrate[x^3/ArcTanh[Tanh[a + b*x]],x]
```

output
```
x^3/(3*b) - (x^2*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/(2*b^2) + (x*(-(b*x) +
 ArcTanh[Tanh[a + b*x]])^2)/b^3 - ((-(b*x) + ArcTanh[Tanh[a + b*x]])^3*Log
[ArcTanh[Tanh[a + b*x]]])/b^4
```

3.86.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2590, 2590, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\text{arctanh}(\tanh(a+bx))}dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{x^2}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{x^3}{3b}$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{x}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{x^2}{2b}\right)}{b}+\frac{x^3}{3b}$$

$$\Big\downarrow \text{2589}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}+\frac{x^3}{3b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int\frac{1}{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+\right.}{b^2}\right.}{b}\right.}{b}$$

$$\frac{x^3}{3b}$$

$$\Big\downarrow \; 14$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\log(\text{arctanh}(\tanh(a+bx)))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}+$$

$$\frac{x^3}{3b}$$

input
```
Int[x^3/ArcTanh[Tanh[a + b*x]],x]
```

output
```
x^3/(3*b) + ((b*x - ArcTanh[Tanh[a + b*x]])*(x^2/(2*b) + ((b*x - ArcTanh[T
anh[a + b*x]])*(x/b + ((b*x - ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a +
 b*x]]])/b^2))/b))/b
```

3.86.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

3.86.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(77) = 154.

Time = 0.95 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.01

method	result
default	$\frac{\frac{b^2x^3}{3}-\frac{ab\,x^2}{2}-\frac{x^2b(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{2}+a^2x+2xa(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+x(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2}{b^3}+\frac{\left(-a^3\right.}{}$
risch	Expression too large to display

input
```
int(x^3/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*b^2*x^3-1/2*a*b*x^2-1/2*x^2*b*(arctanh(tanh(b*x+a))-b*x-a)+a^2*
x+2*x*a*(arctanh(tanh(b*x+a))-b*x-a)+x*(arctanh(tanh(b*x+a))-b*x-a)^2)+(-a
^3-3*a^2*(arctanh(tanh(b*x+a))-b*x-a)-3*a*(arctanh(tanh(b*x+a))-b*x-a)^2-(
arctanh(tanh(b*x+a))-b*x-a)^3)/b^4*ln(arctanh(tanh(b*x+a)))
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.51

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log{(bx+a)}}{6\,b^4}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

3.86.6 Sympy [F]

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int \frac{x^3}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**3/atanh(tanh(b*x+a)),x)
```

output
```
Integral(x**3/atanh(tanh(a + b*x)), x)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = -\frac{a^3\log\left(bx+a\right)}{b^4} + \frac{2\,b^2x^3 - 3\,abx^2 + 6\,a^2x}{6\,b^3}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-a^3*log(b*x + a)/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.53

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = -\frac{a^3\log\left(|bx+a|\right)}{b^4} + \frac{2\,b^2x^3 - 3\,abx^2 + 6\,a^2x}{6\,b^3}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
-a^3*log(abs(b*x + a))/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

3.86.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 354, normalized size of antiderivative = 4.37

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{x^3}{3\,b} + \frac{x^2\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)}{4\,b^2}$$
$$+ \frac{x\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2}{4\,b^3}$$
$$+ \frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\left(\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^3 - 8\,a^3 - 6\,a\,\left(\right.}{8}$$

input
```
int(x^3/atanh(tanh(a + b*x)),x)
```

output
```
x^3/(3*b) + (x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(4*b^2) + (x*(log(2/(exp(2*a)*
exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x)^2)/(4*b^3) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*((2*a - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)))/(8*b^4)
```

3.87 $\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))} \, dx$

3.87.1 Optimal result

Integrand size = 13, antiderivative size = 56

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))} \, dx = \frac{x^2}{2b} + \frac{x(bx-\operatorname{arctanh}(\tanh(a+bx)))}{b^2} + \frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))^2 \log(\operatorname{arctanh}(\tanh(a+bx)))}{b^3}$$

output
```
1/2*x^2/b+x*(b*x-arctanh(tanh(b*x+a)))/b^2+(b*x-arctanh(tanh(b*x+a)))^2*ln
(arctanh(tanh(b*x+a)))/b^3
```

3.87.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))} \, dx = \frac{x^2}{2b} - \frac{x(-bx+\operatorname{arctanh}(\tanh(a+bx)))}{b^2} + \frac{(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2 \log(\operatorname{arctanh}(\tanh(a+bx)))}{b^3}$$

input
```
Integrate[x^2/ArcTanh[Tanh[a + b*x]],x]
```

output
```
x^2/(2*b) - (x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^2 + ((-(b*x) + ArcTanh
[Tanh[a + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^3
```

3.87.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2590, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\text{arctanh}(\tanh(a+bx))}dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{x}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{x^2}{2b}$$

$$\Big\downarrow \text{2589}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}$$

$$\Big\downarrow \text{14}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\log(\text{arctanh}(\tanh(a+bx)))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}$$

input
```
Int[x^2/ArcTanh[Tanh[a + b*x]],x]
```

output
```
x^2/(2*b) + ((b*x - ArcTanh[Tanh[a + b*x]])*(x/b + ((b*x - ArcTanh[Tanh[a
+ b*x]])*Log[ArcTanh[Tanh[a + b*x]]])/b^2))/b
```

3.87.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

3.87.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.54

method	result
default	$\frac{\frac{b x^2}{2}-ax-x(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{b^2}+\frac{\left(a^2+2a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2\right)\ln(\operatorname{arctanh}}{b^3}$
risch	Expression too large to display

input
```
int(x^2/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/2*b*x^2-a*x-x*(arctanh(tanh(b*x+a))-b*x-a))+(a^2+2*a*(arctanh(tan
h(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/b^3*ln(arctanh(tanh(b*x+a
)))
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.52

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{b^2x^2 - 2\,abx + 2\,a^2\log\left(bx+a\right)}{2\,b^3}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

3.87.6 Sympy [F]

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int \frac{x^2}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**2/atanh(tanh(b*x+a)),x)
```

output
```
Integral(x**2/atanh(tanh(a + b*x)), x)
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.52

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{a^2\log\left(bx+a\right)}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.54

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{a^2\log(|bx+a|)}{b^3} + \frac{bx^2-2\,ax}{2\,b^2}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

3.87.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 234, normalized size of antiderivative = 4.18

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{x^2}{2\,b} + \frac{x\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b^2} + \frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\left(\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2-4\,a\left(2\,a-\mathrm{l}\right.\right.}{4\,b^3}$$

input
```
int(x^2/atanh(tanh(a + b*x)),x)
```

output
```
x^2/(2*b) + (x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b^2) + (log(log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*
((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) +
 4*a^2))/(4*b^3)
```

3.88 $\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))}\,dx$

3.88.1 Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{x}{b}+\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^2}$$

output
```
x/b+(b*x-arctanh(tanh(b*x+a)))*ln(arctanh(tanh(b*x+a)))/b^2
```

3.88.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{x}{b}-\frac{(-bx+\operatorname{arctanh}(\tanh(a+bx)))\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^2}$$

input
```
Integrate[x/ArcTanh[Tanh[a + b*x]],x]
```

output
```
x/b - ((-(b*x) + ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]])/b^2
```

3.88.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\text{arctanh}(\tanh(a+bx))} \, dx$$

$$\Big\downarrow \text{ 2589}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{x}{b}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+bx))}{b^2}+\frac{x}{b}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\log(\text{arctanh}(\tanh(a+bx)))}{b^2}+\frac{x}{b}$$

input
```
Int[x/ArcTanh[Tanh[a + b*x]],x]
```

output
```
x/b + ((b*x - ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]])/b^2
```

3.88.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

3.88.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{x}{b}+\frac{(bx-\operatorname{arctanh}(\tanh(bx+a)))\ln(\operatorname{arctanh}(\tanh(bx+a)))}{b^2}$	32
risch	Expression too large to display	2837

input
```
int(x/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
x/b+(b*x-arctanh(tanh(b*x+a)))*ln(arctanh(tanh(b*x+a)))/b^2
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{bx-a\log(bx+a)}{b^2}$$

input
```
integrate(x/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
(b*x - a*log(b*x + a))/b^2
```

3.88.6 Sympy [F]

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int \frac{x}{\operatorname{atanh}(\tanh(a+bx))}\,dx$$

input
```
integrate(x/atanh(tanh(b*x+a)),x)
```

output
```
Integral(x/atanh(tanh(a + b*x)), x)
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{x}{b} - \frac{a\log(bx+a)}{b^2}$$

input
```
integrate(x/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
x/b - a*log(b*x + a)/b^2
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{x}{b} - \frac{a\log(|bx+a|)}{b^2}$$

input
```
integrate(x/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
x/b - a*log(abs(b*x + a))/b^2
```

3.88.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.48

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))}\,dx$$
$$= \frac{x}{b} + \frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)}{2\,b^2}$$

input
```
int(x/atanh(tanh(a + b*x)),x)
```

output
```
x/b + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/
(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b^2)
```

3.89 $\int \frac{1}{\textbf{arctanh}(\tanh(a+bx))}\,dx$

3.89.1 Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{\log(\operatorname{arctanh}(\tanh(a+bx)))}{b}$$

output
```
ln(arctanh(tanh(b*x+a)))/b
```

3.89.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{\log(\operatorname{arctanh}(\tanh(a+bx)))}{b}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(-1),x]
```

output
```
Log[ArcTanh[Tanh[a + b*x]]]/b
```

3.89.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))}\,dx$$

$$\Big\downarrow \; 2588$$

$$\frac{\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))}d\operatorname{arctanh}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \; 14$$

$$\frac{\log(\operatorname{arctanh}(\tanh(a+bx)))}{b}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(-1),x]
```

output
```
Log[ArcTanh[Tanh[a + b*x]]]/b
```

3.89.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.89.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))}{b}$
default	$\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))}{b}$
parallelrisch	$\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))}{b}$
risch	$\ln\left(\ln(\mathrm{e}^{bx+a})-\frac{i\pi\left(\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)-\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}+\operatorname{csgn}\left(i\mathrm{e}^{bx}\right.\right.}{}\right.$

input
```
int(1/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
ln(arctanh(tanh(b*x+a)))/b
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{\log{(bx+a)}}{b}$$

input
```
integrate(1/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
log(b*x + a)/b
```

3.89.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \begin{cases} \frac{\log{(\operatorname{atanh}{(\tanh{(a+bx)})})}}{b} & \text{for } b \neq 0 \\ \frac{x}{\operatorname{atanh}{(\tanh{(a)})}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/atanh(tanh(b*x+a)),x)
```

output
```
Piecewise((log(atanh(tanh(a + b*x)))/b, Ne(b, 0)), (x/atanh(tanh(a)), True
))
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{\text{arctanh}(\tanh(a+bx))}\,dx = \frac{\log(-bx-a)}{b}$$

input
```
integrate(1/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
log(-b*x - a)/b
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{\text{arctanh}(\tanh(a+bx))}\,dx = \frac{\log(|bx+a|)}{b}$$

input
```
integrate(1/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
log(abs(b*x + a))/b
```

3.89.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\text{arctanh}(\tanh(a+bx))}\,dx = \frac{\ln(\text{atanh}(\tanh(a+b\,x)))}{b}$$

input
```
int(1/atanh(tanh(a + b*x)),x)
```

output
```
log(atanh(tanh(a + b*x)))/b
```

3.90 $\int \frac{1}{x\textbf{arctanh}(\tanh(a+bx))}\,dx$

3.90.1 Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))}\,dx = -\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{\log(\text{arctanh}(\tanh(a+bx)))}{bx-\text{arctanh}(\tanh(a+bx))}$$

output
```
-ln(x)/(b*x-arctanh(tanh(b*x+a)))+ln(arctanh(tanh(b*x+a)))/(b*x-arctanh(ta
nh(b*x+a)))
```

3.90.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))}\,dx = \frac{-\log(x)+\log(\text{arctanh}(\tanh(a+bx)))}{bx-\text{arctanh}(\tanh(a+bx))}$$

input
```
Integrate[1/(x*ArcTanh[Tanh[a + b*x]]),x]
```

output
```
(-Log[x] + Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTanh[Tanh[a + b*x]])
```

3.90.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))}dx$$

$$\Big\downarrow \text{2591}$$

$$\frac{b\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

$$\frac{b\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \frac{1}{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+bx))}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

$$\frac{\log(\text{arctanh}(\tanh(a+bx)))}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}$$

input
```
Int[1/(x*ArcTanh[Tanh[a + b*x]]),x]
```

output
```
-(Log[x]/(b*x - ArcTanh[Tanh[a + b*x]])) + Log[ArcTanh[Tanh[a + b*x]]]/(b*
x - ArcTanh[Tanh[a + b*x]])
```

3.90.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

3.90.4 Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

method	result
default	$-\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))}{\operatorname{arctanh}(\tanh(bx+a))-bx}+\frac{\ln(x)}{\operatorname{arctanh}(\tanh(bx+a))-bx}$
risch	$\frac{4}{\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right) \operatorname{csgn}\left(i e^{2bx+2a}\right) \operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)-\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right) \operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^{2}+\pi \operatorname{csgn}\left(i e^{2bx+2a}\right)^{3}-2\pi \operatorname{csgn}(i e^{bx}}$

input
```
int(1/x/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/(arctanh(tanh(b*x+a))-b*x)*ln(arctanh(tanh(b*x+a)))+1/(arctanh(tanh(b*x
+a))-b*x)*ln(x)
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.36

$$\int \frac{1}{x\operatorname{arctanh}(\tanh(a+bx))}\,dx=-\frac{\log(bx+a)-\log(x)}{a}$$

input
```
integrate(1/x/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
-(log(b*x + a) - log(x))/a
```

3.90.6 Sympy [F]

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))}\,dx = \int \frac{1}{x\,\text{atanh}\,(\tanh\,(a+bx))}\,dx$$

input
```
integrate(1/x/atanh(tanh(b*x+a)),x)
```

output
```
Integral(1/(x*atanh(tanh(a + b*x))), x)
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.41

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))}\,dx = -\frac{\log\,(bx+a)}{a} + \frac{\log\,(x)}{a}$$

input
```
integrate(1/x/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-log(b*x + a)/a + log(x)/a
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.45

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))}\,dx = -\frac{\log\,(|bx+a|)}{a} + \frac{\log\,(|x|)}{a}$$

input
```
integrate(1/x/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/a + log(abs(x))/a
```

3.90.9 Mupad [B] (verification not implemented)

Time = 6.01 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.43

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))}\,dx = -\frac{4\,\text{atanh}\left(\frac{4\,b\,x}{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}-1\right)}{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}$$

input
```
int(1/(x*atanh(tanh(a + b*x))),x)
```

output
```
-(4*atanh((4*b*x)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 1))/(log(1/(exp(2*a)*exp(2*b*x
) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
```

3.91 $\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))}\,dx$

3.91.1 Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))}\,dx = \frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))} - \frac{b\log(x)}{(bx-\text{arctanh}(\tanh(a+bx)))^2} + \frac{b\log(\text{arctanh}(\tanh(a+bx)))}{(bx-\text{arctanh}(\tanh(a+bx)))^2}$$

output
```
1/x/(b*x-arctanh(tanh(b*x+a)))-b*ln(x)/(b*x-arctanh(tanh(b*x+a)))^2+b*ln(a
rctanh(tanh(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^2
```

3.91.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))}\,dx = \frac{-\text{arctanh}(\tanh(a+bx))+bx(1-\log(x)+\log(\text{arctanh}(\tanh(a+bx))))}{x(-bx+\text{arctanh}(\tanh(a+bx)))^2}$$

input
```
Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]),x]
```

output
```
(-ArcTanh[Tanh[a + b*x]] + b*x*(1 - Log[x] + Log[ArcTanh[Tanh[a + b*x]]]))
/(x*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

3.91.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))}dx$$

$$\Big\downarrow \text{2594}$$

$$\frac{b\int \frac{1}{x\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2591}$$

$$\frac{b\left(\frac{b\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\text{arctanh}(\tanh(a+bx))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{14}$$

$$\frac{b\left(\frac{b\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2588}$$

$$\frac{b\left(\frac{\int \frac{1}{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+bx))}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{b\left(\frac{\log(\text{arctanh}(\tanh(a+bx)))}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}$$

input
```
Int[1/(x^2*ArcTanh[Tanh[a + b*x]]),x]
```

output
```
1/(x*(b*x - ArcTanh[Tanh[a + b*x]])) + (b*(-(Log[x]/(b*x - ArcTanh[Tanh[a
+ b*x]])) + Log[ArcTanh[Tanh[a + b*x]]]/(b*x - ArcTanh[Tanh[a + b*x]])))/(
b*x - ArcTanh[Tanh[a + b*x]])
```

3.91.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

3.91.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$-\frac{1}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x}-\frac{b\ln\left(x\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}}+\frac{b\ln\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}}$$

input
```
int(1/x^2/arctanh(tanh(b*x+a)),x)
```

output
```
-1/(arctanh(tanh(b*x+a))-b*x)/x-1/(arctanh(tanh(b*x+a))-b*x)^2*b*ln(x)+1/(
arctanh(tanh(b*x+a))-b*x)^2*b*ln(arctanh(tanh(b*x+a)))
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))}\,dx = \frac{bx\log(bx+a) - bx\log(x) - a}{a^2x}$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
(b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

3.91.6 Sympy [F]

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))}\,dx = \int \frac{1}{x^2\,\text{atanh}\,(\tanh\,(a+bx))}\,dx$$

input
```
integrate(1/x**2/atanh(tanh(b*x+a)),x)
```

output
```
Integral(1/(x**2*atanh(tanh(a + b*x))), x)
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))}\,dx = \frac{b\log(bx+a)}{a^2} - \frac{b\log(x)}{a^2} - \frac{1}{ax}$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.46

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))}\,dx = \frac{b\log(|bx+a|)}{a^2} - \frac{b\log(|x|)}{a^2} - \frac{1}{ax}$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

3.91.9 Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.23

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-2\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+4\,b\,x+b\,x\,\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,1\mathrm{i}+\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,1\mathrm{i}+b\,x\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}\right)\,8\mathrm{i}}{x\,{\left(\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}$$

input
```
int(1/(x^2*atanh(tanh(a + b*x))),x)
```

output
```
(2*log(1/(exp(2*a)*exp(2*b*x) + 1)) - 2*log((exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 4*b*x + b*x*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(
1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x))*8i)/(x*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
```

3.92 $\int \frac{1}{x^3\textbf{arctanh}(\tanh(a+bx))}\,dx$

3.92.1 Optimal result

Integrand size = 13, antiderivative size = 92

$$\begin{aligned}\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))}\,dx &= \frac{b}{x(bx-\text{arctanh}(\tanh(a+bx)))^2} \\ &\quad + \frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))} \\ &\quad - \frac{b^2\log(x)}{(bx-\text{arctanh}(\tanh(a+bx)))^3} \\ &\quad + \frac{b^2\log(\text{arctanh}(\tanh(a+bx)))}{(bx-\text{arctanh}(\tanh(a+bx)))^3}\end{aligned}$$

output
```
b/x/(b*x-arctanh(tanh(b*x+a)))^2+1/2/x^2/(b*x-arctanh(tanh(b*x+a)))-b^2*ln
(x)/(b*x-arctanh(tanh(b*x+a)))^3+b^2*ln(arctanh(tanh(b*x+a)))/(b*x-arctanh
(tanh(b*x+a)))^3
```

3.92.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))}\,dx = \frac{-4bx\text{arctanh}(\tanh(a+bx)) + \text{arctanh}(\tanh(a+bx))^2 + b^2x^2(3-2\log(x)+2\log(\text{arctanh}(\tanh(a+bx}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))^3}$$

input
```
Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]),x]
```

output
```
(-4*b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2 + b^2*x^2*(3 - 2
*Log[x] + 2*Log[ArcTanh[Tanh[a + b*x]]]))/(2*x^2*(b*x - ArcTanh[Tanh[a + b
*x]])^3)
```

3.92.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2594, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))}dx$$

$$\Big\downarrow \text{2594}$$

$$\frac{b\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2594}$$

$$\frac{b\left(\frac{b\int \frac{1}{x\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2591}$$

$$\frac{b\left(\frac{b\left(\frac{b\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\text{arctanh}(\tanh(a+bx))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{14}$$

$$\frac{b\left(\frac{b\left(\frac{b\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))}dx}{bx-\operatorname{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\operatorname{arctanh}(\tanh(a+bx)))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{1}{2x^2(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2588}$$

$$\frac{b\left(\frac{b\left(\frac{\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))}d\operatorname{arctanh}(\tanh(a+bx))}{bx-\operatorname{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\operatorname{arctanh}(\tanh(a+bx)))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{1}{2x^2(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2x^2(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{b\left(\frac{1}{x(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{b\left(\frac{\log(\operatorname{arctanh}(\tanh(a+bx)))}{bx-\operatorname{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}$$

input
```
Int[1/(x^3*ArcTanh[Tanh[a + b*x]]),x]
```

output
```
1/(2*x^2*(b*x - ArcTanh[Tanh[a + b*x]])) + (b*(1/(x*(b*x - ArcTanh[Tanh[a
+ b*x]])) + (b*(-(Log[x]/(b*x - ArcTanh[Tanh[a + b*x]])) + Log[ArcTanh[Tan
h[a + b*x]]]/(b*x - ArcTanh[Tanh[a + b*x]])))/(b*x - ArcTanh[Tanh[a + b*x]
])))/(b*x - ArcTanh[Tanh[a + b*x]])
```

3.92.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

3.92.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

$$-\frac{1}{2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{2}}+\frac{b^{2}\ln\left(x\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}}+\frac{b}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}x}$$

input
```
int(1/x^3/arctanh(tanh(b*x+a)),x)
```

output
```
-1/2/(arctanh(tanh(b*x+a))-b*x)/x^2+1/(arctanh(tanh(b*x+a))-b*x)^3*b^2*ln(
x)+1/(arctanh(tanh(b*x+a))-b*x)^2*b/x-1/(arctanh(tanh(b*x+a))-b*x)^3*b^2*l
n(arctanh(tanh(b*x+a)))
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.45

$$\int\frac{1}{x^{3}\operatorname{arctanh}(\tanh(a+bx))}\,dx=-\frac{2\,b^{2}x^{2}\log\left(bx+a\right)-2\,b^{2}x^{2}\log\left(x\right)-2\,abx+a^{2}}{2\,a^{3}x^{2}}$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

3.92.6 Sympy [F]

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))}\,dx = \int \frac{1}{x^3\,\text{atanh}\,(\tanh\,(a+bx))}\,dx$$

input
```
integrate(1/x**3/atanh(tanh(b*x+a)),x)
```

output
```
Integral(1/(x**3*atanh(tanh(a + b*x))), x)
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))}\,dx = -\frac{b^2\log\,(bx+a)}{a^3} + \frac{b^2\log\,(x)}{a^3} + \frac{2\,bx-a}{2\,a^2x^2}$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))}\,dx = -\frac{b^2\log\,(|bx+a|)}{a^3} + \frac{b^2\log\,(|x|)}{a^3} + \frac{2\,abx-a^2}{2\,a^3x^2}$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
-b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^
3*x^2)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.11

$$\int \frac{1}{x^3 \text{arctanh}(\tanh(a+bx))}\,dx$$

$$= \frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)^2 - \ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\left(2\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)+8\,b\,x\right) + \ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)^2 + 12\,b^2\,x^2 + 8\,b\,x\,\ln\left(\frac{}{e^{2a}\,e}\right.}{x^2\left(\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right) - \ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}$$

input
```
int(1/(x^3*atanh(tanh(a + b*x))),x)
```

output
```
(log(1/(exp(2*a)*exp(2*b*x) + 1))^2 - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1))*(2*log(1/(exp(2*a)*exp(2*b*x) + 1)) + 8*b*x) + log((exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 + 12*b^2*x^2 + 8*b*x*log(1/(e
xp(2*a)*exp(2*b*x) + 1)) + b^2*x^2*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(lo
g(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x))*16i)/(x^2*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)
```

3.93 $\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx$

3.93.1 Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{x^m}{b\operatorname{arctanh}(\tanh(a+bx))} - \frac{x^m \text{Hypergeometric2F1}\left(1,m,1+m,\frac{bx}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)}{b(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

output
```
-x^m/b/arctanh(tanh(b*x+a))-x^m*hypergeom([1, m],[1+m],b*x/(b*x-arctanh(ta
nh(b*x+a))))/b/(b*x-arctanh(tanh(b*x+a)))
```

3.93.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(2,1+m,2+m,-\frac{bx}{-bx+\operatorname{arctanh}(\tanh(a+bx))}\right)}{(1+m)(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2}$$

input
```
Integrate[x^m/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcTanh[Ta
nh[a + b*x]]))])/((1 + m)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

3.93.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2599, 2595}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))^2} \, dx$$
$$\Big\downarrow \text{2599}$$
$$\frac{m\int \frac{x^{m-1}}{\operatorname{arctanh}(\tanh(a+bx))}dx}{b} - \frac{x^m}{b\operatorname{arctanh}(\tanh(a+bx))}$$
$$\Big\downarrow \text{2595}$$
$$-\frac{x^m \text{Hypergeometric2F1}\left(1,m,m+1,\frac{bx}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)}{b(bx-\operatorname{arctanh}(\tanh(a+bx)))} - \frac{x^m}{b\operatorname{arctanh}(\tanh(a+bx))}$$

input
```
Int[x^m/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
-(x^m/(b*ArcTanh[Tanh[a + b*x]])) - (x^m*Hypergeometric2F1[1, m, 1 + m, (b
*x)/(b*x - ArcTanh[Tanh[a + b*x]])])/(b*(b*x - ArcTanh[Tanh[a + b*x]]))
```

3.93.3.1 Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] && !IntegerQ[n]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.93.4 Maple [F]

$$\int \frac{x^m}{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}dx$$

input
```
int(x^m/arctanh(tanh(b*x+a))^2,x)
```

output
```
int(x^m/arctanh(tanh(b*x+a))^2,x)
```

3.93.5 Fricas [F]

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \int \frac{x^m}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}\,dx$$

input
```
integrate(x^m/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
integral(x^m/arctanh(tanh(b*x + a))^2, x)
```

3.93.6 Sympy [F]

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \int \frac{x^m}{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**m/atanh(tanh(b*x+a))**2,x)
```

output
```
Integral(x**m/atanh(tanh(a + b*x))**2, x)
```

3.93.7 Maxima [F]

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \int \frac{x^m}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}\,dx$$

input
```
integrate(x^m/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
integrate(x^m/arctanh(tanh(b*x + a))^2, x)
```

3.93.8 Giac [F]

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \int \frac{x^m}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}\,dx$$

input
```
integrate(x^m/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
integrate(x^m/arctanh(tanh(b*x + a))^2, x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \int \frac{x^m}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}\,dx$$

input
```
int(x^m/atanh(tanh(a + b*x))^2,x)
```

output
```
int(x^m/atanh(tanh(a + b*x))^2, x)
```

3.94 $\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx$

3.94.1 Optimal result

Integrand size = 13, antiderivative size = 98

$$\begin{aligned}
&\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx \\
&= \frac{4x^3}{3b^2} + \frac{2x^2(bx-\operatorname{arctanh}(\tanh(a+bx)))}{b^3} \\
&\quad + \frac{4x(bx-\operatorname{arctanh}(\tanh(a+bx)))^2}{b^4} - \frac{x^4}{b\operatorname{arctanh}(\tanh(a+bx))} \\
&\quad + \frac{4(bx-\operatorname{arctanh}(\tanh(a+bx)))^3\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^5}
\end{aligned}$$

output
```
4/3*x^3/b^2+2*x^2*(b*x-arctanh(tanh(b*x+a)))/b^3+4*x*(b*x-arctanh(tanh(b*x
+a)))^2/b^4-x^4/b/arctanh(tanh(b*x+a))+4*(b*x-arctanh(tanh(b*x+a)))^3*ln(a
rctanh(tanh(b*x+a)))/b^5
```

3.94.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{x^3}{3b^2} - \frac{x^2(-bx+\operatorname{arctanh}(\tanh(a+bx)))}{b^3} + \frac{3x(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2}{b^4} - \frac{(-bx+\operatorname{arctanh}(\tanh(a+bx)))^4}{b^5\operatorname{arctanh}(\tanh(a+bx))} - \frac{4(-bx+\operatorname{arctanh}(\tanh(a+bx)))^3\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^5}$$

input
```
Integrate[x^4/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
x^3/(3*b^2) - (x^2*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^3 + (3*x*(-(b*x) +
 ArcTanh[Tanh[a + b*x]])^2)/b^4 - (-(b*x) + ArcTanh[Tanh[a + b*x]])^4/(b^5
*ArcTanh[Tanh[a + b*x]]) - (4*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3*Log[ArcT
anh[Tanh[a + b*x]]])/b^5
```

3.94.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2599, 2590, 2590, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{4\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))}dx}{b} - \frac{x^4}{b\operatorname{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2590}$$

$$\frac{4\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))}dx}{b} + \frac{x^3}{3b}\right)}{b} - \frac{x^4}{b\operatorname{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2590}$$

$$\frac{4\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{x}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{x^2}{2b}\right)}{b}+\frac{x^3}{3b}\right)}{b}-\frac{x^4}{b\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2589}$$

$$\frac{4\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}+\frac{x^3}{3b}\right)}{b}-\frac{x^4}{b\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2588}$$

$$\frac{4\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}\right.}{b}\right.}{b}\qquad \frac{x^4}{b\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

3.94. $\int \frac{x^4}{\text{arctanh}(\tanh(a+bx))^2}\,dx$

$$\frac{4\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}+\frac{x^3}{3b}\right)}{b}-\frac{x^4}{b\operatorname{arctanh}(\tanh(a+bx))}$$

input
```
Int[x^4/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
-(x^4/(b*ArcTanh[Tanh[a + b*x]])) + (4*(x^3/(3*b) + ((b*x - ArcTanh[Tanh[a
 + b*x]])*(x^2/(2*b) + ((b*x - ArcTanh[Tanh[a + b*x]])*(x/b + ((b*x - ArcT
anh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]])/b^2))/b))/b))/b
```

3.94.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.94.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(96) = 192$.

Time = 0.98 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.67

method	result
default	$\frac{\frac{b^2x^3}{3}-ab\,x^2-x^2b(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+3a^2x+6xa(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+3x(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2}{b^4}+$
risch	Expression too large to display

input
```
int(x^4/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/3*b^2*x^3-a*b*x^2-x^2*b*(arctanh(tanh(b*x+a))-b*x-a)+3*a^2*x+6*x*
a*(arctanh(tanh(b*x+a))-b*x-a)+3*x*(arctanh(tanh(b*x+a))-b*x-a)^2)+(-4*a^3
-12*a^2*(arctanh(tanh(b*x+a))-b*x-a)-12*a*(arctanh(tanh(b*x+a))-b*x-a)^2-4
*(arctanh(tanh(b*x+a))-b*x-a)^3)/b^5*ln(arctanh(tanh(b*x+a)))-(a^4+4*a^3*(
arctanh(tanh(b*x+a))-b*x-a)+6*a^2*(arctanh(tanh(b*x+a))-b*x-a)^2+4*a*(arct
anh(tanh(b*x+a))-b*x-a)^3+(arctanh(tanh(b*x+a))-b*x-a)^4)/b^5/arctanh(tanh
(b*x+a))
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{b^4x^4-2\,ab^3x^3+6\,a^2b^2x^2+9\,a^3bx-3\,a^4-12\,(a^3bx+a^4)\log{(bx+a)}}{3\,(b^6x+ab^5)}$$

input
```
integrate(x^4/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b
*x + a^4)*log(b*x + a))/(b^6*x + a*b^5)
```

3.94.6 Sympy [F]

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \int \frac{x^4}{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**4/atanh(tanh(b*x+a))**2,x)
```

output
```
Integral(x**4/atanh(tanh(a + b*x))**2, x)
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{b^4x^4 - 2\,ab^3x^3 + 6\,a^2b^2x^2 + 9\,a^3bx - 3\,a^4}{3\,(b^6x + ab^5)} - \frac{4\,a^3\log\left(bx+a\right)}{b^5}$$

input
```
integrate(x^4/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4)/(b^6*x + a
*b^5) - 4*a^3*log(b*x + a)/b^5
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.63

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{4\,a^3\log\left(|bx+a|\right)}{b^5} - \frac{a^4}{(bx+a)b^5} + \frac{b^4x^3 - 3\,ab^3x^2 + 9\,a^2b^2x}{3\,b^6}$$

input
```
integrate(x^4/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
-4*a^3*log(abs(b*x + a))/b^5 - a^4/((b*x + a)*b^5) + 1/3*(b^4*x^3 - 3*a*b^
3*x^2 + 9*a^2*b^2*x)/b^6
```

3.94.9 Mupad [B] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 669, normalized size of antiderivative = 6.83

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{x^3}{3\,b^2}$$
$$-\frac{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^4+24\,a^2\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right.}{2\,b\left(8\,a\,b^4+8\,b^5\,x-4\,b^4\left(2\right.\right.}$$
$$+\frac{x^2\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b^3}$$
$$+\frac{3\,x\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{4\,b^4}$$
$$+\frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\left(\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3-8\,a^3-6\,a\left(\right.\right.}{2}$$

input
```
int(x^4/atanh(tanh(a + b*x))^2,x)
```

output
```
x^3/(3*b^2) - ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^2 + 16*a^4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 32*a^
3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b*(8*a*b^4 + 8*b^5*x - 4*b^4*(2*a -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x))) + (x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b^3) + (3*
x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)^2)/(4*b^4) + (log(log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*((2*a - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 +
12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log
(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(2*b^5)
```

3.95 $\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^2} \, dx$

3.95.1 Optimal result

Integrand size = 13, antiderivative size = 75

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^2} \, dx = \frac{3x^2}{2b^2} + \frac{3x(bx-\operatorname{arctanh}(\tanh(a+bx)))}{b^3} - \frac{x^3}{b\operatorname{arctanh}(\tanh(a+bx))} + \frac{3(bx-\operatorname{arctanh}(\tanh(a+bx)))^2\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^4}$$

output
```
3/2*x^2/b^2+3*x*(b*x-arctanh(tanh(b*x+a)))/b^3-x^3/b/arctanh(tanh(b*x+a))+
3*(b*x-arctanh(tanh(b*x+a)))^2*ln(arctanh(tanh(b*x+a)))/b^4
```

3.95.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^2} \, dx = \frac{x^2}{2b^2} - \frac{2x(-bx+\operatorname{arctanh}(\tanh(a+bx)))}{b^3} + \frac{(-bx+\operatorname{arctanh}(\tanh(a+bx)))^3}{b^4\operatorname{arctanh}(\tanh(a+bx))} + \frac{3(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^4}$$

input
```
Integrate[x^3/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
x^2/(2*b^2) - (2*x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^3 + (-(b*x) + ArcT
anh[Tanh[a + b*x]])^3/(b^4*ArcTanh[Tanh[a + b*x]]) + (3*(-(b*x) + ArcTanh[
Tanh[a + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^4
```

3.95.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2599, 2590, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\text{arctanh}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{3\int \frac{x^2}{\text{arctanh}(\tanh(a+bx))}dx}{b}-\frac{x^3}{b\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2590}$$

$$\frac{3\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{x}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2589}$$

$$\frac{3\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2588}$$

$$\frac{3\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int\frac{1}{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \; 14$$

$$\frac{3\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\log(\text{arctanh}(\tanh(a+bx)))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\text{arctanh}(\tanh(a+bx))}$$

input
```
Int[x^3/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
-(x^3/(b*ArcTanh[Tanh[a + b*x]])) + (3*(x^2/(2*b) + ((b*x - ArcTanh[Tanh[a
 + b*x]])*(x/b + ((b*x - ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]
]])/b^2))/b))/b
```

3.95.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.95.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(73) = 146$.

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.25

method	result
default	$\frac{\frac{b\,x^2}{2}-2ax-2x(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{b^3}+\frac{\left(3a^2+6a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+3(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2\right)\ln(\mathrm{arc}}{b^4}$
risch	Expression too large to display

input
```
int(x^3/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/2*b*x^2-2*a*x-2*x*(arctanh(tanh(b*x+a))-b*x-a))+(3*a^2+6*a*(arcta
nh(tanh(b*x+a))-b*x-a)+3*(arctanh(tanh(b*x+a))-b*x-a)^2)/b^4*ln(arctanh(ta
nh(b*x+a)))-1/b^4*(-a^3-3*a^2*(arctanh(tanh(b*x+a))-b*x-a)-3*a*(arctanh(ta
nh(b*x+a))-b*x-a)^2-(arctanh(tanh(b*x+a))-b*x-a)^3)/arctanh(tanh(b*x+a))
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{b^3x^3 - 3\,ab^2x^2 - 4\,a^2bx + 2\,a^3 + 6\,(a^2bx + a^3)\log(bx+a)}{2\,(b^5x + ab^4)}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
 + a))/(b^5*x + a*b^4)
```

3.95.6 Sympy [F]

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \int \frac{x^3}{\operatorname{atanh}^2(\tanh(a+bx))}\,dx$$

input
```
integrate(x**3/atanh(tanh(b*x+a))**2,x)
```

output
```
Integral(x**3/atanh(tanh(a + b*x))**2, x)
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{b^3x^3 - 3\,ab^2x^2 - 4\,a^2bx + 2\,a^3}{2\,(b^5x + ab^4)} + \frac{3\,a^2\log(bx+a)}{b^4}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3)/(b^5*x + a*b^4) + 3*a^2*lo
g(b*x + a)/b^4
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{3\,a^2\log\left(|bx+a|\right)}{b^4} + \frac{a^3}{(bx+a)b^4} + \frac{b^2x^2-4\,abx}{2\,b^4}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
3*a^2*log(abs(b*x + a))/b^4 + a^3/((b*x + a)*b^4) + 1/2*(b^2*x^2 - 4*a*b*x
)/b^4
```

3.95.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 490, normalized size of antiderivative = 6.53

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{x^2}{2\,b^2}$$
$$+ \frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\,\left(3\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2-12\,a\,\left(2\,a \cdots\right.\right.}{4\,b^4}$$
$$- \frac{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3-8\,a^3-6\,a\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\cdots\right.}{4\,b\,\left(2\,a\,b^3+2\,b^4\,x-b^3\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a} \cdots}\right.\right.\right.}$$
$$+ \frac{x\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b^3}$$

input
```
int(x^3/atanh(tanh(a + b*x))^2,x)
```

output
```
x^2/(2*b^2) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
- log(2/(exp(2*a)*exp(2*b*x) + 1)))*(3*(2*a - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 -
 12*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(
2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 12*a^2))/(4*b^4) - ((2*a - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*
x) + 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a
^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(4*b*(2*a*b^3 + 2*b^4*x - b^3*(2*a - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x))) + (x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^3
```

3.95. $\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx$

3.96 $\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx$

3.96.1 Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{2x}{b^2}-\frac{x^2}{b\operatorname{arctanh}(\tanh(a+bx))}+\frac{2(bx-\operatorname{arctanh}(\tanh(a+bx)))\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^3}$$

output
```
2*x/b^2-x^2/b/arctanh(tanh(b*x+a))+2*(b*x-arctanh(tanh(b*x+a)))*ln(arctanh
(tanh(b*x+a)))/b^3
```

3.96.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{bx-\frac{(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2}{\operatorname{arctanh}(\tanh(a+bx))}+2(bx-\operatorname{arctanh}(\tanh(a+bx)))\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^3}$$

input
```
Integrate[x^2/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(b*x - (-(b*x) + ArcTanh[Tanh[a + b*x]])^2/ArcTanh[Tanh[a + b*x]] + 2*(b*x
 - ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]])/b^3
```

3.96.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))}dx}{b}-\frac{x^2}{b\operatorname{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2589}$$

$$\frac{2\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\operatorname{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2588}$$

$$\frac{2\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))}d\operatorname{arctanh}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\operatorname{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

$$\frac{2\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^2}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\operatorname{arctanh}(\tanh(a+bx))}$$

input
```
Int[x^2/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
-(x^2/(b*ArcTanh[Tanh[a + b*x]])) + (2*(x/b + ((b*x - ArcTanh[Tanh[a + b*x
]])*Log[ArcTanh[Tanh[a + b*x]]])/b^2))/b
```

3.96.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.96.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.72

method	result
default	$\frac{x}{b^2} - \frac{a^2+2a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2}{b^3 \operatorname{arctanh}(\tanh(bx+a))} + \frac{(-2 \operatorname{arctanh}(\tanh(bx+a))+2bx)\ln(\operatorname{arctanh}(\tanh}{b^3}$
risch	Expression too large to display

input
```
int(x^2/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
x/b^2-(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2
)/b^3/arctanh(tanh(b*x+a))+(-2*arctanh(tanh(b*x+a))+2*b*x)/b^3*ln(arctanh(
tanh(b*x+a)))
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{b^2x^2 + abx - a^2 - 2\,(abx + a^2)\log{(bx + a)}}{b^4x + ab^3}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
(b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

3.96.6 Sympy [F]

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \int \frac{x^2}{\operatorname{atanh}^2{(\tanh{(a + bx)})}}\,dx$$

input
```
integrate(x**2/atanh(tanh(b*x+a))**2,x)
```

output
```
Integral(x**2/atanh(tanh(a + b*x))**2, x)
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{b^2x^2 + abx - a^2}{b^4x + ab^3} - \frac{2\,a\log{(bx + a)}}{b^3}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
(b^2*x^2 + a*b*x - a^2)/(b^4*x + a*b^3) - 2*a*log(b*x + a)/b^3
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{x}{b^2} - \frac{2\,a\log\left(|bx+a|\right)}{b^3} - \frac{a^2}{(bx+a)b^3}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
x/b^2 - 2*a*log(abs(b*x + a))/b^3 - a^2/((b*x + a)*b^3)
```

3.96.9 Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 302, normalized size of antiderivative = 6.04

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{x}{b^2} - \frac{\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2-4\,a\,\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b\,\left(2\,a\,b^2+2\,b^3\,x-b^2\,\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)\right)} + \frac{\ln\left(\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\right)\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b^3}$$

input
```
int(x^2/atanh(tanh(a + b*x))^2,x)
```

output
```
x/b^2 - ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + l
og(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) +
2*b*x) + 4*a^2)/(2*b*(2*a*b^2 + 2*b^3*x - b^2*(2*a - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b
*x))) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(
2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^3
```

3.97 $\int \frac{x}{\textbf{arctanh}(\tanh(a+bx))^2}\,dx$

3.97.1 Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{x}{b\operatorname{arctanh}(\tanh(a+bx))} + \frac{\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^2}$$

output
```
-x/b/arctanh(tanh(b*x+a))+ln(arctanh(tanh(b*x+a)))/b^2
```

3.97.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{1 - \frac{bx}{\operatorname{arctanh}(\tanh(a+bx))} + \log(\operatorname{arctanh}(\tanh(a+bx)))}{b^2}$$

input
```
Integrate[x/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(1 - (b*x)/ArcTanh[Tanh[a + b*x]] + Log[ArcTanh[Tanh[a + b*x]]])/b^2
```

3.97.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2599, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^2}dx$$
$$\Big\downarrow \text{ 2599}$$
$$\frac{\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))}dx}{b}-\frac{x}{b\operatorname{arctanh}(\tanh(a+bx))}$$
$$\Big\downarrow \text{ 2588}$$
$$\frac{\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))}d\operatorname{arctanh}(\tanh(a+bx))}{b^2}-\frac{x}{b\operatorname{arctanh}(\tanh(a+bx))}$$
$$\Big\downarrow \text{ 14}$$
$$\frac{\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^2}-\frac{x}{b\operatorname{arctanh}(\tanh(a+bx))}$$

input
```
Int[x/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
-(x/(b*ArcTanh[Tanh[a + b*x]])) + Log[ArcTanh[Tanh[a + b*x]]]/b^2
```

3.97.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.97.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

method	result
parallelrisch	$\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))\operatorname{arctanh}(\tanh(bx+a))-bx}{b^2\operatorname{arctanh}(\tanh(bx+a))}$
default	$\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))}{b^2}-\frac{bx-\operatorname{arctanh}(\tanh(bx+a))}{b^2\operatorname{arctanh}(\tanh(bx+a))}$
risch	$-\frac{}{b\left(\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(ie^{2bx+2a}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)-\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2+\pi\operatorname{csgn}\left(ie^{bx+a}\right)^2\operatorname{csgn}\right.}$

input
```
int(x/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
(ln(arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))-b*x)/b^2/arctanh(tanh(b*x+a
))
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx=\frac{(bx+a)\log(bx+a)+a}{b^3x+ab^2}$$

input
```
integrate(x/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

3.97.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(24) = 48$.

Time = 14.80 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.36

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \begin{cases} \frac{x^2}{2\operatorname{atanh}^2(\tanh(a))} & \text{for } b=0 \\ \frac{x^2}{2\operatorname{atanh}^2(\tanh(bx+\log(-e^{-bx})))} & \text{for } a=\log(-e^{-bx}) \\ \frac{x^2}{2\operatorname{atanh}^2(\tanh(bx+\log(e^{-bx})))} & \text{for } a=\log(e^{-bx}) \\ -\frac{x}{b\operatorname{atanh}(\tanh(a+bx))}+\frac{\log(\operatorname{atanh}(\tanh(a+bx)))}{b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(x/atanh(tanh(b*x+a))**2,x)
```

output
```
Piecewise((x**2/(2*atanh(tanh(a))**2), Eq(b, 0)), (x**2/(2*atanh(tanh(b*x
+ log(-exp(-b*x))))**2), Eq(a, log(-exp(-b*x)))), (x**2/(2*atanh(tanh(b*x
+ log(exp(-b*x))))**2), Eq(a, log(exp(-b*x)))), (-x/(b*atanh(tanh(a + b*x)
)) + log(atanh(tanh(a + b*x)))/b**2, True))
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{a}{b^3x+ab^2} + \frac{\log(bx+a)}{b^2}$$

input
```
integrate(x/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{\log(|bx+a|)}{b^2} + \frac{a}{(bx+a)b^2}$$

input
```
integrate(x/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
log(abs(b*x + a))/b^2 + a/((b*x + a)*b^2)
```

3.97.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{\ln(\operatorname{atanh}(\tanh(a+b\,x)))}{b^2} - \frac{x}{b\,\operatorname{atanh}(\tanh(a+b\,x))}$$

input
```
int(x/atanh(tanh(a + b*x))^2,x)
```

output
```
log(atanh(tanh(a + b*x)))/b^2 - x/(b*atanh(tanh(a + b*x)))
```

3.98 $\int \frac{1}{\textbf{arctanh}(\tanh(a+bx))^2} \, dx$

3.98.1 Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^2} \, dx = -\frac{1}{b\operatorname{arctanh}(\tanh(a+bx))}$$

output
```
-1/b/arctanh(tanh(b*x+a))
```

3.98.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^2} \, dx = -\frac{1}{b\operatorname{arctanh}(\tanh(a+bx))}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(-2),x]
```

output
```
-(1/(b*ArcTanh[Tanh[a + b*x]]))
```

3.98.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^2}d\operatorname{arctanh}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{1}{b\operatorname{arctanh}(\tanh(a+bx))}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(-2),x]
```

output
```
-(1/(b*ArcTanh[Tanh[a + b*x]]))
```

3.98.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.98.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result
derivativedivides	$-\frac{1}{b \operatorname{arctanh}(\tanh(bx+a))}$
default	$-\frac{1}{b \operatorname{arctanh}(\tanh(bx+a))}$
parallelrisch	$-\frac{1}{b \operatorname{arctanh}(\tanh(bx+a))}$
risch	$-\frac{}{b\left(\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right) \operatorname{csgn}\left(i e^{2bx+2a}\right) \operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)-\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right) \operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2+\pi \operatorname{csgn}\left(i e^{bx+a}\right.}$

input
```
int(1/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/arctanh(tanh(b*x+a))
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{1}{b^2x+ab}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
-1/(b^2*x + a*b)
```

3.98.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(12) = 24.

Time = 14.68 (sec) , antiderivative size = 94, normalized size of antiderivative = 6.71

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \begin{cases} \frac{x}{\operatorname{atanh}^2(\tanh(a))} & \text{for } b=0 \\ -\frac{\log(-e^{-bx})}{b \operatorname{atanh}^2(\tanh(bx+\log(-e^{-bx})))} & \text{for } a=\log(-e^{-bx}) \\ -\frac{\log(e^{-bx})}{b \operatorname{atanh}^2(\tanh(bx+\log(e^{-bx})))} & \text{for } a=\log(e^{-bx}) \\ -\frac{1}{b \operatorname{atanh}(\tanh(a+bx))} & \text{otherwise} \end{cases}$$

input
```
integrate(1/atanh(tanh(b*x+a))**2,x)
```

output
```
Piecewise((x/atanh(tanh(a))**2, Eq(b, 0)), (-log(-exp(-b*x))/(b*atanh(tanh
(b*x + log(-exp(-b*x))))**2), Eq(a, log(-exp(-b*x)))), (-log(exp(-b*x))/(b
*atanh(tanh(b*x + log(exp(-b*x))))**2), Eq(a, log(exp(-b*x)))), (-1/(b*ata
nh(tanh(a + b*x))), True))
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{1}{(bx+a)b}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
-1/((b*x + a)*b)
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{1}{(bx+a)b}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
-1/((b*x + a)*b)
```

3.98.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{1}{b\,\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}$$

input
```
int(1/atanh(tanh(a + b*x))^2,x)
```

output
```
-1/(b*atanh(tanh(a + b*x)))
```

3.99 $\int \frac{1}{x\textbf{arctanh}(\tanh(a+bx))^2}\,dx$

3.99.1 Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))} + \frac{\log(x)}{(bx-\text{arctanh}(\tanh(a+bx)))^2} - \frac{\log(\text{arctanh}(\tanh(a+bx)))}{(bx-\text{arctanh}(\tanh(a+bx)))^2}$$

output
```
-1/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))+ln(x)/(b*x-arctanh(tanh
(b*x+a)))^2-ln(arctanh(tanh(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^2
```

3.99.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^2}\,dx = \frac{-bx+\text{arctanh}(\tanh(a+bx))(1+\log(bx)-\log(\text{arctanh}(\tanh(a+bx))))}{\text{arctanh}(\tanh(a+bx))(-bx+\text{arctanh}(\tanh(a+bx)))^2}$$

input
```
Integrate[1/(x*ArcTanh[Tanh[a + b*x]]^2),x]
```

output
```
(-(b*x) + ArcTanh[Tanh[a + b*x]]*(1 + Log[b*x] - Log[ArcTanh[Tanh[a + b*x]
]]))/(ArcTanh[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

3.99.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \text{ 2594}$$

$$-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{ 2591}$$

$$-\frac{\frac{b\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{ 14}$$

$$-\frac{\frac{b\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{ 2588}$$

$$-\frac{\frac{\int \frac{1}{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+bx))}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{ 14}$$

$$-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}-\frac{\frac{\log(\text{arctanh}(\tanh(a+bx)))}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}$$

input
```
Int[1/(x*ArcTanh[Tanh[a + b*x]]^2),x]
```

output
```
-(1/((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]])) - (-(Log[x]/(
b*x - ArcTanh[Tanh[a + b*x]])) + Log[ArcTanh[Tanh[a + b*x]]]/(b*x - ArcTan
h[Tanh[a + b*x]]))/(b*x - ArcTanh[Tanh[a + b*x]])
```

3.99.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

3.99.4 Maple [A] (verified)

Time = 270.43 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^2}+\frac{1}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))}+\frac{\ln(x)}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^2}$	67
risch	Expression too large to display	1382

input
```
int(1/x/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(arctanh(tanh(b*x+a))-b*x)^2*ln(arctanh(tanh(b*x+a)))+1/(arctanh(tanh(b
*x+a))-b*x)/arctanh(tanh(b*x+a))+1/(arctanh(tanh(b*x+a))-b*x)^2*ln(x)
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.56

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{(bx+a)\log(bx+a) - (bx+a)\log(x) - a}{a^2bx + a^3}$$

input
```
integrate(1/x/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
-((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

3.99.6 Sympy [F]

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^2}\,dx = \int \frac{1}{x\,\text{atanh}^2(\tanh(a+bx))}\,dx$$

input
```
integrate(1/x/atanh(tanh(b*x+a))**2,x)
```

output
```
Integral(1/(x*atanh(tanh(a + b*x))**2), x)
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.40

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^2}\,dx = \frac{1}{abx+a^2} - \frac{\log(bx+a)}{a^2} + \frac{\log(x)}{a^2}$$

input
```
integrate(1/x/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.44

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{\log(|bx+a|)}{a^2} + \frac{\log(|x|)}{a^2} + \frac{1}{(bx+a)a}$$

input
```
integrate(1/x/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/a^2 + log(abs(x))/a^2 + 1/((b*x + a)*a)
```

3.99.9 Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 359, normalized size of antiderivative = 5.13

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^2}\,dx$$
$$= \frac{8\,b\,x - \ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\left(-4+\text{atan}\left(\frac{-\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,1\text{i}+\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,1\text{i}+b\,x\,2\text{i}}{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}\right)\,8\text{i}\right)+\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\left(-4+ \text{a}\right.}{\left(\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\,\left(\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right.\right.}$$

input
```
int(1/(x*atanh(tanh(a + b*x))^2),x)
```

output
```
(8*b*x - log(1/(exp(2*a)*exp(2*b*x) + 1))*(atan((log((exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x
*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x))*8i - 4) + log((exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))*(atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*e
xp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x))*8i - 4))/((log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)))*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
```

3.100 $\int \frac{1}{x^2\mathbf{arctanh}(\tanh(a+bx))^2}\,dx$

3.100.1 Optimal result

Integrand size = 13, antiderivative size = 102

$$\begin{aligned}\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^2}\,dx = &-\frac{2b}{(bx-\text{arctanh}(\tanh(a+bx)))^2\text{arctanh}(\tanh(a+bx))}\\ &+\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}\\ &+\frac{2b\log(x)}{(bx-\text{arctanh}(\tanh(a+bx)))^3}\\ &-\frac{2b\log(\text{arctanh}(\tanh(a+bx)))}{(bx-\text{arctanh}(\tanh(a+bx)))^3}\end{aligned}$$

output
```
-2*b/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))+1/x/(b*x-arctanh(ta
nh(b*x+a)))/arctanh(tanh(b*x+a))+2*b*ln(x)/(b*x-arctanh(tanh(b*x+a)))^3-2*
b*ln(arctanh(tanh(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^3
```

3.100.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

$$\begin{aligned}&\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^2}\,dx\\ &=\frac{-b^2x^2+\text{arctanh}(\tanh(a+bx))^2+2bx\text{arctanh}(\tanh(a+bx))(\log(x)-\log(\text{arctanh}(\tanh(a+bx))))}{x(bx-\text{arctanh}(\tanh(a+bx)))^3\text{arctanh}(\tanh(a+bx))}\end{aligned}$$

input
```
Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]^2),x]
```

output
```
(-(b^2*x^2) + ArcTanh[Tanh[a + b*x]]^2 + 2*b*x*ArcTanh[Tanh[a + b*x]]*(Log
[x] - Log[ArcTanh[Tanh[a + b*x]]]))/(x*(b*x - ArcTanh[Tanh[a + b*x]])^3*Ar
cTanh[Tanh[a + b*x]])
```

3.100.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2602, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{2b\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^2}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2594}$$

$$\frac{2b\left(-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2591}$$

$$\frac{2b\left(-\frac{\frac{b\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

$$\frac{2b\left(-\frac{\frac{b\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))}dx}{bx-\operatorname{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\operatorname{arctanh}(\tanh(a+bx))}}{bx-\operatorname{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2588}$$

$$\frac{2b\left(-\frac{\frac{\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))}d\operatorname{arctanh}(\tanh(a+bx))}{bx-\operatorname{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\operatorname{arctanh}(\tanh(a+bx))}}{bx-\operatorname{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{x(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))}+\frac{2b\left(-\frac{1}{(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))}-\frac{\frac{\log(\operatorname{arctanh}(\tanh(a+bx)))}{bx-\operatorname{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\operatorname{arctanh}(\tanh(a+bx))}}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}$$

input
```
Int[1/(x^2*ArcTanh[Tanh[a + b*x]]^2),x]
```

output
```
1/(x*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]) + (2*b*(-(1/((
b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]])) - (-(Log[x]/(b*x -
ArcTanh[Tanh[a + b*x]])) + Log[ArcTanh[Tanh[a + b*x]]]/(b*x - ArcTanh[Tanh
[a + b*x]]))/(b*x - ArcTanh[Tanh[a + b*x]])))/(b*x - ArcTanh[Tanh[a + b*x]
])
```

3.100. $\int \frac{1}{x^2\operatorname{arctanh}(\tanh(a+bx))^2}\,dx$

3.100.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.100.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

$$-\frac{b}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}+\frac{2b\ln\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}}-\frac{}{\left(\operatorname{arctanh}\left(\tan\right.\right.}$$

input
```
int(1/x^2/arctanh(tanh(b*x+a))^2,x)
```

output
```
-1/(arctanh(tanh(b*x+a))-b*x)^2*b/arctanh(tanh(b*x+a))+2/(arctanh(tanh(b*x
+a))-b*x)^3*b*ln(arctanh(tanh(b*x+a)))-1/(arctanh(tanh(b*x+a))-b*x)^2/x-2/
(arctanh(tanh(b*x+a))-b*x)^3*b*ln(x)
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^2 \text{arctanh}(\tanh(a+bx))^2} \, dx = -\frac{2\,abx + a^2 - 2\,(b^2x^2 + abx)\log(bx+a) + 2\,(b^2x^2 + abx)\log(x)}{a^3bx^2 + a^4x}$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
-(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*l
og(x))/(a^3*b*x^2 + a^4*x)
```

3.100.6 Sympy [F]

$$\int \frac{1}{x^2 \text{arctanh}(\tanh(a+bx))^2} \, dx = \int \frac{1}{x^2 \operatorname{atanh}^2(\tanh(a+bx))} \, dx$$

input
```
integrate(1/x**2/atanh(tanh(b*x+a))**2,x)
```

output
```
Integral(1/(x**2*atanh(tanh(a + b*x))**2), x)
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^2 \text{arctanh}(\tanh(a+bx))^2} \, dx = -\frac{2\,bx + a}{a^2bx^2 + a^3x} + \frac{2\,b\log(bx+a)}{a^3} - \frac{2\,b\log(x)}{a^3}$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
-(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^2}\,dx = \frac{2\,b\log\left(|bx+a|\right)}{a^3} - \frac{2\,b\log\left(|x|\right)}{a^3} - \frac{2\,bx+a}{(bx^2+ax)a^2}$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
2*b*log(abs(b*x + a))/a^3 - 2*b*log(abs(x))/a^3 - (2*b*x + a)/((b*x^2 + a*
x)*a^2)
```

3.100.9 Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.24

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{4\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)^2 - \ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\left(8\,\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right) + b\,x\,\text{atan}\left(\frac{-\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)1\text{i}+\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)1\text{i}+b\,x\,2\text{i}}{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\right.\right.}{x\left(\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\right)\left(\right.}$$

input
```
int(1/(x^2*atanh(tanh(a + b*x))^2),x)
```

output
```
-(4*log(1/(exp(2*a)*exp(2*b*x) + 1))^2 - log((exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1))*(8*log(1/(exp(2*a)*exp(2*b*x) + 1)) + b*x*atan((log((e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*
b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*32i) + 4*log((exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 - 16*b^2*x^2 + b*x*log(1/(exp(2*a)*
exp(2*b*x) + 1))*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
)*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(
2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x))*32i)/(x*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)))*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)
```

3.101 $\int \frac{1}{x^3 \operatorname{arctanh}(\tanh(a+bx))^2} \, dx$

3.101.1 Optimal result

Integrand size = 13, antiderivative size = 143

$$
\begin{aligned}
&\int \frac{1}{x^3 \operatorname{arctanh}(\tanh(a+bx))^2} \, dx \\
&= -\frac{3b^2}{(bx-\operatorname{arctanh}(\tanh(a+bx)))^3 \operatorname{arctanh}(\tanh(a+bx))} \\
&\quad + \frac{3b}{2x(bx-\operatorname{arctanh}(\tanh(a+bx)))^2 \operatorname{arctanh}(\tanh(a+bx))} \\
&\quad + \frac{1}{2x^2(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))} \\
&\quad + \frac{3b^2\log(x)}{(bx-\operatorname{arctanh}(\tanh(a+bx)))^4} - \frac{3b^2\log(\operatorname{arctanh}(\tanh(a+bx)))}{(bx-\operatorname{arctanh}(\tanh(a+bx)))^4}
\end{aligned}
$$

output
```
-3*b^2/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))+3/2*b/x/(b*x-arct
anh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))+1/2/x^2/(b*x-arctanh(tanh(b*x+a))
)/arctanh(tanh(b*x+a))+3*b^2*ln(x)/(b*x-arctanh(tanh(b*x+a)))^4-3*b^2*ln(a
rctanh(tanh(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^4
```

3.101.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{2b^3x^3-6bx\text{arctanh}(\tanh(a+bx))^2+\text{arctanh}(\tanh(a+bx))^3-3b^2x^2\text{arctanh}(\tanh(a+bx))(-1+2l}{2x^2\text{arctanh}(\tanh(a+bx))(-bx+\text{arctanh}(\tanh(a+bx)))^4}$$

input
```
Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]^2),x]
```

output
```
-1/2*(2*b^3*x^3 - 6*b*x*ArcTanh[Tanh[a + b*x]]^2 + ArcTanh[Tanh[a + b*x]]^
3 - 3*b^2*x^2*ArcTanh[Tanh[a + b*x]]*(-1 + 2*Log[x] - 2*Log[ArcTanh[Tanh[a
 + b*x]]]))/(x^2*ArcTanh[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^
4)
```

3.101.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2602, 2602, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \ 2602$$

$$\frac{3b\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^2}dx}{2(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \ 2602$$

$$\frac{3b\left(\frac{2b\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^2}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}\right)}{2(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \ 2594$$

$$\frac{3b\left(\frac{2b\left(-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(}\right.}{2(bx-\text{arctanh}(\tanh(a+bx)))}$$
$$\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}$$

$\downarrow$ 2591

$$\frac{3b\left(\frac{2b\left(-\frac{\frac{b\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{}{x(bx-\text{arctanh}(\text{t}}\right.}{2(bx-\text{arctanh}(\tanh(a+bx)))}$$
$$\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}$$

$\downarrow$ 14

$$\frac{3b\left(\frac{2b\left(-\frac{\frac{b\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{}{x(bx-\text{arctanh}(\text{t}}\right.}{2(bx-\text{arctanh}(\tanh(a+bx)))}$$
$$\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}$$

$\downarrow$ 2588

3.101. $\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^2}\,dx$

$$\frac{3b\left(\frac{2b\left(-\frac{\frac{\int \frac{1}{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+bx))}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\right.}{2(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}+$$

$$\frac{3b\left(\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}+\frac{2b\left(-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}-\frac{\frac{\log(\text{arctanh}(\tanh(a+}{bx-\text{arctanh}(\tanh(a+}}{bx-\text{arc}}\right.}{bx-\text{arctanh}(\tanh(a+bx))}\right.}{2(bx-\text{arctanh}(\tanh(a+bx)))}$$

input
```
Int[1/(x^3*ArcTanh[Tanh[a + b*x]]^2),x]
```

output
```
1/(2*x^2*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]) + (3*b*(1/
(x*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]) + (2*b*(-(1/((b*
x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]])) - (-(Log[x]/(b*x - Ar
cTanh[Tanh[a + b*x]])) + Log[ArcTanh[Tanh[a + b*x]]]/(b*x - ArcTanh[Tanh[a
 + b*x]]))/(b*x - ArcTanh[Tanh[a + b*x]])))/(b*x - ArcTanh[Tanh[a + b*x]])
))/(2*(b*x - ArcTanh[Tanh[a + b*x]]))
```

3.101.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.101.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.81

$$-\frac{3b^2 \ln \left(\operatorname{arctanh}\left(\tanh \left(bx+a\right)\right)\right)}{\left(\operatorname{arctanh}\left(\tanh \left(bx+a\right)\right)-bx\right)^4}+\frac{b^2}{\left(\operatorname{arctanh}\left(\tanh \left(bx+a\right)\right)-bx\right)^3 \operatorname{arctanh}\left(\tanh \left(bx+a\right)\right)}-\frac{}{2 \left(\operatorname{arctanh}\left(\mathrm{ta}\right.\right.}$$

input
```
int(1/x^3/arctanh(tanh(b*x+a))^2,x)
```

output
```
-3/(arctanh(tanh(b*x+a))-b*x)^4*b^2*ln(arctanh(tanh(b*x+a)))+1/(arctanh(ta
nh(b*x+a))-b*x)^3*b^2/arctanh(tanh(b*x+a))-1/2/(arctanh(tanh(b*x+a))-b*x)^
2/x^2+3/(arctanh(tanh(b*x+a))-b*x)^4*b^2*ln(x)+2/(arctanh(tanh(b*x+a))-b*x
)^3*b/x
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^3 \operatorname{arctanh}(\tanh(a+bx))^2} dx = \frac{6\,ab^2x^2 + 3\,a^2bx - a^3 - 6\,(b^3x^3 + ab^2x^2)\log(bx+a) + 6\,(b^3x^3 + ab^2x^2)\log(x)}{2\,(a^4bx^3 + a^5x^2)}$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a)
+ 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

3.101.6 Sympy [F]

$$\int \frac{1}{x^3 \operatorname{arctanh}(\tanh(a+bx))^2} dx = \int \frac{1}{x^3 \operatorname{atanh}^2(\tanh(a+bx))} dx$$

input
```
integrate(1/x**3/atanh(tanh(b*x+a))**2,x)
```

output
```
Integral(1/(x**3*atanh(tanh(a + b*x))**2), x)
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^3 \operatorname{arctanh}(\tanh(a+bx))^2} dx = \frac{6\,b^2x^2 + 3\,abx - a^2}{2\,(a^3bx^3 + a^4x^2)} - \frac{3\,b^2\log(bx+a)}{a^4} + \frac{3\,b^2\log(x)}{a^4}$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)
/a^4 + 3*b^2*log(x)/a^4
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^3 \operatorname{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{3\,b^2\log\left(|bx+a|\right)}{a^4}+\frac{3\,b^2\log\left(|x|\right)}{a^4}+\frac{6\,ab^2x^2+3\,a^2bx-a^3}{2\,(bx+a)a^4x^2}$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
-3*b^2*log(abs(b*x + a))/a^4 + 3*b^2*log(abs(x))/a^4 + 1/2*(6*a*b^2*x^2 +
3*a^2*b*x - a^3)/((b*x + a)*a^4*x^2)
```

3.101.9 Mupad [B] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 660, normalized size of antiderivative = 4.62

$$\int \frac{1}{x^3 \operatorname{arctanh}(\tanh(a+bx))^2}\,dx =$$

$$-\frac{6\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2-6\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3-2\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right.}{}$$

input
```
int(1/(x^3*atanh(tanh(a + b*x))^2),x)
```

output

```
-(6*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))^2 - 6*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*log(1/(exp(2*a)*exp(2*b*x) + 1))^3
 - 2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3 - 32*b^3*x^3 +
 24*b*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 + 24*b^2*x^
2*log(1/(exp(2*a)*exp(2*b*x) + 1)) - 24*b^2*x^2*log((exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 24*b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))^2 + b
^2*x^2*log(1/(exp(2*a)*exp(2*b*x) + 1))*atan((log((exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i
)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x))*96i - b^2*x^2*log((exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1))*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*e
xp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x))*96i - 48*b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)))/(x^2*(log(1/(exp(2*a)*exp(2*b*x) + 1)) -
log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))*(log(1/(exp(2*a)*exp
(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b
*x)^4)
```

3.101. $\int \frac{1}{x^3 \text{arctanh}(\tanh(a+bx))^2} \, dx$

3.102 $\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$

3.102.1 Optimal result

Integrand size = 13, antiderivative size = 94

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{x^m}{2b\operatorname{arctanh}(\tanh(a+bx))^2} - \frac{mx^{-1+m}}{2b^2\operatorname{arctanh}(\tanh(a+bx))} - \frac{mx^{-1+m}\,\text{Hypergeometric2F1}\left(1,-1+m,m,\frac{bx}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)}{2b^2(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

output
```
-1/2*x^m/b/arctanh(tanh(b*x+a))^2-1/2*m*x^(-1+m)/b^2/arctanh(tanh(b*x+a))-
1/2*m*x^(-1+m)*hypergeom([1, -1+m],[m],b*x/(b*x-arctanh(tanh(b*x+a))))/b^2
/(b*x-arctanh(tanh(b*x+a)))
```

3.102.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.54

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(3,1+m,2+m,-\frac{bx}{-bx+\operatorname{arctanh}(\tanh(a+bx))}\right)}{(1+m)(-bx+\operatorname{arctanh}(\tanh(a+bx)))^3}$$

input
```
Integrate[x^m/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcTanh[Ta
nh[a + b*x]]))])/((1 + m)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3)
```

3.102.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2599, 2599, 2595}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\text{arctanh}(\tanh(a+bx))^3}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{m\int \frac{x^{m-1}}{\text{arctanh}(\tanh(a+bx))^2}dx}{2b}-\frac{x^m}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2599}$$

$$\frac{m\left(-\frac{(1-m)\int \frac{x^{m-2}}{\text{arctanh}(\tanh(a+bx))}dx}{b}-\frac{x^{m-1}}{b\text{arctanh}(\tanh(a+bx))}\right)}{2b}-\frac{x^m}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2595}$$

$$\frac{m\left(-\frac{x^{m-1}\text{Hypergeometric2F1}\left(1,m-1,m,\frac{bx}{bx-\text{arctanh}(\tanh(a+bx))}\right)}{b(bx-\text{arctanh}(\tanh(a+bx)))}-\frac{x^{m-1}}{b\text{arctanh}(\tanh(a+bx))}\right)}{2b}-\frac{x^m}{2b\text{arctanh}(\tanh(a+bx))^2}$$

input
```
Int[x^m/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*x^m/(b*ArcTanh[Tanh[a + b*x]]^2) + (m*(-(x^(-1 + m)/(b*ArcTanh[Tanh[a
 + b*x]])) - (x^(-1 + m)*Hypergeometric2F1[1, -1 + m, m, (b*x)/(b*x - ArcT
anh[Tanh[a + b*x]])])/(b*(b*x - ArcTanh[Tanh[a + b*x]]))))/(2*b)
```

3.102.3.1 Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.102.4 Maple [F]

$$\int \frac{x^m}{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^3}dx$$

input
```
int(x^m/arctanh(tanh(b*x+a))^3,x)
```

output
```
int(x^m/arctanh(tanh(b*x+a))^3,x)
```

3.102.5 Fricas [F]

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx=\int \frac{x^m}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

input
```
integrate(x^m/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
integral(x^m/arctanh(tanh(b*x + a))^3, x)
```

3.102.6 Sympy [F]

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \int \frac{x^m}{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**m/atanh(tanh(b*x+a))**3,x)
```

output
```
Integral(x**m/atanh(tanh(a + b*x))**3, x)
```

3.102.7 Maxima [F]

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \int \frac{x^m}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

input
```
integrate(x^m/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
integrate(x^m/arctanh(tanh(b*x + a))^3, x)
```

3.102.8 Giac [F]

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \int \frac{x^m}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

input
```
integrate(x^m/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
integrate(x^m/arctanh(tanh(b*x + a))^3, x)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \int \frac{x^m}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^3}\,dx$$

input
```
int(x^m/atanh(tanh(a + b*x))^3,x)
```

output
```
int(x^m/atanh(tanh(a + b*x))^3, x)
```

3.103 $\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^3} dx$

3.103.1 Optimal result

Integrand size = 13, antiderivative size = 92

$$\begin{aligned}\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^3} dx &= \frac{3x^2}{b^3} + \frac{6x(bx - \operatorname{arctanh}(\tanh(a+bx)))}{b^4} \\ &\quad - \frac{x^4}{2b\operatorname{arctanh}(\tanh(a+bx))^2} - \frac{2x^3}{b^2\operatorname{arctanh}(\tanh(a+bx))} \\ &\quad + \frac{6(bx - \operatorname{arctanh}(\tanh(a+bx)))^2 \log(\operatorname{arctanh}(\tanh(a+bx)))}{b^5}\end{aligned}$$

output
```
3*x^2/b^3+6*x*(b*x-arctanh(tanh(b*x+a)))/b^4-1/2*x^4/b/arctanh(tanh(b*x+a)
)^2-2*x^3/b^2/arctanh(tanh(b*x+a))+6*(b*x-arctanh(tanh(b*x+a)))^2*ln(arcta
nh(tanh(b*x+a)))/b^5
```

3.103.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$$
$$= \frac{x^2}{2b^3} - \frac{3x(-bx+\operatorname{arctanh}(\tanh(a+bx)))}{b^4} + \frac{4(-bx+\operatorname{arctanh}(\tanh(a+bx)))^3}{b^5\operatorname{arctanh}(\tanh(a+bx))} - \frac{(-bx+\operatorname{arctanh}(\tanh(a+bx)))^4}{2b^5\operatorname{arctanh}(\tanh(a+bx))^2} + \frac{6(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^5}$$

input
```
Integrate[x^4/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
x^2/(2*b^3) - (3*x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^4 + (4*(-(b*x) + A
rcTanh[Tanh[a + b*x]])^3)/(b^5*ArcTanh[Tanh[a + b*x]]) - (-(b*x) + ArcTanh
[Tanh[a + b*x]])^4/(2*b^5*ArcTanh[Tanh[a + b*x]]^2) + (6*(-(b*x) + ArcTanh
[Tanh[a + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^5
```

3.103.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2599, 2599, 2590, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$$
$$\Big\downarrow \text{ 2599}$$
$$\frac{2\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^2}dx}{b} - \frac{x^4}{2b\operatorname{arctanh}(\tanh(a+bx))^2}$$
$$\Big\downarrow \text{ 2599}$$
$$\frac{2\left(\frac{3\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))}dx}{b} - \frac{x^3}{b\operatorname{arctanh}(\tanh(a+bx))}\right)}{b} - \frac{x^4}{2b\operatorname{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 2590

$$\frac{2\left(\frac{3\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int\frac{x}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\text{arctanh}(\tanh(a+bx))}\right)}{b}-\frac{x^4}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 2589

$$\frac{2\left(\frac{3\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int\frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\text{arctanh}(\tanh(a+bx))}\right)}{b}-\frac{x^4}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 2588

$$\frac{2\left(\frac{3\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int\frac{1}{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\text{arctanh}(\tanh(a+bx))}\right)}{b}-\frac{x^4}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 14

3.103. $\int \frac{x^4}{\text{arctanh}(\tanh(a+bx))^3}\,dx$

$$\frac{2\left(\frac{3\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\log(\text{arctanh}(\tanh(a+bx)))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\text{arctanh}(\tanh(a+bx))}\right)}{b}-\frac{x^4}{2b\text{arctanh}(\tanh(a+bx))^2}$$

input
```
Int[x^4/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*x^4/(b*ArcTanh[Tanh[a + b*x]]^2) + (2*(-(x^3/(b*ArcTanh[Tanh[a + b*x]
])) + (3*(x^2/(2*b) + ((b*x - ArcTanh[Tanh[a + b*x]])*(x/b + ((b*x - ArcTa
nh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]])/b^2))/b))/b))/b
```

3.103.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.103.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(90) = 180$.

Time = 0.29 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.89

method	result
default	$\frac{\frac{b x^2}{2}-3ax-3x(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{b^4} - \frac{-4a^3-12a^2(\operatorname{arctanh}(\tanh(bx+a))-bx-a)-12a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2-}{b^5 \operatorname{arctanh}(\tanh(bx+a))}$
risch	Expression too large to display

input
```
int(x^4/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/2*b*x^2-3*a*x-3*x*(arctanh(tanh(b*x+a))-b*x-a))-(-4*a^3-12*a^2*(a
rctanh(tanh(b*x+a))-b*x-a)-12*a*(arctanh(tanh(b*x+a))-b*x-a)^2-4*(arctanh(
tanh(b*x+a))-b*x-a)^3)/b^5/arctanh(tanh(b*x+a))+(6*a^2+12*a*(arctanh(tanh(
b*x+a))-b*x-a)+6*(arctanh(tanh(b*x+a))-b*x-a)^2)/b^5*ln(arctanh(tanh(b*x+a
)))-1/2*(a^4+4*a^3*(arctanh(tanh(b*x+a))-b*x-a)+6*a^2*(arctanh(tanh(b*x+a)
)-b*x-a)^2+4*a*(arctanh(tanh(b*x+a))-b*x-a)^3+(arctanh(tanh(b*x+a))-b*x-a)
^4)/b^5/arctanh(tanh(b*x+a))^2
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{b^4x^4 - 4\,ab^3x^3 - 11\,a^2b^2x^2 + 2\,a^3bx + 7\,a^4 + 12\,(a^2b^2x^2 + 2\,a^3bx + a^4)\log(bx+a)}{2\,(b^7x^2 + 2\,ab^6x + a^2b^5)}$$

input
```
integrate(x^4/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/2*(b^4*x^4 - 4*a*b^3*x^3 - 11*a^2*b^2*x^2 + 2*a^3*b*x + 7*a^4 + 12*(a^2*
b^2*x^2 + 2*a^3*b*x + a^4)*log(b*x + a))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

3.103.6 Sympy [F]

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \int \frac{x^4}{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**4/atanh(tanh(b*x+a))**3,x)
```

output
```
Integral(x**4/atanh(tanh(a + b*x))**3, x)
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{b^4x^4 - 4\,ab^3x^3 - 11\,a^2b^2x^2 + 2\,a^3bx + 7\,a^4}{2\,(b^7x^2 + 2\,ab^6x + a^2b^5)} + \frac{6\,a^2\log\left(bx+a\right)}{b^5}$$

input
```
integrate(x^4/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
1/2*(b^4*x^4 - 4*a*b^3*x^3 - 11*a^2*b^2*x^2 + 2*a^3*b*x + 7*a^4)/(b^7*x^2
+ 2*a*b^6*x + a^2*b^5) + 6*a^2*log(b*x + a)/b^5
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{6\,a^2\log\left(|bx+a|\right)}{b^5} + \frac{b^3x^2 - 6\,ab^2x}{2\,b^6} + \frac{8\,a^3bx + 7\,a^4}{2\,(bx+a)^2b^5}$$

input
```
integrate(x^4/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
6*a^2*log(abs(b*x + a))/b^5 + 1/2*(b^3*x^2 - 6*a*b^2*x)/b^6 + 1/2*(8*a^3*b
*x + 7*a^4)/((b*x + a)^2*b^5)
```

3.103.9 Mupad [B] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 867, normalized size of antiderivative = 9.42

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \text{Too large to display}$$

input
```
int(x^4/atanh(tanh(a + b*x))^3,x)
```

output
```
((7*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^2 + 16*a^4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 32*a^3*(2*a - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)))/(4*b) - x*(4*(2*a - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 -
 32*a^3 - 24*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 48*a^2*(2*a - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)))/(2*b^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + x*(16*a*b^5 -
 8*b^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log
(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + 8*a^2*b^4 + 8*b^6*x^2 - 8*a*b^4*
(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + x^2/(2*b^3) + (log(log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(3*
(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 12*a*(2*a - log((2*exp(2*a)*exp(2*b...
```

3.104 $\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$

3.104.1 Optimal result

Integrand size = 13, antiderivative size = 71

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{3x}{b^3} - \frac{x^3}{2b\operatorname{arctanh}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\operatorname{arctanh}(\tanh(a+bx))} + \frac{3(bx-\operatorname{arctanh}(\tanh(a+bx)))\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^4}$$

output
```
3*x/b^3-1/2*x^3/b/arctanh(tanh(b*x+a))^2-3/2*x^2/b^2/arctanh(tanh(b*x+a))+
3*(b*x-arctanh(tanh(b*x+a)))*ln(arctanh(tanh(b*x+a)))/b^4
```

3.104.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{b^3x^3 + 3b^2x^2\operatorname{arctanh}(\tanh(a+bx)) + \operatorname{arctanh}(\tanh(a+bx))^3(5+6\log(\operatorname{arctanh}(\tanh(a+bx)))) - bx}{2b^4\operatorname{arctanh}(\tanh(a+bx))^2}$$

input
```
Integrate[x^3/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*(b^3*x^3 + 3*b^2*x^2*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^
3*(5 + 6*Log[ArcTanh[Tanh[a + b*x]]]) - b*x*ArcTanh[Tanh[a + b*x]]^2*(11 +
 6*Log[ArcTanh[Tanh[a + b*x]]]))/(b^4*ArcTanh[Tanh[a + b*x]]^2)
```

3.104.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2599, 2599, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\text{arctanh}(\tanh(a+bx))^3}\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{3\int \frac{x^2}{\text{arctanh}(\tanh(a+bx))^2}dx}{2b}-\frac{x^3}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{3\left(\frac{2\int \frac{x}{\text{arctanh}(\tanh(a+bx))}dx}{b}-\frac{x^2}{b\text{arctanh}(\tanh(a+bx))}\right)}{2b}-\frac{x^3}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{ 2589}$$

$$\frac{3\left(\frac{2\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\text{arctanh}(\tanh(a+bx))}\right)}{2b}-\frac{x^3}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{3\left(\frac{2\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\int\frac{1}{\operatorname{arctanh}(\tanh(a+bx))}d\operatorname{arctanh}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\operatorname{arctanh}(\tanh(a+bx))}\right)}{2b}-\frac{x^3}{2b\operatorname{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \ 14$$

$$\frac{3\left(\frac{2\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^2}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\operatorname{arctanh}(\tanh(a+bx))}\right)}{2b}-\frac{x^3}{2b\operatorname{arctanh}(\tanh(a+bx))^2}$$

input
```
Int[x^3/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*x^3/(b*ArcTanh[Tanh[a + b*x]]^2) + (3*(-(x^2/(b*ArcTanh[Tanh[a + b*x]
])) + (2*(x/b + ((b*x - ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]
])/b^2))/b))/(2*b)
```

3.104.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.104.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(67) = 134$.

Time = 0.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.38

method	result
default	$\frac{x}{b^3} + \frac{(-3 \operatorname{arctanh}(\tanh(bx+a))+3bx) \ln(\operatorname{arctanh}(\tanh(bx+a)))}{b^4} - \frac{-a^3-3a^2(\operatorname{arctanh}(\tanh(bx+a))-bx-a)-3a(\operatorname{arctanh}(\tanh(bx}{2b^4 \operatorname{arctanh}(\tanh(bx}$
risch	Expression too large to display

input
```
int(x^3/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*x+(-3*arctanh(tanh(b*x+a))+3*b*x)/b^4*ln(arctanh(tanh(b*x+a)))-1/2*(
-a^3-3*a^2*(arctanh(tanh(b*x+a))-b*x-a)-3*a*(arctanh(tanh(b*x+a))-b*x-a)^2
-(arctanh(tanh(b*x+a))-b*x-a)^3)/b^4/arctanh(tanh(b*x+a))^2-(3*a^2+6*a*(ar
ctanh(tanh(b*x+a))-b*x-a)+3*(arctanh(tanh(b*x+a))-b*x-a)^2)/b^4/arctanh(ta
nh(b*x+a))
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{2\,b^3x^3 + 4\,ab^2x^2 - 4\,a^2bx - 5\,a^3 - 6\,(ab^2x^2 + 2\,a^2bx + a^3)\log(bx+a)}{2\,(b^6x^2 + 2\,ab^5x + a^2b^4)}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/2*(2*b^3*x^3 + 4*a*b^2*x^2 - 4*a^2*b*x - 5*a^3 - 6*(a*b^2*x^2 + 2*a^2*b*
x + a^3)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

3.104.6 Sympy [F]

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \int \frac{x^3}{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**3/atanh(tanh(b*x+a))**3,x)
```

output
```
Integral(x**3/atanh(tanh(a + b*x))**3, x)
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{2\,b^3x^3+4\,ab^2x^2-4\,a^2bx-5\,a^3}{2\left(b^6x^2+2\,ab^5x+a^2b^4\right)} - \frac{3\,a\log\left(bx+a\right)}{b^4}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
1/2*(2*b^3*x^3 + 4*a*b^2*x^2 - 4*a^2*b*x - 5*a^3)/(b^6*x^2 + 2*a*b^5*x + a
^2*b^4) - 3*a*log(b*x + a)/b^4
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{x}{b^3} - \frac{3\,a\log\left(|bx+a|\right)}{b^4} - \frac{6\,a^2bx+5\,a^3}{2\left(bx+a\right)^2b^4}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
x/b^3 - 3*a*log(abs(b*x + a))/b^4 - 1/2*(6*a^2*b*x + 5*a^3)/((b*x + a)^2*b
^4)
```

3.104.9 Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 620, normalized size of antiderivative = 8.73

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{x}{b^3}$$

$$-\frac{x\left(3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2-12a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+\right.}{b^3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2+x\left(8ab^4-4b^4\left(2a-\right.\right.}$$

$$+\frac{\ln\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)\right)\left(3\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-3\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+6bx\right)}{2b^4}$$

input
```
int(x^3/atanh(tanh(a + b*x))^3,x)
```

output
```
x/b^3 - (x*(3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 12*a*(2*a - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x) + 12*a^2) - (5*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 8*a^3 - 6
*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b
*x)))/(4*b))/(b^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + x*(8*a*b^4 - 4*b^4*(
2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)) + 4*a^2*b^3 + 4*b^5*x^2 - 4*a*b^3*(2*a - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(3*log(2/(exp(2*a)*exp(2*
b*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*
b*x))/(2*b^4)
```

3.105 $\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^3} \, dx$

3.105.1 Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^3} \, dx = -\frac{x^2}{2b\operatorname{arctanh}(\tanh(a+bx))^2} - \frac{x}{b^2\operatorname{arctanh}(\tanh(a+bx))} + \frac{\log(\operatorname{arctanh}(\tanh(a+bx)))}{b^3}$$

output
```
-1/2*x^2/b/arctanh(tanh(b*x+a))^2-x/b^2/arctanh(tanh(b*x+a))+ln(arctanh(ta
nh(b*x+a)))/b^3
```

3.105.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^3} \, dx = \frac{3 - \frac{b^2x^2}{\operatorname{arctanh}(\tanh(a+bx))^2} - \frac{2bx}{\operatorname{arctanh}(\tanh(a+bx))} + 2\log(\operatorname{arctanh}(\tanh(a+bx)))}{2b^3}$$

input
```
Integrate[x^2/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(3 - (b^2*x^2)/ArcTanh[Tanh[a + b*x]]^2 - (2*b*x)/ArcTanh[Tanh[a + b*x]] +
 2*Log[ArcTanh[Tanh[a + b*x]]])/(2*b^3)
```

3.105.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2599, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\text{arctanh}(\tanh(a+bx))^3}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{\int \frac{x}{\text{arctanh}(\tanh(a+bx))^2}dx}{b}-\frac{x^2}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2599}$$

$$\frac{\frac{\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{b}-\frac{x}{b\text{arctanh}(\tanh(a+bx))}}{b}-\frac{x^2}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2588}$$

$$\frac{\frac{\int \frac{1}{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+bx))}{b^2}-\frac{x}{b\text{arctanh}(\tanh(a+bx))}}{b}-\frac{x^2}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{\log(\text{arctanh}(\tanh(a+bx)))}{b^2}-\frac{x}{b\text{arctanh}(\tanh(a+bx))}}{b}-\frac{x^2}{2b\text{arctanh}(\tanh(a+bx))^2}$$

input
```
Int[x^2/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*x^2/(b*ArcTanh[Tanh[a + b*x]]^2) + (-(x/(b*ArcTanh[Tanh[a + b*x]])) +
 Log[ArcTanh[Tanh[a + b*x]]]/b^2)/b
```

3.105.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.105.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

method	result
parallelrisch	$\frac{-b^2x^2+2\ln(\operatorname{arctanh}(\tanh(bx+a)))\operatorname{arctanh}(\tanh(bx+a))^2-2bx\operatorname{arctanh}(\tanh(bx+a))}{2b^3\operatorname{arctanh}(\tanh(bx+a))^2}$
default	$-\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)^2}{2b^3\operatorname{arctanh}(\tanh(bx+a))^2}-\frac{-2\operatorname{arctanh}(\tanh(bx+a))+2bx}{b^3\operatorname{arctanh}(\tanh(bx+a))}+\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))}{b^3}$
risch	$-\frac{4i\left(\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(ie^{2bx+2a}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)x-\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2x+\pi\operatorname{csgn}\left(ie^{bx+a}\right)^2 c}{b^2\left(\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(ie^{2bx+2a}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)-\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2+\pi\operatorname{csgn}\left(ie^{bx}\right.\right.}$

input
```
int(x^2/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b^2*x^2+2*ln(arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^2-2*b*x*arct
anh(tanh(b*x+a)))/b^3/arctanh(tanh(b*x+a))^2
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{4\,abx + 3\,a^2 + 2\,(b^2x^2 + 2\,abx + a^2)\log(bx+a)}{2\,(b^5x^2 + 2\,ab^4x + a^2b^3)}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/2*(4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3)
```

3.105.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(42) = 84.

Time = 28.82 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.38

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$$
$$= \begin{cases} \frac{x^3}{3\operatorname{atanh}^3(\tanh(a))} & \text{for } b=0 \\ \frac{x^3}{3\operatorname{atanh}^3\left(\tanh\left(bx+\log\left(-e^{-bx}\right)\right)\right)} & \text{for } a=\log\left(-e^{-bx}\right) \\ \frac{x^3}{3\operatorname{atanh}^3\left(\tanh\left(bx+\log\left(e^{-bx}\right)\right)\right)} & \text{for } a=\log\left(e^{-bx}\right) \\ -\frac{x^2}{2b\operatorname{atanh}^2(\tanh(a+bx))} - \frac{x}{b^2\operatorname{atanh}(\tanh(a+bx))} + \frac{\log(\operatorname{atanh}(\tanh(a+bx)))}{b^3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/atanh(tanh(b*x+a))**3,x)
```

output
```
Piecewise((x**3/(3*atanh(tanh(a))**3), Eq(b, 0)), (x**3/(3*atanh(tanh(b*x
+ log(-exp(-b*x))))**3), Eq(a, log(-exp(-b*x)))), (x**3/(3*atanh(tanh(b*x
+ log(exp(-b*x))))**3), Eq(a, log(exp(-b*x)))), (-x**2/(2*b*atanh(tanh(a +
 b*x))**2) - x/(b**2*atanh(tanh(a + b*x))) + log(atanh(tanh(a + b*x)))/b**
3, True))
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{4\,abx+3\,a^2}{2\,(b^5x^2+2\,ab^4x+a^2b^3)} + \frac{\log(bx+a)}{b^3}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
1/2*(4*a*b*x + 3*a^2)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) + log(b*x + a)/b^3
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{\log(|bx+a|)}{b^3} + \frac{4\,ax+\frac{3\,a^2}{b}}{2\,(bx+a)^2b^2}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
log(abs(b*x + a))/b^3 + 1/2*(4*a*x + 3*a^2/b)/((b*x + a)^2*b^2)
```

3.105.9 Mupad [B] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{\ln(\operatorname{atanh}(\tanh(a+b\,x)))}{b^3} - \frac{\frac{b^2\,x^2}{2}+b\,x\operatorname{atanh}(\tanh(a+b\,x))}{b^3\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}$$

input
```
int(x^2/atanh(tanh(a + b*x))^3,x)
```

output
```
log(atanh(tanh(a + b*x)))/b^3 - ((b^2*x^2)/2 + b*x*atanh(tanh(a + b*x)))/(
b^3*atanh(tanh(a + b*x))^2)
```

3.106 $\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^3} dx$

3.106.1 Optimal result

Integrand size = 11, antiderivative size = 34

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^3} dx = -\frac{x}{2b\operatorname{arctanh}(\tanh(a+bx))^2} - \frac{1}{2b^2\operatorname{arctanh}(\tanh(a+bx))}$$

output
```
-1/2*x/b/arctanh(tanh(b*x+a))^2-1/2/b^2/arctanh(tanh(b*x+a))
```

3.106.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^3} dx = -\frac{bx+\operatorname{arctanh}(\tanh(a+bx))}{2b^2\operatorname{arctanh}(\tanh(a+bx))^2}$$

input
```
Integrate[x/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*(b*x + ArcTanh[Tanh[a + b*x]])/(b^2*ArcTanh[Tanh[a + b*x]]^2)
```

3.106.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\text{arctanh}(\tanh(a+bx))^3}\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{\int \frac{1}{\text{arctanh}(\tanh(a+bx))^2}dx}{2b}-\frac{x}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \frac{1}{\text{arctanh}(\tanh(a+bx))^2}d\text{arctanh}(\tanh(a+bx))}{2b^2}-\frac{x}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{2b^2\text{arctanh}(\tanh(a+bx))}-\frac{x}{2b\text{arctanh}(\tanh(a+bx))^2}$$

input
```
Int[x/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*x/(b*ArcTanh[Tanh[a + b*x]]^2) - 1/(2*b^2*ArcTanh[Tanh[a + b*x]])
```

3.106.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.106.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

method	result
parallelrisch	$-\frac{bx+\operatorname{arctanh}(\tanh(bx+a))}{2b^2 \operatorname{arctanh}(\tanh(bx+a))^2}$
default	$-\frac{bx-\operatorname{arctanh}(\tanh(bx+a))}{2b^2 \operatorname{arctanh}(\tanh(bx+a))^2}-\frac{1}{b^2 \operatorname{arctanh}(\tanh(bx+a))}$
risch	$-\frac{2i\left(\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(i e^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)-\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2+\pi \operatorname{csgn}\left(i e^{bx+a}\right)^2 \operatorname{csg}}{b^2\left(\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(i e^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)-\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2+\pi \operatorname{csgn}\left(i e^{bx+a}\right)^2 \operatorname{c}}$

input
```
int(x/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x+arctanh(tanh(b*x+a)))/b^2/arctanh(tanh(b*x+a))^2
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{2\,bx+a}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

input
```
integrate(x/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

3.106.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(31) = 62.

Time = 29.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.94

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \begin{cases} \frac{x^2}{2\operatorname{atanh}^3(\tanh(a))} & \text{for } b=0 \\ \frac{x^2}{2\operatorname{atanh}^3(\tanh(bx+\log(-e^{-bx})))} & \text{for } a=\log(-e^{-bx}) \\ \frac{x^2}{2\operatorname{atanh}^3(\tanh(bx+\log(e^{-bx})))} & \text{for } a=\log(e^{-bx}) \\ -\frac{x}{2b\operatorname{atanh}^2(\tanh(a+bx))} - \frac{1}{2b^2\operatorname{atanh}(\tanh(a+bx))} & \text{otherwise} \end{cases}$$

input
```
integrate(x/atanh(tanh(b*x+a))**3,x)
```

output
```
Piecewise((x**2/(2*atanh(tanh(a))**3), Eq(b, 0)), (x**2/(2*atanh(tanh(b*x
+ log(-exp(-b*x))))**3), Eq(a, log(-exp(-b*x)))), (x**2/(2*atanh(tanh(b*x
+ log(exp(-b*x))))**3), Eq(a, log(exp(-b*x)))), (-x/(2*b*atanh(tanh(a + b*
x))**2) - 1/(2*b**2*atanh(tanh(a + b*x))), True))
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{2\,bx+a}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

input
```
integrate(x/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.53

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{2\,bx+a}{2\,(bx+a)^2b^2}$$

input
```
integrate(x/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
-1/2*(2*b*x + a)/((b*x + a)^2*b^2)
```

3.106.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{\operatorname{atanh}(\tanh(a+b\,x))+b\,x}{2\,b^2\,\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}$$

input
```
int(x/atanh(tanh(a + b*x))^3,x)
```

output
```
-(atanh(tanh(a + b*x)) + b*x)/(2*b^2*atanh(tanh(a + b*x))^2)
```

3.107 $\int \frac{1}{\mathbf{arctanh}(\tanh(a+bx))^3}\,dx$

3.107.1 Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{1}{2b\operatorname{arctanh}(\tanh(a+bx))^2}$$

output
```
-1/2/b/arctanh(tanh(b*x+a))^2
```

3.107.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{1}{2b\operatorname{arctanh}(\tanh(a+bx))^2}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(-3),x]
```

output
```
-1/2*1/(b*ArcTanh[Tanh[a + b*x]]^2)
```

3.107.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arctanh}(\tanh(a+bx))^3}dx$$

$$\Big\downarrow \ 2588$$

$$\frac{\int \frac{1}{\text{arctanh}(\tanh(a+bx))^3}d\text{arctanh}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \ 15$$

$$-\frac{1}{2b\text{arctanh}(\tanh(a+bx))^2}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(-3),x]
```

output
```
-1/2*1/(b*ArcTanh[Tanh[a + b*x]]^2)
```

3.107.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.107.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{1}{2b\,\mathrm{arctanh}(\tanh(bx+a))^2}$
default	$-\frac{1}{2b\,\mathrm{arctanh}(\tanh(bx+a))^2}$
parallelrisch	$-\frac{1}{2b\,\mathrm{arctanh}(\tanh(bx+a))^2}$
risch	$\frac{}{b\left(\pi\ \mathrm{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\mathrm{csgn}\left(ie^{2bx+2a}\right)\mathrm{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)-\pi\ \mathrm{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\mathrm{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2+\pi\ \mathrm{csgn}\left(ie^{bx+a}\right)^2}$

input
```
int(1/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/arctanh(tanh(b*x+a))^2
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{1}{\mathrm{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{1}{2\left(b^3x^2+2\,ab^2x+a^2b\right)}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
-1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

3.107.6 Sympy [A] (verification not implemented)

Time = 28.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{1}{\mathrm{arctanh}(\tanh(a+bx))^3}\,dx = \begin{cases} -\frac{1}{2b\,\mathrm{atanh}^2\left(\tanh\left(a+bx\right)\right)} & \text{for } b\neq 0 \\ \frac{x}{\mathrm{atanh}^3\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/atanh(tanh(b*x+a))**3,x)
```

output
```
Piecewise((-1/(2*b*atanh(tanh(a + b*x))**2), Ne(b, 0)), (x/atanh(tanh(a))*
*3, True))
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{1}{2\,(bx+a)^2b}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-1/2/((b*x + a)^2*b)
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{1}{2\,(bx+a)^2b}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
-1/2/((b*x + a)^2*b)
```

3.107.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{1}{2\,b\,\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}$$

input
```
int(1/atanh(tanh(a + b*x))^3,x)
```

output
```
-1/(2*b*atanh(tanh(a + b*x))^2)
```

3.108 $\int \frac{1}{x\mathbf{arctanh}(\tanh(a+bx))^3}\,dx$

3.108.1 Optimal result

Integrand size = 13, antiderivative size = 97

$$\begin{aligned}\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^3}\,dx = &-\frac{1}{2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}\\&+\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))^2\text{arctanh}(\tanh(a+bx))}\\&-\frac{\log(x)}{(bx-\text{arctanh}(\tanh(a+bx)))^3}\\&+\frac{\log(\text{arctanh}(\tanh(a+bx)))}{(bx-\text{arctanh}(\tanh(a+bx)))^3}\end{aligned}$$

output
```
-1/2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^2+1/(b*x-arctanh(tanh
(b*x+a)))^2/arctanh(tanh(b*x+a))-ln(x)/(b*x-arctanh(tanh(b*x+a)))^3+ln(arc
tanh(tanh(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^3
```

3.108.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^3}\,dx \\ = \frac{b^2x^2-4bx\text{arctanh}(\tanh(a+bx))+\text{arctanh}(\tanh(a+bx))^2(3+2\log(bx)-2\log(\text{arctanh}(\tanh(a+bx))}{2\text{arctanh}(\tanh(a+bx))^2(-bx+\text{arctanh}(\tanh(a+bx)))^3}$$

input
```
Integrate[1/(x*ArcTanh[Tanh[a + b*x]]^3),x]
```

output
```
(b^2*x^2 - 4*b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2*(3 + 2*
Log[b*x] - 2*Log[ArcTanh[Tanh[a + b*x]]]))/(2*ArcTanh[Tanh[a + b*x]]^2*(-(
b*x) + ArcTanh[Tanh[a + b*x]])^3)
```

3.108.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2594, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^3}dx$$

$$\Big\downarrow \text{2594}$$

$$-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^2}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2591}$$

$$-\frac{-\frac{\frac{b\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{14}$$

$$-\frac{-\frac{\frac{b\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2588}$$

$$-\frac{-\frac{\frac{\int \frac{1}{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+bx))}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{14}$$

$$-\frac{1}{2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}-\frac{-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}-\frac{\frac{\log(\text{arctanh}(\tanh(a+bx)))}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}$$

input
```
Int[1/(x*ArcTanh[Tanh[a + b*x]]^3),x]
```

output
```
-1/2*1/((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) - (-(1/((
b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]])) - (-(Log[x]/(b*x -
ArcTanh[Tanh[a + b*x]])) + Log[ArcTanh[Tanh[a + b*x]]]/(b*x - ArcTanh[Tanh
[a + b*x]]))/(b*x - ArcTanh[Tanh[a + b*x]]))/(b*x - ArcTanh[Tanh[a + b*x]]
)
```

3.108.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

3.108.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

$$-\frac{\ln\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}}+\frac{1}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}+\frac{}{2\left(\operatorname{arctanh}\left(\mathrm{ta}\right.\right.}$$

input
```
int(1/x/arctanh(tanh(b*x+a))^3,x)
```

output
```
-1/(arctanh(tanh(b*x+a))-b*x)^3*ln(arctanh(tanh(b*x+a)))+1/(arctanh(tanh(b
*x+a))-b*x)^2/arctanh(tanh(b*x+a))+1/2/(arctanh(tanh(b*x+a))-b*x)/arctanh(
tanh(b*x+a))^2+1/(arctanh(tanh(b*x+a))-b*x)^3*ln(x)
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\operatorname{arctanh}(\tanh(a+bx))^{3}}\,dx = \frac{2\,abx+3\,a^{2}-2\,(b^{2}x^{2}+2\,abx+a^{2})\log\left(bx+a\right)+2\,(b^{2}x^{2}+2\,abx+a^{2})\log\left(x\right)}{2\,(a^{3}b^{2}x^{2}+2\,a^{4}bx+a^{5})}$$

input
```
integrate(1/x/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*x + 3*a^2 - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) + 2*(b^2*x
^2 + 2*a*b*x + a^2)*log(x))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)
```

3.108.6 Sympy [F]

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^3}\,dx = \int \frac{1}{x\,\text{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x/atanh(tanh(b*x+a))**3,x)
```

output
```
Integral(1/(x*atanh(tanh(a + b*x))**3), x)
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.53

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^3}\,dx = \frac{2\,bx+3\,a}{2\,(a^2b^2x^2+2\,a^3bx+a^4)} - \frac{\log\left(bx+a\right)}{a^3} + \frac{\log\left(x\right)}{a^3}$$

input
```
integrate(1/x/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
1/2*(2*b*x + 3*a)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) - log(b*x + a)/a^3 + log
(x)/a^3
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.44

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{\log\left(|bx+a|\right)}{a^3} + \frac{\log\left(|x|\right)}{a^3} + \frac{2\,abx+3\,a^2}{2\,(bx+a)^2a^3}$$

input
```
integrate(1/x/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/a^3 + log(abs(x))/a^3 + 1/2*(2*a*b*x + 3*a^2)/((b*x + a
)^2*a^3)
```

3.108.9 Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 645, normalized size of antiderivative = 6.65

$$\int \frac{1}{x\mathrm{arctanh}(\tanh(a+bx))^3}\,dx =$$

$$-\frac{12\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)^2 - 24\,\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)\,\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right) + 12\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)^2 + 16\,b^2\,x^2 + b\,x\,\left(32\,\ln\left(\frac{}{e^{2a}\cdots}\right.\right.}{}$$

input
```
int(1/(x*atanh(tanh(a + b*x))^3),x)
```

output
```
-(12*log(1/(exp(2*a)*exp(2*b*x) + 1))^2 - 24*log(1/(exp(2*a)*exp(2*b*x) +
1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2*atan((log((exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i
)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x))*16i - log(1/(exp(2*a)*exp(2*b*x) + 1))^2*atan((
log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*
exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*16i + 12*log((exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 + 16*b^2*x^2 + log(1/(exp(2*a
)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*at
an((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2
*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log(
(exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*32i + b*x*(32*l
og(1/(exp(2*a)*exp(2*b*x) + 1)) - 32*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))))/((log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)))^2*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - l
og((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)
```

3.109 $\int \frac{1}{x^2 \mathbf{arctanh}(\tanh(a+bx))^3} \, dx$

3.109.1 Optimal result

Integrand size = 13, antiderivative size = 131

$$\int \frac{1}{x^2 \text{arctanh}(\tanh(a+bx))^3} \, dx = -\frac{3b}{2(bx-\text{arctanh}(\tanh(a+bx)))^2\text{arctanh}(\tanh(a+bx))^2} + \frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2} + \frac{3b}{(bx-\text{arctanh}(\tanh(a+bx)))^3\text{arctanh}(\tanh(a+bx))} - \frac{3b\log(x)}{(bx-\text{arctanh}(\tanh(a+bx)))^4} + \frac{3b\log(\text{arctanh}(\tanh(a+bx)))}{(bx-\text{arctanh}(\tanh(a+bx)))^4}$$

output
```
-3/2*b/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^2+1/x/(b*x-arctan
h(tanh(b*x+a)))/arctanh(tanh(b*x+a))^2+3*b/(b*x-arctanh(tanh(b*x+a)))^3/ar
ctanh(tanh(b*x+a))-3*b*ln(x)/(b*x-arctanh(tanh(b*x+a)))^4+3*b*ln(arctanh(t
anh(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^4
```

3.109.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{b^3x^3-6b^2x^2\text{arctanh}(\tanh(a+bx))+2\text{arctanh}(\tanh(a+bx))^3+3bx\text{arctanh}(\tanh(a+bx))^2(1+2\log}{2x\text{arctanh}(\tanh(a+bx))^2(-bx+\text{arctanh}(\tanh(a+bx)))^4}$$

input
```
Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]^3),x]
```

output
```
-1/2*(b^3*x^3 - 6*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 2*ArcTanh[Tanh[a + b*x]
]^3 + 3*b*x*ArcTanh[Tanh[a + b*x]]^2*(1 + 2*Log[x] - 2*Log[ArcTanh[Tanh[a
+ b*x]]]))/(x*ArcTanh[Tanh[a + b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4
)
```

3.109.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.38, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2602, 2594, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^3}dx$$

$$\Big\downarrow \text{ 2602}$$

$$\frac{3b\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^3}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{ 2594}$$

$$\frac{3b\left(-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^2}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{ 2594}$$

$$3b\left(-\frac{-\frac{\int \frac{1}{x\,\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+\ldots}\right.$$

$$\overline{\qquad bx-\text{arctanh}(\tanh(a+bx))\qquad}$$

$$\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 2591$$

$$3b\left(-\frac{-\frac{\frac{b\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\ldots}{2(bx-\text{arctanh}(\tanh(\ldots}\right.$$

$$\overline{\qquad bx-\text{arctanh}(\tanh(a+bx))\qquad}$$

$$\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 14$$

$$3b\left(-\frac{-\frac{\frac{b\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\ldots}{2(bx-\text{arctanh}(\tanh(\ldots}\right.$$

$$\overline{\qquad bx-\text{arctanh}(\tanh(a+bx))\qquad}$$

$$\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 2588$$

$$3b\left(-\frac{-\frac{\frac{\int \frac{1}{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+bx))}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\ldots}{2(\ldots}\right.$$

$$\overline{\qquad bx-\text{arctanh}(\tanh(a+bx))\qquad}$$

$$\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 14$$

3.109. $\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^3}\,dx$

$$\frac{1}{x(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))^2}+$$

$$\frac{3b\left(-\frac{1}{2(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))^2}-\frac{-\frac{1}{(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))}-\frac{\frac{\log(\operatorname{arctanh}(\tanh(a+b}{bx-\operatorname{arctanh}(\tanh(a+b}}{bx-\operatorname{arct}}}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right.}{bx-\operatorname{arctanh}(\tanh(a+bx))}$$

input
```
Int[1/(x^2*ArcTanh[Tanh[a + b*x]]^3),x]
```

output
```
1/(x*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) + (3*b*(-1/2
*1/((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) - (-(1/((b*x
- ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]])) - (-(Log[x]/(b*x - ArcT
anh[Tanh[a + b*x]])) + Log[ArcTanh[Tanh[a + b*x]]]/(b*x - ArcTanh[Tanh[a +
 b*x]]))/(b*x - ArcTanh[Tanh[a + b*x]]))/(b*x - ArcTanh[Tanh[a + b*x]])))/
(b*x - ArcTanh[Tanh[a + b*x]])
```

3.109.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.109.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

$$-\frac{b}{2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{2}}+\frac{3b\ln\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{4}}-\frac{}{(\operatorname{arctanh}(t}$$

input
```
int(1/x^2/arctanh(tanh(b*x+a))^3,x)
```

output
```
-1/2/(arctanh(tanh(b*x+a))-b*x)^2/arctanh(tanh(b*x+a))^2*b+3/(arctanh(tanh
(b*x+a))-b*x)^4*b*ln(arctanh(tanh(b*x+a)))-2/(arctanh(tanh(b*x+a))-b*x)^3*
b/arctanh(tanh(b*x+a))-1/(arctanh(tanh(b*x+a))-b*x)^3/x-3/(arctanh(tanh(b*
x+a))-b*x)^4*b*ln(x)
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{6\,ab^2x^2+9\,a^2bx+2\,a^3-6\,(b^3x^3+2\,ab^2x^2+a^2bx)\log\left(bx+a\right)+6\,(b^3x^3+2\,ab^2x^2+a^2bx)\log\left(x\right)}{2\,(a^4b^2x^3+2\,a^5bx^2+a^6x)}$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
-1/2*(6*a*b^2*x^2 + 9*a^2*b*x + 2*a^3 - 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x
)*log(b*x + a) + 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*log(x))/(a^4*b^2*x^3
+ 2*a^5*b*x^2 + a^6*x)
```

3.109.6 Sympy [F]

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^3}\,dx = \int \frac{1}{x^2\,\text{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**2/atanh(tanh(b*x+a))**3,x)
```

output
```
Integral(1/(x**2*atanh(tanh(a + b*x))**3), x)
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{6\,b^2x^2+9\,abx+2\,a^2}{2\,(a^3b^2x^3+2\,a^4bx^2+a^5x)}+\frac{3\,b\log\left(bx+a\right)}{a^4}-\frac{3\,b\log\left(x\right)}{a^4}$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-1/2*(6*b^2*x^2 + 9*a*b*x + 2*a^2)/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x) + 3
*b*log(b*x + a)/a^4 - 3*b*log(x)/a^4
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.46

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^3}\,dx = \frac{3\,b\log\left(|bx+a|\right)}{a^4}-\frac{3\,b\log\left(|x|\right)}{a^4}-\frac{6\,ab^2x^2+9\,a^2bx+2\,a^3}{2\,(bx+a)^2a^4x}$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
3*b*log(abs(b*x + a))/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(6*a*b^2*x^2 + 9*a^2
*b*x + 2*a^3)/((b*x + a)^2*a^4*x)
```

3.109.9 Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 804, normalized size of antiderivative = 6.14

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*atanh(tanh(a + b*x))^3),x)
```

output
```
-(24*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) - 24*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 - 8*log(1/(exp(2*a)*exp(2*b*x) + 1))
^3 + 8*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3 + 32*b^3*x^3
 + 24*b*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 + 96*b^2*
x^2*log(1/(exp(2*a)*exp(2*b*x) + 1)) - 96*b^2*x^2*log((exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 24*b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))^2 -
 b*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2*atan((log((exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*
x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*96i - b*x*log(1/(exp(2*a)*e
xp(2*b*x) + 1))^2*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp
(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b
*x))*96i - 48*b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*atan((log((exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i
+ b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*192i)/(x*(log(1/(exp(2*a)*exp(2*b*x)...
```

3.110 $\int \frac{1}{x^3\textbf{arctanh}(\tanh(a+bx))^3}\,dx$

3.110.1 Optimal result

Integrand size = 13, antiderivative size = 170

$$\begin{aligned}\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^3}\,dx &= -\frac{3b^2}{(bx-\text{arctanh}(\tanh(a+bx)))^3\text{arctanh}(\tanh(a+bx))^2} \\ &\quad+\frac{2b}{x(bx-\text{arctanh}(\tanh(a+bx)))^2\text{arctanh}(\tanh(a+bx))^2} \\ &\quad+\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2} \\ &\quad+\frac{6b^2}{(bx-\text{arctanh}(\tanh(a+bx)))^4\text{arctanh}(\tanh(a+bx))} \\ &\quad-\frac{6b^2\log(x)}{(bx-\text{arctanh}(\tanh(a+bx)))^5}+\frac{6b^2\log(\text{arctanh}(\tanh(a+bx)))}{(bx-\text{arctanh}(\tanh(a+bx)))^5}\end{aligned}$$

output
```
-3*b^2/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^2+2*b/x/(b*x-arct
anh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^2+1/2/x^2/(b*x-arctanh(tanh(b*x+a
)))/arctanh(tanh(b*x+a))^2+6*b^2/(b*x-arctanh(tanh(b*x+a)))^4/arctanh(tanh
(b*x+a))-6*b^2*ln(x)/(b*x-arctanh(tanh(b*x+a)))^5+6*b^2*ln(arctanh(tanh(b*
x+a)))/(b*x-arctanh(tanh(b*x+a)))^5
```

3.110.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^3}\,dx = \frac{-b^4x^4+8b^3x^3\text{arctanh}(\tanh(a+bx))-8bx\text{arctanh}(\tanh(a+bx))^3+\text{arctanh}(\tanh(a+bx))^4-12b^2x^2\text{a}}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))^5\text{arctanh}(\tanh(a}$$

input
```
Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]^3),x]
```

output
```
(-(b^4*x^4) + 8*b^3*x^3*ArcTanh[Tanh[a + b*x]] - 8*b*x*ArcTanh[Tanh[a + b*
x]]^3 + ArcTanh[Tanh[a + b*x]]^4 - 12*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2*(Lo
g[x] - Log[ArcTanh[Tanh[a + b*x]]]))/(2*x^2*(b*x - ArcTanh[Tanh[a + b*x]])
^5*ArcTanh[Tanh[a + b*x]]^2)
```

3.110.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {2602, 2602, 2594, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^3}\,dx$$

$$\Big\downarrow \; 2602$$

$$\frac{2b\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^3}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 2602$$

$$\frac{2b\left(\frac{3b\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^3}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 2594$$

$$2b\left(\frac{3b\left(-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^2}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\text{ta}}\right.$$
$$\overline{\qquad bx-\text{arctanh}(\tanh(a+bx)) \qquad}$$
$$\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 2594

$$2b\left(\frac{3b\left(-\frac{-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx)}\right.}{bx-\text{arctanh}(\tanh(a+bx))}\right.$$
$$\overline{\qquad bx-\text{arctanh}(\tanh(a+bx)) \qquad}$$
$$\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 2591

$$2b\left(\frac{3b\left(-\frac{-\frac{\frac{b\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{2(bx-\text{arctanh}(\tanh}\right.}{bx-\text{arctanh}(\tanh(a+bx))}\right.$$
$$\overline{\qquad bx-\text{arctanh}(\tanh(a+bx)) \qquad}$$
$$\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 14

3.110. $\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^3}\,dx$

$$2b\left(\frac{3b\left(-\frac{-\frac{\frac{b\int \frac{1}{\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{2(bx-\text{arctanh}(\tanh\cdots}\right.}{bx-\text{arctanh}(\tanh(a+bx))}\right.$$

$$\overline{\qquad\qquad bx-\text{arctanh}(\tanh(a+bx))}$$

$$\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 2588$$

$$2b\left(\frac{3b\left(-\frac{-\frac{\frac{\int \frac{1}{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+bx))}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\log(x)}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{1}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{2\cdots}\right.}{bx-\text{arctanh}(\tanh(a+bx))}\right.$$

$$\overline{\qquad\qquad bx-\text{arctanh}(\tanh(a+b\cdots}$$

$$\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 14$$

$$\frac{1}{2x^2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}+$$

$$2b\left(\frac{1}{x(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}+\frac{3b\left(-\frac{1}{2(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^2}-\frac{-\cdots}{(bx-\text{arctanh}(\text{ta}\cdots}\right.}{bx-\text{arc}\cdots}\right.$$

$$\overline{\qquad\qquad bx-\text{arctanh}(\tanh(a+bx))}$$

3.110. $\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^3}\,dx$

input
```
Int[1/(x^3*ArcTanh[Tanh[a + b*x]]^3),x]
```

output
```
1/(2*x^2*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) + (2*b*(
1/(x*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) + (3*b*(-1/2
*1/((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) - (-(1/((b*x
- ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]])) - (-(Log[x]/(b*x - ArcT
anh[Tanh[a + b*x]])) + Log[ArcTanh[Tanh[a + b*x]]]/(b*x - ArcTanh[Tanh[a +
 b*x]]))/(b*x - ArcTanh[Tanh[a + b*x]]))/(b*x - ArcTanh[Tanh[a + b*x]])))/
(b*x - ArcTanh[Tanh[a + b*x]])))/(b*x - ArcTanh[Tanh[a + b*x]])
```

3.110.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.110.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

$$-\frac{1}{2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}x^{2}}+\frac{6b^{2}\ln\left(x\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{5}}+\frac{3b}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{4}x}$$

input
```
int(1/x^3/arctanh(tanh(b*x+a))^3,x)
```

output
```
-1/2/(arctanh(tanh(b*x+a))-b*x)^3/x^2+6/(arctanh(tanh(b*x+a))-b*x)^5*b^2*l
n(x)+3/(arctanh(tanh(b*x+a))-b*x)^4*b/x-6/(arctanh(tanh(b*x+a))-b*x)^5*b^2
*ln(arctanh(tanh(b*x+a)))+3/(arctanh(tanh(b*x+a))-b*x)^4*b^2/arctanh(tanh(
b*x+a))+1/2/(arctanh(tanh(b*x+a))-b*x)^3*b^2/arctanh(tanh(b*x+a))^2
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^3}\,dx$$
$$=\frac{12\,ab^{3}x^{3}+18\,a^{2}b^{2}x^{2}+4\,a^{3}bx-a^{4}-12\left(b^{4}x^{4}+2\,ab^{3}x^{3}+a^{2}b^{2}x^{2}\right)\log\left(bx+a\right)+12\left(b^{4}x^{4}+2\,ab^{3}x^{3}+a^{2}b^{2}\right.}{2\left(a^{5}b^{2}x^{4}+2\,a^{6}bx^{3}+a^{7}x^{2}\right)}$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4 - 12*(b^4*x^4 + 2*a*b
^3*x^3 + a^2*b^2*x^2)*log(b*x + a) + 12*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x
^2)*log(x))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)
```

3.110.6 Sympy [F]

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^3}\,dx=\int \frac{1}{x^{3}\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**3/atanh(tanh(b*x+a))**3,x)
```

output
```
Integral(1/(x**3*atanh(tanh(a + b*x))**3), x)
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^3}\,dx = \frac{12\,b^3x^3+18\,ab^2x^2+4\,a^2bx-a^3}{2\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)} - \frac{6\,b^2\log(bx+a)}{a^5} + \frac{6\,b^2\log(x)}{a^5}$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
1/2*(12*b^3*x^3 + 18*a*b^2*x^2 + 4*a^2*b*x - a^3)/(a^4*b^2*x^4 + 2*a^5*b*x
^3 + a^6*x^2) - 6*b^2*log(b*x + a)/a^5 + 6*b^2*log(x)/a^5
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{6\,b^2\log(|bx+a|)}{a^5} + \frac{6\,b^2\log(|x|)}{a^5} + \frac{12\,b^3x^3+18\,ab^2x^2+4\,a^2bx-a^3}{2\,(bx^2+ax)^2a^4}$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
-6*b^2*log(abs(b*x + a))/a^5 + 6*b^2*log(abs(x))/a^5 + 1/2*(12*b^3*x^3 + 1
8*a*b^2*x^2 + 4*a^2*b*x - a^3)/((b*x^2 + a*x)^2*a^4)
```

3.110.9 Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 909, normalized size of antiderivative = 5.35

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^3*atanh(tanh(a + b*x))^3),x)
```

output

```
(4*log(1/(exp(2*a)*exp(2*b*x) + 1))^4 - 16*log(1/(exp(2*a)*exp(2*b*x) + 1)
)^3*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - 16*log(1/(exp(2
*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^
3 + 4*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^4 + 24*log(1/(e
xp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1))^2 - 64*b^4*x^4 - 64*b*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1))^3 - 256*b^3*x^3*log(1/(exp(2*a)*exp(2*b*x) + 1)) + 256*b^3*x^3*lo
g((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 64*b*x*log(1/(exp(2*a
)*exp(2*b*x) + 1))^3 + 192*b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 - 192*b*x*log(1/(exp(2*a)*exp
(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + b^2
*x^2*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*atan((log((exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i
)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x))*384i + b^2*x^2*log((exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1))^2*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a
)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x))*384i - b^2*x^2*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*atan((log((exp(2*a)*exp(2*b*x))/(e...
```

3.110. $\int \frac{1}{x^3 \operatorname{arctanh}(\tanh(a+bx))^3}\,dx$

3.111 $\int x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx$

3.111.1 Optimal result

Integrand size = 15, antiderivative size = 101

$$\begin{aligned}\int x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx &= \frac{2x^4\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3b} \\ &\quad - \frac{16x^3\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{15b^2} \\ &\quad + \frac{32x^2\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{35b^3} \\ &\quad - \frac{128x\operatorname{arctanh}(\tanh(a+bx))^{9/2}}{315b^4} \\ &\quad + \frac{256\operatorname{arctanh}(\tanh(a+bx))^{11/2}}{3465b^5}\end{aligned}$$

output
```
2/3*x^4*arctanh(tanh(b*x+a))^(3/2)/b-16/15*x^3*arctanh(tanh(b*x+a))^(5/2)/
b^2+32/35*x^2*arctanh(tanh(b*x+a))^(7/2)/b^3-128/315*x*arctanh(tanh(b*x+a)
)^(9/2)/b^4+256/3465*arctanh(tanh(b*x+a))^(11/2)/b^5
```

3.111.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int x^4\sqrt{\text{arctanh}(\tanh(a+bx))}\,dx$$
$$=\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}\left(1155b^4x^4-1848b^3x^3\text{arctanh}(\tanh(a+bx))+1584b^2x^2\text{arctanh}(\tanh(a+bx)\right.}{3465b^5}$$

input
```
Integrate[x^4*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(3/2)*(1155*b^4*x^4 - 1848*b^3*x^3*ArcTanh[Tanh[
a + b*x]] + 1584*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 704*b*x*ArcTanh[Tanh[a
 + b*x]]^3 + 128*ArcTanh[Tanh[a + b*x]]^4))/(3465*b^5)
```

3.111.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2599, 2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{\text{arctanh}(\tanh(a+bx))}\,dx$$
$$\Big\downarrow \text{2599}$$
$$\frac{2x^4\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{8\int x^3\text{arctanh}(\tanh(a+bx))^{3/2}dx}{3b}$$
$$\Big\downarrow \text{2599}$$
$$\frac{2x^4\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{8\left(\frac{2x^3\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{6\int x^2\text{arctanh}(\tanh(a+bx))^{5/2}dx}{5b}\right)}{3b}$$
$$\Big\downarrow \text{2599}$$
$$\frac{2x^4\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{8\left(\frac{2x^3\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\int x\text{arctanh}(\tanh(a+bx))^{7/2}dx}{7b}\right)}{5b}\right)}{3b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^4\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{8\left(\frac{2x^3\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{9/2}}{9b}-\frac{2\int\text{arctanh}(\tanh(a+bx))^{9/2}dx}{9b}\right)}{7b}\right)}{5b}\right)}{3b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{2x^4\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{8\left(\frac{2x^3\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{9/2}}{9b}-\frac{2\int\text{arctanh}(\tanh(a+bx))^{9/2}d\text{arctanh}(\tanh(a+bx))}{9b^2}\right)}{7b}\right)}{5b}\right)}{3b}$$

$$\Big\downarrow \text{15}$$

$$\frac{2x^4\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{8\left(\frac{2x^3\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{9/2}}{9b}-\frac{4\text{arctanh}(\tanh(a+bx))^{11/2}}{99b^2}\right)}{7b}\right)}{5b}\right)}{3b}$$

input

```
Int[x^4*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output

```
(2*x^4*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (8*((2*x^3*ArcTanh[Tanh[a + b
*x]]^(5/2))/(5*b) - (6*((2*x^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (4*((
2*x*ArcTanh[Tanh[a + b*x]]^(9/2))/(9*b) - (4*ArcTanh[Tanh[a + b*x]]^(11/2)
)/(99*b^2)))/(7*b)))/(5*b)))/(3*b)
```

3.111. $\int x^4\sqrt{\text{arctanh}(\tanh(a+bx))}\,dx$

3.111.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.111.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.52

method	result
default	$\frac{\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{11}{2}}}{11}+\frac{2(-4 \operatorname{arctanh}(\tanh(bx+a))+4bx) \operatorname{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2\left(2(bx-\operatorname{arctanh}(\tanh(bx+a)))^{2}+(-2 \operatorname{arctanh}(\tanh(bx+a))\right.}{7}}{}$

input
```
int(x^4*arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^5*(1/11*arctanh(tanh(b*x+a))^(11/2)+1/9*(-4*arctanh(tanh(b*x+a))+4*b*x
)*arctanh(tanh(b*x+a))^(9/2)+1/7*(2*(b*x-arctanh(tanh(b*x+a)))^2+(-2*arcta
nh(tanh(b*x+a))+2*b*x)^2)*arctanh(tanh(b*x+a))^(7/2)+2/5*(b*x-arctanh(tanh
(b*x+a)))^2*(-2*arctanh(tanh(b*x+a))+2*b*x)*arctanh(tanh(b*x+a))^(5/2)+1/3
*(b*x-arctanh(tanh(b*x+a)))^4*arctanh(tanh(b*x+a))^(3/2))
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\left(315\,b^5x^5+35\,ab^4x^4-40\,a^2b^3x^3+48\,a^3b^2x^2-64\,a^4bx+128\,a^5\right)\sqrt{bx+a}}{3465\,b^5}$$

input
```
integrate(x^4*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/3465*(315*b^5*x^5 + 35*a*b^4*x^4 - 40*a^2*b^3*x^3 + 48*a^3*b^2*x^2 - 64*
a^4*b*x + 128*a^5)*sqrt(b*x + a)/b^5
```

3.111.6 Sympy [F]

$$\int x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int x^4\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**4*atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(x**4*sqrt(atanh(tanh(a + b*x))), x)
```

3.111.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\left(315\,b^5x^5+35\,ab^4x^4-40\,a^2b^3x^3+48\,a^3b^2x^2-64\,a^4bx+128\,a^5\right)\sqrt{bx+a}}{3465\,b^5}$$

input
```
integrate(x^4*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
2/3465*(315*b^5*x^5 + 35*a*b^4*x^4 - 40*a^2*b^3*x^3 + 48*a^3*b^2*x^2 - 64*
a^4*b*x + 128*a^5)*sqrt(b*x + a)/b^5
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.49

$$\int x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx$$
$$=\frac{\sqrt{2}\left(\frac{11\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}a^4\right)a}{b^4}+\frac{5\sqrt{2}\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+\right.}{}\right.}{3465\,b}$$

input
```
integrate(x^4*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
1/3465*sqrt(2)*(11*sqrt(2)*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 3
78*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*
a/b^4 + 5*sqrt(2)*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x
+ a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693
*sqrt(b*x + a)*a^5)/b^4)/b
```

3.111.9 Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 811, normalized size of antiderivative = 8.03

$$\int x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \text{Too large to display}$$

input
```
int(x^4*atanh(tanh(a + b*x))^(1/2),x)
```

output
```
(2*x^5*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(
exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/11 - (x^4*(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1))/11 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1))/11 + (2*b*x)/11))/(9*b) - (128*(log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(
1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))/2 + b*x)^4*(log(2/(exp(2*a)*exp(2*b*x) + 1))/11 -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/11 + (2*b*x)/11))/(
315*b^5) - (8*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/
2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x)
+ 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)*
(log(2/(exp(2*a)*exp(2*b*x) + 1))/11 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1))/11 + (2*b*x)/11))/(63*b^2) - (64*x*(log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/
2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^3*(log(2/(exp(2*a)*exp(2*b*x) + 1))/1
1 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/11 + (2*b*x)/11
))/(315*b^4) - (16*x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp...
```

3.111. $\int x^4 \sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx$

3.112 $\int x^3 \sqrt{\textbf{arctanh}(\tanh(a+bx))}\,dx$

3.112.1 Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2x^3\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3b} - \frac{4x^2\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{5b^2} + \frac{16x\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{35b^3} - \frac{32\operatorname{arctanh}(\tanh(a+bx))^{9/2}}{315b^4}$$

output
```
2/3*x^3*arctanh(tanh(b*x+a))^(3/2)/b-4/5*x^2*arctanh(tanh(b*x+a))^(5/2)/b^
2+16/35*x*arctanh(tanh(b*x+a))^(7/2)/b^3-32/315*arctanh(tanh(b*x+a))^(9/2)
/b^4
```

3.112.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\operatorname{arctanh}(\tanh(a+bx))^{3/2}\left(105b^3x^3 - 126b^2x^2\operatorname{arctanh}(\tanh(a+bx)) + 72bx\operatorname{arctanh}(\tanh(a+bx))^2 - 1}{315b^4}$$

input
```
Integrate[x^3*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(3/2)*(105*b^3*x^3 - 126*b^2*x^2*ArcTanh[Tanh[a
+ b*x]] + 72*b*x*ArcTanh[Tanh[a + b*x]]^2 - 16*ArcTanh[Tanh[a + b*x]]^3))/
(315*b^4)
```

3.112.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt{\text{arctanh}(\tanh(a+bx))} \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^3 \text{arctanh}(\tanh(a+bx))^{3/2}}{3b} - \frac{2\int x^2 \text{arctanh}(\tanh(a+bx))^{3/2}dx}{b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^3 \text{arctanh}(\tanh(a+bx))^{3/2}}{3b} - \frac{2\left(\frac{2x^2 \text{arctanh}(\tanh(a+bx))^{5/2}}{5b} - \frac{4\int x \text{arctanh}(\tanh(a+bx))^{5/2}dx}{5b}\right)}{b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^3 \text{arctanh}(\tanh(a+bx))^{3/2}}{3b} - \frac{2\left(\frac{2x^2 \text{arctanh}(\tanh(a+bx))^{5/2}}{5b} - \frac{4\left(\frac{2x \text{arctanh}(\tanh(a+bx))^{7/2}}{7b} - \frac{2\int \text{arctanh}(\tanh(a+bx))^{7/2}dx}{7b}\right)}{5b}\right)}{b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{2x^3 \text{arctanh}(\tanh(a+bx))^{3/2}}{3b} - \frac{2\left(\frac{2x^2 \text{arctanh}(\tanh(a+bx))^{5/2}}{5b} - \frac{4\left(\frac{2x \text{arctanh}(\tanh(a+bx))^{7/2}}{7b} - \frac{2\int \text{arctanh}(\tanh(a+bx))^{7/2}d\text{arctanh}(\tanh(a+bx))}{7b^2}\right)}{5b}\right)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{2x^3\text{arctanh}(\tanh(a+bx))^{3/2}}{3b} - \frac{2\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{5/2}}{5b} - \frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{7/2}}{7b} - \frac{4\text{arctanh}(\tanh(a+bx))^{9/2}}{63b^2}\right)}{5b}\right)}{b}$$

input
```
Int[x^3*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*x^3*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (2*((2*x^2*ArcTanh[Tanh[a + b
*x]]^(5/2))/(5*b) - (4*((2*x*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (4*ArcT
anh[Tanh[a + b*x]]^(9/2))/(63*b^2)))/(5*b)))/b
```

3.112.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.112.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.55

method	result
default	$\frac{\frac{2\,\text{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2(-3\,\text{arctanh}(\tanh(bx+a))+3bx)\,\text{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}+\frac{2\left((bx-\text{arctanh}(\tanh(bx+a)))(-2\,\text{arctanh}(\tanh(bx+a))+2bx\right.}{5}}{b^4}$

input
```
int(x^3*arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^4*(1/9*arctanh(tanh(b*x+a))^(9/2)+1/7*(-3*arctanh(tanh(b*x+a))+3*b*x)*
arctanh(tanh(b*x+a))^(7/2)+1/5*((b*x-arctanh(tanh(b*x+a)))*(-2*arctanh(tan
h(b*x+a))+2*b*x)+(b*x-arctanh(tanh(b*x+a)))^2)*arctanh(tanh(b*x+a))^(5/2)+
1/3*(b*x-arctanh(tanh(b*x+a)))^3*arctanh(tanh(b*x+a))^(3/2))
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

$$\int x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\left(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4\right)\sqrt{bx+a}}{315\,b^4}$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt
(b*x + a)/b^4
```

3.112.6 Sympy [F]

$$\int x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int x^3\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**3*atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(x**3*sqrt(atanh(tanh(a + b*x))), x)
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

$$\int x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4)\sqrt{bx+a}}{315\,b^4}$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt
(b*x + a)/b^4
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.56

$$\int x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx$$
$$= \frac{\sqrt{2}\left(\frac{9\,\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a}{b^3}+\frac{\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+\right.}{b^3}\right.}{315\,b}$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
1/315*sqrt(2)*(9*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b
*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a/b^3 + sqrt(2)*(35*(b*x + a)^(9
/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2
)*a^3 + 315*sqrt(b*x + a)*a^4)/b^3)/b
```

3.112.9 Mupad [B] (verification not implemented)

Time = 3.53 (sec) , antiderivative size = 648, normalized size of antiderivative = 8.10

$$\int x^3 \sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,x^4\,\sqrt{\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}}}{9}$$

$$-\frac{x^3\,\sqrt{\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{9}-\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{9}+\frac{2\,b\,x}{9}\right)}{7\,b}$$

$$-\frac{16\,\sqrt{\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}}\,{\left(\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}+b\,x\right)}^3\,\left(\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{9}-\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{9}\right.}{35\,b^4}$$

$$-\frac{6\,x^2\,\sqrt{\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}+b\,x\right)\,\left(\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{9}-\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{9}\right.}{35\,b^2}$$

$$-\frac{8\,x\,\sqrt{\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}}\,{\left(\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}+b\,x\right)}^2\,\left(\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{9}-\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{9}\right.}{35\,b^3}$$

input
```
int(x^3*atanh(tanh(a + b*x))^(1/2),x)
```

output
```
(2*x^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(
exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/9 - (x^3*(log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1))/9 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1))/9 + (2*b*x)/9))/(7*b) - (16*(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))/2 + b*x)^3*(log(2/(exp(2*a)*exp(2*b*x) + 1))/9 - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/9 + (2*b*x)/9))/(35*b^4)
- (6*x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2
/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)*(log(2/(e
xp(2*a)*exp(2*b*x) + 1))/9 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1))/9 + (2*b*x)/9))/(35*b^2) - (8*x*(log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log
(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1))/2 + b*x)^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/9 - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/9 + (2*b*x)/9))/(35*b^3)
```

3.113 $\int x^2\sqrt{\textbf{arctanh}(\tanh(a+bx))}\,dx$

3.113.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2x^2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3b} - \frac{8x\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{16\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{105b^3}$$

output
```
2/3*x^2*arctanh(tanh(b*x+a))^(3/2)/b-8/15*x*arctanh(tanh(b*x+a))^(5/2)/b^2
+16/105*arctanh(tanh(b*x+a))^(7/2)/b^3
```

3.113.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\operatorname{arctanh}(\tanh(a+bx))^{3/2}\left(35b^2x^2-28bx\operatorname{arctanh}(\tanh(a+bx))+8\operatorname{arctanh}(\tanh(a+bx))^2\right)}{105b^3}$$

input
```
Integrate[x^2*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(3/2)*(35*b^2*x^2 - 28*b*x*ArcTanh[Tanh[a + b*x]
] + 8*ArcTanh[Tanh[a + b*x]]^2))/(105*b^3)
```

3.113.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{\text{arctanh}(\tanh(a+bx))}\,dx$$

$$\Big\downarrow \; 2599$$

$$\frac{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\int x\text{arctanh}(\tanh(a+bx))^{3/2}dx}{3b}$$

$$\Big\downarrow \; 2599$$

$$\frac{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{5/2}dx}{5b}\right)}{3b}$$

$$\Big\downarrow \; 2588$$

$$\frac{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{5/2}d\text{arctanh}(\tanh(a+bx))}{5b^2}\right)}{3b}$$

$$\Big\downarrow \; 15$$

$$\frac{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\text{arctanh}(\tanh(a+bx))^{7/2}}{35b^2}\right)}{3b}$$

input
```
Int[x^2*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*x^2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (4*((2*x*ArcTanh[Tanh[a + b*x
]]^(5/2))/(5*b) - (4*ArcTanh[Tanh[a + b*x]]^(7/2))/(35*b^2)))/(3*b)
```

3.113.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.113.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

method	result	siz
default	$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}+\frac{2(-2\operatorname{arctanh}(\tanh(bx+a))+2bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}+\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))^{2}\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}}{b^{3}}$	69

input
```
int(x^2*arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^3*(1/7*arctanh(tanh(b*x+a))^(7/2)+1/5*(-2*arctanh(tanh(b*x+a))+2*b*x)*
arctanh(tanh(b*x+a))^(5/2)+1/3*(b*x-arctanh(tanh(b*x+a)))^2*arctanh(tanh(b
*x+a))^(3/2))
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx+a}}{105\,b^3}$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

3.113.6 Sympy [F]

$$\int x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int x^2\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**2*atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(x**2*sqrt(atanh(tanh(a + b*x))), x)
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx+a}}{105\,b^3}$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

3.113.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(47) = 94$.

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.73

$$\int x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{\sqrt{2}\left(\frac{7\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)a}{b^2}+\frac{3\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)}{b^2}\right)}{105\,b}$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
1/105*sqrt(2)*(7*sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sq
rt(b*x + a)*a^2)*a/b^2 + 3*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)
*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)/b^2)/b
```

3.113.9 Mupad [B] (verification not implemented)

Time = 3.58 (sec) , antiderivative size = 485, normalized size of antiderivative = 8.22

$$\int x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,x^3\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{7}$$

$$-\frac{x^2\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}+\frac{2\,b\,x}{7}\right)}{5\,b}$$

$$-\frac{8\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)^2\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}-\right.}{15\,b^3}$$

$$-\frac{4\,x\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}\right.}{15\,b^2}$$

input
```
int(x^2*atanh(tanh(a + b*x))^(1/2),x)
```

output
```
(2*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(
exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/7 - (x^2*(log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1))/7 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1))/7 + (2*b*x)/7))/(5*b) - (8*(log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1))/2 + b*x)^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/7 - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/7 + (2*b*x)/7))/(15*b^3) -
 (4*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)*(log(2/(exp(
2*a)*exp(2*b*x) + 1))/7 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1))/7 + (2*b*x)/7))/(15*b^2)
```

3.113. $\int x^2 \sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx$

3.114 $\int x\sqrt{\text{arctanh}(\tanh(a+bx))}\,dx$

3.114.1 Optimal result

Integrand size = 13, antiderivative size = 38

$$\int x\sqrt{\text{arctanh}(\tanh(a+bx))}\,dx=\frac{2x\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\text{arctanh}(\tanh(a+bx))^{5/2}}{15b^2}$$

output
```
2/3*x*arctanh(tanh(b*x+a))^(3/2)/b-4/15*arctanh(tanh(b*x+a))^(5/2)/b^2
```

3.114.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int x\sqrt{\text{arctanh}(\tanh(a+bx))}\,dx=\frac{2(5bx-2\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}{15b^2}$$

input
```
Integrate[x*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*(5*b*x - 2*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))/(15*b^
2)
```

3.114.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\text{arctanh}(\tanh(a+bx))}\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2x\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{3/2}dx}{3b}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{2x\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{3/2}d\text{arctanh}(\tanh(a+bx))}{3b^2}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2x\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\text{arctanh}(\tanh(a+bx))^{5/2}}{15b^2}$$

input
```
Int[x*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*x*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (4*ArcTanh[Tanh[a + b*x]]^(5/2)
)/(15*b^2)
```

3.114.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.114.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}+\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a))) \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}}{b^2}$	42

input
```
int(x*arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^2*(1/5*arctanh(tanh(b*x+a))^(5/2)+1/3*(b*x-arctanh(tanh(b*x+a)))*arcta
nh(tanh(b*x+a))^(3/2))
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int x\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx+a}}{15\,b^2}$$

input
```
integrate(x*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

3.114.6 Sympy [F]

$$\int x\sqrt{\text{arctanh}(\tanh(a+bx))}\,dx=\int x\sqrt{\text{atanh}\,(\tanh\,(a+bx))}\,dx$$

input
```
integrate(x*atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(x*sqrt(atanh(tanh(a + b*x))), x)
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int x\sqrt{\text{arctanh}(\tanh(a+bx))}\,dx=\frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx+a}}{15\,b^2}$$

input
```
integrate(x*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

3.114.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(30)=60$.

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.97

$$\int x\sqrt{\text{arctanh}(\tanh(a+bx))}\,dx=\frac{\sqrt{2}\left(\frac{5\,\sqrt{2}\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)a}{b}+\frac{\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)}{b}\right)}{15\,b}$$

input
```
integrate(x*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
1/15*sqrt(2)*(5*sqrt(2)*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a/b + sqrt(2
)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b)/b
```

3.114.9 Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.97

$$\int x\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx$$
$$=\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+5\,b}{15\,b^2}$$

input
```
int(x*atanh(tanh(a + b*x))^(1/2),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 5*b*x))/
(15*b^2)
```

3.115 $\int \sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx$

3.115.1 Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3b}$$

output
```
2/3*arctanh(tanh(b*x+a))^(3/2)/b
```

3.115.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3b}$$

input
```
Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b)
```

3.115.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\text{arctanh}(\tanh(a+bx))}\,dx$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \sqrt{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}$$

input
```
Int[Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b)
```

3.115.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.115.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3b}$	15
default	$\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3b}$	15

input
```
int(arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*arctanh(tanh(b*x+a))^(3/2)/b
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \sqrt{\operatorname{arctanh}(\tanh(a+bx))}\, dx = \frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(b*x + a)^(3/2)/b
```

3.115.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \sqrt{\operatorname{arctanh}(\tanh(a+bx))}\, dx = \begin{cases} \frac{2\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}{3b} & \text{for}\ b \neq 0 \\ x\sqrt{\operatorname{atanh}(\tanh(a))} & \text{otherwise} \end{cases}$$

input
```
integrate(atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Piecewise((2*atanh(tanh(a + b*x))**(3/2)/(3*b), Ne(b, 0)), (x*sqrt(atanh(t
anh(a))), True))
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(b*x + a)^(3/2)/b
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{\sqrt{2}(2\,bx+2\,a)^{\frac{3}{2}}}{6\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*(2*b*x + 2*a)^(3/2)/b
```

3.115.9 Mupad [B] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 95, normalized size of antiderivative = 5.28

$$\int \sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx$$
$$= -\frac{\left(\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\sqrt{\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{3\,b}$$

input
```
int(atanh(tanh(a + b*x))^(1/2),x)
```

output
```
-((log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
- log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*b)
```

3.116 $\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x}\,dx$

3.116.1 Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x}\,dx = -2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)\sqrt{bx-\text{arctanh}(\tanh(a+bx))} + 2\sqrt{\text{arctanh}(\tanh(a+bx))}$$

output
```
-2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*
x-arctanh(tanh(b*x+a)))^(1/2)+2*arctanh(tanh(b*x+a))^(1/2)
```

3.116.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x}\,dx = 2\sqrt{\text{arctanh}(\tanh(a+bx))} - 2\text{arctanh}\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}$$

input
```
Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x,x]
```

output
```
2*Sqrt[ArcTanh[Tanh[a + b*x]]] - 2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sq
rt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
```

3.116.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x}\,dx$$

$$\Big\downarrow \ 2590$$

$$2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}-(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{1}{x\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}dx$$

$$\Big\downarrow \ 2592$$

$$2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}-2\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}\arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)$$

input
```
Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x,x]
```

output
```
-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]
*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]] + 2*Sqrt[ArcTanh[Tanh[a + b*x]]]
```

3.116.3.1 Defintions of rubi rules used

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

3.116.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

method	result
default	$2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}-2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx}\ \operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)$

input
```
int(arctanh(tanh(b*x+a))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*arctanh(tanh(b*x+a))^(1/2)-2*(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(ar
ctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x}\,dx = \left[\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right) + 2\sqrt{bx+a}, 2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + 2\sqrt{bx+a}\right]$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x,x, algorithm="fricas")
```

output
```
[sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a), 2
*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)]
```

3.116.6 Sympy [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x}\,dx = \int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(1/2)/x,x)
```

output
```
Integral(sqrt(atanh(tanh(a + b*x)))/x, x)
```

3.116.7 Maxima [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x}\,dx = \int \frac{\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}{x}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(arctanh(tanh(b*x + a)))/x, x)
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x}\,dx = \sqrt{2}\left(\frac{\sqrt{2}a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{2}\sqrt{bx+a}\right)$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x,x, algorithm="giac")
```

output
```
sqrt(2)*(sqrt(2)*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + sqrt(2)*sqrt(
b*x + a))
```

3.116.9 Mupad [B] (verification not implemented)

Time = 5.03 (sec) , antiderivative size = 308, normalized size of antiderivative = 4.89

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x}\,dx = 2\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}} + \ln\left(-\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\sqrt{\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\sqrt{\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{x\sqrt{\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-b\,x}}\right.$$

input

```
int(atanh(tanh(a + b*x))^(1/2)/x,x)
```

output

```
2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2) + log(-(log(1/(exp(2*a)*exp(2*b*x) + 1)) - l
og((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*(log((exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2
)^(1/2)*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(e
xp(2*a)*exp(2*b*x) + 1))/2 - b*x)^(1/2) + b*x)/(x*(log((exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2 - b*x
)^(1/2)))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/
(exp(2*a)*exp(2*b*x) + 1))/2 - b*x)^(1/2)
```

3.117 $\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^2} \, dx$

3.117.1 Optimal result

Integrand size = 15, antiderivative size = 66

$$\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^2} \, dx = \frac{b \arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}} - \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x}$$

output
```
b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x
-arctanh(tanh(b*x+a)))^(1/2)-arctanh(tanh(b*x+a))^(1/2)/x
```

3.117.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^2} \, dx = -\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x} - \frac{b\text{arctanh}\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}$$

input
```
Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^2,x]
```

output
```
-(Sqrt[ArcTanh[Tanh[a + b*x]]]/x) - (b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]
]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b
*x]]]
```

3.117.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2599, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^2}\,dx$$

$$\Big\downarrow \ 2599$$

$$\frac{1}{2}b\int \frac{1}{x\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}dx - \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x}$$

$$\Big\downarrow \ 2592$$

$$\frac{b\arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}} - \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x}$$

input
```
Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^2,x]
```

output
```
(b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]
)/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]] - Sqrt[ArcTanh[Tanh[a + b*x]]]/x
```

3.117.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.117.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

method	result	size
default	$2b\left(-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)$	63

input
```
int(arctanh(tanh(b*x+a))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2*b*(-1/2*arctanh(tanh(b*x+a))^(1/2)/x/b-1/2/(arctanh(tanh(b*x+a))-b*x)^(1
/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^2}\,dx = \left[\frac{\sqrt{a}bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}a}{2ax}, \frac{\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{bx+a}a}{ax}\right]$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*
x + a)*a)/(a*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(b*x
 + a)*a)/(a*x)]
```

3.117.6 Sympy [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^2}\,dx = \int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x^2}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(atanh(tanh(a + b*x)))/x**2, x)
```

3.117.7 Maxima [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^2}\,dx = \int \frac{\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}{x^2}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(arctanh(tanh(b*x + a)))/x^2, x)
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^2}\,dx = \frac{\sqrt{2}\left(\frac{\sqrt{2}b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{2}\sqrt{bx+a}b}{x}\right)}{2\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^2,x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(sqrt(2)*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(2)
*sqrt(b*x + a)*b/x)/b
```

3.117.9 Mupad [B] (verification not implemented)

Time = 9.96 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.17

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^2}\,dx = -\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{x} + \frac{\sqrt{2}\,b\,\ln\left(\frac{\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}\,\left(\sqrt{2}\,b\,x-\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)+\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln}{}}\right.}{x}\right.}{2\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}$$

input
```
int(atanh(tanh(a + b*x))^(1/2)/x^2,x)
```

output
```
(2^(1/2)*b*log(((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*((log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/
2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
+ 2^(1/2)*b*x)*1i)/x)*1i)/(2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)) - (log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*
x) + 1))/2)^(1/2)/x
```

3.118 $\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^3}\,dx$

3.118.1 Optimal result

Integrand size = 15, antiderivative size = 125

$$\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^3}\,dx = \frac{b^2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{4(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}} - \frac{b}{4x\sqrt{\text{arctanh}(\tanh(a+bx))}} + \frac{b^2}{4(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}} - \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{2x^2}$$

output
```
1/4*b^2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2)
)/(b*x-arctanh(tanh(b*x+a)))^(3/2)-1/4*b/x/arctanh(tanh(b*x+a))^(1/2)+1/4*
b^2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)-1/2*arctanh(tanh
(b*x+a))^(1/2)/x^2
```

3.118.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^3}\,dx = \frac{1}{4}\left(\frac{\left(-2+\frac{bx}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^2} + \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\operatorname{arctanh}(\tanh(a+bx)))^{3/2}}\right)$$

input
```
Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^3,x]
```

output
```
(((-2 + (b*x)/(b*x - ArcTanh[Tanh[a + b*x]]))*Sqrt[ArcTanh[Tanh[a + b*x]]]
)/x^2 + (b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Ta
nh[a + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2))/4
```

3.118.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2599, 2599, 2594, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^3}\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{4}b\int \frac{1}{x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}dx - \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{2x^2}$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{4}b\left(-\frac{1}{2}b\int\frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}dx-\frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)-\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{2x^2}$$

$\Big\downarrow$ 2594

$$\frac{1}{4}b\left(-\frac{1}{2}b\left(-\frac{\int\frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)-\frac{1}{x\sqrt{\text{arcta}}}\right.$$
$$\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{2x^2}$$

$\Big\downarrow$ 2592

$$\frac{1}{4}b\left(-\frac{1}{2}b\left(-\frac{2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)-\right.$$
$$\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{2x^2}$$

input
```
Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^3,x]
```

output
```
(b*(-1/2*(b*((-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Ta
nh[a + b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2/((b*x - ArcTanh[T
anh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]]))) - 1/(x*Sqrt[ArcTanh[Tanh[a
+ b*x]]])))/4 - Sqrt[ArcTanh[Tanh[a + b*x]]]/(2*x^2)
```

3.118. $\int\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^3}dx$

3.118.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.118.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.74

method	result	size
default	$2b^{2}\left(\frac{-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{8(\operatorname{arctanh}(\tanh(bx+a))-bx)}-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{8}}{x^{2}b^{2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{8(\operatorname{arctanh}(\tanh(bx+a))-bx)^{\frac{3}{2}}}\right)$	92

input
```
int(arctanh(tanh(b*x+a))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2*b^2*((-1/8/(arctanh(tanh(b*x+a))-b*x)*arctanh(tanh(b*x+a))^(3/2)-1/8*arc
tanh(tanh(b*x+a))^(1/2))/x^2/b^2+1/8/(arctanh(tanh(b*x+a))-b*x)^(3/2)*arct
anh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^3}\,dx = \left[\frac{\sqrt{a}b^2x^2\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(abx+2a^2\right)\sqrt{bx+a}}{8a^2x^2}, -\frac{\sqrt{-a}b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(abx+2a^2\right)\sqrt{bx+a}}{4a^2x^2}\right]$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*b^2*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(a*
b*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2), -1/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(
b*x + a)*sqrt(-a)/a) + (a*b*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2)]
```

3.118.6 Sympy [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^3}\,dx = \int \frac{\sqrt{\operatorname{atanh}(\tanh(a+bx))}}{x^3}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(atanh(tanh(a + b*x)))/x**3, x)
```

3.118.7 Maxima [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^3}\,dx = \int \frac{\sqrt{\operatorname{artanh}(\tanh(bx+a))}}{x^3}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(arctanh(tanh(b*x + a)))/x^3, x)
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^3}\,dx = -\frac{\sqrt{2}\left(\frac{\sqrt{2}b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{\sqrt{2}\left((bx+a)^{\frac{3}{2}}b^3+\sqrt{bx+a}ab^3\right)}{ab^2x^2}\right)}{8\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*(sqrt(2)*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sq
rt(2)*((b*x + a)^(3/2)*b^3 + sqrt(b*x + a)*a*b^3)/(a*b^2*x^2))/b
```

3.118.9 Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 741, normalized size of antiderivative = 5.93

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^3}\,dx = \frac{b\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{2\,x\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}$$
$$-\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{x^2\,\left(2\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-2\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+4\,b\,x\right)}$$
$$+\frac{\sqrt{2}\,b^2\,\ln\left(\frac{\left(2\,\sqrt{2}\,a-\sqrt{2}\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)+\sqrt{2}\,b\,x+\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}\right.}}\right.}{}\right.}{}$$

input
```
int(atanh(tanh(a + b*x))^(1/2)/x^3,x)
```

output
```
(b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2))/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - ((log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2
*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(x^2*(2*log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 4*b*x)) + (2^(1/2)*b^2*log(((2*2^(1/2)*a + (log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(2*a - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*4i)/(x
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*1i)/(4*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/...
```

3.118. $\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^3}\,dx$

3.119 $\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^4}\,dx$

3.119.1 Optimal result

Integrand size = 15, antiderivative size = 179

$$\begin{aligned}\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^4}\,dx &= \frac{b^3\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{8(bx-\text{arctanh}(\tanh(a+bx)))^{5/2}} + \frac{b^2}{24x\text{arctanh}(\tanh(a+bx))^{3/2}} \\ &\quad - \frac{b^3}{24(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}} \\ &\quad - \frac{b}{12x^2\sqrt{\text{arctanh}(\tanh(a+bx))}} \\ &\quad + \frac{b^3}{8(bx-\text{arctanh}(\tanh(a+bx)))^2\sqrt{\text{arctanh}(\tanh(a+bx))}} - \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{3x^3}\end{aligned}$$

output
```
1/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2)
)/(b*x-arctanh(tanh(b*x+a)))^(5/2)+1/24*b^2/x/arctanh(tanh(b*x+a))^(3/2)-1
/24*b^3/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)-1/12*b/x^2/a
rctanh(tanh(b*x+a))^(1/2)+1/8*b^3/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tan
h(b*x+a))^(1/2)-1/3*arctanh(tanh(b*x+a))^(1/2)/x^3
```

3.119.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^4}\,dx = \frac{1}{24}\left(-\frac{3b^3\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\operatorname{arctanh}(\tanh(a+bx)))^{5/2}} + \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}(-3b^2x^2+14bx\operatorname{arctanh}(\tanh(a+bx))-8\operatorname{arctanh}(\tanh(a+bx))^2)}{x^3(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2}\right)$$

input
```
Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^4,x]
```

output
```
((-3*b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
 + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) + (Sqrt[ArcTanh[Tanh[a
 + b*x]]]*(-3*b^2*x^2 + 14*b*x*ArcTanh[Tanh[a + b*x]] - 8*ArcTanh[Tanh[a +
 b*x]]^2))/(x^3*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2))/24
```

3.119.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2599, 2599, 2599, 2594, 2594, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^4}dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{6}b\int \frac{1}{x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}dx - \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{3x^3}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{6}b\left(-\frac{1}{4}b\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{3/2}}dx-\frac{1}{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)-\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{3x^3}$$

$\downarrow$ 2599

$$\frac{1}{6}b\left(-\frac{1}{4}b\left(-\frac{3}{2}b\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}dx-\frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}\right)-\frac{1}{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)-\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{3x^3}$$

$\downarrow$ 2594

$$\frac{1}{6}b\left(-\frac{1}{4}b\left(-\frac{3}{2}b\left(-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}\right)-\ldots\right.\right.$$

$$\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{3x^3}$$

$\downarrow$ 2594

$$\frac{1}{6}b\left(-\frac{1}{4}b\left(-\frac{3}{2}b\left(-\frac{-\frac{\int \frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\ldots}{3(bx-\text{arctanh}(\text{ta}\ldots}\right.\right.\right.$$

$$\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{3x^3}$$

$\downarrow$ 2592

$$\frac{1}{6}b\left(-\frac{1}{4}b\left(-\frac{3}{2}b\left(-\frac{-\frac{2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\ldots}{3(bx-\text{ar}\ldots}\right.\right.\right.$$

$$\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{3x^3}$$

input
```
Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^4,x]
```

3.119. $\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^4}\,dx$

output
```
(b*(-1/4*(b*((-3*b*(-(((-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x -
ArcTanh[Tanh[a + b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2/((b*x -
 ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]]))/(b*x - ArcTanh[Tan
h[a + b*x]])) - 2/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]
^(3/2))))/2 - 1/(x*ArcTanh[Tanh[a + b*x]]^(3/2)))) - 1/(2*x^2*Sqrt[ArcTanh
[Tanh[a + b*x]]])))/6 - Sqrt[ArcTanh[Tanh[a + b*x]]]/(3*x^3)
```

3.119.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.119.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.03

method	result
default	$2b^3\left(\frac{\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{16a^2+32a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+16(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2}-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{6(\operatorname{arctanh}(\tanh(bx+a))-bx)}-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{16}}{x^3b^3}-\frac{}{1}\right.$

input
```
int(arctanh(tanh(b*x+a))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2*b^3*((1/16/(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b
*x-a)^2)*arctanh(tanh(b*x+a))^(5/2)-1/6/(arctanh(tanh(b*x+a))-b*x)*arctanh
(tanh(b*x+a))^(3/2)-1/16*arctanh(tanh(b*x+a))^(1/2))/x^3/b^3-1/16/(a^2+2*a
*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/(arctanh(tan
h(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+
a))-b*x)^(1/2)))
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^4}\,dx$$
$$= \left[\frac{3\sqrt{a}b^3x^3\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(3ab^2x^2-2a^2bx-8a^3\right)\sqrt{bx+a}}{48a^3x^3}, \frac{3\sqrt{-a}b^3x^3\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+}{24}\right.$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(3*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*
(3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x + a))/(a^3*x^3), 1/24*(3*sqrt(-
a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 - 2*a^2*b*x - 8
*a^3)*sqrt(b*x + a))/(a^3*x^3)]
```

3.119.6 Sympy [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^4}\,dx = \int \frac{\sqrt{\operatorname{atanh}(\tanh(a+bx))}}{x^4}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(1/2)/x**4,x)
```

output
```
Integral(sqrt(atanh(tanh(a + b*x)))/x**4, x)
```

3.119.7 Maxima [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^4}\,dx = \int \frac{\sqrt{\operatorname{artanh}(\tanh(bx+a))}}{x^4}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(arctanh(tanh(b*x + a)))/x^4, x)
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^4}\,dx = \frac{\sqrt{2}\left(\frac{3\sqrt{2}b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{\sqrt{2}\left(3(bx+a)^{\frac{5}{2}}b^4 - 8(bx+a)^{\frac{3}{2}}ab^4 - 3\sqrt{bx+a}a^2b^4\right)}{a^2b^3x^3}\right)}{48\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/48*sqrt(2)*(3*sqrt(2)*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2)
+ sqrt(2)*(3*(b*x + a)^(5/2)*b^4 - 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x +
a)*a^2*b^4)/(a^2*b^3*x^3))/b
```

3.119.9 Mupad [B] (verification not implemented)

Time = 8.04 (sec) , antiderivative size = 964, normalized size of antiderivative = 5.39

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^4}\,dx = \text{Too large to display}$$

input
```
int(atanh(tanh(a + b*x))^(1/2)/x^4,x)
```

output
```
(b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*x^2*(2*log(2/(exp(2*a)*exp(2*b*x) + 1))
 - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x)) + (b
^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2))/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (2^(1/
2)*b^3*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2
i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x)^5 + 40*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 80*a^3*(2*a - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^2 - 32*a^5 - 10*a*(2*a - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^
4 + 80*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*4i)/(x*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2...
```

3.119. $\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^4}\,dx$

3.120 $\int x^4 \textbf{arctanh}(\tanh(a+bx))^{3/2}\,dx$

3.120.1 Optimal result

Integrand size = 15, antiderivative size = 101

$$\int x^4\text{arctanh}(\tanh(a+bx))^{3/2}\,dx=\frac{2x^4\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{16x^3\text{arctanh}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{32x^2\text{arctanh}(\tanh(a+bx))^{9/2}}{105b^3}-\frac{128x\text{arctanh}(\tanh(a+bx))^{11/2}}{1155b^4}+\frac{256\text{arctanh}(\tanh(a+bx))^{13/2}}{15015b^5}$$

output
```
2/5*x^4*arctanh(tanh(b*x+a))^(5/2)/b-16/35*x^3*arctanh(tanh(b*x+a))^(7/2)/
b^2+32/105*x^2*arctanh(tanh(b*x+a))^(9/2)/b^3-128/1155*x*arctanh(tanh(b*x+
a))^(11/2)/b^4+256/15015*arctanh(tanh(b*x+a))^(13/2)/b^5
```

3.120.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int x^4\text{arctanh}(\tanh(a+bx))^{3/2}\,dx=\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}\left(3003b^4x^4-3432b^3x^3\text{arctanh}(\tanh(a+bx))+2288b^2x^2\text{arctanh}(\text{t}\right.}{15015b^5}$$

input
```
Integrate[x^4*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(5/2)*(3003*b^4*x^4 - 3432*b^3*x^3*ArcTanh[Tanh[
a + b*x]] + 2288*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 832*b*x*ArcTanh[Tanh[a
 + b*x]]^3 + 128*ArcTanh[Tanh[a + b*x]]^4))/(15015*b^5)
```

3.120.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2599, 2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{arctanh}(\tanh(a+bx))^{3/2} \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^4 \text{arctanh}(\tanh(a+bx))^{5/2}}{5b} - \frac{8 \int x^3 \text{arctanh}(\tanh(a+bx))^{5/2} dx}{5b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^4 \text{arctanh}(\tanh(a+bx))^{5/2}}{5b} - \frac{8 \left(\frac{2x^3 \text{arctanh}(\tanh(a+bx))^{7/2}}{7b} - \frac{6 \int x^2 \text{arctanh}(\tanh(a+bx))^{7/2} dx}{7b}\right)}{5b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^4 \text{arctanh}(\tanh(a+bx))^{5/2}}{5b} - \frac{8 \left(\frac{2x^3 \text{arctanh}(\tanh(a+bx))^{7/2}}{7b} - \frac{6 \left(\frac{2x^2 \text{arctanh}(\tanh(a+bx))^{9/2}}{9b} - \frac{4 \int x \text{arctanh}(\tanh(a+bx))^{9/2} dx}{9b}\right)}{7b}\right)}{5b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^4 \text{arctanh}(\tanh(a+bx))^{5/2}}{5b} - \frac{8 \left(\frac{2x^3 \text{arctanh}(\tanh(a+bx))^{7/2}}{7b} - \frac{6 \left(\frac{2x^2 \text{arctanh}(\tanh(a+bx))^{9/2}}{9b} - \frac{4 \left(\frac{2x \text{arctanh}(\tanh(a+bx))^{11/2}}{11b} - \frac{2 \int \text{arctanh}(\tanh(a+bx))^{11/2} dx}{11b}\right)}{9b}\right)}{7b}\right)}{5b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{2x^4\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{8\left(\frac{2x^3\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{9/2}}{9b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{11/2}}{11b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{11/2}d\text{arctan}}{11b^2}\right.}{9b}\right.}{7b}\right.}{5b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2x^4\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{8\left(\frac{2x^3\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{9/2}}{9b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{11/2}}{11b}-\frac{4\text{arctanh}(\tanh(a+bx))^{13/2}}{143b^2}\right)}{9b}\right)}{7b}\right)}{5b}$$

input
```
Int[x^4*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(2*x^4*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (8*((2*x^3*ArcTanh[Tanh[a + b
*x]]^(7/2))/(7*b) - (6*((2*x^2*ArcTanh[Tanh[a + b*x]]^(9/2))/(9*b) - (4*((
2*x*ArcTanh[Tanh[a + b*x]]^(11/2))/(11*b) - (4*ArcTanh[Tanh[a + b*x]]^(13/
2))/(143*b^2)))/(9*b)))/(7*b)))/(5*b)
```

3.120.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.120.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.52

method	result
default	$\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{13}{2}}}{13}+\frac{2(-4 \operatorname{arctanh}(\tanh(bx+a))+4bx) \operatorname{arctanh}(\tanh(bx+a))^{\frac{11}{2}}}{11}+\frac{2\left(2(bx-\operatorname{arctanh}(\tanh(bx+a)))^2+(-2 \operatorname{arctanh}(\tanh(bx+a)\right.}{9}$

input
```
int(x^4*arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^5*(1/13*arctanh(tanh(b*x+a))^(13/2)+1/11*(-4*arctanh(tanh(b*x+a))+4*b*
x)*arctanh(tanh(b*x+a))^(11/2)+1/9*(2*(b*x-arctanh(tanh(b*x+a)))^2+(-2*arc
tanh(tanh(b*x+a))+2*b*x)^2)*arctanh(tanh(b*x+a))^(9/2)+2/7*(b*x-arctanh(ta
nh(b*x+a)))^2*(-2*arctanh(tanh(b*x+a))+2*b*x)*arctanh(tanh(b*x+a))^(7/2)+1
/5*(b*x-arctanh(tanh(b*x+a)))^4*arctanh(tanh(b*x+a))^(5/2))
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int x^4 \operatorname{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{2\left(1155\,b^6x^6+1470\,ab^5x^5+35\,a^2b^4x^4-40\,a^3b^3x^3+48\,a^4b^2x^2-64\,a^5bx+128\,a^6\right)\sqrt{bx+a}}{15015\,b^5}$$

input
```
integrate(x^4*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
 48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x + a)/b^5
```

3.120.6 Sympy [F]

$$\int x^4 \text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \int x^4 \operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))\,dx$$

input
```
integrate(x**4*atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(x**4*atanh(tanh(a + b*x))**(3/2), x)
```

3.120.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int x^4 \text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{2\,(1155\,b^5x^5 + 315\,ab^4x^4 - 280\,a^2b^3x^3 + 240\,a^3b^2x^2 - 192\,a^4bx + 128\,a^5)(bx+a)^{\frac{3}{2}}}{15015\,b^5}$$

input
```
integrate(x^4*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/15015*(1155*b^5*x^5 + 315*a*b^4*x^4 - 280*a^2*b^3*x^3 + 240*a^3*b^2*x^2
- 192*a^4*b*x + 128*a^5)*(b*x + a)^(3/2)/b^5
```

3.120.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(81) = 162.

Time = 0.27 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.39

$$\int x^4 \text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{\sqrt{2}\left(\frac{143\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}} - 180\,(bx+a)^{\frac{7}{2}}a + 378\,(bx+a)^{\frac{5}{2}}a^2 - 420\,(bx+a)^{\frac{3}{2}}a^3 + 315\,\sqrt{bx+a}\,a^4\right)a^2}{b^4} + \frac{130\sqrt{2}\left(63\,(bx+a)^{\frac{11}{2}} - 385\right.}{}\right.}{}$$

input
```
integrate(x^4*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
1/45045*sqrt(2)*(143*sqrt(2)*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a +
 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4
)*a^2/b^4 + 130*sqrt(2)*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990
*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4
 - 693*sqrt(b*x + a)*a^5)*a/b^4 + 15*sqrt(2)*(231*(b*x + a)^(13/2) - 1638*
(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 +
 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*
a^6)/b^4)/b
```

3.120.9 Mupad [B] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 1813, normalized size of antiderivative = 17.95

$$\int x^4 \text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \text{Too large to display}$$

input
```
int(x^4*atanh(tanh(a + b*x))^(3/2),x)
```

output
```
(2*b*x^6*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2
/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/13 + (x^5*(log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2
)*((24*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/13 - 2*b*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))
/(11*b) + (x^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 -
 log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) +
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2
+ (10*((24*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/13 - 2*b*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b)))/(9*b) + (128*(log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/
2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2 + (10*((24*b*(log(2/(exp(2*a)*exp(
2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
+ b*x))/13 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x)...
```

3.121 $\int x^3 \textbf{arctanh}(\tanh(a+bx))^{3/2}\,dx$

3.121.1 Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x^3\text{arctanh}(\tanh(a+bx))^{3/2}\,dx=\frac{2x^3\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{12x^2\text{arctanh}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{16x\text{arctanh}(\tanh(a+bx))^{9/2}}{105b^3}-\frac{32\text{arctanh}(\tanh(a+bx))^{11/2}}{1155b^4}$$

output
```
2/5*x^3*arctanh(tanh(b*x+a))^(5/2)/b-12/35*x^2*arctanh(tanh(b*x+a))^(7/2)/
b^2+16/105*x*arctanh(tanh(b*x+a))^(9/2)/b^3-32/1155*arctanh(tanh(b*x+a))^(
11/2)/b^4
```

3.121.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int x^3\text{arctanh}(\tanh(a+bx))^{3/2}\,dx=\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}\left(231b^3x^3-198b^2x^2\text{arctanh}(\tanh(a+bx))+88bx\text{arctanh}(\tanh(a\right.}{1155b^4}$$

input
```
Integrate[x^3*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(5/2)*(231*b^3*x^3 - 198*b^2*x^2*ArcTanh[Tanh[a
+ b*x]] + 88*b*x*ArcTanh[Tanh[a + b*x]]^2 - 16*ArcTanh[Tanh[a + b*x]]^3))/
(1155*b^4)
```

3.121.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{arctanh}(\tanh(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^3\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{6\int x^2\text{arctanh}(\tanh(a+bx))^{5/2}dx}{5b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^3\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\int x\text{arctanh}(\tanh(a+bx))^{7/2}dx}{7b}\right)}{5b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^3\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{9/2}}{9b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{9/2}dx}{9b}\right)}{7b}\right)}{5b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{2x^3\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{9/2}}{9b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{9/2}d\text{arctanh}(\tanh(a+bx))}{9b^2}\right)}{7b}\right)}{5b}$$

$$\Big\downarrow \text{15}$$

$$\frac{2x^3\text{arctanh}(\tanh(a+bx))^{5/2}}{5b} - \frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{7/2}}{7b} - \frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{9/2}}{9b} - \frac{4\text{arctanh}(\tanh(a+bx))^{11/2}}{99b^2}\right)}{7b}\right)}{5b}$$

input
```
Int[x^3*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(2*x^3*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (6*((2*x^2*ArcTanh[Tanh[a + b
*x]]^(7/2))/(7*b) - (4*((2*x*ArcTanh[Tanh[a + b*x]]^(9/2))/(9*b) - (4*ArcT
anh[Tanh[a + b*x]]^(11/2))/(99*b^2)))/(7*b)))/(5*b)
```

3.121.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.121.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.55

method	result
default	$\frac{\frac{2\,\text{arctanh}(\tanh(bx+a))^{\frac{11}{2}}}{11}+\frac{2(-3\,\text{arctanh}(\tanh(bx+a))+3bx)\,\text{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2\left((bx-\text{arctanh}(\tanh(bx+a)))(-2\,\text{arctanh}(\tanh(bx+a))+2bx\right.}{7}}{b^4}$

input
```
int(x^3*arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^4*(1/11*arctanh(tanh(b*x+a))^(11/2)+1/9*(-3*arctanh(tanh(b*x+a))+3*b*x
)*arctanh(tanh(b*x+a))^(9/2)+1/7*((b*x-arctanh(tanh(b*x+a)))*(-2*arctanh(t
anh(b*x+a))+2*b*x)+(b*x-arctanh(tanh(b*x+a)))^2)*arctanh(tanh(b*x+a))^(7/2
)+1/5*(b*x-arctanh(tanh(b*x+a)))^3*arctanh(tanh(b*x+a))^(5/2))
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int x^3\text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{2\,(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5)\sqrt{bx+a}}{1155\,b^4}$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^
4*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

3.121.6 Sympy [F]

$$\int x^3\text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \int x^3\,\text{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)\,dx$$

input
```
integrate(x**3*atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(x**3*atanh(tanh(a + b*x))**(3/2), x)
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

$$\int x^3 \text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{2\,(105\,b^4x^4+35\,ab^3x^3-30\,a^2b^2x^2+24\,a^3bx-16\,a^4)(bx+a)^{\frac{3}{2}}}{1155\,b^4}$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/1155*(105*b^4*x^4 + 35*a*b^3*x^3 - 30*a^2*b^2*x^2 + 24*a^3*b*x - 16*a^4)
*(b*x + a)^(3/2)/b^4
```

3.121.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(64) = 128$.

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.56

$$\int x^3 \text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{\sqrt{2}\left(\frac{99\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a^2}{b^3}+\frac{22\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}\right.}{b^3}\right.}{}$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
1/3465*sqrt(2)*(99*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*
(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2/b^3 + 22*sqrt(2)*(35*(b*x
+ a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x +
a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a/b^3 + 5*sqrt(2)*(63*(b*x + a)^(11/
2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2
)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)/b^3)/b
```

3.121.9 Mupad [B] (verification not implemented)

Time = 3.53 (sec) , antiderivative size = 1483, normalized size of antiderivative = 18.54

$$\int x^3 \text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \text{Too large to display}$$

input
```
int(x^3*atanh(tanh(a + b*x))^(3/2),x)
```

output
```
(2*b*x^5*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2
/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/11 + (x^4*(log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2
)*((20*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/11 - 2*b*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))
/(9*b) + (x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 -
log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2 +
 (8*((20*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/11 - 2*b*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1))/2 + b*x))/(9*b)))/(7*b) + (16*(log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(
1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2 + (8*((20*b*(log(2/(exp(2*a)*exp(2*b*x
) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x
))/11 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)...
```

3.122 $\int x^2 \textbf{arctanh}(\tanh(a+bx))^{3/2}\,dx$

3.122.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^2\text{arctanh}(\tanh(a+bx))^{3/2}\,dx=\frac{2x^2\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{8x\text{arctanh}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{16\text{arctanh}(\tanh(a+bx))^{9/2}}{315b^3}$$

output
```
2/5*x^2*arctanh(tanh(b*x+a))^(5/2)/b-8/35*x*arctanh(tanh(b*x+a))^(7/2)/b^2
+16/315*arctanh(tanh(b*x+a))^(9/2)/b^3
```

3.122.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int x^2\text{arctanh}(\tanh(a+bx))^{3/2}\,dx=\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}\left(63b^2x^2-36bx\text{arctanh}(\tanh(a+bx))+8\text{arctanh}(\tanh(a+bx))^2\right)}{315b^3}$$

input
```
Integrate[x^2*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(5/2)*(63*b^2*x^2 - 36*b*x*ArcTanh[Tanh[a + b*x]
] + 8*ArcTanh[Tanh[a + b*x]]^2))/(315*b^3)
```

3.122.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arctanh}(\tanh(a+bx))^{3/2} \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2x^2\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\int x\text{arctanh}(\tanh(a+bx))^{5/2}dx}{5b}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2x^2\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{7/2}dx}{7b}\right)}{5b}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{2x^2\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{7/2}d\text{arctanh}(\tanh(a+bx))}{7b^2}\right)}{5b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2x^2\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\text{arctanh}(\tanh(a+bx))^{9/2}}{63b^2}\right)}{5b}$$

input
```
Int[x^2*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(2*x^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (4*((2*x*ArcTanh[Tanh[a + b*x
]]^(7/2))/(7*b) - (4*ArcTanh[Tanh[a + b*x]]^(9/2))/(63*b^2)))/(5*b)
```

3.122.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.122.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

method	result	siz
default	$\frac{\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2(-2 \operatorname{arctanh}(\tanh(bx+a))+2bx) \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}+\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))^{2} \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}}{b^{3}}$	69

input
```
int(x^2*arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^3*(1/9*arctanh(tanh(b*x+a))^(9/2)+1/7*(-2*arctanh(tanh(b*x+a))+2*b*x)*
arctanh(tanh(b*x+a))^(7/2)+1/5*(b*x-arctanh(tanh(b*x+a)))^2*arctanh(tanh(b
*x+a))^(5/2))
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x^2 \text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{2\,(35\,b^4x^4 + 50\,ab^3x^3 + 3\,a^2b^2x^2 - 4\,a^3bx + 8\,a^4)\sqrt{bx+a}}{315\,b^3}$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt
(b*x + a)/b^3
```

3.122.6 Sympy [F]

$$\int x^2 \text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \int x^2 \operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))\,dx$$

input
```
integrate(x**2*atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(x**2*atanh(tanh(a + b*x))**(3/2), x)
```

3.122.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int x^2 \text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{2\,(35\,b^3x^3 + 15\,ab^2x^2 - 12\,a^2bx + 8\,a^3)(bx+a)^{\frac{3}{2}}}{315\,b^3}$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/315*(35*b^3*x^3 + 15*a*b^2*x^2 - 12*a^2*b*x + 8*a^3)*(b*x + a)^(3/2)/b^3
```

3.122.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(47) = 94.

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.85

$$\int x^2\text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{\sqrt{2}\left(\frac{21\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)a^2}{b^2}+\frac{18\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a}{b^2}\right.}{315\,b}$$

input

```
integrate(x^2*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output

```
1/315*sqrt(2)*(21*sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*s
qrt(b*x + a)*a^2)*a^2/b^2 + 18*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(
5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a/b^2 + sqrt(2)*(3
5*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*
(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b^2)/b
```

3.122.9 Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 1153, normalized size of antiderivative = 19.54

$$\int x^2\text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \text{Too large to display}$$

input

```
int(x^2*atanh(tanh(a + b*x))^(3/2),x)
```

output
```
(2*b*x^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2
/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/9 + (x^3*(log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)
*((16*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/9 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(
7*b) + (x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - lo
g(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2 + (
6*((16*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/9 - 2*b*(log(2/(exp(2*a)*exp(2*b*x)
+ 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(
log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1))/2 + b*x))/(7*b)))/(5*b) + (8*(log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)
*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)^2/2 + (6*((16*b*(log(2/(exp(2*a)*exp(2*b*x) +
1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/9
 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 -...
```

3.122. $\int x^2 \text{arctanh}(\tanh(a+bx))^{3/2}\,dx$

3.123 $\int x\textbf{arctanh}(\tanh(a+bx))^{3/2}\,dx$

3.123.1 Optimal result

Integrand size = 13, antiderivative size = 38

$$\int x\text{arctanh}(\tanh(a+bx))^{3/2}\,dx=\frac{2x\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\text{arctanh}(\tanh(a+bx))^{7/2}}{35b^2}$$

output
```
2/5*x*arctanh(tanh(b*x+a))^(5/2)/b-4/35*arctanh(tanh(b*x+a))^(7/2)/b^2
```

3.123.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int x\text{arctanh}(\tanh(a+bx))^{3/2}\,dx=\frac{2(7bx-2\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{5/2}}{35b^2}$$

input
```
Integrate[x*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(2*(7*b*x - 2*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(5/2))/(35*b^
2)
```

3.123.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(\tanh(a+bx))^{3/2}\,dx$$
$$\Big\downarrow \text{ 2599}$$
$$\frac{2x\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{5/2}dx}{5b}$$
$$\Big\downarrow \text{ 2588}$$
$$\frac{2x\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{5/2}d\text{arctanh}(\tanh(a+bx))}{5b^2}$$
$$\Big\downarrow \text{ 15}$$
$$\frac{2x\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\text{arctanh}(\tanh(a+bx))^{7/2}}{35b^2}$$

input
```
Int[x*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(2*x*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (4*ArcTanh[Tanh[a + b*x]]^(7/2)
)/(35*b^2)
```

3.123.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.123.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}+\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a))) \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}}{b^2}$	42

input
```
int(x*arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^2*(1/7*arctanh(tanh(b*x+a))^(7/2)+1/5*(b*x-arctanh(tanh(b*x+a)))*arcta
nh(tanh(b*x+a))^(5/2))
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int x\operatorname{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{2\,(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3)\sqrt{bx+a}}{35\,b^2}$$

input
```
integrate(x*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

3.123.6 Sympy [A] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int x\text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \begin{cases} \frac{2x\,\text{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}{5b} - \frac{4\,\text{atanh}^{\frac{7}{2}}\left(\tanh\left(a+bx\right)\right)}{35b^2} & \text{for } b \neq 0 \\ \frac{x^2\,\text{atanh}^{\frac{3}{2}}\left(\tanh\left(a\right)\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Piecewise((2*x*atanh(tanh(a + b*x))**(5/2)/(5*b) - 4*atanh(tanh(a + b*x))*
*(7/2)/(35*b**2), Ne(b, 0)), (x**2*atanh(tanh(a))**(3/2)/2, True))
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int x\text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{2\,(5\,b^2x^2+3\,abx-2\,a^2)(bx+a)^{\frac{3}{2}}}{35\,b^2}$$

input
```
integrate(x*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/35*(5*b^2*x^2 + 3*a*b*x - 2*a^2)*(b*x + a)^(3/2)/b^2
```

3.123.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(30) = 60.

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.45

$$\int x\text{arctanh}(\tanh(a +bx))^{3/2}\,dx = \frac{\sqrt{2}\left(\frac{35\,\sqrt{2}\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)a^2}{b}+\frac{14\,\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)a}{b}+\frac{3\,\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}\right.}{}\right.}{105\,b}$$

input
```
integrate(x*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
1/105*sqrt(2)*(35*sqrt(2)*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^2/b + 14
*sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)
*a/b + 3*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^
(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)/b)/b
```

3.123.9 Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 823, normalized size of antiderivative = 21.66

$$\int x\text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \text{Too large to display}$$

input
```
int(x*atanh(tanh(a + b*x))^(3/2),x)
```

output
```
(2*b*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2
/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/7 + (x^2*(log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)
*((12*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/7 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(
5*b) + (x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(
2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2 + (4*
((12*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/7 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) +
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))/2 + b*x))/(5*b)))/(3*b) + (2*(log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^2/2 + (4*((12*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)
)/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/7 -
 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - l...
```

3.124 $\int \operatorname{arctanh}(\tanh(a+bx))^{3/2}\,dx$

3.124.1 Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \operatorname{arctanh}(\tanh(a+bx))^{3/2}\,dx=\frac{2\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{5b}$$

output
```
2/5*arctanh(tanh(b*x+a))^(5/2)/b
```

3.124.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \operatorname{arctanh}(\tanh(a+bx))^{3/2}\,dx=\frac{2\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{5b}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b)
```

3.124.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(\tanh(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \operatorname{arctanh}(\tanh(a+bx))^{3/2}d\operatorname{arctanh}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{5b}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b)
```

3.124.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.124.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5b}$	15
default	$\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5b}$	15

input
```
int(arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*arctanh(tanh(b*x+a))^(5/2)/b
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \operatorname{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{2\,(b^2x^2+2\,abx+a^2)\sqrt{bx+a}}{5\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

3.124.6 Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \operatorname{arctanh}(\tanh(a+bx))^{3/2}\,dx = \begin{cases} \frac{2\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}{5b} & \text{for } b\neq 0 \\ x\operatorname{atanh}^{\frac{3}{2}}(\tanh(a)) & \text{otherwise} \end{cases}$$

input
```
integrate(atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Piecewise((2*atanh(tanh(a + b*x))**(5/2)/(5*b), Ne(b, 0)), (x*atanh(tanh(a
))**(3/2), True))
```

3.124.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \operatorname{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/5*(b*x + a)^(5/2)/b
```

3.124.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 4.67

$$\int \operatorname{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{\sqrt{2}\left(15\,\sqrt{2}\sqrt{bx+a}a^2+10\,\sqrt{2}\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)a+\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a\right.\right.}{15\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
1/15*sqrt(2)*(15*sqrt(2)*sqrt(b*x + a)*a^2 + 10*sqrt(2)*((b*x + a)^(3/2) -
 3*sqrt(b*x + a)*a)*a + sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a
+ 15*sqrt(b*x + a)*a^2))/b
```

3.124.9 Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 97, normalized size of antiderivative = 5.39

$$\int \operatorname{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{\left(\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)^2\sqrt{\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{10\,b}$$

input
```
int(atanh(tanh(a + b*x))^(3/2),x)
```

output
```
((log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)))^2*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(10*b)
```

3.124. $\int \operatorname{arctanh}(\tanh(a+bx))^{3/2}\,dx$

3.125 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x}\,dx$

3.125.1 Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x}\,dx = 2\arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx - \operatorname{arctanh}(\tanh(a+bx)))^{3/2} - 2(bx-\operatorname{arctanh}(\tanh(a+bx)))\sqrt{\operatorname{arctanh}(\tanh(a+bx))} + \frac{2}{3}\operatorname{arctanh}(\tanh(a+bx))^{3/2}$$

output
```
2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*x
-arctanh(tanh(b*x+a)))^(3/2)+2/3*arctanh(tanh(b*x+a))^(3/2)-2*(b*x-arctanh
(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(1/2)
```

3.125.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x}\,dx = -\frac{2}{3}\left(3bx\sqrt{\operatorname{arctanh}(\tanh(a+bx))} - 4\operatorname{arctanh}(\tanh(a+bx))^{3/2} + 3\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)(-bx+\operatorname{arctanh}(\tanh(a+bx)))^{3/2}\right)$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x,x]
```

output
```
(-2*(3*b*x*Sqrt[ArcTanh[Tanh[a + b*x]]] - 4*ArcTanh[Tanh[a + b*x]]^(3/2) +
 3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x
]]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2)))/3
```

3.125.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2590, 2590, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x}\,dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{2}{3}\text{arctanh}(\tanh(a+bx))^{3/2}-(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x}dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{2}{3}\text{arctanh}(\tanh(a+bx))^{3/2}-(bx-\text{arctanh}(\tanh(a+bx)))\left(2\sqrt{\text{arctanh}(\tanh(a+bx))}-(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx\right)$$

$$\Big\downarrow \text{2592}$$

$$\frac{2}{3}\text{arctanh}(\tanh(a+bx))^{3/2}-(bx-\text{arctanh}(\tanh(a+bx)))\left(2\sqrt{\text{arctanh}(\tanh(a+bx))}-2\sqrt{bx-\text{arctanh}(\tanh(a+bx))}\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)\right)$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x,x]
```

output
```
-((-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]] + 2*Sqrt[ArcTanh[Tanh[a + b*x]]])*(
b*x - ArcTanh[Tanh[a + b*x]])) + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/3
```

3.125.3.1 Defintions of rubi rules used

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

3.125.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.44

method	result
default	$\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3} + 2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}\, a + 2(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right) - bx - a)\sqrt{\operatorname{arct}}$

input
```
int(arctanh(tanh(b*x+a))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3*arctanh(tanh(b*x+a))^(3/2)+2*arctanh(tanh(b*x+a))^(1/2)*a+2*(arctanh(t
anh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)-2*(a^2+2*a*(arctanh(tanh(b*x
+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/(arctanh(tanh(b*x+a))-b*x)^(1/
2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x}\, dx = \left[a^{\frac{3}{2}} \log\left(\frac{bx - 2\sqrt{bx+a}\sqrt{a} + 2a}{x}\right) + \frac{2}{3}(bx+4a)\sqrt{bx+a}, 2\sqrt{-a}a \arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + \frac{2}{3}(bx+4a)\sqrt{bx+a}\right]$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x,x, algorithm="fricas")
```

output
```
[a^(3/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/3*(b*x + 4*a)*sq
rt(b*x + a), 2*sqrt(-a)*a*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2/3*(b*x + 4*
a)*sqrt(b*x + a)]
```

3.125.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x}\,dx = \int \frac{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}{x}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(3/2)/x,x)
```

output
```
Integral(atanh(tanh(a + b*x))**(3/2)/x, x)
```

3.125.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x}\,dx = \int \frac{\operatorname{artanh}(\tanh(bx+a))^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^(3/2)/x, x)
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.63

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x}\,dx = \frac{1}{3}\sqrt{2}\left(\frac{3\sqrt{2}a^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{2}(bx+a)^{\frac{3}{2}} + 3\sqrt{2}\sqrt{bx+a}\,a\right)$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x,x, algorithm="giac")
```

output
```
1/3*sqrt(2)*(3*sqrt(2)*a^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + sqrt(
2)*(b*x + a)^(3/2) + 3*sqrt(2)*sqrt(b*x + a)*a)
```

3.125.9 Mupad [B] (verification not implemented)

Time = 8.14 (sec) , antiderivative size = 501, normalized size of antiderivative = 5.51

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{4\,b\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}{3}-2\ldots}{b}$$

$$+\frac{2\,b\,x\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{3}$$

$$+\frac{\sqrt{2}\,\ln\left(\frac{\left(\sqrt{2}\,b\,x-\sqrt{2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)+\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+\ldots}\right.}\ldots}{x\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}\right.\ldots}{4}$$

input
```
int(atanh(tanh(a + b*x))^(3/2)/x,x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((4*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/3 - 2*b*(l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)))/b + (2^(1/2)*log((((log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) +
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^
(1/2)*b*x)*4i)/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b
*x)^(3/2)*1i)/4 + (2*b*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/3
```

3.126 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^2}\,dx$

3.126.1 Optimal result

Integrand size = 15, antiderivative size = 81

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^2}\,dx = -3b\arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))} + 3b\sqrt{\operatorname{arctanh}(\tanh(a+bx))} - \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x}$$

output
```
-arctanh(tanh(b*x+a))^(3/2)/x-3*b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-a
rctanh(tanh(b*x+a)))^(1/2))*(b*x-arctanh(tanh(b*x+a)))^(1/2)+3*b*arctanh(t
anh(b*x+a))^(1/2)
```

3.126.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^2}\,dx = 3b\sqrt{\operatorname{arctanh}(\tanh(a+bx))} - \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x} - 3b\text{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^2,x]
```

output
```
3*b*Sqrt[ArcTanh[Tanh[a + b*x]]] - ArcTanh[Tanh[a + b*x]]^(3/2)/x - 3*b*Ar
cTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*
Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
```

3.126.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2599, 2590, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^2} \, dx$$

$$\Big\downarrow \; 2599$$

$$\frac{3}{2}b\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x}dx-\frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x}$$

$$\Big\downarrow \; 2590$$

$$\frac{3}{2}b\left(2\sqrt{\text{arctanh}(\tanh(a+bx))}-(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx\right)-\frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x}$$

$$\Big\downarrow \; 2592$$

$$\frac{3}{2}b\left(2\sqrt{\text{arctanh}(\tanh(a+bx))}-2\sqrt{bx-\text{arctanh}(\tanh(a+bx))}\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)\right)-\frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^2,x]
```

output
```
(3*b*(-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b
*x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]] + 2*Sqrt[ArcTanh[Tanh[a + b*x]]]
))/2 - ArcTanh[Tanh[a + b*x]]^(3/2)/x
```

3.126.3.1 Defintions of rubi rules used

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.126.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

method	result
default	$2b\left(\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}+\frac{\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{2}+\frac{bx}{2}\right)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{xb}-\frac{3\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}{}\right.$

input
```
int(arctanh(tanh(b*x+a))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2*b*(arctanh(tanh(b*x+a))^(1/2)+(-1/2*arctanh(tanh(b*x+a))+1/2*b*x)*arctan
h(tanh(b*x+a))^(1/2)/x/b-3/2*(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arct
anh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^2}\,dx = \left[\frac{3\sqrt{a}bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2(2bx-a)\sqrt{bx+a}}{2x}, \frac{3\sqrt{-a}bx\arctan\left(\right.}{}\right.$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(3*sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b*
x - a)*sqrt(b*x + a))/x, (3*sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a)
+ (2*b*x - a)*sqrt(b*x + a))/x]
```

3.126.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^2}\,dx = \int \frac{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}{x^2}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(3/2)/x**2,x)
```

output
```
Integral(atanh(tanh(a + b*x))**(3/2)/x**2, x)
```

3.126.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^2}\,dx = \int \frac{\operatorname{artanh}(\tanh(bx+a))^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^(3/2)/x^2, x)
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^2}\,dx = \frac{\sqrt{2}\left(\frac{3\sqrt{2}ab^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{2}\sqrt{bx+a}b^2 - \frac{\sqrt{2}\sqrt{bx+a}ab}{x}\right)}{2b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(3*sqrt(2)*a*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*s
qrt(2)*sqrt(b*x + a)*b^2 - sqrt(2)*sqrt(b*x + a)*a*b/x)/b
```

3.126.9 Mupad [B] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 459, normalized size of antiderivative = 5.67

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^2}\,dx = 3b\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2} - \frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}$$
$$+ \frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2} - \frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{2x}$$
$$- \frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2} - \frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{2x}$$
$$+b\ln\left(-\frac{4\sqrt{2}\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right) - 4\sqrt{2}\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right) + 8\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2} - \frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}\sqrt{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right) -}}{x\sqrt{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right) - \ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right) - 2bx}}\right.$$

input
```
int(atanh(tanh(a + b*x))^(3/2)/x^2,x)
```

output
```
3*b*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2) + (log(1/(exp(2*a)*exp(2*b*x) + 1))*(log((ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*
x) + 1))/2)^(1/2))/(2*x) - (log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(ex
p(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(2*x) + b*log(-(4*2^(1/2)*log(1/(exp(2*a
)*exp(2*b*x) + 1)) - 4*2^(1/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 8*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - lo
g(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log((exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) - log(1/(exp(2*a)*exp(2*b*x) + 1)) - 2*b*x)^(1/2) + 4*
2^(1/2)*b*x)/(x*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - lo
g(1/(exp(2*a)*exp(2*b*x) + 1)) - 2*b*x)^(1/2)))*((9*log((exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)))/8 - (9*log(1/(exp(2*a)*exp(2*b*x) + 1)))/8
 - (9*b*x)/4)^(1/2)
```

3.126. $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^2}\,dx$

3.127 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^3}\,dx$

3.127.1 Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^3}\,dx = \frac{3b^2\arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{4\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}} - \frac{3b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{4x} - \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{2x^2}$$

output
```
-1/2*arctanh(tanh(b*x+a))^(3/2)/x^2+3/4*b^2*arctan(arctanh(tanh(b*x+a))^(1
/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-arctanh(tanh(b*x+a)))^(1/2)-3/4
*b*arctanh(tanh(b*x+a))^(1/2)/x
```

3.127.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^3}\,dx = \frac{1}{4}\left(-\frac{3b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x} - \frac{2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^2} - \frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^3,x]
```

output
```
((-3*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/x - (2*ArcTanh[Tanh[a + b*x]]^(3/2))/
x^2 - (3*b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Ta
nh[a + b*x]]]])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])/4
```

3.127.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2599, 2599, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^3} \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{3}{4} b \int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^2} dx - \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{2x^2}$$

$$\Big\downarrow \text{2599}$$

$$\frac{3}{4} b \left(\frac{1}{2} b \int \frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}} dx - \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x}\right) - \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{2x^2}$$

$$\Big\downarrow \text{2592}$$

$$\frac{3}{4} b \left(\frac{b \arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}} - \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x}\right) - \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{2x^2}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^3,x]
```

output
```
(3*b*((b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b
*x]]]])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]] - Sqrt[ArcTanh[Tanh[a + b*x]]]/
x))/4 - ArcTanh[Tanh[a + b*x]]^(3/2)/(2*x^2)
```

3.127.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.127.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

method	result
default	$2b^2\left(\frac{-\frac{5\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{8}+\left(\frac{3\operatorname{arctanh}(\tanh(bx+a))}{8}-\frac{3bx}{8}\right)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{x^2b^2}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{8\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)$

input
```
int(arctanh(tanh(b*x+a))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2*b^2*((-5/8*arctanh(tanh(b*x+a))^(3/2)+(3/8*arctanh(tanh(b*x+a))-3/8*b*x)
*arctanh(tanh(b*x+a))^(1/2))/x^2/b^2-3/8/(arctanh(tanh(b*x+a))-b*x)^(1/2)*
arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^3}\,dx = \left[\frac{3\sqrt{a}b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(5abx+2a^2\right)\sqrt{bx+a}}{8ax^2}, \frac{3\sqrt{-a}b^2x^2\right.$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(
5*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sq
rt(b*x + a)*sqrt(-a)/a) - (5*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2)]
```

3.127.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^3}\,dx = \int \frac{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}{x^3}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(3/2)/x**3,x)
```

output
```
Integral(atanh(tanh(a + b*x))**(3/2)/x**3, x)
```

3.127.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^3}\,dx = \int \frac{\operatorname{artanh}(\tanh(bx+a))^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^(3/2)/x^3, x)
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.79

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^3}\,dx = \frac{\sqrt{2}\left(\frac{3\sqrt{2}b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{2}\left(5\,(bx+a)^{\frac{3}{2}}b^3 - 3\sqrt{bx+a}ab^3\right)}{b^2x^2}\right)}{8\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(3*sqrt(2)*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(
2)*(5*(b*x + a)^(3/2)*b^3 - 3*sqrt(b*x + a)*a*b^3)/(b^2*x^2))/b
```

3.127.9 Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 609, normalized size of antiderivative = 6.62

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^3}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)}{2\,x^2\,\left(2\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - 2\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 4\,b\,x\right)}$$

$$- \frac{b\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{5\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{4} - \frac{5\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{4} + \frac{5\,b\,x}{2}\right)}{x\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)}$$

$$+ \frac{\sqrt{2}\,b^2\,\ln\left(\frac{\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x}\,\left(\sqrt{2}\,b\,x - \sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right) + \sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} - \frac{\ln\left(\cdots\right)}{\cdots}}\right.}{x}\cdots}{8\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x}}$$

input
```
int(atanh(tanh(a + b*x))^(3/2)/x^3,x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*x^2*(2*log(2/
(exp(2*a)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 4*b*x)) + (2^(1/2)*b^2*log(((log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*16i)/x)*3i)/(8*(log(2/(exp(2*a)*e
xp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^(1/2)) - (b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
)/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((5*log(2/(exp(2*a)*exp(2*
b*x) + 1)))/4 - (5*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))
/4 + (5*b*x)/2))/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))
```

3.127. $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^3}\,dx$

3.128 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^4}\,dx$

3.128.1 Optimal result

Integrand size = 15, antiderivative size = 146

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^4}\,dx = \frac{b^3\arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))^{3/2}} - \frac{b^2}{8x\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} + \frac{b^3}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} - \frac{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{4x^2} - \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3x^3}$$

output
```
1/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2)
)/(b*x-arctanh(tanh(b*x+a)))^(3/2)-1/3*arctanh(tanh(b*x+a))^(3/2)/x^3-1/8*
b^2/x/arctanh(tanh(b*x+a))^(1/2)+1/8*b^3/(b*x-arctanh(tanh(b*x+a)))/arctan
h(tanh(b*x+a))^(1/2)-1/4*b*arctanh(tanh(b*x+a))^(1/2)/x^2
```

3.128.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^4}\,dx = \frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{8(-bx+\operatorname{arctanh}(\tanh(a+bx)))^{3/2}} + \sqrt{\operatorname{arctanh}(\tanh(a+bx))}\left(-\frac{7b}{12x^2}-\frac{b^2}{8x(-bx+\operatorname{arctanh}(\tanh(a+bx)))}-\frac{-bx+\operatorname{arctanh}(\tanh(a+bx))}{3x^3}\right)$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^4,x]
```

output
```
(b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b
*x]]]])/(8*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2)) + Sqrt[ArcTanh[Tanh[a
+ b*x]]]*((-7*b)/(12*x^2) - b^2/(8*x*(-(b*x) + ArcTanh[Tanh[a + b*x]])) -
(-(b*x) + ArcTanh[Tanh[a + b*x]])/(3*x^3))
```

3.128.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2599, 2599, 2599, 2594, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^4}\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{2}b\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^3}dx-\frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3x^3}$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{2}b\left(\frac{1}{4}b\int \frac{1}{x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}dx-\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{2x^2}\right)-\frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3x^3}$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{1}{2}b\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}dx-\frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)-\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{2x^2}\right)-\frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{3x^3}$$

$\downarrow$ 2594

$$\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{1}{2}b\left(-\frac{\int \frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)-\frac{}{x\sqrt{\cdots}}\right.\right.$$
$$\frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{3x^3}$$

$\downarrow$ 2592

$$\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{1}{2}b\left(-\frac{2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}\right.\right.\right.$$
$$\frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{3x^3}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^4,x]
```

output
```
(b*((b*(-1/2*(b*((-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTan
h[Tanh[a + b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2/((b*x - ArcTa
nh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]]))) - 1/(x*Sqrt[ArcTanh[Tan
h[a + b*x]]])))/4 - Sqrt[ArcTanh[Tanh[a + b*x]]]/(2*x^2)))/2 - ArcTanh[Tan
h[a + b*x]]^(3/2)/(3*x^3)
```

3.128.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.128.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79

method	result
default	$2b^3\left(\frac{-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{16(\operatorname{arctanh}(\tanh(bx+a))-bx)}-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{6}+\left(\frac{\operatorname{arctanh}(\tanh(bx+a))}{16}-\frac{bx}{16}\right)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{x^3b^3}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arc}}}{\sqrt{\operatorname{arcta}}}\right.}{16(\operatorname{arctanh}(\operatorname{ta}}\right.$

input
```
int(arctanh(tanh(b*x+a))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2*b^3*((-1/16/(arctanh(tanh(b*x+a))-b*x)*arctanh(tanh(b*x+a))^(5/2)-1/6*ar
ctanh(tanh(b*x+a))^(3/2)+(1/16*arctanh(tanh(b*x+a))-1/16*b*x)*arctanh(tanh
(b*x+a))^(1/2))/x^3/b^3+1/16/(arctanh(tanh(b*x+a))-b*x)^(3/2)*arctanh(arct
anh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

3.128.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^4}\,dx = \left[\frac{3\sqrt{a}b^3x^3\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\,(3\,ab^2x^2+14\,a^2bx+8\,a^3)\sqrt{bx+a}}{48\,a^2x^3},\right.$$
$$\left.-\frac{3\sqrt{-a}b^3x^3\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+(3\,ab^2x^2+14\,a^2bx+8\,a^3)\sqrt{bx+a}}{24\,a^2x^3}\right]$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(3*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*
(3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^2*x^3), -1/24*(3*sqrt
(-a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 + 14*a^2*b*x
+ 8*a^3)*sqrt(b*x + a))/(a^2*x^3)]
```

3.128.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^4}\,dx = \int \frac{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}{x^4}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(3/2)/x**4,x)
```

output
```
Integral(atanh(tanh(a + b*x))**(3/2)/x**4, x)
```

3.128.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^4}\,dx = \int \frac{\operatorname{artanh}(\tanh(bx+a))^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^(3/2)/x^4, x)
```

3.128.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.64

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^4}\,dx = -\frac{\sqrt{2}\left(\frac{3\sqrt{2}b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{\sqrt{2}\left(3(bx+a)^{\frac{5}{2}}b^4 + 8(bx+a)^{\frac{3}{2}}ab^4 - 3\sqrt{bx+a}a^2b^4\right)}{ab^3x^3}\right)}{48\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^4,x, algorithm="giac")
```

output
```
-1/48*sqrt(2)*(3*sqrt(2)*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) +
 sqrt(2)*(3*(b*x + a)^(5/2)*b^4 + 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a
)*a^2*b^4)/(a*b^3*x^3))/b
```

3.128.9 Mupad [B] (verification not implemented)

Time = 8.19 (sec) , antiderivative size = 1019, normalized size of antiderivative = 6.98

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^4}\,dx = \text{Too large to display}$$

input
```
int(atanh(tanh(a + b*x))^(3/2)/x^4,x)
```

output
```
(11*b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/
(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(12*x*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + ((
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)
*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*x^3*(3*log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 6*b*x)) - (2*b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(x*(3*log(2/(exp
(2*a)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 6*b*x)) + (2^(1/2)*b^3*log(((2*2^(1/2)*a + (log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)
^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(2*a - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1))
 + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 8*a^3 - 6*a*
(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b...
```

3.129 $\int x^4 \mathbf{arctanh}(\tanh(a+bx))^{5/2}\,dx$

3.129.1 Optimal result

Integrand size = 15, antiderivative size = 101

$$\int x^4 \text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{2x^4\text{arctanh}(\tanh(a+bx))^{7/2}}{7b} - \frac{16x^3\text{arctanh}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{32x^2\text{arctanh}(\tanh(a+bx))^{11/2}}{231b^3} - \frac{128x\text{arctanh}(\tanh(a+bx))^{13/2}}{3003b^4} + \frac{256\text{arctanh}(\tanh(a+bx))^{15/2}}{45045b^5}$$

output
```
2/7*x^4*arctanh(tanh(b*x+a))^(7/2)/b-16/63*x^3*arctanh(tanh(b*x+a))^(9/2)/
b^2+32/231*x^2*arctanh(tanh(b*x+a))^(11/2)/b^3-128/3003*x*arctanh(tanh(b*x
+a))^(13/2)/b^4+256/45045*arctanh(tanh(b*x+a))^(15/2)/b^5
```

3.129.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int x^4\text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{2\text{arctanh}(\tanh(a+bx))^{7/2}\left(6435b^4x^4 - 5720b^3x^3\text{arctanh}(\tanh(a+bx)) + 3120b^2x^2\text{arctanh}(\text{t}\right.}{45045b^5}$$

input
```
Integrate[x^4*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(7/2)*(6435*b^4*x^4 - 5720*b^3*x^3*ArcTanh[Tanh[
a + b*x]] + 3120*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 960*b*x*ArcTanh[Tanh[a
 + b*x]]^3 + 128*ArcTanh[Tanh[a + b*x]]^4))/(45045*b^5)
```

3.129.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2599, 2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{arctanh}(\tanh(a+bx))^{5/2} \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^4 \text{arctanh}(\tanh(a+bx))^{7/2}}{7b} - \frac{8\int x^3 \text{arctanh}(\tanh(a+bx))^{7/2}dx}{7b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^4 \text{arctanh}(\tanh(a+bx))^{7/2}}{7b} - \frac{8\left(\frac{2x^3 \text{arctanh}(\tanh(a+bx))^{9/2}}{9b} - \frac{2\int x^2 \text{arctanh}(\tanh(a+bx))^{9/2}dx}{3b}\right)}{7b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^4 \text{arctanh}(\tanh(a+bx))^{7/2}}{7b} - \frac{8\left(\frac{2x^3 \text{arctanh}(\tanh(a+bx))^{9/2}}{9b} - \frac{2\left(\frac{2x^2 \text{arctanh}(\tanh(a+bx))^{11/2}}{11b} - \frac{4\int x \text{arctanh}(\tanh(a+bx))^{11/2}dx}{11b}\right)}{3b}\right)}{7b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^4 \text{arctanh}(\tanh(a+bx))^{7/2}}{7b} - \frac{8\left(\frac{2x^3 \text{arctanh}(\tanh(a+bx))^{9/2}}{9b} - \frac{2\left(\frac{2x^2 \text{arctanh}(\tanh(a+bx))^{11/2}}{11b} - \frac{4\left(\frac{2x \text{arctanh}(\tanh(a+bx))^{13/2}}{13b} - \frac{2\int \text{arctanh}(\tanh(a+bx))^{13/2}dx}{13b}\right)}{11b}\right)}{3b}\right)}{7b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{2x^4\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{8\left(\frac{2x^3\text{arctanh}(\tanh(a+bx))^{9/2}}{9b}-\frac{2\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{11/2}}{11b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{13/2}}{13b}-\frac{2\int\text{arctanh}(\tanh(a+bx))^{13/2}d\text{arcta}}{13b^2}\right.}{11b}\right.}{3b}\right.}{7b}$$

$\downarrow$ 15

$$\frac{2x^4\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{8\left(\frac{2x^3\text{arctanh}(\tanh(a+bx))^{9/2}}{9b}-\frac{2\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{11/2}}{11b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{13/2}}{13b}-\frac{4\text{arctanh}(\tanh(a+bx))^{15/2}}{195b^2}\right)}{11b}\right)}{3b}\right)}{7b}$$

input
```
Int[x^4*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(2*x^4*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (8*((2*x^3*ArcTanh[Tanh[a + b
*x]]^(9/2))/(9*b) - (2*((2*x^2*ArcTanh[Tanh[a + b*x]]^(11/2))/(11*b) - (4*
((2*x*ArcTanh[Tanh[a + b*x]]^(13/2))/(13*b) - (4*ArcTanh[Tanh[a + b*x]]^(1
5/2))/(195*b^2)))/(11*b)))/(3*b)))/(7*b)
```

3.129.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.129. $\int x^4\text{arctanh}(\tanh(a+bx))^{5/2}\,dx$

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.129.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.52

method	result
default	$\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{15}{2}}}{15}+\frac{2(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{13}{2}}}{13}+\frac{2\left(2(bx-\operatorname{arctanh}(\tanh(bx+a)))^{2}+(-2\operatorname{arctanh}(\tanh(bx+a)\right.}{11}$

input
```
int(x^4*arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^5*(1/15*arctanh(tanh(b*x+a))^(15/2)+1/13*(-4*arctanh(tanh(b*x+a))+4*b*
x)*arctanh(tanh(b*x+a))^(13/2)+1/11*(2*(b*x-arctanh(tanh(b*x+a)))^2+(-2*ar
ctanh(tanh(b*x+a))+2*b*x)^2)*arctanh(tanh(b*x+a))^(11/2)+2/9*(b*x-arctanh(
tanh(b*x+a)))^2*(-2*arctanh(tanh(b*x+a))+2*b*x)*arctanh(tanh(b*x+a))^(9/2)
+1/7*(b*x-arctanh(tanh(b*x+a)))^4*arctanh(tanh(b*x+a))^(7/2))
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.85

$$\int x^4\operatorname{arctanh}(\tanh(a+bx))^{5/2}\,dx=\frac{2\left(3003\,b^7x^7+7161\,ab^6x^6+4473\,a^2b^5x^5+35\,a^3b^4x^4-40\,a^4b^3x^3+48\,a^5b^2x^2-64\,a^6bx+128\right.}{45045\,b^5}$$

input
```
integrate(x^4*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/45045*(3003*b^7*x^7 + 7161*a*b^6*x^6 + 4473*a^2*b^5*x^5 + 35*a^3*b^4*x^4
 - 40*a^4*b^3*x^3 + 48*a^5*b^2*x^2 - 64*a^6*b*x + 128*a^7)*sqrt(b*x + a)/b
^5
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int x^4 \operatorname{arctanh}(\tanh(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int x^4 \operatorname{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{2\,(3003\,b^5x^5+1155\,ab^4x^4-840\,a^2b^3x^3+560\,a^3b^2x^2-320\,a^4bx+128\,a^5)(bx+a)^{\frac{5}{2}}}{45045\,b^5}$$

input
```
integrate(x^4*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
2/45045*(3003*b^5*x^5 + 1155*a*b^4*x^4 - 840*a^2*b^3*x^3 + 560*a^3*b^2*x^2
 - 320*a^4*b*x + 128*a^5)*(b*x + a)^(5/2)/b^5
```

3.129.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(81) = 162$.

Time = 0.27 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.41

$$\int x^4 \operatorname{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{\sqrt{2}\left(\frac{143\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)a^3}{b^4}+\frac{195\sqrt{2}\left(63\,(bx+a)^{\frac{11}{2}}-385\right.}{}\right.}{}$$

input
```
integrate(x^4*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
1/45045*sqrt(2)*(143*sqrt(2)*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a +
 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4
)*a^3/b^4 + 195*sqrt(2)*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990
*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4
 - 693*sqrt(b*x + a)*a^5)*a^2/b^4 + 45*sqrt(2)*(231*(b*x + a)^(13/2) - 163
8*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3
 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a
)*a^6)*a/b^4 + 7*sqrt(2)*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a +
 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^
(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*s
qrt(b*x + a)*a^7)/b^4)/b
```

3.129.9 Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 2681, normalized size of antiderivative = 26.54

$$\int x^4 \text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \text{Too large to display}$$

input
```
int(x^4*atanh(tanh(a + b*x))^(5/2),x)
```

output
```
(2*b^2*x^7*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log
(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/15 + (x^5*(log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1
/2)*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (12*(3*b^2*(log(2/(exp(2*a)*exp
(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x) - (28*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/15)*(log(2/(exp(2*a)*exp(2*b*
x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*
x))/(13*b)))/(11*b) - (x^6*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (28*b^2*(log
(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1))/2 + b*x))/15)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(13*b) - (x^4*(l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*
exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/4 - (10*((3*b*(log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x)^2)/2 - (12*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (28*...
```

3.129. $\int x^4 \text{arctanh}(\tanh(a+bx))^{5/2}\,dx$

3.130 $\int x^3 \textbf{arctanh}(\tanh(a+bx))^{5/2}\,dx$

3.130.1 Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x^3\text{arctanh}(\tanh(a+bx))^{5/2}\,dx=\frac{2x^3\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{4x^2\text{arctanh}(\tanh(a+bx))^{9/2}}{21b^2}+\frac{16x\text{arctanh}(\tanh(a+bx))^{11/2}}{231b^3}-\frac{32\text{arctanh}(\tanh(a+bx))^{13/2}}{3003b^4}$$

output
```
2/7*x^3*arctanh(tanh(b*x+a))^(7/2)/b-4/21*x^2*arctanh(tanh(b*x+a))^(9/2)/b
^2+16/231*x*arctanh(tanh(b*x+a))^(11/2)/b^3-32/3003*arctanh(tanh(b*x+a))^(
13/2)/b^4
```

3.130.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int x^3\text{arctanh}(\tanh(a+bx))^{5/2}\,dx=\frac{2\text{arctanh}(\tanh(a+bx))^{7/2}\left(429b^3x^3-286b^2x^2\text{arctanh}(\tanh(a+bx))+104bx\text{arctanh}(\tanh(a\right.}{3003b^4}$$

input
```
Integrate[x^3*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(7/2)*(429*b^3*x^3 - 286*b^2*x^2*ArcTanh[Tanh[a
+ b*x]] + 104*b*x*ArcTanh[Tanh[a + b*x]]^2 - 16*ArcTanh[Tanh[a + b*x]]^3))
/(3003*b^4)
```

3.130.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{arctanh}(\tanh(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^3\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{6\int x^2\text{arctanh}(\tanh(a+bx))^{7/2}dx}{7b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^3\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{9/2}}{9b}-\frac{4\int x\text{arctanh}(\tanh(a+bx))^{9/2}dx}{9b}\right)}{7b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^3\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{9/2}}{9b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{11/2}}{11b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{11/2}dx}{11b}\right)}{9b}\right)}{7b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{2x^3\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{9/2}}{9b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{11/2}}{11b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{11/2}d\text{arctanh}(\tanh(a+bx))}{11b^2}\right)}{9b}\right)}{7b}$$

$$\Big\downarrow \text{15}$$

$$\frac{2x^3\text{arctanh}(\tanh(a+bx))^{7/2}}{7b} - \frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{9/2}}{9b} - \frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{11/2}}{11b} - \frac{4\text{arctanh}(\tanh(a+bx))^{13/2}}{143b^2}\right)}{9b}\right)}{7b}$$

input
```
Int[x^3*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(2*x^3*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (6*((2*x^2*ArcTanh[Tanh[a + b
*x]]^(9/2))/(9*b) - (4*((2*x*ArcTanh[Tanh[a + b*x]]^(11/2))/(11*b) - (4*Ar
cTanh[Tanh[a + b*x]]^(13/2))/(143*b^2)))/(9*b)))/(7*b)
```

3.130.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.130.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.55

method	result
default	$\frac{\frac{2\,\text{arctanh}(\tanh(bx+a))^{\frac{13}{2}}}{13}+\frac{2(-3\,\text{arctanh}(\tanh(bx+a))+3bx)\,\text{arctanh}(\tanh(bx+a))^{\frac{11}{2}}}{11}+\frac{2\left((bx-\text{arctanh}(\tanh(bx+a)))(-2\,\text{arctanh}(\tanh(bx+a))+2b\cdots\right.}{9\cdots}}{b^4}$

input
```
int(x^3*arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^4*(1/13*arctanh(tanh(b*x+a))^(13/2)+1/11*(-3*arctanh(tanh(b*x+a))+3*b*
x)*arctanh(tanh(b*x+a))^(11/2)+1/9*((b*x-arctanh(tanh(b*x+a)))*(-2*arctanh
(tanh(b*x+a))+2*b*x)+(b*x-arctanh(tanh(b*x+a)))^2)*arctanh(tanh(b*x+a))^(9
/2)+1/7*(b*x-arctanh(tanh(b*x+a)))^3*arctanh(tanh(b*x+a))^(7/2))
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int x^3\text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{2\,(231\,b^6x^6+567\,ab^5x^5+371\,a^2b^4x^4+5\,a^3b^3x^3-6\,a^4b^2x^2+8\,a^5bx-16\,a^6)\sqrt{bx+a}}{3003\,b^4}$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/3003*(231*b^6*x^6 + 567*a*b^5*x^5 + 371*a^2*b^4*x^4 + 5*a^3*b^3*x^3 - 6*
a^4*b^2*x^2 + 8*a^5*b*x - 16*a^6)*sqrt(b*x + a)/b^4
```

3.130.6 Sympy [A] (verification not implemented)

Time = 76.87 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18

$$\int x^3\text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \begin{cases} \frac{2x^3\,\text{atanh}^{\frac{7}{2}}\,(\tanh{(a+bx)})}{7b} - \frac{4x^2\,\text{atanh}^{\frac{9}{2}}\,(\tanh{(a+bx)})}{21b^2} + \frac{16x\,\text{atanh}^{\frac{11}{2}}\,(\tanh{(a+bx)})}{231b^3} - \frac{32\,\text{atanh}^{\frac{13}{2}}\,(\tanh{(a+bx)})}{3003b^4} & \text{fo} \\ \frac{x^4\,\text{atanh}^{\frac{5}{2}}\,(\tanh{(a)})}{4} & \text{ot} \end{cases}$$

input
```
integrate(x**3*atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Piecewise((2*x**3*atanh(tanh(a + b*x))**(7/2)/(7*b) - 4*x**2*atanh(tanh(a
+ b*x))**(9/2)/(21*b**2) + 16*x*atanh(tanh(a + b*x))**(11/2)/(231*b**3) -
32*atanh(tanh(a + b*x))**(13/2)/(3003*b**4), Ne(b, 0)), (x**4*atanh(tanh(a
))**(5/2)/4, True))
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

$$\int x^3 \operatorname{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{2\,(231\,b^4x^4 + 105\,ab^3x^3 - 70\,a^2b^2x^2 + 40\,a^3bx - 16\,a^4)(bx+a)^{\frac{5}{2}}}{3003\,b^4}$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
2/3003*(231*b^4*x^4 + 105*a*b^3*x^3 - 70*a^2*b^2*x^2 + 40*a^3*b*x - 16*a^4
)*(b*x + a)^(5/2)/b^4
```

3.130.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(64) = 128.

Time = 0.27 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.70

$$\int x^3 \operatorname{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{\sqrt{2}\left(\frac{429\,\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}} - 21\,(bx+a)^{\frac{5}{2}}a + 35\,(bx+a)^{\frac{3}{2}}a^2 - 35\,\sqrt{bx+a}a^3\right)a^3}{b^3} + \frac{143\,\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}} - 180\,(bx+a)^{\frac{7}{2}}a + 378\,(bx+a\right.}{b^3}\right.}{}$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
1/15015*sqrt(2)*(429*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 3
5*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^3/b^3 + 143*sqrt(2)*(35*(b
*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x
 + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a^2/b^3 + 65*sqrt(2)*(63*(b*x + a
)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a
)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a/b^3 + 5*
sqrt(2)*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(
9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x
 + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)/b^3)/b
```

3.130.9 Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 2235, normalized size of antiderivative = 27.94

$$\int x^3 \text{arctanh}(\tanh(a + bx))^{5/2} \, dx = \text{Too large to display}$$

input
```
int(x^3*atanh(tanh(a + b*x))^(5/2),x)
```

output
```
(2*b^2*x^6*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log
(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/13 + (x^4*(log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1
/2)*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (10*(3*b^2*(log(2/(exp(2*a)*exp
(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x) - (24*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/13)*(log(2/(exp(2*a)*exp(2*b*
x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*
x))/(11*b)))/(9*b) - (x^5*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (24*b^2*(log(
2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1))/2 + b*x))/13)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(11*b) - (x^3*(lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*e
xp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/4 - (8*((3*b*(log(2/
(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^2)/2 - (10*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (24*b^...
```

3.131 $\int x^2 \mathbf{arctanh}(\tanh(a+bx))^{5/2}\,dx$

3.131.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^2 \text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{2x^2\text{arctanh}(\tanh(a+bx))^{7/2}}{7b} - \frac{8x\text{arctanh}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{16\text{arctanh}(\tanh(a+bx))^{11/2}}{693b^3}$$

output
```
2/7*x^2*arctanh(tanh(b*x+a))^(7/2)/b-8/63*x*arctanh(tanh(b*x+a))^(9/2)/b^2
+16/693*arctanh(tanh(b*x+a))^(11/2)/b^3
```

3.131.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int x^2 \text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{2\text{arctanh}(\tanh(a+bx))^{7/2}\left(99b^2x^2 - 44bx\text{arctanh}(\tanh(a+bx)) + 8\text{arctanh}(\tanh(a+bx))^2\right)}{693b^3}$$

input
```
Integrate[x^2*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(7/2)*(99*b^2*x^2 - 44*b*x*ArcTanh[Tanh[a + b*x]
] + 8*ArcTanh[Tanh[a + b*x]]^2))/(693*b^3)
```

3.131.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arctanh}(\tanh(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2x^2\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\int x\text{arctanh}(\tanh(a+bx))^{7/2}dx}{7b}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2x^2\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{9/2}}{9b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{9/2}dx}{9b}\right)}{7b}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{2x^2\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{9/2}}{9b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{9/2}d\text{arctanh}(\tanh(a+bx))}{9b^2}\right)}{7b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2x^2\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{9/2}}{9b}-\frac{4\text{arctanh}(\tanh(a+bx))^{11/2}}{99b^2}\right)}{7b}$$

input
```
Int[x^2*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(2*x^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (4*((2*x*ArcTanh[Tanh[a + b*x
]]^(9/2))/(9*b) - (4*ArcTanh[Tanh[a + b*x]]^(11/2))/(99*b^2)))/(7*b)
```

3.131.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.131.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

method	result	si
default	$\frac{\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{11}{2}}}{11}+\frac{2(-2 \operatorname{arctanh}(\tanh(bx+a))+2bx) \operatorname{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))^{2} \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}}{b^{3}}$	69

input
```
int(x^2*arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^3*(1/11*arctanh(tanh(b*x+a))^(11/2)+1/9*(-2*arctanh(tanh(b*x+a))+2*b*x
)*arctanh(tanh(b*x+a))^(9/2)+1/7*(b*x-arctanh(tanh(b*x+a)))^2*arctanh(tanh
(b*x+a))^(7/2))
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int x^2 \text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{2\,(63\,b^5x^5+161\,ab^4x^4+113\,a^2b^3x^3+3\,a^3b^2x^2-4\,a^4bx+8\,a^5)\sqrt{bx+a}}{693\,b^3}$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/693*(63*b^5*x^5 + 161*a*b^4*x^4 + 113*a^2*b^3*x^3 + 3*a^3*b^2*x^2 - 4*a^
4*b*x + 8*a^5)*sqrt(b*x + a)/b^3
```

3.131.6 Sympy [A] (verification not implemented)

Time = 40.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int x^2 \text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \begin{cases} \frac{2x^2 \operatorname{atanh}^{\frac{7}{2}}(\tanh(a+bx))}{7b} - \frac{8x \operatorname{atanh}^{\frac{9}{2}}(\tanh(a+bx))}{63b^2} + \frac{16 \operatorname{atanh}^{\frac{11}{2}}(\tanh(a+bx))}{693b^3} & \text{for } b \neq 0 \\ \frac{x^3 \operatorname{atanh}^{\frac{5}{2}}(\tanh(a))}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Piecewise((2*x**2*atanh(tanh(a + b*x))**(7/2)/(7*b) - 8*x*atanh(tanh(a + b
*x))**(9/2)/(63*b**2) + 16*atanh(tanh(a + b*x))**(11/2)/(693*b**3), Ne(b,
0)), (x**3*atanh(tanh(a))**(5/2)/3, True))
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int x^2 \text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{2\,(63\,b^3x^3+35\,ab^2x^2-20\,a^2bx+8\,a^3)(bx+a)^{\frac{5}{2}}}{693\,b^3}$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
2/693*(63*b^3*x^3 + 35*a*b^2*x^2 - 20*a^2*b*x + 8*a^3)*(b*x + a)^(5/2)/b^3
```

3.131.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(47) = 94$.

Time = 0.27 (sec) , antiderivative size = 248, normalized size of antiderivative = 4.20

$$\int x^2 \operatorname{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{\sqrt{2}\left(\frac{231\sqrt{2}\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)a^3}{b^2}+\frac{297\sqrt{2}\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}a^3\right.}{b^2}\right.}{}$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
1/3465*sqrt(2)*(231*sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15
*sqrt(b*x + a)*a^2)*a^3/b^2 + 297*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a
)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2/b^2 + 33*sq
rt(2)*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^
2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a/b^2 + 5*sqrt(2)*(63
*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386
*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)/b
^2)/b
```

3.131.9 Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 1789, normalized size of antiderivative = 30.32

$$\int x^2 \operatorname{arctanh}(\tanh(a+bx))^{5/2}\,dx = \text{Too large to display}$$

input
```
int(x^2*atanh(tanh(a + b*x))^(5/2),x)
```

output
```
(2*b^2*x^5*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log
(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/11 + (x^3*(log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1
/2)*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (8*(3*b^2*(log(2/(exp(2*a)*exp(
2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*
b*x) - (20*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/11)*(log(2/(exp(2*a)*exp(2*b*x
) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x
))/(9*b)))/(7*b) - (x^4*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (20*b^2*(log(2/
(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1))/2 + b*x))/11)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(9*b) - (x^2*(log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(
2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/4 - (6*((3*b*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^2)/2 - (8*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (20*b^2*(l...
```

3.131. $\int x^2 \text{arctanh}(\tanh(a+bx))^{5/2}\,dx$

3.132 $\int x\textbf{arctanh}(\tanh(a+bx))^{5/2}\,dx$

3.132.1 Optimal result

Integrand size = 13, antiderivative size = 38

$$\int x\text{arctanh}(\tanh(a+bx))^{5/2}\,dx=\frac{2x\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\text{arctanh}(\tanh(a+bx))^{9/2}}{63b^2}$$

output
```
2/7*x*arctanh(tanh(b*x+a))^(7/2)/b-4/63*arctanh(tanh(b*x+a))^(9/2)/b^2
```

3.132.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int x\text{arctanh}(\tanh(a+bx))^{5/2}\,dx=\frac{2(9bx-2\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{7/2}}{63b^2}$$

input
```
Integrate[x*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(2*(9*b*x - 2*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(7/2))/(63*b^
2)
```

3.132.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(\tanh(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{7/2}dx}{7b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{2x\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{7/2}d\text{arctanh}(\tanh(a+bx))}{7b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{2x\text{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\text{arctanh}(\tanh(a+bx))^{9/2}}{63b^2}$$

input
```
Int[x*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(2*x*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (4*ArcTanh[Tanh[a + b*x]]^(9/2)
)/(63*b^2)
```

3.132.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.132.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2(bx-\mathrm{arctanh}(\tanh(bx+a)))\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}}{b^2}$	42

input
```
int(x*arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^2*(1/9*arctanh(tanh(b*x+a))^(9/2)+1/7*(b*x-arctanh(tanh(b*x+a)))*arcta
nh(tanh(b*x+a))^(7/2))
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int x\mathrm{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{2\,(7\,b^4x^4+19\,ab^3x^3+15\,a^2b^2x^2+a^3bx-2\,a^4)\sqrt{bx+a}}{63\,b^2}$$

input
```
integrate(x*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/63*(7*b^4*x^4 + 19*a*b^3*x^3 + 15*a^2*b^2*x^2 + a^3*b*x - 2*a^4)*sqrt(b*
x + a)/b^2
```

3.132.6 Sympy [A] (verification not implemented)

Time = 22.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int x\text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \begin{cases} \frac{2x\,\text{atanh}^{\frac{7}{2}}\left(\tanh\left(a+bx\right)\right)}{7b} - \frac{4\,\text{atanh}^{\frac{9}{2}}\left(\tanh\left(a+bx\right)\right)}{63b^2} & \text{for } b\neq 0 \\ \frac{x^2\,\text{atanh}^{\frac{5}{2}}\left(\tanh\left(a\right)\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Piecewise((2*x*atanh(tanh(a + b*x))**(7/2)/(7*b) - 4*atanh(tanh(a + b*x))*
*(9/2)/(63*b**2), Ne(b, 0)), (x**2*atanh(tanh(a))**(5/2)/2, True))
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int x\text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{2\,(7\,b^2x^2+5\,abx-2\,a^2)(bx+a)^{\frac{5}{2}}}{63\,b^2}$$

input
```
integrate(x*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
2/63*(7*b^2*x^2 + 5*a*b*x - 2*a^2)*(b*x + a)^(5/2)/b^2
```

3.132.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(30) = 60.

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.18

$$\int x\text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{\sqrt{2}\left(\frac{105\,\sqrt{2}\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)a^3}{b}+\frac{63\,\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)a^2}{b}+\frac{27\,\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+\cdots\right.}{\cdots}\right.}{31\cdots}$$

input
```
integrate(x*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
1/315*sqrt(2)*(105*sqrt(2)*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^3/b + 6
3*sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2
)*a^2/b + 27*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x +
 a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a/b + sqrt(2)*(35*(b*x + a)^(9/2) -
180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3
+ 315*sqrt(b*x + a)*a^4)/b)/b
```

3.132.9 Mupad [B] (verification not implemented)

Time = 3.65 (sec) , antiderivative size = 773, normalized size of antiderivative = 20.34

$$\int x\text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \text{Too large to display}$$

input
```
int(x*atanh(tanh(a + b*x))^(5/2),x)
```

output
```
(log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1))^3*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(63*b^2) - (log((exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))^4*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(252*b^2) -
(log(1/(exp(2*a)*exp(2*b*x) + 1))^4*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(252*b^2) +
 (log(1/(exp(2*a)*exp(2*b*x) + 1))^3*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 -
log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(63*b^2) - (x*log(1/(exp(2*a)*e
xp(2*b*x) + 1))^3*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
- log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(28*b) + (x*log((exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(28*b) -
(log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1))^2*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 -
 log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(42*b^2) - (3*x*log(1/(exp(2*a
)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2*
(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*
exp(2*b*x) + 1))/2)^(1/2))/(28*b) + (3*x*log(1/(exp(2*a)*exp(2*b*x) + 1...
```

3.133 $\int \textbf{arctanh}(\tanh(a+bx))^{5/2}\,dx$

3.133.1 Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \operatorname{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{2\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{7b}$$

output
```
2/7*arctanh(tanh(b*x+a))^(7/2)/b
```

3.133.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \operatorname{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{2\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{7b}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b)
```

3.133.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(\tanh(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \operatorname{arctanh}(\tanh(a+bx))^{5/2}d\operatorname{arctanh}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{2\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{7b}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b)
```

3.133.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.133.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7b}$	15
default	$\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7b}$	15

input
```
int(arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*arctanh(tanh(b*x+a))^(7/2)/b
```

3.133.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(14) = 28.

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int \operatorname{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{2\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\sqrt{bx+a}}{7\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)/b
```

3.133.6 Sympy [A] (verification not implemented)

Time = 11.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \operatorname{arctanh}(\tanh(a+bx))^{5/2}\,dx = \begin{cases} \frac{2\operatorname{atanh}^{\frac{7}{2}}(\tanh(a+bx))}{7b} & \text{for } b \neq 0 \\ x\operatorname{atanh}^{\frac{5}{2}}(\tanh(a)) & \text{otherwise} \end{cases}$$

input
```
integrate(atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Piecewise((2*atanh(tanh(a + b*x))**(7/2)/(7*b), Ne(b, 0)), (x*atanh(tanh(a
))**(5/2), True))
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \operatorname{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
2/7*(b*x + a)^(7/2)/b
```

3.133.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(14) = 28.

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 7.56

$$\int \operatorname{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{\sqrt{2}\left(35\,\sqrt{2}\sqrt{bx+a}a^3+35\,\sqrt{2}\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)a^2+7\,\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)\right.}{35}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
1/35*sqrt(2)*(35*sqrt(2)*sqrt(b*x + a)*a^3 + 35*sqrt(2)*((b*x + a)^(3/2) -
 3*sqrt(b*x + a)*a)*a^2 + 7*sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2
)*a + 15*sqrt(b*x + a)*a^2)*a + sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^
(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3))/b
```

3.133.9 Mupad [B] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 337, normalized size of antiderivative = 18.72

$$\int \operatorname{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^3\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{28\,b}$$
$$-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^3\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{28\,b}$$
$$-\frac{3\,\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)\,\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)^2\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{28\,b}$$
$$+\frac{3\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)^2\,\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{28\,b}$$

input
```
int(atanh(tanh(a + b*x))^(5/2),x)
```

output
```
(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3*(log((exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2
)^(1/2))/(28*b) - (log(1/(exp(2*a)*exp(2*b*x) + 1))^3*(log((exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^
(1/2))/(28*b) - (3*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))^2*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(28*b) + (3*lo
g(1/(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(
1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(28*b)
```

3.134 $\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x} dx$

3.134.1 Optimal result

Integrand size = 15, antiderivative size = 121

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x} dx = -2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))^{5/2} + 2(bx-\text{arctanh}(\tanh(a+bx)))^2\sqrt{\text{arctanh}(\tanh(a+bx))} - \frac{2}{3}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2} + \frac{2}{5}\text{arctanh}(\tanh(a+bx))^{5/2}$$

output
```
-2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*
x-arctanh(tanh(b*x+a)))^(5/2)-2/3*(b*x-arctanh(tanh(b*x+a)))*arctanh(tanh(
b*x+a))^(3/2)+2/5*arctanh(tanh(b*x+a))^(5/2)+2*(b*x-arctanh(tanh(b*x+a)))^
2*arctanh(tanh(b*x+a))^(1/2)
```

3.134.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x} dx = \frac{2}{15}\Bigg(15b^2x^2\sqrt{\text{arctanh}(\tanh(a+bx))} - 35bx\text{arctanh}(\tanh(a+bx))^{3/2} + 23\text{arctanh}(\tanh(a+bx))^{5/2} - 15\text{arctanh}\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)(-bx+\text{arctanh}(\tanh(a+bx)))$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x,x]
```

output
```
(2*(15*b^2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]] - 35*b*x*ArcTanh[Tanh[a + b*x]
]^(3/2) + 23*ArcTanh[Tanh[a + b*x]]^(5/2) - 15*ArcTanh[Sqrt[ArcTanh[Tanh[a
 + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) + ArcTanh[Tanh[a
+ b*x]])^(5/2)))/15
```

3.134.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2590, 2590, 2590, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x} \, dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{2}{5}\operatorname{arctanh}(\tanh(a+bx))^{5/2} - (bx - \operatorname{arctanh}(\tanh(a+bx))) \int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x} dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{2}{5}\operatorname{arctanh}(\tanh(a+bx))^{5/2} - (bx - \operatorname{arctanh}(\tanh(a+ bx))) \left(\frac{2}{3}\operatorname{arctanh}(\tanh(a+bx))^{3/2} - (bx - \operatorname{arctanh}(\tanh(a+bx))) \int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x} dx\right)$$

$$\Big\downarrow \text{2590}$$

$$\frac{2}{5}\operatorname{arctanh}(\tanh(a+bx))^{5/2} - (bx - \operatorname{arctanh}(\tanh(a+ bx))) \left(\frac{2}{3}\operatorname{arctanh}(\tanh(a+bx))^{3/2} - (bx - \operatorname{arctanh}(\tanh(a+bx))) \left(2\sqrt{\operatorname{arctanh}(\tanh(a+bx))} - (bx - \operatorname{arctanh}(\right.\right.$$

$$\Big\downarrow \text{2592}$$

$$\frac{2}{5}\operatorname{arctanh}(\tanh(a+bx))^{5/2} - (bx - \operatorname{arctanh}(\tanh(a+ bx))) \left(\frac{2}{3}\operatorname{arctanh}(\tanh(a+bx))^{3/2} - (bx - \operatorname{arctanh}(\tanh(a+bx))) \left(2\sqrt{\operatorname{arctanh}(\tanh(a+bx))} - 2\sqrt{bx - \operatorname{arctan}\right.\right.$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x,x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(5/2))/5 - (b*x - ArcTanh[Tanh[a + b*x]])*(-((-2
*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*S
qrt[b*x - ArcTanh[Tanh[a + b*x]]] + 2*Sqrt[ArcTanh[Tanh[a + b*x]]])*(b*x -
 ArcTanh[Tanh[a + b*x]])) + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/3)
```

3.134.3.1 Defintions of rubi rules used

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

3.134.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(105) = 210$.

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.83

method	result
default	$\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}+\frac{2a\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}+\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{3}+2a^2\sqrt{\text{ar}$

input
```
int(arctanh(tanh(b*x+a))^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/5*arctanh(tanh(b*x+a))^(5/2)+2/3*a*arctanh(tanh(b*x+a))^(3/2)+2/3*arctan
h(tanh(b*x+a))^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)+2*a^2*arctanh(tanh(b*x+a
))^(1/2)+4*a*(arctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)+2*(ar
ctanh(tanh(b*x+a))-b*x-a)^2*arctanh(tanh(b*x+a))^(1/2)-2*(a^3+3*a^2*(arcta
nh(tanh(b*x+a))-b*x-a)+3*a*(arctanh(tanh(b*x+a))-b*x-a)^2+(arctanh(tanh(b*
x+a))-b*x-a)^3)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+
a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x}\,dx = \left[a^{\frac{5}{2}}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) + \frac{2}{15}\left(3b^2x^2+11abx+23a^2\right)\sqrt{bx+a}, 2\sqrt{-a}a^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + \frac{2}{15}\left(3b^2x^2+11abx+23a^2\right)\sqrt{bx+a}\right]$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x,x, algorithm="fricas")
```

output
```
[a^(5/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/15*(3*b^2*x^2 +
11*a*b*x + 23*a^2)*sqrt(b*x + a), 2*sqrt(-a)*a^2*arctan(sqrt(b*x + a)*sqrt
(-a)/a) + 2/15*(3*b^2*x^2 + 11*a*b*x + 23*a^2)*sqrt(b*x + a)]
```

3.134.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x}\,dx = \int \frac{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}{x}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(5/2)/x,x)
```

output
```
Integral(atanh(tanh(a + b*x))**(5/2)/x, x)
```

3.134.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x}\,dx = \int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^(5/2)/x, x)
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.60

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x}\,dx = \frac{1}{15}\sqrt{2}\left(\frac{15\sqrt{2}a^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 3\sqrt{2}(bx+a)^{\frac{5}{2}} + 5\sqrt{2}(bx+a)^{\frac{3}{2}}a + 1$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x,x, algorithm="giac")
```

output
```
1/15*sqrt(2)*(15*sqrt(2)*a^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 3*s
qrt(2)*(b*x + a)^(5/2) + 5*sqrt(2)*(b*x + a)^(3/2)*a + 15*sqrt(2)*sqrt(b*x
 + a)*a^2)
```

3.134.9 Mupad [B] (verification not implemented)

Time = 8.33 (sec) , antiderivative size = 789, normalized size of antiderivative = 6.52

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x}\,dx = \text{Too large to display}$$

input
```
int(atanh(tanh(a + b*x))^(5/2)/x,x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (2*(3*
b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x) - (8*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/5)*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))/2 + b*x))/(3*b)))/b + (2*b^2*x^2*(log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/
2))/5 + (2^(1/2)*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^(1/2)*2i + 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 2^(1/2)*b*x)*16i)/(x*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(5/2)*1i)/8 - (x*
(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (8*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)
)/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/...
```

3.134. $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x}\,dx$

3.135 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^2} dx$

3.135.1 Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^2} dx = 5b \arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))^{3/2} - 5b(bx-\operatorname{arctanh}(\tanh(a+bx)))\sqrt{\operatorname{arctanh}(\tanh(a+bx))} + \frac{5}{3}b\operatorname{arctanh}(\tanh(a+bx))^{3/2} - \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x}$$

output
```
5*b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b
*x-arctanh(tanh(b*x+a)))^(3/2)+5/3*b*arctanh(tanh(b*x+a))^(3/2)-arctanh(ta
nh(b*x+a))^(5/2)/x-5*b*(b*x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(1/
2)
```

3.135.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^2} dx = -5b\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)(-bx+\operatorname{arctanh}(\tanh(a+bx)))^{3/2} + \sqrt{\operatorname{arctanh}(\tanh(a+bx))}\left(\frac{2b^2x}{3}+\frac{14}{3}b(-bx+\operatorname{arctanh}(\tanh(a+bx)))-\frac{(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2}{x}\right)$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^2,x]
```

output
```
-5*b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b
*x]]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2) + Sqrt[ArcTanh[Tanh[a + b*x
]]]*((2*b^2*x)/3 + (14*b*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/3 - (-(b*x) +
ArcTanh[Tanh[a + b*x]])^2/x)
```

3.135.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2599, 2590, 2590, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^2}\,dx$$

$$\Big\downarrow \; 2599$$

$$\frac{5}{2}b\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x}dx-\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x}$$

$$\Big\downarrow \; 2590$$

$$\frac{5}{2}b\left(\frac{2}{3}\text{arctanh}(\tanh(a+bx))^{3/2}-(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x}dx\right)-\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x}$$

$$\Big\downarrow \; 2590$$

$$\frac{5}{2}b\left(\frac{2}{3}\text{arctanh}(\tanh(a+bx))^{3/2}-(bx-\text{arctanh}(\tanh(a+bx)))\left(2\sqrt{\text{arctanh}(\tanh(a+bx))}-(bx-\text{arctan}\right.\right.$$
$$\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x}$$

$$\Big\downarrow \; 2592$$

$$\frac{5}{2}b\left(\frac{2}{3}\text{arctanh}(\tanh(a+bx))^{3/2}-(bx-\text{arctanh}(\tanh(a+bx)))\left(2\sqrt{\text{arctanh}(\tanh(a+bx))}-2\sqrt{bx-\text{arctanh}(\right.\right.$$
$$\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^2,x]
```

output
```
-(ArcTanh[Tanh[a + b*x]]^(5/2)/x) + (5*b*(-((-2*ArcTan[Sqrt[ArcTanh[Tanh[a
 + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*Sqrt[b*x - ArcTanh[Tanh[a +
b*x]]] + 2*Sqrt[ArcTanh[Tanh[a + b*x]]])*(b*x - ArcTanh[Tanh[a + b*x]])) +
 (2*ArcTanh[Tanh[a + b*x]]^(3/2))/3))/2
```

3.135.3.1 Defintions of rubi rules used

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.135.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.75

method	result
default	$2b\left(\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}+2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}\,a+2(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a)\,\sqrt{a}\right.$

input
```
int(arctanh(tanh(b*x+a))^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2*b*(1/3*arctanh(tanh(b*x+a))^(3/2)+2*arctanh(tanh(b*x+a))^(1/2)*a+2*(arct
anh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)+(-1/2*a^2-a*(arctanh(ta
nh(b*x+a))-b*x-a)-1/2*(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))
^(1/2)/x/b-5/2*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))
-b*x-a)^2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(
1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^2}\,dx = \left[\frac{15\,a^{\frac{3}{2}}bx\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(2\,b^2x^2+14\,abx-3\,a^2)\sqrt{bx+a}}{6\,x}, \frac{15}{}\right.$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^2,x, algorithm="fricas")
```

output
```
[1/6*(15*a^(3/2)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b
^2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x, 1/3*(15*sqrt(-a)*a*b*x*arctan
(sqrt(b*x + a)*sqrt(-a)/a) + (2*b^2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))
/x]
```

3.135.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^2}\,dx = \int \frac{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}{x^2}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(5/2)/x**2,x)
```

output
```
Integral(atanh(tanh(a + b*x))**(5/2)/x**2, x)
```

3.135.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^2}\,dx = \int \frac{\operatorname{artanh}(\tanh(bx+a))^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^(5/2)/x^2, x)
```

3.135.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.81

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^2}\,dx = \frac{\sqrt{2}\left(\frac{15\sqrt{2}a^2b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{2}(bx+a)^{\frac{3}{2}}b^2 + 12\sqrt{2}\sqrt{bx+a}\,ab^2 - \frac{3\sqrt{2}\sqrt{\ldots}}{}\right.}{6\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^2,x, algorithm="giac")
```

output
```
1/6*sqrt(2)*(15*sqrt(2)*a^2*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) +
2*sqrt(2)*(b*x + a)^(3/2)*b^2 + 12*sqrt(2)*sqrt(b*x + a)*a*b^2 - 3*sqrt(2)
*sqrt(b*x + a)*a^2*b/x)/b
```

3.135.9 Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 616, normalized size of antiderivative = 5.60

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^2}\,dx = \frac{2\,b^2\,x\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{3}$$

$$-\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{4\,x}$$

$$-\frac{\left(3\,b^2\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-\frac{4\,b^2\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}{3}\right)\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{b}$$

$$+\frac{\sqrt{2}\,b\,\ln\left(\frac{\left(\sqrt{2}\,b\,x-\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)+\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}}\right.}}{x\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}\right.}{8}$$

input
```
int(atanh(tanh(a + b*x))^(5/2)/x^2,x)
```

output
```
(2*b^2*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2
/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/3 - ((log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)^2)/(4*x) - ((3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (4*b
^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1))/2 + b*x))/3)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/b + (2^(1/
2)*b*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log
(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i
- 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*16i)/(x*(log(2/(exp(2*a
)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2)*5i)/8
```

3.136 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^3}\,dx$

3.136.1 Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^3}\,dx = -\frac{15}{4}b^2\arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))} + \frac{15}{4}b^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))} - \frac{5b\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{4x} - \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{2x^2}$$

output
```
-5/4*b*arctanh(tanh(b*x+a))^(3/2)/x-1/2*arctanh(tanh(b*x+a))^(5/2)/x^2-15/
4*b^2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*
(b*x-arctanh(tanh(b*x+a)))^(1/2)+15/4*b^2*arctanh(tanh(b*x+a))^(1/2)
```

3.136.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^3}\,dx = -\frac{-15b^2x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))} + 5bx\operatorname{arctanh}(\tanh(a+bx))^{3/2} + 2\operatorname{arctanh}(\tanh(a+bx))^{5/2} + 15b^2x^2a}{4x^2}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^3,x]
```

output
```
-1/4*(-15*b^2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]] + 5*b*x*ArcTanh[Tanh[a + b*
x]]^(3/2) + 2*ArcTanh[Tanh[a + b*x]]^(5/2) + 15*b^2*x^2*ArcTanh[Sqrt[ArcTa
nh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*Sqrt[-(b*x) + Ar
cTanh[Tanh[a + b*x]]])/x^2
```

3.136.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2599, 2599, 2590, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^3} \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{5}{4} b \int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^2}dx-\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{2x^2}$$

$$\Big\downarrow \text{2599}$$

$$\frac{5}{4} b \left(\frac{3}{2} b \int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x}dx-\frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x}\right)-\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{2x^2}$$

$$\Big\downarrow \text{2590}$$

$$\frac{5}{4} b \left(\frac{3}{2} b \left(2 \sqrt{\text{arctanh}(\tanh(a+bx))}-(bx-\text{arctanh}(\tanh(a+bx))) \int \frac{1}{x \sqrt{\text{arctanh}(\tanh(a+bx))}}dx\right)-\frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x}\right)-\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{2x^2}$$

$$\Big\downarrow \text{2592}$$

$$\frac{5}{4} b \left(\frac{3}{2} b \left(2 \sqrt{\text{arctanh}(\tanh(a+bx))}-2 \sqrt{bx-\text{arctanh}(\tanh(a+bx))} \arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)\right)-\frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x}\right)-\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{2x^2}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^3,x]
```

output
```
-1/2*ArcTanh[Tanh[a + b*x]]^(5/2)/x^2 + (5*b*((3*b*(-2*ArcTan[Sqrt[ArcTanh
[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*Sqrt[b*x - ArcTanh[Ta
nh[a + b*x]]] + 2*Sqrt[ArcTanh[Tanh[a + b*x]]]))/2 - ArcTanh[Tanh[a + b*x]
]^(3/2)/x))/4
```

3.136.3.1 Defintions of rubi rules used

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.136.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.29

method	result
default	$2b^2\left(\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}+\frac{\left(-\frac{9\operatorname{arctanh}(\tanh(bx+a))}{8}+\frac{9bx}{8}\right)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}+\left(\frac{7a^2}{8}+\frac{7a(\operatorname{arctanh}(\tanh(bx+a}{4}\right.}{x^2b^2}\right.$

input
```
int(arctanh(tanh(b*x+a))^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2*b^2*(arctanh(tanh(b*x+a))^(1/2)+((-9/8*arctanh(tanh(b*x+a))+9/8*b*x)*arc
tanh(tanh(b*x+a))^(3/2)+(7/8*a^2+7/4*a*(arctanh(tanh(b*x+a))-b*x-a)+7/8*(a
rctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))^(1/2))/x^2/b^2-15/8*(ar
ctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(
tanh(b*x+a))-b*x)^(1/2)))
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.21

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^3}\,dx = \left[\frac{15\sqrt{a}b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(8b^2x^2-9abx-2a^2\right)\sqrt{bx+a}}{8x^2}, \right.$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(15*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*
(8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x + a))/x^2, 1/4*(15*sqrt(-a)*b^2*x^2
*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x
 + a))/x^2]
```

3.136.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^3}\,dx = \int \frac{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}{x^3}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(5/2)/x**3,x)
```

output
```
Integral(atanh(tanh(a + b*x))**(5/2)/x**3, x)
```

3.136.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^3}\,dx = \int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^(5/2)/x^3, x)
```

3.136.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^3}\,dx = \frac{\sqrt{2}\left(\frac{15\,\sqrt{2}ab^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 8\,\sqrt{2}\sqrt{bx+a}b^3 - \frac{\sqrt{2}\left(9\,(bx+a)^{\frac{3}{2}}ab^3 - 7\,\sqrt{bx+a}a^2b^3\right)}{b^2x^2}\right)}{8\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^3,x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(15*sqrt(2)*a*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 8*
sqrt(2)*sqrt(b*x + a)*b^3 - sqrt(2)*(9*(b*x + a)^(3/2)*a*b^3 - 7*sqrt(b*x
+ a)*a^2*b^3)/(b^2*x^2))/b
```

3.136.9 Mupad [B] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 614, normalized size of antiderivative = 5.58

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^3}\,dx = 2\,b^2\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}$$

$$+b^2\ln\left(\frac{64\left(2\sqrt{2}\,a-2\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\sqrt{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-2\,b\,x}-\sqrt{2}\left(2\,a\right.\right.}{x\sqrt{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-2\,b\,x}}\right.$$

$$-\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{4\,x^2\left(2\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-2\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+4\,b\,x\right)}$$

$$+\frac{9\,b\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{8\,x}$$

input
```
int(atanh(tanh(a + b*x))^(5/2)/x^3,x)
```

output
```
2*b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2) + b^2*log((64*(2*2^(1/2)*a - 2*(log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b
*x) + 1))/2)^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 - log(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*b*x)^(1/2) - 2^(1/2)*(2*a - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x) + 2^(1/2)*b*x))/(x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*b*x)^(1/2)))*((
225*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/128 - (225*log
(2/(exp(2*a)*exp(2*b*x) + 1)))/128 - (225*b*x)/64)^(1/2) - ((log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) +
 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(4*x^2*(2*log(2/(exp(2*a)*exp(2
*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4
*b*x)) + (9*b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 -
log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(8*x)
```

3.137 $\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^4} dx$

3.137.1 Optimal result

Integrand size = 15, antiderivative size = 113

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^4} dx = \frac{5b^3 \arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{8\sqrt{bx-\text{arctanh}(\tanh(a+bx))}} - \frac{5b^2\sqrt{\text{arctanh}(\tanh(a+bx))}}{8x} - \frac{5b\text{arctanh}(\tanh(a+bx))^{3/2}}{12x^2} - \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{3x^3}$$

output
```
-5/12*b*arctanh(tanh(b*x+a))^(3/2)/x^2-1/3*arctanh(tanh(b*x+a))^(5/2)/x^3+
5/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2)
)/(b*x-arctanh(tanh(b*x+a)))^(1/2)-5/8*b^2*arctanh(tanh(b*x+a))^(1/2)/x
```

3.137.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^4}\,dx = \frac{1}{24}\left(-\frac{15b^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x} - \frac{10b\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^2} - \frac{8\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^3} - \frac{15b^3\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^4,x]
```

output
```
((-15*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/x - (10*b*ArcTanh[Tanh[a + b*x]]^(
3/2))/x^2 - (8*ArcTanh[Tanh[a + b*x]]^(5/2))/x^3 - (15*b^3*ArcTanh[Sqrt[Ar
cTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])/Sqrt[-(b*x)
+ ArcTanh[Tanh[a + b*x]]])/24
```

3.137.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2599, 2599, 2599, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^4}\,dx$$

$$\Big\downarrow \; 2599$$

$$\frac{5}{6}b\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^3}dx - \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{3x^3}$$

$$\Big\downarrow \; 2599$$

$$\frac{5}{6}b\left(\frac{3}{4}b\int\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^2}dx-\frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{2x^2}\right)-\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{3x^3}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{5}{6}b\left(\frac{3}{4}b\left(\frac{1}{2}b\int\frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx-\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x}\right)-\frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{2x^2}\right)-\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{3x^3}$$

$$\Big\downarrow \text{ 2592}$$

$$\frac{5}{6}b\left(\frac{3}{4}b\left(\frac{b\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}-\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x}\right)-\frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{2x^2}\right)-\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{3x^3}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^4,x]
```

output
```
-1/3*ArcTanh[Tanh[a + b*x]]^(5/2)/x^3 + (5*b*((3*b*((b*ArcTan[Sqrt[ArcTanh
[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/Sqrt[b*x - ArcTanh[T
anh[a + b*x]]] - Sqrt[ArcTanh[Tanh[a + b*x]]]/x))/4 - ArcTanh[Tanh[a + b*x
]]^(3/2)/(2*x^2)))/6
```

3.137.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.137.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.27

method	result
default	$2b^3\left(\frac{-\frac{11\,\text{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{16}+\left(\frac{5\,\text{arctanh}(\tanh(bx+a))}{6}-\frac{5bx}{6}\right)\text{arctanh}(\tanh(bx+a))^{\frac{3}{2}}+\left(-\frac{5a^2}{16}-\frac{5a(\text{arctanh}(\tanh(bx+a))-bx-a)}{8}-5\right.}{x^3b^3}\right.$

input
```
int(arctanh(tanh(b*x+a))^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2*b^3*((-11/16*arctanh(tanh(b*x+a))^(5/2)+(5/6*arctanh(tanh(b*x+a))-5/6*b*
x)*arctanh(tanh(b*x+a))^(3/2)+(-5/16*a^2-5/8*a*(arctanh(tanh(b*x+a))-b*x-a
)-5/16*(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))^(1/2))/x^3/b^3
-5/16/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/
(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.29

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^4}\,dx = \left[\frac{15\,\sqrt{a}b^3x^3\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,(33\,ab^2x^2+26\,a^2bx+8\,a^3)\sqrt{bx+}}{48\,ax^3}\right.$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(15*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2
*(33*a*b^2*x^2 + 26*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a*x^3), 1/24*(15*sqrt
(-a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) - (33*a*b^2*x^2 + 26*a^2*b*x
 + 8*a^3)*sqrt(b*x + a))/(a*x^3)]
```

3.137.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^4}\,dx = \int \frac{\operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}{x^4}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(5/2)/x**4,x)
```

output
```
Integral(atanh(tanh(a + b*x))**(5/2)/x**4, x)
```

3.137.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^4}\,dx = \int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{x^4}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^(5/2)/x^4, x)
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.78

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^4}\,dx = \frac{\sqrt{2}\left(\frac{15\sqrt{2}b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{2}\left(33\,(bx+a)^{\frac{5}{2}}b^4 - 40\,(bx+a)^{\frac{3}{2}}ab^4 + 15\,\sqrt{bx+a}a^2b^4\right)}{b^3x^3}\right)}{48\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^4,x, algorithm="giac")
```

output
```
1/48*sqrt(2)*(15*sqrt(2)*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqr
t(2)*(33*(b*x + a)^(5/2)*b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 15*sqrt(b*x + a)
*a^2*b^4)/(b^3*x^3))/b
```

3.137.9 Mupad [B] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 669, normalized size of antiderivative = 5.92

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^4}\,dx = \frac{13\,b\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2}{12\,x^2\,\left(2\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-2\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+4\,b\,x\right)}$$

$$-\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{4\,x^3\,\left(3\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-3\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+6\,b\,x\right)}$$

$$-\frac{11\,b^2\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{8\,x}$$

$$+\frac{\sqrt{2}\,b^3\,\ln\left(\frac{\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}\,\left(\sqrt{2}\,b\,x-\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)+\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{x}\right)}{16\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}$$

input
```
int(atanh(tanh(a + b*x))^(5/2)/x^4,x)
```

output

```
(2^(1/2)*b^3*log(((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*((log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(
1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
) + 2^(1/2)*b*x)*64i)/x)*5i)/(16*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)) - ((log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp
(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(4*x^3*(3*log(2/(exp(2
*a)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 6*b*x)) - (11*b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(8*x) + (13*b*(log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp
(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(12*x^2*(2*log(2/(exp(
2*a)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 4*b*x))
```

3.137. $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^4}\,dx$

3.138 $\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^5} dx$

3.138.1 Optimal result

Integrand size = 15, antiderivative size = 167

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^5} dx = \frac{5b^4 \arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{64(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}} - \frac{5b^3}{64x\sqrt{\text{arctanh}(\tanh(a+bx))}} + \frac{5b^4}{64(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}} - \frac{5b^2\sqrt{\text{arctanh}(\tanh(a+bx))}}{32x^2} - \frac{5b\text{arctanh}(\tanh(a+bx))^{3/2}}{24x^3} - \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{4x^4}$$

output
```
5/64*b^4*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2
))/(b*x-arctanh(tanh(b*x+a)))^(3/2)-5/24*b*arctanh(tanh(b*x+a))^(3/2)/x^3-
1/4*arctanh(tanh(b*x+a))^(5/2)/x^4-5/64*b^3/x/arctanh(tanh(b*x+a))^(1/2)+5
/64*b^4/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)-5/32*b^2*arc
tanh(tanh(b*x+a))^(1/2)/x^2
```

3.138.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^5}\,dx = \frac{5b^4\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{64(-bx+\operatorname{arctanh}(\tanh(a+bx)))^{3/2}}$$
$$-\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}(15b^3x^3+10b^2x^2\operatorname{arctanh}(\tanh(a+bx))+8bx\operatorname{arctanh}(\tanh(a+bx))^2-48\operatorname{arcta}}{192x^4(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^5,x]
```

output
```
(5*b^4*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a +
 b*x]]]])/(64*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2)) - (Sqrt[ArcTanh[Tan
h[a + b*x]]]*(15*b^3*x^3 + 10*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 8*b*x*ArcTa
nh[Tanh[a + b*x]]^2 - 48*ArcTanh[Tanh[a + b*x]]^3))/(192*x^4*(b*x - ArcTan
h[Tanh[a + b*x]]))
```

3.138.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2599, 2599, 2599, 2599, 2594, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^5}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{5}{8}b\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^4}dx-\frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{4x^4}$$

$$\Big\downarrow \text{2599}$$

$$\frac{5}{8}b\left(\frac{1}{2}b\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^3}dx-\frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3x^3}\right)-\frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{4x^4}$$

$$\Big\downarrow \text{2599}$$

$$\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\int\frac{1}{x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}dx-\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{2x^2}\right)-\frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{3x^3}\right)-\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{4x^4}$$

$$\Big\downarrow \text{2599}$$

$$\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{1}{2}b\int\frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}dx-\frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)-\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{2x^2}\right)\right.$$
$$\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{4x^4}$$

$$\Big\downarrow \text{2594}$$

$$\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{1}{2}b\left(-\frac{\int\frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)-\right.\right.\right.$$
$$\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{4x^4}$$

$$\Big\downarrow \text{2592}$$

$$\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{1}{2}b\left(-\frac{2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}\right.\right.\right.\right.$$
$$\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{4x^4}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^5,x]
```

output
```
-1/4*ArcTanh[Tanh[a + b*x]]^(5/2)/x^4 + (5*b*((b*((b*(-1/2*(b*((-2*ArcTan[
Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(b*x - A
rcTanh[Tanh[a + b*x]])^(3/2) - 2/((b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcT
anh[Tanh[a + b*x]]]))) - 1/(x*Sqrt[ArcTanh[Tanh[a + b*x]]])))/4 - Sqrt[Arc
Tanh[Tanh[a + b*x]]]/(2*x^2)))/2 - ArcTanh[Tanh[a + b*x]]^(3/2)/(3*x^3)))/
8
```

3.138. $\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^5}\,dx$

3.138.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.138.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.01

method	result
default	$2b^4\left(\frac{-\frac{5\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{128(\operatorname{arctanh}(\tanh(bx+a))-bx)}-\frac{73\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{384}+\left(\frac{55\,\operatorname{arctanh}(\tanh(bx+a))}{384}-\frac{55bx}{384}\right)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}+\left(-\frac{5a^2}{128}-\right.}{x^4b^4}\right.$

input
```
int(arctanh(tanh(b*x+a))^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2*b^4*((-5/128/(arctanh(tanh(b*x+a))-b*x)*arctanh(tanh(b*x+a))^(7/2)-73/38
4*arctanh(tanh(b*x+a))^(5/2)+(55/384*arctanh(tanh(b*x+a))-55/384*b*x)*arct
anh(tanh(b*x+a))^(3/2)+(-5/128*a^2-5/64*a*(arctanh(tanh(b*x+a))-b*x-a)-5/1
28*(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))^(1/2))/x^4/b^4+5/1
28/(arctanh(tanh(b*x+a))-b*x)^(3/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(ar
ctanh(tanh(b*x+a))-b*x)^(1/2)))
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^5}\,dx = \left[\frac{15\sqrt{a}b^4x^4\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) - 2\left(15ab^3x^3+118a^2b^2x^2+136a^3bx\right.}{384a^2x^4}\right.$$
$$\left. - \frac{15\sqrt{-a}b^4x^4\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + \left(15ab^3x^3+118a^2b^2x^2+136a^3bx+48a^4\right)\sqrt{bx+a}}{192a^2x^4}\right]$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^5,x, algorithm="fricas")
```

output
```
[1/384*(15*sqrt(a)*b^4*x^4*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) -
2*(15*a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(
a^2*x^4), -1/192*(15*sqrt(-a)*b^4*x^4*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (
15*a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^2
*x^4)]
```

3.138.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^5}\,dx = \int \frac{\operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}{x^5}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(5/2)/x**5,x)
```

output
```
Integral(atanh(tanh(a + b*x))**(5/2)/x**5, x)
```

3.138.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^5}\,dx = \int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{x^5}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^(5/2)/x^5, x)
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.65

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^5}\,dx = -\frac{\sqrt{2}\left(\frac{15\sqrt{2}b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{2}\left(15\,(bx+a)^{\frac{7}{2}}b^5+73\,(bx+a)^{\frac{5}{2}}ab^5-55\,(bx+a)^{\frac{3}{2}}a^2b^5+15\sqrt{bx+a}a^3b^5\right)}{ab^4x^4}\right)}{384\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^5,x, algorithm="giac")
```

output
```
-1/384*sqrt(2)*(15*sqrt(2)*b^5*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a)
 + sqrt(2)*(15*(b*x + a)^(7/2)*b^5 + 73*(b*x + a)^(5/2)*a*b^5 - 55*(b*x +
a)^(3/2)*a^2*b^5 + 15*sqrt(b*x + a)*a^3*b^5)/(a*b^4*x^4))/b
```

3.138.9 Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 1069, normalized size of antiderivative = 6.40

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^5}\,dx = \text{Too large to display}$$

input
```
int(atanh(tanh(a + b*x))^(5/2)/x^5,x)
```

output
```
(5*b^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(
exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(32*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - ((l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*
exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(4*x^4*(4*log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - 4*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + 8*b*x)) + (2^(1/2)*b^4*log(((2*2^(1/2)*a + (log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2
)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(2*a - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 8*a^3 - 6*a
*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
))*1024i)/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*5i)/(64*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))...
```

3.138. $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^5}\,dx$

3.139 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^6}\,dx$

3.139.1 Optimal result

Integrand size = 15, antiderivative size = 221

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^6}\,dx = \frac{3b^5\arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{128(bx-\operatorname{arctanh}(\tanh(a+bx)))^{5/2}} + \frac{b^4}{128x\operatorname{arctanh}(\tanh(a+bx))^{3/2}} - \frac{b^5}{128(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))^{3/2}} - \frac{b^3}{64x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} + \frac{3b^5}{128(bx-\operatorname{arctanh}(\tanh(a+bx)))^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} - \frac{b^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{16x^3} - \frac{b\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{8x^4} - \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{5x^5}$$

output
```
3/128*b^5*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/
2))/(b*x-arctanh(tanh(b*x+a)))^(5/2)+1/128*b^4/x/arctanh(tanh(b*x+a))^(3/2
)-1/128*b^5/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)-1/8*b*ar
ctanh(tanh(b*x+a))^(3/2)/x^4-1/5*arctanh(tanh(b*x+a))^(5/2)/x^5-1/64*b^3/x
^2/arctanh(tanh(b*x+a))^(1/2)+3/128*b^5/(b*x-arctanh(tanh(b*x+a)))^2/arcta
nh(tanh(b*x+a))^(1/2)-1/16*b^2*arctanh(tanh(b*x+a))^(1/2)/x^3
```

3.139.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.68

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^6}\,dx = \frac{1}{640}\left(-\frac{15b^5\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\operatorname{arctanh}(\tanh(a+bx)))^{5/2}}\right.$$

$$-\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}(15b^4x^4+10b^3x^3\operatorname{arctanh}(\tanh(a+bx))+8b^2x^2\operatorname{arctanh}(\tanh(a+bx))^2-176bx}{x^5(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^6,x]
```

output
```
((-15*b^5*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[
a + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) - (Sqrt[ArcTanh[Tanh[
a + b*x]]]*(15*b^4*x^4 + 10*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 8*b^2*x^2*Arc
Tanh[Tanh[a + b*x]]^2 - 176*b*x*ArcTanh[Tanh[a + b*x]]^3 + 128*ArcTanh[Tan
h[a + b*x]]^4))/(x^5*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2))/640
```

3.139.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2599, 2599, 2599, 2599, 2599, 2594, 2594, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^6}\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{2}b\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^5}dx-\frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{5x^5}$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^4}dx-\frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{4x^4}\right)-\frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{5x^5}$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\int \frac{1}{x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}dx-\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{3x^3}\right)-\frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{4x^4}\right)-\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{5x^5}$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{1}{4}b\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{3/2}}dx-\frac{1}{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)-\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{3x^3}\right.\right.$$
$$\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{5x^5}$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{1}{4}b\left(-\frac{3}{2}b\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}dx-\frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}\right)-\frac{1}{2x^2\sqrt{\text{arctanh}(\tanh(a}\right.\right.\right.$$
$$\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{5x^5}$$

$$\Big\downarrow \text{2594}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{1}{4}b\left(-\frac{3}{2}b\left(-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx)}\right.\right.\right.\right.\right.$$
$$\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{5x^5}$$

$$\Big\downarrow \text{2594}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{1}{4}b\left(-\frac{3}{2}b\left(-\frac{-\frac{\int \frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{3(bx-a}\right.\right.\right.\right.\right.$$
$$\frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{5x^5}$$

$$\Big\downarrow \text{2592}$$

3.139. $\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^6}\,dx$

$$\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{1}{4}b\left(-\frac{3}{2}b\left(-\frac{-\frac{2\arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{(bx-\operatorname{arctanh}(\tanh(a+bx)))^{3/2}}-\frac{2}{(bx-\operatorname{arctanh}(\tanh(a+bx)))\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}}{bx-\operatorname{arctanh}(\tanh(a+bx))}-\right.\right.\right.\right.\right.$$

$$\frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{5x^5}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^6,x]
```

output
```
-1/5*ArcTanh[Tanh[a + b*x]]^(5/2)/x^5 + (b*((3*b*((b*(-1/4*(b*((-3*b*(-(((
-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]
)/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2/((b*x - ArcTanh[Tanh[a + b*x]])
*Sqrt[ArcTanh[Tanh[a + b*x]]]))/(b*x - ArcTanh[Tanh[a + b*x]])) - 2/(3*(b*
x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))))/2 - 1/(x*ArcTa
nh[Tanh[a + b*x]]^(3/2)))) - 1/(2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]])))/6 -
Sqrt[ArcTanh[Tanh[a + b*x]]]/(3*x^3)))/8 - ArcTanh[Tanh[a + b*x]]^(3/2)/(4
*x^4)))/2
```

3.139.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.139.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.19

method	result
default	$2b^5\left(\frac{3\operatorname{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{256\left(a^2+2a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2\right)}-\frac{7\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{128(\operatorname{arctanh}(\tanh(bx+a))-bx)}-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{10}+\left(\right.\right.$

input
```
int(arctanh(tanh(b*x+a))^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
2*b^5*((3/256/(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-
b*x-a)^2)*arctanh(tanh(b*x+a))^(9/2)-7/128/(arctanh(tanh(b*x+a))-b*x)*arct
anh(tanh(b*x+a))^(7/2)-1/10*arctanh(tanh(b*x+a))^(5/2)+(7/128*arctanh(tanh
(b*x+a))-7/128*b*x)*arctanh(tanh(b*x+a))^(3/2)+(-3/256*a^2-3/128*a*(arctan
h(tanh(b*x+a))-b*x-a)-3/256*(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b
*x+a))^(1/2))/x^5/b^5-3/256/(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh
(tanh(b*x+a))-b*x-a)^2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(t
anh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^6}\,dx = \left[\frac{15\sqrt{a}b^5x^5\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(15ab^4x^4-10a^2b^3x^3-248a^3b^2x^2\right.}{1280a^3x^5}\right.$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^6,x, algorithm="fricas")
```

output
```
[1/1280*(15*sqrt(a)*b^5*x^5*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) +
 2*(15*a*b^4*x^4 - 10*a^2*b^3*x^3 - 248*a^3*b^2*x^2 - 336*a^4*b*x - 128*a^
5)*sqrt(b*x + a))/(a^3*x^5), 1/640*(15*sqrt(-a)*b^5*x^5*arctan(sqrt(b*x +
a)*sqrt(-a)/a) + (15*a*b^4*x^4 - 10*a^2*b^3*x^3 - 248*a^3*b^2*x^2 - 336*a^
4*b*x - 128*a^5)*sqrt(b*x + a))/(a^3*x^5)]
```

3.139.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^6}\,dx = \int \frac{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}{x^6}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(5/2)/x**6,x)
```

output
```
Integral(atanh(tanh(a + b*x))**(5/2)/x**6, x)
```

3.139.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^6}\,dx = \int \frac{\operatorname{artanh}(\tanh(bx+a))^{\frac{5}{2}}}{x^6}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^(5/2)/x^6, x)
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.56

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^6}\,dx = \frac{\sqrt{2}\left(\frac{15\sqrt{2}b^6\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{\sqrt{2}\left(15(bx+a)^{\frac{9}{2}}b^6 - 70(bx+a)^{\frac{7}{2}}ab^6 - 128(bx+a)^{\frac{5}{2}}a^2b^6 + 70(bx+\right.}{a^2b^5x^5}\right.}{1280\,b}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^6,x, algorithm="giac")
```

output
```
1/1280*sqrt(2)*(15*sqrt(2)*b^6*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^
2) + sqrt(2)*(15*(b*x + a)^(9/2)*b^6 - 70*(b*x + a)^(7/2)*a*b^6 - 128*(b*x
 + a)^(5/2)*a^2*b^6 + 70*(b*x + a)^(3/2)*a^3*b^6 - 15*sqrt(b*x + a)*a^4*b^
6)/(a^2*b^5*x^5))/b
```

3.139.9 Mupad [B] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 1292, normalized size of antiderivative = 5.85

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^6}\,dx = \text{Too large to display}$$

input
```
int(atanh(tanh(a + b*x))^(5/2)/x^6,x)
```

output
```
(3*b^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(
exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(32*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) - (
(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(4*x^5*(5*log(2/(
exp(2*a)*exp(2*b*x) + 1)) - 5*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 10*b*x)) + (b^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(16*x^2*(2*log(2
/(exp(2*a)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 4*b*x)) + (2^(1/2)*b^5*log((((log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(
1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 + 40*a^2*(2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x)^3 - 80*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 32*a^5 - 10*...
```

3.140 $\int \frac{x^4}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx$

3.140.1 Optimal result

Integrand size = 15, antiderivative size = 99

$$\int \frac{x^4}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx = \frac{2x^4\sqrt{\text{arctanh}(\tanh(a+bx))}}{b} - \frac{16x^3\text{arctanh}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{32x^2\text{arctanh}(\tanh(a+bx))^{5/2}}{5b^3} - \frac{128x\text{arctanh}(\tanh(a+bx))^{7/2}}{35b^4} + \frac{256\text{arctanh}(\tanh(a+bx))^{9/2}}{315b^5}$$

output
```
-16/3*x^3*arctanh(tanh(b*x+a))^(3/2)/b^2+32/5*x^2*arctanh(tanh(b*x+a))^(5/
2)/b^3-128/35*x*arctanh(tanh(b*x+a))^(7/2)/b^4+256/315*arctanh(tanh(b*x+a)
)^(9/2)/b^5+2*x^4*arctanh(tanh(b*x+a))^(1/2)/b
```

3.140.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$$
$$= \frac{2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}(315b^4x^4 - 840b^3x^3\operatorname{arctanh}(\tanh(a+bx)) + 1008b^2x^2\operatorname{arctanh}(\tanh(a+bx))^2}{315b^5}$$

input
```
Integrate[x^4/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(315*b^4*x^4 - 840*b^3*x^3*ArcTanh[Tanh[a
+ b*x]] + 1008*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 576*b*x*ArcTanh[Tanh[a +
 b*x]]^3 + 128*ArcTanh[Tanh[a + b*x]]^4))/(315*b^5)
```

3.140.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2599, 2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$$

$$\Big\downarrow \ 2599$$

$$\frac{2x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b} - \frac{8\int x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}dx}{b}$$

$$\Big\downarrow \ 2599$$

$$\frac{2x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b} - \frac{8\left(\frac{2x^3\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3b} - \frac{2\int x^2\operatorname{arctanh}(\tanh(a+bx))^{3/2}dx}{b}\right)}{b}$$

$$\Big\downarrow \ 2599$$

$$\frac{2x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b}-\frac{8\left(\frac{2x^3\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{2\left(\frac{2x^2\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\int x\operatorname{arctanh}(\tanh(a+bx))^{5/2}dx}{5b}\right)}{b}\right)}{b}$$

$\big\downarrow$ 2599

$$\frac{2x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b}-\frac{8\left(\frac{2x^3\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{2\left(\frac{2x^2\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\left(\frac{2x\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{2\int \operatorname{arctanh}(\tanh(a+bx))^{7/2}dx}{7b}\right)}{5b}\right)}{b}\right)}{b}$$

$\big\downarrow$ 2588

$$\frac{2x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b}-\frac{8\left(\frac{2x^3\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{2\left(\frac{2x^2\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\left(\frac{2x\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{2\int \operatorname{arctanh}(\tanh(a+bx))^{7/2}d\operatorname{arctanh}}{7b^2}\right.}{5b}\right.}{b}\right.}{b}$$

$\big\downarrow$ 15

$$\frac{2x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b}-\frac{8\left(\frac{2x^3\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{2\left(\frac{2x^2\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\left(\frac{2x\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\operatorname{arctanh}(\tanh(a+bx))^{9/2}}{63b^2}\right)}{5b}\right)}{b}\right)}{b}$$

input
```
Int[x^4/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

3.140. $\int \frac{x^4}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$

output
```
(2*x^4*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (8*((2*x^3*ArcTanh[Tanh[a + b*x]]
^(3/2))/(3*b) - (2*((2*x^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (4*((2*x*
ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (4*ArcTanh[Tanh[a + b*x]]^(9/2))/(63
*b^2)))/(5*b)))/b))/b
```

3.140.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.140.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.55

method	result
default	$\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}+\frac{2\left(2(bx-\operatorname{arctanh}(\tanh(bx+a)))^{2}+(-2\operatorname{arctanh}(\tanh(bx+a))+\right.}{5}$

input
```
int(x^4/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^5*(1/9*arctanh(tanh(b*x+a))^(9/2)+1/7*(-4*arctanh(tanh(b*x+a))+4*b*x)*
arctanh(tanh(b*x+a))^(7/2)+1/5*(2*(b*x-arctanh(tanh(b*x+a)))^2+(-2*arctanh
(tanh(b*x+a))+2*b*x)^2)*arctanh(tanh(b*x+a))^(5/2)+2/3*(b*x-arctanh(tanh(b
*x+a)))^2*(-2*arctanh(tanh(b*x+a))+2*b*x)*arctanh(tanh(b*x+a))^(3/2)+(b*x-
arctanh(tanh(b*x+a)))^4*arctanh(tanh(b*x+a))^(1/2))
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.54

$$\int \frac{x^4}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\,(35\,b^4x^4 - 40\,ab^3x^3 + 48\,a^2b^2x^2 - 64\,a^3bx + 128\,a^4)\sqrt{bx+a}}{315\,b^5}$$

input
```
integrate(x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^4*x^4 - 40*a*b^3*x^3 + 48*a^2*b^2*x^2 - 64*a^3*b*x + 128*a^4)*
sqrt(b*x + a)/b^5
```

3.140.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

input
```
integrate(x**4/atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(x**4/sqrt(atanh(tanh(a + b*x))), x)
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

$$\int \frac{x^4}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\,(35\,b^5x^5 - 5\,ab^4x^4 + 8\,a^2b^3x^3 - 16\,a^3b^2x^2 + 64\,a^4bx + 128\,a^5)}{315\,\sqrt{bx+a}b^5}$$

input
```
integrate(x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*b^5*x^5 - 5*a*b^4*x^4 + 8*a^2*b^3*x^3 - 16*a^3*b^2*x^2 + 64*a^4*
b*x + 128*a^5)/(sqrt(b*x + a)*b^5)
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.62

$$\int \frac{x^4}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\left(35\,(bx+a)^{\frac{9}{2}} - 180\,(bx+a)^{\frac{7}{2}}a + 378\,(bx+a)^{\frac{5}{2}}a^2 - 420\,(bx+a)^{\frac{3}{2}}a^3 + 315\,\sqrt{bx+a}a^4\right)}{315\,b^5}$$

input
```
integrate(x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
2/315*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^
2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b^5
```

3.140.9 Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 496, normalized size of antiderivative = 5.01

$$\int \frac{x^4}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\,x^4\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{9\,b} + \frac{256\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,{\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}^4}{315\,b^5} + \frac{16\,x^3\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}{63\,b^2} + \frac{128\,x\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,{\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}^3}{315\,b^4} + \frac{32\,x^2\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,{\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}^2}{105\,b^3}$$

input
```
int(x^4/atanh(tanh(a + b*x))^(1/2),x)
```

output
```
(2*x^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(
exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(9*b) + (256*(log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/
2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1))/2 + b*x)^4)/(315*b^5) + (16*x^3*(log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2
)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(63*b^2) + (128*x*(log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/
2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^3)/(315*b^4) + (32*x^2*(log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) +
 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^2)/(105*b^3)
```

3.140. $\int \frac{x^4}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$

3.141 $\int \frac{x^3}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$

3.141.1 Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \frac{x^3}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b} - \frac{4x^2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{b^2} + \frac{16x\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{5b^3} - \frac{32\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{35b^4}$$

output
```
-4*x^2*arctanh(tanh(b*x+a))^(3/2)/b^2+16/5*x*arctanh(tanh(b*x+a))^(5/2)/b^
3-32/35*arctanh(tanh(b*x+a))^(7/2)/b^4+2*x^3*arctanh(tanh(b*x+a))^(1/2)/b
```

3.141.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}(35b^3x^3 - 70b^2x^2\operatorname{arctanh}(\tanh(a+bx)) + 56bx\operatorname{arctanh}(\tanh(a+bx))^2 - 16\operatorname{ar}}{35b^4}$$

input
```
Integrate[x^3/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(35*b^3*x^3 - 70*b^2*x^2*ArcTanh[Tanh[a +
b*x]] + 56*b*x*ArcTanh[Tanh[a + b*x]]^2 - 16*ArcTanh[Tanh[a + b*x]]^3))/(3
5*b^4)
```

3.141.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{6\int x^2\sqrt{\text{arctanh}(\tanh(a+bx))}dx}{b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\int x\text{arctanh}(\tanh(a+bx))^{3/2}dx}{3b}\right)}{b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{5/2}dx}{5b}\right)}{3b}\right)}{b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{2x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{5/2}d\text{arctanh}(\tanh(a+bx))}{5b^2}\right)}{3b}\right)}{b}$$

$$\downarrow 15$$

$$\frac{2x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\text{arctanh}(\tanh(a+bx))^{7/2}}{35b^2}\right)}{3b}\right)}{b}$$

input
```
Int[x^3/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*x^3*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (6*((2*x^2*ArcTanh[Tanh[a + b*x]]
^(3/2))/(3*b) - (4*((2*x*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (4*ArcTanh[
Tanh[a + b*x]]^(7/2))/(35*b^2)))/(3*b)))/b
```

3.141.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.141.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.62

method	result
default	$\frac{\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}+\frac{2(-3 \operatorname{arctanh}(\tanh(bx+a))+3bx) \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}+\frac{2\left((bx-\operatorname{arctanh}(\tanh(bx+a)))(-2 \operatorname{arctanh}(\tanh(bx+a))+2bx\right.}{3}}{b^4}$

input
```
int(x^3/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^4*(1/7*arctanh(tanh(b*x+a))^(7/2)+1/5*(-3*arctanh(tanh(b*x+a))+3*b*x)*
arctanh(tanh(b*x+a))^(5/2)+1/3*((b*x-arctanh(tanh(b*x+a)))*(-2*arctanh(tan
h(b*x+a))+2*b*x)+(b*x-arctanh(tanh(b*x+a)))^2)*arctanh(tanh(b*x+a))^(3/2)+
(b*x-arctanh(tanh(b*x+a)))^3*arctanh(tanh(b*x+a))^(1/2))
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int \frac{x^3}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\,(5\,b^3x^3 - 6\,ab^2x^2 + 8\,a^2bx - 16\,a^3)\sqrt{bx+a}}{35\,b^4}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x + a)/b^4
```

3.141.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{x^3}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

input
```
integrate(x**3/atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(x**3/sqrt(atanh(tanh(a + b*x))), x)
```

3.141.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \frac{x^3}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\,(5\,b^4x^4 - ab^3x^3 + 2\,a^2b^2x^2 - 8\,a^3bx - 16\,a^4)}{35\,\sqrt{bx+a}b^4}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
2/35*(5*b^4*x^4 - a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)/(sqrt(b*
x + a)*b^4)
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\left(5\,(bx+a)^{\frac{7}{2}} - 21\,(bx+a)^{\frac{5}{2}}a + 35\,(bx+a)^{\frac{3}{2}}a^2 - 35\,\sqrt{bx+a}a^3\right)}{35\,b^4}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 -
35*sqrt(b*x + a)*a^3)/b^4
```

3.141.9 Mupad [B] (verification not implemented)

Time = 3.57 (sec) , antiderivative size = 385, normalized size of antiderivative = 5.07

$$\int \frac{x^3}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$$
$$= \frac{2\,x^3\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{7\,b}$$
$$+\frac{32\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)^3}{35\,b^4}$$
$$+\frac{12\,x^2\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}{35\,b^2}$$
$$+\frac{16\,x\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)^2}{35\,b^3}$$

input
```
int(x^3/atanh(tanh(a + b*x))^(1/2),x)
```

output
```
(2*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(
exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(7*b) + (32*(log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2
)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1))/2 + b*x)^3)/(35*b^4) + (12*x^2*(log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^
(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(35*b^2) + (16*x*(log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^
(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1))/2 + b*x)^2)/(35*b^3)
```

3.142 $\int \frac{x^2}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx$

3.142.1 Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{x^2}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx = \frac{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}{b} - \frac{8x\text{arctanh}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{16\text{arctanh}(\tanh(a+bx))^{5/2}}{15b^3}$$

output
```
-8/3*x*arctanh(tanh(b*x+a))^(3/2)/b^2+16/15*arctanh(tanh(b*x+a))^(5/2)/b^3
+2*x^2*arctanh(tanh(b*x+a))^(1/2)/b
```

3.142.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}(15b^2x^2 - 20bx\text{arctanh}(\tanh(a+bx)) + 8\text{arctanh}(\tanh(a+bx))^2)}{15b^3}$$

input
```
Integrate[x^2/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 20*b*x*ArcTanh[Tanh[a + b*x]
] + 8*ArcTanh[Tanh[a + b*x]]^2))/(15*b^3)
```

3.142.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{4\int x\sqrt{\text{arctanh}(\tanh(a+bx))}dx}{b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{3/2}dx}{3b}\right)}{b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{3/2}d\text{arctanh}(\tanh(a+bx))}{3b^2}\right)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\text{arctanh}(\tanh(a+bx))^{5/2}}{15b^2}\right)}{b}$$

input
```
Int[x^2/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (4*((2*x*ArcTanh[Tanh[a + b*x]]^(
3/2))/(3*b) - (4*ArcTanh[Tanh[a + b*x]]^(5/2))/(15*b^2)))/b
```

3.142.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.142.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

method	result
default	$\frac{\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}+\frac{2(-2 \operatorname{arctanh}(\tanh(bx+a))+2bx) \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}+2(bx-\operatorname{arctanh}(\tanh(bx+a)))^{2} \sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{b^{3}}$

input
```
int(x^2/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^3*(1/5*arctanh(tanh(b*x+a))^(5/2)+1/3*(-2*arctanh(tanh(b*x+a))+2*b*x)*
arctanh(tanh(b*x+a))^(3/2)+(b*x-arctanh(tanh(b*x+a)))^2*arctanh(tanh(b*x+a
))^(1/2))
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.54

$$\int \frac{x^2}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\,(3\,b^2x^2 - 4\,abx + 8\,a^2)\sqrt{bx+a}}{15\,b^3}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x + a)/b^3
```

3.142.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

input
```
integrate(x**2/atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(x**2/sqrt(atanh(tanh(a + b*x))), x)
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\,(3\,b^3x^3 - ab^2x^2 + 4\,a^2bx + 8\,a^3)}{15\,\sqrt{bx+a}b^3}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*b^3*x^3 - a*b^2*x^2 + 4*a^2*b*x + 8*a^3)/(sqrt(b*x + a)*b^3)
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}a^2\right)}{15\,b^3}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b^3
```

3.142.9 Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.70

$$\int \frac{x^2}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$$
$$= \frac{2\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(15\,b^2\,x^2 - 10\,b\,x\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 10\,b\,x\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b}}{e^{2\,a}\,e^{2\,b\,x}+}\right.\right.}{15\,b^3}$$

input
```
int(x^2/atanh(tanh(a + b*x))^(1/2),x)
```

output
```
(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2)*(2*log(2/(exp(2*a)*exp(2*b*x) + 1))^2 - 4*l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(2/(exp(2*a)*exp(
2*b*x) + 1)) + 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2
+ 15*b^2*x^2 - 10*b*x*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 10*b*x*log(2/(exp(2*a)*exp(2*b*x) + 1))))/(15*b^3)
```

3.143 $\int \frac{x}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx$

3.143.1 Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{x}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx=\frac{2x\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{4\text{arctanh}(\tanh(a+bx))^{3/2}}{3b^2}$$

output
```
-4/3*arctanh(tanh(b*x+a))^(3/2)/b^2+2*x*arctanh(tanh(b*x+a))^(1/2)/b
```

3.143.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx=\frac{2(3bx-2\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}{3b^2}$$

input
```
Integrate[x/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*(3*b*x - 2*ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*b^2
)
```

3.143.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2x\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{2\int \sqrt{\text{arctanh}(\tanh(a+bx))}dx}{b}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{2x\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{2\int \sqrt{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+bx))}{b^2}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2x\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{4\text{arctanh}(\tanh(a+bx))^{3/2}}{3b^2}$$

input
```
Int[x/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*x*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (4*ArcTanh[Tanh[a + b*x]]^(3/2))/(3
*b^2)
```

3.143.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.143.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

method	result	size
default	$\frac{\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}-2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}\, a-2(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{b^2}$	56

input
```
int(x/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^2*(1/3*arctanh(tanh(b*x+a))^(3/2)-arctanh(tanh(b*x+a))^(1/2)*a-(arctan
h(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2))
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \frac{x}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\, dx = \frac{2\sqrt{bx+a}(bx-2\,a)}{3\,b^2}$$

input
```
integrate(x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x + a)*(b*x - 2*a)/b^2
```

3.143.6 Sympy [F]

$$\int \frac{x}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{x}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

input
```
integrate(x/atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(x/sqrt(atanh(tanh(a + b*x))), x)
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\left(b^2x^2-abx-2\,a^2\right)}{3\,\sqrt{bx+a}b^2}$$

input
```
integrate(x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(b^2*x^2 - a*b*x - 2*a^2)/(sqrt(b*x + a)*b^2)
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{x}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)}{3\,b^2}$$

input
```
integrate(x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/b^2
```

3.143.9 Mupad [B] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.92

$$\int \frac{x}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+3\,b\,x\right)}{3\,b^2}$$

input
```
int(x/atanh(tanh(a + b*x))^(1/2),x)
```

output
```
(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 3*b*x))/(3*b^2)
```

3.144 $\int \frac{1}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx$

3.144.1 Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{1}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}$$

output
```
2*arctanh(tanh(b*x+a))^(1/2)/b
```

3.144.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}$$

input
```
Integrate[1/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b
```

3.144.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \frac{1}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}d\operatorname{arctanh}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b}$$

input
```
Int[1/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b
```

3.144.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.144.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{b}$	15
default	$\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{b}$	15

input
```
int(1/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arctanh(tanh(b*x+a))^(1/2)/b
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{bx+a}}{b}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(b*x + a)/b
```

3.144.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \begin{cases} \frac{2\sqrt{\operatorname{atanh}(\tanh(a+bx))}}{b} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{\operatorname{atanh}(\tanh(a))}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Piecewise((2*sqrt(atanh(tanh(a + b*x)))/b, Ne(b, 0)), (x/sqrt(atanh(tanh(a
))), True))
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{bx+a}}{b}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(b*x + a)/b
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{bx+a}}{b}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(b*x + a)/b
```

3.144.9 Mupad [B] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 52, normalized size of antiderivative = 3.25

$$\int \frac{1}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\frac{\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2} - \frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)}{2}}}{b}$$

input
```
int(1/atanh(tanh(a + b*x))^(1/2),x)
```

output
```
(2*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2))/b
```

3.145 $\int \frac{1}{x\sqrt{\textbf{arctanh}(\tanh(a+bx))}}\,dx$

3.145.1 Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{x\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx=\frac{2\arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}$$

output
```
2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x
-arctanh(tanh(b*x+a)))^(1/2)
```

3.145.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}$$

input
```
Integrate[1/(x*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
(-2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*
x]]]])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
```

3.145.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$$

$$\Big\downarrow \text{ 2592}$$

$$\frac{2\arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}$$

input
```
Int[1/(x*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
(2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]
)/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
```

3.145.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

3.145.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}$	42

input
```
int(1/x/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(ar
ctanh(tanh(b*x+a))-b*x)^(1/2))
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

input
```
integrate(1/x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(s
qrt(b*x + a)*sqrt(-a)/a)/a]
```

3.145.6 Sympy [F]

$$\int \frac{1}{x\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{1}{x\sqrt{\operatorname{atanh}(\tanh(a+bx))}}\,dx$$

input
```
integrate(1/x/atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(atanh(tanh(a + b*x)))), x)
```

3.145.7 Maxima [F]

$$\int \frac{1}{x\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{1}{x\sqrt{\operatorname{artanh}(\tanh(bx+a))}}\,dx$$

input
```
integrate(1/x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sqrt(arctanh(tanh(b*x + a)))), x)
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.43

$$\int \frac{1}{x\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(1/x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

3.145.9 Mupad [B] (verification not implemented)

Time = 10.01 (sec) , antiderivative size = 285, normalized size of antiderivative = 5.82

$$\int \frac{1}{x\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{\sqrt{2}\,\ln\left(\frac{\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\left(\frac{\sqrt{2}\,b\,x}{2}-\frac{\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}{2}+\sqrt{\frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{\cdots}{e^{2a}\,e^{\cdots}}\right.}{2}}\cdots\right.}{x}\cdots\right.}{\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}}$$

input
```
int(1/(x*atanh(tanh(a + b*x))^(1/2)),x)
```

output
```
(2^(1/2)*log(((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*((log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*1i - (2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/
2 + (2^(1/2)*b*x)/2)*1i)/x)*1i)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)
```

3.146 $\int \frac{1}{x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx$

3.146.1 Optimal result

Integrand size = 15, antiderivative size = 94

$$\int \frac{1}{x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx = \frac{b\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}} - \frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}} + \frac{b}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

output
```
b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x
-arctanh(tanh(b*x+a)))^(3/2)-1/x/arctanh(tanh(b*x+a))^(1/2)+b/(b*x-arctanh
(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)
```

3.146.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx = \frac{b\text{arctanh}\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\text{arctanh}(\tanh(a+bx)))^{3/2}} - \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x(-bx+\text{arctanh}(\tanh(a+bx)))}$$

input
```
Integrate[1/(x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
(b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x
]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2) - Sqrt[ArcTanh[Tanh[a + b*x]
]]/(x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))
```

3.146.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2599, 2594, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}dx$$

$$\Big\downarrow \text{2599}$$

$$-\frac{1}{2}b\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}dx-\frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{1}{2}b\left(-\frac{\int \frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)-\frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2592}$$

$$-\frac{1}{2}b\left(-\frac{2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)-\frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

input
```
Int[1/(x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
-1/2*(b*((-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a
 + b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2/((b*x - ArcTanh[Tanh[
a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]]))) - 1/(x*Sqrt[ArcTanh[Tanh[a + b*
x]]])
```

3.146.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.146.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

method	result	size
default	$2b\left(\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)xb}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)$	95

input
```
int(1/x^2/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*b*(2*arctanh(tanh(b*x+a))^(1/2)/(-4*arctanh(tanh(b*x+a))+4*b*x)/x/b-2/(-
4*arctanh(tanh(b*x+a))+4*b*x)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arc
tanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \left[\frac{\sqrt{a}bx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}a}{2a^2x}, -\frac{\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\sqrt{bx+a}a}{a^2x}\right]$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*
x + a)*a)/(a^2*x), -(sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + sqrt(
b*x + a)*a)/(a^2*x)]
```

3.146.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{1}{x^2\sqrt{\operatorname{atanh}(\tanh(a+bx))}}\,dx$$

input
```
integrate(1/x**2/atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(atanh(tanh(a + b*x)))), x)
```

3.146.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{1}{x^2\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}\,dx$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*sqrt(arctanh(tanh(b*x + a)))), x)
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = -\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{bx+a}b}{ax}}{b}$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))
/b
```

3.146.9 Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 570, normalized size of antiderivative = 6.06

$$\int \frac{1}{x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{x\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}$$

$$+\frac{\sqrt{2}\,b\,\ln\left(\frac{\left(\sqrt{2}\,b\,x-\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)+\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2}}{e^{2\,a}\,e}\right.}}{}\right.}{}$$

input
```
int(1/(x^2*atanh(tanh(a + b*x))^(1/2)),x)
```

output
```
(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2))/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (2^(1/2)*b
*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(
exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^
(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*1i)/(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)^(3/2)
```

3.146. $\int \frac{1}{x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$

3.147 $\int \frac{1}{x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$

3.147.1 Optimal result

Integrand size = 15, antiderivative size = 158

$$\int \frac{1}{x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{3b^2\arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{4(bx-\operatorname{arctanh}(\tanh(a+bx)))^{5/2}}+\frac{b}{4x\operatorname{arctanh}(\tanh(a+bx))^{3/2}}-\frac{b^2}{4(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))^{3/2}}-\frac{1}{2x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}+\frac{3b^2}{4(bx-\operatorname{arctanh}(\tanh(a+bx)))^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

output
```
3/4*b^2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2)
)/(b*x-arctanh(tanh(b*x+a)))^(5/2)+1/4*b/x/arctanh(tanh(b*x+a))^(3/2)-1/4*
b^2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)-1/2/x^2/arctanh(
tanh(b*x+a))^(1/2)+3/4*b^2/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a
))^(1/2)
```

3.147.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{1}{4}\left(-\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\operatorname{arctanh}(\tanh(a+bx)))^{5/2}} + \frac{(5bx-2\operatorname{arctanh}(\tanh(a+bx)))\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^2(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2}\right)$$

input
```
Integrate[1/(x^3*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
((-3*b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
 + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) + ((5*b*x - 2*ArcTanh[
Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(x^2*(-(b*x) + ArcTanh[Tanh[
a + b*x]])^2))/4
```

3.147.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2599, 2599, 2594, 2594, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$$

$$\Big\downarrow \text{2599}$$

$$-\frac{1}{4}b\int \frac{1}{x^2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}dx-\frac{1}{2x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2599}$$

$$-\frac{1}{4}b\left(-\frac{3}{2}b\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}dx-\frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}\right)-\frac{1}{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$\Big\downarrow$ 2594

$$-\frac{1}{4}b\left(-\frac{3}{2}b\left(-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}\right)-\frac{}{x\text{arct}}\right.$$
$$\frac{1}{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$\Big\downarrow$ 2594

$$-\frac{1}{4}b\left(-\frac{3}{2}b\left(-\frac{-\frac{\int \frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{3(bx-\text{arctanh}(\tanh(a}\right.\right.$$
$$\frac{1}{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$\Big\downarrow$ 2592

$$-\frac{1}{4}b\left(-\frac{3}{2}b\left(-\frac{-\frac{2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{3(bx-\text{arctanh}}\right.\right.$$
$$\frac{1}{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

input
```
Int[1/(x^3*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
-1/4*(b*((-3*b*(-(((-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcT
anh[Tanh[a + b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2/((b*x - Arc
Tanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]]))/(b*x - ArcTanh[Tanh[a
+ b*x]])) - 2/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/
2))))/2 - 1/(x*ArcTanh[Tanh[a + b*x]]^(3/2)))) - 1/(2*x^2*Sqrt[ArcTanh[Tan
h[a + b*x]]])
```

3.147. $\int \frac{1}{x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx$

3.147.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.147.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.94

method	result
default	$2b^2\left(\frac{\sqrt{\mathrm{arctanh}(\tanh(bx+a))}}{(-4\ \mathrm{arctanh}(\tanh(bx+a))+4bx)x^2b^2}+\frac{\frac{6\sqrt{\mathrm{arctanh}(\tanh(bx+a))}}{(-4\ \mathrm{arctanh}(\tanh(bx+a))+4bx)xb}-\frac{6\ \mathrm{arctanh}\left(\frac{\sqrt{\mathrm{arctanh}(\tanh(bx+a))}}{\sqrt{\mathrm{arctanh}(\tanh(bx+a))-bx}}\right)}{(-4\ \mathrm{arctanh}(\tanh(bx+a))+4bx)\sqrt{\mathrm{arctanh}(\tanh(bx+a))-\cdots}}}{-4\ \mathrm{arctanh}(\tanh(bx+a))+4bx}\right.$

input
```
int(1/x^3/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*b^2*(arctanh(tanh(b*x+a))^(1/2)/(-4*arctanh(tanh(b*x+a))+4*b*x)/x^2/b^2+
3/(-4*arctanh(tanh(b*x+a))+4*b*x)*(2*arctanh(tanh(b*x+a))^(1/2)/(-4*arctan
h(tanh(b*x+a))+4*b*x)/x/b-2/(-4*arctanh(tanh(b*x+a))+4*b*x)/(arctanh(tanh(
b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a)
)-b*x)^(1/2))))
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$$
$$= \left[\frac{3\sqrt{a}b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2(3abx-2a^2)\sqrt{bx+a}}{8a^3x^2}, \frac{3\sqrt{-a}b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+(3abx-2\right.$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
3*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(
sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2)]
```

3.147.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{1}{x^3\sqrt{\operatorname{atanh}(\tanh(a+bx))}}\,dx$$

input
```
integrate(1/x**3/atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(atanh(tanh(a + b*x)))), x)
```

3.147.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{1}{x^3\sqrt{\operatorname{artanh}(\tanh(bx+a))}}\,dx$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x^3*sqrt(arctanh(tanh(b*x + a)))), x)
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3\,(bx+a)^{\frac{3}{2}}b^3-5\,\sqrt{bx+a}ab^3}{a^2b^2x^2}}{4\,b}$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(3
/2)*b^3 - 5*sqrt(b*x + a)*a*b^3)/(a^2*b^2*x^2))/b
```

3.147.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 802, normalized size of antiderivative = 5.08

$$\int \frac{1}{x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \text{Too large to display}$$

input
```
int(1/(x^3*atanh(tanh(a + b*x))^(1/2)),x)
```

output
```
(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2))/(x^2*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x)) + (3*b
*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2))/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (2^(1/2)*b
^2*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2
/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i -
2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^5 + 40*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 80*a^3*(2*a - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)^2 - 32*a^5 - 10*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 +
80*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log
(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(x*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x...
```

3.147. $\int \frac{1}{x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$

3.148 $\int \frac{1}{x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$

3.148.1 Optimal result

Integrand size = 15, antiderivative size = 212

$$\begin{aligned}\int \frac{1}{x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx &= \frac{5b^3\arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))^{7/2}}-\frac{b^2}{8x\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\\&\quad+\frac{b^3}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\\&\quad+\frac{b}{12x^2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\\&\quad-\frac{5b^3}{24(bx-\operatorname{arctanh}(\tanh(a+bx)))^2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\\&\quad-\frac{1}{3x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\\&\quad+\frac{5b^3}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\end{aligned}$$

output
```
5/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2)
)/(b*x-arctanh(tanh(b*x+a)))^(7/2)-1/8*b^2/x/arctanh(tanh(b*x+a))^(5/2)+1/
8*b^3/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(5/2)+1/12*b/x^2/arc
tanh(tanh(b*x+a))^(3/2)-5/24*b^3/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh
(b*x+a))^(3/2)-1/3/x^3/arctanh(tanh(b*x+a))^(1/2)+5/8*b^3/(b*x-arctanh(tan
h(b*x+a)))^3/arctanh(tanh(b*x+a))^(1/2)
```

3.148.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{5b^3\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{8(-bx+\operatorname{arctanh}(\tanh(a+bx)))^{7/2}} + \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}(33b^2x^2-26bx\operatorname{arctanh}(\tanh(a+bx))+8\operatorname{arctanh}(\tanh(a+bx))^2)}{24x^3(bx-\operatorname{arctanh}(\tanh(a+bx)))^3}$$

input
```
Integrate[1/(x^4*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
(5*b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a +
 b*x]]]])/(8*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2)) + (Sqrt[ArcTanh[Tanh
[a + b*x]]]*(33*b^2*x^2 - 26*b*x*ArcTanh[Tanh[a + b*x]] + 8*ArcTanh[Tanh[a
 + b*x]]^2))/(24*x^3*(b*x - ArcTanh[Tanh[a + b*x]])^3)
```

3.148.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2599, 2599, 2599, 2594, 2594, 2594, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}dx$$

$$\Big\downarrow \text{2599}$$

$$-\frac{1}{6}b\int \frac{1}{x^3\operatorname{arctanh}(\tanh(a+bx))^{3/2}}dx-\frac{1}{3x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2599}$$

$$-\frac{1}{6}b\left(-\frac{3}{4}b\int \frac{1}{x^2\operatorname{arctanh}(\tanh(a+bx))^{5/2}}dx-\frac{1}{2x^2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\right)-\frac{1}{3x^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2599}$$

$$-\frac{1}{6}b\left(-\frac{3}{4}b\left(-\frac{5}{2}b\int\frac{1}{x\text{arctanh}(\tanh(a+bx))^{7/2}}dx-\frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}\right)-\frac{1}{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}}\right.$$
$$\frac{1}{3x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$\downarrow$ 2594

$$-\frac{1}{6}b\left(-\frac{3}{4}b\left(-\frac{5}{2}b\left(-\frac{\int\frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{5(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{5/2}}\right)-\right.\right.$$
$$\frac{1}{3x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$\downarrow$ 2594

$$-\frac{1}{6}b\left(-\frac{3}{4}b\left(-\frac{5}{2}b\left(-\frac{-\frac{\int\frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{5(bx-\text{arctar}}\right.\right.\right.$$
$$\frac{1}{3x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$\downarrow$ 2594

$$-\frac{1}{6}b\left(-\frac{3}{4}b\left(-\frac{5}{2}b\left(-\frac{-\frac{-\frac{\int\frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{3(bx-\text{arctanh}(\tanh(a+}}{bx-\text{arctanh}(\tanh(a+bx))}\right.\right.\right.$$
$$\frac{1}{3x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$\downarrow$ 2592

$$-\frac{1}{6}b\left(-\frac{3}{4}b\left(-\frac{5}{2}b\left(-\frac{-\frac{-\frac{2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{3(bx-\text{arctanh}}}{bx-\text{arctanh}(\tanh(a+bx))}\right.\right.\right.$$
$$\frac{1}{3x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

3.148. $\int\frac{1}{x^4\sqrt{\text{arctanh}(\tanh(a+bx))}}dx$

input
```
Int[1/(x^4*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
-1/6*(b*((-3*b*((-5*b*(-((-(((-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[
b*x - ArcTanh[Tanh[a + b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2/(
(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]]))/(b*x - ArcTa
nh[Tanh[a + b*x]])) - 2/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a +
 b*x]]^(3/2)))/(b*x - ArcTanh[Tanh[a + b*x]])) - 2/(5*(b*x - ArcTanh[Tanh[
a + b*x]])*ArcTanh[Tanh[a + b*x]]^(5/2))))/2 - 1/(x*ArcTanh[Tanh[a + b*x]]
^(5/2))))/4 - 1/(2*x^2*ArcTanh[Tanh[a + b*x]]^(3/2)))) - 1/(3*x^3*Sqrt[Arc
Tanh[Tanh[a + b*x]]])
```

3.148.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.148.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.94

method	result
default	$2b^3\left(\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)x^3b^3}+\frac{\frac{10\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)x^2b^2}+\frac{10\left(\frac{6\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)xb}-\frac{}{(-4\operatorname{arcta}}\right.}{3(-4\operatorname{arctanh}(\mathrm{ta}}}{-4\operatorname{arctanh}(\tanh(bx+a))+4bx}\right.$

input
```
int(1/x^4/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*b^3*(2/3*arctanh(tanh(b*x+a))^(1/2)/(-4*arctanh(tanh(b*x+a))+4*b*x)/x^3/
b^3+10/3/(-4*arctanh(tanh(b*x+a))+4*b*x)*(arctanh(tanh(b*x+a))^(1/2)/(-4*a
rctanh(tanh(b*x+a))+4*b*x)/x^2/b^2+3/(-4*arctanh(tanh(b*x+a))+4*b*x)*(2*ar
ctanh(tanh(b*x+a))^(1/2)/(-4*arctanh(tanh(b*x+a))+4*b*x)/x/b-2/(-4*arctanh
(tanh(b*x+a))+4*b*x)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh
(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))))
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \left[\frac{15\sqrt{a}b^3x^3\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(15\,ab^2x^2-10\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{48\,a^4x^3}, -\frac{15\sqrt{-a}b^3x^3\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(15\,ab^2x^2-10\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{24\,a^4x^3}\right]$$

input
```
integrate(1/x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2
*(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^4*x^3), -1/24*(15*s
qrt(-a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 - 10*a^2*
b*x + 8*a^3)*sqrt(b*x + a))/(a^4*x^3)]
```

3.148.6 Sympy [F]

$$\int \frac{1}{x^4 \sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{1}{x^4 \sqrt{\operatorname{atanh}(\tanh(a+bx))}}\,dx$$

input
```
integrate(1/x**4/atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(atanh(tanh(a + b*x)))), x)
```

3.148.7 Maxima [F]

$$\int \frac{1}{x^4 \sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{1}{x^4 \sqrt{\operatorname{artanh}(\tanh(bx+a))}}\,dx$$

input
```
integrate(1/x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x^4*sqrt(arctanh(tanh(b*x + a)))), x)
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^4 \sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = -\frac{\frac{15\, b^4 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} + \frac{15\,(bx+a)^{\frac{5}{2}}b^4 - 40\,(bx+a)^{\frac{3}{2}}ab^4 + 33\sqrt{bx+a}a^2b^4}{a^3b^3x^3}}{24\,b}$$

input
```
integrate(1/x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-1/24*(15*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x + a
)^(5/2)*b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 33*sqrt(b*x + a)*a^2*b^4)/(a^3*b^
3*x^3))/b
```

3.148.9 Mupad [B] (verification not implemented)

Time = 8.10 (sec) , antiderivative size = 1086, normalized size of antiderivative = 5.12

$$\int \frac{1}{x^4\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx = \text{Too large to display}$$

input
```
int(1/(x^4*atanh(tanh(a + b*x))^(1/2)),x)
```

output
```
(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2))/(x^3*(3*log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x)) + (5*b
^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2))/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (2^(1/2)
*b^3*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log
(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i
- 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1))
 + 2*b*x)^7 + 84*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 280*a^3*(2*a - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)^4 + 560*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 672
*a^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 128*a^7 - 14*a*(2*a - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x...
```

3.149 $\int \frac{x^4}{\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.149.1 Optimal result

Integrand size = 15, antiderivative size = 95

$$\int \frac{x^4}{\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^4}{b\sqrt{\text{arctanh}(\tanh(a+bx))}} + \frac{16x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}{b^2} - \frac{32x^2\text{arctanh}(\tanh(a+bx))^{3/2}}{b^3} + \frac{128x\text{arctanh}(\tanh(a+bx))^{5/2}}{5b^4} - \frac{256\text{arctanh}(\tanh(a+bx))^{7/2}}{35b^5}$$

output
```
-32*x^2*arctanh(tanh(b*x+a))^(3/2)/b^3+128/5*x*arctanh(tanh(b*x+a))^(5/2)/
b^4-256/35*arctanh(tanh(b*x+a))^(7/2)/b^5-2*x^4/b/arctanh(tanh(b*x+a))^(1/
2)+16*x^3*arctanh(tanh(b*x+a))^(1/2)/b^2
```

3.149.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \frac{x^4}{\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2(35b^4x^4 - 280b^3x^3\text{arctanh}(\tanh(a+bx)) + 560b^2x^2\text{arctanh}(\tanh(a+bx))^2 - 448bx\text{arctanh}(\tanh(a+b}{35b^5\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

input
```
Integrate[x^4/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(-2*(35*b^4*x^4 - 280*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 560*b^2*x^2*ArcTanh
[Tanh[a + b*x]]^2 - 448*b*x*ArcTanh[Tanh[a + b*x]]^3 + 128*ArcTanh[Tanh[a
+ b*x]]^4))/(35*b^5*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

3.149.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2599, 2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{8\int \frac{x^3}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{b}-\frac{2x^4}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2599}$$

$$\frac{8\left(\frac{2x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{6\int x^2\sqrt{\text{arctanh}(\tanh(a+bx))}dx}{b}\right)}{b}-\frac{2x^4}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2599}$$

$$\frac{8\left(\frac{2x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\int x\,\text{arctanh}(\tanh(a+bx))^{3/2}dx}{3b}\right)}{b}\right)}{b}-\frac{2x^4}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2599}$$

$$\frac{8\left(\frac{2x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{5/2}dx}{5b}\right)}{3b}\right)}{b}\right)}{b}-\frac{2x^4}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$\Big\downarrow$ 2588

$$\frac{8\left(\frac{2x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{5/2}d\text{arctanh}(\ldots}{5b^2}\right.}{3b}\right.}{b}\right.}{b}-\frac{2x^4}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$\Big\downarrow$ 15

$$\frac{8\left(\frac{2x^3\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{6\left(\frac{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\text{arctanh}(\tanh(a+bx))^{7/2}}{35b^2}\right)}{3b}\right)}{b}\right)}{b}-\frac{2x^4}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

input
```
Int[x^4/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(-2*x^4)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (8*((2*x^3*Sqrt[ArcTanh[Tanh[a
 + b*x]]])/b - (6*((2*x^2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (4*((2*x*A
rcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (4*ArcTanh[Tanh[a + b*x]]^(7/2))/(35*
b^2)))/(3*b)))/b))/b
```

3.149. $\int \frac{x^4}{\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.149.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.149.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(81) = 162$.

Time = 0.09 (sec) , antiderivative size = 319, normalized size of antiderivative = 3.36

method	result
default	$\frac{\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}-\frac{8 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}} a}{5}-\frac{8 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}} (\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{5}+4a^2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}+8}{}$

input
```
int(x^4/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^5*(1/7*arctanh(tanh(b*x+a))^(7/2)-4/5*arctanh(tanh(b*x+a))^(5/2)*a-4/5
*arctanh(tanh(b*x+a))^(5/2)*(arctanh(tanh(b*x+a))-b*x-a)+2*a^2*arctanh(tan
h(b*x+a))^(3/2)+4*arctanh(tanh(b*x+a))^(3/2)*a*(arctanh(tanh(b*x+a))-b*x-a
)+2*arctanh(tanh(b*x+a))^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)^2-4*a^3*arctan
h(tanh(b*x+a))^(1/2)-12*a^2*(arctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+
a))^(1/2)-12*a*(arctanh(tanh(b*x+a))-b*x-a)^2*arctanh(tanh(b*x+a))^(1/2)-4
*(arctanh(tanh(b*x+a))-b*x-a)^3*arctanh(tanh(b*x+a))^(1/2)-(a^4+4*a^3*(arc
tanh(tanh(b*x+a))-b*x-a)+6*a^2*(arctanh(tanh(b*x+a))-b*x-a)^2+4*a*(arctanh
(tanh(b*x+a))-b*x-a)^3+(arctanh(tanh(b*x+a))-b*x-a)^4)/arctanh(tanh(b*x+a)
)^(1/2))
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2\,(5\,b^4x^4 - 8\,ab^3x^3 + 16\,a^2b^2x^2 - 64\,a^3bx - 128\,a^4)\sqrt{bx+a}}{35\,(b^6x + ab^5)}$$

input
```
integrate(x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/35*(5*b^4*x^4 - 8*a*b^3*x^3 + 16*a^2*b^2*x^2 - 64*a^3*b*x - 128*a^4)*sqr
t(b*x + a)/(b^6*x + a*b^5)
```

3.149.6 Sympy [F]

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{x^4}{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

input
```
integrate(x**4/atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(x**4/atanh(tanh(a + b*x))**(3/2), x)
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2\,(5\,b^5x^5 - 3\,ab^4x^4 + 8\,a^2b^3x^3 - 48\,a^3b^2x^2 - 192\,a^4bx - 128\,a^5)}{35\,(bx+a)^{\frac{3}{2}}b^5}$$

input
```
integrate(x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/35*(5*b^5*x^5 - 3*a*b^4*x^4 + 8*a^2*b^3*x^3 - 48*a^3*b^2*x^2 - 192*a^4*b
*x - 128*a^5)/((b*x + a)^(3/2)*b^5)
```

3.149.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2\,a^4}{\sqrt{bx+a}b^5} + \frac{2\left(5\,(bx+a)^{\frac{7}{2}}b^{30} - 28\,(bx+a)^{\frac{5}{2}}ab^{30} + 70\,(bx+a)^{\frac{3}{2}}a^2b^{30} - 140\,\sqrt{bx+a}a^3b^{30}\right)}{35\,b^{35}}$$

input
```
integrate(x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-2*a^4/(sqrt(b*x + a)*b^5) + 2/35*(5*(b*x + a)^(7/2)*b^30 - 28*(b*x + a)^(
5/2)*a*b^30 + 70*(b*x + a)^(3/2)*a^2*b^30 - 140*sqrt(b*x + a)*a^3*b^30)/b^
35
```

3.149.9 Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 1057, normalized size of antiderivative = 11.13

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^4/atanh(tanh(a + b*x))^(3/2),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/(4*b^4) + (2*((l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^2/(2*b^3) + (4*((log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)/b^2 + (1
2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1))/2 + b*x))/(7*b^2))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/
2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(5*b)
)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1))/2 + b*x))/(3*b)))/b + (2*x^3*(log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1
/2))/(7*b^2) + (x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/
2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2
/(2*b^3) + (4*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)/b^2 + (12*(log(2/(exp(2*a)*exp(2*
b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 +
b*x))/(7*b^2))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(5*b)))/(3*b) - ((log((2*ex...
```

3.149. $\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.150 $\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}} \, dx$

3.150.1 Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}} \, dx = -\frac{2x^3}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} + \frac{12x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b^2} - \frac{16x\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{b^3} + \frac{32\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{5b^4}$$

output
```
-16*x*arctanh(tanh(b*x+a))^(3/2)/b^3+32/5*arctanh(tanh(b*x+a))^(5/2)/b^4-2
*x^3/b/arctanh(tanh(b*x+a))^(1/2)+12*x^2*arctanh(tanh(b*x+a))^(1/2)/b^2
```

3.150.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}} \, dx = \frac{2(-5b^3x^3 + 30b^2x^2\operatorname{arctanh}(\tanh(a+bx)) - 40bx\operatorname{arctanh}(\tanh(a+bx))^2}{5b^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

input
```
Integrate[x^3/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(2*(-5*b^3*x^3 + 30*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 40*b*x*ArcTanh[Tanh[a
 + b*x]]^2 + 16*ArcTanh[Tanh[a + b*x]]^3))/(5*b^4*Sqrt[ArcTanh[Tanh[a + b*
x]]])
```

3.150.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\text{arctanh}(\tanh(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{6\int \frac{x^2}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{b}-\frac{2x^3}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{6\left(\frac{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{4\int x\sqrt{\text{arctanh}(\tanh(a+bx))}dx}{b}\right)}{b}-\frac{2x^3}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{6\left(\frac{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{3/2}dx}{3b}\right)}{b}\right)}{b}-\frac{2x^3}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{6\left(\frac{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{3/2}d\text{arctanh}(\tanh(a+bx))}{3b^2}\right)}{b}\right)}{b}-\frac{2x^3}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{6\left(\frac{2x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b}-\frac{4\left(\frac{2x\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{15b^2}\right)}{b}\right)}{b}-\frac{2x^3}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

input
```
Int[x^3/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(-2*x^3)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (6*((2*x^2*Sqrt[ArcTanh[Tanh[a
 + b*x]]])/b - (4*((2*x*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (4*ArcTanh[T
anh[a + b*x]]^(5/2))/(15*b^2)))/b))/b
```

3.150.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.150. $\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.150.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(64) = 128$.

Time = 0.09 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.72

method	result
default	$\frac{\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5} - 2a \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}} - 2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}} (\operatorname{arctanh}(\tanh(bx+a)) - bx - a) + 6a^2 \sqrt{\operatorname{arctanh}(\tanh(b}}{}$

input
```
int(x^3/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^4*(1/5*arctanh(tanh(b*x+a))^(5/2)-a*arctanh(tanh(b*x+a))^(3/2)-arctanh
(tanh(b*x+a))^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)+3*a^2*arctanh(tanh(b*x+a)
)^(1/2)+6*a*(arctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)+3*(arc
tanh(tanh(b*x+a))-b*x-a)^2*arctanh(tanh(b*x+a))^(1/2)-(-a^3-3*a^2*(arctanh
(tanh(b*x+a))-b*x-a)-3*a*(arctanh(tanh(b*x+a))-b*x-a)^2-(arctanh(tanh(b*x+
a))-b*x-a)^3)/arctanh(tanh(b*x+a))^(1/2))
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2\,(b^3x^3 - 2\,ab^2x^2 + 8\,a^2bx + 16\,a^3)\sqrt{bx+a}}{5\,(b^5x + ab^4)}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x + a)/(b^5*x + a*
b^4)
```

3.150.6 Sympy [F]

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{x^3}{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

input
```
integrate(x**3/atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(x**3/atanh(tanh(a + b*x))**(3/2), x)
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2\,(b^4x^4 - ab^3x^3 + 6\,a^2b^2x^2 + 24\,a^3bx + 16\,a^4)}{5\,(bx+a)^{\frac{3}{2}}b^4}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/5*(b^4*x^4 - a*b^3*x^3 + 6*a^2*b^2*x^2 + 24*a^3*b*x + 16*a^4)/((b*x + a)
^(3/2)*b^4)
```

3.150.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2\,a^3}{\sqrt{bx+a}b^4} + \frac{2\left((bx+a)^{\frac{5}{2}}b^{16} - 5\,(bx+a)^{\frac{3}{2}}ab^{16} + 15\,\sqrt{bx+a}a^2b^{16}\right)}{5\,b^{20}}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2*a^3/(sqrt(b*x + a)*b^4) + 2/5*((b*x + a)^(5/2)*b^16 - 5*(b*x + a)^(3/2)*
a*b^16 + 15*sqrt(b*x + a)*a^2*b^16)/b^20
```

3.150.9 Mupad [B] (verification not implemented)

Time = 3.83 (sec) , antiderivative size = 660, normalized size of antiderivative = 8.92

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{2\,b^3}+\cdots\right.}{\cdots}$$
$$+\frac{2\,x^2\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{5\,b^2}$$
$$+\frac{x\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}{b^2}+\frac{8\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}{5\,b^2}\right)\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{3\,b}$$
$$-\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{2\,b^4\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}$$

input
```
int(x^3/atanh(tanh(a + b*x))^(3/2),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/(2*b^3) + (2*((l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)/b^2 + (8*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(5*b^2))*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))/2 + b*x))/(3*b)))/b + (2*x^2*(log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/
(5*b^2) + (x*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)/b^2 + (8*(log(2/(exp(2*a)*exp(2*b*
x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*
x))/(5*b^2))*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*b) - ((log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(
1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(2*b^4*(log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))
```

3.150. $\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.151 $\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.151.1 Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^2}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} + \frac{8x\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b^2} - \frac{16\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3b^3}$$

output
```
-16/3*arctanh(tanh(b*x+a))^(3/2)/b^3-2*x^2/b/arctanh(tanh(b*x+a))^(1/2)+8*
x*arctanh(tanh(b*x+a))^(1/2)/b^2
```

3.151.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2(3b^2x^2-12bx\operatorname{arctanh}(\tanh(a+bx))+8\operatorname{arctanh}(\tanh(a+bx))^2)}{3b^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

input
```
Integrate[x^2/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(-2*(3*b^2*x^2 - 12*b*x*ArcTanh[Tanh[a + b*x]] + 8*ArcTanh[Tanh[a + b*x]]^
2))/(3*b^3*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

3.151.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\text{arctanh}(\tanh(a+bx))^{3/2}} \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{4\int \frac{x}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{b} - \frac{2x^2}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2599}$$

$$\frac{4\left(\frac{2x\sqrt{\text{arctanh}(\tanh(a+bx))}}{b} - \frac{2\int \sqrt{\text{arctanh}(\tanh(a+bx))}dx}{b}\right)}{b} - \frac{2x^2}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2588}$$

$$\frac{4\left(\frac{2x\sqrt{\text{arctanh}(\tanh(a+bx))}}{b} - \frac{2\int \sqrt{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+bx))}{b^2}\right)}{b} - \frac{2x^2}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{15}$$

$$\frac{4\left(\frac{2x\sqrt{\text{arctanh}(\tanh(a+bx))}}{b} - \frac{4\text{arctanh}(\tanh(a+bx))^{3/2}}{3b^2}\right)}{b} - \frac{2x^2}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

input
```
Int[x^2/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(-2*x^2)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (4*((2*x*Sqrt[ArcTanh[Tanh[a +
 b*x]]])/b - (4*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b^2)))/b
```

3.151.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.151.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(47) = 94$.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.93

method	result
default	$\frac{\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}-4\sqrt{\mathrm{arctanh}(\tanh(bx+a))}\,a-4(\mathrm{arctanh}(\tanh(bx+a))-bx-a)\sqrt{\mathrm{arctanh}(\tanh(bx+a))}-\frac{2\left(a^2+2a(\mathrm{arctanh}(\tanh\right.}{}}{b^3}$

input
```
int(x^2/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^3*(1/3*arctanh(tanh(b*x+a))^(3/2)-2*arctanh(tanh(b*x+a))^(1/2)*a-2*(ar
ctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)-(a^2+2*a*(arctanh(tan
h(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/arctanh(tanh(b*x+a))^(1/2
))
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2\,(b^2x^2-4\,abx-8\,a^2)\sqrt{bx+a}}{3\,(b^4x+ab^3)}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x + a)/(b^4*x + a*b^3)
```

3.151.6 Sympy [F]

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{x^2}{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

input
```
integrate(x**2/atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(x**2/atanh(tanh(a + b*x))**(3/2), x)
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2\,(b^3x^3-3\,ab^2x^2-12\,a^2bx-8\,a^3)}{3\,(bx+a)^{\frac{3}{2}}b^3}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/3*(b^3*x^3 - 3*a*b^2*x^2 - 12*a^2*b*x - 8*a^3)/((b*x + a)^(3/2)*b^3)
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2\,a^2}{\sqrt{bx+a}b^3} + \frac{2\left((bx+a)^{\frac{3}{2}}b^6 - 6\sqrt{bx+a}ab^6\right)}{3\,b^9}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-2*a^2/(sqrt(b*x + a)*b^3) + 2/3*((b*x + a)^(3/2)*b^6 - 6*sqrt(b*x + a)*a*
b^6)/b^9
```

3.151.9 Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 259, normalized size of antiderivative = 4.71

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx =$$

$$-\frac{4\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(3\,b^2\,x^2 - 6\,b\,x\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 6\,b\,x\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2 \cdots\right.}{3\,b^3\,\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}$$

input
```
int(x^2/atanh(tanh(a + b*x))^(3/2),x)
```

output
```
-(4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp
(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(2*log(2/(exp(2*a)*exp(2*b*x) + 1))^2 - 4*
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(2/(exp(2*a)*exp
(2*b*x) + 1)) + 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2
 + 3*b^2*x^2 - 6*b*x*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 6*b*x*log(2/(exp(2*a)*exp(2*b*x) + 1))))/(3*b^3*(log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))
```

3.152 $\int \frac{x}{\textbf{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.152.1 Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx=-\frac{2x}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}+\frac{4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b^2}$$

output
```
-2*x/b/arctanh(tanh(b*x+a))^(1/2)+4*arctanh(tanh(b*x+a))^(1/2)/b^2
```

3.152.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx=\frac{-2bx+4\operatorname{arctanh}(\tanh(a+bx))}{b^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

input
```
Integrate[x/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(-2*b*x + 4*ArcTanh[Tanh[a + b*x]])/(b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

3.152.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2\int \frac{1}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}dx}{b}-\frac{2x}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{2\int \frac{1}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}d\operatorname{arctanh}(\tanh(a+bx))}{b^2}-\frac{2x}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b^2}-\frac{2x}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

input
```
Int[x/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(-2*x)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (4*Sqrt[ArcTanh[Tanh[a + b*x]]])
/b^2
```

3.152.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.152.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}-\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}}{b^{2}}$	40

input
```
int(x/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^2*(arctanh(tanh(b*x+a))^(1/2)-(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(
b*x+a))^(1/2))
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2\,(bx+2\,a)\sqrt{bx+a}}{b^{3}x+ab^{2}}$$

input
```
integrate(x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2*(b*x + 2*a)*sqrt(b*x + a)/(b^3*x + a*b^2)
```

3.152.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \begin{cases} -\frac{2x}{b\sqrt{\operatorname{atanh}(\tanh(a+bx))}} + \frac{4\sqrt{\operatorname{atanh}(\tanh(a+bx))}}{b^2} & \text{for } b \neq 0 \\ \frac{x^2}{2\operatorname{atanh}^{\frac{3}{2}}(\tanh(a))} & \text{otherwise} \end{cases}$$

input
```
integrate(x/atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Piecewise((-2*x/(b*sqrt(atanh(tanh(a + b*x)))) + 4*sqrt(atanh(tanh(a + b*x
)))/b**2, Ne(b, 0)), (x**2/(2*atanh(tanh(a))**(3/2)), True))
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2\,(b^2x^2 + 3\,abx + 2\,a^2)}{(bx+a)^{\frac{3}{2}}b^2}$$

input
```
integrate(x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2*(b^2*x^2 + 3*a*b*x + 2*a^2)/((b*x + a)^(3/2)*b^2)
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{bx+a}}{b} + \frac{a}{\sqrt{bx+a}b}\right)}{b}$$

input
```
integrate(x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2*(sqrt(b*x + a)/b + a/(sqrt(b*x + a)*b))/b
```

3.152.9 Mupad [B] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 152, normalized size of antiderivative = 4.47

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{4\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+b\,x\right)}{b^2\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}$$

input
```
int(x/atanh(tanh(a + b*x))^(3/2),x)
```

output
```
-(4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp
(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + b*x))/(b^2*(log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) +
 1))))
```

3.153 $\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.153.1 Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

output
```
-2/b/arctanh(tanh(b*x+a))^(1/2)
```

3.153.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(-3/2),x]
```

output
```
-2/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

3.153.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \frac{1}{\text{arctanh}(\tanh(a+bx))^{3/2}}d\text{arctanh}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{2}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(-3/2),x]
```

output
```
-2/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

3.153.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.153.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$-\frac{2}{b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}$	15
default	$-\frac{2}{b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}$	15

input
```
int(1/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b/arctanh(tanh(b*x+a))^(1/2)
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2\sqrt{bx+a}}{b^2x+ab}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(b*x + a)/(b^2*x + a*b)
```

3.153.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \begin{cases} -\frac{2}{b\sqrt{\operatorname{atanh}(\tanh(a+bx))}} & \text{for } b \neq 0 \\ \frac{x}{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a))} & \text{otherwise} \end{cases}$$

input
```
integrate(1/atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Piecewise((-2/(b*sqrt(atanh(tanh(a + b*x)))), Ne(b, 0)), (x/atanh(tanh(a))
**(3/2), True))
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2}{\sqrt{bx+a}b}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2/(sqrt(b*x + a)*b)
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2}{\sqrt{bx+a}b}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-2/(sqrt(b*x + a)*b)
```

3.153.9 Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.06

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{4\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{b\left(\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)\right)}$$

input
```
int(1/atanh(tanh(a + b*x))^(3/2),x)
```

output
```
(4*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2))/(b*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))))
```

3.154 $\int \frac{1}{x\mathbf{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.154.1 Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}} - \frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

output
```
-2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*
x-arctanh(tanh(b*x+a)))^(3/2)-2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*
x+a))^(1/2)
```

3.154.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\text{arctanh}(\tanh(a+bx)))^{3/2}} + \frac{2}{\sqrt{\text{arctanh}(\tanh(a+bx))}(-bx+\text{arctanh}(\tanh(a+bx)))}$$

input
```
Integrate[1/(x*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

output
```
(-2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*
x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2) + 2/(Sqrt[ArcTanh[Tanh[a +
b*x]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]]))
```

3.154.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2594, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{2594}$$

$$-\frac{\int \frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2592}$$

$$-\frac{2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

input
```
Int[1/(x*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

output
```
(-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2/((b*x - ArcTanh[Tanh[a + b*x]]
)*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

3.154.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

3.154.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{2}{\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)^{\frac{3}{2}}}$	68

input
```
int(1/x/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(1/2)-2/(arctanh(tanh(b*
x+a))-b*x)^(3/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-
b*x)^(1/2))
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.41

$$\int \frac{1}{x\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \left[\frac{(bx+a)\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}a}{a^2bx+a^3}, \frac{2\left((bx+a)\sqrt{-a}\operatorname{arc}\right.}{}\right.$$

input
```
integrate(1/x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
[((b*x + a)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(
b*x + a)*a)/(a^2*b*x + a^3), 2*((b*x + a)*sqrt(-a)*arctan(sqrt(b*x + a)*sq
rt(-a)/a) + sqrt(b*x + a)*a)/(a^2*b*x + a^3)]
```

3.154.6 Sympy [F]

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{1}{x\,\text{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x/atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(1/(x*atanh(tanh(a + b*x))**(3/2)), x)
```

3.154.7 Maxima [F]

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{1}{x\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*arctanh(tanh(b*x + a))^(3/2)), x)
```

3.154.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.47

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{2}{\sqrt{bx+a}a}$$

input
```
integrate(1/x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + 2/(sqrt(b*x + a)*a)
```

3.154.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 614, normalized size of antiderivative = 7.87

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx =$$

$$-\frac{8\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}$$

$$+\frac{\sqrt{2}\,\ln\left(\frac{\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-\sqrt{2}\,b\,x+\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+}\right.}\right.}{}\right.}{\left(\ln\left(\right.\right.}$$

input
```
int(1/(x*atanh(tanh(a + b*x))^(3/2)),x)
```

output
```
(2^(1/2)*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 -
 log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)
*2i + 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 2^(1/2)*b*x)*((2*a - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(
2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))
*2i)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2) - (8*(log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/((l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*ex
p(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))
```

3.155 $\int \frac{1}{x^2\textbf{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.155.1 Optimal result

Integrand size = 15, antiderivative size = 124

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{3b\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{5/2}} - \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}} + \frac{b}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}} - \frac{3b}{(bx-\text{arctanh}(\tanh(a+bx)))^2\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

output
```
-3*b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(
b*x-arctanh(tanh(b*x+a)))^(5/2)-1/x/arctanh(tanh(b*x+a))^(3/2)+b/(b*x-arct
anh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)-3*b/(b*x-arctanh(tanh(b*x+a))
)^2/arctanh(tanh(b*x+a))^(1/2)
```

3.155.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{3b\text{arctanh}\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\text{arctanh}(\tanh(a+bx)))^{5/2}} - \frac{2bx+\text{arctanh}(\tanh(a+bx))}{x\sqrt{\text{arctanh}(\tanh(a+bx))}(-bx+\text{arctanh}(\tanh(a+bx)))^2}$$

input
```
Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

output
```
(3*b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b
*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) - (2*b*x + ArcTanh[Tanh[a
+ b*x]])/(x*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])
^2)
```

3.155.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2599, 2594, 2594, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$-\frac{3}{2}b\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}dx - \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{ 2594}$$

$$-\frac{3}{2}b\left(-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))} - \frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}\right) - \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{ 2594}$$

$$-\frac{3}{2}b\left(-\frac{-\frac{\int \frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)$$

$$\frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$\downarrow$ 2592

$$-\frac{3}{2}b\left(-\frac{-\frac{2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{3(bx-\text{arctanh}(\tanh$$

$$\frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Int[1/(x^2*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

output
```
(-3*b*(-(((-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[
a + b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2/((b*x - ArcTanh[Tanh
[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]]))/(b*x - ArcTanh[Tanh[a + b*x]]))
 - 2/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))))/2 -
 1/(x*ArcTanh[Tanh[a + b*x]]^(3/2))
```

3.155.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

3.155. $\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.155.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.85

method	result
default	$2b\left(-\frac{1}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{\frac{\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{2xb}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx}}\right)}{2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx}}}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}}\right)$

input
```
int(1/x^2/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*b*(-1/(arctanh(tanh(b*x+a))-b*x)^2/arctanh(tanh(b*x+a))^(1/2)-1/(arctanh
(tanh(b*x+a))-b*x)^2*(1/2*arctanh(tanh(b*x+a))^(1/2)/x/b-3/2/(arctanh(tanh
(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a
))-b*x)^(1/2))))
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \left[\frac{3\left(b^{2}x^{2}+abx\right)\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(3abx+a^{2}\right)\sqrt{bx+a}}{2\left(a^{3}bx^{2}+a^{4}x\right)},\right.$$
$$\left.-\frac{3\left(b^{2}x^{2}+abx\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3abx+a^{2}\right)\sqrt{bx+a}}{a^{3}bx^{2}+a^{4}x}\right]$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*(b^2*x^2 + a*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a
)/x) - 2*(3*a*b*x + a^2)*sqrt(b*x + a))/(a^3*b*x^2 + a^4*x), -(3*(b^2*x^2
+ a*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b*x + a^2)*sqrt(
b*x + a))/(a^3*b*x^2 + a^4*x)]
```

3.155.6 Sympy [F]

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{1}{x^2\,\text{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**2/atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(1/(x**2*atanh(tanh(a + b*x))**(3/2)), x)
```

3.155.7 Maxima [F]

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{1}{x^2\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*arctanh(tanh(b*x + a))^(3/2)), x)
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{3\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} - \frac{3\,(bx+a)b-2\,ab}{\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}a\right)a^2}$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-3*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) - (3*(b*x + a)*b - 2*a*
b)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)*a^2)
```

3.155.9 Mupad [B] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 807, normalized size of antiderivative = 6.51

$$\int \frac{1}{x^2 \text{arctanh}(\tanh(a+bx))^{3/2}} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*atanh(tanh(a + b*x))^(3/2)),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*(4/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (24*b*x)/(log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x)^2))/(2*b*x^2 - x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (2^(1/2
)*b*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(
2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i +
 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x)^5 + 40*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 80*a^3*(2*a - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x)^2 - 32*a^5 - 10*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 +
 80*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*...
```

3.156 $\int \frac{1}{x^3\textbf{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.156.1 Optimal result

Integrand size = 15, antiderivative size = 191

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{15b^2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{4(bx-\text{arctanh}(\tanh(a+bx)))^{7/2}}+\frac{3b}{4x\text{arctanh}(\tanh(a+bx))^{5/2}} - \frac{3b^2}{4(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{5/2}} - \frac{1}{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}} + \frac{5b^2}{4(bx-\text{arctanh}(\tanh(a+bx)))^2\text{arctanh}(\tanh(a+bx))^{3/2}} - \frac{15b^2}{4(bx-\text{arctanh}(\tanh(a+bx)))^3\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

output
```
-15/4*b^2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/
2))/(b*x-arctanh(tanh(b*x+a)))^(7/2)+3/4*b/x/arctanh(tanh(b*x+a))^(5/2)-3/
4*b^2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(5/2)-1/2/x^2/arctan
h(tanh(b*x+a))^(3/2)+5/4*b^2/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x
+a))^(3/2)-15/4*b^2/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(1/2
)
```

3.156.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^3 \text{arctanh}(\tanh(a+bx))^{3/2}} \, dx = \frac{1}{4}\left(-\frac{15b^2 \text{arctanh}\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\text{arctanh}(\tanh(a+bx)))^{7/2}} + \frac{8b^2x^2+9bx\text{arctanh}(\tanh(a+bx))-2\text{arctanh}(\tanh(a+bx))^2}{x^2\sqrt{\text{arctanh}(\tanh(a+bx))}(-bx+\text{arctanh}(\tanh(a+bx)))^3}\right)$$

input
```
Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

output
```
((-15*b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[
a + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2) + (8*b^2*x^2 + 9*b*x*
ArcTanh[Tanh[a + b*x]] - 2*ArcTanh[Tanh[a + b*x]]^2)/(x^2*Sqrt[ArcTanh[Tan
h[a + b*x]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3))/4
```

3.156.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2599, 2599, 2594, 2594, 2594, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \text{arctanh}(\tanh(a+bx))^{3/2}} \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$-\frac{3}{4}b\int \frac{1}{x^2 \text{arctanh}(\tanh(a+bx))^{5/2}}dx - \frac{1}{2x^2 \text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{ 2599}$$

$$-\frac{3}{4}b\left(-\frac{5}{2}b\int \frac{1}{x \text{arctanh}(\tanh(a+bx))^{7/2}}dx - \frac{1}{x \text{arctanh}(\tanh(a+bx))^{5/2}}\right) - \frac{1}{2x^2 \text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \; 2594$$

$$-\frac{3}{4}b\left(-\frac{5}{2}b\left(-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{5(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{5/2}}\right)-\frac{}{x\text{arct}}\right.$$
$$\frac{1}{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \; 2594$$

$$-\frac{3}{4}b\left(-\frac{5}{2}b\left(-\frac{-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{5(bx-\text{arctanh}(\tan}\right.\right.$$
$$\frac{1}{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \; 2594$$

$$-\frac{3}{4}b\left(-\frac{5}{2}b\left(-\frac{-\frac{-\frac{\int \frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{ar}}}{bx-\text{arctanh}(\tanh(a+bx))}\right.\right.$$
$$\frac{1}{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \; 2592$$

$$-\frac{3}{4}b\left(-\frac{5}{2}b\left(-\frac{-\frac{-\frac{2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{3(bx-\text{arctanh}(\tanh(a-}}{bx-\text{arctanh}(\tanh(a+bx))}\right.\right.$$
$$\frac{1}{2x^2\text{arctanh}(\tanh(a+bx))^{3/2}}$$

input

```
Int[1/(x^3*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

3.156. $\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

output
```
(-3*b*((-5*b*(-((-(((-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - Arc
Tanh[Tanh[a + b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2/((b*x - Ar
cTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]]))/(b*x - ArcTanh[Tanh[a
 + b*x]])) - 2/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3
/2)))/(b*x - ArcTanh[Tanh[a + b*x]])) - 2/(5*(b*x - ArcTanh[Tanh[a + b*x]]
)*ArcTanh[Tanh[a + b*x]]^(5/2))))/2 - 1/(x*ArcTanh[Tanh[a + b*x]]^(5/2))))
/4 - 1/(2*x^2*ArcTanh[Tanh[a + b*x]]^(3/2))
```

3.156.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.156.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.69

method	result
default	$2b^2\left(\frac{\frac{\frac{7\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{8}+\left(-\frac{9\,\mathrm{arctanh}(\tanh(bx+a))}{8}+\frac{9bx}{8}\right)\sqrt{\mathrm{arctanh}(\tanh(bx+a))}}{x^2b^2}-\frac{15\,\mathrm{arctanh}\left(\frac{\sqrt{\mathrm{arctanh}(\tanh(bx+a))}}{\sqrt{\mathrm{arctanh}(\tanh(bx+a))-bx}}\right)}{8\sqrt{\mathrm{arctanh}(\tanh(bx+a))-bx}}}{(\mathrm{arctanh}(\tanh(bx+a))-bx)^3}+\frac{}{(\mathrm{arc}}\right.$

input
```
int(1/x^3/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*b^2*(1/(arctanh(tanh(b*x+a))-b*x)^3*((7/8*arctanh(tanh(b*x+a))^(3/2)+(-9
/8*arctanh(tanh(b*x+a))+9/8*b*x)*arctanh(tanh(b*x+a))^(1/2))/x^2/b^2-15/8/
(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arcta
nh(tanh(b*x+a))-b*x)^(1/2)))+1/(arctanh(tanh(b*x+a))-b*x)^3/arctanh(tanh(b
*x+a))^(1/2))
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \left[\frac{15\,(b^3x^3+ab^2x^2)\sqrt{a}\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(15\,ab^2x^2+5\,a^2bx-2\right.}{8\,(a^4bx^3+a^5x^2)}$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a)
+ 2*a)/x) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3
 + a^5*x^2), 1/4*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*s
qrt(-a)/a) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3
+ a^5*x^2)]
```

3.156.6 Sympy [F]

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{1}{x^3\,\text{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**3/atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(1/(x**3*atanh(tanh(a + b*x))**(3/2)), x)
```

3.156.7 Maxima [F]

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{1}{x^3\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x^3*arctanh(tanh(b*x + a))^(3/2)), x)
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{15\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^3} + \frac{2\,b^2}{\sqrt{bx+a}a^3} + \frac{7\,(bx+a)^{\frac{3}{2}}b^2 - 9\,\sqrt{bx+a}ab^2}{4\,a^3b^2x^2}$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
15/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2*b^2/(sqrt(b*x +
 a)*a^3) + 1/4*(7*(b*x + a)^(3/2)*b^2 - 9*sqrt(b*x + a)*a*b^2)/(a^3*b^2*x^
2)
```

3.156.9 Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 1028, normalized size of antiderivative = 5.38

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(x^3*atanh(tanh(a + b*x))^(3/2)),x)
```

output
```
(2^(1/2)*b^2*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(
1/2)*2i + 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 2^(1/2)*b*x)*((2*a - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*
x) + 1)) + 2*b*x)^7 + 84*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 280*a^3*(
2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 560*a^4*(2*a - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^3 - 672*a^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 128*a^7 - 14*a*(2*a - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^6 + 448*a^6*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(2*
x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*15i)/(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(7/2) -
 (2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(...
```

3.156. $\int \frac{1}{x^3 \operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.157 $\int \frac{1}{x^4 \operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.157.1 Optimal result

Integrand size = 15, antiderivative size = 245

$$\begin{aligned}
\int \frac{1}{x^4 \operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = &-\frac{35b^3 \arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))^{9/2}} - \frac{5b^2}{8x\operatorname{arctanh}(\tanh(a+bx))^{7/2}} \\
&+ \frac{5b^3}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))^{7/2}} \\
&+ \frac{b}{4x^2\operatorname{arctanh}(\tanh(a+bx))^{5/2}} \\
&- \frac{7b^3}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))^2\operatorname{arctanh}(\tanh(a+bx))^{5/2}} \\
&- \frac{1}{3x^3\operatorname{arctanh}(\tanh(a+bx))^{3/2}} \\
&+ \frac{35b^3}{24(bx-\operatorname{arctanh}(\tanh(a+bx)))^3\operatorname{arctanh}(\tanh(a+bx))^{3/2}} \\
&- \frac{35b^3}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}
\end{aligned}$$

output
```
-35/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/
2))/(b*x-arctanh(tanh(b*x+a)))^(9/2)-5/8*b^2/x/arctanh(tanh(b*x+a))^(7/2)+
5/8*b^3/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(7/2)+1/4*b/x^2/ar
ctanh(tanh(b*x+a))^(5/2)-7/8*b^3/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh
(b*x+a))^(5/2)-1/3/x^3/arctanh(tanh(b*x+a))^(3/2)+35/24*b^3/(b*x-arctanh(t
anh(b*x+a)))^3/arctanh(tanh(b*x+a))^(3/2)-35/8*b^3/(b*x-arctanh(tanh(b*x+a
)))^4/arctanh(tanh(b*x+a))^(1/2)
```

3.157.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^4\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{35b^3\text{arctanh}\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)}{8(-bx+\text{arctanh}(\tanh(a+bx)))^{9/2}} - \frac{48b^3x^3+87b^2x^2\text{arctanh}(\tanh(a+bx))-38bx\text{arctanh}(\tanh(a+bx))^2+8\text{arctanh}(\tanh(a+bx))^3}{24x^3\sqrt{\text{arctanh}(\tanh(a+bx))}(-bx+\text{arctanh}(\tanh(a+bx)))^4}$$

input
```
Integrate[1/(x^4*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

output
```
(35*b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
+ b*x]]]])/(8*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(9/2)) - (48*b^3*x^3 + 87*
b^2*x^2*ArcTanh[Tanh[a + b*x]] - 38*b*x*ArcTanh[Tanh[a + b*x]]^2 + 8*ArcTa
nh[Tanh[a + b*x]]^3)/(24*x^3*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-(b*x) + ArcTan
h[Tanh[a + b*x]])^4)
```

3.157.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2599, 2599, 2599, 2594, 2594, 2594, 2594, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2599}$$

$$-\frac{1}{2}b\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^{5/2}}dx-\frac{1}{3x^3\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$\big\downarrow$ 2599

$$-\frac{1}{2}b\left(-\frac{5}{4}b\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{7/2}}dx-\frac{1}{2x^2\text{arctanh}(\tanh(a+bx))^{5/2}}\right)-\frac{1}{3x^3\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$\big\downarrow$ 2599

$$-\frac{1}{2}b\left(-\frac{5}{4}b\left(-\frac{7}{2}b\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{9/2}}dx-\frac{1}{x\text{arctanh}(\tanh(a+bx))^{7/2}}\right)-\frac{1}{2x^2\text{arctanh}(\tanh(a+bx))^{5/}}\right.$$
$$\frac{1}{3x^3\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$\big\downarrow$ 2594

$$-\frac{1}{2}b\left(-\frac{5}{4}b\left(-\frac{7}{2}b\left(-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{7/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{7(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{7/2}}\right)-\right.\right.$$
$$\frac{1}{3x^3\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$\big\downarrow$ 2594

$$-\frac{1}{2}b\left(-\frac{5}{4}b\left(-\frac{7}{2}b\left(-\frac{-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{5(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{5/2}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{7(bx-\text{arctar}}\right.\right.\right.$$
$$\frac{1}{3x^3\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$\big\downarrow$ 2594

$$-\frac{1}{2}b\left(-\frac{5}{4}b\left(-\frac{7}{2}b\left(-\frac{-\frac{-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{5(bx-\text{arctanh}(\tanh}}{bx-\text{arctanh}(\tanh(a+bx))}\right.\right.\right.$$
$$\frac{1}{3x^3\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$\big\downarrow$ 2594

3.157. $\int \frac{1}{x^4\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

$$-\frac{1}{2}b\left(-\frac{5}{4}b\left(-\frac{7}{2}b\left(-\frac{-\frac{-\frac{-\frac{\int\frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\cdots}{3(bx-\text{arctanh}(\tanh(a+b\cdots}}{bx-\text{arctanh}(\tanh(a+bx))}\cdots}{bx-\text{arctanh}(\tanh(a+b\cdots}\cdots\right.\right.\right.$$

$$\frac{1}{3x^3\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$\big\downarrow$ 2592

$$-\frac{1}{2}b\left(-\frac{5}{4}b\left(-\frac{7}{2}b\left(-\frac{-\frac{-\frac{-\frac{2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\cdots}{3(bx-\text{arctanh}(\cdots}}{bx-\text{arctanh}(\tanh(a+bx))}\cdots}{bx-\text{arctanh}(\tanh(a\cdots}\cdots\right.\right.\right.$$

$$\frac{1}{3x^3\text{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Int[1/(x^4*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

output
```
-1/2*(b*((-5*b*((-7*b*(-((-((-(((-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sq
rt[b*x - ArcTanh[Tanh[a + b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) -
2/((b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]]))/(b*x - Ar
cTanh[Tanh[a + b*x]])) - 2/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[
a + b*x]]^(3/2)))/(b*x - ArcTanh[Tanh[a + b*x]])) - 2/(5*(b*x - ArcTanh[Ta
nh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(5/2)))/(b*x - ArcTanh[Tanh[a + b*x]]
)) - 2/(7*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(7/2))))/2
 - 1/(x*ArcTanh[Tanh[a + b*x]]^(7/2))))/4 - 1/(2*x^2*ArcTanh[Tanh[a + b*x]
]^(5/2)))) - 1/(3*x^3*ArcTanh[Tanh[a + b*x]]^(3/2))
```

3.157. $\int \frac{1}{x^4\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.157.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.157.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.76

method	result
default	$2b^3\left(-\frac{\frac{\frac{19\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{16}+\left(-\frac{17\,\mathrm{arctanh}(\tanh(bx+a))}{6}+\frac{17bx}{6}\right)\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}+\left(\frac{29a^2}{16}+\frac{29a(\mathrm{arctanh}(\tanh(bx+a))-bx-a)}{8}+\right.}{x^3b^3}}{(\mathrm{arctanh}(\tanh(bx+a))-bx)^4}\right.$

input
```
int(1/x^4/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*b^3*(-1/(arctanh(tanh(b*x+a))-b*x)^4*((19/16*arctanh(tanh(b*x+a))^(5/2)+
(-17/6*arctanh(tanh(b*x+a))+17/6*b*x)*arctanh(tanh(b*x+a))^(3/2)+(29/16*a^
2+29/8*a*(arctanh(tanh(b*x+a))-b*x-a)+29/16*(arctanh(tanh(b*x+a))-b*x-a)^2
)*arctanh(tanh(b*x+a))^(1/2))/x^3/b^3-35/16/(arctanh(tanh(b*x+a))-b*x)^(1/
2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))-1
/(arctanh(tanh(b*x+a))-b*x)^4/arctanh(tanh(b*x+a))^(1/2))
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^4\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \left[\frac{105\,(b^4x^4+ab^3x^3)\sqrt{a}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,(105\,ab^3x^3+35\,a^2b^2}{48\,(a^5bx^4+a^6x^3)}\right.$$

$$\left.-\frac{105\,(b^4x^4+ab^3x^3)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+(105\,ab^3x^3+35\,a^2b^2x^2-14\,a^3bx+8\,a^4)\sqrt{bx+a}}{24\,(a^5bx^4+a^6x^3)}\right]$$

input
```
integrate(1/x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(105*(b^4*x^4 + a*b^3*x^3)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a
) + 2*a)/x) - 2*(105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8*a^4)*sqrt
(b*x + a))/(a^5*b*x^4 + a^6*x^3), -1/24*(105*(b^4*x^4 + a*b^3*x^3)*sqrt(-a
)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*
a^3*b*x + 8*a^4)*sqrt(b*x + a))/(a^5*b*x^4 + a^6*x^3)]
```

3.157.6 Sympy [F]

$$\int \frac{1}{x^4\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{1}{x^4\,\text{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**4/atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(1/(x**4*atanh(tanh(a + b*x))**(3/2)), x)
```

3.157.7 Maxima [F]

$$\int \frac{1}{x^4\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{1}{x^4\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x^4*arctanh(tanh(b*x + a))^(3/2)), x)
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^4 \text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{35\,b^3 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^4} - \frac{2\,b^3}{\sqrt{bx+a}a^4} - \frac{57\,(bx+a)^{\frac{5}{2}}b^3 - 136\,(bx+a)^{\frac{3}{2}}ab^3 + 87\,\sqrt{bx+a}a^2b^3}{24\,a^4b^3x^3}$$

input
```
integrate(1/x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-35/8*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4) - 2*b^3/(sqrt(b*x
+ a)*a^4) - 1/24*(57*(b*x + a)^(5/2)*b^3 - 136*(b*x + a)^(3/2)*a*b^3 + 87*
sqrt(b*x + a)*a^2*b^3)/(a^4*b^3*x^3)
```

3.157.9 Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 1258, normalized size of antiderivative = 5.13

$$\int \frac{1}{x^4 \text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(x^4*atanh(tanh(a + b*x))^(3/2)),x)
```

output
```
(((38*b^2)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - (140*b^3*x)/(log(2/(exp(2*a)*exp(
2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*
b*x)^4)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/
(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(x*(log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))) - (4*(log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*
x) + 1))/2)^(1/2))/(3*x^3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (2^(1/2)*b^3*log(
(((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i + 2^(1/2)
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x) - 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^9 + 144*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^7 - 672*a^3*(2*a - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^6 + 2016*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 4032*a^...
```

3.157. $\int \frac{1}{x^4 \operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.158 $\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.158.1 Optimal result

Integrand size = 15, antiderivative size = 99

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^4}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}} - \frac{16x^3}{3b^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} + \frac{32x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b^3} - \frac{128x\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3b^4} + \frac{256\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{15b^5}$$

output
```
-2/3*x^4/b/arctanh(tanh(b*x+a))^(3/2)-128/3*x*arctanh(tanh(b*x+a))^(3/2)/b
^4+256/15*arctanh(tanh(b*x+a))^(5/2)/b^5-16/3*x^3/b^2/arctanh(tanh(b*x+a))
^(1/2)+32*x^2*arctanh(tanh(b*x+a))^(1/2)/b^3
```

3.158.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2(5b^4x^4 + 40b^3x^3\operatorname{arctanh}(\tanh(a+bx)) - 240b^2x^2\operatorname{arctanh}(\tanh(a+bx))^2 + 320bx\operatorname{arctanh}(\tanh(a+bx))}{15b^5\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Integrate[x^4/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(-2*(5*b^4*x^4 + 40*b^3*x^3*ArcTanh[Tanh[a + b*x]] - 240*b^2*x^2*ArcTanh[T
anh[a + b*x]]^2 + 320*b*x*ArcTanh[Tanh[a + b*x]]^3 - 128*ArcTanh[Tanh[a +
b*x]]^4))/(15*b^5*ArcTanh[Tanh[a + b*x]]^(3/2))
```

3.158.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2599, 2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\text{arctanh}(\tanh(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{8\int \frac{x^3}{\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{3b}-\frac{2x^4}{3b\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2599}$$

$$\frac{8\left(\frac{6\int \frac{x^2}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{b}-\frac{2x^3}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{3b}-\frac{2x^4}{3b\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2599}$$

$$\frac{8\left(\frac{6\left(\frac{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{4\int x\sqrt{\text{arctanh}(\tanh(a+bx))}dx}{b}\right)}{b}-\frac{2x^3}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{3b}-\frac{2x^4}{3b\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2599}$$

$$\frac{8\left(\frac{6\left(\frac{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{3/2}dx}{3b}\right)}{b}\right)}{b}-\frac{2x^3}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{3b}-\frac{2x^4}{3b\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$\Big\downarrow$ 2588

$$\frac{8\left(\frac{6\left(\frac{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{2\int \text{arctanh}(\tanh(a+bx))^{3/2}d\text{arctanh}(\tanh(a+bx))}{3b^2}\right)}{b}\right)}{b}-\frac{2x^3}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{3b}-\frac{2x^4}{3b\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$\Big\downarrow$ 15

$$\frac{8\left(\frac{6\left(\frac{2x^2\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{4\left(\frac{2x\text{arctanh}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\text{arctanh}(\tanh(a+bx))^{5/2}}{15b^2}\right)}{b}\right)}{b}-\frac{2x^3}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{3b}-\frac{2x^4}{3b\text{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Int[x^4/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(-2*x^4)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) + (8*((-2*x^3)/(b*Sqrt[ArcTanh
[Tanh[a + b*x]]]) + (6*((2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (4*((2*x*
ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (4*ArcTanh[Tanh[a + b*x]]^(5/2))/(15
*b^2)))/b))/b))/(3*b)
```

3.158. $\int \frac{x^4}{\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.158.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.158.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(81) = 162$.

Time = 0.10 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.98

method	result
default	$\frac{\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}-\frac{8a \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}-\frac{8 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{3}+12a^2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}+}{}$

input
```
int(x^4/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^5*(1/5*arctanh(tanh(b*x+a))^(5/2)-4/3*a*arctanh(tanh(b*x+a))^(3/2)-4/3
*arctanh(tanh(b*x+a))^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)+6*a^2*arctanh(tan
h(b*x+a))^(1/2)+12*a*(arctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/
2)+6*(arctanh(tanh(b*x+a))-b*x-a)^2*arctanh(tanh(b*x+a))^(1/2)-(-4*a^3-12*
a^2*(arctanh(tanh(b*x+a))-b*x-a)-12*a*(arctanh(tanh(b*x+a))-b*x-a)^2-4*(ar
ctanh(tanh(b*x+a))-b*x-a)^3)/arctanh(tanh(b*x+a))^(1/2)-1/3*(a^4+4*a^3*(ar
ctanh(tanh(b*x+a))-b*x-a)+6*a^2*(arctanh(tanh(b*x+a))-b*x-a)^2+4*a*(arctan
h(tanh(b*x+a))-b*x-a)^3+(arctanh(tanh(b*x+a))-b*x-a)^4)/arctanh(tanh(b*x+a
))^(3/2))
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2\,(3\,b^4x^4 - 8\,ab^3x^3 + 48\,a^2b^2x^2 + 192\,a^3bx + 128\,a^4)\sqrt{bx+a}}{15\,(b^7x^2 + 2\,ab^6x + a^2b^5)}$$

input
```
integrate(x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^4*x^4 - 8*a*b^3*x^3 + 48*a^2*b^2*x^2 + 192*a^3*b*x + 128*a^4)*sq
rt(b*x + a)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

3.158.6 Sympy [F]

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{x^4}{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}\,dx$$

input
```
integrate(x**4/atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Integral(x**4/atanh(tanh(a + b*x))**(5/2), x)
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2\,(3\,b^5x^5 - 5\,ab^4x^4 + 40\,a^2b^3x^3 + 240\,a^3b^2x^2 + 320\,a^4bx + 128\,a^5)}{15\,(bx+a)^{\frac{5}{2}}b^5}$$

input
```
integrate(x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
2/15*(3*b^5*x^5 - 5*a*b^4*x^4 + 40*a^2*b^3*x^3 + 240*a^3*b^2*x^2 + 320*a^4
*b*x + 128*a^5)/((b*x + a)^(5/2)*b^5)
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2\,(12\,(bx+a)a^3-a^4)}{3\,(bx+a)^{\frac{3}{2}}b^5} + \frac{2\left(3\,(bx+a)^{\frac{5}{2}}b^{20}-20\,(bx+a)^{\frac{3}{2}}ab^{20}+90\,\sqrt{bx+a}a^2b^{20}\right)}{15\,b^{25}}$$

input
```
integrate(x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
2/3*(12*(b*x + a)*a^3 - a^4)/((b*x + a)^(3/2)*b^5) + 2/15*(3*(b*x + a)^(5/
2)*b^20 - 20*(b*x + a)^(3/2)*a*b^20 + 90*sqrt(b*x + a)*a^2*b^20)/b^25
```

3.158.9 Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 817, normalized size of antiderivative = 8.25

$$\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(x^4/atanh(tanh(a + b*x))^(5/2),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*b^4) + (2
*((2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^3 + (8*(log(2/(exp(2*a)*exp(2*b*x) + 1))
/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(5*b
^3))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))/2 + b*x))/(3*b)))/b + (2*x^2*(log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)
^(1/2))/(5*b^3) + (x*((2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^3 + (8*(log(2/(exp(2
*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1))/2 + b*x))/(5*b^3))*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*b) - (2*(log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b
*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(b^5*(log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))) -
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2...
```

3.158. $\int \frac{x^4}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.159 $\int \frac{x^3}{\mathbf{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.159.1 Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^3}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}} - \frac{4x^2}{b^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} + \frac{16x\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b^3} - \frac{32\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3b^4}$$

output
```
-2/3*x^3/b/arctanh(tanh(b*x+a))^(3/2)-32/3*arctanh(tanh(b*x+a))^(3/2)/b^4-
4*x^2/b^2/arctanh(tanh(b*x+a))^(1/2)+16*x*arctanh(tanh(b*x+a))^(1/2)/b^3
```

3.159.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2(b^3x^3 + 6b^2x^2\operatorname{arctanh}(\tanh(a+bx)) - 24bx\operatorname{arctanh}(\tanh(a+bx))^2 + 16\operatorname{arctanh}(\tanh(a+bx))^3)}{3b^4\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Integrate[x^3/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(-2*(b^3*x^3 + 6*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 24*b*x*ArcTanh[Tanh[a +
b*x]]^2 + 16*ArcTanh[Tanh[a + b*x]]^3))/(3*b^4*ArcTanh[Tanh[a + b*x]]^(3/2
))
```

3.159.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\text{arctanh}(\tanh(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2\int \frac{x^2}{\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{b}-\frac{2x^3}{3b\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2\left(\frac{4\int \frac{x}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{b}-\frac{2x^2}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{b}-\frac{2x^3}{3b\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2\left(\frac{4\left(\frac{2x\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{2\int \sqrt{\text{arctanh}(\tanh(a+bx))}dx}{b}\right)}{b}-\frac{2x^2}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{b}-\frac{2x^3}{3b\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2588}$$

$$\frac{2\left(\frac{4\left(\frac{2x\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}-\frac{2\int \sqrt{\text{arctanh}(\tanh(a+bx))}d\text{arctanh}(\tanh(a+bx))}{b^2}\right)}{b}-\frac{2x^2}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{b}-\frac{2x^3}{3b\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{15}$$

$$\frac{2\left(\frac{4\left(\frac{2x\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b}-\frac{4\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3b^2}\right)}{b}-\frac{2x^2}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)}{b}-\frac{2x^3}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Int[x^3/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(-2*x^3)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) + (2*((-2*x^2)/(b*Sqrt[ArcTanh
[Tanh[a + b*x]]]) + (4*((2*x*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (4*ArcTanh[
Tanh[a + b*x]]^(3/2))/(3*b^2)))/b))/b
```

3.159.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.159. $\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.159.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(64) = 128$.

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.45

method	result
default	$\frac{\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3} - 6\sqrt{\operatorname{arctanh}(\tanh(bx+a))}\, a - 6(\operatorname{arctanh}(\tanh(bx+a)) - bx - a)\sqrt{\operatorname{arctanh}(\tanh(bx+a))} - \frac{2\left(3a^2+6a(\operatorname{arctanh}(\tan\right.}{}}{}$

input
```
int(x^3/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^4*(1/3*arctanh(tanh(b*x+a))^(3/2)-3*arctanh(tanh(b*x+a))^(1/2)*a-3*(ar
ctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)-(3*a^2+6*a*(arctanh(t
anh(b*x+a))-b*x-a)+3*(arctanh(tanh(b*x+a))-b*x-a)^2)/arctanh(tanh(b*x+a))^
(1/2)-1/3*(-a^3-3*a^2*(arctanh(tanh(b*x+a))-b*x-a)-3*a*(arctanh(tanh(b*x+a
))-b*x-a)^2-(arctanh(tanh(b*x+a))-b*x-a)^3)/arctanh(tanh(b*x+a))^(3/2))
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\, dx = \frac{2\,(b^3x^3 - 6\,ab^2x^2 - 24\,a^2bx - 16\,a^3)\sqrt{bx+a}}{3\,(b^6x^2 + 2\,ab^5x + a^2b^4)}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/3*(b^3*x^3 - 6*a*b^2*x^2 - 24*a^2*b*x - 16*a^3)*sqrt(b*x + a)/(b^6*x^2 +
 2*a*b^5*x + a^2*b^4)
```

3.159.6 Sympy [A] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\, dx = \begin{cases} -\frac{2x^3}{3b\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))} - \frac{4x^2}{b^2\sqrt{\operatorname{atanh}(\tanh(a+bx))}} + \frac{16x\sqrt{\operatorname{atanh}(\tanh(a+bx))}}{b^3} - \frac{32\,\mathrm{at}}{} \\ \frac{x^4}{4\operatorname{atanh}^{\frac{5}{2}}(\tanh(a))} \end{cases}$$

input
```
integrate(x**3/atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Piecewise((-2*x**3/(3*b*atanh(tanh(a + b*x))**(3/2)) - 4*x**2/(b**2*sqrt(a
tanh(tanh(a + b*x)))) + 16*x*sqrt(atanh(tanh(a + b*x)))/b**3 - 32*atanh(ta
nh(a + b*x))**(3/2)/(3*b**4), Ne(b, 0)), (x**4/(4*atanh(tanh(a))**(5/2)),
True))
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2\,(b^4x^4-5\,ab^3x^3-30\,a^2b^2x^2-40\,a^3bx-16\,a^4)}{3\,(bx+a)^{\frac{5}{2}}b^4}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
2/3*(b^4*x^4 - 5*a*b^3*x^3 - 30*a^2*b^2*x^2 - 40*a^3*b*x - 16*a^4)/((b*x +
 a)^(5/2)*b^4)
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\,(9\,(bx+a)a^2-a^3)}{3\,(bx+a)^{\frac{3}{2}}b^4}+\frac{2\left((bx+a)^{\frac{3}{2}}b^8-9\,\sqrt{bx+a}ab^8\right)}{3\,b^{12}}$$

input
```
integrate(x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
-2/3*(9*(b*x + a)*a^2 - a^3)/((b*x + a)^(3/2)*b^4) + 2/3*((b*x + a)^(3/2)*
b^8 - 9*sqrt(b*x + a)*a*b^8)/b^12
```

3.159.9 Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 533, normalized size of antiderivative = 7.01

$$
\int \frac{x^3}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2\,x\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{3\,b^3}
$$

$$
+\frac{\left(\frac{2\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b^3}+\frac{4\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}{3\,b^3}\right)\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{b}
$$

$$
-\frac{3\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{b^4\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}
$$

$$
-\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{3\,b^4\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)^2}
$$

input
```
int(x^3/atanh(tanh(a + b*x))^(5/2),x)
```

output
```
(2*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(ex
p(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*b^3) + (((2*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
)/b^3 + (4*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(3*b^3))*(log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/
2))/b - (3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log
(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(b^4*(
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*e
xp(2*b*x) + 1)))) - ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^3)/(3*b^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2
/(exp(2*a)*exp(2*b*x) + 1)))^2)
```

3.160 $\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.160.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^2}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}} - \frac{8x}{3b^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} + \frac{16\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{3b^3}$$

output
```
-2/3*x^2/b/arctanh(tanh(b*x+a))^(3/2)-8/3*x/b^2/arctanh(tanh(b*x+a))^(1/2)
+16/3*arctanh(tanh(b*x+a))^(1/2)/b^3
```

3.160.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2(b^2x^2+4bx\operatorname{arctanh}(\tanh(a+bx))-8\operatorname{arctanh}(\tanh(a+bx))^2)}{3b^3\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Integrate[x^2/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(-2*(b^2*x^2 + 4*b*x*ArcTanh[Tanh[a + b*x]] - 8*ArcTanh[Tanh[a + b*x]]^2))
/(3*b^3*ArcTanh[Tanh[a + b*x]]^(3/2))
```

3.160.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\text{arctanh}(\tanh(a+bx))^{5/2}} \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{4\int \frac{x}{\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{3b}-\frac{2x^2}{3b\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2599}$$

$$\frac{4\left(\frac{2\int \frac{1}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{b}-\frac{2x}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{3b}-\frac{2x^2}{3b\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2588}$$

$$\frac{4\left(\frac{2\int \frac{1}{\sqrt{\text{arctanh}(\tanh(a+bx))}}d\text{arctanh}(\tanh(a+bx))}{b^2}-\frac{2x}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{3b}-\frac{2x^2}{3b\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{15}$$

$$\frac{4\left(\frac{4\sqrt{\text{arctanh}(\tanh(a+bx))}}{b^2}-\frac{2x}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{3b}-\frac{2x^2}{3b\text{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Int[x^2/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(4*((-2*x)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (4*Sqrt[ArcTanh[Tanh[a + b*x
]]])/b^2))/(3*b) - (2*x^2)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2))
```

3.160.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.160.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.54

method	result	siz
default	$\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}-\frac{2(-2\operatorname{arctanh}(\tanh(bx+a))+2bx)}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\frac{2\left(a^2+2a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2\right)}{3\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}}{b^3}$	91

input
```
int(x^2/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^3*(arctanh(tanh(b*x+a))^(1/2)-(-2*arctanh(tanh(b*x+a))+2*b*x)/arctanh(
tanh(b*x+a))^(1/2)-1/3*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh
(b*x+a))-b*x-a)^2)/arctanh(tanh(b*x+a))^(3/2))
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2\,(3\,b^2x^2+12\,abx+8\,a^2)\sqrt{bx+a}}{3\,(b^5x^2+2\,ab^4x+a^2b^3)}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*b^2*x^2 + 12*a*b*x + 8*a^2)*sqrt(b*x + a)/(b^5*x^2 + 2*a*b^4*x + a^
2*b^3)
```

3.160.6 Sympy [A] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \begin{cases} -\frac{2x^2}{3b\,\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))} - \frac{8x}{3b^2\sqrt{\operatorname{atanh}(\tanh(a+bx))}} + \frac{16\sqrt{\operatorname{atanh}(\tanh(a+bx))}}{3b^3} & \text{for } b \\ \frac{x^3}{3\,\operatorname{atanh}^{\frac{5}{2}}(\tanh(a))} & \text{othe} \end{cases}$$

input
```
integrate(x**2/atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Piecewise((-2*x**2/(3*b*atanh(tanh(a + b*x))**(3/2)) - 8*x/(3*b**2*sqrt(at
anh(tanh(a + b*x)))) + 16*sqrt(atanh(tanh(a + b*x)))/(3*b**3), Ne(b, 0)),
(x**3/(3*atanh(tanh(a))**(5/2)), True))
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2\,(3\,b^3x^3+15\,ab^2x^2+20\,a^2bx+8\,a^3)}{3\,(bx+a)^{\frac{5}{2}}b^3}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
2/3*(3*b^3*x^3 + 15*a*b^2*x^2 + 20*a^2*b*x + 8*a^3)/((b*x + a)^(5/2)*b^3)
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2\sqrt{bx+a}}{b^3} + \frac{2\,(6\,(bx+a)a-a^2)}{3\,(bx+a)^{\frac{3}{2}}b^3}$$

input
```
integrate(x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
2*sqrt(b*x + a)/b^3 + 2/3*(6*(b*x + a)*a - a^2)/((b*x + a)^(3/2)*b^3)
```

3.160.9 Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 259, normalized size of antiderivative = 4.39

$$\int \frac{x^2}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{8\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(-b^2\,x^2-2\,b\,x\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\,\ln \cdots\right.}{3\,b^3\left(\ln\left(\frac{2\,e \cdots}{e^{2\,a} \cdots}\right.\right.}$$

input
```
int(x^2/atanh(tanh(a + b*x))^(5/2),x)
```

output
```
(8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2)*(2*log(2/(exp(2*a)*exp(2*b*x) + 1))^2 - 4*l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(2/(exp(2*a)*exp(
2*b*x) + 1)) + 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2
- b^2*x^2 - 2*b*x*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x*log(2/(exp(2*a)*exp(2*b*x) + 1))))/(3*b^3*(log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2)
```

3.161 $\int \frac{x}{\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.161.1 Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{x}{\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x}{3b\text{arctanh}(\tanh(a+bx))^{3/2}} - \frac{4}{3b^2\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

output
```
-2/3*x/b/arctanh(tanh(b*x+a))^(3/2)-4/3/b^2/arctanh(tanh(b*x+a))^(1/2)
```

3.161.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{x}{\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2(bx+2\text{arctanh}(\tanh(a+bx)))}{3b^2\text{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Integrate[x/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(-2*(b*x + 2*ArcTanh[Tanh[a + b*x]]))/(3*b^2*ArcTanh[Tanh[a + b*x]]^(3/2))
```

3.161.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}dx}{3b}-\frac{2x}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{2\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}d\operatorname{arctanh}(\tanh(a+bx))}{3b^2}-\frac{2x}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{4}{3b^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}-\frac{2x}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Int[x/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(-2*x)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - 4/(3*b^2*Sqrt[ArcTanh[Tanh[a +
 b*x]]])
```

3.161.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.161.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{-\frac{2}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))}{3\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}}{b^2}$	42

input
```
int(x/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^2*(-1/arctanh(tanh(b*x+a))^(1/2)-1/3*(b*x-arctanh(tanh(b*x+a)))/arctan
h(tanh(b*x+a))^(3/2))
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\,(3\,bx+2\,a)\sqrt{bx+a}}{3\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

input
```
integrate(x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(3*b*x + 2*a)*sqrt(b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

3.161.6 Sympy [A] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \begin{cases} -\frac{2x}{3b\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))} - \frac{4}{3b^2\sqrt{\operatorname{atanh}(\tanh(a+bx))}} & \text{for } b \neq 0 \\ \frac{x^2}{2\operatorname{atanh}^{\frac{5}{2}}(\tanh(a))} & \text{otherwise} \end{cases}$$

input
```
integrate(x/atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Piecewise((-2*x/(3*b*atanh(tanh(a + b*x))**(3/2)) - 4/(3*b**2*sqrt(atanh(t
anh(a + b*x)))), Ne(b, 0)), (x**2/(2*atanh(tanh(a))**(5/2)), True))
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\,(3\,b^2x^2+5\,abx+2\,a^2)}{3\,(bx+a)^{\frac{5}{2}}b^2}$$

input
```
integrate(x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(3*b^2*x^2 + 5*a*b*x + 2*a^2)/((b*x + a)^(5/2)*b^2)
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\,(3\,bx+2\,a)}{3\,(bx+a)^{\frac{3}{2}}b^2}$$

input
```
integrate(x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
-2/3*(3*b*x + 2*a)/((b*x + a)^(3/2)*b^2)
```

3.161.9 Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 4.00

$$\int \frac{x}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{8\sqrt{\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+b\,x\right)}{3\,b^2\,\left(\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)\right)^2}$$

input
```
int(x/atanh(tanh(a + b*x))^(5/2),x)
```

output
```
-(8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp
(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)) + b*x))/(3*b^2*(log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x)
 + 1)))^2)
```

3.162 $\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.162.1 Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

output
```
-2/3/b/arctanh(tanh(b*x+a))^(3/2)
```

3.162.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(-5/2),x]
```

output
```
-2/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2))
```

3.162.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{arctanh}(\tanh(a+bx))^{5/2}} \, dx$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \frac{1}{\text{arctanh}(\tanh(a+bx))^{5/2}} d\text{arctanh}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{2}{3b\text{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(-5/2),x]
```

output
```
-2/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2))
```

3.162.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.162.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2}{3b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}$	15
default	$-\frac{2}{3b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}$	15

input
```
int(1/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b/arctanh(tanh(b*x+a))^(3/2)
```

3.162.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(14) = 28.

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{1}{\mathrm{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\sqrt{bx+a}}{3\,(b^3x^2+2\,ab^2x+a^2b)}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(b*x + a)/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

3.162.6 Sympy [A] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{1}{\mathrm{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \begin{cases} -\frac{2}{3b\,\mathrm{atanh}^{\frac{3}{2}}(\tanh(a+bx))} & \text{for } b \neq 0 \\ \frac{x}{\mathrm{atanh}^{\frac{5}{2}}(\tanh(a))} & \text{otherwise} \end{cases}$$

input
```
integrate(1/atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Piecewise((-2/(3*b*atanh(tanh(a + b*x))**(3/2)), Ne(b, 0)), (x/atanh(tanh(
a))**(5/2), True))
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2}{3\,(bx+a)^{\frac{3}{2}}b}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-2/3/((b*x + a)^(3/2)*b)
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2}{3\,(bx+a)^{\frac{3}{2}}b}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
-2/3/((b*x + a)^(3/2)*b)
```

3.162.9 Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 5.72

$$\int \frac{1}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{8\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{3\,b\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)^2}$$

input
```
int(1/atanh(tanh(a + b*x))^(5/2),x)
```

output
```
-(8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp
(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2)
```

3.163 $\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.163.1 Optimal result

Integrand size = 15, antiderivative size = 108

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{5/2}} - \frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}} + \frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))^2\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

output
```
2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x
-arctanh(tanh(b*x+a)))^(5/2)-2/3/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b
*x+a))^(3/2)+2/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(1/2)
```

3.163.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\text{arctanh}(\tanh(a+bx)))^{5/2}} + \frac{2(-bx+4\text{arctanh}(\tanh(a+bx)))}{3\text{arctanh}(\tanh(a+bx))^{3/2}(-bx+\text{arctanh}(\tanh(a+bx)))^2}$$

input
```
Integrate[1/(x*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

output
```
(-2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*
x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) + (2*(-(b*x) + 4*ArcTanh[Ta
nh[a + b*x]]))/(3*ArcTanh[Tanh[a + b*x]]^(3/2)*(-(b*x) + ArcTanh[Tanh[a +
b*x]])^2)
```

3.163.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2594, 2594, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{2594}$$

$$-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{-\frac{\int \frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2592}$$

$$-\frac{-\frac{2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Int[1/(x*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

output
```
-(((-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x
]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2/((b*x - ArcTanh[Tanh[a + b*
x]])*Sqrt[ArcTanh[Tanh[a + b*x]]]))/(b*x - ArcTanh[Tanh[a + b*x]])) - 2/(3
*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))
```

3.163.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

3.163.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.86

method	result
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{\mathrm{arctanh}(\tanh(bx+a))}}{\sqrt{\mathrm{arctanh}(\tanh(bx+a))-bx}}\right)}{(\mathrm{arctanh}(\tanh(bx+a))-bx)^{\frac{5}{2}}}+\frac{2}{(\mathrm{arctanh}(\tanh(bx+a))-bx)^{2}\sqrt{\mathrm{arctanh}(\tanh(bx+a))}}+\frac{2}{3(\mathrm{arctanh}(\tanh(bx+a))-bx)}$

input
```
int(1/x/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(arctanh(tanh(b*x+a))-b*x)^(5/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(ar
ctanh(tanh(b*x+a))-b*x)^(1/2))+2/(arctanh(tanh(b*x+a))-b*x)^2/arctanh(tanh
(b*x+a))^(1/2)+2/3/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.64

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \left[\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{a}\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(3\,abx+4\,a^2)\sqrt{bx+}}{3\,(a^3b^2x^2+2\,a^4bx+a^5)}\right.$$

input
```
integrate(1/x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(
a) + 2*a)/x) + 2*(3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x
 + a^5), 2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sq
rt(-a)/a) + (3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x + a^
5)]
```

3.163.6 Sympy [F]

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{1}{x\,\text{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x/atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Integral(1/(x*atanh(tanh(a + b*x))**(5/2)), x)
```

3.163.7 Maxima [F]

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{1}{x\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*arctanh(tanh(b*x + a))^(5/2)), x)
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.42

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{2\,(3\,bx+4\,a)}{3\,(bx+a)^{\frac{3}{2}}a^2}$$

input
```
integrate(1/x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + 2/3*(3*b*x + 4*a)/((b*x
+ a)^(3/2)*a^2)
```

3.163.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 886, normalized size of antiderivative = 8.20

$$\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(x*atanh(tanh(a + b*x))^(5/2)),x)
```

output
```
(2^(1/2)*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 -
 log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)
*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x)^5 + 40*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 80*a^3*(2*a -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^2 - 32*a^5 - 10*a*(2*a - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)^4 + 80*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
2*b*x)^(1/2)))*4i)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(5/2) + (16*(log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1)
)/2)^(1/2))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log
(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) - (16*(log(...
```

3.163. $\int \frac{1}{x \text{arctanh}(\tanh(a+bx))^{5/2}} \, dx$

3.164 $\int \frac{1}{x^2\textbf{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.164.1 Optimal result

Integrand size = 15, antiderivative size = 155

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{5b\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{7/2}} - \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}} + \frac{b}{(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{5/2}} - \frac{5b}{3(bx-\text{arctanh}(\tanh(a+bx)))^2\text{arctanh}(\tanh(a+bx))^{3/2}} + \frac{5b}{(bx-\text{arctanh}(\tanh(a+bx)))^3\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

output
```
5*b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b
*x-arctanh(tanh(b*x+a)))^(7/2)-1/x/arctanh(tanh(b*x+a))^(5/2)+b/(b*x-arcta
nh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(5/2)-5/3*b/(b*x-arctanh(tanh(b*x+a)
))^2/arctanh(tanh(b*x+a))^(3/2)+5*b/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(t
anh(b*x+a))^(1/2)
```

3.164.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{5b\text{arctanh}\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\text{arctanh}(\tanh(a+bx)))^{7/2}} + \frac{-2b^2x^2+14bx\text{arctanh}(\tanh(a+bx))+3\text{arctanh}(\tanh(a+bx))^2}{3x(bx-\text{arctanh}(\tanh(a+bx)))^3\text{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

output
```
(5*b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b
*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2) + (-2*b^2*x^2 + 14*b*x*Arc
Tanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2)/(3*x*(b*x - ArcTanh[Tanh
[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]^(3/2))
```

3.164.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.26, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2599, 2594, 2594, 2594, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{2599}$$

$$-\frac{5}{2}b\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{7/2}}dx - \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{5}{2}b\left(-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))} - \frac{2}{5(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{5/2}}\right) - \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{5}{2}b\left(-\frac{-\frac{\int \frac{1}{x\,\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{5(bx-\text{arctanh}(\tanh(a+\right.$$

$$\frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$\Big\downarrow$ 2594

$$-\frac{5}{2}b\left(-\frac{-\frac{-\frac{\int \frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(}}{bx-\text{arctanh}(\tanh(a+bx))}\right.$$

$$\frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$\Big\downarrow$ 2592

$$-\frac{5}{2}b\left(-\frac{-\frac{-\frac{2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{ar}}}{bx-\text{arctanh}(\tanh(a+bx))}\right.$$

$$\frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}$$

input
```
Int[1/(x^2*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

output
```
(-5*b*(-((-(((-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Ta
nh[a + b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2/((b*x - ArcTanh[T
anh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]]))/(b*x - ArcTanh[Tanh[a + b*x]
])) - 2/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2)))/(
b*x - ArcTanh[Tanh[a + b*x]])) - 2/(5*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTa
nh[Tanh[a + b*x]]^(5/2))))/2 - 1/(x*ArcTanh[Tanh[a + b*x]]^(5/2))
```

3.164. $\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.164.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.164.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.84

method	result
default	$2b\left(-\frac{\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2xb}-\frac{5\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^3}-\frac{1}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\right.$

input
```
int(1/x^2/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*b*(-1/(arctanh(tanh(b*x+a))-b*x)^3*(1/2*arctanh(tanh(b*x+a))^(1/2)/x/b-5
/2/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(ar
ctanh(tanh(b*x+a))-b*x)^(1/2)))-1/3/(arctanh(tanh(b*x+a))-b*x)^2/arctanh(t
anh(b*x+a))^(3/2)-2/(arctanh(tanh(b*x+a))-b*x)^3/arctanh(tanh(b*x+a))^(1/2
))
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \left[\frac{15\left(b^3x^3+2\,ab^2x^2+a^2bx\right)\sqrt{a}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\left(15\,ab^2x^2+2\right.}{6\left(a^4b^2x^3+2\,a^5bx^2+a^6x\right)}\right.$$

$$\left.-\frac{15\left(b^3x^3+2\,ab^2x^2+a^2bx\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(15\,ab^2x^2+20\,a^2bx+3\,a^3\right)\sqrt{bx+a}}{3\left(a^4b^2x^3+2\,a^5bx^2+a^6x\right)}\right]$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x +
 a)*sqrt(a) + 2*a)/x) - 2*(15*a*b^2*x^2 + 20*a^2*b*x + 3*a^3)*sqrt(b*x + a
))/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x), -1/3*(15*(b^3*x^3 + 2*a*b^2*x^2 +
a^2*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 + 20*a^
2*b*x + 3*a^3)*sqrt(b*x + a))/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x)]
```

3.164.6 Sympy [F]

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{1}{x^2\,\text{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**2/atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Integral(1/(x**2*atanh(tanh(a + b*x))**(5/2)), x)
```

3.164.7 Maxima [F]

$$\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{1}{x^2\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*arctanh(tanh(b*x + a))^(5/2)), x)
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^2 \text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{5\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} - \frac{2\,(6\,(bx+a)b+ab)}{3\,(bx+a)^{\frac{3}{2}}a^3} - \frac{\sqrt{bx+a}}{a^3x}$$

input
```
integrate(1/x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
-5*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) - 2/3*(6*(b*x + a)*b +
a*b)/((b*x + a)^(3/2)*a^3) - sqrt(b*x + a)/(a^3*x)
```

3.164.9 Mupad [B] (verification not implemented)

Time = 10.73 (sec) , antiderivative size = 1230, normalized size of antiderivative = 7.94

$$\int \frac{1}{x^2 \text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*atanh(tanh(a + b*x))^(5/2)),x)
```

output
```
(2^(1/2)*b*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/
2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^7 + 84*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 280*a^3*(2*
a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 560*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3
 - 672*a^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 128*a^7 - 14*a*(2*a - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)^6 + 448*a^6*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(2*x*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*20i)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(7/2) - (
32*b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/...
```

3.164. $\int \frac{1}{x^2 \operatorname{arctanh}(\tanh(a+bx))^{5/2}} \, dx$

3.165 $\int \frac{1}{x^3\mathbf{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.165.1 Optimal result

Integrand size = 15, antiderivative size = 224

$$\begin{aligned}\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx &= \frac{35b^2\arctan\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{4(bx-\text{arctanh}(\tanh(a+bx)))^{9/2}} \\ &+ \frac{5b}{4x\text{arctanh}(\tanh(a+bx))^{7/2}} \\ &- \frac{5b^2}{4(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{7/2}} \\ &- \frac{1}{2x^2\text{arctanh}(\tanh(a+bx))^{5/2}} \\ &+ \frac{7b^2}{4(bx-\text{arctanh}(\tanh(a+bx)))^2\text{arctanh}(\tanh(a+bx))^{5/2}} \\ &- \frac{35b^2}{12(bx-\text{arctanh}(\tanh(a+bx)))^3\text{arctanh}(\tanh(a+bx))^{3/2}} \\ &+ \frac{35b^2}{4(bx-\text{arctanh}(\tanh(a+bx)))^4\sqrt{\text{arctanh}(\tanh(a+bx))}}\end{aligned}$$

output
```
35/4*b^2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2
))/(b*x-arctanh(tanh(b*x+a)))^(9/2)+5/4*b/x/arctanh(tanh(b*x+a))^(7/2)-5/4
*b^2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(7/2)-1/2/x^2/arctanh
(tanh(b*x+a))^(5/2)+7/4*b^2/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+
a))^(5/2)-35/12*b^2/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(3/2
)+35/4*b^2/(b*x-arctanh(tanh(b*x+a)))^4/arctanh(tanh(b*x+a))^(1/2)
```

3.165.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{35b^2\text{arctanh}\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)}{4(-bx+\text{arctanh}(\tanh(a+bx)))^{9/2}} + \frac{-8b^3x^3+80b^2x^2\text{arctanh}(\tanh(a+bx))+39bx\text{arctanh}(\tanh(a+bx))^2-6\text{arctanh}(\tanh(a+bx))^3}{12x^2\text{arctanh}(\tanh(a+bx))^{3/2}(-bx+\text{arctanh}(\tanh(a+bx)))^4}$$

input
```
Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

output
```
(-35*b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
 + b*x]]]])/(4*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(9/2)) + (-8*b^3*x^3 + 80
*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 39*b*x*ArcTanh[Tanh[a + b*x]]^2 - 6*ArcT
anh[Tanh[a + b*x]]^3)/(12*x^2*ArcTanh[Tanh[a + b*x]]^(3/2)*(-(b*x) + ArcTa
nh[Tanh[a + b*x]])^4)
```

3.165.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2599, 2599, 2594, 2594, 2594, 2594, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{ 2599}$$

$$-\frac{5}{4}b\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{7/2}}dx-\frac{1}{2x^2\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$$\Big\downarrow \text{ 2599}$$

$$-\frac{5}{4}b\left(-\frac{7}{2}b\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{9/2}}dx-\frac{1}{x\text{arctanh}(\tanh(a+bx))^{7/2}}\right)-\frac{1}{2x^2\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$$\Big\downarrow \text{ 2594}$$

$$-\frac{5}{4}b\left(-\frac{7}{2}b\left(-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{7/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{7(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{7/2}}\right)-\frac{}{x\text{arct}}\right.$$

$$\frac{1}{2x^2\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{5}{4}b\left(-\frac{7}{2}b\left(-\frac{-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{5(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{5/2}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{7(bx-\text{arctanh}(\tan}\right.\right.$$

$$\frac{1}{2x^2\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{5}{4}b\left(-\frac{7}{2}b\left(-\frac{-\frac{-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{5(bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}\right.\right.$$

$$\frac{1}{2x^2\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{5}{4}b\left(-\frac{7}{2}b\left(-\frac{-\frac{-\frac{-\frac{\int \frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arct}}}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}\right.\right.$$

$$\frac{1}{2x^2\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$$\Big\downarrow \text{2592}$$

3.165. $\displaystyle\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

$$-\frac{5}{4}b\left(-\frac{7}{2}b\left(-\frac{-\frac{-\frac{-\frac{2\arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{(bx-\operatorname{arctanh}(\tanh(a+bx)))^{3/2}}-\frac{2}{(bx-\operatorname{arctanh}(\tanh(a+bx)))\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}}{bx-\operatorname{arctanh}(\tanh(a+bx))}-\frac{}{3(bx-\operatorname{arctanh}(\tanh(a+b}}{bx-\operatorname{arctanh}(\tanh(a+bx))}}{bx-\operatorname{arctanh}(\tanh(a+bx)}\right.\right.$$

$$\frac{1}{2x^2\operatorname{arctanh}(\tanh(a+bx))^{5/2}}$$

input
```
Int[1/(x^3*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

output
```
(-5*b*((-7*b*(-((-((-(((-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x -
ArcTanh[Tanh[a + b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2/((b*x -
 ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]]))/(b*x - ArcTanh[Tan
h[a + b*x]])) - 2/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]
^(3/2)))/(b*x - ArcTanh[Tanh[a + b*x]])) - 2/(5*(b*x - ArcTanh[Tanh[a + b*
x]])*ArcTanh[Tanh[a + b*x]]^(5/2)))/(b*x - ArcTanh[Tanh[a + b*x]])) - 2/(7
*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(7/2))))/2 - 1/(x*A
rcTanh[Tanh[a + b*x]]^(7/2))))/4 - 1/(2*x^2*ArcTanh[Tanh[a + b*x]]^(5/2))
```

3.165.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.165.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.70

method	result
default	$2b^2 \left(\frac{\frac{\frac{11 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{8} + \left(-\frac{13 \operatorname{arctanh}(\tanh(bx+a))}{8} + \frac{13bx}{8}\right)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{x^2 b^2} - \frac{35 \operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{8\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^4} + \right.$

input
```
int(1/x^3/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*b^2*(1/(arctanh(tanh(b*x+a))-b*x)^4*((11/8*arctanh(tanh(b*x+a))^(3/2)+(-
13/8*arctanh(tanh(b*x+a))+13/8*b*x)*arctanh(tanh(b*x+a))^(1/2))/x^2/b^2-35
/8/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(ar
ctanh(tanh(b*x+a))-b*x)^(1/2)))+3/(arctanh(tanh(b*x+a))-b*x)^4/arctanh(tan
h(b*x+a))^(1/2)+1/3/(arctanh(tanh(b*x+a))-b*x)^3/arctanh(tanh(b*x+a))^(3/2
))
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^3 \operatorname{arctanh}(\tanh(a+bx))^{5/2}}\, dx = \left[\frac{105\left(b^4x^4 + 2\,ab^3x^3 + a^2b^2x^2\right)\sqrt{a}\log\left(\frac{bx - 2\sqrt{bx+a}\sqrt{a} + 2\,a}{x}\right) + 2\left(105\,ab^3x\right.}{24\left(a^5b^2x^4 + 2\,a^6bx^3 + a^7x^2\right)} \right.$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(105*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt
(b*x + a)*sqrt(a) + 2*a)/x) + 2*(105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*
b*x - 6*a^4)*sqrt(b*x + a))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2), 1/12*(1
05*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqr
t(-a)/a) + (105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*x - 6*a^4)*sqrt(b*x
 + a))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)]
```

3.165.6 Sympy [F]

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{1}{x^3\,\text{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**3/atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Integral(1/(x**3*atanh(tanh(a + b*x))**(5/2)), x)
```

3.165.7 Maxima [F]

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{1}{x^3\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(x^3*arctanh(tanh(b*x + a))^(5/2)), x)
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{35\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^4} + \frac{2\left(9\left(bx+a\right)b^2+ab^2\right)}{3\left(bx+a\right)^{\frac{3}{2}}a^4} + \frac{11\left(bx+a\right)^{\frac{3}{2}}b^2-13\sqrt{bx+a}ab^2}{4\,a^4b^2x^2}$$

input
```
integrate(1/x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
35/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4) + 2/3*(9*(b*x + a)*
b^2 + a*b^2)/((b*x + a)^(3/2)*a^4) + 1/4*(11*(b*x + a)^(3/2)*b^2 - 13*sqrt
(b*x + a)*a*b^2)/(a^4*b^2*x^2)
```

3.165.9 Mupad [B] (verification not implemented)

Time = 10.94 (sec) , antiderivative size = 1514, normalized size of antiderivative = 6.76

$$\int \frac{1}{x^3 \text{arctanh}(\tanh(a+bx))^{5/2}} \, dx = \text{Too large to display}$$

input
```
int(1/(x^3*atanh(tanh(a + b*x))^(5/2)),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((4*(2*b*(2*log(2/(exp(2*a)*exp(2*b*x) + 1))
- 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x) - 7*b*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)))/(3*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)) + (56*b^2*x)/(3*(2*a*b - b*(2*a - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))))/(2*b*x^2 - x*(log(2/(exp(2*a)*e
xp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x))^2 - ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 -
log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((140*b)/(3*(log(2/(exp(2*a)*exp
(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^3) - (280*b^2*x)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4))/(2*b*x^2 - x*(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + 2*b*x)) + (2^(1/2)*b^2*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(lo...
```

3.166 $\int \frac{1}{x^4 \operatorname{arctanh}(\tanh(a+bx))^{5/2}} \, dx$

3.166.1 Optimal result

Integrand size = 15, antiderivative size = 278

$$\begin{aligned}
\int \frac{1}{x^4 \operatorname{arctanh}(\tanh(a+bx))^{5/2}} \, dx &= \frac{105 b^3 \arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))^{11/2}} \\
&\quad - \frac{35 b^2}{24 x \operatorname{arctanh}(\tanh(a+bx))^{9/2}} \\
&\quad + \frac{35 b^3}{24(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))^{9/2}} \\
&\quad + \frac{5b}{12 x^2 \operatorname{arctanh}(\tanh(a+bx))^{7/2}} \\
&\quad - \frac{15 b^3}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))^2 \operatorname{arctanh}(\tanh(a+bx))^{7/2}} \\
&\quad - \frac{1}{3 x^3 \operatorname{arctanh}(\tanh(a+bx))^{5/2}} \\
&\quad + \frac{21 b^3}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))^3 \operatorname{arctanh}(\tanh(a+bx))^{5/2}} \\
&\quad - \frac{35 b^3}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))^4 \operatorname{arctanh}(\tanh(a+bx))^{3/2}} \\
&\quad + \frac{105 b^3}{8(bx-\operatorname{arctanh}(\tanh(a+bx)))^5 \sqrt{\operatorname{arctanh}(\tanh(a+bx))}}
\end{aligned}$$

output
```
105/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/
2))/(b*x-arctanh(tanh(b*x+a)))^(11/2)-35/24*b^2/x/arctanh(tanh(b*x+a))^(9/
2)+35/24*b^3/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(9/2)+5/12*b/
x^2/arctanh(tanh(b*x+a))^(7/2)-15/8*b^3/(b*x-arctanh(tanh(b*x+a)))^2/arcta
nh(tanh(b*x+a))^(7/2)-1/3/x^3/arctanh(tanh(b*x+a))^(5/2)+21/8*b^3/(b*x-arc
tanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(5/2)-35/8*b^3/(b*x-arctanh(tanh
(b*x+a)))^4/arctanh(tanh(b*x+a))^(3/2)+105/8*b^3/(b*x-arctanh(tanh(b*x+a))
)^5/arctanh(tanh(b*x+a))^(1/2)
```

3.166.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^4 \text{arctanh}(\tanh(a+bx))^{5/2}}\, dx = \frac{1}{24}\left(\frac{315b^3\text{arctanh}\left(\frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\text{arctanh}(\tanh(a+bx)))^{11/2}}\right.$$

$$+\frac{-16b^4x^4+208b^3x^3\text{arctanh}(\tanh(a+bx))+165b^2x^2\text{arctanh}(\tanh(a+bx))^2-50bx\text{arctanh}(\tanh(a+bx)}{x^3(bx-\text{arctanh}(\tanh(a+bx)))^5\text{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Integrate[1/(x^4*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

output
```
((315*b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[
a + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(11/2) + (-16*b^4*x^4 + 208
*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 165*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 5
0*b*x*ArcTanh[Tanh[a + b*x]]^3 + 8*ArcTanh[Tanh[a + b*x]]^4)/(x^3*(b*x - A
rcTanh[Tanh[a + b*x]])^5*ArcTanh[Tanh[a + b*x]]^(3/2)))/24
```

3.166.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2599, 2599, 2599, 2594, 2594, 2594, 2594, 2594, 2592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\text{arctanh}(\tanh(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{2599}$$

$$-\frac{5}{6}b\int \frac{1}{x^3\text{arctanh}(\tanh(a+bx))^{7/2}}dx-\frac{1}{3x^3\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$$\Big\downarrow \text{2599}$$

$$-\frac{5}{6}b\left(-\frac{7}{4}b\int \frac{1}{x^2\text{arctanh}(\tanh(a+bx))^{9/2}}dx-\frac{1}{2x^2\text{arctanh}(\tanh(a+bx))^{7/2}}\right)-\frac{1}{3x^3\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$$\Big\downarrow \text{2599}$$

$$-\frac{5}{6}b\left(-\frac{7}{4}b\left(-\frac{9}{2}b\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{11/2}}dx-\frac{1}{x\text{arctanh}(\tanh(a+bx))^{9/2}}\right)-\frac{1}{2x^2\text{arctanh}(\tanh(a+bx))^{7/}}\right.$$
$$\frac{1}{3x^3\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{5}{6}b\left(-\frac{7}{4}b\left(-\frac{9}{2}b\left(-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{9/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{9(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{9/2}}\right)-\right.\right.$$
$$\frac{1}{3x^3\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{5}{6}b\left(-\frac{7}{4}b\left(-\frac{9}{2}b\left(-\frac{-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{7/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{7(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{7/2}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{9(bx-\text{arctan}}\right.\right.\right.$$
$$\frac{1}{3x^3\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{5}{6}b\left(-\frac{7}{4}b\left(-\frac{9}{2}b\left(-\frac{-\frac{-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{5/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{5(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{5/2}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{7(bx-\text{arctanh}(\tanh}}{bx-\text{arctanh}(\tanh(a+bx))}\right.\right.\right.$$

$$\frac{1}{3x^3\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{5}{6}b\left(-\frac{7}{4}b\left(-\frac{9}{2}b\left(-\frac{-\frac{-\frac{-\frac{\int \frac{1}{x\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{5(bx-\text{arctanh}(\tanh(}}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+}\right.\right.\right.$$

$$\frac{1}{3x^3\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{5}{6}b\left(-\frac{7}{4}b\left(-\frac{9}{2}b\left(-\frac{-\frac{-\frac{-\frac{-\frac{\int \frac{1}{x\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{2}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{}{3(bx-\text{arctanh}(\tanh(a+}}{bx-\text{arctanh}(\tanh(a+bx))}}{bx-\text{arctanh}(\tanh(a+bx))}}{bx}\right.\right.\right.$$

$$\frac{1}{3x^3\text{arctanh}(\tanh(a+bx))^{5/2}}$$

$$\Big\downarrow \text{2592}$$

3.166. $\int \frac{1}{x^4\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

$$-\frac{5}{6}b\left(-\frac{7}{4}b\left(-\frac{9}{2}b\left(-\frac{-\frac{-\frac{-\frac{2\arctan\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{(bx-\operatorname{arctanh}(\tanh(a+bx)))^{3/2}}-\frac{2}{(bx-\operatorname{arctanh}(\tanh(a+bx)))\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}}{bx-\operatorname{arctanh}(\tanh(a+bx))}-\frac{}{3(bx-\operatorname{arctanh}}}{bx-\operatorname{arctanh}(\tanh(a+bx))}}{bx-\operatorname{arctanh}(\tanh(a+}\right.\right.\right.$$

$$\frac{1}{3x^3\operatorname{arctanh}(\tanh(a+bx))^{5/2}}$$

input
```
Int[1/(x^4*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

output
```
(-5*b*((-7*b*((-9*b*(-((-((-((-(((-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/S
qrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) -
 2/((b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]]))/(b*x - A
rcTanh[Tanh[a + b*x]])) - 2/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh
[a + b*x]]^(3/2)))/(b*x - ArcTanh[Tanh[a + b*x]])) - 2/(5*(b*x - ArcTanh[T
anh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(5/2)))/(b*x - ArcTanh[Tanh[a + b*x]
])) - 2/(7*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(7/2)))/(
b*x - ArcTanh[Tanh[a + b*x]])) - 2/(9*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTa
nh[Tanh[a + b*x]]^(9/2))))/2 - 1/(x*ArcTanh[Tanh[a + b*x]]^(9/2))))/4 - 1/
(2*x^2*ArcTanh[Tanh[a + b*x]]^(7/2))))/6 - 1/(3*x^3*ArcTanh[Tanh[a + b*x]]
^(5/2))
```

3.166.3.1 Defintions of rubi rules used

rule 2592
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[2*(ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]]/(a*Rt[(b*u - a*v
)/a, 2])), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.166.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.76

method	result
default	$2b^3\left(-\frac{\frac{\frac{41\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{16}+\left(-\frac{35\,\mathrm{arctanh}(\tanh(bx+a))}{6}+\frac{35bx}{6}\right)\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}+\left(\frac{55a^2}{16}+\frac{55a(\mathrm{arctanh}(\tanh(bx+a))-bx-a)}{8}+\right.}{x^3b^3}}{(\mathrm{arctanh}(\tanh(bx+a))-bx)^5}\right.$

input
```
int(1/x^4/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*b^3*(-1/(arctanh(tanh(b*x+a))-b*x)^5*((41/16*arctanh(tanh(b*x+a))^(5/2)+
(-35/6*arctanh(tanh(b*x+a))+35/6*b*x)*arctanh(tanh(b*x+a))^(3/2)+(55/16*a^
2+55/8*a*(arctanh(tanh(b*x+a))-b*x-a)+55/16*(arctanh(tanh(b*x+a))-b*x-a)^2
)*arctanh(tanh(b*x+a))^(1/2))/x^3/b^3-105/16/(arctanh(tanh(b*x+a))-b*x)^(1
/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))-
1/3/(arctanh(tanh(b*x+a))-b*x)^4/arctanh(tanh(b*x+a))^(3/2)-4/(arctanh(tan
h(b*x+a))-b*x)^5/arctanh(tanh(b*x+a))^(1/2))
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\mathrm{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \left[\frac{315\,(b^5x^5+2\,ab^4x^4+a^2b^3x^3)\sqrt{a}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,(315\,ab^4x}{48\,(a^6b^2x^5+2\,a^7bx^4+}\right.$$

$$-\frac{315\,(b^5x^5+2\,ab^4x^4+a^2b^3x^3)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+(315\,ab^4x^4+420\,a^2b^3x^3+63\,a^3b^2x^2-18\,a^4bx+}{24\,(a^6b^2x^5+2\,a^7bx^4+a^8x^3)}$$

input
```
integrate(1/x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(315*(b^5*x^5 + 2*a*b^4*x^4 + a^2*b^3*x^3)*sqrt(a)*log((b*x + 2*sqrt
(b*x + a)*sqrt(a) + 2*a)/x) - 2*(315*a*b^4*x^4 + 420*a^2*b^3*x^3 + 63*a^3*
b^2*x^2 - 18*a^4*b*x + 8*a^5)*sqrt(b*x + a))/(a^6*b^2*x^5 + 2*a^7*b*x^4 +
a^8*x^3), -1/24*(315*(b^5*x^5 + 2*a*b^4*x^4 + a^2*b^3*x^3)*sqrt(-a)*arctan
(sqrt(b*x + a)*sqrt(-a)/a) + (315*a*b^4*x^4 + 420*a^2*b^3*x^3 + 63*a^3*b^2
*x^2 - 18*a^4*b*x + 8*a^5)*sqrt(b*x + a))/(a^6*b^2*x^5 + 2*a^7*b*x^4 + a^8
*x^3)]
```

3.166.6 Sympy [F]

$$\int \frac{1}{x^4\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{1}{x^4\,\text{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**4/atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Integral(1/(x**4*atanh(tanh(a + b*x))**(5/2)), x)
```

3.166.7 Maxima [F]

$$\int \frac{1}{x^4\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{1}{x^4\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(x^4*arctanh(tanh(b*x + a))^(5/2)), x)
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^4\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{105\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^5} - \frac{315\,(bx+a)^4b^3 - 840\,(bx+a)^3ab^3 + 693\,(bx+a)^2a^2b^3 - 144\,(bx+a)a^3b^3 - 16\,a^4b^3}{24\,\left((bx+a)^{\frac{3}{2}} - \sqrt{bx+a}a\right)^3a^5}$$

input
```
integrate(1/x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
-105/8*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^5) - 1/24*(315*(b*x
+ a)^4*b^3 - 840*(b*x + a)^3*a*b^3 + 693*(b*x + a)^2*a^2*b^3 - 144*(b*x +
a)*a^3*b^3 - 16*a^4*b^3)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)^3*a^5)
```

3.166.9 Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 2359, normalized size of antiderivative = 8.49

$$\int \frac{1}{x^4\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(x^4*atanh(tanh(a + b*x))^(5/2)),x)
```

output
```
(8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*x^3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (((1
40*b^2)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 - (840*b^3*x)/(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)^5)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(ex
p(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))) - ((log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) +
1))/2)^(1/2)*(x*((((1232*b^4)/(9*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (80*b^3*(2*b*(2*log(2/(exp(2*a)*exp(2*b*
x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*
x) - 7*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(9*(2*a*b - b*(2*a - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4))*(log(2/(exp(2*a)*exp(2*b*x) + 1...
```

3.166. $\int \frac{1}{x^4 \operatorname{arctanh}(\tanh(a+bx))^{5/2}} \, dx$

3.167 $\int x^{7/2}\textbf{arctanh}(\tanh(a+bx))\,dx$

3.167.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))\,dx=-\frac{4}{99}bx^{11/2}+\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))$$

output
```
-4/99*b*x^(11/2)+2/9*x^(9/2)*arctanh(tanh(b*x+a))
```

3.167.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))\,dx=\frac{2}{99}x^{9/2}(-2bx+11\text{arctanh}(\tanh(a+bx)))$$

input
```
Integrate[x^(7/2)*ArcTanh[Tanh[a + b*x]],x]
```

output
```
(2*x^(9/2)*(-2*b*x + 11*ArcTanh[Tanh[a + b*x]]))/99
```

3.167.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))-\frac{2}{9}b\int x^{9/2}dx$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))-\frac{4}{99}bx^{11/2}$$

input
```
Int[x^(7/2)*ArcTanh[Tanh[a + b*x]],x]
```

output
```
(-4*b*x^(11/2))/99 + (2*x^(9/2)*ArcTanh[Tanh[a + b*x]])/9
```

3.167.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.167.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{4b\,x^{\frac{11}{2}}}{99}+\frac{2x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))}{9}$
default	$-\frac{4b\,x^{\frac{11}{2}}}{99}+\frac{2x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))}{9}$
risch	$\frac{2x^{\frac{9}{2}}\ln\left(\mathrm{e}^{bx+a}\right)}{9}-\frac{\left(11i\pi\,x^{4}\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{3}+11i\pi\,x^{4}\,\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{3}+11i\pi\,x^{4}\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\right.}{}$

input
```
int(x^(7/2)*arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-4/99*b*x^(11/2)+2/9*x^(9/2)*arctanh(tanh(b*x+a))
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int x^{7/2}\operatorname{arctanh}(\tanh(a+bx))\,dx=\frac{2}{99}\left(9\,bx^{5}+11\,ax^{4}\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
2/99*(9*b*x^5 + 11*a*x^4)*sqrt(x)
```

3.167.6 Sympy [A] (verification not implemented)

Time = 141.79 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int x^{7/2}\operatorname{arctanh}(\tanh(a+bx))\,dx=-\frac{4bx^{\frac{11}{2}}}{99}+\frac{2x^{\frac{9}{2}}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{9}$$

input
```
integrate(x**(7/2)*atanh(tanh(b*x+a)),x)
```

output
```
-4*b*x**(11/2)/99 + 2*x**(9/2)*atanh(tanh(a + b*x))/9
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))\,dx = -\frac{4}{99}\,bx^{\frac{11}{2}} + \frac{2}{9}\,x^{\frac{9}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)$$

input
```
integrate(x^(7/2)*arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-4/99*b*x^(11/2) + 2/9*x^(9/2)*arctanh(tanh(b*x + a))
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))\,dx = \frac{2}{11}\,bx^{\frac{11}{2}} + \frac{2}{9}\,ax^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
2/11*b*x^(11/2) + 2/9*a*x^(9/2)
```

3.167.9 Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))\,dx = \frac{x^{9/2}\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9} - \frac{4\,b\,x^{11/2}}{99} - \frac{x^{9/2}\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}$$

input
```
int(x^(7/2)*atanh(tanh(a + b*x)),x)
```

output
```
(x^(9/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/9 - (4*b*x^
(11/2))/99 - (x^(9/2)*log(1/(exp(2*a)*exp(2*b*x) + 1)))/9
```

3.168 $\int x^{5/2}\textbf{arctanh}(\tanh(a+bx))\,dx$

3.168.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))\,dx=-\frac{4}{63}bx^{9/2}+\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))$$

output
```
-4/63*b*x^(9/2)+2/7*x^(7/2)*arctanh(tanh(b*x+a))
```

3.168.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))\,dx=\frac{2}{63}x^{7/2}(-2bx+9\text{arctanh}(\tanh(a+bx)))$$

input
```
Integrate[x^(5/2)*ArcTanh[Tanh[a + b*x]],x]
```

output
```
(2*x^(7/2)*(-2*b*x + 9*ArcTanh[Tanh[a + b*x]]))/63
```

3.168.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))-\frac{2}{7}b\int x^{7/2}dx$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))-\frac{4}{63}bx^{9/2}$$

input
```
Int[x^(5/2)*ArcTanh[Tanh[a + b*x]],x]
```

output
```
(-4*b*x^(9/2))/63 + (2*x^(7/2)*ArcTanh[Tanh[a + b*x]])/7
```

3.168.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.168.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{4b\,x^{\frac{9}{2}}}{63}+\frac{2x^{\frac{7}{2}}\,\mathrm{arctanh}(\tanh(bx+a))}{7}$
default	$-\frac{4b\,x^{\frac{9}{2}}}{63}+\frac{2x^{\frac{7}{2}}\,\mathrm{arctanh}(\tanh(bx+a))}{7}$
risch	$\frac{2x^{\frac{7}{2}}\,\ln(\mathrm{e}^{bx+a})}{7}-\frac{\left(9i\pi\,x^{3}\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)+9i\pi\,x^{3}\,\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)^{2}\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right.}{}$

input
```
int(x^(5/2)*arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-4/63*b*x^(9/2)+2/7*x^(7/2)*arctanh(tanh(b*x+a))
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int x^{5/2}\mathrm{arctanh}(\tanh(a+bx))\,dx=\frac{2}{63}\left(7\,bx^{4}+9\,ax^{3}\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
2/63*(7*b*x^4 + 9*a*x^3)*sqrt(x)
```

3.168.6 Sympy [A] (verification not implemented)

Time = 12.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int x^{5/2}\mathrm{arctanh}(\tanh(a+bx))\,dx=-\frac{4bx^{\frac{9}{2}}}{63}+\frac{2x^{\frac{7}{2}}\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}{7}$$

input
```
integrate(x**(5/2)*atanh(tanh(b*x+a)),x)
```

output
```
-4*b*x**(9/2)/63 + 2*x**(7/2)*atanh(tanh(a + b*x))/7
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))\,dx = -\frac{4}{63}\,bx^{\frac{9}{2}} + \frac{2}{7}\,x^{\frac{7}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)$$

input
```
integrate(x^(5/2)*arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-4/63*b*x^(9/2) + 2/7*x^(7/2)*arctanh(tanh(b*x + a))
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))\,dx = \frac{2}{9}\,bx^{\frac{9}{2}} + \frac{2}{7}\,ax^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
2/9*b*x^(9/2) + 2/7*a*x^(7/2)
```

3.168.9 Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))\,dx = \frac{x^{7/2}\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7} - \frac{4\,b\,x^{9/2}}{63} - \frac{x^{7/2}\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}$$

input
```
int(x^(5/2)*atanh(tanh(a + b*x)),x)
```

output
```
(x^(7/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/7 - (4*b*x^
(9/2))/63 - (x^(7/2)*log(1/(exp(2*a)*exp(2*b*x) + 1)))/7
```

3.169 $\int x^{3/2}\textbf{arctanh}(\tanh(a+bx))\,dx$

3.169.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))\,dx=-\frac{4}{35}bx^{7/2}+\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))$$

output
```
-4/35*b*x^(7/2)+2/5*x^(5/2)*arctanh(tanh(b*x+a))
```

3.169.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))\,dx=\frac{2}{35}x^{5/2}(-2bx+7\text{arctanh}(\tanh(a+bx)))$$

input
```
Integrate[x^(3/2)*ArcTanh[Tanh[a + b*x]],x]
```

output
```
(2*x^(5/2)*(-2*b*x + 7*ArcTanh[Tanh[a + b*x]]))/35
```

3.169.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))-\frac{2}{5}b\int x^{5/2}dx$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))-\frac{4}{35}bx^{7/2}$$

input
```
Int[x^(3/2)*ArcTanh[Tanh[a + b*x]],x]
```

output
```
(-4*b*x^(7/2))/35 + (2*x^(5/2)*ArcTanh[Tanh[a + b*x]])/5
```

3.169.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.169.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{4b\,x^{\frac{7}{2}}}{35}+\frac{2x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))}{5}$
default	$-\frac{4b\,x^{\frac{7}{2}}}{35}+\frac{2x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))}{5}$
risch	$\frac{2x^{\frac{5}{2}}\ln(\mathrm{e}^{bx+a})}{5}-\frac{\left(7i\pi\,x^{2}\operatorname{csgn}(i\mathrm{e}^{bx+a})^{2}\operatorname{csgn}(i\mathrm{e}^{2bx+2a})-14i\pi\,x^{2}\operatorname{csgn}(i\mathrm{e}^{bx+a})\operatorname{csgn}(i\mathrm{e}^{2bx+2a})^{2}+7i\pi\,x^{2}\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx}}{\mathrm{e}^{2bx+2}}\right.\right.}{}$

input
```
int(x^(3/2)*arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-4/35*b*x^(7/2)+2/5*x^(5/2)*arctanh(tanh(b*x+a))
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))\,dx=\frac{2}{35}\left(5\,bx^{3}+7\,ax^{2}\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
2/35*(5*b*x^3 + 7*a*x^2)*sqrt(x)
```

3.169.6 Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))\,dx=-\frac{4bx^{\frac{7}{2}}}{35}+\frac{2x^{\frac{5}{2}}\,\text{atanh}\left(\tanh\left(a+bx\right)\right)}{5}$$

input
```
integrate(x**(3/2)*atanh(tanh(b*x+a)),x)
```

output
```
-4*b*x**(7/2)/35 + 2*x**(5/2)*atanh(tanh(a + b*x))/5
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))\,dx = -\frac{4}{35}\,bx^{\frac{7}{2}} + \frac{2}{5}\,x^{\frac{5}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)$$

input
```
integrate(x^(3/2)*arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-4/35*b*x^(7/2) + 2/5*x^(5/2)*arctanh(tanh(b*x + a))
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))\,dx = \frac{2}{7}\,bx^{\frac{7}{2}} + \frac{2}{5}\,ax^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
2/7*b*x^(7/2) + 2/5*a*x^(5/2)
```

3.169.9 Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))\,dx = \frac{x^{5/2}\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{5} - \frac{4\,b\,x^{7/2}}{35} - \frac{x^{5/2}\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{5}$$

input
```
int(x^(3/2)*atanh(tanh(a + b*x)),x)
```

output
```
(x^(5/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/5 - (4*b*x^
(7/2))/35 - (x^(5/2)*log(1/(exp(2*a)*exp(2*b*x) + 1)))/5
```

3.170 $\int \sqrt{x}\text{arctanh}(\tanh(a+bx))\,dx$

3.170.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))\,dx=-\frac{4}{15}bx^{5/2}+\frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))$$

output
```
-4/15*b*x^(5/2)+2/3*x^(3/2)*arctanh(tanh(b*x+a))
```

3.170.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))\,dx=\frac{2}{15}x^{3/2}(-2bx+5\text{arctanh}(\tanh(a+bx)))$$

input
```
Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]],x]
```

output
```
(2*x^(3/2)*(-2*b*x + 5*ArcTanh[Tanh[a + b*x]]))/15
```

3.170.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))-\frac{2}{3}b\int x^{3/2}dx$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))-\frac{4}{15}bx^{5/2}$$

input
```
Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]],x]
```

output
```
(-4*b*x^(5/2))/15 + (2*x^(3/2)*ArcTanh[Tanh[a + b*x]])/3
```

3.170.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.170.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{4b\,x^{\frac{5}{2}}}{15}+\frac{2x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))}{3}$
default	$-\frac{4b\,x^{\frac{5}{2}}}{15}+\frac{2x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))}{3}$
risch	$\frac{2x^{\frac{3}{2}}\ln\left(\mathrm{e}^{bx+a}\right)}{3}-\frac{\left(-5i\pi x\ \operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}+5i\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(i\mathrm{e}^{bx+a}\right)^{2}\pi x-10i\pi x\ \operatorname{csgn}(i\right.}{}$

input
```
int(arctanh(tanh(b*x+a))*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/15*b*x^(5/2)+2/3*x^(3/2)*arctanh(tanh(b*x+a))
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))\,dx=\frac{2}{15}\left(3\,bx^{2}+5\,ax\right)\sqrt{x}$$

input
```
integrate(arctanh(tanh(b*x+a))*x^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b*x^2 + 5*a*x)*sqrt(x)
```

3.170.6 Sympy [F]

$$\int\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))\,dx=\int\sqrt{x}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)\,dx$$

input
```
integrate(atanh(tanh(b*x+a))*x**(1/2),x)
```

output
```
Integral(sqrt(x)*atanh(tanh(a + b*x)), x)
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))\,dx = -\frac{4}{15}\,bx^{\frac{5}{2}} + \frac{2}{3}\,x^{\frac{3}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)$$

input
```
integrate(arctanh(tanh(b*x+a))*x^(1/2),x, algorithm="maxima")
```

output
```
-4/15*b*x^(5/2) + 2/3*x^(3/2)*arctanh(tanh(b*x + a))
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))\,dx = \frac{2}{5}\,bx^{\frac{5}{2}} + \frac{2}{3}\,ax^{\frac{3}{2}}$$

input
```
integrate(arctanh(tanh(b*x+a))*x^(1/2),x, algorithm="giac")
```

output
```
2/5*b*x^(5/2) + 2/3*a*x^(3/2)
```

3.170.9 Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))\,dx = \frac{x^{3/2}\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{3} - \frac{4\,b\,x^{5/2}}{15} - \frac{x^{3/2}\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{3}$$

input
```
int(x^(1/2)*atanh(tanh(a + b*x)),x)
```

output
```
(x^(3/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/3 - (4*b*x^
(5/2))/15 - (x^(3/2)*log(1/(exp(2*a)*exp(2*b*x) + 1)))/3
```

3.171 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{\sqrt{x}}\,dx$

3.171.1 Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{\sqrt{x}}\,dx = -\frac{4}{3}bx^{3/2}+2\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))$$

output
```
-4/3*b*x^(3/2)+2*arctanh(tanh(b*x+a))*x^(1/2)
```

3.171.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{\sqrt{x}}\,dx = \frac{2}{3}\sqrt{x}(-2bx+3\operatorname{arctanh}(\tanh(a+bx)))$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(-2*b*x + 3*ArcTanh[Tanh[a + b*x]]))/3
```

3.171.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$2\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))-2b\int \sqrt{x}dx$$

$$\Big\downarrow \text{ 15}$$

$$2\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))-\frac{4}{3}bx^{3/2}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]/Sqrt[x],x]
```

output
```
(-4*b*x^(3/2))/3 + 2*Sqrt[x]*ArcTanh[Tanh[a + b*x]]
```

3.171.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.171.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{4b\,x^{\frac{3}{2}}}{3}+2\,\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\sqrt{x}$
default	$-\frac{4b\,x^{\frac{3}{2}}}{3}+2\,\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\sqrt{x}$
risch	$2\sqrt{x}\,\ln\left(\mathrm{e}^{bx+a}\right)-\frac{\left(3i\pi\,\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)+3i\pi\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{3}+3i\pi\operatorname{csgn}\left(\ldots\right.}{\ldots}$

input
```
int(arctanh(tanh(b*x+a))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*b*x^(3/2)+2*arctanh(tanh(b*x+a))*x^(1/2)
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.48

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{\sqrt{x}}\,dx = \frac{2}{3}\,(bx+3\,a)\sqrt{x}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="fricas")
```

output
```
2/3*(b*x + 3*a)*sqrt(x)
```

3.171.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{\sqrt{x}}\,dx = \int \frac{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{\sqrt{x}}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))/x**(1/2),x)
```

output
```
Integral(atanh(tanh(a + b*x))/sqrt(x), x)
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{\sqrt{x}}\,dx = -\frac{4}{3}\,bx^{\frac{3}{2}} + 2\,\sqrt{x}\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)$$

input
```
integrate(arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="maxima")
```

output
```
-4/3*b*x^(3/2) + 2*sqrt(x)*arctanh(tanh(b*x + a))
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{\sqrt{x}}\,dx = \frac{2}{3}\,bx^{\frac{3}{2}} + 2\,a\sqrt{x}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="giac")
```

output
```
2/3*b*x^(3/2) + 2*a*sqrt(x)
```

3.171.9 Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.24

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{\sqrt{x}}\,dx = \sqrt{x}\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \frac{4\,b\,x^{3/2}}{3} - \sqrt{x}\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)$$

input
```
int(atanh(tanh(a + b*x))/x^(1/2),x)
```

output
```
x^(1/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - (4*b*x^(3/2
))/3 - x^(1/2)*log(1/(exp(2*a)*exp(2*b*x) + 1))
```

3.172 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^{3/2}} \, dx$

3.172.1 Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^{3/2}} \, dx = 4b\sqrt{x} - \frac{2\operatorname{arctanh}(\tanh(a+bx))}{\sqrt{x}}$$

output
```
-2*arctanh(tanh(b*x+a))/x^(1/2)+4*b*x^(1/2)
```

3.172.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^{3/2}} \, dx = \frac{4bx - 2\operatorname{arctanh}(\tanh(a+bx))}{\sqrt{x}}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]/x^(3/2),x]
```

output
```
(4*b*x - 2*ArcTanh[Tanh[a + b*x]])/Sqrt[x]
```

3.172.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))}{x^{3/2}} dx$$

$$\Big\downarrow \ 2599$$

$$2b\int \frac{1}{\sqrt{x}}dx - \frac{2\text{arctanh}(\tanh(a+bx))}{\sqrt{x}}$$

$$\Big\downarrow \ 15$$

$$4b\sqrt{x} - \frac{2\text{arctanh}(\tanh(a+bx))}{\sqrt{x}}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]/x^(3/2),x]
```

output
```
4*b*Sqrt[x] - (2*ArcTanh[Tanh[a + b*x]])/Sqrt[x]
```

3.172.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.172.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))}{\sqrt{x}}+4b\sqrt{x}$
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))}{\sqrt{x}}+4b\sqrt{x}$
risch	$-\frac{2\ln(\mathrm{e}^{bx+a})}{\sqrt{x}}+\frac{i\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)+i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{3}+i\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{3}}{}$

input
```
int(arctanh(tanh(b*x+a))/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh(tanh(b*x+a))/x^(1/2)+4*b*x^(1/2)
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{\mathrm{arctanh}(\tanh(a+bx))}{x^{3/2}}\,dx=\frac{2\,(bx-a)}{\sqrt{x}}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^(3/2),x, algorithm="fricas")
```

output
```
2*(b*x - a)/sqrt(x)
```

3.172.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\mathrm{arctanh}(\tanh(a+bx))}{x^{3/2}}\,dx=4b\sqrt{x}-\frac{2\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}{\sqrt{x}}$$

input
```
integrate(atanh(tanh(b*x+a))/x**(3/2),x)
```

output
```
4*b*sqrt(x) - 2*atanh(tanh(a + b*x))/sqrt(x)
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^{3/2}}\,dx = 4\,b\sqrt{x} - \frac{2\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{\sqrt{x}}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^(3/2),x, algorithm="maxima")
```

output
```
4*b*sqrt(x) - 2*arctanh(tanh(b*x + a))/sqrt(x)
```

3.172.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^{3/2}}\,dx = 2\,b\sqrt{x} - \frac{2\,a}{\sqrt{x}}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^(3/2),x, algorithm="giac")
```

output
```
2*b*sqrt(x) - 2*a/sqrt(x)
```

3.172.9 Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.43

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^{3/2}}\,dx = \frac{\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{\sqrt{x}} + 4\,b\,\sqrt{x} - \frac{\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{\sqrt{x}}$$

input
```
int(atanh(tanh(a + b*x))/x^(3/2),x)
```

output
```
log(1/(exp(2*a)*exp(2*b*x) + 1))/x^(1/2) + 4*b*x^(1/2) - log((exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/x^(1/2)
```

3.173 $\int \frac{\text{arctanh}(\tanh(a+bx))}{x^{5/2}} dx$

3.173.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{\text{arctanh}(\tanh(a+bx))}{x^{5/2}} dx = -\frac{4b}{3\sqrt{x}} - \frac{2\text{arctanh}(\tanh(a+bx))}{3x^{3/2}}$$

output
```
-2/3*arctanh(tanh(b*x+a))/x^(3/2)-4/3*b/x^(1/2)
```

3.173.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\text{arctanh}(\tanh(a+bx))}{x^{5/2}} dx = -\frac{2(2bx + \text{arctanh}(\tanh(a+bx)))}{3x^{3/2}}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]/x^(5/2),x]
```

output
```
(-2*(2*b*x + ArcTanh[Tanh[a + b*x]]))/(3*x^(3/2))
```

3.173.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^{5/2}} \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2}{3} b \int \frac{1}{x^{3/2}} dx - \frac{2 \operatorname{arctanh}(\tanh(a+bx))}{3x^{3/2}}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2 \operatorname{arctanh}(\tanh(a+bx))}{3x^{3/2}} - \frac{4b}{3\sqrt{x}}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]/x^(5/2),x]
```

output
```
(-4*b)/(3*Sqrt[x]) - (2*ArcTanh[Tanh[a + b*x]])/(3*x^(3/2))
```

3.173.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.173.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))}{3x^{\frac{3}{2}}}-\frac{4b}{3\sqrt{x}}$
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))}{3x^{\frac{3}{2}}}-\frac{4b}{3\sqrt{x}}$
risch	$-\frac{2\ln\left(\mathrm{e}^{bx+a}\right)}{3x^{\frac{3}{2}}}-\frac{-i\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)-i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{3}-i\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right.}{}$

input
```
int(arctanh(tanh(b*x+a))/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*arctanh(tanh(b*x+a))/x^(3/2)-4/3*b/x^(1/2)
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.41

$$\int \frac{\mathrm{arctanh}(\tanh(a+bx))}{x^{5/2}}\,dx = -\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(3*b*x + a)/x^(3/2)
```

3.173.6 Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\mathrm{arctanh}(\tanh(a+bx))}{x^{5/2}}\,dx = -\frac{4b}{3\sqrt{x}}-\frac{2\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}{3x^{\frac{3}{2}}}$$

input
```
integrate(atanh(tanh(b*x+a))/x**(5/2),x)
```

output
```
-4*b/(3*sqrt(x)) - 2*atanh(tanh(a + b*x))/(3*x**(3/2))
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^{5/2}}\,dx = -\frac{4\,b}{3\,\sqrt{x}} - \frac{2\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^(5/2),x, algorithm="maxima")
```

output
```
-4/3*b/sqrt(x) - 2/3*arctanh(tanh(b*x + a))/x^(3/2)
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.41

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^{5/2}}\,dx = -\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^(5/2),x, algorithm="giac")
```

output
```
-2/3*(3*b*x + a)/x^(3/2)
```

3.173.9 Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^{5/2}}\,dx = -\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 4\,b\,x}{3\,x^{3/2}}$$

input
```
int(atanh(tanh(a + b*x))/x^(5/2),x)
```

output
```
-(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(1/(exp(2*a)*e
xp(2*b*x) + 1)) + 4*b*x)/(3*x^(3/2))
```

3.174 $\int \frac{\mathbf{arctanh}(\tanh(a+bx))}{x^{7/2}} \, dx$

3.174.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{\text{arctanh}(\tanh(a+bx))}{x^{7/2}} \, dx = -\frac{4b}{15x^{3/2}} - \frac{2\text{arctanh}(\tanh(a+bx))}{5x^{5/2}}$$

output
```
-4/15*b/x^(3/2)-2/5*arctanh(tanh(b*x+a))/x^(5/2)
```

3.174.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{\text{arctanh}(\tanh(a+bx))}{x^{7/2}} \, dx = -\frac{2(2bx + 3\text{arctanh}(\tanh(a+bx)))}{15x^{5/2}}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]/x^(7/2),x]
```

output
```
(-2*(2*b*x + 3*ArcTanh[Tanh[a + b*x]]))/(15*x^(5/2))
```

3.174.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))}{x^{7/2}} \, dx$$

$$\Big\downarrow \; 2599$$

$$\frac{2}{5} b \int \frac{1}{x^{5/2}} dx - \frac{2\text{arctanh}(\tanh(a+bx))}{5x^{5/2}}$$

$$\Big\downarrow \; 15$$

$$-\frac{2\text{arctanh}(\tanh(a+bx))}{5x^{5/2}} - \frac{4b}{15x^{3/2}}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]/x^(7/2),x]
```

output
```
(-4*b)/(15*x^(3/2)) - (2*ArcTanh[Tanh[a + b*x]])/(5*x^(5/2))
```

3.174.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.174.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{4b}{15x^{\frac{3}{2}}}-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))}{5x^{\frac{5}{2}}}$
default	$-\frac{4b}{15x^{\frac{3}{2}}}-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))}{5x^{\frac{5}{2}}}$
risch	$-\frac{2\ln(\mathrm{e}^{bx+a})}{5x^{\frac{5}{2}}}-\frac{-3i\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)-3i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{3}-3i\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+}}{\mathrm{e}^{2bx+2c}}\right.}{}$

input
```
int(arctanh(tanh(b*x+a))/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-4/15*b/x^(3/2)-2/5*arctanh(tanh(b*x+a))/x^(5/2)
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \frac{\mathrm{arctanh}(\tanh(a+bx))}{x^{7/2}}\,dx=-\frac{2\,(5\,bx+3\,a)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(5*b*x + 3*a)/x^(5/2)
```

3.174.6 Sympy [A] (verification not implemented)

Time = 13.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\mathrm{arctanh}(\tanh(a+bx))}{x^{7/2}}\,dx=-\frac{4b}{15x^{\frac{3}{2}}}-\frac{2\,\mathrm{atanh}\,(\tanh\,(a+bx))}{5x^{\frac{5}{2}}}$$

input
```
integrate(atanh(tanh(b*x+a))/x**(7/2),x)
```

output
```
-4*b/(15*x**(3/2)) - 2*atanh(tanh(a + b*x))/(5*x**(5/2))
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^{7/2}}\,dx = -\frac{4\,b}{15\,x^{\frac{3}{2}}} - \frac{2\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^(7/2),x, algorithm="maxima")
```

output
```
-4/15*b/x^(3/2) - 2/5*arctanh(tanh(b*x + a))/x^(5/2)
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^{7/2}}\,dx = -\frac{2\left(5\,bx+3\,a\right)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))/x^(7/2),x, algorithm="giac")
```

output
```
-2/15*(5*b*x + 3*a)/x^(5/2)
```

3.174.9 Mupad [B] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^{7/2}}\,dx = \frac{\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{5\,x^{5/2}} - \frac{4\,b}{15\,x^{3/2}} - \frac{\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{5\,x^{5/2}}$$

input
```
int(atanh(tanh(a + b*x))/x^(7/2),x)
```

output
```
log(1/(exp(2*a)*exp(2*b*x) + 1))/(5*x^(5/2)) - (4*b)/(15*x^(3/2)) - log((e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/(5*x^(5/2))
```

3.175 $\int x^{7/2}\textbf{arctanh}(\tanh(a+bx))^2\,dx$

3.175.1 Optimal result

Integrand size = 15, antiderivative size = 48

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))^2\,dx=\frac{16b^2x^{13/2}}{1287}-\frac{8}{99}bx^{11/2}\text{arctanh}(\tanh(a+bx))+\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))^2$$

output
```
16/1287*b^2*x^(13/2)-8/99*b*x^(11/2)*arctanh(tanh(b*x+a))+2/9*x^(9/2)*arct
anh(tanh(b*x+a))^2
```

3.175.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))^2\,dx=\frac{2x^{9/2}(8b^2x^2-52bx\text{arctanh}(\tanh(a+bx))+143\text{arctanh}(\tanh(a+bx))^2}{1287}$$

input
```
Integrate[x^(7/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(2*x^(9/2)*(8*b^2*x^2 - 52*b*x*ArcTanh[Tanh[a + b*x]] + 143*ArcTanh[Tanh[a
 + b*x]]^2))/1287
```

3.175.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))^2\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{9}b\int x^{9/2}\text{arctanh}(\tanh(a+bx))dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{9}b\left(\frac{2}{11}x^{11/2}\text{arctanh}(\tanh(a+bx))-\frac{2}{11}b\int x^{11/2}dx\right)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{9}b\left(\frac{2}{11}x^{11/2}\text{arctanh}(\tanh(a+bx))-\frac{4}{143}bx^{13/2}\right)$$

input
```
Int[x^(7/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(2*x^(9/2)*ArcTanh[Tanh[a + b*x]]^2)/9 - (4*b*((-4*b*x^(13/2))/143 + (2*x^
(11/2)*ArcTanh[Tanh[a + b*x]])/11))/9
```

3.175.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.175.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{2x^{\frac{9}{2}} \operatorname{arctanh}(\tanh(bx+a))^2}{9} - \frac{8b\left(\frac{x^{\frac{11}{2}} \operatorname{arctanh}(\tanh(bx+a))}{11} - \frac{2x^{\frac{13}{2}} b}{143}\right)}{9}$	38
default	$\frac{2x^{\frac{9}{2}} \operatorname{arctanh}(\tanh(bx+a))^2}{9} - \frac{8b\left(\frac{x^{\frac{11}{2}} \operatorname{arctanh}(\tanh(bx+a))}{11} - \frac{2x^{\frac{13}{2}} b}{143}\right)}{9}$	38
risch	Expression too large to display	2093

input
```
int(x^(7/2)*arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
2/9*x^(9/2)*arctanh(tanh(b*x+a))^2-8/9*b*(1/11*x^(11/2)*arctanh(tanh(b*x+a
))-2/143*x^(13/2)*b)
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))^2\,dx = \frac{2}{1287}\left(99\,b^2x^6+234\,abx^5+143\,a^2x^4\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
2/1287*(99*b^2*x^6 + 234*a*b*x^5 + 143*a^2*x^4)*sqrt(x)
```

3.175.6 Sympy [F(-1)]

Timed out.

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))^2\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*atanh(tanh(b*x+a))**2,x)
```

output
```
Timed out
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))^2\,dx = \frac{16}{1287}\,b^2x^{\frac{13}{2}} - \frac{8}{99}\,bx^{\frac{11}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right) + \frac{2}{9}\,x^{\frac{9}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

input
```
integrate(x^(7/2)*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
16/1287*b^2*x^(13/2) - 8/99*b*x^(11/2)*arctanh(tanh(b*x + a)) + 2/9*x^(9/2
)*arctanh(tanh(b*x + a))^2
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.50

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))^2\,dx=\frac{2}{13}\,b^2x^{\frac{13}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{9}\,a^2x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
2/13*b^2*x^(13/2) + 4/11*a*b*x^(11/2) + 2/9*a^2*x^(9/2)
```

3.175.9 Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.54

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))^2\,dx=\frac{x^{9/2}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^2}{18}+\frac{2\,b^2\,x^{13/2}}{13}-\frac{2\,b\,x^{11/2}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}{11}$$

input
```
int(x^(7/2)*atanh(tanh(a + b*x))^2,x)
```

output
```
(x^(9/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/18 + (2*b^2*x^(13/2))/13 - (2*b*x^(1
1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x))/11
```

3.176 $\int x^{5/2}\mathbf{arctanh}(\tanh(a+bx))^2\,dx$

3.176.1 Optimal result

Integrand size = 15, antiderivative size = 48

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))^2\,dx=\frac{16}{693}b^2x^{11/2}-\frac{8}{63}bx^{9/2}\text{arctanh}(\tanh(a+bx))+\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))^2$$

output
```
16/693*b^2*x^(11/2)-8/63*b*x^(9/2)*arctanh(tanh(b*x+a))+2/7*x^(7/2)*arctan
h(tanh(b*x+a))^2
```

3.176.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))^2\,dx=\frac{2}{693}x^{7/2}\left(8b^2x^2-44bx\text{arctanh}(\tanh(a+bx))+99\text{arctanh}(\tanh(a+bx))^2\right)$$

input
```
Integrate[x^(5/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(2*x^(7/2)*(8*b^2*x^2 - 44*b*x*ArcTanh[Tanh[a + b*x]] + 99*ArcTanh[Tanh[a
+ b*x]]^2))/693
```

3.176.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))^2\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{7}b\int x^{7/2}\text{arctanh}(\tanh(a+bx))dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{7}b\left(\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))-\frac{2}{9}b\int x^{9/2}dx\right)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{7}b\left(\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))-\frac{4}{99}bx^{11/2}\right)$$

input
```
Int[x^(5/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(2*x^(7/2)*ArcTanh[Tanh[a + b*x]]^2)/7 - (4*b*((-4*b*x^(11/2))/99 + (2*x^(
9/2)*ArcTanh[Tanh[a + b*x]])/9))/7
```

3.176.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.176.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{2x^{\frac{7}{2}} \operatorname{arctanh}(\tanh(bx+a))^2}{7} - \frac{8b\left(\frac{x^{\frac{9}{2}} \operatorname{arctanh}(\tanh(bx+a))}{9} - \frac{2b\,x^{\frac{11}{2}}}{99}\right)}{7}$	38
default	$\frac{2x^{\frac{7}{2}} \operatorname{arctanh}(\tanh(bx+a))^2}{7} - \frac{8b\left(\frac{x^{\frac{9}{2}} \operatorname{arctanh}(\tanh(bx+a))}{9} - \frac{2b\,x^{\frac{11}{2}}}{99}\right)}{7}$	38
risch	Expression too large to display	2093

input
```
int(x^(5/2)*arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
2/7*x^(7/2)*arctanh(tanh(b*x+a))^2-8/7*b*(1/9*x^(9/2)*arctanh(tanh(b*x+a))
-2/99*b*x^(11/2))
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))^2\,dx = \frac{2}{693}\left(63\,b^2x^5+154\,abx^4+99\,a^2x^3\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
2/693*(63*b^2*x^5 + 154*a*b*x^4 + 99*a^2*x^3)*sqrt(x)
```

3.176.6 Sympy [A] (verification not implemented)

Time = 22.48 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))^2\,dx = \frac{16b^2x^{\frac{11}{2}}}{693} - \frac{8bx^{\frac{9}{2}}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{63} + \frac{2x^{\frac{7}{2}}\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{7}$$

input
```
integrate(x**(5/2)*atanh(tanh(b*x+a))**2,x)
```

output
```
16*b**2*x**(11/2)/693 - 8*b*x**(9/2)*atanh(tanh(a + b*x))/63 + 2*x**(7/2)*
atanh(tanh(a + b*x))**2/7
```

3.176.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))^2\,dx = \frac{16}{693}\,b^2x^{\frac{11}{2}} - \frac{8}{63}\,bx^{\frac{9}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right) + \frac{2}{7}\,x^{\frac{7}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

input
```
integrate(x^(5/2)*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
16/693*b^2*x^(11/2) - 8/63*b*x^(9/2)*arctanh(tanh(b*x + a)) + 2/7*x^(7/2)*
arctanh(tanh(b*x + a))^2
```

3.176.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.50

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))^2\,dx = \frac{2}{11}\,b^2x^{\frac{11}{2}} + \frac{4}{9}\,abx^{\frac{9}{2}} + \frac{2}{7}\,a^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
2/11*b^2*x^(11/2) + 4/9*a*b*x^(9/2) + 2/7*a^2*x^(7/2)
```

3.176.9 Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.54

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))^2\,dx = \frac{x^{7/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{14} + \frac{2\,b^2\,x^{11/2}}{11} - \frac{2\,b\,x^{9/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{9}$$

input
```
int(x^(5/2)*atanh(tanh(a + b*x))^2,x)
```

output
```
(x^(7/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/14 + (2*b^2*x^(11/2))/11 - (2*b*x^(9
/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x))/9
```

3.177 $\int x^{3/2}\textbf{arctanh}(\tanh(a+bx))^2\,dx$

3.177.1 Optimal result

Integrand size = 15, antiderivative size = 48

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^2\,dx=\frac{16}{315}b^2x^{9/2}-\frac{8}{35}bx^{7/2}\text{arctanh}(\tanh(a+bx))+\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))^2$$

output
```
16/315*b^2*x^(9/2)-8/35*b*x^(7/2)*arctanh(tanh(b*x+a))+2/5*x^(5/2)*arctanh
(tanh(b*x+a))^2
```

3.177.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^2\,dx=\frac{2}{315}x^{5/2}\left(8b^2x^2-36bx\text{arctanh}(\tanh(a+bx))+63\text{arctanh}(\tanh(a+bx))^2\right)$$

input
```
Integrate[x^(3/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(2*x^(5/2)*(8*b^2*x^2 - 36*b*x*ArcTanh[Tanh[a + b*x]] + 63*ArcTanh[Tanh[a
+ b*x]]^2))/315
```

3.177.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^2\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{5}b\int x^{5/2}\text{arctanh}(\tanh(a+bx))dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{5}b\left(\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))-\frac{2}{7}b\int x^{7/2}dx\right)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{5}b\left(\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))-\frac{4}{63}bx^{9/2}\right)$$

input
```
Int[x^(3/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(2*x^(5/2)*ArcTanh[Tanh[a + b*x]]^2)/5 - (4*b*((-4*b*x^(9/2))/63 + (2*x^(7
/2)*ArcTanh[Tanh[a + b*x]])/7))/5
```

3.177.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.177.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{2x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))^{2}}{5}-\frac{8b\left(\frac{x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))}{7}-\frac{2b\,x^{\frac{9}{2}}}{63}\right)}{5}$	38
default	$\frac{2x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))^{2}}{5}-\frac{8b\left(\frac{x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))}{7}-\frac{2b\,x^{\frac{9}{2}}}{63}\right)}{5}$	38
risch	Expression too large to display	2093

input
```
int(x^(3/2)*arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
2/5*x^(5/2)*arctanh(tanh(b*x+a))^2-8/5*b*(1/7*x^(7/2)*arctanh(tanh(b*x+a))
-2/63*b*x^(9/2))
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^2\,dx = \frac{2}{315}\left(35\,b^2x^4+90\,abx^3+63\,a^2x^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
2/315*(35*b^2*x^4 + 90*a*b*x^3 + 63*a^2*x^2)*sqrt(x)
```

3.177.6 Sympy [F]

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^2\,dx = \int x^{\frac{3}{2}}\,\text{atanh}^2\left(\tanh\left(a+bx\right)\right)\,dx$$

input
```
integrate(x**(3/2)*atanh(tanh(b*x+a))**2,x)
```

output
```
Integral(x**(3/2)*atanh(tanh(a + b*x))**2, x)
```

3.177.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^2\,dx = \frac{16}{315}\,b^2x^{\frac{9}{2}} - \frac{8}{35}\,bx^{\frac{7}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right) + \frac{2}{5}\,x^{\frac{5}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

input
```
integrate(x^(3/2)*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
16/315*b^2*x^(9/2) - 8/35*b*x^(7/2)*arctanh(tanh(b*x + a)) + 2/5*x^(5/2)*a
rctanh(tanh(b*x + a))^2
```

3.177.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.50

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^2\,dx = \frac{2}{9}\,b^2x^{\frac{9}{2}} + \frac{4}{7}\,abx^{\frac{7}{2}} + \frac{2}{5}\,a^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
2/9*b^2*x^(9/2) + 4/7*a*b*x^(7/2) + 2/5*a^2*x^(5/2)
```

3.177.9 Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.54

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^2\,dx = \frac{x^{5/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{10} + \frac{2\,b^2\,x^{9/2}}{9} - \frac{2\,b\,x^{7/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{7}$$

input
```
int(x^(3/2)*atanh(tanh(a + b*x))^2,x)
```

output
```
(x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/10 + (2*b^2*x^(9/2))/9 - (2*b*x^(7/2
)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x))/7
```

3.178 $\int \sqrt{x}\textbf{arctanh}(\tanh(a+bx))^2\,dx$

3.178.1 Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^2\,dx = \frac{16}{105}b^2x^{7/2} - \frac{8}{15}bx^{5/2}\text{arctanh}(\tanh(a+bx)) + \frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^2$$

output
```
16/105*b^2*x^(7/2)-8/15*b*x^(5/2)*arctanh(tanh(b*x+a))+2/3*x^(3/2)*arctanh
(tanh(b*x+a))^2
```

3.178.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^2\,dx = \frac{2}{105}x^{3/2}\left(8b^2x^2 - 28bx\text{arctanh}(\tanh(a+bx)) + 35\text{arctanh}(\tanh(a+bx))^2\right)$$

input
```
Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(2*x^(3/2)*(8*b^2*x^2 - 28*b*x*ArcTanh[Tanh[a + b*x]] + 35*ArcTanh[Tanh[a
+ b*x]]^2))/105
```

3.178.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^2\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{3}b\int x^{3/2}\text{arctanh}(\tanh(a+bx))dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{3}b\left(\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))-\frac{2}{5}b\int x^{5/2}dx\right)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{3}b\left(\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))-\frac{4}{35}bx^{7/2}\right)$$

input
```
Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(2*x^(3/2)*ArcTanh[Tanh[a + b*x]]^2)/3 - (4*b*((-4*b*x^(7/2))/35 + (2*x^(5
/2)*ArcTanh[Tanh[a + b*x]])/5))/3
```

3.178.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.178.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}} \operatorname{arctanh}(\tanh(bx+a))^{2}}{3}-\frac{8b\left(\frac{x^{\frac{5}{2}} \operatorname{arctanh}(\tanh(bx+a))}{5}-\frac{2b\,x^{\frac{7}{2}}}{35}\right)}{3}$	38
default	$\frac{2x^{\frac{3}{2}} \operatorname{arctanh}(\tanh(bx+a))^{2}}{3}-\frac{8b\left(\frac{x^{\frac{5}{2}} \operatorname{arctanh}(\tanh(bx+a))}{5}-\frac{2b\,x^{\frac{7}{2}}}{35}\right)}{3}$	38
risch	Expression too large to display	2027

input
```
int(arctanh(tanh(b*x+a))^2*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)*arctanh(tanh(b*x+a))^2-8/3*b*(1/5*x^(5/2)*arctanh(tanh(b*x+a))
-2/35*b*x^(7/2))
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.56

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^2\,dx = \frac{2}{105}\left(15\,b^2x^3+42\,abx^2+35\,a^2x\right)\sqrt{x}$$

input
```
integrate(arctanh(tanh(b*x+a))^2*x^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*b^2*x^3 + 42*a*b*x^2 + 35*a^2*x)*sqrt(x)
```

3.178.6 Sympy [F]

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^2\,dx = \int \sqrt{x}\,\text{atanh}^2\left(\tanh\left(a+bx\right)\right)\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**2*x**(1/2),x)
```

output
```
Integral(sqrt(x)*atanh(tanh(a + b*x))**2, x)
```

3.178.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^2\,dx = \frac{16}{105}\,b^2x^{\frac{7}{2}} - \frac{8}{15}\,bx^{\frac{5}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right) + \frac{2}{3}\,x^{\frac{3}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

input
```
integrate(arctanh(tanh(b*x+a))^2*x^(1/2),x, algorithm="maxima")
```

output
```
16/105*b^2*x^(7/2) - 8/15*b*x^(5/2)*arctanh(tanh(b*x + a)) + 2/3*x^(3/2)*a
rctanh(tanh(b*x + a))^2
```

3.178.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.50

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^2\,dx = \frac{2}{7}\,b^2x^{\frac{7}{2}} + \frac{4}{5}\,abx^{\frac{5}{2}} + \frac{2}{3}\,a^2x^{\frac{3}{2}}$$

input
```
integrate(arctanh(tanh(b*x+a))^2*x^(1/2),x, algorithm="giac")
```

output
```
2/7*b^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*a^2*x^(3/2)
```

3.178.9 Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.54

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^2\,dx = \frac{x^{3/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{6} + \frac{2\,b^2\,x^{7/2}}{7} - \frac{2\,b\,x^{5/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{5}$$

input
```
int(x^(1/2)*atanh(tanh(a + b*x))^2,x)
```

output
```
(x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/6 + (2*b^2*x^(7/2))/7 - (2*b*x^(5/2)
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x))/5
```

3.179 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{\sqrt{x}}\,dx$

3.179.1 Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{\sqrt{x}}\,dx = \frac{16}{15}b^2x^{5/2} - \frac{8}{3}bx^{3/2}\operatorname{arctanh}(\tanh(a+bx)) + 2\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^2$$

output
```
16/15*b^2*x^(5/2)-8/3*b*x^(3/2)*arctanh(tanh(b*x+a))+2*arctanh(tanh(b*x+a)
)^2*x^(1/2)
```

3.179.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{\sqrt{x}}\,dx = \frac{2}{15}\sqrt{x}\left(8b^2x^2 - 20bx\operatorname{arctanh}(\tanh(a+bx)) + 15\operatorname{arctanh}(\tanh(a+bx))^2\right)$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^2/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(8*b^2*x^2 - 20*b*x*ArcTanh[Tanh[a + b*x]] + 15*ArcTanh[Tanh[a
+ b*x]]^2))/15
```

3.179.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^2}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{2599}$$

$$2\sqrt{x}\text{arctanh}(\tanh(a+bx))^2-4b\int \sqrt{x}\text{arctanh}(\tanh(a+bx))dx$$

$$\Big\downarrow \text{2599}$$

$$2\sqrt{x}\text{arctanh}(\tanh(a+bx))^2-4b\left(\frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))-\frac{2}{3}b\int x^{3/2}dx\right)$$

$$\Big\downarrow \text{15}$$

$$2\sqrt{x}\text{arctanh}(\tanh(a+bx))^2-4b\left(\frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))-\frac{4}{15}bx^{5/2}\right)$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^2/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2 - 4*b*((-4*b*x^(5/2))/15 + (2*x^(3/2)*A
rcTanh[Tanh[a + b*x]])/3)
```

3.179.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.179.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{4(\operatorname{arctanh}(\tanh(bx+a))-bx)b\,x^{\frac{3}{2}}}{3}+2(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\sqrt{x}$	47
default	$\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{4(\operatorname{arctanh}(\tanh(bx+a))-bx)b\,x^{\frac{3}{2}}}{3}+2(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\sqrt{x}$	47
risch	Expression too large to display	1978

input
```
int(arctanh(tanh(b*x+a))^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*b^2*x^(5/2)+4/3*(arctanh(tanh(b*x+a))-b*x)*b*x^(3/2)+2*(arctanh(tanh(b
*x+a))-b*x)^2*x^(1/2)
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{\sqrt{x}}\,dx=\frac{2}{15}\left(3\,b^2x^2+10\,abx+15\,a^2\right)\sqrt{x}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^2 + 10*a*b*x + 15*a^2)*sqrt(x)
```

3.179.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{\sqrt{x}}\,dx = \int \frac{\operatorname{atanh}^2(\tanh(a+bx))}{\sqrt{x}}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**2/x**(1/2),x)
```

output
```
Integral(atanh(tanh(a + b*x))**2/sqrt(x), x)
```

3.179.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{\sqrt{x}}\,dx = \frac{16}{15}\,b^2x^{\frac{5}{2}} - \frac{8}{3}\,bx^{\frac{3}{2}}\operatorname{artanh}(\tanh(bx+a)) + 2\sqrt{x}\operatorname{artanh}(\tanh(bx+a))^2$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="maxima")
```

output
```
16/15*b^2*x^(5/2) - 8/3*b*x^(3/2)*arctanh(tanh(b*x + a)) + 2*sqrt(x)*arcta
nh(tanh(b*x + a))^2
```

3.179.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{\sqrt{x}}\,dx = \frac{2}{5}\,b^2x^{\frac{5}{2}} + \frac{4}{3}\,abx^{\frac{3}{2}} + 2\,a^2\sqrt{x}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="giac")
```

output
```
2/5*b^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*a^2*sqrt(x)
```

3.179.9 Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.65

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{\sqrt{x}}\,dx = \frac{\sqrt{x}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^2}{2}+\frac{2\,b^2\,x^{5/2}}{5} - \frac{2\,b\,x^{3/2}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}{3}$$

input
```
int(atanh(tanh(a + b*x))^2/x^(1/2),x)
```

output
```
(x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 + (2*b^2*x^(5/2))/5 - (2*b*x^(3/2)
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x))/3
```

3.180 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{3/2}} \, dx$

3.180.1 Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{3/2}} \, dx = -\frac{16}{3}b^2x^{3/2} + 8b\sqrt{x}\operatorname{arctanh}(\tanh(a+bx)) - \frac{2\operatorname{arctanh}(\tanh(a+bx))^2}{\sqrt{x}}$$

output
```
-16/3*b^2*x^(3/2)-2*arctanh(tanh(b*x+a))^2/x^(1/2)+8*b*arctanh(tanh(b*x+a)
)*x^(1/2)
```

3.180.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{3/2}} \, dx = -\frac{2(8b^2x^2 - 12bx\operatorname{arctanh}(\tanh(a+bx)) + 3\operatorname{arctanh}(\tanh(a+bx))^2)}{3\sqrt{x}}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^2/x^(3/2),x]
```

output
```
(-2*(8*b^2*x^2 - 12*b*x*ArcTanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^
2))/(3*Sqrt[x])
```

3.180.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^{3/2}} \, dx$$

$$\Big\downarrow \text{2599}$$

$$4b \int \frac{\text{arctanh}(\tanh(a+bx))}{\sqrt{x}} dx - \frac{2\text{arctanh}(\tanh(a+bx))^2}{\sqrt{x}}$$

$$\Big\downarrow \text{2599}$$

$$4b\left(2\sqrt{x}\text{arctanh}(\tanh(a+bx)) - 2b\int \sqrt{x}dx\right) - \frac{2\text{arctanh}(\tanh(a+bx))^2}{\sqrt{x}}$$

$$\Big\downarrow \text{15}$$

$$4b\left(2\sqrt{x}\text{arctanh}(\tanh(a+bx)) - \frac{4}{3}bx^{3/2}\right) - \frac{2\text{arctanh}(\tanh(a+bx))^2}{\sqrt{x}}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^2/x^(3/2),x]
```

output
```
(-2*ArcTanh[Tanh[a + b*x]]^2)/Sqrt[x] + 4*b*((-4*b*x^(3/2))/3 + 2*Sqrt[x]*
ArcTanh[Tanh[a + b*x]])
```

3.180.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.180.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^2}{\sqrt{x}}+8b\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\sqrt{x}-\frac{2b\,x^{\frac{3}{2}}}{3}\right)$	37
default	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^2}{\sqrt{x}}+8b\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\sqrt{x}-\frac{2b\,x^{\frac{3}{2}}}{3}\right)$	37
risch	Expression too large to display	1977

input
```
int(arctanh(tanh(b*x+a))^2/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh(tanh(b*x+a))^2/x^(1/2)+8*b*(arctanh(tanh(b*x+a))*x^(1/2)-2/3*b*
x^(3/2))
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.52

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{3/2}}\,dx = \frac{2\,(b^2x^2+6\,abx-3\,a^2)}{3\,\sqrt{x}}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^(3/2),x, algorithm="fricas")
```

output
```
2/3*(b^2*x^2 + 6*a*b*x - 3*a^2)/sqrt(x)
```

3.180.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{3/2}}\,dx = \int \frac{\operatorname{atanh}^2(\tanh(a+bx))}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**2/x**(3/2),x)
```

output
```
Integral(atanh(tanh(a + b*x))**2/x**(3/2), x)
```

3.180.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{3/2}}\,dx = -\frac{16}{3}\,b^2x^{\frac{3}{2}} + 8\,b\sqrt{x}\operatorname{artanh}(\tanh(bx+a)) - \frac{2\operatorname{artanh}(\tanh(bx+a))^2}{\sqrt{x}}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^(3/2),x, algorithm="maxima")
```

output
```
-16/3*b^2*x^(3/2) + 8*b*sqrt(x)*arctanh(tanh(b*x + a)) - 2*arctanh(tanh(b*
x + a))^2/sqrt(x)
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{3/2}}\,dx = \frac{2}{3}\,b^2x^{\frac{3}{2}} + 4\,ab\sqrt{x} - \frac{2\,a^2}{\sqrt{x}}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^(3/2),x, algorithm="giac")
```

output
```
2/3*b^2*x^(3/2) + 4*a*b*sqrt(x) - 2*a^2/sqrt(x)
```

3.180.9 Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.77

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{3/2}}\,dx = \frac{2\,b^2\,x^{3/2}}{3} - \frac{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2}{2\,\sqrt{x}}$$
$$- 2\,b\,\sqrt{x}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)$$

input

```
int(atanh(tanh(a + b*x))^2/x^(3/2),x)
```

output

```
(2*b^2*x^(3/2))/3 - (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/(2*x^(1/2)) - 2*b*x^(1/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)
```

3.181 $\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^{5/2}} dx$

3.181.1 Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^{5/2}} dx = \frac{16b^2\sqrt{x}}{3} - \frac{8b\text{arctanh}(\tanh(a+bx))}{3\sqrt{x}} - \frac{2\text{arctanh}(\tanh(a+bx))^2}{3x^{3/2}}$$

output
```
-2/3*arctanh(tanh(b*x+a))^2/x^(3/2)-8/3*b*arctanh(tanh(b*x+a))/x^(1/2)+16/
3*b^2*x^(1/2)
```

3.181.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^{5/2}} dx = \frac{2(8b^2x^2 - 4bx\text{arctanh}(\tanh(a+bx)) - \text{arctanh}(\tanh(a+bx))^2)}{3x^{3/2}}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^2/x^(5/2),x]
```

output
```
(2*(8*b^2*x^2 - 4*b*x*ArcTanh[Tanh[a + b*x]] - ArcTanh[Tanh[a + b*x]]^2))/
(3*x^(3/2))
```

3.181.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^{5/2}} \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{4}{3}b\int \frac{\text{arctanh}(\tanh(a+bx))}{x^{3/2}}dx-\frac{2\text{arctanh}(\tanh(a+bx))^2}{3x^{3/2}}$$

$$\Big\downarrow \text{2599}$$

$$\frac{4}{3}b\left(2b\int \frac{1}{\sqrt{x}}dx-\frac{2\text{arctanh}(\tanh(a+bx))}{\sqrt{x}}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^2}{3x^{3/2}}$$

$$\Big\downarrow \text{15}$$

$$\frac{4}{3}b\left(4b\sqrt{x}-\frac{2\text{arctanh}(\tanh(a+bx))}{\sqrt{x}}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^2}{3x^{3/2}}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^2/x^(5/2),x]
```

output
```
(-2*ArcTanh[Tanh[a + b*x]]^2)/(3*x^(3/2)) + (4*b*(4*b*Sqrt[x] - (2*ArcTanh
[Tanh[a + b*x]])/Sqrt[x]))/3
```

3.181.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.181.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^2}{3x^{\frac{3}{2}}}+\frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{\sqrt{x}}+2b\sqrt{x}\right)}{3}$	38
default	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^2}{3x^{\frac{3}{2}}}+\frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{\sqrt{x}}+2b\sqrt{x}\right)}{3}$	38
risch	Expression too large to display	1972

input
```
int(arctanh(tanh(b*x+a))^2/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*arctanh(tanh(b*x+a))^2/x^(3/2)+8/3*b*(-arctanh(tanh(b*x+a))/x^(1/2)+2
*b*x^(1/2))
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.50

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{5/2}}\,dx=\frac{2\,(3\,b^2x^2-6\,abx-a^2)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*b^2*x^2 - 6*a*b*x - a^2)/x^(3/2)
```

3.181.6 Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{5/2}}\,dx = \frac{16b^2\sqrt{x}}{3} - \frac{8b\operatorname{atanh}(\tanh(a+bx))}{3\sqrt{x}} - \frac{2\operatorname{atanh}^2(\tanh(a+bx))}{3x^{\frac{3}{2}}}$$

input
```
integrate(atanh(tanh(b*x+a))**2/x**(5/2),x)
```

output
```
16*b**2*sqrt(x)/3 - 8*b*atanh(tanh(a + b*x))/(3*sqrt(x)) - 2*atanh(tanh(a
+ b*x))**2/(3*x**(3/2))
```

3.181.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{5/2}}\,dx = \frac{16}{3}\,b^2\sqrt{x} - \frac{8\,b\operatorname{artanh}(\tanh(bx+a))}{3\sqrt{x}} - \frac{2\operatorname{artanh}(\tanh(bx+a))^2}{3\,x^{\frac{3}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^(5/2),x, algorithm="maxima")
```

output
```
16/3*b^2*sqrt(x) - 8/3*b*arctanh(tanh(b*x + a))/sqrt(x) - 2/3*arctanh(tanh
(b*x + a))^2/x^(3/2)
```

3.181.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.48

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{5/2}}\,dx = 2\,b^2\sqrt{x} - \frac{2\,(6\,abx+a^2)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^(5/2),x, algorithm="giac")
```

output
```
2*b^2*sqrt(x) - 2/3*(6*a*b*x + a^2)/x^(3/2)
```

3.181.9 Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.54

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{5/2}}\,dx = 2\,b^2\,\sqrt{x} - \frac{\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2}{6\,x^{3/2}} + \frac{2\,b\,\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right) + 2\,b\,x\right)}{\sqrt{x}}$$

input
```
int(atanh(tanh(a + b*x))^2/x^(5/2),x)
```

output
```
2*b^2*x^(1/2) - (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/(6*x^(3/2)) + (2*b*(log(2/(exp
(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x))/x^(1/2)
```

3.182 $\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^{7/2}} \, dx$

3.182.1 Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^{7/2}} \, dx = -\frac{16b^2}{15\sqrt{x}} - \frac{8b\text{arctanh}(\tanh(a+bx))}{15x^{3/2}} - \frac{2\text{arctanh}(\tanh(a+bx))^2}{5x^{5/2}}$$

output
```
-8/15*b*arctanh(tanh(b*x+a))/x^(3/2)-2/5*arctanh(tanh(b*x+a))^2/x^(5/2)-16
/15*b^2/x^(1/2)
```

3.182.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^{7/2}} \, dx = -\frac{2(8b^2x^2 + 4bx\text{arctanh}(\tanh(a+bx)) + 3\text{arctanh}(\tanh(a+bx))^2)}{15x^{5/2}}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^2/x^(7/2),x]
```

output
```
(-2*(8*b^2*x^2 + 4*b*x*ArcTanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2
))/(15*x^(5/2))
```

3.182.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{7/2}}\,dx$$

$$\Big\downarrow \; 2599$$

$$\frac{4}{5}b\int \frac{\operatorname{arctanh}(\tanh(a+bx))}{x^{5/2}}dx-\frac{2\operatorname{arctanh}(\tanh(a+bx))^2}{5x^{5/2}}$$

$$\Big\downarrow \; 2599$$

$$\frac{4}{5}b\left(\frac{2}{3}b\int \frac{1}{x^{3/2}}dx-\frac{2\operatorname{arctanh}(\tanh(a+bx))}{3x^{3/2}}\right)-\frac{2\operatorname{arctanh}(\tanh(a+bx))^2}{5x^{5/2}}$$

$$\Big\downarrow \; 15$$

$$\frac{4}{5}b\left(-\frac{2\operatorname{arctanh}(\tanh(a+bx))}{3x^{3/2}}-\frac{4b}{3\sqrt{x}}\right)-\frac{2\operatorname{arctanh}(\tanh(a+bx))^2}{5x^{5/2}}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^2/x^(7/2),x]
```

output
```
(-2*ArcTanh[Tanh[a + b*x]]^2)/(5*x^(5/2)) + (4*b*((-4*b)/(3*Sqrt[x]) - (2*
ArcTanh[Tanh[a + b*x]])/(3*x^(3/2))))/5
```

3.182.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.182.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^2}{5x^{\frac{5}{2}}}+\frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{3x^{\frac{3}{2}}}-\frac{2b}{3\sqrt{x}}\right)}{5}$	38
default	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^2}{5x^{\frac{5}{2}}}+\frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{3x^{\frac{3}{2}}}-\frac{2b}{3\sqrt{x}}\right)}{5}$	38
risch	Expression too large to display	1978

input
```
int(arctanh(tanh(b*x+a))^2/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*arctanh(tanh(b*x+a))^2/x^(5/2)+8/5*b*(-1/3*arctanh(tanh(b*x+a))/x^(3/
2)-2/3*b/x^(1/2))
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.50

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{7/2}}\,dx = -\frac{2\,(15\,b^2x^2+10\,abx+3\,a^2)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(15*b^2*x^2 + 10*a*b*x + 3*a^2)/x^(5/2)
```

3.182.6 Sympy [A] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{7/2}}\,dx = -\frac{16b^2}{15\sqrt{x}} - \frac{8b\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{15x^{\frac{3}{2}}} - \frac{2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{5x^{\frac{5}{2}}}$$

input
```
integrate(atanh(tanh(b*x+a))**2/x**(7/2),x)
```

output
```
-16*b**2/(15*sqrt(x)) - 8*b*atanh(tanh(a + b*x))/(15*x**(3/2)) - 2*atanh(t
anh(a + b*x))**2/(5*x**(5/2))
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{7/2}}\,dx = -\frac{16\,b^2}{15\,\sqrt{x}} - \frac{8\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{15\,x^{\frac{3}{2}}} - \frac{2\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{5\,x^{\frac{5}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^(7/2),x, algorithm="maxima")
```

output
```
-16/15*b^2/sqrt(x) - 8/15*b*arctanh(tanh(b*x + a))/x^(3/2) - 2/5*arctanh(t
anh(b*x + a))^2/x^(5/2)
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.50

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{7/2}}\,dx = -\frac{2\,(15\,b^2x^2+10\,abx+3\,a^2)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^2/x^(7/2),x, algorithm="giac")
```

output
```
-2/15*(15*b^2*x^2 + 10*a*b*x + 3*a^2)/x^(5/2)
```

3.182.9 Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.54

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^2}{x^{7/2}}\,dx = \frac{2\,b\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{3\,x^{3/2}} - \frac{2\,b^2}{\sqrt{x}} - \frac{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{10\,x^{5/2}}$$

input
```
int(atanh(tanh(a + b*x))^2/x^(7/2),x)
```

output
```
(2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x))/(3*x^(3/2)) - (2*b^2)/x^(1/2) - (log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^2/(10*x^(5/2))
```

3.183 $\int x^{7/2}\textbf{arctanh}(\tanh(a+bx))^3\,dx$

3.183.1 Optimal result

Integrand size = 15, antiderivative size = 69

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))^3\,dx=-\frac{32b^3x^{15/2}}{6435}+\frac{16}{429}b^2x^{13/2}\text{arctanh}(\tanh(a+bx))-\frac{4}{33}bx^{11/2}\text{arctanh}(\tanh(a+bx))^2+\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))^3$$

output
```
-32/6435*b^3*x^(15/2)+16/429*b^2*x^(13/2)*arctanh(tanh(b*x+a))-4/33*b*x^(1
1/2)*arctanh(tanh(b*x+a))^2+2/9*x^(9/2)*arctanh(tanh(b*x+a))^3
```

3.183.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))^3\,dx=-\frac{2x^{9/2}(16b^3x^3-120b^2x^2\text{arctanh}(\tanh(a+bx))+390bx\text{arctanh}(\tanh(a+bx))^2-715\text{arctanh}(\tanh(a+b}{6435}$$

input
```
Integrate[x^(7/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(-2*x^(9/2)*(16*b^3*x^3 - 120*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 390*b*x*Arc
Tanh[Tanh[a + b*x]]^2 - 715*ArcTanh[Tanh[a + b*x]]^3))/6435
```

3.183.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))^3\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))^3-\frac{2}{3}b\int x^{9/2}\text{arctanh}(\tanh(a+bx))^2dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))^3- \frac{2}{3}b\left(\frac{2}{11}x^{11/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{11}b\int x^{11/2}\text{arctanh}(\tanh(a+bx))dx\right)$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))^3- \frac{2}{3}b\left(\frac{2}{11}x^{11/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{11}b\left(\frac{2}{13}x^{13/2}\text{arctanh}(\tanh(a+bx))-\frac{2}{13}b\int x^{13/2}dx\right)\right)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))^3- \frac{2}{3}b\left(\frac{2}{11}x^{11/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{11}b\left(\frac{2}{13}x^{13/2}\text{arctanh}(\tanh(a+bx))-\frac{4}{195}bx^{15/2}\right)\right)$$

input
```
Int[x^(7/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(2*x^(9/2)*ArcTanh[Tanh[a + b*x]]^3)/9 - (2*b*((2*x^(11/2)*ArcTanh[Tanh[a
+ b*x]]^2)/11 - (4*b*((-4*b*x^(15/2))/195 + (2*x^(13/2)*ArcTanh[Tanh[a + b
*x]])/13))/11))/3
```

3.183.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.183.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2x^{\frac{9}{2}} \operatorname{arctanh}(\tanh(bx+a))^3}{9} - \frac{4b\left(\frac{x^{\frac{11}{2}} \operatorname{arctanh}(\tanh(bx+a))^2}{11} - \frac{4b\left(\frac{x^{\frac{13}{2}} \operatorname{arctanh}(\tanh(bx+a))}{13} - \frac{2x^{\frac{15}{2}} b}{195}\right)}{11}\right)}{3}$	56
default	$\frac{2x^{\frac{9}{2}} \operatorname{arctanh}(\tanh(bx+a))^3}{9} - \frac{4b\left(\frac{x^{\frac{11}{2}} \operatorname{arctanh}(\tanh(bx+a))^2}{11} - \frac{4b\left(\frac{x^{\frac{13}{2}} \operatorname{arctanh}(\tanh(bx+a))}{13} - \frac{2x^{\frac{15}{2}} b}{195}\right)}{11}\right)}{3}$	56
risch	Expression too large to display	8179

input
```
int(x^(7/2)*arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
2/9*x^(9/2)*arctanh(tanh(b*x+a))^3-4/3*b*(1/11*x^(11/2)*arctanh(tanh(b*x+a
))^2-4/11*b*(1/13*x^(13/2)*arctanh(tanh(b*x+a))-2/195*x^(15/2)*b))
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.58

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))^3\,dx = \frac{2}{6435}\left(429\,b^3x^7 + 1485\,ab^2x^6 + 1755\,a^2bx^5 + 715\,a^3x^4\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
2/6435*(429*b^3*x^7 + 1485*a*b^2*x^6 + 1755*a^2*b*x^5 + 715*a^3*x^4)*sqrt(
x)
```

3.183.6 Sympy [F(-1)]

Timed out.

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))^3\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*atanh(tanh(b*x+a))**3,x)
```

output
```
Timed out
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))^3\,dx = -\frac{4}{33}\,bx^{\frac{11}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2 + \frac{2}{9}\,x^{\frac{9}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^3 - \frac{16}{6435}\left(2\,b^2x^{\frac{15}{2}} - 15\,bx^{\frac{13}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

input
```
integrate(x^(7/2)*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-4/33*b*x^(11/2)*arctanh(tanh(b*x + a))^2 + 2/9*x^(9/2)*arctanh(tanh(b*x +
 a))^3 - 16/6435*(2*b^2*x^(15/2) - 15*b*x^(13/2)*arctanh(tanh(b*x + a)))*b
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

$$\int x^{7/2}\text{arctanh}(\tanh(a+bx))^3\,dx = \frac{2}{15}\,b^3x^{\frac{15}{2}} + \frac{6}{13}\,ab^2x^{\frac{13}{2}} + \frac{6}{11}\,a^2bx^{\frac{11}{2}} + \frac{2}{9}\,a^3x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
2/15*b^3*x^(15/2) + 6/13*a*b^2*x^(13/2) + 6/11*a^2*b*x^(11/2) + 2/9*a^3*x^
(9/2)
```

3.183.9 Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.64

$$\begin{aligned}\int x^{7/2}\text{arctanh}(\tanh(a+bx))^3\,dx &= \frac{2\,b^3\,x^{15/2}}{15}\\ &- \frac{x^{9/2}\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{36}\\ &+ \frac{3\,b\,x^{11/2}\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{22}\\ &- \frac{3\,b^2\,x^{13/2}\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{13}\end{aligned}$$

input
```
int(x^(7/2)*atanh(tanh(a + b*x))^3,x)
```

output
```
(2*b^3*x^(15/2))/15 - (x^(9/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/36 + (3*b*x^(1
1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/22 - (3*b^2*x^(13/2)*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
2*b*x))/13
```

3.184 $\int x^{5/2}\textbf{arctanh}(\tanh(a+bx))^3\,dx$

3.184.1 Optimal result

Integrand size = 15, antiderivative size = 69

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))^3\,dx = -\frac{32b^3x^{13/2}}{3003} + \frac{16}{231}b^2x^{11/2}\text{arctanh}(\tanh(a+bx)) - \frac{4}{21}bx^{9/2}\text{arctanh}(\tanh(a+bx))^2 + \frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))^3$$

output
```
-32/3003*b^3*x^(13/2)+16/231*b^2*x^(11/2)*arctanh(tanh(b*x+a))-4/21*b*x^(9
/2)*arctanh(tanh(b*x+a))^2+2/7*x^(7/2)*arctanh(tanh(b*x+a))^3
```

3.184.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))^3\,dx = \frac{2x^{7/2}(-16b^3x^3 + 104b^2x^2\text{arctanh}(\tanh(a+bx)) - 286bx\text{arctanh}(\tanh(a}{3003}$$

input
```
Integrate[x^(5/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(2*x^(7/2)*(-16*b^3*x^3 + 104*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 286*b*x*Arc
Tanh[Tanh[a + b*x]]^2 + 429*ArcTanh[Tanh[a + b*x]]^3))/3003
```

3.184.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))^3\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))^3-\frac{6}{7}b\int x^{7/2}\text{arctanh}(\tanh(a+bx))^2dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))^3-\frac{6}{7}b\left(\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{9}b\int x^{9/2}\text{arctanh}(\tanh(a+bx))dx\right)$$

$$\Big\downarrow \text{2599}$$

$$\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))^3-\frac{6}{7}b\left(\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{9}b\left(\frac{2}{11}x^{11/2}\text{arctanh}(\tanh(a+bx))-\frac{2}{11}b\int x^{11/2}dx\right)\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))^3-\frac{6}{7}b\left(\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{9}b\left(\frac{2}{11}x^{11/2}\text{arctanh}(\tanh(a+bx))-\frac{4}{143}bx^{13/2}\right)\right)$$

input
```
Int[x^(5/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(2*x^(7/2)*ArcTanh[Tanh[a + b*x]]^3)/7 - (6*b*((2*x^(9/2)*ArcTanh[Tanh[a +
 b*x]]^2)/9 - (4*b*((-4*b*x^(13/2))/143 + (2*x^(11/2)*ArcTanh[Tanh[a + b*x
]])/11))/9))/7
```

3.184.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.184.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))^3}{7}-\frac{12b\left(\frac{x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{9}-\frac{4b\left(\frac{x^{\frac{11}{2}}\operatorname{arctanh}(\tanh(bx+a))}{11}-\frac{2x^{\frac{13}{2}}b}{143}\right)}{9}\right)}{7}$	56
default	$\frac{2x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))^3}{7}-\frac{12b\left(\frac{x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{9}-\frac{4b\left(\frac{x^{\frac{11}{2}}\operatorname{arctanh}(\tanh(bx+a))}{11}-\frac{2x^{\frac{13}{2}}b}{143}\right)}{9}\right)}{7}$	56
risch	Expression too large to display	8179

input
```
int(x^(5/2)*arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
2/7*x^(7/2)*arctanh(tanh(b*x+a))^3-12/7*b*(1/9*x^(9/2)*arctanh(tanh(b*x+a)
)^2-4/9*b*(1/11*x^(11/2)*arctanh(tanh(b*x+a))-2/143*x^(13/2)*b))
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.58

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))^3\,dx = \frac{2}{3003}\left(231\,b^3x^6 + 819\,ab^2x^5 + 1001\,a^2bx^4 + 429\,a^3x^3\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
2/3003*(231*b^3*x^6 + 819*a*b^2*x^5 + 1001*a^2*b*x^4 + 429*a^3*x^3)*sqrt(x
)
```

3.184.6 Sympy [A] (verification not implemented)

Time = 40.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))^3\,dx = -\frac{32b^3x^{\frac{13}{2}}}{3003} + \frac{16b^2x^{\frac{11}{2}}\operatorname{atanh}(\tanh(a+bx))}{231} - \frac{4bx^{\frac{9}{2}}\operatorname{atanh}^2(\tanh(a+bx))}{21} + \frac{2x^{\frac{7}{2}}\operatorname{atanh}^3(\tanh(a+bx))}{7}$$

input
```
integrate(x**(5/2)*atanh(tanh(b*x+a))**3,x)
```

output
```
-32*b**3*x**(13/2)/3003 + 16*b**2*x**(11/2)*atanh(tanh(a + b*x))/231 - 4*b
*x**(9/2)*atanh(tanh(a + b*x))**2/21 + 2*x**(7/2)*atanh(tanh(a + b*x))**3/
7
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int x^{5/2}\text{arctanh}(\tanh(a+bx))^3\,dx = -\frac{4}{21}\,bx^{\frac{9}{2}}\,\text{artanh}(\tanh(bx+a))^2 + \frac{2}{7}\,x^{\frac{7}{2}}\,\text{artanh}(\tanh(bx+a))^3 - \frac{16}{3003}\left(2\,b^2x^{\frac{13}{2}} - 13\,bx^{\frac{11}{2}}\,\text{artanh}(\tanh(bx+a))\right)b$$

input
```
integrate(x^(5/2)*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-4/21*b*x^(9/2)*arctanh(tanh(b*x + a))^2 + 2/7*x^(7/2)*arctanh(tanh(b*x +
a))^3 - 16/3003*(2*b^2*x^(13/2) - 13*b*x^(11/2)*arctanh(tanh(b*x + a)))*b
```

3.184.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

$$\int x^{5/2}\operatorname{arctanh}(\tanh(a+bx))^3\,dx = \frac{2}{13}\,b^3x^{\frac{13}{2}} + \frac{6}{11}\,ab^2x^{\frac{11}{2}} + \frac{2}{3}\,a^2bx^{\frac{9}{2}} + \frac{2}{7}\,a^3x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
2/13*b^3*x^(13/2) + 6/11*a*b^2*x^(11/2) + 2/3*a^2*b*x^(9/2) + 2/7*a^3*x^(7
/2)
```

3.184.9 Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.64

$$\begin{aligned}\int x^{5/2}\operatorname{arctanh}(\tanh(a+bx))^3\,dx &= \frac{2\,b^3\,x^{13/2}}{13} \\ &- \frac{x^{7/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{28} \\ &+ \frac{b\,x^{9/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{6} \\ &- \frac{3\,b^2\,x^{11/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{11}\end{aligned}$$

input
```
int(x^(5/2)*atanh(tanh(a + b*x))^3,x)
```

output
```
(2*b^3*x^(13/2))/13 - (x^(7/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/28 + (b*x^(9/2
)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)^2)/6 - (3*b^2*x^(11/2)*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x))/11
```

3.185 $\int x^{3/2}\textbf{arctanh}(\tanh(a+bx))^3\,dx$

3.185.1 Optimal result

Integrand size = 15, antiderivative size = 69

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^3\,dx=-\frac{32b^3x^{11/2}}{1155}+\frac{16}{105}b^2x^{9/2}\text{arctanh}(\tanh(a+bx))-\frac{12}{35}bx^{7/2}\text{arctanh}(\tanh(a+bx))^2+\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))^3$$

output
```
-32/1155*b^3*x^(11/2)+16/105*b^2*x^(9/2)*arctanh(tanh(b*x+a))-12/35*b*x^(7
/2)*arctanh(tanh(b*x+a))^2+2/5*x^(5/2)*arctanh(tanh(b*x+a))^3
```

3.185.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^3\,dx=-\frac{2x^{5/2}(16b^3x^3-88b^2x^2\text{arctanh}(\tanh(a+bx))+198bx\text{arctanh}(\tanh(a+bx))^2-231\text{arctanh}(\tanh(a+bx}{1155}$$

input
```
Integrate[x^(3/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(-2*x^(5/2)*(16*b^3*x^3 - 88*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 198*b*x*ArcT
anh[Tanh[a + b*x]]^2 - 231*ArcTanh[Tanh[a + b*x]]^3))/1155
```

3.185.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^3\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))^3-\frac{6}{5}b\int x^{5/2}\text{arctanh}(\tanh(a+bx))^2dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))^3- \frac{6}{5}b\left(\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{7}b\int x^{7/2}\text{arctanh}(\tanh(a+bx))dx\right)$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))^3- \frac{6}{5}b\left(\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{7}b\left(\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))-\frac{2}{9}b\int x^{9/2}dx\right)\right)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))^3- \frac{6}{5}b\left(\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{7}b\left(\frac{2}{9}x^{9/2}\text{arctanh}(\tanh(a+bx))-\frac{4}{99}bx^{11/2}\right)\right)$$

input
```
Int[x^(3/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(2*x^(5/2)*ArcTanh[Tanh[a + b*x]]^3)/5 - (6*b*((2*x^(7/2)*ArcTanh[Tanh[a +
 b*x]]^2)/7 - (4*b*((-4*b*x^(11/2))/99 + (2*x^(9/2)*ArcTanh[Tanh[a + b*x]]
)/9))/7))/5
```

3.185.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.185.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))^{3}}{5}-\frac{12b\left(\frac{x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))^{2}}{7}-\frac{4b\left(\frac{x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))}{9}-\frac{2b\,x^{\frac{11}{2}}}{99}\right)}{7}\right)}{5}$	56
default	$\frac{2x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))^{3}}{5}-\frac{12b\left(\frac{x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))^{2}}{7}-\frac{4b\left(\frac{x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))}{9}-\frac{2b\,x^{\frac{11}{2}}}{99}\right)}{7}\right)}{5}$	56
risch	Expression too large to display	8179

input
```
int(x^(3/2)*arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*x^(5/2)*arctanh(tanh(b*x+a))^3-12/5*b*(1/7*x^(7/2)*arctanh(tanh(b*x+a)
)^2-4/7*b*(1/9*x^(9/2)*arctanh(tanh(b*x+a))-2/99*b*x^(11/2)))
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.58

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^3\,dx = \frac{2}{1155}\left(105\,b^3x^5+385\,ab^2x^4+495\,a^2bx^3+231\,a^3x^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
2/1155*(105*b^3*x^5 + 385*a*b^2*x^4 + 495*a^2*b*x^3 + 231*a^3*x^2)*sqrt(x)
```

3.185.6 Sympy [F]

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^3\,dx = \int x^{\frac{3}{2}}\,\text{atanh}^3\left(\tanh\left(a+bx\right)\right)dx$$

input
```
integrate(x**(3/2)*atanh(tanh(b*x+a))**3,x)
```

output
```
Integral(x**(3/2)*atanh(tanh(a + b*x))**3, x)
```

3.185.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^3\,dx = -\frac{12}{35}\,bx^{\frac{7}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2 + \frac{2}{5}\,x^{\frac{5}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^3 - \frac{16}{1155}\left(2\,b^2x^{\frac{11}{2}} - 11\,bx^{\frac{9}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

input
```
integrate(x^(3/2)*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-12/35*b*x^(7/2)*arctanh(tanh(b*x + a))^2 + 2/5*x^(5/2)*arctanh(tanh(b*x +
 a))^3 - 16/1155*(2*b^2*x^(11/2) - 11*b*x^(9/2)*arctanh(tanh(b*x + a)))*b
```

3.185.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^3\,dx = \frac{2}{11}\,b^3x^{\frac{11}{2}} + \frac{2}{3}\,ab^2x^{\frac{9}{2}} + \frac{6}{7}\,a^2bx^{\frac{7}{2}} + \frac{2}{5}\,a^3x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
2/11*b^3*x^(11/2) + 2/3*a*b^2*x^(9/2) + 6/7*a^2*b*x^(7/2) + 2/5*a^3*x^(5/2
)
```

3.185.9 Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.64

$$\begin{aligned}\int x^{3/2}\text{arctanh}(\tanh(a+bx))^3\,dx = {} & \frac{2\,b^3\,x^{11/2}}{11} \\ & - \frac{x^{5/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^3}{20} \\ & + \frac{3\,b\,x^{7/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2}{14} \\ & - \frac{b^2\,x^{9/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)}{3}\end{aligned}$$

input
```
int(x^(3/2)*atanh(tanh(a + b*x))^3,x)
```

output
```
(2*b^3*x^(11/2))/11 - (x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/20 + (3*b*x^(7
/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/14 - (b^2*x^(9/2)*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x))/3
```

3.186 $\int \sqrt{x}\textbf{arctanh}(\tanh(a+bx))^3\,dx$

3.186.1 Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^3\,dx = -\frac{32}{315}b^3x^{9/2}+\frac{16}{35}b^2x^{7/2}\operatorname{arctanh}(\tanh(a+bx)) - \frac{4}{5}bx^{5/2}\operatorname{arctanh}(\tanh(a+bx))^2+\frac{2}{3}x^{3/2}\operatorname{arctanh}(\tanh(a+bx))^3$$

output
```
-32/315*b^3*x^(9/2)+16/35*b^2*x^(7/2)*arctanh(tanh(b*x+a))-4/5*b*x^(5/2)*a
rctanh(tanh(b*x+a))^2+2/3*x^(3/2)*arctanh(tanh(b*x+a))^3
```

3.186.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^3\,dx = -\frac{2}{315}x^{3/2}\big(16b^3x^3-72b^2x^2\operatorname{arctanh}(\tanh(a+bx)) + 126bx\operatorname{arctanh}(\tanh(a+bx))^2 - 105\operatorname{arctanh}(\tanh(a+bx))^3\big)$$

input
```
Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(-2*x^(3/2)*(16*b^3*x^3 - 72*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 126*b*x*ArcT
anh[Tanh[a + b*x]]^2 - 105*ArcTanh[Tanh[a + b*x]]^3))/315
```

3.186.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^3\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^3-2b\int x^{3/2}\text{arctanh}(\tanh(a+bx))^2dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^3-2b\left(\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{5}b\int x^{5/2}\text{arctanh}(\tanh(a+bx))dx\right)$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^3-2b\left(\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{5}b\left(\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))-\frac{2}{7}b\int x^{7/2}dx\right)\right)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^3-2b\left(\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{5}b\left(\frac{2}{7}x^{7/2}\text{arctanh}(\tanh(a+bx))-\frac{4}{63}bx^{9/2}\right)\right)$$

input
```
Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(2*x^(3/2)*ArcTanh[Tanh[a + b*x]]^3)/3 - 2*b*((2*x^(5/2)*ArcTanh[Tanh[a +
b*x]]^2)/5 - (4*b*((-4*b*x^(9/2))/63 + (2*x^(7/2)*ArcTanh[Tanh[a + b*x]])/
7))/5)
```

3.186.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.186.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}} \operatorname{arctanh}(\tanh(bx+a))^{3}}{3}-4b\left(\frac{x^{\frac{5}{2}} \operatorname{arctanh}(\tanh(bx+a))^{2}}{5}-\frac{4b\left(\frac{x^{\frac{7}{2}} \operatorname{arctanh}(\tanh(bx+a))}{7}-\frac{2b\,x^{\frac{9}{2}}}{63}\right)}{5}\right)$	56
default	$\frac{2x^{\frac{3}{2}} \operatorname{arctanh}(\tanh(bx+a))^{3}}{3}-4b\left(\frac{x^{\frac{5}{2}} \operatorname{arctanh}(\tanh(bx+a))^{2}}{5}-\frac{4b\left(\frac{x^{\frac{7}{2}} \operatorname{arctanh}(\tanh(bx+a))}{7}-\frac{2b\,x^{\frac{9}{2}}}{63}\right)}{5}\right)$	56
risch	Expression too large to display	7981

input
```
int(arctanh(tanh(b*x+a))^3*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)*arctanh(tanh(b*x+a))^3-4*b*(1/5*x^(5/2)*arctanh(tanh(b*x+a))^2
-4/5*b*(1/7*x^(7/2)*arctanh(tanh(b*x+a))-2/63*b*x^(9/2)))
```

3.186.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.55

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^3\,dx = \frac{2}{315}\left(35\,b^3x^4+135\,ab^2x^3+189\,a^2bx^2+105\,a^3x\right)\sqrt{x}$$

input
```
integrate(arctanh(tanh(b*x+a))^3*x^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^3*x^4 + 135*a*b^2*x^3 + 189*a^2*b*x^2 + 105*a^3*x)*sqrt(x)
```

3.186.6 Sympy [F]

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^3\,dx = \int \sqrt{x}\,\text{atanh}^3\left(\tanh\left(a+bx\right)\right)\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**3*x**(1/2),x)
```

output
```
Integral(sqrt(x)*atanh(tanh(a + b*x))**3, x)
```

3.186.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^3\,dx = -\frac{4}{5}\,bx^{\frac{5}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2 + \frac{2}{3}\,x^{\frac{3}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^3 - \frac{16}{315}\left(2\,b^2x^{\frac{9}{2}} - 9\,bx^{\frac{7}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

input
```
integrate(arctanh(tanh(b*x+a))^3*x^(1/2),x, algorithm="maxima")
```

output
```
-4/5*b*x^(5/2)*arctanh(tanh(b*x + a))^2 + 2/3*x^(3/2)*arctanh(tanh(b*x + a
))^3 - 16/315*(2*b^2*x^(9/2) - 9*b*x^(7/2)*arctanh(tanh(b*x + a)))*b
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

$$\int \sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^3\,dx = \frac{2}{9}\,b^3x^{\frac{9}{2}} + \frac{6}{7}\,ab^2x^{\frac{7}{2}} + \frac{6}{5}\,a^2bx^{\frac{5}{2}} + \frac{2}{3}\,a^3x^{\frac{3}{2}}$$

input
```
integrate(arctanh(tanh(b*x+a))^3*x^(1/2),x, algorithm="giac")
```

output
```
2/9*b^3*x^(9/2) + 6/7*a*b^2*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2/3*a^3*x^(3/2)
```

3.186.9 Mupad [B] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.64

$$\int \sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^3\,dx = \frac{2\,b^3\,x^{9/2}}{9} - \frac{x^{3/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{12} + \frac{3\,b\,x^{5/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{10} - \frac{3\,b^2\,x^{7/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{7}$$

input
```
int(x^(1/2)*atanh(tanh(a + b*x))^3,x)
```

output
```
(2*b^3*x^(9/2))/9 - (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/12 + (3*b*x^(5/2
)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)^2)/10 - (3*b^2*x^(7/2)*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x))/7
```

3.187 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{\sqrt{x}}\,dx$

3.187.1 Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{\sqrt{x}}\,dx = -\frac{32}{35}b^3x^{7/2} + \frac{16}{5}b^2x^{5/2}\operatorname{arctanh}(\tanh(a+bx)) - 4bx^{3/2}\operatorname{arctanh}(\tanh(a+bx))^2 + 2\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^3$$

output
```
-32/35*b^3*x^(7/2)+16/5*b^2*x^(5/2)*arctanh(tanh(b*x+a))-4*b*x^(3/2)*arcta
nh(tanh(b*x+a))^2+2*arctanh(tanh(b*x+a))^3*x^(1/2)
```

3.187.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{\sqrt{x}}\,dx = \frac{2}{35}\sqrt{x}\left(-16b^3x^3 + 56b^2x^2\operatorname{arctanh}(\tanh(a+bx)) - 70bx\operatorname{arctanh}(\tanh(a+bx))^2 + 35\operatorname{arctanh}(\tanh(a+bx))^3\right)$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^3/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(-16*b^3*x^3 + 56*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 70*b*x*ArcTa
nh[Tanh[a + b*x]]^2 + 35*ArcTanh[Tanh[a + b*x]]^3))/35
```

3.187.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^3}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 2599}$$

$$2\sqrt{x}\text{arctanh}(\tanh(a+bx))^3-6b\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^2dx$$

$$\Big\downarrow \text{ 2599}$$

$$2\sqrt{x}\text{arctanh}(\tanh(a+bx))^3-6b\left(\frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{3}b\int x^{3/2}\text{arctanh}(\tanh(a+bx))dx\right)$$

$$\Big\downarrow \text{ 2599}$$

$$2\sqrt{x}\text{arctanh}(\tanh(a+bx))^3-6b\left(\frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{3}b\left(\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))-\frac{2}{5}b\int x^{5/2}dx\right)\right)$$

$$\Big\downarrow \text{ 15}$$

$$2\sqrt{x}\text{arctanh}(\tanh(a+bx))^3-6b\left(\frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^2-\frac{4}{3}b\left(\frac{2}{5}x^{5/2}\text{arctanh}(\tanh(a+bx))-\frac{4}{35}bx^{7/2}\right)\right)$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^3/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3 - 6*b*((2*x^(3/2)*ArcTanh[Tanh[a + b*x]
]^2)/3 - (4*b*((-4*b*x^(7/2))/35 + (2*x^(5/2)*ArcTanh[Tanh[a + b*x]])/5))/
3)
```

3.187.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.187.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{2b^3x^{\frac{7}{2}}}{7}+\frac{6(\operatorname{arctanh}(\tanh(bx+a))-bx)b^2x^{\frac{5}{2}}}{5}+2(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\,b\,x^{\frac{3}{2}}+2(\operatorname{arctanh}$
default	$\frac{2b^3x^{\frac{7}{2}}}{7}+\frac{6(\operatorname{arctanh}(\tanh(bx+a))-bx)b^2x^{\frac{5}{2}}}{5}+2(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\,b\,x^{\frac{3}{2}}+2(\operatorname{arctanh}$
risch	Expression too large to display

input
```
int(arctanh(tanh(b*x+a))^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*b^3*x^(7/2)+6/5*(arctanh(tanh(b*x+a))-b*x)*b^2*x^(5/2)+2*(arctanh(tanh
(b*x+a))-b*x)^2*b*x^(3/2)+2*(arctanh(tanh(b*x+a))-b*x)^3*x^(1/2)
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.54

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{\sqrt{x}}\,dx=\frac{2}{35}\left(5\,b^3x^3+21\,ab^2x^2+35\,a^2bx+35\,a^3\right)\sqrt{x}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="fricas")
```

output
```
2/35*(5*b^3*x^3 + 21*a*b^2*x^2 + 35*a^2*b*x + 35*a^3)*sqrt(x)
```

3.187.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{\sqrt{x}}\,dx = \int \frac{\operatorname{atanh}^3(\tanh(a+bx))}{\sqrt{x}}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**3/x**(1/2),x)
```

output
```
Integral(atanh(tanh(a + b*x))**3/sqrt(x), x)
```

3.187.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{\sqrt{x}}\,dx = -4\,bx^{\frac{3}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2 + 2\,\sqrt{x}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3 - \frac{16}{35}\left(2\,b^2x^{\frac{7}{2}} - 7\,bx^{\frac{5}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="maxima")
```

output
```
-4*b*x^(3/2)*arctanh(tanh(b*x + a))^2 + 2*sqrt(x)*arctanh(tanh(b*x + a))^3
 - 16/35*(2*b^2*x^(7/2) - 7*b*x^(5/2)*arctanh(tanh(b*x + a)))*b
```

3.187.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.54

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{\sqrt{x}}\,dx = \frac{2}{7}\,b^3x^{\frac{7}{2}} + \frac{6}{5}\,ab^2x^{\frac{5}{2}} + 2\,a^2bx^{\frac{3}{2}} + 2\,a^3\sqrt{x}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="giac")
```

output
```
2/7*b^3*x^(7/2) + 6/5*a*b^2*x^(5/2) + 2*a^2*b*x^(3/2) + 2*a^3*sqrt(x)
```

3.187.9 Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.80

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{\sqrt{x}}\,dx = \frac{2\,b^3\,x^{7/2}}{7} - \frac{\sqrt{x}\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{4} + \frac{b\,x^{3/2}\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{2} - \frac{3\,b^2\,x^{5/2}\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{5}$$

input
```
int(atanh(tanh(a + b*x))^3/x^(1/2),x)
```

output
```
(2*b^3*x^(7/2))/7 - (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/4 + (b*x^(3/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^2)/2 - (3*b^2*x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/
5
```

3.188 $\int \frac{\text{arctanh}(\tanh(a+bx))^3}{x^{3/2}}\,dx$

3.188.1 Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \frac{\text{arctanh}(\tanh(a+bx))^3}{x^{3/2}}\,dx = \frac{32}{5}b^3x^{5/2} - 16b^2x^{3/2}\text{arctanh}(\tanh(a+bx)) + 12b\sqrt{x}\,\text{arctanh}(\tanh(a+bx))^2 - \frac{2\text{arctanh}(\tanh(a+bx))^3}{\sqrt{x}}$$

output
```
32/5*b^3*x^(5/2)-16*b^2*x^(3/2)*arctanh(tanh(b*x+a))-2*arctanh(tanh(b*x+a)
)^3/x^(1/2)+12*b*arctanh(tanh(b*x+a))^2*x^(1/2)
```

3.188.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{\text{arctanh}(\tanh(a+bx))^3}{x^{3/2}}\,dx = \frac{2(16b^3x^3 - 40b^2x^2\text{arctanh}(\tanh(a+bx)) + 30bx\,\text{arctanh}(\tanh(a+bx))^2 -}{5\sqrt{x}}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^3/x^(3/2),x]
```

output
```
(2*(16*b^3*x^3 - 40*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 30*b*x*ArcTanh[Tanh[a
 + b*x]]^2 - 5*ArcTanh[Tanh[a + b*x]]^3))/(5*Sqrt[x])
```

3.188.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^3}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$6b\int \frac{\text{arctanh}(\tanh(a+bx))^2}{\sqrt{x}}dx-\frac{2\text{arctanh}(\tanh(a+bx))^3}{\sqrt{x}}$$

$$\Big\downarrow \text{ 2599}$$

$$6b\left(2\sqrt{x}\text{arctanh}(\tanh(a+bx))^2-4b\int \sqrt{x}\text{arctanh}(\tanh(a+bx))dx\right)-\frac{2\text{arctanh}(\tanh(a+bx))^3}{\sqrt{x}}$$

$$\Big\downarrow \text{ 2599}$$

$$6b\left(2\sqrt{x}\text{arctanh}(\tanh(a+bx))^2-4b\left(\frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))-\frac{2}{3}b\int x^{3/2}dx\right)\right)-\frac{2\text{arctanh}(\tanh(a+bx))^3}{\sqrt{x}}$$

$$\Big\downarrow \text{ 15}$$

$$6b\left(2\sqrt{x}\text{arctanh}(\tanh(a+bx))^2-4b\left(\frac{2}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))-\frac{4}{15}bx^{5/2}\right)\right)-\frac{2\text{arctanh}(\tanh(a+bx))^3}{\sqrt{x}}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^3/x^(3/2),x]
```

output
```
(-2*ArcTanh[Tanh[a + b*x]]^3)/Sqrt[x] + 6*b*(2*Sqrt[x]*ArcTanh[Tanh[a + b*
x]]^2 - 4*b*((-4*b*x^(5/2))/15 + (2*x^(3/2)*ArcTanh[Tanh[a + b*x]])/3))
```

3.188.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.188.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

method	result
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^3}{\sqrt{x}}+12b\left(\frac{b^2x^{\frac{5}{2}}}{5}+\frac{2(\mathrm{arctanh}(\tanh(bx+a))-bx)b\,x^{\frac{3}{2}}}{3}+(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right.\right.$
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^3}{\sqrt{x}}+12b\left(\frac{b^2x^{\frac{5}{2}}}{5}+\frac{2(\mathrm{arctanh}(\tanh(bx+a))-bx)b\,x^{\frac{3}{2}}}{3}+(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right.\right.$
risch	Expression too large to display

input
```
int(arctanh(tanh(b*x+a))^3/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh(tanh(b*x+a))^3/x^(1/2)+12*b*(1/5*b^2*x^(5/2)+2/3*(arctanh(tanh(
b*x+a))-b*x)*b*x^(3/2)+(arctanh(tanh(b*x+a))-b*x)^2*x^(1/2))
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.54

$$\int \frac{\mathrm{arctanh}(\tanh(a+bx))^3}{x^{3/2}}\,dx=\frac{2\left(b^3x^3+5\,ab^2x^2+15\,a^2bx-5\,a^3\right)}{5\,\sqrt{x}}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^(3/2),x, algorithm="fricas")
```

output
```
2/5*(b^3*x^3 + 5*a*b^2*x^2 + 15*a^2*b*x - 5*a^3)/sqrt(x)
```

3.188.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^{3/2}}\,dx = \int \frac{\operatorname{atanh}^3(\tanh(a+bx))}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**3/x**(3/2),x)
```

output
```
Integral(atanh(tanh(a + b*x))**3/x**(3/2), x)
```

3.188.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^{3/2}}\,dx = 12\,b\sqrt{x}\operatorname{artanh}(\tanh(bx+a))^2 - \frac{2\operatorname{artanh}(\tanh(bx+a))^3}{\sqrt{x}} + \frac{16}{5}\left(2\,b^2x^{\frac{5}{2}} - 5\,bx^{\frac{3}{2}}\operatorname{artanh}(\tanh(bx+a))\right)b$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^(3/2),x, algorithm="maxima")
```

output
```
12*b*sqrt(x)*arctanh(tanh(b*x + a))^2 - 2*arctanh(tanh(b*x + a))^3/sqrt(x)
 + 16/5*(2*b^2*x^(5/2) - 5*b*x^(3/2)*arctanh(tanh(b*x + a)))*b
```

3.188.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^{3/2}}\,dx = \frac{2}{5}\,b^3x^{\frac{5}{2}} + 2\,ab^2x^{\frac{3}{2}} + 6\,a^2b\sqrt{x} - \frac{2\,a^3}{\sqrt{x}}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^(3/2),x, algorithm="giac")
```

output
```
2/5*b^3*x^(5/2) + 2*a*b^2*x^(3/2) + 6*a^2*b*sqrt(x) - 2*a^3/sqrt(x)
```

3.188.9 Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.89

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^{3/2}}\,dx = \frac{\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^3}{4\,\sqrt{x}} + \frac{2\,b^3\,x^{5/2}}{5} + \frac{3\,b\,\sqrt{x}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^2}{2} - b^2\,x^{3/2}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)$$

input
```
int(atanh(tanh(a + b*x))^3/x^(3/2),x)
```

output
```
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^3/(4*x^(1/2)) + (2*b^3*x^(5/2))/5 + (3*b*x^(1/2)
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - b^2*x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x)
+ 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
```

3.189 $\int \frac{\text{arctanh}(\tanh(a+bx))^3}{x^{5/2}} \, dx$

3.189.1 Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \frac{\text{arctanh}(\tanh(a+bx))^3}{x^{5/2}} \, dx = -\frac{32}{3}b^3x^{3/2} + 16b^2\sqrt{x}\text{arctanh}(\tanh(a+bx)) - \frac{4b\text{arctanh}(\tanh(a+bx))^2}{\sqrt{x}} - \frac{2\text{arctanh}(\tanh(a+bx))^3}{3x^{3/2}}$$

output
```
-32/3*b^3*x^(3/2)-2/3*arctanh(tanh(b*x+a))^3/x^(3/2)-4*b*arctanh(tanh(b*x+
a))^2/x^(1/2)+16*b^2*arctanh(tanh(b*x+a))*x^(1/2)
```

3.189.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{\text{arctanh}(\tanh(a+bx))^3}{x^{5/2}} \, dx = -\frac{2(16b^3x^3 - 24b^2x^2\text{arctanh}(\tanh(a+bx)) + 6bx\text{arctanh}(\tanh(a+bx))^2 + \text{arctanh}(\tanh(a+bx))^3)}{3x^{3/2}}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^3/x^(5/2),x]
```

output
```
(-2*(16*b^3*x^3 - 24*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 6*b*x*ArcTanh[Tanh[a
 + b*x]]^2 + ArcTanh[Tanh[a + b*x]]^3))/(3*x^(3/2))
```

3.189.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^3}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$2b\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^{3/2}}dx-\frac{2\text{arctanh}(\tanh(a+bx))^3}{3x^{3/2}}$$

$$\Big\downarrow \text{ 2599}$$

$$2b\left(4b\int \frac{\text{arctanh}(\tanh(a+bx))}{\sqrt{x}}dx-\frac{2\text{arctanh}(\tanh(a+bx))^2}{\sqrt{x}}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^3}{3x^{3/2}}$$

$$\Big\downarrow \text{ 2599}$$

$$2b\left(4b\left(2\sqrt{x}\text{arctanh}(\tanh(a+bx))-2b\int \sqrt{x}dx\right)-\frac{2\text{arctanh}(\tanh(a+bx))^2}{\sqrt{x}}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^3}{3x^{3/2}}$$

$$\Big\downarrow \text{ 15}$$

$$2b\left(4b\left(2\sqrt{x}\text{arctanh}(\tanh(a+bx))-\frac{4}{3}bx^{3/2}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^2}{\sqrt{x}}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^3}{3x^{3/2}}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^3/x^(5/2),x]
```

output
```
(-2*ArcTanh[Tanh[a + b*x]]^3)/(3*x^(3/2)) + 2*b*((-2*ArcTanh[Tanh[a + b*x]
]^2)/Sqrt[x] + 4*b*((-4*b*x^(3/2))/3 + 2*Sqrt[x]*ArcTanh[Tanh[a + b*x]]))
```

3.189.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.189.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

method	result
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^3}{3x^{\frac{3}{2}}}+4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{\sqrt{x}}+4b\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\sqrt{x}-\frac{2bx^{\frac{3}{2}}}{3}\right.\right.$
default	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^3}{3x^{\frac{3}{2}}}+4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{\sqrt{x}}+4b\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\sqrt{x}-\frac{2bx^{\frac{3}{2}}}{3}\right.\right.$
risch	Expression too large to display

input
```
int(arctanh(tanh(b*x+a))^3/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*arctanh(tanh(b*x+a))^3/x^(3/2)+4*b*(-arctanh(tanh(b*x+a))^2/x^(1/2)+4
*b*(arctanh(tanh(b*x+a))*x^(1/2)-2/3*b*x^(3/2)))
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^{5/2}}\,dx = \frac{2\left(b^3x^3+9\,ab^2x^2-9\,a^2bx-a^3\right)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^(5/2),x, algorithm="fricas")
```

output
```
2/3*(b^3*x^3 + 9*a*b^2*x^2 - 9*a^2*b*x - a^3)/x^(3/2)
```

3.189.6 Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^{5/2}}\,dx = -\frac{32b^3x^{\frac{3}{2}}}{3} + 16b^2\sqrt{x}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right) - \frac{4b\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{\sqrt{x}} - \frac{2\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{3x^{\frac{3}{2}}}$$

input
```
integrate(atanh(tanh(b*x+a))**3/x**(5/2),x)
```

output
```
-32*b**3*x**(3/2)/3 + 16*b**2*sqrt(x)*atanh(tanh(a + b*x)) - 4*b*atanh(tan
h(a + b*x))**2/sqrt(x) - 2*atanh(tanh(a + b*x))**3/(3*x**(3/2))
```

3.189.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^{5/2}}\,dx = -\frac{4\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{\sqrt{x}} - \frac{16}{3}\left(2\,b^2x^{\frac{3}{2}} - 3\,b\sqrt{x}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\right)b - \frac{2\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}{3\,x^{\frac{3}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^(5/2),x, algorithm="maxima")
```

output
```
-4*b*arctanh(tanh(b*x + a))^2/sqrt(x) - 16/3*(2*b^2*x^(3/2) - 3*b*sqrt(x)*
arctanh(tanh(b*x + a)))*b - 2/3*arctanh(tanh(b*x + a))^3/x^(3/2)
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^{5/2}}\,dx = \frac{2}{3}\,b^3x^{\frac{3}{2}} + 6\,ab^2\sqrt{x} - \frac{2\,(9\,a^2bx + a^3)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^(5/2),x, algorithm="giac")
```

output
```
2/3*b^3*x^(3/2) + 6*a*b^2*sqrt(x) - 2/3*(9*a^2*b*x + a^3)/x^(3/2)
```

3.189.9 Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.80

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^{5/2}}\,dx = \frac{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^3}{12\,x^{3/2}} + \frac{2\,b^3\,x^{3/2}}{3} - \frac{3\,b\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2}{2\,\sqrt{x}} - 3\,b^2\,\sqrt{x}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)$$

input
```
int(atanh(tanh(a + b*x))^3/x^(5/2),x)
```

output
```
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^3/(12*x^(3/2)) + (2*b^3*x^(3/2))/3 - (3*b*(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)^2)/(2*x^(1/2)) - 3*b^2*x^(1/2)*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)
```

3.190 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^{7/2}}\,dx$

3.190.1 Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^{7/2}}\,dx = \frac{32b^3\sqrt{x}}{5} - \frac{16b^2\operatorname{arctanh}(\tanh(a+bx))}{5\sqrt{x}} - \frac{4b\operatorname{arctanh}(\tanh(a+bx))^2}{5x^{3/2}} - \frac{2\operatorname{arctanh}(\tanh(a+bx))^3}{5x^{5/2}}$$

output
```
-4/5*b*arctanh(tanh(b*x+a))^2/x^(3/2)-2/5*arctanh(tanh(b*x+a))^3/x^(5/2)-1
6/5*b^2*arctanh(tanh(b*x+a))/x^(1/2)+32/5*b^3*x^(1/2)
```

3.190.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^{7/2}}\,dx = \frac{2(16b^3x^3 - 8b^2x^2\operatorname{arctanh}(\tanh(a+bx)) - 2bx\operatorname{arctanh}(\tanh(a+bx))^2 - \operatorname{ar}}{5x^{5/2}}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^3/x^(7/2),x]
```

output
```
(2*(16*b^3*x^3 - 8*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 2*b*x*ArcTanh[Tanh[a +
 b*x]]^2 - ArcTanh[Tanh[a + b*x]]^3))/(5*x^(5/2))
```

3.190.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^3}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{6}{5}b\int \frac{\text{arctanh}(\tanh(a+bx))^2}{x^{5/2}}dx-\frac{2\text{arctanh}(\tanh(a+bx))^3}{5x^{5/2}}$$

$$\Big\downarrow \text{2599}$$

$$\frac{6}{5}b\left(\frac{4}{3}b\int \frac{\text{arctanh}(\tanh(a+bx))}{x^{3/2}}dx-\frac{2\text{arctanh}(\tanh(a+bx))^2}{3x^{3/2}}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^3}{5x^{5/2}}$$

$$\Big\downarrow \text{2599}$$

$$\frac{6}{5}b\left(\frac{4}{3}b\left(2b\int \frac{1}{\sqrt{x}}dx-\frac{2\text{arctanh}(\tanh(a+bx))}{\sqrt{x}}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^2}{3x^{3/2}}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^3}{5x^{5/2}}$$

$$\Big\downarrow \text{15}$$

$$\frac{6}{5}b\left(\frac{4}{3}b\left(4b\sqrt{x}-\frac{2\text{arctanh}(\tanh(a+bx))}{\sqrt{x}}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^2}{3x^{3/2}}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^3}{5x^{5/2}}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^3/x^(7/2),x]
```

output
```
(-2*ArcTanh[Tanh[a + b*x]]^3)/(5*x^(5/2)) + (6*b*((-2*ArcTanh[Tanh[a + b*x
]]^2)/(3*x^(3/2)) + (4*b*(4*b*Sqrt[x] - (2*ArcTanh[Tanh[a + b*x]])/Sqrt[x]
))/3))/5
```

3.190.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.190.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{3}}{5 x^{\frac{5}{2}}}+\frac{12 b \left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{2}}{3 x^{\frac{3}{2}}}+\frac{4 b \left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{\sqrt{x}}+2 b \sqrt{x}\right)}{3}\right)}{5}$	56
default	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{3}}{5 x^{\frac{5}{2}}}+\frac{12 b \left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{2}}{3 x^{\frac{3}{2}}}+\frac{4 b \left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{\sqrt{x}}+2 b \sqrt{x}\right)}{3}\right)}{5}$	56
risch	Expression too large to display	7814

input
```
int(arctanh(tanh(b*x+a))^3/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*arctanh(tanh(b*x+a))^3/x^(5/2)+12/5*b*(-1/3*arctanh(tanh(b*x+a))^2/x^
(3/2)+4/3*b*(-arctanh(tanh(b*x+a))/x^(1/2)+2*b*x^(1/2)))
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^{7/2}}\,dx = \frac{2\,(5\,b^3x^3 - 15\,ab^2x^2 - 5\,a^2bx - a^3)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^(7/2),x, algorithm="fricas")
```

output
```
2/5*(5*b^3*x^3 - 15*a*b^2*x^2 - 5*a^2*b*x - a^3)/x^(5/2)
```

3.190.6 Sympy [A] (verification not implemented)

Time = 12.91 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^{7/2}}\,dx = \frac{32b^3\sqrt{x}}{5} - \frac{16b^2\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{5\sqrt{x}} - \frac{4b\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{5x^{\frac{3}{2}}} - \frac{2\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{5x^{\frac{5}{2}}}$$

input
```
integrate(atanh(tanh(b*x+a))**3/x**(7/2),x)
```

output
```
32*b**3*sqrt(x)/5 - 16*b**2*atanh(tanh(a + b*x))/(5*sqrt(x)) - 4*b*atanh(t
anh(a + b*x))**2/(5*x**(3/2)) - 2*atanh(tanh(a + b*x))**3/(5*x**(5/2))
```

3.190.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^{7/2}}\,dx = \frac{16}{5}\left(2\,b^2\sqrt{x} - \frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{\sqrt{x}}\right)b - \frac{4\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{5\,x^{\frac{3}{2}}} - \frac{2\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}{5\,x^{\frac{5}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^(7/2),x, algorithm="maxima")
```

output
```
16/5*(2*b^2*sqrt(x) - b*arctanh(tanh(b*x + a))/sqrt(x))*b - 4/5*b*arctanh(
tanh(b*x + a))^2/x^(3/2) - 2/5*arctanh(tanh(b*x + a))^3/x^(5/2)
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.49

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^{7/2}}\,dx = 2\,b^3\sqrt{x} - \frac{2\,(15\,ab^2x^2 + 5\,a^2bx + a^3)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^3/x^(7/2),x, algorithm="giac")
```

output
```
2*b^3*sqrt(x) - 2/5*(15*a*b^2*x^2 + 5*a^2*b*x + a^3)/x^(5/2)
```

3.190.9 Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.64

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^3}{x^{7/2}}\,dx = \frac{\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right) - \ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right) + 2\,b\,x\right)^3}{20\,x^{5/2}} + 2\,b^3\,\sqrt{x} + \frac{3\,b^2\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right) - \ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right) + 2\,b\,x\right)}{\sqrt{x}} - \frac{b\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right) - \ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right) + 2\,b\,x\right)^2}{2\,x^{3/2}}$$

input
```
int(atanh(tanh(a + b*x))^3/x^(7/2),x)
```

output
```
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^3/(20*x^(5/2)) + 2*b^3*x^(1/2) + (3*b^2*(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + 2*b*x))/x^(1/2) - (b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*x^(3/2))
```

3.191 $\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx$

3.191.1 Optimal result

Integrand size = 15, antiderivative size = 143

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2x^{7/2}}{7b}+\frac{2x^{5/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}{5b^2}+\frac{2x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2}{3b^3}+\frac{2\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))^3}{b^4}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))^{7/2}}{b^{9/2}}$$

output
```
2/7*x^(7/2)/b+2/5*x^(5/2)*(b*x-arctanh(tanh(b*x+a)))/b^2+2/3*x^(3/2)*(b*x-
arctanh(tanh(b*x+a)))^2/b^3-2*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*
x+a)))^(1/2))*(b*x-arctanh(tanh(b*x+a)))^(7/2)/b^(9/2)+2*(b*x-arctanh(tanh
(b*x+a)))^3*x^(1/2)/b^4
```

3.191.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\left(176b^{7/2}x^{7/2}-406b^{5/2}x^{5/2}\operatorname{arctanh}(\tanh(a+bx))+350b^{3/2}x^{3/2}\operatorname{arctanh}(\mathrm{ta}\right.}{}$$

input
```
Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]],x]
```

output
```
(2*(176*b^(7/2)*x^(7/2) - 406*b^(5/2)*x^(5/2)*ArcTanh[Tanh[a + b*x]] + 350
*b^(3/2)*x^(3/2)*ArcTanh[Tanh[a + b*x]]^2 - 105*Sqrt[b]*Sqrt[x]*ArcTanh[Ta
nh[a + b*x]]^3 + 105*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a
 + b*x]]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2)))/(105*b^(9/2))
```

3.191.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2590, 2590, 2590, 2590, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{\text{arctanh}(\tanh(a+bx))}dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{x^{5/2}}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2x^{7/2}}{7b}$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{x^{3/2}}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2x^{5/2}}{5b}\right)}{b}+\frac{2x^{7/2}}{7b}$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{\sqrt{x}}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2x^{3/2}}{3b}\right)}{b}+\frac{2x^{5/2}}{5b}\right)}{b}+\frac{2x^{7/2}}{7b}$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\text{arctanh}}}{b}\right.}{b}\right.}{b}\right.}{b}$$

$$\frac{2x^{7/2}}{7b}$$

$\Big\downarrow$ 2593

$$\frac{\left(\frac{\left(\frac{\left(\frac{2\sqrt{x}}{b}-\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}{b^{3/2}}\right)(bx-\text{arctanh}(\tanh(a+bx)))}{b}+\frac{2x^{3/2}}{3b}\right)(bx-\text{arctanh}(\tanh(a}{b}\right.}{b}$$

$$\frac{2x^{7/2}}{7b}$$

input
```
Int[x^(7/2)/ArcTanh[Tanh[a + b*x]],x]
```

output
```
(2*x^(7/2))/(7*b) + (((2*x^(5/2))/(5*b) + (((2*x^(3/2))/(3*b) + (((2*Sqrt[
x])/b - (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*S
qrt[b*x - ArcTanh[Tanh[a + b*x]]])/b^(3/2))*(b*x - ArcTanh[Tanh[a + b*x]])
)/b)*(b*x - ArcTanh[Tanh[a + b*x]]))/b)*(b*x - ArcTanh[Tanh[a + b*x]]))/b
```

3.191. $\int \frac{x^{7/2}}{\text{arctanh}(\tanh(a+bx))}\,dx$

3.191.3.1 Defintions of rubi rules used

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

3.191.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(119) = 238$.

Time = 0.16 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.83

method	result
derivativedivides	$-\frac{2\left(-\frac{b^3 x^{\frac{7}{2}}}{7}+\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)b^2 x^{\frac{5}{2}}}{5}-\frac{\left(a^2+2a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2\right)x^{\frac{3}{2}}b}{3}+(\ldots}{b^4}$
default	$-\frac{2\left(-\frac{b^3 x^{\frac{7}{2}}}{7}+\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)b^2 x^{\frac{5}{2}}}{5}-\frac{\left(a^2+2a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2\right)x^{\frac{3}{2}}b}{3}+(\ldots}{b^4}$

input
```
int(x^(7/2)/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-2/b^4*(-1/7*b^3*x^(7/2)+1/5*(arctanh(tanh(b*x+a))-b*x)*b^2*x^(5/2)-1/3*(a
^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)*x^(3/2
)*b+(arctanh(tanh(b*x+a))-b*x)*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arct
anh(tanh(b*x+a))-b*x-a)^2)*x^(1/2))+2*(a^4+4*a^3*(arctanh(tanh(b*x+a))-b*x
-a)+6*a^2*(arctanh(tanh(b*x+a))-b*x-a)^2+4*a*(arctanh(tanh(b*x+a))-b*x-a)^
3+(arctanh(tanh(b*x+a))-b*x-a)^4)/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)
*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

3.191.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.07

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \left[\frac{105\,a^3\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(15\,b^3x^3-21\,ab^2x^2+35\,a^2bx-105}{105\,b^4}\right.$$

input
```
integrate(x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
[1/105*(105*a^3*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a
)) + 2*(15*b^3*x^3 - 21*a*b^2*x^2 + 35*a^2*b*x - 105*a^3)*sqrt(x))/b^4, 2/
105*(105*a^3*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (15*b^3*x^3 - 21*a*
b^2*x^2 + 35*a^2*b*x - 105*a^3)*sqrt(x))/b^4]
```

3.191.6 Sympy [F]

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int \frac{x^{\frac{7}{2}}}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**(7/2)/atanh(tanh(b*x+a)),x)
```

output
```
Integral(x**(7/2)/atanh(tanh(a + b*x)), x)
```

3.191.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.45

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,a^4\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{2\left(15\,b^3x^{\frac{7}{2}}-21\,ab^2x^{\frac{5}{2}}+35\,a^2bx^{\frac{3}{2}}-105\,a^3\sqrt{x}\right)}{105\,b^4}$$

input
```
integrate(x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
2*a^4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/105*(15*b^3*x^(7/2)
- 21*a*b^2*x^(5/2) + 35*a^2*b*x^(3/2) - 105*a^3*sqrt(x))/b^4
```

3.191.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.49

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,a^4\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{2\left(15\,b^6x^{\frac{7}{2}} - 21\,ab^5x^{\frac{5}{2}} + 35\,a^2b^4x^{\frac{3}{2}} - 105\,a^3b^3\sqrt{x}\right)}{105\,b^7}$$

input
```
integrate(x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
2*a^4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/105*(15*b^6*x^(7/2)
- 21*a*b^5*x^(5/2) + 35*a^2*b^4*x^(3/2) - 105*a^3*b^3*sqrt(x))/b^7
```

3.191.9 Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.32

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,x^{7/2}}{7\,b} + \frac{x^{5/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)}{5\,b^2} + \frac{x^{3/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2}{6\,b^3} + \frac{\sqrt{x}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^3}{4\,b^4} + \frac{\sqrt{2}\,\ln\left(\frac{64\,b^{19/2}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}\right)\left(\ln \right.}{16\,b^{9/2}}$$

input
```
int(x^(7/2)/atanh(tanh(a + b*x)),x)
```

output
```
(2*x^(7/2))/(7*b) + (x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(5*b^2) + (x^(3/2)
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^2)/(6*b^3) + (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^3)/(4*b^4) + (2^(1/2)*log((64*b^(19/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
 - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*ex
p(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)^(7/2))/(16*b^(9/2))
```

3.191. $\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))} \, dx$

3.192 $\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx$

3.192.1 Optimal result

Integrand size = 15, antiderivative size = 116

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2x^{5/2}}{5b} + \frac{2x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}{3b^2} + \frac{2\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2}{b^3} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))^{5/2}}{b^{7/2}}$$

output
```
2/5*x^(5/2)/b+2/3*x^(3/2)*(b*x-arctanh(tanh(b*x+a)))/b^2-2*arctanh(b^(1/2)
*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*x-arctanh(tanh(b*x+a)))^(5/2
)/b^(7/2)+2*(b*x-arctanh(tanh(b*x+a)))^2*x^(1/2)/b^3
```

3.192.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\Big(23b^{5/2}x^{5/2} - 35b^{3/2}x^{3/2}\operatorname{arctanh}(\tanh(a+bx)) + 15\sqrt{b}\sqrt{x}\operatorname{arctanh}(\tanh(a}{}$$

input
```
Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]],x]
```

output
```
(2*(23*b^(5/2)*x^(5/2) - 35*b^(3/2)*x^(3/2)*ArcTanh[Tanh[a + b*x]] + 15*Sq
rt[b]*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2 - 15*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[
-(b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2))
)/(15*b^(7/2))
```

3.192.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2590, 2590, 2590, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\text{arctanh}(\tanh(a+bx))}dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{x^{3/2}}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2x^{5/2}}{5b}$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{\sqrt{x}}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2x^{3/2}}{3b}\right)}{b}+\frac{2x^{5/2}}{5b}$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2\sqrt{x}}{b}\right)}{b}+\frac{2x^{3/2}}{3b}\right)}{b}+\frac{2x^{5/2}}{5b}$$

$$\Big\downarrow \text{2593}$$

$$\frac{\left(\frac{\left(\frac{2\sqrt{x}}{b}-\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}{b^{3/2}}\right)(bx-\text{arctanh}(\tanh(a+bx)))}{b}+\frac{2x^{3/2}}{3b}\right)(bx-\text{arctanh}}{b}$$

$$\frac{2x^{5/2}}{5b}$$

input
```
Int[x^(5/2)/ArcTanh[Tanh[a + b*x]],x]
```

output
```
(2*x^(5/2))/(5*b) + (((2*x^(3/2))/(3*b) + (((2*Sqrt[x])/b - (2*ArcTanh[(Sq
rt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*Sqrt[b*x - ArcTanh[Tanh
[a + b*x]]])/b^(3/2))*(b*x - ArcTanh[Tanh[a + b*x]]))/b)*(b*x - ArcTanh[Ta
nh[a + b*x]]))/b
```

3.192.3.1 Defintions of rubi rules used

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

3.192.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(96) = 192$.

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.73

method	result
derivativedivides	$\frac{\frac{2b^2x^{\frac{5}{2}}}{5}-\frac{2bx^{\frac{3}{2}}a}{3}-\frac{2bx^{\frac{3}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{3}+2a^2\sqrt{x}+4a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}+2(\operatorname{arctanh}(\tanh(bx+}{b^3}$
default	$\frac{\frac{2b^2x^{\frac{5}{2}}}{5}-\frac{2bx^{\frac{3}{2}}a}{3}-\frac{2bx^{\frac{3}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{3}+2a^2\sqrt{x}+4a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}+2(\operatorname{arctanh}(\tanh(bx+}{b^3}$

input
```
int(x^(5/2)/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
2/b^3*(1/5*b^2*x^(5/2)-1/3*b*x^(3/2)*a-1/3*b*x^(3/2)*(arctanh(tanh(b*x+a))
-b*x-a)+a^2*x^(1/2)+2*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)+(arctanh(tanh
(b*x+a))-b*x-a)^2*x^(1/2))+2*(-a^3-3*a^2*(arctanh(tanh(b*x+a))-b*x-a)-3*a*
(arctanh(tanh(b*x+a))-b*x-a)^2-(arctanh(tanh(b*x+a))-b*x-a)^3)/b^3/((arcta
nh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)
*b)^(1/2))
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.14

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \left[\frac{15\,a^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(3\,b^2x^2-5\,abx+15\,a^2)\sqrt{x}}{15\,b^3},\right.$$
$$\left.-\frac{2\left(15\,a^2\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(3\,b^2x^2-5\,abx+15\,a^2)\sqrt{x}\right)}{15\,b^3}\right]$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
[1/15*(15*a^2*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a))
 + 2*(3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3, -2/15*(15*a^2*sqrt(a/b)*
arctan(b*sqrt(x)*sqrt(a/b)/a) - (3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^
3]
```

3.192.6 Sympy [F]

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int \frac{x^{\frac{5}{2}}}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**(5/2)/atanh(tanh(b*x+a)),x)
```

output
```
Integral(x**(5/2)/atanh(tanh(a + b*x)), x)
```

3.192.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = -\frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{2\left(3\,b^2x^{\frac{5}{2}} - 5\,abx^{\frac{3}{2}} + 15\,a^2\sqrt{x}\right)}{15\,b^3}$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^2*x^(5/2) -
 5*a*b*x^(3/2) + 15*a^2*sqrt(x))/b^3
```

3.192.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.51

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = -\frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{2\left(3\,b^4x^{\frac{5}{2}} - 5\,ab^3x^{\frac{3}{2}} + 15\,a^2b^2\sqrt{x}\right)}{15\,b^5}$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
-2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^4*x^(5/2) -
 5*a*b^3*x^(3/2) + 15*a^2*b^2*sqrt(x))/b^5
```

3.192.9 Mupad [B] (verification not implemented)

Time = 4.50 (sec) , antiderivative size = 415, normalized size of antiderivative = 3.58

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,x^{5/2}}{5\,b} + \frac{x^{3/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)}{3\,b^2}$$
$$+ \frac{\sqrt{x}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2}{2\,b^3}$$
$$+ \frac{\sqrt{2}\,\ln\left(\frac{16\,b^{15/2}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}\right)\left(\ln \cdots\right.}{8\,b^{7/2}}$$

input
```
int(x^(5/2)/atanh(tanh(a + b*x)),x)
```

output
```
(2*x^(5/2))/(5*b) + (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(3*b^2) + (x^(1/2)
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*b^3) + (2^(1/2)*log((16*b^(15/2)*(2^(1/2)
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(
1/2) + 2*2^(1/2)*b*x))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2))
)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)^(5/2))/(8*b^(7/2))
```

3.193 $\int \frac{x^{3/2}}{\text{arctanh}(\tanh(a+bx))} \, dx$

3.193.1 Optimal result

Integrand size = 15, antiderivative size = 89

$$\int \frac{x^{3/2}}{\text{arctanh}(\tanh(a+bx))} \, dx = \frac{2x^{3/2}}{3b} + \frac{2\sqrt{x}(bx - \text{arctanh}(\tanh(a+bx)))}{b^2} - \frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)(bx - \text{arctanh}(\tanh(a+bx)))^{3/2}}{b^{5/2}}$$

output
```
2/3*x^(3/2)/b-2*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*
(b*x-arctanh(tanh(b*x+a)))^(3/2)/b^(5/2)+2*(b*x-arctanh(tanh(b*x+a)))*x^(1
/2)/b^2
```

3.193.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int \frac{x^{3/2}}{\text{arctanh}(\tanh(a+bx))} \, dx = \frac{2x^{3/2}}{3b} - \frac{2\sqrt{x}(-bx + \text{arctanh}(\tanh(a+bx)))}{b^2} + \frac{2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)(-bx + \text{arctanh}(\tanh(a+bx)))^{3/2}}{b^{5/2}}$$

input
```
Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]],x]
```

output
```
(2*x^(3/2))/(3*b) - (2*Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^2 + (2
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) +
 ArcTanh[Tanh[a + b*x]])^(3/2))/b^(5/2)
```

3.193.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2590, 2590, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))}dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2x^{3/2}}{3b}$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2\sqrt{x}}{b}\right)}{b}+\frac{2x^{3/2}}{3b}$$

$$\Big\downarrow \text{2593}$$

$$\frac{\left(\frac{2\sqrt{x}}{b}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}{b^{3/2}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))}{b}+\frac{2x^{3/2}}{3b}$$

input
```
Int[x^(3/2)/ArcTanh[Tanh[a + b*x]],x]
```

output
```
(2*x^(3/2))/(3*b) + (((2*Sqrt[x])/b - (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*
x - ArcTanh[Tanh[a + b*x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/b^(3/2))*
(b*x - ArcTanh[Tanh[a + b*x]]))/b
```

3.193.3.1 Defintions of rubi rules used

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

3.193.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.35

method	result
derivativedivides	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+a\sqrt{x}+(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}\right)}{b^{2}}+\frac{2\left(a^{2}+2a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+(\operatorname{arctanh}(\tanh(b\right.}{b^{2}\sqrt{(\operatorname{arctanh}(\tan}$
default	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+a\sqrt{x}+(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}\right)}{b^{2}}+\frac{2\left(a^{2}+2a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+(\operatorname{arctanh}(\tanh(b\right.}{b^{2}\sqrt{(\operatorname{arctanh}(\tan}$

input
```
int(x^(3/2)/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-2/b^2*(-1/3*b*x^(3/2)+a*x^(1/2)+(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2))+2*(
a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/b^2/(
(arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a)
)-b*x)*b)^(1/2))
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \left[\frac{3\,a\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(bx-3\,a)\sqrt{x}}{3\,b^2}, \frac{2\left(3\,a\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{}}{a}\right.\right.}{3\,b^2}\right.$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
[1/3*(3*a*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2
*(b*x - 3*a)*sqrt(x))/b^2, 2/3*(3*a*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a
) + (b*x - 3*a)*sqrt(x))/b^2]
```

3.193.6 Sympy [F]

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int \frac{x^{\frac{3}{2}}}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**(3/2)/atanh(tanh(b*x+a)),x)
```

output
```
Integral(x**(3/2)/atanh(tanh(a + b*x)), x)
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.47

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\left(bx^{\frac{3}{2}}-3\,a\sqrt{x}\right)}{3\,b^2}$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b*x^(3/2) - 3*a*s
qrt(x))/b^2
```

3.193.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.51

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\left(b^2x^{\frac{3}{2}} - 3\,ab\sqrt{x}\right)}{3\,b^3}$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b^2*x^(3/2) - 3*a
*b*sqrt(x))/b^3
```

3.193.9 Mupad [B] (verification not implemented)

Time = 4.69 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.98

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,x^{3/2}}{3\,b} + \frac{\sqrt{x}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)}{b^2} + \frac{\sqrt{2}\,\ln\left(\frac{4\,b^{11/2}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}\right)\left(\ln\left(\right.\right.}{4\,b^{5/2}}$$

input
```
int(x^(3/2)/atanh(tanh(a + b*x)),x)
```

output
```
(2*x^(3/2))/(3*b) + (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^2 + (2^(1/2)*log
((4*b^(11/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp
(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2))/(4*b^(5/2))
```

3.194 $\int \frac{\sqrt{x}}{\textbf{arctanh}(\tanh(a+bx))}\,dx$

3.194.1 Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\sqrt{x}}{b} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}{b^{3/2}}$$

output
```
2*x^(1/2)/b-2*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b
*x-arctanh(tanh(b*x+a)))^(1/2)/b^(3/2)
```

3.194.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\sqrt{x}}{b} - \frac{2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}{b^{3/2}}$$

input
```
Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]],x]
```

output
```
(2*Sqrt[x])/b - (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a +
 b*x]]]]*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])/b^(3/2)
```

3.194.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\text{arctanh}(\tanh(a+bx))}\,dx$$

$$\Big\downarrow \; 2590$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2\sqrt{x}}{b}$$

$$\Big\downarrow \; 2593$$

$$\frac{2\sqrt{x}}{b}-\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}{b^{3/2}}$$

input
```
Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]],x]
```

output
```
(2*Sqrt[x])/b - (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b
*x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/b^(3/2)
```

3.194.3.1 Defintions of rubi rules used

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

3.194.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b}+\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{b\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}$	66
default	$\frac{2\sqrt{x}}{b}+\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{b\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}$	66

input
```
int(x^(1/2)/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/b+2*(b*x-arctanh(tanh(b*x+a)))/b/((arctanh(tanh(b*x+a))-b*x)*b)^
(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx=\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,\sqrt{x}}{b},\ -\frac{2\left(\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{x}\right)}{b}\right]$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
[(sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*sqrt(x)
)/b, -2*(sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) - sqrt(x))/b]
```

3.194.6 Sympy [F]

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int \frac{\sqrt{x}}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**(1/2)/atanh(tanh(b*x+a)),x)
```

output
```
Integral(sqrt(x)/atanh(tanh(a + b*x)), x)
```

3.194.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = -\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{2\,\sqrt{x}}{b}$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

3.194.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = -\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{2\,\sqrt{x}}{b}$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
-2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

3.194.9 Mupad [B] (verification not implemented)

Time = 4.96 (sec) , antiderivative size = 296, normalized size of antiderivative = 4.62

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{2\sqrt{x}}{b} + \frac{\sqrt{2}\,\ln\left(\frac{b^{7/2}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}}\right)\sqrt{\ln}}{2\,b^{3/2}}$$

input
```
int(x^(1/2)/atanh(tanh(a + b*x)),x)
```

output
```
(2*x^(1/2))/b + (2^(1/2)*log((b^(7/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) -
 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(
2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^
(1/2))/(2*b^(3/2))
```

3.195 $\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}\,dx$

3.195.1 Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{\sqrt{b}\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}$$

output
```
-2*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/b^(1/2)/(b*x-
arctanh(tanh(b*x+a)))^(1/2)
```

3.195.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}\,dx = \frac{2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)}{\sqrt{b}\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}$$

input
```
Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]),x]
```

output
```
(2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])/(Sqrt[
b]*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])
```

3.195.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}\,dx$$

$$\Big\downarrow \ 2593$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{\sqrt{b}\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}$$

input
```
Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]),x]
```

output
```
(-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(Sqrt[b
]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])
```

3.195.3.1 Defintions of rubi rules used

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

3.195.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\text{arctanh}(\tanh(bx+a))-bx)b}}\right)}{\sqrt{(\text{arctanh}(\tanh(bx+a))-bx)b}}$	41
default	$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\text{arctanh}(\tanh(bx+a))-bx)b}}\right)}{\sqrt{(\text{arctanh}(\tanh(bx+a))-bx)b}}$	41

input
```
int(1/arctanh(tanh(b*x+a))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x
+a))-b*x)*b)^(1/2))
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{ab}, -\frac{2\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab}\right]$$

input
```
integrate(1/arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="fricas")
```

output
```
[-sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a))/(a*b), -2*sqr
t(a*b)*arctan(sqrt(a*b)/(b*sqrt(x)))/(a*b)]
```

3.195.6 Sympy [F]

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}\,dx = \int \frac{1}{\sqrt{x}\,\text{atanh}\,(\tanh\,(a+bx))}\,dx$$

input
```
integrate(1/atanh(tanh(b*x+a))/x**(1/2),x)
```

output
```
Integral(1/(sqrt(x)*atanh(tanh(a + b*x))), x)
```

3.195.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}\,dx = \frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="maxima")
```

output
```
2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

3.195.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}\,dx = \frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="giac")
```

output
```
2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

3.195.9 Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 347, normalized size of antiderivative = 6.55

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}\,dx$$

$$= \frac{\sqrt{2}\,\ln\left(\frac{b^2\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)\left(2\sqrt{2}\,a+4\sqrt{x}\sqrt{b\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}-\sqrt{2}\left(2\,a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right.\right.\right.}{2\sqrt{b\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)}}{\sqrt{b\,\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-b\,\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b^2\,x}}$$

input
```
int(1/(x^(1/2)*atanh(tanh(a + b*x))),x)
```

output
```
(2^(1/2)*log((b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)*(2*2^(1/2)*a + 4*x^(1/2)*(b*(l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x))^(1/2) - 2^(1/2)*(2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) -
 2*2^(1/2)*b*x))/(2*(b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))^(1/2)*(log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))
))/(b*log(1/(exp(2*a)*exp(2*b*x) + 1)) - b*log((exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b^2*x)^(1/2)
```

3.195. $\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}\,dx$

3.196 $\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}\,dx$

3.196.1 Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}\,dx = -\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}} + \frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}$$

output
```
-2*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*b^(1/2)/(b*x-
arctanh(tanh(b*x+a)))^(3/2)+2/(b*x-arctanh(tanh(b*x+a)))/x^(1/2)
```

3.196.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}\,dx = -\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\text{arctanh}(\tanh(a+bx)))^{3/2}} - \frac{2}{\sqrt{x}(-bx+\text{arctanh}(\tanh(a+bx)))}$$

input
```
Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]),x]
```

output
```
(-2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2) - 2/(Sqrt[x]*(-(b*x) + ArcTanh[
Tanh[a + b*x]]))
```

3.196.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2594, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}\,dx$$

$$\Big\downarrow \text{2594}$$

$$\frac{b\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2593}$$

$$\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}-\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}$$

input
```
Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]),x]
```

output
```
(-2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a +
 b*x]]))
```

3.196.3.1 Defintions of rubi rules used

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

3.196.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}-\frac{2}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}$	76
default	$-\frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}-\frac{2}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}$	76

input
```
int(1/x^(3/2)/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-2*b/(arctanh(tanh(b*x+a))-b*x)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arcta
n(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))-2/(arctanh(tanh(b*x+a))-
b*x)/x^(1/2)
```

3.196.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^{3/2}\operatorname{arctanh}(\tanh(a+bx))}\,dx=\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\sqrt{x}}{ax},\frac{2\left(x\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\sqrt{x}\right)}{ax}\right.$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
[(x*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) - 2*sqrt(
x))/(a*x), 2*(x*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - sqrt(x))/(a*x)
]
```

3.196.6 Sympy [F]

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}\,dx = \int \frac{1}{x^{\frac{3}{2}}\,\text{atanh}\,(\tanh\,(a+bx))}\,dx$$

input
```
integrate(1/x**(3/2)/atanh(tanh(b*x+a)),x)
```

output
```
Integral(1/(x**(3/2)*atanh(tanh(a + b*x))), x)
```

3.196.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}\,dx = -\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{2}{a\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

3.196.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}\,dx = -\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{2}{a\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
-2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

3.196.9 Mupad [B] (verification not implemented)

Time = 5.06 (sec) , antiderivative size = 464, normalized size of antiderivative = 6.11

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}\,dx = \frac{4}{\sqrt{x}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}$$
$$+\frac{2\,\sqrt{2}\,\sqrt{b}\,\ln\left(\frac{\sqrt{b}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\ldots}\right.}{2\,\left(\ln\left(\frac{2}{e^{2\ldots}}\ldots\right.\right.}\right.}{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\ldots\right.}$$

input
```
int(1/(x^(3/2)*atanh(tanh(a + b*x))),x)
```

output
```
4/(x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (2*2^(1/2)*b^(1/2)*log((b^(1/2)*(l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/
2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*((2*a - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 4*a^2))/(2*(l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*ex
p(2*b*x) + 1))))))/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2)
```

3.197 $\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}\,dx$

3.197.1 Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}\,dx = -\frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{5/2}} + \frac{2b}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))^2} + \frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

output
```
-2*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-
arctanh(tanh(b*x+a)))^(5/2)+2/3/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))+2*b/(b*
x-arctanh(tanh(b*x+a)))^2/x^(1/2)
```

3.197.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}\,dx = \frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\text{arctanh}(\tanh(a+bx)))^{5/2}} + \frac{2(4bx-\text{arctanh}(\tanh(a+bx)))}{3x^{3/2}(-bx+\text{arctanh}(\tanh(a+bx)))^2}$$

input
```
Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]),x]
```

output
```
(2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]
)/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) + (2*(4*b*x - ArcTanh[Tanh[a + b
*x]]))/(3*x^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

3.197.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2594, 2594, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}dx$$

$$\Big\downarrow \text{2594}$$

$$\frac{b\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2594}$$

$$\frac{b\left(\frac{b\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2593}$$

$$\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{b\left(\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}-\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}\right)}{bx-\text{arctanh}(\tanh(a+bx))}$$

input
```
Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]),x]
```

output
```
2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])) + (b*((-2*Sqrt[b]*ArcTanh[(Sq
rt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(b*x - ArcTanh[Tanh[a
+ b*x]])^(3/2) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))))/(b*x - ArcTa
nh[Tanh[a + b*x]])
```

3.197.3.1 Defintions of rubi rules used

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

3.197.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-\frac{2}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}+\frac{2b}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}\sqrt{x}}+\frac{2b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))}}\right.}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}\sqrt{(\operatorname{arctanh}(\text{ta}}}$
default	$-\frac{2}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}+\frac{2b}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}\sqrt{x}}+\frac{2b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))}}\right.}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}\sqrt{(\operatorname{arctanh}(\text{ta}}}$

input
```
int(1/x^(5/2)/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)+2/(arctanh(tanh(b*x+a))-b*x)^2*b/x
^(1/2)+2*b^2/(arctanh(tanh(b*x+a))-b*x)^2/((arctanh(tanh(b*x+a))-b*x)*b)^(
1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

3.197.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}\,dx = \left[\frac{3\,bx^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(3\,bx-a)\sqrt{x}}{3\,a^2x^2},\right.$$
$$\left.-\frac{2\left(3\,bx^2\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(3\,bx-a)\sqrt{x}\right)}{3\,a^2x^2}\right]$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
[1/3*(3*b*x^2*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a))
 + 2*(3*b*x - a)*sqrt(x))/(a^2*x^2), -2/3*(3*b*x^2*sqrt(b/a)*arctan(a*sqrt
(b/a)/(b*sqrt(x))) - (3*b*x - a)*sqrt(x))/(a^2*x^2)]
```

3.197.6 Sympy [F]

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}\,dx = \int \frac{1}{x^{\frac{5}{2}}\,\text{atanh}\,(\tanh\,(a+bx))}\,dx$$

input
```
integrate(1/x**(5/2)/atanh(tanh(b*x+a)),x)
```

output
```
Integral(1/(x**(5/2)*atanh(tanh(a + b*x))), x)
```

3.197.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}+\frac{2\,(3\,bx-a)}{3\,a^2x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x
^(3/2))
```

3.197.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}\,dx = \frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{2\,(3\,bx-a)}{3\,a^2x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x
^(3/2))
```

3.197.9 Mupad [B] (verification not implemented)

Time = 5.10 (sec) , antiderivative size = 642, normalized size of antiderivative = 6.36

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}\,dx = \frac{4}{3\,x^{3/2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}$$
$$+ \frac{8\,b}{\sqrt{x}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}$$
$$+ \frac{4\,\sqrt{2}\,b^{3/2}\,\ln\left(\frac{\sqrt{b}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}\right.}{}\right.}{}$$

input
```
int(1/(x^(5/2)*atanh(tanh(a + b*x))),x)
```

output
```
4/(3*x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (8*b)/(x^(1/2)*(log(2/(exp(2*a)*
exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x)^2) + (4*2^(1/2)*b^(3/2)*log((b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x)
+ 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1
/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a
)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 24*
a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 16*a^4 - 8*a*(2*a - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)^3 - 32*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(2*(log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)
)))))/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(5/2)
```

3.197. $\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}\,dx$

3.198 $\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))}\,dx$

3.198.1 Optimal result

Integrand size = 15, antiderivative size = 128

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))}\,dx = -\frac{2b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{7/2}} + \frac{2b^2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))^3} + \frac{2b}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))^2} + \frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

output
```
-2*b^(5/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-
arctanh(tanh(b*x+a)))^(7/2)+2/3*b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/5
/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))+2*b^2/(b*x-arctanh(tanh(b*x+a)))^3/x^(
1/2)
```

3.198.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))}\,dx = -\frac{2b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\text{arctanh}(\tanh(a+bx)))^{7/2}} + \frac{2(23b^2x^2-11bx\text{arctanh}(\tanh(a+bx))+3\text{arctanh}(\tanh(a+bx))^2)}{15x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))^3}$$

input
```
Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]),x]
```

output
```
(-2*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]
]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2) + (2*(23*b^2*x^2 - 11*b*x*ArcTa
nh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2))/(15*x^(5/2)*(b*x - ArcTan
h[Tanh[a + b*x]])^3)
```

3.198.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2594, 2594, 2594, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))}dx$$

$$\Big\downarrow \text{2594}$$

$$\frac{b\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2594}$$

$$\frac{b\left(\frac{b\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2594}$$

$$\frac{b\left(\frac{b\left(\frac{b\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2593}$$

$$\frac{b\left(\frac{2}{3x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{b\left(\frac{2}{\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))}-\frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{(bx-\operatorname{arctanh}(\tanh(a+bx)))^{3/2}}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

input
```
Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]),x]
```

output
```
2/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])) + (b*(2/(3*x^(3/2)*(b*x - Arc
Tanh[Tanh[a + b*x]])) + (b*((-2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x
 - ArcTanh[Tanh[a + b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) + 2/(Sqr
t[x]*(b*x - ArcTanh[Tanh[a + b*x]]))))/(b*x - ArcTanh[Tanh[a + b*x]])))/(b
*x - ArcTanh[Tanh[a + b*x]])
```

3.198.3.1 Defintions of rubi rules used

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

3.198.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{2b^3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^3\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}-\frac{2}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}-\frac{2}{(\operatorname{arctanh}(\tanh($
default	$-\frac{2b^3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^3\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}-\frac{2}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}-\frac{2}{(\operatorname{arctanh}(\tanh($

input
```
int(1/x^(7/2)/arctanh(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-2*b^3/(arctanh(tanh(b*x+a))-b*x)^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*a
rctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))-2/5/(arctanh(tanh(b*
x+a))-b*x)/x^(5/2)-2/(arctanh(tanh(b*x+a))-b*x)^3*b^2/x^(1/2)+2/3/(arctanh
(tanh(b*x+a))-b*x)^2*b/x^(3/2)
```

3.198.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^{7/2}\operatorname{arctanh}(\tanh(a+bx))}\,dx = \left[\frac{15\,b^2x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(15\,b^2x^2-5\,abx+3\,a^2\right)\sqrt{x}}{15\,a^3x^3}, \frac{2\,(\ldots}{\ldots}\right.$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
[1/15*(15*b^2*x^3*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x +
 a)) - 2*(15*b^2*x^2 - 5*a*b*x + 3*a^2)*sqrt(x))/(a^3*x^3), 2/15*(15*b^2*x
^3*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 - 5*a*b*x + 3*a
^2)*sqrt(x))/(a^3*x^3)]
```

3.198.6 Sympy [F]

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))}\,dx = \int \frac{1}{x^{\frac{7}{2}}\,\text{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**(7/2)/atanh(tanh(b*x+a)),x)
```

output
```
Integral(1/(x**(7/2)*atanh(tanh(a + b*x))), x)
```

3.198.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))}\,dx = -\frac{2\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{2\,(15\,b^2x^2 - 5\,abx + 3\,a^2)}{15\,a^3x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*
a*b*x + 3*a^2)/(a^3*x^(5/2))
```

3.198.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))}\,dx = -\frac{2\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{2\,(15\,b^2x^2 - 5\,abx + 3\,a^2)}{15\,a^3x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

output
```
-2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*
a*b*x + 3*a^2)/(a^3*x^(5/2))
```

3.198.9 Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 822, normalized size of antiderivative = 6.42

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(7/2)*atanh(tanh(a + b*x))),x)
```

output
```
4/(5*x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (8*b)/(3*x^(3/2)*(log(2/(exp(2*a
)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)^2) + (16*b^2)/(x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (8*2^(1/2
)*b^(5/2)*log((b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^
(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 + 60*a^2*(2*a - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x)^4 - 160*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + 240*a^4*(2*a
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^2 + 64*a^6 - 12*a*(2*a - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^5 - 192*a^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*ex...
```

3.199 $\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^2} \, dx$

3.199.1 Optimal result

Integrand size = 15, antiderivative size = 135

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^2} \, dx = \frac{7x^{5/2}}{5b^2} + \frac{7x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}{3b^3} + \frac{7\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2}{b^4} - \frac{7\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))^{5/2}}{b^{9/2}} - \frac{x^{7/2}}{b\operatorname{arctanh}(\tanh(a+bx))}$$

output
```
7/5*x^(5/2)/b^2+7/3*x^(3/2)*(b*x-arctanh(tanh(b*x+a)))/b^3-7*arctanh(b^(1/
2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*x-arctanh(tanh(b*x+a)))^(5
/2)/b^(9/2)-x^(7/2)/b/arctanh(tanh(b*x+a))+7*(b*x-arctanh(tanh(b*x+a)))^2*
x^(1/2)/b^4
```

3.199.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{2x^{5/2}}{5b^2} - \frac{4x^{3/2}(-bx+\operatorname{arctanh}(\tanh(a+bx)))}{3b^3} + \frac{6\sqrt{x}(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2}{b^4} - \frac{7\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)(-bx+\operatorname{arctanh}(\tanh(a+bx)))^{5/2}}{b^{9/2}} + \frac{\sqrt{x}(-bx+\operatorname{arctanh}(\tanh(a+bx)))^3}{b^4\operatorname{arctanh}(\tanh(a+bx))}$$

input
```
Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(2*x^(5/2))/(5*b^2) - (4*x^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/(3*b^3
) + (6*Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)/b^4 - (7*ArcTan[(Sqrt[
b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) + ArcTanh[Tanh[
a + b*x]])^(5/2))/b^(9/2) + (Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3)/
(b^4*ArcTanh[Tanh[a + b*x]])
```

3.199.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2599, 2590, 2590, 2590, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx$$

$$\Big\downarrow \; 2599$$

$$\frac{7\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))}dx}{2b} - \frac{x^{7/2}}{b\operatorname{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \; 2590$$

$$\frac{7\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\int\frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2x^{5/2}}{5b}\right)}{2b}-\frac{x^{7/2}}{b\operatorname{arctanh}(\tanh(a+bx))}$$

$\downarrow$ 2590

$$\frac{7\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\int\frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2x^{3/2}}{3b}\right)}{b}+\frac{2x^{5/2}}{5b}\right)}{2b}-\frac{x^{7/2}}{b\operatorname{arctanh}(\tanh(a+bx))}$$

$\downarrow$ 2590

$$\frac{7\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\int\frac{1}{\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2\sqrt{x}}{b}\right)}{b}+\frac{2x^{3/2}}{3b}\right)}{b}+\right.}{2b}\frac{x^{7/2}}{b\operatorname{arctanh}(\tanh(a+bx))}$$

$\downarrow$ 2593

3.199. $\int\frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx$

$$\frac{7\left(\frac{\left(\frac{\left(\frac{2\sqrt{x}}{b}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}{b^{3/2}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))}{b}+\frac{2x^{3/2}}{3b}\right)(bx-\operatorname{arctanh}(\tanh}{b}\right.}{2b}$$

$$\frac{x^{7/2}}{b\operatorname{arctanh}(\tanh(a+bx))}$$

input
```
Int[x^(7/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(7*((2*x^(5/2))/(5*b) + (((2*x^(3/2))/(3*b) + (((2*Sqrt[x])/b - (2*ArcTanh
[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*Sqrt[b*x - ArcTanh[
Tanh[a + b*x]]])/b^(3/2))*(b*x - ArcTanh[Tanh[a + b*x]]))/b)*(b*x - ArcTan
h[Tanh[a + b*x]]))/b))/(2*b) - x^(7/2)/(b*ArcTanh[Tanh[a + b*x]])
```

3.199.3.1 Defintions of rubi rules used

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.199.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(113) = 226$.

Time = 0.78 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.07

method	result
derivativedivides	$\frac{\frac{2b^2 x^{\frac{5}{2}}}{5}-\frac{4b\,x^{\frac{3}{2}} a}{3}-\frac{4b\,x^{\frac{3}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{3}+6a^2\sqrt{x}+12a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}+6(\operatorname{arctanh}(\tanh(bx}{b^4}$
default	$\frac{\frac{2b^2 x^{\frac{5}{2}}}{5}-\frac{4b\,x^{\frac{3}{2}} a}{3}-\frac{4b\,x^{\frac{3}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{3}+6a^2\sqrt{x}+12a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}+6(\operatorname{arctanh}(\tanh(bx}{b^4}$
risch	Expression too large to display

input
```
int(x^(7/2)/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
2/b^4*(1/5*b^2*x^(5/2)-2/3*b*x^(3/2)*a-2/3*b*x^(3/2)*(arctanh(tanh(b*x+a))
-b*x-a)+3*a^2*x^(1/2)+6*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)+3*(arctanh(
tanh(b*x+a))-b*x-a)^2*x^(1/2))-2/b^4*((-1/2*a^3-3/2*a^2*(arctanh(tanh(b*x+
a))-b*x-a)-3/2*a*(arctanh(tanh(b*x+a))-b*x-a)^2-1/2*(arctanh(tanh(b*x+a))-
b*x-a)^3)*x^(1/2)/arctanh(tanh(b*x+a))+7/2*(a^3+3*a^2*(arctanh(tanh(b*x+a)
)-b*x-a)+3*a*(arctanh(tanh(b*x+a))-b*x-a)^2+(arctanh(tanh(b*x+a))-b*x-a)^3
)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x
+a))-b*x)*b)^(1/2)))
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.39

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \left[\frac{105\,(a^2bx+a^3)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(6\,b^3x^3-14\,ab^2x^2+70\,a^2 \ldots}{30\,(b^5x+ab^4)}\right.$$

$$\left.-\frac{105\,(a^2bx+a^3)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(6\,b^3x^3-14\,ab^2x^2+70\,a^2bx+105\,a^3)\sqrt{x}}{15\,(b^5x+ab^4)}\right]$$

input
```
integrate(x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
[1/30*(105*(a^2*b*x + a^3)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) -
a)/(b*x + a)) + 2*(6*b^3*x^3 - 14*a*b^2*x^2 + 70*a^2*b*x + 105*a^3)*sqrt(x
))/(b^5*x + a*b^4), -1/15*(105*(a^2*b*x + a^3)*sqrt(a/b)*arctan(b*sqrt(x)*
sqrt(a/b)/a) - (6*b^3*x^3 - 14*a*b^2*x^2 + 70*a^2*b*x + 105*a^3)*sqrt(x))/
(b^5*x + a*b^4)]
```

3.199.6 Sympy [F]

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \int \frac{x^{\frac{7}{2}}}{\operatorname{atanh}^2(\tanh(a+bx))}\,dx$$

input
```
integrate(x**(7/2)/atanh(tanh(b*x+a))**2,x)
```

output
```
Integral(x**(7/2)/atanh(tanh(a + b*x))**2, x)
```

3.199.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.56

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{6\,b^3x^{\frac{7}{2}} - 14\,ab^2x^{\frac{5}{2}} + 70\,a^2bx^{\frac{3}{2}} + 105\,a^3\sqrt{x}}{15\,(b^5x + ab^4)} - \frac{7\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^4}$$

input
```
integrate(x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/15*(6*b^3*x^(7/2) - 14*a*b^2*x^(5/2) + 70*a^2*b*x^(3/2) + 105*a^3*sqrt(x
))/(b^5*x + a*b^4) - 7*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4)
```

3.199.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.56

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{7\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{a^3\sqrt{x}}{(bx+a)b^4} + \frac{2\left(3\,b^8x^{\frac{5}{2}} - 10\,ab^7x^{\frac{3}{2}} + 45\,a^2b^6\sqrt{x}\right)}{15\,b^{10}}$$

input
```
integrate(x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
-7*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + a^3*sqrt(x)/((b*x + a
)*b^4) + 2/15*(3*b^8*x^(5/2) - 10*a*b^7*x^(3/2) + 45*a^2*b^6*sqrt(x))/b^10
```

3.199.9 Mupad [B] (verification not implemented)

Time = 4.78 (sec) , antiderivative size = 523, normalized size of antiderivative = 3.87

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{2\,x^{5/2}}{5\,b^2} + \frac{2\,x^{3/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)}{3\,b^3}$$
$$+ \frac{3\,\sqrt{x}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2}{2\,b^4}$$
$$+ \frac{7\,\sqrt{2}\,\ln\left(\frac{64\,b^{19/2}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}\right)\left(\ln\right.}{16\,b^{9/2}}$$
$$- \frac{\sqrt{x}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^3}{4\,b^4\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}$$

input
```
int(x^(7/2)/atanh(tanh(a + b*x))^2,x)
```

output
```
(2*x^(5/2))/(5*b^2) + (2*x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(3*b^3) + (3*x
^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*b^4) + (7*2^(1/2)*log((64*b^(19/2)*
(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(5/2))/(16*b^(9/2)) - (x^(1/2)*(log(2/
(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^3)/(4*b^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))
```

3.200 $\int \frac{x^{5/2}}{\mathbf{arctanh}(\tanh(a+bx))^2}\,dx$

3.200.1 Optimal result

Integrand size = 15, antiderivative size = 108

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{5x^{3/2}}{3b^2} + \frac{5\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))}{b^3} - \frac{5\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))^{3/2}}{b^{7/2}} - \frac{x^{5/2}}{b\operatorname{arctanh}(\tanh(a+bx))}$$

output
```
5/3*x^(3/2)/b^2-5*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2)
)*(b*x-arctanh(tanh(b*x+a)))^(3/2)/b^(7/2)-x^(5/2)/b/arctanh(tanh(b*x+a))+
5*(b*x-arctanh(tanh(b*x+a)))*x^(1/2)/b^3
```

3.200.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{2x^{3/2}}{3b^2} - \frac{4\sqrt{x}(-bx+\operatorname{arctanh}(\tanh(a+bx)))}{b^3} + \frac{5\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)(-bx+\operatorname{arctanh}(\tanh(a+bx)))^{3/2}}{b^{7/2}} - \frac{\sqrt{x}(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2}{b^3\operatorname{arctanh}(\tanh(a+bx))}$$

input
```
Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(2*x^(3/2))/(3*b^2) - (4*Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^3 +
(5*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x)
 + ArcTanh[Tanh[a + b*x]])^(3/2))/b^(7/2) - (Sqrt[x]*(-(b*x) + ArcTanh[Tan
h[a + b*x]])^2)/(b^3*ArcTanh[Tanh[a + b*x]])
```

3.200.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2599, 2590, 2590, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\text{arctanh}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{5\int \frac{x^{3/2}}{\text{arctanh}(\tanh(a+bx))}dx}{2b}-\frac{x^{5/2}}{b\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{ 2590}$$

$$\frac{5\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{\sqrt{x}}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2x^{3/2}}{3b}\right)}{2b}-\frac{x^{5/2}}{b\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{ 2590}$$

$$\frac{5\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2\sqrt{x}}{b}\right)}{b}+\frac{2x^{3/2}}{3b}\right)}{2b}-\frac{x^{5/2}}{b\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{ 2593}$$

$$\frac{5\left(\frac{\left(\frac{2\sqrt{x}}{b}-\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}{b^{3/2}}\right)(bx-\text{arctanh}(\tanh(a+bx)))}{b}+\frac{2x^{3/2}}{3b}\right)}{2b}-\frac{x^{5/2}}{b\text{arctanh}(\tanh(a+bx))}$$

input
```
Int[x^(5/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(5*((2*x^(3/2))/(3*b) + (((2*Sqrt[x])/b - (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqr
t[b*x - ArcTanh[Tanh[a + b*x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/b^(3/
2))*(b*x - ArcTanh[Tanh[a + b*x]]))/b))/(2*b) - x^(5/2)/(b*ArcTanh[Tanh[a
+ b*x]])
```

3.200.3.1 Defintions of rubi rules used

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.200.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.68

method	result
derivativedivides	$-\frac{2\left(-\frac{b x^{\frac{3}{2}}}{3}+2a\sqrt{x}+2(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}\right)}{b^3}+\frac{2\left(-\frac{a^2}{2}-a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)-\frac{(\operatorname{arctanh}(\tanh(bx+\cdots}{2}\cdots\right.}{\operatorname{arctanh}(\tanh(bx+a))}\cdots$
default	$-\frac{2\left(-\frac{b x^{\frac{3}{2}}}{3}+2a\sqrt{x}+2(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}\right)}{b^3}+\frac{2\left(-\frac{a^2}{2}-a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)-\frac{(\operatorname{arctanh}(\tanh(bx+\cdots}{2}\cdots\right.}{\operatorname{arctanh}(\tanh(bx+a))}\cdots$
risch	Expression too large to display

input
```
int(x^(5/2)/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/b^3*(-1/3*b*x^(3/2)+2*a*x^(1/2)+2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2))
+2/b^3*((-1/2*a^2-a*(arctanh(tanh(b*x+a))-b*x-a)-1/2*(arctanh(tanh(b*x+a))
-b*x-a)^2)*x^(1/2)/arctanh(tanh(b*x+a))+5/2*(a^2+2*a*(arctanh(tanh(b*x+a))
-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/
2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)))
```

3.200.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.49

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \left[\frac{15\,(abx+a^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(2\,b^2x^2-10\,abx-15\,a^2)\sqrt{x}\cdots}{6\,(b^4x+ab^3)}\cdots\right.$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
[1/6*(15*(a*b*x + a^2)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(
b*x + a)) + 2*(2*b^2*x^2 - 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3), 1/
3*(15*(a*b*x + a^2)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (2*b^2*x^2 -
 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3)]
```

3.200.6 Sympy [F]

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \int \frac{x^{\frac{5}{2}}}{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**(5/2)/atanh(tanh(b*x+a))**2,x)
```

output
```
Integral(x**(5/2)/atanh(tanh(a + b*x))**2, x)
```

3.200.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{2\,b^2x^{\frac{5}{2}} - 10\,abx^{\frac{3}{2}} - 15\,a^2\sqrt{x}}{3\,(b^4x + ab^3)} + \frac{5\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3}$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/3*(2*b^2*x^(5/2) - 10*a*b*x^(3/2) - 15*a^2*sqrt(x))/(b^4*x + a*b^3) + 5*
a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3)
```

3.200.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{5\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{a^2\sqrt{x}}{(bx+a)b^3} + \frac{2\left(b^4x^{\frac{3}{2}} - 6\,ab^3\sqrt{x}\right)}{3\,b^6}$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
5*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - a^2*sqrt(x)/((b*x + a)
*b^3) + 2/3*(b^4*x^(3/2) - 6*a*b^3*sqrt(x))/b^6
```

3.200.9 Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 463, normalized size of antiderivative = 4.29

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{2\,x^{3/2}}{3\,b^2} + \frac{2\,\sqrt{x}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)}{b^3}$$
$$+ \frac{5\,\sqrt{2}\,\ln\left(\frac{16\,b^{15/2}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}\right)\,\left(\ln\right.}{8\,b^{7/2}}$$
$$- \frac{\sqrt{x}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2}{2\,b^3\,\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}$$

input
```
int(x^(5/2)/atanh(tanh(a + b*x))^2,x)
```

output
```
(2*x^(3/2))/(3*b^2) + (2*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^3 + (5*2^(1/
2)*log((16*b^(15/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log
(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2))/(8*b^(7/2))
 - (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*b^3*(log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))
```

3.201 $\int \frac{x^{3/2}}{\text{arctanh}(\tanh(a+bx))^2} \, dx$

3.201.1 Optimal result

Integrand size = 15, antiderivative size = 83

$$\int \frac{x^{3/2}}{\text{arctanh}(\tanh(a+bx))^2} \, dx = \frac{3\sqrt{x}}{b^2} - \frac{3\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}{b^{5/2}} - \frac{x^{3/2}}{b\text{arctanh}(\tanh(a+bx))}$$

output
```
-x^(3/2)/b/arctanh(tanh(b*x+a))+3*x^(1/2)/b^2-3*arctanh(b^(1/2)*x^(1/2)/(b
*x-arctanh(tanh(b*x+a)))^(1/2))*(b*x-arctanh(tanh(b*x+a)))^(1/2)/b^(5/2)
```

3.201.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int \frac{x^{3/2}}{\text{arctanh}(\tanh(a+bx))^2} \, dx = \frac{3\sqrt{x}}{b^2} - \frac{x^{3/2}}{b\text{arctanh}(\tanh(a+bx))} - \frac{3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}{b^{5/2}}$$

input
```
Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(3*Sqrt[x])/b^2 - x^(3/2)/(b*ArcTanh[Tanh[a + b*x]]) - (3*ArcTan[(Sqrt[b]*
Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*Sqrt[-(b*x) + ArcTanh[Tanh
[a + b*x]]])/b^(5/2)
```

3.201.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2599, 2590, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{3\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))}dx}{2b}-\frac{x^{3/2}}{b\operatorname{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2590}$$

$$\frac{3\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2\sqrt{x}}{b}\right)}{2b}-\frac{x^{3/2}}{b\operatorname{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2593}$$

$$\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}{b^{3/2}}\right)}{2b}-\frac{x^{3/2}}{b\operatorname{arctanh}(\tanh(a+bx))}$$

input
```
Int[x^(3/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
(3*((2*Sqrt[x])/b - (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a
 + b*x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/b^(3/2)))/(2*b) - x^(3/2)/(
b*ArcTanh[Tanh[a + b*x]])
```

3.201.3.1 Defintions of rubi rules used

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.201.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b^2}-\frac{2\left(\frac{\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{2}+\frac{bx}{2}\right)\sqrt{x}}{\operatorname{arctanh}(\tanh(bx+a))}+\frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{2\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{b^2}$	95
default	$\frac{2\sqrt{x}}{b^2}-\frac{2\left(\frac{\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{2}+\frac{bx}{2}\right)\sqrt{x}}{\operatorname{arctanh}(\tanh(bx+a))}+\frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{2\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{b^2}$	95
risch	Expression too large to display	108

input
```
int(x^(3/2)/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/b^2-2/b^2*((-1/2*arctanh(tanh(b*x+a))+1/2*b*x)*x^(1/2)/arctanh(t
anh(b*x+a))+3/2*(arctanh(tanh(b*x+a))-b*x)/((arctanh(tanh(b*x+a))-b*x)*b)^
(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)))
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.61

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \left[\frac{3\,(bx+a)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(2\,bx+3\,a)\sqrt{x}}{2\,(b^3x+ab^2)},\right.$$
$$\left.-\frac{3\,(bx+a)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(2\,bx+3\,a)\sqrt{x}}{b^3x+ab^2}\right]$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
[1/2*(3*(b*x + a)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x +
 a)) + 2*(2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2), -(3*(b*x + a)*sqrt(a/b)*a
rctan(b*sqrt(x)*sqrt(a/b)/a) - (2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2)]
```

3.201.6 Sympy [F]

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \int \frac{x^{\frac{3}{2}}}{\operatorname{atanh}^2(\tanh(a+bx))}\,dx$$

input
```
integrate(x**(3/2)/atanh(tanh(b*x+a))**2,x)
```

output
```
Integral(x**(3/2)/atanh(tanh(a + b*x))**2, x)
```

3.201.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.60

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{2\,bx^{\frac{3}{2}}+3\,a\sqrt{x}}{b^3x+ab^2}-\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
(2*b*x^(3/2) + 3*a*sqrt(x))/(b^3*x + a*b^2) - 3*a*arctan(b*sqrt(x)/sqrt(a*
b))/(sqrt(a*b)*b^2)
```

3.201.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.55

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{a\sqrt{x}}{(bx+a)b^2} + \frac{2\,\sqrt{x}}{b^2}$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
-3*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + a*sqrt(x)/((b*x + a)*b^
2) + 2*sqrt(x)/b^2
```

3.201.9 Mupad [B] (verification not implemented)

Time = 4.64 (sec) , antiderivative size = 403, normalized size of antiderivative = 4.86

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{2\,\sqrt{x}}{b^2} - \frac{\sqrt{x}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)}{b^2\,\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}$$
$$+ \frac{3\,\sqrt{2}\,\ln\left(\frac{4\,b^{11/2}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}\right)\,\sqrt{\ln \cdots}}{4\,b^{5/2}}$$

input
```
int(x^(3/2)/atanh(tanh(a + b*x))^2,x)
```

output
```
(2*x^(1/2))/b^2 - (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(b^2*(log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)
))) + (3*2^(1/2)*log((4*b^(11/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^
(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x
) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)
)/(4*b^(5/2))
```

3.201. $\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx$

3.202 $\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx$

3.202.1 Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{b^{3/2}\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}} - \frac{\sqrt{x}}{b\operatorname{arctanh}(\tanh(a+bx))}$$

output
```
-x^(1/2)/b/arctanh(tanh(b*x+a))-arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(
b*x+a)))^(1/2))/b^(3/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2)
```

3.202.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{\sqrt{x}}{b\operatorname{arctanh}(\tanh(a+bx))} + \frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{b^{3/2}\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}$$

input
```
Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
-(Sqrt[x]/(b*ArcTanh[Tanh[a + b*x]])) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*
x) + ArcTanh[Tanh[a + b*x]]]]/(b^(3/2)*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]
]])
```

3.202.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2599, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^2} \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{\int \frac{1}{\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))}dx}{2b} - \frac{\sqrt{x}}{b\operatorname{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{ 2593}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{b^{3/2}\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}} - \frac{\sqrt{x}}{b\operatorname{arctanh}(\tanh(a+bx))}$$

input
```
Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^2,x]
```

output
```
-(ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]/(b^(3/2)*S
qrt[b*x - ArcTanh[Tanh[a + b*x]]])) - Sqrt[x]/(b*ArcTanh[Tanh[a + b*x]])
```

3.202.3.1 Defintions of rubi rules used

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.202.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{\sqrt{x}}{b\,\mathrm{arctanh}(\tanh(bx+a))}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)}{b\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}$	61
default	$-\frac{\sqrt{x}}{b\,\mathrm{arctanh}(\tanh(bx+a))}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)}{b\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}$	61
risch	Expression too large to display	805

input
```
int(x^(1/2)/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
-x^(1/2)/b/arctanh(tanh(b*x+a))+1/b/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*a
rctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{x}}{\mathrm{arctanh}(\tanh(a+bx))^2}\,dx = \left[-\frac{2\,ab\sqrt{x}+\sqrt{-ab}(bx+a)\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{2\,(ab^3x+a^2b^2)},\ -\frac{ab\sqrt{x}+\sqrt{ab}(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab^3x+a^2b^2}\right]$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*a*b*sqrt(x) + sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sq
rt(x))/(b*x + a)))/(a*b^3*x + a^2*b^2), -(a*b*sqrt(x) + sqrt(a*b)*(b*x + a
)*arctan(sqrt(a*b)/(b*sqrt(x))))/(a*b^3*x + a^2*b^2)]
```

3.202.6 Sympy [F]

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \int \frac{\sqrt{x}}{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**(1/2)/atanh(tanh(b*x+a))**2,x)
```

output
```
Integral(sqrt(x)/atanh(tanh(a + b*x))**2, x)
```

3.202.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{\sqrt{x}}{b^2x+ab} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
-sqrt(x)/(b^2*x + a*b) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b)
```

3.202.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{\sqrt{x}}{(bx+a)b}$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) - sqrt(x)/((b*x + a)*b)
```

3.202.9 Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 344, normalized size of antiderivative = 4.71

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^2}\,dx$$
$$= \frac{\sqrt{2}\,\ln\left(\frac{b^{7/2}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\,\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)-4\sqrt{b}\sqrt{x}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln(\cdots}\right.}{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}\right.}{2b^{3/2}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}}$$
$$-\frac{2\sqrt{x}}{b\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)\right)}$$

input
```
int(x^(1/2)/atanh(tanh(a + b*x))^2,x)
```

output
```
(2^(1/2)*log((b^(7/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp
(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(
1/2)*b*x))/(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2
/(exp(2*a)*exp(2*b*x) + 1)))))/(2*b^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2))
 - (2*x^(1/2))/(b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
- log(2/(exp(2*a)*exp(2*b*x) + 1))))
```

3.203 $\int \frac{1}{\sqrt{x}\mathbf{arctanh}(\tanh(a+bx))^2}\,dx$

3.203.1 Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \frac{1}{\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{\sqrt{b}(bx-\operatorname{arctanh}(\tanh(a+bx)))^{3/2}} - \frac{1}{b\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))} - \frac{1}{b\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))}$$

output
```
arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-arctanh(tan
h(b*x+a)))^(3/2)/b^(1/2)-1/b/(b*x-arctanh(tanh(b*x+a)))/x^(1/2)-1/b/arctan
h(tanh(b*x+a))/x^(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{\sqrt{b}(-bx+\operatorname{arctanh}(\tanh(a+bx)))^{3/2}} + \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))(-bx+\operatorname{arctanh}(\tanh(a+bx)))}$$

input
```
Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]/(Sqrt[b]*(
-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2)) + Sqrt[x]/(ArcTanh[Tanh[a + b*x]]*
(-(b*x) + ArcTanh[Tanh[a + b*x]]))
```

3.203.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2599, 2594, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \text{2599}$$

$$-\frac{\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}dx}{2b}-\frac{1}{b\sqrt{x}\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{\frac{b\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}}{2b}-\frac{1}{b\sqrt{x}\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2593}$$

$$-\frac{\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}-\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}}{2b}-\frac{1}{b\sqrt{x}\text{arctanh}(\tanh(a+bx))}$$

input
```
Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2),x]
```

output
```
-1/2*((-2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) + 2/(Sqrt[x]*(b*x - ArcTanh[Ta
nh[a + b*x]])))/b - 1/(b*Sqrt[x]*ArcTanh[Tanh[a + b*x]])
```

3.203.3.1 Defintions of rubi rules used

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.203.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

method	result	si
derivativedivides	$\frac{\sqrt{x}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}$	82
default	$\frac{\sqrt{x}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}$	82
risch	Expression too large to display	13

input
```
int(1/arctanh(tanh(b*x+a))^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))+1/(arctanh(tanh(b*
x+a))-b*x)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh
(tanh(b*x+a))-b*x)*b)^(1/2))
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^2}\,dx = \left[\frac{2\,ab\sqrt{x}-\sqrt{-ab}(bx+a)\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{2\,(a^2b^2x+a^3b)}, \frac{ab\sqrt{x}-\sqrt{ab}(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{a^2b^2x+a^3b}\right]$$

input
```
integrate(1/arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*b*sqrt(x) - sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)))/(a^2*b^2*x + a^3*b), (a*b*sqrt(x) - sqrt(a*b)*(b*x + a)*
arctan(sqrt(a*b)/(b*sqrt(x))))/(a^2*b^2*x + a^3*b)]
```

3.203.6 Sympy [F]

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^2}\,dx = \int \frac{1}{\sqrt{x}\,\text{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/atanh(tanh(b*x+a))**2/x**(1/2),x)
```

output
```
Integral(1/(sqrt(x)*atanh(tanh(a + b*x))**2), x)
```

3.203.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.36

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^2}\,dx = \frac{\sqrt{x}}{abx+a^2} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x)/(a*b*x + a^2) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a)
```

3.203.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.36

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^2}\,dx = \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{\sqrt{x}}{(bx+a)a}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="giac")
```

output
```
arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) + sqrt(x)/((b*x + a)*a)
```

3.203.9 Mupad [B] (verification not implemented)

Time = 5.23 (sec) , antiderivative size = 516, normalized size of antiderivative = 5.32

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^2}\,dx$$
$$= \frac{\sqrt{2}\ln\left(-\frac{\sqrt{b}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)+4\sqrt{b}\sqrt{x}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\cdots}\right.\cdots}{2\left(\ln\left(\frac{2e\cdots}{e^{2a}\cdots}\right.\right.\cdots}\cdots\right.}{\sqrt{b}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\cdots\right.}$$
$$- \frac{4\sqrt{x}}{\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)\right)\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}$$

input
```
int(1/(x^(1/2)*atanh(tanh(a + b*x))^2),x)
```

output
```
(2^(1/2)*log(-(b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x) + 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^
(1/2)*b*x)*(4*a^2*b + b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 4*a*b*(2*a - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(b^(1/2)*(log(2/(exp(2*a
)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)^(3/2)) - (4*x^(1/2))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x))
```

3.203. $\int \frac{1}{\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^2}\,dx$

3.204 $\int \frac{1}{x^{3/2}\operatorname{arctanh}(\tanh(a+bx))^2}\,dx$

3.204.1 Optimal result

Integrand size = 15, antiderivative size = 120

$$\int \frac{1}{x^{3/2}\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{3\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{(bx-\operatorname{arctanh}(\tanh(a+bx)))^{5/2}} - \frac{3}{\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2} - \frac{1}{bx^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))} - \frac{1}{bx^{3/2}\operatorname{arctanh}(\tanh(a+bx))}$$

output
```
-1/b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))-1/b/x^(3/2)/arctanh(tanh(b*x+a))+3
*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*b^(1/2)/(b*x-ar
ctanh(tanh(b*x+a)))^(5/2)-3/(b*x-arctanh(tanh(b*x+a)))^2/x^(1/2)
```

3.204.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^{3/2}\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\operatorname{arctanh}(\tanh(a+bx)))^{5/2}} - \frac{2}{\sqrt{x}(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2} - \frac{b\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2}$$

input
```
Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

output
```
(-3*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) - 2/(Sqrt[x]*(-(b*x) + ArcTanh[
Tanh[a + b*x]])^2) - (b*Sqrt[x])/(ArcTanh[Tanh[a + b*x]]*(-(b*x) + ArcTanh
[Tanh[a + b*x]])^2)
```

3.204.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2599, 2594, 2594, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \text{2599}$$

$$-\frac{3\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}dx}{2b}-\frac{1}{bx^{3/2}\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{3\left(\frac{b\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{2b}-\frac{1}{bx^{3/2}\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{3\left(\frac{b\left(\frac{b\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{2b}-\frac{1}{bx^{3/2}\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2593}$$

$$-\frac{3\left(\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{b\left(\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}-\frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}\right)}{bx-\text{arctanh}(\tanh(a+bx))}\right)}{2b}-\frac{1}{bx^{3/2}\text{arctanh}(\tanh(a+bx))}$$

input
```
Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

output
```
(-3*(2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])) + (b*((-2*Sqrt[b]*ArcTan
h[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(b*x - ArcTanh[Ta
nh[a + b*x]])^(3/2) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))))/(b*x -
ArcTanh[Tanh[a + b*x]])))/(2*b) - 1/(b*x^(3/2)*ArcTanh[Tanh[a + b*x]])
```

3.204.3.1 Defintions of rubi rules used

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.204.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2b\left(\frac{\sqrt{x}}{2\,\mathrm{arctanh}(\tanh(bx+a))}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)}{2\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\mathrm{arctanh}(\tanh(bx+a))-bx)^2}-\frac{2}{(\mathrm{arctanh}(\tanh(bx+a))-bx)^2\sqrt{x}}$	93
default	$-\frac{2b\left(\frac{\sqrt{x}}{2\,\mathrm{arctanh}(\tanh(bx+a))}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)}{2\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\mathrm{arctanh}(\tanh(bx+a))-bx)^2}-\frac{2}{(\mathrm{arctanh}(\tanh(bx+a))-bx)^2\sqrt{x}}$	93
risch	Expression too large to display	1582

input
```
int(1/x^(3/2)/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/(arctanh(tanh(b*x+a))-b*x)^2*b*(1/2*x^(1/2)/arctanh(tanh(b*x+a))+3/2/((
arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))
-b*x)*b)^(1/2)))-2/(arctanh(tanh(b*x+a))-b*x)^2/x^(1/2)
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^{3/2}\mathrm{arctanh}(\tanh(a+bx))^2}\,dx=\left[\frac{3\,(bx^2+ax)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,(3\,bx+2\,a)\sqrt{x}}{2\,(a^2bx^2+a^3x)},\frac{3\,(bx^2+\right.$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
[1/2*(3*(b*x^2 + a*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b
*x + a)) - 2*(3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2 + a^3*x), (3*(b*x^2 + a*x)*
sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (3*b*x + 2*a)*sqrt(x))/(a^2*b*
x^2 + a^3*x)]
```

3.204.6 Sympy [F]

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^2}\,dx=\int \frac{1}{x^{\frac{3}{2}}\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**(3/2)/atanh(tanh(b*x+a))**2,x)
```

output
```
Integral(1/(x**(3/2)*atanh(tanh(a + b*x))**2), x)
```

3.204.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^2}\,dx=-\frac{3\,bx+2\,a}{a^2bx^{\frac{3}{2}}+a^3\sqrt{x}}-\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
-(3*b*x + 2*a)/(a^2*b*x^(3/2) + a^3*sqrt(x)) - 3*b*arctan(b*sqrt(x)/sqrt(a
*b))/(sqrt(a*b)*a^2)
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^2}\,dx=-\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}-\frac{3\,bx+2\,a}{\left(bx^{\frac{3}{2}}+a\sqrt{x}\right)a^2}$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
-3*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) - (3*b*x + 2*a)/((b*x^(3/
2) + a*sqrt(x))*a^2)
```

3.204.9 Mupad [B] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 705, normalized size of antiderivative = 5.88

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^2}\,dx = \frac{\sqrt{x}\left(\frac{8}{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx} - \frac{24bx}{\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right.}\right.}{2bx^2 - x\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}$$

$$+\frac{6\sqrt{2}\sqrt{b}\ln\left(-\frac{\sqrt{b}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)+4\sqrt{b}\sqrt{x}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right.}\right.}{}\right.}{}$$

input
```
int(1/(x^(3/2)*atanh(tanh(a + b*x))^2),x)
```

output
```
(x^(1/2)*(8/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (24*b*x)/(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^2))/(2*b*x^2 - x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (6*2^(1/2)*b^(1/2)*log(-(b^
(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) +
 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*((2*a
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 +
16*a^4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 32*a^3*(2*a - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(5/2)
```

3.205 $\int \frac{1}{x^{5/2}\operatorname{arctanh}(\tanh(a+bx))^2}\,dx$

3.205.1 Optimal result

Integrand size = 15, antiderivative size = 145

$$\int \frac{1}{x^{5/2}\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{5b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{(bx-\operatorname{arctanh}(\tanh(a+bx)))^{7/2}} - \frac{5b}{\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))^3} - \frac{5}{3x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2} - \frac{1}{bx^{5/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))} - \frac{1}{bx^{5/2}\operatorname{arctanh}(\tanh(a+bx))}$$

output
```
5*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-a
rctanh(tanh(b*x+a)))^(7/2)-5/3/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2-1/b/x^
(5/2)/(b*x-arctanh(tanh(b*x+a)))-1/b/x^(5/2)/arctanh(tanh(b*x+a))-5*b/(b*x
-arctanh(tanh(b*x+a)))^3/x^(1/2)
```

3.205.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{5/2}\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{2(-7bx+\operatorname{arctanh}(\tanh(a+bx)))}{3x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^3} + \frac{5b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\operatorname{arctanh}(\tanh(a+bx)))^{7/2}} + \frac{b^2\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))(-bx+\operatorname{arctanh}(\tanh(a+bx)))^3}$$

input
```
Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

output
```
(2*(-7*b*x + ArcTanh[Tanh[a + b*x]]))/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b
*x]])^3) + (5*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[
a + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2) + (b^2*Sqrt[x])/(ArcT
anh[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3)
```

3.205.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2599, 2594, 2594, 2594, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \text{2599}$$

$$-\frac{5\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))}dx}{2b}-\frac{1}{bx^{5/2}\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{5\left(\frac{b\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{2b}-\frac{1}{bx^{5/2}\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{5\left(\frac{b\left(\frac{b\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{2b}-\frac{1}{bx^{5/2}\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{5\left(\frac{b\left(\frac{b\left(\frac{b\int \frac{1}{\sqrt{x}\,\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{2b}-\frac{1}{bx^{5/2}\text{arctanh}(\tanh(a+bx))}$$

$\Big\downarrow$ 2593

$$-\frac{5\left(\frac{b\left(\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{b\left(\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}-\frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}\right)}{bx-\text{arctanh}(\tanh(a+bx))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{2b}-\frac{1}{bx^{5/2}\text{arctanh}(\tanh(a+bx))}$$

input
```
Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

output
```
(-5*(2/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])) + (b*(2/(3*x^(3/2)*(b*x
- ArcTanh[Tanh[a + b*x]])) + (b*((-2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqr
t[b*x - ArcTanh[Tanh[a + b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) + 2
/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))))/(b*x - ArcTanh[Tanh[a + b*x]])
))/(b*x - ArcTanh[Tanh[a + b*x]])))/(2*b) - 1/(b*x^(5/2)*ArcTanh[Tanh[a +
b*x]])
```

3.205. $\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^2}\,dx$

3.205.3.1 Defintions of rubi rules used

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.205.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{2}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^2 x^{\frac{3}{2}}}+\frac{4b}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^3\sqrt{x}}+\frac{2b^2\left(\frac{\sqrt{x}}{2\operatorname{arctanh}(\tanh(bx+a))}+\frac{5\arctan\left(\frac{\ldots}{\sqrt{(\operatorname{ar}\ldots}}\right)}{2\sqrt{(\operatorname{arcta}\ldots}}\right.}{(\operatorname{arctanh}(\tanh(bx+a))\ldots}$
default	$-\frac{2}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^2 x^{\frac{3}{2}}}+\frac{4b}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^3\sqrt{x}}+\frac{2b^2\left(\frac{\sqrt{x}}{2\operatorname{arctanh}(\tanh(bx+a))}+\frac{5\arctan\left(\frac{\ldots}{\sqrt{(\operatorname{ar}\ldots}}\right)}{2\sqrt{(\operatorname{arcta}\ldots}}\right.}{(\operatorname{arctanh}(\tanh(bx+a))\ldots}$
risch	Expression too large to display

input
```
int(1/x^(5/2)/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3/(arctanh(tanh(b*x+a))-b*x)^2/x^(3/2)+4/(arctanh(tanh(b*x+a))-b*x)^3*b
/x^(1/2)+2/(arctanh(tanh(b*x+a))-b*x)^3*b^2*(1/2*x^(1/2)/arctanh(tanh(b*x+
a))+5/2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(ta
nh(b*x+a))-b*x)*b)^(1/2)))
```

3.205.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^2}\,dx = \left[\frac{15\,(b^2x^3+abx^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(15\,b^2x^2+10\,abx-2}{6\,(a^3bx^3+a^4x^2)}\right.$$

$$\left.-\frac{15\,(b^2x^3+abx^2)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(15\,b^2x^2+10\,abx-2\,a^2)\sqrt{x}}{3\,(a^3bx^3+a^4x^2)}\right]$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
[1/6*(15*(b^2*x^3 + a*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a)
- a)/(b*x + a)) + 2*(15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 +
a^4*x^2), -1/3*(15*(b^2*x^3 + a*b*x^2)*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqr
t(x))) - (15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^4*x^2)]
```

3.205.6 Sympy [F]

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^2}\,dx = \int \frac{1}{x^{\frac{5}{2}}\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**(5/2)/atanh(tanh(b*x+a))**2,x)
```

output
```
Integral(1/(x**(5/2)*atanh(tanh(a + b*x))**2), x)
```

3.205.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^2}\,dx = \frac{15\,b^2x^2+10\,abx-2\,a^2}{3\left(a^3bx^{\frac{5}{2}}+a^4x^{\frac{3}{2}}\right)}+\frac{5\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3}$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/3*(15*b^2*x^2 + 10*a*b*x - 2*a^2)/(a^3*b*x^(5/2) + a^4*x^(3/2)) + 5*b^2*
arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

3.205.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^2}\,dx = \frac{5\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3}+\frac{b^2\sqrt{x}}{(bx+a)a^3}+\frac{2\,(6\,bx-a)}{3\,a^3x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
5*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) + b^2*sqrt(x)/((b*x + a)
*a^3) + 2/3*(6*b*x - a)/(a^3*x^(3/2))
```

3.205.9 Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 871, normalized size of antiderivative = 6.01

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(5/2)*atanh(tanh(a + b*x))^2),x)
```

output
```
((32*b)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - (80*b^2*x)/(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^3)/(x^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log
(2/(exp(2*a)*exp(2*b*x) + 1)))) - 8/(3*x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2
) + (20*2^(1/2)*b^(3/2)*log(-(b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(
1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x) + 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^(1/2) + 2*2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 + 60*a^2*(2*a
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)^4 - 160*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3
+ 240*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 64*a^6 - 12*a*(2*a - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*
x) + 1)) + 2*b*x)^5 - 192*a^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2...
```

3.205. $\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^2}\,dx$

3.206 $\int \frac{1}{x^{7/2}\mathbf{arctanh}(\tanh(a+bx))^2}\,dx$

3.206.1 Optimal result

Integrand size = 15, antiderivative size = 172

$$\int \frac{1}{x^{7/2}\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = \frac{7b^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{(bx-\operatorname{arctanh}(\tanh(a+bx)))^{9/2}} - \frac{7b^2}{\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))^4} - \frac{7b}{3x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^3} - \frac{7}{5x^{5/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2} - \frac{1}{bx^{7/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))} - \frac{1}{bx^{7/2}\operatorname{arctanh}(\tanh(a+bx))}$$

output
```
7*b^(5/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-a
rctanh(tanh(b*x+a)))^(9/2)-7/3*b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^3-7/5/
x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^2-1/b/x^(7/2)/(b*x-arctanh(tanh(b*x+a))
)-1/b/x^(7/2)/arctanh(tanh(b*x+a))-7*b^2/(b*x-arctanh(tanh(b*x+a)))^4/x^(1
/2)
```

3.206.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^{7/2}\operatorname{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{7b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\operatorname{arctanh}(\tanh(a+bx)))^{9/2}} - \frac{b^3\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))(-bx+\operatorname{arctanh}(\tanh(a+bx)))^4} - \frac{2(58b^2x^2-16bx\operatorname{arctanh}(\tanh(a+bx))+3\operatorname{arctanh}(\tanh(a+bx))^2)}{15x^{5/2}(-bx+\operatorname{arctanh}(\tanh(a+bx)))^4}$$

input
```
Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

output
```
(-7*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(9/2) - (b^3*Sqrt[x])/(ArcTanh[Tanh[a
 + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4) - (2*(58*b^2*x^2 - 16*b*x*Ar
cTanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2))/(15*x^(5/2)*(-(b*x) +
ArcTanh[Tanh[a + b*x]])^4)
```

3.206.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.33, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2599, 2594, 2594, 2594, 2594, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\operatorname{arctanh}(\tanh(a+bx))^2}\,dx$$

$$\Big\downarrow \text{2599}$$

$$-\frac{7\int \frac{1}{x^{9/2}\operatorname{arctanh}(\tanh(a+bx))}dx}{2b} - \frac{1}{bx^{7/2}\operatorname{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{7\left(\frac{b\int \frac{1}{x^{7/2}\operatorname{arctanh}(\tanh(a+bx))}dx}{bx-\operatorname{arctanh}(\tanh(a+bx))} + \frac{2}{7x^{7/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}\right)}{2b} - \frac{1}{bx^{7/2}\operatorname{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \; 2594$$

$$-\frac{7\left(\frac{b\left(\frac{b\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{2b}-\frac{1}{bx^{7/2}\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \; 2594$$

$$-\frac{7\left(\frac{b\left(\frac{b\left(\frac{b\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{7x^{7/2}(bx-\text{arctanh}(\cdots}\cdots\right)}{2b}\cdots$$

$$\frac{1}{bx^{7/2}\text{arctanh}(\tanh(a+bx))}$$

$$\Big\downarrow \; 2594$$

$$-\frac{7\left(\frac{b\left(\frac{b\left(\frac{b\left(\frac{b\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(\cdots}\cdots\right)}{bx-\text{arctanh}(\tanh(a+bx))}\cdots\right)}{2b}\cdots$$

$$\frac{1}{bx^{7/2}\text{arctanh}(\tanh(a+bx))}$$

3.206. $\displaystyle\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^2}\,dx$

$\downarrow$ 2593

$$-\frac{7\left(\frac{b\left(\frac{b\left(\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{b\left(\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}-\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}\right)}{bx-\text{arctanh}(\tanh(a+bx))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{\cdots}{5x^{5/2}(bx-\cdots}\right)}{bx-\text{arctanh}(\tanh(a+bx))}\cdots}{2b}$$

$$\frac{1}{bx^{7/2}\text{arctanh}(\tanh(a+bx))}$$

input
```
Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

output
```
(-7*(2/(7*x^(7/2)*(b*x - ArcTanh[Tanh[a + b*x]])) + (b*(2/(5*x^(5/2)*(b*x
- ArcTanh[Tanh[a + b*x]])) + (b*(2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]
])) + (b*((-2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a
+ b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) + 2/(Sqrt[x]*(b*x - ArcTan
h[Tanh[a + b*x]]))))/(b*x - ArcTanh[Tanh[a + b*x]])))/(b*x - ArcTanh[Tanh[
a + b*x]])))/(b*x - ArcTanh[Tanh[a + b*x]])))/(2*b) - 1/(b*x^(7/2)*ArcTanh
[Tanh[a + b*x]])
```

3.206. $\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^2}\,dx$

3.206.3.1 Defintions of rubi rules used

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.206.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{2}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)^2 x^{\frac{5}{2}}}-\frac{6b^2}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^4\sqrt{x}}+\frac{4b}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^3 x^{\frac{3}{2}}}-\frac{2b^3}{}$
default	$-\frac{2}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)^2 x^{\frac{5}{2}}}-\frac{6b^2}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^4\sqrt{x}}+\frac{4b}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^3 x^{\frac{3}{2}}}-\frac{2b^3}{}$
risch	Expression too large to display

input
```
int(1/x^(7/2)/arctanh(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/5/(arctanh(tanh(b*x+a))-b*x)^2/x^(5/2)-6/(arctanh(tanh(b*x+a))-b*x)^4*b
^2/x^(1/2)+4/3/(arctanh(tanh(b*x+a))-b*x)^3*b/x^(3/2)-2/(arctanh(tanh(b*x+
a))-b*x)^4*b^3*(1/2*x^(1/2)/arctanh(tanh(b*x+a))+7/2/((arctanh(tanh(b*x+a)
)-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)))
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^2}\,dx = \left[\frac{105\,(b^3x^4+ab^2x^3)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,(105\,b^3x^3+70\,ab^2x}{30\,(a^4bx^4+a^5x^3)}\right.$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
[1/30*(105*(b^3*x^4 + a*b^2*x^3)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b
/a) - a)/(b*x + a)) - 2*(105*b^3*x^3 + 70*a*b^2*x^2 - 14*a^2*b*x + 6*a^3)*
sqrt(x))/(a^4*b*x^4 + a^5*x^3), 1/15*(105*(b^3*x^4 + a*b^2*x^3)*sqrt(b/a)*
arctan(a*sqrt(b/a)/(b*sqrt(x))) - (105*b^3*x^3 + 70*a*b^2*x^2 - 14*a^2*b*x
 + 6*a^3)*sqrt(x))/(a^4*b*x^4 + a^5*x^3)]
```

3.206.6 Sympy [F]

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^2}\,dx = \int \frac{1}{x^{\frac{7}{2}}\,\text{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**(7/2)/atanh(tanh(b*x+a))**2,x)
```

output
```
Integral(1/(x**(7/2)*atanh(tanh(a + b*x))**2), x)
```

3.206.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{105\,b^3x^3+70\,ab^2x^2-14\,a^2bx+6\,a^3}{15\left(a^4bx^{\frac{7}{2}}+a^5x^{\frac{5}{2}}\right)}-\frac{7\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^4}$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
-1/15*(105*b^3*x^3 + 70*a*b^2*x^2 - 14*a^2*b*x + 6*a^3)/(a^4*b*x^(7/2) + a
^5*x^(5/2)) - 7*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4)
```

3.206.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^2}\,dx = -\frac{7\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} - \frac{b^3\sqrt{x}}{(bx+a)a^4} - \frac{2\left(45\,b^2x^2-10\,abx+3\,a^2\right)}{15\,a^4x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
-7*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) - b^3*sqrt(x)/((b*x + a
)*a^4) - 2/15*(45*b^2*x^2 - 10*a*b*x + 3*a^2)/(a^4*x^(5/2))
```

3.206.9 Mupad [B] (verification not implemented)

Time = 4.97 (sec) , antiderivative size = 1051, normalized size of antiderivative = 6.11

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(7/2)*atanh(tanh(a + b*x))^2),x)
```

output
```
((96*b^2)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - (224*b^3*x)/(log(2/(exp(2*a)*exp(2
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b
*x)^4)/(x^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) -
log(2/(exp(2*a)*exp(2*b*x) + 1)))) - (32*b)/(3*x^(3/2)*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
2*b*x)^3) - 8/(5*x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (56*2^(1/2)*b^(5/2
)*log(-(b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*
exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x) + 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b
*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^8 + 112*a^2*(2*a - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) +
2*b*x)^6 - 448*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 + 1120*a^4*(2*a - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^4 - 1792*a^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(...
```

3.206. $\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^2}\,dx$

3.207 $\int \frac{x^{7/2}}{\textbf{arctanh}(\tanh(a+bx))^3}\,dx$

3.207.1 Optimal result

Integrand size = 15, antiderivative size = 135

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{35x^{3/2}}{12b^3} + \frac{35\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))}{4b^4} - \frac{35\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))^{3/2}}{4b^{9/2}} - \frac{x^{7/2}}{2b\operatorname{arctanh}(\tanh(a+bx))^2} - \frac{7x^{5/2}}{4b^2\operatorname{arctanh}(\tanh(a+bx))}$$

output
```
35/12*x^(3/2)/b^3-35/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^
(1/2))*(b*x-arctanh(tanh(b*x+a)))^(3/2)/b^(9/2)-1/2*x^(7/2)/b/arctanh(tanh
(b*x+a))^2-7/4*x^(5/2)/b^2/arctanh(tanh(b*x+a))+35/4*(b*x-arctanh(tanh(b*x
+a)))*x^(1/2)/b^4
```

3.207.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.09

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx =$$

$$-\frac{6b^{7/2}x^{7/2} + 21b^{5/2}x^{5/2}\operatorname{arctanh}(\tanh(a+bx)) - 140b^{3/2}x^{3/2}\operatorname{arctanh}(\tanh(a+bx))^2 + 105\sqrt{b}\sqrt{x}\operatorname{arctanh}(\mathrm{ta}}{12b^{9/2}\mathrm{arcta}}$$

input
```
Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
-1/12*(6*b^(7/2)*x^(7/2) + 21*b^(5/2)*x^(5/2)*ArcTanh[Tanh[a + b*x]] - 140
*b^(3/2)*x^(3/2)*ArcTanh[Tanh[a + b*x]]^2 + 105*Sqrt[b]*Sqrt[x]*ArcTanh[Ta
nh[a + b*x]]^3 - 105*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a
 + b*x]]]]*ArcTanh[Tanh[a + b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2
))/(b^(9/2)*ArcTanh[Tanh[a + b*x]]^2)
```

3.207.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2599, 2599, 2590, 2590, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{\text{arctanh}(\tanh(a+bx))^3}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{7\int \frac{x^{5/2}}{\text{arctanh}(\tanh(a+bx))^2}dx}{4b}-\frac{x^{7/2}}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2599}$$

$$\frac{7\left(\frac{5\int \frac{x^{3/2}}{\text{arctanh}(\tanh(a+bx))}dx}{2b}-\frac{x^{5/2}}{b\text{arctanh}(\tanh(a+bx))}\right)}{4b}-\frac{x^{7/2}}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2590}$$

$$\frac{7\left(\frac{5\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{\sqrt{x}}{\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2x^{3/2}}{3b}\right)}{2b}-\frac{x^{5/2}}{b\text{arctanh}(\tanh(a+bx))}\right)}{4b}-\frac{x^{7/2}}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2590}$$

$$\frac{7\left(\frac{5\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int\frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2\sqrt{x}}{b}\right)}{b}+\frac{2x^{3/2}}{3b}\right)}{2b}-\frac{x^{5/2}}{b\text{arctanh}(\tanh(a+bx))}\right)}{4b}-\frac{x^{7/2}}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 2593

$$\frac{7\left(\frac{5\left(\frac{\left(\frac{2\sqrt{x}}{b}-\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}{b^{3/2}}\right)(bx-\text{arctanh}(\tanh(a+bx)))}{b}+\frac{2x^{3/2}}{3b}\right)}{2b}-\frac{x^{5/2}}{b\text{arctan}(\tan}\right)}{4b}-\frac{x^{7/2}}{2b\text{arctanh}(\tanh(a+bx))^2}$$

input
```
Int[x^(7/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(7*((5*((2*x^(3/2))/(3*b) + (((2*Sqrt[x])/b - (2*ArcTanh[(Sqrt[b]*Sqrt[x])
/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/b
^(3/2))*(b*x - ArcTanh[Tanh[a + b*x]]))/b))/(2*b) - x^(5/2)/(b*ArcTanh[Tan
h[a + b*x]])))/(4*b) - x^(7/2)/(2*b*ArcTanh[Tanh[a + b*x]]^2)
```

3.207. $\int \frac{x^{7/2}}{\text{arctanh}(\tanh(a+bx))^3}\,dx$

3.207.3.1 Defintions of rubi rules used

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.207.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(107) = 214$.

Time = 0.59 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.89

method	result
derivativedivides	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+3a\sqrt{x}+3(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}\right)}{b^{4}}+\frac{2\left(\left(-\frac{13b\,a^{2}}{8}-\frac{13ab(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{4}-13b(\mathrm{arcta}\ldots\right.\right.}{\ldots}$
default	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+3a\sqrt{x}+3(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\sqrt{x}\right)}{b^{4}}+\frac{2\left(\left(-\frac{13b\,a^{2}}{8}-\frac{13ab(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{4}-13b(\mathrm{arcta}\ldots\right.\right.}{\ldots}$
risch	Expression too large to display

input
```
int(x^(7/2)/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/b^4*(-1/3*b*x^(3/2)+3*a*x^(1/2)+3*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2))
+2/b^4*(((-13/8*b*a^2-13/4*a*b*(arctanh(tanh(b*x+a))-b*x-a)-13/8*b*(arctan
h(tanh(b*x+a))-b*x-a)^2)*x^(3/2)+(-11/8*a^3-33/8*a^2*(arctanh(tanh(b*x+a))
-b*x-a)-33/8*a*(arctanh(tanh(b*x+a))-b*x-a)^2-11/8*(arctanh(tanh(b*x+a))-b
*x-a)^3)*x^(1/2))/arctanh(tanh(b*x+a))^2+35/8*(a^2+2*a*(arctanh(tanh(b*x+a
))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/((arctanh(tanh(b*x+a))-b*x)*b)^(
1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)))
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.68

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \left[\frac{105\,(ab^2x^2+2\,a^2bx+a^3)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(8\,b^3x^3-56\,ab^2}{24\,(b^6x^2+2\,ab^5x+a^2b^4)}\right.$$

input
```
integrate(x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
[1/24*(105*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)
*sqrt(-a/b) - a)/(b*x + a)) + 2*(8*b^3*x^3 - 56*a*b^2*x^2 - 175*a^2*b*x -
105*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/12*(105*(a*b^2*x^2 +
2*a^2*b*x + a^3)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (8*b^3*x^3 - 56
*a*b^2*x^2 - 175*a^2*b*x - 105*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^
4)]
```

3.207.6 Sympy [F]

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \int \frac{x^{\frac{7}{2}}}{\operatorname{atanh}^3(\tanh(a+bx))}\,dx$$

input
```
integrate(x**(7/2)/atanh(tanh(b*x+a))**3,x)
```

output
```
Integral(x**(7/2)/atanh(tanh(a + b*x))**3, x)
```

3.207.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.64

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{8\,b^3x^{\frac{7}{2}} - 56\,ab^2x^{\frac{5}{2}} - 175\,a^2bx^{\frac{3}{2}} - 105\,a^3\sqrt{x}}{12\,(b^6x^2 + 2\,ab^5x + a^2b^4)} + \frac{35\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^4}$$

input
```
integrate(x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
1/12*(8*b^3*x^(7/2) - 56*a*b^2*x^(5/2) - 175*a^2*b*x^(3/2) - 105*a^3*sqrt(
x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) + 35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))
/(sqrt(a*b)*b^4)
```

3.207.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.57

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{35\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^4} - \frac{13\,a^2bx^{\frac{3}{2}} + 11\,a^3\sqrt{x}}{4\,(bx+a)^2b^4} + \frac{2\,\left(b^6x^{\frac{3}{2}} - 9\,ab^5\sqrt{x}\right)}{3\,b^9}$$

input
```
integrate(x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/4*(13*a^2*b*x^(3/
2) + 11*a^3*sqrt(x))/((b*x + a)^2*b^4) + 2/3*(b^6*x^(3/2) - 9*a*b^5*sqrt(x
))/b^9
```

3.207.9 Mupad [B] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 571, normalized size of antiderivative = 4.23

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{2\,x^{3/2}}{3\,b^3} + \frac{3\,\sqrt{x}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b^4}$$
$$+\frac{35\,\sqrt{2}\,\ln\left(\frac{256\,b^{19/2}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}\right)\,(}{32\,b^{9/2}}$$
$$-\frac{13\,\sqrt{x}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{8\,b^4\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}$$
$$-\frac{\sqrt{x}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{4\,b^4\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)^2}$$

input
```
int(x^(7/2)/atanh(tanh(a + b*x))^3,x)
```

output
```
(2*x^(3/2))/(3*b^3) + (3*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^4 + (35*2^(1
/2)*log((256*b^(19/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/
2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2))/(32*b^(9/
2)) - (13*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(8*b^4*(log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))) -
 (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(4*b^4*(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2)
```

3.208 $\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^3} \, dx$

3.208.1 Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^3} \, dx = \frac{15\sqrt{x}}{4b^3} - \frac{15\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}{4b^{7/2}} - \frac{x^{5/2}}{2b\operatorname{arctanh}(\tanh(a+bx))^2} - \frac{5x^{3/2}}{4b^2\operatorname{arctanh}(\tanh(a+bx))}$$

output
```
-1/2*x^(5/2)/b/arctanh(tanh(b*x+a))^2-5/4*x^(3/2)/b^2/arctanh(tanh(b*x+a))
+15/4*x^(1/2)/b^3-15/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^
(1/2))*(b*x-arctanh(tanh(b*x+a)))^(1/2)/b^(7/2)
```

3.208.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{1}{4}\left(\frac{15\sqrt{x}}{b^3} - \frac{2x^{5/2}}{b\operatorname{arctanh}(\tanh(a+bx))^2} - \frac{5x^{3/2}}{b^2\operatorname{arctanh}(\tanh(a+bx))} - \frac{15\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}{b^{7/2}}\right)$$

input
```
Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
((15*Sqrt[x])/b^3 - (2*x^(5/2))/(b*ArcTanh[Tanh[a + b*x]]^2) - (5*x^(3/2))
/(b^2*ArcTanh[Tanh[a + b*x]]) - (15*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) +
 ArcTanh[Tanh[a + b*x]]]]*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])/b^(7/2))/
4
```

3.208.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2599, 2599, 2590, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{5\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^2}dx}{4b} - \frac{x^{5/2}}{2b\operatorname{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2599}$$

$$\frac{5\left(\frac{3\int \frac{\sqrt{x}}{\text{arctanh}(\tanh(a+bx))}dx}{2b}-\frac{x^{3/2}}{b\text{arctanh}(\tanh(a+bx))}\right)}{4b}-\frac{x^{5/2}}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \textbf{2590}$$

$$\frac{5\left(\frac{3\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}dx}{b}+\frac{2\sqrt{x}}{b}\right)}{2b}-\frac{x^{3/2}}{b\text{arctanh}(\tanh(a+bx))}\right)}{4b}-\frac{x^{5/2}}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \textbf{2593}$$

$$\frac{5\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}{b^{3/2}}\right)}{2b}-\frac{x^{3/2}}{b\text{arctanh}(\tanh(a+bx))}\right)}{4b}-\frac{x^{5/2}}{2b\text{arctanh}(\tanh(a+bx))^2}$$

input
```
Int[x^(5/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(5*((3*((2*Sqrt[x])/b - (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Ta
nh[a + b*x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/b^(3/2)))/(2*b) - x^(3/
2)/(b*ArcTanh[Tanh[a + b*x]])))/(4*b) - x^(5/2)/(2*b*ArcTanh[Tanh[a + b*x]
]^2)
```

3.208.3.1 Defintions of rubi rules used

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

3.208. $\int \frac{x^{5/2}}{\text{arctanh}(\tanh(a+bx))^3}\,dx$

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.208.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{2\sqrt{x}}{b^3} - \frac{2\left(\frac{\left(-\frac{9ab}{8}-\frac{9b(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{8}\right)x^{\frac{3}{2}}+\left(-\frac{7a^2}{8}-\frac{7a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{4}-\frac{7(\operatorname{arctanh}(\tanh(bx+a))-\cdots}{8}\cdots\right.}{\operatorname{arctanh}(\tanh(bx+a))^2}\cdots\right.}{b^3}$
default	$\frac{2\sqrt{x}}{b^3} - \frac{2\left(\frac{\left(-\frac{9ab}{8}-\frac{9b(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{8}\right)x^{\frac{3}{2}}+\left(-\frac{7a^2}{8}-\frac{7a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{4}-\frac{7(\operatorname{arctanh}(\tanh(bx+a))-\cdots}{8}\cdots\right.}{\operatorname{arctanh}(\tanh(bx+a))^2}\cdots\right.}{b^3}$
risch	Expression too large to display

input
```
int(x^(5/2)/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/b^3-2/b^3*(((-9/8*a*b-9/8*b*(arctanh(tanh(b*x+a))-b*x-a))*x^(3/2
)+(-7/8*a^2-7/4*a*(arctanh(tanh(b*x+a))-b*x-a)-7/8*(arctanh(tanh(b*x+a))-b
*x-a)^2)*x^(1/2))/arctanh(tanh(b*x+a))^2+15/8*(arctanh(tanh(b*x+a))-b*x)/(
(arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a)
)-b*x)*b)^(1/2)))
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.82

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \left[\frac{15\,(b^2x^2+2\,abx+a^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(8\,b^2x^2+25\,abx+\right.}{8\,(b^5x^2+2\,ab^4x+a^2b^3)}$$

$$\left.-\frac{15\,(b^2x^2+2\,abx+a^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(8\,b^2x^2+25\,abx+15\,a^2)\sqrt{x}}{4\,(b^5x^2+2\,ab^4x+a^2b^3)}\right]$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
[1/8*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(
-a/b) - a)/(b*x + a)) + 2*(8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^
2 + 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a/b)*arc
tan(b*sqrt(x)*sqrt(a/b)/a) - (8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5
*x^2 + 2*a*b^4*x + a^2*b^3)]
```

3.208.6 Sympy [F]

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \int \frac{x^{\frac{5}{2}}}{\operatorname{atanh}^3(\tanh(a+bx))}\,dx$$

input
```
integrate(x**(5/2)/atanh(tanh(b*x+a))**3,x)
```

output
```
Integral(x**(5/2)/atanh(tanh(a + b*x))**3, x)
```

3.208.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.66

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{8\,b^2 x^{\frac{5}{2}} + 25\,abx^{\frac{3}{2}} + 15\,a^2\sqrt{x}}{4\,(b^5x^2 + 2\,ab^4x + a^2b^3)} - \frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^3}$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
1/4*(8*b^2*x^(5/2) + 25*a*b*x^(3/2) + 15*a^2*sqrt(x))/(b^5*x^2 + 2*a*b^4*x
 + a^2*b^3) - 15/4*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3)
```

3.208.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.54

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^3} + \frac{2\,\sqrt{x}}{b^3} + \frac{9\,abx^{\frac{3}{2}} + 7\,a^2\sqrt{x}}{4\,(bx+a)^2b^3}$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
-15/4*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2*sqrt(x)/b^3 + 1/4*
(9*a*b*x^(3/2) + 7*a^2*sqrt(x))/((b*x + a)^2*b^3)
```

3.208.9 Mupad [B] (verification not implemented)

Time = 4.68 (sec) , antiderivative size = 511, normalized size of antiderivative = 4.65

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{2\,\sqrt{x}}{b^3} - \frac{9\,\sqrt{x}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)}{4\,b^3\,\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}$$
$$+ \frac{15\,\sqrt{2}\,\ln\left(\frac{64\,b^{15/2}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}+2\,\sqrt{2}\,b\,x\right)}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}\right)\sqrt{}}{16\,b^{7/2}}$$
$$- \frac{\sqrt{x}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2}{2\,b^3\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)^2}$$

input
```
int(x^(5/2)/atanh(tanh(a + b*x))^3,x)
```

output
```
(2*x^(1/2))/b^3 - (9*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(4*b^3*(log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x)
+ 1)))) + (15*2^(1/2)*log((64*b^(15/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
- 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp
(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^(1/2))/(16*b^(7/2)) - (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*b^3*(log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*
b*x) + 1)))^2)
```

3.208. $\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$

3.209 $\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$

3.209.1 Optimal result

Integrand size = 15, antiderivative size = 98

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{3\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{4b^{5/2}\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}} - \frac{x^{3/2}}{2b\operatorname{arctanh}(\tanh(a+bx))^2} - \frac{3\sqrt{x}}{4b^2\operatorname{arctanh}(\tanh(a+bx))}$$

output
```
-1/2*x^(3/2)/b/arctanh(tanh(b*x+a))^2-3/4*x^(1/2)/b^2/arctanh(tanh(b*x+a))
-3/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/b^(5/2)/(b*
x-arctanh(tanh(b*x+a)))^(1/2)
```

3.209.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{x^{3/2}}{2b\operatorname{arctanh}(\tanh(a+bx))^2} - \frac{3\sqrt{x}}{4b^2\operatorname{arctanh}(\tanh(a+bx))} + \frac{3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{4b^{5/2}\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}$$

input
```
Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*x^(3/2)/(b*ArcTanh[Tanh[a + b*x]]^2) - (3*Sqrt[x])/(4*b^2*ArcTanh[Tan
h[a + b*x]]) + (3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a +
b*x]]]])/(4*b^(5/2)*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])
```

3.209.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2599, 2599, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{3\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^2}dx}{4b}-\frac{x^{3/2}}{2b\operatorname{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2599}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))}dx}{2b}-\frac{\sqrt{x}}{b\operatorname{arctanh}(\tanh(a+bx))}\right)}{4b}-\frac{x^{3/2}}{2b\operatorname{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2593}$$

$$\frac{3\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{b^{3/2}\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}-\frac{\sqrt{x}}{b\operatorname{arctanh}(\tanh(a+bx))}\right)}{4b}-\frac{x^{3/2}}{2b\operatorname{arctanh}(\tanh(a+bx))^2}$$

input
```
Int[x^(3/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(3*(-(ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]/(b^(3/
2)*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])) - Sqrt[x]/(b*ArcTanh[Tanh[a + b*x]
])))/(4*b) - x^(3/2)/(2*b*ArcTanh[Tanh[a + b*x]]^2)
```

3.209.3.1 Defintions of rubi rules used

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.209.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{-\frac{5x^{\frac{3}{2}}}{4b}-\frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}{4b^2}}{\operatorname{arctanh}(\tanh(bx+a))^2}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{4b^2\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}$	85
default	$\frac{-\frac{5x^{\frac{3}{2}}}{4b}-\frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}{4b^2}}{\operatorname{arctanh}(\tanh(bx+a))^2}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{4b^2\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}$	85
risch	Expression too large to display	1075

input
```
int(x^(3/2)/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-5/8*x^(3/2)/b-3/8*(arctanh(tanh(b*x+a))-b*x)/b^2*x^(1/2))/arctanh(tanh
(b*x+a))^2+3/4/b^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/(
(arctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.89

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \left[-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{-ab}\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)+2\,(5\,ab^2x+3\,a^2b)\sqrt{x}}{8\,(ab^5x^2+2\,a^2b^4x+a^3b^3)}, -\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+(5\,ab^2x+3\,a^2b)\sqrt{x}}{4\,(ab^5x^2+2\,a^2b^4x+a^3b^3)}\right]$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
[-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*
sqrt(x))/(b*x + a)) + 2*(5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*
b^4*x + a^3*b^3), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(
a*b)/(b*sqrt(x))) + (5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^4*
x + a^3*b^3)]
```

3.209.6 Sympy [F]

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \int \frac{x^{\frac{3}{2}}}{\operatorname{atanh}^3(\tanh(a+bx))}\,dx$$

input
```
integrate(x**(3/2)/atanh(tanh(b*x+a))**3,x)
```

output
```
Integral(x**(3/2)/atanh(tanh(a + b*x))**3, x)
```

3.209.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.62

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{5\,bx^{\frac{3}{2}}+3\,a\sqrt{x}}{4\,(b^4x^2+2\,ab^3x+a^2b^2)}+\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^2}$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-1/4*(5*b*x^(3/2) + 3*a*sqrt(x))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 3/4*arc
tan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2)
```

3.209.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.48

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}b^2} - \frac{5\,bx^{\frac{3}{2}} + 3\,a\sqrt{x}}{4\,(bx+a)^2b^2}$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/4*(5*b*x^(3/2) + 3*a*s
qrt(x))/((b*x + a)^2*b^2)
```

3.209.9 Mupad [B] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 667, normalized size of antiderivative = 6.81

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{3\sqrt{2}\ln\left(\frac{16\,b^{11/2}\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right.\right.\right.\cdots}{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\cdots\right.}\cdots\right.}{8\,b^{5/2}\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\cdots}}$$

$$-\frac{\sqrt{x}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}{b^2\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)^2}$$

$$-\frac{\sqrt{x}\left(\frac{1}{b\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}+\frac{8\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-8\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+16\,b\,x}{2\,b\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^2}\right)\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right.\right.\cdots}{2\,b\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)}$$

input
```
int(x^(3/2)/atanh(tanh(a + b*x))^3,x)
```

output
```
(3*2^(1/2)*log((16*b^(11/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) +
 2*2^(1/2)*b*x))/(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) -
 log(2/(exp(2*a)*exp(2*b*x) + 1)))))/(8*b^(5/2)*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^
(1/2)) - (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(b^2*(log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2) - (
x^(1/2)*(1/(b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (8*log(2/(exp(2*a)*exp(2*b*x) +
 1)) - 8*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 16*b*x)/
(2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^2))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b*(log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b
*x) + 1))))
```

3.209. $\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^3} \, dx$

3.210 $\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^3} \, dx$

3.210.1 Optimal result

Integrand size = 15, antiderivative size = 125

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^3} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{4b^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^{3/2}} - \frac{1}{4b^2\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))} - \frac{\sqrt{x}}{2b\operatorname{arctanh}(\tanh(a+bx))^2} - \frac{1}{4b^2\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))}$$

output
```
1/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/b^(3/2)/(b*x
-arctanh(tanh(b*x+a)))^(3/2)-1/4/b^2/(b*x-arctanh(tanh(b*x+a)))/x^(1/2)-1/
4/b^2/arctanh(tanh(b*x+a))/x^(1/2)-1/2*x^(1/2)/b/arctanh(tanh(b*x+a))^2
```

3.210.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{1}{4}\left(-\frac{2\sqrt{x}}{b\operatorname{arctanh}(\tanh(a+bx))^2} + \frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{b^{3/2}(-bx+\operatorname{arctanh}(\tanh(a+bx)))^{3/2}} + \frac{\sqrt{x}}{-b^2x\operatorname{arctanh}(\tanh(a+bx))+b\operatorname{arctanh}(\tanh(a+bx))^2}\right)$$

input
```
Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
((-2*Sqrt[x])/(b*ArcTanh[Tanh[a + b*x]]^2) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt
[-(b*x) + ArcTanh[Tanh[a + b*x]]]]/(b^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x
]])^(3/2)) + Sqrt[x]/(-(b^2*x*ArcTanh[Tanh[a + b*x]]) + b*ArcTanh[Tanh[a +
 b*x]]^2))/4
```

3.210.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2599, 2599, 2594, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{\int \frac{1}{\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^2}dx}{4b} - \frac{\sqrt{x}}{2b\operatorname{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{ 2599}$$

$$-\frac{-\frac{\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}dx}{2b}-\frac{1}{b\sqrt{x}\text{arctanh}(\tanh(a+bx))}}{4b}-\frac{\sqrt{x}}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{-\frac{\frac{b\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}}{2b}-\frac{1}{b\sqrt{x}\text{arctanh}(\tanh(a+bx))}}{4b}-\frac{\sqrt{x}}{2b\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2593}$$

$$-\frac{-\frac{\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}-\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}}{2b}-\frac{1}{b\sqrt{x}\text{arctanh}(\tanh(a+bx))}}{4b}-\frac{\sqrt{x}}{2b\text{arctanh}(\tanh(a+bx))^2}$$

input
```
Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^3,x]
```

output
```
(-1/2*((-2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b
*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) + 2/(Sqrt[x]*(b*x - ArcTanh[T
anh[a + b*x]])))/b - 1/(b*Sqrt[x]*ArcTanh[Tanh[a + b*x]]))/(4*b) - Sqrt[x]
/(2*b*ArcTanh[Tanh[a + b*x]]^2)
```

3.210.3.1 Defintions of rubi rules used

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.210.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{2x^{\frac{3}{2}}}{8\operatorname{arctanh}(\tanh(bx+a))-8bx}-\frac{\sqrt{x}}{4b}}{\operatorname{arctanh}(\tanh(bx+a))^{2}}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{4(\operatorname{arctanh}(\tanh(bx+a))-bx)b\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}$	98
default	$\frac{\frac{2x^{\frac{3}{2}}}{8\operatorname{arctanh}(\tanh(bx+a))-8bx}-\frac{\sqrt{x}}{4b}}{\operatorname{arctanh}(\tanh(bx+a))^{2}}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{4(\operatorname{arctanh}(\tanh(bx+a))-bx)b\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}$	98
risch	Expression too large to display	1595

input
```
int(x^(1/2)/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/8/(arctanh(tanh(b*x+a))-b*x)*x^(3/2)-1/8*x^(1/2)/b)/arctanh(tanh(b*x+
a))^2+1/4/(arctanh(tanh(b*x+a))-b*x)/b/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2
)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \left[-\frac{(b^2x^2+2\,abx+a^2)\sqrt{-ab}\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)-2\,(ab^2x-a^2b)\sqrt{x}}{8\,(a^2b^4x^2+2\,a^3b^3x+a^4b^2)}, -\frac{(b^2x^2+2\,abx+a^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-(ab^2x-a^2b)\sqrt{x}}{4\,(a^2b^4x^2+2\,a^3b^3x+a^4b^2)}\right]$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
[-1/8*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sq
rt(x))/(b*x + a)) - 2*(a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*
x + a^4*b^2), -1/4*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)/(
b*sqrt(x))) - (a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x + a^4*
b^2)]
```

3.210.6 Sympy [F]

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \int \frac{\sqrt{x}}{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**(1/2)/atanh(tanh(b*x+a))**3,x)
```

output
```
Integral(sqrt(x)/atanh(tanh(a + b*x))**3, x)
```

3.210.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{bx^{\frac{3}{2}} - a\sqrt{x}}{4\left(ab^3x^2 + 2\,a^2b^2x + a^3b\right)} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}ab}$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
1/4*(b*x^(3/2) - a*sqrt(x))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/4*arctan
(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b)
```

3.210.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}ab} + \frac{bx^{\frac{3}{2}} - a\sqrt{x}}{4\,(bx+a)^2ab}$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
1/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/4*(b*x^(3/2) - a*sqrt(
x))/((b*x + a)^2*a*b)
```

3.210.9 Mupad [B] (verification not implemented)

Time = 4.68 (sec) , antiderivative size = 580, normalized size of antiderivative = 4.64

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$$

$$= \frac{\sqrt{2}\,\ln\left(-\frac{4\sqrt{b}\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)+4\sqrt{b}\sqrt{x}\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\cdots}}{\ln\left(\cdots\right.}\right.}{4\,b^{3/2}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\cdots\right.\right.}$$

$$-\frac{2\sqrt{x}}{b\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)^2}$$

$$-\frac{\sqrt{x}}{b\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}$$

input
```
int(x^(1/2)/atanh(tanh(a + b*x))^3,x)
```

output
```
(2^(1/2)*log(-(4*b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + 2*b*x) + 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*
2^(1/2)*b*x)*(b^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 4*a^2*b^3 - 4*a*b^3*
(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))))/(4*b^(3/2)*(log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x)^(3/2)) - (2*x^(1/2))/(b*(log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2) - x^(1/2)/(b
*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)
*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))
```

3.210. $\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$

3.211 $\int \frac{1}{\sqrt{x}\mathbf{arctanh}(\tanh(a+bx))^3}\,dx$

3.211.1 Optimal result

Integrand size = 15, antiderivative size = 152

$$\begin{aligned}\int \frac{1}{\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = &-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{4\sqrt{b}(bx-\operatorname{arctanh}(\tanh(a+bx)))^{5/2}}\\&+\frac{3}{4b\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2}\\&+\frac{1}{4b^2x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}\\&-\frac{1}{2b\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^2}\\&+\frac{1}{4b^2x^{3/2}\operatorname{arctanh}(\tanh(a+bx))}\end{aligned}$$

output
```
1/4/b^2/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))+1/4/b^2/x^(3/2)/arctanh(tanh(b*
x+a))-3/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-a
rctanh(tanh(b*x+a)))^(5/2)/b^(1/2)+3/4/b/(b*x-arctanh(tanh(b*x+a)))^2/x^(1
/2)-1/2/b/arctanh(tanh(b*x+a))^2/x^(1/2)
```

3.211.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \frac{3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\operatorname{arctanh}(\tanh(a+bx))}}\right)}{4\sqrt{b}(-bx+\operatorname{arctanh}(\tanh(a+bx)))^{5/2}} + \frac{3\sqrt{x}}{4\operatorname{arctanh}(\tanh(a+bx))(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2} + \frac{\sqrt{x}}{2\operatorname{arctanh}(\tanh(a+bx))^2(-bx+\operatorname{arctanh}(\tanh(a+bx)))}$$

input
```
Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3),x]
```

output
```
(3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])/(4*Sqr
t[b]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2)) + (3*Sqrt[x])/(4*ArcTanh[Tan
h[a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2) + Sqrt[x]/(2*ArcTanh[Tanh
[a + b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]]))
```

3.211.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2599, 2599, 2594, 2594, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$$

$$\Big\downarrow \text{2599}$$

$$-\frac{\int \frac{1}{x^{3/2}\operatorname{arctanh}(\tanh(a+bx))^2}dx}{4b} - \frac{1}{2b\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2599}$$

$$-\frac{-\frac{3\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}dx}{2b}-\frac{1}{bx^{3/2}\text{arctanh}(\tanh(a+bx))}}{4b}-\frac{1}{2b\sqrt{x}\text{arctanh}(\tanh(a+bx))^2}$$

$\Big\downarrow$ 2594

$$-\frac{-\frac{3\left(\frac{b\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{2b}-\frac{1}{bx^{3/2}\text{arctanh}(\tanh(a+bx))}}{4b}-\frac{1}{2b\sqrt{x}\text{arctanh}(\tanh(a+bx))^2}$$

$\Big\downarrow$ 2594

$$-\frac{-\frac{3\left(\frac{b\left(\frac{b\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{2b}-\frac{1}{bx^{3/2}\text{arctanh}(\tanh(a+bx))}}{4b}-\frac{1}{2b\sqrt{x}\text{arctanh}(\tanh(a+bx))^2}$$

$\Big\downarrow$ 2593

$$-\frac{-\frac{3\left(\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{b\left(\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}-\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{(bx-\text{arctanh}(\tanh(a+bx)))^{3/2}}\right)}{bx-\text{arctanh}(\tanh(a+bx))}\right)}{2b}-\frac{1}{bx^{3/2}\text{arctanh}(\tanh(a+bx))}}{4b}-\frac{1}{2b\sqrt{x}\text{arctanh}(\tanh(a+bx))^2}$$

input
```
Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3),x]
```

output
```
-1/4*((-3*(2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])) + (b*((-2*Sqrt[b]*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(b*x - ArcT
anh[Tanh[a + b*x]])^(3/2) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))))/(
b*x - ArcTanh[Tanh[a + b*x]])))/(2*b) - 1/(b*x^(3/2)*ArcTanh[Tanh[a + b*x]
]))/b - 1/(2*b*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2)
```

3.211. $\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^3}\,dx$

3.211.3.1 Defintions of rubi rules used

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.211.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{\sqrt{x}}{2(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^2}+\frac{3\sqrt{x}}{4(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\operatorname{arctanh}(\tanh(bx+a))}+\frac{}{4(\operatorname{arct}}$
default	$\frac{\sqrt{x}}{2(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^2}+\frac{3\sqrt{x}}{4(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\operatorname{arctanh}(\tanh(bx+a))}+\frac{}{4(\operatorname{arct}}$
risch	Expression too large to display

input
```
int(1/arctanh(tanh(b*x+a))^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^2+3/4/(arctanh
(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))+3/4/(arctanh(tanh(b*x+a)
)-b*x)^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(t
anh(b*x+a))-b*x)*b)^(1/2))
```

3.211.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^3}\,dx = \left[-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{-ab}\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)-2\,(3\,ab^2x+5\,a^2b)\sqrt{x}}{8\,(a^3b^3x^2+2\,a^4b^2x+a^5b)}, -\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-(3\,ab^2x+5\,a^2b)\sqrt{x}}{4\,(a^3b^3x^2+2\,a^4b^2x+a^5b)}\right]$$

input
```
integrate(1/arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*
sqrt(x))/(b*x + a)) - 2*(3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^
4*b^2*x + a^5*b), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(
a*b)/(b*sqrt(x))) - (3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*b^
2*x + a^5*b)]
```

3.211.6 Sympy [F]

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^3}\,dx = \int \frac{1}{\sqrt{x}\,\text{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/atanh(tanh(b*x+a))**3/x**(1/2),x)
```

output
```
Integral(1/(sqrt(x)*atanh(tanh(a + b*x))**3), x)
```

3.211.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.39

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^3}\,dx = \frac{3\,bx^{\frac{3}{2}}+5\,a\sqrt{x}}{4\,(a^2b^2x^2+2\,a^3bx+a^4)}+\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^2}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="maxima")
```

output
```
1/4*(3*b*x^(3/2) + 5*a*sqrt(x))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 3/4*arct
an(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2)
```

3.211.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.31

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^3}\,dx = \frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^2}+\frac{3\,bx^{\frac{3}{2}}+5\,a\sqrt{x}}{4\,(bx+a)^2a^2}$$

input
```
integrate(1/arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="giac")
```

output
```
3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/4*(3*b*x^(3/2) + 5*a*s
qrt(x))/((b*x + a)^2*a^2)
```

3.211.9 Mupad [B] (verification not implemented)

Time = 4.63 (sec) , antiderivative size = 741, normalized size of antiderivative = 4.88

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^3}\,dx$$

$$= \frac{6\sqrt{x}}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}$$

$$- \frac{4\sqrt{x}}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)^2\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}$$

$$+ \frac{3\sqrt{2}\,\ln\left(\frac{\sqrt{b}\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\sqrt{b}\sqrt{x}\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
int(1/(x^(1/2)*atanh(tanh(a + b*x))^3),x)
```

output
```
(6*x^(1/2))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log
(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) - (4*x^(1/2))/
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)
*exp(2*b*x) + 1)))^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (3*2^(1/2)*log((b^(1/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(
1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*(16*a^4*b +
b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 - 8*a*b*(2*a - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)^3 - 32*a^3*b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 24*a^2*b*(2*a - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^2))/(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) - log(2/(exp(2*a)*exp(2*b*x) + 1)))))/(2*b^(1/2)*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2...
```

3.211. $\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^3}\,dx$

3.212 $\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^3}\,dx$

3.212.1 Optimal result

Integrand size = 15, antiderivative size = 176

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{15\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{4(bx-\text{arctanh}(\tanh(a+bx)))^{7/2}} + \frac{15}{4\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))^3} + \frac{5}{4bx^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))^2} + \frac{3}{4b^2x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))} - \frac{1}{2bx^{3/2}\text{arctanh}(\tanh(a+bx))^2} + \frac{3}{4b^2x^{5/2}\text{arctanh}(\tanh(a+bx))}$$

output
```
5/4/b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2+3/4/b^2/x^(5/2)/(b*x-arctanh(ta
nh(b*x+a)))-1/2/b/x^(3/2)/arctanh(tanh(b*x+a))^2+3/4/b^2/x^(5/2)/arctanh(t
anh(b*x+a))-15/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))
*b^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(7/2)+15/4/(b*x-arctanh(tanh(b*x+a)))^
3/x^(1/2)
```

3.212.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)}{4(-bx+\text{arctanh}(\tanh(a+bx)))^{7/2}} - \frac{2}{\sqrt{x}(-bx+\text{arctanh}(\tanh(a+bx)))^3} - \frac{7b\sqrt{x}}{4\text{arctanh}(\tanh(a+bx))(-bx+\text{arctanh}(\tanh(a+bx)))^3} - \frac{b\sqrt{x}}{2\text{arctanh}(\tanh(a+bx))^2(-bx+\text{arctanh}(\tanh(a+bx)))^2}$$

input
```
Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

output
```
(-15*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]
]])/(4*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2)) - 2/(Sqrt[x]*(-(b*x) + Arc
Tanh[Tanh[a + b*x]])^3) - (7*b*Sqrt[x])/(4*ArcTanh[Tanh[a + b*x]]*(-(b*x)
+ ArcTanh[Tanh[a + b*x]])^3) - (b*Sqrt[x])/(2*ArcTanh[Tanh[a + b*x]]^2*(-(
b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

3.212.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2599, 2599, 2594, 2594, 2594, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^3}\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$-\frac{3\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^2}dx}{4b} - \frac{1}{2bx^{3/2}\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{ 2599}$$

$$-\frac{3\left(-\frac{5\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))}dx}{2b}-\frac{1}{bx^{5/2}\text{arctanh}(\tanh(a+bx))}\right)}{4b}-\frac{1}{2bx^{3/2}\text{arctanh}(\tanh(a+bx))^2}$$

$\Big\downarrow$ 2594

$$-\frac{3\left(-\frac{5\left(\frac{b\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{2b}-\frac{1}{bx^{5/2}\text{arctanh}(\tanh(a+bx))}\right)}{4b}-\frac{1}{2bx^{3/2}\text{arctanh}(\tanh(a+bx))^2}$$

$\Big\downarrow$ 2594

$$-\frac{3\left(-\frac{5\left(\frac{b\left(\frac{b\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{2b}-\frac{1}{bx^{5/2}\text{arctanh}(\tan}\right)}{4b}-\frac{1}{2bx^{3/2}\text{arctanh}(\tanh(a+bx))^2}$$

$\Big\downarrow$ 2594

3.212. $\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^3}\,dx$

$$-\frac{3\left(-\frac{5\left(\frac{b\left(\frac{b\left(\frac{b\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tan}\right)}{2b}}{4b}$$

$$\frac{1}{2bx^{3/2}\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 2593$$

3.212. $\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^3}\,dx$

$$-\frac{3\left(-\frac{5\left(\frac{b\left(\frac{2}{3x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{b\left(\frac{2}{\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))}-\frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{(bx-\operatorname{arctanh}(\tanh(a+bx)))^{3/2}}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{}{5x^{5/2}(bx}\right.}{2b}}{4b}$$

$$\frac{1}{2bx^{3/2}\operatorname{arctanh}(\tanh(a+bx))^2}$$

input
```
Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

output
```
(-3*((-5*(2/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])) + (b*(2/(3*x^(3/2)*
(b*x - ArcTanh[Tanh[a + b*x]])) + (b*((-2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x]
)/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2
) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))))/(b*x - ArcTanh[Tanh[a + b
*x]])))/(b*x - ArcTanh[Tanh[a + b*x]])))/(2*b) - 1/(b*x^(5/2)*ArcTanh[Tanh
[a + b*x]])))/(4*b) - 1/(2*b*x^(3/2)*ArcTanh[Tanh[a + b*x]]^2)
```

3.212. $\int \frac{1}{x^{3/2}\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$

3.212.3.1 Defintions of rubi rules used

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.212.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.65

method	result
derivativedivides	$-\frac{2}{\left(\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)-b x \right)^{3} \sqrt{x}}-\frac{2 b \left(\frac{\frac{7 b \,x^{\frac{3}{2}}}{8}+\left(\frac{9 \,\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)}{8}-\frac{9 b x}{8}\right) \sqrt{x}}{\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{2}}+\frac{15 \arctan \left(\frac{b \sqrt{x}}{\sqrt{\left(\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)-\cdots}}\right.}{8 \sqrt{\left(\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)-b x \right) b\cdots}}\right.}{\left(\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)-b x \right)^{3}}$
default	$-\frac{2}{\left(\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)-b x \right)^{3} \sqrt{x}}-\frac{2 b \left(\frac{\frac{7 b \,x^{\frac{3}{2}}}{8}+\left(\frac{9 \,\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)}{8}-\frac{9 b x}{8}\right) \sqrt{x}}{\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)^{2}}+\frac{15 \arctan \left(\frac{b \sqrt{x}}{\sqrt{\left(\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)-\cdots}}\right.}{8 \sqrt{\left(\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)-b x \right) b\cdots}}\right.}{\left(\operatorname{arctanh}\left(\tanh \left(b x +a \right)\right)-b x \right)^{3}}$
risch	Expression too large to display

input
```
int(1/x^(3/2)/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/(arctanh(tanh(b*x+a))-b*x)^3/x^(1/2)-2/(arctanh(tanh(b*x+a))-b*x)^3*b*(
(7/8*b*x^(3/2)+(9/8*arctanh(tanh(b*x+a))-9/8*b*x)*x^(1/2))/arctanh(tanh(b*
x+a))^2+15/8/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arcta
nh(tanh(b*x+a))-b*x)*b)^(1/2)))
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = \left[\frac{15\left(b^2x^3+2\,abx^2+a^2x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(15\,b^2x^2+25\right.}{8\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)}\right.$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
[1/8*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*s
qrt(-b/a) - a)/(b*x + a)) - 2*(15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt(x))/(a^
3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x), 1/4*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sq
rt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 + 25*a*b*x + 8*a^2)*
sqrt(x))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)]
```

3.212.6 Sympy [F]

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = \int \frac{1}{x^{\frac{3}{2}}\,\text{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**(3/2)/atanh(tanh(b*x+a))**3,x)
```

output
```
Integral(1/(x**(3/2)*atanh(tanh(a + b*x))**3), x)
```

3.212.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{15\,b^2x^2+25\,abx+8\,a^2}{4\left(a^3b^2x^{\frac{5}{2}}+2\,a^4bx^{\frac{3}{2}}+a^5\sqrt{x}\right)}-\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^3}$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-1/4*(15*b^2*x^2 + 25*a*b*x + 8*a^2)/(a^3*b^2*x^(5/2) + 2*a^4*b*x^(3/2) +
a^5*sqrt(x)) - 15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

3.212.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^3}-\frac{2}{a^3\sqrt{x}}-\frac{7\,b^2x^{\frac{3}{2}}+9\,ab\sqrt{x}}{4\,(bx+a)^2a^3}$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
-15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/(a^3*sqrt(x)) - 1/
4*(7*b^2*x^(3/2) + 9*a*b*sqrt(x))/((b*x + a)^2*a^3)
```

3.212.9 Mupad [B] (verification not implemented)

Time = 5.02 (sec) , antiderivative size = 1077, normalized size of antiderivative = 6.12

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(3/2)*atanh(tanh(a + b*x))^3),x)
```

output
```
(x*((12*b)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + (3*b*(16*log(2/(exp(2*a)*exp(2*b*
x) + 1)) - 16*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 32*
b*x))/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4) - (16*log(2/(exp(2*a)*exp(2*b*x) + 1))
- 16*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 32*b*x)/(log
(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^3)/(x^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))) + (15*2^(1/2)*b^(1/2)
*log((b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x
)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 + 60*a^2*(2*a - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b
*x)^4 - 160*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + 240*a^4*(2*a - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2...
```

3.212. $\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^3}\,dx$

3.213 $\int \frac{1}{x^{5/2}\textbf{arctanh}(\tanh(a+bx))^3}\,dx$

3.213.1 Optimal result

Integrand size = 15, antiderivative size = 201

$$\begin{aligned}\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^3}\,dx &= -\frac{35b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{4(bx-\text{arctanh}(\tanh(a+bx)))^{9/2}} \\ &+ \frac{35b}{4\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))^4} + \frac{35}{12x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))^3} \\ &+ \frac{7}{4bx^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))^2} + \frac{5}{4b^2x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))} \\ &- \frac{1}{2bx^{5/2}\text{arctanh}(\tanh(a+bx))^2} + \frac{5}{4b^2x^{7/2}\text{arctanh}(\tanh(a+bx))}\end{aligned}$$

output
```
-35/4*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b
*x-arctanh(tanh(b*x+a)))^(9/2)+35/12/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^3+
7/4/b/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^2+5/4/b^2/x^(7/2)/(b*x-arctanh(ta
nh(b*x+a)))-1/2/b/x^(5/2)/arctanh(tanh(b*x+a))^2+5/4/b^2/x^(7/2)/arctanh(t
anh(b*x+a))+35/4*b/(b*x-arctanh(tanh(b*x+a)))^4/x^(1/2)
```

3.213.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = \frac{1}{12}\left(\frac{105b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\text{arctanh}(\tanh(a+bx)))^{9/2}} + \frac{80bx-8\text{arctanh}(\tanh(a+bx))}{x^{3/2}(-bx+\text{arctanh}(\tanh(a+bx)))^4} + \frac{33b^2\sqrt{x}}{\text{arctanh}(\tanh(a+bx))(-bx+\text{arctanh}(\tanh(a+bx)))^4} + \frac{6b^2\sqrt{x}}{\text{arctanh}(\tanh(a+bx))^2(-bx+\text{arctanh}(\tanh(a+bx)))^3}\right)$$

input
```
Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

output
```
((105*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]
]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(9/2) + (80*b*x - 8*ArcTanh[Tanh[a
+ b*x]])/(x^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4) + (33*b^2*Sqrt[x])/
(ArcTanh[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4) + (6*b^2*Sqrt
[x])/(ArcTanh[Tanh[a + b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3))/12
```

3.213.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2599, 2599, 2594, 2594, 2594, 2594, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^3}\,dx$$

$$\Big\downarrow \; 2599$$

$$-\frac{5\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^2}dx}{4b} - \frac{1}{2bx^{5/2}\text{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 2599

$$-\frac{5\left(-\frac{7\int \frac{1}{x^{9/2}\text{arctanh}(\tanh(a+bx))}dx}{2b}-\frac{1}{bx^{7/2}\text{arctanh}(\tanh(a+bx))}\right)}{4b}-\frac{1}{2bx^{5/2}\text{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 2594

$$-\frac{5\left(-\frac{7\left(\frac{b\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{2b}-\frac{1}{bx^{7/2}\text{arctanh}(\tanh(a+bx))}\right)}{4b}-\frac{1}{2bx^{5/2}\text{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 2594

$$-\frac{5\left(-\frac{7\left(\frac{b\left(\frac{b\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{2b}-\frac{1}{bx^{7/2}\text{arctanh}(\tanh(a+bx))}\right)}{4b}-\frac{1}{2bx^{5/2}\text{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 2594

3.213. $\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^3}\,dx$

$$-\frac{5\left(-\frac{7\left(\frac{b\left(\frac{b\left(\frac{b\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{7x^{7/2}(bx-\text{arctanh}}\right.}{2b}\right.}{4b}$$

$$\frac{1}{2bx^{5/2}\text{arctanh}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 2594$$

3.213. $\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^3}\,dx$

$$-\frac{5\left(-\frac{7\left(\frac{b\left(\frac{b\left(\frac{b\left(\frac{b\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}\cdots}}{bx-\text{arctanh}(\tanh(a+bx))}}{2b}\cdots\right.}{4b}\cdots$$

$$\frac{1}{2bx^{5/2}\text{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 2593

3.213. $\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^3}\,dx$

$$5\left(-\frac{7\left(\frac{b\left(\frac{b\left(-\frac{2}{3x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{b\left(-\frac{2}{\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))}-\frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{(bx-\operatorname{arctanh}(\tanh(a+bx)))^{3/2}}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{}{5x^{5/2}}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}}{2b}\right.$$

3.213. $\int \frac{1}{x^{5/2}\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$

input
```
Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

output
```
(-5*((-7*(2/(7*x^(7/2)*(b*x - ArcTanh[Tanh[a + b*x]])) + (b*(2/(5*x^(5/2)*
(b*x - ArcTanh[Tanh[a + b*x]])) + (b*(2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a +
 b*x]])) + (b*((-2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Ta
nh[a + b*x]]]])/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) + 2/(Sqrt[x]*(b*x - A
rcTanh[Tanh[a + b*x]]))))/(b*x - ArcTanh[Tanh[a + b*x]])))/(b*x - ArcTanh[
Tanh[a + b*x]])))/(b*x - ArcTanh[Tanh[a + b*x]])))/(2*b) - 1/(b*x^(7/2)*Ar
cTanh[Tanh[a + b*x]])))/(4*b) - 1/(2*b*x^(5/2)*ArcTanh[Tanh[a + b*x]]^2)
```

3.213.3.1 Defintions of rubi rules used

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.213.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-\frac{2}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^3 x^{\frac{3}{2}}}+\frac{6b}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^4\sqrt{x}}+\frac{2b^2\left(\frac{\frac{11bx^{\frac{3}{2}}}{8}+\left(\frac{13\operatorname{arctanh}(\tanh(bx+a))}{8}-\frac{13}{8}\right.}{\operatorname{arctanh}(\tanh(bx+a))^2}\right.}{(\operatorname{arctanh}(t}$
default	$-\frac{2}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^3 x^{\frac{3}{2}}}+\frac{6b}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^4\sqrt{x}}+\frac{2b^2\left(\frac{\frac{11bx^{\frac{3}{2}}}{8}+\left(\frac{13\operatorname{arctanh}(\tanh(bx+a))}{8}-\frac{13}{8}\right.}{\operatorname{arctanh}(\tanh(bx+a))^2}\right.}{(\operatorname{arctanh}(t}$
risch	Expression too large to display

input
```
int(1/x^(5/2)/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3/(arctanh(tanh(b*x+a))-b*x)^3/x^(3/2)+6/(arctanh(tanh(b*x+a))-b*x)^4*b
/x^(1/2)+2/(arctanh(tanh(b*x+a))-b*x)^4*b^2*((11/8*b*x^(3/2)+(13/8*arctanh
(tanh(b*x+a))-13/8*b*x)*x^(1/2))/arctanh(tanh(b*x+a))^2+35/8/((arctanh(tan
h(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1
/2)))
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^{5/2}\operatorname{arctanh}(\tanh(a+bx))^3}\,dx = \left[\frac{105\,(b^3x^4+2\,ab^2x^3+a^2bx^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(105\,b^3x^3}{24\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)}\right.$$

$$\left.-\frac{105\,(b^3x^4+2\,ab^2x^3+a^2bx^2)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(105\,b^3x^3+175\,ab^2x^2+56\,a^2bx-8\,a^3)\sqrt{x}}{12\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)}\right]$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
[1/24*(105*(b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*s
qrt(x)*sqrt(-b/a) - a)/(b*x + a)) + 2*(105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^
2*b*x - 8*a^3)*sqrt(x))/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2), -1/12*(105*
(b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x
))) - (105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*b*x - 8*a^3)*sqrt(x))/(a^4*b^2
*x^4 + 2*a^5*b*x^3 + a^6*x^2)]
```

3.213.6 Sympy [F]

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = \int \frac{1}{x^{\frac{5}{2}}\,\text{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**(5/2)/atanh(tanh(b*x+a))**3,x)
```

output
```
Integral(1/(x**(5/2)*atanh(tanh(a + b*x))**3), x)
```

3.213.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = \frac{105\,b^3x^3 + 175\,ab^2x^2 + 56\,a^2bx - 8\,a^3}{12\left(a^4b^2x^{\frac{7}{2}} + 2\,a^5bx^{\frac{5}{2}} + a^6x^{\frac{3}{2}}\right)} + \frac{35\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}a^4}$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
1/12*(105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*b*x - 8*a^3)/(a^4*b^2*x^(7/2) +
 2*a^5*b*x^(5/2) + a^6*x^(3/2)) + 35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sq
rt(a*b)*a^4)
```

3.213.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = \frac{35\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^4} + \frac{2\,(9\,bx-a)}{3\,a^4x^{\frac{3}{2}}} + \frac{11\,b^3x^{\frac{3}{2}}+13\,ab^2\sqrt{x}}{4\,(bx+a)^2a^4}$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + 2/3*(9*b*x - a)/(a^
4*x^(3/2)) + 1/4*(11*b^3*x^(3/2) + 13*a*b^2*sqrt(x))/((b*x + a)^2*a^4)
```

3.213.9 Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 1362, normalized size of antiderivative = 6.78

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(5/2)*atanh(tanh(a + b*x))^3),x)
```

output
```
(x^(1/2)*((2*(2*b*(3*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x) - 14*b*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
2*b*x)))/(3*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*e
xp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)) + (56*b^2*x)/(3*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x))))/(2*b*x^2 - x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))^2 - (x^(1/
2)*((280*b)/(3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (280*b^2*x)/(log(2/(exp(2*a)
*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + 2*b*x)^4))/(2*b*x^2 - x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (70*2^(1/2)*b^(3/2
)*log((b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*e
xp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*...
```

3.213. $\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^3}\,dx$

3.214 $\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^3}\,dx$

3.214.1 Optimal result

Integrand size = 15, antiderivative size = 228

$$\begin{aligned}\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = &-\frac{63b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\text{arctanh}(\tanh(a+bx))}}\right)}{4(bx-\text{arctanh}(\tanh(a+bx)))^{11/2}}\\ &+\frac{63b^2}{4\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))^5}+\frac{21b}{4x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))^4}\\ &+\frac{63}{20x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))^3}\\ &+\frac{9}{4bx^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))^2}+\frac{7}{4b^2x^{9/2}(bx-\text{arctanh}(\tanh(a+bx)))}\\ &-\frac{1}{2bx^{7/2}\text{arctanh}(\tanh(a+bx))^2}+\frac{7}{4b^2x^{9/2}\text{arctanh}(\tanh(a+bx))}\end{aligned}$$

output
```
-63/4*b^(5/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b
*x-arctanh(tanh(b*x+a)))^(11/2)+21/4*b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^
4+63/20/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^3+9/4/b/x^(7/2)/(b*x-arctanh(ta
nh(b*x+a)))^2+7/4/b^2/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))-1/2/b/x^(7/2)/arc
tanh(tanh(b*x+a))^2+7/4/b^2/x^(9/2)/arctanh(tanh(b*x+a))+63/4*b^2/(b*x-arc
tanh(tanh(b*x+a)))^5/x^(1/2)
```

3.214.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = \frac{1}{20}\left(\frac{75b^3\sqrt{x}}{(bx-\text{arctanh}(\tanh(a+bx)))^5\text{arctanh}(\tanh(a+bx))} - \frac{315b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-bx+\text{arctanh}(\tanh(a+bx))}}\right)}{(-bx+\text{arctanh}(\tanh(a+bx)))^{11/2}} - \frac{10b^3\sqrt{x}}{\text{arctanh}(\tanh(a+bx))^2(-bx+\text{arctanh}(\tanh(a+bx)))^4} + \frac{8(36b^2x^2-7bx\text{arctanh}(\tanh(a+bx))+\text{arctanh}(\tanh(a+bx))^2)}{x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))^5}\right)$$

input
```
Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

output
```
((75*b^3*Sqrt[x])/((b*x - ArcTanh[Tanh[a + b*x]])^5*ArcTanh[Tanh[a + b*x]]
) - (315*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b
*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(11/2) - (10*b^3*Sqrt[x])/(ArcTa
nh[Tanh[a + b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4) + (8*(36*b^2*x^2
- 7*b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2))/(x^(5/2)*(b*x
- ArcTanh[Tanh[a + b*x]])^5))/20
```

3.214.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2599, 2599, 2594, 2594, 2594, 2594, 2594, 2593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^3}\,dx$$

$$\Big\downarrow \; 2599$$

$$-\frac{7\int \frac{1}{x^{9/2}\text{arctanh}(\tanh(a+bx))^2}dx}{4b}-\frac{1}{2bx^{7/2}\text{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 2599

$$-\frac{7\left(-\frac{9\int \frac{1}{x^{11/2}\text{arctanh}(\tanh(a+bx))}dx}{2b}-\frac{1}{bx^{9/2}\text{arctanh}(\tanh(a+bx))}\right)}{4b}-\frac{1}{2bx^{7/2}\text{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 2594

$$-\frac{7\left(-\frac{9\left(\frac{b\int \frac{1}{x^{9/2}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{9x^{9/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{2b}-\frac{1}{bx^{9/2}\text{arctanh}(\tanh(a+bx))}\right)}{4b}-\frac{1}{2bx^{7/2}\text{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 2594

$$-\frac{7\left(-\frac{9\left(\frac{b\left(\frac{b\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{9x^{9/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{2b}-\frac{1}{bx^{9/2}\text{arctanh}(\tan}\right.}{4b}\frac{1}{2bx^{7/2}\text{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 2594

3.214. $\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^3}\,dx$

$$-\frac{7\left(-\frac{9\left(\frac{b\left(\frac{b\left(\frac{b\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{9x^{9/2}(bx-\text{arctan}\ldots}\right.}{2b}\right.}{4b}$$

$$\frac{1}{2bx^{7/2}\text{arctanh}(\tanh(a+bx))^2}$$

$$\downarrow \text{2594}$$

3.214. $\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^3}\,dx$

$$-\frac{7\left(-\frac{9\left(\frac{b\left(\frac{b\left(\frac{b\int \frac{1}{x^{3/2}\operatorname{arctanh}(\tanh(a+bx))}dx}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{2}{7x^{7/2}(bx-\operatorname{arcta}\cdots}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}\cdots}{2b}\cdots\right)}{4b}\cdots$$

$$\frac{1}{2bx^{7/2}\operatorname{arctanh}(\tanh(a+bx))^2}$$

$\downarrow$ 2594

3.214. $\int \frac{1}{x^{7/2}\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$

$$7\left(-\frac{9\left(\frac{b\left(\frac{b\left(\frac{b\left(\frac{b\left(\frac{b\int\frac{1}{\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))}dx}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{2}{5x^{5/2}(bx-\operatorname{arctan}\cdots}\cdots}{bx-\operatorname{arctanh}(\tanh(a+bx))}\cdots}{bx-\operatorname{arctanh}(\tanh(a+bx))}\cdots\right)}{2b}\cdots\right.$$

3.214. $\int \frac{1}{x^{7/2}\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$

$\downarrow$ 2593

3.214. $\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^3}\,dx$

$$\left(9\left(\frac{b\left(\frac{b\left(\frac{b\left(\frac{2}{3x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{b\left(\frac{2}{\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))}-\frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\operatorname{arctanh}(\tanh(a+bx))}}\right)}{(bx-\operatorname{arctanh}(\tanh(a+bx)))^{3/2}}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{\cdots}{5x^{5\cdots}}\cdots\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}\cdots\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}\cdots\right)\right.$$

3.214. $\int \frac{1}{x^{7/2}\operatorname{arctanh}(\tanh(a+bx))^3}\,dx$

input
```
Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

output
```
(-7*((-9*(2/(9*x^(9/2)*(b*x - ArcTanh[Tanh[a + b*x]])) + (b*(2/(7*x^(7/2)*
(b*x - ArcTanh[Tanh[a + b*x]])) + (b*(2/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a +
 b*x]])) + (b*(2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])) + (b*((-2*Sqrt
[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(b*x -
ArcTanh[Tanh[a + b*x]])^(3/2) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))
))/(b*x - ArcTanh[Tanh[a + b*x]])))/(b*x - ArcTanh[Tanh[a + b*x]])))/(b*x
- ArcTanh[Tanh[a + b*x]])))/(b*x - ArcTanh[Tanh[a + b*x]])))/(2*b) - 1/(b*
x^(9/2)*ArcTanh[Tanh[a + b*x]])))/(4*b) - 1/(2*b*x^(7/2)*ArcTanh[Tanh[a +
b*x]]^2)
```

3.214.3.1 Defintions of rubi rules used

rule 2593
```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[-2*(ArcTanh[Sqrt[v]/Rt[-(b*u - a*v)/a, 2]]/(a*Rt[-(b*u -
 a*v)/a, 2])), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piecewise
LinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.214.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.69

method	result
derivativedivides	$-\frac{2b^3\left(\frac{\frac{15b\,x^{\frac{3}{2}}}{8}+\left(\frac{17\,\mathrm{arctanh}(\tanh(bx+a))}{8}-\frac{17bx}{8}\right)\sqrt{x}}{\mathrm{arctanh}(\tanh(bx+a))^2}+\frac{63\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)}{8\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\mathrm{arctanh}(\tanh(bx+a))-bx)^5}-\frac{2}{5(\mathrm{arctanh}(\tanh(bx+}$
default	$-\frac{2b^3\left(\frac{\frac{15b\,x^{\frac{3}{2}}}{8}+\left(\frac{17\,\mathrm{arctanh}(\tanh(bx+a))}{8}-\frac{17bx}{8}\right)\sqrt{x}}{\mathrm{arctanh}(\tanh(bx+a))^2}+\frac{63\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)}{8\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\mathrm{arctanh}(\tanh(bx+a))-bx)^5}-\frac{2}{5(\mathrm{arctanh}(\tanh(bx+}$
risch	Expression too large to display

input
```
int(1/x^(7/2)/arctanh(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/(arctanh(tanh(b*x+a))-b*x)^5*b^3*((15/8*b*x^(3/2)+(17/8*arctanh(tanh(b*
x+a))-17/8*b*x)*x^(1/2))/arctanh(tanh(b*x+a))^2+63/8/((arctanh(tanh(b*x+a)
)-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)))-2/
5/(arctanh(tanh(b*x+a))-b*x)^3/x^(5/2)-12/(arctanh(tanh(b*x+a))-b*x)^5*b^2
/x^(1/2)+2/(arctanh(tanh(b*x+a))-b*x)^4*b/x^(3/2)
```

3.214.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^{7/2}\mathrm{arctanh}(\tanh(a+bx))^3}\,dx = \left[\frac{315\left(b^4x^5+2\,ab^3x^4+a^2b^2x^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(315\,b^4x\right.}{40\left(a^5b^2x^5+2\,a^6bx^4+a\right.}\right.$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
[1/40*(315*(b^4*x^5 + 2*a*b^3*x^4 + a^2*b^2*x^3)*sqrt(-b/a)*log((b*x - 2*a
*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) - 2*(315*b^4*x^4 + 525*a*b^3*x^3 + 168
*a^2*b^2*x^2 - 24*a^3*b*x + 8*a^4)*sqrt(x))/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a
^7*x^3), 1/20*(315*(b^4*x^5 + 2*a*b^3*x^4 + a^2*b^2*x^3)*sqrt(b/a)*arctan(
a*sqrt(b/a)/(b*sqrt(x))) - (315*b^4*x^4 + 525*a*b^3*x^3 + 168*a^2*b^2*x^2
- 24*a^3*b*x + 8*a^4)*sqrt(x))/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3)]
```

3.214.6 Sympy [F]

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = \int \frac{1}{x^{\frac{7}{2}}\,\text{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**(7/2)/atanh(tanh(b*x+a))**3,x)
```

output
```
Integral(1/(x**(7/2)*atanh(tanh(a + b*x))**3), x)
```

3.214.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{315\,b^4x^4+525\,ab^3x^3+168\,a^2b^2x^2-24\,a^3bx+8\,a^4}{20\left(a^5b^2x^{\frac{9}{2}}+2\,a^6bx^{\frac{7}{2}}+a^7x^{\frac{5}{2}}\right)} - \frac{63\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^5}$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-1/20*(315*b^4*x^4 + 525*a*b^3*x^3 + 168*a^2*b^2*x^2 - 24*a^3*b*x + 8*a^4)
/(a^5*b^2*x^(9/2) + 2*a^6*b*x^(7/2) + a^7*x^(5/2)) - 63/4*b^3*arctan(b*sqr
t(x)/sqrt(a*b))/(sqrt(a*b)*a^5)
```

3.214.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = -\frac{63\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^5} - \frac{15\,b^4x^{\frac{3}{2}}+17\,ab^3\sqrt{x}}{4\left(bx+a\right)^2a^5} - \frac{2\left(30\,b^2x^2-5\,abx+a^2\right)}{5\,a^5x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
-63/4*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5) - 1/4*(15*b^4*x^(3/2
) + 17*a*b^3*sqrt(x))/((b*x + a)^2*a^5) - 2/5*(30*b^2*x^2 - 5*a*b*x + a^2)
/(a^5*x^(5/2))
```

3.214.9 Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 2151, normalized size of antiderivative = 9.43

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(7/2)*atanh(tanh(a + b*x))^3),x)
```

output
```
16/(5*x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (x*((((448*b^4)/(3*(2*a*b - b
*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (16*b^3*(
2*b*(3*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 6*b*x) - 14*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(3*(2
*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + l
og(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4))*(l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x))/(2*b) - (112*b^3)/((2*a*b - b*(2*a - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (8*b^2*(2*b*(3*log(2/(exp(2*a
)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1)) + 6*b*x) - 14*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/((2*a*b - b*(2*a - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2...
```

3.215 $\int x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx$

3.215.1 Optimal result

Integrand size = 17, antiderivative size = 142

$$\int x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))^3}{8b^{5/2}} + \frac{1}{3}x^{5/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))} - \frac{x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{12b} - \frac{\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{8b^2}$$

output
```
-1/8*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh
(b*x+a)))^3/b^(5/2)+1/3*x^(5/2)*arctanh(tanh(b*x+a))^(1/2)-1/12*x^(3/2)*(b
*x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(1/2)/b-1/8*(b*x-arctanh(tan
h(b*x+a)))^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^2
```

3.215.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.73

$$\int x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}\,dx = \frac{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}(3b^2x^2+8bx\text{arctanh}(\tanh(a+bx))-3a}{24b^2}$$
$$+\frac{(-bx+\text{arctanh}(\tanh(a+bx)))^3\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\text{arctanh}(\tanh(a+bx))}\right)}{8b^{5/2}}$$

input
```
Integrate[x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(3*b^2*x^2 + 8*b*x*ArcTanh[Tanh[a +
b*x]] - 3*ArcTanh[Tanh[a + b*x]]^2))/(24*b^2) + ((-(b*x) + ArcTanh[Tanh[a
+ b*x]])^3*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*b^(5/
2))
```

3.215.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2600, 2600, 2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}\,dx$$

$$\Big\downarrow \text{2600}$$

$$\frac{1}{3}x^{5/2}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{6}(bx-\text{arctanh}(\tanh(a+bx)))\int\frac{x^{3/2}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx$$

$$\Big\downarrow \text{2600}$$

$$\frac{1}{3}x^{5/2}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{6}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{3(bx-\text{arctanh}(\tanh(a+bx)))\int\frac{\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{4b}+\frac{x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}{2b}\right)$$

$$\Big\downarrow \text{2600}$$

$$\frac{1}{3}x^{5/2}\sqrt{\text{arctanh}(\tanh(a+bx))} - \frac{1}{6}(bx - \text{arctanh}(\tanh(a+bx)))\left(\frac{3(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{2b} + \frac{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}\right.}{4b}\right.$$

$\downarrow$ 2596

$$\frac{1}{3}x^{5/2}\sqrt{\text{arctanh}(\tanh(a+bx))} - \frac{1}{6}\left(\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))}{b^{3/2}} + \frac{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}\right)(bx - \text{arctanh}(\tanh(a+bx)))}{4b}\right.$$

input
```
Int[x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
-1/6*(((3*((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x -
 ArcTanh[Tanh[a + b*x]]))/b^(3/2) + (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])
/b)*(b*x - ArcTanh[Tanh[a + b*x]]))/(4*b) + (x^(3/2)*Sqrt[ArcTanh[Tanh[a +
 b*x]]])/(2*b))*(b*x - ArcTanh[Tanh[a + b*x]])) + (x^(5/2)*Sqrt[ArcTanh[Ta
nh[a + b*x]]])/3
```

3.215.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.215. $\int x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}\,dx$

3.215.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{x^{\frac{3}{2}} \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3b} - \frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\, \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{4b} - \frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\sqrt{x}\,\sqrt{\right.}}{}\right.}{b}$
default	$\frac{x^{\frac{3}{2}} \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3b} - \frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\, \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{4b} - \frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\sqrt{x}\,\sqrt{\right.}}{}\right.}{b}$

input
```
int(x^(3/2)*arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(3/2)*arctanh(tanh(b*x+a))^(3/2)/b-(arctanh(tanh(b*x+a))-b*x)/b*(1/4
*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)/b-1/4*(arctanh(tanh(b*x+a))-b*x)/b*(1/
2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/2/b^(1/2)*(arctanh(tanh(b*x+a))-b*x
)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))))
```

3.215.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \left[\frac{3\,a^3\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(8\,b^3x^2+2\,ab^2x-3\,a^2}{48\,b^3}\right.$$
$$\left.-\frac{3\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-(8\,b^3x^2+2\,ab^2x-3\,a^2b)\sqrt{bx+a}\sqrt{x}}{24\,b^3}\right]$$

input
```
integrate(x^(3/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 + 2*a*b^2*x - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/24*(3*a^3
*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 2*a*b^
2*x - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3]
```

3.215.6 Sympy [F]

$$\int x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx=\int x^{\frac{3}{2}}\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**(3/2)*atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(x**(3/2)*sqrt(atanh(tanh(a + b*x))), x)
```

3.215.7 Maxima [F]

$$\int x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx=\int x^{\frac{3}{2}}\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}\,dx$$

input
```
integrate(x^(3/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)*sqrt(arctanh(tanh(b*x + a))), x)
```

3.215.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

$$\int x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx=\frac{1}{24}\sqrt{bx+a}\left(2\left(4x+\frac{a}{b}\right)x-\frac{3a^2}{b^2}\right)\sqrt{x}-\frac{a^3\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{8b^{\frac{5}{2}}}$$

input
```
integrate(x^(3/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x + a)*(2*(4*x + a/b)*x - 3*a^2/b^2)*sqrt(x) - 1/8*a^3*log(abs
(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(5/2)
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx=\int x^{3/2}\,\sqrt{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}\,dx$$

input

```
int(x^(3/2)*atanh(tanh(a + b*x))^(1/2),x)
```

output

```
int(x^(3/2)*atanh(tanh(a + b*x))^(1/2), x)
```

3.216 $\int \sqrt{x}\sqrt{\mathbf{arctanh}(\tanh(a+bx))}\,dx$

3.216.1 Optimal result

Integrand size = 17, antiderivative size = 104

$$\int \sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))^2}{4b^{3/2}} + \frac{1}{2}x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))} - \frac{\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{4b}$$

output
```
-1/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh
(b*x+a)))^2/b^(3/2)+1/2*x^(3/2)*arctanh(tanh(b*x+a))^(1/2)-1/4*(b*x-arctan
h(tanh(b*x+a)))*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b
```

3.216.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int \sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}(bx+\operatorname{arctanh}(\tanh(a+bx)))}{4b} - \frac{(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\right)}{4b^{3/2}}$$

input
```
Integrate[Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(b*x + ArcTanh[Tanh[a + b*x]]))/(4*b
) - ((-(b*x) + ArcTanh[Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcT
anh[Tanh[a + b*x]]]])/(4*b^(3/2))
```

3.216.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2600, 2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}\,dx$$

$$\Big\downarrow \; 2600$$

$$\frac{1}{2}x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{4}(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx$$

$$\Big\downarrow \; 2600$$

$$\frac{1}{2}x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{4}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{2b}+\frac{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}\right)$$

$$\Big\downarrow \; 2596$$

$$\frac{1}{2}x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{4}\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))}{b^{3/2}}+\frac{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}\right)(bx-\text{arctanh}(\tanh(a+bx)))$$

input
```
Int[Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
-1/4*(((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - Arc
Tanh[Tanh[a + b*x]]))/b^(3/2) + (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b)*
(b*x - ArcTanh[Tanh[a + b*x]])) + (x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/2
```

3.216.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.216.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{2b}-\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2}+\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\ln\left(\sqrt{b}\right.}{2\sqrt{b}}\right.}{2b}$
default	$\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{2b}-\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2}+\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\ln\left(\sqrt{b}\right.}{2\sqrt{b}}\right.}{2b}$

input
```
int(x^(1/2)*arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)/b-1/2*(arctanh(tanh(b*x+a))-b*x)/b*
(1/2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/2/b^(1/2)*(arctanh(tanh(b*x+a))-
b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2)))
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10

$$\int \sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx$$
$$= \left[\frac{a^2\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(2\,b^2x+ab)\sqrt{bx+a}\sqrt{x}}{8\,b^2}, \frac{a^2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+(2\,b^2\right.$$

input
```
integrate(x^(1/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(a^2*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2*
b^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/4*(a^2*sqrt(-b)*arctan(sqrt(b*x
 + a)*sqrt(-b)/(b*sqrt(x))) + (2*b^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

3.216.6 Sympy [F]

$$\int \sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int \sqrt{x}\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**(1/2)*atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(x)*sqrt(atanh(tanh(a + b*x))), x)
```

3.216.7 Maxima [F]

$$\int \sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int \sqrt{x}\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}\,dx$$

input
```
integrate(x^(1/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)*sqrt(arctanh(tanh(b*x + a))), x)
```

3.216.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.46

$$\int \sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \frac{1}{4}\sqrt{bx+a}\left(2x+\frac{a}{b}\right)\sqrt{x}+\frac{a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{4b^{\frac{3}{2}}}$$

input
```
integrate(x^(1/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*x + a)*(2*x + a/b)*sqrt(x) + 1/4*a^2*log(abs(-sqrt(b)*sqrt(x) +
 sqrt(b*x + a)))/b^(3/2)
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\,dx = \int \sqrt{x}\,\sqrt{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}\,dx$$

input
```
int(x^(1/2)*atanh(tanh(a + b*x))^(1/2),x)
```

output
```
int(x^(1/2)*atanh(tanh(a + b*x))^(1/2), x)
```

3.217 $\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{x}}\,dx$

3.217.1 Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{x}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))}{\sqrt{b}} + \sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}$$

output
```
-arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b*x
+a)))/b^(1/2)+x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

3.217.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))} + \frac{(-bx+\operatorname{arctanh}(\tanh(a+bx)))\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\right)}{\sqrt{b}}$$

input
```
Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[x],x]
```

output
```
Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]] + ((-(b*x) + ArcTanh[Tanh[a + b*x]])*
Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/Sqrt[b]
```

3.217.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 2600}$$

$$\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{2}(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx$$

$$\Big\downarrow \text{ 2596}$$

$$\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))}{\sqrt{b}}$$

input
```
Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[x],x]
```

output
```
-((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]]))/Sqrt[b]) + Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]
```

3.217.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.217.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\sqrt{x}\,\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}+\frac{\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh(bx+a))}\right)}{\sqrt{b}}$	49
default	$\sqrt{x}\,\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}+\frac{\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh(bx+a))}\right)}{\sqrt{b}}$	49

input
```
int(arctanh(tanh(b*x+a))^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/b^(1/2)*(arctanh(tanh(b*x+a))-b*x)*ln
(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.52

$$\int\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{x}}\,dx=\left[\frac{a\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,\sqrt{bx+a}b\sqrt{x}}{2\,b},\ -\frac{a\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+a}b\sqrt{x}}{b}\right]$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(a*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(
b*x + a)*b*sqrt(x))/b, -(a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(
x))) - sqrt(b*x + a)*b*sqrt(x))/b]
```

3.217.6 Sympy [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{x}}\,dx = \int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{\sqrt{x}}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(1/2)/x**(1/2),x)
```

output
```
Integral(sqrt(atanh(tanh(a + b*x)))/sqrt(x), x)
```

3.217.7 Maxima [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{x}}\,dx = \int \frac{\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}{\sqrt{x}}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(arctanh(tanh(b*x + a)))/sqrt(x), x)
```

3.217.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{x}}\,dx = -\frac{a\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{\sqrt{b}} + \sqrt{bx+a}\sqrt{x}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
-a*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/sqrt(b) + sqrt(b*x + a)*sqrt
(x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{x}}\,dx = \int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}}{\sqrt{x}}\,dx$$

input
```
int(atanh(tanh(a + b*x))^(1/2)/x^(1/2),x)
```

output
```
int(atanh(tanh(a + b*x))^(1/2)/x^(1/2), x)
```

3.218 $\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^{3/2}}\,dx$

3.218.1 Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^{3/2}}\,dx = 2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right) - \frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{x}}$$

output
```
2*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*b^(1/2)-2*arctanh(ta
nh(b*x+a))^(1/2)/x^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^{3/2}}\,dx = -\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{x}} + 2\sqrt{b}\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\text{arctanh}(\tanh(a+bx))}\right)$$

input
```
Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(3/2),x]
```

output
```
(-2*Sqrt[ArcTanh[Tanh[a + b*x]]])/Sqrt[x] + 2*Sqrt[b]*Log[b*Sqrt[x] + Sqrt
[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]]
```

3.218.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2599, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$b\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx-\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{x}}$$

$$\Big\downarrow \text{ 2596}$$

$$2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)-\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{x}}$$

input
```
Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(3/2),x]
```

output
```
2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]] - (2*Sqr
t[ArcTanh[Tanh[a + b*x]]])/Sqrt[x]
```

3.218.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.218.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(37) = 74.

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.98

method	result
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}+\frac{4b\left(\frac{\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2}+\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh(b\right.}}{2\sqrt{b}}\right.}{\operatorname{arctanh}(\tanh(bx+a))-bx}$
default	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}+\frac{4b\left(\frac{\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2}+\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh(b\right.}}{2\sqrt{b}}\right.}{\operatorname{arctanh}(\tanh(bx+a))-bx}$

input
```
int(arctanh(tanh(b*x+a))^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+4*b/(arct
anh(tanh(b*x+a))-b*x)*(1/2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/2/b^(1/2)*
(arctanh(tanh(b*x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2)))
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.82

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{3/2}}\,dx = \left[\frac{\sqrt{b}x\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,\sqrt{bx+a}\sqrt{x}}{x},\right.$$
$$\left.-\frac{2\left(\sqrt{-b}x\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\sqrt{x}\right)}{x}\right]$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[(sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sqrt(b*x
+ a)*sqrt(x))/x, -2*(sqrt(-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))
 + sqrt(b*x + a)*sqrt(x))/x]
```

3.218.6 Sympy [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(1/2)/x**(3/2),x)
```

output
```
Integral(sqrt(atanh(tanh(a + b*x)))/x**(3/2), x)
```

3.218.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{3/2}}\,dx = 2\sqrt{b}\log\left(\frac{b\sqrt{x}}{\sqrt{ab}} + \sqrt{\frac{bx}{a}+1}\right) - \frac{2\sqrt{bx+a}}{\sqrt{x}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(3/2),x, algorithm="maxima")
```

output
```
2*sqrt(b)*log(b*sqrt(x)/sqrt(a*b) + sqrt(b*x/a + 1)) - 2*sqrt(b*x + a)/sqr
t(x)
```

3.218.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{3/2}}\,dx = -\sqrt{b}\log\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2\right) + \frac{4a\sqrt{b}}{\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(3/2),x, algorithm="giac")
```

output
```
-sqrt(b)*log((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2) + 4*a*sqrt(b)/((sqrt(b)*
sqrt(x) - sqrt(b*x + a))^2 - a)
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}}{x^{3/2}}\,dx$$

input
```
int(atanh(tanh(a + b*x))^(1/2)/x^(3/2),x)
```

output
```
int(atanh(tanh(a + b*x))^(1/2)/x^(3/2), x)
```

3.219 $\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{5/2}} dx$

3.219.1 Optimal result

Integrand size = 17, antiderivative size = 35

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{5/2}} dx = \frac{2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{3x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

output
```
2/3*arctanh(tanh(b*x+a))^(3/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))
```

3.219.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{5/2}} dx = \frac{2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{3/2}(3bx-3\operatorname{arctanh}(\tanh(a+bx)))}$$

input
```
Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(5/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(3/2))/(x^(3/2)*(3*b*x - 3*ArcTanh[Tanh[a + b*x]
]))
```

3.219.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^{5/2}} \, dx$$

$$\Big\downarrow \text{ 2598}$$

$$\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

input
```
Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(5/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])
)
```

3.219.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

3.219.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2\,\text{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3(\text{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}$	29
default	$-\frac{2\,\text{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3(\text{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}$	29

input
```
int(arctanh(tanh(b*x+a))^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(3/2)
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{5/2}}\,dx = -\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

3.219.6 Sympy [F]

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(1/2)/x**(5/2),x)
```

output
```
Integral(sqrt(atanh(tanh(a + b*x)))/x**(5/2), x)
```

3.219.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{5/2}}\,dx = -\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

3.219.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{5/2}}\,dx = \frac{4\left(3\,b^{\frac{3}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{4}+a^{2}b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{3}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(5/2),x, algorithm="giac")
```

output
```
4/3*(3*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 + a^2*b^(3/2))/((sqrt(b
)*sqrt(x) - sqrt(b*x + a))^2 - a)^3
```

3.219.9 Mupad [B] (verification not implemented)

Time = 4.88 (sec) , antiderivative size = 210, normalized size of antiderivative = 6.00

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{5/2}}\,dx = \frac{2\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}-2\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\sqrt{\frac{\ln(\cdots}{}}}{\sqrt{x}\,\left(3\,x\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-3\,x\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+6\,b\cdots\right.}$$

input
```
int(atanh(tanh(a + b*x))^(1/2)/x^(5/2),x)
```

output
```
(2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*(log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1
))/2)^(1/2) - 2*log(2/(exp(2*a)*exp(2*b*x) + 1))*(log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/
2))/(x^(1/2)*(3*x*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3*x*log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x^2))
```

3.220 $\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^{7/2}} \, dx$

3.220.1 Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^{7/2}} \, dx = \frac{4b\text{arctanh}(\tanh(a+bx))^{3/2}}{15x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))^2} + \frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

output
```
4/15*b*arctanh(tanh(b*x+a))^(3/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/5
*arctanh(tanh(b*x+a))^(3/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))
```

3.220.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^{7/2}} \, dx = \frac{2(5bx-3\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}{15x^{5/2}(-bx+\text{arctanh}(\tanh(a+bx)))^2}$$

input
```
Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(7/2),x]
```

output
```
(2*(5*b*x - 3*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))/(15*x^
(5/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

3.220.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{ 2602}$$

$$\frac{2b\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^{5/2}}dx}{5(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{ 2598}$$

$$\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{4b\text{arctanh}(\tanh(a+bx))^{3/2}}{15x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))^2}$$

input
```
Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(7/2),x]
```

output
```
(4*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(15*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x
]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a
 + b*x]]))
```

3.220.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.220.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}+\frac{4b \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{15(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}x^{\frac{3}{2}}}$	59
default	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}+\frac{4b \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{15(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}x^{\frac{3}{2}}}$	59

input
```
int(arctanh(tanh(b*x+a))^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(3/2)+4/15*b/
(arctanh(tanh(b*x+a))-b*x)^2/x^(3/2)*arctanh(tanh(b*x+a))^(3/2)
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{7/2}}\,dx = \frac{2\,(2\,b^2x^2 - abx - 3\,a^2)\sqrt{bx+a}}{15\,a^2x^{\frac{5}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(7/2),x, algorithm="fricas")
```

output
```
2/15*(2*b^2*x^2 - a*b*x - 3*a^2)*sqrt(b*x + a)/(a^2*x^(5/2))
```

3.220.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(atanh(tanh(b*x+a))**(1/2)/x**(7/2),x)
```

output
```
Timed out
```

3.220.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{7/2}}\,dx = \frac{2\,(2\,b^2x^2 - abx - 3\,a^2)\sqrt{bx+a}}{15\,a^2x^{\frac{5}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(7/2),x, algorithm="maxima")
```

output
```
2/15*(2*b^2*x^2 - a*b*x - 3*a^2)*sqrt(b*x + a)/(a^2*x^(5/2))
```

3.220.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{7/2}}\,dx = \frac{8\left(15\,b^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^6 + 5\,ab^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4 + 5\,a^2b^{\frac{5}{2}}\left(\sqrt{b}\right.\right.}{15\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^5}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(7/2),x, algorithm="giac")
```

output
```
8/15*(15*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 + 5*a*b^(5/2)*(sqrt(b
)*sqrt(x) - sqrt(b*x + a))^4 + 5*a^2*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x +
 a))^2 - a^3*b^(5/2))/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5
```

3.220.9 Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.42

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{7/2}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{16\,b^2\,x^2}{15\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}+\frac{}{15}\right.}{x^{5/2}}$$

input
```
int(atanh(tanh(a + b*x))^(1/2)/x^(7/2),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((16*b^2*x^2)/(15*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2
) + (4*b*x)/(15*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - 2/5))/x^(5/2)
```

3.220. $\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{7/2}}\,dx$

3.221 $\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{9/2}}\,dx$

3.221.1 Optimal result

Integrand size = 17, antiderivative size = 110

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{9/2}}\,dx = \frac{16b^2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{105x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^3} + \frac{8b\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{35x^{5/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2} + \frac{2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{7x^{7/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

output
```
16/105*b^2*arctanh(tanh(b*x+a))^(3/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^3
+8/35*b*arctanh(tanh(b*x+a))^(3/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/
7*arctanh(tanh(b*x+a))^(3/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))
```

3.221.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{9/2}}\,dx = \frac{2\operatorname{arctanh}(\tanh(a+bx))^{3/2}\,(35b^2x^2-42bx\operatorname{arctanh}(\tanh(a+bx))+15\text{arc}}{105x^{7/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^3}$$

input
```
Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(9/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(3/2)*(35*b^2*x^2 - 42*b*x*ArcTanh[Tanh[a + b*x]
] + 15*ArcTanh[Tanh[a + b*x]]^2))/(105*x^(7/2)*(b*x - ArcTanh[Tanh[a + b*x
]])^3)
```

3.221.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2602, 2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^{9/2}} \, dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{4b\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^{7/2}}dx}{7(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2602}$$

$$\frac{4b\left(\frac{2b\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^{5/2}}dx}{5(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{7(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2598}$$

$$\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{4b\left(\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{4b\text{arctanh}(\tanh(a+bx))^{3/2}}{15x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))^2}\right)}{7(bx-\text{arctanh}(\tanh(a+bx)))}$$

input
```
Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(9/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(3/2))/(7*x^(7/2)*(b*x - ArcTanh[Tanh[a + b*x]])
) + (4*b*((4*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(15*x^(3/2)*(b*x - ArcTanh[Ta
nh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(5*x^(5/2)*(b*x - ArcT
anh[Tanh[a + b*x]]))))/(7*(b*x - ArcTanh[Tanh[a + b*x]]))
```

3.221.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.221.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}-\frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}+\frac{2b \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{15(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}x^{\frac{3}{2}}}\right)}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)}$	105
default	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}-\frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}+\frac{2b \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{15(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}x^{\frac{3}{2}}}\right)}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)}$	105

input
```
int(arctanh(tanh(b*x+a))^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(3/2)-8/7*b/(
arctanh(tanh(b*x+a))-b*x)*(-1/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh
(tanh(b*x+a))^(3/2)+2/15*b/(arctanh(tanh(b*x+a))-b*x)^2/x^(3/2)*arctanh(ta
nh(b*x+a))^(3/2))
```

3.221.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{9/2}}\,dx = -\frac{2\,(8\,b^3x^3 - 4\,ab^2x^2 + 3\,a^2bx + 15\,a^3)\sqrt{bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(9/2),x, algorithm="fricas")
```

output
```
-2/105*(8*b^3*x^3 - 4*a*b^2*x^2 + 3*a^2*b*x + 15*a^3)*sqrt(b*x + a)/(a^3*x
^(7/2))
```

3.221.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(atanh(tanh(b*x+a))**(1/2)/x**(9/2),x)
```

output
```
Timed out
```

3.221.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{9/2}}\,dx = -\frac{2\,(8\,b^3x^3 - 4\,ab^2x^2 + 3\,a^2bx + 15\,a^3)\sqrt{bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(9/2),x, algorithm="maxima")
```

output
```
-2/105*(8*b^3*x^3 - 4*a*b^2*x^2 + 3*a^2*b*x + 15*a^3)*sqrt(b*x + a)/(a^3*x
^(7/2))
```

3.221.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{9/2}}\,dx = \frac{32\left(70\,b^{\frac{7}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{8}+35\,ab^{\frac{7}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{6}+21\,a^{2}b^{\frac{7}{2}}\left(\cdots\right.\right.}{105\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+\cdots}\right.\right.}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(9/2),x, algorithm="giac")
```

output
```
32/105*(70*b^(7/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^8 + 35*a*b^(7/2)*(sqr
t(b)*sqrt(x) - sqrt(b*x + a))^6 + 21*a^2*b^(7/2)*(sqrt(b)*sqrt(x) - sqrt(b
*x + a))^4 - 7*a^3*b^(7/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 + a^4*b^(7/
2))/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^7
```

3.221.9 Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.13

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{9/2}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}}\left(\frac{32\,b^{2}\,x^{2}}{105\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^{2}}+\cdots\right.}{\cdots}$$

input
```
int(atanh(tanh(a + b*x))^(1/2)/x^(9/2),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((32*b^2*x^2)/(105*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^
2) + (128*b^3*x^3)/(105*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (4*b*x)/(35*(log(2/
(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)) - 2/7))/x^(7/2)
```

3.222 $\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{11/2}} \, dx$

3.222.1 Optimal result

Integrand size = 17, antiderivative size = 148

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{11/2}} \, dx = \frac{32b^3 \operatorname{arctanh}(\tanh(a+bx))^{3/2}}{315x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^4} + \frac{16b^2 \operatorname{arctanh}(\tanh(a+bx))^{3/2}}{105x^{5/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^3} + \frac{4b \operatorname{arctanh}(\tanh(a+bx))^{3/2}}{21x^{7/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2} + \frac{2 \operatorname{arctanh}(\tanh(a+bx))^{3/2}}{9x^{9/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

output
```
32/315*b^3*arctanh(tanh(b*x+a))^(3/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^4
+16/105*b^2*arctanh(tanh(b*x+a))^(3/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^
3+4/21*b*arctanh(tanh(b*x+a))^(3/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))^2+2
/9*arctanh(tanh(b*x+a))^(3/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))
```

3.222.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{11/2}} \, dx = \frac{2 \operatorname{arctanh}(\tanh(a+bx))^{3/2} \left(105b^3x^3 - 189b^2x^2 \operatorname{arctanh}(\tanh(a+bx)) + 1\right.}{315x^{9/2}(-bx + \operatorname{arctanh}(\tan}$$

input
```
Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(11/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(3/2)*(105*b^3*x^3 - 189*b^2*x^2*ArcTanh[Tanh[a
+ b*x]] + 135*b*x*ArcTanh[Tanh[a + b*x]]^2 - 35*ArcTanh[Tanh[a + b*x]]^3))
/(315*x^(9/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

3.222.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2602, 2602, 2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{11/2}}dx$$

$$\Big\downarrow \text{ 2602}$$

$$\frac{2b\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{9/2}}dx}{3(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{9x^{9/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{ 2602}$$

$$\frac{2b\left(\frac{4b\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{7/2}}dx}{7(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{7x^{7/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}\right)}{3(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{9x^{9/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{ 2602}$$

$$\frac{2b\left(\frac{4b\left(\frac{2b\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{5/2}}dx}{5(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{5x^{5/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}\right)}{7(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{7x^{7/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}\right)}{3(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{9x^{9/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{ 2598}$$

$$\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{9x^{9/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2b\left(\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{4b\left(\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{4b\text{arctanh}(\tanh(a+bx))^{3/2}}{15x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))^2}\right)}{7(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{3(bx-\text{arctanh}(\tanh(a+bx)))}$$

input
```
Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(11/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(3/2))/(9*x^(9/2)*(b*x - ArcTanh[Tanh[a + b*x]])
) + (2*b*((2*ArcTanh[Tanh[a + b*x]]^(3/2))/(7*x^(7/2)*(b*x - ArcTanh[Tanh[
a + b*x]])) + (4*b*((4*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(15*x^(3/2)*(b*x -
ArcTanh[Tanh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(5*x^(5/2)*(
b*x - ArcTanh[Tanh[a + b*x]]))))/(7*(b*x - ArcTanh[Tanh[a + b*x]]))))/(3*(
b*x - ArcTanh[Tanh[a + b*x]]))
```

3.222.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.222. $\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^{11/2}}\,dx$

3.222.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.02

method	result
derivativedivides	$-\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{9(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}-\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}-\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}+\frac{2b\,a}{15(\operatorname{arcta}}\right.}{7(\operatorname{arctanh}(\tanh(bx+a))-}\right.}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)}$
default	$-\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{9(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}-\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}-\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}+\frac{2b\,a}{15(\operatorname{arcta}}\right.}{7(\operatorname{arctanh}(\tanh(bx+a))-}\right.}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)}$

input
```
int(arctanh(tanh(b*x+a))^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arctanh(tanh(b*x+a))^(3/2)-4/3*b/(
arctanh(tanh(b*x+a))-b*x)*(-1/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh
(tanh(b*x+a))^(3/2)-4/7*b/(arctanh(tanh(b*x+a))-b*x)*(-1/5/(arctanh(tanh(b
*x+a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(3/2)+2/15*b/(arctanh(tanh(b*x+a)
)-b*x)^2/x^(3/2)*arctanh(tanh(b*x+a))^(3/2)))
```

3.222.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{11/2}}\,dx=\frac{2\,(16\,b^4x^4-8\,ab^3x^3+6\,a^2b^2x^2-5\,a^3bx-35\,a^4)\sqrt{bx+a}}{315\,a^4x^{\frac{9}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(11/2),x, algorithm="fricas")
```

output
```
2/315*(16*b^4*x^4 - 8*a*b^3*x^3 + 6*a^2*b^2*x^2 - 5*a^3*b*x - 35*a^4)*sqrt
(b*x + a)/(a^4*x^(9/2))
```

3.222.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate(atanh(tanh(b*x+a))**(1/2)/x**(11/2),x)
```

output
```
Timed out
```

3.222.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{11/2}}\,dx = \frac{2\,(16\,b^4x^4 - 8\,ab^3x^3 + 6\,a^2b^2x^2 - 5\,a^3bx - 35\,a^4)\sqrt{bx+a}}{315\,a^4x^{\frac{9}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(11/2),x, algorithm="maxima")
```

output
```
2/315*(16*b^4*x^4 - 8*a*b^3*x^3 + 6*a^2*b^2*x^2 - 5*a^3*b*x - 35*a^4)*sqrt
(b*x + a)/(a^4*x^(9/2))
```

3.222.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{11/2}}\,dx = \frac{64\left(315\,b^{\frac{9}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{10} + 189\,ab^{\frac{9}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{8} + 84\,a^2\cdots\right.}{315\left(\cdots\right.}$$

input
```
integrate(arctanh(tanh(b*x+a))^(1/2)/x^(11/2),x, algorithm="giac")
```

output
```
64/315*(315*b^(9/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^10 + 189*a*b^(9/2)*(
sqrt(b)*sqrt(x) - sqrt(b*x + a))^8 + 84*a^2*b^(9/2)*(sqrt(b)*sqrt(x) - sqr
t(b*x + a))^6 - 36*a^3*b^(9/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 + 9*a^4
*b^(9/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a^5*b^(9/2))/((sqrt(b)*sqrt
(x) - sqrt(b*x + a))^2 - a)^9
```

3.222.9 Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.99

$$\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{x^{11/2}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{16\,b^2\,x^2}{105\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}+\frac{}{3}\right.}{}$$

input
```
int(atanh(tanh(a + b*x))^(1/2)/x^(11/2),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((16*b^2*x^2)/(105*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^
2) + (128*b^3*x^3)/(315*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (512*b^4*x^4)/(315*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^4) + (4*b*x)/(63*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - 2/
9))/x^(9/2)
```

3.223 $\int x^{3/2}\textbf{arctanh}(\tanh(a+bx))^{3/2}\,dx$

3.223.1 Optimal result

Integrand size = 17, antiderivative size = 177

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}\,dx=\frac{3\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))^4}{64b^{5/2}}$$
$$-\frac{1}{8}x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}+\frac{x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))^2\sqrt{\text{arctan}}}{32b}$$

output
```
3/64*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh
(b*x+a)))^4/b^(5/2)+1/4*x^(5/2)*arctanh(tanh(b*x+a))^(3/2)-1/8*x^(5/2)*(b*
x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(1/2)+1/32*x^(3/2)*(b*x-arcta
nh(tanh(b*x+a)))^2*arctanh(tanh(b*x+a))^(1/2)/b+3/64*(b*x-arctanh(tanh(b*x
+a)))^3*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^2
```

3.223.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.69

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}\,dx=\frac{\sqrt{b}\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}(-3b^3x^3+11b^2x^2\text{arctanh}(\tanh(a+bx}{}$$

input
```
Integrate[x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-3*b^3*x^3 + 11*b^2*x^2*Arc
Tanh[Tanh[a + b*x]] + 11*b*x*ArcTanh[Tanh[a + b*x]]^2 - 3*ArcTanh[Tanh[a +
 b*x]]^3) + 3*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4*Log[b*Sqrt[x] + Sqrt[b]*
Sqrt[ArcTanh[Tanh[a + b*x]]]])/(64*b^(5/2))
```

3.223.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2600, 2600, 2600, 2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \; 2600$$

$$\frac{1}{4}x^{5/2}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{3}{8}(bx-\text{arctanh}(\tanh(a+bx)))\int x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}dx$$

$$\Big\downarrow \; 2600$$

$$\frac{1}{4}x^{5/2}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{3}{8}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{3}x^{5/2}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{6}(bx-\text{arctanh}(\tanh(a+bx)))\int\frac{x^{3/2}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx\right)$$

$$\Big\downarrow \; 2600$$

$$\frac{1}{4}x^{5/2}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{3}{8}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{3}x^{5/2}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{6}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{3(bx-\text{arctanh}(\tanh(a+bx)))\int\frac{}{\sqrt{\text{arc}}}}{4b}\right.\right.$$

$$\Big\downarrow \; 2600$$

$$\frac{1}{4}x^{5/2}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{3}{8}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{3}x^{5/2}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{6}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{3(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{a}}{}\right.}{}\right.\right.$$

$$\Big\downarrow \text{2596}$$

$$\frac{1}{4}x^{5/2}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{3}{8}\left(\frac{1}{3}x^{5/2}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{6}\left(\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))}{b^{3/2}}+\frac{\sqrt{x}\sqrt{\text{arctanh}}}{b}\right.}{4b}\right.\right.$$

$$\text{arctanh}(\tanh(a+bx)))$$

input
```
Int[x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(-3*(-1/6*(((3*((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(
b*x - ArcTanh[Tanh[a + b*x]]))/b^(3/2) + (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*
x]]])/b)*(b*x - ArcTanh[Tanh[a + b*x]]))/(4*b) + (x^(3/2)*Sqrt[ArcTanh[Tan
h[a + b*x]]])/(2*b))*(b*x - ArcTanh[Tanh[a + b*x]])) + (x^(5/2)*Sqrt[ArcTa
nh[Tanh[a + b*x]]])/3)*(b*x - ArcTanh[Tanh[a + b*x]]))/8 + (x^(5/2)*ArcTan
h[Tanh[a + b*x]]^(3/2))/4
```

3.223.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.223.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{x^{\frac{3}{2}} \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{4b} - \frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\, \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{6b} - \frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\sqrt{x}\right.}{}\right.}{}$
default	$\frac{x^{\frac{3}{2}} \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{4b} - \frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\, \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{6b} - \frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\sqrt{x}\right.}{}\right.}{}$

input
```
int(x^(3/2)*arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^(3/2)*arctanh(tanh(b*x+a))^(5/2)/b-3/4*(arctanh(tanh(b*x+a))-b*x)/b*
(1/6*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)/b-1/6*(arctanh(tanh(b*x+a))-b*x)/b
*(1/4*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+3/4*(arctanh(tanh(b*x+a))-b*x)*(1
/2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/2/b^(1/2)*(arctanh(tanh(b*x+a))-b*
x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2)))))
```

3.223.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.92

$$\int x^{3/2}\operatorname{arctanh}(\tanh(a+bx))^{3/2}\,dx = \left[\frac{3\,a^4\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(16\,b^4x^3+24\,ab^3x^2+2}{128\,b^3}\right.$$

input
```
integrate(x^(3/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
[1/128*(3*a^4*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(16*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x)
)/b^3, -1/64*(3*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) -
(16*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))
/b^3]
```

3.223.6 Sympy [F]

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \int x^{\frac{3}{2}}\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)dx$$

input
```
integrate(x**(3/2)*atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(x**(3/2)*atanh(tanh(a + b*x))**(3/2), x)
```

3.223.7 Maxima [F]

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \int x^{\frac{3}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)*arctanh(tanh(b*x + a))^(3/2), x)
```

3.223.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.83

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{1}{384}\sqrt{2}\left(8\sqrt{2}\left(\sqrt{bx+a}\left(2\left(4x+\frac{a}{b}\right)x-\frac{3a^2}{b^2}\right)\sqrt{x}-\frac{3a^3\log\left(\left|-\sqrt{b}\right.\right.}{}\right.\right.$$

input
```
integrate(x^(3/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
1/384*sqrt(2)*(8*sqrt(2)*(sqrt(b*x + a)*(2*(4*x + a/b)*x - 3*a^2/b^2)*sqrt
(x) - 3*a^3*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(5/2))*a + sqrt(2
)*((2*(4*(6*x + a/b)*x - 5*a^2/b^2)*x + 15*a^3/b^3)*sqrt(b*x + a)*sqrt(x)
+ 15*a^4*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2))*b)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \int x^{3/2}\,\text{atanh}(\tanh(a+b\,x))^{3/2}\,dx$$

input
```
int(x^(3/2)*atanh(tanh(a + b*x))^(3/2),x)
```

output
```
int(x^(3/2)*atanh(tanh(a + b*x))^(3/2), x)
```

3.224 $\int \sqrt{x}\,\text{arctanh}(\tanh(a+bx))^{3/2}\,dx$

3.224.1 Optimal result

Integrand size = 17, antiderivative size = 139

$$\int \sqrt{x}\,\text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))^3}{8b^{3/2}} - \frac{1}{4}x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))} + \frac{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))^2\sqrt{\text{arctanh}}}{8b}$$

output
```
1/8*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^3/b^(3/2)+1/3*x^(3/2)*arctanh(tanh(b*x+a))^(3/2)-1/4*x^(3/2)*(b*x
-arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(1/2)+1/8*(b*x-arctanh(tanh(b*
x+a)))^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b
```

3.224.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.76

$$\int \sqrt{x}\,\text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}(-3b^2x^2+8bx\,\text{arctanh}(\tanh(a+bx))+3\,\text{arctanh}(\tanh(a+bx))^2}{24b} + \frac{(bx-\text{arctanh}(\tanh(a+bx)))^3\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\text{arctanh}(\tanh(a+bx))}\right)}{8b^{3/2}}$$

input
```
Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-3*b^2*x^2 + 8*b*x*ArcTanh[Tanh[a +
 b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2))/(24*b) + ((b*x - ArcTanh[Tanh[a + b*
x]])^3*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*b^(3/2))
```

3.224.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2600, 2600, 2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{2600}$$

$$\frac{1}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{1}{2}(bx-\text{arctanh}(\tanh(a+bx)))\int \sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}dx$$

$$\Big\downarrow \text{2600}$$

$$\frac{1}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{1}{2}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{2}x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{4}(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx\right)$$

$$\Big\downarrow \text{2600}$$

$$\frac{1}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{1}{2}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{2}x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{4}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{}{\sqrt{x}\sqrt{\text{a}}}}{2b}\right.\right.$$

$$\Big\downarrow \text{2596}$$

$$\frac{1}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2} -$$
$$\frac{1}{2}\left(\frac{1}{2}x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))} - \frac{1}{4}\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))}{b^{3/2}} + \frac{\sqrt{x}\sqrt{a}}{}\right.\right.$$
$$\text{arctanh}(\tanh(a+bx)))$$

input
```
Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
-1/2*((-1/4*(((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*
x - ArcTanh[Tanh[a + b*x]]))/b^(3/2) + (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]
]])/b)*(b*x - ArcTanh[Tanh[a + b*x]])) + (x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*
x]]])/2)*(b*x - ArcTanh[Tanh[a + b*x]])) + (x^(3/2)*ArcTanh[Tanh[a + b*x]]
^(3/2))/3
```

3.224.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.224.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\sqrt{x}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{3b} - \frac{(\mathrm{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{4} + \frac{3(\mathrm{arctanh}(\tanh(bx+a))-bx)\left(\sqrt{x}\right.}{}\right.}{3b}$
default	$\frac{\sqrt{x}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{3b} - \frac{(\mathrm{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{4} + \frac{3(\mathrm{arctanh}(\tanh(bx+a))-bx)\left(\sqrt{x}\right.}{}\right.}{3b}$

input
```
int(x^(1/2)*arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)/b-1/3*(arctanh(tanh(b*x+a))-b*x)/b*
(1/4*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+3/4*(arctanh(tanh(b*x+a))-b*x)*(1/
2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/2/b^(1/2)*(arctanh(tanh(b*x+a))-b*x
)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))))
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01

$$\int \sqrt{x}\mathrm{arctanh}(\tanh(a+bx))^{3/2}\,dx = \left[\frac{3\,a^3\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(8\,b^3x^2+14\,ab^2x+3\,a^2b)\sqrt{bx+a}\sqrt{x}}{48\,b^2}, \frac{3\,a^3\sqrt{-}}{}\right.$$

input
```
integrate(x^(1/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(3*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 + 14*a*b^2*x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/24*(3*a^3
*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 + 14*a*b
^2*x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

3.224.6 Sympy [F]

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \int \sqrt{x}\,\text{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)dx$$

input
```
integrate(x**(1/2)*atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(sqrt(x)*atanh(tanh(a + b*x))**(3/2), x)
```

3.224.7 Maxima [F]

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \int \sqrt{x}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(1/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)*arctanh(tanh(b*x + a))^(3/2), x)
```

3.224.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.88

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \frac{1}{48}\sqrt{2}\left(6\sqrt{2}\left(\sqrt{bx+a}\left(2x+\frac{a}{b}\right)\sqrt{x}+\frac{a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}\right)a+\sqrt{2}\left(\sqrt{bx+a}\left(2\right.\right.\right.$$

input
```
integrate(x^(1/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
1/48*sqrt(2)*(6*sqrt(2)*(sqrt(b*x + a)*(2*x + a/b)*sqrt(x) + a^2*log(abs(-
sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2))*a + sqrt(2)*(sqrt(b*x + a)*(2*(
4*x + a/b)*x - 3*a^2/b^2)*sqrt(x) - 3*a^3*log(abs(-sqrt(b)*sqrt(x) + sqrt(
b*x + a)))/b^(5/2))*b)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}\,dx = \int \sqrt{x}\,\text{atanh}(\tanh(a+b\,x))^{3/2}\,dx$$

input
```
int(x^(1/2)*atanh(tanh(a + b*x))^(3/2),x)
```

output
```
int(x^(1/2)*atanh(tanh(a + b*x))^(3/2), x)
```

3.225 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\,dx$

3.225.1 Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\,dx = \frac{3\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))^2}{4\sqrt{b}} - \frac{3}{4}\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))\sqrt{\operatorname{arctanh}(\tanh(a+bx))} + \frac{1}{2}\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^{3/2}$$

output
```
3/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^2/b^(1/2)+1/2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)-3/4*(b*x-arctanh
(tanh(b*x+a)))*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

3.225.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\,dx = \frac{1}{4}\left(\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}(-3bx+5\operatorname{arctanh}(\tanh(a+bx))) + \frac{3(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\right)}{\sqrt{b}}\right)$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/Sqrt[x],x]
```

output
```
(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-3*b*x + 5*ArcTanh[Tanh[a + b*x]])
+ (3*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcT
anh[Tanh[a + b*x]]]])/Sqrt[b])/4
```

3.225.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2600, 2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{2600}$$

$$\frac{1}{2}\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{3}{4}(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{2600}$$

$$\frac{1}{2}\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{3}{4}(bx-\text{arctanh}(\tanh(a+bx)))\left(\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{2}(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx\right)$$

$$\Big\downarrow \text{2596}$$

$$\frac{1}{2}\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{3}{4}\left(\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))}{\sqrt{b}}\right)(bx-\text{arctanh}(\tanh(a+bx)))$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(3/2)/Sqrt[x],x]
```

output
```
(-3*(-((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - Arc
Tanh[Tanh[a + b*x]]))/Sqrt[b]) + Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])*(b*
x - ArcTanh[Tanh[a + b*x]]))/4 + (Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2))/2
```

3.225.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.225.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{2}+\frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2}+\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\ln\left(\sqrt{\right.}}{2\sqrt{}}\right.}{2}$
default	$\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{2}+\frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2}+\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\ln\left(\sqrt{\right.}}{2\sqrt{}}\right.}{2}$

input
```
int(arctanh(tanh(b*x+a))^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+3/2*(arctanh(tanh(b*x+a))-b*x)*(1/2
*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/2/b^(1/2)*(arctanh(tanh(b*x+a))-b*x)
*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2)))
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.18

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\,dx = \left[\frac{3\,a^2\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(2\,b^2x+5\,ab)\sqrt{bx+a}\sqrt{x}}{8\,b}, -\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-(2\,b^2x+5\,ab)\sqrt{bx+a}\sqrt{x}}{4\,b}\right]$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
2*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b, -1/4*(3*a^2*sqrt(-b)*arctan(sqr
t(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x)
)/b]
```

3.225.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\,dx = \int \frac{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{\sqrt{x}}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(3/2)/x**(1/2),x)
```

output
```
Integral(atanh(tanh(a + b*x))**(3/2)/sqrt(x), x)
```

3.225.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\,dx = \int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^(3/2)/sqrt(x), x)
```

3.225.8 Giac [A] (verification not implemented)

Time = 75.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\,dx = -\frac{\sqrt{2}\left(\frac{3\sqrt{2}a^2\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{\sqrt{b}} - \sqrt{(bx+a)b-ab}\sqrt{bx+a}\left(\frac{2\sqrt{2}(bx+a)}{b}+\frac{3\sqrt{2}a}{b}\right)\right)b}{8\,|b|}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(1/2),x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*(3*sqrt(2)*a^2*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a
)*b - a*b)))/sqrt(b) - sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*sqrt(2)*(b
*x + a)/b + 3*sqrt(2)*a/b))*b/abs(b)
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\,dx = \int \frac{\operatorname{atanh}(\tanh(a+b\,x))^{3/2}}{\sqrt{x}}\,dx$$

input
```
int(atanh(tanh(a + b*x))^(3/2)/x^(1/2),x)
```

output
```
int(atanh(tanh(a + b*x))^(3/2)/x^(1/2), x)
```

3.226 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{3/2}} dx$

3.226.1 Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{3/2}} dx = -3\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx))) + 3b\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))} - \frac{2\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{\sqrt{x}}$$

output
```
-3*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b
*x+a)))*b^(1/2)-2*arctanh(tanh(b*x+a))^(3/2)/x^(1/2)+3*b*x^(1/2)*arctanh(t
anh(b*x+a))^(1/2)
```

3.226.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{3/2}} dx = \frac{(3bx-2\operatorname{arctanh}(\tanh(a+bx)))\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{x}} + 3\sqrt{b}(-bx+\operatorname{arctanh}(\tanh(a+bx)))\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\right)$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(3/2),x]
```

output
```
((3*b*x - 2*ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/Sqrt[x]
+ 3*Sqrt[b]*(-(b*x) + ArcTanh[Tanh[a + b*x]])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt
[ArcTanh[Tanh[a + b*x]]]]
```

3.226.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2599, 2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{2599}$$

$$3b\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{x}}dx-\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{2600}$$

$$3b\left(\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{2}(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx\right)-\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{2596}$$

$$3b\left(\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))}{\sqrt{b}}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{\sqrt{x}}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(3/2),x]
```

output
```
3*b*(-((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - Arc
Tanh[Tanh[a + b*x]]))/Sqrt[b]) + Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]) - (
2*ArcTanh[Tanh[a + b*x]]^(3/2))/Sqrt[x]
```

3.226.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.226.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

method	result
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}+\frac{8b\left(\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{4}+\frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a}{2}\right.}{}\right.}{\operatorname{arctanh}(\tanh(bx+a)}$
default	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}+\frac{8b\left(\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{4}+\frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a}{2}\right.}{}\right.}{\operatorname{arctanh}(\tanh(bx+a)}$

input
```
int(arctanh(tanh(b*x+a))^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+8*b/(arct
anh(tanh(b*x+a))-b*x)*(1/4*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+3/4*(arctanh
(tanh(b*x+a))-b*x)*(1/2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/2/b^(1/2)*(ar
ctanh(tanh(b*x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))))
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{3/2}}\,dx = \left[\frac{3\,a\sqrt{b}x\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,\sqrt{bx+a}(bx-2\,a)\sqrt{x}}{2\,x}, -\right.$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*a*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*s
qrt(b*x + a)*(b*x - 2*a)*sqrt(x))/x, -(3*a*sqrt(-b)*x*arctan(sqrt(b*x + a)
*sqrt(-b)/(b*sqrt(x))) - sqrt(b*x + a)*(b*x - 2*a)*sqrt(x))/x]
```

3.226.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{3/2}}\,dx = \int \frac{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(3/2)/x**(3/2),x)
```

output
```
Integral(atanh(tanh(a + b*x))**(3/2)/x**(3/2), x)
```

3.226.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{3/2}}\,dx = \int \frac{\operatorname{artanh}(\tanh(bx+a))^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^(3/2)/x^(3/2), x)
```

3.226.8 Giac [A] (verification not implemented)

Time = 74.97 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{3/2}}\,dx = -\frac{\sqrt{2}\left(\frac{3\sqrt{2}a\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{\sqrt{b}} - \frac{\sqrt{bx+a}\left(\sqrt{2}(bx+a)-3\sqrt{2}a\right)}{\sqrt{(bx+a)b-ab}}\right)b^2}{2\,|b|}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(3*sqrt(2)*a*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*
b - a*b)))/sqrt(b) - sqrt(b*x + a)*(sqrt(2)*(b*x + a) - 3*sqrt(2)*a)/sqrt(
(b*x + a)*b - a*b))*b^2/abs(b)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{3/2}}\,dx = \int \frac{\operatorname{atanh}(\tanh(a+b\,x))^{3/2}}{x^{3/2}}\,dx$$

input
```
int(atanh(tanh(a + b*x))^(3/2)/x^(3/2),x)
```

output
```
int(atanh(tanh(a + b*x))^(3/2)/x^(3/2), x)
```

3.227 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{5/2}} dx$

3.227.1 Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{5/2}} dx = 2b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right) - \frac{2b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{x}} -$$

output
```
2*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))-2/3*arctanh(
tanh(b*x+a))^(3/2)/x^(3/2)-2*b*arctanh(tanh(b*x+a))^(1/2)/x^(1/2)
```

3.227.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{5/2}} dx = -\frac{2\left(3bx\sqrt{\operatorname{arctanh}(\tanh(a+bx))} + \operatorname{arctanh}(\tanh(a+bx))^{3/2} - 3b^{3/2}x^{3/2}\log\left(b\sqrt{x} + \sqrt{b}\sqrt{\operatorname{arctanh}(\tanh(a\right.\right.}{3x^{3/2}}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(5/2),x]
```

output
```
(-2*(3*b*x*Sqrt[ArcTanh[Tanh[a + b*x]]] + ArcTanh[Tanh[a + b*x]]^(3/2) - 3
*b^(3/2)*x^(3/2)*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]]))/(
3*x^(3/2))
```

3.227.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2599, 2599, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{5/2}} \, dx$$

$$\Big\downarrow \text{2599}$$

$$b\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^{3/2}}dx - \frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{2599}$$

$$b\left(b\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx - \frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{x}}\right) - \frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{2596}$$

$$b\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right) - \frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{x}}\right) - \frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{3x^{3/2}}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(5/2),x]
```

output
```
b*(2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]] - (2*
Sqrt[ArcTanh[Tanh[a + b*x]]])/Sqrt[x]) - (2*ArcTanh[Tanh[a + b*x]]^(3/2))/
(3*x^(3/2))
```

3.227.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.227.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(52) = 104$.

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.46

method	result
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}+\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}+\frac{4b\left(\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{4}+\frac{3(\operatorname{arctanh}(\tanh}{}\right.}{}\right.}{3(a}$
default	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}+\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}+\frac{4b\left(\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{4}+\frac{3(\operatorname{arctanh}(\tanh}{}\right.}{}\right.}{3(a}$

input
```
int(arctanh(tanh(b*x+a))^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(5/2)+4/3*b/(
arctanh(tanh(b*x+a))-b*x)*(-1/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(t
anh(b*x+a))^(5/2)+4*b/(arctanh(tanh(b*x+a))-b*x)*(1/4*x^(1/2)*arctanh(tanh
(b*x+a))^(3/2)+3/4*(arctanh(tanh(b*x+a))-b*x)*(1/2*x^(1/2)*arctanh(tanh(b*
x+a))^(1/2)+1/2/b^(1/2)*(arctanh(tanh(b*x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arct
anh(tanh(b*x+a))^(1/2)))))
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{5/2}}\,dx = \left[\frac{3\,b^{\frac{3}{2}}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(4\,bx+a)\sqrt{bx+a}\sqrt{x}}{3\,x^2}, -\frac{2 \cdots}{\cdots}\right.$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*(
4*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2, -2/3*(3*sqrt(-b)*b*x^2*arctan(sqrt(
b*x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2]
```

3.227.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{5/2}}\,dx = \int \frac{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(3/2)/x**(5/2),x)
```

output
```
Integral(atanh(tanh(a + b*x))**(3/2)/x**(5/2), x)
```

3.227.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{5/2}}\,dx = \int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(5/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^(3/2)/x^(5/2), x)
```

3.227.8 Giac [A] (verification not implemented)

Time = 74.83 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{5/2}}\,dx = -\frac{\sqrt{2}\left(3\sqrt{2}b^{\frac{3}{2}}\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)+\frac{\left(4\sqrt{2}(bx+a)b^3-3\sqrt{2}ab^3\right)\sqrt{bx+a}}{((bx+a)b-ab)^{\frac{3}{2}}}\right)b}{3\,|b|}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(5/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(2)*(3*sqrt(2)*b^(3/2)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x
 + a)*b - a*b))) + (4*sqrt(2)*(b*x + a)*b^3 - 3*sqrt(2)*a*b^3)*sqrt(b*x +
a)/((b*x + a)*b - a*b)^(3/2))*b/abs(b)
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{5/2}}\,dx = \int \frac{\operatorname{atanh}(\tanh(a+b\,x))^{3/2}}{x^{5/2}}\,dx$$

input
```
int(atanh(tanh(a + b*x))^(3/2)/x^(5/2),x)
```

output
```
int(atanh(tanh(a + b*x))^(3/2)/x^(5/2), x)
```

3.228 $\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{7/2}}\,dx$

3.228.1 Optimal result

Integrand size = 17, antiderivative size = 35

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{7/2}}\,dx = \frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

output
```
2/5*arctanh(tanh(b*x+a))^(5/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))
```

3.228.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{7/2}}\,dx = \frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{5/2}(5bx-5\text{arctanh}(\tanh(a+bx)))}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(7/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(5/2))/(x^(5/2)*(5*b*x - 5*ArcTanh[Tanh[a + b*x]
]))
```

3.228.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{7/2}}\,dx$$

$$\Big\downarrow \; 2598$$

$$\frac{2\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{5x^{5/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(7/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])
)
```

3.228.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

3.228.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}$	29
default	$-\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}$	29

input
```
int(arctanh(tanh(b*x+a))^(3/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(5/2)
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{7/2}}\,dx = -\frac{2\,(b^2x^2+2\,abx+a^2)\sqrt{bx+a}}{5\,ax^{\frac{5}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/(a*x^(5/2))
```

3.228.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(atanh(tanh(b*x+a))**(3/2)/x**(7/2),x)
```

output
```
Timed out
```

3.228.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.43

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{7/2}}\,dx = -\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,ax^{\frac{5}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(7/2),x, algorithm="maxima")
```

output
```
-2/5*(b*x + a)^(5/2)/(a*x^(5/2))
```

3.228.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{7/2}}\,dx = -\frac{2\,(bx+a)^{\frac{5}{2}}b^6}{5\,((bx+a)b-ab)^{\frac{5}{2}}a|b|}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(7/2),x, algorithm="giac")
```

output
```
-2/5*(b*x + a)^(5/2)*b^6/(((b*x + a)*b - a*b)^(5/2)*a*abs(b))
```

3.228.9 Mupad [B] (verification not implemented)

Time = 4.55 (sec) , antiderivative size = 332, normalized size of antiderivative = 9.49

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{7/2}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{5\,\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-5\,\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+10\,b\,x}+\frac{\cdots}{5\,\ln\cdots}\cdots\right.}{x^{5\cdots}}$$

input
```
int(atanh(tanh(a + b*x))^(3/2)/x^(7/2),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/(5*log(2/(exp(2*
a)*exp(2*b*x) + 1)) - 5*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1)) + 10*b*x) + (4*b^2*x^2)/(5*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 5*log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 10*b*x) - (4*b*x*(log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x))/(5*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 5*log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 10*b*x)))/x^(5/2)
```

3.229 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{9/2}}\,dx$

3.229.1 Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{9/2}}\,dx = \frac{4b\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{35x^{5/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2} + \frac{2\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{7x^{7/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

output
```
4/35*b*arctanh(tanh(b*x+a))^(5/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/7
*arctanh(tanh(b*x+a))^(5/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))
```

3.229.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{9/2}}\,dx = \frac{2(7bx-5\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{35x^{7/2}(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(9/2),x]
```

output
```
(2*(7*b*x - 5*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(5/2))/(35*x^
(7/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

3.229.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{9/2}} \, dx$$

$$\Big\downarrow \text{ 2602}$$

$$\frac{2b\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{7/2}}dx}{7(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{ 2598}$$

$$\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{4b\text{arctanh}(\tanh(a+bx))^{5/2}}{35x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))^2}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(9/2),x]
```

output
```
(4*b*ArcTanh[Tanh[a + b*x]]^(5/2))/(35*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x
]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(7*x^(7/2)*(b*x - ArcTanh[Tanh[a
 + b*x]]))
```

3.229.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.229.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{7(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}+\frac{4b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{35(\mathrm{arctanh}(\tanh(bx+a))-bx)^{2}x^{\frac{5}{2}}}$	59
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{7(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}+\frac{4b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{35(\mathrm{arctanh}(\tanh(bx+a))-bx)^{2}x^{\frac{5}{2}}}$	59

input
```
int(arctanh(tanh(b*x+a))^(3/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(5/2)+4/35*b/
(arctanh(tanh(b*x+a))-b*x)^2/x^(5/2)*arctanh(tanh(b*x+a))^(5/2)
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int \frac{\mathrm{arctanh}(\tanh(a+bx))^{3/2}}{x^{9/2}}\,dx=\frac{2\,(2\,b^3x^3-ab^2x^2-8\,a^2bx-5\,a^3)\sqrt{bx+a}}{35\,a^2x^{\frac{7}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(9/2),x, algorithm="fricas")
```

output
```
2/35*(2*b^3*x^3 - a*b^2*x^2 - 8*a^2*b*x - 5*a^3)*sqrt(b*x + a)/(a^2*x^(7/2
))
```

3.229.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\mathrm{arctanh}(\tanh(a+bx))^{3/2}}{x^{9/2}}\,dx=\text{Timed out}$$

input
```
integrate(atanh(tanh(b*x+a))**(3/2)/x**(9/2),x)
```

output
```
Timed out
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.47

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{9/2}}\,dx = \frac{2\,(2\,b^2x^2-3\,abx-5\,a^2)(bx+a)^{\frac{3}{2}}}{35\,a^2x^{\frac{7}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(9/2),x, algorithm="maxima")
```

output
```
2/35*(2*b^2*x^2 - 3*a*b*x - 5*a^2)*(b*x + a)^(3/2)/(a^2*x^(7/2))
```

3.229.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{9/2}}\,dx = \frac{\sqrt{2}\left(\frac{2\sqrt{2}(bx+a)b^7}{a^2}-\frac{7\sqrt{2}b^7}{a}\right)(bx+a)^{\frac{5}{2}}b}{35\left((bx+a)b-ab\right)^{\frac{7}{2}}|b|}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(9/2),x, algorithm="giac")
```

output
```
1/35*sqrt(2)*(2*sqrt(2)*(b*x + a)*b^7/a^2 - 7*sqrt(2)*b^7/a)*(b*x + a)^(5/
2)*b/(((b*x + a)*b - a*b)^(7/2)*abs(b))
```

3.229.9 Mupad [B] (verification not implemented)

Time = 4.47 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.17

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{9/2}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}-\frac{6\,b\,x}{35}+\frac{\quad}{35}\right.}{x^7}$$

input
```
int(atanh(tanh(a + b*x))^(3/2)/x^(9/2),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/7 - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/7 - (6*b*x)/35 + (4*b^2*x^
2)/(35*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (16*b^3*x^3)/(35*(log(2/(exp(2*a)*exp(
2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*
b*x)^2)))/x^(7/2)
```

3.229. $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{9/2}}\,dx$

3.230 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{11/2}}\,dx$

3.230.1 Optimal result

Integrand size = 17, antiderivative size = 110

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{11/2}}\,dx = \frac{16b^2\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{315x^{5/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^3} + \frac{8b\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{63x^{7/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2} + \frac{2\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{9x^{9/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

output
```
16/315*b^2*arctanh(tanh(b*x+a))^(5/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^3
+8/63*b*arctanh(tanh(b*x+a))^(5/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/
9*arctanh(tanh(b*x+a))^(5/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))
```

3.230.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.60

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{11/2}}\,dx = \frac{2\operatorname{arctanh}(\tanh(a+bx))^{5/2}\left(63b^2x^2-90bx\operatorname{arctanh}(\tanh(a+bx))+35\operatorname{arc}\right.}{315x^{9/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^3}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(11/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(5/2)*(63*b^2*x^2 - 90*b*x*ArcTanh[Tanh[a + b*x]
] + 35*ArcTanh[Tanh[a + b*x]]^2))/(315*x^(9/2)*(b*x - ArcTanh[Tanh[a + b*x
]])^3)
```

3.230.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2602, 2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{11/2}}\,dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{4b\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{9/2}}dx}{9(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{9x^{9/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2602}$$

$$\frac{4b\left(\frac{2b\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{7/2}}dx}{7(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{9(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{9x^{9/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2598}$$

$$\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{9x^{9/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{4b\left(\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{4b\text{arctanh}(\tanh(a+bx))^{5/2}}{35x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))^2}\right)}{9(bx-\text{arctanh}(\tanh(a+bx)))}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(11/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(5/2))/(9*x^(9/2)*(b*x - ArcTanh[Tanh[a + b*x]])
) + (4*b*((4*b*ArcTanh[Tanh[a + b*x]]^(5/2))/(35*x^(5/2)*(b*x - ArcTanh[Ta
nh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(7*x^(7/2)*(b*x - ArcT
anh[Tanh[a + b*x]]))))/(9*(b*x - ArcTanh[Tanh[a + b*x]]))
```

3.230.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.230.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{9(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}-\frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}+\frac{2b \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{35(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}x^{\frac{5}{2}}}\right)}{9(\operatorname{arctanh}(\tanh(bx+a))-bx)}$	105
default	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{9(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}-\frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}+\frac{2b \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{35(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}x^{\frac{5}{2}}}\right)}{9(\operatorname{arctanh}(\tanh(bx+a))-bx)}$	105

input
```
int(arctanh(tanh(b*x+a))^(3/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arctanh(tanh(b*x+a))^(5/2)-8/9*b/(
arctanh(tanh(b*x+a))-b*x)*(-1/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh
(tanh(b*x+a))^(5/2)+2/35*b/(arctanh(tanh(b*x+a))-b*x)^2/x^(5/2)*arctanh(ta
nh(b*x+a))^(5/2))
```

3.230.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.51

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{11/2}}\,dx = -\frac{2\,(8\,b^4x^4 - 4\,ab^3x^3 + 3\,a^2b^2x^2 + 50\,a^3bx + 35\,a^4)\sqrt{bx+a}}{315\,a^3x^{\frac{9}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(11/2),x, algorithm="fricas")
```

output
```
-2/315*(8*b^4*x^4 - 4*a*b^3*x^3 + 3*a^2*b^2*x^2 + 50*a^3*b*x + 35*a^4)*sqr
t(b*x + a)/(a^3*x^(9/2))
```

3.230.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate(atanh(tanh(b*x+a))**(3/2)/x**(11/2),x)
```

output
```
Timed out
```

3.230.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.41

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{11/2}}\,dx = -\frac{2\,(8\,b^3x^3 - 12\,ab^2x^2 + 15\,a^2bx + 35\,a^3)(bx+a)^{\frac{3}{2}}}{315\,a^3x^{\frac{9}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(11/2),x, algorithm="maxima")
```

output
```
-2/315*(8*b^3*x^3 - 12*a*b^2*x^2 + 15*a^2*b*x + 35*a^3)*(b*x + a)^(3/2)/(a
^3*x^(9/2))
```

3.230.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{11/2}}\,dx = -\frac{\sqrt{2}\left(\frac{63\sqrt{2}b^9}{a}+4\left(\frac{2\sqrt{2}(bx+a)b^9}{a^3}-\frac{9\sqrt{2}b^9}{a^2}\right)(bx+a)\right)(bx+a)^{\frac{5}{2}}b}{315\left((bx+a)b-ab\right)^{\frac{9}{2}}|b|}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(11/2),x, algorithm="giac")
```

output
```
-1/315*sqrt(2)*(63*sqrt(2)*b^9/a + 4*(2*sqrt(2)*(b*x + a)*b^9/a^3 - 9*sqrt
(2)*b^9/a^2)*(b*x + a))*(b*x + a)^(5/2)*b/(((b*x + a)*b - a*b)^(9/2)*abs(b
))
```

3.230.9 Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.62

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{11/2}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}-\frac{2\,b\,x}{21}+\frac{}{105}\right.}{}$$

input
```
int(atanh(tanh(a + b*x))^(3/2)/x^(11/2),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/9 - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/9 - (2*b*x)/21 + (4*b^2*x^
2)/(105*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (32*b^3*x^3)/(315*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
2*b*x)^2) + (128*b^4*x^4)/(315*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)))/x^(9/2)
```

3.231 $\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{13/2}}\,dx$

3.231.1 Optimal result

Integrand size = 17, antiderivative size = 148

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{13/2}}\,dx = \frac{32b^3\text{arctanh}(\tanh(a+bx))^{5/2}}{1155x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))^4} + \frac{16b^2\text{arctanh}(\tanh(a+bx))^{5/2}}{231x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))^3} + \frac{4b\text{arctanh}(\tanh(a+bx))^{5/2}}{33x^{9/2}(bx-\text{arctanh}(\tanh(a+bx)))^2} + \frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{11x^{11/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

output
```
32/1155*b^3*arctanh(tanh(b*x+a))^(5/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^
4+16/231*b^2*arctanh(tanh(b*x+a))^(5/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))
^3+4/33*b*arctanh(tanh(b*x+a))^(5/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))^2+
2/11*arctanh(tanh(b*x+a))^(5/2)/x^(11/2)/(b*x-arctanh(tanh(b*x+a)))
```

3.231.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.55

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{13/2}}\,dx = \frac{2\text{arctanh}(\tanh(a+bx))^{5/2}\left(231b^3x^3 - 495b^2x^2\text{arctanh}(\tanh(a+bx)) + 3\right.}{1155x^{11/2}(-bx+\text{arctanh}(t}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(13/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(5/2)*(231*b^3*x^3 - 495*b^2*x^2*ArcTanh[Tanh[a
+ b*x]] + 385*b*x*ArcTanh[Tanh[a + b*x]]^2 - 105*ArcTanh[Tanh[a + b*x]]^3)
)/(1155*x^(11/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

3.231.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2602, 2602, 2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{13/2}}\,dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{6b\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{11/2}}dx}{11(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{11x^{11/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2602}$$

$$\frac{6b\left(\frac{4b\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{9/2}}dx}{9(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{9x^{9/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{11(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{11x^{11/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2602}$$

$$\frac{6b\left(\frac{4b\left(\frac{2b\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{7/2}}dx}{7(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{9(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{9x^{9/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{11(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{11x^{11/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2598}$$

$$\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{11x^{11/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{6b\left(\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{9x^{9/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{4b\left(\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{4b\text{arctanh}(\tanh(a+bx))^{5/2}}{35x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))^2}\right)}{9(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{11(bx-\text{arctanh}(\tanh(a+bx)))}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(13/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(5/2))/(11*x^(11/2)*(b*x - ArcTanh[Tanh[a + b*x]
])) + (6*b*((2*ArcTanh[Tanh[a + b*x]]^(5/2))/(9*x^(9/2)*(b*x - ArcTanh[Tan
h[a + b*x]])) + (4*b*((4*b*ArcTanh[Tanh[a + b*x]]^(5/2))/(35*x^(5/2)*(b*x
- ArcTanh[Tanh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(7*x^(7/2)
*(b*x - ArcTanh[Tanh[a + b*x]]))))/(9*(b*x - ArcTanh[Tanh[a + b*x]]))))/(1
1*(b*x - ArcTanh[Tanh[a + b*x]]))
```

3.231.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.231. $\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{13/2}}\,dx$

3.231.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.02

method	result
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{11(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{11}{2}}}-\frac{12b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{9(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}-\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}+\frac{}{35(a}\right.}{9(\operatorname{arctanh}(\tanh(bx+}\right.}{11(\operatorname{arctanh}(\tanh(bx+a))-bx)}$
default	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{11(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{11}{2}}}-\frac{12b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{9(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}-\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}+\frac{}{35(a}\right.}{9(\operatorname{arctanh}(\tanh(bx+}\right.}{11(\operatorname{arctanh}(\tanh(bx+a))-bx)}$

input
```
int(arctanh(tanh(b*x+a))^(3/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/11/(arctanh(tanh(b*x+a))-b*x)/x^(11/2)*arctanh(tanh(b*x+a))^(5/2)-12/11
*b/(arctanh(tanh(b*x+a))-b*x)*(-1/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arc
tanh(tanh(b*x+a))^(5/2)-4/9*b/(arctanh(tanh(b*x+a))-b*x)*(-1/7/(arctanh(ta
nh(b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(5/2)+2/35*b/(arctanh(tanh(b*
x+a))-b*x)^2/x^(5/2)*arctanh(tanh(b*x+a))^(5/2)))
```

3.231.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.45

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{13/2}}\,dx = \frac{2\,(16\,b^5x^5-8\,ab^4x^4+6\,a^2b^3x^3-5\,a^3b^2x^2-140\,a^4bx-105\,a^5)\sqrt{bx+a}}{1155\,a^4x^{\frac{11}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(13/2),x, algorithm="fricas")
```

output
```
2/1155*(16*b^5*x^5 - 8*a*b^4*x^4 + 6*a^2*b^3*x^3 - 5*a^3*b^2*x^2 - 140*a^4
*b*x - 105*a^5)*sqrt(b*x + a)/(a^4*x^(11/2))
```

3.231.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate(atanh(tanh(b*x+a))**(3/2)/x**(13/2),x)
```

output
```
Timed out
```

3.231.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.38

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{13/2}}\,dx = \frac{2\,(16\,b^4x^4-24\,ab^3x^3+30\,a^2b^2x^2-35\,a^3bx-105\,a^4)(bx+a)^{\frac{3}{2}}}{1155\,a^4x^{\frac{11}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(13/2),x, algorithm="maxima")
```

output
```
2/1155*(16*b^4*x^4 - 24*a*b^3*x^3 + 30*a^2*b^2*x^2 - 35*a^3*b*x - 105*a^4)
*(b*x + a)^(3/2)/(a^4*x^(11/2))
```

3.231.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{13/2}}\,dx = -\frac{\sqrt{2}\left(\frac{231\,\sqrt{2}b^{11}}{a}-2\left(\frac{99\,\sqrt{2}b^{11}}{a^2}+4\left(\frac{2\,\sqrt{2}(bx+a)b^{11}}{a^4}-\frac{11\,\sqrt{2}b^{11}}{a^3}\right)(bx+a)\right)(bx+a)\right)(bx+a)^{\frac{5}{2}}b}{1155\,((bx+a)b-ab)^{\frac{11}{2}}\,|b|}$$

input
```
integrate(arctanh(tanh(b*x+a))^(3/2)/x^(13/2),x, algorithm="giac")
```

output
```
-1/1155*sqrt(2)*(231*sqrt(2)*b^11/a - 2*(99*sqrt(2)*b^11/a^2 + 4*(2*sqrt(2
)*(b*x + a)*b^11/a^4 - 11*sqrt(2)*b^11/a^3)*(b*x + a))*(b*x + a))*(b*x + a
)^(5/2)*b/(((b*x + a)*b - a*b)^(11/2)*abs(b))
```

3.231.9 Mupad [B] (verification not implemented)

Time = 4.63 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.35

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{x^{13/2}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{11}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{11}-\frac{2\,b\,x}{33}+\frac{\cdots}{231\cdots}\right.}{\cdots}$$

input
```
int(atanh(tanh(a + b*x))^(3/2)/x^(13/2),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/11 - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/11 - (2*b*x)/33 + (4*b^2*
x^2)/(231*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (16*b^3*x^3)/(385*(log(2/(exp(2*a)*
exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x)^2) + (128*b^4*x^4)/(1155*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (512*b^5*
x^5)/(1155*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)))/x^(11/2)
```

3.232 $\int \sqrt{x}\,\mathrm{arctanh}(\tanh(a+bx))^{5/2}\,dx$

3.232.1 Optimal result

Integrand size = 17, antiderivative size = 174

$$\int \sqrt{x}\,\mathrm{arctanh}(\tanh(a+bx))^{5/2}\,dx = -\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\mathrm{arctanh}(\tanh(a+bx))}}\right)(bx-\mathrm{arctanh}(\tanh(a+bx)))^4}{64b^{3/2}} + \frac{5}{32}x^{3/2}(bx-\mathrm{arctanh}(\tanh(a+bx)))^2\sqrt{\mathrm{arctanh}(\tanh(a+bx))} - \frac{5\sqrt{x}(bx-\mathrm{arctanh}(\tanh(a+bx)))^3\sqrt{\mathrm{arcta}}}{64b}$$

output
```
-5/64*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tan
h(b*x+a)))^4/b^(3/2)-5/24*x^(3/2)*(b*x-arctanh(tanh(b*x+a)))*arctanh(tanh(
b*x+a))^(3/2)+1/4*x^(3/2)*arctanh(tanh(b*x+a))^(5/2)+5/32*x^(3/2)*(b*x-arc
tanh(tanh(b*x+a)))^2*arctanh(tanh(b*x+a))^(1/2)-5/64*(b*x-arctanh(tanh(b*x
+a)))^3*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b
```

3.232.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.70

$$\int \sqrt{x}\,\mathrm{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{\sqrt{x}\sqrt{\mathrm{arctanh}(\tanh(a+bx))}(15b^3x^3-55b^2x^2\mathrm{arctanh}(\tanh(a+bx))+73bx\,\mathrm{arctanh}(\tanh(a+}{192b} - \frac{5(-bx+\mathrm{arctanh}(\tanh(a+bx)))^4\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\mathrm{arctanh}(\tanh(a+bx))}\right)}{64b^{3/2}}$$

input
```
Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^3*x^3 - 55*b^2*x^2*ArcTanh[Tan
h[a + b*x]] + 73*b*x*ArcTanh[Tanh[a + b*x]]^2 + 15*ArcTanh[Tanh[a + b*x]]^
3))/(192*b) - (5*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4*Log[b*Sqrt[x] + Sqrt[
b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(64*b^(3/2))
```

3.232.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2600, 2600, 2600, 2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{2600}$$

$$\frac{1}{4}x^{3/2}\text{arctanh}(\tanh(a+bx))^{5/2}-\frac{5}{8}(bx-\text{arctanh}(\tanh(a+bx)))\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{2600}$$

$$\frac{1}{4}x^{3/2}\text{arctanh}(\tanh(a+bx))^{5/2}-\frac{5}{8}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{1}{2}(bx-\text{arctanh}(\tanh(a+bx)))\int \sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}dx\right)$$

$$\Big\downarrow \text{2600}$$

$$\frac{1}{4}x^{3/2}\text{arctanh}(\tanh(a+bx))^{5/2}-\frac{5}{8}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{1}{2}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{2}x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{4}(bx\right.\right.$$

$$\Big\downarrow \text{2600}$$

$$\frac{1}{4}x^{3/2}\text{arctanh}(\tanh(a+bx))^{5/2}-\frac{5}{8}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{1}{2}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{2}x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{4}(bx\right.\right.$$

$$\Big\downarrow \text{2596}$$

$$\frac{1}{4}x^{3/2}\text{arctanh}(\tanh(a+bx))^{5/2}-\frac{5}{8}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{3}x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{1}{2}\left(\frac{1}{2}x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{4}\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right.}{b}\right.\right.\right.$$

input
```
Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(x^(3/2)*ArcTanh[Tanh[a + b*x]]^(5/2))/4 - (5*(b*x - ArcTanh[Tanh[a + b*x]
])*(-1/2*((-1/4*(((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]
*(b*x - ArcTanh[Tanh[a + b*x]]))/b^(3/2) + (Sqrt[x]*Sqrt[ArcTanh[Tanh[a +
b*x]]])/b)*(b*x - ArcTanh[Tanh[a + b*x]])) + (x^(3/2)*Sqrt[ArcTanh[Tanh[a
+ b*x]]])/2)*(b*x - ArcTanh[Tanh[a + b*x]])) + (x^(3/2)*ArcTanh[Tanh[a + b
*x]]^(3/2))/3))/8
```

3.232.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.232.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{4b} - \frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{6} + \frac{5(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\sqrt{x}\right.}{}\right.}{}$
default	$\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{4b} - \frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{6} + \frac{5(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\sqrt{x}\right.}{}\right.}{}$

input
```
int(x^(1/2)*arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^(1/2)*arctanh(tanh(b*x+a))^(7/2)/b-1/4*(arctanh(tanh(b*x+a))-b*x)/b*
(1/6*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+5/6*(arctanh(tanh(b*x+a))-b*x)*(1/
4*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+3/4*(arctanh(tanh(b*x+a))-b*x)*(1/2*x
^(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/2/b^(1/2)*(arctanh(tanh(b*x+a))-b*x)*l
n(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2)))))
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.93

$$\int \sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^{5/2}\,dx = \left[\frac{15\,a^4\sqrt{b}\log\left(2\,bx - 2\,\sqrt{bx+a}\sqrt{b}\sqrt{x} + a\right) + 2\,(48\,b^4x^3 + 136\,ab^3x^2 + 118\,a^2b^2x + 15\,a^3b)\sqrt{b}}{384\,b^2}\right.$$

input
```
integrate(x^(1/2)*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
[1/384*(15*a^4*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sq
rt(x))/b^2, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x
))) + (48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a
)*sqrt(x))/b^2]
```

3.232.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)*atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.232.7 Maxima [F]

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \int \sqrt{x}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^(1/2)*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)*arctanh(tanh(b*x + a))^(5/2), x)
```

3.232.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.17

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \frac{1}{384}\sqrt{2}\left(48\sqrt{2}\left(\sqrt{bx+a}\left(2x+\frac{a}{b}\right)\sqrt{x}+\frac{a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}\right)a^2+16\sqrt{2}\left(\sqrt{bx\right.\right.$$

input
```
integrate(x^(1/2)*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
1/384*sqrt(2)*(48*sqrt(2)*(sqrt(b*x + a)*(2*x + a/b)*sqrt(x) + a^2*log(abs
(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2))*a^2 + 16*sqrt(2)*(sqrt(b*x +
a)*(2*(4*x + a/b)*x - 3*a^2/b^2)*sqrt(x) - 3*a^3*log(abs(-sqrt(b)*sqrt(x)
+ sqrt(b*x + a)))/b^(5/2))*a*b + sqrt(2)*((2*(4*(6*x + a/b)*x - 5*a^2/b^2)
*x + 15*a^3/b^3)*sqrt(b*x + a)*sqrt(x) + 15*a^4*log(abs(-sqrt(b)*sqrt(x) +
 sqrt(b*x + a)))/b^(7/2))*b^2)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\text{arctanh}(\tanh(a+bx))^{5/2}\,dx = \int \sqrt{x}\,\text{atanh}(\tanh(a+b\,x))^{5/2}\,dx$$

input
```
int(x^(1/2)*atanh(tanh(a + b*x))^(5/2),x)
```

output
```
int(x^(1/2)*atanh(tanh(a + b*x))^(5/2), x)
```

3.233 $\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{\sqrt{x}}\,dx$

3.233.1 Optimal result

Integrand size = 17, antiderivative size = 136

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{\sqrt{x}}\,dx = -\frac{5\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))^3}{8\sqrt{b}} + \frac{5}{8}\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))^2\sqrt{\text{arctanh}(\tanh(a+bx))} - \frac{5}{12}\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2} + \frac{1}{3}\sqrt{x}\text{arctanh}(\tanh(a+bx))^{5/2}$$

output
```
-5/8*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh
(b*x+a)))^3/b^(1/2)-5/12*(b*x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(
3/2)*x^(1/2)+1/3*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+5/8*(b*x-arctanh(tanh(
b*x+a)))^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

3.233.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.74

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{\sqrt{x}}\,dx = \frac{1}{24}\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\big(15b^2x^2 - 40bx\operatorname{arctanh}(\tanh(a+bx)) + 33\operatorname{arctanh}(\tanh(a+bx))^2\big) + \frac{5(-bx+\operatorname{arctanh}(\tanh(a+bx)))^3\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\right)}{8\sqrt{b}}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/Sqrt[x],x]
```

output
```
(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 40*b*x*ArcTanh[Tanh[a
+ b*x]] + 33*ArcTanh[Tanh[a + b*x]]^2))/24 + (5*(-(b*x) + ArcTanh[Tanh[a +
 b*x]])^3*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*Sqrt[b
])
```

3.233.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2600, 2600, 2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{\sqrt{x}}\,dx$$

$$\Big\downarrow \; 2600$$

$$\frac{1}{3}\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^{5/2} - \frac{5}{6}(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}{\sqrt{x}}dx$$

$$\Big\downarrow \; 2600$$

$$\frac{1}{3}\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^{5/2} - \frac{5}{6}(bx-\operatorname{arctanh}(\tanh(a+bx)))\left(\frac{1}{2}\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^{3/2} - \frac{3}{4}(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{x}}dx\right)$$

$$\Big\downarrow \; 2600$$

$$\frac{1}{3}\sqrt{x}\text{arctanh}(\tanh(a+bx))^{5/2}-\frac{5}{6}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{2}\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{3}{4}(bx-\text{arctanh}(\tanh(a+bx)))\left(\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{2}(bx-\right.\right.$$

$$\Big\downarrow \text{2596}$$

$$\frac{1}{3}\sqrt{x}\text{arctanh}(\tanh(a+bx))^{5/2}-\frac{5}{6}(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{1}{2}\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{3}{4}\left(\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-}{\sqrt{b}}\right.\right.$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(5/2)/Sqrt[x],x]
```

output
```
(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2))/3 - (5*(b*x - ArcTanh[Tanh[a + b*x]
])*((-3*(-((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x -
 ArcTanh[Tanh[a + b*x]]))/Sqrt[b]) + Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])
*(b*x - ArcTanh[Tanh[a + b*x]]))/4 + (Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2)
)/2))/6
```

3.233.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.233. $\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{\sqrt{x}}\,dx$

3.233.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\sqrt{x}\, \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{3} + \frac{5(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\, \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{4} + \frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\sqrt{\right.}}{}\right.}{3}$
default	$\frac{\sqrt{x}\, \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{3} + \frac{5(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\, \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{4} + \frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\sqrt{\right.}}{}\right.}{3}$

input
```
int(arctanh(tanh(b*x+a))^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+5/3*(arctanh(tanh(b*x+a))-b*x)*(1/4
*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+3/4*(arctanh(tanh(b*x+a))-b*x)*(1/2*x^
(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/2/b^(1/2)*(arctanh(tanh(b*x+a))-b*x)*ln
(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))))
```

3.233.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{\sqrt{x}}\,dx = \left[\frac{15\,a^3\sqrt{b}\log\left(2\,bx + 2\,\sqrt{bx+a}\sqrt{b}\sqrt{x} + a\right) + 2\,(8\,b^3x^2 + 26\,ab^2x + 33\,a^2 \cdots}{48\,b}\right.$$
$$\left. -\frac{15\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right) - (8\,b^3x^2 + 26\,ab^2x + 33\,a^2b)\sqrt{bx+a}\sqrt{x}}{24\,b}\right]$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(8*b^3*x^2 + 26*a*b^2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b, -1/24*(15*a
^3*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 26*a
*b^2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b]
```

3.233.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{\sqrt{x}}\,dx = \int \frac{\operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}{\sqrt{x}}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(5/2)/x**(1/2),x)
```

output
```
Integral(atanh(tanh(a + b*x))**(5/2)/sqrt(x), x)
```

3.233.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{\sqrt{x}}\,dx = \int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{\sqrt{x}}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^(5/2)/sqrt(x), x)
```

3.233.8 Giac [A] (verification not implemented)

Time = 74.70 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{\sqrt{x}}\,dx =$$
$$-\frac{\sqrt{2}\left(\frac{15\sqrt{2}a^3\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{\sqrt{b}} - \sqrt{(bx+a)b-ab}\sqrt{bx+a}\left(2\left(bx+a\right)\left(\frac{4\sqrt{2}(bx+a)}{b}+\frac{5\sqrt{2}a}{b}\right)+\frac{15\sqrt{2}a}{b}\right.\right.}{48\left|b\right|}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(1/2),x, algorithm="giac")
```

output
```
-1/48*sqrt(2)*(15*sqrt(2)*a^3*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x +
 a)*b - a*b)))/sqrt(b) - sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a
)*(4*sqrt(2)*(b*x + a)/b + 5*sqrt(2)*a/b) + 15*sqrt(2)*a^2/b))*b/abs(b)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{\sqrt{x}}\,dx = \int \frac{\operatorname{atanh}(\tanh(a+b\,x))^{5/2}}{\sqrt{x}}\,dx$$

input
```
int(atanh(tanh(a + b*x))^(5/2)/x^(1/2),x)
```

output
```
int(atanh(tanh(a + b*x))^(5/2)/x^(1/2), x)
```

3.234 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{3/2}}\,dx$

3.234.1 Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{3/2}}\,dx = \frac{15}{4}\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))^2 - \frac{15}{4}b\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))\sqrt{\operatorname{arctanh}(\tanh(a+bx))} + \frac{5}{2}b\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^{3/2} - \frac{2\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{\sqrt{x}}$$

output
```
15/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh
(b*x+a)))^2*b^(1/2)-2*arctanh(tanh(b*x+a))^(5/2)/x^(1/2)+5/2*b*arctanh(tan
h(b*x+a))^(3/2)*x^(1/2)-15/4*b*(b*x-arctanh(tanh(b*x+a)))*x^(1/2)*arctanh(
tanh(b*x+a))^(1/2)
```

3.234.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{3/2}}\,dx = -\frac{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}(15b^2x^2 - 25bx\operatorname{arctanh}(\tanh(a+bx)) + 8\operatorname{arctanh}(\tanh(a+bx))^2)}{4\sqrt{x}} + \frac{15}{4}\sqrt{b}(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\right)$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(3/2),x]
```

output
```
-1/4*(Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 25*b*x*ArcTanh[Tanh[a + b
*x]] + 8*ArcTanh[Tanh[a + b*x]]^2))/Sqrt[x] + (15*Sqrt[b]*(-(b*x) + ArcTan
h[Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])
/4
```

3.234.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2599, 2600, 2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$5b\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{\sqrt{x}}dx-\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{ 2600}$$

$$5b\left(\frac{1}{2}\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{3}{4}(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{x}}dx\right)-\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{ 2600}$$

$$5b\left(\frac{1}{2}\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{3}{4}(bx-\text{arctanh}(\tanh(a+bx)))\left(\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{2}(bx-\text{arc}\right.\right.$$
$$\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{ 2596}$$

$$5b\left(\frac{1}{2}\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}-\frac{3}{4}\left(\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{ar}}{\sqrt{b}}\right.\right.$$

$$\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{\sqrt{x}}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(3/2),x]
```

output
```
(-2*ArcTanh[Tanh[a + b*x]]^(5/2))/Sqrt[x] + 5*b*((-3*(-((ArcTanh[(Sqrt[b]*
Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tanh[a + b*x]]))/Sqr
t[b]) + Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])*(b*x - ArcTanh[Tanh[a + b*x]
]))/4 + (Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2))/2)
```

3.234.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.234.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.28

method	result
derivativedivides	$-\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}+\frac{12b\left(\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{6}+\frac{5(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a)}{4}\right.}{}\right.}{}$
default	$-\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}+\frac{12b\left(\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{6}+\frac{5(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a)}{4}\right.}{}\right.}{}$

input
```
int(arctanh(tanh(b*x+a))^(5/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(tanh(b*x+a))^(7/2)+12*b/(arc
tanh(tanh(b*x+a))-b*x)*(1/6*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+5/6*(arctan
h(tanh(b*x+a))-b*x)*(1/4*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+3/4*(arctanh(t
anh(b*x+a))-b*x)*(1/2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/2/b^(1/2)*(arct
anh(tanh(b*x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2)))))
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{3/2}}\,dx = \left[\frac{15\,a^2\sqrt{b}x\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(2\,b^2x^2+9\,abx-8\,a^2)\sqrt{}}{8\,x}\right.$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(15*a^2*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(2*b^2*x^2 + 9*a*b*x - 8*a^2)*sqrt(b*x + a)*sqrt(x))/x, -1/4*(15*a^2*sqr
t(-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x^2 + 9*a*b*x
- 8*a^2)*sqrt(b*x + a)*sqrt(x))/x]
```

3.234.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{3/2}}\,dx = \int \frac{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(5/2)/x**(3/2),x)
```

output
```
Integral(atanh(tanh(a + b*x))**(5/2)/x**(3/2), x)
```

3.234.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{3/2}}\,dx = \int \frac{\operatorname{artanh}(\tanh(bx+a))^{\frac{5}{2}}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^(5/2)/x^(3/2), x)
```

3.234.8 Giac [A] (verification not implemented)

Time = 76.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{3/2}}\,dx = -\frac{\sqrt{2}\left(\frac{15\sqrt{2}a^2\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{\sqrt{b}}+\frac{\left(15\sqrt{2}a^2-(bx+a)\left(2\sqrt{2}(bx+a)+5\sqrt{2}a\right)\right)\sqrt{bx+a}}{\sqrt{(bx+a)b-ab}}\right)b^2}{8\,|b|}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(3/2),x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*(15*sqrt(2)*a^2*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x +
a)*b - a*b)))/sqrt(b) + (15*sqrt(2)*a^2 - (b*x + a)*(2*sqrt(2)*(b*x + a) +
 5*sqrt(2)*a))*sqrt(b*x + a)/sqrt((b*x + a)*b - a*b))*b^2/abs(b)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{3/2}}\,dx = \int \frac{\operatorname{atanh}(\tanh(a+b\,x))^{5/2}}{x^{3/2}}\,dx$$

input
```
int(atanh(tanh(a + b*x))^(5/2)/x^(3/2),x)
```

output
```
int(atanh(tanh(a + b*x))^(5/2)/x^(3/2), x)
```

3.235 $\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{5/2}} dx$

3.235.1 Optimal result

Integrand size = 17, antiderivative size = 106

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{5/2}} dx =$$

$$-5b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))+5b^2\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}-$$

output
```
-5*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctan
h(tanh(b*x+a)))-2/3*arctanh(tanh(b*x+a))^(5/2)/x^(3/2)-10/3*b*arctanh(tanh
(b*x+a))^(3/2)/x^(1/2)+5*b^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

3.235.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{5/2}} dx = \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}(15b^2x^2 - 10bx\text{arctanh}(\tanh(a+bx)) - 2\text{arctar}}{3x^{3/2}}$$

$$+ 5b^{3/2}(-bx + \text{arctanh}(\tanh(a+bx)))\log\left(b\sqrt{x} + \sqrt{b}\sqrt{\text{arctanh}(\tanh(a+bx))}\right)$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(5/2),x]
```

output
```
(Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 10*b*x*ArcTanh[Tanh[a + b*x]]
- 2*ArcTanh[Tanh[a + b*x]]^2))/(3*x^(3/2)) + 5*b^(3/2)*(-(b*x) + ArcTanh[T
anh[a + b*x]])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]]
```

3.235.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2599, 2599, 2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{5/2}} \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{5}{3} b \int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{3/2}}dx-\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{2599}$$

$$\frac{5}{3} b \left(3b\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{x}}dx-\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{2600}$$

$$\frac{5}{3} b \left(3b\left(\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{1}{2}(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx\right)-2\text{ar}\right.$$
$$\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{2596}$$

$$\frac{5}{3} b \left(3b\left(\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))}{\sqrt{b}}\right)-2\text{arct}\right.$$
$$\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{3x^{3/2}}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(5/2),x]
```

3.235. $\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{5/2}} \, dx$

output
```
(-2*ArcTanh[Tanh[a + b*x]]^(5/2))/(3*x^(3/2)) + (5*b*(3*b*(-((ArcTanh[(Sqr
t[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tanh[a + b*x]])
)/Sqrt[b]) + Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]) - (2*ArcTanh[Tanh[a + b
*x]]^(3/2))/Sqrt[x]))/3
```

3.235.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.235. $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{5/2}} dx$

3.235.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(82) = 164$.

Time = 0.18 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.90

method	result
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}} + \frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}} + \frac{6b\left(\frac{\sqrt{x}\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{6} + \frac{5(\operatorname{arctanh}(\tanh \cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}$
default	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}} + \frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}} + \frac{6b\left(\frac{\sqrt{x}\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{6} + \frac{5(\operatorname{arctanh}(\tanh \cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}$

input
```
int(arctanh(tanh(b*x+a))^(5/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(7/2)+8/3*b/(
arctanh(tanh(b*x+a))-b*x)*(-1/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(t
anh(b*x+a))^(7/2)+6*b/(arctanh(tanh(b*x+a))-b*x)*(1/6*x^(1/2)*arctanh(tanh
(b*x+a))^(5/2)+5/6*(arctanh(tanh(b*x+a))-b*x)*(1/4*x^(1/2)*arctanh(tanh(b*
x+a))^(3/2)+3/4*(arctanh(tanh(b*x+a))-b*x)*(1/2*x^(1/2)*arctanh(tanh(b*x+a
))^(1/2)+1/2/b^(1/2)*(arctanh(tanh(b*x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh
(tanh(b*x+a))^(1/2))))))
```

3.235.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.30

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{5/2}}\,dx = \left[\frac{15\,ab^{\frac{3}{2}}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(3\,b^2x^2-14\,abx-2\,a^2)}{6\,x^2}\right.$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(15*a*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(3*b^2*x^2 - 14*a*b*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2, -1/3*(15*a*sq
rt(-b)*b*x^2*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (3*b^2*x^2 - 14*
a*b*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2]
```

3.235.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{5/2}}\,dx = \int \frac{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**(5/2)/x**(5/2),x)
```

output
```
Integral(atanh(tanh(a + b*x))**(5/2)/x**(5/2), x)
```

3.235.7 Maxima [F]

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{5/2}}\,dx = \int \frac{\text{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(5/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^(5/2)/x^(5/2), x)
```

3.235.8 Giac [A] (verification not implemented)

Time = 75.41 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{5/2}}\,dx = -\frac{\sqrt{2}\left(15\sqrt{2}ab^{\frac{3}{2}}\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)-\frac{\left(15\sqrt{2}a^2b^3+\left(3\sqrt{2}(bx+a)b^3-20\sqrt{2}ab^3\right)(bx+a)\right)\sqrt{bx+a}}{((bx+a)b-ab)^{\frac{3}{2}}}\right)b}{6\,|b|}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(5/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(2)*(15*sqrt(2)*a*b^(3/2)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((
b*x + a)*b - a*b))) - (15*sqrt(2)*a^2*b^3 + (3*sqrt(2)*(b*x + a)*b^3 - 20*
sqrt(2)*a*b^3)*(b*x + a))*sqrt(b*x + a)/((b*x + a)*b - a*b)^(3/2))*b/abs(b
)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{5/2}}\,dx = \int \frac{\text{atanh}(\tanh(a+b\,x))^{5/2}}{x^{5/2}}\,dx$$

input
```
int(atanh(tanh(a + b*x))^(5/2)/x^(5/2),x)
```

output
```
int(atanh(tanh(a + b*x))^(5/2)/x^(5/2), x)
```

3.236 $\int \frac{\textbf{arctanh}(\tanh(a+bx))^{5/2}}{x^{7/2}}\,dx$

3.236.1 Optimal result

Integrand size = 17, antiderivative size = 93

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{7/2}}\,dx = 2b^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right) - \frac{2b^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{x}}$$

output
```
2*b^(5/2)*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))-2/3*b*arctan
h(tanh(b*x+a))^(3/2)/x^(3/2)-2/5*arctanh(tanh(b*x+a))^(5/2)/x^(5/2)-2*b^2*
arctanh(tanh(b*x+a))^(1/2)/x^(1/2)
```

3.236.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{7/2}}\,dx =$$

$$-\frac{2\left(15b^2x^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))} + 5bx\operatorname{arctanh}(\tanh(a+bx))^{3/2} + 3\operatorname{arctanh}(\tanh(a+bx))^{5/2} - 15b^{5/2}x\right.}{15x^{5/2}}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(7/2),x]
```

output
```
(-2*(15*b^2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]] + 5*b*x*ArcTanh[Tanh[a + b*x]
]^(3/2) + 3*ArcTanh[Tanh[a + b*x]]^(5/2) - 15*b^(5/2)*x^(5/2)*Log[b*Sqrt[x
] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]]))/(15*x^(5/2))
```

3.236.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2599, 2599, 2599, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{7/2}}dx$$

$$\Big\downarrow \text{2599}$$

$$b\int \frac{\text{arctanh}(\tanh(a+bx))^{3/2}}{x^{5/2}}dx-\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{5x^{5/2}}$$

$$\Big\downarrow \text{2599}$$

$$b\left(b\int \frac{\sqrt{\text{arctanh}(\tanh(a+bx))}}{x^{3/2}}dx-\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{3x^{3/2}}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{5x^{5/2}}$$

$$\Big\downarrow \text{2599}$$

$$b\left(b\left(b\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx-\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{x}}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{3x^{3/2}}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{5x^{5/2}}$$

$$\Big\downarrow \text{2596}$$

$$b\left(b\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)-\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{x}}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^{3/2}}{3x^{3/2}}\right)-\frac{2\text{arctanh}(\tanh(a+bx))^{5/2}}{5x^{5/2}}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(7/2),x]
```

output
```
(-2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*x^(5/2)) + b*(b*(2*Sqrt[b]*ArcTanh[(S
qrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]] - (2*Sqrt[ArcTanh[Tanh[a + b
*x]]])/Sqrt[x]) - (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*x^(3/2)))
```

3.236.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.236.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(69) = 138$.

Time = 0.18 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.66

method	result
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}+\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}+\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}+\frac{6b\left(\frac{\sqrt{x}\ a}{}\right.}{}\right.}{}\right.}{}$ ... $\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}+\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}+\frac{6b\left(\frac{\sqrt{x}\ a}{}\right.}{}\right.}{}\right.}{}$

3.236. $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{7/2}}\,dx$

input
```
int(arctanh(tanh(b*x+a))^(5/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(7/2)+4/5*b/(
arctanh(tanh(b*x+a))-b*x)*(-1/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh
(tanh(b*x+a))^(7/2)+4/3*b/(arctanh(tanh(b*x+a))-b*x)*(-1/(arctanh(tanh(b*x
+a))-b*x)/x^(1/2)*arctanh(tanh(b*x+a))^(7/2)+6*b/(arctanh(tanh(b*x+a))-b*x
)*(1/6*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+5/6*(arctanh(tanh(b*x+a))-b*x)*(
1/4*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+3/4*(arctanh(tanh(b*x+a))-b*x)*(1/2
*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/2/b^(1/2)*(arctanh(tanh(b*x+a))-b*x)
*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2)))))))
```

3.236.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.47

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{7/2}}\,dx = \left[\frac{15\,b^{\frac{5}{2}}x^3\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(23\,b^2x^2+11\,abx+3\,a^2)}{15\,x^3}\right.$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(7/2),x, algorithm="fricas")
```

output
```
[1/15*(15*b^(5/2)*x^3*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2
*(23*b^2*x^2 + 11*a*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x))/x^3, -2/15*(15*sqr
t(-b)*b^2*x^3*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (23*b^2*x^2 + 1
1*a*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x))/x^3]
```

3.236.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(atanh(tanh(b*x+a))**(5/2)/x**(7/2),x)
```

output
```
Timed out
```

3.236.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{7/2}}\,dx = \int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(7/2),x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^(5/2)/x^(7/2), x)
```

3.236.8 Giac [A] (verification not implemented)

Time = 75.95 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.19

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{7/2}}\,dx =$$
$$-\frac{\sqrt{2}\left(15\sqrt{2}b^{\frac{5}{2}}\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)+\frac{\left(15\sqrt{2}a^2b^5+\left(23\sqrt{2}(bx+a)b^5-35\sqrt{2}ab^5\right)(bx+a)\right)\sqrt{bx+a}}{((bx+a)b-ab)^{\frac{5}{2}}}\right)b}{15\,|b|}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(7/2),x, algorithm="giac")
```

output
```
-1/15*sqrt(2)*(15*sqrt(2)*b^(5/2)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b
*x + a)*b - a*b))) + (15*sqrt(2)*a^2*b^5 + (23*sqrt(2)*(b*x + a)*b^5 - 35*
sqrt(2)*a*b^5)*(b*x + a))*sqrt(b*x + a)/((b*x + a)*b - a*b)^(5/2))*b/abs(b
)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{7/2}}\,dx = \int \frac{\operatorname{atanh}(\tanh(a+b\,x))^{5/2}}{x^{7/2}}\,dx$$

input
```
int(atanh(tanh(a + b*x))^(5/2)/x^(7/2),x)
```

output
```
int(atanh(tanh(a + b*x))^(5/2)/x^(7/2), x)
```

3.237 $\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{9/2}} dx$

3.237.1 Optimal result

Integrand size = 17, antiderivative size = 35

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{9/2}} dx = \frac{2\text{arctanh}(\tanh(a+bx))^{7/2}}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

output
```
2/7*arctanh(tanh(b*x+a))^(7/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))
```

3.237.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{9/2}} dx = \frac{2\text{arctanh}(\tanh(a+bx))^{7/2}}{x^{7/2}(7bx-7\text{arctanh}(\tanh(a+bx)))}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(9/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(7/2))/(x^(7/2)*(7*b*x - 7*ArcTanh[Tanh[a + b*x]
]))
```

3.237.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{9/2}}\,dx$$

$$\Big\downarrow \text{2598}$$

$$\frac{2\text{arctanh}(\tanh(a+bx))^{7/2}}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(9/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*x^(7/2)*(b*x - ArcTanh[Tanh[a + b*x]])
)
```

3.237.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

3.237.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2\,\text{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7(\text{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}$	29
default	$-\frac{2\,\text{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7(\text{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}$	29

input
```
int(arctanh(tanh(b*x+a))^(5/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(7/2)
```

3.237.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{9/2}}\,dx = -\frac{2\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\sqrt{bx+a}}{7\,ax^{\frac{7}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(9/2),x, algorithm="fricas")
```

output
```
-2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)/(a*x^(7/2))
```

3.237.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(atanh(tanh(b*x+a))**(5/2)/x**(9/2),x)
```

output
```
Timed out
```

3.237.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.43

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{9/2}}\,dx = -\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,ax^{\frac{7}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(9/2),x, algorithm="maxima")
```

output
```
-2/7*(b*x + a)^(7/2)/(a*x^(7/2))
```

3.237.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{9/2}}\,dx = -\frac{2\,(bx+a)^{\frac{7}{2}}b^8}{7\,((bx+a)b-ab)^{\frac{7}{2}}a|b|}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(9/2),x, algorithm="giac")
```

output
```
-2/7*(b*x + a)^(7/2)*b^8/(((b*x + a)*b - a*b)^(7/2)*a*abs(b))
```

3.237.9 Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 396, normalized size of antiderivative = 11.31

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{9/2}}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}\left(\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{14\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-14\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+28\,b\,x}-\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{14\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-14\,\ln\left(\frac{e^{2\,a}\,e^{2}}{e^{2\,a}\,e^{2\,b}}\right.}\right.}{2\,x^{7/2}}$$

input
```
int(atanh(tanh(a + b*x))^(5/2)/x^(9/2),x)
```

output
```
-(2^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/
(exp(2*a)*exp(2*b*x) + 1)))^(1/2)*(log(1/(exp(2*a)*exp(2*b*x) + 1))^3/(14*
log(1/(exp(2*a)*exp(2*b*x) + 1)) - 14*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 28*b*x) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1))^3/(14*log(1/(exp(2*a)*exp(2*b*x) + 1)) - 14*log((exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 28*b*x) + (3*log(1/(exp(2*a)*exp(2*b*x) +
 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(14*log(1/(ex
p(2*a)*exp(2*b*x) + 1)) - 14*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 28*b*x) - (3*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/(14*log(1/(exp(2*a)*exp(2*b*x) + 1))
 - 14*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 28*b*x)))/(2*
x^(7/2))
```

3.238 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{11/2}}\,dx$

3.238.1 Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{11/2}}\,dx = \frac{4b\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{63x^{7/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2} + \frac{2\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{9x^{9/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

output
```
4/63*b*arctanh(tanh(b*x+a))^(7/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/9
*arctanh(tanh(b*x+a))^(7/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))
```

3.238.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{11/2}}\,dx = \frac{2(9bx-7\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{63x^{9/2}(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(11/2),x]
```

output
```
(2*(9*b*x - 7*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(7/2))/(63*x^
(9/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

3.238.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{11/2}}\,dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{2b\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{9/2}}dx}{9(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{2\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{9x^{9/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2598}$$

$$\frac{2\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{9x^{9/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{4b\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{63x^{7/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(11/2),x]
```

output
```
(4*b*ArcTanh[Tanh[a + b*x]]^(7/2))/(63*x^(7/2)*(b*x - ArcTanh[Tanh[a + b*x
]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(9*x^(9/2)*(b*x - ArcTanh[Tanh[a
 + b*x]]))
```

3.238.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.238.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{9(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}+\frac{4b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{63(\mathrm{arctanh}(\tanh(bx+a))-bx)^{2}x^{\frac{7}{2}}}$	59
default	$-\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{9(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}+\frac{4b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{63(\mathrm{arctanh}(\tanh(bx+a))-bx)^{2}x^{\frac{7}{2}}}$	59

input
```
int(arctanh(tanh(b*x+a))^(5/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arctanh(tanh(b*x+a))^(7/2)+4/63*b/
(arctanh(tanh(b*x+a))-b*x)^2/x^(7/2)*arctanh(tanh(b*x+a))^(7/2)
```

3.238.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{\mathrm{arctanh}(\tanh(a+bx))^{5/2}}{x^{11/2}}\,dx=\frac{2\,(2\,b^4x^4-ab^3x^3-15\,a^2b^2x^2-19\,a^3bx-7\,a^4)\sqrt{bx+a}}{63\,a^2x^{\frac{9}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(11/2),x, algorithm="fricas")
```

output
```
2/63*(2*b^4*x^4 - a*b^3*x^3 - 15*a^2*b^2*x^2 - 19*a^3*b*x - 7*a^4)*sqrt(b*
x + a)/(a^2*x^(9/2))
```

3.238.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\mathrm{arctanh}(\tanh(a+bx))^{5/2}}{x^{11/2}}\,dx=\text{Timed out}$$

input
```
integrate(atanh(tanh(b*x+a))**(5/2)/x**(11/2),x)
```

output
```
Timed out
```

3.238.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.47

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{11/2}}\,dx = \frac{2\,(2\,b^2x^2 - 5\,abx - 7\,a^2)(bx+a)^{\frac{5}{2}}}{63\,a^2x^{\frac{9}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(11/2),x, algorithm="maxima")
```

output
```
2/63*(2*b^2*x^2 - 5*a*b*x - 7*a^2)*(b*x + a)^(5/2)/(a^2*x^(9/2))
```

3.238.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{11/2}}\,dx = \frac{\sqrt{2}\left(\frac{2\sqrt{2}(bx+a)b^9}{a^2} - \frac{9\sqrt{2}b^9}{a}\right)(bx+a)^{\frac{7}{2}}b}{63\,((bx+a)b-ab)^{\frac{9}{2}}|b|}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(11/2),x, algorithm="giac")
```

output
```
1/63*sqrt(2)*(2*sqrt(2)*(b*x + a)*b^9/a^2 - 9*sqrt(2)*b^9/a)*(b*x + a)^(7/
2)*b/((((b*x + a)*b - a*b)^(9/2)*abs(b))
```

3.238.9 Mupad [B] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 293, normalized size of antiderivative = 4.07

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{11/2}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{19\,b\,x\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{63} - \right.}{}$$

input
```
int(atanh(tanh(a + b*x))^(5/2)/x^(11/2),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((19*b*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/63 - (10*
b^2*x^2)/21 - (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/18 + (4*b^3*x^3)/(63*(log(2/(exp
(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)) + (16*b^4*x^4)/(63*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)))/x^(9/2)
```

3.238. $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{11/2}}\,dx$

3.239 $\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{13/2}}\,dx$

3.239.1 Optimal result

Integrand size = 17, antiderivative size = 110

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{13/2}}\,dx = \frac{16b^2\text{arctanh}(\tanh(a+bx))^{7/2}}{693x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))^3} + \frac{8b\text{arctanh}(\tanh(a+bx))^{7/2}}{99x^{9/2}(bx-\text{arctanh}(\tanh(a+bx)))^2} + \frac{2\text{arctanh}(\tanh(a+bx))^{7/2}}{11x^{11/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

output
```
16/693*b^2*arctanh(tanh(b*x+a))^(7/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))^3
+8/99*b*arctanh(tanh(b*x+a))^(7/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/
11*arctanh(tanh(b*x+a))^(7/2)/x^(11/2)/(b*x-arctanh(tanh(b*x+a)))
```

3.239.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.60

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{13/2}}\,dx = \frac{2\text{arctanh}(\tanh(a+bx))^{7/2}\left(99b^2x^2-154bx\text{arctanh}(\tanh(a+bx))+63\text{ar}\right.}{693x^{11/2}(bx-\text{arctanh}(\tanh(a+bx)))^3}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(13/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(7/2)*(99*b^2*x^2 - 154*b*x*ArcTanh[Tanh[a + b*x
]] + 63*ArcTanh[Tanh[a + b*x]]^2))/(693*x^(11/2)*(b*x - ArcTanh[Tanh[a + b
*x]])^3)
```

3.239.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2602, 2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{13/2}}\,dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{4b\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{11/2}}dx}{11(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{2\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{11x^{11/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2602}$$

$$\frac{4b\left(\frac{2b\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{9/2}}dx}{9(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{2\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{9x^{9/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}\right)}{11(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{2\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{11x^{11/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2598}$$

$$\frac{2\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{11x^{11/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{4b\left(\frac{2\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{9x^{9/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{4b\operatorname{arctanh}(\tanh(a+bx))^{7/2}}{63x^{7/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2}\right)}{11(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(13/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(7/2))/(11*x^(11/2)*(b*x - ArcTanh[Tanh[a + b*x]
])) + (4*b*((4*b*ArcTanh[Tanh[a + b*x]]^(7/2))/(63*x^(7/2)*(b*x - ArcTanh[
Tanh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(9*x^(9/2)*(b*x - Ar
cTanh[Tanh[a + b*x]]))))/(11*(b*x - ArcTanh[Tanh[a + b*x]]))
```

3.239.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.239.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{11(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{11}{2}}}-\frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{9(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}+\frac{2b \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{63(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}x^{\frac{7}{2}}}\right)}{11(\operatorname{arctanh}(\tanh(bx+a))-bx)}$	105
default	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{11(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{11}{2}}}-\frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{9(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}+\frac{2b \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{63(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}x^{\frac{7}{2}}}\right)}{11(\operatorname{arctanh}(\tanh(bx+a))-bx)}$	105

input
```
int(arctanh(tanh(b*x+a))^(5/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/11/(arctanh(tanh(b*x+a))-b*x)/x^(11/2)*arctanh(tanh(b*x+a))^(7/2)-8/11*
b/(arctanh(tanh(b*x+a))-b*x)*(-1/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arct
anh(tanh(b*x+a))^(7/2)+2/63*b/(arctanh(tanh(b*x+a))-b*x)^2/x^(7/2)*arctanh
(tanh(b*x+a))^(7/2))
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.61

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{13/2}}\,dx = -\frac{2\,(8\,b^5x^5 - 4\,ab^4x^4 + 3\,a^2b^3x^3 + 113\,a^3b^2x^2 + 161\,a^4bx + 63\,a^5)\sqrt{bx+a}}{693\,a^3x^{\frac{11}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(13/2),x, algorithm="fricas")
```

output
```
-2/693*(8*b^5*x^5 - 4*a*b^4*x^4 + 3*a^2*b^3*x^3 + 113*a^3*b^2*x^2 + 161*a^
4*b*x + 63*a^5)*sqrt(b*x + a)/(a^3*x^(11/2))
```

3.239.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate(atanh(tanh(b*x+a))**(5/2)/x**(13/2),x)
```

output
```
Timed out
```

3.239.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.41

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{13/2}}\,dx = -\frac{2\,(8\,b^3x^3 - 20\,ab^2x^2 + 35\,a^2bx + 63\,a^3)(bx+a)^{\frac{5}{2}}}{693\,a^3x^{\frac{11}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(13/2),x, algorithm="maxima")
```

output
```
-2/693*(8*b^3*x^3 - 20*a*b^2*x^2 + 35*a^2*b*x + 63*a^3)*(b*x + a)^(5/2)/(a
^3*x^(11/2))
```

3.239.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{13/2}}\,dx = -\frac{\sqrt{2}\left(\frac{99\sqrt{2}b^{11}}{a}+4\left(\frac{2\sqrt{2}(bx+a)b^{11}}{a^3}-\frac{11\sqrt{2}b^{11}}{a^2}\right)(bx+a)\right)(bx+a)^{\frac{7}{2}}b}{693\left((bx+a)b-ab\right)^{\frac{11}{2}}|b|}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(13/2),x, algorithm="giac")
```

output
```
-1/693*sqrt(2)*(99*sqrt(2)*b^11/a + 4*(2*sqrt(2)*(b*x + a)*b^11/a^3 - 11*s
qrt(2)*b^11/a^2)*(b*x + a))*(b*x + a)^(7/2)*b/(((b*x + a)*b - a*b)^(11/2)*
abs(b))
```

3.239.9 Mupad [B] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.21

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{13/2}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{23\,b\,x\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{99}-\right.}{}$$

input
```
int(atanh(tanh(a + b*x))^(5/2)/x^(13/2),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((23*b*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/99 - (226
*b^2*x^2)/693 - (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/22 + (4*b^3*x^3)/(231*(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + 2*b*x)) + (32*b^4*x^4)/(693*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (128
*b^5*x^5)/(693*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)))/x^(11/2)
```

3.240 $\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{15/2}} dx$

3.240.1 Optimal result

Integrand size = 17, antiderivative size = 148

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{15/2}} dx = \frac{32b^3\text{arctanh}(\tanh(a+bx))^{7/2}}{3003x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))^4} + \frac{16b^2\text{arctanh}(\tanh(a+bx))^{7/2}}{429x^{9/2}(bx-\text{arctanh}(\tanh(a+bx)))^3} + \frac{12b\text{arctanh}(\tanh(a+bx))^{7/2}}{143x^{11/2}(bx-\text{arctanh}(\tanh(a+bx)))^2} + \frac{2\text{arctanh}(\tanh(a+bx))^{7/2}}{13x^{13/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

output
```
32/3003*b^3*arctanh(tanh(b*x+a))^(7/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))^
4+16/429*b^2*arctanh(tanh(b*x+a))^(7/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))
^3+12/143*b*arctanh(tanh(b*x+a))^(7/2)/x^(11/2)/(b*x-arctanh(tanh(b*x+a)))
^2+2/13*arctanh(tanh(b*x+a))^(7/2)/x^(13/2)/(b*x-arctanh(tanh(b*x+a)))
```

3.240.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.55

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{15/2}} dx = \frac{2\text{arctanh}(\tanh(a+bx))^{7/2} (429b^3x^3 - 1001b^2x^2\text{arctanh}(\tanh(a+bx)) +}{3003x^{13/2}(-bx + \text{arctanh}(\text{t}}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(15/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(7/2)*(429*b^3*x^3 - 1001*b^2*x^2*ArcTanh[Tanh[a
 + b*x]] + 819*b*x*ArcTanh[Tanh[a + b*x]]^2 - 231*ArcTanh[Tanh[a + b*x]]^3
))/(3003*x^(13/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

3.240.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2602, 2602, 2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{15/2}}\,dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{6b\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{13/2}}dx}{13(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{7/2}}{13x^{13/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2602}$$

$$\frac{6b\left(\frac{4b\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{11/2}}dx}{11(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{7/2}}{11x^{11/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{13(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{7/2}}{13x^{13/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2602}$$

$$\frac{6b\left(\frac{4b\left(\frac{2b\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{9/2}}dx}{9(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{7/2}}{9x^{9/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{11(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{7/2}}{11x^{11/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{13(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\text{arctanh}(\tanh(a+bx))^{7/2}}{13x^{13/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2598}$$

$$\frac{2\text{arctanh}(\tanh(a+bx))^{7/2}}{13x^{13/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{6b\left(\frac{2\text{arctanh}(\tanh(a+bx))^{7/2}}{11x^{11/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{4b\left(\frac{2\text{arctanh}(\tanh(a+bx))^{7/2}}{9x^{9/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{4b\text{arctanh}(\tanh(a+bx))^{7/2}}{63x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))^2}\right)}{11(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{13(bx-\text{arctanh}(\tanh(a+bx)))}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(15/2),x]
```

output
```
(2*ArcTanh[Tanh[a + b*x]]^(7/2))/(13*x^(13/2)*(b*x - ArcTanh[Tanh[a + b*x]
])) + (6*b*((2*ArcTanh[Tanh[a + b*x]]^(7/2))/(11*x^(11/2)*(b*x - ArcTanh[T
anh[a + b*x]])) + (4*b*((4*b*ArcTanh[Tanh[a + b*x]]^(7/2))/(63*x^(7/2)*(b*
x - ArcTanh[Tanh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(9*x^(9/
2)*(b*x - ArcTanh[Tanh[a + b*x]]))))/(11*(b*x - ArcTanh[Tanh[a + b*x]]))))
/(13*(b*x - ArcTanh[Tanh[a + b*x]]))
```

3.240.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.240. $\int \frac{\text{arctanh}(\tanh(a+bx))^{5/2}}{x^{15/2}}\,dx$

3.240.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.02

method	result
derivativedivides	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{13(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{13}{2}}}-\frac{12b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{11(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{11}{2}}}-\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{9(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}+\frac{}{63}\right.}{11(\operatorname{arctanh}(\tanh(b}\right.}{13(\operatorname{arctanh}(\tanh(bx+a))-bx)}$
default	$-\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{13(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{13}{2}}}-\frac{12b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{11(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{11}{2}}}-\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{9(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}+\frac{}{63}\right.}{11(\operatorname{arctanh}(\tanh(b}\right.}{13(\operatorname{arctanh}(\tanh(bx+a))-bx)}$

input
```
int(arctanh(tanh(b*x+a))^(5/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

output
```
-2/13/(arctanh(tanh(b*x+a))-b*x)/x^(13/2)*arctanh(tanh(b*x+a))^(7/2)-12/13
*b/(arctanh(tanh(b*x+a))-b*x)*(-1/11/(arctanh(tanh(b*x+a))-b*x)/x^(11/2)*a
rctanh(tanh(b*x+a))^(7/2)-4/11*b/(arctanh(tanh(b*x+a))-b*x)*(-1/9/(arctanh
(tanh(b*x+a))-b*x)/x^(9/2)*arctanh(tanh(b*x+a))^(7/2)+2/63*b/(arctanh(tanh
(b*x+a))-b*x)^2/x^(7/2)*arctanh(tanh(b*x+a))^(7/2)))
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.53

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{15/2}}\,dx = \frac{2\,(16\,b^6x^6 - 8\,ab^5x^5 + 6\,a^2b^4x^4 - 5\,a^3b^3x^3 - 371\,a^4b^2x^2 - 567\,a^5bx - 231}{3003\,a^4x^{\frac{13}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(15/2),x, algorithm="fricas")
```

output
```
2/3003*(16*b^6*x^6 - 8*a*b^5*x^5 + 6*a^2*b^4*x^4 - 5*a^3*b^3*x^3 - 371*a^4
*b^2*x^2 - 567*a^5*b*x - 231*a^6)*sqrt(b*x + a)/(a^4*x^(13/2))
```

3.240.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate(atanh(tanh(b*x+a))**(5/2)/x**(15/2),x)
```

output
```
Timed out
```

3.240.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.38

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{15/2}}\,dx = \frac{2\,(16\,b^4x^4-40\,ab^3x^3+70\,a^2b^2x^2-105\,a^3bx-231\,a^4)(bx+a)^{\frac{5}{2}}}{3003\,a^4x^{\frac{13}{2}}}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(15/2),x, algorithm="maxima")
```

output
```
2/3003*(16*b^4*x^4 - 40*a*b^3*x^3 + 70*a^2*b^2*x^2 - 105*a^3*b*x - 231*a^4
)*(b*x + a)^(5/2)/(a^4*x^(13/2))
```

3.240.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{15/2}}\,dx = -\frac{\sqrt{2}\left(\frac{429\,\sqrt{2}b^{13}}{a}-2\left(\frac{143\,\sqrt{2}b^{13}}{a^2}+4\left(\frac{2\,\sqrt{2}(bx+a)b^{13}}{a^4}-\frac{13\,\sqrt{2}b^{13}}{a^3}\right)(bx+a)\right)(bx+a)\right)(bx+a)^{\frac{7}{2}}b}{3003\,((bx+a)b-ab)^{\frac{13}{2}}\,|b|}$$

input
```
integrate(arctanh(tanh(b*x+a))^(5/2)/x^(15/2),x, algorithm="giac")
```

output
```
-1/3003*sqrt(2)*(429*sqrt(2)*b^13/a - 2*(143*sqrt(2)*b^13/a^2 + 4*(2*sqrt(
2)*(b*x + a)*b^13/a^4 - 13*sqrt(2)*b^13/a^3)*(b*x + a))*(b*x + a))*(b*x +
a)^(7/2)*b/(((b*x + a)*b - a*b)^(13/2)*abs(b))
```

3.240.9 Mupad [B] (verification not implemented)

Time = 4.50 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.79

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}{x^{15/2}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{27\,b\,x\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{143}-\right.}{}$$

input
```
int(atanh(tanh(a + b*x))^(5/2)/x^(15/2),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((27*b*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/143 - (10
6*b^2*x^2)/429 - (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/26 + (20*b^3*x^3)/(3003*(log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x)) + (16*b^4*x^4)/(1001*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) +
(128*b^5*x^5)/(3003*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (512*b^6*x^6)/(3003*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)^4)))/x^(13/2)
```

3.241 $\int \frac{x^{5/2}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx$

3.241.1 Optimal result

Integrand size = 17, antiderivative size = 145

$$\int \frac{x^{5/2}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx = \frac{5\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))^3}{8b^{7/2}} + \frac{x^{5/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}{3b} + \frac{5x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}{12b^2} + \frac{5\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))^2\sqrt{\text{arctanh}(\tanh(a+bx))}}{8b^3}$$

output
```
5/8*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^3/b^(7/2)+1/3*x^(5/2)*arctanh(tanh(b*x+a))^(1/2)/b+5/12*x^(3/2)*(
b*x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(1/2)/b^2+5/8*(b*x-arctanh(
tanh(b*x+a)))^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^3
```

3.241.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.72

$$\int \frac{x^{5/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}(33b^2x^2-40bx\operatorname{arctanh}(\tanh(a+bx))+15a}{24b^3}$$
$$+\frac{5(bx-\operatorname{arctanh}(\tanh(a+bx)))^3\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\right)}{8b^{7/2}}$$

input
```
Integrate[x^(5/2)/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(33*b^2*x^2 - 40*b*x*ArcTanh[Tanh[a
+ b*x]] + 15*ArcTanh[Tanh[a + b*x]]^2))/(24*b^3) + (5*(b*x - ArcTanh[Tanh[
a + b*x]])^3*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*b^(
7/2))
```

3.241.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2600, 2600, 2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}dx$$
$$\Big\downarrow \text{ 2600}$$
$$\frac{5(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{x^{3/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}dx}{6b}+\frac{x^{5/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{3b}$$
$$\Big\downarrow \text{ 2600}$$
$$\frac{5(bx-\operatorname{arctanh}(\tanh(a+bx)))\left(\frac{3(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}dx}{4b}+\frac{x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{2b}\right)}{6b}+\frac{x^{5/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{3b}$$

$\downarrow$ 2600

$$\frac{5(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{3(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{2b}+\frac{\sqrt{x}\sqrt{\text{arc}}}{}\right.}{4b}\right.}{6b}$$

$$\frac{x^{5/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}{3b}$$

$\downarrow$ 2596

$$\frac{5\left(\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))}{b^{3/2}}+\frac{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}\right)(bx-\text{arctanh}(\tanh(a+bx)))}{4b}+\frac{x^{3/2}\sqrt{}}{}\right.}{6b}$$

$$\frac{x^{5/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}{3b}$$

input
```
Int[x^(5/2)/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(5*((3*((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - Ar
cTanh[Tanh[a + b*x]]))/b^(3/2) + (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b)
*(b*x - ArcTanh[Tanh[a + b*x]]))/(4*b) + (x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*
x]]])/(2*b))*(b*x - ArcTanh[Tanh[a + b*x]]))/(6*b) + (x^(5/2)*Sqrt[ArcTanh
[Tanh[a + b*x]]])/(3*b)
```

3.241. $\int \frac{x^{5/2}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx$

3.241.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.241.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{x^{\frac{5}{2}}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3b}-\frac{5(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{x^{\frac{3}{2}}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{4b}-\frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\sqrt{x}\ldots\right.}{\ldots}\right)}{3b}$
default	$\frac{x^{\frac{5}{2}}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3b}-\frac{5(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{x^{\frac{3}{2}}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{4b}-\frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\sqrt{x}\ldots\right.}{\ldots}\right)}{3b}$

input
```
int(x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(5/2)*arctanh(tanh(b*x+a))^(1/2)/b-5/3*(arctanh(tanh(b*x+a))-b*x)/b*
(1/4*x^(3/2)*arctanh(tanh(b*x+a))^(1/2)/b-3/4*(arctanh(tanh(b*x+a))-b*x)/b
*(1/2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b-1/2*(arctanh(tanh(b*x+a))-b*x)/
b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))))
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.97

$$\int \frac{x^{5/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \left[\frac{15\,a^3\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(8\,b^3x^2-10\,ab^2x+15\,a^2}{48\,b^4}\right.$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4, 1/24*(15*
a^3*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 - 10*
a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4]
```

3.241.6 Sympy [F]

$$\int \frac{x^{5/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

input
```
integrate(x**(5/2)/atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(x**(5/2)/sqrt(atanh(tanh(a + b*x))), x)
```

3.241.7 Maxima [F]

$$\int \frac{x^{5/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}\,dx$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/sqrt(arctanh(tanh(b*x + a))), x)
```

3.241.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.44

$$\int \frac{x^{5/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{1}{24}\sqrt{bx+a}\left(2\,x\left(\frac{4\,x}{b}-\frac{5\,a}{b^2}\right)+\frac{15\,a^2}{b^3}\right)\sqrt{x} + \frac{5\,a^3\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x + a)*(2*x*(4*x/b - 5*a/b^2) + 15*a^2/b^3)*sqrt(x) + 5/8*a^3*
log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{x^{5/2}}{\sqrt{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}}\,dx$$

input
```
int(x^(5/2)/atanh(tanh(a + b*x))^(1/2),x)
```

output
```
int(x^(5/2)/atanh(tanh(a + b*x))^(1/2), x)
```

3.242 $\int \frac{x^{3/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$

3.242.1 Optimal result

Integrand size = 17, antiderivative size = 107

$$\int \frac{x^{3/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{3\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))^2}{4b^{5/2}} + \frac{x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{2b} + \frac{3\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{4b^2}$$

output
```
3/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^2/b^(5/2)+1/2*x^(3/2)*arctanh(tanh(b*x+a))^(1/2)/b+3/4*(b*x-arcta
nh(tanh(b*x+a)))*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^2
```

3.242.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.82

$$\int \frac{x^{3/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{\sqrt{b}\sqrt{x}(5bx-3\operatorname{arctanh}(\tanh(a+bx)))\sqrt{\operatorname{arctanh}(\tanh(a+bx))}+3(-bx}{4b^{5/}}$$

input
```
Integrate[x^(3/2)/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(Sqrt[b]*Sqrt[x]*(5*b*x - 3*ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a +
b*x]]] + 3*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqr
t[ArcTanh[Tanh[a + b*x]]]])/(4*b^(5/2))
```

3.242.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2600, 2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx$$

$$\Big\downarrow \text{2600}$$

$$\frac{3(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{4b}+\frac{x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}{2b}$$

$$\Big\downarrow \text{2600}$$

$$\frac{3(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{2b}+\frac{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}\right)}{4b}+\frac{x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}{2b}$$

$$\Big\downarrow \text{2596}$$

$$\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))}{b^{3/2}}+\frac{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}\right)(bx-\text{arctanh}(\tanh(a+bx)}{4b}+\frac{x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}{2b}$$

input
```
Int[x^(3/2)/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(3*((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTan
h[Tanh[a + b*x]]))/b^(3/2) + (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b)*(b*
x - ArcTanh[Tanh[a + b*x]]))/(4*b) + (x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]
)/(2*b)
```

3.242.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.242.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2b} - \frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2b} - \frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\ln(\sqrt{b}}{2b^{\frac{3}{2}}}\right.}{2b}$
default	$\frac{x^{\frac{3}{2}}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2b} - \frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2b} - \frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\ln(\sqrt{b}}{2b^{\frac{3}{2}}}\right.}{2b}$

input
```
int(x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(3/2)*arctanh(tanh(b*x+a))^(1/2)/b-3/2*(arctanh(tanh(b*x+a))-b*x)/b*
(1/2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b-1/2*(arctanh(tanh(b*x+a))-b*x)/b
^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2)))
```

3.242.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.11

$$\int \frac{x^{3/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \left[\frac{3\,a^2\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(2\,b^2x-3\,ab)\sqrt{bx+a}\sqrt{x}}{8\,b^3},\ -\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-(2\,b^2x-3\,ab)\sqrt{bx+a}\sqrt{x}}{4\,b^3}\right]$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
2*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/4*(3*a^2*sqrt(-b)*arctan(s
qrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(
x))/b^3]
```

3.242.6 Sympy [F]

$$\int \frac{x^{3/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

input
```
integrate(x**(3/2)/atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(x**(3/2)/sqrt(atanh(tanh(a + b*x))), x)
```

3.242.7 Maxima [F]

$$\int \frac{x^{3/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}\,dx$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/sqrt(arctanh(tanh(b*x + a))), x)
```

3.242.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.49

$$\int \frac{x^{3/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{1}{4}\sqrt{bx+a}\sqrt{x}\left(\frac{2\,x}{b}-\frac{3\,a}{b^2}\right)-\frac{3\,a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{4\,b^{\frac{5}{2}}}$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*x + a)*sqrt(x)*(2*x/b - 3*a/b^2) - 3/4*a^2*log(abs(-sqrt(b)*sqr
t(x) + sqrt(b*x + a)))/b^(5/2)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{x^{3/2}}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}}\,dx$$

input
```
int(x^(3/2)/atanh(tanh(a + b*x))^(1/2),x)
```

output
```
int(x^(3/2)/atanh(tanh(a + b*x))^(1/2), x)
```

3.243 $\int \frac{\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} \, dx$

3.243.1 Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))}{b^{3/2}} + \frac{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b}$$

output
```
arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b*x+
a)))/b^(3/2)+x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b
```

3.243.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} \, dx = \frac{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b} - \frac{(-bx+\operatorname{arctanh}(\tanh(a+bx)))\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\right)}{b^{3/2}}$$

input
```
Integrate[Sqrt[x]/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - ((-(b*x) + ArcTanh[Tanh[a + b*x
]])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(3/2)
```

3.243.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx$$

$$\Big\downarrow \text{ 2600}$$

$$\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{2b}+\frac{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}$$

$$\Big\downarrow \text{ 2596}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))}{b^{3/2}}+\frac{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}$$

input
```
Int[Sqrt[x]/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

output
```
(ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Ta
nh[a + b*x]]))/b^(3/2) + (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b
```

3.243.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.243.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{b}-\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh(bx+a))}\right)}{b^{\frac{3}{2}}}$	53
default	$\frac{\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{b}-\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh(bx+a))}\right)}{b^{\frac{3}{2}}}$	53

input
```
int(x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b-(arctanh(tanh(b*x+a))-b*x)/b^(3/2)*ln
(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$$
$$=\left[\frac{a\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,\sqrt{bx+a}b\sqrt{x}}{2\,b^2},\ \frac{a\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}b\sqrt{x}}{b^2}\right]$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(a*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(
b*x + a)*b*sqrt(x))/b^2, (a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt
(x))) + sqrt(b*x + a)*b*sqrt(x))/b^2]
```

3.243.6 Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{\sqrt{x}}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

input
```
integrate(x**(1/2)/atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(x)/sqrt(atanh(tanh(a + b*x))), x)
```

3.243.7 Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{\sqrt{x}}{\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}\,dx$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/sqrt(arctanh(tanh(b*x + a))), x)
```

3.243.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{a\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}} + \frac{\sqrt{bx+a}\sqrt{x}}{b}$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
a*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2) + sqrt(b*x + a)*sqrt(
x)/b
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{\sqrt{x}}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}}\,dx$$

input
```
int(x^(1/2)/atanh(tanh(a + b*x))^(1/2),x)
```

output
```
int(x^(1/2)/atanh(tanh(a + b*x))^(1/2), x)
```

3.244 $\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx$

3.244.1 Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{\sqrt{b}}$$

output
```
2*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))/b^(1/2)
```

3.244.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\text{arctanh}(\tanh(a+bx))}\right)}{\sqrt{b}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
(2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/Sqrt[b]
```

3.244.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx$$

$$\Big\downarrow \text{2596}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{\sqrt{b}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
(2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]])/Sqrt[b]
```

3.244.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

3.244.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{2\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\text{arctanh}(\tanh(bx+a))}\right)}{\sqrt{b}}$	24
default	$\frac{2\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\text{arctanh}(\tanh(bx+a))}\right)}{\sqrt{b}}$	24

input
```
int(1/x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))/b^(1/2)
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \left[\frac{\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

input
```
integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a)/sqrt(b), -2*sqrt(-b)*arc
tan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))/b]
```

3.244.6 Sympy [F]

$$\int \frac{1}{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{1}{\sqrt{x}\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

input
```
integrate(1/x**(1/2)/atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(1/(sqrt(x)*sqrt(atanh(tanh(a + b*x)))), x)
```

3.244.7 Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{1}{\sqrt{x}\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}\,dx$$

input
```
integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x)*sqrt(arctanh(tanh(b*x + a)))), x)
```

3.244.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = -\frac{2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{\sqrt{b}}$$

input
```
integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/sqrt(b)
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{1}{\sqrt{x}\,\sqrt{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}}\,dx$$

input
```
int(1/(x^(1/2)*atanh(tanh(a + b*x))^(1/2)),x)
```

output
```
int(1/(x^(1/2)*atanh(tanh(a + b*x))^(1/2)), x)
```

3.245 $\int \frac{1}{x^{3/2}\sqrt{\mathbf{arctanh}(\tanh(a+bx))}}\,dx$

3.245.1 Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \frac{1}{x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

output
```
2*arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))/x^(1/2)
```

3.245.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = -\frac{2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{\sqrt{x}(-bx+\operatorname{arctanh}(\tanh(a+bx)))}$$

input
```
Integrate[1/(x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
(-2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]
]))
```

3.245.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx$$

$$\Big\downarrow \text{2598}$$

$$\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))}$$

input
```
Int[1/(x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
(2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))
```

3.245.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

3.245.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{2\sqrt{\text{arctanh}(\tanh(bx+a))}}{(\text{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}$	29
default	$-\frac{2\sqrt{\text{arctanh}(\tanh(bx+a))}}{(\text{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}$	29

input
```
int(1/x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = -\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(b*x + a)/(a*sqrt(x))
```

3.245.6 Sympy [F]

$$\int \frac{1}{x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

input
```
integrate(1/x**(3/2)/atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(1/(x**(3/2)*sqrt(atanh(tanh(a + b*x)))), x)
```

3.245.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = -\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(b*x + a)/(a*sqrt(x))
```

3.245.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{4\sqrt{b}}{\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a}$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
4*sqrt(b)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)
```

3.245.9 Mupad [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.06

$$\int \frac{1}{x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{4\sqrt{\frac{\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)}{2}}}{\sqrt{x}\,\left(\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}$$

input
```
int(1/(x^(3/2)*atanh(tanh(a + b*x))^(1/2)),x)
```

output
```
(4*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2))/(x^(1/2)*(log(1/(exp(2*a)*exp(2*b*x) + 1)) -
 log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))
```

3.246 $\int \frac{1}{x^{5/2}\sqrt{\textbf{arctanh}(\tanh(a+bx))}}\,dx$

3.246.1 Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \frac{1}{x^{5/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{4b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{3\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2} + \frac{2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{3x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

output
```
2/3*arctanh(tanh(b*x+a))^(1/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))+4/3*b*ar
ctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^2/x^(1/2)
```

3.246.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^{5/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = -\frac{2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}(-3bx+\operatorname{arctanh}(\tanh(a+bx)))}{3x^{3/2}(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2}$$

input
```
Integrate[1/(x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
(-2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-3*b*x + ArcTanh[Tanh[a + b*x]]))/(3*x^(
3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

3.246.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx$$
$$\Big\downarrow \text{ 2602}$$
$$\frac{2b\int \frac{1}{x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{3(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$
$$\Big\downarrow \text{ 2598}$$
$$\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{4b\sqrt{\text{arctanh}(\tanh(a+bx))}}{3\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))^2}$$

input
```
Int[1/(x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
(4*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]
])^2) + (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a
+ b*x]]))
```

3.246.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.246.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}+\frac{4b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}\sqrt{x}}$	59
default	$-\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}+\frac{4b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}\sqrt{x}}$	59

input
```
int(1/x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(1/2)+4/3*b/(
arctanh(tanh(b*x+a))-b*x)^2/x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

3.246.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.32

$$\int \frac{1}{x^{5/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx=\frac{2\,(2\,bx-a)\sqrt{bx+a}}{3\,a^{2}x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(2*b*x - a)*sqrt(b*x + a)/(a^2*x^(3/2))
```

3.246.6 Sympy [F]

$$\int \frac{1}{x^{5/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx=\int \frac{1}{x^{\frac{5}{2}}\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

input
```
integrate(1/x**(5/2)/atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Integral(1/(x**(5/2)*sqrt(atanh(tanh(a + b*x)))), x)
```

3.246.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.46

$$\int \frac{1}{x^{5/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\,(2\,b^2x^2+abx-a^2)}{3\,\sqrt{bx+a}a^2x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(2*b^2*x^2 + a*b*x - a^2)/(sqrt(b*x + a)*a^2*x^(3/2))
```

3.246.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{5/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{8\left(3\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^3}$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
8/3*(3*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)*b^(3/2)/((sqrt(b)*sqrt(x)
- sqrt(b*x + a))^2 - a)^3
```

3.246.9 Mupad [B] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.03

$$\int \frac{1}{x^{5/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{\sqrt{2}\left(\frac{\frac{4\,\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{3}-\frac{4\,\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{3}+\frac{8\,b\,x}{3}}{{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}+\frac{16\,b\,x}{3\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\right.}\right.}{2\,x^{3/2}}$$

input
```
int(1/(x^(5/2)*atanh(tanh(a + b*x))^(1/2)),x)
```

output
```
(2^(1/2)*(((4*log(2/(exp(2*a)*exp(2*b*x) + 1)))/3 - (4*log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/3 + (8*b*x)/3)/(log(2/(exp(2*a)*exp(2
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b
*x)^2 + (16*b*x)/(3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2))*(log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^(1/2))
/(2*x^(3/2))
```

3.246. $\int \frac{1}{x^{5/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$

3.247 $\int \frac{1}{x^{7/2}\sqrt{\textbf{arctanh}(\tanh(a+bx))}}\,dx$

3.247.1 Optimal result

Integrand size = 17, antiderivative size = 110

$$\int \frac{1}{x^{7/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx = \frac{16b^2\sqrt{\text{arctanh}(\tanh(a+bx))}}{15\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))^3} + \frac{8b\sqrt{\text{arctanh}(\tanh(a+bx))}}{15x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))^2} + \frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

output
```
8/15*b*arctanh(tanh(b*x+a))^(1/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/5
*arctanh(tanh(b*x+a))^(1/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))+16/15*b^2*a
rctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^3/x^(1/2)
```

3.247.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^{7/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}(15b^2x^2-10bx\text{arctanh}(\tanh(a+bx))+3a}{15x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))^3}$$

input
```
Integrate[1/(x^(7/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
(2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 10*b*x*ArcTanh[Tanh[a + b*x]
] + 3*ArcTanh[Tanh[a + b*x]]^2))/(15*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]]
)^3)
```

3.247.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2602, 2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx$$

$$\Big\downarrow \text{ 2602}$$

$$\frac{4b\int \frac{1}{x^{5/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{5(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{ 2602}$$

$$\frac{4b\left(\frac{2b\int \frac{1}{x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{3(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{5(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{ 2598}$$

$$\frac{4b\left(\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{4b\sqrt{\text{arctanh}(\tanh(a+bx))}}{3\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))^2}\right)}{5(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

input
```
Int[1/(x^(7/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
(4*b*((4*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*Sqrt[x]*(b*x - ArcTanh[Tanh[a
+ b*x]])^2) + (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*x^(3/2)*(b*x - ArcTanh[T
anh[a + b*x]]))))/(5*(b*x - ArcTanh[Tanh[a + b*x]])) + (2*Sqrt[ArcTanh[Tan
h[a + b*x]]])/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

3.247.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.247.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}-\frac{8b\left(-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}+\frac{2b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}\sqrt{x}}\right)}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)}$	105
default	$-\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}-\frac{8b\left(-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}+\frac{2b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}\sqrt{x}}\right)}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)}$	105

input
```
int(1/x^(7/2)/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(1/2)-8/5*b/(
arctanh(tanh(b*x+a))-b*x)*(-1/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh
(tanh(b*x+a))^(1/2)+2/3*b/(arctanh(tanh(b*x+a))-b*x)^2/x^(1/2)*arctanh(tan
h(b*x+a))^(1/2))
```

3.247.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^{7/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = -\frac{2\,(8\,b^2x^2-4\,abx+3\,a^2)\sqrt{bx+a}}{15\,a^3x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x + a)/(a^3*x^(5/2))
```

3.247.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(7/2)/atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Timed out
```

3.247.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^{7/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = -\frac{2\,(8\,b^3x^3+4\,ab^2x^2-a^2bx+3\,a^3)}{15\,\sqrt{bx+a}a^3x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-2/15*(8*b^3*x^3 + 4*a*b^2*x^2 - a^2*b*x + 3*a^3)/(sqrt(b*x + a)*a^3*x^(5/
2))
```

3.247.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^{7/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{32\left(10\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4-5\,a\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2+a^2\right)b^{\frac{5}{2}}}{15\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^5}$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
32/15*(10*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 5*a*(sqrt(b)*sqrt(x) - sqr
t(b*x + a))^2 + a^2)*b^(5/2)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5
```

3.247.9 Mupad [B] (verification not implemented)

Time = 4.59 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.06

$$\int \frac{1}{x^{7/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{4}{5\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\right.}{}$$

input
```
int(1/(x^(7/2)*atanh(tanh(a + b*x))^(1/2)),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*(4/(5*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (128*b^2*x
^2)/(15*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (32*b*x)/(15*(log(2/(exp(2*a)*exp(2
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b
*x)^2)))/x^(5/2)
```

3.248 $\int \frac{1}{x^{9/2}\sqrt{\textbf{arctanh}(\tanh(a+bx))}}\,dx$

3.248.1 Optimal result

Integrand size = 17, antiderivative size = 148

$$\int \frac{1}{x^{9/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx = \frac{32b^3\sqrt{\text{arctanh}(\tanh(a+bx))}}{35\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))^4} + \frac{16b^2\sqrt{\text{arctanh}(\tanh(a+bx))}}{35x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))^3} + \frac{12b\sqrt{\text{arctanh}(\tanh(a+bx))}}{35x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))^2} + \frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

output
```
16/35*b^2*arctanh(tanh(b*x+a))^(1/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^3+
12/35*b*arctanh(tanh(b*x+a))^(1/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/
7*arctanh(tanh(b*x+a))^(1/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))+32/35*b^3*
arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^4/x^(1/2)
```

3.248.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{9/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}(35b^3x^3-35b^2x^2\text{arctanh}(\tanh(a+bx))+}{35x^{7/2}(-bx+\text{arctanh}(\text{ta}}$$

input
```
Integrate[1/(x^(9/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
(2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(35*b^3*x^3 - 35*b^2*x^2*ArcTanh[Tanh[a +
b*x]] + 21*b*x*ArcTanh[Tanh[a + b*x]]^2 - 5*ArcTanh[Tanh[a + b*x]]^3))/(35
*x^(7/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

3.248.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2602, 2602, 2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{9/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{6b\int \frac{1}{x^{7/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{7(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2602}$$

$$\frac{6b\left(\frac{4b\int \frac{1}{x^{5/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{5(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{7(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2602}$$

$$\frac{6b\left(\frac{4b\left(\frac{2b\int \frac{1}{x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{3(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{5(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))}\right)}{7(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2\sqrt{\text{arctanh}(\tanh(a+bx))}}{7x^{7/2}(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\Big\downarrow \text{2598}$$

$$\frac{6b\left(\frac{4b\left(\frac{2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{3x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{4b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{3\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2}\right)}{5(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{5x^{5/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}\right)}{7(bx-\operatorname{arctanh}(\tanh(a+bx)))}+\frac{2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{7x^{7/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

input
```
Int[1/(x^(9/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

output
```
(6*b*((4*b*((4*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*Sqrt[x]*(b*x - ArcTanh[T
anh[a + b*x]])^2) + (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*x^(3/2)*(b*x - Arc
Tanh[Tanh[a + b*x]]))))/(5*(b*x - ArcTanh[Tanh[a + b*x]])) + (2*Sqrt[ArcTa
nh[Tanh[a + b*x]]])/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]]))))/(7*(b*x -
 ArcTanh[Tanh[a + b*x]])) + (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(7*x^(7/2)*(b
*x - ArcTanh[Tanh[a + b*x]]))
```

3.248.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.248. $\int \frac{1}{x^{9/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx$

3.248.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.02

method	result
derivativedivides	$-\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}-\frac{12b\left(-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}-\frac{4b\left(-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}+\frac{2b}{3(\operatorname{arct}}\right.}{5(\operatorname{arctanh}(\tanh(bx+a))}\right.}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)}$
default	$-\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}-\frac{12b\left(-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}-\frac{4b\left(-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}+\frac{2b}{3(\operatorname{arct}}\right.}{5(\operatorname{arctanh}(\tanh(bx+a))}\right.}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)}$

input
```
int(1/x^(9/2)/arctanh(tanh(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(1/2)-12/7*b/
(arctanh(tanh(b*x+a))-b*x)*(-1/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctan
h(tanh(b*x+a))^(1/2)-4/5*b/(arctanh(tanh(b*x+a))-b*x)*(-1/3/(arctanh(tanh(
b*x+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(1/2)+2/3*b/(arctanh(tanh(b*x+a)
)-b*x)^2/x^(1/2)*arctanh(tanh(b*x+a))^(1/2)))
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^{9/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx=\frac{2\left(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3\right)\sqrt{bx+a}}{35\,a^4x^{\frac{7}{2}}}$$

input
```
integrate(1/x^(9/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x + a)/(a^4*x^(
7/2))
```

3.248.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{9/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(9/2)/atanh(tanh(b*x+a))**(1/2),x)
```

output
```
Timed out
```

3.248.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^{9/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{2\,(16\,b^4x^4+8\,ab^3x^3-2\,a^2b^2x^2+a^3bx-5\,a^4)}{35\,\sqrt{bx+a}a^4x^{\frac{7}{2}}}$$

input
```
integrate(1/x^(9/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
2/35*(16*b^4*x^4 + 8*a*b^3*x^3 - 2*a^2*b^2*x^2 + a^3*b*x - 5*a^4)/(sqrt(b*
x + a)*a^4*x^(7/2))
```

3.248.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^{9/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{64\left(35\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^6-21\,a\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4+7\,a^2\left(\sqrt{b}\cdots\right.\right.}{35\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^7}$$

input
```
integrate(1/x^(9/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
64/35*(35*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 - 21*a*(sqrt(b)*sqrt(x) - sq
rt(b*x + a))^4 + 7*a^2*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a^3)*b^(7/2)/
((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^7
```

3.248.9 Mupad [B] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.94

$$\int \frac{1}{x^{9/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{4}{7\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\right.}{}$$

input
```
int(1/(x^(9/2)*atanh(tanh(a + b*x))^(1/2)),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*(4/(7*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (128*b^2*x
^2)/(35*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (512*b^3*x^3)/(35*(log(2/(exp(2*a)*
exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x)^4) + (48*b*x)/(35*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)))/x^(7/2)
```

3.249 $\int \frac{x^{7/2}}{\mathbf{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.249.1 Optimal result

Integrand size = 17, antiderivative size = 166

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{35\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))^3}{8b^{9/2}} - \frac{2x^{7/2}}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} + \frac{7x^{5/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{3b^2} + \frac{35x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{12b^3} + \frac{35\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{8b^4}$$

output
```
35/8*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh
(b*x+a)))^3/b^(9/2)-2*x^(7/2)/b/arctanh(tanh(b*x+a))^(1/2)+7/3*x^(5/2)*arc
tanh(tanh(b*x+a))^(1/2)/b^2+35/12*x^(3/2)*(b*x-arctanh(tanh(b*x+a)))*arcta
nh(tanh(b*x+a))^(1/2)/b^3+35/8*(b*x-arctanh(tanh(b*x+a)))^2*x^(1/2)*arctan
h(tanh(b*x+a))^(1/2)/b^4
```

3.249.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{\sqrt{x}(-48b^3x^3+231b^2x^2\operatorname{arctanh}(\tanh(a+bx))-280bx\operatorname{arctanh}(\tanh(a+ }{24b^4\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$
$$+\frac{35(bx-\operatorname{arctanh}(\tanh(a+bx)))^3\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\right)}{8b^{9/2}}$$

input
```
Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(Sqrt[x]*(-48*b^3*x^3 + 231*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 280*b*x*ArcTa
nh[Tanh[a + b*x]]^2 + 105*ArcTanh[Tanh[a + b*x]]^3))/(24*b^4*Sqrt[ArcTanh[
Tanh[a + b*x]]]) + (35*(b*x - ArcTanh[Tanh[a + b*x]])^3*Log[b*Sqrt[x] + Sq
rt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*b^(9/2))
```

3.249.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2599, 2600, 2600, 2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx$$
$$\Big\downarrow \text{2599}$$
$$\frac{7\int \frac{x^{5/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}dx}{b}-\frac{2x^{7/2}}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$
$$\Big\downarrow \text{2600}$$
$$\frac{7\left(\frac{5(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{x^{3/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}dx}{6b}+\frac{x^{5/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{3b}\right)}{b}-\frac{2x^{7/2}}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

$\Big\downarrow$ 2600

$$7\left(\frac{5(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{3(bx-\text{arctanh}(\tanh(a+bx)))\int\frac{\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{4b}+\frac{x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}{2b}\right)}{6b}+\frac{x^{5/2}\sqrt{\text{arcta}}}{}\right.$$
$$\overline{\qquad\qquad b\qquad\qquad}$$

$$\frac{2x^{7/2}}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$\Big\downarrow$ 2600

$$7\left(\frac{5(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{3(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int\frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{2b}+\frac{\sqrt{x}\sqrt{\text{arctanh}(\text{t}}}{b}\right.}{4b}\right.}{6b}\right.$$
$$\overline{\qquad\qquad b\qquad\qquad}$$

$$\frac{2x^{7/2}}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$\Big\downarrow$ 2596

3.249. $\int\frac{x^{7/2}}{\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))}{b^{3/2}}+\frac{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))}{4b}+\frac{x^{3/2}\sqrt{\operatorname{ar}}}{}}{6b}\right.}{b}$$

$$\frac{2x^{7/2}}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

input
```
Int[x^(7/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(7*((5*((3*((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x
- ArcTanh[Tanh[a + b*x]]))/b^(3/2) + (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]
)/b)*(b*x - ArcTanh[Tanh[a + b*x]]))/(4*b) + (x^(3/2)*Sqrt[ArcTanh[Tanh[a
+ b*x]]])/(2*b))*(b*x - ArcTanh[Tanh[a + b*x]]))/(6*b) + (x^(5/2)*Sqrt[Arc
Tanh[Tanh[a + b*x]]])/(3*b)))/b - (2*x^(7/2))/(b*Sqrt[ArcTanh[Tanh[a + b*x
]]])
```

3.249.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.249. $\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.249.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{x^{\frac{7}{2}}}{3b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\frac{7(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{x^{\frac{5}{2}}}{4b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\frac{5(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{}{2b\sqrt{a}}\right.}{}\right.}{}$
default	$\frac{x^{\frac{7}{2}}}{3b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\frac{7(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{x^{\frac{5}{2}}}{4b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\frac{5(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{}{2b\sqrt{a}}\right.}{}\right.}{}$

input
```
int(x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(7/2)/b/arctanh(tanh(b*x+a))^(1/2)-7/3*(arctanh(tanh(b*x+a))-b*x)/b*
(1/4*x^(5/2)/b/arctanh(tanh(b*x+a))^(1/2)-5/4*(arctanh(tanh(b*x+a))-b*x)/b
*(1/2*x^(3/2)/b/arctanh(tanh(b*x+a))^(1/2)-3/2*(arctanh(tanh(b*x+a))-b*x)/
b*(-x^(1/2)/b/arctanh(tanh(b*x+a))^(1/2)+1/b^(3/2)*ln(b^(1/2)*x^(1/2)+arct
anh(tanh(b*x+a))^(1/2)))))
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.18

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \left[\frac{105\,(a^3bx+a^4)\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(8\,b^4x^3-14\,ab\right.$$

$$48\,(b^6x+ab^5)$$

input
```
integrate(x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(105*(a^3*b*x + a^4)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqr
t(x) + a) + 2*(8*b^4*x^3 - 14*a*b^3*x^2 + 35*a^2*b^2*x + 105*a^3*b)*sqrt(b
*x + a)*sqrt(x))/(b^6*x + a*b^5), 1/24*(105*(a^3*b*x + a^4)*sqrt(-b)*arcta
n(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^4*x^3 - 14*a*b^3*x^2 + 35*a^2
*b^2*x + 105*a^3*b)*sqrt(b*x + a)*sqrt(x))/(b^6*x + a*b^5)]
```

3.249.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)/atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Timed out
```

3.249.7 Maxima [F]

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{x^{\frac{7}{2}}}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(7/2)/arctanh(tanh(b*x + a))^(3/2), x)
```

3.249.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{\left(\left(2\,x\left(\frac{4\,x}{b}-\frac{7\,a}{b^2}\right)+\frac{35\,a^2}{b^3}\right)x+\frac{105\,a^3}{b^4}\right)\sqrt{x}}{24\,\sqrt{bx+a}} + \frac{35\,a^3\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{8\,b^{\frac{9}{2}}}$$

input
```
integrate(x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
1/24*((2*x*(4*x/b - 7*a/b^2) + 35*a^2/b^3)*x + 105*a^3/b^4)*sqrt(x)/sqrt(b
*x + a) + 35/8*a^3*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(9/2)
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{x^{7/2}}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(x^(7/2)/atanh(tanh(a + b*x))^(3/2),x)
```

output
```
int(x^(7/2)/atanh(tanh(a + b*x))^(3/2), x)
```

3.250 $\int \frac{x^{5/2}}{\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.250.1 Optimal result

Integrand size = 17, antiderivative size = 128

$$\int \frac{x^{5/2}}{\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{15\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))^2}{4b^{7/2}} - \frac{2x^{5/2}}{b\sqrt{\text{arctanh}(\tanh(a+bx))}} + \frac{5x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}{2b^2} + \frac{15\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}{4b^3}$$

output
```
15/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh
(b*x+a)))^2/b^(7/2)-2*x^(5/2)/b/arctanh(tanh(b*x+a))^(1/2)+5/2*x^(3/2)*arc
tanh(tanh(b*x+a))^(1/2)/b^2+15/4*(b*x-arctanh(tanh(b*x+a)))*x^(1/2)*arctan
h(tanh(b*x+a))^(1/2)/b^3
```

3.250.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.81

$$\int \frac{x^{5/2}}{\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{\sqrt{x}(8b^2x^2-25bx\text{arctanh}(\tanh(a+bx))+15\text{arctanh}(\tanh(a+bx))^2)}{4b^3\sqrt{\text{arctanh}(\tanh(a+bx))}} + \frac{15(-bx+\text{arctanh}(\tanh(a+bx)))^2\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\text{arctanh}(\tanh(a+bx))}\right)}{4b^{7/2}}$$

input
```
Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
-1/4*(Sqrt[x]*(8*b^2*x^2 - 25*b*x*ArcTanh[Tanh[a + b*x]] + 15*ArcTanh[Tanh
[a + b*x]]^2))/(b^3*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (15*(-(b*x) + ArcTanh[
Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(
4*b^(7/2))
```

3.250.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2599, 2600, 2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \; 2599$$

$$\frac{5\int \frac{x^{3/2}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{b}-\frac{2x^{5/2}}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \; 2600$$

$$\frac{5\left(\frac{3(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{4b}+\frac{x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}{2b}\right)}{b}-\frac{2x^{5/2}}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \; 2600$$

$$\frac{5\left(\frac{3(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{2b}+\frac{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}\right)}{4b}+\frac{x^{3/2}\sqrt{\text{arcta}}}{}\right.}{b}$$

$$\frac{2x^{5/2}}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2596}$$

$$\frac{5\left(\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))}{b^{3/2}}+\frac{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))}{4b}+\frac{x^{3/2}\sqrt{}}{}\right.}{b}$$

$$\frac{2x^{5/2}}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

input
```
Int[x^(5/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(5*((3*((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - Ar
cTanh[Tanh[a + b*x]]))/b^(3/2) + (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b)
*(b*x - ArcTanh[Tanh[a + b*x]]))/(4*b) + (x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*
x]]])/(2*b)))/b - (2*x^(5/2))/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

3.250.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.250. $\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.250.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{x^{\frac{5}{2}}}{2b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}} - \frac{5(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{x^{\frac{3}{2}}}{2b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}} - \frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(-\frac{}{b\sqrt{}}\right.}{}\right.}{2b}$
default	$\frac{x^{\frac{5}{2}}}{2b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}} - \frac{5(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{x^{\frac{3}{2}}}{2b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}} - \frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(-\frac{}{b\sqrt{}}\right.}{}\right.}{2b}$

input
```
int(x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(5/2)/b/arctanh(tanh(b*x+a))^(1/2)-5/2*(arctanh(tanh(b*x+a))-b*x)/b*
(1/2*x^(3/2)/b/arctanh(tanh(b*x+a))^(1/2)-3/2*(arctanh(tanh(b*x+a))-b*x)/b
*(-x^(1/2)/b/arctanh(tanh(b*x+a))^(1/2)+1/b^(3/2)*ln(b^(1/2)*x^(1/2)+arcta
nh(tanh(b*x+a))^(1/2))))
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.37

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \left[\frac{15\,(a^2bx+a^3)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(2\,b^3x^2-5\,ab^2x}{8\,(b^5x+ab^4)}\right.$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(15*(a^2*b*x + a^3)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(
x) + a) + 2*(2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^5
*x + a*b^4), -1/4*(15*(a^2*b*x + a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-
b)/(b*sqrt(x))) - (2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x)
)/(b^5*x + a*b^4)]
```

3.250.6 Sympy [F]

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{x^{\frac{5}{2}}}{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**(5/2)/atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(x**(5/2)/atanh(tanh(a + b*x))**(3/2), x)
```

3.250.7 Maxima [F]

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{x^{\frac{5}{2}}}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/arctanh(tanh(b*x + a))^(3/2), x)
```

3.250.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.49

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{\left(x\left(\frac{2x}{b}-\frac{5a}{b^2}\right)-\frac{15a^2}{b^3}\right)\sqrt{x}}{4\sqrt{bx+a}} - \frac{15\,a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{4\,b^{\frac{7}{2}}}$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
1/4*(x*(2*x/b - 5*a/b^2) - 15*a^2/b^3)*sqrt(x)/sqrt(b*x + a) - 15/4*a^2*lo
g(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{x^{5/2}}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(x^(5/2)/atanh(tanh(a + b*x))^(3/2),x)
```

output
```
int(x^(5/2)/atanh(tanh(a + b*x))^(3/2), x)
```

3.251 $\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.251.1 Optimal result

Integrand size = 17, antiderivative size = 86

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{3\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))}{b^{5/2}} - \frac{2x^{3/2}}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} + \frac{3\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b^2}$$

output
```
3*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b*
x+a)))/b^(5/2)-2*x^(3/2)/b/arctanh(tanh(b*x+a))^(1/2)+3*x^(1/2)*arctanh(ta
nh(b*x+a))^(1/2)/b^2
```

3.251.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{\sqrt{x}(-2bx+3\operatorname{arctanh}(\tanh(a+bx)))}{b^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} + \frac{3(bx-\operatorname{arctanh}(\tanh(a+bx)))\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\right)}{b^{5/2}}$$

input
```
Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(Sqrt[x]*(-2*b*x + 3*ArcTanh[Tanh[a + b*x]]))/(b^2*Sqrt[ArcTanh[Tanh[a + b
*x]]]) + (3*(b*x - ArcTanh[Tanh[a + b*x]])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[Ar
cTanh[Tanh[a + b*x]]]])/b^(5/2)
```

3.251.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2599, 2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\text{arctanh}(\tanh(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{3\int \frac{\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{b}-\frac{2x^{3/2}}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2600}$$

$$\frac{3\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{2b}+\frac{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2596}$$

$$\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)(bx-\text{arctanh}(\tanh(a+bx)))}{b^{3/2}}+\frac{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

input
```
Int[x^(3/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(3*((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTan
h[Tanh[a + b*x]]))/b^(3/2) + (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b))/b
- (2*x^(3/2))/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

3.251.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.251.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}}{b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(-\frac{\sqrt{x}}{b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}+\frac{\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh(bx+a))}\right)}{b^{\frac{3}{2}}}\right.}{b}$
default	$\frac{x^{\frac{3}{2}}}{b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\frac{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(-\frac{\sqrt{x}}{b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}+\frac{\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh(bx+a))}\right)}{b^{\frac{3}{2}}}\right.}{b}$

input
```
int(x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x^(3/2)/b/arctanh(tanh(b*x+a))^(1/2)-3*(arctanh(tanh(b*x+a))-b*x)/b*(-x^(1
/2)/b/arctanh(tanh(b*x+a))^(1/2)+1/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh
(b*x+a))^(1/2)))
```

3.251.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.69

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \left[\frac{3\left(abx+a^2\right)\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(b^2x+3\,ab\right)\sqrt{bx-}}{2\left(b^4x+ab^3\right)}\right.$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*(a*b*x + a^2)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x)
+ a) + 2*(b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3), (3*(a*b*x
 + a^2)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (b^2*x + 3*a
*b)*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3)]
```

3.251.6 Sympy [F]

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**(3/2)/atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(x**(3/2)/atanh(tanh(a + b*x))**(3/2), x)
```

3.251.7 Maxima [F]

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/arctanh(tanh(b*x + a))^(3/2), x)
```

3.251.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{\sqrt{x}\left(\frac{x}{b}+\frac{3\,a}{b^2}\right)}{\sqrt{bx+a}} + \frac{3\,a\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
sqrt(x)*(x/b + 3*a/b^2)/sqrt(b*x + a) + 3*a*log(abs(-sqrt(b)*sqrt(x) + sqr
t(b*x + a)))/b^(5/2)
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{x^{3/2}}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(x^(3/2)/atanh(tanh(a + b*x))^(3/2),x)
```

output
```
int(x^(3/2)/atanh(tanh(a + b*x))^(3/2), x)
```

3.252 $\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.252.1 Optimal result

Integrand size = 17, antiderivative size = 52

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

output
```
2*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))/b^(3/2)-2*x^(1/2)/b/
arctanh(tanh(b*x+a))^(1/2)
```

3.252.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx=-\frac{2\sqrt{x}}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}+\frac{2\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\right)}{b^{3/2}}$$

input
```
Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(-2*Sqrt[x])/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (2*Log[b*Sqrt[x] + Sqrt[b]
*Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(3/2)
```

3.252.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2599, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}} \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{\int \frac{1}{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}dx}{b} - \frac{2\sqrt{x}}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{ 2596}$$

$$\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)}{b^{3/2}} - \frac{2\sqrt{x}}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

input
```
Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

output
```
(2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(3/2) - (2*S
qrt[x])/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

3.252.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.252.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{2\sqrt{x}}{b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}+\frac{2\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh(bx+a))}\right)}{b^{\frac{3}{2}}}$	42
default	$-\frac{2\sqrt{x}}{b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}+\frac{2\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh(bx+a))}\right)}{b^{\frac{3}{2}}}$	42

input
```
int(x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*x^(1/2)/b/arctanh(tanh(b*x+a))^(1/2)+2/b^(3/2)*ln(b^(1/2)*x^(1/2)+arcta
nh(tanh(b*x+a))^(1/2))
```

3.252.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.29

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \left[\frac{(bx+a)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,\sqrt{bx+a}b\sqrt{x}}{b^3x+ab^2},\right.$$
$$\left.-\frac{2\left((bx+a)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}b\sqrt{x}\right)}{b^3x+ab^2}\right]$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
[((b*x + a)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*s
qrt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2), -2*((b*x + a)*sqrt(-b)*arctan(sqr
t(b*x + a)*sqrt(-b)/(b*sqrt(x))) + sqrt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2
)]
```

3.252.6 Sympy [F]

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{x}}{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**(1/2)/atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(sqrt(x)/atanh(tanh(a + b*x))**(3/2), x)
```

3.252.7 Maxima [F]

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{x}}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/arctanh(tanh(b*x + a))^(3/2), x)
```

3.252.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}} - \frac{2\sqrt{x}}{\sqrt{bx+a}b}$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2) - 2*sqrt(x)/(sqrt(b*
x + a)*b)
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{x}}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(x^(1/2)/atanh(tanh(a + b*x))^(3/2),x)
```

output
```
int(x^(1/2)/atanh(tanh(a + b*x))^(3/2), x)
```

3.253 $\int \frac{1}{\sqrt{x}\mathbf{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.253.1 Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2\sqrt{x}}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

output
```
-2*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)
```

3.253.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}(-bx+\text{arctanh}(\tanh(a+bx)))}$$

input
```
Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

output
```
(2*Sqrt[x])/(Sqrt[ArcTanh[Tanh[a + b*x]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]]
))
```

3.253.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2598}$$

$$-\frac{2\sqrt{x}}{(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

input
```
Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

output
```
(-2*Sqrt[x])/((b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

3.253.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

3.253.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{(\text{arctanh}(\tanh(bx+a))-bx)\sqrt{\text{arctanh}(\tanh(bx+a))}}$	29
default	$\frac{2\sqrt{x}}{(\text{arctanh}(\tanh(bx+a))-bx)\sqrt{\text{arctanh}(\tanh(bx+a))}}$	29

input
```
int(1/x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(1/2)
```

3.253.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2\sqrt{bx+a}\sqrt{x}}{abx+a^2}$$

input
```
integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2*sqrt(b*x + a)*sqrt(x)/(a*b*x + a^2)
```

3.253.6 Sympy [F]

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{1}{\sqrt{x}\,\text{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

input
```
integrate(1/x**(1/2)/atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(1/(sqrt(x)*atanh(tanh(a + b*x))**(3/2)), x)
```

3.253.7 Maxima [F]

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{1}{\sqrt{x}\,\text{artanh}(\tanh(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x)*arctanh(tanh(b*x + a))^(3/2)), x)
```

3.253.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2\,\sqrt{x}}{\sqrt{bx+a}a}$$

input
```
integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2*sqrt(x)/(sqrt(b*x + a)*a)
```

3.253.9 Mupad [B] (verification not implemented)

Time = 4.85 (sec) , antiderivative size = 163, normalized size of antiderivative = 4.94

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{4\,x\,\sqrt{\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{\left(\frac{\sqrt{x}\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2\,b}-\frac{\sqrt{x}\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2\,b}\right)\,\left(b\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-b\,\ln\left(\frac{e^{2\,a}\,e}{e^{2\,a}\,e^{2}}\right.\right.}$$

input
```
int(1/(x^(1/2)*atanh(tanh(a + b*x))^(3/2)),x)
```

output
```
(4*x*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2))/(((x^(1/2)*log(1/(exp(2*a)*exp(2*b*x) + 1)
))/(2*b) - (x^(1/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/
(2*b))*(b*log(1/(exp(2*a)*exp(2*b*x) + 1)) - b*log((exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b^2*x))
```

3.254 $\int \frac{1}{x^{3/2}\textbf{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.254.1 Optimal result

Integrand size = 17, antiderivative size = 68

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{4b\sqrt{x}}{(bx-\text{arctanh}(\tanh(a+bx)))^2\sqrt{\text{arctanh}(\tanh(a+bx))}} + \frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

output
```
2/(b*x-arctanh(tanh(b*x+a)))/x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-4*b*x^(1/2
)/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(1/2)
```

3.254.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2(bx+\text{arctanh}(\tanh(a+bx)))}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}(-bx+\text{arctanh}(\tanh(a+bx)))^2}$$

input
```
Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

output
```
(-2*(b*x + ArcTanh[Tanh[a + b*x]]))/(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*
(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

3.254.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{2b\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2598}$$

$$\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}-\frac{4b\sqrt{x}}{(bx-\text{arctanh}(\tanh(a+bx)))^2\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

input
```
Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

output
```
(-4*b*Sqrt[x])/((b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x
]]]) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x
]]])
```

3.254.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.254.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

method	result
derivativedivides	$-\frac{2}{\sqrt{x}\,(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\frac{4b\sqrt{x}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}$
default	$-\frac{2}{\sqrt{x}\,(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\frac{4b\sqrt{x}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}$

input
```
int(1/x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(1/2)-4*b/(arct
anh(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)
```

3.254.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^{3/2}\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx=-\frac{2\,(2\,bx+a)\sqrt{bx+a}\sqrt{x}}{a^2bx^2+a^3x}$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2*(2*b*x + a)*sqrt(b*x + a)*sqrt(x)/(a^2*b*x^2 + a^3*x)
```

3.254.6 Sympy [F]

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\,\text{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**(3/2)/atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(1/(x**(3/2)*atanh(tanh(a + b*x))**(3/2)), x)
```

3.254.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2\,(2\,b^2x^2+3\,abx+a^2)}{(bx+a)^{\frac{3}{2}}a^2\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2*(2*b^2*x^2 + 3*a*b*x + a^2)/((b*x + a)^(3/2)*a^2*sqrt(x))
```

3.254.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2\,b\sqrt{x}}{\sqrt{bx+a}a^2} + \frac{4\,\sqrt{b}}{\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)a}$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-2*b*sqrt(x)/(sqrt(b*x + a)*a^2) + 4*sqrt(b)/(((sqrt(b)*sqrt(x) - sqrt(b*x
 + a))^2 - a)*a)
```

3.254.9 Mupad [B] (verification not implemented)

Time = 4.43 (sec) , antiderivative size = 281, normalized size of antiderivative = 4.13

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{16\,x}{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}-\frac{8\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-8\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+16\,b\,x}{2\,b\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}\right)}{x^{3/2}-\frac{\sqrt{x}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b}}$$

input
```
int(1/(x^(3/2)*atanh(tanh(a + b*x))^(3/2)),x)
```

output
```
-((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2)*((16*x)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - (8*log
(2/(exp(2*a)*exp(2*b*x) + 1)) - 8*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 16*b*x)/(2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)))/(x^(3/2) - (x
^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b))
```

3.255 $\int \frac{1}{x^{5/2}\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx$

3.255.1 Optimal result

Integrand size = 17, antiderivative size = 110

$$\int \frac{1}{x^{5/2}\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{16b^2\sqrt{x}}{3(bx-\operatorname{arctanh}(\tanh(a+bx)))^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} + \frac{8b}{3\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} + \frac{2}{3x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

output
```
2/3/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)+8/3*b/(b
*x-arctanh(tanh(b*x+a)))^2/x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-16/3*b^2*x^(
1/2)/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(1/2)
```

3.255.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^{5/2}\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2(-3b^2x^2-6bx\operatorname{arctanh}(\tanh(a+bx))+\operatorname{arctanh}(\tanh(a+bx))^2)}{3x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))^3\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

input
```
Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

output
```
(2*(-3*b^2*x^2 - 6*b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2))
/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

3.255.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2602, 2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{4b\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{3(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2602}$$

$$\frac{4b\left(\frac{2b\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{3(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2598}$$

$$\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}+\frac{4b\left(\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}-\frac{4b\sqrt{x}}{(bx-\text{arctanh}(\tanh(a+bx)))^2\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{3(bx-\text{arctanh}(\tanh(a+bx)))}$$

input
```
Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

output
```
(4*b*((-4*b*Sqrt[x])/((b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a
 + b*x]]]) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a
 + b*x]]])))/(3*(b*x - ArcTanh[Tanh[a + b*x]])) + 2/(3*x^(3/2)*(b*x - ArcT
anh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

3.255.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.255.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

method	result
derivativedivides	$-\frac{2}{3x^{\frac{3}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\frac{8b\left(-\frac{1}{\sqrt{x}\,(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\ldots\right.}{3(\operatorname{arctanh}(\tanh(b\ldots}$
default	$-\frac{2}{3x^{\frac{3}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\frac{8b\left(-\frac{1}{\sqrt{x}\,(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\ldots\right.}{3(\operatorname{arctanh}(\tanh(b\ldots}$

input
```
int(1/x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/x^(3/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(1/2)-8/3*b/(
arctanh(tanh(b*x+a))-b*x)*(-1/x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(t
anh(b*x+a))^(1/2)-2*b/(arctanh(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*
x+a))^(1/2))
```

3.255.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2\,(8\,b^2x^2+4\,abx-a^2)\sqrt{bx+a}\sqrt{x}}{3\,(a^3bx^3+a^4x^2)}$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(8*b^2*x^2 + 4*a*b*x - a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b*x^3 + a^4*x^2
)
```

3.255.6 Sympy [F]

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \int \frac{1}{x^{\frac{5}{2}}\,\text{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**(5/2)/atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Integral(1/(x**(5/2)*atanh(tanh(a + b*x))**(3/2)), x)
```

3.255.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2\,(8\,b^3x^3+12\,ab^2x^2+3\,a^2bx-a^3)}{3\,(bx+a)^{\frac{3}{2}}a^3x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/3*(8*b^3*x^3 + 12*a*b^2*x^2 + 3*a^2*b*x - a^3)/((b*x + a)^(3/2)*a^3*x^(3
/2))
```

3.255.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{2\,b^2\sqrt{x}}{\sqrt{bx+a}a^3} - \frac{4\left(3\,b^{\frac{3}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4 - 12\,ab^{\frac{3}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2 + 5\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^3a^2}$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2*b^2*sqrt(x)/(sqrt(b*x + a)*a^3) - 4/3*(3*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt
(b*x + a))^4 - 12*a*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 + 5*a^2*b^
(3/2))/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^3*a^2)
```

3.255.9 Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.60

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{32\,x}{3\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2} + \cdots\right.}{x^{5/2} - \frac{x^{3/2}\left(\ln\left(\frac{\cdots}{e^{2\,a}\,\cdots}\right.\right.}{\cdots}}$$

input
```
int(1/(x^(5/2)*atanh(tanh(a + b*x))^(3/2)),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((32*x)/(3*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + 4/(
3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)) - (128*b*x^2)/(3*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^
3)))/(x^(5/2) - (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b))
```

3.256 $\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^{3/2}} \, dx$

3.256.1 Optimal result

Integrand size = 17, antiderivative size = 148

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^{3/2}} \, dx = -\frac{32b^3\sqrt{x}}{5(bx-\text{arctanh}(\tanh(a+bx)))^4\sqrt{\text{arctanh}(\tanh(a+bx))}} + \frac{16b^2}{5\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))^3\sqrt{\text{arctanh}(\tanh(a+bx))}} + \frac{4b}{5x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))^2\sqrt{\text{arctanh}(\tanh(a+bx))}} + \frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

output
```
4/5*b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(1/2)+2/5/
x^(5/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)+16/5*b^2/(b*
x-arctanh(tanh(b*x+a)))^3/x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-32/5*b^3*x^(1
/2)/(b*x-arctanh(tanh(b*x+a)))^4/arctanh(tanh(b*x+a))^(1/2)
```

3.256.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2(5b^3x^3+15b^2x^2\text{arctanh}(\tanh(a+bx))-5bx\text{arctanh}(\tanh(a+bx))^2+\text{arctanh}(\tanh(a+bx))^3)}{5x^{5/2}\sqrt{\text{arctanh}(\tanh(a+bx))}(-bx+\text{arctanh}(\tanh(a+bx)))^4}$$

input
```
Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

output
```
(-2*(5*b^3*x^3 + 15*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 5*b*x*ArcTanh[Tanh[a
+ b*x]]^2 + ArcTanh[Tanh[a + b*x]]^3))/(5*x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*
x]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

3.256.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2602, 2602, 2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{6b\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{5(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2602}$$

$$\frac{6b\left(\frac{4b\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{3(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{5(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2602}$$

$$\frac{6b\left(\frac{4b\left(\frac{2b\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{3(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{}}\right.}{5(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

$$\Big\downarrow \text{2598}$$

$$\frac{6b\left(\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}+\frac{4b\left(\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}-\frac{}{(bx-\text{arctan}}\right.}{3(bx-\text{arctanh}(\tanh(a+bx)))}\right.}{5(bx-\text{arctanh}(\tanh(a+bx)))}$$

$$\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

input
```
Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

output
```
(6*b*((4*b*((-4*b*Sqrt[x])/((b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[
Tanh[a + b*x]]]) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[
Tanh[a + b*x]]])))/(3*(b*x - ArcTanh[Tanh[a + b*x]])) + 2/(3*x^(3/2)*(b*x
- ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])))/(5*(b*x - ArcTan
h[Tanh[a + b*x]])) + 2/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcT
anh[Tanh[a + b*x]]])
```

3.256.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.256.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.02

method	result
derivativedivides	$-\frac{2}{5x^{\frac{5}{2}}\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\frac{12b\left(-\frac{1}{3x^{\frac{3}{2}}\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)\sqrt{\operatorname{arctanh}(\tanh(bx+a)}}\right.}{}$
default	$-\frac{2}{5x^{\frac{5}{2}}\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\frac{12b\left(-\frac{1}{3x^{\frac{3}{2}}\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)\sqrt{\operatorname{arctanh}(\tanh(bx+a)}}\right.}{}$

input
```
int(1/x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/x^(5/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(1/2)-12/5*b/
(arctanh(tanh(b*x+a))-b*x)*(-1/3/x^(3/2)/(arctanh(tanh(b*x+a))-b*x)/arctan
h(tanh(b*x+a))^(1/2)-4/3*b/(arctanh(tanh(b*x+a))-b*x)*(-1/x^(1/2)/(arctanh
(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(1/2)-2*b/(arctanh(tanh(b*x+a))-b*
x)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)))
```

3.256.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^{7/2}\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\,dx=-\frac{2\left(16\,b^3x^3+8\,ab^2x^2-2\,a^2bx+a^3\right)\sqrt{bx+a}\sqrt{x}}{5\left(a^4bx^4+a^5x^3\right)}$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2/5*(16*b^3*x^3 + 8*a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(b*x + a)*sqrt(x)/(a
^4*b*x^4 + a^5*x^3)
```

3.256.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(7/2)/atanh(tanh(b*x+a))**(3/2),x)
```

output
```
Timed out
```

3.256.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2\,(16\,b^4x^4+24\,ab^3x^3+6\,a^2b^2x^2-a^3bx+a^4)}{5\,(bx+a)^{\frac{3}{2}}a^4x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2/5*(16*b^4*x^4 + 24*a*b^3*x^3 + 6*a^2*b^2*x^2 - a^3*b*x + a^4)/((b*x + a
)^(3/2)*a^4*x^(5/2))
```

3.256.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2\,b^3\sqrt{x}}{\sqrt{bx+a}a^4} + \frac{4\left(5\,b^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^8-30\,ab^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^6+80\,a^2b^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4-50\,a^3b^{\frac{5}{2}}\left(\sqrt{b}\right.\right.}{5\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^5a^3}$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-2*b^3*sqrt(x)/(sqrt(b*x + a)*a^4) + 4/5*(5*b^(5/2)*(sqrt(b)*sqrt(x) - sqr
t(b*x + a))^8 - 30*a*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 + 80*a^2*
b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 50*a^3*b^(5/2)*(sqrt(b)*sqrt
(x) - sqrt(b*x + a))^2 + 11*a^4*b^(5/2))/(((sqrt(b)*sqrt(x) - sqrt(b*x + a
))^2 - a)^5*a^3)
```

3.256.9 Mupad [B] (verification not implemented)

Time = 4.85 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.34

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^{3/2}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{16\,x}{5\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}+\right.}{}$$

input
```
int(1/(x^(7/2)*atanh(tanh(a + b*x))^(3/2)),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((16*x)/(5*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + 4/(
5*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)) + (128*b*x^2)/(5*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^
3) - (512*b^2*x^3)/(5*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)))/(x^(7/2) - (x^(5/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x))/(2*b))
```

3.257 $\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}} \, dx$

3.257.1 Optimal result

Integrand size = 17, antiderivative size = 153

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}} \, dx = \frac{35\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))^2}{4b^{9/2}} - \frac{2x^{7/2}}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}} - \frac{14x^{5/2}}{3b^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} + \frac{35x^{3/2}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{6b^3} + \frac{35\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{4b^4}$$

output
```
35/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh
(b*x+a)))^2/b^(9/2)-2/3*x^(7/2)/b/arctanh(tanh(b*x+a))^(3/2)-14/3*x^(5/2)/
b^2/arctanh(tanh(b*x+a))^(1/2)+35/6*x^(3/2)*arctanh(tanh(b*x+a))^(1/2)/b^3
+35/4*(b*x-arctanh(tanh(b*x+a)))*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^4
```

3.257.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}} \, dx = -\frac{\sqrt{x}(8b^3x^3+56b^2x^2\operatorname{arctanh}(\tanh(a+bx))-175bx\operatorname{arctanh}(\tanh(a+bx))^2+105\operatorname{arctanh}(\tanh(a+bx))^3)}{12b^4\operatorname{arctanh}(\tanh(a+bx))^{3/2}} + \frac{35(-bx+\operatorname{arctanh}(\tanh(a+bx)))^2\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\right)}{4b^{9/2}}$$

input
```
Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
-1/12*(Sqrt[x]*(8*b^3*x^3 + 56*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 175*b*x*Ar
cTanh[Tanh[a + b*x]]^2 + 105*ArcTanh[Tanh[a + b*x]]^3))/(b^4*ArcTanh[Tanh[
a + b*x]]^(3/2)) + (35*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2*Log[b*Sqrt[x] +
 Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(4*b^(9/2))
```

3.257.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2599, 2599, 2600, 2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}} \, dx$$

$$\Big\downarrow \ 2599$$

$$\frac{7\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}dx}{3b} - \frac{2x^{7/2}}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \ 2599$$

$$\frac{7\left(\frac{5\int \frac{x^{3/2}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}dx}{b} - \frac{2x^{5/2}}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)}{3b} - \frac{2x^{7/2}}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

$\Big\downarrow$ 2600

$$\frac{7\left(\frac{5\left(\frac{3(bx-\text{arctanh}(\tanh(a+bx)))\int\frac{\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{4b}+\frac{x^{3/2}\sqrt{\text{arctanh}(\tanh(a+bx))}}{2b}\right)}{b}-\frac{2x^{5/2}}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{3b}-\frac{2x^{7/2}}{3b\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$\Big\downarrow$ 2600

$$\frac{7\left(\frac{5\left(\frac{3(bx-\text{arctanh}(\tanh(a+bx)))\left(\frac{(bx-\text{arctanh}(\tanh(a+bx)))\int\frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{2b}+\frac{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}{b}\right)}{4b}+\frac{x^{3/2}\sqrt{\text{arctan}}}{2b}\right.}{b}}{3b}$$

$$\frac{2x^{7/2}}{3b\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$\Big\downarrow$ 2596

3.257. $\int\frac{x^{7/2}}{\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

$$7\left(\frac{5\left(\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))}{b^{3/2}}+\frac{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))}{4b}+\frac{x^{3/2}\sqrt{\operatorname{ar}}}{}}{b}\right.$$

$$\overline{3b}$$

$$\frac{2x^{7/2}}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Int[x^(7/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(7*((5*((3*((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x
- ArcTanh[Tanh[a + b*x]]))/b^(3/2) + (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]
)/b)*(b*x - ArcTanh[Tanh[a + b*x]]))/(4*b) + (x^(3/2)*Sqrt[ArcTanh[Tanh[a
+ b*x]]])/(2*b)))/b - (2*x^(5/2))/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])))/(3*b)
 - (2*x^(7/2))/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2))
```

3.257.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.257. $\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.257.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{x^{\frac{7}{2}}}{2b\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{7(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{x^{\frac{5}{2}}}{2b\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{5(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(-\frac{\cdots}{3b\,a\cdots}\right.}{\cdots}\right.}{2b\cdots}$
default	$\frac{x^{\frac{7}{2}}}{2b\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{7(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{x^{\frac{5}{2}}}{2b\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{5(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(-\frac{\cdots}{3b\,a\cdots}\right.}{\cdots}\right.}{2b\cdots}$

input
```
int(x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(7/2)/b/arctanh(tanh(b*x+a))^(3/2)-7/2*(arctanh(tanh(b*x+a))-b*x)/b*
(1/2*x^(5/2)/b/arctanh(tanh(b*x+a))^(3/2)-5/2*(arctanh(tanh(b*x+a))-b*x)/b
*(-1/3*x^(3/2)/b/arctanh(tanh(b*x+a))^(3/2)+1/b*(-x^(1/2)/b/arctanh(tanh(b
*x+a))^(1/2)+1/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2)))))
```

3.257.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.58

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \left[\frac{105\,(a^2b^2x^2+2\,a^3bx+a^4)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(6\,b}{24\,(b^7x^2+2\,ab^6x+a^2b}\right.$$

input
```
integrate(x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(105*(a^2*b^2*x^2 + 2*a^3*b*x + a^4)*sqrt(b)*log(2*b*x + 2*sqrt(b*x
+ a)*sqrt(b)*sqrt(x) + a) + 2*(6*b^4*x^3 - 21*a*b^3*x^2 - 140*a^2*b^2*x -
105*a^3*b)*sqrt(b*x + a)*sqrt(x))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5), -1/12*(
105*(a^2*b^2*x^2 + 2*a^3*b*x + a^4)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)
/(b*sqrt(x))) - (6*b^4*x^3 - 21*a*b^3*x^2 - 140*a^2*b^2*x - 105*a^3*b)*sqr
t(b*x + a)*sqrt(x))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)]
```

3.257.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)/atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.257.7 Maxima [F]

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{x^{\frac{7}{2}}}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^(7/2)/arctanh(tanh(b*x + a))^(5/2), x)
```

3.257.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.49

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{\left(\left(3\,x\left(\frac{2\,x}{b}-\frac{7\,a}{b^2}\right)-\frac{140\,a^2}{b^3}\right)x-\frac{105\,a^3}{b^4}\right)\sqrt{x}}{12\,(bx+a)^{\frac{3}{2}}} - \frac{35\,a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{4\,b^{\frac{9}{2}}}$$

input
```
integrate(x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
1/12*((3*x*(2*x/b - 7*a/b^2) - 140*a^2/b^3)*x - 105*a^3/b^4)*sqrt(x)/(b*x
+ a)^(3/2) - 35/4*a^2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(9/2)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{x^{7/2}}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{5/2}}\,dx$$

input
```
int(x^(7/2)/atanh(tanh(a + b*x))^(5/2),x)
```

output
```
int(x^(7/2)/atanh(tanh(a + b*x))^(5/2), x)
```

3.258 $\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}} dx$

3.258.1 Optimal result

Integrand size = 17, antiderivative size = 111

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}} dx = \frac{5\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))}{b^{7/2}} - \frac{2x^{5/2}}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} + \frac{5\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b^3}$$

output
```
5*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b*
x+a)))/b^(7/2)-2/3*x^(5/2)/b/arctanh(tanh(b*x+a))^(3/2)-10/3*x^(3/2)/b^2/a
rctanh(tanh(b*x+a))^(1/2)+5*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^3
```

3.258.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.91

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}} dx = -\frac{\sqrt{x}(2b^2x^2+10bx\operatorname{arctanh}(\tanh(a+bx))-15\operatorname{arctanh}(\tanh(a+bx))^2)}{3b^3\operatorname{arctanh}(\tanh(a+bx))^{3/2}} + \frac{5(bx-\operatorname{arctanh}(\tanh(a+bx)))\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\right)}{b^{7/2}}$$

input
```
Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
-1/3*(Sqrt[x]*(2*b^2*x^2 + 10*b*x*ArcTanh[Tanh[a + b*x]] - 15*ArcTanh[Tanh
[a + b*x]]^2))/(b^3*ArcTanh[Tanh[a + b*x]]^(3/2)) + (5*(b*x - ArcTanh[Tanh
[a + b*x]])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(7/2)
```

3.258.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2599, 2599, 2600, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{5\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^{3/2}}dx}{3b}-\frac{2x^{5/2}}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2599}$$

$$\frac{5\left(\frac{3\int \frac{\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}dx}{b}-\frac{2x^{3/2}}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)}{3b}-\frac{2x^{5/2}}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2600}$$

$$\frac{5\left(\frac{3\left(\frac{(bx-\operatorname{arctanh}(\tanh(a+bx)))\int \frac{1}{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}dx}{2b}+\frac{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)}{3b}-\frac{2x^{5/2}}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2596}$$

$$\frac{5\left(\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)(bx-\operatorname{arctanh}(\tanh(a+bx)))}{b^{3/2}}+\frac{\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}{b}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)}{3b}-\frac{2x^{5/2}}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Int[x^(5/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(5*((3*((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - Ar
cTanh[Tanh[a + b*x]]))/b^(3/2) + (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b)
)/b - (2*x^(3/2))/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])))/(3*b) - (2*x^(5/2))/(
3*b*ArcTanh[Tanh[a + b*x]]^(3/2))
```

3.258.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

rule 2600
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + n + 1))), x] - Simp[n*((b*u -
a*v)/(a*(m + n + 1)))   Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /;
PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n +
1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
 -2]
```

3.258. $\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.258.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{x^{\frac{5}{2}}}{b \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}} - \frac{5(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(-\frac{x^{\frac{3}{2}}}{3b \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}} + \frac{-\frac{\sqrt{x}}{b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}} + \frac{\ln\left(\sqrt{b}\right.}{}}{b}\right.}{b}$
default	$\frac{x^{\frac{5}{2}}}{b \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}} - \frac{5(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(-\frac{x^{\frac{3}{2}}}{3b \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}} + \frac{-\frac{\sqrt{x}}{b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}} + \frac{\ln\left(\sqrt{b}\right.}{}}{b}\right.}{b}$

input
```
int(x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x^(5/2)/b/arctanh(tanh(b*x+a))^(3/2)-5*(arctanh(tanh(b*x+a))-b*x)/b*(-1/3*
x^(3/2)/b/arctanh(tanh(b*x+a))^(3/2)+1/b*(-x^(1/2)/b/arctanh(tanh(b*x+a))^
(1/2)+1/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))))
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.93

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \left[\frac{15\,(ab^2x^2+2\,a^2bx+a^3)\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(3\,b^3x}{6\,(b^6x^2+2\,ab^5x+a^2b^4)}\right.$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(15*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)
*sqrt(b)*sqrt(x) + a) + 2*(3*b^3*x^2 + 20*a*b^2*x + 15*a^2*b)*sqrt(b*x + a
)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/3*(15*(a*b^2*x^2 + 2*a^2*b*x
 + a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (3*b^3*x^2 +
 20*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*
b^4)]
```

3.258.6 Sympy [F]

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{x^{\frac{5}{2}}}{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}\,dx$$

input
```
integrate(x**(5/2)/atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Integral(x**(5/2)/atanh(tanh(a + b*x))**(5/2), x)
```

3.258.7 Maxima [F]

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{x^{\frac{5}{2}}}{\operatorname{artanh}(\tanh(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/arctanh(tanh(b*x + a))^(5/2), x)
```

3.258.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.55

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{\left(x\left(\frac{3x}{b}+\frac{20a}{b^2}\right)+\frac{15a^2}{b^3}\right)\sqrt{x}}{3(bx+a)^{\frac{3}{2}}} + \frac{5a\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{7}{2}}}$$

input
```
integrate(x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
1/3*(x*(3*x/b + 20*a/b^2) + 15*a^2/b^3)*sqrt(x)/(b*x + a)^(3/2) + 5*a*log(
abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{x^{5/2}}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{5/2}}\,dx$$

input
```
int(x^(5/2)/atanh(tanh(a + b*x))^(5/2),x)
```

output
```
int(x^(5/2)/atanh(tanh(a + b*x))^(5/2), x)
```

3.259 $\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.259.1 Optimal result

Integrand size = 17, antiderivative size = 75

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}\right)}{b^{5/2}} - \frac{2x^{3/2}}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}}$$

output
```
2*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))/b^(5/2)-2/3*x^(3/2)/
b/arctanh(tanh(b*x+a))^(3/2)-2*x^(1/2)/b^2/arctanh(tanh(b*x+a))^(1/2)
```

3.259.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^{3/2}}{3b\operatorname{arctanh}(\tanh(a+bx))^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{\operatorname{arctanh}(\tanh(a+bx))}} + \frac{2\log\left(b\sqrt{x}+\sqrt{b}\sqrt{\operatorname{arctanh}(\tanh(a+bx))}\right)}{b^{5/2}}$$

input
```
Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(-2*x^(3/2))/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[Ar
cTanh[Tanh[a + b*x]]]) + (2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a +
b*x]]]])/b^(5/2)
```

3.259.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2599, 2599, 2596}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \; 2599$$

$$\frac{\int \frac{\sqrt{x}}{\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{b}-\frac{2x^{3/2}}{3b\,\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \; 2599$$

$$\frac{\frac{\int \frac{1}{\sqrt{x}\sqrt{\text{arctanh}(\tanh(a+bx))}}dx}{b}-\frac{2\sqrt{x}}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}}{b}-\frac{2x^{3/2}}{3b\,\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \; 2596$$

$$\frac{\frac{2\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\text{arctanh}(\tanh(a+bx))}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{\text{arctanh}(\tanh(a+bx))}}}{b}-\frac{2x^{3/2}}{3b\,\text{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Int[x^(3/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
((2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(3/2) - (2*
Sqrt[x])/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]))/b - (2*x^(3/2))/(3*b*ArcTanh[Ta
nh[a + b*x]]^(3/2))
```

3.259.3.1 Defintions of rubi rules used

rule 2596
```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2/Rt[a*b, 2])*ArcTanh[Rt[a*b, 2]*(Sqrt[u]/(a*Sqrt[v
]))], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.259.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{2x^{\frac{3}{2}}}{3b\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{2\sqrt{x}}{b^{2}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}+\frac{2\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh(bx+a))}\right)}{b^{\frac{5}{2}}}$	59
default	$-\frac{2x^{\frac{3}{2}}}{3b\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{2\sqrt{x}}{b^{2}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}+\frac{2\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh(bx+a))}\right)}{b^{\frac{5}{2}}}$	59

input
```
int(x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x^(3/2)/b/arctanh(tanh(b*x+a))^(3/2)-2*x^(1/2)/b^2/arctanh(tanh(b*x+a
))^(1/2)+2/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.48

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx=\left[\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(4\,b^2x+}{3\,(b^5x^2+2\,ab^4x+a^2b^3)}\right.$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt
(b)*sqrt(x) + a) - 2*(4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x^2 + 2
*a*b^4*x + a^2*b^3), -2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-b)*arctan(sqr
t(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x)
)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)]
```

3.259.6 Sympy [F]

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{\operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**(3/2)/atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Integral(x**(3/2)/atanh(tanh(a + b*x))**(5/2), x)
```

3.259.7 Maxima [F]

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/arctanh(tanh(b*x + a))^(5/2), x)
```

3.259.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.65

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\sqrt{x}\left(\frac{4x}{b}+\frac{3a}{b^2}\right)}{3\left(bx+a\right)^{\frac{3}{2}}} - \frac{2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(x)*(4*x/b + 3*a/b^2)/(b*x + a)^(3/2) - 2*log(abs(-sqrt(b)*sqrt(x
) + sqrt(b*x + a)))/b^(5/2)
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{x^{3/2}}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{5/2}}\,dx$$

input
```
int(x^(3/2)/atanh(tanh(a + b*x))^(5/2),x)
```

output
```
int(x^(3/2)/atanh(tanh(a + b*x))^(5/2), x)
```

3.260 $\int \frac{\sqrt{x}}{\mathbf{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.260.1 Optimal result

Integrand size = 17, antiderivative size = 35

$$\int \frac{\sqrt{x}}{\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^{3/2}}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}$$

output
```
-2/3*x^(3/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)
```

3.260.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{x}}{\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2x^{3/2}}{3\text{arctanh}(\tanh(a+bx))^{3/2}(-bx+\text{arctanh}(\tanh(a+bx)))}$$

input
```
Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(2*x^(3/2))/(3*ArcTanh[Tanh[a + b*x]]^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x
]]))
```

3.260.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \; 2598$$

$$-\frac{2x^{3/2}}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

output
```
(-2*x^(3/2))/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2
))
```

3.260.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

3.260.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(29) = 58.

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.63

method	result
derivativedivides	$-\frac{\sqrt{x}}{b\,\text{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{(\text{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}}{3(\text{arctanh}(\tanh(bx+a))-bx)\,\text{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{\cdots}{3(\text{arctanh}\cdots}\right.}{b}$
default	$-\frac{\sqrt{x}}{b\,\text{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{(\text{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}}{3(\text{arctanh}(\tanh(bx+a))-bx)\,\text{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{\cdots}{3(\text{arctanh}\cdots}\right.}{b}$

input
```
int(x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-x^(1/2)/b/arctanh(tanh(b*x+a))^(3/2)+(arctanh(tanh(b*x+a))-b*x)/b*(1/3*x^
(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)+2/3/(arctanh(t
anh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2\sqrt{bx+a}x^{\frac{3}{2}}}{3\,(ab^2x^2+2\,a^2bx+a^3)}$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x + a)*x^(3/2)/(a*b^2*x^2 + 2*a^2*b*x + a^3)
```

3.260.6 Sympy [F]

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{\sqrt{x}}{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}\,dx$$

input
```
integrate(x**(1/2)/atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Integral(sqrt(x)/atanh(tanh(a + b*x))**(5/2), x)
```

3.260.7 Maxima [F]

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{\sqrt{x}}{\operatorname{artanh}(\tanh(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/arctanh(tanh(b*x + a))^(5/2), x)
```

3.260.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2\,x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a}$$

input
```
integrate(x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
2/3*x^(3/2)/((b*x + a)^(3/2)*a)
```

3.260.9 Mupad [B] (verification not implemented)

Time = 4.74 (sec) , antiderivative size = 229, normalized size of antiderivative = 6.54

$$\int \frac{\sqrt{x}}{\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx =$$

$$-\frac{4\,x^{3/2}\,\sqrt{\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{2}}}{3\,b^2\,\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)\,\left(\frac{\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{4\,b^2}+x^2-\frac{x\,\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)\right.}{}\right.}$$

input
```
int(x^(1/2)/atanh(tanh(a + b*x))^(5/2),x)
```

output
```
-(4*x^(3/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - lo
g(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*b^2*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)*
((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^2/(4*b^2) + x^2 - (x*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))
/b))
```

3.261 $\int \frac{1}{\sqrt{x}\textbf{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.261.1 Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\sqrt{x}}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}} + \frac{4\sqrt{x}}{3(bx-\text{arctanh}(\tanh(a+bx)))^2\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

output
```
-2/3*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)+4/3*x^(
1/2)/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(1/2)
```

3.261.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\sqrt{x}(bx-3\text{arctanh}(\tanh(a+bx)))}{3\text{arctanh}(\tanh(a+bx))^{3/2}(-bx+\text{arctanh}(\tanh(a+bx)))^2}$$

input
```
Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

output
```
(-2*Sqrt[x]*(b*x - 3*ArcTanh[Tanh[a + b*x]]))/(3*ArcTanh[Tanh[a + b*x]]^(3
/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

3.261.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{2602}$$

$$-\frac{2\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{3(bx-\text{arctanh}(\tanh(a+bx)))}-\frac{2\sqrt{x}}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2598}$$

$$\frac{4\sqrt{x}}{3(bx-\text{arctanh}(\tanh(a+bx)))^2\sqrt{\text{arctanh}(\tanh(a+bx))}}-\frac{2\sqrt{x}}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

output
```
(-2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2
)) + (4*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a +
 b*x]]])
```

3.261.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.261.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}$	58
default	$\frac{2\sqrt{x}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}$	58

input
```
int(1/x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)+4/3/(arc
tanh(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{1}{\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx=\frac{2\,(2\,bx+3\,a)\sqrt{bx+a}\sqrt{x}}{3\,(a^2b^2x^2+2\,a^3bx+a^4)}$$

input
```
integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/3*(2*b*x + 3*a)*sqrt(b*x + a)*sqrt(x)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4)
```

3.261.6 Sympy [F]

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{1}{\sqrt{x}\,\text{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**(1/2)/atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Integral(1/(sqrt(x)*atanh(tanh(a + b*x))**(5/2)), x)
```

3.261.7 Maxima [F]

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{1}{\sqrt{x}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x)*arctanh(tanh(b*x + a))^(5/2)), x)
```

3.261.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.35

$$\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2\sqrt{x}\left(\frac{2\,bx}{a^2}+\frac{3}{a}\right)}{3\left(bx+a\right)^{\frac{3}{2}}}$$

input
```
integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
2/3*sqrt(x)*(2*b*x/a^2 + 3/a)/(b*x + a)^(3/2)
```

3.261.9 Mupad [B] (verification not implemented)

Time = 4.61 (sec) , antiderivative size = 346, normalized size of antiderivative = 4.87

$$\int \frac{1}{\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)}{2}}\left(\frac{16\,x^2}{3\,b\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^2} - \cdots\right.}{x^{5/2}-\frac{x^{3/2}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}{b}+\frac{\sqrt{x}\left(\ln\left(\frac{\cdots}{e^{2\cdots}}\right.\right.}{\cdots}}$$

input
```
int(1/(x^(1/2)*atanh(tanh(a + b*x))^(5/2)),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((16*x^2)/(3*b*(log(2/(exp(2*a)*exp(2*b*x) +
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) -
 (x*(48*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 48*log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 96*b*x))/(12*b^2*(log(2/(exp(2*a)*exp(2*b*x)
+ 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
))/(x^(5/2) - (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b + (x^(1/2)*(log(2/(exp
(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^2)/(4*b^2))
```

3.262 $\int \frac{1}{x^{3/2}\mathbf{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.262.1 Optimal result

Integrand size = 17, antiderivative size = 106

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{8b\sqrt{x}}{3(bx-\text{arctanh}(\tanh(a+bx)))^2\text{arctanh}(\tanh(a+bx))^{3/2}} + \frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}} + \frac{16b\sqrt{x}}{3(bx-\text{arctanh}(\tanh(a+bx)))^3\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

output
```
2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)/x^(1/2)-8/3*b*x^(1
/2)/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(3/2)+16/3*b*x^(1/2)
/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(1/2)
```

3.262.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2(-b^2x^2+6bx\text{arctanh}(\tanh(a+bx))+3\text{arctanh}(\tanh(a+bx))^2)}{3\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))^3\text{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

output
```
(2*(-(b^2*x^2) + 6*b*x*ArcTanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2
))/(3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]^(3/2
))
```

3.262.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2602, 2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{4b\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{5/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2602}$$

$$\frac{4b\left(-\frac{2\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{3(bx-\text{arctanh}(\tanh(a+bx)))}-\frac{2\sqrt{x}}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2598}$$

$$\frac{4b\left(\frac{4\sqrt{x}}{3(bx-\text{arctanh}(\tanh(a+bx)))^2\sqrt{\text{arctanh}(\tanh(a+bx))}}-\frac{2\sqrt{x}}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

output
```
(4*b*((-2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]
]^(3/2)) + (4*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Ta
nh[a + b*x]]])))/(b*x - ArcTanh[Tanh[a + b*x]]) + 2/(Sqrt[x]*(b*x - ArcTan
h[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))
```

3.262.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.262.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{2}{\sqrt{x}\,(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{8b\left(\frac{\sqrt{x}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{}{3(\mathrm{ar}}\right.}{\operatorname{arctanh}(\tanh(bx+}$
default	$-\frac{2}{\sqrt{x}\,(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{8b\left(\frac{\sqrt{x}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{}{3(\mathrm{ar}}\right.}{\operatorname{arctanh}(\tanh(bx+}$

input
```
int(1/x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)-8*b/(arct
anh(tanh(b*x+a))-b*x)*(1/3*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh
(b*x+a))^(3/2)+2/3/(arctanh(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a
))^(1/2))
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\,(8\,b^2x^2+12\,abx+3\,a^2)\sqrt{bx+a}\sqrt{x}}{3\,(a^3b^2x^3+2\,a^4bx^2+a^5x)}$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(8*b^2*x^2 + 12*a*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b^2*x^3 + 2
*a^4*b*x^2 + a^5*x)
```

3.262.6 Sympy [F]

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\,\text{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**(3/2)/atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Integral(1/(x**(3/2)*atanh(tanh(a + b*x))**(5/2)), x)
```

3.262.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\,(8\,b^3x^3+20\,ab^2x^2+15\,a^2bx+3\,a^3)}{3\,(bx+a)^{\frac{5}{2}}a^3\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(8*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 3*a^3)/((b*x + a)^(5/2)*a^3*
sqrt(x))
```

3.262.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\sqrt{x}\left(\frac{5b^2x}{a^3}+\frac{6b}{a^2}\right)}{3(bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{b}}{\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)a^2}$$

input
```
integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(x)*(5*b^2*x/a^3 + 6*b/a^2)/(b*x + a)^(3/2) + 4*sqrt(b)/(((sqrt(b
)*sqrt(x) - sqrt(b*x + a))^2 - a)*a^2)
```

3.262.9 Mupad [B] (verification not implemented)

Time = 4.58 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.28

$$\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{4}{b^2\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}+\cdots\right)}{x^{5/2}-\frac{x^{3/2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\cdots\right)}{b}\cdots}$$

input
```
int(1/(x^(3/2)*atanh(tanh(a + b*x))^(5/2)),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*(4/(b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (128*x^2
)/(3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (32*x)/(b*(log(2/(exp(2*a)*exp(2*b*x)
+ 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
))/(x^(5/2) - (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b + (x^(1/2)*(log(2/(exp
(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^2)/(4*b^2))
```

3.263 $\int \frac{1}{x^{5/2}\textbf{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.263.1 Optimal result

Integrand size = 17, antiderivative size = 146

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{16b^2\sqrt{x}}{3(bx-\text{arctanh}(\tanh(a+bx)))^3\text{arctanh}(\tanh(a+bx))^{3/2}} + \frac{4b}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))^2\text{arctanh}(\tanh(a+bx))^{3/2}} + \frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}} + \frac{32b^2\sqrt{x}}{3(bx-\text{arctanh}(\tanh(a+bx)))^4\sqrt{\text{arctanh}(\tanh(a+bx))}}$$

output
```
2/3/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)+4*b/(b*x
-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(3/2)/x^(1/2)-16/3*b^2*x^(1/
2)/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(3/2)+32/3*b^2*x^(1/2
)/(b*x-arctanh(tanh(b*x+a)))^4/arctanh(tanh(b*x+a))^(1/2)
```

3.263.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^{5/2}\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2(b^3x^3-9b^2x^2\operatorname{arctanh}(\tanh(a+bx))-9bx\operatorname{arctanh}(\tanh(a+bx))^2+\operatorname{arctanh}(\tanh(a+bx))^3)}{3x^{3/2}\operatorname{arctanh}(\tanh(a+bx))^{3/2}(-bx+\operatorname{arctanh}(\tanh(a+bx)))^4}$$

input
```
Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

output
```
(-2*(b^3*x^3 - 9*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 9*b*x*ArcTanh[Tanh[a + b
*x]]^2 + ArcTanh[Tanh[a + b*x]]^3))/(3*x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2
)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

3.263.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2602, 2602, 2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{2b\int \frac{1}{x^{3/2}\operatorname{arctanh}(\tanh(a+bx))^{5/2}}dx}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2602}$$

$$\frac{2b\left(\frac{4b\int \frac{1}{\sqrt{x}\operatorname{arctanh}(\tanh(a+bx))^{5/2}}dx}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))^{3/2}}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\operatorname{arctanh}(\tanh(a+bx)))\operatorname{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2602}$$

$$\frac{2b\left(\frac{4b\left(-\frac{2\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{3(bx-\text{arctanh}(\tanh(a+bx)))}-\frac{2\sqrt{x}}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))\text{arc}}\right.}{bx-\text{arctanh}(\tanh(a+bx))}$$

$$\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \text{2598}$$

$$\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}+$$

$$\frac{2b\left(\frac{4b\left(\frac{4\sqrt{x}}{3(bx-\text{arctanh}(\tanh(a+bx)))^2\sqrt{\text{arctanh}(\tanh(a+bx))}}-\frac{2\sqrt{x}}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{}{\sqrt{x}(bx-\text{arctanh}(\right.}{bx-\text{arctanh}(\tanh(a+bx))}$$

input
```
Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

output
```
(2*b*((4*b*((-2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a
+ b*x]]^(3/2)) + (4*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcT
anh[Tanh[a + b*x]]])))/(b*x - ArcTanh[Tanh[a + b*x]]) + 2/(Sqrt[x]*(b*x -
ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))))/(b*x - ArcTanh[Tan
h[a + b*x]]) + 2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a
+ b*x]]^(3/2))
```

3.263.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.263. $\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.263.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.03

method	result
derivativedivides	$-\frac{2}{3x^{\frac{3}{2}}\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{4b\left(-\frac{1}{\sqrt{x}\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}\right.}{}$
default	$-\frac{2}{3x^{\frac{3}{2}}\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{4b\left(-\frac{1}{\sqrt{x}\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}\right.}{}$

input
```
int(1/x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/x^(3/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)-4*b/(ar
ctanh(tanh(b*x+a))-b*x)*(-1/x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tan
h(b*x+a))^(3/2)-4*b/(arctanh(tanh(b*x+a))-b*x)*(1/3*x^(1/2)/(arctanh(tanh(
b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)+2/3/(arctanh(tanh(b*x+a))-b*x)^2*x
^(1/2)/arctanh(tanh(b*x+a))^(1/2)))
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^{5/2}\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx=\frac{2\,(16\,b^3x^3+24\,ab^2x^2+6\,a^2bx-a^3)\sqrt{bx+a}\sqrt{x}}{3\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)}$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/3*(16*b^3*x^3 + 24*a*b^2*x^2 + 6*a^2*b*x - a^3)*sqrt(b*x + a)*sqrt(x)/(a
^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2)
```

3.263.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(5/2)/atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.263.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2\,(16\,b^4x^4+40\,ab^3x^3+30\,a^2b^2x^2+5\,a^3bx-a^4)}{3\,(bx+a)^{\frac{5}{2}}a^4x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
2/3*(16*b^4*x^4 + 40*a*b^3*x^3 + 30*a^2*b^2*x^2 + 5*a^3*b*x - a^4)/((b*x +
 a)^(5/2)*a^4*x^(3/2))
```

3.263.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2\sqrt{x}\left(\frac{8\,b^3x}{a^4}+\frac{9\,b^2}{a^3}\right)}{3\,(bx+a)^{\frac{3}{2}}} - \frac{8\left(3\,b^{\frac{3}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4-9\,ab^{\frac{3}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2+4\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^3a^3}$$

input
```
integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
2/3*sqrt(x)*(8*b^3*x/a^4 + 9*b^2/a^3)/(b*x + a)^(3/2) - 8/3*(3*b^(3/2)*(sq
rt(b)*sqrt(x) - sqrt(b*x + a))^4 - 9*a*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(b*x
 + a))^2 + 4*a^2*b^(3/2))/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^3*a^3
)
```

3.263.9 Mupad [B] (verification not implemented)

Time = 4.76 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.78

$$\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{4}{3\,b^2\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}\right.}{x^{7/2}-\frac{x^{5/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b}}\right.}{}}$$

input
```
int(1/(x^(5/2)*atanh(tanh(a + b*x))^(5/2)),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*(4/(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - (128*x
^2)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + (16*x)/(b*(log(2/(exp(2*a)*exp(2*b*x) +
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) +
 (512*b*x^3)/(3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)))/(x^(7/2) - (x^(5/2)*(log(2/
(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x))/b + (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(4*b^2))
```

3.264 $\int \frac{1}{x^{7/2}\mathbf{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

3.264.1 Optimal result

Integrand size = 17, antiderivative size = 186

$$\begin{aligned}\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \\ & -\frac{128b^3\sqrt{x}}{15(bx-\text{arctanh}(\tanh(a+bx)))^4\text{arctanh}(\tanh(a+bx))^{3/2}} \\ & +\frac{32b^2}{5\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))^3\text{arctanh}(\tanh(a+bx))^{3/2}} \\ & +\frac{16b}{15x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))^2\text{arctanh}(\tanh(a+bx))^{3/2}} \\ & +\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}} \\ & +\frac{256b^3\sqrt{x}}{15(bx-\text{arctanh}(\tanh(a+bx)))^5\sqrt{\text{arctanh}(\tanh(a+bx))}}\end{aligned}$$

output
```
16/15*b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(3/2)+2/
5/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)+32/5*b^2/(
b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(3/2)/x^(1/2)-128/15*b^3*
x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^4/arctanh(tanh(b*x+a))^(3/2)+256/15*b^3
*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^5/arctanh(tanh(b*x+a))^(1/2)
```

3.264.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{2(-5b^4x^4+60b^3x^3\text{arctanh}(\tanh(a+bx))+90b^2x^2\text{arctanh}(\tanh(a+}{15x^{5/2}(bx-\text{arctanh}(\tanh(a+bx}$$

input
```
Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

output
```
(2*(-5*b^4*x^4 + 60*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 90*b^2*x^2*ArcTanh[Ta
nh[a + b*x]]^2 - 20*b*x*ArcTanh[Tanh[a + b*x]]^3 + 3*ArcTanh[Tanh[a + b*x]
]^4))/(15*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^5*ArcTanh[Tanh[a + b*x]]^
(3/2))
```

3.264.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2602, 2602, 2602, 2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \; 2602$$

$$\frac{8b\int \frac{1}{x^{5/2}\text{arctanh}(\tanh(a+bx))^{5/2}}dx}{5(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \; 2602$$

$$\frac{8b\left(\frac{2b\int \frac{1}{x^{3/2}\text{arctanh}(\tanh(a+bx))^{5/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}\right)}{5(bx-\text{arctanh}(\tanh(a+bx)))}+\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$$\Big\downarrow \; 2602$$

$$8b\left(\frac{2b\left(\frac{4b\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{5/2}}dx}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))\text{a}}\right.$$
$$\frac{}{5(bx-\text{arctanh}(\tanh(a+bx)))}$$
$$\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$\big\downarrow$ 2602

$$8b\left(\frac{2b\left(\frac{4b\left(-\frac{2\int \frac{1}{\sqrt{x}\text{arctanh}(\tanh(a+bx))^{3/2}}dx}{3(bx-\text{arctanh}(\tanh(a+bx)))}-\frac{2\sqrt{x}}{3(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}\right)}{bx-\text{arctanh}(\tanh(a+bx))}+\frac{2}{\sqrt{x}(bx-\text{arctanh}(\tanh(a+bx)))\text{arct}}\right.}{bx-\text{arctanh}(\tanh(a+bx))}\right.$$
$$\frac{}{5(bx-\text{arctanh}(\tanh(a+bx)))}$$
$$\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}$$

$\big\downarrow$ 2598

$$8b\left(\frac{2}{3x^{3/2}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}+\frac{2b\left(\frac{4b\left(\frac{4\sqrt{x}}{3(bx-\text{arctanh}(\tanh(a+bx)))^2\sqrt{\text{arctanh}(\tanh(a+bx))}}-\frac{}{3(bx-\text{arc}}\right.}{bx-\text{arctanh}(\tanh(a+bx}\right.}{}\right.$$
$$\frac{}{5(bx-\text{arctanh}(\tanh(a+bx)}$$
$$\frac{2}{5x^{5/2}(bx-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{3/2}}$$

input
```
Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

3.264. $\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx$

output
```
(8*b*((2*b*((4*b*((-2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[T
anh[a + b*x]]^(3/2)) + (4*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqr
t[ArcTanh[Tanh[a + b*x]]])))/(b*x - ArcTanh[Tanh[a + b*x]]) + 2/(Sqrt[x]*(
b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))))/(b*x - ArcTa
nh[Tanh[a + b*x]]) + 2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[T
anh[a + b*x]]^(3/2))))/(5*(b*x - ArcTanh[Tanh[a + b*x]])) + 2/(5*x^(5/2)*(
b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))
```

3.264.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.264.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-\frac{2}{5x^{\frac{5}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{16b\left(-\frac{1}{3x^{\frac{3}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}\right.}{}$
default	$-\frac{2}{5x^{\frac{5}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{16b\left(-\frac{1}{3x^{\frac{3}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}\right.}{}$

input
```
int(1/x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/x^(5/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)-16/5*b/
(arctanh(tanh(b*x+a))-b*x)*(-1/3/x^(3/2)/(arctanh(tanh(b*x+a))-b*x)/arctan
h(tanh(b*x+a))^(3/2)-2*b/(arctanh(tanh(b*x+a))-b*x)*(-1/x^(1/2)/(arctanh(t
anh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)-4*b/(arctanh(tanh(b*x+a))-b*x)
*(1/3*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)+2/3/(a
rctanh(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))))
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^{7/2}\operatorname{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\,(128\,b^4x^4+192\,ab^3x^3+48\,a^2b^2x^2-8\,a^3bx+3\,a^4)\sqrt{bx+a}\sqrt{x}}{15\,(a^5b^2x^5+2\,a^6bx^4+a^7x^3)}$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(128*b^4*x^4 + 192*a*b^3*x^3 + 48*a^2*b^2*x^2 - 8*a^3*b*x + 3*a^4)*s
qrt(b*x + a)*sqrt(x)/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3)
```

3.264.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(7/2)/atanh(tanh(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.264.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\left(128\,b^5x^5+320\,ab^4x^4+240\,a^2b^3x^3+40\,a^3b^2x^2-5\,a^4bx+3\,a^5\right)}{15\,(bx+a)^{\frac{5}{2}}a^5x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-2/15*(128*b^5*x^5 + 320*a*b^4*x^4 + 240*a^2*b^3*x^3 + 40*a^3*b^2*x^2 - 5*
a^4*b*x + 3*a^5)/((b*x + a)^(5/2)*a^5*x^(5/2))
```

3.264.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\sqrt{x}\left(\frac{11\,b^4x}{a^5}+\frac{12\,b^3}{a^4}\right)}{3\,(bx+a)^{\frac{3}{2}}} + \frac{4\left(45\,b^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^8-240\,ab^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^6+490\,a^2b^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4-320\,a^3b^{\frac{5}{2}}\right.}{15\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^5a^4}$$

input
```
integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(x)*(11*b^4*x/a^5 + 12*b^3/a^4)/(b*x + a)^(3/2) + 4/15*(45*b^(5/2
)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^8 - 240*a*b^(5/2)*(sqrt(b)*sqrt(x) - s
qrt(b*x + a))^6 + 490*a^2*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 32
0*a^3*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 + 73*a^4*b^(5/2))/(((sqr
t(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5*a^4)
```

3.264.9 Mupad [B] (verification not implemented)

Time = 5.03 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.51

$$\int \frac{1}{x^{7/2}\text{arctanh}(\tanh(a+bx))^{5/2}}\,dx = \frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{4}{5\,b^2\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}\right.}{}$$

input
```
int(1/(x^(7/2)*atanh(tanh(a + b*x))^(5/2)),x)
```

output
```
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*(4/(5*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (256*x
^2)/(5*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (64*x)/(15*b*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^2) - (2048*b*x^3)/(5*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4) + (8192*b^2*x^4)/(15*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^5)))/(x^(9/2) - (x^(7/2)*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x))/b + (x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(4*b^2))
```

3.265 $\int x^m \textbf{arctanh}(\tanh(a+bx))^n\,dx$

3.265.1 Optimal result

Integrand size = 13, antiderivative size = 79

$$\int x^m \text{arctanh}(\tanh(a+bx))^n\,dx$$
$$= \frac{x^m\left(\frac{bx}{bx-\text{arctanh}(\tanh(a+bx))}\right)^{-m}\text{arctanh}(\tanh(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(-m,1+n,2+n,-\frac{\text{arct}}{bx-\text{arc}}\right.}{b(1+n)}$$

output
```
x^m*arctanh(tanh(b*x+a))^(1+n)*hypergeom([-m, 1+n],[2+n],-arctanh(tanh(b*x
+a))/(b*x-arctanh(tanh(b*x+a))))/b/(1+n)/((b*x/(b*x-arctanh(tanh(b*x+a))))
^m)
```

3.265.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int x^m \text{arctanh}(\tanh(a+bx))^n\,dx$$
$$= \frac{x^{1+m}\text{arctanh}(\tanh(a+bx))^n\left(1+\frac{bx}{-bx+\text{arctanh}(\tanh(a+bx))}\right)^{-n}\text{Hypergeometric2F1}\left(1+m,-n,2+m,-\right.}{1+m}$$

input
```
Integrate[x^m*ArcTanh[Tanh[a + b*x]]^n,x]
```

output
```
(x^(1 + m)*ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2F1[1 + m, -n, 2 + m, -(
(b*x)/(-(b*x) + ArcTanh[Tanh[a + b*x]]))])/((1 + m)*(1 + (b*x)/(-(b*x) + A
rcTanh[Tanh[a + b*x]]))^n)
```

3.265.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2604}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \text{arctanh}(\tanh(a+bx))^n \, dx$$

$$\Big\downarrow \text{2604}$$

$$\frac{x^m \left(\frac{bx}{bx-\text{arctanh}(\tanh(a+bx))}\right)^{-m} \text{arctanh}(\tanh(a+bx))^{n+1} \text{Hypergeometric2F1}\left(-m,n+1,n+2,-\frac{\text{arctanh}(\tan\ldots}{bx-\text{arctanh}(\text{ta}\ldots}\right.}{b(n+1)}$$

input
```
Int[x^m*ArcTanh[Tanh[a + b*x]]^n,x]
```

output
```
(x^m*ArcTanh[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, -(
ArcTanh[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/(b*(1 + n)*((b*x)
/(b*x - ArcTanh[Tanh[a + b*x]]))^m)
```

3.265.3.1 Defintions of rubi rules used

rule 2604
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[u^m*(v^(n + 1)/(b*(n + 1)*(b*(u/(b*u - a*v)))^m))*Hype
rgeometric2F1[-m, n + 1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v,
 0]] /; PiecewiseLinearQ[u, v, x] && !IntegerQ[m] && !IntegerQ[n]
```

3.265.4 Maple [F]

$$\int x^m \operatorname{arctanh}(\tanh(bx+a))^n\,dx$$

input
```
int(x^m*arctanh(tanh(b*x+a))^n,x)
```

output
```
int(x^m*arctanh(tanh(b*x+a))^n,x)
```

3.265.5 Fricas [F]

$$\int x^m \operatorname{arctanh}(\tanh(a+bx))^n\,dx = \int x^m \operatorname{artanh}(\tanh(bx+a))^n\,dx$$

input
```
integrate(x^m*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral(x^m*arctanh(tanh(b*x + a))^n, x)
```

3.265.6 Sympy [F]

$$\int x^m \operatorname{arctanh}(\tanh(a+bx))^n\,dx = \int x^m \operatorname{atanh}^n(\tanh(a+bx))\,dx$$

input
```
integrate(x**m*atanh(tanh(b*x+a))**n,x)
```

output
```
Integral(x**m*atanh(tanh(a + b*x))**n, x)
```

3.265.7 Maxima [F]

$$\int x^m \operatorname{arctanh}(\tanh(a+bx))^n\,dx = \int x^m \operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^n\,dx$$

input
```
integrate(x^m*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate(x^m*arctanh(tanh(b*x + a))^n, x)
```

3.265.8 Giac [F]

$$\int x^m \operatorname{arctanh}(\tanh(a+bx))^n\,dx = \int x^m \operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^n\,dx$$

input
```
integrate(x^m*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate(x^m*arctanh(tanh(b*x + a))^n, x)
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int x^m \operatorname{arctanh}(\tanh(a+bx))^n\,dx = \int x^m \operatorname{atanh}(\tanh(a+b\,x))^n\,dx$$

input
```
int(x^m*atanh(tanh(a + b*x))^n,x)
```

output
```
int(x^m*atanh(tanh(a + b*x))^n, x)
```

3.266 $\int x^4 \mathbf{arctanh}(\tanh(a+bx))^n\,dx$

3.266.1 Optimal result

Integrand size = 13, antiderivative size = 165

$$\begin{aligned}\int x^4\text{arctanh}(\tanh(a+bx))^n\,dx=&\frac{x^4\text{arctanh}(\tanh(a+bx))^{1+n}}{b(1+n)}\\&-\frac{4x^3\text{arctanh}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}\\&+\frac{12x^2\text{arctanh}(\tanh(a+bx))^{3+n}}{b^3(1+n)(2+n)(3+n)}\\&-\frac{24x\text{arctanh}(\tanh(a+bx))^{4+n}}{b^4(1+n)(2+n)(3+n)(4+n)}\\&+\frac{24\text{arctanh}(\tanh(a+bx))^{5+n}}{b^5(1+n)(2+n)(3+n)(4+n)(5+n)}\end{aligned}$$

output
```
x^4*arctanh(tanh(b*x+a))^(1+n)/b/(1+n)-4*x^3*arctanh(tanh(b*x+a))^(2+n)/b^
2/(1+n)/(2+n)+12*x^2*arctanh(tanh(b*x+a))^(3+n)/b^3/(3+n)/(n^2+3*n+2)-24*x
*arctanh(tanh(b*x+a))^(4+n)/b^4/(n^2+5*n+4)/(n^2+5*n+6)+24*arctanh(tanh(b*
x+a))^(5+n)/b^5/(n^2+7*n+12)/(n^3+8*n^2+17*n+10)
```

3.266.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int x^4 \text{arctanh}(\tanh(a+bx))^n\,dx$$
$$= \frac{\text{arctanh}(\tanh(a+bx))^{1+n}\left(b^4(120+154n+71n^2+14n^3+n^4)\,x^4-4b^3(60+47n+12n^2+n^3)\,x^3\text{arctan}\right.}{b^5(1}$$

input
```
Integrate[x^4*ArcTanh[Tanh[a + b*x]]^n,x]
```

output
```
(ArcTanh[Tanh[a + b*x]]^(1 + n)*(b^4*(120 + 154*n + 71*n^2 + 14*n^3 + n^4)
*x^4 - 4*b^3*(60 + 47*n + 12*n^2 + n^3)*x^3*ArcTanh[Tanh[a + b*x]] + 12*b^
2*(20 + 9*n + n^2)*x^2*ArcTanh[Tanh[a + b*x]]^2 - 24*b*(5 + n)*x*ArcTanh[T
anh[a + b*x]]^3 + 24*ArcTanh[Tanh[a + b*x]]^4))/(b^5*(1 + n)*(2 + n)*(3 +
n)*(4 + n)*(5 + n))
```

3.266.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2599, 2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{arctanh}(\tanh(a+bx))^n\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^4\text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{4\int x^3\text{arctanh}(\tanh(a+bx))^{n+1}dx}{b(n+1)}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^4\text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{4\left(\frac{x^3\text{arctanh}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{3\int x^2\text{arctanh}(\tanh(a+bx))^{n+2}dx}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^4\text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)} - \frac{4\left(\frac{x^3\text{arctanh}(\tanh(a+bx))^{n+2}}{b(n+2)} - \frac{3\left(\frac{x^2\text{arctanh}(\tanh(a+bx))^{n+3}}{b(n+3)} - \frac{2\int x\text{arctanh}(\tanh(a+bx))^{n+3}dx}{b(n+3)}\right)}{b(n+2)}\right)}{b(n+1)}$$

$\Big\downarrow$ 2599

$$\frac{x^4\text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)} - \frac{4\left(\frac{x^3\text{arctanh}(\tanh(a+bx))^{n+2}}{b(n+2)} - \frac{3\left(\frac{x^2\text{arctanh}(\tanh(a+bx))^{n+3}}{b(n+3)} - \frac{2\left(\frac{x\text{arctanh}(\tanh(a+bx))^{n+4}}{b(n+4)} - \frac{\int \text{arctanh}(\tanh(a+bx))^{n+4}dx}{b(n+4)}\right)}{b(n+3)}\right)}{b(n+2)}\right)}{b(n+1)}$$

$\Big\downarrow$ 2588

$$\frac{x^4\text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)} - \frac{4\left(\frac{x^3\text{arctanh}(\tanh(a+bx))^{n+2}}{b(n+2)} - \frac{3\left(\frac{x^2\text{arctanh}(\tanh(a+bx))^{n+3}}{b(n+3)} - \frac{2\left(\frac{x\text{arctanh}(\tanh(a+bx))^{n+4}}{b(n+4)} - \frac{\int \text{arctanh}(\tanh(a+bx))^{n+4}d\text{arctanh}(}{b^2(n+4)}\right.}{b(n+3)}\right.}{b(n+2)}\right.}{b(n+1)}$$

$\Big\downarrow$ 15

$$\frac{x^4\text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)} - \frac{4\left(\frac{x^3\text{arctanh}(\tanh(a+bx))^{n+2}}{b(n+2)} - \frac{3\left(\frac{x^2\text{arctanh}(\tanh(a+bx))^{n+3}}{b(n+3)} - \frac{2\left(\frac{x\text{arctanh}(\tanh(a+bx))^{n+4}}{b(n+4)} - \frac{\text{arctanh}(\tanh(a+bx))^{n+5}}{b^2(n+4)(n+5)}\right)}{b(n+3)}\right)}{b(n+2)}\right)}{b(n+1)}$$

input
```
Int[x^4*ArcTanh[Tanh[a + b*x]]^n,x]
```

3.266. $\int x^4\text{arctanh}(\tanh(a+bx))^n\,dx$

output
```
(x^4*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (4*((x^3*ArcTanh[Tanh[a
 + b*x]]^(2 + n))/(b*(2 + n)) - (3*((x^2*ArcTanh[Tanh[a + b*x]]^(3 + n))/(
b*(3 + n)) - (2*((x*ArcTanh[Tanh[a + b*x]]^(4 + n))/(b*(4 + n)) - ArcTanh[
Tanh[a + b*x]]^(5 + n)/(b^2*(4 + n)*(5 + n))))/(b*(3 + n))))/(b*(2 + n))))
/(b*(1 + n))
```

3.266.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.266.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(165) = 330$.

Time = 3.06 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.46

method	result
default	$\frac{x^{5} \mathrm{e}^{n \ln(\operatorname{arctanh}(\tanh(bx+a)))}}{5+n}+\frac{n(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{4} \mathrm{e}^{n \ln(\operatorname{arctanh}(\tanh(bx+a)))}}{b(n^{2}+9n+20)}-\frac{4n\left(a^{2}+2a(\operatorname{arctanh}(\tanh(bx+a)\right.}{}$
parallelrisch	$-\frac{48x^{3} \operatorname{arctanh}(\tanh(bx+a))^{2} \operatorname{arctanh}(\tanh(bx+a))^{n} b^{3} n^{2}-x^{4} \operatorname{arctanh}(\tanh(bx+a)) \operatorname{arctanh}(\tanh(bx+a))^{n} b^{4} n^{4}+188x^{3} \operatorname{arct}}{}$
risch	Expression too large to display

input
```
int(x^4*arctanh(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
1/(5+n)*x^5*exp(n*ln(arctanh(tanh(b*x+a))))+n/b*(arctanh(tanh(b*x+a))-b*x)
/(n^2+9*n+20)*x^4*exp(n*ln(arctanh(tanh(b*x+a))))-4*n*(a^2+2*a*(arctanh(ta
nh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/b^2/(n^3+12*n^2+47*n+60)
*x^3*exp(n*ln(arctanh(tanh(b*x+a))))+24*(arctanh(tanh(b*x+a))-b*x)*(a^2+2*
a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)^2/b^5/(n^3+
12*n^2+47*n+60)/(n^2+3*n+2)*exp(n*ln(arctanh(tanh(b*x+a))))-24*(a^2+2*a*(a
rctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)^2*n/b^4/(n^3+12
*n^2+47*n+60)/(n^2+3*n+2)*x*exp(n*ln(arctanh(tanh(b*x+a))))+12/b^3*(arctan
h(tanh(b*x+a))-b*x)*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*
x+a))-b*x-a)^2)*n/(2+n)/(n^3+12*n^2+47*n+60)*x^2*exp(n*ln(arctanh(tanh(b*x
+a))))
```

3.266.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(165) = 330$.

Time = 0.26 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.27

$$\int x^4 \text{arctanh}(\tanh(a+bx))^n\,dx =$$
$$-\frac{(24\,a^4bnx - (b^5n^4 + 10\,b^5n^3 + 35\,b^5n^2 + 50\,b^5n + 24\,b^5)x^5 - 24\,a^5 - (ab^4n^4 + 6\,ab^4n^3 + 11\,ab^4n^2 + 6\,ab}{}$$

input
```
integrate(x^4*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
-((24*a^4*b*n*x - (b^5*n^4 + 10*b^5*n^3 + 35*b^5*n^2 + 50*b^5*n + 24*b^5)*
x^5 - 24*a^5 - (a*b^4*n^4 + 6*a*b^4*n^3 + 11*a*b^4*n^2 + 6*a*b^4*n)*x^4 +
4*(a^2*b^3*n^3 + 3*a^2*b^3*n^2 + 2*a^2*b^3*n)*x^3 - 12*(a^3*b^2*n^2 + a^3*
b^2*n)*x^2)*cosh(n*log(b*x + a)) + (24*a^4*b*n*x - (b^5*n^4 + 10*b^5*n^3 +
 35*b^5*n^2 + 50*b^5*n + 24*b^5)*x^5 - 24*a^5 - (a*b^4*n^4 + 6*a*b^4*n^3 +
 11*a*b^4*n^2 + 6*a*b^4*n)*x^4 + 4*(a^2*b^3*n^3 + 3*a^2*b^3*n^2 + 2*a^2*b^
3*n)*x^3 - 12*(a^3*b^2*n^2 + a^3*b^2*n)*x^2)*sinh(n*log(b*x + a)))/(b^5*n^
5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

3.266.6 Sympy [F]

$$\int x^4 \text{arctanh}(\tanh(a+bx))^n \, dx = \text{Too large to display}$$

input
```
integrate(x**4*atanh(tanh(b*x+a))**n,x)
```

output
```
Piecewise((x**5*atanh(tanh(a))**n/5, Eq(b, 0)), (-x**4/(4*b*atanh(tanh(a +
 b*x))**4) - x**3/(3*b**2*atanh(tanh(a + b*x))**3) - x**2/(2*b**3*atanh(ta
nh(a + b*x))**2) - x/(b**4*atanh(tanh(a + b*x))) + log(atanh(tanh(a + b*x)
))/b**5, Eq(n, -5)), (Integral(x**4/atanh(tanh(a + b*x))**4, x), Eq(n, -4)
), (Integral(x**4/atanh(tanh(a + b*x))**3, x), Eq(n, -3)), (Integral(x**4/
atanh(tanh(a + b*x))**2, x), Eq(n, -2)), (Integral(x**4/atanh(tanh(a + b*x
)), x), Eq(n, -1)), (b**4*n**4*x**4*atanh(tanh(a + b*x))*atanh(tanh(a + b*
x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5
*n + 120*b**5) + 14*b**4*n**3*x**4*atanh(tanh(a + b*x))*atanh(tanh(a + b*x
))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) + 71*b**4*n**2*x**4*atanh(tanh(a + b*x))*atanh(tanh(a + b*x)
)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n
 + 120*b**5) + 154*b**4*n*x**4*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**
n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
120*b**5) + 120*b**4*x**4*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b*
*5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b
**5) - 4*b**3*n**3*x**3*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x))**n/(b
**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*
b**5) - 48*b**3*n**2*x**3*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x))**n/
(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +...
```

3.266.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.84

$$\int x^4 \text{arctanh}(\tanh(a+bx))^n \, dx = \frac{((n^4+10\,n^3+35\,n^2+50\,n+24)b^5x^5+(n^4+6\,n^3+11\,n^2+6\,n)ab^4x^4-4\,(n^3+3\,n^2+2\,n)a^2b^3x^3+12}{(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^5}$$

input
```
integrate(x^4*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n
)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2
 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 +
274*n + 120)*b^5)
```

3.266.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(165) = 330$.

Time = 0.26 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.01

$$\int x^4 \text{arctanh}(\tanh(a+bx))^n\,dx$$
$$= \frac{(bx+a)^n b^5 n^4 x^5 + (bx+a)^n ab^4 n^4 x^4 + 10\,(bx+a)^n b^5 n^3 x^5 + 6\,(bx+a)^n ab^4 n^3 x^4 + 35\,(bx+a)^n b^5 n^2 x^5 - 4}{}$$

input
```
integrate(x^4*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^5*n^4*x^5 + (b*x + a)^n*a*b^4*n^4*x^4 + 10*(b*x + a)^n*b^5*
n^3*x^5 + 6*(b*x + a)^n*a*b^4*n^3*x^4 + 35*(b*x + a)^n*b^5*n^2*x^5 - 4*(b*
x + a)^n*a^2*b^3*n^3*x^3 + 11*(b*x + a)^n*a*b^4*n^2*x^4 + 50*(b*x + a)^n*b
^5*n*x^5 - 12*(b*x + a)^n*a^2*b^3*n^2*x^3 + 6*(b*x + a)^n*a*b^4*n*x^4 + 24
*(b*x + a)^n*b^5*x^5 + 12*(b*x + a)^n*a^3*b^2*n^2*x^2 - 8*(b*x + a)^n*a^2*
b^3*n*x^3 + 12*(b*x + a)^n*a^3*b^2*n*x^2 - 24*(b*x + a)^n*a^4*b*n*x + 24*(
b*x + a)^n*a^5)/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5
*n + 120*b^5)
```

3.266.9 Mupad [B] (verification not implemented)

Time = 4.71 (sec) , antiderivative size = 546, normalized size of antiderivative = 3.31

$$\int x^4 \operatorname{arctanh}(\tanh(a+bx))^n\,dx = -\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}\right)^n\left(\frac{3\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^5}{4\,b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}-\frac{x^5\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}+\frac{3\,n\,x\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^4}{2\,b^4\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{n\,x^4\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)\,\left(n^3+6\,n^2+11\,n+6\right)}{2\,b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{3\,n\,x^2\,\left(n+1\right)\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{2\,b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{n\,x^3\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2\,\left(n^2+3\,n+2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)$$

input
```
int(x^4*atanh(tanh(a + b*x))^n,x)
```

output
```
-(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^n*((3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5)/(4*b^5*(274*n +
225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (x^5*(50*n + 35*n^2 + 10*n^3 + n
^4 + 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (3*n*x*(log(2/
(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^4)/(2*b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 12
0)) + (n*x^4*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)*(11*n + 6*n^2 + n^3 + 6))/(2*b*(274
*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (3*n*x^2*(n + 1)*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^3)/(2*b^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
 + (n*x^3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2*(3*n + n^2 + 2))/(b^2*(274*n + 225*n
^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
```

3.267 $\int x^3 \mathbf{arctanh}(\tanh(a+bx))^n\,dx$

3.267.1 Optimal result

Integrand size = 13, antiderivative size = 121

$$\begin{aligned}\int x^3\text{arctanh}(\tanh(a+bx))^n\,dx=&\frac{x^3\text{arctanh}(\tanh(a+bx))^{1+n}}{b(1+n)}\\&-\frac{3x^2\text{arctanh}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}\\&+\frac{6x\text{arctanh}(\tanh(a+bx))^{3+n}}{b^3(1+n)(2+n)(3+n)}\\&-\frac{6\text{arctanh}(\tanh(a+bx))^{4+n}}{b^4(1+n)(2+n)(3+n)(4+n)}\end{aligned}$$

output
```
x^3*arctanh(tanh(b*x+a))^(1+n)/b/(1+n)-3*x^2*arctanh(tanh(b*x+a))^(2+n)/b^
2/(1+n)/(2+n)+6*x*arctanh(tanh(b*x+a))^(3+n)/b^3/(3+n)/(n^2+3*n+2)-6*arcta
nh(tanh(b*x+a))^(4+n)/b^4/(n^2+5*n+4)/(n^2+5*n+6)
```

3.267.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int x^3\text{arctanh}(\tanh(a+bx))^n\,dx$$
$$=\frac{\text{arctanh}(\tanh(a+bx))^{1+n}\left(b^3(24+26n+9n^2+n^3)\,x^3-3b^2(12+7n+n^2)\,x^2\text{arctanh}(\tanh(a+bx))+6\right.}{b^4(1+n)(2+n)(3+n)(4+n)}$$

input
```
Integrate[x^3*ArcTanh[Tanh[a + b*x]]^n,x]
```

output
```
(ArcTanh[Tanh[a + b*x]]^(1 + n)*(b^3*(24 + 26*n + 9*n^2 + n^3)*x^3 - 3*b^2
*(12 + 7*n + n^2)*x^2*ArcTanh[Tanh[a + b*x]] + 6*b*(4 + n)*x*ArcTanh[Tanh[
a + b*x]]^2 - 6*ArcTanh[Tanh[a + b*x]]^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4
 + n))
```

3.267.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{arctanh}(\tanh(a+bx))^n \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^3 \text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{3\int x^2 \text{arctanh}(\tanh(a+bx))^{n+1}dx}{b(n+1)}$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^3 \text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{3\left(\frac{x^2 \text{arctanh}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{2\int x \text{arctanh}(\tanh(a+bx))^{n+2}dx}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^3 \text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{3\left(\frac{x^2 \text{arctanh}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{2\left(\frac{x \text{arctanh}(\tanh(a+bx))^{n+3}}{b(n+3)}-\frac{\int \text{arctanh}(\tanh(a+bx))^{n+3}dx}{b(n+3)}\right)}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \text{2588}$$

$$\frac{x^3 \text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{3\left(\frac{x^2 \text{arctanh}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{2\left(\frac{x \text{arctanh}(\tanh(a+bx))^{n+3}}{b(n+3)}-\frac{\int \text{arctanh}(\tanh(a+bx))^{n+3}d\text{arctanh}(\tanh(a+bx))}{b^2(n+3)}\right)}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow 15$$

$$\frac{x^3\text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{3\left(\frac{x^2\text{arctanh}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{2\left(\frac{x\text{arctanh}(\tanh(a+bx))^{n+3}}{b(n+3)}-\frac{\text{arctanh}(\tanh(a+bx))^{n+4}}{b^2(n+3)(n+4)}\right)}{b(n+2)}\right)}{b(n+1)}$$

input
```
Int[x^3*ArcTanh[Tanh[a + b*x]]^n,x]
```

output
```
(x^3*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (3*((x^2*ArcTanh[Tanh[a
 + b*x]]^(2 + n))/(b*(2 + n)) - (2*((x*ArcTanh[Tanh[a + b*x]]^(3 + n))/(b*
(3 + n)) - ArcTanh[Tanh[a + b*x]]^(4 + n)/(b^2*(3 + n)*(4 + n))))/(b*(2 +
n))))/(b*(1 + n))
```

3.267.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.267.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(121) = 242.

Time = 1.21 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.29

method	result
parallelrisch	$-\frac{-x^3 \operatorname{arctanh}(\tanh(bx+a)) \operatorname{arctanh}(\tanh(bx+a))^n b^3 n^3 - 9x^3 \operatorname{arctanh}(\tanh(bx+a)) \operatorname{arctanh}(\tanh(bx+a))^n b^3 n^2 - 26x^3 \operatorname{arctan}}{}$
default	$\frac{x^4 e^{n \ln(\operatorname{arctanh}(\tanh(bx+a)))}}{4+n} + \frac{n(\operatorname{arctanh}(\tanh(bx+a))-bx)x^3 e^{n \ln(\operatorname{arctanh}(\tanh(bx+a)))}}{b(n^2+7n+12)} - \frac{6(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(a^3\right.}{}$
risch	Expression too large to display

input
```
int(x^3*arctanh(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
-(-x^3*arctanh(tanh(b*x+a))*arctanh(tanh(b*x+a))^n*b^3*n^3-9*x^3*arctanh(t
anh(b*x+a))*arctanh(tanh(b*x+a))^n*b^3*n^2-26*x^3*arctanh(tanh(b*x+a))*arc
tanh(tanh(b*x+a))^n*b^3*n+3*x^2*arctanh(tanh(b*x+a))^2*arctanh(tanh(b*x+a)
)^n*b^2*n^2+21*x^2*arctanh(tanh(b*x+a))^2*arctanh(tanh(b*x+a))^n*b^2*n-6*x
*arctanh(tanh(b*x+a))^3*arctanh(tanh(b*x+a))^n*b*n-24*arctanh(tanh(b*x+a))
^n*arctanh(tanh(b*x+a))^3*x*b-24*arctanh(tanh(b*x+a))^n*arctanh(tanh(b*x+a
))*x^3*b^3+36*arctanh(tanh(b*x+a))^n*arctanh(tanh(b*x+a))^2*x^2*b^2+6*arct
anh(tanh(b*x+a))^n*arctanh(tanh(b*x+a))^4)/(n^2+3*n+2)/(3+n)/(4+n)/b^4
```

3.267.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(121) = 242.

Time = 0.25 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.11

$$\int x^3 \operatorname{arctanh}(\tanh(a+bx))^n \, dx$$
$$= \frac{(6\,a^3bnx + (b^4n^3 + 6\,b^4n^2 + 11\,b^4n + 6\,b^4)x^4 - 6\,a^4 + (ab^3n^3 + 3\,ab^3n^2 + 2\,ab^3n)x^3 - 3\,(a^2b^2n^2 + a^2b^2n)x}{}$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
((6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*
b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*
cosh(n*log(b*x + a)) + (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*
b^4)*x^4 - 6*a^4 + (a*b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*
n^2 + a^2*b^2*n)*x^2)*sinh(n*log(b*x + a)))/(b^4*n^4 + 10*b^4*n^3 + 35*b^4
*n^2 + 50*b^4*n + 24*b^4)
```

3.267.6 Sympy [F]

$$\int x^3 \text{arctanh}(\tanh(a+bx))^n \, dx = \text{Too large to display}$$

input
```
integrate(x**3*atanh(tanh(b*x+a))**n,x)
```

output
```
Piecewise((x**4*atanh(tanh(a))**n/4, Eq(b, 0)), (-x**3/(3*b*atanh(tanh(a +
 b*x))**3) - x**2/(2*b**2*atanh(tanh(a + b*x))**2) - x/(b**3*atanh(tanh(a
+ b*x))) + log(atanh(tanh(a + b*x)))/b**4, Eq(n, -4)), (Integral(x**3/atan
h(tanh(a + b*x))**3, x), Eq(n, -3)), (Integral(x**3/atanh(tanh(a + b*x))**
2, x), Eq(n, -2)), (Integral(x**3/atanh(tanh(a + b*x)), x), Eq(n, -1)), (b
**3*n**3*x**3*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10
*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*b**3*n**2*x**3*atanh(
tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4
*n**2 + 50*b**4*n + 24*b**4) + 26*b**3*n*x**3*atanh(tanh(a + b*x))*atanh(t
anh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24
*b**4) + 24*b**3*x**3*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b**4*n
**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*b**2*n**2*x**
2*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**
3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 21*b**2*n*x**2*atanh(tanh(a + b*
x))**2*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 +
50*b**4*n + 24*b**4) - 36*b**2*x**2*atanh(tanh(a + b*x))**2*atanh(tanh(a +
 b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
+ 6*b*n*x*atanh(tanh(a + b*x))**3*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10*
b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 24*b*x*atanh(tanh(a + b*
x))**3*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2...
```

3.267.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int x^3 \operatorname{arctanh}(\tanh(a+bx))^n\,dx$$
$$= \frac{((n^3+6\,n^2+11\,n+6)b^4x^4+(n^3+3\,n^2+2\,n)ab^3x^3-3\,(n^2+n)a^2b^2x^2+6\,a^3bnx-6\,a^4)(bx+a)^n}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4}$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n
^2 + 50*n + 24)*b^4)
```

3.267.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.87

$$\int x^3 \operatorname{arctanh}(\tanh(a+bx))^n\,dx$$
$$= \frac{(bx+a)^n b^4n^3x^4+(bx+a)^n ab^3n^3x^3+6\,(bx+a)^n b^4n^2x^4+3\,(bx+a)^n ab^3n^2x^3+11\,(bx+a)^n b^4nx^4-3\,(}{b^4n^4+10\,b^4n^3+}$$

input
```
integrate(x^3*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^4*n^3*x^4 + (b*x + a)^n*a*b^3*n^3*x^3 + 6*(b*x + a)^n*b^4*n
^2*x^4 + 3*(b*x + a)^n*a*b^3*n^2*x^3 + 11*(b*x + a)^n*b^4*n*x^4 - 3*(b*x +
 a)^n*a^2*b^2*n^2*x^2 + 2*(b*x + a)^n*a*b^3*n*x^3 + 6*(b*x + a)^n*b^4*x^4
- 3*(b*x + a)^n*a^2*b^2*n*x^2 + 6*(b*x + a)^n*a^3*b*n*x - 6*(b*x + a)^n*a^
4)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

3.267.9 Mupad [B] (verification not implemented)

Time = 4.60 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.45

$$\int x^3\text{arctanh}(\tanh(a+bx))^n\,dx$$
$$= -\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}\right)^n\left(\frac{3\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^4}{8\,b^4\,(n^4+10\,n^3+35\,n^2+50\,n+24)}\right.$$
$$-\frac{x^4\,(n^3+6\,n^2+11\,n+6)}{n^4+10\,n^3+35\,n^2+50\,n+24}+\frac{3\,n\,x\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{4\,b^3\,(n^4+10\,n^3+35\,n^2+50\,n+24)}$$
$$+\frac{n\,x^3\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)(n^2+3\,n+2)}{2\,b\,(n^4+10\,n^3+35\,n^2+50\,n+24)}$$
$$\left.+\frac{3\,n\,x^2\,(n+1)\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{4\,b^2\,(n^4+10\,n^3+35\,n^2+50\,n+24)}\right)$$

input
```
int(x^3*atanh(tanh(a + b*x))^n,x)
```

output
```
-(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^n*((3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)/(8*b^4*(50*n + 3
5*n^2 + 10*n^3 + n^4 + 24)) - (x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^
2 + 10*n^3 + n^4 + 24) + (3*n*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(4*b^3*(50*n
+ 35*n^2 + 10*n^3 + n^4 + 24)) + (n*x^3*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)*(3*n + n
^2 + 2))/(2*b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (3*n*x^2*(n + 1)*(log
(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^2)/(4*b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)))
```

3.268 $\int x^2 \mathbf{arctanh}(\tanh(a+bx))^n\,dx$

3.268.1 Optimal result

Integrand size = 13, antiderivative size = 82

$$\int x^2\text{arctanh}(\tanh(a+bx))^n\,dx = \frac{x^2\text{arctanh}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2x\text{arctanh}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2\text{arctanh}(\tanh(a+bx))^{3+n}}{b^3(1+n)(2+n)(3+n)}$$

output
```
x^2*arctanh(tanh(b*x+a))^(1+n)/b/(1+n)-2*x*arctanh(tanh(b*x+a))^(2+n)/b^2/
(1+n)/(2+n)+2*arctanh(tanh(b*x+a))^(3+n)/b^3/(3+n)/(n^2+3*n+2)
```

3.268.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int x^2\text{arctanh}(\tanh(a+bx))^n\,dx = \frac{\text{arctanh}(\tanh(a+bx))^{1+n}\left(b^2(6+5n+n^2)x^2-2b(3+n)x\text{arctanh}(\tanh(a+bx))+2\text{arctanh}(\tanh(a+\right.}{b^3(1+n)(2+n)(3+n)}$$

input
```
Integrate[x^2*ArcTanh[Tanh[a + b*x]]^n,x]
```

output
```
(ArcTanh[Tanh[a + b*x]]^(1 + n)*(b^2*(6 + 5*n + n^2)*x^2 - 2*b*(3 + n)*x*A
rcTanh[Tanh[a + b*x]] + 2*ArcTanh[Tanh[a + b*x]]^2))/(b^3*(1 + n)*(2 + n)*
(3 + n))
```

3.268.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arctanh}(\tanh(a+bx))^n\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^2 \text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{2\int x \text{arctanh}(\tanh(a+bx))^{n+1}dx}{b(n+1)}$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^2 \text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{2\left(\frac{x \text{arctanh}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{\int \text{arctanh}(\tanh(a+bx))^{n+2}dx}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \text{2588}$$

$$\frac{x^2 \text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{2\left(\frac{x \text{arctanh}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{\int \text{arctanh}(\tanh(a+bx))^{n+2}d\text{arctanh}(\tanh(a+bx))}{b^2(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \text{15}$$

$$\frac{x^2 \text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{2\left(\frac{x \text{arctanh}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{\text{arctanh}(\tanh(a+bx))^{n+3}}{b^2(n+2)(n+3)}\right)}{b(n+1)}$$

input
```
Int[x^2*ArcTanh[Tanh[a + b*x]]^n,x]
```

output
```
(x^2*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (2*((x*ArcTanh[Tanh[a +
 b*x]]^(2 + n))/(b*(2 + n)) - ArcTanh[Tanh[a + b*x]]^(3 + n)/(b^2*(2 + n)*
(3 + n))))/(b*(1 + n))
```

3.268.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.268.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.99

method	result
parallelrisch	$-\frac{-2\operatorname{arctanh}(\tanh(bx+a))^{n}\operatorname{arctanh}(\tanh(bx+a))^{3}+6\operatorname{arctanh}(\tanh(bx+a))^{n}\operatorname{arctanh}(\tanh(bx+a))^{2}xb-6\operatorname{arctanh}(\tanh(bx+}{}$
default	$\frac{x^{3}\mathrm{e}^{n\ln(\operatorname{arctanh}(\tanh(bx+a)))}}{3+n}+\frac{n(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{2}\mathrm{e}^{n\ln(\operatorname{arctanh}(\tanh(bx+a)))}}{b(n^{2}+5n+6)}+\frac{2(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(a^{2}\right.}{}$
risch	Expression too large to display

input
```
int(x^2*arctanh(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
-(-2*arctanh(tanh(b*x+a))^n*arctanh(tanh(b*x+a))^3+6*arctanh(tanh(b*x+a))^
n*arctanh(tanh(b*x+a))^2*x*b-6*arctanh(tanh(b*x+a))^n*x^2*arctanh(tanh(b*x
+a))*b^2-x^2*arctanh(tanh(b*x+a))*arctanh(tanh(b*x+a))^n*b^2*n^2-5*x^2*arc
tanh(tanh(b*x+a))*arctanh(tanh(b*x+a))^n*b^2*n+2*x*arctanh(tanh(b*x+a))^2*
arctanh(tanh(b*x+a))^n*b*n)/(1+n)/(n^2+5*n+6)/b^3
```

3.268.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(82) = 164$.

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.05

$$\int x^2 \operatorname{arctanh}(\tanh(a+bx))^n\,dx = -\frac{(2\,a^2bnx - (b^3n^2 + 3\,b^3n + 2\,b^3)x^3 - 2\,a^3 - (ab^2n^2 + ab^2n)x^2)\cosh(n\log(bx+a)) + (2\,a^2bnx - (b^3n^2}{b^3n^3 + 6\,b^3n^2 + 11\,b^3n + 6\,b^3}$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
-((2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*
b^2*n)*x^2)*cosh(n*log(b*x + a)) + (2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b
^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^2*n)*x^2)*sinh(n*log(b*x + a)))/(b^3*n^
3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

3.268.6 Sympy [F]

$$\int x^2 \operatorname{arctanh}(\tanh(a+bx))^n\,dx$$

$$= \begin{cases} \frac{x^3 \operatorname{atanh}^n(\tanh(a))}{3} \\ -\frac{x^2}{2b\operatorname{atanh}^2(\tanh(a+bx))} - \frac{x}{b^2\operatorname{atanh}(\tanh(a+bx))} + \frac{\log(\operatorname{atanh}(\tanh(a+bx)))}{b^3} \\ \int \frac{x^2}{\operatorname{atanh}^2(\tanh(a+bx))}\,dx \\ \int \frac{x^2}{\operatorname{atanh}(\tanh(a+bx))}\,dx \\ \frac{b^2n^2x^2\operatorname{atanh}(\tanh(a+bx))\operatorname{atanh}^n(\tanh(a+bx))}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{5b^2nx^2\operatorname{atanh}(\tanh(a+bx))\operatorname{atanh}^n(\tanh(a+bx))}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{6b^2x^2\operatorname{atanh}(\tanh(a+bx))\operatorname{atan}}{b^3n^3+6b^3n^2+11b^3n} \end{cases}$$

input
```
integrate(x**2*atanh(tanh(b*x+a))**n,x)
```

output
```
Piecewise((x**3*atanh(tanh(a))**n/3, Eq(b, 0)), (-x**2/(2*b*atanh(tanh(a +
 b*x))**2) - x/(b**2*atanh(tanh(a + b*x))) + log(atanh(tanh(a + b*x)))/b**
3, Eq(n, -3)), (Integral(x**2/atanh(tanh(a + b*x))**2, x), Eq(n, -2)), (In
tegral(x**2/atanh(tanh(a + b*x)), x), Eq(n, -1)), (b**2*n**2*x**2*atanh(ta
nh(a + b*x))*atanh(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n
+ 6*b**3) + 5*b**2*n*x**2*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b*
*3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 6*b**2*x**2*atanh(tanh(a + b
*x))*atanh(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3
) - 2*b*n*x*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x))**n/(b**3*n**3 + 6
*b**3*n**2 + 11*b**3*n + 6*b**3) - 6*b*x*atanh(tanh(a + b*x))**2*atanh(tan
h(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*atanh(ta
nh(a + b*x))**3*atanh(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3
*n + 6*b**3), True))
```

3.268.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^2 \text{arctanh}(\tanh(a+bx))^n\,dx = \frac{((n^2+3\,n+2)b^3x^3+(n^2+n)ab^2x^2-2\,a^2bnx+2\,a^3)(bx+a)^n}{(n^3+6\,n^2+11\,n+6)b^3}$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
 + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

3.268.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.71

$$\int x^2 \text{arctanh}(\tanh(a+bx))^n\,dx = \frac{(bx+a)^n b^3n^2x^3+(bx+a)^n ab^2n^2x^2+3\,(bx+a)^n b^3nx^3+(bx+a)^n ab^2nx^2+2\,(bx+a)^n b^3x^3-2\,(bx+a}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}$$

input
```
integrate(x^2*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^3*n^2*x^3 + (b*x + a)^n*a*b^2*n^2*x^2 + 3*(b*x + a)^n*b^3*n
*x^3 + (b*x + a)^n*a*b^2*n*x^2 + 2*(b*x + a)^n*b^3*x^3 - 2*(b*x + a)^n*a^2
*b*n*x + 2*(b*x + a)^n*a^3)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

3.268.9 Mupad [B] (verification not implemented)

Time = 4.61 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.71

$$\int x^2\text{arctanh}(\tanh(a+bx))^n\,dx$$
$$= -\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}\right)^n\left(\frac{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{4\,b^3\,(n^3+6\,n^2+11\,n+6)}\right.$$
$$-\frac{x^3\,(n^2+3\,n+2)}{n^3+6\,n^2+11\,n+6}+\frac{n\,x\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{2\,b^2\,(n^3+6\,n^2+11\,n+6)}$$
$$\left.+\frac{n\,x^2\,(n+1)\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b\,(n^3+6\,n^2+11\,n+6)}\right)$$

input
```
int(x^2*atanh(tanh(a + b*x))^n,x)
```

output
```
-(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^n*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/(4*b^3*(11*n + 6*n^2
 + n^3 + 6)) - (x^3*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (n*x*(log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x)^2)/(2*b^2*(11*n + 6*n^2 + n^3 + 6)) + (n*x^2*(n + 1)*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x))/(2*b*(11*n + 6*n^2 + n^3 + 6)))
```

3.269 $\int x\text{arctanh}(\tanh(a+bx))^n\,dx$

3.269.1 Optimal result

Integrand size = 11, antiderivative size = 48

$$\int x\text{arctanh}(\tanh(a+bx))^n\,dx=\frac{x\text{arctanh}(\tanh(a+bx))^{1+n}}{b(1+n)}-\frac{\text{arctanh}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}$$

output
```
x*arctanh(tanh(b*x+a))^(1+n)/b/(1+n)-arctanh(tanh(b*x+a))^(2+n)/b^2/(1+n)/
(2+n)
```

3.269.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int x\text{arctanh}(\tanh(a+bx))^n\,dx=\frac{(b(2+n)x-\text{arctanh}(\tanh(a+bx)))\text{arctanh}(\tanh(a+bx))^{1+n}}{b^2(1+n)(2+n)}$$

input
```
Integrate[x*ArcTanh[Tanh[a + b*x]]^n,x]
```

output
```
((b*(2 + n)*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b
^2*(1 + n)*(2 + n))
```

3.269.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(\tanh(a+bx))^n\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x\text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{\int \text{arctanh}(\tanh(a+bx))^{n+1}dx}{b(n+1)}$$

$$\Big\downarrow \text{2588}$$

$$\frac{x\text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{\int \text{arctanh}(\tanh(a+bx))^{n+1}d\text{arctanh}(\tanh(a+bx))}{b^2(n+1)}$$

$$\Big\downarrow \text{15}$$

$$\frac{x\text{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{\text{arctanh}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}$$

input
```
Int[x*ArcTanh[Tanh[a + b*x]]^n,x]
```

output
```
(x*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - ArcTanh[Tanh[a + b*x]]^(2
 + n)/(b^2*(1 + n)*(2 + n))
```

3.269.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.269.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.60

method	result
parallelrisch	$-\frac{\operatorname{arctanh}(\tanh(bx+a))^{n}\operatorname{arctanh}(\tanh(bx+a))^{2}-2\operatorname{arctanh}(\tanh(bx+a))^{n}x\operatorname{arctanh}(\tanh(bx+a))b-x\operatorname{arctanh}(\tanh(bx+a))}{b^{2}(1+n)(2+n)}$
default	$\frac{x^{2}\mathrm{e}^{n\ln(\operatorname{arctanh}(\tanh(bx+a)))}}{2+n}+\frac{n(\operatorname{arctanh}(\tanh(bx+a))-bx)x\,\mathrm{e}^{n\ln(\operatorname{arctanh}(\tanh(bx+a)))}}{b(n^{2}+3n+2)}-\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)^{2}\mathrm{e}^{n\ln}}{b^{2}(n^{2}+3n+2}$
risch	$\frac{\left(\frac{1}{2}\right)^{n}\left(2\ln(\mathrm{e}^{bx+a})-\frac{i\pi\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\left(-\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)+\operatorname{csgn}\left(i\mathrm{e}^{bx+a}\right)\right)^{2}}{2}-\frac{i\pi\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)\left(-\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)+\operatorname{csgn}\left(i\mathrm{e}\right.\right.}{}\right.}{2b(1+n)}$

input
```
int(x*arctanh(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
-(arctanh(tanh(b*x+a))^n*arctanh(tanh(b*x+a))^2-2*arctanh(tanh(b*x+a))^n*x
*arctanh(tanh(b*x+a))*b-x*arctanh(tanh(b*x+a))*arctanh(tanh(b*x+a))^n*b*n)
/b^2/(1+n)/(2+n)
```

3.269.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.90

$$\int x\operatorname{arctanh}(\tanh(a+bx))^{n}\,dx = \frac{(abnx+(b^{2}n+b^{2})x^{2}-a^{2})\cosh(n\log(bx+a))+(abnx+(b^{2}n+b^{2})x^{2}-a^{2})\sinh(n\log(bx+a))}{b^{2}n^{2}+3\,b^{2}n+2\,b^{2}}$$

input
```
integrate(x*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
((a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*cosh(n*log(b*x + a)) + (a*b*n*x + (b^
2*n + b^2)*x^2 - a^2)*sinh(n*log(b*x + a)))/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

3.269.6 Sympy [F]

$$\int x\operatorname{arctanh}(\tanh(a+bx))^n\,dx$$
$$= \begin{cases} \frac{x^2\operatorname{atanh}^n(\tanh(a))}{2} \\ -\frac{x}{b\operatorname{atanh}(\tanh(a+bx))}+\frac{\log(\operatorname{atanh}(\tanh(a+bx)))}{b^2} \\ \int\frac{x}{\operatorname{atanh}(\tanh(a+bx))}\,dx \\ \frac{bnx\operatorname{atanh}(\tanh(a+bx))\operatorname{atanh}^n(\tanh(a+bx))}{b^2n^2+3b^2n+2b^2}+\frac{2bx\operatorname{atanh}(\tanh(a+bx))\operatorname{atanh}^n(\tanh(a+bx))}{b^2n^2+3b^2n+2b^2}-\frac{\operatorname{atanh}^2(\tanh(a+bx))\operatorname{atanh}^n(\tanh(a+ \end{cases}$$

input
```
integrate(x*atanh(tanh(b*x+a))**n,x)
```

output
```
Piecewise((x**2*atanh(tanh(a))**n/2, Eq(b, 0)), (-x/(b*atanh(tanh(a + b*x)
)) + log(atanh(tanh(a + b*x)))/b**2, Eq(n, -2)), (Integral(x/atanh(tanh(a
+ b*x)), x), Eq(n, -1)), (b*n*x*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))*
*n/(b**2*n**2 + 3*b**2*n + 2*b**2) + 2*b*x*atanh(tanh(a + b*x))*atanh(tanh
(a + b*x))**n/(b**2*n**2 + 3*b**2*n + 2*b**2) - atanh(tanh(a + b*x))**2*at
anh(tanh(a + b*x))**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True))
```

3.269.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int x\operatorname{arctanh}(\tanh(a+bx))^n\,dx = \frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^n}{(n^2+3\,n+2)b^2}$$

input
```
integrate(x*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
(b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2)
```

3.269.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.58

$$\int x\text{arctanh}(\tanh(a+bx))^n\,dx = \frac{(bx+a)^n b^2 n x^2 + (bx+a)^n abnx + (bx+a)^n b^2 x^2 - (bx+a)^n a^2}{b^2 n^2 + 3\,b^2 n + 2\,b^2}$$

input
```
integrate(x*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^2*n*x^2 + (b*x + a)^n*a*b*n*x + (b*x + a)^n*b^2*x^2 - (b*x
+ a)^n*a^2)/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

3.269.9 Mupad [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 205, normalized size of antiderivative = 4.27

$$\int x\text{arctanh}(\tanh(a+bx))^n\,dx = -\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}\right)^n \left(\frac{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2}{4\,b^2\,(n^2+3\,n+2)} - \frac{x^2\,(n+1)}{n^2+3\,n+2} + \frac{n\,x\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)}{2\,b\,(n^2+3\,n+2)}\right)$$

input
```
int(x*atanh(tanh(a + b*x))^n,x)
```

output
```
-(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^n*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/(4*b^2*(3*n + n^2 +
2)) - (x^2*(n + 1))/(3*n + n^2 + 2) + (n*x*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b
*(3*n + n^2 + 2)))
```

3.270 $\int \textbf{arctanh}(\tanh(a+bx))^n\,dx$

3.270.1 Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \operatorname{arctanh}(\tanh(a+bx))^n\,dx=\frac{\operatorname{arctanh}(\tanh(a+bx))^{1+n}}{b(1+n)}$$

output
```
arctanh(tanh(b*x+a))^(1+n)/b/(1+n)
```

3.270.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \operatorname{arctanh}(\tanh(a+bx))^n\,dx=\frac{\operatorname{arctanh}(\tanh(a+bx))^{1+n}}{b(1+n)}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^n,x]
```

output
```
ArcTanh[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

3.270.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(\tanh(a+bx))^n\,dx$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \operatorname{arctanh}(\tanh(a+bx))^n d\operatorname{arctanh}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\operatorname{arctanh}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^n,x]
```

output
```
ArcTanh[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

3.270.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.270.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\operatorname{arctanh}(\tanh(bx+a))^{1+n}}{b(1+n)}$
default	$\frac{\operatorname{arctanh}(\tanh(bx+a))^{1+n}}{b(1+n)}$
parallelrisch	$\frac{\operatorname{arctanh}(\tanh(bx+a))^{n} \operatorname{arctanh}(\tanh(bx+a))}{b(1+n)}$
risch	$\frac{\left(\frac{1}{2}\right)^{n} \left(2\ln(\mathrm{e}^{bx+a})-\frac{i\pi\ \mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\left(-\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)+\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)\right)^{2}}{2}-\frac{i\pi\ \mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)\left(-\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)+\cdots\right.}{2b(1+n)}$

input
```
int(arctanh(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
arctanh(tanh(b*x+a))^(1+n)/b/(1+n)
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \operatorname{arctanh}(\tanh(a+bx))^n\,dx = \frac{(bx+a)\cosh\left(n\log\left(bx+a\right)\right)+(bx+a)\sinh\left(n\log\left(bx+a\right)\right)}{bn+b}$$

input
```
integrate(arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
((b*x + a)*cosh(n*log(b*x + a)) + (b*x + a)*sinh(n*log(b*x + a)))/(b*n + b
)
```

3.270.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \operatorname{arctanh}(\tanh(a+bx))^n\,dx = \begin{cases} \frac{x}{\operatorname{atanh}\left(\tanh\left(a\right)\right)} & \text{for}\ b=0 \wedge n=-1 \\ x\operatorname{atanh}^{n}\left(\tanh\left(a\right)\right) & \text{for}\ b=0 \\ \frac{\log\left(\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)\right)}{b} & \text{for}\ n=-1 \\ \frac{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)\operatorname{atanh}^{n}\left(\tanh\left(a+bx\right)\right)}{bn+b} & \text{otherwise} \end{cases}$$

input
```
integrate(atanh(tanh(b*x+a))**n,x)
```

output
```
Piecewise((x/atanh(tanh(a)), Eq(b, 0) & Eq(n, -1)), (x*atanh(tanh(a))**n,
Eq(b, 0)), (log(atanh(tanh(a + b*x)))/b, Eq(n, -1)), (atanh(tanh(a + b*x))
*atanh(tanh(a + b*x))**n/(b*n + b), True))
```

3.270.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \operatorname{arctanh}(\tanh(a+bx))^n\,dx = \frac{(bx+a)(bx+a)^n}{b(n+1)}$$

input
```
integrate(arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
(b*x + a)*(b*x + a)^n/(b*(n + 1))
```

3.270.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \operatorname{arctanh}(\tanh(a+bx))^n\,dx = \frac{(bx+a)^n bx + (bx+a)^n a}{bn+b}$$

input
```
integrate(arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
((b*x + a)^n*b*x + (b*x + a)^n*a)/(b*n + b)
```

3.270.9 Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 121, normalized size of antiderivative = 6.05

$$\int \operatorname{arctanh}(\tanh(a+bx))^n\,dx$$
$$= \left(\frac{1}{2}\right)^n \left(\frac{x}{n+1} - \frac{\frac{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)}{2} - \frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2} + b\,x}{b\,(n+1)}\right)\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right) - \ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)^n$$

input
```
int(atanh(tanh(a + b*x))^n,x)
```

output
```
(1/2)^n*(x/(n + 1) - (log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)/(b*(n + 1)))*(log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) +
 1)))^n
```

3.271 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x} \, dx$

3.271.1 Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x} \, dx = \frac{\operatorname{arctanh}(\tanh(a+bx))^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,-\frac{\operatorname{arctanh}(\tanh(a+bx))}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)}{(1+n)(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

output
```
arctanh(tanh(b*x+a))^(1+n)*hypergeom([1, 1+n],[2+n],-arctanh(tanh(b*x+a))/
(b*x-arctanh(tanh(b*x+a))))/(1+n)/(b*x-arctanh(tanh(b*x+a)))
```

3.271.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x} \, dx = \frac{\operatorname{arctanh}(\tanh(a+bx))^n \left(\frac{\operatorname{arctanh}(\tanh(a+bx))}{bx}\right)^{-n} \text{Hypergeometric2F1}\left(-n,-n,1-n,1-\frac{\operatorname{arctanh}(\tanh(a+}{bx}\right.}{n}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^n/x,x]
```

output
```
(ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2F1[-n, -n, 1 - n, 1 - ArcTanh[Tan
h[a + b*x]]/(b*x)])/(n*(ArcTanh[Tanh[a + b*x]]/(b*x))^n)
```

3.271.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2595}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x} \, dx$$
$$\Big\downarrow \; 2595$$
$$\frac{\operatorname{arctanh}(\tanh(a+bx))^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,-\frac{\operatorname{arctanh}(\tanh(a+bx))}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)}{(n+1)(bx-\operatorname{arctanh}(\tanh(a+bx)))}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^n/x,x]
```

output
```
(ArcTanh[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, -(ArcTa
nh[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/((1 + n)*(b*x - ArcTan
h[Tanh[a + b*x]]))
```

3.271.3.1 Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] && !IntegerQ[n]
```

3.271.4 Maple [F]

$$\int \frac{\operatorname{arctanh}(\tanh(bx+a))^n}{x} dx$$

input
```
int(arctanh(tanh(b*x+a))^n/x,x)
```

output
```
int(arctanh(tanh(b*x+a))^n/x,x)
```

3.271.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x}\,dx = \int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^n}{x}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^n/x,x, algorithm="fricas")
```

output
```
integral(arctanh(tanh(b*x + a))^n/x, x)
```

3.271.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x}\,dx = \int \frac{\operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{x}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**n/x,x)
```

output
```
Integral(atanh(tanh(a + b*x))**n/x, x)
```

3.271.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x}\,dx = \int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^n}{x}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^n/x,x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^n/x, x)
```

3.271.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x}\,dx = \int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^n}{x}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^n/x,x, algorithm="giac")
```

output
```
integrate(arctanh(tanh(b*x + a))^n/x, x)
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x}\,dx = \int \frac{\operatorname{atanh}(\tanh(a+b\,x))^n}{x}\,dx$$

input
```
int(atanh(tanh(a + b*x))^n/x,x)
```

output
```
int(atanh(tanh(a + b*x))^n/x, x)
```

3.272 $\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x^2} dx$

3.272.1 Optimal result

Integrand size = 13, antiderivative size = 71

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x^2} dx = -\frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x} + \frac{b\operatorname{arctanh}(\tanh(a+bx))^n \operatorname{Hypergeometric2F1}\left(1,n,1+n,-\frac{\operatorname{arctanh}(\tanh(a+bx))}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))}$$

output
```
-arctanh(tanh(b*x+a))^n/x+b*arctanh(tanh(b*x+a))^n*hypergeom([1, n],[1+n],
-arctanh(tanh(b*x+a))/(b*x-arctanh(tanh(b*x+a))))/(b*x-arctanh(tanh(b*x+a)
))
```

3.272.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x^2} dx = \frac{\operatorname{arctanh}(\tanh(a+bx))^n \left(\frac{\operatorname{arctanh}(\tanh(a+bx))}{bx}\right)^{-n} \operatorname{Hypergeometric2F1}\left(1-n,-n,2-n,1-\frac{\operatorname{arctanh}(\tanh(}{bx}\right.}{(-1+n)x}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^n/x^2,x]
```

output
```
(ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2F1[1 - n, -n, 2 - n, 1 - ArcTanh[
Tanh[a + b*x]]/(b*x)])/((-1 + n)*x*(ArcTanh[Tanh[a + b*x]]/(b*x))^n)
```

3.272.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2599, 2595}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x^2} \, dx$$

$$\Big\downarrow \text{2599}$$

$$bn \int \frac{\operatorname{arctanh}(\tanh(a+bx))^{n-1}}{x} dx - \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x}$$

$$\Big\downarrow \text{2595}$$

$$\frac{b\operatorname{arctanh}(\tanh(a+bx))^n \text{Hypergeometric2F1}\left(1,n,n+1,-\frac{\operatorname{arctanh}(\tanh(a+bx))}{bx-\operatorname{arctanh}(\tanh(a+bx))}\right)}{bx-\operatorname{arctanh}(\tanh(a+bx))} - \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^n/x^2,x]
```

output
```
-(ArcTanh[Tanh[a + b*x]]^n/x) + (b*ArcTanh[Tanh[a + b*x]]^n*Hypergeometric
2F1[1, n, 1 + n, -(ArcTanh[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))]
)/(b*x - ArcTanh[Tanh[a + b*x]])
```

3.272.3.1 Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.272.4 Maple [F]

$$\int \frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{2}}dx$$

input
```
int(arctanh(tanh(b*x+a))^n/x^2,x)
```

output
```
int(arctanh(tanh(b*x+a))^n/x^2,x)
```

3.272.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x^2}\,dx = \int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{2}}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^n/x^2,x, algorithm="fricas")
```

output
```
integral(arctanh(tanh(b*x + a))^n/x^2, x)
```

3.272.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x^2}\,dx = \int \frac{\operatorname{atanh}^n\left(\tanh\left(a+bx\right)\right)}{x^2}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**n/x**2,x)
```

output
```
Integral(atanh(tanh(a + b*x))**n/x**2, x)
```

3.272.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x^2}\,dx = \int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^n}{x^2}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^n/x^2,x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^n/x^2, x)
```

3.272.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x^2}\,dx = \int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^n}{x^2}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^n/x^2,x, algorithm="giac")
```

output
```
integrate(arctanh(tanh(b*x + a))^n/x^2, x)
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x^2}\,dx = \int \frac{\operatorname{atanh}(\tanh(a+b\,x))^n}{x^2}\,dx$$

input
```
int(atanh(tanh(a + b*x))^n/x^2,x)
```

output
```
int(atanh(tanh(a + b*x))^n/x^2, x)
```

3.273 $\int \frac{\text{arctanh}(\tanh(a+bx))^n}{x^3} dx$

3.273.1 Optimal result

Integrand size = 13, antiderivative size = 101

$$\int \frac{\text{arctanh}(\tanh(a+bx))^n}{x^3} dx = -\frac{bn\text{arctanh}(\tanh(a+bx))^{-1+n}}{2x} - \frac{\text{arctanh}(\tanh(a+bx))^n}{2x^2} + \frac{b^2n\text{arctanh}(\tanh(a+bx))^{-1+n} \text{Hypergeometric2F1}\left(1,-1+n,n,-\frac{\text{arctanh}(\tanh(a+bx))}{bx-\text{arctanh}(\tanh(a+bx))}\right)}{2(bx-\text{arctanh}(\tanh(a+bx)))}$$

output
```
-1/2*b*n*arctanh(tanh(b*x+a))^(-1+n)/x-1/2*arctanh(tanh(b*x+a))^n/x^2+1/2*
b^2*n*arctanh(tanh(b*x+a))^(-1+n)*hypergeom([1, -1+n],[n],-arctanh(tanh(b*
x+a))/(b*x-arctanh(tanh(b*x+a))))/(b*x-arctanh(tanh(b*x+a)))
```

3.273.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.66

$$\int \frac{\text{arctanh}(\tanh(a+bx))^n}{x^3} dx = \frac{\text{arctanh}(\tanh(a+bx))^n \left(\frac{\text{arctanh}(\tanh(a+bx))}{bx}\right)^{-n} \text{Hypergeometric2F1}\left(2-n,-n,3-n,1-\frac{\text{arctanh}(\tanh(a+bx))}{bx}\right)}{(-2+n)x^2}$$

input
```
Integrate[ArcTanh[Tanh[a + b*x]]^n/x^3,x]
```

output
```
(ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2F1[2 - n, -n, 3 - n, 1 - ArcTanh[
Tanh[a + b*x]]/(b*x)])/((-2 + n)*x^2*(ArcTanh[Tanh[a + b*x]]/(b*x))^n)
```

3.273.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2599, 2599, 2595}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\tanh(a+bx))^n}{x^3}\,dx$$

$$\Big\downarrow \; 2599$$

$$\frac{1}{2}bn\int \frac{\text{arctanh}(\tanh(a+bx))^{n-1}}{x^2}dx-\frac{\text{arctanh}(\tanh(a+bx))^n}{2x^2}$$

$$\Big\downarrow \; 2599$$

$$\frac{1}{2}bn\left(-b(1-n)\int \frac{\text{arctanh}(\tanh(a+bx))^{n-2}}{x}dx-\frac{\text{arctanh}(\tanh(a+bx))^{n-1}}{x}\right)-\frac{\text{arctanh}(\tanh(a+bx))^n}{2x^2}$$

$$\Big\downarrow \; 2595$$

$$\frac{1}{2}bn\left(\frac{b\text{arctanh}(\tanh(a+bx))^{n-1}\text{Hypergeometric2F1}\left(1,n-1,n,-\frac{\text{arctanh}(\tanh(a+bx))}{bx-\text{arctanh}(\tanh(a+bx))}\right)}{bx-\text{arctanh}(\tanh(a+bx))}-\frac{\text{arctanh}(\tanh(a+}{x}\right.$$

$$\frac{\text{arctanh}(\tanh(a+bx))^n}{2x^2}$$

input
```
Int[ArcTanh[Tanh[a + b*x]]^n/x^3,x]
```

output
```
-1/2*ArcTanh[Tanh[a + b*x]]^n/x^2 + (b*n*(-(ArcTanh[Tanh[a + b*x]]^(-1 + n
)/x) + (b*ArcTanh[Tanh[a + b*x]]^(-1 + n)*Hypergeometric2F1[1, -1 + n, n,
-(ArcTanh[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/(b*x - ArcTanh[
Tanh[a + b*x]])))/2
```

3.273.3.1 Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] && !IntegerQ[n]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILt
Q[m, 0] && !IntegerQ[n]))
```

3.273.4 Maple [F]

$$\int \frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{3}}dx$$

input
```
int(arctanh(tanh(b*x+a))^n/x^3,x)
```

output
```
int(arctanh(tanh(b*x+a))^n/x^3,x)
```

3.273.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x^3}\,dx = \int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{3}}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^n/x^3,x, algorithm="fricas")
```

output
```
integral(arctanh(tanh(b*x + a))^n/x^3, x)
```

3.273.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x^3}\,dx = \int \frac{\operatorname{atanh}^n(\tanh(a+bx))}{x^3}\,dx$$

input
```
integrate(atanh(tanh(b*x+a))**n/x**3,x)
```

output
```
Integral(atanh(tanh(a + b*x))**n/x**3, x)
```

3.273.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x^3}\,dx = \int \frac{\operatorname{artanh}(\tanh(bx+a))^n}{x^3}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^n/x^3,x, algorithm="maxima")
```

output
```
integrate(arctanh(tanh(b*x + a))^n/x^3, x)
```

3.273.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x^3}\,dx = \int \frac{\operatorname{artanh}(\tanh(bx+a))^n}{x^3}\,dx$$

input
```
integrate(arctanh(tanh(b*x+a))^n/x^3,x, algorithm="giac")
```

output
```
integrate(arctanh(tanh(b*x + a))^n/x^3, x)
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(\tanh(a+bx))^n}{x^3}\,dx = \int \frac{\operatorname{atanh}(\tanh(a+b\,x))^n}{x^3}\,dx$$

input
```
int(atanh(tanh(a + b*x))^n/x^3,x)
```

output
```
int(atanh(tanh(a + b*x))^n/x^3, x)
```

3.274 $\int x^m \coth^{-1}(\tanh(a+bx))\,dx$

3.274.1 Optimal result

Integrand size = 11, antiderivative size = 37

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}$$

output
```
-b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*arccoth(tanh(b*x+a))/(1+m)
```

3.274.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = x^m\left(\frac{bx^2}{2+m}+\frac{x(-bx+\coth^{-1}(\tanh(a+bx)))}{1+m}\right)$$

input
```
Integrate[x^m*ArcCoth[Tanh[a + b*x]],x]
```

output
```
x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/(1 + m))
```

3.274.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{b\int x^{m+1}dx}{m+1}$$

$$\Big\downarrow \text{15}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{(m+1)(m+2)}$$

input
```
Int[x^m*ArcCoth[Tanh[a + b*x]],x]
```

output
```
-((b*x^(2 + m))/((1 + m)*(2 + m))) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]])/(1
 + m)
```

3.274.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.274.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

method	result
parallelrisch	$-\frac{-2\,\mathrm{arccoth}(\tanh(bx+a))x^m x+b\,x^2 x^m-x\,x^m\,\mathrm{arccoth}(\tanh(bx+a))m}{(1+m)(2+m)}$
risch	$\frac{x\,x^m\ln(\mathrm{e}^{bx+a})}{1+m}-\frac{x\left(4bx-i\pi\,\mathrm{csgn}(i\mathrm{e}^{2bx+2a})\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^2 m+2i\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^3-4i\pi\,\mathrm{csgn}(i\mathrm{e}^{bx+a})\,\mathrm{csgn}(i\mathrm{e}^{2bx+}\right.}{}$

input
```
int(x^m*arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-(-2*arccoth(tanh(b*x+a))*x^m*x+b*x^2*x^m-x*x^m*arccoth(tanh(b*x+a))*m)/(1
+m)/(2+m)
```

3.274.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.19

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx$$
$$=\frac{(i\,\pi(m+2)x+2\,(bm+b)x^2+2\,(am+2\,a)x)\cosh\left(m\log\left(x\right)\right)+(i\,\pi(m+2)x+2\,(bm+b)x^2+2\,(am+}{2\,(m^2+3\,m+2)}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*((I*pi*(m + 2)*x + 2*(b*m + b)*x^2 + 2*(a*m + 2*a)*x)*cosh(m*log(x)) +
 (I*pi*(m + 2)*x + 2*(b*m + b)*x^2 + 2*(a*m + 2*a)*x)*sinh(m*log(x)))/(m^2
 + 3*m + 2)
```

3.274.6 Sympy [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx$$
$$= \begin{cases} b\log(x) - \frac{\operatorname{acoth}(\tanh(a+bx))}{x} & \text{for } m=-2 \\ \int \frac{\operatorname{acoth}(\tanh(a+bx))}{x}\,dx & \text{for } m=-1 \\ -\frac{bx^2x^m}{m^2+3m+2} + \frac{mxx^m \operatorname{acoth}(\tanh(a+bx))}{m^2+3m+2} + \frac{2xx^m \operatorname{acoth}(\tanh(a+bx))}{m^2+3m+2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*acoth(tanh(b*x+a)),x)
```

output
```
Piecewise((b*log(x) - acoth(tanh(a + b*x))/x, Eq(m, -2)), (Integral(acoth(
tanh(a + b*x))/x, x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x*x**
m*acoth(tanh(a + b*x))/(m**2 + 3*m + 2) + 2*x*x**m*acoth(tanh(a + b*x))/(m
**2 + 3*m + 2), True))
```

3.274.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^2x^m}{(m+2)(m+1)} + \frac{x^{m+1}\operatorname{arcoth}(\tanh(bx+a))}{m+1}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-b*x^2*x^m/((m + 2)*(m + 1)) + x^(m + 1)*arccoth(tanh(b*x + a))/(m + 1)
```

3.274.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(37) = 74.

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = \frac{x^{m+1}\log\left(-\frac{\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1}+1}{\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1}-1}\right)}{2(m+1)} - \frac{bx^{m+2}}{(m+2)(m+1)}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
1/2*x^(m + 1)*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((e^(
2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/(m + 1) - b*x^(m + 2)/((m +
2)*(m + 1))
```

3.274.9 Mupad [B] (verification not implemented)

Time = 4.72 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.59

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx$$
$$= \frac{2\,b\,x^m\,x^2\,(m+1)}{2\,m^2+6\,m+4} - \frac{x\,x^m\,(m+2)\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2\,m^2+6\,m+4}$$

input
```
int(x^m*acoth(tanh(a + b*x)),x)
```

output
```
(2*b*x^m*x^2*(m + 1))/(6*m + 2*m^2 + 4) - (x*x^m*(m + 2)*(log(-2/(exp(2*a)
*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))
 + 2*b*x))/(6*m + 2*m^2 + 4)
```

3.275 $\int x^2\textbf{arctanh}(\coth(a+bx))\,dx$

3.275.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int x^2\text{arctanh}(\coth(a+bx))\,dx=-\frac{bx^4}{12}+\frac{1}{3}x^3\text{arctanh}(\coth(a+bx))$$

output
```
-1/12*b*x^4+1/3*x^3*arctanh(coth(b*x+a))
```

3.275.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x^2\text{arctanh}(\coth(a+bx))\,dx=-\frac{1}{12}x^3(bx-4\text{arctanh}(\coth(a+bx)))$$

input
```
Integrate[x^2*ArcTanh[Coth[a + b*x]],x]
```

output
```
-1/12*(x^3*(b*x - 4*ArcTanh[Coth[a + b*x]]))
```

3.275.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arctanh}(\coth(a+bx))\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{3}x^3\text{arctanh}(\coth(a+bx))-\frac{b\int x^3dx}{3}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{3}x^3\text{arctanh}(\coth(a+bx))-\frac{bx^4}{12}$$

input
```
Int[x^2*ArcTanh[Coth[a + b*x]],x]
```

output
```
-1/12*(b*x^4) + (x^3*ArcTanh[Coth[a + b*x]])/3
```

3.275.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.275.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{b\,x^{4}}{12}+\frac{x^{3} \operatorname{arctanh}\left(\coth\left(bx+a\right)\right)}{3}$
parts	$-\frac{b\,x^{4}}{12}+\frac{x^{3} \operatorname{arctanh}\left(\coth\left(bx+a\right)\right)}{3}$
parallelrisch	$-\frac{x^{3} \left(bx-4 \operatorname{arctanh}\left(\frac{1}{\tanh\left(bx+a\right)}\right)\right)}{12}$
risch	$\frac{x^{3} \ln\left(\mathrm{e}^{bx+a}\right)}{3}-\frac{b\,x^{4}}{12}+\frac{i \pi \,x^{3} \operatorname{csgn}\left(i \mathrm{e}^{2bx+2a}\right) \operatorname{csgn}\left(\frac{i \mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)^{2}}{12}-\frac{i \pi \,x^{3}}{6}+\frac{i \pi \,x^{3} \operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}-1}\right) \operatorname{csgn}\left(\frac{i \mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)^{2}}{12}$

input
```
int(x^2*arctanh(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*b*x^4+1/3*x^3*arctanh(coth(b*x+a))
```

3.275.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \operatorname{arctanh}(\coth(a+bx))\,dx = \frac{1}{4}\,bx^4 + \frac{1}{6}i\,\pi x^3 + \frac{1}{3}\,ax^3$$

input
```
integrate(x^2*arctanh(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*b*x^4 + 1/6*I*pi*x^3 + 1/3*a*x^3
```

3.275.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(19) = 38.

Time = 4.43 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.30

$$\int x^2 \operatorname{arctanh}(\coth(a+bx))\,dx = \begin{cases} \frac{x^3 \operatorname{atanh}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{3} & \text{for}\: a=\log\left(-e^{-bx}\right) \\ \frac{x^3 \operatorname{atanh}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right)}{3} & \text{for}\: a=\log\left(e^{-bx}\right) \\ -\frac{bx^4}{12}+\frac{x^3 \operatorname{atanh}\left(\frac{1}{\tanh\left(a+bx\right)}\right)}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*atanh(coth(b*x+a)),x)
```

output
```
Piecewise((x**3*atanh(coth(b*x + log(-exp(-b*x))))/3, Eq(a, log(-exp(-b*x)
))), (x**3*atanh(coth(b*x + log(exp(-b*x))))/3, Eq(a, log(exp(-b*x)))), (-
b*x**4/12 + x**3*atanh(1/tanh(a + b*x))/3, True))
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \text{arctanh}(\coth(a+bx))\,dx = -\frac{1}{12}\,bx^4 + \frac{1}{3}\,x^3\,\text{artanh}\left(\coth\left(bx+a\right)\right)$$

input
```
integrate(x^2*arctanh(coth(b*x+a)),x, algorithm="maxima")
```

output
```
-1/12*b*x^4 + 1/3*x^3*arctanh(coth(b*x + a))
```

3.275.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(19) = 38.

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int x^2 \text{arctanh}(\coth(a+bx))\,dx = -\frac{1}{12}\,bx^4 + \frac{1}{6}\,x^3\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)$$

input
```
integrate(x^2*arctanh(coth(b*x+a)),x, algorithm="giac")
```

output
```
-1/12*b*x^4 + 1/6*x^3*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) +
1)/((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

3.275.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \operatorname{arctanh}(\coth(a+bx))\,dx = \frac{x^3 \operatorname{atanh}(\coth(a+b\,x))}{3} - \frac{b\,x^4}{12}$$

input
```
int(x^2*atanh(coth(a + b*x)),x)
```

output
```
(x^3*atanh(coth(a + b*x)))/3 - (b*x^4)/12
```

3.276 $\int x\text{arctanh}(\coth(a+bx))\,dx$

3.276.1 Optimal result

Integrand size = 9, antiderivative size = 23

$$\int x\text{arctanh}(\coth(a+bx))\,dx=-\frac{bx^3}{6}+\frac{1}{2}x^2\text{arctanh}(\coth(a+bx))$$

output
```
-1/6*b*x^3+1/2*x^2*arctanh(coth(b*x+a))
```

3.276.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x\text{arctanh}(\coth(a+bx))\,dx=-\frac{1}{6}x^2(bx-3\text{arctanh}(\coth(a+bx)))$$

input
```
Integrate[x*ArcTanh[Coth[a + b*x]],x]
```

output
```
-1/6*(x^2*(b*x - 3*ArcTanh[Coth[a + b*x]]))
```

3.276.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6795, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(\coth (a+b x)) \, dx$$

$$\Big\downarrow \text{6795}$$

$$\frac{1}{2} b \int -x^2dx+\frac{1}{2} x^2 \text{arctanh}(\coth (a+b x))$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2} x^2 \text{arctanh}(\coth (a+b x))-\frac{b x^3}{6}$$

input
```
Int[x*ArcTanh[Coth[a + b*x]],x]
```

output
```
-1/6*(b*x^3) + (x^2*ArcTanh[Coth[a + b*x]])/2
```

3.276.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6795
```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

3.276.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{b\,x^{3}}{6}+\frac{x^{2} \operatorname{arctanh}\left(\coth \left(b x+a \right)\right)}{2}$
parts	$-\frac{b\,x^{3}}{6}+\frac{x^{2} \operatorname{arctanh}\left(\coth \left(b x+a \right)\right)}{2}$
parallelrisch	$-\frac{x^{2} \left(b x-3 \,\operatorname{arctanh}\left(\frac{1}{\tanh \left(b x+a \right)}\right)\right)}{6}$
risch	$\frac{x^{2} \ln \left({\mathrm e}^{b x+a}\right)}{2}-\frac{b\,x^{3}}{6}+\frac{i \pi \,x^{2} \operatorname{csgn}\left(\frac{i}{{\mathrm e}^{2 b x+2 a}-1}\right) \operatorname{csgn}\left(\frac{i {\mathrm e}^{2 b x+2 a}}{{\mathrm e}^{2 b x+2 a}-1}\right)^{2}}{8}-\frac{i \pi \,x^{2} \operatorname{csgn}\left(\frac{i}{{\mathrm e}^{2 b x+2 a}-1}\right)^{3}}{4}-\frac{i \pi \,x^{2} \operatorname{csgn}\left(\frac{i {\mathrm e}^{2 b x+2 a}}{{\mathrm e}^{2 b x+2 a}-}\right.}{8}$

input
```
int(x*arctanh(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*b*x^3+1/2*x^2*arctanh(coth(b*x+a))
```

3.276.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x\text{arctanh}(\coth(a+bx))\,dx = \frac{1}{3}\,bx^3 + \frac{1}{4}i\,\pi x^2 + \frac{1}{2}\,ax^2$$

input
```
integrate(x*arctanh(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/3*b*x^3 + 1/4*I*pi*x^2 + 1/2*a*x^2
```

3.276.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(19) = 38$.

Time = 2.64 (sec) , antiderivative size = 180, normalized size of antiderivative = 7.83

$$\int x\operatorname{arctanh}(\coth(a+bx))\,dx = \begin{cases} \frac{x^2 \operatorname{atanh}(\coth(a))}{2} & \text{for } b=0 \\ -\frac{x\log(-e^{-bx})\operatorname{atanh}(\coth(bx+\log(-e^{-bx})))}{b} - \frac{\log(-e^{-bx})^2\operatorname{atanh}(\coth(bx+\log(-e^{-bx})))}{2b^2} & \text{for } a=\log(-e^{-bx}) \\ -\frac{x\log(e^{-bx})\operatorname{atanh}(\coth(bx+\log(e^{-bx})))}{b} - \frac{\log(e^{-bx})^2\operatorname{atanh}(\coth(bx+\log(e^{-bx})))}{2b^2} & \text{for } a=\log(e^{-bx}) \\ \frac{x\operatorname{atanh}^2\left(\frac{1}{\tanh(a+bx)}\right)}{2b} - \frac{\operatorname{atanh}^3\left(\frac{1}{\tanh(a+bx)}\right)}{6b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*atanh(coth(b*x+a)),x)
```

output
```
Piecewise((x**2*atanh(coth(a))/2, Eq(b, 0)), (-x*log(-exp(-b*x))*atanh(cot
h(b*x + log(-exp(-b*x))))/b - log(-exp(-b*x))**2*atanh(coth(b*x + log(-exp
(-b*x))))/(2*b**2), Eq(a, log(-exp(-b*x)))), (-x*log(exp(-b*x))*atanh(coth
(b*x + log(exp(-b*x))))/b - log(exp(-b*x))**2*atanh(coth(b*x + log(exp(-b*
x))))/(2*b**2), Eq(a, log(exp(-b*x)))), (x*atanh(1/tanh(a + b*x))**2/(2*b)
 - atanh(1/tanh(a + b*x))**3/(6*b**2), True))
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x\operatorname{arctanh}(\coth(a+bx))\,dx = -\frac{1}{6}\,bx^3 + \frac{1}{2}\,x^2\operatorname{artanh}(\coth(bx+a))$$

input
```
integrate(x*arctanh(coth(b*x+a)),x, algorithm="maxima")
```

output
```
-1/6*b*x^3 + 1/2*x^2*arctanh(coth(b*x + a))
```

3.276.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(19) = 38$.

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int x\text{arctanh}(\coth(a+bx))\,dx = -\frac{1}{6}\,bx^3 + \frac{1}{4}\,x^2\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)$$

input
```
integrate(x*arctanh(coth(b*x+a)),x, algorithm="giac")
```

output
```
-1/6*b*x^3 + 1/4*x^2*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1
)/((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

3.276.9 Mupad [B] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x\text{arctanh}(\coth(a+bx))\,dx = \frac{x^2\,\text{atanh}(\coth(a+b\,x))}{2} - \frac{b\,x^3}{6}$$

input
```
int(x*atanh(coth(a + b*x)),x)
```

output
```
(x^2*atanh(coth(a + b*x)))/2 - (b*x^3)/6
```

3.277 $\int \operatorname{arctanh}(\coth(a+bx))\,dx$

3.277.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \operatorname{arctanh}(\coth(a+bx))\,dx=\frac{\operatorname{arctanh}(\coth(a+bx))^2}{2b}$$

output
```
1/2*arctanh(coth(b*x+a))^2/b
```

3.277.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \operatorname{arctanh}(\coth(a+bx))\,dx=-\frac{bx^2}{2}+x\operatorname{arctanh}(\coth(a+bx))$$

input
```
Integrate[ArcTanh[Coth[a + b*x]],x]
```

output
```
-1/2*(b*x^2) + x*ArcTanh[Coth[a + b*x]]
```

3.277.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arctanh}(\coth(a+bx))\,dx$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \text{arctanh}(\coth(a+bx))d\text{arctanh}(\coth(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\text{arctanh}(\coth(a+bx))^2}{2b}$$

input
```
Int[ArcTanh[Coth[a + b*x]],x]
```

output
```
ArcTanh[Coth[a + b*x]]^2/(2*b)
```

3.277.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.277.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\operatorname{arctanh}(\coth(bx+a))^2}{2b}$
default	$\frac{\operatorname{arctanh}(\coth(bx+a))^2}{2b}$
parts	$x \operatorname{arctanh}(\coth(bx+a)) - \frac{b\,x^2}{2}$
parallelrisch	$-\frac{x\left(bx-2\operatorname{arctanh}\left(\frac{1}{\tanh(bx+a)}\right)\right)}{2}$
risch	$x\ln\left(\mathrm{e}^{bx+a}\right) - \frac{i\,\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)^2 \pi x}{4} + \frac{i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^2 x}{2} - \frac{i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^3 x}{4}$

input
```
int(arctanh(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(coth(b*x+a))^2/b
```

3.277.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \operatorname{arctanh}(\coth(a+bx))\,dx = \frac{1}{2}\,bx^2 + \frac{1}{2}i\,\pi x + ax$$

input
```
integrate(arctanh(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*b*x^2 + 1/2*I*pi*x + a*x
```

3.277.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(12) = 24$.

Time = 1.54 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.88

$$\int \operatorname{arctanh}(\coth(a+bx))\,dx = \begin{cases} x \operatorname{atanh}\left(\coth\left(a\right)\right) & \text{for}\ b = 0 \\ x \operatorname{atanh}\left(\coth\left(bx + \log\left(-e^{-bx}\right)\right)\right) & \text{for}\ a = \log\left(-e^{-bx}\right) \\ -\frac{\log\left(e^{-bx}\right) \operatorname{atanh}\left(\coth\left(bx + \log\left(e^{-bx}\right)\right)\right)}{b} & \text{for}\ a = \log\left(e^{-bx}\right) \\ \frac{\operatorname{atanh}^{2}\left(\frac{1}{\tanh\left(a+bx\right)}\right)}{2b} & \text{otherwise} \end{cases}$$

input
```
integrate(atanh(coth(b*x+a)),x)
```

output
```
Piecewise((x*atanh(coth(a)), Eq(b, 0)), (x*atanh(coth(b*x + log(-exp(-b*x)
))), Eq(a, log(-exp(-b*x)))), (-log(exp(-b*x))*atanh(coth(b*x + log(exp(-b
*x))))/b, Eq(a, log(exp(-b*x)))), (atanh(1/tanh(a + b*x))**2/(2*b), True))
```

3.277.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \operatorname{arctanh}(\coth(a+bx))\,dx = -\frac{1}{2}\,bx^{2} + x \operatorname{artanh}\left(\coth\left(bx + a\right)\right)$$

input
```
integrate(arctanh(coth(b*x+a)),x, algorithm="maxima")
```

output
```
-1/2*b*x^2 + x*arctanh(coth(b*x + a))
```

3.277.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 4.31

$$\int \operatorname{arctanh}(\coth(a+bx))\,dx = -\frac{1}{2}\,bx^{2} + \frac{1}{2}\,x \log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1} + 1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1} - 1}\right)$$

input
```
integrate(arctanh(coth(b*x+a)),x, algorithm="giac")
```

output
```
-1/2*b*x^2 + 1/2*x*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/
((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

3.277.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \operatorname{arctanh}(\coth(a+bx))\,dx = x\,\operatorname{atanh}(\coth(a+b\,x)) - \frac{b\,x^2}{2}$$

input
```
int(atanh(coth(a + b*x)),x)
```

output
```
x*atanh(coth(a + b*x)) - (b*x^2)/2
```

3.278 $\int \frac{\text{arctanh}(\coth(a+bx))}{x} \, dx$

3.278.1 Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \frac{\text{arctanh}(\coth(a+bx))}{x} \, dx = bx - (bx - \text{arctanh}(\coth(a+bx))) \log(x)$$

output
```
b*x-(b*x-arctanh(coth(b*x+a)))*ln(x)
```

3.278.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\text{arctanh}(\coth(a+bx))}{x} \, dx = bx + (-bx + \text{arctanh}(\coth(a+bx))) \log(x)$$

input
```
Integrate[ArcTanh[Coth[a + b*x]]/x,x]
```

output
```
b*x + (-(b*x) + ArcTanh[Coth[a + b*x]])*Log[x]
```

3.278.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\coth(a+bx))}{x} \, dx$$

$$\Big\downarrow \; 2589$$

$$bx-(bx-\operatorname{arctanh}(\coth(a+bx)))\int \frac{1}{x}dx$$

$$\Big\downarrow \; 14$$

$$bx-\log(x)(bx-\operatorname{arctanh}(\coth(a+bx)))$$

input
```
Int[ArcTanh[Coth[a + b*x]]/x,x]
```

output
```
b*x - (b*x - ArcTanh[Coth[a + b*x]])*Log[x]
```

3.278.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

3.278.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

method	result
default	$\ln(x)\operatorname{arctanh}(\coth(bx+a)) - b(x\ln(x) - x)$
parts	$\ln(x)\operatorname{arctanh}(\coth(bx+a)) - b(x\ln(x) - x)$
risch	$\ln(x)\ln\left(\mathrm{e}^{bx+a}\right) - b\ln(x)x + bx - \frac{i\pi\left(\operatorname{csgn}\left(i\mathrm{e}^{bx+a}\right)^2\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right) - 2\operatorname{csgn}\left(i\mathrm{e}^{bx+a}\right)\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^2 + \operatorname{csgn}\left(i\mathrm{e}^{2bx+2}\right.\right.}{}$

input
```
int(arctanh(coth(b*x+a))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arctanh(coth(b*x+a))-b*(x*ln(x)-x)
```

3.278.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{arctanh}(\coth(a+bx))}{x}\,dx = bx + \frac{1}{2}\,(i\,\pi + 2\,a)\log(x)$$

input
```
integrate(arctanh(coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
b*x + 1/2*(I*pi + 2*a)*log(x)
```

3.278.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{atanh}(\coth(a+bx))}{x}\,dx$$

input
```
integrate(atanh(coth(b*x+a))/x,x)
```

output
```
Integral(atanh(coth(a + b*x))/x, x)
```

3.278.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{\operatorname{arctanh}(\coth(a+bx))}{x}\,dx = -b\left(x+\frac{a}{b}\right)\log(x) + b\left(x+\frac{a\log(x)}{b}\right) + \operatorname{artanh}(\coth(bx+a))\log(x)$$

input
```
integrate(arctanh(coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
-b*(x + a/b)*log(x) + b*(x + a*log(x)/b) + arctanh(coth(b*x + a))*log(x)
```

3.278.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{arctanh}(\coth(a+bx))}{x}\,dx = bx + \frac{1}{2}(i\pi + 2a)\log(x)$$

input
```
integrate(arctanh(coth(b*x+a))/x,x, algorithm="giac")
```

output
```
b*x + 1/2*(I*pi + 2*a)*log(x)
```

3.278.9 Mupad [B] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.81

$$\int \frac{\operatorname{arctanh}(\coth(a+bx))}{x}\,dx = b\,x - \ln(x)\left(\frac{\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)}{2} - \frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)}{2} + b\,x\right)$$

input
```
int(atanh(coth(a + b*x))/x,x)
```

output
```
b*x - log(x)*(log(-2/(exp(2*a)*exp(2*b*x) - 1))/2 - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 + b*x)
```

3.279 $\int \frac{\operatorname{arctanh}(\coth(a+bx))}{x^2}\,dx$

3.279.1 Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{\operatorname{arctanh}(\coth(a+bx))}{x^2}\,dx = -\frac{\operatorname{arctanh}(\coth(a+bx))}{x} + b\log(x)$$

output
```
-arctanh(coth(b*x+a))/x+b*ln(x)
```

3.279.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{arctanh}(\coth(a+bx))}{x^2}\,dx = b - \frac{\operatorname{arctanh}(\coth(a+bx))}{x} + b\log(x)$$

input
```
Integrate[ArcTanh[Coth[a + b*x]]/x^2,x]
```

output
```
b - ArcTanh[Coth[a + b*x]]/x + b*Log[x]
```

3.279.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\coth(a+bx))}{x^2} \, dx$$

$$\Big\downarrow \; 2599$$

$$b \int \frac{1}{x} dx - \frac{\text{arctanh}(\coth(a+bx))}{x}$$

$$\Big\downarrow \; 14$$

$$b \log(x) - \frac{\text{arctanh}(\coth(a+bx))}{x}$$

input
```
Int[ArcTanh[Coth[a + b*x]]/x^2,x]
```

output
```
-(ArcTanh[Coth[a + b*x]]/x) + b*Log[x]
```

3.279.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.279.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result
default	$-\frac{\operatorname{arctanh}(\coth(bx+a))}{x}+b\ln(x)$
parts	$-\frac{\operatorname{arctanh}(\coth(bx+a))}{x}+b\ln(x)$
parallelrisch	$\frac{b\ln(x)x-\operatorname{arctanh}\left(\frac{1}{\tanh(bx+a)}\right)}{x}$
risch	$-\frac{\ln(e^{bx+a})}{x}+\frac{-2i\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}-1}\right)^2+i\pi\operatorname{csgn}(ie^{2bx+2a})\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}-1}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}-1}\right)+i\pi\operatorname{csgn}(ie^{bx+a})^2\operatorname{csgn}(\ldots}{\ldots}$

input
```
int(arctanh(coth(b*x+a))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arctanh(coth(b*x+a))/x+b*ln(x)
```

3.279.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{arctanh}(\coth(a+bx))}{x^2}\,dx=\frac{-i\,\pi+2\,bx\log(x)-2\,a}{2\,x}$$

input
```
integrate(arctanh(coth(b*x+a))/x^2,x, algorithm="fricas")
```

output
```
1/2*(-I*pi + 2*b*x*log(x) - 2*a)/x
```

3.279.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(14) = 28.

Time = 2.62 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.00

$$\int \frac{\operatorname{arctanh}(\coth(a+bx))}{x^2}\,dx=\begin{cases}-\frac{\operatorname{atanh}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{x} & \text{for } a=\log\left(-e^{-bx}\right)\\ -\frac{\operatorname{atanh}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right)}{x} & \text{for } a=\log\left(e^{-bx}\right)\\ b\log(x)-\frac{\operatorname{atanh}\left(\frac{1}{\tanh(a+bx)}\right)}{x} & \text{otherwise}\end{cases}$$

input
```
integrate(atanh(coth(b*x+a))/x**2,x)
```

output
```
Piecewise((-atanh(coth(b*x + log(-exp(-b*x))))/x, Eq(a, log(-exp(-b*x)))),
 (-atanh(coth(b*x + log(exp(-b*x))))/x, Eq(a, log(exp(-b*x)))), (b*log(x)
- atanh(1/tanh(a + b*x))/x, True))
```

3.279.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(\coth(a+bx))}{x^2}\,dx = b\log(x) - \frac{\operatorname{artanh}(\coth(bx+a))}{x}$$

input
```
integrate(arctanh(coth(b*x+a))/x^2,x, algorithm="maxima")
```

output
```
b*log(x) - arctanh(coth(b*x + a))/x
```

3.279.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.12

$$\int \frac{\operatorname{arctanh}(\coth(a+bx))}{x^2}\,dx = b\log(|x|) - \frac{\log\left(-\frac{\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1}+1}{\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1}-1}\right)}{2x}$$

input
```
integrate(arctanh(coth(b*x+a))/x^2,x, algorithm="giac")
```

output
```
b*log(abs(x)) - 1/2*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)
/((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/x
```

3.279.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(\coth(a+bx))}{x^2}\,dx = b\ln(x) - \frac{\operatorname{atanh}(\coth(a+b\,x))}{x}$$

input
```
int(atanh(coth(a + b*x))/x^2,x)
```

output
```
b*log(x) - atanh(coth(a + b*x))/x
```

3.280 $\int \frac{\text{arctanh}(\coth(a+bx))}{x^3}\,dx$

3.280.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{\text{arctanh}(\coth(a+bx))}{x^3}\,dx = -\frac{b}{2x} - \frac{\text{arctanh}(\coth(a+bx))}{2x^2}$$

output
```
-1/2*b/x-1/2*arctanh(coth(b*x+a))/x^2
```

3.280.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{\text{arctanh}(\coth(a+bx))}{x^3}\,dx = -\frac{bx+\text{arctanh}(\coth(a+bx))}{2x^2}$$

input
```
Integrate[ArcTanh[Coth[a + b*x]]/x^3,x]
```

output
```
-1/2*(b*x + ArcTanh[Coth[a + b*x]])/x^2
```

3.280.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\coth(a+bx))}{x^3} dx$$

$$\Big\downarrow \; 2599$$

$$\frac{1}{2}b\int \frac{1}{x^2}dx-\frac{\text{arctanh}(\coth(a+bx))}{2x^2}$$

$$\Big\downarrow \; 15$$

$$-\frac{\text{arctanh}(\coth(a+bx))}{2x^2}-\frac{b}{2x}$$

input
```
Int[ArcTanh[Coth[a + b*x]]/x^3,x]
```

output
```
-1/2*b/x - ArcTanh[Coth[a + b*x]]/(2*x^2)
```

3.280.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.280.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{b}{2x}-\frac{\operatorname{arctanh}(\coth(bx+a))}{2x^2}$
parts	$-\frac{b}{2x}-\frac{\operatorname{arctanh}(\coth(bx+a))}{2x^2}$
parallelrisch	$\frac{-bx-\operatorname{arctanh}\left(\frac{1}{\tanh(bx+a)}\right)}{2x^2}$
risch	$-\frac{\ln\left(e^{bx+a}\right)}{2x^2}-\frac{4bx-i\pi\ \operatorname{csgn}\left(i e^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}-1}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}-1}\right)+2i\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}-1}\right)^2-i\pi\operatorname{csgn}\left(i e^{bx+a}\right)^2 \operatorname{csg}}{}$

input
```
int(arctanh(coth(b*x+a))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/x-1/2*arctanh(coth(b*x+a))/x^2
```

3.280.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\operatorname{arctanh}(\coth(a+bx))}{x^3}\,dx=\frac{-i\,\pi-4\,bx-2\,a}{4\,x^2}$$

input
```
integrate(arctanh(coth(b*x+a))/x^3,x, algorithm="fricas")
```

output
```
1/4*(-I*pi - 4*b*x - 2*a)/x^2
```

3.280.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(19) = 38.

Time = 4.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.48

$$\int \frac{\operatorname{arctanh}(\coth(a+bx))}{x^3}\,dx=\begin{cases}-\frac{\operatorname{atanh}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{2x^2} & \text{for } a=\log\left(-e^{-bx}\right)\\ -\frac{\operatorname{atanh}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right)}{2x^2} & \text{for } a=\log\left(e^{-bx}\right)\\ -\frac{b}{2x}-\frac{\operatorname{atanh}\left(\frac{1}{\tanh(a+bx)}\right)}{2x^2} & \text{otherwise}\end{cases}$$

input
```
integrate(atanh(coth(b*x+a))/x**3,x)
```

output
```
Piecewise((-atanh(coth(b*x + log(-exp(-b*x))))/(2*x**2), Eq(a, log(-exp(-b
*x)))), (-atanh(coth(b*x + log(exp(-b*x))))/(2*x**2), Eq(a, log(exp(-b*x))
)), (-b/(2*x) - atanh(1/tanh(a + b*x))/(2*x**2), True))
```

3.280.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\text{arctanh}(\coth(a+bx))}{x^3}\,dx = -\frac{b}{2\,x} - \frac{\text{artanh}\left(\coth\left(bx+a\right)\right)}{2\,x^2}$$

input
```
integrate(arctanh(coth(b*x+a))/x^3,x, algorithm="maxima")
```

output
```
-1/2*b/x - 1/2*arctanh(coth(b*x + a))/x^2
```

3.280.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(19) = 38.

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int \frac{\text{arctanh}(\coth(a+bx))}{x^3}\,dx = -\frac{b}{2\,x} - \frac{\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{4\,x^2}$$

input
```
integrate(arctanh(coth(b*x+a))/x^3,x, algorithm="giac")
```

output
```
-1/2*b/x - 1/4*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((e^
(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/x^2
```

3.280.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\text{arctanh}(\coth(a+bx))}{x^3}\,dx = -\frac{\text{atanh}(\coth(a+b\,x))+b\,x}{2\,x^2}$$

input
```
int(atanh(coth(a + b*x))/x^3,x)
```

output
```
-(atanh(coth(a + b*x)) + b*x)/(2*x^2)
```

3.281 $\int \operatorname{arctanh}(\cosh(x))\,dx$

3.281.1 Optimal result

Integrand size = 3, antiderivative size = 27

$$\int \operatorname{arctanh}(\cosh(x))\,dx = -2x\operatorname{arctanh}(e^x) + x\operatorname{arctanh}(\cosh(x)) - \operatorname{PolyLog}(2,-e^x) + \operatorname{PolyLog}(2,e^x)$$

output
```
-2*x*arctanh(exp(x))+x*arctanh(cosh(x))-polylog(2,-exp(x))+polylog(2,exp(x
))
```

3.281.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \operatorname{arctanh}(\cosh(x))\,dx = x\operatorname{arctanh}(\cosh(x)) + x(\log(1-e^x) - \log(1+e^x)) - \operatorname{PolyLog}(2,-e^x) + \operatorname{PolyLog}(2,e^x)$$

input
```
Integrate[ArcTanh[Cosh[x]],x]
```

output
```
x*ArcTanh[Cosh[x]] + x*(Log[1 - E^x] - Log[1 + E^x]) - PolyLog[2, -E^x] +
PolyLog[2, E^x]
```

3.281.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.333, Rules used = {6825, 25, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arctanh}(\cosh(x))\,dx$$

$$\Big\downarrow \text{6825}$$

$$x\text{arctanh}(\cosh(x))-\int -x\text{csch}(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int x\text{csch}(x)dx+x\text{arctanh}(\cosh(x))$$

$$\Big\downarrow \text{3042}$$

$$x\text{arctanh}(\cosh(x))+\int ix\csc(ix)dx$$

$$\Big\downarrow \text{26}$$

$$x\text{arctanh}(\cosh(x))+i\int x\csc(ix)dx$$

$$\Big\downarrow \text{4670}$$

$$x\text{arctanh}(\cosh(x))+i\left(i\int \log\left(1-e^x\right)dx-i\int \log\left(1+e^x\right)dx+2ix\text{arctanh}(e^x)\right)$$

$$\Big\downarrow \text{2715}$$

$$x\text{arctanh}(\cosh(x))+i\left(i\int e^{-x}\log\left(1-e^x\right)de^x-i\int e^{-x}\log\left(1+e^x\right)de^x+2ix\text{arctanh}(e^x)\right)$$

$$\Big\downarrow \text{2838}$$

$$x\text{arctanh}(\cosh(x))+i(2ix\text{arctanh}(e^x)+i\,\text{PolyLog}\left(2,-e^x\right)-i\,\text{PolyLog}\left(2,e^x\right))$$

input
```
Int[ArcTanh[Cosh[x]],x]
```

output
```
x*ArcTanh[Cosh[x]] + I*((2*I)*x*ArcTanh[E^x] + I*PolyLog[2, -E^x] - I*Poly
Log[2, E^x])
```

3.281.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6825
```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

3.281.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

method	result
default	$x \operatorname{arctanh}(\cosh(x)) + x \ln(1 - e^x) + \operatorname{polylog}(2, e^x) - x \ln(1 + e^x) - \operatorname{polylog}(2, -e^x)$
parts	$x \operatorname{arctanh}(\cosh(x)) + x \ln(1 - e^x) + \operatorname{polylog}(2, e^x) - x \ln(1 + e^x) - \operatorname{polylog}(2, -e^x)$
risch	$-\frac{i\pi \operatorname{csgn}\left(i(e^x-1)^2\right) \operatorname{csgn}\left(ie^{-x}(e^x-1)^2\right)^2 x}{4} + \frac{i\pi \operatorname{csgn}\left(i(e^x-1)^2\right) \operatorname{csgn}\left(ie^{-x}\right) \operatorname{csgn}\left(ie^{-x}(e^x-1)^2\right) x}{4} - \frac{i\pi x}{2} - \frac{i\pi \operatorname{csgn}\left(i(1+e^x\right.}{4}$

input
```
int(arctanh(cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
x*arctanh(cosh(x))+x*ln(1-exp(x))+polylog(2,exp(x))-x*ln(1+exp(x))-polylog
(2,-exp(x))
```

3.281.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(22) = 44.

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\begin{aligned}\int \operatorname{arctanh}(\cosh(x))\, dx = & \frac{1}{2} x \log\left(-\frac{\cosh(x)+1}{\cosh(x)-1}\right) - x \log(\cosh(x) + \sinh(x) + 1) \\ & + x \log(-\cosh(x) - \sinh(x) + 1) \\ & + \operatorname{Li}_2(\cosh(x) + \sinh(x)) - \operatorname{Li}_2(-\cosh(x) - \sinh(x))\end{aligned}$$

input
```
integrate(arctanh(cosh(x)),x, algorithm="fricas")
```

output
```
1/2*x*log(-(cosh(x) + 1)/(cosh(x) - 1)) - x*log(cosh(x) + sinh(x) + 1) + x
*log(-cosh(x) - sinh(x) + 1) + dilog(cosh(x) + sinh(x)) - dilog(-cosh(x) -
 sinh(x))
```

3.281.6 Sympy [F]

$$\int \operatorname{arctanh}(\cosh(x))\,dx = \int \operatorname{atanh}(\cosh(x))\,dx$$

input
```
integrate(atanh(cosh(x)),x)
```

output
```
Integral(atanh(cosh(x)), x)
```

3.281.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \operatorname{arctanh}(\cosh(x))\,dx = x \operatorname{artanh}(\cosh(x)) - x\log(e^x+1) + x\log(-e^x+1) - \operatorname{Li}_2(-e^x) + \operatorname{Li}_2(e^x)$$

input
```
integrate(arctanh(cosh(x)),x, algorithm="maxima")
```

output
```
x*arctanh(cosh(x)) - x*log(e^x + 1) + x*log(-e^x + 1) - dilog(-e^x) + dilo
g(e^x)
```

3.281.8 Giac [F]

$$\int \operatorname{arctanh}(\cosh(x))\,dx = \int \operatorname{artanh}(\cosh(x))\,dx$$

input
```
integrate(arctanh(cosh(x)),x, algorithm="giac")
```

output
```
integrate(arctanh(cosh(x)), x)
```

3.281.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}(\cosh(x))\,dx = \int \operatorname{atanh}(\cosh(x))\,dx$$

input
```
int(atanh(cosh(x)),x)
```

output
```
int(atanh(cosh(x)), x)
```

3.282 $\int x \text{arctanh}(\cosh(x))\,dx$

3.282.1 Optimal result

Integrand size = 5, antiderivative size = 51

$$\int x \text{arctanh}(\cosh(x))\,dx = -x^2\text{arctanh}(e^x) + \frac{1}{2}x^2\text{arctanh}(\cosh(x)) - x\,\text{PolyLog}(2,-e^x) + x\,\text{PolyLog}(2,e^x) + \text{PolyLog}(3,-e^x) - \text{PolyLog}(3,e^x)$$

output
```
-x^2*arctanh(exp(x))+1/2*x^2*arctanh(cosh(x))-x*polylog(2,-exp(x))+x*polyl
og(2,exp(x))+polylog(3,-exp(x))-polylog(3,exp(x))
```

3.282.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int x \text{arctanh}(\cosh(x))\,dx = \frac{1}{2}x^2\text{arctanh}(\cosh(x)) + \frac{1}{2}x^2\log(1-e^x) - \frac{1}{2}x^2\log(1+e^x) - x\,\text{PolyLog}(2,-e^x) + x\,\text{PolyLog}(2,e^x) + \text{PolyLog}(3,-e^x) - \text{PolyLog}(3,e^x)$$

input
```
Integrate[x*ArcTanh[Cosh[x]],x]
```

output
```
(x^2*ArcTanh[Cosh[x]])/2 + (x^2*Log[1 - E^x])/2 - (x^2*Log[1 + E^x])/2 - x
*PolyLog[2, -E^x] + x*PolyLog[2, E^x] + PolyLog[3, -E^x] - PolyLog[3, E^x]
```

3.282.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.600$, Rules used = {6827, 25, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(\cosh(x))\,dx$$

$$\Big\downarrow \text{6827}$$

$$\frac{1}{2}x^2\text{arctanh}(\cosh(x)) - \frac{1}{2}\int -x^2\text{csch}(x)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int x^2\text{csch}(x)dx + \frac{1}{2}x^2\text{arctanh}(\cosh(x))$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x^2\text{arctanh}(\cosh(x)) + \frac{1}{2}\int ix^2\csc(ix)dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}x^2\text{arctanh}(\cosh(x)) + \frac{1}{2}i\int x^2\csc(ix)dx$$

$$\Big\downarrow \text{4670}$$

$$\frac{1}{2}x^2\text{arctanh}(\cosh(x)) + \frac{1}{2}i\left(2i\int x\log(1-e^x)\,dx - 2i\int x\log(1+e^x)\,dx + 2ix^2\text{arctanh}(e^x)\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}x^2\text{arctanh}(\cosh(x)) + \frac{1}{2}i\left(-2i\left(\int \text{PolyLog}(2,-e^x)\,dx - x\,\text{PolyLog}(2,-e^x)\right) + 2i\left(\int \text{PolyLog}(2,e^x)\,dx - x\,\text{PolyLog}(2,e^x)\right) + 2ix^2\text{ar}$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}x^2\text{arctanh}(\cosh(x)) + \frac{1}{2}i\left(-2i\left(\int e^{-x}\,\text{PolyLog}(2,-e^x)\,de^x - x\,\text{PolyLog}(2,-e^x)\right) + 2i\left(\int e^{-x}\,\text{PolyLog}(2,e^x)\,de^x - x\,\text{PolyLog}(2,e^x)\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x^2\text{arctanh}(\cosh(x)) + \frac{1}{2}i\left(2ix^2\text{arctanh}(e^x) - 2i(\text{PolyLog}(3,-e^x) - x\,\text{PolyLog}(2,-e^x)) + 2i(\text{PolyLog}(3,e^x) - x\,\text{PolyLog}(2,e^x))\right)$$

input
```
Int[x*ArcTanh[Cosh[x]],x]
```

output
```
(x^2*ArcTanh[Cosh[x]])/2 + (I/2)*((2*I)*x^2*ArcTanh[E^x] - (2*I)*(-(x*Poly
Log[2, -E^x]) + PolyLog[3, -E^x]) + (2*I)*(-(x*PolyLog[2, E^x]) + PolyLog[
3, E^x]))
```

3.282.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6827
```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x],
x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
 &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
 + 1, x]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.282.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 400, normalized size of antiderivative = 7.84

method	result
risch	$-\frac{x^2 \ln(\mathrm{e}^x-1)}{2}+\frac{i\pi\left(2\,\mathrm{csgn}\left(i\mathrm{e}^{-x}(\mathrm{e}^x-1)^2\right)^2-\mathrm{csgn}(i(1+\mathrm{e}^x))^2\,\mathrm{csgn}\left(i(1+\mathrm{e}^x)^2\right)+2\,\mathrm{csgn}(i(1+\mathrm{e}^x))\mathrm{csgn}\left(i(1+\mathrm{e}^x)^2\right)^2-\mathrm{csgn}\left(i(1+\mathrm{e}^x)\right.}{}$

input
```
int(x*arctanh(cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2*ln(exp(x)-1)+1/8*I*Pi*(2*csgn(I*exp(-x)*(exp(x)-1)^2)^2-csgn(I*(1
+exp(x)))^2*csgn(I*(1+exp(x))^2)+2*csgn(I*(1+exp(x)))*csgn(I*(1+exp(x))^2)
^2-csgn(I*(1+exp(x))^2)^3-csgn(I*(1+exp(x))^2)*csgn(I*exp(-x))*csgn(I*exp(
-x)*(1+exp(x))^2)+csgn(I*(1+exp(x))^2)*csgn(I*exp(-x)*(1+exp(x))^2)^2+csgn
(I*(exp(x)-1))^2*csgn(I*(exp(x)-1)^2)-2*csgn(I*(exp(x)-1))*csgn(I*(exp(x)-
1)^2)^2+csgn(I*(exp(x)-1)^2)^3+csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x))*csgn(I
*exp(-x)*(exp(x)-1)^2)-csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x)*(exp(x)-1)^2)^2
+csgn(I*exp(-x))*csgn(I*exp(-x)*(1+exp(x))^2)^2-csgn(I*exp(-x))*csgn(I*exp
(-x)*(exp(x)-1)^2)^2-csgn(I*exp(-x)*(1+exp(x))^2)^3-csgn(I*exp(-x)*(exp(x)
-1)^2)^3-2)*x^2-x*polylog(2,-exp(x))+polylog(3,-exp(x))+1/2*x^2*ln(1-exp(x
))+x*polylog(2,exp(x))-polylog(3,exp(x))
```

3.282.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(42) = 84$.

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.73

$$\int x\text{arctanh}(\cosh(x))\,dx = \frac{1}{4}x^2\log\left(-\frac{\cosh(x)+1}{\cosh(x)-1}\right) - \frac{1}{2}x^2\log(\cosh(x)+\sinh(x)+1) + \frac{1}{2}x^2\log(-\cosh(x)-\sinh(x)+1) + x\text{Li}_2(\cosh(x)+\sinh(x)) - x\text{Li}_2(-\cosh(x)-\sinh(x)) - \text{polylog}(3,\cosh(x)+\sinh(x)) + \text{polylog}(3,-\cosh(x)-\sinh(x))$$

input
```
integrate(x*arctanh(cosh(x)),x, algorithm="fricas")
```

output
```
1/4*x^2*log(-(cosh(x) + 1)/(cosh(x) - 1)) - 1/2*x^2*log(cosh(x) + sinh(x)
+ 1) + 1/2*x^2*log(-cosh(x) - sinh(x) + 1) + x*dilog(cosh(x) + sinh(x)) -
x*dilog(-cosh(x) - sinh(x)) - polylog(3, cosh(x) + sinh(x)) + polylog(3, -
cosh(x) - sinh(x))
```

3.282.6 Sympy [F]

$$\int x\text{arctanh}(\cosh(x))\,dx = \int x\,\text{atanh}\,(\cosh\,(x))\,dx$$

input
```
integrate(x*atanh(cosh(x)),x)
```

output
```
Integral(x*atanh(cosh(x)), x)
```

3.282.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int x\text{arctanh}(\cosh(x))\,dx = \frac{1}{2}\,x^2\,\text{artanh}\,(\cosh\,(x)) - \frac{1}{2}\,x^2\log\,(e^x+1) + \frac{1}{2}\,x^2\log\,(-e^x+1) - x\text{Li}_2(-e^x) + x\text{Li}_2(e^x) + \text{Li}_3(-e^x) - \text{Li}_3(e^x)$$

input
```
integrate(x*arctanh(cosh(x)),x, algorithm="maxima")
```

output
```
1/2*x^2*arctanh(cosh(x)) - 1/2*x^2*log(e^x + 1) + 1/2*x^2*log(-e^x + 1) -
x*dilog(-e^x) + x*dilog(e^x) + polylog(3, -e^x) - polylog(3, e^x)
```

3.282.8 Giac [F]

$$\int x\text{arctanh}(\cosh(x))\,dx = \int x\,\text{artanh}\,(\cosh\,(x))\;dx$$

input
```
integrate(x*arctanh(cosh(x)),x, algorithm="giac")
```

output
```
integrate(x*arctanh(cosh(x)), x)
```

3.282.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arctanh}(\cosh(x))\,dx=\int x\,\text{atanh}(\cosh(x))\,dx$$

input
```
int(x*atanh(cosh(x)),x)
```

output
```
int(x*atanh(cosh(x)), x)
```

3.283 $\int x^2 \operatorname{arctanh}(\cosh(x))\,dx$

3.283.1 Optimal result

Integrand size = 7, antiderivative size = 77

$$\int x^2 \operatorname{arctanh}(\cosh(x))\,dx = -\frac{2}{3}x^3\operatorname{arctanh}(e^x) + \frac{1}{3}x^3\operatorname{arctanh}(\cosh(x)) - x^2 \operatorname{PolyLog}(2,-e^x) + x^2 \operatorname{PolyLog}(2,e^x) + 2x \operatorname{PolyLog}(3,-e^x) - 2x \operatorname{PolyLog}(3,e^x) - 2 \operatorname{PolyLog}(4,-e^x) + 2 \operatorname{PolyLog}(4,e^x)$$

output
```
-2/3*x^3*arctanh(exp(x))+1/3*x^3*arctanh(cosh(x))-x^2*polylog(2,-exp(x))+x
^2*polylog(2,exp(x))+2*x*polylog(3,-exp(x))-2*x*polylog(3,exp(x))-2*polylo
g(4,-exp(x))+2*polylog(4,exp(x))
```

3.283.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18

$$\int x^2 \operatorname{arctanh}(\cosh(x))\,dx = \frac{1}{3}\left(x^3\operatorname{arctanh}(\cosh(x)) + x^3 \log(1-e^x) - x^3 \log(1+e^x) - 3x^2 \operatorname{PolyLog}(2,-e^x) + 3x^2 \operatorname{PolyLog}(2,e^x) + 6x \operatorname{PolyLog}(3,-e^x) - 6x \operatorname{PolyLog}(3,e^x) - 6 \operatorname{PolyLog}(4,-e^x) + 6 \operatorname{PolyLog}(4,e^x)\right)$$

input
```
Integrate[x^2*ArcTanh[Cosh[x]],x]
```

output
```
(x^3*ArcTanh[Cosh[x]] + x^3*Log[1 - E^x] - x^3*Log[1 + E^x] - 3*x^2*PolyLo
g[2, -E^x] + 3*x^2*PolyLog[2, E^x] + 6*x*PolyLog[3, -E^x] - 6*x*PolyLog[3,
 E^x] - 6*PolyLog[4, -E^x] + 6*PolyLog[4, E^x])/3
```

3.283.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.29, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {6827, 25, 3042, 26, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arctanh}(\cosh(x))\,dx$$

$$\Big\downarrow \text{6827}$$

$$\frac{1}{3}x^3\text{arctanh}(\cosh(x))-\frac{1}{3}\int -x^3\text{csch}(x)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int x^3\text{csch}(x)dx+\frac{1}{3}x^3\text{arctanh}(\cosh(x))$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}x^3\text{arctanh}(\cosh(x))+\frac{1}{3}\int ix^3\csc(ix)dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{3}x^3\text{arctanh}(\cosh(x))+\frac{1}{3}i\int x^3\csc(ix)dx$$

$$\Big\downarrow \text{4670}$$

$$\frac{1}{3}x^3\text{arctanh}(\cosh(x))+\frac{1}{3}i\left(3i\int x^2\log\left(1-e^x\right)dx-3i\int x^2\log\left(1+e^x\right)dx+2ix^3\text{arctanh}(e^x)\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}x^3\text{arctanh}(\cosh(x))+$$
$$\frac{1}{3}i\left(-3i\left(2\int x\,\text{PolyLog}\left(2,-e^x\right)dx-x^2\,\text{PolyLog}\left(2,-e^x\right)\right)+3i\left(2\int x\,\text{PolyLog}\left(2,e^x\right)dx-x^2\,\text{PolyLog}\left(2,e^x\right)\right)\right.$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3}x^3\text{arctanh}(\cosh(x))+\frac{1}{3}i\left(-3i\left(2\left(x\,\text{PolyLog}\left(3,-e^x\right)-\int \text{PolyLog}\left(3,-e^x\right)dx\right)-x^2\,\text{PolyLog}\left(2,-e^x\right)\right)+3i\left(2\left(x\,\text{PolyLog}\left(3,e^x\right)-\int\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3}x^3\text{arctanh}(\cosh(x))+\frac{1}{3}i\left(-3i\left(2\left(x\,\text{PolyLog}\left(3,-e^x\right)-\int e^{-x}\,\text{PolyLog}\left(3,-e^x\right)de^x\right)-x^2\,\text{PolyLog}\left(2,-e^x\right)\right)+3i\left(2\left(x\,\text{PolyLog}\left(3,e^x\right)\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{3}x^3\text{arctanh}(\cosh(x))+\frac{1}{3}i(2ix^3\text{arctanh}(e^x)-3i(2(x\,\text{PolyLog}\left(3,-e^x\right)-\text{PolyLog}\left(4,-e^x\right))-x^2\,\text{PolyLog}\left(2,-e^x\right))+3i(2(x\,\text{PolyLog}\left(3,e\right.$$

input
```
Int[x^2*ArcTanh[Cosh[x]],x]
```

output
```
(x^3*ArcTanh[Cosh[x]])/3 + (I/3)*((2*I)*x^3*ArcTanh[E^x] - (3*I)*(-(x^2*Po
lyLog[2, -E^x]) + 2*(x*PolyLog[3, -E^x] - PolyLog[4, -E^x])) + (3*I)*(-(x^
2*PolyLog[2, E^x]) + 2*(x*PolyLog[3, E^x] - PolyLog[4, E^x])))
```

3.283.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6827
```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x],
x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
 &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
 + 1, x]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.283. $\int x^2 \operatorname{arctanh}(\cosh(x))\, dx$

3.283.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 422, normalized size of antiderivative = 5.48

method	result
risch	$-\frac{x^3 \ln(e^x-1)}{3} + \frac{i\pi\left(2\operatorname{csgn}\left(i e^{-x}(e^x-1)^2\right)^2 - \operatorname{csgn}(i(1+e^x))^2 \operatorname{csgn}\left(i(1+e^x)^2\right) + 2\operatorname{csgn}(i(1+e^x))\operatorname{csgn}\left(i(1+e^x)^2\right)^2 - \operatorname{csgn}\left(i(1+e^x)\right.}{}$

input
```
int(x^2*arctanh(cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3*ln(exp(x)-1)+1/12*I*Pi*(2*csgn(I*exp(-x)*(exp(x)-1)^2)^2-csgn(I*(
1+exp(x)))^2*csgn(I*(1+exp(x))^2)+2*csgn(I*(1+exp(x)))*csgn(I*(1+exp(x))^2
)^2-csgn(I*(1+exp(x))^2)^3-csgn(I*(1+exp(x))^2)*csgn(I*exp(-x))*csgn(I*exp
(-x)*(1+exp(x))^2)+csgn(I*(1+exp(x))^2)*csgn(I*exp(-x)*(1+exp(x))^2)^2+csg
n(I*(exp(x)-1))^2*csgn(I*(exp(x)-1)^2)-2*csgn(I*(exp(x)-1))*csgn(I*(exp(x)
-1)^2)^2+csgn(I*(exp(x)-1)^2)^3+csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x))*csgn(
I*exp(-x)*(exp(x)-1)^2)-csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x)*(exp(x)-1)^2)^
2+csgn(I*exp(-x))*csgn(I*exp(-x)*(1+exp(x))^2)^2-csgn(I*exp(-x))*csgn(I*ex
p(-x)*(exp(x)-1)^2)^2-csgn(I*exp(-x)*(1+exp(x))^2)^3-csgn(I*exp(-x)*(exp(x
)-1)^2)^3-2)*x^3-x^2*polylog(2,-exp(x))+2*x*polylog(3,-exp(x))-2*polylog(4
,-exp(x))+1/3*x^3*ln(1-exp(x))+x^2*polylog(2,exp(x))-2*x*polylog(3,exp(x))
+2*polylog(4,exp(x))
```

3.283.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.53

$$\begin{aligned}\int x^2 \operatorname{arctanh}(\cosh(x))\,dx = {} & \frac{1}{6}x^3 \log\left(-\frac{\cosh(x)+1}{\cosh(x)-1}\right) - \frac{1}{3}x^3 \log(\cosh(x)+\sinh(x)+1) \\ & + \frac{1}{3}x^3 \log(-\cosh(x)-\sinh(x)+1) + x^2 \mathrm{Li}_2(\cosh(x)+\sinh(x)) \\ & - x^2 \mathrm{Li}_2(-\cosh(x)-\sinh(x)) - 2x \operatorname{polylog}(3,\cosh(x)+\sinh(x)) \\ & + 2x \operatorname{polylog}(3,-\cosh(x)-\sinh(x)) \\ & + 2\operatorname{polylog}(4,\cosh(x)+\sinh(x)) \\ & - 2\operatorname{polylog}(4,-\cosh(x)-\sinh(x))\end{aligned}$$

input
```
integrate(x^2*arctanh(cosh(x)),x, algorithm="fricas")
```

output
```
1/6*x^3*log(-(cosh(x) + 1)/(cosh(x) - 1)) - 1/3*x^3*log(cosh(x) + sinh(x)
+ 1) + 1/3*x^3*log(-cosh(x) - sinh(x) + 1) + x^2*dilog(cosh(x) + sinh(x))
- x^2*dilog(-cosh(x) - sinh(x)) - 2*x*polylog(3, cosh(x) + sinh(x)) + 2*x*
polylog(3, -cosh(x) - sinh(x)) + 2*polylog(4, cosh(x) + sinh(x)) - 2*polyl
og(4, -cosh(x) - sinh(x))
```

3.283.6 Sympy [F]

$$\int x^2\text{arctanh}(\cosh(x))\,dx = \int x^2\,\text{atanh}\,(\cosh{(x)})\,dx$$

input
```
integrate(x**2*atanh(cosh(x)),x)
```

output
```
Integral(x**2*atanh(cosh(x)), x)
```

3.283.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\begin{aligned}\int x^2\text{arctanh}(\cosh(x))\,dx =& \frac{1}{3}\,x^3\,\text{artanh}\,(\cosh{(x)}) - \frac{1}{3}\,x^3\log{(e^x+1)}\\&+ \frac{1}{3}\,x^3\log{(-e^x+1)} - x^2\text{Li}_2(-e^x) + x^2\text{Li}_2(e^x)\\&+ 2\,x\text{Li}_3(-e^x) - 2\,x\text{Li}_3(e^x) - 2\,\text{Li}_4(-e^x) + 2\,\text{Li}_4(e^x)\end{aligned}$$

input
```
integrate(x^2*arctanh(cosh(x)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctanh(cosh(x)) - 1/3*x^3*log(e^x + 1) + 1/3*x^3*log(-e^x + 1) -
x^2*dilog(-e^x) + x^2*dilog(e^x) + 2*x*polylog(3, -e^x) - 2*x*polylog(3, e
^x) - 2*polylog(4, -e^x) + 2*polylog(4, e^x)
```

3.283.8 Giac [F]

$$\int x^2 \text{arctanh}(\cosh(x))\,dx = \int x^2\,\text{artanh}\left(\cosh\left(x\right)\right)\,dx$$

input
```
integrate(x^2*arctanh(cosh(x)),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(cosh(x)), x)
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arctanh}(\cosh(x))\,dx = \int x^2\,\text{atanh}(\cosh(x))\,dx$$

input
```
int(x^2*atanh(cosh(x)),x)
```

output
```
int(x^2*atanh(cosh(x)), x)
```

3.284 $\int x^2\mathbf{arctanh}(c + d\tanh(a + bx))\,dx$

3.284.1 Optimal result

Integrand size = 15, antiderivative size = 307

$$\begin{aligned}\int x^2\text{arctanh}(c+d\tanh(a+bx))\,dx =& \frac{1}{3}x^3\text{arctanh}(c+d\tanh(a+bx)) \\ &+\frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) \\ &-\frac{1}{6}x^3\log\left(1+\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right) \\ &+\frac{x^2\operatorname{PolyLog}\left(2,-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b} \\ &-\frac{x^2\operatorname{PolyLog}\left(2,-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b} \\ &-\frac{x\operatorname{PolyLog}\left(3,-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b^2} \\ &+\frac{x\operatorname{PolyLog}\left(3,-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b^2} \\ &+\frac{\operatorname{PolyLog}\left(4,-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{8b^3} \\ &-\frac{\operatorname{PolyLog}\left(4,-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arctanh(c+d*tanh(b*x+a))+1/6*x^3*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+
d))-1/6*x^3*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x^2*polylog(2,-(1-c-d
)*exp(2*b*x+2*a)/(1-c+d))/b-1/4*x^2*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c
-d))/b-1/4*x*polylog(3,-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/4*x*polylog(
3,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^2+1/8*polylog(4,-(1-c-d)*exp(2*b*x+2*
a)/(1-c+d))/b^3-1/8*polylog(4,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^3
```

3.284.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.86

$$\int x^2\operatorname{arctanh}(c+d\tanh(a+bx))\,dx=\frac{1}{3}x^3\operatorname{arctanh}(c+d\tanh(a+bx))$$
$$+\frac{4b^3x^3\log\left(1+\frac{(-1+c-d)e^{-2(a+bx)}}{-1+c+d}\right)-4b^3x^3\log\left(1+\frac{(1+c-d)e^{-2(a+bx)}}{1+c+d}\right)-6b^2x^2\operatorname{PolyLog}\left(2,\frac{(1-c+d)e^{-2(a+bx)}}{-1+c+d}\right)}{}$$

input
```
Integrate[x^2*ArcTanh[c + d*Tanh[a + b*x]],x]
```

output
```
(x^3*ArcTanh[c + d*Tanh[a + b*x]])/3 + (4*b^3*x^3*Log[1 + (-1 + c - d)/((-
1 + c + d)*E^(2*(a + b*x)))] - 4*b^3*x^3*Log[1 + (1 + c - d)/((1 + c + d)*
E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (1 - c + d)/((-1 + c + d)*E^(2*(a
 + b*x)))] + 6*b^2*x^2*PolyLog[2, (-1 - c + d)/((1 + c + d)*E^(2*(a + b*x)
))] - 6*b*x*PolyLog[3, (1 - c + d)/((-1 + c + d)*E^(2*(a + b*x)))] + 6*b*x
*PolyLog[3, (-1 - c + d)/((1 + c + d)*E^(2*(a + b*x)))] - 3*PolyLog[4, (1
- c + d)/((-1 + c + d)*E^(2*(a + b*x)))] + 3*PolyLog[4, (-1 - c + d)/((1 +
 c + d)*E^(2*(a + b*x)))])/(24*b^3)
```

3.284.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6797, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\operatorname{arctanh}(d\tanh(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6797}$$

$$\frac{1}{3}b(-c-d+1)\int \frac{e^{2a+2bx}x^3}{-c+(-c-d+1)e^{2a+2bx}+d+1}dx-\frac{1}{3}b(c+d+1)\int \frac{e^{2a+2bx}x^3}{c+(c+d+1)e^{2a+2bx}-d+1}dx+\frac{1}{3}x^3\text{arctanh}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}b(-c-d+1)\left(\frac{x^3\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{3\int x^2\log\left(\frac{e^{2a+2bx}(-c-d+1)}{-c+d+1}+1\right)dx}{2b(-c-d+1)}\right)-\frac{1}{3}b(c+d+1)\left(\frac{x^3\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{3\int x^2\log\left(\frac{e^{2a+2bx}(c+d+1)}{c-d+1}+1\right)dx}{2b(c+d+1)}\right)+\frac{1}{3}x^3\text{arctanh}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}b(-c-d+1)\left(\frac{x^3\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}\right)}{2b(-c-d+1)}\right)-\frac{1}{3}b(c+d+1)\left(\frac{x^3\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}\right)}{2b(c+d+1)}\right)+\frac{1}{3}x^3\text{arctanh}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{7163}$$

3.284. $\int x^2\text{arctanh}(c+d\tanh(a+bx))\,dx$

$$\frac{1}{3}b(-c-d+$$

$$1)\left(\frac{x^3\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(-}{}\right.}{2b}\right.}{2b(-c-d+1)}\right.$$

$$\frac{1}{3}b(c+d+$$

$$1)\left(\frac{x^3\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(c+d+1)}{c-}\right.}{2b}\right.}{2b(c+d+1)}\right.$$

$$\frac{1}{3}x^3\text{arctanh}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{3}b(-c-d+$$

$$1)\left(\frac{x^3\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2}{}\right.}{2b(-c-d+1)}\right.$$

$$\frac{1}{3}b(c+d+$$

$$1)\left(\frac{x^3\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyL}}{}\right.}{2b(c+d+1)}\right.$$

$$\frac{1}{3}x^3\text{arctanh}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \; 7143$$

3.284. $\int x^2\text{arctanh}(c+d\tanh(a+bx))\,dx$

$$\frac{1}{3}x^3\text{arctanh}(d\tanh(a+bx)+c)+\frac{1}{3}b(-c-d+1)\left(\frac{x^3\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\text{PolyLog}\left(4,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(-c-d\cdots}{-\cdots}\right.}{2b}\cdots\right.}{2b(-c-d+1)}\right.$$

$$\frac{1}{3}b(c+d+1)\left(\frac{x^3\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\text{PolyLog}\left(4,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a\cdots}}{c-d+1}\right.}{2b}\cdots\right.}{2b(c+d+1)}\right.$$

input
```
Int[x^2*ArcTanh[c + d*Tanh[a + b*x]],x]
```

output
```
(x^3*ArcTanh[c + d*Tanh[a + b*x]])/3 + (b*(1 - c - d)*((x^3*Log[1 + ((1 -
c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(2*b*(1 - c - d)) - (3*(-1/2*(x^2*Po
lyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/b + ((x*PolyLog[3,
 -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(2*b) - PolyLog[4, -(((1 -
 c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b^2))/b))/(2*b*(1 - c - d))))/3
- (b*(1 + c + d)*((x^3*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])
/(2*b*(1 + c + d)) - (3*(-1/2*(x^2*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*
x))/(1 + c - d))])/b + ((x*PolyLog[3, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 +
 c - d))])/(2*b) - PolyLog[4, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))
]/(4*b^2))/b))/(2*b*(1 + c + d))))/3
```

3.284.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6797
```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + (Simp[b*((1 - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x))), x], x] - Simp[b*
((1 + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c -
 d + (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.284. $\int x^2 \operatorname{arctanh}(c + d\tanh(a + bx))\,dx$

3.284.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.68 (sec) , antiderivative size = 5320, normalized size of antiderivative = 17.33

method	result	size
risch	Expression too large to display	5320

input
```
int(x^2*arctanh(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.284.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(263) = 526$.

Time = 0.29 (sec) , antiderivative size = 900, normalized size of antiderivative = 2.93

$$\int x^2 \text{arctanh}(c + d \tanh(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate(x^2*arctanh(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(b^3*x^3*log(-((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(
b*x + a) + d*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt(-(c + d + 1)/(c - d +
1))*(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt(-(c + d + 1)/
(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt(-(c +
 d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-s
qrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2
*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*s
qrt(-(c + d + 1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(
c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) -
 a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c
- d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x
+ a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c -
d - 1))) + 6*b*x*polylog(3, sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a)
+ sinh(b*x + a))) + 6*b*x*polylog(3, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh
(b*x + a) + sinh(b*x + a))) - 6*b*x*polylog(3, sqrt(-(c + d - 1)/(c - d -
1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*b*x*polylog(3, -sqrt(-(c + d - 1)
/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt(
-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3
+ a^3)*log(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))
 + 1) + (b^3*x^3 + a^3)*log(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + ...
```

3.284.6 Sympy [F]

$$\int x^2 \operatorname{arctanh}(c + d\tanh(a + bx))\, dx = \int x^2 \operatorname{atanh}(c + d\tanh(a + bx))\, dx$$

input
```
integrate(x**2*atanh(c+d*tanh(b*x+a)),x)
```

output
```
Integral(x**2*atanh(c + d*tanh(a + b*x)), x)
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.92

$$\int x^2\text{arctanh}(c+d\tanh(a+bx))\,dx = \frac{1}{3}x^3\,\text{artanh}\left(d\tanh\left(bx+a\right)+c\right) - \frac{1}{18}\,bd\left(\frac{4\,b^3x^3\log\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+6\,b^2x^2\text{Li}_2\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-6\,bx\text{Li}_3\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)+3\,\text{I}}{b^4d}\right.$$

input
```
integrate(x^2*arctanh(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctanh(d*tanh(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log((c + d + 1
)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 6*b^2*x^2*dilog(-(c + d + 1)*e^(2*b*x
 + 2*a)/(c - d + 1)) - 6*b*x*polylog(3, -(c + d + 1)*e^(2*b*x + 2*a)/(c -
d + 1)) + 3*polylog(4, -(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d)
- (4*b^3*x^3*log((c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*
dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, -(c + d
 - 1)*e^(2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, -(c + d - 1)*e^(2*b*x +
2*a)/(c - d - 1)))/(b^4*d))
```

3.284.8 Giac [F]

$$\int x^2\text{arctanh}(c+d\tanh(a+bx))\,dx = \int x^2\,\text{artanh}\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x^2*arctanh(c+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(d*tanh(b*x + a) + c), x)
```

3.284.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arctanh}(c + d\tanh(a + bx))\, dx = \int x^2 \,\text{atanh}(c + d\tanh(a + b\,x))\, dx$$

input
```
int(x^2*atanh(c + d*tanh(a + b*x)),x)
```

output
```
int(x^2*atanh(c + d*tanh(a + b*x)), x)
```

3.285 $\int x\text{arctanh}(c + d\tanh(a + bx))\,dx$

3.285.1 Optimal result

Integrand size = 13, antiderivative size = 231

$$\begin{aligned}\int x\text{arctanh}(c+d\tanh(a+bx))\,dx =& \frac{1}{2}x^2\text{arctanh}(c+d\tanh(a+bx)) \\ &+ \frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) \\ &- \frac{1}{4}x^2\log\left(1+\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right) \\ &+ \frac{x\,\text{PolyLog}\left(2,-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b} \\ &- \frac{x\,\text{PolyLog}\left(2,-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b} \\ &- \frac{\text{PolyLog}\left(3,-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{8b^2} \\ &+ \frac{\text{PolyLog}\left(3,-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{8b^2}\end{aligned}$$

output
```
1/2*x^2*arctanh(c+d*tanh(b*x+a))+1/4*x^2*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+
d))-1/4*x^2*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x*polylog(2,-(1-c-d)*
exp(2*b*x+2*a)/(1-c+d))/b-1/4*x*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))
/b-1/8*polylog(3,-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/8*polylog(3,-(1+c+
d)*exp(2*b*x+2*a)/(1+c-d))/b^2
```

3.285.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.86

$$\int x\text{arctanh}(c+d\tanh(a+bx))\,dx$$
$$=\frac{4b^2x^2\text{arctanh}(c+d\tanh(a+bx))+2b^2x^2\log\left(1+\frac{(-1+c-d)e^{-2(a+bx)}}{-1+c+d}\right)-2b^2x^2\log\left(1+\frac{(1+c-d)e^{-2(a+bx)}}{1+c+d}\right)-$$

input
```
Integrate[x*ArcTanh[c + d*Tanh[a + b*x]],x]
```

output
```
(4*b^2*x^2*ArcTanh[c + d*Tanh[a + b*x]] + 2*b^2*x^2*Log[1 + (-1 + c - d)/(
(-1 + c + d)*E^(2*(a + b*x)))] - 2*b^2*x^2*Log[1 + (1 + c - d)/((1 + c + d
)*E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, (1 - c + d)/((-1 + c + d)*E^(2*(a +
 b*x)))] + 2*b*x*PolyLog[2, (-1 - c + d)/((1 + c + d)*E^(2*(a + b*x)))] -
PolyLog[3, (1 - c + d)/((-1 + c + d)*E^(2*(a + b*x)))] + PolyLog[3, (-1 -
c + d)/((1 + c + d)*E^(2*(a + b*x)))])/(8*b^2)
```

3.285.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6797, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(d\tanh(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6797}$$

$$\frac{1}{2}b(-c-d+1)\int\frac{e^{2a+2bx}x^2}{-c+(-c-d+1)e^{2a+2bx}+d+1}dx-\frac{1}{2}b(c+d+1)\int\frac{e^{2a+2bx}x^2}{c+(c+d+1)e^{2a+2bx}-d+1}dx+\frac{1}{2}x^2\text{arctanh}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}b(-c-d+1)\left(\frac{x^2\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{\int x\log\left(\frac{e^{2a+2bx}(-c-d+1)}{-c+d+1}+1\right)dx}{b(-c-d+1)}\right)-\frac{1}{2}b(c+d+1)\left(\frac{x^2\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{\int x\log\left(\frac{e^{2a+2bx}(c+d+1)}{c-d+1}+1\right)dx}{b(c+d+1)}\right)+\frac{1}{2}x^2\text{arctanh}(d\tanh(a+bx)+c)$$

$\downarrow$ 3011

$$\frac{1}{2}b(-c-d+1)\left(\frac{x^2\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{\frac{\int \text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}\right)-\frac{1}{2}b(c+d+1)\left(\frac{x^2\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{\frac{\int \text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}\right)+\frac{1}{2}x^2\text{arctanh}(d\tanh(a+bx)+c)$$

$\downarrow$ 2720

$$\frac{1}{2}b(-c-d+1)\left(\frac{x^2\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}\right)-\frac{1}{2}b(c+d+1)\left(\frac{x^2\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}\right)+\frac{1}{2}x^2\text{arctanh}(d\tanh(a+bx)+c)$$

$\downarrow$ 7143

3.285. $\int x\text{arctanh}(c+d\tanh(a+bx))\,dx$

$$\frac{1}{2}x^2\text{arctanh}(d\tanh(a+bx)+c)+\frac{1}{2}b(-c-d+1)\left(\frac{x^2\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{\frac{\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}\right)-\frac{1}{2}b(c+d+1)\left(\frac{x^2\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{\frac{\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}\right)$$

input
```
Int[x*ArcTanh[c + d*Tanh[a + b*x]],x]
```

output
```
(x^2*ArcTanh[c + d*Tanh[a + b*x]])/2 + (b*(1 - c - d)*((x^2*Log[1 + ((1 -
c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(2*b*(1 - c - d)) - (-1/2*(x*PolyLog
[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/b + PolyLog[3, -(((1 -
c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b^2))/(b*(1 - c - d))))/2 - (b*(1
 + c + d)*((x^2*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(2*b*(
1 + c + d)) - (-1/2*(x*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c -
 d))])/b + PolyLog[3, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(4*b^2
))/(b*(1 + c + d))))/2
```

3.285.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6797
```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + (Simp[b*((1 - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x))), x], x] - Simp[b*
((1 + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c -
 d + (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.285.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.25 (sec) , antiderivative size = 5016, normalized size of antiderivative = 21.71

method	result	size
risch	Expression too large to display	5016

input
```
int(x*arctanh(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.285.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(197) = 394$.

Time = 0.28 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.23

$$\int x\operatorname{arctanh}(c + d\tanh(a + bx))\,dx$$
$$= \frac{b^2x^2\log\left(-\frac{(c+1)\cosh(bx+a)+d\sinh(bx+a)}{(c-1)\cosh(bx+a)+d\sinh(bx+a)}\right) - 2\,bx\mathrm{Li}_2\left(\sqrt{-\frac{c+d+1}{c-d+1}}(\cosh(bx+a)+\sinh(bx+a))\right) - 2\,bx\mathrm{Li}_2\left(-\sqrt{}\right.}{}$$

input
```
integrate(x*arctanh(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(b^2*x^2*log(-((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(
b*x + a) + d*sinh(b*x + a))) - 2*b*x*dilog(sqrt(-(c + d + 1)/(c - d + 1))*
(cosh(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt(-(c + d + 1)/(c - d +
 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt(-(c + d - 1)/(c -
 d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt(-(c + d - 1)
/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cos
h(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)
/(c - d + 1))) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(
b*x + a) - 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a^2*log(2*(c +
d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(
c + d - 1)/(c - d - 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d
- 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - (b^2*
x^2 - a^2)*log(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x +
a)) + 1) - (b^2*x^2 - a^2)*log(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x +
 a) + sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(sqrt(-(c + d - 1)/(c - d -
 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt(-(c
+ d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3,
sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*polylo
g(3, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*
polylog(3, sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a...
```

3.285.6 Sympy [F]

$$\int x\text{arctanh}(c+d\tanh(a+bx))\,dx=\int x\,\text{atanh}\,(c+d\tanh\,(a+bx))\,dx$$

input
```
integrate(x*atanh(c+d*tanh(b*x+a)),x)
```

output
```
Integral(x*atanh(c + d*tanh(a + b*x)), x)
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.93

$$\int x\text{arctanh}(c+d\tanh(a+bx))\,dx=$$
$$-\frac{1}{8}\,bd\left(\frac{2\,b^2x^2\log\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+2\,bx\text{Li}_2\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-\text{Li}_3(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1})}{b^3d}-\frac{2\,b^2x^2\log}{}\right.$$
$$+\frac{1}{2}\,x^2\,\text{artanh}\,(d\tanh\,(bx+a)+c)$$

input
```
integrate(x*arctanh(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*b*d*((2*b^2*x^2*log((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 2*
b*x*dilog(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - polylog(3, -(c + d +
 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^3*d) - (2*b^2*x^2*log((c + d - 1)*e^(
2*b*x + 2*a)/(c - d - 1) + 1) + 2*b*x*dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(
c - d - 1)) - polylog(3, -(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^3*d
)) + 1/2*x^2*arctanh(d*tanh(b*x + a) + c)
```

3.285.8 Giac [F]

$$\int x\text{arctanh}(c+d\tanh(a+bx))\,dx=\int x\,\text{artanh}\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arctanh(c+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctanh(d*tanh(b*x + a) + c), x)
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arctanh}(c+d\tanh(a+bx))\,dx=\int x\,\text{atanh}(c+d\tanh(a+b\,x))\,dx$$

input
```
int(x*atanh(c + d*tanh(a + b*x)),x)
```

output
```
int(x*atanh(c + d*tanh(a + b*x)), x)
```

3.286 $\int \operatorname{arctanh}(c + d\tanh(a + bx))\,dx$

3.286.1 Optimal result

Integrand size = 11, antiderivative size = 150

$$\int \operatorname{arctanh}(c+d\tanh(a+bx))\,dx = x\operatorname{arctanh}(c+d\tanh(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log\left(1+\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right) + \frac{\operatorname{PolyLog}\left(2,-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b} - \frac{\operatorname{PolyLog}\left(2,-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b}$$

output
```
x*arctanh(c+d*tanh(b*x+a))+1/2*x*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+d))-1/2*
x*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*polylog(2,-(1-c-d)*exp(2*b*x+2*
a)/(1-c+d))/b-1/4*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b
```

3.286.2 Mathematica [A] (verified)

Time = 3.59 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \operatorname{arctanh}(c+d\tanh(a+bx))\,dx = x\operatorname{arctanh}(c+d\tanh(a+bx)) + \frac{2bx\left(\log\left(1+\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)-\log\left(1+\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)\right)+\operatorname{PolyLog}\left(2,-\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)-\operatorname{PolyLog}}{4b}$$

input
```
Integrate[ArcTanh[c + d*Tanh[a + b*x]],x]
```

output
```
x*ArcTanh[c + d*Tanh[a + b*x]] + (2*b*x*(Log[1 + ((-1 + c + d)*E^(2*(a + b
*x)))/(-1 + c - d)] - Log[1 + ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)])
+ PolyLog[2, -(((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d))] - PolyLog[2,
-(((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d))])/(4*b)
```

3.286.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.38, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6789, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(d\tanh(a+bx)+c)\,dx$$

$$\Big\downarrow \; 6789$$

$$b(-c-d+1)\int \frac{e^{2a+2bx}x}{-c+(-c-d+1)e^{2a+2bx}+d+1}dx - b(c+d+1)\int \frac{e^{2a+2bx}x}{c+(c+d+1)e^{2a+2bx}-d+1}dx + x\operatorname{arctanh}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \; 2620$$

$$b(-c-d+1)\left(\frac{x\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{\int \log\left(\frac{e^{2a+2bx}(-c-d+1)}{-c+d+1}+1\right)dx}{2b(-c-d+1)}\right) - b(c+d+1)\left(\frac{x\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{\int \log\left(\frac{e^{2a+2bx}(c+d+1)}{c-d+1}+1\right)dx}{2b(c+d+1)}\right) + x\operatorname{arctanh}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \ 2715$$

$$b(-c-d+1)\left(\frac{x\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{\int e^{-2a-2bx}\log\left(\frac{e^{2a+2bx}(-c-d+1)}{-c+d+1}+1\right)de^{2a+2bx}}{4b^2(-c-d+1)}\right)-b(c+d+1)\left(\frac{x\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{\int e^{-2a-2bx}\log\left(\frac{e^{2a+2bx}(c+d+1)}{c-d+1}+1\right)de^{2a+2bx}}{4b^2(c+d+1)}\right)+x\text{arctanh}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \ 2838$$

$$x\text{arctanh}(d\tanh(a+bx)+c)+b(-c-d+1)\left(\frac{\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2(-c-d+1)}+\frac{x\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}\right)-b(c+d+1)\left(\frac{\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2(c+d+1)}+\frac{x\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}\right)$$

input
```
Int[ArcTanh[c + d*Tanh[a + b*x]],x]
```

output
```
x*ArcTanh[c + d*Tanh[a + b*x]] + b*(1 - c - d)*((x*Log[1 + ((1 - c - d)*E^
(2*a + 2*b*x))/(1 - c + d)])/(2*b*(1 - c - d)) + PolyLog[2, -(((1 - c - d)
*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b^2*(1 - c - d))) - b*(1 + c + d)*((x*L
og[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(2*b*(1 + c + d)) + Pol
yLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(4*b^2*(1 + c + d)))
```

3.286.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6789
```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Tanh[a + b*x]], x] + (Simp[b*(1 - c - d)   Int[x*(E^(2*a + 2*b*
x)/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x))), x], x] - Simp[b*(1 + c + d)
  Int[x*(E^(2*a + 2*b*x)/(1 + c - d + (1 + c + d)*E^(2*a + 2*b*x))), x], x]
) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

3.286.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(138) = 276$.

Time = 1.89 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.32

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arctanh}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}+\frac{d^2\left(-\frac{\operatorname{dilog}\left(\frac{-d\tanh(bx+a}{-1-c+c}\right.}{2}\right.}{}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arctanh}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}+\frac{d^2\left(-\frac{\operatorname{dilog}\left(\frac{-d\tanh(bx+a}{-1-c+c}\right.}{2}\right.}{}}{}$
risch	Expression too large to display

input
```
int(arctanh(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-1/2*arctanh(c+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)+1/2*arctanh(c+
d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)-d)+1/2*d^2*(1/d*(-1/2*dilog((-d*tanh(b*
x+a)-c-1)/(-1-c+d))-1/2*ln(-d*tanh(b*x+a)-d)*ln((-d*tanh(b*x+a)-c-1)/(-1-c
+d))+1/2*dilog((-d*tanh(b*x+a)-c+1)/(1-c+d))+1/2*ln(-d*tanh(b*x+a)-d)*ln((
-d*tanh(b*x+a)-c+1)/(1-c+d)))-1/d*(1/2*dilog((-d*tanh(b*x+a)-c+1)/(1-c-d))
+1/2*ln(-d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)-c+1)/(1-c-d))-1/2*dilog((-d*t
anh(b*x+a)-c-1)/(-1-c-d))-1/2*ln(-d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)-c-1)
/(-1-c-d)))))
```

3.286.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(128) = 256.

Time = 0.30 (sec) , antiderivative size = 552, normalized size of antiderivative = 3.68

$$\int \operatorname{arctanh}(c+d\tanh(a+bx))\,dx$$
$$= \frac{bx\log\left(-\frac{(c+1)\cosh(bx+a)+d\sinh(bx+a)}{(c-1)\cosh(bx+a)+d\sinh(bx+a)}\right)+a\log\left(2\,(c+d+1)\cosh\,(bx+a)+2\,(c+d+1)\sinh\,(bx+a)+2\,(\right.}{}$$

input
```
integrate(arctanh(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b*x*log(-((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b*x
+ a) + d*sinh(b*x + a))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d +
1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a*log(2
*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*s
qrt(-(c + d + 1)/(c - d + 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c
+ d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - a
*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d
- 1)*sqrt(-(c + d - 1)/(c - d - 1))) - (b*x + a)*log(sqrt(-(c + d + 1)/(c
- d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(-(c +
 d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(
sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x
 + a)*log(-sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))
+ 1) - dilog(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)
)) - dilog(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))
) + dilog(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)))
+ dilog(-sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/
b
```

3.286.6 Sympy [F]

$$\int \operatorname{arctanh}(c+d\tanh(a+bx))\,dx = \int \operatorname{atanh}\left(c+d\tanh\left(a+bx\right)\right)dx$$

input
```
integrate(atanh(c+d*tanh(b*x+a)),x)
```

output
```
Integral(atanh(c + d*tanh(a + b*x)), x)
```

3.286.7 Maxima [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \operatorname{arctanh}(c+d\tanh(a+bx))\,dx = -\frac{1}{4}bd\left(\frac{2\,bx\log\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+\mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^2d} - \frac{2\,bx\log\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+\mathrm{Li}_2\left(-\right.}{b^2d}\right. + x\,\mathrm{artanh}\left(d\tanh\left(bx+a\right)+c\right)$$

input
```
integrate(arctanh(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/4*b*d*((2*b*x*log((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + dilog(
-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^2*d) - (2*b*x*log((c + d - 1
)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(c
 - d - 1)))/(b^2*d)) + x*arctanh(d*tanh(b*x + a) + c)
```

3.286.8 Giac [F]

$$\int \operatorname{arctanh}(c+d\tanh(a+bx))\,dx = \int \mathrm{artanh}\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arctanh(c+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctanh(d*tanh(b*x + a) + c), x)
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}(c+d\tanh(a+bx))\,dx = \int \mathrm{atanh}(c+d\tanh(a+b\,x))\,dx$$

input
```
int(atanh(c + d*tanh(a + b*x)),x)
```

output
```
int(atanh(c + d*tanh(a + b*x)), x)
```

3.287 $\int \frac{\operatorname{arctanh}(c+d\tanh(a+bx))}{x}\,dx$

3.287.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\operatorname{arctanh}(c+d\tanh(a+bx))}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{arctanh}(c+d\tanh(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arctanh(c+d*tanh(b*x+a))/x,x)
```

3.287.2 Mathematica [N/A]

Not integrable

Time = 10.64 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arctanh}(c+d\tanh(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTanh[c + d*Tanh[a + b*x]]/x,x]
```

output
```
Integrate[ArcTanh[c + d*Tanh[a + b*x]]/x, x]
```

3.287.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(d\tanh(a+bx)+c)}{x}\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\text{arctanh}(d\tanh(a+bx)+c)}{x}dx$$

input
```
Int[ArcTanh[c + d*Tanh[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.287.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.287.4 Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\text{arctanh}\left(c+d\tanh\left(bx+a\right)\right)}{x}dx$$

input
```
int(arctanh(c+d*tanh(b*x+a))/x,x)
```

output
```
int(arctanh(c+d*tanh(b*x+a))/x,x)
```

3.287.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c+d\tanh(a+bx))}{x}\,dx=\int \frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctanh(c+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arctanh(d*tanh(b*x + a) + c)/x, x)
```

3.287.6 Sympy [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arctanh}(c+d\tanh(a+bx))}{x}\,dx=\int \frac{\operatorname{atanh}\left(c+d\tanh\left(a+bx\right)\right)}{x}\,dx$$

input
```
integrate(atanh(c+d*tanh(b*x+a))/x,x)
```

output
```
Integral(atanh(c + d*tanh(a + b*x))/x, x)
```

3.287.7 Maxima [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c+d\tanh(a+bx))}{x}\,dx=\int \frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctanh(c+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arctanh(d*tanh(b*x + a) + c)/x, x)
```

3.287.8 Giac [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c + d\tanh(a + bx))}{x}\,dx = \int \frac{\operatorname{artanh}\left(d\tanh\left(bx + a\right) + c\right)}{x}\,dx$$

input
```
integrate(arctanh(c+d*tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctanh(d*tanh(b*x + a) + c)/x, x)
```

3.287.9 Mupad [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c + d\tanh(a + bx))}{x}\,dx = \int \frac{\operatorname{atanh}(c + d\tanh(a + b\,x))}{x}\,dx$$

input
```
int(atanh(c + d*tanh(a + b*x))/x,x)
```

output
```
int(atanh(c + d*tanh(a + b*x))/x, x)
```

3.288 $\int x^3 \textbf{arctanh}(1+d+d\tanh(a+bx))\,dx$

3.288.1 Optimal result

Integrand size = 16, antiderivative size = 155

$$\begin{aligned}\int x^3\text{arctanh}(1+d+d\tanh(a+bx))\,dx =& \frac{bx^5}{20}+\frac{1}{4}x^4\text{arctanh}(1+d+d\tanh(a+bx)) \\ &-\frac{1}{8}x^4\log\left(1+(1+d)e^{2a+2bx}\right) \\ &-\frac{x^3\,\text{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b} \\ &+\frac{3x^2\,\text{PolyLog}\left(3,-\left((1+d)e^{2a+2bx}\right)\right)}{8b^2} \\ &-\frac{3x\,\text{PolyLog}\left(4,-\left((1+d)e^{2a+2bx}\right)\right)}{8b^3} \\ &+\frac{3\,\text{PolyLog}\left(5,-\left((1+d)e^{2a+2bx}\right)\right)}{16b^4}\end{aligned}$$

output
```
1/20*b*x^5+1/4*x^4*arctanh(1+d+d*tanh(b*x+a))-1/8*x^4*ln(1+(1+d)*exp(2*b*x
+2*a))-1/4*x^3*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,-(1+d)
*exp(2*b*x+2*a))/b^2-3/8*x*polylog(4,-(1+d)*exp(2*b*x+2*a))/b^3+3/16*polyl
og(5,-(1+d)*exp(2*b*x+2*a))/b^4
```

3.288.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int x^3\text{arctanh}(1+d+d\tanh(a+bx))\,dx$$
$$=\frac{4b^4x^4\text{arctanh}(1+d+d\tanh(a+bx))-2b^4x^4\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right)+4b^3x^3\,\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right)+6b^2x}{16b^4}$$

input
```
Integrate[x^3*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

output
```
(4*b^4*x^4*ArcTanh[1 + d + d*Tanh[a + b*x]] - 2*b^4*x^4*Log[1 + 1/((1 + d)
*E^(2*(a + b*x)))] + 4*b^3*x^3*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))]
+ 6*b^2*x^2*PolyLog[3, -(1/((1 + d)*E^(2*(a + b*x))))] + 6*b*x*PolyLog[4,
-(1/((1 + d)*E^(2*(a + b*x))))] + 3*PolyLog[5, -(1/((1 + d)*E^(2*(a + b*x)
)))])/(16*b^4)
```

3.288.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6793, 2615, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{arctanh}(d\tanh(a+bx)+d+1)\,dx$$
$$\Big\downarrow \text{ 6793}$$
$$\frac{1}{4}b\int\frac{x^4}{e^{2a+2bx}(d+1)+1}dx+\frac{1}{4}x^4\text{arctanh}(d\tanh(a+bx)+d+1)$$
$$\Big\downarrow \text{ 2615}$$
$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\int\frac{e^{2a+2bx}x^4}{e^{2a+2bx}(d+1)+1}dx\right)+\frac{1}{4}x^4\text{arctanh}(d\tanh(a+bx)+d+1)$$
$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\int x^3\log\left(e^{2a+2bx}(d+1)+1\right)dx}{b(d+1)}\right)\right)+\frac{1}{4}x^4\text{arctanh}(d\tanh(a+bx)+d+1)$$

$\downarrow$ 3011

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\left(\frac{3\int x^2\,\text{PolyLog}(2,-((d+1)e^{2a+2bx}))dx}{2b}-\frac{x^3\,\text{PolyLog}(2,-((d+1)e^{2a+2bx}))}{2b}\right.}{b(d+1)}\right.\right.$$
$$\frac{1}{4}x^4\text{arctanh}(d\tanh(a+bx)+d+1)$$

$\downarrow$ 7163

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\int x\,\text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)dx}{b}\right)}{2b}-\right.}{b(d+1)}\right.\right.$$
$$\frac{1}{4}x^4\text{arctanh}(d\tanh(a+bx)+d+1)$$

$\downarrow$ 7163

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\,\text{PolyLog}\left(4,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\int \text{PolyLo}}{}}{b}\right.}{2b}\right.}{b(d+1)}\right.\right.$$
$$\frac{1}{4}x^4\text{arctanh}(d\tanh(a+bx)+d+1)$$

$\downarrow$ 2720

3.288. $\int x^3\text{arctanh}(1+d+d\tanh(a+bx))\,dx$

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\int e^{-2a-}}{}}{}\right)}{2b}}{b(d}\right.\right.$$

$$\frac{1}{4}x^4\text{arctanh}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{1}{4}x^4\text{arctanh}(d\tanh(a+bx)+d+1)+$$

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\operatorname{PolyLog}}{}}{b}\right)}{2b}}{b(d+1)}\right.\right.$$

input
```
Int[x^3*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

output
```
(x^4*ArcTanh[1 + d + d*Tanh[a + b*x]])/4 + (b*(x^5/5 - (1 + d)*((x^4*Log[1
 + (1 + d)*E^(2*a + 2*b*x)])/(2*b*(1 + d)) - (2*(-1/2*(x^3*PolyLog[2, -((1
 + d)*E^(2*a + 2*b*x))])/b + (3*((x^2*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x)
)])/(2*b) - ((x*PolyLog[4, -((1 + d)*E^(2*a + 2*b*x))])/(2*b) - PolyLog[5,
 -((1 + d)*E^(2*a + 2*b*x))]/(4*b^2))/b))/(2*b)))/(b*(1 + d)))))/4
```

3.288. $\int x^3\text{arctanh}(1+d+d\tanh(a+bx))\,dx$

3.288.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6793
```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.288. $\int x^3\text{arctanh}(1+d+d\tanh(a+bx))\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.288.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.39 (sec) , antiderivative size = 1721, normalized size of antiderivative = 11.10

method	result	size
risch	Expression too large to display	1721

input
```
int(x^3*arctanh(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^3*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x*a^3+1/2/b^3*d*a^3/(1+d)*ln(1
+exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/
2))*x-3/8/b^4*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^4+3/8/b^2*d/(1+d)*polyl
og(3,-(1+d)*exp(2*b*x+2*a))*x^2-1/4/b^4*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x
+2*a))*a^3-3/8/b^3*d/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2*a))*x+1/2/b^3*a^3/
(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(-
d-1)^(1/2))*x+1/2/b^4*d*a^4/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^4*d*
a^4/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))+1/2/b^4*d*a^3/(1+d)*dilog(1+exp(b*
x+a)*(-d-1)^(1/2))+1/2/b^4*d*a^3/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))+1/
20*b*x^5-1/4/b/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^3-3/8/b^4/(1+d)*ln
(1+(1+d)*exp(2*b*x+2*a))*a^4+3/8/b^2/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a)
)*x^2-1/4/b^4/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a^3-3/8/b^3/(1+d)*pol
ylog(4,-(1+d)*exp(2*b*x+2*a))*x+3/16/b^4*d/(1+d)*polylog(5,-(1+d)*exp(2*b*
x+2*a))+1/2/b^4*a^4/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^4*a^4/(1+d)*
ln(1-exp(b*x+a)*(-d-1)^(1/2))+1/2/b^4*a^3/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^
(1/2))+1/2/b^4*a^3/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))-1/8*d/(1+d)*ln(1
+(1+d)*exp(2*b*x+2*a))*x^4-1/8/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^4+3/16/b
^4/(1+d)*polylog(5,-(1+d)*exp(2*b*x+2*a))-1/2/b^3/(1+d)*ln(1+(1+d)*exp(2*b
*x+2*a))*x*a^3+1/8*x^4*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)-1/4*x^4*ln(ex
p(b*x+a))-1/16*(-I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+I*Pi*...
```

3.288.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(135) = 270$.

Time = 0.27 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.91

$$\int x^3 \text{arctanh}(1+d+d\tanh(a+bx))\,dx$$
$$= \frac{2\,b^5x^5 + 5\,b^4x^4\log\left(-\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 20\,b^3x^3\text{Li}_2\left(\frac{1}{2}\sqrt{-4\,d-4}(\cosh(bx+a)+\sinh(bx+a))\right.}{}$$

input
```
integrate(x^3*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/40*(2*b^5*x^5 + 5*b^4*x^4*log(-((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))
/(d*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(1/2*sqrt(-4*d - 4
)*(cosh(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-1/2*sqrt(-4*d - 4)*
(cosh(b*x + a) + sinh(b*x + a))) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(
d + 1)*sinh(b*x + a) + sqrt(-4*d - 4)) - 5*a^4*log(2*(d + 1)*cosh(b*x + a)
 + 2*(d + 1)*sinh(b*x + a) - sqrt(-4*d - 4)) + 60*b^2*x^2*polylog(3, 1/2*s
qrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -1/
2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, 1/2
*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, -1/2
*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(b^4*x^4 - a^4)*log(1
/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^4*x^4 - a^4)
*log(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*polylo
g(5, 1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5,
-1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

3.288.6 Sympy [F]

$$\int x^3 \text{arctanh}(1+d+d\tanh(a+bx))\,dx = \int x^3 \operatorname{atanh}(d\tanh(a+bx)+d+1)\,dx$$

input
```
integrate(x**3*atanh(1+d+d*tanh(b*x+a)),x)
```

output
```
Integral(x**3*atanh(d*tanh(a + b*x) + d + 1), x)
```

3.288.7 Maxima [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

$$\int x^3\text{arctanh}(1+d+d\tanh(a+bx))\,dx = \frac{1}{4}x^4\,\text{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)$$
$$+\frac{1}{40}\left(\frac{2x^5}{d}-\frac{5\left(2b^4x^4\log\left((d+1)e^{(2bx+2a)}+1\right)+4b^3x^3\text{Li}_2\left(-(d+1)e^{(2bx+2a)}\right)-6b^2x^2\text{Li}_3(-(d+1)e^{(2\right.}{b^5d}\right.$$

input
```
integrate(x^3*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*x^4*arctanh(d*tanh(b*x + a) + d + 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*lo
g((d + 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog(-(d + 1)*e^(2*b*x + 2*a))
 - 6*b^2*x^2*polylog(3, -(d + 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, -(d +
 1)*e^(2*b*x + 2*a)) - 3*polylog(5, -(d + 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*
d
```

3.288.8 Giac [F]

$$\int x^3\text{arctanh}(1+d+d\tanh(a+bx))\,dx = \int x^3\,\text{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(x^3*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^3*arctanh(d*tanh(b*x + a) + d + 1), x)
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int x^3\text{arctanh}(1+d+d\tanh(a+bx))\,dx = \int x^3\,\text{atanh}(d+d\tanh(a+b\,x)+1)\,dx$$

input
```
int(x^3*atanh(d + d*tanh(a + b*x) + 1),x)
```

output
```
int(x^3*atanh(d + d*tanh(a + b*x) + 1), x)
```

3.289 $\int x^2 \mathbf{arctanh}(1+d+d\tanh(a+bx))\,dx$

3.289.1 Optimal result

Integrand size = 16, antiderivative size = 128

$$\begin{aligned}\int x^2\text{arctanh}(1+d+d\tanh(a+bx))\,dx =& \frac{bx^4}{12}+\frac{1}{3}x^3\text{arctanh}(1+d+d\tanh(a+bx)) \\ &-\frac{1}{6}x^3\log\left(1+(1+d)e^{2a+2bx}\right) \\ &-\frac{x^2\,\text{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b} \\ &+\frac{x\,\text{PolyLog}\left(3,-\left((1+d)e^{2a+2bx}\right)\right)}{4b^2} \\ &-\frac{\text{PolyLog}\left(4,-\left((1+d)e^{2a+2bx}\right)\right)}{8b^3}\end{aligned}$$

output
```
1/12*b*x^4+1/3*x^3*arctanh(1+d+d*tanh(b*x+a))-1/6*x^3*ln(1+(1+d)*exp(2*b*x
+2*a))-1/4*x^2*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,-(1+d)*e
xp(2*b*x+2*a))/b^2-1/8*polylog(4,-(1+d)*exp(2*b*x+2*a))/b^3
```

3.289.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int x^2\text{arctanh}(1+d+d\tanh(a+bx))\,dx$$
$$=\frac{8b^3x^3\text{arctanh}(1+d+d\tanh(a+bx))-4b^3x^3\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right)+6b^2x^2\,\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right)+6bx\,\text{I}}{24b^3}$$

input
```
Integrate[x^2*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcTanh[1 + d + d*Tanh[a + b*x]] - 4*b^3*x^3*Log[1 + 1/((1 + d)
*E^(2*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))]
+ 6*b*x*PolyLog[3, -(1/((1 + d)*E^(2*(a + b*x))))] + 3*PolyLog[4, -(1/((1
+ d)*E^(2*(a + b*x))))])/(24*b^3)
```

3.289.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6793, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arctanh}(d\tanh(a+bx)+d+1)\,dx$$
$$\Big\downarrow \text{6793}$$
$$\frac{1}{3}b\int\frac{x^3}{e^{2a+2bx}(d+1)+1}dx+\frac{1}{3}x^3\text{arctanh}(d\tanh(a+bx)+d+1)$$
$$\Big\downarrow \text{2615}$$
$$\frac{1}{3}b\left(\frac{x^4}{4}-(d+1)\int\frac{e^{2a+2bx}x^3}{e^{2a+2bx}(d+1)+1}dx\right)+\frac{1}{3}x^3\text{arctanh}(d\tanh(a+bx)+d+1)$$
$$\Big\downarrow \text{2620}$$
$$\frac{1}{3}b\left(\frac{x^4}{4}-(d+1)\left(\frac{x^3\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{3\int x^2\log\left(e^{2a+2bx}(d+1)+1\right)dx}{2b(d+1)}\right)\right)+$$
$$\frac{1}{3}x^3\text{arctanh}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}b\left(\frac{x^4}{4}-(d+1)\left(\frac{x^3\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-\left((d+1)e^{2a+2bx}\right)\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}\right)}{2b(d+1)}\right)\right.$$

$$\frac{1}{3}x^3\text{arctanh}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3}b\left(\frac{x^4}{4}-(d+1)\left(\frac{x^3\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)dx}{2b}}{b}-\frac{x^2\,\text{Poly}\ldots}{\ldots}\right.}{2b(d+1)}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3}b\left(\frac{x^4}{4}-(d+1)\left(\frac{x^3\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)de^{2a-\ldots}}{4b^2}}{b}\ldots\right.}{2b(d+1)}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{3}x^3\text{arctanh}(d\tanh(a+bx)+d+1)+$$

$$\frac{1}{3}b\left(\frac{x^4}{4}-(d+1)\left(\frac{x^3\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\text{PolyLog}\left(4,-\left((d+1)e^{2a+2bx}\right)\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\ldots}{\ldots}\right.}{2b(d+1)}\right.\right.$$

input
```
Int[x^2*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

3.289. $\int x^2\text{arctanh}(1+d+d\tanh(a+bx))\,dx$

output
```
(x^3*ArcTanh[1 + d + d*Tanh[a + b*x]])/3 + (b*(x^4/4 - (1 + d)*((x^3*Log[1
 + (1 + d)*E^(2*a + 2*b*x)])/(2*b*(1 + d)) - (3*(-1/2*(x^2*PolyLog[2, -((1
 + d)*E^(2*a + 2*b*x))])/b + ((x*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(
2*b) - PolyLog[4, -((1 + d)*E^(2*a + 2*b*x))]/(4*b^2))/b))/(2*b*(1 + d))))
)/3
```

3.289.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6793
```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.289.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.06 (sec) , antiderivative size = 1662, normalized size of antiderivative = 12.98

method	result	size
risch	Expression too large to display	1662

input
```
int(x^2*arctanh(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^3*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)+1/12*b*x^4-1/3*x^3*ln(exp(b*
x+a))-1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*a^2/(1+d)*di
log(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*a^2/(1+d)*dilog(1-exp(b*x+a)*(-d-1)
^(1/2))-1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))-1/4/b/(1+d)*polylo
g(2,-(1+d)*exp(2*b*x+2*a))*x^2+1/3/b^3/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^
3+1/4/b^3/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2/(1+d)*polylog
(3,-(1+d)*exp(2*b*x+2*a))*x-1/8/b^3*d/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2*a
))+1/6/b^3*a^3/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)-1/6*d/(1+d)*ln(
1+(1+d)*exp(2*b*x+2*a))*x^3-1/2/b^2*a^2/(1+d)*x*ln(1+exp(b*x+a)*(-d-1)^(1/
2))-1/2/b^2*a^2/(1+d)*x*ln(1-exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2/(1+d)*ln(1+(
1+d)*exp(2*b*x+2*a))*a^2*x-1/4/b*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*
x^2+1/3/b^3*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^3+1/4/b^3*d/(1+d)*polylog
(2,-(1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2
*a))*x+1/6/b^3*d*a^3/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)-1/2/b^3*a
^3*d/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*a^3*d/(1+d)*ln(1-exp(b*x+
a)*(-d-1)^(1/2))-1/2/b^3*a^2*d/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/
b^3*a^2*d/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*d/(1+d)*ln(1+(1+d
)*exp(2*b*x+2*a))*a^2*x-1/2/b^2*a^2*d/(1+d)*x*ln(1+exp(b*x+a)*(-d-1)^(1/2)
)-1/2/b^2*a^2*d/(1+d)*x*ln(1-exp(b*x+a)*(-d-1)^(1/2))-1/6/(1+d)*ln(1+(1+d)
*exp(2*b*x+2*a))*x^3-1/8/b^3/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2*a))-1/1...
```

3.289.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(111) = 222$.

Time = 0.26 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.98

$$\int x^2 \operatorname{arctanh}(1+d+d\tanh(a+bx))\,dx$$

$$= \frac{b^4x^4 + 2\,b^3x^3\log\left(-\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2\operatorname{Li}_2\left(\frac{1}{2}\,\sqrt{-4\,d-4}(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right) -}{}$$

input
```
integrate(x^2*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(b^4*x^4 + 2*b^3*x^3*log(-((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(
d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d - 4)*(
cosh(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d - 4)*(cos
h(b*x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d +
1)*sinh(b*x + a) + sqrt(-4*d - 4)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2
*(d + 1)*sinh(b*x + a) - sqrt(-4*d - 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d
 - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d
- 4)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*
d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*s
qrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqr
t(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*
d - 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

3.289.6 Sympy [F]

$$\int x^2 \operatorname{arctanh}(1+d+d\tanh(a+bx))\,dx = \int x^2 \operatorname{atanh}(d\tanh(a+bx)+d+1)\,dx$$

input
```
integrate(x**2*atanh(1+d+d*tanh(b*x+a)),x)
```

output
```
Integral(x**2*atanh(d*tanh(a + b*x) + d + 1), x)
```

3.289.7 Maxima [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

$$\int x^2 \operatorname{arctanh}(1+d+d\tanh(a+bx))\,dx = \frac{1}{3}x^3 \operatorname{artanh}(d\tanh(bx+a)+d+1) + \frac{1}{36}\left(\frac{3x^4}{d} - \frac{2\left(4b^3x^3\log\left((d+1)e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(-(d+1)e^{(2bx+2a)}\right)-6bx\mathrm{Li}_3\left(-(d+1)e^{(2bx}\right.\right.}{b^4d}\right.$$

input
```
integrate(x^2*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctanh(d*tanh(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*lo
g((d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-(d + 1)*e^(2*b*x + 2*a))
 - 6*b*x*polylog(3, -(d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, -(d + 1)*e^(2
*b*x + 2*a)))/(b^4*d))*b*d
```

3.289.8 Giac [F]

$$\int x^2 \mathrm{arctanh}(1+d+d\tanh(a+bx))\,dx = \int x^2\,\mathrm{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(x^2*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(d*tanh(b*x + a) + d + 1), x)
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \mathrm{arctanh}(1+d+d\tanh(a+bx))\,dx = \int x^2\,\mathrm{atanh}(d+d\tanh(a+b\,x)+1)\,dx$$

input
```
int(x^2*atanh(d + d*tanh(a + b*x) + 1),x)
```

output
```
int(x^2*atanh(d + d*tanh(a + b*x) + 1), x)
```

3.290 $\int x\text{arctanh}(1+d+d\tanh(a+bx))\,dx$

3.290.1 Optimal result

Integrand size = 14, antiderivative size = 101

$$\int x\text{arctanh}(1+d+d\tanh(a+bx))\,dx = \frac{bx^3}{6}+\frac{1}{2}x^2\text{arctanh}(1+d+d\tanh(a+bx)) - \frac{1}{4}x^2\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{x\,\text{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b} + \frac{\text{PolyLog}\left(3,-\left((1+d)e^{2a+2bx}\right)\right)}{8b^2}$$

output
```
1/6*b*x^3+1/2*x^2*arctanh(1+d+d*tanh(b*x+a))-1/4*x^2*ln(1+(1+d)*exp(2*b*x+
2*a))-1/4*x*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+1/8*polylog(3,-(1+d)*exp(2*
b*x+2*a))/b^2
```

3.290.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int x\text{arctanh}(1+d+d\tanh(a+bx))\,dx = \frac{2b^2x^2\left(2\text{arctanh}(1+d+d\tanh(a+bx))-\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right)\right)+2bx\,\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right)+\text{PolyLog}}{8b^2}$$

input
```
Integrate[x*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcTanh[1 + d + d*Tanh[a + b*x]] - Log[1 + 1/((1 + d)*E^(2*(
a + b*x)))]) + 2*b*x*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))] + PolyLog[
3, -(1/((1 + d)*E^(2*(a + b*x))))])/(8*b^2)
```

3.290.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6793, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(d\tanh(a+bx)+d+1)\,dx$$

$$\Big\downarrow \text{ 6793}$$

$$\frac{1}{2}b\int \frac{x^2}{e^{2a+2bx}(d+1)+1}dx+\frac{1}{2}x^2\text{arctanh}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(d+1)\int \frac{e^{2a+2bx}x^2}{e^{2a+2bx}(d+1)+1}dx\right)+\frac{1}{2}x^2\text{arctanh}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(d+1)\left(\frac{x^2\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\int x\log\left(e^{2a+2bx}(d+1)+1\right)dx}{b(d+1)}\right)\right)+\frac{1}{2}x^2\text{arctanh}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(d+1)\left(\frac{x^2\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\frac{\int \text{PolyLog}\left(2,-\left((d+1)e^{2a+2bx}\right)\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}}{b(d+1)}\right)\right)+\frac{1}{2}x^2\text{arctanh}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(d+1)\left(\frac{x^2\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\frac{\int e^{-2a-2bx}\operatorname{PolyLog}(2,-((d+1)e^{2a+2bx}))de^{2a+2bx}}{4b^2}-\frac{x\operatorname{PolyLog}(2,-((d+1)e^{2a}}{2b}}{b(d+1)}\right.\right.$$
$$\frac{1}{2}x^2\text{arctanh}(d\tanh(a+bx)+d+1)$$

$\downarrow$ 7143

$$\frac{1}{2}x^2\text{arctanh}(d\tanh(a+bx)+d+1)+$$
$$\frac{1}{2}b\left(\frac{x^3}{3}-(d+1)\left(\frac{x^2\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\frac{\operatorname{PolyLog}(3,-((d+1)e^{2a+2bx}))}{4b^2}-\frac{x\operatorname{PolyLog}(2,-((d+1)e^{2a+2bx}))}{2b}}{b(d+1)}\right)\right)$$

input
```
Int[x*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

output
```
(x^2*ArcTanh[1 + d + d*Tanh[a + b*x]])/2 + (b*(x^3/3 - (1 + d)*((x^2*Log[1
 + (1 + d)*E^(2*a + 2*b*x)])/(2*b*(1 + d)) - (-1/2*(x*PolyLog[2, -((1 + d)
*E^(2*a + 2*b*x))])/b + PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))]/(4*b^2))/(b
*(1 + d)))))/2
```

3.290.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6793
```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.290.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.86 (sec) , antiderivative size = 1579, normalized size of antiderivative = 15.63

method	result	size
risch	Expression too large to display	1579

input
```
int(x*arctanh(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2*ln(exp(b*x+a))-1/4/b^2*a^2*d/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+
2*a)+1)+1/2/b^2*a^2*d/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*a^2*d/(1
+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*a*d/(1+d)*dilog(1+exp(b*x+a)*(-d
-1)^(1/2))+1/2/b^2*a*d/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))-1/2/b/(1+d)*
ln(1+(1+d)*exp(2*b*x+2*a))*a*x-1/4/b^2*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*
a^2+1/6*b*x^3-1/2/b*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a*x+1/2/b*a*d/(1+d)
*x*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b*a*d/(1+d)*x*ln(1-exp(b*x+a)*(-d-1)^
(1/2))-1/4*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^2-1/4/b^2*a^2/(1+d)*ln(d*e
xp(2*b*x+2*a)+exp(2*b*x+2*a)+1)-1/4/b^2/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a
^2-1/4/b/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x-1/4/b^2/(1+d)*polylog(2,
-(1+d)*exp(2*b*x+2*a))*a+1/8/b^2*d/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))+
1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*a^2/(1+d)*ln(1-exp
(b*x+a)*(-d-1)^(1/2))+1/2/b^2*a/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))+1/2
/b^2*a/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))-1/4/(1+d)*ln(1+(1+d)*exp(2*b
*x+2*a))*x^2+1/8/b^2/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))-1/4/b*d/(1+d)*
polylog(2,-(1+d)*exp(2*b*x+2*a))*x-1/4/b^2*d/(1+d)*polylog(2,-(1+d)*exp(2*
b*x+2*a))*a+1/2/b*a/(1+d)*x*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b*a/(1+d)*x*
ln(1-exp(b*x+a)*(-d-1)^(1/2))+1/4*x^2*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1
)-1/8*(-I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+I*Pi*csgn(I/(exp(
2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*I*Pi*csgn(...
```

3.290.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(87) = 174$.

Time = 0.27 (sec) , antiderivative size = 323, normalized size of antiderivative = 3.20

$$\int x\text{arctanh}(1 + d + d\tanh(a + bx))\,dx$$

$$= \frac{2\,b^3x^3 + 3\,b^2x^2\log\left(-\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d-4}(\cosh(bx+a) + \sinh(bx+a))\right) -}{}$$

input
```
integrate(x*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*x^3 + 3*b^2*x^2*log(-((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))
/(d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(1/2*sqrt(-4*d - 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 6*b*x*dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*x
+ a) + sinh(b*x + a))) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sin
h(b*x + a) + sqrt(-4*d - 4)) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d +
1)*sinh(b*x + a) - sqrt(-4*d - 4)) - 3*(b^2*x^2 - a^2)*log(1/2*sqrt(-4*d -
 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*sqrt
(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*polylog(3, 1/2*sqrt(-4
*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(3, -1/2*sqrt(-4*d - 4
)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

3.290.6 Sympy [F]

$$\int x\text{arctanh}(1+d+d\tanh(a+bx))\,dx=\int x\,\text{atanh}\left(d\tanh\left(a+bx\right)+d+1\right)dx$$

input
```
integrate(x*atanh(1+d+d*tanh(b*x+a)),x)
```

output
```
Integral(x*atanh(d*tanh(a + b*x) + d + 1), x)
```

3.290.7 Maxima [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int x\text{arctanh}(1+d+d\tanh(a+bx))\,dx\\&=\frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left((d+1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\text{Li}_2\left(-(d+1)e^{(2\,bx+2\,a)}\right)-\text{Li}_3(-(d+1)e^{(2\,bx+2\,a)})\right)}{b^3d}\right)\\&\quad+\frac{1}{2}\,x^2\,\text{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)\end{aligned}$$

input
```
integrate(x*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(4*x^3/d - 3*(2*b^2*x^2*log((d + 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g(-(d + 1)*e^(2*b*x + 2*a)) - polylog(3, -(d + 1)*e^(2*b*x + 2*a)))/(b^3*d
))*b*d + 1/2*x^2*arctanh(d*tanh(b*x + a) + d + 1)
```

3.290.8 Giac [F]

$$\int x\text{arctanh}(1+d+d\tanh(a+bx))\,dx=\int x\,\text{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(x*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctanh(d*tanh(b*x + a) + d + 1), x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arctanh}(1+d+d\tanh(a+bx))\,dx=\int x\,\text{atanh}(d+d\tanh(a+b\,x)+1)\,dx$$

input
```
int(x*atanh(d + d*tanh(a + b*x) + 1),x)
```

output
```
int(x*atanh(d + d*tanh(a + b*x) + 1), x)
```

3.291 $\int \operatorname{arctanh}(1+d+d\tanh(a+bx))\,dx$

3.291.1 Optimal result

Integrand size = 12, antiderivative size = 69

$$\int \operatorname{arctanh}(1+d+d\tanh(a+bx))\,dx=\frac{bx^2}{2}+x\operatorname{arctanh}(1+d+d\tanh(a+bx))-\frac{1}{2}x\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{\operatorname{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b}$$

output
```
1/2*b*x^2+x*arctanh(1+d+d*tanh(b*x+a))-1/2*x*ln(1+(1+d)*exp(2*b*x+2*a))-1/
4*polylog(2,-(1+d)*exp(2*b*x+2*a))/b
```

3.291.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \operatorname{arctanh}(1+d+d\tanh(a+bx))\,dx=x\operatorname{arctanh}(1+d+d\tanh(a+bx))+\frac{-2bx\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right)+\operatorname{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right)}{4b}$$

input
```
Integrate[ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

output
```
x*ArcTanh[1 + d + d*Tanh[a + b*x]] + (-2*b*x*Log[1 + 1/((1 + d)*E^(2*(a +
b*x)))] + PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))])/(4*b)
```

3.291.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6785, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(d\tanh(a+bx)+d+1)\,dx$$

$$\Big\downarrow \text{ 6785}$$

$$b\int \frac{x}{e^{2a+2bx}(d+1)+1}dx+x\operatorname{arctanh}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{ 2615}$$

$$b\left(\frac{x^2}{2}-(d+1)\int \frac{e^{2a+2bx}x}{e^{2a+2bx}(d+1)+1}dx\right)+x\operatorname{arctanh}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{ 2620}$$

$$b\left(\frac{x^2}{2}-(d+1)\left(\frac{x\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\int \log\left(e^{2a+2bx}(d+1)+1\right)dx}{2b(d+1)}\right)\right)+ x\operatorname{arctanh}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{ 2715}$$

$$b\left(\frac{x^2}{2}-(d+1)\left(\frac{x\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\int e^{-2a-2bx}\log\left(e^{2a+2bx}(d+1)+1\right)de^{2a+2bx}}{4b^2(d+1)}\right)\right)+ x\operatorname{arctanh}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{ 2838}$$

$$x\operatorname{arctanh}(d\tanh(a+bx)+d+1)+ b\left(\frac{x^2}{2}-(d+1)\left(\frac{\operatorname{PolyLog}\left(2,-\left((d+1)e^{2a+2bx}\right)\right)}{4b^2(d+1)}+\frac{x\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}\right)\right)$$

input
```
Int[ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

output
```
x*ArcTanh[1 + d + d*Tanh[a + b*x]] + b*(x^2/2 - (1 + d)*((x*Log[1 + (1 + d
)*E^(2*a + 2*b*x)])/(2*b*(1 + d)) + PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))]
/(4*b^2*(1 + d))))
```

3.291.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6785
```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Tanh[a + b*x]], x] + Simp[b   Int[x/(c - d + c*E^(2*a + 2*b*x))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

3.291.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(61) = 122$.

Time = 0.85 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.70

method	result
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(1+d+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arctanh}(1+d+d\tanh(bx+a))d\ln(d+d\tanh(bx+a))}{2}-\frac{d^2\left(-\frac{\operatorname{dilog}\left(\frac{-d\tanh(b}{-2}\right.}{2}\right.}{}}{}$
default	$\frac{-\frac{\operatorname{arctanh}(1+d+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arctanh}(1+d+d\tanh(bx+a))d\ln(d+d\tanh(bx+a))}{2}-\frac{d^2\left(-\frac{\operatorname{dilog}\left(\frac{-d\tanh(b}{-2}\right.}{2}\right.}{}}{}$
risch	Expression too large to display

input
```
int(arctanh(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-1/2*arctanh(1+d+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)+1/2*arctanh(
1+d+d*tanh(b*x+a))*d*ln(d+d*tanh(b*x+a))-1/2*d^2*(1/d*(-1/2*dilog((-d*tanh
(b*x+a)-d-2)/(-2*d-2))-1/2*ln(-d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)-d-2)/(-
2*d-2))+1/2*dilog(-1/2*(-d*tanh(b*x+a)-d)/d)+1/2*ln(-d*tanh(b*x+a)+d)*ln(-
1/2*(-d*tanh(b*x+a)-d)/d))-1/d*(-1/2*dilog(1/2*d*tanh(b*x+a)+1/2*d+1)-1/2*
ln(d+d*tanh(b*x+a))*ln(1/2*d*tanh(b*x+a)+1/2*d+1)+1/4*ln(d+d*tanh(b*x+a))^
2)))
```

3.291.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(60) = 120$.

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.46

$$\int \operatorname{arctanh}(1+d+d\tanh(a+bx))\,dx$$

$$= \frac{b^2x^2 + bx\log\left(-\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) + a\log\left(2\,(d+1)\cosh\left(bx+a\right) + 2\,(d+1)\sinh\left(bx+a\right) + \sqrt{-}\right.}{}$$

input
```
integrate(arctanh(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 + b*x*log(-((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(
b*x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*s
inh(b*x + a) + sqrt(-4*d - 4)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)
*sinh(b*x + a) - sqrt(-4*d - 4)) - (b*x + a)*log(1/2*sqrt(-4*d - 4)*(cosh(
b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-1/2*sqrt(-4*d - 4)*(cosh(b
*x + a) + sinh(b*x + a)) + 1) - dilog(1/2*sqrt(-4*d - 4)*(cosh(b*x + a) +
sinh(b*x + a))) - dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)
)))/b
```

3.291.6 Sympy [F]

$$\int \operatorname{arctanh}(1+d+d\tanh(a+bx))\,dx = \int \operatorname{atanh}\left(d\tanh\left(a+bx\right)+d+1\right)dx$$

input
```
integrate(atanh(1+d+d*tanh(b*x+a)),x)
```

output
```
Integral(atanh(d*tanh(a + b*x) + d + 1), x)
```

3.291.7 Maxima [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \operatorname{arctanh}(1+d+d\tanh(a+bx))\,dx = \frac{1}{4}bd\left(\frac{2x^2}{d}-\frac{2bx\log\left((d+1)e^{(2bx+2a)}+1\right)+\operatorname{Li}_2\left(-(d+1)e^{(2bx+2a)}\right)}{b^2d}\right) + x\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)$$

input
```
integrate(arctanh(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*b*d*(2*x^2/d - (2*b*x*log((d + 1)*e^(2*b*x + 2*a) + 1) + dilog(-(d + 1
)*e^(2*b*x + 2*a)))/(b^2*d)) + x*arctanh(d*tanh(b*x + a) + d + 1)
```

3.291.8 Giac [F]

$$\int \operatorname{arctanh}(1+d+d\tanh(a+bx))\,dx = \int \operatorname{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(arctanh(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctanh(d*tanh(b*x + a) + d + 1), x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}(1+d+d\tanh(a+bx))\,dx = \int \operatorname{atanh}(d+d\tanh(a+b\,x)+1)\,dx$$

input
```
int(atanh(d + d*tanh(a + b*x) + 1),x)
```

output
```
int(atanh(d + d*tanh(a + b*x) + 1), x)
```

3.292 $\int \frac{\operatorname{arctanh}(1+d+d\tanh(a+bx))}{x}\,dx$

3.292.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{arctanh}(1+d+d\tanh(a+bx))}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{arctanh}(1+d+d\tanh(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arctanh(1+d+d*tanh(b*x+a))/x,x)
```

3.292.2 Mathematica [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{arctanh}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arctanh}(1+d+d\tanh(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTanh[1 + d + d*Tanh[a + b*x]]/x,x]
```

output
```
Integrate[ArcTanh[1 + d + d*Tanh[a + b*x]]/x, x]
```

3.292.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(d\tanh(a+bx)+d+1)}{x}\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\operatorname{arctanh}(d\tanh(a+bx)+d+1)}{x}dx$$

input
```
Int[ArcTanh[1 + d + d*Tanh[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.292.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.292.4 Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}\left(1+d+d\tanh\left(bx+a\right)\right)}{x}dx$$

input
```
int(arctanh(1+d+d*tanh(b*x+a))/x,x)
```

output
```
int(arctanh(1+d+d*tanh(b*x+a))/x,x)
```

3.292.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{arctanh}(1+d+d\tanh(a+bx))}{x}\,dx=\int \frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)}{x}\,dx$$

input
```
integrate(arctanh(1+d+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arctanh(d*tanh(b*x + a) + d + 1)/x, x)
```

3.292.6 Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{arctanh}(1+d+d\tanh(a+bx))}{x}\,dx=\int \frac{\operatorname{atanh}\left(d\tanh\left(a+bx\right)+d+1\right)}{x}\,dx$$

input
```
integrate(atanh(1+d+d*tanh(b*x+a))/x,x)
```

output
```
Integral(atanh(d*tanh(a + b*x) + d + 1)/x, x)
```

3.292.7 Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{arctanh}(1+d+d\tanh(a+bx))}{x}\,dx=\int \frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)}{x}\,dx$$

input
```
integrate(arctanh(1+d+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arctanh(d*tanh(b*x + a) + d + 1)/x, x)
```

3.292.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{arctanh}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)}{x}\,dx$$

input
```
integrate(arctanh(1+d+d*tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctanh(d*tanh(b*x + a) + d + 1)/x, x)
```

3.292.9 Mupad [N/A]

Not integrable

Time = 4.63 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{arctanh}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{atanh}(d+d\tanh(a+b\,x)+1)}{x}\,dx$$

input
```
int(atanh(d + d*tanh(a + b*x) + 1)/x,x)
```

output
```
int(atanh(d + d*tanh(a + b*x) + 1)/x, x)
```

3.293 $\int x^3 \operatorname{arctanh}(1-d-d\tanh(a+bx))\,dx$

3.293.1 Optimal result

Integrand size = 19, antiderivative size = 168

$$\begin{aligned}\int x^3 \operatorname{arctanh}(1-d-d\tanh(a+bx))\,dx &= \frac{bx^5}{20}+\frac{1}{4}x^4\operatorname{arctanh}(1-d-d\tanh(a+bx))\\ &\quad-\frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2bx}\right)\\ &\quad-\frac{x^3\operatorname{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b}\\ &\quad+\frac{3x^2\operatorname{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{8b^2}\\ &\quad-\frac{3x\operatorname{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{8b^3}\\ &\quad+\frac{3\operatorname{PolyLog}\left(5,-\left((1-d)e^{2a+2bx}\right)\right)}{16b^4}\end{aligned}$$

output
```
1/20*b*x^5-1/4*x^4*arctanh(-1+d+d*tanh(b*x+a))-1/8*x^4*ln(1+(1-d)*exp(2*b*
x+2*a))-1/4*x^3*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,-(1-d
)*exp(2*b*x+2*a))/b^2-3/8*x*polylog(4,-(1-d)*exp(2*b*x+2*a))/b^3+3/16*poly
log(5,-(1-d)*exp(2*b*x+2*a))/b^4
```

3.293.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.88

$$\int x^3 \text{arctanh}(1-d-d\tanh(a+bx))\,dx = \frac{4b^4x^4\text{arctanh}(1-d-d\tanh(a+bx)) - 2b^4x^4\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right) + 4b^3x^3\,\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right) + 6b^2x^2\,\text{I}}{16b^4}$$

input
```
Integrate[x^3*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

output
```
(4*b^4*x^4*ArcTanh[1 - d - d*Tanh[a + b*x]] - 2*b^4*x^4*Log[1 - 1/((-1 + d
)*E^(2*(a + b*x)))] + 4*b^3*x^3*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))] +
 6*b^2*x^2*PolyLog[3, 1/((-1 + d)*E^(2*(a + b*x)))] + 6*b*x*PolyLog[4, 1/(
(-1 + d)*E^(2*(a + b*x)))] + 3*PolyLog[5, 1/((-1 + d)*E^(2*(a + b*x)))])/(
16*b^4)
```

3.293.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {6793, 2615, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{arctanh}(d(-\tanh(a+bx))-d+1)\,dx$$

$$\Big\downarrow \; 6793$$

$$\frac{1}{4}b\int \frac{x^4}{e^{2a+2bx}(1-d)+1}dx + \frac{1}{4}x^4\text{arctanh}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \; 2615$$

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\int \frac{e^{2a+2bx}x^4}{e^{2a+2bx}(1-d)+1}dx\right) + \frac{1}{4}x^4\text{arctanh}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \; 2620$$

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\int x^3\log\left(e^{2a+2bx}(1-d)+1\right)dx}{b(1-d)}\right)\right)+\frac{1}{4}x^4\text{arctanh}(d(-\tanh(a+bx))-d+1)$$

$\big\downarrow$ 3011

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\left(\frac{3\int x^2\,\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)dx}{2b}-\frac{x^3\,\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}\right.}{b(1-d)}\right.\right.$$
$$\frac{1}{4}x^4\text{arctanh}(d(-\tanh(a+bx))-d+1)$$

$\big\downarrow$ 7163

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\int x\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)dx}{b}\right)}{2b}-\right.}{b(1-d)}\right.\right.$$
$$\frac{1}{4}x^4\text{arctanh}(d(-\tanh(a+bx))-d+1)$$

$\big\downarrow$ 7163

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\,\text{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\int \text{PolyLo}}{}}{b}\right.}{2b}\right.}{b(1-d)}\right.\right.$$
$$\frac{1}{4}x^4\text{arctanh}(d(-\tanh(a+bx))-d+1)$$

$\big\downarrow$ 2720

3.293. $\int x^3\text{arctanh}(1-d-d\tanh(a+bx))\,dx$

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\int e^{-2a-}}{}}{}\right)}{2b}\right.}{b(1}\right.\right.$$

$$\frac{1}{4}x^4\text{arctanh}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{4}x^4\text{arctanh}(d(-\tanh(a+bx))-d+1)+$$

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\operatorname{PolyLog}}{}}{b}\right)}{2b}\right.}{b(1-d)}\right.\right.$$

input
```
Int[x^3*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

output
```
(x^4*ArcTanh[1 - d - d*Tanh[a + b*x]])/4 + (b*(x^5/5 - (1 - d)*((x^4*Log[1
 + (1 - d)*E^(2*a + 2*b*x)])/(2*b*(1 - d)) - (2*(-1/2*(x^3*PolyLog[2, -((1
 - d)*E^(2*a + 2*b*x))])/b + (3*((x^2*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x)
)])/(2*b) - ((x*PolyLog[4, -((1 - d)*E^(2*a + 2*b*x))])/(2*b) - PolyLog[5,
 -((1 - d)*E^(2*a + 2*b*x))]/(4*b^2))/b))/(2*b)))/(b*(1 - d)))))/4
```

3.293. $\int x^3\text{arctanh}(1-d-d\tanh(a+bx))\,dx$

3.293.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6793
```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.293.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.32 (sec) , antiderivative size = 1725, normalized size of antiderivative = 10.27

method	result	size
risch	Expression too large to display	1725

input
```
int(-x^3*arctanh(-1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/8*x^4*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1)+1/2/b^3/(d-1)*ln(1-(d-1)*exp
(2*b*x+2*a))*x*a^3-1/4/b*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x^3-3/8/b
^4*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^4+3/8/b^2*d/(d-1)*polylog(3,(d-1)*
exp(2*b*x+2*a))*x^2-1/4/b^4*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a^3-3/
8/b^3*d/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))*x-1/2/b^3*a^3/(d-1)*ln(1-exp
(b*x+a)*(d-1)^(1/2))*x-1/2/b^3*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x+1/
2/b^4*d*a^4/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b^4*d*a^4/(d-1)*ln(1+ex
p(b*x+a)*(d-1)^(1/2))+1/2/b^4*d*a^3/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))+
1/20*b*x^5-1/8/b^4*d*a^4/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1)-1/8*d
/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^4-1/2/b^4*a^4/(d-1)*ln(1+exp(b*x+a)*(d
-1)^(1/2))-1/2/b^4*a^3/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^4*a^3/(
d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))+3/16/b^4*d/(d-1)*polylog(5,(d-1)*exp(
2*b*x+2*a))+1/4/b/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x^3+3/8/b^4/(d-1)*
ln(1-(d-1)*exp(2*b*x+2*a))*a^4-3/8/b^2/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a
))*x^2+1/4/b^4/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a^3+3/8/b^3/(d-1)*pol
ylog(4,(d-1)*exp(2*b*x+2*a))*x-1/2/b^4*a^4/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/
2))+1/8/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^4-3/16/b^4/(d-1)*polylog(5,(d-1
)*exp(2*b*x+2*a))+1/8/b^4*a^4/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1)-
1/16*(I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2+2*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+2*I*Pi*csgn(I/(e...
```

3.293.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(132) = 264.

Time = 0.26 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.52

$$\int x^3 \operatorname{arctanh}(1-d-d\tanh(a+bx))\,dx$$
$$= \frac{2\,b^5x^5 - 5\,b^4x^4\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right) - 20\,b^3x^3\mathrm{Li}_2\left(\sqrt{d-1}(\cosh(bx+a)+\sinh(bx+a))\right) - 2}{}$$

input
```
integrate(-x^3*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/40*(2*b^5*x^5 - 5*b^4*x^4*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/((d -
 2)*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(sqrt(d - 1)*(cosh
(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-sqrt(d - 1)*(cosh(b*x + a)
 + sinh(b*x + a))) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*
x + a) + 2*sqrt(d - 1)) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*si
nh(b*x + a) - 2*sqrt(d - 1)) + 60*b^2*x^2*polylog(3, sqrt(d - 1)*(cosh(b*x
 + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -sqrt(d - 1)*(cosh(b*x + a
) + sinh(b*x + a))) - 120*b*x*polylog(4, sqrt(d - 1)*(cosh(b*x + a) + sinh
(b*x + a))) - 120*b*x*polylog(4, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x +
a))) - 5*(b^4*x^4 - a^4)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) +
 1) - 5*(b^4*x^4 - a^4)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) +
 1) + 120*polylog(5, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) + 120*po
lylog(5, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

3.293.6 Sympy [F]

$$\int x^3 \operatorname{arctanh}(1-d-d\tanh(a+bx))\,dx = -\int x^3 \operatorname{atanh}(d\tanh(a+bx)+d-1)\,dx$$

input
```
integrate(-x**3*atanh(-1+d+d*tanh(b*x+a)),x)
```

output
```
-Integral(x**3*atanh(d*tanh(a + b*x) + d - 1), x)
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.87

$$\int x^3\text{arctanh}(1-d-d\tanh(a+bx))\,dx=-\frac{1}{4}\,x^4\,\text{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)$$
$$+\frac{1}{40}\left(\frac{2\,x^5}{d}-\frac{5\left(2\,b^4x^4\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+4\,b^3x^3\text{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right)-6\,b^2x^2\text{Li}_3((d-1)e^{(2\,b}\right.}{b^5d}\right.$$

input
```
integrate(-x^3*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/4*x^4*arctanh(d*tanh(b*x + a) + d - 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*l
og(-(d - 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog((d - 1)*e^(2*b*x + 2*a)
) - 6*b^2*x^2*polylog(3, (d - 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, (d -
1)*e^(2*b*x + 2*a)) - 3*polylog(5, (d - 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

3.293.8 Giac [F]

$$\int x^3\text{arctanh}(1-d-d\tanh(a+bx))\,dx=\int -x^3\,\text{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)\,dx$$

input
```
integrate(-x^3*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-x^3*arctanh(d*tanh(b*x + a) + d - 1), x)
```

3.293.9 Mupad [F(-1)]

Timed out.

$$\int x^3\text{arctanh}(1-d-d\tanh(a+bx))\,dx=\int -x^3\,\text{atanh}(d+d\tanh(a+b\,x)-1)\,dx$$

input
```
int(-x^3*atanh(d + d*tanh(a + b*x) - 1),x)
```

output
```
int(-x^3*atanh(d + d*tanh(a + b*x) - 1), x)
```

3.294 $\int x^2 \textbf{arctanh}(1-d-d\tanh(a+bx))\,dx$

3.294.1 Optimal result

Integrand size = 19, antiderivative size = 139

$$\int x^2\text{arctanh}(1-d-d\tanh(a+bx))\,dx=\frac{bx^4}{12}+\frac{1}{3}x^3\text{arctanh}(1-d-d\tanh(a+bx))-\frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2bx}\right)-\frac{x^2\operatorname{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b}+\frac{x\operatorname{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{4b^2}-\frac{\operatorname{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{8b^3}$$

output
```
1/12*b*x^4-1/3*x^3*arctanh(-1+d+d*tanh(b*x+a))-1/6*x^3*ln(1+(1-d)*exp(2*b*
x+2*a))-1/4*x^2*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,-(1-d)*
exp(2*b*x+2*a))/b^2-1/8*polylog(4,-(1-d)*exp(2*b*x+2*a))/b^3
```

3.294.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int x^2\text{arctanh}(1-d-d\tanh(a+bx))\,dx$$
$$=\frac{8b^3x^3\text{arctanh}(1-d-d\tanh(a+bx))-4b^3x^3\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right)+6b^2x^2\operatorname{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right)+6bx\operatorname{Po}}{24b^3}$$

input
```
Integrate[x^2*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcTanh[1 - d - d*Tanh[a + b*x]] - 4*b^3*x^3*Log[1 - 1/((-1 + d
)*E^(2*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))] +
 6*b*x*PolyLog[3, 1/((-1 + d)*E^(2*(a + b*x)))] + 3*PolyLog[4, 1/((-1 + d)
*E^(2*(a + b*x)))])/(24*b^3)
```

3.294.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6793, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arctanh}(d(-\tanh(a+bx))-d+1)\,dx$$
$$\Big\downarrow \text{ 6793}$$
$$\frac{1}{3}b\int\frac{x^3}{e^{2a+2bx}(1-d)+1}dx+\frac{1}{3}x^3\text{arctanh}(d(-\tanh(a+bx))-d+1)$$
$$\Big\downarrow \text{ 2615}$$
$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\int\frac{e^{2a+2bx}x^3}{e^{2a+2bx}(1-d)+1}dx\right)+\frac{1}{3}x^3\text{arctanh}(d(-\tanh(a+bx))-d+1)$$
$$\Big\downarrow \text{ 2620}$$
$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\left(\frac{x^3\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{3\int x^2\log\left(e^{2a+2bx}(1-d)+1\right)dx}{2b(1-d)}\right)\right)+$$
$$\frac{1}{3}x^3\text{arctanh}(d(-\tanh(a+bx))-d+1)$$

$\downarrow$ 3011

$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\left(\frac{x^3\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}\right)}{2b(1-d)}\right.\right.$$
$$\frac{1}{3}x^3\text{arctanh}(d(-\tanh(a+bx))-d+1)$$

$\downarrow$ 7163

$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\left(\frac{x^3\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyL}}{}\right.}{2b(1-d)}\right.\right.$$
$$\frac{1}{3}x^3\text{arctanh}(d(-\tanh(a+bx))-d+1)$$

$\downarrow$ 2720

$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\left(\frac{x^3\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)de^{2a-}}{4b^2}}{b}\right.}{2b(1-d)}\right.\right.$$
$$\frac{1}{3}x^3\text{arctanh}(d(-\tanh(a+bx))-d+1)$$

$\downarrow$ 7143

$$\frac{1}{3}x^3\text{arctanh}(d(-\tanh(a+bx))-d+1)+$$
$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\left(\frac{x^3\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\text{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}}{}\right.}{2b(1-d)}\right.\right.$$

input
```
Int[x^2*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

3.294. $\int x^2\text{arctanh}(1-d-d\tanh(a+bx))\,dx$

output
```
(x^3*ArcTanh[1 - d - d*Tanh[a + b*x]])/3 + (b*(x^4/4 - (1 - d)*((x^3*Log[1
 + (1 - d)*E^(2*a + 2*b*x)])/(2*b*(1 - d)) - (3*(-1/2*(x^2*PolyLog[2, -((1
 - d)*E^(2*a + 2*b*x))])/b + ((x*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))])/(
2*b) - PolyLog[4, -((1 - d)*E^(2*a + 2*b*x))]/(4*b^2))/b))/(2*b*(1 - d))))
)/3
```

3.294.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6793
```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.294.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.16 (sec) , antiderivative size = 1668, normalized size of antiderivative = 12.00

method	result	size
risch	Expression too large to display	1668

input
```
int(-x^2*arctanh(-1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))^2+2*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+2*I*Pi*csgn(I/(exp
(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^2+I*Pi*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*d*exp(2*b*x+2*a))^2-I
*Pi*csgn(I*exp(2*b*x+2*a))^3-I*Pi*csgn(I/(exp(2*b*x+2*a)+1)*d*exp(2*b*x+2*
a))^3-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-I*Pi*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d)*csgn(I/(exp(2*b*x+2*a)+1)*d*exp(2*b*
x+2*a))-I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))+I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(d*e
xp(2*b*x+2*a)-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*
a)-exp(2*b*x+2*a)-1))-I*Pi*csgn(I*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))*csg
n(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^2+I*Pi*csgn(I*
d)*csgn(I/(exp(2*b*x+2*a)+1)*d*exp(2*b*x+2*a))^2+2*ln(d)+I*Pi*csgn(I/(exp(
2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*I*Pi-I*Pi*csg
n(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^3-I*Pi*csgn(I/
(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+
2*a)-1))^2-I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a)))*x^3-1/2/b^3*a
^2*d/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))+1/12*b*x^4-1/3*x^3*ln(exp(b*x+a
))-1/8/b^3*d/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))+1/4/b/(d-1)*polylog(2,(
d-1)*exp(2*b*x+2*a))*x^2-1/3/b^3/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^3-1...
```

3.294.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(109) = 218$.

Time = 0.27 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.59

$$\int x^2 \operatorname{arctanh}(1-d-d\tanh(a+bx))\,dx$$
$$= \frac{b^4x^4 - 2\,b^3x^3\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\sqrt{d-1}(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,b^2\cdots}{\cdots}$$

input
```
integrate(-x^2*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(b^4*x^4 - 2*b^3*x^3*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2
)*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d - 1)*(cosh(b*
x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d - 1)*(cosh(b*x + a) + s
inh(b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x +
a) + 2*sqrt(d - 1)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b
*x + a) - 2*sqrt(d - 1)) + 12*b*x*polylog(3, sqrt(d - 1)*(cosh(b*x + a) +
sinh(b*x + a))) + 12*b*x*polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x
 + a))) - 2*(b^3*x^3 + a^3)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)
) + 1) - 2*(b^3*x^3 + a^3)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)
) + 1) - 12*polylog(4, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*p
olylog(4, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

3.294.6 Sympy [F]

$$\int x^2 \operatorname{arctanh}(1-d-d\tanh(a+bx))\,dx = -\int x^2 \operatorname{atanh}\left(d\tanh\left(a+bx\right)+d-1\right)dx$$

input
```
integrate(-x**2*atanh(-1+d+d*tanh(b*x+a)),x)
```

output
```
-Integral(x**2*atanh(d*tanh(a + b*x) + d - 1), x)
```

3.294.7 Maxima [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int x^2 \operatorname{arctanh}(1-d-d\tanh(a+bx))\,dx = -\frac{1}{3}x^3 \operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right) + \frac{1}{36}\left(\frac{3\,x^4}{d} - \frac{2\left(4\,b^3x^3\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3\left((d-1)e^{(2\,bx+\right.}\right.}{b^4d}\right.$$

input
```
integrate(-x^2*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/3*x^3*arctanh(d*tanh(b*x + a) + d - 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*l
og(-(d - 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d - 1)*e^(2*b*x + 2*a)
) - 6*b*x*polylog(3, (d - 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d - 1)*e^(2*
b*x + 2*a)))/(b^4*d))*b*d
```

3.294.8 Giac [F]

$$\int x^2 \text{arctanh}(1-d-d\tanh(a+bx))\,dx = \int -x^2 \,\text{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)\,dx$$

input
```
integrate(-x^2*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-x^2*arctanh(d*tanh(b*x + a) + d - 1), x)
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arctanh}(1-d-d\tanh(a+bx))\,dx = \int -x^2 \,\text{atanh}(d+d\tanh(a+b\,x)-1)\,dx$$

input
```
int(-x^2*atanh(d + d*tanh(a + b*x) - 1),x)
```

output
```
int(-x^2*atanh(d + d*tanh(a + b*x) - 1), x)
```

3.295 $\int x\text{arctanh}(1-d-d\tanh(a+bx))\,dx$

3.295.1 Optimal result

Integrand size = 17, antiderivative size = 110

$$\int x\text{arctanh}(1-d-d\tanh(a+bx))\,dx=\frac{bx^3}{6}+\frac{1}{2}x^2\text{arctanh}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d)e^{2a+2bx}\right)-\frac{x\,\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b}+\frac{\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{8b^2}$$

output
```
1/6*b*x^3-1/2*x^2*arctanh(-1+d+d*tanh(b*x+a))-1/4*x^2*ln(1+(1-d)*exp(2*b*x
+2*a))-1/4*x*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+1/8*polylog(3,-(1-d)*exp(2
*b*x+2*a))/b^2
```

3.295.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int x\text{arctanh}(1-d-d\tanh(a+bx))\,dx$$
$$=\frac{2b^2x^2\left(2\text{arctanh}(1-d-d\tanh(a+bx))-\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right)\right)+2bx\,\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right)+\text{PolyLog}\left(\right.}{8b^2}$$

input
```
Integrate[x*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcTanh[1 - d - d*Tanh[a + b*x]] - Log[1 - 1/((-1 + d)*E^(2*
(a + b*x)))]) + 2*b*x*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))] + PolyLog[3
, 1/((-1 + d)*E^(2*(a + b*x)))])/(8*b^2)
```

3.295.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6793, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(d(-\tanh(a+bx))-d+1)\,dx$$

$$\Big\downarrow \ 6793$$

$$\frac{1}{2}b\int \frac{x^2}{e^{2a+2bx}(1-d)+1}dx+\frac{1}{2}x^2\text{arctanh}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \ 2615$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(1-d)\int \frac{e^{2a+2bx}x^2}{e^{2a+2bx}(1-d)+1}dx\right)+\frac{1}{2}x^2\text{arctanh}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \ 2620$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(1-d)\left(\frac{x^2\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\int x\log\left(e^{2a+2bx}(1-d)+1\right)dx}{b(1-d)}\right)\right)+\frac{1}{2}x^2\text{arctanh}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \ 3011$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(1-d)\left(\frac{x^2\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\frac{\int \text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}}{b(1-d)}\right)\right)+\frac{1}{2}x^2\text{arctanh}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \ 2720$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(1-d)\left(\frac{x^2\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\frac{\int e^{-2a-2bx}\operatorname{PolyLog}(2,-((1-d)e^{2a+2bx}))de^{2a+2bx}}{4b^2}-\frac{x\operatorname{PolyLog}(2,-((1-d)e^{2a}}{2b}}{b(1-d)}\right.\right.$$

$$\frac{1}{2}x^2\operatorname{arctanh}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x^2\operatorname{arctanh}(d(-\tanh(a+bx))-d+1)+$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(1-d)\left(\frac{x^2\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\frac{\operatorname{PolyLog}(3,-((1-d)e^{2a+2bx}))}{4b^2}-\frac{x\operatorname{PolyLog}(2,-((1-d)e^{2a+2bx}))}{2b}}{b(1-d)}\right)\right)$$

input
```
Int[x*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

output
```
(x^2*ArcTanh[1 - d - d*Tanh[a + b*x]])/2 + (b*(x^3/3 - (1 - d)*((x^2*Log[1
 + (1 - d)*E^(2*a + 2*b*x)])/(2*b*(1 - d)) - (-1/2*(x*PolyLog[2, -((1 - d)
*E^(2*a + 2*b*x))])/b + PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))]/(4*b^2))/(b
*(1 - d)))))/2
```

3.295.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6793
```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.295.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.89 (sec) , antiderivative size = 1587, normalized size of antiderivative = 14.43

method	result	size
risch	Expression too large to display	1587

input
```
int(-x*arctanh(-1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*a*d/(d-1)*x*ln(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b*a*d/(d-1)*x*ln(1+exp(
b*x+a)*(d-1)^(1/2))-1/2/b*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a*x-1/4/b^2*d
/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a+1/2/b/(d-1)*ln(1-(d-1)*exp(2*b*x+
2*a))*a*x-1/2/b*a/(d-1)*x*ln(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b*a/(d-1)*x*ln(
1+exp(b*x+a)*(d-1)^(1/2))-1/4/b^2*a^2*d/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*
x+2*a)-1)+1/2/b^2*a^2*d/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b^2*a^2*d/(
d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))+1/2/b^2*a*d/(d-1)*dilog(1-exp(b*x+a)*(d-
1)^(1/2))+1/2/b^2*a*d/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))-1/4/b^2*d/(d-1
)*ln(1-(d-1)*exp(2*b*x+2*a))*a^2-1/4/b*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2
*a))*x-1/8/b^2/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))+1/4/(d-1)*ln(1-(d-1)*
exp(2*b*x+2*a))*x^2-1/2*x^2*ln(exp(b*x+a))+1/6*b*x^3-1/8*(I*Pi*csgn(I*exp(
2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*I*Pi*csgn(I*exp(
b*x+a))*csgn(I*exp(2*b*x+2*a))^2+2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2
*b*x+2*a)-exp(2*b*x+2*a)-1))^2+I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))*csgn(I/(exp(2*b*x+2*a)+1)*d*exp(2*b*x+2*a))^2-I*Pi*csgn(I*exp(2*b*x+2*
a))^3-I*Pi*csgn(I/(exp(2*b*x+2*a)+1)*d*exp(2*b*x+2*a))^3-I*Pi*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))*csgn(I*d)*csgn(I/(exp(2*b*x+2*a)+1)*d*exp(2*b*x+2*a))-I*Pi*csgn(I/(ex
p(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)+1))+I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(d*exp(2*b*x+2*a)-exp(2*...
```

3.295.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(86) = 172$.

Time = 0.26 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.78

$$\int x\text{arctanh}(1-d-d\tanh(a+bx))\,dx$$
$$= \frac{2\,b^3x^3 - 3\,b^2x^2\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,bx\text{Li}_2\left(\sqrt{d-1}(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,bx}{}$$

input
```
integrate(-x*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*x^3 - 3*b^2*x^2*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/((d -
 2)*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(sqrt(d - 1)*(cosh(b*x
+ a) + sinh(b*x + a))) - 6*b*x*dilog(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*
x + a))) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) + 2
*sqrt(d - 1)) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a
) - 2*sqrt(d - 1)) - 3*(b^2*x^2 - a^2)*log(sqrt(d - 1)*(cosh(b*x + a) + si
nh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-sqrt(d - 1)*(cosh(b*x + a) + si
nh(b*x + a)) + 1) + 6*polylog(3, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a
))) + 6*polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

3.295.6 Sympy [F]

$$\int x\text{arctanh}(1-d-d\tanh(a+bx))\,dx=-\int x\,\text{atanh}\left(d\tanh\left(a+bx\right)+d-1\right)dx$$

input
```
integrate(-x*atanh(-1+d+d*tanh(b*x+a)),x)
```

output
```
-Integral(x*atanh(d*tanh(a + b*x) + d - 1), x)
```

3.295.7 Maxima [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

$$\int x\text{arctanh}(1-d-d\tanh(a+bx))\,dx$$
$$=\frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\text{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right)-\text{Li}_3\left((d-1)e^{(2\,bx+2\,a)}\right)\right)}{b^3d}\right)b$$
$$-\frac{1}{2}x^2\,\text{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)$$

input
```
integrate(-x*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(4*x^3/d - 3*(2*b^2*x^2*log(-(d - 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dil
og((d - 1)*e^(2*b*x + 2*a)) - polylog(3, (d - 1)*e^(2*b*x + 2*a)))/(b^3*d)
)*b*d - 1/2*x^2*arctanh(d*tanh(b*x + a) + d - 1)
```

3.295.8 Giac [F]

$$\int x\text{arctanh}(1-d-d\tanh(a+bx))\,dx=\int -x\,\text{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)\,dx$$

input
```
integrate(-x*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-x*arctanh(d*tanh(b*x + a) + d - 1), x)
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arctanh}(1-d-d\tanh(a+bx))\,dx=\int -x\,\text{atanh}(d+d\tanh(a+b\,x)-1)\,dx$$

input
```
int(-x*atanh(d + d*tanh(a + b*x) - 1),x)
```

output
```
int(-x*atanh(d + d*tanh(a + b*x) - 1), x)
```

3.296 $\int \operatorname{arctanh}(1-d-d\tanh(a+bx))\,dx$

3.296.1 Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \operatorname{arctanh}(1-d-d\tanh(a+bx))\,dx=\frac{bx^2}{2}+x\operatorname{arctanh}(1-d-d\tanh(a+bx))-\frac{1}{2}x\log\left(1+(1-d)e^{2a+2bx}\right)-\frac{\operatorname{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b}$$

output
```
1/2*b*x^2-x*arctanh(-1+d+d*tanh(b*x+a))-1/2*x*ln(1+(1-d)*exp(2*b*x+2*a))-1
/4*polylog(2,-(1-d)*exp(2*b*x+2*a))/b
```

3.296.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \operatorname{arctanh}(1-d-d\tanh(a+bx))\,dx=x\operatorname{arctanh}(1-d-d\tanh(a+bx))+\frac{-2bx\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right)+\operatorname{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right)}{4b}$$

input
```
Integrate[ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

output
```
x*ArcTanh[1 - d - d*Tanh[a + b*x]] + (-2*b*x*Log[1 - 1/((-1 + d)*E^(2*(a +
 b*x)))] + PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))])/(4*b)
```

3.296.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6785, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(d(-\tanh(a+bx))-d+1)\,dx$$

$$\Big\downarrow \text{6785}$$

$$b\int \frac{x}{e^{2a+2bx}(1-d)+1}dx+x\operatorname{arctanh}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{2615}$$

$$b\left(\frac{x^2}{2}-(1-d)\int \frac{e^{2a+2bx}x}{e^{2a+2bx}(1-d)+1}dx\right)+x\operatorname{arctanh}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{2620}$$

$$b\left(\frac{x^2}{2}-(1-d)\left(\frac{x\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\int \log\left(e^{2a+2bx}(1-d)+1\right)dx}{2b(1-d)}\right)\right)+x\operatorname{arctanh}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{2715}$$

$$b\left(\frac{x^2}{2}-(1-d)\left(\frac{x\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\int e^{-2a-2bx}\log\left(e^{2a+2bx}(1-d)+1\right)de^{2a+2bx}}{4b^2(1-d)}\right)\right)+x\operatorname{arctanh}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{2838}$$

$$x\operatorname{arctanh}(d(-\tanh(a+bx))-d+1)+b\left(\frac{x^2}{2}-(1-d)\left(\frac{\operatorname{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b^2(1-d)}+\frac{x\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}\right)\right)$$

input
```
Int[ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

output
```
x*ArcTanh[1 - d - d*Tanh[a + b*x]] + b*(x^2/2 - (1 - d)*((x*Log[1 + (1 - d
)*E^(2*a + 2*b*x)])/(2*b*(1 - d)) + PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))]
/(4*b^2*(1 - d))))
```

3.296.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6785
```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Tanh[a + b*x]], x] + Simp[b   Int[x/(c - d + c*E^(2*a + 2*b*x))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

3.296.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(66) = 132$.

Time = 0.86 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.58

method	result
derivativedivides	$-\frac{\frac{\operatorname{arctanh}(-1+d+d\tanh(bx+a))d\ln(d+d\tanh(bx+a))}{2}-\frac{\operatorname{arctanh}(-1+d+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{-d\tanh}{}\right.}{}\right.}{}}{}$
default	$-\frac{\frac{\operatorname{arctanh}(-1+d+d\tanh(bx+a))d\ln(d+d\tanh(bx+a))}{2}-\frac{\operatorname{arctanh}(-1+d+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{-d\tanh}{}\right.}{}\right.}{}}{}$
risch	Expression too large to display

input
```
int(-arctanh(-1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/b/d*(1/2*arctanh(-1+d+d*tanh(b*x+a))*d*ln(d+d*tanh(b*x+a))-1/2*arctanh(
-1+d+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)-1/2*d^2*(1/d*(1/2*dilog((-d*tan
h(b*x+a)-d+2)/(-2*d+2))+1/2*ln(-d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)-d+2)/(
-2*d+2))-1/2*dilog(-1/2*(-d*tanh(b*x+a)-d)/d)-1/2*ln(-d*tanh(b*x+a)+d)*ln(
-1/2*(-d*tanh(b*x+a)-d)/d))-1/d*(1/2*(ln(d+d*tanh(b*x+a))-ln(1/2*d*tanh(b*
x+a)+1/2*d))*ln(-1/2*d*tanh(b*x+a)-1/2*d+1)-1/2*dilog(1/2*d*tanh(b*x+a)+1/
2*d)-1/4*ln(d+d*tanh(b*x+a))^2)))
```

3.296.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(61) = 122.

Time = 0.28 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.00

$$\int \operatorname{arctanh}(1-d-d\tanh(a+bx))\,dx$$
$$=\frac{b^2x^2-bx\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right)+a\log\left(2\left(d-1\right)\cosh\left(bx+a\right)+2\left(d-1\right)\sinh\left(bx+a\right)+2\sqrt{}\right.}{}$$

input
```
integrate(-arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 - b*x*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)*cosh(
b*x + a) + d*sinh(b*x + a))) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*s
inh(b*x + a) + 2*sqrt(d - 1)) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*
sinh(b*x + a) - 2*sqrt(d - 1)) - (b*x + a)*log(sqrt(d - 1)*(cosh(b*x + a)
+ sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b
*x + a)) + 1) - dilog(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog
(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

3.296.6 Sympy [F]

$$\int \operatorname{arctanh}(1-d-d\tanh(a+bx))\,dx = -\int \operatorname{atanh}\left(d\tanh\left(a+bx\right)+d-1\right)dx$$

input
```
integrate(-atanh(-1+d+d*tanh(b*x+a)),x)
```

output
```
-Integral(atanh(d*tanh(a + b*x) + d - 1), x)
```

3.296.7 Maxima [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \operatorname{arctanh}(1-d-d\tanh(a+bx))\,dx = \frac{1}{4}bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+\operatorname{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right) - x\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)$$

input
```
integrate(-arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*b*d*(2*x^2/d - (2*b*x*log(-(d - 1)*e^(2*b*x + 2*a) + 1) + dilog((d - 1
)*e^(2*b*x + 2*a)))/(b^2*d)) - x*arctanh(d*tanh(b*x + a) + d - 1)
```

3.296.8 Giac [F]

$$\int \operatorname{arctanh}(1-d-d\tanh(a+bx))\,dx=\int -\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)\,dx$$

input
```
integrate(-arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-arctanh(d*tanh(b*x + a) + d - 1), x)
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}(1-d-d\tanh(a+bx))\,dx=\int -\operatorname{atanh}(d+d\tanh(a+b\,x)-1)\,dx$$

input
```
int(-atanh(d + d*tanh(a + b*x) - 1),x)
```

output
```
int(-atanh(d + d*tanh(a + b*x) - 1), x)
```

3.297 $\int \frac{\operatorname{arctanh}(1-d-d\tanh(a+bx))}{x}\,dx$

3.297.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\operatorname{arctanh}(1-d-d\tanh(a+bx))}{x}\,dx=\operatorname{Int}\left(\frac{\operatorname{arctanh}(1-d-d\tanh(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(-arctanh(-1+d+d*tanh(b*x+a))/x,x)
```

3.297.2 Mathematica [N/A]

Not integrable

Time = 4.69 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{arctanh}(1-d-d\tanh(a+bx))}{x}\,dx=\int \frac{\operatorname{arctanh}(1-d-d\tanh(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTanh[1 - d - d*Tanh[a + b*x]]/x,x]
```

output
```
Integrate[ArcTanh[1 - d - d*Tanh[a + b*x]]/x, x]
```

3.297.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(d(-\tanh(a+bx))-d+1)}{x}\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\text{arctanh}(d(-\tanh(a+bx))-d+1)}{x}dx$$

input
```
Int[ArcTanh[1 - d - d*Tanh[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.297.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.297.4 Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int -\frac{\text{arctanh}\left(-1+d+d\tanh\left(bx+a\right)\right)}{x}dx$$

input
```
int(-arctanh(-1+d+d*tanh(b*x+a))/x,x)
```

output
```
int(-arctanh(-1+d+d*tanh(b*x+a))/x,x)
```

3.297.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(1-d-d\tanh(a+bx))}{x}\,dx = \int -\frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)}{x}\,dx$$

input
```
integrate(-arctanh(-1+d+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(-arctanh(d*tanh(b*x + a) + d - 1)/x, x)
```

3.297.6 Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{arctanh}(1-d-d\tanh(a+bx))}{x}\,dx = -\int \frac{\operatorname{atanh}\left(d\tanh\left(a+bx\right)+d-1\right)}{x}\,dx$$

input
```
integrate(-atanh(-1+d+d*tanh(b*x+a))/x,x)
```

output
```
-Integral(atanh(d*tanh(a + b*x) + d - 1)/x, x)
```

3.297.7 Maxima [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\operatorname{arctanh}(1-d-d\tanh(a+bx))}{x}\,dx = \int -\frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)}{x}\,dx$$

input
```
integrate(-arctanh(-1+d+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
-integrate(arctanh(d*tanh(b*x + a) + d - 1)/x, x)
```

3.297.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(1-d-d\tanh(a+bx))}{x}\,dx=\int -\frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)}{x}\,dx$$

input
```
integrate(-arctanh(-1+d+d*tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(-arctanh(d*tanh(b*x + a) + d - 1)/x, x)
```

3.297.9 Mupad [N/A]

Not integrable

Time = 4.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(1-d-d\tanh(a+bx))}{x}\,dx=\int -\frac{\operatorname{atanh}(d+d\tanh(a+b\,x)-1)}{x}\,dx$$

input
```
int(-atanh(d + d*tanh(a + b*x) - 1)/x,x)
```

output
```
int(-atanh(d + d*tanh(a + b*x) - 1)/x, x)
```

3.298 $\int x^2 \mathbf{arctanh}(c + d\coth(a + bx))\,dx$

3.298.1 Optimal result

Integrand size = 15, antiderivative size = 303

$$\begin{aligned}\int x^2\text{arctanh}(c+d\coth(a+bx))\,dx &= \frac{1}{3}x^3\text{arctanh}(c+d\coth(a+bx)) \\ &+ \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) \\ &- \frac{1}{6}x^3\log\left(1-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right) \\ &+ \frac{x^2\,\text{PolyLog}\left(2,\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b} \\ &- \frac{x^2\,\text{PolyLog}\left(2,\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b} \\ &- \frac{x\,\text{PolyLog}\left(3,\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b^2} \\ &+ \frac{x\,\text{PolyLog}\left(3,\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b^2} \\ &+ \frac{\text{PolyLog}\left(4,\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{8b^3} \\ &- \frac{\text{PolyLog}\left(4,\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arctanh(c+d*coth(b*x+a))+1/6*x^3*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+
d))-1/6*x^3*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x^2*polylog(2,(1-c-d)
*exp(2*b*x+2*a)/(1-c+d))/b-1/4*x^2*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d
))/b-1/4*x*polylog(3,(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/4*x*polylog(3,(
1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^2+1/8*polylog(4,(1-c-d)*exp(2*b*x+2*a)/(1
-c+d))/b^3-1/8*polylog(4,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^3
```

3.298.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.87

$$\int x^2 \text{arctanh}(c+d\coth(a+bx))\,dx = \frac{1}{3}x^3\text{arctanh}(c+d\coth(a+bx)) - \frac{-4b^3x^3\log\left(1+\frac{(1-c+d)e^{-2(a+bx)}}{-1+c+d}\right)+4b^3x^3\log\left(1+\frac{(-1-c+d)e^{-2(a+bx)}}{1+c+d}\right)+6b^2x^2\,\text{PolyLog}\left(2,\frac{(-1+c-d)e^{-2(a+b}}{-1+c+d}\right.}{}$$

input
```
Integrate[x^2*ArcTanh[c + d*Coth[a + b*x]],x]
```

output
```
(x^3*ArcTanh[c + d*Coth[a + b*x]])/3 - (-4*b^3*x^3*Log[1 + (1 - c + d)/((-
1 + c + d)*E^(2*(a + b*x)))] + 4*b^3*x^3*Log[1 + (-1 - c + d)/((1 + c + d)
*E^(2*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, (-1 + c - d)/((-1 + c + d)*E^(2*
(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (1 + c - d)/((1 + c + d)*E^(2*(a + b*x
)))] + 6*b*x*PolyLog[3, (-1 + c - d)/((-1 + c + d)*E^(2*(a + b*x)))] - 6*b
*x*PolyLog[3, (1 + c - d)/((1 + c + d)*E^(2*(a + b*x)))] + 3*PolyLog[4, (-
1 + c - d)/((-1 + c + d)*E^(2*(a + b*x)))] - 3*PolyLog[4, (1 + c - d)/((1
+ c + d)*E^(2*(a + b*x)))])/(24*b^3)
```

3.298.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6799, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arctanh}(d\coth(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6799}$$

$$-\frac{1}{3}b(-c-d+1)\int \frac{e^{2a+2bx}x^3}{-c-(-c-d+1)e^{2a+2bx}+d+1}dx+\frac{1}{3}b(c+d+1)\int \frac{e^{2a+2bx}x^3}{c-(c+d+1)e^{2a+2bx}-d+1}dx+\frac{1}{3}x^3\text{arctanh}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$-\frac{1}{3}b(-c-d+1)\left(\frac{3\int x^2\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b(-c-d+1)}-\frac{x^3\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{3}b(c+d+1)\left(\frac{3\int x^2\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b(c+d+1)}-\frac{x^3\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+\frac{1}{3}x^3\text{arctanh}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$-\frac{1}{3}b(-c-d+1)\left(\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}\right)}{2b(-c-d+1)}-\frac{x^3\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{3}b(c+d+1)\left(\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}\right)}{2b(c+d+1)}-\frac{x^3\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+\frac{1}{3}x^3\text{arctanh}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{7163}$$

3.298. $\int x^2\text{arctanh}(c+d\coth(a+bx))\,dx$

$$-\frac{1}{3}b(-c-d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}\right)}{2b(-c-d+1)}-\frac{x^3\log\left(1-\frac{(-c-\cdots}{-\cdots}\right.}{2b(-c-d\cdots}\right.$$

$$\frac{1}{3}b(c+d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}\right)}{2b(c+d+1)}-\frac{x^3\log\left(1-\frac{(c+d+1)e^{2\cdots}}{c-d+\cdots}\right.}{2b(c+d+1)\cdots}\right.$$

$$\frac{1}{3}x^3\text{arctanh}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2720}$$

$$-\frac{1}{3}b(-c-d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}\right)}{2b(-c-d+1)}-\frac{x^3\,\text{l}\cdots}{\cdots}\right.$$

$$\frac{1}{3}b(c+d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}\right)}{2b(c+d+1)}-\frac{x^3\log\left(\cdots\right.}{2\cdots}\right.$$

$$\frac{1}{3}x^3\text{arctanh}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{7143}$$

3.298. $\int x^2\text{arctanh}(c+d\coth(a+bx))\,dx$

$$\frac{1}{3}x^3\text{arctanh}(d\coth(a+bx)+c)-\frac{1}{3}b(-c-d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\text{PolyLog}\left(4,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}\right)}{2b(-c-d+1)}-\frac{x^3\log\left(1-\frac{(-c-d+1}{-c+}\right.}{2b(-c-d+}\right.$$

$$\frac{1}{3}b(c+d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\text{PolyLog}\left(4,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}\right)}{2b(c+d+1)}-\frac{x^3\log\left(1-\frac{(c+d+1)e^{2a+2}}{c-d+1}\right.}{2b(c+d+1)}\right.$$

input
```
Int[x^2*ArcTanh[c + d*Coth[a + b*x]],x]
```

output
```
(x^3*ArcTanh[c + d*Coth[a + b*x]])/3 - (b*(1 - c - d)*(-1/2*(x^3*Log[1 - (
(1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(b*(1 - c - d)) + (3*(-1/2*(x^2
*PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/b + ((x*PolyLog[3,
 ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(2*b) - PolyLog[4, ((1 - c -
d)*E^(2*a + 2*b*x))/(1 - c + d)]/(4*b^2))/b))/(2*b*(1 - c - d))))/3 + (b*(
1 + c + d)*(-1/2*(x^3*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/
(b*(1 + c + d)) + (3*(-1/2*(x^2*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(
1 + c - d)])/b + ((x*PolyLog[3, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]
)/(2*b) - PolyLog[4, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(4*b^2))/b
))/(2*b*(1 + c + d))))/3
```

3.298.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6799
```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + (-Simp[b*((1 - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(
E^(2*a + 2*b*x)/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x))), x], x] + Simp[b
*((1 + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c
- d - (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.298. $\int x^2 \text{arctanh}(c + d\coth(a + bx))\, dx$

3.298.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.16 (sec) , antiderivative size = 5248, normalized size of antiderivative = 17.32

method	result	size
risch	Expression too large to display	5248

input
```
int(x^2*arctanh(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.298.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 880 vs. 2(259) = 518.

Time = 0.30 (sec) , antiderivative size = 880, normalized size of antiderivative = 2.90

$$\int x^2 \text{arctanh}(c + d \coth(a + bx)) \, dx = \text{Too large to display}$$

input
```
integrate(x^2*arctanh(c+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(b^3*x^3*log(-(d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x +
a) + (c - 1)*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt((c + d + 1)/(c - d + 1
))*(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt((c + d + 1)/(c
 - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt((c + d
- 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-sqrt
((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2*(c
+ d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(
(c + d + 1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d
 + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) - a^3*l
og(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d -
1)*sqrt((c + d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x + a) +
2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1)))
 + 6*b*x*polylog(3, sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*
x + a))) + 6*b*x*polylog(3, -sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a)
+ sinh(b*x + a))) - 6*b*x*polylog(3, sqrt((c + d - 1)/(c - d - 1))*(cosh(b
*x + a) + sinh(b*x + a))) - 6*b*x*polylog(3, -sqrt((c + d - 1)/(c - d - 1)
)*(cosh(b*x + a) + sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt((c + d + 1)/
(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3 + a^3)*log(-s
qrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x
^3 + a^3)*log(sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x +...
```

3.298.6 Sympy [F]

$$\int x^2 \text{arctanh}(c+d\coth(a+bx))\,dx = \int x^2 \operatorname{atanh}(c+d\coth(a+bx))\,dx$$

input
```
integrate(x**2*atanh(c+d*coth(b*x+a)),x)
```

output
```
Integral(x**2*atanh(c + d*coth(a + b*x)), x)
```

3.298.7 Maxima [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.91

$$\int x^2 \text{arctanh}(c + d \coth(a + bx)) \, dx = \frac{1}{3} x^3 \operatorname{artanh}\left(d \coth\left(bx + a\right) + c\right) - \frac{1}{18} bd \left(\frac{4 b^3 x^3 \log\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1} + 1\right) + 6 b^2 x^2 \operatorname{Li}_2\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right) - 6 bx \operatorname{Li}_3\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right) + 3 \operatorname{Li}}{b^4 d} \right.$$

input
```
integrate(x^2*arctanh(c+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctanh(d*coth(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log(-(c + d +
1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 6*b^2*x^2*dilog((c + d + 1)*e^(2*b*x
 + 2*a)/(c - d + 1)) - 6*b*x*polylog(3, (c + d + 1)*e^(2*b*x + 2*a)/(c - d
 + 1)) + 3*polylog(4, (c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d) -
(4*b^3*x^3*log(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*d
ilog((c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, (c + d -
1)*e^(2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, (c + d - 1)*e^(2*b*x + 2*a)
/(c - d - 1)))/(b^4*d))
```

3.298.8 Giac [F]

$$\int x^2 \text{arctanh}(c + d \coth(a + bx)) \, dx = \int x^2 \operatorname{artanh}\left(d \coth\left(bx + a\right) + c\right) \, dx$$

input
```
integrate(x^2*arctanh(c+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(d*coth(b*x + a) + c), x)
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{arctanh}(c + d\coth(a + bx))\,dx = \int x^2 \operatorname{atanh}(c + d\coth(a + b\,x))\,dx$$

input
```
int(x^2*atanh(c + d*coth(a + b*x)),x)
```

output
```
int(x^2*atanh(c + d*coth(a + b*x)), x)
```

3.299 $\int x\text{arctanh}(c+d\coth(a+bx))\,dx$

3.299.1 Optimal result

Integrand size = 13, antiderivative size = 229

$$\begin{aligned}\int x\text{arctanh}(c+d\coth(a+bx))\,dx &= \frac{1}{2}x^2\text{arctanh}(c+d\coth(a+bx)) \\ &+ \frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) \\ &- \frac{1}{4}x^2\log\left(1-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right) \\ &+ \frac{x\,\text{PolyLog}\left(2,\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b} \\ &- \frac{x\,\text{PolyLog}\left(2,\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b} \\ &- \frac{\text{PolyLog}\left(3,\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{8b^2} \\ &+ \frac{\text{PolyLog}\left(3,\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{8b^2}\end{aligned}$$

output
```
1/2*x^2*arctanh(c+d*coth(b*x+a))+1/4*x^2*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+
d))-1/4*x^2*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x*polylog(2,(1-c-d)*e
xp(2*b*x+2*a)/(1-c+d))/b-1/4*x*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b
-1/8*polylog(3,(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/8*polylog(3,(1+c+d)*e
xp(2*b*x+2*a)/(1+c-d))/b^2
```

3.299.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.87

$$\int x\text{arctanh}(c+d\coth(a+bx))\,dx$$

$$= \frac{4b^2x^2\text{arctanh}(c+d\coth(a+bx))+2b^2x^2\log\left(1+\frac{(1-c+d)e^{-2(a+bx)}}{-1+c+d}\right)-2b^2x^2\log\left(1+\frac{(-1-c+d)e^{-2(a+bx)}}{1+c+d}\right)-}{}$$

input
```
Integrate[x*ArcTanh[c + d*Coth[a + b*x]],x]
```

output
```
(4*b^2*x^2*ArcTanh[c + d*Coth[a + b*x]] + 2*b^2*x^2*Log[1 + (1 - c + d)/((
-1 + c + d)*E^(2*(a + b*x)))] - 2*b^2*x^2*Log[1 + (-1 - c + d)/((1 + c + d
)*E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, (-1 + c - d)/((-1 + c + d)*E^(2*(a
+ b*x)))] + 2*b*x*PolyLog[2, (1 + c - d)/((1 + c + d)*E^(2*(a + b*x)))] -
PolyLog[3, (-1 + c - d)/((-1 + c + d)*E^(2*(a + b*x)))] + PolyLog[3, (1 +
c - d)/((1 + c + d)*E^(2*(a + b*x)))])/(8*b^2)
```

3.299.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6799, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(d\coth(a+bx)+c)\,dx$$

$$\Big\downarrow \; 6799$$

$$-\frac{1}{2}b(-c-d+1)\int \frac{e^{2a+2bx}x^2}{-c-(-c-d+1)e^{2a+2bx}+d+1}dx+\frac{1}{2}b(c+d+1)\int \frac{e^{2a+2bx}x^2}{c-(c+d+1)e^{2a+2bx}-d+1}dx+\frac{1}{2}x^2\text{arctanh}(d\coth(a+bx)+c)$$

$$\Big\downarrow \; 2620$$

$$-\frac{1}{2}b(-c-d+1)\left(\frac{\int x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{b(-c-d+1)}-\frac{x^2\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{2}b(c+d+1)\left(\frac{\int x\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{b(c+d+1)}-\frac{x^2\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+\frac{1}{2}x^2\text{arctanh}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$-\frac{1}{2}b(-c-d+1)\left(\frac{\frac{\int \text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}-\frac{x^2\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{2}b(c+d+1)\left(\frac{\frac{\int \text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}-\frac{x^2\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+\frac{1}{2}x^2\text{arctanh}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2720}$$

$$-\frac{1}{2}b(-c-d+1)\left(\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}-\frac{x^2\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{2}b(c+d+1)\left(\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}-\frac{x^2\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+\frac{1}{2}x^2\text{arctanh}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{7143}$$

3.299. $\int x\text{arctanh}(c+d\coth(a+bx))\,dx$

$$\frac{1}{2}x^2\text{arctanh}(d\coth(a+bx)+c)-\frac{1}{2}b(-c-d+1)\left(\frac{\frac{\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}-\frac{x\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}-\frac{x^2\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{2}b(c+d+1)\left(\frac{\frac{\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}-\frac{x\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}-\frac{x^2\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)$$

input
```
Int[x*ArcTanh[c + d*Coth[a + b*x]],x]
```

output
```
(x^2*ArcTanh[c + d*Coth[a + b*x]])/2 - (b*(1 - c - d)*(-1/2*(x^2*Log[1 - (
(1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(b*(1 - c - d)) + (-1/2*(x*Poly
Log[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/b + PolyLog[3, ((1 - c
- d)*E^(2*a + 2*b*x))/(1 - c + d)]/(4*b^2))/(b*(1 - c - d))))/2 + (b*(1 +
c + d)*(-1/2*(x^2*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(b*(
1 + c + d)) + (-1/2*(x*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d
)])/b + PolyLog[3, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(4*b^2))/(b*
(1 + c + d))))/2
```

3.299.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6799
```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + (-Simp[b*((1 - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(
E^(2*a + 2*b*x)/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x))), x], x] + Simp[b
*((1 + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c
- d - (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.299.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.59 (sec) , antiderivative size = 4944, normalized size of antiderivative = 21.59

method	result	size
risch	Expression too large to display	4944

input
```
int(x*arctanh(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/4*c/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x^2+1/4*d/(c+d-1)*ln(1-
(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x^2+1/8*I*Pi*(csgn(I/(exp(2*b*x+2*a)-1))*c
sgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1))*csgn(I
*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1)/(exp(2*b*x+2
*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x
+2*a)+1)*d+exp(2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)
+1)*d+exp(2*b*x+2*a)-1)/(exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csg
n(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1)/(exp(2*b*
x+2*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(
2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1)/(exp(2*b*x+2*a)-1))^2-csgn(I*((exp(2*b*x
+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1))*csgn(I*((exp(2*b*x+2*a)
-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)-1))^2+csgn(I*
((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1))*csgn(I*((exp
(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1)/(exp(2*b*x+2*a)-1)
)^2-csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1)/(e
xp(2*b*x+2*a)-1))^3+2*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-ex
p(2*b*x+2*a)+1)/(exp(2*b*x+2*a)-1))^2-csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*
b*x+2*a)+1)*d+exp(2*b*x+2*a)-1)/(exp(2*b*x+2*a)-1))^3-2)*x^2+1/2/b*a/(c+d-
1)*x*ln((-exp(b*x+a)*c-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((
c-d-1)*(c+d-1))^(1/2))+1/2/b*a/(c+d-1)*x*ln((exp(b*x+a)*c+exp(b*x+a)*d+...
```

3.299.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. 2(195) = 390.

Time = 0.29 (sec) , antiderivative size = 730, normalized size of antiderivative = 3.19

$$\int x\text{arctanh}(c + d\coth(a + bx))\, dx$$
$$= \frac{b^2x^2\log\left(-\frac{d\cosh(bx+a)+(c+1)\sinh(bx+a)}{d\cosh(bx+a)+(c-1)\sinh(bx+a)}\right) - 2\,bx\text{Li}_2\left(\sqrt{\frac{c+d+1}{c-d+1}}(\cosh(bx+a)+\sinh(bx+a))\right) - 2\,bx\text{Li}_2\left(-\sqrt{\cdots}\right.}{\cdots}$$

input
```
integrate(x*arctanh(c+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(b^2*x^2*log(-(d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x +
a) + (c - 1)*sinh(b*x + a))) - 2*b*x*dilog(sqrt((c + d + 1)/(c - d + 1))*(
cosh(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt((c + d + 1)/(c - d + 1
))*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt((c + d - 1)/(c - d
- 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt((c + d - 1)/(c
- d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cosh(b*
x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d + 1)/(c -
 d + 1))) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x +
 a) - 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) + a^2*log(2*(c + d - 1)
*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c + d -
 1)/(c - d - 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*si
nh(b*x + a) - 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) - (b^2*x^2 - a^
2)*log(sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1)
- (b^2*x^2 - a^2)*log(-sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh
(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(sqrt((c + d - 1)/(c - d - 1))*(cosh(
b*x + a) + sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt((c + d - 1)/(c
- d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3, sqrt((c + d
+ 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*polylog(3, -sqrt((c
 + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, sqr
t((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog...
```

3.299.6 Sympy [F]

$$\int x\text{arctanh}(c+d\coth(a+bx))\,dx=\int x\,\text{atanh}\,(c+d\coth\,(a+bx))\,dx$$

input
```
integrate(x*atanh(c+d*coth(b*x+a)),x)
```

output
```
Integral(x*atanh(c + d*coth(a + b*x)), x)
```

3.299.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.93

$$\int x\text{arctanh}(c+d\coth(a+bx))\,dx =$$
$$-\frac{1}{8}\,bd\left(\frac{2\,b^2x^2\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+2\,bx\text{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-\text{Li}_3\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^3d}-\frac{2\,b^2x^2\log\left(\right.}{}\right.$$
$$+\frac{1}{2}\,x^2\,\text{artanh}\left(d\coth\left(bx+a\right)+c\right)$$

input
```
integrate(x*arctanh(c+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*b*d*((2*b^2*x^2*log(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 2
*b*x*dilog((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - polylog(3, (c + d +
1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^3*d) - (2*b^2*x^2*log(-(c + d - 1)*e^(
2*b*x + 2*a)/(c - d - 1) + 1) + 2*b*x*dilog((c + d - 1)*e^(2*b*x + 2*a)/(c
 - d - 1)) - polylog(3, (c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^3*d))
 + 1/2*x^2*arctanh(d*coth(b*x + a) + c)
```

3.299.8 Giac [F]

$$\int x\text{arctanh}(c+d\coth(a+bx))\,dx=\int x\,\text{artanh}\left(d\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arctanh(c+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctanh(d*coth(b*x + a) + c), x)
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arctanh}(c+d\coth(a+bx))\,dx=\int x\,\text{atanh}(c+d\coth(a+b\,x))\,dx$$

input
```
int(x*atanh(c + d*coth(a + b*x)),x)
```

output
```
int(x*atanh(c + d*coth(a + b*x)), x)
```

3.300 $\int \textbf{arctanh}(c + d\coth(a + bx))\,dx$

3.300.1 Optimal result

Integrand size = 11, antiderivative size = 150

$$\begin{aligned}\int \operatorname{arctanh}(c+d\coth(a+bx))\,dx = {} & x\operatorname{arctanh}(c+d\coth(a+bx)) \\ & + \frac{1}{2}x\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) \\ & - \frac{1}{2}x\log\left(1-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right) \\ & + \frac{\operatorname{PolyLog}\left(2,\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b} \\ & - \frac{\operatorname{PolyLog}\left(2,\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b}\end{aligned}$$

output
```
x*arctanh(c+d*coth(b*x+a))+1/2*x*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))-1/2*
x*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*polylog(2,(1-c-d)*exp(2*b*x+2*a
)/(1-c+d))/b-1/4*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b
```

3.300.2 Mathematica [A] (verified)

Time = 3.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \operatorname{arctanh}(c+d\coth(a+bx))\,dx = x\operatorname{arctanh}(c+d\coth(a+bx)) - \frac{-2bx\left(\log\left(1-\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)-\log\left(1-\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)\right)-\operatorname{PolyLog}\left(2,\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)+\operatorname{PolyLog}}{4b}$$

input
```
Integrate[ArcTanh[c + d*Coth[a + b*x]],x]
```

output
```
x*ArcTanh[c + d*Coth[a + b*x]] - (-2*b*x*(Log[1 - ((-1 + c + d)*E^(2*(a +
b*x)))/(-1 + c - d)] - Log[1 - ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)])
 - PolyLog[2, ((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d)] + PolyLog[2, ((
1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)])/(4*b)
```

3.300.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.38, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6791, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(d\coth(a+bx)+c)\,dx$$

$$\Big\downarrow \text{ 6791}$$

$$-b(-c-d+1)\int \frac{e^{2a+2bx}x}{-c-(-c-d+1)e^{2a+2bx}+d+1}dx+b(c+d+1)\int \frac{e^{2a+2bx}x}{c-(c+d+1)e^{2a+2bx}-d+1}dx+x\operatorname{arctanh}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{ 2620}$$

$$-b(-c-d+1)\left(\frac{\int \log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b(-c-d+1)}-\frac{x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+b(c+d+1)\left(\frac{\int \log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b(c+d+1)}-\frac{x\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+x\operatorname{arctanh}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{ 2715}$$

$$-b(-c-d+1)\left(\frac{\int e^{-2a-2bx}\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)de^{2a+2bx}}{4b^2(-c-d+1)}-\frac{x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+b(c+d+1)\left(\frac{\int e^{-2a-2bx}\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)de^{2a+2bx}}{4b^2(c+d+1)}-\frac{x\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+x\text{arctanh}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{ 2838}$$

$$x\text{arctanh}(d\coth(a+bx)+c)-b(-c-d+1)\left(-\frac{\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2(-c-d+1)}-\frac{x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+b(c+d+1)\left(-\frac{\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2(c+d+1)}-\frac{x\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)$$

input
```
Int[ArcTanh[c + d*Coth[a + b*x]],x]
```

output
```
x*ArcTanh[c + d*Coth[a + b*x]] - b*(1 - c - d)*(-1/2*(x*Log[1 - ((1 - c -
d)*E^(2*a + 2*b*x))/(1 - c + d)])/(b*(1 - c - d)) - PolyLog[2, ((1 - c - d
)*E^(2*a + 2*b*x))/(1 - c + d)]/(4*b^2*(1 - c - d))) + b*(1 + c + d)*(-1/2
*(x*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(b*(1 + c + d)) -
PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(4*b^2*(1 + c + d)))
```

3.300.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6791
```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Coth[a + b*x]], x] + (-Simp[b*(1 - c - d)   Int[x*(E^(2*a + 2*b
*x)/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x))), x], x] + Simp[b*(1 + c + d)
   Int[x*(E^(2*a + 2*b*x)/(1 + c - d - (1 + c + d)*E^(2*a + 2*b*x))), x], x
]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

3.300.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(138) = 276$.

Time = 2.00 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.32

method	result
derivativedivides	$-\frac{\operatorname{arctanh}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arctanh}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}+\frac{d^2\left(-\frac{\operatorname{dilog}\left(\frac{-d\coth(bx+a)}{-1-c+d}\right)}{2}\right.}{}$
default	$-\frac{\operatorname{arctanh}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arctanh}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}+\frac{d^2\left(-\frac{\operatorname{dilog}\left(\frac{-d\coth(bx+a)}{-1-c+d}\right)}{2}\right.}{}$
risch	Expression too large to display

input
```
int(arctanh(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-1/2*arctanh(c+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)+1/2*arctanh(c+
d*coth(b*x+a))*d*ln(-d*coth(b*x+a)-d)+1/2*d^2*(1/d*(-1/2*dilog((-d*coth(b*
x+a)-c-1)/(-1-c+d))-1/2*ln(-d*coth(b*x+a)-d)*ln((-d*coth(b*x+a)-c-1)/(-1-c
+d))+1/2*dilog((-d*coth(b*x+a)-c+1)/(1-c+d))+1/2*ln(-d*coth(b*x+a)-d)*ln((
-d*coth(b*x+a)-c+1)/(1-c+d)))-1/d*(1/2*dilog((-d*coth(b*x+a)-c+1)/(1-c-d))
+1/2*ln(-d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)-c+1)/(1-c-d))-1/2*dilog((-d*c
oth(b*x+a)-c-1)/(-1-c-d))-1/2*ln(-d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)-c-1)
/(-1-c-d)))))
```

3.300.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(128) = 256$.

Time = 0.28 (sec) , antiderivative size = 540, normalized size of antiderivative = 3.60

$$\int \operatorname{arctanh}(c + d\coth(a + bx))\, dx$$
$$= \frac{bx \log\left(-\frac{d\cosh(bx+a)+(c+1)\sinh(bx+a)}{d\cosh(bx+a)+(c-1)\sinh(bx+a)}\right) + a\log\left(2\,(c+d+1)\cosh\,(bx+a) + 2\,(c+d+1)\sinh\,(bx+a) + 2\,(\right.}{}$$

input
```
integrate(arctanh(c+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b*x*log(-(d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a) +
 (c - 1)*sinh(b*x + a))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d +
1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) + a*log(2*
(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sq
rt((c + d + 1)/(c - d + 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c +
d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) - a*lo
g(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1
)*sqrt((c + d - 1)/(c - d - 1))) - (b*x + a)*log(sqrt((c + d + 1)/(c - d +
 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt((c + d + 1
)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(sqrt((
c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*l
og(-sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - d
ilog(sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - dilo
g(-sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(
sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(-sq
rt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

3.300.6 Sympy [F]

$$\int \operatorname{arctanh}(c + d\coth(a + bx))\, dx = \int \operatorname{atanh}\left(c + d\coth\left(a + bx\right)\right) dx$$

input
```
integrate(atanh(c+d*coth(b*x+a)),x)
```

output
```
Integral(atanh(c + d*coth(a + b*x)), x)
```

3.300.7 Maxima [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \operatorname{arctanh}(c+d\coth(a+bx))\,dx =$$

$$-\frac{1}{4}bd\left(\frac{2\,bx\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^2d}-\frac{2\,bx\log\left(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+\mathrm{Li}_2\left(\right.}{b^2d}\right.$$

$$+x\,\operatorname{artanh}\left(d\coth\left(bx+a\right)+c\right)$$

input
```
integrate(arctanh(c+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
-1/4*b*d*((2*b*x*log(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + dilog
((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^2*d) - (2*b*x*log(-(c + d -
1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + dilog((c + d - 1)*e^(2*b*x + 2*a)/(c
 - d - 1)))/(b^2*d)) + x*arctanh(d*coth(b*x + a) + c)
```

3.300.8 Giac [F]

$$\int \operatorname{arctanh}(c+d\coth(a+bx))\,dx = \int \operatorname{artanh}\left(d\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arctanh(c+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctanh(d*coth(b*x + a) + c), x)
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}(c+d\coth(a+bx))\,dx = \int \operatorname{atanh}(c+d\coth(a+b\,x))\,dx$$

input
```
int(atanh(c + d*coth(a + b*x)),x)
```

output
```
int(atanh(c + d*coth(a + b*x)), x)
```

3.301 $\int \frac{\operatorname{arctanh}(c+d\coth(a+bx))}{x}\,dx$

3.301.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\operatorname{arctanh}(c+d\coth(a+bx))}{x}\,dx=\operatorname{Int}\left(\frac{\operatorname{arctanh}(c+d\coth(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arctanh(c+d*coth(b*x+a))/x,x)
```

3.301.2 Mathematica [N/A]

Not integrable

Time = 10.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{arctanh}(c+d\coth(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTanh[c + d*Coth[a + b*x]]/x,x]
```

output
```
Integrate[ArcTanh[c + d*Coth[a + b*x]]/x, x]
```

3.301.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(d\coth(a+bx)+c)}{x}\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{\operatorname{arctanh}(d\coth(a+bx)+c)}{x}dx$$

input
```
Int[ArcTanh[c + d*Coth[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.301.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.301.4 Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}\left(c+d\coth\left(bx+a\right)\right)}{x}dx$$

input
```
int(arctanh(c+d*coth(b*x+a))/x,x)
```

output
```
int(arctanh(c+d*coth(b*x+a))/x,x)
```

3.301.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\text{arctanh}(c + d \coth(a + bx))}{x} \, dx = \int \frac{\text{artanh}\left(d \coth\left(bx + a\right) + c\right)}{x} \, dx$$

input
```
integrate(arctanh(c+d*coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arctanh(d*coth(b*x + a) + c)/x, x)
```

3.301.6 Sympy [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\text{arctanh}(c + d \coth(a + bx))}{x} \, dx = \int \frac{\text{atanh}\left(c + d \coth\left(a + bx\right)\right)}{x} \, dx$$

input
```
integrate(atanh(c+d*coth(b*x+a))/x,x)
```

output
```
Integral(atanh(c + d*coth(a + b*x))/x, x)
```

3.301.7 Maxima [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\text{arctanh}(c + d \coth(a + bx))}{x} \, dx = \int \frac{\text{artanh}\left(d \coth\left(bx + a\right) + c\right)}{x} \, dx$$

input
```
integrate(arctanh(c+d*coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arctanh(d*coth(b*x + a) + c)/x, x)
```

3.301.8 Giac [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c + d\coth(a + bx))}{x} \, dx = \int \frac{\operatorname{artanh}\left(d\coth\left(bx + a\right) + c\right)}{x} \, dx$$

input
```
integrate(arctanh(c+d*coth(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctanh(d*coth(b*x + a) + c)/x, x)
```

3.301.9 Mupad [N/A]

Not integrable

Time = 4.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c + d\coth(a + bx))}{x} \, dx = \int \frac{\operatorname{atanh}(c + d\coth(a + b\,x))}{x} \, dx$$

input
```
int(atanh(c + d*coth(a + b*x))/x,x)
```

output
```
int(atanh(c + d*coth(a + b*x))/x, x)
```

3.302 $\int x^3 \text{arctanh}(1+d+d\coth(a+bx))\,dx$

3.302.1 Optimal result

Integrand size = 16, antiderivative size = 152

$$\begin{aligned}\int x^3 \text{arctanh}(1+d+d\coth(a+bx))\,dx &= \frac{bx^5}{20}+\frac{1}{4}x^4\text{arctanh}(1+d+d\coth(a+bx))\\ &\quad-\frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2bx}\right)\\ &\quad-\frac{x^3\operatorname{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b}\\ &\quad+\frac{3x^2\operatorname{PolyLog}\left(3,(1+d)e^{2a+2bx}\right)}{8b^2}\\ &\quad-\frac{3x\operatorname{PolyLog}\left(4,(1+d)e^{2a+2bx}\right)}{8b^3}\\ &\quad+\frac{3\operatorname{PolyLog}\left(5,(1+d)e^{2a+2bx}\right)}{16b^4}\end{aligned}$$

output
```
1/20*b*x^5+1/4*x^4*arctanh(1+d+d*coth(b*x+a))-1/8*x^4*ln(1-(1+d)*exp(2*b*x
+2*a))-1/4*x^3*polylog(2,(1+d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,(1+d)*e
xp(2*b*x+2*a))/b^2-3/8*x*polylog(4,(1+d)*exp(2*b*x+2*a))/b^3+3/16*polylog(
5,(1+d)*exp(2*b*x+2*a))/b^4
```

3.302.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int x^3\text{arctanh}(1+d+d\coth(a+bx))\,dx$$
$$=\frac{4b^4x^4\text{arctanh}(1+d+d\coth(a+bx))-2b^4x^4\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right)+4b^3x^3\,\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right)+6b^2x^2\,\text{I}}{16b^4}$$

input
```
Integrate[x^3*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

output
```
(4*b^4*x^4*ArcTanh[1 + d + d*Coth[a + b*x]] - 2*b^4*x^4*Log[1 - 1/((1 + d)
*E^(2*(a + b*x)))] + 4*b^3*x^3*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))] + 6
*b^2*x^2*PolyLog[3, 1/((1 + d)*E^(2*(a + b*x)))] + 6*b*x*PolyLog[4, 1/((1
+ d)*E^(2*(a + b*x)))] + 3*PolyLog[5, 1/((1 + d)*E^(2*(a + b*x)))])/(16*b^
4)
```

3.302.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6795, 2615, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{arctanh}(d\coth(a+bx)+d+1)\,dx$$
$$\Big\downarrow \ 6795$$
$$\frac{1}{4}b\int\frac{x^4}{1-(d+1)e^{2a+2bx}}dx+\frac{1}{4}x^4\text{arctanh}(d\coth(a+bx)+d+1)$$
$$\Big\downarrow \ 2615$$
$$\frac{1}{4}b\left((d+1)\int\frac{e^{2a+2bx}x^4}{1-(d+1)e^{2a+2bx}}dx+\frac{x^5}{5}\right)+\frac{1}{4}x^4\text{arctanh}(d\coth(a+bx)+d+1)$$
$$\Big\downarrow \ 2620$$

$$\frac{1}{4}b\left((d+1)\left(\frac{2\int x^3\log\left(1-(d+1)e^{2a+2bx}\right)dx}{b(d+1)}-\frac{x^4\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^5}{5}\right)+\frac{1}{4}x^4\text{arctanh}(d\coth(a+bx)+d+1)$$

$\big\downarrow$ 3011

$$\frac{1}{4}b\left((d+1)\left(\frac{2\left(\frac{3\int x^2\,\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)dx}{2b}-\frac{x^3\,\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{2b}\right)}{b(d+1)}-\frac{x^4\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^5}{5}\right)+\frac{1}{4}x^4\text{arctanh}(d\coth(a+bx)+d+1)$$

$\big\downarrow$ 7163

$$\frac{1}{4}b\left((d+1)\left(\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\int x\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)dx}{b}\right)}{2b}-\frac{x^3\,\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{2b}\right)}{b(d+1)}-\frac{x^4\log\left(1-(d\right.}{2b(d}\right.\right.$$

$$\frac{1}{4}x^4\text{arctanh}(d\coth(a+bx)+d+1)$$

$\big\downarrow$ 7163

$$\frac{1}{4}b\left((d+1)\left(\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\frac{x\,\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\int\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)dx}{2b}}{b}\right)}{2b}-\frac{x^3\,\text{PolyLog}\left(2,(d+1)e^{2a+2\right.}{2b}\right.}{b(d+1)}\right.\right.$$

$$\frac{1}{4}x^4\text{arctanh}(d\coth(a+bx)+d+1)$$

$\big\downarrow$ 2720

3.302. $\int x^3\text{arctanh}(1+d+d\coth(a+bx))\,dx$

$$\frac{1}{4}b\left((d+1)\left(\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\frac{x\,\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}\right)}{2b}-\frac{x^3\,\text{PolyLo}\ldots}{\ldots}\right.}{b(d+1)}\right.\right.$$

$$\frac{1}{4}x^4\text{arctanh}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{4}x^4\text{arctanh}(d\coth(a+bx)+d+1)+$$

$$\frac{1}{4}b\left((d+1)\left(\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\frac{x\,\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\text{PolyLog}\left(5,(d+1)e^{2a+2bx}\right)}{4b^2}}{b}\right)}{2b}-\frac{x^3\,\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{2b}\ldots\right.}{b(d+1)}\right.\right.$$

input
```
Int[x^3*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

output
```
(x^4*ArcTanh[1 + d + d*Coth[a + b*x]])/4 + (b*(x^5/5 + (1 + d)*(-1/2*(x^4*
Log[1 - (1 + d)*E^(2*a + 2*b*x)])/(b*(1 + d)) + (2*(-1/2*(x^3*PolyLog[2, (
1 + d)*E^(2*a + 2*b*x)])/b + (3*((x^2*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])
/(2*b) - ((x*PolyLog[4, (1 + d)*E^(2*a + 2*b*x)])/(2*b) - PolyLog[5, (1 +
d)*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b)))/(b*(1 + d)))))/4
```

3.302. $\int x^3\text{arctanh}(1+d+d\coth(a+bx))\,dx$

3.302.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6795
```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.302. $\int x^3 \text{arctanh}(1 + d + d\coth(a + bx))\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.302.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.40 (sec) , antiderivative size = 1693, normalized size of antiderivative = 11.14

method	result	size
risch	Expression too large to display	1693

input
```
int(x^3*arctanh(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^4*d*a^4/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+3/16/b^4/(1+d)*polylog(5,
(1+d)*exp(2*b*x+2*a))-1/8/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^4+1/8*x^4*ln(
d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)+1/2/b^4*d*a^4/(1+d)*ln(1+exp(b*x+a)*(1+
d)^(1/2))+1/2/b^4*d*a^3/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^4*d*a^
3/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/4/b^4*d/(1+d)*polylog(2,(1+d)*ex
p(2*b*x+2*a))*a^3-3/8/b^3*d/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))*x+1/2/b^
3*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+
a)*(1+d)^(1/2))*x-1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x^3-3/8/b^
4*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^4+3/8/b^2*d/(1+d)*polylog(3,(1+d)*e
xp(2*b*x+2*a))*x^2-1/2/b^3*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a^3+1/2/b^
3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+1/2/b^3*d*a^3/(1+d)*ln(1+exp(
b*x+a)*(1+d)^(1/2))*x+1/20*b*x^5-1/16*(I*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(d*e
xp(2*b*x+2*a)+exp(2*b*x+2*a)-1))^3+I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-I*Pi*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x
+2*a))+I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1
))^2-I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-I*Pi*csgn(I*exp(2*b*
x+2*a))^3-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3+I*Pi*csgn(I*d/(
exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3+2*ln(d)+2*I*Pi*csgn(I*exp(b*x+a))*csgn
(I*exp(2*b*x+2*a))^2-I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)...
```

3.302.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(132) = 264.

Time = 0.27 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.79

$$\int x^3 \text{arctanh}(1+d+d\coth(a+bx))\,dx$$
$$= \frac{2\,b^5x^5 + 5\,b^4x^4\log\left(-\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 20\,b^3x^3\text{Li}_2\left(\sqrt{d+1}(\cosh(bx+a)+\sinh(bx+a))\right) - 2}{}$$

input
```
integrate(x^3*arctanh(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/40*(2*b^5*x^5 + 5*b^4*x^4*log(-(d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))
/(d*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(sqrt(d + 1)*(cosh
(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-sqrt(d + 1)*(cosh(b*x + a)
 + sinh(b*x + a))) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*
x + a) + 2*sqrt(d + 1)) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*si
nh(b*x + a) - 2*sqrt(d + 1)) + 60*b^2*x^2*polylog(3, sqrt(d + 1)*(cosh(b*x
 + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -sqrt(d + 1)*(cosh(b*x + a
) + sinh(b*x + a))) - 120*b*x*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh
(b*x + a))) - 120*b*x*polylog(4, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x +
a))) - 5*(b^4*x^4 - a^4)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) +
 1) - 5*(b^4*x^4 - a^4)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) +
 1) + 120*polylog(5, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) + 120*po
lylog(5, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

3.302.6 Sympy [F]

$$\int x^3 \text{arctanh}(1+d+d\coth(a+bx))\,dx = \int x^3 \operatorname{atanh}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

input
```
integrate(x**3*atanh(1+d+d*coth(b*x+a)),x)
```

output
```
Integral(x**3*atanh(d*coth(a + b*x) + d + 1), x)
```

3.302.7 Maxima [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.96

$$\int x^3 \operatorname{arctanh}(1+d+d\coth(a+bx))\,dx = \frac{1}{4}x^4 \operatorname{artanh}\left(d\coth\left(bx+a\right)+d+1\right) + \frac{1}{40}\left(\frac{2x^5}{d} - \frac{5\left(2b^4x^4\log\left(-(d+1)e^{(2bx+2a)}+1\right)+4b^3x^3\mathrm{Li}_2\left((d+1)e^{(2bx+2a)}\right)-6b^2x^2\mathrm{Li}_3\left((d+1)e^{(2bx}\right.\right.}{b^5d}\right.$$

input
```
integrate(x^3*arctanh(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*x^4*arctanh(d*coth(b*x + a) + d + 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*lo
g(-(d + 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog((d + 1)*e^(2*b*x + 2*a))
 - 6*b^2*x^2*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, (d + 1
)*e^(2*b*x + 2*a)) - 3*polylog(5, (d + 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

3.302.8 Giac [F]

$$\int x^3 \operatorname{arctanh}(1+d+d\coth(a+bx))\,dx = \int x^3 \operatorname{artanh}\left(d\coth\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(x^3*arctanh(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^3*arctanh(d*coth(b*x + a) + d + 1), x)
```

3.302.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{arctanh}(1+d+d\coth(a+bx))\,dx = \int x^3\,\operatorname{atanh}(d+d\coth(a+b\,x)+1)\,dx$$

input
```
int(x^3*atanh(d + d*coth(a + b*x) + 1),x)
```

output
```
int(x^3*atanh(d + d*coth(a + b*x) + 1), x)
```

3.303 $\int x^2 \mathbf{arctanh}(1 + d + d\coth(a + bx))\, dx$

3.303.1 Optimal result

Integrand size = 16, antiderivative size = 126

$$\int x^2 \text{arctanh}(1 + d + d\coth(a + bx))\, dx = \frac{bx^4}{12} + \frac{1}{3}x^3 \text{arctanh}(1 + d + d\coth(a + bx)) - \frac{1}{6}x^3 \log\left(1 - (1 + d)e^{2a+2bx}\right) - \frac{x^2 \operatorname{PolyLog}\left(2, (1 + d)e^{2a+2bx}\right)}{4b} + \frac{x \operatorname{PolyLog}\left(3, (1 + d)e^{2a+2bx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4, (1 + d)e^{2a+2bx}\right)}{8b^3}$$

output
```
1/12*b*x^4+1/3*x^3*arctanh(1+d+d*coth(b*x+a))-1/6*x^3*ln(1-(1+d)*exp(2*b*x
+2*a))-1/4*x^2*polylog(2,(1+d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,(1+d)*exp
(2*b*x+2*a))/b^2-1/8*polylog(4,(1+d)*exp(2*b*x+2*a))/b^3
```

3.303.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95

$$\int x^2 \text{arctanh}(1+d+d\coth(a+bx))\,dx$$
$$= \frac{8b^3x^3\text{arctanh}(1+d+d\coth(a+bx)) - 4b^3x^3\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right) + 6b^2x^2\,\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right) + 6bx\,\text{Po}}{24b^3}$$

input
```
Integrate[x^2*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcTanh[1 + d + d*Coth[a + b*x]] - 4*b^3*x^3*Log[1 - 1/((1 + d)
*E^(2*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))] + 6
*b*x*PolyLog[3, 1/((1 + d)*E^(2*(a + b*x)))] + 3*PolyLog[4, 1/((1 + d)*E^(
2*(a + b*x)))])/(24*b^3)
```

3.303.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6795, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arctanh}(d\coth(a+bx)+d+1)\,dx$$
$$\Big\downarrow \text{6795}$$
$$\frac{1}{3}b\int \frac{x^3}{1-(d+1)e^{2a+2bx}}dx + \frac{1}{3}x^3\text{arctanh}(d\coth(a+bx)+d+1)$$
$$\Big\downarrow \text{2615}$$
$$\frac{1}{3}b\left((d+1)\int \frac{e^{2a+2bx}x^3}{1-(d+1)e^{2a+2bx}}dx + \frac{x^4}{4}\right) + \frac{1}{3}x^3\text{arctanh}(d\coth(a+bx)+d+1)$$
$$\Big\downarrow \text{2620}$$
$$\frac{1}{3}b\left((d+1)\left(\frac{3\int x^2\log\left(1-(d+1)e^{2a+2bx}\right)dx}{2b(d+1)} - \frac{x^3\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right) + \frac{x^4}{4}\right) + \frac{1}{3}x^3\text{arctanh}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}b\left((d+1)\left(\frac{3\left(\frac{\int x\,\text{PolyLog}(2,(d+1)e^{2a+2bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,(d+1)e^{2a+2bx})}{2b}\right)}{2b(d+1)}-\frac{x^3\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^4}{4}\right)+$$
$$\frac{1}{3}x^3\text{arctanh}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3}b\left((d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}(2,(d+1)e^{2a+2bx})}{2b}\right)}{2b(d+1)}-\frac{x^3\log\left(1-(d+1)e^{\right.}}{2b(d+1)}\right.\right.$$
$$\frac{1}{3}x^3\text{arctanh}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3}b\left((d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}(2,(d+1)e^{2a+2bx})}{2b}\right)}{2b(d+1)}-\frac{x^3\log\left(\right.}{}\right.\right.$$
$$\frac{1}{3}x^3\text{arctanh}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{3}x^3\text{arctanh}(d\coth(a+bx)+d+1)+$$
$$\frac{1}{3}b\left((d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}(2,(d+1)e^{2a+2bx})}{2b}\right)}{2b(d+1)}-\frac{x^3\log\left(1-(d+1)e^{2a+\right.}}{2b(d+1)}\right.\right.$$

input
```
Int[x^2*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

output
```
(x^3*ArcTanh[1 + d + d*Coth[a + b*x]])/3 + (b*(x^4/4 + (1 + d)*(-1/2*(x^3*
Log[1 - (1 + d)*E^(2*a + 2*b*x)])/(b*(1 + d)) + (3*(-1/2*(x^2*PolyLog[2, (
1 + d)*E^(2*a + 2*b*x)])/b + ((x*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])/(2*b
) - PolyLog[4, (1 + d)*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b*(1 + d)))))/3
```

3.303. $\int x^2\text{arctanh}(1+d+d\coth(a+bx))\,dx$

3.303.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6795
```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.303. $\int x^2 \text{arctanh}(1+d+d\coth(a+bx))\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.303.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.18 (sec) , antiderivative size = 1636, normalized size of antiderivative = 12.98

method	result	size
risch	Expression too large to display	1636

input
```
int(x^2*arctanh(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^2*a^2*d/(1+d)*x*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^2*a^2*d/(1+d)*x*
ln(1+exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^
2*x+1/12*b*x^4-1/3*x^3*ln(exp(b*x+a))-1/8/b^3*d/(1+d)*polylog(4,(1+d)*exp(
2*b*x+2*a))-1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*a^3/(1+
d)*ln(1+exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*a^2/(1+d)*dilog(1-exp(b*x+a)*(1+d)
^(1/2))-1/2/b^3*a^2/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/4/b/(1+d)*poly
log(2,(1+d)*exp(2*b*x+2*a))*x^2+1/3/b^3/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a
^3+1/4/b^3/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2/(1+d)*polylog
(3,(1+d)*exp(2*b*x+2*a))*x+1/6/b^3*a^3/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x
+2*a)-1)-1/6*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^3-1/6/(1+d)*ln(1-(1+d)*e
xp(2*b*x+2*a))*x^3-1/8/b^3/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))-1/2/b^2*a
^2/(1+d)*x*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^2*a^2/(1+d)*x*ln(1+exp(b*x+a
)*(1+d)^(1/2))-1/2/b^3*a^3*d/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*a^
3*d/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*a^2*d/(1+d)*dilog(1-exp(b*x
+a)*(1+d)^(1/2))-1/2/b^3*a^2*d/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))+1/6/b
^3*d*a^3/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)+1/2/b^2/(1+d)*ln(1-(1
+d)*exp(2*b*x+2*a))*a^2*x-1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x^
2+1/3/b^3*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^3+1/4/b^3*d/(1+d)*polylog(2
,(1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))
*x+1/6*x^3*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)-1/12*(I*Pi*csgn(I/(exp...
```

3.303.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(109) = 218$.

Time = 0.26 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.86

$$\int x^2 \text{arctanh}(1 + d + d\coth(a + bx))\,dx$$
$$= \frac{b^4x^4 + 2\,b^3x^3\log\left(-\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2\text{Li}_2\left(\sqrt{d+1}(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,b^2 \cdots}{}$$

input
```
integrate(x^2*arctanh(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(b^4*x^4 + 2*b^3*x^3*log(-(d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(
d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d + 1)*(cosh(b*
x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d + 1)*(cosh(b*x + a) + s
inh(b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x +
a) + 2*sqrt(d + 1)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b
*x + a) - 2*sqrt(d + 1)) + 12*b*x*polylog(3, sqrt(d + 1)*(cosh(b*x + a) +
sinh(b*x + a))) + 12*b*x*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x
 + a))) - 2*(b^3*x^3 + a^3)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)
) + 1) - 2*(b^3*x^3 + a^3)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)
) + 1) - 12*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*p
olylog(4, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

3.303.6 Sympy [F]

$$\int x^2 \text{arctanh}(1 + d + d\coth(a + bx))\,dx = \int x^2 \operatorname{atanh}(d\coth(a + bx) + d + 1)\,dx$$

input
```
integrate(x**2*atanh(1+d+d*coth(b*x+a)),x)
```

output
```
Integral(x**2*atanh(d*coth(a + b*x) + d + 1), x)
```

3.303.7 Maxima [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

$$\int x^2 \text{arctanh}(1+d+d\coth(a+bx))\,dx = \frac{1}{3}x^3 \,\text{artanh}\left(d\coth(bx+a)+d+1\right)$$
$$+\frac{1}{36}\left(\frac{3x^4}{d}-\frac{2\left(4b^3x^3\log\left(-(d+1)e^{(2bx+2a)}+1\right)+6b^2x^2\text{Li}_2\left((d+1)e^{(2bx+2a)}\right)-6bx\text{Li}_3\left((d+1)e^{(2bx+}\right.\right.}{b^4d}\right.$$

input
```
integrate(x^2*arctanh(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctanh(d*coth(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*lo
g(-(d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d + 1)*e^(2*b*x + 2*a))
 - 6*b*x*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d + 1)*e^(2*b
*x + 2*a)))/(b^4*d))*b*d
```

3.303.8 Giac [F]

$$\int x^2 \text{arctanh}(1+d+d\coth(a+bx))\,dx = \int x^2 \,\text{artanh}\left(d\coth(bx+a)+d+1\right)\,dx$$

input
```
integrate(x^2*arctanh(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(d*coth(b*x + a) + d + 1), x)
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arctanh}(1+d+d\coth(a+bx))\,dx = \int x^2 \,\text{atanh}(d+d\coth(a+bx)+1)\,dx$$

input
```
int(x^2*atanh(d + d*coth(a + b*x) + 1),x)
```

output
```
int(x^2*atanh(d + d*coth(a + b*x) + 1), x)
```

3.304 $\int x\text{arctanh}(1+d+d\coth(a+bx))\,dx$

3.304.1 Optimal result

Integrand size = 14, antiderivative size = 100

$$\int x\text{arctanh}(1+d+d\coth(a+bx))\,dx=\frac{bx^3}{6}+\frac{1}{2}x^2\text{arctanh}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)e^{2a+2bx}\right)-\frac{x\,\text{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b}+\frac{\text{PolyLog}\left(3,(1+d)e^{2a+2bx}\right)}{8b^2}$$

output
```
1/6*b*x^3+1/2*x^2*arctanh(1+d+d*coth(b*x+a))-1/4*x^2*ln(1-(1+d)*exp(2*b*x+
2*a))-1/4*x*polylog(2,(1+d)*exp(2*b*x+2*a))/b+1/8*polylog(3,(1+d)*exp(2*b*
x+2*a))/b^2
```

3.304.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int x\text{arctanh}(1+d+d\coth(a+bx))\,dx$$
$$=\frac{2b^2x^2\left(2\text{arctanh}(1+d+d\coth(a+bx))-\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right)\right)+2bx\,\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right)+\text{PolyLog}\left(\right.}{8b^2}$$

input
```
Integrate[x*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcTanh[1 + d + d*Coth[a + b*x]] - Log[1 - 1/((1 + d)*E^(2*(
a + b*x)))]) + 2*b*x*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))] + PolyLog[3,
1/((1 + d)*E^(2*(a + b*x)))])/(8*b^2)
```

3.304.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6795, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(d\coth(a+bx)+d+1)\,dx$$

$$\Big\downarrow \text{ 6795}$$

$$\frac{1}{2}b\int \frac{x^2}{1-(d+1)e^{2a+2bx}}dx+\frac{1}{2}x^2\text{arctanh}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{1}{2}b\left((d+1)\int \frac{e^{2a+2bx}x^2}{1-(d+1)e^{2a+2bx}}dx+\frac{x^3}{3}\right)+\frac{1}{2}x^2\text{arctanh}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{2}b\left((d+1)\left(\frac{\int x\log\left(1-(d+1)e^{2a+2bx}\right)dx}{b(d+1)}-\frac{x^2\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^3}{3}\right)+\frac{1}{2}x^2\text{arctanh}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{1}{2}b\left((d+1)\left(\frac{\frac{\int \text{PolyLog}(2,(d+1)e^{2a+2bx})dx}{2b}-\frac{x\,\text{PolyLog}(2,(d+1)e^{2a+2bx})}{2b}}{b(d+1)}-\frac{x^2\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^3}{3}\right)+\frac{1}{2}x^2\text{arctanh}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{2}b\left((d+1)\left(\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}(2,(d+1)e^{2a+2bx})de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}(2,(d+1)e^{2a+2bx})}{2b}}{b(d+1)}-\frac{x^2\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x}{\ }\right.$$

$$\frac{1}{2}x^2\text{arctanh}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x^2\text{arctanh}(d\coth(a+bx)+d+1)+$$

$$\frac{1}{2}b\left((d+1)\left(\frac{\frac{\text{PolyLog}(3,(d+1)e^{2a+2bx})}{4b^2}-\frac{x\,\text{PolyLog}(2,(d+1)e^{2a+2bx})}{2b}}{b(d+1)}-\frac{x^2\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^3}{3}\right)$$

input
```
Int[x*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

output
```
(x^2*ArcTanh[1 + d + d*Coth[a + b*x]])/2 + (b*(x^3/3 + (1 + d)*(-1/2*(x^2*
Log[1 - (1 + d)*E^(2*a + 2*b*x)])/(b*(1 + d)) + (-1/2*(x*PolyLog[2, (1 + d
)*E^(2*a + 2*b*x)])/b + PolyLog[3, (1 + d)*E^(2*a + 2*b*x)]/(4*b^2))/(b*(1
 + d)))))/2
```

3.304.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6795
```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.304.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.92 (sec) , antiderivative size = 1555, normalized size of antiderivative = 15.55

method	result	size
risch	Expression too large to display	1555

input
```
int(x*arctanh(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/8/b^2/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))-1/4/(1+d)*ln(1-(1+d)*exp(2*b
*x+2*a))*x^2-1/2*x^2*ln(exp(b*x+a))+1/6*b*x^3-1/2/b*d/(1+d)*ln(1-(1+d)*exp
(2*b*x+2*a))*a*x+1/2/b*a*d/(1+d)*x*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b*a*d/
(1+d)*x*ln(1+exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(1+
d)^(1/2))+1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a/(1+d)*d
ilog(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1
/2))-1/4/b^2*a^2/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)-1/4/b^2/(1+d)
*ln(1-(1+d)*exp(2*b*x+2*a))*a^2-1/4/b/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a)
)*x-1/4/b^2/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a-1/4*d/(1+d)*ln(1-(1+d)
*exp(2*b*x+2*a))*x^2+1/8/b^2*d/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))-1/4/b
^2*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^2-1/4/b*d/(1+d)*polylog(2,(1+d)*ex
p(2*b*x+2*a))*x-1/4/b^2*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a-1/2/b/(1
+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a*x+1/2/b^2*a*d/(1+d)*dilog(1-exp(b*x+a)*(1
+d)^(1/2))+1/2/b^2*a*d/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))+1/2/b*a/(1+d)
*x*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b*a/(1+d)*x*ln(1+exp(b*x+a)*(1+d)^(1/2
))+1/2/b^2*a^2*d/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a^2*d/(1+d)*ln
(1+exp(b*x+a)*(1+d)^(1/2))-1/4/b^2*a^2*d/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b
*x+2*a)-1)+1/4*x^2*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)-1/8*(I*Pi*csgn(I/
(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))^3+I*Pi*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*...
```

3.304.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(86) = 172$.

Time = 0.27 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.06

$$\int x\text{arctanh}(1 + d + d\coth(a + bx))\, dx$$

$$= \frac{2\, b^3x^3 + 3\, b^2x^2 \log\left(-\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\, bx\text{Li}_2\left(\sqrt{d+1}(\cosh(bx+a) + \sinh(bx+a))\right) - 6\, bx}{}$$

input
```
integrate(x*arctanh(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*x^3 + 3*b^2*x^2*log(-(d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))
/(d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(sqrt(d + 1)*(cosh(b*x
+ a) + sinh(b*x + a))) - 6*b*x*dilog(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*
x + a))) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) + 2
*sqrt(d + 1)) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a
) - 2*sqrt(d + 1)) - 3*(b^2*x^2 - a^2)*log(sqrt(d + 1)*(cosh(b*x + a) + si
nh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-sqrt(d + 1)*(cosh(b*x + a) + si
nh(b*x + a)) + 1) + 6*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a
))) + 6*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

3.304.6 Sympy [F]

$$\int x\text{arctanh}(1+d+d\coth(a+bx))\,dx=\int x\,\text{atanh}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

input
```
integrate(x*atanh(1+d+d*coth(b*x+a)),x)
```

output
```
Integral(x*atanh(d*coth(a + b*x) + d + 1), x)
```

3.304.7 Maxima [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int x\text{arctanh}(1+d+d\coth(a+bx))\,dx$$
$$=\frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\text{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)-\text{Li}_3\left((d+1)e^{(2\,bx+2\,a)}\right)\right)}{b^3d}\right)b$$
$$+\frac{1}{2}\,x^2\,\text{artanh}\left(d\coth\left(bx+a\right)+d+1\right)$$

input
```
integrate(x*arctanh(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(4*x^3/d - 3*(2*b^2*x^2*log(-(d + 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dil
og((d + 1)*e^(2*b*x + 2*a)) - polylog(3, (d + 1)*e^(2*b*x + 2*a)))/(b^3*d)
)*b*d + 1/2*x^2*arctanh(d*coth(b*x + a) + d + 1)
```

3.304.8 Giac [F]

$$\int x\text{arctanh}(1+d+d\coth(a+bx))\,dx=\int x\,\text{artanh}\left(d\coth\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(x*arctanh(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctanh(d*coth(b*x + a) + d + 1), x)
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arctanh}(1+d+d\coth(a+bx))\,dx=\int x\,\text{atanh}(d+d\coth(a+b\,x)+1)\,dx$$

input
```
int(x*atanh(d + d*coth(a + b*x) + 1),x)
```

output
```
int(x*atanh(d + d*coth(a + b*x) + 1), x)
```

3.305 $\int \operatorname{arctanh}(1+d+d\coth(a+bx))\,dx$

3.305.1 Optimal result

Integrand size = 12, antiderivative size = 69

$$\int \operatorname{arctanh}(1+d+d\coth(a+bx))\,dx = \frac{bx^2}{2} + x\operatorname{arctanh}(1+d+d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{\operatorname{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b}$$

output
```
1/2*b*x^2+x*arctanh(1+d+d*coth(b*x+a))-1/2*x*ln(1-(1+d)*exp(2*b*x+2*a))-1/
4*polylog(2,(1+d)*exp(2*b*x+2*a))/b
```

3.305.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \operatorname{arctanh}(1+d+d\coth(a+bx))\,dx = x\operatorname{arctanh}(1+d+d\coth(a+bx)) + \frac{-2bx\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right)+\operatorname{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right)}{4b}$$

input
```
Integrate[ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

output
```
x*ArcTanh[1 + d + d*Coth[a + b*x]] + (-2*b*x*Log[1 - 1/((1 + d)*E^(2*(a +
b*x)))] + PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))])/(4*b)
```

3.305.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6787, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(d\coth(a+bx)+d+1)\,dx$$

$$\Big\downarrow \text{6787}$$

$$b\int \frac{x}{1-(d+1)e^{2a+2bx}}dx+x\operatorname{arctanh}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{2615}$$

$$b\left((d+1)\int \frac{e^{2a+2bx}x}{1-(d+1)e^{2a+2bx}}dx+\frac{x^2}{2}\right)+x\operatorname{arctanh}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{2620}$$

$$b\left((d+1)\left(\frac{\int \log\left(1-(d+1)e^{2a+2bx}\right)dx}{2b(d+1)}-\frac{x\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^2}{2}\right)+ x\operatorname{arctanh}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{2715}$$

$$b\left((d+1)\left(\frac{\int e^{-2a-2bx}\log\left(1-(d+1)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2(d+1)}-\frac{x\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^2}{2}\right)+ x\operatorname{arctanh}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{2838}$$

$$x\operatorname{arctanh}(d\coth(a+bx)+d+1)+ b\left((d+1)\left(-\frac{\operatorname{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b^2(d+1)}-\frac{x\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^2}{2}\right)$$

input
```
Int[ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

output
```
x*ArcTanh[1 + d + d*Coth[a + b*x]] + b*(x^2/2 + (1 + d)*(-1/2*(x*Log[1 - (
1 + d)*E^(2*a + 2*b*x)])/(b*(1 + d)) - PolyLog[2, (1 + d)*E^(2*a + 2*b*x)]
/(4*b^2*(1 + d))))
```

3.305.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6787
```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Coth[a + b*x]], x] + Simp[b   Int[x/(c - d - c*E^(2*a + 2*b*x))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

3.305.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(61) = 122$.

Time = 0.94 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.70

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(1+d+d\coth(bx+a))d\ln(d+d\coth(bx+a))}{2}-\frac{\operatorname{arctanh}(1+d+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}-\frac{d^2\left(-\frac{\operatorname{dilog}\left(\frac{-d\coth(bx+\dots}{-2d-\dots}\right)}{2}\right.}{\dots}}{\dots}$
default	$\frac{\frac{\operatorname{arctanh}(1+d+d\coth(bx+a))d\ln(d+d\coth(bx+a))}{2}-\frac{\operatorname{arctanh}(1+d+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}-\frac{d^2\left(-\frac{\operatorname{dilog}\left(\frac{-d\coth(bx+\dots}{-2d-\dots}\right)}{2}\right.}{\dots}}{\dots}$
risch	Expression too large to display

input
```
int(arctanh(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(1/2*arctanh(1+d+d*coth(b*x+a))*d*ln(d+d*coth(b*x+a))-1/2*arctanh(1+
d+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)-1/2*d^2*(1/d*(-1/2*dilog((-d*coth(
b*x+a)-d-2)/(-2*d-2))-1/2*ln(-d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)-d-2)/(-2
*d-2))+1/2*dilog(-1/2*(-d*coth(b*x+a)-d)/d)+1/2*ln(-d*coth(b*x+a)+d)*ln(-1
/2*(-d*coth(b*x+a)-d)/d))-1/d*(1/4*ln(d+d*coth(b*x+a))^2-1/2*dilog(1/2*d*c
oth(b*x+a)+1/2*d+1)-1/2*ln(d+d*coth(b*x+a))*ln(1/2*d*coth(b*x+a)+1/2*d+1))
))
```

3.305.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(60) = 120$.

Time = 0.26 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.29

$$\int \operatorname{arctanh}(1+d+d\coth(a+bx))\,dx$$
$$=\frac{b^2x^2+bx\log\left(-\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)+a\log\left(2\left(d+1\right)\cosh\left(bx+a\right)+2\left(d+1\right)\sinh\left(bx+a\right)+2\sqrt{\dots}\right.}{\dots}$$

input
```
integrate(arctanh(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 + b*x*log(-(d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d*cosh(
b*x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*s
inh(b*x + a) + 2*sqrt(d + 1)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*
sinh(b*x + a) - 2*sqrt(d + 1)) - (b*x + a)*log(sqrt(d + 1)*(cosh(b*x + a)
+ sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b
*x + a)) + 1) - dilog(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog
(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

3.305.6 Sympy [F]

$$\int \operatorname{arctanh}(1+d+d\coth(a+bx))\,dx = \int \operatorname{atanh}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

input
```
integrate(atanh(1+d+d*coth(b*x+a)),x)
```

output
```
Integral(atanh(d*coth(a + b*x) + d + 1), x)
```

3.305.7 Maxima [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \operatorname{arctanh}(1+d+d\coth(a+bx))\,dx = \frac{1}{4}bd\left(\frac{2x^2}{d}-\frac{2bx\log\left(-(d+1)e^{(2bx+2a)}+1\right)+\operatorname{Li}_2\left((d+1)e^{(2bx+2a)}\right)}{b^2d}\right) + x\operatorname{artanh}\left(d\coth\left(bx+a\right)+d+1\right)$$

input
```
integrate(arctanh(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*b*d*(2*x^2/d - (2*b*x*log(-(d + 1)*e^(2*b*x + 2*a) + 1) + dilog((d + 1
)*e^(2*b*x + 2*a)))/(b^2*d)) + x*arctanh(d*coth(b*x + a) + d + 1)
```

3.305.8 Giac [F]

$$\int \operatorname{arctanh}(1+d+d\coth(a+bx))\,dx = \int \operatorname{artanh}\left(d\coth\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(arctanh(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctanh(d*coth(b*x + a) + d + 1), x)
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}(1+d+d\coth(a+bx))\,dx = \int \operatorname{atanh}(d+d\coth(a+b\,x)+1)\,dx$$

input
```
int(atanh(d + d*coth(a + b*x) + 1),x)
```

output
```
int(atanh(d + d*coth(a + b*x) + 1), x)
```

3.306 $\int \frac{\operatorname{arctanh}(1+d+d\coth(a+bx))}{x}\,dx$

3.306.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{arctanh}(1+d+d\coth(a+bx))}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{arctanh}(1+d+d\coth(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arctanh(1+d+d*coth(b*x+a))/x,x)
```

3.306.2 Mathematica [N/A]

Not integrable

Time = 3.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{arctanh}(1+d+d\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arctanh}(1+d+d\coth(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTanh[1 + d + d*Coth[a + b*x]]/x,x]
```

output
```
Integrate[ArcTanh[1 + d + d*Coth[a + b*x]]/x, x]
```

3.306.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(d\coth(a+bx)+d+1)}{x}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\text{arctanh}(d\coth(a+bx)+d+1)}{x}dx$$

input
```
Int[ArcTanh[1 + d + d*Coth[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.306.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.306.4 Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\text{arctanh}\left(1+d+d\coth\left(bx+a\right)\right)}{x}dx$$

input
```
int(arctanh(1+d+d*coth(b*x+a))/x,x)
```

output
```
int(arctanh(1+d+d*coth(b*x+a))/x,x)
```

3.306.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{arctanh}(1+d+d\coth(a+bx))}{x}\,dx = \int \frac{\text{artanh}\left(d\coth\left(bx+a\right)+d+1\right)}{x}\,dx$$

input
```
integrate(arctanh(1+d+d*coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arctanh(d*coth(b*x + a) + d + 1)/x, x)
```

3.306.6 Sympy [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\text{arctanh}(1+d+d\coth(a+bx))}{x}\,dx = \int \frac{\text{atanh}\left(d\coth\left(a+bx\right)+d+1\right)}{x}\,dx$$

input
```
integrate(atanh(1+d+d*coth(b*x+a))/x,x)
```

output
```
Integral(atanh(d*coth(a + b*x) + d + 1)/x, x)
```

3.306.7 Maxima [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{arctanh}(1+d+d\coth(a+bx))}{x}\,dx = \int \frac{\text{artanh}\left(d\coth\left(bx+a\right)+d+1\right)}{x}\,dx$$

input
```
integrate(arctanh(1+d+d*coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arctanh(d*coth(b*x + a) + d + 1)/x, x)
```

3.306.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{arctanh}(1+d+d\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{artanh}\left(d\coth\left(bx+a\right)+d+1\right)}{x}\,dx$$

input
```
integrate(arctanh(1+d+d*coth(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctanh(d*coth(b*x + a) + d + 1)/x, x)
```

3.306.9 Mupad [N/A]

Not integrable

Time = 4.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{arctanh}(1+d+d\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{atanh}(d+d\coth(a+b\,x)+1)}{x}\,dx$$

input
```
int(atanh(d + d*coth(a + b*x) + 1)/x,x)
```

output
```
int(atanh(d + d*coth(a + b*x) + 1)/x, x)
```

3.307 $\int x^3 \text{arctanh}(1-d-d\coth(a+bx))\,dx$

3.307.1 Optimal result

Integrand size = 19, antiderivative size = 165

$$\int x^3 \text{arctanh}(1-d-d\coth(a+bx))\,dx = \frac{bx^5}{20} + \frac{1}{4}x^4 \text{arctanh}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4 \log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x^3 \operatorname{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b} + \frac{3x^2 \operatorname{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{8b^2} - \frac{3x \operatorname{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{8b^3} + \frac{3 \operatorname{PolyLog}\left(5,(1-d)e^{2a+2bx}\right)}{16b^4}$$

output
```
1/20*b*x^5-1/4*x^4*arctanh(-1+d+d*coth(b*x+a))-1/8*x^4*ln(1-(1-d)*exp(2*b*
x+2*a))-1/4*x^3*polylog(2,(1-d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,(1-d)*
exp(2*b*x+2*a))/b^2-3/8*x*polylog(4,(1-d)*exp(2*b*x+2*a))/b^3+3/16*polylog
(5,(1-d)*exp(2*b*x+2*a))/b^4
```

3.307.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92

$$\int x^3 \operatorname{arctanh}(1-d-d\coth(a+bx))\,dx$$
$$= \frac{4b^4x^4\operatorname{arctanh}(1-d-d\coth(a+bx)) - 2b^4x^4\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right) + 4b^3x^3\operatorname{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right) + 6b^2x^2 \cdots}{16b^4}$$

input
```
Integrate[x^3*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

output
```
(4*b^4*x^4*ArcTanh[1 - d - d*Coth[a + b*x]] - 2*b^4*x^4*Log[1 + 1/((-1 + d
)*E^(2*(a + b*x)))] + 4*b^3*x^3*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))
] + 6*b^2*x^2*PolyLog[3, -(1/((-1 + d)*E^(2*(a + b*x))))] + 6*b*x*PolyLog[
4, -(1/((-1 + d)*E^(2*(a + b*x))))] + 3*PolyLog[5, -(1/((-1 + d)*E^(2*(a +
 b*x))))])/(16*b^4)
```

3.307.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {6795, 2615, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \operatorname{arctanh}(d(-\coth(a+bx))-d+1)\,dx$$
$$\Big\downarrow \text{ 6795}$$
$$\frac{1}{4}b\int \frac{x^4}{1-(1-d)e^{2a+2bx}}dx + \frac{1}{4}x^4\operatorname{arctanh}(d(-\coth(a+bx))-d+1)$$
$$\Big\downarrow \text{ 2615}$$
$$\frac{1}{4}b\left((1-d)\int \frac{e^{2a+2bx}x^4}{1-(1-d)e^{2a+2bx}}dx + \frac{x^5}{5}\right) + \frac{1}{4}x^4\operatorname{arctanh}(d(-\coth(a+bx))-d+1)$$
$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{4}b\left((1-d)\left(\frac{2\int x^3\log\left(1-(1-d)e^{2a+2bx}\right)dx}{b(1-d)}-\frac{x^4\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^5}{5}\right)+\frac{1}{4}x^4\text{arctanh}(d(-\coth(a+bx))-d+1)$$

$\big\downarrow$ 3011

$$\frac{1}{4}b\Bigg((1-d)\Bigg(\frac{2\left(\frac{3\int x^2\,\text{PolyLog}(2,(1-d)e^{2a+2bx})dx}{2b}-\frac{x^3\,\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}\right)}{b(1-d)}-\frac{x^4\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\Bigg)+\frac{x^5}{5}\Bigg)$$

$$\frac{1}{4}x^4\text{arctanh}(d(-\coth(a+bx))-d+1)$$

$\big\downarrow$ 7163

$$\frac{1}{4}b\Bigg((1-d)\Bigg(\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\int x\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)dx}{b}\right)}{2b}-\frac{x^3\,\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}\right)}{b(1-d)}-\frac{x^4\log(1-(1}{2b(1}$$

$$\frac{1}{4}x^4\text{arctanh}(d(-\coth(a+bx))-d+1)$$

$\big\downarrow$ 7163

$$\frac{1}{4}b\Bigg((1-d)\Bigg(\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\frac{x\,\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\int\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)dx}{2b}}{b}\right)}{2b}-\frac{x^3\,\text{PolyLog}(2,(1-d)e^{2a+2}}{2b}}{b(1-d)}$$

$$\frac{1}{4}x^4\text{arctanh}(d(-\coth(a+bx))-d+1)$$

$\big\downarrow$ 2720

3.307. $\int x^3\text{arctanh}(1-d-d\coth(a+bx))\,dx$

$$\frac{1}{4}b\left((1-d)\left(\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\frac{x\,\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}\right)}{2b}-\frac{x^3\,\text{PolyLo}}{}\right.}{b(1-d)}\right.\right.$$

$$\frac{1}{4}x^4\text{arctanh}(d(-\coth(a+bx))-d+1)$$

$\downarrow$ 7143

$$\frac{1}{4}x^4\text{arctanh}(d(-\coth(a+bx))-d+1)+$$

$$\frac{1}{4}b\left((1-d)\left(\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\frac{x\,\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\text{PolyLog}\left(5,(1-d)e^{2a+2bx}\right)}{4b^2}}{b}\right)}{2b}-\frac{x^3\,\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{2b}\right.}{b(1-d)}\right.\right.$$

input
```
Int[x^3*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

output
```
(x^4*ArcTanh[1 - d - d*Coth[a + b*x]])/4 + (b*(x^5/5 + (1 - d)*(-1/2*(x^4*
Log[1 - (1 - d)*E^(2*a + 2*b*x)])/(b*(1 - d)) + (2*(-1/2*(x^3*PolyLog[2, (
1 - d)*E^(2*a + 2*b*x)])/b + (3*((x^2*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])
/(2*b) - ((x*PolyLog[4, (1 - d)*E^(2*a + 2*b*x)])/(2*b) - PolyLog[5, (1 -
d)*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b)))/(b*(1 - d)))))/4
```

3.307. $\int x^3\text{arctanh}(1-d-d\coth(a+bx))\,dx$

3.307.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6795
```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.307. $\int x^3 \text{arctanh}(1-d-d\coth(a+bx))\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.307.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.46 (sec) , antiderivative size = 1753, normalized size of antiderivative = 10.62

method	result	size
risch	Expression too large to display	1753

input
```
int(-x^3*arctanh(-1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^4*d*a^3/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^4*a^3/(d-1)*dilo
g(1+exp(b*x+a)*(1-d)^(1/2))-1/8*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^4-1/2
/b^4*a^4/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^4*a^4/(d-1)*ln(1-exp(b*x
+a)*(1-d)^(1/2))+3/16/b^4*d/(d-1)*polylog(5,-(d-1)*exp(2*b*x+2*a))+1/4/b/(
d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^3+3/8/b^4/(d-1)*ln(1+(d-1)*exp(2*b
*x+2*a))*a^4-3/8/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x^2+1/4/b^4/(d
-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a^3-1/2/b^4*a^3/(d-1)*dilog(1-exp(b*x
+a)*(1-d)^(1/2))+3/8/b^3/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))*x+1/2/b^4*
d*a^3/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))+1/2/b^3/(d-1)*ln(1+(d-1)*exp(2
*b*x+2*a))*x*a^3-1/4/b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^3-3/8/b^
4*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^4+3/8/b^2*d/(d-1)*polylog(3,-(d-1)*
exp(2*b*x+2*a))*x^2-1/4/b^4*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a^3-1
/2/b^3*a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x-1/2/b^3*a^3/(d-1)*ln(1-exp
(b*x+a)*(1-d)^(1/2))*x-3/8/b^3*d/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))*x+
1/2/b^4*d*a^4/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^4*d*a^4/(d-1)*ln(1-
exp(b*x+a)*(1-d)^(1/2))+1/8/b^4*a^4/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*
a)+1)+1/20*b*x^5-1/16*(I*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*
x+2*a))^2-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d)*csgn(I*
d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))-I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(e
xp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))-I*Pi*csgn(I...
```

3.307.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(135) = 270$.

Time = 0.27 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.73

$$\int x^3 \text{arctanh}(1-d-d\coth(a+bx))\,dx$$
$$= \frac{2\,b^5x^5 - 5\,b^4x^4\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) - 20\,b^3x^3\text{Li}_2\left(\frac{1}{2}\sqrt{-4\,d+4}(\cosh(bx+a)+\sinh(bx+a)\right)}{}$$

input
```
integrate(-x^3*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/40*(2*b^5*x^5 - 5*b^4*x^4*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/(d*co
sh(b*x + a) + (d - 2)*sinh(b*x + a))) - 20*b^3*x^3*dilog(1/2*sqrt(-4*d + 4
)*(cosh(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-1/2*sqrt(-4*d + 4)*
(cosh(b*x + a) + sinh(b*x + a))) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(
d - 1)*sinh(b*x + a) + sqrt(-4*d + 4)) - 5*a^4*log(2*(d - 1)*cosh(b*x + a)
 + 2*(d - 1)*sinh(b*x + a) - sqrt(-4*d + 4)) + 60*b^2*x^2*polylog(3, 1/2*s
qrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -1/
2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, 1/2
*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, -1/2
*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(b^4*x^4 - a^4)*log(1
/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^4*x^4 - a^4)
*log(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*polylo
g(5, 1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5,
-1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

3.307.6 Sympy [F]

$$\int x^3 \text{arctanh}(1-d-d\coth(a+bx))\,dx = -\int x^3 \operatorname{atanh}\left(d\coth\left(a+bx\right)+d-1\right)dx$$

input
```
integrate(-x**3*atanh(-1+d+d*coth(b*x+a)),x)
```

output
```
-Integral(x**3*atanh(d*coth(a + b*x) + d - 1), x)
```

3.307.7 Maxima [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.90

$$\int x^3\text{arctanh}(1-d-d\coth(a+bx))\,dx = -\frac{1}{4}x^4\,\text{artanh}\,(d\coth\,(bx+a)+d-1)$$
$$+\frac{1}{40}\left(\frac{2\,x^5}{d}-\frac{5\left(2\,b^4x^4\log\left((d-1)e^{(2\,bx+2\,a)}+1\right)+4\,b^3x^3\text{Li}_2\left(-(d-1)e^{(2\,bx+2\,a)}\right)-6\,b^2x^2\text{Li}_3(-(d-1)e^{(\cdots}}{b^5d}\right.$$

input
```
integrate(-x^3*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
-1/4*x^4*arctanh(d*coth(b*x + a) + d - 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*l
og((d - 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog(-(d - 1)*e^(2*b*x + 2*a)
) - 6*b^2*x^2*polylog(3, -(d - 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, -(d
- 1)*e^(2*b*x + 2*a)) - 3*polylog(5, -(d - 1)*e^(2*b*x + 2*a)))/(b^5*d))*b
*d
```

3.307.8 Giac [F]

$$\int x^3\text{arctanh}(1-d-d\coth(a+bx))\,dx = \int -x^3\,\text{artanh}\,(d\coth\,(bx+a)+d-1)\;dx$$

input
```
integrate(-x^3*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-x^3*arctanh(d*coth(b*x + a) + d - 1), x)
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int x^3\text{arctanh}(1-d-d\coth(a+bx))\,dx = \int -x^3\,\text{atanh}(d+d\coth(a+b\,x)-1)\;dx$$

input
```
int(-x^3*atanh(d + d*coth(a + b*x) - 1),x)
```

output
```
int(-x^3*atanh(d + d*coth(a + b*x) - 1), x)
```

3.308 $\int x^2 \mathbf{arctanh}(1-d-d\coth(a+bx))\,dx$

3.308.1 Optimal result

Integrand size = 19, antiderivative size = 137

$$\int x^2\text{arctanh}(1-d-d\coth(a+bx))\,dx = \frac{bx^4}{12}+\frac{1}{3}x^3\text{arctanh}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x^2\,\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b} + \frac{x\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{4b^2} - \frac{\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{8b^3}$$

output
```
1/12*b*x^4-1/3*x^3*arctanh(-1+d+d*coth(b*x+a))-1/6*x^3*ln(1-(1-d)*exp(2*b*
x+2*a))-1/4*x^2*polylog(2,(1-d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,(1-d)*ex
p(2*b*x+2*a))/b^2-1/8*polylog(4,(1-d)*exp(2*b*x+2*a))/b^3
```

3.308.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int x^2\text{arctanh}(1-d-d\coth(a+bx))\,dx$$
$$=\frac{8b^3x^3\text{arctanh}(1-d-d\coth(a+bx))-4b^3x^3\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right)+6b^2x^2\,\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right)+6bx\,\text{F}}{24b^3}$$

input
```
Integrate[x^2*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcTanh[1 - d - d*Coth[a + b*x]] - 4*b^3*x^3*Log[1 + 1/((-1 + d
)*E^(2*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))
] + 6*b*x*PolyLog[3, -(1/((-1 + d)*E^(2*(a + b*x))))] + 3*PolyLog[4, -(1/(
(-1 + d)*E^(2*(a + b*x))))])/(24*b^3)
```

3.308.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6795, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arctanh}(d(-\coth(a+bx))-d+1)\,dx$$
$$\Big\downarrow \text{6795}$$
$$\frac{1}{3}b\int\frac{x^3}{1-(1-d)e^{2a+2bx}}dx+\frac{1}{3}x^3\text{arctanh}(d(-\coth(a+bx))-d+1)$$
$$\Big\downarrow \text{2615}$$
$$\frac{1}{3}b\left((1-d)\int\frac{e^{2a+2bx}x^3}{1-(1-d)e^{2a+2bx}}dx+\frac{x^4}{4}\right)+\frac{1}{3}x^3\text{arctanh}(d(-\coth(a+bx))-d+1)$$
$$\Big\downarrow \text{2620}$$
$$\frac{1}{3}b\left((1-d)\left(\frac{3\int x^2\log\left(1-(1-d)e^{2a+2bx}\right)dx}{2b(1-d)}-\frac{x^3\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^4}{4}\right)+$$
$$\frac{1}{3}x^3\text{arctanh}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \ 3011$$

$$\frac{1}{3}b\left((1-d)\left(\frac{3\left(\frac{\int x\,\text{PolyLog}(2,(1-d)e^{2a+2bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}\right)}{2b(1-d)}-\frac{x^3\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^4}{4}\right)+\frac{1}{3}x^3\text{arctanh}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \ 7163$$

$$\frac{1}{3}b\left((1-d)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}\right)}{2b(1-d)}-\frac{x^3\log\left(1-(1-d)e\right.}{2b(1-d)}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \ 2720$$

$$\frac{1}{3}b\left((1-d)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}\right)}{2b(1-d)}-\frac{x^3\log\left(\right.}{}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \ 7143$$

$$\frac{1}{3}x^3\text{arctanh}(d(-\coth(a+bx))-d+1)+$$

$$\frac{1}{3}b\left((1-d)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}\right)}{2b(1-d)}-\frac{x^3\log\left(1-(1-d)e^{2a+}\right.}{2b(1-d)}\right.\right.$$

input
```
Int[x^2*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

output
```
(x^3*ArcTanh[1 - d - d*Coth[a + b*x]])/3 + (b*(x^4/4 + (1 - d)*(-1/2*(x^3*
Log[1 - (1 - d)*E^(2*a + 2*b*x)])/(b*(1 - d)) + (3*(-1/2*(x^2*PolyLog[2, (
1 - d)*E^(2*a + 2*b*x)])/b + ((x*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])/(2*b
) - PolyLog[4, (1 - d)*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b*(1 - d)))))/3
```

3.308. $\int x^2\text{arctanh}(1-d-d\coth(a+bx))\,dx$

3.308.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6795
```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.308. $\int x^2 \text{arctanh}(1-d-d\coth(a+bx))\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.308.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.20 (sec) , antiderivative size = 1694, normalized size of antiderivative = 12.36

method	result	size
risch	Expression too large to display	1694

input
```
int(-x^2*arctanh(-1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*a^2/(d-1)*x*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^2*a^2/(d-1)*x*ln(1-
exp(b*x+a)*(1-d)^(1/2))-1/2/b^3*a^3*d/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))-1
/2/b^3*a^3*d/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))-1/2/b^3*a^2*d/(d-1)*dilog(
1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^3*a^2*d/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/
2))+1/6/b^3*d*a^3/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1)-1/4/b*d/(d-1
)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^2+1/3/b^3*d/(d-1)*ln(1+(d-1)*exp(2*b*
x+2*a))*a^3+1/4/b^3*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a^2+1/4/b^2*d
/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x-1/2/b^2/(d-1)*ln(1+(d-1)*exp(2*b
*x+2*a))*a^2*x+1/12*b*x^4+1/8/b^3/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))+1
/6/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^3-1/3*x^3*ln(exp(b*x+a))-1/6*d/(d-1)
*ln(1+(d-1)*exp(2*b*x+2*a))*x^3-1/8/b^3*d/(d-1)*polylog(4,-(d-1)*exp(2*b*x
+2*a))+1/4/b/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^2-1/3/b^3/(d-1)*ln(1
+(d-1)*exp(2*b*x+2*a))*a^3-1/4/b^3/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*
a^2-1/4/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x+1/2/b^3*a^3/(d-1)*ln(
1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))+1
/2/b^3*a^2/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^3*a^2/(d-1)*dilog(1
-exp(b*x+a)*(1-d)^(1/2))-1/6/b^3*a^3/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2
*a)+1)+1/2/b^2*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^2*x-1/2/b^2*a^2*d/(d-1
)*x*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^2*a^2*d/(d-1)*x*ln(1-exp(b*x+a)*(1-
d)^(1/2))-1/12*(I*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*...
```

3.308.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(111) = 222$.

Time = 0.26 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.79

$$\int x^2 \text{arctanh}(1-d-d\coth(a+bx))\,dx$$
$$= \frac{b^4x^4 - 2\,b^3x^3\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) - 6\,b^2x^2\text{Li}_2\left(\frac{1}{2}\sqrt{-4\,d+4}(\cosh(bx+a)+\sinh(bx+a))\right) -}{}$$

input
```
integrate(-x^2*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(b^4*x^4 - 2*b^3*x^3*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh
(b*x + a) + (d - 2)*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d + 4)*(
cosh(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d + 4)*(cos
h(b*x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d -
1)*sinh(b*x + a) + sqrt(-4*d + 4)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2
*(d - 1)*sinh(b*x + a) - sqrt(-4*d + 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d
 + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d
+ 4)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*
d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*s
qrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqr
t(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*
d + 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

3.308.6 Sympy [F]

$$\int x^2 \text{arctanh}(1-d-d\coth(a+bx))\,dx = -\int x^2 \operatorname{atanh}\left(d\coth\left(a+bx\right)+d-1\right)dx$$

input
```
integrate(-x**2*atanh(-1+d+d*coth(b*x+a)),x)
```

output
```
-Integral(x**2*atanh(d*coth(a + b*x) + d - 1), x)
```

3.308.7 Maxima [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int x^2\text{arctanh}(1-d-d\coth(a+bx))\,dx = -\frac{1}{3}x^3\,\text{artanh}\left(d\coth\left(bx+a\right)+d-1\right) + \frac{1}{36}\left(\frac{3x^4}{d} - \frac{2\left(4b^3x^3\log\left((d-1)e^{(2bx+2a)}+1\right)+6b^2x^2\text{Li}_2\left(-(d-1)e^{(2bx+2a)}\right)-6bx\text{Li}_3(-(d-1)e^{(2bx}}{b^4d}\right.$$

input
```
integrate(-x^2*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
-1/3*x^3*arctanh(d*coth(b*x + a) + d - 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*l
og((d - 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-(d - 1)*e^(2*b*x + 2*a)
) - 6*b*x*polylog(3, -(d - 1)*e^(2*b*x + 2*a)) + 3*polylog(4, -(d - 1)*e^(
2*b*x + 2*a)))/(b^4*d))*b*d
```

3.308.8 Giac [F]

$$\int x^2\text{arctanh}(1-d-d\coth(a+bx))\,dx = \int -x^2\,\text{artanh}\left(d\coth\left(bx+a\right)+d-1\right)\,dx$$

input
```
integrate(-x^2*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-x^2*arctanh(d*coth(b*x + a) + d - 1), x)
```

3.308.9 Mupad [F(-1)]

Timed out.

$$\int x^2\text{arctanh}(1-d-d\coth(a+bx))\,dx = \int -x^2\,\text{atanh}(d+d\coth(a+b\,x)-1)\,dx$$

input
```
int(-x^2*atanh(d + d*coth(a + b*x) - 1),x)
```

output
```
int(-x^2*atanh(d + d*coth(a + b*x) - 1), x)
```

3.309 $\int x\text{arctanh}(1-d-d\coth(a+bx))\,dx$

3.309.1 Optimal result

Integrand size = 17, antiderivative size = 109

$$\int x\text{arctanh}(1-d-d\coth(a+bx))\,dx = \frac{bx^3}{6}+\frac{1}{2}x^2\text{arctanh}(1-d-d\coth(a+bx)) - \frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x\,\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b} + \frac{\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{8b^2}$$

output
```
1/6*b*x^3-1/2*x^2*arctanh(-1+d+d*coth(b*x+a))-1/4*x^2*ln(1-(1-d)*exp(2*b*x
+2*a))-1/4*x*polylog(2,(1-d)*exp(2*b*x+2*a))/b+1/8*polylog(3,(1-d)*exp(2*b
*x+2*a))/b^2
```

3.309.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int x\text{arctanh}(1-d-d\coth(a+bx))\,dx = \frac{2b^2x^2\left(2\text{arctanh}(1-d-d\coth(a+bx))-\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right)\right)+2bx\,\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right)+\text{PolyLog}}{8b^2}$$

input
```
Integrate[x*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcTanh[1 - d - d*Coth[a + b*x]] - Log[1 + 1/((-1 + d)*E^(2*
(a + b*x)))]) + 2*b*x*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))] + PolyLo
g[3, -(1/((-1 + d)*E^(2*(a + b*x))))])/(8*b^2)
```

3.309.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6795, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(d(-\coth(a+bx))-d+1)\,dx$$

$$\Big\downarrow \; 6795$$

$$\frac{1}{2}b\int \frac{x^2}{1-(1-d)e^{2a+2bx}}dx+\frac{1}{2}x^2\text{arctanh}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \; 2615$$

$$\frac{1}{2}b\left((1-d)\int \frac{e^{2a+2bx}x^2}{1-(1-d)e^{2a+2bx}}dx+\frac{x^3}{3}\right)+\frac{1}{2}x^2\text{arctanh}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \; 2620$$

$$\frac{1}{2}b\left((1-d)\left(\frac{\int x\log\left(1-(1-d)e^{2a+2bx}\right)dx}{b(1-d)}-\frac{x^2\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^3}{3}\right)+ \frac{1}{2}x^2\text{arctanh}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \; 3011$$

$$\frac{1}{2}b\left((1-d)\left(\frac{\frac{\int \text{PolyLog}(2,(1-d)e^{2a+2bx})dx}{2b}-\frac{x\,\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}}{b(1-d)}-\frac{x^2\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^3}{3}\right)+ \frac{1}{2}x^2\text{arctanh}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{2}b\left((1-d)\left(\frac{\frac{\int e^{-2a-2bx}\text{PolyLog}(2,(1-d)e^{2a+2bx})de^{2a+2bx}}{4b^2}-\frac{x\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}}{b(1-d)}-\frac{x^2\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x}{\ }\right.$$
$$\frac{1}{2}x^2\text{arctanh}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x^2\text{arctanh}(d(-\coth(a+bx))-d+1)+$$
$$\frac{1}{2}b\left((1-d)\left(\frac{\frac{\text{PolyLog}(3,(1-d)e^{2a+2bx})}{4b^2}-\frac{x\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}}{b(1-d)}-\frac{x^2\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^3}{3}\right)$$

input
```
Int[x*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

output
```
(x^2*ArcTanh[1 - d - d*Coth[a + b*x]])/2 + (b*(x^3/3 + (1 - d)*(-1/2*(x^2*
Log[1 - (1 - d)*E^(2*a + 2*b*x)])/(b*(1 - d)) + (-1/2*(x*PolyLog[2, (1 - d
)*E^(2*a + 2*b*x)])/b + PolyLog[3, (1 - d)*E^(2*a + 2*b*x)]/(4*b^2))/(b*(1
 - d)))))/2
```

3.309.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6795
```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.309.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.97 (sec) , antiderivative size = 1611, normalized size of antiderivative = 14.78

method	result	size
risch	Expression too large to display	1611

input
```
int(-x*arctanh(-1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/8/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))+1/4/(d-1)*ln(1+(d-1)*exp(2
*b*x+2*a))*x^2+1/2/b*a*d/(d-1)*x*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b*a*d/(d
-1)*x*ln(1-exp(b*x+a)*(1-d)^(1/2))-1/2/b*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a)
)*a*x-1/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^2*a^2/(d-1)*ln(
1-exp(b*x+a)*(1-d)^(1/2))-1/2/b^2*a/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))-
1/2/b^2*a/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))+1/4/b^2*a^2/(d-1)*ln(d*exp
(2*b*x+2*a)-exp(2*b*x+2*a)+1)+1/8/b^2*d/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2
*a))+1/4/b^2/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^2+1/4/b/(d-1)*polylog(2,-(
d-1)*exp(2*b*x+2*a))*x+1/4/b^2/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a-1/
4*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^2-1/2*x^2*ln(exp(b*x+a))+1/6*b*x^3-
1/2/b*a/(d-1)*x*ln(1-exp(b*x+a)*(1-d)^(1/2))+1/2/b^2*a*d/(d-1)*dilog(1-exp
(b*x+a)*(1-d)^(1/2))-1/4/b^2*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^2-1/4/b*
d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x-1/4/b^2*d/(d-1)*polylog(2,-(d-1
)*exp(2*b*x+2*a))*a-1/4/b^2*a^2*d/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)
+1)+1/2/b^2*a^2*d/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^2*a^2*d/(d-1)*l
n(1-exp(b*x+a)*(1-d)^(1/2))+1/2/b^2*a*d/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/
2))+1/2/b/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a*x-1/2/b*a/(d-1)*x*ln(1+exp(b*
x+a)*(1-d)^(1/2))-1/8*(I*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*
x+2*a))^2-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d)*csgn(I*
d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))-I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I...
```

3.309.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(87) = 174$.

Time = 0.26 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.96

$$\int x\text{arctanh}(1-d-d\coth(a+bx))\,dx$$
$$= \frac{2\,b^3x^3 - 3\,b^2x^2\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) - 6\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d+4}(\cosh(bx+a)+\sinh(bx+a))\right) - \cdots}{\cdots}$$

input
```
integrate(-x*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*x^3 - 3*b^2*x^2*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/(d*co
sh(b*x + a) + (d - 2)*sinh(b*x + a))) - 6*b*x*dilog(1/2*sqrt(-4*d + 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 6*b*x*dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*x
+ a) + sinh(b*x + a))) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sin
h(b*x + a) + sqrt(-4*d + 4)) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d -
1)*sinh(b*x + a) - sqrt(-4*d + 4)) - 3*(b^2*x^2 - a^2)*log(1/2*sqrt(-4*d +
 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*sqrt
(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*polylog(3, 1/2*sqrt(-4
*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(3, -1/2*sqrt(-4*d + 4
)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

3.309.6 Sympy [F]

$$\int x\text{arctanh}(1-d-d\coth(a+bx))\,dx=-\int x\,\text{atanh}\,(d\coth\,(a+bx)+d-1)\,dx$$

input
```
integrate(-x*atanh(-1+d+d*coth(b*x+a)),x)
```

output
```
-Integral(x*atanh(d*coth(a + b*x) + d - 1), x)
```

3.309.7 Maxima [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x\text{arctanh}(1-d-d\coth(a+bx))\,dx$$
$$=\frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left((d-1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\text{Li}_2\left(-(d-1)e^{(2\,bx+2\,a)}\right)-\text{Li}_3(-(d-1)e^{(2\,bx+2\,a)})\right)}{b^3d}\right)$$
$$-\frac{1}{2}\,x^2\,\text{artanh}\,(d\coth\,(bx+a)+d-1)$$

input
```
integrate(-x*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(4*x^3/d - 3*(2*b^2*x^2*log((d - 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g(-(d - 1)*e^(2*b*x + 2*a)) - polylog(3, -(d - 1)*e^(2*b*x + 2*a)))/(b^3*d
))*b*d - 1/2*x^2*arctanh(d*coth(b*x + a) + d - 1)
```

3.309.8 Giac [F]

$$\int x\text{arctanh}(1-d-d\coth(a+bx))\,dx = \int -x\,\text{artanh}\left(d\coth\left(bx+a\right)+d-1\right)\,dx$$

input
```
integrate(-x*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-x*arctanh(d*coth(b*x + a) + d - 1), x)
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arctanh}(1-d-d\coth(a+bx))\,dx = \int -x\,\text{atanh}(d+d\coth(a+b\,x)-1)\,dx$$

input
```
int(-x*atanh(d + d*coth(a + b*x) - 1),x)
```

output
```
int(-x*atanh(d + d*coth(a + b*x) - 1), x)
```

3.310 $\int \operatorname{arctanh}(1-d-d\coth(a+bx))\,dx$

3.310.1 Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \operatorname{arctanh}(1-d-d\coth(a+bx))\,dx = \frac{bx^2}{2}+x\operatorname{arctanh}(1-d-d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{\operatorname{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}$$

output
```
1/2*b*x^2-x*arctanh(-1+d+d*coth(b*x+a))-1/2*x*ln(1-(1-d)*exp(2*b*x+2*a))-1
/4*polylog(2,(1-d)*exp(2*b*x+2*a))/b
```

3.310.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \operatorname{arctanh}(1-d-d\coth(a+bx))\,dx = x\operatorname{arctanh}(1-d-d\coth(a+bx)) + \frac{-2bx\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right)+\operatorname{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{4b}$$

input
```
Integrate[ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

output
```
x*ArcTanh[1 - d - d*Coth[a + b*x]] + (-2*b*x*Log[1 + 1/((-1 + d)*E^(2*(a +
 b*x)))] + PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))])/(4*b)
```

3.310.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.34, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6787, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(d(-\coth(a+bx))-d+1)\,dx$$

$$\Big\downarrow \text{6787}$$

$$b\int \frac{x}{1-(1-d)e^{2a+2bx}}dx+x\operatorname{arctanh}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{2615}$$

$$b\left((1-d)\int \frac{e^{2a+2bx}x}{1-(1-d)e^{2a+2bx}}dx+\frac{x^2}{2}\right)+x\operatorname{arctanh}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{2620}$$

$$b\left((1-d)\left(\frac{\int \log\left(1-(1-d)e^{2a+2bx}\right)dx}{2b(1-d)}-\frac{x\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^2}{2}\right)+x\operatorname{arctanh}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{2715}$$

$$b\left((1-d)\left(\frac{\int e^{-2a-2bx}\log\left(1-(1-d)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2(1-d)}-\frac{x\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^2}{2}\right)+x\operatorname{arctanh}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{2838}$$

$$x\operatorname{arctanh}(d(-\coth(a+bx))-d+1)+b\left((1-d)\left(-\frac{\operatorname{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b^2(1-d)}-\frac{x\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^2}{2}\right)$$

input
```
Int[ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

output
```
x*ArcTanh[1 - d - d*Coth[a + b*x]] + b*(x^2/2 + (1 - d)*(-1/2*(x*Log[1 - (
1 - d)*E^(2*a + 2*b*x)])/(b*(1 - d)) - PolyLog[2, (1 - d)*E^(2*a + 2*b*x)]
/(4*b^2*(1 - d))))
```

3.310.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6787
```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Coth[a + b*x]], x] + Simp[b   Int[x/(c - d - c*E^(2*a + 2*b*x))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

3.310.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(66) = 132$.

Time = 0.91 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.58

method	result
derivativedivides	$-\frac{-\frac{\operatorname{arctanh}(-1+d+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arctanh}(-1+d+d\coth(bx+a))d\ln(d+d\coth(bx+a))}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{-d\operatorname{cot}}{}\right.}{}\right.}{}}{}$
default	$-\frac{-\frac{\operatorname{arctanh}(-1+d+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arctanh}(-1+d+d\coth(bx+a))d\ln(d+d\coth(bx+a))}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{-d\operatorname{cot}}{}\right.}{}\right.}{}}{}$
risch	Expression too large to display

input
```
int(-arctanh(-1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/b/d*(-1/2*arctanh(-1+d+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)+1/2*arctan
h(-1+d+d*coth(b*x+a))*d*ln(d+d*coth(b*x+a))-1/2*d^2*(1/d*(1/2*dilog((-d*co
th(b*x+a)-d+2)/(-2*d+2))+1/2*ln(-d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)-d+2)/
(-2*d+2))-1/2*dilog(-1/2*(-d*coth(b*x+a)-d)/d)-1/2*ln(-d*coth(b*x+a)+d)*ln
(-1/2*(-d*coth(b*x+a)-d)/d))-1/d*(1/2*(ln(d+d*coth(b*x+a))-ln(1/2*d*coth(b
*x+a)+1/2*d))*ln(-1/2*d*coth(b*x+a)-1/2*d+1)-1/2*dilog(1/2*d*coth(b*x+a)+1
/2*d)-1/4*ln(d+d*coth(b*x+a))^2)))
```

3.310.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(61) = 122$.

Time = 0.25 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.16

$$\int \operatorname{arctanh}(1-d-d\coth(a+bx))\,dx$$
$$= \frac{b^2x^2 - bx\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) + a\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)+\sqrt{-}\right.}{}$$

input
```
integrate(-arctanh(-1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 - b*x*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b*x +
a) + (d - 2)*sinh(b*x + a))) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*s
inh(b*x + a) + sqrt(-4*d + 4)) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)
*sinh(b*x + a) - sqrt(-4*d + 4)) - (b*x + a)*log(1/2*sqrt(-4*d + 4)*(cosh(
b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-1/2*sqrt(-4*d + 4)*(cosh(b
*x + a) + sinh(b*x + a)) + 1) - dilog(1/2*sqrt(-4*d + 4)*(cosh(b*x + a) +
sinh(b*x + a))) - dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)
)))/b
```

3.310.6 Sympy [F]

$$\int \operatorname{arctanh}(1-d-d\coth(a+bx))\,dx = -\int \operatorname{atanh}\left(d\coth\left(a+bx\right)+d-1\right)dx$$

input
```
integrate(-atanh(-1+d+d*coth(b*x+a)),x)
```

output
```
-Integral(atanh(d*coth(a + b*x) + d - 1), x)
```

3.310.7 Maxima [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \operatorname{arctanh}(1-d-d\coth(a+bx))\,dx = \frac{1}{4}bd\left(\frac{2x^2}{d}-\frac{2bx\log\left((d-1)e^{(2bx+2a)}+1\right)+\operatorname{Li}_2\left(-(d-1)e^{(2bx+2a)}\right)}{b^2d}\right) - x\operatorname{artanh}\left(d\coth\left(bx+a\right)+d-1\right)$$

input
```
integrate(-arctanh(-1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*b*d*(2*x^2/d - (2*b*x*log((d - 1)*e^(2*b*x + 2*a) + 1) + dilog(-(d - 1
)*e^(2*b*x + 2*a)))/(b^2*d)) - x*arctanh(d*coth(b*x + a) + d - 1)
```

3.310.8 Giac [F]

$$\int \operatorname{arctanh}(1-d-d\coth(a+bx))\,dx=\int -\operatorname{artanh}\left(d\coth\left(bx+a\right)+d-1\right)\,dx$$

input
```
integrate(-arctanh(-1+d+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-arctanh(d*coth(b*x + a) + d - 1), x)
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}(1-d-d\coth(a+bx))\,dx=\int -\operatorname{atanh}(d+d\coth(a+b\,x)-1)\,dx$$

input
```
int(-atanh(d + d*coth(a + b*x) - 1),x)
```

output
```
int(-atanh(d + d*coth(a + b*x) - 1), x)
```

3.311 $\int \frac{\operatorname{arctanh}(1-d-d\coth(a+bx))}{x}\,dx$

3.311.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\operatorname{arctanh}(1-d-d\coth(a+bx))}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{arctanh}(1-d-d\coth(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(-arctanh(-1+d+d*coth(b*x+a))/x,x)
```

3.311.2 Mathematica [N/A]

Not integrable

Time = 3.90 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{arctanh}(1-d-d\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arctanh}(1-d-d\coth(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTanh[1 - d - d*Coth[a + b*x]]/x,x]
```

output
```
Integrate[ArcTanh[1 - d - d*Coth[a + b*x]]/x, x]
```

3.311.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(d(-\coth(a+bx))-d+1)}{x} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\text{arctanh}(d(-\coth(a+bx))-d+1)}{x}dx$$

input
```
Int[ArcTanh[1 - d - d*Coth[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.311.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.311.4 Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int -\frac{\text{arctanh}\left(-1+d+d\coth\left(bx+a\right)\right)}{x}dx$$

input
```
int(-arctanh(-1+d+d*coth(b*x+a))/x,x)
```

output
```
int(-arctanh(-1+d+d*coth(b*x+a))/x,x)
```

3.311.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(1-d-d\coth(a+bx))}{x}\,dx = \int -\frac{\operatorname{artanh}\left(d\coth\left(bx+a\right)+d-1\right)}{x}\,dx$$

input
```
integrate(-arctanh(-1+d+d*coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(-arctanh(d*coth(b*x + a) + d - 1)/x, x)
```

3.311.6 Sympy [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{arctanh}(1-d-d\coth(a+bx))}{x}\,dx = -\int \frac{\operatorname{atanh}\left(d\coth\left(a+bx\right)+d-1\right)}{x}\,dx$$

input
```
integrate(-atanh(-1+d+d*coth(b*x+a))/x,x)
```

output
```
-Integral(atanh(d*coth(a + b*x) + d - 1)/x, x)
```

3.311.7 Maxima [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\operatorname{arctanh}(1-d-d\coth(a+bx))}{x}\,dx = \int -\frac{\operatorname{artanh}\left(d\coth\left(bx+a\right)+d-1\right)}{x}\,dx$$

input
```
integrate(-arctanh(-1+d+d*coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
-integrate(arctanh(d*coth(b*x + a) + d - 1)/x, x)
```

3.311.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(1-d-d\coth(a+bx))}{x}\,dx = \int -\frac{\operatorname{artanh}\left(d\coth\left(bx+a\right)+d-1\right)}{x}\,dx$$

input
```
integrate(-arctanh(-1+d+d*coth(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(-arctanh(d*coth(b*x + a) + d - 1)/x, x)
```

3.311.9 Mupad [N/A]

Not integrable

Time = 4.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(1-d-d\coth(a+bx))}{x}\,dx = \int -\frac{\operatorname{atanh}(d+d\coth(a+b\,x)-1)}{x}\,dx$$

input
```
int(-atanh(d + d*coth(a + b*x) - 1)/x,x)
```

output
```
int(-atanh(d + d*coth(a + b*x) - 1)/x, x)
```

3.312 $\int (e+fx)^3 \operatorname{arctanh}(\tan(a+bx))\,dx$

3.312.1 Optimal result

Integrand size = 15, antiderivative size = 302

$$\begin{aligned}\int (e+fx)^3\text{arctanh}(\tan(a+bx))\,dx =& \frac{i(e+fx)^4\arctan\left(e^{2i(a+bx)}\right)}{4f} \\ &+ \frac{(e+fx)^4\text{arctanh}(\tan(a+bx))}{4f} \\ &- \frac{i(e+fx)^3\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} \\ &+ \frac{i(e+fx)^3\,\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b} \\ &+ \frac{3f(e+fx)^2\,\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2} \\ &- \frac{3f(e+fx)^2\,\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2} \\ &+ \frac{3if^2(e+fx)\,\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3} \\ &- \frac{3if^2(e+fx)\,\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3} \\ &- \frac{3f^3\,\text{PolyLog}\left(5,-ie^{2i(a+bx)}\right)}{16b^4} \\ &+ \frac{3f^3\,\text{PolyLog}\left(5,ie^{2i(a+bx)}\right)}{16b^4}\end{aligned}$$

output
```
1/4*I*(f*x+e)^4*arctan(exp(2*I*(b*x+a)))/f+1/4*(f*x+e)^4*arctanh(tan(b*x+a
))/f-1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^3*poly
log(2,I*exp(2*I*(b*x+a)))/b+3/8*f*(f*x+e)^2*polylog(3,-I*exp(2*I*(b*x+a)))
/b^2-3/8*f*(f*x+e)^2*polylog(3,I*exp(2*I*(b*x+a)))/b^2+3/8*I*f^2*(f*x+e)*p
olylog(4,-I*exp(2*I*(b*x+a)))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*I*(b
*x+a)))/b^3-3/16*f^3*polylog(5,-I*exp(2*I*(b*x+a)))/b^4+3/16*f^3*polylog(5
,I*exp(2*I*(b*x+a)))/b^4
```

3.312.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 654 vs. 2(302) = 604.

Time = 1.06 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.17

$$\int (e+fx)^3 \operatorname{arctanh}(\tan(a+bx))\,dx = \frac{1}{4}x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\operatorname{arctanh}(\tan(a+bx)) + \frac{-8b^4e^3x\log\left(1-ie^{2i(a+bx)}\right)-12b^4e^2fx^2\log\left(1-ie^{2i(a+bx)}\right)-8b^4ef^2x^3\log\left(1-ie^{2i(a+bx)}\right)-2b^4f^3x^4\operatorname{lo}}{}$$

input
```
Integrate[(e + f*x)^3*ArcTanh[Tan[a + b*x]],x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTanh[Tan[a + b*x]])/4 +
(-8*b^4*e^3*x*Log[1 - I*E^((2*I)*(a + b*x))] - 12*b^4*e^2*f*x^2*Log[1 - I*
E^((2*I)*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] - 2*
b^4*f^3*x^4*Log[1 - I*E^((2*I)*(a + b*x))] + 8*b^4*e^3*x*Log[1 + I*E^((2*I
)*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 8*b^4*e*
f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] + 2*b^4*f^3*x^4*Log[1 + I*E^((2*I)*
(a + b*x))] - (4*I)*b^3*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))] +
 (4*I)*b^3*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))] + 6*b^2*e^2*f*Pol
yLog[3, (-I)*E^((2*I)*(a + b*x))] + 12*b^2*e*f^2*x*PolyLog[3, (-I)*E^((2*I
)*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b^2
*e^2*f*PolyLog[3, I*E^((2*I)*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3, I*E^(
(2*I)*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^((2*I)*(a + b*x))] + (6*I
)*b*e*f^2*PolyLog[4, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b*f^3*x*PolyLog[4,
(-I)*E^((2*I)*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))
] - (6*I)*b*f^3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I
)*E^((2*I)*(a + b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

3.312.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6805, 3042, 4669, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3\text{arctanh}(\tan(a+bx))\,dx$$

$$\Big\downarrow \text{6805}$$

$$\frac{(e+fx)^4\text{arctanh}(\tan(a+bx))}{4f}-\frac{b\int (e+fx)^4\sec(2a+2bx)dx}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^4\text{arctanh}(\tan(a+bx))}{4f}-\frac{b\int (e+fx)^4\csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{4f}$$

$$\Big\downarrow \text{4669}$$

$$\frac{(e+fx)^4\text{arctanh}(\tan(a+bx))}{4f}-\frac{b\left(-\frac{2f\int (e+fx)^3\log\left(1-ie^{2i(a+bx)}\right)dx}{b}+\frac{2f\int (e+fx)^3\log\left(1+ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^4\arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{4f}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(e+fx)^4\text{arctanh}(\tan(a+bx))}{4f}-\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\int (e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{2f\left(\frac{i(e+fx)^3\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{3if\int (e+fx)^2\text{PolyLog}\left(2\right.}{2b}\right.}{b}\right.}{4f}$$

$$\Big\downarrow \text{7163}$$

$$\frac{(e+fx)^4\text{arctanh}(\tan(a+bx))}{4f}-\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\int (e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^2\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{2b}\right)}{b}-\frac{2f\left(\frac{i(e+fx)^3\text{PolyLo}}{2}\right.}{}\right.}{4f}$$

$$\Big\downarrow \text{7163}$$

$$\frac{(e+fx)^4\operatorname{arctanh}(\tan(a+bx))}{4f} - \frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b} - \frac{3if\left(\frac{if\left(\frac{if\int \operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)dx}{2b} - \frac{i(e+fx)\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right)}{b} - \frac{i(e+fx)^2\operatorname{PolyLog}\left(3,-ie^{2i(a+b}}{2b}}{2b}\right)}{b}\right.}{}$$

$$\Big\downarrow \text{2720}$$

$$\frac{(e+fx)^4\operatorname{arctanh}(\tan(a+bx))}{4f} - \frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b} - \frac{3if\left(\frac{if\left(\frac{f\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2} - \frac{i(e+fx)\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right)}{b} - \frac{i(e+fx)^2\,\mathrm{P}}{}}{2b}\right)}{b}\right.}{}$$

$$\Big\downarrow \text{7143}$$

3.312. $\int (e+fx)^3\operatorname{arctanh}(\tan(a+bx))\,dx$

$$\frac{(e+fx)^4\text{arctanh}(\tan(a+bx))}{4f} - \frac{b\left(-\frac{i(e+fx)^4\arctan\left(e^{2i(a+bx)}\right)}{b} + \frac{2f\left(\frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b} - \frac{3if\left(\frac{if\left(\frac{f\operatorname{PolyLog}\left(5,-ie^{2i(a+bx)}\right)}{4b^2} - \frac{i(e+fx)\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\cdots\right)}{2b}\cdots\right)}{b}\cdots\right)}{\cdots}$$

input
```
Int[(e + f*x)^3*ArcTanh[Tan[a + b*x]],x]
```

output
```
((e + f*x)^4*ArcTanh[Tan[a + b*x]])/(4*f) - (b*(((-I)*(e + f*x)^4*ArcTan[E
^((2*I)*(a + b*x))])/b + (2*f*(((I/2)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)
*(a + b*x))])/b - (((3*I)/2)*f*(((-1/2*I)*(e + f*x)^2*PolyLog[3, (-I)*E^((
2*I)*(a + b*x))])/b + (I*f*(((-1/2*I)*(e + f*x)*PolyLog[4, (-I)*E^((2*I)*(
a + b*x))])/b + (f*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b))
/b - (2*f*(((I/2)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (((3*
I)/2)*f*(((-1/2*I)*(e + f*x)^2*PolyLog[3, I*E^((2*I)*(a + b*x))])/b + (I*f
*(((-1/2*I)*(e + f*x)*PolyLog[4, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[5,
 I*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b))/b))/(4*f)
```

3.312.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6805
```
Int[ArcTanh[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTanh[Tan[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.312. $\int (e+fx)^3 \operatorname{arctanh}(\tan(a+bx))\,dx$

3.312.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.45 (sec) , antiderivative size = 3640, normalized size of antiderivative = 12.05

method	result	size
risch	Expression too large to display	3640

input
```
int((f*x+e)^3*arctanh(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*e^3*a*ln(exp(2*I*(b*x+a))+I)-3/4*f*e^2*ln(-I*exp(2*I*(b*x+a))+1)*x^2
-3/8*f^3/b^2*polylog(3,I*exp(2*I*(b*x+a)))*x^2-3/8*f/b^2*e^2*polylog(3,I*e
xp(2*I*(b*x+a)))+3/8*f/b^2*e^2*polylog(3,-I*exp(2*I*(b*x+a)))+3/8*f^3/b^2*
polylog(3,-I*exp(2*I*(b*x+a)))*x^2-3/16*f^3*polylog(5,-I*exp(2*I*(b*x+a)))
/b^4+3/16*f^3*polylog(5,I*exp(2*I*(b*x+a)))/b^4-1/2*f^3/b^3*a^3*ln(1+exp(I
*(b*x+a))*(-1)^(3/4))*x-1/2*f^3/b^3*a^3*ln(1-exp(I*(b*x+a))*(-1)^(3/4))*x-
f^2/b^3*e*ln(1+I*exp(2*I*(b*x+a)))*a^3+1/2*f^3/b^3*ln(1+I*exp(2*I*(b*x+a))
)*x*a^3-1/2*f^2/b^3*a^3*e*ln(-exp(2*I*(b*x+a))+I)+3/4*f/b^2*a^2*e^2*ln(-ex
p(2*I*(b*x+a))+I)-3/4*I*f^2/b*e*polylog(2,-I*exp(2*I*(b*x+a)))*x^2+3/4*I*f
^2/b^3*e*polylog(2,-I*exp(2*I*(b*x+a)))*a^2+3/4*I*f^2/b*e*polylog(2,I*exp(
2*I*(b*x+a)))*x^2-3/8*I*f^2/b^3*e*polylog(4,I*exp(2*I*(b*x+a)))+1/2*I*f^3/
b^4*a^3*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))+1/2*I*f^3/b^4*a^3*dilog(1-exp(I
*(b*x+a))*(-1)^(3/4))-1/4*I*f^3/b*polylog(2,-I*exp(2*I*(b*x+a)))*x^3-1/4*I
*f^3/b^4*polylog(2,-I*exp(2*I*(b*x+a)))*a^3+1/2*e^3*ln(1+exp(I*(b*x+a))*(-
1)^(3/4))*x+1/2*e^3*ln(1-exp(I*(b*x+a))*(-1)^(3/4))*x+1/8/f*e^4*ln(-exp(2*
I*(b*x+a))+I)+1/8*f^3*ln(1+I*exp(2*I*(b*x+a)))*x^4-3/4*f/b^2*e^2*ln(-I*exp
(2*I*(b*x+a))+1)*a^2+3/2*f^2/b^3*a^3*e*ln(1+exp(I*(b*x+a))*(-1)^(3/4))+3/2
*f^2/b^3*a^3*e*ln(1-exp(I*(b*x+a))*(-1)^(3/4))-3/2*f/b^2*a^2*e^2*ln(1+exp(
I*(b*x+a))*(-1)^(3/4))-3/2*f/b^2*a^2*e^2*ln(1-exp(I*(b*x+a))*(-1)^(3/4))+3
/4*f/b^2*e^2*ln(1+I*exp(2*I*(b*x+a)))*a^2-1/2*f^2*ln(exp(2*I*(b*x+a))-I...
```

3.312.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1809 vs. 2(236) = 472.

Time = 0.33 (sec) , antiderivative size = 1809, normalized size of antiderivative = 5.99

$$\int (e + fx)^3 \operatorname{arctanh}(\tan(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*arctanh(tan(b*x+a)),x, algorithm="fricas")
```

output
```
-1/32*(3*f^3*polylog(5, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x +
 a)^2 + 1)) - 3*f^3*polylog(5, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/(ta
n(b*x + a)^2 + 1)) + 3*f^3*polylog(5, (-I*tan(b*x + a)^2 + 2*tan(b*x + a)
+ I)/(tan(b*x + a)^2 + 1)) - 3*f^3*polylog(5, (-I*tan(b*x + a)^2 - 2*tan(b
*x + a) + I)/(tan(b*x + a)^2 + 1)) + 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2
 - 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x
 + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f
^2*x^2 - 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(-((I + 1)*tan(b*x + a)^2 - 2*t
an(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + 4*(I*b^3*f^3*x^3 + 3*I*b^
3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(-(-(I - 1)*tan(b*x + a)^2
 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) + 4*(I*b^3*f^3*x^3 +
3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(-(-(I - 1)*tan(b*x
+ a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^4*f^3*x^
4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*
b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 + 2*tan(b
*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) - 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f
+ 4*a^3*b*e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a)
+ I - 1)/(tan(b*x + a)^2 + 1)) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*
b*e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)
/(tan(b*x + a)^2 + 1)) - 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f...
```

3.312.6 Sympy [F]

$$\int (e+fx)^3\text{arctanh}(\tan(a+bx))\,dx = \int (e+fx)^3\operatorname{atanh}(\tan(a+bx))\,dx$$

input
```
integrate((f*x+e)**3*atanh(tan(b*x+a)),x)
```

output
```
Integral((e + f*x)**3*atanh(tan(a + b*x)), x)
```

3.312.7 Maxima [F]

$$\int (e+fx)^3 \text{arctanh}(\tan(a+bx))\,dx = \int (fx+e)^3 \,\text{artanh}\left(\tan\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)^3*arctanh(tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/16*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a
)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*e
*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x +
 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*e*f^2*
x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^
3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*sin(4*b*x + 4*a)*sin(2*
b*x + 2*a) + (b*f^3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(2
*b*x + 2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)
 + 1), x)
```

3.312.8 Giac [F]

$$\int (e+fx)^3 \text{arctanh}(\tan(a+bx))\,dx = \int (fx+e)^3 \,\text{artanh}\left(\tan\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)^3*arctanh(tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*arctanh(tan(b*x + a)), x)
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3 \text{arctanh}(\tan(a+bx))\,dx = \int \text{atanh}(\tan(a+b\,x))\,\left(e+f\,x\right)^3\,dx$$

input
```
int(atanh(tan(a + b*x))*(e + f*x)^3,x)
```

output
```
int(atanh(tan(a + b*x))*(e + f*x)^3, x)
```

3.313 $\int (e+fx)^2 \textbf{arctanh}(\tan(a+bx))\,dx$

3.313.1 Optimal result

Integrand size = 15, antiderivative size = 234

$$\int (e+fx)^2\text{arctanh}(\tan(a+bx))\,dx = \frac{i(e+fx)^3\arctan\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3\text{arctanh}(\tan(a+bx))}{3f} - \frac{i(e+fx)^2\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} + \frac{i(e+fx)^2\,\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b} + \frac{f(e+fx)\,\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2} - \frac{f(e+fx)\,\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{4b^2} + \frac{if^2\,\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3} - \frac{if^2\,\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}$$

output
```
1/3*I*(f*x+e)^3*arctan(exp(2*I*(b*x+a)))/f+1/3*(f*x+e)^3*arctanh(tan(b*x+a
))/f-1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^2*poly
log(2,I*exp(2*I*(b*x+a)))/b+1/4*f*(f*x+e)*polylog(3,-I*exp(2*I*(b*x+a)))/b
^2-1/4*f*(f*x+e)*polylog(3,I*exp(2*I*(b*x+a)))/b^2+1/8*I*f^2*polylog(4,-I*
exp(2*I*(b*x+a)))/b^3-1/8*I*f^2*polylog(4,I*exp(2*I*(b*x+a)))/b^3
```

3.313.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.75

$$\int (e+fx)^2\text{arctanh}(\tan(a+bx))\,dx = \frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\text{arctanh}(\tan(a+bx)) + \frac{-12b^3e^2x\log\left(1-ie^{2i(a+bx)}\right)-12b^3efx^2\log\left(1-ie^{2i(a+bx)}\right)-4b^3f^2x^3\log\left(1-ie^{2i(a+bx)}\right)+12b^3e^2x\log}{}$$

input
```
Integrate[(e + f*x)^2*ArcTanh[Tan[a + b*x]],x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTanh[Tan[a + b*x]])/3 + (-12*b^3*e^2*x*L
og[1 - I*E^((2*I)*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 - I*E^((2*I)*(a + b*x
))] - 4*b^3*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] + 12*b^3*e^2*x*Log[1 +
I*E^((2*I)*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 4
*b^3*f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] - (6*I)*b^2*(e + f*x)^2*PolyLo
g[2, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b^2*(e + f*x)^2*PolyLog[2, I*E^((2*
I)*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] + 6*b*f^2*x*
PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b*e*f*PolyLog[3, I*E^((2*I)*(a +
b*x))] - 6*b*f^2*x*PolyLog[3, I*E^((2*I)*(a + b*x))] + (3*I)*f^2*PolyLog[4
, (-I)*E^((2*I)*(a + b*x))] - (3*I)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])
/(24*b^3)
```

3.313.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6805, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2\text{arctanh}(\tan(a+bx))\,dx$$

$$\Big\downarrow \text{6805}$$

$$\frac{(e+fx)^3\text{arctanh}(\tan(a+bx))}{3f}-\frac{b\int (e+fx)^3\sec(2a+2bx)dx}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3\text{arctanh}(\tan(a+bx))}{3f}-\frac{b\int (e+fx)^3\csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{3f}$$

$\downarrow$ 4669

$$\frac{(e+fx)^3\text{arctanh}(\tan(a+bx))}{3f}-\frac{b\left(-\frac{3f\int (e+fx)^2\log\left(1-ie^{2i(a+bx)}\right)dx}{2b}+\frac{3f\int (e+fx)^2\log\left(1+ie^{2i(a+bx)}\right)dx}{2b}-\frac{i(e+fx)^3\arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{3f}$$

$\downarrow$ 3011

$$\frac{(e+fx)^3\text{arctanh}(\tan(a+bx))}{3f}-\frac{b\left(\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\int (e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{if\int (e+fx)\text{PolyLog}\left(2,ie^{2i\ldots}\right.}{b}\right.}{2b}\ldots}{3f}$$

$\downarrow$ 7163

$$\frac{(e+fx)^3\text{arctanh}(\tan(a+bx))}{3f}-\frac{b\left(\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\left(\frac{if\int \text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)dx}{2b}-\frac{i(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,ie^{2i(a\ldots}\right.}{2b}\right.}{}\ldots}{3f}$$

$\downarrow$ 2720

$$\frac{(e+fx)^3\text{arctanh}(\tan(a+bx))}{3f}-\frac{b\left(\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\left(\frac{f\int e^{-2i(a+bx)}\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{3f\left(\frac{i(e+f\ldots}{}\right.}{}\ldots}{3\ldots}$$

$\downarrow$ 7143

3.313. $\int (e+fx)^2\text{arctanh}(\tan(a+bx))\,dx$

$$\frac{\frac{(e+fx)^3\operatorname{arctanh}(\tan(a+bx))}{3f}-}{}$$

$$\frac{b\left(-\frac{i(e+fx)^3\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\left(\frac{f\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\right.}{3f}$$

input
```
Int[(e + f*x)^2*ArcTanh[Tan[a + b*x]],x]
```

output
```
((e + f*x)^3*ArcTanh[Tan[a + b*x]])/(3*f) - (b*(((-I)*(e + f*x)^3*ArcTan[E
^((2*I)*(a + b*x))])/b + (3*f*(((I/2)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)
*(a + b*x))])/b - (I*f*(((-1/2*I)*(e + f*x)*PolyLog[3, (-I)*E^((2*I)*(a +
b*x))])/b + (f*PolyLog[4, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/(2*b) -
 (3*f*(((I/2)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (I*f*(((-
1/2*I)*(e + f*x)*PolyLog[3, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[4, I*E^
((2*I)*(a + b*x))])/(4*b^2)))/b))/(2*b)))/(3*f)
```

3.313.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6805
```
Int[ArcTanh[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTanh[Tan[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.313.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.25 (sec) , antiderivative size = 2719, normalized size of antiderivative = 11.62

method	result	size
risch	Expression too large to display	2719

input
```
int((f*x+e)^2*arctanh(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*f^2*ln(-I*exp(2*I*(b*x+a))+1)*x^3-1/6/f*e^3*ln(exp(2*I*(b*x+a))+I)-1/
2*e^2*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))*x-1/2*e^2*ln(((-I)^(1/2)+
exp(I*(b*x+a)))/(-I)^(1/2))*x-1/8*I*f^2*polylog(4,I*exp(2*I*(b*x+a)))/b^3+
1/8*I*f^2*polylog(4,-I*exp(2*I*(b*x+a)))/b^3-f*e/b*ln(-I*exp(2*I*(b*x+a))+
1)*a*x+f/b*a*e*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))*x+f/b*a*e*ln(((-
I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))*x-I*f/b^2*a*e*dilog(((-I)^(1/2)+exp(I
*(b*x+a)))/(-I)^(1/2))+1/2*I*f*e/b*polylog(2,I*exp(2*I*(b*x+a)))*x+1/2*I*f
*e/b^2*polylog(2,I*exp(2*I*(b*x+a)))*a-I*f/b^2*a*e*dilog(((-I)^(1/2)-exp(I
*(b*x+a)))/(-I)^(1/2))-f/b*a*e*ln(1+exp(I*(b*x+a))*(-1)^(3/4))*x-f/b*a*e*l
n(1-exp(I*(b*x+a))*(-1)^(3/4))*x+f*e/b*ln(1+I*exp(2*I*(b*x+a)))*a*x+I*f/b^
2*a*e*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))+I*f/b^2*a*e*dilog(1-exp(I*(b*x+a)
)*(-1)^(3/4))-1/2*I*f*e/b*polylog(2,-I*exp(2*I*(b*x+a)))*x-1/2*I*f*e/b^2*p
olylog(2,-I*exp(2*I*(b*x+a)))*a+1/6/f*e^3*ln(-exp(2*I*(b*x+a))+I)+1/6*f^2*
ln(1+I*exp(2*I*(b*x+a)))*x^3+1/2*e^2*ln(1-exp(I*(b*x+a))*(-1)^(3/4))*x+1/2
*e^2*ln(1+exp(I*(b*x+a))*(-1)^(3/4))*x-1/2*f*ln(exp(2*I*(b*x+a))-I)*x^2*e-
1/2*f^2/b^2*ln(1+I*exp(2*I*(b*x+a)))*a^2*x+1/2*f/b^2*a^2*e*ln(-exp(2*I*(b*
x+a))+I)+1/4*I*f^2*a^2/b^3*polylog(2,-I*exp(2*I*(b*x+a)))-1/2*I*f^2/b^3*a^
2*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I*f^2/b^3*a^2*dilog(1-exp(I*(b*x+
a))*(-1)^(3/4))-1/4*I*f^2/b*polylog(2,-I*exp(2*I*(b*x+a)))*x^2-1/3*f^2/b^3
*ln(1+I*exp(2*I*(b*x+a)))*a^3+1/2*f*e*ln(1+I*exp(2*I*(b*x+a)))*x^2+1/2*...
```

3.313.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1283 vs. 2(180) = 360.

Time = 0.30 (sec) , antiderivative size = 1283, normalized size of antiderivative = 5.48

$$\int (e+fx)^2 \text{arctanh}(\tan(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*arctanh(tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/48*(3*I*f^2*polylog(4, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x
+ a)^2 + 1)) + 3*I*f^2*polylog(4, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/
(tan(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 + 2*tan(b*x
+ a) + I)/(tan(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 -
2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2*e*
f*x - I*b^2*e^2)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/
(tan(b*x + a)^2 + 1) + 1) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)
*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2
+ 1) + 1) - 6*(I*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-(-(I - 1)
*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(I
*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-(-(I - 1)*tan(b*x + a)^2
- 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - 4*(b^3*f^2*x^3 + 3*b
^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(((I +
1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 4*(3*a
*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*
x + a) + I - 1)/(tan(b*x + a)^2 + 1)) - 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3
*f^2)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a
)^2 + 1)) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3
*a^2*b*e*f + a^3*f^2)*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1
)/(tan(b*x + a)^2 + 1)) - 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x ...
```

3.313.6 Sympy [F]

$$\int (e+fx)^2\text{arctanh}(\tan(a+bx))\,dx=\int (e+fx)^2\,\text{atanh}\,(\tan\,(a+bx))\,dx$$

input
```
integrate((f*x+e)**2*atanh(tan(b*x+a)),x)
```

output
```
Integral((e + f*x)**2*atanh(tan(a + b*x)), x)
```

3.313.7 Maxima [F]

$$\int (e+fx)^2\text{arctanh}(\tan(a+bx))\,dx = \int (fx+e)^2\,\text{artanh}\left(\tan\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)^2*arctanh(tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*
x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x
)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2
) - integrate(2/3*((b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(4*b*x + 4*a)*
cos(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(2*b*x + 2*a))
/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

3.313.8 Giac [F]

$$\int (e+fx)^2\text{arctanh}(\tan(a+bx))\,dx = \int (fx+e)^2\,\text{artanh}\left(\tan\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)^2*arctanh(tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*arctanh(tan(b*x + a)), x)
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\text{arctanh}(\tan(a+bx))\,dx = \int \text{atanh}(\tan(a+b\,x))\,\left(e+f\,x\right)^2\,dx$$

input
```
int(atanh(tan(a + b*x))*(e + f*x)^2,x)
```

output
```
int(atanh(tan(a + b*x))*(e + f*x)^2, x)
```

3.314 $\int (e+fx)\text{arctanh}(\tan(a+bx))\,dx$

3.314.1 Optimal result

Integrand size = 13, antiderivative size = 162

$$\begin{aligned}\int (e+fx)\text{arctanh}(\tan(a+bx))\,dx &= \frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{2f} \\ &\quad+\frac{(e+fx)^2\text{arctanh}(\tan(a+bx))}{2f} \\ &\quad-\frac{i(e+fx)\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} \\ &\quad+\frac{i(e+fx)\,\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b} \\ &\quad+\frac{f\,\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2} \\ &\quad-\frac{f\,\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2}\end{aligned}$$

output
```
1/2*I*(f*x+e)^2*arctan(exp(2*I*(b*x+a)))/f+1/2*(f*x+e)^2*arctanh(tan(b*x+a
))/f-1/4*I*(f*x+e)*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)*polylog(
2,I*exp(2*I*(b*x+a)))/b+1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*pol
ylog(3,I*exp(2*I*(b*x+a)))/b^2
```

3.314.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.82

$$\int (e+fx)\text{arctanh}(\tan(a+bx))\,dx = ex\text{arctanh}(\tan(a+bx)) + \frac{1}{2}fx^2\text{arctanh}(\tan(a+bx))$$
$$-\frac{e\left((-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+\right.}{8b}$$
$$+\frac{f(4ib^2x^2\arctan(\cos(2(a+bx))+i\sin(2(a+bx)))+2ibx\,\text{PolyLog}(2,i\cos(2(a+bx))-\sin(2(a+bx))}{}$$

input
```
Integrate[(e + f*x)*ArcTanh[Tan[a + b*x]],x]
```

output
```
e*x*ArcTanh[Tan[a + b*x]] + (f*x^2*ArcTanh[Tan[a + b*x]])/2 - (e*((-4*a +
Pi - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*x))] - Log[1 + I/E^((2*I)*(a + b*x)
)]) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*x]] + (2*I)*(PolyLog[2, (-I)/E^((2*
I)*(a + b*x))] - PolyLog[2, I/E^((2*I)*(a + b*x))])))/(8*b) + (f*((4*I)*b^
2*x^2*ArcTan[Cos[2*(a + b*x)] + I*Sin[2*(a + b*x)]] + (2*I)*b*x*PolyLog[2,
 I*Cos[2*(a + b*x)] - Sin[2*(a + b*x)]] - (2*I)*b*x*PolyLog[2, (-I)*Cos[2*
(a + b*x)] + Sin[2*(a + b*x)]] - PolyLog[3, I*Cos[2*(a + b*x)] - Sin[2*(a
+ b*x)]] + PolyLog[3, (-I)*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]]))/(8*b^2)
```

3.314.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {6805, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\text{arctanh}(\tan(a+bx))\,dx$$
$$\Big\downarrow \text{6805}$$
$$\frac{(e+fx)^2\text{arctanh}(\tan(a+bx))}{2f} - \frac{b\int (e+fx)^2\sec(2a+2bx)dx}{2f}$$
$$\Big\downarrow \text{3042}$$
$$\frac{(e+fx)^2\text{arctanh}(\tan(a+bx))}{2f} - \frac{b\int (e+fx)^2\csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{2f}$$

$\downarrow$ 4669

$$\frac{(e+fx)^2\text{arctanh}(\tan(a+bx))}{2f}-\frac{b\left(-\frac{f\int(e+fx)\log\left(1-ie^{2i(a+bx)}\right)dx}{b}+\frac{f\int(e+fx)\log\left(1+ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{2f}$$

$\downarrow$ 3011

$$\frac{(e+fx)^2\text{arctanh}(\tan(a+bx))}{2f}-\frac{b\left(\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\int\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{if\int\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{i(\ldots}{}\right.}{2f}$$

$\downarrow$ 2720

$$\frac{(e+fx)^2\text{arctanh}(\tan(a+bx))}{2f}-\frac{b\left(\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{f\int e^{-2i(a+bx)}\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{f\int e^{-2i(a+bx)}\text{Po}\ldots}{}\right.}{b}\right.}{2f}$$

$\downarrow$ 7143

$$\frac{(e+fx)^2\text{arctanh}(\tan(a+bx))}{2f}-\frac{b\left(-\frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{f\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2}\right)}{b}-\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{f\text{Poly}\ldots}{}\right.}{b}\right.}{2f}$$

input
```
Int[(e + f*x)*ArcTanh[Tan[a + b*x]],x]
```

output
```
((e + f*x)^2*ArcTanh[Tan[a + b*x]])/(2*f) - (b*(((-I)*(e + f*x)^2*ArcTan[E
^((2*I)*(a + b*x))])/b + (f*(((I/2)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a
+ b*x))])/b - (f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b - (f*((
(I/2)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (f*PolyLog[3, I*E^(
(2*I)*(a + b*x))])/(4*b^2)))/b))/(2*f)
```

3.314. $\int(e+fx)\text{arctanh}(\tan(a+bx))\,dx$

3.314.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6805
```
Int[ArcTanh[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTanh[Tan[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.314.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.08 (sec) , antiderivative size = 1818, normalized size of antiderivative = 11.22

method	result	size
risch	Expression too large to display	1818

input
```
int((f*x+e)*arctanh(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*e*a*ln(exp(2*I*(b*x+a))+I)-1/2*e/b*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-
I)^(1/2))*a-1/2*e/b*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))*a+1/2*I*e/b
*dilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I*e/b*dilog(((-I)^(1/2)
+exp(I*(b*x+a)))/(-I)^(1/2))+1/2/b^2*f*a^2*ln(((-I)^(1/2)-exp(I*(b*x+a)))/
(-I)^(1/2))+1/2/b^2*f*a^2*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-1/4*l
n(exp(2*I*(b*x+a))-I)*x^2*f-1/2*ln(exp(2*I*(b*x+a))-I)*e*x+1/8*f*polylog(3
,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylog(3,I*exp(2*I*(b*x+a)))/b^2+1/2*(1/2
*f*x^2+e*x)*ln(exp(2*I*(b*x+a))+I)+1/4/b^2*f*a^2*ln(-exp(2*I*(b*x+a))+I)-1
/2*e/b*a*ln(-exp(2*I*(b*x+a))+I)-1/4*f*ln(-I*exp(2*I*(b*x+a))+1)*x^2-1/2*e
*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))*x-1/2*e*ln(((-I)^(1/2)+exp(I*(
b*x+a)))/(-I)^(1/2))*x+1/2*e*ln(1+exp(I*(b*x+a))*(-1)^(3/4))*x+1/2*e*ln(1-
exp(I*(b*x+a))*(-1)^(3/4))*x+1/4*f*ln(1+I*exp(2*I*(b*x+a)))*x^2+1/4*I*Pi*(
csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((1-I)*(exp(2*I*(b*x
+a))-I)/(exp(2*I*(b*x+a))+1))+csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*
x+a))+1))^2+csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))-I))*csgn
(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))-csgn(I/(exp(2*I*(b*x+a))+1))
*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a)
)+1))-csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b
*x+a))+1))^2+csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp
(2*I*(b*x+a))+1))^2-csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a...
```

3.314.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 835 vs. 2(130) = 260.

Time = 0.29 (sec) , antiderivative size = 835, normalized size of antiderivative = 5.15

$$\int (e + fx)\text{arctanh}(\tan(a + bx))\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*arctanh(tan(b*x+a)),x, algorithm="fricas")
```

output
```
-1/16*(2*(-I*b*f*x - I*b*e)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a
) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(-I*b*f*x - I*b*e)*dilog(-((I + 1
)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(
I*b*f*x + I*b*e)*dilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)
/(tan(b*x + a)^2 + 1) + 1) + 2*(I*b*f*x + I*b*e)*dilog(-(-(I - 1)*tan(b*x
+ a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^2*f*x^2
+ 2*b^2*e*x + 2*a*b*e - a^2*f)*log(((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a
) - I + 1)/(tan(b*x + a)^2 + 1)) - 2*(2*a*b*e - a^2*f)*log(((I + 1)*tan(b*
x + a)^2 + 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 + 1)) + 2*(2*a*b*e -
a^2*f)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x +
a)^2 + 1)) - 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(((I + 1)*tan(
b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*f*x^2
+ 2*b^2*e*x + 2*a*b*e - a^2*f)*log((-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x +
a) + I + 1)/(tan(b*x + a)^2 + 1)) - 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a
^2*f)*log((-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)
^2 + 1)) - 2*(2*a*b*e - a^2*f)*log(((I - 1)*tan(b*x + a)^2 + 2*I*tan(b*x +
 a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*(2*a*b*e - a^2*f)*log(((I - 1)*tan(
b*x + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 4*(b^2*f*x^
2 + 2*b^2*e*x)*log(-(tan(b*x + a) + 1)/(tan(b*x + a) - 1)) - f*polylog(3,
(I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + f*polyl...
```

3.314.6 Sympy [F]

$$\int (e+fx)\text{arctanh}(\tan(a+bx))\,dx = \int (e+fx)\,\text{atanh}\,(\tan{(a+bx)})\,dx$$

input
```
integrate((f*x+e)*atanh(tan(b*x+a)),x)
```

output
```
Integral((e + f*x)*atanh(tan(a + b*x)), x)
```

3.314.7 Maxima [F]

$$\int (e+fx)\text{arctanh}(\tan(a+bx))\,dx = \int (fx+e)\,\text{artanh}\,(\tan\,(bx+a))\;dx$$

input
```
integrate((f*x+e)*arctanh(tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*si
n(2*b*x + 2*a) + 2) - 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin
(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(((b*f*x^2 + 2*b*e*x)
*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)
^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

3.314.8 Giac [F]

$$\int (e+fx)\text{arctanh}(\tan(a+bx))\,dx = \int (fx+e)\,\text{artanh}\,(\tan\,(bx+a))\;dx$$

input
```
integrate((f*x+e)*arctanh(tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*arctanh(tan(b*x + a)), x)
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)\text{arctanh}(\tan(a+bx))\,dx = \int \text{atanh}(\tan(a+b\,x))\;(e+f\,x)\;dx$$

input
```
int(atanh(tan(a + b*x))*(e + f*x),x)
```

output
```
int(atanh(tan(a + b*x))*(e + f*x), x)
```

3.315 $\int \operatorname{arctanh}(\tan(a+bx))\,dx$

3.315.1 Optimal result

Integrand size = 7, antiderivative size = 79

$$\int \operatorname{arctanh}(\tan(a+bx))\,dx = ix\arctan\left(e^{2i(a+bx)}\right)+x\operatorname{arctanh}(\tan(a+bx)) - \frac{i\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i\,\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}$$

output
```
I*x*arctan(exp(2*I*(b*x+a)))+x*arctanh(tan(b*x+a))-1/4*I*polylog(2,-I*exp(
2*I*(b*x+a)))/b+1/4*I*polylog(2,I*exp(2*I*(b*x+a)))/b
```

3.315.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.61

$$\int \operatorname{arctanh}(\tan(a+bx))\,dx = x\operatorname{arctanh}(\tan(a+bx)) - \frac{(-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+2}{8b}$$

input
```
Integrate[ArcTanh[Tan[a + b*x]],x]
```

output
```
x*ArcTanh[Tan[a + b*x]] - ((-4*a + Pi - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*
x))] - Log[1 + I/E^((2*I)*(a + b*x))]) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*
x]] + (2*I)*(PolyLog[2, (-I)/E^((2*I)*(a + b*x))] - PolyLog[2, I/E^((2*I)*
(a + b*x))]))/(8*b)
```

3.315.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6801, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(\tan(a+bx))\,dx$$

$$\Big\downarrow \text{6801}$$

$$x\operatorname{arctanh}(\tan(a+bx))-b\int x\sec(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$x\operatorname{arctanh}(\tan(a+bx))-b\int x\csc\left(2a+2bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4669}$$

$$x\operatorname{arctanh}(\tan(a+bx))-b\left(-\frac{\int \log\left(1-ie^{2i(a+bx)}\right)dx}{2b}+\frac{\int \log\left(1+ie^{2i(a+bx)}\right)dx}{2b}-\frac{ix\arctan\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\operatorname{arctanh}(\tan(a+bx))-b\left(\frac{i\int e^{-2i(a+bx)}\log\left(1-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i\int e^{-2i(a+bx)}\log\left(1+ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{ix\arctan\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\operatorname{arctanh}(\tan(a+bx))-b\left(-\frac{ix\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{i\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{i\operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b^2}\right)$$

input
```
Int[ArcTanh[Tan[a + b*x]],x]
```

output
```
x*ArcTanh[Tan[a + b*x]] - b*(((-I)*x*ArcTan[E^((2*I)*(a + b*x))])/b + ((I/
4)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])/b^2 - ((I/4)*PolyLog[2, I*E^((2*I
)*(a + b*x))])/b^2)
```

3.315.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6801
```
Int[ArcTanh[Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTanh[Tan[a + b
*x]], x] - Simp[b   Int[x*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

3.315.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(64) = 128$.

Time = 0.75 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.14

method	result
derivativedivides	$\frac{\arctan(\tan(bx+a))\operatorname{arctanh}(\tan(bx+a))+\frac{\arctan(\tan(bx+a))\ln\left(1+\frac{i(1+i\tan(bx+a))^2}{1+\tan(bx+a)^2}\right)}{2}-\frac{\arctan(\tan(bx+a))\ln\left(1-\frac{i(1+i\tan(b}{1+\tan(b}\right.}{2}}{b}$
default	$\frac{\arctan(\tan(bx+a))\operatorname{arctanh}(\tan(bx+a))+\frac{\arctan(\tan(bx+a))\ln\left(1+\frac{i(1+i\tan(bx+a))^2}{1+\tan(bx+a)^2}\right)}{2}-\frac{\arctan(\tan(bx+a))\ln\left(1-\frac{i(1+i\tan(b}{1+\tan(b}\right.}{2}}{b}$
risch	Expression too large to display

input
```
int(arctanh(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(arctan(tan(b*x+a))*arctanh(tan(b*x+a))+1/2*arctan(tan(b*x+a))*ln(1+I*
(1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2))-1/2*arctan(tan(b*x+a))*ln(1-I*(1+I*ta
n(b*x+a))^2/(1+tan(b*x+a)^2))-1/4*I*dilog(1+I*(1+I*tan(b*x+a))^2/(1+tan(b*
x+a)^2))+1/4*I*dilog(1-I*(1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2)))
```

3.315.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(57) = 114$.

Time = 0.28 (sec) , antiderivative size = 499, normalized size of antiderivative = 6.32

$$\int \operatorname{arctanh}(\tan(a+bx))\,dx$$
$$= \frac{4\,bx\log\left(-\frac{\tan(bx+a)+1}{\tan(bx+a)-1}\right) - 2\,(bx+a)\log\left(\frac{(i+1)\tan(bx+a)^2+2\tan(bx+a)-i+1}{\tan(bx+a)^2+1}\right) + 2\,a\log\left(\frac{(i+1)\tan(bx+a)^2+2i\tan(bx+a)}{\tan(bx+a)^2+1}\right.}{}$$

input
```
integrate(arctanh(tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(4*b*x*log(-(tan(b*x + a) + 1)/(tan(b*x + a) - 1)) - 2*(b*x + a)*log((
(I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2
*a*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2
 + 1)) - 2*a*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(
b*x + a)^2 + 1)) + 2*(b*x + a)*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a
) - I + 1)/(tan(b*x + a)^2 + 1)) - 2*(b*x + a)*log((-(I - 1)*tan(b*x + a)^
2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*(b*x + a)*log((-(I -
 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*a*l
og(((I - 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1
)) - 2*a*log(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x
+ a)^2 + 1)) + I*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/
(tan(b*x + a)^2 + 1) + 1) + I*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x +
 a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*tan(b*x + a)^2
 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*
tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1))/b
```

3.315.6 Sympy [F]

$$\int \operatorname{arctanh}(\tan(a+bx))\,dx = \int \operatorname{atanh}(\tan(a+bx))\,dx$$

input
```
integrate(atanh(tan(b*x+a)),x)
```

output
```
Integral(atanh(tan(a + b*x)), x)
```

3.315.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(57) = 114$.

Time = 0.33 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.30

$$\int \operatorname{arctanh}(\tan(a+bx))\,dx = \frac{4(bx+a)\operatorname{artanh}(\tan(bx+a)) + \left(\arctan\left(\frac{1}{2}\tan(bx+a)+\frac{1}{2},\frac{1}{2}\tan(bx+a)+\frac{1}{2}\right) - \arctan\left(\frac{1}{2}\tan(bx+\right.\right.}{}$$

input
```
integrate(arctanh(tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*(4*(b*x + a)*arctanh(tan(b*x + a)) + (arctan2(1/2*tan(b*x + a) + 1/2,
1/2*tan(b*x + a) + 1/2) - arctan2(1/2*tan(b*x + a) - 1/2, -1/2*tan(b*x + a
) + 1/2))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log(1/2*tan(b*x + a)^2 + tan
(b*x + a) + 1/2) + (b*x + a)*log(1/2*tan(b*x + a)^2 - tan(b*x + a) + 1/2)
- I*dilog((1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2) + I*dilog(-(1/2*I - 1/
2)*tan(b*x + a) + 1/2*I + 1/2) + I*dilog((1/2*I - 1/2)*tan(b*x + a) + 1/2*
I + 1/2) - I*dilog(-(1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2))/b
```

3.315.8 Giac [F]

$$\int \operatorname{arctanh}(\tan(a+bx))\,dx = \int \operatorname{artanh}\left(\tan\left(bx+a\right)\right)\,dx$$

input
```
integrate(arctanh(tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctanh(tan(b*x + a)), x)
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}(\tan(a+bx))\,dx = \int \operatorname{atanh}(\tan(a+b\,x))\,dx$$

input
```
int(atanh(tan(a + b*x)),x)
```

output
```
int(atanh(tan(a + b*x)), x)
```

3.316 $\int \frac{\operatorname{arctanh}(\tan(a+bx))}{e+fx} dx$

3.316.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\operatorname{arctanh}(\tan(a+bx))}{e+fx} dx = \operatorname{Int}\left(\frac{\operatorname{arctanh}(\tan(a+bx))}{e+fx}, x\right)$$

output
```
CannotIntegrate(arctanh(tan(b*x+a))/(f*x+e),x)
```

3.316.2 Mathematica [N/A]

Not integrable

Time = 7.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(\tan(a+bx))}{e+fx} dx = \int \frac{\operatorname{arctanh}(\tan(a+bx))}{e+fx} dx$$

input
```
Integrate[ArcTanh[Tan[a + b*x]]/(e + f*x),x]
```

output
```
Integrate[ArcTanh[Tan[a + b*x]]/(e + f*x), x]
```

3.316.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(\tan(a+bx))}{e+fx} \, dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\operatorname{arctanh}(\tan(a+bx))}{e+fx}dx$$

input
```
Int[ArcTanh[Tan[a + b*x]]/(e + f*x),x]
```

output
```
$Aborted
```

3.316.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.316.4 Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}\left(\tan \left(bx+a\right)\right)}{fx+e}dx$$

input
```
int(arctanh(tan(b*x+a))/(f*x+e),x)
```

output
```
int(arctanh(tan(b*x+a))/(f*x+e),x)
```

3.316.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(\tan(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{artanh}(\tan(bx+a))}{fx+e}\,dx$$

input
```
integrate(arctanh(tan(b*x+a))/(f*x+e),x, algorithm="fricas")
```

output
```
integral(arctanh(tan(b*x + a))/(f*x + e), x)
```

3.316.6 Sympy [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arctanh}(\tan(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{atanh}(\tan(a+bx))}{e+fx}\,dx$$

input
```
integrate(atanh(tan(b*x+a))/(f*x+e),x)
```

output
```
Integral(atanh(tan(a + b*x))/(e + f*x), x)
```

3.316.7 Maxima [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(\tan(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{artanh}(\tan(bx+a))}{fx+e}\,dx$$

input
```
integrate(arctanh(tan(b*x+a))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(arctanh(tan(b*x + a))/(f*x + e), x)
```

3.316.8 Giac [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(\tan(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{artanh}\left(\tan\left(bx+a\right)\right)}{fx+e}\,dx$$

input
```
integrate(arctanh(tan(b*x+a))/(f*x+e),x, algorithm="giac")
```

output
```
integrate(arctanh(tan(b*x + a))/(f*x + e), x)
```

3.316.9 Mupad [N/A]

Not integrable

Time = 4.84 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(\tan(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{atanh}(\tan(a+b\,x))}{e+f\,x}\,dx$$

input
```
int(atanh(tan(a + b*x))/(e + f*x),x)
```

output
```
int(atanh(tan(a + b*x))/(e + f*x), x)
```

3.317 $\int x^2 \textbf{arctanh}(c + d\tan(a + bx))\,dx$

3.317.1 Optimal result

Integrand size = 15, antiderivative size = 395

$$\begin{aligned}\int x^2\text{arctanh}(c+d\tan(a+bx))\,dx &= \frac{1}{3}x^3\text{arctanh}(c+d\tan(a+bx)) \\ &+ \frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) \\ &- \frac{1}{6}x^3\log\left(1+\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right) \\ &- \frac{ix^2\,\text{PolyLog}\left(2,-\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{4b} \\ &+ \frac{ix^2\,\text{PolyLog}\left(2,-\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{4b} \\ &+ \frac{x\,\text{PolyLog}\left(3,-\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{4b^2} \\ &- \frac{x\,\text{PolyLog}\left(3,-\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{4b^2} \\ &+ \frac{i\,\text{PolyLog}\left(4,-\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{8b^3} \\ &- \frac{i\,\text{PolyLog}\left(4,-\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arctanh(c+d*tan(b*x+a))+1/6*x^3*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/
(1-c-I*d))-1/6*x^3*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*x^2*
polylog(2,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*x^2*polylog(2,-
(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b+1/4*x*polylog(3,-(1-c+I*d)*exp(2
*I*a+2*I*b*x)/(1-c-I*d))/b^2-1/4*x*polylog(3,-(1+c-I*d)*exp(2*I*a+2*I*b*x)
/(1+c+I*d))/b^2+1/8*I*polylog(4,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b
^3-1/8*I*polylog(4,-(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b^3
```

3.317.2 Mathematica [A] (verified)

Time = 3.61 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.88

$$\int x^2\text{arctanh}(c+d\tan(a+bx))\,dx$$
$$=\frac{8b^3x^3\text{arctanh}(c+d\tan(a+bx))+4b^3x^3\log\left(1+\frac{(-1+c+id)e^{-2i(a+bx)}}{-1+c-id}\right)-4b^3x^3\log\left(1+\frac{(1+c+id)e^{-2i(a+bx)}}{1+c-id}\right)+}{}$$

input
```
Integrate[x^2*ArcTanh[c + d*Tan[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcTanh[c + d*Tan[a + b*x]] + 4*b^3*x^3*Log[1 + (-1 + c + I*d)/
((-1 + c - I*d)*E^((2*I)*(a + b*x)))] - 4*b^3*x^3*Log[1 + (1 + c + I*d)/((
1 + c - I*d)*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (1 - c - I*d
)/((-1 + c - I*d)*E^((2*I)*(a + b*x)))] - (6*I)*b^2*x^2*PolyLog[2, (-1 - c
 - I*d)/((1 + c - I*d)*E^((2*I)*(a + b*x)))] + 6*b*x*PolyLog[3, (1 - c - I
*d)/((-1 + c - I*d)*E^((2*I)*(a + b*x)))] - 6*b*x*PolyLog[3, (-1 - c - I*d
)/((1 + c - I*d)*E^((2*I)*(a + b*x)))] - (3*I)*PolyLog[4, (1 - c - I*d)/((
-1 + c - I*d)*E^((2*I)*(a + b*x)))] + (3*I)*PolyLog[4, (-1 - c - I*d)/((1
+ c - I*d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.317.3 Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6821, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arctanh}(d\tan(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6821}$$

$$-\frac{1}{3}b(ic+d+i)\int \frac{e^{2ia+2ibx}x^3}{c+(c-id+1)e^{2ia+2ibx}+id+1}dx+\frac{1}{3}b(-d+i(1-c))\int \frac{e^{2ia+2ibx}x^3}{-c+(-c+id+1)e^{2ia+2ibx}-id+1}dx+\frac{1}{3}x^3\text{arctanh}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}b(-d+i(1-c))\left(\frac{x^3\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{3\int x^2\log\left(\frac{e^{2ia+2ibx}(-c+id+1)}{-c-id+1}+1\right)dx}{2b(-d+i(1-c))}\right)-\frac{1}{3}b(ic+d+i)\left(\frac{x^3\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{3\int x^2\log\left(\frac{e^{2ia+2ibx}(c-id+1)}{c+id+1}+1\right)dx}{2(bd+i(bc+b))}\right)+\frac{1}{3}x^3\text{arctanh}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}b(-d+i(1-c))\left(\frac{x^3\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{i\int x\,\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)dx}{b}\right)}{2b(-d+i(1-c))}\right)-\frac{1}{3}b(ic+d+i)\left(\frac{x^3\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{i\int x\,\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)dx}{b}\right)}{2(bd+i(bc+b))}\right)+\frac{1}{3}x^3\text{arctanh}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{7163}$$

3.317. $\int x^2\text{arctanh}(c+d\tan(a+bx))\,dx$

$$\frac{1}{3}b(-d+i(1-$$

$$c))\left(\frac{x^3\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)dx}{2b}-\frac{ix\operatorname{Po}}{}\right.}{b}\right.}{2b(-d+i(1-c))}\right.$$

$$\frac{1}{3}b(ic+d+$$

$$i)\left(\frac{x^3\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}(}{}\right.}{b}\right.}{2(bd+i(bc+b))}\right.$$

$$\frac{1}{3}x^3\text{arctanh}(d\tan(a+bx)+c)$$

$$\Big\downarrow \; 2720$$

3.317. $\int x^2\text{arctanh}(c+d\tan(a+bx))\,dx$

$$\frac{1}{3}b(-d+i(1-$$

$$c))\left(\frac{x^3\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right.}{4b^2}\right.}{}\right.}{2b(-d+i(1-c))}\right.$$

$$\frac{1}{3}b(ic+d+$$

$$i)\left(\frac{x^3\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)de^{2ia}}{4b^2}\right.}{b}\right.}{2(bd+i(bc+b))}\right.$$

$$\frac{1}{3}x^3\operatorname{arctanh}(d\tan(a+bx)+c)$$

$$\downarrow 7143$$

3.317. $\int x^2\operatorname{arctanh}(c+d\tan(a+bx))\,dx$

$$\frac{1}{3}x^3\text{arctanh}(d\tan(a+bx)+c)+\frac{1}{3}b(-d+i(1-$$

$$c))\left(\frac{x^3\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{i\left(\frac{\text{PolyLog}\left(4,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2}-\frac{ix\,\text{PolyLog}}{}\right.}{b}\right.}{2b(-d+i(1-c))}\right.$$

$$\frac{1}{3}b(ic+d+$$

$$i)\left(\frac{x^3\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{i\left(\frac{\text{PolyLog}\left(4,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2}-\frac{ix\,\text{PolyLog}\left(3,-\right.}{}\right.}{b}\right.}{2(bd+i(bc+b))}\right.$$

input
```
Int[x^2*ArcTanh[c + d*Tan[a + b*x]],x]
```

output
```
(x^3*ArcTanh[c + d*Tan[a + b*x]])/3 + (b*(I*(1 - c) - d)*((x^3*Log[1 + ((1
 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d)])/(2*b*(I*(1 - c) - d))
 - (3*(((I/2)*x^2*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1
- c - I*d))])/b - (I*(((-1/2*I)*x*PolyLog[3, -(((1 - c + I*d)*E^((2*I)*a +
 (2*I)*b*x))/(1 - c - I*d))])/b + PolyLog[4, -(((1 - c + I*d)*E^((2*I)*a +
 (2*I)*b*x))/(1 - c - I*d))]/(4*b^2)))/b))/(2*b*(I*(1 - c) - d))))/3 - (b*
(I + I*c + d)*((x^3*Log[1 + ((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c
 + I*d)])/(2*(I*(b + b*c) + b*d)) - (3*(((I/2)*x^2*PolyLog[2, -(((1 + c -
I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b - (I*(((-1/2*I)*x*PolyLog
[3, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + PolyLog
[4, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))]/(4*b^2)))/b)
)/(2*(I*(b + b*c) + b*d))))/3
```

3.317. $\int x^2\text{arctanh}(c+d\tan(a+bx))\,dx$

3.317.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6821
```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + (-Simp[I*b*((1 + c - I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)
*(E^(2*I*a + 2*I*b*x)/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x))), x
], x] + Simp[I*b*((1 - c + I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*
I*a + 2*I*b*x)/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x))), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.317.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 21.30 (sec) , antiderivative size = 6921, normalized size of antiderivative = 17.52

method	result	size
risch	Expression too large to display	6921

input
```
int(x^2*arctanh(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.317.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2165 vs. 2(279) = 558.

Time = 0.34 (sec) , antiderivative size = 2165, normalized size of antiderivative = 5.48

$$\int x^2 \text{arctanh}(c + d\tan(a + bx))\,dx = \text{Too large to display}$$

input
```
integrate(x^2*arctanh(c+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/48*(8*b^3*x^3*log(-(d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) -
6*I*b^2*x^2*dilog(2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*
d + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^
2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 6*I*b^2*x^
2*dilog(2*((-I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c
^2 - 2*(c + 1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2
+ 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 6*I*b^2*x^2*dilog(
2*((I*(c - 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c
- 1)*d - I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)
*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) - 6*I*b^2*x^2*dilog(2*((-I*(c
- 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d +
 I*d^2 + 2*I*c - I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x
 + a)^2 + c^2 + d^2 - 2*c + 1) + 1) + 4*a^3*log(((I*(c + 1)*d + d^2)*tan(b
*x + a)^2 - c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) -
 2*c - 1)/(tan(b*x + a)^2 + 1)) + 4*a^3*log(((I*(c + 1)*d - d^2)*tan(b*x +
 a)^2 + c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c
 + 1)/(tan(b*x + a)^2 + 1)) - 4*a^3*log(((I*(c - 1)*d + d^2)*tan(b*x + a)^
2 - c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1
)/(tan(b*x + a)^2 + 1)) - 4*a^3*log(((I*(c - 1)*d - d^2)*tan(b*x + a)^2 +
c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) - 2*c + 1)...
```

3.317.6 Sympy [F]

$$\int x^2\text{arctanh}(c+d\tan(a+bx))\,dx=\int x^2\operatorname{atanh}\left(c+d\tan\left(a+bx\right)\right)dx$$

input
```
integrate(x**2*atanh(c+d*tan(b*x+a)),x)
```

output
```
Integral(x**2*atanh(c + d*tan(a + b*x)), x)
```

3.317.7 Maxima [F]

$$\int x^2 \operatorname{arctanh}(c + d\tan(a + bx))\, dx = \int x^2 \operatorname{artanh}\left(d\tan\left(bx + a\right) + c\right)\, dx$$

input
```
integrate(x^2*arctanh(c+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*x^3*log((c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*
b*x + 2*a) + (c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2
 - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/12*x^3*log((c^2 + d^2 -
2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 -
2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*
x + 2*a) - 2*c + 1) - 4*b*d*integrate(-1/3*(2*(c^2 + d^2 - 1)*x^3*cos(2*b*
x + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^3*sin(2*b*x
+ 2*a)^2 + (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2
*a) - (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*
cos(2*b*x + 2*a) + (c^2 - d^2 - 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))
/(c^4 + d^4 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*
cos(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x
 + 2*a)^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 +
 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 +
2*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 - 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2
*b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*
a) + 4*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)
*d - 2*(c*d^3 + (c^3 - c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*si
n(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a
) + 1), x)
```

3.317.8 Giac [F]

$$\int x^2 \operatorname{arctanh}(c + d\tan(a + bx))\, dx = \int x^2 \operatorname{artanh}\left(d\tan\left(bx + a\right) + c\right)\, dx$$

input
```
integrate(x^2*arctanh(c+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(d*tan(b*x + a) + c), x)
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arctanh}(c + d\tan(a + bx))\, dx = \int x^2\, \text{atanh}(c + d\tan(a + b\,x))\, dx$$

input
```
int(x^2*atanh(c + d*tan(a + b*x)),x)
```

output
```
int(x^2*atanh(c + d*tan(a + b*x)), x)
```

3.318 $\int x\text{arctanh}(c+d\tan(a+bx))\,dx$

3.318.1 Optimal result

Integrand size = 13, antiderivative size = 295

$$\int x\text{arctanh}(c+d\tan(a+bx))\,dx = \frac{1}{2}x^2\text{arctanh}(c+d\tan(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{4}x^2\log\left(1+\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right) - \frac{ix\,\text{PolyLog}\left(2,-\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{4b} + \frac{ix\,\text{PolyLog}\left(2,-\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{4b} + \frac{\text{PolyLog}\left(3,-\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{8b^2} - \frac{\text{PolyLog}\left(3,-\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{8b^2}$$

output
```
1/2*x^2*arctanh(c+d*tan(b*x+a))+1/4*x^2*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/
(1-c-I*d))-1/4*x^2*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*x*po
lylog(2,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*x*polylog(2,-(1+c
-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b+1/8*polylog(3,-(1-c+I*d)*exp(2*I*a+2
*I*b*x)/(1-c-I*d))/b^2-1/8*polylog(3,-(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*
d))/b^2
```

3.318.2 Mathematica [A] (verified)

Time = 2.86 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.88

$$\int x\text{arctanh}(c+d\tan(a+bx))\,dx$$
$$=\frac{4b^2x^2\text{arctanh}(c+d\tan(a+bx))+2b^2x^2\log\left(1+\frac{(-1+c+id)e^{-2i(a+bx)}}{-1+c-id}\right)-2b^2x^2\log\left(1+\frac{(1+c+id)e^{-2i(a+bx)}}{1+c-id}\right)+}{}$$

input
```
Integrate[x*ArcTanh[c + d*Tan[a + b*x]],x]
```

output
```
(4*b^2*x^2*ArcTanh[c + d*Tan[a + b*x]] + 2*b^2*x^2*Log[1 + (-1 + c + I*d)/
((-1 + c - I*d)*E^((2*I)*(a + b*x)))] - 2*b^2*x^2*Log[1 + (1 + c + I*d)/((
1 + c - I*d)*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (1 - c - I*d)/((
-1 + c - I*d)*E^((2*I)*(a + b*x)))] - (2*I)*b*x*PolyLog[2, (-1 - c - I*d)/
((1 + c - I*d)*E^((2*I)*(a + b*x)))] + PolyLog[3, (1 - c - I*d)/((-1 + c -
 I*d)*E^((2*I)*(a + b*x)))] - PolyLog[3, (-1 - c - I*d)/((1 + c - I*d)*E^(
(2*I)*(a + b*x)))])/(8*b^2)
```

3.318.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6821, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(d\tan(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6821}$$

$$-\frac{1}{2}b(ic+d+i)\int\frac{e^{2ia+2ibx}x^2}{c+(c-id+1)e^{2ia+2ibx}+id+1}dx+\frac{1}{2}b(-d+i(1-c))\int\frac{e^{2ia+2ibx}x^2}{-c+(-c+id+1)e^{2ia+2ibx}-id+1}dx+\frac{1}{2}x^2\text{arctanh}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}b(-d+i(1-c))\left(\frac{x^2\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{\int x\log\left(\frac{e^{2ia+2ibx}(-c+id+1)}{-c-id+1}+1\right)dx}{b(-d+i(1-c))}\right)-\frac{1}{2}b(ic+d+i)\left(\frac{x^2\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{\int x\log\left(\frac{e^{2ia+2ibx}(c-id+1)}{c+id+1}+1\right)dx}{bd+i(bc+b)}\right)+\frac{1}{2}x^2\text{arctanh}(d\tan(a+bx)+c)$$

$$\Big\downarrow \ 3011$$

$$\frac{1}{2}b(-d+i(1-c))\left(\frac{x^2\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{i\int\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)dx}{2b}}{b(-d+i(1-c))}\right)-\frac{1}{2}b(ic+d+i)\left(\frac{x^2\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{i\int\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)dx}{2b}}{bd+i(bc+b)}\right)+\frac{1}{2}x^2\text{arctanh}(d\tan(a+bx)+c)$$

$$\Big\downarrow \ 2720$$

$$\frac{1}{2}b(-d+i(1-c))\left(\frac{x^2\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)de^{2i}}{4b^2}}{b(-d+i(1-c))}\right.$$

$$\frac{1}{2}b(ic+d+i)\left(\frac{x^2\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)de^{2ia+2ibx}}{4b^2}}{bd+i(bc+b)}\right.$$

$$\frac{1}{2}x^2\text{arctanh}(d\tan(a+bx)+c)$$

$$\Big\downarrow \ 7143$$

3.318. $\int x\text{arctanh}(c+d\tan(a+bx))\,dx$

$$\frac{1}{2}x^2\text{arctanh}(d\tan(a+bx)+c)+\frac{1}{2}b(-d+i(1-c))\left(\frac{x^2\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{\text{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2}}{b(-d+i(1-c))}\right)-\frac{1}{2}b(ic+d+i)\left(\frac{x^2\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{\text{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2}}{bd+i(bc+b)}\right)$$

input
```
Int[x*ArcTanh[c + d*Tan[a + b*x]],x]
```

output
```
(x^2*ArcTanh[c + d*Tan[a + b*x]])/2 + (b*(I*(1 - c) - d)*((x^2*Log[1 + ((1
 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d)])/(2*b*(I*(1 - c) - d))
 - (((I/2)*x*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c -
 I*d))])/b - PolyLog[3, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c -
 I*d))]/(4*b^2))/(b*(I*(1 - c) - d))))/2 - (b*(I + I*c + d)*((x^2*Log[1 +
((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d)])/(2*(I*(b + b*c) +
b*d)) - (((I/2)*x*PolyLog[2, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1
+ c + I*d))])/b - PolyLog[3, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1
+ c + I*d))]/(4*b^2))/(I*(b + b*c) + b*d)))/2
```

3.318.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6821
```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + (-Simp[I*b*((1 + c - I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)
*(E^(2*I*a + 2*I*b*x)/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x))), x
], x] + Simp[I*b*((1 - c + I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*
I*a + 2*I*b*x)/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x))), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.318.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.60 (sec) , antiderivative size = 6547, normalized size of antiderivative = 22.19

method	result	size
risch	Expression too large to display	6547

input
```
int(x*arctanh(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.318.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1689 vs. $2(209) = 418$.

Time = 0.35 (sec) , antiderivative size = 1689, normalized size of antiderivative = 5.73

$$\int x\text{arctanh}(c + d\tan(a + bx))\,dx = \text{Too large to display}$$

input
```
integrate(x*arctanh(c+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/16*(4*b^2*x^2*log(-(d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) -
2*I*b*x*dilog(2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d +
(I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 +
d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 2*I*b*x*dilog(
2*((-I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^2 - 2*(
c + 1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c +
1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 2*I*b*x*dilog(2*((I*(c - 1
)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c - 1)*d - I*d
^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a
)^2 + c^2 + d^2 - 2*c + 1) + 1) - 2*I*b*x*dilog(2*((-I*(c - 1)*d - d^2)*ta
n(b*x + a)^2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c -
 I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 +
d^2 - 2*c + 1) + 1) - 2*a^2*log(((I*(c + 1)*d + d^2)*tan(b*x + a)^2 - c^2
+ I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/(tan(b
*x + a)^2 + 1)) - 2*a^2*log(((I*(c + 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*
(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c + 1)/(tan(b*x +
 a)^2 + 1)) + 2*a^2*log(((I*(c - 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c -
 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/(tan(b*x + a)^
2 + 1)) + 2*a^2*log(((I*(c - 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c - 1)*
d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) - 2*c + 1)/(tan(b*x + a)^2...
```

3.318.6 Sympy [F]

$$\int x\text{arctanh}(c + d\tan(a + bx))\,dx = \int x\,\text{atanh}\,(c + d\tan\,(a + bx))\,dx$$

input
```
integrate(x*atanh(c+d*tan(b*x+a)),x)
```

output
```
Integral(x*atanh(c + d*tan(a + b*x)), x)
```

3.318.7 Maxima [F]

$$\int x\text{arctanh}(c+d\tan(a+bx))\,dx=\int x\,\text{artanh}\,(d\tan\,(bx+a)+c)\;dx$$

input
```
integrate(x*arctanh(c+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-2*b*d*integrate(-(2*(c^2 + d^2 - 1)*x^2*cos(2*b*x + 2*a)^2 + 2*c*d*x^2*si
n(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*sin(2*b*x + 2*a)^2 + (c^2 - d^2 - 1
)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) - (c^2 - d^2 - 1)*x^2
*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*x + 2*a) + (c^2 -
 d^2 - 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)
*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c
^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (c^4 + d^4 +
2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 -
1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 -
 1)*d^2 - 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(c*d^3 +
(c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 - d^4 - 2*c^2
 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)*d - 2*(c*d^3 + (c^3 - c)
*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin(4*b*x
 + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*log(
(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b*x + 2*a) +
(c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 + 2*c
+ 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/8*x^2*log((c^2 + d^2 - 2*c + 1)*cos(2
*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*sin(2
*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) - 2*c
 + 1)
```

3.318.8 Giac [F]

$$\int x\text{arctanh}(c+d\tan(a+bx))\,dx=\int x\,\text{artanh}\,(d\tan\,(bx+a)+c)\;dx$$

input
```
integrate(x*arctanh(c+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctanh(d*tan(b*x + a) + c), x)
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arctanh}(c+d\tan(a+bx))\,dx=\int x\,\text{atanh}(c+d\tan(a+b\,x))\;dx$$

input
```
int(x*atanh(c + d*tan(a + b*x)),x)
```

output
```
int(x*atanh(c + d*tan(a + b*x)), x)
```

3.319 $\int \operatorname{arctanh}(c+d\tan(a+bx))\,dx$

3.319.1 Optimal result

Integrand size = 11, antiderivative size = 194

$$\begin{aligned}\int \operatorname{arctanh}(c+d\tan(a+bx))\,dx = {} & x\operatorname{arctanh}(c+d\tan(a+bx)) \\ & + \frac{1}{2}x\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) \\ & - \frac{1}{2}x\log\left(1+\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right) \\ & - \frac{i\operatorname{PolyLog}\left(2,-\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{4b} \\ & + \frac{i\operatorname{PolyLog}\left(2,-\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{4b}\end{aligned}$$

output
```
x*arctanh(c+d*tan(b*x+a))+1/2*x*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d
))-1/2*x*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*polylog(2,-(1-
c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*polylog(2,-(1+c-I*d)*exp(2*I*
a+2*I*b*x)/(1+c+I*d))/b
```

3.319.2 Mathematica [A] (warning: unable to verify)

Time = 4.01 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.88

$$\int \operatorname{arctanh}(c+d\tan(a+bx))\,dx = x\Bigg(\operatorname{arctanh}(c+d\tan(a+bx)) + \frac{2a\log(1-c-d\tan(a+bx)) - i\log(1-i\tan(a+bx))\log\left(\frac{-1+c+d\tan(a+bx)}{-1+c-id}\right) + i\log(1+i\tan(a+bx)}{}$$

input
```
Integrate[ArcTanh[c + d*Tan[a + b*x]],x]
```

output
```
x*(ArcTanh[c + d*Tan[a + b*x]] + (2*a*Log[1 - c - d*Tan[a + b*x]] - I*Log[
1 - I*Tan[a + b*x]]*Log[(-1 + c + d*Tan[a + b*x])/(-1 + c - I*d)] + I*Log[
1 + I*Tan[a + b*x]]*Log[(-1 + c + d*Tan[a + b*x])/(-1 + c + I*d)] - 2*a*Lo
g[1 + c + d*Tan[a + b*x]] + I*Log[1 - I*Tan[a + b*x]]*Log[(1 + c + d*Tan[a
 + b*x])/(1 + c - I*d)] - I*Log[1 + I*Tan[a + b*x]]*Log[(1 + c + d*Tan[a +
 b*x])/(1 + c + I*d)] + I*PolyLog[2, -((d*(-I + Tan[a + b*x]))/(-1 + c + I
*d))] - I*PolyLog[2, -((d*(-I + Tan[a + b*x]))/(1 + c + I*d))] - I*PolyLog
[2, -((d*(I + Tan[a + b*x]))/(-1 + c - I*d))] + I*PolyLog[2, -((d*(I + Tan
[a + b*x]))/(1 + c - I*d))])/(4*a - (2*I)*Log[1 - I*Tan[a + b*x]] + (2*I)*
Log[1 + I*Tan[a + b*x]]))
```

3.319.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.48, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6813, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(d\tan(a+bx)+c)\,dx$$

$$\Big\downarrow \; 6813$$

$$-b(ic+d+i)\int \frac{e^{2ia+2ibx}x}{c+(c-id+1)e^{2ia+2ibx}+id+1}dx + b(-d+i(1-c))\int \frac{e^{2ia+2ibx}x}{-c+(-c+id+1)e^{2ia+2ibx}-id+1}dx + x\operatorname{arctanh}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$b(-d+i(1-c))\left(\frac{x\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{\int\log\left(\frac{e^{2ia+2ibx}(-c+id+1)}{-c-id+1}+1\right)dx}{2b(-d+i(1-c))}\right)-b(ic+d+i)\left(\frac{x\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{\int\log\left(\frac{e^{2ia+2ibx}(c-id+1)}{c+id+1}+1\right)dx}{2(bd+i(bc+b))}\right)+x\text{arctanh}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{2715}$$

$$b(-d+i(1-c))\left(\frac{i\int e^{-2ia-2ibx}\log\left(\frac{e^{2ia+2ibx}(-c+id+1)}{-c-id+1}+1\right)de^{2ia+2ibx}}{4b^2(-d+i(1-c))}+\frac{x\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}\right)-b(ic+d+i)\left(\frac{i\int e^{-2ia-2ibx}\log\left(\frac{e^{2ia+2ibx}(c-id+1)}{c+id+1}+1\right)de^{2ia+2ibx}}{4b(bd+i(bc+b))}+\frac{x\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}\right)+x\text{arctanh}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{2838}$$

$$x\text{arctanh}(d\tan(a+bx)+c)+b(-d+i(1-c))\left(\frac{x\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{i\,\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2(-d+i(1-c))}\right)-b(ic+d+i)\left(\frac{x\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{i\,\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b(bd+i(bc+b))}\right)$$

input
```
Int[ArcTanh[c + d*Tan[a + b*x]],x]
```

output
```
x*ArcTanh[c + d*Tan[a + b*x]] + b*(I*(1 - c) - d)*((x*Log[1 + ((1 - c + I*
d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d)])/(2*b*(I*(1 - c) - d)) - ((I/4)
*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/(b^
2*(I*(1 - c) - d))) - b*(I + I*c + d)*((x*Log[1 + ((1 + c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 + c + I*d)])/(2*(I*(b + b*c) + b*d)) - ((I/4)*PolyLog[2
, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/(b*(I*(b + b*
c) + b*d)))
```

3.319. $\int \text{arctanh}(c+d\tan(a+bx))\,dx$

3.319.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6813
```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Tan[a + b*x]], x] + (-Simp[I*b*(1 + c - I*d)   Int[x*(E^(2*I*a +
 2*I*b*x)/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x))), x], x] + Simp
[I*b*(1 - c + I*d)   Int[x*(E^(2*I*a + 2*I*b*x)/(1 - c - I*d + (1 - c + I*d
)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)
^2, 1]
```

3.319.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(164) = 328$.

Time = 1.98 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.87

method	result
derivativedivides	$d\arctan(\tan(bx+a))\operatorname{arctanh}(c+d\tan(bx+a))-d^2\left(\frac{\arctan\left(-\frac{c+d\tan(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)+c-1\right)}{2d}-\frac{\arctan \ldots}{}\right.$
default	$d\arctan(\tan(bx+a))\operatorname{arctanh}(c+d\tan(bx+a))-d^2\left(\frac{\arctan\left(-\frac{c+d\tan(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)+c-1\right)}{2d}-\frac{\arctan \ldots}{}\right.$
risch	Expression too large to display

input
```
int(arctanh(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(d*arctan(tan(b*x+a))*arctanh(c+d*tan(b*x+a))-d^2*(1/2*arctan(-(c+d*
tan(b*x+a))/d+c/d)/d*ln(d*((c+d*tan(b*x+a))/d-c/d)+c-1)-1/2*arctan(-(c+d*t
an(b*x+a))/d+c/d)/d*ln(d*((c+d*tan(b*x+a))/d-c/d)+c+1)+1/4*I*ln(d*((c+d*ta
n(b*x+a))/d-c/d)+c+1)*(ln((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(1+c+I*d))-ln((
I*d+d*((c+d*tan(b*x+a))/d-c/d))/(I*d-c-1)))/d+1/4*I*(dilog((I*d-d*((c+d*ta
n(b*x+a))/d-c/d))/(1+c+I*d))-dilog((I*d+d*((c+d*tan(b*x+a))/d-c/d))/(I*d-c
-1)))/d-1/4*I*ln(d*((c+d*tan(b*x+a))/d-c/d)+c-1)*(ln((I*d-d*((c+d*tan(b*x+
a))/d-c/d))/(I*d+c-1))-ln((I*d+d*((c+d*tan(b*x+a))/d-c/d))/(1-c+I*d)))/d-1
/4*I*(dilog((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(I*d+c-1))-dilog((I*d+d*((c+d
*tan(b*x+a))/d-c/d))/(1-c+I*d)))/d))
```

3.319.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1185 vs. $2(136) = 272$.

Time = 0.32 (sec) , antiderivative size = 1185, normalized size of antiderivative = 6.11

$$\int \operatorname{arctanh}(c+d\tan(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate(arctanh(c+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(4*b*x*log(-(d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) - 2*(b*
x + a)*log(-2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d + (I
*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^
2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 2*(b*x + a)*log(-2*(
(-I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^2 - 2*(c +
 1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*
tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*log(-2*((I*(c - 1)*d
- d^2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c - 1)*d - I*d^2 -
 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2
+ c^2 + d^2 - 2*c + 1)) + 2*(b*x + a)*log(-2*((-I*(c - 1)*d - d^2)*tan(b*x
 + a)^2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c - I)*t
an(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 -
 2*c + 1)) + 2*a*log(((I*(c + 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)
*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/(tan(b*x + a)^2 +
 1)) + 2*a*log(((I*(c + 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c + 1)*d + (
I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c + 1)/(tan(b*x + a)^2 + 1)) -
 2*a*log(((I*(c - 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (I*c^2
+ I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/(tan(b*x + a)^2 + 1)) - 2*a*l
og(((I*(c - 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c - 1)*d + (I*c^2 + I*d^
2 - 2*I*c + I)*tan(b*x + a) - 2*c + 1)/(tan(b*x + a)^2 + 1)) - I*dilog(...
```

3.319.6 Sympy [F]

$$\int \operatorname{arctanh}(c+d\tan(a+bx))\,dx=\int \operatorname{atanh}\left(c+d\tan\left(a+bx\right)\right)dx$$

input
```
integrate(atanh(c+d*tan(b*x+a)),x)
```

output
```
Integral(atanh(c + d*tan(a + b*x)), x)
```

3.319.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(136) = 272.

Time = 0.35 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.92

$$\int \operatorname{arctanh}(c + d\tan(a + bx))\,dx$$
$$= \frac{4\,(bx + a)\,\operatorname{artanh}\left(d\tan\left(bx + a\right) + c\right) + \left(\arctan\left(\frac{d^2\tan(bx+a)+(c+1)d}{c^2+d^2+2\,c+1}, \frac{(c+1)d\tan(bx+a)+c^2+2\,c+1}{c^2+d^2+2\,c+1}\right) - \arctan\left(\frac{d^2 \mathrm{t}}{}\right.\right.}{}$$

input
```
integrate(arctanh(c+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*(4*(b*x + a)*arctanh(d*tan(b*x + a) + c) + (arctan2((d^2*tan(b*x + a)
+ (c + 1)*d)/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*tan(b*x + a) + c^2 + 2*c +
1)/(c^2 + d^2 + 2*c + 1)) - arctan2((d^2*tan(b*x + a) + (c - 1)*d)/(c^2 +
d^2 - 2*c + 1), ((c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d^2 - 2*c
+ 1)))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log((d^2*tan(b*x + a)^2 + 2*(c
+ 1)*d*tan(b*x + a) + c^2 + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + (b*x + a)*lo
g((d^2*tan(b*x + a)^2 + 2*(c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d
^2 - 2*c + 1)) - I*dilog(-(I*d*tan(b*x + a) - d)/(I*c + d + I)) + I*dilog(
-(I*d*tan(b*x + a) - d)/(I*c + d - I)) - I*dilog((I*d*tan(b*x + a) + d)/(-
I*c + d + I)) + I*dilog((I*d*tan(b*x + a) + d)/(-I*c + d - I)))/b
```

3.319.8 Giac [F]

$$\int \operatorname{arctanh}(c + d\tan(a + bx))\,dx = \int \operatorname{artanh}\left(d\tan\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(arctanh(c+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctanh(d*tan(b*x + a) + c), x)
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}(c+d\tan(a+bx))\,dx=\int \operatorname{atanh}(c+d\tan(a+b\,x))\ dx$$

input
```
int(atanh(c + d*tan(a + b*x)),x)
```

output
```
int(atanh(c + d*tan(a + b*x)), x)
```

3.320 $\int \frac{\operatorname{arctanh}(c+d\tan(a+bx))}{x}\,dx$

3.320.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\operatorname{arctanh}(c+d\tan(a+bx))}{x}\,dx=\operatorname{Int}\left(\frac{\operatorname{arctanh}(c+d\tan(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arctanh(c+d*tan(b*x+a))/x,x)
```

3.320.2 Mathematica [N/A]

Not integrable

Time = 2.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arctanh}(c+d\tan(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTanh[c + d*Tan[a + b*x]]/x,x]
```

output
```
Integrate[ArcTanh[c + d*Tan[a + b*x]]/x, x]
```

3.320.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(d\tan(a+bx)+c)}{x}\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\text{arctanh}(d\tan(a+bx)+c)}{x}dx$$

input
```
Int[ArcTanh[c + d*Tan[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.320.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.320.4 Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\text{arctanh}\left(c+d\tan\left(bx+a\right)\right)}{x}dx$$

input
```
int(arctanh(c+d*tan(b*x+a))/x,x)
```

output
```
int(arctanh(c+d*tan(b*x+a))/x,x)
```

3.320.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c + d\tan(a + bx))}{x}\, dx = \int \frac{\operatorname{artanh}\left(d\tan\left(bx + a\right) + c\right)}{x}\, dx$$

input
```
integrate(arctanh(c+d*tan(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arctanh(d*tan(b*x + a) + c)/x, x)
```

3.320.6 Sympy [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arctanh}(c + d\tan(a + bx))}{x}\, dx = \int \frac{\operatorname{atanh}\left(c + d\tan\left(a + bx\right)\right)}{x}\, dx$$

input
```
integrate(atanh(c+d*tan(b*x+a))/x,x)
```

output
```
Integral(atanh(c + d*tan(a + b*x))/x, x)
```

3.320.7 Maxima [N/A]

Not integrable

Time = 3.42 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c + d\tan(a + bx))}{x}\, dx = \int \frac{\operatorname{artanh}\left(d\tan\left(bx + a\right) + c\right)}{x}\, dx$$

input
```
integrate(arctanh(c+d*tan(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arctanh(d*tan(b*x + a) + c)/x, x)
```

3.320.8 Giac [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c + d\tan(a + bx))}{x}\,dx = \int \frac{\operatorname{artanh}\left(d\tan\left(bx + a\right) + c\right)}{x}\,dx$$

input
```
integrate(arctanh(c+d*tan(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctanh(d*tan(b*x + a) + c)/x, x)
```

3.320.9 Mupad [N/A]

Not integrable

Time = 5.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c + d\tan(a + bx))}{x}\,dx = \int \frac{\operatorname{atanh}(c + d\tan(a + b\,x))}{x}\,dx$$

input
```
int(atanh(c + d*tan(a + b*x))/x,x)
```

output
```
int(atanh(c + d*tan(a + b*x))/x, x)
```

3.321 $\int x^2\textbf{arctanh}(1 - id + d\tan(a + bx))\,dx$

3.321.1 Optimal result

Integrand size = 20, antiderivative size = 170

$$\int x^2\text{arctanh}(1-id+d\tan(a+bx))\,dx=\frac{1}{12}ibx^4+\frac{1}{3}x^3\text{arctanh}(1-id+d\tan(a+bx))-\frac{1}{6}x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{ix^2\,\text{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b}-\frac{x\,\text{PolyLog}\left(3,-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b^2}-\frac{i\,\text{PolyLog}\left(4,-\left((1-id)e^{2ia+2ibx}\right)\right)}{8b^3}$$

output
```
1/12*I*b*x^4+1/3*x^3*arctanh(1-I*d+d*tan(b*x+a))-1/6*x^3*ln(1+(1-I*d)*exp(
2*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*p
olylog(3,-(1-I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,-(1-I*d)*exp(2*I
*a+2*I*b*x))/b^3
```

3.321.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.91

$$\int x^2\text{arctanh}(1-id+d\tan(a+bx))\,dx=\frac{1}{3}x^3\text{arctanh}(1-id+d\tan(a+bx))$$
$$-\frac{4b^3x^3\log\left(1+\frac{ie^{-2i(a+bx)}}{i+d}\right)+6ib^2x^2\,\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{i+d}\right)+6bx\,\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{i+d}\right)-3i\,\text{PolyL}}{24b^3}$$

input
```
Integrate[x^2*ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

output
```
(x^3*ArcTanh[1 - I*d + d*Tan[a + b*x]])/3 - (4*b^3*x^3*Log[1 + I/((I + d)*
E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a
 + b*x)))] + 6*b*x*PolyLog[3, (-I)/((I + d)*E^((2*I)*(a + b*x)))] - (3*I)*
PolyLog[4, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.321.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6817, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arctanh}(d\tan(a+bx)-id+1)\,dx$$

$$\Big\downarrow \; 6817$$

$$\frac{1}{3}ib\int\frac{x^3}{e^{2ia+2ibx}(1-id)+1}dx+\frac{1}{3}x^3\text{arctanh}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \; 2615$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1-id)\int\frac{e^{2ia+2ibx}x^3}{e^{2ia+2ibx}(1-id)+1}dx\right)+\frac{1}{3}x^3\text{arctanh}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \; 2620$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1-id)\left(\frac{x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{3\int x^2\log\left(e^{2ia+2ibx}(1-id)+1\right)dx}{2b(d+i)}\right)\right)+$$
$$\frac{1}{3}x^3\text{arctanh}(d\tan(a+bx)-id+1)$$

$\big\downarrow$ 3011

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1-id)\left(\frac{x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,-\left((1-id)e^{2ia+2\ldots}\right.\right.}{b}\right.}{2b(d+i)}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(d\tan(a+bx)-id+1)$$

$\big\downarrow$ 7163

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1-id)\left(\frac{x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,-\left((1-id)e^{2ia+2\ldots}\right.\right.}{2b}\right.}{\ldots}\right.}{2b(d+i)}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(d\tan(a+bx)-id+1)$$

$\big\downarrow$ 2720

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1-id)\left(\frac{x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,-\left((1-\ldots\right.\right.}{4b^2}\right.}{\ldots}\right.}{2b(d+i)}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(d\tan(a+bx)-id+1)$$

$\big\downarrow$ 7143

$$\frac{1}{3}x^3\text{arctanh}(d\tan(a+bx)-id+1)+$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1-id)\left(\frac{x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,-\left((1-id)e^{2ia+2ibx}\right.\right.}{4b^2}\right.}{\ldots}\right.}{2b(d+i)}\right.\right.$$

input
```
Int[x^2*ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

3.321. $\int x^2\text{arctanh}(1-id+d\tan(a+bx))\,dx$

output
```
(x^3*ArcTanh[1 - I*d + d*Tan[a + b*x]])/3 + (I/3)*b*(x^4/4 - (1 - I*d)*((x
^3*Log[1 + (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I + d)) - (3*(((I/2)*
x^2*PolyLog[2, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))])/b - (I*(((-1/2*I)*x*
PolyLog[3, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))])/b + PolyLog[4, -((1 - I*
d)*E^((2*I)*a + (2*I)*b*x))]/(4*b^2)))/b))/(2*b*(I + d))))
```

3.321.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6817
```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.321.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.74 (sec) , antiderivative size = 2277, normalized size of antiderivative = 13.39

method	result	size
risch	Expression too large to display	2277

input
```
int(x^2*arctanh(1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/8/b^3/(I+d)*polylog(4,I*(I+d)*exp(2*I*(b*x+a)))+1/4*I/b*d/(I+d)*polylog(
2,I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/2*I/b^3*d*a^2/(I+d)*dilog(1-I*exp(I*(b*x
+a))*(-I*(I+d))^(1/2))+1/2*I/b^3*d*a^2/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*
(I+d))^(1/2))-1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*
x-1/4/b/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I+d)*polylo
g(2,I*(I+d)*exp(2*I*(b*x+a)))*a^2-1/2/b^3*a^2/(I+d)*dilog(1-I*exp(I*(b*x+a
))*(-I*(I+d))^(1/2))-1/2/b^3*a^2/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))
^(1/2))-1/6*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^3-1/6*I/(I+d)*ln(1-I*
(I+d)*exp(2*I*(b*x+a)))*x^3+1/12*I*b*x^4-1/2/b^2*d*a^2/(I+d)*ln(1-I*exp(I*
(b*x+a))*(-I*(I+d))^(1/2))*x-1/2/b^2*d*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I
*(I+d))^(1/2))*x+1/2/b^2*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^2*x-1/3*
x^3*ln(exp(I*(b*x+a)))-1/4/b^2*d/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))
*x+1/3/b^3*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/2/b^3*d*a^3/(I+d)*
ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2/b^3*d*a^3/(I+d)*ln(1+I*exp(I*(
b*x+a))*(-I*(I+d))^(1/2))-1/2*I/b^3*a^3/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I
+d))^(1/2))-1/2*I/b^3*a^3/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/
6*I/b^3*a^3/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)-1/4*I/b^2/(I
+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))*x-1/8*I/b^3*d/(I+d)*polylog(4,I*(I
+d)*exp(2*I*(b*x+a)))+1/3*I/b^3/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^3+1
/6/b^3*a^3*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)-1/2*I/b^...
```

3.321.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(118) = 236$.

Time = 0.27 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.03

$$\int x^2 \operatorname{arctanh}(1 - id + d\tan(a + bx))\,dx = \frac{i\,b^4x^4 + 2\,b^3x^3\log\left(-\frac{((d+i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,d-4}e^{(i\,bx+i\,a)}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i}\right.}{}$$

input
```
integrate(x^2*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(I*b^4*x^4 + 2*b^3*x^3*log(-((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I
*b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a))
+ 6*I*b^2*x^2*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*
log(1/2*(2*(d + I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(d + I)) + 2*a^3*l
og(1/2*(2*(d + I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(d + I)) - 12*b*x*p
olylog(3, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2*sq
rt(4*I*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(4*I*d - 4)
*e^(I*b*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*
x + I*a) + 1) - 12*I*polylog(4, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*
I*polylog(4, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

3.321.6 Sympy [F]

$$\int x^2 \text{arctanh}(1 - id + d\tan(a + bx))\,dx = \int x^2 \,\text{atanh}\,(d\tan\,(a + bx) - id + 1)\,dx$$

input
```
integrate(x**2*atanh(1-I*d+d*tan(b*x+a)),x)
```

output
```
Integral(x**2*atanh(d*tan(a + b*x) - I*d + 1), x)
```

3.321.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(118) = 236$.

Time = 0.24 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.02

$$\int x^2 \text{arctanh}(1 - id + d\tan(a + bx))\,dx$$
$$= \frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\text{artanh}(d\tan(bx+a)-i\,d+1)}{b^2} - \frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(-4i\,(bx+a)^3+9i\,(bx+a\right.}{}}{}$$

input
```
integrate(x^2*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctanh(d*tan(b
*x + a) - I*d + 1)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*
x + a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^2
)*arctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) + cos
(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog
((I*d - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*
x + a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^
2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog
(3, (I*d - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d - 1)*e^(2*I*b*x +
 2*I*a)))/b^2)/b
```

3.321.8 Giac [F]

$$\int x^2 \text{arctanh}(1 - id + d\tan(a + bx))\,dx = \int x^2\,\text{artanh}\left(d\tan\left(bx + a\right) - i\,d + 1\right)\,dx$$

input
```
integrate(x^2*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(d*tan(b*x + a) - I*d + 1), x)
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arctanh}(1 - id + d\tan(a + bx))\,dx = \int x^2\,\text{atanh}(d\tan(a + b\,x) + 1 - d\,1\text{i})\,dx$$

input
```
int(x^2*atanh(d*tan(a + b*x) - d*1i + 1),x)
```

output
```
int(x^2*atanh(d*tan(a + b*x) - d*1i + 1), x)
```

3.322 $\int x\textbf{arctanh}(1 - id + d\tan(a + bx))\,dx$

3.322.1 Optimal result

Integrand size = 18, antiderivative size = 133

$$\begin{aligned}\int x\text{arctanh}(1 - id + d\tan(a + bx))\,dx = {} & \frac{1}{6}ibx^3 + \frac{1}{2}x^2\text{arctanh}(1 - id + d\tan(a + bx)) \\ & - \frac{1}{4}x^2\log\left(1 + (1 - id)e^{2ia+2ibx}\right) \\ & + \frac{ix\,\text{PolyLog}\left(2, -\left((1 - id)e^{2ia+2ibx}\right)\right)}{4b} \\ & - \frac{\text{PolyLog}\left(3, -\left((1 - id)e^{2ia+2ibx}\right)\right)}{8b^2}\end{aligned}$$

output
```
1/6*I*b*x^3+1/2*x^2*arctanh(1-I*d+d*tan(b*x+a))-1/4*x^2*ln(1+(1-I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylo
g(3,-(1-I*d)*exp(2*I*a+2*I*b*x))/b^2
```

3.322.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\begin{aligned}&\int x\text{arctanh}(1 - id + d\tan(a + bx))\,dx \\ &= \frac{1}{2}x^2\text{arctanh}(1 - id + d\tan(a + bx)) \\ &\quad - \frac{2b^2x^2\log\left(1 + \frac{ie^{-2i(a+bx)}}{i+d}\right) + 2ibx\,\text{PolyLog}\left(2, -\frac{ie^{-2i(a+bx)}}{i+d}\right) + \text{PolyLog}\left(3, -\frac{ie^{-2i(a+bx)}}{i+d}\right)}{8b^2}\end{aligned}$$

input
```
Integrate[x*ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

output
```
(x^2*ArcTanh[1 - I*d + d*Tan[a + b*x]])/2 - (2*b^2*x^2*Log[1 + I/((I + d)*
E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a + b
*x)))] + PolyLog[3, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

3.322.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6817, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(d\tan(a+bx)-id+1)\,dx$$

$$\Big\downarrow \ 6817$$

$$\frac{1}{2}ib\int \frac{x^2}{e^{2ia+2ibx}(1-id)+1}dx+\frac{1}{2}x^2\text{arctanh}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \ 2615$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1-id)\int \frac{e^{2ia+2ibx}x^2}{e^{2ia+2ibx}(1-id)+1}dx\right)+\frac{1}{2}x^2\text{arctanh}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \ 2620$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1-id)\left(\frac{x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{\int x\log\left(e^{2ia+2ibx}(1-id)+1\right)dx}{b(d+i)}\right)\right)+\frac{1}{2}x^2\text{arctanh}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \ 3011$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1-id)\left(\frac{x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)d}{2b}}{b(d+i)}\right.\right.$$
$$\frac{1}{2}x^2\text{arctanh}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \ 2720$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1-id)\left(\frac{x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{\frac{ix\operatorname{PolyLog}(2,-((1-id)e^{2ia+2ibx}))}{2b}-\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}(2,-((1-id)e^{2}}{4b^2}}{b(d+i)}\right.\right.$$

$$\frac{1}{2}x^2\text{arctanh}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \; 7143$$

$$\frac{1}{2}x^2\text{arctanh}(d\tan(a+bx)-id+1)+$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1-id)\left(\frac{x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{\frac{ix\operatorname{PolyLog}(2,-((1-id)e^{2ia+2ibx}))}{2b}-\frac{\operatorname{PolyLog}(3,-((1-id)e^{2ia+2ibx}))}{4b^2}}{b(d+i)}\right)\right)$$

input
```
Int[x*ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

output
```
(x^2*ArcTanh[1 - I*d + d*Tan[a + b*x]])/2 + (I/2)*b*(x^3/3 - (1 - I*d)*((x
^2*Log[1 + (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I + d)) - (((I/2)*x*P
olyLog[2, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 - I*d
)*E^((2*I)*a + (2*I)*b*x))]/(4*b^2))/(b*(I + d))))
```

3.322.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6817
```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.322.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.14 (sec) , antiderivative size = 2187, normalized size of antiderivative = 16.44

method	result	size
risch	Expression too large to display	2187

input
```
int(x*arctanh(1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*a*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/2/b*a*d/(I+d)*
ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/2/b*d/(I+d)*ln(1-I*(I+d)*exp(2
*I*(b*x+a)))*a*x+1/2*I/b*a/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x
-1/2*I/b/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a*x+1/4*I/b*d/(I+d)*polylog(
2,I*(I+d)*exp(2*I*(b*x+a)))*x+1/4*I/b^2*d/(I+d)*polylog(2,I*(I+d)*exp(2*I*
(b*x+a)))*a-1/2*I/b^2*a*d/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))
+1/6*I*b*x^3-1/2*I/b^2*a*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2)
)+1/2*I/b*a/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/4/b^2*d/(I+d
)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^2-1/4/b^2*a^2*d/(I+d)*ln(I*exp(2*I*(b*x
+a))+exp(2*I*(b*x+a))*d+I)+1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(
I+d))^(1/2))+1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1
/2/b^2*a/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/2/b^2*a/(I+d)*
dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/4/b/(I+d)*polylog(2,I*(I+d)*e
xp(2*I*(b*x+a)))*x-1/4/b^2/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*a-1/8
/b^2*d/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))-1/4*d/(I+d)*ln(1-I*(I+d)*
exp(2*I*(b*x+a)))*x^2-1/4*I/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^2-1/8*I
/b^2/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))-1/4*I/b^2*a^2/(I+d)*ln(I*ex
p(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)-1/4*I/b^2/(I+d)*ln(1-I*(I+d)*exp(2*I*
(b*x+a)))*a^2+1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+
1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2*x^2*ln(...
```

3.322.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(93) = 186$.

Time = 0.28 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.20

$$\int x\text{arctanh}(1 - id + d\tan(a + bx))\,dx$$
$$= \frac{2i\,b^3x^3 + 3\,b^2x^2\log\left(-\frac{((d+i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right) + 2i\,a^3 + 6i\,bx\text{Li}_2\left(\frac{1}{2}\sqrt{4i\,d-4}e^{(i\,bx+i\,a)}\right) + 6i\,bx\text{Li}_2\left(-\frac{1}{2}\right.}{}$$

input
```
integrate(x*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*I*b^3*x^3 + 3*b^2*x^2*log(-((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2
*I*b*x - 2*I*a)/d) + 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x
+ I*a)) + 6*I*b*x*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 3*a^2*log(
1/2*(2*(d + I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(d + I)) - 3*a^2*log(1
/2*(2*(d + I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(d + I)) - 3*(b^2*x^2 -
 a^2)*log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 3*(b^2*x^2 - a^2)*log
(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 6*polylog(3, 1/2*sqrt(4*I*d -
 4)*e^(I*b*x + I*a)) - 6*polylog(3, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))
/b^2
```

3.322.6 Sympy [F]

$$\int x\text{arctanh}(1-id+d\tan(a+bx))\,dx=\int x\,\text{atanh}\,(d\tan\,(a+bx)-id+1)\,dx$$

input
```
integrate(x*atanh(1-I*d+d*tan(b*x+a)),x)
```

output
```
Integral(x*atanh(d*tan(a + b*x) - I*d + 1), x)
```

3.322.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(93) = 186.

Time = 0.24 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.86

$$\int x\text{arctanh}(1-id+d\tan(a+bx))\,dx = \frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\text{artanh}(d\tan(bx+a)-i\,d+1)}{b}-\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\text{Li}_2\left((i\,d-1)e^{(2i\,bx+2i\,a)}\right)-6\left(-i\,(bx+a)^2+2i\,(bx+a\right.}{}}{}$$

input
```
integrate(x*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arctanh(d*tan(b*x + a) - I*d + 1)/b
 - (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((I*d - 1)*e^(2*I
*b*x + 2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(-d*cos(2*b*x
 + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1) + 3
*((b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1
)*sin(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*
polylog(3, (I*d - 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

3.322.8 Giac [F]

$$\int x\text{arctanh}(1-id+d\tan(a+bx))\,dx=\int x\,\text{artanh}\left(d\tan\left(bx+a\right)-i\,d+1\right)\,dx$$

input
```
integrate(x*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctanh(d*tan(b*x + a) - I*d + 1), x)
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arctanh}(1-id+d\tan(a+bx))\,dx=\int x\,\text{atanh}(d\tan(a+b\,x)+1-d\,1\text{i})\,dx$$

input
```
int(x*atanh(d*tan(a + b*x) - d*1i + 1),x)
```

output
```
int(x*atanh(d*tan(a + b*x) - d*1i + 1), x)
```

3.323 $\int \operatorname{arctanh}(1 - id + d\tan(a + bx))\,dx$

3.323.1 Optimal result

Integrand size = 16, antiderivative size = 93

$$\int \operatorname{arctanh}(1 - id + d\tan(a + bx))\,dx = \frac{1}{2}ibx^2 + x\operatorname{arctanh}(1 - id + d\tan(a + bx)) - \frac{1}{2}x\log\left(1 + (1 - id)e^{2ia+2ibx}\right) + \frac{i\operatorname{PolyLog}\left(2, -\left((1 - id)e^{2ia+2ibx}\right)\right)}{4b}$$

output
```
1/2*I*b*x^2+x*arctanh(1-I*d+d*tan(b*x+a))-1/2*x*ln(1+(1-I*d)*exp(2*I*a+2*I
*b*x))+1/4*I*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b
```

3.323.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 766 vs. 2(93) = 186.

Time = 8.09 (sec) , antiderivative size = 766, normalized size of antiderivative = 8.24

$$\int \operatorname{arctanh}(1 - id + d\tan(a + bx))\,dx = x\operatorname{arctanh}(1 - id + d\tan(a + bx)) + \frac{x\left(2ibx\log(2\cos(bx)(\cos(bx) - i\sin(bx))) - \log\left(\frac{\sec(bx)(\cos(a)-i\sin(a))((2i+d)\cos(a+bx)+id\sin(a+bx))}{2(i+d)}\right)\log(1 - i\right.}{((2i + d)\cos(a + bx) + id\sin(a + bx))\left(\frac{i\log(1+i\tan(bx))\sec(bx)(d\cos(a)+i(2i+d)\sin(a))}{(2i+d)\cos(a+bx)+id\sin(a+bx)} + \frac{\log(}{}\right.}$$

input
```
Integrate[ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

output
```
x*ArcTanh[1 - I*d + d*Tan[a + b*x]] + (x*((2*I)*b*x*Log[2*Cos[b*x]*(Cos[b*
x] - I*Sin[b*x])] - Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((2*I + d)*Cos[a + b
*x] + I*d*Sin[a + b*x]))/(2*(I + d))]*Log[1 - I*Tan[b*x]] + Log[(Sec[b*x]*
((2 - I*d)*Cos[a + b*x] + d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Log[
1 + I*Tan[b*x]] - PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - PolyLog[2, (Sec
[b*x]*(d*Cos[a] + I*(2*I + d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*
(I + d))] + PolyLog[2, -1/2*((Cos[a] + I*Sin[a])*(d*Cos[a] + I*(2*I + d)*S
in[a])*(-I + Tan[b*x]))])*Sec[a + b*x]^2*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*
x] + Sin[b*x])*((2 - I*d)*Cos[a + b*x] + d*Sin[a + b*x]))/(((2*I + d)*Cos[
a + b*x] + I*d*Sin[a + b*x])*((I*Log[1 + I*Tan[b*x]]*Sec[b*x]*(d*Cos[a] +
I*(2*I + d)*Sin[a]))/((2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]) + (Log[1
- I*Tan[b*x]]*Sec[b*x]*((-I)*d*Cos[a] + (2*I + d)*Sin[a]))/((2*I + d)*Cos[
a + b*x] + I*d*Sin[a + b*x]) + 2*b*x*(1 - I*Tan[b*x]) + (Log[(Sec[b*x]*((2
 - I*d)*Cos[a + b*x] + d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*x
]^2)/(-I + Tan[b*x]) - (Log[1 + ((Cos[a] + I*Sin[a])*(d*Cos[a] + I*(2*I +
d)*Sin[a])*(-I + Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + Log[(Sec[b*x]
*(Cos[a] - I*Sin[a])*((2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]))/(2*(I +
d))]*(-I + Tan[b*x]) - (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((2*I + d)*Cos[a
 + b*x] + I*d*Sin[a + b*x]))/(2*(I + d))]*Sec[b*x]^2)/(I + Tan[b*x]))*(-I
+ Tan[a + b*x])*(2 - I*d + d*Tan[a + b*x]))
```

3.323.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6809, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(d\tan(a+bx)-id+1)\,dx$$

$$\Big\downarrow \text{6809}$$

$$ib\int \frac{x}{e^{2ia+2ibx}(1-id)+1}dx+x\operatorname{arctanh}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{2615}$$

$$ib\left(\frac{x^2}{2}-(1-id)\int \frac{e^{2ia+2ibx}x}{e^{2ia+2ibx}(1-id)+1}dx\right)+x\text{arctanh}(d\tan(a+bx)-id+1)$$

$\big\downarrow$ 2620

$$ib\left(\frac{x^2}{2}-(1-id)\left(\frac{x\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{\int \log\left(e^{2ia+2ibx}(1-id)+1\right)dx}{2b(d+i)}\right)\right)+ x\text{arctanh}(d\tan(a+bx)-id+1)$$

$\big\downarrow$ 2715

$$ib\left(\frac{x^2}{2}-(1-id)\left(\frac{i\int e^{-2ia-2ibx}\log\left(e^{2ia+2ibx}(1-id)+1\right)de^{2ia+2ibx}}{4b^2(d+i)}+\frac{x\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+ x\text{arctanh}(d\tan(a+bx)-id+1)$$

$\big\downarrow$ 2838

$$x\text{arctanh}(d\tan(a+bx)-id+1)+ ib\left(\frac{x^2}{2}-(1-id)\left(\frac{x\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{i\,\text{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b^2(d+i)}\right)\right)$$

input
```
Int[ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

output
```
x*ArcTanh[1 - I*d + d*Tan[a + b*x]] + I*b*(x^2/2 - (1 - I*d)*((x*Log[1 + (
1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I + d)) - ((I/4)*PolyLog[2, -((1
- I*d)*E^((2*I)*a + (2*I)*b*x))])/(b^2*(I + d))))
```

3.323.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6809
```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Tan[a + b*x]], x] + Simp[I*b   Int[x/(c + I*d + c*E^(2*I*a + 2*I
*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, 1]
```

3.323.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(76) = 152$.

Time = 1.25 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.30

method	result
derivativedivides	$\frac{-\frac{i \operatorname{arctanh}(1-id+d\tan(bx+a))d\ln(-id+d\tan(bx+a))}{2}+\frac{i \operatorname{arctanh}(1-id+d\tan(bx+a))d\ln(id+d\tan(bx+a))}{2}-\frac{d^2\left(-\frac{i\left(\operatorname{dilog}\left(\frac{i(-id-\cdots}{\cdots}\right)\cdots\right)}{\cdots}\cdots\right)}{\cdots}}{\cdots}$
default	$\frac{-\frac{i \operatorname{arctanh}(1-id+d\tan(bx+a))d\ln(-id+d\tan(bx+a))}{2}+\frac{i \operatorname{arctanh}(1-id+d\tan(bx+a))d\ln(id+d\tan(bx+a))}{2}-\frac{d^2\left(-\frac{i\left(\operatorname{dilog}\left(\frac{i(-id-\cdots}{\cdots}\right)\cdots\right)}{\cdots}\cdots\right)}{\cdots}}{\cdots}$
risch	Expression too large to display

input
```
int(arctanh(1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-1/2*I*arctanh(1-I*d+d*tan(b*x+a))*d*ln(-I*d+d*tan(b*x+a))+1/2*I*ar
ctanh(1-I*d+d*tan(b*x+a))*d*ln(I*d+d*tan(b*x+a))-1/2*d^2*(-I/d*(1/2*dilog(
1/2*I*(-I*d+d*tan(b*x+a))/d)+1/2*ln(I*d+d*tan(b*x+a))*ln(1/2*I*(-I*d+d*tan
(b*x+a))/d)-1/2*dilog(I*(I*d+d*tan(b*x+a)-I*(2*I+2*d))/(2*I+2*d))-1/2*ln(I
*d+d*tan(b*x+a))*ln(I*(I*d+d*tan(b*x+a)-I*(2*I+2*d))/(2*I+2*d)))+I/d*(-1/2
*dilog(1-1/2*I*d+1/2*d*tan(b*x+a))-1/2*ln(-I*d+d*tan(b*x+a))*ln(1-1/2*I*d+
1/2*d*tan(b*x+a))+1/4*ln(-I*d+d*tan(b*x+a))^2)))
```

3.323.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(65) = 130.

Time = 0.27 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.34

$$\int \operatorname{arctanh}(1-id+d\tan(a+bx))\,dx = \frac{i\,b^2x^2 + bx\log\left(-\frac{\left((d+i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right) - i\,a^2 - (bx+a)\log\left(\frac{1}{2}\sqrt{4i\,d-4}e^{(i\,bx+i\,a)}+1\right) - (bx+a)\,\cdots}{\cdots}$$

input
```
integrate(arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(I*b^2*x^2 + b*x*log(-((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x -
2*I*a)/d) - I*a^2 - (b*x + a)*log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1)
 - (b*x + a)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) + a*log(1/2*(2*
(d + I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(d + I)) + a*log(1/2*(2*(d +
I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(d + I)) + I*dilog(1/2*sqrt(4*I*d
- 4)*e^(I*b*x + I*a)) + I*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b
```

3.323.6 Sympy [F]

$$\int \operatorname{arctanh}(1-id+d\tan(a+bx))\,dx = \int \operatorname{atanh}\left(d\tan\left(a+bx\right)-id+1\right)dx$$

input
```
integrate(atanh(1-I*d+d*tan(b*x+a)),x)
```

output
```
Integral(atanh(d*tan(a + b*x) - I*d + 1), x)
```

3.323.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(65) = 130$.

Time = 0.29 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.80

$$\int \operatorname{arctanh}(1 - id + d\tan(a + bx))\,dx =$$
$$-\frac{4\,(bx+a)d\left(\frac{\log(d\tan(bx+a)-i\,d+2)}{d} - \frac{\log(\tan(bx+a)-i)}{d}\right) + d\left(-\frac{2i\left(\log(d\tan(bx+a)-i\,d+2)\log\left(-\frac{i\,d\tan(bx+a)+d+2i}{2\,(d+i)}+1\right)+\mathrm{Li}_2\right.}{d}\right.}{}$$

input
```
integrate(arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*(4*(b*x + a)*d*(log(d*tan(b*x + a) - I*d + 2)/d - log(tan(b*x + a) -
I)/d) + d*(-2*I*(log(d*tan(b*x + a) - I*d + 2)*log(-1/2*(I*d*tan(b*x + a)
+ d + 2*I)/(d + I) + 1) + dilog(1/2*(I*d*tan(b*x + a) + d + 2*I)/(d + I)))
/d + (2*I*log(d*tan(b*x + a) - I*d + 2)*log(tan(b*x + a) - I) - I*log(tan(
b*x + a) - I)^2)/d - 2*I*(log(1/2*d*tan(b*x + a) - 1/2*I*d + 1)*log(tan(b*
x + a) - I) + dilog(-1/2*d*tan(b*x + a) + 1/2*I*d))/d + 2*I*(log(tan(b*x +
 a) - I)*log(-1/2*I*tan(b*x + a) + 1/2) + dilog(1/2*I*tan(b*x + a) + 1/2))
/d) - 8*(b*x + a)*arctanh(d*tan(b*x + a) - I*d + 1))/b
```

3.323.8 Giac [F]

$$\int \operatorname{arctanh}(1 - id + d\tan(a + bx))\,dx = \int \operatorname{artanh}\left(d\tan\left(bx + a\right) - i\,d + 1\right)\,dx$$

input
```
integrate(arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctanh(d*tan(b*x + a) - I*d + 1), x)
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}(1 - id + d\tan(a + bx))\, dx = \int \operatorname{atanh}(d\tan(a + b\,x) + 1 - d\,1\mathrm{i})\, dx$$

input
```
int(atanh(d*tan(a + b*x) - d*1i + 1),x)
```

output
```
int(atanh(d*tan(a + b*x) - d*1i + 1), x)
```

3.324 $\int \frac{\operatorname{arctanh}(1-id+d\tan(a+bx))}{x}\,dx$

3.324.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\operatorname{arctanh}(1-id+d\tan(a+bx))}{x}\,dx=\operatorname{Int}\left(\frac{\operatorname{arctanh}(1-id+d\tan(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arctanh(1-I*d+d*tan(b*x+a))/x,x)
```

3.324.2 Mathematica [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{arctanh}(1-id+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arctanh}(1-id+d\tan(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTanh[1 - I*d + d*Tan[a + b*x]]/x,x]
```

output
```
Integrate[ArcTanh[1 - I*d + d*Tan[a + b*x]]/x, x]
```

3.324.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(d\tan(a+bx)-id+1)}{x}\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\text{arctanh}(d\tan(a+bx)-id+1)}{x}dx$$

input
```
Int[ArcTanh[1 - I*d + d*Tan[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.324.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.324.4 Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{arctanh}\left(1-id+d\tan\left(bx+a\right)\right)}{x}dx$$

input
```
int(arctanh(1-I*d+d*tan(b*x+a))/x,x)
```

output
```
int(arctanh(1-I*d+d*tan(b*x+a))/x,x)
```

3.324.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{\operatorname{arctanh}(1-id+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{artanh}\left(d\tan\left(bx+a\right)-i\,d+1\right)}{x}\,dx$$

input
```
integrate(arctanh(1-I*d+d*tan(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*log(-((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d
)/x, x)
```

3.324.6 Sympy [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arctanh}(1-id+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{atanh}\left(d\tan\left(a+bx\right)-id+1\right)}{x}\,dx$$

input
```
integrate(atanh(1-I*d+d*tan(b*x+a))/x,x)
```

output
```
Integral(atanh(d*tan(a + b*x) - I*d + 1)/x, x)
```

3.324.7 Maxima [N/A]

Not integrable

Time = 4.15 (sec) , antiderivative size = 144, normalized size of antiderivative = 7.20

$$\int \frac{\operatorname{arctanh}(1-id+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{artanh}\left(d\tan\left(bx+a\right)-i\,d+1\right)}{x}\,dx$$

input
```
integrate(arctanh(1-I*d+d*tan(b*x+a))/x,x, algorithm="maxima")
```

output
```
-I*b*x + 1/4*(I*pi - 4*I*a - 2*log(d))*log(x) - 1/2*I*integrate(arctan2(-d
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*
a) - 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)
*sin(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1)/x, x)
```

3.324.8 Giac [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(1-id+d\tan(a+bx))}{x}\,dx = \int \frac{\operatorname{artanh}\left(d\tan\left(bx+a\right)-i\,d+1\right)}{x}\,dx$$

input
```
integrate(arctanh(1-I*d+d*tan(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctanh(d*tan(b*x + a) - I*d + 1)/x, x)
```

3.324.9 Mupad [N/A]

Not integrable

Time = 5.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\operatorname{arctanh}(1-id+d\tan(a+bx))}{x}\,dx = \int \frac{\operatorname{atanh}(d\tan(a+b\,x)+1-d\,1\mathrm{i})}{x}\,dx$$

input
```
int(atanh(d*tan(a + b*x) - d*1i + 1)/x,x)
```

output
```
int(atanh(d*tan(a + b*x) - d*1i + 1)/x, x)
```

3.325 $\int x^2 \textbf{arctanh}(1 + id - d\tan(a + bx))\,dx$

3.325.1 Optimal result

Integrand size = 21, antiderivative size = 171

$$\begin{aligned}\int x^2\text{arctanh}(1+id-d\tan(a+bx))\,dx =& \frac{1}{12}ibx^4+\frac{1}{3}x^3\text{arctanh}(1+id-d\tan(a+bx))\\ &-\frac{1}{6}x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)\\ &+\frac{ix^2\,\text{PolyLog}\left(2,-\left((1+id)e^{2ia+2ibx}\right)\right)}{4b}\\ &-\frac{x\,\text{PolyLog}\left(3,-\left((1+id)e^{2ia+2ibx}\right)\right)}{4b^2}\\ &-\frac{i\,\text{PolyLog}\left(4,-\left((1+id)e^{2ia+2ibx}\right)\right)}{8b^3}\end{aligned}$$

output
```
1/12*I*b*x^4-1/3*x^3*arctanh(-1-I*d+d*tan(b*x+a))-1/6*x^3*ln(1+(1+I*d)*exp
(2*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*
polylog(3,-(1+I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,-(1+I*d)*exp(2*
I*a+2*I*b*x))/b^3
```

3.325.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.91

$$\int x^2\text{arctanh}(1+id-d\tan(a+bx))\,dx = \frac{1}{3}x^3\text{arctanh}(1+id-d\tan(a+bx)) - \frac{4b^3x^3\log\left(1-\frac{ie^{-2i(a+bx)}}{-i+d}\right)+6ib^2x^2\,\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{-i+d}\right)+6bx\,\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{-i+d}\right)-3i\,\text{PolyLog}\left(\right.}{24b^3}$$

input

```
Integrate[x^2*ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

output

```
(x^3*ArcTanh[1 + I*d - d*Tan[a + b*x]])/3 - (4*b^3*x^3*Log[1 - I/((-I + d)
*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/((-I + d)*E^((2*I)*(a
+ b*x)))] + 6*b*x*PolyLog[3, I/((-I + d)*E^((2*I)*(a + b*x)))] - (3*I)*Pol
yLog[4, I/((-I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.325.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6817, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arctanh}(d(-\tan(a+bx))+id+1)\,dx$$

$$\Big\downarrow \ 6817$$

$$\frac{1}{3}ib\int \frac{x^3}{e^{2ia+2ibx}(id+1)+1}dx+\frac{1}{3}x^3\text{arctanh}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \ 2615$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\int \frac{e^{2ia+2ibx}x^3}{e^{2ia+2ibx}(id+1)+1}dx\right)+\frac{1}{3}x^3\text{arctanh}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \ 2620$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\left(\frac{x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{3\int x^2\log\left(e^{2ia+2ibx}(id+1)+1\right)dx}{2b(-d+i)}\right)\right)+ \frac{1}{3}x^3\text{arctanh}(d(-\tan(a+bx))+id+1)$$

$\downarrow$ 3011

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\left(\frac{x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}(2,-((id+1)e^{2ia+2ibx}))}{2b}-\frac{i\int x\operatorname{PolyLog}(2,-((id+1)e^{2ia+2}}{b}\right.}{2b(-d+i)}\right.\right.$$

$$\frac{1}{3}x^3\operatorname{arctanh}(d(-\tan(a+bx))+id+1)$$

$\downarrow$ 7163

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\left(\frac{x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}(2,-((id+1)e^{2ia+2ibx}))}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,-\left((id+1)e^{2ia+2i}\right.\right.}{2b}\right.}{}\right.}{2b(-d+i)}\right.\right.$$

$$\frac{1}{3}x^3\operatorname{arctanh}(d(-\tan(a+bx))+id+1)$$

$\downarrow$ 2720

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\left(\frac{x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}(2,-((id+1)e^{2ia+2ibx}))}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,-\left((id\right.\right.}{4b^2}\right.}{}\right.}{2b(-d+i}\right.\right.$$

$$\frac{1}{3}x^3\operatorname{arctanh}(d(-\tan(a+bx))+id+1)$$

$\downarrow$ 7143

$$\frac{1}{3}x^3\operatorname{arctanh}(d(-\tan(a+bx))+id+1)+$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\left(\frac{x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}(2,-((id+1)e^{2ia+2ibx}))}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,-\left((id+1)e^{2ia+2ibx}\right.\right.}{4b^2}\right.}{}\right.}{2b(-d+i)}\right.\right.$$

input
```
Int[x^2*ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

3.325. $\int x^2\operatorname{arctanh}(1+id-d\tan(a+bx))\,dx$

output
```
(x^3*ArcTanh[1 + I*d - d*Tan[a + b*x]])/3 + (I/3)*b*(x^4/4 - (1 + I*d)*((x
^3*Log[1 + (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I - d)) - (3*(((I/2)*
x^2*PolyLog[2, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))])/b - (I*(((-1/2*I)*x*
PolyLog[3, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))])/b + PolyLog[4, -((1 + I*
d)*E^((2*I)*a + (2*I)*b*x))]/(4*b^2)))/b))/(2*b*(I - d))))
```

3.325.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6817
```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.325.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.59 (sec) , antiderivative size = 2387, normalized size of antiderivative = 13.96

method	result	size
risch	Expression too large to display	2387

input
```
int(-x^2*arctanh(-1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^2*d/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*a^2*x+1/4*I/b^3*a^2*d/(I-d
)*polylog(2,I*(I-d)*exp(2*I*(b*x+a)))-1/2*I/b^2*a^2/(I-d)*ln(1+I*exp(I*(b*
x+a))*(-I*(I-d))^(1/2))*x-1/2*I/b^2*a^2/(I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I
-d))^(1/2))*x-1/2*I/b^3*d*a^2/(I-d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1
/2))-1/2*I/b^3*d*a^2/(I-d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+1/2*
I/b^2/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*a^2*x-1/4*I/b*d/(I-d)*polylog(2
,I*(I-d)*exp(2*I*(b*x+a)))*x^2+1/12*I*b*x^4+1/2/b^2*d*a^2/(I-d)*ln(1+I*exp
(I*(b*x+a))*(-I*(I-d))^(1/2))*x+1/2/b^2*d*a^2/(I-d)*ln(1-I*exp(I*(b*x+a))*
(-I*(I-d))^(1/2))*x-1/3*x^3*ln(exp(I*(b*x+a)))-1/3/b^3*d/(I-d)*ln(1-I*(I-d
)*exp(2*I*(b*x+a)))*a^3+1/2/b^3*d*a^3/(I-d)*ln(1+I*exp(I*(b*x+a))*(-I*(I-d
))^(1/2))+1/2/b^3*d*a^3/(I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+1/4/
b^2*d/(I-d)*polylog(3,I*(I-d)*exp(2*I*(b*x+a)))*x-1/6/b^3*a^3*d/(I-d)*ln(I
*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d+I)+1/3*I/b^3/(I-d)*ln(1-I*(I-d)*exp(2
*I*(b*x+a)))*a^3+1/8*I/b^3*d/(I-d)*polylog(4,I*(I-d)*exp(2*I*(b*x+a)))-1/4
*I/b^2/(I-d)*polylog(3,I*(I-d)*exp(2*I*(b*x+a)))*x-1/2*I/b^3*a^3/(I-d)*ln(
1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/2*I/b^3*a^3/(I-d)*ln(1-I*exp(I*(b*x
+a))*(-I*(I-d))^(1/2))+1/6*I/b^3*a^3/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(
b*x+a))*d+I)+1/6*d/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*x^3-1/2/b^3*a^2/(I
-d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/2/b^3*a^2/(I-d)*dilog(1-I
*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/4/b/(I-d)*polylog(2,I*(I-d)*exp(2*I...
```

3.325.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(118) = 236.

Time = 0.27 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.02

$$\int x^2 \text{arctanh}(1 + id - d\tan(a + bx))\, dx$$

$$= \frac{i\, b^4 x^4 - 2\, b^3 x^3 \log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right) + 6i\, b^2 x^2 \text{Li}_2\left(\frac{1}{2}\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}\right) + 6i\, b^2 x^2 \text{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,d-4}e^{(\cdots}\right.}{}$$

input
```
integrate(-x^2*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(I*b^4*x^4 - 2*b^3*x^3*log(-d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x
 + 2*I*a) - I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a))
+ 6*I*b^2*x^2*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3
*log(1/2*(2*(d - I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(d - I)) + 2*a^3
*log(1/2*(2*(d - I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(d - I)) - 12*b*
x*polylog(3, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/
2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*I*
d - 4)*e^(I*b*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*I*d - 4)*
e^(I*b*x + I*a) + 1) - 12*I*polylog(4, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a
)) - 12*I*polylog(4, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

3.325.6 Sympy [F]

$$\int x^2 \text{arctanh}(1+id-d\tan(a+bx))\,dx = \int x^2 \operatorname{atanh}\left(-d\tan\left(a+bx\right)+id+1\right)dx$$

input
```
integrate(-x**2*atanh(-1-I*d+d*tan(b*x+a)),x)
```

output
```
Integral(x**2*atanh(-d*tan(a + b*x) + I*d + 1), x)
```

3.325.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(118) = 236.

Time = 0.24 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.00

$$\int x^2 \text{arctanh}(1+id-d\tan(a+bx))\,dx =$$

$$-\frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\text{artanh}(d\tan(bx+a)-i\,d-1)}{b^2}+\frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(-4i\,(bx+a)^3+9i\,(b\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(-x^2*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctanh(d*tan(
b*x + a) - I*d - 1)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b
*x + a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^
2)*arctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + co
s(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilo
g((-I*d - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(
b*x + a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a
)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polyl
og(3, (-I*d - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d - 1)*e^(2*I*b
*x + 2*I*a)))/b^2)/b
```

3.325.8 Giac [F]

$$\int x^2 \text{arctanh}(1+id-d\tan(a+bx))\,dx = \int -x^2\,\text{artanh}\left(d\tan\left(bx+a\right)-i\,d-1\right)\,dx$$

input
```
integrate(-x^2*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-x^2*arctanh(d*tan(b*x + a) - I*d - 1), x)
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arctanh}(1+id-d\tan(a+bx))\,dx = \int x^2\,\text{atanh}(1-d\tan(a+b\,x)+d\,1\text{i})\,dx$$

input
```
int(x^2*atanh(d*1i - d*tan(a + b*x) + 1),x)
```

output
```
int(x^2*atanh(d*1i - d*tan(a + b*x) + 1), x)
```

3.326 $\int x\text{arctanh}(1+id-d\tan(a+bx))\,dx$

3.326.1 Optimal result

Integrand size = 19, antiderivative size = 134

$$\begin{aligned}\int x\text{arctanh}(1+id-d\tan(a+bx))\,dx &= \frac{1}{6}ibx^3+\frac{1}{2}x^2\text{arctanh}(1+id-d\tan(a+bx))\\&\quad-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)\\&\quad+\frac{ix\,\text{PolyLog}\left(2,-\left((1+id)e^{2ia+2ibx}\right)\right)}{4b}\\&\quad-\frac{\text{PolyLog}\left(3,-\left((1+id)e^{2ia+2ibx}\right)\right)}{8b^2}\end{aligned}$$

output
```
1/6*I*b*x^3-1/2*x^2*arctanh(-1-I*d+d*tan(b*x+a))-1/4*x^2*ln(1+(1+I*d)*exp(
2*I*a+2*I*b*x))+1/4*I*x*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polyl
og(3,-(1+I*d)*exp(2*I*a+2*I*b*x))/b^2
```

3.326.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\begin{aligned}&\int x\text{arctanh}(1+id-d\tan(a+bx))\,dx\\&=\frac{1}{2}x^2\text{arctanh}(1+id-d\tan(a+bx))\\&\quad-\frac{2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{-i+d}\right)+2ibx\,\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{-i+d}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{8b^2}\end{aligned}$$

input
```
Integrate[x*ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

output
```
(x^2*ArcTanh[1 + I*d - d*Tan[a + b*x]])/2 - (2*b^2*x^2*Log[1 - I/((-I + d)
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/((-I + d)*E^((2*I)*(a + b*
x)))] + PolyLog[3, I/((-I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

3.326.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6817, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(d(-\tan(a+bx))+id+1)\,dx$$

$$\Big\downarrow \text{6817}$$

$$\frac{1}{2}ib\int \frac{x^2}{e^{2ia+2ibx}(id+1)+1}dx+\frac{1}{2}x^2\text{arctanh}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1+id)\int \frac{e^{2ia+2ibx}x^2}{e^{2ia+2ibx}(id+1)+1}dx\right)+\frac{1}{2}x^2\text{arctanh}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1+id)\left(\frac{x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{\int x\log\left(e^{2ia+2ibx}(id+1)+1\right)dx}{b(-d+i)}\right)\right)+\frac{1}{2}x^2\text{arctanh}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1+id)\left(\frac{x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)d}{2b}}{b(-d+i)}\right.\right.$$
$$\frac{1}{2}x^2\text{arctanh}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1+id)\left(\frac{x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{\frac{ix\operatorname{PolyLog}(2,-((id+1)e^{2ia+2ibx}))}{2b}-\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}(2,-((id+1)e^{2}}{4b^2}}{b(-d+i)}\right.\right.$$
$$\frac{1}{2}x^2\text{arctanh}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x^2\text{arctanh}(d(-\tan(a+bx))+id+1)+$$
$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1+id)\left(\frac{x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{\frac{ix\operatorname{PolyLog}(2,-((id+1)e^{2ia+2ibx}))}{2b}-\frac{\operatorname{PolyLog}(3,-((id+1)e^{2ia+2ibx}))}{4b^2}}{b(-d+i)}\right)\right)$$

input
```
Int[x*ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

output
```
(x^2*ArcTanh[1 + I*d - d*Tan[a + b*x]])/2 + (I/2)*b*(x^3/3 - (1 + I*d)*((x
^2*Log[1 + (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I - d)) - (((I/2)*x*P
olyLog[2, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 + I*d
)*E^((2*I)*a + (2*I)*b*x))]/(4*b^2))/(b*(I - d))))
```

3.326.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6817
```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.326.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.12 (sec) , antiderivative size = 2289, normalized size of antiderivative = 17.08

method	result	size
risch	Expression too large to display	2289

input
```
int(-x*arctanh(-1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/6*I*b*x^3-1/4/b/(I-d)*polylog(2,I*(I-d)*exp(2*I*(b*x+a)))*x-1/4/b^2/(I-d
)*polylog(2,I*(I-d)*exp(2*I*(b*x+a)))*a+1/2/b^2*a/(I-d)*dilog(1+I*exp(I*(b
*x+a))*(-I*(I-d))^(1/2))+1/2/b^2*a/(I-d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I-d
))^(1/2))+1/8/b^2*d/(I-d)*polylog(3,I*(I-d)*exp(2*I*(b*x+a)))+1/4*d/(I-d)*
ln(1-I*(I-d)*exp(2*I*(b*x+a)))*x^2-1/8*I/b^2/(I-d)*polylog(3,I*(I-d)*exp(2
*I*(b*x+a)))-1/4*I/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*x^2-1/8*(-2*I*Pi-I
*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a
))+1)*exp(2*I*(b*x+a)))+I*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)
))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-I*Pi*csgn(I*exp(I*(b*x+
a)))^2*csgn(I*exp(2*I*(b*x+a)))-I*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(ex
p(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))+2*ln(d)
+I*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1
))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))+I*
Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+
a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))
+1))+I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b
*x+a))+1))^2-I*Pi*csgn(I*exp(2*I*(b*x+a)))^3-I*Pi*csgn(I*exp(2*I*(b*x+a))/
(exp(2*I*(b*x+a))+1))^3+I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+
a))/(exp(2*I*(b*x+a))+1))^2+I*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))+1)*e
xp(2*I*(b*x+a)))^2+I*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*c...
```

3.326.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(93) = 186.

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.19

$$\int x\text{arctanh}(1+id-d\tan(a+bx))\,dx$$
$$= \frac{2i\,b^3x^3-3\,b^2x^2\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right)+2i\,a^3+6i\,bx\text{Li}_2\left(\frac{1}{2}\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}\right)+6i\,bx\text{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,d-\right.}{}$$

input
```
integrate(-x*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*I*b^3*x^3 - 3*b^2*x^2*log(-d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b
*x + 2*I*a) - I)) + 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(-4*I*d - 4)*e^(I*b*x
+ I*a)) + 6*I*b*x*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 3*a^2*log
(1/2*(2*(d - I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(d - I)) - 3*a^2*log
(1/2*(2*(d - I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(d - I)) - 3*(b^2*x^
2 - a^2)*log(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 3*(b^2*x^2 - a^2)
*log(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 6*polylog(3, 1/2*sqrt(-4
*I*d - 4)*e^(I*b*x + I*a)) - 6*polylog(3, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x +
 I*a)))/b^2
```

3.326.6 Sympy [F]

$$\int x\text{arctanh}(1+id-d\tan(a+bx))\,dx=\int x\,\text{atanh}\left(-d\tan\left(a+bx\right)+id+1\right)dx$$

input
```
integrate(-x*atanh(-1-I*d+d*tan(b*x+a)),x)
```

output
```
Integral(x*atanh(-d*tan(a + b*x) + I*d + 1), x)
```

3.326.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(93) = 186.

Time = 0.26 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.84

$$\int x\text{arctanh}(1+id-d\tan(a+bx))\,dx=$$

$$-\frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\text{artanh}(d\tan(bx+a)-i\,d-1)}{b}+\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\text{Li}_2\left((-i\,d-1)e^{(2i\,bx+2i\,a)}\right)-6\left(-i\,(bx+a)^2+2i\,(\ldots}{\ldots}}{\ldots}$$

input
```
integrate(-x*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arctanh(d*tan(b*x + a) - I*d - 1)/
b + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((-I*d - 1)*e^(2
*I*b*x + 2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(d*cos(2*b*
x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1) +
 3*((b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 +
 1)*sin(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) +
3*polylog(3, (-I*d - 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

3.326.8 Giac [F]

$$\int x\text{arctanh}(1+id-d\tan(a+bx))\,dx=\int -x\,\text{artanh}\left(d\tan\left(bx+a\right)-i\,d-1\right)\,dx$$

input
```
integrate(-x*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-x*arctanh(d*tan(b*x + a) - I*d - 1), x)
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arctanh}(1+id-d\tan(a+bx))\,dx=\int x\,\text{atanh}(1-d\tan(a+b\,x)+d\,1\text{i})\,dx$$

input
```
int(x*atanh(d*1i - d*tan(a + b*x) + 1),x)
```

output
```
int(x*atanh(d*1i - d*tan(a + b*x) + 1), x)
```

3.327 $\int \operatorname{arctanh}(1+id-d\tan(a+bx))\,dx$

3.327.1 Optimal result

Integrand size = 17, antiderivative size = 94

$$\int \operatorname{arctanh}(1+id-d\tan(a+bx))\,dx = \frac{1}{2}ibx^2 + x\operatorname{arctanh}(1+id-d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right) + \frac{i\operatorname{PolyLog}\left(2,-\left((1+id)e^{2ia+2ibx}\right)\right)}{4b}$$

output
```
1/2*I*b*x^2-x*arctanh(-1-I*d+d*tan(b*x+a))-1/2*x*ln(1+(1+I*d)*exp(2*I*a+2*
I*b*x))+1/4*I*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b
```

3.327.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 723 vs. 2(94) = 188.

Time = 5.13 (sec) , antiderivative size = 723, normalized size of antiderivative = 7.69

$$\int \operatorname{arctanh}(1+id-d\tan(a+bx))\,dx = x\operatorname{arctanh}(1+id-d\tan(a+bx)) - \frac{x\left(-2ibx\log(2\cos(bx)(\cos(bx)-i\sin(bx))) + \log\left(\frac{\sec(bx)(\cos(a)-i\sin(a))((-2i+d)\cos(a+bx)+id\sin(a+bx))}{2(-i+d)}\right)\log(1\right.}{((-2i+d)\cos(a+bx)+id\sin(a+bx))\left(\frac{i\log(1-i\tan(bx))\sec(bx)(d\cos(a)+(2+id)\sin(a))}{(-2i+d)\cos(a+bx)+id\sin(a+bx)} + \frac{\log(1+i\tan(bx)}{(-2i+}\right.}$$

input
```
Integrate[ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

output
```
x*ArcTanh[1 + I*d - d*Tan[a + b*x]] - (x*((-2*I)*b*x*Log[2*Cos[b*x]*(Cos[b
*x] - I*Sin[b*x])] + Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-2*I + d)*Cos[a +
 b*x] + I*d*Sin[a + b*x]))/(2*(-I + d))]*Log[1 - I*Tan[b*x]] - Log[(Sec[b*
x]*((2 + I*d)*Cos[a + b*x] - d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*L
og[1 + I*Tan[b*x]] + PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] + PolyLog[2, (
Sec[b*x]*(d*Cos[a] + (2 + I*d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2
*(-I + d))] - PolyLog[2, ((Cos[a] + I*Sin[a])*(d*Cos[a] + (2 + I*d)*Sin[a]
)*(-I + Tan[b*x]))/2])*Sec[a + b*x]*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*x] +
Sin[b*x]))/(((-2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x])*((I*Log[1 - I*Tan
[b*x]]*Sec[b*x]*(d*Cos[a] + (2 + I*d)*Sin[a]))/((-2*I + d)*Cos[a + b*x] +
I*d*Sin[a + b*x]) + (Log[1 + I*Tan[b*x]]*Sec[b*x]*((-I)*d*Cos[a] + (-2*I +
 d)*Sin[a]))/((-2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]) - (Log[(Sec[b*x]
*((2 + I*d)*Cos[a + b*x] - d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec
[b*x]^2)/(-I + Tan[b*x]) + (Log[1 - ((Cos[a] + I*Sin[a])*(d*Cos[a] + (2 +
I*d)*Sin[a])*(-I + Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) - Log[1 - (Se
c[b*x]*(d*Cos[a] + (2 + I*d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*(
-I + d))]*(-I + Tan[b*x]) + (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-2*I + d)
*Cos[a + b*x] + I*d*Sin[a + b*x]))/(2*(-I + d))]*Sec[b*x]^2)/(I + Tan[b*x]
) + (2*I)*b*x*(I + Tan[b*x]))*(-I + Tan[a + b*x]))
```

3.327.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6809, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arctanh}(d(-\tan(a+bx))+id+1)\,dx$$

$$\Big\downarrow \text{6809}$$

$$ib\int \frac{x}{e^{2ia+2ibx}(id+1)+1}dx+x\text{arctanh}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{2615}$$

$$ib\left(\frac{x^2}{2}-(1+id)\int \frac{e^{2ia+2ibx}x}{e^{2ia+2ibx}(id+1)+1}dx\right)+x\text{arctanh}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{2620}$$

$$ib\left(\frac{x^2}{2}-(1+id)\left(\frac{x\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{\int\log\left(e^{2ia+2ibx}(id+1)+1\right)dx}{2b(-d+i)}\right)\right)+ x\text{arctanh}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{2715}$$

$$ib\left(\frac{x^2}{2}-(1+id)\left(\frac{i\int e^{-2ia-2ibx}\log\left(e^{2ia+2ibx}(id+1)+1\right)de^{2ia+2ibx}}{4b^2(-d+i)}+\frac{x\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+ x\text{arctanh}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{2838}$$

$$x\text{arctanh}(d(-\tan(a+bx))+id+1)+ ib\left(\frac{x^2}{2}-(1+id)\left(\frac{x\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{i\,\text{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)}{4b^2(-d+i)}\right)\right)$$

input
```
Int[ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

output
```
x*ArcTanh[1 + I*d - d*Tan[a + b*x]] + I*b*(x^2/2 - (1 + I*d)*((x*Log[1 + (
1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I - d)) - ((I/4)*PolyLog[2, -((1
+ I*d)*E^((2*I)*a + (2*I)*b*x))])/(b^2*(I - d))))
```

3.327.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6809
```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Tan[a + b*x]], x] + Simp[I*b   Int[x/(c + I*d + c*E^(2*I*a + 2*I
*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, 1]
```

3.327.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(77) = 154$.

Time = 1.27 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.53

method	result
derivativedivides	$-\frac{-\frac{i \operatorname{arctanh}(-1-id+d\tan(bx+a))d\ln(-id+d\tan(bx+a))}{2}+\frac{i \operatorname{arctanh}(-1-id+d\tan(bx+a))d\ln(-id-d\tan(bx+a))}{2}-\frac{d^2\left(-\frac{i\left(-\frac{\text{dil}\ldots}{}\right.}{}\right.}{}}{}$
default	$-\frac{-\frac{i \operatorname{arctanh}(-1-id+d\tan(bx+a))d\ln(-id+d\tan(bx+a))}{2}+\frac{i \operatorname{arctanh}(-1-id+d\tan(bx+a))d\ln(-id-d\tan(bx+a))}{2}-\frac{d^2\left(-\frac{i\left(-\frac{\text{dil}\ldots}{}\right.}{}\right.}{}}{}$
risch	Expression too large to display

input
```
int(-arctanh(-1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/b/d*(-1/2*I*arctanh(-1-I*d+d*tan(b*x+a))*d*ln(-I*d+d*tan(b*x+a))+1/2*I*
arctanh(-1-I*d+d*tan(b*x+a))*d*ln(-I*d-d*tan(b*x+a))-1/2*d^2*(-I/d*(-1/2*d
ilog(-1/2*I*(I*d-d*tan(b*x+a))/d)-1/2*ln(-I*d-d*tan(b*x+a))*ln(-1/2*I*(I*d
-d*tan(b*x+a))/d)+1/2*dilog(I*(-I*d-d*tan(b*x+a)-I*(2*I-2*d))/(2*I-2*d))+1
/2*ln(-I*d-d*tan(b*x+a))*ln(I*(-I*d-d*tan(b*x+a)-I*(2*I-2*d))/(2*I-2*d))+
I/d*(-1/4*ln(-I*d+d*tan(b*x+a))^2+1/2*(ln(-I*d+d*tan(b*x+a))-ln(-1/2*I*d+1
/2*d*tan(b*x+a)))*ln(1+1/2*I*d-1/2*d*tan(b*x+a))-1/2*dilog(-1/2*I*d+1/2*d*
tan(b*x+a)))))
```

3.327.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(66) = 132.

Time = 0.27 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.33

$$\int \operatorname{arctanh}(1 + id - d\tan(a + bx))\, dx$$

$$= \frac{i\,b^2x^2 - bx\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right) - i\,a^2 - (bx+a)\log\left(\frac{1}{2}\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}+1\right) - (bx+a)\log\left(-\frac{1}{2}\sqrt{\ldots}\right.}{\ldots}$$

input
```
integrate(-arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(I*b^2*x^2 - b*x*log(-d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x + 2*I*
a) - I)) - I*a^2 - (b*x + a)*log(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1)
 - (b*x + a)*log(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) + a*log(1/2*(2
*(d - I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(d - I)) + a*log(1/2*(2*(d
- I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(d - I)) + I*dilog(1/2*sqrt(-4*
I*d - 4)*e^(I*b*x + I*a)) + I*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a))
)/b
```

3.327.6 Sympy [F]

$$\int \operatorname{arctanh}(1+id-d\tan(a+bx))\,dx=\int \operatorname{atanh}\left(-d\tan\left(a+bx\right)+id+1\right)dx$$

input
```
integrate(-atanh(-1-I*d+d*tan(b*x+a)),x)
```

output
```
Integral(atanh(-d*tan(a + b*x) + I*d + 1), x)
```

3.327.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(66) = 132.

Time = 0.29 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.79

$$\int \operatorname{arctanh}(1+id-d\tan(a+bx))\,dx = -\frac{4\,(bx+a)d\left(\frac{\log(d\tan(bx+a)-i\,d-2)}{d}-\frac{\log(\tan(bx+a)-i)}{d}\right)-d\left(\frac{2i\left(\log(d\tan(bx+a)-i\,d-2)\log\left(-\frac{i\,d\tan(bx+a)+d-2i}{2\,(d-i)}+1\right)+\mathrm{Li}_2\left(\frac{i}{\cdots}\right.\right.}{d}\right.}{\cdots}$$

input
```
integrate(-arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*(4*(b*x + a)*d*(log(d*tan(b*x + a) - I*d - 2)/d - log(tan(b*x + a) -
I)/d) - d*(2*I*(log(d*tan(b*x + a) - I*d - 2)*log(-1/2*(I*d*tan(b*x + a) +
 d - 2*I)/(d - I) + 1) + dilog(1/2*(I*d*tan(b*x + a) + d - 2*I)/(d - I)))/
d - (2*I*log(d*tan(b*x + a) - I*d - 2)*log(tan(b*x + a) - I) - I*log(tan(b
*x + a) - I)^2)/d + 2*I*(log(-1/2*d*tan(b*x + a) + 1/2*I*d + 1)*log(tan(b*
x + a) - I) + dilog(1/2*d*tan(b*x + a) - 1/2*I*d))/d - 2*I*(log(tan(b*x +
a) - I)*log(-1/2*I*tan(b*x + a) + 1/2) + dilog(1/2*I*tan(b*x + a) + 1/2))/
d) + 8*(b*x + a)*arctanh(d*tan(b*x + a) - I*d - 1))/b
```

3.327.8 Giac [F]

$$\int \operatorname{arctanh}(1+id-d\tan(a+bx))\,dx=\int -\operatorname{artanh}\left(d\tan\left(bx+a\right)-i\,d-1\right)\,dx$$

input
```
integrate(-arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-arctanh(d*tan(b*x + a) - I*d - 1), x)
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}(1+id-d\tan(a+bx))\,dx=\int \operatorname{atanh}(1-d\tan(a+b\,x)+d\,1\mathrm{i})\,dx$$

input
```
int(atanh(d*1i - d*tan(a + b*x) + 1),x)
```

output
```
int(atanh(d*1i - d*tan(a + b*x) + 1), x)
```

3.328 $\int \frac{\operatorname{arctanh}(1+id-d\tan(a+bx))}{x}\,dx$

3.328.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\operatorname{arctanh}(1+id-d\tan(a+bx))}{x}\,dx=\operatorname{Int}\left(\frac{\operatorname{arctanh}(1+id-d\tan(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(-arctanh(-1-I*d+d*tan(b*x+a))/x,x)
```

3.328.2 Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{arctanh}(1+id-d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arctanh}(1+id-d\tan(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTanh[1 + I*d - d*Tan[a + b*x]]/x,x]
```

output
```
Integrate[ArcTanh[1 + I*d - d*Tan[a + b*x]]/x, x]
```

3.328.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(d(-\tan(a+bx))+id+1)}{x} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\text{arctanh}(d(-\tan(a+bx))+id+1)}{x}dx$$

input
```
Int[ArcTanh[1 + I*d - d*Tan[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.328.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.328.4 Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int -\frac{\text{arctanh}\left(-1-id+d\tan\left(bx+a\right)\right)}{x}dx$$

input
```
int(-arctanh(-1-I*d+d*tan(b*x+a))/x,x)
```

output
```
int(-arctanh(-1-I*d+d*tan(b*x+a))/x,x)
```

3.328.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{\operatorname{arctanh}(1+id-d\tan(a+bx))}{x}\,dx = \int -\frac{\operatorname{artanh}\left(d\tan\left(bx+a\right)-i\,d-1\right)}{x}\,dx$$

input
```
integrate(-arctanh(-1-I*d+d*tan(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(-1/2*log(-d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x + 2*I*a) - I)
)/x, x)
```

3.328.6 Sympy [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\operatorname{arctanh}(1+id-d\tan(a+bx))}{x}\,dx = \int \frac{\operatorname{atanh}\left(-d\tan\left(a+bx\right)+id+1\right)}{x}\,dx$$

input
```
integrate(-atanh(-1-I*d+d*tan(b*x+a))/x,x)
```

output
```
Integral(atanh(-d*tan(a + b*x) + I*d + 1)/x, x)
```

3.328.7 Maxima [N/A]

Not integrable

Time = 4.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 6.81

$$\int \frac{\operatorname{arctanh}(1+id-d\tan(a+bx))}{x}\,dx = \int -\frac{\operatorname{artanh}\left(d\tan\left(bx+a\right)-i\,d-1\right)}{x}\,dx$$

input
```
integrate(-arctanh(-1-I*d+d*tan(b*x+a))/x,x, algorithm="maxima")
```

output
```
-I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(-d))*log(x) + 1/2*I*integrate(arctan2(
d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2
*a) + 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1
)*sin(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1)/x, x
)
```

3.328.8 Giac [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\text{arctanh}(1+id-d\tan(a+bx))}{x}\,dx = \int -\frac{\text{artanh}\left(d\tan\left(bx+a\right)-i\,d-1\right)}{x}\,dx$$

input
```
integrate(-arctanh(-1-I*d+d*tan(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(-arctanh(d*tan(b*x + a) - I*d - 1)/x, x)
```

3.328.9 Mupad [N/A]

Not integrable

Time = 4.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\text{arctanh}(1+id-d\tan(a+bx))}{x}\,dx = \int \frac{\text{atanh}(1-d\tan(a+b\,x)+d\,1\text{i})}{x}\,dx$$

input
```
int(atanh(d*1i - d*tan(a + b*x) + 1)/x,x)
```

output
```
int(atanh(d*1i - d*tan(a + b*x) + 1)/x, x)
```

3.329 $\int (e+fx)^3 \textbf{arctanh}(\cot(a+bx))\,dx$

3.329.1 Optimal result

Integrand size = 15, antiderivative size = 302

$$
\begin{aligned}
\int (e+fx)^3\text{arctanh}(\cot(a+bx))\,dx = & \frac{i(e+fx)^4\arctan\left(e^{2i(a+bx)}\right)}{4f} \\
& + \frac{(e+fx)^4\text{arctanh}(\cot(a+bx))}{4f} \\
& - \frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} \\
& + \frac{i(e+fx)^3\operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b} \\
& + \frac{3f(e+fx)^2\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2} \\
& - \frac{3f(e+fx)^2\operatorname{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2} \\
& + \frac{3if^2(e+fx)\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3} \\
& - \frac{3if^2(e+fx)\operatorname{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3} \\
& - \frac{3f^3\operatorname{PolyLog}\left(5,-ie^{2i(a+bx)}\right)}{16b^4} \\
& + \frac{3f^3\operatorname{PolyLog}\left(5,ie^{2i(a+bx)}\right)}{16b^4}
\end{aligned}
$$

output
```
1/4*I*(f*x+e)^4*arctan(exp(2*I*(b*x+a)))/f+1/4*(f*x+e)^4*arctanh(cot(b*x+a
))/f-1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^3*poly
log(2,I*exp(2*I*(b*x+a)))/b+3/8*f*(f*x+e)^2*polylog(3,-I*exp(2*I*(b*x+a)))
/b^2-3/8*f*(f*x+e)^2*polylog(3,I*exp(2*I*(b*x+a)))/b^2+3/8*I*f^2*(f*x+e)*p
olylog(4,-I*exp(2*I*(b*x+a)))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*I*(b
*x+a)))/b^3-3/16*f^3*polylog(5,-I*exp(2*I*(b*x+a)))/b^4+3/16*f^3*polylog(5
,I*exp(2*I*(b*x+a)))/b^4
```

3.329.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 654 vs. 2(302) = 604.

Time = 0.20 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.17

$$\int (e+fx)^3\text{arctanh}(\cot(a+bx))\,dx = \frac{1}{4}x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\text{arctanh}(\cot(a+bx)) + \frac{-8b^4e^3x\log\left(1-ie^{2i(a+bx)}\right)-12b^4e^2fx^2\log\left(1-ie^{2i(a+bx)}\right)-8b^4ef^2x^3\log\left(1-ie^{2i(a+bx)}\right)-2b^4f^3x^4\text{lo}}{}$$

input
```
Integrate[(e + f*x)^3*ArcTanh[Cot[a + b*x]],x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTanh[Cot[a + b*x]])/4 +
(-8*b^4*e^3*x*Log[1 - I*E^((2*I)*(a + b*x))] - 12*b^4*e^2*f*x^2*Log[1 - I*
E^((2*I)*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] - 2*
b^4*f^3*x^4*Log[1 - I*E^((2*I)*(a + b*x))] + 8*b^4*e^3*x*Log[1 + I*E^((2*I
)*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 8*b^4*e*
f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] + 2*b^4*f^3*x^4*Log[1 + I*E^((2*I)*
(a + b*x))] - (4*I)*b^3*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))] +
 (4*I)*b^3*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))] + 6*b^2*e^2*f*Pol
yLog[3, (-I)*E^((2*I)*(a + b*x))] + 12*b^2*e*f^2*x*PolyLog[3, (-I)*E^((2*I
)*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b^2
*e^2*f*PolyLog[3, I*E^((2*I)*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3, I*E^(
(2*I)*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^((2*I)*(a + b*x))] + (6*I
)*b*e*f^2*PolyLog[4, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b*f^3*x*PolyLog[4,
(-I)*E^((2*I)*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))
] - (6*I)*b*f^3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I
)*E^((2*I)*(a + b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

3.329.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6807, 3042, 4669, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3\text{arctanh}(\cot(a+bx))\,dx$$

$$\Big\downarrow \text{ 6807}$$

$$\frac{(e+fx)^4\text{arctanh}(\cot(a+bx))}{4f}-\frac{b\int (e+fx)^4\sec(2a+2bx)dx}{4f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e+fx)^4\text{arctanh}(\cot(a+bx))}{4f}-\frac{b\int (e+fx)^4\csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{4f}$$

$$\Big\downarrow \text{ 4669}$$

$$\frac{(e+fx)^4\text{arctanh}(\cot(a+bx))}{4f}-\frac{b\left(-\frac{2f\int (e+fx)^3\log\left(1-ie^{2i(a+bx)}\right)dx}{b}+\frac{2f\int (e+fx)^3\log\left(1+ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^4\arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{4f}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{(e+fx)^4\text{arctanh}(\cot(a+bx))}{4f}-\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\int (e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{2f\left(\frac{i(e+fx)^3\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{3if\int (e+fx)^2\text{PolyLog}\left(2 \cdots\right.}{2b}\right.}{b}\cdots\right.}{4f}$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{(e+fx)^4\text{arctanh}(\cot(a+bx))}{4f}-\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\int (e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^2\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{2b}\right)}{b}-\frac{2f\left(\frac{i(e+fx)^3\text{PolyLo}\cdots}{2\cdots}\right.}{}\cdots\right.}{4f}$$

$$\Big\downarrow \; 7163$$

$$\frac{(e+fx)^4\text{arctanh}(\cot(a+bx))}{4f}-\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\left(\frac{if\int\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)dx}{2b}-\frac{i(e+fx)\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}-\frac{i(e+fx)^2\text{PolyLog}\left(3,-ie^{2i(a+b}\right.}{2b}}{2b}\right)}{b}$$

$$\Big\downarrow \; 2720$$

$$\frac{(e+fx)^4\text{arctanh}(\cot(a+bx))}{4f}-\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\left(\frac{f\int e^{-2i(a+bx)}\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(e+fx)\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}-\frac{i(e+fx)^2\text{P}}{}}{2b}\right)}{b}$$

$$\Big\downarrow \; 7143$$

3.329. $\int (e+fx)^3\text{arctanh}(\cot(a+bx))\,dx$

$$\frac{(e+fx)^4\text{arctanh}(\cot(a+bx))}{4f}-\frac{b\left(-\frac{i(e+fx)^4\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{2f\left(\frac{i(e+fx)^3\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\left(\frac{f\text{PolyLog}\left(5,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{i(e+fx)\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\cdots\right)}{2b}\right)}{b}\cdots\right)}{\cdots}$$

input
```
Int[(e + f*x)^3*ArcTanh[Cot[a + b*x]],x]
```

output
```
((e + f*x)^4*ArcTanh[Cot[a + b*x]])/(4*f) - (b*(((-I)*(e + f*x)^4*ArcTan[E
^((2*I)*(a + b*x))])/b + (2*f*(((I/2)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)
*(a + b*x))])/b - (((3*I)/2)*f*(((-1/2*I)*(e + f*x)^2*PolyLog[3, (-I)*E^((
2*I)*(a + b*x))])/b + (I*f*(((-1/2*I)*(e + f*x)*PolyLog[4, (-I)*E^((2*I)*(
a + b*x))])/b + (f*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b))
/b - (2*f*(((I/2)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (((3*
I)/2)*f*(((-1/2*I)*(e + f*x)^2*PolyLog[3, I*E^((2*I)*(a + b*x))])/b + (I*f
*(((-1/2*I)*(e + f*x)*PolyLog[4, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[5,
 I*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b))/b))/(4*f)
```

3.329.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6807
```
Int[ArcTanh[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTanh[Cot[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.329. $\int (e+fx)^3 \text{arctanh}(\cot(a+bx))\,dx$

3.329.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.00 (sec) , antiderivative size = 3640, normalized size of antiderivative = 12.05

method	result	size
risch	Expression too large to display	3640

input
```
int((f*x+e)^3*arctanh(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*f^3/b^4*a^3*dilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I*f^3
/b^4*a^3*dilog(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-3/2*I*f/b^2*e^2*a*d
ilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-3/2*I*f/b^2*e^2*a*dilog(((-I)
^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-3/2*f^2/b^2*e*a^2*ln(((-I)^(1/2)+exp(I*
(b*x+a)))/(-I)^(1/2))*x+1/2/b*e^3*a*ln(exp(2*I*(b*x+a))+I)-3/4*f*e^2*ln(-I
*exp(2*I*(b*x+a))+1)*x^2-3/8*f^3/b^2*polylog(3,I*exp(2*I*(b*x+a)))*x^2-3/8
*f/b^2*e^2*polylog(3,I*exp(2*I*(b*x+a)))+3/8*f/b^2*e^2*polylog(3,-I*exp(2*
I*(b*x+a)))+3/8*f^3/b^2*polylog(3,-I*exp(2*I*(b*x+a)))*x^2-3/16*f^3*polylo
g(5,-I*exp(2*I*(b*x+a)))/b^4+3/16*f^3*polylog(5,I*exp(2*I*(b*x+a)))/b^4-f^
2/b^3*e*ln(1+I*exp(2*I*(b*x+a)))*a^3+1/2*f^3/b^3*ln(1+I*exp(2*I*(b*x+a)))*
x*a^3-1/2*f^2/b^3*a^3*e*ln(-exp(2*I*(b*x+a))+I)+3/4*f/b^2*a^2*e^2*ln(-exp(
2*I*(b*x+a))+I)-3/4*I*f^2/b*e*polylog(2,-I*exp(2*I*(b*x+a)))*x^2+3/4*I*f^2
/b^3*e*polylog(2,-I*exp(2*I*(b*x+a)))*a^2+3/4*I*f^2/b*e*polylog(2,I*exp(2*
I*(b*x+a)))*x^2-3/8*I*f^2/b^3*e*polylog(4,I*exp(2*I*(b*x+a)))-1/4*I*f^3/b*
polylog(2,-I*exp(2*I*(b*x+a)))*x^3-1/4*I*f^3/b^4*polylog(2,-I*exp(2*I*(b*x
+a)))*a^3+1/8/f*e^4*ln(-exp(2*I*(b*x+a))+I)+1/8*f^3*ln(1+I*exp(2*I*(b*x+a)
))*x^4+3/2*f/b*e^2*a*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))*x+3/2*f/b*
e^2*a*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))*x-1/8/f*e^4*ln(exp(2*I*(b
*x+a))+I)-1/2*e^3*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))*x-1/2*e^3*ln(
((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))*x+1/8*(f*x+e)^4/f*ln(exp(2*I*(b...
```

3.329.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1567 vs. 2(236) = 472.

Time = 0.35 (sec) , antiderivative size = 1567, normalized size of antiderivative = 5.19

$$\int (e+fx)^3 \text{arctanh}(\cot(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*arctanh(cot(b*x+a)),x, algorithm="fricas")
```

output
```
-1/32*(3*f^3*polylog(5, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*f^3*pol
ylog(5, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 3*f^3*polylog(5, -I*cos(2
*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*f^3*polylog(5, -I*cos(2*b*x + 2*a) - s
in(2*b*x + 2*a)) + 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f*x
 - I*b^3*e^3)*dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 4*(-I*b^3*f^3
*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(I*cos(2*b*x
+ 2*a) - sin(2*b*x + 2*a)) + 4*(I*b^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^
3*e^2*f*x + I*b^3*e^3)*dilog(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 4*(
I*b^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(-I*
cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 4*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 +
6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*log(-(cos(2*b*x + 2*a) + sin(2*b*x + 2*a) +
 1)/(cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1)) - 2*(4*a*b^3*e^3 - 6*a^2*b^
2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*
a) + I) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(
cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*
x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^
3*b*e*f^2 - a^4*f^3)*log(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) - 2*(b
^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3
 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(I*cos(2*b*x + 2*a) - sin
(2*b*x + 2*a) + 1) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2...
```

3.329.6 Sympy [F]

$$\int (e+fx)^3\text{arctanh}(\cot(a+bx))\,dx=\int (e+fx)^3\,\text{atanh}\,(\cot\,(a+bx))\,dx$$

input
```
integrate((f*x+e)**3*atanh(cot(b*x+a)),x)
```

output
```
Integral((e + f*x)**3*atanh(cot(a + b*x)), x)
```

3.329.7 Maxima [F]

$$\int (e+fx)^3\text{arctanh}(\cot(a+bx))\,dx=\int (fx+e)^3\,\text{artanh}\left(\cot\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)^3*arctanh(cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/16*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a
)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*e
*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x +
 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*e*f^2*
x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^
3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*sin(4*b*x + 4*a)*sin(2*
b*x + 2*a) + (b*f^3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(2
*b*x + 2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)
 + 1), x)
```

3.329.8 Giac [F]

$$\int (e+fx)^3\text{arctanh}(\cot(a+bx))\,dx=\int (fx+e)^3\,\text{artanh}\left(\cot\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)^3*arctanh(cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*arctanh(cot(b*x + a)), x)
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3\text{arctanh}(\cot(a+bx))\,dx=\int \text{atanh}(\cot(a+b\,x))\,\left(e+f\,x\right)^3\,dx$$

input
```
int(atanh(cot(a + b*x))*(e + f*x)^3,x)
```

output
```
int(atanh(cot(a + b*x))*(e + f*x)^3, x)
```

3.330 $\int (e+fx)^2 \text{arctanh}(\cot(a+bx))\,dx$

3.330.1 Optimal result

Integrand size = 15, antiderivative size = 234

$$\begin{aligned}\int (e+fx)^2\text{arctanh}(\cot(a+bx))\,dx =& \frac{i(e+fx)^3\arctan\left(e^{2i(a+bx)}\right)}{3f}\\&+\frac{(e+fx)^3\text{arctanh}(\cot(a+bx))}{3f}\\&-\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}\\&+\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}\\&+\frac{f(e+fx)\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2}\\&-\frac{f(e+fx)\operatorname{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{4b^2}\\&+\frac{if^2\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}\\&-\frac{if^2\operatorname{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}\end{aligned}$$

output
```
1/3*I*(f*x+e)^3*arctan(exp(2*I*(b*x+a)))/f+1/3*(f*x+e)^3*arctanh(cot(b*x+a
))/f-1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^2*poly
log(2,I*exp(2*I*(b*x+a)))/b+1/4*f*(f*x+e)*polylog(3,-I*exp(2*I*(b*x+a)))/b
^2-1/4*f*(f*x+e)*polylog(3,I*exp(2*I*(b*x+a)))/b^2+1/8*I*f^2*polylog(4,-I*
exp(2*I*(b*x+a)))/b^3-1/8*I*f^2*polylog(4,I*exp(2*I*(b*x+a)))/b^3
```

3.330.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.75

$$\int (e+fx)^2\text{arctanh}(\cot(a+bx))\,dx = \frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\text{arctanh}(\cot(a+bx)) + \frac{-12b^3e^2x\log\left(1-ie^{2i(a+bx)}\right)-12b^3efx^2\log\left(1-ie^{2i(a+bx)}\right)-4b^3f^2x^3\log\left(1-ie^{2i(a+bx)}\right)+12b^3e^2x\log}{}$$

input
```
Integrate[(e + f*x)^2*ArcTanh[Cot[a + b*x]],x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTanh[Cot[a + b*x]])/3 + (-12*b^3*e^2*x*L
og[1 - I*E^((2*I)*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 - I*E^((2*I)*(a + b*x
))] - 4*b^3*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] + 12*b^3*e^2*x*Log[1 +
I*E^((2*I)*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 4
*b^3*f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] - (6*I)*b^2*(e + f*x)^2*PolyLo
g[2, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b^2*(e + f*x)^2*PolyLog[2, I*E^((2*
I)*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] + 6*b*f^2*x*
PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b*e*f*PolyLog[3, I*E^((2*I)*(a +
b*x))] - 6*b*f^2*x*PolyLog[3, I*E^((2*I)*(a + b*x))] + (3*I)*f^2*PolyLog[4
, (-I)*E^((2*I)*(a + b*x))] - (3*I)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])
/(24*b^3)
```

3.330.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6807, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2\text{arctanh}(\cot(a+bx))\,dx$$

$$\Big\downarrow \text{6807}$$

$$\frac{(e+fx)^3\text{arctanh}(\cot(a+bx))}{3f} - \frac{b\int (e+fx)^3\sec(2a+2bx)dx}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3\text{arctanh}(\cot(a+bx))}{3f}-\frac{b\int(e+fx)^3\csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{3f}$$

$\Big\downarrow$ 4669

$$\frac{(e+fx)^3\text{arctanh}(\cot(a+bx))}{3f}-\frac{b\left(-\frac{3f\int(e+fx)^2\log\left(1-ie^{2i(a+bx)}\right)dx}{2b}+\frac{3f\int(e+fx)^2\log\left(1+ie^{2i(a+bx)}\right)dx}{2b}-\frac{i(e+fx)^3\arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{3f}$$

$\Big\downarrow$ 3011

$$\frac{(e+fx)^3\text{arctanh}(\cot(a+bx))}{3f}-\frac{b\left(\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\int(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{if\int(e+fx)\text{PolyLog}\left(2,ie^{2i}\right.}{b}\right.}{2b}\right.}{3f}$$

$\Big\downarrow$ 7163

$$\frac{(e+fx)^3\text{arctanh}(\cot(a+bx))}{3f}-\frac{b\left(\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\left(\frac{if\int\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)dx}{2b}-\frac{i(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,ie^{2i(a}\right.}{2b}\right.}{}\right.}{3f}$$

$\Big\downarrow$ 2720

$$\frac{(e+fx)^3\text{arctanh}(\cot(a+bx))}{3f}-\frac{b\left(\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\left(\frac{f\int e^{-2i(a+bx)}\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{3f\left(\frac{i(e+f}{}\right.}{}\right.}{3}$$

$\Big\downarrow$ 7143

3.330. $\int(e+fx)^2\text{arctanh}(\cot(a+bx))\,dx$

$$\frac{(e+fx)^3\text{arctanh}(\cot(a+bx))}{3f} - \frac{b\left(-\frac{i(e+fx)^3\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\left(\frac{f\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b} - \right.}{3f}$$

input
```
Int[(e + f*x)^2*ArcTanh[Cot[a + b*x]],x]
```

output
```
((e + f*x)^3*ArcTanh[Cot[a + b*x]])/(3*f) - (b*(((-I)*(e + f*x)^3*ArcTan[E
^((2*I)*(a + b*x))])/b + (3*f*(((I/2)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)
*(a + b*x))])/b - (I*f*(((-1/2*I)*(e + f*x)*PolyLog[3, (-I)*E^((2*I)*(a +
b*x))])/b + (f*PolyLog[4, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/(2*b) -
 (3*f*(((I/2)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (I*f*(((-
1/2*I)*(e + f*x)*PolyLog[3, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[4, I*E^
((2*I)*(a + b*x))])/(4*b^2)))/b))/(2*b)))/(3*f)
```

3.330.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6807
```
Int[ArcTanh[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTanh[Cot[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.330.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.21 (sec) , antiderivative size = 2719, normalized size of antiderivative = 11.62

method	result	size
risch	Expression too large to display	2719

input
```
int((f*x+e)^2*arctanh(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*f^2*ln(-I*exp(2*I*(b*x+a))+1)*x^3+1/6*(f*x+e)^3/f*ln(exp(2*I*(b*x+a))
+I)-1/8*I*f^2*polylog(4,I*exp(2*I*(b*x+a)))/b^3+1/8*I*f^2*polylog(4,-I*exp
(2*I*(b*x+a)))/b^3-f*e/b*ln(-I*exp(2*I*(b*x+a))+1)*a*x+1/2*I*f*e/b*polylog
(2,I*exp(2*I*(b*x+a)))*x+1/2*I*f*e/b^2*polylog(2,I*exp(2*I*(b*x+a)))*a+f*e
/b*ln(1+I*exp(2*I*(b*x+a)))*a*x-1/2*I*f*e/b*polylog(2,-I*exp(2*I*(b*x+a)))
*x-1/2*I*f*e/b^2*polylog(2,-I*exp(2*I*(b*x+a)))*a-I*f/b^2*a*e*dilog(((-I)^
(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-I*f/b^2*a*e*dilog(((-I)^(1/2)+exp(I*(b*x
+a)))/(-I)^(1/2))+f/b*a*e*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))*x+f/b
*a*e*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))*x-f/b*a*e*ln(1+exp(I*(b*x+
a))*(-1)^(3/4))*x-f/b*a*e*ln(1-exp(I*(b*x+a))*(-1)^(3/4))*x+I*f/b^2*a*e*di
log(1+exp(I*(b*x+a))*(-1)^(3/4))+I*f/b^2*a*e*dilog(1-exp(I*(b*x+a))*(-1)^(
3/4))+1/6/f*e^3*ln(-exp(2*I*(b*x+a))+I)+1/6*f^2*ln(1+I*exp(2*I*(b*x+a)))*x
^3-1/2*f*ln(exp(2*I*(b*x+a))-I)*x^2*e-1/2*f^2/b^2*ln(1+I*exp(2*I*(b*x+a)))
*a^2*x+1/2*f/b^2*a^2*e*ln(-exp(2*I*(b*x+a))+I)+1/4*I*f^2*a^2/b^3*polylog(2
,-I*exp(2*I*(b*x+a)))-1/4*I*f^2/b*polylog(2,-I*exp(2*I*(b*x+a)))*x^2-1/3*f
^2/b^3*ln(1+I*exp(2*I*(b*x+a)))*a^3+1/2*f*e*ln(1+I*exp(2*I*(b*x+a)))*x^2+1
/4*f*e/b^2*polylog(3,-I*exp(2*I*(b*x+a)))-1/6*f^2/b^3*a^3*ln(-exp(2*I*(b*x
+a))+I)+1/4*f^2/b^2*polylog(3,-I*exp(2*I*(b*x+a)))*x-1/2/b*a*e^2*ln(-exp(2
*I*(b*x+a))+I)-1/2*f^2*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))/b^3*a^3-
1/2*f^2*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))/b^3*a^3-1/2*ln(((-I)...
```

3.330.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(180) = 360$.

Time = 0.34 (sec) , antiderivative size = 1085, normalized size of antiderivative = 4.64

$$\int (e+fx)^2 \operatorname{arctanh}(\cot(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*arctanh(cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/48*(-3*I*f^2*polylog(4, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*I*f^2
*polylog(4, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I*cos(2*b*x +
2*a) - sin(2*b*x + 2*a)) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)*
dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2
*e*f*x - I*b^2*e^2)*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 6*(I*b^
2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-I*cos(2*b*x + 2*a) + sin(2*b
*x + 2*a)) - 6*(I*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-I*cos(2*
b*x + 2*a) - sin(2*b*x + 2*a)) + 8*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^
2*x)*log(-(cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a) - si
n(2*b*x + 2*a) + 1)) + 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(cos(2*b
*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f
^2)*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - 4*(b^3*f^2*x^3 + 3*b^
3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(I*cos(2
*b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b
^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(I*cos(2*b*x + 2*a) - s
in(2*b*x + 2*a) + 1) - 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*
b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a
) + 1) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^
2*b*e*f + a^3*f^2)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) + 4*...
```

3.330.6 Sympy [F]

$$\int (e+fx)^2\text{arctanh}(\cot(a+bx))\,dx=\int (e+fx)^2\,\text{atanh}\,(\cot\,(a+bx))\,dx$$

input
```
integrate((f*x+e)**2*atanh(cot(b*x+a)),x)
```

output
```
Integral((e + f*x)**2*atanh(cot(a + b*x)), x)
```

3.330.7 Maxima [F]

$$\int (e+fx)^2\text{arctanh}(\cot(a+bx))\,dx = \int (fx+e)^2 \operatorname{artanh}(\cot(bx+a))\,dx$$

input
```
integrate((f*x+e)^2*arctanh(cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*
x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x
)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2
) - integrate(2/3*((b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(4*b*x + 4*a)*
cos(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(2*b*x + 2*a))
/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

3.330.8 Giac [F]

$$\int (e+fx)^2\text{arctanh}(\cot(a+bx))\,dx = \int (fx+e)^2 \operatorname{artanh}(\cot(bx+a))\,dx$$

input
```
integrate((f*x+e)^2*arctanh(cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*arctanh(cot(b*x + a)), x)
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\text{arctanh}(\cot(a+bx))\,dx = \int \operatorname{atanh}(\cot(a+b\,x))\,(e+f\,x)^2\,dx$$

input
```
int(atanh(cot(a + b*x))*(e + f*x)^2,x)
```

output
```
int(atanh(cot(a + b*x))*(e + f*x)^2, x)
```

3.331 $\int (e+fx)\textbf{arctanh}(\cot(a+bx))\,dx$

3.331.1 Optimal result

Integrand size = 13, antiderivative size = 162

$$\int (e+fx)\text{arctanh}(\cot(a+bx))\,dx = \frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\text{arctanh}(\cot(a+bx))}{2f} - \frac{i(e+fx)\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} + \frac{i(e+fx)\,\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b} + \frac{f\,\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2} - \frac{f\,\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2}$$

output
```
1/2*I*(f*x+e)^2*arctan(exp(2*I*(b*x+a)))/f+1/2*(f*x+e)^2*arctanh(cot(b*x+a
))/f-1/4*I*(f*x+e)*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)*polylog(
2,I*exp(2*I*(b*x+a)))/b+1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*pol
ylog(3,I*exp(2*I*(b*x+a)))/b^2
```

3.331.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.82

$$\int (e+fx)\text{arctanh}(\cot(a+bx))\,dx = ex\text{arctanh}(\cot(a+bx)) + \frac{1}{2}fx^2\text{arctanh}(\cot(a+bx)) - \frac{e\left((-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+\right.}{8b} + \frac{f(4ib^2x^2\arctan(\cos(2(a+bx))+i\sin(2(a+bx)))+2ibx\,\text{PolyLog}(2,i\cos(2(a+bx))-\sin(2(a+bx))}{}$$

input
```
Integrate[(e + f*x)*ArcTanh[Cot[a + b*x]],x]
```

output
```
e*x*ArcTanh[Cot[a + b*x]] + (f*x^2*ArcTanh[Cot[a + b*x]])/2 - (e*((-4*a +
Pi - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*x))] - Log[1 + I/E^((2*I)*(a + b*x)
)]) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*x]] + (2*I)*(PolyLog[2, (-I)/E^((2*
I)*(a + b*x))] - PolyLog[2, I/E^((2*I)*(a + b*x))])))/(8*b) + (f*((4*I)*b^
2*x^2*ArcTan[Cos[2*(a + b*x)] + I*Sin[2*(a + b*x)]] + (2*I)*b*x*PolyLog[2,
 I*Cos[2*(a + b*x)] - Sin[2*(a + b*x)]] - (2*I)*b*x*PolyLog[2, (-I)*Cos[2*
(a + b*x)] + Sin[2*(a + b*x)]] - PolyLog[3, I*Cos[2*(a + b*x)] - Sin[2*(a
+ b*x)]] + PolyLog[3, (-I)*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]]))/(8*b^2)
```

3.331.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6807, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\text{arctanh}(\cot(a+bx))\,dx$$

$$\Big\downarrow \ 6807$$

$$\frac{(e+fx)^2\text{arctanh}(\cot(a+bx))}{2f} - \frac{b\int (e+fx)^2\sec(2a+2bx)dx}{2f}$$

$$\Big\downarrow \ 3042$$

$$\frac{(e+fx)^2\text{arctanh}(\cot(a+bx))}{2f} - \frac{b\int (e+fx)^2\csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{2f}$$

$$\Big\downarrow \; 4669$$

$$\frac{(e+fx)^2\text{arctanh}(\cot(a+bx))}{2f}-\frac{b\left(-\frac{f\int (e+fx)\log\left(1-ie^{2i(a+bx)}\right)dx}{b}+\frac{f\int (e+fx)\log\left(1+ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{2f}$$

$$\Big\downarrow \; 3011$$

$$\frac{(e+fx)^2\text{arctanh}(\cot(a+bx))}{2f}-\frac{b\left(\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\int \text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{if\int \text{PolyLog}\left(2,ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{i(\ldots}{}\right.}{2f}$$

$$\Big\downarrow \; 2720$$

$$\frac{(e+fx)^2\text{arctanh}(\cot(a+bx))}{2f}-\frac{b\left(\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{f\int e^{-2i(a+bx)}\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{f\int e^{-2i(a+bx)}\text{Po}\ldots}{}\right.}{b}\right.}{2f}$$

$$\Big\downarrow \; 7143$$

$$\frac{(e+fx)^2\text{arctanh}(\cot(a+bx))}{2f}-\frac{b\left(-\frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{f\,\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2}\right)}{b}-\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{f\,\text{Poly}\ldots}{}\right.}{b}\right.}{2f}$$

input
```
Int[(e + f*x)*ArcTanh[Cot[a + b*x]],x]
```

output
```
((e + f*x)^2*ArcTanh[Cot[a + b*x]])/(2*f) - (b*(((-I)*(e + f*x)^2*ArcTan[E
^((2*I)*(a + b*x))])/b + (f*(((I/2)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a
+ b*x))])/b - (f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b - (f*((
(I/2)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (f*PolyLog[3, I*E^(
(2*I)*(a + b*x))])/(4*b^2)))/b))/(2*f)
```

3.331. $\int (e+fx)\text{arctanh}(\cot(a+bx))\,dx$

3.331.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6807
```
Int[ArcTanh[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTanh[Cot[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.331. $\int (e+fx)\text{arctanh}(\cot(a+bx))\,dx$

3.331.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.12 (sec) , antiderivative size = 1819, normalized size of antiderivative = 11.23

method	result	size
risch	Expression too large to display	1819

input
```
int((f*x+e)*arctanh(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*e/b*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))*a-1/2*e/b*ln(((-I)^(1/
2)+exp(I*(b*x+a)))/(-I)^(1/2))*a+1/2*I*e/b*dilog(((-I)^(1/2)-exp(I*(b*x+a)
))/(-I)^(1/2))+1/2*I*e/b*dilog(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/2
/b^2*f*a^2*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2/b^2*f*a^2*ln(((-
I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I/b^2*f*a*dilog(1+exp(I*(b*x+a))*
(-1)^(3/4))+1/2*I/b^2*f*a*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))-1/2/b*f*a*ln(
1+exp(I*(b*x+a))*(-1)^(3/4))*x-1/2/b*f*a*ln(1-exp(I*(b*x+a))*(-1)^(3/4))*x
+1/2*e/b*ln(1+exp(I*(b*x+a))*(-1)^(3/4))*a+1/2*e/b*ln(1-exp(I*(b*x+a))*(-1
)^(3/4))*a-1/2*I*e/b*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I*e/b*dilog(1-
exp(I*(b*x+a))*(-1)^(3/4))-1/2/b^2*f*a^2*ln(1+exp(I*(b*x+a))*(-1)^(3/4))-1
/2/b^2*f*a^2*ln(1-exp(I*(b*x+a))*(-1)^(3/4))+1/2/b*e*a*ln(exp(2*I*(b*x+a))
+I)-1/4*ln(exp(2*I*(b*x+a))-I)*x^2*f-1/2*ln(exp(2*I*(b*x+a))-I)*e*x-1/4*I*
Pi*(csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*
(b*x+a))+I)/(exp(2*I*(b*x+a))-1))+csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I
*(b*x+a))-1))^2-csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I/(exp(2*I*(b*x+a))-1))*
csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))+csgn(I*(exp(2*I*(b*x+a))
-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2+csgn(I*(exp(2*I*(
b*x+a))+I))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(
2*I*(b*x+a))-1))-csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/
(exp(2*I*(b*x+a))-1))^2-csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1...
```

3.331.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 681 vs. 2(130) = 260.

Time = 0.31 (sec) , antiderivative size = 681, normalized size of antiderivative = 4.20

$$\int (e+fx)\operatorname{arctanh}(\cot(a+bx))\,dx =$$

$$-\frac{2\,(-i\,bfx-i\,be)\mathrm{Li}_2(i\,\cos\left(2\,bx+2\,a\right)+\sin\left(2\,bx+2\,a\right))+2\,(-i\,bfx-i\,be)\mathrm{Li}_2(i\,\cos\left(2\,bx+2\,a\right)-\sin\left(\right.}{}$$

input
```
integrate((f*x+e)*arctanh(cot(b*x+a)),x, algorithm="fricas")
```

output
```
-1/16*(2*(-I*b*f*x - I*b*e)*dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) +
 2*(-I*b*f*x - I*b*e)*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 2*(I*
b*f*x + I*b*e)*dilog(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 2*(I*b*f*x
+ I*b*e)*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 4*(b^2*f*x^2 + 2*
b^2*e*x)*log(-(cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a)
- sin(2*b*x + 2*a) + 1)) - 2*(2*a*b*e - a^2*f)*log(cos(2*b*x + 2*a) + I*si
n(2*b*x + 2*a) + I) + 2*(2*a*b*e - a^2*f)*log(cos(2*b*x + 2*a) - I*sin(2*b
*x + 2*a) + I) + 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(I*cos(2*b
*x + 2*a) + sin(2*b*x + 2*a) + 1) - 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a
^2*f)*log(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) + 2*(b^2*f*x^2 + 2*b^
2*e*x + 2*a*b*e - a^2*f)*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) -
 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(-I*cos(2*b*x + 2*a) - sin
(2*b*x + 2*a) + 1) - 2*(2*a*b*e - a^2*f)*log(-cos(2*b*x + 2*a) + I*sin(2*b
*x + 2*a) + I) + 2*(2*a*b*e - a^2*f)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x +
 2*a) + I) - f*polylog(3, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + f*polyl
og(3, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - f*polylog(3, -I*cos(2*b*x +
 2*a) + sin(2*b*x + 2*a)) + f*polylog(3, -I*cos(2*b*x + 2*a) - sin(2*b*x +
 2*a)))/b^2
```

3.331.6 Sympy [F]

$$\int (e+fx)\text{arctanh}(\cot(a+bx))\,dx = \int (e+fx)\,\text{atanh}\,(\cot\,(a+bx))\,dx$$

input
```
integrate((f*x+e)*atanh(cot(b*x+a)),x)
```

output
```
Integral((e + f*x)*atanh(cot(a + b*x)), x)
```

3.331.7 Maxima [F]

$$\int (e+fx)\text{arctanh}(\cot(a+bx))\,dx=\int (fx+e)\,\text{artanh}\left(\cot\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)*arctanh(cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*si
n(2*b*x + 2*a) + 2) - 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin
(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(((b*f*x^2 + 2*b*e*x)
*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)
^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

3.331.8 Giac [F]

$$\int (e+fx)\text{arctanh}(\cot(a+bx))\,dx=\int (fx+e)\,\text{artanh}\left(\cot\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)*arctanh(cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*arctanh(cot(b*x + a)), x)
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)\text{arctanh}(\cot(a+bx))\,dx=\int \text{atanh}(\cot(a+b\,x))\,\left(e+f\,x\right)\,dx$$

input
```
int(atanh(cot(a + b*x))*(e + f*x),x)
```

output
```
int(atanh(cot(a + b*x))*(e + f*x), x)
```

3.332 $\int \operatorname{arctanh}(\cot(a+bx))\,dx$

3.332.1 Optimal result

Integrand size = 7, antiderivative size = 79

$$\int \operatorname{arctanh}(\cot(a+bx))\,dx = ix\arctan\left(e^{2i(a+bx)}\right) + x\operatorname{arctanh}(\cot(a+bx)) - \frac{i\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} + \frac{i\operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}$$

output
```
I*x*arctan(exp(2*I*(b*x+a)))+x*arctanh(cot(b*x+a))-1/4*I*polylog(2,-I*exp(
2*I*(b*x+a)))/b+1/4*I*polylog(2,I*exp(2*I*(b*x+a)))/b
```

3.332.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.61

$$\int \operatorname{arctanh}(\cot(a+bx))\,dx = x\operatorname{arctanh}(\cot(a+bx)) - \frac{(-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+2i}{8b}$$

input
```
Integrate[ArcTanh[Cot[a + b*x]],x]
```

output
```
x*ArcTanh[Cot[a + b*x]] - ((-4*a + Pi - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*
x))] - Log[1 + I/E^((2*I)*(a + b*x))]) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*
x]] + (2*I)*(PolyLog[2, (-I)/E^((2*I)*(a + b*x))] - PolyLog[2, I/E^((2*I)*
(a + b*x))]))/(8*b)
```

3.332.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6803, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(\cot(a+bx))\,dx$$

$$\Big\downarrow \text{6803}$$

$$x\operatorname{arctanh}(\cot(a+bx))-b\int x\sec(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$x\operatorname{arctanh}(\cot(a+bx))-b\int x\csc\left(2a+2bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4669}$$

$$x\operatorname{arctanh}(\cot(a+bx))-b\left(-\frac{\int \log\left(1-ie^{2i(a+bx)}\right)dx}{2b}+\frac{\int \log\left(1+ie^{2i(a+bx)}\right)dx}{2b}-\frac{ix\arctan\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\operatorname{arctanh}(\cot(a+bx))-b\left(\frac{i\int e^{-2i(a+bx)}\log\left(1-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i\int e^{-2i(a+bx)}\log\left(1+ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{ix\arctan\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\operatorname{arctanh}(\cot(a+bx))-b\left(-\frac{ix\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{i\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{i\operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b^2}\right)$$

input
```
Int[ArcTanh[Cot[a + b*x]],x]
```

output
```
x*ArcTanh[Cot[a + b*x]] - b*(((-I)*x*ArcTan[E^((2*I)*(a + b*x))])/b + ((I/
4)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])/b^2 - ((I/4)*PolyLog[2, I*E^((2*I
)*(a + b*x))])/b^2)
```

3.332.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6803
```
Int[ArcTanh[Cot[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTanh[Cot[a + b
*x]], x] - Simp[b   Int[x*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

3.332.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(64) = 128$.

Time = 0.75 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.84

method	result
derivativedivides	$-\frac{\frac{i\,\mathrm{arctanh}(\cot(bx+a))\left(\ln\left(1-\frac{i(\cot(bx+a)+1)^2}{-\cot(bx+a)^2+1}\right)-\ln\left(1+\frac{i(\cot(bx+a)+1)^2}{-\cot(bx+a)^2+1}\right)\right)}{2}+\frac{i\,\mathrm{dilog}\left(1+\frac{i(\cot(bx+a)+1)^2}{-\cot(bx+a)^2+1}\right)}{4}-\frac{i\,\mathrm{dilog}\left(1-\frac{i(\cot(b\ldots}{-\cot(\ldots}\right.}{4}\ldots}{b}$
default	$-\frac{\frac{i\,\mathrm{arctanh}(\cot(bx+a))\left(\ln\left(1-\frac{i(\cot(bx+a)+1)^2}{-\cot(bx+a)^2+1}\right)-\ln\left(1+\frac{i(\cot(bx+a)+1)^2}{-\cot(bx+a)^2+1}\right)\right)}{2}+\frac{i\,\mathrm{dilog}\left(1+\frac{i(\cot(bx+a)+1)^2}{-\cot(bx+a)^2+1}\right)}{4}-\frac{i\,\mathrm{dilog}\left(1-\frac{i(\cot(b\ldots}{-\cot(\ldots}\right.}{4}\ldots}{b}$
risch	Expression too large to display

input
```
int(arctanh(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*I*arctanh(cot(b*x+a))*(ln(1-I*(cot(b*x+a)+1)^2/(-cot(b*x+a)^2+1)
)-ln(1+I*(cot(b*x+a)+1)^2/(-cot(b*x+a)^2+1)))+1/4*I*dilog(1+I*(cot(b*x+a)+
1)^2/(-cot(b*x+a)^2+1))-1/4*I*dilog(1-I*(cot(b*x+a)+1)^2/(-cot(b*x+a)^2+1)
))
```

3.332.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(57) = 114$.

Time = 0.28 (sec) , antiderivative size = 389, normalized size of antiderivative = 4.92

$$\int \operatorname{arctanh}(\cot(a+bx))\,dx$$
$$= \frac{4\,bx\log\left(-\frac{\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a)+1}{\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a)+1}\right)+2\,a\log\left(\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right)+i\right)-2\,a\log\left(\cos\left(2\,bx\right.\right.}{}$$

input
```
integrate(arctanh(cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(4*b*x*log(-(cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a
) - sin(2*b*x + 2*a) + 1)) + 2*a*log(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)
 + I) - 2*a*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - 2*(b*x + a)*l
og(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(b*x + a)*log(I*cos(2*b*
x + 2*a) - sin(2*b*x + 2*a) + 1) - 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) + s
in(2*b*x + 2*a) + 1) + 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2
*a) + 1) + 2*a*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 2*a*log(-
cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) + I*dilog(I*cos(2*b*x + 2*a) +
sin(2*b*x + 2*a)) + I*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - I*dil
og(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - I*dilog(-I*cos(2*b*x + 2*a) -
 sin(2*b*x + 2*a)))/b
```

3.332.6 Sympy [F]

$$\int \operatorname{arctanh}(\cot(a+bx))\,dx = \int \operatorname{atanh}\left(\cot\left(a+bx\right)\right)dx$$

input
```
integrate(atanh(cot(b*x+a)),x)
```

output
```
Integral(atanh(cot(a + b*x)), x)
```

3.332.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(57) = 114.

Time = 0.33 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.33

$$\int \operatorname{arctanh}(\cot(a+bx))\,dx$$
$$= \frac{4\left(bx+a\right)\operatorname{artanh}\left(\frac{1}{\tan(bx+a)}\right) + \left(\arctan\left(\frac{1}{2}\tan\left(bx+a\right)+\frac{1}{2}, \frac{1}{2}\tan\left(bx+a\right)+\frac{1}{2}\right) - \arctan\left(\frac{1}{2}\tan\left(bx+a\right)\right.}{}$$

input
```
integrate(arctanh(cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*(4*(b*x + a)*arctanh(1/tan(b*x + a)) + (arctan2(1/2*tan(b*x + a) + 1/2
, 1/2*tan(b*x + a) + 1/2) - arctan2(1/2*tan(b*x + a) - 1/2, -1/2*tan(b*x +
 a) + 1/2))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log(1/2*tan(b*x + a)^2 + t
an(b*x + a) + 1/2) + (b*x + a)*log(1/2*tan(b*x + a)^2 - tan(b*x + a) + 1/2
) - I*dilog((1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2) + I*dilog(-(1/2*I -
1/2)*tan(b*x + a) + 1/2*I + 1/2) + I*dilog((1/2*I - 1/2)*tan(b*x + a) + 1/
2*I + 1/2) - I*dilog(-(1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2))/b
```

3.332.8 Giac [F]

$$\int \operatorname{arctanh}(\cot(a+bx))\,dx=\int \operatorname{artanh}\left(\cot\left(bx+a\right)\right)\,dx$$

input
```
integrate(arctanh(cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctanh(cot(b*x + a)), x)
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}(\cot(a+bx))\,dx=\int \operatorname{atanh}(\cot(a+b\,x))\,dx$$

input
```
int(atanh(cot(a + b*x)),x)
```

output
```
int(atanh(cot(a + b*x)), x)
```

3.333 $\int \frac{\text{arctanh}(\cot(a+bx))}{e+fx} dx$

3.333.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\text{arctanh}(\cot(a+bx))}{e+fx} dx = \text{Int}\left(\frac{\text{arctanh}(\cot(a+bx))}{e+fx}, x\right)$$

output
```
CannotIntegrate(arctanh(cot(b*x+a))/(f*x+e),x)
```

3.333.2 Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\text{arctanh}(\cot(a+bx))}{e+fx} dx = \int \frac{\text{arctanh}(\cot(a+bx))}{e+fx} dx$$

input
```
Integrate[ArcTanh[Cot[a + b*x]]/(e + f*x),x]
```

output
```
Integrate[ArcTanh[Cot[a + b*x]]/(e + f*x), x]
```

3.333.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\cot(a+bx))}{e+fx}\,dx$$
$$\Big\downarrow \ 7299$$
$$\int \frac{\text{arctanh}(\cot(a+bx))}{e+fx}dx$$

input
```
Int[ArcTanh[Cot[a + b*x]]/(e + f*x),x]
```

output
```
$Aborted
```

3.333.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.333.4 Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\text{arctanh}\left(\cot\left(bx+a\right)\right)}{fx+e}dx$$

input
```
int(arctanh(cot(b*x+a))/(f*x+e),x)
```

output
```
int(arctanh(cot(b*x+a))/(f*x+e),x)
```

3.333.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(\cot(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{artanh}\left(\cot\left(bx+a\right)\right)}{fx+e}\,dx$$

input
```
integrate(arctanh(cot(b*x+a))/(f*x+e),x, algorithm="fricas")
```

output
```
integral(arctanh(cot(b*x + a))/(f*x + e), x)
```

3.333.6 Sympy [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arctanh}(\cot(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{atanh}\left(\cot\left(a+bx\right)\right)}{e+fx}\,dx$$

input
```
integrate(atanh(cot(b*x+a))/(f*x+e),x)
```

output
```
Integral(atanh(cot(a + b*x))/(e + f*x), x)
```

3.333.7 Maxima [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(\cot(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{artanh}\left(\cot\left(bx+a\right)\right)}{fx+e}\,dx$$

input
```
integrate(arctanh(cot(b*x+a))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(arctanh(cot(b*x + a))/(f*x + e), x)
```

3.333.8 Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(\cot(a+bx))}{e+fx}\,dx=\int \frac{\operatorname{artanh}\left(\cot\left(bx+a\right)\right)}{fx+e}\,dx$$

input
```
integrate(arctanh(cot(b*x+a))/(f*x+e),x, algorithm="giac")
```

output
```
integrate(arctanh(cot(b*x + a))/(f*x + e), x)
```

3.333.9 Mupad [N/A]

Not integrable

Time = 5.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(\cot(a+bx))}{e+fx}\,dx=\int \frac{\mathrm{atanh}(\cot(a+b\,x))}{e+f\,x}\,dx$$

input
```
int(atanh(cot(a + b*x))/(e + f*x),x)
```

output
```
int(atanh(cot(a + b*x))/(e + f*x), x)
```

3.334 $\int x^2 \operatorname{arctanh}(c + d\cot(a + bx))\,dx$

3.334.1 Optimal result

Integrand size = 15, antiderivative size = 391

$$\begin{aligned}\int x^2 \operatorname{arctanh}(c + d\cot(a + bx))\,dx =& \frac{1}{3}x^3 \operatorname{arctanh}(c + d\cot(a + bx)) \\ &+ \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) \\ &- \frac{1}{6}x^3 \log\left(1 - \frac{(1 + c + id)e^{2ia+2ibx}}{1 + c - id}\right) \\ &- \frac{ix^2 \operatorname{PolyLog}\left(2, \frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{4b} \\ &+ \frac{ix^2 \operatorname{PolyLog}\left(2, \frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{4b} \\ &+ \frac{x \operatorname{PolyLog}\left(3, \frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{4b^2} \\ &- \frac{x \operatorname{PolyLog}\left(3, \frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{4b^2} \\ &+ \frac{i \operatorname{PolyLog}\left(4, \frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{8b^3} \\ &- \frac{i \operatorname{PolyLog}\left(4, \frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arctanh(c+d*cot(b*x+a))+1/6*x^3*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/
(1-c+I*d))-1/6*x^3*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*x^2*
polylog(2,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*x^2*polylog(2,(1
+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b+1/4*x*polylog(3,(1-c-I*d)*exp(2*I*
a+2*I*b*x)/(1-c+I*d))/b^2-1/4*x*polylog(3,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+
c-I*d))/b^2+1/8*I*polylog(4,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b^3-1/
8*I*polylog(4,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b^3
```

3.334.2 Mathematica [A] (verified)

Time = 3.51 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.87

$$\int x^2 \text{arctanh}(c + d\cot(a + bx))\, dx$$

$$= \frac{8b^3x^3\text{arctanh}(c + d\cot(a + bx)) + 4b^3x^3\log\left(1 + \frac{(1-c+id)e^{-2i(a+bx)}}{-1+c+id}\right) - 4b^3x^3\log\left(1 + \frac{(-1-c+id)e^{-2i(a+bx)}}{1+c+id}\right) + \cdots}{}$$

input
```
Integrate[x^2*ArcTanh[c + d*Cot[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcTanh[c + d*Cot[a + b*x]] + 4*b^3*x^3*Log[1 + (1 - c + I*d)/(
(-1 + c + I*d)*E^((2*I)*(a + b*x)))] - 4*b^3*x^3*Log[1 + (-1 - c + I*d)/((
1 + c + I*d)*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-1 + c - I*
d)/((-1 + c + I*d)*E^((2*I)*(a + b*x)))] - (6*I)*b^2*x^2*PolyLog[2, (1 + c
 - I*d)/((1 + c + I*d)*E^((2*I)*(a + b*x)))] + 6*b*x*PolyLog[3, (-1 + c -
I*d)/((-1 + c + I*d)*E^((2*I)*(a + b*x)))] - 6*b*x*PolyLog[3, (1 + c - I*d
)/((1 + c + I*d)*E^((2*I)*(a + b*x)))] - (3*I)*PolyLog[4, (-1 + c - I*d)/(
(-1 + c + I*d)*E^((2*I)*(a + b*x)))] + (3*I)*PolyLog[4, (1 + c - I*d)/((1
+ c + I*d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.334.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6823, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arctanh}(d\cot(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6823}$$

$$-\frac{1}{3}b(-ic+d+i)\int\frac{e^{2ia+2ibx}x^3}{-c-(-c-id+1)e^{2ia+2ibx}+id+1}dx+\frac{1}{3}b(-d+i(c+1))\int\frac{e^{2ia+2ibx}x^3}{c-(c+id+1)e^{2ia+2ibx}-id+1}dx+\frac{1}{3}x^3\text{arctanh}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$-\frac{1}{3}b(-ic+d+i)\left(\frac{3\int x^2\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{2b(d+i(1-c))}-\frac{x^3\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+\frac{1}{3}b(-d+i(c+1))\left(\frac{3\int x^2\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{2(-bd+i(bc+b))}-\frac{x^3\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+\frac{1}{3}x^3\text{arctanh}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$-\frac{1}{3}b(-ic+d+i)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{b}\right)}{2b(d+i(1-c))}-\frac{x^3\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+\frac{1}{3}b(-d+i(c+1))\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{b}\right)}{2(-bd+i(bc+b))}-\frac{x^3\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+\frac{1}{3}x^3\text{arctanh}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{7163}$$

3.334. $\int x^2\text{arctanh}(c+d\cot(a+bx))\,dx$

$$-\frac{1}{3}b(-ic+d+$$

$$i)\left(\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{i\left(\frac{i\int \text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{2b}-\frac{ix\,\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}\right)}{b}\right)}{2b(d+i(1-c))}-\frac{x^3\log\right.$$

$$\frac{1}{3}b(-d+i(c+$$

$$1))\left(\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{i\left(\frac{i\int \text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{2b}-\frac{ix\,\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}\right)}{b}\right)}{2(-bd+i(bc+b))}-\frac{x^3\log\left(1\right.}{2(-}\right.$$

$$\frac{1}{3}x^3\text{arctanh}(d\cot(a+bx)+c)$$

$$\Big\downarrow \; 2720$$

3.334. $\int x^2\text{arctanh}(c+d\cot(a+bx))\,dx$

$$-\frac{1}{3}b(-ic+d+$$

$$i)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right.}{2b}\right.}{b}\right.}{2b(d+i(1-c))}\right.$$

$$\frac{1}{3}b(-d+i(c+$$

$$1))\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}\right)}{b}\right.}{2(-bd+i(bc+b))}\right.$$

$$\frac{1}{3}x^3\operatorname{arctanh}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{7143}$$

3.334. $\int x^2\operatorname{arctanh}(c+d\cot(a+bx))\,dx$

$$\frac{1}{3}x^3\text{arctanh}(d\cot(a+bx)+c)-\frac{1}{3}b(-ic+d+i)\left(\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{i\left(\frac{\text{PolyLog}\left(4,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2}-\frac{ix\,\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}\right)}{b}\right)}{2b(d+i(1-c))}-\frac{x^3\log\left(1\ \cdots\right.}{2b(\cdots}\right.$$

$$\frac{1}{3}b(-d+i(c+1))\left(\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{i\left(\frac{\text{PolyLog}\left(4,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2}-\frac{ix\,\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}\right)}{b}\right)}{2(-bd+i(bc+b))}-\frac{x^3\log\left(1-\cdots\right.}{2(-bd\cdots}\right.$$

input
```
Int[x^2*ArcTanh[c + d*Cot[a + b*x]],x]
```

output
```
(x^3*ArcTanh[c + d*Cot[a + b*x]])/3 - (b*(I - I*c + d)*(-1/2*(x^3*Log[1 -
((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(b*(I*(1 - c) + d)
) + (3*(((I/2)*x^2*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 -
 c + I*d)])/b - (I*(((-1/2*I)*x*PolyLog[3, ((1 - c - I*d)*E^((2*I)*a + (2*
I)*b*x))/(1 - c + I*d)])/b + PolyLog[4, ((1 - c - I*d)*E^((2*I)*a + (2*I)*
b*x))/(1 - c + I*d)]/(4*b^2)))/b))/(2*b*(I*(1 - c) + d))))/3 + (b*(I*(1 +
c) - d)*(-1/2*(x^3*Log[1 - ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c
- I*d)])/(I*(b + b*c) - b*d) + (3*(((I/2)*x^2*PolyLog[2, ((1 + c + I*d)*E^
((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b - (I*(((-1/2*I)*x*PolyLog[3, ((1
+ c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + PolyLog[4, ((1 + c
 + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)]/(4*b^2)))/b))/(2*(I*(b + b
*c) - b*d))))/3
```

3.334. $\int x^2\text{arctanh}(c+d\cot(a+bx))\,dx$

3.334.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6823
```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + (-Simp[I*b*((1 - c - I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)
*(E^(2*I*a + 2*I*b*x)/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x))), x
], x] + Simp[I*b*((1 + c + I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*
I*a + 2*I*b*x)/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x))), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.334.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 24.30 (sec) , antiderivative size = 6693, normalized size of antiderivative = 17.12

method	result	size
risch	Expression too large to display	6693

input
```
int(x^2*arctanh(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.334.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1799 vs. 2(275) = 550.

Time = 0.46 (sec) , antiderivative size = 1799, normalized size of antiderivative = 4.60

$$\int x^2 \text{arctanh}(c + d\cot(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate(x^2*arctanh(c+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/48*(8*b^3*x^3*log(-(d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(
d*cos(2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 6*I*b^2*x^2*dilog(-(
c^2 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c
^2 + 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d
^2 + 2*c + 1) + 1) - 6*I*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d
- d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c +
 I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 6*I*b^2*x^2*d
ilog(-(c^2 + d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a)
+ (-I*c^2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(
c^2 + d^2 - 2*c + 1) + 1) + 6*I*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c
- 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 -
2*I*c + I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) + 4*a^3*
log(1/2*c^2 + I*(c + 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x
+ 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) - 4*a
^3*log(1/2*c^2 + I*(c - 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b
*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) +
4*a^3*log(-1/2*c^2 + I*(c + 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos
(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2
) - 4*a^3*log(-1/2*c^2 + I*(c - 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)
*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) + ...
```

3.334.6 Sympy [F]

$$\int x^2 \text{arctanh}(c + d\cot(a + bx))\,dx = \int x^2 \operatorname{atanh}(c + d\cot(a + bx))\,dx$$

input
```
integrate(x**2*atanh(c+d*cot(b*x+a)),x)
```

output
```
Integral(x**2*atanh(c + d*cot(a + b*x)), x)
```

3.334.7 Maxima [F]

$$\int x^2 \text{arctanh}(c + d\cot(a + bx))\,dx = \int x^2\, \text{artanh}\,(d\cot\,(bx + a) + c)\,\,dx$$

input
```
integrate(x^2*arctanh(c+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*x^3*log((c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*
b*x + 2*a) + (c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2
 - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/12*x^3*log((c^2 + d^2 -
2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 -
2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*
x + 2*a) - 2*c + 1) - 4*b*d*integrate(1/3*(2*(c^2 + d^2 - 1)*x^3*cos(2*b*x
 + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^3*sin(2*b*x +
 2*a)^2 - (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*
a) + (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*c
os(2*b*x + 2*a) - (c^2 - d^2 - 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/
(c^4 + d^4 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*c
os(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x
+ 2*a)^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 +
4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2
*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 - 2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*
b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a
) - 4*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*
d + 2*(c*d^3 + (c^3 - c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin
(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a)
 + 1), x)
```

3.334.8 Giac [F]

$$\int x^2 \text{arctanh}(c + d\cot(a + bx))\,dx = \int x^2\, \text{artanh}\,(d\cot\,(bx + a) + c)\,\,dx$$

input
```
integrate(x^2*arctanh(c+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(d*cot(b*x + a) + c), x)
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{arctanh}(c + d\cot(a + bx))\, dx = \int x^2 \operatorname{atanh}(c + d\cot(a + b\,x))\ dx$$

input
```
int(x^2*atanh(c + d*cot(a + b*x)),x)
```

output
```
int(x^2*atanh(c + d*cot(a + b*x)), x)
```

3.335 $\int x\text{arctanh}(c+d\cot(a+bx))\,dx$

3.335.1 Optimal result

Integrand size = 13, antiderivative size = 293

$$\int x\text{arctanh}(c+d\cot(a+bx))\,dx = \frac{1}{2}x^2\text{arctanh}(c+d\cot(a+bx)) + \frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{4}x^2\log\left(1-\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right) - \frac{ix\,\text{PolyLog}\left(2,\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{4b} + \frac{ix\,\text{PolyLog}\left(2,\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{4b} + \frac{\text{PolyLog}\left(3,\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{8b^2} - \frac{\text{PolyLog}\left(3,\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{8b^2}$$

output
```
1/2*x^2*arctanh(c+d*cot(b*x+a))+1/4*x^2*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/
(1-c+I*d))-1/4*x^2*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*x*po
lylog(2,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*x*polylog(2,(1+c+I
*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b+1/8*polylog(3,(1-c-I*d)*exp(2*I*a+2*I*
b*x)/(1-c+I*d))/b^2-1/8*polylog(3,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/
b^2
```

3.335.2 Mathematica [A] (verified)

Time = 2.82 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.87

$$\int x\text{arctanh}(c+d\cot(a+bx))\,dx$$
$$=\frac{4b^2x^2\text{arctanh}(c+d\cot(a+bx))+2b^2x^2\log\left(1+\frac{(1-c+id)e^{-2i(a+bx)}}{-1+c+id}\right)-2b^2x^2\log\left(1+\frac{(-1-c+id)e^{-2i(a+bx)}}{1+c+id}\right)+$$

input
```
Integrate[x*ArcTanh[c + d*Cot[a + b*x]],x]
```

output
```
(4*b^2*x^2*ArcTanh[c + d*Cot[a + b*x]] + 2*b^2*x^2*Log[1 + (1 - c + I*d)/(
(-1 + c + I*d)*E^((2*I)*(a + b*x)))] - 2*b^2*x^2*Log[1 + (-1 - c + I*d)/((
1 + c + I*d)*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-1 + c - I*d)/(
(-1 + c + I*d)*E^((2*I)*(a + b*x)))] - (2*I)*b*x*PolyLog[2, (1 + c - I*d)/
((1 + c + I*d)*E^((2*I)*(a + b*x)))] + PolyLog[3, (-1 + c - I*d)/((-1 + c
+ I*d)*E^((2*I)*(a + b*x)))] - PolyLog[3, (1 + c - I*d)/((1 + c + I*d)*E^(
(2*I)*(a + b*x)))])/(8*b^2)
```

3.335.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.34, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6823, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(d\cot(a+bx)+c)\,dx$$
$$\Big\downarrow \text{6823}$$
$$-\frac{1}{2}b(-ic+d+i)\int\frac{e^{2ia+2ibx}x^2}{-c-(-c-id+1)e^{2ia+2ibx}+id+1}dx+\frac{1}{2}b(-d+i(c+1))\int\frac{e^{2ia+2ibx}x^2}{c-(c+id+1)e^{2ia+2ibx}-id+1}dx+\frac{1}{2}x^2\text{arctanh}(d\cot(a+bx)+c)$$
$$\Big\downarrow \text{2620}$$

$$-\frac{1}{2}b(-ic+d+i)\left(\frac{\int x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{b(d+i(1-c))}-\frac{x^2\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+\frac{1}{2}b(-d+i(c+1))\left(\frac{\int x\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{-bd+i(bc+b)}-\frac{x^2\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+\frac{1}{2}x^2\text{arctanh}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{1}{2}b(-ic+d+i)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{i\int\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{2b}}{b(d+i(1-c))}-\frac{x^2\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+\frac{1}{2}b(-d+i(c+1))\left(\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{i\int\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{2b}}{-bd+i(bc+b)}-\frac{x^2\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+\frac{1}{2}x^2\text{arctanh}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{ 2720}$$

$$-\frac{1}{2}b(-ic+d+i)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)de^{2ia+2ibx}}{4b^2}}{b(d+i(1-c))}-\frac{x^2\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+\frac{1}{2}b(-d+i(c+1))\left(\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)de^{2ia+2ibx}}{4b^2}}{-bd+i(bc+b)}-\frac{x^2\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+\frac{1}{2}x^2\text{arctanh}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{ 7143}$$

3.335. $\int x\text{arctanh}(c+d\cot(a+bx))\,dx$

$$\frac{1}{2}x^2\text{arctanh}(d\cot(a+bx)+c)-\frac{1}{2}b(-ic+d+i)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2}}{b(d+i(1-c))}-\frac{x^2\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+\frac{1}{2}b(-d+i(c+1))\left(\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2}}{-bd+i(bc+b)}-\frac{x^2\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)$$

input
```
Int[x*ArcTanh[c + d*Cot[a + b*x]],x]
```

output
```
(x^2*ArcTanh[c + d*Cot[a + b*x]])/2 - (b*(I - I*c + d)*(-1/2*(x^2*Log[1 -
((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(b*(I*(1 - c) + d)
) + (((I/2)*x*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c +
I*d)])/b - PolyLog[3, ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d
)]/(4*b^2))/(b*(I*(1 - c) + d))))/2 + (b*(I*(1 + c) - d)*(-1/2*(x^2*Log[1
- ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/(I*(b + b*c) - b
*d) + (((I/2)*x*PolyLog[2, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c
- I*d)])/b - PolyLog[3, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I
*d)]/(4*b^2))/(I*(b + b*c) - b*d)))/2
```

3.335.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6823
```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + (-Simp[I*b*((1 - c - I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)
*(E^(2*I*a + 2*I*b*x)/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x))), x
], x] + Simp[I*b*((1 + c + I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*
I*a + 2*I*b*x)/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x))), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.335.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.45 (sec) , antiderivative size = 6343, normalized size of antiderivative = 21.65

method	result	size
risch	Expression too large to display	6343

input
```
int(x*arctanh(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.335.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1463 vs. $2(207) = 414$.

Time = 0.41 (sec) , antiderivative size = 1463, normalized size of antiderivative = 4.99

$$\int x\text{arctanh}(c + d\cot(a + bx))\,dx = \text{Too large to display}$$

input
```
integrate(x*arctanh(c+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/16*(4*b^2*x^2*log(-(d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(
d*cos(2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 2*I*b*x*dilog(-(c^2
+ d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 +
 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 +
 2*c + 1) + 1) - 2*I*b*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 +
2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(
2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 2*I*b*x*dilog(-(c^2 +
 d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 +
2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 -
2*c + 1) + 1) + 2*I*b*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2
*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2
*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) - 2*a^2*log(1/2*c^2 + I*
(c + 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*
c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) + 2*a^2*log(1/2*c^2 +
 I*(c - 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*
(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) - 2*a^2*log(-1/2*c
^2 + I*(c + 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) +
1/2*(I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) + 2*a^2*log(-1
/2*c^2 + I*(c - 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a
) + 1/2*(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 2*(b^...
```

3.335.6 Sympy [F]

$$\int x\text{arctanh}(c + d\cot(a + bx))\,dx = \int x\,\text{atanh}\,(c + d\cot\,(a + bx))\,dx$$

input
```
integrate(x*atanh(c+d*cot(b*x+a)),x)
```

output
```
Integral(x*atanh(c + d*cot(a + b*x)), x)
```

3.335.7 Maxima [F]

$$\int x\text{arctanh}(c+d\cot(a+bx))\,dx=\int x\,\text{artanh}\left(d\cot\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arctanh(c+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-2*b*d*integrate((2*(c^2 + d^2 - 1)*x^2*cos(2*b*x + 2*a)^2 + 2*c*d*x^2*sin
(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*sin(2*b*x + 2*a)^2 - (c^2 - d^2 - 1)
*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) + (c^2 - d^2 - 1)*x^2*
cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*x + 2*a) - (c^2 -
d^2 - 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)*
d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c^
4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (c^4 + d^4 + 2
*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1
)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 -
1)*d^2 - 2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(c*d^3 + (
c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4 - d^4 - 2*c^2
+ 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*d + 2*(c*d^3 + (c^3 - c)*
d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin(4*b*x
+ 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*log((
c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b*x + 2*a) + (
c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 + 2*c +
 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/8*x^2*log((c^2 + d^2 - 2*c + 1)*cos(2*
b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*sin(2*
b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) - 2*c
+ 1)
```

3.335.8 Giac [F]

$$\int x\text{arctanh}(c+d\cot(a+bx))\,dx=\int x\,\text{artanh}\left(d\cot\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arctanh(c+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctanh(d*cot(b*x + a) + c), x)
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arctanh}(c+d\cot(a+bx))\,dx=\int x\,\mathrm{atanh}(c+d\cot(a+b\,x))\,dx$$

input
```
int(x*atanh(c + d*cot(a + b*x)),x)
```

output
```
int(x*atanh(c + d*cot(a + b*x)), x)
```

3.336 $\int \operatorname{arctanh}(c + d\cot(a + bx))\,dx$

3.336.1 Optimal result

Integrand size = 11, antiderivative size = 194

$$\int \operatorname{arctanh}(c+d\cot(a+bx))\,dx = x\operatorname{arctanh}(c+d\cot(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{2}x\log\left(1-\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right) - \frac{i\,\text{PolyLog}\left(2,\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{4b} + \frac{i\,\text{PolyLog}\left(2,\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{4b}$$

output
```
x*arctanh(c+d*cot(b*x+a))+1/2*x*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d
))-1/2*x*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*polylog(2,(1-c
-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*polylog(2,(1+c+I*d)*exp(2*I*a+
2*I*b*x)/(1+c-I*d))/b
```

3.336.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 390 vs. 2(194) = 388.

Time = 3.95 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.01

$$\int \operatorname{arctanh}(c+d\cot(a+bx))\,dx = x\left(\operatorname{arctanh}(c+d\cot(a+bx))\right.$$

$$+\frac{2a\log(d+(-1+c)\tan(a+bx))+i\log(1+i\tan(a+bx))\log\left(-\frac{i(d+(-1+c)\tan(a+bx))}{-1+c-id}\right)-i\log(1-i\tan}{}$$

input
```
Integrate[ArcTanh[c + d*Cot[a + b*x]],x]
```

output
```
x*(ArcTanh[c + d*Cot[a + b*x]] + (2*a*Log[d + (-1 + c)*Tan[a + b*x]] + I*L
og[1 + I*Tan[a + b*x]]*Log[((-I)*(d + (-1 + c)*Tan[a + b*x]))/(-1 + c - I*
d)] - I*Log[1 - I*Tan[a + b*x]]*Log[(I*(d + (-1 + c)*Tan[a + b*x]))/(-1 +
c + I*d)] - 2*a*Log[d + (1 + c)*Tan[a + b*x]] + I*Log[1 - I*Tan[a + b*x]]*
Log[(I*(d + (1 + c)*Tan[a + b*x]))/(1 + c + I*d)] - I*Log[1 + I*Tan[a + b*
x]]*Log[(d + (1 + c)*Tan[a + b*x])/(I*(1 + c) + d)] - I*PolyLog[2, ((-1 +
c)*(1 - I*Tan[a + b*x]))/(-1 + c + I*d)] + I*PolyLog[2, ((1 + c)*(1 - I*Ta
n[a + b*x]))/(1 + c + I*d)] + I*PolyLog[2, ((-1 + c)*(1 + I*Tan[a + b*x]))
/(-1 + c - I*d)] - I*PolyLog[2, ((1 + c)*(1 + I*Tan[a + b*x]))/(1 + c - I*
d)])/(4*a - (2*I)*Log[1 - I*Tan[a + b*x]] + (2*I)*Log[1 + I*Tan[a + b*x]])
)
```

3.336.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.46, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6815, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(d\cot(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6815}$$

$$-b(-ic+d+i)\int \frac{e^{2ia+2ibx}x}{-c-(-c-id+1)e^{2ia+2ibx}+id+1}dx+b(-d+i(c+1))\int \frac{e^{2ia+2ibx}x}{c-(c+id+1)e^{2ia+2ibx}-id+1}dx+x\text{arctanh}(d\cot(a+bx)+c)$$

$$\Big\downarrow \; 2620$$

$$-b(-ic+d+i)\left(\frac{\int \log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{2b(d+i(1-c))}-\frac{x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+b(-d+i(c+1))\left(\frac{\int \log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{2(-bd+i(bc+b))}-\frac{x\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+x\text{arctanh}(d\cot(a+bx)+c)$$

$$\Big\downarrow \; 2715$$

$$-b(-ic+d+i)\left(-\frac{i\int e^{-2ia-2ibx}\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)de^{2ia+2ibx}}{4b^2(d+i(1-c))}-\frac{x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+b(-d+i(c+1))\left(-\frac{i\int e^{-2ia-2ibx}\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)de^{2ia+2ibx}}{4b(-bd+i(bc+b))}-\frac{x\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+x\text{arctanh}(d\cot(a+bx)+c)$$

$$\Big\downarrow \; 2838$$

$$x\text{arctanh}(d\cot(a+bx)+c)-b(-ic+d+i)\left(\frac{i\,\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2(d+i(1-c))}-\frac{x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+b(-d+i(c+1))\left(\frac{i\,\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b(-bd+i(bc+b))}-\frac{x\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)$$

input
```
Int[ArcTanh[c + d*Cot[a + b*x]],x]
```

output
```
x*ArcTanh[c + d*Cot[a + b*x]] - b*(I - I*c + d)*(-1/2*(x*Log[1 - ((1 - c -
 I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(b*(I*(1 - c) + d)) + ((I/4
)*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(b^2*
(I*(1 - c) + d))) + b*(I*(1 + c) - d)*(-1/2*(x*Log[1 - ((1 + c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/(I*(b + b*c) - b*d) + ((I/4)*PolyLog[
2, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/(b*(I*(b + b*c)
 - b*d)))
```

3.336. $\int \text{arctanh}(c+d\cot(a+bx))\,dx$

3.336.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6815
```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Cot[a + b*x]], x] + (-Simp[I*b*(1 - c - I*d)   Int[x*(E^(2*I*a +
 2*I*b*x)/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x))), x], x] + Simp
[I*b*(1 + c + I*d)   Int[x*(E^(2*I*a + 2*I*b*x)/(1 + c - I*d - (1 + c + I*d
)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - I*d)
^2, 1]
```

3.336.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(164) = 328$.

Time = 2.41 (sec) , antiderivative size = 563, normalized size of antiderivative = 2.90

method	result
derivativedivides	$\frac{-d\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arctanh}(c+d\cot(bx+a))+d^2\left(-\frac{\arctan\left(-\frac{c+d\cot(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\cot(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2d}\right.}{}$
default	$\frac{-d\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arctanh}(c+d\cot(bx+a))+d^2\left(-\frac{\arctan\left(-\frac{c+d\cot(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\cot(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2d}\right.}{}$
risch	Expression too large to display

input
```
int(arctanh(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-d*(1/2*Pi-arccot(cot(b*x+a)))*arctanh(c+d*cot(b*x+a))+d^2*(-1/2*ar
ctan(-(c+d*cot(b*x+a))/d+c/d)/d*ln(d*((c+d*cot(b*x+a))/d-c/d)+c+1)+1/2*arc
tan(-(c+d*cot(b*x+a))/d+c/d)/d*ln(d*((c+d*cot(b*x+a))/d-c/d)+c-1)+1/4*I*ln
(d*((c+d*cot(b*x+a))/d-c/d)+c+1)*(ln((I*d-d*((c+d*cot(b*x+a))/d-c/d))/(1+c
+I*d))-ln((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(I*d-c-1)))/d+1/4*I*(dilog((I*d
-d*((c+d*cot(b*x+a))/d-c/d))/(1+c+I*d))-dilog((I*d+d*((c+d*cot(b*x+a))/d-c
/d))/(I*d-c-1)))/d-1/4*I*ln(d*((c+d*cot(b*x+a))/d-c/d)+c-1)*(ln((I*d-d*((c
+d*cot(b*x+a))/d-c/d))/(I*d+c-1))-ln((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(1-c
+I*d)))/d-1/4*I*(dilog((I*d-d*((c+d*cot(b*x+a))/d-c/d))/(I*d+c-1))-dilog((
I*d+d*((c+d*cot(b*x+a))/d-c/d))/(1-c+I*d)))/d))
```

3.336.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1099 vs. 2(136) = 272.

Time = 0.38 (sec) , antiderivative size = 1099, normalized size of antiderivative = 5.66

$$\int \operatorname{arctanh}(c+d\cot(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate(arctanh(c+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(4*b*x*log(-(d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d*cos
(2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 2*a*log(1/2*c^2 + I*(c +
1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 +
 I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) - 2*a*log(1/2*c^2 + I*(c -
 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2
+ I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) + 2*a*log(-1/2*c^2 + I*(c
 + 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^
2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) - 2*a*log(-1/2*c^2 + I*
(c - 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*
c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 2*(b*x + a)*log((c^
2 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2
 + 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2
 + 2*c + 1)) - 2*(b*x + a)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2
*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2
*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*log((c^2 + d^2
 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c
 - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c
+ 1)) + 2*(b*x + a)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)
*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x +
2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) + I*dilog(-(c^2 + d^2 - (c^2 + 2...
```

3.336.6 Sympy [F]

$$\int \operatorname{arctanh}(c+d\cot(a+bx))\,dx=\int \operatorname{atanh}\left(c+d\cot\left(a+bx\right)\right)dx$$

input
```
integrate(atanh(c+d*cot(b*x+a)),x)
```

output
```
Integral(atanh(c + d*cot(a + b*x)), x)
```

3.336.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(136) = 272$.

Time = 0.37 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.02

$$\int \operatorname{arctanh}(c + d\cot(a + bx))\,dx$$
$$= \frac{4\,(bx+a)\operatorname{artanh}\left(c+\frac{d}{\tan(bx+a)}\right)+\left(\arctan\left(\frac{(c+1)d+(c^2+2\,c+1)\tan(bx+a)}{c^2+d^2+2\,c+1},\frac{(c+1)d\tan(bx+a)+d^2}{c^2+d^2+2\,c+1}\right)-\arctan\left(\frac{(c-1)d\cdots}{}\right.\right.}{}$$

input
```
integrate(arctanh(c+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*(4*(b*x + a)*arctanh(c + d/tan(b*x + a)) + (arctan2(((c + 1)*d + (c^2
+ 2*c + 1)*tan(b*x + a))/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*tan(b*x + a) +
d^2)/(c^2 + d^2 + 2*c + 1)) - arctan2(((c - 1)*d + (c^2 - 2*c + 1)*tan(b*x
 + a))/(c^2 + d^2 - 2*c + 1), ((c - 1)*d*tan(b*x + a) + d^2)/(c^2 + d^2 -
2*c + 1)))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log((2*(c + 1)*d*tan(b*x +
a) + (c^2 + 2*c + 1)*tan(b*x + a)^2 + d^2)/(c^2 + d^2 + 2*c + 1)) + (b*x +
 a)*log((2*(c - 1)*d*tan(b*x + a) + (c^2 - 2*c + 1)*tan(b*x + a)^2 + d^2)/
(c^2 + d^2 - 2*c + 1)) + I*dilog(-((c + 1)*tan(b*x + a) - I*c - I)/(I*c +
d + I)) - I*dilog(-((c - 1)*tan(b*x + a) - I*c + I)/(I*c + d - I)) + I*dil
og(-((c - 1)*tan(b*x + a) + I*c - I)/(-I*c + d + I)) - I*dilog(-((c + 1)*t
an(b*x + a) + I*c + I)/(-I*c + d - I)))/b
```

3.336.8 Giac [F]

$$\int \operatorname{arctanh}(c + d\cot(a + bx))\,dx = \int \operatorname{artanh}\left(d\cot\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arctanh(c+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctanh(d*cot(b*x + a) + c), x)
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}(c + d\cot(a + bx))\,dx = \int \operatorname{atanh}(c + d\cot(a + b\,x))\,dx$$

input
```
int(atanh(c + d*cot(a + b*x)),x)
```

output
```
int(atanh(c + d*cot(a + b*x)), x)
```

3.337 $\int \frac{\operatorname{arctanh}(c+d\cot(a+bx))}{x}\,dx$

3.337.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\operatorname{arctanh}(c+d\cot(a+bx))}{x}\,dx=\operatorname{Int}\left(\frac{\operatorname{arctanh}(c+d\cot(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arctanh(c+d*cot(b*x+a))/x,x)
```

3.337.2 Mathematica [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c+d\cot(a+bx))}{x}\,dx=\int \frac{\operatorname{arctanh}(c+d\cot(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTanh[c + d*Cot[a + b*x]]/x,x]
```

output
```
Integrate[ArcTanh[c + d*Cot[a + b*x]]/x, x]
```

3.337.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{7299\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(d\cot(a+bx)+c)}{x} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\text{arctanh}(d\cot(a+bx)+c)}{x} dx$$

input
```
Int[ArcTanh[c + d*Cot[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.337.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.337.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\text{arctanh}\left(c+d\cot\left(bx+a\right)\right)}{x} dx$$

input
```
int(arctanh(c+d*cot(b*x+a))/x,x)
```

output
```
int(arctanh(c+d*cot(b*x+a))/x,x)
```

3.337.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{artanh}\left(d\cot\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctanh(c+d*cot(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arctanh(d*cot(b*x + a) + c)/x, x)
```

3.337.6 Sympy [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{arctanh}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{atanh}\left(c+d\cot\left(a+bx\right)\right)}{x}\,dx$$

input
```
integrate(atanh(c+d*cot(b*x+a))/x,x)
```

output
```
Integral(atanh(c + d*cot(a + b*x))/x, x)
```

3.337.7 Maxima [N/A]

Not integrable

Time = 3.39 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{artanh}\left(d\cot\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctanh(c+d*cot(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arctanh(d*cot(b*x + a) + c)/x, x)
```

3.337.8 Giac [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c + d\cot(a + bx))}{x} \, dx = \int \frac{\operatorname{artanh}(d\cot(bx + a) + c)}{x} \, dx$$

input
```
integrate(arctanh(c+d*cot(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctanh(d*cot(b*x + a) + c)/x, x)
```

3.337.9 Mupad [N/A]

Not integrable

Time = 5.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{arctanh}(c + d\cot(a + bx))}{x} \, dx = \int \frac{\operatorname{atanh}(c + d\cot(a + b\,x))}{x} \, dx$$

input
```
int(atanh(c + d*cot(a + b*x))/x,x)
```

output
```
int(atanh(c + d*cot(a + b*x))/x, x)
```

3.338 $\int x^2 \mathbf{arctanh}(1 + id + d\cot(a + bx))\,dx$

3.338.1 Optimal result

Integrand size = 20, antiderivative size = 168

$$\begin{aligned}\int x^2\text{arctanh}(1+id+d\cot(a+bx))\,dx &= \frac{1}{12}ibx^4+\frac{1}{3}x^3\text{arctanh}(1+id+d\cot(a+bx)) \\ &\quad -\frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right) \\ &\quad +\frac{ix^2\,\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b} \\ &\quad -\frac{x\,\text{PolyLog}\left(3,(1+id)e^{2ia+2ibx}\right)}{4b^2} \\ &\quad -\frac{i\,\text{PolyLog}\left(4,(1+id)e^{2ia+2ibx}\right)}{8b^3}\end{aligned}$$

output
```
1/12*I*b*x^4+1/3*x^3*arctanh(1+I*d+d*cot(b*x+a))-1/6*x^3*ln(1-(1+I*d)*exp(
2*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*po
lylog(3,(1+I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,(1+I*d)*exp(2*I*a+
2*I*b*x))/b^3
```

3.338.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.92

$$\int x^2\text{arctanh}(1+id+d\cot(a+bx))\,dx=\frac{1}{3}x^3\text{arctanh}(1+id+d\cot(a+bx))$$
$$-\frac{4b^3x^3\log\left(1+\frac{ie^{-2i(a+bx)}}{-i+d}\right)+6ib^2x^2\operatorname{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)+6bx\operatorname{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)-3i\operatorname{PolyL}}{24b^3}$$

input
```
Integrate[x^2*ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

output
```
(x^3*ArcTanh[1 + I*d + d*Cot[a + b*x]])/3 - (4*b^3*x^3*Log[1 + I/((-I + d)
*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/((-I + d)*E^((2*I)*
(a + b*x)))] + 6*b*x*PolyLog[3, (-I)/((-I + d)*E^((2*I)*(a + b*x)))] - (3*
I)*PolyLog[4, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.338.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6819, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arctanh}(d\cot(a+bx)+id+1)\,dx$$

$$\Big\downarrow \text{6819}$$

$$\frac{1}{3}ib\int\frac{x^3}{1-(id+1)e^{2ia+2ibx}}dx+\frac{1}{3}x^3\text{arctanh}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\int\frac{e^{2ia+2ibx}x^3}{1-(id+1)e^{2ia+2ibx}}dx\right)+\frac{1}{3}x^3\text{arctanh}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\left(\frac{3\int x^2\log\left(1-(id+1)e^{2ia+2ibx}\right)dx}{2b(-d+i)}-\frac{x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+$$
$$\frac{1}{3}x^3\text{arctanh}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \; 3011$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}(2,(id+1)e^{2ia+2ibx})}{2b}-\frac{i\int x\operatorname{PolyLog}(2,(id+1)e^{2ia+2ibx})dx}{b}\right)}{2b(-d+i)}-\frac{x^3\log\left(1-(1+id)e^{2ia+2ibx}\right.}{2b(-d+i)}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \; 7163$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}(2,(id+1)e^{2ia+2ibx})}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,(id+1)e^{2ia+2ibx}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,(id+1)e^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2b(-d+i)}-\frac{x}{}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}(2,(id+1)e^{2ia+2ibx})}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,(id+1)e^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,(id+1)e^{2i}\right.}{2b}\right.}{b}\right.}{2b(-d+i)}\right.\right.$$

$$\frac{1}{3}x^3\text{arctanh}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \; 7143$$

$$\frac{1}{3}x^3\text{arctanh}(d\cot(a+bx)+id+1)+$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}(2,(id+1)e^{2ia+2ibx})}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,(id+1)e^{2ia+2ibx}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,(id+1)e^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2b(-d+i)}-\frac{x^3\log}{}\right.\right.$$

input
```
Int[x^2*ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

3.338. $\int x^2\text{arctanh}(1+id+d\cot(a+bx))\,dx$

output
```
(x^3*ArcTanh[1 + I*d + d*Cot[a + b*x]])/3 + (I/3)*b*(x^4/4 + (1 + I*d)*(-1
/2*(x^3*Log[1 - (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I - d)) + (3*(((I/
2)*x^2*PolyLog[2, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/b - (I*(((-1/2*I)*x*
PolyLog[3, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/b + PolyLog[4, (1 + I*d)*E^
((2*I)*a + (2*I)*b*x)]/(4*b^2)))/b))/(2*b*(I - d))))
```

3.338.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6819
```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.338.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.03 (sec) , antiderivative size = 2387, normalized size of antiderivative = 14.21

method	result	size
risch	Expression too large to display	2387

input
```
int(x^2*arctanh(1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b^3*a^3/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/6*I/b^3*a^3/
(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d-I)+1/12*I*b*x^4+1/2/b^3*d*a
^3/(I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/2/b^3*d*a^3/(I-d)*ln(1+I
*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/4/b^2*d/(I-d)*polylog(3,-I*(I-d)*exp(2*
I*(b*x+a)))*x-1/6/b^3*a^3*d/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d
-I)-1/3/b^3*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^3+1/8*I/b^3*d/(I-d)*p
olylog(4,-I*(I-d)*exp(2*I*(b*x+a)))+1/3*I/b^3/(I-d)*ln(1+I*(I-d)*exp(2*I*(
b*x+a)))*a^3-1/4*I/b^2/(I-d)*polylog(3,-I*(I-d)*exp(2*I*(b*x+a)))*x-1/2*I/
b^3*a^3/(I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/6*d/(I-d)*ln(1+I*(I
-d)*exp(2*I*(b*x+a)))*x^3-1/4/b/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))
*x^2+1/4/b^3/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*a^2-1/2/b^3*a^2/(I
-d)*dilog(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/2/b^3*a^2/(I-d)*dilog(1+I*
exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/6*I/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))
*x^3-1/12*(-I*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(1/(e
xp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a))*d)+I*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*e
xp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*
I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))+I*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp
(2*I*(b*x+a)))*csgn(1/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a))*d)^2+I*Pi*csgn
(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-2*I*P
i-I*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))...
```

3.338.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.07

$$\int x^2\text{arctanh}(1+id+d\cot(a+bx))\,dx = \frac{2i\,b^4x^4+4\,b^3x^3\log\left(-\frac{((d-i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right)+6i\,b^2x^2\text{Li}_2\left(-(-i\,d-1)e^{(2i\,bx+2i\,a)}\right)-2i\,a^4+4\,a^3\log}{}$$

input
```
integrate(x^2*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/24*(2*I*b^4*x^4 + 4*b^3*x^3*log(-((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2
*I*b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(-(-I*d - 1)*e^(2*I*b*x + 2*I*a)) -
2*I*a^4 + 4*a^3*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*b*x*pol
ylog(3, (I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((-I*d - 1)*
e^(2*I*b*x + 2*I*a) + 1) - 3*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/
b^3
```

3.338.6 Sympy [F]

$$\int x^2 \text{arctanh}(1 + id + d\cot(a + bx))\,dx = \int x^2 \operatorname{atanh}\left(d\cot\left(a + bx\right) + id + 1\right)dx$$

input
```
integrate(x**2*atanh(1+I*d+d*cot(b*x+a)),x)
```

output
```
Integral(x**2*atanh(d*cot(a + b*x) + I*d + 1), x)
```

3.338.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(119) = 238.

Time = 0.22 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.05

$$\int x^2 \text{arctanh}(1 + id + d\cot(a + bx))\,dx$$
$$= \frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\text{artanh}(d\cot(bx+a)+i\,d+1)}{b^2} - \frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(4i\,(bx+a)^3-9i\,(bx+a)^2 \cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(x^2*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctanh(d*cot(b
*x + a) + I*d + 1)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*
x + a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2)
*arctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) - cos(2
*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog((
I*d + 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x
+ a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2
+ 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3
, (I*d + 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2
*I*a)))/b^2)/b
```

3.338.8 Giac [F]

$$\int x^2 \operatorname{arctanh}(1 + id + d\cot(a + bx))\,dx = \int x^2 \operatorname{artanh}\left(d\cot\left(bx + a\right) + i\,d + 1\right)\,dx$$

input
```
integrate(x^2*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(d*cot(b*x + a) + I*d + 1), x)
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{arctanh}(1 + id + d\cot(a + bx))\,dx = \int x^2 \operatorname{atanh}(d\cot(a + b\,x) + 1 + d\,1\mathrm{i})\,dx$$

input
```
int(x^2*atanh(d*1i + d*cot(a + b*x) + 1),x)
```

output
```
int(x^2*atanh(d*1i + d*cot(a + b*x) + 1), x)
```

3.339 $\int x\text{arctanh}(1+id+d\cot(a+bx))\,dx$

3.339.1 Optimal result

Integrand size = 18, antiderivative size = 132

$$\int x\text{arctanh}(1+id+d\cot(a+bx))\,dx = \frac{1}{6}ibx^3+\frac{1}{2}x^2\text{arctanh}(1+id+d\cot(a+bx)) - \frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right) + \frac{ix\,\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b} - \frac{\text{PolyLog}\left(3,(1+id)e^{2ia+2ibx}\right)}{8b^2}$$

output
```
1/6*I*b*x^3+1/2*x^2*arctanh(1+I*d+d*cot(b*x+a))-1/4*x^2*ln(1-(1+I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylog
(3,(1+I*d)*exp(2*I*a+2*I*b*x))/b^2
```

3.339.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.90

$$\int x\text{arctanh}(1+id+d\cot(a+bx))\,dx = \frac{1}{2}x^2\text{arctanh}(1+id+d\cot(a+bx)) - \frac{2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{-i+d}\right)+2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{8b^2}$$

input
```
Integrate[x*ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

output
```
(x^2*ArcTanh[1 + I*d + d*Cot[a + b*x]])/2 - (2*b^2*x^2*Log[1 + I/((-I + d)
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/((-I + d)*E^((2*I)*(a +
 b*x)))] + PolyLog[3, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

3.339.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6819, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(d\cot(a+bx)+id+1)\,dx$$

$$\Big\downarrow \text{ 6819}$$

$$\frac{1}{2}ib\int \frac{x^2}{1-(id+1)e^{2ia+2ibx}}dx+\frac{1}{2}x^2\text{arctanh}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1+id)\int \frac{e^{2ia+2ibx}x^2}{1-(id+1)e^{2ia+2ibx}}dx\right)+\frac{1}{2}x^2\text{arctanh}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1+id)\left(\frac{\int x\log\left(1-(id+1)e^{2ia+2ibx}\right)dx}{b(-d+i)}-\frac{x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+\frac{1}{2}x^2\text{arctanh}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1+id)\left(\frac{\frac{ix\,\text{PolyLog}(2,(id+1)e^{2ia+2ibx})}{2b}-\frac{i\int \text{PolyLog}(2,(id+1)e^{2ia+2ibx})dx}{2b}}{b(-d+i)}-\frac{x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)$$
$$\frac{1}{2}x^2\text{arctanh}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1+id)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,(id+1)e^{2ia+2ibx}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,(id+1)e^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}}{b(-d+i)}-\frac{x^2\log\left(1-(1+id)\right.}{2b(-d+i}\right.\right.$$

$$\frac{1}{2}x^2\text{arctanh}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x^2\text{arctanh}(d\cot(a+bx)+id+1)+$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1+id)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,(id+1)e^{2ia+2ibx}\right)}{2b}-\frac{\text{PolyLog}\left(3,(id+1)e^{2ia+2ibx}\right)}{4b^2}}{b(-d+i)}-\frac{x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)$$

input
```
Int[x*ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

output
```
(x^2*ArcTanh[1 + I*d + d*Cot[a + b*x]])/2 + (I/2)*b*(x^3/3 + (1 + I*d)*(-1
/2*(x^2*Log[1 - (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I - d)) + (((I/2)*
x*PolyLog[2, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/b - PolyLog[3, (1 + I*d)*
E^((2*I)*a + (2*I)*b*x)]/(4*b^2))/(b*(I - d))))
```

3.339.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6819
```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.339.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.42 (sec) , antiderivative size = 2289, normalized size of antiderivative = 17.34

method	result	size
risch	Expression too large to display	2289

input
```
int(x*arctanh(1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/4*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x^2+1/2/b^2*a/(I-d)*dilog(1-I*e
xp(I*(b*x+a))*(I*(I-d))^(1/2))+1/2/b^2*a/(I-d)*dilog(1+I*exp(I*(b*x+a))*(I
*(I-d))^(1/2))+1/8/b^2*d/(I-d)*polylog(3,-I*(I-d)*exp(2*I*(b*x+a)))-1/4/b/
(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*x-1/4/b^2/(I-d)*polylog(2,-I*(I
-d)*exp(2*I*(b*x+a)))*a-1/8*I/b^2/(I-d)*polylog(3,-I*(I-d)*exp(2*I*(b*x+a)
))-1/2/b^2*a^2*d/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/4/b^2*a^2*
d/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d-I)-1/4*I/b^2/(I-d)*ln(1+I
*(I-d)*exp(2*I*(b*x+a)))*a^2+1/2*I/b^2*a^2/(I-d)*ln(1-I*exp(I*(b*x+a))*(I*
(I-d))^(1/2))+1/2*I/b^2*a^2/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1
/4*I/b^2*a^2/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d-I)+1/6*I*b*x^3
+1/4/b^2*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^2-1/2/b^2*a^2*d/(I-d)*ln
(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/2/b*a*d/(I-d)*ln(1-I*exp(I*(b*x+a))
*(I*(I-d))^(1/2))*x-1/2/b*a*d/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))
*x+1/2/b*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a*x+1/2*I/b^2*a*d/(I-d)*di
log(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/2*I/b^2*a*d/(I-d)*dilog(1+I*exp(
I*(b*x+a))*(I*(I-d))^(1/2))+1/2*I/b*a/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d)
)^(1/2))*x-1/4*I/b*d/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*x-1/4*I/b^
2*d/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*a-1/2*I/b/(I-d)*ln(1+I*(I-d
)*exp(2*I*(b*x+a)))*a*x+1/2*I/b*a/(I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1
/2))*x-1/8*(-I*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(...
```

3.339.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.19

$$\int x\text{arctanh}(1+id+d\cot(a+bx))\,dx = \frac{4i\,b^3x^3+6\,b^2x^2\log\left(-\frac{((d-i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right)+4i\,a^3+6i\,bx\text{Li}_2\left(-(-i\,d-1)e^{(2i\,bx+2i\,a)}\right)-6\,a^2\log\left(\right.}{24\,b^2}$$

input
```
integrate(x*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/24*(4*I*b^3*x^3 + 6*b^2*x^2*log(-((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2
*I*b*x - 2*I*a)/d) + 4*I*a^3 + 6*I*b*x*dilog(-(-I*d - 1)*e^(2*I*b*x + 2*I*
a)) - 6*a^2*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*(b^2*x^2 -
a^2)*log((-I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) - 3*polylog(3, (I*d + 1)*e^(2
*I*b*x + 2*I*a)))/b^2
```

3.339.6 Sympy [F]

$$\int x\text{arctanh}(1 + id + d\cot(a + bx))\,dx = \int x\,\text{atanh}\left(d\cot\left(a + bx\right) + id + 1\right)dx$$

input
```
integrate(x*atanh(1+I*d+d*cot(b*x+a)),x)
```

output
```
Integral(x*atanh(d*cot(a + b*x) + I*d + 1), x)
```

3.339.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(94) = 188.

Time = 0.21 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.89

$$\int x\text{arctanh}(1 + id + d\cot(a + bx))\,dx$$
$$= \frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\text{artanh}(d\cot(bx+a)+i\,d+1)}{b} - \frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\text{Li}_2\left((i\,d+1)e^{(2i\,bx+2i\,a)}\right)-6\left(i\,(bx+a)^2-2i\,(bx+a)a\right.}{}}{}$$

input
```
integrate(x*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arctanh(d*cot(b*x + a) + I*d + 1)/b
 - (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((I*d + 1)*e^(2*I
*b*x + 2*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(d*cos(2*b*x +
 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1) + 3*(
(b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*
sin(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*po
lylog(3, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

3.339.8 Giac [F]

$$\int x\text{arctanh}(1+id+d\cot(a+bx))\,dx=\int x\,\text{artanh}\left(d\cot\left(bx+a\right)+i\,d+1\right)\,dx$$

input
```
integrate(x*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctanh(d*cot(b*x + a) + I*d + 1), x)
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arctanh}(1+id+d\cot(a+bx))\,dx=\int x\,\text{atanh}(d\cot(a+b\,x)+1+d\,1\text{i})\,dx$$

input
```
int(x*atanh(d*1i + d*cot(a + b*x) + 1),x)
```

output
```
int(x*atanh(d*1i + d*cot(a + b*x) + 1), x)
```

3.340 $\int \operatorname{arctanh}(1+id+d\cot(a+bx))\,dx$

3.340.1 Optimal result

Integrand size = 16, antiderivative size = 93

$$\int \operatorname{arctanh}(1+id+d\cot(a+bx))\,dx = \frac{1}{2}ibx^2 + x\operatorname{arctanh}(1+id+d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right) + \frac{i\operatorname{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}$$

output
```
1/2*I*b*x^2+x*arctanh(1+I*d+d*cot(b*x+a))-1/2*x*ln(1-(1+I*d)*exp(2*I*a+2*I
*b*x))+1/4*I*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b
```

3.340.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 709 vs. 2(93) = 186.

Time = 10.66 (sec) , antiderivative size = 709, normalized size of antiderivative = 7.62

$$\int \operatorname{arctanh}(1+id+d\cot(a+bx))\,dx = x\operatorname{arctanh}(1+id+d\cot(a+bx)) + \frac{x\csc^2(a+bx)\left(2bx\log(2\cos(bx)(\cos(bx)-i\sin(bx)))+i\right.}{(i+\cot(a+bx))(2+id+d\cot(a+bx))\left(2ibx+\log\left(1+\frac{1}{2}\sec(bx)((-2-id)\cos(a)+d\sin(a))(\cos(a\right.\right.}$$

input
```
Integrate[ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

output
```
x*ArcTanh[1 + I*d + d*Cot[a + b*x]] + (x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos[b
*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*(d*Cos[
a + b*x] + (2 + I*d)*Sin[a + b*x]))/(2*(-I + d))]*Log[1 - I*Tan[b*x]] - I*
Log[(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b
*x]))/2]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] -
I*PolyLog[2, (Sec[b*x]*((2 + I*d)*Cos[a] - d*Sin[a])*(Cos[a + b*x] + I*Sin
[a + b*x]))/2] + I*PolyLog[2, ((Cos[a] - I*Sin[a])*((-2 - I*d)*Cos[a] + d*
Sin[a])*(I + Tan[b*x]))/(2*(-I + d))])*(Cos[b*x] - I*Sin[b*x])*(Cos[b*x] +
 I*Sin[b*x]))/((I + Cot[a + b*x])*(2 + I*d + d*Cot[a + b*x])*((2*I)*b*x +
Log[1 + (Sec[b*x]*((-2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a +
 b*x]))/2] - Log[(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I
*d)*Sin[a + b*x]))/2] + ((-2*I + d)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] - Lo
g[1 + I*Tan[b*x]]))/(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x]) + (d*(Log[1
- I*Tan[b*x]] - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/((-I)*d*Cos[a + b*x] +
(-2*I + d)*Sin[a + b*x]) + 2*b*x*Tan[b*x] - I*Log[1 + (Sec[b*x]*((-2 - I*d
)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*Tan[b*x] + I*Log[
(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x])
)/2]*Tan[b*x] - I*Log[1 - I*Tan[b*x]]*Tan[b*x] + I*Log[1 + I*Tan[b*x]]*Tan
[b*x]))
```

3.340.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6811, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arctanh}(d\cot(a+bx)+id+1)\,dx$$

$$\Big\downarrow \text{6811}$$

$$ib\int \frac{x}{1-(id+1)e^{2ia+2ibx}}dx+x\text{arctanh}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2615}$$

$$ib\left(\frac{x^2}{2}+(1+id)\int \frac{e^{2ia+2ibx}x}{1-(id+1)e^{2ia+2ibx}}dx\right)+x\text{arctanh}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2620}$$

$$ib\left(\frac{x^2}{2}+(1+id)\left(\frac{\int \log\left(1-(id+1)e^{2ia+2ibx}\right)dx}{2b(-d+i)}-\frac{x\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+ x\text{arctanh}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2715}$$

$$ib\left(\frac{x^2}{2}+(1+id)\left(-\frac{i\int e^{-2ia-2ibx}\log\left(1-(id+1)e^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2(-d+i)}-\frac{x\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+ x\text{arctanh}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2838}$$

$$x\text{arctanh}(d\cot(a+bx)+id+1)+ ib\left(\frac{x^2}{2}+(1+id)\left(\frac{i\,\text{PolyLog}\left(2,(id+1)e^{2ia+2ibx}\right)}{4b^2(-d+i)}-\frac{x\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)$$

input
```
Int[ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

output
```
x*ArcTanh[1 + I*d + d*Cot[a + b*x]] + I*b*(x^2/2 + (1 + I*d)*(-1/2*(x*Log[
1 - (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I - d)) + ((I/4)*PolyLog[2, (1
 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(b^2*(I - d))))
```

3.340.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6811
```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Cot[a + b*x]], x] + Simp[I*b   Int[x/(c - I*d - c*E^(2*I*a + 2*I
*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, 1]
```

3.340.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(76) = 152.

Time = 1.50 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.30

method	result
derivativedivides	$\frac{i\operatorname{arctanh}(1+id+d\cot(bx+a))d\ln(-id+d\cot(bx+a))}{2} - \frac{i\operatorname{arctanh}(1+id+d\cot(bx+a))d\ln(id+d\cot(bx+a))}{2} - \frac{d^2\left(-\frac{i\left(-\frac{\operatorname{dilog}\left(\frac{i(-id+\cdots}{\cdots}\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$
default	$\frac{i\operatorname{arctanh}(1+id+d\cot(bx+a))d\ln(-id+d\cot(bx+a))}{2} - \frac{i\operatorname{arctanh}(1+id+d\cot(bx+a))d\ln(id+d\cot(bx+a))}{2} - \frac{d^2\left(-\frac{i\left(-\frac{\operatorname{dilog}\left(\frac{i(-id+\cdots}{\cdots}\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$
risch	Expression too large to display

input
```
int(arctanh(1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(1/2*I*arctanh(1+I*d+d*cot(b*x+a))*d*ln(-I*d+d*cot(b*x+a))-1/2*I*arc
tanh(1+I*d+d*cot(b*x+a))*d*ln(I*d+d*cot(b*x+a))-1/2*d^2*(-I/d*(-1/2*dilog(
I*(-I*d+d*cot(b*x+a)-I*(2*I-2*d))/(2*I-2*d))-1/2*ln(-I*d+d*cot(b*x+a))*ln(
I*(-I*d+d*cot(b*x+a)-I*(2*I-2*d))/(2*I-2*d))+1/2*dilog(-1/2*I*(I*d+d*cot(b
*x+a))/d)+1/2*ln(-I*d+d*cot(b*x+a))*ln(-1/2*I*(I*d+d*cot(b*x+a))/d))+I/d*(
1/4*ln(I*d+d*cot(b*x+a))^2-1/2*dilog(1+1/2*I*d+1/2*d*cot(b*x+a))-1/2*ln(I*
d+d*cot(b*x+a))*ln(1+1/2*I*d+1/2*d*cot(b*x+a))))
```

3.340.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.31

$$\int \operatorname{arctanh}(1+id+d\cot(a+bx))\,dx = \frac{2i\,b^2x^2+2\,bx\log\left(-\frac{((d-i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right)-2i\,a^2-2\,(bx+a)\log\left((-i\,d-1)e^{(2i\,bx+2i\,a)}+1\right)+2\,a}{4\,b}$$

input
```
integrate(arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*I*b^2*x^2 + 2*b*x*log(-((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*
x - 2*I*a)/d) - 2*I*a^2 - 2*(b*x + a)*log((-I*d - 1)*e^(2*I*b*x + 2*I*a) +
 1) + 2*a*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) + I*dilog(-(-I*d
- 1)*e^(2*I*b*x + 2*I*a)))/b
```

3.340.6 Sympy [F]

$$\int \operatorname{arctanh}(1+id+d\cot(a+bx))\,dx = \int \operatorname{atanh}\left(d\cot\left(a+bx\right)+id+1\right)dx$$

input
```
integrate(atanh(1+I*d+d*cot(b*x+a)),x)
```

output
```
Integral(atanh(d*cot(a + b*x) + I*d + 1), x)
```

3.340.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(66) = 132$.

Time = 0.28 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.10

$$\int \operatorname{arctanh}(1+id+d\cot(a+bx))\,dx =$$

$$-\frac{4\,(bx+a)d\left(\frac{\log((i\,d+2)\tan(bx+a)+d)}{d}-\frac{\log(i\,\tan(bx+a)+1)}{d}\right)-d\left(\frac{2i\left(\log((i\,d+2)\tan(bx+a)+d)\log\left(\frac{(d-2i)\tan(bx+a)-i\,d}{2i\,d+2}+1\right)+\mathrm{I}\right.}{d}\right.}{}$$

input
```
integrate(arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*(4*(b*x + a)*d*(log((I*d + 2)*tan(b*x + a) + d)/d - log(I*tan(b*x + a
) + 1)/d) - d*(2*I*(log((I*d + 2)*tan(b*x + a) + d)*log(((d - 2*I)*tan(b*x
 + a) - I*d)/(2*I*d + 2) + 1) + dilog(-((d - 2*I)*tan(b*x + a) - I*d)/(2*I
*d + 2)))/d + 2*I*(log(1/2*(d - 2*I)*tan(b*x + a) - 1/2*I*d)*log(I*tan(b*x
 + a) + 1) + dilog(-1/2*(d - 2*I)*tan(b*x + a) + 1/2*I*d + 1))/d - (2*I*lo
g((I*d + 2)*tan(b*x + a) + d)*log(I*tan(b*x + a) + 1) - I*log(I*tan(b*x +
a) + 1)^2)/d - 2*I*(log(I*tan(b*x + a) + 1)*log(-1/2*I*tan(b*x + a) + 1/2)
 + dilog(1/2*I*tan(b*x + a) + 1/2))/d) - 8*(b*x + a)*arctanh(I*d + d/tan(b
*x + a) + 1))/b
```

3.340.8 Giac [F]

$$\int \operatorname{arctanh}(1+id+d\cot(a+bx))\,dx = \int \operatorname{artanh}\left(d\cot\left(bx+a\right)+i\,d+1\right)\,dx$$

input
```
integrate(arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctanh(d*cot(b*x + a) + I*d + 1), x)
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}(1 + id + d\cot(a + bx))\,dx = \int \operatorname{atanh}(d\cot(a + b\,x) + 1 + d\,1\mathrm{i})\;dx$$

input
```
int(atanh(d*1i + d*cot(a + b*x) + 1),x)
```

output
```
int(atanh(d*1i + d*cot(a + b*x) + 1), x)
```

3.341 $\int \frac{\operatorname{arctanh}(1+id+d\cot(a+bx))}{x}\,dx$

3.341.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\operatorname{arctanh}(1+id+d\cot(a+bx))}{x}\,dx=\operatorname{Int}\left(\frac{\operatorname{arctanh}(1+id+d\cot(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arctanh(1+I*d+d*cot(b*x+a))/x,x)
```

3.341.2 Mathematica [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{arctanh}(1+id+d\cot(a+bx))}{x}\,dx=\int \frac{\operatorname{arctanh}(1+id+d\cot(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTanh[1 + I*d + d*Cot[a + b*x]]/x,x]
```

output
```
Integrate[ArcTanh[1 + I*d + d*Cot[a + b*x]]/x, x]
```

3.341.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(d\cot(a+bx)+id+1)}{x}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\operatorname{arctanh}(d\cot(a+bx)+id+1)}{x}dx$$

input
```
Int[ArcTanh[1 + I*d + d*Cot[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.341.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.341.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arctanh}\left(1+id+d\cot\left(bx+a\right)\right)}{x}dx$$

input
```
int(arctanh(1+I*d+d*cot(b*x+a))/x,x)
```

output
```
int(arctanh(1+I*d+d*cot(b*x+a))/x,x)
```

3.341.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{\operatorname{arctanh}(1 + id + d\cot(a + bx))}{x}\,dx = \int \frac{\operatorname{artanh}\left(d\cot\left(bx + a\right) + i\,d + 1\right)}{x}\,dx$$

input
```
integrate(arctanh(1+I*d+d*cot(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*log(-((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d
)/x, x)
```

3.341.6 Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{arctanh}(1 + id + d\cot(a + bx))}{x}\,dx = \int \frac{\operatorname{atanh}\left(d\cot\left(a + bx\right) + id + 1\right)}{x}\,dx$$

input
```
integrate(atanh(1+I*d+d*cot(b*x+a))/x,x)
```

output
```
Integral(atanh(d*cot(a + b*x) + I*d + 1)/x, x)
```

3.341.7 Maxima [N/A]

Not integrable

Time = 4.75 (sec) , antiderivative size = 143, normalized size of antiderivative = 7.15

$$\int \frac{\operatorname{arctanh}(1 + id + d\cot(a + bx))}{x}\,dx = \int \frac{\operatorname{artanh}\left(d\cot\left(bx + a\right) + i\,d + 1\right)}{x}\,dx$$

input
```
integrate(arctanh(1+I*d+d*cot(b*x+a))/x,x, algorithm="maxima")
```

output
```
-I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(-d))*log(x) + 1/2*I*integrate(arctan2(
d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2
*a) - 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1
)*sin(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1)/x, x
)
```

3.341.8 Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(1+id+d\cot(a+bx))}{x}\,dx=\int \frac{\operatorname{artanh}\left(d\cot\left(bx+a\right)+i\,d+1\right)}{x}\,dx$$

input
```
integrate(arctanh(1+I*d+d*cot(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctanh(d*cot(b*x + a) + I*d + 1)/x, x)
```

3.341.9 Mupad [N/A]

Not integrable

Time = 4.94 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\operatorname{arctanh}(1+id+d\cot(a+bx))}{x}\,dx=\int \frac{\operatorname{atanh}(d\cot(a+b\,x)+1+d\,1\mathrm{i})}{x}\,dx$$

input
```
int(atanh(d*1i + d*cot(a + b*x) + 1)/x,x)
```

output
```
int(atanh(d*1i + d*cot(a + b*x) + 1)/x, x)
```

3.342 $\int x^2 \textbf{arctanh}(1 - id - d\cot(a + bx))\,dx$

3.342.1 Optimal result

Integrand size = 21, antiderivative size = 169

$$\begin{aligned}\int x^2 \operatorname{arctanh}(1-id-d\cot(a+bx))\,dx =& \frac{1}{12}ibx^4+\frac{1}{3}x^3\operatorname{arctanh}(1-id-d\cot(a+bx))\\ &-\frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+2ibx}\right)\\ &+\frac{ix^2\operatorname{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}\\ &-\frac{x\operatorname{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{4b^2}\\ &-\frac{i\operatorname{PolyLog}\left(4,(1-id)e^{2ia+2ibx}\right)}{8b^3}\end{aligned}$$

output
```
1/12*I*b*x^4-1/3*x^3*arctanh(-1+I*d+d*cot(b*x+a))-1/6*x^3*ln(1-(1-I*d)*exp
(2*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*p
olylog(3,(1-I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,(1-I*d)*exp(2*I*a
+2*I*b*x))/b^3
```

3.342.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.92

$$\int x^2\text{arctanh}(1-id-d\cot(a+bx))\,dx=\frac{1}{3}x^3\text{arctanh}(1-id-d\cot(a+bx))$$
$$-\frac{4b^3x^3\log\left(1+\frac{e^{-2i(a+bx)}}{-1+id}\right)+6ib^2x^2\,\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{i+d}\right)+6bx\,\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{i+d}\right)-3i\,\text{PolyLog}\left(\right.}{24b^3}$$

input
```
Integrate[x^2*ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

output
```
(x^3*ArcTanh[1 - I*d - d*Cot[a + b*x]])/3 - (4*b^3*x^3*Log[1 + 1/((-1 + I*
d)*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/((I + d)*E^((2*I)*(a
 + b*x)))] + 6*b*x*PolyLog[3, I/((I + d)*E^((2*I)*(a + b*x)))] - (3*I)*Pol
yLog[4, I/((I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.342.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6819, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arctanh}(d(-\cot(a+bx))-id+1)\,dx$$
$$\Big\downarrow \text{6819}$$
$$\frac{1}{3}ib\int\frac{x^3}{1-(1-id)e^{2ia+2ibx}}dx+\frac{1}{3}x^3\text{arctanh}(d(-\cot(a+bx))-id+1)$$
$$\Big\downarrow \text{2615}$$
$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\int\frac{e^{2ia+2ibx}x^3}{1-(1-id)e^{2ia+2ibx}}dx\right)+\frac{1}{3}x^3\text{arctanh}(d(-\cot(a+bx))-id+1)$$
$$\Big\downarrow \text{2620}$$
$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\left(\frac{3\int x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)dx}{2b(d+i)}-\frac{x^3\log\left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+$$
$$\frac{1}{3}x^3\text{arctanh}(d(-\cot(a+bx))-id+1)$$

$\big\downarrow$ 3011

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)dx}{b}\right)}{2b(d+i)}-\frac{x^3\log\left(1-(1-id)e^{2ia+2ibx}\right.}{2b(d+i)}\right.\right.$$

$$\frac{1}{3}x^3\operatorname{arctanh}(d(-\cot(a+bx))-id+1)$$

$\big\downarrow$ 7163

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2b(d+i)}-\frac{x}{\ }\right.\right.$$

$$\frac{1}{3}x^3\operatorname{arctanh}(d(-\cot(a+bx))-id+1)$$

$\big\downarrow$ 2720

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,(1-id)e^{2i\ldots}\right.}{2b}\right.}{b}\right.}{2b(d+i)}\right.\right.$$

$$\frac{1}{3}x^3\operatorname{arctanh}(d(-\cot(a+bx))-id+1)$$

$\big\downarrow$ 7143

$$\frac{1}{3}x^3\operatorname{arctanh}(d(-\cot(a+bx))-id+1)+$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,(1-id)e^{2ia+2ibx}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2b(d+i)}-\frac{x^3\log\ldots}{\ }\right.\right.$$

input
```
Int[x^2*ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

3.342. $\int x^2\operatorname{arctanh}(1-id-d\cot(a+bx))\,dx$

output
```
(x^3*ArcTanh[1 - I*d - d*Cot[a + b*x]])/3 + (I/3)*b*(x^4/4 + (1 - I*d)*(-1
/2*(x^3*Log[1 - (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I + d)) + (3*(((I/
2)*x^2*PolyLog[2, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/b - (I*(((-1/2*I)*x*
PolyLog[3, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/b + PolyLog[4, (1 - I*d)*E^
((2*I)*a + (2*I)*b*x)]/(4*b^2)))/b))/(2*b*(I + d))))
```

3.342.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6819
```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.342.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.80 (sec) , antiderivative size = 2277, normalized size of antiderivative = 13.47

method	result	size
risch	Expression too large to display	2277

input
```
int(-x^2*arctanh(-1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I+d)*polylo
g(2,-I*(I+d)*exp(2*I*(b*x+a)))*a^2-1/2/b^3*a^2/(I+d)*dilog(1+I*exp(I*(b*x+
a))*(I*(I+d))^(1/2))-1/2/b^3*a^2/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^
(1/2))-1/6*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x^3+1/6*x^3*ln(I*exp(2*I
*(b*x+a))+exp(2*I*(b*x+a))*d-I)-1/2/b^2*d*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*
(I*(I+d))^(1/2))*x+1/2/b^2*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a^2*x-1/
2/b^2*d*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x+1/2*I/b^2/(I+d)
*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a^2*x+1/2*I/b^3*d*a^2/(I+d)*dilog(1+I*exp(
I*(b*x+a))*(I*(I+d))^(1/2))+1/12*I*b*x^4+1/3/b^3*d/(I+d)*ln(1+I*(I+d)*exp(
2*I*(b*x+a)))*a^3-1/4/b^2*d/(I+d)*polylog(3,-I*(I+d)*exp(2*I*(b*x+a)))*x-1
/2/b^3*d*a^3/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/2/b^3*d*a^3/(I
+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/6/b^3*a^3*d/(I+d)*ln(I*exp(2*
I*(b*x+a))+exp(2*I*(b*x+a))*d-I)-1/8*I/b^3*d/(I+d)*polylog(4,-I*(I+d)*exp(
2*I*(b*x+a)))-1/2*I/b^3*a^3/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1
/3*I/b^3/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/2*I/b^3*a^3/(I+d)*ln(1
-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/6*I/b^3*a^3/(I+d)*ln(I*exp(2*I*(b*x+a
))+exp(2*I*(b*x+a))*d-I)-1/4*I/b^2/(I+d)*polylog(3,-I*(I+d)*exp(2*I*(b*x+a
)))*x-1/3*x^3*ln(exp(I*(b*x+a)))+1/2*I/b^3*d*a^2/(I+d)*dilog(1-I*exp(I*(b*
x+a))*(I*(I+d))^(1/2))-1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))
^(1/2))*x-1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x-...
```

3.342.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.07

$$\int x^2 \text{arctanh}(1 - id - d\cot(a + bx))\, dx = \frac{2i\, b^4 x^4 - 4\, b^3 x^3 \log\left(-\frac{d e^{(2i\, bx + 2i\, a)}}{(d+i)e^{(2i\, bx+2i\, a)} - i}\right) + 6i\, b^2 x^2 \text{Li}_2\left(-(i\, d - 1)e^{(2i\, bx + 2i\, a)}\right) - 2i\, a^4 + 4\, a^3 \log\left(\frac{(d+i)e^{(2i\, bx+2i}}{d+i}\right.}{}$$

input
```
integrate(-x^2*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/24*(2*I*b^4*x^4 - 4*b^3*x^3*log(-d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b
*x + 2*I*a) - I)) + 6*I*b^2*x^2*dilog(-(I*d - 1)*e^(2*I*b*x + 2*I*a)) - 2*
I*a^4 + 4*a^3*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) - 6*b*x*polyl
og(3, (-I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((I*d - 1)*e^
(2*I*b*x + 2*I*a) + 1) - 3*I*polylog(4, (-I*d + 1)*e^(2*I*b*x + 2*I*a)))/b
^3
```

3.342.6 Sympy [F]

$$\int x^2\text{arctanh}(1-id-d\cot(a+bx))\,dx=-\int x^2\,\text{atanh}\,(d\cot\,(a+bx)+id-1)\,dx$$

input
```
integrate(-x**2*atanh(-1+I*d+d*cot(b*x+a)),x)
```

output
```
-Integral(x**2*atanh(d*cot(a + b*x) + I*d - 1), x)
```

3.342.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(119) = 238.

Time = 0.23 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.04

$$\int x^2\text{arctanh}(1-id-d\cot(a+bx))\,dx=$$

$$-\frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\text{artanh}(d\cot(bx+a)+i\,d-1)}{b^2}+\frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(4i\,(bx+a)^3-9i\,(bx+\right.}{}}{}$$

input
```
integrate(-x^2*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctanh(d*cot(
b*x + a) + I*d - 1)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b
*x + a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2
)*arctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - co
s(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilo
g((-I*d + 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(
b*x + a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a
)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polyl
og(3, (-I*d + 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d + 1)*e^(2*I*b
*x + 2*I*a)))/b^2)/b
```

3.342.8 Giac [F]

$$\int x^2\text{arctanh}(1-id-d\cot(a+bx))\,dx=\int -x^2\,\text{artanh}\left(d\cot\left(bx+a\right)+i\,d-1\right)\,dx$$

input
```
integrate(-x^2*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-x^2*arctanh(d*cot(b*x + a) + I*d - 1), x)
```

3.342.9 Mupad [F(-1)]

Timed out.

$$\int x^2\text{arctanh}(1-id-d\cot(a+bx))\,dx=\int -x^2\,\text{atanh}(d\cot(a+b\,x)-1+d\,1\text{i})\,dx$$

input
```
int(-x^2*atanh(d*1i + d*cot(a + b*x) - 1),x)
```

output
```
int(-x^2*atanh(d*1i + d*cot(a + b*x) - 1), x)
```

3.343 $\int x\text{arctanh}(1 - id - d\cot(a + bx))\,dx$

3.343.1 Optimal result

Integrand size = 19, antiderivative size = 133

$$\int x\text{arctanh}(1 - id - d\cot(a + bx))\,dx = \frac{1}{6}ibx^3 + \frac{1}{2}x^2\text{arctanh}(1 - id - d\cot(a + bx)) - \frac{1}{4}x^2\log\left(1 - (1 - id)e^{2ia+2ibx}\right) + \frac{ix\,\text{PolyLog}\left(2, (1 - id)e^{2ia+2ibx}\right)}{4b} - \frac{\text{PolyLog}\left(3, (1 - id)e^{2ia+2ibx}\right)}{8b^2}$$

output
```
1/6*I*b*x^3-1/2*x^2*arctanh(-1+I*d+d*cot(b*x+a))-1/4*x^2*ln(1-(1-I*d)*exp(
2*I*a+2*I*b*x))+1/4*I*x*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylo
g(3,(1-I*d)*exp(2*I*a+2*I*b*x))/b^2
```

3.343.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\int x\text{arctanh}(1 - id - d\cot(a + bx))\,dx = \frac{1}{2}x^2\text{arctanh}(1 - id - d\cot(a + bx)) - \frac{2b^2x^2\log\left(1 + \frac{e^{-2i(a+bx)}}{-1+id}\right) + 2ibx\,\text{PolyLog}\left(2, \frac{ie^{-2i(a+bx)}}{i+d}\right) + \text{PolyLog}\left(3, \frac{ie^{-2i(a+bx)}}{i+d}\right)}{8b^2}$$

input
```
Integrate[x*ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

output
```
(x^2*ArcTanh[1 - I*d - d*Cot[a + b*x]])/2 - (2*b^2*x^2*Log[1 + 1/((-1 + I*
d)*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/((I + d)*E^((2*I)*(a + b
*x)))] + PolyLog[3, I/((I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

3.343.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6819, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(d(-\cot(a+bx))-id+1)\,dx$$

$$\Big\downarrow \text{6819}$$

$$\frac{1}{2}ib\int \frac{x^2}{1-(1-id)e^{2ia+2ibx}}dx+\frac{1}{2}x^2\text{arctanh}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1-id)\int \frac{e^{2ia+2ibx}x^2}{1-(1-id)e^{2ia+2ibx}}dx\right)+\frac{1}{2}x^2\text{arctanh}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1-id)\left(\frac{\int x\log\left(1-(1-id)e^{2ia+2ibx}\right)dx}{b(d+i)}-\frac{x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+\frac{1}{2}x^2\text{arctanh}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1-id)\left(\frac{\frac{ix\,\text{PolyLog}(2,(1-id)e^{2ia+2ibx})}{2b}-\frac{i\int \text{PolyLog}(2,(1-id)e^{2ia+2ibx})dx}{2b}}{b(d+i)}-\frac{x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+\frac{1}{2}x^2\text{arctanh}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1-id)\left(\frac{\frac{ix\,\text{PolyLog}(2,(1-id)e^{2ia+2ibx})}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}(2,(1-id)e^{2ia+2ibx})de^{2ia+2ibx}}{4b^2}}{b(d+i)}-\frac{x^2\log\left(1-(1-id)\right.}{2b(d+i)}\right.\right.$$

$$\frac{1}{2}x^2\text{arctanh}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x^2\text{arctanh}(d(-\cot(a+bx))-id+1)+$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1-id)\left(\frac{\frac{ix\,\text{PolyLog}(2,(1-id)e^{2ia+2ibx})}{2b}-\frac{\text{PolyLog}(3,(1-id)e^{2ia+2ibx})}{4b^2}}{b(d+i)}-\frac{x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)$$

input
```
Int[x*ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

output
```
(x^2*ArcTanh[1 - I*d - d*Cot[a + b*x]])/2 + (I/2)*b*(x^3/3 + (1 - I*d)*(-1
/2*(x^2*Log[1 - (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I + d)) + (((I/2)*
x*PolyLog[2, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/b - PolyLog[3, (1 - I*d)*
E^((2*I)*a + (2*I)*b*x)]/(4*b^2))/(b*(I + d))))
```

3.343.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6819
```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTanh[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.343.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.27 (sec) , antiderivative size = 2187, normalized size of antiderivative = 16.44

method	result	size
risch	Expression too large to display	2187

input
```
int(-x*arctanh(-1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x-1/4/b^2/(I+d)*polylog(
2,-I*(I+d)*exp(2*I*(b*x+a)))*a-1/8/b^2*d/(I+d)*polylog(3,-I*(I+d)*exp(2*I*
(b*x+a)))+1/2/b^2*a/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/6*I*
b*x^3-1/4/b^2*a^2*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)-1/4/
b^2*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a^2+1/2/b^2*a^2*d/(I+d)*ln(1+I*
exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a))*
(I*(I+d))^(1/2))+1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2)
)+1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/2*I/b^2*a*d
/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/2*I/b^2*a*d/(I+d)*dilog
(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/2*I/b*a/(I+d)*ln(1-I*exp(I*(b*x+a))
*(I*(I+d))^(1/2))*x+1/4*I/b*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x
+1/4*I/b^2*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*a+1/2*I/b*a/(I+d)*
ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x-1/2*I/b/(I+d)*ln(1+I*(I+d)*exp(2*
I*(b*x+a)))*a*x-1/8*(-I*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))
*csgn(1/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a))*d)+I*Pi*csgn(I*d/(exp(2*I*(b
*x+a))-1)*exp(2*I*(b*x+a)))*csgn(1/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a))*d
)^2+I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x
+a)))^2-I*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+
a))-1))^2+2*I*Pi-I*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*cs
gn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2+...
```

3.343.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.18

$$\int x\operatorname{arctanh}(1-id-d\cot(a+bx))\,dx = \frac{4i\,b^3x^3-6\,b^2x^2\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right)+4i\,a^3+6i\,bx\mathrm{Li}_2\left(-(i\,d-1)e^{(2i\,bx+2i\,a)}\right)-6\,a^2\log\left(\frac{(d+i)e^{(2i\,bx+2i\,a)}\ldots}{d+i}\right.\ldots}{24\,b^2}$$

input
```
integrate(-x*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/24*(4*I*b^3*x^3 - 6*b^2*x^2*log(-d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b
*x + 2*I*a) - I)) + 4*I*a^3 + 6*I*b*x*dilog(-(I*d - 1)*e^(2*I*b*x + 2*I*a)
) - 6*a^2*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) - 6*(b^2*x^2 - a^
2)*log((I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) - 3*polylog(3, (-I*d + 1)*e^(2*I
*b*x + 2*I*a)))/b^2
```

3.343.6 Sympy [F]

$$\int x\text{arctanh}(1-id-d\cot(a+bx))\,dx=-\int x\,\text{atanh}\,(d\cot\,(a+bx)+id-1)\,dx$$

input
```
integrate(-x*atanh(-1+I*d+d*cot(b*x+a)),x)
```

output
```
-Integral(x*atanh(d*cot(a + b*x) + I*d - 1), x)
```

3.343.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(94) = 188$.

Time = 0.22 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.88

$$\int x\text{arctanh}(1-id-d\cot(a+bx))\,dx=$$
$$-\frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\text{artanh}(d\cot(bx+a)+i\,d-1)}{b}+\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\text{Li}_2\left((-i\,d+1)e^{(2i\,bx+2i\,a)}\right)-6\left(i\,(bx+a)^2-2i\,(bx}{}}{}$$

input
```
integrate(-x*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arctanh(d*cot(b*x + a) + I*d - 1)/
b + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((-I*d + 1)*e^(2
*I*b*x + 2*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(-d*cos(2*b*
x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1) +
 3*((b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 +
 1)*sin(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) +
3*polylog(3, (-I*d + 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

3.343.8 Giac [F]

$$\int x\operatorname{arctanh}(1-id-d\cot(a+bx))\,dx=\int -x\operatorname{artanh}\left(d\cot\left(bx+a\right)+i\,d-1\right)\,dx$$

input
```
integrate(-x*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-x*arctanh(d*cot(b*x + a) + I*d - 1), x)
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int x\operatorname{arctanh}(1-id-d\cot(a+bx))\,dx=\int -x\operatorname{atanh}(d\cot(a+b\,x)-1+d\,1\mathrm{i})\,dx$$

input
```
int(-x*atanh(d*1i + d*cot(a + b*x) - 1),x)
```

output
```
int(-x*atanh(d*1i + d*cot(a + b*x) - 1), x)
```

3.344 $\int \operatorname{arctanh}(1 - id - d\cot(a + bx))\, dx$

3.344.1 Optimal result

Integrand size = 17, antiderivative size = 94

$$\int \operatorname{arctanh}(1 - id - d\cot(a + bx))\, dx = \frac{1}{2} ibx^2 + x\operatorname{arctanh}(1 - id - d\cot(a + bx)) - \frac{1}{2} x \log\left(1 - (1 - id)e^{2ia+2ibx}\right) + \frac{i\,\operatorname{PolyLog}\left(2, (1 - id)e^{2ia+2ibx}\right)}{4b}$$

output
```
1/2*I*b*x^2-x*arctanh(-1+I*d+d*cot(b*x+a))-1/2*x*ln(1-(1-I*d)*exp(2*I*a+2*
I*b*x))+1/4*I*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b
```

3.344.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 605 vs. 2(94) = 188.

Time = 9.50 (sec) , antiderivative size = 605, normalized size of antiderivative = 6.44

$$\int \operatorname{arctanh}(1 - id - d\cot(a + bx))\, dx = x\operatorname{arctanh}(1 - id - d\cot(a + bx)) + \frac{x\csc^2(a + bx)\left(2bx\log(2\cos(bx)(\cos(bx) - i\sin(bx))) + i\log\left(\frac{\sec(bx)(\cos(a) - i\sin(a))(d\cos(a+bx) + i(2i+d)\sin(a+b}{2(i+d)}\right.\right.}{(i + \cot(a + bx))(-2 + id + d\cot(a +}$$

input
```
Integrate[ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

output
```
x*ArcTanh[1 - I*d - d*Cot[a + b*x]] + (x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos[b
*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*(d*Cos[
a + b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*(I + d))]*Log[1 - I*Tan[b*x]] - I
*Log[(I*Sec[b*x]*(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*Cos[a] -
(2*I)*Sin[a])]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*
x]] - I*PolyLog[2, (Sec[b*x]*((2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] +
 I*Sin[a + b*x]))/2] + I*PolyLog[2, ((Cos[a] - I*Sin[a])*((2 - I*d)*Cos[a]
 + d*Sin[a])*(I + Tan[b*x]))/(2*(I + d))])*(Cos[b*x] - I*Sin[b*x])*(Cos[b*
x] + I*Sin[b*x]))/((I + Cot[a + b*x])*(-2 + I*d + d*Cot[a + b*x])*(-((Log[
1 - I*Tan[b*x]]*Sec[b*x]*((2*I + d)*Cos[a] + I*d*Sin[a]))/(d*Cos[a + b*x]
+ I*(2*I + d)*Sin[a + b*x])) + (Log[1 + I*Tan[b*x]]*Sec[b*x]*((2*I + d)*Co
s[a] + I*d*Sin[a]))/(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]) + (Log[(I*
Sec[b*x]*(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Si
n[a])]*Sec[b*x]^2)/(1 + I*Tan[b*x]) - 2*b*x*(I + Tan[b*x]) + I*Log[1 - (Se
c[b*x]*((2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*(
I + Tan[b*x])))
```

3.344.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6811, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(d(-\cot(a+bx))-id+1)\,dx$$

$$\Big\downarrow \text{6811}$$

$$ib\int \frac{x}{1-(1-id)e^{2ia+2ibx}}dx+x\operatorname{arctanh}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2615}$$

$$ib\left(\frac{x^2}{2}+(1-id)\int \frac{e^{2ia+2ibx}x}{1-(1-id)e^{2ia+2ibx}}dx\right)+x\operatorname{arctanh}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2620}$$

$$ib\left(\frac{x^2}{2}+(1-id)\left(\frac{\int \log \left(1-(1-id)e^{2ia+2ibx}\right)dx}{2b(d+i)}-\frac{x\log \left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+ x\text{arctanh}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2715}$$

$$ib\left(\frac{x^2}{2}+(1-id)\left(-\frac{i\int e^{-2ia-2ibx}\log \left(1-(1-id)e^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2(d+i)}-\frac{x\log \left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+ x\text{arctanh}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2838}$$

$$x\text{arctanh}(d(-\cot(a+bx))-id+1)+ ib\left(\frac{x^2}{2}+(1-id)\left(\frac{i\,\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b^2(d+i)}-\frac{x\log \left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)$$

input
```
Int[ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

output
```
x*ArcTanh[1 - I*d - d*Cot[a + b*x]] + I*b*(x^2/2 + (1 - I*d)*(-1/2*(x*Log[
1 - (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I + d)) + ((I/4)*PolyLog[2, (1
 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(b^2*(I + d))))
```

3.344.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6811
```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Cot[a + b*x]], x] + Simp[I*b   Int[x/(c - I*d - c*E^(2*I*a + 2*I
*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, 1]
```

3.344.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(77) = 154$.

Time = 1.43 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.53

method	result
derivativedivides	$-\frac{\frac{i \operatorname{arctanh}(-1+id+d\cot(bx+a))d\ln(id-d\cot(bx+a))}{2}-\frac{i \operatorname{arctanh}(-1+id+d\cot(bx+a))d\ln(id+d\cot(bx+a))}{2}-\frac{d^2\left(-\frac{i\left(-\frac{\operatorname{dilog}\left(\frac{i(-}{}\right.}{}\right.}{}\right.}{}}{}$
default	$-\frac{\frac{i \operatorname{arctanh}(-1+id+d\cot(bx+a))d\ln(id-d\cot(bx+a))}{2}-\frac{i \operatorname{arctanh}(-1+id+d\cot(bx+a))d\ln(id+d\cot(bx+a))}{2}-\frac{d^2\left(-\frac{i\left(-\frac{\operatorname{dilog}\left(\frac{i(-}{}\right.}{}\right.}{}\right.}{}}{}$
risch	Expression too large to display

input
```
int(-arctanh(-1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/b/d*(1/2*I*arctanh(-1+I*d+d*cot(b*x+a))*d*ln(I*d-d*cot(b*x+a))-1/2*I*ar
ctanh(-1+I*d+d*cot(b*x+a))*d*ln(I*d+d*cot(b*x+a))-1/2*d^2*(-I/d*(-1/2*dilo
g(1/2*I*(-I*d-d*cot(b*x+a))/d)-1/2*ln(I*d-d*cot(b*x+a))*ln(1/2*I*(-I*d-d*c
ot(b*x+a))/d)+1/2*dilog(I*(I*d-d*cot(b*x+a)-I*(2*I+2*d))/(2*I+2*d))+1/2*ln
(I*d-d*cot(b*x+a))*ln(I*(I*d-d*cot(b*x+a)-I*(2*I+2*d))/(2*I+2*d)))+I/d*(1/
2*(ln(I*d+d*cot(b*x+a))-ln(1/2*I*d+1/2*d*cot(b*x+a)))*ln(1-1/2*I*d-1/2*d*c
ot(b*x+a))-1/2*dilog(1/2*I*d+1/2*d*cot(b*x+a))-1/4*ln(I*d+d*cot(b*x+a))^2)
))
```

3.344.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30

$$\int \operatorname{arctanh}(1 - id - d\cot(a + bx))\,dx = \frac{2i\,b^2x^2 - 2\,bx\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right) - 2i\,a^2 - 2\,(bx+a)\log\left((i\,d-1)e^{(2i\,bx+2i\,a)}+1\right) + 2\,a\log\left(\frac{(d+i)e^{(2i\,b}}{d+}\right.}{4\,b}$$

input
```
integrate(-arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*I*b^2*x^2 - 2*b*x*log(-d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x +
2*I*a) - I)) - 2*I*a^2 - 2*(b*x + a)*log((I*d - 1)*e^(2*I*b*x + 2*I*a) + 1
) + 2*a*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) + I*dilog(-(I*d - 1
)*e^(2*I*b*x + 2*I*a)))/b
```

3.344.6 Sympy [F]

$$\int \operatorname{arctanh}(1 - id - d\cot(a + bx))\,dx = -\int \operatorname{atanh}\left(d\cot\left(a + bx\right) + id - 1\right)dx$$

input
```
integrate(-atanh(-1+I*d+d*cot(b*x+a)),x)
```

output
```
-Integral(atanh(d*cot(a + b*x) + I*d - 1), x)
```

3.344.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(67) = 134.

Time = 0.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.04

$$\int \operatorname{arctanh}(1 - id - d\cot(a + bx))\,dx = -\frac{4\,(bx+a)d\left(\frac{\log((i\,d-2)\tan(bx+a)+d)}{d} - \frac{\log(i\,\tan(bx+a)+1)}{d}\right) + d\left(-\frac{2i\left(\log((i\,d-2)\tan(bx+a)+d)\log\left(\frac{(d+2i)\tan(bx+a)-i\,d}{2i\,d-2}+1\right)\right.}{d}\right.}{}$$

input
```
integrate(-arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*(4*(b*x + a)*d*(log((I*d - 2)*tan(b*x + a) + d)/d - log(I*tan(b*x + a
) + 1)/d) + d*(-2*I*(log((I*d - 2)*tan(b*x + a) + d)*log(((d + 2*I)*tan(b*
x + a) - I*d)/(2*I*d - 2) + 1) + dilog(-((d + 2*I)*tan(b*x + a) - I*d)/(2*
I*d - 2)))/d - 2*I*(log(-1/2*(d + 2*I)*tan(b*x + a) + 1/2*I*d)*log(I*tan(b
*x + a) + 1) + dilog(1/2*(d + 2*I)*tan(b*x + a) - 1/2*I*d + 1))/d + (2*I*l
og((I*d - 2)*tan(b*x + a) + d)*log(I*tan(b*x + a) + 1) - I*log(I*tan(b*x +
 a) + 1)^2)/d + 2*I*(log(I*tan(b*x + a) + 1)*log(-1/2*I*tan(b*x + a) + 1/2
) + dilog(1/2*I*tan(b*x + a) + 1/2))/d) + 8*(b*x + a)*arctanh(I*d + d/tan(
b*x + a) - 1))/b
```

3.344.8 Giac [F]

$$\int \operatorname{arctanh}(1-id-d\cot(a+bx))\,dx = \int -\operatorname{artanh}\left(d\cot\left(bx+a\right)+i\,d-1\right)\,dx$$

input
```
integrate(-arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-arctanh(d*cot(b*x + a) + I*d - 1), x)
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}(1-id-d\cot(a+bx))\,dx = \int -\operatorname{atanh}(d\cot(a+b\,x)-1+d\,1\mathrm{i})\,dx$$

input
```
int(-atanh(d*1i + d*cot(a + b*x) - 1),x)
```

output
```
int(-atanh(d*1i + d*cot(a + b*x) - 1), x)
```

3.345 $\int \frac{\operatorname{arctanh}(1-id-d\cot(a+bx))}{x}\,dx$

3.345.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\operatorname{arctanh}(1-id-d\cot(a+bx))}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{arctanh}(1-id-d\cot(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(-arctanh(-1+I*d+d*cot(b*x+a))/x,x)
```

3.345.2 Mathematica [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{arctanh}(1-id-d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{arctanh}(1-id-d\cot(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTanh[1 - I*d - d*Cot[a + b*x]]/x,x]
```

output
```
Integrate[ArcTanh[1 - I*d - d*Cot[a + b*x]]/x, x]
```

3.345.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(d(-\cot(a+bx))-id+1)}{x}\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\text{arctanh}(d(-\cot(a+bx))-id+1)}{x}dx$$

input
```
Int[ArcTanh[1 - I*d - d*Cot[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.345.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.345.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int -\frac{\text{arctanh}\left(-1+id+d\cot\left(bx+a\right)\right)}{x}dx$$

input
```
int(-arctanh(-1+I*d+d*cot(b*x+a))/x,x)
```

output
```
int(-arctanh(-1+I*d+d*cot(b*x+a))/x,x)
```

3.345.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{\operatorname{arctanh}(1-id-d\cot(a+bx))}{x}\,dx = \int -\frac{\operatorname{artanh}\left(d\cot\left(bx+a\right)+i\,d-1\right)}{x}\,dx$$

input
```
integrate(-arctanh(-1+I*d+d*cot(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(-1/2*log(-d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x + 2*I*a) - I)
)/x, x)
```

3.345.6 Sympy [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arctanh}(1-id-d\cot(a+bx))}{x}\,dx = -\int \frac{\operatorname{atanh}\left(d\cot\left(a+bx\right)+id-1\right)}{x}\,dx$$

input
```
integrate(-atanh(-1+I*d+d*cot(b*x+a))/x,x)
```

output
```
-Integral(atanh(d*cot(a + b*x) + I*d - 1)/x, x)
```

3.345.7 Maxima [N/A]

Not integrable

Time = 5.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 6.86

$$\int \frac{\operatorname{arctanh}(1-id-d\cot(a+bx))}{x}\,dx = \int -\frac{\operatorname{artanh}\left(d\cot\left(bx+a\right)+i\,d-1\right)}{x}\,dx$$

input
```
integrate(-arctanh(-1+I*d+d*cot(b*x+a))/x,x, algorithm="maxima")
```

output
```
-I*b*x + 1/4*(I*pi - 4*I*a - 2*log(d))*log(x) - 1/2*I*integrate(arctan2(-d
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*
a) + 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)
*sin(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1)/x, x)
```

3.345.8 Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\text{arctanh}(1 - id - d\cot(a + bx))}{x}\,dx = \int -\frac{\text{artanh}\left(d\cot\left(bx + a\right) + i\,d - 1\right)}{x}\,dx$$

input
```
integrate(-arctanh(-1+I*d+d*cot(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(-arctanh(d*cot(b*x + a) + I*d - 1)/x, x)
```

3.345.9 Mupad [N/A]

Not integrable

Time = 4.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\text{arctanh}(1 - id - d\cot(a + bx))}{x}\,dx = \int -\frac{\text{atanh}(d\cot(a + b\,x) - 1 + d\,1\text{i})}{x}\,dx$$

input
```
int(-atanh(d*1i + d*cot(a + b*x) - 1)/x,x)
```

output
```
int(-atanh(d*1i + d*cot(a + b*x) - 1)/x, x)
```

3.346 $\int \operatorname{arctanh}(e^x)\,dx$

3.346.1 Optimal result

Integrand size = 4, antiderivative size = 21

$$\int \operatorname{arctanh}(e^x)\,dx = -\frac{\operatorname{PolyLog}(2,-e^x)}{2} + \frac{\operatorname{PolyLog}(2,e^x)}{2}$$

output
```
-1/2*polylog(2,-exp(x))+1/2*polylog(2,exp(x))
```

3.346.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.43

$$\int \operatorname{arctanh}(e^x)\,dx = x\operatorname{arctanh}(e^x) + \frac{1}{2}x\log(1-e^x) - \frac{1}{2}x\log(1+e^x) - \frac{\operatorname{PolyLog}(2,-e^x)}{2} + \frac{\operatorname{PolyLog}(2,e^x)}{2}$$

input
```
Integrate[ArcTanh[E^x],x]
```

output
```
x*ArcTanh[E^x] + (x*Log[1 - E^x])/2 - (x*Log[1 + E^x])/2 - PolyLog[2, -E^x
]/2 + PolyLog[2, E^x]/2
```

3.346.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2720, 6446}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(e^x)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int e^{-x}\operatorname{arctanh}(e^x)\,de^x$$

$$\Big\downarrow \text{ 6446}$$

$$\frac{\operatorname{PolyLog}(2,e^x)}{2}-\frac{\operatorname{PolyLog}(2,-e^x)}{2}$$

input
```
Int[ArcTanh[E^x],x]
```

output
```
-1/2*PolyLog[2, -E^x] + PolyLog[2, E^x]/2
```

3.346.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

3.346.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
risch	$-\frac{\operatorname{dilog}(1+e^x)}{2}+\frac{\operatorname{dilog}(1-e^x)}{2}$	18
derivativedivides	$\ln(e^x)\operatorname{arctanh}(e^x)-\frac{\operatorname{dilog}(1+e^x)}{2}-\frac{\ln(e^x)\ln(1+e^x)}{2}-\frac{\operatorname{dilog}(e^x)}{2}$	31
default	$\ln(e^x)\operatorname{arctanh}(e^x)-\frac{\operatorname{dilog}(1+e^x)}{2}-\frac{\ln(e^x)\ln(1+e^x)}{2}-\frac{\operatorname{dilog}(e^x)}{2}$	31
parts	$x\operatorname{arctanh}(e^x)-\frac{x\ln(1+e^x)}{2}-\frac{\operatorname{polylog}(2,-e^x)}{2}+\frac{x\ln(1-e^x)}{2}+\frac{\operatorname{polylog}(2,e^x)}{2}$	39

input
```
int(arctanh(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*dilog(1+exp(x))+1/2*dilog(1-exp(x))
```

3.346.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.10

$$\int \operatorname{arctanh}(e^x)\,dx = \frac{1}{2}x\log\left(-\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right)-\frac{1}{2}x\log(\cosh(x)+\sinh(x)+1) + \frac{1}{2}x\log(-\cosh(x)-\sinh(x)+1) + \frac{1}{2}\operatorname{Li}_2(\cosh(x)+\sinh(x))-\frac{1}{2}\operatorname{Li}_2(-\cosh(x)-\sinh(x))$$

input
```
integrate(arctanh(exp(x)),x, algorithm="fricas")
```

output
```
1/2*x*log(-(cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/2*x*log(co
sh(x) + sinh(x) + 1) + 1/2*x*log(-cosh(x) - sinh(x) + 1) + 1/2*dilog(cosh(
x) + sinh(x)) - 1/2*dilog(-cosh(x) - sinh(x))
```

3.346.6 Sympy [F]

$$\int \operatorname{arctanh}(e^x)\, dx = \int \operatorname{atanh}(e^x)\, dx$$

input
```
integrate(atanh(exp(x)),x)
```

output
```
Integral(atanh(exp(x)), x)
```

3.346.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(13) = 26.

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.76

$$\begin{aligned}\int \operatorname{arctanh}(e^x)\, dx = & -\frac{1}{2}x(\log(e^x+1) - \log(e^x-1)) + x\operatorname{artanh}(e^x) \\ & + \frac{1}{2}\log(-e^x)\log(e^x+1) - \frac{1}{2}x\log(e^x-1) \\ & + \frac{1}{2}\operatorname{Li}_2(e^x+1) - \frac{1}{2}\operatorname{Li}_2(-e^x+1)\end{aligned}$$

input
```
integrate(arctanh(exp(x)),x, algorithm="maxima")
```

output
```
-1/2*x*(log(e^x + 1) - log(e^x - 1)) + x*arctanh(e^x) + 1/2*log(-e^x)*log(
e^x + 1) - 1/2*x*log(e^x - 1) + 1/2*dilog(e^x + 1) - 1/2*dilog(-e^x + 1)
```

3.346.8 Giac [F]

$$\int \operatorname{arctanh}(e^x)\, dx = \int \operatorname{artanh}(e^x)\, dx$$

input
```
integrate(arctanh(exp(x)),x, algorithm="giac")
```

output
```
integrate(arctanh(e^x), x)
```

3.346.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}(e^x)\, dx = \int \operatorname{atanh}(\mathrm{e}^x)\, dx$$

input
```
int(atanh(exp(x)),x)
```

output
```
int(atanh(exp(x)), x)
```

3.347 $\int x\text{arctanh}(e^x)\,dx$

3.347.1 Optimal result

Integrand size = 6, antiderivative size = 43

$$\int x\text{arctanh}(e^x)\,dx = -\frac{1}{2}x\,\text{PolyLog}\left(2,-e^x\right)+\frac{x\,\text{PolyLog}\left(2,e^x\right)}{2}+\frac{\text{PolyLog}\left(3,-e^x\right)}{2}-\frac{\text{PolyLog}\left(3,e^x\right)}{2}$$

output
```
-1/2*x*polylog(2,-exp(x))+1/2*x*polylog(2,exp(x))+1/2*polylog(3,-exp(x))-1
/2*polylog(3,exp(x))
```

3.347.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int x\text{arctanh}(e^x)\,dx = \frac{1}{4}\left(2x^2\text{arctanh}(e^x)+x^2\log\left(1-e^x\right)-x^2\log\left(1+e^x\right)-2x\,\text{PolyLog}\left(2,-e^x\right)+2x\,\text{PolyLog}\left(2,e^x\right)+2\,\text{PolyLog}\left(3,-e^x\right)-2\,\text{PolyLog}\left(3,e^x\right)\right)$$

input
```
Integrate[x*ArcTanh[E^x],x]
```

output
```
(2*x^2*ArcTanh[E^x] + x^2*Log[1 - E^x] - x^2*Log[1 + E^x] - 2*x*PolyLog[2,
 -E^x] + 2*x*PolyLog[2, E^x] + 2*PolyLog[3, -E^x] - 2*PolyLog[3, E^x])/4
```

3.347.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6767, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(e^x)\,dx$$

$$\Big\downarrow \text{6767}$$

$$\frac{1}{2}\int x\log(1+e^x)\,dx-\frac{1}{2}\int x\log(1-e^x)\,dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}\left(\int \text{PolyLog}(2,-e^x)\,dx-x\,\text{PolyLog}(2,-e^x)\right)+\frac{1}{2}\left(x\,\text{PolyLog}(2,e^x)-\int \text{PolyLog}(2,e^x)\,dx\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}\left(\int e^{-x}\,\text{PolyLog}(2,-e^x)\,de^x-x\,\text{PolyLog}(2,-e^x)\right)+\frac{1}{2}\left(x\,\text{PolyLog}(2,e^x)-\int e^{-x}\,\text{PolyLog}(2,e^x)\,de^x\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}(\text{PolyLog}(3,-e^x)-x\,\text{PolyLog}(2,-e^x))+\frac{1}{2}(x\,\text{PolyLog}(2,e^x)-\text{PolyLog}(3,e^x))$$

input
```
Int[x*ArcTanh[E^x],x]
```

output
```
(-(x*PolyLog[2, -E^x]) + PolyLog[3, -E^x])/2 + (x*PolyLog[2, E^x] - PolyLo
g[3, E^x])/2
```

3.347.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6767
```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Simp[1/2   Int[x
^m*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[
m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.347.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{x\,\mathrm{polylog}(2,-\mathrm{e}^{x})}{2}+\frac{x\,\mathrm{polylog}(2,\mathrm{e}^{x})}{2}+\frac{\mathrm{polylog}(3,-\mathrm{e}^{x})}{2}-\frac{\mathrm{polylog}(3,\mathrm{e}^{x})}{2}$
default	$\frac{x^{2}\,\mathrm{arctanh}(\mathrm{e}^{x})}{2}-\frac{x^{2}\ln(1+\mathrm{e}^{x})}{4}-\frac{x\,\mathrm{polylog}(2,-\mathrm{e}^{x})}{2}+\frac{\mathrm{polylog}(3,-\mathrm{e}^{x})}{2}+\frac{x^{2}\ln(1-\mathrm{e}^{x})}{4}+\frac{x\,\mathrm{polylog}(2,\mathrm{e}^{x})}{2}-\frac{\mathrm{polylog}(3,\mathrm{e}^{x})}{2}$
parts	$\frac{x^{2}\,\mathrm{arctanh}(\mathrm{e}^{x})}{2}-\frac{x^{2}\ln(1+\mathrm{e}^{x})}{4}-\frac{x\,\mathrm{polylog}(2,-\mathrm{e}^{x})}{2}+\frac{\mathrm{polylog}(3,-\mathrm{e}^{x})}{2}+\frac{x^{2}\ln(1-\mathrm{e}^{x})}{4}+\frac{x\,\mathrm{polylog}(2,\mathrm{e}^{x})}{2}-\frac{\mathrm{polylog}(3,\mathrm{e}^{x})}{2}$

input
```
int(x*arctanh(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*polylog(2,-exp(x))+1/2*x*polylog(2,exp(x))+1/2*polylog(3,-exp(x))-1
/2*polylog(3,exp(x))
```

3.347.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(29) = 58$.

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.21

$$\begin{aligned}\int x\text{arctanh}(e^x)\,dx &= \frac{1}{4}x^2\log\left(-\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right)-\frac{1}{4}x^2\log(\cosh(x)+\sinh(x)+1)\\&\quad+\frac{1}{4}x^2\log(-\cosh(x)-\sinh(x)+1)+\frac{1}{2}x\text{Li}_2(\cosh(x)+\sinh(x))\\&\quad-\frac{1}{2}x\text{Li}_2(-\cosh(x)-\sinh(x))-\frac{1}{2}\text{polylog}(3,\cosh(x)+\sinh(x))\\&\quad+\frac{1}{2}\text{polylog}(3,-\cosh(x)-\sinh(x))\end{aligned}$$

input
```
integrate(x*arctanh(exp(x)),x, algorithm="fricas")
```

output
```
1/4*x^2*log(-(cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/4*x^2*lo
g(cosh(x) + sinh(x) + 1) + 1/4*x^2*log(-cosh(x) - sinh(x) + 1) + 1/2*x*dil
og(cosh(x) + sinh(x)) - 1/2*x*dilog(-cosh(x) - sinh(x)) - 1/2*polylog(3, c
osh(x) + sinh(x)) + 1/2*polylog(3, -cosh(x) - sinh(x))
```

3.347.6 Sympy [F]

$$\int x\text{arctanh}(e^x)\,dx = \int x\operatorname{atanh}(e^x)\,dx$$

input
```
integrate(x*atanh(exp(x)),x)
```

output
```
Integral(x*atanh(exp(x)), x)
```

3.347.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(29) = 58$.

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

$$\int x\text{arctanh}(e^x)\,dx = \frac{1}{2}x^2\,\text{artanh}\,(e^x) - \frac{1}{4}x^2\log\,(e^x+1) + \frac{1}{4}x^2\log\,(-e^x+1) - \frac{1}{2}x\text{Li}_2(-e^x) + \frac{1}{2}x\text{Li}_2(e^x) + \frac{1}{2}\text{Li}_3(-e^x) - \frac{1}{2}\text{Li}_3(e^x)$$

input
```
integrate(x*arctanh(exp(x)),x, algorithm="maxima")
```

output
```
1/2*x^2*arctanh(e^x) - 1/4*x^2*log(e^x + 1) + 1/4*x^2*log(-e^x + 1) - 1/2*
x*dilog(-e^x) + 1/2*x*dilog(e^x) + 1/2*polylog(3, -e^x) - 1/2*polylog(3, e
^x)
```

3.347.8 Giac [F]

$$\int x\text{arctanh}(e^x)\,dx = \int x\,\text{artanh}\,(e^x)\,dx$$

input
```
integrate(x*arctanh(exp(x)),x, algorithm="giac")
```

output
```
integrate(x*arctanh(e^x), x)
```

3.347.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arctanh}(e^x)\,dx = \int x\,\text{atanh}(\text{e}^x)\,dx$$

input
```
int(x*atanh(exp(x)),x)
```

output
```
int(x*atanh(exp(x)), x)
```

3.348 $\int x^2\text{arctanh}(e^x)\,dx$

3.348.1 Optimal result

Integrand size = 8, antiderivative size = 58

$$\int x^2\text{arctanh}(e^x)\,dx = -\frac{1}{2}x^2\operatorname{PolyLog}(2,-e^x)+\frac{1}{2}x^2\operatorname{PolyLog}(2,e^x)+x\operatorname{PolyLog}(3,-e^x)-x\operatorname{PolyLog}(3,e^x)-\operatorname{PolyLog}(4,-e^x)+\operatorname{PolyLog}(4,e^x)$$

output
```
-1/2*x^2*polylog(2,-exp(x))+1/2*x^2*polylog(2,exp(x))+x*polylog(3,-exp(x))
-x*polylog(3,exp(x))-polylog(4,-exp(x))+polylog(4,exp(x))
```

3.348.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.60

$$\int x^2\text{arctanh}(e^x)\,dx = \frac{1}{6}\left(2x^3\text{arctanh}(e^x)+x^3\log(1-e^x)-x^3\log(1+e^x)-3x^2\operatorname{PolyLog}(2,-e^x)+3x^2\operatorname{PolyLog}(2,e^x)+6x\operatorname{PolyLog}(3,-e^x)-6x\operatorname{PolyLog}(3,e^x)-6\operatorname{PolyLog}(4,-e^x)+6\operatorname{PolyLog}(4,e^x)\right)$$

input
```
Integrate[x^2*ArcTanh[E^x],x]
```

output
```
(2*x^3*ArcTanh[E^x] + x^3*Log[1 - E^x] - x^3*Log[1 + E^x] - 3*x^2*PolyLog[
2, -E^x] + 3*x^2*PolyLog[2, E^x] + 6*x*PolyLog[3, -E^x] - 6*x*PolyLog[3, E
^x] - 6*PolyLog[4, -E^x] + 6*PolyLog[4, E^x])/6
```

3.348.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6767, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arctanh}(e^x)\,dx$$

$$\Big\downarrow \text{6767}$$

$$\frac{1}{2}\int x^2\log(1+e^x)\,dx-\frac{1}{2}\int x^2\log(1-e^x)\,dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}\left(2\int x\,\text{PolyLog}(2,-e^x)\,dx-x^2\,\text{PolyLog}(2,-e^x)\right)+\frac{1}{2}\left(x^2\,\text{PolyLog}(2,e^x)-2\int x\,\text{PolyLog}(2,e^x)\,dx\right)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{2}\left(2\left(x\,\text{PolyLog}(3,-e^x)-\int \text{PolyLog}(3,-e^x)\,dx\right)-x^2\,\text{PolyLog}(2,-e^x)\right)+\frac{1}{2}\left(x^2\,\text{PolyLog}(2,e^x)-2\left(x\,\text{PolyLog}(3,e^x)-\int \text{PolyLog}(3,e^x)\,dx\right)\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}\left(2\left(x\,\text{PolyLog}(3,-e^x)-\int e^{-x}\,\text{PolyLog}(3,-e^x)\,de^x\right)-x^2\,\text{PolyLog}(2,-e^x)\right)+\frac{1}{2}\left(x^2\,\text{PolyLog}(2,e^x)-2\left(x\,\text{PolyLog}(3,e^x)-\int e^{-x}\,\text{PolyLog}(3,e^x)\,de^x\right)\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}(2(x\,\text{PolyLog}(3,-e^x)-\text{PolyLog}(4,-e^x))-x^2\,\text{PolyLog}(2,-e^x))+\frac{1}{2}(x^2\,\text{PolyLog}(2,e^x)-2(x\,\text{PolyLog}(3,e^x)-\text{PolyLog}(4,e^x)))$$

input
```
Int[x^2*ArcTanh[E^x],x]
```

output
```
(-(x^2*PolyLog[2, -E^x]) + 2*(x*PolyLog[3, -E^x] - PolyLog[4, -E^x]))/2 +
(x^2*PolyLog[2, E^x] - 2*(x*PolyLog[3, E^x] - PolyLog[4, E^x]))/2
```

3.348.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6767
```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Simp[1/2   Int[x
^m*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[
m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.348.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{x^2 \operatorname{polylog}(2,-e^x)}{2}+\frac{x^2 \operatorname{polylog}(2,e^x)}{2}+x \operatorname{polylog}(3,-e^x)-x \operatorname{polylog}(3,e^x)-\operatorname{polylog}(4,-e^x)+\operatorname{poly}$
default	$\frac{x^3 \operatorname{arctanh}(e^x)}{3}-\frac{x^3 \ln(1+e^x)}{6}-\frac{x^2 \operatorname{polylog}(2,-e^x)}{2}+x \operatorname{polylog}(3,-e^x)-\operatorname{polylog}(4,-e^x)+\frac{x^3 \ln(1-e^x)}{6}+\frac{x}{}$
parts	$\frac{x^3 \operatorname{arctanh}(e^x)}{3}-\frac{x^3 \ln(1+e^x)}{6}-\frac{x^2 \operatorname{polylog}(2,-e^x)}{2}+x \operatorname{polylog}(3,-e^x)-\operatorname{polylog}(4,-e^x)+\frac{x^3 \ln(1-e^x)}{6}+\frac{x}{}$

input
```
int(x^2*arctanh(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2*polylog(2,-exp(x))+1/2*x^2*polylog(2,exp(x))+x*polylog(3,-exp(x))
-x*polylog(3,exp(x))-polylog(4,-exp(x))+polylog(4,exp(x))
```

3.348.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(46) = 92$.

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.07

$$\begin{aligned}\int x^2\operatorname{arctanh}(e^x)\,dx &= \frac{1}{6}\,x^3\log\left(-\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right)\\ &\quad-\frac{1}{6}\,x^3\log(\cosh(x)+\sinh(x)+1)+\frac{1}{6}\,x^3\log(-\cosh(x)-\sinh(x)+1)\\ &\quad+\frac{1}{2}\,x^2\mathrm{Li}_2(\cosh(x)+\sinh(x))-\frac{1}{2}\,x^2\mathrm{Li}_2(-\cosh(x)-\sinh(x))\\ &\quad-x\operatorname{polylog}(3,\cosh(x)+\sinh(x))+x\operatorname{polylog}(3,-\cosh(x)-\sinh(x))\\ &\quad+\operatorname{polylog}(4,\cosh(x)+\sinh(x))-\operatorname{polylog}(4,-\cosh(x)-\sinh(x))\end{aligned}$$

input
```
integrate(x^2*arctanh(exp(x)),x, algorithm="fricas")
```

output
```
1/6*x^3*log(-(cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/6*x^3*lo
g(cosh(x) + sinh(x) + 1) + 1/6*x^3*log(-cosh(x) - sinh(x) + 1) + 1/2*x^2*d
ilog(cosh(x) + sinh(x)) - 1/2*x^2*dilog(-cosh(x) - sinh(x)) - x*polylog(3,
 cosh(x) + sinh(x)) + x*polylog(3, -cosh(x) - sinh(x)) + polylog(4, cosh(x
) + sinh(x)) - polylog(4, -cosh(x) - sinh(x))
```

3.348.6 Sympy [F]

$$\int x^2 \text{arctanh}(e^x)\,dx = \int x^2 \operatorname{atanh}(e^x)\,dx$$

input
```
integrate(x**2*atanh(exp(x)),x)
```

output
```
Integral(x**2*atanh(exp(x)), x)
```

3.348.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.31

$$\int x^2 \text{arctanh}(e^x)\,dx = \frac{1}{3}x^3 \operatorname{artanh}(e^x) - \frac{1}{6}x^3 \log(e^x+1) + \frac{1}{6}x^3 \log(-e^x+1) - \frac{1}{2}x^2 \text{Li}_2(-e^x) + \frac{1}{2}x^2 \text{Li}_2(e^x) + x\text{Li}_3(-e^x) - x\text{Li}_3(e^x) - \text{Li}_4(-e^x) + \text{Li}_4(e^x)$$

input
```
integrate(x^2*arctanh(exp(x)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctanh(e^x) - 1/6*x^3*log(e^x + 1) + 1/6*x^3*log(-e^x + 1) - 1/2*
x^2*dilog(-e^x) + 1/2*x^2*dilog(e^x) + x*polylog(3, -e^x) - x*polylog(3, e
^x) - polylog(4, -e^x) + polylog(4, e^x)
```

3.348.8 Giac [F]

$$\int x^2 \text{arctanh}(e^x)\,dx = \int x^2 \operatorname{artanh}(e^x)\,dx$$

input
```
integrate(x^2*arctanh(exp(x)),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(e^x), x)
```

3.348.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arctanh}(e^x)\,dx = \int x^2\,\text{atanh}(\text{e}^x)\,dx$$

input
```
int(x^2*atanh(exp(x)),x)
```

output
```
int(x^2*atanh(exp(x)), x)
```

3.349 $\int \operatorname{arctanh}\left(e^{a+bx}\right) dx$

3.349.1 Optimal result

Integrand size = 8, antiderivative size = 35

$$\int \operatorname{arctanh}\left(e^{a+bx}\right) dx = -\frac{\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{2b} + \frac{\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{2b}$$

output
```
-1/2*polylog(2,-exp(b*x+a))/b+1/2*polylog(2,exp(b*x+a))/b
```

3.349.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.94

$$\int \operatorname{arctanh}\left(e^{a+bx}\right) dx = \frac{bx\left(2\operatorname{arctanh}\left(e^{a+bx}\right)+\log\left(1-e^{a+bx}\right)-\log\left(1+e^{a+bx}\right)\right)-\operatorname{PolyLog}\left(2,-e^{a+bx}\right)+\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{2b}$$

input
```
Integrate[ArcTanh[E^(a + b*x)],x]
```

output
```
(b*x*(2*ArcTanh[E^(a + b*x)] + Log[1 - E^(a + b*x)] - Log[1 + E^(a + b*x)]
) - PolyLog[2, -E^(a + b*x)] + PolyLog[2, E^(a + b*x)])/(2*b)
```

3.349.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 6446}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arctanh}\left(e^{a+bx}\right) dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int e^{-a-bx}\text{arctanh}\left(e^{a+bx}\right) de^{a+bx}}{b}$$

$$\Big\downarrow \text{6446}$$

$$\frac{\frac{1}{2}\text{PolyLog}\left(2,e^{a+bx}\right)-\frac{1}{2}\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}$$

input
```
Int[ArcTanh[E^(a + b*x)],x]
```

output
```
(-1/2*PolyLog[2, -E^(a + b*x)] + PolyLog[2, E^(a + b*x)]/2)/b
```

3.349.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

3.349.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{\operatorname{dilog}(\mathrm{e}^{bx+a}+1)}{2b}+\frac{\operatorname{dilog}(1-\mathrm{e}^{bx+a})}{2b}$
derivativedivides	$\frac{\ln(\mathrm{e}^{bx+a})\operatorname{arctanh}(\mathrm{e}^{bx+a})-\frac{\operatorname{dilog}\left(\mathrm{e}^{bx+a}\right)}{2}-\frac{\operatorname{dilog}\left(\mathrm{e}^{bx+a}+1\right)}{2}-\frac{\ln\left(\mathrm{e}^{bx+a}\right)\ln\left(\mathrm{e}^{bx+a}+1\right)}{2}}{b}$
default	$\frac{\ln(\mathrm{e}^{bx+a})\operatorname{arctanh}(\mathrm{e}^{bx+a})-\frac{\operatorname{dilog}\left(\mathrm{e}^{bx+a}\right)}{2}-\frac{\operatorname{dilog}\left(\mathrm{e}^{bx+a}+1\right)}{2}-\frac{\ln\left(\mathrm{e}^{bx+a}\right)\ln\left(\mathrm{e}^{bx+a}+1\right)}{2}}{b}$
parts	$x\operatorname{arctanh}\left(\mathrm{e}^{bx+a}\right)-\frac{\frac{(bx+a)\ln\left(\mathrm{e}^{bx+a}+1\right)}{2}+\frac{\operatorname{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{2}-\frac{(bx+a)\ln\left(1-\mathrm{e}^{bx+a}\right)}{2}-\frac{\operatorname{polylog}\left(2,\mathrm{e}^{bx+a}\right)}{2}-a\operatorname{arctan}}{b}$

input
```
int(arctanh(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*dilog(exp(b*x+a)+1)+1/2/b*dilog(1-exp(b*x+a))
```

3.349.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(27) = 54.

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.94

$$\int \operatorname{arctanh}\left(e^{a+bx}\right)\,dx$$
$$=\frac{bx\log\left(-\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right)-bx\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-a\log\left(\cosh\left(bx+a\right)+\sinh\right.}{}$$

input
```
integrate(arctanh(exp(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b*x*log(-(cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh(b*
x + a) - 1)) - b*x*log(cosh(b*x + a) + sinh(b*x + a) + 1) - a*log(cosh(b*x
 + a) + sinh(b*x + a) - 1) + (b*x + a)*log(-cosh(b*x + a) - sinh(b*x + a)
+ 1) + dilog(cosh(b*x + a) + sinh(b*x + a)) - dilog(-cosh(b*x + a) - sinh(
b*x + a)))/b
```

3.349.6 Sympy [F]

$$\int \operatorname{arctanh}\left(e^{a+bx}\right)\,dx=\int \operatorname{atanh}\left(e^{a+bx}\right)dx$$

input
```
integrate(atanh(exp(b*x+a)),x)
```

output
```
Integral(atanh(exp(a + b*x)), x)
```

3.349.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(27)=54$.

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.06

$$\int \operatorname{arctanh}\left(e^{a+bx}\right)\,dx=\frac{(bx+a)\operatorname{artanh}\left(e^{(bx+a)}\right)}{b}-\frac{(bx+a)\left(\log\left(e^{(bx+a)}+1\right)-\log\left(e^{(bx+a)}-1\right)\right)-\log\left(-e^{(bx+a)}\right)\log\left(e^{(bx+a)}+1\right)+(bx+a)\log\left(e^{(bx+a)}\right.}{2b}$$

input
```
integrate(arctanh(exp(b*x+a)),x, algorithm="maxima")
```

output
```
(b*x + a)*arctanh(e^(b*x + a))/b - 1/2*((b*x + a)*(log(e^(b*x + a) + 1) -
log(e^(b*x + a) - 1)) - log(-e^(b*x + a))*log(e^(b*x + a) + 1) + (b*x + a)
*log(e^(b*x + a) - 1) - dilog(e^(b*x + a) + 1) + dilog(-e^(b*x + a) + 1))/
b
```

3.349.8 Giac [F]

$$\int \operatorname{arctanh}\left(e^{a+bx}\right)\,dx=\int \operatorname{artanh}\left(e^{(bx+a)}\right)\,dx$$

input
```
integrate(arctanh(exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctanh(e^(b*x + a)), x)
```

3.349.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}\left(e^{a+bx}\right)\,dx=\int \operatorname{atanh}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

input
```
int(atanh(exp(a + b*x)),x)
```

output
```
int(atanh(exp(a + b*x)), x)
```

3.350 $\int x\text{arctanh}\left(e^{a+bx}\right)\,dx$

3.350.1 Optimal result

Integrand size = 10, antiderivative size = 71

$$\int x\text{arctanh}\left(e^{a+bx}\right)\,dx=-\frac{x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b}+\frac{x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{2b}+\frac{\text{PolyLog}\left(3,-e^{a+bx}\right)}{2b^2}-\frac{\text{PolyLog}\left(3,e^{a+bx}\right)}{2b^2}$$

output
```
-1/2*x*polylog(2,-exp(b*x+a))/b+1/2*x*polylog(2,exp(b*x+a))/b+1/2*polylog(
3,-exp(b*x+a))/b^2-1/2*polylog(3,exp(b*x+a))/b^2
```

3.350.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.59

$$\int x\text{arctanh}\left(e^{a+bx}\right)\,dx$$
$$=\frac{2b^2x^2\text{arctanh}\left(e^{a+bx}\right)+b^2x^2\log\left(1-e^{a+bx}\right)-b^2x^2\log\left(1+e^{a+bx}\right)-2bx\,\text{PolyLog}\left(2,-e^{a+bx}\right)+2bx\,\text{PolyI}}{4b^2}$$

input
```
Integrate[x*ArcTanh[E^(a + b*x)],x]
```

output
```
(2*b^2*x^2*ArcTanh[E^(a + b*x)] + b^2*x^2*Log[1 - E^(a + b*x)] - b^2*x^2*L
og[1 + E^(a + b*x)] - 2*b*x*PolyLog[2, -E^(a + b*x)] + 2*b*x*PolyLog[2, E^
(a + b*x)] + 2*PolyLog[3, -E^(a + b*x)] - 2*PolyLog[3, E^(a + b*x)])/(4*b^
2)
```

3.350.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6767, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}\left(e^{a+bx}\right)\,dx$$

$$\Big\downarrow \text{6767}$$

$$\frac{1}{2}\int x\log\left(1+e^{a+bx}\right)dx-\frac{1}{2}\int x\log\left(1-e^{a+bx}\right)dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}\left(\frac{\int \text{PolyLog}\left(2,-e^{a+bx}\right)dx}{b}-\frac{x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)+\frac{1}{2}\left(\frac{x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b}-\frac{\int \text{PolyLog}\left(2,e^{a+bx}\right)dx}{b}\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}\left(\frac{\int e^{-a-bx}\,\text{PolyLog}\left(2,-e^{a+bx}\right)de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)+\frac{1}{2}\left(\frac{x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b}-\frac{\int e^{-a-bx}\,\text{PolyLog}\left(2,e^{a+bx}\right)de^{a+bx}}{b^2}\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}\left(\frac{\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)+\frac{1}{2}\left(\frac{x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b}-\frac{\text{PolyLog}\left(3,e^{a+bx}\right)}{b^2}\right)$$

input
```
Int[x*ArcTanh[E^(a + b*x)],x]
```

output
```
(-((x*PolyLog[2, -E^(a + b*x)])/b) + PolyLog[3, -E^(a + b*x)]/b^2)/2 + ((x
*PolyLog[2, E^(a + b*x)])/b - PolyLog[3, E^(a + b*x)]/b^2)/2
```

3.350.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6767
```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Simp[1/2   Int[x
^m*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[
m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.350.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(59) = 118$.

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.18

method	result
risch	$\frac{\ln\left(1-e^{bx+a}\right)ax}{2b}+\frac{a^{2}\ln\left(1-e^{bx+a}\right)}{2b^{2}}+\frac{x\,\mathrm{polylog}\left(2,e^{bx+a}\right)}{2b}+\frac{\mathrm{polylog}\left(2,e^{bx+a}\right)a}{2b^{2}}+\frac{a\,\mathrm{dilog}\left(e^{bx+a}\right)}{2b^{2}}-\frac{\mathrm{polylog}\left(3,e^{bx+a}\right)}{2b^{2}}-x$ …
default	$\frac{x^{2}\,\mathrm{arctanh}\left(e^{bx+a}\right)}{2}-\frac{a^{2}\,\mathrm{arctanh}\left(e^{bx+a}\right)+\frac{(bx+a)^{2}\ln\left(e^{bx+a}+1\right)}{2}+(bx+a)\,\mathrm{polylog}\left(2,-e^{bx+a}\right)-\mathrm{polylog}\left(3,-e^{bx+a}\right)-\frac{(bx+a)^{2}\ln\left(1-e^{b}\right.}{2}}{}$ …
parts	$\frac{x^{2}\,\mathrm{arctanh}\left(e^{bx+a}\right)}{2}-\frac{a^{2}\,\mathrm{arctanh}\left(e^{bx+a}\right)+\frac{(bx+a)^{2}\ln\left(e^{bx+a}+1\right)}{2}+(bx+a)\,\mathrm{polylog}\left(2,-e^{bx+a}\right)-\mathrm{polylog}\left(3,-e^{bx+a}\right)-\frac{(bx+a)^{2}\ln\left(1-e^{b}\right.}{2}}{}$ …

input
```
int(x*arctanh(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*ln(1-exp(b*x+a))*a*x+1/2/b^2*a^2*ln(1-exp(b*x+a))+1/2*x*polylog(2,ex
p(b*x+a))/b+1/2/b^2*polylog(2,exp(b*x+a))*a+1/2/b^2*a*dilog(exp(b*x+a))-1/
2*polylog(3,exp(b*x+a))/b^2-1/2*x*polylog(2,-exp(b*x+a))/b+1/2/b^2*dilog(e
xp(b*x+a)+1)*a-1/2/b^2*polylog(2,-exp(b*x+a))*a+1/2*polylog(3,-exp(b*x+a))
/b^2
```

3.350.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(57) = 114$.

Time = 0.25 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.80

$$\int x\text{arctanh}\left(e^{a+bx}\right)\,dx$$
$$= \frac{b^2x^2\log\left(-\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right) - b^2x^2\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right) + 2\,bx\text{Li}_2(\cosh\left(bx+a\right) -}{}$$

input
```
integrate(x*arctanh(exp(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(b^2*x^2*log(-(cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sin
h(b*x + a) - 1)) - b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*b*x*
dilog(cosh(b*x + a) + sinh(b*x + a)) - 2*b*x*dilog(-cosh(b*x + a) - sinh(b
*x + a)) + a^2*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (b^2*x^2 - a^2)*lo
g(-cosh(b*x + a) - sinh(b*x + a) + 1) - 2*polylog(3, cosh(b*x + a) + sinh(
b*x + a)) + 2*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^2
```

3.350.6 Sympy [F]

$$\int x\text{arctanh}\left(e^{a+bx}\right)\,dx = \int x\,\text{atanh}\left(e^{a}e^{bx}\right)dx$$

input
```
integrate(x*atanh(exp(b*x+a)),x)
```

output
```
Integral(x*atanh(exp(a)*exp(b*x)), x)
```

3.350.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.52

$$\int x\text{arctanh}\left(e^{a+bx}\right) dx = \frac{1}{2}x^2 \text{artanh}\left(e^{(bx+a)}\right) - \frac{1}{4}b\left(\frac{b^2x^2 \log\left(e^{(bx+a)}+1\right) + 2\,bx\text{Li}_2\left(-e^{(bx+a)}\right) - 2\,\text{Li}_3\left(-e^{(bx+a)}\right)}{b^3} - \frac{b^2x^2 \log\left(-e^{(bx+a)}+1\right) + 2\,bx\text{Li}_2\left(e^{(bx+a)}\right) \ldots}{b^3}\right.$$

input
```
integrate(x*arctanh(exp(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*x^2*arctanh(e^(b*x + a)) - 1/4*b*((b^2*x^2*log(e^(b*x + a) + 1) + 2*b*
x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 - (b^2*x^2*log(-e^
(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
)
```

3.350.8 Giac [F]

$$\int x\text{arctanh}\left(e^{a+bx}\right) dx = \int x\,\text{artanh}\left(e^{(bx+a)}\right) dx$$

input
```
integrate(x*arctanh(exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctanh(e^(b*x + a)), x)
```

3.350.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arctanh}\left(e^{a+bx}\right) dx = \int x\,\text{atanh}\left(e^{a+b\,x}\right) dx$$

input
```
int(x*atanh(exp(a + b*x)),x)
```

output
```
int(x*atanh(exp(a + b*x)), x)
```

3.351 $\int x^2 \mathbf{arctanh}\left(e^{a+bx}\right)\,dx$

3.351.1 Optimal result

Integrand size = 12, antiderivative size = 101

$$\int x^2 \text{arctanh}\left(e^{a+bx}\right)\,dx = -\frac{x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b} + \frac{x^2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{2b} + \frac{x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^2} - \frac{x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^2} - \frac{\text{PolyLog}\left(4,-e^{a+bx}\right)}{b^3} + \frac{\text{PolyLog}\left(4,e^{a+bx}\right)}{b^3}$$

output
```
-1/2*x^2*polylog(2,-exp(b*x+a))/b+1/2*x^2*polylog(2,exp(b*x+a))/b+x*polylo
g(3,-exp(b*x+a))/b^2-x*polylog(3,exp(b*x+a))/b^2-polylog(4,-exp(b*x+a))/b^
3+polylog(4,exp(b*x+a))/b^3
```

3.351.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.48

$$\int x^2 \text{arctanh}\left(e^{a+bx}\right)\,dx = \frac{2b^3x^3\text{arctanh}\left(e^{a+bx}\right) + b^3x^3\log\left(1-e^{a+bx}\right) - b^3x^3\log\left(1+e^{a+bx}\right) - 3b^2x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right) + 3b^2x^2\,\text{P}}{}$$

input
```
Integrate[x^2*ArcTanh[E^(a + b*x)],x]
```

output
```
(2*b^3*x^3*ArcTanh[E^(a + b*x)] + b^3*x^3*Log[1 - E^(a + b*x)] - b^3*x^3*L
og[1 + E^(a + b*x)] - 3*b^2*x^2*PolyLog[2, -E^(a + b*x)] + 3*b^2*x^2*PolyL
og[2, E^(a + b*x)] + 6*b*x*PolyLog[3, -E^(a + b*x)] - 6*b*x*PolyLog[3, E^(
a + b*x)] - 6*PolyLog[4, -E^(a + b*x)] + 6*PolyLog[4, E^(a + b*x)])/(6*b^3
)
```

3.351.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6767, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{arctanh}\left(e^{a+bx}\right)dx$$

$$\Big\downarrow \text{ 6767}$$

$$\frac{1}{2}\int x^2\log\left(1+e^{a+bx}\right)dx-\frac{1}{2}\int x^2\log\left(1-e^{a+bx}\right)dx$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{1}{2}\left(\frac{2\int x\,\text{PolyLog}\left(2,-e^{a+bx}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)+\frac{1}{2}\left(\frac{x^2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b}-\frac{2\int x\,\text{PolyLog}\left(2,e^{a+bx}\right)dx}{b}\right)$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{1}{2}\left(\frac{2\left(\frac{x\,\text{PolyLog}(3,-e^{a+bx})}{b}-\frac{\int \text{PolyLog}(3,-e^{a+bx})dx}{b}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)+\frac{1}{2}\left(\frac{x^2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b}-\frac{2\left(\frac{x\,\text{PolyLog}(3,e^{a+bx})}{b}-\frac{\int \text{PolyLog}(3,e^{a+bx})dx}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{2}\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}(3,-e^{a+bx})}{b}-\frac{\int e^{-a-bx}\,\mathrm{PolyLog}(3,-e^{a+bx})de^{a+bx}}{b^2}\right)}{b}-\frac{x^2\,\mathrm{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)+\frac{1}{2}\left(\frac{x^2\,\mathrm{PolyLog}\left(2,e^{a+bx}\right)}{b}-\frac{2\left(\frac{x\,\mathrm{PolyLog}(3,e^{a+bx})}{b}-\frac{\int e^{-a-bx}\,\mathrm{PolyLog}(3,e^{a+bx})de^{a+bx}}{b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{1}{2}\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}(3,-e^{a+bx})}{b}-\frac{\mathrm{PolyLog}(4,-e^{a+bx})}{b^2}\right)}{b}-\frac{x^2\,\mathrm{PolyLog}\left(2,-e^{a+bx}\right)}{b}\right)+\frac{1}{2}\left(\frac{x^2\,\mathrm{PolyLog}\left(2,e^{a+bx}\right)}{b}-\frac{2\left(\frac{x\,\mathrm{PolyLog}(3,e^{a+bx})}{b}-\frac{\mathrm{PolyLog}(4,e^{a+bx})}{b^2}\right)}{b}\right)$$

input
```
Int[x^2*ArcTanh[E^(a + b*x)],x]
```

output
```
(-((x^2*PolyLog[2, -E^(a + b*x)])/b) + (2*((x*PolyLog[3, -E^(a + b*x)])/b
- PolyLog[4, -E^(a + b*x)]/b^2))/b)/2 + ((x^2*PolyLog[2, E^(a + b*x)])/b -
 (2*((x*PolyLog[3, E^(a + b*x)])/b - PolyLog[4, E^(a + b*x)]/b^2))/b)/2
```

3.351.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6767
```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Simp[1/2   Int[x
^m*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[
m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.351.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(91) = 182$.

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.95

method	result
risch	$\frac{x^2 \operatorname{polylog}(2,e^{bx+a})}{2b} - \frac{\ln(1-e^{bx+a}) x a^2}{2b^2} - \frac{a^3 \ln(1-e^{bx+a})}{2b^3} - \frac{x \operatorname{polylog}(3,e^{bx+a})}{b^2} - \frac{\operatorname{polylog}(2,e^{bx+a}) a^2}{2b^3} - \frac{a^2 \operatorname{dilog}(e^{bx+a})}{2b^3}$
default	$\frac{x^3 \operatorname{arctanh}(e^{bx+a})}{3} - \frac{\frac{(bx+a)^3 \ln(e^{bx+a}+1)}{2} + \frac{3(bx+a)^2 \operatorname{polylog}(2,-e^{bx+a})}{2} - 3(bx+a) \operatorname{polylog}(3,-e^{bx+a}) + 3 \operatorname{polylog}(4,-e^{bx+a}) - \frac{(bx-}{}}{}$
parts	$\frac{x^3 \operatorname{arctanh}(e^{bx+a})}{3} - \frac{\frac{(bx+a)^3 \ln(e^{bx+a}+1)}{2} + \frac{3(bx+a)^2 \operatorname{polylog}(2,-e^{bx+a})}{2} - 3(bx+a) \operatorname{polylog}(3,-e^{bx+a}) + 3 \operatorname{polylog}(4,-e^{bx+a}) - \frac{(bx-}{}}{}$

input
```
int(x^2*arctanh(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*polylog(2,exp(b*x+a))/b-1/2/b^2*ln(1-exp(b*x+a))*x*a^2-1/2/b^3*a^3
*ln(1-exp(b*x+a))-x*polylog(3,exp(b*x+a))/b^2-1/2/b^3*polylog(2,exp(b*x+a)
)*a^2-1/2/b^3*a^2*dilog(exp(b*x+a))+polylog(4,exp(b*x+a))/b^3-1/2*x^2*poly
log(2,-exp(b*x+a))/b+x*polylog(3,-exp(b*x+a))/b^2+1/2/b^3*polylog(2,-exp(b
*x+a))*a^2-1/2/b^3*dilog(exp(b*x+a)+1)*a^2-polylog(4,-exp(b*x+a))/b^3
```

3.351.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(89) = 178.

Time = 0.27 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.46

$$\int x^2 \text{arctanh}\left(e^{a+bx}\right) \, dx$$
$$= \frac{b^3x^3 \log\left(-\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right) - b^3x^3 \log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right) + 3\,b^2x^2\text{Li}_2(\cosh\left(bx+a\right)}{}$$

input
```
integrate(x^2*arctanh(exp(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(b^3*x^3*log(-(cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sin
h(b*x + a) - 1)) - b^3*x^3*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*b^2*
x^2*dilog(cosh(b*x + a) + sinh(b*x + a)) - 3*b^2*x^2*dilog(-cosh(b*x + a)
- sinh(b*x + a)) - a^3*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 6*b*x*poly
log(3, cosh(b*x + a) + sinh(b*x + a)) + 6*b*x*polylog(3, -cosh(b*x + a) -
sinh(b*x + a)) + (b^3*x^3 + a^3)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) +
 6*polylog(4, cosh(b*x + a) + sinh(b*x + a)) - 6*polylog(4, -cosh(b*x + a)
 - sinh(b*x + a)))/b^3
```

3.351.6 Sympy [F]

$$\int x^2 \text{arctanh}\left(e^{a+bx}\right) \, dx = \int x^2 \operatorname{atanh}\left(e^a e^{bx}\right) dx$$

input
```
integrate(x**2*atanh(exp(b*x+a)),x)
```

output
```
Integral(x**2*atanh(exp(a)*exp(b*x)), x)
```

3.351.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.41

$$\int x^2\text{arctanh}\left(e^{a+bx}\right)\,dx=\frac{1}{3}\,x^3\,\text{artanh}\left(e^{(bx+a)}\right)$$
$$-\frac{1}{6}\,b\left(\frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\text{Li}_2\left(-e^{(bx+a)}\right)-6\,bx\text{Li}_3\left(-e^{(bx+a)}\right)+6\,\text{Li}_4\left(-e^{(bx+a)}\right)}{b^4}-\frac{b^3x^3\log\left(-e^{(\right.}}{}\right.$$

input
```
integrate(x^2*arctanh(exp(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctanh(e^(b*x + a)) - 1/6*b*((b^3*x^3*log(e^(b*x + a) + 1) + 3*b^
2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4,
-e^(b*x + a)))/b^4 - (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b
*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4)
```

3.351.8 Giac [F]

$$\int x^2\text{arctanh}\left(e^{a+bx}\right)\,dx=\int x^2\,\text{artanh}\left(e^{(bx+a)}\right)\,dx$$

input
```
integrate(x^2*arctanh(exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(e^(b*x + a)), x)
```

3.351.9 Mupad [F(-1)]

Timed out.

$$\int x^2\text{arctanh}\left(e^{a+bx}\right)\,dx=\int x^2\,\text{atanh}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

input
```
int(x^2*atanh(exp(a + b*x)),x)
```

output
```
int(x^2*atanh(exp(a + b*x)), x)
```

3.352 $\int \operatorname{arctanh}\left(a+bf^{c+dx}\right)\,dx$

3.352.1 Optimal result

Integrand size = 12, antiderivative size = 168

$$\int \operatorname{arctanh}\left(a+bf^{c+dx}\right)\,dx = -\frac{\operatorname{arctanh}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)} + \frac{\operatorname{arctanh}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)\left(1+a+bf^{c+dx}\right)}\right)}{d\log(f)} + \frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1+a+bf^{c+dx}}\right)}{2d\log(f)} - \frac{\operatorname{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(1-a)\left(1+a+bf^{c+dx}\right)}\right)}{2d\log(f)}$$

output
```
-arctanh(a+b*f^(d*x+c))*ln(2/(1+a+b*f^(d*x+c)))/d/ln(f)+arctanh(a+b*f^(d*x
+c))*ln(2*b*f^(d*x+c)/(1-a)/(1+a+b*f^(d*x+c)))/d/ln(f)+1/2*polylog(2,1-2/(
1+a+b*f^(d*x+c)))/d/ln(f)-1/2*polylog(2,1-2*b*f^(d*x+c)/(1-a)/(1+a+b*f^(d*
x+c)))/d/ln(f)
```

3.352.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.64

$$\int \operatorname{arctanh}\left(a+bf^{c+dx}\right)\,dx$$
$$= \frac{dx\log(f)\left(2\operatorname{arctanh}\left(a+bf^{c+dx}\right)+\log\left(\frac{-1+a+bf^{c+dx}}{-1+a}\right)-\log\left(\frac{1+a+bf^{c+dx}}{1+a}\right)\right)+\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{-1+a}\right)-\operatorname{Pol}}{2d\log(f)}$$

input
```
Integrate[ArcTanh[a + b*f^(c + d*x)],x]
```

output
```
(d*x*Log[f]*(2*ArcTanh[a + b*f^(c + d*x)] + Log[(-1 + a + b*f^(c + d*x))/(
-1 + a)] - Log[(1 + a + b*f^(c + d*x))/(1 + a)]) + PolyLog[2, -((b*f^(c +
d*x))/(-1 + a))] - PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f])
```

3.352.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2720, 6661, 25, 27, 6472, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}\left(a+bf^{c+dx}\right)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int f^{-c-dx}\operatorname{arctanh}\left(bf^{c+dx}+a\right)df^{c+dx}}{d\log(f)}$$

$$\Big\downarrow \text{ 6661}$$

$$\frac{\int f^{-c-dx}\operatorname{arctanh}\left(bf^{c+dx}+a\right)d\left(bf^{c+dx}+a\right)}{bd\log(f)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -f^{-c-dx}\operatorname{arctanh}\left(bf^{c+dx}+a\right)d\left(bf^{c+dx}+a\right)}{bd\log(f)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int -\frac{f^{-c-dx}\text{arctanh}(bf^{c+dx}+a)}{b}d(bf^{c+dx}+a)}{d\log(f)}$$

$\downarrow$ 6472

$$-\frac{-\int \frac{\log\left(\frac{2}{bf^{c+dx}+a+1}\right)}{1-f^{2c+2dx}}d(bf^{c+dx}+a)+\int \frac{\log\left(\frac{2bf^{c+dx}}{(1-a)\left(bf^{c+dx}+a+1\right)}\right)}{1-f^{2c+2dx}}d(bf^{c+dx}+a)+\text{arctanh}(a+bf^{c+dx})\log\left(\frac{2}{a+bf^{c+dx}\ldots}\right.}{d\log(f)}$$

$\downarrow$ 2849

$$-\frac{-\int \frac{\log\left(\frac{2}{bf^{c+dx}+a+1}\right)}{1-\frac{2}{bf^{c+dx}+a+1}}d\frac{1}{bf^{c+dx}+a+1}+\int \frac{\log\left(\frac{2bf^{c+dx}}{(1-a)\left(bf^{c+dx}+a+1\right)}\right)}{1-f^{2c+2dx}}d(bf^{c+dx}+a)+\text{arctanh}(a+bf^{c+dx})\log\left(\frac{2}{a+bf^{c+dx}+1}\right.}{d\log(f)}$$

$\downarrow$ 2752

$$-\frac{\int \frac{\log\left(\frac{2bf^{c+dx}}{(1-a)\left(bf^{c+dx}+a+1\right)}\right)}{1-f^{2c+2dx}}d(bf^{c+dx}+a)+\text{arctanh}(a+bf^{c+dx})\log\left(\frac{2}{a+bf^{c+dx}+1}\right)-\text{arctanh}(a+bf^{c+dx})\log\left(\frac{\ldots}{(1-\ldots}\right.}{d\log(f)}$$

$\downarrow$ 2897

$$-\frac{\text{arctanh}(a+bf^{c+dx})\log\left(\frac{2}{a+bf^{c+dx}+1}\right)-\text{arctanh}(a+bf^{c+dx})\log\left(\frac{2bf^{c+dx}}{(1-a)(a+bf^{c+dx}+1)}\right)-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{\ldots}{bf^{c+\ldots}}\right.}{d\log(f)}$$

input
```
Int[ArcTanh[a + b*f^(c + d*x)],x]
```

output
```
-((ArcTanh[a + b*f^(c + d*x)]*Log[2/(1 + a + b*f^(c + d*x))] - ArcTanh[a +
 b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c + d*x)))] -
 PolyLog[2, 1 - 2/(1 + a + b*f^(c + d*x))]/2 + PolyLog[2, 1 - (2*b*f^(c +
d*x))/((1 - a)*(1 + a + b*f^(c + d*x)))]/2)/(d*Log[f]))
```

3.352. $\int \text{arctanh}(a+bf^{c+dx})\,dx$

3.352.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6472
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*ArcTanh
[c*x])*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(c/e)
Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Simp[b*(c/e)   Int[Log[2*c*((d
 + e*x)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x]) /; FreeQ[{a, b, c, d
, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.352.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\ln\left(-b\,f^{dx+c}\right)\operatorname{arctanh}\left(a+b\,f^{dx+c}\right)+\frac{\operatorname{dilog}\left(\frac{1-a-b\,f^{dx+c}}{1-a}\right)}{2}+\frac{\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{1-a-b\,f^{dx+c}}{1-a}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{-b\,f^{dx+c}-a-1}{-1-a}\right)}{2}-}{d\ln(f)}$
default	$\frac{\ln\left(-b\,f^{dx+c}\right)\operatorname{arctanh}\left(a+b\,f^{dx+c}\right)+\frac{\operatorname{dilog}\left(\frac{1-a-b\,f^{dx+c}}{1-a}\right)}{2}+\frac{\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{1-a-b\,f^{dx+c}}{1-a}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{-b\,f^{dx+c}-a-1}{-1-a}\right)}{2}-}{d\ln(f)}$
risch	$\frac{x\ln\left(1+a+b\,f^{dx+c}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{1+a+f^{dx}f^{c}b}{1+a}\right)}{2\ln(f)d}-\frac{\ln\left(\frac{1+a+f^{dx}f^{c}b}{1+a}\right)x}{2}-\frac{\ln\left(\frac{1+a+f^{dx}f^{c}b}{1+a}\right)c}{2d}+\frac{c\ln\left(1+a+f^{dx}f^{c}b\right)}{2d}-$

input
```
int(arctanh(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/ln(f)*(ln(-b*f^(d*x+c))*arctanh(a+b*f^(d*x+c))+1/2*dilog((1-a-b*f^(d*x
+c))/(1-a))+1/2*ln(-b*f^(d*x+c))*ln((1-a-b*f^(d*x+c))/(1-a))-1/2*dilog((-b
*f^(d*x+c)-a-1)/(-1-a))-1/2*ln(-b*f^(d*x+c))*ln((-b*f^(d*x+c)-a-1)/(-1-a))
)
```

3.352.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.69

$$\int \operatorname{arctanh}\left(a+bf^{c+dx}\right)\,dx$$
$$=\frac{dx\log(f)\log\left(-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1}{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1}\right)+c\log\left(b\cosh\left((dx+c)\log(f)\right)+b\sinh\left((dx+c\right.\right.}{}$$

input
```
integrate(arctanh(a+b*f^(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(d*x*log(f)*log(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f))
+ a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)) +
c*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log(f)
- c*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)*log(f
) - (d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(
f)) + a + 1)/(a + 1)) + (d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b
*sinh((d*x + c)*log(f)) + a - 1)/(a - 1)) - dilog(-(b*cosh((d*x + c)*log(f
)) + b*sinh((d*x + c)*log(f)) + a + 1)/(a + 1) + 1) + dilog(-(b*cosh((d*x
+ c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1))/(d*log(f))
```

3.352.6 Sympy [F]

$$\int \operatorname{arctanh}\left(a+bf^{c+dx}\right)\,dx=\int \operatorname{atanh}\left(a+bf^{c+dx}\right)dx$$

input
```
integrate(atanh(a+b*f**(d*x+c)),x)
```

output
```
Integral(atanh(a + b*f**(c + d*x)), x)
```

3.352.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.20

$$\int \operatorname{arctanh}\left(a+bf^{c+dx}\right)\,dx=\frac{(dx+c)\operatorname{artanh}\left(bf^{dx+c}+a\right)}{d}-\frac{(dx+c)b\left(\frac{\log\left(bf^{dx+c}+a+1\right)}{b}-\frac{\log\left(bf^{dx+c}+a-1\right)}{b}\right)\log\left(f\right)-b\left(\frac{\log\left(bf^{dx+c}+a+1\right)\log\left(-\frac{bf^{dx+c}+a+1}{a+1}+1\right)+\operatorname{Li}_2\left(\frac{bf^{dx+c}+a+1}{a+1}\right)}{b}\right.}{2\,d\log\left(f\right)}$$

input
```
integrate(arctanh(a+b*f^(d*x+c)),x, algorithm="maxima")
```

output
```
(d*x + c)*arctanh(b*f^(d*x + c) + a)/d - 1/2*((d*x + c)*b*(log(b*f^(d*x +
c) + a + 1)/b - log(b*f^(d*x + c) + a - 1)/b)*log(f) - b*((log(b*f^(d*x +
c) + a + 1)*log(-(b*f^(d*x + c) + a + 1)/(a + 1) + 1) + dilog((b*f^(d*x +
c) + a + 1)/(a + 1)))/b - (log(b*f^(d*x + c) + a - 1)*log(-(b*f^(d*x + c)
+ a - 1)/(a - 1) + 1) + dilog((b*f^(d*x + c) + a - 1)/(a - 1)))/b))/(d*log
(f))
```

3.352.8 Giac [F(-2)]

Exception generated.

$$\int \operatorname{arctanh}\left(a + bf^{c+dx}\right) dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctanh(a+b*f^(d*x+c)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,2,0,0,0]%%%}+%%%{2,[0,1,1,1,1,0]%%%}+%%%{-2,[0,1,1,0,0,0]%%%}+%%%{1,[0,
1,0,2,0,1]%%
```

3.352.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}\left(a + bf^{c+dx}\right) dx = \int \operatorname{atanh}\left(a + b\,f^{c+d\,x}\right) dx$$

input
```
int(atanh(a + b*f^(c + d*x)),x)
```

output
```
int(atanh(a + b*f^(c + d*x)), x)
```

3.353 $\int x\text{arctanh}\left(a+bf^{c+dx}\right)\,dx$

3.353.1 Optimal result

Integrand size = 14, antiderivative size = 211

$$\begin{aligned}\int x\text{arctanh}\left(a+bf^{c+dx}\right)\,dx=&-\frac{1}{4}x^2\log\left(1-a-bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1+a+bf^{c+dx}\right)\\&+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\left(1+\frac{bf^{c+dx}}{1+a}\right)\\&+\frac{x\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{2d\log(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{1+a}\right)}{2d\log(f)}\\&-\frac{\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{2d^2\log^2(f)}+\frac{\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{1+a}\right)}{2d^2\log^2(f)}\end{aligned}$$

output
```
-1/4*x^2*ln(1-a-b*f^(d*x+c))+1/4*x^2*ln(1+a+b*f^(d*x+c))+1/4*x^2*ln(1-b*f^
(d*x+c)/(1-a))-1/4*x^2*ln(1+b*f^(d*x+c)/(1+a))+1/2*x*polylog(2,b*f^(d*x+c)
/(1-a))/d/ln(f)-1/2*x*polylog(2,-b*f^(d*x+c)/(1+a))/d/ln(f)-1/2*polylog(3,
b*f^(d*x+c)/(1-a))/d^2/ln(f)^2+1/2*polylog(3,-b*f^(d*x+c)/(1+a))/d^2/ln(f)
^2
```

3.353.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.84

$$\int x\text{arctanh}\left(a+bf^{c+dx}\right)\,dx = \frac{2d^2x^2\text{arctanh}\left(a+bf^{c+dx}\right)\log^2(f)+d^2x^2\log^2(f)\log\left(1+\frac{bf^{c+dx}}{-1+a}\right)-d^2x^2\log^2(f)\log\left(1+\frac{bf^{c+dx}}{1+a}\right)+2dx\,\text{l}}{}$$

input
```
Integrate[x*ArcTanh[a + b*f^(c + d*x)],x]
```

output
```
(2*d^2*x^2*ArcTanh[a + b*f^(c + d*x)]*Log[f]^2 + d^2*x^2*Log[f]^2*Log[1 +
(b*f^(c + d*x))/(-1 + a)] - d^2*x^2*Log[f]^2*Log[1 + (b*f^(c + d*x))/(1 +
a)] + 2*d*x*Log[f]*PolyLog[2, -((b*f^(c + d*x))/(-1 + a))] - 2*d*x*Log[f]*
PolyLog[2, -((b*f^(c + d*x))/(1 + a))] - 2*PolyLog[3, -((b*f^(c + d*x))/(-
1 + a))] + 2*PolyLog[3, -((b*f^(c + d*x))/(1 + a))])/(4*d^2*Log[f]^2)
```

3.353.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6767, 3012, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}\left(a+bf^{c+dx}\right)\,dx$$

$$\Big\downarrow \text{ 6767}$$

$$\frac{1}{2}\int x\log\left(bf^{c+dx}+a+1\right)dx-\frac{1}{2}\int x\log\left(-bf^{c+dx}-a+1\right)dx$$

$$\Big\downarrow \text{ 3012}$$

$$\frac{1}{2}\left(-\int x\log\left(1-\frac{bf^{c+dx}}{1-a}\right)dx-\frac{1}{2}x^2\log\left(-a-bf^{c+dx}+1\right)+\frac{1}{2}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)\right)+\frac{1}{2}\left(\int x\log\left(\frac{bf^{c+dx}}{a+1}+1\right)dx+\frac{1}{2}x^2\log\left(a+bf^{c+dx}+1\right)-\frac{1}{2}x^2\log\left(\frac{bf^{c+dx}}{a+1}+1\right)\right)$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{1}{2}\left(-\frac{\int \text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)dx}{d\log(f)}+\frac{x\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{d\log(f)}-\frac{1}{2}x^2\log\left(-a-bf^{c+dx}+1\right)+\frac{1}{2}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)\right)$$
$$\frac{1}{2}\left(\frac{\int \text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)dx}{d\log(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)}{d\log(f)}+\frac{1}{2}x^2\log\left(a+bf^{c+dx}+1\right)-\frac{1}{2}x^2\log\left(\frac{bf^{c+dx}}{a+1}+1\right)\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}\left(-\frac{\int f^{-c-dx}\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)df^{c+dx}}{d^2\log^2(f)}+\frac{x\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{d\log(f)}-\frac{1}{2}x^2\log\left(-a-bf^{c+dx}+1\right)+\frac{1}{2}x^2\log\left(1-\right.\right.$$
$$\frac{1}{2}\left(\frac{\int f^{-c-dx}\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)df^{c+dx}}{d^2\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)}{d\log(f)}+\frac{1}{2}x^2\log\left(a+bf^{c+dx}+1\right)-\frac{1}{2}x^2\log\left(\frac{bf^{c}}{a-}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}\left(-\frac{\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{d^2\log^2(f)}+\frac{x\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{d\log(f)}-\frac{1}{2}x^2\log\left(-a-bf^{c+dx}+1\right)+\frac{1}{2}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)\right)+$$
$$\frac{1}{2}\left(\frac{\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)}{d^2\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)}{d\log(f)}+\frac{1}{2}x^2\log\left(a+bf^{c+dx}+1\right)-\frac{1}{2}x^2\log\left(\frac{bf^{c+dx}}{a+1}+1\right)\right)$$

input
```
Int[x*ArcTanh[a + b*f^(c + d*x)],x]
```

output
```
(-1/2*(x^2*Log[1 - a - b*f^(c + d*x)]) + (x^2*Log[1 - (b*f^(c + d*x))/(1 -
 a)])/2 + (x*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(d*Log[f]) - PolyLog[3,
(b*f^(c + d*x))/(1 - a)]/(d^2*Log[f]^2))/2 + ((x^2*Log[1 + a + b*f^(c + d*
x)])/2 - (x^2*Log[1 + (b*f^(c + d*x))/(1 + a)])/2 - (x*PolyLog[2, -((b*f^(
c + d*x))/(1 + a))])/(d*Log[f]) + PolyLog[3, -((b*f^(c + d*x))/(1 + a))]/(
d^2*Log[f]^2))/2
```

3.353. $\int x\operatorname{arctanh}\left(a+bf^{c+dx}\right)\,dx$

3.353.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3012
```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

rule 6767
```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Simp[1/2   Int[x
^m*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[
m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.353. $\int x\text{arctanh}\left(a+bf^{c+dx}\right)\,dx$

3.353.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(195) = 390.

Time = 0.69 (sec) , antiderivative size = 596, normalized size of antiderivative = 2.82

method	result
risch	$\frac{x^2 \ln(1+a+b f^{dx+c})}{4} - \frac{x^2 \ln(1-a-b f^{dx+c})}{4} + \frac{\ln\left(1-\frac{b f^{dx} f^c}{1-a}\right) x^2}{4} + \frac{\ln\left(1-\frac{b f^{dx} f^c}{1-a}\right) c x}{2d} + \frac{\ln\left(1-\frac{b f^{dx} f^c}{1-a}\right) c^2}{4d^2} + \frac{\mathrm{polylog}\left(2, \cdot\right.}{2\ln(}$

input
```
int(x*arctanh(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2*ln(1+a+b*f^(d*x+c))-1/4*x^2*ln(1-a-b*f^(d*x+c))+1/4*ln(1-b*f^(d*x)
*f^c/(1-a))*x^2+1/2/d*ln(1-b*f^(d*x)*f^c/(1-a))*c*x+1/4/d^2*ln(1-b*f^(d*x)
*f^c/(1-a))*c^2+1/2/ln(f)/d*polylog(2,b*f^(d*x)*f^c/(1-a))*x+1/2/ln(f)/d^2
*polylog(2,b*f^(d*x)*f^c/(1-a))*c-1/2/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^c/
(1-a))+1/4/d^2*c^2*ln(1-a-f^(d*x)*f^c*b)-1/2/ln(f)/d^2*c*dilog((f^(d*x)*f^
c*b+a-1)/(-1+a))-1/2/d*c*ln((f^(d*x)*f^c*b+a-1)/(-1+a))*x-1/2/d^2*c^2*ln((
f^(d*x)*f^c*b+a-1)/(-1+a))-1/4*ln(1-b*f^(d*x)*f^c/(-1-a))*x^2-1/2/d*ln(1-b
*f^(d*x)*f^c/(-1-a))*c*x-1/4/d^2*ln(1-b*f^(d*x)*f^c/(-1-a))*c^2-1/2/ln(f)/
d*polylog(2,b*f^(d*x)*f^c/(-1-a))*x-1/2/ln(f)/d^2*polylog(2,b*f^(d*x)*f^c/
(-1-a))*c+1/2/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^c/(-1-a))-1/4/d^2*c^2*ln(1
+a+f^(d*x)*f^c*b)+1/2/ln(f)/d^2*c*dilog((1+a+f^(d*x)*f^c*b)/(1+a))+1/2/d*c
*ln((1+a+f^(d*x)*f^c*b)/(1+a))*x+1/2/d^2*c^2*ln((1+a+f^(d*x)*f^c*b)/(1+a))
```

3.353.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(188) = 376.

Time = 0.26 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.88

$$\int x \operatorname{arctanh}\left(a + b f^{c+dx}\right) \, dx$$
$$= \frac{d^2 x^2 \log(f)^2 \log\left(-\frac{b \cosh((dx+c)\log(f)) + b \sinh((dx+c)\log(f)) + a + 1}{b \cosh((dx+c)\log(f)) + b \sinh((dx+c)\log(f)) + a - 1}\right) - c^2 \log\left(b \cosh\left((dx+c)\log(f)\right) + b \sinh\left((dx\right.\right.}{}$$

input
```
integrate(x*arctanh(a+b*f^(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(d^2*x^2*log(f)^2*log(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*lo
g(f)) + a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a -
1)) - c^2*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)
*log(f)^2 + c^2*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
a - 1)*log(f)^2 - 2*d*x*dilog(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c
)*log(f)) + a + 1)/(a + 1) + 1)*log(f) + 2*d*x*dilog(-(b*cosh((d*x + c)*lo
g(f)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1)*log(f) - (d^2*x^2 -
 c^2)*log(f)^2*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
a + 1)/(a + 1)) + (d^2*x^2 - c^2)*log(f)^2*log((b*cosh((d*x + c)*log(f)) +
 b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1)) + 2*polylog(3, -(b*cosh((d*x +
 c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a + 1)) - 2*polylog(3, -(b*cosh((
d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a - 1)))/(d^2*log(f)^2)
```

3.353.6 Sympy [F]

$$\int x\text{arctanh}\left(a+bf^{c+dx}\right)\,dx=\int x\,\text{atanh}\left(a+bf^{c+dx}\right)dx$$

input
```
integrate(x*atanh(a+b*f**(d*x+c)),x)
```

output
```
Integral(x*atanh(a + b*f**(c + d*x)), x)
```

3.353.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.92

$$\int x\text{arctanh}\left(a+bf^{c+dx}\right)\,dx=$$
$$-\frac{1}{4}bd\left(\frac{d^2x^2\log\left(\frac{bf^{dx}f^c}{a+1}+1\right)\log(f)^2+2\,dx\text{Li}_2\left(-\frac{bf^{dx}f^c}{a+1}\right)\log(f)-2\,\text{Li}_3\left(-\frac{bf^{dx}f^c}{a+1}\right)}{bd^3\log(f)^3}-\frac{d^2x^2\log\left(\frac{bf^{dx}f^c}{a-1}+1\right.}{}\right.$$
$$+\frac{1}{2}x^2\,\text{artanh}\left(bf^{dx+c}+a\right)$$

input
```
integrate(x*arctanh(a+b*f^(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*b*d*((d^2*x^2*log(b*f^(d*x)*f^c/(a + 1) + 1)*log(f)^2 + 2*d*x*dilog(-
b*f^(d*x)*f^c/(a + 1))*log(f) - 2*polylog(3, -b*f^(d*x)*f^c/(a + 1)))/(b*d
^3*log(f)^3) - (d^2*x^2*log(b*f^(d*x)*f^c/(a - 1) + 1)*log(f)^2 + 2*d*x*di
log(-b*f^(d*x)*f^c/(a - 1))*log(f) - 2*polylog(3, -b*f^(d*x)*f^c/(a - 1)))
/(b*d^3*log(f)^3))*log(f) + 1/2*x^2*arctanh(b*f^(d*x + c) + a)
```

3.353.8 Giac [F]

$$\int x\text{arctanh}\left(a+bf^{c+dx}\right)\,dx=\int x\,\text{artanh}\left(bf^{dx+c}+a\right)\,dx$$

input
```
integrate(x*arctanh(a+b*f^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x*arctanh(b*f^(d*x + c) + a), x)
```

3.353.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arctanh}\left(a+bf^{c+dx}\right)\,dx=\int x\,\text{atanh}\left(a+b\,f^{c+d\,x}\right)\,dx$$

input
```
int(x*atanh(a + b*f^(c + d*x)),x)
```

output
```
int(x*atanh(a + b*f^(c + d*x)), x)
```

3.354 $\int x^2 \text{arctanh}\left(a+bf^{c+dx}\right)\,dx$

3.354.1 Optimal result

Integrand size = 16, antiderivative size = 264

$$\int x^2 \text{arctanh}\left(a+bf^{c+dx}\right)\,dx = -\frac{1}{6}x^3\log\left(1-a-bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1+a+bf^{c+dx}\right) + \frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log\left(1+\frac{bf^{c+dx}}{1+a}\right) + \frac{x^2\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{2d\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{1+a}\right)}{2d\log(f)} - \frac{x\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{d^2\log^2(f)}+\frac{x\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{1+a}\right)}{d^2\log^2(f)} + \frac{\text{PolyLog}\left(4,\frac{bf^{c+dx}}{1-a}\right)}{d^3\log^3(f)}-\frac{\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{1+a}\right)}{d^3\log^3(f)}$$

output
```
-1/6*x^3*ln(1-a-b*f^(d*x+c))+1/6*x^3*ln(1+a+b*f^(d*x+c))+1/6*x^3*ln(1-b*f^
(d*x+c)/(1-a))-1/6*x^3*ln(1+b*f^(d*x+c)/(1+a))+1/2*x^2*polylog(2,b*f^(d*x+
c)/(1-a))/d/ln(f)-1/2*x^2*polylog(2,-b*f^(d*x+c)/(1+a))/d/ln(f)-x*polylog(
3,b*f^(d*x+c)/(1-a))/d^2/ln(f)^2+x*polylog(3,-b*f^(d*x+c)/(1+a))/d^2/ln(f)
^2+polylog(4,b*f^(d*x+c)/(1-a))/d^3/ln(f)^3-polylog(4,-b*f^(d*x+c)/(1+a))/
d^3/ln(f)^3
```

3.354.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.89

$$\int x^2 \operatorname{arctanh}\left(a+bf^{c+dx}\right) dx$$
$$= \frac{2d^3x^3\operatorname{arctanh}\left(a+bf^{c+dx}\right)\log^3(f)+d^3x^3\log^3(f)\log\left(1+\frac{bf^{c+dx}}{-1+a}\right)-d^3x^3\log^3(f)\log\left(1+\frac{bf^{c+dx}}{1+a}\right)+3d^2x^2\cdots}{\cdots}$$

input
```
Integrate[x^2*ArcTanh[a + b*f^(c + d*x)],x]
```

output
```
(2*d^3*x^3*ArcTanh[a + b*f^(c + d*x)]*Log[f]^3 + d^3*x^3*Log[f]^3*Log[1 +
(b*f^(c + d*x))/(-1 + a)] - d^3*x^3*Log[f]^3*Log[1 + (b*f^(c + d*x))/(1 +
a)] + 3*d^2*x^2*Log[f]^2*PolyLog[2, -((b*f^(c + d*x))/(-1 + a))] - 3*d^2*x
^2*Log[f]^2*PolyLog[2, -((b*f^(c + d*x))/(1 + a))] - 6*d*x*Log[f]*PolyLog[
3, -((b*f^(c + d*x))/(-1 + a))] + 6*d*x*Log[f]*PolyLog[3, -((b*f^(c + d*x)
)/(1 + a))] + 6*PolyLog[4, -((b*f^(c + d*x))/(-1 + a))] - 6*PolyLog[4, -((
b*f^(c + d*x))/(1 + a))])/(6*d^3*Log[f]^3)
```

3.354.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6767, 3012, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \operatorname{arctanh}\left(a+bf^{c+dx}\right) dx$$
$$\Big\downarrow \; 6767$$
$$\frac{1}{2}\int x^2\log\left(bf^{c+dx}+a+1\right)dx-\frac{1}{2}\int x^2\log\left(-bf^{c+dx}-a+1\right)dx$$
$$\Big\downarrow \; 3012$$
$$\frac{1}{2}\left(-\int x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)dx-\frac{1}{3}x^3\log\left(-a-bf^{c+dx}+1\right)+\frac{1}{3}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)\right)+\frac{1}{2}\left(\int x^2\log\left(\frac{bf^{c+dx}}{a+1}+1\right)dx+\frac{1}{3}x^3\log\left(a+bf^{c+dx}+1\right)-\frac{1}{3}x^3\log\left(\frac{bf^{c+dx}}{a+1}+1\right)\right)$$

$$\Big\downarrow \; 3011$$

$$\frac{1}{2}\left(-\frac{2\int x\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)dx}{d\log(f)}+\frac{x^2\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{d\log(f)}-\frac{1}{3}x^3\log\left(-a-bf^{c+dx}+1\right)+\frac{1}{3}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right.\right.$$
$$\frac{1}{2}\left(\frac{2\int x\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)dx}{d\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)}{d\log(f)}+\frac{1}{3}x^3\log\left(a+bf^{c+dx}+1\right)-\frac{1}{3}x^3\log\left(\frac{bf^{c+dx}}{a+1}+\right.\right.$$

$$\Big\downarrow \; 7163$$

$$\frac{1}{2}\left(-\frac{2\left(\frac{x\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{d\log(f)}-\frac{\int \text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)dx}{d\log(f)}\right)}{d\log(f)}+\frac{x^2\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{d\log(f)}-\frac{1}{3}x^3\log\left(-a-bf^{c+dx}+1\right)+\frac{1}{3}\right.$$
$$\frac{1}{2}\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)}{d\log(f)}-\frac{\int \text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)dx}{d\log(f)}\right)}{d\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)}{d\log(f)}+\frac{1}{3}x^3\log\left(a+bf^{c+dx}+1\right)-\right.$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{2}\left(-\frac{2\left(\frac{x\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{d\log(f)}-\frac{\int f^{-c-dx}\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)df^{c+dx}}{d^2\log^2(f)}\right)}{d\log(f)}+\frac{x^2\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{d\log(f)}-\frac{1}{3}x^3\log\left(-a-bf^{c+dx}\right.\right.$$
$$\frac{1}{2}\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)}{d\log(f)}-\frac{\int f^{-c-dx}\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)df^{c+dx}}{d^2\log^2(f)}\right)}{d\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)}{d\log(f)}+\frac{1}{3}x^3\log\left(a+bf^{c+dx}\right.\right.$$

$$\Big\downarrow \; 7143$$

3.354. $\int x^2\text{arctanh}\left(a+bf^{c+dx}\right)\,dx$

$$\frac{1}{2}\left(-\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{d\log(f)}-\frac{\mathrm{PolyLog}\left(4,\frac{bf^{c+dx}}{1-a}\right)}{d^2\log^2(f)}\right)}{d\log(f)}+\frac{x^2\,\mathrm{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{d\log(f)}-\frac{1}{3}x^3\log\left(-a-bf^{c+dx}+1\right)+\frac{1}{3}x^3\right.$$

$$\frac{1}{2}\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)}{d\log(f)}-\frac{\mathrm{PolyLog}\left(4,-\frac{bf^{c+dx}}{a+1}\right)}{d^2\log^2(f)}\right)}{d\log(f)}-\frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)}{d\log(f)}+\frac{1}{3}x^3\log\left(a+bf^{c+dx}+1\right)-\frac{1}{3}x^3\right.$$

input
```
Int[x^2*ArcTanh[a + b*f^(c + d*x)],x]
```

output
```
(-1/3*(x^3*Log[1 - a - b*f^(c + d*x)]) + (x^3*Log[1 - (b*f^(c + d*x))/(1 -
 a)])/3 + (x^2*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(d*Log[f]) - (2*((x*Po
lyLog[3, (b*f^(c + d*x))/(1 - a)])/(d*Log[f]) - PolyLog[4, (b*f^(c + d*x))
/(1 - a)]/(d^2*Log[f]^2)))/(d*Log[f]))/2 + ((x^3*Log[1 + a + b*f^(c + d*x)
])/3 - (x^3*Log[1 + (b*f^(c + d*x))/(1 + a)])/3 - (x^2*PolyLog[2, -((b*f^(
c + d*x))/(1 + a))])/(d*Log[f]) + (2*((x*PolyLog[3, -((b*f^(c + d*x))/(1 +
 a))])/(d*Log[f]) - PolyLog[4, -((b*f^(c + d*x))/(1 + a))]/(d^2*Log[f]^2))
)/(d*Log[f]))/2
```

3.354.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3012
```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

rule 6767
```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Simp[1/2   Int[x
^m*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[
m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.354.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(252) = 504$.

Time = 1.14 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.55

method	result
risch	$\frac{x^3 \ln(1+a+b f^{dx+c})}{6} - \frac{x^3 \ln(1-a-b f^{dx+c})}{6} + \frac{\ln\left(1-\frac{b f^{dx} f^{c}}{1-a}\right) x^3}{6} - \frac{\ln\left(1-\frac{b f^{dx} f^{c}}{1-a}\right) x c^2}{2d^2} - \frac{\ln\left(1-\frac{b f^{dx} f^{c}}{1-a}\right) c^3}{3d^3} + \frac{\mathrm{polylog}\left(2 \right.}{2\mathrm{l}}$

input
```
int(x^2*arctanh(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^3*ln(1+a+b*f^(d*x+c))-1/6*x^3*ln(1-a-b*f^(d*x+c))+1/6*ln(1-b*f^(d*x)
*f^c/(1-a))*x^3-1/2/d^2*ln(1-b*f^(d*x)*f^c/(1-a))*x*c^2-1/3/d^3*ln(1-b*f^(
d*x)*f^c/(1-a))*c^3+1/2/ln(f)/d*polylog(2,b*f^(d*x)*f^c/(1-a))*x^2-1/2/ln(
f)/d^3*polylog(2,b*f^(d*x)*f^c/(1-a))*c^2-1/ln(f)^2/d^2*polylog(3,b*f^(d*x
)*f^c/(1-a))*x+1/ln(f)^3/d^3*polylog(4,b*f^(d*x)*f^c/(1-a))-1/6/d^3*c^3*ln
(1-a-f^(d*x)*f^c*b)+1/2/ln(f)/d^3*c^2*dilog((f^(d*x)*f^c*b+a-1)/(-1+a))+1/
2/d^2*c^2*ln((f^(d*x)*f^c*b+a-1)/(-1+a))*x+1/2/d^3*c^3*ln((f^(d*x)*f^c*b+a
-1)/(-1+a))-1/6*ln(1-b*f^(d*x)*f^c/(-1-a))*x^3+1/2/d^2*ln(1-b*f^(d*x)*f^c/
(-1-a))*x*c^2+1/3/d^3*ln(1-b*f^(d*x)*f^c/(-1-a))*c^3-1/2/ln(f)/d*polylog(2
,b*f^(d*x)*f^c/(-1-a))*x^2+1/2/ln(f)/d^3*polylog(2,b*f^(d*x)*f^c/(-1-a))*c
^2+1/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^c/(-1-a))*x-1/ln(f)^3/d^3*polylog(4
,b*f^(d*x)*f^c/(-1-a))+1/6/d^3*c^3*ln(1+a+f^(d*x)*f^c*b)-1/2/ln(f)/d^3*c^2
*dilog((1+a+f^(d*x)*f^c*b)/(1+a))-1/2/d^2*c^2*ln((1+a+f^(d*x)*f^c*b)/(1+a)
)*x-1/2/d^3*c^3*ln((1+a+f^(d*x)*f^c*b)/(1+a))
```

3.354.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.82

$$\int x^2 \text{arctanh}\left(a + bf^{c+dx}\right)\, dx$$
$$= \frac{d^3x^3\log(f)^3\log\left(-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1}{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1}\right) - 3\,d^2x^2\text{Li}_2\left(-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f}{a+1}\right.}{}$$

input
```
integrate(x^2*arctanh(a+b*f^(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(d^3*x^3*log(f)^3*log(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*lo
g(f)) + a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a -
1)) - 3*d^2*x^2*dilog(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)
) + a + 1)/(a + 1) + 1)*log(f)^2 + 3*d^2*x^2*dilog(-(b*cosh((d*x + c)*log(
f)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1)*log(f)^2 + c^3*log(b*
cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log(f)^3 - c^3*
log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)*log(f)^3
- (d^3*x^3 + c^3)*log(f)^3*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c
)*log(f)) + a + 1)/(a + 1)) + (d^3*x^3 + c^3)*log(f)^3*log((b*cosh((d*x +
c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1)) + 6*d*x*log(f)*pol
ylog(3, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a + 1)) -
6*d*x*log(f)*polylog(3, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(
f)))/(a - 1)) - 6*polylog(4, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)
*log(f)))/(a + 1)) + 6*polylog(4, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x
 + c)*log(f)))/(a - 1)))/(d^3*log(f)^3)
```

3.354.6 Sympy [F]

$$\int x^2 \text{arctanh}\left(a+bf^{c+dx}\right)\,dx=\int x^2\operatorname{atanh}\left(a+bf^{c+dx}\right)dx$$

input
```
integrate(x**2*atanh(a+b*f**(d*x+c)),x)
```

output
```
Integral(x**2*atanh(a + b*f**(c + d*x)), x)
```

3.354.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.96

$$\int x^2 \text{arctanh}\left(a+bf^{c+dx}\right)\,dx=\frac{1}{3}\,x^3\,\text{artanh}\left(bf^{dx+c}+a\right)$$
$$-\frac{1}{6}\,bd\left(\frac{d^3x^3\log\left(\frac{bf^{dx}f^c}{a+1}+1\right)\log\left(f\right)^3+3\,d^2x^2\text{Li}_2\left(-\frac{bf^{dx}f^c}{a+1}\right)\log\left(f\right)^2-6\,dx\log\left(f\right)\text{Li}_3\left(-\frac{bf^{dx}f^c}{a+1}\right)+6\,\text{Li}_4\left(-\right.}{bd^4\log\left(f\right)^4}\right.$$

input
```
integrate(x^2*arctanh(a+b*f^(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctanh(b*f^(d*x + c) + a) - 1/6*b*d*((d^3*x^3*log(b*f^(d*x)*f^c/(
a + 1) + 1)*log(f)^3 + 3*d^2*x^2*dilog(-b*f^(d*x)*f^c/(a + 1))*log(f)^2 -
6*d*x*log(f)*polylog(3, -b*f^(d*x)*f^c/(a + 1)) + 6*polylog(4, -b*f^(d*x)*
f^c/(a + 1)))/(b*d^4*log(f)^4) - (d^3*x^3*log(b*f^(d*x)*f^c/(a - 1) + 1)*l
og(f)^3 + 3*d^2*x^2*dilog(-b*f^(d*x)*f^c/(a - 1))*log(f)^2 - 6*d*x*log(f)*
polylog(3, -b*f^(d*x)*f^c/(a - 1)) + 6*polylog(4, -b*f^(d*x)*f^c/(a - 1)))
/(b*d^4*log(f)^4))*log(f)
```

3.354.8 Giac [F]

$$\int x^2 \text{arctanh}\left(a + bf^{c+dx}\right) \, dx = \int x^2 \, \text{artanh}\left(bf^{dx+c} + a\right) \, dx$$

input
```
integrate(x^2*arctanh(a+b*f^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(b*f^(d*x + c) + a), x)
```

3.354.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arctanh}\left(a + bf^{c+dx}\right) \, dx = \int x^2 \, \text{atanh}\left(a + b \, f^{c+d\,x}\right) \, dx$$

input
```
int(x^2*atanh(a + b*f^(c + d*x)),x)
```

output
```
int(x^2*atanh(a + b*f^(c + d*x)), x)
```

3.355 $\int e^{c(a+bx)}\textbf{arctanh}(\sinh(ac+bcx))\,dx$

3.355.1 Optimal result

Integrand size = 20, antiderivative size = 107

$$\int e^{c(a+bx)}\text{arctanh}(\sinh(ac+bcx))\,dx=\frac{e^{ac+bcx}\text{arctanh}(\sinh(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}-e^{2c(a+bx)}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(3+2\sqrt{2}-e^{2c(a+bx)}\right)}{2bc}$$

output
```
exp(b*c*x+a*c)*arctanh(sinh(c*(b*x+a)))/b/c+1/2*ln(3-exp(2*c*(b*x+a))-2*2^
(1/2))*(1-2^(1/2))/b/c+1/2*ln(3-exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/
c
```

3.355.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.43

$$\int e^{c(a+bx)}\text{arctanh}(\sinh(ac+bcx))\,dx=\frac{-2e^{c(a+bx)}\text{arctanh}\left(\frac{1}{2}e^{-c(a+bx)}-\frac{1}{2}e^{c(a+bx)}\right)-2\sqrt{2}\text{arctanh}\left(\frac{-1+e^{c(a+bx)}}{\sqrt{2}}\right)+2\sqrt{2}\text{arctanh}\left(\frac{1+e^{c(a+bx)}}{\sqrt{2}}\right)+\log\left(1\right.}{2bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcTanh[Sinh[a*c + b*c*x]],x]
```

output
```
(-2*E^(c*(a + b*x))*ArcTanh[1/(2*E^(c*(a + b*x))) - E^(c*(a + b*x))/2] - 2
*Sqrt[2]*ArcTanh[(-1 + E^(c*(a + b*x)))/Sqrt[2]] + 2*Sqrt[2]*ArcTanh[(1 +
E^(c*(a + b*x)))/Sqrt[2]] + Log[1 - 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))]
 + Log[1 + 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))])/(2*b*c)
```

3.355.3 Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {7281, 6829, 2720, 27, 1576, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\text{arctanh}(\sinh(ac+bcx))\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc}\text{arctanh}(\sinh(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{6829}$$

$$\frac{e^{ac+bcx}\text{arctanh}(\sinh(ac+bcx))-\int \frac{e^{ac+bxc}\cosh(ac+bxc)}{1-\sinh^2(ac+bxc)}d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{2720}$$

$$\frac{e^{ac+bcx}\text{arctanh}(\sinh(ac+bcx))-\int -\frac{2e^{ac+bxc}\left(1+e^{2ac+2bxc}\right)}{1-6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{e^{ac+bxc}\left(1+e^{2ac+2bxc}\right)}{1-6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc}+e^{ac+bcx}\text{arctanh}(\sinh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{1576}$$

$$\frac{\int \frac{1+e^{2ac+2bxc}}{1-5e^{2ac+2bxc}}de^{2ac+2bxc}+e^{ac+bcx}\text{arctanh}(\sinh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{1141}$$

$$\frac{\int \left(-\frac{1+\sqrt{2}}{2\left(-ac-bxc+2\sqrt{2}+3\right)}-\frac{1-\sqrt{2}}{2\left(-ac-bxc-2\sqrt{2}+3\right)}\right)de^{2ac+2bxc}+e^{ac+bcx}\text{arctanh}(\sinh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{ac+bcx}\text{arctanh}(\sinh(ac+bcx))+\frac{1}{2}\left(1-\sqrt{2}\right)\log\left(-ac-bcx-2\sqrt{2}+3\right)+\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(-ac-bcx+2\sqrt{2}+3\right)}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcTanh[Sinh[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcTanh[Sinh[a*c + b*c*x]] + ((1 - Sqrt[2])*Log[3 - 2*Sqr
t[2] - a*c - b*c*x])/2 + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2] - a*c - b*c*x])/
2)/(b*c)
```

3.355.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.355. $\int e^{c(a+bx)}\text{arctanh}(\sinh(ac+bcx))\,dx$

rule 6829
```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcTanh[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.355.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.45 (sec) , antiderivative size = 868, normalized size of antiderivative = 8.11

method	result	size
risch	Expression too large to display	868

input
```
int(exp(c*(b*x+a))*arctanh(sinh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)+1/4*I/b/c*P
i*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^3*exp(c*(
b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I
*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a
))+1))*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+
a))+1))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^2*e
xp(c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp
(c*(b*x+a))+1))^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn
(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))
-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)
)^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-
1))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*
(b*x+a))-1/2*I/b/c*exp(c*(b*x+a))*Pi-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*
csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^2*exp(c*(b*
x+a))-1/4*I/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I*ex
p(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1
))*exp(c*(b*x+a))-1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-2*exp(c*(b*x+
a))-1)+1/2/b/c*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^2)*2^(1/2)-1/2/b/c*ln(exp(2
*c*(b*x+a))-(2^(1/2)-1)^2)*2^(1/2)-2*a/b+1/2/b/c*ln(exp(2*c*(b*x+a))-(1+2^
(1/2))^2)+1/2/b/c*ln(exp(2*c*(b*x+a))-(2^(1/2)-1)^2)
```

3.355.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(90) = 180$.

Time = 0.25 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.19

$$\int e^{c(a+bx)}\text{arctanh}(\sinh(ac+bcx))\,dx$$

$$= \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(-\frac{\sinh(bcx+ac)+1}{\sinh(bcx+ac)-1}\right)+\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh(bcx+ac)^2-4\left(3\sqrt{2}+4\right)\cosh(bcx\cdots}{\cosh(bcx+ac)^2\cdots}\right.}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(sinh(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log(-(sinh(b*c*x + a*c) + 1)/
(sinh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c
)^2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2)
 + 3)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c
*x + a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/
(cosh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x
+ a*c)^2)))/(b*c)
```

3.355.6 Sympy [F]

$$\int e^{c(a+bx)}\text{arctanh}(\sinh(ac+bcx))\,dx = e^{ac}\int e^{bcx}\,\text{atanh}\left(\sinh\left(ac+bcx\right)\right)dx$$

input
```
integrate(exp(c*(b*x+a))*atanh(sinh(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*atanh(sinh(a*c + b*c*x)), x)
```

3.355.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(90) = 180$.

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.72

$$\int e^{c(a+bx)}\text{arctanh}(\sinh(ac+bcx))\,dx = \frac{\text{artanh}\left(\sinh\left(bcx+ac\right)\right)e^{((bx+a)c)}}{bc} + \frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}+1}{\sqrt{2}+e^{(bcx+ac)}-1}\right)}{2\,bc} - \frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}-1}{\sqrt{2}+e^{(bcx+ac)}+1}\right)}{2\,bc} + \frac{\log\left(e^{(2\,bcx+2\,ac)}+2\,e^{(bcx+ac)}-1\right)}{2\,bc} + \frac{\log\left(e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}-1\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(sinh(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arctanh(sinh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(sqrt(
2) - e^(b*c*x + a*c) + 1)/(sqrt(2) + e^(b*c*x + a*c) - 1))/(b*c) - 1/2*sqr
t(2)*log(-(sqrt(2) - e^(b*c*x + a*c) - 1)/(sqrt(2) + e^(b*c*x + a*c) + 1))
/(b*c) + 1/2*log(e^(2*b*c*x + 2*a*c) + 2*e^(b*c*x + a*c) - 1)/(b*c) + 1/2*
log(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) - 1)/(b*c)
```

3.355.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.47

$$\int e^{c(a+bx)}\text{arctanh}(\sinh(ac+bcx))\,dx = \frac{e^{((bx+a)c)}\log\left(-\frac{e^{(bcx+ac)}-e^{(-bcx-ac)}+2}{e^{(bcx+ac)}-e^{(-bcx-ac)}-2}\right)}{2\,bc} + \frac{\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}{\left|4\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}\right)+\log\left(\left|e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right|\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(sinh(b*c*x+a*c)),x, algorithm="giac")
```

output
```
1/2*e^((b*x + a)*c)*log(-(e^(b*c*x + a*c) - e^(-b*c*x - a*c) + 2)/(e^(b*c*
x + a*c) - e^(-b*c*x - a*c) - 2))/(b*c) + 1/2*(sqrt(2)*log(abs(-4*sqrt(2)
+ 2*e^(2*b*c*x + 2*a*c) - 6)/abs(4*sqrt(2) + 2*e^(2*b*c*x + 2*a*c) - 6)) +
 log(abs(e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)))/(b*c)
```

3.355.9 Mupad [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.67

$$\int e^{c(a+bx)}\text{arctanh}(\sinh(ac+bcx))\,dx = \frac{e^{a\,c+b\,c\,x}\,\ln\left(\frac{e^{b\,c\,x}\,e^{a\,c}}{2}-\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}+1\right)}{2\,b\,c} - \frac{e^{a\,c+b\,c\,x}\,\ln\left(\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}-\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+1\right)}{2\,b\,c} + \frac{\ln\left(6\sqrt{2}\,e^{2\,c\,(a+b\,x)}-2\sqrt{2}-8\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}+1\right)}{2\,b\,c} - \frac{\ln\left(2\sqrt{2}-8\,e^{2\,c\,(a+b\,x)}-6\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}-1\right)}{2\,b\,c}$$

input
```
int(exp(c*(a + b*x))*atanh(sinh(a*c + b*c*x)),x)
```

output
```
(exp(a*c + b*c*x)*log((exp(b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2
+ 1))/(2*b*c) - (exp(a*c + b*c*x)*log((exp(-b*c*x)*exp(-a*c))/2 - (exp(b*c
*x)*exp(a*c))/2 + 1))/(2*b*c) + (log(6*2^(1/2)*exp(2*c*(a + b*x)) - 2*2^(1
/2) - 8*exp(2*c*(a + b*x)))*(2^(1/2) + 1))/(2*b*c) - (log(2*2^(1/2) - 8*ex
p(2*c*(a + b*x)) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c)
```

3.355. $\int e^{c(a+bx)}\text{arctanh}(\sinh(ac+bcx))\,dx$

3.356 $\int e^{c(a+bx)}\textbf{arctanh}(\cosh(ac+bcx))\,dx$

3.356.1 Optimal result

Integrand size = 20, antiderivative size = 49

$$\int e^{c(a+bx)}\text{arctanh}(\cosh(ac+bcx))\,dx=\frac{e^{ac+bcx}\text{arctanh}(\cosh(c(a+bx)))}{bc}+\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

output
```
exp(b*c*x+a*c)*arctanh(cosh(c*(b*x+a)))/b/c+ln(1-exp(2*c*(b*x+a)))/b/c
```

3.356.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int e^{c(a+bx)}\text{arctanh}(\cosh(ac+bcx))\,dx =\frac{e^{c(a+bx)}\text{arctanh}\left(\frac{1}{2}e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)\right)+\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcTanh[Cosh[a*c + b*c*x]],x]
```

output
```
(E^(c*(a + b*x))*ArcTanh[(1 + E^(2*c*(a + b*x)))/(2*E^(c*(a + b*x)))] + Lo
g[1 - E^(2*c*(a + b*x))])/(b*c)
```

3.356.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7281, 6829, 25, 2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\text{arctanh}(\cosh(ac+bcx))\,dx$$

$$\Big\downarrow \; 7281$$

$$\frac{\int e^{ac+bxc}\text{arctanh}(\cosh(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \; 6829$$

$$\frac{e^{ac+bcx}\text{arctanh}(\cosh(ac+bcx))-\int -e^{ac+bxc}\text{csch}(ac+bxc)d(ac+bxc)}{bc}$$

$$\Big\downarrow \; 25$$

$$\frac{\int e^{ac+bxc}\text{csch}(ac+bxc)d(ac+bxc)+e^{ac+bcx}\text{arctanh}(\cosh(ac+bcx))}{bc}$$

$$\Big\downarrow \; 2720$$

$$\frac{\int -\frac{2e^{ac+bxc}}{1-e^{2ac+2bxc}}de^{ac+bxc}+e^{ac+bcx}\text{arctanh}(\cosh(ac+bcx))}{bc}$$

$$\Big\downarrow \; 27$$

$$\frac{e^{ac+bcx}\text{arctanh}(\cosh(ac+bcx))-2\int \frac{e^{ac+bxc}}{1-e^{2ac+2bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \; 240$$

$$\frac{e^{ac+bcx}\text{arctanh}(\cosh(ac+bcx))+\log\left(1-e^{2ac+2bcx}\right)}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcTanh[Cosh[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcTanh[Cosh[a*c + b*c*x]] + Log[1 - E^(2*a*c + 2*b*c*x)]
)/(b*c)
```

3.356.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6829
```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcTanh[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.356.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.32 (sec) , antiderivative size = 887, normalized size of antiderivative = 18.10

method	result	size
risch	Expression too large to display	887

input
```
int(exp(c*(b*x+a))*arctanh(cosh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
1/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+1)-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+
a))+1)^2)^3*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*(ex
p(c*(b*x+a))+1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+1)^2)*exp(c*(b*x
+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b
*x+a))-1)^2)^2*exp(c*(b*x+a))-1/2*I/b/c*exp(c*(b*x+a))*Pi-1/4*I/b/c*Pi*csg
n(I*(exp(c*(b*x+a))-1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)^2*e
xp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)^3*exp(c*(b*x+a))+1
/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I*ex
p(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)*exp(c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*(ex
p(c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))-1/4*I/b/c*P
i*csgn(I*(exp(c*(b*x+a))+1))^2*csgn(I*(exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a))
+1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a
))+1)^2)^2*exp(c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+
a))-1)^2)^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x
+a))+1)^2)^3*exp(c*(b*x+a))-1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1))*csgn(I
*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a)
)+1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))-1/4*
I/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)^3*exp(c*(b*x+a))+1/4
*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1))^2*csgn(I*(exp(c*(b*x+a))-1)^2)*exp(c*
(b*x+a))-1/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-1)-2*a/b+1/b/c*ln(exp(2...
```

3.356.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.90

$$\int e^{c(a+bx)}\text{arctanh}(\cosh(ac+bcx))\,dx = \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(-\frac{\cosh(bcx+ac)+1}{\cosh(bcx+ac)-1}\right)+2\log\left(\frac{2\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(cosh(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log(-(cosh(b*c*x + a*c) + 1)/
(cosh(b*c*x + a*c) - 1)) + 2*log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) -
sinh(b*c*x + a*c))))/(b*c)
```

3.356.6 Sympy [F]

$$\int e^{c(a+bx)}\text{arctanh}(\cosh(ac+bcx))\,dx = e^{ac}\int e^{bcx}\,\text{atanh}\,(\cosh\,(ac+bcx))\,dx$$

input
```
integrate(exp(c*(b*x+a))*atanh(cosh(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*atanh(cosh(a*c + b*c*x)), x)
```

3.356.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int e^{c(a+bx)}\text{arctanh}(\cosh(ac+bcx))\,dx = \frac{\text{artanh}\,(\cosh\,(bcx+ac))\,e^{((bx+a)c)}}{bc} + \frac{\log\left(e^{(bcx+ac)}+1\right)}{bc} + \frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(cosh(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arctanh(cosh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(b*c*x + a*c) + 1
)/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

3.356.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(47) = 94.

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.00

$$\int e^{c(a+bx)}\text{arctanh}(\cosh(ac+bcx))\,dx = \frac{\left(e^{(bcx)}\log\left(-\frac{e^{(2\,bcx+2\,ac)}}{e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}+1} - \frac{2\,e^{(bcx+ac)}}{e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}+1} - \frac{1}{e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}+1}\right) + 2\,e^{(-ac)}\log\left(\left|e^{(2\,bcx-\right.\right.}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(cosh(b*c*x+a*c)),x, algorithm="giac")
```

output
```
1/2*(e^(b*c*x)*log(-e^(2*b*c*x + 2*a*c)/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x
+ a*c) + 1) - 2*e^(b*c*x + a*c)/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) +
 1) - 1/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) + 1)) + 2*e^(-a*c)*log(ab
s(e^(2*b*c*x + 2*a*c) - 1)))*e^(a*c)/(b*c)
```

3.356.9 Mupad [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.27

$$\int e^{c(a+bx)}\mathrm{arctanh}(\cosh(ac+bcx))\,dx = \frac{\ln\left(\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}-1\right)}{b\,c} - \frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,\ln\left(1-\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}-\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}\right)}{2\,b\,c} + \frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,\ln\left(\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}+\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}+1\right)}{2\,b\,c}$$

input
```
int(exp(c*(a + b*x))*atanh(cosh(a*c + b*c*x)),x)
```

output
```
log(exp(2*b*c*x)*exp(2*a*c) - 1)/(b*c) - (exp(b*c*x)*exp(a*c)*log(1 - (exp
(-b*c*x)*exp(-a*c))/2 - (exp(b*c*x)*exp(a*c))/2))/(2*b*c) + (exp(b*c*x)*ex
p(a*c)*log((exp(b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2 + 1))/(2*b*
c)
```

3.357 $\int e^{c(a+bx)}\mathbf{arctanh}(\tanh(ac+bcx))\,dx$

3.357.1 Optimal result

Integrand size = 20, antiderivative size = 45

$$\int e^{c(a+bx)}\text{arctanh}(\tanh(ac+bcx))\,dx=-\frac{e^{ac+bcx}}{bc}+\frac{e^{ac+bcx}\text{arctanh}(\tanh(c(a+bx)))}{bc}$$

output
```
-exp(b*c*x+a*c)/b/c+exp(b*c*x+a*c)*arctanh(tanh(c*(b*x+a)))/b/c
```

3.357.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int e^{c(a+bx)}\text{arctanh}(\tanh(ac+bcx))\,dx=\frac{e^{c(a+bx)}\left(-1+\text{arctanh}\left(\frac{-1+e^{2c(a+bx)}}{1+e^{2c(a+bx)}}\right)\right)}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcTanh[Tanh[a*c + b*c*x]],x]
```

output
```
(E^(c*(a + b*x))*(-1 + ArcTanh[(-1 + E^(2*c*(a + b*x)))/(1 + E^(2*c*(a + b
*x)))]))/(b*c)
```

3.357.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7281, 6829, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\text{arctanh}(\tanh(ac+bcx))\,dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int e^{ac+bxc}\text{arctanh}(\tanh(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{ 6829}$$

$$\frac{e^{ac+bcx}\text{arctanh}(\tanh(ac+bcx))-\int e^{ac+bxc}d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{e^{ac+bcx}\text{arctanh}(\tanh(ac+bcx))-e^{ac+bcx}}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcTanh[Tanh[a*c + b*c*x]],x]
```

output
```
(-E^(a*c + b*c*x) + E^(a*c + b*c*x)*ArcTanh[Tanh[a*c + b*c*x]])/(b*c)
```

3.357.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 6829
```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcTanh[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.357.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result
parallelrisch	$-\frac{-e^{c(bx+a)}\operatorname{arctanh}(\tanh(c(bx+a)))+e^{c(bx+a)}}{bc}$
default	$\frac{e^{bcx+ac}(bcx+ac)-e^{bcx+ac}+e^{bcx+ac}(\operatorname{arctanh}(\tanh(bcx+ac))-bcx-ac)}{bc}$
risch	$\frac{e^{c(bx+a)}\ln\left(e^{c(bx+a)}\right)}{bc} - \frac{i\left(\pi \operatorname{csgn}\left(\frac{i}{e^{2c(bx+a)}+1}\right)\operatorname{csgn}\left(ie^{2c(bx+a)}\right)\operatorname{csgn}\left(\frac{ie^{2c(bx+a)}}{e^{2c(bx+a)}+1}\right)-\pi \operatorname{csgn}\left(\frac{i}{e^{2c(bx+a)}+1}\right)\operatorname{csgn}\left(\frac{ie^{2c}}{e^{2c(b}}\right.\right.}{}$

input
```
int(exp(c*(b*x+a))*arctanh(tanh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
-(-exp(c*(b*x+a))*arctanh(tanh(c*(b*x+a)))+exp(c*(b*x+a)))/b/c
```

3.357.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int e^{c(a+bx)}\operatorname{arctanh}(\tanh(ac+bcx))\,dx = \frac{(bcx+ac-1)\cosh(bcx+ac)+(bcx+ac-1)\sinh(bcx+ac)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(tanh(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
((b*c*x + a*c - 1)*cosh(b*c*x + a*c) + (b*c*x + a*c - 1)*sinh(b*c*x + a*c)
)/(b*c)
```

3.357.6 Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int e^{c(a+bx)}\text{arctanh}(\tanh(ac+bcx))\,dx=\begin{cases}0 & \text{for } b=0\wedge c=0\\ xe^{ac}\,\text{atanh}\,(\tanh\,(ac)) & \text{for } b=0\\ 0 & \text{for } c=0\\ \frac{e^{ac}e^{bcx}\,\text{atanh}\,(\tanh\,(ac+bcx))}{bc}-\frac{e^{ac}e^{bcx}}{bc} & \text{otherwise}\end{cases}$$

input
```
integrate(exp(c*(b*x+a))*atanh(tanh(b*c*x+a*c)),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0)), (x*exp(a*c)*atanh(tanh(a*c)), Eq(b, 0)
), (0, Eq(c, 0)), (exp(a*c)*exp(b*c*x)*atanh(tanh(a*c + b*c*x))/(b*c) - ex
p(a*c)*exp(b*c*x)/(b*c), True))
```

3.357.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int e^{c(a+bx)}\text{arctanh}(\tanh(ac+bcx))\,dx=\frac{\text{artanh}\,(\tanh\,(bcx+ac))\,e^{((bx+a)c)}}{bc}-\frac{e^{(bcx+ac)}}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(tanh(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arctanh(tanh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - e^(b*c*x + a*c)/(b*c)
```

3.357.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int e^{c(a+bx)}\text{arctanh}(\tanh(ac+bcx))\,dx=\frac{(b^2c^2x+abc^2-bc)e^{(bcx+ac)}}{b^2c^2}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(tanh(b*c*x+a*c)),x, algorithm="giac")
```

output
```
(b^2*c^2*x + a*b*c^2 - b*c)*e^(b*c*x + a*c)/(b^2*c^2)
```

3.357.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.62

$$\int e^{c(a+bx)}\text{arctanh}(\tanh(ac+bcx))\,dx=\frac{\mathrm{e}^{a\,c+b\,c\,x}\,\left(\mathrm{atanh}(\tanh(a\,c+b\,c\,x))-1\right)}{b\,c}$$

input
```
int(exp(c*(a + b*x))*atanh(tanh(a*c + b*c*x)),x)
```

output
```
(exp(a*c + b*c*x)*(atanh(tanh(a*c + b*c*x)) - 1))/(b*c)
```

3.358 $\int e^{c(a+bx)}\textbf{arctanh}(\coth(ac+bcx))\,dx$

3.358.1 Optimal result

Integrand size = 20, antiderivative size = 45

$$\int e^{c(a+bx)}\text{arctanh}(\coth(ac+bcx))\,dx=-\frac{e^{ac+bcx}}{bc}+\frac{e^{ac+bcx}\text{arctanh}(\coth(c(a+bx)))}{bc}$$

output
```
-exp(b*c*x+a*c)/b/c+exp(b*c*x+a*c)*arctanh(coth(c*(b*x+a)))/b/c
```

3.358.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int e^{c(a+bx)}\text{arctanh}(\coth(ac+bcx))\,dx=\frac{e^{c(a+bx)}\left(-1+\text{arctanh}\left(\frac{1+e^{2c(a+bx)}}{-1+e^{2c(a+bx)}}\right)\right)}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcTanh[Coth[a*c + b*c*x]],x]
```

output
```
(E^(c*(a + b*x))*(-1 + ArcTanh[(1 + E^(2*c*(a + b*x)))/(-1 + E^(2*c*(a + b
*x)))]))/(b*c)
```

3.358.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7281, 6829, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\text{arctanh}(\coth(ac+bcx))\,dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int e^{ac+bxc}\text{arctanh}(\coth(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{ 6829}$$

$$\frac{e^{ac+bcx}\text{arctanh}(\coth(ac+bcx))-\int e^{ac+bxc}d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{e^{ac+bcx}\text{arctanh}(\coth(ac+bcx))-e^{ac+bcx}}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcTanh[Coth[a*c + b*c*x]],x]
```

output
```
(-E^(a*c + b*c*x) + E^(a*c + b*c*x)*ArcTanh[Coth[a*c + b*c*x]])/(b*c)
```

3.358.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 6829
```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcTanh[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.358.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

method	result
parallelrisch	$\frac{e^{c(bx+a)}\left(\operatorname{arctanh}\left(\frac{1}{\tanh(c(bx+a))}\right)-1\right)}{bc}$
risch	$\frac{e^{c(bx+a)}\ln\left(e^{c(bx+a)}\right)}{bc} - \frac{i\left(\pi \operatorname{csgn}\left(ie^{c(bx+a)}\right)^2 \operatorname{csgn}\left(ie^{2c(bx+a)}\right)-2\pi \operatorname{csgn}\left(ie^{c(bx+a)}\right) \operatorname{csgn}\left(ie^{2c(bx+a)}\right)^2+\pi \operatorname{csgn}\left(ie^{2c(bx+a)}\right)^3\right.}{}$

input
```
int(exp(c*(b*x+a))*arctanh(coth(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
exp(c*(b*x+a))*(arctanh(1/tanh(c*(b*x+a)))-1)/b/c
```

3.358.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int e^{c(a+bx)}\operatorname{arctanh}(\coth(ac+bcx))\,dx = \frac{(i\,\pi+2\,bcx+2\,ac-2)\cosh\left(bcx+ac\right)+(i\,\pi+2\,bcx+2\,ac-2)\sinh\left(bcx+ac\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(coth(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*((I*pi + 2*b*c*x + 2*a*c - 2)*cosh(b*c*x + a*c) + (I*pi + 2*b*c*x + 2*
a*c - 2)*sinh(b*c*x + a*c))/(b*c)
```

3.358.6 Sympy [F]

$$\int e^{c(a+bx)}\text{arctanh}(\coth(ac+bcx))\,dx = e^{ac}\int e^{bcx}\,\text{atanh}\left(\coth\left(ac+bcx\right)\right)dx$$

input
```
integrate(exp(c*(b*x+a))*atanh(coth(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*atanh(coth(a*c + b*c*x)), x)
```

3.358.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int e^{c(a+bx)}\text{arctanh}(\coth(ac+bcx))\,dx = \frac{\text{artanh}\left(\coth\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} - \frac{e^{(bcx+ac)}}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(coth(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arctanh(coth(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - e^(b*c*x + a*c)/(b*c)
```

3.358.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int e^{c(a+bx)}\text{arctanh}(\coth(ac+bcx))\,dx = \frac{\left(e^{(bcx)}\log\left(-e^{(2\,bcx+2\,ac)}\right) - 2\,e^{(bcx)}\right)e^{(ac)}}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(coth(b*c*x+a*c)),x, algorithm="giac")
```

output
```
1/2*(e^(b*c*x)*log(-e^(2*b*c*x + 2*a*c)) - 2*e^(b*c*x))*e^(a*c)/(b*c)
```

3.358.9 Mupad [B] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.62

$$\int e^{c(a+bx)}\text{arctanh}(\coth(ac+bcx))\,dx = \frac{e^{a\,c+b\,c\,x}\,(\text{atanh}(\coth(a\,c+b\,c\,x))-1)}{b\,c}$$

input
```
int(exp(c*(a + b*x))*atanh(coth(a*c + b*c*x)),x)
```

output
```
(exp(a*c + b*c*x)*(atanh(coth(a*c + b*c*x)) - 1))/(b*c)
```

3.359 $\int e^{c(a+bx)}\text{arctanh}(\text{sech}(ac+bcx))\,dx$

3.359.1 Optimal result

Integrand size = 20, antiderivative size = 49

$$\int e^{c(a+bx)}\text{arctanh}(\text{sech}(ac+bcx))\,dx=\frac{e^{ac+bcx}\text{arctanh}(\text{sech}(c(a+bx)))}{bc}+\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

output
```
exp(b*c*x+a*c)*arctanh(sech(c*(b*x+a)))/b/c+ln(1-exp(2*c*(b*x+a)))/b/c
```

3.359.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20

$$\int e^{c(a+bx)}\text{arctanh}(\text{sech}(ac+bcx))\,dx=\frac{e^{c(a+bx)}\text{arctanh}\left(\frac{2e^{c(a+bx)}}{1+e^{2c(a+bx)}}\right)+\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcTanh[Sech[a*c + b*c*x]],x]
```

output
```
(E^(c*(a + b*x))*ArcTanh[(2*E^(c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))] + Lo
g[1 - E^(2*c*(a + b*x))])/(b*c)
```

3.359.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7281, 6829, 25, 2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\text{arctanh}(\text{sech}(ac+bcx))\,dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int e^{ac+bxc}\text{arctanh}(\text{sech}(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{ 6829}$$

$$\frac{e^{ac+bcx}\text{arctanh}(\text{sech}(ac+bcx))-\int -e^{ac+bxc}\text{csch}(ac+bxc)d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int e^{ac+bxc}\text{csch}(ac+bxc)d(ac+bxc)+e^{ac+bcx}\text{arctanh}(\text{sech}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int -\frac{2e^{ac+bxc}}{1-e^{2ac+2bxc}}de^{ac+bxc}+e^{ac+bcx}\text{arctanh}(\text{sech}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^{ac+bcx}\text{arctanh}(\text{sech}(ac+bcx))-2\int \frac{e^{ac+bxc}}{1-e^{2ac+2bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{e^{ac+bcx}\text{arctanh}(\text{sech}(ac+bcx))+\log\left(1-e^{2ac+2bcx}\right)}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcTanh[Sech[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcTanh[Sech[a*c + b*c*x]] + Log[1 - E^(2*a*c + 2*b*c*x)]
)/(b*c)
```

3.359.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6829
```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcTanh[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.359.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.34 (sec) , antiderivative size = 872, normalized size of antiderivative = 17.80

method	result	size
risch	Expression too large to display	872

input
```
int(exp(c*(b*x+a))*arctanh(sech(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I/(exp(2*c*(b*x+a))+1))*csg
n(I*(exp(c*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))*exp(c*(b*x+a))-1/4*I/b/c*Pi
*csgn(I*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I/(exp(2*
c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(
b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)*csgn(I*(exp(c*(b*x+a))+1
)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x
+a))+1)^2)*csgn(I/(exp(2*c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2/(exp(2
*c*(b*x+a))+1))*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1))^2*c
sgn(I*(exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x
+a))-1))^2*csgn(I*(exp(c*(b*x+a))-1)^2)*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I
*(exp(c*(b*x+a))-1)^2)^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+
a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))
+1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2)^2*ex
p(c*(b*x+a))-1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a)
)-1)^2)^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2/(exp(2*c
*(b*x+a))+1))^3*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2/(e
xp(2*c*(b*x+a))+1))^3*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1
)^2)*csgn(I*(exp(c*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))-1/
b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-1)+1/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x
+a))+1)-2*a/b+1/b/c*ln(exp(2*c*(b*x+a))-1)
```

3.359.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.88

$$\int e^{c(a+bx)}\text{arctanh}(\text{sech}(ac+bcx))\,dx = \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(\frac{\cosh(bcx+ac)+1}{\cosh(bcx+ac)-1}\right)+2\,\log\left(\frac{2\,\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(sech(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log((cosh(b*c*x + a*c) + 1)/(
cosh(b*c*x + a*c) - 1)) + 2*log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - s
inh(b*c*x + a*c))))/(b*c)
```

3.359.6 Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\text{arctanh}(\text{sech}(ac+bcx))\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))*atanh(sech(b*c*x+a*c)),x)
```

output
```
Timed out
```

3.359.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int e^{c(a+bx)}\text{arctanh}(\text{sech}(ac+bcx))\,dx = \frac{\text{artanh}\left(\text{sech}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} + \frac{\log\left(e^{(bcx+ac)}+1\right)}{bc} + \frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(sech(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arctanh(sech(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(b*c*x + a*c) + 1
)/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

3.359.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(47) = 94$.

Time = 0.37 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.00

$$\int e^{c(a+bx)}\text{arctanh}(\text{sech}(ac+bcx))\,dx = \frac{e^{\left((bx+a)c\right)}\log\left(-\frac{\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}+1}{\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}-1}\right)}{2\,bc} + \frac{\log\left(\left|e^{(2\,bcx+2\,ac)}-1\right|\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(sech(b*c*x+a*c)),x, algorithm="giac")
```

output
```
1/2*e^((b*x + a)*c)*log(-(2/(e^(b*c*x + a*c) + e^(-b*c*x - a*c)) + 1)/(2/(
e^(b*c*x + a*c) + e^(-b*c*x - a*c)) - 1))/(b*c) + log(abs(e^(2*b*c*x + 2*a
*c) - 1))/(b*c)
```

3.359.9 Mupad [B] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.43

$$\int e^{c(a+bx)}\mathrm{arctanh}(\mathrm{sech}(ac+bcx))\,dx = \frac{\ln\left(e^{2\,b\,c\,x}\,e^{2\,a\,c}-1\right)}{b\,c} - \frac{e^{a\,c+b\,c\,x}\,\ln\left(1-\frac{1}{\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}}\right)}{2\,b\,c} + \frac{\ln\left(\frac{1}{\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}}+1\right)\,e^{a\,c+b\,c\,x}}{2\,b\,c}$$

input
```
int(atanh(1/cosh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

output
```
log(exp(2*b*c*x)*exp(2*a*c) - 1)/(b*c) - (exp(a*c + b*c*x)*log(1 - 1/((exp
(b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2)))/(2*b*c) + (log(1/((exp(
b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2) + 1)*exp(a*c + b*c*x))/(2*
b*c)
```

3.360 $\int e^{c(a+bx)}\text{arctanh}(\text{csch}(ac+bcx))\,dx$

3.360.1 Optimal result

Integrand size = 20, antiderivative size = 107

$$\int e^{c(a+bx)}\text{arctanh}(\text{csch}(ac+bcx))\,dx = \frac{e^{ac+bcx}\text{arctanh}(\text{csch}(c(a+bx)))}{bc} + \frac{(1-\sqrt{2})\log\left(3-2\sqrt{2}-e^{2c(a+bx)}\right)}{2bc} + \frac{(1+\sqrt{2})\log\left(3+2\sqrt{2}-e^{2c(a+bx)}\right)}{2bc}$$

output
```
exp(b*c*x+a*c)*arctanh(csch(c*(b*x+a)))/b/c+1/2*ln(3-exp(2*c*(b*x+a))-2*2^
(1/2))*(1-2^(1/2))/b/c+1/2*ln(3-exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/
c
```

3.360.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.40

$$\int e^{c(a+bx)}\text{arctanh}(\text{csch}(ac+bcx))\,dx = \frac{-2\sqrt{2}\text{arctanh}\left(\frac{-1+e^{c(a+bx)}}{\sqrt{2}}\right)+2\sqrt{2}\text{arctanh}\left(\frac{1+e^{c(a+bx)}}{\sqrt{2}}\right)+2e^{c(a+bx)}\text{arctanh}\left(\frac{2e^{c(a+bx)}}{-1+e^{2c(a+bx)}}\right)+\log\left(1-2e^{c(a+b}\right.}{2bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcTanh[Csch[a*c + b*c*x]],x]
```

output
```
(-2*Sqrt[2]*ArcTanh[(-1 + E^(c*(a + b*x)))/Sqrt[2]] + 2*Sqrt[2]*ArcTanh[(1
 + E^(c*(a + b*x)))/Sqrt[2]] + 2*E^(c*(a + b*x))*ArcTanh[(2*E^(c*(a + b*x)
))/(-1 + E^(2*c*(a + b*x)))] + Log[1 - 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x
))] + Log[1 + 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))])/(2*b*c)
```

3.360.3 Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {7281, 6829, 25, 2720, 27, 1576, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\text{arctanh}(\text{csch}(ac+bcx))\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc}\text{arctanh}(\text{csch}(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{6829}$$

$$\frac{e^{ac+bcx}\text{arctanh}(\text{csch}(ac+bcx))-\int -\frac{e^{ac+bxc}\coth(ac+bxc)\text{csch}(ac+bxc)}{1-\text{csch}^2(ac+bxc)}d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{e^{ac+bxc}\coth(ac+bxc)\text{csch}(ac+bxc)}{1-\text{csch}^2(ac+bxc)}d(ac+bxc)+e^{ac+bcx}\text{arctanh}(\text{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{2e^{ac+bxc}\left(1+e^{2ac+2bxc}\right)}{1-6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc}+e^{ac+bcx}\text{arctanh}(\text{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{e^{ac+bxc}\left(1+e^{2ac+2bxc}\right)}{1-6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc}+e^{ac+bcx}\text{arctanh}(\text{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{1576}$$

$$\frac{\int \frac{1+e^{2ac+2bxc}}{1-5e^{2ac+2bxc}}de^{2ac+2bxc}+e^{ac+bcx}\text{arctanh}(\text{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \ 1141$$

$$\frac{\int \left(-\frac{1+\sqrt{2}}{2\left(-ac-bxc+2\sqrt{2}+3\right)}-\frac{1-\sqrt{2}}{2\left(-ac-bxc-2\sqrt{2}+3\right)}\right)de^{2ac+2bxc}+e^{ac+bcx}\text{arctanh}(\text{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \ 2009$$

$$\frac{e^{ac+bcx}\text{arctanh}(\text{csch}(ac+bcx))+\frac{1}{2}\left(1-\sqrt{2}\right)\log\left(-ac-bcx-2\sqrt{2}+3\right)+\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(-ac-bcx+2\sqrt{2}+3\right)}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcTanh[Csch[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcTanh[Csch[a*c + b*c*x]] + ((1 - Sqrt[2])*Log[3 - 2*Sqr
t[2] - a*c - b*c*x])/2 + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2] - a*c - b*c*x])/
2)/(b*c)
```

3.360.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.360. $\int e^{c(a+bx)}\text{arctanh}(\text{csch}(ac+bcx))\,dx$

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6829
```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcTanh[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.360.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.42 (sec) , antiderivative size = 842, normalized size of antiderivative = 7.87

method	result	size
risch	Expression too large to display	842

input
```
int(exp(c*(b*x+a))*arctanh(csch(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)+1/4*I/b/c*P
i*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I/(exp(2*c*(b*x+a))-
1))*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))*ex
p(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*c
sgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^2*exp(c
*(b*x+a))-1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(-exp(2*c*(b*x+
a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/b/c*P
i*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-1
))*csgn(I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*exp(
c*(b*x+a))+1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+
a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/b/c*P
i*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^3*ex
p(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*cs
gn(I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(
b*x+a))-1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c
*(b*x+a))-1))^3*exp(c*(b*x+a))-1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-
2*exp(c*(b*x+a))-1)+1/2/b/c*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^2)*2^(1/2)-1/2
/b/c*ln(exp(2*c*(b*x+a))-(2^(1/2)-1)^2)*2^(1/2)-2*a/b+1/2/b/c*ln(exp(2*c*(
b*x+a))-(1+2^(1/2))^2)+1/2/b/c*ln(exp(2*c*(b*x+a))-(2^(1/2)-1)^2)
```

3.360.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(90) = 180$.

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.18

$$\int e^{c(a+bx)}\operatorname{arctanh}(\operatorname{csch}(ac+bcx))\,dx$$
$$= \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(\frac{\sinh(bcx+ac)+1}{\sinh(bcx+ac)-1}\right)+\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh(bcx+ac)^2-4\left(3\sqrt{2}+4\right)\cosh(bcx+a}{\cosh(bcx+ac)^2+}\right.}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(csch(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log((sinh(b*c*x + a*c) + 1)/(
sinh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c)
^2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2)
+ 3)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*
x + a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/(
cosh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x +
 a*c)^2)))/(b*c)
```

3.360.6 Sympy [F]

$$\int e^{c(a+bx)}\text{arctanh}(\text{csch}(ac+bcx))\,dx = e^{ac}\int e^{bcx}\,\text{atanh}\left(\text{csch}\left(ac+bcx\right)\right)dx$$

input
```
integrate(exp(c*(b*x+a))*atanh(csch(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*atanh(csch(a*c + b*c*x)), x)
```

3.360.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(90) = 180.

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.72

$$\begin{aligned}\int e^{c(a+bx)}\text{arctanh}(\text{csch}(ac+bcx))\,dx =& \frac{\text{artanh}\left(\text{csch}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} \\ &+ \frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}+1}{\sqrt{2}+e^{(bcx+ac)}-1}\right)}{2\,bc} \\ &- \frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}-1}{\sqrt{2}+e^{(bcx+ac)}+1}\right)}{2\,bc} \\ &+ \frac{\log\left(e^{(2\,bcx+2\,ac)}+2\,e^{(bcx+ac)}-1\right)}{2\,bc} \\ &+ \frac{\log\left(e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}-1\right)}{2\,bc}\end{aligned}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(csch(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arctanh(csch(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(sqrt(
2) - e^(b*c*x + a*c) + 1)/(sqrt(2) + e^(b*c*x + a*c) - 1))/(b*c) - 1/2*sqr
t(2)*log(-(sqrt(2) - e^(b*c*x + a*c) - 1)/(sqrt(2) + e^(b*c*x + a*c) + 1))
/(b*c) + 1/2*log(e^(2*b*c*x + 2*a*c) + 2*e^(b*c*x + a*c) - 1)/(b*c) + 1/2*
log(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) - 1)/(b*c)
```

3.360.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.56

$$\int e^{c(a+bx)}\operatorname{arctanh}(\operatorname{csch}(ac+bcx))\,dx$$
$$=\frac{e^{((bx+a)c)}\log\left(-\frac{\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}+1}{\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}-1}\right)}{2\,bc}$$
$$+\frac{\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}{\left|4\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}\right)+\log\left(\left|e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right|\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arctanh(csch(b*c*x+a*c)),x, algorithm="giac")
```

output
```
1/2*e^((b*x + a)*c)*log(-(2/(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 1)/(2/(
e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 1))/(b*c) + 1/2*(sqrt(2)*log(abs(-4*
sqrt(2) + 2*e^(2*b*c*x + 2*a*c) - 6)/abs(4*sqrt(2) + 2*e^(2*b*c*x + 2*a*c)
 - 6)) + log(abs(e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)))/(b*c)
```

3.360.9 Mupad [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.75

$$\int e^{c(a+bx)}\operatorname{arctanh}(\operatorname{csch}(ac+bcx))\,dx=\frac{\ln\left(6\sqrt{2}\,\mathrm{e}^{2\,c\,(a+b\,x)}-2\sqrt{2}-8\,\mathrm{e}^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}+1\right)}{2\,b\,c}$$
$$-\frac{\mathrm{e}^{a\,c+b\,c\,x}\,\ln\left(1-\frac{1}{\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}-\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}\right)}{2\,b\,c}$$
$$-\frac{\ln\left(2\sqrt{2}-8\,\mathrm{e}^{2\,c\,(a+b\,x)}-6\sqrt{2}\,\mathrm{e}^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}-1\right)}{2\,b\,c}$$
$$+\frac{\ln\left(\frac{1}{\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}-\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}+1\right)\,\mathrm{e}^{a\,c+b\,c\,x}}{2\,b\,c}$$

input
```
int(atanh(1/sinh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

output
```
(log(6*2^(1/2)*exp(2*c*(a + b*x)) - 2*2^(1/2) - 8*exp(2*c*(a + b*x)))*(2^(
1/2) + 1))/(2*b*c) - (exp(a*c + b*c*x)*log(1 - 1/((exp(b*c*x)*exp(a*c))/2
- (exp(-b*c*x)*exp(-a*c))/2)))/(2*b*c) - (log(2*2^(1/2) - 8*exp(2*c*(a + b
*x)) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c) + (log(1/((exp
(b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2) + 1)*exp(a*c + b*c*x))/(2
*b*c)
```

3.360. $\int e^{c(a+bx)}\operatorname{arctanh}(\operatorname{csch}(ac+bcx))\,dx$

3.361 $\int \frac{(a+b\,\text{arctanh}(cx^n))(d+e\log(fx^m))}{x}\,dx$

3.361.1 Optimal result

Integrand size = 24, antiderivative size = 136

$$\begin{aligned}\int \frac{(a+b\,\text{arctanh}(cx^n))\,(d+e\log(fx^m))}{x}\,dx &= ad\log(x)+\frac{ae\log^2(fx^m)}{2m}-\frac{bd\,\text{PolyLog}(2,-cx^n)}{2n}\\ &\quad-\frac{be\log(fx^m)\,\text{PolyLog}(2,-cx^n)}{2n}+\frac{bd\,\text{PolyLog}(2,cx^n)}{2n}\\ &\quad+\frac{be\log(fx^m)\,\text{PolyLog}(2,cx^n)}{2n}+\frac{bem\,\text{PolyLog}(3,-cx^n)}{2n^2}-\frac{bem\,\text{PolyLog}(3,cx^n)}{2n^2}\end{aligned}$$

output
```
a*d*ln(x)+1/2*a*e*ln(f*x^m)^2/m-1/2*b*d*polylog(2,-c*x^n)/n-1/2*b*e*ln(f*x
^m)*polylog(2,-c*x^n)/n+1/2*b*d*polylog(2,c*x^n)/n+1/2*b*e*ln(f*x^m)*polyl
og(2,c*x^n)/n+1/2*b*e*m*polylog(3,-c*x^n)/n^2-1/2*b*e*m*polylog(3,c*x^n)/n
^2
```

3.361.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\operatorname{arctanh}(cx^n))(d+e\log(fx^m))}{x}\,dx$$
$$= -\frac{bcemx^n\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2},\frac{3}{2};c^2x^{2n}\right)}{n^2}+\frac{bcx^n\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2};c^2x^{2n}\right)(d+e\log(fx^m))}{n}$$
$$+\frac{1}{2}a\log(x)(2d-em\log(x)+2e\log(fx^m))$$

input
```
Integrate[((a + b*ArcTanh[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

output
```
-((b*c*e*m*x^n*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, c^2*
x^(2*n)])/n^2) + (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, c^2
*x^(2*n)]*(d + e*Log[f*x^m]))/n + (a*Log[x]*(2*d - e*m*Log[x] + 2*e*Log[f*
x^m]))/2
```

3.361.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\operatorname{arctanh}(cx^n))(d+e\log(fx^m))}{x}\,dx$$
$$\Big\downarrow \; 7293$$
$$\int \left(\frac{d(a+b\operatorname{arctanh}(cx^n))}{x}+\frac{e\log(fx^m)(a+b\operatorname{arctanh}(cx^n))}{x}\right)dx$$
$$\Big\downarrow \; 2009$$
$$ad\log(x)+\frac{ae\log^2(fx^m)}{2m}-\frac{bd\operatorname{PolyLog}(2,-cx^n)}{2n}+\frac{bd\operatorname{PolyLog}(2,cx^n)}{2n}-\frac{be\operatorname{PolyLog}(2,-cx^n)\log(fx^m)}{2n}+\frac{be\operatorname{PolyLog}(2,cx^n)\log(fx^m)}{2n}+\frac{bem\operatorname{PolyLog}(3,-cx^n)}{2n^2}-\frac{bem\operatorname{PolyLog}(3,cx^n)}{2n^2}$$

input
```
Int[((a + b*ArcTanh[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

output
```
a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) - (b*d*PolyLog[2, -(c*x^n)])/(2*n) -
 (b*e*Log[f*x^m]*PolyLog[2, -(c*x^n)])/(2*n) + (b*d*PolyLog[2, c*x^n])/(2*
n) + (b*e*Log[f*x^m]*PolyLog[2, c*x^n])/(2*n) + (b*e*m*PolyLog[3, -(c*x^n)
])/(2*n^2) - (b*e*m*PolyLog[3, c*x^n])/(2*n^2)
```

3.361.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.361.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 150.80 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.40

method	result
risch	$\frac{\left(-\frac{ie\pi\,\mathrm{csgn}(if)\,\mathrm{csgn}(ix^m)\,\mathrm{csgn}(if\,x^m)}{4}+\frac{ie\pi\,\mathrm{csgn}(if)\,\mathrm{csgn}(if\,x^m)^2}{4}+\frac{ie\pi\,\mathrm{csgn}(ix^m)\,\mathrm{csgn}(if\,x^m)^2}{4}-\frac{ie\pi\,\mathrm{csgn}(if\,x^m)^3}{4}+\frac{e\ln(f)}{2}+\frac{d}{2}\right)(-b\,\mathrm{dilog}}{n}$

input
```
int((a+b*arctanh(c*x^n))*(d+e*ln(f*x^m))/x,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*I*e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/4*I*e*Pi*csgn(I*f)*csgn
(I*f*x^m)^2+1/4*I*e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/4*I*e*Pi*csgn(I*f*x^m
)^3+1/2*e*ln(f)+1/2*d)/n*(-b*dilog(c*x^n+1)+2*a*ln(x^n)+b*dilog(1-c*x^n))-
1/2*e*b*m/n*ln(x)*polylog(2,-c*x^n)+1/2*b*e*m*polylog(3,-c*x^n)/n^2+1/2*e*
b/n*dilog(c*x^n+1)*m*ln(x)-1/2*e*b/n*dilog(c*x^n+1)*ln(x^m)+1/2*e*a/m*ln(x
^m)^2+1/2*e*b*m/n*ln(x)*polylog(2,c*x^n)-1/2*b*e*m*polylog(3,c*x^n)/n^2+1/
2*e*b/n*ln(1-c*x^n)*ln(c*x^n)*m*ln(x)-1/2*e*b/n*ln(1-c*x^n)*ln(c*x^n)*ln(x
^m)+1/2*e*b/n*dilog(c*x^n)*m*ln(x)-1/2*e*b/n*dilog(c*x^n)*ln(x^m)
```

3.361.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(118) = 236$.

Time = 0.28 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.40

$$\int \frac{(a + b\text{arctanh}(cx^n))\,(d + e\log{(fx^m)})}{x}\,dx$$
$$= \frac{2\,aemn^2\log{(x)}^2 - 2\,bem\text{polylog}(3, c\cosh{(n\log{(x)})} + c\sinh{(n\log{(x)})}) + 2\,bem\text{polylog}(3, -c\cosh{(n\log}}{}$$

input
```
integrate((a+b*arctanh(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="fricas")
```

output
```
1/4*(2*a*e*m*n^2*log(x)^2 - 2*b*e*m*polylog(3, c*cosh(n*log(x)) + c*sinh(n
*log(x))) + 2*b*e*m*polylog(3, -c*cosh(n*log(x)) - c*sinh(n*log(x))) + 2*(
b*e*m*n*log(x) + b*e*n*log(f) + b*d*n)*dilog(c*cosh(n*log(x)) + c*sinh(n*l
og(x))) - 2*(b*e*m*n*log(x) + b*e*n*log(f) + b*d*n)*dilog(-c*cosh(n*log(x)
) - c*sinh(n*log(x))) - (b*e*m*n^2*log(x)^2 + 2*(b*e*n^2*log(f) + b*d*n^2)
*log(x))*log(c*cosh(n*log(x)) + c*sinh(n*log(x)) + 1) + (b*e*m*n^2*log(x)^
2 + 2*(b*e*n^2*log(f) + b*d*n^2)*log(x))*log(-c*cosh(n*log(x)) - c*sinh(n*
log(x)) + 1) + 4*(a*e*n^2*log(f) + a*d*n^2)*log(x) + (b*e*m*n^2*log(x)^2 +
 2*(b*e*n^2*log(f) + b*d*n^2)*log(x))*log(-(c*cosh(n*log(x)) + c*sinh(n*lo
g(x)) + 1)/(c*cosh(n*log(x)) + c*sinh(n*log(x)) - 1)))/n^2
```

3.361.6 Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx^n))\,(d + e\log{(fx^m)})}{x}\,dx = \int \frac{(a + b\,\text{atanh}\,(cx^n))\,(d + e\log{(fx^m)})}{x}\,dx$$

input
```
integrate((a+b*atanh(c*x**n))*(d+e*ln(f*x**m))/x,x)
```

output
```
Integral((a + b*atanh(c*x**n))*(d + e*log(f*x**m))/x, x)
```

3.361.7 Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx^n))\,(d + e\log(fx^m))}{x}\,dx = \int \frac{(b\,\text{artanh}(cx^n) + a)(e\log(fx^m) + d)}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="maxima")
```

output
```
1/2*a*e*log(f*x^m)^2/m + a*d*log(x) - 1/4*(b*e*m*log(x)^2 - 2*b*e*log(x)*l
og(x^m) - 2*(e*log(f) + d)*b*log(x))*log(c*x^n + 1) + 1/4*(b*e*m*log(x)^2
- 2*b*e*log(x)*log(x^m) - 2*(e*log(f) + d)*b*log(x))*log(-c*x^n + 1) + int
egrate(1/2*(2*b*c*e*n*x^n*log(x)*log(x^m) - (b*c*e*m*n*log(x)^2 - 2*(e*n*l
og(f) + d*n)*b*c*log(x))*x^n)/(c^2*x*x^(2*n) - x), x)
```

3.361.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx^n))\,(d + e\log(fx^m))}{x}\,dx = \int \frac{(b\,\text{artanh}(cx^n) + a)(e\log(fx^m) + d)}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^n) + a)*(e*log(f*x^m) + d)/x, x)
```

3.361.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx^n))\,(d + e\log(fx^m))}{x}\,dx = \int \frac{(a + b\,\text{atanh}(c\,x^n))\,(d + e\,\ln(f\,x^m))}{x}\,dx$$

input
```
int(((a + b*atanh(c*x^n))*(d + e*log(f*x^m)))/x,x)
```

output
```
int(((a + b*atanh(c*x^n))*(d + e*log(f*x^m)))/x, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```