Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

7-Inverse-hyperbolic-functions/7.4-Inverse-hyperbolic-cotangent/198-7.4.1-Inverse-hyperbolic-cotangent-functions

Nasser M. Abbasi

December 9, 2023 Compiled on December 9, 2023 at 6:44am

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [300]. This is test number [198].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (300)	0.00 (0)
Mathematica	98.00 (294)	2.00 (6)
Maple	91.00 (273)	9.00 (27)
Maxima	82.00 (246)	18.00 (54)
Fricas	74.33 (223)	25.67 (77)
Mupad	51.00 (153)	49.00 (147)
Giac	50.67 (152)	49.33 (148)
Sympy	36.00 (108)	64.00 (192)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

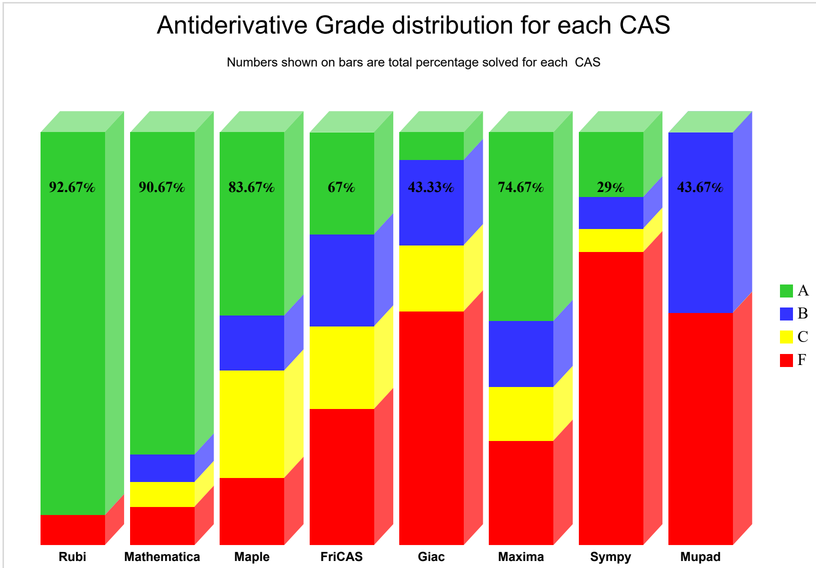
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

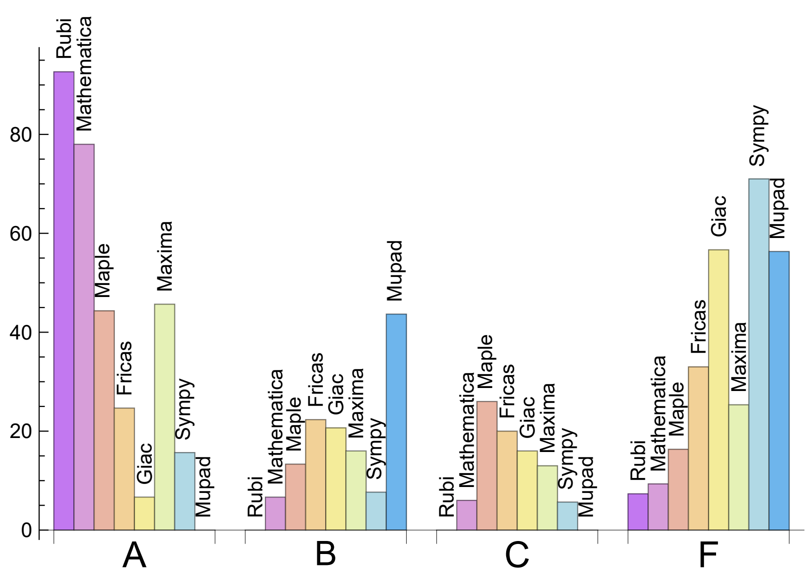
System	% A grade	% B grade	% C grade	% F grade
Rubi	91.000	0.667	1.000	7.333
Mathematica	78.000	6.667	6.000	9.333
Maxima	45.667	16.000	13.000	25.333
Maple	44.333	13.333	26.000	16.333
Fricas	24.667	22.333	20.000	33.000
Sympy	15.667	7.667	5.667	71.000
Giac	6.667	20.667	16.000	56.667
Mupad	0.000	43.667	0.000	56.333

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	6	100.00	0.00	0.00
Maple	27	70.37	29.63	0.00
Maxima	54	98.15	0.00	1.85
Fricas	77	100.00	0.00	0.00
Mupad	147	0.00	100.00	0.00
Giac	148	97.97	0.00	2.03
Sympy	192	93.23	6.25	0.52

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.27
Giac	0.33
Maxima	0.47
Rubi	0.57
Mathematica	0.60
Maple	3.09
Sympy	3.62
Mupad	4.11

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	135.78	1.64	96.00	1.15
Rubi	145.27	1.10	86.50	1.01
Mupad	149.18	2.05	48.00	1.13
Giac	167.72	2.51	77.00	1.73
Mathematica	177.10	1.30	71.50	1.00
Fricas	233.18	2.04	87.00	1.44
Sympy	279.66	2.76	46.00	1.15
Maple	2099.93	21.37	167.00	1.33

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

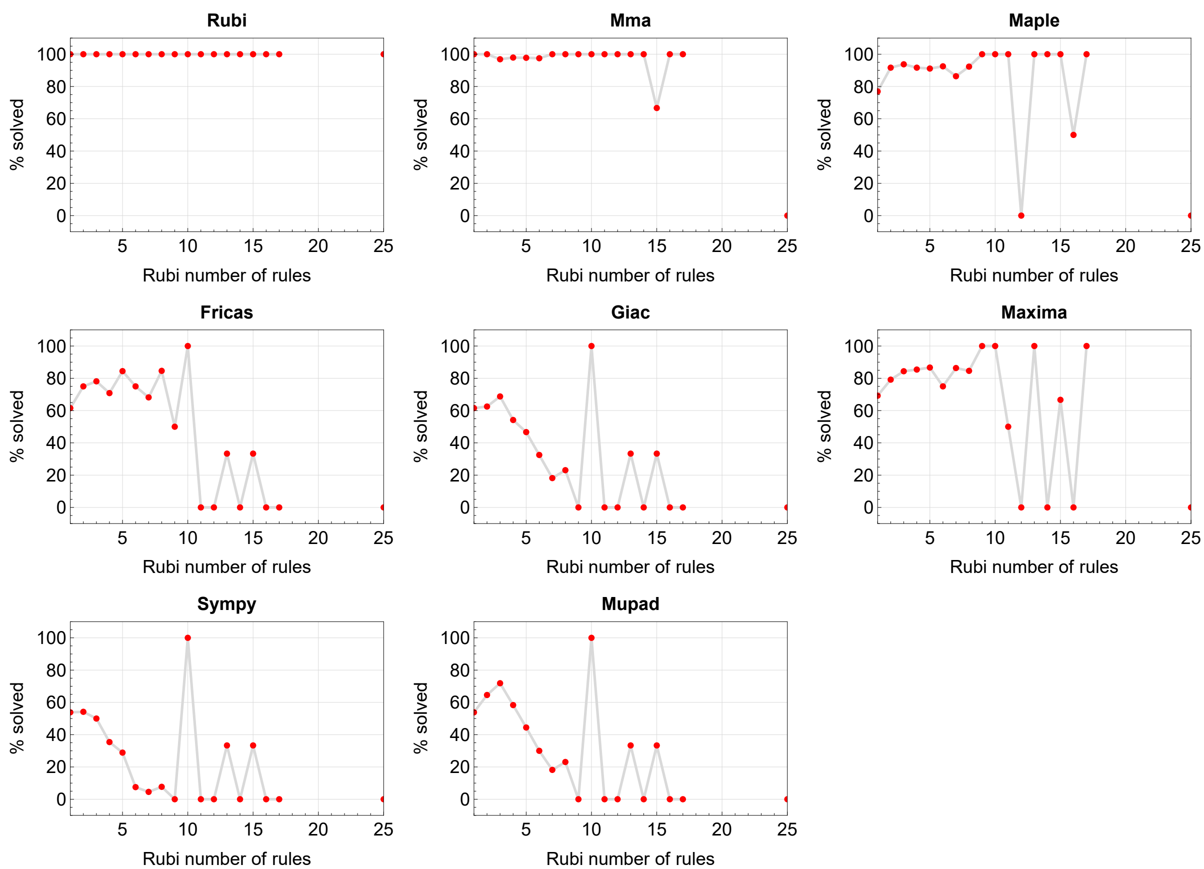


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

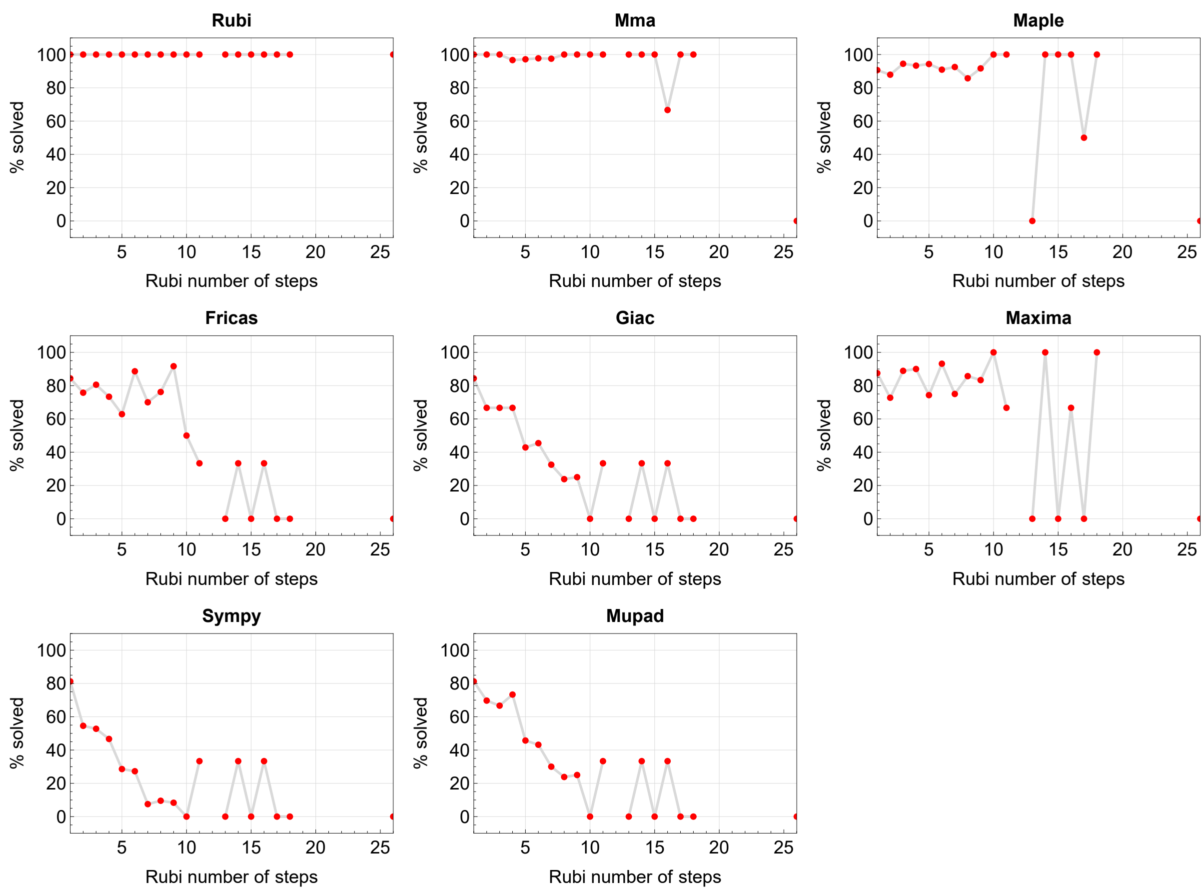


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

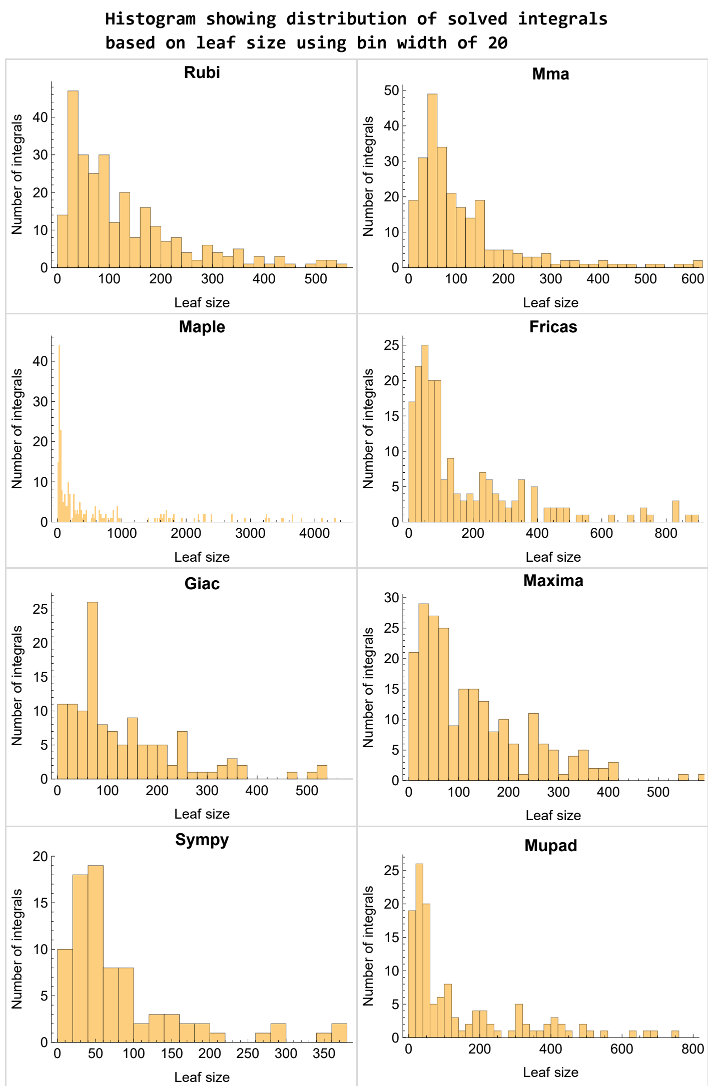


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

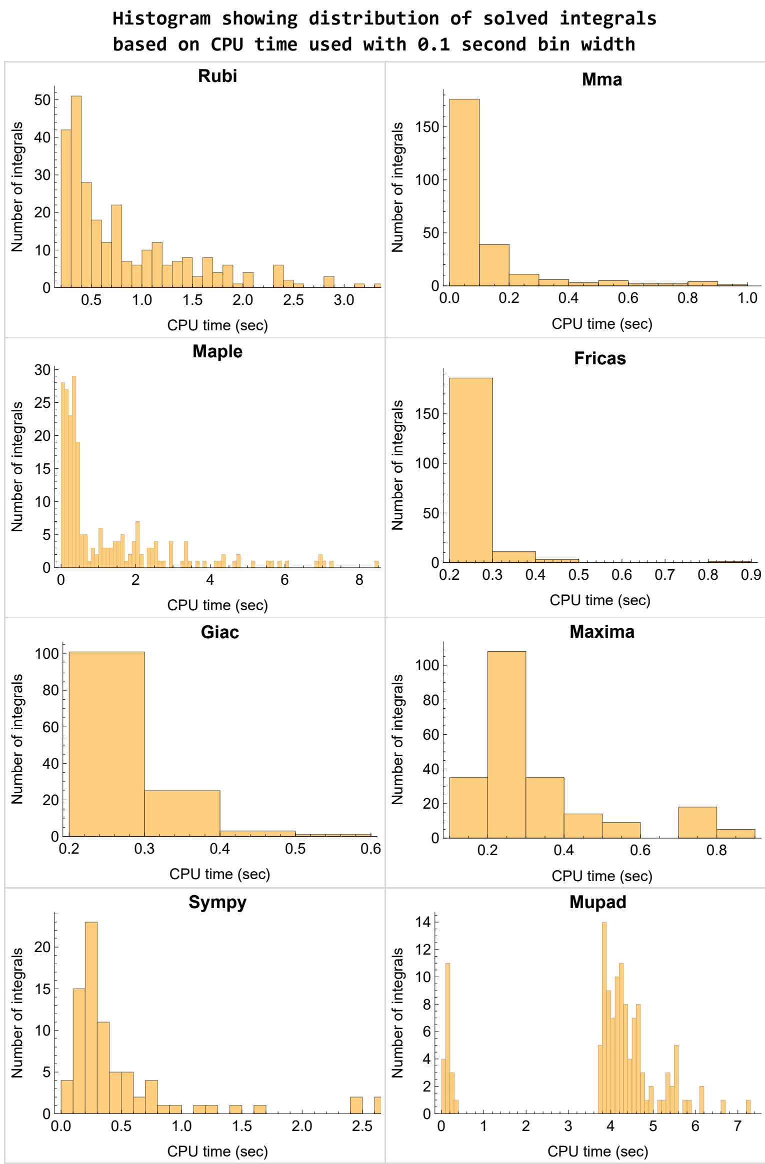


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

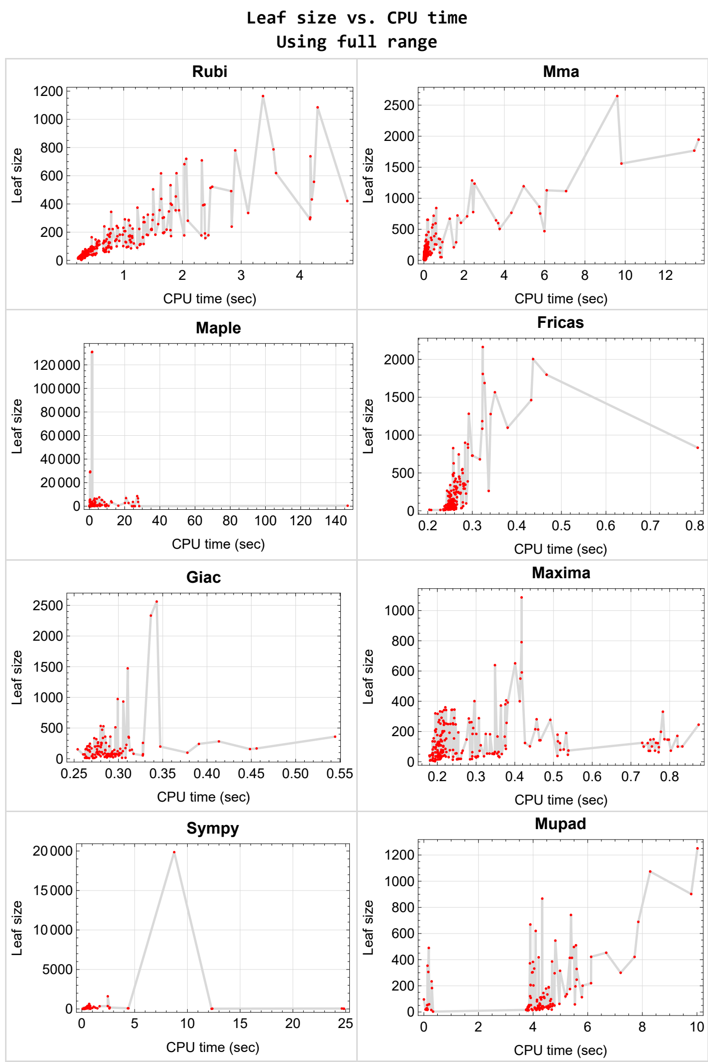


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{42, 43, 120, 121, 122, 126, 127, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 260, 264, 280}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {46, 97, 99, 275, 276, 277, 293, 294, 295, 300}

Mathematica {34, 41, 70, 73, 109, 112, 114, 118, 238, 242, 246, 255, 259, 263}

Maple {18, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 73, 112, 114, 115, 117, 118, 141, 142, 152, 153, 154, 159, 160, 161, 163, 167, 168, 169, 173, 176, 177, 181, 201, 202, 203, 204, 207, 208, 209, 212, 213, 214, 217, 218, 221, 222, 223, 226, 227, 228, 231, 232, 233, 236, 237, 240, 241, 244, 245, 248, 249, 250, 253, 254, 257, 258, 261, 262, 265, 269, 279, 295, 296, 299, 300}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

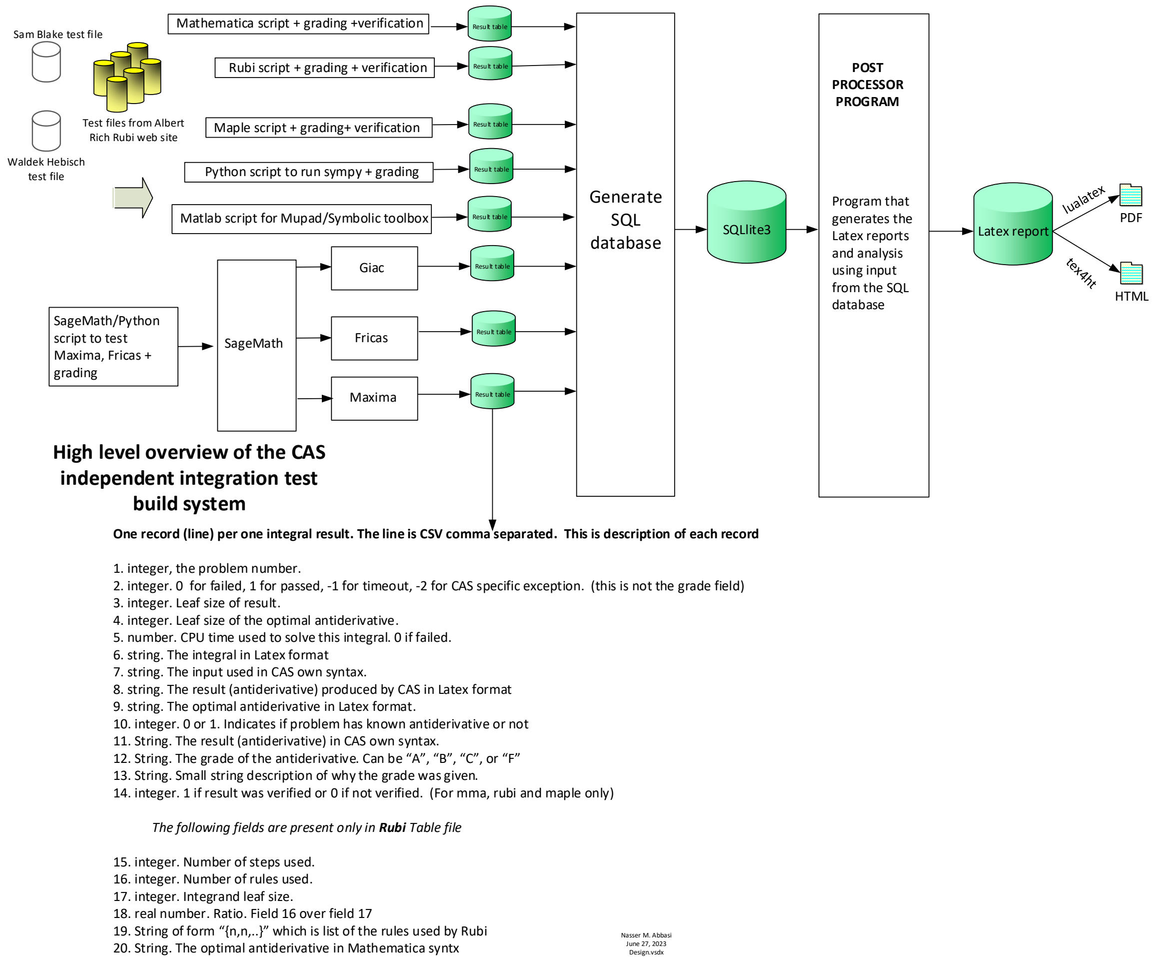
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 203, 204, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 226, 227, 228, 229, 231, 232, 233, 234, 236, 237, 238, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251, 253, 254, 255, 257, 258, 259, 261, 262, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

B grade { 23, 291 }

C grade { 200, 201, 202 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 71, 72, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 99, 102, 103, 104, 105, 106, 107, 108, 110, 111, 125, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 226, 227, 228, 229, 232, 233, 234, 236, 237, 238, 240, 241, 244, 245, 249, 250, 251, 253, 254, 257, 258, 261, 262, 266, 267, 268, 270, 271, 272, 273, 274, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

B grade { 41, 70, 98, 100, 101, 109, 139, 150, 231, 242, 246, 248, 255, 259, 263, 275, 276, 279, 281, 283 }

C grade { 24, 26, 34, 66, 73, 74, 75, 78, 95, 112, 113, 114, 115, 116, 118, 265, 278, 282 }

F normal fail { 117, 119, 123, 124, 269, 277 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 22, 35, 36, 37, 38, 39, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 104, 105, 106, 107, 108, 111, 113, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 143, 144, 145, 146, 150, 151, 155, 156, 157, 162, 170, 171, 178, 179, 180, 187, 188, 189, 193, 194, 195, 196, 197, 198, 199, 200, 266, 267, 268, 272, 273, 274, 282, 283, 284, 285, 286, 289, 292, 293, 297, 298 }

B grade { 19, 21, 28, 40, 41, 55, 59, 61, 82, 86, 102, 103, 109, 110, 116, 123, 124, 125, 185, 186, 205, 210, 215, 219, 224, 229, 234, 238, 242, 246, 251, 255, 259, 263, 278, 287, 288, 290, 291, 294 }

C grade { 18, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 73, 93, 112, 114, 115, 117, 118, 141, 142, 152, 153, 154, 159, 160, 161, 163, 167, 168, 169, 173, 176, 177, 181, 201, 202, 203, 204, 207, 208, 209, 212, 213, 214, 217, 218, 221, 222, 223, 226, 227, 228, 231, 232, 233, 236, 237, 240, 241, 244, 245, 248, 249, 250, 253, 254, 257, 258, 261, 262, 265, 269, 279, 295, 296, 299, 300 }

F normal fail { 44, 45, 46, 47, 119, 158, 165, 166, 175, 184, 190, 191, 192, 270, 271, 275, 276, 277, 281 }

F(-1) timedout fail { 147, 148, 149, 164, 172, 174, 182, 183 }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 35, 36, 37, 38, 50, 51, 52, 55, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 104, 105, 194, 195, 196, 197, 198, 199, 202, 257, 258, 259, 261, 262, 263, 266, 267, 268, 272, 273, 274, 289, 291, 292, 293, 296, 298, 299 }

B grade { 44, 45, 46, 47, 53, 54, 57, 95, 102, 103, 107, 108, 200, 201, 203, 204, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 226, 227, 228, 229, 231, 232, 233, 234, 236, 237, 238, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251, 253, 254, 255, 265, 283, 284, 285, 286, 287, 288, 290, 294, 295, 300 }

C grade { 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 193, 297 }

F normal fail { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 39, 40, 41, 48, 49, 56, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 93, 98, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 158, 166, 175, 184, 190, 191, 192, 269, 270, 271, 275, 276, 277, 278, 279, 281, 282 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 20, 21, 23, 35, 36, 37, 38, 39, 40, 50, 51, 52, 53, 54, 57, 58, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 77, 78, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 103, 104, 105, 107, 108, 110, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 142, 143, 144, 145, 146, 147, 148, 149, 150, 154, 155, 156, 157, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 226, 227, 228, 229, 268, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299 }

B grade { 7, 16, 17, 19, 22, 25, 27, 31, 33, 41, 44, 45, 46, 47, 55, 56, 59, 61, 86, 93, 95, 98, 100, 101, 102, 109, 140, 151, 234, 238, 240, 241, 242, 244, 245, 246, 251, 255, 257, 258, 259, 261, 262, 263, 283, 286, 295, 300 }

C grade { 76, 79, 141, 152, 153, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 266, 267, 269, 272, 273, 274 }

F normal fail { 18, 24, 26, 28, 29, 30, 32, 34, 48, 49, 73, 80, 81, 106, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 158, 166, 175, 184, 190, 191, 192, 231, 232, 233, 236, 237, 248, 249, 250, 253, 254, 265, 270, 271, 275, 276, 277, 278, 279, 281, 282 }

F(-1) timedout fail { }

F(-2) exception fail { 82 }

2.1.6 Giac

A grade { 44, 45, 46, 47, 50, 51, 52, 54, 55, 189, 194, 195, 196, 197, 198, 199, 295, 297, 298, 300 }

B grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 35, 36, 37, 38, 53, 57, 58, 59, 60, 62, 63, 64, 65, 67, 68, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 102, 103, 104, 105, 107, 108, 128, 129, 130, 131, 133, 134, 135, 193, 292, 293, 294, 296, 299 }

C grade { 132, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 268, 272, 273, 274 }

F normal fail { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 39, 40, 41, 48, 49, 56, 61, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 93, 95, 98, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 136, 146, 158, 166, 175, 184, 185, 186, 187, 188, 190, 191, 192, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 226, 227, 228, 229, 231, 232, 233, 234, 236, 237, 238, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251, 253, 254, 255, 257, 258, 259, 261, 262, 263, 265, 269, 270, 271, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291 }

F(-1) timedout fail { }

F(-2) exception fail { 266, 267, 289 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 35, 36, 37, 38, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 83, 84, 85, 87, 88, 89, 91, 92, 94, 96, 97, 99, 102, 103, 104, 105, 107, 108, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 193, 194, 195, 196, 197, 198, 199, 266, 267, 268, 272, 273, 274, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

C grade { }

F normal fail { }

F(-1) timedout fail { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 90, 93, 95, 98, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 158, 166, 175, 184, 190, 191, 192, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 226, 227, 228, 229, 231, 232, 233, 234, 236, 237, 238, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251, 253, 254, 255, 257, 258, 259, 261, 262, 263, 265, 269, 270, 271, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 8, 9, 10, 11, 12, 14, 16, 20, 22, 53, 54, 57, 58, 62, 63, 64, 65, 94, 96, 105, 129, 131, 133, 134, 135, 137, 138, 139, 140, 143, 144, 145, 147, 148, 149, 150, 151, 155, 157, 162, 170, 171, 178, 179, 180, 292, 293 }

B grade { 60, 67, 68, 87, 88, 89, 91, 92, 97, 99, 102, 103, 104, 107, 108, 130, 156, 189, 194, 195, 196, 198, 199 }

C grade { 1, 2, 3, 4, 5, 6, 35, 36, 37, 38, 266, 267, 268, 272, 273, 274, 297 }

F normal fail { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 39, 40, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 59, 61, 66, 69, 70, 71, 72, 73, 74, 75, 77, 78, 83, 84, 85, 86, 90, 93, 95, 98, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 128, 132, 136, 141, 142, 146, 152, 153, 154, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 172, 173, 174, 175, 176, 177, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 197, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 226, 227, 228, 229, 231, 232, 233, 234, 236, 237, 238, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251, 253, 254, 255, 257, 258, 259, 261, 262, 263, 265, 269, 270, 271, 275, 276, 277, 283, 284, 285, 286, 287, 288, 289, 290, 291, 295, 296, 298, 300 }

F(-1) timedout fail { 41, 76, 79, 80, 81, 82, 121, 278, 279, 281, 282, 299 }

F(-2) exception fail { 294 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	52	67	45	61	51	49	245	41
N.S.	1	1.02	1.31	0.88	1.20	1.00	0.96	4.80	0.80
time (sec)	N/A	0.216	0.009	0.160	0.212	0.244	0.333	0.268	4.487

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	52	50	47	46	55	54	255	43
N.S.	1	1.04	1.00	0.94	0.92	1.10	1.08	5.10	0.86
time (sec)	N/A	0.234	0.008	0.099	0.185	0.252	0.301	0.277	4.323

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	42	57	37	52	43	41	195	33
N.S.	1	1.02	1.39	0.90	1.27	1.05	1.00	4.76	0.80
time (sec)	N/A	0.223	0.011	0.077	0.188	0.260	0.266	0.268	4.532

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	42	40	38	35	44	46	206	35
N.S.	1	1.05	1.00	0.95	0.88	1.10	1.15	5.15	0.88
time (sec)	N/A	0.217	0.008	0.060	0.209	0.248	0.231	0.276	4.403

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	32	47	27	41	34	32	144	26
N.S.	1	1.03	1.52	0.87	1.32	1.10	1.03	4.65	0.84
time (sec)	N/A	0.190	0.008	0.059	0.180	0.246	0.196	0.265	4.125

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	25	33	27	153	22
N.S.	1	1.00	1.00	0.92	1.00	1.32	1.08	6.12	0.88
time (sec)	N/A	0.173	0.002	0.070	0.208	0.241	0.113	0.254	4.099

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	28	86	0	0	0	0
N.S.	1	1.00	0.93	1.00	3.07	0.00	0.00	0.00	0.00
time (sec)	N/A	0.183	0.008	0.073	0.195	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	33	30	35	30	39	26	143	27
N.S.	1	1.10	1.00	1.17	1.00	1.30	0.87	4.77	0.90
time (sec)	N/A	0.192	0.008	0.064	0.229	0.254	0.128	0.285	4.169

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	29	47	26	36	35	24	140	40
N.S.	1	0.94	1.52	0.84	1.16	1.13	0.77	4.52	1.29
time (sec)	N/A	0.190	0.008	0.071	0.180	0.259	0.194	0.267	4.194

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	48	40	50	46	209	39
N.S.	1	1.00	1.00	1.02	0.85	1.06	0.98	4.45	0.83
time (sec)	N/A	0.222	0.010	0.085	0.188	0.256	0.227	0.266	4.610

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	57	36	51	43	32	205	60
N.S.	1	1.00	1.39	0.88	1.24	1.05	0.78	5.00	1.46
time (sec)	N/A	0.209	0.008	0.095	0.188	0.248	0.257	0.278	4.635

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	177	80	92	135	98	114	534	85
N.S.	1	1.69	0.76	0.88	1.29	0.93	1.09	5.09	0.81
time (sec)	N/A	1.250	0.018	0.300	0.188	0.257	0.400	0.280	4.627

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	172	87	164	155	0	0	0	0
N.S.	1	1.35	0.69	1.29	1.22	0.00	0.00	0.00	0.00
time (sec)	N/A	1.131	0.331	0.473	0.191	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	115	62	72	118	81	90	335	65
N.S.	1	1.42	0.77	0.89	1.46	1.00	1.11	4.14	0.80
time (sec)	N/A	0.813	0.014	0.201	0.204	0.260	0.324	0.273	4.240

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	120	66	144	134	0	0	0	0
N.S.	1	1.17	0.64	1.40	1.30	0.00	0.00	0.00	0.00
time (sec)	N/A	0.732	0.179	0.486	0.193	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	61	43	49	97	62	60	154	44
N.S.	1	1.13	0.80	0.91	1.80	1.15	1.11	2.85	0.81
time (sec)	N/A	0.457	0.010	0.166	0.196	0.253	0.234	0.276	4.359

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	71	46	116	135	0	0	0	0
N.S.	1	1.22	0.79	2.00	2.33	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	0.063	0.500	0.212	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	97	128	114	459	0	0	0	0	0
N.S.	1	1.32	1.18	4.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.707	0.051	3.368	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	60	49	145	146	0	0	0	0
N.S.	1	1.09	0.89	2.64	2.65	0.00	0.00	0.00	0.00
time (sec)	N/A	0.424	0.081	0.082	0.209	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	60	57	72	96	79	56	137	145
N.S.	1	0.98	0.93	1.18	1.57	1.30	0.92	2.25	2.38
time (sec)	N/A	0.466	0.014	0.090	0.190	0.245	0.210	0.266	4.376

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	97	87	184	176	0	0	0	0
N.S.	1	0.94	0.84	1.79	1.71	0.00	0.00	0.00	0.00
time (sec)	N/A	0.692	0.166	0.117	0.200	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	114	82	101	154	97	90	319	196
N.S.	1	1.27	0.91	1.12	1.71	1.08	1.00	3.54	2.18
time (sec)	N/A	0.796	0.018	0.109	0.243	0.286	0.261	0.275	5.550

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	186	421	117	2133	289	0	0	0	0
N.S.	1	2.26	0.63	11.47	1.55	0.00	0.00	0.00	0.00
time (sec)	N/A	2.904	0.392	5.819	0.244	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	196	304	178	737	0	0	0	0	0
N.S.	1	1.55	0.91	3.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.571	0.506	5.114	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	139	239	88	871	262	0	0	0	0
N.S.	1	1.72	0.63	6.27	1.88	0.00	0.00	0.00	0.00
time (sec)	N/A	1.748	0.234	4.399	0.203	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	149	177	143	681	0	0	0	0	0
N.S.	1	1.19	0.96	4.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.489	0.308	3.880	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	95	109	68	2910	215	0	0	0	0
N.S.	1	1.15	0.72	30.63	2.26	0.00	0.00	0.00	0.00
time (sec)	N/A	0.801	0.110	3.285	0.197	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	101	79	168	0	0	0	0	0
N.S.	1	1.19	0.93	1.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.636	0.087	0.399	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	150	182	156	536	0	0	0	0	0
N.S.	1	1.21	1.04	3.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.984	0.061	3.673	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	79	93	72	718	0	0	0	0	0
N.S.	1	1.18	0.91	9.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.662	0.101	4.614	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	95	90	79	3498	252	0	0	0	0
N.S.	1	0.95	0.83	36.82	2.65	0.00	0.00	0.00	0.00
time (sec)	N/A	0.792	0.125	3.316	0.223	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	154	158	142	840	0	0	0	0	0
N.S.	1	1.03	0.92	5.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.514	0.163	6.865	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	141	191	118	657	342	0	0	0	0
N.S.	1	1.35	0.84	4.66	2.43	0.00	0.00	0.00	0.00
time (sec)	N/A	1.442	0.169	4.726	0.217	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	164	164	864	869	0	0	0	0	0
N.S.	1	1.00	5.27	5.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.296	5.734	8.678	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	247	213	245	276	247	427	1473	296
N.S.	1	1.01	0.87	1.00	1.13	1.01	1.74	6.01	1.21
time (sec)	N/A	0.676	0.067	0.606	0.205	0.247	0.640	0.310	4.789

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	171	150	167	198	177	282	932	190
N.S.	1	1.01	0.89	0.99	1.17	1.05	1.67	5.51	1.12
time (sec)	N/A	0.567	0.043	0.482	0.196	0.249	0.468	0.306	4.570

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	112	98	105	131	118	182	529	115
N.S.	1	1.02	0.89	0.95	1.19	1.07	1.65	4.81	1.05
time (sec)	N/A	0.389	0.033	0.485	0.205	0.254	0.357	0.283	4.398

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	59	69	55	65	64	87	268	60
N.S.	1	1.04	1.21	0.96	1.14	1.12	1.53	4.70	1.05
time (sec)	N/A	0.269	0.019	0.085	0.201	0.262	0.259	0.277	4.248

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	617	671	358	406	0	0	0	0
N.S.	1	1.58	1.72	0.92	1.04	0.00	0.00	0.00	0.00
time (sec)	N/A	1.043	1.286	1.046	0.378	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	590	618	755	1926	550	0	0	0	0
N.S.	1	1.05	1.28	3.26	0.93	0.00	0.00	0.00	0.00
time (sec)	N/A	1.333	5.781	1.213	0.414	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	657	682	1559	3791	1087	0	0	0	0
N.S.	1	1.04	2.37	5.77	1.65	0.00	0.00	0.00	0.00
time (sec)	N/A	1.343	9.814	1.477	0.418	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	16	15	16	16
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.94	1.00	1.00
time (sec)	N/A	0.200	4.335	0.387	0.587	0.252	0.567	0.299	4.242

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	16	15	16	16
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.94	1.00	1.00
time (sec)	N/A	0.208	3.127	0.293	0.641	0.260	0.412	0.293	4.834

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	119	0	153	354	0	79	0
N.S.	1	1.00	1.92	0.00	2.47	5.71	0.00	1.27	0.00
time (sec)	N/A	0.291	0.087	0.000	0.223	0.279	0.000	0.293	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	130	226	0	223	728	0	143	0
N.S.	1	1.02	1.77	0.00	1.74	5.69	0.00	1.12	0.00
time (sec)	N/A	0.347	0.197	0.000	0.293	0.300	0.000	0.295	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F	A	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	200	194	329	0	401	1278	0	226	0
N.S.	1	0.97	1.64	0.00	2.00	6.39	0.00	1.13	0.00
time (sec)	N/A	1.095	0.386	0.000	0.296	0.341	0.000	0.309	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	283	281	431	0	639	2004	0	357	0
N.S.	1	0.99	1.52	0.00	2.26	7.08	0.00	1.26	0.00
time (sec)	N/A	1.298	0.596	0.000	0.349	0.436	0.000	0.312	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	139	125	199	0	0	0	0	0
N.S.	1	0.75	0.67	1.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.416	0.789	0.484	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	99	77	190	0	0	0	0	0
N.S.	1	0.69	0.53	1.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.300	0.143	0.422	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	30	45	63	41	0	58	0
N.S.	1	1.00	0.81	1.22	1.70	1.11	0.00	1.57	0.00
time (sec)	N/A	0.210	0.054	0.359	0.290	0.258	0.000	0.293	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	86	45	81	67	61	0	90	0
N.S.	1	1.04	0.54	0.98	0.81	0.73	0.00	1.08	0.00
time (sec)	N/A	0.329	0.060	0.598	0.214	0.279	0.000	0.303	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	135	55	100	99	81	0	122	0
N.S.	1	1.09	0.44	0.81	0.80	0.65	0.00	0.98	0.00
time (sec)	N/A	0.446	0.059	0.584	0.215	0.251	0.000	0.299	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	11	3	12	3
N.S.	1	1.00	1.00	1.33	1.00	3.67	1.00	4.00	1.00
time (sec)	N/A	0.183	0.069	0.343	0.194	0.240	0.106	0.273	0.338

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	62	15	22	22
N.S.	1	1.00	1.00	1.08	1.00	5.17	1.25	1.83	1.83
time (sec)	N/A	0.201	0.009	0.432	0.196	0.254	0.806	0.294	4.341

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	67	61	159	171	63	0	53	201
N.S.	1	1.08	0.98	2.56	2.76	1.02	0.00	0.85	3.24
time (sec)	N/A	0.304	0.052	1.026	0.207	0.246	0.000	0.327	5.813

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	39	35	59	76	0	0	0	0
N.S.	1	1.05	0.95	1.59	2.05	0.00	0.00	0.00	0.00
time (sec)	N/A	0.338	0.060	0.361	0.201	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	13	6	14	5	14	21
N.S.	1	1.00	1.00	1.62	0.75	1.75	0.62	1.75	2.62
time (sec)	N/A	0.177	0.005	0.360	0.190	0.259	0.282	0.273	4.092

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	41	44	22	34	29	31	101	21
N.S.	1	1.14	1.22	0.61	0.94	0.81	0.86	2.81	0.58
time (sec)	N/A	0.217	0.029	0.335	0.211	0.246	0.207	0.275	0.108

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	28	80	76	42	0	80	81
N.S.	1	1.00	0.74	2.11	2.00	1.11	0.00	2.11	2.13
time (sec)	N/A	0.199	0.031	0.355	0.232	0.264	0.000	0.317	4.141

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	60	50	37	47	47	88	154	34
N.S.	1	1.20	1.00	0.74	0.94	0.94	1.76	3.08	0.68
time (sec)	N/A	0.228	0.051	0.355	0.202	0.250	0.292	0.263	4.502

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	72	43	128	118	66	0	0	112
N.S.	1	1.07	0.64	1.91	1.76	0.99	0.00	0.00	1.67
time (sec)	N/A	0.282	0.041	0.365	0.205	0.254	0.000	0.000	4.297

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	99	81	106	106	112	153	512	134
N.S.	1	0.98	0.80	1.05	1.05	1.11	1.51	5.07	1.33
time (sec)	N/A	0.339	0.031	0.118	0.204	0.255	0.444	0.297	4.523

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	82	92	85	79	84	117	360	98
N.S.	1	1.05	1.18	1.09	1.01	1.08	1.50	4.62	1.26
time (sec)	N/A	0.318	0.020	0.102	0.210	0.266	0.376	0.291	4.654

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	71	56	63	61	66	76	259	62
N.S.	1	1.09	0.86	0.97	0.94	1.02	1.17	3.98	0.95
time (sec)	N/A	0.303	0.019	0.094	0.199	0.248	0.321	0.281	4.964

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	33	43	30	31	48	41	197	42
N.S.	1	0.94	1.23	0.86	0.89	1.37	1.17	5.63	1.20
time (sec)	N/A	0.224	0.013	0.069	0.202	0.249	0.179	0.274	4.635

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	259	68	128	0	0	0	0
N.S.	1	1.00	2.82	0.74	1.39	0.00	0.00	0.00	0.00
time (sec)	N/A	0.479	0.123	0.343	0.201	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	67	55	61	54	68	144	259	62
N.S.	1	1.05	0.86	0.95	0.84	1.06	2.25	4.05	0.97
time (sec)	N/A	0.280	0.044	0.098	0.205	0.262	0.520	0.286	4.721

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	93	76	78	85	111	410	360	247
N.S.	1	1.03	0.84	0.87	0.94	1.23	4.56	4.00	2.74
time (sec)	N/A	0.307	0.084	0.128	0.221	0.279	0.782	0.285	5.595

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	244	210	448	320	0	0	0	0
N.S.	1	0.93	0.80	1.70	1.22	0.00	0.00	0.00	0.00
time (sec)	N/A	0.569	1.486	0.241	0.195	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	204	193	644	350	259	0	0	0	0
N.S.	1	0.95	3.16	1.72	1.27	0.00	0.00	0.00	0.00
time (sec)	N/A	0.500	3.586	0.188	0.217	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	127	106	244	202	0	0	0	0
N.S.	1	0.93	0.78	1.79	1.49	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	0.242	0.171	0.213	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	76	55	133	139	0	0	0	0
N.S.	1	0.94	0.68	1.64	1.72	0.00	0.00	0.00	0.00
time (sec)	N/A	0.447	0.069	0.246	0.211	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	148	148	777	866	0	0	0	0	0
N.S.	1	1.00	5.25	5.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.326	2.449	5.624	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	257	206	299	244	0	0	0	0
N.S.	1	1.02	0.82	1.19	0.97	0.00	0.00	0.00	0.00
time (sec)	N/A	1.018	0.824	0.144	0.217	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	347	291	386	360	0	0	0	0
N.S.	1	0.94	0.79	1.04	0.97	0.00	0.00	0.00	0.00
time (sec)	N/A	1.126	1.604	0.187	0.221	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	673	709	529	426	591	0	0	0	0
N.S.	1	1.05	0.79	0.63	0.88	0.00	0.00	0.00	0.00
time (sec)	N/A	1.468	0.410	0.889	0.418	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	138	185	164	192	0	0	0	0
N.S.	1	1.15	1.54	1.37	1.60	0.00	0.00	0.00	0.00
time (sec)	N/A	0.535	0.048	1.302	0.231	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	436	507	259	192	0	0	0	0
N.S.	1	1.49	1.74	0.89	0.66	0.00	0.00	0.00	0.00
time (sec)	N/A	1.033	3.764	1.165	0.213	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	738	1165	843	554	651	0	0	0	0
N.S.	1	1.58	1.14	0.75	0.88	0.00	0.00	0.00	0.00
time (sec)	N/A	2.176	0.616	1.472	0.401	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	619	619	575	646	0	0	0	0	0
N.S.	1	1.00	0.93	1.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.375	0.448	0.407	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	738	738	719	752	0	0	0	0	0
N.S.	1	1.00	0.97	1.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.665	0.487	0.391	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	356	596	750	0	0	0	0	0
N.S.	1	1.06	1.78	2.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.059	0.581	1.383	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	52	59	42	41	38	0	164	31
N.S.	1	1.02	1.16	0.82	0.80	0.75	0.00	3.22	0.61
time (sec)	N/A	0.204	0.015	0.039	0.203	0.251	0.000	0.264	4.301

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	43	52	37	36	31	0	114	26
N.S.	1	1.02	1.24	0.88	0.86	0.74	0.00	2.71	0.62
time (sec)	N/A	0.199	0.012	0.040	0.198	0.255	0.000	0.279	4.246

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	29	22	27	26	24	0	65	16
N.S.	1	1.32	1.00	1.23	1.18	1.09	0.00	2.95	0.73
time (sec)	N/A	0.182	0.017	0.038	0.209	0.264	0.000	0.274	4.236

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	27	19	33	66	0	0	0	0
N.S.	1	1.42	1.00	1.74	3.47	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	0.009	0.162	0.209	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	32	45	32	31	30	92	65	18
N.S.	1	1.28	1.80	1.28	1.24	1.20	3.68	2.60	0.72
time (sec)	N/A	0.192	0.015	0.048	0.201	0.252	0.552	0.278	4.683

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	43	58	37	36	38	160	114	45
N.S.	1	1.02	1.38	0.88	0.86	0.90	3.81	2.71	1.07
time (sec)	N/A	0.192	0.017	0.043	0.195	0.276	1.281	0.268	4.700

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	35	31	35	24	35	121	168	24
N.S.	1	0.92	0.82	0.92	0.63	0.92	3.18	4.42	0.63
time (sec)	N/A	0.199	0.012	0.043	0.189	0.275	0.994	0.270	4.562

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	28	25	30	19	30	0	119	0
N.S.	1	0.90	0.81	0.97	0.61	0.97	0.00	3.84	0.00
time (sec)	N/A	0.196	0.011	0.038	0.195	0.257	0.000	0.276	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	16	24	87	70	14
N.S.	1	1.00	1.00	0.75	0.80	1.20	4.35	3.50	0.70
time (sec)	N/A	0.173	0.007	0.037	0.206	0.255	0.201	0.262	3.981

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	29	18	36	126	70	22
N.S.	1	1.00	1.00	1.21	0.75	1.50	5.25	2.92	0.92
time (sec)	N/A	0.179	0.015	0.042	0.207	0.262	0.400	0.260	3.944

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	32	26	95	104	0	0	0	0
N.S.	1	1.14	0.93	3.39	3.71	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	0.015	0.181	0.216	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	15	23	15	104	26
N.S.	1	1.00	1.00	0.95	0.79	1.21	0.79	5.47	1.37
time (sec)	N/A	0.176	0.001	0.147	0.201	0.257	0.084	0.281	4.150

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	36	52	45	147	128	0	0	0
N.S.	1	0.95	1.37	1.18	3.87	3.37	0.00	0.00	0.00
time (sec)	N/A	0.233	0.039	0.285	0.275	0.257	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	66	46	62	44	56	188	50
N.S.	1	1.00	1.69	1.18	1.59	1.13	1.44	4.82	1.28
time (sec)	N/A	0.234	0.032	0.103	0.194	0.253	0.220	0.280	4.782

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	41	42	48	81	86	97	255	114
N.S.	1	0.76	0.78	0.89	1.50	1.59	1.80	4.72	2.11
time (sec)	N/A	0.268	0.033	0.446	0.188	0.258	0.310	0.281	4.381

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	33	144	43	112	0	0	0	0
N.S.	1	0.94	4.11	1.23	3.20	0.00	0.00	0.00	0.00
time (sec)	N/A	0.235	0.030	0.453	0.198	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	45	43	45	53	67	136	198	93
N.S.	1	0.94	0.90	0.94	1.10	1.40	2.83	4.12	1.94
time (sec)	N/A	0.259	0.033	0.484	0.204	0.250	0.524	0.274	4.143

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	29	117	22	58	0	0	0	0
N.S.	1	1.16	4.68	0.88	2.32	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	0.014	0.361	0.188	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	33	144	43	132	0	0	0	0
N.S.	1	0.94	4.11	1.23	3.77	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	0.019	0.452	0.203	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	167	270	599	333	385	644	2333	742
N.S.	1	0.99	1.61	3.57	1.98	2.29	3.83	13.89	4.42
time (sec)	N/A	0.463	0.142	0.559	0.207	0.273	0.697	0.337	5.386

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	128	174	351	207	241	369	973	386
N.S.	1	1.07	1.45	2.92	1.72	2.01	3.08	8.11	3.22
time (sec)	N/A	0.402	0.085	0.485	0.194	0.269	0.520	0.299	4.694

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	106	138	121	109	133	173	338	136
N.S.	1	1.09	1.42	1.25	1.12	1.37	1.78	3.48	1.40
time (sec)	N/A	0.351	0.043	0.177	0.208	0.261	0.406	0.283	5.235

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	48	35	36	60	46	202	48
N.S.	1	1.00	1.20	0.88	0.90	1.50	1.15	5.05	1.20
time (sec)	N/A	0.182	0.010	0.104	0.200	0.260	0.191	0.265	4.561

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	148	206	191	0	0	0	0	0
N.S.	1	1.14	1.58	1.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.552	0.074	1.223	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	121	125	137	121	262	1605	472	175
N.S.	1	1.05	1.09	1.19	1.05	2.28	13.96	4.10	1.52
time (sec)	N/A	0.417	0.131	0.633	0.201	0.337	2.457	0.282	5.351

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	170	174	198	291	833	19859	2562	422
N.S.	1	1.02	1.04	1.19	1.74	4.99	118.92	15.34	2.53
time (sec)	N/A	0.488	0.205	1.049	0.211	0.806	8.752	0.343	6.126

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	374	374	1115	1411	791	0	0	0	0
N.S.	1	1.00	2.98	3.77	2.11	0.00	0.00	0.00	0.00
time (sec)	N/A	0.800	7.064	0.671	0.417	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	231	295	450	400	0	0	0	0
N.S.	1	1.05	1.33	2.04	1.81	0.00	0.00	0.00	0.00
time (sec)	N/A	0.636	0.922	0.362	0.413	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	93	111	174	0	0	0	0	0
N.S.	1	0.96	1.14	1.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.503	0.141	0.439	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	214	241	1767	1603	0	0	0	0	0
N.S.	1	1.13	8.26	7.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.418	13.432	9.717	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	480	491	470	590	0	0	0	0	0
N.S.	1	1.02	0.98	1.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.807	5.990	1.184	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	546	533	2646	8597	0	0	0	0	0
N.S.	1	0.98	4.85	15.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.132	9.608	27.127	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	326	324	600	7528	0	0	0	0	0
N.S.	1	0.99	1.84	23.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.899	3.692	5.556	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	130	208	372	0	0	0	0	0
N.S.	1	0.98	1.58	2.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.729	0.240	0.851	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	308	344	0	3250	0	0	0	0	0
N.S.	1	1.12	0.00	10.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.477	0.000	12.702	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1089	1085	1945	4101	0	0	0	0	0
N.S.	1	1.00	1.79	3.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.793	13.648	12.353	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	162	222	0	0	0	0	0	0	0
N.S.	1	1.37	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.470	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	253	36	19	22	22
N.S.	1	1.00	1.10	1.00	12.65	1.80	0.95	1.10	1.10
time (sec)	N/A	0.313	2.092	0.361	2.550	0.255	85.904	0.342	3.874

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	418	52	0	22	22
N.S.	1	1.00	1.10	1.00	20.90	2.60	0.00	1.10	1.10
time (sec)	N/A	0.306	0.316	0.358	4.130	0.257	0.000	0.375	3.862

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	38	34	38	39
N.S.	1	1.00	1.05	0.90	0.98	0.95	0.85	0.95	0.98
time (sec)	N/A	0.250	0.136	0.654	0.799	0.261	8.499	0.730	4.272

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	460	453	0	1681	0	0	0	0	0
N.S.	1	0.98	0.00	3.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.198	0.000	1.381	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	302	305	0	926	0	0	0	0	0
N.S.	1	1.01	0.00	3.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.844	0.000	0.898	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	84	98	372	0	0	0	0	0
N.S.	1	0.94	1.10	4.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.292	0.344	0.671	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	48	61	38	39
N.S.	1	1.00	1.05	0.90	0.98	1.20	1.52	0.95	0.98
time (sec)	N/A	0.245	0.206	0.566	0.372	0.235	3.650	0.429	3.988

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	246	91	126	38	39
N.S.	1	1.00	1.05	0.90	6.15	2.28	3.15	0.95	0.98
time (sec)	N/A	0.236	0.864	0.566	0.426	0.264	10.502	0.824	5.007

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	49	38	81	0	90	96
N.S.	1	1.00	0.92	1.32	1.03	2.19	0.00	2.43	2.59
time (sec)	N/A	0.181	0.033	0.210	0.206	0.249	0.000	0.277	4.297

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	19	19	71	19
N.S.	1	1.00	0.87	0.87	0.83	0.83	0.83	3.09	0.83
time (sec)	N/A	0.159	0.013	0.487	0.253	0.251	0.104	0.287	0.141

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	19	39	71	19
N.S.	1	1.00	0.87	0.87	0.83	0.83	1.70	3.09	0.83
time (sec)	N/A	0.179	0.012	0.421	0.254	0.248	0.158	0.278	4.101

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	17	16	14	19	69	16
N.S.	1	1.00	1.12	1.06	1.00	0.88	1.19	4.31	1.00
time (sec)	N/A	0.146	0.007	0.555	0.251	0.246	0.072	0.262	3.747

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	21	34	15	0	15	59
N.S.	1	1.00	0.90	1.00	1.62	0.71	0.00	0.71	2.81
time (sec)	N/A	0.176	0.014	0.243	0.222	0.240	0.000	0.308	0.194

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	20	17	18	14	70	17
N.S.	1	1.00	1.06	1.18	1.00	1.06	0.82	4.12	1.00
time (sec)	N/A	0.149	0.014	0.319	0.253	0.251	0.086	0.287	0.094

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	17	19	16	19	71	16
N.S.	1	1.00	0.78	0.74	0.83	0.70	0.83	3.09	0.70
time (sec)	N/A	0.150	0.012	0.306	0.257	0.239	0.177	0.283	3.753

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	19	19	16	20	71	19
N.S.	1	1.00	0.87	0.83	0.83	0.70	0.87	3.09	0.83
time (sec)	N/A	0.153	0.012	0.325	0.261	0.259	0.234	0.271	3.767

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	66	62	112	73	264	0	0	203
N.S.	1	0.93	0.87	1.58	1.03	3.72	0.00	0.00	2.86
time (sec)	N/A	0.217	0.122	0.733	0.265	0.253	0.000	0.000	4.013

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	45	37	37	36	48	37	41	36
N.S.	1	1.07	0.88	0.88	0.86	1.14	0.88	0.98	0.86
time (sec)	N/A	0.197	0.028	28.279	0.303	0.249	0.206	0.291	3.839

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	45	37	37	36	48	37	41	36
N.S.	1	1.07	0.88	0.88	0.86	1.14	0.88	0.98	0.86
time (sec)	N/A	0.201	0.034	25.125	0.290	0.249	0.145	0.286	4.218

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	74	37	36	48	41	41	36
N.S.	1	1.00	2.18	1.09	1.06	1.41	1.21	1.21	1.06
time (sec)	N/A	0.193	0.164	24.160	0.304	0.259	0.194	0.290	3.804

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	33	39	20	39	33
N.S.	1	1.00	1.00	0.94	2.06	2.44	1.25	2.44	2.06
time (sec)	N/A	0.153	0.006	0.422	0.298	0.240	0.091	0.280	4.174

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	C	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	49	50	53	664	38	36	0	37	183
N.S.	1	1.02	1.08	13.55	0.78	0.73	0.00	0.76	3.73
time (sec)	N/A	0.211	0.083	0.181	0.510	0.248	0.000	0.287	0.297

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F	C	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	39	39	37	647	54	41	0	36	207
N.S.	1	1.00	0.95	16.59	1.38	1.05	0.00	0.92	5.31
time (sec)	N/A	0.197	0.033	0.152	0.247	0.245	0.000	0.291	3.905

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	42	39	34	38	32	37	34
N.S.	1	1.00	1.17	1.08	0.94	1.06	0.89	1.03	0.94
time (sec)	N/A	0.199	0.030	0.293	0.304	0.247	0.184	0.297	4.060

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	34	33	36	40	37	38	32
N.S.	1	1.00	1.10	1.06	1.16	1.29	1.19	1.23	1.03
time (sec)	N/A	0.167	0.029	0.257	0.305	0.240	0.226	0.287	3.810

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	37	36	36	40	39	38	36
N.S.	1	1.00	0.58	0.56	0.56	0.62	0.61	0.59	0.56
time (sec)	N/A	0.223	0.021	0.297	0.320	0.245	0.299	0.292	3.809

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	95	97	197	109	627	0	0	332
N.S.	1	0.86	0.88	1.79	0.99	5.70	0.00	0.00	3.02
time (sec)	N/A	0.289	0.112	6.026	0.312	0.258	0.000	0.000	4.041

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	C	A	C	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	61	67	54	0	54	88	56	77	53
N.S.	1	1.10	0.89	0.00	0.89	1.44	0.92	1.26	0.87
time (sec)	N/A	0.242	0.023	0.000	0.335	0.248	0.444	0.288	3.921

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	C	A	C	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	61	67	54	0	54	88	58	77	53
N.S.	1	1.10	0.89	0.00	0.89	1.44	0.95	1.26	0.87
time (sec)	N/A	0.246	0.019	0.000	0.328	0.251	0.314	0.297	4.245

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	C	A	C	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	61	54	0	54	88	56	77	53
N.S.	1	1.15	1.02	0.00	1.02	1.66	1.06	1.45	1.00
time (sec)	N/A	0.238	0.017	0.000	0.346	0.260	0.209	0.298	3.853

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	99	54	54	87	41	77	53
N.S.	1	1.00	2.91	1.59	1.59	2.56	1.21	2.26	1.56
time (sec)	N/A	0.186	0.173	25.289	0.343	0.257	0.261	0.289	0.124

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	51	76	20	75	47
N.S.	1	1.00	1.00	0.94	3.19	4.75	1.25	4.69	2.94
time (sec)	N/A	0.153	0.006	25.865	0.346	0.241	0.118	0.277	3.833

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	C	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	77	79	104	3294	74	75	0	74	306
N.S.	1	1.03	1.35	42.78	0.96	0.97	0.00	0.96	3.97
time (sec)	N/A	0.260	0.098	0.287	0.539	0.247	0.000	0.297	0.163

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	C	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	68	68	62	3248	124	79	0	74	372
N.S.	1	1.00	0.91	47.76	1.82	1.16	0.00	1.09	5.47
time (sec)	N/A	0.253	0.028	0.256	0.426	0.251	0.000	0.295	3.884

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F	C	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	60	61	66	3227	72	77	0	71	383
N.S.	1	1.02	1.10	53.78	1.20	1.28	0.00	1.18	6.38
time (sec)	N/A	0.246	0.033	0.275	0.309	0.251	0.000	0.300	3.995

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	60	56	52	75	51	73	51
N.S.	1	1.00	1.09	1.02	0.95	1.36	0.93	1.33	0.93
time (sec)	N/A	0.244	0.023	0.993	0.347	0.242	0.231	0.294	3.859

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	50	49	53	75	56	74	48
N.S.	1	1.00	1.61	1.58	1.71	2.42	1.81	2.39	1.55
time (sec)	N/A	0.173	0.028	1.015	0.342	0.260	0.293	0.298	4.180

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	54	53	54	75	60	74	53
N.S.	1	1.00	0.84	0.83	0.84	1.17	0.94	1.16	0.83
time (sec)	N/A	0.237	0.032	0.962	0.354	0.251	0.402	0.328	0.133

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	0.060	0.000	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	C	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	81	87	79	130774	85	90	0	81	354
N.S.	1	1.07	0.98	1614.49	1.05	1.11	0.00	1.00	4.37
time (sec)	N/A	0.305	0.034	1.417	0.377	0.245	0.000	0.295	0.135

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	C	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	56	59	55	28786	51	51	0	50	234
N.S.	1	1.05	0.98	514.04	0.91	0.91	0.00	0.89	4.18
time (sec)	N/A	0.247	0.030	0.370	0.366	0.248	0.000	0.300	0.286

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	C	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	31	31	31	4303	30	30	0	28	108
N.S.	1	1.00	1.00	138.81	0.97	0.97	0.00	0.90	3.48
time (sec)	N/A	0.192	0.024	0.197	0.360	0.255	0.000	0.285	3.914

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	16	17	14	12
N.S.	1	1.00	1.00	1.08	1.33	1.33	1.42	1.17	1.00
time (sec)	N/A	0.149	0.093	0.215	0.284	0.249	0.198	0.304	3.802

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	C	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	44	44	29	972	37	28	0	31	113
N.S.	1	1.00	0.66	22.09	0.84	0.64	0.00	0.70	2.57
time (sec)	N/A	0.215	0.021	7.070	0.358	0.253	0.000	0.287	5.786

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	C	C	F	C	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	81	45	0	65	50	0	62	220
N.S.	1	1.25	0.69	0.00	1.00	0.77	0.00	0.95	3.38
time (sec)	N/A	0.267	0.022	0.000	0.358	0.249	0.000	0.296	6.122

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	C	F	C	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	121	66	0	106	95	0	107	300
N.S.	1	1.32	0.72	0.00	1.15	1.03	0.00	1.16	3.26
time (sec)	N/A	0.348	0.023	0.000	0.376	0.247	0.000	0.281	7.206

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	51	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	0.825	0.000	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	C	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	98	110	106	131085	179	212	0	135	669
N.S.	1	1.12	1.08	1337.60	1.83	2.16	0.00	1.38	6.83
time (sec)	N/A	0.364	0.075	1.565	0.510	0.252	0.000	0.313	3.899

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	C	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	75	82	83	29109	123	151	0	97	490
N.S.	1	1.09	1.11	388.12	1.64	2.01	0.00	1.29	6.53
time (sec)	N/A	0.294	0.041	0.383	0.517	0.253	0.000	0.293	0.181

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	C	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	50	54	56	4626	81	95	0	66	302
N.S.	1	1.08	1.12	92.52	1.62	1.90	0.00	1.32	6.04
time (sec)	N/A	0.237	0.055	0.204	0.527	0.254	0.000	0.287	3.962

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	35	46	53	36	47	28
N.S.	1	1.00	0.96	1.25	1.64	1.89	1.29	1.68	1.00
time (sec)	N/A	0.186	0.119	0.142	0.537	0.256	12.365	0.289	0.100

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	18	19	20	19	14
N.S.	1	1.00	1.00	1.07	1.29	1.36	1.43	1.36	1.00
time (sec)	N/A	0.150	0.006	0.181	0.289	0.235	12.295	0.287	3.839

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	C	C	F	C	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	88	53	0	78	93	0	78	421
N.S.	1	1.26	0.76	0.00	1.11	1.33	0.00	1.11	6.01
time (sec)	N/A	0.286	0.112	0.000	0.518	0.246	0.000	0.291	7.715

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	C	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	102	135	70	5357	135	157	0	136	453
N.S.	1	1.32	0.69	52.52	1.32	1.54	0.00	1.33	4.44
time (sec)	N/A	0.354	0.047	0.352	0.511	0.257	0.000	0.285	6.677

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	C	C	F	C	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	187	92	0	190	245	0	203	689
N.S.	1	1.31	0.64	0.00	1.33	1.71	0.00	1.42	4.82
time (sec)	N/A	0.462	0.032	0.000	0.533	0.268	0.000	0.290	7.852

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	97	51	0	0	0	0	0	0
N.S.	1	1.03	0.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.296	0.878	0.000	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	C	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	92	107	114	29456	198	265	0	163	867
N.S.	1	1.16	1.24	320.17	2.15	2.88	0.00	1.77	9.42
time (sec)	N/A	0.354	0.041	0.433	0.777	0.243	0.000	0.302	4.336

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	C	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	71	81	86	4977	146	195	0	123	620
N.S.	1	1.14	1.21	70.10	2.06	2.75	0.00	1.73	8.73
time (sec)	N/A	0.296	0.048	0.245	0.798	0.246	0.000	0.305	4.096

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	52	49	54	96	123	54	92	46
N.S.	1	1.11	1.04	1.15	2.04	2.62	1.15	1.96	0.98
time (sec)	N/A	0.232	0.032	0.172	0.772	0.251	24.640	0.301	3.859

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	27	26	63	60	42	61	25
N.S.	1	1.00	0.79	0.76	1.85	1.76	1.24	1.79	0.74
time (sec)	N/A	0.187	0.105	0.156	0.771	0.238	24.808	0.301	0.098

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	28	44	24	44	14
N.S.	1	1.00	1.00	0.94	1.75	2.75	1.50	2.75	0.88
time (sec)	N/A	0.152	0.006	0.211	0.300	0.244	24.853	0.287	3.803

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	C	F	C	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	97	134	74	5655	171	231	0	173	902
N.S.	1	1.38	0.76	58.30	1.76	2.38	0.00	1.78	9.30
time (sec)	N/A	0.360	0.141	0.353	0.820	0.249	0.000	0.308	9.788

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	C	C	F	C	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	181	93	0	245	340	0	258	1074
N.S.	1	1.38	0.71	0.00	1.87	2.60	0.00	1.97	8.20
time (sec)	N/A	0.458	0.040	0.000	0.875	0.250	0.000	0.328	8.287

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	C	C	F	C	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	170	231	107	0	331	461	0	343	1251
N.S.	1	1.36	0.63	0.00	1.95	2.71	0.00	2.02	7.36
time (sec)	N/A	0.573	0.031	0.000	0.783	0.257	0.000	0.312	10.019

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	71	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.225	0.104	0.000	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	150	146	420	380	828	0	0	546
N.S.	1	0.91	0.88	2.55	2.30	5.02	0.00	0.00	3.31
time (sec)	N/A	0.442	0.088	16.412	0.376	0.257	0.000	0.000	4.815

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	116	106	277	257	498	0	0	418
N.S.	1	0.96	0.88	2.29	2.12	4.12	0.00	0.00	3.45
time (sec)	N/A	0.346	0.052	10.861	0.379	0.259	0.000	0.000	4.204

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	71	163	166	276	0	0	304
N.S.	1	1.00	0.87	1.99	2.02	3.37	0.00	0.00	3.71
time (sec)	N/A	0.275	0.053	9.523	0.374	0.258	0.000	0.000	3.984

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	41	77	102	130	0	0	205
N.S.	1	1.00	0.85	1.60	2.12	2.71	0.00	0.00	4.27
time (sec)	N/A	0.206	0.036	9.098	0.439	0.246	0.000	0.000	3.909

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	65	58	51	27	121
N.S.	1	1.00	1.00	1.05	3.25	2.90	2.55	1.35	6.05
time (sec)	N/A	0.157	0.012	9.137	0.349	0.249	0.268	0.275	3.891

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	60	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	0.061	0.000	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	67	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.029	0.000	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	98	67	0	0	0	0	0	0
N.S.	1	0.97	0.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	49	38	81	0	90	96
N.S.	1	1.00	0.92	1.32	1.03	2.19	0.00	2.43	2.59
time (sec)	N/A	0.175	0.018	0.148	0.215	0.245	0.000	0.285	0.002

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	13	13	76	13	19
N.S.	1	1.00	0.87	0.87	0.57	0.57	3.30	0.57	0.83
time (sec)	N/A	0.163	0.016	0.309	0.240	0.239	4.413	0.262	3.755

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	13	13	180	13	19
N.S.	1	1.00	0.87	0.87	0.57	0.57	7.83	0.57	0.83
time (sec)	N/A	0.179	0.012	0.280	0.195	0.204	2.642	0.265	0.059

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	19	10	10	78	10	16
N.S.	1	1.00	1.12	1.19	0.62	0.62	4.88	0.62	1.00
time (sec)	N/A	0.151	0.006	0.355	0.185	0.208	1.471	0.271	0.128

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	21	8	8	0	9	58
N.S.	1	1.00	0.90	1.00	0.38	0.38	0.00	0.43	2.76
time (sec)	N/A	0.174	0.014	0.171	0.181	0.236	0.000	0.267	4.257

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	21	11	13	68	12	17
N.S.	1	1.00	1.06	1.24	0.65	0.76	4.00	0.71	1.00
time (sec)	N/A	0.159	0.013	0.262	0.182	0.237	2.611	0.264	3.750

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	20	11	11	80	11	16
N.S.	1	1.00	0.78	0.87	0.48	0.48	3.48	0.48	0.70
time (sec)	N/A	0.158	0.011	0.255	0.187	0.228	4.361	0.272	4.130

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	40	39	36	33	57	0	0	0
N.S.	1	1.48	1.44	1.33	1.22	2.11	0.00	0.00	0.00
time (sec)	N/A	0.303	0.011	0.295	0.241	0.252	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	51	69	70	379	56	87	0	0	0
N.S.	1	1.35	1.37	7.43	1.10	1.71	0.00	0.00	0.00
time (sec)	N/A	0.439	0.010	0.325	0.249	0.253	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	77	99	91	401	78	117	0	0	0
N.S.	1	1.29	1.18	5.21	1.01	1.52	0.00	0.00	0.00
time (sec)	N/A	0.570	0.014	0.337	0.241	0.256	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	307	395	265	5257	281	899	0	0	0
N.S.	1	1.29	0.86	17.12	0.92	2.93	0.00	0.00	0.00
time (sec)	N/A	1.421	0.205	6.914	0.456	0.284	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	231	305	199	4953	215	745	0	0	0
N.S.	1	1.32	0.86	21.44	0.93	3.23	0.00	0.00	0.00
time (sec)	N/A	1.025	0.132	2.481	0.453	0.270	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	207	131	348	142	551	0	0	0
N.S.	1	1.38	0.87	2.32	0.95	3.67	0.00	0.00	0.00
time (sec)	N/A	0.629	0.145	2.750	0.466	0.278	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13
time (sec)	N/A	0.291	3.698	0.112	1.042	0.246	0.794	0.747	4.270

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	155	199	148	1684	149	450	0	0	0
N.S.	1	1.28	0.95	10.86	0.96	2.90	0.00	0.00	0.00
time (sec)	N/A	1.023	0.119	2.093	0.787	0.271	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	128	165	122	1625	125	381	0	0	0
N.S.	1	1.29	0.95	12.70	0.98	2.98	0.00	0.00	0.00
time (sec)	N/A	0.813	0.063	1.904	0.729	0.259	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	101	131	91	1542	101	322	0	0	0
N.S.	1	1.30	0.90	15.27	1.00	3.19	0.00	0.00	0.00
time (sec)	N/A	0.658	0.064	1.515	0.754	0.260	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	90	63	255	72	238	0	0	0
N.S.	1	1.30	0.91	3.70	1.04	3.45	0.00	0.00	0.00
time (sec)	N/A	0.459	0.078	1.597	0.746	0.274	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12
time (sec)	N/A	0.248	2.825	0.116	0.921	0.252	0.529	0.342	4.214

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	168	218	148	1754	146	423	0	0	0
N.S.	1	1.30	0.88	10.44	0.87	2.52	0.00	0.00	0.00
time (sec)	N/A	1.101	0.127	2.024	0.795	0.271	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	139	182	123	1697	123	359	0	0	0
N.S.	1	1.31	0.88	12.21	0.88	2.58	0.00	0.00	0.00
time (sec)	N/A	0.888	0.064	1.942	0.743	0.280	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	110	146	93	1616	100	305	0	0	0
N.S.	1	1.33	0.85	14.69	0.91	2.77	0.00	0.00	0.00
time (sec)	N/A	0.686	0.065	1.615	0.733	0.263	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	103	66	271	73	227	0	0	0
N.S.	1	1.36	0.87	3.57	0.96	2.99	0.00	0.00	0.00
time (sec)	N/A	0.471	0.085	1.633	0.762	0.286	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	20	19	17	21	19
N.S.	1	1.00	1.11	1.00	1.05	1.00	0.89	1.11	1.00
time (sec)	N/A	0.235	2.835	0.115	0.970	0.251	0.517	0.342	4.187

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	303	391	265	5185	277	879	0	0	0
N.S.	1	1.29	0.87	17.11	0.91	2.90	0.00	0.00	0.00
time (sec)	N/A	1.434	0.204	7.202	0.492	0.290	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	301	199	4881	213	729	0	0	0
N.S.	1	1.31	0.87	21.31	0.93	3.18	0.00	0.00	0.00
time (sec)	N/A	1.024	0.127	2.948	0.460	0.300	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	207	131	348	142	539	0	0	0
N.S.	1	1.38	0.87	2.32	0.95	3.59	0.00	0.00	0.00
time (sec)	N/A	0.636	0.166	2.950	0.464	0.277	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13
time (sec)	N/A	0.291	3.890	0.133	0.975	0.259	1.846	0.685	4.455

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	152	195	145	1656	146	423	0	0	0
N.S.	1	1.28	0.95	10.89	0.96	2.78	0.00	0.00	0.00
time (sec)	N/A	1.029	0.113	2.055	0.750	0.263	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	126	162	120	1599	123	359	0	0	0
N.S.	1	1.29	0.95	12.69	0.98	2.85	0.00	0.00	0.00
time (sec)	N/A	0.848	0.070	1.979	0.769	0.282	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	100	128	90	1518	100	305	0	0	0
N.S.	1	1.28	0.90	15.18	1.00	3.05	0.00	0.00	0.00
time (sec)	N/A	0.664	0.066	1.641	0.821	0.266	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	89	63	255	72	226	0	0	0
N.S.	1	1.29	0.91	3.70	1.04	3.28	0.00	0.00	0.00
time (sec)	N/A	0.453	0.089	1.700	0.749	0.273	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12
time (sec)	N/A	0.243	2.891	0.105	0.939	0.252	1.610	0.332	4.311

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	165	214	151	1782	149	450	0	0	0
N.S.	1	1.30	0.92	10.80	0.90	2.73	0.00	0.00	0.00
time (sec)	N/A	1.085	0.122	2.017	0.759	0.270	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	137	179	125	1723	125	381	0	0	0
N.S.	1	1.31	0.91	12.58	0.91	2.78	0.00	0.00	0.00
time (sec)	N/A	0.876	0.064	1.882	0.765	0.267	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	109	143	94	1640	101	322	0	0	0
N.S.	1	1.31	0.86	15.05	0.93	2.95	0.00	0.00	0.00
time (sec)	N/A	0.693	0.067	1.572	0.833	0.263	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	102	66	271	73	239	0	0	0
N.S.	1	1.34	0.87	3.57	0.96	3.14	0.00	0.00	0.00
time (sec)	N/A	0.474	0.065	1.663	0.803	0.259	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	20	19	17	21	19
N.S.	1	1.00	1.11	1.00	1.05	1.00	0.89	1.11	1.00
time (sec)	N/A	0.252	2.911	0.111	1.004	0.250	1.645	0.352	4.462

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	302	355	654	3640	0	1808	0	0	0
N.S.	1	1.18	2.17	12.05	0.00	5.99	0.00	0.00	0.00
time (sec)	N/A	1.160	0.201	24.711	0.000	0.324	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	234	271	409	2719	0	1282	0	0	0
N.S.	1	1.16	1.75	11.62	0.00	5.48	0.00	0.00	0.00
time (sec)	N/A	0.861	0.138	20.177	0.000	0.292	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	162	182	295	1818	0	834	0	0	0
N.S.	1	1.12	1.82	11.22	0.00	5.15	0.00	0.00	0.00
time (sec)	N/A	0.604	0.206	1.830	0.000	0.290	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	86	127	169	182	498	0	0	0
N.S.	1	1.09	1.61	2.14	2.30	6.30	0.00	0.00	0.00
time (sec)	N/A	0.347	0.018	1.060	0.336	0.278	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13
time (sec)	N/A	0.214	0.486	0.159	1.539	0.250	0.528	0.991	4.424

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	395	522	349	6855	0	2164	0	0	0
N.S.	1	1.32	0.88	17.35	0.00	5.48	0.00	0.00	0.00
time (sec)	N/A	1.510	0.747	20.796	0.000	0.323	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	295	402	259	6481	0	1688	0	0	0
N.S.	1	1.36	0.88	21.97	0.00	5.72	0.00	0.00	0.00
time (sec)	N/A	1.092	0.552	2.923	0.000	0.328	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	194	287	365	556	372	1184	0	0	0
N.S.	1	1.48	1.88	2.87	1.92	6.10	0.00	0.00	0.00
time (sec)	N/A	0.675	0.353	2.644	0.364	0.322	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13
time (sec)	N/A	0.297	0.336	0.102	3.479	0.258	0.698	1.494	5.967

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	170	218	155	2273	343	344	0	0	0
N.S.	1	1.28	0.91	13.37	2.02	2.02	0.00	0.00	0.00
time (sec)	N/A	0.900	0.158	2.313	0.237	0.284	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	133	171	119	2183	248	292	0	0	0
N.S.	1	1.29	0.89	16.41	1.86	2.20	0.00	0.00	0.00
time (sec)	N/A	0.687	0.076	1.977	0.241	0.283	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	93	123	766	307	262	217	0	0	0
N.S.	1	1.32	8.24	3.30	2.82	2.33	0.00	0.00	0.00
time (sec)	N/A	0.479	4.347	2.322	0.288	0.262	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	144	36	17	20	21
N.S.	1	1.00	1.10	0.90	7.20	1.80	0.85	1.00	1.05
time (sec)	N/A	0.261	0.641	0.129	4.086	0.249	0.719	0.772	5.375

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	171	223	156	2383	342	344	0	0	0
N.S.	1	1.30	0.91	13.94	2.00	2.01	0.00	0.00	0.00
time (sec)	N/A	0.904	0.147	2.414	0.223	0.273	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	134	176	120	2285	247	292	0	0	0
N.S.	1	1.31	0.90	17.05	1.84	2.18	0.00	0.00	0.00
time (sec)	N/A	0.702	0.082	2.162	0.247	0.281	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	94	128	723	320	260	218	0	0	0
N.S.	1	1.36	7.69	3.40	2.77	2.32	0.00	0.00	0.00
time (sec)	N/A	0.477	1.669	2.500	0.283	0.261	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	141	36	17	21	22
N.S.	1	1.00	1.10	0.90	6.71	1.71	0.81	1.00	1.05
time (sec)	N/A	0.259	0.621	0.127	4.389	0.252	0.758	0.850	4.818

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	302	355	654	3640	0	1566	0	0	0
N.S.	1	1.18	2.17	12.05	0.00	5.19	0.00	0.00	0.00
time (sec)	N/A	1.179	0.173	27.658	0.000	0.351	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	234	271	409	2719	0	1084	0	0	0
N.S.	1	1.16	1.75	11.62	0.00	4.63	0.00	0.00	0.00
time (sec)	N/A	0.844	0.119	22.217	0.000	0.322	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	162	182	295	1818	0	680	0	0	0
N.S.	1	1.12	1.82	11.22	0.00	4.20	0.00	0.00	0.00
time (sec)	N/A	0.609	0.219	2.089	0.000	0.317	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	86	127	188	184	388	0	0	0
N.S.	1	1.09	1.61	2.38	2.33	4.91	0.00	0.00	0.00
time (sec)	N/A	0.346	0.017	1.434	0.325	0.290	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13
time (sec)	N/A	0.215	0.085	0.178	1.600	0.260	0.660	0.443	4.319

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	391	514	341	6661	0	1798	0	0	0
N.S.	1	1.31	0.87	17.04	0.00	4.60	0.00	0.00	0.00
time (sec)	N/A	1.503	0.809	27.599	0.000	0.467	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	293	394	255	6311	0	1462	0	0	0
N.S.	1	1.34	0.87	21.54	0.00	4.99	0.00	0.00	0.00
time (sec)	N/A	1.108	0.622	4.183	0.000	0.432	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	194	283	390	564	392	1098	0	0	0
N.S.	1	1.46	2.01	2.91	2.02	5.66	0.00	0.00	0.00
time (sec)	N/A	0.677	0.517	2.993	0.382	0.379	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13
time (sec)	N/A	0.296	0.341	0.204	3.571	0.257	0.982	0.712	6.148

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	168	219	155	2383	344	179	0	0	0
N.S.	1	1.30	0.92	14.18	2.05	1.07	0.00	0.00	0.00
time (sec)	N/A	0.900	0.146	2.530	0.238	0.266	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	132	172	119	2285	249	156	0	0	0
N.S.	1	1.30	0.90	17.31	1.89	1.18	0.00	0.00	0.00
time (sec)	N/A	0.693	0.091	2.094	0.228	0.258	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	93	126	709	307	286	121	0	0	0
N.S.	1	1.35	7.62	3.30	3.08	1.30	0.00	0.00	0.00
time (sec)	N/A	0.486	2.148	2.550	0.281	0.250	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	141	36	17	20	21
N.S.	1	1.00	1.10	0.90	7.05	1.80	0.85	1.00	1.05
time (sec)	N/A	0.253	0.583	0.169	5.127	0.262	1.004	0.475	5.263

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	169	216	155	2273	345	179	0	0	0
N.S.	1	1.28	0.92	13.45	2.04	1.06	0.00	0.00	0.00
time (sec)	N/A	0.901	0.138	2.491	0.244	0.266	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	133	169	119	2183	250	156	0	0	0
N.S.	1	1.27	0.89	16.41	1.88	1.17	0.00	0.00	0.00
time (sec)	N/A	0.696	0.082	2.071	0.234	0.258	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	94	123	605	320	288	121	0	0	0
N.S.	1	1.31	6.44	3.40	3.06	1.29	0.00	0.00	0.00
time (sec)	N/A	0.483	1.842	2.378	0.307	0.265	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	144	36	19	21	22
N.S.	1	1.00	1.10	0.90	6.86	1.71	0.90	1.00	1.05
time (sec)	N/A	0.259	0.628	0.173	4.999	0.253	1.016	0.477	5.276

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	160	160	131	414	0	326	0	0	0
N.S.	1	1.00	0.82	2.59	0.00	2.04	0.00	0.00	0.00
time (sec)	N/A	0.776	0.218	146.704	0.000	0.280	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	C	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	291	236	314	331	246	362	0	510
N.S.	1	0.98	0.79	1.06	1.11	0.83	1.22	0.00	1.72
time (sec)	N/A	0.656	0.094	4.222	0.213	0.268	2.453	0.000	5.570

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	C	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	222	192	249	271	195	286	0	414
N.S.	1	0.99	0.85	1.11	1.20	0.87	1.27	0.00	1.84
time (sec)	N/A	0.521	0.078	2.507	0.215	0.247	1.123	0.000	5.422

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	129	174	171	138	209	241	329
N.S.	1	1.00	0.92	1.24	1.22	0.99	1.49	1.72	2.35
time (sec)	N/A	0.356	0.057	1.631	0.208	0.248	0.558	0.391	5.596

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	C	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	381	336	0	589	167	0	0	0	0
N.S.	1	0.88	0.00	1.55	0.44	0.00	0.00	0.00	0.00
time (sec)	N/A	1.916	0.000	4.721	0.355	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	247	225	161	0	0	0	0	0	0
N.S.	1	0.91	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.737	0.094	0.000	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	339	325	307	0	0	0	0	0	0
N.S.	1	0.96	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.994	0.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	286	236	296	317	249	345	359	497
N.S.	1	0.91	0.75	0.94	1.01	0.79	1.10	1.14	1.58
time (sec)	N/A	0.983	0.081	3.315	0.201	0.264	1.684	0.545	5.495

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	223	183	229	252	198	265	282	414
N.S.	1	0.90	0.74	0.93	1.02	0.80	1.07	1.14	1.68
time (sec)	N/A	0.847	0.068	2.116	0.208	0.252	0.792	0.413	5.345

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	116	144	141	178	130	155	198	315
N.S.	1	1.12	1.38	1.36	1.71	1.25	1.49	1.90	3.03
time (sec)	N/A	0.856	0.021	1.250	0.210	0.252	0.357	0.347	4.987

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	85	332	0	0	0	0	0	0
N.S.	1	0.81	3.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.709	0.112	0.000	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	197	178	457	0	0	0	0	0	0
N.S.	1	0.90	2.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.543	0.230	0.000	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	256	293	0	0	0	0	0	0	0
N.S.	1	1.14	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.545	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	512	504	1128	952	0	0	0	0	0
N.S.	1	0.98	2.20	1.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.003	6.103	6.951	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	546	787	1287	3508	0	0	0	0	0
N.S.	1	1.44	2.36	6.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.206	2.394	8.478	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	93	37	22	26	26
N.S.	1	1.00	1.08	1.00	3.88	1.54	0.92	1.08	1.08
time (sec)	N/A	0.793	0.151	0.566	0.398	0.260	113.024	0.300	5.667

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	560	780	1236	0	0	0	0	0	0
N.S.	1	1.39	2.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.834	2.518	0.000	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	712	720	1193	937	0	0	0	0	0
N.S.	1	1.01	1.68	1.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.326	4.958	9.688	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	51	22	58	64	0	0	0
N.S.	1	1.00	2.04	0.88	2.32	2.56	0.00	0.00	0.00
time (sec)	N/A	0.208	0.003	0.098	0.263	0.257	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	50	71	54	59	94	0	0	0
N.S.	1	0.98	1.39	1.06	1.16	1.84	0.00	0.00	0.00
time (sec)	N/A	0.349	0.020	0.077	0.212	0.257	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	84	93	71	76	119	0	0	0
N.S.	1	1.20	1.33	1.01	1.09	1.70	0.00	0.00	0.00
time (sec)	N/A	0.484	0.020	0.092	0.215	0.257	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	39	68	49	107	137	0	0	0
N.S.	1	0.95	1.66	1.20	2.61	3.34	0.00	0.00	0.00
time (sec)	N/A	0.207	0.018	0.258	0.203	0.253	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	113	179	108	198	0	0	0
N.S.	1	1.00	1.36	2.16	1.30	2.39	0.00	0.00	0.00
time (sec)	N/A	0.405	0.031	0.249	0.210	0.252	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	138	149	244	142	247	0	0	0
N.S.	1	1.16	1.25	2.05	1.19	2.08	0.00	0.00	0.00
time (sec)	N/A	0.579	0.028	0.316	0.219	0.269	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	149	108	160	202	283	0	0	0
N.S.	1	0.89	0.64	0.95	1.20	1.68	0.00	0.00	0.00
time (sec)	N/A	0.545	0.046	1.161	0.218	0.264	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	432	177	590	194	395	0	0	0
N.S.	1	2.00	0.82	2.73	0.90	1.83	0.00	0.00	0.00
time (sec)	N/A	2.703	0.073	0.645	0.254	0.261	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	557	235	666	254	479	0	0	0
N.S.	1	2.07	0.87	2.48	0.94	1.78	0.00	0.00	0.00
time (sec)	N/A	2.694	0.045	1.040	0.240	0.256	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	21	21	14	44	15
N.S.	1	1.00	1.00	0.82	1.24	1.24	0.82	2.59	0.88
time (sec)	N/A	0.216	0.078	0.418	0.209	0.253	0.242	0.271	0.274

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	36	39	37	37	58	60	225	107
N.S.	1	0.82	0.89	0.84	0.84	1.32	1.36	5.11	2.43
time (sec)	N/A	0.294	0.014	0.300	0.200	0.252	0.709	0.281	4.415

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	38	42	118	40	108	0	119	58
N.S.	1	0.81	0.89	2.51	0.85	2.30	0.00	2.53	1.23
time (sec)	N/A	0.306	0.027	4.395	0.204	0.256	0.000	0.293	5.530

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	107	91	153	794	184	233	0	167	187
N.S.	1	0.85	1.43	7.42	1.72	2.18	0.00	1.56	1.75
time (sec)	N/A	0.484	0.102	3.447	0.291	0.271	0.000	0.456	4.574

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	49	46	60	824	64	92	0	98	119
N.S.	1	0.94	1.22	16.82	1.31	1.88	0.00	2.00	2.43
time (sec)	N/A	0.317	0.045	0.356	0.215	0.252	0.000	0.378	5.185

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	41	46	37	43	57	66	40	28
N.S.	1	0.91	1.02	0.82	0.96	1.27	1.47	0.89	0.62
time (sec)	N/A	0.302	0.046	0.181	0.219	0.249	0.715	0.270	0.119

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	41	46	29	42	25	0	35	28
N.S.	1	0.91	1.02	0.64	0.93	0.56	0.00	0.78	0.62
time (sec)	N/A	0.298	0.043	0.244	0.209	0.253	0.000	0.264	4.063

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	49	46	59	939	64	93	0	147	111
N.S.	1	0.94	1.20	19.16	1.31	1.90	0.00	3.00	2.27
time (sec)	N/A	0.324	0.046	0.354	0.223	0.251	0.000	0.315	4.263

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	107	91	150	920	184	234	0	157	179
N.S.	1	0.85	1.40	8.60	1.72	2.19	0.00	1.47	1.67
time (sec)	N/A	0.500	0.089	3.326	0.301	0.261	0.000	0.449	4.474

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [200] had the largest ratio of [2.33333000000000013]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.02	8	0.375
2	A	5	4	1.04	8	0.500
3	A	3	3	1.02	8	0.375
4	A	5	4	1.05	8	0.500
5	A	3	3	1.03	6	0.500
6	A	2	2	1.00	4	0.500
7	A	1	1	1.00	8	0.125
8	A	6	5	1.10	8	0.625
9	A	3	3	0.94	8	0.375
10	A	5	4	1.00	8	0.500
11	A	4	4	1.00	8	0.500
12	A	16	15	1.69	10	1.500
13	A	14	13	1.35	10	1.300
14	A	11	10	1.42	10	1.000
15	A	10	9	1.17	10	0.900
16	A	5	5	1.13	8	0.625
17	A	6	5	1.22	6	0.833
18	A	4	4	1.32	10	0.400
19	A	4	4	1.09	10	0.400
20	A	9	8	0.98	10	0.800
21	A	8	8	0.94	10	0.800
22	A	14	13	1.27	10	1.300

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	B	18	17	2.26	10	1.700
24	A	17	16	1.55	10	1.600
25	A	14	13	1.72	10	1.300
26	A	11	11	1.19	10	1.100
27	A	9	8	1.15	8	1.000
28	A	5	5	1.19	6	0.833
29	A	5	5	1.21	10	0.500
30	A	5	5	1.18	10	0.500
31	A	7	7	0.95	10	0.700
32	A	15	14	1.03	10	1.400
33	A	11	11	1.35	10	1.100
34	A	1	1	1.00	14	0.071
35	A	6	5	1.01	14	0.357
36	A	6	5	1.01	14	0.357
37	A	6	5	1.02	14	0.357
38	A	6	5	1.04	12	0.417
39	A	6	6	1.58	14	0.429
40	A	4	4	1.05	14	0.286
41	A	4	4	1.04	14	0.286
42	N/A	1	0	1.00	16	0.000
43	N/A	1	0	1.00	16	0.000
44	A	6	5	1.00	16	0.312
45	A	7	6	1.02	16	0.375
46	A	7	6	0.97	16	0.375
47	A	6	5	0.99	16	0.312
48	A	3	3	0.75	15	0.200
49	A	2	2	0.69	15	0.133
50	A	1	1	1.00	15	0.067
51	A	2	2	1.04	15	0.133
52	A	3	3	1.09	15	0.200
53	A	1	1	1.00	14	0.071
54	A	1	1	1.00	14	0.071
55	A	4	4	1.08	14	0.286

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
56	A	5	4	1.05	13	0.308
57	A	1	1	1.00	12	0.083
58	A	3	3	1.14	13	0.231
59	A	2	2	1.00	12	0.167
60	A	4	4	1.20	13	0.308
61	A	3	3	1.07	12	0.250
62	A	7	6	0.98	10	0.600
63	A	6	5	1.05	10	0.500
64	A	7	6	1.09	8	0.750
65	A	4	3	0.94	6	0.500
66	A	8	7	1.00	10	0.700
67	A	6	5	1.05	10	0.500
68	A	5	4	1.03	10	0.400
69	A	6	5	0.93	12	0.417
70	A	5	4	0.95	12	0.333
71	A	6	5	0.93	10	0.500
72	A	7	6	0.94	8	0.750
73	A	5	4	1.00	12	0.333
74	A	8	7	1.02	12	0.583
75	A	7	6	0.94	12	0.500
76	A	8	7	1.05	16	0.438
77	A	7	6	1.15	14	0.429
78	A	7	7	1.49	16	0.438
79	A	7	7	1.58	16	0.438
80	A	5	4	1.00	18	0.222
81	A	5	4	1.00	18	0.222
82	A	2	2	1.06	19	0.105
83	A	7	6	1.02	10	0.600
84	A	6	5	1.02	8	0.625
85	A	5	4	1.32	6	0.667
86	A	3	2	1.42	10	0.200
87	A	5	4	1.28	10	0.400
88	A	6	5	1.02	10	0.500
89	A	3	3	0.92	12	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
90	A	3	3	0.90	12	0.250
91	A	2	2	1.00	12	0.167
92	A	4	4	1.00	12	0.333
93	A	3	2	1.14	10	0.200
94	A	3	3	1.00	4	0.750
95	A	3	2	0.95	10	0.200
96	A	5	4	1.00	12	0.333
97	A	6	5	0.76	14	0.357
98	A	3	2	0.94	14	0.143
99	A	7	6	0.94	14	0.429
100	A	4	3	1.16	12	0.250
101	A	4	3	0.94	19	0.158
102	A	6	5	0.99	18	0.278
103	A	6	5	1.07	18	0.278
104	A	6	5	1.09	16	0.312
105	A	1	1	1.00	10	0.100
106	A	7	6	1.14	18	0.333
107	A	4	4	1.05	18	0.222
108	A	4	4	1.02	18	0.222
109	A	5	4	1.00	20	0.200
110	A	5	4	1.05	18	0.222
111	A	7	6	0.96	12	0.500
112	A	4	3	1.13	20	0.150
113	A	7	6	1.02	20	0.300
114	A	5	4	0.98	20	0.200
115	A	5	4	0.99	18	0.222
116	A	7	6	0.98	12	0.500
117	A	4	3	1.12	20	0.150
118	A	7	6	1.00	20	0.300
119	A	5	4	1.37	18	0.222
120	N/A	3	0	1.00	20	0.000
121	N/A	3	0	1.00	20	0.000
122	N/A	1	0	1.00	40	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
123	A	7	6	0.98	40	0.150
124	A	6	5	1.01	40	0.125
125	A	3	2	0.94	38	0.053
126	N/A	1	0	1.00	40	0.000
127	N/A	1	0	1.00	40	0.000
128	A	2	2	1.00	11	0.182
129	A	2	2	1.00	11	0.182
130	A	2	2	1.00	9	0.222
131	A	3	2	1.00	7	0.286
132	A	2	2	1.00	11	0.182
133	A	2	2	1.00	11	0.182
134	A	2	2	1.00	11	0.182
135	A	2	2	1.00	11	0.182
136	A	3	3	0.93	13	0.231
137	A	3	3	1.07	13	0.231
138	A	3	3	1.07	13	0.231
139	A	4	3	1.00	11	0.273
140	A	3	2	1.00	9	0.222
141	A	3	3	1.02	13	0.231
142	A	3	3	1.00	13	0.231
143	A	3	3	1.00	13	0.231
144	A	1	1	1.00	13	0.077
145	A	2	2	1.00	13	0.154
146	A	4	4	0.86	13	0.308
147	A	4	4	1.10	13	0.308
148	A	4	4	1.10	13	0.308
149	A	5	4	1.15	13	0.308
150	A	4	3	1.00	11	0.273
151	A	3	2	1.00	9	0.222
152	A	4	4	1.03	13	0.308
153	A	4	4	1.00	13	0.308
154	A	4	4	1.02	13	0.308
155	A	4	4	1.00	13	0.308

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
156	A	1	1	1.00	13	0.077
157	A	2	2	1.00	13	0.154
158	A	1	1	1.00	13	0.077
159	A	6	5	1.07	13	0.385
160	A	5	4	1.05	13	0.308
161	A	4	3	1.00	11	0.273
162	A	3	2	1.00	9	0.222
163	A	5	4	1.00	13	0.308
164	A	6	5	1.25	13	0.385
165	A	7	6	1.32	13	0.462
166	A	2	2	1.00	13	0.154
167	A	7	6	1.12	13	0.462
168	A	6	5	1.09	13	0.385
169	A	5	4	1.08	13	0.308
170	A	4	3	1.00	11	0.273
171	A	3	2	1.00	9	0.222
172	A	6	5	1.26	13	0.385
173	A	7	6	1.32	13	0.462
174	A	8	7	1.31	13	0.538
175	A	3	3	1.03	13	0.231
176	A	7	6	1.16	13	0.462
177	A	6	5	1.14	13	0.385
178	A	5	4	1.11	13	0.308
179	A	4	3	1.00	11	0.273
180	A	3	2	1.00	9	0.222
181	A	7	6	1.38	13	0.462
182	A	8	7	1.38	13	0.538
183	A	9	8	1.36	13	0.615
184	A	1	1	1.00	13	0.077
185	A	7	6	0.91	13	0.462
186	A	6	5	0.96	13	0.385
187	A	5	4	1.00	13	0.308
188	A	4	3	1.00	11	0.273
189	A	3	2	1.00	9	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
190	A	1	1	1.00	13	0.077
191	A	2	2	1.00	13	0.154
192	A	3	3	0.97	13	0.231
193	A	2	2	1.00	11	0.182
194	A	2	2	1.00	11	0.182
195	A	2	2	1.00	9	0.222
196	A	3	2	1.00	7	0.286
197	A	2	2	1.00	11	0.182
198	A	2	2	1.00	11	0.182
199	A	2	2	1.00	11	0.182
200	C	8	7	1.48	3	2.333
201	C	9	8	1.35	5	1.600
202	C	10	9	1.29	7	1.286
203	A	7	6	1.29	15	0.400
204	A	6	5	1.32	13	0.385
205	A	5	4	1.38	11	0.364
206	N/A	1	0	1.00	15	0.000
207	A	9	8	1.28	16	0.500
208	A	8	7	1.29	16	0.438
209	A	7	6	1.30	14	0.429
210	A	6	5	1.30	12	0.417
211	N/A	1	0	1.00	16	0.000
212	A	9	8	1.30	19	0.421
213	A	8	7	1.31	19	0.368
214	A	7	6	1.33	17	0.353
215	A	6	5	1.36	15	0.333
216	N/A	1	0	1.00	19	0.000
217	A	7	6	1.29	15	0.400
218	A	6	5	1.31	13	0.385
219	A	5	4	1.38	11	0.364
220	N/A	1	0	1.00	15	0.000
221	A	9	8	1.28	16	0.500
222	A	8	7	1.29	16	0.438

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
223	A	7	6	1.28	14	0.429
224	A	6	5	1.29	12	0.417
225	N/A	1	0	1.00	16	0.000
226	A	9	8	1.30	19	0.421
227	A	8	7	1.31	19	0.368
228	A	7	6	1.31	17	0.353
229	A	6	5	1.34	15	0.333
230	N/A	1	0	1.00	19	0.000
231	A	9	8	1.18	15	0.533
232	A	8	7	1.16	15	0.467
233	A	7	6	1.12	13	0.462
234	A	6	5	1.09	7	0.714
235	N/A	1	0	1.00	15	0.000
236	A	7	6	1.32	15	0.400
237	A	6	5	1.36	13	0.385
238	A	5	4	1.48	11	0.364
239	N/A	1	0	1.00	15	0.000
240	A	8	7	1.28	20	0.350
241	A	7	6	1.29	18	0.333
242	A	6	5	1.32	16	0.312
243	N/A	1	0	1.00	20	0.000
244	A	8	7	1.30	21	0.333
245	A	7	6	1.31	19	0.316
246	A	6	5	1.36	17	0.294
247	N/A	1	0	1.00	21	0.000
248	A	9	8	1.18	15	0.533
249	A	8	7	1.16	15	0.467
250	A	7	6	1.12	13	0.462
251	A	6	5	1.09	7	0.714
252	N/A	1	0	1.00	15	0.000
253	A	7	6	1.31	15	0.400
254	A	6	5	1.34	13	0.385
255	A	5	4	1.46	11	0.364

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
256	N/A	1	0	1.00	15	0.000
257	A	8	7	1.30	20	0.350
258	A	7	6	1.30	18	0.333
259	A	6	5	1.35	16	0.312
260	N/A	1	0	1.00	20	0.000
261	A	8	7	1.28	21	0.333
262	A	7	6	1.27	19	0.316
263	A	6	5	1.31	17	0.294
264	N/A	1	0	1.00	21	0.000
265	A	2	2	1.00	24	0.083
266	A	2	2	0.98	27	0.074
267	A	2	2	0.99	27	0.074
268	A	2	2	1.00	25	0.080
269	A	16	15	0.88	27	0.556
270	A	2	2	0.91	27	0.074
271	A	2	2	0.96	27	0.074
272	A	2	2	0.91	27	0.074
273	A	2	2	0.90	27	0.074
274	A	8	7	1.12	24	0.292
275	A	8	7	0.81	27	0.259
276	A	17	16	0.90	27	0.593
277	A	26	25	1.14	27	0.926
278	A	2	2	0.98	22	0.091
279	A	16	15	1.44	21	0.714
280	N/A	8	0	1.00	24	0.000
281	A	13	12	1.39	24	0.500
282	A	2	2	1.01	24	0.083
283	A	3	2	1.00	4	0.500
284	A	5	4	0.98	6	0.667
285	A	6	5	1.20	8	0.625
286	A	3	2	0.95	8	0.250
287	A	5	4	1.00	10	0.400
288	A	6	5	1.16	12	0.417

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
289	A	9	8	0.89	12	0.667
290	A	6	6	2.00	14	0.429
291	B	6	6	2.07	16	0.375
292	A	1	1	1.00	20	0.050
293	A	5	4	0.82	12	0.333
294	A	5	4	0.81	14	0.286
295	A	8	7	0.85	20	0.350
296	A	7	6	0.94	20	0.300
297	A	4	3	0.91	20	0.150
298	A	4	3	0.91	20	0.150
299	A	7	6	0.94	20	0.300
300	A	9	8	0.85	20	0.400

CHAPTER 3

Listing of integrals

3.1 $\int x^5 \coth^{-1}(ax)\,dx$

3.1.1 Optimal result

Integrand size = 8, antiderivative size = 51

$$\int x^5 \coth^{-1}(ax)\,dx = \frac{x}{6a^5} + \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6\coth^{-1}(ax) - \frac{\operatorname{arctanh}(ax)}{6a^6}$$

output
```
1/6*x/a^5+1/18*x^3/a^3+1/30*x^5/a+1/6*x^6*arccoth(a*x)-1/6*arctanh(a*x)/a^
6
```

3.1.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int x^5 \coth^{-1}(ax)\,dx = \frac{x}{6a^5} + \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6\coth^{-1}(ax) + \frac{\log(1-ax)}{12a^6} - \frac{\log(1+ax)}{12a^6}$$

input
```
Integrate[x^5*ArcCoth[a*x],x]
```

output
```
x/(6*a^5) + x^3/(18*a^3) + x^5/(30*a) + (x^6*ArcCoth[a*x])/6 + Log[1 - a*x
]/(12*a^6) - Log[1 + a*x]/(12*a^6)
```

3.1.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6453, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \coth^{-1}(ax)\,dx$$

$$\Big\downarrow \; 6453$$

$$\frac{1}{6}x^6 \coth^{-1}(ax) - \frac{1}{6}a\int \frac{x^6}{1-a^2x^2}dx$$

$$\Big\downarrow \; 254$$

$$\frac{1}{6}x^6 \coth^{-1}(ax) - \frac{1}{6}a\int \left(-\frac{x^4}{a^2} - \frac{x^2}{a^4} + \frac{1}{a^6(1-a^2x^2)} - \frac{1}{a^6}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{6}x^6 \coth^{-1}(ax) - \frac{1}{6}a\left(\frac{\text{arctanh}(ax)}{a^7} - \frac{x}{a^6} - \frac{x^3}{3a^4} - \frac{x^5}{5a^2}\right)$$

input
```
Int[x^5*ArcCoth[a*x],x]
```

output
```
(x^6*ArcCoth[a*x])/6 - (a*(-(x/a^6) - x^3/(3*a^4) - x^5/(5*a^2) + ArcTanh[
a*x]/a^7))/6
```

3.1.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.1.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

method	result	size
parallelrisch	$-\frac{-15a^6x^6\,\mathrm{arccoth}(ax)-3a^5x^5-5a^3x^3-15ax+15\,\mathrm{arccoth}(ax)}{90a^6}$	45
derivativedivides	$\frac{\frac{a^6x^6\,\mathrm{arccoth}(ax)}{6}+\frac{a^5x^5}{30}+\frac{a^3x^3}{18}+\frac{ax}{6}+\frac{\ln(ax-1)}{12}-\frac{\ln(ax+1)}{12}}{a^6}$	54
default	$\frac{\frac{a^6x^6\,\mathrm{arccoth}(ax)}{6}+\frac{a^5x^5}{30}+\frac{a^3x^3}{18}+\frac{ax}{6}+\frac{\ln(ax-1)}{12}-\frac{\ln(ax+1)}{12}}{a^6}$	54
parts	$\frac{x^6\,\mathrm{arccoth}(ax)}{6}+\frac{a\left(-\frac{\ln(ax+1)}{2a^7}+\frac{\frac{1}{5}a^4x^5+\frac{1}{3}a^2x^3+x}{a^6}+\frac{\ln(ax-1)}{2a^7}\right)}{6}$	59
risch	$\frac{x^6\ln(ax+1)}{12}-\frac{\ln(ax-1)x^6}{12}+\frac{x^5}{30a}+\frac{x^3}{18a^3}+\frac{x}{6a^5}-\frac{\ln(ax+1)}{12a^6}+\frac{\ln(-ax+1)}{12a^6}$	69

input
```
int(x^5*arccoth(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/90*(-15*a^6*x^6*arccoth(a*x)-3*a^5*x^5-5*a^3*x^3-15*a*x+15*arccoth(a*x)
)/a^6
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^5\coth^{-1}(ax)\,dx=\frac{6\,a^5x^5+10\,a^3x^3+30\,ax+15\,(a^6x^6-1)\log\left(\frac{ax+1}{ax-1}\right)}{180\,a^6}$$

input
```
integrate(x^5*arccoth(a*x),x, algorithm="fricas")
```

output
```
1/180*(6*a^5*x^5 + 10*a^3*x^3 + 30*a*x + 15*(a^6*x^6 - 1)*log((a*x + 1)/(a
*x - 1)))/a^6
```

3.1.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int x^5 \coth^{-1}(ax)\,dx = \begin{cases} \frac{x^6 \operatorname{acoth}(ax)}{6} + \frac{x^5}{30a} + \frac{x^3}{18a^3} + \frac{x}{6a^5} - \frac{\operatorname{acoth}(ax)}{6a^6} & \text{for } a \neq 0 \\ \frac{i\pi x^6}{12} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*acoth(a*x),x)
```

output
```
Piecewise((x**6*acoth(a*x)/6 + x**5/(30*a) + x**3/(18*a**3) + x/(6*a**5) -
 acoth(a*x)/(6*a**6), Ne(a, 0)), (I*pi*x**6/12, True))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int x^5 \coth^{-1}(ax)\,dx = \frac{1}{6}\,x^6 \operatorname{arcoth}(ax) + \frac{1}{180}\,a\left(\frac{2\,(3\,a^4x^5 + 5\,a^2x^3 + 15\,x)}{a^6} - \frac{15\,\log(ax+1)}{a^7} + \frac{15\,\log(ax-1)}{a^7}\right)$$

input
```
integrate(x^5*arccoth(a*x),x, algorithm="maxima")
```

output
```
1/6*x^6*arccoth(a*x) + 1/180*a*(2*(3*a^4*x^5 + 5*a^2*x^3 + 15*x)/a^6 - 15*
log(a*x + 1)/a^7 + 15*log(a*x - 1)/a^7)
```

3.1.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(41) = 82.

Time = 0.27 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.80

$$\int x^5 \coth^{-1}(ax)\,dx$$
$$= \frac{1}{45}a\left(\frac{\frac{45\,(ax+1)^4}{(ax-1)^4}-\frac{90\,(ax+1)^3}{(ax-1)^3}+\frac{140\,(ax+1)^2}{(ax-1)^2}-\frac{70\,(ax+1)}{ax-1}+23}{a^7\left(\frac{ax+1}{ax-1}-1\right)^5}+\frac{15\left(\frac{3\,(ax+1)^5}{(ax-1)^5}+\frac{10\,(ax+1)^3}{(ax-1)^3}+\frac{3\,(ax+1)}{ax-1}\right)\log\left(-\cdots\right)}{a^7\left(\frac{ax+1}{ax-1}-1\right)^6}\right.$$

input
```
integrate(x^5*arccoth(a*x),x, algorithm="giac")
```

output
```
1/45*a*((45*(a*x + 1)^4/(a*x - 1)^4 - 90*(a*x + 1)^3/(a*x - 1)^3 + 140*(a*
x + 1)^2/(a*x - 1)^2 - 70*(a*x + 1)/(a*x - 1) + 23)/(a^7*((a*x + 1)/(a*x -
 1) - 1)^5) + 15*(3*(a*x + 1)^5/(a*x - 1)^5 + 10*(a*x + 1)^3/(a*x - 1)^3 +
 3*(a*x + 1)/(a*x - 1))*log(-(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a
*x - 1) + 1)) + 1)/(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) +
1)) - 1))/(a^7*((a*x + 1)/(a*x - 1) - 1)^6))
```

3.1.9 Mupad [B] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int x^5 \coth^{-1}(ax)\,dx = \frac{\frac{a\,x}{6}-\frac{\operatorname{acoth}(a\,x)}{6}+\frac{a^3\,x^3}{18}+\frac{a^5\,x^5}{30}}{a^6}+\frac{x^6\operatorname{acoth}(a\,x)}{6}$$

input
```
int(x^5*acoth(a*x),x)
```

output
```
((a*x)/6 - acoth(a*x)/6 + (a^3*x^3)/18 + (a^5*x^5)/30)/a^6 + (x^6*acoth(a*
x))/6
```

3.2 $\int x^4 \coth^{-1}(ax)\,dx$

3.2.1 Optimal result

Integrand size = 8, antiderivative size = 50

$$\int x^4 \coth^{-1}(ax)\,dx = \frac{x^2}{10a^3} + \frac{x^4}{20a} + \frac{1}{5}x^5 \coth^{-1}(ax) + \frac{\log(1-a^2x^2)}{10a^5}$$

output
```
1/10*x^2/a^3+1/20*x^4/a+1/5*x^5*arccoth(a*x)+1/10*ln(-a^2*x^2+1)/a^5
```

3.2.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int x^4 \coth^{-1}(ax)\,dx = \frac{x^2}{10a^3} + \frac{x^4}{20a} + \frac{1}{5}x^5 \coth^{-1}(ax) + \frac{\log(1-a^2x^2)}{10a^5}$$

input
```
Integrate[x^4*ArcCoth[a*x],x]
```

output
```
x^2/(10*a^3) + x^4/(20*a) + (x^5*ArcCoth[a*x])/5 + Log[1 - a^2*x^2]/(10*a^
5)
```

3.2.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6453, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \coth^{-1}(ax) \, dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{5}x^5 \coth^{-1}(ax) - \frac{1}{5}a \int \frac{x^5}{1-a^2x^2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{5}x^5 \coth^{-1}(ax) - \frac{1}{10}a \int \frac{x^4}{1-a^2x^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{5}x^5 \coth^{-1}(ax) - \frac{1}{10}a \int \left(-\frac{x^2}{a^2} - \frac{1}{a^4(a^2x^2-1)} - \frac{1}{a^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5 \coth^{-1}(ax) - \frac{1}{10}a\left(-\frac{x^2}{a^4} - \frac{x^4}{2a^2} - \frac{\log(1-a^2x^2)}{a^6}\right)$$

input
```
Int[x^4*ArcCoth[a*x],x]
```

output
```
(x^5*ArcCoth[a*x])/5 - (a*(-(x^2/a^4) - x^4/(2*a^2) - Log[1 - a^2*x^2]/a^6
))/10
```

3.2.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.2.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

method	result	size
parts	$\frac{x^{5} \operatorname{arccoth}\left(a x \right)}{5}+\frac{a \left(\frac{\frac{1}{2} x^{4} a^{2}+x^{2}}{2 a^{4}}+\frac{\ln \left(a^{2} x^{2}-1\right)}{2 a^{6}}\right)}{5}$	47
derivativedivides	$\frac{\frac{a^{5} x^{5} \operatorname{arccoth}\left(a x \right)}{5}+\frac{a^{4} x^{4}}{20}+\frac{a^{2} x^{2}}{10}+\frac{\ln \left(a x -1\right)}{10}+\frac{\ln \left(a x +1\right)}{10}}{a^{5}}$	50
default	$\frac{\frac{a^{5} x^{5} \operatorname{arccoth}\left(a x \right)}{5}+\frac{a^{4} x^{4}}{20}+\frac{a^{2} x^{2}}{10}+\frac{\ln \left(a x -1\right)}{10}+\frac{\ln \left(a x +1\right)}{10}}{a^{5}}$	50
parallelrisch	$-\frac{-4 a^{5} x^{5} \operatorname{arccoth}\left(a x \right)-a^{4} x^{4}-2-2 a^{2} x^{2}-4 \ln \left(a x -1\right)-4 \operatorname{arccoth}\left(a x \right)}{20 a^{5}}$	50
risch	$\frac{x^{5} \ln \left(a x +1\right)}{10}-\frac{\ln \left(a x -1\right) x^{5}}{10}+\frac{x^{4}}{20 a}+\frac{x^{2}}{10 a^{3}}+\frac{\ln \left(a^{2} x^{2}-1\right)}{10 a^{5}}+\frac{1}{20 a^{5}}$	60

input
```
int(x^4*arccoth(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*arccoth(a*x)+1/5*a*(1/2/a^4*(1/2*x^4*a^2+x^2)+1/2/a^6*ln(a^2*x^2-1
))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int x^4 \coth^{-1}(ax)\,dx = \frac{2\,a^5x^5\log\left(\frac{ax+1}{ax-1}\right) + a^4x^4 + 2\,a^2x^2 + 2\,\log\left(a^2x^2-1\right)}{20\,a^5}$$

input
```
integrate(x^4*arccoth(a*x),x, algorithm="fricas")
```

output
```
1/20*(2*a^5*x^5*log((a*x + 1)/(a*x - 1)) + a^4*x^4 + 2*a^2*x^2 + 2*log(a^2
*x^2 - 1))/a^5
```

3.2.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int x^4 \coth^{-1}(ax)\,dx = \begin{cases} \frac{x^5\operatorname{acoth}(ax)}{5} + \frac{x^4}{20a} + \frac{x^2}{10a^3} + \frac{\log(ax+1)}{5a^5} - \frac{\operatorname{acoth}(ax)}{5a^5} & \text{for } a \neq 0 \\ \frac{i\pi x^5}{10} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*acoth(a*x),x)
```

output
```
Piecewise((x**5*acoth(a*x)/5 + x**4/(20*a) + x**2/(10*a**3) + log(a*x + 1)
/(5*a**5) - acoth(a*x)/(5*a**5), Ne(a, 0)), (I*pi*x**5/10, True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int x^4 \coth^{-1}(ax)\,dx = \frac{1}{5}\,x^5\operatorname{arcoth}(ax) + \frac{1}{20}\,a\left(\frac{a^2x^4+2\,x^2}{a^4} + \frac{2\,\log\left(a^2x^2-1\right)}{a^6}\right)$$

input
```
integrate(x^4*arccoth(a*x),x, algorithm="maxima")
```

output
```
1/5*x^5*arccoth(a*x) + 1/20*a*((a^2*x^4 + 2*x^2)/a^4 + 2*log(a^2*x^2 - 1)/
a^6)
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(42) = 84$.

Time = 0.28 (sec) , antiderivative size = 255, normalized size of antiderivative = 5.10

$$\int x^4 \coth^{-1}(ax)\,dx = \frac{1}{5}a\left(\frac{\log\left(\frac{|ax+1|}{|ax-1|}\right)}{a^6} - \frac{\log\left(\left|\frac{ax+1}{ax-1}-1\right|\right)}{a^6} + \frac{4\left(\frac{(ax+1)^3}{(ax-1)^3} - \frac{(ax+1)^2}{(ax-1)^2} + \frac{ax+1}{ax-1}\right)}{a^6\left(\frac{ax+1}{ax-1}-1\right)^4} + \frac{\left(\frac{5(ax+1)^4}{(ax-1)^4} + \frac{10(ax+1)^2}{(ax-1)^2} + 1\right)\log\left(-\right.}{a^6\left(\frac{ax+1}{ax-1}-1\right)^5}\right.$$

input
```
integrate(x^4*arccoth(a*x),x, algorithm="giac")
```

output
```
1/5*a*(log(abs(a*x + 1)/abs(a*x - 1))/a^6 - log(abs((a*x + 1)/(a*x - 1) -
1))/a^6 + 4*((a*x + 1)^3/(a*x - 1)^3 - (a*x + 1)^2/(a*x - 1)^2 + (a*x + 1)
/(a*x - 1))/(a^6*((a*x + 1)/(a*x - 1) - 1)^4) + (5*(a*x + 1)^4/(a*x - 1)^4
 + 10*(a*x + 1)^2/(a*x - 1)^2 + 1)*log(-(((a*x + 1)*a/(a*x - 1) - a)/(a*((
a*x + 1)/(a*x - 1) + 1)) + 1)/(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(
a*x - 1) + 1)) - 1))/(a^6*((a*x + 1)/(a*x - 1) - 1)^5))
```

3.2.9 Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int x^4 \coth^{-1}(ax)\,dx = \frac{\frac{\ln(a^2x^2-1)}{10} + \frac{a^2x^2}{10} + \frac{a^4x^4}{20}}{a^5} + \frac{x^5\,\mathrm{acoth}(a\,x)}{5}$$

input
```
int(x^4*acoth(a*x),x)
```

output
```
(log(a^2*x^2 - 1)/10 + (a^2*x^2)/10 + (a^4*x^4)/20)/a^5 + (x^5*acoth(a*x))
/5
```

3.3 $\int x^3 \coth^{-1}(ax)\,dx$

3.3.1 Optimal result

Integrand size = 8, antiderivative size = 41

$$\int x^3 \coth^{-1}(ax)\,dx = \frac{x}{4a^3} + \frac{x^3}{12a} + \frac{1}{4}x^4 \coth^{-1}(ax) - \frac{\operatorname{arctanh}(ax)}{4a^4}$$

output
```
1/4*x/a^3+1/12*x^3/a+1/4*x^4*arccoth(a*x)-1/4*arctanh(a*x)/a^4
```

3.3.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int x^3 \coth^{-1}(ax)\,dx = \frac{x}{4a^3} + \frac{x^3}{12a} + \frac{1}{4}x^4 \coth^{-1}(ax) + \frac{\log(1-ax)}{8a^4} - \frac{\log(1+ax)}{8a^4}$$

input
```
Integrate[x^3*ArcCoth[a*x],x]
```

output
```
x/(4*a^3) + x^3/(12*a) + (x^4*ArcCoth[a*x])/4 + Log[1 - a*x]/(8*a^4) - Log
[1 + a*x]/(8*a^4)
```

3.3.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6453, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(ax)\,dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{4}x^4\coth^{-1}(ax)-\frac{1}{4}a\int \frac{x^4}{1-a^2x^2}dx$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{4}x^4\coth^{-1}(ax)-\frac{1}{4}a\int \left(-\frac{x^2}{a^2}+\frac{1}{a^4\left(1-a^2x^2\right)}-\frac{1}{a^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4\coth^{-1}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right)$$

input
```
Int[x^3*ArcCoth[a*x],x]
```

output
```
(x^4*ArcCoth[a*x])/4 - (a*(-(x/a^4) - x^3/(3*a^2) + ArcTanh[a*x]/a^5))/4
```

3.3.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.3.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$-\frac{-3a^4x^4\operatorname{arccoth}(ax)-a^3x^3-3ax+3\operatorname{arccoth}(ax)}{12a^4}$	37
derivativedivides	$\frac{\frac{a^4x^4\operatorname{arccoth}(ax)}{4}+\frac{a^3x^3}{12}+\frac{ax}{4}+\frac{\ln(ax-1)}{8}-\frac{\ln(ax+1)}{8}}{a^4}$	46
default	$\frac{\frac{a^4x^4\operatorname{arccoth}(ax)}{4}+\frac{a^3x^3}{12}+\frac{ax}{4}+\frac{\ln(ax-1)}{8}-\frac{\ln(ax+1)}{8}}{a^4}$	46
parts	$\frac{x^4\operatorname{arccoth}(ax)}{4}+\frac{a\left(-\frac{\ln(ax+1)}{2a^5}+\frac{\frac{1}{3}a^2x^3+x}{a^4}+\frac{\ln(ax-1)}{2a^5}\right)}{4}$	51
risch	$\frac{x^4\ln(ax+1)}{8}-\frac{\ln(ax-1)x^4}{8}+\frac{x^3}{12a}+\frac{x}{4a^3}+\frac{\ln(-ax+1)}{8a^4}-\frac{\ln(ax+1)}{8a^4}$	61

input
```
int(x^3*arccoth(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-3*a^4*x^4*arccoth(a*x)-a^3*x^3-3*a*x+3*arccoth(a*x))/a^4
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int x^3\coth^{-1}(ax)\,dx=\frac{2\,a^3x^3+6\,ax+3\,(a^4x^4-1)\log\left(\frac{ax+1}{ax-1}\right)}{24\,a^4}$$

input
```
integrate(x^3*arccoth(a*x),x, algorithm="fricas")
```

output
```
1/24*(2*a^3*x^3 + 6*a*x + 3*(a^4*x^4 - 1)*log((a*x + 1)/(a*x - 1)))/a^4
```

3.3.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int x^3 \coth^{-1}(ax)\, dx = \begin{cases} \frac{x^4 \operatorname{acoth}(ax)}{4} + \frac{x^3}{12a} + \frac{x}{4a^3} - \frac{\operatorname{acoth}(ax)}{4a^4} & \text{for } a \neq 0 \\ \frac{i\pi x^4}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acoth(a*x),x)
```

output
```
Piecewise((x**4*acoth(a*x)/4 + x**3/(12*a) + x/(4*a**3) - acoth(a*x)/(4*a*
*4), Ne(a, 0)), (I*pi*x**4/8, True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

$$\int x^3 \coth^{-1}(ax)\, dx = \frac{1}{4} x^4 \operatorname{arcoth}(ax) + \frac{1}{24} a \left(\frac{2 (a^2 x^3 + 3x)}{a^4} - \frac{3 \log(ax+1)}{a^5} + \frac{3 \log(ax-1)}{a^5} \right)$$

input
```
integrate(x^3*arccoth(a*x),x, algorithm="maxima")
```

output
```
1/4*x^4*arccoth(a*x) + 1/24*a*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a^5
+ 3*log(a*x - 1)/a^5)
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(33) = 66.

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 4.76

$$\int x^3 \coth^{-1}(ax)\,dx$$
$$= \frac{1}{3}a\left(\frac{\frac{3(ax+1)^2}{(ax-1)^2} - \frac{3(ax+1)}{ax-1} + 2}{a^5\left(\frac{ax+1}{ax-1}-1\right)^3} + \frac{3\left(\frac{(ax+1)^3}{(ax-1)^3} + \frac{ax+1}{ax-1}\right)\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}+1}{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}-1}\right)}{a^5\left(\frac{ax+1}{ax-1}-1\right)^4}\right)$$

input
```
integrate(x^3*arccoth(a*x),x, algorithm="giac")
```

output
```
1/3*a*((3*(a*x + 1)^2/(a*x - 1)^2 - 3*(a*x + 1)/(a*x - 1) + 2)/(a^5*((a*x
+ 1)/(a*x - 1) - 1)^3) + 3*((a*x + 1)^3/(a*x - 1)^3 + (a*x + 1)/(a*x - 1))
*log(-(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) + 1)/(((a
*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) - 1))/(a^5*((a*x +
1)/(a*x - 1) - 1)^4))
```

3.3.9 Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int x^3 \coth^{-1}(ax)\,dx = \frac{\frac{a\,x}{4} - \frac{\mathrm{acoth}(a\,x)}{4} + \frac{a^3\,x^3}{12}}{a^4} + \frac{x^4\,\mathrm{acoth}(a\,x)}{4}$$

input
```
int(x^3*acoth(a*x),x)
```

output
```
((a*x)/4 - acoth(a*x)/4 + (a^3*x^3)/12)/a^4 + (x^4*acoth(a*x))/4
```

3.4 $\int x^2 \coth^{-1}(ax) \, dx$

3.4.1 Optimal result

Integrand size = 8, antiderivative size = 40

$$\int x^2 \coth^{-1}(ax) \, dx = \frac{x^2}{6a} + \frac{1}{3}x^3 \coth^{-1}(ax) + \frac{\log(1-a^2x^2)}{6a^3}$$

output
```
1/6*x^2/a+1/3*x^3*arccoth(a*x)+1/6*ln(-a^2*x^2+1)/a^3
```

3.4.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int x^2 \coth^{-1}(ax) \, dx = \frac{x^2}{6a} + \frac{1}{3}x^3 \coth^{-1}(ax) + \frac{\log(1-a^2x^2)}{6a^3}$$

input
```
Integrate[x^2*ArcCoth[a*x],x]
```

output
```
x^2/(6*a) + (x^3*ArcCoth[a*x])/3 + Log[1 - a^2*x^2]/(6*a^3)
```

3.4.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6453, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(ax) \, dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{3}x^3 \coth^{-1}(ax) - \frac{1}{3}a \int \frac{x^3}{1-a^2x^2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{3}x^3 \coth^{-1}(ax) - \frac{1}{6}a \int \frac{x^2}{1-a^2x^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}x^3 \coth^{-1}(ax) - \frac{1}{6}a \int \left(-\frac{1}{a^2} - \frac{1}{a^2(a^2x^2-1)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3 \coth^{-1}(ax) - \frac{1}{6}a\left(-\frac{x^2}{a^2} - \frac{\log(1-a^2x^2)}{a^4}\right)$$

input
```
Int[x^2*ArcCoth[a*x],x]
```

output
```
(x^3*ArcCoth[a*x])/3 - (a*(-(x^2/a^2) - Log[1 - a^2*x^2]/a^4))/6
```

3.4.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.4.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

method	result	size
parts	$\frac{x^{3} \operatorname{arccoth}\left(a x \right)}{3}+\frac{a \left(\frac{x^{2}}{2 a^{2}}+\frac{\ln \left(a^{2} x^{2}-1\right)}{2 a^{4}}\right)}{3}$	38
parallelrisch	$-\frac{-2 a^{3} x^{3} \operatorname{arccoth}\left(a x \right)-a^{2} x^{2}-2 \ln \left(a x -1\right)-2 \,\operatorname{arccoth}\left(a x \right)}{6 a^{3}}$	41
derivativedivides	$\frac{\frac{a^{3} x^{3} \operatorname{arccoth}\left(a x \right)}{3}+\frac{a^{2} x^{2}}{6}+\frac{\ln \left(a x -1\right)}{6}+\frac{\ln \left(a x +1\right)}{6}}{a^{3}}$	42
default	$\frac{\frac{a^{3} x^{3} \operatorname{arccoth}\left(a x \right)}{3}+\frac{a^{2} x^{2}}{6}+\frac{\ln \left(a x -1\right)}{6}+\frac{\ln \left(a x +1\right)}{6}}{a^{3}}$	42
risch	$\frac{x^{3} \ln \left(a x +1\right)}{6}-\frac{\ln \left(a x -1\right) x^{3}}{6}+\frac{x^{2}}{6 a}+\frac{\ln \left(a^{2} x^{2}-1\right)}{6 a^{3}}$	47

input
```
int(x^2*arccoth(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccoth(a*x)+1/3*a*(1/2*x^2/a^2+1/2/a^4*ln(a^2*x^2-1))
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int x^2 \coth^{-1}(ax)\, dx = \frac{a^3x^3 \log\left(\frac{ax+1}{ax-1}\right) + a^2x^2 + \log\left(a^2x^2 - 1\right)}{6\, a^3}$$

input
```
integrate(x^2*arccoth(a*x),x, algorithm="fricas")
```

output
```
1/6*(a^3*x^3*log((a*x + 1)/(a*x - 1)) + a^2*x^2 + log(a^2*x^2 - 1))/a^3
```

3.4.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int x^2 \coth^{-1}(ax)\, dx = \begin{cases} \frac{x^3 \operatorname{acoth}(ax)}{3} + \frac{x^2}{6a} + \frac{\log(ax+1)}{3a^3} - \frac{\operatorname{acoth}(ax)}{3a^3} & \text{for}\ a \neq 0 \\ \frac{i\pi x^3}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*acoth(a*x),x)
```

output
```
Piecewise((x**3*acoth(a*x)/3 + x**2/(6*a) + log(a*x + 1)/(3*a**3) - acoth(
a*x)/(3*a**3), Ne(a, 0)), (I*pi*x**3/6, True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int x^2 \coth^{-1}(ax)\, dx = \frac{1}{3}\, x^3 \operatorname{arcoth}(ax) + \frac{1}{6}\, a\left(\frac{x^2}{a^2} + \frac{\log\left(a^2x^2 - 1\right)}{a^4}\right)$$

input
```
integrate(x^2*arccoth(a*x),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(a*x) + 1/6*a*(x^2/a^2 + log(a^2*x^2 - 1)/a^4)
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(34) = 68.

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 5.15

$$\int x^2 \coth^{-1}(ax)\,dx$$
$$= \frac{1}{3}\,a\left(\frac{\log\left(\frac{|ax+1|}{|ax-1|}\right)}{a^4} - \frac{\log\left(\left|\frac{ax+1}{ax-1}-1\right|\right)}{a^4} + \frac{\left(\frac{3\,(ax+1)^2}{(ax-1)^2}+1\right)\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}+1}{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}-1}\right)}{a^4\left(\frac{ax+1}{ax-1}-1\right)^3} + \frac{2\,(ax+1)}{(ax-1)a^4\left(\frac{ax+1}{ax-1}-1\right)^2}\right)$$

input
```
integrate(x^2*arccoth(a*x),x, algorithm="giac")
```

output
```
1/3*a*(log(abs(a*x + 1)/abs(a*x - 1))/a^4 - log(abs((a*x + 1)/(a*x - 1) -
1))/a^4 + (3*(a*x + 1)^2/(a*x - 1)^2 + 1)*log(-(((a*x + 1)*a/(a*x - 1) - a
)/(a*((a*x + 1)/(a*x - 1) + 1)) + 1)/(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x
 + 1)/(a*x - 1) + 1)) - 1))/(a^4*((a*x + 1)/(a*x - 1) - 1)^3) + 2*(a*x + 1
)/((a*x - 1)*a^4*((a*x + 1)/(a*x - 1) - 1)^2))
```

3.4.9 Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int x^2 \coth^{-1}(ax)\,dx = \frac{\frac{\ln(a^2\,x^2-1)}{6}+\frac{a^2\,x^2}{6}}{a^3} + \frac{x^3\,\mathrm{acoth}(a\,x)}{3}$$

input
```
int(x^2*acoth(a*x),x)
```

output
```
(log(a^2*x^2 - 1)/6 + (a^2*x^2)/6)/a^3 + (x^3*acoth(a*x))/3
```

3.5 $\int x \coth^{-1}(ax)\,dx$

3.5.1 Optimal result

Integrand size = 6, antiderivative size = 31

$$\int x \coth^{-1}(ax)\,dx = \frac{x}{2a} + \frac{1}{2}x^2 \coth^{-1}(ax) - \frac{\operatorname{arctanh}(ax)}{2a^2}$$

output
```
1/2*x/a+1/2*x^2*arccoth(a*x)-1/2*arctanh(a*x)/a^2
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int x \coth^{-1}(ax)\,dx = \frac{x}{2a} + \frac{1}{2}x^2 \coth^{-1}(ax) + \frac{\log(1-ax)}{4a^2} - \frac{\log(1+ax)}{4a^2}$$

input
```
Integrate[x*ArcCoth[a*x],x]
```

output
```
x/(2*a) + (x^2*ArcCoth[a*x])/2 + Log[1 - a*x]/(4*a^2) - Log[1 + a*x]/(4*a^
2)
```

3.5.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6453, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(ax) \, dx$$

$$\Big\downarrow \text{ 6453}$$

$$\frac{1}{2}x^2 \coth^{-1}(ax) - \frac{1}{2}a \int \frac{x^2}{1-a^2x^2}dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{2}x^2 \coth^{-1}(ax) - \frac{1}{2}a\left(\frac{\int \frac{1}{1-a^2x^2}dx}{a^2} - \frac{x}{a^2}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}x^2 \coth^{-1}(ax) - \frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3} - \frac{x}{a^2}\right)$$

input
```
Int[x*ArcCoth[a*x],x]
```

output
```
(x^2*ArcCoth[a*x])/2 - (a*(-(x/a^2) + ArcTanh[a*x]/a^3))/2
```

3.5.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.5.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
parallelrisch	$-\frac{-\operatorname{arccoth}(ax)a^2x^2-ax+\operatorname{arccoth}(ax)}{2a^2}$	27
derivativedivides	$\frac{\frac{\operatorname{arccoth}(ax)a^2x^2}{2}+\frac{ax}{2}+\frac{\ln(ax-1)}{4}-\frac{\ln(ax+1)}{4}}{a^2}$	38
default	$\frac{\frac{\operatorname{arccoth}(ax)a^2x^2}{2}+\frac{ax}{2}+\frac{\ln(ax-1)}{4}-\frac{\ln(ax+1)}{4}}{a^2}$	38
parts	$\frac{x^2\operatorname{arccoth}(ax)}{2}+\frac{a\left(\frac{x}{a^2}-\frac{\ln(ax+1)}{2a^3}+\frac{\ln(ax-1)}{2a^3}\right)}{2}$	42
risch	$\frac{x^2\ln(ax+1)}{4}-\frac{\ln(ax-1)x^2}{4}+\frac{x}{2a}+\frac{\ln(-ax+1)}{4a^2}-\frac{\ln(ax+1)}{4a^2}$	53

input
```
int(x*arccoth(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-arccoth(a*x)*a^2*x^2-a*x+arccoth(a*x))/a^2
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int x\coth^{-1}(ax)\,dx=\frac{2\,ax+(a^2x^2-1)\log\left(\frac{ax+1}{ax-1}\right)}{4\,a^2}$$

input
```
integrate(x*arccoth(a*x),x, algorithm="fricas")
```

output
```
1/4*(2*a*x + (a^2*x^2 - 1)*log((a*x + 1)/(a*x - 1)))/a^2
```

3.5.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int x\coth^{-1}(ax)\,dx = \begin{cases} \frac{x^2\operatorname{acoth}(ax)}{2} + \frac{x}{2a} - \frac{\operatorname{acoth}(ax)}{2a^2} & \text{for } a \neq 0 \\ \frac{i\pi x^2}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acoth(a*x),x)
```

output
```
Piecewise((x**2*acoth(a*x)/2 + x/(2*a) - acoth(a*x)/(2*a**2), Ne(a, 0)), (
I*pi*x**2/4, True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int x\coth^{-1}(ax)\,dx = \frac{1}{2}x^2\operatorname{arcoth}(ax) + \frac{1}{4}a\left(\frac{2x}{a^2} - \frac{\log(ax+1)}{a^3} + \frac{\log(ax-1)}{a^3}\right)$$

input
```
integrate(x*arccoth(a*x),x, algorithm="maxima")
```

output
```
1/2*x^2*arccoth(a*x) + 1/4*a*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a^
3)
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(25) = 50.

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 4.65

$$\int x\coth^{-1}(ax)\,dx = a\left(\frac{1}{a^3\left(\frac{ax+1}{ax-1}-1\right)} + \frac{(ax+1)\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}+1}{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}-1}\right)}{(ax-1)a^3\left(\frac{ax+1}{ax-1}-1\right)^2}\right)$$

input
```
integrate(x*arccoth(a*x),x, algorithm="giac")
```

output
```
a*(1/(a^3*((a*x + 1)/(a*x - 1) - 1)) + (a*x + 1)*log(-(((a*x + 1)*a/(a*x -
 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) + 1)/(((a*x + 1)*a/(a*x - 1) - a)/(
a*((a*x + 1)/(a*x - 1) + 1)) - 1))/((a*x - 1)*a^3*((a*x + 1)/(a*x - 1) - 1
)^2))
```

3.5.9 Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int x \coth^{-1}(ax)\,dx = \frac{x^2\,\mathrm{acoth}(a\,x)}{2} - \frac{\frac{\mathrm{acoth}(a\,x)}{2} - \frac{a\,x}{2}}{a^2}$$

input
```
int(x*acoth(a*x),x)
```

output
```
(x^2*acoth(a*x))/2 - (acoth(a*x)/2 - (a*x)/2)/a^2
```

3.6 $\int \coth^{-1}(ax)\,dx$

3.6.1 Optimal result

Integrand size = 4, antiderivative size = 25

$$\int \coth^{-1}(ax)\,dx = x\coth^{-1}(ax) + \frac{\log(1-a^2x^2)}{2a}$$

output
```
x*arccoth(a*x)+1/2*ln(-a^2*x^2+1)/a
```

3.6.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \coth^{-1}(ax)\,dx = x\coth^{-1}(ax) + \frac{\log(1-a^2x^2)}{2a}$$

input
```
Integrate[ArcCoth[a*x],x]
```

output
```
x*ArcCoth[a*x] + Log[1 - a^2*x^2]/(2*a)
```

3.6.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6437, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(ax)\,dx$$

$$\Big\downarrow \; 6437$$

$$x\coth^{-1}(ax) - a\int \frac{x}{1-a^2x^2}dx$$

$$\Big\downarrow \; 240$$

$$\frac{\log(1-a^2x^2)}{2a} + x\coth^{-1}(ax)$$

input
```
Int[ArcCoth[a*x],x]
```

output
```
x*ArcCoth[a*x] + Log[1 - a^2*x^2]/(2*a)
```

3.6.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

3.6.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result	size
parts	$x \operatorname{arccoth}(ax)+\frac{\ln(a^2x^2-1)}{2a}$	23
derivativedivides	$\frac{ax \operatorname{arccoth}(ax)+\frac{\ln\left(a^2x^2-1\right)}{2}}{a}$	25
default	$\frac{ax \operatorname{arccoth}(ax)+\frac{\ln\left(a^2x^2-1\right)}{2}}{a}$	25
parallelrisch	$-\frac{-ax \operatorname{arccoth}(ax)-\ln(ax-1)-\operatorname{arccoth}(ax)}{a}$	29
risch	$\frac{x\ln(ax+1)}{2}-\frac{\ln(ax-1)x}{2}+\frac{\ln(a^2x^2-1)}{2a}$	35

input
```
int(arccoth(a*x),x,method=_RETURNVERBOSE)
```

output
```
x*arccoth(a*x)+1/2/a*ln(a^2*x^2-1)
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \coth^{-1}(ax)\,dx=\frac{ax\log\left(\frac{ax+1}{ax-1}\right)+\log\left(a^2x^2-1\right)}{2\,a}$$

input
```
integrate(arccoth(a*x),x, algorithm="fricas")
```

output
```
1/2*(a*x*log((a*x + 1)/(a*x - 1)) + log(a^2*x^2 - 1))/a
```

3.6.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \coth^{-1}(ax)\,dx=\begin{cases} x\operatorname{acoth}(ax)+\frac{\log(ax+1)}{a}-\frac{\operatorname{acoth}(ax)}{a} & \text{for } a\neq 0\\ \frac{i\pi x}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(acoth(a*x),x)
```

output
```
Piecewise((x*acoth(a*x) + log(a*x + 1)/a - acoth(a*x)/a, Ne(a, 0)), (I*pi*
x/2, True))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \coth^{-1}(ax)\,dx = \frac{2\,ax\,\mathrm{arcoth}\,(ax) + \log\left(-a^2x^2 + 1\right)}{2\,a}$$

input
```
integrate(arccoth(a*x),x, algorithm="maxima")
```

output
```
1/2*(2*a*x*arccoth(a*x) + log(-a^2*x^2 + 1))/a
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 6.12

$$\int \coth^{-1}(ax)\,dx = a\left(\frac{\log\left(\frac{|ax+1|}{|ax-1|}\right)}{a^2} - \frac{\log\left(\left|\frac{ax+1}{ax-1} - 1\right|\right)}{a^2} + \frac{\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1} - a}{a\left(\frac{ax+1}{ax-1}+1\right)} + 1}{\frac{\frac{(ax+1)a}{ax-1} - a}{a\left(\frac{ax+1}{ax-1}+1\right)} - 1}\right)}{a^2\left(\frac{ax+1}{ax-1} - 1\right)}\right)$$

input
```
integrate(arccoth(a*x),x, algorithm="giac")
```

output
```
a*(log(abs(a*x + 1)/abs(a*x - 1))/a^2 - log(abs((a*x + 1)/(a*x - 1) - 1))/
a^2 + log(-(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) + 1)
/(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) - 1))/(a^2*((a
*x + 1)/(a*x - 1) - 1)))
```

3.6.9 Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \coth^{-1}(ax)\,dx = x\,\mathrm{acoth}(a\,x) + \frac{\ln\left(a^2\,x^2 - 1\right)}{2\,a}$$

input
```
int(acoth(a*x),x)
```

output
```
x*acoth(a*x) + log(a^2*x^2 - 1)/(2*a)
```

3.7 $\int \frac{\coth^{-1}(ax)}{x} \, dx$

3.7.1 Optimal result

Integrand size = 8, antiderivative size = 28

$$\int \frac{\coth^{-1}(ax)}{x} \, dx = \frac{1}{2}\text{PolyLog}\left(2,-\frac{1}{ax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{1}{ax}\right)$$

output
```
1/2*polylog(2,-1/a/x)-1/2*polylog(2,1/a/x)
```

3.7.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(ax)}{x} \, dx = \frac{1}{2}\left(\text{PolyLog}\left(2,-\frac{1}{ax}\right)-\text{PolyLog}\left(2,\frac{1}{ax}\right)\right)$$

input
```
Integrate[ArcCoth[a*x]/x,x]
```

output
```
(PolyLog[2, -(1/(a*x))] - PolyLog[2, 1/(a*x)])/2
```

3.7.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6447}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)}{x} \, dx$$

$$\Big\downarrow \; 6447$$

$$\frac{1}{2} \text{PolyLog}\left(2,-\frac{1}{ax}\right)-\frac{1}{2} \text{PolyLog}\left(2,\frac{1}{ax}\right)$$

input
```
Int[ArcCoth[a*x]/x,x]
```

output
```
PolyLog[2, -(1/(a*x))]/2 - PolyLog[2, 1/(a*x)]/2
```

3.7.3.1 Defintions of rubi rules used

rule 6447
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

3.7.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{\operatorname{dilog}(ax+1)}{2}-\frac{\operatorname{dilog}(ax)}{2}-\frac{\ln(ax-1)\ln(ax)}{2}$
derivativedivides	$\ln(ax)\operatorname{arccoth}(ax)-\frac{\operatorname{dilog}(ax+1)}{2}-\frac{\ln(ax)\ln(ax+1)}{2}-\frac{\operatorname{dilog}(ax)}{2}$
default	$\ln(ax)\operatorname{arccoth}(ax)-\frac{\operatorname{dilog}(ax+1)}{2}-\frac{\ln(ax)\ln(ax+1)}{2}-\frac{\operatorname{dilog}(ax)}{2}$
parts	$\ln(x)\operatorname{arccoth}(ax)+a\left(-\frac{\operatorname{dilog}(ax+1)}{2a}-\frac{\ln(x)\ln(ax+1)}{2a}+\frac{(\ln(x)-\ln(ax))\ln(-ax+1)}{2a}-\frac{\operatorname{dilog}(ax)}{2a}\right)$

input
```
int(arccoth(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*dilog(a*x+1)-1/2*dilog(a*x)-1/2*ln(a*x-1)*ln(a*x)
```

3.7.5 Fricas [F]

$$\int \frac{\coth^{-1}(ax)}{x}\,dx = \int \frac{\operatorname{arcoth}(ax)}{x}\,dx$$

input
```
integrate(arccoth(a*x)/x,x, algorithm="fricas")
```

output
```
integral(arccoth(a*x)/x, x)
```

3.7.6 Sympy [F]

$$\int \frac{\coth^{-1}(ax)}{x}\,dx = \int \frac{\operatorname{acoth}(ax)}{x}\,dx$$

input
```
integrate(acoth(a*x)/x,x)
```

output
```
Integral(acoth(a*x)/x, x)
```

3.7.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(22) = 44.

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.07

$$\begin{aligned}
&\int \frac{\coth^{-1}(ax)}{x}\,dx \\
&= -\frac{1}{2}a\left(\frac{\log(ax+1)}{a} - \frac{\log(ax-1)}{a}\right)\log(x) \\
&\quad - \frac{1}{2}a\left(\frac{\log(ax-1)\log(ax) + \mathrm{Li}_2(-ax+1)}{a} - \frac{\log(ax+1)\log(-ax) + \mathrm{Li}_2(ax+1)}{a}\right) \\
&\quad + \operatorname{arcoth}(ax)\log(x)
\end{aligned}$$

input
```
integrate(arccoth(a*x)/x,x, algorithm="maxima")
```

output
```
-1/2*a*(log(a*x + 1)/a - log(a*x - 1)/a)*log(x) - 1/2*a*((log(a*x - 1)*log
(a*x) + dilog(-a*x + 1))/a - (log(a*x + 1)*log(-a*x) + dilog(a*x + 1))/a)
+ arccoth(a*x)*log(x)
```

3.7.8 Giac [F]

$$\int \frac{\coth^{-1}(ax)}{x} dx = \int \frac{\operatorname{arcoth}(ax)}{x} dx$$

input
```
integrate(arccoth(a*x)/x,x, algorithm="giac")
```

output
```
integrate(arccoth(a*x)/x, x)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax)}{x} dx = \int \frac{\operatorname{acoth}(a\,x)}{x} dx$$

input
```
int(acoth(a*x)/x,x)
```

output
```
int(acoth(a*x)/x, x)
```

3.8 $\int \frac{\coth^{-1}(ax)}{x^2} \, dx$

3.8.1 Optimal result

Integrand size = 8, antiderivative size = 30

$$\int \frac{\coth^{-1}(ax)}{x^2} \, dx = -\frac{\coth^{-1}(ax)}{x} + a\log(x) - \frac{1}{2}a\log\left(1 - a^2x^2\right)$$

output
```
-arccoth(a*x)/x+a*ln(x)-1/2*a*ln(-a^2*x^2+1)
```

3.8.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(ax)}{x^2} \, dx = -\frac{\coth^{-1}(ax)}{x} + a\log(x) - \frac{1}{2}a\log\left(1 - a^2x^2\right)$$

input
```
Integrate[ArcCoth[a*x]/x^2,x]
```

output
```
-(ArcCoth[a*x]/x) + a*Log[x] - (a*Log[1 - a^2*x^2])/2
```

3.8.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6453, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)}{x^2} \, dx$$

$$\Big\downarrow \text{6453}$$

$$a \int \frac{1}{x(1-a^2x^2)}dx - \frac{\coth^{-1}(ax)}{x}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a \int \frac{1}{x^2(1-a^2x^2)}dx^2 - \frac{\coth^{-1}(ax)}{x}$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}a\left(a^2 \int \frac{1}{1-a^2x^2}dx^2 + \int \frac{1}{x^2}dx^2\right) - \frac{\coth^{-1}(ax)}{x}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}a\left(a^2 \int \frac{1}{1-a^2x^2}dx^2 + \log\left(x^2\right)\right) - \frac{\coth^{-1}(ax)}{x}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}a(\log\left(x^2\right) - \log\left(1-a^2x^2\right)) - \frac{\coth^{-1}(ax)}{x}$$

input
```
Int[ArcCoth[a*x]/x^2,x]
```

output
```
-(ArcCoth[a*x]/x) + (a*(Log[x^2] - Log[1 - a^2*x^2]))/2
```

3.8.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.8.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

method	result	size
parallelrisch	$\frac{a\ln(x)x-a\ln(ax-1)x-ax\ \operatorname{arccoth}(ax)-\operatorname{arccoth}(ax)}{x}$	35
parts	$-\frac{\operatorname{arccoth}(ax)}{x}-a\left(\frac{\ln(ax+1)}{2}-\ln\left(x\right)+\frac{\ln(ax-1)}{2}\right)$	35
derivativedivides	$a\left(-\frac{\operatorname{arccoth}(ax)}{ax}-\frac{\ln(ax+1)}{2}-\frac{\ln(ax-1)}{2}+\ln\left(ax\right)\right)$	36
default	$a\left(-\frac{\operatorname{arccoth}(ax)}{ax}-\frac{\ln(ax+1)}{2}-\frac{\ln(ax-1)}{2}+\ln\left(ax\right)\right)$	36
risch	$-\frac{\ln(ax+1)}{2x}+\frac{2a\ln(x)x-a\ln(a^2x^2-1)x+\ln(ax-1)}{2x}$	45

input
```
int(arccoth(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(a*ln(x)*x-a*ln(a*x-1)*x-a*x*arccoth(a*x)-arccoth(a*x))/x
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$\int \frac{\coth^{-1}(ax)}{x^2}\,dx = -\frac{ax\log\left(a^2x^2-1\right)-2\,ax\log\left(x\right)+\log\left(\frac{ax+1}{ax-1}\right)}{2\,x}$$

input
```
integrate(arccoth(a*x)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(a*x*log(a^2*x^2 - 1) - 2*a*x*log(x) + log((a*x + 1)/(a*x - 1)))/x
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\coth^{-1}(ax)}{x^2}\,dx = a\log\left(x\right)-a\log\left(ax+1\right)+a\,\mathrm{acoth}\left(ax\right)-\frac{\mathrm{acoth}\left(ax\right)}{x}$$

input
```
integrate(acoth(a*x)/x**2,x)
```

output
```
a*log(x) - a*log(a*x + 1) + a*acoth(a*x) - acoth(a*x)/x
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(ax)}{x^2}\,dx = -\frac{1}{2}\,a\left(\log\left(a^2x^2-1\right)-\log\left(x^2\right)\right)-\frac{\mathrm{arcoth}\left(ax\right)}{x}$$

input
```
integrate(arccoth(a*x)/x^2,x, algorithm="maxima")
```

output
```
-1/2*a*(log(a^2*x^2 - 1) - log(x^2)) - arccoth(a*x)/x
```

3.8.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(28) = 56.

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 4.77

$$\int \frac{\coth^{-1}(ax)}{x^2}\,dx = a\left(\frac{\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}+1}{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}-1}\right)}{\frac{ax+1}{ax-1}+1} - \log\left(\frac{|ax+1|}{|ax-1|}\right) + \log\left(\left|\frac{ax+1}{ax-1}+1\right|\right)\right)$$

input
```
integrate(arccoth(a*x)/x^2,x, algorithm="giac")
```

output
```
a*(log(-(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) + 1)/((
(a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) - 1))/((a*x + 1)/
(a*x - 1) + 1) - log(abs(a*x + 1)/abs(a*x - 1)) + log(abs((a*x + 1)/(a*x -
 1) + 1)))
```

3.8.9 Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{\coth^{-1}(ax)}{x^2}\,dx = a\,\ln(x) - \frac{a\,\ln(a^2\,x^2-1)}{2} - \frac{\mathrm{acoth}(a\,x)}{x}$$

input
```
int(acoth(a*x)/x^2,x)
```

output
```
a*log(x) - (a*log(a^2*x^2 - 1))/2 - acoth(a*x)/x
```

3.9 $\int \frac{\coth^{-1}(ax)}{x^3}\,dx$

3.9.1 Optimal result

Integrand size = 8, antiderivative size = 31

$$\int \frac{\coth^{-1}(ax)}{x^3}\,dx = -\frac{a}{2x} - \frac{\coth^{-1}(ax)}{2x^2} + \frac{1}{2}a^2\text{arctanh}(ax)$$

output
```
-1/2*a/x-1/2*arccoth(a*x)/x^2+1/2*a^2*arctanh(a*x)
```

3.9.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \frac{\coth^{-1}(ax)}{x^3}\,dx = -\frac{a}{2x} - \frac{\coth^{-1}(ax)}{2x^2} - \frac{1}{4}a^2\log(1-ax) + \frac{1}{4}a^2\log(1+ax)$$

input
```
Integrate[ArcCoth[a*x]/x^3,x]
```

output
```
-1/2*a/x - ArcCoth[a*x]/(2*x^2) - (a^2*Log[1 - a*x])/4 + (a^2*Log[1 + a*x]
)/4
```

3.9.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6453, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)}{x^3} \, dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{2}a\int \frac{1}{x^2\left(1-a^2x^2\right)}dx-\frac{\coth^{-1}(ax)}{2x^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx-\frac{1}{x}\right)-\frac{\coth^{-1}(ax)}{2x^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)-\frac{\coth^{-1}(ax)}{2x^2}$$

input
```
Int[ArcCoth[a*x]/x^3,x]
```

output
```
-1/2*ArcCoth[a*x]/x^2 + (a*(-x^(-1) + a*ArcTanh[a*x]))/2
```

3.9.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.9.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
parallelrisch	$-\frac{-\operatorname{arccoth}(ax)a^2x^2+ax+\operatorname{arccoth}(ax)}{2x^2}$	26
parts	$-\frac{\operatorname{arccoth}(ax)}{2x^2}-\frac{a\left(-\frac{a\ln(ax+1)}{2}+\frac{1}{x}+\frac{a\ln(ax-1)}{2}\right)}{2}$	36
derivativedivides	$a^2\left(-\frac{\operatorname{arccoth}(ax)}{2a^2x^2}+\frac{\ln(ax+1)}{4}-\frac{\ln(ax-1)}{4}-\frac{1}{2ax}\right)$	42
default	$a^2\left(-\frac{\operatorname{arccoth}(ax)}{2a^2x^2}+\frac{\ln(ax+1)}{4}-\frac{\ln(ax-1)}{4}-\frac{1}{2ax}\right)$	42
risch	$-\frac{\ln(ax+1)}{4x^2}-\frac{\ln(-ax+1)a^2x^2-\ln(-ax-1)a^2x^2+2ax-\ln(ax-1)}{4x^2}$	60

input
```
int(arccoth(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-arccoth(a*x)*a^2*x^2+a*x+arccoth(a*x))/x^2
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(ax)}{x^3}\,dx = -\frac{2\,ax-(a^2x^2-1)\log\left(\frac{ax+1}{ax-1}\right)}{4\,x^2}$$

input
```
integrate(arccoth(a*x)/x^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a*x - (a^2*x^2 - 1)*log((a*x + 1)/(a*x - 1)))/x^2
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\coth^{-1}(ax)}{x^3}\,dx = \frac{a^2\operatorname{acoth}(ax)}{2} - \frac{a}{2x} - \frac{\operatorname{acoth}(ax)}{2x^2}$$

input
```
integrate(acoth(a*x)/x**3,x)
```

output
```
a**2*acoth(a*x)/2 - a/(2*x) - acoth(a*x)/(2*x**2)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{\coth^{-1}(ax)}{x^3}\,dx = \frac{1}{4}\left(a\log(ax+1) - a\log(ax-1) - \frac{2}{x}\right)a - \frac{\operatorname{arcoth}(ax)}{2\,x^2}$$

input
```
integrate(arccoth(a*x)/x^3,x, algorithm="maxima")
```

output
```
1/4*(a*log(a*x + 1) - a*log(a*x - 1) - 2/x)*a - 1/2*arccoth(a*x)/x^2
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(25) = 50.

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 4.52

$$\int \frac{\coth^{-1}(ax)}{x^3}\,dx = a\left(\frac{a}{\frac{ax+1}{ax-1}+1} + \frac{(ax+1)a\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}+1}{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}-1}\right)}{(ax-1)\left(\frac{ax+1}{ax-1}+1\right)^2}\right)$$

input
```
integrate(arccoth(a*x)/x^3,x, algorithm="giac")
```

output
```
a*(a/((a*x + 1)/(a*x - 1) + 1) + (a*x + 1)*a*log(-(((a*x + 1)*a/(a*x - 1)
- a)/(a*((a*x + 1)/(a*x - 1) + 1)) + 1)/(((a*x + 1)*a/(a*x - 1) - a)/(a*((
a*x + 1)/(a*x - 1) + 1)) - 1))/((a*x - 1)*((a*x + 1)/(a*x - 1) + 1)^2))
```

3.9.9 Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{\coth^{-1}(ax)}{x^3} \, dx = \frac{a \operatorname{atan}\left(\frac{a^2 x}{\sqrt{-a^2}}\right) \sqrt{-a^2}}{2} - \frac{\frac{\operatorname{acoth}(a\,x)}{2} + \frac{a\,x}{2}}{x^2}$$

input
```
int(acoth(a*x)/x^3,x)
```

output
```
(a*atan((a^2*x)/(-a^2)^(1/2))*(-a^2)^(1/2))/2 - (acoth(a*x)/2 + (a*x)/2)/x
^2
```

3.10 $\int \frac{\coth^{-1}(ax)}{x^4} dx$

3.10.1 Optimal result

Integrand size = 8, antiderivative size = 47

$$\int \frac{\coth^{-1}(ax)}{x^4} dx = -\frac{a}{6x^2} - \frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{3}a^3 \log(x) - \frac{1}{6}a^3 \log(1 - a^2x^2)$$

output
```
-1/6*a/x^2-1/3*arccoth(a*x)/x^3+1/3*a^3*ln(x)-1/6*a^3*ln(-a^2*x^2+1)
```

3.10.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(ax)}{x^4} dx = -\frac{a}{6x^2} - \frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{3}a^3 \log(x) - \frac{1}{6}a^3 \log(1 - a^2x^2)$$

input
```
Integrate[ArcCoth[a*x]/x^4,x]
```

output
```
-1/6*a/x^2 - ArcCoth[a*x]/(3*x^3) + (a^3*Log[x])/3 - (a^3*Log[1 - a^2*x^2]
)/6
```

3.10.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6453, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)}{x^4} \, dx$$

$$\Big\downarrow \text{ 6453}$$

$$\frac{1}{3}a\int \frac{1}{x^3\left(1-a^2x^2\right)}dx-\frac{\coth^{-1}(ax)}{3x^3}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{6}a\int \frac{1}{x^4\left(1-a^2x^2\right)}dx^2-\frac{\coth^{-1}(ax)}{3x^3}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{6}a\int \left(-\frac{a^4}{a^2x^2-1}+\frac{a^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{\coth^{-1}(ax)}{3x^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6}a\left(a^2\log\left(x^2\right)-a^2\log\left(1-a^2x^2\right)-\frac{1}{x^2}\right)-\frac{\coth^{-1}(ax)}{3x^3}$$

input
```
Int[ArcCoth[a*x]/x^4,x]
```

output
```
-1/3*ArcCoth[a*x]/x^3 + (a*(-x^(-2) + a^2*Log[x^2] - a^2*Log[1 - a^2*x^2])
)/6
```

3.10.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.10.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$a^3\left(-\frac{\operatorname{arccoth}(ax)}{3a^3x^3}-\frac{\ln(ax-1)}{6}-\frac{\ln(ax+1)}{6}-\frac{1}{6a^2x^2}+\frac{\ln(ax)}{3}\right)$	48
default	$a^3\left(-\frac{\operatorname{arccoth}(ax)}{3a^3x^3}-\frac{\ln(ax-1)}{6}-\frac{\ln(ax+1)}{6}-\frac{1}{6a^2x^2}+\frac{\ln(ax)}{3}\right)$	48
parts	$-\frac{\operatorname{arccoth}(ax)}{3x^3}-\frac{a\left(\frac{a^2\ln(ax+1)}{2}+\frac{1}{2x^2}-a^2\ln(x)+\frac{a^2\ln(ax-1)}{2}\right)}{3}$	49
risch	$-\frac{\ln(ax+1)}{6x^3}+\frac{2a^3\ln(x)x^3-\ln(a^2x^2-1)a^3x^3-ax+\ln(ax-1)}{6x^3}$	57
parallelrisch	$\frac{2a^3\ln(x)x^3-2a^3\ln(ax-1)x^3-2a^3x^3\operatorname{arccoth}(ax)-a^3x^3-ax-2\operatorname{arccoth}(ax)}{6x^3}$	61

input
```
int(arccoth(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/3/a^3/x^3*arccoth(a*x)-1/6*ln(a*x-1)-1/6*ln(a*x+1)-1/6/a^2/x^2+1/3
*ln(a*x))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{\coth^{-1}(ax)}{x^4}\,dx = -\frac{a^3x^3\log\left(a^2x^2-1\right) - 2\,a^3x^3\log\left(x\right) + ax + \log\left(\frac{ax+1}{ax-1}\right)}{6\,x^3}$$

input
```
integrate(arccoth(a*x)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(a^3*x^3*log(a^2*x^2 - 1) - 2*a^3*x^3*log(x) + a*x + log((a*x + 1)/(a
*x - 1)))/x^3
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{\coth^{-1}(ax)}{x^4}\,dx = \frac{a^3\log\left(x\right)}{3} - \frac{a^3\log\left(ax+1\right)}{3} + \frac{a^3\operatorname{acoth}\left(ax\right)}{3} - \frac{a}{6x^2} - \frac{\operatorname{acoth}\left(ax\right)}{3x^3}$$

input
```
integrate(acoth(a*x)/x**4,x)
```

output
```
a**3*log(x)/3 - a**3*log(a*x + 1)/3 + a**3*acoth(a*x)/3 - a/(6*x**2) - aco
th(a*x)/(3*x**3)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{\coth^{-1}(ax)}{x^4}\,dx = -\frac{1}{6}\left(a^2\log\left(a^2x^2-1\right) - a^2\log\left(x^2\right) + \frac{1}{x^2}\right)a - \frac{\operatorname{arcoth}\left(ax\right)}{3\,x^3}$$

input
```
integrate(arccoth(a*x)/x^4,x, algorithm="maxima")
```

output
```
-1/6*(a^2*log(a^2*x^2 - 1) - a^2*log(x^2) + 1/x^2)*a - 1/3*arccoth(a*x)/x^
3
```

3.10.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(39) = 78$.

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 4.45

$$\int \frac{\coth^{-1}(ax)}{x^4}\,dx =$$

$$-\frac{1}{3}\left(a^2\log\left(\frac{|ax+1|}{|ax-1|}\right)-a^2\log\left(\left|\frac{ax+1}{ax-1}+1\right|\right)-\frac{2\,(ax+1)a^2}{(ax-1)\left(\frac{ax+1}{ax-1}+1\right)^2}-\frac{\left(\frac{3\,(ax+1)^2a^2}{(ax-1)^2}+a^2\right)\log\left(-\cdots\right)}{\left(\frac{ax+1}{ax-1}+1\right)^3}\cdots\right.$$

input
```
integrate(arccoth(a*x)/x^4,x, algorithm="giac")
```

output
```
-1/3*(a^2*log(abs(a*x + 1)/abs(a*x - 1)) - a^2*log(abs((a*x + 1)/(a*x - 1)
 + 1)) - 2*(a*x + 1)*a^2/((a*x - 1)*((a*x + 1)/(a*x - 1) + 1)^2) - (3*(a*x
 + 1)^2*a^2/(a*x - 1)^2 + a^2)*log(-(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x
+ 1)/(a*x - 1) + 1)) + 1)/(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x
- 1) + 1)) - 1))/((a*x + 1)/(a*x - 1) + 1)^3)*a
```

3.10.9 Mupad [B] (verification not implemented)

Time = 4.61 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{\coth^{-1}(ax)}{x^4}\,dx = \frac{a^3\,\ln(x)}{3}-\frac{\frac{\mathrm{acoth}(a\,x)}{3}+\frac{a\,x}{6}}{x^3}-\frac{a^3\,\ln\left(a^2\,x^2-1\right)}{6}$$

input
```
int(acoth(a*x)/x^4,x)
```

output
```
(a^3*log(x))/3 - (acoth(a*x)/3 + (a*x)/6)/x^3 - (a^3*log(a^2*x^2 - 1))/6
```

3.11 $\int \frac{\coth^{-1}(ax)}{x^5}\,dx$

3.11.1 Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \frac{\coth^{-1}(ax)}{x^5}\,dx = -\frac{a}{12x^3} - \frac{a^3}{4x} - \frac{\coth^{-1}(ax)}{4x^4} + \frac{1}{4}a^4\operatorname{arctanh}(ax)$$

output
```
-1/12*a/x^3-1/4*a^3/x-1/4*arccoth(a*x)/x^4+1/4*a^4*arctanh(a*x)
```

3.11.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \frac{\coth^{-1}(ax)}{x^5}\,dx = -\frac{a}{12x^3} - \frac{a^3}{4x} - \frac{\coth^{-1}(ax)}{4x^4} - \frac{1}{8}a^4\log(1-ax) + \frac{1}{8}a^4\log(1+ax)$$

input
```
Integrate[ArcCoth[a*x]/x^5,x]
```

output
```
-1/12*a/x^3 - a^3/(4*x) - ArcCoth[a*x]/(4*x^4) - (a^4*Log[1 - a*x])/8 + (a
^4*Log[1 + a*x])/8
```

3.11.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6453, 264, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)}{x^5} \, dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{4} a \int \frac{1}{x^4 (1-a^2x^2)}dx-\frac{\coth^{-1}(ax)}{4x^4}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{4} a \left(a^2 \int \frac{1}{x^2 (1-a^2x^2)}dx-\frac{1}{3x^3}\right)-\frac{\coth^{-1}(ax)}{4x^4}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{4} a \left(a^2 \left(a^2 \int \frac{1}{1-a^2x^2}dx-\frac{1}{x}\right)-\frac{1}{3x^3}\right)-\frac{\coth^{-1}(ax)}{4x^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4} a \left(a^2 \left(a \text{arctanh}(ax)-\frac{1}{x}\right)-\frac{1}{3x^3}\right)-\frac{\coth^{-1}(ax)}{4x^4}$$

input
```
Int[ArcCoth[a*x]/x^5,x]
```

output
```
-1/4*ArcCoth[a*x]/x^4 + (a*(-1/3*1/x^3 + a^2*(-x^(-1) + a*ArcTanh[a*x])))/
4
```

3.11.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.11.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
parallelrisch	$-\frac{-3a^4x^4\,\mathrm{arccoth}(ax)+3a^3x^3+ax+3\,\mathrm{arccoth}(ax)}{12x^4}$	36
parts	$-\frac{\mathrm{arccoth}(ax)}{4x^4}-\frac{a\left(-\frac{a^3\ln(ax+1)}{2}+\frac{1}{3x^3}+\frac{a^2}{x}+\frac{a^3\ln(ax-1)}{2}\right)}{4}$	49
derivativedivides	$a^4\left(-\frac{\mathrm{arccoth}(ax)}{4a^4x^4}-\frac{\ln(ax-1)}{8}+\frac{\ln(ax+1)}{8}-\frac{1}{12a^3x^3}-\frac{1}{4ax}\right)$	50
default	$a^4\left(-\frac{\mathrm{arccoth}(ax)}{4a^4x^4}-\frac{\ln(ax-1)}{8}+\frac{\ln(ax+1)}{8}-\frac{1}{12a^3x^3}-\frac{1}{4ax}\right)$	50
risch	$-\frac{\ln(ax+1)}{8x^4}+\frac{3\ln(-ax-1)a^4x^4-3\ln(-ax+1)a^4x^4-6a^3x^3-2ax+3\ln(ax-1)}{24x^4}$	69

input
```
int(arccoth(a*x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-3*a^4*x^4*arccoth(a*x)+3*a^3*x^3+a*x+3*arccoth(a*x))/x^4
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(ax)}{x^5}\,dx = -\frac{6\,a^3x^3 + 2\,ax - 3\,(a^4x^4 - 1)\log\left(\frac{ax+1}{ax-1}\right)}{24\,x^4}$$

input
```
integrate(arccoth(a*x)/x^5,x, algorithm="fricas")
```

output
```
-1/24*(6*a^3*x^3 + 2*a*x - 3*(a^4*x^4 - 1)*log((a*x + 1)/(a*x - 1)))/x^4
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{\coth^{-1}(ax)}{x^5}\,dx = \frac{a^4\,\operatorname{acoth}(ax)}{4} - \frac{a^3}{4x} - \frac{a}{12x^3} - \frac{\operatorname{acoth}(ax)}{4x^4}$$

input
```
integrate(acoth(a*x)/x**5,x)
```

output
```
a**4*acoth(a*x)/4 - a**3/(4*x) - a/(12*x**3) - acoth(a*x)/(4*x**4)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \frac{\coth^{-1}(ax)}{x^5}\,dx$$
$$= \frac{1}{24}\left(3\,a^3\log(ax+1) - 3\,a^3\log(ax-1) - \frac{2\,(3\,a^2x^2+1)}{x^3}\right)a - \frac{\operatorname{arcoth}(ax)}{4\,x^4}$$

input
```
integrate(arccoth(a*x)/x^5,x, algorithm="maxima")
```

output
```
1/24*(3*a^3*log(a*x + 1) - 3*a^3*log(a*x - 1) - 2*(3*a^2*x^2 + 1)/x^3)*a -
 1/4*arccoth(a*x)/x^4
```

3.11.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(33) = 66$.

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 5.00

$$\int \frac{\coth^{-1}(ax)}{x^5}\,dx = \frac{1}{3}a\left(\frac{\frac{3\,(ax+1)^2a^3}{(ax-1)^2}+\frac{3\,(ax+1)a^3}{ax-1}+2\,a^3}{\left(\frac{ax+1}{ax-1}+1\right)^3}+\frac{3\left(\frac{(ax+1)^3a^3}{(ax-1)^3}+\frac{(ax+1)a^3}{ax-1}\right)\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}+1}{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}-1}\right)}{\left(\frac{ax+1}{ax-1}+1\right)^4}\right)$$

input
```
integrate(arccoth(a*x)/x^5,x, algorithm="giac")
```

output
```
1/3*a*((3*(a*x + 1)^2*a^3/(a*x - 1)^2 + 3*(a*x + 1)*a^3/(a*x - 1) + 2*a^3)
/((a*x + 1)/(a*x - 1) + 1)^3 + 3*((a*x + 1)^3*a^3/(a*x - 1)^3 + (a*x + 1)*
a^3/(a*x - 1))*log(-(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) +
 1)) + 1)/(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) - 1))
/((a*x + 1)/(a*x - 1) + 1)^4)
```

3.11.9 Mupad [B] (verification not implemented)

Time = 4.63 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \frac{\coth^{-1}(ax)}{x^5}\,dx = \frac{\ln\left(1-\frac{1}{a\,x}\right)}{8\,x^4}-\frac{\ln\left(\frac{1}{a\,x}+1\right)}{8\,x^4}-\frac{a^3\,x^2+\frac{a}{3}}{4\,x^3}-\frac{a^4\,\mathrm{atan}(a\,x\,1\mathrm{i})\,1\mathrm{i}}{4}$$

input
```
int(acoth(a*x)/x^5,x)
```

output
```
log(1 - 1/(a*x))/(8*x^4) - (a^4*atan(a*x*1i)*1i)/4 - log(1/(a*x) + 1)/(8*x
^4) - (a/3 + a^3*x^2)/(4*x^3)
```

3.12 $\int x^5 \coth^{-1}(ax)^2 \, dx$

3.12.1 Optimal result

Integrand size = 10, antiderivative size = 105

$$\int x^5 \coth^{-1}(ax)^2 \, dx = \frac{4x^2}{45a^4} + \frac{x^4}{60a^2} + \frac{x \coth^{-1}(ax)}{3a^5} + \frac{x^3 \coth^{-1}(ax)}{9a^3} + \frac{x^5 \coth^{-1}(ax)}{15a} - \frac{\coth^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6 \coth^{-1}(ax)^2 + \frac{23 \log(1-a^2x^2)}{90a^6}$$

output
```
4/45*x^2/a^4+1/60*x^4/a^2+1/3*x*arccoth(a*x)/a^5+1/9*x^3*arccoth(a*x)/a^3+
1/15*x^5*arccoth(a*x)/a-1/6*arccoth(a*x)^2/a^6+1/6*x^6*arccoth(a*x)^2+23/9
0*ln(-a^2*x^2+1)/a^6
```

3.12.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

$$\int x^5 \coth^{-1}(ax)^2 \, dx = \frac{16a^2x^2 + 3a^4x^4 + 4ax(15 + 5a^2x^2 + 3a^4x^4) \coth^{-1}(ax) + 30(-1 + a^6x^6) \coth^{-1}(ax)^2 + 46 \log(1-a^2x^2)}{180a^6}$$

input
```
Integrate[x^5*ArcCoth[a*x]^2,x]
```

output
```
(16*a^2*x^2 + 3*a^4*x^4 + 4*a*x*(15 + 5*a^2*x^2 + 3*a^4*x^4)*ArcCoth[a*x]
+ 30*(-1 + a^6*x^6)*ArcCoth[a*x]^2 + 46*Log[1 - a^2*x^2])/(180*a^6)
```

3.12.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.69, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {6453, 6543, 6453, 243, 49, 2009, 6543, 6453, 243, 49, 2009, 6543, 6437, 240, 6511}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \coth^{-1}(ax)^2 \, dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{6}x^6 \coth^{-1}(ax)^2 - \frac{1}{3}a\int \frac{x^6 \coth^{-1}(ax)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{6543}$$

$$\frac{1}{6}x^6 \coth^{-1}(ax)^2 - \frac{1}{3}a\left(\frac{\int \frac{x^4 \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\int x^4 \coth^{-1}(ax)dx}{a^2}\right)$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{6}x^6 \coth^{-1}(ax)^2 - \frac{1}{3}a\left(\frac{\int \frac{x^4 \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{5}x^5 \coth^{-1}(ax) - \frac{1}{5}a\int \frac{x^5}{1-a^2x^2}dx}{a^2}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{6}x^6 \coth^{-1}(ax)^2 - \frac{1}{3}a\left(\frac{\int \frac{x^4 \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{5}x^5 \coth^{-1}(ax) - \frac{1}{10}a\int \frac{x^4}{1-a^2x^2}dx^2}{a^2}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{6}x^6 \coth^{-1}(ax)^2 - \frac{1}{3}a\left(\frac{\int \frac{x^4 \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{5}x^5 \coth^{-1}(ax) - \frac{1}{10}a\int \left(-\frac{x^2}{a^2} - \frac{1}{a^4(a^2x^2-1)} - \frac{1}{a^4}\right)dx^2}{a^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}x^6 \coth^{-1}(ax)^2 - \frac{1}{3}a\left(\frac{\int \frac{x^4 \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{5}x^5 \coth^{-1}(ax) - \frac{1}{10}a\left(-\frac{x^2}{a^4} - \frac{x^4}{2a^2} - \frac{\log(1-a^2x^2)}{a^6}\right)}{a^2}\right)$$

$$\Big\downarrow \text{6543}$$

$$\frac{1}{6}x^6\coth^{-1}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\int\frac{x^2\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x^2\coth^{-1}(ax)dx}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\coth^{-1}(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}-\frac{x^4}{2a^2}-\frac{\log(1-a^2x^2)}{a^6}\right)}{a^2}\right)$$

$\Big\downarrow$ 6453

$$\frac{1}{6}x^6\coth^{-1}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\int\frac{x^2\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{3}a\int\frac{x^3}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\coth^{-1}(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}-\frac{x^4}{2a^2}-\frac{\log(1-a^2x^2)}{a^6}\right)}{a^2}\right)$$

$\Big\downarrow$ 243

$$\frac{1}{6}x^6\coth^{-1}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\int\frac{x^2\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{6}a\int\frac{x^2}{1-a^2x^2}dx^2}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\coth^{-1}(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}-\frac{x^4}{2a^2}-\frac{\log(1-a^2x^2)}{a^6}\right)}{a^2}\right)$$

$\Big\downarrow$ 49

$$\frac{1}{6}x^6\coth^{-1}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\int\frac{x^2\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{6}a\int\left(-\frac{1}{a^2}-\frac{1}{a^2(a^2x^2-1)}\right)dx^2}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\coth^{-1}(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}-\frac{x^4}{2a^2}-\frac{\log(1-a^2x^2)}{a^6}\right)}{a^2}\right)$$

$\Big\downarrow$ 2009

$$\frac{1}{6}x^6\coth^{-1}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\int\frac{x^2\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\coth^{-1}(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}-\frac{x^4}{2a^2}-\frac{\log(1-a^2x^2)}{a^6}\right)}{a^2}\right)$$

$\Big\downarrow$ 6543

$$\frac{1}{6}x^6\coth^{-1}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\frac{\int\frac{\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int\coth^{-1}(ax)dx}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\coth^{-1}(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}-\frac{x^4}{2a^2}-\frac{\log(1-\cdots}{a\cdots}\right.}{a^2}\right.$$

3.12. $\int x^5\coth^{-1}(ax)^2\,dx$

$\downarrow$ 6437

$$\frac{1}{6}x^6\coth^{-1}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\frac{\int\frac{\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{x\coth^{-1}(ax)-a\int\frac{x}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log\left(1-a^2x^2\right)}{a^4}\right)}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\coth^{-1}(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}-\frac{x}{2a}\right.}{a^2}\right.$$

$\downarrow$ 240

$$\frac{1}{6}x^6\coth^{-1}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\frac{\int\frac{\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{\log\left(1-a^2x^2\right)}{2a}+x\coth^{-1}(ax)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log\left(1-a^2x^2\right)}{a^4}\right)}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\coth^{-1}(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}-\frac{x}{2a}\right.}{a^2}\right.$$

$\downarrow$ 6511

$$\frac{1}{6}x^6\coth^{-1}(ax)^2-\frac{1}{3}a\left(\frac{\frac{\frac{\coth^{-1}(ax)^2}{2a^3}-\frac{\frac{\log\left(1-a^2x^2\right)}{2a}+x\coth^{-1}(ax)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log\left(1-a^2x^2\right)}{a^4}\right)}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\coth^{-1}(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}-\frac{x^4}{2a^2}-\right.}{a^2}\right.$$

input
```
Int[x^5*ArcCoth[a*x]^2,x]
```

output
```
(x^6*ArcCoth[a*x]^2)/6 - (a*(-(((x^5*ArcCoth[a*x])/5 - (a*(-(x^2/a^4) - x^
4/(2*a^2) - Log[1 - a^2*x^2]/a^6))/10)/a^2) + (-(((x^3*ArcCoth[a*x])/3 - (
a*(-(x^2/a^2) - Log[1 - a^2*x^2]/a^4))/6)/a^2) + (ArcCoth[a*x]^2/(2*a^3) -
 (x*ArcCoth[a*x] + Log[1 - a^2*x^2]/(2*a))/a^2)/a^2)/a^2))/3
```

3.12.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6543
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.12. $\int x^5 \coth^{-1}(ax)^2 \, dx$

3.12.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

method	result
parallelrisch	$-\frac{-30a^6x^6 \operatorname{arccoth}(ax)^2-12a^5x^5 \operatorname{arccoth}(ax)-16-3a^4x^4-20a^3x^3 \operatorname{arccoth}(ax)-16a^2x^2-60ax \operatorname{arccoth}(ax)+30 \operatorname{arcco}}{180a^6}$
parts	$\frac{x^6 \operatorname{arccoth}(ax)^2}{6}+\frac{\frac{a^5x^5 \operatorname{arccoth}(ax)}{5}+\frac{a^3x^3 \operatorname{arccoth}(ax)}{3}+ax \operatorname{arccoth}(ax)+\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{2}-\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{2}-\frac{\ln}{}}{}$
derivativedivides	$\frac{\frac{a^6x^6 \operatorname{arccoth}(ax)^2}{6}+\frac{a^5x^5 \operatorname{arccoth}(ax)}{15}+\frac{a^3x^3 \operatorname{arccoth}(ax)}{9}+\frac{ax \operatorname{arccoth}(ax)}{3}+\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{6}-\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{6}-\frac{\ln(ax}{}}{a^6}$
default	$\frac{\frac{a^6x^6 \operatorname{arccoth}(ax)^2}{6}+\frac{a^5x^5 \operatorname{arccoth}(ax)}{15}+\frac{a^3x^3 \operatorname{arccoth}(ax)}{9}+\frac{ax \operatorname{arccoth}(ax)}{3}+\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{6}-\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{6}-\frac{\ln(ax}{}}{a^6}$
risch	$\frac{(a^6x^6-1)\ln(ax+1)^2}{24a^6}-\frac{(15\ln(ax-1)x^6a^6-6a^5x^5-10a^3x^3-30ax-15\ln(ax-1))\ln(ax+1)}{180a^6}+\frac{x^6\ln(ax-1)^2}{24}-\frac{\ln(a}{}$

input
```
int(x^5*arccoth(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/180*(-30*a^6*x^6*arccoth(a*x)^2-12*a^5*x^5*arccoth(a*x)-16-3*a^4*x^4-20
*a^3*x^3*arccoth(a*x)-16*a^2*x^2-60*a*x*arccoth(a*x)+30*arccoth(a*x)^2-92*
ln(a*x-1)-92*arccoth(a*x))/a^6
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93

$$\int x^5 \coth^{-1}(ax)^2 \, dx = \frac{6\,a^4x^4+32\,a^2x^2+15\,(a^6x^6-1)\log\left(\frac{ax+1}{ax-1}\right)^2+4\,(3\,a^5x^5+5\,a^3x^3+15\,ax)\log\left(\frac{ax+1}{ax-1}\right)+92\,\log\left(a^2x^2-1\right)}{360\,a^6}$$

input
```
integrate(x^5*arccoth(a*x)^2,x, algorithm="fricas")
```

output
```
1/360*(6*a^4*x^4 + 32*a^2*x^2 + 15*(a^6*x^6 - 1)*log((a*x + 1)/(a*x - 1))^
2 + 4*(3*a^5*x^5 + 5*a^3*x^3 + 15*a*x)*log((a*x + 1)/(a*x - 1)) + 92*log(a
^2*x^2 - 1))/a^6
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09

$$\int x^5 \coth^{-1}(ax)^2\,dx$$
$$= \begin{cases} \frac{x^6 \operatorname{acoth}^2(ax)}{6} + \frac{x^5 \operatorname{acoth}(ax)}{15a} + \frac{x^4}{60a^2} + \frac{x^3 \operatorname{acoth}(ax)}{9a^3} + \frac{4x^2}{45a^4} + \frac{x \operatorname{acoth}(ax)}{3a^5} + \frac{23\log(ax+1)}{45a^6} - \frac{\operatorname{acoth}^2(ax)}{6a^6} - \frac{23\operatorname{acoth}(ax)}{45a^6} \\ -\frac{\pi^2 x^6}{24} \end{cases}$$

input
```
integrate(x**5*acoth(a*x)**2,x)
```

output
```
Piecewise((x**6*acoth(a*x)**2/6 + x**5*acoth(a*x)/(15*a) + x**4/(60*a**2)
+ x**3*acoth(a*x)/(9*a**3) + 4*x**2/(45*a**4) + x*acoth(a*x)/(3*a**5) + 23
*log(a*x + 1)/(45*a**6) - acoth(a*x)**2/(6*a**6) - 23*acoth(a*x)/(45*a**6)
, Ne(a, 0)), (-pi**2*x**6/24, True))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.29

$$\int x^5 \coth^{-1}(ax)^2\,dx = \frac{1}{6}x^6 \operatorname{arcoth}(ax)^2$$
$$+ \frac{1}{90}a\left(\frac{2\,(3\,a^4x^5 + 5\,a^2x^3 + 15\,x)}{a^6} - \frac{15\,\log(ax+1)}{a^7} + \frac{15\,\log(ax-1)}{a^7}\right)\operatorname{arcoth}(ax)$$
$$+ \frac{6\,a^4x^4 + 32\,a^2x^2 - 2\,(15\,\log(ax-1) - 46)\log(ax+1) + 15\,\log(ax+1)^2 + 15\,\log(ax-1)^2 + 92\,\log}{360\,a^6}$$

input
```
integrate(x^5*arccoth(a*x)^2,x, algorithm="maxima")
```

output
```
1/6*x^6*arccoth(a*x)^2 + 1/90*a*(2*(3*a^4*x^5 + 5*a^2*x^3 + 15*x)/a^6 - 15
*log(a*x + 1)/a^7 + 15*log(a*x - 1)/a^7)*arccoth(a*x) + 1/360*(6*a^4*x^4 +
 32*a^2*x^2 - 2*(15*log(a*x - 1) - 46)*log(a*x + 1) + 15*log(a*x + 1)^2 +
15*log(a*x - 1)^2 + 92*log(a*x - 1))/a^6
```

3.12.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(89) = 178$.

Time = 0.28 (sec) , antiderivative size = 534, normalized size of antiderivative = 5.09

$$\int x^5 \coth^{-1}(ax)^2\,dx$$
$$= \frac{1}{90}\left(\frac{15\left(\frac{3\,(ax+1)^5}{(ax-1)^5}+\frac{10\,(ax+1)^3}{(ax-1)^3}+\frac{3\,(ax+1)}{ax-1}\right)\log\left(\frac{ax+1}{ax-1}\right)^2}{\frac{(ax+1)^6a^7}{(ax-1)^6}-\frac{6\,(ax+1)^5a^7}{(ax-1)^5}+\frac{15\,(ax+1)^4a^7}{(ax-1)^4}-\frac{20\,(ax+1)^3a^7}{(ax-1)^3}+\frac{15\,(ax+1)^2a^7}{(ax-1)^2}-\frac{6\,(ax+1)a^7}{ax-1}+a^7}+\frac{2\left(\frac{45\,(ax+1)^4}{(ax-1)^4}-\frac{90\,(a}{(ax}\right.}{\frac{(ax+1)^5a^7}{(ax-1)^5}-\frac{5\,(ax+1)^4}{(ax-1)}}\right.$$

input
```
integrate(x^5*arccoth(a*x)^2,x, algorithm="giac")
```

output
```
1/90*(15*(3*(a*x + 1)^5/(a*x - 1)^5 + 10*(a*x + 1)^3/(a*x - 1)^3 + 3*(a*x
+ 1)/(a*x - 1))*log((a*x + 1)/(a*x - 1))^2/((a*x + 1)^6*a^7/(a*x - 1)^6 -
6*(a*x + 1)^5*a^7/(a*x - 1)^5 + 15*(a*x + 1)^4*a^7/(a*x - 1)^4 - 20*(a*x +
 1)^3*a^7/(a*x - 1)^3 + 15*(a*x + 1)^2*a^7/(a*x - 1)^2 - 6*(a*x + 1)*a^7/(
a*x - 1) + a^7) + 2*(45*(a*x + 1)^4/(a*x - 1)^4 - 90*(a*x + 1)^3/(a*x - 1)
^3 + 140*(a*x + 1)^2/(a*x - 1)^2 - 70*(a*x + 1)/(a*x - 1) + 23)*log((a*x +
 1)/(a*x - 1))/((a*x + 1)^5*a^7/(a*x - 1)^5 - 5*(a*x + 1)^4*a^7/(a*x - 1)^
4 + 10*(a*x + 1)^3*a^7/(a*x - 1)^3 - 10*(a*x + 1)^2*a^7/(a*x - 1)^2 + 5*(a
*x + 1)*a^7/(a*x - 1) - a^7) + 4*(11*(a*x + 1)^3/(a*x - 1)^3 - 16*(a*x + 1
)^2/(a*x - 1)^2 + 11*(a*x + 1)/(a*x - 1))/((a*x + 1)^4*a^7/(a*x - 1)^4 - 4
*(a*x + 1)^3*a^7/(a*x - 1)^3 + 6*(a*x + 1)^2*a^7/(a*x - 1)^2 - 4*(a*x + 1)
*a^7/(a*x - 1) + a^7) - 46*log((a*x + 1)/(a*x - 1) - 1)/a^7 + 46*log((a*x
+ 1)/(a*x - 1))/a^7)*a
```

3.12.9 Mupad [B] (verification not implemented)

Time = 4.63 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81

$$\int x^5 \coth^{-1}(ax)^2\,dx$$
$$= \frac{x^6\,\mathrm{acoth}(a\,x)^2}{6} + \frac{\frac{23\,\ln(a^2\,x^2-1)}{90}+\frac{4\,a^2\,x^2}{45}+\frac{a^4\,x^4}{60}-\frac{\mathrm{acoth}(a\,x)^2}{6}+\frac{a^3\,x^3\,\mathrm{acoth}(a\,x)}{9}+\frac{a^5\,x^5\,\mathrm{acoth}(a\,x)}{15}+\frac{a\,x\,\mathrm{acoth}(a\,x)}{3}}{a^6}$$

input
```
int(x^5*acoth(a*x)^2,x)
```

output
```
(x^6*acoth(a*x)^2)/6 + ((23*log(a^2*x^2 - 1))/90 + (4*a^2*x^2)/45 + (a^4*x
^4)/60 - acoth(a*x)^2/6 + (a^3*x^3*acoth(a*x))/9 + (a^5*x^5*acoth(a*x))/15
 + (a*x*acoth(a*x))/3)/a^6
```

3.12. $\int x^5 \coth^{-1}(ax)^2 \, dx$

3.13 $\int x^4 \coth^{-1}(ax)^2 \, dx$

3.13.1 Optimal result

Integrand size = 10, antiderivative size = 127

$$\int x^4 \coth^{-1}(ax)^2 \, dx = \frac{3x}{10a^4} + \frac{x^3}{30a^2} + \frac{x^2 \coth^{-1}(ax)}{5a^3} + \frac{x^4 \coth^{-1}(ax)}{10a} + \frac{\coth^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{3\text{arctanh}(ax)}{10a^5} - \frac{2\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{5a^5} - \frac{\text{PolyLog}\left(2, 1-\frac{2}{1-ax}\right)}{5a^5}$$

output
```
3/10*x/a^4+1/30*x^3/a^2+1/5*x^2*arccoth(a*x)/a^3+1/10*x^4*arccoth(a*x)/a+1
/5*arccoth(a*x)^2/a^5+1/5*x^5*arccoth(a*x)^2-3/10*arctanh(a*x)/a^5-2/5*arc
coth(a*x)*ln(2/(-a*x+1))/a^5-1/5*polylog(2,1-2/(-a*x+1))/a^5
```

3.13.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

$$\int x^4 \coth^{-1}(ax)^2 \, dx = \frac{ax(9+a^2x^2) + 6(-1+a^5x^5)\coth^{-1}(ax)^2 + 3\coth^{-1}(ax)\left(-3+2a^2x^2+a^4x^4-4\log\left(1-e^{-2\coth^{-1}(ax)}\right)\right.}{30a^5}$$

input
```
Integrate[x^4*ArcCoth[a*x]^2,x]
```

output
```
(a*x*(9 + a^2*x^2) + 6*(-1 + a^5*x^5)*ArcCoth[a*x]^2 + 3*ArcCoth[a*x]*(-3
+ 2*a^2*x^2 + a^4*x^4 - 4*Log[1 - E^(-2*ArcCoth[a*x])]) + 6*PolyLog[2, E^(
-2*ArcCoth[a*x])])/(30*a^5)
```

3.13.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.35, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {6453, 6543, 6453, 254, 2009, 6543, 6453, 262, 219, 6547, 6471, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \coth^{-1}(ax)^2 \, dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{2}{5}a \int \frac{x^5 \coth^{-1}(ax)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{6543}$$

$$\frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{2}{5}a \left(\frac{\int \frac{x^3 \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\int x^3 \coth^{-1}(ax)dx}{a^2}\right)$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{2}{5}a \left(\frac{\int \frac{x^3 \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{4}x^4 \coth^{-1}(ax) - \frac{1}{4}a \int \frac{x^4}{1-a^2x^2}dx}{a^2}\right)$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{2}{5}a \left(\frac{\int \frac{x^3 \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{4}x^4 \coth^{-1}(ax) - \frac{1}{4}a \int \left(-\frac{x^2}{a^2} + \frac{1}{a^4(1-a^2x^2)} - \frac{1}{a^4}\right)dx}{a^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{2}{5}a \left(\frac{\int \frac{x^3 \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{4}x^4 \coth^{-1}(ax) - \frac{1}{4}a \left(\frac{\text{arctanh}(ax)}{a^5} - \frac{x}{a^4} - \frac{x^3}{3a^2}\right)}{a^2}\right)$$

$$\Big\downarrow \text{6543}$$

$$\frac{1}{5}x^5\coth^{-1}(ax)^2-\frac{2}{5}a\left(\frac{\frac{\int\frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x\coth^{-1}(ax)dx}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right)}{a^2}\right)$$

$$\Big\downarrow \; 6453$$

$$\frac{1}{5}x^5\coth^{-1}(ax)^2-\frac{2}{5}a\left(\frac{\frac{\int\frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\int\frac{x^2}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right)}{a^2}\right)$$

$$\Big\downarrow \; 262$$

$$\frac{1}{5}x^5\coth^{-1}(ax)^2-\frac{2}{5}a\left(\frac{\frac{\int\frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\left(\frac{\int\frac{1}{1-a^2x^2}dx}{a^2}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right)}{a^2}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{5}x^5\coth^{-1}(ax)^2-\frac{2}{5}a\left(\frac{\frac{\int\frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x^3}{3a^2}\right)}{a^2}\right)$$

$$\Big\downarrow \; 6547$$

$$\frac{1}{5}x^5\coth^{-1}(ax)^2-\frac{2}{5}a\left(\frac{\frac{\frac{\int\frac{\coth^{-1}(ax)}{1-ax}dx}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)-\frac{1}{4}a\left(\frac{\text{arctanh}(ax)}{a^5}-\frac{x}{a^4}-\frac{x}{3a}\cdots\right.}{a^2}\right.$$

$$\Big\downarrow \; 6471$$

$$\frac{1}{5}x^5\coth^{-1}(ax)^2-\frac{2}{5}a\left(\frac{\frac{\frac{\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}-\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)-\frac{1}{4}a\left(\text{arc}\cdots\right.}{a^2}\right.$$

3.13. $\int x^4\coth^{-1}(ax)^2\,dx$

$$\Big\downarrow \text{2849}$$

$$\frac{1}{5}x^5\coth^{-1}(ax)^2-\frac{2}{5}a\left(\frac{\frac{\frac{\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)-\frac{1}{4}a(\right.$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{5}x^5\coth^{-1}(ax)^2-\frac{2}{5}a\left(\frac{\frac{\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}+\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)-\frac{1}{4}a(\right.$$

input
```
Int[x^4*ArcCoth[a*x]^2,x]
```

output
```
(x^5*ArcCoth[a*x]^2)/5 - (2*a*(-(((x^4*ArcCoth[a*x])/4 - (a*(-(x/a^4) - x^
3/(3*a^2) + ArcTanh[a*x]/a^5))/4)/a^2) + (-(((x^2*ArcCoth[a*x])/2 - (a*(-(
x/a^2) + ArcTanh[a*x]/a^3))/2)/a^2) + (-1/2*ArcCoth[a*x]^2/a^2 + ((ArcCoth
[a*x]*Log[2/(1 - a*x)])/a + PolyLog[2, 1 - 2/(1 - a*x)]/(2*a))/a)/a^2)/a^2
))/5
```

3.13.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6471
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6543
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.13. $\int x^4 \coth^{-1}(ax)^2 \, dx$

rule 6547
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.13.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.29

method	result
parts	$\frac{x^5 \operatorname{arccoth}(ax)^2}{5} + \frac{\frac{a^4x^4 \operatorname{arccoth}(ax)}{10} + \frac{\operatorname{arccoth}(ax)a^2x^2}{5} + \frac{\operatorname{arccoth}(ax)\ln(ax-1)}{5} + \frac{\operatorname{arccoth}(ax)\ln(ax+1)}{5} + \frac{a^3x^3}{30} + \frac{3ax}{10} + \frac{3\ln(ax\cdots}{20}\cdots}{\cdots}$
derivativedivides	$\frac{\frac{a^5x^5 \operatorname{arccoth}(ax)^2}{5} + \frac{a^4x^4 \operatorname{arccoth}(ax)}{10} + \frac{\operatorname{arccoth}(ax)a^2x^2}{5} + \frac{\operatorname{arccoth}(ax)\ln(ax-1)}{5} + \frac{\operatorname{arccoth}(ax)\ln(ax+1)}{5} + \frac{a^3x^3}{30} + \frac{3ax}{10} + \frac{3\ln(ax-\cdots}{20}\cdots}{a^5}$
default	$\frac{\frac{a^5x^5 \operatorname{arccoth}(ax)^2}{5} + \frac{a^4x^4 \operatorname{arccoth}(ax)}{10} + \frac{\operatorname{arccoth}(ax)a^2x^2}{5} + \frac{\operatorname{arccoth}(ax)\ln(ax-1)}{5} + \frac{\operatorname{arccoth}(ax)\ln(ax+1)}{5} + \frac{a^3x^3}{30} + \frac{3ax}{10} + \frac{3\ln(ax-\cdots}{20}\cdots}{a^5}$
risch	$\frac{413}{2250a^5} + \frac{x^3}{30a^2} + \frac{47\ln(ax+1)}{600a^5} + \frac{\ln(ax+1)x^4}{40a} - \frac{\ln(ax+1)x^3}{30a^2} + \frac{\ln(ax+1)x^2}{20a^3} - \frac{\ln(ax+1)x}{10a^4} - \frac{\ln(ax-1)x^4}{40a} - \cdots$

input
```
int(x^4*arccoth(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*arccoth(a*x)^2+2/5/a^5*(1/4*a^4*x^4*arccoth(a*x)+1/2*arccoth(a*x)*
a^2*x^2+1/2*arccoth(a*x)*ln(a*x-1)+1/2*arccoth(a*x)*ln(a*x+1)+1/12*a^3*x^3
+3/4*a*x+3/8*ln(a*x-1)-3/8*ln(a*x+1)-1/2*dilog(1/2*a*x+1/2)-1/4*ln(a*x-1)*
ln(1/2*a*x+1/2)+1/8*ln(a*x-1)^2+1/4*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*
x+1/2)-1/8*ln(a*x+1)^2)
```

3.13.5 Fricas [F]

$$\int x^4 \coth^{-1}(ax)^2 \, dx = \int x^4 \operatorname{arcoth}(ax)^2 \, dx$$

input
```
integrate(x^4*arccoth(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^4*arccoth(a*x)^2, x)
```

3.13.6 Sympy [F]

$$\int x^4 \coth^{-1}(ax)^2 \, dx = \int x^4 \operatorname{acoth}^2(ax) \, dx$$

input
```
integrate(x**4*acoth(a*x)**2,x)
```

output
```
Integral(x**4*acoth(a*x)**2, x)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.22

$$\int x^4 \coth^{-1}(ax)^2 \, dx = \frac{1}{5} x^5 \operatorname{arcoth}(ax)^2 + \frac{1}{60} a^2 \left(\frac{2 a^3 x^3 + 18 ax - 3 \log(ax+1)^2 + 6 \log(ax+1) \log(ax-1) + 3 \log(ax-1)^2 + 9 \log(ax-1)}{a^7} \right. + \frac{1}{10} a \left(\frac{a^2 x^4 + 2 x^2}{a^4} + \frac{2 \log(a^2 x^2 - 1)}{a^6} \right) \operatorname{arcoth}(ax)$$

input
```
integrate(x^4*arccoth(a*x)^2,x, algorithm="maxima")
```

output
```
1/5*x^5*arccoth(a*x)^2 + 1/60*a^2*((2*a^3*x^3 + 18*a*x - 3*log(a*x + 1)^2
+ 6*log(a*x + 1)*log(a*x - 1) + 3*log(a*x - 1)^2 + 9*log(a*x - 1))/a^7 - 1
2*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^7 - 9*log(a*
x + 1)/a^7) + 1/10*a*((a^2*x^4 + 2*x^2)/a^4 + 2*log(a^2*x^2 - 1)/a^6)*arcc
oth(a*x)
```

3.13.8 Giac [F]

$$\int x^4 \coth^{-1}(ax)^2 \, dx = \int x^4 \operatorname{arcoth}(ax)^2 \, dx$$

input
```
integrate(x^4*arccoth(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^4*arccoth(a*x)^2, x)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \coth^{-1}(ax)^2 \, dx = \int x^4 \operatorname{acoth}(a\,x)^2 \, dx$$

input
```
int(x^4*acoth(a*x)^2,x)
```

output
```
int(x^4*acoth(a*x)^2, x)
```

3.14 $\int x^3 \coth^{-1}(ax)^2 \, dx$

3.14.1 Optimal result

Integrand size = 10, antiderivative size = 81

$$\int x^3 \coth^{-1}(ax)^2 \, dx = \frac{x^2}{12a^2} + \frac{x \coth^{-1}(ax)}{2a^3} + \frac{x^3 \coth^{-1}(ax)}{6a} - \frac{\coth^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^2 + \frac{\log(1-a^2x^2)}{3a^4}$$

output
```
1/12*x^2/a^2+1/2*x*arccoth(a*x)/a^3+1/6*x^3*arccoth(a*x)/a-1/4*arccoth(a*x
)^2/a^4+1/4*x^4*arccoth(a*x)^2+1/3*ln(-a^2*x^2+1)/a^4
```

3.14.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

$$\int x^3 \coth^{-1}(ax)^2 \, dx = \frac{a^2x^2 + 2ax(3+a^2x^2)\coth^{-1}(ax) + 3(-1+a^4x^4)\coth^{-1}(ax)^2 + 4\log(1-a^2x^2)}{12a^4}$$

input
```
Integrate[x^3*ArcCoth[a*x]^2,x]
```

output
```
(a^2*x^2 + 2*a*x*(3 + a^2*x^2)*ArcCoth[a*x] + 3*(-1 + a^4*x^4)*ArcCoth[a*x
]^2 + 4*Log[1 - a^2*x^2])/(12*a^4)
```

3.14.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.42, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6453, 6543, 6453, 243, 49, 2009, 6543, 6437, 240, 6511}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(ax)^2 \, dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{4}x^4 \coth^{-1}(ax)^2 - \frac{1}{2}a \int \frac{x^4 \coth^{-1}(ax)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{6543}$$

$$\frac{1}{4}x^4 \coth^{-1}(ax)^2 - \frac{1}{2}a \left(\frac{\int \frac{x^2 \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\int x^2 \coth^{-1}(ax)dx}{a^2}\right)$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{4}x^4 \coth^{-1}(ax)^2 - \frac{1}{2}a \left(\frac{\int \frac{x^2 \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{3}x^3 \coth^{-1}(ax) - \frac{1}{3}a \int \frac{x^3}{1-a^2x^2}dx}{a^2}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{4}x^4 \coth^{-1}(ax)^2 - \frac{1}{2}a \left(\frac{\int \frac{x^2 \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{3}x^3 \coth^{-1}(ax) - \frac{1}{6}a \int \frac{x^2}{1-a^2x^2}dx^2}{a^2}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}x^4 \coth^{-1}(ax)^2 - \frac{1}{2}a \left(\frac{\int \frac{x^2 \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{3}x^3 \coth^{-1}(ax) - \frac{1}{6}a \int \left(-\frac{1}{a^2} - \frac{1}{a^2(a^2x^2-1)}\right)dx^2}{a^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4 \coth^{-1}(ax)^2 - \frac{1}{2}a \left(\frac{\int \frac{x^2 \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{3}x^3 \coth^{-1}(ax) - \frac{1}{6}a \left(-\frac{x^2}{a^2} - \frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)$$

$$\Big\downarrow \text{6543}$$

$$\frac{1}{4}x^4\coth^{-1}(ax)^2-\frac{1}{2}a\left(\frac{\frac{\int\frac{\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int\coth^{-1}(ax)dx}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)$$

$$\Big\downarrow \text{6437}$$

$$\frac{1}{4}x^4\coth^{-1}(ax)^2-\frac{1}{2}a\left(\frac{\frac{\int\frac{\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{x\coth^{-1}(ax)-a\int\frac{x}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{4}x^4\coth^{-1}(ax)^2-\frac{1}{2}a\left(\frac{\frac{\int\frac{\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{\log\left(1-a^2x^2\right)}{2a}+x\coth^{-1}(ax)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)$$

$$\Big\downarrow \text{6511}$$

$$\frac{1}{4}x^4\coth^{-1}(ax)^2-\frac{1}{2}a\left(\frac{\frac{\coth^{-1}(ax)^2}{2a^3}-\frac{\frac{\log\left(1-a^2x^2\right)}{2a}+x\coth^{-1}(ax)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{6}a\left(-\frac{x^2}{a^2}-\frac{\log(1-a^2x^2)}{a^4}\right)}{a^2}\right)$$

input
```
Int[x^3*ArcCoth[a*x]^2,x]
```

output
```
(x^4*ArcCoth[a*x]^2)/4 - (a*(-(((x^3*ArcCoth[a*x])/3 - (a*(-(x^2/a^2) - Lo
g[1 - a^2*x^2]/a^4))/6)/a^2) + (ArcCoth[a*x]^2/(2*a^3) - (x*ArcCoth[a*x] +
 Log[1 - a^2*x^2]/(2*a))/a^2)/a^2))/2
```

3.14. $\int x^3\coth^{-1}(ax)^2\,dx$

3.14.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6543
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.14.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

method	result
parallelrisch	$-\frac{-3a^4x^4\operatorname{arccoth}(ax)^2-2a^3x^3\operatorname{arccoth}(ax)-1-a^2x^2-6ax\operatorname{arccoth}(ax)+3\operatorname{arccoth}(ax)^2-8\ln(ax-1)-8\operatorname{arccoth}(ax)}{12a^4}$
risch	$\frac{(a^4x^4-1)\ln(ax+1)^2}{16a^4}-\frac{(3\ln(ax-1)x^4a^4-2a^3x^3-6ax-3\ln(ax-1))\ln(ax+1)}{24a^4}+\frac{x^4\ln(ax-1)^2}{16}-\frac{\ln(ax-1)x^3}{12a}+$
parts	$\frac{x^4\operatorname{arccoth}(ax)^2}{4}+\frac{\frac{a^3x^3\operatorname{arccoth}(ax)}{3}+ax\operatorname{arccoth}(ax)+\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{2}-\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{2}-\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}}{2a^4}$
derivativedivides	$\frac{\frac{a^4x^4\operatorname{arccoth}(ax)^2}{4}+\frac{a^3x^3\operatorname{arccoth}(ax)}{6}+\frac{ax\operatorname{arccoth}(ax)}{2}+\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{4}-\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{4}-\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{8}+\ln}{a^4}$
default	$\frac{\frac{a^4x^4\operatorname{arccoth}(ax)^2}{4}+\frac{a^3x^3\operatorname{arccoth}(ax)}{6}+\frac{ax\operatorname{arccoth}(ax)}{2}+\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{4}-\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{4}-\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{8}+\ln}{a^4}$

input
```
int(x^3*arccoth(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-3*a^4*x^4*arccoth(a*x)^2-2*a^3*x^3*arccoth(a*x)-1-a^2*x^2-6*a*x*ar
ccoth(a*x)+3*arccoth(a*x)^2-8*ln(a*x-1)-8*arccoth(a*x))/a^4
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int x^3\coth^{-1}(ax)^2\,dx = \frac{4\,a^2x^2+3\,(a^4x^4-1)\log\left(\frac{ax+1}{ax-1}\right)^2+4\,(a^3x^3+3\,ax)\log\left(\frac{ax+1}{ax-1}\right)+16\,\log\left(a^2x^2-1\right)}{48\,a^4}$$

input
```
integrate(x^3*arccoth(a*x)^2,x, algorithm="fricas")
```

output
```
1/48*(4*a^2*x^2 + 3*(a^4*x^4 - 1)*log((a*x + 1)/(a*x - 1))^2 + 4*(a^3*x^3
+ 3*a*x)*log((a*x + 1)/(a*x - 1)) + 16*log(a^2*x^2 - 1))/a^4
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int x^3 \coth^{-1}(ax)^2 \, dx$$
$$= \begin{cases} \frac{x^4 \operatorname{acoth}^2(ax)}{4} + \frac{x^3 \operatorname{acoth}(ax)}{6a} + \frac{x^2}{12a^2} + \frac{x \operatorname{acoth}(ax)}{2a^3} + \frac{2\log(ax+1)}{3a^4} - \frac{\operatorname{acoth}^2(ax)}{4a^4} - \frac{2\operatorname{acoth}(ax)}{3a^4} & \text{for } a \neq 0 \\ -\frac{\pi^2 x^4}{16} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acoth(a*x)**2,x)
```

output
```
Piecewise((x**4*acoth(a*x)**2/4 + x**3*acoth(a*x)/(6*a) + x**2/(12*a**2) +
 x*acoth(a*x)/(2*a**3) + 2*log(a*x + 1)/(3*a**4) - acoth(a*x)**2/(4*a**4)
- 2*acoth(a*x)/(3*a**4), Ne(a, 0)), (-pi**2*x**4/16, True))
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.46

$$\int x^3 \coth^{-1}(ax)^2 \, dx$$
$$= \frac{1}{4}x^4 \operatorname{arcoth}(ax)^2 + \frac{1}{12}a\left(\frac{2(a^2x^3 + 3x)}{a^4} - \frac{3\log(ax+1)}{a^5} + \frac{3\log(ax-1)}{a^5}\right)\operatorname{arcoth}(ax)$$
$$+ \frac{4a^2x^2 - 2(3\log(ax-1) - 8)\log(ax+1) + 3\log(ax+1)^2 + 3\log(ax-1)^2 + 16\log(ax-1)}{48a^4}$$

input
```
integrate(x^3*arccoth(a*x)^2,x, algorithm="maxima")
```

output
```
1/4*x^4*arccoth(a*x)^2 + 1/12*a*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a^
5 + 3*log(a*x - 1)/a^5)*arccoth(a*x) + 1/48*(4*a^2*x^2 - 2*(3*log(a*x - 1)
 - 8)*log(a*x + 1) + 3*log(a*x + 1)^2 + 3*log(a*x - 1)^2 + 16*log(a*x - 1)
)/a^4
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(69) = 138$.

Time = 0.27 (sec) , antiderivative size = 335, normalized size of antiderivative = 4.14

$$\int x^3 \coth^{-1}(ax)^2\,dx = \frac{1}{6}\left(\frac{3\left(\frac{(ax+1)^3}{(ax-1)^3}+\frac{ax+1}{ax-1}\right)\log\left(\frac{ax+1}{ax-1}\right)^2}{\frac{(ax+1)^4a^5}{(ax-1)^4}-\frac{4(ax+1)^3a^5}{(ax-1)^3}+\frac{6(ax+1)^2a^5}{(ax-1)^2}-\frac{4(ax+1)a^5}{ax-1}+a^5}+\frac{2\left(\frac{3(ax+1)^2}{(ax-1)^2}-\frac{3(ax+1)}{ax-1}+2\right)\log\left(\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)^3a^5}{(ax-1)^3}-\frac{3(ax+1)^2a^5}{(ax-1)^2}+\frac{3(ax+1)a^5}{ax-1}-a^5}+\frac{}{\left(\frac{(ax+1}{(ax-}\right.}\right.$$

input
```
integrate(x^3*arccoth(a*x)^2,x, algorithm="giac")
```

output
```
1/6*(3*((a*x + 1)^3/(a*x - 1)^3 + (a*x + 1)/(a*x - 1))*log((a*x + 1)/(a*x
- 1))^2/((a*x + 1)^4*a^5/(a*x - 1)^4 - 4*(a*x + 1)^3*a^5/(a*x - 1)^3 + 6*(
a*x + 1)^2*a^5/(a*x - 1)^2 - 4*(a*x + 1)*a^5/(a*x - 1) + a^5) + 2*(3*(a*x
+ 1)^2/(a*x - 1)^2 - 3*(a*x + 1)/(a*x - 1) + 2)*log((a*x + 1)/(a*x - 1))/(
(a*x + 1)^3*a^5/(a*x - 1)^3 - 3*(a*x + 1)^2*a^5/(a*x - 1)^2 + 3*(a*x + 1)*
a^5/(a*x - 1) - a^5) + 2*(a*x + 1)/(((a*x + 1)^2*a^5/(a*x - 1)^2 - 2*(a*x
+ 1)*a^5/(a*x - 1) + a^5)*(a*x - 1)) - 4*log((a*x + 1)/(a*x - 1) - 1)/a^5
+ 4*log((a*x + 1)/(a*x - 1))/a^5)*a
```

3.14.9 Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

$$\int x^3 \coth^{-1}(ax)^2\,dx = \frac{x^4\,\mathrm{acoth}(a\,x)^2}{4} + \frac{\frac{\ln(a^2\,x^2-1)}{3}+\frac{a^2\,x^2}{12}-\frac{\mathrm{acoth}(a\,x)^2}{4}+\frac{a^3\,x^3\,\mathrm{acoth}(a\,x)}{6}+\frac{a\,x\,\mathrm{acoth}(a\,x)}{2}}{a^4}$$

input
```
int(x^3*acoth(a*x)^2,x)
```

output
```
(x^4*acoth(a*x)^2)/4 + (log(a^2*x^2 - 1)/3 + (a^2*x^2)/12 - acoth(a*x)^2/4
 + (a^3*x^3*acoth(a*x))/6 + (a*x*acoth(a*x))/2)/a^4
```

3.15 $\int x^2 \coth^{-1}(ax)^2 \, dx$

3.15.1 Optimal result

Integrand size = 10, antiderivative size = 103

$$\int x^2 \coth^{-1}(ax)^2 \, dx = \frac{x}{3a^2} + \frac{x^2 \coth^{-1}(ax)}{3a} + \frac{\coth^{-1}(ax)^2}{3a^3} + \frac{1}{3} x^3 \coth^{-1}(ax)^2 - \frac{\operatorname{arctanh}(ax)}{3a^3} - \frac{2 \coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{3a^3} - \frac{\operatorname{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{3a^3}$$

output
```
1/3*x/a^2+1/3*x^2*arccoth(a*x)/a+1/3*arccoth(a*x)^2/a^3+1/3*x^3*arccoth(a*
x)^2-1/3*arctanh(a*x)/a^3-2/3*arccoth(a*x)*ln(2/(-a*x+1))/a^3-1/3*polylog(
2,1-2/(-a*x+1))/a^3
```

3.15.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.64

$$\int x^2 \coth^{-1}(ax)^2 \, dx = \frac{ax + (-1 + a^3x^3) \coth^{-1}(ax)^2 + \coth^{-1}(ax) \left(-1 + a^2x^2 - 2 \log\left(1 - e^{-2\coth^{-1}(ax)}\right)\right) + \operatorname{PolyLog}\left(2, e^{-2\coth^{-1}(ax)}\right)}{3a^3}$$

input
```
Integrate[x^2*ArcCoth[a*x]^2,x]
```

output
```
(a*x + (-1 + a^3*x^3)*ArcCoth[a*x]^2 + ArcCoth[a*x]*(-1 + a^2*x^2 - 2*Log[
1 - E^(-2*ArcCoth[a*x])]) + PolyLog[2, E^(-2*ArcCoth[a*x])])/(3*a^3)
```

3.15.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6453, 6543, 6453, 262, 219, 6547, 6471, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(ax)^2 \, dx$$

$$\Big\downarrow \text{ 6453}$$

$$\frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{2}{3}a \int \frac{x^3 \coth^{-1}(ax)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{ 6543}$$

$$\frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{2}{3}a \left(\frac{\int \frac{x \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\int x \coth^{-1}(ax)dx}{a^2}\right)$$

$$\Big\downarrow \text{ 6453}$$

$$\frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{2}{3}a \left(\frac{\int \frac{x \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{2}x^2 \coth^{-1}(ax) - \frac{1}{2}a \int \frac{x^2}{1-a^2x^2}dx}{a^2}\right)$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{2}{3}a \left(\frac{\int \frac{x \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{2}x^2 \coth^{-1}(ax) - \frac{1}{2}a \left(\frac{\int \frac{1}{1-a^2x^2}dx}{a^2} - \frac{x}{a^2}\right)}{a^2}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{2}{3}a \left(\frac{\int \frac{x \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{2}x^2 \coth^{-1}(ax) - \frac{1}{2}a \left(\frac{\text{arctanh}(ax)}{a^3} - \frac{x}{a^2}\right)}{a^2}\right)$$

$$\Big\downarrow \text{ 6547}$$

$$\frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{2}{3}a \left(\frac{\frac{\int \frac{\coth^{-1}(ax)}{1-ax}dx}{a} - \frac{\coth^{-1}(ax)^2}{2a^2}}{a^2} - \frac{\frac{1}{2}x^2 \coth^{-1}(ax) - \frac{1}{2}a \left(\frac{\text{arctanh}(ax)}{a^3} - \frac{x}{a^2}\right)}{a^2}\right)$$

$$\Big\downarrow \text{ 6471}$$

$$\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\left(\frac{\frac{\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}-\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\left(\frac{\operatorname{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}\right)$$

$$\Big\downarrow \text{2849}$$

$$\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\left(\frac{\frac{\frac{\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\left(\frac{\operatorname{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\left(\frac{\frac{\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}+\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\left(\frac{\operatorname{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}\right)$$

input
```
Int[x^2*ArcCoth[a*x]^2,x]
```

output
```
(x^3*ArcCoth[a*x]^2)/3 - (2*a*(-(((x^2*ArcCoth[a*x])/2 - (a*(-(x/a^2) + Ar
cTanh[a*x]/a^3))/2)/a^2) + (-1/2*ArcCoth[a*x]^2/a^2 + ((ArcCoth[a*x]*Log[2
/(1 - a*x)])/a + PolyLog[2, 1 - 2/(1 - a*x)]/(2*a))/a)/a^2))/3
```

3.15.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6471
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6543
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6547
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.15. $\int x^2 \coth^{-1}(ax)^2 \, dx$

3.15.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.40

method	result
parts	$\frac{x^3 \operatorname{arccoth}(ax)^2}{3} + \frac{\frac{\operatorname{arccoth}(ax) a^2 x^2}{3} + \frac{\operatorname{arccoth}(ax) \ln(ax-1)}{3} + \frac{\operatorname{arccoth}(ax) \ln(ax+1)}{3} + \frac{ax}{3} + \frac{\ln(ax-1)}{6} - \frac{\ln(ax+1)}{6} - \frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{3}}{a^3}$
derivativedivides	$\frac{\frac{a^3 x^3 \operatorname{arccoth}(ax)^2}{3} + \frac{\operatorname{arccoth}(ax) a^2 x^2}{3} + \frac{\operatorname{arccoth}(ax) \ln(ax-1)}{3} + \frac{\operatorname{arccoth}(ax) \ln(ax+1)}{3} + \frac{ax}{3} + \frac{\ln(ax-1)}{6} - \frac{\ln(ax+1)}{6} - \frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{3}}{a^3}$
default	$\frac{\frac{a^3 x^3 \operatorname{arccoth}(ax)^2}{3} + \frac{\operatorname{arccoth}(ax) a^2 x^2}{3} + \frac{\operatorname{arccoth}(ax) \ln(ax-1)}{3} + \frac{\operatorname{arccoth}(ax) \ln(ax+1)}{3} + \frac{ax}{3} + \frac{\ln(ax-1)}{6} - \frac{\ln(ax+1)}{6} - \frac{\operatorname{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{3}}{a^3}$
risch	$\frac{\ln(ax-1)^2 x^3}{12} - \frac{\ln(ax-1)^2}{12a^3} - \frac{\ln(ax-1) x^3}{18} - \frac{\ln(ax-1) x^2}{12a} - \frac{\ln(ax-1) x}{6a^2} + \frac{11 \ln(ax-1)}{36a^3} + \frac{x}{3a^2} + \frac{\ln(ax+1)^2 x^3}{12}$

input
```
int(x^2*arccoth(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccoth(a*x)^2+2/3/a^3*(1/2*arccoth(a*x)*a^2*x^2+1/2*arccoth(a*x)*
ln(a*x-1)+1/2*arccoth(a*x)*ln(a*x+1)+1/2*a*x+1/4*ln(a*x-1)-1/4*ln(a*x+1)-1
/2*dilog(1/2*a*x+1/2)-1/4*ln(a*x-1)*ln(1/2*a*x+1/2)+1/8*ln(a*x-1)^2-1/8*ln
(a*x+1)^2+1/4*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2))
```

3.15.5 Fricas [F]

$$\int x^2 \coth^{-1}(ax)^2 \, dx = \int x^2 \operatorname{arcoth}(ax)^2 \, dx$$

input
```
integrate(x^2*arccoth(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^2*arccoth(a*x)^2, x)
```

3.15.6 Sympy [F]

$$\int x^2 \coth^{-1}(ax)^2 \, dx = \int x^2 \operatorname{acoth}^2{(ax)}\, dx$$

input
```
integrate(x**2*acoth(a*x)**2,x)
```

output
```
Integral(x**2*acoth(a*x)**2, x)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.30

$$\int x^2 \coth^{-1}(ax)^2 \, dx = \frac{1}{3}\,x^3 \operatorname{arcoth}{(ax)}^2 + \frac{1}{12}\,a^2\left(\frac{4\,ax - \log{(ax+1)}^2 + 2\,\log{(ax+1)}\log{(ax-1)} + \log{(ax-1)}^2 + 2\,\log{(ax-1)}}{a^5} - \frac{4\,(\log{(ax}\right. + \frac{1}{3}\,a\left(\frac{x^2}{a^2} + \frac{\log{(a^2x^2-1)}}{a^4}\right)\operatorname{arcoth}{(ax)}$$

input
```
integrate(x^2*arccoth(a*x)^2,x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(a*x)^2 + 1/12*a^2*((4*a*x - log(a*x + 1)^2 + 2*log(a*x + 1
)*log(a*x - 1) + log(a*x - 1)^2 + 2*log(a*x - 1))/a^5 - 4*(log(a*x - 1)*lo
g(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^5 - 2*log(a*x + 1)/a^5) + 1/3*
a*(x^2/a^2 + log(a^2*x^2 - 1)/a^4)*arccoth(a*x)
```

3.15.8 Giac [F]

$$\int x^2 \coth^{-1}(ax)^2 \, dx = \int x^2 \operatorname{arcoth}{(ax)}^2 \, dx$$

input
```
integrate(x^2*arccoth(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(a*x)^2, x)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(ax)^2 \, dx = \int x^2 \operatorname{acoth}(a\,x)^2 \, dx$$

input
```
int(x^2*acoth(a*x)^2,x)
```

output
```
int(x^2*acoth(a*x)^2, x)
```

3.16 $\int x \coth^{-1}(ax)^2 \, dx$

3.16.1 Optimal result

Integrand size = 8, antiderivative size = 54

$$\int x \coth^{-1}(ax)^2 \, dx = \frac{x \coth^{-1}(ax)}{a} - \frac{\coth^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^2 + \frac{\log(1-a^2x^2)}{2a^2}$$

output
```
x*arccoth(a*x)/a-1/2*arccoth(a*x)^2/a^2+1/2*x^2*arccoth(a*x)^2+1/2*ln(-a^2
*x^2+1)/a^2
```

3.16.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int x \coth^{-1}(ax)^2 \, dx = \frac{2ax \coth^{-1}(ax) + (-1+a^2x^2) \coth^{-1}(ax)^2 + \log(1-a^2x^2)}{2a^2}$$

input
```
Integrate[x*ArcCoth[a*x]^2,x]
```

output
```
(2*a*x*ArcCoth[a*x] + (-1 + a^2*x^2)*ArcCoth[a*x]^2 + Log[1 - a^2*x^2])/(2
*a^2)
```

3.16.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6453, 6543, 6437, 240, 6511}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(ax)^2 \, dx$$

$$\Big\downarrow \text{ 6453}$$

$$\frac{1}{2}x^2 \coth^{-1}(ax)^2 - a \int \frac{x^2 \coth^{-1}(ax)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{ 6543}$$

$$\frac{1}{2}x^2 \coth^{-1}(ax)^2 - a \left(\frac{\int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\int \coth^{-1}(ax)dx}{a^2}\right)$$

$$\Big\downarrow \text{ 6437}$$

$$\frac{1}{2}x^2 \coth^{-1}(ax)^2 - a \left(\frac{\int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{x \coth^{-1}(ax) - a \int \frac{x}{1-a^2x^2}dx}{a^2}\right)$$

$$\Big\downarrow \text{ 240}$$

$$\frac{1}{2}x^2 \coth^{-1}(ax)^2 - a \left(\frac{\int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{\log(1-a^2x^2)}{2a} + x \coth^{-1}(ax)}{a^2}\right)$$

$$\Big\downarrow \text{ 6511}$$

$$\frac{1}{2}x^2 \coth^{-1}(ax)^2 - a \left(\frac{\coth^{-1}(ax)^2}{2a^3} - \frac{\frac{\log(1-a^2x^2)}{2a} + x \coth^{-1}(ax)}{a^2}\right)$$

input
```
Int[x*ArcCoth[a*x]^2,x]
```

output
```
(x^2*ArcCoth[a*x]^2)/2 - a*(ArcCoth[a*x]^2/(2*a^3) - (x*ArcCoth[a*x] + Log
[1 - a^2*x^2]/(2*a))/a^2)
```

3.16.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6543
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.16.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

method	result
parallelrisch	$-\frac{-a^2x^2 \operatorname{arccoth}(ax)^2-2ax \operatorname{arccoth}(ax)+\operatorname{arccoth}(ax)^2-2\ln(ax-1)-2 \operatorname{arccoth}(ax)}{2a^2}$
risch	$\frac{(a^2x^2-1)\ln(ax+1)^2}{8a^2}-\frac{(a^2\ln(ax-1)x^2-2ax-\ln(ax-1))\ln(ax+1)}{4a^2}+\frac{x^2\ln(ax-1)^2}{8}-\frac{\ln(ax-1)x}{2a}-\frac{\ln(ax-1)^2}{8a^2}$
parts	$\frac{x^2 \operatorname{arccoth}(ax)^2}{2}+\frac{ax \operatorname{arccoth}(ax)+\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{2}-\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{2}-\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}+\frac{\ln(ax-1)^2}{8}+\frac{\ln(ax}{2}}{a^2}$
derivativedivides	$\frac{\frac{a^2x^2 \operatorname{arccoth}(ax)^2}{2}+ax \operatorname{arccoth}(ax)+\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{2}-\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{2}-\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}+\frac{\ln(ax-1)^2}{8}+\frac{\ln(ax-}{2}}{a^2}$
default	$\frac{\frac{a^2x^2 \operatorname{arccoth}(ax)^2}{2}+ax \operatorname{arccoth}(ax)+\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{2}-\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{2}-\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}+\frac{\ln(ax-1)^2}{8}+\frac{\ln(ax-}{2}}{a^2}$

input
```
int(x*arccoth(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-a^2*x^2*arccoth(a*x)^2-2*a*x*arccoth(a*x)+arccoth(a*x)^2-2*ln(a*x-1
)-2*arccoth(a*x))/a^2
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int x\coth^{-1}(ax)^2\,dx=\frac{4\,ax\log\left(\frac{ax+1}{ax-1}\right)+(a^2x^2-1)\log\left(\frac{ax+1}{ax-1}\right)^2+4\,\log\left(a^2x^2-1\right)}{8\,a^2}$$

input
```
integrate(x*arccoth(a*x)^2,x, algorithm="fricas")
```

output
```
1/8*(4*a*x*log((a*x + 1)/(a*x - 1)) + (a^2*x^2 - 1)*log((a*x + 1)/(a*x - 1
))^2 + 4*log(a^2*x^2 - 1))/a^2
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int x\coth^{-1}(ax)^2\,dx = \begin{cases} \frac{x^2\operatorname{acoth}^2(ax)}{2}+\frac{x\operatorname{acoth}(ax)}{a}+\frac{\log(ax+1)}{a^2}-\frac{\operatorname{acoth}^2(ax)}{2a^2}-\frac{\operatorname{acoth}(ax)}{a^2} & \text{for } a\neq 0\\ -\frac{\pi^2x^2}{8} & \text{otherwise}\end{cases}$$

input
```
integrate(x*acoth(a*x)**2,x)
```

output
```
Piecewise((x**2*acoth(a*x)**2/2 + x*acoth(a*x)/a + log(a*x + 1)/a**2 - aco
th(a*x)**2/(2*a**2) - acoth(a*x)/a**2, Ne(a, 0)), (-pi**2*x**2/8, True))
```

3.16.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(48) = 96.

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.80

$$\int x\coth^{-1}(ax)^2\,dx = \frac{1}{2}x^2\operatorname{arcoth}(ax)^2+\frac{1}{2}a\left(\frac{2x}{a^2}-\frac{\log(ax+1)}{a^3}+\frac{\log(ax-1)}{a^3}\right)\operatorname{arcoth}(ax) - \frac{2(\log(ax-1)-2)\log(ax+1)-\log(ax+1)^2-\log(ax-1)^2-4\log(ax-1)}{8a^2}$$

input
```
integrate(x*arccoth(a*x)^2,x, algorithm="maxima")
```

output
```
1/2*x^2*arccoth(a*x)^2 + 1/2*a*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/
a^3)*arccoth(a*x) - 1/8*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^
2 - log(a*x - 1)^2 - 4*log(a*x - 1))/a^2
```

3.16.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(48) = 96$.

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.85

$$\int x\coth^{-1}(ax)^2\,dx$$
$$=\frac{1}{2}a\left(\frac{(ax+1)\log\left(\frac{ax+1}{ax-1}\right)^2}{\left(\frac{(ax+1)^2a^3}{(ax-1)^2}-\frac{2\,(ax+1)a^3}{ax-1}+a^3\right)(ax-1)}+\frac{2\,\log\left(\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)a^3}{ax-1}-a^3}-\frac{2\,\log\left(\frac{ax+1}{ax-1}-1\right)}{a^3}+\frac{2\,\log\left(\frac{ax+1}{ax-1}\right)}{a^3}\right)$$

input
```
integrate(x*arccoth(a*x)^2,x, algorithm="giac")
```

output
```
1/2*a*((a*x + 1)*log((a*x + 1)/(a*x - 1))^2/(((a*x + 1)^2*a^3/(a*x - 1)^2
- 2*(a*x + 1)*a^3/(a*x - 1) + a^3)*(a*x - 1)) + 2*log((a*x + 1)/(a*x - 1))
/((a*x + 1)*a^3/(a*x - 1) - a^3) - 2*log((a*x + 1)/(a*x - 1) - 1)/a^3 + 2*
log((a*x + 1)/(a*x - 1))/a^3)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x\coth^{-1}(ax)^2\,dx=\frac{x^2\,\mathrm{acoth}(a\,x)^2}{2}+\frac{-\frac{\mathrm{acoth}(a\,x)^2}{2}+a\,x\,\mathrm{acoth}(a\,x)+\frac{\ln(a^2\,x^2-1)}{2}}{a^2}$$

input
```
int(x*acoth(a*x)^2,x)
```

output
```
(x^2*acoth(a*x)^2)/2 + (log(a^2*x^2 - 1)/2 - acoth(a*x)^2/2 + a*x*acoth(a*
x))/a^2
```

3.17 $\int \coth^{-1}(ax)^2\,dx$

3.17.1 Optimal result

Integrand size = 6, antiderivative size = 58

$$\int \coth^{-1}(ax)^2\,dx = \frac{\coth^{-1}(ax)^2}{a} + x\coth^{-1}(ax)^2 - \frac{2\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a} - \frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a}$$

output
```
arccoth(a*x)^2/a+x*arccoth(a*x)^2-2*arccoth(a*x)*ln(2/(-a*x+1))/a-polylog(
2,1-2/(-a*x+1))/a
```

3.17.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \coth^{-1}(ax)^2\,dx = \frac{\coth^{-1}(ax)\left((-1+ax)\coth^{-1}(ax) - 2\log\left(1-e^{-2\coth^{-1}(ax)}\right)\right) + \text{PolyLog}\left(2,e^{-2\coth^{-1}(ax)}\right)}{a}$$

input
```
Integrate[ArcCoth[a*x]^2,x]
```

output
```
(ArcCoth[a*x]*((-1 + a*x)*ArcCoth[a*x] - 2*Log[1 - E^(-2*ArcCoth[a*x])]) +
 PolyLog[2, E^(-2*ArcCoth[a*x])])/a
```

3.17.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6437, 6547, 6471, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(ax)^2\,dx$$

$$\Big\downarrow \text{6437}$$

$$x\coth^{-1}(ax)^2-2a\int \frac{x\coth^{-1}(ax)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{6547}$$

$$x\coth^{-1}(ax)^2-2a\left(\frac{\int \frac{\coth^{-1}(ax)}{1-ax}dx}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}\right)$$

$$\Big\downarrow \text{6471}$$

$$x\coth^{-1}(ax)^2-2a\left(\frac{\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}-\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}\right)$$

$$\Big\downarrow \text{2849}$$

$$x\coth^{-1}(ax)^2-2a\left(\frac{\frac{\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}\right)$$

$$\Big\downarrow \text{2752}$$

$$x\coth^{-1}(ax)^2-2a\left(\frac{\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}+\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}\right)$$

input
```
Int[ArcCoth[a*x]^2,x]
```

output
```
x*ArcCoth[a*x]^2 - 2*a*(-1/2*ArcCoth[a*x]^2/a^2 + ((ArcCoth[a*x]*Log[2/(1
- a*x)])/a + PolyLog[2, 1 - 2/(1 - a*x)]/(2*a))/a)
```

3.17.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6471
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6547
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.17.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.00

method	result
derivativedivides	$\frac{\operatorname{arccoth}(ax)^2(ax-1)+2\operatorname{arccoth}(ax)^2-2\operatorname{arccoth}(ax)\ln\left(1-\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)-2\operatorname{polylog}\left(2,\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)-2\operatorname{arccoth}(ax)\ln\left(1+\right.}{a}$
default	$\frac{\operatorname{arccoth}(ax)^2(ax-1)+2\operatorname{arccoth}(ax)^2-2\operatorname{arccoth}(ax)\ln\left(1-\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)-2\operatorname{polylog}\left(2,\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)-2\operatorname{arccoth}(ax)\ln\left(1+\right.}{a}$
risch	$\frac{\ln(ax-1)^2x}{4}-\frac{\ln(ax-1)x}{2}-\frac{\ln(ax-1)^2}{4a}+\frac{\ln(ax-1)}{2a}+\frac{1}{a}+\frac{\ln(ax+1)^2x}{4}-\frac{x\ln(ax+1)}{2}+\frac{\ln(ax+1)^2}{4a}+\ln(a$

input
```
int(arccoth(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(arccoth(a*x)^2*(a*x-1)+2*arccoth(a*x)^2-2*arccoth(a*x)*ln(1-1/((a*x-1
)/(a*x+1))^(1/2))-2*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))-2*arccoth(a*x)*ln
(1+1/((a*x-1)/(a*x+1))^(1/2))-2*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2)))
```

3.17.5 Fricas [F]

$$\int \coth^{-1}(ax)^2\,dx = \int \operatorname{arcoth}\left(ax\right)^2\,dx$$

input
```
integrate(arccoth(a*x)^2,x, algorithm="fricas")
```

output
```
integral(arccoth(a*x)^2, x)
```

3.17.6 Sympy [F]

$$\int \coth^{-1}(ax)^2\,dx = \int \operatorname{acoth}^2\left(ax\right)dx$$

input
```
integrate(acoth(a*x)**2,x)
```

output
```
Integral(acoth(a*x)**2, x)
```

3.17.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(55) = 110$.

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.33

$$\int \coth^{-1}(ax)^2\,dx = x\operatorname{arcoth}\left(ax\right)^2 + \frac{1}{4}\left(a\left(\frac{\log\left(ax+1\right)^2 + 2\log\left(ax+1\right)\log\left(ax-1\right) - \log\left(ax-1\right)^2}{a^3} - \frac{4\left(\log\left(ax-1\right)\log\left(\frac{1}{2}ax+\frac{1}{2}\right) + \mathrm{Li}\right.}{a^3}\right.\right.$$
$$+ \frac{\operatorname{arcoth}\left(ax\right)\log\left(a^2x^2-1\right)}{a}$$

input
```
integrate(arccoth(a*x)^2,x, algorithm="maxima")
```

output
```
x*arccoth(a*x)^2 + 1/4*(a*((log(a*x + 1)^2 + 2*log(a*x + 1)*log(a*x - 1) -
 log(a*x - 1)^2)/a^3 - 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x
 + 1/2))/a^3) - 2*(log(a*x + 1)/a - log(a*x - 1)/a)*log(a^2*x^2 - 1)/a)*a
+ arccoth(a*x)*log(a^2*x^2 - 1)/a
```

3.17.8 Giac [F]

$$\int \coth^{-1}(ax)^2\,dx = \int \operatorname{arcoth}(ax)^2\,dx$$

input
```
integrate(arccoth(a*x)^2,x, algorithm="giac")
```

output
```
integrate(arccoth(a*x)^2, x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(ax)^2\,dx = \int \operatorname{acoth}(a\,x)^2\,dx$$

input
```
int(acoth(a*x)^2,x)
```

output
```
int(acoth(a*x)^2, x)
```

3.18 $\int \frac{\coth^{-1}(ax)^2}{x}\,dx$

3.18.1 Optimal result

Integrand size = 10, antiderivative size = 97

$$\begin{aligned}\int \frac{\coth^{-1}(ax)^2}{x}\,dx &= 2\coth^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1-ax}\right)\\ &+\coth^{-1}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1+ax}\right)\\ &-\coth^{-1}(ax)\,\text{PolyLog}\left(2,1-\frac{2ax}{1+ax}\right)\\ &+\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2}{1+ax}\right)-\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2ax}{1+ax}\right)\end{aligned}$$

output
```
2*arccoth(a*x)^2*arccoth(1-2/(-a*x+1))+arccoth(a*x)*polylog(2,1-2/(a*x+1))
-arccoth(a*x)*polylog(2,1-2*a*x/(a*x+1))+1/2*polylog(3,1-2/(a*x+1))-1/2*po
lylog(3,1-2*a*x/(a*x+1))
```

3.18.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.18

$$\int \frac{\coth^{-1}(ax)^2}{x}\,dx = \frac{2}{3}\coth^{-1}(ax)^3 + \coth^{-1}(ax)^2\log\left(1+e^{-2\coth^{-1}(ax)}\right) - \coth^{-1}(ax)^2\log\left(1-e^{2\coth^{-1}(ax)}\right) - \coth^{-1}(ax)\,\text{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right) - \coth^{-1}(ax)\,\text{PolyLog}\left(2,e^{2\coth^{-1}(ax)}\right) - \frac{1}{2}\text{PolyLog}\left(3,-e^{-2\coth^{-1}(ax)}\right) + \frac{1}{2}\text{PolyLog}\left(3,e^{2\coth^{-1}(ax)}\right)$$

input
```
Integrate[ArcCoth[a*x]^2/x,x]
```

output
```
(2*ArcCoth[a*x]^3)/3 + ArcCoth[a*x]^2*Log[1 + E^(-2*ArcCoth[a*x])] - ArcCo
th[a*x]^2*Log[1 - E^(2*ArcCoth[a*x])] - ArcCoth[a*x]*PolyLog[2, -E^(-2*Arc
Coth[a*x])] - ArcCoth[a*x]*PolyLog[2, E^(2*ArcCoth[a*x])] - PolyLog[3, -E^
(-2*ArcCoth[a*x])]/2 + PolyLog[3, E^(2*ArcCoth[a*x])]/2
```

3.18.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6449, 6615, 6619, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)^2}{x}\,dx$$

$$\Big\downarrow \text{6449}$$

$$2\coth^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1-ax}\right) - 4a\int \frac{\coth^{-1}(ax)\coth^{-1}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{6615}$$

$$2\coth^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1-ax}\right)-$$
$$4a\left(\frac{1}{2}\int\frac{\coth^{-1}(ax)\log\left(\frac{2ax}{ax+1}\right)}{1-a^2x^2}dx-\frac{1}{2}\int\frac{\coth^{-1}(ax)\log\left(\frac{2}{ax+1}\right)}{1-a^2x^2}dx\right)$$

$\downarrow$ 6619

$$2\coth^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1-ax}\right)-$$
$$4a\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{1-a^2x^2}dx-\frac{\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)\coth^{-1}(ax)}{2a}\right)+\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2ax}{ax+1}\right)\text{coth}^{\text{-}}}{2a}\right.\right.$$

$\downarrow$ 7164

$$2\coth^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1-ax}\right)-$$
$$4a\left(\frac{1}{2}\left(-\frac{\text{PolyLog}\left(3,1-\frac{2}{ax+1}\right)}{4a}-\frac{\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)\coth^{-1}(ax)}{2a}\right)+\frac{1}{2}\left(\frac{\text{PolyLog}\left(3,1-\frac{2ax}{ax+1}\right)}{4a}+\frac{\text{PolyLo}}{}\right.\right.$$

input
```
Int[ArcCoth[a*x]^2/x,x]
```

output
```
2*ArcCoth[a*x]^2*ArcCoth[1 - 2/(1 - a*x)] - 4*a*((-1/2*(ArcCoth[a*x]*PolyL
og[2, 1 - 2/(1 + a*x)])/a - PolyLog[3, 1 - 2/(1 + a*x)]/(4*a))/2 + ((ArcCo
th[a*x]*PolyLog[2, 1 - (2*a*x)/(1 + a*x)])/(2*a) + PolyLog[3, 1 - (2*a*x)/
(1 + a*x)]/(4*a))/2)
```

3.18.3.1 Defintions of rubi rules used

rule 6449
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcCoth[c*x])^p*ArcCoth[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcCoth[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6615
```
Int[(ArcCoth[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a
+ b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] - Simp[1/2   Int[Log[SimplifyInteg
rand[1 - 1/u, x]]*((a + b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1
- c*x))^2, 0]
```

rule 6619
```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.18.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.37 (sec) , antiderivative size = 459, normalized size of antiderivative = 4.73

method	result
derivativedivides	$\ln(ax)\operatorname{arccoth}(ax)^2+\frac{i\pi\operatorname{csgn}\left(\frac{i\left(1+\frac{ax+1}{ax-1}\right)}{\frac{ax+1}{ax-1}-1}\right)\left(\operatorname{csgn}\left(\frac{i}{\frac{ax+1}{ax-1}-1}\right)\operatorname{csgn}\left(i\left(1+\frac{ax+1}{ax-1}\right)\right)-\operatorname{csgn}\left(\frac{i}{\frac{ax+1}{ax-1}-1}\right)\operatorname{csgn}\left(\cdots\right.\right.}{\cdots}$
default	$\ln(ax)\operatorname{arccoth}(ax)^2+\frac{i\pi\operatorname{csgn}\left(\frac{i\left(1+\frac{ax+1}{ax-1}\right)}{\frac{ax+1}{ax-1}-1}\right)\left(\operatorname{csgn}\left(\frac{i}{\frac{ax+1}{ax-1}-1}\right)\operatorname{csgn}\left(i\left(1+\frac{ax+1}{ax-1}\right)\right)-\operatorname{csgn}\left(\frac{i}{\frac{ax+1}{ax-1}-1}\right)\operatorname{csgn}\left(\cdots\right.\right.}{\cdots}$
parts	$\ln(x)\operatorname{arccoth}(ax)^2+2a\left(\frac{\left(i\pi\operatorname{csgn}\left(\frac{i}{a}\right)\operatorname{csgn}\left(\frac{i\left(1+\frac{ax+1}{ax-1}\right)}{\frac{ax+1}{ax-1}-1}\right)\operatorname{csgn}\left(\frac{i\left(1+\frac{ax+1}{ax-1}\right)}{a\left(\frac{ax+1}{ax-1}-1\right)}\right)-i\pi\operatorname{csgn}\left(\frac{i}{a}\right)\operatorname{csgn}\left(\cdots\right.\right.}{\cdots}\right.$

input
```
int(arccoth(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
ln(a*x)*arccoth(a*x)^2+1/2*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1)*(1+(a*x+1)/(a*x
-1)))*(csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(1+(a*x+1)/(a*x-1)))-csgn(I/((a*
x+1)/(a*x-1)-1))*csgn(I/((a*x+1)/(a*x-1)-1)*(1+(a*x+1)/(a*x-1)))-csgn(I/((
a*x+1)/(a*x-1)-1)*(1+(a*x+1)/(a*x-1)))*csgn(I*(1+(a*x+1)/(a*x-1)))+csgn(I/
((a*x+1)/(a*x-1)-1)*(1+(a*x+1)/(a*x-1)))^2)*arccoth(a*x)^2+arccoth(a*x)^2*
ln((a*x+1)/(a*x-1)-1)-arccoth(a*x)^2*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-2*arc
coth(a*x)*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+2*polylog(3,1/((a*x-1)/(a*x
+1))^(1/2))-arccoth(a*x)^2*ln(1+1/((a*x-1)/(a*x+1))^(1/2))-2*arccoth(a*x)*
polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+2*polylog(3,-1/((a*x-1)/(a*x+1))^(1/
2))+arccoth(a*x)*polylog(2,-(a*x+1)/(a*x-1))-1/2*polylog(3,-(a*x+1)/(a*x-1
))
```

3.18.5 Fricas [F]

$$\int \frac{\coth^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{arcoth}(ax)^2}{x}\,dx$$

input
```
integrate(arccoth(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral(arccoth(a*x)^2/x, x)
```

3.18.6 Sympy [F]

$$\int \frac{\coth^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{acoth}^2(ax)}{x}\,dx$$

input
```
integrate(acoth(a*x)**2/x,x)
```

output
```
Integral(acoth(a*x)**2/x, x)
```

3.18.7 Maxima [F]

$$\int \frac{\coth^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{arcoth}(ax)^2}{x}\,dx$$

input
```
integrate(arccoth(a*x)^2/x,x, algorithm="maxima")
```

output
```
integrate(arccoth(a*x)^2/x, x)
```

3.18.8 Giac [F]

$$\int \frac{\coth^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{arcoth}(ax)^2}{x}\,dx$$

input
```
integrate(arccoth(a*x)^2/x,x, algorithm="giac")
```

output
```
integrate(arccoth(a*x)^2/x, x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{acoth}(a\,x)^2}{x}\,dx$$

input
```
int(acoth(a*x)^2/x,x)
```

output
```
int(acoth(a*x)^2/x, x)
```

3.19 $\int \frac{\coth^{-1}(ax)^2}{x^2}\,dx$

3.19.1 Optimal result

Integrand size = 10, antiderivative size = 55

$$\int \frac{\coth^{-1}(ax)^2}{x^2}\,dx = a\coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{x} + 2a\coth^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right) - a\,\text{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)$$

output
```
a*arccoth(a*x)^2-arccoth(a*x)^2/x+2*a*arccoth(a*x)*ln(2-2/(a*x+1))-a*polyl
og(2,-1+2/(a*x+1))
```

3.19.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{\coth^{-1}(ax)^2}{x^2}\,dx = \frac{(-1+ax)\coth^{-1}(ax)^2}{x} + 2a\coth^{-1}(ax)\log\left(1+e^{-2\coth^{-1}(ax)}\right) - a\,\text{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right)$$

input
```
Integrate[ArcCoth[a*x]^2/x^2,x]
```

output
```
((-1 + a*x)*ArcCoth[a*x]^2)/x + 2*a*ArcCoth[a*x]*Log[1 + E^(-2*ArcCoth[a*x
])] - a*PolyLog[2, -E^(-2*ArcCoth[a*x])]
```

3.19.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6453, 6551, 6495, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)^2}{x^2} \, dx$$

$$\Big\downarrow \text{6453}$$

$$2a \int \frac{\coth^{-1}(ax)}{x\left(1-a^2x^2\right)}dx-\frac{\coth^{-1}(ax)^2}{x}$$

$$\Big\downarrow \text{6551}$$

$$2a\left(\int \frac{\coth^{-1}(ax)}{x(ax+1)}dx+\frac{1}{2}\coth^{-1}(ax)^2\right)-\frac{\coth^{-1}(ax)^2}{x}$$

$$\Big\downarrow \text{6495}$$

$$2a\left(-a\int \frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\coth^{-1}(ax)^2+\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)\right)-\frac{\coth^{-1}(ax)^2}{x}$$

$$\Big\downarrow \text{2897}$$

$$2a\left(-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{2}\coth^{-1}(ax)^2+\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)\right)-\frac{\coth^{-1}(ax)^2}{x}$$

input
```
Int[ArcCoth[a*x]^2/x^2,x]
```

output
```
-(ArcCoth[a*x]^2/x) + 2*a*(ArcCoth[a*x]^2/2 + ArcCoth[a*x]*Log[2 - 2/(1 +
a*x)] - PolyLog[2, -1 + 2/(1 + a*x)]/2)
```

3.19.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6495
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6551
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.19.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(55) = 110$.

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.64

method	result
derivativedivides	$a\left(-\frac{\operatorname{arccoth}(ax)^2}{ax} - \operatorname{arccoth}(ax)\ln(ax+1) + 2\ln(ax)\operatorname{arccoth}(ax) - \operatorname{arccoth}(ax)\ln(a$
default	$a\left(-\frac{\operatorname{arccoth}(ax)^2}{ax} - \operatorname{arccoth}(ax)\ln(ax+1) + 2\ln(ax)\operatorname{arccoth}(ax) - \operatorname{arccoth}(ax)\ln(a$
parts	$-\frac{\operatorname{arccoth}(ax)^2}{x} - 2a\left(\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{2} - \ln(ax)\operatorname{arccoth}(ax) + \frac{\operatorname{arccoth}(ax)\ln(ax-1)}{2} - \frac{\operatorname{dilog}(\frac{ax}{2}}{2}$

input
```
int(arccoth(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/a/x*arccoth(a*x)^2-arccoth(a*x)*ln(a*x+1)+2*ln(a*x)*arccoth(a*x)-arc
coth(a*x)*ln(a*x-1)+dilog(1/2*a*x+1/2)+1/2*ln(a*x-1)*ln(1/2*a*x+1/2)-1/4*l
n(a*x-1)^2+1/4*ln(a*x+1)^2-1/2*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2
)-dilog(a*x+1)-ln(a*x)*ln(a*x+1)-dilog(a*x))
```

3.19.5 Fricas [F]

$$\int \frac{\coth^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{arcoth}(ax)^2}{x^2}\,dx$$

input
```
integrate(arccoth(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral(arccoth(a*x)^2/x^2, x)
```

3.19.6 Sympy [F]

$$\int \frac{\coth^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{acoth}^2(ax)}{x^2}\,dx$$

input
```
integrate(acoth(a*x)**2/x**2,x)
```

output
```
Integral(acoth(a*x)**2/x**2, x)
```

3.19.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(54) = 108.

Time = 0.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.65

$$\int \frac{\coth^{-1}(ax)^2}{x^2}\,dx$$
$$= \frac{1}{4}a^2\left(\frac{\log(ax+1)^2 - 2\log(ax+1)\log(ax-1) - \log(ax-1)^2}{a} + \frac{4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right) + \mathrm{Li}_2(-\right.}{a}\right.$$
$$- a\left(\log\left(a^2x^2-1\right) - \log\left(x^2\right)\right)\operatorname{arcoth}(ax) - \frac{\operatorname{arcoth}(ax)^2}{x}$$

input
```
integrate(arccoth(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
1/4*a^2*((log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) - log(a*x - 1)^2)/a
 + 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 4*(log(
a*x + 1)*log(x) + dilog(-a*x))/a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))/a
) - a*(log(a^2*x^2 - 1) - log(x^2))*arccoth(a*x) - arccoth(a*x)^2/x
```

3.19.8 Giac [F]

$$\int \frac{\coth^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{arcoth}(ax)^2}{x^2}\,dx$$

input
```
integrate(arccoth(a*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate(arccoth(a*x)^2/x^2, x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{acoth}(a\,x)^2}{x^2}\,dx$$

input
```
int(acoth(a*x)^2/x^2,x)
```

output
```
int(acoth(a*x)^2/x^2, x)
```

3.20 $\int \frac{\coth^{-1}(ax)^2}{x^3} \, dx$

3.20.1 Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \frac{\coth^{-1}(ax)^2}{x^3} \, dx = -\frac{a\coth^{-1}(ax)}{x} + \frac{1}{2}a^2\coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{2x^2} + a^2\log(x) - \frac{1}{2}a^2\log\left(1-a^2x^2\right)$$

output
```
-a*arccoth(a*x)/x+1/2*a^2*arccoth(a*x)^2-1/2*arccoth(a*x)^2/x^2+a^2*ln(x)-
1/2*a^2*ln(-a^2*x^2+1)
```

3.20.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(ax)^2}{x^3} \, dx = -\frac{a\coth^{-1}(ax)}{x} + \frac{(-1+a^2x^2)\coth^{-1}(ax)^2}{2x^2} + a^2\log(x) - \frac{1}{2}a^2\log\left(1-a^2x^2\right)$$

input
```
Integrate[ArcCoth[a*x]^2/x^3,x]
```

output
```
-((a*ArcCoth[a*x])/x) + ((-1 + a^2*x^2)*ArcCoth[a*x]^2)/(2*x^2) + a^2*Log[
x] - (a^2*Log[1 - a^2*x^2])/2
```

3.20.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6453, 6545, 6453, 243, 47, 14, 16, 6511}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)^2}{x^3} \, dx$$

$$\Big\downarrow \text{6453}$$

$$a \int \frac{\coth^{-1}(ax)}{x^2 (1-a^2x^2)}dx - \frac{\coth^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6545}$$

$$a \left(a^2 \int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx + \int \frac{\coth^{-1}(ax)}{x^2}dx\right) - \frac{\coth^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6453}$$

$$a \left(a \int \frac{1}{x (1-a^2x^2)}dx + a^2 \int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx - \frac{\coth^{-1}(ax)}{x}\right) - \frac{\coth^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$a \left(\frac{1}{2} a \int \frac{1}{x^2 (1-a^2x^2)}dx^2 + a^2 \int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx - \frac{\coth^{-1}(ax)}{x}\right) - \frac{\coth^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{47}$$

$$a \left(\frac{1}{2} a \left(a^2 \int \frac{1}{1-a^2x^2}dx^2 + \int \frac{1}{x^2}dx^2\right) + a^2 \int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx - \frac{\coth^{-1}(ax)}{x}\right) - \frac{\coth^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$a \left(\frac{1}{2} a \left(a^2 \int \frac{1}{1-a^2x^2}dx^2 + \log \left(x^2\right)\right) + a^2 \int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx - \frac{\coth^{-1}(ax)}{x}\right) - \frac{\coth^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$a \left(a^2 \int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx + \frac{1}{2} a \left(\log \left(x^2\right) - \log \left(1-a^2x^2\right)\right) - \frac{\coth^{-1}(ax)}{x}\right) - \frac{\coth^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{6511}$$

$$a \left(\frac{1}{2} a \left(\log \left(x^2\right) - \log \left(1-a^2x^2\right)\right) + \frac{1}{2} a \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)}{x}\right) - \frac{\coth^{-1}(ax)^2}{2x^2}$$

input
```
Int[ArcCoth[a*x]^2/x^3,x]
```

output
```
-1/2*ArcCoth[a*x]^2/x^2 + a*(-(ArcCoth[a*x]/x) + (a*ArcCoth[a*x]^2)/2 + (a
*(Log[x^2] - Log[1 - a^2*x^2]))/2)
```

3.20.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6545
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.20.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

method	result
parallelrisch	$\frac{a^2x^2 \operatorname{arccoth}(ax)^2+2a^2\ln(x)x^2-2a^2\ln(ax-1)x^2-2\operatorname{arccoth}(ax)a^2x^2-2ax\operatorname{arccoth}(ax)-\operatorname{arccoth}(ax)^2}{2x^2}$
risch	$\frac{(a^2x^2-1)\ln(ax+1)^2}{8x^2}-\frac{(a^2\ln(ax-1)x^2+2ax-\ln(ax-1))\ln(ax+1)}{4x^2}+\frac{a^2x^2\ln(ax-1)^2+8a^2\ln(x)x^2-4a^2\ln(a^2x^2-}{8x^2}$
derivativedivides	$a^2\left(-\frac{\operatorname{arccoth}(ax)^2}{2a^2x^2}-\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{2}+\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{2}-\frac{\operatorname{arccoth}(ax)}{ax}+\frac{\ln(ax-1)\ln(\frac{ax}{2}+\frac{1}{2})}{4}-\right.$
default	$a^2\left(-\frac{\operatorname{arccoth}(ax)^2}{2a^2x^2}-\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{2}+\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{2}-\frac{\operatorname{arccoth}(ax)}{ax}+\frac{\ln(ax-1)\ln(\frac{ax}{2}+\frac{1}{2})}{4}-\right.$
parts	$-\frac{\operatorname{arccoth}(ax)^2}{2x^2}-a^2\left(\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{2}-\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{2}+\frac{\operatorname{arccoth}(ax)}{ax}-\frac{\ln(ax-1)\ln(\frac{ax}{2}+\frac{1}{2})}{4}+\right.$

input
```
int(arccoth(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a^2*x^2*arccoth(a*x)^2+2*a^2*ln(x)*x^2-2*a^2*ln(a*x-1)*x^2-2*arccoth(
a*x)*a^2*x^2-2*a*x*arccoth(a*x)-arccoth(a*x)^2)/x^2
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.30

$$\int \frac{\coth^{-1}(ax)^2}{x^3}\,dx = -\frac{4\,a^2x^2\log\left(a^2x^2-1\right)-8\,a^2x^2\log\left(x\right)+4\,ax\log\left(\frac{ax+1}{ax-1}\right)-\left(a^2x^2-1\right)\log\left(\frac{ax+1}{ax-1}\right)^2}{8\,x^2}$$

input
```
integrate(arccoth(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
-1/8*(4*a^2*x^2*log(a^2*x^2 - 1) - 8*a^2*x^2*log(x) + 4*a*x*log((a*x + 1)/
(a*x - 1)) - (a^2*x^2 - 1)*log((a*x + 1)/(a*x - 1))^2)/x^2
```

3.20.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{\coth^{-1}(ax)^2}{x^3}\,dx = a^2\log(x) - a^2\log(ax+1) + \frac{a^2\operatorname{acoth}^2(ax)}{2} + a^2\operatorname{acoth}(ax) - \frac{a\operatorname{acoth}(ax)}{x} - \frac{\operatorname{acoth}^2(ax)}{2x^2}$$

input
```
integrate(acoth(a*x)**2/x**3,x)
```

output
```
a**2*log(x) - a**2*log(a*x + 1) + a**2*acoth(a*x)**2/2 + a**2*acoth(a*x) -
 a*acoth(a*x)/x - acoth(a*x)**2/(2*x**2)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.57

$$\int \frac{\coth^{-1}(ax)^2}{x^3}\,dx = \frac{1}{8}\left(2\left(\log(ax-1)-2\right)\log(ax+1) - \log(ax+1)^2 - \log(ax-1)^2 - 4\log(ax-1) + 8\log(x)\right)a^2 + \frac{1}{2}\left(a\log(ax+1) - a\log(ax-1) - \frac{2}{x}\right)a\operatorname{arcoth}(ax) - \frac{\operatorname{arcoth}(ax)^2}{2x^2}$$

input
```
integrate(arccoth(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
1/8*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 -
 4*log(a*x - 1) + 8*log(x))*a^2 + 1/2*(a*log(a*x + 1) - a*log(a*x - 1) - 2
/x)*a*arccoth(a*x) - 1/2*arccoth(a*x)^2/x^2
```

3.20.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(55) = 110.

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.25

$$\int \frac{\coth^{-1}(ax)^2}{x^3}\,dx$$
$$=\frac{1}{2}\left(2\,a\log\left(\frac{ax+1}{ax-1}+1\right)-2\,a\log\left(\frac{ax+1}{ax-1}\right)+\frac{(ax+1)a\log\left(\frac{ax+1}{ax-1}\right)^2}{(ax-1)\left(\frac{(ax+1)^2}{(ax-1)^2}+\frac{2\,(ax+1)}{ax-1}+1\right)}+\frac{2\,a\log\left(\frac{ax+1}{ax-1}\right)}{\frac{ax+1}{ax-1}+1}\right)a$$

input
```
integrate(arccoth(a*x)^2/x^3,x, algorithm="giac")
```

output
```
1/2*(2*a*log((a*x + 1)/(a*x - 1) + 1) - 2*a*log((a*x + 1)/(a*x - 1)) + (a*
x + 1)*a*log((a*x + 1)/(a*x - 1))^2/((a*x - 1)*((a*x + 1)^2/(a*x - 1)^2 +
2*(a*x + 1)/(a*x - 1) + 1)) + 2*a*log((a*x + 1)/(a*x - 1))/((a*x + 1)/(a*x
 - 1) + 1))*a
```

3.20.9 Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.38

$$\int \frac{\coth^{-1}(ax)^2}{x^3}\,dx = a^2\,\ln(x)+\ln\left(\frac{1}{a\,x}+1\right)^2\left(\frac{a^2}{8}-\frac{1}{8\,x^2}\right)$$
$$+\ln\left(1-\frac{1}{a\,x}\right)^2\left(\frac{a^2}{8}-\frac{1}{8\,x^2}\right)-\frac{a^2\,\ln(a^2\,x^2-1)}{2}$$
$$+\ln\left(1-\frac{1}{a\,x}\right)\left(\frac{4\,a\,x-2}{16\,x^2}+\frac{4\,a\,x+2}{16\,x^2}-\ln\left(\frac{1}{a\,x}+1\right)\left(\frac{a^2}{4}-\frac{1}{4\,x^2}\right)\right)$$
$$-\frac{a\,\ln\left(\frac{1}{a\,x}+1\right)}{2\,x}$$

input
```
int(acoth(a*x)^2/x^3,x)
```

output
```
a^2*log(x) + log(1/(a*x) + 1)^2*(a^2/8 - 1/(8*x^2)) + log(1 - 1/(a*x))^2*(
a^2/8 - 1/(8*x^2)) - (a^2*log(a^2*x^2 - 1))/2 + log(1 - 1/(a*x))*((4*a*x -
 2)/(16*x^2) + (4*a*x + 2)/(16*x^2) - log(1/(a*x) + 1)*(a^2/4 - 1/(4*x^2))
) - (a*log(1/(a*x) + 1))/(2*x)
```

3.21 $\int \frac{\coth^{-1}(ax)^2}{x^4}\,dx$

3.21.1 Optimal result

Integrand size = 10, antiderivative size = 103

$$\int \frac{\coth^{-1}(ax)^2}{x^4}\,dx = -\frac{a^2}{3x}-\frac{a\coth^{-1}(ax)}{3x^2}+\frac{1}{3}a^3\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{3x^3}+\frac{1}{3}a^3\text{arctanh}(ax)+\frac{2}{3}a^3\coth^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)-\frac{1}{3}a^3\,\text{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)$$

output
```
-1/3*a^2/x-1/3*a*arccoth(a*x)/x^2+1/3*a^3*arccoth(a*x)^2-1/3*arccoth(a*x)^
2/x^3+1/3*a^3*arctanh(a*x)+2/3*a^3*arccoth(a*x)*ln(2-2/(a*x+1))-1/3*a^3*po
lylog(2,-1+2/(a*x+1))
```

3.21.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{\coth^{-1}(ax)^2}{x^4}\,dx = \frac{-a^2x^2+(-1+a^3x^3)\coth^{-1}(ax)^2+ax\coth^{-1}(ax)\left(-1+a^2x^2+2a^2x^2\log\left(1+e^{-2\coth^{-1}(ax)}\right)\right)-a^3x^3\,\text{P}}{3x^3}$$

input
```
Integrate[ArcCoth[a*x]^2/x^4,x]
```

output
```
(-(a^2*x^2) + (-1 + a^3*x^3)*ArcCoth[a*x]^2 + a*x*ArcCoth[a*x]*(-1 + a^2*x
^2 + 2*a^2*x^2*Log[1 + E^(-2*ArcCoth[a*x])]) - a^3*x^3*PolyLog[2, -E^(-2*A
rcCoth[a*x])])/(3*x^3)
```

3.21.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6453, 6545, 6453, 264, 219, 6551, 6495, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)^2}{x^4} \, dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{2}{3}a\int \frac{\coth^{-1}(ax)}{x^3(1-a^2x^2)}dx-\frac{\coth^{-1}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{6545}$$

$$\frac{2}{3}a\left(a^2\int \frac{\coth^{-1}(ax)}{x(1-a^2x^2)}dx+\int \frac{\coth^{-1}(ax)}{x^3}dx\right)-\frac{\coth^{-1}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{6453}$$

$$\frac{2}{3}a\left(\frac{1}{2}a\int \frac{1}{x^2(1-a^2x^2)}dx+a^2\int \frac{\coth^{-1}(ax)}{x(1-a^2x^2)}dx-\frac{\coth^{-1}(ax)}{2x^2}\right)-\frac{\coth^{-1}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{3}a\left(\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx-\frac{1}{x}\right)+a^2\int \frac{\coth^{-1}(ax)}{x(1-a^2x^2)}dx-\frac{\coth^{-1}(ax)}{2x^2}\right)-\frac{\coth^{-1}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{2}{3}a\left(a^2\int \frac{\coth^{-1}(ax)}{x(1-a^2x^2)}dx+\frac{1}{2}a\left(a\,\text{arctanh}(ax)-\frac{1}{x}\right)-\frac{\coth^{-1}(ax)}{2x^2}\right)-\frac{\coth^{-1}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{6551}$$

$$\frac{2}{3}a\left(a^2\left(\int \frac{\coth^{-1}(ax)}{x(ax+1)}dx+\frac{1}{2}\coth^{-1}(ax)^2\right)+\frac{1}{2}a\left(a\,\text{arctanh}(ax)-\frac{1}{x}\right)-\frac{\coth^{-1}(ax)}{2x^2}\right)-\frac{\coth^{-1}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{6495}$$

$$\frac{2}{3}a\left(a^2\left(-a\int\frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\coth^{-1}(ax)^2+\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)\right)+\frac{1}{2}a\left(a\text{arctanh}(ax)-\frac{1}{x}\right)\right.$$
$$\frac{\coth^{-1}(ax)^2}{3x^3}$$
$$\Big\downarrow \text{2897}$$
$$\frac{2}{3}a\left(a^2\left(-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{2}\coth^{-1}(ax)^2+\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)\right)+\frac{1}{2}a\left(a\text{arctanh}(ax)-\right.\right.$$
$$\frac{\coth^{-1}(ax)^2}{3x^3}$$

input
```
Int[ArcCoth[a*x]^2/x^4,x]
```

output
```
-1/3*ArcCoth[a*x]^2/x^3 + (2*a*(-1/2*ArcCoth[a*x]/x^2 + (a*(-x^(-1) + a*Ar
cTanh[a*x]))/2 + a^2*(ArcCoth[a*x]^2/2 + ArcCoth[a*x]*Log[2 - 2/(1 + a*x)]
 - PolyLog[2, -1 + 2/(1 + a*x)]/2)))/3
```

3.21.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6495
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6545
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6551
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.21.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(89) = 178$.

Time = 0.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.79

method	result
parts	$-\frac{\operatorname{arccoth}(ax)^2}{3x^3} - \frac{2a^3\left(\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{2} + \frac{\operatorname{arccoth}(ax)}{2a^2x^2} - \ln(ax)\operatorname{arccoth}(ax) + \frac{\operatorname{arccoth}(ax)\ln(ax+1)}{2} + \frac{\ln(ax-1)}{4} - \frac{\ln(ax-}{4}\right.}{}$
derivativedivides	$a^3\left(-\frac{\operatorname{arccoth}(ax)^2}{3a^3x^3} - \frac{\operatorname{arccoth}(ax)\ln(ax-1)}{3} - \frac{\operatorname{arccoth}(ax)}{3a^2x^2} + \frac{2\ln(ax)\operatorname{arccoth}(ax)}{3} - \frac{\operatorname{arccoth}(ax)\ln(ax+1)}{3} -\right.$
default	$a^3\left(-\frac{\operatorname{arccoth}(ax)^2}{3a^3x^3} - \frac{\operatorname{arccoth}(ax)\ln(ax-1)}{3} - \frac{\operatorname{arccoth}(ax)}{3a^2x^2} + \frac{2\ln(ax)\operatorname{arccoth}(ax)}{3} - \frac{\operatorname{arccoth}(ax)\ln(ax+1)}{3} -\right.$

input
```
int(arccoth(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arccoth(a*x)^2/x^3-2/3*a^3*(1/2*arccoth(a*x)*ln(a*x-1)+1/2*arccoth(a*
x)/a^2/x^2-ln(a*x)*arccoth(a*x)+1/2*arccoth(a*x)*ln(a*x+1)+1/4*ln(a*x-1)-1
/4*ln(a*x+1)+1/2/a/x-1/2*dilog(1/2*a*x+1/2)-1/4*ln(a*x-1)*ln(1/2*a*x+1/2)+
1/8*ln(a*x-1)^2-1/8*ln(a*x+1)^2+1/4*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*
x+1/2)+1/2*dilog(a*x+1)+1/2*ln(a*x)*ln(a*x+1)+1/2*dilog(a*x))
```

3.21.5 Fricas [F]

$$\int \frac{\coth^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arcoth}\left(ax\right)^2}{x^4}\,dx$$

input
```
integrate(arccoth(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral(arccoth(a*x)^2/x^4, x)
```

3.21.6 Sympy [F]

$$\int \frac{\coth^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{acoth}^2\left(ax\right)}{x^4}\,dx$$

input
```
integrate(acoth(a*x)**2/x***4,x)
```

output
```
Integral(acoth(a*x)**2/x***4, x)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.71

$$\int \frac{\coth^{-1}(ax)^2}{x^4}\,dx$$
$$= \frac{1}{12}\left(4\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)a-4\left(\log\left(ax+1\right)\log\left(x\right)+\mathrm{Li}_2(-ax)\right)a+4\,(\right.$$
$$-\frac{1}{3}\left(a^2\log\left(a^2x^2-1\right)-a^2\log\left(x^2\right)+\frac{1}{x^2}\right)a\operatorname{arcoth}\left(ax\right)-\frac{\operatorname{arcoth}\left(ax\right)^2}{3\,x^3}$$

input
```
integrate(arccoth(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
1/12*(4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 4*(l
og(a*x + 1)*log(x) + dilog(-a*x))*a + 4*(log(-a*x + 1)*log(x) + dilog(a*x)
)*a + 2*a*log(a*x + 1) - 2*a*log(a*x - 1) + (a*x*log(a*x + 1)^2 - 2*a*x*lo
g(a*x + 1)*log(a*x - 1) - a*x*log(a*x - 1)^2 - 4)/x)*a^2 - 1/3*(a^2*log(a^
2*x^2 - 1) - a^2*log(x^2) + 1/x^2)*a*arccoth(a*x) - 1/3*arccoth(a*x)^2/x^3
```

3.21.8 Giac [F]

$$\int \frac{\coth^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arcoth}(ax)^2}{x^4}\,dx$$

input
```
integrate(arccoth(a*x)^2/x^4,x, algorithm="giac")
```

output
```
integrate(arccoth(a*x)^2/x^4, x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{acoth}(a\,x)^2}{x^4}\,dx$$

input
```
int(acoth(a*x)^2/x^4,x)
```

output
```
int(acoth(a*x)^2/x^4, x)
```

3.22 $\int \frac{\coth^{-1}(ax)^2}{x^5} dx$

3.22.1 Optimal result

Integrand size = 10, antiderivative size = 90

$$\int \frac{\coth^{-1}(ax)^2}{x^5} dx = -\frac{a^2}{12x^2} - \frac{a\coth^{-1}(ax)}{6x^3} - \frac{a^3\coth^{-1}(ax)}{2x} + \frac{1}{4}a^4\coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{4x^4} + \frac{2}{3}a^4\log(x) - \frac{1}{3}a^4\log\left(1-a^2x^2\right)$$

output
```
-1/12*a^2/x^2-1/6*a*arccoth(a*x)/x^3-1/2*a^3*arccoth(a*x)/x+1/4*a^4*arccot
h(a*x)^2-1/4*arccoth(a*x)^2/x^4+2/3*a^4*ln(x)-1/3*a^4*ln(-a^2*x^2+1)
```

3.22.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{\coth^{-1}(ax)^2}{x^5} dx = -\frac{a^2}{12x^2} - \frac{a(1+3a^2x^2)\coth^{-1}(ax)}{6x^3} + \frac{(-1+a^4x^4)\coth^{-1}(ax)^2}{4x^4} + \frac{2}{3}a^4\log(x) - \frac{1}{3}a^4\log\left(1-a^2x^2\right)$$

input
```
Integrate[ArcCoth[a*x]^2/x^5,x]
```

output
```
-1/12*a^2/x^2 - (a*(1 + 3*a^2*x^2)*ArcCoth[a*x])/(6*x^3) + ((-1 + a^4*x^4)
*ArcCoth[a*x]^2)/(4*x^4) + (2*a^4*Log[x])/3 - (a^4*Log[1 - a^2*x^2])/3
```

3.22.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.27, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {6453, 6545, 6453, 243, 54, 2009, 6545, 6453, 243, 47, 14, 16, 6511}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)^2}{x^5} \, dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{2}a\int \frac{\coth^{-1}(ax)}{x^4(1-a^2x^2)}dx-\frac{\coth^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{6545}$$

$$\frac{1}{2}a\left(a^2\int \frac{\coth^{-1}(ax)}{x^2(1-a^2x^2)}dx+\int \frac{\coth^{-1}(ax)}{x^4}dx\right)-\frac{\coth^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{2}a\left(a^2\int \frac{\coth^{-1}(ax)}{x^2(1-a^2x^2)}dx+\frac{1}{3}a\int \frac{1}{x^3(1-a^2x^2)}dx-\frac{\coth^{-1}(ax)}{3x^3}\right)-\frac{\coth^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\left(a^2\int \frac{\coth^{-1}(ax)}{x^2(1-a^2x^2)}dx+\frac{1}{6}a\int \frac{1}{x^4(1-a^2x^2)}dx^2-\frac{\coth^{-1}(ax)}{3x^3}\right)-\frac{\coth^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}a\left(a^2\int \frac{\coth^{-1}(ax)}{x^2(1-a^2x^2)}dx+\frac{1}{6}a\int \left(-\frac{a^4}{a^2x^2-1}+\frac{a^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{\coth^{-1}(ax)}{3x^3}\right)-\frac{\coth^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a\left(a^2\int \frac{\coth^{-1}(ax)}{x^2(1-a^2x^2)}dx+\frac{1}{6}a\left(a^2\log\left(x^2\right)-a^2\log\left(1-a^2x^2\right)-\frac{1}{x^2}\right)-\frac{\coth^{-1}(ax)}{3x^3}\right)-\frac{\coth^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{6545}$$

$$\frac{1}{2}a\left(a^2\left(a^2\int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx+\int \frac{\coth^{-1}(ax)}{x^2}dx\right)+\frac{1}{6}a\left(a^2\log\left(x^2\right)-a^2\log\left(1-a^2x^2\right)-\frac{1}{x^2}\right)-\frac{\coth^{-1}(ax)}{3x^3}\right)-\frac{\coth^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{2}a\left(a^2\left(a\int \frac{1}{x(1-a^2x^2)}dx+a^2\int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx-\frac{\coth^{-1}(ax)}{x}\right)+\frac{1}{6}a\left(a^2\log\left(x^2\right)-a^2\log\left(1-a^2x^2\right)-\frac{1}{x^2}\right)\right.$$
$$\frac{\coth^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\left(a^2\left(\frac{1}{2}a\int \frac{1}{x^2(1-a^2x^2)}dx^2+a^2\int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx-\frac{\coth^{-1}(ax)}{x}\right)+\frac{1}{6}a\left(a^2\log\left(x^2\right)-a^2\log\left(1-a^2x^2\right)-\frac{1}{x}\right.\right.$$
$$\frac{\coth^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}a\left(a^2\left(\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx^2+\int \frac{1}{x^2}dx^2\right)+a^2\int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx-\frac{\coth^{-1}(ax)}{x}\right)+\frac{1}{6}a\left(a^2\log\left(x^2\right)-a^2\log\left(1\right.\right.\right.$$
$$\frac{\coth^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}a\left(a^2\left(\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx^2+\log\left(x^2\right)\right)+a^2\int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx-\frac{\coth^{-1}(ax)}{x}\right)+\frac{1}{6}a\left(a^2\log\left(x^2\right)-a^2\log\left(1-\right.\right.\right.$$
$$\frac{\coth^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}a\left(a^2\left(a^2\int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)-\frac{\coth^{-1}(ax)}{x}\right)+\frac{1}{6}a\left(a^2\log\left(x^2\right)-a^2\log\left(1-a^2x\right.\right.\right.$$
$$\frac{\coth^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{6511}$$

$$\frac{1}{2}a\left(\frac{1}{6}a\left(a^2\log\left(x^2\right)-a^2\log\left(1-a^2x^2\right)-\frac{1}{x^2}\right)+a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\coth^{-1}(ax)^2-\frac{\coth^{-1}(\right.\right.}{x}$$
$$\frac{\coth^{-1}(ax)^2}{4x^4}$$

input
```
Int[ArcCoth[a*x]^2/x^5,x]
```

3.22. $\int \frac{\coth^{-1}(ax)^2}{x^5}\,dx$

output
```
-1/4*ArcCoth[a*x]^2/x^4 + (a*(-1/3*ArcCoth[a*x]/x^3 + a^2*(-(ArcCoth[a*x]/
x) + (a*ArcCoth[a*x]^2)/2 + (a*(Log[x^2] - Log[1 - a^2*x^2]))/2) + (a*(-x^
(-2) + a^2*Log[x^2] - a^2*Log[1 - a^2*x^2]))/6))/2
```

3.22.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6545
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.22.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.12

method	result
parallelrisch	$\frac{3a^4x^4\operatorname{arccoth}(ax)^2+8\ln(x)x^4a^4-8\ln(ax-1)x^4a^4-8a^4x^4\operatorname{arccoth}(ax)-a^4x^4-6a^3x^3\operatorname{arccoth}(ax)-a^2x^2-2ax\operatorname{arccoth}}{12x^4}$
parts	$-\frac{\operatorname{arccoth}(ax)^2}{4x^4}-\frac{a^4\left(\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{2}+\frac{\operatorname{arccoth}(ax)}{3a^3x^3}+\frac{\operatorname{arccoth}(ax)}{ax}-\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{2}-\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}+\ln(a\right.}{2}$
derivativedivides	$a^4\left(-\frac{\operatorname{arccoth}(ax)^2}{4a^4x^4}-\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{4}-\frac{\operatorname{arccoth}(ax)}{6a^3x^3}-\frac{\operatorname{arccoth}(ax)}{2ax}+\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{4}+\frac{\ln(ax-}{}\right.$
default	$a^4\left(-\frac{\operatorname{arccoth}(ax)^2}{4a^4x^4}-\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{4}-\frac{\operatorname{arccoth}(ax)}{6a^3x^3}-\frac{\operatorname{arccoth}(ax)}{2ax}+\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{4}+\frac{\ln(ax-}{}\right.$
risch	$\frac{(a^4x^4-1)\ln(ax+1)^2}{16x^4}-\frac{(3\ln(ax-1)x^4a^4+6a^3x^3+2ax-3\ln(ax-1))\ln(ax+1)}{24x^4}+\frac{3a^4x^4\ln(ax-1)^2+32\ln(x)x^4a^4-1}{}$

input
```
int(arccoth(a*x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*a^4*x^4*arccoth(a*x)^2+8*ln(x)*x^4*a^4-8*ln(a*x-1)*x^4*a^4-8*a^4*x
^4*arccoth(a*x)-a^4*x^4-6*a^3*x^3*arccoth(a*x)-a^2*x^2-2*a*x*arccoth(a*x)-
3*arccoth(a*x)^2)/x^4
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{\coth^{-1}(ax)^2}{x^5}\,dx = -\frac{16\,a^4x^4\log\left(a^2x^2-1\right)-32\,a^4x^4\log\left(x\right)+4\,a^2x^2-3\left(a^4x^4-1\right)\log\left(\frac{ax+1}{ax-1}\right)^2+4\left(3\,a^3x^3+ax\right)\log\left(\frac{ax+1}{ax-1}\right)}{48\,x^4}$$

input
```
integrate(arccoth(a*x)^2/x^5,x, algorithm="fricas")
```

output
```
-1/48*(16*a^4*x^4*log(a^2*x^2 - 1) - 32*a^4*x^4*log(x) + 4*a^2*x^2 - 3*(a^
4*x^4 - 1)*log((a*x + 1)/(a*x - 1))^2 + 4*(3*a^3*x^3 + a*x)*log((a*x + 1)/
(a*x - 1)))/x^4
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(ax)^2}{x^5}\,dx = \frac{2a^4\log(x)}{3} - \frac{2a^4\log(ax+1)}{3} + \frac{a^4\operatorname{acoth}^2(ax)}{4} + \frac{2a^4\operatorname{acoth}(ax)}{3} - \frac{a^3\operatorname{acoth}(ax)}{2x} - \frac{a^2}{12x^2} - \frac{a\operatorname{acoth}(ax)}{6x^3} - \frac{\operatorname{acoth}^2(ax)}{4x^4}$$

input
```
integrate(acoth(a*x)**2/x**5,x)
```

output
```
2*a**4*log(x)/3 - 2*a**4*log(a*x + 1)/3 + a**4*acoth(a*x)**2/4 + 2*a**4*ac
oth(a*x)/3 - a**3*acoth(a*x)/(2*x) - a**2/(12*x**2) - a*acoth(a*x)/(6*x**3
) - acoth(a*x)**2/(4*x**4)
```

3.22.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(76) = 152.

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.71

$$\int \frac{\coth^{-1}(ax)^2}{x^5}\,dx = \frac{1}{48}\left(32\,a^2\log(x) - \frac{3\,a^2x^2\log(ax+1)^2 + 3\,a^2x^2\log(ax-1)^2 + 16\,a^2x^2\log(ax-1) - 2\,(3\,a^2x^2\log(ax -}{x^2}\right. + \frac{1}{12}\left(3\,a^3\log(ax+1) - 3\,a^3\log(ax-1) - \frac{2\,(3\,a^2x^2+1)}{x^3}\right)a\operatorname{arcoth}(ax) - \frac{\operatorname{arcoth}(ax)^2}{4\,x^4}$$

input
```
integrate(arccoth(a*x)^2/x^5,x, algorithm="maxima")
```

output
```
1/48*(32*a^2*log(x) - (3*a^2*x^2*log(a*x + 1)^2 + 3*a^2*x^2*log(a*x - 1)^2
 + 16*a^2*x^2*log(a*x - 1) - 2*(3*a^2*x^2*log(a*x - 1) - 8*a^2*x^2)*log(a*
x + 1) + 4)/x^2)*a^2 + 1/12*(3*a^3*log(a*x + 1) - 3*a^3*log(a*x - 1) - 2*(
3*a^2*x^2 + 1)/x^3)*a*arccoth(a*x) - 1/4*arccoth(a*x)^2/x^4
```

3.22.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(76) = 152$.

Time = 0.28 (sec) , antiderivative size = 319, normalized size of antiderivative = 3.54

$$\int \frac{\coth^{-1}(ax)^2}{x^5}\,dx$$
$$= \frac{1}{6}\left(4\,a^3\log\left(\frac{ax+1}{ax-1}+1\right)-4\,a^3\log\left(\frac{ax+1}{ax-1}\right)+\frac{2\,(ax+1)a^3}{(ax-1)\left(\frac{(ax+1)^2}{(ax-1)^2}+\frac{2\,(ax+1)}{ax-1}+1\right)}+\frac{3\left(\frac{(ax+1)^3a^3}{(ax-1)^3}+\cdots\right.}{\frac{(ax+1)^4}{(ax-1)^4}+\frac{4\,(ax+1)^3}{(ax-1)^3}\cdots}\right.$$

input
```
integrate(arccoth(a*x)^2/x^5,x, algorithm="giac")
```

output
```
1/6*(4*a^3*log((a*x + 1)/(a*x - 1) + 1) - 4*a^3*log((a*x + 1)/(a*x - 1)) +
 2*(a*x + 1)*a^3/((a*x - 1)*((a*x + 1)^2/(a*x - 1)^2 + 2*(a*x + 1)/(a*x -
1) + 1)) + 3*((a*x + 1)^3*a^3/(a*x - 1)^3 + (a*x + 1)*a^3/(a*x - 1))*log((
a*x + 1)/(a*x - 1))^2/((a*x + 1)^4/(a*x - 1)^4 + 4*(a*x + 1)^3/(a*x - 1)^3
 + 6*(a*x + 1)^2/(a*x - 1)^2 + 4*(a*x + 1)/(a*x - 1) + 1) + 2*(3*(a*x + 1)
^2*a^3/(a*x - 1)^2 + 3*(a*x + 1)*a^3/(a*x - 1) + 2*a^3)*log((a*x + 1)/(a*x
 - 1))/((a*x + 1)^3/(a*x - 1)^3 + 3*(a*x + 1)^2/(a*x - 1)^2 + 3*(a*x + 1)/
(a*x - 1) + 1))*a
```

3.22.9 Mupad [B] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.18

$$\int \frac{\coth^{-1}(ax)^2}{x^5}\,dx = \frac{2\,a^4\,\ln(x)}{3}+\ln\left(\frac{1}{a\,x}+1\right)^2\left(\frac{a^4}{16}-\frac{1}{16\,x^4}\right)+\ln\left(1-\frac{1}{a\,x}\right)^2\left(\frac{a^4}{16}-\frac{1}{16\,x^4}\right)+\ln\left(1-\frac{1}{a\,x}\right)\left(\frac{24\,a^3\,x^3-12\,a^2\,x^2+8\,a\,x-6}{192\,x^4}+\frac{24\,a^3\,x^3+12\,a^2\,x^2+8\,a\,x+6}{192\,x^4}-\ln\left(\frac{1}{a\,x}+1\right)\left(\frac{a^4}{8}-\frac{1}{8\,x^4}\right)\right)-\frac{a^4\,\ln\left(a^2\,x^2-1\right)}{3}-\frac{a^2}{12\,x^2}-\frac{a\,\ln\left(\frac{1}{a\,x}+1\right)\left(\frac{a^2\,x^2}{4}+\frac{1}{12}\right)}{x^3}$$

input
```
int(acoth(a*x)^2/x^5,x)
```

output
```
(2*a^4*log(x))/3 + log(1/(a*x) + 1)^2*(a^4/16 - 1/(16*x^4)) + log(1 - 1/(a
*x))^2*(a^4/16 - 1/(16*x^4)) + log(1 - 1/(a*x))*((8*a*x - 12*a^2*x^2 + 24*
a^3*x^3 - 6)/(192*x^4) + (8*a*x + 12*a^2*x^2 + 24*a^3*x^3 + 6)/(192*x^4) -
 log(1/(a*x) + 1)*(a^4/8 - 1/(8*x^4))) - (a^4*log(a^2*x^2 - 1))/3 - a^2/(1
2*x^2) - (a*log(1/(a*x) + 1)*((a^2*x^2)/4 + 1/12))/x^3
```

3.22. $\int \frac{\coth^{-1}(ax)^2}{x^5}\,dx$

3.23 $\int x^5 \coth^{-1}(ax)^3 \, dx$

3.23.1 Optimal result

Integrand size = 10, antiderivative size = 186

$$\int x^5 \coth^{-1}(ax)^3 \, dx = \frac{19x}{60a^5} + \frac{x^3}{60a^3} + \frac{4x^2 \coth^{-1}(ax)}{15a^4} + \frac{x^4 \coth^{-1}(ax)}{20a^2} + \frac{23 \coth^{-1}(ax)^2}{30a^6} + \frac{x \coth^{-1}(ax)^2}{2a^5} + \frac{x^3 \coth^{-1}(ax)^2}{6a^3} + \frac{x^5 \coth^{-1}(ax)^2}{10a} - \frac{\coth^{-1}(ax)^3}{6a^6} + \frac{1}{6}x^6 \coth^{-1}(ax)^3 - \frac{19\text{arctanh}(ax)}{60a^6} - \frac{23 \coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{15a^6} - \frac{23 \,\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{30a^6}$$

output
```
19/60*x/a^5+1/60*x^3/a^3+4/15*x^2*arccoth(a*x)/a^4+1/20*x^4*arccoth(a*x)/a
^2+23/30*arccoth(a*x)^2/a^6+1/2*x*arccoth(a*x)^2/a^5+1/6*x^3*arccoth(a*x)^
2/a^3+1/10*x^5*arccoth(a*x)^2/a-1/6*arccoth(a*x)^3/a^6+1/6*x^6*arccoth(a*x
)^3-19/60*arctanh(a*x)/a^6-23/15*arccoth(a*x)*ln(2/(-a*x+1))/a^6-23/30*pol
ylog(2,1-2/(-a*x+1))/a^6
```

3.23.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.63

$$\int x^5 \coth^{-1}(ax)^3 \, dx$$
$$= \frac{ax(19 + a^2x^2) + 2(-23 + 15ax + 5a^3x^3 + 3a^5x^5) \coth^{-1}(ax)^2 + 10(-1 + a^6x^6) \coth^{-1}(ax)^3 + \coth^{-1}(ax}{60a^6}$$

input
```
Integrate[x^5*ArcCoth[a*x]^3,x]
```

output
```
(a*x*(19 + a^2*x^2) + 2*(-23 + 15*a*x + 5*a^3*x^3 + 3*a^5*x^5)*ArcCoth[a*x
]^2 + 10*(-1 + a^6*x^6)*ArcCoth[a*x]^3 + ArcCoth[a*x]*(-19 + 16*a^2*x^2 +
3*a^4*x^4 - 92*Log[1 - E^(-2*ArcCoth[a*x])]) + 46*PolyLog[2, E^(-2*ArcCoth
[a*x])])/(60*a^6)
```

3.23.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 421 vs. 2(186) = 372.

Time = 2.90 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.26, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.700, Rules used = {6453, 6543, 6453, 6543, 6453, 254, 2009, 6543, 6437, 6453, 262, 219, 6511, 6547, 6471, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \coth^{-1}(ax)^3 \, dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{6}x^6 \coth^{-1}(ax)^3 - \frac{1}{2}a \int \frac{x^6 \coth^{-1}(ax)^2}{1 - a^2x^2} dx$$

$$\Big\downarrow \text{6543}$$

$$\frac{1}{6}x^6 \coth^{-1}(ax)^3 - \frac{1}{2}a \left(\frac{\int \frac{x^4 \coth^{-1}(ax)^2}{1-a^2x^2} dx}{a^2} - \frac{\int x^4 \coth^{-1}(ax)^2 dx}{a^2} \right)$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{6}x^6\coth^{-1}(ax)^3-\frac{1}{2}a\left(\frac{\int\frac{x^4\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{5}x^5\coth^{-1}(ax)^2-\frac{2}{5}a\int\frac{x^5\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}\right)$$

$\downarrow$ 6543

$$\frac{1}{6}x^6\coth^{-1}(ax)^3-\frac{1}{2}a\left(\frac{\frac{\int\frac{x^2\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\int x^2\coth^{-1}(ax)^2dx}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\coth^{-1}(ax)^2-\frac{2}{5}a\left(\frac{\int\frac{x^3\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x^3\coth^{-1}(ax)dx}{a^2}\right)}{a^2}\right)$$

$\downarrow$ 6453

$$\frac{1}{6}x^6\coth^{-1}(ax)^3-\frac{1}{2}a\left(\frac{\frac{\int\frac{x^2\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\int\frac{x^3\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\coth^{-1}(ax)^2-\frac{2}{5}a\left(\frac{\int\frac{x^3\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)\cdots}{a^2}\right.}{a^2}\right.$$

$\downarrow$ 254

$$\frac{1}{6}x^6\coth^{-1}(ax)^3-\frac{1}{2}a\left(\frac{\frac{\int\frac{x^2\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\int\frac{x^3\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\coth^{-1}(ax)^2-\frac{2}{5}a\left(\frac{\int\frac{x^3\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)\cdots}{\cdots}\right.}{a^2}\right.$$

$\downarrow$ 2009

$$\frac{1}{6}x^6\coth^{-1}(ax)^3-\frac{1}{2}a\left(\frac{\frac{\int\frac{x^2\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\int\frac{x^3\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\coth^{-1}(ax)^2-\frac{2}{5}a\left(\frac{\int\frac{x^3\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)\cdots}{\cdots}\right.}{a^2}\right.$$

$\downarrow$ 6543

3.23. $\int x^5\coth^{-1}(ax)^3\,dx$

$$\frac{1}{6}x^6\coth^{-1}(ax)^3-\frac{1}{2}a\left(\frac{\frac{\frac{\int\frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\int\coth^{-1}(ax)^2dx}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\left(\frac{\int\frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x\coth^{-1}(ax)dx}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\coth^{-1}(ax)^2-\frac{2}{5}a\left(\cdots\right.}{\cdots}\right.$$

$\downarrow$ 6437

$$\frac{1}{6}x^6\coth^{-1}(ax)^3-\frac{1}{2}a\left(\frac{\frac{\frac{\int\frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{x\coth^{-1}(ax)^2-2a\int\frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\left(\frac{\int\frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x\coth^{-1}(ax)dx}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{5}x^5\coth^{-1}(\cdots}{\cdots}\right.$$

$\downarrow$ 6453

$$\frac{1}{6}x^6\coth^{-1}(ax)^3-\frac{1}{2}a\left(\frac{\frac{\frac{\int\frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{x\coth^{-1}(ax)^2-2a\int\frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\left(\frac{\int\frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\int\frac{x^2}{1-a^2x^2}dx}{a^2}\right)}{a^2}}{a^2}-\cdots\right.$$

$\downarrow$ 262

$$\frac{1}{6}x^6\coth^{-1}(ax)^3-\frac{1}{2}a\left(\frac{\frac{\frac{\int\frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{x\coth^{-1}(ax)^2-2a\int\frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\left(\frac{\int\frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\left(\frac{\int\frac{1}{1-a^2x^2}dx}{a^2}-\frac{x}{a^2}\right)}{a^2}\cdots\right.}{a^2}}{a^2}\cdots\right.$$

$\downarrow$ 219

3.23. $\int x^5\coth^{-1}(ax)^3\,dx$

$$\frac{1}{6}x^6\coth^{-1}(ax)^3-\frac{1}{2}a\left(\frac{\frac{\frac{\int\frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{x\coth^{-1}(ax)^2-2a\int\frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\left(\frac{\int\frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}\right.}{a^2}\cdots}{a^2}\right.$$

$\big\downarrow$ 6511

$$\frac{1}{6}x^6\coth^{-1}(ax)^3-\frac{1}{2}a\left(\frac{\frac{\frac{\coth^{-1}(ax)^3}{3a^3}-\frac{x\coth^{-1}(ax)^2-2a\int\frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\left(\frac{\int\frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{a^2}\right)}{a^2}\cdots}{a^2}\right.$$

$\big\downarrow$ 6547

$$\frac{1}{6}x^6\coth^{-1}(ax)^3-\frac{1}{2}a\left(\frac{\frac{\frac{\coth^{-1}(ax)^3}{3a^3}-\frac{x\coth^{-1}(ax)^2-2a\left(\frac{\int\frac{\coth^{-1}(ax)}{1-ax}dx}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\left(\frac{\frac{\int\frac{\coth^{-1}(ax)}{1-ax}dx}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax\cdots}{\cdots}\right.}{a^2}\cdots}{a^2}\right.$$

$\big\downarrow$ 6471

$$\frac{1}{6}x^6\coth^{-1}(ax)^3-\frac{1}{2}a\left(\frac{\frac{\frac{\coth^{-1}(ax)^3}{3a^3}-\frac{x\coth^{-1}(ax)^2-2a\left(\frac{\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}-\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\left(\frac{\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)\cdots}{a}}{a}\cdots\right.}{a^2}\cdots}{\cdots}\right.$$

3.23. $\int x^5\coth^{-1}(ax)^3\,dx$

$\Big\downarrow$ 2849

$$\frac{1}{6}x^6\coth^{-1}(ax)^3-\frac{1}{2}a\left(\frac{\frac{\coth^{-1}(ax)^3}{3a^3}-\frac{x\coth^{-1}(ax)^2-2a\left(\frac{\frac{\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\left(\frac{\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}\cdots\right.}{a^2}\cdots\right.$$

$\Big\downarrow$ 2752

$$\frac{1}{6}x^6\coth^{-1}(ax)^3-\frac{1}{2}a\left(\frac{\frac{\coth^{-1}(ax)^3}{3a^3}-\frac{x\coth^{-1}(ax)^2-2a\left(\frac{\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}+\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}\cdots\right.}{a^2}\cdots\right.$$

input

```
Int[x^5*ArcCoth[a*x]^3,x]
```

output

```
(x^6*ArcCoth[a*x]^3)/6 - (a*(-(((x^5*ArcCoth[a*x]^2)/5 - (2*a*(-(((x^4*Arc
Coth[a*x])/4 - (a*(-(x/a^4) - x^3/(3*a^2) + ArcTanh[a*x]/a^5))/4)/a^2) + (
-(((x^2*ArcCoth[a*x])/2 - (a*(-(x/a^2) + ArcTanh[a*x]/a^3))/2)/a^2) + (-1/
2*ArcCoth[a*x]^2/a^2 + ((ArcCoth[a*x]*Log[2/(1 - a*x)])/a + PolyLog[2, 1 -
 2/(1 - a*x)]/(2*a))/a)/a^2)/a^2))/5)/a^2) + (-(((x^3*ArcCoth[a*x]^2)/3 -
(2*a*(-(((x^2*ArcCoth[a*x])/2 - (a*(-(x/a^2) + ArcTanh[a*x]/a^3))/2)/a^2)
+ (-1/2*ArcCoth[a*x]^2/a^2 + ((ArcCoth[a*x]*Log[2/(1 - a*x)])/a + PolyLog[
2, 1 - 2/(1 - a*x)]/(2*a))/a)/a^2))/3)/a^2) + (ArcCoth[a*x]^3/(3*a^3) - (x
*ArcCoth[a*x]^2 - 2*a*(-1/2*ArcCoth[a*x]^2/a^2 + ((ArcCoth[a*x]*Log[2/(1 -
 a*x)])/a + PolyLog[2, 1 - 2/(1 - a*x)]/(2*a))/a))/a^2)/a^2)/a^2))/2
```

3.23. $\int x^5\coth^{-1}(ax)^3\,dx$

3.23.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6471
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6543
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6547
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.23.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.82 (sec) , antiderivative size = 2133, normalized size of antiderivative = 11.47

method	result	size
parts	Expression too large to display	2133
derivativedivides	Expression too large to display	2135
default	Expression too large to display	2135

input
```
int(x^5*arccoth(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*arccoth(a*x)^3+1/2/a^6*(arccoth(a*x)^2*a*x+3/20*(2*((a*x-1)/(a*x+1
))^(1/2)*a^2*x^2+2*((a*x-1)/(a*x+1))^(1/2)*a*x-2*a^2*x^2+1)*arccoth(a*x)*(
a*x+1)*(a*x-1)-3/20*(2*((a*x-1)/(a*x+1))^(1/2)*a^2*x^2+2*((a*x-1)/(a*x+1))
^(1/2)*a*x+2*a^2*x^2-1)*arccoth(a*x)*(a*x+1)*(a*x-1)+1/10*(2*((a*x-1)/(a*x
+1))^(1/2)*a^2*x^2+2*a^2*x^2-((a*x-1)/(a*x+1))^(1/2)-2*a*x)*(a*x-1)*arccot
h(a*x)*(a*x+1)+1/3*a^3*x^3*arccoth(a*x)^2+1/5*a^5*x^5*arccoth(a*x)^2-1/4*I
*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*csgn(I/(a*x-1)*(a*
x+1)/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2-46/15*arccoth(a*x)*ln(1+1/((a*x-1
)/(a*x+1))^(1/2))-1/3*arccoth(a*x)^3-41/120*(((a*x-1)/(a*x+1))^(1/2)*a*x+(
(a*x-1)/(a*x+1))^(1/2)-a*x-1)*arccoth(a*x)-1/40*(a*x-1)/(((a*x-1)/(a*x+1))
^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)-a*x)-41/60/(((a*x-1)/(a*x+1))^(1/2)-1)*
((a*x-1)/(a*x+1))^(1/2)+41/120*(((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+
1))^(1/2)+a*x+1)*arccoth(a*x)-41/60/(((a*x-1)/(a*x+1))^(1/2)+1)*((a*x-1)/(
a*x+1))^(1/2)+1/40*(a*x-1)/(((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^
(1/2)+a*x)+23/15*arccoth(a*x)^2-1/4*I*Pi*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(
a*x-1)-1))^3*arccoth(a*x)^2+1/4*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I/(a
*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))^2*arccoth(a*x)^2+1/2*I*Pi*csgn(I/((a*x-
1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arccoth(a*x)^2-1/4*I*Pi*csgn(
I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)^2+1/4*I*
Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))^...
```

3.23.5 Fricas [F]

$$\int x^5 \coth^{-1}(ax)^3 \, dx = \int x^5 \operatorname{arcoth}(ax)^3 \, dx$$

input
```
integrate(x^5*arccoth(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^5*arccoth(a*x)^3, x)
```

3.23.6 Sympy [F]

$$\int x^5 \coth^{-1}(ax)^3 \, dx = \int x^5 \operatorname{acoth}^3(ax) \, dx$$

input
```
integrate(x**5*acoth(a*x)**3,x)
```

output
```
Integral(x**5*acoth(a*x)**3, x)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.55

$$\int x^5 \coth^{-1}(ax)^3 \, dx = \frac{1}{6} x^6 \operatorname{arcoth}(ax)^3 + \frac{1}{60} a \left(\frac{2(3a^4x^5 + 5a^2x^3 + 15x)}{a^6} - \frac{15 \log(ax+1)}{a^7} + \frac{15 \log(ax-1)}{a^7} \right) \operatorname{arcoth}(ax)^2 + \frac{1}{240} a \left(\frac{\frac{4a^3x^3 + (15 \log(ax-1) - 46)\log(ax+1)^2 - 5\log(ax+1)^3 + 5\log(ax-1)^3 + 76ax - \left(15\log(ax-1)^2 - 92\log(ax-1)\right)\log(ax+1) + 46 \log}{a}}{a^6} \right.$$

input
```
integrate(x^5*arccoth(a*x)^3,x, algorithm="maxima")
```

output
```
1/6*x^6*arccoth(a*x)^3 + 1/60*a*(2*(3*a^4*x^5 + 5*a^2*x^3 + 15*x)/a^6 - 15
*log(a*x + 1)/a^7 + 15*log(a*x - 1)/a^7)*arccoth(a*x)^2 + 1/240*a*(((4*a^3
*x^3 + (15*log(a*x - 1) - 46)*log(a*x + 1)^2 - 5*log(a*x + 1)^3 + 5*log(a*
x - 1)^3 + 76*a*x - (15*log(a*x - 1)^2 - 92*log(a*x - 1))*log(a*x + 1) + 4
6*log(a*x - 1)^2 + 38*log(a*x - 1))/a - 184*(log(a*x - 1)*log(1/2*a*x + 1/
2) + dilog(-1/2*a*x + 1/2))/a - 38*log(a*x + 1)/a)/a^6 + 2*(6*a^4*x^4 + 32
*a^2*x^2 - 2*(15*log(a*x - 1) - 46)*log(a*x + 1) + 15*log(a*x + 1)^2 + 15*
log(a*x - 1)^2 + 92*log(a*x - 1))*arccoth(a*x)/a^7)
```

3.23.8 Giac [F]

$$\int x^5 \coth^{-1}(ax)^3 \, dx = \int x^5 \operatorname{arcoth}(ax)^3 \, dx$$

input
```
integrate(x^5*arccoth(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^5*arccoth(a*x)^3, x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int x^5 \coth^{-1}(ax)^3 \, dx = \int x^5 \operatorname{acoth}(a\,x)^3 \, dx$$

input
```
int(x^5*acoth(a*x)^3,x)
```

output
```
int(x^5*acoth(a*x)^3, x)
```

3.24 $\int x^4 \coth^{-1}(ax)^3 \, dx$

3.24.1 Optimal result

Integrand size = 10, antiderivative size = 196

$$\begin{aligned}\int x^4 \coth^{-1}(ax)^3 \, dx = {} & \frac{x^2}{20a^3} + \frac{9x \coth^{-1}(ax)}{10a^4} + \frac{x^3 \coth^{-1}(ax)}{10a^2} - \frac{9 \coth^{-1}(ax)^2}{20a^5} \\ & + \frac{3x^2 \coth^{-1}(ax)^2}{10a^3} + \frac{3x^4 \coth^{-1}(ax)^2}{20a} + \frac{\coth^{-1}(ax)^3}{5a^5} \\ & + \frac{1}{5}x^5 \coth^{-1}(ax)^3 - \frac{3 \coth^{-1}(ax)^2 \log\left(\frac{2}{1-ax}\right)}{5a^5} + \frac{\log(1-a^2x^2)}{2a^5} \\ & - \frac{3 \coth^{-1}(ax) \operatorname{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{5a^5} + \frac{3 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1-ax}\right)}{10a^5}\end{aligned}$$

output

```
1/20*x^2/a^3+9/10*x*arccoth(a*x)/a^4+1/10*x^3*arccoth(a*x)/a^2-9/20*arccot
h(a*x)^2/a^5+3/10*x^2*arccoth(a*x)^2/a^3+3/20*x^4*arccoth(a*x)^2/a+1/5*arc
coth(a*x)^3/a^5+1/5*x^5*arccoth(a*x)^3-3/5*arccoth(a*x)^2*ln(2/(-a*x+1))/a
^5+1/2*ln(-a^2*x^2+1)/a^5-3/5*arccoth(a*x)*polylog(2,1-2/(-a*x+1))/a^5+3/1
0*polylog(3,1-2/(-a*x+1))/a^5
```

3.24.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.91

$$\int x^4 \coth^{-1}(ax)^3 \, dx$$

$$= \frac{-2 - i\pi^3 + 2a^2x^2 + 36ax\coth^{-1}(ax) + 4a^3x^3\coth^{-1}(ax) - 18\coth^{-1}(ax)^2 + 12a^2x^2\coth^{-1}(ax)^2 + 6a^4x}{}$$

input
```
Integrate[x^4*ArcCoth[a*x]^3,x]
```

output
```
(-2 - I*Pi^3 + 2*a^2*x^2 + 36*a*x*ArcCoth[a*x] + 4*a^3*x^3*ArcCoth[a*x] -
18*ArcCoth[a*x]^2 + 12*a^2*x^2*ArcCoth[a*x]^2 + 6*a^4*x^4*ArcCoth[a*x]^2 +
 8*ArcCoth[a*x]^3 + 8*a^5*x^5*ArcCoth[a*x]^3 - 24*ArcCoth[a*x]^2*Log[1 - E
^(2*ArcCoth[a*x])] - 40*Log[1/Sqrt[1 - 1/(a^2*x^2)]] - 40*Log[1/(a*x)] - 2
4*ArcCoth[a*x]*PolyLog[2, E^(2*ArcCoth[a*x])] + 12*PolyLog[3, E^(2*ArcCoth
[a*x])])/(40*a^5)
```

3.24.3 Rubi [A] (verified)

Time = 2.57 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.55, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.600, Rules used = {6453, 6543, 6453, 6543, 6453, 243, 49, 2009, 6543, 6437, 240, 6511, 6547, 6471, 6621, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \coth^{-1}(ax)^3 \, dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{5}x^5\coth^{-1}(ax)^3 - \frac{3}{5}a\int \frac{x^5\coth^{-1}(ax)^2}{1-a^2x^2}dx$$

$$\Big\downarrow \text{6543}$$

$$\frac{1}{5}x^5\coth^{-1}(ax)^3 - \frac{3}{5}a\left(\frac{\int \frac{x^3\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2} - \frac{\int x^3\coth^{-1}(ax)^2dx}{a^2}\right)$$

$\downarrow$ 6453

$$\frac{1}{5}x^5\coth^{-1}(ax)^3-\frac{3}{5}a\left(\frac{\int\frac{x^3\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)^2-\frac{1}{2}a\int\frac{x^4\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}\right)$$

$\downarrow$ 6543

$$\frac{1}{5}x^5\coth^{-1}(ax)^3-\frac{3}{5}a\left(\frac{\frac{\int\frac{x\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\int x\coth^{-1}(ax)^2dx}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)^2-\frac{1}{2}a\left(\frac{\int\frac{x^2\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int x^2\coth^{-1}(ax)dx}{a^2}\right)}{a^2}\right)$$

$\downarrow$ 6453

$$\frac{1}{5}x^5\coth^{-1}(ax)^3-\frac{3}{5}a\left(\frac{\frac{\int\frac{x\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)^2-a\int\frac{x^2\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)^2-\frac{1}{2}a\left(\frac{\int\frac{x^2\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{3}\cdots}{a^2}\right.\cdots}{a^2}\cdots\right.$$

$\downarrow$ 243

$$\frac{1}{5}x^5\coth^{-1}(ax)^3-\frac{3}{5}a\left(\frac{\frac{\int\frac{x\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)^2-a\int\frac{x^2\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)^2-\frac{1}{2}a\left(\frac{\int\frac{x^2\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{6}\cdots}{a^2}\right.\cdots}{a^2}\cdots\right.$$

$\downarrow$ 49

$$\frac{1}{5}x^5\coth^{-1}(ax)^3-\frac{3}{5}a\left(\frac{\frac{\int\frac{x\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)^2-a\int\frac{x^2\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)^2-\frac{1}{2}a\left(\frac{\int\frac{x^2\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)-\cdots}{\cdots}\right.\cdots}{a^2}\cdots\right.$$

$\downarrow$ 2009

3.24. $\int x^4\coth^{-1}(ax)^3\,dx$

$$\frac{1}{5}x^5\coth^{-1}(ax)^3-$$
$$\frac{3}{5}a\left(\frac{\frac{\int\frac{x\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)^2-a\int\frac{x^2\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)^2-\frac{1}{2}a\left(\frac{\int\frac{x^2\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)-\ldots}{\ldots}\right.}{a^2}\ldots\right.$$

$\big\downarrow$ 6543

$$\frac{1}{5}x^5\coth^{-1}(ax)^3-$$
$$\frac{3}{5}a\left(\frac{\frac{\int\frac{x\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)^2-a\left(\frac{\int\frac{\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int\coth^{-1}(ax)dx}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)^2-\frac{1}{2}a\left(\frac{\frac{\int\frac{\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\int\coth\ldots}{\ldots}}{a^2}\ldots\right.}{\ldots}\right.$$

$\big\downarrow$ 6437

$$\frac{1}{5}x^5\coth^{-1}(ax)^3-$$
$$\frac{3}{5}a\left(\frac{\frac{\int\frac{x\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)^2-a\left(\frac{\int\frac{\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{x\coth^{-1}(ax)-a\int\frac{x}{1-a^2x^2}dx}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)^2-\frac{1}{2}a\left(\frac{\frac{\int\frac{\coth^{-1}(ax)}{1-a^2x^2}\ldots}{\ldots}}{a^2}\ldots\right.}{\ldots}\right.$$

$\big\downarrow$ 240

$$\frac{1}{5}x^5\coth^{-1}(ax)^3-$$
$$\frac{3}{5}a\left(\frac{\frac{\int\frac{x\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)^2-a\left(\frac{\int\frac{\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{\log\left(1-a^2x^2\right)}{2a}+x\coth^{-1}(ax)}{a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{4}x^4\coth^{-1}(ax)^2-\frac{1}{2}a\left(\frac{\frac{\int\frac{\coth^{-1}(ax\ldots}{1-a^2x^2}\ldots}{\ldots}}{a^2}\ldots\right.}{\ldots}\right.$$

$\big\downarrow$ 6511

3.24. $\int x^4\coth^{-1}(ax)^3\,dx$

$$\frac{1}{5}x^5\coth^{-1}(ax)^3 - \frac{3}{5}a\left(\frac{\frac{\int\frac{x\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{2}x^2\coth^{-1}(ax)^2 - a\left(\frac{\coth^{-1}(ax)^2}{2a^3} - \frac{\frac{\log(1-a^2x^2)}{2a}+x\coth^{-1}(ax)}{a^2}\right)}{a^2}}{a^2} - \frac{\frac{1}{4}x^4\coth^{-1}(ax)^2 - \frac{1}{2}a\left(\frac{\coth^{-1}(ax)^2}{2a^3} - \cdots\right.}{\cdots}\right.$$

$\downarrow$ 6547

$$\frac{1}{5}x^5\coth^{-1}(ax)^3 - \frac{3}{5}a\left(\frac{\frac{\frac{\int\frac{\coth^{-1}(ax)^2}{1-ax}dx}{a} - \frac{\coth^{-1}(ax)^3}{3a^2}}{a^2} - \frac{\frac{1}{2}x^2\coth^{-1}(ax)^2 - a\left(\frac{\coth^{-1}(ax)^2}{2a^3} - \frac{\frac{\log(1-a^2x^2)}{2a}+x\coth^{-1}(ax)}{a^2}\right)}{a^2}}{a^2} - \frac{\frac{1}{4}x^4\coth^{-1}(ax)^2 - \frac{1}{2}a\left(\cdots\right.}{\cdots}\right.$$

$\downarrow$ 6471

$$\frac{1}{5}x^5\coth^{-1}(ax)^3 - \frac{3}{5}a\left(\frac{\frac{\frac{\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)^2}{a} - 2\int\frac{\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a} - \frac{\coth^{-1}(ax)^3}{3a^2}}{a^2} - \frac{\frac{1}{2}x^2\coth^{-1}(ax)^2 - a\left(\frac{\coth^{-1}(ax)^2}{2a^3} - \frac{\frac{\log(1-a^2x^2)}{2a}+x\coth^{-1}(ax)}{a^2}\right)}{a^2}}{a^2} - \cdots\right.$$

$\downarrow$ 6621

$$\frac{1}{5}x^5\coth^{-1}(ax)^3 - \frac{3}{5}a\left(\frac{\frac{\frac{\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)^2}{a} - 2\left(\frac{1}{2}\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx - \frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)\coth^{-1}(ax)}{2a}\right)}{a} - \frac{\coth^{-1}(ax)^3}{3a^2}}{a^2} - \frac{\frac{1}{2}x^2\coth^{-1}(ax)^2 - a\left(\frac{\coth^{-1}(ax\cdots}{2a^3}\cdots\right.}{a\cdots}}{a^2}\right.$$

$\downarrow$ 7164

3.24. $\int x^4\coth^{-1}(ax)^3\,dx$

$$\frac{1}{5}x^5\coth^{-1}(ax)^3 -$$

$$\frac{3}{5}a\left(\frac{\frac{\frac{\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)^2}{a}-2\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{4a}-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)\coth^{-1}(ax)}{2a}\right)}{a}-\frac{\coth^{-1}(ax)^3}{3a^2}}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)^2-a\left(\frac{\coth^{-1}(ax)^2}{2a^3}-\cdots\right.}{a^2}}{a^2}\right.$$

input
```
Int[x^4*ArcCoth[a*x]^3,x]
```

output
```
(x^5*ArcCoth[a*x]^3)/5 - (3*a*(-(((x^4*ArcCoth[a*x]^2)/4 - (a*(-(((x^3*Arc
Coth[a*x])/3 - (a*(-(x^2/a^2) - Log[1 - a^2*x^2]/a^4))/6)/a^2) + (ArcCoth[
a*x]^2/(2*a^3) - (x*ArcCoth[a*x] + Log[1 - a^2*x^2]/(2*a))/a^2)/a^2))/2)/a
^2) + (-(((x^2*ArcCoth[a*x]^2)/2 - a*(ArcCoth[a*x]^2/(2*a^3) - (x*ArcCoth[
a*x] + Log[1 - a^2*x^2]/(2*a))/a^2))/a^2) + (-1/3*ArcCoth[a*x]^3/a^2 + ((A
rcCoth[a*x]^2*Log[2/(1 - a*x)])/a - 2*(-1/2*(ArcCoth[a*x]*PolyLog[2, 1 - 2
/(1 - a*x)])/a + PolyLog[3, 1 - 2/(1 - a*x)]/(4*a)))/a)/a^2)/a^2))/5
```

3.24.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6471
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6543
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6547
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.24. $\int x^4 \coth^{-1}(ax)^3 \, dx$

rule 6621
```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.24.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.11 (sec) , antiderivative size = 737, normalized size of antiderivative = 3.76

method	result	size
derivativedivides	Expression too large to display	737
default	Expression too large to display	737
parts	Expression too large to display	737

input
```
int(x^4*arccoth(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(6/5*polylog(3,1/((a*x-1)/(a*x+1))^(1/2))+6/5*polylog(3,-1/((a*x-1)/
(a*x+1))^(1/2))+9/10*a*x*arccoth(a*x)+3/10*a^2*x^2*arccoth(a*x)^2+1/10*a^3
*x^3*arccoth(a*x)+3/20*a^4*x^4*arccoth(a*x)^2+3/20*I*csgn(I/(a*x-1)*(a*x+1
)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))
*arccoth(a*x)^2*Pi-1/20+1/20*a^2*x^2-ln(1+1/((a*x-1)/(a*x+1))^(1/2))+1/5*a
rccoth(a*x)^3+arccoth(a*x)-ln(1/((a*x-1)/(a*x+1))^(1/2)-1)-9/20*arccoth(a*
x)^2+1/5*x^5*arccoth(a*x)^3*a^5+3/5*arccoth(a*x)^2*ln((a*x+1)/(a*x-1)-1)-3
/5*arccoth(a*x)^2*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-6/5*arccoth(a*x)*polylog
(2,1/((a*x-1)/(a*x+1))^(1/2))-3/5*arccoth(a*x)^2*ln(1+1/((a*x-1)/(a*x+1))^
(1/2))-6/5*arccoth(a*x)*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+3/20*I*csgn(
I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)^2*Pi-3/1
0*I*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arccoth(a*x)
^2*Pi-3/20*I*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/
(a*x-1))*arccoth(a*x)^2*Pi-3/20*I*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-
1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2*Pi+3/10*arccoth(a*x)^2*ln
((a*x-1)/(a*x+1))+3/10*arccoth(a*x)^2*ln(a*x-1)+3/10*arccoth(a*x)^2*ln(a*x
+1)+3/20*I*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))^3*arccoth(a*x)^2*Pi
+3/20*I*csgn(I*(a*x+1)/(a*x-1))^3*arccoth(a*x)^2*Pi-3/5*arccoth(a*x)^2*ln(
2))
```

3.24.5 Fricas [F]

$$\int x^4 \coth^{-1}(ax)^3 \, dx = \int x^4 \operatorname{arcoth}(ax)^3 \, dx$$

input
```
integrate(x^4*arccoth(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^4*arccoth(a*x)^3, x)
```

3.24.6 Sympy [F]

$$\int x^4 \coth^{-1}(ax)^3 \, dx = \int x^4 \operatorname{acoth}^3(ax) \, dx$$

input
```
integrate(x**4*acoth(a*x)**3,x)
```

output
```
Integral(x**4*acoth(a*x)**3, x)
```

3.24.7 Maxima [F]

$$\int x^4 \coth^{-1}(ax)^3 \, dx = \int x^4 \operatorname{arcoth}(ax)^3 \, dx$$

input
```
integrate(x^4*arccoth(a*x)^3,x, algorithm="maxima")
```

output
```
1/80*(2*(a^5*x^5 + 1)*log(a*x + 1)^3 + 3*(a^4*x^4 + 2*a^2*x^2 - 2*(a^5*x^5
 - 1)*log(a*x - 1))*log(a*x + 1)^2)/a^5 + 1/8*integrate(-1/5*(5*(a^5*x^5 +
 a^4*x^4)*log(a*x - 1)^3 + 3*(a^4*x^4 + 2*a^2*x^2 - 5*(a^5*x^5 + a^4*x^4)*
log(a*x - 1)^2 - 2*(a^5*x^5 - 1)*log(a*x - 1))*log(a*x + 1))/(a^5*x + a^4)
, x)
```

3.24.8 Giac [F]

$$\int x^4 \coth^{-1}(ax)^3 \, dx = \int x^4 \operatorname{arcoth}(ax)^3 \, dx$$

input
```
integrate(x^4*arccoth(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^4*arccoth(a*x)^3, x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \coth^{-1}(ax)^3 \, dx = \int x^4 \operatorname{acoth}(a\,x)^3 \, dx$$

input
```
int(x^4*acoth(a*x)^3,x)
```

output
```
int(x^4*acoth(a*x)^3, x)
```

3.25 $\int x^3 \coth^{-1}(ax)^3 \, dx$

3.25.1 Optimal result

Integrand size = 10, antiderivative size = 139

$$\int x^3 \coth^{-1}(ax)^3 \, dx = \frac{x}{4a^3} + \frac{x^2 \coth^{-1}(ax)}{4a^2} + \frac{\coth^{-1}(ax)^2}{a^4} + \frac{3x \coth^{-1}(ax)^2}{4a^3} + \frac{x^3 \coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^3 - \frac{\operatorname{arctanh}(ax)}{4a^4} - \frac{2\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4} - \frac{\operatorname{PolyLog}\left(2, 1-\frac{2}{1-ax}\right)}{a^4}$$

output
```
1/4*x/a^3+1/4*x^2*arccoth(a*x)/a^2+arccoth(a*x)^2/a^4+3/4*x*arccoth(a*x)^2
/a^3+1/4*x^3*arccoth(a*x)^2/a-1/4*arccoth(a*x)^3/a^4+1/4*x^4*arccoth(a*x)^
3-1/4*arctanh(a*x)/a^4-2*arccoth(a*x)*ln(2/(-a*x+1))/a^4-polylog(2,1-2/(-a
*x+1))/a^4
```

3.25.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.63

$$\int x^3 \coth^{-1}(ax)^3 \, dx = \frac{ax + (-4 + 3ax + a^3x^3)\coth^{-1}(ax)^2 + (-1 + a^4x^4)\coth^{-1}(ax)^3 + \coth^{-1}(ax)\left(-1 + a^2x^2 - 8\log\left(1 - \right.\right.}{4a^4}$$

input
```
Integrate[x^3*ArcCoth[a*x]^3,x]
```

output
```
(a*x + (-4 + 3*a*x + a^3*x^3)*ArcCoth[a*x]^2 + (-1 + a^4*x^4)*ArcCoth[a*x]
^3 + ArcCoth[a*x]*(-1 + a^2*x^2 - 8*Log[1 - E^(-2*ArcCoth[a*x])]) + 4*Poly
Log[2, E^(-2*ArcCoth[a*x])])/(4*a^4)
```

3.25.3 Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.72, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.300$, Rules used = {6453, 6543, 6453, 6543, 6437, 6453, 262, 219, 6511, 6547, 6471, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(ax)^3 \, dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{4}x^4 \coth^{-1}(ax)^3 - \frac{3}{4}a \int \frac{x^4 \coth^{-1}(ax)^2}{1-a^2x^2}dx$$

$$\Big\downarrow \text{6543}$$

$$\frac{1}{4}x^4 \coth^{-1}(ax)^3 - \frac{3}{4}a \left(\frac{\int \frac{x^2 \coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2} - \frac{\int x^2 \coth^{-1}(ax)^2dx}{a^2}\right)$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{4}x^4 \coth^{-1}(ax)^3 - \frac{3}{4}a \left(\frac{\int \frac{x^2 \coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{2}{3}a \int \frac{x^3 \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}\right)$$

$$\Big\downarrow \text{6543}$$

$$\frac{1}{4}x^4 \coth^{-1}(ax)^3 - \frac{3}{4}a \left(\frac{\frac{\int \frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2} - \frac{\int \coth^{-1}(ax)^2dx}{a^2}}{a^2} - \frac{\frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{2}{3}a \left(\frac{\int \frac{x \coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\int x \coth^{-1}(ax)dx}{a^2}\right)}{a^2}\right)$$

$$\Big\downarrow \text{6437}$$

$$\frac{1}{4}x^4\coth^{-1}(ax)^3 - \frac{3}{4}a\left(\frac{\frac{\int \frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2} - \frac{x\coth^{-1}(ax)^2-2a\int \frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}}{a^2} - \frac{\frac{1}{3}x^3\coth^{-1}(ax)^2 - \frac{2}{3}a\left(\frac{\int \frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\int x\coth^{-1}(ax)dx}{a^2}\right)}{a^2}\right)$$

$\Big\downarrow$ 6453

$$\frac{1}{4}x^4\coth^{-1}(ax)^3 - \frac{3}{4}a\left(\frac{\frac{\int \frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2} - \frac{x\coth^{-1}(ax)^2-2a\int \frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}}{a^2} - \frac{\frac{1}{3}x^3\coth^{-1}(ax)^2 - \frac{2}{3}a\left(\frac{\int \frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\int \cdots}{a^2} \cdots\right.}{a^2} \cdots\right.$$

$\Big\downarrow$ 262

$$\frac{1}{4}x^4\coth^{-1}(ax)^3 - \frac{3}{4}a\left(\frac{\frac{\int \frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2} - \frac{x\coth^{-1}(ax)^2-2a\int \frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}}{a^2} - \frac{\frac{1}{3}x^3\coth^{-1}(ax)^2 - \frac{2}{3}a\left(\frac{\int \frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\left(\int \cdots\right.}{a^2} \cdots\right.}{a^2} \cdots\right.$$

$\Big\downarrow$ 219

$$\frac{1}{4}x^4\coth^{-1}(ax)^3 - \frac{3}{4}a\left(\frac{\frac{\int \frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2} - \frac{x\coth^{-1}(ax)^2-2a\int \frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}}{a^2} - \frac{\frac{1}{3}x^3\coth^{-1}(ax)^2 - \frac{2}{3}a\left(\frac{\int \frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\left(a \cdots\right.}{a^2} \cdots\right.}{a^2} \cdots\right.$$

$\Big\downarrow$ 6511

$$\frac{1}{4}x^4\coth^{-1}(ax)^3 - \frac{3}{4}a\left(\frac{\frac{\coth^{-1}(ax)^3}{3a^3} - \frac{x\coth^{-1}(ax)^2-2a\int \frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}}{a^2} - \frac{\frac{1}{3}x^3\coth^{-1}(ax)^2 - \frac{2}{3}a\left(\frac{\int \frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\left(\text{arct}\cdots\right.}{a^2} \cdots\right.}{a^2} \cdots\right.$$

3.25. $\int x^3\coth^{-1}(ax)^3\,dx$

$\downarrow$ 6547

$$\frac{1}{4}x^4\coth^{-1}(ax)^3-$$

$$\frac{3}{4}a\left(\frac{\frac{\coth^{-1}(ax)^3}{3a^3}-\frac{x\coth^{-1}(ax)^2-2a\left(\frac{\int\frac{\coth^{-1}(ax)}{1-ax}dx}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\left(\frac{\frac{\int\frac{\coth^{-1}(ax)}{1-ax}dx}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}}{a^2}-\right.}{a^2}\right.$$

$\downarrow$ 6471

$$\frac{1}{4}x^4\coth^{-1}(ax)^3-$$

$$\frac{3}{4}a\left(\frac{\frac{\coth^{-1}(ax)^3}{3a^3}-\frac{x\coth^{-1}(ax)^2-2a\left(\frac{\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}-\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\left(\frac{\log\left(\frac{2}{1-ax}\right.}{}\right.}{}\right.$$

$\downarrow$ 2849

$$\frac{1}{4}x^4\coth^{-1}(ax)^3-$$

$$\frac{3}{4}a\left(\frac{\frac{\coth^{-1}(ax)^3}{3a^3}-\frac{x\coth^{-1}(ax)^2-2a\left(\frac{\frac{\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a}+\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\left(\frac{\int\frac{\log}{1}}{}\right.}{}\right.$$

$\downarrow$ 2752

$$\frac{1}{4}x^4\coth^{-1}(ax)^3-$$

$$\frac{3}{4}a\left(\frac{\frac{\coth^{-1}(ax)^3}{3a^3}-\frac{x\coth^{-1}(ax)^2-2a\left(\frac{\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a}+\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}\right)}{a^2}}{a^2}-\frac{\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{2}{3}a\left(\frac{\text{PolyL}}{}\right.}{}\right.$$

3.25. $\int x^3\coth^{-1}(ax)^3\,dx$

input
```
Int[x^3*ArcCoth[a*x]^3,x]
```

output
```
(x^4*ArcCoth[a*x]^3)/4 - (3*a*(-(((x^3*ArcCoth[a*x]^2)/3 - (2*a*(-(((x^2*A
rcCoth[a*x])/2 - (a*(-(x/a^2) + ArcTanh[a*x]/a^3))/2)/a^2) + (-1/2*ArcCoth
[a*x]^2/a^2 + ((ArcCoth[a*x]*Log[2/(1 - a*x)])/a + PolyLog[2, 1 - 2/(1 - a
*x)]/(2*a))/a)/a^2))/3)/a^2) + (ArcCoth[a*x]^3/(3*a^3) - (x*ArcCoth[a*x]^2
 - 2*a*(-1/2*ArcCoth[a*x]^2/a^2 + ((ArcCoth[a*x]*Log[2/(1 - a*x)])/a + Pol
yLog[2, 1 - 2/(1 - a*x)]/(2*a))/a))/a^2)/a^2))/4
```

3.25.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6471
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6543
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6547
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.25.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.40 (sec) , antiderivative size = 871, normalized size of antiderivative = 6.27

method	result	size
derivativedivides	Expression too large to display	871
default	Expression too large to display	871
parts	Expression too large to display	871

input
```
int(x^3*arccoth(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(3/16*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I/(a*x-1)*(a*x+1)/((a*x+
1)/(a*x-1)-1))^2*arccoth(a*x)^2+3/16*I*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I/(
a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))^2*arccoth(a*x)^2-3/16*I*Pi*csgn(I/((a*
x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)^2+3/8*I*Pi*csg
n(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arccoth(a*x)^2+3/4*
arccoth(a*x)^2*a*x+1/4*a^3*x^3*arccoth(a*x)^2-3/16*I*Pi*csgn(I/((a*x+1)/(a
*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-
1))*arccoth(a*x)^2-2*arccoth(a*x)*ln(1+1/((a*x-1)/(a*x+1))^(1/2))-1/4*arcc
oth(a*x)^3-1/8*(((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)-a*x-1)
*arccoth(a*x)-1/4/(((a*x-1)/(a*x+1))^(1/2)-1)*((a*x-1)/(a*x+1))^(1/2)+1/8*
(((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)+a*x+1)*arccoth(a*x)-1
/4/(((a*x-1)/(a*x+1))^(1/2)+1)*((a*x-1)/(a*x+1))^(1/2)+arccoth(a*x)^2+1/4*
a^4*x^4*arccoth(a*x)^3-1/4*(((a*x-1)/(a*x+1))^(1/2)*a*x-a*x+1)*arccoth(a*x
)*(a*x+1)+1/8*(((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)-a*x)*ar
ccoth(a*x)*(a*x+1)+1/4*(((a*x-1)/(a*x+1))^(1/2)*a*x+a*x-1)*arccoth(a*x)*(a
*x+1)-1/8*(((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)+a*x)*arccot
h(a*x)*(a*x+1)-3/16*I*Pi*csgn(I*(a*x+1)/(a*x-1))^3*arccoth(a*x)^2-3/16*I*P
i*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))^3*arccoth(a*x)^2+2*dilog(1/(
(a*x-1)/(a*x+1))^(1/2))-2*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))-3/8*arccoth(a
*x)^2*ln((a*x-1)/(a*x+1))+3/8*arccoth(a*x)^2*ln(a*x-1)-3/8*arccoth(a*x)...
```

3.25.5 Fricas [F]

$$\int x^3 \coth^{-1}(ax)^3 \, dx = \int x^3 \operatorname{arcoth}(ax)^3 \, dx$$

input
```
integrate(x^3*arccoth(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^3*arccoth(a*x)^3, x)
```

3.25.6 Sympy [F]

$$\int x^3 \coth^{-1}(ax)^3 \, dx = \int x^3 \operatorname{acoth}^3(ax) \, dx$$

input
```
integrate(x**3*acoth(a*x)**3,x)
```

output
```
Integral(x**3*acoth(a*x)**3, x)
```

3.25.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(122) = 244$.

Time = 0.20 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.88

$$\int x^3 \coth^{-1}(ax)^3 \, dx$$
$$= \frac{1}{4} x^4 \operatorname{arcoth}(ax)^3 + \frac{1}{8} a \left(\frac{2(a^2x^3 + 3x)}{a^4} - \frac{3 \log(ax+1)}{a^5} + \frac{3 \log(ax-1)}{a^5} \right) \operatorname{arcoth}(ax)^2$$
$$+ \frac{1}{32} a \left(\frac{\frac{(3 \log(ax-1)-8) \log(ax+1)^2 - \log(ax+1)^3 + \log(ax-1)^3 + 8ax - \left(3 \log(ax-1)^2 - 16 \log(ax-1)\right) \log(ax+1) + 8 \log(ax-1)^2 + 4 \log(ax}{a}}{a^4} \right.$$

input
```
integrate(x^3*arccoth(a*x)^3,x, algorithm="maxima")
```

output
```
1/4*x^4*arccoth(a*x)^3 + 1/8*a*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a^5
 + 3*log(a*x - 1)/a^5)*arccoth(a*x)^2 + 1/32*a*((((3*log(a*x - 1) - 8)*log
(a*x + 1)^2 - log(a*x + 1)^3 + log(a*x - 1)^3 + 8*a*x - (3*log(a*x - 1)^2
- 16*log(a*x - 1))*log(a*x + 1) + 8*log(a*x - 1)^2 + 4*log(a*x - 1))/a - 3
2*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 4*log(a*x
+ 1)/a)/a^4 + 2*(4*a^2*x^2 - 2*(3*log(a*x - 1) - 8)*log(a*x + 1) + 3*log(a
*x + 1)^2 + 3*log(a*x - 1)^2 + 16*log(a*x - 1))*arccoth(a*x)/a^5)
```

3.25.8 Giac [F]

$$\int x^3 \coth^{-1}(ax)^3 \, dx = \int x^3 \operatorname{arcoth}\left(ax\right)^3 \, dx$$

input
```
integrate(x^3*arccoth(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^3*arccoth(a*x)^3, x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^{-1}(ax)^3 \, dx = \int x^3 \operatorname{acoth}(a\,x)^3 \, dx$$

input
```
int(x^3*acoth(a*x)^3,x)
```

output
```
int(x^3*acoth(a*x)^3, x)
```

3.26 $\int x^2 \coth^{-1}(ax)^3 \, dx$

3.26.1 Optimal result

Integrand size = 10, antiderivative size = 149

$$\int x^2 \coth^{-1}(ax)^3 \, dx = \frac{x \coth^{-1}(ax)}{a^2} - \frac{\coth^{-1}(ax)^2}{2a^3} + \frac{x^2 \coth^{-1}(ax)^2}{2a} + \frac{\coth^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^2 \log\left(\frac{2}{1-ax}\right)}{a^3} + \frac{\log(1-a^2x^2)}{2a^3} - \frac{\coth^{-1}(ax) \operatorname{PolyLog}\left(2, 1-\frac{2}{1-ax}\right)}{a^3} + \frac{\operatorname{PolyLog}\left(3, 1-\frac{2}{1-ax}\right)}{2a^3}$$

output
```
x*arccoth(a*x)/a^2-1/2*arccoth(a*x)^2/a^3+1/2*x^2*arccoth(a*x)^2/a+1/3*arc
coth(a*x)^3/a^3+1/3*x^3*arccoth(a*x)^3-arccoth(a*x)^2*ln(2/(-a*x+1))/a^3+1
/2*ln(-a^2*x^2+1)/a^3-arccoth(a*x)*polylog(2,1-2/(-a*x+1))/a^3+1/2*polylog
(3,1-2/(-a*x+1))/a^3
```

3.26.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.96

$$\int x^2 \coth^{-1}(ax)^3 \, dx$$

$$= \frac{-i\pi^3 + 24ax \coth^{-1}(ax) - 12\coth^{-1}(ax)^2 + 12a^2x^2 \coth^{-1}(ax)^2 + 8\coth^{-1}(ax)^3 + 8a^3x^3 \coth^{-1}(ax)^3 -}{}$$

input
```
Integrate[x^2*ArcCoth[a*x]^3,x]
```

output
```
((-I)*Pi^3 + 24*a*x*ArcCoth[a*x] - 12*ArcCoth[a*x]^2 + 12*a^2*x^2*ArcCoth[
a*x]^2 + 8*ArcCoth[a*x]^3 + 8*a^3*x^3*ArcCoth[a*x]^3 - 24*ArcCoth[a*x]^2*L
og[1 - E^(2*ArcCoth[a*x])] - 24*Log[1/Sqrt[1 - 1/(a^2*x^2)]] - 24*Log[1/(a
*x)] - 24*ArcCoth[a*x]*PolyLog[2, E^(2*ArcCoth[a*x])] + 12*PolyLog[3, E^(2
*ArcCoth[a*x])])/(24*a^3)
```

3.26.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6453, 6543, 6453, 6543, 6437, 240, 6511, 6547, 6471, 6621, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(ax)^3 \, dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{3}x^3 \coth^{-1}(ax)^3 - a\int \frac{x^3 \coth^{-1}(ax)^2}{1-a^2x^2}dx$$

$$\Big\downarrow \text{6543}$$

$$\frac{1}{3}x^3 \coth^{-1}(ax)^3 - a\left(\frac{\int \frac{x\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2} - \frac{\int x\coth^{-1}(ax)^2dx}{a^2}\right)$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{3}x^3 \coth^{-1}(ax)^3 - a\left(\frac{\int \frac{x\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{2}x^2\coth^{-1}(ax)^2 - a\int \frac{x^2\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}\right)$$

$$\Big\downarrow \text{6543}$$

$$\frac{1}{3}x^3 \coth^{-1}(ax)^3 - a\left(\frac{\int \frac{x\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2} - \frac{\frac{1}{2}x^2\coth^{-1}(ax)^2 - a\left(\frac{\int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2} - \frac{\int \coth^{-1}(ax)dx}{a^2}\right)}{a^2}\right)$$

$$\Big\downarrow \text{6437}$$

$$\frac{1}{3}x^3\coth^{-1}(ax)^3-a\left(\frac{\int\frac{x\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)^2-a\left(\frac{\int\frac{\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{x\coth^{-1}(ax)-a\int\frac{x}{1-a^2x^2}dx}{a^2}\right)}{a^2}\right)$$

$\downarrow$ 240

$$\frac{1}{3}x^3\coth^{-1}(ax)^3-a\left(\frac{\int\frac{x\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)^2-a\left(\frac{\int\frac{\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}-\frac{\frac{\log(1-a^2x^2)}{2a}+x\coth^{-1}(ax)}{a^2}\right)}{a^2}\right)$$

$\downarrow$ 6511

$$\frac{1}{3}x^3\coth^{-1}(ax)^3-a\left(\frac{\int\frac{x\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)^2-a\left(\frac{\coth^{-1}(ax)^2}{2a^3}-\frac{\frac{\log(1-a^2x^2)}{2a}+x\coth^{-1}(ax)}{a^2}\right)}{a^2}\right)$$

$\downarrow$ 6547

$$\frac{1}{3}x^3\coth^{-1}(ax)^3-a\left(\frac{\frac{\int\frac{\coth^{-1}(ax)^2}{1-ax}dx}{a}-\frac{\coth^{-1}(ax)^3}{3a^2}}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)^2-a\left(\frac{\coth^{-1}(ax)^2}{2a^3}-\frac{\frac{\log(1-a^2x^2)}{2a}+x\coth^{-1}(ax)}{a^2}\right)}{a^2}\right)$$

$\downarrow$ 6471

$$\frac{1}{3}x^3\coth^{-1}(ax)^3-a\left(\frac{\frac{\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)^2}{a}-2\int\frac{\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\coth^{-1}(ax)^3}{3a^2}}{a^2}-\frac{\frac{1}{2}x^2\coth^{-1}(ax)^2-a\left(\frac{\coth^{-1}(ax)^2}{2a^3}-\frac{\frac{\log(1-a^2x^2)}{2a}+x\,\mathrm{coth}}{a^2}\right.}{a^2}\right.$$

$\downarrow$ 6621

3.26. $\int x^2\coth^{-1}(ax)^3\,dx$

$$\frac{1}{3}x^3\coth^{-1}(ax)^3 - a\left(\frac{\frac{\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)^2}{a}-2\left(\frac{1}{2}\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)\coth^{-1}(ax)}{2a}\right)}{a}-\frac{\coth^{-1}(ax)^3}{3a^2}}{a^2} - \frac{\frac{1}{2}x^2\coth^{-1}(ax)^2-a\left(\frac{c\ldots}{}\right.}{}\right.$$

$\big\downarrow$ 7164

$$\frac{1}{3}x^3\coth^{-1}(ax)^3 - a\left(\frac{\frac{\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)^2}{a}-2\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{4a}-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)\coth^{-1}(ax)}{2a}\right)}{a}-\frac{\coth^{-1}(ax)^3}{3a^2}}{a^2} - \frac{\frac{1}{2}x^2\coth^{-1}(ax)^2-a\left(\frac{\coth^{-1}(\ldots}{2a^3\ldots}\right.}{}\right.$$

input
```
Int[x^2*ArcCoth[a*x]^3,x]
```

output
```
(x^3*ArcCoth[a*x]^3)/3 - a*(-(((x^2*ArcCoth[a*x]^2)/2 - a*(ArcCoth[a*x]^2/
(2*a^3) - (x*ArcCoth[a*x] + Log[1 - a^2*x^2]/(2*a))/a^2))/a^2) + (-1/3*Arc
Coth[a*x]^3/a^2 + ((ArcCoth[a*x]^2*Log[2/(1 - a*x)])/a - 2*(-1/2*(ArcCoth[
a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a + PolyLog[3, 1 - 2/(1 - a*x)]/(4*a)))/
a)/a^2)
```

3.26.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6471
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6543
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6547
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6621
```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.26. $\int x^2 \coth^{-1}(ax)^3 \, dx$

3.26.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.88 (sec) , antiderivative size = 681, normalized size of antiderivative = 4.57

method	result
parts	$\frac{x^3 \operatorname{arccoth}(ax)^3}{3} + \frac{\frac{a^2x^2 \operatorname{arccoth}(ax)^2}{2} + \frac{\operatorname{arccoth}(ax)^2 \ln(ax-1)}{2} + \frac{\operatorname{arccoth}(ax)^2 \ln(ax+1)}{2} + \frac{\operatorname{arccoth}(ax)^2 \ln\left(\frac{ax-1}{ax+1}\right)}{2} + \operatorname{arccoth}(a}{}$
derivativedivides	$\frac{\frac{\operatorname{arccoth}(ax)^3 a^3 x^3}{3} + \frac{a^2x^2 \operatorname{arccoth}(ax)^2}{2} + \frac{\operatorname{arccoth}(ax)^2 \ln(ax-1)}{2} + \frac{\operatorname{arccoth}(ax)^2 \ln(ax+1)}{2} + \frac{\operatorname{arccoth}(ax)^2 \ln\left(\frac{ax-1}{ax+1}\right)}{2} + \operatorname{arccoth}(ax)}{}$
default	$\frac{\frac{\operatorname{arccoth}(ax)^3 a^3 x^3}{3} + \frac{a^2x^2 \operatorname{arccoth}(ax)^2}{2} + \frac{\operatorname{arccoth}(ax)^2 \ln(ax-1)}{2} + \frac{\operatorname{arccoth}(ax)^2 \ln(ax+1)}{2} + \frac{\operatorname{arccoth}(ax)^2 \ln\left(\frac{ax-1}{ax+1}\right)}{2} + \operatorname{arccoth}(ax)}{}$

input
```
int(x^2*arccoth(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccoth(a*x)^3+1/a^3*(1/2*a^2*x^2*arccoth(a*x)^2+1/2*arccoth(a*x)^
2*ln(a*x-1)+1/2*arccoth(a*x)^2*ln(a*x+1)+1/2*arccoth(a*x)^2*ln((a*x-1)/(a*
x+1))+arccoth(a*x)^2*ln((a*x+1)/(a*x-1)-1)+1/12*arccoth(a*x)*(3*I*arccoth(
a*x)*Pi*csgn(I*(a*x+1)/(a*x-1))^3-6*I*arccoth(a*x)*Pi*csgn(I*(a*x+1)/(a*x-
1))^2*csgn(I/((a*x-1)/(a*x+1))^(1/2))+3*I*arccoth(a*x)*Pi*csgn(I*(a*x+1)/(
a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1
)-1))-3*I*arccoth(a*x)*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I/(a*x-1)*(a*x+1)/(
(a*x+1)/(a*x-1)-1))^2+3*I*arccoth(a*x)*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I/(
(a*x-1)/(a*x+1))^(1/2))^2-3*I*arccoth(a*x)*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*
csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))^2+3*I*arccoth(a*x)*Pi*csgn(I/(
a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))^3+4*arccoth(a*x)^2-12*arccoth(a*x)*ln(
2)-6*arccoth(a*x)+12*a*x+12)-ln(1/((a*x-1)/(a*x+1))^(1/2)-1)-ln(1+1/((a*x-
1)/(a*x+1))^(1/2))-arccoth(a*x)^2*ln(1+1/((a*x-1)/(a*x+1))^(1/2))-2*arccot
h(a*x)*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+2*polylog(3,-1/((a*x-1)/(a*x+
1))^(1/2))-arccoth(a*x)^2*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-2*arccoth(a*x)*p
olylog(2,1/((a*x-1)/(a*x+1))^(1/2))+2*polylog(3,1/((a*x-1)/(a*x+1))^(1/2))
)
```

3.26.5 Fricas [F]

$$\int x^2 \coth^{-1}(ax)^3 \, dx = \int x^2 \operatorname{arcoth}(ax)^3 \, dx$$

input
```
integrate(x^2*arccoth(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^2*arccoth(a*x)^3, x)
```

3.26.6 Sympy [F]

$$\int x^2 \coth^{-1}(ax)^3 \, dx = \int x^2 \operatorname{acoth}^3(ax) \, dx$$

input
```
integrate(x**2*acoth(a*x)**3,x)
```

output
```
Integral(x**2*acoth(a*x)**3, x)
```

3.26.7 Maxima [F]

$$\int x^2 \coth^{-1}(ax)^3 \, dx = \int x^2 \operatorname{arcoth}(ax)^3 \, dx$$

input
```
integrate(x^2*arccoth(a*x)^3,x, algorithm="maxima")
```

output
```
1/24*((a^3*x^3 + 1)*log(a*x + 1)^3 + 3*(a^2*x^2 - (a^3*x^3 - 1)*log(a*x -
1))*log(a*x + 1)^2)/a^3 + 1/8*integrate(-((a^3*x^3 + a^2*x^2)*log(a*x - 1)
^3 + (2*a^2*x^2 - 3*(a^3*x^3 + a^2*x^2)*log(a*x - 1)^2 - 2*(a^3*x^3 - 1)*l
og(a*x - 1))*log(a*x + 1))/(a^3*x + a^2), x)
```

3.26.8 Giac [F]

$$\int x^2 \coth^{-1}(ax)^3 \, dx = \int x^2 \operatorname{arcoth}(ax)^3 \, dx$$

input
```
integrate(x^2*arccoth(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(a*x)^3, x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(ax)^3 \, dx = \int x^2 \operatorname{acoth}(a\,x)^3 \, dx$$

input
```
int(x^2*acoth(a*x)^3,x)
```

output
```
int(x^2*acoth(a*x)^3, x)
```

3.27 $\int x \coth^{-1}(ax)^3\,dx$

3.27.1 Optimal result

Integrand size = 8, antiderivative size = 95

$$\int x\coth^{-1}(ax)^3\,dx=\frac{3\coth^{-1}(ax)^2}{2a^2}+\frac{3x\coth^{-1}(ax)^2}{2a}-\frac{\coth^{-1}(ax)^3}{2a^2}+\frac{1}{2}x^2\coth^{-1}(ax)^3-\frac{3\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^2}-\frac{3\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^2}$$

output
```
3/2*arccoth(a*x)^2/a^2+3/2*x*arccoth(a*x)^2/a-1/2*arccoth(a*x)^3/a^2+1/2*x
^2*arccoth(a*x)^3-3*arccoth(a*x)*ln(2/(-a*x+1))/a^2-3/2*polylog(2,1-2/(-a*
x+1))/a^2
```

3.27.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int x\coth^{-1}(ax)^3\,dx$$
$$=\frac{\coth^{-1}(ax)\left(3(-1+ax)\coth^{-1}(ax)+(-1+a^2x^2)\coth^{-1}(ax)^2-6\log\left(1-e^{-2\coth^{-1}(ax)}\right)\right)+3\,\text{PolyLog}}{2a^2}$$

input
```
Integrate[x*ArcCoth[a*x]^3,x]
```

output
```
(ArcCoth[a*x]*(3*(-1 + a*x)*ArcCoth[a*x] + (-1 + a^2*x^2)*ArcCoth[a*x]^2 -
 6*Log[1 - E^(-2*ArcCoth[a*x])]) + 3*PolyLog[2, E^(-2*ArcCoth[a*x])])/(2*a
^2)
```

3.27.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6453, 6543, 6437, 6511, 6547, 6471, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(ax)^3\,dx$$

$$\Big\downarrow \text{ 6453}$$

$$\frac{1}{2}x^2\coth^{-1}(ax)^3-\frac{3}{2}a\int\frac{x^2\coth^{-1}(ax)^2}{1-a^2x^2}dx$$

$$\Big\downarrow \text{ 6543}$$

$$\frac{1}{2}x^2\coth^{-1}(ax)^3-\frac{3}{2}a\left(\frac{\int\frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{\int\coth^{-1}(ax)^2dx}{a^2}\right)$$

$$\Big\downarrow \text{ 6437}$$

$$\frac{1}{2}x^2\coth^{-1}(ax)^3-\frac{3}{2}a\left(\frac{\int\frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx}{a^2}-\frac{x\coth^{-1}(ax)^2-2a\int\frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}\right)$$

$$\Big\downarrow \text{ 6511}$$

$$\frac{1}{2}x^2\coth^{-1}(ax)^3-\frac{3}{2}a\left(\frac{\coth^{-1}(ax)^3}{3a^3}-\frac{x\coth^{-1}(ax)^2-2a\int\frac{x\coth^{-1}(ax)}{1-a^2x^2}dx}{a^2}\right)$$

$$\Big\downarrow \text{ 6547}$$

$$\frac{1}{2}x^2\coth^{-1}(ax)^3-\frac{3}{2}a\left(\frac{\coth^{-1}(ax)^3}{3a^3}-\frac{x\coth^{-1}(ax)^2-2a\left(\frac{\int\frac{\coth^{-1}(ax)}{1-ax}dx}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}\right)}{a^2}\right)$$

$$\Big\downarrow \text{ 6471}$$

$$\frac{1}{2}x^2\coth^{-1}(ax)^3 - \frac{3}{2}a\left(\frac{\coth^{-1}(ax)^3}{3a^3} - \frac{x\coth^{-1}(ax)^2 - 2a\left(\frac{\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a} - \int\frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a} - \frac{\coth^{-1}(ax)^2}{2a^2}\right)}{a^2}\right)$$

$$\Big\downarrow \text{2849}$$

$$\frac{1}{2}x^2\coth^{-1}(ax)^3 - \frac{3}{2}a\left(\frac{\coth^{-1}(ax)^3}{3a^3} - \frac{x\coth^{-1}(ax)^2 - 2a\left(\frac{\frac{\int\frac{\log\left(\frac{2}{1-ax}\right)}{1-\frac{2}{1-ax}}d\frac{1}{1-ax}}{a} + \frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}}{a} - \frac{\coth^{-1}(ax)^2}{2a^2}\right)}{a^2}\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{2}x^2\coth^{-1}(ax)^3 - \frac{3}{2}a\left(\frac{\coth^{-1}(ax)^3}{3a^3} - \frac{x\coth^{-1}(ax)^2 - 2a\left(\frac{\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a} + \frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}}{a} - \frac{\coth^{-1}(ax)^2}{2a^2}\right)}{a^2}\right)$$

input
```
Int[x*ArcCoth[a*x]^3,x]
```

output
```
(x^2*ArcCoth[a*x]^3)/2 - (3*a*(ArcCoth[a*x]^3/(3*a^3) - (x*ArcCoth[a*x]^2
- 2*a*(-1/2*ArcCoth[a*x]^2/a^2 + ((ArcCoth[a*x]*Log[2/(1 - a*x)])/a + Poly
Log[2, 1 - 2/(1 - a*x)]/(2*a))/a))/a^2))/2
```

3.27. $\int x\coth^{-1}(ax)^3\,dx$

3.27.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6471
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6543
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6547
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.27.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.28 (sec) , antiderivative size = 2910, normalized size of antiderivative = 30.63

method	result	size
parts	Expression too large to display	2910
derivativedivides	Expression too large to display	2916
default	Expression too large to display	2916

input
```
int(x*arccoth(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arccoth(a*x)^3+3/2/a^2*(arccoth(a*x)^2*a*x+1/4*I*Pi*csgn(I/((a*x+1
)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x
-1)-1))*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+1/4*I*Pi*csgn(I/((a*x+1)/(a*x
-1)-1))*csgn(I*(a*x+1)/(a*x-1))*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1)
)*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+1/4*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1
))*csgn(I*(a*x+1)/(a*x-1))*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))*dil
og(1/((a*x-1)/(a*x+1))^(1/2))-1/4*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*
(a*x+1)/(a*x-1))*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))*dilog(1+1/((a
*x-1)/(a*x+1))^(1/2))-1/4*I*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I/(a*x-1)*(a*x
+1)/((a*x+1)/(a*x-1)-1))^2*arccoth(a*x)*ln(1-1/((a*x-1)/(a*x+1))^(1/2))+1/
4*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a
*x)*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-1/4*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*c
sgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))^2*arccoth(a*x)*ln(1-1/((a*x-1)/
(a*x+1))^(1/2))-1/4*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1
))*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2-1/4*I*Pi*csg
n(I*(a*x+1)/(a*x-1))^3*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))+1/4*I*Pi*csgn(I*
(a*x+1)/(a*x-1))^3*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+1/4*I*Pi*csgn(I*(a
*x+1)/(a*x-1))^3*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+1/4*I*Pi*csgn(I/((a
*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/
(a*x-1)-1))*arccoth(a*x)*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-polylog(2,1/((...
```

3.27.5 Fricas [F]

$$\int x \coth^{-1}(ax)^3\, dx = \int x\,\mathrm{arcoth}\,(ax)^3\, dx$$

input
```
integrate(x*arccoth(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x*arccoth(a*x)^3, x)
```

3.27.6 Sympy [F]

$$\int x \coth^{-1}(ax)^3\, dx = \int x\,\mathrm{acoth}^3\,(ax)\, dx$$

input
```
integrate(x*acoth(a*x)**3,x)
```

output
```
Integral(x*acoth(a*x)**3, x)
```

3.27.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(82) = 164$.

Time = 0.20 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.26

$$\int x \coth^{-1}(ax)^3\, dx$$
$$= \frac{1}{2}\,x^2\,\mathrm{arcoth}\,(ax)^3 + \frac{3}{4}\,a\left(\frac{2\,x}{a^2} - \frac{\log\,(ax+1)}{a^3} + \frac{\log\,(ax-1)}{a^3}\right)\mathrm{arcoth}\,(ax)^2$$
$$+ \frac{1}{16}\,a\left(\frac{\frac{3\,(\log(ax-1)-2)\,\log(ax+1)^2-\log(ax+1)^3+\log(ax-1)^3-3\,\left(\log(ax-1)^2-4\,\log(ax-1)\right)\,\log(ax+1)+6\,\log(ax-1)^2}{a}}{a^2} - \frac{24\,(\log(ax-1}{}\right.$$

input
```
integrate(x*arccoth(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*x^2*arccoth(a*x)^3 + 3/4*a*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/
a^3)*arccoth(a*x)^2 + 1/16*a*(((3*(log(a*x - 1) - 2)*log(a*x + 1)^2 - log(
a*x + 1)^3 + log(a*x - 1)^3 - 3*(log(a*x - 1)^2 - 4*log(a*x - 1))*log(a*x
+ 1) + 6*log(a*x - 1)^2)/a - 24*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-
1/2*a*x + 1/2))/a)/a^2 - 6*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x +
1)^2 - log(a*x - 1)^2 - 4*log(a*x - 1))*arccoth(a*x)/a^3)
```

3.27.8 Giac [F]

$$\int x \coth^{-1}(ax)^3 \, dx = \int x \operatorname{arcoth}(ax)^3 \, dx$$

input
```
integrate(x*arccoth(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x*arccoth(a*x)^3, x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(ax)^3 \, dx = \int x \operatorname{acoth}(a\,x)^3 \, dx$$

input
```
int(x*acoth(a*x)^3,x)
```

output
```
int(x*acoth(a*x)^3, x)
```

3.28 $\int \coth^{-1}(ax)^3\,dx$

3.28.1 Optimal result

Integrand size = 6, antiderivative size = 85

$$\int \coth^{-1}(ax)^3\,dx = \frac{\coth^{-1}(ax)^3}{a} + x\coth^{-1}(ax)^3 - \frac{3\coth^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a} - \frac{3\coth^{-1}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a} + \frac{3\,\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}$$

output
```
arccoth(a*x)^3/a+x*arccoth(a*x)^3-3*arccoth(a*x)^2*ln(2/(-a*x+1))/a-3*arcc
oth(a*x)*polylog(2,1-2/(-a*x+1))/a+3/2*polylog(3,1-2/(-a*x+1))/a
```

3.28.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int \coth^{-1}(ax)^3\,dx = \frac{\coth^{-1}(ax)^3}{a} + x\coth^{-1}(ax)^3 - \frac{3\coth^{-1}(ax)^2\log\left(1-e^{2\coth^{-1}(ax)}\right)}{a} - \frac{3\coth^{-1}(ax)\,\text{PolyLog}\left(2,e^{2\coth^{-1}(ax)}\right)}{a} + \frac{3\,\text{PolyLog}\left(3,e^{2\coth^{-1}(ax)}\right)}{2a}$$

input
```
Integrate[ArcCoth[a*x]^3,x]
```

output
```
ArcCoth[a*x]^3/a + x*ArcCoth[a*x]^3 - (3*ArcCoth[a*x]^2*Log[1 - E^(2*ArcCo
th[a*x])])/a - (3*ArcCoth[a*x]*PolyLog[2, E^(2*ArcCoth[a*x])])/a + (3*Poly
Log[3, E^(2*ArcCoth[a*x])])/(2*a)
```

3.28.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6437, 6547, 6471, 6621, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(ax)^3\,dx$$

$$\Big\downarrow \text{6437}$$

$$x\coth^{-1}(ax)^3-3a\int \frac{x\coth^{-1}(ax)^2}{1-a^2x^2}dx$$

$$\Big\downarrow \text{6547}$$

$$x\coth^{-1}(ax)^3-3a\left(\frac{\int \frac{\coth^{-1}(ax)^2}{1-ax}dx}{a}-\frac{\coth^{-1}(ax)^3}{3a^2}\right)$$

$$\Big\downarrow \text{6471}$$

$$x\coth^{-1}(ax)^3-3a\left(\frac{\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)^2}{a}-2\int \frac{\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}dx}{a}-\frac{\coth^{-1}(ax)^3}{3a^2}\right)$$

$$\Big\downarrow \text{6621}$$

$$x\coth^{-1}(ax)^3-3a\left(\frac{\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)^2}{a}-2\left(\frac{1}{2}\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{1-a^2x^2}dx-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)\coth^{-1}(ax)}{2a}\right)}{a}-\frac{\coth^{-1}(ax)^3}{3a^2}\right)$$

$$\Big\downarrow \text{7164}$$

$$x\coth^{-1}(ax)^3-3a\left(\frac{\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)^2}{a}-2\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{4a}-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)\coth^{-1}(ax)}{2a}\right)}{a}-\frac{\coth^{-1}(ax)^3}{3a^2}\right)$$

input
```
Int[ArcCoth[a*x]^3,x]
```

output
```
x*ArcCoth[a*x]^3 - 3*a*(-1/3*ArcCoth[a*x]^3/a^2 + ((ArcCoth[a*x]^2*Log[2/(
1 - a*x)])/a - 2*(-1/2*(ArcCoth[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a + Poly
Log[3, 1 - 2/(1 - a*x)]/(4*a)))/a)
```

3.28.3.1 Defintions of rubi rules used

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6471
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6547
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6621
```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.28.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(83) = 166$.

Time = 0.40 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.98

method	result
derivativedivides	$\frac{\operatorname{arccoth}(ax)^3(ax-1)+2\operatorname{arccoth}(ax)^3-3\operatorname{arccoth}(ax)^2\ln\left(1+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)-6\operatorname{arccoth}(ax)\operatorname{polylog}\left(2,-\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)+6\operatorname{polyl}}{a}$
default	$\frac{\operatorname{arccoth}(ax)^3(ax-1)+2\operatorname{arccoth}(ax)^3-3\operatorname{arccoth}(ax)^2\ln\left(1+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)-6\operatorname{arccoth}(ax)\operatorname{polylog}\left(2,-\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)+6\operatorname{polyl}}{a}$

input
```
int(arccoth(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(arccoth(a*x)^3*(a*x-1)+2*arccoth(a*x)^3-3*arccoth(a*x)^2*ln(1+1/((a*x
-1)/(a*x+1))^(1/2))-6*arccoth(a*x)*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+6
*polylog(3,-1/((a*x-1)/(a*x+1))^(1/2))-3*arccoth(a*x)^2*ln(1-1/((a*x-1)/(a
*x+1))^(1/2))-6*arccoth(a*x)*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+6*polylo
g(3,1/((a*x-1)/(a*x+1))^(1/2)))
```

3.28.5 Fricas [F]

$$\int \coth^{-1}(ax)^3\,dx = \int \operatorname{arcoth}(ax)^3\,dx$$

input
```
integrate(arccoth(a*x)^3,x, algorithm="fricas")
```

output
```
integral(arccoth(a*x)^3, x)
```

3.28.6 Sympy [F]

$$\int \coth^{-1}(ax)^3 \, dx = \int \operatorname{acoth}^3(ax) \, dx$$

input
```
integrate(acoth(a*x)**3,x)
```

output
```
Integral(acoth(a*x)**3, x)
```

3.28.7 Maxima [F]

$$\int \coth^{-1}(ax)^3 \, dx = \int \operatorname{arcoth}(ax)^3 \, dx$$

input
```
integrate(arccoth(a*x)^3,x, algorithm="maxima")
```

output
```
1/8*((a*x + 1)*log(a*x + 1)^3 - 3*(a*x - 1)*log(a*x + 1)^2*log(a*x - 1))/a
 + 1/8*integrate(-((a*x + 1)*log(a*x - 1)^3 - 3*((a*x + 1)*log(a*x - 1)^2
+ 2*(a*x - 1)*log(a*x - 1))*log(a*x + 1))/(a*x + 1), x)
```

3.28.8 Giac [F]

$$\int \coth^{-1}(ax)^3 \, dx = \int \operatorname{arcoth}(ax)^3 \, dx$$

input
```
integrate(arccoth(a*x)^3,x, algorithm="giac")
```

output
```
integrate(arccoth(a*x)^3, x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(ax)^3 \, dx = \int \operatorname{acoth}(a\,x)^3 \, dx$$

input
```
int(acoth(a*x)^3,x)
```

output
```
int(acoth(a*x)^3, x)
```

3.29 $\int \frac{\coth^{-1}(ax)^3}{x} dx$

3.29.1 Optimal result

Integrand size = 10, antiderivative size = 150

$$\begin{aligned}\int \frac{\coth^{-1}(ax)^3}{x} dx &= 2\coth^{-1}(ax)^3 \coth^{-1}\left(1-\frac{2}{1-ax}\right) \\ &+ \frac{3}{2}\coth^{-1}(ax)^2 \operatorname{PolyLog}\left(2,1-\frac{2}{1+ax}\right) \\ &- \frac{3}{2}\coth^{-1}(ax)^2 \operatorname{PolyLog}\left(2,1-\frac{2ax}{1+ax}\right) \\ &+ \frac{3}{2}\coth^{-1}(ax) \operatorname{PolyLog}\left(3,1-\frac{2}{1+ax}\right) \\ &- \frac{3}{2}\coth^{-1}(ax) \operatorname{PolyLog}\left(3,1-\frac{2ax}{1+ax}\right) \\ &+ \frac{3}{4}\operatorname{PolyLog}\left(4,1-\frac{2}{1+ax}\right) - \frac{3}{4}\operatorname{PolyLog}\left(4,1-\frac{2ax}{1+ax}\right)\end{aligned}$$

output
```
2*arccoth(a*x)^3*arccoth(1-2/(-a*x+1))+3/2*arccoth(a*x)^2*polylog(2,1-2/(a
*x+1))-3/2*arccoth(a*x)^2*polylog(2,1-2*a*x/(a*x+1))+3/2*arccoth(a*x)*poly
log(3,1-2/(a*x+1))-3/2*arccoth(a*x)*polylog(3,1-2*a*x/(a*x+1))+3/4*polylog
(4,1-2/(a*x+1))-3/4*polylog(4,1-2*a*x/(a*x+1))
```

3.29.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.04

$$\int \frac{\coth^{-1}(ax)^3}{x} dx = \frac{1}{64}\Big(-\pi^4 + 32\coth^{-1}(ax)^4 + 64\coth^{-1}(ax)^3 \log\left(1 + e^{-2\coth^{-1}(ax)}\right) - 64\coth^{-1}(ax)^3 \log\left(1 - e^{2\coth^{-1}(ax)}\right) - 96\coth^{-1}(ax)^2 \text{PolyLog}\left(2, -e^{-2\coth^{-1}(ax)}\right) - 96\coth^{-1}(ax)^2 \text{PolyLog}\left(2, e^{2\coth^{-1}(ax)}\right) - 96\coth^{-1}(ax) \text{PolyLog}\left(3, -e^{-2\coth^{-1}(ax)}\right) + 96\coth^{-1}(ax) \text{PolyLog}\left(3, e^{2\coth^{-1}(ax)}\right) - 48 \text{PolyLog}\left(4, -e^{-2\coth^{-1}(ax)}\right) - 48 \text{PolyLog}\left(4, e^{2\coth^{-1}(ax)}\right)\Big)$$

input
```
Integrate[ArcCoth[a*x]^3/x,x]
```

output
```
(-Pi^4 + 32*ArcCoth[a*x]^4 + 64*ArcCoth[a*x]^3*Log[1 + E^(-2*ArcCoth[a*x])
] - 64*ArcCoth[a*x]^3*Log[1 - E^(2*ArcCoth[a*x])] - 96*ArcCoth[a*x]^2*Poly
Log[2, -E^(-2*ArcCoth[a*x])] - 96*ArcCoth[a*x]^2*PolyLog[2, E^(2*ArcCoth[a
*x])] - 96*ArcCoth[a*x]*PolyLog[3, -E^(-2*ArcCoth[a*x])] + 96*ArcCoth[a*x]
*PolyLog[3, E^(2*ArcCoth[a*x])] - 48*PolyLog[4, -E^(-2*ArcCoth[a*x])] - 48
*PolyLog[4, E^(2*ArcCoth[a*x])])/64
```

3.29.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6449, 6615, 6619, 6623, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)^3}{x} dx$$

$$\Big\downarrow \ 6449$$

$$2\coth^{-1}(ax)^3 \coth^{-1}\left(1 - \frac{2}{1-ax}\right) - 6a\int \frac{\coth^{-1}(ax)^2 \coth^{-1}\left(1 - \frac{2}{1-ax}\right)}{1 - a^2x^2} dx$$

$$\Big\downarrow \text{6615}$$

$$2\coth^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1-ax}\right)-6a\left(\frac{1}{2}\int\frac{\coth^{-1}(ax)^2\log\left(\frac{2ax}{ax+1}\right)}{1-a^2x^2}dx-\frac{1}{2}\int\frac{\coth^{-1}(ax)^2\log\left(\frac{2}{ax+1}\right)}{1-a^2x^2}dx\right)$$

$$\Big\downarrow \text{6619}$$

$$2\coth^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1-ax}\right)-6a\left(\frac{1}{2}\left(\int\frac{\coth^{-1}(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{1-a^2x^2}dx-\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{ax+1}\right)\coth^{-1}(ax)^2}{2a}\right)+\frac{1}{2}\left(\frac{\operatorname{PolyLog}\left(2,1-\right.}{2}\right.\right.$$

$$\Big\downarrow \text{6623}$$

$$2\coth^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1-ax}\right)-6a\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{ax+1}\right)}{1-a^2x^2}dx-\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{ax+1}\right)\coth^{-1}(ax)^2}{2a}-\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{ax+1}\right)\coth^{-1}(a}{2a}\right.\right.$$

$$\Big\downarrow \text{7164}$$

$$2\coth^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1-ax}\right)-6a\left(\frac{1}{2}\left(-\frac{\operatorname{PolyLog}\left(4,1-\frac{2}{ax+1}\right)}{4a}-\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{ax+1}\right)\coth^{-1}(ax)^2}{2a}-\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{ax+1}\right)\coth^{-1}(ax)}{2a}\right)+\right.$$

input
```
Int[ArcCoth[a*x]^3/x,x]
```

output
```
2*ArcCoth[a*x]^3*ArcCoth[1 - 2/(1 - a*x)] - 6*a*((-1/2*(ArcCoth[a*x]^2*Pol
yLog[2, 1 - 2/(1 + a*x)])/a - (ArcCoth[a*x]*PolyLog[3, 1 - 2/(1 + a*x)])/(
2*a) - PolyLog[4, 1 - 2/(1 + a*x)]/(4*a))/2 + ((ArcCoth[a*x]^2*PolyLog[2,
1 - (2*a*x)/(1 + a*x)])/(2*a) + (ArcCoth[a*x]*PolyLog[3, 1 - (2*a*x)/(1 +
a*x)])/(2*a) + PolyLog[4, 1 - (2*a*x)/(1 + a*x)]/(4*a))/2)
```

3.29. $\int \frac{\coth^{-1}(ax)^3}{x}\,dx$

3.29.3.1 Defintions of rubi rules used

rule 6449
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcCoth[c*x])^p*ArcCoth[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcCoth[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6615
```
Int[(ArcCoth[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a
+ b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] - Simp[1/2   Int[Log[SimplifyInteg
rand[1 - 1/u, x]]*((a + b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1
- c*x))^2, 0]
```

rule 6619
```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6623
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.29.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.67 (sec) , antiderivative size = 536, normalized size of antiderivative = 3.57

method	result
derivativedivides	$\ln(ax)\operatorname{arccoth}(ax)^3 + \frac{i\pi \operatorname{csgn}\left(\frac{i\left(1+\frac{ax+1}{ax-1}\right)}{\frac{ax+1}{ax-1}-1}\right)\left(\operatorname{csgn}\left(\frac{i}{\frac{ax+1}{ax-1}-1}\right)\operatorname{csgn}\left(i\left(1+\frac{ax+1}{ax-1}\right)\right)-\operatorname{csgn}\left(\frac{i}{\frac{ax+1}{ax-1}-1}\right)\operatorname{csgn}\right.}{}$
default	$\ln(ax)\operatorname{arccoth}(ax)^3 + \frac{i\pi \operatorname{csgn}\left(\frac{i\left(1+\frac{ax+1}{ax-1}\right)}{\frac{ax+1}{ax-1}-1}\right)\left(\operatorname{csgn}\left(\frac{i}{\frac{ax+1}{ax-1}-1}\right)\operatorname{csgn}\left(i\left(1+\frac{ax+1}{ax-1}\right)\right)-\operatorname{csgn}\left(\frac{i}{\frac{ax+1}{ax-1}-1}\right)\operatorname{csgn}\right.}{}$
parts	$\ln(x)\operatorname{arccoth}(ax)^3 + 3a\left(\frac{\left(i\pi \operatorname{csgn}\left(\frac{i}{a}\right)\operatorname{csgn}\left(\frac{i\left(1+\frac{ax+1}{ax-1}\right)}{\frac{ax+1}{ax-1}-1}\right)\operatorname{csgn}\left(\frac{i\left(1+\frac{ax+1}{ax-1}\right)}{a\left(\frac{ax+1}{ax-1}-1\right)}\right)-i\pi \operatorname{csgn}\left(\frac{i}{a}\right)\operatorname{csgn}\right.}{}\right.$

input
```
int(arccoth(a*x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
ln(a*x)*arccoth(a*x)^3+1/2*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1)*(1+(a*x+1)/(a*x
-1)))*(csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(1+(a*x+1)/(a*x-1)))-csgn(I/((a*
x+1)/(a*x-1)-1))*csgn(I/((a*x+1)/(a*x-1)-1)*(1+(a*x+1)/(a*x-1)))-csgn(I/((
a*x+1)/(a*x-1)-1)*(1+(a*x+1)/(a*x-1)))*csgn(I*(1+(a*x+1)/(a*x-1)))+csgn(I/
((a*x+1)/(a*x-1)-1)*(1+(a*x+1)/(a*x-1)))^2)*arccoth(a*x)^3+arccoth(a*x)^3*
ln((a*x+1)/(a*x-1)-1)-arccoth(a*x)^3*ln(1+1/((a*x-1)/(a*x+1))^(1/2))-3*arc
coth(a*x)^2*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+6*arccoth(a*x)*polylog(3
,-1/((a*x-1)/(a*x+1))^(1/2))-6*polylog(4,-1/((a*x-1)/(a*x+1))^(1/2))-arcco
th(a*x)^3*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-3*arccoth(a*x)^2*polylog(2,1/((a
*x-1)/(a*x+1))^(1/2))+6*arccoth(a*x)*polylog(3,1/((a*x-1)/(a*x+1))^(1/2))-
6*polylog(4,1/((a*x-1)/(a*x+1))^(1/2))+3/2*arccoth(a*x)^2*polylog(2,-(a*x+
1)/(a*x-1))-3/2*arccoth(a*x)*polylog(3,-(a*x+1)/(a*x-1))+3/4*polylog(4,-(a
*x+1)/(a*x-1))
```

3.29.5 Fricas [F]

$$\int \frac{\coth^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{arccoth}(ax)^3}{x}\,dx$$

input
```
integrate(arccoth(a*x)^3/x,x, algorithm="fricas")
```

output
```
integral(arccoth(a*x)^3/x, x)
```

3.29.6 Sympy [F]

$$\int \frac{\coth^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{acoth}^3(ax)}{x}\,dx$$

input
```
integrate(acoth(a*x)**3/x,x)
```

output
```
Integral(acoth(a*x)**3/x, x)
```

3.29.7 Maxima [F]

$$\int \frac{\coth^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{arcoth}(ax)^3}{x}\,dx$$

input
```
integrate(arccoth(a*x)^3/x,x, algorithm="maxima")
```

output
```
integrate(arccoth(a*x)^3/x, x)
```

3.29.8 Giac [F]

$$\int \frac{\coth^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{arcoth}(ax)^3}{x}\,dx$$

input
```
integrate(arccoth(a*x)^3/x,x, algorithm="giac")
```

output
```
integrate(arccoth(a*x)^3/x, x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax)^3}{x}\,dx = \int \frac{\mathrm{acoth}(a\,x)^3}{x}\,dx$$

input
```
int(acoth(a*x)^3/x,x)
```

output
```
int(acoth(a*x)^3/x, x)
```

3.30 $\int \frac{\coth^{-1}(ax)^3}{x^2}\,dx$

3.30.1 Optimal result

Integrand size = 10, antiderivative size = 79

$$\int \frac{\coth^{-1}(ax)^3}{x^2}\,dx = a\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{x} + 3a\coth^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - 3a\coth^{-1}(ax)\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1+ax}\right) - \frac{3}{2}a\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)$$

output
```
a*arccoth(a*x)^3-arccoth(a*x)^3/x+3*a*arccoth(a*x)^2*ln(2-2/(a*x+1))-3*a*a
rccoth(a*x)*polylog(2,-1+2/(a*x+1))-3/2*a*polylog(3,-1+2/(a*x+1))
```

3.30.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \frac{\coth^{-1}(ax)^3}{x^2}\,dx = \frac{(-1+ax)\coth^{-1}(ax)^3}{x} + 3a\coth^{-1}(ax)^2\log\left(1+e^{-2\coth^{-1}(ax)}\right) - 3a\coth^{-1}(ax)\,\mathrm{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right) - \frac{3}{2}a\,\mathrm{PolyLog}\left(3,-e^{-2\coth^{-1}(ax)}\right)$$

input
```
Integrate[ArcCoth[a*x]^3/x^2,x]
```

output
```
((-1 + a*x)*ArcCoth[a*x]^3)/x + 3*a*ArcCoth[a*x]^2*Log[1 + E^(-2*ArcCoth[a
*x])] - 3*a*ArcCoth[a*x]*PolyLog[2, -E^(-2*ArcCoth[a*x])] - (3*a*PolyLog[3
, -E^(-2*ArcCoth[a*x])])/2
```

3.30.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6453, 6551, 6495, 6619, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)^3}{x^2}\,dx$$

$$\Big\downarrow \text{6453}$$

$$3a\int \frac{\coth^{-1}(ax)^2}{x(1-a^2x^2)}dx-\frac{\coth^{-1}(ax)^3}{x}$$

$$\Big\downarrow \text{6551}$$

$$3a\left(\int \frac{\coth^{-1}(ax)^2}{x(ax+1)}dx+\frac{1}{3}\coth^{-1}(ax)^3\right)-\frac{\coth^{-1}(ax)^3}{x}$$

$$\Big\downarrow \text{6495}$$

$$3a\left(-2a\int \frac{\coth^{-1}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\coth^{-1}(ax)^3+\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)^2\right)-\frac{\coth^{-1}(ax)^3}{x}$$

$$\Big\downarrow \text{6619}$$

$$3a\left(-2a\left(\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\coth^{-1}(ax)}{2a}-\frac{1}{2}\int \frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{3}\coth^{-1}(ax)^3+\log\left(2-\frac{}{ax}\right.\right.$$
$$\frac{\coth^{-1}(ax)^3}{x}$$

$$\Big\downarrow \text{7164}$$

$$3a\left(-2a\left(\frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{4a}+\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\coth^{-1}(ax)}{2a}\right)+\frac{1}{3}\coth^{-1}(ax)^3+\log\left(2-\frac{2}{ax+1}\right)\text{c}\right.$$
$$\frac{\coth^{-1}(ax)^3}{x}$$

input
```
Int[ArcCoth[a*x]^3/x^2,x]
```

output
```
-(ArcCoth[a*x]^3/x) + 3*a*(ArcCoth[a*x]^3/3 + ArcCoth[a*x]^2*Log[2 - 2/(1
+ a*x)] - 2*a*((ArcCoth[a*x]*PolyLog[2, -1 + 2/(1 + a*x)])/(2*a) + PolyLog
[3, -1 + 2/(1 + a*x)]/(4*a)))
```

3.30.3.1 Defintions of rubi rules used

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6495
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6551
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6619
```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.30.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.61 (sec) , antiderivative size = 718, normalized size of antiderivative = 9.09

method	result
parts	$-\frac{\operatorname{arccoth}(ax)^3}{x} - 3a\left(\frac{\operatorname{arccoth}(ax)^2 \ln(ax+1)}{2} + \frac{\operatorname{arccoth}(ax)^2 \ln(ax-1)}{2} - \ln(ax)\operatorname{arccoth}(ax)^2 + \frac{\operatorname{arcc}}{}\right.$
derivativedivides	$a\left(-\frac{\operatorname{arccoth}(ax)^3}{ax} - \frac{3\operatorname{arccoth}(ax)^2 \ln(ax+1)}{2} - \frac{3\operatorname{arccoth}(ax)^2 \ln(ax-1)}{2} + 3\ln(ax)\operatorname{arccoth}(ax)^2 - \frac{3}{}\right.$
default	$a\left(-\frac{\operatorname{arccoth}(ax)^3}{ax} - \frac{3\operatorname{arccoth}(ax)^2 \ln(ax+1)}{2} - \frac{3\operatorname{arccoth}(ax)^2 \ln(ax-1)}{2} + 3\ln(ax)\operatorname{arccoth}(ax)^2 - \frac{3}{}\right.$

input
```
int(arccoth(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccoth(a*x)^3/x-3*a*(1/2*arccoth(a*x)^2*ln(a*x+1)+1/2*arccoth(a*x)^2*ln(
a*x-1)-ln(a*x)*arccoth(a*x)^2+1/2*arccoth(a*x)^2*ln((a*x-1)/(a*x+1))+1/3*a
rccoth(a*x)^3-1/4*(-I*Pi*csgn(I*(a*x+1)/(a*x-1))^3-I*Pi*csgn(I/(a*x-1)*(a*
x+1)/((a*x+1)/(a*x-1)-1))^3+I*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I/(a*x-1)*(a
*x+1)/((a*x+1)/(a*x-1)-1))^2-I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+
1)/(a*x-1))*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))+2*I*Pi*csgn(I/((a*
x+1)/(a*x-1)-1))*csgn(I*(1+(a*x+1)/(a*x-1)))*csgn(I/((a*x+1)/(a*x-1)-1)*(1
+(a*x+1)/(a*x-1)))-2*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I/((a*x+1)/(a*x
-1)-1)*(1+(a*x+1)/(a*x-1)))^2+2*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1)*(1+(a*x+1)
/(a*x-1)))^3+I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I/(a*x-1)*(a*x+1)/((a*x
+1)/(a*x-1)-1))^2-I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a
*x-1))-2*I*Pi*csgn(I*(1+(a*x+1)/(a*x-1)))*csgn(I/((a*x+1)/(a*x-1)-1)*(1+(a
*x+1)/(a*x-1)))^2+2*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a
*x-1))^2+4*ln(2))*arccoth(a*x)^2-arccoth(a*x)*polylog(2,-(a*x+1)/(a*x-1))+
1/2*polylog(3,-(a*x+1)/(a*x-1)))
```

3.30.5 Fricas [F]

$$\int \frac{\coth^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{arcoth}(ax)^3}{x^2}\,dx$$

input
```
integrate(arccoth(a*x)^3/x^2,x, algorithm="fricas")
```

output
```
integral(arccoth(a*x)^3/x^2, x)
```

3.30.6 Sympy [F]

$$\int \frac{\coth^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{acoth}^3(ax)}{x^2}\,dx$$

input
```
integrate(acoth(a*x)**3/x**2,x)
```

output
```
Integral(acoth(a*x)**3/x**2, x)
```

3.30.7 Maxima [F]

$$\int \frac{\coth^{-1}(ax)^3}{x^2} \, dx = \int \frac{\operatorname{arcoth}(ax)^3}{x^2} \, dx$$

input
```
integrate(arccoth(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
1/8*a*(log(a*x + 1) - log(x))*log(a)^3 + 3/8*a*integrate(x*log(a*x - 1)/(a
*x^3 + x^2), x)*log(a)^2 - 3/8*a*integrate(x*log(x)/(a*x^3 + x^2), x)*log(
a)^2 - 1/8*(a*log(a*x + 1) - a*log(x) - 1/x)*log(a)^3 + 3/4*a^2*integrate(
x^2*log(a*x + 1)*log(a*x - 1)/(a*x^3 + x^2), x) - 3/2*a^2*integrate(x^2*lo
g(a*x + 1)*log(x)/(a*x^3 + x^2), x) + 3/4*a*integrate(x*log(a*x - 1)*log(x
)/(a*x^3 + x^2), x)*log(a) - 3/8*a*integrate(x*log(x)^2/(a*x^3 + x^2), x)*
log(a) + 3/8*integrate(log(a*x - 1)/(a*x^3 + x^2), x)*log(a)^2 - 3/8*integ
rate(log(x)/(a*x^3 + x^2), x)*log(a)^2 + 3/8*a*integrate(x*log(a*x + 1)*lo
g(a*x - 1)^2/(a*x^3 + x^2), x) - 3/8*a*integrate(x*log(a*x - 1)^2*log(x)/(
a*x^3 + x^2), x) + 3/8*a*integrate(x*log(a*x - 1)*log(x)^2/(a*x^3 + x^2),
x) - 1/8*a*integrate(x*log(x)^3/(a*x^3 + x^2), x) - 3/4*a*integrate(x*log(
a*x + 1)*log(a*x - 1)/(a*x^3 + x^2), x) - 3/8*integrate(a*x*log(a*x - 1)^2
/(a*x^3 + x^2), x)*log(a) - 3/8*integrate(log(a*x - 1)^2/(a*x^3 + x^2), x)
*log(a) + 3/4*integrate(log(a*x - 1)*log(x)/(a*x^3 + x^2), x)*log(a) - 3/8
*integrate(log(x)^2/(a*x^3 + x^2), x)*log(a) - 3/8*(a^2*log(a*x - 1) - a^2
*log(x) + a/x)*log(-1/(a*x) + 1)^2/a + 1/8*log(-1/(a*x) + 1)^3/x - 1/8*((a
*x + 1)*log(a*x + 1)^3 - 3*(2*a*x*log(x) - (a*x - 1)*log(a*x - 1))*log(a*x
 + 1)^2)/x + 1/8*(3*(a^3*x*log(a*x - 1)^2 + a^3*x*log(x)^2 - 2*a^3*x*log(x
) + 2*a^2 - 2*(a^3*x*log(x) - a^3*x)*log(a*x - 1))*log(-1/(a*x) + 1)/(a*x)
 - (a^4*x*log(a*x - 1)^3 - a^4*x*log(x)^3 + 3*a^4*x*log(x)^2 - 6*a^4*x*...
```

3.30.8 Giac [F]

$$\int \frac{\coth^{-1}(ax)^3}{x^2} \, dx = \int \frac{\operatorname{arcoth}(ax)^3}{x^2} \, dx$$

input
```
integrate(arccoth(a*x)^3/x^2,x, algorithm="giac")
```

output
```
integrate(arccoth(a*x)^3/x^2, x)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax)^3}{x^2}\,dx = \int \frac{\mathrm{acoth}(a\,x)^3}{x^2}\,dx$$

input
```
int(acoth(a*x)^3/x^2,x)
```

output
```
int(acoth(a*x)^3/x^2, x)
```

3.31 $\int \frac{\coth^{-1}(ax)^3}{x^3}\,dx$

3.31.1 Optimal result

Integrand size = 10, antiderivative size = 95

$$\int \frac{\coth^{-1}(ax)^3}{x^3}\,dx=\frac{3}{2}a^2\coth^{-1}(ax)^2-\frac{3a\coth^{-1}(ax)^2}{2x}+\frac{1}{2}a^2\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{2x^2}+3a^2\coth^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)-\frac{3}{2}a^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)$$

output
```
3/2*a^2*arccoth(a*x)^2-3/2*a*arccoth(a*x)^2/x+1/2*a^2*arccoth(a*x)^3-1/2*a
rccoth(a*x)^3/x^2+3*a^2*arccoth(a*x)*ln(2-2/(a*x+1))-3/2*a^2*polylog(2,-1+
2/(a*x+1))
```

3.31.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{\coth^{-1}(ax)^3}{x^3}\,dx=\frac{1}{2}\left(\frac{\coth^{-1}(ax)\left(3ax(-1+ax)\coth^{-1}(ax)+(-1+a^2x^2)\coth^{-1}(ax)^2+6a^2x^2\log\left(1+e^{-2\coth^{-1}(ax)}\right)\right)}{x^2}-3a^2\operatorname{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[ArcCoth[a*x]^3/x^3,x]
```

output
```
((ArcCoth[a*x]*(3*a*x*(-1 + a*x)*ArcCoth[a*x] + (-1 + a^2*x^2)*ArcCoth[a*x
]^2 + 6*a^2*x^2*Log[1 + E^(-2*ArcCoth[a*x])]))/x^2 - 3*a^2*PolyLog[2, -E^(
-2*ArcCoth[a*x])])/2
```

3.31.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6453, 6545, 6453, 6511, 6551, 6495, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)^3}{x^3} \, dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{3}{2}a\int \frac{\coth^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)}dx-\frac{\coth^{-1}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6545}$$

$$\frac{3}{2}a\left(a^2\int \frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx+\int \frac{\coth^{-1}(ax)^2}{x^2}dx\right)-\frac{\coth^{-1}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6453}$$

$$\frac{3}{2}a\left(a^2\int \frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx+2a\int \frac{\coth^{-1}(ax)}{x\left(1-a^2x^2\right)}dx-\frac{\coth^{-1}(ax)^2}{x}\right)-\frac{\coth^{-1}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6511}$$

$$\frac{3}{2}a\left(2a\int \frac{\coth^{-1}(ax)}{x\left(1-a^2x^2\right)}dx+\frac{1}{3}a\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^2}{x}\right)-\frac{\coth^{-1}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6551}$$

$$\frac{3}{2}a\left(2a\left(\int \frac{\coth^{-1}(ax)}{x(ax+1)}dx+\frac{1}{2}\coth^{-1}(ax)^2\right)+\frac{1}{3}a\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^2}{x}\right)-\frac{\coth^{-1}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{6495}$$

$$\frac{3}{2}a\left(2a\left(-a\int\frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\coth^{-1}(ax)^2+\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)\right)+\frac{1}{3}a\coth^{-1}(ax)^3-\frac{\coth^{-}}{x}\right.$$
$$\frac{\coth^{-1}(ax)^3}{2x^2}$$

$\downarrow$ 2897

$$\frac{3}{2}a\left(2a\left(-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{2}\coth^{-1}(ax)^2+\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)\right)+\frac{1}{3}a\coth^{-1}(ax)^3-\frac{\text{co}}{}\right.$$
$$\frac{\coth^{-1}(ax)^3}{2x^2}$$

input
```
Int[ArcCoth[a*x]^3/x^3,x]
```

output
```
-1/2*ArcCoth[a*x]^3/x^2 + (3*a*(-(ArcCoth[a*x]^2/x) + (a*ArcCoth[a*x]^3)/3
 + 2*a*(ArcCoth[a*x]^2/2 + ArcCoth[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2,
-1 + 2/(1 + a*x)]/2)))/2
```

3.31.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6495
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6545
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6551
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.31.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.32 (sec) , antiderivative size = 3498, normalized size of antiderivative = 36.82

method	result	size
parts	Expression too large to display	3498
derivativedivides	Expression too large to display	3502
default	Expression too large to display	3502

input
```
int(arccoth(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccoth(a*x)^3/x^2-3/2*a^2*(-1/2*polylog(2,-(a*x+1)/(a*x-1))+1/a/x*ar
ccoth(a*x)^2-1/4*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*
csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2+1/4*I*Pi*csgn(I
/((a*x+1)/(a*x-1)-1))*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))^2*arccot
h(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2))-1/2*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(
1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arccoth(a*x)*ln(1+(a*x+1)/(a*x-1))+1/4*I*P
i*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)*l
n(1+(a*x+1)/(a*x-1))-1/4*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I/(a*x-1)*(
a*x+1)/((a*x+1)/(a*x-1)-1))^2*arccoth(a*x)*ln(1+(a*x+1)/(a*x-1))-1/4*I*Pi*
csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)*ln(
1+I/((a*x-1)/(a*x+1))^(1/2))-1/4*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*cs
gn(I*(a*x+1)/(a*x-1))*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2))+1/4*I*P
i*csgn(I*(a*x+1)/(a*x-1))*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))^2*ar
ccoth(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))+1/4*I*Pi*csgn(I*(a*x+1)/(a*x-1)
)*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))^2*arccoth(a*x)*ln(1-I/((a*x-
1)/(a*x+1))^(1/2))+1/2*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)
/(a*x-1))^2*arccoth(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))+1/2*I*Pi*csgn(I/(
(a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arccoth(a*x)*ln(1-I/((a*
x-1)/(a*x+1))^(1/2))-1/4*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(
a*x-1))*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))*dilog(1+I/((a*x-1)/...
```

3.31.5 Fricas [F]

$$\int \frac{\coth^{-1}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{arcoth}(ax)^3}{x^3}\,dx$$

input
```
integrate(arccoth(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
integral(arccoth(a*x)^3/x^3, x)
```

3.31.6 Sympy [F]

$$\int \frac{\coth^{-1}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{acoth}^3(ax)}{x^3}\,dx$$

input
```
integrate(acoth(a*x)**3/x**3,x)
```

output
```
Integral(acoth(a*x)**3/x**3, x)
```

3.31.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(84) = 168.

Time = 0.22 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.65

$$\int \frac{\coth^{-1}(ax)^3}{x^3}\,dx = \frac{3}{4}\left(a\log(ax+1) - a\log(ax-1) - \frac{2}{x}\right)a\operatorname{arcoth}(ax)^2$$
$$-\frac{1}{16}\left(a^2\left(\frac{3(\log(ax-1)-2)\log(ax+1)^2 - \log(ax+1)^3 + \log(ax-1)^3 - 3\left(\log(ax-1)^2 - 4\log(a\right.}{a}\right.\right.$$
$$-\frac{\operatorname{arcoth}(ax)^3}{2x^2}$$

input
```
integrate(arccoth(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
3/4*(a*log(a*x + 1) - a*log(a*x - 1) - 2/x)*a*arccoth(a*x)^2 - 1/16*(a^2*(
(3*(log(a*x - 1) - 2)*log(a*x + 1)^2 - log(a*x + 1)^3 + log(a*x - 1)^3 - 3
*(log(a*x - 1)^2 - 4*log(a*x - 1))*log(a*x + 1) + 6*log(a*x - 1)^2)/a - 24
*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a + 24*(log(a*x
 + 1)*log(x) + dilog(-a*x))/a - 24*(log(-a*x + 1)*log(x) + dilog(a*x))/a)
- 6*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 -
 4*log(a*x - 1) + 8*log(x))*a*arccoth(a*x))*a - 1/2*arccoth(a*x)^3/x^2
```

3.31.8 Giac [F]

$$\int \frac{\coth^{-1}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{arcoth}\left(ax\right)^3}{x^3}\,dx$$

input
```
integrate(arccoth(a*x)^3/x^3,x, algorithm="giac")
```

output
```
integrate(arccoth(a*x)^3/x^3, x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{acoth}(a\,x)^3}{x^3}\,dx$$

input
```
int(acoth(a*x)^3/x^3,x)
```

output
```
int(acoth(a*x)^3/x^3, x)
```

3.32 $\int \frac{\coth^{-1}(ax)^3}{x^4} dx$

3.32.1 Optimal result

Integrand size = 10, antiderivative size = 154

$$\begin{aligned}\int \frac{\coth^{-1}(ax)^3}{x^4} dx = & -\frac{a^2\coth^{-1}(ax)}{x}+\frac{1}{2}a^3\coth^{-1}(ax)^2-\frac{a\coth^{-1}(ax)^2}{2x^2} \\ & +\frac{1}{3}a^3\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{3x^3}+a^3\log(x) \\ & -\frac{1}{2}a^3\log\left(1-a^2x^2\right)+a^3\coth^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) \\ & -a^3\coth^{-1}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right) \\ & -\frac{1}{2}a^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1+ax}\right)\end{aligned}$$

output
```
-a^2*arccoth(a*x)/x+1/2*a^3*arccoth(a*x)^2-1/2*a*arccoth(a*x)^2/x^2+1/3*a^
3*arccoth(a*x)^3-1/3*arccoth(a*x)^3/x^3+a^3*ln(x)-1/2*a^3*ln(-a^2*x^2+1)+a
^3*arccoth(a*x)^2*ln(2-2/(a*x+1))-a^3*arccoth(a*x)*polylog(2,-1+2/(a*x+1))
-1/2*a^3*polylog(3,-1+2/(a*x+1))
```

3.32.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.92

$$\int \frac{\coth^{-1}(ax)^3}{x^4}\,dx = \frac{1}{6}\left(-\frac{6a^2\coth^{-1}(ax)}{x}+3a^3\coth^{-1}(ax)^2-\frac{3a\coth^{-1}(ax)^2}{x^2}+2a^3\coth^{-1}(ax)^3-\frac{2\coth^{-1}(ax)^3}{x^3}+6a^3\coth^{-1}(ax)^2\log\left(1+e^{-2\coth^{-1}(ax)}\right)+6a^3\log\left(\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}\right)-6a^3\coth^{-1}(ax)\,\mathrm{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right)-3a^3\,\mathrm{PolyLog}\left(3,-e^{-2\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[ArcCoth[a*x]^3/x^4,x]
```

output
```
((-6*a^2*ArcCoth[a*x])/x + 3*a^3*ArcCoth[a*x]^2 - (3*a*ArcCoth[a*x]^2)/x^2
 + 2*a^3*ArcCoth[a*x]^3 - (2*ArcCoth[a*x]^3)/x^3 + 6*a^3*ArcCoth[a*x]^2*Lo
g[1 + E^(-2*ArcCoth[a*x])] + 6*a^3*Log[1/Sqrt[1 - 1/(a^2*x^2)]] - 6*a^3*Ar
cCoth[a*x]*PolyLog[2, -E^(-2*ArcCoth[a*x])] - 3*a^3*PolyLog[3, -E^(-2*ArcC
oth[a*x])])/6
```

3.32.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {6453, 6545, 6453, 6545, 6453, 243, 47, 14, 16, 6511, 6551, 6495, 6619, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)^3}{x^4}\,dx$$

$$\Big\downarrow \ 6453$$

$$a\int \frac{\coth^{-1}(ax)^2}{x^3\left(1-a^2x^2\right)}dx-\frac{\coth^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \text{6545}$$

$$a\left(a^2\int \frac{\coth^{-1}(ax)^2}{x\left(1-a^2x^2\right)}dx+\int \frac{\coth^{-1}(ax)^2}{x^3}dx\right)-\frac{\coth^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \text{6453}$$

$$a\left(a^2\int \frac{\coth^{-1}(ax)^2}{x\left(1-a^2x^2\right)}dx+a\int \frac{\coth^{-1}(ax)}{x^2\left(1-a^2x^2\right)}dx-\frac{\coth^{-1}(ax)^2}{2x^2}\right)-\frac{\coth^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \text{6545}$$

$$a\left(a^2\int \frac{\coth^{-1}(ax)^2}{x\left(1-a^2x^2\right)}dx+a\left(a^2\int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx+\int \frac{\coth^{-1}(ax)}{x^2}dx\right)-\frac{\coth^{-1}(ax)^2}{2x^2}\right)-\frac{\coth^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \text{6453}$$

$$a\left(a^2\int \frac{\coth^{-1}(ax)^2}{x\left(1-a^2x^2\right)}dx+a\left(a\int \frac{1}{x\left(1-a^2x^2\right)}dx+a^2\int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx-\frac{\coth^{-1}(ax)}{x}\right)-\frac{\coth^{-1}(ax)^2}{2x^2}\right)-\frac{\coth^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$a\left(a^2\int \frac{\coth^{-1}(ax)^2}{x\left(1-a^2x^2\right)}dx+a\left(\frac{1}{2}a\int \frac{1}{x^2\left(1-a^2x^2\right)}dx^2+a^2\int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx-\frac{\coth^{-1}(ax)}{x}\right)-\frac{\coth^{-1}(ax)^2}{2x^2}\right)-\frac{\coth^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \text{47}$$

$$a\left(a^2\int \frac{\coth^{-1}(ax)^2}{x\left(1-a^2x^2\right)}dx+a\left(\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx^2+\int \frac{1}{x^2}dx^2\right)+a^2\int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx-\frac{\coth^{-1}(ax)}{x}\right)-\ldots\right.\;\frac{\coth^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \text{14}$$

$$a\left(a^2\int \frac{\coth^{-1}(ax)^2}{x\left(1-a^2x^2\right)}dx+a\left(\frac{1}{2}a\left(a^2\int \frac{1}{1-a^2x^2}dx^2+\log\left(x^2\right)\right)+a^2\int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx-\frac{\coth^{-1}(ax)}{x}\right)-\ldots\right.\;\frac{\coth^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \text{16}$$

3.32. $\int \frac{\coth^{-1}(ax)^3}{x^4}\,dx$

$$a\left(a^2\int \frac{\coth^{-1}(ax)^2}{x\left(1-a^2x^2\right)}dx+a\left(a^2\int \frac{\coth^{-1}(ax)}{1-a^2x^2}dx+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)-\frac{\coth^{-1}(ax)}{x}\right)-\frac{\coth^{-1}(ax)^2}{2x^2}$$
$$\frac{\coth^{-1}(ax)^3}{3x^3}$$

$\big\downarrow$ 6511

$$a\left(a^2\int \frac{\coth^{-1}(ax)^2}{x\left(1-a^2x^2\right)}dx+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)}{x}\right)-\frac{\coth^{-1}(ax)^2}{2x^2}\right)-$$
$$\frac{\coth^{-1}(ax)^3}{3x^3}$$

$\big\downarrow$ 6551

$$a\left(a^2\left(\int \frac{\coth^{-1}(ax)^2}{x(ax+1)}dx+\frac{1}{3}\coth^{-1}(ax)^3\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(1-a^2x^2\right)\right)+\frac{1}{2}a\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)}{x}\right.\right.$$
$$\frac{\coth^{-1}(ax)^3}{3x^3}$$

$\big\downarrow$ 6495

$$a\left(a^2\left(-2a\int \frac{\coth^{-1}(ax)\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{3}\coth^{-1}(ax)^3+\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)^2\right)+a\left(\frac{1}{2}a\left(\log\left(x\right.\right.\right.\right.$$
$$\frac{\coth^{-1}(ax)^3}{3x^3}$$

$\big\downarrow$ 6619

$$a\left(a^2\left(-2a\left(\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\coth^{-1}(ax)}{2a}-\frac{1}{2}\int \frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{1-a^2x^2}dx\right)+\frac{1}{3}\coth^{-1}(ax)^3+\log\left(2-\right.\right.\right.$$
$$\frac{\coth^{-1}(ax)^3}{3x^3}$$

$\big\downarrow$ 7164

$$a\left(a^2\left(-2a\left(\frac{\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{4a}+\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)\coth^{-1}(ax)}{2a}\right)+\frac{1}{3}\coth^{-1}(ax)^3+\log\left(2-\frac{2}{ax+1}\right.\right.\right.$$
$$\frac{\coth^{-1}(ax)^3}{3x^3}$$

input
```
Int[ArcCoth[a*x]^3/x^4,x]
```

3.32. $\int \frac{\coth^{-1}(ax)^3}{x^4}\,dx$

output
```
-1/3*ArcCoth[a*x]^3/x^3 + a*(-1/2*ArcCoth[a*x]^2/x^2 + a*(-(ArcCoth[a*x]/x
) + (a*ArcCoth[a*x]^2)/2 + (a*(Log[x^2] - Log[1 - a^2*x^2]))/2) + a^2*(Arc
Coth[a*x]^3/3 + ArcCoth[a*x]^2*Log[2 - 2/(1 + a*x)] - 2*a*((ArcCoth[a*x]*P
olyLog[2, -1 + 2/(1 + a*x)])/(2*a) + PolyLog[3, -1 + 2/(1 + a*x)]/(4*a)))
```

3.32.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6495
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6545
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6551
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6619
```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.32.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.86 (sec) , antiderivative size = 840, normalized size of antiderivative = 5.45

method	result	size
derivativedivides	Expression too large to display	840
default	Expression too large to display	840
parts	Expression too large to display	843

input
```
int(arccoth(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/3/a^3/x^3*arccoth(a*x)^3-1/2*arccoth(a*x)^2*ln(a*x-1)-1/2*arccoth(
a*x)^2*ln(a*x+1)-1/2*arccoth(a*x)^2/a^2/x^2+ln(a*x)*arccoth(a*x)^2-1/2*arc
coth(a*x)^2*ln((a*x-1)/(a*x+1))+arccoth(a*x)*polylog(2,-(a*x+1)/(a*x-1))-1
/2*polylog(3,-(a*x+1)/(a*x-1))+1/12*arccoth(a*x)*(-3*I*csgn(I/((a*x+1)/(a*
x-1)-1))*arccoth(a*x)*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I/(a*x-1)*(a*x+1)/((
a*x+1)/(a*x-1)-1))*a*x+6*I*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(1+(a*x+1)/(
a*x-1)))*arccoth(a*x)*Pi*csgn(I/((a*x+1)/(a*x-1)-1)*(1+(a*x+1)/(a*x-1)))*a
*x+3*I*arccoth(a*x)*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I/(a*x-1)*(a*x+1)/((a*
x+1)/(a*x-1)-1))^2*a*x-3*I*arccoth(a*x)*Pi*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)
/(a*x-1)-1))^3*a*x+6*I*arccoth(a*x)*Pi*csgn(I/((a*x+1)/(a*x-1)-1)*(1+(a*x+
1)/(a*x-1)))^3*a*x-3*I*arccoth(a*x)*Pi*csgn(I*(a*x+1)/(a*x-1))^3*a*x+6*I*a
rccoth(a*x)*Pi*csgn(I*(a*x+1)/(a*x-1))^2*csgn(I/((a*x-1)/(a*x+1))^(1/2))*a
*x-6*I*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)*Pi*csgn(I/((a*x+1)/(a*x-1)
-1)*(1+(a*x+1)/(a*x-1)))^2*a*x-6*I*csgn(I*(1+(a*x+1)/(a*x-1)))*arccoth(a*x
)*Pi*csgn(I/((a*x+1)/(a*x-1)-1)*(1+(a*x+1)/(a*x-1)))^2*a*x-3*I*arccoth(a*x
)*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*a*x+3*I*csg
n(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)*Pi*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(
a*x-1)-1))^2*a*x-4*arccoth(a*x)^2*a*x+12*arccoth(a*x)*ln(2)*a*x+6*a*x*arcc
oth(a*x)-12*a*x-12)/a/x+ln(1+(a*x+1)/(a*x-1)))
```

3.32.5 Fricas [F]

$$\int \frac{\coth^{-1}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{arcoth}(ax)^3}{x^4}\,dx$$

input
```
integrate(arccoth(a*x)^3/x^4,x, algorithm="fricas")
```

output
```
integral(arccoth(a*x)^3/x^4, x)
```

3.32.6 Sympy [F]

$$\int \frac{\coth^{-1}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{acoth}^3(ax)}{x^4}\,dx$$

input
```
integrate(acoth(a*x)**3/x**4,x)
```

output
```
Integral(acoth(a*x)**3/x**4, x)
```

3.32.7 Maxima [F]

$$\int \frac{\coth^{-1}(ax)^3}{x^4} dx = \int \frac{\operatorname{arcoth}(ax)^3}{x^4} dx$$

input
```
integrate(arccoth(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
1/4*a^4*integrate(x^4*log(a*x + 1)*log(a*x - 1)/(a*x^5 + x^4), x) - 1/2*a^
4*integrate(x^4*log(a*x + 1)*log(x)/(a*x^5 + x^4), x) + 1/16*(2*a^2*log(a*
x + 1) - 2*a^2*log(x) - (2*a*x - 1)/x^2)*a*log(a)^3 + 3/8*a*integrate(x*lo
g(a*x - 1)/(a*x^5 + x^4), x)*log(a)^2 - 3/8*a*integrate(x*log(x)/(a*x^5 +
x^4), x)*log(a)^2 - 1/48*(6*a^3*log(a*x + 1) - 6*a^3*log(x) - (6*a^2*x^2 -
 3*a*x + 2)/x^3)*log(a)^3 + 1/4*a^2*integrate(x^2*log(a*x + 1)/(a*x^5 + x^
4), x) + 3/4*a*integrate(x*log(a*x - 1)*log(x)/(a*x^5 + x^4), x)*log(a) -
3/8*a*integrate(x*log(x)^2/(a*x^5 + x^4), x)*log(a) + 3/8*integrate(log(a*
x - 1)/(a*x^5 + x^4), x)*log(a)^2 - 3/8*integrate(log(x)/(a*x^5 + x^4), x)
*log(a)^2 + 3/8*a*integrate(x*log(a*x + 1)*log(a*x - 1)^2/(a*x^5 + x^4), x
) - 3/8*a*integrate(x*log(a*x - 1)^2*log(x)/(a*x^5 + x^4), x) + 3/8*a*inte
grate(x*log(a*x - 1)*log(x)^2/(a*x^5 + x^4), x) - 1/8*a*integrate(x*log(x)
^3/(a*x^5 + x^4), x) - 1/4*a*integrate(x*log(a*x + 1)*log(a*x - 1)/(a*x^5
+ x^4), x) - 3/8*integrate(a*x*log(a*x - 1)^2/(a*x^5 + x^4), x)*log(a) - 3
/8*integrate(log(a*x - 1)^2/(a*x^5 + x^4), x)*log(a) + 3/4*integrate(log(a
*x - 1)*log(x)/(a*x^5 + x^4), x)*log(a) - 3/8*integrate(log(x)^2/(a*x^5 +
x^4), x)*log(a) - 1/48*(6*a^4*log(a*x - 1) - 6*a^4*log(x) + (6*a^3*x^2 + 3
*a^2*x + 2*a)/x^3)*log(-1/(a*x) + 1)^2/a + 1/864*(6*(18*a^5*x^3*log(a*x -
1)^2 + 18*a^5*x^3*log(x)^2 - 66*a^5*x^3*log(x) + 66*a^4*x^2 + 15*a^3*x + 4
*a^2 - 6*(6*a^5*x^3*log(x) - 11*a^5*x^3)*log(a*x - 1))*log(-1/(a*x) + 1...
```

3.32.8 Giac [F]

$$\int \frac{\coth^{-1}(ax)^3}{x^4} dx = \int \frac{\operatorname{arcoth}(ax)^3}{x^4} dx$$

input
```
integrate(arccoth(a*x)^3/x^4,x, algorithm="giac")
```

output
```
integrate(arccoth(a*x)^3/x^4, x)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax)^3}{x^4}\,dx = \int \frac{\mathrm{acoth}(a\,x)^3}{x^4}\,dx$$

input
```
int(acoth(a*x)^3/x^4,x)
```

output
```
int(acoth(a*x)^3/x^4, x)
```

3.33 $\int \frac{\coth^{-1}(ax)^3}{x^5}\,dx$

3.33.1 Optimal result

Integrand size = 10, antiderivative size = 141

$$\int \frac{\coth^{-1}(ax)^3}{x^5}\,dx = -\frac{a^3}{4x} - \frac{a^2\coth^{-1}(ax)}{4x^2} + a^4\coth^{-1}(ax)^2 - \frac{a\coth^{-1}(ax)^2}{4x^3} - \frac{3a^3\coth^{-1}(ax)^2}{4x} + \frac{1}{4}a^4\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{4x^4} + \frac{1}{4}a^4\operatorname{arctanh}(ax) + 2a^4\coth^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right) - a^4\operatorname{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)$$

output
```
-1/4*a^3/x-1/4*a^2*arccoth(a*x)/x^2+a^4*arccoth(a*x)^2-1/4*a*arccoth(a*x)^
2/x^3-3/4*a^3*arccoth(a*x)^2/x+1/4*a^4*arccoth(a*x)^3-1/4*arccoth(a*x)^3/x
^4+1/4*a^4*arctanh(a*x)+2*a^4*arccoth(a*x)*ln(2-2/(a*x+1))-a^4*polylog(2,-
1+2/(a*x+1))
```

3.33.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

$$\int \frac{\coth^{-1}(ax)^3}{x^5}\,dx$$
$$= \frac{-a^3x^3 + ax(-1-3a^2x^2+4a^3x^3)\coth^{-1}(ax)^2 + (-1+a^4x^4)\coth^{-1}(ax)^3 + a^2x^2\coth^{-1}(ax)\left(-1+a^2x^2\right.}{4x^4}$$

input
```
Integrate[ArcCoth[a*x]^3/x^5,x]
```

output
```
(-(a^3*x^3) + a*x*(-1 - 3*a^2*x^2 + 4*a^3*x^3)*ArcCoth[a*x]^2 + (-1 + a^4*
x^4)*ArcCoth[a*x]^3 + a^2*x^2*ArcCoth[a*x]*(-1 + a^2*x^2 + 8*a^2*x^2*Log[1
 + E^(-2*ArcCoth[a*x])]) - 4*a^4*x^4*PolyLog[2, -E^(-2*ArcCoth[a*x])])/(4*
x^4)
```

3.33.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.35, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6453, 6545, 6453, 6545, 6453, 264, 219, 6511, 6551, 6495, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)^3}{x^5}dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{3}{4}a\int \frac{\coth^{-1}(ax)^2}{x^4(1-a^2x^2)}dx-\frac{\coth^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{6545}$$

$$\frac{3}{4}a\left(a^2\int \frac{\coth^{-1}(ax)^2}{x^2(1-a^2x^2)}dx+\int \frac{\coth^{-1}(ax)^2}{x^4}dx\right)-\frac{\coth^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{6453}$$

$$\frac{3}{4}a\left(a^2\int \frac{\coth^{-1}(ax)^2}{x^2(1-a^2x^2)}dx+\frac{2}{3}a\int \frac{\coth^{-1}(ax)}{x^3(1-a^2x^2)}dx-\frac{\coth^{-1}(ax)^2}{3x^3}\right)-\frac{\coth^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{6545}$$

$$\frac{3}{4}a\left(a^2\left(a^2\int \frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx+\int \frac{\coth^{-1}(ax)^2}{x^2}dx\right)+\frac{2}{3}a\left(a^2\int \frac{\coth^{-1}(ax)}{x(1-a^2x^2)}dx+\int \frac{\coth^{-1}(ax)}{x^3}dx\right)-\frac{\coth^{-1}(ax)^2}{3x^3}\right)-\frac{\coth^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{6453}$$

$$\frac{3}{4}a\Bigg(a^2\Bigg(a^2\int\frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx+2a\int\frac{\coth^{-1}(ax)}{x\left(1-a^2x^2\right)}dx-\frac{\coth^{-1}(ax)^2}{x}\Bigg)+\frac{2}{3}a\Bigg(\frac{1}{2}a\int\frac{1}{x^2\left(1-a^2x^2\right)}dx+a^2\int\frac{\text{coth}}{x\,(1}$$

$$\frac{\coth^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow\ 264$$

$$\frac{3}{4}a\Bigg(a^2\Bigg(a^2\int\frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx+2a\int\frac{\coth^{-1}(ax)}{x\left(1-a^2x^2\right)}dx-\frac{\coth^{-1}(ax)^2}{x}\Bigg)+\frac{2}{3}a\Bigg(\frac{1}{2}a\Bigg(a^2\int\frac{1}{1-a^2x^2}dx-\frac{1}{x}\Bigg)+a^2\int$$

$$\frac{\coth^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow\ 219$$

$$\frac{3}{4}a\Bigg(\frac{2}{3}a\Bigg(a^2\int\frac{\coth^{-1}(ax)}{x\left(1-a^2x^2\right)}dx+\frac{1}{2}a\Bigg(a\text{arctanh}(ax)-\frac{1}{x}\Bigg)-\frac{\coth^{-1}(ax)}{2x^2}\Bigg)+a^2\Bigg(a^2\int\frac{\coth^{-1}(ax)^2}{1-a^2x^2}dx+2a\int\frac{\text{cot}}{x\,(1}$$

$$\frac{\coth^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow\ 6511$$

$$\frac{3}{4}a\Bigg(\frac{2}{3}a\Bigg(a^2\int\frac{\coth^{-1}(ax)}{x\left(1-a^2x^2\right)}dx+\frac{1}{2}a\Bigg(a\text{arctanh}(ax)-\frac{1}{x}\Bigg)-\frac{\coth^{-1}(ax)}{2x^2}\Bigg)+a^2\Bigg(2a\int\frac{\coth^{-1}(ax)}{x\left(1-a^2x^2\right)}dx+\frac{1}{3}a\coth^{-}$$

$$\frac{\coth^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow\ 6551$$

$$\frac{3}{4}a\Bigg(\frac{2}{3}a\Bigg(a^2\Bigg(\int\frac{\coth^{-1}(ax)}{x(ax+1)}dx+\frac{1}{2}\coth^{-1}(ax)^2\Bigg)+\frac{1}{2}a\Bigg(a\text{arctanh}(ax)-\frac{1}{x}\Bigg)-\frac{\coth^{-1}(ax)}{2x^2}\Bigg)+a^2\Bigg(2a\Bigg(\int\frac{\text{coth}^{-}}{x(ax}$$

$$\frac{\coth^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow\ 6495$$

$$\frac{3}{4}a\left(\frac{2}{3}a\left(a^2\left(-a\int\frac{\log\left(2-\frac{2}{ax+1}\right)}{1-a^2x^2}dx+\frac{1}{2}\coth^{-1}(ax)^2+\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)\right)+\frac{1}{2}a\Bigg(a\text{arctanh}(ax)-\right.\right.$$

$$\frac{\coth^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow\ 2897$$

3.33. $\int \frac{\coth^{-1}(ax)^3}{x^5}\,dx$

$$\frac{3}{4}a\left(\frac{2}{3}a\left(a^2\left(-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{2}\coth^{-1}(ax)^2+\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)\right)+\frac{1}{2}a\left(a\operatorname{arctanh}(a\right.\right.\right.$$

$$\frac{\coth^{-1}(ax)^3}{4x^4}$$

input
```
Int[ArcCoth[a*x]^3/x^5,x]
```

output
```
-1/4*ArcCoth[a*x]^3/x^4 + (3*a*(-1/3*ArcCoth[a*x]^2/x^3 + a^2*(-(ArcCoth[a
*x]^2/x) + (a*ArcCoth[a*x]^3)/3 + 2*a*(ArcCoth[a*x]^2/2 + ArcCoth[a*x]*Log
[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(1 + a*x)]/2)) + (2*a*(-1/2*ArcCoth[
a*x]/x^2 + (a*(-x^(-1) + a*ArcTanh[a*x]))/2 + a^2*(ArcCoth[a*x]^2/2 + ArcC
oth[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(1 + a*x)]/2)))/3))/4
```

3.33.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6495
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6545
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6551
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.33.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.73 (sec) , antiderivative size = 657, normalized size of antiderivative = 4.66

method	result
parts	$-\frac{\operatorname{arccoth}(ax)^3}{4x^4}-\frac{3a^4\left(\frac{\operatorname{arccoth}(ax)^2}{3a^3x^3}+\frac{\operatorname{arccoth}(ax)^2}{ax}-\frac{\operatorname{arccoth}(ax)^2\ln(ax+1)}{2}+\frac{\operatorname{arccoth}(ax)^2\ln(ax-1)}{2}-\frac{8\operatorname{arccoth}(ax)\ln\left(1+\right.}{3}\right.}{}$
derivativedivides	$a^4\left(-\frac{\operatorname{arccoth}(ax)^3}{4a^4x^4}-\frac{\operatorname{arccoth}(ax)^2}{4a^3x^3}-\frac{3\operatorname{arccoth}(ax)^2}{4ax}+\frac{3\operatorname{arccoth}(ax)^2\ln(ax+1)}{8}-\frac{3\operatorname{arccoth}(ax)^2\ln(ax-1)}{8}\right.$
default	$a^4\left(-\frac{\operatorname{arccoth}(ax)^3}{4a^4x^4}-\frac{\operatorname{arccoth}(ax)^2}{4a^3x^3}-\frac{3\operatorname{arccoth}(ax)^2}{4ax}+\frac{3\operatorname{arccoth}(ax)^2\ln(ax+1)}{8}-\frac{3\operatorname{arccoth}(ax)^2\ln(ax-1)}{8}\right.$

input
```
int(arccoth(a*x)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arccoth(a*x)^3/x^4-3/4*a^4*(1/3/a^3/x^3*arccoth(a*x)^2+1/a/x*arccoth(
a*x)^2-1/2*arccoth(a*x)^2*ln(a*x+1)+1/2*arccoth(a*x)^2*ln(a*x-1)-8/3*arcco
th(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))-8/3*arccoth(a*x)*ln(1-I/((a*x-1)/(
a*x+1))^(1/2))-8/3*dilog(1+I/((a*x-1)/(a*x+1))^(1/2))-8/3*dilog(1-I/((a*x-
1)/(a*x+1))^(1/2))-1/3*(a*x-1)/a/x-1/3*arccoth(a*x)^3+2/3*arccoth(a*x)*(a*
x+1)/a/x+4/3*arccoth(a*x)^2+1/4*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I/(a
*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))^2*arccoth(a*x)^2-1/3*arccoth(a*x)*(a*x+
1)^2/a^2/x^2-2/3*arccoth(a*x)*(a*x-1)*(a*x+1)/a^2/x^2+1/2*I*Pi*csgn(I/((a*
x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arccoth(a*x)^2+1/4*I*Pi*csg
n(I*(a*x+1)/(a*x-1))*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))^2*arccoth
(a*x)^2-1/2*arccoth(a*x)^2*ln((a*x-1)/(a*x+1))-1/4*I*Pi*csgn(I/((a*x-1)/(a
*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)^2-1/4*I*Pi*csgn(I/(a*
x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))^3*arccoth(a*x)^2-1/4*I*Pi*csgn(I*(a*x+1)
/(a*x-1))^3*arccoth(a*x)^2-1/4*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*
x+1)/(a*x-1))*csgn(I/(a*x-1)*(a*x+1)/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2)
```

3.33. $\int \frac{\coth^{-1}(ax)^3}{x^5}\,dx$

3.33.5 Fricas [F]

$$\int \frac{\coth^{-1}(ax)^3}{x^5}\,dx = \int \frac{\operatorname{arcoth}(ax)^3}{x^5}\,dx$$

input
```
integrate(arccoth(a*x)^3/x^5,x, algorithm="fricas")
```

output
```
integral(arccoth(a*x)^3/x^5, x)
```

3.33.6 Sympy [F]

$$\int \frac{\coth^{-1}(ax)^3}{x^5}\,dx = \int \frac{\operatorname{acoth}^3(ax)}{x^5}\,dx$$

input
```
integrate(acoth(a*x)**3/x**5,x)
```

output
```
Integral(acoth(a*x)**3/x**5, x)
```

3.33.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(126) = 252.

Time = 0.22 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.43

$$\int \frac{\coth^{-1}(ax)^3}{x^5}\,dx = \frac{1}{8}\left(3\,a^3\log(ax+1) - 3\,a^3\log(ax-1) - \frac{2\,(3\,a^2x^2+1)}{x^3}\right)a\operatorname{arcoth}(ax)^2$$
$$+\frac{1}{32}\left(\left(32\left(\log(ax-1)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)a - 32\left(\log(ax+1)\log(x)+\operatorname{Li}_2(-ax)\right)a\right.\right.$$
$$-\frac{\operatorname{arcoth}(ax)^3}{4\,x^4}$$

input
```
integrate(arccoth(a*x)^3/x^5,x, algorithm="maxima")
```

output
```
1/8*(3*a^3*log(a*x + 1) - 3*a^3*log(a*x - 1) - 2*(3*a^2*x^2 + 1)/x^3)*a*ar
ccoth(a*x)^2 + 1/32*((32*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x
 + 1/2))*a - 32*(log(a*x + 1)*log(x) + dilog(-a*x))*a + 32*(log(-a*x + 1)*
log(x) + dilog(a*x))*a + 4*a*log(a*x + 1) - 4*a*log(a*x - 1) + (a*x*log(a*
x + 1)^3 - a*x*log(a*x - 1)^3 - 8*a*x*log(a*x - 1)^2 - (3*a*x*log(a*x - 1)
 - 8*a*x)*log(a*x + 1)^2 + (3*a*x*log(a*x - 1)^2 - 16*a*x*log(a*x - 1))*lo
g(a*x + 1) - 8)/x)*a^2 + 2*(32*a^2*log(x) - (3*a^2*x^2*log(a*x + 1)^2 + 3*
a^2*x^2*log(a*x - 1)^2 + 16*a^2*x^2*log(a*x - 1) - 2*(3*a^2*x^2*log(a*x -
1) - 8*a^2*x^2)*log(a*x + 1) + 4)/x^2)*a*arccoth(a*x))*a - 1/4*arccoth(a*x
)^3/x^4
```

3.33.8 Giac [F]

$$\int \frac{\coth^{-1}(ax)^3}{x^5} dx = \int \frac{\text{arcoth}(ax)^3}{x^5} dx$$

input
```
integrate(arccoth(a*x)^3/x^5,x, algorithm="giac")
```

output
```
integrate(arccoth(a*x)^3/x^5, x)
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax)^3}{x^5} dx = \int \frac{\text{acoth}(a\,x)^3}{x^5} dx$$

input
```
int(acoth(a*x)^3/x^5,x)
```

output
```
int(acoth(a*x)^3/x^5, x)
```

3.34 $\int \frac{\coth^{-1}(cx)^2}{d+ex}\,dx$

3.34.1 Optimal result

Integrand size = 14, antiderivative size = 164

$$\int \frac{\coth^{-1}(cx)^2}{d+ex}\,dx = -\frac{\coth^{-1}(cx)^2\log\left(\frac{2}{1+cx}\right)}{e}+\frac{\coth^{-1}(cx)^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e}+\frac{\coth^{-1}(cx)\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{e}-\frac{\coth^{-1}(cx)\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e}+\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{1+cx}\right)}{2e}-\frac{\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{2e}$$

output
```
-arccoth(c*x)^2*ln(2/(c*x+1))/e+arccoth(c*x)^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x
+1))/e+arccoth(c*x)*polylog(2,1-2/(c*x+1))/e-arccoth(c*x)*polylog(2,1-2*c*
(e*x+d)/(c*d+e)/(c*x+1))/e+1/2*polylog(3,1-2/(c*x+1))/e-1/2*polylog(3,1-2*
c*(e*x+d)/(c*d+e)/(c*x+1))/e
```

3.34.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.73 (sec) , antiderivative size = 864, normalized size of antiderivative = 5.27

$$\int \frac{\coth^{-1}(cx)^2}{d+ex}\,dx = \text{Too large to display}$$

input
```
Integrate[ArcCoth[c*x]^2/(d + e*x),x]
```

output
```
((-I)*e*Pi^3 + 8*c*d*ArcCoth[c*x]^3 + 8*e*ArcCoth[c*x]^3 - 24*e*ArcCoth[c*
x]^2*Log[1 - E^(2*ArcCoth[c*x])] - 24*e*ArcCoth[c*x]*PolyLog[2, E^(2*ArcCo
th[c*x])] + 12*e*PolyLog[3, E^(2*ArcCoth[c*x])] + (24*(-(c*d) + e)*(c*d +
e)*(-2*c*d*ArcCoth[c*x]^3 + 6*e*ArcCoth[c*x]^3 + (4*c*d*Sqrt[1 - e^2/(c^2*
d^2)]*ArcCoth[c*x]^3)/E^ArcTanh[e/(c*d)] + (6*I)*e*Pi*ArcCoth[c*x]*Log[(E^
(-ArcCoth[c*x]) + E^ArcCoth[c*x])/2] + 6*e*ArcCoth[c*x]^2*Log[1 - (Sqrt[c*
d + e]*E^ArcCoth[c*x])/Sqrt[c*d - e]] + 6*e*ArcCoth[c*x]^2*Log[1 + (Sqrt[c
*d + e]*E^ArcCoth[c*x])/Sqrt[c*d - e]] - 6*e*ArcCoth[c*x]^2*Log[1 - E^(Arc
Coth[c*x] + ArcTanh[e/(c*d)])] - 6*e*ArcCoth[c*x]^2*Log[1 + E^(ArcCoth[c*x
] + ArcTanh[e/(c*d)])] - 6*e*ArcCoth[c*x]^2*Log[1 - E^(2*(ArcCoth[c*x] + A
rcTanh[e/(c*d)]))] - 12*e*ArcCoth[c*x]*ArcTanh[e/(c*d)]*Log[(I/2)*E^(-ArcC
oth[c*x] - ArcTanh[e/(c*d)])*(-1 + E^(2*(ArcCoth[c*x] + ArcTanh[e/(c*d)]))
)] - 6*e*ArcCoth[c*x]^2*Log[(c*d*(-1 + E^(2*ArcCoth[c*x])) + e*(1 + E^(2*A
rcCoth[c*x])))/(2*E^ArcCoth[c*x])] - (6*I)*e*Pi*ArcCoth[c*x]*Log[1/Sqrt[1
- 1/(c^2*x^2)]] + 6*e*ArcCoth[c*x]^2*Log[(d + e*x)/(Sqrt[1 - 1/(c^2*x^2)]*
x)] + 12*e*ArcCoth[c*x]*ArcTanh[e/(c*d)]*Log[I*Sinh[ArcCoth[c*x] + ArcTanh
[e/(c*d)]]] + 12*e*ArcCoth[c*x]*PolyLog[2, -((Sqrt[c*d + e]*E^ArcCoth[c*x]
)/Sqrt[c*d - e])] + 12*e*ArcCoth[c*x]*PolyLog[2, (Sqrt[c*d + e]*E^ArcCoth[
c*x])/Sqrt[c*d - e]] - 12*e*ArcCoth[c*x]*PolyLog[2, -E^(ArcCoth[c*x] + Arc
Tanh[e/(c*d)])] - 12*e*ArcCoth[c*x]*PolyLog[2, E^(ArcCoth[c*x] + ArcTan...
```

3.34.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {6475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(cx)^2}{d+ex}\,dx$$
$$\Big\downarrow \text{ 6475}$$
$$-\frac{\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e}-\frac{\coth^{-1}(cx)\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{e}+\frac{\coth^{-1}(cx)^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}+\frac{\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2e}+\frac{\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\coth^{-1}(cx)}{e}-\frac{\log\left(\frac{2}{cx+1}\right)\coth^{-1}(cx)^2}{e}$$

input
```
Int[ArcCoth[c*x]^2/(d + e*x),x]
```

output
```
-((ArcCoth[c*x]^2*Log[2/(1 + c*x)])/e) + (ArcCoth[c*x]^2*Log[(2*c*(d + e*x
))/((c*d + e)*(1 + c*x))])/e + (ArcCoth[c*x]*PolyLog[2, 1 - 2/(1 + c*x)])/
e - (ArcCoth[c*x]*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e
 + PolyLog[3, 1 - 2/(1 + c*x)]/(2*e) - PolyLog[3, 1 - (2*c*(d + e*x))/((c*
d + e)*(1 + c*x))]/(2*e)
```

3.34.3.1 Defintions of rubi rules used

rule 6475
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcCoth[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Coth[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcCoth[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcC
oth[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

3.34.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.68 (sec) , antiderivative size = 869, normalized size of antiderivative = 5.30

method	result
derivativedivides	$\frac{c\ln(cex+cd)\,\mathrm{arccoth}(cx)^2}{e}+\frac{2c\left(-\frac{\mathrm{arccoth}(cx)^2\ln\left(dc\left(\frac{cx+1}{cx-1}-1\right)+e\left(\frac{cx+1}{cx-1}+1\right)\right)}{2}+\frac{i\pi\,\mathrm{csgn}\left(\frac{i\left(dc\left(\frac{cx+1}{cx-1}-1\right)+e\left(\frac{cx+1}{cx-1}+1\right)\right)}{\frac{cx+1}{cx-1}-1}\right)\left(\mathrm{csgn}\right.}{}\right.}{}$
default	$\frac{c\ln(cex+cd)\,\mathrm{arccoth}(cx)^2}{e}+\frac{2c\left(-\frac{\mathrm{arccoth}(cx)^2\ln\left(dc\left(\frac{cx+1}{cx-1}-1\right)+e\left(\frac{cx+1}{cx-1}+1\right)\right)}{2}+\frac{i\pi\,\mathrm{csgn}\left(\frac{i\left(dc\left(\frac{cx+1}{cx-1}-1\right)+e\left(\frac{cx+1}{cx-1}+1\right)\right)}{\frac{cx+1}{cx-1}-1}\right)\left(\mathrm{csgn}\right.}{}\right.}{}$
parts	Expression too large to display

input
```
int(arccoth(c*x)^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/c*(c*ln(c*e*x+c*d)/e*arccoth(c*x)^2+2*c/e*(-1/2*arccoth(c*x)^2*ln(d*c*(1
/(c*x-1)*(c*x+1)-1)+e*(1/(c*x-1)*(c*x+1)+1))+1/4*I*Pi*csgn(I*(d*c*(1/(c*x-
1)*(c*x+1)-1)+e*(1/(c*x-1)*(c*x+1)+1))/(1/(c*x-1)*(c*x+1)-1))*(csgn(I*(d*c
*(1/(c*x-1)*(c*x+1)-1)+e*(1/(c*x-1)*(c*x+1)+1)))*csgn(I/(1/(c*x-1)*(c*x+1)
-1))-csgn(I*(d*c*(1/(c*x-1)*(c*x+1)-1)+e*(1/(c*x-1)*(c*x+1)+1))/(1/(c*x-1)
*(c*x+1)-1))*csgn(I/(1/(c*x-1)*(c*x+1)-1))-csgn(I*(d*c*(1/(c*x-1)*(c*x+1)-
1)+e*(1/(c*x-1)*(c*x+1)+1)))*csgn(I*(d*c*(1/(c*x-1)*(c*x+1)-1)+e*(1/(c*x-1
)*(c*x+1)+1))/(1/(c*x-1)*(c*x+1)-1))+csgn(I*(d*c*(1/(c*x-1)*(c*x+1)-1)+e*(
1/(c*x-1)*(c*x+1)+1))/(1/(c*x-1)*(c*x+1)-1))^2)*arccoth(c*x)^2+1/2*arccoth
(c*x)^2*ln(1/(c*x-1)*(c*x+1)-1)-1/2*arccoth(c*x)^2*ln(1-1/((c*x-1)/(c*x+1)
)^(1/2))-arccoth(c*x)*polylog(2,1/((c*x-1)/(c*x+1))^(1/2))+polylog(3,1/((c
*x-1)/(c*x+1))^(1/2))-1/2*arccoth(c*x)^2*ln(1+1/((c*x-1)/(c*x+1))^(1/2))-a
rccoth(c*x)*polylog(2,-1/((c*x-1)/(c*x+1))^(1/2))+polylog(3,-1/((c*x-1)/(c
*x+1))^(1/2))+1/2*e/(c*d+e)*arccoth(c*x)^2*ln(1-(c*d+e)/(c*x-1)*(c*x+1)/(c
*d-e))+1/2*e/(c*d+e)*arccoth(c*x)*polylog(2,(c*d+e)/(c*x-1)*(c*x+1)/(c*d-e
))-1/4*e/(c*d+e)*polylog(3,(c*d+e)/(c*x-1)*(c*x+1)/(c*d-e))+1/2*d*c/(c*d+e
)*arccoth(c*x)^2*ln(1-(c*d+e)/(c*x-1)*(c*x+1)/(c*d-e))+1/2*d*c/(c*d+e)*arc
coth(c*x)*polylog(2,(c*d+e)/(c*x-1)*(c*x+1)/(c*d-e))-1/4*d*c/(c*d+e)*polyl
og(3,(c*d+e)/(c*x-1)*(c*x+1)/(c*d-e)))
```

3.34.5 Fricas [F]

$$\int \frac{\coth^{-1}(cx)^2}{d+ex}\,dx = \int \frac{\operatorname{arcoth}(cx)^2}{ex+d}\,dx$$

input
```
integrate(arccoth(c*x)^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral(arccoth(c*x)^2/(e*x + d), x)
```

3.34.6 Sympy [F]

$$\int \frac{\coth^{-1}(cx)^2}{d+ex}\,dx = \int \frac{\operatorname{acoth}^2(cx)}{d+ex}\,dx$$

input
```
integrate(acoth(c*x)**2/(e*x+d),x)
```

output
```
Integral(acoth(c*x)**2/(d + e*x), x)
```

3.34.7 Maxima [F]

$$\int \frac{\coth^{-1}(cx)^2}{d+ex}\,dx = \int \frac{\operatorname{arcoth}(cx)^2}{ex+d}\,dx$$

input
```
integrate(arccoth(c*x)^2/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(arccoth(c*x)^2/(e*x + d), x)
```

3.34.8 Giac [F]

$$\int \frac{\coth^{-1}(cx)^2}{d+ex}\,dx = \int \frac{\operatorname{arcoth}(cx)^2}{ex+d}\,dx$$

input
```
integrate(arccoth(c*x)^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate(arccoth(c*x)^2/(e*x + d), x)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(cx)^2}{d+ex}\,dx = \int \frac{\operatorname{acoth}(c\,x)^2}{d+e\,x}\,dx$$

input
```
int(acoth(c*x)^2/(d + e*x),x)
```

output
```
int(acoth(c*x)^2/(d + e*x), x)
```

3.35 $\int (c+dx^2)^4 \coth^{-1}(ax)\,dx$

3.35.1 Optimal result

Integrand size = 14, antiderivative size = 245

$$\begin{aligned}&\int (c+dx^2)^4 \coth^{-1}(ax)\,dx\\ &= \frac{d(420a^6c^3+378a^4c^2d+180a^2cd^2+35d^3)\,x^2}{630a^7}+\frac{d^2(378a^4c^2+180a^2cd+35d^2)\,x^4}{1260a^5}\\ &+\frac{d^3(36a^2c+7d)\,x^6}{378a^3}+\frac{d^4x^8}{72a}+c^4x\coth^{-1}(ax)+\frac{4}{3}c^3dx^3\coth^{-1}(ax)\\ &+\frac{6}{5}c^2d^2x^5\coth^{-1}(ax)+\frac{4}{7}cd^3x^7\coth^{-1}(ax)+\frac{1}{9}d^4x^9\coth^{-1}(ax)\\ &+\frac{(315a^8c^4+420a^6c^3d+378a^4c^2d^2+180a^2cd^3+35d^4)\log(1-a^2x^2)}{630a^9}\end{aligned}$$

output
```
1/630*d*(420*a^6*c^3+378*a^4*c^2*d+180*a^2*c*d^2+35*d^3)*x^2/a^7+1/1260*d^
2*(378*a^4*c^2+180*a^2*c*d+35*d^2)*x^4/a^5+1/378*d^3*(36*a^2*c+7*d)*x^6/a^
3+1/72*d^4*x^8/a+c^4*x*arccoth(a*x)+4/3*c^3*d*x^3*arccoth(a*x)+6/5*c^2*d^2
*x^5*arccoth(a*x)+4/7*c*d^3*x^7*arccoth(a*x)+1/9*d^4*x^9*arccoth(a*x)+1/63
0*(315*a^8*c^4+420*a^6*c^3*d+378*a^4*c^2*d^2+180*a^2*c*d^3+35*d^4)*ln(-a^2
*x^2+1)/a^9
```

3.35.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.87

$$\int \left(c+dx^2\right)^4 \coth^{-1}(ax)\,dx$$
$$=\frac{a^2dx^2(420d^3+30a^2d^2(72c+7dx^2)+4a^4d(1134c^2+270cdx^2+35d^2x^4)+3a^6(1680c^3+756c^2dx^2+240c}{}$$

input
```
Integrate[(c + d*x^2)^4*ArcCoth[a*x],x]
```

output
```
(a^2*d*x^2*(420*d^3 + 30*a^2*d^2*(72*c + 7*d*x^2) + 4*a^4*d*(1134*c^2 + 27
0*c*d*x^2 + 35*d^2*x^4) + 3*a^6*(1680*c^3 + 756*c^2*d*x^2 + 240*c*d^2*x^4
+ 35*d^3*x^6)) + 24*a^9*x*(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*x^4 + 180
*c*d^3*x^6 + 35*d^4*x^8)*ArcCoth[a*x] + 12*(315*a^8*c^4 + 420*a^6*c^3*d +
378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*Log[1 - a^2*x^2])/(7560*a^9)
```

3.35.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6539, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(ax)\left(c+dx^2\right)^4dx$$

$$\Big\downarrow \text{6539}$$

$$-a\int \frac{x\left(35d^4x^8+180cd^3x^6+378c^2d^2x^4+420c^3dx^2+315c^4\right)}{315\left(1-a^2x^2\right)}dx+c^4x\coth^{-1}(ax)+\frac{4}{3}c^3dx^3\coth^{-1}(ax)+\frac{6}{5}c^2d^2x^5\coth^{-1}(ax)+\frac{4}{7}cd^3x^7\coth^{-1}(ax)+\frac{1}{9}d^4x^9\coth^{-1}(ax)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{315}a\int \frac{x\left(35d^4x^8+180cd^3x^6+378c^2d^2x^4+420c^3dx^2+315c^4\right)}{1-a^2x^2}dx+c^4x\coth^{-1}(ax)+\frac{4}{3}c^3dx^3\coth^{-1}(ax)+\frac{6}{5}c^2d^2x^5\coth^{-1}(ax)+\frac{4}{7}cd^3x^7\coth^{-1}(ax)+\frac{1}{9}d^4x^9\coth^{-1}(ax)$$

$$\Big\downarrow \text{2331}$$

$$-\frac{1}{630}a\int \frac{35d^4x^8+180cd^3x^6+378c^2d^2x^4+420c^3dx^2+315c^4}{1-a^2x^2}dx^2+c^4x\coth^{-1}(ax)+\frac{4}{3}c^3dx^3\coth^{-1}(ax)+\frac{6}{5}c^2d^2x^5\coth^{-1}(ax)+\frac{4}{7}cd^3x^7\coth^{-1}(ax)+\frac{1}{9}d^4x^9\coth^{-1}(ax)$$

$\downarrow$ 2389

$$-\frac{1}{630}a\int \left(-\frac{35d^4x^6}{a^2}-\frac{5d^3(36ca^2+7d)x^4}{a^4}-\frac{d^2(378c^2a^4+180cda^2+35d^2)x^2}{a^6}-\frac{d(420c^3a^6+378c^2da^4+180c\right.$$
$$c^4x\coth^{-1}(ax)+\frac{4}{3}c^3dx^3\coth^{-1}(ax)+\frac{6}{5}c^2d^2x^5\coth^{-1}(ax)+\frac{4}{7}cd^3x^7\coth^{-1}(ax)+\frac{1}{9}d^4x^9\coth^{-1}(ax)$$

$\downarrow$ 2009

$$-\frac{1}{630}a\left(-\frac{35d^4x^8}{4a^2}-\frac{5d^3x^6(36a^2c+7d)}{3a^4}-\frac{d^2x^4(378a^4c^2+180a^2cd+35d^2)}{2a^6}-\frac{dx^2(420a^6c^3+378a^4c^2d+180a^2c\right.$$
$$c^4x\coth^{-1}(ax)+\frac{4}{3}c^3dx^3\coth^{-1}(ax)+\frac{6}{5}c^2d^2x^5\coth^{-1}(ax)+\frac{4}{7}cd^3x^7\coth^{-1}(ax)+\frac{1}{9}d^4x^9\coth^{-1}(ax)$$

input
```
Int[(c + d*x^2)^4*ArcCoth[a*x],x]
```

output
```
c^4*x*ArcCoth[a*x] + (4*c^3*d*x^3*ArcCoth[a*x])/3 + (6*c^2*d^2*x^5*ArcCoth
[a*x])/5 + (4*c*d^3*x^7*ArcCoth[a*x])/7 + (d^4*x^9*ArcCoth[a*x])/9 - (a*(-
((d*(420*a^6*c^3 + 378*a^4*c^2*d + 180*a^2*c*d^2 + 35*d^3)*x^2)/a^8) - (d^
2*(378*a^4*c^2 + 180*a^2*c*d + 35*d^2)*x^4)/(2*a^6) - (5*d^3*(36*a^2*c + 7
*d)*x^6)/(3*a^4) - (35*d^4*x^8)/(4*a^2) - ((315*a^8*c^4 + 420*a^6*c^3*d +
378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*Log[1 - a^2*x^2])/a^10))/630
```

3.35.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 6539
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCoth[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.35.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00

method	result
parts	$\frac{d^4x^9 \operatorname{arccoth}(ax)}{9}+\frac{4c\,d^3x^7 \operatorname{arccoth}(ax)}{7}+\frac{6c^2d^2x^5 \operatorname{arccoth}(ax)}{5}+\frac{4c^3d\,x^3 \operatorname{arccoth}(ax)}{3}+c^4x \operatorname{arccoth}(ax)$
derivativedivides	$\frac{\operatorname{arccoth}(ax)c^4ax+\frac{4a \operatorname{arccoth}(ax)c^3d\,x^3}{3}+\frac{6a \operatorname{arccoth}(ax)c^2d^2x^5}{5}+\frac{4a \operatorname{arccoth}(ax)c\,d^3x^7}{7}+\frac{a \operatorname{arccoth}(ax)d^4x^9}{9}+-\frac{\left(-315a^8c^4-42\right.}{}}{}$
default	$\frac{\operatorname{arccoth}(ax)c^4ax+\frac{4a \operatorname{arccoth}(ax)c^3d\,x^3}{3}+\frac{6a \operatorname{arccoth}(ax)c^2d^2x^5}{5}+\frac{4a \operatorname{arccoth}(ax)c\,d^3x^7}{7}+\frac{a \operatorname{arccoth}(ax)d^4x^9}{9}+-\frac{\left(-315a^8c^4-42\right.}{}}{}$
parallelrisch	$-\frac{-140d^4a^6x^6-7560c^4 \operatorname{arccoth}(ax)x\,a^9-840x^9 \operatorname{arccoth}(ax)a^9d^4-10080 \operatorname{arccoth}(ax)a^6c^3d-9072 \operatorname{arccoth}(ax)a^4c^2d^2-}{}$
risch	$\left(\frac{1}{18}d^4x^9+\frac{2}{7}c\,d^3x^7+\frac{3}{5}c^2d^2x^5+\frac{2}{3}c^3d\,x^3+\frac{1}{2}c^4x\right)\ln(ax+1)-\frac{d^4x^9\ln(ax-1)}{18}-\frac{2c\,d^3x^7\ln(ax-}{7}$

input
```
int((d*x^2+c)^4*arccoth(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/9*d^4*x^9*arccoth(a*x)+4/7*c*d^3*x^7*arccoth(a*x)+6/5*c^2*d^2*x^5*arccot
h(a*x)+4/3*c^3*d*x^3*arccoth(a*x)+c^4*x*arccoth(a*x)+1/315*a*(1/2*d/a^8*(3
5/4*a^6*d^3*x^8+60*a^6*c*d^2*x^6+189*a^6*c^2*d*x^4+420*a^6*c^3*x^2+35/3*a^
4*d^3*x^6+90*a^4*c*d^2*x^4+378*a^4*c^2*d*x^2+35/2*a^2*d^3*x^4+180*a^2*c*d^
2*x^2+35*d^3*x^2)+1/2*(315*a^8*c^4+420*a^6*c^3*d+378*a^4*c^2*d^2+180*a^2*c
*d^3+35*d^4)/a^10*ln(a^2*x^2-1))
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.01

$$\int \left(c+dx^2\right)^4 \coth^{-1}(ax)\,dx$$
$$= \frac{105\,a^8d^4x^8 + 20\,(36\,a^8cd^3 + 7\,a^6d^4)x^6 + 6\,(378\,a^8c^2d^2 + 180\,a^6cd^3 + 35\,a^4d^4)x^4 + 12\,(420\,a^8c^3d + 378\,a^6c^2 \cdots}{\cdots}$$

input
```
integrate((d*x^2+c)^4*arccoth(a*x),x, algorithm="fricas")
```

output
```
1/7560*(105*a^8*d^4*x^8 + 20*(36*a^8*c*d^3 + 7*a^6*d^4)*x^6 + 6*(378*a^8*c
^2*d^2 + 180*a^6*c*d^3 + 35*a^4*d^4)*x^4 + 12*(420*a^8*c^3*d + 378*a^6*c^2
*d^2 + 180*a^4*c*d^3 + 35*a^2*d^4)*x^2 + 12*(315*a^8*c^4 + 420*a^6*c^3*d +
 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a^2*x^2 - 1) + 12*(35*a^9*d
^4*x^9 + 180*a^9*c*d^3*x^7 + 378*a^9*c^2*d^2*x^5 + 420*a^9*c^3*d*x^3 + 315
*a^9*c^4*x)*log((a*x + 1)/(a*x - 1)))/a^9
```

3.35.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.74

$$\int \left(c+dx^2\right)^4 \coth^{-1}(ax)\,dx$$
$$= \begin{cases} c^4x \operatorname{acoth}(ax) + \frac{4c^3dx^3 \operatorname{acoth}(ax)}{3} + \frac{6c^2d^2x^5 \operatorname{acoth}(ax)}{5} + \frac{4cd^3x^7 \operatorname{acoth}(ax)}{7} + \frac{d^4x^9 \operatorname{acoth}(ax)}{9} + \frac{c^4 \log\left(x-\frac{1}{a}\right)}{a} + \frac{c^4 \operatorname{acoth}(a \cdots}{a} \cdots \\ \frac{i\pi\left(c^4x+\frac{4c^3dx^3}{3}+\frac{6c^2d^2x^5}{5}+\frac{4cd^3x^7}{7}+\frac{d^4x^9}{9}\right)}{2} \end{cases}$$

input
```
integrate((d*x**2+c)**4*acoth(a*x),x)
```

output
```
Piecewise((c**4*x*acoth(a*x) + 4*c**3*d*x**3*acoth(a*x)/3 + 6*c**2*d**2*x*
*5*acoth(a*x)/5 + 4*c*d**3*x**7*acoth(a*x)/7 + d**4*x**9*acoth(a*x)/9 + c*
*4*log(x - 1/a)/a + c**4*acoth(a*x)/a + 2*c**3*d*x**2/(3*a) + 3*c**2*d**2*
x**4/(10*a) + 2*c*d**3*x**6/(21*a) + d**4*x**8/(72*a) + 4*c**3*d*log(x - 1
/a)/(3*a**3) + 4*c**3*d*acoth(a*x)/(3*a**3) + 3*c**2*d**2*x**2/(5*a**3) +
c*d**3*x**4/(7*a**3) + d**4*x**6/(54*a**3) + 6*c**2*d**2*log(x - 1/a)/(5*a
**5) + 6*c**2*d**2*acoth(a*x)/(5*a**5) + 2*c*d**3*x**2/(7*a**5) + d**4*x**
4/(36*a**5) + 4*c*d**3*log(x - 1/a)/(7*a**7) + 4*c*d**3*acoth(a*x)/(7*a**7
) + d**4*x**2/(18*a**7) + d**4*log(x - 1/a)/(9*a**9) + d**4*acoth(a*x)/(9*
a**9), Ne(a, 0)), (I*pi*(c**4*x + 4*c**3*d*x**3/3 + 6*c**2*d**2*x**5/5 + 4
*c*d**3*x**7/7 + d**4*x**9/9)/2, True))
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.13

$$\int \left(c+dx^2\right)^4 \coth^{-1}(ax)\,dx$$
$$=\frac{1}{7560}a\left(\frac{105\,a^6d^4x^8+20\,(36\,a^6cd^3+7\,a^4d^4)x^6+6\,(378\,a^6c^2d^2+180\,a^4cd^3+35\,a^2d^4)x^4+12\,(420\,a^6c^3d+\right.}{a^8}$$
$$+\frac{1}{315}\left(35\,d^4x^9+180\,cd^3x^7+378\,c^2d^2x^5+420\,c^3dx^3+315\,c^4x\right)\operatorname{arcoth}(ax)$$

input
```
integrate((d*x^2+c)^4*arccoth(a*x),x, algorithm="maxima")
```

output
```
1/7560*a*((105*a^6*d^4*x^8 + 20*(36*a^6*c*d^3 + 7*a^4*d^4)*x^6 + 6*(378*a^
6*c^2*d^2 + 180*a^4*c*d^3 + 35*a^2*d^4)*x^4 + 12*(420*a^6*c^3*d + 378*a^4*
c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*x^2)/a^8 + 12*(315*a^8*c^4 + 420*a^6*c^3
*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a*x + 1)/a^10 + 12*(315
*a^8*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a
*x - 1)/a^10) + 1/315*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*
c^3*d*x^3 + 315*c^4*x)*arccoth(a*x)
```

3.35.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1473 vs. $2(227) = 454$.

Time = 0.31 (sec) , antiderivative size = 1473, normalized size of antiderivative = 6.01

$$\int \left(c+dx^2\right)^4 \coth^{-1}(ax)\,dx = \text{Too large to display}$$

input

```
integrate((d*x^2+c)^4*arccoth(a*x),x, algorithm="giac")
```

output

```
1/945*a*(3*(315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3
+ 35*d^4)*log(abs(a*x + 1)/abs(a*x - 1))/a^10 - 3*(315*a^8*c^4 + 420*a^6*c
^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(abs((a*x + 1)/(a*x -
1) - 1))/a^10 + 8*(3*(105*a^6*c^3*d + 189*a^4*c^2*d^2 + 135*a^2*c*d^3 + 35
*d^4)*(a*x + 1)^7/(a*x - 1)^7 - 45*(42*a^6*c^3*d + 63*a^4*c^2*d^2 + 36*a^2
*c*d^3 + 7*d^4)*(a*x + 1)^6/(a*x - 1)^6 + (4725*a^6*c^3*d + 6237*a^4*c^2*d
^2 + 3555*a^2*c*d^3 + 875*d^4)*(a*x + 1)^5/(a*x - 1)^5 - 2*(3150*a^6*c^3*d
 + 3969*a^4*c^2*d^2 + 2340*a^2*c*d^3 + 455*d^4)*(a*x + 1)^4/(a*x - 1)^4 +
(4725*a^6*c^3*d + 6237*a^4*c^2*d^2 + 3555*a^2*c*d^3 + 875*d^4)*(a*x + 1)^3
/(a*x - 1)^3 - 45*(42*a^6*c^3*d + 63*a^4*c^2*d^2 + 36*a^2*c*d^3 + 7*d^4)*(
a*x + 1)^2/(a*x - 1)^2 + 3*(105*a^6*c^3*d + 189*a^4*c^2*d^2 + 135*a^2*c*d^
3 + 35*d^4)*(a*x + 1)/(a*x - 1))/(a^10*((a*x + 1)/(a*x - 1) - 1)^8) + 3*(3
15*(a*x + 1)^8*a^8*c^4/(a*x - 1)^8 - 2520*(a*x + 1)^7*a^8*c^4/(a*x - 1)^7
+ 8820*(a*x + 1)^6*a^8*c^4/(a*x - 1)^6 - 17640*(a*x + 1)^5*a^8*c^4/(a*x -
1)^5 + 22050*(a*x + 1)^4*a^8*c^4/(a*x - 1)^4 - 17640*(a*x + 1)^3*a^8*c^4/(
a*x - 1)^3 + 8820*(a*x + 1)^2*a^8*c^4/(a*x - 1)^2 - 2520*(a*x + 1)*a^8*c^4
/(a*x - 1) + 315*a^8*c^4 + 1260*(a*x + 1)^8*a^6*c^3*d/(a*x - 1)^8 - 7560*(
a*x + 1)^7*a^6*c^3*d/(a*x - 1)^7 + 19320*(a*x + 1)^6*a^6*c^3*d/(a*x - 1)^6
 - 27720*(a*x + 1)^5*a^6*c^3*d/(a*x - 1)^5 + 25200*(a*x + 1)^4*a^6*c^3*d/(
a*x - 1)^4 - 15960*(a*x + 1)^3*a^6*c^3*d/(a*x - 1)^3 + 7560*(a*x + 1)^2...
```

3.35.9 Mupad [B] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.21

$$\int \left(c+dx^2\right)^4 \coth^{-1}(ax)\,dx$$
$$= \ln\left(\frac{1}{a\,x}+1\right)\left(\frac{c^4\,x}{2}+\frac{2\,c^3\,d\,x^3}{3}+\frac{3\,c^2\,d^2\,x^5}{5}+\frac{2\,c\,d^3\,x^7}{7}+\frac{d^4\,x^9}{18}\right)$$
$$-\ln\left(1-\frac{1}{a\,x}\right)\left(\frac{c^4\,x}{2}+\frac{2\,c^3\,d\,x^3}{3}+\frac{3\,c^2\,d^2\,x^5}{5}+\frac{2\,c\,d^3\,x^7}{7}+\frac{d^4\,x^9}{18}\right)$$
$$+x^2\left(\frac{\frac{\frac{d^4}{9\,a^3}+\frac{4\,c\,d^3}{7\,a}}{a^2}+\frac{6\,c^2\,d^2}{5\,a}}{2\,a^2}+\frac{2\,c^3\,d}{3\,a}\right)+x^6\left(\frac{d^4}{54\,a^3}+\frac{2\,c\,d^3}{21\,a}\right)+x^4\left(\frac{\frac{d^4}{9\,a^3}+\frac{4\,c\,d^3}{7\,a}}{4\,a^2}+\frac{3\,c^2\,d^2}{10\,a}\right)$$
$$+\frac{\ln\left(a^2\,x^2-1\right)\left(315\,a^8\,c^4+420\,a^6\,c^3\,d+378\,a^4\,c^2\,d^2+180\,a^2\,c\,d^3+35\,d^4\right)}{630\,a^9}+\frac{d^4\,x^8}{72\,a}$$

input
```
int(acoth(a*x)*(c + d*x^2)^4,x)
```

output
```
log(1/(a*x) + 1)*((c^4*x)/2 + (d^4*x^9)/18 + (2*c^3*d*x^3)/3 + (2*c*d^3*x^
7)/7 + (3*c^2*d^2*x^5)/5) - log(1 - 1/(a*x))*((c^4*x)/2 + (d^4*x^9)/18 + (
2*c^3*d*x^3)/3 + (2*c*d^3*x^7)/7 + (3*c^2*d^2*x^5)/5) + x^2*(((d^4/(9*a^3)
 + (4*c*d^3)/(7*a))/a^2 + (6*c^2*d^2)/(5*a))/(2*a^2) + (2*c^3*d)/(3*a)) +
x^6*(d^4/(54*a^3) + (2*c*d^3)/(21*a)) + x^4*((d^4/(9*a^3) + (4*c*d^3)/(7*a
))/(4*a^2) + (3*c^2*d^2)/(10*a)) + (log(a^2*x^2 - 1)*(35*d^4 + 315*a^8*c^4
 + 180*a^2*c*d^3 + 420*a^6*c^3*d + 378*a^4*c^2*d^2))/(630*a^9) + (d^4*x^8)
/(72*a)
```

3.36 $\int (c + dx^2)^3 \coth^{-1}(ax)\, dx$

3.36.1 Optimal result

Integrand size = 14, antiderivative size = 169

$$\int (c + dx^2)^3 \coth^{-1}(ax)\, dx = \frac{d(35a^4c^2 + 21a^2cd + 5d^2)\, x^2}{70a^5} + \frac{d^2(21a^2c + 5d)\, x^4}{140a^3} + \frac{d^3x^6}{42a} + c^3x \coth^{-1}(ax) + c^2dx^3 \coth^{-1}(ax) + \frac{3}{5}cd^2x^5 \coth^{-1}(ax) + \frac{1}{7}d^3x^7 \coth^{-1}(ax) + \frac{(35a^6c^3 + 35a^4c^2d + 21a^2cd^2 + 5d^3) \log(1 - a^2x^2)}{70a^7}$$

output
```
1/70*d*(35*a^4*c^2+21*a^2*c*d+5*d^2)*x^2/a^5+1/140*d^2*(21*a^2*c+5*d)*x^4/
a^3+1/42*d^3*x^6/a+c^3*x*arccoth(a*x)+c^2*d*x^3*arccoth(a*x)+3/5*c*d^2*x^5
*arccoth(a*x)+1/7*d^3*x^7*arccoth(a*x)+1/70*(35*a^6*c^3+35*a^4*c^2*d+21*a^
2*c*d^2+5*d^3)*ln(-a^2*x^2+1)/a^7
```

3.36.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int (c + dx^2)^3 \coth^{-1}(ax)\, dx = \frac{a^2dx^2(30d^2 + 3a^2d(42c + 5dx^2) + a^4(210c^2 + 63cdx^2 + 10d^2x^4)) + 12a^7x(35c^3 + 35c^2dx^2 + 21cd^2x^4 + 5a}{420a^7}$$

input
```
Integrate[(c + d*x^2)^3*ArcCoth[a*x],x]
```

output
```
(a^2*d*x^2*(30*d^2 + 3*a^2*d*(42*c + 5*d*x^2) + a^4*(210*c^2 + 63*c*d*x^2
+ 10*d^2*x^4)) + 12*a^7*x*(35*c^3 + 35*c^2*d*x^2 + 21*c*d^2*x^4 + 5*d^3*x^
6)*ArcCoth[a*x] + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log
[1 - a^2*x^2])/(420*a^7)
```

3.36.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6539, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(ax)\left(c+dx^2\right)^3 dx$$

$$\Big\downarrow \text{6539}$$

$$-a\int \frac{x\left(5d^3x^6+21cd^2x^4+35c^2dx^2+35c^3\right)}{35\left(1-a^2x^2\right)}dx+c^3x\coth^{-1}(ax)+c^2dx^3\coth^{-1}(ax)+\frac{3}{5}cd^2x^5\coth^{-1}(ax)+\frac{1}{7}d^3x^7\coth^{-1}(ax)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{35}a\int \frac{x\left(5d^3x^6+21cd^2x^4+35c^2dx^2+35c^3\right)}{1-a^2x^2}dx+c^3x\coth^{-1}(ax)+c^2dx^3\coth^{-1}(ax)+\frac{3}{5}cd^2x^5\coth^{-1}(ax)+\frac{1}{7}d^3x^7\coth^{-1}(ax)$$

$$\Big\downarrow \text{2331}$$

$$-\frac{1}{70}a\int \frac{5d^3x^6+21cd^2x^4+35c^2dx^2+35c^3}{1-a^2x^2}dx^2+c^3x\coth^{-1}(ax)+c^2dx^3\coth^{-1}(ax)+\frac{3}{5}cd^2x^5\coth^{-1}(ax)+\frac{1}{7}d^3x^7\coth^{-1}(ax)$$

$$\Big\downarrow \text{2389}$$

$$-\frac{1}{70}a\int \left(-\frac{5d^3x^4}{a^2}-\frac{d^2\left(21ca^2+5d\right)x^2}{a^4}-\frac{d\left(35c^2a^4+21cda^2+5d^2\right)}{a^6}+\frac{-35c^3a^6-35c^2da^4-21cd^2a^2-5d^3}{a^6\left(a^2x^2-1\right)}\right)dx^2+c^3x\coth^{-1}(ax)+c^2dx^3\coth^{-1}(ax)+\frac{3}{5}cd^2x^5\coth^{-1}(ax)+\frac{1}{7}d^3x^7\coth^{-1}(ax)$$

$$\downarrow 2009$$

$$-\frac{1}{70}a\left(-\frac{5d^3x^6}{3a^2}-\frac{d^2x^4(21a^2c+5d)}{2a^4}-\frac{dx^2(35a^4c^2+21a^2cd+5d^2)}{a^6}-\frac{(35a^6c^3+35a^4c^2d+21a^2cd^2+5d^3)\log(}{a^8}\right.$$
$$c^3x\coth^{-1}(ax)+c^2dx^3\coth^{-1}(ax)+\frac{3}{5}cd^2x^5\coth^{-1}(ax)+\frac{1}{7}d^3x^7\coth^{-1}(ax)$$

input
```
Int[(c + d*x^2)^3*ArcCoth[a*x],x]
```

output
```
c^3*x*ArcCoth[a*x] + c^2*d*x^3*ArcCoth[a*x] + (3*c*d^2*x^5*ArcCoth[a*x])/5
 + (d^3*x^7*ArcCoth[a*x])/7 - (a*(-((d*(35*a^4*c^2 + 21*a^2*c*d + 5*d^2)*x
^2)/a^6) - (d^2*(21*a^2*c + 5*d)*x^4)/(2*a^4) - (5*d^3*x^6)/(3*a^2) - ((35
*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log[1 - a^2*x^2])/a^8))/70
```

3.36.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 6539
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCoth[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.36.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

method	result
parts	$\frac{d^3x^7\operatorname{arccoth}(ax)}{7}+\frac{3c\,d^2x^5\operatorname{arccoth}(ax)}{5}+c^2d\,x^3\operatorname{arccoth}(ax)+c^3x\operatorname{arccoth}(ax)+\frac{a\left(\frac{d\left(\frac{5}{3}a^4d^2x^6+\right.}{}\right.}{}$
derivativedivides	$\frac{\operatorname{arccoth}(ax)c^3ax+a\operatorname{arccoth}(ax)c^2d\,x^3+\frac{3a\operatorname{arccoth}(ax)c\,d^2x^5}{5}+\frac{a\operatorname{arccoth}(ax)d^3x^7}{7}+\frac{\frac{21c\,a^4d^2x^2}{2}+\frac{21c\,a^6d^2x^4}{4}+\frac{35c^2a^6d\,x^2}{2}+}{a}$
default	$\frac{\operatorname{arccoth}(ax)c^3ax+a\operatorname{arccoth}(ax)c^2d\,x^3+\frac{3a\operatorname{arccoth}(ax)c\,d^2x^5}{5}+\frac{a\operatorname{arccoth}(ax)d^3x^7}{7}+\frac{\frac{21c\,a^4d^2x^2}{2}+\frac{21c\,a^6d^2x^4}{4}+\frac{35c^2a^6d\,x^2}{2}+}{a}$
parallelrisch	$-\frac{-60x^7\operatorname{arccoth}(ax)a^7d^3-252x^5\operatorname{arccoth}(ax)a^7c\,d^2-10d^3a^6x^6-420x^3\operatorname{arccoth}(ax)a^7c^2d-63c\,a^6d^2x^4-420c^3x\operatorname{arccot}}{}$
risch	$\left(\frac{1}{14}d^3x^7+\frac{3}{10}c\,d^2x^5+\frac{1}{2}c^2d\,x^3+\frac{1}{2}c^3x\right)\ln(ax+1)-\frac{d^3x^7\ln(ax-1)}{14}-\frac{3c\,d^2x^5\ln(ax-1)}{10}+\frac{d^3x^6}{42a}$

input
```
int((d*x^2+c)^3*arccoth(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/7*d^3*x^7*arccoth(a*x)+3/5*c*d^2*x^5*arccoth(a*x)+c^2*d*x^3*arccoth(a*x)
+c^3*x*arccoth(a*x)+1/35*a*(1/2*d/a^6*(5/3*a^4*d^2*x^6+21/2*a^4*c*d*x^4+35
*a^4*c^2*x^2+5/2*a^2*d^2*x^4+21*a^2*c*d*x^2+5*d^2*x^2)+1/2*(35*a^6*c^3+35*
a^4*c^2*d+21*a^2*c*d^2+5*d^3)/a^8*ln(a^2*x^2-1))
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.05

$$\int \left(c+dx^2\right)^3\coth^{-1}(ax)\,dx$$
$$=\frac{10\,a^6d^3x^6+3\left(21\,a^6cd^2+5\,a^4d^3\right)x^4+6\left(35\,a^6c^2d+21\,a^4cd^2+5\,a^2d^3\right)x^2+6\left(35\,a^6c^3+35\,a^4c^2d+21\,a^2cd\right.}{420\,a^7}$$

input
```
integrate((d*x^2+c)^3*arccoth(a*x),x, algorithm="fricas")
```

output
```
1/420*(10*a^6*d^3*x^6 + 3*(21*a^6*c*d^2 + 5*a^4*d^3)*x^4 + 6*(35*a^6*c^2*d
 + 21*a^4*c*d^2 + 5*a^2*d^3)*x^2 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c
*d^2 + 5*d^3)*log(a^2*x^2 - 1) + 6*(5*a^7*d^3*x^7 + 21*a^7*c*d^2*x^5 + 35*
a^7*c^2*d*x^3 + 35*a^7*c^3*x)*log((a*x + 1)/(a*x - 1)))/a^7
```

3.36.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.67

$$\int \left(c+dx^2\right)^3 \coth^{-1}(ax)\,dx$$
$$= \begin{cases} c^3x\,\mathrm{acoth}\,(ax)+c^2dx^3\,\mathrm{acoth}\,(ax)+\frac{3cd^2x^5\,\mathrm{acoth}\,(ax)}{5}+\frac{d^3x^7\,\mathrm{acoth}\,(ax)}{7}+\frac{c^3\log\left(x-\frac{1}{a}\right)}{a}+\frac{c^3\,\mathrm{acoth}\,(ax)}{a}+\frac{c^2dx^2}{2a}+\frac{3cd}{2} \\ \frac{i\pi\left(c^3x+c^2dx^3+\frac{3cd^2x^5}{5}+\frac{d^3x^7}{7}\right)}{2} \end{cases}$$

input
```
integrate((d*x**2+c)**3*acoth(a*x),x)
```

output
```
Piecewise((c**3*x*acoth(a*x) + c**2*d*x**3*acoth(a*x) + 3*c*d**2*x**5*acot
h(a*x)/5 + d**3*x**7*acoth(a*x)/7 + c**3*log(x - 1/a)/a + c**3*acoth(a*x)/
a + c**2*d*x**2/(2*a) + 3*c*d**2*x**4/(20*a) + d**3*x**6/(42*a) + c**2*d*l
og(x - 1/a)/a**3 + c**2*d*acoth(a*x)/a**3 + 3*c*d**2*x**2/(10*a**3) + d**3
*x**4/(28*a**3) + 3*c*d**2*log(x - 1/a)/(5*a**5) + 3*c*d**2*acoth(a*x)/(5*
a**5) + d**3*x**2/(14*a**5) + d**3*log(x - 1/a)/(7*a**7) + d**3*acoth(a*x)
/(7*a**7), Ne(a, 0)), (I*pi*(c**3*x + c**2*d*x**3 + 3*c*d**2*x**5/5 + d**3
*x**7/7)/2, True))
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.17

$$\int \left(c+dx^2\right)^3 \coth^{-1}(ax)\,dx$$
$$= \frac{1}{420}\,a\left(\frac{10\,a^4d^3x^6+3\,(21\,a^4cd^2+5\,a^2d^3)x^4+6\,(35\,a^4c^2d+21\,a^2cd^2+5\,d^3)x^2}{a^6}+\frac{6\,(35\,a^6c^3+35\,a^4c^2d+2\right.$$
$$+\frac{1}{35}\left(5\,d^3x^7+21\,cd^2x^5+35\,c^2dx^3+35\,c^3x\right)\mathrm{arcoth}\,(ax)$$

input
```
integrate((d*x^2+c)^3*arccoth(a*x),x, algorithm="maxima")
```

output
```
1/420*a*((10*a^4*d^3*x^6 + 3*(21*a^4*c*d^2 + 5*a^2*d^3)*x^4 + 6*(35*a^4*c^
2*d + 21*a^2*c*d^2 + 5*d^3)*x^2)/a^6 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a
^2*c*d^2 + 5*d^3)*log(a*x + 1)/a^8 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2
*c*d^2 + 5*d^3)*log(a*x - 1)/a^8) + 1/35*(5*d^3*x^7 + 21*c*d^2*x^5 + 35*c^
2*d*x^3 + 35*c^3*x)*arccoth(a*x)
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(157) = 314$.

Time = 0.31 (sec) , antiderivative size = 932, normalized size of antiderivative = 5.51

$$\int (c+dx^2)^3 \coth^{-1}(ax)\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3*arccoth(a*x),x, algorithm="giac")
```

output
```
1/105*a*(3*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*log(abs(a*x
+ 1)/abs(a*x - 1))/a^8 - 3*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d
^3)*log(abs((a*x + 1)/(a*x - 1) - 1))/a^8 + 2*(3*(35*a^4*c^2*d + 42*a^2*c*
d^2 + 15*d^3)*(a*x + 1)^5/(a*x - 1)^5 - 6*(70*a^4*c^2*d + 63*a^2*c*d^2 + 1
5*d^3)*(a*x + 1)^4/(a*x - 1)^4 + 2*(315*a^4*c^2*d + 252*a^2*c*d^2 + 85*d^3
)*(a*x + 1)^3/(a*x - 1)^3 - 6*(70*a^4*c^2*d + 63*a^2*c*d^2 + 15*d^3)*(a*x
+ 1)^2/(a*x - 1)^2 + 3*(35*a^4*c^2*d + 42*a^2*c*d^2 + 15*d^3)*(a*x + 1)/(a
*x - 1))/(a^8*((a*x + 1)/(a*x - 1) - 1)^6) + 3*(35*(a*x + 1)^6*a^6*c^3/(a*
x - 1)^6 - 210*(a*x + 1)^5*a^6*c^3/(a*x - 1)^5 + 525*(a*x + 1)^4*a^6*c^3/(
a*x - 1)^4 - 700*(a*x + 1)^3*a^6*c^3/(a*x - 1)^3 + 525*(a*x + 1)^2*a^6*c^3
/(a*x - 1)^2 - 210*(a*x + 1)*a^6*c^3/(a*x - 1) + 35*a^6*c^3 + 105*(a*x + 1
)^6*a^4*c^2*d/(a*x - 1)^6 - 420*(a*x + 1)^5*a^4*c^2*d/(a*x - 1)^5 + 665*(a
*x + 1)^4*a^4*c^2*d/(a*x - 1)^4 - 560*(a*x + 1)^3*a^4*c^2*d/(a*x - 1)^3 +
315*(a*x + 1)^2*a^4*c^2*d/(a*x - 1)^2 - 140*(a*x + 1)*a^4*c^2*d/(a*x - 1)
+ 35*a^4*c^2*d + 105*(a*x + 1)^6*a^2*c*d^2/(a*x - 1)^6 - 210*(a*x + 1)^5*a
^2*c*d^2/(a*x - 1)^5 + 315*(a*x + 1)^4*a^2*c*d^2/(a*x - 1)^4 - 420*(a*x +
1)^3*a^2*c*d^2/(a*x - 1)^3 + 231*(a*x + 1)^2*a^2*c*d^2/(a*x - 1)^2 - 42*(a
*x + 1)*a^2*c*d^2/(a*x - 1) + 21*a^2*c*d^2 + 35*(a*x + 1)^6*d^3/(a*x - 1)^
6 + 175*(a*x + 1)^4*d^3/(a*x - 1)^4 + 105*(a*x + 1)^2*d^3/(a*x - 1)^2 + 5*
d^3)*log(-(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) + ...
```

3.36.9 Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.12

$$\begin{aligned}\int (c+dx^2)^3 \coth^{-1}(ax)\,dx = {} & c^3\,x\,\mathrm{acoth}(a\,x) + \frac{d^3\,x^7\,\mathrm{acoth}(a\,x)}{7} + \frac{c^3\,\ln\left(a^2\,x^2-1\right)}{2\,a} \\ & + \frac{d^3\,\ln\left(a^2\,x^2-1\right)}{14\,a^7} + \frac{d^3\,x^6}{42\,a} + \frac{d^3\,x^4}{28\,a^3} + \frac{d^3\,x^2}{14\,a^5} \\ & + \frac{c^2\,d\,\ln\left(a^2\,x^2-1\right)}{2\,a^3} + \frac{3\,c\,d^2\,\ln\left(a^2\,x^2-1\right)}{10\,a^5} + \frac{c^2\,d\,x^2}{2\,a} \\ & + \frac{3\,c\,d^2\,x^4}{20\,a} + \frac{3\,c\,d^2\,x^2}{10\,a^3} + c^2\,d\,x^3\,\mathrm{acoth}(a\,x) + \frac{3\,c\,d^2\,x^5\,\mathrm{acoth}(a\,x)}{5}\end{aligned}$$

input
```
int(acoth(a*x)*(c + d*x^2)^3,x)
```

output
```
c^3*x*acoth(a*x) + (d^3*x^7*acoth(a*x))/7 + (c^3*log(a^2*x^2 - 1))/(2*a) +
 (d^3*log(a^2*x^2 - 1))/(14*a^7) + (d^3*x^6)/(42*a) + (d^3*x^4)/(28*a^3) +
 (d^3*x^2)/(14*a^5) + (c^2*d*log(a^2*x^2 - 1))/(2*a^3) + (3*c*d^2*log(a^2*
x^2 - 1))/(10*a^5) + (c^2*d*x^2)/(2*a) + (3*c*d^2*x^4)/(20*a) + (3*c*d^2*x
^2)/(10*a^3) + c^2*d*x^3*acoth(a*x) + (3*c*d^2*x^5*acoth(a*x))/5
```

3.36. $\int (c + dx^2)^3 \coth^{-1}(ax)\, dx$

3.37 $\int \left(c+dx^2\right)^2 \coth^{-1}(ax)\,dx$

3.37.1 Optimal result

Integrand size = 14, antiderivative size = 110

$$\int \left(c+dx^2\right)^2 \coth^{-1}(ax)\,dx = \frac{d(10a^2c+3d)\,x^2}{30a^3}+\frac{d^2x^4}{20a}+c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth^{-1}(ax)+\frac{(15a^4c^2+10a^2cd+3d^2)\log\left(1-a^2x^2\right)}{30a^5}$$

output
```
1/30*d*(10*a^2*c+3*d)*x^2/a^3+1/20*d^2*x^4/a+c^2*x*arccoth(a*x)+2/3*c*d*x^
3*arccoth(a*x)+1/5*d^2*x^5*arccoth(a*x)+1/30*(15*a^4*c^2+10*a^2*c*d+3*d^2)
*ln(-a^2*x^2+1)/a^5
```

3.37.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.89

$$\int \left(c+dx^2\right)^2 \coth^{-1}(ax)\,dx = \frac{a^2dx^2(6d+a^2(20c+3dx^2))+4a^5x(15c^2+10cdx^2+3d^2x^4)\coth^{-1}(ax)+(30a^4c^2+20a^2cd+6d^2)\log(1}{60a^5}$$

input
```
Integrate[(c + d*x^2)^2*ArcCoth[a*x],x]
```

output
```
(a^2*d*x^2*(6*d + a^2*(20*c + 3*d*x^2)) + 4*a^5*x*(15*c^2 + 10*c*d*x^2 + 3
*d^2*x^4)*ArcCoth[a*x] + (30*a^4*c^2 + 20*a^2*c*d + 6*d^2)*Log[1 - a^2*x^2
])/(60*a^5)
```

3.37.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6539, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(ax)\left(c+dx^2\right)^2 dx$$

$$\Big\downarrow \text{6539}$$

$$-a\int \frac{x\left(3d^2x^4+10cdx^2+15c^2\right)}{15\left(1-a^2x^2\right)}dx+c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth^{-1}(ax)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{15}a\int \frac{x\left(3d^2x^4+10cdx^2+15c^2\right)}{1-a^2x^2}dx+c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth^{-1}(ax)$$

$$\Big\downarrow \text{1576}$$

$$-\frac{1}{30}a\int \frac{3d^2x^4+10cdx^2+15c^2}{1-a^2x^2}dx^2+c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth^{-1}(ax)$$

$$\Big\downarrow \text{1140}$$

$$-\frac{1}{30}a\int \left(-\frac{3d^2x^2}{a^2}-\frac{d\left(10ca^2+3d\right)}{a^4}+\frac{-15c^2a^4-10cda^2-3d^2}{a^4\left(a^2x^2-1\right)}\right)dx^2+c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth^{-1}(ax)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{30}a\left(-\frac{3d^2x^4}{2a^2}-\frac{dx^2\left(10a^2c+3d\right)}{a^4}-\frac{\left(15a^4c^2+10a^2cd+3d^2\right)\log\left(1-a^2x^2\right)}{a^6}\right)+c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth^{-1}(ax)$$

input
```
Int[(c + d*x^2)^2*ArcCoth[a*x],x]
```

output
```
c^2*x*ArcCoth[a*x] + (2*c*d*x^3*ArcCoth[a*x])/3 + (d^2*x^5*ArcCoth[a*x])/5
 - (a*(-((d*(10*a^2*c + 3*d)*x^2)/a^4) - (3*d^2*x^4)/(2*a^2) - ((15*a^4*c^
2 + 10*a^2*c*d + 3*d^2)*Log[1 - a^2*x^2])/a^6))/30
```

3.37.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6539
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCoth[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.37.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

method	result
parts	$\frac{d^{2}x^{5}\operatorname{arccoth}(ax)}{5}+\frac{2cd\,x^{3}\operatorname{arccoth}(ax)}{3}+c^{2}x\,\operatorname{arccoth}(ax)+\frac{a\left(\frac{d\left(\frac{3}{2}a^{2}d\,x^{4}+10a^{2}c\,x^{2}+3d\,x^{2}\right)}{2a^{4}}+\frac{\left(15a^{4}c^{2}+10a^{2}c\right.}{}\right.}{15}$
derivativedivides	$\frac{\operatorname{arccoth}(ax)c^{2}ax+\frac{2a\,\operatorname{arccoth}(ax)cd\,x^{3}}{3}+\frac{a\,\operatorname{arccoth}(ax)d^{2}x^{5}}{5}+\frac{5c\,a^{4}d\,x^{2}+\frac{3d^{2}a^{4}x^{4}}{4}+\frac{3a^{2}d^{2}x^{2}}{2}+\frac{\left(15a^{4}c^{2}+10a^{2}cd+3d^{2}\right)\ln(ax-1)}{2}}{15a^{4}}}{a}$
default	$\frac{\operatorname{arccoth}(ax)c^{2}ax+\frac{2a\,\operatorname{arccoth}(ax)cd\,x^{3}}{3}+\frac{a\,\operatorname{arccoth}(ax)d^{2}x^{5}}{5}+\frac{5c\,a^{4}d\,x^{2}+\frac{3d^{2}a^{4}x^{4}}{4}+\frac{3a^{2}d^{2}x^{2}}{2}+\frac{\left(15a^{4}c^{2}+10a^{2}cd+3d^{2}\right)\ln(ax-1)}{2}}{15a^{4}}}{a}$
parallelrisch	$-\frac{-12x^{5}\operatorname{arccoth}(ax)a^{5}d^{2}-40x^{3}\operatorname{arccoth}(ax)a^{5}cd-3d^{2}a^{4}x^{4}-60c^{2}x\,\operatorname{arccoth}(ax)a^{5}-20c\,a^{4}d\,x^{2}-60\ln(ax-1)a^{4}c^{2}-60}{60}$
risch	$\left(\frac{1}{10}d^{2}x^{5}+\frac{1}{3}cd\,x^{3}+\frac{1}{2}c^{2}x\right)\ln(ax+1)-\frac{d^{2}x^{5}\ln(ax-1)}{10}-\frac{cd\,x^{3}\ln(ax-1)}{3}+\frac{d^{2}x^{4}}{20a}-\frac{c^{2}x\ln(ax-1)}{2}-$

input
```
int((d*x^2+c)^2*arccoth(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/5*d^2*x^5*arccoth(a*x)+2/3*c*d*x^3*arccoth(a*x)+c^2*x*arccoth(a*x)+1/15*
a*(1/2*d/a^4*(3/2*a^2*d*x^4+10*a^2*c*x^2+3*d*x^2)+1/2*(15*a^4*c^2+10*a^2*c
*d+3*d^2)/a^6*ln(a^2*x^2-1))
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07

$$\int \left(c+dx^2\right)^2 \coth^{-1}(ax)\,dx = \frac{3\,a^4d^2x^4+2\,(10\,a^4cd+3\,a^2d^2)x^2+2\,(15\,a^4c^2+10\,a^2cd+3\,d^2)\log\left(a^2x^2-1\right)+2\,(3\,a^5d^2x^5+10\,a^5cdx^3+}{60\,a^5}$$

input
```
integrate((d*x^2+c)^2*arccoth(a*x),x, algorithm="fricas")
```

output
```
1/60*(3*a^4*d^2*x^4 + 2*(10*a^4*c*d + 3*a^2*d^2)*x^2 + 2*(15*a^4*c^2 + 10*
a^2*c*d + 3*d^2)*log(a^2*x^2 - 1) + 2*(3*a^5*d^2*x^5 + 10*a^5*c*d*x^3 + 15
*a^5*c^2*x)*log((a*x + 1)/(a*x - 1)))/a^5
```

3.37.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.65

$$\int \left(c+dx^2\right)^2 \coth^{-1}(ax)\,dx = \begin{cases} c^2x\,\mathrm{acoth}\,(ax)+\frac{2cdx^3\,\mathrm{acoth}\,(ax)}{3}+\frac{d^2x^5\,\mathrm{acoth}\,(ax)}{5}+\frac{c^2\log\left(x-\frac{1}{a}\right)}{a}+\frac{c^2\,\mathrm{acoth}\,(ax)}{a}+\frac{cdx^2}{3a}+\frac{d^2x^4}{20a}+\frac{2cd\log\left(x-\frac{1}{a}\right)}{3a^3}+ \\ \frac{i\pi\left(c^2x+\frac{2cdx^3}{3}+\frac{d^2x^5}{5}\right)}{2} \end{cases}$$

input
```
integrate((d*x**2+c)**2*acoth(a*x),x)
```

output
```
Piecewise((c**2*x*acoth(a*x) + 2*c*d*x**3*acoth(a*x)/3 + d**2*x**5*acoth(a
*x)/5 + c**2*log(x - 1/a)/a + c**2*acoth(a*x)/a + c*d*x**2/(3*a) + d**2*x*
*4/(20*a) + 2*c*d*log(x - 1/a)/(3*a**3) + 2*c*d*acoth(a*x)/(3*a**3) + d**2
*x**2/(10*a**3) + d**2*log(x - 1/a)/(5*a**5) + d**2*acoth(a*x)/(5*a**5), N
e(a, 0)), (I*pi*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5)/2, True))
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.19

$$\int (c+dx^2)^2 \coth^{-1}(ax)\,dx$$
$$= \frac{1}{60}a\left(\frac{3a^2d^2x^4 + 2(10a^2cd + 3d^2)x^2}{a^4} + \frac{2(15a^4c^2 + 10a^2cd + 3d^2)\log(ax+1)}{a^6} + \frac{2(15a^4c^2 + 10a^2cd + \right.$$
$$+ \frac{1}{15}(3d^2x^5 + 10cdx^3 + 15c^2x)\,\mathrm{arcoth}(ax)$$

input
```
integrate((d*x^2+c)^2*arccoth(a*x),x, algorithm="maxima")
```

output
```
1/60*a*((3*a^2*d^2*x^4 + 2*(10*a^2*c*d + 3*d^2)*x^2)/a^4 + 2*(15*a^4*c^2 +
 10*a^2*c*d + 3*d^2)*log(a*x + 1)/a^6 + 2*(15*a^4*c^2 + 10*a^2*c*d + 3*d^2
)*log(a*x - 1)/a^6) + 1/15*(3*d^2*x^5 + 10*c*d*x^3 + 15*c^2*x)*arccoth(a*x
)
```

3.37.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(100) = 200$.

Time = 0.28 (sec) , antiderivative size = 529, normalized size of antiderivative = 4.81

$$\int (c+dx^2)^2 \coth^{-1}(ax)\,dx$$
$$= \frac{1}{15}a\left(\frac{(15a^4c^2 + 10a^2cd + 3d^2)\log\left(\frac{|ax+1|}{|ax-1|}\right)}{a^6} - \frac{(15a^4c^2 + 10a^2cd + 3d^2)\log\left(\left|\frac{ax+1}{ax-1} - 1\right|\right)}{a^6} + \frac{4\left(\frac{(5a^2cd+3d}{(ax-}\right.}{}\right.$$

input
```
integrate((d*x^2+c)^2*arccoth(a*x),x, algorithm="giac")
```

output
```
1/15*a*((15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*log(abs(a*x + 1)/abs(a*x - 1))/a
^6 - (15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*log(abs((a*x + 1)/(a*x - 1) - 1))/a
^6 + 4*((5*a^2*c*d + 3*d^2)*(a*x + 1)^3/(a*x - 1)^3 - (10*a^2*c*d + 3*d^2)
*(a*x + 1)^2/(a*x - 1)^2 + (5*a^2*c*d + 3*d^2)*(a*x + 1)/(a*x - 1))/(a^6*(
(a*x + 1)/(a*x - 1) - 1)^4) + (15*(a*x + 1)^4*a^4*c^2/(a*x - 1)^4 - 60*(a*
x + 1)^3*a^4*c^2/(a*x - 1)^3 + 90*(a*x + 1)^2*a^4*c^2/(a*x - 1)^2 - 60*(a*
x + 1)*a^4*c^2/(a*x - 1) + 15*a^4*c^2 + 30*(a*x + 1)^4*a^2*c*d/(a*x - 1)^4
 - 60*(a*x + 1)^3*a^2*c*d/(a*x - 1)^3 + 40*(a*x + 1)^2*a^2*c*d/(a*x - 1)^2
 - 20*(a*x + 1)*a^2*c*d/(a*x - 1) + 10*a^2*c*d + 15*(a*x + 1)^4*d^2/(a*x -
 1)^4 + 30*(a*x + 1)^2*d^2/(a*x - 1)^2 + 3*d^2)*log(-(((a*x + 1)*a/(a*x -
1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) + 1)/(((a*x + 1)*a/(a*x - 1) - a)/(a
*((a*x + 1)/(a*x - 1) + 1)) - 1))/(a^6*((a*x + 1)/(a*x - 1) - 1)^5))
```

3.37.9 Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int (c+dx^2)^2 \coth^{-1}(ax)\,dx = \frac{a^4\left(\frac{c^2\,\ln(a^2\,x^2-1)}{2}+\frac{d^2\,x^4}{20}+\frac{c\,d\,x^2}{3}\right)+a^2\left(\frac{d^2\,x^2}{10}+\frac{c\,d\,\ln(a^2\,x^2-1)}{3}\right)+\frac{d^2\,\ln(a^2\,x^2-1)}{10}}{a^5} + c^2\,x\,\mathrm{acoth}(a\,x)+\frac{d^2\,x^5\,\mathrm{acoth}(a\,x)}{5}+\frac{2\,c\,d\,x^3\,\mathrm{acoth}(a\,x)}{3}$$

input
```
int(acoth(a*x)*(c + d*x^2)^2,x)
```

output
```
(a^4*((c^2*log(a^2*x^2 - 1))/2 + (d^2*x^4)/20 + (c*d*x^2)/3) + a^2*((d^2*x
^2)/10 + (c*d*log(a^2*x^2 - 1))/3) + (d^2*log(a^2*x^2 - 1))/10)/a^5 + c^2*
x*acoth(a*x) + (d^2*x^5*acoth(a*x))/5 + (2*c*d*x^3*acoth(a*x))/3
```

3.38 $\int (c+dx^2)\coth^{-1}(ax)\,dx$

3.38.1 Optimal result

Integrand size = 12, antiderivative size = 57

$$\int (c+dx^2)\coth^{-1}(ax)\,dx = \frac{dx^2}{6a}+cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)+\frac{(3a^2c+d)\log(1-a^2x^2)}{6a^3}$$

output
```
1/6*d*x^2/a+c*x*arccoth(a*x)+1/3*d*x^3*arccoth(a*x)+1/6*(3*a^2*c+d)*ln(-a^
2*x^2+1)/a^3
```

3.38.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int (c+dx^2)\coth^{-1}(ax)\,dx = \frac{dx^2}{6a}+cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)+\frac{c\log(1-a^2x^2)}{2a}+\frac{d\log(1-a^2x^2)}{6a^3}$$

input
```
Integrate[(c + d*x^2)*ArcCoth[a*x],x]
```

output
```
(d*x^2)/(6*a) + c*x*ArcCoth[a*x] + (d*x^3*ArcCoth[a*x])/3 + (c*Log[1 - a^2
*x^2])/(2*a) + (d*Log[1 - a^2*x^2])/(6*a^3)
```

3.38.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6539, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(ax)\left(c+dx^2\right) \, dx$$

$$\Big\downarrow \; 6539$$

$$-a\int \frac{x\left(dx^2+3c\right)}{3\left(1-a^2x^2\right)}dx+cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{3}a\int \frac{x\left(dx^2+3c\right)}{1-a^2x^2}dx+cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)$$

$$\Big\downarrow \; 353$$

$$-\frac{1}{6}a\int \frac{dx^2+3c}{1-a^2x^2}dx^2+cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)$$

$$\Big\downarrow \; 49$$

$$-\frac{1}{6}a\int \left(\frac{-3ca^2-d}{a^2\left(a^2x^2-1\right)}-\frac{d}{a^2}\right)dx^2+cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{6}a\left(-\frac{dx^2}{a^2}-\frac{\left(3a^2c+d\right)\log\left(1-a^2x^2\right)}{a^4}\right)+cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)$$

input
```
Int[(c + d*x^2)*ArcCoth[a*x],x]
```

output
```
c*x*ArcCoth[a*x] + (d*x^3*ArcCoth[a*x])/3 - (a*(-((d*x^2)/a^2) - ((3*a^2*c
 + d)*Log[1 - a^2*x^2])/a^4))/6
```

3.38.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6539
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCoth[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.38.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

method	result
parts	$\frac{d\,x^{3} \operatorname{arccoth}(a x)}{3}+c x \,\operatorname{arccoth}\left(a x \right)+\frac{a \left(\frac{d\,x^{2}}{2 a^{2}}+\frac{\left(3 a^{2} c +d \right) \ln \left(a^{2} x^{2}-1\right)}{2 a^{4}}\right)}{3}$
derivativedivides	$\frac{\operatorname{arccoth}(a x) c a x +\frac{a \,\operatorname{arccoth}(a x) d\,x^{3}}{3}+\frac{\frac{a^{2} d\,x^{2}}{2}+\frac{\left(3 a^{2} c +d \right) \ln (a x -1)}{2}-\frac{\left(-3 a^{2} c -d \right) \ln (a x +1)}{2}}{3 a^{2}}}{a}$
default	$\frac{\operatorname{arccoth}(a x) c a x +\frac{a \,\operatorname{arccoth}(a x) d\,x^{3}}{3}+\frac{\frac{a^{2} d\,x^{2}}{2}+\frac{\left(3 a^{2} c +d \right) \ln (a x -1)}{2}-\frac{\left(-3 a^{2} c -d \right) \ln (a x +1)}{2}}{3 a^{2}}}{a}$
parallelrisch	$-\frac{-2 x^{3} \operatorname{arccoth}(a x) a^{3} d -6 c \,\operatorname{arccoth}(a x) x \,a^{3}-a^{2} d\,x^{2}-6 \ln (a x -1) a^{2} c -6 \,\operatorname{arccoth}(a x) a^{2} c -2 \ln (a x -1) d -2 \,\operatorname{arccoth}(a x)}{6 a^{3}}$
risch	$\left(\frac{1}{6} d\,x^{3}+\frac{1}{2} c x \right) \ln \left(a x +1\right)-\frac{d\,x^{3} \ln (a x -1)}{6}-\frac{c x \ln (a x -1)}{2}+\frac{d\,x^{2}}{6 a}+\frac{\ln \left(a^{2} x^{2}-1\right) c}{2 a}+\frac{\ln \left(a^{2} x^{2}-1\right) d}{6 a^{3}}$

input
```
int((d*x^2+c)*arccoth(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*d*x^3*arccoth(a*x)+c*x*arccoth(a*x)+1/3*a*(1/2*d/a^2*x^2+1/2*(3*a^2*c+
d)/a^4*ln(a^2*x^2-1))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12

$$\int (c+dx^2)\coth^{-1}(ax)\,dx = \frac{a^2dx^2 + (3\,a^2c+d)\log(a^2x^2-1) + (a^3dx^3+3\,a^3cx)\log\left(\frac{ax+1}{ax-1}\right)}{6\,a^3}$$

input
```
integrate((d*x^2+c)*arccoth(a*x),x, algorithm="fricas")
```

output
```
1/6*(a^2*d*x^2 + (3*a^2*c + d)*log(a^2*x^2 - 1) + (a^3*d*x^3 + 3*a^3*c*x)*
log((a*x + 1)/(a*x - 1)))/a^3
```

3.38.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.53

$$\int (c+dx^2)\coth^{-1}(ax)\,dx$$
$$= \begin{cases} cx\,\mathrm{acoth}\,(ax) + \frac{dx^3\,\mathrm{acoth}\,(ax)}{3} + \frac{c\log\left(x-\frac{1}{a}\right)}{a} + \frac{c\,\mathrm{acoth}\,(ax)}{a} + \frac{dx^2}{6a} + \frac{d\log\left(x-\frac{1}{a}\right)}{3a^3} + \frac{d\,\mathrm{acoth}\,(ax)}{3a^3} & \text{for } a \neq 0 \\ \frac{i\pi\left(cx+\frac{dx^3}{3}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**2+c)*acoth(a*x),x)
```

output
```
Piecewise((c*x*acoth(a*x) + d*x**3*acoth(a*x)/3 + c*log(x - 1/a)/a + c*aco
th(a*x)/a + d*x**2/(6*a) + d*log(x - 1/a)/(3*a**3) + d*acoth(a*x)/(3*a**3)
, Ne(a, 0)), (I*pi*(c*x + d*x**3/3)/2, True))
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

$$\int \left(c+dx^2\right)\coth^{-1}(ax)\,dx = \frac{1}{6}\,a\left(\frac{dx^2}{a^2} + \frac{\left(3\,a^2c+d\right)\log\left(ax+1\right)}{a^4} + \frac{\left(3\,a^2c+d\right)\log\left(ax-1\right)}{a^4}\right) + \frac{1}{3}\left(dx^3+3\,cx\right)\operatorname{arcoth}\left(ax\right)$$

input
```
integrate((d*x^2+c)*arccoth(a*x),x, algorithm="maxima")
```

output
```
1/6*a*(d*x^2/a^2 + (3*a^2*c + d)*log(a*x + 1)/a^4 + (3*a^2*c + d)*log(a*x
- 1)/a^4) + 1/3*(d*x^3 + 3*c*x)*arccoth(a*x)
```

3.38.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(51) = 102.

Time = 0.28 (sec) , antiderivative size = 268, normalized size of antiderivative = 4.70

$$\int \left(c+dx^2\right)\coth^{-1}(ax)\,dx$$

$$= \frac{1}{3}\,a\left(\frac{\left(3\,a^2c+d\right)\log\left(\frac{|ax+1|}{|ax-1|}\right)}{a^4} - \frac{\left(3\,a^2c+d\right)\log\left(\left|\frac{ax+1}{ax-1}-1\right|\right)}{a^4} + \frac{2\,(ax+1)d}{(ax-1)a^4\left(\frac{ax+1}{ax-1}-1\right)^2} + \frac{\left(\frac{3\,(ax+1)^2a^2c}{(ax-1)^2} - 6(\right.}{}\right.$$

input
```
integrate((d*x^2+c)*arccoth(a*x),x, algorithm="giac")
```

output
```
1/3*a*((3*a^2*c + d)*log(abs(a*x + 1)/abs(a*x - 1))/a^4 - (3*a^2*c + d)*lo
g(abs((a*x + 1)/(a*x - 1) - 1))/a^4 + 2*(a*x + 1)*d/((a*x - 1)*a^4*((a*x +
 1)/(a*x - 1) - 1)^2) + (3*(a*x + 1)^2*a^2*c/(a*x - 1)^2 - 6*(a*x + 1)*a^2
*c/(a*x - 1) + 3*a^2*c + 3*(a*x + 1)^2*d/(a*x - 1)^2 + d)*log(-(((a*x + 1)
*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) + 1)/(((a*x + 1)*a/(a*x -
1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) - 1))/(a^4*((a*x + 1)/(a*x - 1) - 1)
^3))
```

3.38.9 Mupad [B] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int (c+dx^2)\coth^{-1}(ax)\,dx = \frac{\frac{d\ln(a^2x^2-1)}{6}+a^2\left(\frac{c\ln(a^2x^2-1)}{2}+\frac{dx^2}{6}\right)}{a^3} + \frac{dx^3\operatorname{acoth}(ax)}{3}+cx\operatorname{acoth}(ax)$$

input
```
int(acoth(a*x)*(c + d*x^2),x)
```

output
```
((d*log(a^2*x^2 - 1))/6 + a^2*((c*log(a^2*x^2 - 1))/2 + (d*x^2)/6))/a^3 +
(d*x^3*acoth(a*x))/3 + c*x*acoth(a*x)
```

3.39 $\int \frac{\coth^{-1}(ax)}{c+dx^2}\,dx$

3.39.1 Optimal result

Integrand size = 14, antiderivative size = 390

$$\int \frac{\coth^{-1}(ax)}{c+dx^2}\,dx = -\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}}$$
$$+ \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(-\frac{2\sqrt{c}\sqrt{d}(1-ax)}{(ia\sqrt{c}-\sqrt{d})(\sqrt{c}-i\sqrt{d}x)}\right)}{2\sqrt{c}\sqrt{d}}$$
$$- \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}\sqrt{d}(1+ax)}{(ia\sqrt{c}+\sqrt{d})(\sqrt{c}-i\sqrt{d}x)}\right)}{2\sqrt{c}\sqrt{d}}$$
$$- \frac{i\,\mathrm{PolyLog}\left(2, 1+\frac{2\sqrt{c}\sqrt{d}(1-ax)}{(ia\sqrt{c}-\sqrt{d})(\sqrt{c}-i\sqrt{d}x)}\right)}{4\sqrt{c}\sqrt{d}}$$
$$+ \frac{i\,\mathrm{PolyLog}\left(2, 1-\frac{2\sqrt{c}\sqrt{d}(1+ax)}{(ia\sqrt{c}+\sqrt{d})(\sqrt{c}-i\sqrt{d}x)}\right)}{4\sqrt{c}\sqrt{d}}$$

output
```
-1/2*arctan(x*d^(1/2)/c^(1/2))*ln(1-1/a/x)/c^(1/2)/d^(1/2)+1/2*arctan(x*d^
(1/2)/c^(1/2))*ln(1+1/a/x)/c^(1/2)/d^(1/2)+1/2*arctan(x*d^(1/2)/c^(1/2))*l
n(-2*(-a*x+1)*c^(1/2)*d^(1/2)/(I*a*c^(1/2)-d^(1/2))/(c^(1/2)-I*x*d^(1/2)))
/c^(1/2)/d^(1/2)-1/2*arctan(x*d^(1/2)/c^(1/2))*ln(2*(a*x+1)*c^(1/2)*d^(1/2
)/(I*a*c^(1/2)+d^(1/2))/(c^(1/2)-I*x*d^(1/2)))/c^(1/2)/d^(1/2)-1/4*I*polyl
og(2,1+2*(-a*x+1)*c^(1/2)*d^(1/2)/(I*a*c^(1/2)-d^(1/2))/(c^(1/2)-I*x*d^(1/
2)))/c^(1/2)/d^(1/2)+1/4*I*polylog(2,1-2*(a*x+1)*c^(1/2)*d^(1/2)/(I*a*c^(1
/2)+d^(1/2))/(c^(1/2)-I*x*d^(1/2)))/c^(1/2)/d^(1/2)
```

3.39.2 Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.72

$$\int \frac{\coth^{-1}(ax)}{c+dx^2}\,dx$$
$$= \frac{a\left(-2i\arccos\left(\frac{a^2c-d}{a^2c+d}\right)\arctan\left(\frac{ac}{\sqrt{a^2cdx}}\right)+4\coth^{-1}(ax)\arctan\left(\frac{adx}{\sqrt{a^2cd}}\right)-\left(\arccos\left(\frac{a^2c-d}{a^2c+d}\right)+2\arctan\left(\frac{a}{\sqrt{a^2}}\right.\right.\right.}{}$$

input
```
Integrate[ArcCoth[a*x]/(c + d*x^2),x]
```

output
```
(a*((-2*I)*ArcCos[(a^2*c - d)/(a^2*c + d)]*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]
 + 4*ArcCoth[a*x]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]] - (ArcCos[(a^2*c - d)/(a^2
*c + d)] + 2*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)])*Log[(2*d*(a^2*c - I*Sqrt[a^2
*c*d])*(-1 + a*x))/((a^2*c + d)*(I*Sqrt[a^2*c*d] + a*d*x))] - (ArcCos[(a^2
*c - d)/(a^2*c + d)] - 2*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)])*Log[(2*d*(a^2*c
+ I*Sqrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(I*Sqrt[a^2*c*d] + a*d*x))] + (
ArcCos[(a^2*c - d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + Arc
Tan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c + d]*
E^ArcCoth[a*x]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]]])] + (
ArcCos[(a^2*c - d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + Arc
Tan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcCoth[a*x])/(S
qrt[a^2*c + d]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]]])] + I
*(-PolyLog[2, ((a^2*c - d - (2*I)*Sqrt[a^2*c*d])*(Sqrt[a^2*c*d] + I*a*d*x)
)/((a^2*c + d)*(Sqrt[a^2*c*d] - I*a*d*x))] + PolyLog[2, ((a^2*c - d + (2*I
)*Sqrt[a^2*c*d])*(Sqrt[a^2*c*d] + I*a*d*x))/((a^2*c + d)*(Sqrt[a^2*c*d] -
I*a*d*x))])))/(4*Sqrt[a^2*c*d])
```

3.39.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.58, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6535, 2920, 27, 2005, 5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)}{c+dx^2}\,dx$$

$$\Big\downarrow \text{ 6535}$$

$$\frac{1}{2}\int \frac{\log\left(1+\frac{1}{ax}\right)}{dx^2+c}dx-\frac{1}{2}\int \frac{\log\left(1-\frac{1}{ax}\right)}{dx^2+c}dx$$

$$\Big\downarrow \text{ 2920}$$

$$\frac{1}{2}\left(\frac{\int \frac{a\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}\left(a-\frac{1}{x}\right)x^2}dx}{a}-\frac{\log\left(1-\frac{1}{ax}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\int \frac{a\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}\left(a+\frac{1}{x}\right)x^2}dx}{a}+\frac{\log\left(\frac{1}{ax}+1\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{\int \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\left(a-\frac{1}{x}\right)x^2}dx}{\sqrt{c}\sqrt{d}}-\frac{\log\left(1-\frac{1}{ax}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\int \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\left(a+\frac{1}{x}\right)x^2}dx}{\sqrt{c}\sqrt{d}}+\frac{\log\left(\frac{1}{ax}+1\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}\right)$$

$$\Big\downarrow \text{ 2005}$$

$$\frac{1}{2}\left(\frac{\int \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x(ax-1)}dx}{\sqrt{c}\sqrt{d}}-\frac{\log\left(1-\frac{1}{ax}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\int \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x(ax+1)}dx}{\sqrt{c}\sqrt{d}}+\frac{\log\left(\frac{1}{ax}+1\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}\right)$$

$$\Big\downarrow \text{5411}$$

$$\frac{1}{2}\left(\frac{\int \left(\frac{a \arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{ax-1}-\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x}\right)dx}{\sqrt{c}\sqrt{d}}-\frac{\log\left(1-\frac{1}{ax}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\int \left(\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x}-\frac{a \arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{ax+1}\right)dx}{\sqrt{c}\sqrt{d}}+\frac{\log\left(\frac{1}{ax}+1\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\log\left(1-\frac{1}{ax}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}+\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(-\frac{2\sqrt{c}\sqrt{d}(1-ax)}{\left(-\sqrt{d}+ia\sqrt{c}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{2\sqrt{c}\sqrt{d}(1-a}{\left(ia\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}-\right.}\right.}{}\right.$$

$$\frac{1}{2}\left(\frac{\log\left(\frac{1}{ax}+1\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}+\frac{-\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}\sqrt{d}(ax+1)}{\left(\sqrt{d}+ia\sqrt{c}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2\sqrt{c}\sqrt{d}(ax-}{\left(i\sqrt{c}a+\sqrt{d}\right)\left(\sqrt{c}\right.}\right.}{}\right.$$

input

```
Int[ArcCoth[a*x]/(c + d*x^2),x]
```

output

```
(-((ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[1 - 1/(a*x)])/(Sqrt[c]*Sqrt[d])) + (-(
ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*Sqrt[d]*x)]) + Ar
cTan[(Sqrt[d]*x)/Sqrt[c]]*Log[(-2*Sqrt[c]*Sqrt[d]*(1 - a*x))/((I*a*Sqrt[c]
 - Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))] - (I/2)*PolyLog[2, ((-I)*Sqrt[d]*x)/
Sqrt[c]] + (I/2)*PolyLog[2, (I*Sqrt[d]*x)/Sqrt[c]] + (I/2)*PolyLog[2, 1 -
(2*Sqrt[c])/(Sqrt[c] - I*Sqrt[d]*x)] - (I/2)*PolyLog[2, 1 + (2*Sqrt[c]*Sqr
t[d]*(1 - a*x))/((I*a*Sqrt[c] - Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))])/(Sqrt[
c]*Sqrt[d]))/2 + ((ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[1 + 1/(a*x)])/(Sqrt[c]*
Sqrt[d]) + (ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*Sqrt[
d]*x)] - ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[(2*Sqrt[c]*Sqrt[d]*(1 + a*x))/((I
*a*Sqrt[c] + Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))] + (I/2)*PolyLog[2, ((-I)*S
qrt[d]*x)/Sqrt[c]] - (I/2)*PolyLog[2, (I*Sqrt[d]*x)/Sqrt[c]] - (I/2)*PolyL
og[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*Sqrt[d]*x)] + (I/2)*PolyLog[2, 1 - (2*S
qrt[c]*Sqrt[d]*(1 + a*x))/((I*a*Sqrt[c] + Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x)
)])/(Sqrt[c]*Sqrt[d]))/2
```

3.39. $\int \frac{\coth^{-1}(ax)}{c+dx^2}\,dx$

3.39.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

rule 6535
```
Int[ArcCoth[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[1/2   Int
[Log[1 + 1/(c*x)]/(d + e*x^2), x], x] - Simp[1/2   Int[Log[1 - 1/(c*x)]/(d
+ e*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

3.39.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{\ln(ax-1)\ln\left(\frac{a\sqrt{-cd}-(ax-1)d-d}{a\sqrt{-cd}-d}\right)}{4\sqrt{-cd}}+\frac{\ln(ax-1)\ln\left(\frac{a\sqrt{-cd}+(ax-1)d+d}{a\sqrt{-cd}+d}\right)}{4\sqrt{-cd}}-\frac{\mathrm{dilog}\left(\frac{a\sqrt{-cd}-(ax-1)d-d}{a\sqrt{-cd}-d}\right)}{4\sqrt{-cd}}+\mathrm{dilog}\Big(a$
derivativedivides	$\frac{\sqrt{-a^2cd}\,\mathrm{arccoth}(ax)\ln\left(1-\frac{(a^2c+d)(ax+1)}{(ax-1)\left(a^2c+2\sqrt{-a^2cd}-d\right)}\right)}{2cd}-\frac{\sqrt{-a^2cd}\,\mathrm{arccoth}(ax)^2}{2cd}+\frac{\sqrt{-a^2cd}\,\mathrm{polylog}\left(2,\frac{(a^2c+d)(ax+1)}{(ax-1)\left(a^2c+2\sqrt{-a^2cd}-\right.}\right.}{4cd}$
default	$\frac{\sqrt{-a^2cd}\,\mathrm{arccoth}(ax)\ln\left(1-\frac{(a^2c+d)(ax+1)}{(ax-1)\left(a^2c+2\sqrt{-a^2cd}-d\right)}\right)}{2cd}-\frac{\sqrt{-a^2cd}\,\mathrm{arccoth}(ax)^2}{2cd}+\frac{\sqrt{-a^2cd}\,\mathrm{polylog}\left(2,\frac{(a^2c+d)(ax+1)}{(ax-1)\left(a^2c+2\sqrt{-a^2cd}-\right.}\right.}{4cd}$

input
```
int(arccoth(a*x)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(a*x-1)/(-c*d)^(1/2)*ln((a*(-c*d)^(1/2)-(a*x-1)*d-d)/(a*(-c*d)^(1/2
)-d))+1/4*ln(a*x-1)/(-c*d)^(1/2)*ln((a*(-c*d)^(1/2)+(a*x-1)*d+d)/(a*(-c*d)
^(1/2)+d))-1/4/(-c*d)^(1/2)*dilog((a*(-c*d)^(1/2)-(a*x-1)*d-d)/(a*(-c*d)^(
1/2)-d))+1/4/(-c*d)^(1/2)*dilog((a*(-c*d)^(1/2)+(a*x-1)*d+d)/(a*(-c*d)^(1/
2)+d))+1/4*ln(a*x+1)/(-c*d)^(1/2)*ln((a*(-c*d)^(1/2)-(a*x+1)*d+d)/(a*(-c*d
)^(1/2)+d))-1/4*ln(a*x+1)/(-c*d)^(1/2)*ln((a*(-c*d)^(1/2)+(a*x+1)*d-d)/(a*
(-c*d)^(1/2)-d))+1/4/(-c*d)^(1/2)*dilog((a*(-c*d)^(1/2)-(a*x+1)*d+d)/(a*(-
c*d)^(1/2)+d))-1/4/(-c*d)^(1/2)*dilog((a*(-c*d)^(1/2)+(a*x+1)*d-d)/(a*(-c*
d)^(1/2)-d))
```

3.39.5 Fricas [F]

$$\int \frac{\coth^{-1}(ax)}{c+dx^2}\,dx=\int \frac{\mathrm{arcoth}\,(ax)}{dx^2+c}\,dx$$

input
```
integrate(arccoth(a*x)/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral(arccoth(a*x)/(d*x^2 + c), x)
```

3.39.6 Sympy [F]

$$\int \frac{\coth^{-1}(ax)}{c+dx^2}\,dx=\int \frac{\operatorname{acoth}(ax)}{c+dx^2}\,dx$$

input
```
integrate(acoth(a*x)/(d*x**2+c),x)
```

output
```
Integral(acoth(a*x)/(c + d*x**2), x)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.04

$$\int \frac{\coth^{-1}(ax)}{c+dx^2}\,dx=\frac{\operatorname{arcoth}(ax)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}} + \frac{\left(\arctan\left(\frac{(a^2x+a)\sqrt{c}\sqrt{d}}{a^2c+d},\frac{adx+d}{a^2c+d}\right)-\arctan\left(\frac{(a^2x-a)\sqrt{c}\sqrt{d}}{a^2c+d},-\frac{adx-d}{a^2c+d}\right)\right)\log\left(dx^2+c\right)-\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{a^2dx^2}{a\cdots}\right.}{\cdots}$$

input
```
integrate(arccoth(a*x)/(d*x^2+c),x, algorithm="maxima")
```

output
```
arccoth(a*x)*arctan(d*x/sqrt(c*d))/sqrt(c*d) + 1/4*((arctan2((a^2*x + a)*s
qrt(c)*sqrt(d)/(a^2*c + d), (a*d*x + d)/(a^2*c + d)) - arctan2((a^2*x - a)
*sqrt(c)*sqrt(d)/(a^2*c + d), -(a*d*x - d)/(a^2*c + d)))*log(d*x^2 + c) -
arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 + 2*a*d*x + d)/(a^2*c + d)) + arc
tan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 - 2*a*d*x + d)/(a^2*c + d)) - I*dilo
g((a^2*c + a*d*x - (I*a^2*x - I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)
*sqrt(d) - d)) - I*dilog((a^2*c - a*d*x + (I*a^2*x + I*a)*sqrt(c)*sqrt(d))
/(a^2*c + 2*I*a*sqrt(c)*sqrt(d) - d)) + I*dilog((a^2*c + a*d*x + (I*a^2*x
- I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)) + I*dilog((a^
2*c - a*d*x - (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt
(d) - d)))/sqrt(c*d)
```

3.39.8 Giac [F]

$$\int \frac{\coth^{-1}(ax)}{c+dx^2}\,dx = \int \frac{\operatorname{arcoth}(ax)}{dx^2+c}\,dx$$

input
```
integrate(arccoth(a*x)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(arccoth(a*x)/(d*x^2 + c), x)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax)}{c+dx^2}\,dx = \int \frac{\operatorname{acoth}(a\,x)}{d\,x^2+c}\,dx$$

input
```
int(acoth(a*x)/(c + d*x^2),x)
```

output
```
int(acoth(a*x)/(c + d*x^2), x)
```

3.40 $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^2}\,dx$

3.40.1 Optimal result

Integrand size = 14, antiderivative size = 590

$$
\begin{aligned}
\int \frac{\coth^{-1}(ax)}{(c+dx^2)^2}\,dx = {} & \frac{x\coth^{-1}(ax)}{2c\,(c+dx^2)} + \frac{\coth^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} \\
& + \frac{i\log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i\log\left(-\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} \\
& - \frac{i\log\left(-\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}-\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} + \frac{i\log\left(\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} \\
& + \frac{a\log(1-a^2x^2)}{4c\,(a^2c+d)} - \frac{a\log(c+dx^2)}{4c\,(a^2c+d)} \\
& + \frac{i\,\mathrm{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i\,\mathrm{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} \\
& + \frac{i\,\mathrm{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i\,\mathrm{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}
\end{aligned}
$$

output
```
1/2*x*arccoth(a*x)/c/(d*x^2+c)+1/4*a*ln(-a^2*x^2+1)/c/(a^2*c+d)-1/4*a*ln(d
*x^2+c)/c/(a^2*c+d)+1/2*arccoth(a*x)*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/d^(
1/2)-1/8*I*ln(-(a*x+1)*d^(1/2)/(I*a*c^(1/2)-d^(1/2)))*ln(1-I*x*d^(1/2)/c^(
1/2))/c^(3/2)/d^(1/2)+1/8*I*ln((-a*x+1)*d^(1/2)/(I*a*c^(1/2)+d^(1/2)))*ln(
1-I*x*d^(1/2)/c^(1/2))/c^(3/2)/d^(1/2)-1/8*I*ln(-(-a*x+1)*d^(1/2)/(I*a*c^(
1/2)-d^(1/2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(3/2)/d^(1/2)+1/8*I*ln((a*x+1)*
d^(1/2)/(I*a*c^(1/2)+d^(1/2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(3/2)/d^(1/2)+1
/8*I*polylog(2,a*(c^(1/2)-I*x*d^(1/2))/(a*c^(1/2)-I*d^(1/2)))/c^(3/2)/d^(1
/2)-1/8*I*polylog(2,a*(c^(1/2)-I*x*d^(1/2))/(a*c^(1/2)+I*d^(1/2)))/c^(3/2)
/d^(1/2)+1/8*I*polylog(2,a*(c^(1/2)+I*x*d^(1/2))/(a*c^(1/2)-I*d^(1/2)))/c^
(3/2)/d^(1/2)-1/8*I*polylog(2,a*(c^(1/2)+I*x*d^(1/2))/(a*c^(1/2)+I*d^(1/2)
))/c^(3/2)/d^(1/2)
```

3.40.2 Mathematica [A] (verified)

Time = 5.78 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.28

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^2}\,dx = -\frac{a\left(\frac{2\log\left(1-\frac{(a^2c+d)\cosh\left(2\coth^{-1}(ax)\right)}{a^2c-d}\right)}{a^2c+d}+\frac{2i\arccos\left(\frac{a^2c-d}{a^2c+d}\right)\arctan\left(\frac{ac}{\sqrt{a^2cd}x}\right)-4\coth^{-1}(ax)\arctan\left(\frac{adx}{\sqrt{a^2cd}}\right)+\left(\arccos\left(\frac{a^2c-d}{a^2c+d}\right)+2\,\mathrm{ar}\right.}{}\right.}{}$$

input
```
Integrate[ArcCoth[a*x]/(c + d*x^2)^2,x]
```

output
```
-1/8*(a*((2*Log[1 - ((a^2*c + d)*Cosh[2*ArcCoth[a*x]])/(a^2*c - d)])/(a^2*
c + d) + ((2*I)*ArcCos[(a^2*c - d)/(a^2*c + d)]*ArcTan[(a*c)/(Sqrt[a^2*c*d
]*x)] - 4*ArcCoth[a*x]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]] + (ArcCos[(a^2*c - d)
/(a^2*c + d)] + 2*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)])*Log[(2*d*(a^2*c - I*Sqr
t[a^2*c*d])*(-1 + a*x))/((a^2*c + d)*(I*Sqrt[a^2*c*d] + a*d*x))] + (ArcCos
[(a^2*c - d)/(a^2*c + d)] - 2*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)])*Log[(2*d*(a
^2*c + I*Sqrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(I*Sqrt[a^2*c*d] + a*d*x))
] - (ArcCos[(a^2*c - d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]
+ ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c
+ d]*E^ArcCoth[a*x]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]]])
] - (ArcCos[(a^2*c - d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]
+ ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcCoth[a*x
])/(Sqrt[a^2*c + d]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]]])
] + I*(PolyLog[2, ((a^2*c - d - (2*I)*Sqrt[a^2*c*d])*(Sqrt[a^2*c*d] + I*a*
d*x))/((a^2*c + d)*(Sqrt[a^2*c*d] - I*a*d*x))] - PolyLog[2, ((a^2*c - d +
(2*I)*Sqrt[a^2*c*d])*(Sqrt[a^2*c*d] + I*a*d*x))/((a^2*c + d)*(Sqrt[a^2*c*d
] - I*a*d*x))]))/Sqrt[a^2*c*d] - (4*ArcCoth[a*x]*Sinh[2*ArcCoth[a*x]])/(-(
a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]])))/c
```

3.40.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 618, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6539, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^2}dx$$

$$\Big\downarrow \text{6539}$$

$$-a\int \frac{\frac{x}{c(dx^2+c)}+\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}\sqrt{d}}}{2(1-a^2x^2)}dx+\frac{\coth^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{x\coth^{-1}(ax)}{2c(c+dx^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}a\int \frac{\frac{x}{c(dx^2+c)}+\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}\sqrt{d}}}{1-a^2x^2}dx+\frac{\coth^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{x\coth^{-1}(ax)}{2c(c+dx^2)}$$

$$\Big\downarrow \text{7276}$$

$$-\frac{1}{2}a\int\left(-\frac{x}{c(ax-1)(ax+1)\left(dx^2+c\right)}-\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}\sqrt{d}\left(a^2x^2-1\right)}\right)dx+\frac{\coth^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{x\coth^{-1}(ax)}{2c\left(c+dx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}a\left(-\frac{\log\left(1-a^2x^2\right)}{2c\left(a^2c+d\right)}+\frac{\log\left(c+dx^2\right)}{2c\left(a^2c+d\right)}-\frac{i\operatorname{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{4ac^{3/2}\sqrt{d}}+\frac{i\operatorname{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{\sqrt{c}a+i\sqrt{d}}\right)}{4ac^{3/2}\sqrt{d}}-\frac{i\operatorname{PolyLo}}{}\right.$$

$$\frac{\coth^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{x\coth^{-1}(ax)}{2c\left(c+dx^2\right)}$$

input
```
Int[ArcCoth[a*x]/(c + d*x^2)^2,x]
```

output
```
(x*ArcCoth[a*x])/(2*c*(c + d*x^2)) + (ArcCoth[a*x]*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(2*c^(3/2)*Sqrt[d]) - (a*(((-1/4*I)*Log[(Sqrt[d]*(1 - a*x))/(I*a*Sqr
t[c] + Sqrt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(a*c^(3/2)*Sqrt[d]) + ((I
/4)*Log[-((Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d]
*x)/Sqrt[c]])/(a*c^(3/2)*Sqrt[d]) + ((I/4)*Log[-((Sqrt[d]*(1 - a*x))/(I*a*
Sqrt[c] - Sqrt[d]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(a*c^(3/2)*Sqrt[d]) -
 ((I/4)*Log[(Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])]*Log[1 + (I*Sqrt[d
]*x)/Sqrt[c]])/(a*c^(3/2)*Sqrt[d]) - Log[1 - a^2*x^2]/(2*c*(a^2*c + d)) +
Log[c + d*x^2]/(2*c*(a^2*c + d)) - ((I/4)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[
d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/(a*c^(3/2)*Sqrt[d]) + ((I/4)*PolyLog[2, (
a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(a*c^(3/2)*Sqrt[d]) -
 ((I/4)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/(
a*c^(3/2)*Sqrt[d]) + ((I/4)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt
[c] + I*Sqrt[d])])/(a*c^(3/2)*Sqrt[d])))/2
```

3.40. $\int \frac{\coth^{-1}(ax)}{\left(c+dx^2\right)^2}\,dx$

3.40.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6539
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCoth[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.40.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1925 vs. $2(430) = 860$.

Time = 1.21 (sec) , antiderivative size = 1926, normalized size of antiderivative = 3.26

method	result	size
risch	Expression too large to display	1926
derivativedivides	Expression too large to display	2071
default	Expression too large to display	2071

input
```
int(arccoth(a*x)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a/c/(a^2*c+d)*ln((a*x+1)^2*d+a^2*c-2*(a*x+1)*d+d)-1/4*a^2/(a^2*c+d)/(
c*d)^(1/2)*arctan(1/2*(2*(a*x+1)*d-2*d)/a/(c*d)^(1/2))+1/4*a^3*ln(a*x+1)/(
a^2*c+d)/(a^2*d*x^2+a^2*c)-1/8/c/(-c*d)^(1/2)*dilog((a*(-c*d)^(1/2)-(a*x-1
)*d-d)/(a*(-c*d)^(1/2)-d))+1/8/c/(-c*d)^(1/2)*dilog((a*(-c*d)^(1/2)+(a*x-1
)*d+d)/(a*(-c*d)^(1/2)+d))-1/8*a/c/(a^2*c+d)*ln((a*x-1)^2*d+a^2*c+2*(a*x-1
)*d+d)+1/4*a^2/(a^2*c+d)/(c*d)^(1/2)*arctan(1/2*(2*(a*x-1)*d+2*d)/a/(c*d)^
(1/2))+1/4*a^3*ln(a*x-1)/(a^2*c+d)/(a^2*d*x^2+a^2*c)-1/4*a^4*ln(a*x-1)/(a^
2*c+d)/(a^2*d*x^2+a^2*c)*x+1/8/c/(-c*d)^(1/2)*dilog((a*(-c*d)^(1/2)-(a*x+1
)*d+d)/(a*(-c*d)^(1/2)+d))-1/8/c/(-c*d)^(1/2)*dilog((a*(-c*d)^(1/2)+(a*x+1
)*d-d)/(a*(-c*d)^(1/2)-d))+1/8*a^4*ln(a*x+1)/(a^2*c+d)/(a^2*d*x^2+a^2*c)/(
-c*d)^(1/2)*ln((a*(-c*d)^(1/2)-(a*x+1)*d+d)/(a*(-c*d)^(1/2)+d))*d*x^2-1/8*
a^4*ln(a*x+1)/(a^2*c+d)/(a^2*d*x^2+a^2*c)/(-c*d)^(1/2)*ln((a*(-c*d)^(1/2)+
(a*x+1)*d-d)/(a*(-c*d)^(1/2)-d))*d*x^2-1/8*a^4*ln(a*x-1)/(a^2*c+d)/(a^2*d*
x^2+a^2*c)/(-c*d)^(1/2)*ln((a*(-c*d)^(1/2)-(a*x-1)*d-d)/(a*(-c*d)^(1/2)-d)
)*d*x^2+1/8*a^4*ln(a*x-1)/(a^2*c+d)/(a^2*d*x^2+a^2*c)/(-c*d)^(1/2)*ln((a*(
-c*d)^(1/2)+(a*x-1)*d+d)/(a*(-c*d)^(1/2)+d))*d*x^2+1/4*a^3*ln(a*x+1)/c/(a^
2*c+d)/(a^2*d*x^2+a^2*c)*d*x^2+1/8*a^4*ln(a*x+1)*c/(a^2*c+d)/(a^2*d*x^2+a^
2*c)/(-c*d)^(1/2)*ln((a*(-c*d)^(1/2)-(a*x+1)*d+d)/(a*(-c*d)^(1/2)+d))-1/8*
a^4*ln(a*x+1)*c/(a^2*c+d)/(a^2*d*x^2+a^2*c)/(-c*d)^(1/2)*ln((a*(-c*d)^(1/2
)+(a*x+1)*d-d)/(a*(-c*d)^(1/2)-d))+1/4*a^2*ln(a*x+1)/c/(a^2*c+d)/(a^2*d...
```

3.40.5 Fricas [F]

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^2}\,dx=\int \frac{\operatorname{arcoth}(ax)}{(dx^2+c)^2}\,dx$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arccoth(a*x)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

3.40.6 Sympy [F]

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^2}\,dx = \int \frac{\operatorname{acoth}(ax)}{(c+dx^2)^2}\,dx$$

input
```
integrate(acoth(a*x)/(d*x**2+c)**2,x)
```

output
```
Integral(acoth(a*x)/(c + d*x**2)**2, x)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 550, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^2}\,dx = \frac{1}{2}\left(\frac{x}{cdx^2+c^2} + \frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}c}\right)\operatorname{arcoth}(ax)$$
$$- \frac{\left(2\,acd\log\left(dx^2+c\right) - 2\,acd\log\left(ax+1\right) - 2\,acd\log\left(ax-1\right) + \left(\left(a^2c+d\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{a^2dx^2+2\,adx}{a^2c+d}\right.\right.\right.}{}$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(x/(c*d*x^2 + c^2) + arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c))*arccoth(a*x)
 - 1/8*(2*a*c*d*log(d*x^2 + c) - 2*a*c*d*log(a*x + 1) - 2*a*c*d*log(a*x -
1) + ((a^2*c + d)*arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 + 2*a*d*x + d)/
(a^2*c + d)) - (a^2*c + d)*arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 - 2*a*
d*x + d)/(a^2*c + d)) + (I*a^2*c + I*d)*dilog((a^2*c + a*d*x - (I*a^2*x -
I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)*sqrt(d) - d)) + (I*a^2*c + I*
d)*dilog((a^2*c - a*d*x + (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*
sqrt(c)*sqrt(d) - d)) + (-I*a^2*c - I*d)*dilog((a^2*c + a*d*x + (I*a^2*x -
 I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)) + (-I*a^2*c -
I*d)*dilog((a^2*c - a*d*x - (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*
a*sqrt(c)*sqrt(d) - d)) - ((a^2*c + d)*arctan2((a^2*x + a)*sqrt(c)*sqrt(d)
/(a^2*c + d), (a*d*x + d)/(a^2*c + d)) - (a^2*c + d)*arctan2((a^2*x - a)*s
qrt(c)*sqrt(d)/(a^2*c + d), -(a*d*x - d)/(a^2*c + d)))*log(d*x^2 + c))*sqr
t(c)*sqrt(d))*a/(a^3*c^3*d + a*c^2*d^2)
```

3.40.8 Giac [F]

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^2}\,dx = \int \frac{\operatorname{arcoth}(ax)}{(dx^2+c)^2}\,dx$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arccoth(a*x)/(d*x^2 + c)^2, x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^2}\,dx = \int \frac{\operatorname{acoth}(a\,x)}{(d\,x^2+c)^2}\,dx$$

input
```
int(acoth(a*x)/(c + d*x^2)^2,x)
```

output
```
int(acoth(a*x)/(c + d*x^2)^2, x)
```

3.41 $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^3}\,dx$

3.41.1 Optimal result

Integrand size = 14, antiderivative size = 657

$$\begin{aligned}\int \frac{\coth^{-1}(ax)}{(c+dx^2)^3}\,dx &= \frac{a}{8c(a^2c+d)(c+dx^2)}+\frac{x\coth^{-1}(ax)}{4c(c+dx^2)^2}\\
&+\frac{3x\coth^{-1}(ax)}{8c^2(c+dx^2)}+\frac{3\coth^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}\\
&+\frac{3i\log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\log\left(-\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{32c^{5/2}\sqrt{d}}\\
&-\frac{3i\log\left(-\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}-\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{32c^{5/2}\sqrt{d}}+\frac{3i\log\left(\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1+\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{32c^{5/2}\sqrt{d}}\\
&+\frac{a(5a^2c+3d)\log(1-a^2x^2)}{16c^2(a^2c+d)^2}-\frac{a(5a^2c+3d)\log(c+dx^2)}{16c^2(a^2c+d)^2}\\
&+\frac{3i\,\mathrm{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\,\mathrm{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}\\
&+\frac{3i\,\mathrm{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\,\mathrm{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}\end{aligned}$$

output
```
1/8*a/c/(a^2*c+d)/(d*x^2+c)+1/4*x*arccoth(a*x)/c/(d*x^2+c)^2+3/8*x*arccoth
(a*x)/c^2/(d*x^2+c)+1/16*a*(5*a^2*c+3*d)*ln(-a^2*x^2+1)/c^2/(a^2*c+d)^2-1/
16*a*(5*a^2*c+3*d)*ln(d*x^2+c)/c^2/(a^2*c+d)^2+3/8*arccoth(a*x)*arctan(x*d
^(1/2)/c^(1/2))/c^(5/2)/d^(1/2)-3/32*I*ln(-(a*x+1)*d^(1/2)/(I*a*c^(1/2)-d^
(1/2)))*ln(1-I*x*d^(1/2)/c^(1/2))/c^(5/2)/d^(1/2)+3/32*I*ln((-a*x+1)*d^(1/
2)/(I*a*c^(1/2)+d^(1/2)))*ln(1-I*x*d^(1/2)/c^(1/2))/c^(5/2)/d^(1/2)-3/32*I
*ln(-(-a*x+1)*d^(1/2)/(I*a*c^(1/2)-d^(1/2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(
5/2)/d^(1/2)+3/32*I*ln((a*x+1)*d^(1/2)/(I*a*c^(1/2)+d^(1/2)))*ln(1+I*x*d^(
1/2)/c^(1/2))/c^(5/2)/d^(1/2)+3/32*I*polylog(2,a*(c^(1/2)-I*x*d^(1/2))/(a*
c^(1/2)-I*d^(1/2)))/c^(5/2)/d^(1/2)-3/32*I*polylog(2,a*(c^(1/2)-I*x*d^(1/2
))/(a*c^(1/2)+I*d^(1/2)))/c^(5/2)/d^(1/2)+3/32*I*polylog(2,a*(c^(1/2)+I*x*
d^(1/2))/(a*c^(1/2)-I*d^(1/2)))/c^(5/2)/d^(1/2)-3/32*I*polylog(2,a*(c^(1/2
)+I*x*d^(1/2))/(a*c^(1/2)+I*d^(1/2)))/c^(5/2)/d^(1/2)
```

3.41.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1559 vs. 2(657) = 1314.

Time = 9.81 (sec) , antiderivative size = 1559, normalized size of antiderivative = 2.37

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[ArcCoth[a*x]/(c + d*x^2)^3,x]
```

output
```
-1/32*(a*(10*a^2*c*Log[1 - ((a^2*c + d)*Cosh[2*ArcCoth[a*x]])/(a^2*c - d)]
 + 6*d*Log[1 - ((a^2*c + d)*Cosh[2*ArcCoth[a*x]])/(a^2*c - d)] - (3*d*(a^2
*c + d)*((-2*I)*ArcCos[(a^2*c - d)/(a^2*c + d)]*ArcTan[(a*c)/(Sqrt[a^2*c*d
]*x)] + 4*ArcCoth[a*x]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]] - (ArcCos[(a^2*c - d)
/(a^2*c + d)] + 2*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)])*Log[(2*d*(a^2*c - I*Sqr
t[a^2*c*d])*(-1 + a*x))/((a^2*c + d)*(I*Sqrt[a^2*c*d] + a*d*x))] - (ArcCos
[(a^2*c - d)/(a^2*c + d)] - 2*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)])*Log[(2*d*(a
^2*c + I*Sqrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(I*Sqrt[a^2*c*d] + a*d*x))
] + (ArcCos[(a^2*c - d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]
+ ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c
+ d]*E^ArcCoth[a*x]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]]])
] + (ArcCos[(a^2*c - d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]
+ ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcCoth[a*x
])/(Sqrt[a^2*c + d]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]]])
] + I*(-PolyLog[2, ((a^2*c - d - (2*I)*Sqrt[a^2*c*d])*(Sqrt[a^2*c*d] + I*a
*d*x))/((a^2*c + d)*(Sqrt[a^2*c*d] - I*a*d*x))] + PolyLog[2, ((a^2*c - d +
 (2*I)*Sqrt[a^2*c*d])*(Sqrt[a^2*c*d] + I*a*d*x))/((a^2*c + d)*(Sqrt[a^2*c*
d] - I*a*d*x))])))/Sqrt[a^2*c*d] - (3*Sqrt[a^2*c*d]*(a^2*c + d)*((-2*I)*Ar
cCos[(a^2*c - d)/(a^2*c + d)]*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + 4*ArcCoth[
a*x]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]] - (ArcCos[(a^2*c - d)/(a^2*c + d)] +...
```

3.41.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 682, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6539, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^3}\,dx$$

$$\Big\downarrow \ 6539$$

$$-a\int \frac{\frac{3dx^3+5cx}{c^2(dx^2+c)^2}+\frac{3\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}}{8(1-a^2x^2)}dx+\frac{3\coth^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{3x\coth^{-1}(ax)}{8c^2(c+dx^2)}+\frac{x\coth^{-1}(ax)}{4c(c+dx^2)^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{8}a\int \frac{\frac{3dx^3+5cx}{c^2(dx^2+c)^2}+\frac{3\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}}{1-a^2x^2}dx+\frac{3\coth^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{3x\coth^{-1}(ax)}{8c^2(c+dx^2)}+\frac{x\coth^{-1}(ax)}{4c(c+dx^2)^2}$$

$$\Big\downarrow \text{7276}$$

$$-\frac{1}{8}a\int \left(-\frac{x(3dx^2+5c)}{c^2(a^2x^2-1)(dx^2+c)^2}-\frac{3\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}(a^2x^2-1)}\right)dx+\frac{3\coth^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{3x\coth^{-1}(ax)}{8c^2(c+dx^2)}+\frac{x\coth^{-1}(ax)}{4c(c+dx^2)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{8}a\left(-\frac{(5a^2c+3d)\log(1-a^2x^2)}{2c^2(a^2c+d)^2}+\frac{(5a^2c+3d)\log(c+dx^2)}{2c^2(a^2c+d)^2}-\frac{1}{c(a^2c+d)(c+dx^2)}-\frac{3i\,\text{PolyLog}\left(2,\frac{a(\sqrt{c}-i}{a\sqrt{c}-i}\right.}{4ac^{5/2}\sqrt{d}}\right.$$

$$\frac{3\coth^{-1}(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{3x\coth^{-1}(ax)}{8c^2(c+dx^2)}+\frac{x\coth^{-1}(ax)}{4c(c+dx^2)^2}$$

input
```
Int[ArcCoth[a*x]/(c + d*x^2)^3,x]
```

output
```
(x*ArcCoth[a*x])/(4*c*(c + d*x^2)^2) + (3*x*ArcCoth[a*x])/(8*c^2*(c + d*x^
2)) + (3*ArcCoth[a*x]*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*Sqrt[d]) - (
a*(-(1/(c*(a^2*c + d)*(c + d*x^2))) - (((3*I)/4)*Log[(Sqrt[d]*(1 - a*x))/(
I*a*Sqrt[c] + Sqrt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(a*c^(5/2)*Sqrt[d]
) + (((3*I)/4)*Log[-((Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 -
 (I*Sqrt[d]*x)/Sqrt[c]])/(a*c^(5/2)*Sqrt[d]) + (((3*I)/4)*Log[-((Sqrt[d]*(
1 - a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(a*c^(
5/2)*Sqrt[d]) - (((3*I)/4)*Log[(Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])
]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(a*c^(5/2)*Sqrt[d]) - ((5*a^2*c + 3*d)*L
og[1 - a^2*x^2])/(2*c^2*(a^2*c + d)^2) + ((5*a^2*c + 3*d)*Log[c + d*x^2])/
(2*c^2*(a^2*c + d)^2) - (((3*I)/4)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/
(a*Sqrt[c] - I*Sqrt[d])])/(a*c^(5/2)*Sqrt[d]) + (((3*I)/4)*PolyLog[2, (a*(
Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(a*c^(5/2)*Sqrt[d]) - ((
(3*I)/4)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/
(a*c^(5/2)*Sqrt[d]) + (((3*I)/4)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a
*Sqrt[c] + I*Sqrt[d])])/(a*c^(5/2)*Sqrt[d])))/8
```

3.41. $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^3}\,dx$

3.41.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6539
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCoth[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.41.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3790 vs. 2(493) = 986.

Time = 1.48 (sec) , antiderivative size = 3791, normalized size of antiderivative = 5.77

method	result	size
derivativedivides	Expression too large to display	3791
default	Expression too large to display	3791
risch	Expression too large to display	4508

input
```
int(arccoth(a*x)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(3/8*(a^2*c+2*(-a^2*c*d)^(1/2)-d)*a^2*d^2*arccoth(a*x)^2/c^2/(a^4*c^2+
2*a^2*c*d+d^2)^2-3/8/c^2/(a^4*c^2+2*a^2*c*d+d^2)*a^2*d^2/(a^2*c+d)*ln((a*x
-1)/(a*x+1))-1/2/c/(a^4*c^2+2*a^2*c*d+d^2)*a^4*d/(a^2*c+d)*ln(a^2*c/(a*x-1
)^2*(a*x+1)^2-2*a^2*c/(a*x-1)*(a*x+1)+d/(a*x-1)^2*(a*x+1)^2+a^2*c+2/(a*x-1
)*(a*x+1)*d+d)+1/8*a^2*(5*arccoth(a*x)*a^6*c^3+3*arccoth(a*x)*a^6*c^2*d*x^
2-7*arccoth(a*x)*a^5*c^2*d*x-5*arccoth(a*x)*a^5*c*d^2*x^3+3*arccoth(a*x)*a
^4*c^2*d+arccoth(a*x)*a^4*c*d^2*x^2-a^5*c^2*d*x-a^5*c*d^2*x^3-5*arccoth(a*
x)*a^3*c*d^2*x-3*arccoth(a*x)*d^3*a^3*x^3-a^4*c^2*d-c*a^4*d^2*x^2)*(a*x-1)
/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c^2+3/16*(-a^2*c*d)^(1/2)/c^3
*d/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x)*ln(1-(a^2*c+d)/(a*x-1)*(a*x+1)/(a^
2*c+2*(-a^2*c*d)^(1/2)-d))+5/16*(c*d)^(1/2)/d*a^7*arctan(1/4*(2*(a^2*c+d)/
(a*x-1)*(a*x+1)-2*a^2*c+2*d)/a/(c*d)^(1/2))/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*c
+d)-3/32*(-(-a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(-a^2*c*d)^(1/2)*d)/c^2*a^2*d/
(a^4*c^2+2*a^2*c*d+d^2)^2*polylog(2,(a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a
^2*c*d)^(1/2)-d))+3/8*(-a^2*c*d)^(1/2)/c^2*a^2/(a^4*c^2+2*a^2*c*d+d^2)*arc
coth(a*x)*ln(1-(a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c+2*(-a^2*c*d)^(1/2)-d))+3/1
6*(c*d)^(1/2)/c*a^5*arctan(1/4*(2*(a^2*c+d)/(a*x-1)*(a*x+1)-2*a^2*c+2*d)/a
/(c*d)^(1/2))/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*c+d)+3/16*(-(-a^2*c*d)^(1/2)*a^
2*c+2*a^2*c*d+(-a^2*c*d)^(1/2)*d)*a^6/d/(a^4*c^2+2*a^2*c*d+d^2)^2*ln(1-(a^
2*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*arccoth(a*x)+3/16*...
```

3.41.5 Fricas [F]

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^3}\,dx = \int \frac{\operatorname{arcoth}(ax)}{(dx^2+c)^3}\,dx$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arccoth(a*x)/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

3.41.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(acoth(a*x)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.41.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1087 vs. $2(463) = 926$.

Time = 0.42 (sec) , antiderivative size = 1087, normalized size of antiderivative = 1.65

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/8*((3*d*x^3 + 5*c*x)/(c^2*d^2*x^4 + 2*c^3*d*x^2 + c^4) + 3*arctan(d*x/sq
rt(c*d))/(sqrt(c*d)*c^2))*arccoth(a*x) + 1/32*(4*a^3*c^3*d + 4*a*c^2*d^2 -
 3*((a^4*c^3 + 2*a^2*c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*
arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 + 2*a*d*x + d)/(a^2*c + d)) - (a^
4*c^3 + 2*a^2*c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arctan(
sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 - 2*a*d*x + d)/(a^2*c + d)) - (-I*a^4*c^
3 - 2*I*a^2*c^2*d - I*c*d^2 + (-I*a^4*c^2*d - 2*I*a^2*c*d^2 - I*d^3)*x^2)*
dilog((a^2*c + a*d*x - (I*a^2*x - I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqr
t(c)*sqrt(d) - d)) - (-I*a^4*c^3 - 2*I*a^2*c^2*d - I*c*d^2 + (-I*a^4*c^2*d
 - 2*I*a^2*c*d^2 - I*d^3)*x^2)*dilog((a^2*c - a*d*x + (I*a^2*x + I*a)*sqrt
(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)*sqrt(d) - d)) - (I*a^4*c^3 + 2*I*a^2*c
^2*d + I*c*d^2 + (I*a^4*c^2*d + 2*I*a^2*c*d^2 + I*d^3)*x^2)*dilog((a^2*c +
 a*d*x + (I*a^2*x - I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) -
 d)) - (I*a^4*c^3 + 2*I*a^2*c^2*d + I*c*d^2 + (I*a^4*c^2*d + 2*I*a^2*c*d^2
 + I*d^3)*x^2)*dilog((a^2*c - a*d*x - (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^
2*c - 2*I*a*sqrt(c)*sqrt(d) - d)) - ((a^4*c^3 + 2*a^2*c^2*d + c*d^2 + (a^4
*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arctan2((a^2*x + a)*sqrt(c)*sqrt(d)/(a^2*
c + d), (a*d*x + d)/(a^2*c + d)) - (a^4*c^3 + 2*a^2*c^2*d + c*d^2 + (a^4*c
^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arctan2((a^2*x - a)*sqrt(c)*sqrt(d)/(a^2*c
+ d), -(a*d*x - d)/(a^2*c + d)))*log(d*x^2 + c))*sqrt(c)*sqrt(d) - 2*(5...
```

3.41.8 Giac [F]

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^3}\,dx = \int \frac{\operatorname{arcoth}(ax)}{(dx^2+c)^3}\,dx$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arccoth(a*x)/(d*x^2 + c)^3, x)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^3}\,dx = \int \frac{\mathrm{acoth}(a\,x)}{(d\,x^2+c)^3}\,dx$$

input
```
int(acoth(a*x)/(c + d*x^2)^3,x)
```

output
```
int(acoth(a*x)/(c + d*x^2)^3, x)
```

3.42 $\int \sqrt{c+dx^2} \coth^{-1}(ax) \, dx$

3.42.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \sqrt{c+dx^2} \coth^{-1}(ax) \, dx = \text{Int}\left(\sqrt{c+dx^2} \coth^{-1}(ax), x\right)$$

output
```
Unintegrable((d*x^2+c)^(1/2)*arccoth(a*x),x)
```

3.42.2 Mathematica [N/A]

Not integrable

Time = 4.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \sqrt{c+dx^2} \coth^{-1}(ax) \, dx = \int \sqrt{c+dx^2} \coth^{-1}(ax) \, dx$$

input
```
Integrate[Sqrt[c + d*x^2]*ArcCoth[a*x],x]
```

output
```
Integrate[Sqrt[c + d*x^2]*ArcCoth[a*x], x]
```

3.42.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6652}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(ax)\sqrt{c+dx^2}\,dx$$

$$\Big\downarrow \; 6652$$

$$\int \coth^{-1}(ax)\sqrt{c+dx^2}dx$$

input
```
Int[Sqrt[c + d*x^2]*ArcCoth[a*x],x]
```

output
```
$Aborted
```

3.42.3.1 Defintions of rubi rules used

rule 6652
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCoth[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] ||
 MatchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f
_.)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || Match
Q[u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*
x)^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.42.4 Maple [N/A] (verified)

Not integrable

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \sqrt{d\,x^2+c}\;\operatorname{arccoth}\left(ax\right)dx$$

input
```
int((d*x^2+c)^(1/2)*arccoth(a*x),x)
```

output
```
int((d*x^2+c)^(1/2)*arccoth(a*x),x)
```

3.42.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx^2}\coth^{-1}(ax)\,dx = \int \sqrt{dx^2+c}\,\mathrm{arcoth}\,(ax)\,\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*arccoth(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^2 + c)*arccoth(a*x), x)
```

3.42.6 Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{c+dx^2}\coth^{-1}(ax)\,dx = \int \sqrt{c+dx^2}\,\mathrm{acoth}\,(ax)\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*acoth(a*x),x)
```

output
```
Integral(sqrt(c + d*x**2)*acoth(a*x), x)
```

3.42.7 Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx^2}\coth^{-1}(ax)\,dx = \int \sqrt{dx^2+c}\,\mathrm{arcoth}\,(ax)\,\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*arccoth(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)*arccoth(a*x), x)
```

3.42.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx^2}\coth^{-1}(ax)\,dx = \int \sqrt{dx^2+c}\,\mathrm{arcoth}\left(ax\right)\,dx$$

input
```
integrate((d*x^2+c)^(1/2)*arccoth(a*x),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^2 + c)*arccoth(a*x), x)
```

3.42.9 Mupad [N/A]

Not integrable

Time = 4.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx^2}\coth^{-1}(ax)\,dx = \int \mathrm{acoth}(a\,x)\,\sqrt{d\,x^2+c}\,dx$$

input
```
int(acoth(a*x)*(c + d*x^2)^(1/2),x)
```

output
```
int(acoth(a*x)*(c + d*x^2)^(1/2), x)
```

3.43 $\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$

3.43.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \text{Int}\left(\frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}},x\right)$$

output
```
Unintegrable(arccoth(a*x)/(d*x^2+c)^(1/2),x)
```

3.43.2 Mathematica [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[ArcCoth[a*x]/Sqrt[c + d*x^2],x]
```

output
```
Integrate[ArcCoth[a*x]/Sqrt[c + d*x^2], x]
```

3.43.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6652}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \; 6652$$
$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}dx$$

input
```
Int[ArcCoth[a*x]/Sqrt[c + d*x^2],x]
```

output
```
$Aborted
```

3.43.3.1 Defintions of rubi rules used

rule 6652
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCoth[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] ||
 MatchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f
_.)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || Match
Q[u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*
x)^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.43.4 Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{arccoth}\left(a x\right)}{\sqrt{d\,x^{2}+c}}dx$$

input
```
int(arccoth(a*x)/(d*x^2+c)^(1/2),x)
```

output
```
int(arccoth(a*x)/(d*x^2+c)^(1/2),x)
```

3.43.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{arcoth}(ax)}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(arccoth(a*x)/sqrt(d*x^2 + c), x)
```

3.43.6 Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{acoth}(ax)}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(acoth(a*x)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(acoth(a*x)/sqrt(c + d*x**2), x)
```

3.43.7 Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{arcoth}(ax)}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccoth(a*x)/sqrt(d*x^2 + c), x)
```

3.43.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\operatorname{arcoth}(ax)}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccoth(a*x)/sqrt(d*x^2 + c), x)
```

3.43.9 Mupad [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\mathrm{acoth}(a\,x)}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(acoth(a*x)/(c + d*x^2)^(1/2),x)
```

output
```
int(acoth(a*x)/(c + d*x^2)^(1/2), x)
```

3.44 $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx$

3.44.1 Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx=\frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

output
```
-arctanh(a*(d*x^2+c)^(1/2)/(a^2*c+d)^(1/2))/c/(a^2*c+d)^(1/2)+x*arccoth(a*
x)/c/(d*x^2+c)^(1/2)
```

3.44.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.92

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx=\frac{\frac{2x\coth^{-1}(ax)}{\sqrt{c+dx^2}}+\frac{\log(1-ax)+\log(1+ax)-\log\left(ac-dx+\sqrt{a^2c+d}\sqrt{c+dx^2}\right)-\log\left(ac+dx+\sqrt{a^2c+d}\sqrt{c+dx^2}\right)}{\sqrt{a^2c+d}}}{2c}$$

input
```
Integrate[ArcCoth[a*x]/(c + d*x^2)^(3/2),x]
```

output
```
((2*x*ArcCoth[a*x])/Sqrt[c + d*x^2] + (Log[1 - a*x] + Log[1 + a*x] - Log[a
*c - d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]] - Log[a*c + d*x + Sqrt[a^2*c +
 d]*Sqrt[c + d*x^2]])/Sqrt[a^2*c + d])/(2*c)
```

3.44.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6539, 27, 353, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6539}$$

$$\frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}}-a\int \frac{x}{c\left(1-a^2x^2\right)\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{a\int \frac{x}{(1-a^2x^2)\sqrt{dx^2+c}}dx}{c}$$

$$\Big\downarrow \text{353}$$

$$\frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{a\int \frac{1}{(1-a^2x^2)\sqrt{dx^2+c}}dx^2}{2c}$$

$$\Big\downarrow \text{73}$$

$$\frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{a\int \frac{1}{-\frac{a^2x^4}{d}+\frac{a^2c}{d}+1}d\sqrt{dx^2+c}}{cd}$$

$$\Big\downarrow \text{221}$$

$$\frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

input
```
Int[ArcCoth[a*x]/(c + d*x^2)^(3/2),x]
```

output
```
(x*ArcCoth[a*x])/(c*Sqrt[c + d*x^2]) - ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^
2*c + d]]/(c*Sqrt[a^2*c + d])
```

3.44.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 6539
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCoth[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.44.4 Maple [F]

$$\int \frac{\operatorname{arccoth}(ax)}{(d\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int(arccoth(a*x)/(d*x^2+c)^(3/2),x)
```

output
```
int(arccoth(a*x)/(d*x^2+c)^(3/2),x)
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(54) = 108$.

Time = 0.28 (sec) , antiderivative size = 354, normalized size of antiderivative = 5.71

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \left[\frac{2\,(a^2c+d)\sqrt{dx^2+c}x\log\left(\frac{ax+1}{ax-1}\right)+\sqrt{a^2c+d}(dx^2+c)\log\left(\frac{a^4d^2x^4+8\,a^4c^2+8\,a^2cd+2\,(4\,a^4cd+}{\quad}\right.}{4\,(a^2c^3+c^2d+(a^2c^2d+cd^2)x^2)}\right.$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*(a^2*c + d)*sqrt(d*x^2 + c)*x*log((a*x + 1)/(a*x - 1)) + sqrt(a^2*
c + d)*(d*x^2 + c)*log((a^4*d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d
 + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x
^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2 + 1)))/(a^2*c^3 + c^2*d + (a^2*c^2*d +
 c*d^2)*x^2), 1/2*((a^2*c + d)*sqrt(d*x^2 + c)*x*log((a*x + 1)/(a*x - 1))
+ sqrt(-a^2*c - d)*(d*x^2 + c)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-
a^2*c - d)*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)))/(a^
2*c^3 + c^2*d + (a^2*c^2*d + c*d^2)*x^2)]
```

3.44.6 Sympy [F]

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\operatorname{acoth}(ax)}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(acoth(a*x)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(acoth(a*x)/(c + d*x**2)**(3/2), x)
```

3.44.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(54) = 108$.

Time = 0.22 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.47

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \frac{a^2\left(\frac{\operatorname{arsinh}\left(-\frac{2\,a^2c}{\sqrt{cd}\left|2\,a^2x+2\,a\right|}+\frac{2\,adx}{\sqrt{cd}\left|2\,a^2x+2\,a\right|}\right)}{a^3\sqrt{c+\frac{d}{a^2}}}-\frac{\operatorname{arsinh}\left(\frac{2\,a^2c}{\sqrt{cd}\left|2\,a^2x-2\,a\right|}+\frac{2\,adx}{\sqrt{cd}\left|2\,a^2x-2\,a\right|}\right)}{a^3\sqrt{c+\frac{d}{a^2}}}\right)}{2\,c} + \frac{x\,\operatorname{arcoth}\left(ax\right)}{\sqrt{dx^2+cc}}$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
1/2*a^2*(arcsinh(-2*a^2*c/(sqrt(c*d)*abs(2*a^2*x + 2*a)) + 2*a*d*x/(sqrt(c
*d)*abs(2*a^2*x + 2*a)))/(a^3*sqrt(c + d/a^2)) - arcsinh(2*a^2*c/(sqrt(c*d
)*abs(2*a^2*x - 2*a)) + 2*a*d*x/(sqrt(c*d)*abs(2*a^2*x - 2*a)))/(a^3*sqrt(
c + d/a^2)))/c + x*arccoth(a*x)/(sqrt(d*x^2 + c)*c)
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.27

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \frac{x\log\left(-\frac{\frac{1}{ax}+1}{\frac{1}{ax}-1}\right)}{2\sqrt{dx^2+cc}} + \frac{\arctan\left(\frac{\sqrt{dx^2+ca}}{\sqrt{-a^2c-d}}\right)}{\sqrt{-a^2c-dc}}$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
1/2*x*log(-(1/(a*x) + 1)/(1/(a*x) - 1))/(sqrt(d*x^2 + c)*c) + arctan(sqrt(
d*x^2 + c)*a/sqrt(-a^2*c - d))/(sqrt(-a^2*c - d)*c)
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\operatorname{acoth}(a\,x)}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(acoth(a*x)/(c + d*x^2)^(3/2),x)
```

output
```
int(acoth(a*x)/(c + d*x^2)^(3/2), x)
```

3.45 $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx$

3.45.1 Optimal result

Integrand size = 16, antiderivative size = 128

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \frac{a}{3c(a^2c+d)\sqrt{c+dx^2}} + \frac{x\coth^{-1}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(3a^2c+2d)\operatorname{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2(a^2c+d)^{3/2}}$$

output
```
1/3*x*arccoth(a*x)/c/(d*x^2+c)^(3/2)-1/3*(3*a^2*c+2*d)*arctanh(a*(d*x^2+c)
^(1/2)/(a^2*c+d)^(1/2))/c^2/(a^2*c+d)^(3/2)+1/3*a/c/(a^2*c+d)/(d*x^2+c)^(1
/2)+2/3*x*arccoth(a*x)/c^2/(d*x^2+c)^(1/2)
```

3.45.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.77

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \frac{\frac{2ac}{(a^2c+d)\sqrt{c+dx^2}} + \frac{2x(3c+2dx^2)\coth^{-1}(ax)}{(c+dx^2)^{3/2}} + \frac{(3a^2c+2d)\log(1-ax)}{(a^2c+d)^{3/2}} + \frac{(3a^2c+2d)\log(1+ax)}{(a^2c+d)^{3/2}} - \frac{(3a^2c+2d)\log\cdots}{\cdots}}{6c^2}$$

input
```
Integrate[ArcCoth[a*x]/(c + d*x^2)^(5/2),x]
```

output
```
((2*a*c)/((a^2*c + d)*Sqrt[c + d*x^2]) + (2*x*(3*c + 2*d*x^2)*ArcCoth[a*x]
)/(c + d*x^2)^(3/2) + ((3*a^2*c + 2*d)*Log[1 - a*x])/(a^2*c + d)^(3/2) + (
(3*a^2*c + 2*d)*Log[1 + a*x])/(a^2*c + d)^(3/2) - ((3*a^2*c + 2*d)*Log[a*c
 - d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(a^2*c + d)^(3/2) - ((3*a^2*c +
 2*d)*Log[a*c + d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(a^2*c + d)^(3/2))
/(6*c^2)
```

3.45.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6539, 27, 435, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6539}$$

$$-a\int \frac{x(2dx^2+3c)}{3c^2(1-a^2x^2)(dx^2+c)^{3/2}}dx+\frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\coth^{-1}(ax)}{3c(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{x(2dx^2+3c)}{(1-a^2x^2)(dx^2+c)^{3/2}}dx}{3c^2}+\frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\coth^{-1}(ax)}{3c(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{435}$$

$$-\frac{a\int \frac{2dx^2+3c}{(1-a^2x^2)(dx^2+c)^{3/2}}dx^2}{6c^2}+\frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\coth^{-1}(ax)}{3c(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{a\left(\frac{(3a^2c+2d)\int \frac{1}{(1-a^2x^2)\sqrt{dx^2+c}}dx^2}{a^2c+d}-\frac{2c}{(a^2c+d)\sqrt{c+dx^2}}\right)}{6c^2}+\frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\coth^{-1}(ax)}{3c(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a\left(\frac{2(3a^2c+2d)\int \frac{1}{-\frac{a^2x^4}{d}+\frac{a^2c}{d}+1}d\sqrt{dx^2+c}}{d(a^2c+d)}-\frac{2c}{(a^2c+d)\sqrt{c+dx^2}}\right)}{6c^2}+\frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\coth^{-1}(ax)}{3c(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{a\left(\frac{2(3a^2c+2d)\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{a(a^2c+d)^{3/2}}-\frac{2c}{(a^2c+d)\sqrt{c+dx^2}}\right)}{6c^2}+\frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\coth^{-1}(ax)}{3c(c+dx^2)^{3/2}}$$

input
```
Int[ArcCoth[a*x]/(c + d*x^2)^(5/2),x]
```

output
```
(x*ArcCoth[a*x])/(3*c*(c + d*x^2)^(3/2)) + (2*x*ArcCoth[a*x])/(3*c^2*Sqrt[
c + d*x^2]) - (a*((-2*c)/((a^2*c + d)*Sqrt[c + d*x^2]) + (2*(3*a^2*c + 2*d
)*ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2*c + d]])/(a*(a^2*c + d)^(3/2))))/(6
*c^2)
```

3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 6539
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCoth[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.45.4 Maple [F]

$$\int \frac{\operatorname{arccoth}(ax)}{(d\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(arccoth(a*x)/(d*x^2+c)^(5/2),x)
```

output
```
int(arccoth(a*x)/(d*x^2+c)^(5/2),x)
```

3.45.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(108) = 216.

Time = 0.30 (sec) , antiderivative size = 728, normalized size of antiderivative = 5.69

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \left[\frac{(3\,a^2c^3+(3\,a^2cd^2+2\,d^3)x^4+2\,c^2d+2\,(3\,a^2c^2d+2\,cd^2)x^2)\sqrt{a^2c+d}\log\left(\frac{a^4d^2x^4+8\,a^4c^2\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*((3*a^2*c^3 + (3*a^2*c*d^2 + 2*d^3)*x^4 + 2*c^2*d + 2*(3*a^2*c^2*d +
 2*c*d^2)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d +
2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c + a*d)*sqrt(a^2*c +
 d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2 + 1)) + 2*(2*a^3*c^3 + 2*a
*c^2*d + 2*(a^3*c^2*d + a*c*d^2)*x^2 + (2*(a^4*c^2*d + 2*a^2*c*d^2 + d^3)*
x^3 + 3*(a^4*c^3 + 2*a^2*c^2*d + c*d^2)*x)*log((a*x + 1)/(a*x - 1)))*sqrt(
d*x^2 + c))/(a^4*c^6 + 2*a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 + 2*a^2*c^3*d^
3 + c^2*d^4)*x^4 + 2*(a^4*c^5*d + 2*a^2*c^4*d^2 + c^3*d^3)*x^2), 1/6*((3*a
^2*c^3 + (3*a^2*c*d^2 + 2*d^3)*x^4 + 2*c^2*d + 2*(3*a^2*c^2*d + 2*c*d^2)*x
^2)*sqrt(-a^2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c - d)
*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)) + (2*a^3*c^3 +
 2*a*c^2*d + 2*(a^3*c^2*d + a*c*d^2)*x^2 + (2*(a^4*c^2*d + 2*a^2*c*d^2 + d
^3)*x^3 + 3*(a^4*c^3 + 2*a^2*c^2*d + c*d^2)*x)*log((a*x + 1)/(a*x - 1)))*s
qrt(d*x^2 + c))/(a^4*c^6 + 2*a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 + 2*a^2*c^
3*d^3 + c^2*d^4)*x^4 + 2*(a^4*c^5*d + 2*a^2*c^4*d^2 + c^3*d^3)*x^2)]
```

3.45.6 Sympy [F]

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \int \frac{\operatorname{acoth}(ax)}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(acoth(a*x)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(acoth(a*x)/(c + d*x**2)**(5/2), x)
```

3.45.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(108) = 216$.

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.74

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \frac{1}{6}\,a\left(\frac{\frac{ad\log\left(\frac{\sqrt{dx^2+c}a^2-\sqrt{a^2c+d}a}{\sqrt{dx^2+c}a^2+\sqrt{a^2c+d}a}\right)}{(a^2c^2+cd)\sqrt{a^2c+d}}+\frac{2\,d}{(a^2c^2+cd)\sqrt{dx^2+c}}}{d}+\frac{2\,\log\left(\frac{\sqrt{dx^2+c}a^2-\sqrt{a^2c+d}a}{\sqrt{dx^2+c}a^2+\sqrt{a^2c+d}a}\right)}{\sqrt{a^2c+d}ac^2}\right) + \frac{1}{3}\left(\frac{2\,x}{\sqrt{dx^2+c}c^2}+\frac{x}{(dx^2+c)^{\frac{3}{2}}c}\right)\operatorname{arcoth}(ax)$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/6*a*((a*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)
*a^2 + sqrt(a^2*c + d)*a))/((a^2*c^2 + c*d)*sqrt(a^2*c + d)) + 2*d/((a^2*c
^2 + c*d)*sqrt(d*x^2 + c)))/d + 2*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c +
d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/(sqrt(a^2*c + d)*a*c^2))
+ 1/3*(2*x/(sqrt(d*x^2 + c)*c^2) + x/((d*x^2 + c)^(3/2)*c))*arccoth(a*x)
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \frac{1}{3}a\left(\frac{(3a^2c+2d)\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c-d}}\right)}{(a^2c^3+c^2d)\sqrt{-a^2c-da}}+\frac{1}{(a^2c^2+cd)\sqrt{dx^2+c}}\right) + \frac{x\left(\frac{2\,dx^2}{c^2}+\frac{3}{c}\right)\log\left(-\frac{\frac{1}{ax}+1}{\frac{1}{ax}-1}\right)}{6\,(dx^2+c)^{\frac{3}{2}}}$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/3*a*((3*a^2*c + 2*d)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a^2*c - d))/((a^2*c^
3 + c^2*d)*sqrt(-a^2*c - d)*a) + 1/((a^2*c^2 + c*d)*sqrt(d*x^2 + c))) + 1/
6*x*(2*d*x^2/c^2 + 3/c)*log(-(1/(a*x) + 1)/(1/(a*x) - 1))/(d*x^2 + c)^(3/2
)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \int \frac{\mathrm{acoth}(a\,x)}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(acoth(a*x)/(c + d*x^2)^(5/2),x)
```

output
```
int(acoth(a*x)/(c + d*x^2)^(5/2), x)
```

3.46 $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx$

3.46.1 Optimal result

Integrand size = 16, antiderivative size = 200

$$\begin{aligned}\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx &= \frac{a}{15c(a^2c+d)(c+dx^2)^{3/2}} \\ &+ \frac{a(7a^2c+4d)}{15c^2(a^2c+d)^2\sqrt{c+dx^2}} + \frac{x\coth^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} \\ &+ \frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{(15a^4c^2+20a^2cd+8d^2)\,\mathrm{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{15c^3(a^2c+d)^{5/2}}\end{aligned}$$

output
```
1/15*a/c/(a^2*c+d)/(d*x^2+c)^(3/2)+1/5*x*arccoth(a*x)/c/(d*x^2+c)^(5/2)+4/
15*x*arccoth(a*x)/c^2/(d*x^2+c)^(3/2)-1/15*(15*a^4*c^2+20*a^2*c*d+8*d^2)*a
rctanh(a*(d*x^2+c)^(1/2)/(a^2*c+d)^(1/2))/c^3/(a^2*c+d)^(5/2)+1/15*a*(7*a^
2*c+4*d)/c^2/(a^2*c+d)^2/(d*x^2+c)^(1/2)+8/15*x*arccoth(a*x)/c^3/(d*x^2+c)
^(1/2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.64

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{2ac\sqrt{a^2c+d}(c+dx^2)\left(d(5c+4dx^2)+a^2c(8c+7dx^2)\right)+2(a^2c+d)^{5/2}x(15c^2+20cdx$$

input
```
Integrate[ArcCoth[a*x]/(c + d*x^2)^(7/2),x]
```

output
```
(2*a*c*Sqrt[a^2*c + d]*(c + d*x^2)*(d*(5*c + 4*d*x^2) + a^2*c*(8*c + 7*d*x
^2)) + 2*(a^2*c + d)^(5/2)*x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4)*ArcCoth[a*x
] + (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[1 - a*x] + (15
*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[1 + a*x] - (15*a^4*c^
2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[a*c - d*x + Sqrt[a^2*c + d]*
Sqrt[c + d*x^2]] - (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log
[a*c + d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(30*c^3*(a^2*c + d)^(5/2)*(
c + d*x^2)^(5/2))
```

3.46.3 Rubi [A] (warning: unable to verify)

Time = 1.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6539, 27, 7266, 1192, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{6539}$$

$$-a\int \frac{x(8d^2x^4+20cdx^2+15c^2)}{15c^3(1-a^2x^2)(dx^2+c)^{5/2}}dx+\frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}}+\frac{x\coth^{-1}(ax)}{5c(c+dx^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{x(8d^2x^4+20cdx^2+15c^2)}{(1-a^2x^2)(dx^2+c)^{5/2}}dx}{15c^3}+\frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}}+\frac{x\coth^{-1}(ax)}{5c(c+dx^2)^{5/2}}$$

$$\Big\downarrow \text{7266}$$

$$-\frac{a\int \frac{8d^2x^4+20cdx^2+15c^2}{(1-a^2x^2)(dx^2+c)^{5/2}}dx^2}{30c^3}+\frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\coth^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\coth^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

$\big\downarrow$ 1192

$$-\frac{a\int \frac{8d^2x^8+4cd^2x^4+3c^2d^2}{x^8(-a^2x^4+a^2c+d)}d\sqrt{dx^2+c}}{15c^3d^2}+\frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\coth^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\coth^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

$\big\downarrow$ 1584

$$-\frac{a\int \left(\frac{(15c^2a^4+20cda^2+8d^2)d^2}{(ca^2+d)^2(-a^2x^4+a^2c+d)}+\frac{c(7ca^2+4d)d^2}{(ca^2+d)^2x^4}+\frac{3c^2d^2}{(ca^2+d)x^8}\right)d\sqrt{dx^2+c}}{15c^3d^2}+\frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\coth^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\coth^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

$\big\downarrow$ 2009

$$-\frac{a\left(-\frac{c^2d^2}{x^6(a^2c+d)}-\frac{cd^2(7a^2c+4d)}{x^2(a^2c+d)^2}+\frac{d^2(15a^4c^2+20a^2cd+8d^2)\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{a(a^2c+d)^{5/2}}\right)}{15c^3d^2}+\frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\coth^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\coth^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

input
```
Int[ArcCoth[a*x]/(c + d*x^2)^(7/2),x]
```

output
```
(x*ArcCoth[a*x])/(5*c*(c + d*x^2)^(5/2)) + (4*x*ArcCoth[a*x])/(15*c^2*(c +
 d*x^2)^(3/2)) + (8*x*ArcCoth[a*x])/(15*c^3*Sqrt[c + d*x^2]) - (a*(-((c^2*
d^2)/((a^2*c + d)*x^6)) - (c*d^2*(7*a^2*c + 4*d))/((a^2*c + d)^2*x^2) + (d
^2*(15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2*
c + d]])/(a*(a^2*c + d)^(5/2))))/(15*c^3*d^2)
```

3.46. $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx$

3.46.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6539
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCoth[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.46.4 Maple [F]

$$\int \frac{\operatorname{arccoth}(ax)}{(d\,x^2+c)^{\frac{7}{2}}}dx$$

input
```
int(arccoth(a*x)/(d*x^2+c)^(7/2),x)
```

output
```
int(arccoth(a*x)/(d*x^2+c)^(7/2),x)
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(172) = 344$.

Time = 0.34 (sec) , antiderivative size = 1278, normalized size of antiderivative = 6.39

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
[1/60*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a^4*c^2*d^3 + 20*a^2*c*d^4 + 8*d^5
)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*
(15*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(a^2*c + d)*log((a^4*
d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d
*x^2 + 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*
a^2*x^2 + 1)) + 2*(16*a^5*c^5 + 26*a^3*c^4*d + 10*a*c^3*d^2 + 2*(7*a^5*c^3
*d^2 + 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 6*(5*a^5*c^4*d + 8*a^3*c^3*d^2 +
3*a*c^2*d^3)*x^2 + (8*(a^6*c^3*d^2 + 3*a^4*c^2*d^3 + 3*a^2*c*d^4 + d^5)*x^
5 + 20*(a^6*c^4*d + 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 + c*d^4)*x^3 + 15*(a^6*c
^5 + 3*a^4*c^4*d + 3*a^2*c^3*d^2 + c^2*d^3)*x)*log((a*x + 1)/(a*x - 1)))*s
qrt(d*x^2 + c))/(a^6*c^9 + 3*a^4*c^8*d + 3*a^2*c^7*d^2 + c^6*d^3 + (a^6*c^
6*d^3 + 3*a^4*c^5*d^4 + 3*a^2*c^4*d^5 + c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 + 3*
a^4*c^6*d^3 + 3*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^6*c^8*d + 3*a^4*c^7*d^2
+ 3*a^2*c^6*d^3 + c^5*d^4)*x^2), 1/30*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a^
4*c^2*d^3 + 20*a^2*c*d^4 + 8*d^5)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 20
*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*(15*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^3
)*x^2)*sqrt(-a^2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c -
 d)*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)) + (16*a^5*c
^5 + 26*a^3*c^4*d + 10*a*c^3*d^2 + 2*(7*a^5*c^3*d^2 + 11*a^3*c^2*d^3 + 4*a
*c*d^4)*x^4 + 6*(5*a^5*c^4*d + 8*a^3*c^3*d^2 + 3*a*c^2*d^3)*x^2 + (8*(a...
```

3.46.6 Sympy [F]

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \int \frac{\operatorname{acoth}(ax)}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(acoth(a*x)/(d*x**2+c)**(7/2),x)
```

output
```
Integral(acoth(a*x)/(c + d*x**2)**(7/2), x)
```

3.46.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(172) = 344$.

Time = 0.30 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.00

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{1}{30}\,a\left(\frac{\frac{3\,a^3 d\log\left(\frac{\sqrt{dx^2+c}a^2-\sqrt{a^2c+d}a}{\sqrt{dx^2+c}a^2+\sqrt{a^2c+d}a}\right)}{(a^4c^3+2\,a^2c^2d+cd^2)\sqrt{a^2c+d}}+\frac{2\,(3\,(dx^2+c)a^2d+a^2cd+d^2)}{(a^4c^3+2\,a^2c^2d+cd^2)(dx^2+c)^{\frac{3}{2}}}}{d}+\frac{4\left(\frac{ad\log\left(\frac{\sqrt{dx^2+c}a^2-\sqrt{a^2c+d}a}{\sqrt{dx^2+c}a^2+\sqrt{a^2c+d}a}\right)}{(a^2c^3+c^2d)\sqrt{a^2c+d}}\right.}{d}$$

$$+\frac{1}{15}\left(\frac{8\,x}{\sqrt{dx^2+c}c^3}+\frac{4\,x}{(dx^2+c)^{\frac{3}{2}}c^2}+\frac{3\,x}{(dx^2+c)^{\frac{5}{2}}c}\right)\operatorname{arcoth}(ax)$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
1/30*a*((3*a^3*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2
 + c)*a^2 + sqrt(a^2*c + d)*a))/((a^4*c^3 + 2*a^2*c^2*d + c*d^2)*sqrt(a^2*
c + d)) + 2*(3*(d*x^2 + c)*a^2*d + a^2*c*d + d^2)/((a^4*c^3 + 2*a^2*c^2*d
+ c*d^2)*(d*x^2 + c)^(3/2)))/d + 4*(a*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^
2*c + d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/((a^2*c^3 + c^2*d)*
sqrt(a^2*c + d)) + 2*d/((a^2*c^3 + c^2*d)*sqrt(d*x^2 + c)))/d + 8*log((sqr
t(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c +
d)*a))/(sqrt(a^2*c + d)*a*c^3)) + 1/15*(8*x/(sqrt(d*x^2 + c)*c^3) + 4*x/((
d*x^2 + c)^(3/2)*c^2) + 3*x/((d*x^2 + c)^(5/2)*c))*arccoth(a*x)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{1}{15}\,a\left(\frac{(15\,a^4c^2+20\,a^2cd+8\,d^2)\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c-d}}\right)}{(a^4c^5+2\,a^2c^4d+c^3d^2)\sqrt{-a^2c-da}}+\frac{7\,(dx^2+c)a^2c+a^2c^2+4\,(dx^2+}{(a^4c^4+2\,a^2c^3d+c^2d^2)(dx^2+}\right.$$

$$+\frac{\left(4\,x^2\left(\frac{2\,d^2x^2}{c^3}+\frac{5\,d}{c^2}\right)+\frac{15}{c}\right)x\log\left(-\frac{\frac{1}{ax}+1}{\frac{1}{ax}-1}\right)}{30\,(dx^2+c)^{\frac{5}{2}}}$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
1/15*a*((15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a
^2*c - d))/((a^4*c^5 + 2*a^2*c^4*d + c^3*d^2)*sqrt(-a^2*c - d)*a) + (7*(d*
x^2 + c)*a^2*c + a^2*c^2 + 4*(d*x^2 + c)*d + c*d)/((a^4*c^4 + 2*a^2*c^3*d
+ c^2*d^2)*(d*x^2 + c)^(3/2))) + 1/30*(4*x^2*(2*d^2*x^2/c^3 + 5*d/c^2) + 1
5/c)*x*log(-(1/(a*x) + 1)/(1/(a*x) - 1))/(d*x^2 + c)^(5/2)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \int \frac{\mathrm{acoth}(a\,x)}{(d\,x^2+c)^{7/2}}\,dx$$

input
```
int(acoth(a*x)/(c + d*x^2)^(7/2),x)
```

output
```
int(acoth(a*x)/(c + d*x^2)^(7/2), x)
```

3.47 $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx$

3.47.1 Optimal result

Integrand size = 16, antiderivative size = 283

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{a}{35c(a^2c+d)(c+dx^2)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2(a^2c+d)^2(c+dx^2)^{3/2}} + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3(a^2c+d)^3\sqrt{c+dx^2}} + \frac{x\coth^{-1}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x\coth^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x\coth^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x\coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3)\,\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{35c^4(a^2c+d)^{7/2}}$$

output
```
1/35*a/c/(a^2*c+d)/(d*x^2+c)^(5/2)+1/105*a*(11*a^2*c+6*d)/c^2/(a^2*c+d)^2/
(d*x^2+c)^(3/2)+1/7*x*arccoth(a*x)/c/(d*x^2+c)^(7/2)+6/35*x*arccoth(a*x)/c
^2/(d*x^2+c)^(5/2)+8/35*x*arccoth(a*x)/c^3/(d*x^2+c)^(3/2)-1/35*(35*a^6*c^
3+70*a^4*c^2*d+56*a^2*c*d^2+16*d^3)*arctanh(a*(d*x^2+c)^(1/2)/(a^2*c+d)^(1
/2))/c^4/(a^2*c+d)^(7/2)+1/35*a*(19*a^4*c^2+22*a^2*c*d+8*d^2)/c^3/(a^2*c+d
)^3/(d*x^2+c)^(1/2)+16/35*x*arccoth(a*x)/c^4/(d*x^2+c)^(1/2)
```

3.47.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.52

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{2ac\sqrt{a^2c+d}(c+dx^2)\left(3c^2(a^2c+d)^2+c(a^2c+d)\left(11a^2c+6d\right)(c+dx^2)+3(19a^4c^2+\right.}{}$$

input
```
Integrate[ArcCoth[a*x]/(c + d*x^2)^(9/2),x]
```

output
```
(2*a*c*Sqrt[a^2*c + d]*(c + d*x^2)*(3*c^2*(a^2*c + d)^2 + c*(a^2*c + d)*(1
1*a^2*c + 6*d)*(c + d*x^2) + 3*(19*a^4*c^2 + 22*a^2*c*d + 8*d^2)*(c + d*x^
2)^2) + 6*(a^2*c + d)^(7/2)*x*(35*c^3 + 70*c^2*d*x^2 + 56*c*d^2*x^4 + 16*d
^3*x^6)*ArcCoth[a*x] + 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^
3)*(c + d*x^2)^(7/2)*Log[1 - a*x] + 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*
c*d^2 + 16*d^3)*(c + d*x^2)^(7/2)*Log[1 + a*x] - 3*(35*a^6*c^3 + 70*a^4*c^
2*d + 56*a^2*c*d^2 + 16*d^3)*(c + d*x^2)^(7/2)*Log[a*c - d*x + Sqrt[a^2*c
+ d]*Sqrt[c + d*x^2]] - 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d
^3)*(c + d*x^2)^(7/2)*Log[a*c + d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(2
10*c^4*(a^2*c + d)^(7/2)*(c + d*x^2)^(7/2))
```

3.47.3 Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6539, 27, 7266, 2122, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{6539}$$

$$-a\int \frac{x\left(16d^3x^6+56cd^2x^4+70c^2dx^2+35c^3\right)}{35c^4\left(1-a^2x^2\right)\left(dx^2+c\right)^{7/2}}dx+\frac{16x\coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\coth^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}}+\frac{6x\coth^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}}+\frac{x\coth^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{x(16d^3x^6+56cd^2x^4+70c^2dx^2+35c^3)}{(1-a^2x^2)(dx^2+c)^{7/2}}dx}{35c^4}+\frac{16x\coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\coth^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}}+\frac{6x\coth^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}}+\frac{x\coth^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 7266}$$

$$-\frac{a\int \frac{16d^3x^6+56cd^2x^4+70c^2dx^2+35c^3}{(1-a^2x^2)(dx^2+c)^{7/2}}dx^2}{70c^4}+\frac{16x\coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\coth^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}}+\frac{6x\coth^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}}+\frac{x\coth^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 2122}$$

$$-\frac{a\int \left(\frac{5dc^3}{(ca^2+d)(dx^2+c)^{7/2}}+\frac{d(11ca^2+6d)c^2}{(ca^2+d)^2(dx^2+c)^{5/2}}+\frac{d(19c^2a^4+22cda^2+8d^2)c}{(ca^2+d)^3(dx^2+c)^{3/2}}+\frac{-35c^3a^6-70c^2da^4-56cd^2a^2-16d^3}{(ca^2+d)^3(a^2x^2-1)\sqrt{dx^2+c}}\right)dx^2}{70c^4}+\frac{16x\coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\coth^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}}+\frac{6x\coth^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}}+\frac{x\coth^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a\left(-\frac{2c^3}{(a^2c+d)(c+dx^2)^{5/2}}-\frac{2c^2(11a^2c+6d)}{3(a^2c+d)^2(c+dx^2)^{3/2}}-\frac{2c(19a^4c^2+22a^2cd+8d^2)}{(a^2c+d)^3\sqrt{c+dx^2}}+\frac{2(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3)\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{a(a^2c+d)^{7/2}}\right)}{70c^4}+\frac{16x\coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\coth^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}}+\frac{6x\coth^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}}+\frac{x\coth^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}}$$

input
```
Int[ArcCoth[a*x]/(c + d*x^2)^(9/2),x]
```

output
```
(x*ArcCoth[a*x])/(7*c*(c + d*x^2)^(7/2)) + (6*x*ArcCoth[a*x])/(35*c^2*(c +
 d*x^2)^(5/2)) + (8*x*ArcCoth[a*x])/(35*c^3*(c + d*x^2)^(3/2)) + (16*x*Arc
Coth[a*x])/(35*c^4*Sqrt[c + d*x^2]) - (a*((-2*c^3)/((a^2*c + d)*(c + d*x^2
)^(5/2)) - (2*c^2*(11*a^2*c + 6*d))/(3*(a^2*c + d)^2*(c + d*x^2)^(3/2)) -
(2*c*(19*a^4*c^2 + 22*a^2*c*d + 8*d^2))/((a^2*c + d)^3*Sqrt[c + d*x^2]) +
(2*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*ArcTanh[(a*Sqrt[c +
 d*x^2])/Sqrt[a^2*c + d]])/(a*(a^2*c + d)^(7/2))))/(70*c^4)
```

3.47. $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx$

3.47.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2122
```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_))/((a_.) + (b_.)*(x_)), x_Symbol] :> In
t[ExpandIntegrand[1/Sqrt[c + d*x], Px*((c + d*x)^(n + 1/2)/(a + b*x)), x],
x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0]
```

rule 6539
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcCoth[c*x])   u
, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 - c^2*x^2), x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.47.4 Maple [F]

$$\int \frac{\operatorname{arccoth}(ax)}{(d\,x^2+c)^{\frac{9}{2}}}dx$$

input
```
int(arccoth(a*x)/(d*x^2+c)^(9/2),x)
```

output
```
int(arccoth(a*x)/(d*x^2+c)^(9/2),x)
```

3.47.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(247) = 494$.

Time = 0.44 (sec) , antiderivative size = 2004, normalized size of antiderivative = 7.08

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

output
```
[1/420*(3*(35*a^6*c^7 + 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 +
70*a^4*c^2*d^5 + 56*a^2*c*d^6 + 16*d^7)*x^8 + 16*c^4*d^3 + 4*(35*a^6*c^4*d
^3 + 70*a^4*c^3*d^4 + 56*a^2*c^2*d^5 + 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 +
 70*a^4*c^4*d^3 + 56*a^2*c^3*d^4 + 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d + 70*
a^4*c^5*d^2 + 56*a^2*c^4*d^3 + 16*c^3*d^4)*x^2)*sqrt(a^2*c + d)*log((a^4*d
^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*
x^2 + 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a
^2*x^2 + 1)) + 2*(142*a^7*c^7 + 320*a^5*c^6*d + 244*a^3*c^5*d^2 + 66*a*c^4
*d^3 + 6*(19*a^7*c^4*d^3 + 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 + 8*a*c*d^6)*x^
6 + 2*(182*a^7*c^5*d^2 + 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 + 78*a*c^2*d^5)
*x^4 + 2*(196*a^7*c^6*d + 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 + 87*a*c^3*d^4
)*x^2 + 3*(16*(a^8*c^4*d^3 + 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 + 4*a^2*c*d^6 +
 d^7)*x^7 + 56*(a^8*c^5*d^2 + 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 + 4*a^2*c^2*d^
5 + c*d^6)*x^5 + 70*(a^8*c^6*d + 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 + 4*a^2*c^3
*d^4 + c^2*d^5)*x^3 + 35*(a^8*c^7 + 4*a^6*c^6*d + 6*a^4*c^5*d^2 + 4*a^2*c^
4*d^3 + c^3*d^4)*x)*log((a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^8*c^12 +
 4*a^6*c^11*d + 6*a^4*c^10*d^2 + 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 +
4*a^6*c^7*d^5 + 6*a^4*c^6*d^6 + 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*
d^3 + 4*a^6*c^8*d^4 + 6*a^4*c^7*d^5 + 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^
8*c^10*d^2 + 4*a^6*c^9*d^3 + 6*a^4*c^8*d^4 + 4*a^2*c^7*d^5 + c^6*d^6)*x...
```

3.47.6 Sympy [F]

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \int \frac{\operatorname{acoth}(ax)}{(c+dx^2)^{\frac{9}{2}}}\,dx$$

input
```
integrate(acoth(a*x)/(d*x**2+c)**(9/2),x)
```

output
```
Integral(acoth(a*x)/(c + d*x**2)**(9/2), x)
```

3.47.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. 2(247) = 494.

Time = 0.35 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.26

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{1}{210}a\left(\frac{\frac{15\,a^5d\log\left(\frac{\sqrt{dx^2+c}a^2-\sqrt{a^2c+d}a}{\sqrt{dx^2+c}a^2+\sqrt{a^2c+d}a}\right)}{(a^6c^4+3\,a^4c^3d+3\,a^2c^2d^2+cd^3)\sqrt{a^2c+d}}+\frac{2\left(15\,(dx^2+c)^2a^4d+3\,a^4c^2d+6\,a^2cd^2+3\,d^3+5\,(a^4cd+a^2d^2)\,(dx\right.}{(a^6c^4+3\,a^4c^3d+3\,a^2c^2d^2+cd^3)(dx^2+c)^{\frac{5}{2}}}}{d}\right.$$

$$+\frac{1}{35}\left(\frac{16\,x}{\sqrt{dx^2+c}c^4}+\frac{8\,x}{(dx^2+c)^{\frac{3}{2}}c^3}+\frac{6\,x}{(dx^2+c)^{\frac{5}{2}}c^2}+\frac{5\,x}{(dx^2+c)^{\frac{7}{2}}c}\right)\mathrm{arcoth}\,(ax)$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
1/210*a*((15*a^5*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x
^2 + c)*a^2 + sqrt(a^2*c + d)*a))/((a^6*c^4 + 3*a^4*c^3*d + 3*a^2*c^2*d^2
+ c*d^3)*sqrt(a^2*c + d)) + 2*(15*(d*x^2 + c)^2*a^4*d + 3*a^4*c^2*d + 6*a^
2*c*d^2 + 3*d^3 + 5*(a^4*c*d + a^2*d^2)*(d*x^2 + c))/((a^6*c^4 + 3*a^4*c^3
*d + 3*a^2*c^2*d^2 + c*d^3)*(d*x^2 + c)^(5/2)))/d + 6*(3*a^3*d*log((sqrt(d
*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*
a))/((a^4*c^4 + 2*a^2*c^3*d + c^2*d^2)*sqrt(a^2*c + d)) + 2*(3*(d*x^2 + c)
*a^2*d + a^2*c*d + d^2)/((a^4*c^4 + 2*a^2*c^3*d + c^2*d^2)*(d*x^2 + c)^(3/
2)))/d + 24*(a*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2
 + c)*a^2 + sqrt(a^2*c + d)*a))/((a^2*c^4 + c^3*d)*sqrt(a^2*c + d)) + 2*d/
((a^2*c^4 + c^3*d)*sqrt(d*x^2 + c)))/d + 48*log((sqrt(d*x^2 + c)*a^2 - sqr
t(a^2*c + d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/(sqrt(a^2*c + d
)*a*c^4)) + 1/35*(16*x/(sqrt(d*x^2 + c)*c^4) + 8*x/((d*x^2 + c)^(3/2)*c^3)
 + 6*x/((d*x^2 + c)^(5/2)*c^2) + 5*x/((d*x^2 + c)^(7/2)*c))*arccoth(a*x)
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.26

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{1}{105}a\left(\frac{3\,(35\,a^6c^3+70\,a^4c^2d+56\,a^2cd^2+16\,d^3)\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c-d}}\right)}{(a^6c^7+3\,a^4c^6d+3\,a^2c^5d^2+c^4d^3)\sqrt{-a^2c-d}a}+\frac{57\,(dx^2+c)^2a^4c^2\cdots}{\cdots}\right.$$

$$+\frac{\left(2\left(4\,x^2\left(\frac{2\,d^3x^2}{c^4}+\frac{7\,d^2}{c^3}\right)+\frac{35\,d}{c^2}\right)x^2+\frac{35}{c}\right)x\log\left(-\frac{\frac{1}{ax}+1}{\frac{1}{ax}-1}\right)}{70\,(dx^2+c)^{\frac{7}{2}}}$$

input
```
integrate(arccoth(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
1/105*a*(3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*arctan(sqrt
(d*x^2 + c)*a/sqrt(-a^2*c - d))/((a^6*c^7 + 3*a^4*c^6*d + 3*a^2*c^5*d^2 +
c^4*d^3)*sqrt(-a^2*c - d)*a) + (57*(d*x^2 + c)^2*a^4*c^2 + 11*(d*x^2 + c)*
a^4*c^3 + 3*a^4*c^4 + 66*(d*x^2 + c)^2*a^2*c*d + 17*(d*x^2 + c)*a^2*c^2*d
+ 6*a^2*c^3*d + 24*(d*x^2 + c)^2*d^2 + 6*(d*x^2 + c)*c*d^2 + 3*c^2*d^2)/((
a^6*c^6 + 3*a^4*c^5*d + 3*a^2*c^4*d^2 + c^3*d^3)*(d*x^2 + c)^(5/2))) + 1/7
0*(2*(4*x^2*(2*d^3*x^2/c^4 + 7*d^2/c^3) + 35*d/c^2)*x^2 + 35/c)*x*log(-(1/
(a*x) + 1)/(1/(a*x) - 1))/(d*x^2 + c)^(7/2)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \int \frac{\mathrm{acoth}(a\,x)}{(d\,x^2+c)^{9/2}}\,dx$$

input
```
int(acoth(a*x)/(c + d*x^2)^(9/2),x)
```

output
```
int(acoth(a*x)/(c + d*x^2)^(9/2), x)
```

3.48 $\int \sqrt{a-ax^2}\coth^{-1}(x)\,dx$

3.48.1 Optimal result

Integrand size = 15, antiderivative size = 186

$$\int \sqrt{a-ax^2}\coth^{-1}(x)\,dx = \frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\coth^{-1}(x) - \frac{a\sqrt{1-x^2}\coth^{-1}(x)\arctan\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}} - \frac{ia\sqrt{1-x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{2\sqrt{a-ax^2}} + \frac{ia\sqrt{1-x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{2\sqrt{a-ax^2}}$$

output
```
-a*arccoth(x)*arctan((1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a)^(1
/2)-1/2*I*a*polylog(2,-I*(1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a
)^(1/2)+1/2*I*a*polylog(2,I*(1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^
2+a)^(1/2)+1/2*(-a*x^2+a)^(1/2)+1/2*x*arccoth(x)*(-a*x^2+a)^(1/2)
```

3.48.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.67

$$\int \sqrt{a-ax^2}\coth^{-1}(x)\,dx =$$
$$-\frac{\sqrt{a-ax^2}\left(-2\coth\left(\frac{1}{2}\coth^{-1}(x)\right)-\coth^{-1}(x)\operatorname{csch}^2\left(\frac{1}{2}\coth^{-1}(x)\right)-4\coth^{-1}(x)\log\left(1-e^{-\coth^{-1}(x)}\right)+\right.}{}$$

input
```
Integrate[Sqrt[a - a*x^2]*ArcCoth[x],x]
```

output
```
-1/8*(Sqrt[a - a*x^2]*(-2*Coth[ArcCoth[x]/2] - ArcCoth[x]*Csch[ArcCoth[x]/
2]^2 - 4*ArcCoth[x]*Log[1 - E^(-ArcCoth[x])] + 4*ArcCoth[x]*Log[1 + E^(-Ar
cCoth[x])] - 4*PolyLog[2, -E^(-ArcCoth[x])] + 4*PolyLog[2, E^(-ArcCoth[x])
] - ArcCoth[x]*Sech[ArcCoth[x]/2]^2 + 2*Tanh[ArcCoth[x]/2]))/(Sqrt[1 - x^(
-2)]*x)
```

3.48.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6505, 6517, 6513}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-ax^2}\coth^{-1}(x)\,dx$$
$$\Big\downarrow \text{6505}$$
$$\frac{1}{2}a\int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}}dx+\frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\coth^{-1}(x)$$
$$\Big\downarrow \text{6517}$$
$$\frac{a\sqrt{1-x^2}\int \frac{\coth^{-1}(x)}{\sqrt{1-x^2}}dx}{2\sqrt{a-ax^2}}+\frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\coth^{-1}(x)$$
$$\Big\downarrow \text{6513}$$

$$\frac{a\sqrt{1-x^2}\left(-2\arctan\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)\coth^{-1}(x)-i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)+i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)\right)}{2\sqrt{a-ax^2}}+$$
$$\frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\coth^{-1}(x)$$

input
```
Int[Sqrt[a - a*x^2]*ArcCoth[x],x]
```

output
```
Sqrt[a - a*x^2]/2 + (x*Sqrt[a - a*x^2]*ArcCoth[x])/2 + (a*Sqrt[1 - x^2]*(-
2*ArcCoth[x]*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]] - I*PolyLog[2, ((-I)*Sqrt[1 -
 x])/Sqrt[1 + x]] + I*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 + x]]))/(2*Sqrt[a
- a*x^2])
```

3.48.3.1 Defintions of rubi rules used

rule 6505
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)^q
*((a + b*ArcCoth[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d + e
*x^2)^(q - 1)*(a + b*ArcCoth[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

rule 6513
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcCoth[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6517
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcCoth[c*x]
)^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e
, 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.48.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.07

method	result
default	$\frac{(\operatorname{arccoth}(x)x+1)\sqrt{-(1+x)(x-1)a}}{2}+\frac{\sqrt{-(1+x)(x-1)a}\,\sqrt{\frac{x-1}{1+x}}\,\operatorname{arccoth}(x)\ln\left(1-\frac{1}{\sqrt{\frac{x-1}{1+x}}}\right)}{2x-2}+\frac{\sqrt{-(1+x)(x-1)a}\,\sqrt{\frac{x-1}{1+x}}\,\operatorname{polylog}\left(\right.}{2x-2}$

input
```
int((-a*x^2+a)^(1/2)*arccoth(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arccoth(x)*x+1)*(-(1+x)*(x-1)*a)^(1/2)+1/2*(-(1+x)*(x-1)*a)^(1/2)*((x
-1)/(1+x))^(1/2)/(x-1)*arccoth(x)*ln(1-1/((x-1)/(1+x))^(1/2))+1/2*(-(1+x)*
(x-1)*a)^(1/2)*((x-1)/(1+x))^(1/2)/(x-1)*polylog(2,1/((x-1)/(1+x))^(1/2))-
1/2*(-(1+x)*(x-1)*a)^(1/2)*((x-1)/(1+x))^(1/2)/(x-1)*arccoth(x)*ln(1/((x-1
)/(1+x))^(1/2)+1)-1/2*(-(1+x)*(x-1)*a)^(1/2)*((x-1)/(1+x))^(1/2)/(x-1)*pol
ylog(2,-1/((x-1)/(1+x))^(1/2))
```

3.48.5 Fricas [F]

$$\int \sqrt{a-ax^2}\coth^{-1}(x)\,dx=\int \sqrt{-ax^2+a}\,\operatorname{arcoth}(x)\,dx$$

input
```
integrate((-a*x^2+a)^(1/2)*arccoth(x),x, algorithm="fricas")
```

output
```
integral(sqrt(-a*x^2 + a)*arccoth(x), x)
```

3.48.6 Sympy [F]

$$\int \sqrt{a-ax^2}\coth^{-1}(x)\,dx=\int \sqrt{-a(x-1)(x+1)}\,\operatorname{acoth}(x)\,dx$$

input
```
integrate((-a*x**2+a)**(1/2)*acoth(x),x)
```

output
```
Integral(sqrt(-a*(x - 1)*(x + 1))*acoth(x), x)
```

3.48.7 Maxima [F]

$$\int \sqrt{a-ax^2}\coth^{-1}(x)\,dx=\int \sqrt{-ax^2+a}\,\mathrm{arcoth}\,(x)\,\,dx$$

input
```
integrate((-a*x^2+a)^(1/2)*arccoth(x),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*x^2 + a)*arccoth(x), x)
```

3.48.8 Giac [F]

$$\int \sqrt{a-ax^2}\coth^{-1}(x)\,dx=\int \sqrt{-ax^2+a}\,\mathrm{arcoth}\,(x)\,\,dx$$

input
```
integrate((-a*x^2+a)^(1/2)*arccoth(x),x, algorithm="giac")
```

output
```
integrate(sqrt(-a*x^2 + a)*arccoth(x), x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a-ax^2}\coth^{-1}(x)\,dx=\int \mathrm{acoth}(x)\,\sqrt{a-a\,x^2}\,dx$$

input
```
int(acoth(x)*(a - a*x^2)^(1/2),x)
```

output
```
int(acoth(x)*(a - a*x^2)^(1/2), x)
```

3.49 $\int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}}\,dx$

3.49.1 Optimal result

Integrand size = 15, antiderivative size = 144

$$\int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}}\,dx = -\frac{2\sqrt{1-x^2}\coth^{-1}(x)\arctan\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}} - \frac{i\sqrt{1-x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}} + \frac{i\sqrt{1-x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}}$$

output
```
-2*arccoth(x)*arctan((1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a)^(1
/2)-I*polylog(2,-I*(1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a)^(1/2
)+I*polylog(2,I*(1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a)^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.53

$$\int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}}\,dx = \frac{\sqrt{a-ax^2}\left(\coth^{-1}(x)\left(\log\left(1-e^{-\coth^{-1}(x)}\right)-\log\left(1+e^{-\coth^{-1}(x)}\right)\right)+\mathrm{PolyLog}\left(2,-e^{-\coth^{-1}(x)}\right)-\mathrm{Poly}}{a\sqrt{1-\frac{1}{x^2}}x}$$

input
```
Integrate[ArcCoth[x]/Sqrt[a - a*x^2],x]
```

output
```
(Sqrt[a - a*x^2]*(ArcCoth[x]*(Log[1 - E^(-ArcCoth[x])] - Log[1 + E^(-ArcCo
th[x])]) + PolyLog[2, -E^(-ArcCoth[x])] - PolyLog[2, E^(-ArcCoth[x])]))/(a
*Sqrt[1 - x^(-2)]*x)
```

3.49.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.69, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6517, 6513}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}}\,dx$$

$$\Big\downarrow \text{6517}$$

$$\frac{\sqrt{1-x^2}\int \frac{\coth^{-1}(x)}{\sqrt{1-x^2}}dx}{\sqrt{a-ax^2}}$$

$$\Big\downarrow \text{6513}$$

$$\frac{\sqrt{1-x^2}\left(-2\arctan\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)\coth^{-1}(x)-i\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)+i\,\text{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)\right)}{\sqrt{a-ax^2}}$$

input
```
Int[ArcCoth[x]/Sqrt[a - a*x^2],x]
```

output
```
(Sqrt[1 - x^2]*(-2*ArcCoth[x]*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]] - I*PolyLog[
2, ((-I)*Sqrt[1 - x])/Sqrt[1 + x]] + I*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 +
 x]]))/Sqrt[a - a*x^2]
```

3.49.3.1 Defintions of rubi rules used

rule 6513
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcCoth[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

rule 6517
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcCoth[c*x]
)^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e
, 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.49.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.32

method	result
default	$\frac{\ln\left(1-\frac{1}{\sqrt{\frac{x-1}{1+x}}}\right)\operatorname{arccoth}(x)\sqrt{\frac{x-1}{1+x}}\,\sqrt{-(1+x)(x-1)a}}{(x-1)a}+\frac{\operatorname{polylog}\left(2,\frac{1}{\sqrt{\frac{x-1}{1+x}}}\right)\sqrt{\frac{x-1}{1+x}}\,\sqrt{-(1+x)(x-1)a}}{(x-1)a}-\frac{\ln\left(\frac{1}{\sqrt{\frac{x-1}{1+x}}}+1\right)\operatorname{arccoth}(}{(x}$

input
```
int(arccoth(x)/(-a*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(1-1/((x-1)/(1+x))^(1/2))*arccoth(x)*((x-1)/(1+x))^(1/2)*(-(1+x)*(x-1)*a
)^(1/2)/(x-1)/a+polylog(2,1/((x-1)/(1+x))^(1/2))*((x-1)/(1+x))^(1/2)*(-(1+
x)*(x-1)*a)^(1/2)/(x-1)/a-ln(1/((x-1)/(1+x))^(1/2)+1)*arccoth(x)*((x-1)/(1
+x))^(1/2)*(-(1+x)*(x-1)*a)^(1/2)/(x-1)/a-polylog(2,-1/((x-1)/(1+x))^(1/2)
)*((x-1)/(1+x))^(1/2)*(-(1+x)*(x-1)*a)^(1/2)/(x-1)/a
```

3.49.5 Fricas [F]

$$\int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}}\,dx = \int \frac{\operatorname{arcoth}(x)}{\sqrt{-ax^2+a}}\,dx$$

input
```
integrate(arccoth(x)/(-a*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a*x^2 + a)*arccoth(x)/(a*x^2 - a), x)
```

3.49.6 Sympy [F]

$$\int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}}\,dx = \int \frac{\operatorname{acoth}(x)}{\sqrt{-a(x-1)(x+1)}}\,dx$$

input
```
integrate(acoth(x)/(-a*x**2+a)**(1/2),x)
```

output
```
Integral(acoth(x)/sqrt(-a*(x - 1)*(x + 1)), x)
```

3.49.7 Maxima [F]

$$\int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}}\,dx = \int \frac{\operatorname{arcoth}(x)}{\sqrt{-ax^2+a}}\,dx$$

input
```
integrate(arccoth(x)/(-a*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccoth(x)/sqrt(-a*x^2 + a), x)
```

3.49.8 Giac [F]

$$\int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}}\,dx = \int \frac{\operatorname{arcoth}(x)}{\sqrt{-ax^2+a}}\,dx$$

input
```
integrate(arccoth(x)/(-a*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccoth(x)/sqrt(-a*x^2 + a), x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}}\,dx = \int \frac{\operatorname{acoth}(x)}{\sqrt{a-a\,x^2}}\,dx$$

input
```
int(acoth(x)/(a - a*x^2)^(1/2),x)
```

output
```
int(acoth(x)/(a - a*x^2)^(1/2), x)
```

3.50 $\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}}\,dx$

3.50.1 Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}}\,dx = -\frac{1}{a\sqrt{a-ax^2}}+\frac{x\coth^{-1}(x)}{a\sqrt{a-ax^2}}$$

output
```
-1/a/(-a*x^2+a)^(1/2)+x*arccoth(x)/a/(-a*x^2+a)^(1/2)
```

3.50.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}}\,dx = \frac{\sqrt{a-ax^2}\left(1-x\coth^{-1}(x)\right)}{a^2\left(-1+x^2\right)}$$

input
```
Integrate[ArcCoth[x]/(a - a*x^2)^(3/2),x]
```

output
```
(Sqrt[a - a*x^2]*(1 - x*ArcCoth[x]))/(a^2*(-1 + x^2))
```

3.50.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {6521}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 6521$$

$$\frac{x\coth^{-1}(x)}{a\sqrt{a-ax^2}}-\frac{1}{a\sqrt{a-ax^2}}$$

input
```
Int[ArcCoth[x]/(a - a*x^2)^(3/2),x]
```

output
```
-(1/(a*Sqrt[a - a*x^2])) + (x*ArcCoth[x])/(a*Sqrt[a - a*x^2])
```

3.50.3.1 Defintions of rubi rules used

rule 6521
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcCoth[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

3.50.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

method	result	size
risch	$-\frac{x\ln(x-1)}{2a\sqrt{-a(x^2-1)}}+\frac{\ln(1+x)x-2}{2a\sqrt{-a(x^2-1)}}$	45
default	$-\frac{(-1+\operatorname{arccoth}(x))\sqrt{-(1+x)(x-1)a}}{2(x-1)a^2}-\frac{(1+\operatorname{arccoth}(x))\sqrt{-(1+x)(x-1)a}}{2(1+x)a^2}$	52

input
```
int(arccoth(x)/(-a*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a*x/(-a*(x^2-1))^(1/2)*ln(x-1)+1/2/a*(ln(1+x)*x-2)/(-a*(x^2-1))^(1/2)
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}}\,dx = -\frac{\sqrt{-ax^2+a}\left(x\log\left(\frac{x+1}{x-1}\right)-2\right)}{2\left(a^2x^2-a^2\right)}$$

input
```
integrate(arccoth(x)/(-a*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-a*x^2 + a)*(x*log((x + 1)/(x - 1)) - 2)/(a^2*x^2 - a^2)
```

3.50.6 Sympy [F]

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}}\,dx = \int \frac{\operatorname{acoth}(x)}{\left(-a(x-1)(x+1)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(acoth(x)/(-a*x**2+a)**(3/2),x)
```

output
```
Integral(acoth(x)/(-a*(x - 1)*(x + 1))**(3/2), x)
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.70

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}}\,dx = \frac{x\,\mathrm{arcoth}(x)}{\sqrt{-ax^2+a}a} - \frac{\frac{\sqrt{-ax^2+a}}{ax+a} - \frac{\sqrt{-ax^2+a}}{ax-a}}{2a}$$

input
```
integrate(arccoth(x)/(-a*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
x*arccoth(x)/(sqrt(-a*x^2 + a)*a) - 1/2*(sqrt(-a*x^2 + a)/(a*x + a) - sqrt
(-a*x^2 + a)/(a*x - a))/a
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}}\,dx = -\frac{\sqrt{-ax^2+ax}\log\left(-\frac{\frac{1}{x}+1}{\frac{1}{x}-1}\right)}{2\,(ax^2-a)a} - \frac{1}{\sqrt{-ax^2+aa}}$$

input
```
integrate(arccoth(x)/(-a*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-a*x^2 + a)*x*log(-(1/x + 1)/(1/x - 1))/((a*x^2 - a)*a) - 1/(sqr
t(-a*x^2 + a)*a)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}}\,dx = \int \frac{\mathrm{acoth}(x)}{(a-a\,x^2)^{3/2}}\,dx$$

input
```
int(acoth(x)/(a - a*x^2)^(3/2),x)
```

output
```
int(acoth(x)/(a - a*x^2)^(3/2), x)
```

3.51 $\int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}}\,dx$

3.51.1 Optimal result

Integrand size = 15, antiderivative size = 83

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}}\,dx = -\frac{1}{9a(a-ax^2)^{3/2}} - \frac{2}{3a^2\sqrt{a-ax^2}} + \frac{x\coth^{-1}(x)}{3a(a-ax^2)^{3/2}} + \frac{2x\coth^{-1}(x)}{3a^2\sqrt{a-ax^2}}$$

output
```
-1/9/a/(-a*x^2+a)^(3/2)+1/3*x*arccoth(x)/a/(-a*x^2+a)^(3/2)-2/3/a^2/(-a*x^
2+a)^(1/2)+2/3*x*arccoth(x)/a^2/(-a*x^2+a)^(1/2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}}\,dx = -\frac{\sqrt{a-ax^2}\left(7-6x^2+(-9x+6x^3)\coth^{-1}(x)\right)}{9a^3(-1+x^2)^2}$$

input
```
Integrate[ArcCoth[x]/(a - a*x^2)^(5/2),x]
```

output
```
-1/9*(Sqrt[a - a*x^2]*(7 - 6*x^2 + (-9*x + 6*x^3)*ArcCoth[x]))/(a^3*(-1 +
x^2)^2)
```

3.51.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6523, 6521}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}}dx$$

$$\Big\downarrow \text{6523}$$

$$\frac{2\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}}dx}{3a}-\frac{1}{9a(a-ax^2)^{3/2}}+\frac{x\coth^{-1}(x)}{3a(a-ax^2)^{3/2}}$$

$$\Big\downarrow \text{6521}$$

$$-\frac{1}{9a(a-ax^2)^{3/2}}+\frac{x\coth^{-1}(x)}{3a(a-ax^2)^{3/2}}+\frac{2\left(\frac{x\coth^{-1}(x)}{a\sqrt{a-ax^2}}-\frac{1}{a\sqrt{a-ax^2}}\right)}{3a}$$

input
```
Int[ArcCoth[x]/(a - a*x^2)^(5/2),x]
```

output
```
-1/9*1/(a*(a - a*x^2)^(3/2)) + (x*ArcCoth[x])/(3*a*(a - a*x^2)^(3/2)) + (2
*(-(1/(a*Sqrt[a - a*x^2])) + (x*ArcCoth[x])/(a*Sqrt[a - a*x^2])))/(3*a)
```

3.51.3.1 Defintions of rubi rules used

rule 6521
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcCoth[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6523
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcCoth[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcCoth[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

3.51.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{x(2x^2-3)\ln(x-1)}{6a^2(x^2-1)\sqrt{-a(x^2-1)}}+\frac{6x^3\ln(1+x)-12x^2-9\ln(1+x)x+14}{18a^2(x^2-1)\sqrt{-a(x^2-1)}}$
default	$\frac{(1+x)(-1+3\,\text{arccoth}(x))\sqrt{-(1+x)(x-1)a}}{72(x-1)^2a^3}-\frac{3(-1+\text{arccoth}(x))\sqrt{-(1+x)(x-1)a}}{8a^3(x-1)}-\frac{3(1+\text{arccoth}(x))\sqrt{-(1+x)(x-1)a}}{8(1+x)a^3}+\frac{(1+$

input
```
int(arccoth(x)/(-a*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/a^2*x*(2*x^2-3)/(x^2-1)/(-a*(x^2-1))^(1/2)*ln(x-1)+1/18/a^2*(6*x^3*ln
(1+x)-12*x^2-9*ln(1+x)*x+14)/(x^2-1)/(-a*(x^2-1))^(1/2)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}}\,dx=\frac{\sqrt{-ax^2+a}\left(12\,x^2-3\,(2\,x^3-3\,x)\log\left(\frac{x+1}{x-1}\right)-14\right)}{18\,(a^3x^4-2\,a^3x^2+a^3)}$$

input
```
integrate(arccoth(x)/(-a*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/18*sqrt(-a*x^2 + a)*(12*x^2 - 3*(2*x^3 - 3*x)*log((x + 1)/(x - 1)) - 14)
/(a^3*x^4 - 2*a^3*x^2 + a^3)
```

3.51.6 Sympy [F]

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}}\,dx=\int \frac{\operatorname{acoth}(x)}{(-a\,(x-1)\,(x+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(acoth(x)/(-a*x**2+a)**(5/2),x)
```

output
```
Integral(acoth(x)/(-a*(x - 1)*(x + 1))**(5/2), x)
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{2\,x}{\sqrt{-ax^2+a}a^2} + \frac{x}{(-ax^2+a)^{\frac{3}{2}}a}\right)\operatorname{arcoth}(x) - \frac{2}{3\sqrt{-ax^2+a}a^2} - \frac{1}{9\,(-ax^2+a)^{\frac{3}{2}}a}$$

input
```
integrate(arccoth(x)/(-a*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*x/(sqrt(-a*x^2 + a)*a^2) + x/((-a*x^2 + a)^(3/2)*a))*arccoth(x) - 2
/3/(sqrt(-a*x^2 + a)*a^2) - 1/9/((-a*x^2 + a)^(3/2)*a)
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}}\,dx = -\frac{\sqrt{-ax^2+a}x\left(\frac{2\,x^2}{a}-\frac{3}{a}\right)\log\left(-\frac{\frac{1}{x}+1}{\frac{1}{x}-1}\right)}{6\,(ax^2-a)^2} - \frac{6\,ax^2-7\,a}{9\,(ax^2-a)\sqrt{-ax^2+a}a^2}$$

input
```
integrate(arccoth(x)/(-a*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(-a*x^2 + a)*x*(2*x^2/a - 3/a)*log(-(1/x + 1)/(1/x - 1))/(a*x^2 -
 a)^2 - 1/9*(6*a*x^2 - 7*a)/((a*x^2 - a)*sqrt(-a*x^2 + a)*a^2)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}}\,dx = \int \frac{\operatorname{acoth}(x)}{(a-a\,x^2)^{5/2}}\,dx$$

input
```
int(acoth(x)/(a - a*x^2)^(5/2),x)
```

output
```
int(acoth(x)/(a - a*x^2)^(5/2), x)
```

3.52 $\int \frac{\coth^{-1}(x)}{(a-ax^2)^{7/2}}\,dx$

3.52.1 Optimal result

Integrand size = 15, antiderivative size = 124

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{7/2}}\,dx = -\frac{1}{25a(a-ax^2)^{5/2}} - \frac{4}{45a^2(a-ax^2)^{3/2}} - \frac{8}{15a^3\sqrt{a-ax^2}} + \frac{x\coth^{-1}(x)}{5a(a-ax^2)^{5/2}} + \frac{4x\coth^{-1}(x)}{15a^2(a-ax^2)^{3/2}} + \frac{8x\coth^{-1}(x)}{15a^3\sqrt{a-ax^2}}$$

output
```
-1/25/a/(-a*x^2+a)^(5/2)-4/45/a^2/(-a*x^2+a)^(3/2)+1/5*x*arccoth(x)/a/(-a*
x^2+a)^(5/2)+4/15*x*arccoth(x)/a^2/(-a*x^2+a)^(3/2)-8/15/a^3/(-a*x^2+a)^(1
/2)+8/15*x*arccoth(x)/a^3/(-a*x^2+a)^(1/2)
```

3.52.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{7/2}}\,dx = \frac{\sqrt{a-ax^2}\left(149-260x^2+120x^4-15x(15-20x^2+8x^4)\coth^{-1}(x)\right)}{225a^4(-1+x^2)^3}$$

input
```
Integrate[ArcCoth[x]/(a - a*x^2)^(7/2),x]
```

output
```
(Sqrt[a - a*x^2]*(149 - 260*x^2 + 120*x^4 - 15*x*(15 - 20*x^2 + 8*x^4)*Arc
Coth[x]))/(225*a^4*(-1 + x^2)^3)
```

3.52.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6523, 6523, 6521}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{7/2}}\,dx$$

$$\Big\downarrow \ 6523$$

$$\frac{4\int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}}dx}{5a}-\frac{1}{25a\,(a-ax^2)^{5/2}}+\frac{x\coth^{-1}(x)}{5a\,(a-ax^2)^{5/2}}$$

$$\Big\downarrow \ 6523$$

$$\frac{4\left(\frac{2\int \frac{\coth^{-1}(x)}{\left(a-ax^2\right)^{3/2}}dx}{3a}-\frac{1}{9a(a-ax^2)^{3/2}}+\frac{x\coth^{-1}(x)}{3a(a-ax^2)^{3/2}}\right)}{5a}-\frac{1}{25a\,(a-ax^2)^{5/2}}+\frac{x\coth^{-1}(x)}{5a\,(a-ax^2)^{5/2}}$$

$$\Big\downarrow \ 6521$$

$$-\frac{1}{25a\,(a-ax^2)^{5/2}}+\frac{x\coth^{-1}(x)}{5a\,(a-ax^2)^{5/2}}+\frac{4\left(-\frac{1}{9a(a-ax^2)^{3/2}}+\frac{x\coth^{-1}(x)}{3a(a-ax^2)^{3/2}}+\frac{2\left(\frac{x\coth^{-1}(x)}{a\sqrt{a-ax^2}}-\frac{1}{a\sqrt{a-ax^2}}\right)}{3a}\right)}{5a}$$

input
```
Int[ArcCoth[x]/(a - a*x^2)^(7/2),x]
```

output
```
-1/25*1/(a*(a - a*x^2)^(5/2)) + (x*ArcCoth[x])/(5*a*(a - a*x^2)^(5/2)) + (
4*(-1/9*1/(a*(a - a*x^2)^(3/2)) + (x*ArcCoth[x])/(3*a*(a - a*x^2)^(3/2)) +
 (2*(-(1/(a*Sqrt[a - a*x^2])) + (x*ArcCoth[x])/(a*Sqrt[a - a*x^2])))/(3*a)
))/(5*a)
```

3.52.3.1 Defintions of rubi rules used

rule 6521
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcCoth[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

rule 6523
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcCoth[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcCoth[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

3.52.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{x\left(8x^4-20x^2+15\right)\ln(x-1)}{30a^3(x^2-1)^2\sqrt{-a(x^2-1)}}+\frac{120x^5\ln(1+x)-240x^4-300x^3\ln(1+x)+520x^2+225\ln(1+x)x-298}{450a^3(x^2-1)^2\sqrt{-a(x^2-1)}}$
default	$-\frac{(1+x)^2(-1+5\ \text{arccoth}(x))\sqrt{-(1+x)(x-1)a}}{800(x-1)^3a^4}+\frac{5(1+x)(-1+3\ \text{arccoth}(x))\sqrt{-(1+x)(x-1)a}}{288a^4(x-1)^2}-\frac{5(-1+\text{arccoth}(x))\sqrt{-(1+x)(x}}{16(x-1)a^4}$

input
```
int(arccoth(x)/(-a*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30/a^3*x*(8*x^4-20*x^2+15)/(x^2-1)^2/(-a*(x^2-1))^(1/2)*ln(x-1)+1/450/a
^3*(120*x^5*ln(1+x)-240*x^4-300*x^3*ln(1+x)+520*x^2+225*ln(1+x)*x-298)/(x^
2-1)^2/(-a*(x^2-1))^(1/2)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.65

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{7/2}}\,dx = \frac{\left(240\,x^4-520\,x^2-15\left(8\,x^5-20\,x^3+15\,x\right)\log\left(\frac{x+1}{x-1}\right)+298\right)\sqrt{-ax^2+a}}{450\left(a^4x^6-3\,a^4x^4+3\,a^4x^2-a^4\right)}$$

input
```
integrate(arccoth(x)/(-a*x^2+a)^(7/2),x, algorithm="fricas")
```

output
```
1/450*(240*x^4 - 520*x^2 - 15*(8*x^5 - 20*x^3 + 15*x)*log((x + 1)/(x - 1))
 + 298)*sqrt(-a*x^2 + a)/(a^4*x^6 - 3*a^4*x^4 + 3*a^4*x^2 - a^4)
```

3.52.6 Sympy [F]

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{7/2}}\,dx = \int \frac{\operatorname{acoth}(x)}{(-a(x-1)(x+1))^{\frac{7}{2}}}\,dx$$

input
```
integrate(acoth(x)/(-a*x**2+a)**(7/2),x)
```

output
```
Integral(acoth(x)/(-a*(x - 1)*(x + 1))**(7/2), x)
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{7/2}}\,dx = \frac{1}{15}\left(\frac{8x}{\sqrt{-ax^2+a}a^3} + \frac{4x}{(-ax^2+a)^{\frac{3}{2}}a^2} + \frac{3x}{(-ax^2+a)^{\frac{5}{2}}a}\right)\operatorname{arcoth}(x) - \frac{8}{15\sqrt{-ax^2+a}a^3} - \frac{4}{45(-ax^2+a)^{\frac{3}{2}}a^2} - \frac{1}{25(-ax^2+a)^{\frac{5}{2}}a}$$

input
```
integrate(arccoth(x)/(-a*x^2+a)^(7/2),x, algorithm="maxima")
```

output
```
1/15*(8*x/(sqrt(-a*x^2 + a)*a^3) + 4*x/((-a*x^2 + a)^(3/2)*a^2) + 3*x/((-a
*x^2 + a)^(5/2)*a))*arccoth(x) - 8/15/(sqrt(-a*x^2 + a)*a^3) - 4/45/((-a*x
^2 + a)^(3/2)*a^2) - 1/25/((-a*x^2 + a)^(5/2)*a)
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{7/2}}\,dx = -\frac{\sqrt{-ax^2+a}\left(4\,x^2\left(\frac{2\,x^2}{a}-\frac{5}{a}\right)+\frac{15}{a}\right)x\log\left(-\frac{\frac{1}{x}+1}{\frac{1}{x}-1}\right)}{30\,(ax^2-a)^3} - \frac{120\,(ax^2-a)^2-20\,(ax^2-a)a+9\,a^2}{225\,(ax^2-a)^2\sqrt{-ax^2+a}a^3}$$

input
```
integrate(arccoth(x)/(-a*x^2+a)^(7/2),x, algorithm="giac")
```

output
```
-1/30*sqrt(-a*x^2 + a)*(4*x^2*(2*x^2/a - 5/a) + 15/a)*x*log(-(1/x + 1)/(1/
x - 1))/(a*x^2 - a)^3 - 1/225*(120*(a*x^2 - a)^2 - 20*(a*x^2 - a)*a + 9*a^
2)/((a*x^2 - a)^2*sqrt(-a*x^2 + a)*a^3)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{7/2}}\,dx = \int \frac{\mathrm{acoth}(x)}{(a-a\,x^2)^{7/2}}\,dx$$

input
```
int(acoth(x)/(a - a*x^2)^(7/2),x)
```

output
```
int(acoth(x)/(a - a*x^2)^(7/2), x)
```

3.53 $\int \frac{1}{(1-x^2)\coth^{-1}(x)}\,dx$

3.53.1 Optimal result

Integrand size = 14, antiderivative size = 3

$$\int \frac{1}{(1-x^2)\coth^{-1}(x)}\,dx = \log\left(\coth^{-1}(x)\right)$$

output
```
ln(arccoth(x))
```

3.53.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-x^2)\coth^{-1}(x)}\,dx = \log\left(\coth^{-1}(x)\right)$$

input
```
Integrate[1/((1 - x^2)*ArcCoth[x]),x]
```

output
```
Log[ArcCoth[x]]
```

3.53.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {6509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x^2)\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{ 6509}$$

$$\log\left(\coth^{-1}(x)\right)$$

input
```
Int[1/((1 - x^2)*ArcCoth[x]),x]
```

output
```
Log[ArcCoth[x]]
```

3.53.3.1 Defintions of rubi rules used

rule 6509
```
Int[1/(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol
] :> Simp[Log[RemoveContent[a + b*ArcCoth[c*x], x]]/(b*c*d), x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

3.53.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
default	$\ln\left(\operatorname{arccoth}\left(x\right)\right)$	4
parallelrisch	$\ln\left(\operatorname{arccoth}\left(x\right)\right)$	4
risch	$\ln\left(-\ln\left(1+x\right)+\ln\left(x-1\right)\right)$	13

input
```
int(1/(-x^2+1)/arccoth(x),x,method=_RETURNVERBOSE)
```

output
```
ln(arccoth(x))
```

3.53.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(3) = 6.

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \frac{1}{(1-x^2)\coth^{-1}(x)}\,dx = \log\left(\log\left(\frac{x+1}{x-1}\right)\right)$$

input
```
integrate(1/(-x^2+1)/arccoth(x),x, algorithm="fricas")
```

output
```
log(log((x + 1)/(x - 1)))
```

3.53.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-x^2)\coth^{-1}(x)}\,dx = \log\left(\operatorname{acoth}\left(x\right)\right)$$

input
```
integrate(1/(-x**2+1)/acoth(x),x)
```

output
```
log(acoth(x))
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-x^2)\coth^{-1}(x)}\,dx = \log\left(\operatorname{arcoth}\left(x\right)\right)$$

input
```
integrate(1/(-x^2+1)/arccoth(x),x, algorithm="maxima")
```

output
```
log(arccoth(x))
```

3.53.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(3) = 6$.

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

$$\int \frac{1}{(1-x^2)\coth^{-1}(x)}\,dx = \log\left(\left|\log\left(\frac{x+1}{x-1}\right)\right|\right)$$

input
```
integrate(1/(-x^2+1)/arccoth(x),x, algorithm="giac")
```

output
```
log(abs(log((x + 1)/(x - 1))))
```

3.53.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-x^2)\coth^{-1}(x)}\,dx = \ln\left(\operatorname{acoth}(x)\right)$$

input
```
int(-1/(acoth(x)*(x^2 - 1)),x)
```

output
```
log(acoth(x))
```

3.54 $\int \frac{\coth^{-1}(x)^n}{1-x^2} dx$

3.54.1 Optimal result

Integrand size = 14, antiderivative size = 12

$$\int \frac{\coth^{-1}(x)^n}{1-x^2} dx = \frac{\coth^{-1}(x)^{1+n}}{1+n}$$

output
```
arccoth(x)^(1+n)/(1+n)
```

3.54.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(x)^n}{1-x^2} dx = \frac{\coth^{-1}(x)^{1+n}}{1+n}$$

input
```
Integrate[ArcCoth[x]^n/(1 - x^2),x]
```

output
```
ArcCoth[x]^(1 + n)/(1 + n)
```

3.54.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {6511}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(x)^n}{1-x^2} \, dx$$
$$\Big\downarrow \; 6511$$
$$\frac{\coth^{-1}(x)^{n+1}}{n+1}$$

input
```
Int[ArcCoth[x]^n/(1 - x^2),x]
```

output
```
ArcCoth[x]^(1 + n)/(1 + n)
```

3.54.3.1 Defintions of rubi rules used

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

3.54.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\operatorname{arccoth}(x)^{1+n}}{1+n}$	13

input
```
int(arccoth(x)^n/(-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
arccoth(x)^(1+n)/(1+n)
```

3.54.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 5.17

$$\int \frac{\coth^{-1}(x)^n}{1-x^2}\,dx = \frac{\cosh\left(n\log\left(\frac{1}{2}\log\left(\frac{x+1}{x-1}\right)\right)\right)\log\left(\frac{x+1}{x-1}\right)+\log\left(\frac{x+1}{x-1}\right)\sinh\left(n\log\left(\frac{1}{2}\log\left(\frac{x+1}{x-1}\right)\right)\right)}{2\,(n+1)}$$

input
```
integrate(arccoth(x)^n/(-x^2+1),x, algorithm="fricas")
```

output
```
1/2*(cosh(n*log(1/2*log((x + 1)/(x - 1))))*log((x + 1)/(x - 1)) + log((x +
 1)/(x - 1))*sinh(n*log(1/2*log((x + 1)/(x - 1)))))/(n + 1)
```

3.54.6 Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\coth^{-1}(x)^n}{1-x^2}\,dx = \begin{cases} \frac{\operatorname{acoth}^{n+1}(x)}{n+1} & \text{for } n \neq -1 \\ \log\left(\operatorname{acoth}(x)\right) & \text{otherwise} \end{cases}$$

input
```
integrate(acoth(x)**n/(-x**2+1),x)
```

output
```
Piecewise((acoth(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(acoth(x)), True))
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(x)^n}{1-x^2}\,dx = \frac{\operatorname{arcoth}(x)^{n+1}}{n+1}$$

input
```
integrate(arccoth(x)^n/(-x^2+1),x, algorithm="maxima")
```

output
```
arccoth(x)^(n + 1)/(n + 1)
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^{-1}(x)^n}{1-x^2}\,dx = \frac{\left(\frac{1}{2}\log\left(\frac{x+1}{x-1}\right)\right)^{n+1}}{n+1}$$

input
```
integrate(arccoth(x)^n/(-x^2+1),x, algorithm="giac")
```

output
```
(1/2*log((x + 1)/(x - 1)))^(n + 1)/(n + 1)
```

3.54.9 Mupad [B] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^{-1}(x)^n}{1-x^2}\,dx = \begin{cases} \ln\left(\mathrm{acoth}(x)\right) & \text{if } n=-1 \\ \frac{\mathrm{acoth}(x)^{n+1}}{n+1} & \text{if } n\neq -1 \end{cases}$$

input
```
int(-acoth(x)^n/(x^2 - 1),x)
```

output
```
piecewise(n == -1, log(acoth(x)), n ~= -1, acoth(x)^(n + 1)/(n + 1))
```

3.55 $\int \frac{\coth^{-1}(x)^2}{(1-x^2)^2}\,dx$

3.55.1 Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \frac{\coth^{-1}(x)^2}{(1-x^2)^2}\,dx = \frac{x}{4(1-x^2)} - \frac{\coth^{-1}(x)}{2(1-x^2)} + \frac{x\coth^{-1}(x)^2}{2(1-x^2)} + \frac{1}{6}\coth^{-1}(x)^3 + \frac{\operatorname{arctanh}(x)}{4}$$

output
```
1/4*x/(-x^2+1)-1/2*arccoth(x)/(-x^2+1)+1/2*x*arccoth(x)^2/(-x^2+1)+1/6*arc
coth(x)^3+1/4*arctanh(x)
```

3.55.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{\coth^{-1}(x)^2}{(1-x^2)^2}\,dx$$
$$= \frac{-6x + 12\coth^{-1}(x) - 12x\coth^{-1}(x)^2 + 4(-1+x^2)\coth^{-1}(x)^3 - 3(-1+x^2)\log(1-x) + 3(-1+x^2)\log}{24(-1+x^2)}$$

input
```
Integrate[ArcCoth[x]^2/(1 - x^2)^2,x]
```

output
```
(-6*x + 12*ArcCoth[x] - 12*x*ArcCoth[x]^2 + 4*(-1 + x^2)*ArcCoth[x]^3 - 3*
(-1 + x^2)*Log[1 - x] + 3*(-1 + x^2)*Log[1 + x])/(24*(-1 + x^2))
```

3.55.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6519, 6557, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(x)^2}{(1-x^2)^2}dx$$

$$\Big\downarrow \text{ 6519}$$

$$-\int \frac{x\coth^{-1}(x)}{(1-x^2)^2}dx+\frac{x\coth^{-1}(x)^2}{2(1-x^2)}+\frac{1}{6}\coth^{-1}(x)^3$$

$$\Big\downarrow \text{ 6557}$$

$$\frac{1}{2}\int \frac{1}{(1-x^2)^2}dx+\frac{x\coth^{-1}(x)^2}{2(1-x^2)}-\frac{\coth^{-1}(x)}{2(1-x^2)}+\frac{1}{6}\coth^{-1}(x)^3$$

$$\Big\downarrow \text{ 215}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx+\frac{x}{2(1-x^2)}\right)+\frac{x\coth^{-1}(x)^2}{2(1-x^2)}-\frac{\coth^{-1}(x)}{2(1-x^2)}+\frac{1}{6}\coth^{-1}(x)^3$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{\text{arctanh}(x)}{2}+\frac{x}{2(1-x^2)}\right)+\frac{x\coth^{-1}(x)^2}{2(1-x^2)}-\frac{\coth^{-1}(x)}{2(1-x^2)}+\frac{1}{6}\coth^{-1}(x)^3$$

input
```
Int[ArcCoth[x]^2/(1 - x^2)^2,x]
```

output
```
-1/2*ArcCoth[x]/(1 - x^2) + (x*ArcCoth[x]^2)/(2*(1 - x^2)) + ArcCoth[x]^3/
6 + (x/(2*(1 - x^2)) + ArcTanh[x]/2)/2
```

3.55.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6519
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcCoth[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcCoth[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcCoth[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6557
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcCoth[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcCoth[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

3.55.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(52) = 104.

Time = 1.03 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.56

method	result
risch	$-\frac{\ln(x-1)^3}{48}+\frac{(x^2\ln(1+x)-2x-\ln(1+x))\ln(x-1)^2}{16x^2-16}-\frac{\left(x^2\ln(1+x)^2-4\ln(1+x)x-\ln(1+x)^2+4\right)\ln(x-1)}{16(x-1)(1+x)}+\frac{x^2\ln(1+x)^3+6x^2}{}$
default	$-\frac{\operatorname{arccoth}(x)^2}{4(1+x)}+\frac{\operatorname{arccoth}(x)^2\ln(1+x)}{4}-\frac{\operatorname{arccoth}(x)^2}{4(x-1)}-\frac{\operatorname{arccoth}(x)^2\ln(x-1)}{4}+\frac{\operatorname{arccoth}(x)^2\ln\left(\frac{x-1}{1+x}\right)}{4}+\frac{i\operatorname{arccoth}(x)^2\operatorname{csgn}(}{}$
parts	$-\frac{\operatorname{arccoth}(x)^2}{4(1+x)}+\frac{\operatorname{arccoth}(x)^2\ln(1+x)}{4}-\frac{\operatorname{arccoth}(x)^2}{4(x-1)}-\frac{\operatorname{arccoth}(x)^2\ln(x-1)}{4}+\frac{\operatorname{arccoth}(x)^2\ln\left(\frac{x-1}{1+x}\right)}{4}+\frac{i\operatorname{arccoth}(x)^2\operatorname{csgn}(}{}$

input
```
int(arccoth(x)^2/(-x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/48*ln(x-1)^3+1/16*(x^2*ln(1+x)-2*x-ln(1+x))/(x^2-1)*ln(x-1)^2-1/16*(x^2
*ln(1+x)^2-4*ln(1+x)*x-ln(1+x)^2+4)/(x-1)/(1+x)*ln(x-1)+1/48*(x^2*ln(1+x)^
3+6*x^2*ln(1+x)-6*ln(x-1)*x^2-6*x*ln(1+x)^2-ln(1+x)^3+6*ln(1+x)+6*ln(x-1)-
12*x)/(x-1)/(1+x)
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{\coth^{-1}(x)^2}{(1-x^2)^2}\,dx = \frac{(x^2-1)\log\left(\frac{x+1}{x-1}\right)^3 - 6\,x\log\left(\frac{x+1}{x-1}\right)^2 + 6\,(x^2+1)\log\left(\frac{x+1}{x-1}\right) - 12\,x}{48\,(x^2-1)}$$

input
```
integrate(arccoth(x)^2/(-x^2+1)^2,x, algorithm="fricas")
```

output
```
1/48*((x^2 - 1)*log((x + 1)/(x - 1))^3 - 6*x*log((x + 1)/(x - 1))^2 + 6*(x
^2 + 1)*log((x + 1)/(x - 1)) - 12*x)/(x^2 - 1)
```

3.55.6 Sympy [F]

$$\int \frac{\coth^{-1}(x)^2}{(1-x^2)^2}\,dx = \int \frac{\operatorname{acoth}^2(x)}{(x-1)^2\,(x+1)^2}\,dx$$

input
```
integrate(acoth(x)**2/(-x**2+1)**2,x)
```

output
```
Integral(acoth(x)**2/((x - 1)**2*(x + 1)**2), x)
```

3.55.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(46) = 92.

Time = 0.21 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.76

$$\int \frac{\coth^{-1}(x)^2}{(1-x^2)^2}\,dx = -\frac{1}{4}\left(\frac{2\,x}{x^2-1} - \log(x+1) + \log(x-1)\right)\operatorname{arcoth}(x)^2 - \frac{\left((x^2-1)\log(x+1)^2 - 2\,(x^2-1)\log(x+1)\log(x-1) + (x^2-1)\log(x-1)^2 - 4\right)\operatorname{arcoth}(x)}{8\,(x^2-1)} + \frac{(x^2-1)\log(x+1)^3 - 3\,(x^2-1)\log(x+1)^2\log(x-1) - (x^2-1)\log(x-1)^3 + 3\left((x^2-1)\log(x-\right.}{48\,(x^2-1)}$$

input
```
integrate(arccoth(x)^2/(-x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*x/(x^2 - 1) - log(x + 1) + log(x - 1))*arccoth(x)^2 - 1/8*((x^2 -
1)*log(x + 1)^2 - 2*(x^2 - 1)*log(x + 1)*log(x - 1) + (x^2 - 1)*log(x - 1)
^2 - 4)*arccoth(x)/(x^2 - 1) + 1/48*((x^2 - 1)*log(x + 1)^3 - 3*(x^2 - 1)*
log(x + 1)^2*log(x - 1) - (x^2 - 1)*log(x - 1)^3 + 3*((x^2 - 1)*log(x - 1)
^2 + 2*x^2 - 2)*log(x + 1) - 6*(x^2 - 1)*log(x - 1) - 12*x)/(x^2 - 1)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{\coth^{-1}(x)^2}{(1-x^2)^2}\,dx = -\frac{(x-1)\log\left(\frac{x+1}{x-1}\right)^2}{16\,(x+1)} - \frac{(x-1)\log\left(\frac{x+1}{x-1}\right)}{8\,(x+1)} - \frac{x-1}{8\,(x+1)}$$

input
```
integrate(arccoth(x)^2/(-x^2+1)^2,x, algorithm="giac")
```

output
```
-1/16*(x - 1)*log((x + 1)/(x - 1))^2/(x + 1) - 1/8*(x - 1)*log((x + 1)/(x
- 1))/(x + 1) - 1/8*(x - 1)/(x + 1)
```

3.55.9 Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.24

$$\int \frac{\coth^{-1}(x)^2}{(1-x^2)^2}\,dx = \frac{\ln\left(\frac{1}{x}+1\right)^3}{48} - \frac{\ln\left(1-\frac{1}{x}\right)^3}{48} - \frac{x}{4\,(x^2-1)} + \ln\left(1-\frac{1}{x}\right)\left(\frac{\frac{3x}{32}-\frac{1}{8}}{x^2-1} - \frac{\frac{x}{8}+\frac{1}{8}}{x^2-1} - \frac{\ln\left(\frac{1}{x}+1\right)^2}{16} + \frac{x}{32\,(x^2-1)} + \ln\left(\frac{1}{x}+1\right)\left(\frac{\frac{x}{4}+\frac{1}{16}}{x^2-1} - \frac{1}{16\,(x^2-1)}\right)\right) + \ln\left(1-\frac{1}{x}\right)^2\left(\frac{\ln\left(\frac{1}{x}+1\right)}{16} - \frac{x}{8\,(x^2-1)}\right) + \frac{\ln\left(\frac{1}{x}+1\right)}{4\,(x^2-1)} - \frac{x\ln\left(\frac{1}{x}+1\right)^2}{8\,(x^2-1)} - \frac{\mathrm{atan}(x\,1\mathrm{i})\,1\mathrm{i}}{4}$$

input
```
int(acoth(x)^2/(x^2 - 1)^2,x)
```

output
```
log(1/x + 1)^3/48 - (atan(x*1i)*1i)/4 - log(1 - 1/x)^3/48 - x/(4*(x^2 - 1)
) + log(1 - 1/x)*(((3*x)/32 - 1/8)/(x^2 - 1) - (x/8 + 1/8)/(x^2 - 1) - log
(1/x + 1)^2/16 + x/(32*(x^2 - 1)) + log(1/x + 1)*((x/4 + 1/16)/(x^2 - 1) -
 1/(16*(x^2 - 1)))) + log(1 - 1/x)^2*(log(1/x + 1)/16 - x/(8*(x^2 - 1))) +
 log(1/x + 1)/(4*(x^2 - 1)) - (x*log(1/x + 1)^2)/(8*(x^2 - 1))
```

3.56 $\int \frac{x \coth^{-1}(x)}{1-x^2} dx$

3.56.1 Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{x \coth^{-1}(x)}{1-x^2} dx = -\frac{1}{2}\coth^{-1}(x)^2 + \coth^{-1}(x)\log\left(\frac{2}{1-x}\right) + \frac{1}{2}\text{PolyLog}\left(2, \frac{1+x}{-1+x}\right)$$

output
```
-1/2*arccoth(x)^2+arccoth(x)*ln(2/(1-x))+1/2*polylog(2,(1+x)/(-1+x))
```

3.56.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{x \coth^{-1}(x)}{1-x^2} dx = -\frac{1}{2}\coth^{-1}(x)^2 + \coth^{-1}(x)\log\left(1-e^{2\coth^{-1}(x)}\right) + \frac{1}{2}\text{PolyLog}\left(2, e^{2\coth^{-1}(x)}\right)$$

input
```
Integrate[(x*ArcCoth[x])/(1 - x^2),x]
```

output
```
-1/2*ArcCoth[x]^2 + ArcCoth[x]*Log[1 - E^(2*ArcCoth[x])] + PolyLog[2, E^(2
*ArcCoth[x])]/2
```

3.56.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6547, 6471, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \coth^{-1}(x)}{1-x^2} \, dx$$

$$\Big\downarrow \text{6547}$$

$$\int \frac{\coth^{-1}(x)}{1-x}dx-\frac{1}{2}\coth^{-1}(x)^2$$

$$\Big\downarrow \text{6471}$$

$$-\int \frac{\log\left(\frac{2}{1-x}\right)}{1-x^2}dx-\frac{1}{2}\coth^{-1}(x)^2+\log\left(\frac{2}{1-x}\right)\coth^{-1}(x)$$

$$\Big\downarrow \text{2849}$$

$$\int \frac{\log\left(\frac{2}{1-x}\right)}{1-\frac{2}{1-x}}d\frac{1}{1-x}-\frac{1}{2}\coth^{-1}(x)^2+\log\left(\frac{2}{1-x}\right)\coth^{-1}(x)$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{1-x}\right)-\frac{1}{2}\coth^{-1}(x)^2+\log\left(\frac{2}{1-x}\right)\coth^{-1}(x)$$

input
```
Int[(x*ArcCoth[x])/(1 - x^2),x]
```

output
```
-1/2*ArcCoth[x]^2 + ArcCoth[x]*Log[2/(1 - x)] + PolyLog[2, 1 - 2/(1 - x)]/
2
```

3.56.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6471
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6547
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.56.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

method	result
risch	$-\frac{\ln(1+x)^2}{8}-\frac{\left(\ln(1+x)-\ln\left(\frac{1}{2}+\frac{x}{2}\right)\right)\ln\left(\frac{1}{2}-\frac{x}{2}\right)}{4}+\frac{\operatorname{dilog}\left(\frac{1}{2}+\frac{x}{2}\right)}{2}+\frac{\ln(x-1)\ln\left(\frac{1}{2}+\frac{x}{2}\right)}{4}+\frac{\ln(x-1)^2}{8}$
default	$-\frac{\operatorname{arccoth}(x)\ln(x-1)}{2}-\frac{\operatorname{arccoth}(x)\ln(1+x)}{2}+\frac{\ln(1+x)^2}{8}-\frac{\left(\ln(1+x)-\ln\left(\frac{1}{2}+\frac{x}{2}\right)\right)\ln\left(\frac{1}{2}-\frac{x}{2}\right)}{4}+\frac{\operatorname{dilog}\left(\frac{1}{2}+\frac{x}{2}\right)}{2}+\frac{\ln(x-1)\ln(\frac{1}{2}-}{4}$
parts	$-\frac{\operatorname{arccoth}(x)\ln(x-1)}{2}-\frac{\operatorname{arccoth}(x)\ln(1+x)}{2}+\frac{\ln(1+x)^2}{8}-\frac{\left(\ln(1+x)-\ln\left(\frac{1}{2}+\frac{x}{2}\right)\right)\ln\left(\frac{1}{2}-\frac{x}{2}\right)}{4}+\frac{\operatorname{dilog}\left(\frac{1}{2}+\frac{x}{2}\right)}{2}+\frac{\ln(x-1)\ln(\frac{1}{2}-}{4}$

input
```
int(x*arccoth(x)/(-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(1+x)^2-1/4*(ln(1+x)-ln(1/2+1/2*x))*ln(1/2-1/2*x)+1/2*dilog(1/2+1/2
*x)+1/4*ln(x-1)*ln(1/2+1/2*x)+1/8*ln(x-1)^2
```

3.56.5 Fricas [F]

$$\int \frac{x\coth^{-1}(x)}{1-x^2}\,dx = \int -\frac{x\,\text{arcoth}\,(x)}{x^2-1}\,dx$$

input
```
integrate(x*arccoth(x)/(-x^2+1),x, algorithm="fricas")
```

output
```
integral(-x*arccoth(x)/(x^2 - 1), x)
```

3.56.6 Sympy [F]

$$\int \frac{x\coth^{-1}(x)}{1-x^2}\,dx = -\int \frac{x\,\text{acoth}\,(x)}{x^2-1}\,dx$$

input
```
integrate(x*acoth(x)/(-x**2+1),x)
```

output
```
-Integral(x*acoth(x)/(x**2 - 1), x)
```

3.56.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(30) = 60.

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.05

$$\begin{aligned}\int \frac{x\coth^{-1}(x)}{1-x^2}\,dx &= \frac{1}{4}\left(\log\left(x+1\right)-\log\left(x-1\right)\right)\log\left(x^2-1\right)-\frac{1}{2}\,\text{arcoth}\,(x)\log\left(x^2-1\right)\\&\quad-\frac{1}{8}\log\left(x+1\right)^2-\frac{1}{4}\log\left(x+1\right)\log\left(x-1\right)+\frac{1}{8}\log\left(x-1\right)^2\\&\quad+\frac{1}{2}\log\left(x-1\right)\log\left(\frac{1}{2}x+\frac{1}{2}\right)+\frac{1}{2}\,\text{Li}_2\left(-\frac{1}{2}x+\frac{1}{2}\right)\end{aligned}$$

input
```
integrate(x*arccoth(x)/(-x^2+1),x, algorithm="maxima")
```

output
```
1/4*(log(x + 1) - log(x - 1))*log(x^2 - 1) - 1/2*arccoth(x)*log(x^2 - 1) -
 1/8*log(x + 1)^2 - 1/4*log(x + 1)*log(x - 1) + 1/8*log(x - 1)^2 + 1/2*log
(x - 1)*log(1/2*x + 1/2) + 1/2*dilog(-1/2*x + 1/2)
```

3.56.8 Giac [F]

$$\int \frac{x \coth^{-1}(x)}{1-x^2} \, dx = \int -\frac{x \operatorname{arcoth}(x)}{x^2-1} \, dx$$

input
```
integrate(x*arccoth(x)/(-x^2+1),x, algorithm="giac")
```

output
```
integrate(-x*arccoth(x)/(x^2 - 1), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \coth^{-1}(x)}{1-x^2} \, dx = -\int \frac{x \operatorname{acoth}(x)}{x^2-1} \, dx$$

input
```
int(-(x*acoth(x))/(x^2 - 1),x)
```

output
```
-int((x*acoth(x))/(x^2 - 1), x)
```

3.57 $\int \frac{\coth^{-1}(x)}{1-x^2} dx$

3.57.1 Optimal result

Integrand size = 12, antiderivative size = 8

$$\int \frac{\coth^{-1}(x)}{1-x^2} dx = \frac{1}{2}\coth^{-1}(x)^2$$

output
```
1/2*arccoth(x)^2
```

3.57.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(x)}{1-x^2} dx = \frac{1}{2}\coth^{-1}(x)^2$$

input
```
Integrate[ArcCoth[x]/(1 - x^2),x]
```

output
```
ArcCoth[x]^2/2
```

3.57.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6511}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(x)}{1-x^2} \, dx$$

$$\Big\downarrow \; 6511$$

$$\frac{1}{2}\coth^{-1}(x)^2$$

input
```
Int[ArcCoth[x]/(1 - x^2),x]
```

output
```
ArcCoth[x]^2/2
```

3.57.3.1 Defintions of rubi rules used

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

3.57.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.62

method	result	size
default	$\operatorname{arctanh}(x) \operatorname{arccoth}(x) - \frac{\operatorname{arctanh}(x)^2}{2}$	13
parts	$\operatorname{arctanh}(x) \operatorname{arccoth}(x) - \frac{\operatorname{arctanh}(x)^2}{2}$	13
risch	$\frac{\ln(x-1)^2}{8} - \frac{\ln(1+x)\ln(x-1)}{4} + \frac{\ln(1+x)^2}{8}$	28

input
```
int(arccoth(x)/(-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
arctanh(x)*arccoth(x)-1/2*arctanh(x)^2
```

3.57.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{\coth^{-1}(x)}{1-x^2} \, dx = \frac{1}{8} \log \left(\frac{x+1}{x-1}\right)^2$$

input
```
integrate(arccoth(x)/(-x^2+1),x, algorithm="fricas")
```

output
```
1/8*log((x + 1)/(x - 1))^2
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{\coth^{-1}(x)}{1-x^2} \, dx = \frac{\operatorname{acoth}^2(x)}{2}$$

input
```
integrate(acoth(x)/(-x**2+1),x)
```

output
```
acoth(x)**2/2
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\coth^{-1}(x)}{1-x^2} \, dx = \frac{1}{2} \operatorname{arcoth}(x)^2$$

input
```
integrate(arccoth(x)/(-x^2+1),x, algorithm="maxima")
```

output
```
1/2*arccoth(x)^2
```

3.57.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.

Time $= 0.27$ (sec) , antiderivative size $= 14$, normalized size of antiderivative $= 1.75$

$$\int \frac{\coth^{-1}(x)}{1-x^2}\,dx = \frac{1}{8}\log\left(\frac{x+1}{x-1}\right)^2$$

input
```
integrate(arccoth(x)/(-x^2+1),x, algorithm="giac")
```

output
```
1/8*log((x + 1)/(x - 1))^2
```

3.57.9 Mupad [B] (verification not implemented)

Time $= 4.09$ (sec) , antiderivative size $= 21$, normalized size of antiderivative $= 2.62$

$$\int \frac{\coth^{-1}(x)}{1-x^2}\,dx = \frac{\left(\ln\left(1-\frac{1}{x}\right)-\ln\left(\frac{1}{x}+1\right)\right)^2}{8}$$

input
```
int(-acoth(x)/(x^2 - 1),x)
```

output
```
(log(1 - 1/x) - log(1/x + 1))^2/8
```

3.58 $\int \frac{x \coth^{-1}(x)}{(1-x^2)^2} dx$

3.58.1 Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{x \coth^{-1}(x)}{(1-x^2)^2} dx = -\frac{x}{4(1-x^2)} + \frac{\coth^{-1}(x)}{2(1-x^2)} - \frac{\operatorname{arctanh}(x)}{4}$$

output
```
-1/4*x/(-x^2+1)+1/2*arccoth(x)/(-x^2+1)-1/4*arctanh(x)
```

3.58.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{x \coth^{-1}(x)}{(1-x^2)^2} dx = \frac{x}{4(-1+x^2)} - \frac{\coth^{-1}(x)}{2(-1+x^2)} + \frac{1}{8}\log(1-x) - \frac{1}{8}\log(1+x)$$

input
```
Integrate[(x*ArcCoth[x])/(1 - x^2)^2,x]
```

output
```
x/(4*(-1 + x^2)) - ArcCoth[x]/(2*(-1 + x^2)) + Log[1 - x]/8 - Log[1 + x]/8
```

3.58.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6557, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \coth^{-1}(x)}{(1-x^2)^2} dx$$

$$\Big\downarrow \text{6557}$$

$$\frac{\coth^{-1}(x)}{2(1-x^2)} - \frac{1}{2}\int \frac{1}{(1-x^2)^2}dx$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-x^2}dx - \frac{x}{2(1-x^2)}\right) + \frac{\coth^{-1}(x)}{2(1-x^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{\text{arctanh}(x)}{2} - \frac{x}{2(1-x^2)}\right) + \frac{\coth^{-1}(x)}{2(1-x^2)}$$

input
```
Int[(x*ArcCoth[x])/(1 - x^2)^2,x]
```

output
```
ArcCoth[x]/(2*(1 - x^2)) + (-1/2*x/(1 - x^2) - ArcTanh[x]/2)/2
```

3.58.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6557
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcCoth[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcCoth[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

3.58.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

method	result	size
parallelrisch	$-\frac{x^2 \operatorname{arccoth}(x)-x+\operatorname{arccoth}(x)}{4(x^2-1)}$	22
default	$-\frac{\operatorname{arccoth}(x)}{2(x^2-1)}+\frac{1}{8+8x}-\frac{\ln(1+x)}{8}+\frac{1}{8x-8}+\frac{\ln(x-1)}{8}$	39
parts	$-\frac{\operatorname{arccoth}(x)}{2(x^2-1)}+\frac{1}{8+8x}-\frac{\ln(1+x)}{8}+\frac{1}{8x-8}+\frac{\ln(x-1)}{8}$	39
risch	$\frac{\ln(x-1)}{4x^2-4}+\frac{\ln(x-1)x^2-x^2\ln(1+x)-\ln(x-1)-\ln(1+x)+2x}{8(x-1)(1+x)}$	60

input
```
int(x*arccoth(x)/(-x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(x^2*arccoth(x)-x+arccoth(x))/(x^2-1)
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{x \coth^{-1}(x)}{(1-x^2)^2}\, dx = -\frac{(x^2+1)\log\left(\frac{x+1}{x-1}\right)-2\,x}{8\,(x^2-1)}$$

input
```
integrate(x*arccoth(x)/(-x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/8*((x^2 + 1)*log((x + 1)/(x - 1)) - 2*x)/(x^2 - 1)
```

3.58.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{x\coth^{-1}(x)}{(1-x^2)^2}\,dx = -\frac{x^2\operatorname{acoth}(x)}{4x^2-4} + \frac{x}{4x^2-4} - \frac{\operatorname{acoth}(x)}{4x^2-4}$$

input
```
integrate(x*acoth(x)/(-x**2+1)**2,x)
```

output
```
-x**2*acoth(x)/(4*x**2 - 4) + x/(4*x**2 - 4) - acoth(x)/(4*x**2 - 4)
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{x\coth^{-1}(x)}{(1-x^2)^2}\,dx = \frac{x}{4(x^2-1)} - \frac{\operatorname{arcoth}(x)}{2(x^2-1)} - \frac{1}{8}\log(x+1) + \frac{1}{8}\log(x-1)$$

input
```
integrate(x*arccoth(x)/(-x^2+1)^2,x, algorithm="maxima")
```

output
```
1/4*x/(x^2 - 1) - 1/2*arccoth(x)/(x^2 - 1) - 1/8*log(x + 1) + 1/8*log(x -
1)
```

3.58.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(26) = 52$.

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.81

$$\int \frac{x\coth^{-1}(x)}{(1-x^2)^2}\,dx = -\frac{1}{16}\left(\frac{x+1}{x-1} + \frac{x-1}{x+1}\right)\log\left(-\frac{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}+1}{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}-1}\right) + \frac{x+1}{16(x-1)} - \frac{x-1}{16(x+1)}$$

input
```
integrate(x*arccoth(x)/(-x^2+1)^2,x, algorithm="giac")
```

output
```
-1/16*((x + 1)/(x - 1) + (x - 1)/(x + 1))*log(-(((x + 1)/(x - 1) - 1)/((x
+ 1)/(x - 1) + 1) + 1)/(((x + 1)/(x - 1) - 1)/((x + 1)/(x - 1) + 1) - 1))
+ 1/16*(x + 1)/(x - 1) - 1/16*(x - 1)/(x + 1)
```

3.58.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

$$\int \frac{x \coth^{-1}(x)}{(1-x^2)^2} \, dx = \frac{\frac{x}{4} - \frac{\operatorname{acoth}(x)}{2}}{x^2-1} - \frac{\operatorname{acoth}(x)}{4}$$

input
```
int((x*acoth(x))/(x^2 - 1)^2,x)
```

output
```
(x/4 - acoth(x)/2)/(x^2 - 1) - acoth(x)/4
```

3.59 $\int \frac{\coth^{-1}(x)}{(1-x^2)^2}\,dx$

3.59.1 Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^2}\,dx = -\frac{1}{4(1-x^2)} + \frac{x\coth^{-1}(x)}{2(1-x^2)} + \frac{1}{4}\coth^{-1}(x)^2$$

output
```
-1/4/(-x^2+1)+1/2*x*arccoth(x)/(-x^2+1)+1/4*arccoth(x)^2
```

3.59.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^2}\,dx = \frac{1-2x\coth^{-1}(x)+(-1+x^2)\coth^{-1}(x)^2}{4(-1+x^2)}$$

input
```
Integrate[ArcCoth[x]/(1 - x^2)^2,x]
```

output
```
(1 - 2*x*ArcCoth[x] + (-1 + x^2)*ArcCoth[x]^2)/(4*(-1 + x^2))
```

3.59.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6519, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^2} \, dx$$

$$\Big\downarrow \; 6519$$

$$-\frac{1}{2}\int \frac{x}{(1-x^2)^2}dx+\frac{x\coth^{-1}(x)}{2(1-x^2)}+\frac{1}{4}\coth^{-1}(x)^2$$

$$\Big\downarrow \; 241$$

$$-\frac{1}{4(1-x^2)}+\frac{x\coth^{-1}(x)}{2(1-x^2)}+\frac{1}{4}\coth^{-1}(x)^2$$

input
```
Int[ArcCoth[x]/(1 - x^2)^2,x]
```

output
```
-1/4*1/(1 - x^2) + (x*ArcCoth[x])/(2*(1 - x^2)) + ArcCoth[x]^2/4
```

3.59.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6519
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcCoth[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcCoth[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcCoth[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.59.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

method	result
risch	$\frac{\ln(x-1)^2}{16}-\frac{(x^2\ln(1+x)-2x-\ln(1+x))\ln(x-1)}{8(x^2-1)}+\frac{x^2\ln(1+x)^2-4\ln(1+x)x-\ln(1+x)^2+4}{16(x-1)(1+x)}$
default	$-\frac{\operatorname{arccoth}(x)}{4(1+x)}+\frac{\operatorname{arccoth}(x)\ln(1+x)}{4}-\frac{\operatorname{arccoth}(x)}{4(x-1)}-\frac{\operatorname{arccoth}(x)\ln(x-1)}{4}-\frac{\ln(1+x)^2}{16}+\frac{(\ln(1+x)-\ln(\frac{1}{2}+\frac{x}{2}))\ln(\frac{1}{2}-\frac{x}{2})}{8}+$
parts	$-\frac{\operatorname{arccoth}(x)}{4(1+x)}+\frac{\operatorname{arccoth}(x)\ln(1+x)}{4}-\frac{\operatorname{arccoth}(x)}{4(x-1)}-\frac{\operatorname{arccoth}(x)\ln(x-1)}{4}-\frac{\ln(1+x)^2}{16}+\frac{(\ln(1+x)-\ln(\frac{1}{2}+\frac{x}{2}))\ln(\frac{1}{2}-\frac{x}{2})}{8}+$

input
```
int(arccoth(x)/(-x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*ln(x-1)^2-1/8*(x^2*ln(1+x)-2*x-ln(1+x))/(x^2-1)*ln(x-1)+1/16*(x^2*ln(
1+x)^2-4*ln(1+x)*x-ln(1+x)^2+4)/(x-1)/(1+x)
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^2}\,dx = \frac{(x^2-1)\log\left(\frac{x+1}{x-1}\right)^2 - 4\,x\log\left(\frac{x+1}{x-1}\right) + 4}{16\,(x^2-1)}$$

input
```
integrate(arccoth(x)/(-x^2+1)^2,x, algorithm="fricas")
```

output
```
1/16*((x^2 - 1)*log((x + 1)/(x - 1))^2 - 4*x*log((x + 1)/(x - 1)) + 4)/(x^
2 - 1)
```

3.59.6 Sympy [F]

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^2}\,dx = \int \frac{\operatorname{acoth}(x)}{(x-1)^2(x+1)^2}\,dx$$

input
```
integrate(acoth(x)/(-x**2+1)**2,x)
```

output
```
Integral(acoth(x)/((x - 1)**2*(x + 1)**2), x)
```

3.59.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(28) = 56.

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^2}\,dx$$
$$= -\frac{1}{4}\left(\frac{2\,x}{x^2-1} - \log(x+1) + \log(x-1)\right)\operatorname{arcoth}(x)$$
$$- \frac{(x^2-1)\log(x+1)^2 - 2\,(x^2-1)\log(x+1)\log(x-1) + (x^2-1)\log(x-1)^2 - 4}{16\,(x^2-1)}$$

input
```
integrate(arccoth(x)/(-x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*x/(x^2 - 1) - log(x + 1) + log(x - 1))*arccoth(x) - 1/16*((x^2 - 1
)*log(x + 1)^2 - 2*(x^2 - 1)*log(x + 1)*log(x - 1) + (x^2 - 1)*log(x - 1)^
2 - 4)/(x^2 - 1)
```

3.59.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(28) = 56.

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^2}\,dx = -\frac{(x-1)\log\left(-\frac{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}+1}{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}-1}\right)}{8\,(x+1)} - \frac{x-1}{8\,(x+1)}$$

input
```
integrate(arccoth(x)/(-x^2+1)^2,x, algorithm="giac")
```

output
```
-1/8*(x - 1)*log(-(((x + 1)/(x - 1) - 1)/((x + 1)/(x - 1) + 1) + 1)/(((x +
 1)/(x - 1) - 1)/((x + 1)/(x - 1) + 1) - 1))/(x + 1) - 1/8*(x - 1)/(x + 1)
```

3.59.9 Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.13

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^2}\,dx = \frac{\ln\left(\frac{1}{x}+1\right)^2}{16} - \ln\left(1-\frac{1}{x}\right)\left(\frac{\ln\left(\frac{1}{x}+1\right)}{8} - \frac{x}{4\,(x^2-1)}\right) + \frac{\ln\left(1-\frac{1}{x}\right)^2}{16} + \frac{1}{4\,(x^2-1)} - \frac{x\,\ln\left(\frac{1}{x}+1\right)}{4\,(x^2-1)}$$

input
```
int(acoth(x)/(x^2 - 1)^2,x)
```

output
```
log(1/x + 1)^2/16 - log(1 - 1/x)*(log(1/x + 1)/8 - x/(4*(x^2 - 1))) + log(
1 - 1/x)^2/16 + 1/(4*(x^2 - 1)) - (x*log(1/x + 1))/(4*(x^2 - 1))
```

3.60 $\int \frac{x\coth^{-1}(x)}{(1-x^2)^3}\,dx$

3.60.1 Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \frac{x\coth^{-1}(x)}{(1-x^2)^3}\,dx = -\frac{x}{16(1-x^2)^2} - \frac{3x}{32(1-x^2)} + \frac{\coth^{-1}(x)}{4(1-x^2)^2} - \frac{3\text{arctanh}(x)}{32}$$

output
```
-1/16*x/(-x^2+1)^2-3/32*x/(-x^2+1)+1/4*arccoth(x)/(-x^2+1)^2-3/32*arctanh(
x)
```

3.60.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{x\coth^{-1}(x)}{(1-x^2)^3}\,dx = \frac{1}{64}\left(-\frac{4x}{(-1+x^2)^2} + \frac{6x}{-1+x^2} + \frac{16\coth^{-1}(x)}{(-1+x^2)^2} + 3\log(1-x) - 3\log(1+x)\right)$$

input
```
Integrate[(x*ArcCoth[x])/(1 - x^2)^3,x]
```

output
```
((-4*x)/(-1 + x^2)^2 + (6*x)/(-1 + x^2) + (16*ArcCoth[x])/(-1 + x^2)^2 + 3
*Log[1 - x] - 3*Log[1 + x])/64
```

3.60.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6557, 215, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\coth^{-1}(x)}{(1-x^2)^3}dx$$

$$\Big\downarrow \text{6557}$$

$$\frac{\coth^{-1}(x)}{4(1-x^2)^2}-\frac{1}{4}\int \frac{1}{(1-x^2)^3}dx$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{4}\left(-\frac{3}{4}\int \frac{1}{(1-x^2)^2}dx-\frac{x}{4(1-x^2)^2}\right)+\frac{\coth^{-1}(x)}{4(1-x^2)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{4}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx+\frac{x}{2(1-x^2)}\right)-\frac{x}{4(1-x^2)^2}\right)+\frac{\coth^{-1}(x)}{4(1-x^2)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(-\frac{3}{4}\left(\frac{\text{arctanh}(x)}{2}+\frac{x}{2(1-x^2)}\right)-\frac{x}{4(1-x^2)^2}\right)+\frac{\coth^{-1}(x)}{4(1-x^2)^2}$$

input
```
Int[(x*ArcCoth[x])/(1 - x^2)^3,x]
```

output
```
ArcCoth[x]/(4*(1 - x^2)^2) + (-1/4*x/(1 - x^2)^2 - (3*(x/(2*(1 - x^2)) + A
rcTanh[x]/2))/4)/4
```

3.60.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6557
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcCoth[c*x])^p/(2*e*(q
+ 1))), x] + Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcCoth[c*
x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
 GtQ[p, 0] && NeQ[q, -1]
```

3.60.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$-\frac{3\,\mathrm{arccoth}(x)x^4-3x^3-6x^2\,\mathrm{arccoth}(x)+5x-5\,\mathrm{arccoth}(x)}{32(x^2-1)^2}$	37
default	$\frac{\mathrm{arccoth}(x)}{4(x^2-1)^2}+\frac{1}{64(1+x)^2}+\frac{3}{64(1+x)}-\frac{3\ln(1+x)}{64}-\frac{1}{64(x-1)^2}+\frac{3}{64(x-1)}+\frac{3\ln(x-1)}{64}$	53
parts	$\frac{\mathrm{arccoth}(x)}{4(x^2-1)^2}+\frac{1}{64(1+x)^2}+\frac{3}{64(1+x)}-\frac{3\ln(1+x)}{64}-\frac{1}{64(x-1)^2}+\frac{3}{64(x-1)}+\frac{3\ln(x-1)}{64}$	53
risch	$-\frac{\ln(x-1)}{8(x^2-1)^2}-\frac{3x^4\ln(1+x)-3\ln(x-1)x^4-6x^2\ln(1+x)+6\ln(x-1)x^2-6x^3-5\ln(1+x)-3\ln(x-1)+10x}{64(x-1)(1+x)(x^2-1)}$	91

input
```
int(x*arccoth(x)/(-x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/32*(3*arccoth(x)*x^4-3*x^3-6*x^2*arccoth(x)+5*x-5*arccoth(x))/(x^2-1)^2
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{x\coth^{-1}(x)}{(1-x^2)^3}\,dx = \frac{6\,x^3 - (3\,x^4 - 6\,x^2 - 5)\log\left(\frac{x+1}{x-1}\right) - 10\,x}{64\,(x^4 - 2\,x^2 + 1)}$$

input
```
integrate(x*arccoth(x)/(-x^2+1)^3,x, algorithm="fricas")
```

output
```
1/64*(6*x^3 - (3*x^4 - 6*x^2 - 5)*log((x + 1)/(x - 1)) - 10*x)/(x^4 - 2*x^
2 + 1)
```

3.60.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(37) = 74.

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.76

$$\int \frac{x\coth^{-1}(x)}{(1-x^2)^3}\,dx = -\frac{3x^4\operatorname{acoth}(x)}{32x^4-64x^2+32} + \frac{3x^3}{32x^4-64x^2+32} + \frac{6x^2\operatorname{acoth}(x)}{32x^4-64x^2+32} - \frac{5x}{32x^4-64x^2+32} + \frac{5\operatorname{acoth}(x)}{32x^4-64x^2+32}$$

input
```
integrate(x*acoth(x)/(-x**2+1)**3,x)
```

output
```
-3*x**4*acoth(x)/(32*x**4 - 64*x**2 + 32) + 3*x**3/(32*x**4 - 64*x**2 + 32
) + 6*x**2*acoth(x)/(32*x**4 - 64*x**2 + 32) - 5*x/(32*x**4 - 64*x**2 + 32
) + 5*acoth(x)/(32*x**4 - 64*x**2 + 32)
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{x\coth^{-1}(x)}{(1-x^2)^3}\,dx = \frac{3\,x^3 - 5\,x}{32\,(x^4 - 2\,x^2 + 1)} + \frac{\operatorname{arcoth}(x)}{4\,(x^2-1)^2} - \frac{3}{64}\,\log(x+1) + \frac{3}{64}\,\log(x-1)$$

input
```
integrate(x*arccoth(x)/(-x^2+1)^3,x, algorithm="maxima")
```

output
```
1/32*(3*x^3 - 5*x)/(x^4 - 2*x^2 + 1) + 1/4*arccoth(x)/(x^2 - 1)^2 - 3/64*l
og(x + 1) + 3/64*log(x - 1)
```

3.60.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(36) = 72$.

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.08

$$\int \frac{x\coth^{-1}(x)}{(1-x^2)^3}\,dx$$
$$= -\frac{1}{128}\left(\frac{(x-1)^2\left(\frac{4(x+1)}{x-1}-1\right)}{(x+1)^2} - \frac{(x+1)^2}{(x-1)^2} + \frac{4(x+1)}{x-1}\right)\log\left(-\frac{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}+1}{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}-1}\right)$$
$$- \frac{(x-1)^2\left(\frac{8(x+1)}{x-1}-1\right)}{256(x+1)^2} - \frac{(x+1)^2}{256(x-1)^2} + \frac{x+1}{32(x-1)}$$

input
```
integrate(x*arccoth(x)/(-x^2+1)^3,x, algorithm="giac")
```

output
```
-1/128*((x - 1)^2*(4*(x + 1)/(x - 1) - 1)/(x + 1)^2 - (x + 1)^2/(x - 1)^2
+ 4*(x + 1)/(x - 1))*log(-(((x + 1)/(x - 1) - 1)/((x + 1)/(x - 1) + 1) + 1
)/(((x + 1)/(x - 1) - 1)/((x + 1)/(x - 1) + 1) - 1)) - 1/256*(x - 1)^2*(8*
(x + 1)/(x - 1) - 1)/(x + 1)^2 - 1/256*(x + 1)^2/(x - 1)^2 + 1/32*(x + 1)/
(x - 1)
```

3.60.9 Mupad [B] (verification not implemented)

Time = 4.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{x\coth^{-1}(x)}{(1-x^2)^3}\,dx = \frac{3\ln(x-1)}{64} - \frac{3\ln(x+1)}{64} + \frac{\frac{\operatorname{acoth}(x)}{4} - \frac{5x}{32} + \frac{3x^3}{32}}{(x^2-1)^2}$$

input
```
int(-(x*acoth(x))/(x^2 - 1)^3,x)
```

output
```
(3*log(x - 1))/64 - (3*log(x + 1))/64 + (acoth(x)/4 - (5*x)/32 + (3*x^3)/3
2)/(x^2 - 1)^2
```

3.61 $\int \frac{\coth^{-1}(x)}{(1-x^2)^3}\,dx$

3.61.1 Optimal result

Integrand size = 12, antiderivative size = 67

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^3}\,dx = -\frac{1}{16(1-x^2)^2}-\frac{3}{16(1-x^2)}+\frac{x\coth^{-1}(x)}{4(1-x^2)^2}+\frac{3x\coth^{-1}(x)}{8(1-x^2)}+\frac{3}{16}\coth^{-1}(x)^2$$

output
```
-1/16/(-x^2+1)^2-3/16/(-x^2+1)+1/4*x*arccoth(x)/(-x^2+1)^2+3/8*x*arccoth(x
)/(-x^2+1)+3/16*arccoth(x)^2
```

3.61.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^3}\,dx = -\frac{4-3x^2+2x(-5+3x^2)\coth^{-1}(x)-3(-1+x^2)^2\coth^{-1}(x)^2}{16(-1+x^2)^2}$$

input
```
Integrate[ArcCoth[x]/(1 - x^2)^3,x]
```

output
```
-1/16*(4 - 3*x^2 + 2*x*(-5 + 3*x^2)*ArcCoth[x] - 3*(-1 + x^2)^2*ArcCoth[x]
^2)/(-1 + x^2)^2
```

3.61.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6523, 6519, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^3} \, dx$$

$$\Big\downarrow \; 6523$$

$$\frac{3}{4}\int \frac{\coth^{-1}(x)}{(1-x^2)^2}dx-\frac{1}{16(1-x^2)^2}+\frac{x\coth^{-1}(x)}{4(1-x^2)^2}$$

$$\Big\downarrow \; 6519$$

$$\frac{3}{4}\left(-\frac{1}{2}\int \frac{x}{(1-x^2)^2}dx+\frac{x\coth^{-1}(x)}{2(1-x^2)}+\frac{1}{4}\coth^{-1}(x)^2\right)-\frac{1}{16(1-x^2)^2}+\frac{x\coth^{-1}(x)}{4(1-x^2)^2}$$

$$\Big\downarrow \; 241$$

$$-\frac{1}{16(1-x^2)^2}+\frac{x\coth^{-1}(x)}{4(1-x^2)^2}+\frac{3}{4}\left(-\frac{1}{4(1-x^2)}+\frac{x\coth^{-1}(x)}{2(1-x^2)}+\frac{1}{4}\coth^{-1}(x)^2\right)$$

input
```
Int[ArcCoth[x]/(1 - x^2)^3,x]
```

output
```
-1/16*1/(1 - x^2)^2 + (x*ArcCoth[x])/(4*(1 - x^2)^2) + (3*(-1/4*1/(1 - x^2
) + (x*ArcCoth[x])/(2*(1 - x^2)) + ArcCoth[x]^2/4))/4
```

3.61.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6519
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcCoth[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a +
 b*ArcCoth[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*(
(a + b*ArcCoth[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6523
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d +
 e*x^2)^(q + 1)*((a + b*ArcCoth[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(
2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcCoth[c*x]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

3.61.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(57) = 114$.

Time = 0.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.91

method	result
risch	$\frac{3\ln(x-1)^2}{64} - \frac{(3x^4\ln(1+x)-6x^3-6x^2\ln(1+x)+10x+3\ln(1+x))\ln(x-1)}{32(x^2-1)^2} + \frac{3x^4\ln(1+x)^2-12x^3\ln(1+x)-6x^2\ln(1+x)^2+12x^2+}{64(x-1)(1+x)(x^2-1)}$
default	$-\frac{\operatorname{arccoth}(x)}{16(1+x)^2} - \frac{3\operatorname{arccoth}(x)}{16(1+x)} + \frac{3\operatorname{arccoth}(x)\ln(1+x)}{16} + \frac{\operatorname{arccoth}(x)}{16(x-1)^2} - \frac{3\operatorname{arccoth}(x)}{16(x-1)} - \frac{3\operatorname{arccoth}(x)\ln(x-1)}{16} - \frac{3\ln(1+x)^2}{64}$
parts	$-\frac{\operatorname{arccoth}(x)}{16(1+x)^2} - \frac{3\operatorname{arccoth}(x)}{16(1+x)} + \frac{3\operatorname{arccoth}(x)\ln(1+x)}{16} + \frac{\operatorname{arccoth}(x)}{16(x-1)^2} - \frac{3\operatorname{arccoth}(x)}{16(x-1)} - \frac{3\operatorname{arccoth}(x)\ln(x-1)}{16} - \frac{3\ln(1+x)^2}{64}$

input
```
int(arccoth(x)/(-x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
3/64*ln(x-1)^2-1/32*(3*x^4*ln(1+x)-6*x^3-6*x^2*ln(1+x)+10*x+3*ln(1+x))/(x^
2-1)^2*ln(x-1)+1/64*(3*x^4*ln(1+x)^2-12*x^3*ln(1+x)-6*x^2*ln(1+x)^2+12*x^2
+20*ln(1+x)*x+3*ln(1+x)^2-16)/(x-1)/(1+x)/(x^2-1)
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^3}\,dx = \frac{3\,(x^4-2\,x^2+1)\log\left(\frac{x+1}{x-1}\right)^2+12\,x^2-4\,(3\,x^3-5\,x)\log\left(\frac{x+1}{x-1}\right)-16}{64\,(x^4-2\,x^2+1)}$$

input
```
integrate(arccoth(x)/(-x^2+1)^3,x, algorithm="fricas")
```

output
```
1/64*(3*(x^4 - 2*x^2 + 1)*log((x + 1)/(x - 1))^2 + 12*x^2 - 4*(3*x^3 - 5*x
)*log((x + 1)/(x - 1)) - 16)/(x^4 - 2*x^2 + 1)
```

3.61.6 Sympy [F]

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^3}\,dx = -\int \frac{\operatorname{acoth}(x)}{x^6-3x^4+3x^2-1}\,dx$$

input
```
integrate(acoth(x)/(-x**2+1)**3,x)
```

output
```
-Integral(acoth(x)/(x**6 - 3*x**4 + 3*x**2 - 1), x)
```

3.61.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(49) = 98$.

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^3}\,dx = -\frac{1}{16}\left(\frac{2\,(3\,x^3-5\,x)}{x^4-2\,x^2+1} - 3\,\log(x+1) + 3\,\log(x-1)\right)\operatorname{arcoth}(x) - \frac{3\,(x^4-2\,x^2+1)\log(x+1)^2 - 6\,(x^4-2\,x^2+1)\log(x+1)\log(x-1) + 3\,(x^4-2\,x^2+1)\log(x-1)^2}{64\,(x^4-2\,x^2+1)}$$

input
```
integrate(arccoth(x)/(-x^2+1)^3,x, algorithm="maxima")
```

output
```
-1/16*(2*(3*x^3 - 5*x)/(x^4 - 2*x^2 + 1) - 3*log(x + 1) + 3*log(x - 1))*ar
ccoth(x) - 1/64*(3*(x^4 - 2*x^2 + 1)*log(x + 1)^2 - 6*(x^4 - 2*x^2 + 1)*lo
g(x + 1)*log(x - 1) + 3*(x^4 - 2*x^2 + 1)*log(x - 1)^2 - 12*x^2 + 16)/(x^4
 - 2*x^2 + 1)
```

3.61.8 Giac [F]

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^3}\,dx = \int -\frac{\operatorname{arcoth}(x)}{(x^2-1)^3}\,dx$$

input
```
integrate(arccoth(x)/(-x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(-arccoth(x)/(x^2 - 1)^3, x)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.67

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^3}\,dx = \frac{3\ln\left(\frac{1}{x}+1\right)^2}{64} - \ln\left(1-\frac{1}{x}\right)\left(\frac{3\,\ln\left(\frac{1}{x}+1\right)}{32} + \frac{\frac{5\,x}{16}-\frac{3\,x^3}{16}}{x^4-2\,x^2+1}\right) + \frac{3\ln\left(1-\frac{1}{x}\right)^2}{64} + \frac{\frac{3\,x^2}{16}-\frac{1}{4}}{x^4-2\,x^2+1} + \frac{\ln\left(\frac{1}{x}+1\right)\left(\frac{5\,x}{16}-\frac{3\,x^3}{16}\right)}{x^4-2\,x^2+1}$$

input
```
int(-acoth(x)/(x^2 - 1)^3,x)
```

output
```
(3*log(1/x + 1)^2)/64 - log(1 - 1/x)*((3*log(1/x + 1))/32 + ((5*x)/16 - (3
*x^3)/16)/(x^4 - 2*x^2 + 1)) + (3*log(1 - 1/x)^2)/64 + ((3*x^2)/16 - 1/4)/
(x^4 - 2*x^2 + 1) + (log(1/x + 1)*((5*x)/16 - (3*x^3)/16))/(x^4 - 2*x^2 +
1)
```

3.62 $\int x^3 \coth^{-1}(a+bx)\,dx$

3.62.1 Optimal result

Integrand size = 10, antiderivative size = 101

$$\int x^3 \coth^{-1}(a+bx)\,dx = \frac{(1+6a^2)\,x}{4b^3} - \frac{a(a+bx)^2}{2b^4} + \frac{(a+bx)^3}{12b^4} + \frac{1}{4}x^4 \coth^{-1}(a+bx) + \frac{(1-a)^4 \log(1-a-bx)}{8b^4} - \frac{(1+a)^4 \log(1+a+bx)}{8b^4}$$

output
```
1/4*(6*a^2+1)*x/b^3-1/2*a*(b*x+a)^2/b^4+1/12*(b*x+a)^3/b^4+1/4*x^4*arccoth
(b*x+a)+1/8*(1-a)^4*ln(-b*x-a+1)/b^4-1/8*(1+a)^4*ln(b*x+a+1)/b^4
```

3.62.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int x^3 \coth^{-1}(a+bx)\,dx = \frac{6(1+3a^2)\,bx - 6ab^2x^2 + 2b^3x^3 + 6b^4x^4 \coth^{-1}(a+bx) + 3(-1+a)^4 \log(1-a-bx) - 3(1+a)^4 \log(1+}{24b^4}$$

input
```
Integrate[x^3*ArcCoth[a + b*x],x]
```

output
```
(6*(1 + 3*a^2)*b*x - 6*a*b^2*x^2 + 2*b^3*x^3 + 6*b^4*x^4*ArcCoth[a + b*x]
+ 3*(-1 + a)^4*Log[1 - a - b*x] - 3*(1 + a)^4*Log[1 + a + b*x])/(24*b^4)
```

3.62.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6662, 25, 27, 6479, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{6662}$$

$$\frac{\int x^3 \coth^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -x^3 \coth^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -b^3x^3 \coth^{-1}(a+bx)d(a+bx)}{b^4}$$

$$\Big\downarrow \text{6479}$$

$$-\frac{\frac{1}{4}\int \frac{b^4x^4}{1-(a+bx)^2}d(a+bx)-\frac{1}{4}b^4x^4\coth^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \text{477}$$

$$-\frac{\frac{1}{4}\int \left(\frac{(1-a)^4}{2(-a-bx+1)}-6a^2-(a+bx)^2+4a(a+bx)+\frac{(a+1)^4}{2(a+bx+1)}-1\right)d(a+bx)-\frac{1}{4}b^4x^4\coth^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{4}\left(-\left(6a^2+1\right)(a+bx)-\frac{1}{3}(a+bx)^3+2a(a+bx)^2-\frac{1}{2}(1-a)^4\log(-a-bx+1)+\frac{1}{2}(a+1)^4\log(a+bx+1)\right) -}{b^4}$$

input
```
Int[x^3*ArcCoth[a + b*x],x]
```

output
```
-((-1/4*(b^4*x^4*ArcCoth[a + b*x]) + (-((1 + 6*a^2)*(a + b*x)) + 2*a*(a +
b*x)^2 - (a + b*x)^3/3 - ((1 - a)^4*Log[1 - a - b*x])/2 + ((1 + a)^4*Log[1
 + a + b*x])/2)/4)/b^4)
```

3.62.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6479
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.62.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05

method	result
parts	$\frac{x^4 \operatorname{arccoth}(bx+a)}{4}+\frac{b\left(\frac{\frac{1}{3}b^2x^3-ab\,x^2+3a^2x+x}{b^4}+\frac{\left(a^4-4a^3+6a^2-4a+1\right)\ln(bx+a-1)}{2b^5}+\frac{\left(-a^4-4a^3-6a^2-4a-1\right)\ln(bx+a+1)}{2b^5}\right)}{4}$
parallelrisch	$-\frac{-3\operatorname{arccoth}(bx+a)x^4b^4-b^3x^3+3a\,b^2x^2+3\operatorname{arccoth}(bx+a)a^4+12\ln(bx+a-1)a^3-9a^2bx+12\operatorname{arccoth}(bx+a)a^3+18\text{ ar}}{12b^4}$
derivativedivides	$\frac{\frac{\operatorname{arccoth}(bx+a)a^4}{4}-\operatorname{arccoth}(bx+a)a^3(bx+a)+\frac{3\operatorname{arccoth}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccoth}(bx+a)a(bx+a)^3+\frac{\operatorname{arccoth}(bx+a)(bx+a)^4}{4}}{b^4}$
default	$\frac{\frac{\operatorname{arccoth}(bx+a)a^4}{4}-\operatorname{arccoth}(bx+a)a^3(bx+a)+\frac{3\operatorname{arccoth}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccoth}(bx+a)a(bx+a)^3+\frac{\operatorname{arccoth}(bx+a)(bx+a)^4}{4}}{b^4}$
risch	$\frac{x^4\ln(bx+a+1)}{8}-\frac{x^4\ln(bx+a-1)}{8}+\frac{x^3}{12b}-\frac{\ln(bx+a+1)a^4}{8b^4}+\frac{\ln(-bx-a+1)a^4}{8b^4}-\frac{x^2a}{4b^2}-\frac{\ln(bx+a+1)a^3}{2b^4}-\ln($

input
```
int(x^3*arccoth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*arccoth(b*x+a)+1/4*b*(1/b^4*(1/3*b^2*x^3-a*b*x^2+3*a^2*x+x)+1/2*(a
^4-4*a^3+6*a^2-4*a+1)/b^5*ln(b*x+a-1)+1/2*(-a^4-4*a^3-6*a^2-4*a-1)/b^5*ln(
b*x+a+1))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.11

$$\int x^3 \coth^{-1}(a+bx)\,dx$$
$$=\frac{3\,b^4x^4\log\left(\frac{bx+a+1}{bx+a-1}\right)+2\,b^3x^3-6\,ab^2x^2+6\,(3\,a^2+1)bx-3\,(a^4+4\,a^3+6\,a^2+4\,a+1)\log(bx+a+1)+3}{24\,b^4}$$

input
```
integrate(x^3*arccoth(b*x+a),x, algorithm="fricas")
```

output
```
1/24*(3*b^4*x^4*log((b*x + a + 1)/(b*x + a - 1)) + 2*b^3*x^3 - 6*a*b^2*x^2
 + 6*(3*a^2 + 1)*b*x - 3*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1)
+ 3*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*log(b*x + a - 1))/b^4
```

3.62.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.51

$$\int x^3 \coth^{-1}(a+bx)\,dx$$
$$= \begin{cases} -\frac{a^4 \operatorname{acoth}(a+bx)}{4b^4} - \frac{a^3 \log(a+bx+1)}{b^4} + \frac{a^3 \operatorname{acoth}(a+bx)}{b^4} + \frac{3a^2x}{4b^3} - \frac{3a^2 \operatorname{acoth}(a+bx)}{2b^4} - \frac{ax^2}{4b^2} - \frac{a\log(a+bx+1)}{b^4} + \frac{a \operatorname{acoth}(a+bx)}{b^4} \\ \frac{x^4 \operatorname{acoth}(a)}{4} \end{cases}$$

input
```
integrate(x**3*acoth(b*x+a),x)
```

output
```
Piecewise((-a**4*acoth(a + b*x)/(4*b**4) - a**3*log(a + b*x + 1)/b**4 + a*
*3*acoth(a + b*x)/b**4 + 3*a**2*x/(4*b**3) - 3*a**2*acoth(a + b*x)/(2*b**4
) - a*x**2/(4*b**2) - a*log(a + b*x + 1)/b**4 + a*acoth(a + b*x)/b**4 + x*
*4*acoth(a + b*x)/4 + x**3/(12*b) + x/(4*b**3) - acoth(a + b*x)/(4*b**4),
Ne(b, 0)), (x**4*acoth(a)/4, True))
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05

$$\int x^3 \coth^{-1}(a+bx)\,dx = \frac{1}{4}x^4 \operatorname{arcoth}(bx+a)$$
$$+ \frac{1}{24}b\left(\frac{2\,(b^2x^3 - 3\,abx^2 + 3\,(3\,a^2+1)x)}{b^4} - \frac{3\,(a^4 + 4\,a^3 + 6\,a^2 + 4\,a + 1)\log(bx+a+1)}{b^5} + \frac{3\,(a^4 - 4\,a^3}{}\right.$$

input
```
integrate(x^3*arccoth(b*x+a),x, algorithm="maxima")
```

output
```
1/4*x^4*arccoth(b*x + a) + 1/24*b*(2*(b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 + 1)*
x)/b^4 - 3*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1)/b^5 + 3*(a^4 -
 4*a^3 + 6*a^2 - 4*a + 1)*log(b*x + a - 1)/b^5)
```

3.62.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(87) = 174$.

Time = 0.30 (sec) , antiderivative size = 512, normalized size of antiderivative = 5.07

$$\int x^3 \coth^{-1}(a+bx)\,dx = -\frac{1}{6}\left((a+1)b-(a-1)b\right)\left(\frac{3\left(a^3+a\right)\log\left(\frac{|bx+a+1|}{|bx+a-1|}\right)}{b^5} - \frac{3\left(a^3+a\right)\log\left(\left|\frac{bx+a+1}{bx+a-1}-1\right|\right)}{b^5} - \frac{9\,a^2+\frac{3\left(3\,a^2-2\,a+1\right)}{(bx+a-}}{}\right.$$

input
```
integrate(x^3*arccoth(b*x+a),x, algorithm="giac")
```

output
```
-1/6*((a + 1)*b - (a - 1)*b)*(3*(a^3 + a)*log(abs(b*x + a + 1)/abs(b*x + a
 - 1))/b^5 - 3*(a^3 + a)*log(abs((b*x + a + 1)/(b*x + a - 1) - 1))/b^5 - (
9*a^2 + 3*(3*a^2 - 2*a + 1)*(b*x + a + 1)^2/(b*x + a - 1)^2 - 3*(6*a^2 - 2
*a + 1)*(b*x + a + 1)/(b*x + a - 1) + 2)/(b^5*((b*x + a + 1)/(b*x + a - 1)
 - 1)^3) + 3*((b*x + a + 1)^3*a^3/(b*x + a - 1)^3 - 3*(b*x + a + 1)^2*a^3/
(b*x + a - 1)^2 + 3*(b*x + a + 1)*a^3/(b*x + a - 1) - a^3 - 3*(b*x + a + 1
)^3*a^2/(b*x + a - 1)^3 + 6*(b*x + a + 1)^2*a^2/(b*x + a - 1)^2 - 3*(b*x +
 a + 1)*a^2/(b*x + a - 1) + 3*(b*x + a + 1)^3*a/(b*x + a - 1)^3 - 3*(b*x +
 a + 1)^2*a/(b*x + a - 1)^2 + (b*x + a + 1)*a/(b*x + a - 1) - a - (b*x + a
 + 1)^3/(b*x + a - 1)^3 - (b*x + a + 1)/(b*x + a - 1))*log(-(1/(a - ((b*x
+ a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a - 1) -
 b)) + 1)/(1/(a - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x +
a + 1)*b/(b*x + a - 1) - b)) - 1))/(b^5*((b*x + a + 1)/(b*x + a - 1) - 1)^
4))
```

3.62.9 Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.33

$$\int x^3 \coth^{-1}(a+bx)\, dx = \frac{x^4 \ln\left(\frac{1}{a+b\,x}+1\right)}{8} - x\left(\frac{4\,a^2-4}{16\,b^3} - \frac{a^2}{b^3}\right) - \frac{x^4 \ln\left(1-\frac{1}{a+b\,x}\right)}{8} + \frac{x^3}{12\,b} - \frac{a\,x^2}{4\,b^2} + \frac{\ln\left(a+b\,x-1\right)\,\left(a^4-4\,a^3+6\,a^2-4\,a+1\right)}{8\,b^4} - \frac{\ln\left(a+b\,x+1\right)\,\left(a^4+4\,a^3+6\,a^2+4\,a+1\right)}{8\,b^4}$$

input
```
int(x^3*acoth(a + b*x),x)
```

output
```
(x^4*log(1/(a + b*x) + 1))/8 - x*((4*a^2 - 4)/(16*b^3) - a^2/b^3) - (x^4*l
og(1 - 1/(a + b*x)))/8 + x^3/(12*b) - (a*x^2)/(4*b^2) + (log(a + b*x - 1)*
(6*a^2 - 4*a - 4*a^3 + a^4 + 1))/(8*b^4) - (log(a + b*x + 1)*(4*a + 6*a^2
+ 4*a^3 + a^4 + 1))/(8*b^4)
```

3.63 $\int x^2 \coth^{-1}(a+bx)\,dx$

3.63.1 Optimal result

Integrand size = 10, antiderivative size = 78

$$\int x^2 \coth^{-1}(a+bx)\,dx = -\frac{ax}{b^2}+\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\coth^{-1}(a+bx)+\frac{(1-a)^3\log(1-a-bx)}{6b^3}+\frac{(1+a)^3\log(1+a+bx)}{6b^3}$$

output
```
-a*x/b^2+1/6*(b*x+a)^2/b^3+1/3*x^3*arccoth(b*x+a)+1/6*(1-a)^3*ln(-b*x-a+1)
/b^3+1/6*(1+a)^3*ln(b*x+a+1)/b^3
```

3.63.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18

$$\int x^2 \coth^{-1}(a+bx)\,dx = -\frac{2ax}{3b^2}+\frac{x^2}{6b}+\frac{1}{3}x^3\coth^{-1}(a+bx)+\frac{(1-3a+3a^2-a^3)\log(1-a-bx)}{6b^3}+\frac{(1+3a+3a^2+a^3)\log(1+a+bx)}{6b^3}$$

input
```
Integrate[x^2*ArcCoth[a + b*x],x]
```

output
```
(-2*a*x)/(3*b^2) + x^2/(6*b) + (x^3*ArcCoth[a + b*x])/3 + ((1 - 3*a + 3*a^
2 - a^3)*Log[1 - a - b*x])/(6*b^3) + ((1 + 3*a + 3*a^2 + a^3)*Log[1 + a +
b*x])/(6*b^3)
```

3.63.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6662, 27, 6479, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{ 6662}$$

$$\frac{\int x^2 \coth^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int b^2x^2 \coth^{-1}(a+bx)d(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 6479}$$

$$\frac{\frac{1}{3}\int -\frac{b^3x^3}{1-(a+bx)^2}d(a+bx)+\frac{1}{3}b^3x^3\coth^{-1}(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 477}$$

$$\frac{\frac{1}{3}\int \left(-\frac{(1-a)^3}{2(-a-bx+1)}-2a+bx+\frac{(a+1)^3}{2(a+bx+1)}\right)d(a+bx)+\frac{1}{3}b^3x^3\coth^{-1}(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{3}b^3x^3\coth^{-1}(a+bx)+\frac{1}{3}\left(\frac{1}{2}(a+bx)^2-3a(a+bx)+\frac{1}{2}(1-a)^3\log(-a-bx+1)+\frac{1}{2}(a+1)^3\log(a+bx+1)\right)}{b^3}$$

input
```
Int[x^2*ArcCoth[a + b*x],x]
```

output
```
((b^3*x^3*ArcCoth[a + b*x])/3 + (-3*a*(a + b*x) + (a + b*x)^2/2 + ((1 - a)
^3*Log[1 - a - b*x])/2 + ((1 + a)^3*Log[1 + a + b*x])/2)/3)/b^3
```

3.63.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6479
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.63.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09

method	result
parts	$\frac{x^3 \operatorname{arccoth}(bx+a)}{3}+\frac{b\left(-\frac{-\frac{1}{2} b\, x^2+2ax}{b^3}+\frac{\left(-a^3+3a^2-3a+1\right)\ln(bx+a-1)}{2b^4}+\frac{\left(a^3+3a^2+3a+1\right)\ln(bx+a+1)}{2b^4}\right)}{3}$
parallelrisch	$\frac{2\ \operatorname{arccoth}(bx+a)x^3b^3+1+b^2x^2+2\ \operatorname{arccoth}(bx+a)a^3+6\ln(bx+a-1)a^2-4abx+6\ \operatorname{arccoth}(bx+a)a^2+6\ \operatorname{arccoth}(bx+a)a+}{6b^3}$
derivativedivides	$\frac{-\frac{\operatorname{arccoth}(bx+a)a^3}{3}+\operatorname{arccoth}(bx+a)a^2(bx+a)-\operatorname{arccoth}(bx+a)a(bx+a)^2+\frac{\operatorname{arccoth}(bx+a)(bx+a)^3}{3}-(bx+a)a+\frac{(bx+a)^2}{6}-}{b^3}$
default	$\frac{-\frac{\operatorname{arccoth}(bx+a)a^3}{3}+\operatorname{arccoth}(bx+a)a^2(bx+a)-\operatorname{arccoth}(bx+a)a(bx+a)^2+\frac{\operatorname{arccoth}(bx+a)(bx+a)^3}{3}-(bx+a)a+\frac{(bx+a)^2}{6}-}{b^3}$
risch	$\frac{x^3\ln(bx+a+1)}{6}-\frac{\ln(bx+a-1)x^3}{6}+\frac{\ln(-bx-a-1)a^3}{6b^3}-\frac{\ln(bx+a-1)a^3}{6b^3}+\frac{x^2}{6b}+\frac{\ln(-bx-a-1)a^2}{2b^3}+$

input
```
int(x^2*arccoth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccoth(b*x+a)+1/3*b*(-1/b^3*(-1/2*b*x^2+2*a*x)+1/2*(-a^3+3*a^2-3*
a+1)/b^4*ln(b*x+a-1)+1/2*(a^3+3*a^2+3*a+1)/b^4*ln(b*x+a+1))
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

$$\int x^2 \coth^{-1}(a+bx)\,dx$$
$$= \frac{b^3x^3\log\left(\frac{bx+a+1}{bx+a-1}\right) + b^2x^2 - 4\,abx + (a^3+3\,a^2+3\,a+1)\log(bx+a+1) - (a^3-3\,a^2+3\,a-1)\log(bx+}{6\,b^3}$$

input
```
integrate(x^2*arccoth(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(b^3*x^3*log((b*x + a + 1)/(b*x + a - 1)) + b^2*x^2 - 4*a*b*x + (a^3 +
 3*a^2 + 3*a + 1)*log(b*x + a + 1) - (a^3 - 3*a^2 + 3*a - 1)*log(b*x + a -
 1))/b^3
```

3.63.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.50

$$\int x^2 \coth^{-1}(a+bx)\,dx$$
$$= \begin{cases} \frac{a^3\,\mathrm{acoth}\,(a+bx)}{3b^3} + \frac{a^2\log(a+bx+1)}{b^3} - \frac{a^2\,\mathrm{acoth}\,(a+bx)}{b^3} - \frac{2ax}{3b^2} + \frac{a\,\mathrm{acoth}\,(a+bx)}{b^3} + \frac{x^3\,\mathrm{acoth}\,(a+bx)}{3} + \frac{x^2}{6b} + \frac{\log(a+bx+1)}{3b^3} - \mathrm{acot} \\ \frac{x^3\,\mathrm{acoth}\,(a)}{3} \end{cases}$$

input
```
integrate(x**2*acoth(b*x+a),x)
```

output
```
Piecewise((a**3*acoth(a + b*x)/(3*b**3) + a**2*log(a + b*x + 1)/b**3 - a**
2*acoth(a + b*x)/b**3 - 2*a*x/(3*b**2) + a*acoth(a + b*x)/b**3 + x**3*acot
h(a + b*x)/3 + x**2/(6*b) + log(a + b*x + 1)/(3*b**3) - acoth(a + b*x)/(3*
b**3), Ne(b, 0)), (x**3*acoth(a)/3, True))
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int x^2 \coth^{-1}(a+bx)\,dx = \frac{1}{3}x^3 \operatorname{arcoth}(bx+a) + \frac{1}{6}b\left(\frac{bx^2-4ax}{b^3} + \frac{(a^3+3a^2+3a+1)\log(bx+a+1)}{b^4} - \frac{(a^3-3a^2+3a-1)\log(bx+a-1)}{b^4}\right)$$

input
```
integrate(x^2*arccoth(b*x+a),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(b*x + a) + 1/6*b*((b*x^2 - 4*a*x)/b^3 + (a^3 + 3*a^2 + 3*a
 + 1)*log(b*x + a + 1)/b^4 - (a^3 - 3*a^2 + 3*a - 1)*log(b*x + a - 1)/b^4)
```

3.63.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(68) = 136.

Time = 0.29 (sec) , antiderivative size = 360, normalized size of antiderivative = 4.62

$$\int x^2 \coth^{-1}(a+bx)\,dx$$

$$= \frac{1}{6}\left((a+1)b-(a-1)b\right)\left(\frac{(3a^2+1)\log\left(\frac{|bx+a+1|}{|bx+a-1|}\right)}{b^4} - \frac{(3a^2+1)\log\left(\left|\frac{bx+a+1}{bx+a-1}-1\right|\right)}{b^4} - \frac{2\left(\frac{(bx+a+1)(3a-1)}{bx+a-1} - 3\right.}{b^4\left(\frac{bx+a+1}{bx+a-1}-1\right)^2}\right.$$

input
```
integrate(x^2*arccoth(b*x+a),x, algorithm="giac")
```

output
```
1/6*((a + 1)*b - (a - 1)*b)*((3*a^2 + 1)*log(abs(b*x + a + 1)/abs(b*x + a
- 1))/b^4 - (3*a^2 + 1)*log(abs((b*x + a + 1)/(b*x + a - 1) - 1))/b^4 - 2*
((b*x + a + 1)*(3*a - 1)/(b*x + a - 1) - 3*a)/(b^4*((b*x + a + 1)/(b*x + a
 - 1) - 1)^2) + (3*(b*x + a + 1)^2*a^2/(b*x + a - 1)^2 - 6*(b*x + a + 1)*a
^2/(b*x + a - 1) + 3*a^2 - 6*(b*x + a + 1)^2*a/(b*x + a - 1)^2 + 6*(b*x +
a + 1)*a/(b*x + a - 1) + 3*(b*x + a + 1)^2/(b*x + a - 1)^2 + 1)*log(-(1/(a
 - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x +
 a - 1) - b)) + 1)/(1/(a - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b
/((b*x + a + 1)*b/(b*x + a - 1) - b)) - 1))/(b^4*((b*x + a + 1)/(b*x + a -
 1) - 1)^3))
```

3.63.9 Mupad [B] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.26

$$\int x^2 \coth^{-1}(a+bx)\,dx = \frac{x^3\,\ln\left(\frac{1}{a+b\,x}+1\right)}{6}-\frac{x^3\,\ln\left(1-\frac{1}{a+b\,x}\right)}{6}+\frac{x^2}{6\,b} -\frac{\ln\left(a+b\,x-1\right)\,\left(a^3-3\,a^2+3\,a-1\right)}{6\,b^3} +\frac{\ln\left(a+b\,x+1\right)\,\left(a^3+3\,a^2+3\,a+1\right)}{6\,b^3}-\frac{2\,a\,x}{3\,b^2}$$

input
```
int(x^2*acoth(a + b*x),x)
```

output
```
(x^3*log(1/(a + b*x) + 1))/6 - (x^3*log(1 - 1/(a + b*x)))/6 + x^2/(6*b) -
(log(a + b*x - 1)*(3*a - 3*a^2 + a^3 - 1))/(6*b^3) + (log(a + b*x + 1)*(3*
a + 3*a^2 + a^3 + 1))/(6*b^3) - (2*a*x)/(3*b^2)
```

3.64 $\int x \coth^{-1}(a+bx)\,dx$

3.64.1 Optimal result

Integrand size = 8, antiderivative size = 65

$$\int x \coth^{-1}(a+bx)\,dx = \frac{x}{2b}+\frac{1}{2}x^2\coth^{-1}(a+bx)+\frac{(1-a)^2\log(1-a-bx)}{4b^2} - \frac{(1+a)^2\log(1+a+bx)}{4b^2}$$

output
```
1/2*x/b+1/2*x^2*arccoth(b*x+a)+1/4*(1-a)^2*ln(-b*x-a+1)/b^2-1/4*(1+a)^2*ln
(b*x+a+1)/b^2
```

3.64.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int x \coth^{-1}(a+bx)\,dx = \frac{2bx+2b^2x^2\coth^{-1}(a+bx)+(-1+a)^2\log(1-a-bx)-(1+a)^2\log(1+a+bx)}{4b^2}$$

input
```
Integrate[x*ArcCoth[a + b*x],x]
```

output
```
(2*b*x + 2*b^2*x^2*ArcCoth[a + b*x] + (-1 + a)^2*Log[1 - a - b*x] - (1 + a
)^2*Log[1 + a + b*x])/(4*b^2)
```

3.64.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6662, 25, 27, 6479, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{6662}$$

$$\frac{\int x\coth^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -x\coth^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -bx\coth^{-1}(a+bx)d(a+bx)}{b^2}$$

$$\Big\downarrow \text{6479}$$

$$-\frac{\frac{1}{2}\int \frac{b^2x^2}{1-(a+bx)^2}d(a+bx)-\frac{1}{2}b^2x^2\coth^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{477}$$

$$-\frac{\frac{1}{2}\int \left(\frac{(1-a)^2}{2(-a-bx+1)}+\frac{(a+1)^2}{2(a+bx+1)}-1\right)d(a+bx)-\frac{1}{2}b^2x^2\coth^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}\left(-\frac{1}{2}(1-a)^2\log(-a-bx+1)+\frac{1}{2}(a+1)^2\log(a+bx+1)-a-bx\right)-\frac{1}{2}b^2x^2\coth^{-1}(a+bx)}{b^2}$$

input
```
Int[x*ArcCoth[a + b*x],x]
```

output
```
-((-1/2*(b^2*x^2*ArcCoth[a + b*x]) + (-a - b*x - ((1 - a)^2*Log[1 - a - b*
x])/2 + ((1 + a)^2*Log[1 + a + b*x])/2)/2)/b^2)
```

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6479
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.64.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

method	result
parallelrisch	$-\frac{-\operatorname{arccoth}(bx+a)b^2x^2+\operatorname{arccoth}(bx+a)a^2+2\ln(bx+a-1)a-bx+2\operatorname{arccoth}(bx+a)a+\operatorname{arccoth}(bx+a)+2a}{2b^2}$
parts	$\frac{x^2\operatorname{arccoth}(bx+a)}{2}+\frac{b\left(\frac{x}{b^2}+\frac{(a^2-2a+1)\ln(bx+a-1)}{2b^3}+\frac{(-a^2-2a-1)\ln(bx+a+1)}{2b^3}\right)}{2}$
derivativedivides	$\frac{\frac{(bx+a)^2\operatorname{arccoth}(bx+a)}{2}-\operatorname{arccoth}(bx+a)a(bx+a)+\frac{bx}{2}+\frac{a}{2}-\frac{(-1+2a)\ln(bx+a-1)}{4}+\frac{(-2a-1)\ln(bx+a+1)}{4}}{b^2}$
default	$\frac{\frac{(bx+a)^2\operatorname{arccoth}(bx+a)}{2}-\operatorname{arccoth}(bx+a)a(bx+a)+\frac{bx}{2}+\frac{a}{2}-\frac{(-1+2a)\ln(bx+a-1)}{4}+\frac{(-2a-1)\ln(bx+a+1)}{4}}{b^2}$
risch	$\frac{x^2\ln(bx+a+1)}{4}-\frac{x^2\ln(bx+a-1)}{4}-\frac{\ln(bx+a+1)a^2}{4b^2}+\frac{\ln(-bx-a+1)a^2}{4b^2}-\frac{\ln(bx+a+1)a}{2b^2}-\frac{\ln(-bx-a+1)a}{2b^2}+$

input
```
int(x*arccoth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-arccoth(b*x+a)*b^2*x^2+arccoth(b*x+a)*a^2+2*ln(b*x+a-1)*a-b*x+2*arc
coth(b*x+a)*a+arccoth(b*x+a)+2*a)/b^2
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int x\coth^{-1}(a+bx)\,dx = \frac{b^2x^2\log\left(\frac{bx+a+1}{bx+a-1}\right)+2\,bx-(a^2+2\,a+1)\log(bx+a+1)+(a^2-2\,a+1)\log(bx+a-1)}{4\,b^2}$$

input
```
integrate(x*arccoth(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(b^2*x^2*log((b*x + a + 1)/(b*x + a - 1)) + 2*b*x - (a^2 + 2*a + 1)*lo
g(b*x + a + 1) + (a^2 - 2*a + 1)*log(b*x + a - 1))/b^2
```

3.64.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.17

$$\int x\coth^{-1}(a+bx)\,dx$$
$$=\begin{cases} -\frac{a^2\operatorname{acoth}(a+bx)}{2b^2}-\frac{a\log(a+bx+1)}{b^2}+\frac{a\operatorname{acoth}(a+bx)}{b^2}+\frac{x^2\operatorname{acoth}(a+bx)}{2}+\frac{x}{2b}-\frac{\operatorname{acoth}(a+bx)}{2b^2} & \text{for } b\neq 0\\ \frac{x^2\operatorname{acoth}(a)}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*acoth(b*x+a),x)
```

output
```
Piecewise((-a**2*acoth(a + b*x)/(2*b**2) - a*log(a + b*x + 1)/b**2 + a*aco
th(a + b*x)/b**2 + x**2*acoth(a + b*x)/2 + x/(2*b) - acoth(a + b*x)/(2*b**
2), Ne(b, 0)), (x**2*acoth(a)/2, True))
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int x\coth^{-1}(a+bx)\,dx$$
$$=\frac{1}{2}x^2\operatorname{arcoth}(bx+a)+\frac{1}{4}b\left(\frac{2x}{b^2}-\frac{(a^2+2a+1)\log(bx+a+1)}{b^3}+\frac{(a^2-2a+1)\log(bx+a-1)}{b^3}\right)$$

input
```
integrate(x*arccoth(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x^2*arccoth(b*x + a) + 1/4*b*(2*x/b^2 - (a^2 + 2*a + 1)*log(b*x + a +
1)/b^3 + (a^2 - 2*a + 1)*log(b*x + a - 1)/b^3)
```

3.64.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(55) = 110$.

Time = 0.28 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.98

$$\int x\coth^{-1}(a+bx)\,dx = -\frac{1}{2}\left((a+1)b-(a-1)b\right)\left(\frac{a\log\left(\frac{|bx+a+1|}{|bx+a-1|}\right)}{b^3} - \frac{a\log\left(\left|\frac{bx+a+1}{bx+a-1}-1\right|\right)}{b^3} + \frac{\left(\frac{(bx+a+1)a}{bx+a-1}-a-\frac{bx+a+1}{bx+a-1}\right)\log\left(-\cdots\right)}{b^3\left(\frac{bx+a+1}{bx+a-1}-1\cdots\right)}\cdots\right)$$

input

```
integrate(x*arccoth(b*x+a),x, algorithm="giac")
```

output

```
-1/2*((a + 1)*b - (a - 1)*b)*(a*log(abs(b*x + a + 1)/abs(b*x + a - 1))/b^3
 - a*log(abs((b*x + a + 1)/(b*x + a - 1) - 1))/b^3 + ((b*x + a + 1)*a/(b*x
 + a - 1) - a - (b*x + a + 1)/(b*x + a - 1))*log(-(1/(a - ((b*x + a + 1)*(
a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a - 1) - b)) + 1)/
(1/(a - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(
b*x + a - 1) - b)) - 1))/(b^3*((b*x + a + 1)/(b*x + a - 1) - 1)^2) - 1/(b^
3*((b*x + a + 1)/(b*x + a - 1) - 1)))
```

3.64.9 Mupad [B] (verification not implemented)

Time = 4.96 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int x\coth^{-1}(a+bx)\,dx = \frac{x^2\,\mathrm{acoth}(a+b\,x)}{2} - \frac{\frac{\mathrm{acoth}(a+b\,x)}{2}-\frac{b\,x}{2}+\frac{a^2\,\mathrm{acoth}(a+b\,x)}{2}+\frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2-1\right)}{2}}{b^2}$$

input

```
int(x*acoth(a + b*x),x)
```

output

```
(x^2*acoth(a + b*x))/2 - (acoth(a + b*x)/2 - (b*x)/2 + (a^2*acoth(a + b*x)
)/2 + (a*log(a^2 + b^2*x^2 + 2*a*b*x - 1))/2)/b^2
```

3.65 $\int \coth^{-1}(a+bx)\,dx$

3.65.1 Optimal result

Integrand size = 6, antiderivative size = 35

$$\int \coth^{-1}(a+bx)\,dx=\frac{(a+bx)\coth^{-1}(a+bx)}{b}+\frac{\log\left(1-(a+bx)^2\right)}{2b}$$

output
```
(b*x+a)*arccoth(b*x+a)/b+1/2*ln(1-(b*x+a)^2)/b
```

3.65.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \coth^{-1}(a+bx)\,dx=x\coth^{-1}(a+bx)+\frac{-((-1+a)\log(1-a-bx))+(1+a)\log(1+a+bx)}{2b}$$

input
```
Integrate[ArcCoth[a + b*x],x]
```

output
```
x*ArcCoth[a + b*x] + (-((-1 + a)*Log[1 - a - b*x]) + (1 + a)*Log[1 + a + b
*x])/(2*b)
```

3.65.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6654, 6437, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(a+bx)\,dx$$
$$\Big\downarrow \text{6654}$$
$$\frac{\int \coth^{-1}(a+bx)d(a+bx)}{b}$$
$$\Big\downarrow \text{6437}$$
$$\frac{(a+bx)\coth^{-1}(a+bx)-\int \frac{a+bx}{1-(a+bx)^2}d(a+bx)}{b}$$
$$\Big\downarrow \text{240}$$
$$\frac{\frac{1}{2}\log\left(1-(a+bx)^2\right)+(a+bx)\coth^{-1}(a+bx)}{b}$$

input
```
Int[ArcCoth[a + b*x],x]
```

output
```
((a + b*x)*ArcCoth[a + b*x] + Log[1 - (a + b*x)^2]/2)/b
```

3.65.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6654
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
}, x] && IGtQ[p, 0]
```

3.65.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{(bx+a)\operatorname{arccoth}(bx+a)+\frac{\ln\left((bx+a)^2-1\right)}{2}}{b}$	30
default	$\frac{(bx+a)\operatorname{arccoth}(bx+a)+\frac{\ln\left((bx+a)^2-1\right)}{2}}{b}$	30
parts	$x\operatorname{arccoth}(bx+a)+b\left(\frac{(1-a)\ln(bx+a-1)}{2b^2}+\frac{(1+a)\ln(bx+a+1)}{2b^2}\right)$	45
parallelrisch	$-\frac{-\operatorname{arccoth}(bx+a)b^2x-\operatorname{arccoth}(bx+a)ab-b\ln(bx+a-1)-\operatorname{arccoth}(bx+a)b}{b^2}$	48
risch	$\frac{x\ln(bx+a+1)}{2}-\frac{\ln(bx+a-1)x}{2}-\frac{\ln(bx+a-1)a}{2b}+\frac{\ln(-bx-a-1)a}{2b}+\frac{\ln(bx+a-1)}{2b}+\frac{\ln(-bx-a-1)}{2b}$	78

input
```
int(arccoth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*((b*x+a)*arccoth(b*x+a)+1/2*ln((b*x+a)^2-1))
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \coth^{-1}(a+bx)\,dx = \frac{bx\log\left(\frac{bx+a+1}{bx+a-1}\right)+(a+1)\log(bx+a+1)-(a-1)\log(bx+a-1)}{2\,b}$$

input
```
integrate(arccoth(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b*x*log((b*x + a + 1)/(b*x + a - 1)) + (a + 1)*log(b*x + a + 1) - (a
- 1)*log(b*x + a - 1))/b
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \coth^{-1}(a+bx)\,dx = \begin{cases} \frac{a\operatorname{acoth}(a+bx)}{b} + x\operatorname{acoth}(a+bx) + \frac{\log(a+bx+1)}{b} - \frac{\operatorname{acoth}(a+bx)}{b} & \text{for } b \neq 0 \\ x\operatorname{acoth}(a) & \text{otherwise} \end{cases}$$

input
```
integrate(acoth(b*x+a),x)
```

output
```
Piecewise((a*acoth(a + b*x)/b + x*acoth(a + b*x) + log(a + b*x + 1)/b - ac
oth(a + b*x)/b, Ne(b, 0)), (x*acoth(a), True))
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \coth^{-1}(a+bx)\,dx = \frac{2(bx+a)\operatorname{arcoth}(bx+a) + \log\left(-(bx+a)^2+1\right)}{2b}$$

input
```
integrate(arccoth(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*(b*x + a)*arccoth(b*x + a) + log(-(b*x + a)^2 + 1))/b
```

3.65.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(33) = 66.

Time = 0.27 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.63

$$\int \coth^{-1}(a+bx)\,dx$$

$$= \frac{1}{2}\left((a+1)b-(a-1)b\right)\left(\frac{\log\left(\frac{|bx+a+1|}{|bx+a-1|}\right)}{b^2} - \frac{\log\left(\left|\frac{bx+a+1}{bx+a-1}-1\right|\right)}{b^2} + \frac{\log\left(-\frac{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}+1}{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}-1}\right)}{b^2\left(\frac{bx+a+1}{bx+a-1}-1\right)}\right)$$

input
```
integrate(arccoth(b*x+a),x, algorithm="giac")
```

output
```
1/2*((a + 1)*b - (a - 1)*b)*(log(abs(b*x + a + 1)/abs(b*x + a - 1))/b^2 -
log(abs((b*x + a + 1)/(b*x + a - 1) - 1))/b^2 + log(-(1/(a - ((b*x + a + 1
)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a - 1) - b)) +
1)/(1/(a - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*
b/(b*x + a - 1) - b)) - 1))/(b^2*((b*x + a + 1)/(b*x + a - 1) - 1)))
```

3.65.9 Mupad [B] (verification not implemented)

Time = 4.64 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \coth^{-1}(a+bx)\,dx = \frac{\frac{\ln(a^2+2\,a\,b\,x+b^2\,x^2-1)}{2}+a\,\mathrm{acoth}(a+b\,x)}{b}+x\,\mathrm{acoth}(a+b\,x)$$

input
```
int(acoth(a + b*x),x)
```

output
```
(log(a^2 + b^2*x^2 + 2*a*b*x - 1)/2 + a*acoth(a + b*x))/b + x*acoth(a + b*
x)
```

3.66 $\int \frac{\coth^{-1}(a+bx)}{x} dx$

3.66.1 Optimal result

Integrand size = 10, antiderivative size = 92

$$\begin{aligned}\int \frac{\coth^{-1}(a+bx)}{x} dx = & -\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right) \\ & +\coth^{-1}(a+bx)\log\left(\frac{2bx}{(1-a)(1+a+bx)}\right) \\ & +\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{1+a+bx}\right) \\ & -\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2bx}{(1-a)(1+a+bx)}\right)\end{aligned}$$

output
```
-arccoth(b*x+a)*ln(2/(b*x+a+1))+arccoth(b*x+a)*ln(2*b*x/(1-a)/(b*x+a+1))+1
/2*polylog(2,1-2/(b*x+a+1))-1/2*polylog(2,1-2*b*x/(1-a)/(b*x+a+1))
```

3.66.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.82

$$\int \frac{\coth^{-1}(a+bx)}{x}\,dx = \left(\coth^{-1}(a+bx)-\operatorname{arctanh}(a+bx)\right)\log(x) + \operatorname{arctanh}(a+bx)\left(-\log\left(\frac{1}{\sqrt{1-(a+bx)^2}}\right) + \log(-i\sinh(\operatorname{arctanh}(a)-\operatorname{arctanh}(a+bx)))\right) + \frac{1}{8}\left(4(\operatorname{arctanh}(a)-\operatorname{arctanh}(a+bx))^2-(\pi-2i\operatorname{arctanh}(a+bx))^2 - 8(\operatorname{arctanh}(a)-\operatorname{arctanh}(a+bx))\log\left(1-e^{2\operatorname{arctanh}(a)-2\operatorname{arctanh}(a+bx)}\right) - 4i(\pi-2i\operatorname{arctanh}(a+bx))\log\left(1+e^{2\operatorname{arctanh}(a+bx)}\right) + 4(i\pi+2\operatorname{arctanh}(a+bx))\log\left(\frac{2}{\sqrt{1-(a+bx)^2}}\right) + 8(\operatorname{arctanh}(a) - \operatorname{arctanh}(a+bx))\log(-2i\sinh(\operatorname{arctanh}(a)-\operatorname{arctanh}(a+bx))) - 4\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(a)-2\operatorname{arctanh}(a+bx)}\right) - 4\operatorname{PolyLog}\left(2,-e^{2\operatorname{arctanh}(a+bx)}\right)\right)$$

input
```
Integrate[ArcCoth[a + b*x]/x,x]
```

output
```
(ArcCoth[a + b*x] - ArcTanh[a + b*x])*Log[x] + ArcTanh[a + b*x]*(-Log[1/Sq
rt[1 - (a + b*x)^2]] + Log[(-I)*Sinh[ArcTanh[a] - ArcTanh[a + b*x]]]) + (4
*(ArcTanh[a] - ArcTanh[a + b*x])^2 - (Pi - (2*I)*ArcTanh[a + b*x])^2 - 8*(
ArcTanh[a] - ArcTanh[a + b*x])*Log[1 - E^(2*ArcTanh[a] - 2*ArcTanh[a + b*x
])] - (4*I)*(Pi - (2*I)*ArcTanh[a + b*x])*Log[1 + E^(2*ArcTanh[a + b*x])]
+ 4*(I*Pi + 2*ArcTanh[a + b*x])*Log[2/Sqrt[1 - (a + b*x)^2]] + 8*(ArcTanh[
a] - ArcTanh[a + b*x])*Log[(-2*I)*Sinh[ArcTanh[a] - ArcTanh[a + b*x]]] - 4
*PolyLog[2, E^(2*ArcTanh[a] - 2*ArcTanh[a + b*x])] - 4*PolyLog[2, -E^(2*Ar
cTanh[a + b*x])])/8
```

3.66. $\int \frac{\coth^{-1}(a+bx)}{x}\,dx$

3.66.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6662, 25, 27, 6473, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(a+bx)}{x} \, dx$$

$$\Big\downarrow \text{6662}$$

$$\frac{\int \frac{\coth^{-1}(a+bx)}{x} d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{\coth^{-1}(a+bx)}{x} d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\int -\frac{\coth^{-1}(a+bx)}{bx} d(a+bx)$$

$$\Big\downarrow \text{6473}$$

$$\int \frac{\log\left(\frac{2}{a+bx+1}\right)}{1-(a+bx)^2} d(a+bx) - \int \frac{\log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)}{1-(a+bx)^2} d(a+bx) + \log\left(\frac{2}{a+bx+1}\right)\left(-\coth^{-1}(a+bx)\right) + \log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)\coth^{-1}(a+bx)$$

$$\Big\downarrow \text{2849}$$

$$\int \frac{\log\left(\frac{2}{a+bx+1}\right)}{1-\frac{2}{a+bx+1}} d\frac{1}{a+bx+1} - \int \frac{\log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)}{1-(a+bx)^2} d(a+bx) + \log\left(\frac{2}{a+bx+1}\right)\left(-\coth^{-1}(a+bx)\right) + \log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)\coth^{-1}(a+bx)$$

$$\Big\downarrow \text{2752}$$

$$-\int \frac{\log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)}{1-(a+bx)^2} d(a+bx) + \frac{1}{2}\text{PolyLog}\left(2, 1-\frac{2}{a+bx+1}\right) + \log\left(\frac{2}{a+bx+1}\right)\left(-\coth^{-1}(a+bx)\right) + \log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)\coth^{-1}(a+bx)$$

$$\Big\downarrow \text{2897}$$

$$\frac{1}{2}\operatorname{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,1-\frac{2bx}{(1-a)(a+bx+1)}\right)+\log\left(\frac{2}{a+bx+1}\right)\left(-\coth^{-1}(a+bx)\right)+\log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)\coth^{-1}(a+bx)$$

input
```
Int[ArcCoth[a + b*x]/x,x]
```

output
```
-(ArcCoth[a + b*x]*Log[2/(1 + a + b*x)]) + ArcCoth[a + b*x]*Log[(2*b*x)/((
1 - a)*(1 + a + b*x))] + PolyLog[2, 1 - 2/(1 + a + b*x)]/2 - PolyLog[2, 1
- (2*b*x)/((1 - a)*(1 + a + b*x))]/2
```

3.66.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6473
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcCoth[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*ArcCoth
[c*x])*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(c/e)
Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Simp[b*(c/e)   Int[Log[2*c*((d
 + e*x)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x]) /; FreeQ[{a, b, c, d
, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.66.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.74

method	result
risch	$\frac{\operatorname{dilog}\left(\frac{xb}{-1-a}\right)}{2}+\frac{\ln(bx+a+1)\ln\left(\frac{xb}{-1-a}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{bx}{1-a}\right)}{2}-\frac{\ln(bx+a-1)\ln\left(\frac{bx}{1-a}\right)}{2}$
parts	$\ln(x)\operatorname{arccoth}(bx+a)+b\left(\frac{\operatorname{dilog}\left(\frac{bx+a-1}{-1+a}\right)}{2b}+\frac{\ln(x)\ln\left(\frac{bx+a-1}{-1+a}\right)}{2b}-\frac{\operatorname{dilog}\left(\frac{bx+a+1}{1+a}\right)}{2b}-\frac{\ln(x)\ln\left(\frac{bx+a+1}{1+a}\right.}{2b}\right.$
derivativedivides	$\ln(-bx)\operatorname{arccoth}(bx+a)-\frac{\operatorname{dilog}\left(\frac{-bx-a-1}{-1-a}\right)}{2}-\frac{\ln(-bx)\ln\left(\frac{-bx-a-1}{-1-a}\right)}{2}+\frac{\operatorname{dilog}\left(\frac{-bx-a+1}{1-a}\right)}{2}+\frac{\ln(-bx)}{}$
default	$\ln(-bx)\operatorname{arccoth}(bx+a)-\frac{\operatorname{dilog}\left(\frac{-bx-a-1}{-1-a}\right)}{2}-\frac{\ln(-bx)\ln\left(\frac{-bx-a-1}{-1-a}\right)}{2}+\frac{\operatorname{dilog}\left(\frac{-bx-a+1}{1-a}\right)}{2}+\frac{\ln(-bx)}{}$

input
```
int(arccoth(b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*dilog(x*b/(-1-a))+1/2*ln(b*x+a+1)*ln(x*b/(-1-a))-1/2*dilog(b*x/(1-a))-
1/2*ln(b*x+a-1)*ln(b*x/(1-a))
```

3.66.5 Fricas [F]

$$\int \frac{\coth^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{arcoth}(bx+a)}{x}\,dx$$

input
```
integrate(arccoth(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(arccoth(b*x + a)/x, x)
```

3.66.6 Sympy [F]

$$\int \frac{\coth^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{acoth}(a+bx)}{x}\,dx$$

input
```
integrate(acoth(b*x+a)/x,x)
```

output
```
Integral(acoth(a + b*x)/x, x)
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.39

$$\int \frac{\coth^{-1}(a+bx)}{x}\,dx = -\frac{1}{2}b\left(\frac{\log(bx+a+1)}{b} - \frac{\log(bx+a-1)}{b}\right)\log(x) + \frac{1}{2}b\left(\frac{\log(bx+a+1)\log\left(-\frac{bx+a+1}{a+1}+1\right)+\mathrm{Li}_2\left(\frac{bx+a+1}{a+1}\right)}{b} - \frac{\log(bx+a-1)\log\left(-\frac{bx+a-1}{a-1}+1\right)+\mathrm{Li}_2\left(\frac{bx+}{a-}\right.}{b}\right. + \operatorname{arcoth}(bx+a)\log(x)$$

input
```
integrate(arccoth(b*x+a)/x,x, algorithm="maxima")
```

output
```
-1/2*b*(log(b*x + a + 1)/b - log(b*x + a - 1)/b)*log(x) + 1/2*b*((log(b*x
+ a + 1)*log(-(b*x + a + 1)/(a + 1) + 1) + dilog((b*x + a + 1)/(a + 1)))/b
 - (log(b*x + a - 1)*log(-(b*x + a - 1)/(a - 1) + 1) + dilog((b*x + a - 1)
/(a - 1)))/b) + arccoth(b*x + a)*log(x)
```

3.66.8 Giac [F]

$$\int \frac{\coth^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{arcoth}(bx+a)}{x}\,dx$$

input
```
integrate(arccoth(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(arccoth(b*x + a)/x, x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(a+bx)}{x}\,dx = \int \frac{\mathrm{acoth}(a+b\,x)}{x}\,dx$$

input
```
int(acoth(a + b*x)/x,x)
```

output
```
int(acoth(a + b*x)/x, x)
```

3.67 $\int \frac{\coth^{-1}(a+bx)}{x^2} dx$

3.67.1 Optimal result

Integrand size = 10, antiderivative size = 64

$$\int \frac{\coth^{-1}(a+bx)}{x^2} dx = -\frac{\coth^{-1}(a+bx)}{x} + \frac{b\log(x)}{1-a^2} - \frac{b\log(1-a-bx)}{2(1-a)} - \frac{b\log(1+a+bx)}{2(1+a)}$$

output
```
-arccoth(b*x+a)/x+b*ln(x)/(-a^2+1)-1/2*b*ln(-b*x-a+1)/(1-a)-1/2*b*ln(b*x+a
+1)/(1+a)
```

3.67.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{\coth^{-1}(a+bx)}{x^2} dx = -\frac{\coth^{-1}(a+bx)}{x} + \frac{b(-2\log(x) + (1+a)\log(1-a-bx) - (-1+a)\log(1+a+bx))}{2(-1+a^2)}$$

input
```
Integrate[ArcCoth[a + b*x]/x^2,x]
```

output
```
-(ArcCoth[a + b*x]/x) + (b*(-2*Log[x] + (1 + a)*Log[1 - a - b*x] - (-1 + a
)*Log[1 + a + b*x]))/(2*(-1 + a^2))
```

3.67.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6660, 896, 25, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(a+bx)}{x^2} \, dx$$

$$\Big\downarrow \text{6660}$$

$$b\int \frac{1}{x\left(1-(a+bx)^2\right)}dx-\frac{\coth^{-1}(a+bx)}{x}$$

$$\Big\downarrow \text{896}$$

$$b\int \frac{1}{bx\left(1-(a+bx)^2\right)}d(a+bx)-\frac{\coth^{-1}(a+bx)}{x}$$

$$\Big\downarrow \text{25}$$

$$-b\int -\frac{1}{bx\left(1-(a+bx)^2\right)}d(a+bx)-\frac{\coth^{-1}(a+bx)}{x}$$

$$\Big\downarrow \text{477}$$

$$-b\int \left(-\frac{1}{2(1-a)(-a-bx+1)}+\frac{1}{2(a+1)(a+bx+1)}-\frac{1}{(1-a^2)\,bx}\right)d(a+bx)-\frac{\coth^{-1}(a+bx)}{x}$$

$$\Big\downarrow \text{2009}$$

$$b\left(\frac{\log(-bx)}{1-a^2}-\frac{\log(-a-bx+1)}{2(1-a)}-\frac{\log(a+bx+1)}{2(a+1)}\right)-\frac{\coth^{-1}(a+bx)}{x}$$

input
```
Int[ArcCoth[a + b*x]/x^2,x]
```

output
```
-(ArcCoth[a + b*x]/x) + b*(Log[-(b*x)]/(1 - a^2) - Log[1 - a - b*x]/(2*(1
- a)) - Log[1 + a + b*x]/(2*(1 + a)))
```

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6660
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCoth[c + d*x])^p/(f*(m
+ 1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCo
th[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.67.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

method	result
parts	$-\frac{\operatorname{arccoth}(bx+a)}{x}-b\left(\frac{\ln(x)}{(-1+a)(1+a)}-\frac{\ln(bx+a-1)}{-2+2a}+\frac{\ln(bx+a+1)}{2a+2}\right)$
derivativedivides	$b\left(-\frac{\operatorname{arccoth}(bx+a)}{bx}-\frac{\ln(-bx)}{(-1+a)(1+a)}+\frac{\ln(bx+a-1)}{-2+2a}-\frac{\ln(bx+a+1)}{2a+2}\right)$
default	$b\left(-\frac{\operatorname{arccoth}(bx+a)}{bx}-\frac{\ln(-bx)}{(-1+a)(1+a)}+\frac{\ln(bx+a-1)}{-2+2a}-\frac{\ln(bx+a+1)}{2a+2}\right)$
parallelrisch	$-\frac{x\ \operatorname{arccoth}(bx+a)a\,b^3+\ln(x)x\,b^3-\ln(bx+a-1)x\,b^3-x\ \operatorname{arccoth}(bx+a)b^3+\operatorname{arccoth}(bx+a)a^2b^2-\operatorname{arccoth}(bx+a)b^2}{(a^2-1)x\,b^2}$
risch	$-\frac{\ln(bx+a+1)}{2x}-\frac{\ln(bx+a+1)abx-\ln(-bx-a+1)abx-b\ln(bx+a+1)x+2\ln(-x)bx-\ln(-bx-a+1)bx-\ln(bx+a-1)a^2}{2x(-1+a)(1+a)}$

input
```
int(arccoth(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccoth(b*x+a)/x-b*(1/(-1+a)/(1+a)*ln(x)-1/(-2+2*a)*ln(b*x+a-1)+1/(2*a+2)
*ln(b*x+a+1))
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

$$\int \frac{\coth^{-1}(a+bx)}{x^2}\,dx = -\frac{(a-1)bx\log\left(bx+a+1\right)-(a+1)bx\log\left(bx+a-1\right)+2\,bx\log\left(x\right)+(a^2-1)\log\left(\frac{bx+a+1}{bx+a-1}\right)}{2\,(a^2-1)x}$$

input
```
integrate(arccoth(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
-1/2*((a - 1)*b*x*log(b*x + a + 1) - (a + 1)*b*x*log(b*x + a - 1) + 2*b*x*
log(x) + (a^2 - 1)*log((b*x + a + 1)/(b*x + a - 1)))/((a^2 - 1)*x)
```

3.67.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(48) = 96.

Time = 0.52 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.25

$$\int \frac{\coth^{-1}(a+bx)}{x^2}\,dx = \begin{cases} \frac{b\operatorname{acoth}\left(bx-1\right)}{2}-\frac{\operatorname{acoth}\left(bx-1\right)}{x}-\frac{1}{2x} & \text{for}\: a=-1 \\ -\frac{b\operatorname{acoth}\left(bx+1\right)}{2}-\frac{\operatorname{acoth}\left(bx+1\right)}{x}+\frac{1}{2x} & \text{for}\: a=1 \\ -\frac{a^2\operatorname{acoth}\left(a+bx\right)}{a^2x-x}-\frac{abx\operatorname{acoth}\left(a+bx\right)}{a^2x-x}-\frac{bx\log\left(x\right)}{a^2x-x}+\frac{bx\log\left(a+bx+1\right)}{a^2x-x}-\frac{bx\operatorname{acoth}\left(a+bx\right)}{a^2x-x}+\frac{\operatorname{acoth}\left(a+bx\right)}{a^2x-x} & \text{otherwise} \end{cases}$$

input
```
integrate(acoth(b*x+a)/x**2,x)
```

output
```
Piecewise((b*acoth(b*x - 1)/2 - acoth(b*x - 1)/x - 1/(2*x), Eq(a, -1)), (-
b*acoth(b*x + 1)/2 - acoth(b*x + 1)/x + 1/(2*x), Eq(a, 1)), (-a**2*acoth(a
 + b*x)/(a**2*x - x) - a*b*x*acoth(a + b*x)/(a**2*x - x) - b*x*log(x)/(a**
2*x - x) + b*x*log(a + b*x + 1)/(a**2*x - x) - b*x*acoth(a + b*x)/(a**2*x
- x) + acoth(a + b*x)/(a**2*x - x), True))
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{\coth^{-1}(a+bx)}{x^2}\,dx = -\frac{1}{2}b\left(\frac{\log(bx+a+1)}{a+1} - \frac{\log(bx+a-1)}{a-1} + \frac{2\log(x)}{a^2-1}\right) - \frac{\operatorname{arcoth}(bx+a)}{x}$$

input
```
integrate(arccoth(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
-1/2*b*(log(b*x + a + 1)/(a + 1) - log(b*x + a - 1)/(a - 1) + 2*log(x)/(a^
2 - 1)) - arccoth(b*x + a)/x
```

3.67.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(57) = 114$.

Time = 0.29 (sec) , antiderivative size = 259, normalized size of antiderivative = 4.05

$$\int \frac{\coth^{-1}(a+bx)}{x^2}\,dx = -\frac{1}{2}\left((a+1)b-(a-1)b\right)\left(\frac{(a-1)\log\left(\left|\frac{(bx+a+1)a}{bx+a-1} - a - \frac{bx+a+1}{bx+a-1} - 1\right|\right)}{a^3-a^2-a+1} - \frac{\log\left(\frac{|bx+a+1|}{|bx+a-1|}\right)}{a^2-1} - \frac{\log\left(-\cdots\right)}{\left(\frac{(bx+a+1)a}{bx+a-1} - a\cdots\right.}\right.$$

input
```
integrate(arccoth(b*x+a)/x^2,x, algorithm="giac")
```

output
```
-1/2*((a + 1)*b - (a - 1)*b)*((a - 1)*log(abs((b*x + a + 1)*a/(b*x + a - 1
) - a - (b*x + a + 1)/(b*x + a - 1) - 1))/(a^3 - a^2 - a + 1) - log(abs(b*
x + a + 1)/abs(b*x + a - 1))/(a^2 - 1) - log(-(1/(a - ((b*x + a + 1)*(a -
1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a - 1) - b)) + 1)/(1/(
a - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x
+ a - 1) - b)) - 1))/(((b*x + a + 1)*a/(b*x + a - 1) - a - (b*x + a + 1)/(
b*x + a - 1) - 1)*(a - 1)))
```

3.67.9 Mupad [B] (verification not implemented)

Time = 4.72 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{\coth^{-1}(a+bx)}{x^2}\,dx = -\frac{\operatorname{acoth}(a+b\,x)}{x} - \frac{b\,x\,\ln(x) - \frac{b\,x\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2-1\right)}{2} + a\,b\,x\,\operatorname{acoth}(a+b\,x)}{x\,(a^2-1)}$$

input
```
int(acoth(a + b*x)/x^2,x)
```

output
```
- acoth(a + b*x)/x - (b*x*log(x) - (b*x*log(a^2 + b^2*x^2 + 2*a*b*x - 1))/
2 + a*b*x*acoth(a + b*x))/(x*(a^2 - 1))
```

3.68 $\int \frac{\coth^{-1}(a+bx)}{x^3} \, dx$

3.68.1 Optimal result

Integrand size = 10, antiderivative size = 90

$$\int \frac{\coth^{-1}(a+bx)}{x^3} \, dx = -\frac{b}{2(1-a^2)x} - \frac{\coth^{-1}(a+bx)}{2x^2} + \frac{ab^2\log(x)}{(1-a^2)^2} - \frac{b^2\log(1-a-bx)}{4(1-a)^2} + \frac{b^2\log(1+a+bx)}{4(1+a)^2}$$

output
```
-1/2*b/(-a^2+1)/x-1/2*arccoth(b*x+a)/x^2+a*b^2*ln(x)/(-a^2+1)^2-1/4*b^2*ln
(-b*x-a+1)/(1-a)^2+1/4*b^2*ln(b*x+a+1)/(1+a)^2
```

3.68.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \frac{\coth^{-1}(a+bx)}{x^3} \, dx = \frac{1}{4}\left(-\frac{2\coth^{-1}(a+bx)}{x^2} + b\left(\frac{2}{(-1+a^2)x} + \frac{4ab\log(x)}{(-1+a^2)^2} - \frac{b\log(1-a-bx)}{(-1+a)^2} + \frac{b\log(1+a+bx)}{(1+a)^2}\right)\right)$$

input
```
Integrate[ArcCoth[a + b*x]/x^3,x]
```

output
```
((-2*ArcCoth[a + b*x])/x^2 + b*(2/((-1 + a^2)*x) + (4*a*b*Log[x])/(-1 + a^
2)^2 - (b*Log[1 - a - b*x])/(-1 + a)^2 + (b*Log[1 + a + b*x])/(1 + a)^2))/
4
```

3.68.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6660, 896, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(a+bx)}{x^3} \, dx$$

$$\Big\downarrow \text{6660}$$

$$\frac{1}{2} b \int \frac{1}{x^2 \left(1-(a+bx)^2\right)}dx-\frac{\coth^{-1}(a+bx)}{2 x^2}$$

$$\Big\downarrow \text{896}$$

$$\frac{1}{2} b^2 \int \frac{1}{b^2 x^2 \left(1-(a+bx)^2\right)}d(a+bx)-\frac{\coth^{-1}(a+bx)}{2 x^2}$$

$$\Big\downarrow \text{477}$$

$$\frac{1}{2} b^2 \int \left(\frac{2 a}{\left(1-a^2\right)^2 b x}+\frac{1}{2 (1-a)^2 (-a-bx+1)}+\frac{1}{2 (a+1)^2 (a+bx+1)}+\frac{1}{\left(1-a^2\right) b^2 x^2}\right)d(a+bx)-\frac{\coth^{-1}(a+bx)}{2 x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} b^2 \left(-\frac{1}{\left(1-a^2\right) b x}+\frac{2 a \log (-bx)}{\left(1-a^2\right)^2}-\frac{\log (-a-bx+1)}{2 (1-a)^2}+\frac{\log (a+bx+1)}{2 (a+1)^2}\right)-\frac{\coth^{-1}(a+bx)}{2 x^2}$$

input
```
Int[ArcCoth[a + b*x]/x^3,x]
```

output
```
-1/2*ArcCoth[a + b*x]/x^2 + (b^2*(-(1/((1 - a^2)*b*x)) + (2*a*Log[-(b*x)])
/(1 - a^2)^2 - Log[1 - a - b*x]/(2*(1 - a)^2) + Log[1 + a + b*x]/(2*(1 + a
)^2)))/2
```

3.68.3.1 Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6660
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCoth[c + d*x])^p/(f*(m
+ 1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCo
th[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.68.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

method	result
parts	$-\frac{\operatorname{arccoth}(bx+a)}{2x^{2}}-\frac{b\left(-\frac{1}{(-1+a)(1+a)x}-\frac{2ab\ln(x)}{(-1+a)^{2}(1+a)^{2}}+\frac{b\ln(bx+a-1)}{2(-1+a)^{2}}-\frac{b\ln(bx+a+1)}{2(1+a)^{2}}\right)}{2}$
derivativedivides	$b^{2}\left(-\frac{\operatorname{arccoth}(bx+a)}{2b^{2}x^{2}}+\frac{\ln(bx+a+1)}{4(1+a)^{2}}-\frac{\ln(bx+a-1)}{4(-1+a)^{2}}+\frac{1}{2(-1+a)(1+a)bx}+\frac{a\ln(-bx)}{(-1+a)^{2}(1+a)^{2}}\right)$
default	$b^{2}\left(-\frac{\operatorname{arccoth}(bx+a)}{2b^{2}x^{2}}+\frac{\ln(bx+a+1)}{4(1+a)^{2}}-\frac{\ln(bx+a-1)}{4(-1+a)^{2}}+\frac{1}{2(-1+a)(1+a)bx}+\frac{a\ln(-bx)}{(-1+a)^{2}(1+a)^{2}}\right)$
parallelrisch	$\frac{x^{2}\operatorname{arccoth}(bx+a)a^{2}b^{2}+2\ln(x)a\,b^{2}x^{2}-2\ln(bx+a-1)x^{2}a\,b^{2}-2x^{2}\operatorname{arccoth}(bx+a)a\,b^{2}+\operatorname{arccoth}(bx+a)b^{2}x^{2}-2a\,b^{2}x^{2}-\text{arc}}{2x^{2}(a^{2}+2a+1)(a^{2}-2a+1)}$
risch	$-\frac{\ln(bx+a+1)}{4x^{2}}-\frac{\ln(-bx-a+1)a^{2}b^{2}x^{2}-\ln(-bx-a-1)a^{2}b^{2}x^{2}+2\ln(-bx-a+1)a\,b^{2}x^{2}+2\ln(-bx-a-1)a\,b^{2}x^{2}-4\ln(x)}{4x}$

input
```
int(arccoth(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccoth(b*x+a)/x^2-1/2*b*(-1/(-1+a)/(1+a)/x-2*a*b/(-1+a)^2/(1+a)^2*ln
(x)+1/2*b/(-1+a)^2*ln(b*x+a-1)-1/2*b*ln(b*x+a+1)/(1+a)^2)
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.23

$$\int \frac{\coth^{-1}(a+bx)}{x^3}\,dx = \frac{(a^2-2\,a+1)b^2x^2\log\left(bx+a+1\right)-(a^2+2\,a+1)b^2x^2\log\left(bx+a-1\right)+4\,ab^2x^2\log\left(x\right)+2\,(a^2-1)bx-}{4\,(a^4-2\,a^2+1)x^2}$$

input
```
integrate(arccoth(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
1/4*((a^2 - 2*a + 1)*b^2*x^2*log(b*x + a + 1) - (a^2 + 2*a + 1)*b^2*x^2*lo
g(b*x + a - 1) + 4*a*b^2*x^2*log(x) + 2*(a^2 - 1)*b*x - (a^4 - 2*a^2 + 1)*
log((b*x + a + 1)/(b*x + a - 1)))/((a^4 - 2*a^2 + 1)*x^2)
```

3.68.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(73) = 146.

Time = 0.78 (sec) , antiderivative size = 410, normalized size of antiderivative = 4.56

$$\int \frac{\coth^{-1}(a+bx)}{x^3}\,dx = \begin{cases} \frac{b^2\operatorname{acoth}(bx-1)}{8}-\frac{b}{8x}-\frac{\operatorname{acoth}(bx-1)}{2x^2}-\frac{1}{8x^2} \\ \frac{b^2\operatorname{acoth}(bx+1)}{8}-\frac{b}{8x}-\frac{\operatorname{acoth}(bx+1)}{2x^2}+\frac{1}{8x^2} \\ -\frac{a^4\operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2}+\frac{a^2b^2x^2\operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2}+\frac{a^2bx}{2a^4x^2-4a^2x^2+2x^2}+\frac{2a^2\operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2}+\frac{2ab^2x^2\log(x)}{2a^4x^2-4a^2x^2+2x^2}-\frac{2ab^2x^2\log(a+bx}{2a^4x^2-4a^2x^2+2} \end{cases}$$

input
```
integrate(acoth(b*x+a)/x**3,x)
```

output
```
Piecewise((b**2*acoth(b*x - 1)/8 - b/(8*x) - acoth(b*x - 1)/(2*x**2) - 1/(
8*x**2), Eq(a, -1)), (b**2*acoth(b*x + 1)/8 - b/(8*x) - acoth(b*x + 1)/(2*
x**2) + 1/(8*x**2), Eq(a, 1)), (-a**4*acoth(a + b*x)/(2*a**4*x**2 - 4*a**2
*x**2 + 2*x**2) + a**2*b**2*x**2*acoth(a + b*x)/(2*a**4*x**2 - 4*a**2*x**2
 + 2*x**2) + a**2*b*x/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) + 2*a**2*acoth(
a + b*x)/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) + 2*a*b**2*x**2*log(x)/(2*a*
*4*x**2 - 4*a**2*x**2 + 2*x**2) - 2*a*b**2*x**2*log(a + b*x + 1)/(2*a**4*x
**2 - 4*a**2*x**2 + 2*x**2) + 2*a*b**2*x**2*acoth(a + b*x)/(2*a**4*x**2 -
4*a**2*x**2 + 2*x**2) + b**2*x**2*acoth(a + b*x)/(2*a**4*x**2 - 4*a**2*x**
2 + 2*x**2) - b*x/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) - acoth(a + b*x)/(2
*a**4*x**2 - 4*a**2*x**2 + 2*x**2), True))
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int \frac{\coth^{-1}(a+bx)}{x^3}\,dx = \frac{1}{4}\left(\frac{4\,ab\log{(x)}}{a^4-2\,a^2+1}+\frac{b\log{(bx+a+1)}}{a^2+2\,a+1}-\frac{b\log{(bx+a-1)}}{a^2-2\,a+1}+\frac{2}{(a^2-1)x}\right)b - \frac{\operatorname{arcoth}{(bx+a)}}{2\,x^2}$$

input
```
integrate(arccoth(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
1/4*(4*a*b*log(x)/(a^4 - 2*a^2 + 1) + b*log(b*x + a + 1)/(a^2 + 2*a + 1) -
 b*log(b*x + a - 1)/(a^2 - 2*a + 1) + 2/((a^2 - 1)*x))*b - 1/2*arccoth(b*x
 + a)/x^2
```

3.68.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(76) = 152.

Time = 0.29 (sec) , antiderivative size = 360, normalized size of antiderivative = 4.00

$$\int \frac{\coth^{-1}(a+bx)}{x^3}\,dx =$$

$$-\frac{1}{2}\left((a+1)b-(a-1)b\right)\left(\frac{ab\log\left(\frac{|bx+a+1|}{|bx+a-1|}\right)}{a^4-2a^2+1}-\frac{ab\log\left(\left|\frac{(bx+a+1)a}{bx+a-1}-a-\frac{bx+a+1}{bx+a-1}-1\right|\right)}{a^4-2a^2+1}+\frac{\left(\frac{(bx+a+1)ab}{bx+a-1}-ab\right.}{(a^2-2}\right.$$

input
```
integrate(arccoth(b*x+a)/x^3,x, algorithm="giac")
```

output
```
-1/2*((a + 1)*b - (a - 1)*b)*(a*b*log(abs(b*x + a + 1)/abs(b*x + a - 1))/(
a^4 - 2*a^2 + 1) - a*b*log(abs((b*x + a + 1)*a/(b*x + a - 1) - a - (b*x +
a + 1)/(b*x + a - 1) - 1))/(a^4 - 2*a^2 + 1) + ((b*x + a + 1)*a*b/(b*x + a
 - 1) - a*b - (b*x + a + 1)*b/(b*x + a - 1))*log(-(1/(a - ((b*x + a + 1)*(
a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a - 1) - b)) + 1)/
(1/(a - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(
b*x + a - 1) - b)) - 1))/((a^2 - 2*a + 1)*((b*x + a + 1)*a/(b*x + a - 1) -
 a - (b*x + a + 1)/(b*x + a - 1) - 1)^2) + (a*b + b)/(((b*x + a + 1)*a/(b*
x + a - 1) - a - (b*x + a + 1)/(b*x + a - 1) - 1)*(a + 1)^2*(a - 1)^2))
```

3.68.9 Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.74

$$\int \frac{\coth^{-1}(a+bx)}{x^3}\,dx$$

$$= \ln(x)\left(\frac{b^2}{4(a-1)^2}-\frac{b^2}{4(a+1)^2}\right)-\ln\left(a^2+2abx+b^2x^2-1\right)\left(\frac{b^2}{8(a-1)^2}-\frac{b^2}{8(a+1)^2}\right)$$
$$-\frac{\mathrm{acoth}(a+bx)\left(\frac{a^2}{2}-\frac{1}{2}\right)-\frac{bx}{2}+\frac{b^2x^2\,\mathrm{acoth}(a+bx)}{2}+\frac{x^3\,(3a^2b^3+b^3)}{2\,(a^2-1)^2}+\frac{a\,b^4\,x^4}{(a^2-1)^2}+a\,b\,x\,\mathrm{acoth}(a+bx)}{a^2x^2+2abx^3+b^2x^4-x^2}$$
$$-\frac{\mathrm{atan}\left(\frac{2\,x\,b^2+2\,a\,b}{2\sqrt{b^2\,(a^2-1)-a^2\,b^2}}\right)\,(a^2b^3+b^3)}{\sqrt{-b^2}\,(2a^4-4a^2+2)}$$

input
```
int(acoth(a + b*x)/x^3,x)
```

output
```
log(x)*(b^2/(4*(a - 1)^2) - b^2/(4*(a + 1)^2)) - log(a^2 + b^2*x^2 + 2*a*b
*x - 1)*(b^2/(8*(a - 1)^2) - b^2/(8*(a + 1)^2)) - (acoth(a + b*x)*(a^2/2 -
 1/2) - (b*x)/2 + (b^2*x^2*acoth(a + b*x))/2 + (x^3*(b^3 + 3*a^2*b^3))/(2*
(a^2 - 1)^2) + (a*b^4*x^4)/(a^2 - 1)^2 + a*b*x*acoth(a + b*x))/(a^2*x^2 -
x^2 + b^2*x^4 + 2*a*b*x^3) - (atan((2*a*b + 2*b^2*x)/(2*(b^2*(a^2 - 1) - a
^2*b^2)^(1/2)))*(b^3 + a^2*b^3))/((-b^2)^(1/2)*(2*a^4 - 4*a^2 + 2))
```

3.68. $\int \frac{\coth^{-1}(a+bx)}{x^3} \, dx$

3.69 $\int x^3 \coth^{-1}(a+bx)^2 \, dx$

3.69.1 Optimal result

Integrand size = 12, antiderivative size = 263

$$\begin{aligned}\int x^3 \coth^{-1}(a+bx)^2 \, dx = & -\frac{ax}{b^3}+\frac{(a+bx)^2}{12b^4}+\frac{(1+6a^2)(a+bx)\coth^{-1}(a+bx)}{2b^4} \\ & -\frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4}+\frac{(a+bx)^3\coth^{-1}(a+bx)}{6b^4} \\ & -\frac{a(1+a^2)\coth^{-1}(a+bx)^2}{b^4}-\frac{(1+6a^2+a^4)\coth^{-1}(a+bx)^2}{4b^4} \\ & +\frac{1}{4}x^4\coth^{-1}(a+bx)^2+\frac{a\text{arctanh}(a+bx)}{b^4} \\ & +\frac{2a(1+a^2)\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{b^4}+\frac{\log\left(1-(a+bx)^2\right)}{12b^4} \\ & +\frac{(1+6a^2)\log\left(1-(a+bx)^2\right)}{4b^4}+\frac{a(1+a^2)\,\text{PolyLog}\left(2,-\frac{1+a+bx}{1-a-bx}\right)}{b^4}\end{aligned}$$

output
```
-a*x/b^3+1/12*(b*x+a)^2/b^4+1/2*(6*a^2+1)*(b*x+a)*arccoth(b*x+a)/b^4-a*(b*
x+a)^2*arccoth(b*x+a)/b^4+1/6*(b*x+a)^3*arccoth(b*x+a)/b^4-a*(a^2+1)*arcco
th(b*x+a)^2/b^4-1/4*(a^4+6*a^2+1)*arccoth(b*x+a)^2/b^4+1/4*x^4*arccoth(b*x
+a)^2+a*arctanh(b*x+a)/b^4+2*a*(a^2+1)*arccoth(b*x+a)*ln(2/(-b*x-a+1))/b^4
+1/12*ln(1-(b*x+a)^2)/b^4+1/4*(6*a^2+1)*ln(1-(b*x+a)^2)/b^4+a*(a^2+1)*poly
log(2,(-b*x-a-1)/(-b*x-a+1))/b^4
```

3.69.2 Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.80

$$\int x^3 \coth^{-1}(a+bx)^2\,dx =$$

$$-\frac{1+11a^2+10abx-b^2x^2+3(1-4a+6a^2-4a^3+a^4-b^4x^4)\coth^{-1}(a+bx)^2-2\coth^{-1}(a+bx)\Big(9a}{}$$

input
```
Integrate[x^3*ArcCoth[a + b*x]^2,x]
```

output
```
-1/12*(1 + 11*a^2 + 10*a*b*x - b^2*x^2 + 3*(1 - 4*a + 6*a^2 - 4*a^3 + a^4
- b^4*x^4)*ArcCoth[a + b*x]^2 - 2*ArcCoth[a + b*x]*(9*a + 13*a^3 + 3*b*x +
 9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 12*(a + a^3)*Log[1 - E^(-2*ArcCoth[a
+ b*x])]) + 8*Log[(a + b*x)^(-1)] + 36*a^2*Log[(a + b*x)^(-1)] + 8*Log[1/S
qrt[1 - (a + b*x)^(-2)]] + 36*a^2*Log[1/Sqrt[1 - (a + b*x)^(-2)]] + 12*(a
+ a^3)*PolyLog[2, E^(-2*ArcCoth[a + b*x])])/b^4
```

3.69.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6662, 25, 27, 6481, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(a+bx)^2\,dx$$

$$\Big\downarrow \; 6662$$

$$\frac{\int x^3 \coth^{-1}(a+bx)^2 d(a+bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -x^3 \coth^{-1}(a+bx)^2 d(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int -b^3x^3 \coth^{-1}(a+bx)^2 d(a+bx)}{b^4}$$

$\Big\downarrow$ 6481

$$-\frac{\frac{1}{2}\int\left(-\coth^{-1}(a+bx)(a+bx)^2+4a\coth^{-1}(a+bx)(a+bx)-\left(6a^2+1\right)\coth^{-1}(a+bx)+\frac{\left(a^4+6a^2-4\left(a^2+1\right)(a+b\right.}{1-(a+}\right.}{b^4}$$

$\Big\downarrow$ 2009

$$-\frac{\frac{1}{2}\left(-2a\left(a^2+1\right)\text{PolyLog}\left(2,-\frac{a+bx+1}{-a-bx+1}\right)-\frac{1}{2}\left(6a^2+1\right)\log\left(1-(a+bx)^2\right)-\left(6a^2+1\right)(a+bx)\coth^{-1}(a+bx)-\right.}{}$$

input
```
Int[x^3*ArcCoth[a + b*x]^2,x]
```

output
```
-((-1/4*(b^4*x^4*ArcCoth[a + b*x]^2) + (2*a*(a + b*x) - (a + b*x)^2/6 - (1
 + 6*a^2)*(a + b*x)*ArcCoth[a + b*x] + 2*a*(a + b*x)^2*ArcCoth[a + b*x] -
((a + b*x)^3*ArcCoth[a + b*x])/3 + 2*a*(1 + a^2)*ArcCoth[a + b*x]^2 + ((1
+ 6*a^2 + a^4)*ArcCoth[a + b*x]^2)/2 - 2*a*ArcTanh[a + b*x] - 4*a*(1 + a^2
)*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)] - Log[1 - (a + b*x)^2]/6 - ((1 + 6
*a^2)*Log[1 - (a + b*x)^2])/2 - 2*a*(1 + a^2)*PolyLog[2, -((1 + a + b*x)/(
1 - a - b*x))])/2)/b^4)
```

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6481
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.69.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.70

method	result
parts	$\frac{x^4 \operatorname{arccoth}(bx+a)^2}{4} + \frac{6 \operatorname{arccoth}(bx+a)a^2(bx+a) - 2 \operatorname{arccoth}(bx+a)a(bx+a)^2 + \frac{\operatorname{arccoth}(bx+a)(bx+a)^3}{3} + (bx+a) \operatorname{arccoth}(\cdots}{\cdots}$
derivativedivides	$\frac{3 \operatorname{arccoth}(bx+a)a^2(bx+a) + \frac{3 \operatorname{arccoth}(bx+a)^2 a^2 (bx+a)^2}{2} - \operatorname{arccoth}(bx+a)a(bx+a)^2 - \operatorname{arccoth}(bx+a)^2 a(bx+a)^3 - \operatorname{arccoth}(\cdots}{\cdots}$
default	$\frac{3 \operatorname{arccoth}(bx+a)a^2(bx+a) + \frac{3 \operatorname{arccoth}(bx+a)^2 a^2 (bx+a)^2}{2} - \operatorname{arccoth}(bx+a)a(bx+a)^2 - \operatorname{arccoth}(bx+a)^2 a(bx+a)^3 - \operatorname{arccoth}(\cdots}{\cdots}$
risch	$-\frac{1}{12b^4} - \frac{5ax}{6b^3} - \frac{\ln(bx+a-1)x}{4b^3} + \frac{\ln(bx+a-1)^2 a^3}{4b^4} - \frac{3\ln(bx+a-1)^2 a^2}{8b^4} + \frac{\ln(bx+a-1)^2 a}{4b^4} - \frac{\ln(bx+a-1)^2 a^4}{16b^4} - \cdots$

input
```
int(x^3*arccoth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*arccoth(b*x+a)^2+1/2/b^4*(6*arccoth(b*x+a)*a^2*(b*x+a)-2*arccoth(b
*x+a)*a*(b*x+a)^2+1/3*arccoth(b*x+a)*(b*x+a)^3+(b*x+a)*arccoth(b*x+a)+1/2*
arccoth(b*x+a)*ln(b*x+a-1)*a^4-2*arccoth(b*x+a)*ln(b*x+a-1)*a^3+3*arccoth(
b*x+a)*ln(b*x+a-1)*a^2-2*arccoth(b*x+a)*ln(b*x+a-1)*a+1/2*arccoth(b*x+a)*l
n(b*x+a-1)-1/2*arccoth(b*x+a)*ln(b*x+a+1)*a^4-2*arccoth(b*x+a)*ln(b*x+a+1)
*a^3-3*arccoth(b*x+a)*ln(b*x+a+1)*a^2-2*arccoth(b*x+a)*ln(b*x+a+1)*a-1/2*a
rccoth(b*x+a)*ln(b*x+a+1)-2*(b*x+a)*a+1/6*(b*x+a)^2+1/6*(18*a^2-6*a+4)*ln(
b*x+a-1)-1/6*(-18*a^2-6*a-4)*ln(b*x+a+1)+1/6*(3*a^4-12*a^3+18*a^2-12*a+3)*
(1/4*ln(b*x+a-1)^2-1/2*dilog(1/2*b*x+1/2*a+1/2)-1/2*ln(b*x+a-1)*ln(1/2*b*x
+1/2*a+1/2))+1/6*(-3*a^4-12*a^3-18*a^2-12*a-3)*(1/2*(ln(b*x+a+1)-ln(1/2*b*
x+1/2*a+1/2))*ln(-1/2*b*x-1/2*a+1/2)-1/2*dilog(1/2*b*x+1/2*a+1/2)-1/4*ln(b
*x+a+1)^2))
```

3.69.5 Fricas [F]

$$\int x^3 \coth^{-1}(a+bx)^2\,dx = \int x^3 \operatorname{arcoth}\left(bx+a\right)^2\,dx$$

input
```
integrate(x^3*arccoth(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^3*arccoth(b*x + a)^2, x)
```

3.69.6 Sympy [F]

$$\int x^3 \coth^{-1}(a+bx)^2\,dx = \int x^3 \operatorname{acoth}^2\left(a+bx\right)dx$$

input
```
integrate(x**3*acoth(b*x+a)**2,x)
```

output
```
Integral(x**3*acoth(a + b*x)**2, x)
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.22

$$\int x^3 \coth^{-1}(a+bx)^2\,dx = \frac{1}{4}\,x^4 \operatorname{arcoth}\left(bx+a\right)^2 + \frac{1}{48}\,b^2\left(\frac{48\left(a^3+a\right)\left(\log\left(bx+a-1\right)\log\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a+\frac{1}{2}\right)\right)}{b^6} + \frac{4\left(13\,a^3+18\,a^2+\right.}{} \right. + \frac{1}{12}\,b\left(\frac{2\left(b^2x^3-3\,abx^2+3\left(3\,a^2+1\right)x\right)}{b^4} - \frac{3\left(a^4+4\,a^3+6\,a^2+4\,a+1\right)\log\left(bx+a+1\right)}{b^5} + \frac{3\left(a^4-4\,a^3\right.}{}\right. + a)$$

input
```
integrate(x^3*arccoth(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*x^4*arccoth(b*x + a)^2 + 1/48*b^2*(48*(a^3 + a)*(log(b*x + a - 1)*log(
1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^6 + 4*(13*a^3 +
18*a^2 + 9*a + 4)*log(b*x + a + 1)/b^6 + (4*b^2*x^2 - 40*a*b*x + 3*(a^4 +
4*a^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1)^2 - 6*(a^4 + 4*a^3 + 6*a^2 + 4*a
 + 1)*log(b*x + a + 1)*log(b*x + a - 1) + 3*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1
)*log(b*x + a - 1)^2 - 4*(13*a^3 - 18*a^2 + 9*a - 4)*log(b*x + a - 1))/b^6
) + 1/12*b*(2*(b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 + 1)*x)/b^4 - 3*(a^4 + 4*a^3
 + 6*a^2 + 4*a + 1)*log(b*x + a + 1)/b^5 + 3*(a^4 - 4*a^3 + 6*a^2 - 4*a +
1)*log(b*x + a - 1)/b^5)*arccoth(b*x + a)
```

3.69.8 Giac [F]

$$\int x^3 \coth^{-1}(a+bx)^2 \, dx = \int x^3 \operatorname{arcoth}(bx+a)^2 \, dx$$

input
```
integrate(x^3*arccoth(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*arccoth(b*x + a)^2, x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^{-1}(a+bx)^2 \, dx = \int x^3 \operatorname{acoth}(a+b\,x)^2 \, dx$$

input
```
int(x^3*acoth(a + b*x)^2,x)
```

output
```
int(x^3*acoth(a + b*x)^2, x)
```

3.70 $\int x^2 \coth^{-1}(a+bx)^2 \, dx$

3.70.1 Optimal result

Integrand size = 12, antiderivative size = 204

$$\begin{aligned}\int x^2 \coth^{-1}(a+bx)^2 \, dx = {} & \frac{x}{3b^2} - \frac{2a(a+bx)\coth^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\coth^{-1}(a+bx)}{3b^3} \\ & + \frac{a(3+a^2)\coth^{-1}(a+bx)^2}{3b^3} + \frac{(1+3a^2)\coth^{-1}(a+bx)^2}{3b^3} \\ & + \frac{1}{3}x^3\coth^{-1}(a+bx)^2 - \frac{\operatorname{arctanh}(a+bx)}{3b^3} \\ & - \frac{2(1+3a^2)\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{3b^3} \\ & - \frac{a\log\left(1-(a+bx)^2\right)}{b^3} - \frac{(1+3a^2)\operatorname{PolyLog}\left(2,-\frac{1+a+bx}{1-a-bx}\right)}{3b^3}\end{aligned}$$

output
```
1/3*x/b^2-2*a*(b*x+a)*arccoth(b*x+a)/b^3+1/3*(b*x+a)^2*arccoth(b*x+a)/b^3+
1/3*a*(a^2+3)*arccoth(b*x+a)^2/b^3+1/3*(3*a^2+1)*arccoth(b*x+a)^2/b^3+1/3*
x^3*arccoth(b*x+a)^2-1/3*arctanh(b*x+a)/b^3-2/3*(3*a^2+1)*arccoth(b*x+a)*l
n(2/(-b*x-a+1))/b^3-a*ln(1-(b*x+a)^2)/b^3-1/3*(3*a^2+1)*polylog(2,(-b*x-a-
1)/(-b*x-a+1))/b^3
```

3.70.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 644 vs. 2(204) = 408.

Time = 3.59 (sec) , antiderivative size = 644, normalized size of antiderivative = 3.16

$$\int x^2 \coth^{-1}(a+bx)^2 \, dx =$$

$$-\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}(1-(a+bx)^2)\left(\frac{4\coth^{-1}(a+bx)}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}+\frac{3\coth^{-1}(a+bx)^2}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}-\frac{12a\coth^{-1}(a+bx)^2}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}+\frac{9a^2\coth^{-1}}{(a+bx)\sqrt{1}}\right.}{}$$

input
```
Integrate[x^2*ArcCoth[a + b*x]^2,x]
```

output
```
-1/12*((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*(1 - (a + b*x)^2)*((4*ArcCoth[a
+ b*x])/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]) + (3*ArcCoth[a + b*x]^2)/((a
+ b*x)*Sqrt[1 - (a + b*x)^(-2)]) - (12*a*ArcCoth[a + b*x]^2)/((a + b*x)*Sq
rt[1 - (a + b*x)^(-2)]) + (9*a^2*ArcCoth[a + b*x]^2)/((a + b*x)*Sqrt[1 - (
a + b*x)^(-2)]) + (-1 + 6*a*ArcCoth[a + b*x] - 3*(-1 + a^2)*ArcCoth[a + b*
x]^2)/Sqrt[1 - (a + b*x)^(-2)] + Cosh[3*ArcCoth[a + b*x]] - 6*a*ArcCoth[a
+ b*x]*Cosh[3*ArcCoth[a + b*x]] + ArcCoth[a + b*x]^2*Cosh[3*ArcCoth[a + b*
x]] + 3*a^2*ArcCoth[a + b*x]^2*Cosh[3*ArcCoth[a + b*x]] + (6*ArcCoth[a + b
*x]*Log[1 - E^(-2*ArcCoth[a + b*x])])/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)])
 + (18*a^2*ArcCoth[a + b*x]*Log[1 - E^(-2*ArcCoth[a + b*x])])/((a + b*x)*S
qrt[1 - (a + b*x)^(-2)]) - (18*a*Log[(a + b*x)^(-1)])/((a + b*x)*Sqrt[1 -
(a + b*x)^(-2)]) - (18*a*Log[1/Sqrt[1 - (a + b*x)^(-2)]])/((a + b*x)*Sqrt[
1 - (a + b*x)^(-2)]) + (4*(1 + 3*a^2)*PolyLog[2, E^(-2*ArcCoth[a + b*x])])
/((a + b*x)^3*(1 - (a + b*x)^(-2))^(3/2)) - ArcCoth[a + b*x]^2*Sinh[3*ArcC
oth[a + b*x]] - 3*a^2*ArcCoth[a + b*x]^2*Sinh[3*ArcCoth[a + b*x]] - 2*ArcC
oth[a + b*x]*Log[1 - E^(-2*ArcCoth[a + b*x])]*Sinh[3*ArcCoth[a + b*x]] - 6
*a^2*ArcCoth[a + b*x]*Log[1 - E^(-2*ArcCoth[a + b*x])]*Sinh[3*ArcCoth[a +
b*x]] + 6*a*Log[(a + b*x)^(-1)]*Sinh[3*ArcCoth[a + b*x]] + 6*a*Log[1/Sqrt[
1 - (a + b*x)^(-2)]]*Sinh[3*ArcCoth[a + b*x]]))/b^3
```

3.70.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6662, 27, 6481, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(a+bx)^2 \, dx$$

$$\Big\downarrow \; 6662$$

$$\frac{\int x^2 \coth^{-1}(a+bx)^2 d(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int b^2x^2 \coth^{-1}(a+bx)^2 d(a+bx)}{b^3}$$

$$\Big\downarrow \; 6481$$

$$\frac{\frac{2}{3}\int \left(-3a\coth^{-1}(a+bx)+(a+bx)\coth^{-1}(a+bx)+\frac{(a(a^2+3)-(3a^2+1)(a+bx))\coth^{-1}(a+bx)}{1-(a+bx)^2}\right)d(a+bx)+\frac{1}{3}b^3x^3\coth}{b^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{3}\left(-\frac{1}{2}(3a^2+1)\operatorname{PolyLog}\left(2,-\frac{a+bx+1}{-a-bx+1}\right)+\frac{1}{2}a(a^2+3)\coth^{-1}(a+bx)^2+\frac{1}{2}(3a^2+1)\coth^{-1}(a+bx)^2-(3a^2+1$$

input
```
Int[x^2*ArcCoth[a + b*x]^2,x]
```

output
```
((b^3*x^3*ArcCoth[a + b*x]^2)/3 + (2*((a + b*x)/2 - 3*a*(a + b*x)*ArcCoth[
a + b*x] + ((a + b*x)^2*ArcCoth[a + b*x])/2 + (a*(3 + a^2)*ArcCoth[a + b*x
]^2)/2 + ((1 + 3*a^2)*ArcCoth[a + b*x]^2)/2 - ArcTanh[a + b*x]/2 - (1 + 3*
a^2)*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)] - (3*a*Log[1 - (a + b*x)^2])/2
- ((1 + 3*a^2)*PolyLog[2, -((1 + a + b*x)/(1 - a - b*x))])/2))/3)/b^3
```

3.70.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6481
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.70.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.72

method	result
parts	$\frac{x^3 \operatorname{arccoth}(bx+a)^2}{3} + \frac{-2 \operatorname{arccoth}(bx+a)a(bx+a)+\frac{(bx+a)^2 \operatorname{arccoth}(bx+a)}{3}-\frac{\operatorname{arccoth}(bx+a)\ln(bx+a-1)a^3}{3}+\operatorname{arccoth}(bx+a)}{}$
derivativedivides	$\frac{-\frac{\operatorname{arccoth}(bx+a)^2a^3}{3}+\operatorname{arccoth}(bx+a)^2a^2(bx+a)-\operatorname{arccoth}(bx+a)^2a(bx+a)^2+\frac{\operatorname{arccoth}(bx+a)^2(bx+a)^3}{3}-2 \operatorname{arccoth}(bx+a)a(}{}$
default	$\frac{-\frac{\operatorname{arccoth}(bx+a)^2a^3}{3}+\operatorname{arccoth}(bx+a)^2a^2(bx+a)-\operatorname{arccoth}(bx+a)^2a(bx+a)^2+\frac{\operatorname{arccoth}(bx+a)^2(bx+a)^3}{3}-2 \operatorname{arccoth}(bx+a)a(}{}$
risch	$-\frac{1}{3b^3}-\frac{5\ln(bx+a+1)a^2}{6b^3}-\frac{\ln(bx+a+1)a}{b^3}-\frac{\ln(bx+a-1)x^2}{6b}-\frac{\ln(bx+a-1)^2a^2}{4b^3}+\frac{\ln(bx+a-1)^2a}{4b^3}+\frac{5\ln(bx+a}{6b^3}$

input
```
int(x^2*arccoth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccoth(b*x+a)^2+2/3/b^3*(-3*arccoth(b*x+a)*a*(b*x+a)+1/2*(b*x+a)^
2*arccoth(b*x+a)-1/2*arccoth(b*x+a)*ln(b*x+a-1)*a^3+3/2*arccoth(b*x+a)*ln(
b*x+a-1)*a^2-3/2*arccoth(b*x+a)*ln(b*x+a-1)*a+1/2*arccoth(b*x+a)*ln(b*x+a-
1)+1/2*arccoth(b*x+a)*ln(b*x+a+1)*a^3+3/2*arccoth(b*x+a)*ln(b*x+a+1)*a^2+3
/2*arccoth(b*x+a)*ln(b*x+a+1)*a+1/2*arccoth(b*x+a)*ln(b*x+a+1)+1/2*(a^3+3*
a^2+3*a+1)*(1/2*(ln(b*x+a+1)-ln(1/2*b*x+1/2*a+1/2))*ln(-1/2*b*x-1/2*a+1/2)
-1/2*dilog(1/2*b*x+1/2*a+1/2)-1/4*ln(b*x+a+1)^2)+1/2*b*x+1/2*a-1/4*(6*a-1)
*ln(b*x+a-1)+1/4*(-6*a-1)*ln(b*x+a+1)+1/2*(-a^3+3*a^2-3*a+1)*(1/4*ln(b*x+a
-1)^2-1/2*dilog(1/2*b*x+1/2*a+1/2)-1/2*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2)))
```

3.70.5 Fricas [F]

$$\int x^2 \coth^{-1}(a+bx)^2\,dx = \int x^2 \operatorname{arcoth}\left(bx+a\right)^2\,dx$$

input
```
integrate(x^2*arccoth(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^2*arccoth(b*x + a)^2, x)
```

3.70.6 Sympy [F]

$$\int x^2 \coth^{-1}(a+bx)^2\,dx = \int x^2 \operatorname{acoth}^2\left(a+bx\right)dx$$

input
```
integrate(x**2*acoth(b*x+a)**2,x)
```

output
```
Integral(x**2*acoth(a + b*x)**2, x)
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.27

$$\int x^2 \coth^{-1}(a+bx)^2\,dx = \frac{1}{3}\,x^3\,\mathrm{arcoth}\,(bx+a)^2$$
$$-\frac{1}{12}\,b^2\left(\frac{4\,(3\,a^2+1)\left(\log\left(bx+a-1\right)\log\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a+\frac{1}{2}\right)\right)}{b^5}+\frac{2\,(5\,a^2+6\,a+1)\,\mathrm{l}}{b^5}\right.$$
$$+\frac{1}{3}\,b\left(\frac{bx^2-4\,ax}{b^3}+\frac{(a^3+3\,a^2+3\,a+1)\log\left(bx+a+1\right)}{b^4}-\frac{(a^3-3\,a^2+3\,a-1)\log\left(bx+a-1\right)}{b^4}\right)\mathrm{arco}$$
$$+a)$$

input
```
integrate(x^2*arccoth(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(b*x + a)^2 - 1/12*b^2*(4*(3*a^2 + 1)*(log(b*x + a - 1)*log
(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^5 + 2*(5*a^2 +
6*a + 1)*log(b*x + a + 1)/b^5 + ((a^3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1)^
2 - 2*(a^3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1)*log(b*x + a - 1) + (a^3 - 3
*a^2 + 3*a - 1)*log(b*x + a - 1)^2 - 4*b*x - 2*(5*a^2 - 6*a + 1)*log(b*x +
 a - 1))/b^5) + 1/3*b*((b*x^2 - 4*a*x)/b^3 + (a^3 + 3*a^2 + 3*a + 1)*log(b
*x + a + 1)/b^4 - (a^3 - 3*a^2 + 3*a - 1)*log(b*x + a - 1)/b^4)*arccoth(b*
x + a)
```

3.70.8 Giac [F]

$$\int x^2 \coth^{-1}(a+bx)^2\,dx = \int x^2\,\mathrm{arcoth}\,(bx+a)^2\,dx$$

input
```
integrate(x^2*arccoth(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(b*x + a)^2, x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(a+bx)^2\,dx = \int x^2\,\mathrm{acoth}(a+b\,x)^2\,dx$$

input
```
int(x^2*acoth(a + b*x)^2,x)
```

output
```
int(x^2*acoth(a + b*x)^2, x)
```

3.71 $\int x \coth^{-1}(a+bx)^2 \, dx$

3.71.1 Optimal result

Integrand size = 10, antiderivative size = 136

$$\int x \coth^{-1}(a+bx)^2 \, dx = \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} - \frac{a\coth^{-1}(a+bx)^2}{b^2} - \frac{(1+a^2)\coth^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\coth^{-1}(a+bx)^2 + \frac{2a\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{b^2} + \frac{\log\left(1-(a+bx)^2\right)}{2b^2} + \frac{a\,\mathrm{PolyLog}\left(2,-\frac{1+a+bx}{1-a-bx}\right)}{b^2}$$

output
```
(b*x+a)*arccoth(b*x+a)/b^2-a*arccoth(b*x+a)^2/b^2-1/2*(a^2+1)*arccoth(b*x+
a)^2/b^2+1/2*x^2*arccoth(b*x+a)^2+2*a*arccoth(b*x+a)*ln(2/(-b*x-a+1))/b^2+
1/2*ln(1-(b*x+a)^2)/b^2+a*polylog(2,(-b*x-a-1)/(-b*x-a+1))/b^2
```

3.71.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.78

$$\int x \coth^{-1}(a+bx)^2 \, dx = \frac{(-1+2a-a^2+b^2x^2)\coth^{-1}(a+bx)^2 + 2\coth^{-1}(a+bx)\left(a+bx+2a\log\left(1-e^{-2\coth^{-1}(a+bx)}\right)\right) - 2\,\mathrm{lo}}{2b^2}$$

input
```
Integrate[x*ArcCoth[a + b*x]^2,x]
```

output
```
((-1 + 2*a - a^2 + b^2*x^2)*ArcCoth[a + b*x]^2 + 2*ArcCoth[a + b*x]*(a + b
*x + 2*a*Log[1 - E^(-2*ArcCoth[a + b*x])]) - 2*Log[1/((a + b*x)*Sqrt[1 - (
a + b*x)^(-2)])] - 2*a*PolyLog[2, E^(-2*ArcCoth[a + b*x])])/(2*b^2)
```

3.71.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6662, 25, 27, 6481, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(a+bx)^2 \, dx$$

$$\Big\downarrow \text{6662}$$

$$\frac{\int x \coth^{-1}(a+bx)^2 d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -x \coth^{-1}(a+bx)^2 d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -bx \coth^{-1}(a+bx)^2 d(a+bx)}{b^2}$$

$$\Big\downarrow \text{6481}$$

$$-\frac{\int \left(\frac{(a^2-2(a+bx)a+1)\coth^{-1}(a+bx)}{1-(a+bx)^2} - \coth^{-1}(a+bx)\right) d(a+bx) - \frac{1}{2}b^2x^2\coth^{-1}(a+bx)^2}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}(a^2+1)\coth^{-1}(a+bx)^2 - \frac{1}{2}b^2x^2\coth^{-1}(a+bx)^2 - a\,\text{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right) - \frac{1}{2}\log\left(1-(a+bx)^2\right) + a\,\text{cot}}{b^2}$$

input
```
Int[x*ArcCoth[a + b*x]^2,x]
```

output
```
-((-((a + b*x)*ArcCoth[a + b*x]) + a*ArcCoth[a + b*x]^2 + ((1 + a^2)*ArcCo
th[a + b*x]^2)/2 - (b^2*x^2*ArcCoth[a + b*x]^2)/2 - 2*a*ArcCoth[a + b*x]*L
og[2/(1 - a - b*x)] - Log[1 - (a + b*x)^2]/2 - a*PolyLog[2, -((1 + a + b*x
)/(1 - a - b*x))])/b^2)
```

3.71.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6481
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.71.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.79

method	result
derivativedivides	$\frac{\frac{\operatorname{arccoth}(bx+a)^2(bx+a)^2}{2}-\operatorname{arccoth}(bx+a)^2a(bx+a)+(bx+a)\operatorname{arccoth}(bx+a)-\operatorname{arccoth}(bx+a)\ln(bx+a-1)a+\frac{\operatorname{arccoth}(bx+a)}{2}}{}$
default	$\frac{\frac{\operatorname{arccoth}(bx+a)^2(bx+a)^2}{2}-\operatorname{arccoth}(bx+a)^2a(bx+a)+(bx+a)\operatorname{arccoth}(bx+a)-\operatorname{arccoth}(bx+a)\ln(bx+a-1)a+\frac{\operatorname{arccoth}(bx+a)}{2}}{}$
risch	$\frac{\operatorname{dilog}\left(\frac{bx}{2}+\frac{a}{2}+\frac{1}{2}\right)a}{b^2}+\frac{\ln(bx+a+1)a}{2b^2}-\frac{\ln(bx+a-1)x}{2b}-\frac{\ln(bx+a-1)^2a^2}{8b^2}+\frac{\ln(bx+a-1)^2a}{4b^2}-\frac{(-b^2x^2+a^2+2a+1}{8b^2}$
parts	$\frac{x^2\operatorname{arccoth}(bx+a)^2}{2}+\frac{(bx+a)\operatorname{arccoth}(bx+a)+\frac{\operatorname{arccoth}(bx+a)\ln(bx+a-1)a^2}{2}-\operatorname{arccoth}(bx+a)\ln(bx+a-1)a+\frac{\operatorname{arccoth}(bx+a}{}}{}$

input
```
int(x*arccoth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/2*arccoth(b*x+a)^2*(b*x+a)^2-arccoth(b*x+a)^2*a*(b*x+a)+(b*x+a)*a
rccoth(b*x+a)-arccoth(b*x+a)*ln(b*x+a-1)*a+1/2*arccoth(b*x+a)*ln(b*x+a-1)-
arccoth(b*x+a)*ln(b*x+a+1)*a-1/2*arccoth(b*x+a)*ln(b*x+a+1)+1/2*ln(b*x+a-1
)+1/2*ln(b*x+a+1)+1/2*(-2*a+1)*(1/4*ln(b*x+a-1)^2-1/2*dilog(1/2*b*x+1/2*a+
1/2)-1/2*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2))+1/2*(-2*a-1)*(1/2*(ln(b*x+a+1)
-ln(1/2*b*x+1/2*a+1/2))*ln(-1/2*b*x-1/2*a+1/2)-1/2*dilog(1/2*b*x+1/2*a+1/2
)-1/4*ln(b*x+a+1)^2))
```

3.71.5 Fricas [F]

$$\int x\coth^{-1}(a+bx)^2\,dx=\int x\operatorname{arcoth}(bx+a)^2\,dx$$

input
```
integrate(x*arccoth(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x*arccoth(b*x + a)^2, x)
```

3.71.6 Sympy [F]

$$\int x \coth^{-1}(a+bx)^2\,dx = \int x \operatorname{acoth}^2(a+bx)\,dx$$

input
```
integrate(x*acoth(b*x+a)**2,x)
```

output
```
Integral(x*acoth(a + b*x)**2, x)
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.49

$$\int x \coth^{-1}(a+bx)^2\,dx = \frac{1}{2}x^2 \operatorname{arcoth}(bx+a)^2 + \frac{1}{8}b^2\left(\frac{8\left(\log(bx+a-1)\log\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}bx-\frac{1}{2}a+\frac{1}{2}\right)\right)a}{b^4} + \frac{4(a+1)\log(bx+a+1)}{b^4} + \right. + \frac{1}{2}b\left(\frac{2x}{b^2} - \frac{(a^2+2a+1)\log(bx+a+1)}{b^3} + \frac{(a^2-2a+1)\log(bx+a-1)}{b^3}\right)\operatorname{arcoth}(bx + a)$$

input
```
integrate(x*arccoth(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*x^2*arccoth(b*x + a)^2 + 1/8*b^2*(8*(log(b*x + a - 1)*log(1/2*b*x + 1/
2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))*a/b^4 + 4*(a + 1)*log(b*x + a
+ 1)/b^4 + ((a^2 + 2*a + 1)*log(b*x + a + 1)^2 - 2*(a^2 + 2*a + 1)*log(b*x
 + a + 1)*log(b*x + a - 1) + (a^2 - 2*a + 1)*log(b*x + a - 1)^2 - 4*(a - 1
)*log(b*x + a - 1))/b^4) + 1/2*b*(2*x/b^2 - (a^2 + 2*a + 1)*log(b*x + a +
1)/b^3 + (a^2 - 2*a + 1)*log(b*x + a - 1)/b^3)*arccoth(b*x + a)
```

3.71.8 Giac [F]

$$\int x\coth^{-1}(a+bx)^2\,dx=\int x\,\mathrm{arcoth}\,(bx+a)^2\,dx$$

input
```
integrate(x*arccoth(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x*arccoth(b*x + a)^2, x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int x\coth^{-1}(a+bx)^2\,dx=\int x\,\mathrm{acoth}(a+b\,x)^2\,dx$$

input
```
int(x*acoth(a + b*x)^2,x)
```

output
```
int(x*acoth(a + b*x)^2, x)
```

3.72 $\int \coth^{-1}(a+bx)^2\,dx$

3.72.1 Optimal result

Integrand size = 8, antiderivative size = 81

$$\int \coth^{-1}(a+bx)^2\,dx = \frac{\coth^{-1}(a+bx)^2}{b} + \frac{(a+bx)\coth^{-1}(a+bx)^2}{b} - \frac{2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{b} - \frac{\text{PolyLog}\left(2,-\frac{1+a+bx}{1-a-bx}\right)}{b}$$

output
```
arccoth(b*x+a)^2/b+(b*x+a)*arccoth(b*x+a)^2/b-2*arccoth(b*x+a)*ln(2/(-b*x-
a+1))/b-polylog(2,(-b*x-a-1)/(-b*x-a+1))/b
```

3.72.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

$$\int \coth^{-1}(a+bx)^2\,dx = \frac{\coth^{-1}(a+bx)\left((-1+a+bx)\coth^{-1}(a+bx)-2\log\left(1-e^{-2\coth^{-1}(a+bx)}\right)\right)+\text{PolyLog}\left(2,e^{-2\coth^{-1}(a+b}\right.}{b}$$

input
```
Integrate[ArcCoth[a + b*x]^2,x]
```

output
```
(ArcCoth[a + b*x]*((-1 + a + b*x)*ArcCoth[a + b*x] - 2*Log[1 - E^(-2*ArcCo
th[a + b*x])]) + PolyLog[2, E^(-2*ArcCoth[a + b*x])])/b
```

3.72.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6654, 6437, 6547, 6471, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(a+bx)^2\,dx$$

$$\Big\downarrow \text{6654}$$

$$\frac{\int \coth^{-1}(a+bx)^2d(a+bx)}{b}$$

$$\Big\downarrow \text{6437}$$

$$\frac{(a+bx)\coth^{-1}(a+bx)^2-2\int \frac{(a+bx)\coth^{-1}(a+bx)}{1-(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \text{6547}$$

$$\frac{(a+bx)\coth^{-1}(a+bx)^2-2\left(\int \frac{\coth^{-1}(a+bx)}{-a-bx+1}d(a+bx)-\frac{1}{2}\coth^{-1}(a+bx)^2\right)}{b}$$

$$\Big\downarrow \text{6471}$$

$$\frac{(a+bx)\coth^{-1}(a+bx)^2-2\left(-\int \frac{\log\left(\frac{2}{-a-bx+1}\right)}{1-(a+bx)^2}d(a+bx)-\frac{1}{2}\coth^{-1}(a+bx)^2+\log\left(\frac{2}{-a-bx+1}\right)\coth^{-1}(a+bx)\right)}{b}$$

$$\Big\downarrow \text{2849}$$

$$\frac{(a+bx)\coth^{-1}(a+bx)^2-2\left(\int \frac{\log\left(\frac{2}{-a-bx+1}\right)}{1-\frac{2}{-a-bx+1}}d\frac{1}{-a-bx+1}-\frac{1}{2}\coth^{-1}(a+bx)^2+\log\left(\frac{2}{-a-bx+1}\right)\coth^{-1}(a+bx)\right)}{b}$$

$$\Big\downarrow \text{2752}$$

$$\frac{(a+bx)\coth^{-1}(a+bx)^2-2\left(\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{-a-bx+1}\right)-\frac{1}{2}\coth^{-1}(a+bx)^2+\log\left(\frac{2}{-a-bx+1}\right)\coth^{-1}(a+bx)\right)}{b}$$

input
```
Int[ArcCoth[a + b*x]^2,x]
```

output
```
((a + b*x)*ArcCoth[a + b*x]^2 - 2*(-1/2*ArcCoth[a + b*x]^2 + ArcCoth[a + b
*x]*Log[2/(1 - a - b*x)] + PolyLog[2, 1 - 2/(1 - a - b*x)]/2))/b
```

3.72.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6471
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6547
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6654
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
}, x] && IGtQ[p, 0]
```

3.72.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.64

method	result
derivativedivides	$\frac{\operatorname{arccoth}(bx+a)^2(bx+a-1)+2\operatorname{arccoth}(bx+a)^2-2\operatorname{arccoth}(bx+a)\ln\left(1-\frac{1}{\sqrt{\frac{bx+a-1}{bx+a+1}}}\right)-2\operatorname{polylog}\left(2,\frac{1}{\sqrt{\frac{bx+a-1}{bx+a+1}}}\right)-2\operatorname{arc}}{b}$
default	$\frac{\operatorname{arccoth}(bx+a)^2(bx+a-1)+2\operatorname{arccoth}(bx+a)^2-2\operatorname{arccoth}(bx+a)\ln\left(1-\frac{1}{\sqrt{\frac{bx+a-1}{bx+a+1}}}\right)-2\operatorname{polylog}\left(2,\frac{1}{\sqrt{\frac{bx+a-1}{bx+a+1}}}\right)-2\operatorname{arc}}{b}$
risch	$\frac{(bx+a+1)\ln(bx+a+1)^2}{4b}+\left(-\frac{x\ln(bx+a-1)}{2}+\frac{-\ln(bx+a-1)a+\ln(bx+a-1)}{2b}\right)\ln(bx+a+1)+\frac{x\ln(bx+a}{4}$

input
```
int(arccoth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(arccoth(b*x+a)^2*(b*x+a-1)+2*arccoth(b*x+a)^2-2*arccoth(b*x+a)*ln(1-1
/((b*x+a-1)/(b*x+a+1))^(1/2))-2*polylog(2,1/((b*x+a-1)/(b*x+a+1))^(1/2))-2
*arccoth(b*x+a)*ln(1+1/((b*x+a-1)/(b*x+a+1))^(1/2))-2*polylog(2,-1/((b*x+a
-1)/(b*x+a+1))^(1/2)))
```

3.72.5 Fricas [F]

$$\int \coth^{-1}(a+bx)^2\,dx = \int \operatorname{arcoth}(bx+a)^2\,dx$$

input
```
integrate(arccoth(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(arccoth(b*x + a)^2, x)
```

3.72.6 Sympy [F]

$$\int \coth^{-1}(a+bx)^2\,dx=\int \operatorname{acoth}^2(a+bx)\,dx$$

input
```
integrate(acoth(b*x+a)**2,x)
```

output
```
Integral(acoth(a + b*x)**2, x)
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.72

$$\int \coth^{-1}(a+bx)^2\,dx = -\frac{1}{4}b^2\left(\frac{(a+1)\log(bx+a+1)^2-2(a+1)\log(bx+a+1)\log(bx+a-1)+(a-1)\log(bx+a-1)^2}{b^3}\right. + b\left(\frac{(a+1)\log(bx+a+1)}{b^2}-\frac{(a-1)\log(bx+a-1)}{b^2}\right)\operatorname{arcoth}(bx+a) + x\operatorname{arcoth}(bx+a)^2$$

input
```
integrate(arccoth(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*b^2*(((a + 1)*log(b*x + a + 1)^2 - 2*(a + 1)*log(b*x + a + 1)*log(b*x
 + a - 1) + (a - 1)*log(b*x + a - 1)^2)/b^3 + 4*(log(b*x + a - 1)*log(1/2*
b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^3) + b*((a + 1)*log(
b*x + a + 1)/b^2 - (a - 1)*log(b*x + a - 1)/b^2)*arccoth(b*x + a) + x*arcc
oth(b*x + a)^2
```

3.72.8 Giac [F]

$$\int \coth^{-1}(a+bx)^2\,dx = \int \operatorname{arcoth}\left(bx+a\right)^2\,dx$$

input
```
integrate(arccoth(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(arccoth(b*x + a)^2, x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(a+bx)^2\,dx = \int \operatorname{acoth}(a+b\,x)^2\,dx$$

input
```
int(acoth(a + b*x)^2,x)
```

output
```
int(acoth(a + b*x)^2, x)
```

3.73 $\int \frac{\coth^{-1}(a+bx)^2}{x} dx$

3.73.1 Optimal result

Integrand size = 12, antiderivative size = 148

$$\begin{aligned}\int \frac{\coth^{-1}(a+bx)^2}{x} dx = & -\coth^{-1}(a+bx)^2 \log\left(\frac{2}{1+a+bx}\right) \\ & + \coth^{-1}(a+bx)^2 \log\left(\frac{2bx}{(1-a)(1+a+bx)}\right) \\ & + \coth^{-1}(a+bx) \operatorname{PolyLog}\left(2, 1-\frac{2}{1+a+bx}\right) \\ & - \coth^{-1}(a+bx) \operatorname{PolyLog}\left(2, 1-\frac{2bx}{(1-a)(1+a+bx)}\right) \\ & + \frac{1}{2} \operatorname{PolyLog}\left(3, 1-\frac{2}{1+a+bx}\right) \\ & - \frac{1}{2} \operatorname{PolyLog}\left(3, 1-\frac{2bx}{(1-a)(1+a+bx)}\right)\end{aligned}$$

output
```
-arccoth(b*x+a)^2*ln(2/(b*x+a+1))+arccoth(b*x+a)^2*ln(2*b*x/(1-a)/(b*x+a+1
))+arccoth(b*x+a)*polylog(2,1-2/(b*x+a+1))-arccoth(b*x+a)*polylog(2,1-2*b*
x/(1-a)/(b*x+a+1))+1/2*polylog(3,1-2/(b*x+a+1))-1/2*polylog(3,1-2*b*x/(1-a
)/(b*x+a+1))
```

3.73.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.45 (sec) , antiderivative size = 777, normalized size of antiderivative = 5.25

$$
\begin{aligned}
\int \frac{\coth^{-1}(a+bx)^2}{x}\,dx =& -\frac{i\pi^3}{24} - \frac{2}{3}\coth^{-1}(a+bx)^3 - \frac{2}{3}a\coth^{-1}(a+bx)^3 \\
&+ \frac{2}{3}\sqrt{1-\frac{1}{a^2}}ae^{\operatorname{arctanh}\left(\frac{1}{a}\right)}\coth^{-1}(a+bx)^3 \\
&- i\pi\coth^{-1}(a+bx)\log\left(\frac{1}{2}\left(e^{-\coth^{-1}(a+bx)}+e^{\coth^{-1}(a+bx)}\right)\right) \\
&- \coth^{-1}(a+bx)^2\log\left(1-\sqrt{\frac{-1+a}{1+a}}e^{\coth^{-1}(a+bx)}\right) \\
&- \coth^{-1}(a+bx)^2\log\left(1+\sqrt{\frac{-1+a}{1+a}}e^{\coth^{-1}(a+bx)}\right) \\
&- \coth^{-1}(a+bx)^2\log\left(1-e^{2\coth^{-1}(a+bx)}\right) \\
&+ \coth^{-1}(a+bx)^2\log\left(1-e^{2\coth^{-1}(a+bx)-2\operatorname{arctanh}\left(\frac{1}{a}\right)}\right) \\
&+ \coth^{-1}(a+bx)^2\log\left(1-e^{\coth^{-1}(a+bx)-\operatorname{arctanh}\left(\frac{1}{a}\right)}\right) \\
&+ \coth^{-1}(a+bx)^2\log\left(1+e^{\coth^{-1}(a+bx)-\operatorname{arctanh}\left(\frac{1}{a}\right)}\right) \\
&- 2\coth^{-1}(a+bx)\operatorname{arctanh}\left(\frac{1}{a}\right)\log\left(\frac{1}{2}i\left(e^{\coth^{-1}(a+bx)-\operatorname{arctanh}\left(\frac{1}{a}\right)}\right.\right. \\
&\qquad \left.\left.- e^{-\coth^{-1}(a+bx)+\operatorname{arctanh}\left(\frac{1}{a}\right)}\right)\right) \\
&+ \coth^{-1}(a+bx)^2\log\left(\frac{1}{2}e^{-\coth^{-1}(a+bx)}\left(-1-e^{2\coth^{-1}(a+bx)}\right.\right. \\
&\qquad \left.\left.+ a\left(-1+e^{2\coth^{-1}(a+bx)}\right)\right)\right) \\
&+ i\pi\coth^{-1}(a+bx)\log\left(\frac{1}{\sqrt{1-\frac{1}{(a+bx)^2}}}\right) \\
&- \coth^{-1}(a+bx)^2\log\left(-\frac{bx}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}\right) + 2\coth^{-1}(a \\
&+ bx)\operatorname{arctanh}\left(\frac{1}{a}\right)\log\left(i\sinh\left(\coth^{-1}(a+bx)-\operatorname{arctanh}\left(\frac{1}{a}\right)\right)\right) \\
&- 2\coth^{-1}(a+bx)\operatorname{PolyLog}\left(2,-\sqrt{\frac{-1+a}{1+a}}e^{\coth^{-1}(a+bx)}\right) \\
&- 2\coth^{-1}(a+bx)\operatorname{PolyLog}\left(2,\sqrt{\frac{-1+a}{1+a}}e^{\coth^{-1}(a+bx)}\right) \\
&- \coth^{-1}(a+bx)\operatorname{PolyLog}\left(2,e^{2\coth^{-1}(a+bx)}\right) \\
&+ \coth^{-1}(a+bx)\operatorname{PolyLog}\left(2,e^{2\coth^{-1}(a+bx)-2\operatorname{arctanh}\left(\frac{1}{a}\right)}\right) \\
&+ 2\coth^{-1}(a+bx)\operatorname{PolyLog}\left(2,-e^{\coth^{-1}(a+bx)-\operatorname{arctanh}\left(\frac{1}{a}\right)}\right)
\end{aligned}
$$

input
```
Integrate[ArcCoth[a + b*x]^2/x,x]
```

output
```
(-1/24*I)*Pi^3 - (2*ArcCoth[a + b*x]^3)/3 - (2*a*ArcCoth[a + b*x]^3)/3 + (
2*Sqrt[1 - a^(-2)]*a*E^ArcTanh[a^(-1)]*ArcCoth[a + b*x]^3)/3 - I*Pi*ArcCot
h[a + b*x]*Log[(E^(-ArcCoth[a + b*x]) + E^ArcCoth[a + b*x])/2] - ArcCoth[a
 + b*x]^2*Log[1 - Sqrt[(-1 + a)/(1 + a)]*E^ArcCoth[a + b*x]] - ArcCoth[a +
 b*x]^2*Log[1 + Sqrt[(-1 + a)/(1 + a)]*E^ArcCoth[a + b*x]] - ArcCoth[a + b
*x]^2*Log[1 - E^(2*ArcCoth[a + b*x])] + ArcCoth[a + b*x]^2*Log[1 - E^(2*Ar
cCoth[a + b*x] - 2*ArcTanh[a^(-1)])] + ArcCoth[a + b*x]^2*Log[1 - E^(ArcCo
th[a + b*x] - ArcTanh[a^(-1)])] + ArcCoth[a + b*x]^2*Log[1 + E^(ArcCoth[a
+ b*x] - ArcTanh[a^(-1)])] - 2*ArcCoth[a + b*x]*ArcTanh[a^(-1)]*Log[(I/2)*
(E^(ArcCoth[a + b*x] - ArcTanh[a^(-1)]) - E^(-ArcCoth[a + b*x] + ArcTanh[a
^(-1)]))] + ArcCoth[a + b*x]^2*Log[(-1 - E^(2*ArcCoth[a + b*x]) + a*(-1 +
E^(2*ArcCoth[a + b*x])))/(2*E^ArcCoth[a + b*x])] + I*Pi*ArcCoth[a + b*x]*L
og[1/Sqrt[1 - (a + b*x)^(-2)]] - ArcCoth[a + b*x]^2*Log[-((b*x)/((a + b*x)
*Sqrt[1 - (a + b*x)^(-2)]))] + 2*ArcCoth[a + b*x]*ArcTanh[a^(-1)]*Log[I*Si
nh[ArcCoth[a + b*x] - ArcTanh[a^(-1)]]] - 2*ArcCoth[a + b*x]*PolyLog[2, -(
Sqrt[(-1 + a)/(1 + a)]*E^ArcCoth[a + b*x])] - 2*ArcCoth[a + b*x]*PolyLog[2
, Sqrt[(-1 + a)/(1 + a)]*E^ArcCoth[a + b*x]] - ArcCoth[a + b*x]*PolyLog[2,
 E^(2*ArcCoth[a + b*x])] + ArcCoth[a + b*x]*PolyLog[2, E^(2*ArcCoth[a + b*
x] - 2*ArcTanh[a^(-1)])] + 2*ArcCoth[a + b*x]*PolyLog[2, -E^(ArcCoth[a + b
*x] - ArcTanh[a^(-1)])] + 2*ArcCoth[a + b*x]*PolyLog[2, E^(ArcCoth[a + ...
```

3.73.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6662, 25, 27, 6475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(a+bx)^2}{x} \, dx$$

$$\Big\downarrow \text{6662}$$

$$\frac{\int \frac{\coth^{-1}(a+bx)^2}{x}d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{\coth^{-1}(a+bx)^2}{x}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\int -\frac{\coth^{-1}(a+bx)^2}{bx}d(a+bx)$$

$$\Big\downarrow \text{6475}$$

$$\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2}{a+bx+1}\right)-\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2bx}{(1-a)(a+bx+1)}\right)+\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)\coth^{-1}(a+bx)-\text{PolyLog}\left(2,1-\frac{2bx}{(1-a)(a+bx+1)}\right)\coth^{-1}(a+bx)-\log\left(\frac{2}{a+bx+1}\right)\coth^{-1}(a+bx)^2+\log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)\coth^{-1}(a+bx)^2$$

input
```
Int[ArcCoth[a + b*x]^2/x,x]
```

output
```
-(ArcCoth[a + b*x]^2*Log[2/(1 + a + b*x)]) + ArcCoth[a + b*x]^2*Log[(2*b*x
)/((1 - a)*(1 + a + b*x))] + ArcCoth[a + b*x]*PolyLog[2, 1 - 2/(1 + a + b*
x)] - ArcCoth[a + b*x]*PolyLog[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))] + P
olyLog[3, 1 - 2/(1 + a + b*x)]/2 - PolyLog[3, 1 - (2*b*x)/((1 - a)*(1 + a
+ b*x))]/2
```

3.73.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6475
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcCoth[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Coth[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcCoth[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcC
oth[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.73.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.62 (sec) , antiderivative size = 866, normalized size of antiderivative = 5.85

method	result
derivativedivides	$\ln(-bx)\operatorname{arccoth}(bx+a)^2 - \operatorname{arccoth}(bx+a)^2 \ln\left(-\frac{bx+a+1}{bx+a-1} - 1 + a\left(\frac{bx+a+1}{bx+a-1} - 1\right)\right) + \frac{i\pi}{}$
default	$\ln(-bx)\operatorname{arccoth}(bx+a)^2 - \operatorname{arccoth}(bx+a)^2 \ln\left(-\frac{bx+a+1}{bx+a-1} - 1 + a\left(\frac{bx+a+1}{bx+a-1} - 1\right)\right) + \frac{i\pi}{}$
parts	Expression too large to display

input
```
int(arccoth(b*x+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
ln(-b*x)*arccoth(b*x+a)^2-arccoth(b*x+a)^2*ln(-(b*x+a+1)/(b*x+a-1)-1+a*((b
*x+a+1)/(b*x+a-1)-1))+1/2*I*Pi*csgn(I*(-(b*x+a+1)/(b*x+a-1)-1+a*((b*x+a+1)
/(b*x+a-1)-1))/((b*x+a+1)/(b*x+a-1)-1))*(csgn(I*(-(b*x+a+1)/(b*x+a-1)-1+a*
((b*x+a+1)/(b*x+a-1)-1)))*csgn(I/((b*x+a+1)/(b*x+a-1)-1))-csgn(I*(-(b*x+a+
1)/(b*x+a-1)-1+a*((b*x+a+1)/(b*x+a-1)-1))/((b*x+a+1)/(b*x+a-1)-1))*csgn(I/
((b*x+a+1)/(b*x+a-1)-1))-csgn(I*(-(b*x+a+1)/(b*x+a-1)-1+a*((b*x+a+1)/(b*x+
a-1)-1)))*csgn(I*(-(b*x+a+1)/(b*x+a-1)-1+a*((b*x+a+1)/(b*x+a-1)-1))/((b*x+
a+1)/(b*x+a-1)-1))+csgn(I*(-(b*x+a+1)/(b*x+a-1)-1+a*((b*x+a+1)/(b*x+a-1)-1
))/((b*x+a+1)/(b*x+a-1)-1))^2)*arccoth(b*x+a)^2+arccoth(b*x+a)^2*ln((b*x+a
+1)/(b*x+a-1)-1)-arccoth(b*x+a)^2*ln(1+1/((b*x+a-1)/(b*x+a+1))^(1/2))-2*ar
ccoth(b*x+a)*polylog(2,-1/((b*x+a-1)/(b*x+a+1))^(1/2))+2*polylog(3,-1/((b*
x+a-1)/(b*x+a+1))^(1/2))-arccoth(b*x+a)^2*ln(1-1/((b*x+a-1)/(b*x+a+1))^(1/
2))-2*arccoth(b*x+a)*polylog(2,1/((b*x+a-1)/(b*x+a+1))^(1/2))+2*polylog(3,
1/((b*x+a-1)/(b*x+a+1))^(1/2))+a/(-1+a)*arccoth(b*x+a)^2*ln(1-(b*x+a+1)*(-
1+a)/(1+a)/(b*x+a-1))+a/(-1+a)*arccoth(b*x+a)*polylog(2,(b*x+a+1)*(-1+a)/(
1+a)/(b*x+a-1))-1/2*a/(-1+a)*polylog(3,(b*x+a+1)*(-1+a)/(1+a)/(b*x+a-1))-1
/(-1+a)*arccoth(b*x+a)^2*ln(1-(b*x+a+1)*(-1+a)/(1+a)/(b*x+a-1))-1/(-1+a)*a
rccoth(b*x+a)*polylog(2,(b*x+a+1)*(-1+a)/(1+a)/(b*x+a-1))+1/2/(-1+a)*polyl
og(3,(b*x+a+1)*(-1+a)/(1+a)/(b*x+a-1))
```

3.73.5 Fricas [F]

$$\int \frac{\coth^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{arcoth}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(arccoth(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(arccoth(b*x + a)^2/x, x)
```

3.73.6 Sympy [F]

$$\int \frac{\coth^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{acoth}^2\left(a+bx\right)}{x}\,dx$$

input
```
integrate(acoth(b*x+a)**2/x,x)
```

output
```
Integral(acoth(a + b*x)**2/x, x)
```

3.73.7 Maxima [F]

$$\int \frac{\coth^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{arcoth}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(arccoth(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
integrate(arccoth(b*x + a)^2/x, x)
```

3.73.8 Giac [F]

$$\int \frac{\coth^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{arcoth}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(arccoth(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(arccoth(b*x + a)^2/x, x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{acoth}(a+b\,x)^2}{x}\,dx$$

input
```
int(acoth(a + b*x)^2/x,x)
```

output
```
int(acoth(a + b*x)^2/x, x)
```

3.74 $\int \frac{\coth^{-1}(a+bx)^2}{x^2}\,dx$

3.74.1 Optimal result

Integrand size = 12, antiderivative size = 251

$$\begin{aligned}\int \frac{\coth^{-1}(a+bx)^2}{x^2}\,dx = &-\frac{\coth^{-1}(a+bx)^2}{x}+\frac{b\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{1-a}\\ &+\frac{b\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{1+a}-\frac{2b\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{1-a^2}\\ &+\frac{2b\coth^{-1}(a+bx)\log\left(\frac{2bx}{(1-a)(1+a+bx)}\right)}{1-a^2}\\ &+\frac{b\,\mathrm{PolyLog}\left(2,-\frac{1+a+bx}{1-a-bx}\right)}{2(1-a)}-\frac{b\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+a+bx}\right)}{2(1+a)}\\ &+\frac{b\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+a+bx}\right)}{1-a^2}-\frac{b\,\mathrm{PolyLog}\left(2,1-\frac{2bx}{(1-a)(1+a+bx)}\right)}{1-a^2}\end{aligned}$$

output
```
-arccoth(b*x+a)^2/x+b*arccoth(b*x+a)*ln(2/(-b*x-a+1))/(1-a)+b*arccoth(b*x+
a)*ln(2/(b*x+a+1))/(1+a)-2*b*arccoth(b*x+a)*ln(2/(b*x+a+1))/(-a^2+1)+2*b*a
rccoth(b*x+a)*ln(2*b*x/(1-a)/(b*x+a+1))/(-a^2+1)+1/2*b*polylog(2,(-b*x-a-1
)/(-b*x-a+1))/(1-a)-1/2*b*polylog(2,1-2/(b*x+a+1))/(1+a)+b*polylog(2,1-2/(
b*x+a+1))/(-a^2+1)-b*polylog(2,1-2*b*x/(1-a)/(b*x+a+1))/(-a^2+1)
```

3.74.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.82

$$\int \frac{\coth^{-1}(a+bx)^2}{x^2}\,dx = \frac{-\left(\left(-1+a^2+\sqrt{1-\frac{1}{a^2}}abe^{\operatorname{arctanh}\left(\frac{1}{a}\right)}x\right)\coth^{-1}(a+bx)^2\right)+bx\coth^{-1}(a+bx)\left(-i\pi+2\operatorname{arctanh}\left(\frac{1}{a}\right)-2\operatorname{lo}\right.}{}$$

input
```
Integrate[ArcCoth[a + b*x]^2/x^2,x]
```

output
```
(-((-1 + a^2 + Sqrt[1 - a^(-2)]*a*b*E^ArcTanh[a^(-1)]*x)*ArcCoth[a + b*x]^
2) + b*x*ArcCoth[a + b*x]*((-I)*Pi + 2*ArcTanh[a^(-1)] - 2*Log[1 - E^(-2*A
rcCoth[a + b*x] + 2*ArcTanh[a^(-1)])]) + b*x*(I*Pi*(Log[1 + E^(2*ArcCoth[a
 + b*x])] - Log[1/Sqrt[1 - (a + b*x)^(-2)]]) + 2*ArcTanh[a^(-1)]*(Log[1 -
E^(-2*ArcCoth[a + b*x] + 2*ArcTanh[a^(-1)])] - Log[I*Sinh[ArcCoth[a + b*x]
 - ArcTanh[a^(-1)]]])) + b*x*PolyLog[2, E^(-2*ArcCoth[a + b*x] + 2*ArcTanh
[a^(-1)])])/((-1 + a^2)*x)
```

3.74.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6660, 7292, 6672, 25, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(a+bx)^2}{x^2}\,dx$$

$$\Big\downarrow \text{6660}$$

$$2b\int \frac{\coth^{-1}(a+bx)}{x\left(1-(a+bx)^2\right)}dx-\frac{\coth^{-1}(a+bx)^2}{x}$$

$$\Big\downarrow \text{7292}$$

$$2b\int \frac{\coth^{-1}(a+bx)}{x\left(-a^2-2bxa-b^2x^2+1\right)}dx-\frac{\coth^{-1}(a+bx)^2}{x}$$

$$\Big\downarrow \ 6672$$

$$2\int \frac{\coth^{-1}(a+bx)}{x(1-(a+bx)^2)}d(a+bx)-\frac{\coth^{-1}(a+bx)^2}{x}$$

$$\Big\downarrow \ 25$$

$$-2\int -\frac{\coth^{-1}(a+bx)}{x(1-(a+bx)^2)}d(a+bx)-\frac{\coth^{-1}(a+bx)^2}{x}$$

$$\Big\downarrow \ 27$$

$$-2b\int -\frac{\coth^{-1}(a+bx)}{bx(1-(a+bx)^2)}d(a+bx)-\frac{\coth^{-1}(a+bx)^2}{x}$$

$$\Big\downarrow \ 7276$$

$$-2b\int \left(\frac{\coth^{-1}(a+bx)}{(a^2-1)\,bx}-\frac{\coth^{-1}(a+bx)}{2(a-1)(a+bx-1)}+\frac{\coth^{-1}(a+bx)}{2(a+1)(a+bx+1)}\right)d(a+bx)-\frac{\coth^{-1}(a+bx)^2}{x}$$

$$\Big\downarrow \ 2009$$

$$-2b\left(-\frac{\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)}{2(1-a^2)}+\frac{\text{PolyLog}\left(2,1-\frac{2bx}{(1-a)(a+bx+1)}\right)}{2(1-a^2)}+\frac{\log\left(\frac{2}{a+bx+1}\right)\coth^{-1}(a+bx)}{1-a^2}-\frac{\log\left(\frac{2}{(1-a)(\cdots}\right.}{\cdots}\right.$$
$$\frac{\coth^{-1}(a+bx)^2}{x}$$

input
```
Int[ArcCoth[a + b*x]^2/x^2,x]
```

output
```
-(ArcCoth[a + b*x]^2/x) - 2*b*(-1/2*(ArcCoth[a + b*x]*Log[2/(1 - a - b*x)]
)/(1 - a) - (ArcCoth[a + b*x]*Log[2/(1 + a + b*x)])/(2*(1 + a)) + (ArcCoth
[a + b*x]*Log[2/(1 + a + b*x)])/(1 - a^2) - (ArcCoth[a + b*x]*Log[(2*b*x)/
((1 - a)*(1 + a + b*x))])/(1 - a^2) - PolyLog[2, -((1 + a + b*x)/(1 - a -
b*x))]/(4*(1 - a)) + PolyLog[2, 1 - 2/(1 + a + b*x)]/(4*(1 + a)) - PolyLog
[2, 1 - 2/(1 + a + b*x)]/(2*(1 - a^2)) + PolyLog[2, 1 - (2*b*x)/((1 - a)*(
1 + a + b*x))]/(2*(1 - a^2)))
```

3.74. $\int \frac{\coth^{-1}(a+bx)^2}{x^2}\,dx$

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6660
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCoth[c + d*x])^p/(f*(m
+ 1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCo
th[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[p, 0] && ILtQ[m, -1]
```

rule 6672
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Sub
st[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCoth[
x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x
] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.74.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.19

method	result
parts	$-\frac{\operatorname{arccoth}(bx+a)^2}{x}-2b\left(\frac{\operatorname{arccoth}(bx+a)\ln(bx+a+1)}{2a+2}-\frac{\operatorname{arccoth}(bx+a)\ln(bx+a-1)}{-2+2a}+\frac{\operatorname{arccoth}(bx+a)\ln(-bx)}{(-1+a)(1+a)}-\right.$
derivativedivides	$b\left(-\frac{\operatorname{arccoth}(bx+a)^2}{bx}-\frac{2\operatorname{arccoth}(bx+a)\ln(bx+a+1)}{2a+2}+\frac{2\operatorname{arccoth}(bx+a)\ln(bx+a-1)}{-2+2a}-\frac{2\operatorname{arccoth}(bx+a)\ln(-b}{(-1+a)(1+a)}\right.$
default	$b\left(-\frac{\operatorname{arccoth}(bx+a)^2}{bx}-\frac{2\operatorname{arccoth}(bx+a)\ln(bx+a+1)}{2a+2}+\frac{2\operatorname{arccoth}(bx+a)\ln(bx+a-1)}{-2+2a}-\frac{2\operatorname{arccoth}(bx+a)\ln(-b}{(-1+a)(1+a)}\right.$

input
```
int(arccoth(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccoth(b*x+a)^2/x-2*b*(arccoth(b*x+a)/(2*a+2)*ln(b*x+a+1)-arccoth(b*x+a)
/(-2+2*a)*ln(b*x+a-1)+arccoth(b*x+a)/(-1+a)/(1+a)*ln(-b*x)-1/2/(-1+a)*(1/4
*ln(b*x+a-1)^2-1/2*dilog(1/2*b*x+1/2*a+1/2)-1/2*ln(b*x+a-1)*ln(1/2*b*x+1/2
*a+1/2))+1/2/(1+a)*(1/2*(ln(b*x+a+1)-ln(1/2*b*x+1/2*a+1/2))*ln(-1/2*b*x-1/
2*a+1/2)-1/2*dilog(1/2*b*x+1/2*a+1/2)-1/4*ln(b*x+a+1)^2)+1/(-1+a)/(1+a)*(1
/2*dilog(1/(1-a)*(-b*x-a+1))+1/2*ln(-b*x)*ln(1/(1-a)*(-b*x-a+1))-1/2*dilog
((-b*x-a-1)/(-1-a))-1/2*ln(-b*x)*ln((-b*x-a-1)/(-1-a))))
```

3.74.5 Fricas [F]

$$\int \frac{\coth^{-1}(a+bx)^2}{x^2}\,dx=\int \frac{\operatorname{arcoth}(bx+a)^2}{x^2}\,dx$$

input
```
integrate(arccoth(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(arccoth(b*x + a)^2/x^2, x)
```

3.74.6 Sympy [F]

$$\int \frac{\coth^{-1}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{acoth}^2(a+bx)}{x^2}\,dx$$

input

```
integrate(acoth(b*x+a)**2/x**2,x)
```

output

```
Integral(acoth(a + b*x)**2/x**2, x)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.97

$$\int \frac{\coth^{-1}(a+bx)^2}{x^2}\,dx$$
$$= \frac{1}{4}b^2\left(\frac{(a-1)\log(bx+a+1)^2 - 2(a-1)\log(bx+a+1)\log(bx+a-1) + (a+1)\log(bx+a-1)^2}{a^2b-b} - \right.$$
$$- b\left(\frac{\log(bx+a+1)}{a+1} - \frac{\log(bx+a-1)}{a-1} + \frac{2\log(x)}{a^2-1}\right)\operatorname{arcoth}(bx+a) - \frac{\operatorname{arcoth}(bx+a)^2}{x}$$

input

```
integrate(arccoth(b*x+a)^2/x^2,x, algorithm="maxima")
```

output

```
1/4*b^2*(((a - 1)*log(b*x + a + 1)^2 - 2*(a - 1)*log(b*x + a + 1)*log(b*x
+ a - 1) + (a + 1)*log(b*x + a - 1)^2)/(a^2*b - b) - 4*(log(b*x + a - 1)*l
og(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/(a^2*b - b) + 4
*(log(b*x/(a + 1) + 1)*log(x) + dilog(-b*x/(a + 1)))/(a^2*b - b) - 4*(log(
b*x/(a - 1) + 1)*log(x) + dilog(-b*x/(a - 1)))/(a^2*b - b)) - b*(log(b*x +
 a + 1)/(a + 1) - log(b*x + a - 1)/(a - 1) + 2*log(x)/(a^2 - 1))*arccoth(b
*x + a) - arccoth(b*x + a)^2/x
```

3.74.8 Giac [F]

$$\int \frac{\coth^{-1}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{arcoth}\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(arccoth(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(arccoth(b*x + a)^2/x^2, x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{acoth}(a+b\,x)^2}{x^2}\,dx$$

input
```
int(acoth(a + b*x)^2/x^2,x)
```

output
```
int(acoth(a + b*x)^2/x^2, x)
```

3.75 $\int \frac{\coth^{-1}(a+bx)^2}{x^3}\,dx$

3.75.1 Optimal result

Integrand size = 12, antiderivative size = 370

$$\begin{aligned}\int \frac{\coth^{-1}(a+bx)^2}{x^3}\,dx =& -\frac{b\coth^{-1}(a+bx)}{(1-a^2)\,x} - \frac{\coth^{-1}(a+bx)^2}{2x^2} \\ &+ \frac{b^2\log(x)}{(1-a^2)^2} + \frac{b^2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} \\ &- \frac{b^2\log(1-a-bx)}{2(1-a)^2(1+a)} - \frac{b^2\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{2(1+a)^2} \\ &- \frac{2ab^2\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{(1-a^2)^2} \\ &+ \frac{2ab^2\coth^{-1}(a+bx)\log\left(\frac{2bx}{(1-a)(1+a+bx)}\right)}{(1-a^2)^2} - \frac{b^2\log(1+a+bx)}{2(1-a)(1+a)^2} \\ &+ \frac{b^2\operatorname{PolyLog}\left(2,-\frac{1+a+bx}{1-a-bx}\right)}{4(1-a)^2} + \frac{b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+a+bx}\right)}{4(1+a)^2} \\ &+ \frac{ab^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+a+bx}\right)}{(1-a^2)^2} - \frac{ab^2\operatorname{PolyLog}\left(2,1-\frac{2bx}{(1-a)(1+a+bx)}\right)}{(1-a^2)^2}\end{aligned}$$

output
```
-b*arccoth(b*x+a)/(-a^2+1)/x-1/2*arccoth(b*x+a)^2/x^2+b^2*ln(x)/(-a^2+1)^2
+1/2*b^2*arccoth(b*x+a)*ln(2/(-b*x-a+1))/(1-a)^2-1/2*b^2*ln(-b*x-a+1)/(1-a
)^2/(1+a)-1/2*b^2*arccoth(b*x+a)*ln(2/(b*x+a+1))/(1+a)^2-2*a*b^2*arccoth(b
*x+a)*ln(2/(b*x+a+1))/(-a^2+1)^2+2*a*b^2*arccoth(b*x+a)*ln(2*b*x/(1-a)/(b*
x+a+1))/(-a^2+1)^2-1/2*b^2*ln(b*x+a+1)/(1-a)/(1+a)^2+1/4*b^2*polylog(2,(-b
*x-a-1)/(-b*x-a+1))/(1-a)^2+1/4*b^2*polylog(2,1-2/(b*x+a+1))/(1+a)^2+a*b^2
*polylog(2,1-2/(b*x+a+1))/(-a^2+1)^2-a*b^2*polylog(2,1-2*b*x/(1-a)/(b*x+a+
1))/(-a^2+1)^2
```

3.75.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.79

$$\int \frac{\coth^{-1}(a+bx)^2}{x^3}\,dx$$
$$= \frac{\left(-1-a^4+b^2x^2+a^2\left(2+b^2\left(-1+2\sqrt{1-\frac{1}{a^2}}e^{\operatorname{arctanh}\left(\frac{1}{a}\right)}\right)x^2\right)\right)\coth^{-1}(a+bx)^2+2bx\coth^{-1}(a+bx)\left(-\right.}{}$$

input
```
Integrate[ArcCoth[a + b*x]^2/x^3,x]
```

output
```
((-1 - a^4 + b^2*x^2 + a^2*(2 + b^2*(-1 + 2*Sqrt[1 - a^(-2)]*E^ArcTanh[a^(
-1)])*x^2))*ArcCoth[a + b*x]^2 + 2*b*x*ArcCoth[a + b*x]*(-1 + a^2 + a*b*x
+ I*a*b*Pi*x - 2*a*b*x*ArcTanh[a^(-1)] + 2*a*b*x*Log[1 - E^(-2*ArcCoth[a +
 b*x] + 2*ArcTanh[a^(-1)])]) + 2*b^2*x^2*((-I)*a*Pi*Log[1 + E^(2*ArcCoth[a
 + b*x])] + I*a*Pi*Log[1/Sqrt[1 - (a + b*x)^(-2)]] + Log[-((b*x)/((a + b*x
)*Sqrt[1 - (a + b*x)^(-2)]))] - 2*a*ArcTanh[a^(-1)]*(Log[1 - E^(-2*ArcCoth
[a + b*x] + 2*ArcTanh[a^(-1)])] - Log[I*Sinh[ArcCoth[a + b*x] - ArcTanh[a^
(-1)]]])) - 2*a*b^2*x^2*PolyLog[2, E^(-2*ArcCoth[a + b*x] + 2*ArcTanh[a^(-
1)])])/(2*(-1 + a^2)^2*x^2)
```

3.75.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6660, 7292, 6672, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(a+bx)^2}{x^3} \, dx$$

$$\Big\downarrow \text{ 6660}$$

$$b\int \frac{\coth^{-1}(a+bx)}{x^2\left(1-(a+bx)^2\right)}dx-\frac{\coth^{-1}(a+bx)^2}{2x^2}$$

$$\Big\downarrow \text{ 7292}$$

$$b\int \frac{\coth^{-1}(a+bx)}{x^2\left(-a^2-2bxa-b^2x^2+1\right)}dx-\frac{\coth^{-1}(a+bx)^2}{2x^2}$$

$$\Big\downarrow \text{ 6672}$$

$$\int \frac{\coth^{-1}(a+bx)}{x^2\left(1-(a+bx)^2\right)}d(a+bx)-\frac{\coth^{-1}(a+bx)^2}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$b^2\int \frac{\coth^{-1}(a+bx)}{b^2x^2\left(1-(a+bx)^2\right)}d(a+bx)-\frac{\coth^{-1}(a+bx)^2}{2x^2}$$

$$\Big\downarrow \text{ 7276}$$

$$b^2\int \left(\frac{2a\coth^{-1}(a+bx)}{\left(a^2-1\right)^2bx}-\frac{\coth^{-1}(a+bx)}{2(a-1)^2(a+bx-1)}+\frac{\coth^{-1}(a+bx)}{2(a+1)^2(a+bx+1)}-\frac{\coth^{-1}(a+bx)}{\left(a^2-1\right)b^2x^2}\right)d(a+bx)-\frac{\coth^{-1}(a+bx)^2}{2x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$b^2\left(\frac{a\,\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)}{\left(1-a^2\right)^2}-\frac{a\,\text{PolyLog}\left(2,1-\frac{2bx}{(1-a)(a+bx+1)}\right)}{\left(1-a^2\right)^2}+\frac{\log(-bx)}{\left(1-a^2\right)^2}-\frac{\coth^{-1}(a+bx)}{\left(1-a^2\right)bx}-\frac{2a\log\left(\frac{2}{a+bx\ldots}\right.}{(\ldots}\right.$$
$$\frac{\coth^{-1}(a+bx)^2}{2x^2}$$

input
```
Int[ArcCoth[a + b*x]^2/x^3,x]
```

output
```
-1/2*ArcCoth[a + b*x]^2/x^2 + b^2*(-(ArcCoth[a + b*x]/((1 - a^2)*b*x)) + L
og[-(b*x)]/(1 - a^2)^2 + (ArcCoth[a + b*x]*Log[2/(1 - a - b*x)])/(2*(1 - a
)^2) - Log[1 - a - b*x]/(2*(1 - a)^2*(1 + a)) - (ArcCoth[a + b*x]*Log[2/(1
 + a + b*x)])/(2*(1 + a)^2) - (2*a*ArcCoth[a + b*x]*Log[2/(1 + a + b*x)])/
(1 - a^2)^2 + (2*a*ArcCoth[a + b*x]*Log[(2*b*x)/((1 - a)*(1 + a + b*x))])/
(1 - a^2)^2 - Log[1 + a + b*x]/(2*(1 - a)*(1 + a)^2) + PolyLog[2, -((1 + a
 + b*x)/(1 - a - b*x))]/(4*(1 - a)^2) + PolyLog[2, 1 - 2/(1 + a + b*x)]/(4
*(1 + a)^2) + (a*PolyLog[2, 1 - 2/(1 + a + b*x)])/(1 - a^2)^2 - (a*PolyLog
[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))])/(1 - a^2)^2
```

3.75.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6660
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCoth[c + d*x])^p/(f*(m
+ 1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCo
th[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[p, 0] && ILtQ[m, -1]
```

rule 6672
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Sub
st[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCoth[
x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x
] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.75.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.04

method	result
parts	$-\frac{\operatorname{arccoth}(bx+a)^2}{2x^2} - b^2\left(-\frac{\operatorname{arccoth}(bx+a)}{(-1+a)(1+a)bx} - \frac{2\operatorname{arccoth}(bx+a)a\ln(-bx)}{(-1+a)^2(1+a)^2} - \frac{\operatorname{arccoth}(bx+a)\ln(bx+a+1)}{2(1+a)^2} + \mathrm{arc}\right.$
derivativedivides	$b^2\left(-\frac{\operatorname{arccoth}(bx+a)^2}{2b^2x^2} + \frac{\operatorname{arccoth}(bx+a)}{(-1+a)(1+a)bx} + \frac{2\operatorname{arccoth}(bx+a)a\ln(-bx)}{(-1+a)^2(1+a)^2} + \frac{\operatorname{arccoth}(bx+a)\ln(bx+a+1)}{2(1+a)^2} - \mathrm{arccot}\right.$
default	$b^2\left(-\frac{\operatorname{arccoth}(bx+a)^2}{2b^2x^2} + \frac{\operatorname{arccoth}(bx+a)}{(-1+a)(1+a)bx} + \frac{2\operatorname{arccoth}(bx+a)a\ln(-bx)}{(-1+a)^2(1+a)^2} + \frac{\operatorname{arccoth}(bx+a)\ln(bx+a+1)}{2(1+a)^2} - \mathrm{arccot}\right.$

input
```
int(arccoth(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccoth(b*x+a)^2/x^2-b^2*(-arccoth(b*x+a)/(-1+a)/(1+a)/b/x-2*arccoth(
b*x+a)*a/(-1+a)^2/(1+a)^2*ln(-b*x)-1/2*arccoth(b*x+a)/(1+a)^2*ln(b*x+a+1)+
1/2*arccoth(b*x+a)/(-1+a)^2*ln(b*x+a-1)+1/2/(-1+a)^2*(1/4*ln(b*x+a-1)^2-1/
2*dilog(1/2*b*x+1/2*a+1/2)-1/2*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2))-1/2/(1+a
)^2*(1/2*(ln(b*x+a+1)-ln(1/2*b*x+1/2*a+1/2))*ln(-1/2*b*x-1/2*a+1/2)-1/2*di
log(1/2*b*x+1/2*a+1/2)-1/4*ln(b*x+a+1)^2)+1/(-1+a)/(1+a)*(-1/(-1+a)/(1+a)*
ln(-b*x)-1/(2*a+2)*ln(b*x+a+1)+1/(-2+2*a)*ln(b*x+a-1))-2*a/(-1+a)^2/(1+a)^
2*(1/2*dilog(1/(1-a)*(-b*x-a+1))+1/2*ln(-b*x)*ln(1/(1-a)*(-b*x-a+1))-1/2*d
ilog((-b*x-a-1)/(-1-a))-1/2*ln(-b*x)*ln((-b*x-a-1)/(-1-a))))
```

3.75.5 Fricas [F]

$$\int \frac{\coth^{-1}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{arcoth}(bx+a)^2}{x^3}\,dx$$

input
```
integrate(arccoth(b*x+a)^2/x^3,x, algorithm="fricas")
```

output
```
integral(arccoth(b*x + a)^2/x^3, x)
```

3.75.6 Sympy [F]

$$\int \frac{\coth^{-1}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{acoth}^2(a+bx)}{x^3}\,dx$$

input
```
integrate(acoth(b*x+a)**2/x**3,x)
```

output
```
Integral(acoth(a + b*x)**2/x**3, x)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.97

$$\int \frac{\coth^{-1}(a+bx)^2}{x^3}\,dx = \frac{1}{8}\left(\frac{8\left(\log\left(bx+a-1\right)\log\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a+\frac{1}{2}\right)\right)a}{a^4-2\,a^2+1} - \frac{8\left(\log\left(\frac{bx}{a+1}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\right.\right.}{a^4-2\,a^2+1}\right. + \frac{1}{2}\left(\frac{4\,ab\log\left(x\right)}{a^4-2\,a^2+1}+\frac{b\log\left(bx+a+1\right)}{a^2+2\,a+1}-\frac{b\log\left(bx+a-1\right)}{a^2-2\,a+1}+\frac{2}{(a^2-1)x}\right)b\operatorname{arcoth}\left(bx+a\right) - \frac{\operatorname{arcoth}\left(bx+a\right)^2}{2\,x^2}$$

input
```
integrate(arccoth(b*x+a)^2/x^3,x, algorithm="maxima")
```

output
```
1/8*(8*(log(b*x + a - 1)*log(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2
*a + 1/2))*a/(a^4 - 2*a^2 + 1) - 8*(log(b*x/(a + 1) + 1)*log(x) + dilog(-b
*x/(a + 1)))*a/(a^4 - 2*a^2 + 1) + 8*(log(b*x/(a - 1) + 1)*log(x) + dilog(
-b*x/(a - 1)))*a/(a^4 - 2*a^2 + 1) - ((a^2 - 2*a + 1)*log(b*x + a + 1)^2 -
 2*(a^2 - 2*a + 1)*log(b*x + a + 1)*log(b*x + a - 1) + (a^2 + 2*a + 1)*log
(b*x + a - 1)^2)/(a^4 - 2*a^2 + 1) + 4*log(b*x + a + 1)/(a^3 + a^2 - a - 1
) - 4*log(b*x + a - 1)/(a^3 - a^2 - a + 1) + 8*log(x)/(a^4 - 2*a^2 + 1))*b
^2 + 1/2*(4*a*b*log(x)/(a^4 - 2*a^2 + 1) + b*log(b*x + a + 1)/(a^2 + 2*a +
 1) - b*log(b*x + a - 1)/(a^2 - 2*a + 1) + 2/((a^2 - 1)*x))*b*arccoth(b*x
+ a) - 1/2*arccoth(b*x + a)^2/x^2
```

3.75.8 Giac [F]

$$\int \frac{\coth^{-1}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{arcoth}(bx+a)^2}{x^3}\,dx$$

input
```
integrate(arccoth(b*x+a)^2/x^3,x, algorithm="giac")
```

output
```
integrate(arccoth(b*x + a)^2/x^3, x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{acoth}(a+b\,x)^2}{x^3}\,dx$$

input
```
int(acoth(a + b*x)^2/x^3,x)
```

output
```
int(acoth(a + b*x)^2/x^3, x)
```

3.76 $\int \frac{\coth^{-1}(a+bx)}{c+dx^2} \, dx$

3.76.1 Optimal result

Integrand size = 16, antiderivative size = 673

$$\int \frac{\coth^{-1}(a+bx)}{c+dx^2}\,dx = \frac{\log\left(-\frac{1-a-bx}{a+bx}\right)\log\left(1+\frac{(b^2c+a^2d)(1-a-bx)}{\left(b^2c-b\sqrt{-c}\sqrt{d}-(1-a)ad\right)(a+bx)}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{\log\left(-\frac{1-a-bx}{a+bx}\right)\log\left(1+\frac{(b^2c+a^2d)(1-a-bx)}{\left(b^2c+b\sqrt{-c}\sqrt{d}-(1-a)ad\right)(a+bx)}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\log\left(\frac{1+a+bx}{a+bx}\right)\log\left(1-\frac{(b^2c+a^2d)(1+a+bx)}{\left(b^2c-b\sqrt{-c}\sqrt{d}+a(1+a)d\right)(a+bx)}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{\log\left(\frac{1+a+bx}{a+bx}\right)\log\left(1-\frac{(b^2c+a^2d)(1+a+bx)}{\left(b^2c+b\sqrt{-c}\sqrt{d}+a(1+a)d\right)(a+bx)}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\operatorname{PolyLog}\left(2,-\frac{(b^2c+a^2d)(1-a-bx)}{\left(b^2c-b\sqrt{-c}\sqrt{d}-(1-a)ad\right)(a+bx)}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{\operatorname{PolyLog}\left(2,-\frac{(b^2c+a^2d)(1-a-bx)}{\left(b^2c+b\sqrt{-c}\sqrt{d}-(1-a)ad\right)(a+bx)}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\operatorname{PolyLog}\left(2,\frac{(b^2c+a^2d)(1+a+bx)}{\left(b^2c-b\sqrt{-c}\sqrt{d}+a(1+a)d\right)(a+bx)}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{\operatorname{PolyLog}\left(2,\frac{(b^2c+a^2d)(1+a+bx)}{\left(b^2c+b\sqrt{-c}\sqrt{d}+a(1+a)d\right)(a+bx)}\right)}{4\sqrt{-c}\sqrt{d}}$$

output
```
1/4*ln((b*x+a-1)/(b*x+a))*ln(1+(a^2*d+b^2*c)*(-b*x-a+1)/(b*x+a)/(b^2*c-(1-
a)*a*d-b*(-c)^(1/2)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*ln((b*x+a+1)/(b*x+a))
*ln(1-(a^2*d+b^2*c)*(b*x+a+1)/(b*x+a)/(b^2*c+a*(1+a)*d-b*(-c)^(1/2)*d^(1/2
)))/(-c)^(1/2)/d^(1/2)-1/4*ln((b*x+a-1)/(b*x+a))*ln(1+(a^2*d+b^2*c)*(-b*x-
a+1)/(b*x+a)/(b^2*c-(1-a)*a*d+b*(-c)^(1/2)*d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/
4*ln((b*x+a+1)/(b*x+a))*ln(1-(a^2*d+b^2*c)*(b*x+a+1)/(b*x+a)/(b^2*c+a*(1+a
)*d+b*(-c)^(1/2)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*polylog(2,-(a^2*d+b^2*c)
*(-b*x-a+1)/(b*x+a)/(b^2*c-(1-a)*a*d-b*(-c)^(1/2)*d^(1/2)))/(-c)^(1/2)/d^(
1/2)+1/4*polylog(2,(a^2*d+b^2*c)*(b*x+a+1)/(b*x+a)/(b^2*c+a*(1+a)*d-b*(-c)
^(1/2)*d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*polylog(2,-(a^2*d+b^2*c)*(-b*x-a+1
)/(b*x+a)/(b^2*c-(1-a)*a*d+b*(-c)^(1/2)*d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*p
olylog(2,(a^2*d+b^2*c)*(b*x+a+1)/(b*x+a)/(b^2*c+a*(1+a)*d+b*(-c)^(1/2)*d^(
1/2)))/(-c)^(1/2)/d^(1/2)
```

3.76.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 529, normalized size of antiderivative = 0.79

$$\int \frac{\coth^{-1}(a+bx)}{c+dx^2}\,dx$$

$$= \frac{\log\left(\frac{\sqrt{d}(-1+a+bx)}{b\sqrt{-c}+(-1+a)\sqrt{d}}\right)\log\left(\sqrt{-c}-\sqrt{d}x\right)-\log\left(\frac{-1+a+bx}{a+bx}\right)\log\left(\sqrt{-c}-\sqrt{d}x\right)-\log\left(\frac{\sqrt{d}(1+a+bx)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right)\log\left(\sqrt{\cdot}\right.}{}$$

input
```
Integrate[ArcCoth[a + b*x]/(c + d*x^2),x]
```

output
```
(Log[(Sqrt[d]*(-1 + a + b*x))/(b*Sqrt[-c] + (-1 + a)*Sqrt[d])]*Log[Sqrt[-c
] - Sqrt[d]*x] - Log[(-1 + a + b*x)/(a + b*x)]*Log[Sqrt[-c] - Sqrt[d]*x] -
 Log[(Sqrt[d]*(1 + a + b*x))/(b*Sqrt[-c] + (1 + a)*Sqrt[d])]*Log[Sqrt[-c]
- Sqrt[d]*x] + Log[(1 + a + b*x)/(a + b*x)]*Log[Sqrt[-c] - Sqrt[d]*x] - Lo
g[-((Sqrt[d]*(-1 + a + b*x))/(b*Sqrt[-c] - (-1 + a)*Sqrt[d]))]*Log[Sqrt[-c
] + Sqrt[d]*x] + Log[(-1 + a + b*x)/(a + b*x)]*Log[Sqrt[-c] + Sqrt[d]*x] +
 Log[-((Sqrt[d]*(1 + a + b*x))/(b*Sqrt[-c] - (1 + a)*Sqrt[d]))]*Log[Sqrt[-
c] + Sqrt[d]*x] - Log[(1 + a + b*x)/(a + b*x)]*Log[Sqrt[-c] + Sqrt[d]*x] +
 PolyLog[2, (b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (-1 + a)*Sqrt[d])] -
PolyLog[2, (b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (1 + a)*Sqrt[d])] - Po
lyLog[2, (b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (-1 + a)*Sqrt[d])] + Pol
yLog[2, (b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (1 + a)*Sqrt[d])])/(4*Sqr
t[-c]*Sqrt[d])
```

3.76.3 Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6666, 2976, 2804, 2009, 2977, 2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(a+bx)}{c+dx^2}dx$$

$$\Big\downarrow \text{6666}$$

$$\frac{1}{2}\int \frac{\log\left(\frac{a+bx+1}{a+bx}\right)}{dx^2+c}dx-\frac{1}{2}\int \frac{\log\left(-\frac{-a-bx+1}{a+bx}\right)}{dx^2+c}dx$$

$$\Big\downarrow \text{2976}$$

$$-\frac{1}{2}b\int \frac{\log\left(\frac{a+bx+1}{a+bx}\right)}{d(a+1)^2+\frac{(da^2+b^2c)(a+bx+1)^2}{(a+bx)^2}+b^2c-\frac{2(cb^2+a(a+1)d)(a+bx+1)}{a+bx}}d\frac{a+bx+1}{a+bx}-\frac{1}{2}\int \frac{\log\left(-\frac{-a-bx+1}{a+bx}\right)}{dx^2+c}dx$$

$$\Big\downarrow \text{2804}$$

$$-\frac{1}{2}b\int \left(\frac{(da^2+b^2c)\log\left(\frac{a+bx+1}{a+bx}\right)}{b\sqrt{-c}\sqrt{d}\left(2da^2+2da+2b^2c-\frac{2(da^2+b^2c)(a+bx+1)}{a+bx}-2b\sqrt{-c}\sqrt{d}\right)}+\frac{(da^2+b^2c)\log(}{b\sqrt{-c}\sqrt{d}\left(-2da^2-2da-2b^2c+\frac{2(a}{}\right.}\right.$$
$$\frac{1}{2}\int \frac{\log\left(-\frac{-a-bx+1}{a+bx}\right)}{dx^2+c}dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\int \frac{\log\left(-\frac{-a-bx+1}{a+bx}\right)}{dx^2+c}dx-$$
$$\frac{1}{2}b\left(-\frac{\text{PolyLog}\left(2,\frac{(da^2+b^2c)(a+bx+1)}{\left(cb^2-\sqrt{-c}\sqrt{d}b+a(a+1)d\right)(a+bx)}\right)}{2b\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{(da^2+b^2c)(a+bx+1)}{\left(cb^2+\sqrt{-c}\sqrt{d}b+a(a+1)d\right)(a+bx)}\right)}{2b\sqrt{-c}\sqrt{d}}-\frac{\log\left(\frac{a+bx+1}{a+bx}\right)\log}{}\right.$$

$$\Big\downarrow \text{2977}$$

$$\frac{1}{2}b\int \frac{\log\left(-\frac{-a-bx+1}{a+bx}\right)}{d(1-a)^2+b^2c+\frac{2(b^2c-(1-a)ad)(-a-bx+1)}{a+bx}+\frac{(da^2+b^2c)(-a-bx+1)^2}{(a+bx)^2}}d\frac{-a-bx+1}{a+bx}-$$

$$\frac{1}{2}b\left(-\frac{\text{PolyLog}\left(2,\frac{(da^2+b^2c)(a+bx+1)}{\left(cb^2-\sqrt{-c}\sqrt{d}b+a(a+1)d\right)(a+bx)}\right)}{2b\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{(da^2+b^2c)(a+bx+1)}{\left(cb^2+\sqrt{-c}\sqrt{d}b+a(a+1)d\right)(a+bx)}\right)}{2b\sqrt{-c}\sqrt{d}}-\frac{\log\left(\frac{a+bx+1}{a+bx}\right)\log\right.}{}\right.$$

$\downarrow$ 2804

$$\frac{1}{2}b\int\left(\frac{(da^2+b^2c)\log\left(-\frac{-a-bx+1}{a+bx}\right)}{b\sqrt{-c}\sqrt{d}\left(-2da^2+2da-2b^2c-2b\sqrt{-c}\sqrt{d}-\frac{2(da^2+b^2c)(-a-bx+1)}{a+bx}\right)}+\frac{(da^2+b^2c)\log\left(\right.}{b\sqrt{-c}\sqrt{d}\left(2da^2-2da+2b^2c-2b\right.}\right.$$

$$\frac{1}{2}b\left(-\frac{\text{PolyLog}\left(2,\frac{(da^2+b^2c)(a+bx+1)}{\left(cb^2-\sqrt{-c}\sqrt{d}b+a(a+1)d\right)(a+bx)}\right)}{2b\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{(da^2+b^2c)(a+bx+1)}{\left(cb^2+\sqrt{-c}\sqrt{d}b+a(a+1)d\right)(a+bx)}\right)}{2b\sqrt{-c}\sqrt{d}}-\frac{\log\left(\frac{a+bx+1}{a+bx}\right)\log\right.}{}\right.$$

$\downarrow$ 2009

$$\frac{1}{2}b\left(\frac{\text{PolyLog}\left(2,-\frac{(da^2+b^2c)(-a-bx+1)}{\left(cb^2-\sqrt{-c}\sqrt{d}b-(1-a)ad\right)(a+bx)}\right)}{2b\sqrt{-c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,-\frac{(da^2+b^2c)(-a-bx+1)}{\left(cb^2+\sqrt{-c}\sqrt{d}b-(1-a)ad\right)(a+bx)}\right)}{2b\sqrt{-c}\sqrt{d}}+\frac{\log\left(-\frac{-a-bx+}{a+bx}\right.}{}\right.$$

$$\frac{1}{2}b\left(-\frac{\text{PolyLog}\left(2,\frac{(da^2+b^2c)(a+bx+1)}{\left(cb^2-\sqrt{-c}\sqrt{d}b+a(a+1)d\right)(a+bx)}\right)}{2b\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{(da^2+b^2c)(a+bx+1)}{\left(cb^2+\sqrt{-c}\sqrt{d}b+a(a+1)d\right)(a+bx)}\right)}{2b\sqrt{-c}\sqrt{d}}-\frac{\log\left(\frac{a+bx+1}{a+bx}\right)\log\right.}{}\right.$$

input
```
Int[ArcCoth[a + b*x]/(c + d*x^2),x]
```

3.76. $\int \frac{\coth^{-1}(a+bx)}{c+dx^2}\,dx$

output
```
(b*((Log[-((1 - a - b*x)/(a + b*x))]*Log[1 + ((b^2*c + a^2*d)*(1 - a - b*x
))/((b^2*c - b*Sqrt[-c]*Sqrt[d] - (1 - a)*a*d)*(a + b*x))])/(2*b*Sqrt[-c]*
Sqrt[d]) - (Log[-((1 - a - b*x)/(a + b*x))]*Log[1 + ((b^2*c + a^2*d)*(1 -
a - b*x))/((b^2*c + b*Sqrt[-c]*Sqrt[d] - (1 - a)*a*d)*(a + b*x))])/(2*b*Sq
rt[-c]*Sqrt[d]) + PolyLog[2, -(((b^2*c + a^2*d)*(1 - a - b*x))/((b^2*c - b
*Sqrt[-c]*Sqrt[d] - (1 - a)*a*d)*(a + b*x)))]/(2*b*Sqrt[-c]*Sqrt[d]) - Pol
yLog[2, -(((b^2*c + a^2*d)*(1 - a - b*x))/((b^2*c + b*Sqrt[-c]*Sqrt[d] - (
1 - a)*a*d)*(a + b*x)))]/(2*b*Sqrt[-c]*Sqrt[d])))/2 - (b*(-1/2*(Log[(1 + a
 + b*x)/(a + b*x)]*Log[1 - ((b^2*c + a^2*d)*(1 + a + b*x))/((b^2*c - b*Sqr
t[-c]*Sqrt[d] + a*(1 + a)*d)*(a + b*x))])/(b*Sqrt[-c]*Sqrt[d]) + (Log[(1 +
 a + b*x)/(a + b*x)]*Log[1 - ((b^2*c + a^2*d)*(1 + a + b*x))/((b^2*c + b*S
qrt[-c]*Sqrt[d] + a*(1 + a)*d)*(a + b*x))])/(2*b*Sqrt[-c]*Sqrt[d]) - PolyL
og[2, ((b^2*c + a^2*d)*(1 + a + b*x))/((b^2*c - b*Sqrt[-c]*Sqrt[d] + a*(1
+ a)*d)*(a + b*x))]/(2*b*Sqrt[-c]*Sqrt[d]) + PolyLog[2, ((b^2*c + a^2*d)*(
1 + a + b*x))/((b^2*c + b*Sqrt[-c]*Sqrt[d] + a*(1 + a)*d)*(a + b*x))]/(2*b
*Sqrt[-c]*Sqrt[d])))/2
```

3.76.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2804
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

rule 2976
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)   Subst[Int[(b^2*f -
a*b*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^
2*h)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/
(c + d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] &
& NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.76. $\int \frac{\coth^{-1}(a+bx)}{c+dx^2} dx$

rule 2977
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g =
Coeff[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)   Subst[Int[(b^2*
f - a*b*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g
+ c^2*h)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b
*x)/(c + d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x,
2] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] && I
GtQ[p, 0]
```

rule 6666
```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Simp
[1/2   Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Simp[1/2   In
t[Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f},
x] && RationalQ[n]
```

3.76.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{\ln(bx+a-1)\ln\left(\frac{b\sqrt{-cd}-(bx+a-1)d+ad-d}{b\sqrt{-cd}+ad-d}\right)}{4\sqrt{-cd}}+\frac{\ln(bx+a-1)\ln\left(\frac{b\sqrt{-cd}+(bx+a-1)d-ad+d}{b\sqrt{-cd}-ad+d}\right)}{4\sqrt{-cd}}-\frac{\operatorname{dilog}\left(\frac{b\sqrt{-cd}-(bx+a-1}{b\sqrt{-cd}+ad}\right.}{4\sqrt{-cd}}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arccoth(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(b*x+a-1)/(-c*d)^(1/2)*ln((b*(-c*d)^(1/2)-(b*x+a-1)*d+a*d-d)/(b*(-c
*d)^(1/2)+a*d-d))+1/4*ln(b*x+a-1)/(-c*d)^(1/2)*ln((b*(-c*d)^(1/2)+(b*x+a-1
)*d-a*d+d)/(b*(-c*d)^(1/2)-a*d+d))-1/4/(-c*d)^(1/2)*dilog((b*(-c*d)^(1/2)-
(b*x+a-1)*d+a*d-d)/(b*(-c*d)^(1/2)+a*d-d))+1/4/(-c*d)^(1/2)*dilog((b*(-c*d
)^(1/2)+(b*x+a-1)*d-a*d+d)/(b*(-c*d)^(1/2)-a*d+d))+1/4*ln(b*x+a+1)/(-c*d)^
(1/2)*ln((b*(-c*d)^(1/2)-(b*x+a+1)*d+a*d+d)/(b*(-c*d)^(1/2)+a*d+d))-1/4*ln
(b*x+a+1)/(-c*d)^(1/2)*ln((b*(-c*d)^(1/2)+(b*x+a+1)*d-a*d-d)/(b*(-c*d)^(1/
2)-a*d-d))+1/4/(-c*d)^(1/2)*dilog((b*(-c*d)^(1/2)-(b*x+a+1)*d+a*d+d)/(b*(-
c*d)^(1/2)+a*d+d))-1/4/(-c*d)^(1/2)*dilog((b*(-c*d)^(1/2)+(b*x+a+1)*d-a*d-
d)/(b*(-c*d)^(1/2)-a*d-d))
```

3.76.5 Fricas [F]

$$\int \frac{\coth^{-1}(a+bx)}{c+dx^2}\,dx = \int \frac{\operatorname{arcoth}(bx+a)}{dx^2+c}\,dx$$

input
```
integrate(arccoth(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral(arccoth(b*x + a)/(d*x^2 + c), x)
```

3.76.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(a+bx)}{c+dx^2}\,dx = \text{Timed out}$$

input
```
integrate(acoth(b*x+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.76.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 591, normalized size of antiderivative = 0.88

$$\int \frac{\coth^{-1}(a+bx)}{c+dx^2}\,dx = \frac{\operatorname{arcoth}(bx+a)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}} + \frac{\left(\arctan\left(\frac{(b^2x+(a+1)b)\sqrt{c}\sqrt{d}}{b^2c+(a^2+2a+1)d}, \frac{(a+1)bdx+(a^2+2a+1)d}{b^2c+(a^2+2a+1)d}\right) - \arctan\left(\frac{(b^2x+(a-1)b)\sqrt{c}\sqrt{d}}{b^2c+(a^2-2a+1)d}, \frac{(a-1)bdx+(a^2-2a+1)d}{b^2c+(a^2-2a+1)d}\right)\right)\log(dx}{}$$

input
```
integrate(arccoth(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
arccoth(b*x + a)*arctan(d*x/sqrt(c*d))/sqrt(c*d) + 1/4*((arctan2((b^2*x +
(a + 1)*b)*sqrt(c)*sqrt(d)/(b^2*c + (a^2 + 2*a + 1)*d), ((a + 1)*b*d*x + (
a^2 + 2*a + 1)*d)/(b^2*c + (a^2 + 2*a + 1)*d)) - arctan2((b^2*x + (a - 1)*
b)*sqrt(c)*sqrt(d)/(b^2*c + (a^2 - 2*a + 1)*d), ((a - 1)*b*d*x + (a^2 - 2*
a + 1)*d)/(b^2*c + (a^2 - 2*a + 1)*d)))*log(d*x^2 + c) - arctan(sqrt(d)*x/
sqrt(c))*log((b^2*d*x^2 + 2*(a + 1)*b*d*x + (a^2 + 2*a + 1)*d)/(b^2*c + (a
^2 + 2*a + 1)*d)) + arctan(sqrt(d)*x/sqrt(c))*log((b^2*d*x^2 + 2*(a - 1)*b
*d*x + (a^2 - 2*a + 1)*d)/(b^2*c + (a^2 - 2*a + 1)*d)) - I*dilog(((a - 1)*
b*d*x + b^2*c + (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(b^2*c + 2*(-I*a
 + I)*b*sqrt(c)*sqrt(d) - (a^2 - 2*a + 1)*d)) + I*dilog(((a - 1)*b*d*x + b
^2*c - (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(b^2*c - 2*(-I*a + I)*b*s
qrt(c)*sqrt(d) - (a^2 - 2*a + 1)*d)) + I*dilog(((a + 1)*b*d*x + b^2*c + (I
*b^2*x + (-I*a - I)*b)*sqrt(c)*sqrt(d))/(b^2*c + 2*(-I*a - I)*b*sqrt(c)*sq
rt(d) - (a^2 + 2*a + 1)*d)) - I*dilog(((a + 1)*b*d*x + b^2*c - (I*b^2*x +
(-I*a - I)*b)*sqrt(c)*sqrt(d))/(b^2*c - 2*(-I*a - I)*b*sqrt(c)*sqrt(d) - (
a^2 + 2*a + 1)*d)))/sqrt(c*d)
```

3.76.8 Giac [F]

$$\int \frac{\coth^{-1}(a+bx)}{c+dx^2}\,dx=\int \frac{\operatorname{arcoth}(bx+a)}{dx^2+c}\,dx$$

input
```
integrate(arccoth(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(arccoth(b*x + a)/(d*x^2 + c), x)
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(a+bx)}{c+dx^2}\,dx=\int \frac{\operatorname{acoth}(a+b\,x)}{d\,x^2+c}\,dx$$

input
```
int(acoth(a + b*x)/(c + d*x^2),x)
```

output
```
int(acoth(a + b*x)/(c + d*x^2), x)
```

3.77 $\int \frac{\coth^{-1}(a+bx)}{c+dx}\,dx$

3.77.1 Optimal result

Integrand size = 14, antiderivative size = 120

$$\int \frac{\coth^{-1}(a+bx)}{c+dx}\,dx = -\frac{\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\coth^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} + \frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1+a+bx}\right)}{2d} - \frac{\operatorname{PolyLog}\left(2,1-\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{2d}$$

output
```
-arccoth(b*x+a)*ln(2/(b*x+a+1))/d+arccoth(b*x+a)*ln(2*b*(d*x+c)/(-a*d+b*c+
d)/(b*x+a+1))/d+1/2*polylog(2,1-2/(b*x+a+1))/d-1/2*polylog(2,1-2*b*(d*x+c)
/(-a*d+b*c+d)/(b*x+a+1))/d
```

3.77.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.54

$$\int \frac{\coth^{-1}(a+bx)}{c+dx}\,dx = \frac{\log\left(\frac{d(1-a-bx)}{bc+d-ad}\right)\log(c+dx)}{2d} - \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)\log(c+dx)}{2d} - \frac{\log\left(-\frac{d(1+a+bx)}{bc-d-ad}\right)\log(c+dx)}{2d} + \frac{\log\left(\frac{1+a+bx}{a+bx}\right)\log(c+dx)}{2d} - \frac{\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-d-ad}\right)}{2d} + \frac{\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc+d-ad}\right)}{2d}$$

input
```
Integrate[ArcCoth[a + b*x]/(c + d*x),x]
```

output
```
(Log[(d*(1 - a - b*x))/(b*c + d - a*d)]*Log[c + d*x])/(2*d) - (Log[(-1 + a
 + b*x)/(a + b*x)]*Log[c + d*x])/(2*d) - (Log[-((d*(1 + a + b*x))/(b*c - d
 - a*d))]*Log[c + d*x])/(2*d) + (Log[(1 + a + b*x)/(a + b*x)]*Log[c + d*x]
)/(2*d) - PolyLog[2, (b*(c + d*x))/(b*c - d - a*d)]/(2*d) + PolyLog[2, (b*
(c + d*x))/(b*c + d - a*d)]/(2*d)
```

3.77.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6662, 27, 6473, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \; 6662$$

$$\frac{\int \frac{b\coth^{-1}(a+bx)}{b\left(c-\frac{ad}{b}\right)+d(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$\int \frac{\coth^{-1}(a+bx)}{d(a+bx)-ad+bc}d(a+bx)$$

$$\Big\downarrow \; 6473$$

$$-\frac{\int \frac{\log\left(\frac{2(bc-ad+d(a+bx))}{(bc-ad+d)(a+bx+1)}\right)}{1-(a+bx)^2}d(a+bx)}{d}+\frac{\int \frac{\log\left(\frac{2}{a+bx+1}\right)}{1-(a+bx)^2}d(a+bx)}{d}+\frac{\coth^{-1}(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(a+bx+1)(-ad+bc+d)}\right)}{d}-\frac{\log\left(\frac{2}{a+bx+1}\right)\coth^{-1}(a+bx)}{d}$$

$$\Big\downarrow \; 2849$$

$$-\frac{\int \frac{\log\left(\frac{2(bc-ad+d(a+bx))}{(bc-ad+d)(a+bx+1)}\right)}{1-(a+bx)^2}d(a+bx)}{d}+\frac{\int \frac{\log\left(\frac{2}{a+bx+1}\right)}{1-\frac{2}{a+bx+1}}d\frac{1}{a+bx+1}}{d}+\frac{\coth^{-1}(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(a+bx+1)(-ad+bc+d)}\right)}{d}-\frac{\log\left(\frac{2}{a+bx+1}\right)\coth^{-1}(a+bx)}{d}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{\int \frac{\log\left(\frac{2(bc-ad+d(a+bx))}{(bc-ad+d)(a+bx+1)}\right)}{1-(a+bx)^2}d(a+bx)}{d}+\frac{\coth^{-1}(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(a+bx+1)(-ad+bc+d)}\right)}{d}+\frac{\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)}{2d}-\frac{\log\left(\frac{2}{a+bx+1}\right)\coth^{-1}(a+bx)}{d}$$

$$\Big\downarrow \text{2897}$$

$$-\frac{\text{PolyLog}\left(2,1-\frac{2(bc-ad+d(a+bx))}{(bc-ad+d)(a+bx+1)}\right)}{2d}+\frac{\coth^{-1}(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(a+bx+1)(-ad+bc+d)}\right)}{d}+\frac{\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)}{2d}-\frac{\log\left(\frac{2}{a+bx+1}\right)\coth^{-1}(a+bx)}{d}$$

input
```
Int[ArcCoth[a + b*x]/(c + d*x),x]
```

output
```
-((ArcCoth[a + b*x]*Log[2/(1 + a + b*x)])/d) + (ArcCoth[a + b*x]*Log[(2*(b
*c - a*d + d*(a + b*x)))/((b*c + d - a*d)*(1 + a + b*x))])/d + PolyLog[2,
1 - 2/(1 + a + b*x)]/(2*d) - PolyLog[2, 1 - (2*(b*c - a*d + d*(a + b*x)))/
((b*c + d - a*d)*(1 + a + b*x))]/(2*d)
```

3.77.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6473
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcCoth[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*ArcCoth
[c*x])*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(c/e)
Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Simp[b*(c/e)   Int[Log[2*c*((d
 + e*x)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x]) /; FreeQ[{a, b, c, d
, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.77.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{\operatorname{dilog}\left(\frac{(bx+a-1)d-ad+bc+d}{-ad+bc+d}\right)}{2d}-\frac{\ln(bx+a-1)\ln\left(\frac{(bx+a-1)d-ad+bc+d}{-ad+bc+d}\right)}{2d}+\frac{\operatorname{dilog}\left(\frac{(bx+a+1)d-ad+bc-d}{-ad+bc-d}\right)}{2d}+\frac{\ln(bx+a$
parts	$\frac{\ln(dx+c)\operatorname{arccoth}(bx+a)}{d}+\frac{b\left(\frac{d\left(\frac{\operatorname{dilog}\left(\frac{ad-bc+b(dx+c)-d}{ad-bc-d}\right)}{b}+\frac{\ln(dx+c)\ln\left(\frac{ad-bc+b(dx+c)-d}{ad-bc-d}\right)}{b}\right)}{2}-\frac{d\left(\frac{\operatorname{dilog}\left(\frac{ad-bc+b(dx+}{ad-bc+}\right)}{b}\right.}{}\right.}{d^2}$
derivativedivides	$\frac{\frac{b\ln(ad-bc-d(bx+a))\operatorname{arccoth}(bx+a)}{d}-\frac{b\left(\frac{d\left(\operatorname{dilog}\left(\frac{-d(bx+a)-d}{-ad+bc-d}\right)+\ln(ad-bc-d(bx+a))\ln\left(\frac{-d(bx+a)-d}{-ad+bc-d}\right)\right)}{2}-\frac{d\left(\operatorname{dilog}\left(\frac{-d(bx+a)+}{-ad+bc+}\right.\right.}{}\right.}{d^2}}{b}$
default	$\frac{\frac{b\ln(ad-bc-d(bx+a))\operatorname{arccoth}(bx+a)}{d}-\frac{b\left(\frac{d\left(\operatorname{dilog}\left(\frac{-d(bx+a)-d}{-ad+bc-d}\right)+\ln(ad-bc-d(bx+a))\ln\left(\frac{-d(bx+a)-d}{-ad+bc-d}\right)\right)}{2}-\frac{d\left(\operatorname{dilog}\left(\frac{-d(bx+a)+}{-ad+bc+}\right.\right.}{}\right.}{d^2}}{b}$

input
```
int(arccoth(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*dilog(((b*x+a-1)*d-a*d+b*c+d)/(-a*d+b*c+d))/d-1/2*ln(b*x+a-1)*ln(((b*
x+a-1)*d-a*d+b*c+d)/(-a*d+b*c+d))/d+1/2*dilog(((b*x+a+1)*d-a*d+b*c-d)/(-a*
d+b*c-d))/d+1/2*ln(b*x+a+1)*ln(((b*x+a+1)*d-a*d+b*c-d)/(-a*d+b*c-d))/d
```

3.77.5 Fricas [F]

$$\int \frac{\coth^{-1}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{arcoth}(bx+a)}{dx+c}\,dx$$

input
```
integrate(arccoth(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(arccoth(b*x + a)/(d*x + c), x)
```

3.77.6 Sympy [F]

$$\int \frac{\coth^{-1}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{acoth}(a+bx)}{c+dx}\,dx$$

input
```
integrate(acoth(b*x+a)/(d*x+c),x)
```

output
```
Integral(acoth(a + b*x)/(c + d*x), x)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.60

$$\int \frac{\coth^{-1}(a+bx)}{c+dx}\,dx = -\frac{1}{2}b\left(\frac{\log(bx+a-1)\log\left(\frac{bdx+ad-d}{bc-ad+d}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad-d}{bc-ad+d}\right)}{bd}-\frac{\log(bx+a+1)\log\left(\frac{bdx+ad+d}{bc-ad-d}+1\right)+\mathrm{Li}_2(\right.$$
$$-\frac{b\left(\frac{\log(bx+a+1)}{b}-\frac{\log(bx+a-1)}{b}\right)\log(dx+c)}{2\,d}+\frac{\operatorname{arcoth}(bx+a)\log(dx+c)}{d}$$

input
```
integrate(arccoth(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*b*((log(b*x + a - 1)*log((b*d*x + a*d - d)/(b*c - a*d + d) + 1) + dil
og(-(b*d*x + a*d - d)/(b*c - a*d + d)))/(b*d) - (log(b*x + a + 1)*log((b*d
*x + a*d + d)/(b*c - a*d - d) + 1) + dilog(-(b*d*x + a*d + d)/(b*c - a*d -
 d)))/(b*d)) - 1/2*b*(log(b*x + a + 1)/b - log(b*x + a - 1)/b)*log(d*x + c
)/d + arccoth(b*x + a)*log(d*x + c)/d
```

3.77.8 Giac [F]

$$\int \frac{\coth^{-1}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{arcoth}(bx+a)}{dx+c}\,dx$$

input
```
integrate(arccoth(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(arccoth(b*x + a)/(d*x + c), x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{acoth}(a+b\,x)}{c+d\,x}\,dx$$

input
```
int(acoth(a + b*x)/(c + d*x),x)
```

output
```
int(acoth(a + b*x)/(c + d*x), x)
```

3.78 $\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$

3.78.1 Optimal result

Integrand size = 16, antiderivative size = 292

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \frac{(1-a-bx)\log\left(-\frac{1-a-bx}{a+bx}\right)}{2bc}+\frac{\log(a+bx)}{2bc}+\frac{\log(1+a+bx)}{2bc}+\frac{(a+bx)\log\left(\frac{1+a+bx}{a+bx}\right)}{2bc}-\frac{d\log\left(\frac{c(1-a-bx)}{c-ac+bd}\right)\log(d+cx)}{2c^2}+\frac{d\log\left(-\frac{1-a-bx}{a+bx}\right)\log(d+cx)}{2c^2}+\frac{d\log\left(\frac{c(1+a+bx)}{c+ac-bd}\right)\log(d+cx)}{2c^2}-\frac{d\log\left(\frac{1+a+bx}{a+bx}\right)\log(d+cx)}{2c^2}+\frac{d\,\text{PolyLog}\left(2,-\frac{b(d+cx)}{c+ac-bd}\right)}{2c^2}-\frac{d\,\text{PolyLog}\left(2,\frac{b(d+cx)}{c-ac+bd}\right)}{2c^2}$$

output
```
1/2*(-b*x-a+1)*ln((b*x+a-1)/(b*x+a))/b/c+1/2*ln(b*x+a)/b/c+1/2*ln(b*x+a+1)
/b/c+1/2*(b*x+a)*ln((b*x+a+1)/(b*x+a))/b/c-1/2*d*ln(c*(-b*x-a+1)/(-a*c+b*d
+c))*ln(c*x+d)/c^2+1/2*d*ln((b*x+a-1)/(b*x+a))*ln(c*x+d)/c^2+1/2*d*ln(c*(b
*x+a+1)/(a*c-b*d+c))*ln(c*x+d)/c^2-1/2*d*ln((b*x+a+1)/(b*x+a))*ln(c*x+d)/c
^2+1/2*d*polylog(2,-b*(c*x+d)/(a*c-b*d+c))/c^2-1/2*d*polylog(2,b*(c*x+d)/(
-a*c+b*d+c))/c^2
```

3.78.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.76 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.74

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$$
$$= \frac{2ac^2\coth^{-1}(a+bx) - ibcd\pi\coth^{-1}(a+bx) + 2bc^2x\coth^{-1}(a+bx) + bcd\coth^{-1}(a+bx)^2 + abcd\coth^{-1}}{}$$

input
```
Integrate[ArcCoth[a + b*x]/(c + d/x),x]
```

output
```
(2*a*c^2*ArcCoth[a + b*x] - I*b*c*d*Pi*ArcCoth[a + b*x] + 2*b*c^2*x*ArcCot
h[a + b*x] + b*c*d*ArcCoth[a + b*x]^2 + a*b*c*d*ArcCoth[a + b*x]^2 - b^2*d
^2*ArcCoth[a + b*x]^2 - a*b*c*d*Sqrt[1 - c^2/(a*c - b*d)^2]*E^ArcTanh[c/(a
*c - b*d)]*ArcCoth[a + b*x]^2 + b^2*d^2*Sqrt[1 - c^2/(a*c - b*d)^2]*E^ArcT
anh[c/(a*c - b*d)]*ArcCoth[a + b*x]^2 + 2*b*c*d*ArcCoth[a + b*x]*ArcTanh[c
/(a*c - b*d)] + 2*b*c*d*ArcCoth[a + b*x]*Log[1 - E^(-2*ArcCoth[a + b*x])]
+ I*b*c*d*Pi*Log[1 + E^(2*ArcCoth[a + b*x])] - 2*b*c*d*ArcCoth[a + b*x]*Lo
g[1 - E^(-2*ArcCoth[a + b*x] + 2*ArcTanh[c/(a*c - b*d)])] + 2*b*c*d*ArcTan
h[c/(a*c - b*d)]*Log[1 - E^(-2*ArcCoth[a + b*x] + 2*ArcTanh[c/(a*c - b*d)]
)] - 2*c^2*Log[(a + b*x)^(-1)] - 2*c^2*Log[1/Sqrt[1 - (a + b*x)^(-2)]] - I
*b*c*d*Pi*Log[1/Sqrt[1 - (a + b*x)^(-2)]] - 2*b*c*d*ArcTanh[c/(a*c - b*d)]
*Log[I*Sinh[ArcCoth[a + b*x] - ArcTanh[c/(a*c - b*d)]]] - b*c*d*PolyLog[2,
 E^(-2*ArcCoth[a + b*x])] + b*c*d*PolyLog[2, E^(-2*ArcCoth[a + b*x] + 2*Ar
cTanh[c/(a*c - b*d)])])/(2*b*c^3)
```

3.78.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.49, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6666, 2993, 772, 49, 2009, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$$

$\downarrow$ 6666

$$\frac{1}{2}\int \frac{\log\left(\frac{a+bx+1}{a+bx}\right)}{c+\frac{d}{x}}dx-\frac{1}{2}\int \frac{\log\left(-\frac{-a-bx+1}{a+bx}\right)}{c+\frac{d}{x}}dx$$

$\downarrow$ 2993

$$\frac{1}{2}\left(\left(\log(-a-bx+1)-\log\left(-\frac{-a-bx+1}{a+bx}\right)-\log(a+bx)\right)\int \frac{1}{c+\frac{d}{x}}dx-\int \frac{\log(-a-bx+1)}{c+\frac{d}{x}}dx+\int \frac{\log(a+\right.$$
$$\frac{1}{2}\left(\left(\log(a+bx)-\log(a+bx+1)+\log\left(\frac{a+bx+1}{a+bx}\right)\right)\int \frac{1}{c+\frac{d}{x}}dx-\int \frac{\log(a+bx)}{c+\frac{d}{x}}dx+\int \frac{\log(a+bx+1)}{c+\frac{d}{x}}dx\right.$$

$\downarrow$ 772

$$\frac{1}{2}\left(\left(\log(-a-bx+1)-\log\left(-\frac{-a-bx+1}{a+bx}\right)-\log(a+bx)\right)\int \frac{x}{d+cx}dx-\int \frac{\log(-a-bx+1)}{c+\frac{d}{x}}dx+\int \frac{\log(a\right.$$
$$\frac{1}{2}\left(\left(\log(a+bx)-\log(a+bx+1)+\log\left(\frac{a+bx+1}{a+bx}\right)\right)\int \frac{x}{d+cx}dx-\int \frac{\log(a+bx)}{c+\frac{d}{x}}dx+\int \frac{\log(a+bx+1)}{c+\frac{d}{x}}dx\right.$$

$\downarrow$ 49

$$\frac{1}{2}\left(\left(\log(-a-bx+1)-\log\left(-\frac{-a-bx+1}{a+bx}\right)-\log(a+bx)\right)\int \left(\frac{1}{c}-\frac{d}{c(d+cx)}\right)dx-\int \frac{\log(-a-bx+1)}{c+\frac{d}{x}}dx\right.$$
$$\frac{1}{2}\left(\left(\log(a+bx)-\log(a+bx+1)+\log\left(\frac{a+bx+1}{a+bx}\right)\right)\int \left(\frac{1}{c}-\frac{d}{c(d+cx)}\right)dx-\int \frac{\log(a+bx)}{c+\frac{d}{x}}dx+\int \frac{\log(a}{c}\right.$$

$\downarrow$ 2009

$$\frac{1}{2}\left(-\int \frac{\log(-a-bx+1)}{c+\frac{d}{x}}dx+\int \frac{\log(a+bx)}{c+\frac{d}{x}}dx+\left(\log(-a-bx+1)-\log\left(-\frac{-a-bx+1}{a+bx}\right)-\log(a+bx)\right)\left(\right.\right.$$
$$\frac{1}{2}\left(-\int \frac{\log(a+bx)}{c+\frac{d}{x}}dx+\int \frac{\log(a+bx+1)}{c+\frac{d}{x}}dx+\left(\log(a+bx)-\log(a+bx+1)+\log\left(\frac{a+bx+1}{a+bx}\right)\right)\left(\frac{x}{c}-\frac{d\log}{}\right.\right.$$

$\downarrow$ 2856

$$\frac{1}{2}\left(-\int \left(\frac{\log(-a-bx+1)}{c}-\frac{d\log(-a-bx+1)}{c(d+cx)}\right)dx+\int \left(\frac{\log(a+bx)}{c}-\frac{d\log(a+bx)}{c(d+cx)}\right)dx+\left(\log(-a-bx+\right.\right.$$
$$\frac{1}{2}\left(-\int \left(\frac{\log(a+bx)}{c}-\frac{d\log(a+bx)}{c(d+cx)}\right)dx+\int \left(\frac{\log(a+bx+1)}{c}-\frac{d\log(a+bx+1)}{c(d+cx)}\right)dx+\left(\log(a+bx)-\log(\right.\right.$$

$\downarrow$ 2009

3.78. $\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$

$$\frac{1}{2}\left(\frac{d\,\text{PolyLog}\left(2,\frac{c(-a-bx+1)}{-ac+c+bd}\right)}{c^2}-\frac{d\,\text{PolyLog}\left(2,\frac{c(a+bx)}{ac-bd}\right)}{c^2}+\frac{d\log(-a-bx+1)\log\left(\frac{b(cx+d)}{-ac+bd+c}\right)}{c^2}+\left(\log(-a-bx+\right.\right.$$

$$\frac{1}{2}\left(\frac{d\,\text{PolyLog}\left(2,\frac{c(a+bx)}{ac-bd}\right)}{c^2}-\frac{d\,\text{PolyLog}\left(2,\frac{c(a+bx+1)}{ac+c-bd}\right)}{c^2}+\frac{d\log(a+bx)\log\left(-\frac{b(cx+d)}{ac-bd}\right)}{c^2}+\left(\log(a+bx)-\log(a+\right.\right.$$

input
```
Int[ArcCoth[a + b*x]/(c + d/x),x]
```

output
```
(((1 - a - b*x)*Log[1 - a - b*x])/(b*c) + ((a + b*x)*Log[a + b*x])/(b*c) +
 (Log[1 - a - b*x] - Log[-((1 - a - b*x)/(a + b*x))] - Log[a + b*x])*(x/c
- (d*Log[d + c*x])/c^2) - (d*Log[a + b*x]*Log[-((b*(d + c*x))/(a*c - b*d))
])/c^2 + (d*Log[1 - a - b*x]*Log[(b*(d + c*x))/(c - a*c + b*d)])/c^2 + (d*
PolyLog[2, (c*(1 - a - b*x))/(c - a*c + b*d)])/c^2 - (d*PolyLog[2, (c*(a +
 b*x))/(a*c - b*d)])/c^2)/2 + (-(((a + b*x)*Log[a + b*x])/(b*c)) + ((1 + a
 + b*x)*Log[1 + a + b*x])/(b*c) + (Log[a + b*x] - Log[1 + a + b*x] + Log[(
1 + a + b*x)/(a + b*x)])*(x/c - (d*Log[d + c*x])/c^2) + (d*Log[a + b*x]*Lo
g[-((b*(d + c*x))/(a*c - b*d))])/c^2 - (d*Log[1 + a + b*x]*Log[-((b*(d + c
*x))/(c + a*c - b*d))])/c^2 + (d*PolyLog[2, (c*(a + b*x))/(a*c - b*d)])/c^
2 - (d*PolyLog[2, (c*(1 + a + b*x))/(c + a*c - b*d)])/c^2)/2
```

3.78.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

rule 6666
```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Simp
[1/2   Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Simp[1/2   In
t[Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f},
x] && RationalQ[n]
```

3.78.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.89

method	result
parts	$\frac{\operatorname{arccoth}(bx+a)x}{c}-\frac{\operatorname{arccoth}(bx+a)d\ln(cx+d)}{c^{2}}+\frac{b\left(-\frac{(-1-a)\ln(ac-db+b(cx+d)+c)}{2b^{2}}-\frac{(-1+a)\ln(ac-db+b(cx+d)-c)}{2b^{2}}-d\left(\ldots\right.\right.}{\ldots}$
risch	$\frac{\ln(bx+a+1)x}{2c}+\frac{\ln(bx+a+1)a}{2bc}+\frac{\ln(bx+a+1)}{2bc}-\frac{1}{bc}-\frac{d\,\mathrm{dilog}\left(\frac{(bx+a+1)c-ac+db-c}{-ac+db-c}\right)}{2c^{2}}-\frac{d\ln(bx+a+1)\ln\left(\frac{(bx+a+\ldots}{-\ldots}\right.}{2c^{2}}$
derivativedivides	$\frac{\frac{\operatorname{arccoth}(bx+a)(bx+a)}{c}-\frac{\operatorname{arccoth}(bx+a)db\ln(ac-db-c(bx+a))}{c^{2}}-\frac{-\frac{\ln\left(a^{2}c^{2}-2abcd+b^{2}d^{2}-2ac(ac-db-c(bx+a))+2bd(ac-db-c(bx+a\ldots\right.}{2}\ldots}{\ldots}}{\ldots}$
default	$\frac{\frac{\operatorname{arccoth}(bx+a)(bx+a)}{c}-\frac{\operatorname{arccoth}(bx+a)db\ln(ac-db-c(bx+a))}{c^{2}}-\frac{-\frac{\ln\left(a^{2}c^{2}-2abcd+b^{2}d^{2}-2ac(ac-db-c(bx+a))+2bd(ac-db-c(bx+a\ldots\right.}{2}\ldots}{\ldots}}{\ldots}$

input
```
int(arccoth(b*x+a)/(c+d/x),x,method=_RETURNVERBOSE)
```

output
```
arccoth(b*x+a)*x/c-arccoth(b*x+a)/c^2*d*ln(c*x+d)+b/c*(-1/2*(-1-a)/b^2*ln(
a*c-d*b+b*(c*x+d)+c)-1/2*(-1+a)/b^2*ln(a*c-d*b+b*(c*x+d)-c)-d*(-1/2/c*(dil
og((a*c-d*b+b*(c*x+d)+c)/(a*c-b*d+c))/b+ln(c*x+d)*ln((a*c-d*b+b*(c*x+d)+c)
/(a*c-b*d+c))/b)+1/2/c*(dilog((a*c-d*b+b*(c*x+d)-c)/(a*c-b*d-c))/b+ln(c*x+
d)*ln((a*c-d*b+b*(c*x+d)-c)/(a*c-b*d-c))/b)))
```

3.78.5 Fricas [F]

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \int \frac{\operatorname{arcoth}(bx+a)}{c+\frac{d}{x}}\,dx$$

input
```
integrate(arccoth(b*x+a)/(c+d/x),x, algorithm="fricas")
```

output
```
integral(x*arccoth(b*x + a)/(c*x + d), x)
```

3.78.6 Sympy [F]

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \int \frac{x\operatorname{acoth}(a+bx)}{cx+d}\,dx$$

input
```
integrate(acoth(b*x+a)/(c+d/x),x)
```

output
```
Integral(x*acoth(a + b*x)/(c*x + d), x)
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.66

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \frac{1}{2}b\left(\frac{\left(\log(cx+d)\log\left(\frac{bcx+bd}{ac-bd+c}+1\right)+\operatorname{Li}_2\left(-\frac{bcx+bd}{ac-bd+c}\right)\right)d}{bc^2} - \frac{\left(\log(cx+d)\log\left(\frac{bcx+bd}{ac-bd-c}+1\right)+\operatorname{Li}_2\left(-\frac{bcx+bd}{ac-bd-c}\right)\right.}{bc^2}\right. + \left(\frac{x}{c}-\frac{d\log(cx+d)}{c^2}\right)\operatorname{arcoth}(bx+a)$$

input
```
integrate(arccoth(b*x+a)/(c+d/x),x, algorithm="maxima")
```

output
```
1/2*b*((log(c*x + d)*log((b*c*x + b*d)/(a*c - b*d + c) + 1) + dilog(-(b*c*
x + b*d)/(a*c - b*d + c)))*d/(b*c^2) - (log(c*x + d)*log((b*c*x + b*d)/(a*
c - b*d - c) + 1) + dilog(-(b*c*x + b*d)/(a*c - b*d - c)))*d/(b*c^2) + (a
+ 1)*log(b*x + a + 1)/(b^2*c) - (a - 1)*log(b*x + a - 1)/(b^2*c)) + (x/c -
 d*log(c*x + d)/c^2)*arccoth(b*x + a)
```

3.78.8 Giac [F]

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \int \frac{\operatorname{arcoth}(bx+a)}{c+\frac{d}{x}}\,dx$$

input
```
integrate(arccoth(b*x+a)/(c+d/x),x, algorithm="giac")
```

output
```
integrate(arccoth(b*x + a)/(c + d/x), x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \int \frac{\operatorname{acoth}(a+b\,x)}{c+\frac{d}{x}}\,dx$$

input
```
int(acoth(a + b*x)/(c + d/x),x)
```

output
```
int(acoth(a + b*x)/(c + d/x), x)
```

3.79 $\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx$

3.79.1 Optimal result

Integrand size = 16, antiderivative size = 738

$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx
&= \frac{(1-a-bx)\log(-1+a+bx)}{2bc}+\frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+bx)\right)}{2c}\\
&\quad-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+bx)\right)}{2c^{3/2}}\\
&\quad+\frac{(1+a+bx)\log(1+a+bx)}{2bc}+\frac{x\left(\log(a+bx)-\log(1+a+bx)+\log\left(\frac{1+a+bx}{a+bx}\right)\right)}{2c}\\
&\quad-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left(\log(a+bx)-\log(1+a+bx)+\log\left(\frac{1+a+bx}{a+bx}\right)\right)}{2c^{3/2}}\\
&\quad+\frac{\sqrt{d}\log(-1+a+bx)\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{(1-a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\log(1+a+bx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{(1+a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}\\
&\quad+\frac{\sqrt{d}\log(1+a+bx)\log\left(-\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{(1+a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\log(-1+a+bx)\log\left(\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{(1-a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}\\
&\quad+\frac{\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(1-a-bx)}{(1-a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(1-a-bx)}{(1-a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}\\
&\quad+\frac{\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(1+a+bx)}{(1+a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(1+a+bx)}{(1+a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}
\end{aligned}$$

output
```
1/2*(-b*x-a+1)*ln(b*x+a-1)/b/c+1/2*x*(ln(b*x+a-1)-ln((b*x+a-1)/(b*x+a))-ln
(b*x+a))/c+1/2*(b*x+a+1)*ln(b*x+a+1)/b/c+1/2*x*(ln(b*x+a)-ln(b*x+a+1)+ln((
b*x+a+1)/(b*x+a)))/c-1/2*arctan(x*c^(1/2)/d^(1/2))*(ln(b*x+a-1)-ln((b*x+a-
1)/(b*x+a))-ln(b*x+a))*d^(1/2)/c^(3/2)-1/2*arctan(x*c^(1/2)/d^(1/2))*(ln(b
*x+a)-ln(b*x+a+1)+ln((b*x+a+1)/(b*x+a)))*d^(1/2)/c^(3/2)+1/4*ln(b*x+a-1)*l
n(-b*(-x*(-c)^(1/2)+d^(1/2))/((1-a)*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3
/2)+1/4*ln(b*x+a+1)*ln(-b*(x*(-c)^(1/2)+d^(1/2))/((1+a)*(-c)^(1/2)-b*d^(1/
2)))*d^(1/2)/(-c)^(3/2)-1/4*ln(b*x+a-1)*ln(b*(x*(-c)^(1/2)+d^(1/2))/((1-a)
*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/4*ln(b*x+a+1)*ln(b*(-x*(-c)^(
1/2)+d^(1/2))/((1+a)*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+1/4*polylog
(2,(-b*x-a+1)*(-c)^(1/2)/((1-a)*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+
1/4*polylog(2,(b*x+a+1)*(-c)^(1/2)/((1+a)*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(
-c)^(3/2)-1/4*polylog(2,(-b*x-a+1)*(-c)^(1/2)/((1-a)*(-c)^(1/2)+b*d^(1/2))
)*d^(1/2)/(-c)^(3/2)-1/4*polylog(2,(b*x+a+1)*(-c)^(1/2)/((1+a)*(-c)^(1/2)+
b*d^(1/2)))*d^(1/2)/(-c)^(3/2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 843, normalized size of antiderivative = 1.14

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx$$

$$= \frac{2c\log\left(\frac{-1+a+bx}{a+bx}\right)-2ac\log\left(\frac{-1+a+bx}{a+bx}\right)-2bcx\log\left(\frac{-1+a+bx}{a+bx}\right)+4c\log(a+bx)+2c\log\left(\frac{1+a+bx}{a+bx}\right)+2ac\log\left(\frac{1}{}\right.}{}$$

input
```
Integrate[ArcCoth[a + b*x]/(c + d/x^2),x]
```

output
```
(2*c*Log[(-1 + a + b*x)/(a + b*x)] - 2*a*c*Log[(-1 + a + b*x)/(a + b*x)] -
 2*b*c*x*Log[(-1 + a + b*x)/(a + b*x)] + 4*c*Log[a + b*x] + 2*c*Log[(1 + a
 + b*x)/(a + b*x)] + 2*a*c*Log[(1 + a + b*x)/(a + b*x)] + 2*b*c*x*Log[(1 +
 a + b*x)/(a + b*x)] - b*Sqrt[-c]*Sqrt[d]*Log[(Sqrt[-c]*(-1 + a + b*x))/(-
Sqrt[-c] + a*Sqrt[-c] + b*Sqrt[d])]*Log[Sqrt[d] - Sqrt[-c]*x] + b*Sqrt[-c]
*Sqrt[d]*Log[(-1 + a + b*x)/(a + b*x)]*Log[Sqrt[d] - Sqrt[-c]*x] + b*Sqrt[
-c]*Sqrt[d]*Log[(Sqrt[-c]*(1 + a + b*x))/((1 + a)*Sqrt[-c] + b*Sqrt[d])]*L
og[Sqrt[d] - Sqrt[-c]*x] - b*Sqrt[-c]*Sqrt[d]*Log[(1 + a + b*x)/(a + b*x)]
*Log[Sqrt[d] - Sqrt[-c]*x] + b*Sqrt[-c]*Sqrt[d]*Log[(Sqrt[-c]*(1 - a - b*x
))/(-((-1 + a)*Sqrt[-c]) + b*Sqrt[d])]*Log[Sqrt[d] + Sqrt[-c]*x] - b*Sqrt[
-c]*Sqrt[d]*Log[(-1 + a + b*x)/(a + b*x)]*Log[Sqrt[d] + Sqrt[-c]*x] - b*Sq
rt[-c]*Sqrt[d]*Log[(Sqrt[-c]*(1 + a + b*x))/((1 + a)*Sqrt[-c] - b*Sqrt[d])
]*Log[Sqrt[d] + Sqrt[-c]*x] + b*Sqrt[-c]*Sqrt[d]*Log[(1 + a + b*x)/(a + b*
x)]*Log[Sqrt[d] + Sqrt[-c]*x] - b*Sqrt[-c]*Sqrt[d]*PolyLog[2, (b*(Sqrt[d]
- Sqrt[-c]*x))/(-Sqrt[-c] + a*Sqrt[-c] + b*Sqrt[d])] + b*Sqrt[-c]*Sqrt[d]*
PolyLog[2, (b*(Sqrt[d] - Sqrt[-c]*x))/(Sqrt[-c] + a*Sqrt[-c] + b*Sqrt[d])]
 - b*Sqrt[-c]*Sqrt[d]*PolyLog[2, -((b*(Sqrt[d] + Sqrt[-c]*x))/(Sqrt[-c] +
a*Sqrt[-c] - b*Sqrt[d]))] + b*Sqrt[-c]*Sqrt[d]*PolyLog[2, (b*(Sqrt[d] + Sq
rt[-c]*x))/(Sqrt[-c] - a*Sqrt[-c] + b*Sqrt[d])])/(4*b*c^2)
```

3.79.3 Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 1165, normalized size of antiderivative = 1.58, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6666, 2993, 772, 262, 218, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{6666}$$

$$\frac{1}{2}\int \frac{\log\left(\frac{a+bx+1}{a+bx}\right)}{c+\frac{d}{x^2}}dx-\frac{1}{2}\int \frac{\log\left(-\frac{-a-bx+1}{a+bx}\right)}{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{2993}$$

$$\frac{1}{2}\left(\left(\log(-a-bx+1)-\log\left(-\frac{-a-bx+1}{a+bx}\right)-\log(a+bx)\right)\int\frac{1}{c+\frac{d}{x^2}}dx-\int\frac{\log(-a-bx+1)}{c+\frac{d}{x^2}}dx+\int\frac{\log(a-}{c+}\right.$$
$$\frac{1}{2}\left(\left(\log(a+bx)-\log(a+bx+1)+\log\left(\frac{a+bx+1}{a+bx}\right)\right)\int\frac{1}{c+\frac{d}{x^2}}dx-\int\frac{\log(a+bx)}{c+\frac{d}{x^2}}dx+\int\frac{\log(a+bx+1)}{c+\frac{d}{x^2}}dx\right.$$

$\downarrow$ 772

$$\frac{1}{2}\left(\left(\log(-a-bx+1)-\log\left(-\frac{-a-bx+1}{a+bx}\right)-\log(a+bx)\right)\int\frac{x^2}{cx^2+d}dx-\int\frac{\log(-a-bx+1)}{c+\frac{d}{x^2}}dx+\int\frac{\log(a}{c+}\right.$$
$$\frac{1}{2}\left(\left(\log(a+bx)-\log(a+bx+1)+\log\left(\frac{a+bx+1}{a+bx}\right)\right)\int\frac{x^2}{cx^2+d}dx-\int\frac{\log(a+bx)}{c+\frac{d}{x^2}}dx+\int\frac{\log(a+bx+1)}{c+\frac{d}{x^2}}d\right.$$

$\downarrow$ 262

$$\frac{1}{2}\left(\left(\log(-a-bx+1)-\log\left(-\frac{-a-bx+1}{a+bx}\right)-\log(a+bx)\right)\left(\frac{x}{c}-\frac{d\int\frac{1}{cx^2+d}dx}{c}\right)-\int\frac{\log(-a-bx+1)}{c+\frac{d}{x^2}}dx+\right.$$
$$\frac{1}{2}\left(\left(\log(a+bx)-\log(a+bx+1)+\log\left(\frac{a+bx+1}{a+bx}\right)\right)\left(\frac{x}{c}-\frac{d\int\frac{1}{cx^2+d}dx}{c}\right)-\int\frac{\log(a+bx)}{c+\frac{d}{x^2}}dx+\int\frac{\log(a+b}{c+}\right.$$

$\downarrow$ 218

$$\frac{1}{2}\left(-\int\frac{\log(-a-bx+1)}{c+\frac{d}{x^2}}dx+\int\frac{\log(a+bx)}{c+\frac{d}{x^2}}dx+\left(\log(-a-bx+1)-\log\left(-\frac{-a-bx+1}{a+bx}\right)-\log(a+bx)\right)\right($$
$$\frac{1}{2}\left(-\int\frac{\log(a+bx)}{c+\frac{d}{x^2}}dx+\int\frac{\log(a+bx+1)}{c+\frac{d}{x^2}}dx+\left(\log(a+bx)-\log(a+bx+1)+\log\left(\frac{a+bx+1}{a+bx}\right)\right)\left(\frac{x}{c}-\frac{\sqrt{}}{}\right.\right.$$

$\downarrow$ 2856

$$\frac{1}{2}\left(-\int\left(\frac{\log(-a-bx+1)}{c}-\frac{d\log(-a-bx+1)}{c\,(cx^2+d)}\right)dx+\int\left(\frac{\log(a+bx)}{c}-\frac{d\log(a+bx)}{c\,(cx^2+d)}\right)dx+\left(\log(-a-bx+\right.\right.$$
$$\frac{1}{2}\left(-\int\left(\frac{\log(a+bx)}{c}-\frac{d\log(a+bx)}{c\,(cx^2+d)}\right)dx+\int\left(\frac{\log(a+bx+1)}{c}-\frac{d\log(a+bx+1)}{c\,(cx^2+d)}\right)dx+\left(\log(a+bx)-\log\right.\right.$$

$\downarrow$ 2009

3.79. $\int\frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx$

$$\frac{1}{2}\left(\frac{(-a-bx+1)\log(-a-bx+1)}{bc}+\frac{\sqrt{d}\log\left(-\frac{b(\sqrt{d}-\sqrt{-c}x)}{(1-a)\sqrt{-c}-b\sqrt{d}}\right)\log(-a-bx+1)}{2(-c)^{3/2}}-\frac{\sqrt{d}\log\left(\frac{b(\sqrt{-c}x+\sqrt{d})}{\sqrt{-c}(1-a)+b\sqrt{d}}\right)\log\right.$$

$$\frac{1}{2}\left(-\frac{(a+bx)\log(a+bx)}{bc}+\frac{\sqrt{d}\log\left(\frac{b(\sqrt{d}-\sqrt{-c}x)}{\sqrt{-c}a+b\sqrt{d}}\right)\log(a+bx)}{2(-c)^{3/2}}-\frac{\sqrt{d}\log\left(-\frac{b(\sqrt{-c}x+\sqrt{d})}{a\sqrt{-c}-b\sqrt{d}}\right)\log(a+bx)}{2(-c)^{3/2}}+\frac{(a+bx\right.$$

input
```
Int[ArcCoth[a + b*x]/(c + d/x^2),x]
```

output
```
(((1 - a - b*x)*Log[1 - a - b*x])/(b*c) + (x/c - (Sqrt[d]*ArcTan[(Sqrt[c]*
x)/Sqrt[d]])/c^(3/2))*(Log[1 - a - b*x] - Log[-((1 - a - b*x)/(a + b*x))]
- Log[a + b*x]) + ((a + b*x)*Log[a + b*x])/(b*c) + (Sqrt[d]*Log[1 - a - b*
x]*Log[-((b*(Sqrt[d] - Sqrt[-c]*x))/((1 - a)*Sqrt[-c] - b*Sqrt[d]))])/(2*(
-c)^(3/2)) - (Sqrt[d]*Log[a + b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/(a*Sqrt[
-c] + b*Sqrt[d])])/(2*(-c)^(3/2)) + (Sqrt[d]*Log[a + b*x]*Log[-((b*(Sqrt[d
] + Sqrt[-c]*x))/(a*Sqrt[-c] - b*Sqrt[d]))])/(2*(-c)^(3/2)) - (Sqrt[d]*Log
[1 - a - b*x]*Log[(b*(Sqrt[d] + Sqrt[-c]*x))/((1 - a)*Sqrt[-c] + b*Sqrt[d]
)])/(2*(-c)^(3/2)) + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 - a - b*x))/(Sqrt[-c
] - a*Sqrt[-c] - b*Sqrt[d])])/(2*(-c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-
c]*(1 - a - b*x))/((1 - a)*Sqrt[-c] + b*Sqrt[d])])/(2*(-c)^(3/2)) + (Sqrt[
d]*PolyLog[2, (Sqrt[-c]*(a + b*x))/(a*Sqrt[-c] - b*Sqrt[d])])/(2*(-c)^(3/2
)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(a + b*x))/(a*Sqrt[-c] + b*Sqrt[d])])/(
2*(-c)^(3/2)))/2 + (-(((a + b*x)*Log[a + b*x])/(b*c)) + ((1 + a + b*x)*Log
[1 + a + b*x])/(b*c) + (x/c - (Sqrt[d]*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/c^(3/2
))*(Log[a + b*x] - Log[1 + a + b*x] + Log[(1 + a + b*x)/(a + b*x)]) + (Sqr
t[d]*Log[a + b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/(a*Sqrt[-c] + b*Sqrt[d])]
)/(2*(-c)^(3/2)) - (Sqrt[d]*Log[1 + a + b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x)
)/((1 + a)*Sqrt[-c] + b*Sqrt[d])])/(2*(-c)^(3/2)) - (Sqrt[d]*Log[a + b*x]*
Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/(a*Sqrt[-c] - b*Sqrt[d]))])/(2*(-c)^(3...
```

3.79. $\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx$

3.79.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 2993
```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Simp[p*r   Int[RFx*Log[a + b*x], x], x] + (Si
mp[q*r   Int[RFx*Log[c + d*x], x], x] - Simp[(p*r*Log[a + b*x] + q*r*Log[c
+ d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])   Int[RFx, x], x]) /; FreeQ[
{a, b, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a
*d, 0] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[
m, n]]
```

rule 6666
```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Simp
[1/2   Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Simp[1/2   In
t[Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f},
x] && RationalQ[n]
```

3.79.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 554, normalized size of antiderivative = 0.75

method	result
risch	$\frac{\ln(bx+a+1)x}{2c}+\frac{\ln(bx+a+1)a}{2bc}+\frac{\ln(bx+a+1)}{2bc}-\frac{1}{bc}-\frac{d\ln(bx+a+1)\ln\left(\frac{b\sqrt{-cd}-(bx+a+1)c+ac+c}{b\sqrt{-cd}+ac+c}\right)}{4c\sqrt{-cd}}+\frac{d\ln(bx+a+$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arccoth(b*x+a)/(c+d/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/c*ln(b*x+a+1)*x+1/2/b/c*ln(b*x+a+1)*a+1/2/b/c*ln(b*x+a+1)-1/b/c-1/4*d/
c*ln(b*x+a+1)/(-c*d)^(1/2)*ln((b*(-c*d)^(1/2)-(b*x+a+1)*c+a*c+c)/(b*(-c*d)
^(1/2)+a*c+c))+1/4*d/c*ln(b*x+a+1)/(-c*d)^(1/2)*ln((b*(-c*d)^(1/2)+(b*x+a+
1)*c-a*c-c)/(b*(-c*d)^(1/2)-a*c-c))-1/4*d/c/(-c*d)^(1/2)*dilog((b*(-c*d)^(
1/2)-(b*x+a+1)*c+a*c+c)/(b*(-c*d)^(1/2)+a*c+c))+1/4*d/c/(-c*d)^(1/2)*dilog
((b*(-c*d)^(1/2)+(b*x+a+1)*c-a*c-c)/(b*(-c*d)^(1/2)-a*c-c))-1/2/c*ln(b*x+a
-1)*x-1/2/b/c*ln(b*x+a-1)*a+1/2/b/c*ln(b*x+a-1)+1/4*d/c*ln(b*x+a-1)/(-c*d)
^(1/2)*ln((b*(-c*d)^(1/2)-(b*x+a-1)*c+a*c-c)/(b*(-c*d)^(1/2)+a*c-c))-1/4*d
/c*ln(b*x+a-1)/(-c*d)^(1/2)*ln((b*(-c*d)^(1/2)+(b*x+a-1)*c-a*c+c)/(b*(-c*d
)^(1/2)-a*c+c))+1/4*d/c/(-c*d)^(1/2)*dilog((b*(-c*d)^(1/2)-(b*x+a-1)*c+a*c
-c)/(b*(-c*d)^(1/2)+a*c-c))-1/4*d/c/(-c*d)^(1/2)*dilog((b*(-c*d)^(1/2)+(b*
x+a-1)*c-a*c+c)/(b*(-c*d)^(1/2)-a*c+c))
```

3.79.5 Fricas [F]

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx=\int \frac{\operatorname{arcoth}(bx+a)}{c+\frac{d}{x^2}}\,dx$$

input
```
integrate(arccoth(b*x+a)/(c+d/x^2),x, algorithm="fricas")
```

output
```
integral(x^2*arccoth(b*x + a)/(c*x^2 + d), x)
```

3.79.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \text{Timed out}$$

input
```
integrate(acoth(b*x+a)/(c+d/x**2),x)
```

output
```
Timed out
```

3.79.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 651, normalized size of antiderivative = 0.88

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = -\left(\frac{d\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}c} - \frac{x}{c}\right)\operatorname{arcoth}(bx+a) + \frac{2(a+1)c\log(bx+a+1) - 2(a-1)c\log(bx+a-1) + \left(b\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\log\left(\frac{b^2cx^2+2(a+1)bcx+(a^2+2a+1}{b^2d+(a^2+2a+1)c}\right.\right.}{}$$

input
```
integrate(arccoth(b*x+a)/(c+d/x^2),x, algorithm="maxima")
```

output
```
-(d*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c) - x/c)*arccoth(b*x + a) + 1/4*(2*(
a + 1)*c*log(b*x + a + 1) - 2*(a - 1)*c*log(b*x + a - 1) + (b*arctan(sqrt(
c)*x/sqrt(d))*log((b^2*c*x^2 + 2*(a + 1)*b*c*x + (a^2 + 2*a + 1)*c)/(b^2*d
 + (a^2 + 2*a + 1)*c)) - b*arctan(sqrt(c)*x/sqrt(d))*log((b^2*c*x^2 + 2*(a
 - 1)*b*c*x + (a^2 - 2*a + 1)*c)/(b^2*d + (a^2 - 2*a + 1)*c)) + I*b*dilog(
((a - 1)*b*c*x + b^2*d + (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(2*(-I*
a + I)*b*sqrt(c)*sqrt(d) + b^2*d - (a^2 - 2*a + 1)*c)) - I*b*dilog(-((a -
1)*b*c*x + b^2*d - (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(2*(-I*a + I)
*b*sqrt(c)*sqrt(d) - b^2*d + (a^2 - 2*a + 1)*c)) - I*b*dilog(((a + 1)*b*c*
x + b^2*d + (I*b^2*x + (-I*a - I)*b)*sqrt(c)*sqrt(d))/(2*(-I*a - I)*b*sqrt
(c)*sqrt(d) + b^2*d - (a^2 + 2*a + 1)*c)) + I*b*dilog(-((a + 1)*b*c*x + b^
2*d - (I*b^2*x + (-I*a - I)*b)*sqrt(c)*sqrt(d))/(2*(-I*a - I)*b*sqrt(c)*sq
rt(d) - b^2*d + (a^2 + 2*a + 1)*c)) - (b*arctan2((b^2*x + (a + 1)*b)*sqrt(
c)*sqrt(d)/(b^2*d + (a^2 + 2*a + 1)*c), ((a + 1)*b*c*x + (a^2 + 2*a + 1)*c
)/(b^2*d + (a^2 + 2*a + 1)*c)) - b*arctan2((b^2*x + (a - 1)*b)*sqrt(c)*sqr
t(d)/(b^2*d + (a^2 - 2*a + 1)*c), ((a - 1)*b*c*x + (a^2 - 2*a + 1)*c)/(b^2
*d + (a^2 - 2*a + 1)*c)))*log(c*x^2 + d))*sqrt(c)*sqrt(d))/(b*c^2)
```

3.79.8 Giac [F]

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx=\int \frac{\operatorname{arcoth}(bx+a)}{c+\frac{d}{x^2}}\,dx$$

input
```
integrate(arccoth(b*x+a)/(c+d/x^2),x, algorithm="giac")
```

output
```
integrate(arccoth(b*x + a)/(c + d/x^2), x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx=\int \frac{\operatorname{acoth}(a+b\,x)}{c+\frac{d}{x^2}}\,dx$$

input
```
int(acoth(a + b*x)/(c + d/x^2),x)
```

output
```
int(acoth(a + b*x)/(c + d/x^2), x)
```

3.80 $\int \frac{\coth^{-1}(a+bx)}{c+d\sqrt{x}}\,dx$

3.80.1 Optimal result

Integrand size = 18, antiderivative size = 619

$$\begin{aligned}\int \frac{\coth^{-1}(a+bx)}{c+d\sqrt{x}}\,dx =& \frac{2\sqrt{1+a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}d}-\frac{2\sqrt{1-a}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d}\\
&+\frac{c\log\left(\frac{d\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-1-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&-\frac{c\log\left(\frac{d\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{1-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&+\frac{c\log\left(-\frac{d\left(\sqrt{-1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-1-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&-\frac{c\log\left(-\frac{d\left(\sqrt{1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{1-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&-\frac{\sqrt{x}\log\left(-\frac{1-a-bx}{a+bx}\right)}{d}+\frac{c\log\left(c+d\sqrt{x}\right)\log\left(-\frac{1-a-bx}{a+bx}\right)}{d^2}\\
&+\frac{\sqrt{x}\log\left(\frac{1+a+bx}{a+bx}\right)}{d}-\frac{c\log\left(c+d\sqrt{x}\right)\log\left(\frac{1+a+bx}{a+bx}\right)}{d^2}\\
&+\frac{c\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-1-a}d}\right)}{d^2}+\frac{c\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-1-a}d}\right)}{d^2}\\
&-\frac{c\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{1-a}d}\right)}{d^2}-\frac{c\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{1-a}d}\right)}{d^2}\end{aligned}$$

output
```
c*ln((b*x+a-1)/(b*x+a))*ln(c+d*x^(1/2))/d^2-c*ln((b*x+a+1)/(b*x+a))*ln(c+d
*x^(1/2))/d^2+c*ln(c+d*x^(1/2))*ln(d*((-1-a)^(1/2)-b^(1/2)*x^(1/2))/(d*(-1
-a)^(1/2)+c*b^(1/2)))/d^2-c*ln(c+d*x^(1/2))*ln(d*((1-a)^(1/2)-b^(1/2)*x^(1
/2))/(d*(1-a)^(1/2)+c*b^(1/2)))/d^2+c*ln(c+d*x^(1/2))*ln(-d*((-1-a)^(1/2)+
b^(1/2)*x^(1/2))/(-d*(-1-a)^(1/2)+c*b^(1/2)))/d^2-c*ln(c+d*x^(1/2))*ln(-d*
((1-a)^(1/2)+b^(1/2)*x^(1/2))/(-d*(1-a)^(1/2)+c*b^(1/2)))/d^2+c*polylog(2,
b^(1/2)*(c+d*x^(1/2))/(-d*(-1-a)^(1/2)+c*b^(1/2)))/d^2+c*polylog(2,b^(1/2)
*(c+d*x^(1/2))/(d*(-1-a)^(1/2)+c*b^(1/2)))/d^2-c*polylog(2,b^(1/2)*(c+d*x^
(1/2))/(-d*(1-a)^(1/2)+c*b^(1/2)))/d^2-c*polylog(2,b^(1/2)*(c+d*x^(1/2))/(
d*(1-a)^(1/2)+c*b^(1/2)))/d^2-2*arctanh(b^(1/2)*x^(1/2)/(1-a)^(1/2))*(1-a)
^(1/2)/d/b^(1/2)+2*arctan(b^(1/2)*x^(1/2)/(1+a)^(1/2))*(1+a)^(1/2)/d/b^(1/
2)-ln((b*x+a-1)/(b*x+a))*x^(1/2)/d+ln((b*x+a+1)/(b*x+a))*x^(1/2)/d
```

3.80.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 575, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \frac{\frac{2\sqrt{1+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}} - \frac{2\sqrt{1-a}d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}} + c\log\left(\frac{d\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-1-a}d}\right)\log\left(c+d\sqrt{x}\right) - c\log\left(\frac{d\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{1-a}d}\right.}{}$$

input
```
Integrate[ArcCoth[a + b*x]/(c + d*Sqrt[x]),x]
```

output
```
((2*Sqrt[1 + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/Sqrt[b] - (2*Sqrt
[1 - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/Sqrt[b] + c*Log[(d*(Sqrt
[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)]*Log[c + d*Sqrt[
x]] - c*Log[(d*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1 - a]*d
)]*Log[c + d*Sqrt[x]] + c*Log[(d*(Sqrt[-1 - a] + Sqrt[b]*Sqrt[x]))/(-(Sqrt
[b]*c) + Sqrt[-1 - a]*d)]*Log[c + d*Sqrt[x]] - c*Log[(d*(Sqrt[1 - a] + Sqr
t[b]*Sqrt[x]))/(-(Sqrt[b]*c) + Sqrt[1 - a]*d)]*Log[c + d*Sqrt[x]] - d*Sqrt
[x]*Log[(-1 + a + b*x)/(a + b*x)] + c*Log[c + d*Sqrt[x]]*Log[(-1 + a + b*x
)/(a + b*x)] + d*Sqrt[x]*Log[(1 + a + b*x)/(a + b*x)] - c*Log[c + d*Sqrt[x
]]*Log[(1 + a + b*x)/(a + b*x)] + c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(
Sqrt[b]*c - Sqrt[-1 - a]*d)] + c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqr
t[b]*c + Sqrt[-1 - a]*d)] - c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b
]*c - Sqrt[1 - a]*d)] - c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c
+ Sqrt[1 - a]*d)])/d^2
```

3.80.3 Rubi [A] (verified)

Time = 2.37 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6666, 7267, 3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(a+bx)}{c+d\sqrt{x}}dx$$

$$\Big\downarrow \text{6666}$$

$$\frac{1}{2}\int \frac{\log\left(\frac{a+bx+1}{a+bx}\right)}{c+d\sqrt{x}}dx-\frac{1}{2}\int \frac{\log\left(-\frac{-a-bx+1}{a+bx}\right)}{c+d\sqrt{x}}dx$$

$$\Big\downarrow \text{7267}$$

$$\int \frac{\sqrt{x}\log\left(\frac{a+bx+1}{a+bx}\right)}{c+d\sqrt{x}}d\sqrt{x}-\int \frac{\sqrt{x}\log\left(-\frac{-a-bx+1}{a+bx}\right)}{c+d\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{3008}$$

$$\int \left(\frac{\log\left(\frac{a+bx+1}{a+bx}\right)}{d}-\frac{c\log\left(\frac{a+bx+1}{a+bx}\right)}{d\left(c+d\sqrt{x}\right)}\right)d\sqrt{x}-\int \left(\frac{\log\left(-\frac{-a-bx+1}{a+bx}\right)}{d}-\frac{c\log\left(-\frac{-a-bx+1}{a+bx}\right)}{d\left(c+d\sqrt{x}\right)}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{a+1}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+1}}\right)}{\sqrt{b}d}-\frac{2\sqrt{1-a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d}+\frac{c\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-a-1}d}\right)}{d^2}+\frac{c\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-a-1}d}\right)}{d^2}-\frac{c\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{1-a}d}\right)}{d^2}-\frac{c\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{1-a}d}\right)}{d^2}+\frac{c\log\left(c+d\sqrt{x}\right)\log\left(\frac{d\left(\sqrt{-a-1}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-a-1}d+\sqrt{b}c}\right)}{d^2}-\frac{c\log\left(c+d\sqrt{x}\right)\log\left(\frac{d\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}d+\sqrt{b}c}\right)}{d^2}+\frac{c\log\left(c+d\sqrt{x}\right)\log\left(-\frac{d\left(\sqrt{-a-1}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-a-1}d}\right)}{d^2}-\frac{c\log\left(c+d\sqrt{x}\right)\log\left(-\frac{d\left(\sqrt{1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{1-a}d}\right)}{d^2}+\frac{c\log\left(-\frac{-a-bx+1}{a+bx}\right)\log\left(c+d\sqrt{x}\right)}{d^2}-\frac{c\log\left(\frac{a+bx+1}{a+bx}\right)\log\left(c+d\sqrt{x}\right)}{d^2}-\frac{\sqrt{x}\log\left(-\frac{-a-bx+1}{a+bx}\right)}{d}+\frac{\sqrt{x}\log\left(\frac{a+bx+1}{a+bx}\right)}{d}$$

input
```
Int[ArcCoth[a + b*x]/(c + d*Sqrt[x]),x]
```

output
```
(2*Sqrt[1 + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/(Sqrt[b]*d) - (2*Sqr
t[1 - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/(Sqrt[b]*d) + (c*Log[(d*(
Sqrt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)]*Log[c + d*S
qrt[x]])/d^2 - (c*Log[(d*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqr
t[1 - a]*d)]*Log[c + d*Sqrt[x]])/d^2 + (c*Log[-((d*(Sqrt[-1 - a] + Sqrt[b]
*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-1 - a]*d))]*Log[c + d*Sqrt[x]])/d^2 - (c*Log
[-((d*(Sqrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[1 - a]*d))]*Log[c
 + d*Sqrt[x]])/d^2 - (Sqrt[x]*Log[-((1 - a - b*x)/(a + b*x))])/d + (c*Log[
c + d*Sqrt[x]]*Log[-((1 - a - b*x)/(a + b*x))])/d^2 + (Sqrt[x]*Log[(1 + a
+ b*x)/(a + b*x)])/d - (c*Log[c + d*Sqrt[x]]*Log[(1 + a + b*x)/(a + b*x)])
/d^2 + (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-1 - a]*d
)])/d^2 + (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a
]*d)])/d^2 - (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[1 -
 a]*d)])/d^2 - (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1
 - a]*d)])/d^2
```

3.80.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3008
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

rule 6666
```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Simp
[1/2   Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Simp[1/2   In
t[Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f},
x] && RationalQ[n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.80.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{2\,\text{arccoth}(bx+a)\sqrt{x}}{d}-\frac{2\,\text{arccoth}(bx+a)c\ln\left(c+d\sqrt{x}\right)}{d^{2}}+\frac{4b\left(d^{2}\left(\frac{(1+a)\arctan\left(\frac{-2bc+2b(c+d\sqrt{x})}{2\sqrt{ab\,d^{2}+b\,d^{2}}}\right)}{2b\sqrt{ab\,d^{2}+b\,d^{2}}}+\frac{(1-a)\arctan\left(\frac{-2bc}{2\sqrt{}}\right.}{2b\sqrt{ab\,d^{2}}}\right.\right.}{}$
default	$\frac{2\,\text{arccoth}(bx+a)\sqrt{x}}{d}-\frac{2\,\text{arccoth}(bx+a)c\ln\left(c+d\sqrt{x}\right)}{d^{2}}+\frac{4b\left(d^{2}\left(\frac{(1+a)\arctan\left(\frac{-2bc+2b(c+d\sqrt{x})}{2\sqrt{ab\,d^{2}+b\,d^{2}}}\right)}{2b\sqrt{ab\,d^{2}+b\,d^{2}}}+\frac{(1-a)\arctan\left(\frac{-2bc}{2\sqrt{}}\right.}{2b\sqrt{ab\,d^{2}}}\right.\right.}{}$

input
```
int(arccoth(b*x+a)/(c+d*x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*arccoth(b*x+a)/d*x^(1/2)-2*arccoth(b*x+a)*c/d^2*ln(c+d*x^(1/2))+4*b/d^2*
(d^2*(1/2*(1+a)/b/(a*b*d^2+b*d^2)^(1/2)*arctan(1/2*(-2*b*c+2*b*(c+d*x^(1/2
)))/(a*b*d^2+b*d^2)^(1/2))+1/2*(1-a)/b/(a*b*d^2-b*d^2)^(1/2)*arctan(1/2*(-
2*b*c+2*b*(c+d*x^(1/2)))/(a*b*d^2-b*d^2)^(1/2)))+c*d^2*(1/2/d^2*(1/2*ln(c+
d*x^(1/2))*(ln((b*c-b*(c+d*x^(1/2))+(-a*b*d^2-b*d^2)^(1/2))/(b*c+(-a*b*d^2
-b*d^2)^(1/2)))+ln((-b*c+b*(c+d*x^(1/2))+(-a*b*d^2-b*d^2)^(1/2))/(-b*c+(-a
*b*d^2-b*d^2)^(1/2))))/b+1/2*(dilog((b*c-b*(c+d*x^(1/2))+(-a*b*d^2-b*d^2)^
(1/2))/(b*c+(-a*b*d^2-b*d^2)^(1/2)))+dilog((-b*c+b*(c+d*x^(1/2))+(-a*b*d^2
-b*d^2)^(1/2))/(-b*c+(-a*b*d^2-b*d^2)^(1/2))))/b)+1/2/d^2*(-1/2*ln(c+d*x^(
1/2))*(ln((b*c-b*(c+d*x^(1/2))+(-a*b*d^2+b*d^2)^(1/2))/(b*c+(-a*b*d^2+b*d^
2)^(1/2)))+ln((-b*c+b*(c+d*x^(1/2))+(-a*b*d^2+b*d^2)^(1/2))/(-b*c+(-a*b*d^
2+b*d^2)^(1/2))))/b-1/2*(dilog((b*c-b*(c+d*x^(1/2))+(-a*b*d^2+b*d^2)^(1/2)
)/(b*c+(-a*b*d^2+b*d^2)^(1/2)))+dilog((-b*c+b*(c+d*x^(1/2))+(-a*b*d^2+b*d^
2)^(1/2))/(-b*c+(-a*b*d^2+b*d^2)^(1/2))))/b)))
```

3.80.5 Fricas [F]

$$\int \frac{\coth^{-1}(a+bx)}{c+d\sqrt{x}}\,dx=\int \frac{\text{arcoth}\,(bx+a)}{d\sqrt{x}+c}\,dx$$

input
```
integrate(arccoth(b*x+a)/(c+d*x^(1/2)),x, algorithm="fricas")
```

output
```
integral((d*sqrt(x)*arccoth(b*x + a) - c*arccoth(b*x + a))/(d^2*x - c^2),
x)
```

3.80.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \text{Timed out}$$

input
```
integrate(acoth(b*x+a)/(c+d*x**(1/2)),x)
```

output
```
Timed out
```

3.80.7 Maxima [F]

$$\int \frac{\coth^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\operatorname{arcoth}(bx+a)}{d\sqrt{x}+c}\,dx$$

input
```
integrate(arccoth(b*x+a)/(c+d*x^(1/2)),x, algorithm="maxima")
```

output
```
integrate(arccoth(b*x + a)/(d*sqrt(x) + c), x)
```

3.80.8 Giac [F]

$$\int \frac{\coth^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\operatorname{arcoth}(bx+a)}{d\sqrt{x}+c}\,dx$$

input
```
integrate(arccoth(b*x+a)/(c+d*x^(1/2)),x, algorithm="giac")
```

output
```
integrate(arccoth(b*x + a)/(d*sqrt(x) + c), x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\operatorname{acoth}(a+b\,x)}{c+d\,\sqrt{x}}\,dx$$

input
```
int(acoth(a + b*x)/(c + d*x^(1/2)),x)
```

output
```
int(acoth(a + b*x)/(c + d*x^(1/2)), x)
```

3.81 $\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$

3.81.1 Optimal result

Integrand size = 18, antiderivative size = 738

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = -\frac{2\sqrt{1+a}\,d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}c^2} + \frac{2\sqrt{1-a}\,d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2}$$
$$-\frac{d^2\log\left(\frac{c\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-1-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$+\frac{d^2\log\left(\frac{c\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$-\frac{d^2\log\left(\frac{c\left(\sqrt{-1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-1-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$+\frac{d^2\log\left(\frac{c\left(\sqrt{1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} + \frac{(1-a)\log(1-a-bx)}{2bc}$$
$$+\frac{d\sqrt{x}\log\left(-\frac{1-a-bx}{a+bx}\right)}{c^2} - \frac{x\log\left(-\frac{1-a-bx}{a+bx}\right)}{2c}$$
$$-\frac{d^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{1-a-bx}{a+bx}\right)}{c^3} + \frac{(1+a)\log(1+a+bx)}{2bc}$$
$$-\frac{d\sqrt{x}\log\left(\frac{1+a+bx}{a+bx}\right)}{c^2} + \frac{x\log\left(\frac{1+a+bx}{a+bx}\right)}{2c} + \frac{d^2\log\left(d+c\sqrt{x}\right)\log\left(\frac{1+a+bx}{a+bx}\right)}{c^3}$$
$$-\frac{d^2\,\text{PolyLog}\left(2,-\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-1-a}c-\sqrt{b}d}\right)}{c^3} + \frac{d^2\,\text{PolyLog}\left(2,-\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{1-a}c-\sqrt{b}d}\right)}{c^3}$$
$$-\frac{d^2\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-1-a}c+\sqrt{b}d}\right)}{c^3} + \frac{d^2\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{1-a}c+\sqrt{b}d}\right)}{c^3}$$

output
```
1/2*(1-a)*ln(-b*x-a+1)/b/c-1/2*x*ln((b*x+a-1)/(b*x+a))/c+1/2*(1+a)*ln(b*x+
a+1)/b/c+1/2*x*ln((b*x+a+1)/(b*x+a))/c-d^2*ln((b*x+a-1)/(b*x+a))*ln(d+c*x^
(1/2))/c^3+d^2*ln((b*x+a+1)/(b*x+a))*ln(d+c*x^(1/2))/c^3-d^2*ln(d+c*x^(1/2
))*ln(c*((-1-a)^(1/2)-b^(1/2)*x^(1/2))/(c*(-1-a)^(1/2)+d*b^(1/2)))/c^3+d^2
*ln(d+c*x^(1/2))*ln(c*((1-a)^(1/2)-b^(1/2)*x^(1/2))/(c*(1-a)^(1/2)+d*b^(1/
2)))/c^3-d^2*ln(d+c*x^(1/2))*ln(c*((-1-a)^(1/2)+b^(1/2)*x^(1/2))/(c*(-1-a)
^(1/2)-d*b^(1/2)))/c^3+d^2*ln(d+c*x^(1/2))*ln(c*((1-a)^(1/2)+b^(1/2)*x^(1/
2))/(c*(1-a)^(1/2)-d*b^(1/2)))/c^3-d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c
*(-1-a)^(1/2)-d*b^(1/2)))/c^3+d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c*(1-a
)^(1/2)-d*b^(1/2)))/c^3-d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(-1-a)^(1/2
)+d*b^(1/2)))/c^3+d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(1-a)^(1/2)+d*b^(
1/2)))/c^3+2*d*arctanh(b^(1/2)*x^(1/2)/(1-a)^(1/2))*(1-a)^(1/2)/c^2/b^(1/2
)-2*d*arctan(b^(1/2)*x^(1/2)/(1+a)^(1/2))*(1+a)^(1/2)/c^2/b^(1/2)+d*ln((b*
x+a-1)/(b*x+a))*x^(1/2)/c^2-d*ln((b*x+a+1)/(b*x+a))*x^(1/2)/c^2
```

3.81.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 719, normalized size of antiderivative = 0.97

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$$

$$= \frac{-4\sqrt{1+a}\sqrt{b}cd\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)+4\sqrt{1-a}\sqrt{b}cd\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)-2bd^2\log\left(\frac{c\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-1-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{\ldots}\right.}{}$$

input
```
Integrate[ArcCoth[a + b*x]/(c + d/Sqrt[x]),x]
```

output
```
(-4*Sqrt[1 + a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]] + 4*Sqrt
[1 - a]*Sqrt[b]*c*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]] - 2*b*d^2*Log[(
c*(Sqrt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)]*Log[d +
c*Sqrt[x]] + 2*b*d^2*Log[(c*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[1 - a]*
c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]] - 2*b*d^2*Log[(c*(Sqrt[-1 - a] + Sqrt[b
]*Sqrt[x]))/(Sqrt[-1 - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]] + 2*b*d^2*Log
[(c*(Sqrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[1 - a]*c - Sqrt[b]*d)]*Log[d +
c*Sqrt[x]] + c^2*Log[1 - a - b*x] - a*c^2*Log[1 - a - b*x] + 2*b*c*d*Sqrt[
x]*Log[(-1 + a + b*x)/(a + b*x)] - b*c^2*x*Log[(-1 + a + b*x)/(a + b*x)] -
 2*b*d^2*Log[d + c*Sqrt[x]]*Log[(-1 + a + b*x)/(a + b*x)] + c^2*Log[1 + a
+ b*x] + a*c^2*Log[1 + a + b*x] - 2*b*c*d*Sqrt[x]*Log[(1 + a + b*x)/(a + b
*x)] + b*c^2*x*Log[(1 + a + b*x)/(a + b*x)] + 2*b*d^2*Log[d + c*Sqrt[x]]*L
og[(1 + a + b*x)/(a + b*x)] - 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))
/(-(Sqrt[-1 - a]*c) + Sqrt[b]*d)] - 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqr
t[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)] + 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*
Sqrt[x]))/(-(Sqrt[1 - a]*c) + Sqrt[b]*d)] + 2*b*d^2*PolyLog[2, (Sqrt[b]*(d
 + c*Sqrt[x]))/(Sqrt[1 - a]*c + Sqrt[b]*d)])/(2*b*c^3)
```

3.81.3 Rubi [A] (verified)

Time = 2.67 (sec) , antiderivative size = 738, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6666, 7267, 3008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}} \, dx$$

$$\Big\downarrow \text{6666}$$

$$\frac{1}{2}\int \frac{\log\left(\frac{a+bx+1}{a+bx}\right)}{c+\frac{d}{\sqrt{x}}}dx-\frac{1}{2}\int \frac{\log\left(-\frac{-a-bx+1}{a+bx}\right)}{c+\frac{d}{\sqrt{x}}}dx$$

$$\Big\downarrow \text{7267}$$

$$\int \frac{x\log\left(\frac{a+bx+1}{a+bx}\right)}{\sqrt{x}c+d}d\sqrt{x}-\int \frac{x\log\left(-\frac{-a-bx+1}{a+bx}\right)}{\sqrt{x}c+d}d\sqrt{x}$$

$$\Big\downarrow \text{3008}$$

$$\int \left(\frac{\log\left(\frac{a+bx+1}{a+bx}\right)d^2}{c^2\left(\sqrt{x}c+d\right)}-\frac{\log\left(\frac{a+bx+1}{a+bx}\right)d}{c^2}+\frac{\sqrt{x}\log\left(\frac{a+bx+1}{a+bx}\right)}{c}\right)d\sqrt{x}-\int \left(\frac{\log\left(-\frac{-a-bx+1}{a+bx}\right)d^2}{c^2\left(\sqrt{x}c+d\right)}-\frac{\log\left(-\frac{-a-bx+1}{a+bx}\right)d}{c^2}+\frac{\sqrt{x}\log\left(-\frac{-a-bx+1}{a+bx}\right)}{c}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\sqrt{a+1}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+1}}\right)}{\sqrt{b}c^2}+\frac{2\sqrt{1-a}d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2}-\frac{d^2\text{PolyLog}\left(2,-\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{-a-1}c-\sqrt{b}d}\right)}{c^3}+\frac{d^2\text{PolyLog}\left(2,-\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{1-a}c-\sqrt{b}d}\right)}{c^3}-\frac{d^2\text{PolyLog}\left(2,\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{-a-1}c+\sqrt{b}d}\right)}{c^3}+\frac{d^2\text{PolyLog}\left(2,\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{1-a}c+\sqrt{b}d}\right)}{c^3}-\frac{d^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{-a-1}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-a-1}c+\sqrt{b}d}\right)}{c^3}+\frac{d^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}c+\sqrt{b}d}\right)}{c^3}-\frac{d^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{-a-1}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-a-1}c-\sqrt{b}d}\right)}{c^3}+\frac{d^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}c-\sqrt{b}d}\right)}{c^3}-\frac{d^2\log\left(-\frac{-a-bx+1}{a+bx}\right)\log\left(c\sqrt{x}+d\right)}{c^3}+\frac{d^2\log\left(\frac{a+bx+1}{a+bx}\right)\log\left(c\sqrt{x}+d\right)}{c^3}+\frac{d\sqrt{x}\log\left(-\frac{-a-bx+1}{a+bx}\right)}{c^2}-\frac{d\sqrt{x}\log\left(\frac{a+bx+1}{a+bx}\right)}{c^2}+\frac{(1-a)\log(-a-bx+1)}{2bc}-\frac{x\log\left(-\frac{-a-bx+1}{a+bx}\right)}{2c}+\frac{(a+1)\log(a+bx+1)}{2bc}+\frac{x\log\left(\frac{a+bx+1}{a+bx}\right)}{2c}$$

input
```
Int[ArcCoth[a + b*x]/(c + d/Sqrt[x]),x]
```

3.81. $\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$

output
```
(-2*Sqrt[1 + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/(Sqrt[b]*c^2) + (
2*Sqrt[1 - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/(Sqrt[b]*c^2) - (d
^2*Log[(c*(Sqrt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)]*
Log[d + c*Sqrt[x]])/c^3 + (d^2*Log[(c*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sq
rt[1 - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (d^2*Log[(c*(Sqrt[-1 -
 a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/
c^3 + (d^2*Log[(c*(Sqrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[1 - a]*c - Sqrt[b
]*d)]*Log[d + c*Sqrt[x]])/c^3 + ((1 - a)*Log[1 - a - b*x])/(2*b*c) + (d*Sq
rt[x]*Log[-((1 - a - b*x)/(a + b*x))])/c^2 - (x*Log[-((1 - a - b*x)/(a + b
*x))])/(2*c) - (d^2*Log[d + c*Sqrt[x]]*Log[-((1 - a - b*x)/(a + b*x))])/c^
3 + ((1 + a)*Log[1 + a + b*x])/(2*b*c) - (d*Sqrt[x]*Log[(1 + a + b*x)/(a +
 b*x)])/c^2 + (x*Log[(1 + a + b*x)/(a + b*x)])/(2*c) + (d^2*Log[d + c*Sqrt
[x]]*Log[(1 + a + b*x)/(a + b*x)])/c^3 - (d^2*PolyLog[2, -((Sqrt[b]*(d + c
*Sqrt[x]))/(Sqrt[-1 - a]*c - Sqrt[b]*d))])/c^3 + (d^2*PolyLog[2, -((Sqrt[b
]*(d + c*Sqrt[x]))/(Sqrt[1 - a]*c - Sqrt[b]*d))])/c^3 - (d^2*PolyLog[2, (S
qrt[b]*(d + c*Sqrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)])/c^3 + (d^2*PolyLog[
2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[1 - a]*c + Sqrt[b]*d)])/c^3
```

3.81.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3008
```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

rule 6666
```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Simp
[1/2   Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Simp[1/2   In
t[Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f},
x] && RationalQ[n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.81.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 752, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\operatorname{arccoth}(bx+a)x}{c}-\frac{2\operatorname{arccoth}(bx+a)d\sqrt{x}}{c^{2}}+\frac{2\operatorname{arccoth}(bx+a)d^{2}\ln\left(d+c\sqrt{x}\right)}{c^{3}}+\frac{4b\left(-\frac{c\left(-\frac{(-1+a)\left(-\frac{\ln\left(a\,c^{2}+b\,d^{2}-2b \ldots\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}$ …
default	$\frac{\operatorname{arccoth}(bx+a)x}{c}-\frac{2\operatorname{arccoth}(bx+a)d\sqrt{x}}{c^{2}}+\frac{2\operatorname{arccoth}(bx+a)d^{2}\ln\left(d+c\sqrt{x}\right)}{c^{3}}+\frac{4b\left(-\frac{c\left(-\frac{(-1+a)\left(-\frac{\ln\left(a\,c^{2}+b\,d^{2}-2b \ldots\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}$ …

input
```
int(arccoth(b*x+a)/(c+d/x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
arccoth(b*x+a)*x/c-2*arccoth(b*x+a)/c^2*d*x^(1/2)+2*arccoth(b*x+a)*d^2/c^3
*ln(d+c*x^(1/2))+4*b/c^2*(-1/2*c*(-1/2*(-1+a)/b*(-1/2/b*ln(a*c^2+b*d^2-2*b
*d*(d+c*x^(1/2))+b*(d+c*x^(1/2))^2-c^2)+2*d/(a*b*c^2-b*c^2)^(1/2)*arctan(1
/2*(-2*d*b+2*b*(d+c*x^(1/2)))/(a*b*c^2-b*c^2)^(1/2)))+1/2*(1+a)/b*(-1/2/b*
ln(a*c^2+b*d^2-2*b*d*(d+c*x^(1/2))+b*(d+c*x^(1/2))^2+c^2)+2*d/(a*b*c^2+b*c
^2)^(1/2)*arctan(1/2*(-2*d*b+2*b*(d+c*x^(1/2)))/(a*b*c^2+b*c^2)^(1/2))))-c
*d^2*(1/2/c^2*(-1/2*ln(d+c*x^(1/2))*(ln((d*b-b*(d+c*x^(1/2))+(-a*b*c^2+b*c
^2)^(1/2))/(d*b+(-a*b*c^2+b*c^2)^(1/2)))+ln((-d*b+b*(d+c*x^(1/2))+(-a*b*c^
2+b*c^2)^(1/2))/(-d*b+(-a*b*c^2+b*c^2)^(1/2))))/b-1/2*(dilog((d*b-b*(d+c*x
^(1/2))+(-a*b*c^2+b*c^2)^(1/2))/(d*b+(-a*b*c^2+b*c^2)^(1/2)))+dilog((-d*b+
b*(d+c*x^(1/2))+(-a*b*c^2+b*c^2)^(1/2))/(-d*b+(-a*b*c^2+b*c^2)^(1/2))))/b)
+1/2/c^2*(1/2*ln(d+c*x^(1/2))*(ln((d*b-b*(d+c*x^(1/2))+(-a*b*c^2-b*c^2)^(1
/2))/(d*b+(-a*b*c^2-b*c^2)^(1/2)))+ln((-d*b+b*(d+c*x^(1/2))+(-a*b*c^2-b*c^
2)^(1/2))/(-d*b+(-a*b*c^2-b*c^2)^(1/2))))/b+1/2*(dilog((d*b-b*(d+c*x^(1/2)
)+(-a*b*c^2-b*c^2)^(1/2))/(d*b+(-a*b*c^2-b*c^2)^(1/2)))+dilog((-d*b+b*(d+c
*x^(1/2))+(-a*b*c^2-b*c^2)^(1/2))/(-d*b+(-a*b*c^2-b*c^2)^(1/2))))/b))
```

3.81.5 Fricas [F]

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\operatorname{arcoth}(bx+a)}{c+\frac{d}{\sqrt{x}}}\,dx$$

input
```
integrate(arccoth(b*x+a)/(c+d/x^(1/2)),x, algorithm="fricas")
```

output
```
integral((c*x*arccoth(b*x + a) - d*sqrt(x)*arccoth(b*x + a))/(c^2*x - d^2)
, x)
```

3.81.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \text{Timed out}$$

input
```
integrate(acoth(b*x+a)/(c+d/x**(1/2)),x)
```

output
```
Timed out
```

3.81.7 Maxima [F]

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\operatorname{arcoth}(bx+a)}{c+\frac{d}{\sqrt{x}}}\,dx$$

input
```
integrate(arccoth(b*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")
```

output
```
1/2*((b*x + a + 1)*log(b*x + a + 1) - (b*x + a - 1)*log(b*x + a - 1))/(b*c
) - 1/2*integrate((d*log(b*x + a + 1) - d*log(b*x + a - 1))/(c^2*sqrt(x) +
 c*d), x)
```

3.81.8 Giac [F]

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\operatorname{arcoth}(bx+a)}{c+\frac{d}{\sqrt{x}}}\,dx$$

input
```
integrate(arccoth(b*x+a)/(c+d/x^(1/2)),x, algorithm="giac")
```

output
```
integrate(arccoth(b*x + a)/(c + d/sqrt(x)), x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\operatorname{acoth}(a+b\,x)}{c+\frac{d}{\sqrt{x}}}\,dx$$

input
```
int(acoth(a + b*x)/(c + d/x^(1/2)),x)
```

output
```
int(acoth(a + b*x)/(c + d/x^(1/2)), x)
```

3.82 $\int \frac{\coth^{-1}(d+ex)}{a+bx+cx^2}\,dx$

3.82.1 Optimal result

Integrand size = 19, antiderivative size = 335

$$\int \frac{\coth^{-1}(d+ex)}{a+bx+cx^2}\,dx = \frac{\coth^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2c(1-d)+\left(b-\sqrt{b^2-4ac}\right)e\right)(1+d+ex)}\right)}{\sqrt{b^2-4ac}} - \frac{\coth^{-1}(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(1-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1+d+ex)}\right)}{\sqrt{b^2-4ac}} - \frac{\text{PolyLog}\left(2,1+\frac{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c-2cd+be-\sqrt{b^2-4ac}e\right)(1+d+ex)}\right)}{2\sqrt{b^2-4ac}} + \frac{\text{PolyLog}\left(2,1+\frac{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c(1-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1+d+ex)}\right)}{2\sqrt{b^2-4ac}}$$

output
```
arccoth(e*x+d)*ln(2*e*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(e*x+d+1)/(2*c*(1-d)+e*
(b-(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)-arccoth(e*x+d)*ln(2*e*(b+2*c*x
+(-4*a*c+b^2)^(1/2))/(e*x+d+1)/(2*c*(1-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-4*a
*c+b^2)^(1/2)-1/2*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b-(-4*a*c+b^2)^(1/2)
))/(e*x+d+1)/(2*c-2*c*d+b*e-e*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)+1/2*
polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b+(-4*a*c+b^2)^(1/2)))/(e*x+d+1)/(2*c*
(1-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.78

$$\int \frac{\coth^{-1}(d+ex)}{a+bx+cx^2}\,dx$$
$$= \frac{\log\left(b-\sqrt{b^2-4ac}+2cx\right)\log\left(\frac{2c(-1+d+ex)}{2c(-1+d)+\left(-b+\sqrt{b^2-4ac}\right)e}\right)-\log\left(b+\sqrt{b^2-4ac}+2cx\right)\log\left(\frac{2c(-1+d+e}{2c(-1+d)-\left(b+\sqrt{b}\right.}\right.}{}$$

input
```
Integrate[ArcCoth[d + e*x]/(a + b*x + c*x^2),x]
```

output
```
(Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(2*c*(-1 + d + e*x))/(2*c*(-1 + d)
 + (-b + Sqrt[b^2 - 4*a*c])*e)] - Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(
2*c*(-1 + d + e*x))/(2*c*(-1 + d) - (b + Sqrt[b^2 - 4*a*c])*e)] - Log[b -
Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(-1 + d + e*x)/(d + e*x)] + Log[b + Sqrt[b^
2 - 4*a*c] + 2*c*x]*Log[(-1 + d + e*x)/(d + e*x)] - Log[b - Sqrt[b^2 - 4*a
*c] + 2*c*x]*Log[(2*c*(1 + d + e*x))/(2*c*(1 + d) + (-b + Sqrt[b^2 - 4*a*c
])*e)] + Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(2*c*(1 + d + e*x))/(2*c*(
1 + d) - (b + Sqrt[b^2 - 4*a*c])*e)] + Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*
Log[(1 + d + e*x)/(d + e*x)] - Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(1 +
 d + e*x)/(d + e*x)] - PolyLog[2, (e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*
c*(1 + d) + (-b + Sqrt[b^2 - 4*a*c])*e)] + PolyLog[2, (e*(b - Sqrt[b^2 - 4
*a*c] + 2*c*x))/(2*c - 2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e)] - PolyLog[2, (e
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*(-1 + d) + (b + Sqrt[b^2 - 4*a*c])
*e)] + PolyLog[2, (e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*(1 + d) + (b +
 Sqrt[b^2 - 4*a*c])*e)])/(2*Sqrt[b^2 - 4*a*c])
```

3.82.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d+ex)}{a+bx+cx^2}\,dx$$
$$\Big\downarrow \; 7279$$

$$\int \left(\frac{2c\coth^{-1}(d+ex)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}+b+2cx\right)}-\frac{2c\coth^{-1}(d+ex)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b+2cx\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{PolyLog}\left(2,\frac{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(-2dc+2c+be-\sqrt{b^2-4ac}e\right)(d+ex+1)}+1\right)}{2\sqrt{b^2-4ac}}+\frac{\text{PolyLog}\left(2,\frac{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c(1-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(d+ex+1)}+1\right)}{2\sqrt{b^2-4ac}}+\frac{\coth^{-1}(d+ex)\log\left(-\frac{2\left(-e\left(b-\sqrt{b^2-4ac}\right)-2c(d+ex)+2cd\right)}{(d+ex+1)\left(-e\sqrt{b^2-4ac}+be-2cd+2c\right)}\right)}{\sqrt{b^2-4ac}}-\frac{\coth^{-1}(d+ex)\log\left(-\frac{2\left(-e\left(\sqrt{b^2-4ac}+b\right)-2c(d+ex)+2cd\right)}{(d+ex+1)\left(e\left(\sqrt{b^2-4ac}+b\right)+2c(1-d)\right)}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[ArcCoth[d + e*x]/(a + b*x + c*x^2),x]
```

output
```
(ArcCoth[d + e*x]*Log[(-2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*
x)))/((2*c - 2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e)*(1 + d + e*x))]])/Sqrt[b^2
 - 4*a*c] - (ArcCoth[d + e*x]*Log[(-2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e -
2*c*(d + e*x)))/((2*c*(1 - d) + (b + Sqrt[b^2 - 4*a*c])*e)*(1 + d + e*x))]
)/Sqrt[b^2 - 4*a*c] - PolyLog[2, 1 + (2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
 - 2*c*(d + e*x)))/((2*c - 2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e)*(1 + d + e*x
))]/(2*Sqrt[b^2 - 4*a*c]) + PolyLog[2, 1 + (2*(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e - 2*c*(d + e*x)))/((2*c*(1 - d) + (b + Sqrt[b^2 - 4*a*c])*e)*(1 + d
 + e*x))]/(2*Sqrt[b^2 - 4*a*c])
```

3.82.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.82.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(307) = 614$.

Time = 1.38 (sec) , antiderivative size = 750, normalized size of antiderivative = 2.24

method	result
risch	$-\frac{e\ln(ex+d-1)\ln\left(\frac{-2(ex+d-1)c-be+2cd+\sqrt{-4ac\,e^2+b^2e^2}-2c}{-be+2cd+\sqrt{-4ac\,e^2+b^2e^2}-2c}\right)}{2\sqrt{-4ac\,e^2+b^2e^2}}+\frac{e\ln(ex+d-1)\ln\left(\frac{2(ex+d-1)c+be-2cd+\sqrt{-4ac\,e^2+b^2}}{be-2cd+2c+\sqrt{-4ac\,e^2+b^2e^2}}\right.}{2\sqrt{-4ac\,e^2+b^2e^2}}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arccoth(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*e*ln(e*x+d-1)/(-4*a*c*e^2+b^2*e^2)^(1/2)*ln((-2*(e*x+d-1)*c-b*e+2*c*d
+(-4*a*c*e^2+b^2*e^2)^(1/2)-2*c)/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)-2*
c))+1/2*e*ln(e*x+d-1)/(-4*a*c*e^2+b^2*e^2)^(1/2)*ln((2*(e*x+d-1)*c+b*e-2*c
*d+(-4*a*c*e^2+b^2*e^2)^(1/2)+2*c)/(b*e-2*c*d+2*c+(-4*a*c*e^2+b^2*e^2)^(1/
2)))-1/2*e/(-4*a*c*e^2+b^2*e^2)^(1/2)*dilog((-2*(e*x+d-1)*c-b*e+2*c*d+(-4*
a*c*e^2+b^2*e^2)^(1/2)-2*c)/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)-2*c))+1
/2*e/(-4*a*c*e^2+b^2*e^2)^(1/2)*dilog((2*(e*x+d-1)*c+b*e-2*c*d+(-4*a*c*e^2
+b^2*e^2)^(1/2)+2*c)/(b*e-2*c*d+2*c+(-4*a*c*e^2+b^2*e^2)^(1/2)))+1/2*e*ln(
e*x+d+1)/(-4*a*c*e^2+b^2*e^2)^(1/2)*ln((-2*(e*x+d+1)*c-b*e+2*c*d+(-4*a*c*e
^2+b^2*e^2)^(1/2)+2*c)/(-b*e+2*c*d+2*c+(-4*a*c*e^2+b^2*e^2)^(1/2)))-1/2*e*
ln(e*x+d+1)/(-4*a*c*e^2+b^2*e^2)^(1/2)*ln((2*(e*x+d+1)*c+b*e-2*c*d+(-4*a*c
*e^2+b^2*e^2)^(1/2)-2*c)/(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)-2*c))+1/2*e
/(-4*a*c*e^2+b^2*e^2)^(1/2)*dilog((-2*(e*x+d+1)*c-b*e+2*c*d+(-4*a*c*e^2+b^
2*e^2)^(1/2)+2*c)/(-b*e+2*c*d+2*c+(-4*a*c*e^2+b^2*e^2)^(1/2)))-1/2*e/(-4*a
*c*e^2+b^2*e^2)^(1/2)*dilog((2*(e*x+d+1)*c+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^
(1/2)-2*c)/(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)-2*c))
```

3.82.5 Fricas [F]

$$\int \frac{\coth^{-1}(d+ex)}{a+bx+cx^2}\,dx = \int \frac{\operatorname{arcoth}(ex+d)}{cx^2+bx+a}\,dx$$

input
```
integrate(arccoth(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral(arccoth(e*x + d)/(c*x^2 + b*x + a), x)
```

3.82.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(d+ex)}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate(acoth(e*x+d)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.82.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\coth^{-1}(d+ex)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arccoth(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.82.8 Giac [F]

$$\int \frac{\coth^{-1}(d+ex)}{a+bx+cx^2}\,dx = \int \frac{\operatorname{arcoth}(ex+d)}{cx^2+bx+a}\,dx$$

input
```
integrate(arccoth(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate(arccoth(e*x + d)/(c*x^2 + b*x + a), x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(d+ex)}{a+bx+cx^2}\,dx = \int \frac{\mathrm{acoth}(d+e\,x)}{c\,x^2+b\,x+a}\,dx$$

input
```
int(acoth(d + e*x)/(a + b*x + c*x^2),x)
```

output
```
int(acoth(d + e*x)/(a + b*x + c*x^2), x)
```

3.83 $\int x^2 \coth^{-1}\left(\sqrt{x}\right)\,dx$

3.83.1 Optimal result

Integrand size = 10, antiderivative size = 51

$$\int x^2 \coth^{-1}\left(\sqrt{x}\right)\,dx = \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \coth^{-1}\left(\sqrt{x}\right) - \frac{\operatorname{arctanh}\left(\sqrt{x}\right)}{3}$$

output
```
1/9*x^(3/2)+1/15*x^(5/2)+1/3*x^3*arccoth(x^(1/2))-1/3*arctanh(x^(1/2))+1/3
*x^(1/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int x^2 \coth^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{90}\left(30\sqrt{x} + 10x^{3/2} + 6x^{5/2} + 30x^3 \coth^{-1}\left(\sqrt{x}\right) + 15\log\left(1-\sqrt{x}\right) - 15\log\left(1+\sqrt{x}\right)\right)$$

input
```
Integrate[x^2*ArcCoth[Sqrt[x]],x]
```

output
```
(30*Sqrt[x] + 10*x^(3/2) + 6*x^(5/2) + 30*x^3*ArcCoth[Sqrt[x]] + 15*Log[1
- Sqrt[x]] - 15*Log[1 + Sqrt[x]])/90
```

3.83.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6453, 60, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{3}x^3 \coth^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{x^{5/2}}{1-x}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}\left(\frac{2x^{5/2}}{5} - \int \frac{x^{3/2}}{1-x}dx\right) + \frac{1}{3}x^3 \coth^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}\left(-\int \frac{\sqrt{x}}{1-x}dx + \frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3}\right) + \frac{1}{3}x^3 \coth^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}\left(-\int \frac{1}{(1-x)\sqrt{x}}dx + \frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3} + 2\sqrt{x}\right) + \frac{1}{3}x^3 \coth^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(-2\int \frac{1}{1-x}d\sqrt{x} + \frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3} + 2\sqrt{x}\right) + \frac{1}{3}x^3 \coth^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}\left(-2\text{arctanh}\left(\sqrt{x}\right) + \frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3} + 2\sqrt{x}\right) + \frac{1}{3}x^3 \coth^{-1}\left(\sqrt{x}\right)$$

input
```
Int[x^2*ArcCoth[Sqrt[x]],x]
```

output
```
(x^3*ArcCoth[Sqrt[x]])/3 + (2*Sqrt[x] + (2*x^(3/2))/3 + (2*x^(5/2))/5 - 2*
ArcTanh[Sqrt[x]])/6
```

3.83.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.83.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{x^{3} \operatorname{arccoth}\left(\sqrt{x}\right)}{3}+\frac{x^{\frac{5}{2}}}{15}+\frac{x^{\frac{3}{2}}}{9}+\frac{\sqrt{x}}{3}+\frac{\ln\left(\sqrt{x}-1\right)}{6}-\frac{\ln\left(\sqrt{x}+1\right)}{6}$	42
default	$\frac{x^{3} \operatorname{arccoth}\left(\sqrt{x}\right)}{3}+\frac{x^{\frac{5}{2}}}{15}+\frac{x^{\frac{3}{2}}}{9}+\frac{\sqrt{x}}{3}+\frac{\ln\left(\sqrt{x}-1\right)}{6}-\frac{\ln\left(\sqrt{x}+1\right)}{6}$	42
parts	$\frac{x^{3} \operatorname{arccoth}\left(\sqrt{x}\right)}{3}+\frac{x^{\frac{5}{2}}}{15}+\frac{x^{\frac{3}{2}}}{9}+\frac{\sqrt{x}}{3}+\frac{\ln\left(\sqrt{x}-1\right)}{6}-\frac{\ln\left(\sqrt{x}+1\right)}{6}$	42

input
```
int(x^2*arccoth(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccoth(x^(1/2))+1/15*x^(5/2)+1/9*x^(3/2)+1/3*x^(1/2)+1/6*ln(x^(1/
2)-1)-1/6*ln(x^(1/2)+1)
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

$$\int x^2 \coth^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{6}\left(x^3-1\right)\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right) + \frac{1}{45}\left(3x^2+5x+15\right)\sqrt{x}$$

input
```
integrate(x^2*arccoth(x^(1/2)),x, algorithm="fricas")
```

output
```
1/6*(x^3 - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) + 1/45*(3*x^2 + 5*x + 15)*s
qrt(x)
```

3.83.6 Sympy [F]

$$\int x^2 \coth^{-1}\left(\sqrt{x}\right)\,dx = \int x^2 \operatorname{acoth}\left(\sqrt{x}\right)\,dx$$

input
```
integrate(x**2*acoth(x**(1/2)),x)
```

output
```
Integral(x**2*acoth(sqrt(x)), x)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int x^2 \coth^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{3}x^3 \operatorname{arcoth}\left(\sqrt{x}\right) + \frac{1}{15}x^{\frac{5}{2}} + \frac{1}{9}x^{\frac{3}{2}} + \frac{1}{3}\sqrt{x} - \frac{1}{6}\log\left(\sqrt{x}+1\right) + \frac{1}{6}\log\left(\sqrt{x}-1\right)$$

input
```
integrate(x^2*arccoth(x^(1/2)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(sqrt(x)) + 1/15*x^(5/2) + 1/9*x^(3/2) + 1/3*sqrt(x) - 1/6*
log(sqrt(x) + 1) + 1/6*log(sqrt(x) - 1)
```

3.83.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(31) = 62$.

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 3.22

$$\int x^2 \coth^{-1}\left(\sqrt{x}\right)\,dx = \frac{2\left(\frac{45\left(\sqrt{x}+1\right)^4}{\left(\sqrt{x}-1\right)^4} - \frac{90\left(\sqrt{x}+1\right)^3}{\left(\sqrt{x}-1\right)^3} + \frac{140\left(\sqrt{x}+1\right)^2}{\left(\sqrt{x}-1\right)^2} - \frac{70\left(\sqrt{x}+1\right)}{\sqrt{x}-1} + 23\right)}{45\left(\frac{\sqrt{x}+1}{\sqrt{x}-1} - 1\right)^5} + \frac{2\left(\frac{3\left(\sqrt{x}+1\right)^5}{\left(\sqrt{x}-1\right)^5} + \frac{10\left(\sqrt{x}+1\right)^3}{\left(\sqrt{x}-1\right)^3} + \frac{3\left(\sqrt{x}+1\right)}{\sqrt{x}-1}\right)\log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{3\left(\frac{\sqrt{x}+1}{\sqrt{x}-1} - 1\right)^6}$$

input
```
integrate(x^2*arccoth(x^(1/2)),x, algorithm="giac")
```

output
```
2/45*(45*(sqrt(x) + 1)^4/(sqrt(x) - 1)^4 - 90*(sqrt(x) + 1)^3/(sqrt(x) - 1
)^3 + 140*(sqrt(x) + 1)^2/(sqrt(x) - 1)^2 - 70*(sqrt(x) + 1)/(sqrt(x) - 1)
 + 23)/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^5 + 2/3*(3*(sqrt(x) + 1)^5/(sqrt(
x) - 1)^5 + 10*(sqrt(x) + 1)^3/(sqrt(x) - 1)^3 + 3*(sqrt(x) + 1)/(sqrt(x)
- 1))*log((sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^6
```

3.83.9 Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int x^2 \coth^{-1}\left(\sqrt{x}\right)\,dx = \frac{x^3\,\mathrm{acoth}\left(\sqrt{x}\right)}{3} - \frac{\mathrm{acoth}\left(\sqrt{x}\right)}{3} + \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15}$$

input
```
int(x^2*acoth(x^(1/2)),x)
```

output
```
(x^3*acoth(x^(1/2)))/3 - acoth(x^(1/2))/3 + x^(1/2)/3 + x^(3/2)/9 + x^(5/2
)/15
```

3.84 $\int x \coth^{-1}\left(\sqrt{x}\right) \, dx$

3.84.1 Optimal result

Integrand size = 8, antiderivative size = 42

$$\int x \coth^{-1}\left(\sqrt{x}\right) \, dx = \frac{\sqrt{x}}{2} + \frac{x^{3/2}}{6} + \frac{1}{2}x^2 \coth^{-1}\left(\sqrt{x}\right) - \frac{\operatorname{arctanh}\left(\sqrt{x}\right)}{2}$$

output
```
1/6*x^(3/2)+1/2*x^2*arccoth(x^(1/2))-1/2*arctanh(x^(1/2))+1/2*x^(1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int x \coth^{-1}\left(\sqrt{x}\right) \, dx = \frac{1}{12}\left(6\sqrt{x} + 2x^{3/2} + 6x^2 \coth^{-1}\left(\sqrt{x}\right) + 3\log\left(1 - \sqrt{x}\right) - 3\log\left(1 + \sqrt{x}\right)\right)$$

input
```
Integrate[x*ArcCoth[Sqrt[x]],x]
```

output
```
(6*Sqrt[x] + 2*x^(3/2) + 6*x^2*ArcCoth[Sqrt[x]] + 3*Log[1 - Sqrt[x]] - 3*L
og[1 + Sqrt[x]])/12
```

3.84.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6453, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{2}x^2 \coth^{-1}\left(\sqrt{x}\right) - \frac{1}{4}\int \frac{x^{3/2}}{1-x}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{2x^{3/2}}{3} - \int \frac{\sqrt{x}}{1-x}dx\right) + \frac{1}{2}x^2 \coth^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(-\int \frac{1}{(1-x)\sqrt{x}}dx + \frac{2x^{3/2}}{3} + 2\sqrt{x}\right) + \frac{1}{2}x^2 \coth^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-2\int \frac{1}{1-x}d\sqrt{x} + \frac{2x^{3/2}}{3} + 2\sqrt{x}\right) + \frac{1}{2}x^2 \coth^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(-2\text{arctanh}\left(\sqrt{x}\right) + \frac{2x^{3/2}}{3} + 2\sqrt{x}\right) + \frac{1}{2}x^2 \coth^{-1}\left(\sqrt{x}\right)$$

input
```
Int[x*ArcCoth[Sqrt[x]],x]
```

output
```
(x^2*ArcCoth[Sqrt[x]])/2 + (2*Sqrt[x] + (2*x^(3/2))/3 - 2*ArcTanh[Sqrt[x]]
)/4
```

3.84.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.84.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{x^{2} \operatorname{arccoth}\left(\sqrt{x}\right)}{2}+\frac{x^{\frac{3}{2}}}{6}+\frac{\sqrt{x}}{2}+\frac{\ln\left(\sqrt{x}-1\right)}{4}-\frac{\ln\left(\sqrt{x}+1\right)}{4}$	37
default	$\frac{x^{2} \operatorname{arccoth}\left(\sqrt{x}\right)}{2}+\frac{x^{\frac{3}{2}}}{6}+\frac{\sqrt{x}}{2}+\frac{\ln\left(\sqrt{x}-1\right)}{4}-\frac{\ln\left(\sqrt{x}+1\right)}{4}$	37
parts	$\frac{x^{2} \operatorname{arccoth}\left(\sqrt{x}\right)}{2}+\frac{x^{\frac{3}{2}}}{6}+\frac{\sqrt{x}}{2}+\frac{\ln\left(\sqrt{x}-1\right)}{4}-\frac{\ln\left(\sqrt{x}+1\right)}{4}$	37

input
```
int(x*arccoth(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arccoth(x^(1/2))+1/6*x^(3/2)+1/2*x^(1/2)+1/4*ln(x^(1/2)-1)-1/4*ln(
x^(1/2)+1)
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int x\coth^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{4}\left(x^2-1\right)\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)+\frac{1}{6}(x+3)\sqrt{x}$$

input
```
integrate(x*arccoth(x^(1/2)),x, algorithm="fricas")
```

output
```
1/4*(x^2 - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) + 1/6*(x + 3)*sqrt(x)
```

3.84.6 Sympy [F]

$$\int x\coth^{-1}\left(\sqrt{x}\right)\,dx=\int x\,\mathrm{acoth}\left(\sqrt{x}\right)\,dx$$

input
```
integrate(x*acoth(x**(1/2)),x)
```

output
```
Integral(x*acoth(sqrt(x)), x)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int x\coth^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{2}x^2\,\mathrm{arcoth}\left(\sqrt{x}\right)+\frac{1}{6}x^{\frac{3}{2}}+\frac{1}{2}\sqrt{x}-\frac{1}{4}\log\left(\sqrt{x}+1\right)+\frac{1}{4}\log\left(\sqrt{x}-1\right)$$

input
```
integrate(x*arccoth(x^(1/2)),x, algorithm="maxima")
```

output
```
1/2*x^2*arccoth(sqrt(x)) + 1/6*x^(3/2) + 1/2*sqrt(x) - 1/4*log(sqrt(x) + 1
) + 1/4*log(sqrt(x) - 1)
```

3.84.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(26) = 52.

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.71

$$\int x\coth^{-1}\left(\sqrt{x}\right)\,dx=\frac{2\left(\frac{3\left(\sqrt{x}+1\right)^2}{\left(\sqrt{x}-1\right)^2}-\frac{3\left(\sqrt{x}+1\right)}{\sqrt{x}-1}+2\right)}{3\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^3}+\frac{2\left(\frac{\left(\sqrt{x}+1\right)^3}{\left(\sqrt{x}-1\right)^3}+\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)\log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^4}$$

input
```
integrate(x*arccoth(x^(1/2)),x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(x) + 1)^2/(sqrt(x) - 1)^2 - 3*(sqrt(x) + 1)/(sqrt(x) - 1) + 2
)/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^3 + 2*((sqrt(x) + 1)^3/(sqrt(x) - 1)^3
 + (sqrt(x) + 1)/(sqrt(x) - 1))*log((sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x)
 + 1)/(sqrt(x) - 1) - 1)^4
```

3.84.9 Mupad [B] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int x\coth^{-1}\left(\sqrt{x}\right)\,dx=\frac{x^2\,\mathrm{acoth}\left(\sqrt{x}\right)}{2}-\frac{\mathrm{acoth}\left(\sqrt{x}\right)}{2}+\frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}$$

input
```
int(x*acoth(x^(1/2)),x)
```

output
```
(x^2*acoth(x^(1/2)))/2 - acoth(x^(1/2))/2 + x^(1/2)/2 + x^(3/2)/6
```

3.85 $\int \coth^{-1}\left(\sqrt{x}\right)\,dx$

3.85.1 Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \coth^{-1}\left(\sqrt{x}\right)\,dx=\sqrt{x}+x\coth^{-1}\left(\sqrt{x}\right)-\operatorname{arctanh}\left(\sqrt{x}\right)$$

output
```
x*arccoth(x^(1/2))-arctanh(x^(1/2))+x^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \coth^{-1}\left(\sqrt{x}\right)\,dx=\sqrt{x}+x\coth^{-1}\left(\sqrt{x}\right)-\operatorname{arctanh}\left(\sqrt{x}\right)$$

input
```
Integrate[ArcCoth[Sqrt[x]],x]
```

output
```
Sqrt[x] + x*ArcCoth[Sqrt[x]] - ArcTanh[Sqrt[x]]
```

3.85.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6437, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{6437}$$

$$x\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{1-x}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(2\sqrt{x}-\int \frac{1}{(1-x)\sqrt{x}}dx\right)+x\coth^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(2\sqrt{x}-2\int \frac{1}{1-x}d\sqrt{x}\right)+x\coth^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(2\sqrt{x}-2\text{arctanh}\left(\sqrt{x}\right)\right)+x\coth^{-1}\left(\sqrt{x}\right)$$

input
```
Int[ArcCoth[Sqrt[x]],x]
```

output
```
x*ArcCoth[Sqrt[x]] + (2*Sqrt[x] - 2*ArcTanh[Sqrt[x]])/2
```

3.85.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

3.85.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

method	result	size
derivativedivides	$x \operatorname{arccoth}\left(\sqrt{x}\right)+\sqrt{x}+\frac{\ln\left(\sqrt{x}-1\right)}{2}-\frac{\ln\left(\sqrt{x}+1\right)}{2}$	27
default	$x \operatorname{arccoth}\left(\sqrt{x}\right)+\sqrt{x}+\frac{\ln\left(\sqrt{x}-1\right)}{2}-\frac{\ln\left(\sqrt{x}+1\right)}{2}$	27
parts	$x \operatorname{arccoth}\left(\sqrt{x}\right)+\sqrt{x}+\frac{\ln\left(\sqrt{x}-1\right)}{2}-\frac{\ln\left(\sqrt{x}+1\right)}{2}$	27

input
```
int(arccoth(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*arccoth(x^(1/2))+x^(1/2)+1/2*ln(x^(1/2)-1)-1/2*ln(x^(1/2)+1)
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \coth^{-1}\left(\sqrt{x}\right) \, dx = \frac{1}{2}\,(x-1)\log\left(\frac{x+2\,\sqrt{x}+1}{x-1}\right)+\sqrt{x}$$

input
```
integrate(arccoth(x^(1/2)),x, algorithm="fricas")
```

output
```
1/2*(x - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) + sqrt(x)
```

3.85.6 Sympy [F]

$$\int \coth^{-1}\left(\sqrt{x}\right) \, dx = \int \operatorname{acoth}\left(\sqrt{x}\right)\, dx$$

input
```
integrate(acoth(x**(1/2)),x)
```

output
```
Integral(acoth(sqrt(x)), x)
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \coth^{-1}\left(\sqrt{x}\right) \, dx = x\operatorname{arcoth}\left(\sqrt{x}\right)+\sqrt{x}-\frac{1}{2}\,\log\left(\sqrt{x}+1\right)+\frac{1}{2}\,\log\left(\sqrt{x}-1\right)$$

input
```
integrate(arccoth(x^(1/2)),x, algorithm="maxima")
```

output
```
x*arccoth(sqrt(x)) + sqrt(x) - 1/2*log(sqrt(x) + 1) + 1/2*log(sqrt(x) - 1)
```

3.85.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \coth^{-1}\left(\sqrt{x}\right)\,dx = \frac{2}{\frac{\sqrt{x}+1}{\sqrt{x}-1}-1} + \frac{2\left(\sqrt{x}+1\right)\log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\left(\sqrt{x}-1\right)\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^2}$$

input
```
integrate(arccoth(x^(1/2)),x, algorithm="giac")
```

output
```
2/((sqrt(x) + 1)/(sqrt(x) - 1) - 1) + 2*(sqrt(x) + 1)*log((sqrt(x) + 1)/(s
qrt(x) - 1))/((sqrt(x) - 1)*((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^2)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \coth^{-1}\left(\sqrt{x}\right)\,dx = x\,\mathrm{acoth}\left(\sqrt{x}\right) - \mathrm{acoth}\left(\sqrt{x}\right) + \sqrt{x}$$

input
```
int(acoth(x^(1/2)),x)
```

output
```
x*acoth(x^(1/2)) - acoth(x^(1/2)) + x^(1/2)
```

3.86 $\int \frac{\coth^{-1}(\sqrt{x})}{x} dx$

3.86.1 Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x} dx = \text{PolyLog}\left(2, -\frac{1}{\sqrt{x}}\right) - \text{PolyLog}\left(2, \frac{1}{\sqrt{x}}\right)$$

output
```
polylog(2,-1/x^(1/2))-polylog(2,1/x^(1/2))
```

3.86.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x} dx = \text{PolyLog}\left(2, -\frac{1}{\sqrt{x}}\right) - \text{PolyLog}\left(2, \frac{1}{\sqrt{x}}\right)$$

input
```
Integrate[ArcCoth[Sqrt[x]]/x,x]
```

output
```
PolyLog[2, -(1/Sqrt[x])] - PolyLog[2, 1/Sqrt[x]]
```

3.86.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6451, 6447}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x} \, dx$$
$$\Big\downarrow \text{ 6451}$$
$$2\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}d\sqrt{x}$$
$$\Big\downarrow \text{ 6447}$$
$$2\left(\frac{1}{2}\text{PolyLog}\left(2,-\frac{1}{\sqrt{x}}\right)-\frac{\text{PolyLog}\left(2,\frac{1}{\sqrt{x}}\right)}{2}\right)$$

input
```
Int[ArcCoth[Sqrt[x]]/x,x]
```

output
```
2*(PolyLog[2, -(1/Sqrt[x])]/2 - PolyLog[2, 1/Sqrt[x]]/2)
```

3.86.3.1 Defintions of rubi rules used

rule 6447
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

rule 6451
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcCoth[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

3.86.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

method	result	size
derivativedivides	$\ln(x) \operatorname{arccoth}(\sqrt{x}) - \operatorname{dilog}(\sqrt{x}) - \operatorname{dilog}(\sqrt{x}+1) - \frac{\ln(x)\ln(\sqrt{x}+1)}{2}$	33
default	$\ln(x) \operatorname{arccoth}(\sqrt{x}) - \operatorname{dilog}(\sqrt{x}) - \operatorname{dilog}(\sqrt{x}+1) - \frac{\ln(x)\ln(\sqrt{x}+1)}{2}$	33
parts	$\ln(x) \operatorname{arccoth}(\sqrt{x}) - \operatorname{dilog}(\sqrt{x}) - \operatorname{dilog}(\sqrt{x}+1) - \frac{\ln(x)\ln(\sqrt{x}+1)}{2}$	33

input
```
int(arccoth(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arccoth(x^(1/2))-dilog(x^(1/2))-dilog(x^(1/2)+1)-1/2*ln(x)*ln(x^(1/2
)+1)
```

3.86.5 Fricas [F]

$$\int \frac{\coth^{-1}(\sqrt{x})}{x}\,dx = \int \frac{\operatorname{arcoth}(\sqrt{x})}{x}\,dx$$

input
```
integrate(arccoth(x^(1/2))/x,x, algorithm="fricas")
```

output
```
integral(arccoth(sqrt(x))/x, x)
```

3.86.6 Sympy [F]

$$\int \frac{\coth^{-1}(\sqrt{x})}{x}\,dx = \int \frac{\operatorname{acoth}(\sqrt{x})}{x}\,dx$$

input
```
integrate(acoth(x**(1/2))/x,x)
```

output
```
Integral(acoth(sqrt(x))/x, x)
```

3.86.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(13) = 26$.

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.47

$$\int \frac{\coth^{-1}(\sqrt{x})}{x}\,dx = -\frac{1}{2}\left(\log\left(\sqrt{x}+1\right)-\log\left(\sqrt{x}-1\right)\right)\log(x) + \operatorname{arcoth}\left(\sqrt{x}\right)\log(x) + \log\left(-\sqrt{x}\right)\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log(x)\log\left(\sqrt{x}-1\right) + \operatorname{Li}_2\left(\sqrt{x}+1\right) - \operatorname{Li}_2\left(-\sqrt{x}+1\right)$$

input
```
integrate(arccoth(x^(1/2))/x,x, algorithm="maxima")
```

output
```
-1/2*(log(sqrt(x) + 1) - log(sqrt(x) - 1))*log(x) + arccoth(sqrt(x))*log(x
) + log(-sqrt(x))*log(sqrt(x) + 1) - 1/2*log(x)*log(sqrt(x) - 1) + dilog(s
qrt(x) + 1) - dilog(-sqrt(x) + 1)
```

3.86.8 Giac [F]

$$\int \frac{\coth^{-1}(\sqrt{x})}{x}\,dx = \int \frac{\operatorname{arcoth}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(arccoth(x^(1/2))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(sqrt(x))/x, x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(\sqrt{x})}{x}\,dx = \int \frac{\operatorname{acoth}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
int(acoth(x^(1/2))/x,x)
```

output
```
int(acoth(x^(1/2))/x, x)
```

3.87 $\int \frac{\coth^{-1}(\sqrt{x})}{x^2} dx$

3.87.1 Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^2} dx = -\frac{1}{\sqrt{x}} - \frac{\coth^{-1}\left(\sqrt{x}\right)}{x} + \operatorname{arctanh}\left(\sqrt{x}\right)$$

output
```
-arccoth(x^(1/2))/x+arctanh(x^(1/2))-1/x^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^2} dx = -\frac{1}{\sqrt{x}} - \frac{\coth^{-1}\left(\sqrt{x}\right)}{x} - \frac{1}{2}\log\left(1-\sqrt{x}\right) + \frac{1}{2}\log\left(1+\sqrt{x}\right)$$

input
```
Integrate[ArcCoth[Sqrt[x]]/x^2,x]
```

output
```
-(1/Sqrt[x]) - ArcCoth[Sqrt[x]]/x - Log[1 - Sqrt[x]]/2 + Log[1 + Sqrt[x]]/
2
```

3.87.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6453, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^2}dx$$

$$\Big\downarrow \text{ 6453}$$

$$\frac{1}{2}\int \frac{1}{(1-x)x^{3/2}}dx-\frac{\coth^{-1}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{1}{2}\left(\int \frac{1}{(1-x)\sqrt{x}}dx-\frac{2}{\sqrt{x}}\right)-\frac{\coth^{-1}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(2\int \frac{1}{1-x}d\sqrt{x}-\frac{2}{\sqrt{x}}\right)-\frac{\coth^{-1}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(2\text{arctanh}\left(\sqrt{x}\right)-\frac{2}{\sqrt{x}}\right)-\frac{\coth^{-1}\left(\sqrt{x}\right)}{x}$$

input
```
Int[ArcCoth[Sqrt[x]]/x^2,x]
```

output
```
-(ArcCoth[Sqrt[x]]/x) + (-2/Sqrt[x] + 2*ArcTanh[Sqrt[x]])/2
```

3.87.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.87.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

method	result	size
derivativedivides	$-\frac{\operatorname{arccoth}\left(\sqrt{x}\right)}{x}+\frac{\ln\left(\sqrt{x}+1\right)}{2}-\frac{1}{\sqrt{x}}-\frac{\ln\left(\sqrt{x}-1\right)}{2}$	32
default	$-\frac{\operatorname{arccoth}\left(\sqrt{x}\right)}{x}+\frac{\ln\left(\sqrt{x}+1\right)}{2}-\frac{1}{\sqrt{x}}-\frac{\ln\left(\sqrt{x}-1\right)}{2}$	32
parts	$-\frac{\operatorname{arccoth}\left(\sqrt{x}\right)}{x}+\frac{\ln\left(\sqrt{x}+1\right)}{2}-\frac{1}{\sqrt{x}}-\frac{\ln\left(\sqrt{x}-1\right)}{2}$	32

input
```
int(arccoth(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccoth(x^(1/2))/x+1/2*ln(x^(1/2)+1)-1/x^(1/2)-1/2*ln(x^(1/2)-1)
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = \frac{(x-1)\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)-2\sqrt{x}}{2\,x}$$

input
```
integrate(arccoth(x^(1/2))/x^2,x, algorithm="fricas")
```

output
```
1/2*((x - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) - 2*sqrt(x))/x
```

3.87.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(20) = 40.

Time = 0.55 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.68

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = \frac{x^{\frac{5}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}-x^{\frac{3}{2}}} - \frac{2x^{\frac{3}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}-x^{\frac{3}{2}}} + \frac{\sqrt{x}\operatorname{acoth}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}-x^{\frac{3}{2}}} - \frac{x^2}{x^{\frac{5}{2}}-x^{\frac{3}{2}}} + \frac{x}{x^{\frac{5}{2}}-x^{\frac{3}{2}}}$$

input
```
integrate(acoth(x**(1/2))/x**2,x)
```

output
```
x**(5/2)*acoth(sqrt(x))/(x**(5/2) - x**(3/2)) - 2*x**(3/2)*acoth(sqrt(x))/
(x**(5/2) - x**(3/2)) + sqrt(x)*acoth(sqrt(x))/(x**(5/2) - x**(3/2)) - x**
2/(x**(5/2) - x**(3/2)) + x/(x**(5/2) - x**(3/2))
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\operatorname{arcoth}\left(\sqrt{x}\right)}{x} - \frac{1}{\sqrt{x}} + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

input
```
integrate(arccoth(x^(1/2))/x^2,x, algorithm="maxima")
```

output
```
-arccoth(sqrt(x))/x - 1/sqrt(x) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) -
 1)
```

3.87.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(19) = 38$.

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.60

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = \frac{2}{\frac{\sqrt{x}+1}{\sqrt{x}-1}+1} + \frac{2\left(\sqrt{x}+1\right)\log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\left(\sqrt{x}-1\right)\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}+1\right)^2}$$

input
```
integrate(arccoth(x^(1/2))/x^2,x, algorithm="giac")
```

output
```
2/((sqrt(x) + 1)/(sqrt(x) - 1) + 1) + 2*(sqrt(x) + 1)*log((sqrt(x) + 1)/(s
qrt(x) - 1))/((sqrt(x) - 1)*((sqrt(x) + 1)/(sqrt(x) - 1) + 1)^2)
```

3.87.9 Mupad [B] (verification not implemented)

Time = 4.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = \operatorname{atanh}\left(\sqrt{x}\right) - \frac{\operatorname{acoth}\left(\sqrt{x}\right)+\sqrt{x}}{x}$$

input
```
int(acoth(x^(1/2))/x^2,x)
```

output
```
atanh(x^(1/2)) - (acoth(x^(1/2)) + x^(1/2))/x
```

3.88 $\int \frac{\coth^{-1}(\sqrt{x})}{x^3}\,dx$

3.88.1 Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{1}{6x^{3/2}} - \frac{1}{2\sqrt{x}} - \frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{\operatorname{arctanh}\left(\sqrt{x}\right)}{2}$$

output
```
-1/6/x^(3/2)-1/2*arccoth(x^(1/2))/x^2+1/2*arctanh(x^(1/2))-1/2/x^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{1}{6x^{3/2}} - \frac{1}{2\sqrt{x}} - \frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{1}{4}\log\left(1-\sqrt{x}\right) + \frac{1}{4}\log\left(1+\sqrt{x}\right)$$

input
```
Integrate[ArcCoth[Sqrt[x]]/x^3,x]
```

output
```
-1/6*1/x^(3/2) - 1/(2*Sqrt[x]) - ArcCoth[Sqrt[x]]/(2*x^2) - Log[1 - Sqrt[x
]]/4 + Log[1 + Sqrt[x]]/4
```

3.88.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6453, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^3}\,dx$$

$$\Big\downarrow \text{6453}$$

$$\frac{1}{4}\int \frac{1}{(1-x)x^{5/2}}dx-\frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(\int \frac{1}{(1-x)x^{3/2}}dx-\frac{2}{3x^{3/2}}\right)-\frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(\int \frac{1}{(1-x)\sqrt{x}}dx-\frac{2}{3x^{3/2}}-\frac{2}{\sqrt{x}}\right)-\frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(2\int \frac{1}{1-x}d\sqrt{x}-\frac{2}{3x^{3/2}}-\frac{2}{\sqrt{x}}\right)-\frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(2\text{arctanh}\left(\sqrt{x}\right)-\frac{2}{3x^{3/2}}-\frac{2}{\sqrt{x}}\right)-\frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2}$$

input
```
Int[ArcCoth[Sqrt[x]]/x^3,x]
```

output
```
-1/2*ArcCoth[Sqrt[x]]/x^2 + (-2/(3*x^(3/2)) - 2/Sqrt[x] + 2*ArcTanh[Sqrt[x
]])/4
```

3.88.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.88.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\operatorname{arccoth}\left(\sqrt{x}\right)}{2x^{2}}-\frac{1}{6x^{\frac{3}{2}}}-\frac{1}{2\sqrt{x}}-\frac{\ln\left(\sqrt{x}-1\right)}{4}+\frac{\ln\left(\sqrt{x}+1\right)}{4}$	37
default	$-\frac{\operatorname{arccoth}\left(\sqrt{x}\right)}{2x^{2}}-\frac{1}{6x^{\frac{3}{2}}}-\frac{1}{2\sqrt{x}}-\frac{\ln\left(\sqrt{x}-1\right)}{4}+\frac{\ln\left(\sqrt{x}+1\right)}{4}$	37
parts	$-\frac{\operatorname{arccoth}\left(\sqrt{x}\right)}{2x^{2}}-\frac{1}{6x^{\frac{3}{2}}}-\frac{1}{2\sqrt{x}}-\frac{\ln\left(\sqrt{x}-1\right)}{4}+\frac{\ln\left(\sqrt{x}+1\right)}{4}$	37

input
```
int(arccoth(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccoth(x^(1/2))/x^2-1/6/x^(3/2)-1/2/x^(1/2)-1/4*ln(x^(1/2)-1)+1/4*ln
(x^(1/2)+1)
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3\,(x^2-1)\log\left(\frac{x+2\,\sqrt{x}+1}{x-1}\right)-2\,(3\,x+1)\sqrt{x}}{12\,x^2}$$

input
```
integrate(arccoth(x^(1/2))/x^3,x, algorithm="fricas")
```

output
```
1/12*(3*(x^2 - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) - 2*(3*x + 1)*sqrt(x))/
x^2
```

3.88.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(36) = 72.

Time = 1.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.81

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3x^{\frac{7}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3x^{\frac{5}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3x^{\frac{3}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{3\sqrt{x}\operatorname{acoth}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3x^3}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{2x^2}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{x}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}$$

input
```
integrate(acoth(x**(1/2))/x**3,x)
```

output
```
3*x**(7/2)*acoth(sqrt(x))/(6*x**(7/2) - 6*x**(5/2)) - 3*x**(5/2)*acoth(sqr
t(x))/(6*x**(7/2) - 6*x**(5/2)) - 3*x**(3/2)*acoth(sqrt(x))/(6*x**(7/2) -
6*x**(5/2)) + 3*sqrt(x)*acoth(sqrt(x))/(6*x**(7/2) - 6*x**(5/2)) - 3*x**3/
(6*x**(7/2) - 6*x**(5/2)) + 2*x**2/(6*x**(7/2) - 6*x**(5/2)) + x/(6*x**(7/
2) - 6*x**(5/2))
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{3\,x+1}{6\,x^{\frac{3}{2}}} - \frac{\operatorname{arcoth}\left(\sqrt{x}\right)}{2\,x^2} + \frac{1}{4}\,\log\left(\sqrt{x}+1\right) - \frac{1}{4}\,\log\left(\sqrt{x}-1\right)$$

input
```
integrate(arccoth(x^(1/2))/x^3,x, algorithm="maxima")
```

output
```
-1/6*(3*x + 1)/x^(3/2) - 1/2*arccoth(sqrt(x))/x^2 + 1/4*log(sqrt(x) + 1) -
 1/4*log(sqrt(x) - 1)
```

3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.71

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{2\left(\frac{3\left(\sqrt{x}+1\right)^2}{\left(\sqrt{x}-1\right)^2} + \frac{3\left(\sqrt{x}+1\right)}{\sqrt{x}-1} + 2\right)}{3\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}+1\right)^3} + \frac{2\left(\frac{\left(\sqrt{x}+1\right)^3}{\left(\sqrt{x}-1\right)^3} + \frac{\sqrt{x}+1}{\sqrt{x}-1}\right)\log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}+1\right)^4}$$

input
```
integrate(arccoth(x^(1/2))/x^3,x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(x) + 1)^2/(sqrt(x) - 1)^2 + 3*(sqrt(x) + 1)/(sqrt(x) - 1) + 2
)/((sqrt(x) + 1)/(sqrt(x) - 1) + 1)^3 + 2*((sqrt(x) + 1)^3/(sqrt(x) - 1)^3
 + (sqrt(x) + 1)/(sqrt(x) - 1))*log((sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x)
 + 1)/(sqrt(x) - 1) + 1)^4
```

3.88.9 Mupad [B] (verification not implemented)

Time = 4.70 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{\ln\left(1-\frac{1}{\sqrt{x}}\right)}{4\,x^2} - \frac{\frac{x}{2}+\frac{1}{6}}{x^{3/2}} - \frac{\ln\left(\frac{1}{\sqrt{x}}+1\right)}{4\,x^2} - \frac{\operatorname{atan}\left(\sqrt{x}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

input
```
int(acoth(x^(1/2))/x^3,x)
```

output
```
log(1 - 1/x^(1/2))/(4*x^2) - (atan(x^(1/2)*1i)*1i)/2 - (x/2 + 1/6)/x^(3/2)
 - log(1/x^(1/2) + 1)/(4*x^2)
```

3.88. $\int \frac{\coth^{-1}(\sqrt{x})}{x^3} \, dx$

3.89 $\int x^{3/2} \coth^{-1}\left(\sqrt{x}\right) \, dx$

3.89.1 Optimal result

Integrand size = 12, antiderivative size = 38

$$\int x^{3/2} \coth^{-1}\left(\sqrt{x}\right) \, dx = \frac{x}{5} + \frac{x^2}{10} + \frac{2}{5} x^{5/2} \coth^{-1}\left(\sqrt{x}\right) + \frac{1}{5} \log(1-x)$$

output
```
1/5*x+1/10*x^2+2/5*x^(5/2)*arccoth(x^(1/2))+1/5*ln(1-x)
```

3.89.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int x^{3/2} \coth^{-1}\left(\sqrt{x}\right) \, dx = \frac{1}{10}\left(x(2+x) + 4x^{5/2} \coth^{-1}\left(\sqrt{x}\right) + 2\log(1-x)\right)$$

input
```
Integrate[x^(3/2)*ArcCoth[Sqrt[x]],x]
```

output
```
(x*(2 + x) + 4*x^(5/2)*ArcCoth[Sqrt[x]] + 2*Log[1 - x])/10
```

3.89.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6453, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2} \coth^{-1}\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{ 6453}$$

$$\frac{2}{5} x^{5/2} \coth^{-1}\left(\sqrt{x}\right) - \frac{1}{5} \int \frac{x^2}{1-x} dx$$

$$\Big\downarrow \text{ 49}$$

$$\frac{2}{5} x^{5/2} \coth^{-1}\left(\sqrt{x}\right) - \frac{1}{5} \int \left(-x + \frac{1}{1-x} - 1\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{5} x^{5/2} \coth^{-1}\left(\sqrt{x}\right) + \frac{1}{5}\left(\frac{x^2}{2} + x + \log(1-x)\right)$$

input
```
Int[x^(3/2)*ArcCoth[Sqrt[x]],x]
```

output
```
(2*x^(5/2)*ArcCoth[Sqrt[x]])/5 + (x + x^2/2 + Log[1 - x])/5
```

3.89.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.89.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{2x^{\frac{5}{2}}\operatorname{arccoth}\left(\sqrt{x}\right)}{5}+\frac{x^2}{10}+\frac{x}{5}+\frac{\ln\left(\sqrt{x}-1\right)}{5}+\frac{\ln\left(\sqrt{x}+1\right)}{5}$	35
default	$\frac{2x^{\frac{5}{2}}\operatorname{arccoth}\left(\sqrt{x}\right)}{5}+\frac{x^2}{10}+\frac{x}{5}+\frac{\ln\left(\sqrt{x}-1\right)}{5}+\frac{\ln\left(\sqrt{x}+1\right)}{5}$	35

input
```
int(x^(3/2)*arccoth(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^(5/2)*arccoth(x^(1/2))+1/10*x^2+1/5*x+1/5*ln(x^(1/2)-1)+1/5*ln(x^(1/
2)+1)
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int x^{3/2}\coth^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{5}\,x^{\frac{5}{2}}\log\left(\frac{x+2\,\sqrt{x}+1}{x-1}\right)+\frac{1}{10}\,x^2+\frac{1}{5}\,x+\frac{1}{5}\,\log\left(x-1\right)$$

input
```
integrate(x^(3/2)*arccoth(x^(1/2)),x, algorithm="fricas")
```

output
```
1/5*x^(5/2)*log((x + 2*sqrt(x) + 1)/(x - 1)) + 1/10*x^2 + 1/5*x + 1/5*log(
x - 1)
```

3.89.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(29) = 58.

Time = 0.99 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.18

$$\int x^{3/2}\coth^{-1}\left(\sqrt{x}\right)\,dx = \frac{4x^{\frac{7}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{10x-10} - \frac{4x^{\frac{5}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{10x-10} + \frac{x^3}{10x-10} + \frac{x^2}{10x-10} + \frac{4x\log\left(\sqrt{x}+1\right)}{10x-10} - \frac{4x\operatorname{acoth}\left(\sqrt{x}\right)}{10x-10} - \frac{4\log\left(\sqrt{x}+1\right)}{10x-10} + \frac{4\operatorname{acoth}\left(\sqrt{x}\right)}{10x-10} - \frac{2}{10x-10}$$

input
```
integrate(x**(3/2)*acoth(x**(1/2)),x)
```

output
```
4*x**(7/2)*acoth(sqrt(x))/(10*x - 10) - 4*x**(5/2)*acoth(sqrt(x))/(10*x -
10) + x**3/(10*x - 10) + x**2/(10*x - 10) + 4*x*log(sqrt(x) + 1)/(10*x - 1
0) - 4*x*acoth(sqrt(x))/(10*x - 10) - 4*log(sqrt(x) + 1)/(10*x - 10) + 4*a
coth(sqrt(x))/(10*x - 10) - 2/(10*x - 10)
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int x^{3/2}\coth^{-1}\left(\sqrt{x}\right)\,dx = \frac{2}{5}x^{\frac{5}{2}}\operatorname{arcoth}\left(\sqrt{x}\right) + \frac{1}{10}x^2 + \frac{1}{5}x + \frac{1}{5}\log\left(x-1\right)$$

input
```
integrate(x^(3/2)*arccoth(x^(1/2)),x, algorithm="maxima")
```

output
```
2/5*x^(5/2)*arccoth(sqrt(x)) + 1/10*x^2 + 1/5*x + 1/5*log(x - 1)
```

3.89.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 4.42

$$\int x^{3/2}\coth^{-1}\left(\sqrt{x}\right)\,dx = \frac{8\left(\frac{(\sqrt{x}+1)^3}{(\sqrt{x}-1)^3}-\frac{(\sqrt{x}+1)^2}{(\sqrt{x}-1)^2}+\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{5\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^4} + \frac{2\left(\frac{5(\sqrt{x}+1)^4}{(\sqrt{x}-1)^4}+\frac{10(\sqrt{x}+1)^2}{(\sqrt{x}-1)^2}+1\right)\log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{5\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^5} + \frac{2}{5}\log\left(\frac{\sqrt{x}+1}{|\sqrt{x}-1|}\right) - \frac{2}{5}\log\left(\left|\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right|\right)$$

input
```
integrate(x^(3/2)*arccoth(x^(1/2)),x, algorithm="giac")
```

output
```
8/5*((sqrt(x) + 1)^3/(sqrt(x) - 1)^3 - (sqrt(x) + 1)^2/(sqrt(x) - 1)^2 + (
sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^4 + 2/5*(5*(
sqrt(x) + 1)^4/(sqrt(x) - 1)^4 + 10*(sqrt(x) + 1)^2/(sqrt(x) - 1)^2 + 1)*l
og((sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^5 + 2/5*
log((sqrt(x) + 1)/abs(sqrt(x) - 1)) - 2/5*log(abs((sqrt(x) + 1)/(sqrt(x) -
 1) - 1))
```

3.89.9 Mupad [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int x^{3/2}\coth^{-1}\left(\sqrt{x}\right)\,dx = \frac{x}{5} + \frac{\ln(x-1)}{5} + \frac{2\,x^{5/2}\,\mathrm{acoth}\left(\sqrt{x}\right)}{5} + \frac{x^2}{10}$$

input
```
int(x^(3/2)*acoth(x^(1/2)),x)
```

output
```
x/5 + log(x - 1)/5 + (2*x^(5/2)*acoth(x^(1/2)))/5 + x^2/10
```

3.90 $\int \sqrt{x} \coth^{-1}\left(\sqrt{x}\right) \, dx$

3.90.1 Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \sqrt{x} \coth^{-1}\left(\sqrt{x}\right) \, dx = \frac{x}{3} + \frac{2}{3} x^{3/2} \coth^{-1}\left(\sqrt{x}\right) + \frac{1}{3} \log(1-x)$$

output
```
1/3*x+2/3*x^(3/2)*arccoth(x^(1/2))+1/3*ln(1-x)
```

3.90.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sqrt{x} \coth^{-1}\left(\sqrt{x}\right) \, dx = \frac{1}{3}\left(x + 2x^{3/2} \coth^{-1}\left(\sqrt{x}\right) + \log(1-x)\right)$$

input
```
Integrate[Sqrt[x]*ArcCoth[Sqrt[x]],x]
```

output
```
(x + 2*x^(3/2)*ArcCoth[Sqrt[x]] + Log[1 - x])/3
```

3.90.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6453, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\coth^{-1}\left(\sqrt{x}\right)dx$$

$$\Big\downarrow \text{ 6453}$$

$$\frac{2}{3}x^{3/2}\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int \frac{x}{1-x}dx$$

$$\Big\downarrow \text{ 49}$$

$$\frac{2}{3}x^{3/2}\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int \left(\frac{1}{1-x}-1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}x^{3/2}\coth^{-1}\left(\sqrt{x}\right)+\frac{1}{3}(x+\log(1-x))$$

input
```
Int[Sqrt[x]*ArcCoth[Sqrt[x]],x]
```

output
```
(2*x^(3/2)*ArcCoth[Sqrt[x]])/3 + (x + Log[1 - x])/3
```

3.90.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.90.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}\operatorname{arccoth}(\sqrt{x})}{3}+\frac{x}{3}+\frac{\ln(\sqrt{x}-1)}{3}+\frac{\ln(\sqrt{x}+1)}{3}$	30
default	$\frac{2x^{\frac{3}{2}}\operatorname{arccoth}(\sqrt{x})}{3}+\frac{x}{3}+\frac{\ln(\sqrt{x}-1)}{3}+\frac{\ln(\sqrt{x}+1)}{3}$	30

input
```
int(arccoth(x^(1/2))*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)*arccoth(x^(1/2))+1/3*x+1/3*ln(x^(1/2)-1)+1/3*ln(x^(1/2)+1)
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \sqrt{x}\coth^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{3}\,x^{\frac{3}{2}}\log\left(\frac{x+2\,\sqrt{x}+1}{x-1}\right)+\frac{1}{3}\,x+\frac{1}{3}\,\log\left(x-1\right)$$

input
```
integrate(arccoth(x^(1/2))*x^(1/2),x, algorithm="fricas")
```

output
```
1/3*x^(3/2)*log((x + 2*sqrt(x) + 1)/(x - 1)) + 1/3*x + 1/3*log(x - 1)
```

3.90.6 Sympy [F]

$$\int \sqrt{x}\coth^{-1}\left(\sqrt{x}\right)\,dx = \int \sqrt{x}\,\mathrm{acoth}\left(\sqrt{x}\right)dx$$

input
```
integrate(acoth(x**(1/2))*x**(1/2),x)
```

output
```
Integral(sqrt(x)*acoth(sqrt(x)), x)
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \sqrt{x}\coth^{-1}\left(\sqrt{x}\right)\,dx = \frac{2}{3}\,x^{\frac{3}{2}}\,\mathrm{arcoth}\left(\sqrt{x}\right) + \frac{1}{3}\,x + \frac{1}{3}\,\log\left(x-1\right)$$

input
```
integrate(arccoth(x^(1/2))*x^(1/2),x, algorithm="maxima")
```

output
```
2/3*x^(3/2)*arccoth(sqrt(x)) + 1/3*x + 1/3*log(x - 1)
```

3.90.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(21) = 42.

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.84

$$\int \sqrt{x}\coth^{-1}\left(\sqrt{x}\right)\,dx = \frac{2\left(\frac{3\left(\sqrt{x}+1\right)^2}{\left(\sqrt{x}-1\right)^2}+1\right)\log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{3\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^3} + \frac{4\left(\sqrt{x}+1\right)}{3\left(\sqrt{x}-1\right)\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^2} + \frac{2}{3}\log\left(\frac{\sqrt{x}+1}{\left|\sqrt{x}-1\right|}\right) - \frac{2}{3}\log\left(\left|\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right|\right)$$

input
```
integrate(arccoth(x^(1/2))*x^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(x) + 1)^2/(sqrt(x) - 1)^2 + 1)*log((sqrt(x) + 1)/(sqrt(x) - 1
))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^3 + 4/3*(sqrt(x) + 1)/((sqrt(x) - 1)*
((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^2) + 2/3*log((sqrt(x) + 1)/abs(sqrt(x) -
 1)) - 2/3*log(abs((sqrt(x) + 1)/(sqrt(x) - 1) - 1))
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\coth^{-1}\left(\sqrt{x}\right)\,dx = \int \sqrt{x}\,\mathrm{acoth}\left(\sqrt{x}\right)\,dx$$

input
```
int(x^(1/2)*acoth(x^(1/2)),x)
```

output
```
int(x^(1/2)*acoth(x^(1/2)), x)
```

3.91 $\int \frac{\coth^{-1}(\sqrt{x})}{\sqrt{x}}\,dx$

3.91.1 Optimal result

Integrand size = 12, antiderivative size = 20

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\coth^{-1}\left(\sqrt{x}\right) + \log(1-x)$$

output
```
ln(1-x)+2*arccoth(x^(1/2))*x^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\coth^{-1}\left(\sqrt{x}\right) + \log(1-x)$$

input
```
Integrate[ArcCoth[Sqrt[x]]/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcCoth[Sqrt[x]] + Log[1 - x]
```

3.91.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6453, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \; 6453$$

$$2\sqrt{x}\coth^{-1}\left(\sqrt{x}\right)-\int \frac{1}{1-x}dx$$

$$\Big\downarrow \; 16$$

$$\log(1-x)+2\sqrt{x}\coth^{-1}\left(\sqrt{x}\right)$$

input
```
Int[ArcCoth[Sqrt[x]]/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcCoth[Sqrt[x]] + Log[1 - x]
```

3.91.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.91.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$2 \operatorname{arccoth}\left(\sqrt{x}\right) \sqrt{x}+\ln \left(x -1\right)$	15
default	$2 \operatorname{arccoth}\left(\sqrt{x}\right) \sqrt{x}+\ln \left(x -1\right)$	15

input
```
int(arccoth(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arccoth(x^(1/2))*x^(1/2)+ln(x-1)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sqrt{x}\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)+\log\left(x-1\right)$$

input
```
integrate(arccoth(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x)*log((x + 2*sqrt(x) + 1)/(x - 1)) + log(x - 1)
```

3.91.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(17) = 34.

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.35

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2x^{\frac{3}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{x-1}-\frac{2\sqrt{x}\operatorname{acoth}\left(\sqrt{x}\right)}{x-1}+\frac{2x\log\left(\sqrt{x}+1\right)}{x-1}-\frac{2x\operatorname{acoth}\left(\sqrt{x}\right)}{x-1}-\frac{2\log\left(\sqrt{x}+1\right)}{x-1}+\frac{2\operatorname{acoth}\left(\sqrt{x}\right)}{x-1}$$

input
```
integrate(acoth(x**(1/2))/x**(1/2),x)
```

output
```
2*x**(3/2)*acoth(sqrt(x))/(x - 1) - 2*sqrt(x)*acoth(sqrt(x))/(x - 1) + 2*x
*log(sqrt(x) + 1)/(x - 1) - 2*x*acoth(sqrt(x))/(x - 1) - 2*log(sqrt(x) + 1
)/(x - 1) + 2*acoth(sqrt(x))/(x - 1)
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\coth^{-1}(\sqrt{x})}{\sqrt{x}}\,dx = 2\sqrt{x}\,\mathrm{arcoth}\left(\sqrt{x}\right) + \log\left(-x+1\right)$$

input
```
integrate(arccoth(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x)*arccoth(sqrt(x)) + log(-x + 1)
```

3.91.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(16) = 32.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.50

$$\int \frac{\coth^{-1}(\sqrt{x})}{\sqrt{x}}\,dx = \frac{2\log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\frac{\sqrt{x}+1}{\sqrt{x}-1}-1} + 2\log\left(\frac{\sqrt{x}+1}{\left|\sqrt{x}-1\right|}\right) - 2\log\left(\left|\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right|\right)$$

input
```
integrate(arccoth(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
2*log((sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1) + 2*l
og((sqrt(x) + 1)/abs(sqrt(x) - 1)) - 2*log(abs((sqrt(x) + 1)/(sqrt(x) - 1)
 - 1))
```

3.91.9 Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{\coth^{-1}(\sqrt{x})}{\sqrt{x}}\,dx = \ln\left(x-1\right) + 2\sqrt{x}\,\mathrm{acoth}\left(\sqrt{x}\right)$$

input
```
int(acoth(x^(1/2))/x^(1/2),x)
```

output
```
log(x - 1) + 2*x^(1/2)*acoth(x^(1/2))
```

3.92 $\int \frac{\coth^{-1}(\sqrt{x})}{x^{3/2}}\,dx$

3.92.1 Optimal result

Integrand size = 12, antiderivative size = 24

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} - \log(1-x) + \log(x)$$

output
```
-ln(1-x)+ln(x)-2*arccoth(x^(1/2))/x^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} - \log(1-x) + \log(x)$$

input
```
Integrate[ArcCoth[Sqrt[x]]/x^(3/2),x]
```

output
```
(-2*ArcCoth[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]
```

3.92.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6453, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx$$
$$\Big\downarrow \text{ 6453}$$
$$\int \frac{1}{(1-x)x}dx-\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$
$$\Big\downarrow \text{ 47}$$
$$\int \frac{1}{1-x}dx+\int \frac{1}{x}dx-\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$
$$\Big\downarrow \text{ 14}$$
$$\int \frac{1}{1-x}dx+\log(x)-\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$
$$\Big\downarrow \text{ 16}$$
$$-\log(1-x)+\log(x)-\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

input
```
Int[ArcCoth[Sqrt[x]]/x^(3/2),x]
```

output
```
(-2*ArcCoth[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]
```

3.92.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.92.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arccoth}\left(\sqrt{x}\right)}{\sqrt{x}}-\ln\left(\sqrt{x}-1\right)+\ln\left(x\right)-\ln\left(\sqrt{x}+1\right)$	29
default	$-\frac{2\,\mathrm{arccoth}\left(\sqrt{x}\right)}{\sqrt{x}}-\ln\left(\sqrt{x}-1\right)+\ln\left(x\right)-\ln\left(\sqrt{x}+1\right)$	29

input
```
int(arccoth(x^(1/2))/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arccoth(x^(1/2))/x^(1/2)-ln(x^(1/2)-1)+ln(x)-ln(x^(1/2)+1)
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx=-\frac{x\log\left(x-1\right)-x\log\left(x\right)+\sqrt{x}\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)}{x}$$

input
```
integrate(arccoth(x^(1/2))/x^(3/2),x, algorithm="fricas")
```

output
```
-(x*log(x - 1) - x*log(x) + sqrt(x)*log((x + 2*sqrt(x) + 1)/(x - 1)))/x
```

3.92.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(20) = 40$.

Time = 0.40 (sec) , antiderivative size = 126, normalized size of antiderivative = 5.25

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2x^{\frac{3}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{x^2-x}+\frac{2\sqrt{x}\operatorname{acoth}\left(\sqrt{x}\right)}{x^2-x}+\frac{x^2\log\left(x\right)}{x^2-x}-\frac{2x^2\log\left(\sqrt{x}+1\right)}{x^2-x}+\frac{2x^2\operatorname{acoth}\left(\sqrt{x}\right)}{x^2-x}-\frac{x\log\left(x\right)}{x^2-x}+\frac{2x\log\left(\sqrt{x}+1\right)}{x^2-x}-\frac{2x\operatorname{acoth}\left(\sqrt{x}\right)}{x^2-x}$$

input
```
integrate(acoth(x**(1/2))/x**(3/2),x)
```

output
```
-2*x**(3/2)*acoth(sqrt(x))/(x**2 - x) + 2*sqrt(x)*acoth(sqrt(x))/(x**2 - x
) + x**2*log(x)/(x**2 - x) - 2*x**2*log(sqrt(x) + 1)/(x**2 - x) + 2*x**2*a
coth(sqrt(x))/(x**2 - x) - x*log(x)/(x**2 - x) + 2*x*log(sqrt(x) + 1)/(x**
2 - x) - 2*x*acoth(sqrt(x))/(x**2 - x)
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\ \mathrm{arcoth}\left(\sqrt{x}\right)}{\sqrt{x}}-\log\left(x-1\right)+\log\left(x\right)$$

input
```
integrate(arccoth(x^(1/2))/x^(3/2),x, algorithm="maxima")
```

output
```
-2*arccoth(sqrt(x))/sqrt(x) - log(x - 1) + log(x)
```

3.92.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(20) = 40$.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.92

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = \frac{2\ \log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\frac{\sqrt{x}+1}{\sqrt{x}-1}+1}-2\ \log\left(\frac{\sqrt{x}+1}{\left|\sqrt{x}-1\right|}\right)+2\ \log\left(\left|\frac{\sqrt{x}+1}{\sqrt{x}-1}+1\right|\right)$$

input
```
integrate(arccoth(x^(1/2))/x^(3/2),x, algorithm="giac")
```

output
```
2*log((sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) + 1) - 2*l
og((sqrt(x) + 1)/abs(sqrt(x) - 1)) + 2*log(abs((sqrt(x) + 1)/(sqrt(x) - 1)
 + 1))
```

3.92.9 Mupad [B] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = 2\,\ln\left(\sqrt{x}\right) - \ln\left(x-1\right) - \frac{2\,\mathrm{acoth}\left(\sqrt{x}\right)}{\sqrt{x}}$$

input
```
int(acoth(x^(1/2))/x^(3/2),x)
```

output
```
2*log(x^(1/2)) - log(x - 1) - (2*acoth(x^(1/2)))/x^(1/2)
```

3.93 $\int \frac{\coth^{-1}(ax^5)}{x} \, dx$

3.93.1 Optimal result

Integrand size = 10, antiderivative size = 28

$$\int \frac{\coth^{-1}(ax^5)}{x} \, dx = \frac{1}{10}\operatorname{PolyLog}\left(2,-\frac{1}{ax^5}\right) - \frac{1}{10}\operatorname{PolyLog}\left(2,\frac{1}{ax^5}\right)$$

output
```
1/10*polylog(2,-1/a/x^5)-1/10*polylog(2,1/a/x^5)
```

3.93.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(ax^5)}{x} \, dx = \frac{1}{10}\left(\operatorname{PolyLog}\left(2,-\frac{1}{ax^5}\right) - \operatorname{PolyLog}\left(2,\frac{1}{ax^5}\right)\right)$$

input
```
Integrate[ArcCoth[a*x^5]/x,x]
```

output
```
(PolyLog[2, -(1/(a*x^5))] - PolyLog[2, 1/(a*x^5)])/10
```

3.93.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6451, 6447}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax^5)}{x}\,dx$$

$$\Big\downarrow \text{ 6451}$$

$$\frac{1}{5}\int \frac{\coth^{-1}(ax^5)}{x^5}dx^5$$

$$\Big\downarrow \text{ 6447}$$

$$\frac{1}{5}\left(\frac{1}{2}\text{PolyLog}\left(2,-\frac{1}{ax^5}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{1}{ax^5}\right)\right)$$

input
```
Int[ArcCoth[a*x^5]/x,x]
```

output
```
(PolyLog[2, -(1/(a*x^5))]/2 - PolyLog[2, 1/(a*x^5)]/2)/5
```

3.93.3.1 Defintions of rubi rules used

rule 6447
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

rule 6451
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcCoth[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

3.93.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.39

method	result
default	$\ln(x)\operatorname{arccoth}(a\,x^5)+5a\left(-\frac{\sum_{\_R1=\operatorname{RootOf}(a\_Z^5+1)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)}{10a}+\frac{\sum_{\_R1=\operatorname{RootOf}(a\_Z}\right.$
parts	$\ln(x)\operatorname{arccoth}(a\,x^5)+5a\left(-\frac{\sum_{\_R1=\operatorname{RootOf}(a\_Z^5+1)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)}{10a}+\frac{\sum_{\_R1=\operatorname{RootOf}(a\_Z}\right.$
risch	$\frac{\ln(x)\ln(a\,x^5+1)}{2}-\frac{\left(\sum_{\_R1=\operatorname{RootOf}(a\_Z^5+1)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{2}-\frac{\ln(x)\ln(a\,x^5-1)}{2}+\frac{\left(\sum_{\_R1=\operatorname{Roo}}\right.}{}$

input
```
int(arccoth(a*x^5)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arccoth(a*x^5)+5*a*(-1/10/a*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/
_R1),_R1=RootOf(_Z^5*a+1))+1/10/a*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/
_R1),_R1=RootOf(_Z^5*a-1)))
```

3.93.5 Fricas [F]

$$\int \frac{\coth^{-1}(ax^5)}{x}\,dx=\int \frac{\operatorname{arcoth}(ax^5)}{x}\,dx$$

input
```
integrate(arccoth(a*x^5)/x,x, algorithm="fricas")
```

output
```
integral(arccoth(a*x^5)/x, x)
```

3.93.6 Sympy [F]

$$\int \frac{\coth^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{acoth}(ax^5)}{x}\,dx$$

input
```
integrate(acoth(a*x**5)/x,x)
```

output
```
Integral(acoth(a*x**5)/x, x)
```

3.93.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(22) = 44$.

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.71

$$\int \frac{\coth^{-1}(ax^5)}{x}\,dx = -\frac{1}{2}a\left(\frac{\log(ax^5+1)}{a} - \frac{\log(ax^5-1)}{a}\right)\log(x) - \frac{1}{10}a\left(\frac{\log(ax^5-1)\log(ax^5) + \operatorname{Li}_2(-ax^5+1)}{a} - \frac{\log(ax^5+1)\log(-ax^5) + \operatorname{Li}_2(ax^5+1)}{a}\right) + \operatorname{arcoth}(ax^5)\log(x)$$

input
```
integrate(arccoth(a*x^5)/x,x, algorithm="maxima")
```

output
```
-1/2*a*(log(a*x^5 + 1)/a - log(a*x^5 - 1)/a)*log(x) - 1/10*a*((log(a*x^5 -
 1)*log(a*x^5) + dilog(-a*x^5 + 1))/a - (log(a*x^5 + 1)*log(-a*x^5) + dilo
g(a*x^5 + 1))/a) + arccoth(a*x^5)*log(x)
```

3.93.8 Giac [F]

$$\int \frac{\coth^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{arcoth}(ax^5)}{x}\,dx$$

input
```
integrate(arccoth(a*x^5)/x,x, algorithm="giac")
```

output
```
integrate(arccoth(a*x^5)/x, x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax^5)}{x} \, dx = \int \frac{\mathrm{acoth}(a\,x^5)}{x} \, dx$$

input
```
int(acoth(a*x^5)/x,x)
```

output
```
int(acoth(a*x^5)/x, x)
```

3.94 $\int \coth^{-1}\left(\frac{1}{x}\right)\,dx$

3.94.1 Optimal result

Integrand size = 4, antiderivative size = 19

$$\int \coth^{-1}\left(\frac{1}{x}\right)\,dx = x\coth^{-1}\left(\frac{1}{x}\right)+\frac{1}{2}\log\left(1-x^2\right)$$

output
```
x*arccoth(1/x)+1/2*ln(-x^2+1)
```

3.94.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \coth^{-1}\left(\frac{1}{x}\right)\,dx = x\coth^{-1}\left(\frac{1}{x}\right)+\frac{1}{2}\log\left(1-x^2\right)$$

input
```
Integrate[ArcCoth[x^(-1)],x]
```

output
```
x*ArcCoth[x^(-1)] + Log[1 - x^2]/2
```

3.94.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6437, 795, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}\left(\frac{1}{x}\right) dx$$

$$\Big\downarrow \text{ 6437}$$

$$\int \frac{1}{\left(1-\frac{1}{x^2}\right) x}dx + x\coth^{-1}\left(\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x}{x^2-1}dx + x\coth^{-1}\left(\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 240}$$

$$\frac{1}{2}\log\left(1-x^2\right) + x\coth^{-1}\left(\frac{1}{x}\right)$$

input
```
Int[ArcCoth[x^(-1)],x]
```

output
```
x*ArcCoth[x^(-1)] + Log[1 - x^2]/2
```

3.94.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

3.94.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result
parallelrisch	$x \operatorname{arccoth}\left(\frac{1}{x}\right)+\ln\left(1+x\right)-\operatorname{arccoth}\left(\frac{1}{x}\right)$
parts	$x \operatorname{arccoth}\left(\frac{1}{x}\right)+\frac{\ln(x-1)}{2}+\frac{\ln(1+x)}{2}$
derivativedivides	$x \operatorname{arccoth}\left(\frac{1}{x}\right)+\frac{\ln\left(\frac{1}{x}+1\right)}{2}-\ln\left(\frac{1}{x}\right)+\frac{\ln\left(\frac{1}{x}-1\right)}{2}$
default	$x \operatorname{arccoth}\left(\frac{1}{x}\right)+\frac{\ln\left(\frac{1}{x}+1\right)}{2}-\ln\left(\frac{1}{x}\right)+\frac{\ln\left(\frac{1}{x}-1\right)}{2}$
risch	$-\frac{\ln(x-1)x}{2}+\frac{\ln(1+x)x}{2}+\frac{i\pi \operatorname{csgn}\left(\frac{i(1+x)}{x}\right)^{2} \operatorname{csgn}\left(\frac{i}{x}\right)x}{4}-\frac{i\pi \operatorname{csgn}\left(\frac{i(x-1)}{x}\right)^{2} \operatorname{csgn}\left(\frac{i}{x}\right)x}{4}-\frac{i\pi \operatorname{csgn}\left(\frac{i(x-1)}{x}\right)^{3}x}{4}+$

input
```
int(arccoth(1/x),x,method=_RETURNVERBOSE)
```

output
```
x*arccoth(1/x)+ln(1+x)-arccoth(1/x)
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \coth^{-1}\left(\frac{1}{x}\right)\,dx=\frac{1}{2}\,x\log\left(-\frac{x+1}{x-1}\right)+\frac{1}{2}\,\log\left(x^{2}-1\right)$$

input
```
integrate(arccoth(1/x),x, algorithm="fricas")
```

output
```
1/2*x*log(-(x + 1)/(x - 1)) + 1/2*log(x^2 - 1)
```

3.94.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \coth^{-1}\left(\frac{1}{x}\right)\,dx = x\,\mathrm{acoth}\left(\frac{1}{x}\right) + \log\left(x+1\right) - \mathrm{acoth}\left(\frac{1}{x}\right)$$

input
```
integrate(acoth(1/x),x)
```

output
```
x*acoth(1/x) + log(x + 1) - acoth(1/x)
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \coth^{-1}\left(\frac{1}{x}\right)\,dx = x\,\mathrm{arcoth}\left(\frac{1}{x}\right) + \frac{1}{2}\log\left(x^2-1\right)$$

input
```
integrate(arccoth(1/x),x, algorithm="maxima")
```

output
```
x*arccoth(1/x) + 1/2*log(x^2 - 1)
```

3.94.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(17) = 34.

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 5.47

$$\int \coth^{-1}\left(\frac{1}{x}\right)\,dx = \frac{\log\left(-\frac{\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}+1}{\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}-1}\right)}{\frac{x+1}{x-1}-1} + \log\left(\frac{|-x-1|}{|x-1|}\right) - \log\left(\left|-\frac{x+1}{x-1}+1\right|\right)$$

input
```
integrate(arccoth(1/x),x, algorithm="giac")
```

output
```
log(-((((x + 1)/(x - 1) + 1)/((x + 1)/(x - 1) - 1) + 1)/((((x + 1)/(x - 1) +
 1)/((x + 1)/(x - 1) - 1) - 1))/((x + 1)/(x - 1) - 1) + log(abs(-x - 1)/ab
s(x - 1)) - log(abs(-(x + 1)/(x - 1) + 1))
```

3.94.9 Mupad [B] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \coth^{-1}\left(\frac{1}{x}\right) dx = \frac{\ln(x^2-1)}{2} + x\left(\frac{\ln(x+1)}{2} - \frac{\ln(1-x)}{2}\right)$$

input
```
int(acoth(1/x),x)
```

output
```
log(x^2 - 1)/2 + x*(log(x + 1)/2 - log(1 - x)/2)
```

3.95 $\int \frac{\coth^{-1}(ax^n)}{x} dx$

3.95.1 Optimal result

Integrand size = 10, antiderivative size = 38

$$\int \frac{\coth^{-1}(ax^n)}{x} dx = \frac{\text{PolyLog}\left(2, -\frac{x^{-n}}{a}\right)}{2n} - \frac{\text{PolyLog}\left(2, \frac{x^{-n}}{a}\right)}{2n}$$

output
```
1/2*polylog(2,-1/a/(x^n))/n-1/2*polylog(2,1/a/(x^n))/n
```

3.95.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \frac{\coth^{-1}(ax^n)}{x} dx = \frac{ax^n\, {}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}; a^2x^{2n}\right)}{n} + \left(\coth^{-1}(ax^n) - \text{arctanh}(ax^n)\right)\log(x)$$

input
```
Integrate[ArcCoth[a*x^n]/x,x]
```

output
```
(a*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, a^2*x^(2*n)])/n + (Arc
Coth[a*x^n] - ArcTanh[a*x^n])*Log[x]
```

3.95.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6451, 6447}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(ax^n)}{x} \, dx$$

$$\Big\downarrow \text{ 6451}$$

$$\frac{\int x^{-n}\coth^{-1}(ax^n)\,dx^n}{n}$$

$$\Big\downarrow \text{ 6447}$$

$$\frac{\frac{1}{2}\text{PolyLog}\left(2,-\frac{x^{-n}}{a}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{x^{-n}}{a}\right)}{n}$$

input
```
Int[ArcCoth[a*x^n]/x,x]
```

output
```
(PolyLog[2, -(1/(a*x^n))]/2 - PolyLog[2, 1/(a*x^n)]/2)/n
```

3.95.3.1 Defintions of rubi rules used

rule 6447
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

rule 6451
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcCoth[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

3.95.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

method	result	size
risch	$-\frac{\ln(a\,x^n-1)\ln(a\,x^n)}{2n}-\frac{\mathrm{dilog}(a\,x^n)}{2n}-\frac{\mathrm{dilog}(a\,x^n+1)}{2n}$	45
derivativedivides	$\frac{\ln(a\,x^n)\,\mathrm{arccoth}(a\,x^n)-\frac{\mathrm{dilog}(a\,x^n)}{2}-\frac{\mathrm{dilog}(a\,x^n+1)}{2}-\frac{\ln(a\,x^n)\ln(a\,x^n+1)}{2}}{n}$	53
default	$\frac{\ln(a\,x^n)\,\mathrm{arccoth}(a\,x^n)-\frac{\mathrm{dilog}(a\,x^n)}{2}-\frac{\mathrm{dilog}(a\,x^n+1)}{2}-\frac{\ln(a\,x^n)\ln(a\,x^n+1)}{2}}{n}$	53

input
```
int(arccoth(a*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2/n*ln(a*x^n-1)*ln(a*x^n)-1/2/n*dilog(a*x^n)-1/2/n*dilog(a*x^n+1)
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(32) = 64.

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.37

$$\int \frac{\coth^{-1}(ax^n)}{x}\,dx =$$

$$-\frac{n\log\left(a\cosh\left(n\log\left(x\right)\right)+a\sinh\left(n\log\left(x\right)\right)+1\right)\log\left(x\right)-n\log\left(-a\cosh\left(n\log\left(x\right)\right)-a\sinh\left(n\log\left(x\right)\right)\right.}{}$$

input
```
integrate(arccoth(a*x^n)/x,x, algorithm="fricas")
```

output
```
-1/2*(n*log(a*cosh(n*log(x)) + a*sinh(n*log(x)) + 1)*log(x) - n*log(-a*cos
h(n*log(x)) - a*sinh(n*log(x)) + 1)*log(x) - n*log(x)*log((a*cosh(n*log(x)
) + a*sinh(n*log(x)) + 1)/(a*cosh(n*log(x)) + a*sinh(n*log(x)) - 1)) - dil
og(a*cosh(n*log(x)) + a*sinh(n*log(x))) + dilog(-a*cosh(n*log(x)) - a*sinh
(n*log(x))))/n
```

3.95.6 Sympy [F]

$$\int \frac{\coth^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{acoth}(ax^n)}{x}\,dx$$

input
```
integrate(acoth(a*x**n)/x,x)
```

output
```
Integral(acoth(a*x**n)/x, x)
```

3.95.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(32) = 64.

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.87

$$\int \frac{\coth^{-1}(ax^n)}{x}\,dx = -\frac{1}{2}\,an\left(\frac{\log\left(\frac{ax^n+1}{a}\right)}{an} - \frac{\log\left(\frac{ax^n-1}{a}\right)}{an}\right)\log(x) + \frac{1}{2}\,an\left(\frac{\log(ax^n+1)\log(x) - \log(ax^n-1)\log(x)}{an} - \frac{n\log(ax^n+1)\log(x) + \mathrm{Li}_2(-ax^n)}{an^2} + \frac{n\log(-ax^{n}\right. + \operatorname{arcoth}(ax^n)\log(x)$$

input
```
integrate(arccoth(a*x^n)/x,x, algorithm="maxima")
```

output
```
-1/2*a*n*(log((a*x^n + 1)/a)/(a*n) - log((a*x^n - 1)/a)/(a*n))*log(x) + 1/
2*a*n*((log(a*x^n + 1)*log(x) - log(a*x^n - 1)*log(x))/(a*n) - (n*log(a*x^
n + 1)*log(x) + dilog(-a*x^n))/(a*n^2) + (n*log(-a*x^n + 1)*log(x) + dilog
(a*x^n))/(a*n^2)) + arccoth(a*x^n)*log(x)
```

3.95.8 Giac [F]

$$\int \frac{\coth^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{arcoth}(ax^n)}{x}\,dx$$

input
```
integrate(arccoth(a*x^n)/x,x, algorithm="giac")
```

output
```
integrate(arccoth(a*x^n)/x, x)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(ax^n)}{x} \, dx = \int \frac{\mathrm{acoth}(a\, x^n)}{x} \, dx$$

input
```
int(acoth(a*x^n)/x,x)
```

output
```
int(acoth(a*x^n)/x, x)
```

3.96 $\int (a+bx)\coth^{-1}(a+bx)\,dx$

3.96.1 Optimal result

Integrand size = 12, antiderivative size = 39

$$\int (a+bx)\coth^{-1}(a+bx)\,dx = \frac{x}{2}+\frac{(a+bx)^2\coth^{-1}(a+bx)}{2b}-\frac{\text{arctanh}(a+bx)}{2b}$$

output
```
1/2*x+1/2*(b*x+a)^2*arccoth(b*x+a)/b-1/2*arctanh(b*x+a)/b
```

3.96.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.69

$$\int (a+bx)\coth^{-1}(a+bx)\,dx$$
$$=\frac{2bx+2bx(2a+bx)\coth^{-1}(a+bx)-(-1+a^2)\log(1-a-bx)-\log(1+a+bx)+a^2\log(1+a+bx)}{4b}$$

input
```
Integrate[(a + b*x)*ArcCoth[a + b*x],x]
```

output
```
(2*b*x + 2*b*x*(2*a + b*x)*ArcCoth[a + b*x] - (-1 + a^2)*Log[1 - a - b*x]
- Log[1 + a + b*x] + a^2*Log[1 + a + b*x])/(4*b)
```

3.96.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6658, 6453, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\coth^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{6658}$$

$$\frac{\int (a+bx)\coth^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{6453}$$

$$\frac{\frac{1}{2}(a+bx)^2\coth^{-1}(a+bx)-\frac{1}{2}\int \frac{(a+bx)^2}{1-(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{1}{2}\left(-\int \frac{1}{1-(a+bx)^2}d(a+bx)+a+bx\right)+\frac{1}{2}(a+bx)^2\coth^{-1}(a+bx)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}(-\text{arctanh}(a+bx)+a+bx)+\frac{1}{2}(a+bx)^2\coth^{-1}(a+bx)}{b}$$

input
```
Int[(a + b*x)*ArcCoth[a + b*x],x]
```

output
```
(((a + b*x)^2*ArcCoth[a + b*x])/2 + (a + b*x - ArcTanh[a + b*x])/2)/b
```

3.96.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6658
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCoth[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.96.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{(bx+a)^2 \operatorname{arccoth}(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}+\frac{\ln(bx+a-1)}{4}-\frac{\ln(bx+a+1)}{4}}{b}$
default	$\frac{\frac{(bx+a)^2 \operatorname{arccoth}(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}+\frac{\ln(bx+a-1)}{4}-\frac{\ln(bx+a+1)}{4}}{b}$
parallelrisch	$-\frac{-b^3 \operatorname{arccoth}(bx+a)x^2-2a \operatorname{arccoth}(bx+a)x b^2-\operatorname{arccoth}(bx+a)a^2 b-b^2 x+\operatorname{arccoth}(bx+a)b+2ab}{2b^2}$
parts	$\frac{\operatorname{arccoth}(bx+a)b x^2}{2}+\operatorname{arccoth}(bx+a)\,ax+\frac{b\left(\frac{x}{b}+\frac{(-a^2+1)\ln(bx+a-1)}{2b^2}+\frac{(a^2-1)\ln(bx+a+1)}{2b^2}\right)}{2}$
risch	$\left(\frac{1}{4}b x^2+\frac{1}{2}ax\right)\ln(bx+a+1)-\frac{b x^2 \ln(bx+a-1)}{4}-\frac{ax\ln(bx+a-1)}{2}-\frac{\ln(bx+a-1)a^2}{4b}+\frac{\ln(-bx-a-1)}{4b}$

input
```
int((b*x+a)*arccoth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*(b*x+a)^2*arccoth(b*x+a)+1/2*b*x+1/2*a+1/4*ln(b*x+a-1)-1/4*ln(b*x
+a+1))
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int (a+bx)\coth^{-1}(a+bx)\,dx=\frac{2\,bx+(b^2x^2+2\,abx+a^2-1)\log\left(\frac{bx+a+1}{bx+a-1}\right)}{4\,b}$$

input
```
integrate((b*x+a)*arccoth(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(2*b*x + (b^2*x^2 + 2*a*b*x + a^2 - 1)*log((b*x + a + 1)/(b*x + a - 1)
))/b
```

3.96.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int (a+bx)\coth^{-1}(a+bx)\,dx$$
$$=\begin{cases}\frac{a^2\operatorname{acoth}(a+bx)}{2b}+ax\operatorname{acoth}(a+bx)+\frac{bx^2\operatorname{acoth}(a+bx)}{2}+\frac{x}{2}-\frac{\operatorname{acoth}(a+bx)}{2b} & \text{for } b\neq 0\\ ax\operatorname{acoth}(a) & \text{otherwise}\end{cases}$$

input
```
integrate((b*x+a)*acoth(b*x+a),x)
```

output
```
Piecewise((a**2*acoth(a + b*x)/(2*b) + a*x*acoth(a + b*x) + b*x**2*acoth(a
 + b*x)/2 + x/2 - acoth(a + b*x)/(2*b), Ne(b, 0)), (a*x*acoth(a), True))
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.59

$$\int (a+bx)\coth^{-1}(a+bx)\,dx$$
$$=\frac{1}{4}\,b\left(\frac{2\,x}{b}+\frac{(a^2-1)\log(bx+a+1)}{b^2}-\frac{(a^2-1)\log(bx+a-1)}{b^2}\right)$$
$$+\frac{1}{2}\left(bx^2+2\,ax\right)\operatorname{arcoth}(bx+a)$$

input
```
integrate((b*x+a)*arccoth(b*x+a),x, algorithm="maxima")
```

output
```
1/4*b*(2*x/b + (a^2 - 1)*log(b*x + a + 1)/b^2 - (a^2 - 1)*log(b*x + a - 1)
/b^2) + 1/2*(b*x^2 + 2*a*x)*arccoth(b*x + a)
```

3.96.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(33) = 66$.

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 4.82

$$\int (a+bx)\coth^{-1}(a+bx)\,dx$$

$$= \frac{1}{2}\left((a+1)b-(a-1)b\right)\left(\frac{1}{b^2\left(\frac{bx+a+1}{bx+a-1}-1\right)}+\frac{(bx+a+1)\log\left(-\frac{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}+1}{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}-1}\right)}{(bx+a-1)b^2\left(\frac{bx+a+1}{bx+a-1}-1\right)^2}\right)$$

input
```
integrate((b*x+a)*arccoth(b*x+a),x, algorithm="giac")
```

output
```
1/2*((a + 1)*b - (a - 1)*b)*(1/(b^2*((b*x + a + 1)/(b*x + a - 1) - 1)) + (
b*x + a + 1)*log(-(1/(a - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/
((b*x + a + 1)*b/(b*x + a - 1) - b)) + 1)/(1/(a - ((b*x + a + 1)*(a - 1)/(
b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a - 1) - b)) - 1))/((b*x +
 a - 1)*b^2*((b*x + a + 1)/(b*x + a - 1) - 1)^2))
```

3.96.9 Mupad [B] (verification not implemented)

Time = 4.78 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int (a+bx)\coth^{-1}(a+bx)\,dx = \frac{x}{2}-\frac{\frac{\mathrm{acoth}(a+b\,x)}{2}-\frac{a^2\,\mathrm{acoth}(a+b\,x)}{2}}{b} + a\,x\,\mathrm{acoth}(a+b\,x)+\frac{b\,x^2\,\mathrm{acoth}(a+b\,x)}{2}$$

input
```
int(acoth(a + b*x)*(a + b*x),x)
```

output
```
x/2 - (acoth(a + b*x)/2 - (a^2*acoth(a + b*x))/2)/b + a*x*acoth(a + b*x) +
 (b*x^2*acoth(a + b*x))/2
```

3.97 $\int (a+bx)^2 \coth^{-1}(a+bx)\,dx$

3.97.1 Optimal result

Integrand size = 14, antiderivative size = 54

$$\int (a+bx)^2 \coth^{-1}(a+bx)\,dx = \frac{(a+bx)^2}{6b} + \frac{(a+bx)^3 \coth^{-1}(a+bx)}{3b} + \frac{\log\left(1-(a+bx)^2\right)}{6b}$$

output
```
1/6*(b*x+a)^2/b+1/3*(b*x+a)^3*arccoth(b*x+a)/b+1/6*ln(1-(b*x+a)^2)/b
```

3.97.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int (a+bx)^2 \coth^{-1}(a+bx)\,dx = \frac{(a+bx)^2 + 2(a+bx)^3 \coth^{-1}(a+bx) + \log\left(1-(a+bx)^2\right)}{6b}$$

input
```
Integrate[(a + b*x)^2*ArcCoth[a + b*x],x]
```

output
```
((a + b*x)^2 + 2*(a + b*x)^3*ArcCoth[a + b*x] + Log[1 - (a + b*x)^2])/(6*b
)
```

3.97.3 Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6658, 6453, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2 \coth^{-1}(a+bx) \, dx$$

$$\Big\downarrow \text{ 6658}$$

$$\frac{\int (a+bx)^2 \coth^{-1}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{ 6453}$$

$$\frac{\frac{1}{3}(a+bx)^3 \coth^{-1}(a+bx)-\frac{1}{3}\int \frac{(a+bx)^3}{1-(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\frac{1}{3}(a+bx)^3 \coth^{-1}(a+bx)-\frac{1}{6}\int \frac{(a+bx)^2}{-a-bx+1}d(a+bx)^2}{b}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\frac{1}{3}(a+bx)^3 \coth^{-1}(a+bx)-\frac{1}{6}\int \left(\frac{1}{-a-bx+1}-1\right)d(a+bx)^2}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{6}(\log(-a-bx+1)+a+bx)+\frac{1}{3}(a+bx)^3 \coth^{-1}(a+bx)}{b}$$

input
```
Int[(a + b*x)^2*ArcCoth[a + b*x],x]
```

output
```
(((a + b*x)^3*ArcCoth[a + b*x])/3 + (a + b*x + Log[1 - a - b*x])/6)/b
```

3.97.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6658
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCoth[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.97.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{\operatorname{arccoth}(bx+a)(bx+a)^3}{3}+\frac{(bx+a)^2}{6}+\frac{\ln(bx+a-1)}{6}+\frac{\ln(bx+a+1)}{6}}{b}$
default	$\frac{\frac{\operatorname{arccoth}(bx+a)(bx+a)^3}{3}+\frac{(bx+a)^2}{6}+\frac{\ln(bx+a-1)}{6}+\frac{\ln(bx+a+1)}{6}}{b}$
parts	$\frac{\operatorname{arccoth}(bx+a)b^2x^3}{3}+\operatorname{arccoth}(bx+a)\,ba\,x^2+\operatorname{arccoth}(bx+a)\,a^2x+\frac{\operatorname{arccoth}(bx+a)a^3}{3b}+\frac{b\,x^2}{6}+$
parallelrisch	$-\frac{-2b^4\operatorname{arccoth}(bx+a)x^3-6x^2\operatorname{arccoth}(bx+a)a\,b^3-6x\,\operatorname{arccoth}(bx+a)a^2b^2-b^3x^2-2\,\operatorname{arccoth}(bx+a)a^3b-2a\,b^2x+5b\,a^2-}{6b^2}$
risch	$\frac{(bx+a)^3\ln(bx+a+1)}{6b}-\frac{b^2\ln(bx+a-1)x^3}{6}-\frac{b\ln(bx+a-1)x^2a}{2}-\frac{a^2x\ln(bx+a-1)}{2}-\frac{\ln(bx+a-1)a^3}{6b}+\frac{b\,x^2}{6}+$

input
```
int((b*x+a)^2*arccoth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*arccoth(b*x+a)*(b*x+a)^3+1/6*(b*x+a)^2+1/6*ln(b*x+a-1)+1/6*ln(b*x
+a+1))
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.59

$$\int (a+bx)^2 \coth^{-1}(a+bx)\,dx$$
$$= \frac{b^2x^2 + 2\,abx + (a^3+1)\log(bx+a+1) - (a^3-1)\log(bx+a-1) + (b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx)\log\left(\frac{bx+a+}{bx+a-}\right.}{6\,b}$$

input
```
integrate((b*x+a)^2*arccoth(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(b^2*x^2 + 2*a*b*x + (a^3 + 1)*log(b*x + a + 1) - (a^3 - 1)*log(b*x +
a - 1) + (b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)*log((b*x + a + 1)/(b*x + a -
1)))/b
```

3.97.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(39) = 78.

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.80

$$\int (a+bx)^2 \coth^{-1}(a+bx)\,dx$$
$$= \begin{cases} \frac{a^3\operatorname{acoth}(a+bx)}{3b} + a^2x\operatorname{acoth}(a+bx) + abx^2\operatorname{acoth}(a+bx) + \frac{ax}{3} + \frac{b^2x^3\operatorname{acoth}(a+bx)}{3} + \frac{bx^2}{6} + \frac{\log\left(\frac{a}{b}+x+\frac{1}{b}\right)}{3b} - \frac{\operatorname{acot}}{} \\ a^2x\operatorname{acoth}(a) \end{cases}$$

input
```
integrate((b*x+a)**2*acoth(b*x+a),x)
```

output
```
Piecewise((a**3*acoth(a + b*x)/(3*b) + a**2*x*acoth(a + b*x) + a*b*x**2*ac
oth(a + b*x) + a*x/3 + b**2*x**3*acoth(a + b*x)/3 + b*x**2/6 + log(a/b + x
 + 1/b)/(3*b) - acoth(a + b*x)/(3*b), Ne(b, 0)), (a**2*x*acoth(a), True))
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$
\begin{aligned}
&\int (a+bx)^2 \coth^{-1}(a+bx)\,dx \\
&= \frac{1}{6}\,b\left(\frac{bx^2+2\,ax}{b} + \frac{(a^3+1)\log(bx+a+1)}{b^2} - \frac{(a^3-1)\log(bx+a-1)}{b^2}\right) \\
&\quad + \frac{1}{3}\left(b^2x^3+3\,abx^2+3\,a^2x\right)\operatorname{arcoth}(bx+a)
\end{aligned}
$$

input
```
integrate((b*x+a)^2*arccoth(b*x+a),x, algorithm="maxima")
```

output
```
1/6*b*((b*x^2 + 2*a*x)/b + (a^3 + 1)*log(b*x + a + 1)/b^2 - (a^3 - 1)*log(
b*x + a - 1)/b^2) + 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*arccoth(b*x + a)
```

3.97.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(48) = 96$.

Time = 0.28 (sec) , antiderivative size = 255, normalized size of antiderivative = 4.72

$$
\int (a+bx)^2 \coth^{-1}(a+bx)\,dx
$$

$$
= \frac{1}{6}\left((a+1)b-(a-1)b\right)\left(\frac{\log\left(\frac{|bx+a+1|}{|bx+a-1|}\right)}{b^2} - \frac{\log\left(\left|\frac{bx+a+1}{bx+a-1}-1\right|\right)}{b^2} + \frac{\left(\frac{3\,(bx+a+1)^2}{(bx+a-1)^2}+1\right)\log\left(-\frac{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a\cdots\right)}{\frac{(bx+a+1)b}{bx+a-1}-b\cdots}}\cdots}{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a\cdots\right)}{\frac{(bx+a+1)b}{bx+a-1}-b\cdots}}\cdots}\cdots\right.}{b^2\left(\frac{bx+a+1}{bx+a-1}-1\right)^3}\cdots\right.
$$

input
```
integrate((b*x+a)^2*arccoth(b*x+a),x, algorithm="giac")
```

output
```
1/6*((a + 1)*b - (a - 1)*b)*(log(abs(b*x + a + 1)/abs(b*x + a - 1))/b^2 -
log(abs((b*x + a + 1)/(b*x + a - 1) - 1))/b^2 + (3*(b*x + a + 1)^2/(b*x +
a - 1)^2 + 1)*log(-(1/(a - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b
/((b*x + a + 1)*b/(b*x + a - 1) - b)) + 1)/(1/(a - ((b*x + a + 1)*(a - 1)/
(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a - 1) - b)) - 1))/(b^2*(
(b*x + a + 1)/(b*x + a - 1) - 1)^3) + 2*(b*x + a + 1)/((b*x + a - 1)*b^2*(
(b*x + a + 1)/(b*x + a - 1) - 1)^2))
```

3.97.9 Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.11

$$\int (a+bx)^2 \coth^{-1}(a+bx)\,dx = \frac{a\,x}{3} + \ln\left(\frac{1}{a+b\,x}+1\right)\left(\frac{a^2\,x}{2}+\frac{a\,b\,x^2}{2}+\frac{b^2\,x^3}{6}\right) + \frac{b\,x^2}{6} - \ln\left(1-\frac{1}{a+b\,x}\right)\left(\frac{a^2\,x}{2}+\frac{a\,b\,x^2}{2}+\frac{b^2\,x^3}{6}\right) - \frac{\ln\left(a+b\,x-1\right)\,\left(a^3-1\right)}{6\,b} + \frac{\ln\left(a+b\,x+1\right)\,\left(a^3+1\right)}{6\,b}$$

input
```
int(acoth(a + b*x)*(a + b*x)^2,x)
```

output
```
(a*x)/3 + log(1/(a + b*x) + 1)*((a^2*x)/2 + (b^2*x^3)/6 + (a*b*x^2)/2) + (
b*x^2)/6 - log(1 - 1/(a + b*x))*((a^2*x)/2 + (b^2*x^3)/6 + (a*b*x^2)/2) -
(log(a + b*x - 1)*(a^3 - 1))/(6*b) + (log(a + b*x + 1)*(a^3 + 1))/(6*b)
```

3.98 $\int \frac{\coth^{-1}(a+bx)}{a+bx}\,dx$

3.98.1 Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{\coth^{-1}(a+bx)}{a+bx}\,dx=\frac{\operatorname{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{2b}-\frac{\operatorname{PolyLog}\left(2,\frac{1}{a+bx}\right)}{2b}$$

output
```
1/2*polylog(2,-1/(b*x+a))/b-1/2*polylog(2,1/(b*x+a))/b
```

3.98.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 144 vs. 2(35) = 70.

Time = 0.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 4.11

$$\int \frac{\coth^{-1}(a+bx)}{a+bx}\,dx = \frac{\log^2\left(-\frac{1}{a+bx}\right)-2\log(1-a-bx)\log\left(\frac{1}{a+bx}\right)-\log^2\left(\frac{1}{a+bx}\right)+2\log\left(\frac{1}{a+bx}\right)\log\left(\frac{-1+a+bx}{a+bx}\right)+2\log\left(-\frac{1}{a+bx}\right)\log}{4b}$$

input
```
Integrate[ArcCoth[a + b*x]/(a + b*x),x]
```

output
```
(Log[-(a + b*x)^(-1)]^2 - 2*Log[1 - a - b*x]*Log[(a + b*x)^(-1)] - Log[(a
+ b*x)^(-1)]^2 + 2*Log[(a + b*x)^(-1)]*Log[(-1 + a + b*x)/(a + b*x)] + 2*L
og[-(a + b*x)^(-1)]*Log[1 + a + b*x] - 2*Log[-(a + b*x)^(-1)]*Log[(1 + a +
 b*x)/(a + b*x)] - 2*PolyLog[2, -a - b*x] + 2*PolyLog[2, a + b*x])/(4*b)
```

3.98.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6658, 6447}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(a+bx)}{a+bx} \, dx$$

$$\Big\downarrow \; 6658$$

$$\frac{\int \frac{\coth^{-1}(a+bx)}{a+bx}d(a+bx)}{b}$$

$$\Big\downarrow \; 6447$$

$$\frac{\frac{1}{2}\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{b}$$

input
```
Int[ArcCoth[a + b*x]/(a + b*x),x]
```

output
```
(PolyLog[2, -(a + b*x)^(-1)]/2 - PolyLog[2, (a + b*x)^(-1)]/2)/b
```

3.98.3.1 Defintions of rubi rules used

rule 6447
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

rule 6658
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCoth[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.98.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

method	result	size
risch	$-\frac{\ln(bx+a-1)\ln(bx+a)}{2b}-\frac{\operatorname{dilog}(bx+a)}{2b}-\frac{\operatorname{dilog}(bx+a+1)}{2b}$	43
derivativedivides	$\frac{\ln(bx+a)\operatorname{arccoth}(bx+a)-\frac{\operatorname{dilog}(bx+a)}{2}-\frac{\operatorname{dilog}(bx+a+1)}{2}-\frac{\ln(bx+a)\ln(bx+a+1)}{2}}{b}$	51
default	$\frac{\ln(bx+a)\operatorname{arccoth}(bx+a)-\frac{\operatorname{dilog}(bx+a)}{2}-\frac{\operatorname{dilog}(bx+a+1)}{2}-\frac{\ln(bx+a)\ln(bx+a+1)}{2}}{b}$	51
parts	$\frac{\ln(bx+a)\operatorname{arccoth}(bx+a)}{b}+\frac{-\frac{\operatorname{dilog}(bx+a)}{2}-\frac{\operatorname{dilog}(bx+a+1)}{2}-\frac{\ln(bx+a)\ln(bx+a+1)}{2}}{b}$	55

input
```
int(arccoth(b*x+a)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*ln(b*x+a-1)*ln(b*x+a)-1/2/b*dilog(b*x+a)-1/2/b*dilog(b*x+a+1)
```

3.98.5 Fricas [F]

$$\int \frac{\coth^{-1}(a+bx)}{a+bx}\,dx=\int \frac{\operatorname{arcoth}(bx+a)}{bx+a}\,dx$$

input
```
integrate(arccoth(b*x+a)/(b*x+a),x, algorithm="fricas")
```

output
```
integral(arccoth(b*x + a)/(b*x + a), x)
```

3.98.6 Sympy [F]

$$\int \frac{\coth^{-1}(a+bx)}{a+bx}\,dx=\int \frac{\operatorname{acoth}(a+bx)}{a+bx}\,dx$$

input
```
integrate(acoth(b*x+a)/(b*x+a),x)
```

output
```
Integral(acoth(a + b*x)/(a + b*x), x)
```

3.98.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(29) = 58.

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.20

$$\int \frac{\coth^{-1}(a+bx)}{a+bx}\,dx = -\frac{1}{2}b\left(\frac{\log(bx+a)\log(bx+a-1)+\text{Li}_2(-bx-a+1)}{b^2} - \frac{\log(bx+a+1)\log(-bx-a)+\text{Li}_2(bx+a+\right. \\ -\frac{1}{2}\left(\frac{\log(bx+a+1)}{b} - \frac{\log(bx+a-1)}{b}\right)\log(bx+a) + \frac{\text{arcoth}(bx+a)\log(bx+a)}{b}$$

input
```
integrate(arccoth(b*x+a)/(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*b*((log(b*x + a)*log(b*x + a - 1) + dilog(-b*x - a + 1))/b^2 - (log(b
*x + a + 1)*log(-b*x - a) + dilog(b*x + a + 1))/b^2) - 1/2*(log(b*x + a +
1)/b - log(b*x + a - 1)/b)*log(b*x + a) + arccoth(b*x + a)*log(b*x + a)/b
```

3.98.8 Giac [F]

$$\int \frac{\coth^{-1}(a+bx)}{a+bx}\,dx = \int \frac{\text{arcoth}(bx+a)}{bx+a}\,dx$$

input
```
integrate(arccoth(b*x+a)/(b*x+a),x, algorithm="giac")
```

output
```
integrate(arccoth(b*x + a)/(b*x + a), x)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(a+bx)}{a+bx}\,dx = \int \frac{\text{acoth}(a+b\,x)}{a+b\,x}\,dx$$

input
```
int(acoth(a + b*x)/(a + b*x),x)
```

output
```
int(acoth(a + b*x)/(a + b*x), x)
```

3.99 $\int \frac{\coth^{-1}(a+bx)}{(a+bx)^2} dx$

3.99.1 Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{\coth^{-1}(a+bx)}{(a+bx)^2} dx = -\frac{\coth^{-1}(a+bx)}{b(a+bx)} + \frac{\log(a+bx)}{b} - \frac{\log\left(1-(a+bx)^2\right)}{2b}$$

output
```
-arccoth(b*x+a)/b/(b*x+a)+ln(b*x+a)/b-1/2*ln(1-(b*x+a)^2)/b
```

3.99.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{\coth^{-1}(a+bx)}{(a+bx)^2} dx = -\frac{\frac{2\coth^{-1}(a+bx)}{a+bx} - 2\log(a+bx) + \log\left(1-(a+bx)^2\right)}{2b}$$

input
```
Integrate[ArcCoth[a + b*x]/(a + b*x)^2,x]
```

output
```
-1/2*((2*ArcCoth[a + b*x])/(a + b*x) - 2*Log[a + b*x] + Log[1 - (a + b*x)^
2])/b
```

3.99.3 Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6658, 6453, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(a+bx)}{(a+bx)^2}\,dx$$

$$\Big\downarrow \text{ 6658}$$

$$\frac{\int \frac{\coth^{-1}(a+bx)}{(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 6453}$$

$$\frac{\int \frac{1}{(a+bx)(1-(a+bx)^2)}d(a+bx)-\frac{\coth^{-1}(a+bx)}{a+bx}}{b}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\frac{1}{2}\int \frac{1}{(-a-bx+1)(a+bx)^2}d(a+bx)^2-\frac{\coth^{-1}(a+bx)}{a+bx}}{b}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{\frac{1}{2}\left(\int \frac{1}{-a-bx+1}d(a+bx)^2+\int \frac{1}{(a+bx)^2}d(a+bx)^2\right)-\frac{\coth^{-1}(a+bx)}{a+bx}}{b}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\frac{1}{2}\left(\int \frac{1}{-a-bx+1}d(a+bx)^2+\log\left((a+bx)^2\right)\right)-\frac{\coth^{-1}(a+bx)}{a+bx}}{b}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\frac{1}{2}\left(\log\left((a+bx)^2\right)-\log(-a-bx+1)\right)-\frac{\coth^{-1}(a+bx)}{a+bx}}{b}$$

input
```
Int[ArcCoth[a + b*x]/(a + b*x)^2,x]
```

output
```
(-(ArcCoth[a + b*x]/(a + b*x)) + (-Log[1 - a - b*x] + Log[(a + b*x)^2])/2)
/b
```

3.99.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6658
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCoth[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.99. $\int \frac{\coth^{-1}(a+bx)}{(a+bx)^2} dx$

3.99.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccoth}(bx+a)}{bx+a}+\ln(bx+a)-\frac{\ln(bx+a+1)}{2}-\frac{\ln(bx+a-1)}{2}}{b}$
default	$\frac{-\frac{\operatorname{arccoth}(bx+a)}{bx+a}+\ln(bx+a)-\frac{\ln(bx+a+1)}{2}-\frac{\ln(bx+a-1)}{2}}{b}$
parts	$-\frac{\operatorname{arccoth}(bx+a)}{b(bx+a)}-\frac{\ln(bx+a-1)}{2b}+\frac{\ln(bx+a)}{b}-\frac{\ln(bx+a+1)}{2b}$
parallelrisch	$\frac{3\ln(bx+a)xa\,b^2-3\ln(bx+a-1)xa\,b^2-3a\ \operatorname{arccoth}(bx+a)x\,b^2+3\ln(bx+a)a^2b-3\ln(bx+a-1)a^2b-3\ \operatorname{arccoth}(bx+a)a^2b-}{3(bx+a)a\,b^2}$
risch	$-\frac{\ln(bx+a+1)}{2b(bx+a)}+\frac{2\ln(-bx-a)bx-\ln(b^2x^2+2abx+a^2-1)bx+2\ln(-bx-a)a-\ln(b^2x^2+2abx+a^2-1)a+\ln(bx+a-1)}{2(bx+a)b}$

input
```
int(arccoth(b*x+a)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-arccoth(b*x+a)/(b*x+a)+ln(b*x+a)-1/2*ln(b*x+a+1)-1/2*ln(b*x+a-1))
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.40

$$\int \frac{\coth^{-1}(a+bx)}{(a+bx)^2}\,dx = -\frac{(bx+a)\log\left(b^2x^2+2\,abx+a^2-1\right)-2\,(bx+a)\log\left(bx+a\right)+\log\left(\frac{bx+a+1}{bx+a-1}\right)}{2\,(b^2x+ab)}$$

input
```
integrate(arccoth(b*x+a)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*((b*x + a)*log(b^2*x^2 + 2*a*b*x + a^2 - 1) - 2*(b*x + a)*log(b*x + a
) + log((b*x + a + 1)/(b*x + a - 1)))/(b^2*x + a*b)
```

3.99.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(34) = 68.

Time = 0.52 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.83

$$\int \frac{\coth^{-1}(a+bx)}{(a+bx)^2}\,dx$$
$$= \begin{cases} \frac{a\log\left(\frac{a}{b}+x\right)}{ab+b^2x} - \frac{a\log\left(\frac{a}{b}+x+\frac{1}{b}\right)}{ab+b^2x} + \frac{a\,\mathrm{acoth}\,(a+bx)}{ab+b^2x} + \frac{bx\log\left(\frac{a}{b}+x\right)}{ab+b^2x} - \frac{bx\log\left(\frac{a}{b}+x+\frac{1}{b}\right)}{ab+b^2x} + \frac{bx\,\mathrm{acoth}\,(a+bx)}{ab+b^2x} - \frac{\mathrm{acoth}\,(a+bx)}{ab+b^2x} & \text{for } b \neq \\ \frac{x\,\mathrm{acoth}\,(a)}{a^2} & \text{otherw} \end{cases}$$

input
```
integrate(acoth(b*x+a)/(b*x+a)**2,x)
```

output
```
Piecewise((a*log(a/b + x)/(a*b + b**2*x) - a*log(a/b + x + 1/b)/(a*b + b**
2*x) + a*acoth(a + b*x)/(a*b + b**2*x) + b*x*log(a/b + x)/(a*b + b**2*x) -
 b*x*log(a/b + x + 1/b)/(a*b + b**2*x) + b*x*acoth(a + b*x)/(a*b + b**2*x)
 - acoth(a + b*x)/(a*b + b**2*x), Ne(b, 0)), (x*acoth(a)/a**2, True))
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{\coth^{-1}(a+bx)}{(a+bx)^2}\,dx = -\frac{\log(bx+a+1)}{2b} + \frac{\log(bx+a)}{b} - \frac{\log(bx+a-1)}{2b} - \frac{\mathrm{arcoth}\,(bx+a)}{(bx+a)b}$$

input
```
integrate(arccoth(b*x+a)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*log(b*x + a + 1)/b + log(b*x + a)/b - 1/2*log(b*x + a - 1)/b - arccot
h(b*x + a)/((b*x + a)*b)
```

3.99.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(46) = 92$.

Time = 0.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 4.12

$$\int \frac{\coth^{-1}(a+bx)}{(a+bx)^2}\,dx = -\frac{1}{2}\left((a+1)b-(a-1)b\right)\left(\frac{\log\left(\frac{|bx+a+1|}{|bx+a-1|}\right)}{b^2}-\frac{\log\left(\left|\frac{bx+a+1}{bx+a-1}+1\right|\right)}{b^2}-\frac{\log\left(-\frac{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}+1}{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}-1}\right)}{b^2\left(\frac{bx+a+1}{bx+a-1}+1\right)}\right)$$

input
```
integrate(arccoth(b*x+a)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*((a + 1)*b - (a - 1)*b)*(log(abs(b*x + a + 1)/abs(b*x + a - 1))/b^2 -
 log(abs((b*x + a + 1)/(b*x + a - 1) + 1))/b^2 - log(-(1/(a - ((b*x + a +
1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a - 1) - b)) +
 1)/(1/(a - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)
*b/(b*x + a - 1) - b)) - 1))/(b^2*((b*x + a + 1)/(b*x + a - 1) + 1)))
```

3.99.9 Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.94

$$\int \frac{\coth^{-1}(a+bx)}{(a+bx)^2}\,dx = \frac{\ln(a+b\,x)}{b}-\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2-1\right)}{2\,b}-\frac{\ln\left(\frac{a+b\,x+1}{a+b\,x}\right)}{2\,\left(x\,b^2+a\,b\right)}+\frac{\ln\left(\frac{a+b\,x-1}{a+b\,x}\right)}{2\,x\,b^2+2\,a\,b}$$

input
```
int(acoth(a + b*x)/(a + b*x)^2,x)
```

output
```
log(a + b*x)/b - log(a^2 + b^2*x^2 + 2*a*b*x - 1)/(2*b) - log((a + b*x + 1
)/(a + b*x))/(2*(a*b + b^2*x)) + log((a + b*x - 1)/(a + b*x))/(2*a*b + 2*b
^2*x)
```

3.100 $\int \frac{\coth^{-1}(1+x)}{2+2x} dx$

3.100.1 Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{\coth^{-1}(1+x)}{2+2x} dx = \frac{1}{4}\text{PolyLog}\left(2,-\frac{1}{1+x}\right) - \frac{1}{4}\text{PolyLog}\left(2,\frac{1}{1+x}\right)$$

output
```
1/4*polylog(2,-1/(1+x))-1/4*polylog(2,1/(1+x))
```

3.100.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 117 vs. 2(25) = 50.

Time = 0.01 (sec) , antiderivative size = 117, normalized size of antiderivative = 4.68

$$\begin{aligned}\int \frac{\coth^{-1}(1+x)}{2+2x} dx &= \frac{1}{8}\log^2\left(-\frac{1}{1+x}\right) - \frac{1}{4}\log(-x)\log\left(\frac{1}{1+x}\right) \\ &\quad - \frac{1}{8}\log^2\left(\frac{1}{1+x}\right) + \frac{1}{4}\log\left(\frac{1}{1+x}\right)\log\left(\frac{x}{1+x}\right) \\ &\quad + \frac{1}{4}\log\left(-\frac{1}{1+x}\right)\log(2+x) - \frac{1}{4}\log\left(-\frac{1}{1+x}\right)\log\left(\frac{2+x}{1+x}\right) \\ &\quad - \frac{\text{PolyLog}(2,-1-x)}{4} + \frac{\text{PolyLog}(2,1+x)}{4}\end{aligned}$$

input
```
Integrate[ArcCoth[1 + x]/(2 + 2*x),x]
```

output
```
Log[-(1 + x)^(-1)]^2/8 - (Log[-x]*Log[(1 + x)^(-1)])/4 - Log[(1 + x)^(-1)]
^2/8 + (Log[(1 + x)^(-1)]*Log[x/(1 + x)])/4 + (Log[-(1 + x)^(-1)]*Log[2 +
x])/4 - (Log[-(1 + x)^(-1)]*Log[(2 + x)/(1 + x)])/4 - PolyLog[2, -1 - x]/4
 + PolyLog[2, 1 + x]/4
```

3.100.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6658, 27, 6447}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(x+1)}{2x+2}\,dx$$

$$\Big\downarrow \text{6658}$$

$$\int \frac{\coth^{-1}(x+1)}{2(x+1)}d(x+1)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\coth^{-1}(x+1)}{x+1}d(x+1)$$

$$\Big\downarrow \text{6447}$$

$$\frac{1}{2}\left(\frac{1}{2}\text{PolyLog}\left(2,-\frac{1}{x+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{1}{x+1}\right)\right)$$

input
```
Int[ArcCoth[1 + x]/(2 + 2*x),x]
```

output
```
(PolyLog[2, -(1 + x)^(-1)]/2 - PolyLog[2, (1 + x)^(-1)]/2)/2
```

3.100.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6447
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

rule 6658
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCoth[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.100.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
risch	$-\frac{\operatorname{dilog}(1+x)}{4}-\frac{\ln(x)\ln(1+x)}{4}-\frac{\operatorname{dilog}(x+2)}{4}$	22
derivativedivides	$\frac{\ln(1+x)\operatorname{arccoth}(1+x)}{2}-\frac{\operatorname{dilog}(x+2)}{4}-\frac{\ln(1+x)\ln(x+2)}{4}-\frac{\operatorname{dilog}(1+x)}{4}$	34
default	$\frac{\ln(1+x)\operatorname{arccoth}(1+x)}{2}-\frac{\operatorname{dilog}(x+2)}{4}-\frac{\ln(1+x)\ln(x+2)}{4}-\frac{\operatorname{dilog}(1+x)}{4}$	34
parts	$\frac{\ln(1+x)\operatorname{arccoth}(1+x)}{2}-\frac{\operatorname{dilog}(x+2)}{4}-\frac{\ln(1+x)\ln(x+2)}{4}-\frac{\operatorname{dilog}(1+x)}{4}$	34

input
```
int(arccoth(1+x)/(2+2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*dilog(1+x)-1/4*ln(x)*ln(1+x)-1/4*dilog(x+2)
```

3.100.5 Fricas [F]

$$\int \frac{\coth^{-1}(1+x)}{2+2x}\,dx = \int \frac{\operatorname{arcoth}(x+1)}{2(x+1)}\,dx$$

input
```
integrate(arccoth(1+x)/(2+2*x),x, algorithm="fricas")
```

output
```
integral(1/2*arccoth(x + 1)/(x + 1), x)
```

3.100.6 Sympy [F]

$$\int \frac{\coth^{-1}(1+x)}{2+2x}\,dx = \frac{\int \frac{\operatorname{acoth}(x+1)}{x+1}\,dx}{2}$$

input
```
integrate(acoth(1+x)/(2+2*x),x)
```

output
```
Integral(acoth(x + 1)/(x + 1), x)/2
```

3.100.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(19) = 38$.

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \frac{\coth^{-1}(1+x)}{2+2x}\,dx = -\frac{1}{4}(\log(x+2) - \log(x))\log(x+1) + \frac{1}{2}\operatorname{arcoth}(x+1)\log(x+1) - \frac{1}{4}\log(x+1)\log(x) + \frac{1}{4}\log(x+2)\log(-x-1) - \frac{1}{4}\operatorname{Li}_2(-x) + \frac{1}{4}\operatorname{Li}_2(x+2)$$

input
```
integrate(arccoth(1+x)/(2+2*x),x, algorithm="maxima")
```

output
```
-1/4*(log(x + 2) - log(x))*log(x + 1) + 1/2*arccoth(x + 1)*log(x + 1) - 1/
4*log(x + 1)*log(x) + 1/4*log(x + 2)*log(-x - 1) - 1/4*dilog(-x) + 1/4*dil
og(x + 2)
```

3.100.8 Giac [F]

$$\int \frac{\coth^{-1}(1+x)}{2+2x}\,dx = \int \frac{\operatorname{arcoth}(x+1)}{2(x+1)}\,dx$$

input
```
integrate(arccoth(1+x)/(2+2*x),x, algorithm="giac")
```

output
```
integrate(1/2*arccoth(x + 1)/(x + 1), x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(1+x)}{2+2x}\,dx = \int \frac{\operatorname{acoth}(x+1)}{2\,x+2}\,dx$$

input
```
int(acoth(x + 1)/(2*x + 2),x)
```

output
```
int(acoth(x + 1)/(2*x + 2), x)
```

3.101 $\int \frac{\coth^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

3.101.1 Optimal result

Integrand size = 19, antiderivative size = 35

$$\int \frac{\coth^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{2d}-\frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{2d}$$

output
```
1/2*polylog(2,-1/(b*x+a))/d-1/2*polylog(2,1/(b*x+a))/d
```

3.101.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 144 vs. $2(35)=70$.

Time = 0.02 (sec) , antiderivative size = 144, normalized size of antiderivative = 4.11

$$\int \frac{\coth^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$$
$$=\frac{\log^2\left(-\frac{1}{a+bx}\right)-2\log(1-a-bx)\log\left(\frac{1}{a+bx}\right)-\log^2\left(\frac{1}{a+bx}\right)+2\log\left(\frac{1}{a+bx}\right)\log\left(\frac{-1+a+bx}{a+bx}\right)+2\log\left(-\frac{1}{a+bx}\right)\log}{4d}$$

input
```
Integrate[ArcCoth[a + b*x]/((a*d)/b + d*x),x]
```

output
```
(Log[-(a + b*x)^(-1)]^2 - 2*Log[1 - a - b*x]*Log[(a + b*x)^(-1)] - Log[(a
+ b*x)^(-1)]^2 + 2*Log[(a + b*x)^(-1)]*Log[(-1 + a + b*x)/(a + b*x)] + 2*L
og[-(a + b*x)^(-1)]*Log[1 + a + b*x] - 2*Log[-(a + b*x)^(-1)]*Log[(1 + a +
 b*x)/(a + b*x)] - 2*PolyLog[2, -a - b*x] + 2*PolyLog[2, a + b*x])/(4*d)
```

3.101.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6658, 27, 6447}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(a+bx)}{\frac{ad}{b}+dx} \, dx$$

$$\Big\downarrow \ 6658$$

$$\frac{\int \frac{b\coth^{-1}(a+bx)}{d(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\coth^{-1}(a+bx)}{a+bx}d(a+bx)}{d}$$

$$\Big\downarrow \ 6447$$

$$\frac{\frac{1}{2}\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{d}$$

input
```
Int[ArcCoth[a + b*x]/((a*d)/b + d*x),x]
```

output
```
(PolyLog[2, -(a + b*x)^(-1)]/2 - PolyLog[2, (a + b*x)^(-1)]/2)/d
```

3.101.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6447
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

rule 6658
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCoth[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.101.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

method	result	size
risch	$-\frac{\operatorname{dilog}(bx+a+1)}{2d}-\frac{\ln(bx+a-1)\ln(bx+a)}{2d}-\frac{\operatorname{dilog}(bx+a)}{2d}$	43
parts	$\frac{\ln(bx+a)\operatorname{arccoth}(bx+a)}{d}+\frac{-\frac{\operatorname{dilog}(bx+a)}{2}-\frac{\operatorname{dilog}(bx+a+1)}{2}-\frac{\ln(bx+a)\ln(bx+a+1)}{2}}{d}$	55
derivativedivides	$\frac{\frac{b\ln(bx+a)\operatorname{arccoth}(bx+a)}{d}+\frac{b\left(-\frac{\operatorname{dilog}(bx+a)}{2}-\frac{\operatorname{dilog}(bx+a+1)}{2}-\frac{\ln(bx+a)\ln(bx+a+1)}{2}\right)}{d}}{b}$	61
default	$\frac{\frac{b\ln(bx+a)\operatorname{arccoth}(bx+a)}{d}+\frac{b\left(-\frac{\operatorname{dilog}(bx+a)}{2}-\frac{\operatorname{dilog}(bx+a+1)}{2}-\frac{\ln(bx+a)\ln(bx+a+1)}{2}\right)}{d}}{b}$	61

input
```
int(arccoth(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*dilog(b*x+a+1)-1/2/d*ln(b*x+a-1)*ln(b*x+a)-1/2/d*dilog(b*x+a)
```

3.101.5 Fricas [F]

$$\int \frac{\coth^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\operatorname{arcoth}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arccoth(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

output
```
integral(b*arccoth(b*x + a)/(b*d*x + a*d), x)
```

3.101.6 Sympy [F]

$$\int \frac{\coth^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{b\int \frac{\operatorname{acoth}(a+bx)}{a+bx}\,dx}{d}$$

input
```
integrate(acoth(b*x+a)/(a*d/b+d*x),x)
```

output
```
b*Integral(acoth(a + b*x)/(a + b*x), x)/d
```

3.101.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(29) = 58$.

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.77

$$\int \frac{\coth^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = -\frac{1}{2}b\left(\frac{\log(bx+a)\log(bx+a-1)+\operatorname{Li}_2(-bx-a+1)}{bd}-\frac{\log(bx+a+1)\log(-bx-a)+\operatorname{Li}_2(bx+a+\right.$$
$$-\frac{b\left(\frac{\log(bx+a+1)}{b}-\frac{\log(bx+a-1)}{b}\right)\log\left(dx+\frac{ad}{b}\right)}{2\,d}+\frac{\operatorname{arcoth}(bx+a)\log\left(dx+\frac{ad}{b}\right)}{d}$$

input
```
integrate(arccoth(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

output
```
-1/2*b*((log(b*x + a)*log(b*x + a - 1) + dilog(-b*x - a + 1))/(b*d) - (log
(b*x + a + 1)*log(-b*x - a) + dilog(b*x + a + 1))/(b*d)) - 1/2*b*(log(b*x
+ a + 1)/b - log(b*x + a - 1)/b)*log(d*x + a*d/b)/d + arccoth(b*x + a)*log
(d*x + a*d/b)/d
```

3.101.8 Giac [F]

$$\int \frac{\coth^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\operatorname{arcoth}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arccoth(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

output
```
integrate(arccoth(b*x + a)/(d*x + a*d/b), x)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\operatorname{acoth}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

input
```
int(acoth(a + b*x)/(d*x + (a*d)/b),x)
```

output
```
int(acoth(a + b*x)/(d*x + (a*d)/b), x)
```

3.102 $\int (e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right) dx$

3.102.1 Optimal result

Integrand size = 18, antiderivative size = 168

$$\int (e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right) dx = \frac{bf(6d^2e^2-12cdef+(1+6c^2)f^2)x}{4d^3} + \frac{bf^2(de-cf)(c+dx)^2}{2d^4} + \frac{bf^3(c+dx)^3}{12d^4} + \frac{(e+fx)^4\left(a+b\coth^{-1}(c+dx)\right)}{4f} + \frac{b(de+f-cf)^4\log(1-c-dx)}{8d^4f} - \frac{b(de-f-cf)^4\log(1+c+dx)}{8d^4f}$$

output
```
1/4*b*f*(6*d^2*e^2-12*c*d*e*f+(6*c^2+1)*f^2)*x/d^3+1/2*b*f^2*(-c*f+d*e)*(d
*x+c)^2/d^4+1/12*b*f^3*(d*x+c)^3/d^4+1/4*(f*x+e)^4*(a+b*arccoth(d*x+c))/f+
1/8*b*(-c*f+d*e+f)^4*ln(-d*x-c+1)/d^4/f-1/8*b*(-c*f+d*e-f)^4*ln(d*x+c+1)/d
^4/f
```

3.102.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.61

$$\int (e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)\,dx$$
$$=\frac{6d(4ad^3e^3+bf(6d^2e^2-8cdef+(1+3c^2)f^2))x+6d^2f(6ad^2e^2+bf(2de-cf))x^2+2d^3f^2(12ade+bf}{}$$

input
```
Integrate[(e + f*x)^3*(a + b*ArcCoth[c + d*x]),x]
```

output
```
(6*d*(4*a*d^3*e^3 + b*f*(6*d^2*e^2 - 8*c*d*e*f + (1 + 3*c^2)*f^2))*x + 6*d
^2*f*(6*a*d^2*e^2 + b*f*(2*d*e - c*f))*x^2 + 2*d^3*f^2*(12*a*d*e + b*f)*x^
3 + 6*a*d^4*f^3*x^4 + 6*b*d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3
)*ArcCoth[c + d*x] - 3*b*(-1 + c)*(4*d^3*e^3 - 6*(-1 + c)*d^2*e^2*f + 4*(-
1 + c)^2*d*e*f^2 - (-1 + c)^3*f^3)*Log[1 - c - d*x] - 3*b*(1 + c)*(-4*d^3*
e^3 + 6*(1 + c)*d^2*e^2*f - 4*(1 + c)^2*d*e*f^2 + (1 + c)^3*f^3)*Log[1 + c
 + d*x])/(24*d^4)
```

3.102.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6662, 27, 6479, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)\,dx$$
$$\Big\downarrow \ 6662$$
$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^3\left(a+b\coth^{-1}(c+dx)\right)}{d^3}d(c+dx)}{d}$$
$$\Big\downarrow \ 27$$
$$\frac{\int (de-cf+f(c+dx))^3\left(a+b\coth^{-1}(c+dx)\right)d(c+dx)}{d^4}$$
$$\Big\downarrow \ 6479$$

$$\frac{\frac{(f(c+dx)-cf+de)^4(a+b\coth^{-1}(c+dx))}{4f}-\frac{b\int\frac{(de-cf+f(c+dx))^4}{1-(c+dx)^2}d(c+dx)}{4f}}{d^4}$$

$$\Big\downarrow \text{477}$$

$$\frac{\frac{(f(c+dx)-cf+de)^4(a+b\coth^{-1}(c+dx))}{4f}-\frac{b\int\Big(-(c+dx)^2f^4-4(de-cf)(c+dx)f^3-(6d^2e^2-12cdfe+(6c^2+1)f^2)f^2+\frac{(de-cf+f)^4}{2(-c-dx+1)}+\frac{(de-cf-f)^4}{2(c+dx+1)}}{4f}}{d^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(f(c+dx)-cf+de)^4(a+b\coth^{-1}(c+dx))}{4f}-\frac{b(-f^2(c+dx)((6c^2+1)f^2-12cdef+6d^2e^2)-2f^3(c+dx)^2(de-cf)-\frac{1}{2}(-cf+de+f)^4\log(-c-dx+1)+}{4f}}{d^4}$$

input
```
Int[(e + f*x)^3*(a + b*ArcCoth[c + d*x]),x]
```

output
```
(((d*e - c*f + f*(c + d*x))^4*(a + b*ArcCoth[c + d*x]))/(4*f) - (b*(-(f^2*
(6*d^2*e^2 - 12*c*d*e*f + (1 + 6*c^2)*f^2)*(c + d*x)) - 2*f^3*(d*e - c*f)*
(c + d*x)^2 - (f^4*(c + d*x)^3)/3 - ((d*e + f - c*f)^4*Log[1 - c - d*x])/2
 + ((d*e - f - c*f)^4*Log[1 + c + d*x])/2))/(4*f))/d^4
```

3.102.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6479
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.102.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(156) = 312$.

Time = 0.56 (sec) , antiderivative size = 599, normalized size of antiderivative = 3.57

method	result
parallelrisch	$-\frac{12\ln(dx+c-1)bc^3f^3-12\ln(dx+c-1)bd^3e^3+12\ln(dx+c-1)bcf^3+24xbcd^2ef^2-42bc^2def^2+24acd^3e^3+36bcd^2e^2f}{}$
derivativedivides	$\frac{\frac{a(cf-de-f(dx+c))^4}{4d^3f}-\frac{b\left(-\frac{f^3\operatorname{arccoth}(dx+c)c^4}{4}+f^2\operatorname{arccoth}(dx+c)c^3de+f^3\operatorname{arccoth}(dx+c)c^3(dx+c)-\frac{3f\operatorname{arccoth}(dx+c)c^2d^2e^2}{2}-3f\right.}{}}{}$
default	$\frac{\frac{a(cf-de-f(dx+c))^4}{4d^3f}-\frac{b\left(-\frac{f^3\operatorname{arccoth}(dx+c)c^4}{4}+f^2\operatorname{arccoth}(dx+c)c^3de+f^3\operatorname{arccoth}(dx+c)c^3(dx+c)-\frac{3f\operatorname{arccoth}(dx+c)c^2d^2e^2}{2}-3f\right.}{}}{}$
parts	$\frac{a(fx+e)^4}{4f}+\frac{b\left(\frac{f^3\operatorname{arccoth}(dx+c)(dx+c)^4}{4d^3}-\frac{f^3\operatorname{arccoth}(dx+c)(dx+c)^3c}{d^3}+\frac{f^2\operatorname{arccoth}(dx+c)(dx+c)^3e}{d^2}+\frac{3f^3\operatorname{arccoth}(dx+c)(dx+c)}{2d^3}\right.}{}$
risch	$-\frac{be^4\ln(dx+c-1)}{8f}+\frac{f^3ax^4}{4}+\frac{f^3bx^3}{12d}+\frac{f^3bx}{4d^3}+\frac{f^3\ln(-dx-c+1)b}{8d^4}-\frac{f^3\ln(dx+c+1)b}{8d^4}-\frac{be^3x\ln(dx+c-1)}{2}+$

input
```
int((f*x+e)^3*(a+b*arccoth(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(12*ln(d*x+c-1)*b*c^3*f^3-12*ln(d*x+c-1)*b*d^3*e^3+12*ln(d*x+c-1)*b*
c*f^3+24*x*b*c*d^2*e*f^2-42*b*c^2*d*e*f^2+24*a*c*d^3*e^3+36*b*c*d^2*e^2*f+
3*arccoth(d*x+c)*b*f^3+15*b*c^3*f^3-6*b*e*f^2*d+9*b*c*f^3+36*arccoth(d*x+c
)*b*c*d^2*e^2*f-18*a*e^2*f*d^2-12*ln(d*x+c-1)*b*d*e*f^2-36*ln(d*x+c-1)*b*c
^2*d*e*f^2+36*ln(d*x+c-1)*b*c*d^2*e^2*f-3*x^4*a*d^4*f^3-12*x*a*d^4*e^3-3*x
*b*d*f^3-x^3*b*d^3*f^3+3*arccoth(d*x+c)*b*c^4*f^3-12*arccoth(d*x+c)*b*d^3*
e^3+18*arccoth(d*x+c)*b*c^2*f^3+12*arccoth(d*x+c)*b*c*f^3+12*arccoth(d*x+c
)*b*c^3*f^3-12*x^3*a*d^4*e*f^2-12*arccoth(d*x+c)*b*c*d^3*e^3+18*arccoth(d*
x+c)*b*d^2*e^2*f-12*arccoth(d*x+c)*b*d*e*f^2-12*x*arccoth(d*x+c)*b*d^4*e^3
-18*x^2*a*d^4*e^2*f+3*x^2*b*c*d^2*f^3-6*x^2*b*d^3*e*f^2-3*x^4*arccoth(d*x+
c)*b*d^4*f^3-9*x*b*c^2*d*f^3-18*x*b*d^3*e^2*f-12*x^3*arccoth(d*x+c)*b*d^4*
e*f^2+18*a*c^2*d^2*e^2*f-18*x^2*arccoth(d*x+c)*b*d^4*e^2*f+18*arccoth(d*x+
c)*b*c^2*d^2*e^2*f-36*arccoth(d*x+c)*b*c^2*d*e*f^2-36*arccoth(d*x+c)*b*c*d
*e*f^2-12*arccoth(d*x+c)*b*c^3*d*e*f^2)/d^4
```

3.102.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(156) = 312$.

Time = 0.27 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.29

$$\int (e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)\,dx$$
$$= \frac{6\,ad^4f^3x^4 + 2\,(12\,ad^4ef^2 + bd^3f^3)x^3 + 6\,(6\,ad^4e^2f + 2\,bd^3ef^2 - bcd^2f^3)x^2 + 6\,(4\,ad^4e^3 + 6\,bd^3e^2f - 8\,bcc}{}$$

input
```
integrate((f*x+e)^3*(a+b*arccoth(d*x+c)),x, algorithm="fricas")
```

output
```
1/24*(6*a*d^4*f^3*x^4 + 2*(12*a*d^4*e*f^2 + b*d^3*f^3)*x^3 + 6*(6*a*d^4*e^
2*f + 2*b*d^3*e*f^2 - b*c*d^2*f^3)*x^2 + 6*(4*a*d^4*e^3 + 6*b*d^3*e^2*f -
8*b*c*d^2*e*f^2 + (3*b*c^2 + b)*d*f^3)*x + 3*(4*(b*c + b)*d^3*e^3 - 6*(b*c
^2 + 2*b*c + b)*d^2*e^2*f + 4*(b*c^3 + 3*b*c^2 + 3*b*c + b)*d*e*f^2 - (b*c
^4 + 4*b*c^3 + 6*b*c^2 + 4*b*c + b)*f^3)*log(d*x + c + 1) - 3*(4*(b*c - b)
*d^3*e^3 - 6*(b*c^2 - 2*b*c + b)*d^2*e^2*f + 4*(b*c^3 - 3*b*c^2 + 3*b*c -
b)*d*e*f^2 - (b*c^4 - 4*b*c^3 + 6*b*c^2 - 4*b*c + b)*f^3)*log(d*x + c - 1)
 + 3*(b*d^4*f^3*x^4 + 4*b*d^4*e*f^2*x^3 + 6*b*d^4*e^2*f*x^2 + 4*b*d^4*e^3*
x)*log((d*x + c + 1)/(d*x + c - 1)))/d^4
```

3.102.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(151) = 302$.

Time = 0.70 (sec) , antiderivative size = 644, normalized size of antiderivative = 3.83

$$\int (e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)\,dx$$
$$= \begin{cases} ae^3x+\frac{3ae^2fx^2}{2}+aef^2x^3+\frac{af^3x^4}{4}-\frac{bc^4f^3\operatorname{acoth}(c+dx)}{4d^4}+\frac{bc^3ef^2\operatorname{acoth}(c+dx)}{d^3}-\frac{bc^3f^3\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^4}+\frac{bc^3f^3\operatorname{acoth}(c+dx)}{d^4} \\ (a+b\operatorname{acoth}(c))\left(e^3x+\frac{3e^2fx^2}{2}+ef^2x^3+\frac{f^3x^4}{4}\right) \end{cases}$$

input

```
integrate((f*x+e)**3*(a+b*acoth(d*x+c)),x)
```

output

```
Piecewise((a*e**3*x + 3*a*e**2*f*x**2/2 + a*e*f**2*x**3 + a*f**3*x**4/4 -
b*c**4*f**3*acoth(c + d*x)/(4*d**4) + b*c**3*e*f**2*acoth(c + d*x)/d**3 -
b*c**3*f**3*log(c/d + x + 1/d)/d**4 + b*c**3*f**3*acoth(c + d*x)/d**4 - 3*
b*c**2*e**2*f*acoth(c + d*x)/(2*d**2) + 3*b*c**2*e*f**2*log(c/d + x + 1/d)
/d**3 - 3*b*c**2*e*f**2*acoth(c + d*x)/d**3 + 3*b*c**2*f**3*x/(4*d**3) - 3
*b*c**2*f**3*acoth(c + d*x)/(2*d**4) + b*c*e**3*acoth(c + d*x)/d - 3*b*c*e
**2*f*log(c/d + x + 1/d)/d**2 + 3*b*c*e**2*f*acoth(c + d*x)/d**2 - 2*b*c*e
*f**2*x/d**2 - b*c*f**3*x**2/(4*d**2) + 3*b*c*e*f**2*acoth(c + d*x)/d**3 -
 b*c*f**3*log(c/d + x + 1/d)/d**4 + b*c*f**3*acoth(c + d*x)/d**4 + b*e**3*
x*acoth(c + d*x) + 3*b*e**2*f*x**2*acoth(c + d*x)/2 + b*e*f**2*x**3*acoth(
c + d*x) + b*f**3*x**4*acoth(c + d*x)/4 + b*e**3*log(c/d + x + 1/d)/d - b*
e**3*acoth(c + d*x)/d + 3*b*e**2*f*x/(2*d) + b*e*f**2*x**2/(2*d) + b*f**3*
x**3/(12*d) - 3*b*e**2*f*acoth(c + d*x)/(2*d**2) + b*e*f**2*log(c/d + x +
1/d)/d**3 - b*e*f**2*acoth(c + d*x)/d**3 + b*f**3*x/(4*d**3) - b*f**3*acot
h(c + d*x)/(4*d**4), Ne(d, 0)), ((a + b*acoth(c))*(e**3*x + 3*e**2*f*x**2/
2 + e*f**2*x**3 + f**3*x**4/4), True))
```

3.102.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(156) = 312.

Time = 0.21 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.98

$$\int (e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)\,dx = \frac{1}{4}af^3x^4 + aef^2x^3 + \frac{3}{2}ae^2fx^2$$
$$+\frac{3}{4}\left(2\,x^2\operatorname{arcoth}(dx+c) + d\left(\frac{2\,x}{d^2} - \frac{(c^2+2\,c+1)\log(dx+c+1)}{d^3} + \frac{(c^2-2\,c+1)\log(dx+c-1)}{d^3}\right)\right)b$$
$$+\frac{1}{2}\left(2\,x^3\operatorname{arcoth}(dx+c) + d\left(\frac{dx^2-4\,cx}{d^3} + \frac{(c^3+3\,c^2+3\,c+1)\log(dx+c+1)}{d^4} - \frac{(c^3-3\,c^2+3\,c-1)}{d^4}\right.\right.$$
$$+\frac{1}{24}\left(6\,x^4\operatorname{arcoth}(dx+c) + d\left(\frac{2\,(d^2x^3-3\,cdx^2+3\,(3\,c^2+1)x)}{d^4} - \frac{3\,(c^4+4\,c^3+6\,c^2+4\,c+1)\log(dx-}{d^5}\right.\right.$$
$$+ae^3x + \frac{\left(2\,(dx+c)\operatorname{arcoth}(dx+c) + \log\left(-(dx+c)^2+1\right)\right)be^3}{2\,d}$$

input
```
integrate((f*x+e)^3*(a+b*arccoth(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*a*f^3*x^4 + a*e*f^2*x^3 + 3/2*a*e^2*f*x^2 + 3/4*(2*x^2*arccoth(d*x + c
) + d*(2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*lo
g(d*x + c - 1)/d^3))*b*e^2*f + 1/2*(2*x^3*arccoth(d*x + c) + d*((d*x^2 - 4
*c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 +
3*c - 1)*log(d*x + c - 1)/d^4))*b*e*f^2 + 1/24*(6*x^4*arccoth(d*x + c) + d
*(2*(d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 + 1)*x)/d^4 - 3*(c^4 + 4*c^3 + 6*c^2 +
 4*c + 1)*log(d*x + c + 1)/d^5 + 3*(c^4 - 4*c^3 + 6*c^2 - 4*c + 1)*log(d*x
 + c - 1)/d^5))*b*f^3 + a*e^3*x + 1/2*(2*(d*x + c)*arccoth(d*x + c) + log(
-(d*x + c)^2 + 1))*b*e^3/d
```

3.102.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2333 vs. 2(156) = 312.

Time = 0.34 (sec) , antiderivative size = 2333, normalized size of antiderivative = 13.89

$$\int (e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*(a+b*arccoth(d*x+c)),x, algorithm="giac")
```

output
```
1/6*((c + 1)*d - (c - 1)*d)*(3*((d*x + c + 1)^3*b*d^3*e^3/(d*x + c - 1)^3
- 3*(d*x + c + 1)^2*b*d^3*e^3/(d*x + c - 1)^2 + 3*(d*x + c + 1)*b*d^3*e^3/
(d*x + c - 1) - b*d^3*e^3 - 3*(d*x + c + 1)^3*b*c*d^2*e^2*f/(d*x + c - 1)^
3 + 9*(d*x + c + 1)^2*b*c*d^2*e^2*f/(d*x + c - 1)^2 - 9*(d*x + c + 1)*b*c*
d^2*e^2*f/(d*x + c - 1) + 3*b*c*d^2*e^2*f + 3*(d*x + c + 1)^3*b*c^2*d*e*f^
2/(d*x + c - 1)^3 - 9*(d*x + c + 1)^2*b*c^2*d*e*f^2/(d*x + c - 1)^2 + 9*(d
*x + c + 1)*b*c^2*d*e*f^2/(d*x + c - 1) - 3*b*c^2*d*e*f^2 - (d*x + c + 1)^
3*b*c^3*f^3/(d*x + c - 1)^3 + 3*(d*x + c + 1)^2*b*c^3*f^3/(d*x + c - 1)^2
- 3*(d*x + c + 1)*b*c^3*f^3/(d*x + c - 1) + b*c^3*f^3 + 3*(d*x + c + 1)^3*
b*d^2*e^2*f/(d*x + c - 1)^3 - 6*(d*x + c + 1)^2*b*d^2*e^2*f/(d*x + c - 1)^
2 + 3*(d*x + c + 1)*b*d^2*e^2*f/(d*x + c - 1) - 6*(d*x + c + 1)^3*b*c*d*e*
f^2/(d*x + c - 1)^3 + 12*(d*x + c + 1)^2*b*c*d*e*f^2/(d*x + c - 1)^2 - 6*(
d*x + c + 1)*b*c*d*e*f^2/(d*x + c - 1) + 3*(d*x + c + 1)^3*b*c^2*f^3/(d*x
+ c - 1)^3 - 6*(d*x + c + 1)^2*b*c^2*f^3/(d*x + c - 1)^2 + 3*(d*x + c + 1)
*b*c^2*f^3/(d*x + c - 1) + 3*(d*x + c + 1)^3*b*d*e*f^2/(d*x + c - 1)^3 - 3
*(d*x + c + 1)^2*b*d*e*f^2/(d*x + c - 1)^2 + (d*x + c + 1)*b*d*e*f^2/(d*x
+ c - 1) - b*d*e*f^2 - 3*(d*x + c + 1)^3*b*c*f^3/(d*x + c - 1)^3 + 3*(d*x
+ c + 1)^2*b*c*f^3/(d*x + c - 1)^2 - (d*x + c + 1)*b*c*f^3/(d*x + c - 1) +
 b*c*f^3 + (d*x + c + 1)^3*b*f^3/(d*x + c - 1)^3 + (d*x + c + 1)*b*f^3/(d*
x + c - 1))*log((d*x + c + 1)/(d*x + c - 1))/((d*x + c + 1)^4*d^5/(d*x ...
```

3.102. $\int (e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)\, dx$

3.102.9 Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 742, normalized size of antiderivative = 4.42

$$\int (e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)\,dx$$

$$= x\left(\frac{e\,(6\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+2\,a\,d^2\,e^2+3\,b\,d\,e\,f-6\,a\,f^2)}{2\,d^2} - \frac{(4\,c^2-4)\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{4\,d^2} + \frac{2\,c\left(\frac{2\,c\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{d}-\frac{4\,a\,c^2\,f^3+24\,a\,c\,d\,e\,f^2+12\,a\,d^2\,e^2\,f+4\,b\,d\,e\,f^2-4\,a\,f^3}{4\,d^2}+\frac{a\,f^3\,(4\,c^2-4)}{4\,d^2}\right)}{d}\right)$$

$$- \ln\left(1-\frac{1}{c+d\,x}\right)\left(\frac{b\,e^3\,x}{2}+\frac{3\,b\,e^2\,f\,x^2}{4}+\frac{b\,e\,f^2\,x^3}{2}+\frac{b\,f^3\,x^4}{8}\right)$$

$$- x^2\left(\frac{c\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{d} - \frac{4\,a\,c^2\,f^3+24\,a\,c\,d\,e\,f^2+12\,a\,d^2\,e^2\,f+4\,b\,d\,e\,f^2-4\,a\,f^3}{8\,d^2}+\frac{a\,f^3\,(4\,c^2-4)}{8\,d^2}\right)$$

$$+ x^3\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{12\,d}-\frac{2\,a\,c\,f^3}{3\,d}\right)$$

$$+ \ln\left(\frac{1}{c+d\,x}+1\right)\left(\frac{b\,e^3\,x}{2}+\frac{3\,b\,e^2\,f\,x^2}{4}+\frac{b\,e\,f^2\,x^3}{2}+\frac{b\,f^3\,x^4}{8}\right)+\frac{a\,f^3\,x^4}{4}$$

$$+ \frac{\ln(c+d\,x-1)\,(b\,c^4\,f^3-4\,b\,c^3\,d\,e\,f^2-4\,b\,c^3\,f^3+6\,b\,c^2\,d^2\,e^2\,f+12\,b\,c^2\,d\,e\,f^2+6\,b\,c^2\,f^3-4\,b\,c\,d^3\,e^3-}{8\,d^4}$$

$$- \frac{\ln(c+d\,x+1)\,(b\,c^4\,f^3-4\,b\,c^3\,d\,e\,f^2+4\,b\,c^3\,f^3+6\,b\,c^2\,d^2\,e^2\,f-12\,b\,c^2\,d\,e\,f^2+6\,b\,c^2\,f^3-4\,b\,c\,d^3\,e^3+}{8\,d^4}$$

input
```
int((e + f*x)^3*(a + b*acoth(c + d*x)),x)
```

output
```
x*((e*(6*a*c^2*f^2 - 6*a*f^2 + 2*a*d^2*e^2 + 3*b*d*e*f + 12*a*c*d*e*f))/(2
*d^2) - ((4*c^2 - 4)*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)
/d))/(4*d^2) + (2*c*((2*c*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c
*f^3)/d))/d - (4*a*c^2*f^3 - 4*a*f^3 + 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a
*c*d*e*f^2)/(4*d^2) + (a*f^3*(4*c^2 - 4))/(4*d^2)))/d) - log(1 - 1/(c + d*
x))*((b*f^3*x^4)/8 + (b*e^3*x)/2 + (3*b*e^2*f*x^2)/4 + (b*e*f^2*x^3)/2) -
x^2*((c*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/d))/d - (4*a
*c^2*f^3 - 4*a*f^3 + 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*d*e*f^2)/(8*d^2
) + (a*f^3*(4*c^2 - 4))/(8*d^2)) + x^3*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(
12*d) - (2*a*c*f^3)/(3*d)) + log(1/(c + d*x) + 1)*((b*f^3*x^4)/8 + (b*e^3*
x)/2 + (3*b*e^2*f*x^2)/4 + (b*e*f^2*x^3)/2) + (a*f^3*x^4)/4 + (log(c + d*x
 - 1)*(b*f^3 + 6*b*c^2*f^3 - 4*b*c^3*f^3 + 4*b*d^3*e^3 + b*c^4*f^3 - 4*b*c
*f^3 + 4*b*d*e*f^2 - 4*b*c*d^3*e^3 + 6*b*d^2*e^2*f - 12*b*c*d^2*e^2*f + 12
*b*c^2*d*e*f^2 - 4*b*c^3*d*e*f^2 + 6*b*c^2*d^2*e^2*f - 12*b*c*d*e*f^2))/(8
*d^4) - (log(c + d*x + 1)*(b*f^3 + 6*b*c^2*f^3 + 4*b*c^3*f^3 - 4*b*d^3*e^3
 + b*c^4*f^3 + 4*b*c*f^3 - 4*b*d*e*f^2 - 4*b*c*d^3*e^3 + 6*b*d^2*e^2*f + 1
2*b*c*d^2*e^2*f - 12*b*c^2*d*e*f^2 - 4*b*c^3*d*e*f^2 + 6*b*c^2*d^2*e^2*f -
 12*b*c*d*e*f^2))/(8*d^4)
```

3.102. $\int (e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)\,dx$

3.103 $\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right) \, dx$

3.103.1 Optimal result

Integrand size = 18, antiderivative size = 120

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right) \, dx = \frac{bf(de-cf)x}{d^2}+\frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)}{3f} + \frac{b(de+f-cf)^3\log(1-c-dx)}{6d^3f} - \frac{b(de-(1+c)f)^3\log(1+c+dx)}{6d^3f}$$

output
```
b*f*(-c*f+d*e)*x/d^2+1/6*b*f^2*(d*x+c)^2/d^3+1/3*(f*x+e)^3*(a+b*arccoth(d*
x+c))/f+1/6*b*(-c*f+d*e+f)^3*ln(-d*x-c+1)/d^3/f-1/6*b*(d*e-(1+c)*f)^3*ln(d
*x+c+1)/d^3/f
```

3.103.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.45

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right) \, dx$$
$$= \frac{2d(3ad^2e^2+bf(3de-2cf))\,x+d^2f(6ade+bf)x^2+2ad^3f^2x^3+2bd^3x(3e^2+3efx+f^2x^2)\coth^{-1}(c+d}{}$$

input
```
Integrate[(e + f*x)^2*(a + b*ArcCoth[c + d*x]),x]
```

output
```
(2*d*(3*a*d^2*e^2 + b*f*(3*d*e - 2*c*f))*x + d^2*f*(6*a*d*e + b*f)*x^2 + 2
*a*d^3*f^2*x^3 + 2*b*d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCoth[c + d*x] -
b*(-1 + c)*(3*d^2*e^2 - 3*(-1 + c)*d*e*f + (-1 + c)^2*f^2)*Log[1 - c - d*x
] + b*(1 + c)*(3*d^2*e^2 - 3*(1 + c)*d*e*f + (1 + c)^2*f^2)*Log[1 + c + d*
x])/(6*d^3)
```

3.103.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6662, 27, 6479, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right) dx$$

$$\Big\downarrow \text{ 6662}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2\left(a+b\coth^{-1}(c+dx)\right)}{d^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (de-cf+f(c+dx))^2\left(a+b\coth^{-1}(c+dx)\right)d(c+dx)}{d^3}$$

$$\Big\downarrow \text{ 6479}$$

$$\frac{\frac{(f(c+dx)-cf+de)^3\left(a+b\coth^{-1}(c+dx)\right)}{3f}-\frac{b\int \frac{(de-cf+f(c+dx))^3}{1-(c+dx)^2}d(c+dx)}{3f}}{d^3}$$

$$\Big\downarrow \text{ 477}$$

$$\frac{\frac{(f(c+dx)-cf+de)^3\left(a+b\coth^{-1}(c+dx)\right)}{3f}-\frac{b\int \left(-\left((c+dx)f^3\right)-3(de-cf)f^2+\frac{(de-cf+f)^3}{2(-c-dx+1)}+\frac{(de-(c+1)f)^3}{2(c+dx+1)}\right)d(c+dx)}{3f}}{d^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{(f(c+dx)-cf+de)^3\left(a+b\coth^{-1}(c+dx)\right)}{3f}-\frac{b\left(-3f^2(c+dx)(de-cf)-\frac{1}{2}(-cf+de+f)^3\log(-c-dx+1)+\frac{1}{2}(de-(c+1)f)^3\log(c+dx+1)-\frac{1}{2}f^3(c+dx\right.}{3f}}{d^3}$$

input
```
Int[(e + f*x)^2*(a + b*ArcCoth[c + d*x]),x]
```

output
```
(((d*e - c*f + f*(c + d*x))^3*(a + b*ArcCoth[c + d*x]))/(3*f) - (b*(-3*f^2
*(d*e - c*f)*(c + d*x) - (f^3*(c + d*x)^2)/2 - ((d*e + f - c*f)^3*Log[1 -
c - d*x])/2 + ((d*e - (1 + c)*f)^3*Log[1 + c + d*x])/2))/(3*f))/d^3
```

3.103.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6479
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.103.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(112) = 224$.

Time = 0.48 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.92

method	result
parallelrisch	$\frac{6x^2 a d^3 ef + 6x \operatorname{arccoth}(dx+c) b d^3 e^2 + 2x^3 \operatorname{arccoth}(dx+c) b d^3 f^2 + 6 \operatorname{arccoth}(dx+c) bc d^2 e^2 - 6 \operatorname{arccoth}(dx+c) bdef - 4xbcd}{}$
parts	$\frac{a(fx+e)^3}{3f} + \frac{b\left(\frac{f^2 \operatorname{arccoth}(dx+c)(dx+c)^3}{3d^2} - \frac{f^2 \operatorname{arccoth}(dx+c)(dx+c)^2 c}{d^2} + \frac{f \operatorname{arccoth}(dx+c)(dx+c)^2 e}{d} + \frac{f^2 \operatorname{arccoth}(dx+c)(dx+c)c}{d^2}\right.}{}$
derivativedivides	$\frac{-\frac{a(cf-de-f(dx+c))^3}{3d^2 f} + \frac{b\left(-\frac{f^2 \operatorname{arccoth}(dx+c)c^3}{3} + f \operatorname{arccoth}(dx+c)c^2 de + f^2 \operatorname{arccoth}(dx+c)c^2(dx+c) - \operatorname{arccoth}(dx+c)c d^2 e^2 - 2f \operatorname{arc}\right.}{}}{}$
default	$\frac{-\frac{a(cf-de-f(dx+c))^3}{3d^2 f} + \frac{b\left(-\frac{f^2 \operatorname{arccoth}(dx+c)c^3}{3} + f \operatorname{arccoth}(dx+c)c^2 de + f^2 \operatorname{arccoth}(dx+c)c^2(dx+c) - \operatorname{arccoth}(dx+c)c d^2 e^2 - 2f \operatorname{arc}\right.}{}}{}$
risch	$-\frac{\ln(-dx-c-1)b e^3}{6f} - \frac{b e^2 x \ln(dx+c-1)}{2} + \frac{f^2 a x^3}{3} + \frac{f^2 b x^2}{6d} + \frac{f^2 \ln(-dx-c-1)b}{6d^3} + \frac{f^2 \ln(dx+c-1)b}{6d^3} + \frac{\ln(-d}{}$

input
```
int((f*x+e)^2*(a+b*arccoth(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/6*(6*x^2*a*d^3*e*f+6*x*arccoth(d*x+c)*b*d^3*e^2+2*x^3*arccoth(d*x+c)*b*d
^3*f^2+6*arccoth(d*x+c)*b*c*d^2*e^2-6*arccoth(d*x+c)*b*d*e*f-4*x*b*c*d*f^2
+6*x*b*d^2*e*f-6*a*c^2*e*f*d+x^2*b*d^2*f^2+2*arccoth(d*x+c)*b*c^3*f^2+6*ar
ccoth(d*x+c)*b*c^2*f^2+6*arccoth(d*x+c)*b*d^2*e^2+6*arccoth(d*x+c)*b*c*f^2
+6*x*a*d^3*e^2+2*x^3*a*d^3*f^2+6*ln(d*x+c-1)*b*c^2*f^2+6*ln(d*x+c-1)*b*d^2
*e^2-12*a*c*e^2*d^2-12*ln(d*x+c-1)*b*c*d*e*f+6*x^2*arccoth(d*x+c)*b*d^3*e*
f-12*arccoth(d*x+c)*b*c*d*e*f-6*arccoth(d*x+c)*b*c^2*d*e*f+b*f^2+7*b*c^2*f
^2+6*a*e*f*d-12*b*c*d*e*f+2*ln(d*x+c-1)*b*f^2+2*arccoth(d*x+c)*b*f^2)/d^3
```

3.103.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(112) = 224$.

Time = 0.27 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.01

$$\int (e+fx)^2 \left(a + b \coth^{-1}(c+dx)\right) dx$$
$$= \frac{2\,ad^3 f^2 x^3 + (6\,ad^3 ef + bd^2 f^2)x^2 + 2\,(3\,ad^3 e^2 + 3\,bd^2 ef - 2\,bcdf^2)x + (3\,(bc+b)d^2 e^2 - 3\,(bc^2 + 2\,bc + b)d}{}$$

input
```
integrate((f*x+e)^2*(a+b*arccoth(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*a*d^3*f^2*x^3 + (6*a*d^3*e*f + b*d^2*f^2)*x^2 + 2*(3*a*d^3*e^2 + 3*
b*d^2*e*f - 2*b*c*d*f^2)*x + (3*(b*c + b)*d^2*e^2 - 3*(b*c^2 + 2*b*c + b)*
d*e*f + (b*c^3 + 3*b*c^2 + 3*b*c + b)*f^2)*log(d*x + c + 1) - (3*(b*c - b)
*d^2*e^2 - 3*(b*c^2 - 2*b*c + b)*d*e*f + (b*c^3 - 3*b*c^2 + 3*b*c - b)*f^2
)*log(d*x + c - 1) + (b*d^3*f^2*x^3 + 3*b*d^3*e*f*x^2 + 3*b*d^3*e^2*x)*log
((d*x + c + 1)/(d*x + c - 1)))/d^3
```

3.103.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(105) = 210$.

Time = 0.52 (sec) , antiderivative size = 369, normalized size of antiderivative = 3.08

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)\,dx$$
$$= \begin{cases} ae^2x + aefx^2 + \frac{af^2x^3}{3} + \frac{bc^3f^2\operatorname{acoth}(c+dx)}{3d^3} - \frac{bc^2ef\operatorname{acoth}(c+dx)}{d^2} + \frac{bc^2f^2\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^3} - \frac{bc^2f^2\operatorname{acoth}(c+dx)}{d^3} + \frac{bce^2\operatorname{acoth}(}{d} \\ (a+b\operatorname{acoth}(c))\left(e^2x + efx^2 + \frac{f^2x^3}{3}\right) \end{cases}$$

input
```
integrate((f*x+e)**2*(a+b*acoth(d*x+c)),x)
```

output
```
Piecewise((a*e**2*x + a*e*f*x**2 + a*f**2*x**3/3 + b*c**3*f**2*acoth(c + d
*x)/(3*d**3) - b*c**2*e*f*acoth(c + d*x)/d**2 + b*c**2*f**2*log(c/d + x +
1/d)/d**3 - b*c**2*f**2*acoth(c + d*x)/d**3 + b*c*e**2*acoth(c + d*x)/d -
2*b*c*e*f*log(c/d + x + 1/d)/d**2 + 2*b*c*e*f*acoth(c + d*x)/d**2 - 2*b*c*
f**2*x/(3*d**2) + b*c*f**2*acoth(c + d*x)/d**3 + b*e**2*x*acoth(c + d*x) +
 b*e*f*x**2*acoth(c + d*x) + b*f**2*x**3*acoth(c + d*x)/3 + b*e**2*log(c/d
 + x + 1/d)/d - b*e**2*acoth(c + d*x)/d + b*e*f*x/d + b*f**2*x**2/(6*d) -
b*e*f*acoth(c + d*x)/d**2 + b*f**2*log(c/d + x + 1/d)/(3*d**3) - b*f**2*ac
oth(c + d*x)/(3*d**3), Ne(d, 0)), ((a + b*acoth(c))*(e**2*x + e*f*x**2 + f
**2*x**3/3), True))
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.72

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)\,dx = \frac{1}{3}af^2x^3 + aefx^2$$
$$+\frac{1}{2}\left(2\,x^2\,\mathrm{arcoth}\,(dx+c)+d\left(\frac{2\,x}{d^2}-\frac{(c^2+2\,c+1)\log{(dx+c+1)}}{d^3}+\frac{(c^2-2\,c+1)\log{(dx+c-1)}}{d^3}\right)\right)b$$
$$+\frac{1}{6}\left(2\,x^3\,\mathrm{arcoth}\,(dx+c)+d\left(\frac{dx^2-4\,cx}{d^3}+\frac{(c^3+3\,c^2+3\,c+1)\log{(dx+c+1)}}{d^4}-\frac{(c^3-3\,c^2+3\,c-1)}{d^4}\right.\right.$$
$$+ae^2x+\frac{\left(2\,(dx+c)\,\mathrm{arcoth}\,(dx+c)+\log\left(-(dx+c)^2+1\right)\right)be^2}{2\,d}$$

input
```
integrate((f*x+e)^2*(a+b*arccoth(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*a*f^2*x^3 + a*e*f*x^2 + 1/2*(2*x^2*arccoth(d*x + c) + d*(2*x/d^2 - (c^
2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))
*b*e*f + 1/6*(2*x^3*arccoth(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 + (c^3 + 3*c
^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*log(d*x + c -
 1)/d^4))*b*f^2 + a*e^2*x + 1/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x
+ c)^2 + 1))*b*e^2/d
```

3.103.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 973 vs. 2(112) = 224.

Time = 0.30 (sec) , antiderivative size = 973, normalized size of antiderivative = 8.11

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*(a+b*arccoth(d*x+c)),x, algorithm="giac")
```

output
```
1/6*((c + 1)*d - (c - 1)*d)*((3*(d*x + c + 1)^2*b*d^2*e^2/(d*x + c - 1)^2
- 6*(d*x + c + 1)*b*d^2*e^2/(d*x + c - 1) + 3*b*d^2*e^2 - 6*(d*x + c + 1)^
2*b*c*d*e*f/(d*x + c - 1)^2 + 12*(d*x + c + 1)*b*c*d*e*f/(d*x + c - 1) - 6
*b*c*d*e*f + 3*(d*x + c + 1)^2*b*c^2*f^2/(d*x + c - 1)^2 - 6*(d*x + c + 1)
*b*c^2*f^2/(d*x + c - 1) + 3*b*c^2*f^2 + 6*(d*x + c + 1)^2*b*d*e*f/(d*x +
c - 1)^2 - 6*(d*x + c + 1)*b*d*e*f/(d*x + c - 1) - 6*(d*x + c + 1)^2*b*c*f
^2/(d*x + c - 1)^2 + 6*(d*x + c + 1)*b*c*f^2/(d*x + c - 1) + 3*(d*x + c +
1)^2*b*f^2/(d*x + c - 1)^2 + b*f^2)*log((d*x + c + 1)/(d*x + c - 1))/((d*x
 + c + 1)^3*d^4/(d*x + c - 1)^3 - 3*(d*x + c + 1)^2*d^4/(d*x + c - 1)^2 +
3*(d*x + c + 1)*d^4/(d*x + c - 1) - d^4) + 2*(3*(d*x + c + 1)^2*a*d^2*e^2/
(d*x + c - 1)^2 - 6*(d*x + c + 1)*a*d^2*e^2/(d*x + c - 1) + 3*a*d^2*e^2 -
6*(d*x + c + 1)^2*a*c*d*e*f/(d*x + c - 1)^2 + 12*(d*x + c + 1)*a*c*d*e*f/(
d*x + c - 1) - 6*a*c*d*e*f + 3*(d*x + c + 1)^2*a*c^2*f^2/(d*x + c - 1)^2 -
 6*(d*x + c + 1)*a*c^2*f^2/(d*x + c - 1) + 3*a*c^2*f^2 + 6*(d*x + c + 1)^2
*a*d*e*f/(d*x + c - 1)^2 - 6*(d*x + c + 1)*a*d*e*f/(d*x + c - 1) + 3*(d*x
+ c + 1)^2*b*d*e*f/(d*x + c - 1)^2 - 6*(d*x + c + 1)*b*d*e*f/(d*x + c - 1)
 + 3*b*d*e*f - 6*(d*x + c + 1)^2*a*c*f^2/(d*x + c - 1)^2 + 6*(d*x + c + 1)
*a*c*f^2/(d*x + c - 1) - 3*(d*x + c + 1)^2*b*c*f^2/(d*x + c - 1)^2 + 6*(d*
x + c + 1)*b*c*f^2/(d*x + c - 1) - 3*b*c*f^2 + 3*(d*x + c + 1)^2*a*f^2/(d*
x + c - 1)^2 + a*f^2 + (d*x + c + 1)^2*b*f^2/(d*x + c - 1)^2 - (d*x + c...
```

3.103.9 Mupad [B] (verification not implemented)

Time = 4.69 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.22

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)\,dx = x^2\left(\frac{f\,(b\,f+6\,a\,c\,f+6\,a\,d\,e)}{6\,d}-\frac{a\,c\,f^2}{d}\right)
-\ln\left(1-\frac{1}{c+d\,x}\right)\left(\frac{b\,e^2\,x}{2}+\frac{b\,e\,f\,x^2}{2}+\frac{b\,f^2\,x^3}{6}\right)-x\left(\frac{2\,c\left(\frac{f\,(b\,f+6\,a\,c\,f+6\,a\,d\,e)}{3\,d}-\frac{2\,a\,c\,f^2}{d}\right)}{d}
-\frac{3\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+3\,a\,d^2\,e^2+3\,b\,d\,e\,f-3\,a\,f^2}{3\,d^2}+\frac{a\,f^2\,(3\,c^2-3)}{3\,d^2}\right)
+\ln\left(\frac{1}{c+d\,x}+1\right)\left(\frac{b\,e^2\,x}{2}+\frac{b\,e\,f\,x^2}{2}+\frac{b\,f^2\,x^3}{6}\right)+\frac{a\,f^2\,x^3}{3}
+\frac{\ln(c+d\,x-1)\left(\frac{b\,f^2}{6}+d\left(\frac{b\,e\,f\,c^2}{2}-b\,e\,f\,c+\frac{b\,e\,f}{2}\right)+d^2\left(\frac{b\,e^2}{2}-\frac{b\,c\,e^2}{2}\right)+\frac{b\,c^2\,f^2}{2}-\frac{b\,c^3\,f^2}{6}-\frac{b\,c\,f^2}{2}\right)}{d^3}
+\frac{\ln(c+d\,x+1)\left(\frac{b\,f^2}{6}-d\left(\frac{b\,e\,f\,c^2}{2}+b\,e\,f\,c+\frac{b\,e\,f}{2}\right)+d^2\left(\frac{b\,e^2}{2}+\frac{b\,c\,e^2}{2}\right)+\frac{b\,c^2\,f^2}{2}+\frac{b\,c^3\,f^2}{6}+\frac{b\,c\,f^2}{2}\right)}{d^3}$$

input
```
int((e + f*x)^2*(a + b*acoth(c + d*x)),x)
```

output
```
x^2*((f*(b*f + 6*a*c*f + 6*a*d*e))/(6*d) - (a*c*f^2)/d) - log(1 - 1/(c + d
*x))*((b*f^2*x^3)/6 + (b*e^2*x)/2 + (b*e*f*x^2)/2) - x*((2*c*((f*(b*f + 6*
a*c*f + 6*a*d*e))/(3*d) - (2*a*c*f^2)/d))/d - (3*a*c^2*f^2 - 3*a*f^2 + 3*a
*d^2*e^2 + 3*b*d*e*f + 12*a*c*d*e*f)/(3*d^2) + (a*f^2*(3*c^2 - 3))/(3*d^2)
) + log(1/(c + d*x) + 1)*((b*f^2*x^3)/6 + (b*e^2*x)/2 + (b*e*f*x^2)/2) + (
a*f^2*x^3)/3 + (log(c + d*x - 1)*((b*f^2)/6 + d*((b*e*f)/2 + (b*c^2*e*f)/2
 - b*c*e*f) + d^2*((b*e^2)/2 - (b*c*e^2)/2) + (b*c^2*f^2)/2 - (b*c^3*f^2)/
6 - (b*c*f^2)/2))/d^3 + (log(c + d*x + 1)*((b*f^2)/6 - d*((b*e*f)/2 + (b*c
^2*e*f)/2 + b*c*e*f) + d^2*((b*e^2)/2 + (b*c*e^2)/2) + (b*c^2*f^2)/2 + (b*
c^3*f^2)/6 + (b*c*f^2)/2))/d^3
```

3.103. $\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)\,dx$

3.104 $\int (e+fx) \left(a+b \coth^{-1}(c+dx)\right) \, dx$

3.104.1 Optimal result

Integrand size = 16, antiderivative size = 97

$$\int (e+fx) \left(a+b \coth^{-1}(c+dx)\right) \, dx = \frac{bfx}{2d}+\frac{(e+fx)^2 \left(a+b \coth^{-1}(c+dx)\right)}{2f} + \frac{b(de+f-cf)^2 \log(1-c-dx)}{4d^2 f} - \frac{b(de-(1+c)f)^2 \log(1+c+dx)}{4d^2 f}$$

output
```
1/2*b*f*x/d+1/2*(f*x+e)^2*(a+b*arccoth(d*x+c))/f+1/4*b*(-c*f+d*e+f)^2*ln(-
d*x-c+1)/d^2/f-1/4*b*(d*e-(1+c)*f)^2*ln(d*x+c+1)/d^2/f
```

3.104.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.42

$$\begin{aligned}\int (e+fx) \left(a+b \coth^{-1}(c+dx)\right) \, dx &= aex+\frac{bfx}{2d}+\frac{1}{2}afx^2+bex \coth^{-1}(c+dx)+\frac{1}{2}bfx^2 \coth^{-1}(c+dx) \\ &+ \frac{b(1-2c+c^2)\, f \log(1-c-dx)}{4d^2}+\frac{b(-1-2c-c^2)\, f \log(1+c+dx)}{4d^2} \\ &+ \frac{be(-((-1+c) \log(1-c-dx))+(1+c) \log(1+c+dx))}{2d}\end{aligned}$$

input
```
Integrate[(e + f*x)*(a + b*ArcCoth[c + d*x]),x]
```

output
```
a*e*x + (b*f*x)/(2*d) + (a*f*x^2)/2 + b*e*x*ArcCoth[c + d*x] + (b*f*x^2*Ar
cCoth[c + d*x])/2 + (b*(1 - 2*c + c^2)*f*Log[1 - c - d*x])/(4*d^2) + (b*(-
1 - 2*c - c^2)*f*Log[1 + c + d*x])/(4*d^2) + (b*e*(-((-1 + c)*Log[1 - c -
d*x]) + (1 + c)*Log[1 + c + d*x]))/(2*d)
```

3.104.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6662, 27, 6479, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)dx$$

$$\Big\downarrow \text{ 6662}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)\left(a+b\coth^{-1}(c+dx)\right)}{d}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (de-cf+f(c+dx))\left(a+b\coth^{-1}(c+dx)\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \text{ 6479}$$

$$\frac{\frac{(f(c+dx)-cf+de)^2\left(a+b\coth^{-1}(c+dx)\right)}{2f}-\frac{b\int \frac{(de-cf+f(c+dx))^2}{1-(c+dx)^2}d(c+dx)}{2f}}{d^2}$$

$$\Big\downarrow \text{ 477}$$

$$\frac{\frac{(f(c+dx)-cf+de)^2\left(a+b\coth^{-1}(c+dx)\right)}{2f}-\frac{b\int \left(-f^2+\frac{(de-cf+f)^2}{2(-c-dx+1)}+\frac{(de-(c+1)f)^2}{2(c+dx+1)}\right)d(c+dx)}{2f}}{d^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{(f(c+dx)-cf+de)^2\left(a+b\coth^{-1}(c+dx)\right)}{2f}-\frac{b\left(-\frac{1}{2}(-cf+de+f)^2\log(-c-dx+1)+\frac{1}{2}(de-(c+1)f)^2\log(c+dx+1)-\left(f^2(c+dx)\right)\right)}{2f}}{d^2}$$

input
```
Int[(e + f*x)*(a + b*ArcCoth[c + d*x]),x]
```

output
```
(((d*e - c*f + f*(c + d*x))^2*(a + b*ArcCoth[c + d*x]))/(2*f) - (b*(-(f^2*
(c + d*x)) - ((d*e + f - c*f)^2*Log[1 - c - d*x])/2 + ((d*e - (1 + c)*f)^2
*Log[1 + c + d*x])/2))/(2*f))/d^2
```

3.104.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6479
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.104.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.25

method	result
parts	$a\left(\frac{1}{2}f\,x^{2}+ex\right)+\frac{b\left(\frac{\operatorname{arccoth}(dx+c)(dx+c)^{2}f}{2d}-\frac{\operatorname{arccoth}(dx+c)cf(dx+c)}{d}+\operatorname{arccoth}(dx+c)e(dx+c)+\frac{f(dx+c)+\frac{(-2cf+2de \cdots}{\cdots}}{\cdots}\right.}{d}$
derivativedivides	$\frac{-\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^{2}}{2}\right)}{d}-\frac{b\left(\operatorname{arccoth}(dx+c)fc(dx+c)-\operatorname{arccoth}(dx+c)ed(dx+c)-\frac{\operatorname{arccoth}(dx+c)f(dx+c)^{2}}{2}-\frac{f(dx+c)}{2}\cdots\right.}{d}}{d}$
default	$\frac{-\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^{2}}{2}\right)}{d}-\frac{b\left(\operatorname{arccoth}(dx+c)fc(dx+c)-\operatorname{arccoth}(dx+c)ed(dx+c)-\frac{\operatorname{arccoth}(dx+c)f(dx+c)^{2}}{2}-\frac{f(dx+c)}{2}\cdots\right.}{d}}{d}$
parallelrisch	$-\frac{-\operatorname{arccoth}(dx+c)b\,d^{2}f\,x^{2}-a\,d^{2}f\,x^{2}-2x\,\operatorname{arccoth}(dx+c)b\,d^{2}e-2a\,d^{2}ex+\operatorname{arccoth}(dx+c)b\,c^{2}f-2\,\operatorname{arccoth}(dx+c)bcde+2\cdots}{\cdots}$
risch	$\frac{bx(fx+2e)\ln(dx+c+1)}{4}-\frac{bf\,x^{2}\ln(dx+c-1)}{4}-\frac{bex\ln(dx+c-1)}{2}+\frac{af\,x^{2}}{2}+\frac{\ln(-dx-c+1)b\,c^{2}f}{4d^{2}}-\frac{\ln(-dx-c+1)\cdots}{2d}$

input
```
int((f*x+e)*(a+b*arccoth(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*f*x^2+e*x)+b/d*(1/2/d*arccoth(d*x+c)*(d*x+c)^2*f-1/d*arccoth(d*x+c)
*c*f*(d*x+c)+arccoth(d*x+c)*e*(d*x+c)+1/2/d*(f*(d*x+c)+1/2*(-2*c*f+2*d*e+f
)*ln(d*x+c-1)-1/2*(2*c*f-2*d*e+f)*ln(d*x+c+1)))
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.37

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)\,dx$$
$$=\frac{2\,ad^{2}fx^{2}+2\,(2\,ad^{2}e+bdf)x+(2\,(bc+b)de-(bc^{2}+2\,bc+b)f)\log\left(dx+c+1\right)-(2\,(bc-b)de-(bc^{2}-\cdots}{4\,d^{2}}$$

input
```
integrate((f*x+e)*(a+b*arccoth(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*a*d^2*f*x^2 + 2*(2*a*d^2*e + b*d*f)*x + (2*(b*c + b)*d*e - (b*c^2 +
 2*b*c + b)*f)*log(d*x + c + 1) - (2*(b*c - b)*d*e - (b*c^2 - 2*b*c + b)*f
)*log(d*x + c - 1) + (b*d^2*f*x^2 + 2*b*d^2*e*x)*log((d*x + c + 1)/(d*x +
c - 1)))/d^2
```

3.104.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(82) = 164.

Time = 0.41 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.78

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)\,dx = \begin{cases} aex+\frac{afx^2}{2}-\frac{bc^2f\,\mathrm{acoth}\,(c+dx)}{2d^2}+\frac{bce\,\mathrm{acoth}\,(c+dx)}{d}-\frac{bcf\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^2}+\frac{bcf\,\mathrm{acoth}\,(c+dx)}{d^2}+bex\,\mathrm{acoth}\,(c+dx)+\frac{bfx^2}{} \\ (a+b\,\mathrm{acoth}\,(c))\left(ex+\frac{fx^2}{2}\right) \end{cases}$$

input
```
integrate((f*x+e)*(a+b*acoth(d*x+c)),x)
```

output
```
Piecewise((a*e*x + a*f*x**2/2 - b*c**2*f*acoth(c + d*x)/(2*d**2) + b*c*e*a
coth(c + d*x)/d - b*c*f*log(c/d + x + 1/d)/d**2 + b*c*f*acoth(c + d*x)/d**
2 + b*e*x*acoth(c + d*x) + b*f*x**2*acoth(c + d*x)/2 + b*e*log(c/d + x + 1
/d)/d - b*e*acoth(c + d*x)/d + b*f*x/(2*d) - b*f*acoth(c + d*x)/(2*d**2),
Ne(d, 0)), ((a + b*acoth(c))*(e*x + f*x**2/2), True))
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.12

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)\,dx = \frac{1}{2}afx^2 + \frac{1}{4}\left(2x^2\,\mathrm{arcoth}\,(dx+c)+d\left(\frac{2x}{d^2}-\frac{(c^2+2c+1)\log(dx+c+1)}{d^3}+\frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)b + aex + \frac{\left(2(dx+c)\,\mathrm{arcoth}\,(dx+c)+\log\left(-(dx+c)^2+1\right)\right)be}{2d}$$

input
```
integrate((f*x+e)*(a+b*arccoth(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*a*f*x^2 + 1/4*(2*x^2*arccoth(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*l
og(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*b*f + a*e*x +
 1/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*b*e/d
```

3.104.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(89) = 178$.

Time = 0.28 (sec) , antiderivative size = 338, normalized size of antiderivative = 3.48

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)\,dx$$
$$=\frac{1}{2}\left((c+1)d-(c-1)d\right)\left(\frac{\left(\frac{(dx+c+1)bde}{dx+c-1}-bde-\frac{(dx+c+1)bcf}{dx+c-1}+bcf+\frac{(dx+c+1)bf}{dx+c-1}\right)\log\left(\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)^2d^3}{(dx+c-1)^2}-\frac{2\,(dx+c+1)d^3}{dx+c-1}+d^3}+\frac{\frac{2\,(dx+c+1)ade}{dx+c-1}}{}\right.$$

input
```
integrate((f*x+e)*(a+b*arccoth(d*x+c)),x, algorithm="giac")
```

output
```
1/2*((c + 1)*d - (c - 1)*d)*((((d*x + c + 1)*b*d*e/(d*x + c - 1) - b*d*e -
(d*x + c + 1)*b*c*f/(d*x + c - 1) + b*c*f + (d*x + c + 1)*b*f/(d*x + c - 1
))*log((d*x + c + 1)/(d*x + c - 1))/((d*x + c + 1)^2*d^3/(d*x + c - 1)^2 -
 2*(d*x + c + 1)*d^3/(d*x + c - 1) + d^3) + (2*(d*x + c + 1)*a*d*e/(d*x +
c - 1) - 2*a*d*e - 2*(d*x + c + 1)*a*c*f/(d*x + c - 1) + 2*a*c*f + 2*(d*x
+ c + 1)*a*f/(d*x + c - 1) + (d*x + c + 1)*b*f/(d*x + c - 1) - b*f)/((d*x
+ c + 1)^2*d^3/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^3/(d*x + c - 1) + d^3)
- (b*d*e - b*c*f)*log((d*x + c + 1)/(d*x + c - 1) - 1)/d^3 + (b*d*e - b*c*
f)*log((d*x + c + 1)/(d*x + c - 1))/d^3)
```

3.104.9 Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.40

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)\,dx=a\,e\,x+\frac{a\,f\,x^2}{2}+\frac{b\,e\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2-1\right)}{2\,d}-\frac{b\,f\,\mathrm{acoth}(c+d\,x)}{2\,d^2}+\frac{b\,f\,x^2\,\mathrm{acoth}(c+d\,x)}{2}+\frac{b\,f\,x}{2\,d}+b\,e\,x\,\mathrm{acoth}(c+d\,x)-\frac{b\,c^2\,f\,\mathrm{acoth}(c+d\,x)}{2\,d^2}-\frac{b\,c\,f\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2-1\right)}{2\,d^2}+\frac{b\,c\,e\,\mathrm{acoth}(c+d\,x)}{d}$$

input
```
int((e + f*x)*(a + b*acoth(c + d*x)),x)
```

output
```
a*e*x + (a*f*x^2)/2 + (b*e*log(c^2 + d^2*x^2 + 2*c*d*x - 1))/(2*d) - (b*f*
acoth(c + d*x))/(2*d^2) + (b*f*x^2*acoth(c + d*x))/2 + (b*f*x)/(2*d) + b*e
*x*acoth(c + d*x) - (b*c^2*f*acoth(c + d*x))/(2*d^2) - (b*c*f*log(c^2 + d^
2*x^2 + 2*c*d*x - 1))/(2*d^2) + (b*c*e*acoth(c + d*x))/d
```

3.104. $\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)\,dx$

3.105 $\int (a+b\coth^{-1}(c+dx))\,dx$

3.105.1 Optimal result

Integrand size = 10, antiderivative size = 40

$$\int (a+b\coth^{-1}(c+dx))\,dx = ax+\frac{b(c+dx)\coth^{-1}(c+dx)}{d}+\frac{b\log(1-(c+dx)^2)}{2d}$$

output
```
a*x+b*(d*x+c)*arccoth(d*x+c)/d+1/2*b*ln(1-(d*x+c)^2)/d
```

3.105.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int (a+b\coth^{-1}(c+dx))\,dx = ax+bx\coth^{-1}(c+dx) + \frac{b(-((-1+c)\log(1-c-dx))+(1+c)\log(1+c+dx))}{2d}$$

input
```
Integrate[a + b*ArcCoth[c + d*x],x]
```

output
```
a*x + b*x*ArcCoth[c + d*x] + (b*(-((-1 + c)*Log[1 - c - d*x]) + (1 + c)*Lo
g[1 + c + d*x]))/(2*d)
```

3.105.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\coth^{-1}(c+dx)\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$ax+\frac{b\log\left(1-(c+dx)^2\right)}{2d}+\frac{b(c+dx)\coth^{-1}(c+dx)}{d}$$

input
```
Int[a + b*ArcCoth[c + d*x],x]
```

output
```
a*x + (b*(c + d*x)*ArcCoth[c + d*x])/d + (b*Log[1 - (c + d*x)^2])/(2*d)
```

3.105.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.105.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result
default	$ax+\frac{b\left((dx+c)\,\mathrm{arccoth}(dx+c)+\frac{\ln\left((dx+c)^2-1\right)}{2}\right)}{d}$
parts	$ax+\frac{b\left((dx+c)\,\mathrm{arccoth}(dx+c)+\frac{\ln\left((dx+c)^2-1\right)}{2}\right)}{d}$
derivativedivides	$\frac{(dx+c)a+b\left((dx+c)\,\mathrm{arccoth}(dx+c)+\frac{\ln\left((dx+c)^2-1\right)}{2}\right)}{d}$
parallelrisch	$-\frac{b\left(-x\,\mathrm{arccoth}(dx+c)d^2-\mathrm{arccoth}(dx+c)cd-\ln(dx+c-1)d-\mathrm{arccoth}(dx+c)d\right)}{d^2}+ax$
risch	$ax+\frac{bx\ln(dx+c+1)}{2}-\frac{bx\ln(dx+c-1)}{2}-\frac{b\ln(dx+c-1)c}{2d}+\frac{b\ln(-dx-c-1)c}{2d}+\frac{b\ln(dx+c-1)}{2d}+\frac{b\ln(-dx-c-}{2d}$

input
```
int(a+b*arccoth(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+b/d*((d*x+c)*arccoth(d*x+c)+1/2*ln((d*x+c)^2-1))
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int \left(a+b\coth^{-1}(c+dx)\right)\,dx = \frac{bdx\log\left(\frac{dx+c+1}{dx+c-1}\right)+2\,adx+(bc+b)\log(dx+c+1)-(bc-b)\log(dx+c-1)}{2\,d}$$

input
```
integrate(a+b*arccoth(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(b*d*x*log((d*x + c + 1)/(d*x + c - 1)) + 2*a*d*x + (b*c + b)*log(d*x
+ c + 1) - (b*c - b)*log(d*x + c - 1))/d
```

3.105.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \left(a+b\coth^{-1}(c+dx)\right)\,dx = ax+b\left(\begin{cases}\frac{c\,\mathrm{acoth}\,(c+dx)}{d}+x\,\mathrm{acoth}\,(c+dx)+\frac{\log(c+dx+1)}{d}-\frac{\mathrm{acoth}\,(c+dx)}{d} & \text{for } d\neq 0\\ x\,\mathrm{acoth}\,(c) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*acoth(d*x+c),x)
```

output
```
a*x + b*Piecewise((c*acoth(c + d*x)/d + x*acoth(c + d*x) + log(c + d*x + 1
)/d - acoth(c + d*x)/d, Ne(d, 0)), (x*acoth(c), True))
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \left(a+b\coth^{-1}(c+dx)\right)\,dx=ax+\frac{\left(2\,(dx+c)\,\mathrm{arcoth}\,(dx+c)+\log\left(-(dx+c)^2+1\right)\right)b}{2\,d}$$

input
```
integrate(a+b*arccoth(d*x+c),x, algorithm="maxima")
```

output
```
a*x + 1/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*b/d
```

3.105.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(38) = 76.

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 5.05

$$\int \left(a+b\coth^{-1}(c+dx)\right)\,dx$$
$$=\frac{1}{2}\left((c+1)d-(c-1)d\right)b\left(\frac{\log\left(\frac{|dx+c+1|}{|dx+c-1|}\right)}{d^2}-\frac{\log\left(\left|\frac{dx+c+1}{dx+c-1}-1\right|\right)}{d^2}+\frac{\log\left(-\frac{\frac{1}{c-\frac{\left(\frac{(dx+c+1)(c-1)}{dx+c-1}-c-1\right)d}{\frac{(dx+c+1)d}{dx+c-1}-d}}+1}{\frac{1}{c-\frac{\left(\frac{(dx+c+1)(c-1)}{dx+c-1}-c-1\right)d}{\frac{(dx+c+1)d}{dx+c-1}-d}}-1}\right)}{d^2\left(\frac{dx+c+1}{dx+c-1}-1\right)}\right)$$
$$+ax$$

input
```
integrate(a+b*arccoth(d*x+c),x, algorithm="giac")
```

output
```
1/2*((c + 1)*d - (c - 1)*d)*b*(log(abs(d*x + c + 1)/abs(d*x + c - 1))/d^2
- log(abs((d*x + c + 1)/(d*x + c - 1) - 1))/d^2 + log(-(1/(c - ((d*x + c +
 1)*(c - 1)/(d*x + c - 1) - c - 1)*d/((d*x + c + 1)*d/(d*x + c - 1) - d))
+ 1)/(1/(c - ((d*x + c + 1)*(c - 1)/(d*x + c - 1) - c - 1)*d/((d*x + c + 1
)*d/(d*x + c - 1) - d)) - 1))/(d^2*((d*x + c + 1)/(d*x + c - 1) - 1))) + a
*x
```

3.105.9 Mupad [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \left(a+b\coth^{-1}(c+dx)\right)\,dx = a\,x+\frac{\frac{b\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2-1\right)}{2}+b\,c\,\mathrm{acoth}(c+d\,x)}{d}+b\,x\,\mathrm{acoth}(c+d\,x)$$

input
```
int(a + b*acoth(c + d*x),x)
```

output
```
a*x + ((b*log(c^2 + d^2*x^2 + 2*c*d*x - 1))/2 + b*c*acoth(c + d*x))/d + b*
x*acoth(c + d*x)
```

3.106 $\int \frac{a+b\coth^{-1}(c+dx)}{e+fx}\,dx$

3.106.1 Optimal result

Integrand size = 18, antiderivative size = 130

$$\int \frac{a+b\coth^{-1}(c+dx)}{e+fx}\,dx = -\frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f} + \frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{2f} - \frac{b\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{2f}$$

output
```
-(a+b*arccoth(d*x+c))*ln(2/(d*x+c+1))/f+(a+b*arccoth(d*x+c))*ln(2*d*(f*x+e
)/(-c*f+d*e+f)/(d*x+c+1))/f+1/2*b*polylog(2,1-2/(d*x+c+1))/f-1/2*b*polylog
(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f
```

3.106.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.58

$$\int \frac{a+b\coth^{-1}(c+dx)}{e+fx}\,dx = \frac{a\log(e+fx)}{f}+\frac{b\log\left(\frac{f(1-c-dx)}{de+f-cf}\right)\log(e+fx)}{2f} - \frac{b\log\left(-\frac{1-c-dx}{c+dx}\right)\log(e+fx)}{2f} - \frac{b\log\left(-\frac{f(1+c+dx)}{de-f-cf}\right)\log(e+fx)}{2f}+\frac{b\log\left(\frac{1+c+dx}{c+dx}\right)\log(e+fx)}{2f} - \frac{b\,\text{PolyLog}\left(2,\frac{d(e+fx)}{de-f-cf}\right)}{2f}+\frac{b\,\text{PolyLog}\left(2,\frac{d(e+fx)}{de+f-cf}\right)}{2f}$$

input
```
Integrate[(a + b*ArcCoth[c + d*x])/(e + f*x),x]
```

output
```
(a*Log[e + f*x])/f + (b*Log[(f*(1 - c - d*x))/(d*e + f - c*f)]*Log[e + f*x
])/(2*f) - (b*Log[-((1 - c - d*x)/(c + d*x))]*Log[e + f*x])/(2*f) - (b*Log
[-((f*(1 + c + d*x))/(d*e - f - c*f))]*Log[e + f*x])/(2*f) + (b*Log[(1 + c
 + d*x)/(c + d*x)]*Log[e + f*x])/(2*f) - (b*PolyLog[2, (d*(e + f*x))/(d*e
- f - c*f)])/(2*f) + (b*PolyLog[2, (d*(e + f*x))/(d*e + f - c*f)])/(2*f)
```

3.106.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6662, 27, 6473, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\coth^{-1}(c+dx)}{e+fx}\,dx$$

$$\Big\downarrow \text{6662}$$

$$\frac{\int \frac{d(a+b\coth^{-1}(c+dx))}{d\left(e-\frac{cf}{d}\right)+f(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{a+b\coth^{-1}(c+dx)}{f(c+dx)-cf+de}d(c+dx)$$

$$\Big\downarrow \text{6473}$$

$$-\frac{b\int \frac{\log\left(\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{1-(c+dx)^2}d(c+dx)}{f}+\frac{b\int \frac{\log\left(\frac{2}{c+dx+1}\right)}{1-(c+dx)^2}d(c+dx)}{f}+\frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2(f(c+dx)-cf+de)}{(c+dx+1)(-cf+de+f)}\right)}{f}-\frac{\log\left(\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{f}$$

$$\Big\downarrow \text{2849}$$

$$-\frac{b\int \frac{\log\left(\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{1-(c+dx)^2}d(c+dx)}{f}+\frac{b\int \frac{\log\left(\frac{2}{c+dx+1}\right)}{1-\frac{2}{c+dx+1}}d\frac{1}{c+dx+1}}{f}+\frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2(f(c+dx)-cf+de)}{(c+dx+1)(-cf+de+f)}\right)}{f}-\frac{\log\left(\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{f}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{b\int \frac{\log\left(\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{1-(c+dx)^2}d(c+dx)}{f}+\frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2(f(c+dx)-cf+de)}{(c+dx+1)(-cf+de+f)}\right)}{f}-\frac{\log\left(\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{f}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{c+dx+1}\right)}{2f}$$

$$\Big\downarrow \text{2897}$$

$$\frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2(f(c+dx)-cf+de)}{(c+dx+1)(-cf+de+f)}\right)}{f}-\frac{\log\left(\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{f}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{2f}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{c+dx+1}\right)}{2f}$$

input
```
Int[(a + b*ArcCoth[c + d*x])/(e + f*x),x]
```

output
```
-(((a + b*ArcCoth[c + d*x])*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcCoth[c +
 d*x])*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d*x))])
/f + (b*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*f) - (b*PolyLog[2, 1 - (2*(d*e
 - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d*x))])/(2*f)
```

3.106. $\int \frac{a+b\coth^{-1}(c+dx)}{e+fx}\,dx$

3.106.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6473
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcCoth[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*ArcCoth
[c*x])*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(c/e)
Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Simp[b*(c/e)   Int[Log[2*c*((d
 + e*x)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x]) /; FreeQ[{a, b, c, d
, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.106.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.47

method	result
risch	$\frac{a\ln((dx+c-1)f-cf+de+f)}{f}-\frac{b\,\mathrm{dilog}\left(\frac{(dx+c-1)f-cf+de+f}{-cf+de+f}\right)}{2f}-\frac{b\ln(dx+c-1)\ln\left(\frac{(dx+c-1)f-cf+de+f}{-cf+de+f}\right)}{2f}+ b\,\mathrm{dilog}$
parts	$\frac{a\ln(fx+e)}{f}+\frac{b\ln(f(dx+c)-cf+de)\operatorname{arccoth}(dx+c)}{f}+\frac{b\ln(f(dx+c)-cf+de)\ln\left(\frac{f(dx+c)-f}{cf-de-f}\right)}{2f}+\frac{b\,\mathrm{dilog}\left(\frac{f(dx+c)-f}{cf-de-f}\right.}{2f}$
derivativedivides	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}-bd\left(-\frac{\ln(cf-de-f(dx+c))\operatorname{arccoth}(dx+c)}{f}+-\frac{f\left(\mathrm{dilog}\left(\frac{-f(dx+c)+f}{-cf+de+f}\right)+\ln(cf-de-f(dx+c))\ln\left(\frac{-f(dx+}{-cf+de}\right.\right.}{2}\right.}{d}$
default	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}-bd\left(-\frac{\ln(cf-de-f(dx+c))\operatorname{arccoth}(dx+c)}{f}+-\frac{f\left(\mathrm{dilog}\left(\frac{-f(dx+c)+f}{-cf+de+f}\right)+\ln(cf-de-f(dx+c))\ln\left(\frac{-f(dx+}{-cf+de}\right.\right.}{2}\right.}{d}$

input
```
int((a+b*arccoth(d*x+c))/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
a*ln((d*x+c-1)*f-c*f+d*e+f)/f-1/2*b*dilog(((d*x+c-1)*f-c*f+d*e+f)/(-c*f+d*
e+f))/f-1/2*b*ln(d*x+c-1)*ln(((d*x+c-1)*f-c*f+d*e+f)/(-c*f+d*e+f))/f+1/2*b
*dilog(((d*x+c+1)*f-c*f+d*e-f)/(-c*f+d*e-f))/f+1/2*b*ln(d*x+c+1)*ln(((d*x+
c+1)*f-c*f+d*e-f)/(-c*f+d*e-f))/f
```

3.106.5 Fricas [F]

$$\int \frac{a+b\coth^{-1}(c+dx)}{e+fx}\,dx=\int \frac{b\operatorname{arcoth}(dx+c)+a}{fx+e}\,dx$$

input
```
integrate((a+b*arccoth(d*x+c))/(f*x+e),x, algorithm="fricas")
```

output
```
integral((b*arccoth(d*x + c) + a)/(f*x + e), x)
```

3.106.6 Sympy [F]

$$\int \frac{a + b\coth^{-1}(c + dx)}{e + fx}\,dx = \int \frac{a + b\,\mathrm{acoth}\,(c + dx)}{e + fx}\,dx$$

input
```
integrate((a+b*acoth(d*x+c))/(f*x+e),x)
```

output
```
Integral((a + b*acoth(c + d*x))/(e + f*x), x)
```

3.106.7 Maxima [F]

$$\int \frac{a + b\coth^{-1}(c + dx)}{e + fx}\,dx = \int \frac{b\,\mathrm{arcoth}\,(dx + c) + a}{fx + e}\,dx$$

input
```
integrate((a+b*arccoth(d*x+c))/(f*x+e),x, algorithm="maxima")
```

output
```
1/2*b*integrate((log(1/(d*x + c) + 1) - log(-1/(d*x + c) + 1))/(f*x + e),
x) + a*log(f*x + e)/f
```

3.106.8 Giac [F]

$$\int \frac{a + b\coth^{-1}(c + dx)}{e + fx}\,dx = \int \frac{b\,\mathrm{arcoth}\,(dx + c) + a}{fx + e}\,dx$$

input
```
integrate((a+b*arccoth(d*x+c))/(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*arccoth(d*x + c) + a)/(f*x + e), x)
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\coth^{-1}(c+dx)}{e+fx}\,dx=\int \frac{a+b\,\mathrm{acoth}(c+d\,x)}{e+f\,x}\,dx$$

input
```
int((a + b*acoth(c + d*x))/(e + f*x),x)
```

output
```
int((a + b*acoth(c + d*x))/(e + f*x), x)
```

3.107 $\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^2}\,dx$

3.107.1 Optimal result

Integrand size = 18, antiderivative size = 115

$$\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^2}\,dx = -\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)} - \frac{bd\log(1-c-dx)}{2f(de+f-cf)} + \frac{bd\log(1+c+dx)}{2f(de-f-cf)} - \frac{bd\log(e+fx)}{(de+f-cf)(de-(1+c)f)}$$

output
```
(-a-b*arccoth(d*x+c))/f/(f*x+e)-1/2*b*d*ln(-d*x-c+1)/f/(-c*f+d*e+f)+1/2*b*
d*ln(d*x+c+1)/f/(-c*f+d*e-f)-b*d*ln(f*x+e)/(-c*f+d*e-f)/(-c*f+d*e+f)
```

3.107.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^2}\,dx = \frac{1}{2}\left(-\frac{2a}{f(e+fx)} - \frac{2b\coth^{-1}(c+dx)}{f(e+fx)} + \frac{bd\log(1-c-dx)}{f(-de+(-1+c)f)} - \frac{bd\log(1+c+dx)}{f(-de+f+cf)} - \frac{2bd\log(e+fx)}{d^2e^2-2cdef+(-1+c^2)\,f^2}\right)$$

input
```
Integrate[(a + b*ArcCoth[c + d*x])/(e + f*x)^2,x]
```

output
```
((-2*a)/(f*(e + f*x)) - (2*b*ArcCoth[c + d*x])/(f*(e + f*x)) + (b*d*Log[1
- c - d*x])/(f*(-(d*e) + (-1 + c)*f)) - (b*d*Log[1 + c + d*x])/(f*(-(d*e)
+ f + c*f)) - (2*b*d*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2))
/2
```

3.107.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6660, 2081, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^2}\,dx$$

$$\Big\downarrow \text{6660}$$

$$\frac{bd\int \frac{1}{(e+fx)(1-(c+dx)^2)}dx}{f}-\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \text{2081}$$

$$\frac{bd\int \frac{1}{(e+fx)(-c^2-2dxc-d^2x^2+1)}dx}{f}-\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \text{1141}$$

$$-\frac{bd^3\int \left(\frac{f^2}{d^2(de-cf+f)(de-(c+1)f)(e+fx)}-\frac{1}{2d(de-cf+f)(-c-dx+1)}-\frac{1}{2d(de-cf-f)(c+dx+1)}\right)dx}{f}-\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}-\frac{bd^3\left(\frac{\log(-c-dx+1)}{2d^2(-cf+de+f)}-\frac{\log(c+dx+1)}{2d^2(de-(c+1)f)}+\frac{f\log(e+fx)}{d^2(-cf+de+f)(de-(c+1)f)}\right)}{f}$$

input
```
Int[(a + b*ArcCoth[c + d*x])/(e + f*x)^2,x]
```

output
```
-((a + b*ArcCoth[c + d*x])/(f*(e + f*x))) - (b*d^3*(Log[1 - c - d*x]/(2*d^
2*(d*e + f - c*f)) - Log[1 + c + d*x]/(2*d^2*(d*e - (1 + c)*f)) + (f*Log[e
 + f*x])/(d^2*(d*e + f - c*f)*(d*e - (1 + c)*f))))/f
```

3.107.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2081
```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

rule 6660
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCoth[c + d*x])^p/(f*(m
+ 1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCo
th[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.107.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.19

method	result
parts	$-\frac{a}{(fx+e)f}-\frac{bd\,\mathrm{arccoth}(dx+c)}{(dfx+de)f}-\frac{bd\ln(f(dx+c)-cf+de)}{(cf-de-f)(cf-de+f)}+\frac{bd\ln(dx+c-1)}{f(2cf-2de-2f)}-\frac{bd\ln(dx+c+1)}{f(2cf-2de+2f)}$
derivativedivides	$\frac{\frac{a\,d^2}{(cf-de-f(dx+c))f}+b\,d^2\left(\frac{\mathrm{arccoth}(dx+c)}{(cf-de-f(dx+c))f}+\frac{\frac{\ln(dx+c-1)}{2cf-2de-2f}-\frac{f\ln(cf-de-f(dx+c))}{(cf-de-f)(cf-de+f)}-\frac{\ln(dx+c+1)}{2cf-2de+2f}}{f}\right)}{d}$
default	$\frac{\frac{a\,d^2}{(cf-de-f(dx+c))f}+b\,d^2\left(\frac{\mathrm{arccoth}(dx+c)}{(cf-de-f(dx+c))f}+\frac{\frac{\ln(dx+c-1)}{2cf-2de-2f}-\frac{f\ln(cf-de-f(dx+c))}{(cf-de-f)(cf-de+f)}-\frac{\ln(dx+c+1)}{2cf-2de+2f}}{f}\right)}{d}$
parallelrisch	$-\frac{a\,d^4e^2-a\,d^2f^2-2ac\,d^3ef+a\,c^2d^2f^2-x\ \mathrm{arccoth}(dx+c)b\,d^3f^2+\mathrm{arccoth}(dx+c)b\,c^2d^2f^2+x\ \mathrm{arccoth}(dx+c)bc\,d^3f^2-x\ a}{}$
risch	$-\frac{b\ln(dx+c+1)}{2f(fx+e)}-\frac{\ln(dx+c+1)bcd\,f^2x-\ln(dx+c+1)b\,d^2efx-\ln(-dx-c+1)bcd\,f^2x+\ln(-dx-c+1)b\,d^2efx+2\ln(-fx}{}$

input
```
int((a+b*arccoth(d*x+c))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/(f*x+e)/f-b*d/(d*f*x+d*e)/f*arccoth(d*x+c)-b*d/(c*f-d*e-f)/(c*f-d*e+f)*
ln(f*(d*x+c)-c*f+d*e)+b*d/f/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)-b*d/f/(2*c*f-2*d
*e+2*f)*ln(d*x+c+1)
```

3.107.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(112) = 224$.

Time = 0.34 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.28

$$\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^2}\,dx =$$
$$-\frac{2\,ad^2e^2-4\,acdef+2\,(ac^2-a)f^2-(bd^2e^2-(bc-b)def+(bd^2ef-(bc-b)df^2)x)\log(dx+c+1)+}{2\,(d^2e^3f-2\,cd}$$

input
```
integrate((a+b*arccoth(d*x+c))/(f*x+e)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*d^2*e^2 - 4*a*c*d*e*f + 2*(a*c^2 - a)*f^2 - (b*d^2*e^2 - (b*c -
b)*d*e*f + (b*d^2*e*f - (b*c - b)*d*f^2)*x)*log(d*x + c + 1) + (b*d^2*e^2
- (b*c + b)*d*e*f + (b*d^2*e*f - (b*c + b)*d*f^2)*x)*log(d*x + c - 1) + 2*
(b*d*f^2*x + b*d*e*f)*log(f*x + e) + (b*d^2*e^2 - 2*b*c*d*e*f + (b*c^2 - b
)*f^2)*log((d*x + c + 1)/(d*x + c - 1)))/(d^2*e^3*f - 2*c*d*e^2*f^2 + (c^2
 - 1)*e*f^3 + (d^2*e^2*f^2 - 2*c*d*e*f^3 + (c^2 - 1)*f^4)*x)
```

3.107.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1605 vs. $2(92) = 184$.

Time = 2.46 (sec) , antiderivative size = 1605, normalized size of antiderivative = 13.96

$$\int \frac{a + b\coth^{-1}(c + dx)}{(e + fx)^2}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*acoth(d*x+c))/(f*x+e)**2,x)
```

output
```
Piecewise((-(a + b*acoth(c))/(e*f + f**2*x), Eq(d, 0)), ((a*x + b*c*acoth(
c + d*x)/d + b*x*acoth(c + d*x) + b*log(c/d + x + 1/d)/d - b*acoth(c + d*x
)/d)/e**2, Eq(f, 0)), (-2*a*f/(2*e*f**2 + 2*f**3*x) + b*d*e*acoth(d*e/f +
d*x - 1)/(2*e*f**2 + 2*f**3*x) + b*d*f*x*acoth(d*e/f + d*x - 1)/(2*e*f**2
+ 2*f**3*x) - 2*b*f*acoth(d*e/f + d*x - 1)/(2*e*f**2 + 2*f**3*x) - b*f/(2*
e*f**2 + 2*f**3*x), Eq(c, (d*e - f)/f)), (-2*a*f/(2*e*f**2 + 2*f**3*x) - b
*d*e*acoth(d*e/f + d*x + 1)/(2*e*f**2 + 2*f**3*x) - b*d*f*x*acoth(d*e/f +
d*x + 1)/(2*e*f**2 + 2*f**3*x) - 2*b*f*acoth(d*e/f + d*x + 1)/(2*e*f**2 +
2*f**3*x) + b*f/(2*e*f**2 + 2*f**3*x), Eq(c, (d*e + f)/f)), (-a*c**2*f**2/
(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*
f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + 2*a*c*d*e*f/(c**2*e*f**3 + c**2*
f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x
 - e*f**3 - f**4*x) - a*d**2*e**2/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*
f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x)
+ a*f**2/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d
**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - b*c**2*f**2*acoth(c + d
*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e
**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + b*c*d*e*f*acoth(c + d*x)/(c*
*2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f +
 d**2*e**2*f**2*x - e*f**3 - f**4*x) - b*c*d*f**2*x*acoth(c + d*x)/(c**...
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^2}\,dx$$
$$=\frac{1}{2}\left(d\left(\frac{\log(dx+c+1)}{def-(c+1)f^2}-\frac{\log(dx+c-1)}{def-(c-1)f^2}-\frac{2\log(fx+e)}{d^2e^2-2\,cdef+(c^2-1)f^2}\right)-\frac{2\,\mathrm{arcoth}(dx+c)}{f^2x+ef}\right)b-\frac{a}{f^2x+ef}$$

input
```
integrate((a+b*arccoth(d*x+c))/(f*x+e)^2,x, algorithm="maxima")
```

output
```
1/2*(d*(log(d*x + c + 1)/(d*e*f - (c + 1)*f^2) - log(d*x + c - 1)/(d*e*f -
 (c - 1)*f^2) - 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 - 1)*f^2)) - 2*
arccoth(d*x + c)/(f^2*x + e*f))*b - a/(f^2*x + e*f)
```

3.107.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(112) = 224.

Time = 0.28 (sec) , antiderivative size = 472, normalized size of antiderivative = 4.10

$$\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^2}\,dx =$$
$$-\frac{1}{2}\left((c+1)d-(c-1)d\right)\left(\frac{b\log\left(-\frac{(dx+c+1)de}{dx+c-1}+de+\frac{(dx+c+1)cf}{dx+c-1}-cf-\frac{(dx+c+1)f}{dx+c-1}-f\right)}{d^2e^2-2\,cdef+c^2f^2-f^2}-\frac{}{\frac{(dx+c+1)d^2e^2}{dx+c-1}-d}\right.$$

input
```
integrate((a+b*arccoth(d*x+c))/(f*x+e)^2,x, algorithm="giac")
```

output
```
-1/2*((c + 1)*d - (c - 1)*d)*(b*log(-(d*x + c + 1)*d*e/(d*x + c - 1) + d*e
 + (d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*f/(d*x + c - 1) -
 f)/(d^2*e^2 - 2*c*d*e*f + c^2*f^2 - f^2) - b*log((d*x + c + 1)/(d*x + c -
 1))/((d*x + c + 1)*d^2*e^2/(d*x + c - 1) - d^2*e^2 - 2*(d*x + c + 1)*c*d*
e*f/(d*x + c - 1) + 2*c*d*e*f + (d*x + c + 1)*c^2*f^2/(d*x + c - 1) - c^2*
f^2 + 2*(d*x + c + 1)*d*e*f/(d*x + c - 1) - 2*(d*x + c + 1)*c*f^2/(d*x + c
 - 1) + (d*x + c + 1)*f^2/(d*x + c - 1) + f^2) - b*log((d*x + c + 1)/(d*x
+ c - 1))/(d^2*e^2 - 2*c*d*e*f + c^2*f^2 - f^2) - 2*a/((d*x + c + 1)*d^2*e
^2/(d*x + c - 1) - d^2*e^2 - 2*(d*x + c + 1)*c*d*e*f/(d*x + c - 1) + 2*c*d
*e*f + (d*x + c + 1)*c^2*f^2/(d*x + c - 1) - c^2*f^2 + 2*(d*x + c + 1)*d*e
*f/(d*x + c - 1) - 2*(d*x + c + 1)*c*f^2/(d*x + c - 1) + (d*x + c + 1)*f^2
/(d*x + c - 1) + f^2))
```

3.107.9 Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.52

$$\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^2}\,dx = \ln(e+fx)\left(\frac{b(c-1)}{2e\,(d\,e-f\,(c-1))}-\frac{b(c+1)}{2e\,(d\,e-f\,(c+1))}\right) - \frac{a}{x\,f^2+e\,f} - \frac{b\ln\left(\frac{1}{c+d\,x}+1\right)}{2\,f\,(e+f\,x)} - \frac{b\,d\ln(c+d\,x-1)}{2\,f^2-2\,c\,f^2+2\,d\,e\,f} - \frac{b\,d\ln(c+d\,x+1)}{2\,c\,f^2+2\,f^2-2\,d\,e\,f} + \frac{b\ln\left(1-\frac{1}{c+d\,x}\right)}{f\,(2\,e+2\,f\,x)}$$

input
```
int((a + b*acoth(c + d*x))/(e + f*x)^2,x)
```

output
```
log(e + f*x)*((b*(c - 1))/(2*e*(d*e - f*(c - 1))) - (b*(c + 1))/(2*e*(d*e
- f*(c + 1)))) - a/(e*f + f^2*x) - (b*log(1/(c + d*x) + 1))/(2*f*(e + f*x)
) - (b*d*log(c + d*x - 1))/(2*f^2 - 2*c*f^2 + 2*d*e*f) - (b*d*log(c + d*x
+ 1))/(2*c*f^2 + 2*f^2 - 2*d*e*f) + (b*log(1 - 1/(c + d*x)))/(f*(2*e + 2*f
*x))
```

3.108 $\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^3}\,dx$

3.108.1 Optimal result

Integrand size = 18, antiderivative size = 167

$$\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^3}\,dx = \frac{bd}{2(de+f-cf)(de-(1+c)f)(e+fx)} - \frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2\log(1-c-dx)}{4f(de+f-cf)^2} + \frac{bd^2\log(1+c+dx)}{4f(de-f-cf)^2} - \frac{bd^2(de-cf)\log(e+fx)}{(de+f-cf)^2(de-(1+c)f)^2}$$

output
```
1/2*b*d/(-c*f+d*e-f)/(-c*f+d*e+f)/(f*x+e)+1/2*(-a-b*arccoth(d*x+c))/f/(f*x
+e)^2-1/4*b*d^2*ln(-d*x-c+1)/f/(-c*f+d*e+f)^2+1/4*b*d^2*ln(d*x+c+1)/f/(-c*
f+d*e-f)^2-b*d^2*(-c*f+d*e)*ln(f*x+e)/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2
```

3.108.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.04

$$\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^3}\,dx = \frac{1}{4}\left(-\frac{2a}{f(e+fx)^2} + \frac{2bd}{(d^2e^2-2cdef+(-1+c^2)f^2)(e+fx)} - \frac{2b\coth^{-1}(c+dx)}{f(e+fx)^2} - \frac{bd^2\log(1-c-dx)}{f(de+f-cf)^2} + \frac{bd^2\log(1+c+dx)}{f(-de+f+cf)^2} - \frac{4bd^2(de-cf)\log(e+fx)}{(d^2e^2-2cdef+(-1+c^2)f^2)^2}\right)$$

input
```
Integrate[(a + b*ArcCoth[c + d*x])/(e + f*x)^3,x]
```

output
```
((-2*a)/(f*(e + f*x)^2) + (2*b*d)/((d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)*
(e + f*x)) - (2*b*ArcCoth[c + d*x])/(f*(e + f*x)^2) - (b*d^2*Log[1 - c - d
*x])/(f*(d*e + f - c*f)^2) + (b*d^2*Log[1 + c + d*x])/(f*(-(d*e) + f + c*f
)^2) - (4*b*d^2*(d*e - c*f)*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2
)*f^2)^2)/4
```

3.108.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6660, 2081, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^3}\,dx$$

$$\Big\downarrow \; 6660$$

$$\frac{bd\int \frac{1}{(e+fx)^2(1-(c+dx)^2)}dx}{2f}-\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}$$

$$\Big\downarrow \; 2081$$

$$\frac{bd\int \frac{1}{(e+fx)^2(-c^2-2dxc-d^2x^2+1)}dx}{2f}-\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}$$

$$\Big\downarrow \; 1141$$

$$-\frac{bd^3\int \left(\frac{2(de-cf)f^2}{d(de-cf+f)^2(de-(c+1)f)^2(e+fx)}+\frac{f^2}{d^2(de-cf+f)(de-(c+1)f)(e+fx)^2}-\frac{1}{2(de-cf+f)^2(-c-dx+1)}-\frac{1}{2(de-(c+1)f)^2(c+dx+}\right.}{2f}$$

$$\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd^3\left(-\frac{f}{d^2(e+fx)(-cf+de+f)(de-(c+1)f)}+\frac{2f(de-cf)\log(e+fx)}{d(-cf+de+f)^2(de-(c+1)f)^2}+\frac{\log(-c-dx+1)}{2d(-cf+de+f)^2}-\frac{\log(c+dx+1)}{2d(de-(c+1)f)^2}\right)}{2f}$$

input
```
Int[(a + b*ArcCoth[c + d*x])/(e + f*x)^3,x]
```

output
```
-1/2*(a + b*ArcCoth[c + d*x])/(f*(e + f*x)^2) - (b*d^3*(-(f/(d^2*(d*e + f
- c*f)*(d*e - (1 + c)*f)*(e + f*x))) + Log[1 - c - d*x]/(2*d*(d*e + f - c*
f)^2) - Log[1 + c + d*x]/(2*d*(d*e - (1 + c)*f)^2) + (2*f*(d*e - c*f)*Log[
e + f*x])/(d*(d*e + f - c*f)^2*(d*e - (1 + c)*f)^2)))/(2*f)
```

3.108.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2081
```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

rule 6660
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCoth[c + d*x])^p/(f*(m
+ 1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCo
th[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.108.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.19

method	result
parts	$-\frac{a}{2(fx+e)^2 f}+\frac{b\left(-\frac{d^3 \operatorname{arccoth}(dx+c)}{2(f(dx+c)-cf+de)^2 f}-\frac{d^3\left(-\frac{f}{(cf-de-f)(cf-de+f)(f(dx+c)-cf+de)}-\frac{2f(cf-de)\ln(f(dx+c)-cf+de)}{(cf-de-f)^2(cf-de+f)^2}\right.}{2f}\right.}{d}$
derivativedivides	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-b\,d^3\left(\frac{\operatorname{arccoth}(dx+c)}{2(cf-de-f(dx+c))^2 f}+\frac{-\frac{\ln(dx+c+1)}{2(cf-de+f)^2}+\frac{\ln(dx+c-1)}{2(cf-de-f)^2}+\frac{f}{(cf-de-f)(cf-de+f)(cf-de-f(dx+c}}{2f}\right.}{d}$
default	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-b\,d^3\left(\frac{\operatorname{arccoth}(dx+c)}{2(cf-de-f(dx+c))^2 f}+\frac{-\frac{\ln(dx+c+1)}{2(cf-de+f)^2}+\frac{\ln(dx+c-1)}{2(cf-de-f)^2}+\frac{f}{(cf-de-f)(cf-de+f)(cf-de-f(dx+c}}{2f}\right.}{d}$
parallelrisch	$2x \operatorname{arccoth}(dx+c)b\,d^6 e^3 f^2+b\,d^5 e^3 f^2-b\,d^3 e\,f^4-a\,d^6 e^4 f+2a\,d^4 e^2 f^3+4a\,c^3 d^3 e\,f^4-6a\,c^2 d^4 e^2 f^3+4ac\,d^5 e^3 f^2-4ac\,d^3 e\,f$
risch	Expression too large to display

input
```
int((a+b*arccoth(d*x+c))/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/(f*x+e)^2/f+b/d*(-1/2*d^3/(f*(d*x+c)-c*f+d*e)^2/f*arccoth(d*x+c)-1/
2*d^3/f*(-f/(c*f-d*e-f)/(c*f-d*e+f)/(f*(d*x+c)-c*f+d*e)-2*f*(c*f-d*e)/(c*f
-d*e-f)^2/(c*f-d*e+f)^2*ln(f*(d*x+c)-c*f+d*e)+1/2/(c*f-d*e-f)^2*ln(d*x+c-1
)-1/2/(c*f-d*e+f)^2*ln(d*x+c+1)))
```

3.108.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 833 vs. $2(160) = 320$.

Time = 0.81 (sec) , antiderivative size = 833, normalized size of antiderivative = 4.99

$$\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^3}\,dx =$$
$$-\frac{2\,ad^4e^4-2\,(4\,ac+b)d^3e^3f+4\,(3\,ac^2+bc-a)d^2e^2f^2-2\,(4\,ac^3+bc^2-4\,ac-b)def^3+2\,(ac^4-2\,ac^2}{}$$

input
```
integrate((a+b*arccoth(d*x+c))/(f*x+e)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a*d^4*e^4 - 2*(4*a*c + b)*d^3*e^3*f + 4*(3*a*c^2 + b*c - a)*d^2*e^
2*f^2 - 2*(4*a*c^3 + b*c^2 - 4*a*c - b)*d*e*f^3 + 2*(a*c^4 - 2*a*c^2 + a)*
f^4 - 2*(b*d^3*e^2*f^2 - 2*b*c*d^2*e*f^3 + (b*c^2 - b)*d*f^4)*x - (b*d^4*e
^4 - 2*(b*c - b)*d^3*e^3*f + (b*c^2 - 2*b*c + b)*d^2*e^2*f^2 + (b*d^4*e^2*
f^2 - 2*(b*c - b)*d^3*e*f^3 + (b*c^2 - 2*b*c + b)*d^2*f^4)*x^2 + 2*(b*d^4*
e^3*f - 2*(b*c - b)*d^3*e^2*f^2 + (b*c^2 - 2*b*c + b)*d^2*e*f^3)*x)*log(d*
x + c + 1) + (b*d^4*e^4 - 2*(b*c + b)*d^3*e^3*f + (b*c^2 + 2*b*c + b)*d^2*
e^2*f^2 + (b*d^4*e^2*f^2 - 2*(b*c + b)*d^3*e*f^3 + (b*c^2 + 2*b*c + b)*d^2
*f^4)*x^2 + 2*(b*d^4*e^3*f - 2*(b*c + b)*d^3*e^2*f^2 + (b*c^2 + 2*b*c + b)
*d^2*e*f^3)*x)*log(d*x + c - 1) + 4*(b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^
3*e*f^3 - b*c*d^2*f^4)*x^2 + 2*(b*d^3*e^2*f^2 - b*c*d^2*e*f^3)*x)*log(f*x
+ e) + (b*d^4*e^4 - 4*b*c*d^3*e^3*f + 2*(3*b*c^2 - b)*d^2*e^2*f^2 - 4*(b*c
^3 - b*c)*d*e*f^3 + (b*c^4 - 2*b*c^2 + b)*f^4)*log((d*x + c + 1)/(d*x + c
- 1)))/(d^4*e^6*f - 4*c*d^3*e^5*f^2 + 2*(3*c^2 - 1)*d^2*e^4*f^3 - 4*(c^3 -
 c)*d*e^3*f^4 + (c^4 - 2*c^2 + 1)*e^2*f^5 + (d^4*e^4*f^3 - 4*c*d^3*e^3*f^4
 + 2*(3*c^2 - 1)*d^2*e^2*f^5 - 4*(c^3 - c)*d*e*f^6 + (c^4 - 2*c^2 + 1)*f^7
)*x^2 + 2*(d^4*e^5*f^2 - 4*c*d^3*e^4*f^3 + 2*(3*c^2 - 1)*d^2*e^3*f^4 - 4*(
c^3 - c)*d*e^2*f^5 + (c^4 - 2*c^2 + 1)*e*f^6)*x)
```

3.108.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19859 vs. $2(143) = 286$.

Time = 8.75 (sec) , antiderivative size = 19859, normalized size of antiderivative = 118.92

$$\int \frac{a + b\coth^{-1}(c + dx)}{(e + fx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*acoth(d*x+c))/(f*x+e)**3,x)
```

output
```
Piecewise((-(a + b*acoth(c))/(2*e**2*f + 4*e*f**2*x + 2*f**3*x**2), Eq(d,
0)), ((a*x + b*c*acoth(c + d*x)/d + b*x*acoth(c + d*x) + b*log(c/d + x + 1
/d)/d - b*acoth(c + d*x)/d)/e**3, Eq(f, 0)), (-4*a*f**2/(8*e**2*f**3 + 16*
e*f**4*x + 8*f**5*x**2) + b*d**2*e**2*acoth(d*e/f + d*x - 1)/(8*e**2*f**3
+ 16*e*f**4*x + 8*f**5*x**2) + 2*b*d**2*e*f*x*acoth(d*e/f + d*x - 1)/(8*e*
*2*f**3 + 16*e*f**4*x + 8*f**5*x**2) + b*d**2*f**2*x**2*acoth(d*e/f + d*x
- 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) - b*d*e*f/(8*e**2*f**3 + 16
*e*f**4*x + 8*f**5*x**2) - b*d*f**2*x/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*
x**2) - 4*b*f**2*acoth(d*e/f + d*x - 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**
5*x**2) - b*f**2/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2), Eq(c, (d*e - f
)/f)), (-4*a*f**2/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) + b*d**2*e**2*
acoth(d*e/f + d*x + 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) + 2*b*d**
2*e*f*x*acoth(d*e/f + d*x + 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) +
 b*d**2*f**2*x**2*acoth(d*e/f + d*x + 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f*
*5*x**2) - b*d*e*f/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) - b*d*f**2*x/
(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) - 4*b*f**2*acoth(d*e/f + d*x + 1
)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) + b*f**2/(8*e**2*f**3 + 16*e*f
**4*x + 8*f**5*x**2), Eq(c, (d*e + f)/f)), (-a*c**4*f**4/(2*c**4*e**2*f**5
 + 4*c**4*e*f**6*x + 2*c**4*f**7*x**2 - 8*c**3*d*e**3*f**4 - 16*c**3*d*e**
2*f**5*x - 8*c**3*d*e*f**6*x**2 + 12*c**2*d**2*e**4*f**3 + 24*c**2*d**2...
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.74

$$\int \frac{a + b\coth^{-1}(c + dx)}{(e + fx)^3}\,dx$$
$$= \frac{1}{4}\left(d\left(\frac{d\log(dx + c + 1)}{d^2e^2f - 2(c+1)def^2 + (c^2 + 2c + 1)f^3} - \frac{d\log(dx + c - 1)}{d^2e^2f - 2(c-1)def^2 + (c^2 - 2c + 1)f^3} - \frac{}{d^4e^4 - 4c\cdots}\right.\right.$$
$$- \frac{a}{2(f^3x^2 + 2ef^2x + e^2f)}$$

input
```
integrate((a+b*arccoth(d*x+c))/(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/4*(d*(d*log(d*x + c + 1)/(d^2*e^2*f - 2*(c + 1)*d*e*f^2 + (c^2 + 2*c + 1
)*f^3) - d*log(d*x + c - 1)/(d^2*e^2*f - 2*(c - 1)*d*e*f^2 + (c^2 - 2*c +
1)*f^3) - 4*(d^2*e - c*d*f)*log(f*x + e)/(d^4*e^4 - 4*c*d^3*e^3*f + 2*(3*c
^2 - 1)*d^2*e^2*f^2 - 4*(c^3 - c)*d*e*f^3 + (c^4 - 2*c^2 + 1)*f^4) + 2/(d^
2*e^3 - 2*c*d*e^2*f + (c^2 - 1)*e*f^2 + (d^2*e^2*f - 2*c*d*e*f^2 + (c^2 -
1)*f^3)*x)) - 2*arccoth(d*x + c)/(f^3*x^2 + 2*e*f^2*x + e^2*f))*b - 1/2*a/
(f^3*x^2 + 2*e*f^2*x + e^2*f)
```

3.108.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2562 vs. $2(160) = 320$.

Time = 0.34 (sec) , antiderivative size = 2562, normalized size of antiderivative = 15.34

$$\int \frac{a + b\coth^{-1}(c + dx)}{(e + fx)^3} \, dx = \text{Too large to display}$$

input
```
integrate((a+b*arccoth(d*x+c))/(f*x+e)^3,x, algorithm="giac")
```

output
```
-1/2*((c + 1)*d - (c - 1)*d)*((b*d^2*e - b*c*d*f)*log((d*x + c + 1)*d*e/(d
*x + c - 1) - d*e - (d*x + c + 1)*c*f/(d*x + c - 1) + c*f + (d*x + c + 1)*
f/(d*x + c - 1) + f)/(d^4*e^4 - 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2 - 4*c^3*
d*e*f^3 + c^4*f^4 - 2*d^2*e^2*f^2 + 4*c*d*e*f^3 - 2*c^2*f^4 + f^4) - ((d*x
 + c + 1)*b*d^2*e/(d*x + c - 1) - b*d^2*e - (d*x + c + 1)*b*c*d*f/(d*x + c
 - 1) + b*c*d*f + (d*x + c + 1)*b*d*f/(d*x + c - 1))*log((d*x + c + 1)/(d*
x + c - 1))/((d*x + c + 1)^2*d^4*e^4/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^4
*e^4/(d*x + c - 1) + d^4*e^4 - 4*(d*x + c + 1)^2*c*d^3*e^3*f/(d*x + c - 1)
^2 + 8*(d*x + c + 1)*c*d^3*e^3*f/(d*x + c - 1) - 4*c*d^3*e^3*f + 6*(d*x +
c + 1)^2*c^2*d^2*e^2*f^2/(d*x + c - 1)^2 - 12*(d*x + c + 1)*c^2*d^2*e^2*f^
2/(d*x + c - 1) + 6*c^2*d^2*e^2*f^2 - 4*(d*x + c + 1)^2*c^3*d*e*f^3/(d*x +
 c - 1)^2 + 8*(d*x + c + 1)*c^3*d*e*f^3/(d*x + c - 1) - 4*c^3*d*e*f^3 + (d
*x + c + 1)^2*c^4*f^4/(d*x + c - 1)^2 - 2*(d*x + c + 1)*c^4*f^4/(d*x + c -
 1) + c^4*f^4 + 4*(d*x + c + 1)^2*d^3*e^3*f/(d*x + c - 1)^2 - 4*(d*x + c +
 1)*d^3*e^3*f/(d*x + c - 1) - 12*(d*x + c + 1)^2*c*d^2*e^2*f^2/(d*x + c -
1)^2 + 12*(d*x + c + 1)*c*d^2*e^2*f^2/(d*x + c - 1) + 12*(d*x + c + 1)^2*c
^2*d*e*f^3/(d*x + c - 1)^2 - 12*(d*x + c + 1)*c^2*d*e*f^3/(d*x + c - 1) -
4*(d*x + c + 1)^2*c^3*f^4/(d*x + c - 1)^2 + 4*(d*x + c + 1)*c^3*f^4/(d*x +
 c - 1) + 6*(d*x + c + 1)^2*d^2*e^2*f^2/(d*x + c - 1)^2 - 2*d^2*e^2*f^2 -
12*(d*x + c + 1)^2*c*d*e*f^3/(d*x + c - 1)^2 + 4*c*d*e*f^3 + 6*(d*x + c...
```

3.108.9 Mupad [B] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.53

$$\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^3}\,dx = \frac{b\,\ln\left(1-\frac{1}{c+d\,x}\right)}{2\,f\,\left(2\,e^2+4\,e\,f\,x+2\,f^2\,x^2\right)} - \frac{\ln\left(e+f\,x\right)\,\left(b\,d^3\,e-b\,c\,d^2\,f\right)}{c^4\,f^4-4\,c^3\,d\,e\,f^3+6\,c^2\,d^2\,e^2\,f^2-2\,c^2\,f^4-4\,c\,d^3\,e^3\,f+4\,c\,d\,e\,f^3+d^4\,e^4-2\,d^2\,e^2\,f^2+f^4} - \frac{\frac{-a\,c^2\,f^2+2\,a\,c\,d\,e\,f-a\,d^2\,e^2+b\,d\,e\,f+a\,f^2}{-c^2\,f^2+2\,c\,d\,e\,f-d^2\,e^2+f^2}+\frac{b\,d\,f^2\,x}{-c^2\,f^2+2\,c\,d\,e\,f-d^2\,e^2+f^2}}{2\,e^2\,f+4\,e\,f^2\,x+2\,f^3\,x^2} - \frac{b\,d^2\,\ln\left(c+d\,x-1\right)}{4\,c^2\,f^3-8\,c\,d\,e\,f^2-8\,c\,f^3+4\,d^2\,e^2\,f+8\,d\,e\,f^2+4\,f^3} + \frac{b\,d^2\,\ln\left(c+d\,x+1\right)}{4\,c^2\,f^3-8\,c\,d\,e\,f^2+8\,c\,f^3+4\,d^2\,e^2\,f-8\,d\,e\,f^2+4\,f^3} - \frac{b\,\ln\left(\frac{1}{c+d\,x}+1\right)}{4\,f\,\left(e^2+2\,e\,f\,x+f^2\,x^2\right)}$$

input
```
int((a + b*acoth(c + d*x))/(e + f*x)^3,x)
```

output
```
(b*log(1 - 1/(c + d*x)))/(2*f*(2*e^2 + 2*f^2*x^2 + 4*e*f*x)) - (log(e + f*
x)*(b*d^3*e - b*c*d^2*f))/(f^4 - 2*c^2*f^4 + c^4*f^4 + d^4*e^4 - 2*d^2*e^2
*f^2 + 4*c*d*e*f^3 + 6*c^2*d^2*e^2*f^2 - 4*c*d^3*e^3*f - 4*c^3*d*e*f^3) -
((a*f^2 - a*c^2*f^2 - a*d^2*e^2 + b*d*e*f + 2*a*c*d*e*f)/(f^2 - c^2*f^2 -
d^2*e^2 + 2*c*d*e*f) + (b*d*f^2*x)/(f^2 - c^2*f^2 - d^2*e^2 + 2*c*d*e*f))/
(2*e^2*f + 2*f^3*x^2 + 4*e*f^2*x) - (b*d^2*log(c + d*x - 1))/(4*f^3 - 8*c*
f^3 + 4*c^2*f^3 + 4*d^2*e^2*f + 8*d*e*f^2 - 8*c*d*e*f^2) + (b*d^2*log(c +
d*x + 1))/(8*c*f^3 + 4*f^3 + 4*c^2*f^3 + 4*d^2*e^2*f - 8*d*e*f^2 - 8*c*d*e
*f^2) - (b*log(1/(c + d*x) + 1))/(4*f*(e^2 + f^2*x^2 + 2*e*f*x))
```

3.109 $\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^2 dx$

3.109.1 Optimal result

Integrand size = 20, antiderivative size = 374

$$\begin{aligned}
&\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^2 dx \\
&= \frac{b^2f^2x}{3d^2}+\frac{2abf(de-cf)x}{d^2}+\frac{2b^2f(de-cf)(c+dx)\coth^{-1}(c+dx)}{d^3} \\
&\quad+\frac{bf^2(c+dx)^2\left(a+b\coth^{-1}(c+dx)\right)}{3d^3} \\
&\quad-\frac{(de-cf)\left(d^2e^2-2cdef+(3+c^2)f^2\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{3d^3f} \\
&\quad+\frac{\left(3d^2e^2-6cdef+(1+3c^2)f^2\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{3d^3} \\
&\quad+\frac{(e+fx)^3\left(a+b\coth^{-1}(c+dx)\right)^2}{3f}-\frac{b^2f^2\text{arctanh}(c+dx)}{3d^3} \\
&\quad-\frac{2b\left(3d^2e^2-6cdef+(1+3c^2)f^2\right)\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2}{1-c-dx}\right)}{3d^3} \\
&\quad+\frac{b^2f(de-cf)\log\left(1-(c+dx)^2\right)}{d^3} \\
&\quad-\frac{b^2\left(3d^2e^2-6cdef+(1+3c^2)f^2\right)\text{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{3d^3}
\end{aligned}$$

output
```
1/3*b^2*f^2*x/d^2+2*a*b*f*(-c*f+d*e)*x/d^2+2*b^2*f*(-c*f+d*e)*(d*x+c)*arcc
oth(d*x+c)/d^3+1/3*b*f^2*(d*x+c)^2*(a+b*arccoth(d*x+c))/d^3-1/3*(-c*f+d*e)
*(d^2*e^2-2*c*d*e*f+(c^2+3)*f^2)*(a+b*arccoth(d*x+c))^2/d^3/f+1/3*(3*d^2*e
^2-6*c*d*e*f+(3*c^2+1)*f^2)*(a+b*arccoth(d*x+c))^2/d^3+1/3*(f*x+e)^3*(a+b*
arccoth(d*x+c))^2/f-1/3*b^2*f^2*arctanh(d*x+c)/d^3-2/3*b*(3*d^2*e^2-6*c*d*
e*f+(3*c^2+1)*f^2)*(a+b*arccoth(d*x+c))*ln(2/(-d*x-c+1))/d^3+b^2*f*(-c*f+d
*e)*ln(1-(d*x+c)^2)/d^3-1/3*b^2*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*polylo
g(2,(-d*x-c-1)/(-d*x-c+1))/d^3
```

3.109.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1115 vs. $2(374) = 748$.

Time = 7.06 (sec) , antiderivative size = 1115, normalized size of antiderivative = 2.98

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^2 dx$$
$$= a^2e^2x + a^2efx^2 + \frac{1}{3}a^2f^2x^3 + \frac{1}{3}ab\Bigg(2x\left(3e^2+3efx+f^2x^2\right)\coth^{-1}(c+dx)$$
$$+ \frac{dfx(6de-4cf+dfx) - (-1+c)\left(3d^2e^2-3(-1+c)def+(-1+c)^2f^2\right)\log(1-c-dx) + (1+c)\,(3c}{d^3}$$
$$- \frac{2b^2ef\left(1-(c+dx)^2\right)\left(\frac{(c+dx)\coth^{-1}(c+dx)}{d^2} - \frac{c(c+dx)\coth^{-1}(c+dx)^2}{d^2} + \frac{(c+dx)^2\left(1-\frac{1}{(c+dx)^2}\right)\coth^{-1}(c+dx)^2}{2d^2} - \frac{\log\left(\frac{}{(c+dx}\right.}{}\right.}{(c+dx)^2\left(1-\frac{}{(c+}\right.}$$
$$+ \frac{b^2e^2\left(1-(c+dx)^2\right)\left(\coth^{-1}(c+dx)\left(\coth^{-1}(c+dx)-(c+dx)\coth^{-1}(c+dx)+2\log\left(1-e^{-2\coth^{-1}(}\right.\right.\right.}{d(c+dx)^2\left(1-\frac{1}{(c+dx)^2}\right)}$$
$$- \frac{b^2f^2(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}\left(1-(c+dx)^2\right)\left(\frac{4\coth^{-1}(c+dx)}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}} + \frac{3\coth^{-1}(c+dx)^2}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}} - \frac{12c\coth^{-1}(c+dx)^2}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}} + \frac{9c^2}{(c+d}\right.}{}$$

input
```
Integrate[(e + f*x)^2*(a + b*ArcCoth[c + d*x])^2,x]
```

output
```
a^2*e^2*x + a^2*e*f*x^2 + (a^2*f^2*x^3)/3 + (a*b*(2*x*(3*e^2 + 3*e*f*x + f
^2*x^2)*ArcCoth[c + d*x] + (d*f*x*(6*d*e - 4*c*f + d*f*x) - (-1 + c)*(3*d^
2*e^2 - 3*(-1 + c)*d*e*f + (-1 + c)^2*f^2)*Log[1 - c - d*x] + (1 + c)*(3*d
^2*e^2 - 3*(1 + c)*d*e*f + (1 + c)^2*f^2)*Log[1 + c + d*x])/d^3))/3 - (2*b
^2*e*f*(1 - (c + d*x)^2)*(((c + d*x)*ArcCoth[c + d*x])/d^2 - (c*(c + d*x)*
ArcCoth[c + d*x]^2)/d^2 + ((c + d*x)^2*(1 - (c + d*x)^(-2))*ArcCoth[c + d*
x]^2)/(2*d^2) - Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])]/d^2 + (2*c*(Ar
cCoth[c + d*x]^2/2 + ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x])] - P
olyLog[2, E^(-2*ArcCoth[c + d*x])]/2))/d^2))/((c + d*x)^2*(1 - (c + d*x)^(
-2))) + (b^2*e^2*(1 - (c + d*x)^2)*(ArcCoth[c + d*x]*(ArcCoth[c + d*x] - (
c + d*x)*ArcCoth[c + d*x] + 2*Log[1 - E^(-2*ArcCoth[c + d*x])]) - PolyLog[
2, E^(-2*ArcCoth[c + d*x])]))/(d*(c + d*x)^2*(1 - (c + d*x)^(-2))) - (b^2*
f^2*(c + d*x)*Sqrt[1 - (c + d*x)^(-2)]*(1 - (c + d*x)^2)*((4*ArcCoth[c + d
*x])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (3*ArcCoth[c + d*x]^2)/((c + d
*x)*Sqrt[1 - (c + d*x)^(-2)]) - (12*c*ArcCoth[c + d*x]^2)/((c + d*x)*Sqrt[
1 - (c + d*x)^(-2)]) + (9*c^2*ArcCoth[c + d*x]^2)/((c + d*x)*Sqrt[1 - (c +
 d*x)^(-2)]) + (-1 + 6*c*ArcCoth[c + d*x] + 3*ArcCoth[c + d*x]^2 - 3*c^2*A
rcCoth[c + d*x]^2)/Sqrt[1 - (c + d*x)^(-2)] + Cosh[3*ArcCoth[c + d*x]] - 6
*c*ArcCoth[c + d*x]*Cosh[3*ArcCoth[c + d*x]] + ArcCoth[c + d*x]^2*Cosh[3*A
rcCoth[c + d*x]] + 3*c^2*ArcCoth[c + d*x]^2*Cosh[3*ArcCoth[c + d*x]] + ...
```

3.109.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6662, 27, 6481, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{6662}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2\left(a+b\coth^{-1}(c+dx)\right)^2}{d^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (de-cf+f(c+dx))^2\left(a+b\coth^{-1}(c+dx)\right)^2d(c+dx)}{d^3}$$

$$\Big\downarrow \text{6481}$$

$$\frac{(f(c+dx)-cf+de)^3\left(a+b\coth^{-1}(c+dx)\right)^2}{3f}-\frac{2b\int\left(-\left((c+dx)\left(a+b\coth^{-1}(c+dx)\right)f^3\right)-3(de-cf)\left(a+b\coth^{-1}(c+dx)\right)f^2+\frac{\left((de-cf)\left(d^2e^2-2cdfe+\cdots\right.\right.}{\cdots}\right.}{3f \cdots d^3}$$

$\downarrow$ 2009

$$\frac{(f(c+dx)-cf+de)^3\left(a+b\coth^{-1}(c+dx)\right)^2}{3f}-\frac{2b\left(-\frac{f\left(\left(3c^2+1\right)f^2-6cdef+3d^2e^2\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{2b}+\frac{(de-cf)\left(\left(c^2+3\right)f^2-2cdef+d^2e^2\right)\left(a+b\coth^{-1}\cdots\right.}{2b}\cdots\right.}{\cdots}$$

input
```
Int[(e + f*x)^2*(a + b*ArcCoth[c + d*x])^2,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^3*(a + b*ArcCoth[c + d*x])^2)/(3*f) - (2*b*(-1
/2*(b*f^3*(c + d*x)) - 3*a*f^2*(d*e - c*f)*(c + d*x) - 3*b*f^2*(d*e - c*f)
*(c + d*x)*ArcCoth[c + d*x] - (f^3*(c + d*x)^2*(a + b*ArcCoth[c + d*x]))/2
 + ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f + (3 + c^2)*f^2)*(a + b*ArcCoth[c + d
*x])^2)/(2*b) - (f*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a + b*ArcCot
h[c + d*x])^2)/(2*b) + (b*f^3*ArcTanh[c + d*x])/2 + f*(3*d^2*e^2 - 6*c*d*e
*f + (1 + 3*c^2)*f^2)*(a + b*ArcCoth[c + d*x])*Log[2/(1 - c - d*x)] - (3*b
*f^2*(d*e - c*f)*Log[1 - (c + d*x)^2])/2 + (b*f*(3*d^2*e^2 - 6*c*d*e*f + (
1 + 3*c^2)*f^2)*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/2))/(3*f))/d^3
```

3.109.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6481
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.109.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1410 vs. $2(360) = 720$.

Time = 0.67 (sec) , antiderivative size = 1411, normalized size of antiderivative = 3.77

method	result	size
parts	Expression too large to display	1411
derivativedivides	Expression too large to display	1412
default	Expression too large to display	1412
risch	Expression too large to display	1687

input
```
int((f*x+e)^2*(a+b*arccoth(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*(f*x+e)^3/f+b^2/d*(1/3/d^2*f^2*arccoth(d*x+c)^2*(d*x+c)^3-1/d^2*f^
2*arccoth(d*x+c)^2*(d*x+c)^2*c+1/d*f*arccoth(d*x+c)^2*(d*x+c)^2*e+1/d^2*f^
2*arccoth(d*x+c)^2*(d*x+c)*c^2-2/d*f*arccoth(d*x+c)^2*(d*x+c)*c*e+arccoth(
d*x+c)^2*(d*x+c)*e^2-1/3/d^2*f^2*arccoth(d*x+c)^2*c^3+1/d*f*arccoth(d*x+c)
^2*c^2*e-arccoth(d*x+c)^2*c*e^2+1/3*d/f*arccoth(d*x+c)^2*e^3+2/3/d^2/f*(1/
2*arccoth(d*x+c)*f^3*(d*x+c)^2+1/2*arccoth(d*x+c)*ln(d*x+c-1)*f^3+1/2*arcc
oth(d*x+c)*ln(d*x+c+1)*f^3-3/2*arccoth(d*x+c)*ln(d*x+c+1)*c^2*d*e*f^2+3/2*
arccoth(d*x+c)*ln(d*x+c+1)*c*d^2*e^2*f-3*arccoth(d*x+c)*ln(d*x+c+1)*c*d*e*
f^2+3/2*arccoth(d*x+c)*ln(d*x+c+1)*d^2*e^2*f-3/2*arccoth(d*x+c)*ln(d*x+c+1
)*d*e*f^2+3/2*arccoth(d*x+c)*ln(d*x+c-1)*c^2*d*e*f^2-3/2*arccoth(d*x+c)*ln
(d*x+c-1)*c*d^2*e^2*f-3*arccoth(d*x+c)*ln(d*x+c-1)*c*d*e*f^2+3*arccoth(d*x
+c)*d*e*f^2*(d*x+c)+3/2*arccoth(d*x+c)*ln(d*x+c-1)*d^2*e^2*f+3/2*arccoth(d
*x+c)*ln(d*x+c-1)*d*e*f^2-1/2*arccoth(d*x+c)*ln(d*x+c-1)*c^3*f^3+1/2*arcco
th(d*x+c)*ln(d*x+c-1)*d^3*e^3+3/2*arccoth(d*x+c)*ln(d*x+c-1)*c^2*f^3-3/2*a
rccoth(d*x+c)*ln(d*x+c-1)*c*f^3+1/2*arccoth(d*x+c)*ln(d*x+c+1)*c^3*f^3-1/2
*arccoth(d*x+c)*ln(d*x+c+1)*d^3*e^3+3/2*arccoth(d*x+c)*ln(d*x+c+1)*c^2*f^3
+3/2*arccoth(d*x+c)*ln(d*x+c+1)*c*f^3-3*arccoth(d*x+c)*c*f^3*(d*x+c)+1/2*f
^2*(f*(d*x+c)+1/2*(-6*c*f+6*d*e+f)*ln(d*x+c-1)-1/2*(6*c*f-6*d*e+f)*ln(d*x+
c+1))+1/2*(c^3*f^3-3*c^2*d*e*f^2+3*c*d^2*e^2*f-d^3*e^3+3*c^2*f^3-6*c*d*e*f
^2+3*d^2*e^2*f+3*c*f^3-3*d*e*f^2+f^3)*(-1/4*ln(d*x+c+1)^2+1/2*(ln(d*x+c...
```

3.109.5 Fricas [F]

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx = \int (fx+e)^2(b\,\mathrm{arcoth}\,(dx+c)+a)^2 \, dx$$

input
```
integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(a^2*f^2*x^2 + 2*a^2*e*f*x + a^2*e^2 + (b^2*f^2*x^2 + 2*b^2*e*f*x
+ b^2*e^2)*arccoth(d*x + c)^2 + 2*(a*b*f^2*x^2 + 2*a*b*e*f*x + a*b*e^2)*ar
ccoth(d*x + c), x)
```

3.109.6 Sympy [F]

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx = \int (a+b\,\mathrm{acoth}\,(c+dx))^2\,(e+fx)^2 \, dx$$

input
```
integrate((f*x+e)**2*(a+b*acoth(d*x+c))**2,x)
```

output
```
Integral((a + b*acoth(c + d*x))**2*(e + f*x)**2, x)
```

3.109.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(350) = 700$.

Time = 0.42 (sec) , antiderivative size = 791, normalized size of antiderivative = 2.11

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx = \frac{1}{3}\,a^2f^2x^3 + a^2efx^2$$
$$+ \left(2\,x^2\,\mathrm{arcoth}\,(dx+c) + d\left(\frac{2\,x}{d^2} - \frac{(c^2+2\,c+1)\log(dx+c+1)}{d^3} + \frac{(c^2-2\,c+1)\log(dx+c-1)}{d^3}\right)\right)ab$$
$$+\frac{1}{3}\left(2\,x^3\,\mathrm{arcoth}\,(dx+c) + d\left(\frac{dx^2-4\,cx}{d^3} + \frac{(c^3+3\,c^2+3\,c+1)\log(dx+c+1)}{d^4} - \frac{(c^3-3\,c^2+3\,c-1)}{d^4}\right.\right.$$
$$+ a^2e^2x + \frac{\left(2\,(dx+c)\,\mathrm{arcoth}\,(dx+c) + \log\left(-(dx+c)^2+1\right)\right)abe^2}{d}$$
$$- \frac{(3\,d^2e^2 - 6\,cdef + 3\,c^2f^2 + f^2)\left(\log(dx+c-1)\log\left(\frac{1}{2}\,dx + \frac{1}{2}\,c + \frac{1}{2}\right) + \mathrm{Li}_2\left(-\frac{1}{2}\,dx - \frac{1}{2}\,c + \frac{1}{2}\right)\right)b^2}{3\,d^3}$$
$$- \frac{(5\,c^2f^2 - 6\,def - 6\,(def - f^2)c + f^2)b^2\log(dx+c+1)}{6\,d^3}$$
$$+ \frac{4\,b^2df^2x + (b^2d^3f^2x^3 + 3\,b^2d^3efx^2 + 3\,b^2d^3e^2x + (c^3f^2 + 3\,d^2e^2 - 3\,(def - f^2)c^2 - 3\,def + 3\,(d^2e^2 - 2}{}$$

input
```
integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*f^2*x^3 + a^2*e*f*x^2 + (2*x^2*arccoth(d*x + c) + d*(2*x/d^2 - (c^
2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))
*a*b*e*f + 1/3*(2*x^3*arccoth(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 + (c^3 + 3
*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*log(d*x + c
 - 1)/d^4))*a*b*f^2 + a^2*e^2*x + (2*(d*x + c)*arccoth(d*x + c) + log(-(d*
x + c)^2 + 1))*a*b*e^2/d - 1/3*(3*d^2*e^2 - 6*c*d*e*f + 3*c^2*f^2 + f^2)*(
log(d*x + c - 1)*log(1/2*d*x + 1/2*c + 1/2) + dilog(-1/2*d*x - 1/2*c + 1/2
))*b^2/d^3 - 1/6*(5*c^2*f^2 - 6*d*e*f - 6*(d*e*f - f^2)*c + f^2)*b^2*log(d
*x + c + 1)/d^3 + 1/12*(4*b^2*d*f^2*x + (b^2*d^3*f^2*x^3 + 3*b^2*d^3*e*f*x
^2 + 3*b^2*d^3*e^2*x + (c^3*f^2 + 3*d^2*e^2 - 3*(d*e*f - f^2)*c^2 - 3*d*e*
f + 3*(d^2*e^2 - 2*d*e*f + f^2)*c + f^2)*b^2)*log(d*x + c + 1)^2 + (b^2*d^
3*f^2*x^3 + 3*b^2*d^3*e*f*x^2 + 3*b^2*d^3*e^2*x + (c^3*f^2 - 3*d^2*e^2 - 3
*(d*e*f + f^2)*c^2 - 3*d*e*f + 3*(d^2*e^2 + 2*d*e*f + f^2)*c - f^2)*b^2)*l
og(d*x + c - 1)^2 + 2*(b^2*d^2*f^2*x^2 + 2*(3*d^2*e*f - 2*c*d*f^2)*b^2*x -
 (b^2*d^3*f^2*x^3 + 3*b^2*d^3*e*f*x^2 + 3*b^2*d^3*e^2*x + (c^3*f^2 - 3*d^2
*e^2 - 3*(d*e*f + f^2)*c^2 - 3*d*e*f + 3*(d^2*e^2 + 2*d*e*f + f^2)*c - f^2
)*b^2)*log(d*x + c - 1))*log(d*x + c + 1) - 2*(b^2*d^2*f^2*x^2 + 2*(3*d^2*
e*f - 2*c*d*f^2)*b^2*x - (5*c^2*f^2 + 6*d*e*f - 6*(d*e*f + f^2)*c + f^2)*b
^2)*log(d*x + c - 1))/d^3
```

3.109.8 Giac [F]

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx = \int (fx+e)^2 (b \operatorname{arcoth}(dx+c)+a)^2 \, dx$$

input
```
integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*(b*arccoth(d*x + c) + a)^2, x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx = \int (e+f\,x)^2\,(a+b\,\mathrm{acoth}(c+d\,x))^2 \, dx$$

input
```
int((e + f*x)^2*(a + b*acoth(c + d*x))^2,x)
```

output
```
int((e + f*x)^2*(a + b*acoth(c + d*x))^2, x)
```

3.110 $\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^2\,dx$

3.110.1 Optimal result

Integrand size = 18, antiderivative size = 221

$$\begin{aligned}\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^2\,dx &= \frac{abfx}{d}+\frac{b^2f(c+dx)\coth^{-1}(c+dx)}{d^2}+\frac{(de-cf)\left(a+b\coth^{-1}(c+dx)\right)^2}{d^2}\\ &\quad-\frac{\left(d^2e^2-2cdef+(1+c^2)f^2\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2f}\\ &\quad+\frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2f}-\frac{2b(de-cf)\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2}{1-c-dx}\right)}{d^2}\\ &\quad+\frac{b^2f\log\left(1-(c+dx)^2\right)}{2d^2}-\frac{b^2(de-cf)\operatorname{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{d^2}\end{aligned}$$

output
```
a*b*f*x/d+b^2*f*(d*x+c)*arccoth(d*x+c)/d^2+(-c*f+d*e)*(a+b*arccoth(d*x+c))
^2/d^2-1/2*(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)*(a+b*arccoth(d*x+c))^2/d^2/f+1/
2*(f*x+e)^2*(a+b*arccoth(d*x+c))^2/f-2*b*(-c*f+d*e)*(a+b*arccoth(d*x+c))*l
n(2/(-d*x-c+1))/d^2+1/2*b^2*f*ln(1-(d*x+c)^2)/d^2-b^2*(-c*f+d*e)*polylog(2
,(-d*x-c-1)/(-d*x-c+1))/d^2
```

3.110.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.33

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^2\,dx$$

$$=\frac{2a^2cde+2abcf-a^2c^2f+2a^2d^2ex+2abdfx+a^2d^2fx^2+b^2(-1+c+dx)(2de+f-cf+dfx)\coth^{-1}(c}{}$$

input
```
Integrate[(e + f*x)*(a + b*ArcCoth[c + d*x])^2,x]
```

output
```
(2*a^2*c*d*e + 2*a*b*c*f - a^2*c^2*f + 2*a^2*d^2*e*x + 2*a*b*d*f*x + a^2*d
^2*f*x^2 + b^2*(-1 + c + d*x)*(2*d*e + f - c*f + d*f*x)*ArcCoth[c + d*x]^2
 + 2*b*ArcCoth[c + d*x]*(-((c + d*x)*(-(b*f) + a*c*f - a*d*(2*e + f*x))) -
 2*b*(d*e - c*f)*Log[1 - E^(-2*ArcCoth[c + d*x])]) + a*b*f*Log[1 - c - d*x
] - a*b*f*Log[1 + c + d*x] - 4*a*b*d*e*Log[1/((c + d*x)*Sqrt[1 - (c + d*x)
^(-2)])] - 2*b^2*f*Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])] + 4*a*b*c*f
*Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])] + 2*b^2*(d*e - c*f)*PolyLog[2
, E^(-2*ArcCoth[c + d*x])])/(2*d^2)
```

3.110.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6662, 27, 6481, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^2\,dx$$

$$\Big\downarrow \; 6662$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{d}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (de-cf+f(c+dx))\left(a+b\coth^{-1}(c+dx)\right)^2d(c+dx)}{d^2}$$

$\Big\downarrow$ 6481

$$\frac{\frac{(f(c+dx)-cf+de)^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2f}-\frac{b\int\left(\frac{\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2+2f(de-cf)(c+dx)\right)\left(a+b\coth^{-1}(c+dx)\right)}{1-(c+dx)^2}-f^2\left(a+b\coth^{-1}(c+dx)\right)\right)d(c+d}{f}}{d^2}$$

$\Big\downarrow$ 2009

$$\frac{\frac{(f(c+dx)-cf+de)^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2f}-\frac{b\left(\frac{\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{2b}-\frac{f(de-cf)\left(a+b\coth^{-1}(c+dx)\right)^2}{b}+2f(de-cf)\log\left(\frac{2}{-c}\right.\right.}{}}{d^2}$$

input
```
Int[(e + f*x)*(a + b*ArcCoth[c + d*x])^2,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^2*(a + b*ArcCoth[c + d*x])^2)/(2*f) - (b*(-(a*
f^2*(c + d*x)) - b*f^2*(c + d*x)*ArcCoth[c + d*x] - (f*(d*e - c*f)*(a + b*
ArcCoth[c + d*x])^2)/b + ((d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(a + b*Arc
Coth[c + d*x])^2)/(2*b) + 2*f*(d*e - c*f)*(a + b*ArcCoth[c + d*x])*Log[2/(
1 - c - d*x)] - (b*f^2*Log[1 - (c + d*x)^2])/2 + b*f*(d*e - c*f)*PolyLog[2
, -((1 + c + d*x)/(1 - c - d*x))]))/f)/d^2
```

3.110.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6481
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.110.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(217) = 434$.

Time = 0.36 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.04

method	result
parts	$a^2\left(\frac{1}{2}f\,x^2+ex\right)+\frac{b^2\left(\frac{\operatorname{arccoth}(dx+c)^2(dx+c)^2f}{2d}-\frac{\operatorname{arccoth}(dx+c)^2cf(dx+c)}{d}+\operatorname{arccoth}(dx+c)^2e(dx+c)+\frac{\operatorname{arccoth}(dx+c)\ldots}{\ldots}\right.}{\ldots}$
derivativedivides	$-\frac{a^2\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b^2\left(\operatorname{arccoth}(dx+c)^2fc(dx+c)-\operatorname{arccoth}(dx+c)^2ed(dx+c)-\frac{\operatorname{arccoth}(dx+c)^2f(dx+c)^2}{2}-\operatorname{arcc}\ldots\right.}{\ldots}$
default	$-\frac{a^2\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b^2\left(\operatorname{arccoth}(dx+c)^2fc(dx+c)-\operatorname{arccoth}(dx+c)^2ed(dx+c)-\frac{\operatorname{arccoth}(dx+c)^2f(dx+c)^2}{2}-\operatorname{arcc}\ldots\right.}{\ldots}$
risch	$\frac{a^2ce}{d}-\frac{baf}{d^2}+\frac{a^2cf}{d^2}-\frac{a^2f\,c^2}{2d^2}+\frac{b^2\operatorname{dilog}\left(\frac{dx}{2}+\frac{c}{2}+\frac{1}{2}\right)cf}{d^2}-\frac{b^2\ln\left(\frac{dx}{2}+\frac{c}{2}+\frac{1}{2}\right)\ln(dx+c-1)e}{d}+\frac{b\ln(dx+c+1)ae}{d}\ldots$

input
```
int((f*x+e)*(a+b*arccoth(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/2*f*x^2+e*x)+b^2/d*(1/2/d*arccoth(d*x+c)^2*(d*x+c)^2*f-1/d*arccoth(
d*x+c)^2*c*f*(d*x+c)+arccoth(d*x+c)^2*e*(d*x+c)+1/d*(arccoth(d*x+c)*(d*x+c
)*f-arccoth(d*x+c)*ln(d*x+c-1)*c*f+arccoth(d*x+c)*ln(d*x+c-1)*d*e+1/2*arcc
oth(d*x+c)*ln(d*x+c-1)*f-arccoth(d*x+c)*ln(d*x+c+1)*c*f+arccoth(d*x+c)*ln(
d*x+c+1)*d*e-1/2*arccoth(d*x+c)*ln(d*x+c+1)*f+1/2*ln(d*x+c-1)*f+1/2*ln(d*x
+c+1)*f+1/2*(-2*c*f+2*d*e+f)*(1/4*ln(d*x+c-1)^2-1/2*dilog(1/2*d*x+1/2*c+1/
2)-1/2*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2))+1/2*(-2*c*f+2*d*e-f)*(-1/4*ln(d*
x+c+1)^2+1/2*(ln(d*x+c+1)-ln(1/2*d*x+1/2*c+1/2))*ln(-1/2*d*x-1/2*c+1/2)-1/
2*dilog(1/2*d*x+1/2*c+1/2))))+2*a*b/d*(1/2/d*arccoth(d*x+c)*(d*x+c)^2*f-1/
d*arccoth(d*x+c)*c*f*(d*x+c)+arccoth(d*x+c)*e*(d*x+c)+1/2/d*(f*(d*x+c)+1/2
*(-2*c*f+2*d*e+f)*ln(d*x+c-1)-1/2*(2*c*f-2*d*e+f)*ln(d*x+c+1)))
```

3.110.5 Fricas [F]

$$\int (e+fx) \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx = \int (fx+e)(b\operatorname{arcoth}(dx+c)+a)^2 \, dx$$

input
```
integrate((f*x+e)*(a+b*arccoth(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(a^2*f*x + a^2*e + (b^2*f*x + b^2*e)*arccoth(d*x + c)^2 + 2*(a*b*f
*x + a*b*e)*arccoth(d*x + c), x)
```

3.110.6 Sympy [F]

$$\int (e+fx) \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx = \int (a+b\operatorname{acoth}(c+dx))^2 (e+fx) \, dx$$

input
```
integrate((f*x+e)*(a+b*acoth(d*x+c))**2,x)
```

output
```
Integral((a + b*acoth(c + d*x))**2*(e + f*x), x)
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.81

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^2\,dx = \frac{1}{2}a^2fx^2 + \frac{1}{2}\left(2x^2\,\mathrm{arcoth}\,(dx+c)+d\left(\frac{2x}{d^2}-\frac{(c^2+2c+1)\log(dx+c+1)}{d^3}+\frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)c$$
$$+a^2ex+\frac{\left(2(dx+c)\,\mathrm{arcoth}\,(dx+c)+\log\left(-(dx+c)^2+1\right)\right)abe}{d}$$
$$-\frac{(de-cf)\left(\log(dx+c-1)\log\left(\frac{1}{2}dx+\frac{1}{2}c+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}dx-\frac{1}{2}c+\frac{1}{2}\right)\right)b^2}{d^2}$$
$$+\frac{(cf+f)b^2\log(dx+c+1)}{2d^2}$$
$$+\frac{(b^2d^2fx^2+2b^2d^2ex-(c^2f-2(de-f)c-2de+f)b^2)\log(dx+c+1)^2+(b^2d^2fx^2+2b^2d^2ex-(c^2f}{}$$

input
```
integrate((f*x+e)*(a+b*arccoth(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*f*x^2 + 1/2*(2*x^2*arccoth(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)
*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a*b*f + a^2
*e*x + (2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*a*b*e/d - (d
*e - c*f)*(log(d*x + c - 1)*log(1/2*d*x + 1/2*c + 1/2) + dilog(-1/2*d*x -
1/2*c + 1/2))*b^2/d^2 + 1/2*(c*f + f)*b^2*log(d*x + c + 1)/d^2 + 1/8*((b^2
*d^2*f*x^2 + 2*b^2*d^2*e*x - (c^2*f - 2*(d*e - f)*c - 2*d*e + f)*b^2)*log(
d*x + c + 1)^2 + (b^2*d^2*f*x^2 + 2*b^2*d^2*e*x - (c^2*f - 2*(d*e + f)*c +
 2*d*e + f)*b^2)*log(d*x + c - 1)^2 + 2*(2*b^2*d*f*x - (b^2*d^2*f*x^2 + 2*
b^2*d^2*e*x - (c^2*f - 2*(d*e + f)*c + 2*d*e + f)*b^2)*log(d*x + c - 1))*l
og(d*x + c + 1) - 4*(b^2*d*f*x + (c*f - f)*b^2)*log(d*x + c - 1))/d^2
```

3.110.8 Giac [F]

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^2\,dx = \int (fx+e)(b\,\mathrm{arcoth}\,(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)*(a+b*arccoth(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)*(b*arccoth(d*x + c) + a)^2, x)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^2 dx = \int (e+f\,x)\,\left(a+b\,\mathrm{acoth}(c+d\,x)\right)^2 dx$$

input
```
int((e + f*x)*(a + b*acoth(c + d*x))^2,x)
```

output
```
int((e + f*x)*(a + b*acoth(c + d*x))^2, x)
```

3.111 $\int (a+b\coth^{-1}(c+dx))^2\,dx$

3.111.1 Optimal result

Integrand size = 12, antiderivative size = 97

$$\int (a+b\coth^{-1}(c+dx))^2\,dx=\frac{(a+b\coth^{-1}(c+dx))^2}{d}+\frac{(c+dx)(a+b\coth^{-1}(c+dx))^2}{d}-\frac{2b(a+b\coth^{-1}(c+dx))\log\left(\frac{2}{1-c-dx}\right)}{d}-\frac{b^2\,\mathrm{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{d}$$

output
```
(a+b*arccoth(d*x+c))^2/d+(d*x+c)*(a+b*arccoth(d*x+c))^2/d-2*b*(a+b*arccoth
(d*x+c))*ln(2/(-d*x-c+1))/d-b^2*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d
```

3.111.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14

$$\int (a+b\coth^{-1}(c+dx))^2\,dx$$
$$=\frac{b^2(-1+c+dx)\coth^{-1}(c+dx)^2+2b\coth^{-1}(c+dx)\left(ac+adx-b\log\left(1-e^{-2\coth^{-1}(c+dx)}\right)\right)+a\Big(ac+}{d}$$

input
```
Integrate[(a + b*ArcCoth[c + d*x])^2,x]
```

output
```
(b^2*(-1 + c + d*x)*ArcCoth[c + d*x]^2 + 2*b*ArcCoth[c + d*x]*(a*c + a*d*x
 - b*Log[1 - E^(-2*ArcCoth[c + d*x])]) + a*(a*c + a*d*x - 2*b*Log[1/((c +
d*x)*Sqrt[1 - (c + d*x)^(-2)])]) + b^2*PolyLog[2, E^(-2*ArcCoth[c + d*x])]
)/d
```

3.111.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6654, 6437, 6547, 6471, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\coth^{-1}(c+dx)\right)^2dx$$

$$\Big\downarrow \text{6654}$$

$$\frac{\int \left(a+b\coth^{-1}(c+dx)\right)^2d(c+dx)}{d}$$

$$\Big\downarrow \text{6437}$$

$$\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2-2b\int \frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)}{1-(c+dx)^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{6547}$$

$$\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2-2b\left(\int \frac{a+b\coth^{-1}(c+dx)}{-c-dx+1}d(c+dx)-\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{2b}\right)}{d}$$

$$\Big\downarrow \text{6471}$$

$$\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2-2b\left(-b\int \frac{\log\left(\frac{2}{-c-dx+1}\right)}{1-(c+dx)^2}d(c+dx)-\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{2b}+\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\,\text{co}\right.\right.}{d}$$

$$\Big\downarrow \text{2849}$$

$$\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2-2b\left(b\int \frac{\log\left(\frac{2}{-c-dx+1}\right)}{1-\frac{2}{-c-dx+1}}d\frac{1}{-c-dx+1}-\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{2b}+\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\,\text{cot}\right.\right.}{d}$$

$$\Big\downarrow \text{2752}$$

$$\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2-2b\left(-\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{2b}+\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)+\frac{1}{2}b\,\text{PolyLog}\right.}{d}$$

input
```
Int[(a + b*ArcCoth[c + d*x])^2,x]
```

output
```
((c + d*x)*(a + b*ArcCoth[c + d*x])^2 - 2*b*(-1/2*(a + b*ArcCoth[c + d*x])
^2/b + (a + b*ArcCoth[c + d*x])*Log[2/(1 - c - d*x)] + (b*PolyLog[2, 1 - 2
/(1 - c - d*x)])/2))/d
```

3.111.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6471
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6547
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6654
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
}, x] && IGtQ[p, 0]
```

3.111.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.79

method	result
parts	$a^2x + \frac{b^2\left(\operatorname{arccoth}(dx+c)^2(dx+c-1)+2\operatorname{arccoth}(dx+c)^2-2\operatorname{arccoth}(dx+c)\ln\left(1-\frac{1}{\sqrt{\frac{dx+c-1}{dx+c+1}}}\right)-2\operatorname{polylog}\left(2,\frac{1}{\sqrt{\frac{dx+c}{dx+c}}}\right.\right.}{d}$
derivativedivides	$\frac{(dx+c)a^2+b^2\left(\operatorname{arccoth}(dx+c)^2(dx+c-1)+2\operatorname{arccoth}(dx+c)^2-2\operatorname{arccoth}(dx+c)\ln\left(1-\frac{1}{\sqrt{\frac{dx+c-1}{dx+c+1}}}\right)-2\operatorname{polylog}\left(2,\frac{}{\sqrt{\frac{dx}{dx}}}\right.\right.}{}$
default	$\frac{(dx+c)a^2+b^2\left(\operatorname{arccoth}(dx+c)^2(dx+c-1)+2\operatorname{arccoth}(dx+c)^2-2\operatorname{arccoth}(dx+c)\ln\left(1-\frac{1}{\sqrt{\frac{dx+c-1}{dx+c+1}}}\right)-2\operatorname{polylog}\left(2,\frac{}{\sqrt{\frac{dx}{dx}}}\right.\right.}{}$
risch	$a^2x - \frac{b^2\ln(dx+c-1)\ln\left(\frac{dx}{2}+\frac{c}{2}+\frac{1}{2}\right)}{d} + \frac{\ln(dx+c-1)^2b^2c}{4d} - \ln(dx+c-1)\,abx + \frac{\ln(dx+c-1)ab}{d} + \frac{ab\ln(}{}$

input
```
int((a+b*arccoth(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+b^2/d*(arccoth(d*x+c)^2*(d*x+c-1)+2*arccoth(d*x+c)^2-2*arccoth(d*x+c
)*ln(1-1/((d*x+c-1)/(d*x+c+1))^(1/2))-2*polylog(2,1/((d*x+c-1)/(d*x+c+1))^
(1/2))-2*arccoth(d*x+c)*ln(1+1/((d*x+c-1)/(d*x+c+1))^(1/2))-2*polylog(2,-1
/((d*x+c-1)/(d*x+c+1))^(1/2)))+2*a*b/d*((d*x+c)*arccoth(d*x+c)+1/2*ln((d*x
+c)^2-1))
```

3.111.5 Fricas [F]

$$\int \left(a+b\coth^{-1}(c+dx)\right)^2\,dx=\int \left(b\operatorname{arcoth}\left(dx+c\right)+a\right)^2dx$$

input
```
integrate((a+b*arccoth(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arccoth(d*x + c)^2 + 2*a*b*arccoth(d*x + c) + a^2, x)
```

3.111.6 Sympy [F]

$$\int \left(a+b\coth^{-1}(c+dx)\right)^2\,dx=\int \left(a+b\,\mathrm{acoth}\,(c+dx)\right)^2\,dx$$

input
```
integrate((a+b*acoth(d*x+c))**2,x)
```

output
```
Integral((a + b*acoth(c + d*x))**2, x)
```

3.111.7 Maxima [F]

$$\int \left(a+b\coth^{-1}(c+dx)\right)^2\,dx=\int (b\,\mathrm{arcoth}\,(dx+c)+a)^2\,dx$$

input
```
integrate((a+b*arccoth(d*x+c))^2,x, algorithm="maxima")
```

output
```
a^2*x + 1/4*b^2*((d*x*log(d*x + c - 1)^2 + (d*x + c + 1)*log(d*x + c + 1)^
2 - 2*(d*x + c - 1)*log(d*x + c + 1)*log(d*x + c - 1))/d + integrate(2*(c^
2 + (c*d - 3*d)*x - 2*c + 1)*log(d*x + c - 1)/(d^2*x^2 + 2*c*d*x + c^2 - 1
), x)) + (2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*a*b/d
```

3.111.8 Giac [F]

$$\int \left(a+b\coth^{-1}(c+dx)\right)^2\,dx=\int (b\,\mathrm{arcoth}\,(dx+c)+a)^2\,dx$$

input
```
integrate((a+b*arccoth(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*arccoth(d*x + c) + a)^2, x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\coth^{-1}(c+dx)\right)^2\,dx=\int (a+b\,\mathrm{acoth}(c+d\,x))^2\,dx$$

input
```
int((a + b*acoth(c + d*x))^2,x)
```

output
```
int((a + b*acoth(c + d*x))^2, x)
```

3.112 $\int \frac{(a+b\coth^{-1}(c+dx))^2}{e+fx}\,dx$

3.112.1 Optimal result

Integrand size = 20, antiderivative size = 214

$$\begin{aligned}\int \frac{(a+b\coth^{-1}(c+dx))^2}{e+fx}\,dx &= -\frac{(a+b\coth^{-1}(c+dx))^2\log\left(\frac{2}{1+c+dx}\right)}{f}+\frac{(a+b\coth^{-1}(c+dx))^2\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}\\&\quad+\frac{b(a+b\coth^{-1}(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{f}\\&\quad-\frac{b(a+b\coth^{-1}(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}\\&\quad+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+c+dx}\right)}{2f}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{2f}\end{aligned}$$

output
```
-(a+b*arccoth(d*x+c))^2*ln(2/(d*x+c+1))/f+(a+b*arccoth(d*x+c))^2*ln(2*d*(f
*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f+b*(a+b*arccoth(d*x+c))*polylog(2,1-2/(d*x+
c+1))/f-b*(a+b*arccoth(d*x+c))*polylog(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c
+1))/f+1/2*b^2*polylog(3,1-2/(d*x+c+1))/f-1/2*b^2*polylog(3,1-2*d*(f*x+e)/
(-c*f+d*e+f)/(d*x+c+1))/f
```

3.112.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.43 (sec) , antiderivative size = 1767, normalized size of antiderivative = 8.26

$$\int \frac{(a+b\coth^{-1}(c+dx))^2}{e+fx}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCoth[c + d*x])^2/(e + f*x),x]
```

output
```
(a^2*Log[e + f*x])/f + 2*a*b*(((ArcCoth[c + d*x] - ArcTanh[c + d*x])*Log[e
 + f*x])/f - (I*(I*ArcTanh[c + d*x]*(-Log[1/Sqrt[1 - (c + d*x)^2]] + Log[I
*Sinh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]]]) + ((-I)*(I*ArcTanh[(d*e
 - c*f)/f] + I*ArcTanh[c + d*x])^2 - (I/4)*(Pi - (2*I)*ArcTanh[c + d*x])^2
 + 2*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x])*Log[1 - E^((2*I)*(I*A
rcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x]))] + (Pi - (2*I)*ArcTanh[c + d*
x])*Log[1 - E^(I*(Pi - (2*I)*ArcTanh[c + d*x]))] - (Pi - (2*I)*ArcTanh[c +
 d*x])*Log[2*Sin[(Pi - (2*I)*ArcTanh[c + d*x])/2]] - 2*(I*ArcTanh[(d*e - c
*f)/f] + I*ArcTanh[c + d*x])*Log[(2*I)*Sinh[ArcTanh[(d*e - c*f)/f] + ArcTa
nh[c + d*x]]] - I*PolyLog[2, E^((2*I)*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTan
h[c + d*x]))] - I*PolyLog[2, E^(I*(Pi - (2*I)*ArcTanh[c + d*x]))])/2))/f)
- (b^2*(d*e - c*f + f*(c + d*x))*(1 - (c + d*x)^2)*(-1/24*(I*f*Pi^3 - 8*d*
e*ArcCoth[c + d*x]^3 - 8*f*ArcCoth[c + d*x]^3 + 8*c*f*ArcCoth[c + d*x]^3 +
 24*f*ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])] + 24*f*ArcCoth[c
+ d*x]*PolyLog[2, E^(2*ArcCoth[c + d*x])] - 12*f*PolyLog[3, E^(2*ArcCoth[c
 + d*x])])/f^2 + ((-(d*e) - f + c*f)*(-(d*e) + f + c*f)*(2*d*e*ArcCoth[c +
 d*x]^3 - 6*f*ArcCoth[c + d*x]^3 - 2*c*f*ArcCoth[c + d*x]^3 - (4*d*e*Sqrt[
(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f)^2]*ArcCoth[c + d*x]^3)/
E^ArcTanh[f/(d*e - c*f)] + (4*c*f*Sqrt[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f
^2)/(d*e - c*f)^2]*ArcCoth[c + d*x]^3)/E^ArcTanh[f/(d*e - c*f)] + (6*I)...
```

3.112.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6662, 27, 6475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{e+fx}\,dx$$

$$\Big\downarrow \text{ 6662}$$

$$\frac{\int \frac{d\left(a+b\coth^{-1}(c+dx)\right)^2}{d\left(e-\frac{cf}{d}\right)+f(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(c+dx)-cf+de}d(c+dx)$$

$$\Big\downarrow \text{ 6475}$$

$$-\frac{b\left(a+b\coth^{-1}(c+dx)\right)\operatorname{PolyLog}\left(2,1-\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{f}+\frac{\left(a+b\coth^{-1}(c+dx)\right)^2\log\left(\frac{2(f(c+dx)-cf+de)}{(c+dx+1)(-cf+de+f)}\right)}{f}+\frac{b\operatorname{PolyLog}\left(2,1-\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{f}-\frac{\log\left(\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{f}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{2f}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{c+dx+1}\right)}{2f}$$

input
```
Int[(a + b*ArcCoth[c + d*x])^2/(e + f*x),x]
```

output
```
-(((a + b*ArcCoth[c + d*x])^2*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcCoth[c
 + d*x])^2*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d*x
))])/f + (b*(a + b*ArcCoth[c + d*x])*PolyLog[2, 1 - 2/(1 + c + d*x)])/f -
(b*(a + b*ArcCoth[c + d*x])*PolyLog[2, 1 - (2*(d*e - c*f + f*(c + d*x)))/(
(d*e + f - c*f)*(1 + c + d*x))])/f + (b^2*PolyLog[3, 1 - 2/(1 + c + d*x)])
/(2*f) - (b^2*PolyLog[3, 1 - (2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f
)*(1 + c + d*x))])/(2*f)
```

3.112. $\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{e+fx}\,dx$

3.112.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6475
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcCoth[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Coth[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcCoth[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcC
oth[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.112.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.72 (sec) , antiderivative size = 1603, normalized size of antiderivative = 7.49

method	result	size
derivativedivides	Expression too large to display	1603
default	Expression too large to display	1603
parts	Expression too large to display	1684

input
```
int((a+b*arccoth(d*x+c))^2/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*d*ln(c*f-d*e-f*(d*x+c))/f-b^2*d*(-ln(c*f-d*e-f*(d*x+c))/f*arccoth
(d*x+c)^2-2/f*(-1/2*arccoth(d*x+c)^2*ln(f*c*((d*x+c+1)/(d*x+c-1)-1)+(1-(d*
x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)+1/4*I*Pi*csgn(I*(f*c*((d
*x+c+1)/(d*x+c-1)-1)+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*
f)/((d*x+c+1)/(d*x+c-1)-1))*(csgn(I*(f*c*((d*x+c+1)/(d*x+c-1)-1)+(1-(d*x+c
+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*csgn(I/((d*x+c+1)/(d*x+c-1
)-1))-csgn(I*(f*c*((d*x+c+1)/(d*x+c-1)-1)+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d
*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))*csgn(I/((d*x+c+1)/(d*x+c-
1)-1))-csgn(I*(f*c*((d*x+c+1)/(d*x+c-1)-1)+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(
d*x+c+1)/(d*x+c-1)-1)*f))*csgn(I*(f*c*((d*x+c+1)/(d*x+c-1)-1)+(1-(d*x+c+1)
/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))+csgn(
I*(f*c*((d*x+c+1)/(d*x+c-1)-1)+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*
x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2)*arccoth(d*x+c)^2+1/2*arccoth(d*x+
c)^2*ln((d*x+c+1)/(d*x+c-1)-1)-1/2*arccoth(d*x+c)^2*ln(1-1/((d*x+c-1)/(d*x
+c+1))^(1/2))-arccoth(d*x+c)*polylog(2,1/((d*x+c-1)/(d*x+c+1))^(1/2))+poly
log(3,1/((d*x+c-1)/(d*x+c+1))^(1/2))-1/2*arccoth(d*x+c)^2*ln(1+1/((d*x+c-1
)/(d*x+c+1))^(1/2))-arccoth(d*x+c)*polylog(2,-1/((d*x+c-1)/(d*x+c+1))^(1/2
))+polylog(3,-1/((d*x+c-1)/(d*x+c+1))^(1/2))+1/2*f*c/(c*f-d*e-f)*arccoth(d
*x+c)^2*ln(1-(d*x+c+1)*(c*f-d*e-f)/(c*f-d*e+f)/(d*x+c-1))+1/2*f*c/(c*f-d*e
-f)*arccoth(d*x+c)*polylog(2,(d*x+c+1)*(c*f-d*e-f)/(c*f-d*e+f)/(d*x+c-1...
```

3.112.5 Fricas [F]

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{e+fx}\,dx=\int \frac{\left(b\operatorname{arcoth}\left(dx+c\right)+a\right)^2}{fx+e}\,dx$$

input
```
integrate((a+b*arccoth(d*x+c))^2/(f*x+e),x, algorithm="fricas")
```

output
```
integral((b^2*arccoth(d*x + c)^2 + 2*a*b*arccoth(d*x + c) + a^2)/(f*x + e)
, x)
```

3.112.6 Sympy [F]

$$\int \frac{(a+b\coth^{-1}(c+dx))^2}{e+fx}\,dx=\int \frac{(a+b\,\text{acoth}\,(c+dx))^2}{e+fx}\,dx$$

input
```
integrate((a+b*acoth(d*x+c))**2/(f*x+e),x)
```

output
```
Integral((a + b*acoth(c + d*x))**2/(e + f*x), x)
```

3.112.7 Maxima [F]

$$\int \frac{(a+b\coth^{-1}(c+dx))^2}{e+fx}\,dx=\int \frac{(b\,\text{arcoth}\,(dx+c)+a)^2}{fx+e}\,dx$$

input
```
integrate((a+b*arccoth(d*x+c))^2/(f*x+e),x, algorithm="maxima")
```

output
```
a^2*log(f*x + e)/f + integrate(1/4*b^2*(log(1/(d*x + c) + 1) - log(-1/(d*x
 + c) + 1))^2/(f*x + e) + a*b*(log(1/(d*x + c) + 1) - log(-1/(d*x + c) + 1
))/(f*x + e), x)
```

3.112.8 Giac [F]

$$\int \frac{(a+b\coth^{-1}(c+dx))^2}{e+fx}\,dx=\int \frac{(b\,\text{arcoth}\,(dx+c)+a)^2}{fx+e}\,dx$$

input
```
integrate((a+b*arccoth(d*x+c))^2/(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*arccoth(d*x + c) + a)^2/(f*x + e), x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{e+fx}\,dx=\int \frac{\left(a+b\,\mathrm{acoth}(c+d\,x)\right)^2}{e+f\,x}\,dx$$

input
```
int((a + b*acoth(c + d*x))^2/(e + f*x),x)
```

output
```
int((a + b*acoth(c + d*x))^2/(e + f*x), x)
```

3.113 $\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx$

3.113.1 Optimal result

Integrand size = 20, antiderivative size = 480

$$\begin{aligned}\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx = &-\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)}+\frac{b^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)}\\
&-\frac{abd\log(1-c-dx)}{f(de+f-cf)}-\frac{b^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1+c+dx}\right)}{f(de-f-cf)}\\
&+\frac{2b^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1+c+dx}\right)}{(de+f-cf)(de-(1+c)f)}\\
&+\frac{abd\log(1+c+dx)}{f(de-f-cf)}+\frac{2abd\log(e+fx)}{f^2-(de-cf)^2}\\
&-\frac{2b^2d\coth^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{(de+f-cf)(de-(1+c)f)}\\
&+\frac{b^2d\,\mathrm{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{2f(de+f-cf)}+\frac{b^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{2f(de-f-cf)}\\
&-\frac{b^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{(de+f-cf)(de-(1+c)f)}\\
&+\frac{b^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{(de+f-cf)(de-(1+c)f)}\end{aligned}$$

output
```
-(a+b*arccoth(d*x+c))^2/f/(f*x+e)+b^2*d*arccoth(d*x+c)*ln(2/(-d*x-c+1))/f/
(-c*f+d*e+f)-a*b*d*ln(-d*x-c+1)/f/(-c*f+d*e+f)-b^2*d*arccoth(d*x+c)*ln(2/(
d*x+c+1))/f/(-c*f+d*e-f)+2*b^2*d*arccoth(d*x+c)*ln(2/(d*x+c+1))/(-c*f+d*e-
f)/(-c*f+d*e+f)+a*b*d*ln(d*x+c+1)/f/(-c*f+d*e-f)+2*a*b*d*ln(f*x+e)/(f^2-(-
c*f+d*e)^2)-2*b^2*d*arccoth(d*x+c)*ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/
(-c*f+d*e-f)/(-c*f+d*e+f)+1/2*b^2*d*polylog(2,(-d*x-c-1)/(-d*x-c+1))/f/(-c
*f+d*e+f)+1/2*b^2*d*polylog(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-b^2*d*polylog(
2,1-2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+b^2*d*polylog(2,1-2*d*(f*x+e)/(
-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)
```

3.113.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.99 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx$$

$$= \frac{-\frac{a^2}{f} + \frac{2ab\left(\left(f-c^2f+d^2ex+cd(e-fx)\right)\coth^{-1}(c+dx)-d(e+fx)\log\left(-\frac{d(e+fx)}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}}\right)\right)}{(de+f-cf)(de-(1+c)f)} + \frac{b^2d(e+fx)\left(1-(c+dx)^2\right)\left(\frac{e^{\operatorname{arctanh}\left(\frac{f}{-de\cdots}\right)}\cdots}{(-de+cf)\cdots}\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(a + b*ArcCoth[c + d*x])^2/(e + f*x)^2,x]
```

output
```
(-(a^2/f) + (2*a*b*((f - c^2*f + d^2*e*x + c*d*(e - f*x))*ArcCoth[c + d*x]
 - d*(e + f*x)*Log[-((d*(e + f*x))/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))])
)/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (b^2*d*(e + f*x)*(1 - (c + d*x)^2)
*((E^ArcTanh[f/(-(d*e) + c*f)]*ArcCoth[c + d*x]^2)/((-(d*e) + c*f)*Sqrt[1
- f^2/(d*e - c*f)^2]) + ArcCoth[c + d*x]^2/(d*e + d*f*x) + (f*((-I)*Pi*Log
[1 + E^(2*ArcCoth[c + d*x])] - 2*ArcTanh[f/(-(d*e) + c*f)]*Log[1 - E^(-2*(
ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] + ArcCoth[c + d*x]*(I*Pi + 2*
ArcTanh[f/(d*e - c*f)] + 2*Log[1 - E^(-2*(ArcCoth[c + d*x] + ArcTanh[f/(d*
e - c*f)]))]) + I*Pi*Log[1/Sqrt[1 - (c + d*x)^(-2)]] + 2*ArcTanh[f/(-(d*e)
 + c*f)]*Log[I*Sinh[ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]]] - PolyLog[
2, E^(-2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))]))/(d^2*e^2 - 2*c*d*
e*f + (-1 + c^2)*f^2)))/((c + d*x)^2*(f - f/(c + d*x)^2)))/(e + f*x)
```

3.113.3 Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6660, 7292, 6672, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{(e+fx)^2}dx$$

$$\Big\downarrow \; 6660$$

$$\frac{2bd\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)(1-(c+dx)^2)}dx}{f}-\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)}$$

$$\Big\downarrow \; 7292$$

$$\frac{2bd\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)(-c^2-2dxc-d^2x^2+1)}dx}{f}-\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)}$$

$$\Big\downarrow \; 6672$$

$$\frac{2b\int \frac{d(a+b\coth^{-1}(c+dx))}{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)(1-(c+dx)^2)}d(c+dx)}{f}-\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)}$$

$$\Big\downarrow \; 27$$

$$\frac{2bd\int \frac{a+b\coth^{-1}(c+dx)}{(de-cf+f(c+dx))(1-(c+dx)^2)}d(c+dx)}{f}-\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)}$$

$$\Big\downarrow \; 7276$$

$$\frac{2bd\int \left(-\frac{a}{(c+dx-1)(c+dx+1)(de-cf+f(c+dx))}-\frac{b\coth^{-1}(c+dx)}{(c+dx-1)(c+dx+1)(de-cf+f(c+dx))}\right)d(c+dx)}{f}-\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)}$$

$$\Big\downarrow \; 2009$$

$$\frac{2bd\left(-\frac{a\log(-c-dx+1)}{2(-cf+de+f)}+\frac{a\log(c+dx+1)}{2(de-(c+1)f)}-\frac{af\log(f(c+dx)-cf+de)}{(-cf+de+f)(de-(c+1)f)}+\frac{b\,\text{PolyLog}\left(2,-\frac{c+dx+1}{-c-dx+1}\right)}{4(-cf+de+f)}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{c+dx+1}\right)}{4(-cf+de-f)}-\frac{bf\,\text{Po}}{2(-c}\right.}{}-\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)}$$

3.113. $\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{(e+fx)^2}dx$

input
```
Int[(a + b*ArcCoth[c + d*x])^2/(e + f*x)^2,x]
```

output
```
-((a + b*ArcCoth[c + d*x])^2/(f*(e + f*x))) + (2*b*d*((b*ArcCoth[c + d*x]*
Log[2/(1 - c - d*x)])/(2*(d*e + f - c*f)) - (a*Log[1 - c - d*x])/(2*(d*e +
 f - c*f)) - (b*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/(2*(d*e - f - c*f))
 + (b*f*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e - (1
+ c)*f)) + (a*Log[1 + c + d*x])/(2*(d*e - (1 + c)*f)) - (a*f*Log[d*e - c*f
 + f*(c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (b*f*ArcCoth[c + d*
x]*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d*x))])/((d
*e + f - c*f)*(d*e - (1 + c)*f)) + (b*PolyLog[2, -((1 + c + d*x)/(1 - c -
d*x))])/(4*(d*e + f - c*f)) + (b*PolyLog[2, 1 - 2/(1 + c + d*x)])/(4*(d*e
- f - c*f)) - (b*f*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*(d*e + f - c*f)*(d*
e - (1 + c)*f)) + (b*f*PolyLog[2, 1 - (2*(d*e - c*f + f*(c + d*x)))/((d*e
+ f - c*f)*(1 + c + d*x))])/(2*(d*e + f - c*f)*(d*e - (1 + c)*f))))/f
```

3.113.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6660
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCoth[c + d*x])^p/(f*(m
+ 1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCo
th[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[p, 0] && ILtQ[m, -1]
```

rule 6672
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Sub
st[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCoth[
x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x
] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.113.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.23

method	result
parts	$-\frac{a^2}{(fx+e)f}+\frac{b^2\left(-\frac{d^2\operatorname{arccoth}(dx+c)^2}{(f(dx+c)-cf+de)f}-\frac{2d^2\left(\frac{\operatorname{arccoth}(dx+c)f\ln(f(dx+c)-cf+de)}{(cf-de-f)(cf-de+f)}-\frac{\operatorname{arccoth}(dx+c)\ln(dx+c-1)}{2cf-2de-2f}+\frac{\operatorname{arccoth}(dx\cdots}{2cf\cdots}\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{\frac{a^2d^2}{(cf-de-f(dx+c))f}+b^2d^2\left(\frac{\operatorname{arccoth}(dx+c)^2}{(cf-de-f(dx+c))f}+\frac{-\frac{2\operatorname{arccoth}(dx+c)f\ln(cf-de-f(dx+c))}{(cf-de-f)(cf-de+f)}-\frac{2\operatorname{arccoth}(dx+c)\ln(dx+c+1)}{2cf-2de+2f}+\frac{2\operatorname{arcc}\cdots}{\cdots}}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{a^2d^2}{(cf-de-f(dx+c))f}+b^2d^2\left(\frac{\operatorname{arccoth}(dx+c)^2}{(cf-de-f(dx+c))f}+\frac{-\frac{2\operatorname{arccoth}(dx+c)f\ln(cf-de-f(dx+c))}{(cf-de-f)(cf-de+f)}-\frac{2\operatorname{arccoth}(dx+c)\ln(dx+c+1)}{2cf-2de+2f}+\frac{2\operatorname{arcc}\cdots}{\cdots}}{\cdots}\right.}{\cdots}$

input
```
int((a+b*arccoth(d*x+c))^2/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/(f*x+e)/f+b^2/d*(-d^2/(f*(d*x+c)-c*f+d*e)/f*arccoth(d*x+c)^2-2*d^2/f*
(arccoth(d*x+c)*f/(c*f-d*e-f)/(c*f-d*e+f)*ln(f*(d*x+c)-c*f+d*e)-arccoth(d*
x+c)/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)+arccoth(d*x+c)/(2*c*f-2*d*e+2*f)*ln(d*x
+c+1)+1/(c*f-d*e-f)/(c*f-d*e+f)*(1/2*f*(dilog((f*(d*x+c)-f)/(c*f-d*e-f))+l
n(f*(d*x+c)-c*f+d*e)*ln((f*(d*x+c)-f)/(c*f-d*e-f)))-1/2*f*(dilog((f*(d*x+c
)+f)/(c*f-d*e+f))+ln(f*(d*x+c)-c*f+d*e)*ln((f*(d*x+c)+f)/(c*f-d*e+f))))-1/
2/(c*f-d*e-f)*(1/4*ln(d*x+c-1)^2-1/2*dilog(1/2*d*x+1/2*c+1/2)-1/2*ln(d*x+c
-1)*ln(1/2*d*x+1/2*c+1/2))+1/2/(c*f-d*e+f)*(-1/4*ln(d*x+c+1)^2+1/2*(ln(d*x
+c+1)-ln(1/2*d*x+1/2*c+1/2))*ln(-1/2*d*x-1/2*c+1/2)-1/2*dilog(1/2*d*x+1/2*
c+1/2))))-2*a*b*d/(d*f*x+d*e)/f*arccoth(d*x+c)-2*a*b*d/(c*f-d*e-f)/(c*f-d*
e+f)*ln(f*(d*x+c)-c*f+d*e)+2*a*b*d/f/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)-2*a*b*d
/f/(2*c*f-2*d*e+2*f)*ln(d*x+c+1)
```

3.113.5 Fricas [F]

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx=\int \frac{\left(b\operatorname{arcoth}\left(dx+c\right)+a\right)^2}{\left(fx+e\right)^2}\,dx$$

input
```
integrate((a+b*arccoth(d*x+c))^2/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccoth(d*x + c)^2 + 2*a*b*arccoth(d*x + c) + a^2)/(f^2*x^2
+ 2*e*f*x + e^2), x)
```

3.113.6 Sympy [F]

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx=\int \frac{\left(a+b\operatorname{acoth}\left(c+dx\right)\right)^2}{\left(e+fx\right)^2}\,dx$$

input
```
integrate((a+b*acoth(d*x+c))**2/(f*x+e)**2,x)
```

output
```
Integral((a + b*acoth(c + d*x))**2/(e + f*x)**2, x)
```

3.113.7 Maxima [F]

$$\int \frac{(a + b\coth^{-1}(c + dx))^2}{(e + fx)^2} dx = \int \frac{(b\,\mathrm{arcoth}\,(dx + c) + a)^2}{(fx + e)^2} dx$$

input
```
integrate((a+b*arccoth(d*x+c))^2/(f*x+e)^2,x, algorithm="maxima")
```

output
```
(d*(log(d*x + c + 1)/(d*e*f - (c + 1)*f^2) - log(d*x + c - 1)/(d*e*f - (c
- 1)*f^2) - 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 - 1)*f^2)) - 2*arcc
oth(d*x + c)/(f^2*x + e*f))*a*b - 1/4*b^2*(log(d*x + c + 1)^2/(f^2*x + e*f
) + integrate(-((d*f*x + c*f + f)*log(d*x + c - 1)^2 + 2*(d*f*x + d*e - (d
*f*x + c*f + f)*log(d*x + c - 1))*log(d*x + c + 1))/(d*f^3*x^3 + c*e^2*f +
 e^2*f + (2*d*e*f^2 + c*f^3 + f^3)*x^2 + (d*e^2*f + 2*c*e*f^2 + 2*e*f^2)*x
), x)) - a^2/(f^2*x + e*f)
```

3.113.8 Giac [F]

$$\int \frac{(a + b\coth^{-1}(c + dx))^2}{(e + fx)^2} dx = \int \frac{(b\,\mathrm{arcoth}\,(dx + c) + a)^2}{(fx + e)^2} dx$$

input
```
integrate((a+b*arccoth(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*arccoth(d*x + c) + a)^2/(f*x + e)^2, x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\coth^{-1}(c + dx))^2}{(e + fx)^2} dx = \int \frac{(a + b\,\mathrm{acoth}(c + d\,x))^2}{(e + f\,x)^2} dx$$

input
```
int((a + b*acoth(c + d*x))^2/(e + f*x)^2,x)
```

output
```
int((a + b*acoth(c + d*x))^2/(e + f*x)^2, x)
```

3.114 $\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^3 dx$

3.114.1 Optimal result

Integrand size = 20, antiderivative size = 546

$$\begin{aligned}\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^3 dx &= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\coth^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^3} \\ &+ \frac{3bf(de-cf)\left(a+b\coth^{-1}(c+dx)\right)^2}{d^3} \\ &+ \frac{3bf(de-cf)(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{d^3} + \frac{bf^2(c+dx)^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^3} \\ &- \frac{(de-cf)\left(d^2e^2-2cdef+(3+c^2)f^2\right)\left(a+b\coth^{-1}(c+dx)\right)^3}{3d^3f} \\ &+ \frac{\left(3d^2e^2-6cdef+(1+3c^2)f^2\right)\left(a+b\coth^{-1}(c+dx)\right)^3}{3d^3} \\ &+ \frac{(e+fx)^3\left(a+b\coth^{-1}(c+dx)\right)^3}{3f} \\ &- \frac{6b^2f(de-cf)\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2}{1-c-dx}\right)}{d^3} \\ &- \frac{b\left(3d^2e^2-6cdef+(1+3c^2)f^2\right)\left(a+b\coth^{-1}(c+dx)\right)^2\log\left(\frac{2}{1-c-dx}\right)}{d^3} \\ &+ \frac{b^3f^2\log\left(1-(c+dx)^2\right)}{2d^3} - \frac{3b^3f(de-cf)\operatorname{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{d^3} \\ &- \frac{b^2\left(3d^2e^2-6cdef+(1+3c^2)f^2\right)\left(a+b\coth^{-1}(c+dx)\right)\operatorname{PolyLog}\left(2,1-\frac{2}{1-c-dx}\right)}{d^3} \\ &+ \frac{b^3\left(3d^2e^2-6cdef+(1+3c^2)f^2\right)\operatorname{PolyLog}\left(3,1-\frac{2}{1-c-dx}\right)}{2d^3}\end{aligned}$$

output
```
a*b^2*f^2*x/d^2+b^3*f^2*(d*x+c)*arccoth(d*x+c)/d^3-1/2*b*f^2*(a+b*arccoth(
d*x+c))^2/d^3+3*b*f*(-c*f+d*e)*(a+b*arccoth(d*x+c))^2/d^3+3*b*f*(-c*f+d*e)
*(d*x+c)*(a+b*arccoth(d*x+c))^2/d^3+1/2*b*f^2*(d*x+c)^2*(a+b*arccoth(d*x+c
))^2/d^3-1/3*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f+(c^2+3)*f^2)*(a+b*arccoth(d*x+c
))^3/d^3/f+1/3*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*(a+b*arccoth(d*x+c))^3/
d^3+1/3*(f*x+e)^3*(a+b*arccoth(d*x+c))^3/f-6*b^2*f*(-c*f+d*e)*(a+b*arccoth
(d*x+c))*ln(2/(-d*x-c+1))/d^3-b*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*(a+b*a
rccoth(d*x+c))^2*ln(2/(-d*x-c+1))/d^3+1/2*b^3*f^2*ln(1-(d*x+c)^2)/d^3-3*b^
3*f*(-c*f+d*e)*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d^3-b^2*(3*d^2*e^2-6*c*d*e
*f+(3*c^2+1)*f^2)*(a+b*arccoth(d*x+c))*polylog(2,1-2/(-d*x-c+1))/d^3+1/2*b
^3*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*polylog(3,1-2/(-d*x-c+1))/d^3
```

3.114.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.61 (sec) , antiderivative size = 2646, normalized size of antiderivative = 4.85

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx = \text{Result too large to show}$$

input
```
Integrate[(e + f*x)^2*(a + b*ArcCoth[c + d*x])^3,x]
```

output
```
(a^2*(a*d^2*e^2 + 3*b*d*e*f - 2*b*c*f^2)*x)/d^2 + (a^2*f*(2*a*d*e + b*f)*x
^2)/(2*d) + (a^3*f^2*x^3)/3 + a^2*b*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCoth[
c + d*x] + ((3*a^2*b*d^2*e^2 - 3*a^2*b*c*d^2*e^2 + 3*a^2*b*d*e*f - 6*a^2*b
*c*d*e*f + 3*a^2*b*c^2*d*e*f + a^2*b*f^2 - 3*a^2*b*c*f^2 + 3*a^2*b*c^2*f^2
 - a^2*b*c^3*f^2)*Log[1 - c - d*x])/(2*d^3) + ((3*a^2*b*d^2*e^2 + 3*a^2*b*
c*d^2*e^2 - 3*a^2*b*d*e*f - 6*a^2*b*c*d*e*f - 3*a^2*b*c^2*d*e*f + a^2*b*f^
2 + 3*a^2*b*c*f^2 + 3*a^2*b*c^2*f^2 + a^2*b*c^3*f^2)*Log[1 + c + d*x])/(2*
d^3) - (6*a*b^2*e*f*(1 - (c + d*x)^2)*(((c + d*x)*ArcCoth[c + d*x])/d^2 -
(c*(c + d*x)*ArcCoth[c + d*x]^2)/d^2 + ((c + d*x)^2*(1 - (c + d*x)^(-2))*A
rcCoth[c + d*x]^2)/(2*d^2) - Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])]/d
^2 + (2*c*(ArcCoth[c + d*x]^2/2 + ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c
 + d*x])] - PolyLog[2, E^(-2*ArcCoth[c + d*x])]/2))/d^2))/((c + d*x)^2*(1
- (c + d*x)^(-2))) + (3*a*b^2*e^2*(1 - (c + d*x)^2)*(ArcCoth[c + d*x]*(Arc
Coth[c + d*x] - (c + d*x)*ArcCoth[c + d*x] + 2*Log[1 - E^(-2*ArcCoth[c + d
*x])]) - PolyLog[2, E^(-2*ArcCoth[c + d*x])]))/(d*(c + d*x)^2*(1 - (c + d*
x)^(-2))) + (b^3*e^2*(1 - (c + d*x)^2)*(ArcCoth[c + d*x]^2*(ArcCoth[c + d*
x] - (c + d*x)*ArcCoth[c + d*x] + 3*Log[1 - E^(-2*ArcCoth[c + d*x])]) - 3*
ArcCoth[c + d*x]*PolyLog[2, E^(-2*ArcCoth[c + d*x])] - (3*PolyLog[3, E^(-2
*ArcCoth[c + d*x])])/2))/(d*(c + d*x)^2*(1 - (c + d*x)^(-2))) - (b^3*e*f*(
1 - (c + d*x)^2)*(ArcCoth[c + d*x]*(-3*ArcCoth[c + d*x] + 2*c*ArcCoth[c...
```

3.114.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 533, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6662, 27, 6481, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{6662}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2\left(a+b\coth^{-1}(c+dx)\right)^3}{d^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (de-cf+f(c+dx))^2\left(a+b\coth^{-1}(c+dx)\right)^3d(c+dx)}{d^3}$$

$$\Big\downarrow \text{6481}$$

$$\frac{(f(c+dx)-cf+de)^3(a+b\coth^{-1}(c+dx))^3}{3f}-\frac{b\int\left(-\left((c+dx)(a+b\coth^{-1}(c+dx))^2f^3\right)-3(de-cf)(a+b\coth^{-1}(c+dx))^2f^2+\frac{((de-cf)(d^2e^2-2cdf}{f}}{d^3}$$

$\downarrow$ 2009

$$\frac{(f(c+dx)-cf+de)^3(a+b\coth^{-1}(c+dx))^3}{3f}-\frac{b\left(bf((3c^2+1)f^2-6cdef+3d^2e^2)\operatorname{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)(a+b\coth^{-1}(c+dx))-\frac{f((3c^2+1)f^2-6c}{}\right.}{}$$

input
```
Int[(e + f*x)^2*(a + b*ArcCoth[c + d*x])^3,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^3*(a + b*ArcCoth[c + d*x])^3)/(3*f) - (b*(-(a*
b*f^3*(c + d*x)) - b^2*f^3*(c + d*x)*ArcCoth[c + d*x] + (f^3*(a + b*ArcCot
h[c + d*x])^2)/2 - 3*f^2*(d*e - c*f)*(a + b*ArcCoth[c + d*x])^2 - 3*f^2*(d
*e - c*f)*(c + d*x)*(a + b*ArcCoth[c + d*x])^2 - (f^3*(c + d*x)^2*(a + b*A
rcCoth[c + d*x])^2)/2 + ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f + (3 + c^2)*f^2)
*(a + b*ArcCoth[c + d*x])^3)/(3*b) - (f*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^
2)*f^2)*(a + b*ArcCoth[c + d*x])^3)/(3*b) + 6*b*f^2*(d*e - c*f)*(a + b*Arc
Coth[c + d*x])*Log[2/(1 - c - d*x)] + f*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^
2)*f^2)*(a + b*ArcCoth[c + d*x])^2*Log[2/(1 - c - d*x)] - (b^2*f^3*Log[1 -
 (c + d*x)^2])/2 + 3*b^2*f^2*(d*e - c*f)*PolyLog[2, -((1 + c + d*x)/(1 - c
 - d*x))] + b*f*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a + b*ArcCoth[c
 + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)] - (b^2*f*(3*d^2*e^2 - 6*c*d*e*f +
 (1 + 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 - c - d*x)])/2))/f)/d^3
```

3.114.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6481
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.114.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 27.13 (sec) , antiderivative size = 8597, normalized size of antiderivative = 15.75

method	result	size
derivativedivides	Expression too large to display	8597
default	Expression too large to display	8597
parts	Expression too large to display	8610

input
```
int((f*x+e)^2*(a+b*arccoth(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.114.5 Fricas [F]

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx = \int (fx+e)^2 (b\operatorname{arcoth}(dx+c)+a)^3 \, dx$$

input
```
integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(a^3*f^2*x^2 + 2*a^3*e*f*x + a^3*e^2 + (b^3*f^2*x^2 + 2*b^3*e*f*x
+ b^3*e^2)*arccoth(d*x + c)^3 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*e*f*x + a*b^2*e
^2)*arccoth(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x + a^2*b*e^2)*arc
coth(d*x + c), x)
```

3.114.6 Sympy [F]

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx = \int \left(a+b\operatorname{acoth}\left(c+dx\right)\right)^3 (e+fx)^2 \, dx$$

input
```
integrate((f*x+e)**2*(a+b*acoth(d*x+c))**3,x)
```

output
```
Integral((a + b*acoth(c + d*x))**3*(e + f*x)**2, x)
```

3.114.7 Maxima [F]

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx = \int (fx+e)^2 (b\operatorname{arcoth}(dx+c)+a)^3 \, dx$$

input
```
integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/3*a^3*f^2*x^3 + a^3*e*f*x^2 + 3/2*(2*x^2*arccoth(d*x + c) + d*(2*x/d^2 -
 (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d
^3))*a^2*b*e*f + 1/2*(2*x^3*arccoth(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 + (c
^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*log(d
*x + c - 1)/d^4))*a^2*b*f^2 + a^3*e^2*x + 3/2*(2*(d*x + c)*arccoth(d*x + c
) + log(-(d*x + c)^2 + 1))*a^2*b*e^2/d + 1/24*((b^3*d^3*f^2*x^3 + 3*b^3*d^
3*e*f*x^2 + 3*b^3*d^3*e^2*x + (c^3*f^2 + 3*d^2*e^2 - 3*(d*e*f - f^2)*c^2 -
 3*d*e*f + 3*(d^2*e^2 - 2*d*e*f + f^2)*c + f^2)*b^3)*log(d*x + c + 1)^3 +
3*(2*a*b^2*d^3*f^2*x^3 + (6*a*b^2*d^3*e*f + b^3*d^2*f^2)*x^2 + 2*(3*a*b^2*
d^3*e^2 + (3*d^2*e*f - 2*c*d*f^2)*b^3)*x - (b^3*d^3*f^2*x^3 + 3*b^3*d^3*e*
f*x^2 + 3*b^3*d^3*e^2*x + (c^3*f^2 - 3*d^2*e^2 - 3*(d*e*f + f^2)*c^2 - 3*d
*e*f + 3*(d^2*e^2 + 2*d*e*f + f^2)*c - f^2)*b^3)*log(d*x + c - 1))*log(d*x
 + c + 1)^2)/d^3 + integrate(-1/8*((b^3*d^3*f^2*x^3 + (2*d^3*e*f + c*d^2*f
^2 + d^2*f^2)*b^3*x^2 + (d^3*e^2 + 2*c*d^2*e*f + 2*d^2*e*f)*b^3*x + (c*d^2
*e^2 + d^2*e^2)*b^3)*log(d*x + c - 1)^3 - 6*(a*b^2*d^3*f^2*x^3 + (2*d^3*e*
f + c*d^2*f^2 + d^2*f^2)*a*b^2*x^2 + (d^3*e^2 + 2*c*d^2*e*f + 2*d^2*e*f)*a
*b^2*x + (c*d^2*e^2 + d^2*e^2)*a*b^2)*log(d*x + c - 1)^2 + (4*a*b^2*d^3*f^
2*x^3 + 2*(6*a*b^2*d^3*e*f + b^3*d^2*f^2)*x^2 - 3*(b^3*d^3*f^2*x^3 + (2*d^
3*e*f + c*d^2*f^2 + d^2*f^2)*b^3*x^2 + (d^3*e^2 + 2*c*d^2*e*f + 2*d^2*e*f)
*b^3*x + (c*d^2*e^2 + d^2*e^2)*b^3)*log(d*x + c - 1)^2 + 4*(3*a*b^2*d^3...
```

3.114.8 Giac [F]

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx = \int (fx+e)^2 (b\operatorname{arcoth}(dx+c)+a)^3 \, dx$$

input
```
integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*(b*arccoth(d*x + c) + a)^3, x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx = \int (e+f\,x)^2 \,(a+b\operatorname{acoth}(c+d\,x))^3 \, dx$$

input
```
int((e + f*x)^2*(a + b*acoth(c + d*x))^3,x)
```

output
```
int((e + f*x)^2*(a + b*acoth(c + d*x))^3, x)
```

3.115 $\int (e+fx) \left(a+b\coth^{-1}(c+dx)\right)^3 dx$

3.115.1 Optimal result

Integrand size = 18, antiderivative size = 326

$$\begin{aligned}
&\int (e+fx) \left(a+b\coth^{-1}(c+dx)\right)^3 dx \\
&= \frac{3bf\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} \\
&+ \frac{(de-cf)\left(a+b\coth^{-1}(c+dx)\right)^3}{d^2} \\
&- \frac{\left(d^2e^2-2cdef+(1+c^2)f^2\right)\left(a+b\coth^{-1}(c+dx)\right)^3}{2d^2f} \\
&+ \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)^3}{2f} - \frac{3b^2f\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2}{1-c-dx}\right)}{d^2} \\
&- \frac{3b(de-cf)\left(a+b\coth^{-1}(c+dx)\right)^2\log\left(\frac{2}{1-c-dx}\right)}{d^2} - \frac{3b^3f\,\text{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{2d^2} \\
&- \frac{3b^2(de-cf)\left(a+b\coth^{-1}(c+dx)\right)\text{PolyLog}\left(2,1-\frac{2}{1-c-dx}\right)}{d^2} \\
&+ \frac{3b^3(de-cf)\,\text{PolyLog}\left(3,1-\frac{2}{1-c-dx}\right)}{2d^2}
\end{aligned}$$

output
```
3/2*b*f*(a+b*arccoth(d*x+c))^2/d^2+3/2*b*f*(d*x+c)*(a+b*arccoth(d*x+c))^2/
d^2+(-c*f+d*e)*(a+b*arccoth(d*x+c))^3/d^2-1/2*(d^2*e^2-2*c*d*e*f+(c^2+1)*f
^2)*(a+b*arccoth(d*x+c))^3/d^2/f+1/2*(f*x+e)^2*(a+b*arccoth(d*x+c))^3/f-3*
b^2*f*(a+b*arccoth(d*x+c))*ln(2/(-d*x-c+1))/d^2-3*b*(-c*f+d*e)*(a+b*arccot
h(d*x+c))^2*ln(2/(-d*x-c+1))/d^2-3/2*b^3*f*polylog(2,(-d*x-c-1)/(-d*x-c+1)
)/d^2-3*b^2*(-c*f+d*e)*(a+b*arccoth(d*x+c))*polylog(2,1-2/(-d*x-c+1))/d^2+
3/2*b^3*(-c*f+d*e)*polylog(3,1-2/(-d*x-c+1))/d^2
```

3.115.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.69 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.84

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^3 dx$$

$$= \frac{2a^2(2ade+3bf-2acf)(c+dx)+2a^3f(c+dx)^2-6a^2b(c+dx)(cf-d(2e+fx))\coth^{-1}(c+dx)+3a^2}{}$$

input
```
Integrate[(e + f*x)*(a + b*ArcCoth[c + d*x])^3,x]
```

output
```
(2*a^2*(2*a*d*e + 3*b*f - 2*a*c*f)*(c + d*x) + 2*a^3*f*(c + d*x)^2 - 6*a^2
*b*(c + d*x)*(c*f - d*(2*e + f*x))*ArcCoth[c + d*x] + 3*a^2*b*(2*d*e + f -
 2*c*f)*Log[1 - c - d*x] + 3*a^2*b*(2*d*e - (1 + 2*c)*f)*Log[1 + c + d*x]
+ 12*a*b^2*f*((c + d*x)*ArcCoth[c + d*x] + ((-1 + (c + d*x)^2)*ArcCoth[c +
 d*x]^2)/2 - Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])]) + 12*a*b^2*d*e*(
ArcCoth[c + d*x]*((-1 + c + d*x)*ArcCoth[c + d*x] - 2*Log[1 - E^(-2*ArcCot
h[c + d*x])]) + PolyLog[2, E^(-2*ArcCoth[c + d*x])]) - 12*a*b^2*c*f*(ArcCo
th[c + d*x]*((-1 + c + d*x)*ArcCoth[c + d*x] - 2*Log[1 - E^(-2*ArcCoth[c +
 d*x])]) + PolyLog[2, E^(-2*ArcCoth[c + d*x])]) + 2*b^3*f*(ArcCoth[c + d*x
]*(3*(-1 + c + d*x)*ArcCoth[c + d*x] + (-1 + c^2 + 2*c*d*x + d^2*x^2)*ArcC
oth[c + d*x]^2 - 6*Log[1 - E^(-2*ArcCoth[c + d*x])]) + 3*PolyLog[2, E^(-2*
ArcCoth[c + d*x])]) + 4*b^3*d*e*((-1/8*I)*Pi^3 + ArcCoth[c + d*x]^3 + (c +
 d*x)*ArcCoth[c + d*x]^3 - 3*ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c + d
*x])] - 3*ArcCoth[c + d*x]*PolyLog[2, E^(2*ArcCoth[c + d*x])] + (3*PolyLog
[3, E^(2*ArcCoth[c + d*x])])/2) - 4*b^3*c*f*((-1/8*I)*Pi^3 + ArcCoth[c + d
*x]^3 + (c + d*x)*ArcCoth[c + d*x]^3 - 3*ArcCoth[c + d*x]^2*Log[1 - E^(2*A
rcCoth[c + d*x])] - 3*ArcCoth[c + d*x]*PolyLog[2, E^(2*ArcCoth[c + d*x])]
+ (3*PolyLog[3, E^(2*ArcCoth[c + d*x])])/2))/(4*d^2)
```

3.115.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6662, 27, 6481, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{6662}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)\left(a+b\coth^{-1}(c+dx)\right)^3}{d}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (de-cf+f(c+dx))\left(a+b\coth^{-1}(c+dx)\right)^3d(c+dx)}{d^2}$$

$$\Big\downarrow \text{6481}$$

$$\frac{\frac{(f(c+dx)-cf+de)^2\left(a+b\coth^{-1}(c+dx)\right)^3}{2f}-\frac{3b\int \left(\frac{\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2+2f(de-cf)(c+dx)\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{1-(c+dx)^2}-f^2\left(a+b\coth^{-1}(c+dx)\right)^2\right)d(\cdots}{2f}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(f(c+dx)-cf+de)^2\left(a+b\coth^{-1}(c+dx)\right)^3}{2f}-\frac{3b\left(\frac{\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)\left(a+b\coth^{-1}(c+dx)\right)^3}{3b}+2bf(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)\left(a+b\coth\cdots\right.}{\cdots}}{\cdots}$$

input
```
Int[(e + f*x)*(a + b*ArcCoth[c + d*x])^3,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^2*(a + b*ArcCoth[c + d*x])^3)/(2*f) - (3*b*(-(
f^2*(a + b*ArcCoth[c + d*x])^2) - f^2*(c + d*x)*(a + b*ArcCoth[c + d*x])^2
 - (2*f*(d*e - c*f)*(a + b*ArcCoth[c + d*x])^3)/(3*b) + ((d^2*e^2 - 2*c*d*
e*f + (1 + c^2)*f^2)*(a + b*ArcCoth[c + d*x])^3)/(3*b) + 2*b*f^2*(a + b*Ar
cCoth[c + d*x])*Log[2/(1 - c - d*x)] + 2*f*(d*e - c*f)*(a + b*ArcCoth[c +
d*x])^2*Log[2/(1 - c - d*x)] + b^2*f^2*PolyLog[2, -((1 + c + d*x)/(1 - c -
 d*x))] + 2*b*f*(d*e - c*f)*(a + b*ArcCoth[c + d*x])*PolyLog[2, 1 - 2/(1 -
 c - d*x)] - b^2*f*(d*e - c*f)*PolyLog[3, 1 - 2/(1 - c - d*x)]))/(2*f))/d^
2
```

3.115.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6481
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.115.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.56 (sec) , antiderivative size = 7528, normalized size of antiderivative = 23.09

method	result	size
parts	Expression too large to display	7528
derivativedivides	Expression too large to display	7638
default	Expression too large to display	7638

input
```
int((f*x+e)*(a+b*arccoth(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.115.5 Fricas [F]

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^3\,dx=\int (fx+e)(b\,\mathrm{arcoth}\,(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)*(a+b*arccoth(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(a^3*f*x + a^3*e + (b^3*f*x + b^3*e)*arccoth(d*x + c)^3 + 3*(a*b^2
*f*x + a*b^2*e)*arccoth(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*arccoth(d*x +
 c), x)
```

3.115.6 Sympy [F]

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^3\,dx=\int (a+b\,\mathrm{acoth}\,(c+dx))^3\,(e+fx)\,dx$$

input
```
integrate((f*x+e)*(a+b*acoth(d*x+c))**3,x)
```

output
```
Integral((a + b*acoth(c + d*x))**3*(e + f*x), x)
```

3.115.7 Maxima [F]

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^3\,dx=\int (fx+e)(b\operatorname{arcoth}(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)*(a+b*arccoth(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*a^3*f*x^2 + 3/4*(2*x^2*arccoth(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)
*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a^2*b*f + a
^3*e*x + 3/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*a^2*b*
e/d + 1/16*((b^3*d^2*f*x^2 + 2*b^3*d^2*e*x - (c^2*f - 2*(d*e - f)*c - 2*d*
e + f)*b^3)*log(d*x + c + 1)^3 + 3*(2*a*b^2*d^2*f*x^2 + 2*(2*a*b^2*d^2*e +
 b^3*d*f)*x - (b^3*d^2*f*x^2 + 2*b^3*d^2*e*x - (c^2*f - 2*(d*e + f)*c + 2*
d*e + f)*b^3)*log(d*x + c - 1))*log(d*x + c + 1)^2)/d^2 + integrate(-1/8*(
(b^3*d^2*f*x^2 + (d^2*e + c*d*f + d*f)*b^3*x + (c*d*e + d*e)*b^3)*log(d*x
+ c - 1)^3 - 6*(a*b^2*d^2*f*x^2 + (d^2*e + c*d*f + d*f)*a*b^2*x + (c*d*e +
 d*e)*a*b^2)*log(d*x + c - 1)^2 + 3*(2*a*b^2*d^2*f*x^2 - (b^3*d^2*f*x^2 +
(d^2*e + c*d*f + d*f)*b^3*x + (c*d*e + d*e)*b^3)*log(d*x + c - 1)^2 + 2*(2
*a*b^2*d^2*e + b^3*d*f)*x + (4*(c*d*e + d*e)*a*b^2 + (c^2*f - 2*(d*e + f)*
c + 2*d*e + f)*b^3 + (4*a*b^2*d^2*f - b^3*d^2*f)*x^2 - 2*(b^3*d^2*e - 2*(d
^2*e + c*d*f + d*f)*a*b^2)*x)*log(d*x + c - 1))*log(d*x + c + 1))/(d^2*x +
 c*d + d), x)
```

3.115.8 Giac [F]

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^3\,dx=\int (fx+e)(b\operatorname{arcoth}(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)*(a+b*arccoth(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)*(b*arccoth(d*x + c) + a)^3, x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx) \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx = \int (e+f\,x)\,\left(a+b\,\mathrm{acoth}(c+d\,x)\right)^3 \, dx$$

input
```
int((e + f*x)*(a + b*acoth(c + d*x))^3,x)
```

output
```
int((e + f*x)*(a + b*acoth(c + d*x))^3, x)
```

3.116 $\int \left(a+b\coth^{-1}(c+dx)\right)^3\,dx$

3.116.1 Optimal result

Integrand size = 12, antiderivative size = 132

$$\int \left(a+b\coth^{-1}(c+dx)\right)^3\,dx=\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3}{d}-\frac{3b\left(a+b\coth^{-1}(c+dx)\right)^2\log\left(\frac{2}{1-c-dx}\right)}{d}-\frac{3b^2\left(a+b\coth^{-1}(c+dx)\right)\operatorname{PolyLog}\left(2,1-\frac{2}{1-c-dx}\right)}{d}+\frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1-c-dx}\right)}{2d}$$

output
```
(a+b*arccoth(d*x+c))^3/d+(d*x+c)*(a+b*arccoth(d*x+c))^3/d-3*b*(a+b*arccoth
(d*x+c))^2*ln(2/(-d*x-c+1))/d-3*b^2*(a+b*arccoth(d*x+c))*polylog(2,1-2/(-d
*x-c+1))/d+3/2*b^3*polylog(3,1-2/(-d*x-c+1))/d
```

3.116.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.58

$$\int \left(a+b\coth^{-1}(c+dx)\right)^3\,dx$$
$$=\frac{2a^3(c+dx)+6a^2b(c+dx)\coth^{-1}(c+dx)+3a^2b\log\left(1-(c+dx)^2\right)+6ab^2\Big(\coth^{-1}(c+dx)\Big(\big((-1+c+$$

input
```
Integrate[(a + b*ArcCoth[c + d*x])^3,x]
```

output
```
(2*a^3*(c + d*x) + 6*a^2*b*(c + d*x)*ArcCoth[c + d*x] + 3*a^2*b*Log[1 - (c
 + d*x)^2] + 6*a*b^2*(ArcCoth[c + d*x]*((-1 + c + d*x)*ArcCoth[c + d*x] -
2*Log[1 - E^(-2*ArcCoth[c + d*x])]) + PolyLog[2, E^(-2*ArcCoth[c + d*x])])
 + 2*b^3*((-1/8*I)*Pi^3 + ArcCoth[c + d*x]^3 + (c + d*x)*ArcCoth[c + d*x]^
3 - 3*ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])] - 3*ArcCoth[c + d
*x]*PolyLog[2, E^(2*ArcCoth[c + d*x])] + (3*PolyLog[3, E^(2*ArcCoth[c + d*
x])])/2))/(2*d)
```

3.116.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6654, 6437, 6547, 6471, 6621, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\coth^{-1}(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{ 6654}$$

$$\frac{\int \left(a+b\coth^{-1}(c+dx)\right)^3 d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6437}$$

$$\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3-3b\int \frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{1-(c+dx)^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6547}$$

$$\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3-3b\left(\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{-c-dx+1}d(c+dx)-\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{3b}\right)}{d}$$

$$\Big\downarrow \text{ 6471}$$

$$\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3-3b\left(-2b\int \frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2}{-c-dx+1}\right)}{1-(c+dx)^2}d(c+dx)-\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{3b}+\log\left(\right.\right.}{d}$$

$$\Big\downarrow \text{ 6621}$$

$$\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3-3b\left(-2b\left(\frac{1}{2}b\int\frac{\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)}{1-(c+dx)^2}d(c+dx)-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)(a\right.\right.}{d}$$

$\big\downarrow$ 7164

$$\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3-3b\left(-2b\left(\frac{1}{4}b\,\text{PolyLog}\left(3,1-\frac{2}{-c-dx+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)(a+b\,\mathrm{co}\right.\right.}{d}$$

input
```
Int[(a + b*ArcCoth[c + d*x])^3,x]
```

output
```
((c + d*x)*(a + b*ArcCoth[c + d*x])^3 - 3*b*(-1/3*(a + b*ArcCoth[c + d*x])
^3/b + (a + b*ArcCoth[c + d*x])^2*Log[2/(1 - c - d*x)] - 2*b*(-1/2*((a + b
*ArcCoth[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)]) + (b*PolyLog[3, 1 - 2/
(1 - c - d*x)])/4)))/d
```

3.116.3.1 Defintions of rubi rules used

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6471
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6547
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6621
```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6654
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
}, x] && IGtQ[p, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.116.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(130) = 260$.

Time = 0.85 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.82

method	result
derivativedivides	$\frac{(dx+c)a^3+b^3\left(\operatorname{arccoth}(dx+c)^3(dx+c-1)+2\operatorname{arccoth}(dx+c)^3-3\operatorname{arccoth}(dx+c)^2\ln\left(1+\frac{1}{\sqrt{\frac{dx+c-1}{dx+c+1}}}\right)-6\operatorname{arccoth}(dx+c)\right.}{}$
default	$\frac{(dx+c)a^3+b^3\left(\operatorname{arccoth}(dx+c)^3(dx+c-1)+2\operatorname{arccoth}(dx+c)^3-3\operatorname{arccoth}(dx+c)^2\ln\left(1+\frac{1}{\sqrt{\frac{dx+c-1}{dx+c+1}}}\right)-6\operatorname{arccoth}(dx+c)\right.}{}$
parts	$a^3x+\frac{b^3\left(\operatorname{arccoth}(dx+c)^3(dx+c-1)+2\operatorname{arccoth}(dx+c)^3-3\operatorname{arccoth}(dx+c)^2\ln\left(1+\frac{1}{\sqrt{\frac{dx+c-1}{dx+c+1}}}\right)-6\operatorname{arccoth}(dx+c)\,\mathrm{po}\right.}{}$

input
```
int((a+b*arccoth(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((d*x+c)*a^3+b^3*(arccoth(d*x+c)^3*(d*x+c-1)+2*arccoth(d*x+c)^3-3*arcc
oth(d*x+c)^2*ln(1+1/((d*x+c-1)/(d*x+c+1))^(1/2))-6*arccoth(d*x+c)*polylog(
2,-1/((d*x+c-1)/(d*x+c+1))^(1/2))+6*polylog(3,-1/((d*x+c-1)/(d*x+c+1))^(1/
2))-3*arccoth(d*x+c)^2*ln(1-1/((d*x+c-1)/(d*x+c+1))^(1/2))-6*arccoth(d*x+c
)*polylog(2,1/((d*x+c-1)/(d*x+c+1))^(1/2))+6*polylog(3,1/((d*x+c-1)/(d*x+c
+1))^(1/2)))+3*a*b^2*(arccoth(d*x+c)^2*(d*x+c-1)+2*arccoth(d*x+c)^2-2*arcc
oth(d*x+c)*ln(1-1/((d*x+c-1)/(d*x+c+1))^(1/2))-2*polylog(2,1/((d*x+c-1)/(d
*x+c+1))^(1/2))-2*arccoth(d*x+c)*ln(1+1/((d*x+c-1)/(d*x+c+1))^(1/2))-2*pol
ylog(2,-1/((d*x+c-1)/(d*x+c+1))^(1/2)))+3*b*a^2*((d*x+c)*arccoth(d*x+c)+1/
2*ln((d*x+c)^2-1)))
```

3.116.5 Fricas [F]

$$\int \left(a+b\coth^{-1}(c+dx)\right)^3\,dx=\int (b\operatorname{arcoth}(dx+c)+a)^3\,dx$$

input
```
integrate((a+b*arccoth(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(b^3*arccoth(d*x + c)^3 + 3*a*b^2*arccoth(d*x + c)^2 + 3*a^2*b*arc
coth(d*x + c) + a^3, x)
```

3.116.6 Sympy [F]

$$\int \left(a+b\coth^{-1}(c+dx)\right)^3\,dx=\int (a+b\operatorname{acoth}(c+dx))^3\,dx$$

input
```
integrate((a+b*acoth(d*x+c))**3,x)
```

output
```
Integral((a + b*acoth(c + d*x))**3, x)
```

3.116.7 Maxima [F]

$$\int \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx = \int \left(b \operatorname{arcoth}\left(dx+c\right)+a\right)^3 \, dx$$

input
```
integrate((a+b*arccoth(d*x+c))^3,x, algorithm="maxima")
```

output
```
a^3*x + 3/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*a^2*b/d
 + 1/8*((b^3*d*x + b^3*(c + 1))*log(d*x + c + 1)^3 + 3*(2*a*b^2*d*x - (b^3
*d*x + b^3*(c - 1))*log(d*x + c - 1))*log(d*x + c + 1)^2)/d + integrate(-1
/8*((b^3*d*x + b^3*(c + 1))*log(d*x + c - 1)^3 - 6*(a*b^2*d*x + a*b^2*(c +
 1))*log(d*x + c - 1)^2 + 3*(4*a*b^2*d*x - (b^3*d*x + b^3*(c + 1))*log(d*x
 + c - 1)^2 + 2*(2*a*b^2*(c + 1) - b^3*(c - 1) + (2*a*b^2*d - b^3*d)*x)*lo
g(d*x + c - 1))*log(d*x + c + 1))/(d*x + c + 1), x)
```

3.116.8 Giac [F]

$$\int \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx = \int \left(b \operatorname{arcoth}\left(dx+c\right)+a\right)^3 \, dx$$

input
```
integrate((a+b*arccoth(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*arccoth(d*x + c) + a)^3, x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx = \int \left(a+b\,\mathrm{acoth}(c+d\,x)\right)^3 \, dx$$

input
```
int((a + b*acoth(c + d*x))^3,x)
```

output
```
int((a + b*acoth(c + d*x))^3, x)
```

3.117 $\int \frac{(a+b\coth^{-1}(c+dx))^3}{e+fx}\,dx$

3.117.1 Optimal result

Integrand size = 20, antiderivative size = 308

$$\begin{aligned}
\int \frac{(a+b\coth^{-1}(c+dx))^3}{e+fx}\,dx
&= -\frac{(a+b\coth^{-1}(c+dx))^3\log\left(\frac{2}{1+c+dx}\right)}{f}+\frac{(a+b\coth^{-1}(c+dx))^3\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}\\
&\quad+\frac{3b(a+b\coth^{-1}(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{2f}\\
&\quad-\frac{3b(a+b\coth^{-1}(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{2f}\\
&\quad+\frac{3b^2(a+b\coth^{-1}(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2}{1+c+dx}\right)}{2f}\\
&\quad-\frac{3b^2(a+b\coth^{-1}(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{2f}\\
&\quad+\frac{3b^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+c+dx}\right)}{4f}-\frac{3b^3\operatorname{PolyLog}\left(4,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{4f}
\end{aligned}$$

output
```
-(a+b*arccoth(d*x+c))^3*ln(2/(d*x+c+1))/f+(a+b*arccoth(d*x+c))^3*ln(2*d*(f
*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f+3/2*b*(a+b*arccoth(d*x+c))^2*polylog(2,1-2
/(d*x+c+1))/f-3/2*b*(a+b*arccoth(d*x+c))^2*polylog(2,1-2*d*(f*x+e)/(-c*f+d
*e+f)/(d*x+c+1))/f+3/2*b^2*(a+b*arccoth(d*x+c))*polylog(3,1-2/(d*x+c+1))/f
-3/2*b^2*(a+b*arccoth(d*x+c))*polylog(3,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+
1))/f+3/4*b^3*polylog(4,1-2/(d*x+c+1))/f-3/4*b^3*polylog(4,1-2*d*(f*x+e)/(
-c*f+d*e+f)/(d*x+c+1))/f
```

3.117.2 Mathematica [F]

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{e+fx}\,dx=\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{e+fx}\,dx$$

input
```
Integrate[(a + b*ArcCoth[c + d*x])^3/(e + f*x),x]
```

output
```
Integrate[(a + b*ArcCoth[c + d*x])^3/(e + f*x), x]
```

3.117.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6662, 27, 6477}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{e+fx}\,dx$$
$$\Big\downarrow \text{6662}$$
$$\frac{\int \frac{d\left(a+b\coth^{-1}(c+dx)\right)^3}{d\left(e-\frac{cf}{d}\right)+f(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(c+dx)-cf+de}d(c+dx)$$

$\Big\downarrow$ 6477

$$-\frac{3b^2\left(a+b\coth^{-1}(c+dx)\right)\operatorname{PolyLog}\left(3,1-\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{2f}+\frac{3b^2\operatorname{PolyLog}\left(3,1-\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{2f}-\frac{3b\left(a+b\coth^{-1}(c+dx)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{2f}+\frac{\left(a+b\coth^{-1}(c+dx)\right)^3\log\left(\frac{2(f(c+dx)-cf+de)}{(c+dx+1)(-cf+de+f)}\right)}{f}+\frac{3b\operatorname{PolyLog}\left(2,1-\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{2f}-\frac{\log\left(\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)^3}{f}-\frac{3b^3\operatorname{PolyLog}\left(4,1-\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{4f}+\frac{3b^3\operatorname{PolyLog}\left(4,1-\frac{2}{c+dx+1}\right)}{4f}$$

input
```
Int[(a + b*ArcCoth[c + d*x])^3/(e + f*x),x]
```

output
```
-(((a + b*ArcCoth[c + d*x])^3*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcCoth[c
 + d*x])^3*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d*x
))])/f + (3*b*(a + b*ArcCoth[c + d*x])^2*PolyLog[2, 1 - 2/(1 + c + d*x)])/
(2*f) - (3*b*(a + b*ArcCoth[c + d*x])^2*PolyLog[2, 1 - (2*(d*e - c*f + f*(
c + d*x)))/((d*e + f - c*f)*(1 + c + d*x))])/(2*f) + (3*b^2*(a + b*ArcCoth
[c + d*x])*PolyLog[3, 1 - 2/(1 + c + d*x)])/(2*f) - (3*b^2*(a + b*ArcCoth[
c + d*x])*PolyLog[3, 1 - (2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1
 + c + d*x))])/(2*f) + (3*b^3*PolyLog[4, 1 - 2/(1 + c + d*x)])/(4*f) - (3*
b^3*PolyLog[4, 1 - (2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c +
 d*x))])/(4*f)
```

3.117. $\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{e+fx}\,dx$

3.117.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6477
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcCoth[c*x])^3)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Coth[c*x])^3*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[3*b*
(a + b*ArcCoth[c*x])^2*(PolyLog[2, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[3*b*(
a + b*ArcCoth[c*x])^2*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))
]/(2*e)), x] + Simp[3*b^2*(a + b*ArcCoth[c*x])*(PolyLog[3, 1 - 2/(1 + c*x)]
/(2*e)), x] - Simp[3*b^2*(a + b*ArcCoth[c*x])*(PolyLog[3, 1 - 2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(2*e)), x] + Simp[3*b^3*(PolyLog[4, 1 - 2/(1 + c*
x)]/(4*e)), x] - Simp[3*b^3*(PolyLog[4, 1 - 2*c*((d + e*x)/((c*d + e)*(1 +
c*x)))]/(4*e)), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.117.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 12.70 (sec) , antiderivative size = 3250, normalized size of antiderivative = 10.55

method	result	size
derivativedivides	Expression too large to display	3250
default	Expression too large to display	3250
parts	Expression too large to display	3425

input
```
int((a+b*arccoth(d*x+c))^3/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*d*ln(c*f-d*e-f*(d*x+c))/f-b^3*d*(-ln(c*f-d*e-f*(d*x+c))/f*arccoth
(d*x+c)^3-3/f*(-1/3*arccoth(d*x+c)^3*ln(f*c*((d*x+c+1)/(d*x+c-1)-1)+(1-(d*
x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)+1/6*I*Pi*csgn(I*(f*c*((d
*x+c+1)/(d*x+c-1)-1)+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*
f)/((d*x+c+1)/(d*x+c-1)-1))*(csgn(I*(f*c*((d*x+c+1)/(d*x+c-1)-1)+(1-(d*x+c
+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*csgn(I/((d*x+c+1)/(d*x+c-1
)-1))-csgn(I*(f*c*((d*x+c+1)/(d*x+c-1)-1)+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d
*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))*csgn(I/((d*x+c+1)/(d*x+c-
1)-1))-csgn(I*(f*c*((d*x+c+1)/(d*x+c-1)-1)+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(
d*x+c+1)/(d*x+c-1)-1)*f))*csgn(I*(f*c*((d*x+c+1)/(d*x+c-1)-1)+(1-(d*x+c+1)
/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))+csgn(
I*(f*c*((d*x+c+1)/(d*x+c-1)-1)+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*
x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2)*arccoth(d*x+c)^3+1/3*arccoth(d*x+
c)^3*ln((d*x+c+1)/(d*x+c-1)-1)-1/3*arccoth(d*x+c)^3*ln(1+1/((d*x+c-1)/(d*x
+c+1))^(1/2))-arccoth(d*x+c)^2*polylog(2,-1/((d*x+c-1)/(d*x+c+1))^(1/2))+2
*arccoth(d*x+c)*polylog(3,-1/((d*x+c-1)/(d*x+c+1))^(1/2))-2*polylog(4,-1/(
(d*x+c-1)/(d*x+c+1))^(1/2))-1/3*arccoth(d*x+c)^3*ln(1-1/((d*x+c-1)/(d*x+c+
1))^(1/2))-arccoth(d*x+c)^2*polylog(2,1/((d*x+c-1)/(d*x+c+1))^(1/2))+2*arc
coth(d*x+c)*polylog(3,1/((d*x+c-1)/(d*x+c+1))^(1/2))-2*polylog(4,1/((d*x+c
-1)/(d*x+c+1))^(1/2))+1/3*f*c/(c*f-d*e-f)*arccoth(d*x+c)^3*ln(1-(d*x+c+...
```

3.117.5 Fricas [F]

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{e+fx}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(dx+c)+a)^3}{fx+e}\,dx$$

input
```
integrate((a+b*arccoth(d*x+c))^3/(f*x+e),x, algorithm="fricas")
```

output
```
integral((b^3*arccoth(d*x + c)^3 + 3*a*b^2*arccoth(d*x + c)^2 + 3*a^2*b*ar
ccoth(d*x + c) + a^3)/(f*x + e), x)
```

3.117.6 Sympy [F]

$$\int \frac{(a+b\coth^{-1}(c+dx))^3}{e+fx}\,dx = \int \frac{(a+b\operatorname{acoth}(c+dx))^3}{e+fx}\,dx$$

input
```
integrate((a+b*acoth(d*x+c))**3/(f*x+e),x)
```

output
```
Integral((a + b*acoth(c + d*x))**3/(e + f*x), x)
```

3.117.7 Maxima [F]

$$\int \frac{(a+b\coth^{-1}(c+dx))^3}{e+fx}\,dx = \int \frac{(b\operatorname{arcoth}(dx+c)+a)^3}{fx+e}\,dx$$

input
```
integrate((a+b*arccoth(d*x+c))^3/(f*x+e),x, algorithm="maxima")
```

output
```
a^3*log(f*x + e)/f + integrate(1/8*b^3*(log(1/(d*x + c) + 1) - log(-1/(d*x
 + c) + 1))^3/(f*x + e) + 3/4*a*b^2*(log(1/(d*x + c) + 1) - log(-1/(d*x +
c) + 1))^2/(f*x + e) + 3/2*a^2*b*(log(1/(d*x + c) + 1) - log(-1/(d*x + c)
+ 1))/(f*x + e), x)
```

3.117.8 Giac [F]

$$\int \frac{(a+b\coth^{-1}(c+dx))^3}{e+fx}\,dx = \int \frac{(b\operatorname{arcoth}(dx+c)+a)^3}{fx+e}\,dx$$

input
```
integrate((a+b*arccoth(d*x+c))^3/(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*arccoth(d*x + c) + a)^3/(f*x + e), x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{e+fx}\,dx=\int \frac{\left(a+b\,\mathrm{acoth}(c+d\,x)\right)^3}{e+f\,x}\,dx$$

input
```
int((a + b*acoth(c + d*x))^3/(e + f*x),x)
```

output
```
int((a + b*acoth(c + d*x))^3/(e + f*x), x)
```

3.118 $\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$

3.118.1 Optimal result

Integrand size = 20, antiderivative size = 1089

$$
\begin{aligned}
\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx = & -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} \\
& + \frac{3ab^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} \\
& + \frac{3b^3d\coth^{-1}(c+dx)^2\log\left(\frac{2}{1-c-dx}\right)}{2f(de+f-cf)} \\
& - \frac{3a^2bd\log(1-c-dx)}{2f(de+f-cf)} \\
& - \frac{3ab^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1+c+dx}\right)}{f(de-f-cf)} \\
& + \frac{6ab^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1+c+dx}\right)}{(de+f-cf)(de-(1+c)f)} \\
& - \frac{3b^3d\coth^{-1}(c+dx)^2\log\left(\frac{2}{1+c+dx}\right)}{2f(de-f-cf)} \\
& + \frac{3b^3d\coth^{-1}(c+dx)^2\log\left(\frac{2}{1+c+dx}\right)}{(de+f-cf)(de-(1+c)f)} \\
& + \frac{3a^2bd\log(1+c+dx)}{2f(de-f-cf)} + \frac{3a^2bd\log(e+fx)}{f^2-(de-cf)^2} \\
& - \frac{6ab^2d\coth^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{(de+f-cf)(de-(1+c)f)} \\
& - \frac{3b^3d\coth^{-1}(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{(de+f-cf)(de-(1+c)f)} \\
& + \frac{3ab^2d\operatorname{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{2f(de+f-cf)} \\
& + \frac{3b^3d\coth^{-1}(c+dx)\operatorname{PolyLog}\left(2,1-\frac{2}{1-c-dx}\right)}{2f(de+f-cf)} \\
& + \frac{3ab^2d\operatorname{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{2f(de-f-cf)} \\
& - \frac{3ab^2d\operatorname{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{(de+f-cf)(de-(1+c)f)} \\
& + \frac{3b^3d\coth^{-1}(c+dx)\operatorname{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{2f(de-f-cf)} \\
& - \frac{3b^3d\coth^{-1}(c+dx)\operatorname{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{(de+f-cf)(de-(1+c)f)} \\
& + \frac{3ab^2d\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{(de+f-cf)(de-(1+c)f)} \\
& + 3b^3d\coth^{-1}(c+dx)\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)
\end{aligned}
$$

output
```
-(a+b*arccoth(d*x+c))^3/f/(f*x+e)+3*a*b^2*d*arccoth(d*x+c)*ln(2/(-d*x-c+1)
)/f/(-c*f+d*e+f)+3/2*b^3*d*arccoth(d*x+c)^2*ln(2/(-d*x-c+1))/f/(-c*f+d*e+f
)-3/2*a^2*b*d*ln(-d*x-c+1)/f/(-c*f+d*e+f)-3*a*b^2*d*arccoth(d*x+c)*ln(2/(d
*x+c+1))/f/(-c*f+d*e-f)+6*a*b^2*d*arccoth(d*x+c)*ln(2/(d*x+c+1))/(-c*f+d*e
-f)/(-c*f+d*e+f)-3/2*b^3*d*arccoth(d*x+c)^2*ln(2/(d*x+c+1))/f/(-c*f+d*e-f)
+3*b^3*d*arccoth(d*x+c)^2*ln(2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*a^
2*b*d*ln(d*x+c+1)/f/(-c*f+d*e-f)+3*a^2*b*d*ln(f*x+e)/(f^2-(-c*f+d*e)^2)-6*
a*b^2*d*arccoth(d*x+c)*ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)
/(-c*f+d*e+f)-3*b^3*d*arccoth(d*x+c)^2*ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+
1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*a*b^2*d*polylog(2,(-d*x-c-1)/(-d*x-c+1))
/f/(-c*f+d*e+f)+3/2*b^3*d*arccoth(d*x+c)*polylog(2,1-2/(-d*x-c+1))/f/(-c*f
+d*e+f)+3/2*a*b^2*d*polylog(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-3*a*b^2*d*poly
log(2,1-2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*b^3*d*arccoth(d*x+c)*po
lylog(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-3*b^3*d*arccoth(d*x+c)*polylog(2,1-2
/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3*a*b^2*d*polylog(2,1-2*d*(f*x+e)/(-
c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3*b^3*d*arccoth(d*x+c)*pol
ylog(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)-3/4
*b^3*d*polylog(3,1-2/(-d*x-c+1))/f/(-c*f+d*e+f)+3/4*b^3*d*polylog(3,1-2/(d
*x+c+1))/f/(-c*f+d*e-f)-3/2*b^3*d*polylog(3,1-2/(d*x+c+1))/(-c*f+d*e-f)/(-
c*f+d*e+f)+3/2*b^3*d*polylog(3,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-...
```

3.118.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.65 (sec) , antiderivative size = 1945, normalized size of antiderivative = 1.79

$$\int \frac{\left(a + b \coth^{-1}(c + dx)\right)^3}{(e + fx)^2} \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCoth[c + d*x])^3/(e + f*x)^2,x]
```

output
```
-(a^3/(f*(e + f*x))) - (3*a^2*b*ArcCoth[c + d*x])/(f*(e + f*x)) + (3*a^2*b
*d*Log[1 - c - d*x])/(2*f*(-(d*e) - f + c*f)) - (3*a^2*b*d*Log[1 + c + d*x
])/(2*f*(-(d*e) + f + c*f)) - (3*a^2*b*d*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*
f - f^2 + c^2*f^2) + (3*a*b^2*(1 - (c + d*x)^2)*(f/Sqrt[1 - (c + d*x)^(-2)
] + (d*e - c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))^2*((E^ArcTanh[f/(-(d
*e) + c*f)]*ArcCoth[c + d*x]^2)/((-(d*e) + c*f)*Sqrt[1 - f^2/(d*e - c*f)^2
]) + ArcCoth[c + d*x]^2/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]*(f/Sqrt[1 - (c
 + d*x)^(-2)] + (d*e - c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))) + (f*(I
*Pi*ArcCoth[c + d*x] + 2*ArcCoth[c + d*x]*ArcTanh[f/(d*e - c*f)] - I*Pi*Lo
g[1 + E^(2*ArcCoth[c + d*x])] + 2*ArcCoth[c + d*x]*Log[1 - E^(-2*(ArcCoth[
c + d*x] + ArcTanh[f/(d*e - c*f)]))] - 2*ArcTanh[f/(-(d*e) + c*f)]*Log[1 -
 E^(-2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] + I*Pi*Log[1/Sqrt[1 -
 (c + d*x)^(-2)]] + 2*ArcTanh[f/(-(d*e) + c*f)]*Log[I*Sinh[ArcCoth[c + d*x
] + ArcTanh[f/(d*e - c*f)]]] - PolyLog[2, E^(-2*(ArcCoth[c + d*x] + ArcTan
h[f/(d*e - c*f)]))]))/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)))/(d*f*(e + f
*x)^2) - (b^3*(1 - (c + d*x)^2)*(f/Sqrt[1 - (c + d*x)^(-2)] + (d*e - c*f)/
((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))^2*((d*ArcCoth[c + d*x]^3)/(f*(c + d*
x)*Sqrt[1 - (c + d*x)^(-2)]*(-(f/Sqrt[1 - (c + d*x)^(-2)]) - (d*e)/((c + d
*x)*Sqrt[1 - (c + d*x)^(-2)]) + (c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])
)) - (d*(2*d*e*ArcCoth[c + d*x]^3 - 6*f*ArcCoth[c + d*x]^3 - 2*c*f*ArcC...
```

3.118.3 Rubi [A] (verified)

Time = 2.79 (sec) , antiderivative size = 1085, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6660, 7292, 6672, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{(e+fx)^2}dx$$

$$\Big\downarrow \text{6660}$$

$$\frac{3bd\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{(e+fx)(1-(c+dx)^2)}dx}{f}-\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)}$$

$$\Big\downarrow \text{7292}$$

$$\frac{3bd\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{(e+fx)(-c^2-2dxc-d^2x^2+1)}dx}{f}-\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)}$$

$$\Big\downarrow \text{6672}$$

$$\frac{3b\int \frac{d\left(a+b\coth^{-1}(c+dx)\right)^2}{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)(1-(c+dx)^2)}d(c+dx)}{f}-\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3bd\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{(de-cf+f(c+dx))(1-(c+dx)^2)}d(c+dx)}{f}-\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)}$$

$$\Big\downarrow \text{7276}$$

$$\frac{3bd\int \left(-\frac{a^2}{(c+dx-1)(c+dx+1)(de-cf+f(c+dx))}-\frac{2b\coth^{-1}(c+dx)a}{(c+dx-1)(c+dx+1)(de-cf+f(c+dx))}-\frac{b^2\coth^{-1}(c+dx)^2}{(c+dx-1)(c+dx+1)(de-cf+f(c+dx))}\right)d(c\cdot}{f}$$

$$\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)}$$

$$\Big\downarrow \text{2009}$$

$$3bd\left(-\frac{\log(-c-dx+1)a^2}{2(de-cf+f)}+\frac{\log(c+dx+1)a^2}{2(de-(c+1)f)}-\frac{f\log(de-cf+f(c+dx))a^2}{(de-cf+f)(de-(c+1)f)}+\frac{b\coth^{-1}(c+dx)\log\left(\frac{2}{-c-dx+1}\right)a}{de-cf+f}-\frac{b\coth^{-1}(c+dx)\log\left(\frac{2}{c+dx+\cdot}\right.}{de-cf-f}\right.$$

$$\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)}$$

input
```
Int[(a + b*ArcCoth[c + d*x])^3/(e + f*x)^2,x]
```

3.118. $\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$

output
```
-((a + b*ArcCoth[c + d*x])^3/(f*(e + f*x))) + (3*b*d*((a*b*ArcCoth[c + d*x
]*Log[2/(1 - c - d*x)])/(d*e + f - c*f) + (b^2*ArcCoth[c + d*x]^2*Log[2/(1
 - c - d*x)])/(2*(d*e + f - c*f)) - (a^2*Log[1 - c - d*x])/(2*(d*e + f - c
*f)) - (a*b*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/(d*e - f - c*f) + (2*a*
b*f*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c)
*f)) - (b^2*ArcCoth[c + d*x]^2*Log[2/(1 + c + d*x)])/(2*(d*e - f - c*f)) +
 (b^2*f*ArcCoth[c + d*x]^2*Log[2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e - (
1 + c)*f)) + (a^2*Log[1 + c + d*x])/(2*(d*e - (1 + c)*f)) - (a^2*f*Log[d*e
 - c*f + f*(c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (2*a*b*f*ArcC
oth[c + d*x]*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d
*x))])/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (b^2*f*ArcCoth[c + d*x]^2*Log
[(2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f
 - c*f)*(d*e - (1 + c)*f)) + (a*b*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x)
)])/(2*(d*e + f - c*f)) + (b^2*ArcCoth[c + d*x]*PolyLog[2, 1 - 2/(1 - c -
d*x)])/(2*(d*e + f - c*f)) + (a*b*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*(d*e
 - f - c*f)) - (a*b*f*PolyLog[2, 1 - 2/(1 + c + d*x)])/((d*e + f - c*f)*(d
*e - (1 + c)*f)) + (b^2*ArcCoth[c + d*x]*PolyLog[2, 1 - 2/(1 + c + d*x)])/
(2*(d*e - f - c*f)) - (b^2*f*ArcCoth[c + d*x]*PolyLog[2, 1 - 2/(1 + c + d*
x)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (a*b*f*PolyLog[2, 1 - (2*(d*e -
 c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f - c*f)...
```

3.118.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6660
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCoth[c + d*x])^p/(f*(m
+ 1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcCo
th[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.118. $\int \frac{(a+b\coth^{-1}(c+dx))^3}{(e+fx)^2}\,dx$

rule 6672
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Sub
st[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCoth[
x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x
] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.118.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 12.35 (sec) , antiderivative size = 4101, normalized size of antiderivative = 3.77

method	result	size
derivativedivides	Expression too large to display	4101
default	Expression too large to display	4101
parts	Expression too large to display	4252

input
```
int((a+b*arccoth(d*x+c))^3/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*d^2/(c*f-d*e-f*(d*x+c))/f+b^3*d^2*(1/(c*f-d*e-f*(d*x+c))/f*arccot
h(d*x+c)^3+3/f*(-1/(c*f-d*e-f)^2/(c*f-d*e+f)*f^2*c*arccoth(d*x+c)^2*ln(1-(
d*x+c+1)*(c*f-d*e-f)/(c*f-d*e+f)/(d*x+c-1))-1/(c*f-d*e-f)^2/(c*f-d*e+f)*f^
2*c*arccoth(d*x+c)*polylog(2,(d*x+c+1)*(c*f-d*e-f)/(c*f-d*e+f)/(d*x+c-1))-
1/2/(c*f-d*e-f)^2/(c*f-d*e+f)*f*e*d*polylog(3,(d*x+c+1)*(c*f-d*e-f)/(c*f-d
*e+f)/(d*x+c-1))+1/4*I/(c*f-d*e-f)/(c*f-d*e+f)*Pi*csgn(I*(d*x+c+1)/(d*x+c-
1))*csgn(I/((d*x+c+1)/(d*x+c-1)-1))*csgn(I/(d*x+c-1)*(d*x+c+1)/((d*x+c+1)/
(d*x+c-1)-1))*d*e*arccoth(d*x+c)^2-1/4*I/(c*f-d*e-f)/(c*f-d*e+f)*Pi*csgn(I
*(d*x+c+1)/(d*x+c-1))*csgn(I/((d*x+c+1)/(d*x+c-1)-1))*csgn(I/(d*x+c-1)*(d*
x+c+1)/((d*x+c+1)/(d*x+c-1)-1))*c*f*arccoth(d*x+c)^2-1/2*arccoth(d*x+c)^2/
(c*f-d*e+f)*ln((d*x+c-1)/(d*x+c+1))+1/4*I/(c*f-d*e-f)/(c*f-d*e+f)*Pi*csgn(
I*(d*x+c+1)/(d*x+c-1))^3*f*arccoth(d*x+c)^2+1/4*I/(c*f-d*e-f)/(c*f-d*e+f)*
Pi*csgn(I/(d*x+c-1)*(d*x+c+1)/((d*x+c+1)/(d*x+c-1)-1))^3*f*arccoth(d*x+c)^
2-1/2*I/(c*f-d*e-f)/(c*f-d*e+f)*Pi*csgn(I/((d*x+c-1)/(d*x+c+1))^(1/2))*csg
n(I*(d*x+c+1)/(d*x+c-1))^2*f*arccoth(d*x+c)^2-1/4*I/(c*f-d*e-f)/(c*f-d*e+f
)*Pi*csgn(I*(d*x+c+1)/(d*x+c-1))^3*c*f*arccoth(d*x+c)^2+1/(c*f-d*e-f)^2/(c
*f-d*e+f)*f*e*d*arccoth(d*x+c)^2*ln(1-(d*x+c+1)*(c*f-d*e-f)/(c*f-d*e+f)/(d
*x+c-1))+1/(c*f-d*e-f)^2/(c*f-d*e+f)*f*e*d*arccoth(d*x+c)*polylog(2,(d*x+c
+1)*(c*f-d*e-f)/(c*f-d*e+f)/(d*x+c-1))-1/2*I/(c*f-d*e-f)/(c*f-d*e+f)*csgn(
I*(f*c*((d*x+c+1)/(d*x+c-1)-1)+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/...
```

3.118.5 Fricas [F]

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx=\int \frac{\left(b\,\mathrm{arcoth}\,(dx+c)+a\right)^3}{\left(fx+e\right)^2}\,dx$$

input
```
integrate((a+b*arccoth(d*x+c))^3/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b^3*arccoth(d*x + c)^3 + 3*a*b^2*arccoth(d*x + c)^2 + 3*a^2*b*ar
ccoth(d*x + c) + a^3)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

3.118.6 Sympy [F]

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx=\int \frac{\left(a+b\,\text{acoth}\,(c+dx)\right)^3}{\left(e+fx\right)^2}\,dx$$

input
```
integrate((a+b*acoth(d*x+c))**3/(f*x+e)**2,x)
```

output
```
Integral((a + b*acoth(c + d*x))**3/(e + f*x)**2, x)
```

3.118.7 Maxima [F]

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx=\int \frac{\left(b\,\text{arcoth}\,(dx+c)+a\right)^3}{\left(fx+e\right)^2}\,dx$$

input
```
integrate((a+b*arccoth(d*x+c))^3/(f*x+e)^2,x, algorithm="maxima")
```

output
```
3/2*(d*(log(d*x + c + 1)/(d*e*f - (c + 1)*f^2) - log(d*x + c - 1)/(d*e*f -
 (c - 1)*f^2) - 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 - 1)*f^2)) - 2*
arccoth(d*x + c)/(f^2*x + e*f))*a^2*b - a^3/(f^2*x + e*f) + 1/8*(((d^2*e*f
 - c*d*f^2 + d*f^2)*b^3*x + (c*d*e*f - c^2*f^2 + d*e*f + f^2)*b^3)*log(d*x
 + c + 1)^3 - 3*(2*(d^2*e^2 - 2*c*d*e*f + c^2*f^2 - f^2)*a*b^2 + ((d^2*e*f
 - c*d*f^2 - d*f^2)*b^3*x + (c*d*e*f - c^2*f^2 - d*e*f + f^2)*b^3)*log(d*x
 + c - 1))*log(d*x + c + 1)^2)/(d^2*e^3*f - 2*c*d*e^2*f^2 + c^2*e*f^3 - e*
f^3 + (d^2*e^2*f^2 - 2*c*d*e*f^3 + c^2*f^4 - f^4)*x) + integrate(-1/8*(((d
^2*e*f - c*d*f^2 + d*f^2)*b^3*x + (c*d*e*f - c^2*f^2 + d*e*f + f^2)*b^3)*l
og(d*x + c - 1)^3 - 6*((d^2*e*f - c*d*f^2 + d*f^2)*a*b^2*x + (c*d*e*f - c^
2*f^2 + d*e*f + f^2)*a*b^2)*log(d*x + c - 1)^2 - 3*(4*(d^2*e*f - c*d*f^2 +
 d*f^2)*a*b^2*x + 4*(d^2*e^2 - c*d*e*f + d*e*f)*a*b^2 + ((d^2*e*f - c*d*f^
2 + d*f^2)*b^3*x + (c*d*e*f - c^2*f^2 + d*e*f + f^2)*b^3)*log(d*x + c - 1)
^2 + 2*(b^3*d^2*f^2*x^2 - 2*(c*d*e*f - c^2*f^2 + d*e*f + f^2)*a*b^2 + (c*d
*e*f - d*e*f)*b^3 - (2*(d^2*e*f - c*d*f^2 + d*f^2)*a*b^2 - (d^2*e*f + c*d*
f^2 - d*f^2)*b^3)*x)*log(d*x + c - 1))*log(d*x + c + 1))/(c*d*e^3*f - c^2*
e^2*f^2 + d*e^3*f + e^2*f^2 + (d^2*e*f^3 - c*d*f^4 + d*f^4)*x^3 + (2*d^2*e
^2*f^2 - c*d*e*f^3 - c^2*f^4 + 3*d*e*f^3 + f^4)*x^2 + (d^2*e^3*f + c*d*e^2
*f^2 - 2*c^2*e*f^3 + 3*d*e^2*f^2 + 2*e*f^3)*x), x)
```

3.118.8 Giac [F]

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx=\int \frac{\left(b\,\mathrm{arcoth}\left(dx+c\right)+a\right)^3}{\left(fx+e\right)^2}\,dx$$

input
```
integrate((a+b*arccoth(d*x+c))^3/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*arccoth(d*x + c) + a)^3/(f*x + e)^2, x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx=\int \frac{\left(a+b\,\mathrm{acoth}(c+d\,x)\right)^3}{\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b*acoth(c + d*x))^3/(e + f*x)^2,x)
```

output
```
int((a + b*acoth(c + d*x))^3/(e + f*x)^2, x)
```

3.119 $\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)\,dx$

3.119.1 Optimal result

Integrand size = 18, antiderivative size = 162

$$\begin{aligned}\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)\,dx &= \frac{(e+fx)^{1+m}\left(a+b\coth^{-1}(c+dx)\right)}{f(1+m)} \\ &\quad+\frac{bd(e+fx)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de-f-cf}\right)}{2f(de-(1+c)f)(1+m)(2+m)} \\ &\quad-\frac{bd(e+fx)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de+f-cf}\right)}{2f(de+f-cf)(1+m)(2+m)}\end{aligned}$$

output
```
(f*x+e)^(1+m)*(a+b*arccoth(d*x+c))/f/(1+m)+1/2*b*d*(f*x+e)^(2+m)*hypergeom
([1, 2+m],[3+m],d*(f*x+e)/(-c*f+d*e-f))/f/(d*e-(1+c)*f)/(1+m)/(2+m)-1/2*b*
d*(f*x+e)^(2+m)*hypergeom([1, 2+m],[3+m],d*(f*x+e)/(-c*f+d*e+f))/f/(-c*f+d
*e+f)/(1+m)/(2+m)
```

3.119.2 Mathematica [F]

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right) dx = \int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right) dx$$

input
```
Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x]),x]
```

output
```
Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x]), x]
```

3.119.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.37, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6662, 6479, 485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right) dx$$

$$\Big\downarrow \text{6662}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m \left(a+b\coth^{-1}(c+dx)\right) d(c+dx)}{d}$$

$$\Big\downarrow \text{6479}$$

$$\frac{\frac{d\left(a+b\coth^{-1}(c+dx)\right)\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+1}}{f(m+1)}-\frac{bd\int \frac{\left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^{m+1}}{1-(c+dx)^2}d(c+dx)}{f(m+1)}}{d}$$

$$\Big\downarrow \text{485}$$

$$\frac{\frac{d\left(a+b\coth^{-1}(c+dx)\right)\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+1}}{f(m+1)}-\frac{bd\int \left(\frac{\left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^{m+1}}{2(-c-dx+1)}+\frac{\left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^{m+1}}{2(c+dx+1)}\right)d(c+dx)}{f(m+1)}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{d\left(a+b\coth^{-1}(c+dx)\right)\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+1}}{f(m+1)}-\frac{bd\left(\frac{d\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{de-cf+f(c+dx)}{de-cf+f}\right)}{2(m+2)(-cf+de+f)}-\frac{d\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right.}{}\right.}{f(m+1)}}{d}$$

input
```
Int[(e + f*x)^m*(a + b*ArcCoth[c + d*x]),x]
```

output
```
((d*(e - (c*f)/d + (f*(c + d*x))/d)^(1 + m)*(a + b*ArcCoth[c + d*x]))/(f*(
1 + m)) - (b*d*(-1/2*(d*(e - (c*f)/d + (f*(c + d*x))/d)^(2 + m)*Hypergeome
tric2F1[1, 2 + m, 3 + m, (d*e - c*f + f*(c + d*x))/(d*e - f - c*f)])/((d*e
 - (1 + c)*f)*(2 + m)) + (d*(e - (c*f)/d + (f*(c + d*x))/d)^(2 + m)*Hyperg
eometric2F1[1, 2 + m, 3 + m, (d*e - c*f + f*(c + d*x))/(d*e + f - c*f)])/(
2*(d*e + f - c*f)*(2 + m))))/(f*(1 + m)))/d
```

3.119.3.1 Defintions of rubi rules used

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6479
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.119.4 Maple [F]

$$\int (fx+e)^m (a+b\ \mathrm{arccoth}(dx+c))\, dx$$

input
```
int((f*x+e)^m*(a+b*arccoth(d*x+c)),x)
```

output
```
int((f*x+e)^m*(a+b*arccoth(d*x+c)),x)
```

3.119.5 Fricas [F]

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)\, dx = \int (b\,\mathrm{arcoth}\,(dx+c)+a)(fx+e)^m\, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccoth(d*x+c)),x, algorithm="fricas")
```

output
```
integral((b*arccoth(d*x + c) + a)*(f*x + e)^m, x)
```

3.119.6 Sympy [F]

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)\, dx = \int (a+b\,\mathrm{acoth}\,(c+dx))\,(e+fx)^m\, dx$$

input
```
integrate((f*x+e)**m*(a+b*acoth(d*x+c)),x)
```

output
```
Integral((a + b*acoth(c + d*x))*(e + f*x)**m, x)
```

3.119.7 Maxima [F]

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)\, dx = \int (b\,\mathrm{arcoth}\,(dx+c)+a)(fx+e)^m\, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccoth(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*b*((f*x + e)*(f*x + e)^m*log(d*x + c + 1)/(f*(m + 1)) - integrate((d*f
*x + d*e + (d*f*(m + 1)*x + c*f*(m + 1) + f*(m + 1))*log(d*x + c - 1))*(f*
x + e)^m/(d*f*(m + 1)*x + c*f*(m + 1) + f*(m + 1)), x)) + (f*x + e)^(m + 1
)*a/(f*(m + 1))
```

3.119.8 Giac [F]

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)\, dx = \int (b\,\mathrm{arcoth}\,(dx+c)+a)(fx+e)^m\, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccoth(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*arccoth(d*x + c) + a)*(f*x + e)^m, x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)\, dx = \int (e+f\,x)^m\,\left(a+b\,\mathrm{acoth}(c+d\,x)\right)\, dx$$

input
```
int((e + f*x)^m*(a + b*acoth(c + d*x)),x)
```

output
```
int((e + f*x)^m*(a + b*acoth(c + d*x)), x)
```

3.120 $\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2\,dx$

3.120.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2\,dx = \text{Int}\left((e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2,x\right)$$

output
```
Unintegrable((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x)
```

3.120.2 Mathematica [N/A]

Not integrable

Time = 2.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2\,dx = \int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2\,dx$$

input
```
Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^2,x]
```

output
```
Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^2, x]
```

3.120.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6662, 6652}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2dx$$

$$\Big\downarrow \text{6662}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m \left(a+b\coth^{-1}(c+dx)\right)^2d(c+dx)}{d}$$

$$\Big\downarrow \text{6652}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m \left(a+b\coth^{-1}(c+dx)\right)^2d(c+dx)}{d}$$

input
```
Int[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^2,x]
```

output
```
$Aborted
```

3.120.3.1 Defintions of rubi rules used

rule 6652
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCoth[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] ||
 MatchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f
_.)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || Match
Q[u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*
x)^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.120.4 Maple [N/A] (verified)

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (fx+e)^m (a+b \operatorname{arccoth}(dx+c))^2 \, dx$$

input
```
int((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x)
```

output
```
int((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x)
```

3.120.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx = \int (b \operatorname{arcoth}(dx+c)+a)^2 (fx+e)^m \, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccoth(d*x + c)^2 + 2*a*b*arccoth(d*x + c) + a^2)*(f*x + e)
^m, x)
```

3.120.6 Sympy [N/A]

Not integrable

Time = 85.90 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx = \int (a+b \operatorname{acoth}(c+dx))^2 (e+fx)^m \, dx$$

input
```
integrate((f*x+e)**m*(a+b*acoth(d*x+c))**2,x)
```

output
```
Integral((a + b*acoth(c + d*x))**2*(e + f*x)**m, x)
```

3.120.7 Maxima [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 253, normalized size of antiderivative = 12.65

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx = \int (b\,\mathrm{arcoth}\,(dx+c)+a)^2(fx+e)^m \, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*(b^2*f*x + b^2*e)*(f*x + e)^m*log(d*x + c + 1)^2/(f*(m + 1)) + (f*x +
e)^(m + 1)*a^2/(f*(m + 1)) - integrate(-1/4*((b^2*d*f*(m + 1)*x + (c*f*(m
+ 1) + f*(m + 1))*b^2)*log(d*x + c - 1)^2 - 2*(b^2*d*e - 2*(c*f*(m + 1) +
f*(m + 1))*a*b - (2*a*b*d*f*(m + 1) - b^2*d*f)*x + (b^2*d*f*(m + 1)*x + (c
*f*(m + 1) + f*(m + 1))*b^2)*log(d*x + c - 1))*log(d*x + c + 1) - 4*(a*b*d
*f*(m + 1)*x + (c*f*(m + 1) + f*(m + 1))*a*b)*log(d*x + c - 1))*(f*x + e)^
m/(d*f*(m + 1)*x + c*f*(m + 1) + f*(m + 1)), x)
```

3.120.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx = \int (b\,\mathrm{arcoth}\,(dx+c)+a)^2(fx+e)^m \, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*arccoth(d*x + c) + a)^2*(f*x + e)^m, x)
```

3.120.9 Mupad [N/A]

Not integrable

Time = 3.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx = \int (e+f\,x)^m\,{\left(a+b\,\mathrm{acoth}(c+d\,x)\right)}^2 \, dx$$

input
```
int((e + f*x)^m*(a + b*acoth(c + d*x))^2,x)
```

output
```
int((e + f*x)^m*(a + b*acoth(c + d*x))^2, x)
```

3.120. $\int (e + fx)^m \left(a + b\coth^{-1}(c + dx)\right)^2 \, dx$

3.121 $\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx$

3.121.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx = \text{Int}\left((e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3, x\right)$$

output
```
Unintegrable((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x)
```

3.121.2 Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx = \int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx$$

input
```
Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^3,x]
```

output
```
Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^3, x]
```

3.121.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6662, 6652}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{6662}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m \left(a+b\coth^{-1}(c+dx)\right)^3 d(c+dx)}{d}$$

$$\Big\downarrow \text{6652}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m \left(a+b\coth^{-1}(c+dx)\right)^3 d(c+dx)}{d}$$

input
```
Int[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^3,x]
```

output
```
$Aborted
```

3.121.3.1 Defintions of rubi rules used

rule 6652
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCoth[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] ||
 MatchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f
_.)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || Match
Q[u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*
x)^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.121.4 Maple [N/A] (verified)

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (fx+e)^m (a+b\operatorname{arccoth}(dx+c))^3\,dx$$

input
```
int((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x)
```

output
```
int((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x)
```

3.121.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3\,dx = \int (b\operatorname{arccoth}(dx+c)+a)^3(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arccoth(d*x + c)^3 + 3*a*b^2*arccoth(d*x + c)^2 + 3*a^2*b*ar
ccoth(d*x + c) + a^3)*(f*x + e)^m, x)
```

3.121.6 Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**m*(a+b*acoth(d*x+c))**3,x)
```

output
```
Timed out
```

3.121.7 Maxima [N/A]

Not integrable

Time = 4.13 (sec) , antiderivative size = 418, normalized size of antiderivative = 20.90

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx = \int (b \operatorname{arcoth}(dx+c)+a)^3(fx+e)^m \, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/8*(b^3*f*x + b^3*e)*(f*x + e)^m*log(d*x + c + 1)^3/(f*(m + 1)) + (f*x +
e)^(m + 1)*a^3/(f*(m + 1)) - integrate(1/8*((b^3*d*f*(m + 1)*x + (c*f*(m +
 1) + f*(m + 1))*b^3)*log(d*x + c - 1)^3 + 3*(b^3*d*e - 2*(c*f*(m + 1) + f
*(m + 1))*a*b^2 - (2*a*b^2*d*f*(m + 1) - b^3*d*f)*x + (b^3*d*f*(m + 1)*x +
 (c*f*(m + 1) + f*(m + 1))*b^3)*log(d*x + c - 1))*log(d*x + c + 1)^2 - 6*(
a*b^2*d*f*(m + 1)*x + (c*f*(m + 1) + f*(m + 1))*a*b^2)*log(d*x + c - 1)^2
- 3*(4*a^2*b*d*f*(m + 1)*x + 4*(c*f*(m + 1) + f*(m + 1))*a^2*b + (b^3*d*f*
(m + 1)*x + (c*f*(m + 1) + f*(m + 1))*b^3)*log(d*x + c - 1)^2 - 4*(a*b^2*d
*f*(m + 1)*x + (c*f*(m + 1) + f*(m + 1))*a*b^2)*log(d*x + c - 1))*log(d*x
+ c + 1) + 12*(a^2*b*d*f*(m + 1)*x + (c*f*(m + 1) + f*(m + 1))*a^2*b)*log(
d*x + c - 1))*(f*x + e)^m/(d*f*(m + 1)*x + c*f*(m + 1) + f*(m + 1)), x)
```

3.121.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx = \int (b \operatorname{arcoth}(dx+c)+a)^3(fx+e)^m \, dx$$

input
```
integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*arccoth(d*x + c) + a)^3*(f*x + e)^m, x)
```

3.121.9 Mupad [N/A]

Not integrable

Time = 3.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx = \int (e+f\,x)^m\,\left(a+b\,\mathrm{acoth}(c+d\,x)\right)^3 \,dx$$

input
```
int((e + f*x)^m*(a + b*acoth(c + d*x))^3,x)
```

output
```
int((e + f*x)^m*(a + b*acoth(c + d*x))^3, x)
```

3.122 $\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

3.122.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\operatorname{Int}\left(\frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2},x\right)$$

output
```
Unintegrable((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

3.122.2 Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

3.122.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2}dx$$
$$\Big\downarrow \; 7234$$
$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2}dx$$

input
```
Int[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
$Aborted
```

3.122.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] && !IGt
Q[n, 0]
```

3.122.4 Maple [N/A] (verified)

Not integrable

Time = 0.65 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\,\mathrm{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{-c^2x^2+1}dx$$

input
```
int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

output
```
int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

3.122.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, al
gorithm="fricas")
```

output
```
integral(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

3.122.6 Sympy [N/A]

Not integrable

Time = 8.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\int \frac{\left(a+b\operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

output
```
-Integral((a + b*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1)))**n/(c**2*x**2 - 1),
x)
```

3.122.7 Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \int -\frac{\left(b\,\text{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, al
gorithm="maxima")
```

output
```
-integrate((b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1),
x)
```

3.122.8 Giac [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \int -\frac{\left(b\,\text{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, al
gorithm="giac")
```

output
```
integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1),
x)
```

3.122.9 Mupad [N/A]

Not integrable

Time = 4.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = -\int \frac{\left(a+b\,\text{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

output
```
-int((a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.122. $\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

3.123 $\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

3.123.1 Optimal result

Integrand size = 40, antiderivative size = 460

$$\begin{aligned}
&\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx \\
&= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} \\
&\quad-\frac{3b\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&\quad+\frac{3b\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} \\
&\quad-\frac{3b^2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&\quad+\frac{3b^2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(3,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} \\
&\quad-\frac{3b^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{4c}+\frac{3b^3\operatorname{PolyLog}\left(4,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{4c}
\end{aligned}$$

output
```
-2*arccoth(1-2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))*(a+b*arccoth((-c*x+1)^(1/
2)/(c*x+1)^(1/2)))^3/c-3/2*b*(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2
*polylog(2,1-2/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))/c+3/2*b*(a+b*arccoth((-c*
x+1)^(1/2)/(c*x+1)^(1/2)))^2*polylog(2,1-2*(-c*x+1)^(1/2)/((-c*x+1)^(1/2)/
(c*x+1)^(1/2)+1)/(c*x+1)^(1/2))/c-3/2*b^2*(a+b*arccoth((-c*x+1)^(1/2)/(c*x
+1)^(1/2)))*polylog(3,1-2/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))/c+3/2*b^2*(a+b
*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,1-2*(-c*x+1)^(1/2)/((-c*
x+1)^(1/2)/(c*x+1)^(1/2)+1)/(c*x+1)^(1/2))/c-3/4*b^3*polylog(4,1-2/((-c*x+
1)^(1/2)/(c*x+1)^(1/2)+1))/c+3/4*b^3*polylog(4,1-2*(-c*x+1)^(1/2)/((-c*x+1
)^(1/2)/(c*x+1)^(1/2)+1)/(c*x+1)^(1/2))/c
```

3.123.2 Mathematica [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

3.123.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7232, 6449, 6615, 6619, 6623, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{1-c^2x^2}\,dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{6449}$$

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\int\frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$\Big\downarrow$ 6615

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\left(\frac{1}{2}\int\frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2\log\left(\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1\right)}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}-\right.}{c}$$

$\Big\downarrow$ 6619

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\left(\frac{1}{2}\left(b\int\frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-c}}{\sqrt{cx+}}\right.\right.}{}$$

$\Big\downarrow$ 6623

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\left(\frac{1}{2}\left(b\left(\frac{1}{2}b\int\frac{\text{PolyLog}\left(3,1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}-\frac{1}{2}\text{PolyLog}\right.\right.\right.}{}$$

$\Big\downarrow$ 7164

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\left(\frac{1}{2}\left(b\left(-\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.\right.\right.\right.}{}$$

input
```
Int[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

3.123. $\int\frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}dx$

output
```
-((2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcCoth[1 - 2/(1 - Sqr
t[1 - c*x]/Sqrt[1 + c*x])] - 6*b*((-1/2*((a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt
[1 + c*x]])^2*PolyLog[2, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])]) + b*(-1
/2*((a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1 - 2/(1 + Sqr
t[1 - c*x]/Sqrt[1 + c*x])]) - (b*PolyLog[4, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[
1 + c*x])])/4))/2 + (((a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyL
og[2, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x
]))])/2 - b*(-1/2*((a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3,
 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])
 - (b*PolyLog[4, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/S
qrt[1 + c*x]))])/4))/2))/c)
```

3.123.3.1 Defintions of rubi rules used

rule 6449
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcCoth[c*x])^p*ArcCoth[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcCoth[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6615
```
Int[(ArcCoth[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a
+ b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] - Simp[1/2   Int[Log[SimplifyInteg
rand[1 - 1/u, x]]*((a + b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1
- c*x))^2, 0]
```

rule 6619
```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6623
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*(p/2)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

3.123. $\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.123.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1680 vs. $2(388) = 776$.

Time = 1.38 (sec) , antiderivative size = 1681, normalized size of antiderivative = 3.65

method	result	size
default	Expression too large to display	1681
parts	Expression too large to display	1681

input
```
int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_R
ETURNVERBOSE)
```

output
```
-1/2*a^3/c*ln(c*x-1)+1/2*a^3/c*ln(c*x+1)-b^3*(-1/c*arccoth((-c*x+1)^(1/2)/
(c*x+1)^(1/2))^3*ln(1+1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/
(c*x+1)^(1/2)+1))^(1/2))-3/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polyl
og(2,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)
)^(1/2))+6/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-1/(((-c*x+1)
^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))-6/c*polyl
og(4,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)
)^(1/2))+1/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+1/((-c*x+1)^(1/2
)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))+3/2/c*arccoth((-c*x+1
)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-
c*x+1)^(1/2)/(c*x+1)^(1/2)+1))-3/2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))
*polylog(3,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/
2)+1))+3/4/c*polylog(4,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)
/(c*x+1)^(1/2)+1))-1/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1-1/(((-
c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))-3/c
*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,1/(((-c*x+1)^(1/2)/(c*x
+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+6/c*arccoth((-c*x+1)
^(1/2)/(c*x+1)^(1/2))*polylog(3,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x
+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))-6/c*polylog(4,1/(((-c*x+1)^(1/2)/(c*x+1
)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2)))-3*a*b^2*(-1/c*arcc...
```

3.123.5 Fricas [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, al
gorithm="fricas")
```

output
```
integral(-(b^3*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arccoth(s
qrt(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arccoth(sqrt(-c*x + 1)/sqrt(c*x +
 1)) + a^3)/(c^2*x^2 - 1), x)
```

3.123.6 Sympy [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\int \frac{a^3}{c^2x^2-1}\,dx - \int \frac{b^3\operatorname{acoth}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3ab^2\operatorname{acoth}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3a^2b\operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

output
```
-Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*acoth(sqrt(-c*x + 1)/sq
rt(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*acoth(sqrt(-c*x +
1)/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*acoth(sqrt(-c
*x + 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

3.123.7 Maxima [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(b\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, al
gorithm="maxima")
```

output
```
1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) - 1/16*(b^3*log(c*x + 1) - b^3*l
og(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1))^3/c - integrate(1/32*(4
*(sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x + 1) + sqrt(-c*x +
1))^3 + 24*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*log(sqrt(c*x + 1)
+ sqrt(-c*x + 1))^2 + 3*(4*(sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sq
rt(c*x + 1) + sqrt(-c*x + 1)) + (8*a*b^2 - (b^3*c*x - b^3)*log(c*x + 1) +
(b^3*c*x - b^3)*log(-c*x + 1))*sqrt(c*x + 1) - (8*a*b^2 - (b^3*c*x + b^3)*
log(c*x + 1) + (b^3*c*x + b^3)*log(-c*x + 1))*sqrt(-c*x + 1))*log(-sqrt(c*
x + 1) + sqrt(-c*x + 1))^2 + 48*(sqrt(c*x + 1)*a^2*b - sqrt(-c*x + 1)*a^2*
b)*log(sqrt(c*x + 1) + sqrt(-c*x + 1)) - 12*(4*sqrt(c*x + 1)*a^2*b - 4*sqr
t(-c*x + 1)*a^2*b + (sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x
+ 1) + sqrt(-c*x + 1))^2 + 4*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*
log(sqrt(c*x + 1) + sqrt(-c*x + 1)))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1)))
/((c^2*x^2 - 1)*sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)
```

3.123.8 Giac [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(b\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, al
gorithm="giac")
```

output
```
integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1),
x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(a+b\,\mathrm{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.124 $\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

3.124.1 Optimal result

Integrand size = 40, antiderivative size = 302

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{b\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{b\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} + \frac{b^2\,\text{PolyLog}\left(3,1-\frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(1+\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}$$

output
```
-2*arccoth(1-2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))*(a+b*arccoth((-c*x+1)^(1/
2)/(c*x+1)^(1/2)))^2/c-b*(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polyl
og(2,1-2/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))/c+b*(a+b*arccoth((-c*x+1)^(1/2)
/(c*x+1)^(1/2)))*polylog(2,1-2*(-c*x+1)^(1/2)/((-c*x+1)^(1/2)/(c*x+1)^(1/2
)+1)/(c*x+1)^(1/2))/c-1/2*b^2*polylog(3,1-2/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+
1))/c+1/2*b^2*polylog(3,1-2*(-c*x+1)^(1/2)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1
)/(c*x+1)^(1/2))/c
```

3.124.2 Mathematica [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]
```

3.124.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7232, 6449, 6615, 6619, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{1-c^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{6449}$$

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-4b\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{6615}$$

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-4b\left(\frac{1}{2}\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\log\left(\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1\right)}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}-\right.}{c}$$

$\downarrow$ 6619

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-4b\left(\frac{1}{2}\left(\frac{1}{2}b\int\frac{\text{PolyLog}\left(2,1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{1-\frac{1-cx}{cx+1}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}-\frac{1}{2}\text{PolyLog}\left(2\right.\right.\right.}{}$$

$\downarrow$ 7164

$$-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-4b\left(\frac{1}{2}\left(-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-}}{\sqrt{cx-}}\right.\right.\right.\right.}{}$$

input
```
Int[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
-((2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcCoth[1 - 2/(1 - Sqr
t[1 - c*x]/Sqrt[1 + c*x])] - 4*b*((-1/2*((a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt
[1 + c*x]])*PolyLog[2, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])]) - (b*Poly
Log[3, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/4)/2 + (((a + b*ArcCoth[S
qrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*
x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/2 + (b*PolyLog[3, 1 - (2*Sqrt[1 -
c*x])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/4)/2))/c)
```

3.124.3.1 Defintions of rubi rules used

rule 6449
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcCoth[c*x])^p*ArcCoth[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcCoth[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6615
```
Int[(ArcCoth[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a
+ b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] - Simp[1/2   Int[Log[SimplifyInteg
rand[1 - 1/u, x]]*((a + b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1
- c*x))^2, 0]
```

3.124. $\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

rule 6619
```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.124.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(258) = 516$.

Time = 0.90 (sec) , antiderivative size = 926, normalized size of antiderivative = 3.07

method	result
default	$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}-b^2\left(-\frac{\operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(1+\frac{1}{\sqrt{\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}}}\right)}{c}-\frac{2\operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\operatorname{polylog}\left(2,-\right.}{c}\right.$
parts	$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}-b^2\left(-\frac{\operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(1+\frac{1}{\sqrt{\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}}}\right)}{c}-\frac{2\operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\operatorname{polylog}\left(2,-\right.}{c}\right.$

input
```
int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_R
ETURNVERBOSE)
```

output
```
-1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)-b^2*(-1/c*arccoth((-c*x+1)^(1/2)/
(c*x+1)^(1/2))^2*ln(1+1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/
(c*x+1)^(1/2)+1))^(1/2))-2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog
(2,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^
(1/2))+2/c*polylog(3,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/
(c*x+1)^(1/2)+1))^(1/2))+1/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+
1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))+1/c*a
rccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-1/((-c*x+1)^(1/2)/(c*x+1)^
(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))-1/2/c*polylog(3,-1/((-c*x+1)^(1
/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))-1/c*arccoth((-c*x+1
)^(1/2)/(c*x+1)^(1/2))^2*ln(1-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1
)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))-2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))
*polylog(2,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/
2)+1))^(1/2))+2/c*polylog(3,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^
(1/2)/(c*x+1)^(1/2)+1))^(1/2)))-2*a*b*(-1/c*arccoth((-c*x+1)^(1/2)/(c*x+1)
^(1/2))*ln(1+1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(
1/2)+1))^(1/2))-1/c*polylog(2,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+
1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+1/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)
)*ln(1+1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)
)+1/2/c*polylog(2,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(...
```

3.124.5 Fricas [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, al
gorithm="fricas")
```

output
```
integral(-(b^2*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arccoth(sqr
t(-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

3.124.6 Sympy [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\int \frac{a^2}{c^2x^2-1}\,dx - \int \frac{b^2\operatorname{acoth}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{2ab\operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

output
```
-Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*acoth(sqrt(-c*x + 1)/sq
rt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*acoth(sqrt(-c*x + 1)/
sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

3.124.7 Maxima [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(b\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, al
gorithm="maxima")
```

output
```
1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/8*(b^2*log(c*x + 1) - b^2*lo
g(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1))^2/c + integrate(-1/8*(2*
(sqrt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1) + sqrt(-c*x + 1
))^2 + 8*(sqrt(c*x + 1)*a*b - sqrt(-c*x + 1)*a*b)*log(sqrt(c*x + 1) + sqrt
(-c*x + 1)) - (4*(sqrt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1
) + sqrt(-c*x + 1)) + (8*a*b - (b^2*c*x - b^2)*log(c*x + 1) + (b^2*c*x - b
^2)*log(-c*x + 1))*sqrt(c*x + 1) - (8*a*b - (b^2*c*x + b^2)*log(c*x + 1) +
 (b^2*c*x + b^2)*log(-c*x + 1))*sqrt(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(
-c*x + 1)))/((c^2*x^2 - 1)*sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)),
x)
```

3.124.8 Giac [F]

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(b\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, al
gorithm="giac")
```

output
```
integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1),
x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(a+b\,\mathrm{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.125 $\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

3.125.1 Optimal result

Integrand size = 38, antiderivative size = 89

$$\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} - \frac{b\,\text{PolyLog}\left(2,-\frac{\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c} + \frac{b\,\text{PolyLog}\left(2,\frac{\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}$$

output
```
-a*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))/c-1/2*b*polylog(2,-(c*x+1)^(1/2)/(-c*x
+1)^(1/2))/c+1/2*b*polylog(2,(c*x+1)^(1/2)/(-c*x+1)^(1/2))/c
```

3.125.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10

$$\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \frac{a\text{arctanh}(cx)}{c} + \frac{b\left(\text{arctanh}(cx)\left(2\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)+\log\left(1-e^{-\text{arctanh}(cx)}\right)-\log\left(1+e^{-\text{arctanh}(cx)}\right)\right)+\text{PolyLog}\left(2,-e^{-\cdots}\right.\right.}{2c}$$

input
```
Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

output
```
(a*ArcTanh[c*x])/c + (b*(ArcTanh[c*x]*(2*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*
x]] + Log[1 - E^(-ArcTanh[c*x])] - Log[1 + E^(-ArcTanh[c*x])]) + PolyLog[2
, -E^(-ArcTanh[c*x])] - PolyLog[2, E^(-ArcTanh[c*x])]))/(2*c)
```

3.125.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {7232, 6447}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{1-c^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{6447}$$

$$-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{c}$$

input
```
Int[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

output
```
-((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]] + (b*PolyLog[2, -(Sqrt[1 + c*x]/Sqrt
[1 - c*x])])/2 - (b*PolyLog[2, Sqrt[1 + c*x]/Sqrt[1 - c*x]])/2)/c)
```

3.125.3.1 Defintions of rubi rules used

rule 6447
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.125.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(73) = 146$.

Time = 0.67 (sec) , antiderivative size = 372, normalized size of antiderivative = 4.18

method	result
default	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}-b\left(-\frac{\operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1+\frac{1}{\sqrt{\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}}}\right)}{c}-\frac{\operatorname{polylog}\left(2,-\frac{1}{\sqrt{\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}}}\right)}{c}+\frac{\operatorname{arccoth}\left(\right.}{}\right.$
parts	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}-b\left(-\frac{\operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1+\frac{1}{\sqrt{\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}}}\right)}{c}-\frac{\operatorname{polylog}\left(2,-\frac{1}{\sqrt{\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}}}\right)}{c}+\frac{\operatorname{arccoth}\left(\right.}{}\right.$

input
```
int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
```

output
```
-1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)-b*(-1/c*arccoth((-c*x+1)^(1/2)/(c*x+1
)^(1/2))*ln(1+1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^
(1/2)+1))^(1/2))-1/c*polylog(2,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x
+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+1/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2
))*ln(1+1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1
))+1/2/c*polylog(2,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*
x+1)^(1/2)+1))-1/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-1/(((-c*x+1)
^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))-1/c*polyl
og(2,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))
^(1/2)))
```

3.125.5 Fricas [F]

$$\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algo
rithm="fricas")
```

output
```
integral(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

3.125.6 Sympy [F]

$$\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=-\int \frac{a}{c^2x^2-1}\,dx-\int \frac{b\,\mathrm{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

output
```
-Integral(a/(c**2*x**2 - 1), x) - Integral(b*acoth(sqrt(-c*x + 1)/sqrt(c*x
 + 1))/(c**2*x**2 - 1), x)
```

3.125.7 Maxima [F]

$$\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \int -\frac{b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algo
rithm="maxima")
```

output
```
1/4*b*(((log(c*x + 1) - log(-c*x + 1))*log(sqrt(c*x + 1) + sqrt(-c*x + 1))
 - (log(c*x + 1) - log(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1)))/c
- 2*integrate(-1/2*sqrt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2
- 1)*sqrt(c*x + 1) + (c^2*x^2 - 1)*sqrt(-c*x + 1)), x) - 2*integrate(1/2*s
qrt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)*sqrt(c*x + 1) -
 (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)) + 1/2*a*(log(c*x + 1)/c - log(c*x - 1)
/c)
```

3.125.8 Giac [F]

$$\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \int -\frac{b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algo
rithm="giac")
```

output
```
integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \int -\frac{a+b\,\mathrm{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

3.126 $\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

3.126.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)},x\right)$$

output
```
Unintegrable(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

3.126.2 Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x
]
```

output
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),
x]
```

3.126.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$

$$\Big\downarrow \text{7234}$$

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

output
```
$Aborted
```

3.126.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !IGt
Q[n, 0]
```

3.126.4 Maple [N/A] (verified)

Not integrable

Time = 0.57 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\,\text{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}dx$$

input
```
int(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

3.126.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, al
gorithm="fricas")
```

output
```
integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccoth(sqrt(-c*x + 1)/sqrt(c*x +
 1)) - a), x)
```

3.126.6 Sympy [N/A]

Not integrable

Time = 3.65 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$
$$= -\int \frac{1}{ac^2x^2-a+bc^2x^2\,\mathrm{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b\,\mathrm{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

output
```
-Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*acoth(sqrt(-c*x + 1)/sqrt(c*x +
 1)) - b*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

3.126.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, al
gorithm="maxima")
```

output
```
-integrate(1/((c^2*x^2 - 1)*(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a))
, x)
```

3.126.8 Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, al
gorithm="giac")
```

output
```
integrate(-1/((c^2*x^2 - 1)*(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a))
, x)
```

3.126.9 Mupad [N/A]

Not integrable

Time = 3.99 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=-\int \frac{1}{\left(a+b\,\mathrm{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,(c^2\,x^2-1)}\,dx$$

input
```
int(-1/((a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.126. $\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

3.127 $\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

3.127.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,
x)
```

3.127.2 Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
,x]
```

output
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
, x]
```

3.127.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}dx$$
$$\Big\downarrow \text{7234}$$
$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}dx$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

output
```
$Aborted
```

3.127.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !IGt
Q[n, 0]
```

3.127.4 Maple [N/A] (verified)

Not integrable

Time = 0.57 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\,\mathrm{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}dx$$

input
```
int(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

3.127.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.28

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x,
algorithm="fricas")
```

output
```
integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccoth(sqrt(-c*x + 1)/sqrt
(c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccoth(sqrt(-c*x + 1)/sqrt(c*x
 + 1))), x)
```

3.127.6 Sympy [N/A]

Not integrable

Time = 10.50 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.15

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\int \frac{1}{a^2c^2x^2-a^2+2abc^2x^2\,\mathrm{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-2ab\,\mathrm{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+b^2c^2x^2\,\mathrm{acoth}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b^2\,\mathrm{acoth}^2}$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,
x)
```

output
```
-Integral(1/(a**2*c**2*x**2 - a**2 + 2*a*b*c**2*x**2*acoth(sqrt(-c*x + 1)/
sqrt(c*x + 1)) - 2*a*b*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + b**2*c**2*x**
2*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1))**2 - b**2*acoth(sqrt(-c*x + 1)/sqrt(
c*x + 1))**2), x)
```

3.127.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 246, normalized size of antiderivative = 6.15

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\text{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x,
algorithm="maxima")
```

output
```
4*c*x/(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*c*log(sqrt(c*x + 1) + sqrt(-c*x +
1)) - sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*c*log(-sqrt(c*x + 1) + sqrt(-c*x +
1)) + 2*sqrt(c*x + 1)*sqrt(-c*x + 1)*a*b*c) - integrate(-4/((b^2*c^2*x^2 -
 b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(sqrt(c*x + 1) + sqrt(-c*x + 1)) - (
b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(-sqrt(c*x + 1) + sqrt(
-c*x + 1)) + 2*(a*b*c^2*x^2 - a*b)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)
```

3.127.8 Giac [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\,\text{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x,
algorithm="giac")
```

output
```
integrate(-1/((c^2*x^2 - 1)*(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^
2), x)
```

3.127.9 Mupad [N/A]

Not integrable

Time = 5.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\int \frac{1}{\left(a+b\,\mathrm{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,(c^2\,x^2-1)}\,dx$$

input
```
int(-1/((a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x
)
```

3.128 $\int x^m \coth^{-1}(\tanh(a+bx))\,dx$

3.128.1 Optimal result

Integrand size = 11, antiderivative size = 37

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}$$

output
```
-b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*arccoth(tanh(b*x+a))/(1+m)
```

3.128.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = x^m\left(\frac{bx^2}{2+m}+\frac{x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}{1+m}\right)$$

input
```
Integrate[x^m*ArcCoth[Tanh[a + b*x]],x]
```

output
```
x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/(1 + m))
```

3.128.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{b\int x^{m+1}dx}{m+1}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{(m+1)(m+2)}$$

input
```
Int[x^m*ArcCoth[Tanh[a + b*x]],x]
```

output
```
-((b*x^(2 + m))/((1 + m)*(2 + m))) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]])/(1
 + m)
```

3.128.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.128.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

method	result
parallelrisch	$-\frac{-x\,x^{m} \operatorname{arccoth}(\tanh(bx+a))m+b\,x^{2} x^{m}-2 \operatorname{arccoth}(\tanh(bx+a))x\,x^{m}}{(1+m)(2+m)}$
risch	$\frac{x\,x^{m} \ln(\mathrm{e}^{bx+a})}{1+m}-\frac{x\left(4bx-2i\pi \operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)^{2} m+i\pi \operatorname{csgn}(i\mathrm{e}^{bx+a})^{2} \operatorname{csgn}(i\mathrm{e}^{2bx+2a})m-4i\pi \operatorname{csgn}(i\mathrm{e}^{bx+a}) \operatorname{csgn}(i\mathrm{e}^{2bx+2a}\right.}{}$

input
```
int(x^m*arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-(-x*x^m*arccoth(tanh(b*x+a))*m+b*x^2*x^m-2*arccoth(tanh(b*x+a))*x*x^m)/(1
+m)/(2+m)
```

3.128.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.19

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = \frac{(i\,\pi(m+2)x+2\,(bm+b)x^2+2\,(am+2\,a)x)\cosh{(m\log{(x)})}+(i\,\pi(m+2)x+2\,(bm+b)x^2+2\,(am+}{2\,(m^2+3\,m+2)}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*((I*pi*(m + 2)*x + 2*(b*m + b)*x^2 + 2*(a*m + 2*a)*x)*cosh(m*log(x)) +
 (I*pi*(m + 2)*x + 2*(b*m + b)*x^2 + 2*(a*m + 2*a)*x)*sinh(m*log(x)))/(m^2
 + 3*m + 2)
```

3.128.6 Sympy [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = \begin{cases} b\log{(x)} - \frac{\operatorname{acoth}{(\tanh{(a+bx)})}}{x} & \text{for}\ m=-2 \\ \int \frac{\operatorname{acoth}{(\tanh{(a+bx)})}}{x}\,dx & \text{for}\ m=-1 \\ -\frac{bx^2x^m}{m^2+3m+2} + \frac{mxx^m\operatorname{acoth}{(\tanh{(a+bx)})}}{m^2+3m+2} + \frac{2xx^m\operatorname{acoth}{(\tanh{(a+bx)})}}{m^2+3m+2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*acoth(tanh(b*x+a)),x)
```

output
```
Piecewise((b*log(x) - acoth(tanh(a + b*x))/x, Eq(m, -2)), (Integral(acoth(
tanh(a + b*x))/x, x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x*x**
m*acoth(tanh(a + b*x))/(m**2 + 3*m + 2) + 2*x*x**m*acoth(tanh(a + b*x))/(m
**2 + 3*m + 2), True))
```

3.128.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^2x^m}{(m+2)(m+1)} + \frac{x^{m+1}\operatorname{arcoth}{(\tanh{(bx+a)})}}{m+1}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-b*x^2*x^m/((m + 2)*(m + 1)) + x^(m + 1)*arccoth(tanh(b*x + a))/(m + 1)
```

3.128.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = \frac{x^{m+1}\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{2\,(m+1)} - \frac{bx^{m+2}}{(m+2)(m+1)}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
1/2*x^(m + 1)*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((e^(
2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/(m + 1) - b*x^(m + 2)/((m +
2)*(m + 1))
```

3.128.9 Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.59

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx$$
$$= \frac{2\,b\,x^m\,x^2\,(m+1)}{2\,m^2+6\,m+4} - \frac{x\,x^m\,(m+2)\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2\,m^2+6\,m+4}$$

input
```
int(x^m*acoth(tanh(a + b*x)),x)
```

output
```
(2*b*x^m*x^2*(m + 1))/(6*m + 2*m^2 + 4) - (x*x^m*(m + 2)*(log(-2/(exp(2*a)
*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))
 + 2*b*x))/(6*m + 2*m^2 + 4)
```

3.129 $\int x^2 \coth^{-1}(\tanh(a+bx))\,dx$

3.129.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int x^2 \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(\tanh(a+bx))$$

output
```
-1/12*b*x^4+1/3*x^3*arccoth(tanh(b*x+a))
```

3.129.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x^2 \coth^{-1}(\tanh(a+bx))\,dx = -\frac{1}{12}x^3\left(bx - 4\coth^{-1}(\tanh(a+bx))\right)$$

input
```
Integrate[x^2*ArcCoth[Tanh[a + b*x]],x]
```

output
```
-1/12*(x^3*(b*x - 4*ArcCoth[Tanh[a + b*x]]))
```

3.129.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{3}x^3 \coth^{-1}(\tanh(a+bx)) - \frac{b\int x^3dx}{3}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{3}x^3 \coth^{-1}(\tanh(a+bx)) - \frac{bx^4}{12}$$

input
```
Int[x^2*ArcCoth[Tanh[a + b*x]],x]
```

output
```
-1/12*(b*x^4) + (x^3*ArcCoth[Tanh[a + b*x]])/3
```

3.129.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.129.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{b\,x^{4}}{12}+\frac{x^{3} \operatorname{arccoth}\left(\tanh \left(b x+a\right)\right)}{3}$
parallelrisch	$-\frac{b\,x^{4}}{12}+\frac{x^{3} \operatorname{arccoth}\left(\tanh \left(b x+a\right)\right)}{3}$
parts	$-\frac{b\,x^{4}}{12}+\frac{x^{3} \operatorname{arccoth}\left(\tanh \left(b x+a\right)\right)}{3}$
risch	$\frac{x^{3} \ln \left({\mathrm e}^{b x+a}\right)}{3}-\frac{b\,x^{4}}{12}-\frac{i \pi \,x^{3}}{6}-\frac{i \pi \,x^{3} \operatorname{csgn}\left(\frac{i}{{\mathrm e}^{2 b x+2 a}+1}\right) \operatorname{csgn}\left(i {\mathrm e}^{2 b x+2 a}\right) \operatorname{csgn}\left(\frac{i {\mathrm e}^{2 b x+2 a}}{{\mathrm e}^{2 b x+2 a}+1}\right)}{12}-\frac{i \pi \,x^{3} \operatorname{csgn}\left(\frac{i}{{\mathrm e}^{2 b x+2 a}+1}\right)^{3}}{6}$

input
```
int(x^2*arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*b*x^4+1/3*x^3*arccoth(tanh(b*x+a))
```

3.129.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \coth^{-1}(\tanh(a+bx))\,dx = \frac{1}{4}\,bx^4 + \frac{1}{6}i\,\pi x^3 + \frac{1}{3}\,ax^3$$

input
```
integrate(x^2*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*b*x^4 + 1/6*I*pi*x^3 + 1/3*a*x^3
```

3.129.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^4}{12} + \frac{x^3 \operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}{3}$$

input
```
integrate(x**2*acoth(tanh(b*x+a)),x)
```

output
```
-b*x**4/12 + x**3*acoth(tanh(a + b*x))/3
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \coth^{-1}(\tanh(a+bx))\,dx = -\frac{1}{12}\,bx^4 + \frac{1}{3}\,x^3\,\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)$$

input
```
integrate(x^2*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/12*b*x^4 + 1/3*x^3*arccoth(tanh(b*x + a))
```

3.129.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(19) = 38.

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int x^2 \coth^{-1}(\tanh(a+bx))\,dx = -\frac{1}{12}\,bx^4 + \frac{1}{6}\,x^3\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)$$

input
```
integrate(x^2*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
-1/12*b*x^4 + 1/6*x^3*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) +
1)/((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

3.129.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \coth^{-1}(\tanh(a+bx))\,dx = \frac{x^3\,\mathrm{acoth}(\tanh(a+b\,x))}{3} - \frac{b\,x^4}{12}$$

input
```
int(x^2*acoth(tanh(a + b*x)),x)
```

output
```
(x^3*acoth(tanh(a + b*x)))/3 - (b*x^4)/12
```

3.130 $\int x \coth^{-1}(\tanh(a+bx))\,dx$

3.130.1 Optimal result

Integrand size = 9, antiderivative size = 23

$$\int x \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^3}{6} + \frac{1}{2}x^2\coth^{-1}(\tanh(a+bx))$$

output
```
-1/6*b*x^3+1/2*x^2*arccoth(tanh(b*x+a))
```

3.130.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x \coth^{-1}(\tanh(a+bx))\,dx = -\frac{1}{6}x^2\left(bx - 3\coth^{-1}(\tanh(a+bx))\right)$$

input
```
Integrate[x*ArcCoth[Tanh[a + b*x]],x]
```

output
```
-1/6*(x^2*(b*x - 3*ArcCoth[Tanh[a + b*x]]))
```

3.130.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6794, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(\tanh(a+bx)) \, dx$$

$$\Big\downarrow \text{6794}$$

$$\frac{1}{2} b \int -x^2 dx + \frac{1}{2} x^2 \coth^{-1}(\tanh(a+bx))$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2} x^2 \coth^{-1}(\tanh(a+bx)) - \frac{bx^3}{6}$$

input
```
Int[x*ArcCoth[Tanh[a + b*x]],x]
```

output
```
-1/6*(b*x^3) + (x^2*ArcCoth[Tanh[a + b*x]])/2
```

3.130.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6794
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

3.130.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{b\,x^{3}}{6}+\frac{x^{2}\operatorname{arccoth}(\tanh(bx+a))}{2}$
parallelrisch	$-\frac{b\,x^{3}}{6}+\frac{x^{2}\operatorname{arccoth}(\tanh(bx+a))}{2}$
parts	$-\frac{b\,x^{3}}{6}+\frac{x^{2}\operatorname{arccoth}(\tanh(bx+a))}{2}$
risch	$\frac{x^{2}\ln(\mathrm{e}^{bx+a})}{2}-\frac{b\,x^{3}}{6}-\frac{i\pi\,x^{2}\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)}{8}-\frac{i\pi\,x^{2}\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)^{3}}{4}-\frac{i\pi\,x^{2}\,\mathrm{c}}{}$

input
```
int(x*arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*b*x^3+1/2*x^2*arccoth(tanh(b*x+a))
```

3.130.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x\coth^{-1}(\tanh(a+bx))\,dx=\frac{1}{3}\,bx^{3}+\frac{1}{4}i\,\pi x^{2}+\frac{1}{2}\,ax^{2}$$

input
```
integrate(x*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/3*b*x^3 + 1/4*I*pi*x^2 + 1/2*a*x^2
```

3.130.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x\coth^{-1}(\tanh(a+bx))\,dx=\begin{cases}\frac{x\operatorname{acoth}^{2}(\tanh(a+bx))}{2b}-\frac{\operatorname{acoth}^{3}(\tanh(a+bx))}{6b^{2}} & \text{for } b\neq 0\\ \frac{x^{2}\operatorname{acoth}(\tanh(a))}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*acoth(tanh(b*x+a)),x)
```

output
```
Piecewise((x*acoth(tanh(a + b*x))**2/(2*b) - acoth(tanh(a + b*x))**3/(6*b*
*2), Ne(b, 0)), (x**2*acoth(tanh(a))/2, True))
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x \coth^{-1}(\tanh(a+bx))\,dx = -\frac{1}{6}\,bx^3 + \frac{1}{2}\,x^2\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)$$

input
```
integrate(x*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/6*b*x^3 + 1/2*x^2*arccoth(tanh(b*x + a))
```

3.130.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(19) = 38.

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int x \coth^{-1}(\tanh(a+bx))\,dx = -\frac{1}{6}\,bx^3 + \frac{1}{4}\,x^2\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)$$

input
```
integrate(x*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
-1/6*b*x^3 + 1/4*x^2*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1
)/((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

3.130.9 Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x \coth^{-1}(\tanh(a+bx))\,dx = \frac{x^2\,\mathrm{acoth}(\tanh(a+b\,x))}{2} - \frac{b\,x^3}{6}$$

input
```
int(x*acoth(tanh(a + b*x)),x)
```

output
```
(x^2*acoth(tanh(a + b*x)))/2 - (b*x^3)/6
```

3.131 $\int \coth^{-1}(\tanh(a+bx))\,dx$

3.131.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \coth^{-1}(\tanh(a+bx))\,dx = \frac{\coth^{-1}(\tanh(a+bx))^2}{2b}$$

output
```
1/2*arccoth(tanh(b*x+a))^2/b
```

3.131.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^2}{2} + x\coth^{-1}(\tanh(a+bx))$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]],x]
```

output
```
-1/2*(b*x^2) + x*ArcCoth[Tanh[a + b*x]]
```

3.131.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \ 2588$$

$$\frac{\int \coth^{-1}(\tanh(a+bx))d\coth^{-1}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \ 15$$

$$\frac{\coth^{-1}(\tanh(a+bx))^2}{2b}$$

input
```
Int[ArcCoth[Tanh[a + b*x]],x]
```

output
```
ArcCoth[Tanh[a + b*x]]^2/(2*b)
```

3.131.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.131.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result
parallelrisch	$-\frac{b\,x^2}{2} + x \operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)$
derivativedivides	$\frac{\operatorname{arctanh}(\tanh(bx+a))\operatorname{arccoth}(\tanh(bx+a)) - \frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2}}{b}$
default	$\frac{\operatorname{arctanh}(\tanh(bx+a))\operatorname{arccoth}(\tanh(bx+a)) - \frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2}}{b}$
parts	$x \operatorname{arccoth}\left(\tanh\left(bx+a\right)\right) + \frac{-\frac{(bx+a)^2}{2} + (bx+a)a}{b}$
risch	$x \ln\left(e^{bx+a}\right) - \frac{i\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^3 x}{2} - \frac{i\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(i e^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)x}{4} + i\pi \operatorname{csgn}\left(\right.$

input
```
int(arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*x^2+x*arccoth(tanh(b*x+a))
```

3.131.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \coth^{-1}(\tanh(a+bx))\,dx = \frac{1}{2}\,bx^2 + \frac{1}{2}i\,\pi x + ax$$

input
```
integrate(arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*b*x^2 + 1/2*I*pi*x + a*x
```

3.131.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \coth^{-1}(\tanh(a+bx))\,dx = \begin{cases} \frac{\operatorname{acoth}^2(\tanh(a+bx))}{2b} & \text{for } b \neq 0 \\ x \operatorname{acoth}(\tanh(a)) & \text{otherwise} \end{cases}$$

input
```
integrate(acoth(tanh(b*x+a)),x)
```

output
```
Piecewise((acoth(tanh(a + b*x))**2/(2*b), Ne(b, 0)), (x*acoth(tanh(a)), Tr
ue))
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \coth^{-1}(\tanh(a+bx))\,dx = -\frac{1}{2}bx^2 + x \operatorname{arcoth}(\tanh(bx+a))$$

input
```
integrate(arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/2*b*x^2 + x*arccoth(tanh(b*x + a))
```

3.131.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(14) = 28.

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 4.31

$$\int \coth^{-1}(\tanh(a+bx))\,dx = -\frac{1}{2}bx^2 + \frac{1}{2}x\log\left(-\frac{\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1}+1}{\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1}-1}\right)$$

input
```
integrate(arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
-1/2*b*x^2 + 1/2*x*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/
((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

3.131.9 Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \coth^{-1}(\tanh(a+bx))\,dx = x\,\mathrm{acoth}(\tanh(a+b\,x)) - \frac{b\,x^2}{2}$$

input
```
int(acoth(tanh(a + b*x)),x)
```

output
```
x*acoth(tanh(a + b*x)) - (b*x^2)/2
```

3.132 $\int \frac{\coth^{-1}(\tanh(a+bx))}{x} \, dx$

3.132.1 Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x} \, dx = bx - \left(bx - \coth^{-1}(\tanh(a+bx))\right) \log(x)$$

output
```
b*x-(b*x-arccoth(tanh(b*x+a)))*ln(x)
```

3.132.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x} \, dx = bx + \left(-bx + \coth^{-1}(\tanh(a+bx))\right) \log(x)$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]/x,x]
```

output
```
b*x + (-(b*x) + ArcCoth[Tanh[a + b*x]])*Log[x]
```

3.132.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x} \, dx$$

$$\Big\downarrow \text{2589}$$

$$bx - \left(bx - \coth^{-1}(\tanh(a+bx))\right) \int \frac{1}{x}dx$$

$$\Big\downarrow \text{14}$$

$$bx - \log(x) \left(bx - \coth^{-1}(\tanh(a+bx))\right)$$

input
```
Int[ArcCoth[Tanh[a + b*x]]/x,x]
```

output
```
b*x - (b*x - ArcCoth[Tanh[a + b*x]])*Log[x]
```

3.132.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

3.132.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

method	result
default	$\ln(x)\,\mathrm{arccoth}(\tanh(bx+a))+b(-x\ln(x)+x)$
parts	$\ln(x)\,\mathrm{arccoth}(\tanh(bx+a))+b(-x\ln(x)+x)$
risch	$\ln(x)\ln\left(\mathrm{e}^{bx+a}\right)-b\ln(x)\,x+bx-\frac{i\pi\left(\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)-\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(\right.\right.}{}$

input
```
int(arccoth(tanh(b*x+a))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arccoth(tanh(b*x+a))+b*(-x*ln(x)+x)
```

3.132.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x}\,dx = bx+\frac{1}{2}\,(i\,\pi+2\,a)\log(x)$$

input
```
integrate(arccoth(tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
b*x + 1/2*(I*pi + 2*a)*log(x)
```

3.132.6 Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{acoth}(\tanh(a+bx))}{x}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))/x,x)
```

output
```
Integral(acoth(tanh(a + b*x))/x, x)
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x}\,dx = -b\left(x+\frac{a}{b}\right)\log(x) + b\left(x+\frac{a\log(x)}{b}\right) + \operatorname{arcoth}(\tanh(bx+a))\log(x)$$

input
```
integrate(arccoth(tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
-b*(x + a/b)*log(x) + b*(x + a*log(x)/b) + arccoth(tanh(b*x + a))*log(x)
```

3.132.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x}\,dx = bx + \frac{1}{2}\left(i\pi + 2a\right)\log(x)$$

input
```
integrate(arccoth(tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
b*x + 1/2*(I*pi + 2*a)*log(x)
```

3.132.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.81

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x}\,dx = b\,x - \ln(x)\left(\frac{\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)}{2} - \frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)}{2} + b\,x\right)$$

input
```
int(acoth(tanh(a + b*x))/x,x)
```

output
```
b*x - log(x)*(log(-2/(exp(2*a)*exp(2*b*x) - 1))/2 - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 + b*x)
```

3.133 $\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2} dx$

3.133.1 Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2} dx = -\frac{\coth^{-1}(\tanh(a+bx))}{x} + b\log(x)$$

output
```
-arccoth(tanh(b*x+a))/x+b*ln(x)
```

3.133.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2} dx = b - \frac{\coth^{-1}(\tanh(a+bx))}{x} + b\log(x)$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]/x^2,x]
```

output
```
b - ArcCoth[Tanh[a + b*x]]/x + b*Log[x]
```

3.133.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2} \, dx$$

$$\Big\downarrow \; 2599$$

$$b \int \frac{1}{x} dx - \frac{\coth^{-1}(\tanh(a+bx))}{x}$$

$$\Big\downarrow \; 14$$

$$b \log(x) - \frac{\coth^{-1}(\tanh(a+bx))}{x}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]/x^2,x]
```

output
```
-(ArcCoth[Tanh[a + b*x]]/x) + b*Log[x]
```

3.133.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.133.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

method	result
parallelrisch	$\frac{b\ln(x)x-\operatorname{arccoth}(\tanh(bx+a))}{x}$
default	$-\frac{\operatorname{arccoth}(\tanh(bx+a))}{x}+b\ln(-bx)$
parts	$-\frac{\operatorname{arccoth}(\tanh(bx+a))}{x}+b\ln(-bx)$
risch	$-\frac{\ln(\mathrm{e}^{bx+a})}{x}+\frac{2i\pi\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)^{3}+i\pi\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{3}-i\pi\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}+i\pi\operatorname{csgn}(i\mathrm{e}^{bx+a})}{}$

input
```
int(arccoth(tanh(b*x+a))/x^2,x,method=_RETURNVERBOSE)
```

output
```
(b*ln(x)*x-arccoth(tanh(b*x+a)))/x
```

3.133.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}\,dx = \frac{-i\,\pi + 2\,bx\log\left(x\right) - 2\,a}{2\,x}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^2,x, algorithm="fricas")
```

output
```
1/2*(-I*pi + 2*b*x*log(x) - 2*a)/x
```

3.133.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}\,dx = b\log\left(x\right) - \frac{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}{x}$$

input
```
integrate(acoth(tanh(b*x+a))/x**2,x)
```

output
```
b*log(x) - acoth(tanh(a + b*x))/x
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}\,dx = b\log(x) - \frac{\operatorname{arcoth}(\tanh(bx+a))}{x}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^2,x, algorithm="maxima")
```

output
```
b*log(x) - arccoth(tanh(b*x + a))/x
```

3.133.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(17) = 34.

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.12

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}\,dx = b\log(|x|) - \frac{\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{2\,x}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^2,x, algorithm="giac")
```

output
```
b*log(abs(x)) - 1/2*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)
/((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/x
```

3.133.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}\,dx = b\,\ln(x) - \frac{\operatorname{acoth}(\tanh(a+b\,x))}{x}$$

input
```
int(acoth(tanh(a + b*x))/x^2,x)
```

output
```
b*log(x) - acoth(tanh(a + b*x))/x
```

3.134 $\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3} dx$

3.134.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3} dx = -\frac{b}{2x} - \frac{\coth^{-1}(\tanh(a+bx))}{2x^2}$$

output
```
-1/2*b/x-1/2*arccoth(tanh(b*x+a))/x^2
```

3.134.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3} dx = -\frac{bx + \coth^{-1}(\tanh(a+bx))}{2x^2}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]/x^3,x]
```

output
```
-1/2*(b*x + ArcCoth[Tanh[a + b*x]])/x^2
```

3.134.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3} \, dx$$

$$\Big\downarrow \; 2599$$

$$\frac{1}{2}b\int \frac{1}{x^2}dx-\frac{\coth^{-1}(\tanh(a+bx))}{2x^2}$$

$$\Big\downarrow \; 15$$

$$-\frac{\coth^{-1}(\tanh(a+bx))}{2x^2}-\frac{b}{2x}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]/x^3,x]
```

output
```
-1/2*b/x - ArcCoth[Tanh[a + b*x]]/(2*x^2)
```

3.134.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.134.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

method	result
parallelrisch	$-\frac{bx+\operatorname{arccoth}(\tanh(bx+a))}{2x^2}$
default	$-\frac{b}{2x}-\frac{\operatorname{arccoth}(\tanh(bx+a))}{2x^2}$
parts	$-\frac{b}{2x}-\frac{\operatorname{arccoth}(\tanh(bx+a))}{2x^2}$
risch	$-\frac{\ln(\mathrm{e}^{bx+a})}{2x^2}-\frac{4bx-i\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^3+i\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^2-i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)^2\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)+2}{}$

input
```
int(arccoth(tanh(b*x+a))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x+arccoth(tanh(b*x+a)))/x^2
```

3.134.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3}\,dx = \frac{-i\,\pi - 4\,bx - 2\,a}{4\,x^2}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^3,x, algorithm="fricas")
```

output
```
1/4*(-I*pi - 4*b*x - 2*a)/x^2
```

3.134.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3}\,dx = -\frac{b}{2x} - \frac{\operatorname{acoth}(\tanh(a+bx))}{2x^2}$$

input
```
integrate(acoth(tanh(b*x+a))/x**3,x)
```

output
```
-b/(2*x) - acoth(tanh(a + b*x))/(2*x**2)
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3}\,dx = -\frac{b}{2\,x} - \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}{2\,x^2}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^3,x, algorithm="maxima")
```

output
```
-1/2*b/x - 1/2*arccoth(tanh(b*x + a))/x^2
```

3.134.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(19) = 38.

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3}\,dx = -\frac{b}{2\,x} - \frac{\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{4\,x^2}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^3,x, algorithm="giac")
```

output
```
-1/2*b/x - 1/4*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((e^
(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/x^2
```

3.134.9 Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3}\,dx = -\frac{\mathrm{acoth}(\tanh(a+b\,x))+b\,x}{2\,x^2}$$

input
```
int(acoth(tanh(a + b*x))/x^3,x)
```

output
```
-(acoth(tanh(a + b*x)) + b*x)/(2*x^2)
```

3.135 $\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4} dx$

3.135.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4} dx = -\frac{b}{6x^2} - \frac{\coth^{-1}(\tanh(a+bx))}{3x^3}$$

output
```
-1/6*b/x^2-1/3*arccoth(tanh(b*x+a))/x^3
```

3.135.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4} dx = -\frac{bx + 2\coth^{-1}(\tanh(a+bx))}{6x^3}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]/x^4,x]
```

output
```
-1/6*(b*x + 2*ArcCoth[Tanh[a + b*x]])/x^3
```

3.135.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4} \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{3} b \int \frac{1}{x^3} dx - \frac{\coth^{-1}(\tanh(a+bx))}{3x^3}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\coth^{-1}(\tanh(a+bx))}{3x^3} - \frac{b}{6x^2}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]/x^4,x]
```

output
```
-1/6*b/x^2 - ArcCoth[Tanh[a + b*x]]/(3*x^3)
```

3.135.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.135.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result
parallelrisch	$-\frac{bx+2\,\mathrm{arccoth}(\tanh(bx+a))}{6x^3}$
default	$-\frac{b}{6x^2}-\frac{\mathrm{arccoth}(\tanh(bx+a))}{3x^3}$
parts	$-\frac{b}{6x^2}-\frac{\mathrm{arccoth}(\tanh(bx+a))}{3x^3}$
risch	$-\frac{\ln(\mathrm{e}^{bx+a})}{3x^3}-\frac{2bx+2i\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)^2-i\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^3+i\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^2-i\pi\,\mathrm{csgn}(i\mathrm{e}^{b}}{}$

input
```
int(arccoth(tanh(b*x+a))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x+2*arccoth(tanh(b*x+a)))/x^3
```

3.135.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4}\,dx=\frac{-i\,\pi-3\,bx-2\,a}{6\,x^3}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^4,x, algorithm="fricas")
```

output
```
1/6*(-I*pi - 3*b*x - 2*a)/x^3
```

3.135.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4}\,dx=-\frac{b}{6x^2}-\frac{\operatorname{acoth}(\tanh(a+bx))}{3x^3}$$

input
```
integrate(acoth(tanh(b*x+a))/x**4,x)
```

output
```
-b/(6*x**2) - acoth(tanh(a + b*x))/(3*x**3)
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4}\,dx = -\frac{b}{6\,x^2} - \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}{3\,x^3}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^4,x, algorithm="maxima")
```

output
```
-1/6*b/x^2 - 1/3*arccoth(tanh(b*x + a))/x^3
```

3.135.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(19) = 38.

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4}\,dx = -\frac{b}{6\,x^2} - \frac{\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{6\,x^3}$$

input
```
integrate(arccoth(tanh(b*x+a))/x^4,x, algorithm="giac")
```

output
```
-1/6*b/x^2 - 1/6*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((
e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/x^3
```

3.135.9 Mupad [B] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4}\,dx = -\frac{\operatorname{acoth}(\tanh(a+b\,x))}{3\,x^3} - \frac{b}{6\,x^2}$$

input
```
int(acoth(tanh(a + b*x))/x^4,x)
```

output
```
- acoth(tanh(a + b*x))/(3*x^3) - b/(6*x^2)
```

3.136 $\int x^m \coth^{-1}(\tanh(a+bx))^2\,dx$

3.136.1 Optimal result

Integrand size = 13, antiderivative size = 71

$$\int x^m \coth^{-1}(\tanh(a+bx))^2\,dx=\frac{2b^2x^{3+m}}{6+11m+6m^2+m^3}-\frac{2bx^{2+m}\coth^{-1}(\tanh(a+bx))}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))^2}{1+m}$$

output
```
2*b^2*x^(3+m)/(m^3+6*m^2+11*m+6)-2*b*x^(2+m)*arccoth(tanh(b*x+a))/(m^2+3*m
+2)+x^(1+m)*arccoth(tanh(b*x+a))^2/(1+m)
```

3.136.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int x^m \coth^{-1}(\tanh(a+bx))^2\,dx=\frac{x^{1+m}\left(2b^2x^2-2b(3+m)x\coth^{-1}(\tanh(a+bx))+(6+5m+m^2)\coth^{-1}(\tanh(a+bx))^2\right)}{(1+m)(2+m)(3+m)}$$

input
```
Integrate[x^m*ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(x^(1 + m)*(2*b^2*x^2 - 2*b*(3 + m)*x*ArcCoth[Tanh[a + b*x]] + (6 + 5*m +
m^2)*ArcCoth[Tanh[a + b*x]]^2))/((1 + m)*(2 + m)*(3 + m))
```

3.136.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \coth^{-1}(\tanh(a+bx))^2 \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^2}{m+1} - \frac{2b\int x^{m+1}\coth^{-1}(\tanh(a+bx))dx}{m+1}$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^2}{m+1} - \frac{2b\left(\frac{x^{m+2}\coth^{-1}(\tanh(a+bx))}{m+2} - \frac{b\int x^{m+2}dx}{m+2}\right)}{m+1}$$

$$\Big\downarrow \text{15}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^2}{m+1} - \frac{2b\left(\frac{x^{m+2}\coth^{-1}(\tanh(a+bx))}{m+2} - \frac{bx^{m+3}}{(m+2)(m+3)}\right)}{m+1}$$

input
```
Int[x^m*ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(x^(1 + m)*ArcCoth[Tanh[a + b*x]]^2)/(1 + m) - (2*b*(-((b*x^(3 + m))/((2 +
 m)*(3 + m))) + (x^(2 + m)*ArcCoth[Tanh[a + b*x]])/(2 + m)))/(1 + m)
```

3.136.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.136.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.58

method	result
parallelrisch	$-\frac{6b\,\operatorname{arccoth}(\tanh(bx+a))x^m x^2-x\,x^m \operatorname{arccoth}(\tanh(bx+a))^2 m^2-5x\,x^m \operatorname{arccoth}(\tanh(bx+a))^2 m+2x^2x^m \operatorname{arccoth}(\tanh(bx+}{(1+m)(2+m)(3+m)}$
risch	Expression too large to display

input
```
int(x^m*arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
-(6*b*arccoth(tanh(b*x+a))*x^m*x^2-x*x^m*arccoth(tanh(b*x+a))^2*m^2-5*x*x^
m*arccoth(tanh(b*x+a))^2*m+2*x^2*x^m*arccoth(tanh(b*x+a))*b*m-2*b^2*x^m*x^
3-6*arccoth(tanh(b*x+a))^2*x*x^m)/(1+m)/(2+m)/(3+m)
```

3.136.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.72

$$\int x^m \coth^{-1}(\tanh(a+bx))^2\,dx =$$
$$-\frac{(\pi^2(m^2+5\,m+6)x-4\,(b^2m^2+3\,b^2m+2\,b^2)x^3-8\,(abm^2+4\,abm+3\,ab)x^2-4i\,\pi((bm^2+4\,bm+3}{}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
-1/4*((pi^2*(m^2 + 5*m + 6)*x - 4*(b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 - 8*(a*b
*m^2 + 4*a*b*m + 3*a*b)*x^2 - 4*I*pi*((b*m^2 + 4*b*m + 3*b)*x^2 + (a*m^2 +
 5*a*m + 6*a)*x) - 4*(a^2*m^2 + 5*a^2*m + 6*a^2)*x)*cosh(m*log(x)) + (pi^2
*(m^2 + 5*m + 6)*x - 4*(b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 - 8*(a*b*m^2 + 4*a*
b*m + 3*a*b)*x^2 - 4*I*pi*((b*m^2 + 4*b*m + 3*b)*x^2 + (a*m^2 + 5*a*m + 6*
a)*x) - 4*(a^2*m^2 + 5*a^2*m + 6*a^2)*x)*sinh(m*log(x)))/(m^3 + 6*m^2 + 11
*m + 6)
```

3.136.6 Sympy [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))^2\,dx$$

$$= \begin{cases} b^2\log(x) - \frac{b\,\text{acoth}(\tanh(a+bx))}{x} - \frac{\text{acoth}^2(\tanh(a+bx))}{2x^2} \\ \int \frac{\text{acoth}^2(\tanh(a+bx))}{x^2}\,dx \\ \int \frac{\text{acoth}^2(\tanh(a+bx))}{x}\,dx \\ \frac{2b^2x^3x^m}{m^3+6m^2+11m+6} - \frac{2bmx^2x^m\,\text{acoth}(\tanh(a+bx))}{m^3+6m^2+11m+6} - \frac{6bx^2x^m\,\text{acoth}(\tanh(a+bx))}{m^3+6m^2+11m+6} + \frac{m^2xx^m\,\text{acoth}^2(\tanh(a+bx))}{m^3+6m^2+11m+6} + \frac{5mxx^m\,\text{acoth}^2}{m^3+6m^2} \end{cases}$$

input
```
integrate(x**m*acoth(tanh(b*x+a))**2,x)
```

output
```
Piecewise((b**2*log(x) - b*acoth(tanh(a + b*x))/x - acoth(tanh(a + b*x))**
2/(2*x**2), Eq(m, -3)), (Integral(acoth(tanh(a + b*x))**2/x**2, x), Eq(m,
-2)), (Integral(acoth(tanh(a + b*x))**2/x, x), Eq(m, -1)), (2*b**2*x**3*x*
*m/(m**3 + 6*m**2 + 11*m + 6) - 2*b*m*x**2*x**m*acoth(tanh(a + b*x))/(m**3
 + 6*m**2 + 11*m + 6) - 6*b*x**2*x**m*acoth(tanh(a + b*x))/(m**3 + 6*m**2
+ 11*m + 6) + m**2*x*x**m*acoth(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m +
6) + 5*m*x*x**m*acoth(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6) + 6*x**x
**m*acoth(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6), True))
```

3.136.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int x^m \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{2\,b^2x^3x^m}{(m+3)(m+2)(m+1)} - \frac{2\,bx^2x^m\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)}{(m+2)(m+1)} + \frac{x^{m+1}\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{m+1}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
2*b^2*x^3*x^m/((m + 3)*(m + 2)*(m + 1)) - 2*b*x^2*x^m*arccoth(tanh(b*x + a
))/((m + 2)*(m + 1)) + x^(m + 1)*arccoth(tanh(b*x + a))^2/(m + 1)
```

3.136.8 Giac [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))^2\,dx = \int x^m\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2\,dx$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
integrate(x^m*arccoth(tanh(b*x + a))^2, x)
```

3.136.9 Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.86

$$\int x^m \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{4\,b^2\,x^m\,x^3\,\left(m^2+3\,m+2\right)}{4\,m^3+24\,m^2+44\,m+24} + \frac{x\,x^m\,\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)^2\,\left(m^2+5\,m+6\right)}{4\,m^3+24\,m^2+44\,m+24} - \frac{4\,b\,x^m\,x^2\,\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)\,\left(m^2+4\,m+3\right)}{4\,m^3+24\,m^2+44\,m+24}$$

input
```
int(x^m*acoth(tanh(a + b*x))^2,x)
```

output
```
(4*b^2*x^m*x^3*(3*m + m^2 + 2))/(44*m + 24*m^2 + 4*m^3 + 24) + (x*x^m*(log
(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) - 1)) + 2*b*x)^2*(5*m + m^2 + 6))/(44*m + 24*m^2 + 4*m^3 + 24) - (
4*b*x^m*x^2*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)*(4*m + m^2 + 3))/(44*m + 24*m^2 + 4
*m^3 + 24)
```

3.136. $\int x^m \coth^{-1}(\tanh(a+bx))^2 \, dx$

3.137 $\int x^3 \coth^{-1}(\tanh(a+bx))^2 \, dx$

3.137.1 Optimal result

Integrand size = 13, antiderivative size = 42

$$\int x^3 \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{b^2x^6}{60} - \frac{1}{10}bx^5 \coth^{-1}(\tanh(a+bx)) + \frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx))^2$$

output
```
1/60*b^2*x^6-1/10*b*x^5*arccoth(tanh(b*x+a))+1/4*x^4*arccoth(tanh(b*x+a))^
2
```

3.137.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int x^3 \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{1}{60}x^4 \left(b^2x^2 - 6bx \coth^{-1}(\tanh(a+bx)) + 15 \coth^{-1}(\tanh(a+bx))^2\right)$$

input
```
Integrate[x^3*ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(x^4*(b^2*x^2 - 6*b*x*ArcCoth[Tanh[a + b*x]] + 15*ArcCoth[Tanh[a + b*x]]^2
))/60
```

3.137.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(\tanh(a+bx))^2 \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx))^2 - \frac{1}{2}b \int x^4 \coth^{-1}(\tanh(a+bx))dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx))^2 - \frac{1}{2}b\left(\frac{1}{5}x^5 \coth^{-1}(\tanh(a+bx)) - \frac{b\int x^5dx}{5}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx))^2 - \frac{1}{2}b\left(\frac{1}{5}x^5 \coth^{-1}(\tanh(a+bx)) - \frac{bx^6}{30}\right)$$

input
```
Int[x^3*ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(x^4*ArcCoth[Tanh[a + b*x]]^2)/4 - (b*(-1/30*(b*x^6) + (x^5*ArcCoth[Tanh[a
 + b*x]])/5))/2
```

3.137.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.137.4 Maple [A] (verified)

Time = 28.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
parallelrisch	$\frac{b^{2} x^{6}}{60}-\frac{b\,x^{5} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)}{10}+\frac{x^{4} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)^{2}}{4}$	37
risch	Expression too large to display	3418

input
```
int(x^3*arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
1/60*b^2*x^6-1/10*b*x^5*arccoth(tanh(b*x+a))+1/4*x^4*arccoth(tanh(b*x+a))^
2
```

3.137.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int x^3 \coth^{-1}(\tanh(a+bx))^2\,dx=\frac{1}{6}\,b^{2}x^{6}+\frac{2}{5}\,abx^{5}-\frac{1}{16}\,\pi^{2}x^{4}+\frac{1}{4}\,a^{2}x^{4}+\frac{1}{20}i\,\pi\left(4\,bx^{5}+5\,ax^{4}\right)$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/6*b^2*x^6 + 2/5*a*b*x^5 - 1/16*pi^2*x^4 + 1/4*a^2*x^4 + 1/20*I*pi*(4*b*x
^5 + 5*a*x^4)
```

3.137.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int x^3 \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{b^2x^6}{60} - \frac{bx^5 \operatorname{acoth}(\tanh(a+bx))}{10} + \frac{x^4 \operatorname{acoth}^2(\tanh(a+bx))}{4}$$

input
```
integrate(x**3*acoth(tanh(b*x+a))**2,x)
```

output
```
b**2*x**6/60 - b*x**5*acoth(tanh(a + b*x))/10 + x**4*acoth(tanh(a + b*x))*
*2/4
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int x^3 \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{1}{60} b^2x^6 - \frac{1}{10} bx^5 \operatorname{arcoth}(\tanh(bx+a)) + \frac{1}{4} x^4 \operatorname{arcoth}(\tanh(bx+a))^2$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/60*b^2*x^6 - 1/10*b*x^5*arccoth(tanh(b*x + a)) + 1/4*x^4*arccoth(tanh(b*
x + a))^2
```

3.137.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int x^3 \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{6}\,b^2x^6 - \frac{1}{5}\,(-i\,\pi b - 2\,ab)x^5 - \frac{1}{16}\left(\pi^2 - 4i\,\pi a - 4\,a^2\right)x^4$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
1/6*b^2*x^6 - 1/5*(-I*pi*b - 2*a*b)*x^5 - 1/16*(pi^2 - 4*I*pi*a - 4*a^2)*x
^4
```

3.137.9 Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int x^3 \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{b^2\,x^6}{60} - \frac{b\,x^5\,\mathrm{acoth}(\tanh(a+b\,x))}{10} + \frac{x^4\,\mathrm{acoth}(\tanh(a+b\,x))^2}{4}$$

input
```
int(x^3*acoth(tanh(a + b*x))^2,x)
```

output
```
(x^4*acoth(tanh(a + b*x))^2)/4 + (b^2*x^6)/60 - (b*x^5*acoth(tanh(a + b*x)
))/10
```

3.138 $\int x^2 \coth^{-1}(\tanh(a+bx))^2 \, dx$

3.138.1 Optimal result

Integrand size = 13, antiderivative size = 42

$$\int x^2 \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{b^2x^5}{30} - \frac{1}{6}bx^4 \coth^{-1}(\tanh(a+bx)) + \frac{1}{3}x^3 \coth^{-1}(\tanh(a+bx))^2$$

output
```
1/30*b^2*x^5-1/6*b*x^4*arccoth(tanh(b*x+a))+1/3*x^3*arccoth(tanh(b*x+a))^2
```

3.138.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int x^2 \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{1}{30}x^3 \left(b^2x^2 - 5bx \coth^{-1}(\tanh(a+bx)) + 10 \coth^{-1}(\tanh(a+bx))^2\right)$$

input
```
Integrate[x^2*ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(x^3*(b^2*x^2 - 5*b*x*ArcCoth[Tanh[a + b*x]] + 10*ArcCoth[Tanh[a + b*x]]^2
))/30
```

3.138.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(\tanh(a+bx))^2 \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{3}x^3 \coth^{-1}(\tanh(a+bx))^2 - \frac{2}{3}b\int x^3 \coth^{-1}(\tanh(a+bx))dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{3}x^3 \coth^{-1}(\tanh(a+bx))^2 - \frac{2}{3}b\left(\frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx)) - \frac{b\int x^4dx}{4}\right)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{3}x^3 \coth^{-1}(\tanh(a+bx))^2 - \frac{2}{3}b\left(\frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx)) - \frac{bx^5}{20}\right)$$

input
```
Int[x^2*ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(x^3*ArcCoth[Tanh[a + b*x]]^2)/3 - (2*b*(-1/20*(b*x^5) + (x^4*ArcCoth[Tanh
[a + b*x]])/4))/3
```

3.138.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.138.4 Maple [A] (verified)

Time = 25.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
parallelrisch	$\frac{b^2x^5}{30}-\frac{b\,x^4\,\mathrm{arccoth}(\tanh(bx+a))}{6}+\frac{x^3\,\mathrm{arccoth}(\tanh(bx+a))^2}{3}$	37
risch	Expression too large to display	3418

input
```
int(x^2*arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
1/30*b^2*x^5-1/6*b*x^4*arccoth(tanh(b*x+a))+1/3*x^3*arccoth(tanh(b*x+a))^2
```

3.138.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int x^2\coth^{-1}(\tanh(a+bx))^2\,dx=\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4-\frac{1}{12}\,\pi^2x^3+\frac{1}{3}\,a^2x^3+\frac{1}{12}i\,\pi\left(3\,bx^4+4\,ax^3\right)$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/5*b^2*x^5 + 1/2*a*b*x^4 - 1/12*pi^2*x^3 + 1/3*a^2*x^3 + 1/12*I*pi*(3*b*x
^4 + 4*a*x^3)
```

3.138.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int x^2 \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{b^2x^5}{30} - \frac{bx^4 \operatorname{acoth}(\tanh(a+bx))}{6} + \frac{x^3 \operatorname{acoth}^2(\tanh(a+bx))}{3}$$

input
```
integrate(x**2*acoth(tanh(b*x+a))**2,x)
```

output
```
b**2*x**5/30 - b*x**4*acoth(tanh(a + b*x))/6 + x**3*acoth(tanh(a + b*x))**
2/3
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int x^2 \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{30}\,b^2x^5 - \frac{1}{6}\,bx^4 \operatorname{arcoth}(\tanh(bx+a)) + \frac{1}{3}\,x^3 \operatorname{arcoth}(\tanh(bx+a))^2$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/30*b^2*x^5 - 1/6*b*x^4*arccoth(tanh(b*x + a)) + 1/3*x^3*arccoth(tanh(b*x
 + a))^2
```

3.138.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int x^2 \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{5}\,b^2x^5 - \frac{1}{4}\,(-i\,\pi b - 2\,ab)x^4 - \frac{1}{12}\,(\pi^2 - 4i\,\pi a - 4\,a^2)x^3$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
1/5*b^2*x^5 - 1/4*(-I*pi*b - 2*a*b)*x^4 - 1/12*(pi^2 - 4*I*pi*a - 4*a^2)*x
^3
```

3.138.9 Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int x^2 \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{b^2 x^5}{30} - \frac{b x^4 \operatorname{acoth}(\tanh(a + b x))}{6} + \frac{x^3 \operatorname{acoth}(\tanh(a + b x))^2}{3}$$

input
```
int(x^2*acoth(tanh(a + b*x))^2,x)
```

output
```
(x^3*acoth(tanh(a + b*x))^2)/3 + (b^2*x^5)/30 - (b*x^4*acoth(tanh(a + b*x)
))/6
```

3.139 $\int x \coth^{-1}(\tanh(a+bx))^2 \, dx$

3.139.1 Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{x \coth^{-1}(\tanh(a+bx))^3}{3b} - \frac{\coth^{-1}(\tanh(a+bx))^4}{12b^2}$$

output
```
1/3*x*arccoth(tanh(b*x+a))^3/b-1/12*arccoth(tanh(b*x+a))^4/b^2
```

3.139.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 74 vs. 2(34) = 68.

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.18

$$\int x \coth^{-1}(\tanh(a+bx))^2 \, dx$$
$$= \frac{(a+bx)\left(-((3a-bx)(a+bx)^2)+4(2a^2+abx-b^2x^2)\coth^{-1}(\tanh(a+bx))-6(a-bx)\coth^{-1}(\tanh(a\right.}{12b^2}$$

input
```
Integrate[x*ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
((a + b*x)*(-((3*a - b*x)*(a + b*x)^2) + 4*(2*a^2 + a*b*x - b^2*x^2)*ArcCo
th[Tanh[a + b*x]] - 6*(a - b*x)*ArcCoth[Tanh[a + b*x]]^2))/(12*b^2)
```

3.139.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(\tanh(a+bx))^2 \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^3}{3b}-\frac{\int \coth^{-1}(\tanh(a+bx))^3dx}{3b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^3}{3b}-\frac{\int \coth^{-1}(\tanh(a+bx))^3d\coth^{-1}(\tanh(a+bx))}{3b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^3}{3b}-\frac{\coth^{-1}(\tanh(a+bx))^4}{12b^2}$$

input
```
Int[x*ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(x*ArcCoth[Tanh[a + b*x]]^3)/(3*b) - ArcCoth[Tanh[a + b*x]]^4/(12*b^2)
```

3.139.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.139.4 Maple [A] (verified)

Time = 24.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$-\frac{b\,x^{3} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)}{3}+\frac{x^{2} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)^{2}}{2}+\frac{b^{2} x^{4}}{12}$	37
risch	Expression too large to display	3418

input
```
int(x*arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b*x^3*arccoth(tanh(b*x+a))+1/2*x^2*arccoth(tanh(b*x+a))^2+1/12*b^2*x^
4
```

3.139.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int x \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{4}\,b^2x^4 + \frac{2}{3}\,abx^3 - \frac{1}{8}\,\pi^2x^2 + \frac{1}{2}\,a^2x^2 + \frac{1}{6}i\,\pi\left(2\,bx^3 + 3\,ax^2\right)$$

input
```
integrate(x*arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/4*b^2*x^4 + 2/3*a*b*x^3 - 1/8*pi^2*x^2 + 1/2*a^2*x^2 + 1/6*I*pi*(2*b*x^3
 + 3*a*x^2)
```

3.139.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int x \coth^{-1}(\tanh(a+bx))^2 \, dx = \begin{cases} \frac{x \operatorname{acoth}^3(\tanh(a+bx))}{3b} - \frac{\operatorname{acoth}^4(\tanh(a+bx))}{12b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{acoth}^2(\tanh(a))}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acoth(tanh(b*x+a))**2,x)
```

output
```
Piecewise((x*acoth(tanh(a + b*x))**3/(3*b) - acoth(tanh(a + b*x))**4/(12*b
**2), Ne(b, 0)), (x**2*acoth(tanh(a))**2/2, True))
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int x \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{1}{12} b^2 x^4 - \frac{1}{3} b x^3 \operatorname{arcoth}(\tanh(bx+a)) + \frac{1}{2} x^2 \operatorname{arcoth}(\tanh(bx+a))^2$$

input
```
integrate(x*arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/12*b^2*x^4 - 1/3*b*x^3*arccoth(tanh(b*x + a)) + 1/2*x^2*arccoth(tanh(b*x
 + a))^2
```

3.139.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int x \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{1}{4} b^2 x^4 - \frac{1}{3} (-i\pi b - 2ab) x^3 - \frac{1}{8} (\pi^2 - 4i\pi a - 4a^2) x^2$$

input
```
integrate(x*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
1/4*b^2*x^4 - 1/3*(-I*pi*b - 2*a*b)*x^3 - 1/8*(pi^2 - 4*I*pi*a - 4*a^2)*x^
2
```

3.139.9 Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int x \coth^{-1}(\tanh(a+bx))^2 \, dx = \frac{b^2 x^4}{12} - \frac{b x^3 \operatorname{acoth}(\tanh(a+b x))}{3} + \frac{x^2 \operatorname{acoth}(\tanh(a+b x))^2}{2}$$

input
```
int(x*acoth(tanh(a + b*x))^2,x)
```

output
```
(x^2*acoth(tanh(a + b*x))^2)/2 + (b^2*x^4)/12 - (b*x^3*acoth(tanh(a + b*x)
))/3
```

3.140 $\int \coth^{-1}(\tanh(a+bx))^2\,dx$

3.140.1 Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \coth^{-1}(\tanh(a+bx))^2\,dx=\frac{\coth^{-1}(\tanh(a+bx))^3}{3b}$$

output
```
1/3*arccoth(tanh(b*x+a))^3/b
```

3.140.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \coth^{-1}(\tanh(a+bx))^2\,dx=\frac{\coth^{-1}(\tanh(a+bx))^3}{3b}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^3/(3*b)
```

3.140.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(\tanh(a+bx))^2 \, dx$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \coth^{-1}(\tanh(a+bx))^2 d\coth^{-1}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3b}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^3/(3*b)
```

3.140.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.140.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\operatorname{arccoth}(\tanh(bx+a))^3}{3b}$	15
default	$\frac{\operatorname{arccoth}(\tanh(bx+a))^3}{3b}$	15
parallelrisch	$\frac{b^2x^3}{3} - b\,x^2 \operatorname{arccoth}(\tanh(bx+a)) + x \operatorname{arccoth}(\tanh(bx+a))^2$	34
risch	Expression too large to display	14844

input
```
int(arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*arccoth(tanh(b*x+a))^3/b
```

3.140.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{3}b^2x^3 + abx^2 - \frac{1}{4}\pi^2x + a^2x + \frac{1}{2}i\pi\left(bx^2 + 2ax\right)$$

input
```
integrate(arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/3*b^2*x^3 + a*b*x^2 - 1/4*pi^2*x + a^2*x + 1/2*I*pi*(b*x^2 + 2*a*x)
```

3.140.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \coth^{-1}(\tanh(a+bx))^2\,dx = \begin{cases} \frac{\operatorname{acoth}^3(\tanh(a+bx))}{3b} & \text{for } b \neq 0 \\ x \operatorname{acoth}^2(\tanh(a)) & \text{otherwise} \end{cases}$$

input
```
integrate(acoth(tanh(b*x+a))**2,x)
```

output
```
Piecewise((acoth(tanh(a + b*x))**3/(3*b), Ne(b, 0)), (x*acoth(tanh(a))**2,
 True))
```

3.140.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{3}\,b^2x^3 - bx^2\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right) + x\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2$$

input
```
integrate(arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3 - b*x^2*arccoth(tanh(b*x + a)) + x*arccoth(tanh(b*x + a))^2
```

3.140.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{3}\,b^2x^3 - \frac{1}{2}\,(-i\,\pi b - 2\,ab)x^2 - \frac{1}{4}\,(\pi^2 - 4i\,\pi a - 4\,a^2)x$$

input
```
integrate(arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
1/3*b^2*x^3 - 1/2*(-I*pi*b - 2*a*b)*x^2 - 1/4*(pi^2 - 4*I*pi*a - 4*a^2)*x
```

3.140.9 Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \coth^{-1}(\tanh(a+bx))^2\,dx = \frac{b^2\,x^3}{3} - b\,x^2\,\mathrm{acoth}(\tanh(a+b\,x)) + x\,\mathrm{acoth}(\tanh(a+b\,x))^2$$

input
```
int(acoth(tanh(a + b*x))^2,x)
```

output
```
x*acoth(tanh(a + b*x))^2 + (b^2*x^3)/3 - b*x^2*acoth(tanh(a + b*x))
```

3.141 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx$

3.141.1 Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx = -bx(bx-\coth^{-1}(\tanh(a+bx)))+\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2 + (bx-\coth^{-1}(\tanh(a+bx)))^2\log(x)$$

output
```
-b*x*(b*x-arccoth(tanh(b*x+a)))+1/2*arccoth(tanh(b*x+a))^2+(b*x-arccoth(ta
nh(b*x+a)))^2*ln(x)
```

3.141.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx = \frac{1}{2}(a+bx)^2-(a+bx)\left(a+2bx-2\coth^{-1}(\tanh(a+bx))\right) + \left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2\log(bx)$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^2/x,x]
```

output
```
(a + b*x)^2/2 - (a + b*x)*(a + 2*b*x - 2*ArcCoth[Tanh[a + b*x]]) + (-(b*x)
 + ArcCoth[Tanh[a + b*x]])^2*Log[b*x]
```

3.141.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2590, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x} \, dx$$

$$\Big\downarrow \ 2590$$

$$\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{\coth^{-1}(\tanh(a+bx))}{x}dx$$

$$\Big\downarrow \ 2589$$

$$\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2- \left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(bx-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{x}dx\right)$$

$$\Big\downarrow \ 14$$

$$\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(bx-\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)\right)$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^2/x,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^2/2 - (b*x - ArcCoth[Tanh[a + b*x]])*(b*x - (b*x -
ArcCoth[Tanh[a + b*x]])*Log[x])
```

3.141.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

3.141.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 664, normalized size of antiderivative = 13.55

method	result
risch	$\ln(x)\ln\left(e^{bx+a}\right)^2+b^2\ln(x)x^2-\frac{3b^2x^2}{2}-2b\ln\left(e^{bx+a}\right)\ln(x)x+2b\ln\left(e^{bx+a}\right)x-\frac{\pi^2\left(\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right.\right.}{}$

input
```
int(arccoth(tanh(b*x+a))^2/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*ln(exp(b*x+a))^2+b^2*ln(x)*x^2-3/2*b^2*x^2-2*b*ln(exp(b*x+a))*ln(x)*
x+2*b*ln(exp(b*x+a))*x-1/16*Pi^2*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*
b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)
+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*csgn(I/(exp(2*b*x+2*a)+
1))^3-2*csgn(I/(exp(2*b*x+2*a)+1))^2+csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x
+2*a))-2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+csgn(I*exp(2*b*x+2*a)
)^3-csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2)^2*ln(x)-1/2*I*Pi*(csgn(I/(exp(
2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a
)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^2+2*csgn(I/(exp(2*b*x+2*a)+1))^3-2*csgn(I/(exp(2*b*x+2*a)+1))^2+csgn(I*ex
p(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2
*a))^2+csgn(I*exp(2*b*x+2*a))^3-csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)+1))^2+csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2)*(l
n(x)*ln(exp(b*x+a))-b*(x*ln(x)-x))
```

3.141.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx = \frac{1}{2}\,b^2x^2 + i\,\pi bx + 2\,abx - \frac{1}{4}\,(\pi^2 - 4i\,\pi a - 4\,a^2)\log(x)$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x,x, algorithm="fricas")
```

output
```
1/2*b^2*x^2 + I*pi*b*x + 2*a*b*x - 1/4*(pi^2 - 4*I*pi*a - 4*a^2)*log(x)
```

3.141.6 Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx = \int \frac{\operatorname{acoth}^2(\tanh(a+bx))}{x}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))**2/x,x)
```

output
```
Integral(acoth(tanh(a + b*x))**2/x, x)
```

3.141.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx = \frac{1}{2}\,b^2x^2 - (i\,\pi b - 2\,ab)x - \frac{1}{4}\,(\pi^2 + 4i\,\pi a - 4\,a^2)\log(x)$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2 - (I*pi*b - 2*a*b)*x - 1/4*(pi^2 + 4*I*pi*a - 4*a^2)*log(x)
```

3.141.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx = \frac{1}{2}b^2x^2 + (i\pi b + 2ab)x - \frac{1}{4}\left(\pi^2 - 4i\pi a - 4a^2\right)\log(x)$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x,x, algorithm="giac")
```

output
```
1/2*b^2*x^2 + (I*pi*b + 2*a*b)*x - 1/4*(pi^2 - 4*I*pi*a - 4*a^2)*log(x)
```

3.141.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.73

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx = \ln(x)\left(\frac{\left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) + 2bx\right)^2}{4} - a\left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) + 2bx\right) + a^2\right) + \frac{b^2x^2}{2} - bx\left(\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + 2bx\right)$$

input
```
int(acoth(tanh(a + b*x))^2/x,x)
```

output
```
log(x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + lo
g(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2/4 - a*(2*a - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) +
 2*b*x) + a^2) + (b^2*x^2)/2 - b*x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)
```

3.142 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2} dx$

3.142.1 Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2} dx = 2b^2x - \frac{\coth^{-1}(\tanh(a+bx))^2}{x} - 2b\left(bx - \coth^{-1}(\tanh(a+bx))\right)\log(x)$$

output
```
2*b^2*x-arccoth(tanh(b*x+a))^2/x-2*b*(b*x-arccoth(tanh(b*x+a)))*ln(x)
```

3.142.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2} dx = -\frac{\coth^{-1}(\tanh(a+bx))^2}{x} - 2b^2x\log(x) + 2b\coth^{-1}(\tanh(a+bx))(1+\log(x))$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^2/x^2,x]
```

output
```
-(ArcCoth[Tanh[a + b*x]]^2/x) - 2*b^2*x*Log[x] + 2*b*ArcCoth[Tanh[a + b*x]
]*(1 + Log[x])
```

3.142.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2599, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2} dx$$

$$\Big\downarrow \text{ 2599}$$

$$2b\int \frac{\coth^{-1}(\tanh(a+bx))}{x}dx-\frac{\coth^{-1}(\tanh(a+bx))^2}{x}$$

$$\Big\downarrow \text{ 2589}$$

$$2b\left(bx-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{x}dx\right)-\frac{\coth^{-1}(\tanh(a+bx))^2}{x}$$

$$\Big\downarrow \text{ 14}$$

$$2b\left(bx-\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)\right)-\frac{\coth^{-1}(\tanh(a+bx))^2}{x}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^2/x^2,x]
```

output
```
-(ArcCoth[Tanh[a + b*x]]^2/x) + 2*b*(b*x - (b*x - ArcCoth[Tanh[a + b*x]])*
Log[x])
```

3.142.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.142.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 647, normalized size of antiderivative = 16.59

method	result
risch	$-\frac{\ln(\mathrm{e}^{bx+a})^2}{x} - 2\ln(x)\,x\,b^2 + 2\ln(x)\ln(\mathrm{e}^{bx+a})\,b + 2b^2x + \frac{\pi^2\left(\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+\right.}\right.}{}$

input
```
int(arccoth(tanh(b*x+a))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*ln(exp(b*x+a))^2-2*ln(x)*x*b^2+2*ln(x)*ln(exp(b*x+a))*b+2*b^2*x+1/16*
Pi^2*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))^2+2*csgn(I/(exp(2*b*x+2*a)+1))^3-2*csgn(I/(exp(2*b*x+2*
a)+1))^2+csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*csgn(I*exp(b*x+a))*
csgn(I*exp(2*b*x+2*a))^2+csgn(I*exp(2*b*x+2*a))^3-csgn(I*exp(2*b*x+2*a))*c
sgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))^3+2)^2/x-1/2*I*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+
2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*csgn(I/(exp(2*b*x+2*a)+1))^
3-2*csgn(I/(exp(2*b*x+2*a)+1))^2+csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a
))-2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+csgn(I*exp(2*b*x+2*a))^3-
csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2)*(-1/x*ln(exp(b*x+a))+b*ln(x))
```

3.142.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}\,dx = \frac{4\,b^2x^2+\pi^2-4i\,\pi a-4\,a^2-4\left(-i\,\pi bx-2\,abx\right)\log\left(x\right)}{4\,x}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^2,x, algorithm="fricas")
```

output
```
1/4*(4*b^2*x^2 + pi^2 - 4*I*pi*a - 4*a^2 - 4*(-I*pi*b*x - 2*a*b*x)*log(x))
/x
```

3.142.6 Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}\,dx = \int \frac{\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}{x^2}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))**2/x**2,x)
```

output
```
Integral(acoth(tanh(a + b*x))**2/x**2, x)
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}\,dx = 2\,b\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)\log\left(x\right) - 2\left(b\left(x+\frac{a}{b}\right)\log\left(x\right)-b\left(x+\frac{a\log\left(x\right)}{b}\right)\right)b - \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{x}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^2,x, algorithm="maxima")
```

output
```
2*b*arccoth(tanh(b*x + a))*log(x) - 2*(b*(x + a/b)*log(x) - b*(x + a*log(x
)/b))*b - arccoth(tanh(b*x + a))^2/x
```

3.142.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}\,dx = b^2x + (i\pi b + 2\,ab)\log(x) + \frac{\pi^2 - 4i\,\pi a - 4\,a^2}{4\,x}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^2,x, algorithm="giac")
```

output
```
b^2*x + (I*pi*b + 2*a*b)*log(x) + 1/4*(pi^2 - 4*I*pi*a - 4*a^2)/x
```

3.142.9 Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 207, normalized size of antiderivative = 5.31

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}\,dx &= b\,\ln\left(\frac{e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)^2}{4\,x} \\ &\quad - b\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \frac{\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)^2}{4\,x} \\ &\quad + b\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\ln(x) - b\,\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\ln(x) \\ &\quad + \frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2\,x} - 2\,b^2\,x\,\ln(x)\end{aligned}$$

input
```
int(acoth(tanh(a + b*x))^2/x^2,x)
```

output
```
b*log(exp(2*b*x)/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) - 1))^2/(4*x) - b*log(1/(exp(2*a)*exp(2*b*x) - 1)) -
log(-2/(exp(2*a)*exp(2*b*x) - 1))^2/(4*x) + b*log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) - 1))*log(x) - b*log(-2/(exp(2*a)*exp(2*b*x) - 1))*lo
g(x) + (log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*log(-2/(exp
(2*a)*exp(2*b*x) - 1)))/(2*x) - 2*b^2*x*log(x)
```

3.143 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx$

3.143.1 Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx = -\frac{b\coth^{-1}(\tanh(a+bx))}{x} - \frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2} + b^2\log(x)$$

output
```
-b*arccoth(tanh(b*x+a))/x-1/2*arccoth(tanh(b*x+a))^2/x^2+b^2*ln(x)
```

3.143.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx = -\frac{2bx\coth^{-1}(\tanh(a+bx)) + \coth^{-1}(\tanh(a+bx))^2 - b^2x^2(3+2\log(x))}{2x^2}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^2/x^3,x]
```

output
```
-1/2*(2*b*x*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^2 - b^2*x^2*(3
 + 2*Log[x]))/x^2
```

3.143.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2599, 2599, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}dx$$

$$\Big\downarrow \text{ 2599}$$

$$b\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}dx-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}$$

$$\Big\downarrow \text{ 2599}$$

$$b\left(b\int \frac{1}{x}dx-\frac{\coth^{-1}(\tanh(a+bx))}{x}\right)-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}$$

$$\Big\downarrow \text{ 14}$$

$$b\left(b\log(x)-\frac{\coth^{-1}(\tanh(a+bx))}{x}\right)-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^2/x^3,x]
```

output
```
-1/2*ArcCoth[Tanh[a + b*x]]^2/x^2 + b*(-(ArcCoth[Tanh[a + b*x]]/x) + b*Log
[x])
```

3.143.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.143.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

method	result	size
parallelrisch	$\frac{2b^2 \ln(x)x^2 - 2bx \operatorname{arccoth}(\tanh(bx+a)) - \operatorname{arccoth}(\tanh(bx+a))^2}{2x^2}$	39
risch	Expression too large to display	3213

input
```
int(arccoth(tanh(b*x+a))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*b^2*ln(x)*x^2-2*b*x*arccoth(tanh(b*x+a))-arccoth(tanh(b*x+a))^2)/x^
2
```

3.143.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx = \frac{8\,b^2x^2\log(x) - 16\,abx + \pi^2 - 4i\,\pi(2\,bx + a) - 4\,a^2}{8\,x^2}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^3,x, algorithm="fricas")
```

output
```
1/8*(8*b^2*x^2*log(x) - 16*a*b*x + pi^2 - 4*I*pi*(2*b*x + a) - 4*a^2)/x^2
```

3.143.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx = b^2\log(x) - \frac{b\operatorname{acoth}(\tanh(a+bx))}{x} - \frac{\operatorname{acoth}^2(\tanh(a+bx))}{2x^2}$$

input
```
integrate(acoth(tanh(b*x+a))**2/x**3,x)
```

output
```
b**2*log(x) - b*acoth(tanh(a + b*x))/x - acoth(tanh(a + b*x))**2/(2*x**2)
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx = b^2\log(x) - \frac{b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)}{x} - \frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{2\,x^2}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^3,x, algorithm="maxima")
```

output
```
b^2*log(x) - b*arccoth(tanh(b*x + a))/x - 1/2*arccoth(tanh(b*x + a))^2/x^2
```

3.143.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx = b^2\log(x) - \frac{8i\,\pi bx + 16\,abx - \pi^2 + 4i\,\pi a + 4\,a^2}{8\,x^2}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^3,x, algorithm="giac")
```

output
```
b^2*log(x) - 1/8*(8*I*pi*b*x + 16*a*b*x - pi^2 + 4*I*pi*a + 4*a^2)/x^2
```

3.143.9 Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx = b^2\,\ln(x) - \frac{\frac{\mathrm{acoth}(\tanh(a+b\,x))^2}{2} + b\,x\,\mathrm{acoth}(\tanh(a+b\,x))}{x^2}$$

input
```
int(acoth(tanh(a + b*x))^2/x^3,x)
```

output
```
b^2*log(x) - (acoth(tanh(a + b*x))^2/2 + b*x*acoth(tanh(a + b*x)))/x^2
```

3.144 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4} dx$

3.144.1 Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4} dx = \frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

output
```
1/3*arccoth(tanh(b*x+a))^3/x^3/(b*x-arccoth(tanh(b*x+a)))
```

3.144.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4} dx = -\frac{b^2x^2+bx\coth^{-1}(\tanh(a+bx))+\coth^{-1}(\tanh(a+bx))^2}{3x^3}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^2/x^4,x]
```

output
```
-1/3*(b^2*x^2 + b*x*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^2)/x^3
```

3.144.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4} \, dx$$

$$\Big\downarrow \text{2598}$$

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3 \left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^2/x^4,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^3/(3*x^3*(b*x - ArcCoth[Tanh[a + b*x]]))
```

3.144.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

3.144.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

method	result	size
parallelrisch	$-\frac{b^2x^2+bx\,\operatorname{arccoth}(\tanh(bx+a))+\operatorname{arccoth}(\tanh(bx+a))^2}{3x^3}$	33
risch	Expression too large to display	3217

input
```
int(arccoth(tanh(b*x+a))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b^2*x^2+b*x*arccoth(tanh(b*x+a))+arccoth(tanh(b*x+a))^2)/x^3
```

3.144.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}\,dx = -\frac{12\,b^2x^2+12\,abx-\pi^2+2i\,\pi(3\,bx+2\,a)+4\,a^2}{12\,x^3}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^4,x, algorithm="fricas")
```

output
```
-1/12*(12*b^2*x^2 + 12*a*b*x - pi^2 + 2*I*pi*(3*b*x + 2*a) + 4*a^2)/x^3
```

3.144.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}\,dx = -\frac{b^2}{3x}-\frac{b\,\mathrm{acoth}\,(\tanh\,(a+bx))}{3x^2}-\frac{\mathrm{acoth}^2\,(\tanh\,(a+bx))}{3x^3}$$

input
```
integrate(acoth(tanh(b*x+a))**2/x**4,x)
```

output
```
-b**2/(3*x) - b*acoth(tanh(a + b*x))/(3*x**2) - acoth(tanh(a + b*x))**2/(3
*x**3)
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}\,dx = -\frac{b^2}{3\,x}-\frac{b\,\mathrm{arcoth}\,(\tanh\,(bx+a))}{3\,x^2}-\frac{\mathrm{arcoth}\,(\tanh\,(bx+a))^2}{3\,x^3}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^4,x, algorithm="maxima")
```

output
```
-1/3*b^2/x - 1/3*b*arccoth(tanh(b*x + a))/x^2 - 1/3*arccoth(tanh(b*x + a))
^2/x^3
```

3.144.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}\,dx = -\frac{12\,b^2x^2 + 6i\,\pi bx + 12\,abx - \pi^2 + 4i\,\pi a + 4\,a^2}{12\,x^3}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^4,x, algorithm="giac")
```

output
```
-1/12*(12*b^2*x^2 + 6*I*pi*b*x + 12*a*b*x - pi^2 + 4*I*pi*a + 4*a^2)/x^3
```

3.144.9 Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}\,dx = -\frac{b^2\,x^2 + b\,x\,\mathrm{acoth}(\tanh(a+b\,x)) + \mathrm{acoth}(\tanh(a+b\,x))^2}{3\,x^3}$$

input
```
int(acoth(tanh(a + b*x))^2/x^4,x)
```

output
```
-(acoth(tanh(a + b*x))^2 + b^2*x^2 + b*x*acoth(tanh(a + b*x)))/(3*x^3)
```

3.145 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx$

3.145.1 Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx = \frac{b\coth^{-1}(\tanh(a+bx))^3}{12x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2} + \frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

output
```
1/12*b*arccoth(tanh(b*x+a))^3/x^3/(b*x-arccoth(tanh(b*x+a)))^2+1/4*arccoth
(tanh(b*x+a))^3/x^4/(b*x-arccoth(tanh(b*x+a)))
```

3.145.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.58

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{b^2x^2+2bx\coth^{-1}(\tanh(a+bx))+3\coth^{-1}(\tanh(a+bx))^2}{12x^4}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^2/x^5,x]
```

output
```
-1/12*(b^2*x^2 + 2*b*x*ArcCoth[Tanh[a + b*x]] + 3*ArcCoth[Tanh[a + b*x]]^2
)/x^4
```

3.145.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5} \, dx$$

$$\Big\downarrow \; 2602$$

$$\frac{b\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}dx}{4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \; 2598$$

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\coth^{-1}(\tanh(a+bx))^3}{12x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^2/x^5,x]
```

output
```
(b*ArcCoth[Tanh[a + b*x]]^3)/(12*x^3*(b*x - ArcCoth[Tanh[a + b*x]])^2) + A
rcCoth[Tanh[a + b*x]]^3/(4*x^4*(b*x - ArcCoth[Tanh[a + b*x]]))
```

3.145.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.145.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

method	result	size
parallelrisch	$-\frac{b^{2} x^{2}+2 b x \,\operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)+3 \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)^{2}}{12 x^{4}}$	36
risch	Expression too large to display	3217

input
```
int(arccoth(tanh(b*x+a))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b^2*x^2+2*b*x*arccoth(tanh(b*x+a))+3*arccoth(tanh(b*x+a))^2)/x^4
```

3.145.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{24\,b^2x^2 + 32\,abx - 3\,\pi^2 + 4i\,\pi(4\,bx + 3\,a) + 12\,a^2}{48\,x^4}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^5,x, algorithm="fricas")
```

output
```
-1/48*(24*b^2*x^2 + 32*a*b*x - 3*pi^2 + 4*I*pi*(4*b*x + 3*a) + 12*a^2)/x^4
```

3.145.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{b^2}{12x^2} - \frac{b\operatorname{acoth}(\tanh(a+bx))}{6x^3} - \frac{\operatorname{acoth}^2(\tanh(a+bx))}{4x^4}$$

input
```
integrate(acoth(tanh(b*x+a))**2/x**5,x)
```

output
```
-b**2/(12*x**2) - b*acoth(tanh(a + b*x))/(6*x**3) - acoth(tanh(a + b*x))**
2/(4*x**4)
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{b^2}{12\,x^2} - \frac{b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)}{6\,x^3} - \frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{4\,x^4}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^5,x, algorithm="maxima")
```

output
```
-1/12*b^2/x^2 - 1/6*b*arccoth(tanh(b*x + a))/x^3 - 1/4*arccoth(tanh(b*x +
a))^2/x^4
```

3.145.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.59

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{24\,b^2x^2 + 16i\,\pi bx + 32\,abx - 3\,\pi^2 + 12i\,\pi a + 12\,a^2}{48\,x^4}$$

input
```
integrate(arccoth(tanh(b*x+a))^2/x^5,x, algorithm="giac")
```

output
```
-1/48*(24*b^2*x^2 + 16*I*pi*b*x + 32*a*b*x - 3*pi^2 + 12*I*pi*a + 12*a^2)/
x^4
```

3.145.9 Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx = -\frac{\mathrm{acoth}(\tanh(a+b\,x))^2}{4\,x^4} - \frac{b^2}{12\,x^2} - \frac{b\,\mathrm{acoth}(\tanh(a+b\,x))}{6\,x^3}$$

input
```
int(acoth(tanh(a + b*x))^2/x^5,x)
```

output
```
- acoth(tanh(a + b*x))^2/(4*x^4) - b^2/(12*x^2) - (b*acoth(tanh(a + b*x)))
/(6*x^3)
```

3.146 $\int x^m \coth^{-1}(\tanh(a+bx))^3 \, dx$

3.146.1 Optimal result

Integrand size = 13, antiderivative size = 110

$$\int x^m \coth^{-1}(\tanh(a+bx))^3 \, dx = -\frac{6b^3x^{4+m}}{(1+m)\left(24+26m+9m^2+m^3\right)} + \frac{6b^2x^{3+m}\coth^{-1}(\tanh(a+bx))}{6+11m+6m^2+m^3} - \frac{3bx^{2+m}\coth^{-1}(\tanh(a+bx))^2}{2+3m+m^2} + \frac{x^{1+m}\coth^{-1}(\tanh(a+bx))^3}{1+m}$$

output
```
-6*b^3*x^(4+m)/(1+m)/(m^3+9*m^2+26*m+24)+6*b^2*x^(3+m)*arccoth(tanh(b*x+a)
)/(m^3+6*m^2+11*m+6)-3*b*x^(2+m)*arccoth(tanh(b*x+a))^2/(m^2+3*m+2)+x^(1+m
)*arccoth(tanh(b*x+a))^3/(1+m)
```

3.146.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int x^m \coth^{-1}(\tanh(a+bx))^3 \, dx = \frac{x^{1+m}\left(-6b^3x^3+6b^2(4+m)x^2\coth^{-1}(\tanh(a+bx))-3b\left(12+7m+m^2\right)x\coth^{-1}(\tanh(a+bx))^2+(24\right.}{(1+m)(2+m)(3+m)(4+m)}$$

input
```
Integrate[x^m*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
(x^(1 + m)*(-6*b^3*x^3 + 6*b^2*(4 + m)*x^2*ArcCoth[Tanh[a + b*x]] - 3*b*(1
2 + 7*m + m^2)*x*ArcCoth[Tanh[a + b*x]]^2 + (24 + 26*m + 9*m^2 + m^3)*ArcC
oth[Tanh[a + b*x]]^3))/((1 + m)*(2 + m)*(3 + m)*(4 + m))
```

3.146.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \coth^{-1}(\tanh(a+bx))^3 \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^3}{m+1}-\frac{3b\int x^{m+1}\coth^{-1}(\tanh(a+bx))^2dx}{m+1}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^3}{m+1}-\frac{3b\left(\frac{x^{m+2}\coth^{-1}(\tanh(a+bx))^2}{m+2}-\frac{2b\int x^{m+2}\coth^{-1}(\tanh(a+bx))dx}{m+2}\right)}{m+1}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^3}{m+1}-\frac{3b\left(\frac{x^{m+2}\coth^{-1}(\tanh(a+bx))^2}{m+2}-\frac{2b\left(\frac{x^{m+3}\coth^{-1}(\tanh(a+bx))}{m+3}-\frac{b\int x^{m+3}dx}{m+3}\right)}{m+2}\right)}{m+1}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^3}{m+1}-\frac{3b\left(\frac{x^{m+2}\coth^{-1}(\tanh(a+bx))^2}{m+2}-\frac{2b\left(\frac{x^{m+3}\coth^{-1}(\tanh(a+bx))}{m+3}-\frac{bx^{m+4}}{(m+3)(m+4)}\right)}{m+2}\right)}{m+1}$$

input
```
Int[x^m*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
(x^(1 + m)*ArcCoth[Tanh[a + b*x]]^3)/(1 + m) - (3*b*((x^(2 + m)*ArcCoth[Ta
nh[a + b*x]]^2)/(2 + m) - (2*b*(-((b*x^(4 + m))/((3 + m)*(4 + m))) + (x^(3
 + m)*ArcCoth[Tanh[a + b*x]])/(3 + m)))/(2 + m)))/(1 + m)
```

3.146.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.146.4 Maple [A] (verified)

Time = 6.03 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.79

method	result
parallelrisch	$-\frac{36b\,\mathrm{arccoth}(\tanh(bx+a))^2 x^m x^2-24b^2\,\mathrm{arccoth}(\tanh(bx+a))x^m x^3-x\,x^m\,\mathrm{arccoth}(\tanh(bx+a))^3 m^3-9x\,x^m\,\mathrm{arccoth}(\tanh(b$
risch	Expression too large to display

input
```
int(x^m*arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
-(36*b*arccoth(tanh(b*x+a))^2*x^m*x^2-24*b^2*arccoth(tanh(b*x+a))*x^m*x^3-
x*x^m*arccoth(tanh(b*x+a))^3*m^3-9*x*x^m*arccoth(tanh(b*x+a))^3*m^2-26*x*x
^m*arccoth(tanh(b*x+a))^3*m+6*b^3*x^m*x^4-24*arccoth(tanh(b*x+a))^3*x*x^m-
6*x^3*x^m*arccoth(tanh(b*x+a))*b^2*m+3*x^2*x^m*arccoth(tanh(b*x+a))^2*b*m^
2+21*x^2*x^m*arccoth(tanh(b*x+a))^2*b*m)/(1+m)/(m^3+9*m^2+26*m+24)
```

3.146.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 627, normalized size of antiderivative = 5.70

$$\int x^m \coth^{-1}(\tanh(a+bx))^3\,dx$$
$$= \frac{(-i\,\pi^3(m^3+9\,m^2+26\,m+24)x+8\,(b^3m^3+6\,b^3m^2+11\,b^3m+6\,b^3)x^4+24\,(ab^2m^3+7\,ab^2m^2+14\,ab^2}{}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/8*((-I*pi^3*(m^3 + 9*m^2 + 26*m + 24)*x + 8*(b^3*m^3 + 6*b^3*m^2 + 11*b^
3*m + 6*b^3)*x^4 + 24*(a*b^2*m^3 + 7*a*b^2*m^2 + 14*a*b^2*m + 8*a*b^2)*x^3
 - 6*pi^2*((b*m^3 + 8*b*m^2 + 19*b*m + 12*b)*x^2 + (a*m^3 + 9*a*m^2 + 26*a
*m + 24*a)*x) + 24*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12*a^2*b)*x^2 +
 12*I*pi*((b^2*m^3 + 7*b^2*m^2 + 14*b^2*m + 8*b^2)*x^3 + 2*(a*b*m^3 + 8*a*
b*m^2 + 19*a*b*m + 12*a*b)*x^2 + (a^2*m^3 + 9*a^2*m^2 + 26*a^2*m + 24*a^2)
*x) + 8*(a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*cosh(m*log(x)) + (-I*
pi^3*(m^3 + 9*m^2 + 26*m + 24)*x + 8*(b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b
^3)*x^4 + 24*(a*b^2*m^3 + 7*a*b^2*m^2 + 14*a*b^2*m + 8*a*b^2)*x^3 - 6*pi^2
*((b*m^3 + 8*b*m^2 + 19*b*m + 12*b)*x^2 + (a*m^3 + 9*a*m^2 + 26*a*m + 24*a
)*x) + 24*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12*a^2*b)*x^2 + 12*I*pi*
((b^2*m^3 + 7*b^2*m^2 + 14*b^2*m + 8*b^2)*x^3 + 2*(a*b*m^3 + 8*a*b*m^2 + 1
9*a*b*m + 12*a*b)*x^2 + (a^2*m^3 + 9*a^2*m^2 + 26*a^2*m + 24*a^2)*x) + 8*(
a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*sinh(m*log(x)))/(m^4 + 10*m^3
+ 35*m^2 + 50*m + 24)
```

3.146.6 Sympy [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))^3\,dx$$
$$= \begin{cases} b^3\log(x) - \frac{b^2\,\mathrm{acoth}\,(\tanh\,(a+bx))}{x} - \frac{b\,\mathrm{acoth}^2\,(\tanh\,(a+bx))}{2x^2} - \frac{\mathrm{acoth}^3\,(\tanh\,(a+bx))}{3x^3} \\ \int \frac{\mathrm{acoth}^3\,(\tanh\,(a+bx))}{x^3}\,dx \\ \int \frac{\mathrm{acoth}^3\,(\tanh\,(a+bx))}{x^2}\,dx \\ \int \frac{\mathrm{acoth}^3\,(\tanh\,(a+bx))}{x}\,dx \\ -\frac{6b^3x^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{6b^2mx^3x^m\,\mathrm{acoth}\,(\tanh\,(a+bx))}{m^4+10m^3+35m^2+50m+24} + \frac{24b^2x^3x^m\,\mathrm{acoth}\,(\tanh\,(a+bx))}{m^4+10m^3+35m^2+50m+24} - \frac{3bm^2x^2x^m\,\mathrm{acoth}^2\,(\tanh\,(a+bx))}{m^4+10m^3+35m^2+50m+24} \end{cases}$$

input
```
integrate(x**m*acoth(tanh(b*x+a))**3,x)
```

output
```
Piecewise((b**3*log(x) - b**2*acoth(tanh(a + b*x))/x - b*acoth(tanh(a + b*
x))**2/(2*x**2) - acoth(tanh(a + b*x))**3/(3*x**3), Eq(m, -4)), (Integral(
acoth(tanh(a + b*x))**3/x**3, x), Eq(m, -3)), (Integral(acoth(tanh(a + b*x
))**3/x**2, x), Eq(m, -2)), (Integral(acoth(tanh(a + b*x))**3/x, x), Eq(m,
 -1)), (-6*b**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**2*
m*x**3*x**m*acoth(tanh(a + b*x))/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
24*b**2*x**3*x**m*acoth(tanh(a + b*x))/(m**4 + 10*m**3 + 35*m**2 + 50*m +
24) - 3*b*m**2*x**2*x**m*acoth(tanh(a + b*x))**2/(m**4 + 10*m**3 + 35*m**2
 + 50*m + 24) - 21*b*m*x**2*x**m*acoth(tanh(a + b*x))**2/(m**4 + 10*m**3 +
 35*m**2 + 50*m + 24) - 36*b*x**2*x**m*acoth(tanh(a + b*x))**2/(m**4 + 10*
m**3 + 35*m**2 + 50*m + 24) + m**3*x*x**m*acoth(tanh(a + b*x))**3/(m**4 +
10*m**3 + 35*m**2 + 50*m + 24) + 9*m**2*x*x**m*acoth(tanh(a + b*x))**3/(m*
*4 + 10*m**3 + 35*m**2 + 50*m + 24) + 26*m*x*x**m*acoth(tanh(a + b*x))**3/
(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*x*x**m*acoth(tanh(a + b*x))**3
/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24), True))
```

3.146.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.99

$$\int x^m \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{3\,bx^2x^m\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{(m+2)(m+1)} + \frac{x^{m+1}\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^3}{m+1} - \frac{6\left(\frac{b^2x^4x^m}{(m+4)(m+3)(m+2)} - \frac{bx^3x^m\,\mathrm{arcoth}(\tanh(bx+a))}{(m+3)(m+2)}\right)b}{m+1}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-3*b*x^2*x^m*arccoth(tanh(b*x + a))^2/((m + 2)*(m + 1)) + x^(m + 1)*arccot
h(tanh(b*x + a))^3/(m + 1) - 6*(b^2*x^4*x^m/((m + 4)*(m + 3)*(m + 2)) - b*
x^3*x^m*arccoth(tanh(b*x + a))/((m + 3)*(m + 2)))*b/(m + 1)
```

3.146.8 Giac [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))^3\,dx = \int x^m \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^3\,dx$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
integrate(x^m*arccoth(tanh(b*x + a))^3, x)
```

3.146.9 Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.02

$$\begin{aligned}\int x^m \coth^{-1}(\tanh(a+bx))^3\,dx &= \frac{8\,b^3\,x^m\,x^4\,(m^3+6\,m^2+11\,m+6)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192} \\ &\quad - \frac{x\,x^m\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^3(m^3+9\,m^2+26\,m+24)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192} \\ &\quad - \frac{12\,b^2\,x^m\,x^3\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)(m^3+7\,m^2+14\,m+8)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192} \\ &\quad + \frac{6\,b\,x^m\,x^2\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2(m^3+8\,m^2+19\,m+12)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192}\end{aligned}$$

input
```
int(x^m*acoth(tanh(a + b*x))^3,x)
```

output
```
(8*b^3*x^m*x^4*(11*m + 6*m^2 + m^3 + 6))/(400*m + 280*m^2 + 80*m^3 + 8*m^4
 + 192) - (x*x^m*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3*(26*m + 9*m^2 + m^3 + 24))/(
400*m + 280*m^2 + 80*m^3 + 8*m^4 + 192) - (12*b^2*x^m*x^3*(log(-2/(exp(2*a
)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)
) + 2*b*x)*(14*m + 7*m^2 + m^3 + 8))/(400*m + 280*m^2 + 80*m^3 + 8*m^4 + 1
92) + (6*b*x^m*x^2*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2*(19*m + 8*m^2 + m^3 + 12))
/(400*m + 280*m^2 + 80*m^3 + 8*m^4 + 192)
```

3.147 $\int x^4 \coth^{-1}(\tanh(a+bx))^3 \, dx$

3.147.1 Optimal result

Integrand size = 13, antiderivative size = 61

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3 \, dx = -\frac{1}{280}b^3x^8 + \frac{1}{35}b^2x^7\coth^{-1}(\tanh(a+bx)) - \frac{1}{10}bx^6\coth^{-1}(\tanh(a+bx))^2 + \frac{1}{5}x^5\coth^{-1}(\tanh(a+bx))^3$$

output
```
-1/280*b^3*x^8+1/35*b^2*x^7*arccoth(tanh(b*x+a))-1/10*b*x^6*arccoth(tanh(b
*x+a))^2+1/5*x^5*arccoth(tanh(b*x+a))^3
```

3.147.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3 \, dx = -\frac{1}{280}x^5\left(b^3x^3 - 8b^2x^2\coth^{-1}(\tanh(a+bx)) + 28bx\coth^{-1}(\tanh(a+bx))^2 - 56\coth^{-1}(\tanh(a+bx))^3\right)$$

input
```
Integrate[x^4*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/280*(x^5*(b^3*x^3 - 8*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 28*b*x*ArcCoth[T
anh[a + b*x]]^2 - 56*ArcCoth[Tanh[a + b*x]]^3))
```

3.147.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{5}x^5\coth^{-1}(\tanh(a+bx))^3-\frac{3}{5}b\int x^5\coth^{-1}(\tanh(a+bx))^2dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{5}x^5\coth^{-1}(\tanh(a+bx))^3-\frac{3}{5}b\left(\frac{1}{6}x^6\coth^{-1}(\tanh(a+bx))^2-\frac{1}{3}b\int x^6\coth^{-1}(\tanh(a+bx))dx\right)$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{5}x^5\coth^{-1}(\tanh(a+bx))^3-\frac{3}{5}b\left(\frac{1}{6}x^6\coth^{-1}(\tanh(a+bx))^2-\frac{1}{3}b\left(\frac{1}{7}x^7\coth^{-1}(\tanh(a+bx))-\frac{b\int x^7dx}{7}\right)\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{5}x^5\coth^{-1}(\tanh(a+bx))^3-\frac{3}{5}b\left(\frac{1}{6}x^6\coth^{-1}(\tanh(a+bx))^2-\frac{1}{3}b\left(\frac{1}{7}x^7\coth^{-1}(\tanh(a+bx))-\frac{bx^8}{56}\right)\right)$$

input
```
Int[x^4*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
(x^5*ArcCoth[Tanh[a + b*x]]^3)/5 - (3*b*((x^6*ArcCoth[Tanh[a + b*x]]^2)/6
- (b*(-1/56*(b*x^8) + (x^7*ArcCoth[Tanh[a + b*x]])/7))/3))/5
```

3.147.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.147.4 Maple [F(-1)]

Timed out.

$$\int x^4 \operatorname{arccoth}(\tanh(bx+a))^3\,dx$$

input
```
int(x^4*arccoth(tanh(b*x+a))^3,x)
```

output
```
int(x^4*arccoth(tanh(b*x+a))^3,x)
```

3.147.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{1}{8}b^3x^8 + \frac{3}{7}ab^2x^7 + \frac{1}{2}a^2bx^6 - \frac{1}{40}i\pi^3x^5 + \frac{1}{5}a^3x^5 - \frac{1}{40}\pi^2\left(5\,bx^6 + 6\,ax^5\right) + \frac{1}{70}i\pi\left(15\,b^2x^7 + 35\,abx^6 + 21\,a^2x^5\right)$$

input
```
integrate(x^4*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 - 1/40*I*pi^3*x^5 + 1/5*a^3*x^
5 - 1/40*pi^2*(5*b*x^6 + 6*a*x^5) + 1/70*I*pi*(15*b^2*x^7 + 35*a*b*x^6 + 2
1*a^2*x^5)
```

3.147.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{b^3x^8}{280} + \frac{b^2x^7\operatorname{acoth}(\tanh(a+bx))}{35} - \frac{bx^6\operatorname{acoth}^2(\tanh(a+bx))}{10} + \frac{x^5\operatorname{acoth}^3(\tanh(a+bx))}{5}$$

input
```
integrate(x**4*acoth(tanh(b*x+a))**3,x)
```

output
```
-b**3*x**8/280 + b**2*x**7*acoth(tanh(a + b*x))/35 - b*x**6*acoth(tanh(a +
 b*x))**2/10 + x**5*acoth(tanh(a + b*x))**3/5
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{1}{10}\,bx^6\operatorname{arcoth}(\tanh(bx+a))^2 + \frac{1}{5}\,x^5\operatorname{arcoth}(\tanh(bx+a))^3 - \frac{1}{280}\left(b^2x^8 - 8\,bx^7\operatorname{arcoth}(\tanh(bx+a))\right)b$$

input
```
integrate(x^4*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-1/10*b*x^6*arccoth(tanh(b*x + a))^2 + 1/5*x^5*arccoth(tanh(b*x + a))^3 -
1/280*(b^2*x^8 - 8*b*x^7*arccoth(tanh(b*x + a)))*b
```

3.147.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3 \, dx = \frac{1}{8} b^3 x^8 - \frac{3}{14} \left(-i \pi b^2 - 2 ab^2\right) x^7 - \frac{1}{8} \left(\pi^2 b - 4i \pi ab - 4 a^2 b\right) x^6 - \frac{1}{40} \left(i \pi^3 + 6 \pi^2 a - 12i \pi a^2 - 8 a^3\right) x^5$$

input
```
integrate(x^4*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
1/8*b^3*x^8 - 3/14*(-I*pi*b^2 - 2*a*b^2)*x^7 - 1/8*(pi^2*b - 4*I*pi*a*b -
4*a^2*b)*x^6 - 1/40*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*x^5
```

3.147.9 Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3 \, dx = -\frac{b^3 x^8}{280} + \frac{b^2 x^7 \operatorname{acoth}(\tanh(a+b x))}{35} - \frac{b x^6 \operatorname{acoth}(\tanh(a+b x))^2}{10} + \frac{x^5 \operatorname{acoth}(\tanh(a+b x))^3}{5}$$

input
```
int(x^4*acoth(tanh(a + b*x))^3,x)
```

output
```
(x^5*acoth(tanh(a + b*x))^3)/5 - (b^3*x^8)/280 - (b*x^6*acoth(tanh(a + b*x
))^2)/10 + (b^2*x^7*acoth(tanh(a + b*x)))/35
```

3.148 $\int x^3 \coth^{-1}(\tanh(a+bx))^3 \, dx$

3.148.1 Optimal result

Integrand size = 13, antiderivative size = 61

$$\int x^3 \coth^{-1}(\tanh(a+bx))^3 \, dx = -\frac{1}{140}b^3x^7 + \frac{1}{20}b^2x^6 \coth^{-1}(\tanh(a+bx)) - \frac{3}{20}bx^5 \coth^{-1}(\tanh(a+bx))^2 + \frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx))^3$$

output
```
-1/140*b^3*x^7+1/20*b^2*x^6*arccoth(tanh(b*x+a))-3/20*b*x^5*arccoth(tanh(b
*x+a))^2+1/4*x^4*arccoth(tanh(b*x+a))^3
```

3.148.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int x^3 \coth^{-1}(\tanh(a+bx))^3 \, dx = -\frac{1}{140}x^4\left(b^3x^3 - 7b^2x^2 \coth^{-1}(\tanh(a+bx)) + 21bx \coth^{-1}(\tanh(a+bx))^2 - 35 \coth^{-1}(\tanh(a+bx))^3\right)$$

input
```
Integrate[x^3*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/140*(x^4*(b^3*x^3 - 7*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 21*b*x*ArcCoth[T
anh[a + b*x]]^2 - 35*ArcCoth[Tanh[a + b*x]]^3))
```

3.148.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2599, 2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(\tanh(a+bx))^3\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^3-\frac{3}{4}b\int x^4\coth^{-1}(\tanh(a+bx))^2dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^3-\frac{3}{4}b\left(\frac{1}{5}x^5\coth^{-1}(\tanh(a+bx))^2-\frac{2}{5}b\int x^5\coth^{-1}(\tanh(a+bx))dx\right)$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^3-\frac{3}{4}b\left(\frac{1}{5}x^5\coth^{-1}(\tanh(a+bx))^2-\frac{2}{5}b\left(\frac{1}{6}x^6\coth^{-1}(\tanh(a+bx))-\frac{b\int x^6dx}{6}\right)\right)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^3-\frac{3}{4}b\left(\frac{1}{5}x^5\coth^{-1}(\tanh(a+bx))^2-\frac{2}{5}b\left(\frac{1}{6}x^6\coth^{-1}(\tanh(a+bx))-\frac{bx^7}{42}\right)\right)$$

input
```
Int[x^3*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
(x^4*ArcCoth[Tanh[a + b*x]]^3)/4 - (3*b*((x^5*ArcCoth[Tanh[a + b*x]]^2)/5
- (2*b*(-1/42*(b*x^7) + (x^6*ArcCoth[Tanh[a + b*x]])/6))/5))/4
```

3.148.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.148.4 Maple [F(-1)]

Timed out.

$$\int x^3 \operatorname{arccoth}(\tanh(bx+a))^3 \, dx$$

input
```
int(x^3*arccoth(tanh(b*x+a))^3,x)
```

output
```
int(x^3*arccoth(tanh(b*x+a))^3,x)
```

3.148.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\begin{aligned}\int x^3 \coth^{-1}(\tanh(a+bx))^3 \, dx = &\frac{1}{7} b^3x^7 + \frac{1}{2} ab^2x^6 + \frac{3}{5} a^2bx^5 - \frac{1}{32} i \pi^3x^4 \\ &+ \frac{1}{4} a^3x^4 - \frac{3}{80} \pi^2 \left(4 bx^5 + 5 ax^4\right) \\ &+ \frac{1}{40} i \pi \left(10 b^2x^6 + 24 abx^5 + 15 a^2x^4\right)\end{aligned}$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 - 1/32*I*pi^3*x^4 + 1/4*a^3*x^
4 - 3/80*pi^2*(4*b*x^5 + 5*a*x^4) + 1/40*I*pi*(10*b^2*x^6 + 24*a*b*x^5 + 1
5*a^2*x^4)
```

3.148.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int x^3 \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{b^3x^7}{140} + \frac{b^2x^6\operatorname{acoth}(\tanh(a+bx))}{20} - \frac{3bx^5\operatorname{acoth}^2(\tanh(a+bx))}{20} + \frac{x^4\operatorname{acoth}^3(\tanh(a+bx))}{4}$$

input
```
integrate(x**3*acoth(tanh(b*x+a))**3,x)
```

output
```
-b**3*x**7/140 + b**2*x**6*acoth(tanh(a + b*x))/20 - 3*b*x**5*acoth(tanh(a
 + b*x))**2/20 + x**4*acoth(tanh(a + b*x))**3/4
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int x^3 \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{3}{20}\,bx^5\operatorname{arcoth}(\tanh(bx+a))^2 + \frac{1}{4}\,x^4\operatorname{arcoth}(\tanh(bx+a))^3 - \frac{1}{140}\left(b^2x^7 - 7\,bx^6\operatorname{arcoth}(\tanh(bx+a))\right)b$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-3/20*b*x^5*arccoth(tanh(b*x + a))^2 + 1/4*x^4*arccoth(tanh(b*x + a))^3 -
1/140*(b^2*x^7 - 7*b*x^6*arccoth(tanh(b*x + a)))*b
```

3.148.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int x^3 \coth^{-1}(\tanh(a+bx))^3 \, dx = \frac{1}{7} b^3 x^7 - \frac{1}{4} \left(-i \pi b^2 - 2 ab^2\right) x^6 - \frac{3}{20} \left(\pi^2 b - 4i \pi ab - 4 a^2 b\right) x^5 - \frac{1}{32} \left(i \pi^3 + 6 \pi^2 a - 12i \pi a^2 - 8 a^3\right) x^4$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
1/7*b^3*x^7 - 1/4*(-I*pi*b^2 - 2*a*b^2)*x^6 - 3/20*(pi^2*b - 4*I*pi*a*b -
4*a^2*b)*x^5 - 1/32*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*x^4
```

3.148.9 Mupad [B] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int x^3 \coth^{-1}(\tanh(a+bx))^3 \, dx = -\frac{b^3 x^7}{140} + \frac{b^2 x^6 \operatorname{acoth}(\tanh(a+b x))}{20} - \frac{3 b x^5 \operatorname{acoth}(\tanh(a+b x))^2}{20} + \frac{x^4 \operatorname{acoth}(\tanh(a+b x))^3}{4}$$

input
```
int(x^3*acoth(tanh(a + b*x))^3,x)
```

output
```
(x^4*acoth(tanh(a + b*x))^3)/4 - (b^3*x^7)/140 - (3*b*x^5*acoth(tanh(a + b
*x))^2)/20 + (b^2*x^6*acoth(tanh(a + b*x)))/20
```

3.149 $\int x^2 \coth^{-1}(\tanh(a+bx))^3\,dx$

3.149.1 Optimal result

Integrand size = 13, antiderivative size = 53

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{x^2 \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{x \coth^{-1}(\tanh(a+bx))^5}{10b^2} + \frac{\coth^{-1}(\tanh(a+bx))^6}{60b^3}$$

output
```
1/4*x^2*arccoth(tanh(b*x+a))^4/b-1/10*x*arccoth(tanh(b*x+a))^5/b^2+1/60*ar
ccoth(tanh(b*x+a))^6/b^3
```

3.149.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{1}{60}x^3\left(b^3x^3 - 6b^2x^2 \coth^{-1}(\tanh(a+bx)) + 15bx \coth^{-1}(\tanh(a+bx))^2 - 20 \coth^{-1}(\tanh(a+bx))^3\right)$$

input
```
Integrate[x^2*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/60*(x^3*(b^3*x^3 - 6*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 15*b*x*ArcCoth[Ta
nh[a + b*x]]^2 - 20*ArcCoth[Tanh[a + b*x]]^3))
```

3.149.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3 \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^2 \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\int x \coth^{-1}(\tanh(a+bx))^4 dx}{2b}$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^2 \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\frac{x \coth^{-1}(\tanh(a+bx))^5}{5b} - \frac{\int \coth^{-1}(\tanh(a+bx))^5 dx}{5b}}{2b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{x^2 \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\frac{x \coth^{-1}(\tanh(a+bx))^5}{5b} - \frac{\int \coth^{-1}(\tanh(a+bx))^5 d \coth^{-1}(\tanh(a+bx))}{5b^2}}{2b}$$

$$\Big\downarrow \text{15}$$

$$\frac{x^2 \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\frac{x \coth^{-1}(\tanh(a+bx))^5}{5b} - \frac{\coth^{-1}(\tanh(a+bx))^6}{30b^2}}{2b}$$

input
```
Int[x^2*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
(x^2*ArcCoth[Tanh[a + b*x]]^4)/(4*b) - ((x*ArcCoth[Tanh[a + b*x]]^5)/(5*b)
 - ArcCoth[Tanh[a + b*x]]^6/(30*b^2))/(2*b)
```

3.149.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.149.4 Maple [F(-1)]

Timed out.

$$\int x^2 \operatorname{arccoth}(\tanh(bx+a))^3 \, dx$$

input
```
int(x^2*arccoth(tanh(b*x+a))^3,x)
```

output
```
int(x^2*arccoth(tanh(b*x+a))^3,x)
```

3.149.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.66

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3 \, dx = \frac{1}{6} b^3x^6 + \frac{3}{5} ab^2x^5 + \frac{3}{4} a^2bx^4 - \frac{1}{24} i\pi^3x^3 + \frac{1}{3} a^3x^3 - \frac{1}{16} \pi^2 (3bx^4 + 4ax^3) + \frac{1}{20} i\pi (6b^2x^5 + 15abx^4 + 10a^2x^3)$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 - 1/24*I*pi^3*x^3 + 1/3*a^3*x^
3 - 1/16*pi^2*(3*b*x^4 + 4*a*x^3) + 1/20*I*pi*(6*b^2*x^5 + 15*a*b*x^4 + 10
*a^2*x^3)
```

3.149.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3 \, dx = -\frac{b^3x^6}{60} + \frac{b^2x^5 \operatorname{acoth}(\tanh(a+bx))}{10} - \frac{bx^4 \operatorname{acoth}^2(\tanh(a+bx))}{4} + \frac{x^3 \operatorname{acoth}^3(\tanh(a+bx))}{3}$$

input
```
integrate(x**2*acoth(tanh(b*x+a))**3,x)
```

output
```
-b**3*x**6/60 + b**2*x**5*acoth(tanh(a + b*x))/10 - b*x**4*acoth(tanh(a +
b*x))**2/4 + x**3*acoth(tanh(a + b*x))**3/3
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3 \, dx = -\frac{1}{4} bx^4 \operatorname{arcoth}(\tanh(bx+a))^2 + \frac{1}{3} x^3 \operatorname{arcoth}(\tanh(bx+a))^3 - \frac{1}{60} \left(b^2x^6 - 6\, bx^5 \operatorname{arcoth}(\tanh(bx+a))\right) b$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-1/4*b*x^4*arccoth(tanh(b*x + a))^2 + 1/3*x^3*arccoth(tanh(b*x + a))^3 - 1
/60*(b^2*x^6 - 6*b*x^5*arccoth(tanh(b*x + a)))*b
```

3.149.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{1}{6}\,b^3x^6 - \frac{3}{10}\left(-i\,\pi b^2 - 2\,ab^2\right)x^5 - \frac{3}{16}\left(\pi^2 b - 4i\,\pi ab - 4\,a^2 b\right)x^4 - \frac{1}{24}\left(i\,\pi^3 + 6\,\pi^2 a - 12i\,\pi a^2 - 8\,a^3\right)x^3$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
1/6*b^3*x^6 - 3/10*(-I*pi*b^2 - 2*a*b^2)*x^5 - 3/16*(pi^2*b - 4*I*pi*a*b -
 4*a^2*b)*x^4 - 1/24*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*x^3
```

3.149.9 Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{b^3\,x^6}{60} + \frac{b^2\,x^5\,\mathrm{acoth}(\tanh(a+b\,x))}{10} - \frac{b\,x^4\,\mathrm{acoth}(\tanh(a+b\,x))^2}{4} + \frac{x^3\,\mathrm{acoth}(\tanh(a+b\,x))^3}{3}$$

input
```
int(x^2*acoth(tanh(a + b*x))^3,x)
```

output
```
(x^3*acoth(tanh(a + b*x))^3)/3 - (b^3*x^6)/60 - (b*x^4*acoth(tanh(a + b*x)
)^2)/4 + (b^2*x^5*acoth(tanh(a + b*x)))/10
```

3.150 $\int x \coth^{-1}(\tanh(a+bx))^3 \, dx$

3.150.1 Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x \coth^{-1}(\tanh(a+bx))^3 \, dx = \frac{x \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\coth^{-1}(\tanh(a+bx))^5}{20b^2}$$

output
```
1/4*x*arccoth(tanh(b*x+a))^4/b-1/20*arccoth(tanh(b*x+a))^5/b^2
```

3.150.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 99 vs. $2(34) = 68$.

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.91

$$\int x \coth^{-1}(\tanh(a+bx))^3 \, dx = \frac{(a+bx)\left((4a-bx)(a+bx)^3 - 5(3a-bx)(a+bx)^2\coth^{-1}(\tanh(a+bx)) + 10(2a^2+abx-b^2x^2)\coth^{-1}\right.}{20b^2}$$

input
```
Integrate[x*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
((a + b*x)*((4*a - b*x)*(a + b*x)^3 - 5*(3*a - b*x)*(a + b*x)^2*ArcCoth[Ta
nh[a + b*x]] + 10*(2*a^2 + a*b*x - b^2*x^2)*ArcCoth[Tanh[a + b*x]]^2 - 10*
(a - b*x)*ArcCoth[Tanh[a + b*x]]^3))/(20*b^2)
```

3.150.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(\tanh(a+bx))^3 \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\int \coth^{-1}(\tanh(a+bx))^4 dx}{4b}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\int \coth^{-1}(\tanh(a+bx))^4 d \coth^{-1}(\tanh(a+bx))}{4b^2}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\coth^{-1}(\tanh(a+bx))^5}{20b^2}$$

input
```
Int[x*ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
(x*ArcCoth[Tanh[a + b*x]]^4)/(4*b) - ArcCoth[Tanh[a + b*x]]^5/(20*b^2)
```

3.150.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.150.4 Maple [A] (verified)

Time = 25.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

method	result	size
parallelrisch	$-\frac{b^3x^5}{20}+\frac{x^2\operatorname{arccoth}(\tanh(bx+a))^3}{2}+\frac{b^2\operatorname{arccoth}(\tanh(bx+a))x^4}{4}-\frac{b\operatorname{arccoth}(\tanh(bx+a))^2x^3}{2}$	54
risch	Expression too large to display	18111

input
```
int(x*arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/20*b^3*x^5+1/2*x^2*arccoth(tanh(b*x+a))^3+1/4*b^2*arccoth(tanh(b*x+a))*
x^4-1/2*b*arccoth(tanh(b*x+a))^2*x^3
```

3.150.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.56

$$\int x\coth^{-1}(\tanh(a+bx))^3\,dx=\frac{1}{5}\,b^3x^5+\frac{3}{4}\,ab^2x^4+a^2bx^3-\frac{1}{16}i\,\pi^3x^2+\frac{1}{2}\,a^3x^2-\frac{1}{8}\,\pi^2\left(2\,bx^3+3\,ax^2\right)+\frac{1}{8}i\,\pi\left(3\,b^2x^4+8\,abx^3+6\,a^2x^2\right)$$

input
```
integrate(x*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 - 1/16*I*pi^3*x^2 + 1/2*a^3*x^2 -
1/8*pi^2*(2*b*x^3 + 3*a*x^2) + 1/8*I*pi*(3*b^2*x^4 + 8*a*b*x^3 + 6*a^2*x^2
)
```

3.150.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int x\coth^{-1}(\tanh(a+bx))^3\,dx = \begin{cases} \frac{x\,\mathrm{acoth}^4\left(\tanh\left(a+bx\right)\right)}{4b} - \frac{\mathrm{acoth}^5\left(\tanh\left(a+bx\right)\right)}{20b^2} & \text{for } b\neq 0 \\ \frac{x^2\,\mathrm{acoth}^3\left(\tanh\left(a\right)\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acoth(tanh(b*x+a))**3,x)
```

output
```
Piecewise((x*acoth(tanh(a + b*x))**4/(4*b) - acoth(tanh(a + b*x))**5/(20*b
**2), Ne(b, 0)), (x**2*acoth(tanh(a))**3/2, True))
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int x\coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{1}{2}\,bx^3\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2 + \frac{1}{2}\,x^2\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^3 - \frac{1}{20}\left(b^2x^5 - 5\,bx^4\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)\right)b$$

input
```
integrate(x*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-1/2*b*x^3*arccoth(tanh(b*x + a))^2 + 1/2*x^2*arccoth(tanh(b*x + a))^3 - 1
/20*(b^2*x^5 - 5*b*x^4*arccoth(tanh(b*x + a)))*b
```

3.150.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.26

$$\int x\coth^{-1}(\tanh(a+bx))^3\,dx = \frac{1}{5}\,b^3x^5 - \frac{3}{8}\left(-i\,\pi b^2 - 2\,ab^2\right)x^4 - \frac{1}{4}\left(\pi^2 b - 4i\,\pi ab - 4\,a^2b\right)x^3 - \frac{1}{16}\left(i\,\pi^3 + 6\,\pi^2 a - 12i\,\pi a^2 - 8\,a^3\right)x^2$$

input
```
integrate(x*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
1/5*b^3*x^5 - 3/8*(-I*pi*b^2 - 2*a*b^2)*x^4 - 1/4*(pi^2*b - 4*I*pi*a*b - 4
*a^2*b)*x^3 - 1/16*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*x^2
```

3.150.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int x\coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{b^3\,x^5}{20}+\frac{b^2\,x^4\,\mathrm{acoth}(\tanh(a+b\,x))}{4} - \frac{b\,x^3\,\mathrm{acoth}(\tanh(a+b\,x))^2}{2}+\frac{x^2\,\mathrm{acoth}(\tanh(a+b\,x))^3}{2}$$

input
```
int(x*acoth(tanh(a + b*x))^3,x)
```

output
```
(x^2*acoth(tanh(a + b*x))^3)/2 - (b^3*x^5)/20 - (b*x^3*acoth(tanh(a + b*x)
)^2)/2 + (b^2*x^4*acoth(tanh(a + b*x)))/4
```

3.151 $\int \coth^{-1}(\tanh(a+bx))^3\,dx$

3.151.1 Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{\coth^{-1}(\tanh(a+bx))^4}{4b}$$

output
```
1/4*arccoth(tanh(b*x+a))^4/b
```

3.151.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{\coth^{-1}(\tanh(a+bx))^4}{4b}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^4/(4*b)
```

3.151.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(\tanh(a+bx))^3 \, dx$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \coth^{-1}(\tanh(a+bx))^3 d\coth^{-1}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4b}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^4/(4*b)
```

3.151.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.151.4 Maple [A] (verified)

Time = 25.86 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\operatorname{arccoth}(\tanh(bx+a))^4}{4b}$
default	$\frac{\operatorname{arccoth}(\tanh(bx+a))^4}{4b}$
parallelrisch	$b^2 \operatorname{arccoth}(\tanh(bx+a))\, x^3 - \frac{3b \operatorname{arccoth}(\tanh(bx+a))^2 x^2}{2} + x \operatorname{arccoth}(\tanh(bx+a))^3 - \frac{b^3 x}{4}$
risch	Expression too large to display

input
```
int(arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*arccoth(tanh(b*x+a))^4/b
```

3.151.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 4.75

$$\int \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{1}{4}b^3x^4 + ab^2x^3 + \frac{3}{2}a^2bx^2 - \frac{1}{8}i\pi^3x + a^3x - \frac{3}{8}\pi^2\left(bx^2+2\,ax\right) + \frac{1}{2}i\pi\left(b^2x^3+3\,abx^2+3\,a^2x\right)$$

input
```
integrate(arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 - 1/8*I*pi^3*x + a^3*x - 3/8*pi^2*
(b*x^2 + 2*a*x) + 1/2*I*pi*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)
```

3.151.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \coth^{-1}(\tanh(a+bx))^3\,dx = \begin{cases} \frac{\operatorname{acoth}^4(\tanh(a+bx))}{4b} & \text{for } b \neq 0 \\ x\operatorname{acoth}^3(\tanh(a)) & \text{otherwise} \end{cases}$$

input
```
integrate(acoth(tanh(b*x+a))**3,x)
```

output
```
Piecewise((acoth(tanh(a + b*x))**4/(4*b), Ne(b, 0)), (x*acoth(tanh(a))**3,
 True))
```

3.151.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(14) = 28.

Time = 0.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int \coth^{-1}(\tanh(a+bx))^3\,dx = -\frac{3}{2}bx^2\operatorname{arcoth}(\tanh(bx+a))^2 + x\operatorname{arcoth}(\tanh(bx+a))^3 - \frac{1}{4}\left(b^2x^4 - 4bx^3\operatorname{arcoth}(\tanh(bx+a))\right)b$$

input
```
integrate(arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-3/2*b*x^2*arccoth(tanh(b*x + a))^2 + x*arccoth(tanh(b*x + a))^3 - 1/4*(b^
2*x^4 - 4*b*x^3*arccoth(tanh(b*x + a)))*b
```

3.151.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 4.69

$$\int \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{1}{4}b^3x^4 - \frac{1}{2}\left(-i\pi b^2 - 2ab^2\right)x^3 - \frac{3}{8}\left(\pi^2 b - 4i\pi ab - 4a^2b\right)x^2 - \frac{1}{8}\left(i\pi^3 + 6\pi^2 a - 12i\pi a^2 - 8a^3\right)x$$

input
```
integrate(arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
1/4*b^3*x^4 - 1/2*(-I*pi*b^2 - 2*a*b^2)*x^3 - 3/8*(pi^2*b - 4*I*pi*a*b - 4
*a^2*b)*x^2 - 1/8*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*x
```

3.151.9 Mupad [B] (verification not implemented)

Time = 3.83 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.94

$$\int \coth^{-1}(\tanh(a+bx))^3\,dx$$
$$= \frac{x\left(2\,\mathrm{acoth}(\tanh(a+b\,x))-b\,x\right)\left(b^2\,x^2-2\,b\,x\,\mathrm{acoth}(\tanh(a+b\,x))+2\,\mathrm{acoth}(\tanh(a+b\,x))^2\right)}{4}$$

input
```
int(acoth(tanh(a + b*x))^3,x)
```

output
```
(x*(2*acoth(tanh(a + b*x)) - b*x)*(2*acoth(tanh(a + b*x))^2 + b^2*x^2 - 2*
b*x*acoth(tanh(a + b*x))))/4
```

3.152 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x} dx$

3.152.1 Optimal result

Integrand size = 13, antiderivative size = 77

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x} dx = bx\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 - \frac{1}{2}\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2 + \frac{1}{3}\coth^{-1}(\tanh(a+bx))^3 - \left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\log(x)$$

output
```
b*x*(b*x-arccoth(tanh(b*x+a)))^2-1/2*(b*x-arccoth(tanh(b*x+a)))*arccoth(ta
nh(b*x+a))^2+1/3*arccoth(tanh(b*x+a))^3-(b*x-arccoth(tanh(b*x+a)))^3*ln(x)
```

3.152.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x} dx = \frac{1}{3}(a+bx)^3 + (a+bx)\left(a^2 - 3a\left(a+bx-\coth^{-1}(\tanh(a+bx))\right) + 3\left(a+bx-\coth^{-1}(\tanh(a+bx))\right)^2\right) - \frac{1}{2}(a+bx)^2\left(2a+3bx-3\coth^{-1}(\tanh(a+bx))\right) + \left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3\log(bx)$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^3/x,x]
```

output
```
(a + b*x)^3/3 + (a + b*x)*(a^2 - 3*a*(a + b*x - ArcCoth[Tanh[a + b*x]]) +
3*(a + b*x - ArcCoth[Tanh[a + b*x]])^2) - ((a + b*x)^2*(2*a + 3*b*x - 3*Ar
cCoth[Tanh[a + b*x]]))/2 + (-(b*x) + ArcCoth[Tanh[a + b*x]])^3*Log[b*x]
```

3.152.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2590, 2590, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{1}{3}\coth^{-1}(\tanh(a+bx))^3-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{1}{3}\coth^{-1}(\tanh(a+bx))^3-$$
$$\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{\coth^{-1}(\tanh(a+bx))}{x}\right.$$

$$\Big\downarrow \text{2589}$$

$$\frac{1}{3}\coth^{-1}(\tanh(a+bx))^3-$$
$$\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(bx-\left(bx-\coth^{-1}(\tanh\right.\right.\right.$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{3}\coth^{-1}(\tanh(a+bx))^3-$$
$$\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(bx-\log(x)\left(bx-\coth^{-}\right.\right.\right.$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^3/x,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^3/3 - (b*x - ArcCoth[Tanh[a + b*x]])*(ArcCoth[Tanh[
a + b*x]]^2/2 - (b*x - ArcCoth[Tanh[a + b*x]])*(b*x - (b*x - ArcCoth[Tanh[
a + b*x]])*Log[x]))
```

3.152.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

3.152.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.29 (sec) , antiderivative size = 3294, normalized size of antiderivative = 42.78

method	result	size
risch	Expression too large to display	3294

input
```
int(arccoth(tanh(b*x+a))^3/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*ln(exp(b*x+a))^3-9/2*b^2*ln(exp(b*x+a))*x^2-3*b*ln(x)*ln(exp(b*x+a))
^2*x+11/6*b^3*x^3+3*b^2*ln(x)*ln(exp(b*x+a))*x^2+3*b*ln(exp(b*x+a))^2*x-b^
3*ln(x)*x^3+(-3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(
I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+3/4*Pi^2*c
sgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))^3-3/2*Pi^2*csgn(I*exp(b
*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I/(exp(2*b*x+2*a)+1))^2-3/4*Pi^2-3/4*
Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))-3/4*Pi^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))^2*csgn(I/(exp(2*b*x+2*a)+1))^2+3/4*Pi^2*csgn(I*ex
p(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)+1))^2+3/8*Pi^2*c
sgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-3/4*csgn
(I*exp(2*b*x+2*a))*csgn(I*exp(b*x+a))^2*Pi^2+3/2*csgn(I*exp(2*b*x+2*a))^2*
csgn(I*exp(b*x+a))*Pi^2-3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b
*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4-3/8*Pi^2*csgn(I*exp(b
*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^
3+3/4*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)+1))^3-3/16*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4+3/8*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-3/16*Pi^2*csgn(I*exp(b*x+a))^4*csg
n(I*exp(2*b*x+2*a))^2+3/4*Pi^2*csgn(I*exp(b*x+a))^3*csgn(I*exp(2*b*x+2*...
```

3.152.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx = \frac{1}{3}\,b^3x^3 + \frac{3}{2}\,ab^2x^2 - \frac{3}{4}\,\pi^2bx + 3\,a^2bx + \frac{3}{4}i\,\pi\left(b^2x^2 + 4\,abx\right) - \frac{1}{8}\left(i\,\pi^3 + 6\,\pi^2a - 12i\,\pi a^2 - 8\,a^3\right)\log\left(x\right)$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x,x, algorithm="fricas")
```

output
```
1/3*b^3*x^3 + 3/2*a*b^2*x^2 - 3/4*pi^2*b*x + 3*a^2*b*x + 3/4*I*pi*(b^2*x^2
 + 4*a*b*x) - 1/8*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*log(x)
```

3.152.6 Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx = \int \frac{\operatorname{acoth}^3(\tanh(a+bx))}{x}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))**3/x,x)
```

output
```
Integral(acoth(tanh(a + b*x))**3/x, x)
```

3.152.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx = \frac{1}{3}b^3x^3 - \frac{3}{4}\left(i\pi b^2 - 2ab^2\right)x^2 - \frac{3}{4}\left(\pi^2 b + 4i\pi ab - 4a^2 b\right)x + \frac{1}{8}\left(i\pi^3 - 6\pi^2 a - 12i\pi a^2 + 8a^3\right)\log(x)$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x,x, algorithm="maxima")
```

output
```
1/3*b^3*x^3 - 3/4*(I*pi*b^2 - 2*a*b^2)*x^2 - 3/4*(pi^2*b + 4*I*pi*a*b - 4*
a^2*b)*x + 1/8*(I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3)*log(x)
```

3.152.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx = \frac{1}{3}b^3x^3 - \frac{3}{4}\left(-i\pi b^2 - 2ab^2\right)x^2 - \frac{3}{4}\left(\pi^2 b - 4i\pi ab - 4a^2 b\right)x + \frac{1}{8}\left(-i\pi^3 - 6\pi^2 a + 12i\pi a^2 + 8a^3\right)\log(x)$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x,x, algorithm="giac")
```

output
```
1/3*b^3*x^3 - 3/4*(-I*pi*b^2 - 2*a*b^2)*x^2 - 3/4*(pi^2*b - 4*I*pi*a*b - 4
*a^2*b)*x + 1/8*(-I*pi^3 - 6*pi^2*a + 12*I*pi*a^2 + 8*a^3)*log(x)
```

3.152.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.97

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx = \frac{b^3\,x^3}{3} - \ln(x)\left(\frac{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^3}{8} - a^3 - \frac{3\,a\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2}{4} + \frac{3\,a^2\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2}\right) - \frac{3\,b^2\,x^2\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}{4} + \frac{3\,b\,x\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2}{4}$$

input
```
int(acoth(tanh(a + b*x))^3/x,x)
```

output
```
(b^3*x^3)/3 - log(x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3/8 - a^3 - (3*a*(2
*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(
2*a)*exp(2*b*x) - 1)) + 2*b*x)^2)/4 + (3*a^2*(2*a - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b
*x))/2) - (3*b^2*x^2*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/4 + (3*b*x*(log(-2/(exp(2
*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) -
1)) + 2*b*x)^2)/4
```

3.153 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2} dx$

3.153.1 Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2} dx = -3b^2x(bx-\coth^{-1}(\tanh(a+bx))) + \frac{3}{2}b\coth^{-1}(\tanh(a+bx))^2 - \frac{\coth^{-1}(\tanh(a+bx))^3}{x} + 3b(bx-\coth^{-1}(\tanh(a+bx)))^2\log(x)$$

output
```
-3*b^2*x*(b*x-arccoth(tanh(b*x+a)))+3/2*b*arccoth(tanh(b*x+a))^2-arccoth(t
anh(b*x+a))^3/x+3*b*(b*x-arccoth(tanh(b*x+a)))^2*ln(x)
```

3.153.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2} dx = -\frac{\coth^{-1}(\tanh(a+bx))^3}{x} - 6b^2x\coth^{-1}(\tanh(a+bx))\log(x) + 3b\coth^{-1}(\tanh(a+bx))^2(1+\log(x)) + \frac{3}{2}b^3x^2(-1+2\log(x))$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^3/x^2,x]
```

output
```
-(ArcCoth[Tanh[a + b*x]]^3/x) - 6*b^2*x*ArcCoth[Tanh[a + b*x]]*Log[x] + 3*
b*ArcCoth[Tanh[a + b*x]]^2*(1 + Log[x]) + (3*b^3*x^2*(-1 + 2*Log[x]))/2
```

3.153.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2590, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}dx$$

$$\Big\downarrow \text{2599}$$

$$3b\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}dx-\frac{\coth^{-1}(\tanh(a+bx))^3}{x}$$

$$\Big\downarrow \text{2590}$$

$$3b\left(\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{\coth^{-1}(\tanh(a+bx))}{x}dx\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{x}$$

$$\Big\downarrow \text{2589}$$

$$3b\left(\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(bx-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{x}dx\right)\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{x}$$

$$\Big\downarrow \text{14}$$

$$3b\left(\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(bx-\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)\right)\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{x}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^3/x^2,x]
```

output
```
-(ArcCoth[Tanh[a + b*x]]^3/x) + 3*b*(ArcCoth[Tanh[a + b*x]]^2/2 - (b*x - A
rcCoth[Tanh[a + b*x]])*(b*x - (b*x - ArcCoth[Tanh[a + b*x]])*Log[x]))
```

3.153.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.153.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 3248, normalized size of antiderivative = 47.76

method	result	size
risch	Expression too large to display	3248

input
```
int(arccoth(tanh(b*x+a))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*ln(exp(b*x+a))^3+3*ln(x)*ln(exp(b*x+a))^2*b+3*b^3*x^2*ln(x)-9/2*b^3*x
^2-6*b^2*ln(x)*ln(exp(b*x+a))*x+6*b^2*ln(exp(b*x+a))*x+(-3/4*Pi^2*csgn(I/(
exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csg
n(I*exp(2*b*x+2*a))^3-3/2*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2
*csgn(I/(exp(2*b*x+2*a)+1))^2-3/4*Pi^2-3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))
^4*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-3/4*Pi
^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*csgn
(I/(exp(2*b*x+2*a)+1))^2+3/4*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*
a))*csgn(I/(exp(2*b*x+2*a)+1))^2+3/8*Pi^2*csgn(I*exp(2*b*x+2*a))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-3/4*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(b
*x+a))^2*Pi^2+3/2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(b*x+a))*Pi^2-3/4*Pi^
2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/
(exp(2*b*x+2*a)+1))^4-3/8*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))
*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+3/4*Pi^2*csgn(I*exp(b*x+a))*c
sgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-3/16*P
i^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^4+3/8*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*
a)+1))^5-3/16*Pi^2*csgn(I*exp(b*x+a))^4*csgn(I*exp(2*b*x+2*a))^2+3/4*Pi^2*
csgn(I*exp(b*x+a))^3*csgn(I*exp(2*b*x+2*a))^3-3/4*Pi^2*csgn(I/(exp(2*b*...
```

3.153.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.16

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx = \frac{4\,b^3x^3+24\,ab^2x^2+i\,\pi^3+6\,\pi^2a-8\,a^3+12i\,\pi(b^2x^2-a^2)-6\,(\pi^2bx-4i\,\pi abx-4\,a^2bx)\log{(x)}}{8\,x}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^2,x, algorithm="fricas")
```

output
```
1/8*(4*b^3*x^3 + 24*a*b^2*x^2 + I*pi^3 + 6*pi^2*a - 8*a^3 + 12*I*pi*(b^2*x
^2 - a^2) - 6*(pi^2*b*x - 4*I*pi*a*b*x - 4*a^2*b*x)*log(x))/x
```

3.153.6 Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx = \int \frac{\operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}{x^2}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))**3/x**2,x)
```

output
```
Integral(acoth(tanh(a + b*x))**3/x**2, x)
```

3.153.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.82

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx = 3\,b\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2\log\left(x\right) - \frac{3}{2}\left(2\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2\log\left(x\right) - \left(bx^2 - 2\left(-i\pi - 2a\right)x + 2\left(-\frac{i\pi(bx+a)}{b} - \frac{(bx+a)^2}{b}\right)\log\left(x\right.\right.\right. - \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^3}{x}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^2,x, algorithm="maxima")
```

output
```
3*b*arccoth(tanh(b*x + a))^2*log(x) - 3/2*(2*arccoth(tanh(b*x + a))^2*log(
x) - (b*x^2 - 2*(-I*pi - 2*a)*x + 2*(-I*pi*(b*x + a)/b - (b*x + a)^2/b)*lo
g(x) + 2*arccoth(tanh(b*x + a))^2*log(x)/b + 2*(I*pi*a + a^2)*log(x)/b)*b)
*b - arccoth(tanh(b*x + a))^3/x
```

3.153.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx = \frac{1}{2}\,b^3x^2 - \frac{3}{2}\left(-i\,\pi b^2 - 2\,ab^2\right)x - \frac{3}{4}\left(\pi^2 b - 4i\,\pi ab - 4\,a^2 b\right)\log\left(x\right) - \frac{-i\,\pi^3 - 6\,\pi^2 a + 12i\,\pi a^2 + 8\,a^3}{8\,x}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^2,x, algorithm="giac")
```

output
```
1/2*b^3*x^2 - 3/2*(-I*pi*b^2 - 2*a*b^2)*x - 3/4*(pi^2*b - 4*I*pi*a*b - 4*a
^2*b)*log(x) - 1/8*(-I*pi^3 - 6*pi^2*a + 12*I*pi*a^2 + 8*a^3)/x
```

3.153.9 Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 372, normalized size of antiderivative = 5.47

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx$$

$$= \ln(x)\left(3a^2b + \frac{3b\left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) + 2bx\right)^2}{4} - 3ab\left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) + 2bx\right)\right)$$

$$+ \frac{\left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) + 2bx\right)^3 - 8a^3 - 6a\left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1} \cdots\right.\right.}{8x}$$

$$+ \frac{b^3x^2}{2} - \frac{3b^2x\left(\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + 2bx\right)}{2}$$

input
```
int(acoth(tanh(a + b*x))^3/x^2,x)
```

output
```
log(x)*(3*a^2*b + (3*b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2)/4 - 3*a*b*(2*a
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a
)*exp(2*b*x) - 1)) + 2*b*x)) + ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 8*a^3
 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log
(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1))
 + 2*b*x))/(8*x) + (b^3*x^2)/2 - (3*b^2*x*(log(-2/(exp(2*a)*exp(2*b*x) - 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/2
```

3.154 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3} dx$

3.154.1 Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3} dx = 3b^3x - \frac{3b\coth^{-1}(\tanh(a+bx))^2}{2x} - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} - 3b^2\left(bx - \coth^{-1}(\tanh(a+bx))\right)\log(x)$$

output
```
3*b^3*x-3/2*b*arccoth(tanh(b*x+a))^2/x-1/2*arccoth(tanh(b*x+a))^3/x^2-3*b^
2*(b*x-arccoth(tanh(b*x+a)))*ln(x)
```

3.154.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3} dx = b^3x - \frac{3b\left(-bx + \coth^{-1}(\tanh(a+bx))\right)^2}{x} - \frac{\left(-bx + \coth^{-1}(\tanh(a+bx))\right)^3}{2x^2} + 3b^2\left(-bx + \coth^{-1}(\tanh(a+bx))\right)\log(x)$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^3/x^3,x]
```

output
```
b^3*x - (3*b*(-(b*x) + ArcCoth[Tanh[a + b*x]])^2)/x - (-(b*x) + ArcCoth[Ta
nh[a + b*x]])^3/(2*x^2) + 3*b^2*(-(b*x) + ArcCoth[Tanh[a + b*x]])*Log[x]
```

3.154.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2599, 2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{3}{2}b\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}dx-\frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{3}{2}b\left(2b\int \frac{\coth^{-1}(\tanh(a+bx))}{x}dx-\frac{\coth^{-1}(\tanh(a+bx))^2}{x}\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2}$$

$$\Big\downarrow \text{ 2589}$$

$$\frac{3}{2}b\left(2b\left(bx-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{x}dx\right)-\frac{\coth^{-1}(\tanh(a+bx))^2}{x}\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{3}{2}b\left(2b\left(bx-\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)\right)-\frac{\coth^{-1}(\tanh(a+bx))^2}{x}\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^3/x^3,x]
```

output
```
-1/2*ArcCoth[Tanh[a + b*x]]^3/x^2 + (3*b*(-(ArcCoth[Tanh[a + b*x]]^2/x) +
2*b*(b*x - (b*x - ArcCoth[Tanh[a + b*x]])*Log[x])))/2
```

3.154.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.154.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.28 (sec) , antiderivative size = 3227, normalized size of antiderivative = 53.78

method	result	size
risch	Expression too large to display	3227

input
```
int(arccoth(tanh(b*x+a))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2*ln(exp(b*x+a))^3-3/2/x*ln(exp(b*x+a))^2*b-3*ln(x)*x*b^3+3*ln(exp(
b*x+a))*ln(x)*b^2+3*b^3*x+(-3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp
(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2+3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))^3-3/2*Pi^
2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I/(exp(2*b*x+2*a)+1))^2
-3/4*Pi^2-3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b*x+2*a))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-3/4*Pi^2*csgn(I*exp(2*b*x+2*a))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*csgn(I/(exp(2*b*x+2*a)+1))^2+3/4*
Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)+1)
)^2+3/8*Pi^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^5-3/4*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(b*x+a))^2*Pi^2+3/2*csgn(I*exp(
2*b*x+2*a))^2*csgn(I*exp(b*x+a))*Pi^2-3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4-3/8*Pi^
2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)+1))^3+3/4*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(
I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-3/16*Pi^2*csgn(I/(exp(2*b*x+2*a)+1)
)^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4+3/8*Pi^2*csgn(I/(exp(2*b*x
+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-3/16*Pi^2*csgn(I*exp
(b*x+a))^4*csgn(I*exp(2*b*x+2*a))^2+3/4*Pi^2*csgn(I*exp(b*x+a))^3*csgn(I*e
xp(2*b*x+2*a))^3-3/4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x...
```

3.154.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.28

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx$$
$$= \frac{16\,b^3x^3 - 48\,a^2bx + i\,\pi^3 + 6\,\pi^2(2\,bx + a) - 8\,a^3 - 12i\,\pi(4\,abx + a^2) - 24\,(-i\,\pi b^2x^2 - 2\,ab^2x^2)\log(x)}{16\,x^2}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^3,x, algorithm="fricas")
```

output
```
1/16*(16*b^3*x^3 - 48*a^2*b*x + I*pi^3 + 6*pi^2*(2*b*x + a) - 8*a^3 - 12*I
*pi*(4*a*b*x + a^2) - 24*(-I*pi*b^2*x^2 - 2*a*b^2*x^2)*log(x))/x^2
```

3.154.6 Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx = \int \frac{\operatorname{acoth}^3(\tanh(a+bx))}{x^3}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))**3/x**3,x)
```

output
```
Integral(acoth(tanh(a + b*x))**3/x**3, x)
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

$$\begin{aligned}&\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx\\&=3\left(b\operatorname{arcoth}(\tanh(bx+a))\log(x)-\left(b\left(x+\frac{a}{b}\right)\log(x)-b\left(x+\frac{a\log(x)}{b}\right)\right)b\right)b\\&\quad-\frac{3\,b\operatorname{arcoth}(\tanh(bx+a))^2}{2\,x}-\frac{\operatorname{arcoth}(\tanh(bx+a))^3}{2\,x^2}\end{aligned}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^3,x, algorithm="maxima")
```

output
```
3*(b*arccoth(tanh(b*x + a))*log(x) - (b*(x + a/b)*log(x) - b*(x + a*log(x)
/b))*b)*b - 3/2*b*arccoth(tanh(b*x + a))^2/x - 1/2*arccoth(tanh(b*x + a))^
3/x^2
```

3.154.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\begin{aligned}&\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx\\&=b^3x+\frac{3}{2}\left(i\pi b^2+2\,ab^2\right)\log(x)\\&\quad+\frac{12\,\pi^2bx-48i\,\pi abx-48\,a^2bx+i\,\pi^3+6\,\pi^2a-12i\,\pi a^2-8\,a^3}{16\,x^2}\end{aligned}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^3,x, algorithm="giac")
```

output
```
b^3*x + 3/2*(I*pi*b^2 + 2*a*b^2)*log(x) + 1/16*(12*pi^2*b*x - 48*I*pi*a*b*
x - 48*a^2*b*x + I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)/x^2
```

3.154.9 Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 383, normalized size of antiderivative = 6.38

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx = \frac{\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)^3}{16x^2} - \frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)^3}{16x^2} + \frac{9b^2\ln\left(\frac{e^{2bx}}{e^{2a}e^{2bx}-1}\right)}{4} - \frac{9b^2\ln\left(\frac{1}{e^{2a}e^{2bx}-1}\right)}{4} - \frac{3b^3x}{2} - \frac{3b\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)^2}{8x} + \frac{3b^2\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)\ln(x)}{2} - 3b^3x\ln(x) - \frac{3b\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)^2}{8x} - \frac{3b^2\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)\ln(x)}{2} - \frac{3\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)^2}{16x^2} + \frac{3\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)^2\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)}{16x^2} + \frac{3b\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)}{4x}$$

input
```
int(acoth(tanh(a + b*x))^3/x^3,x)
```

output
```
log(-2/(exp(2*a)*exp(2*b*x) - 1))^3/(16*x^2) - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) - 1))^3/(16*x^2) + (9*b^2*log(exp(2*b*x)/(exp(2*a)*e
xp(2*b*x) - 1)))/4 - (9*b^2*log(1/(exp(2*a)*exp(2*b*x) - 1)))/4 - (3*b^3*x
)/2 - (3*b*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2)/(8*x)
 + (3*b^2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*log(x))/2
 - 3*b^3*x*log(x) - (3*b*log(-2/(exp(2*a)*exp(2*b*x) - 1))^2)/(8*x) - (3*b
^2*log(-2/(exp(2*a)*exp(2*b*x) - 1))*log(x))/2 - (3*log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1))*log(-2/(exp(2*a)*exp(2*b*x) - 1))^2)/(16*
x^2) + (3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2*log(-2/
(exp(2*a)*exp(2*b*x) - 1)))/(16*x^2) + (3*b*log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) - 1))*log(-2/(exp(2*a)*exp(2*b*x) - 1)))/(4*x)
```

3.154. $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx$

3.155 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4} dx$

3.155.1 Optimal result

Integrand size = 13, antiderivative size = 55

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4} dx = -\frac{b^2\coth^{-1}(\tanh(a+bx))}{x} - \frac{b\coth^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3} + b^3\log(x)$$

output
```
-b^2*arccoth(tanh(b*x+a))/x-1/2*b*arccoth(tanh(b*x+a))^2/x^2-1/3*arccoth(t
anh(b*x+a))^3/x^3+b^3*ln(x)
```

3.155.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4} dx = \frac{-6b^2x^2\coth^{-1}(\tanh(a+bx)) - 3bx\coth^{-1}(\tanh(a+bx))^2 - 2\coth^{-1}(\tanh(a+bx))^3 + b^3x^3(11+6\log(x))}{6x^3}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^3/x^4,x]
```

output
```
(-6*b^2*x^2*ArcCoth[Tanh[a + b*x]] - 3*b*x*ArcCoth[Tanh[a + b*x]]^2 - 2*Ar
cCoth[Tanh[a + b*x]]^3 + b^3*x^3*(11 + 6*Log[x]))/(6*x^3)
```

3.155.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2599, 2599, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4}dx$$

$$\Big\downarrow \text{2599}$$

$$b\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}dx-\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3}$$

$$\Big\downarrow \text{2599}$$

$$b\left(b\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}dx-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3}$$

$$\Big\downarrow \text{2599}$$

$$b\left(b\left(b\int \frac{1}{x}dx-\frac{\coth^{-1}(\tanh(a+bx))}{x}\right)-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3}$$

$$\Big\downarrow \text{14}$$

$$b\left(b\left(b\log(x)-\frac{\coth^{-1}(\tanh(a+bx))}{x}\right)-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^3/x^4,x]
```

output
```
-1/3*ArcCoth[Tanh[a + b*x]]^3/x^3 + b*(-1/2*ArcCoth[Tanh[a + b*x]]^2/x^2 +
 b*(-(ArcCoth[Tanh[a + b*x]]/x) + b*Log[x]))
```

3.155.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.155.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

method	result	size
parallelrisch	$\frac{6b^3\ln(x)x^3-6b^2x^2\,\mathrm{arccoth}(\tanh(bx+a))-3bx\,\mathrm{arccoth}(\tanh(bx+a))^2-2\,\mathrm{arccoth}(\tanh(bx+a))^3}{6x^3}$	56
risch	Expression too large to display	17237

input
```
int(arccoth(tanh(b*x+a))^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(6*b^3*ln(x)*x^3-6*b^2*x^2*arccoth(tanh(b*x+a))-3*b*x*arccoth(tanh(b*x
+a))^2-2*arccoth(tanh(b*x+a))^3)/x^3
```

3.155.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.36

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4}\,dx = \frac{24\,b^3x^3\log(x) - 72\,ab^2x^2 - 36\,a^2bx + i\,\pi^3 + 3\,\pi^2(3\,bx + 2\,a) - 8\,a^3 - 12i\,\pi(3\,b^2x^2 + 3\,abx + a^2)}{24\,x^3}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^4,x, algorithm="fricas")
```

output
```
1/24*(24*b^3*x^3*log(x) - 72*a*b^2*x^2 - 36*a^2*b*x + I*pi^3 + 3*pi^2*(3*b
*x + 2*a) - 8*a^3 - 12*I*pi*(3*b^2*x^2 + 3*a*b*x + a^2))/x^3
```

3.155.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4} \, dx = b^3 \log(x) - \frac{b^2 \operatorname{acoth}(\tanh(a+bx))}{x} - \frac{b \operatorname{acoth}^2(\tanh(a+bx))}{2x^2} - \frac{\operatorname{acoth}^3(\tanh(a+bx))}{3x^3}$$

input
```
integrate(acoth(tanh(b*x+a))**3/x**4,x)
```

output
```
b**3*log(x) - b**2*acoth(tanh(a + b*x))/x - b*acoth(tanh(a + b*x))**2/(2*x
**2) - acoth(tanh(a + b*x))**3/(3*x**3)
```

3.155.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4} \, dx = \left(b^2 \log(x) - \frac{b \operatorname{arcoth}(\tanh(bx+a))}{x}\right) b - \frac{b \operatorname{arcoth}(\tanh(bx+a))^2}{2 x^2} - \frac{\operatorname{arcoth}(\tanh(bx+a))^3}{3 x^3}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^4,x, algorithm="maxima")
```

output
```
(b^2*log(x) - b*arccoth(tanh(b*x + a))/x)*b - 1/2*b*arccoth(tanh(b*x + a))
^2/x^2 - 1/3*arccoth(tanh(b*x + a))^3/x^3
```

3.155.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4}\,dx$$
$$= b^3 \log(x) - \frac{36i\,\pi b^2x^2 + 72\,ab^2x^2 - 9\,\pi^2bx + 36i\,\pi abx + 36\,a^2bx - i\,\pi^3 - 6\,\pi^2a + 12i\,\pi a^2 + 8\,a^3}{24\,x^3}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^4,x, algorithm="giac")
```

output
```
b^3*log(x) - 1/24*(36*I*pi*b^2*x^2 + 72*a*b^2*x^2 - 9*pi^2*b*x + 36*I*pi*a
*b*x + 36*a^2*b*x - I*pi^3 - 6*pi^2*a + 12*I*pi*a^2 + 8*a^3)/x^3
```

3.155.9 Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4}\,dx$$
$$= b^3 \ln(x) - \frac{b^2\,x^2\,\mathrm{acoth}(\tanh(a+b\,x)) + \frac{b\,x\,\mathrm{acoth}(\tanh(a+b\,x))^2}{2} + \frac{\mathrm{acoth}(\tanh(a+b\,x))^3}{3}}{x^3}$$

input
```
int(acoth(tanh(a + b*x))^3/x^4,x)
```

output
```
b^3*log(x) - (acoth(tanh(a + b*x))^3/3 + (b*x*acoth(tanh(a + b*x))^2)/2 +
b^2*x^2*acoth(tanh(a + b*x)))/x^3
```

3.156 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5} dx$

3.156.1 Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5} dx = \frac{\coth^{-1}(\tanh(a+bx))^4}{4x^4 \left(bx - \coth^{-1}(\tanh(a+bx))\right)}$$

output
```
1/4*arccoth(tanh(b*x+a))^4/x^4/(b*x-arccoth(tanh(b*x+a)))
```

3.156.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5} dx = -\frac{b^3x^3 + b^2x^2\coth^{-1}(\tanh(a+bx)) + bx\coth^{-1}(\tanh(a+bx))^2 + \coth^{-1}(\tanh(a+bx))^3}{4x^4}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^3/x^5,x]
```

output
```
-1/4*(b^3*x^3 + b^2*x^2*ArcCoth[Tanh[a + b*x]] + b*x*ArcCoth[Tanh[a + b*x]
]^2 + ArcCoth[Tanh[a + b*x]]^3)/x^4
```

3.156.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5} \, dx$$

$$\Big\downarrow \text{2598}$$

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4x^4 \left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^3/x^5,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^4/(4*x^4*(b*x - ArcCoth[Tanh[a + b*x]]))
```

3.156.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

3.156.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

method	result	size
parallelrisch	$-\frac{b^3x^3+b^2x^2\,\mathrm{arccoth}(\tanh(bx+a))+bx\,\mathrm{arccoth}(\tanh(bx+a))^2+\mathrm{arccoth}(\tanh(bx+a))^3}{4x^4}$	49
risch	Expression too large to display	17235

input
```
int(arccoth(tanh(b*x+a))^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b^3*x^3+b^2*x^2*arccoth(tanh(b*x+a))+b*x*arccoth(tanh(b*x+a))^2+arcc
oth(tanh(b*x+a))^3)/x^4
```

3.156.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.42

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}\,dx = -\frac{32\,b^3x^3+48\,ab^2x^2+32\,a^2bx-i\,\pi^3-2\,\pi^2(4\,bx+3\,a)+8\,a^3+4i\,\pi(6\,b^2x^2+8\,abx+3\,a^2)}{32\,x^4}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^5,x, algorithm="fricas")
```

output
```
-1/32*(32*b^3*x^3 + 48*a*b^2*x^2 + 32*a^2*b*x - I*pi^3 - 2*pi^2*(4*b*x + 3
*a) + 8*a^3 + 4*I*pi*(6*b^2*x^2 + 8*a*b*x + 3*a^2))/x^4
```

3.156.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(26) = 52.

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}\,dx = -\frac{b^3}{4x}-\frac{b^2\operatorname{acoth}(\tanh(a+bx))}{4x^2}-\frac{b\operatorname{acoth}^2(\tanh(a+bx))}{4x^3}-\frac{\operatorname{acoth}^3(\tanh(a+bx))}{4x^4}$$

input
```
integrate(acoth(tanh(b*x+a))**3/x**5,x)
```

output
```
-b**3/(4*x) - b**2*acoth(tanh(a + b*x))/(4*x**2) - b*acoth(tanh(a + b*x))*
*2/(4*x**3) - acoth(tanh(a + b*x))**3/(4*x**4)
```

3.156.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}\,dx = -\frac{1}{4}\,b\left(\frac{b^2}{x} + \frac{b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)}{x^2}\right) - \frac{b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{4\,x^3} - \frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^3}{4\,x^4}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^5,x, algorithm="maxima")
```

output
```
-1/4*b*(b^2/x + b*arccoth(tanh(b*x + a))/x^2) - 1/4*b*arccoth(tanh(b*x + a
))^2/x^3 - 1/4*arccoth(tanh(b*x + a))^3/x^4
```

3.156.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.39

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}\,dx = -\frac{32\,b^3x^3 + 24i\,\pi b^2x^2 + 48\,ab^2x^2 - 8\,\pi^2bx + 32i\,\pi abx + 32\,a^2bx - i\,\pi^3 - 6\,\pi^2a + 12i\,\pi a^2 + 8\,a^3}{32\,x^4}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^5,x, algorithm="giac")
```

output
```
-1/32*(32*b^3*x^3 + 24*I*pi*b^2*x^2 + 48*a*b^2*x^2 - 8*pi^2*b*x + 32*I*pi*
a*b*x + 32*a^2*b*x - I*pi^3 - 6*pi^2*a + 12*I*pi*a^2 + 8*a^3)/x^4
```

3.156.9 Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}\,dx = -\frac{b^3\,x^3 + b^2\,x^2\,\mathrm{acoth}(\tanh(a+b\,x)) + b\,x\,\mathrm{acoth}(\tanh(a+b\,x))^2 + \mathrm{acoth}(\tanh(a+b\,x))^3}{4\,x^4}$$

input
```
int(acoth(tanh(a + b*x))^3/x^5,x)
```

output
```
-(acoth(tanh(a + b*x))^3 + b^3*x^3 + b*x*acoth(tanh(a + b*x))^2 + b^2*x^2*
acoth(tanh(a + b*x)))/(4*x^4)
```

3.156. $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}\,dx$

3.157 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6} dx$

3.157.1 Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6} dx = \frac{b\coth^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2} + \frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

output
```
1/20*b*arccoth(tanh(b*x+a))^4/x^4/(b*x-arccoth(tanh(b*x+a)))^2+1/5*arccoth
(tanh(b*x+a))^4/x^5/(b*x-arccoth(tanh(b*x+a)))
```

3.157.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6} dx = -\frac{b^3x^3+2b^2x^2\coth^{-1}(\tanh(a+bx))+3bx\coth^{-1}(\tanh(a+bx))^2+4\coth^{-1}(\tanh(a+bx))^3}{20x^5}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^3/x^6,x]
```

output
```
-1/20*(b^3*x^3 + 2*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 3*b*x*ArcCoth[Tanh[a +
 b*x]]^2 + 4*ArcCoth[Tanh[a + b*x]]^3)/x^5
```

3.157.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2602, 2598}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6}dx$$

$$\Big\downarrow \ 2602$$

$$\frac{b\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}dx}{5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \ 2598$$

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\coth^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^3/x^6,x]
```

output
```
(b*ArcCoth[Tanh[a + b*x]]^4)/(20*x^4*(b*x - ArcCoth[Tanh[a + b*x]])^2) + A
rcCoth[Tanh[a + b*x]]^4/(5*x^5*(b*x - ArcCoth[Tanh[a + b*x]]))
```

3.157.3.1 Defintions of rubi rules used

rule 2598
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.157.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

method	result	size
parallelrisch	$-\frac{b^{3} x^{3}+2 b^{2} x^{2} \operatorname{arccoth}\left(\tanh \left(b x+a\right)\right)+3 b x \operatorname{arccoth}\left(\tanh \left(b x+a\right)\right)^{2}+4 \operatorname{arccoth}\left(\tanh \left(b x+a\right)\right)^{3}}{20 x^{5}}$	53
risch	Expression too large to display	17234

input
```
int(arccoth(tanh(b*x+a))^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/20*(b^3*x^3+2*b^2*x^2*arccoth(tanh(b*x+a))+3*b*x*arccoth(tanh(b*x+a))^2
+4*arccoth(tanh(b*x+a))^3)/x^5
```

3.157.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6}\,dx = -\frac{40\,b^3x^3+80\,ab^2x^2+60\,a^2bx-2i\,\pi^3-3\,\pi^2(5\,bx+4\,a)+16\,a^3+4i\,\pi(10\,b^2x^2+15\,abx+6\,a^2)}{80\,x^5}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^6,x, algorithm="fricas")
```

output
```
-1/80*(40*b^3*x^3 + 80*a*b^2*x^2 + 60*a^2*b*x - 2*I*pi^3 - 3*pi^2*(5*b*x +
 4*a) + 16*a^3 + 4*I*pi*(10*b^2*x^2 + 15*a*b*x + 6*a^2))/x^5
```

3.157.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6}\,dx = -\frac{b^3}{20x^2}-\frac{b^2\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}{10x^3}-\frac{3b\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}{20x^4}-\frac{\operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}{5x^5}$$

input
```
integrate(acoth(tanh(b*x+a))**3/x**6,x)
```

output
```
-b**3/(20*x**2) - b**2*acoth(tanh(a + b*x))/(10*x**3) - 3*b*acoth(tanh(a +
 b*x))**2/(20*x**4) - acoth(tanh(a + b*x))**3/(5*x**5)
```

3.157.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6}\,dx = -\frac{1}{20}\,b\left(\frac{b^2}{x^2}+\frac{2\,b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)}{x^3}\right) - \frac{3\,b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{20\,x^4} - \frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^3}{5\,x^5}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^6,x, algorithm="maxima")
```

output
```
-1/20*b*(b^2/x^2 + 2*b*arccoth(tanh(b*x + a))/x^3) - 3/20*b*arccoth(tanh(b
*x + a))^2/x^4 - 1/5*arccoth(tanh(b*x + a))^3/x^5
```

3.157.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6}\,dx = -\frac{40\,b^3x^3+40i\,\pi b^2x^2+80\,ab^2x^2-15\,\pi^2bx+60i\,\pi abx+60\,a^2bx-2i\,\pi^3-12\,\pi^2a+24i\,\pi a^2+16\,a^3}{80\,x^5}$$

input
```
integrate(arccoth(tanh(b*x+a))^3/x^6,x, algorithm="giac")
```

output
```
-1/80*(40*b^3*x^3 + 40*I*pi*b^2*x^2 + 80*a*b^2*x^2 - 15*pi^2*b*x + 60*I*pi
*a*b*x + 60*a^2*b*x - 2*I*pi^3 - 12*pi^2*a + 24*I*pi*a^2 + 16*a^3)/x^5
```

3.157.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6}\,dx = -\frac{\operatorname{acoth}(\tanh(a+b\,x))^3}{5\,x^5} - \frac{b^3}{20\,x^2} - \frac{b^2\operatorname{acoth}(\tanh(a+b\,x))}{10\,x^3} - \frac{3\,b\operatorname{acoth}(\tanh(a+b\,x))^2}{20\,x^4}$$

input
```
int(acoth(tanh(a + b*x))^3/x^6,x)
```

output
```
- acoth(tanh(a + b*x))^3/(5*x^5) - b^3/(20*x^2) - (b^2*acoth(tanh(a + b*x)
))/(10*x^3) - (3*b*acoth(tanh(a + b*x))^2)/(20*x^4)
```

3.158 $\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx$

3.158.1 Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx = -\frac{x^{1+m}\operatorname{Hypergeometric2F1}\left(1,1+m,2+m,\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

output
```
-x^(1+m)*hypergeom([1, 1+m],[2+m],b*x/(b*x-arccoth(tanh(b*x+a))))/(1+m)/(b
*x-arccoth(tanh(b*x+a)))
```

3.158.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x^{1+m}\operatorname{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{bx}{-bx+\coth^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}$$

input
```
Integrate[x^m/ArcCoth[Tanh[a + b*x]],x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcCoth[Ta
nh[a + b*x]]))])/((1 + m)*(-(b*x) + ArcCoth[Tanh[a + b*x]]))
```

3.158.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2595}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))} \, dx$$

$$\Big\downarrow \text{2595}$$

$$-\frac{x^{m+1} \text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(m+1) \left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

input
```
Int[x^m/ArcCoth[Tanh[a + b*x]],x]
```

output
```
-((x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (b*x)/(b*x - ArcCoth[Tanh[
a + b*x]])])/((1 + m)*(b*x - ArcCoth[Tanh[a + b*x]])))
```

3.158.3.1 Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

3.158.4 Maple [F]

$$\int \frac{x^m}{\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)}dx$$

input
```
int(x^m/arccoth(tanh(b*x+a)),x)
```

output
```
int(x^m/arccoth(tanh(b*x+a)),x)
```

3.158.5 Fricas [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x^m}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}\,dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
integral(x^m/arccoth(tanh(b*x + a)), x)
```

3.158.6 Sympy [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x^m}{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**m/acoth(tanh(b*x+a)),x)
```

output
```
Integral(x**m/acoth(tanh(a + b*x)), x)
```

3.158.7 Maxima [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x^m}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}\,dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(x^m/arccoth(tanh(b*x + a)), x)
```

3.158.8 Giac [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x^m}{\operatorname{arcoth}(\tanh(bx+a))}\,dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^m/arccoth(tanh(b*x + a)), x)
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x^m}{\operatorname{acoth}(\tanh(a+b\,x))}\,dx$$

input
```
int(x^m/acoth(tanh(a + b*x)),x)
```

output
```
int(x^m/acoth(tanh(a + b*x)), x)
```

3.159 $\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx$

3.159.1 Optimal result

Integrand size = 13, antiderivative size = 81

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx=\frac{x^3}{3b}+\frac{x^2(bx-\coth^{-1}(\tanh(a+bx)))}{2b^2}+\frac{x(bx-\coth^{-1}(\tanh(a+bx)))^2}{b^3}+\frac{(bx-\coth^{-1}(\tanh(a+bx)))^3\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^4}$$

output
```
1/3*x^3/b+1/2*x^2*(b*x-arccoth(tanh(b*x+a)))/b^2+x*(b*x-arccoth(tanh(b*x+a
)))^2/b^3+(b*x-arccoth(tanh(b*x+a)))^3*ln(arccoth(tanh(b*x+a)))/b^4
```

3.159.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx=\frac{x^3}{3b}-\frac{x^2(-bx+\coth^{-1}(\tanh(a+bx)))}{2b^2}+\frac{x(-bx+\coth^{-1}(\tanh(a+bx)))^2}{b^3}-\frac{(-bx+\coth^{-1}(\tanh(a+bx)))^3\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^4}$$

input
```
Integrate[x^3/ArcCoth[Tanh[a + b*x]],x]
```

output
```
x^3/(3*b) - (x^2*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/(2*b^2) + (x*(-(b*x) +
 ArcCoth[Tanh[a + b*x]])^2)/b^3 - ((-(b*x) + ArcCoth[Tanh[a + b*x]])^3*Log
[ArcCoth[Tanh[a + b*x]]])/b^4
```

3.159.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2590, 2590, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x^3}{3b}$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{x}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x^2}{2b}\right)}{b}+\frac{x^3}{3b}$$

$$\Big\downarrow \text{2589}$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}+\frac{x^3}{3b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}+\right.}{b}$$

$$\frac{x^3}{3b}$$

$$\Big\downarrow \text{14}$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}+$$

$$\frac{x^3}{3b}$$

input
```
Int[x^3/ArcCoth[Tanh[a + b*x]],x]
```

output
```
x^3/(3*b) + ((b*x - ArcCoth[Tanh[a + b*x]])*(x^2/(2*b) + ((b*x - ArcCoth[T
anh[a + b*x]])*(x/b + ((b*x - ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a +
 b*x]]])/b^2))/b))/b
```

3.159.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

3.159. $\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx$

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

3.159.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.42 (sec) , antiderivative size = 130774, normalized size of antiderivative = 1614.49

method	result	size
risch	Expression too large to display	130774

input
```
int(x^3/arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.159.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{8\,b^3x^3 - 12\,ab^2x^2 - 6\,\pi^2bx + 24\,a^2bx - 6i\,\pi(b^2x^2 - 4\,abx) - 3\,(-i\,\pi^3 - 6\,\pi^2a + 12i\,\pi a^2 + 8\,a^3)\log\left(i\,\pi + 2\right.}{24\,b^4}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/24*(8*b^3*x^3 - 12*a*b^2*x^2 - 6*pi^2*b*x + 24*a^2*b*x - 6*I*pi*(b^2*x^2
 - 4*a*b*x) - 3*(-I*pi^3 - 6*pi^2*a + 12*I*pi*a^2 + 8*a^3)*log(I*pi + 2*b*
x + 2*a))/b^4
```

3.159.6 Sympy [F]

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x^3}{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**3/acoth(tanh(b*x+a)),x)
```

output
```
Integral(x**3/acoth(tanh(a + b*x)), x)
```

3.159.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{4\,b^2x^3 - 3\,(-i\,\pi b + 2\,ab)x^2 - 3\,(\pi^2 + 4i\,\pi a - 4\,a^2)x}{12\,b^3} - \frac{(i\,\pi^3 - 6\,\pi^2 a - 12i\,\pi a^2 + 8\,a^3)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{8\,b^4}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(4*b^2*x^3 - 3*(-I*pi*b + 2*a*b)*x^2 - 3*(pi^2 + 4*I*pi*a - 4*a^2)*x)
/b^3 - 1/8*(I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3)*log(-I*pi + 2*b*x + 2
*a)/b^4
```

3.159.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x^3}{3\,b} - \frac{(i\,\pi + 2\,a)x^2}{4\,b^2} - \frac{(\pi^2 - 4i\,\pi a - 4\,a^2)x}{4\,b^3} + \frac{(i\,\pi^3 + 6\,\pi^2 a - 12i\,\pi a^2 - 8\,a^3)\log\left(\pi - 2i\,bx - 2i\,a\right)}{8\,b^4}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
1/3*x^3/b - 1/4*(I*pi + 2*a)*x^2/b^2 - 1/4*(pi^2 - 4*I*pi*a - 4*a^2)*x/b^3
 + 1/8*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*log(pi - 2*I*b*x - 2*I*a)
/b^4
```

3.159.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 354, normalized size of antiderivative = 4.37

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x^3}{3\,b} + \frac{x^2\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)}{4\,b^2}$$
$$+ \frac{x\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)^2}{4\,b^3}$$
$$+ \frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\right)\left(\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)^3 - 8\,a^3 - 6}{}$$

input
```
int(x^3/acoth(tanh(a + b*x)),x)
```

output
```
x^3/(3*b) + (x^2*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(4*b^2) + (x*(log(-2/(exp(2*a
)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)
) + 2*b*x)^2)/(4*b^3) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) - 1)) - log(-2/(exp(2*a)*exp(2*b*x) - 1)))*((2*a - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) +
2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2*(2*a -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*
exp(2*b*x) - 1)) + 2*b*x)))/(8*b^4)
```

3.160 $\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))} dx$

3.160.1 Optimal result

Integrand size = 13, antiderivative size = 56

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))} dx = \frac{x^2}{2b} + \frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}$$

output
```
1/2*x^2/b+x*(b*x-arccoth(tanh(b*x+a)))/b^2+(b*x-arccoth(tanh(b*x+a)))^2*ln
(arccoth(tanh(b*x+a)))/b^3
```

3.160.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))} dx = \frac{x^2}{2b} - \frac{x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2 \log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}$$

input
```
Integrate[x^2/ArcCoth[Tanh[a + b*x]],x]
```

output
```
x^2/(2*b) - (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^2 + ((-(b*x) + ArcCoth
[Tanh[a + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^3
```

3.160.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2590, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}dx$$

$$\Big\downarrow \text{2590}$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{x}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x^2}{2b}$$

$$\Big\downarrow \text{2589}$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}$$

$$\Big\downarrow \text{2588}$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}$$

$$\Big\downarrow \text{14}$$

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\log(\coth^{-1}(\tanh(a+bx)))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}$$

input
```
Int[x^2/ArcCoth[Tanh[a + b*x]],x]
```

output
```
x^2/(2*b) + ((b*x - ArcCoth[Tanh[a + b*x]])*(x/b + ((b*x - ArcCoth[Tanh[a
+ b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^2))/b
```

3.160.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

3.160.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.37 (sec) , antiderivative size = 28786, normalized size of antiderivative = 514.04

method	result	size
risch	Expression too large to display	28786

input
```
int(x^2/arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.160.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{2\,b^2x^2 - 2i\,\pi bx - 4\,abx - (\pi^2 - 4i\,\pi a - 4\,a^2)\log\left(i\,\pi + 2\,bx + 2\,a\right)}{4\,b^3}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*b^2*x^2 - 2*I*pi*b*x - 4*a*b*x - (pi^2 - 4*I*pi*a - 4*a^2)*log(I*pi
 + 2*b*x + 2*a))/b^3
```

3.160.6 Sympy [F]

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x^2}{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**2/acoth(tanh(b*x+a)),x)
```

output
```
Integral(x**2/acoth(tanh(a + b*x)), x)
```

3.160.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{bx^2 + (i\,\pi - 2\,a)x}{2\,b^2} - \frac{(\pi^2 + 4i\,\pi a - 4\,a^2)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{4\,b^3}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*(b*x^2 + (I*pi - 2*a)*x)/b^2 - 1/4*(pi^2 + 4*I*pi*a - 4*a^2)*log(-I*pi
 + 2*b*x + 2*a)/b^3
```

3.160.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x^2}{2\,b} - \frac{(i\,\pi + 2\,a)x}{2\,b^2} - \frac{(\pi^2 - 4i\,\pi a - 4\,a^2)\log\left(\pi - 2i\,bx - 2i\,a\right)}{4\,b^3}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
1/2*x^2/b - 1/2*(I*pi + 2*a)*x/b^2 - 1/4*(pi^2 - 4*I*pi*a - 4*a^2)*log(pi
- 2*I*b*x - 2*I*a)/b^3
```

3.160.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 234, normalized size of antiderivative = 4.18

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x^2}{2\,b} + \frac{x\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)}{2\,b^2}$$
$$+ \frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\right)\left(\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)^2 - 4\,a\,\Big(2\,a}{4\,b^3}$$

input
```
int(x^2/acoth(tanh(a + b*x)),x)
```

output
```
x^2/(2*b) + (x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b^2) + (log(log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-2/(exp(2*a)*exp(2*b*x) - 1))
)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/
(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*
x) + 4*a^2))/(4*b^3)
```

3.161 $\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx$

3.161.1 Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x}{b} + \frac{(bx-\coth^{-1}(\tanh(a+bx)))\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}$$

output
```
x/b+(b*x-arccoth(tanh(b*x+a)))*ln(arccoth(tanh(b*x+a)))/b^2
```

3.161.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x}{b} - \frac{(-bx+\coth^{-1}(\tanh(a+bx)))\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}$$

input
```
Integrate[x/ArcCoth[Tanh[a + b*x]],x]
```

output
```
x/b - ((-(b*x) + ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^2
```

3.161.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))}dx$$
$$\Big\downarrow \text{2589}$$
$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x}{b}$$
$$\Big\downarrow \text{2588}$$
$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}+\frac{x}{b}$$
$$\Big\downarrow \text{14}$$
$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x}{b}$$

input
```
Int[x/ArcCoth[Tanh[a + b*x]],x]
```

output
```
x/b + ((b*x - ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^2
```

3.161.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

3.161.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 4303, normalized size of antiderivative = 138.81

method	result	size
risch	Expression too large to display	4303

input
```
int(x/arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
x/b+1/4*I/b^2*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3-2*
Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2
*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+
2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*(ln(exp(b*x+a))-b*x
-a)+4*I*a+4*I*b*x+2*Pi)*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/4
*I/b^2*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3-2*Pi*csgn
(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*
Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3
-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*(ln(exp(b*x+a))-b*x-a)+4*I
*a+4*I*b*x+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))^2-1/4*I/b^2*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b
*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*
a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+
2*a)+1))^3-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(b*x+a))^2*cs...
```

3.161.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{2\,bx + (-i\,\pi - 2\,a)\log\left(i\,\pi + 2\,bx + 2\,a\right)}{2\,b^2}$$

input
```
integrate(x/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(2*b*x + (-I*pi - 2*a)*log(I*pi + 2*b*x + 2*a))/b^2
```

3.161.6 Sympy [F]

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{x}{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x/acoth(tanh(b*x+a)),x)
```

output
```
Integral(x/acoth(tanh(a + b*x)), x)
```

3.161.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x}{b} - \frac{(-i\,\pi + 2\,a)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{2\,b^2}$$

input
```
integrate(x/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
x/b - 1/2*(-I*pi + 2*a)*log(-I*pi + 2*b*x + 2*a)/b^2
```

3.161.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x}{b} - \frac{(i\,\pi + 2\,a)\log\left(\pi - 2i\,bx - 2i\,a\right)}{2\,b^2}$$

input
```
integrate(x/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
x/b - 1/2*(I*pi + 2*a)*log(pi - 2*I*b*x - 2*I*a)/b^2
```

3.161.9 Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.48

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x}{b} + \frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\right)\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)}{2\,b^2}$$

input
```
int(x/acoth(tanh(a + b*x)),x)
```

output
```
x/b + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-2
/(exp(2*a)*exp(2*b*x) - 1)))*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b^2)
```

3.162 $\int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx$

3.162.1 Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx = \frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b}$$

output
```
ln(arccoth(tanh(b*x+a)))/b
```

3.162.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx = \frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^(-1),x]
```

output
```
Log[ArcCoth[Tanh[a + b*x]]]/b
```

3.162.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^(-1),x]
```

output
```
Log[ArcCoth[Tanh[a + b*x]]]/b
```

3.162.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.162.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\ln(\operatorname{arccoth}(\tanh(bx+a)))}{b}$
default	$\frac{\ln(\operatorname{arccoth}(\tanh(bx+a)))}{b}$
parallelrisch	$\frac{\ln(\operatorname{arccoth}(\tanh(bx+a)))}{b}$
risch	$\ln\left(\ln\left(e^{bx+a}\right)-\frac{i\pi\left(\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(i e^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)-\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^{2}+2\operatorname{csgn}\left(\right.\right.}{}\right.$

input
```
int(1/arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
ln(arccoth(tanh(b*x+a)))/b
```

3.162.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{\log\left(i\,\pi + 2\,bx + 2\,a\right)}{b}$$

input
```
integrate(1/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
log(I*pi + 2*b*x + 2*a)/b
```

3.162.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx = \begin{cases} \frac{\log\left(\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)\right)}{b} & \text{for}\ b \neq 0 \\ \frac{x}{\operatorname{acoth}\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/acoth(tanh(b*x+a)),x)
```

output
```
Piecewise((log(acoth(tanh(a + b*x)))/b, Ne(b, 0)), (x/acoth(tanh(a)), True
))
```

3.162.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{\log\left(-\frac{1}{2}i\,\pi - bx - a\right)}{b}$$

input
```
integrate(1/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
log(-1/2*I*pi - b*x - a)/b
```

3.162.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{\log\left(\pi - 2i\,bx - 2i\,a\right)}{b}$$

input
```
integrate(1/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
log(pi - 2*I*b*x - 2*I*a)/b
```

3.162.9 Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx=\frac{\ln\left(\mathrm{acoth}(\tanh(a+b\,x))\right)}{b}$$

input
```
int(1/acoth(tanh(a + b*x)),x)
```

output
```
log(acoth(tanh(a + b*x)))/b
```

3.163 $\int \frac{1}{x \coth^{-1}(\tanh(a+bx))} \, dx$

3.163.1 Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))} \, dx = -\frac{\log(x)}{bx - \coth^{-1}(\tanh(a+bx))} + \frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx - \coth^{-1}(\tanh(a+bx))}$$

output
```
-ln(x)/(b*x-arccoth(tanh(b*x+a)))+ln(arccoth(tanh(b*x+a)))/(b*x-arccoth(ta
nh(b*x+a)))
```

3.163.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))} \, dx = \frac{-\log(x) + \log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx - \coth^{-1}(\tanh(a+bx))}$$

input
```
Integrate[1/(x*ArcCoth[Tanh[a + b*x]]),x]
```

output
```
(-Log[x] + Log[ArcCoth[Tanh[a + b*x]]])/(b*x - ArcCoth[Tanh[a + b*x]])
```

3.163.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}dx$$

$$\Big\downarrow \text{2591}$$

$$\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

$$\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[1/(x*ArcCoth[Tanh[a + b*x]]),x]
```

output
```
-(Log[x]/(b*x - ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*
x - ArcCoth[Tanh[a + b*x]])
```

3.163.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

3.163.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 7.07 (sec) , antiderivative size = 972, normalized size of antiderivative = 22.09

method	result	size
risch	Expression too large to display	972

input
```
int(1/x/arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-4*I/(2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(ex
p(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp
(2*b*x+2*a))^3-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-Pi*csgn(I*
exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(
b*x+a))^2*csgn(I*exp(2*b*x+2*a))+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^3-4*I*b*x-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+4*I*ln(exp(b*x+a))+2*Pi)*l
n(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3-2*Pi*csgn(I/(exp(
2*b*x+2*a)+1))^2+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(
I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn
(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+4*I*b
*x+2*Pi)+4*I/(2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I/(exp(2*b*x+2*a)+
1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*cs
gn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*cs
gn(I*exp(2*b*x+2*a))^3-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-Pi
*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csg
n(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+Pi*csgn(I*exp(2*b*x+2*a)/(exp(...
```

3.163.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))} \, dx = -\frac{2 \left(\log\left(i \pi + 2 \, bx + 2 \, a\right) - \log\left(x\right)\right)}{i \pi + 2 \, a}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
-2*(log(I*pi + 2*b*x + 2*a) - log(x))/(I*pi + 2*a)
```

3.163.6 Sympy [F]

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{1}{x\,\mathrm{acoth}\,(\tanh\,(a+bx))}\,dx$$

input
```
integrate(1/x/acoth(tanh(b*x+a)),x)
```

output
```
Integral(1/(x*acoth(tanh(a + b*x))), x)
```

3.163.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}\,dx = \frac{2\,\log\left(-i\,\pi+2\,bx+2\,a\right)}{i\,\pi-2\,a} - \frac{2\,\log\left(x\right)}{i\,\pi-2\,a}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
2*log(-I*pi + 2*b*x + 2*a)/(I*pi - 2*a) - 2*log(x)/(I*pi - 2*a)
```

3.163.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}\,dx = \frac{2i\,\log\left(\pi-2i\,bx-2i\,a\right)}{\pi-2i\,a} - \frac{2i\,\log\left(x\right)}{\pi-2i\,a}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
2*I*log(pi - 2*I*b*x - 2*I*a)/(pi - 2*I*a) - 2*I*log(x)/(pi - 2*I*a)
```

3.163.9 Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.57

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}\,dx = -\frac{4\,\mathrm{atanh}\left(\frac{4\,b\,x}{\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x}-1\right)}{\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x}$$

input
```
int(1/(x*acoth(tanh(a + b*x))),x)
```

output
```
-(4*atanh((4*b*x)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) - 1))/(log(-2/(exp(2*a)*exp(2
*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b
*x)
```

3.164 $\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))} dx$

3.164.1 Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))} dx = \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)} - \frac{b\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2} + \frac{b\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

output
```
1/x/(b*x-arccoth(tanh(b*x+a)))-b*ln(x)/(b*x-arccoth(tanh(b*x+a)))^2+b*ln(a
rccoth(tanh(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^2
```

3.164.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))} dx = \frac{-\coth^{-1}(\tanh(a+bx)) + bx\left(1-\log(x)+\log\left(\coth^{-1}(\tanh(a+bx))\right)\right)}{x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2}$$

input
```
Integrate[1/(x^2*ArcCoth[Tanh[a + b*x]]),x]
```

output
```
(-ArcCoth[Tanh[a + b*x]] + b*x*(1 - Log[x] + Log[ArcCoth[Tanh[a + b*x]]]))
/(x*(-(b*x) + ArcCoth[Tanh[a + b*x]])^2)
```

3.164.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))}dx$$
$$\Big\downarrow \text{2594}$$
$$\frac{b\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$
$$\Big\downarrow \text{2591}$$
$$\frac{b\left(\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$
$$\Big\downarrow \text{14}$$
$$\frac{b\left(\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$
$$\Big\downarrow \text{2588}$$
$$\frac{b\left(\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$
$$\Big\downarrow \text{14}$$
$$\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\left(\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[1/(x^2*ArcCoth[Tanh[a + b*x]]),x]
```

output
```
1/(x*(b*x - ArcCoth[Tanh[a + b*x]])) + (b*(-(Log[x]/(b*x - ArcCoth[Tanh[a
+ b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Tanh[a + b*x]])))/(
b*x - ArcCoth[Tanh[a + b*x]])
```

3.164.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

3.164.4 Maple [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \operatorname{arccoth}(\tanh(bx+a))} dx$$

input
```
int(1/x^2/arccoth(tanh(b*x+a)),x)
```

output
```
int(1/x^2/arccoth(tanh(b*x+a)),x)
```

3.164.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))}\,dx = \frac{2\left(i\pi - 2\,bx\log\left(i\pi + 2\,bx + 2\,a\right) + 2\,bx\log\left(x\right) + 2\,a\right)}{\pi^2 x - 4i\,\pi a x - 4\,a^2 x}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
2*(I*pi - 2*b*x*log(I*pi + 2*b*x + 2*a) + 2*b*x*log(x) + 2*a)/(pi^2*x - 4*
I*pi*a*x - 4*a^2*x)
```

3.164.6 Sympy [F]

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{1}{x^2\,\mathrm{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**2/acoth(tanh(b*x+a)),x)
```

output
```
Integral(1/(x**2*acoth(tanh(a + b*x))), x)
```

3.164.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))}\,dx = -\frac{4\,b\log\left(-i\pi + 2\,bx + 2\,a\right)}{\pi^2 + 4i\,\pi a - 4\,a^2} + \frac{4\,b\log\left(x\right)}{\pi^2 + 4i\,\pi a - 4\,a^2} + \frac{2}{(i\pi - 2\,a)x}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-4*b*log(-I*pi + 2*b*x + 2*a)/(pi^2 + 4*I*pi*a - 4*a^2) + 4*b*log(x)/(pi^2
 + 4*I*pi*a - 4*a^2) + 2/((I*pi - 2*a)*x)
```

3.164.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))}\,dx = -\frac{4\,b\log\left(\pi - 2i\,bx - 2i\,a\right)}{\pi^2 - 4i\,\pi a - 4\,a^2} + \frac{4\,b\log\left(x\right)}{\pi^2 - 4i\,\pi a - 4\,a^2} + \frac{2}{-i\,\pi x - 2\,ax}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
-4*b*log(pi - 2*I*b*x - 2*I*a)/(pi^2 - 4*I*pi*a - 4*a^2) + 4*b*log(x)/(pi^
2 - 4*I*pi*a - 4*a^2) + 2/(-I*pi*x - 2*a*x)
```

3.164.9 Mupad [B] (verification not implemented)

Time = 6.12 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.38

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))}\,dx = \frac{2\,\ln\left(-\frac{1}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - 2\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 4\,b\,x + b\,x\,\mathrm{atan}\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\,1\mathrm{i} - \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\,1\mathrm{i} + b\,x\,2\mathrm{i}}{\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x}\right)\,8\mathrm{i}}{x\,\left(\ln\left(-\frac{1}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)^2}$$

input
```
int(1/(x^2*acoth(tanh(a + b*x))),x)
```

output
```
(2*log(-1/(exp(2*a)*exp(2*b*x) - 1)) - 2*log((exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) - 1)) + 4*b*x + b*x*atan((log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) - 1))*1i - log(-2/(exp(2*a)*exp(2*b*x) - 1))*1i + b*x*2i)/(
log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) - 1)) + 2*b*x))*8i)/(x*(log(-1/(exp(2*a)*exp(2*b*x) - 1)) - log
((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2)
```

3.165 $\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))} dx$

3.165.1 Optimal result

Integrand size = 13, antiderivative size = 92

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))} dx = \frac{b}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)} - \frac{b^2\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3} + \frac{b^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3}$$

output
```
b/x/(b*x-arccoth(tanh(b*x+a)))^2+1/2/x^2/(b*x-arccoth(tanh(b*x+a)))-b^2*ln
(x)/(b*x-arccoth(tanh(b*x+a)))^3+b^2*ln(arccoth(tanh(b*x+a)))/(b*x-arccoth
(tanh(b*x+a)))^3
```

3.165.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))} \, dx = \frac{-4bx\coth^{-1}(\tanh(a+bx)) + \coth^{-1}(\tanh(a+bx))^2 + b^2x^2\left(3 - 2\log(x) + 2\log\left(\coth^{-1}(\tanh(a+bx))\right)\right)}{2x^2\left(bx - \coth^{-1}(\tanh(a+bx))\right)^3}$$

input
```
Integrate[1/(x^3*ArcCoth[Tanh[a + b*x]]),x]
```

output
```
(-4*b*x*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^2 + b^2*x^2*(3 - 2
*Log[x] + 2*Log[ArcCoth[Tanh[a + b*x]]]))/(2*x^2*(b*x - ArcCoth[Tanh[a + b
*x]])^3)
```

3.165.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2594, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))} \, dx$$

$$\Big\downarrow \text{2594}$$

$$\frac{b\int \frac{1}{x^2\coth^{-1}(\tanh(a+bx))}dx}{bx - \coth^{-1}(\tanh(a+bx))} + \frac{1}{2x^2\left(bx - \coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \text{2594}$$

$$\frac{b\left(\frac{b\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}dx}{bx - \coth^{-1}(\tanh(a+bx))} + \frac{1}{x\left(bx - \coth^{-1}(\tanh(a+bx))\right)}\right)}{bx - \coth^{-1}(\tanh(a+bx))} + \frac{1}{2x^2\left(bx - \coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \text{2591}$$

$$\frac{b\left(\frac{b\left(\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x(bx-\coth^{-1}(\tanh(a+bx)))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \text{14}$$

$$\frac{b\left(\frac{b\left(\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x(bx-\coth^{-1}(\tanh(a+bx)))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \text{2588}$$

$$\frac{b\left(\frac{b\left(\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x(bx-\coth^{-1}(\tanh(a+bx)))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\left(\frac{1}{x(bx-\coth^{-1}(\tanh(a+bx)))}+\frac{b\left(\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[1/(x^3*ArcCoth[Tanh[a + b*x]]),x]
```

output
```
1/(2*x^2*(b*x - ArcCoth[Tanh[a + b*x]])) + (b*(1/(x*(b*x - ArcCoth[Tanh[a
+ b*x]])) + (b*(-(Log[x]/(b*x - ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tan
h[a + b*x]]]/(b*x - ArcCoth[Tanh[a + b*x]])))/(b*x - ArcCoth[Tanh[a + b*x]
])))/(b*x - ArcCoth[Tanh[a + b*x]])
```

3.165. $\int \frac{1}{x^3\coth^{-1}(\tanh(a+bx))}\,dx$

3.165.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

3.165.4 Maple [F]

$$\int \frac{1}{x^3 \operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)}dx$$

input
```
int(1/x^3/arccoth(tanh(b*x+a)),x)
```

output
```
int(1/x^3/arccoth(tanh(b*x+a)),x)
```

3.165.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))}\,dx = -\frac{8\,b^2x^2\log\left(i\,\pi+2\,bx+2\,a\right)-8\,b^2x^2\log\left(x\right)-8\,abx-\pi^2-4i\,\pi(bx-a)+4\,a^2}{-i\,\pi^3x^2-6\,\pi^2ax^2+12i\,\pi a^2x^2+8\,a^3x^2}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
-(8*b^2*x^2*log(I*pi + 2*b*x + 2*a) - 8*b^2*x^2*log(x) - 8*a*b*x - pi^2 -
4*I*pi*(b*x - a) + 4*a^2)/(-I*pi^3*x^2 - 6*pi^2*a*x^2 + 12*I*pi*a^2*x^2 +
8*a^3*x^2)
```

3.165.6 Sympy [F]

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))}\,dx = \int \frac{1}{x^3 \operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**3/acoth(tanh(b*x+a)),x)
```

output
```
Integral(1/(x**3*acoth(tanh(a + b*x))), x)
```

3.165.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))}\,dx = \frac{8\,b^2 \log\left(-i\,\pi + 2\,bx + 2\,a\right)}{-i\,\pi^3 + 6\,\pi^2 a + 12i\,\pi a^2 - 8\,a^3} - \frac{8\,b^2 \log\left(x\right)}{-i\,\pi^3 + 6\,\pi^2 a + 12i\,\pi a^2 - 8\,a^3} - \frac{i\,\pi + 4\,bx - 2\,a}{(\pi^2 + 4i\,\pi a - 4\,a^2)x^2}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
8*b^2*log(-I*pi + 2*b*x + 2*a)/(-I*pi^3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3)
- 8*b^2*log(x)/(-I*pi^3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3) - (I*pi + 4*b*x
- 2*a)/((pi^2 + 4*I*pi*a - 4*a^2)*x^2)
```

3.165.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))}\,dx = -\frac{8i\,b^2 \log\left(\pi - 2i\,bx - 2i\,a\right)}{\pi^3 - 6i\,\pi^2 a - 12\,\pi a^2 + 8i\,a^3} + \frac{8i\,b^2 \log\left(x\right)}{\pi^3 - 6i\,\pi^2 a - 12\,\pi a^2 + 8i\,a^3} - \frac{-i\,\pi + 4\,bx - 2\,a}{\pi^2 x^2 - 4i\,\pi a x^2 - 4\,a^2 x^2}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
-8*I*b^2*log(pi - 2*I*b*x - 2*I*a)/(pi^3 - 6*I*pi^2*a - 12*pi*a^2 + 8*I*a^
3) + 8*I*b^2*log(x)/(pi^3 - 6*I*pi^2*a - 12*pi*a^2 + 8*I*a^3) - (-I*pi + 4
*b*x - 2*a)/(pi^2*x^2 - 4*I*pi*a*x^2 - 4*a^2*x^2)
```

3.165.9 Mupad [B] (verification not implemented)

Time = 7.21 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.26

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))}\,dx$$

$$= \frac{\ln\left(-\frac{1}{e^{2a}\,e^{2bx}-1}\right)^2 - \ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)\,\left(2\,\ln\left(-\frac{1}{e^{2a}\,e^{2bx}-1}\right) + 8\,b\,x\right) + \ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)^2 + 12\,b^2\,x^2 + 8\,b\,x\,\ln\left(\cdots\right.}{x^2\,\left(\ln\left(-\frac{1}{e^{2a}\,e^{2bx}-1}\right) - \ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right) + 2 \cdots\right.}$$

input
```
int(1/(x^3*acoth(tanh(a + b*x))),x)
```

output
```
(log(-1/(exp(2*a)*exp(2*b*x) - 1))^2 - log((exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) - 1))*(2*log(-1/(exp(2*a)*exp(2*b*x) - 1)) + 8*b*x) + log((exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2 + 12*b^2*x^2 + b^2*x^2*atan
((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1i - log(-2/(exp(
2*a)*exp(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))*16i + 8*b*x
*log(-1/(exp(2*a)*exp(2*b*x) - 1)))/(x^2*(log(-1/(exp(2*a)*exp(2*b*x) - 1)
) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3)
```

3.166 $\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2} \, dx$

3.166.1 Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2} \, dx = -\frac{x^m}{b\coth^{-1}(\tanh(a+bx))} - \frac{x^m \operatorname{Hypergeometric2F1}\left(1,m,1+m,\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

output
```
-x^m/b/arccoth(tanh(b*x+a))-x^m*hypergeom([1, m],[1+m],b*x/(b*x-arccoth(ta
nh(b*x+a))))/b/(b*x-arccoth(tanh(b*x+a)))
```

3.166.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2} \, dx = \frac{x^{1+m} \operatorname{Hypergeometric2F1}\left(2,1+m,2+m,-\frac{bx}{-bx+\coth^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2}$$

input
```
Integrate[x^m/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcCoth[Ta
nh[a + b*x]]))])/((1 + m)*(-(b*x) + ArcCoth[Tanh[a + b*x]])^2)
```

3.166.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2599, 2595}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{m\int \frac{x^{m-1}}{\coth^{-1}(\tanh(a+bx))}dx}{b}-\frac{x^m}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2595}$$

$$-\frac{x^m \text{Hypergeometric2F1}\left(1,m,m+1,\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\coth^{-1}(\tanh(a+bx))\right)}-\frac{x^m}{b\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[x^m/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
-(x^m/(b*ArcCoth[Tanh[a + b*x]])) - (x^m*Hypergeometric2F1[1, m, 1 + m, (b
*x)/(b*x - ArcCoth[Tanh[a + b*x]])])/(b*(b*x - ArcCoth[Tanh[a + b*x]]))
```

3.166.3.1 Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.166.4 Maple [F]

$$\int \frac{x^m}{\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)^2}dx$$

input
```
int(x^m/arccoth(tanh(b*x+a))^2,x)
```

output
```
int(x^m/arccoth(tanh(b*x+a))^2,x)
```

3.166.5 Fricas [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^m}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2}\,dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
integral(x^m/arccoth(tanh(b*x + a))^2, x)
```

3.166.6 Sympy [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^m}{\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**m/acoth(tanh(b*x+a))**2,x)
```

output
```
Integral(x**m/acoth(tanh(a + b*x))**2, x)
```

3.166.7 Maxima [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^m}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2}\,dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
integrate(x^m/arccoth(tanh(b*x + a))^2, x)
```

3.166.8 Giac [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^m}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2}\,dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
integrate(x^m/arccoth(tanh(b*x + a))^2, x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^m}{\operatorname{acoth}\left(\tanh\left(a+b\,x\right)\right)^2}\,dx$$

input
```
int(x^m/acoth(tanh(a + b*x))^2,x)
```

output
```
int(x^m/acoth(tanh(a + b*x))^2, x)
```

3.167 $\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx$

3.167.1 Optimal result

Integrand size = 13, antiderivative size = 98

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{4x^3}{3b^2}+\frac{2x^2(bx-\coth^{-1}(\tanh(a+bx)))}{b^3}+\frac{4x(bx-\coth^{-1}(\tanh(a+bx)))^2}{b^4}-\frac{x^4}{b\coth^{-1}(\tanh(a+bx))}+\frac{4(bx-\coth^{-1}(\tanh(a+bx)))^3\log(\coth^{-1}(\tanh(a+bx)))}{b^5}$$

output
```
4/3*x^3/b^2+2*x^2*(b*x-arccoth(tanh(b*x+a)))/b^3+4*x*(b*x-arccoth(tanh(b*x
+a)))^2/b^4-x^4/b/arccoth(tanh(b*x+a))+4*(b*x-arccoth(tanh(b*x+a)))^3*ln(a
rccoth(tanh(b*x+a)))/b^5
```

3.167.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{x^3}{3b^2}-\frac{x^2(-bx+\coth^{-1}(\tanh(a+bx)))}{b^3}+\frac{3x(-bx+\coth^{-1}(\tanh(a+bx)))^2}{b^4}-\frac{(-bx+\coth^{-1}(\tanh(a+bx)))^4}{b^5\coth^{-1}(\tanh(a+bx))}-\frac{4(-bx+\coth^{-1}(\tanh(a+bx)))^3\log(\coth^{-1}(\tanh(a+bx)))}{b^5}$$

input
```
Integrate[x^4/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
x^3/(3*b^2) - (x^2*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^3 + (3*x*(-(b*x) +
 ArcCoth[Tanh[a + b*x]])^2)/b^4 - (-(b*x) + ArcCoth[Tanh[a + b*x]])^4/(b^5
*ArcCoth[Tanh[a + b*x]]) - (4*(-(b*x) + ArcCoth[Tanh[a + b*x]])^3*Log[ArcC
oth[Tanh[a + b*x]]])/b^5
```

3.167.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2599, 2590, 2590, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{4\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}dx}{b}-\frac{x^4}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2590}$$

$$\frac{4\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x^3}{3b}\right)}{b}-\frac{x^4}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2590}$$

$$\frac{4\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{x}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x^2}{2b}\right)}{b}+\frac{x^3}{3b}\right)}{b}-\frac{x^4}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2589}$$

$$\frac{4\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}+\frac{x^3}{3b}\right)}{b}-\frac{x^4}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \; 2588$$

$$\frac{4\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}+\frac{x^3}{3b}\right)}{b}-\frac{x^4}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \; 14$$

$$\frac{4\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}+\frac{x^3}{3b}\right)}{b}-\frac{x^4}{b\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[x^4/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
-(x^4/(b*ArcCoth[Tanh[a + b*x]])) + (4*(x^3/(3*b) + ((b*x - ArcCoth[Tanh[a
 + b*x]])*(x^2/(2*b) + ((b*x - ArcCoth[Tanh[a + b*x]])*(x/b + ((b*x - ArcC
oth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^2))/b))/b))/b
```

3.167. $\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx$

3.167.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.167.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 131085, normalized size of antiderivative = 1337.60

method	result	size
risch	Expression too large to display	131085

input
```
int(x^4/arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.167.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.16

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{16\,b^4x^4 - 32\,ab^3x^3 + 96\,a^2b^2x^2 + 144\,a^3bx - 3\,\pi^4 - 6i\,\pi^3(3\,bx - 4\,a) - 48\,a^4 - 12\,\pi^2(2\,b^2x^2 + 9\,abx - 6\,a^2}{}$$

input
```
integrate(x^4/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/24*(16*b^4*x^4 - 32*a*b^3*x^3 + 96*a^2*b^2*x^2 + 144*a^3*b*x - 3*pi^4 -
6*I*pi^3*(3*b*x - 4*a) - 48*a^4 - 12*pi^2*(2*b^2*x^2 + 9*a*b*x - 6*a^2) -
8*I*pi*(2*b^3*x^3 - 12*a*b^2*x^2 - 27*a^2*b*x + 12*a^3) - 12*(16*a^3*b*x +
 pi^4 - 2*I*pi^3*(b*x + 4*a) + 16*a^4 - 12*pi^2*(a*b*x + 2*a^2) + 8*I*pi*(
3*a^2*b*x + 4*a^3))*log(I*pi + 2*b*x + 2*a))/(2*b^6*x + I*pi*b^5 + 2*a*b^5
)
```

3.167.6 Sympy [F]

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^4}{\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**4/acoth(tanh(b*x+a))**2,x)
```

output
```
Integral(x**4/acoth(tanh(a + b*x))**2, x)
```

3.167.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.83

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{16\,b^4x^4 - 3\,\pi^4 - 24i\,\pi^3a + 72\,\pi^2a^2 + 96i\,\pi a^3 - 48\,a^4 - 16\,(-i\,\pi b^3 + 2\,ab^3)x^3 - 24\,(\pi^2b^2 + 4i\,\pi ab^2 - 4\,a^2b}{24\,(2\,b^6x - i\,\pi b^5 + 2\,ab^5)} - \frac{(i\,\pi^3 - 6\,\pi^2a - 12i\,\pi a^2 + 8\,a^3)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{2\,b^5}$$

input
```
integrate(x^4/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/24*(16*b^4*x^4 - 3*pi^4 - 24*I*pi^3*a + 72*pi^2*a^2 + 96*I*pi*a^3 - 48*a
^4 - 16*(-I*pi*b^3 + 2*a*b^3)*x^3 - 24*(pi^2*b^2 + 4*I*pi*a*b^2 - 4*a^2*b^
2)*x^2 - 18*(-I*pi^3*b + 6*pi^2*a*b + 12*I*pi*a^2*b - 8*a^3*b)*x)/(2*b^6*x
 - I*pi*b^5 + 2*a*b^5) - 1/2*(I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3)*log
(-I*pi + 2*b*x + 2*a)/b^5
```

3.167.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.38

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2} \, dx = \frac{x^3}{3\,b^2} - \frac{\pi^4 - 8i\,\pi^3 a - 24\,\pi^2 a^2 + 32i\,\pi a^3 + 16\,a^4}{8\,(2\,b^6 x + i\,\pi b^5 + 2\,ab^5)} - \frac{(i\,\pi + 2\,a)x^2}{2\,b^3} - \frac{3\,(\pi^2 - 4i\,\pi a - 4\,a^2)x}{4\,b^4} + \frac{(i\,\pi^3 + 6\,\pi^2 a - 12i\,\pi a^2 - 8\,a^3)\log(i\,\pi + 2\,bx + 2\,a)}{2\,b^5}$$

input
```
integrate(x^4/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
1/3*x^3/b^2 - 1/8*(pi^4 - 8*I*pi^3*a - 24*pi^2*a^2 + 32*I*pi*a^3 + 16*a^4)
/(2*b^6*x + I*pi*b^5 + 2*a*b^5) - 1/2*(I*pi + 2*a)*x^2/b^3 - 3/4*(pi^2 - 4
*I*pi*a - 4*a^2)*x/b^4 + 1/2*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*log
(I*pi + 2*b*x + 2*a)/b^5
```

3.167.9 Mupad [B] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 669, normalized size of antiderivative = 6.83

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{x^3}{3\,b^2}$$

$$-\frac{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^4+24\,a^2\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\right.}{2\,b\left(8\,a\,b^4+8\,b^5\,x-4\,b^4\right.}$$

$$+\frac{x^2\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2\,b^3}$$

$$+\frac{3\,x\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2}{4\,b^4}$$

$$+\frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\right)\left(\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^3-8\,a^3-6\right.}{}$$

input
```
int(x^4/acoth(tanh(a + b*x))^2,x)
```

output
```
x^3/(3*b^2) - ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1
)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*
x) - 1)) + 2*b*x)^2 + 16*a^4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 32
*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-
2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b*(8*a*b^4 + 8*b^5*x - 4*b^4*(2*
a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2
*a)*exp(2*b*x) - 1)) + 2*b*x))) + (x^2*(log(-2/(exp(2*a)*exp(2*b*x) - 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b^3)
 + (3*x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2)/(4*b^4) + (log(log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-2/(exp(2*a)*exp(2*b*x) - 1)))*((2
*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(
2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2
*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) -
 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)))/(2*b^5)
```

3.168 $\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2} dx$

3.168.1 Optimal result

Integrand size = 13, antiderivative size = 75

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2} dx = \frac{3x^2}{2b^2} + \frac{3x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^3}{b\coth^{-1}(\tanh(a+bx))} + \frac{3\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^4}$$

output
```
3/2*x^2/b^2+3*x*(b*x-arccoth(tanh(b*x+a)))/b^3-x^3/b/arccoth(tanh(b*x+a))+
3*(b*x-arccoth(tanh(b*x+a)))^2*ln(arccoth(tanh(b*x+a)))/b^4
```

3.168.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2} dx = \frac{x^2}{2b^2} - \frac{2x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3}{b^4\coth^{-1}(\tanh(a+bx))} + \frac{3\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2 \log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^4}$$

input
```
Integrate[x^3/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
x^2/(2*b^2) - (2*x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^3 + (-(b*x) + ArcC
oth[Tanh[a + b*x]])^3/(b^4*ArcCoth[Tanh[a + b*x]]) + (3*(-(b*x) + ArcCoth[
Tanh[a + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^4
```

3.168.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2599, 2590, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{3\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}dx}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2590}$$

$$\frac{3\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{x}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2589}$$

$$\frac{3\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2588}$$

$$\frac{3\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int\frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow 14$$

$$\frac{3\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[x^3/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
-(x^3/(b*ArcCoth[Tanh[a + b*x]])) + (3*(x^2/(2*b) + ((b*x - ArcCoth[Tanh[a
 + b*x]])*(x/b + ((b*x - ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]
]])/b^2))/b))/b
```

3.168.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.168.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 29109, normalized size of antiderivative = 388.12

method	result	size
risch	Expression too large to display	29109

input
```
int(x^3/arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.168.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.01

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx$$
$$= \frac{4\,b^3x^3 - 12\,ab^2x^2 - 16\,a^2bx - i\,\pi^3 + 2\,\pi^2(2\,bx - 3\,a) + 8\,a^3 - 2i\,\pi(3\,b^2x^2 + 8\,abx - 6\,a^2) + 3\,(8\,a^2bx - i\,\pi}{4\,(2\,b^5x + i\,\pi b^4 + 2\,ab^4)}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/4*(4*b^3*x^3 - 12*a*b^2*x^2 - 16*a^2*b*x - I*pi^3 + 2*pi^2*(2*b*x - 3*a)
 + 8*a^3 - 2*I*pi*(3*b^2*x^2 + 8*a*b*x - 6*a^2) + 3*(8*a^2*b*x - I*pi^3 -
2*pi^2*(b*x + 3*a) + 8*a^3 + 4*I*pi*(2*a*b*x + 3*a^2))*log(I*pi + 2*b*x +
2*a))/(2*b^5*x + I*pi*b^4 + 2*a*b^4)
```

3.168.6 Sympy [F]

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^3}{\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**3/acoth(tanh(b*x+a))**2,x)
```

output
```
Integral(x**3/acoth(tanh(a + b*x))**2, x)
```

3.168.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.64

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx$$
$$= \frac{4\,b^3x^3 + i\,\pi^3 - 6\,\pi^2a - 12i\,\pi a^2 + 8\,a^3 - 6\left(-i\,\pi b^2 + 2\,ab^2\right)x^2 + 4\left(\pi^2b + 4i\,\pi ab - 4\,a^2b\right)x}{4\left(2\,b^5x - i\,\pi b^4 + 2\,ab^4\right)} - \frac{3\left(\pi^2 + 4i\,\pi a - 4\,a^2\right)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{4\,b^4}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/4*(4*b^3*x^3 + I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3 - 6*(-I*pi*b^2 +
2*a*b^2)*x^2 + 4*(pi^2*b + 4*I*pi*a*b - 4*a^2*b)*x)/(2*b^5*x - I*pi*b^4 +
2*a*b^4) - 3/4*(pi^2 + 4*I*pi*a - 4*a^2)*log(-I*pi + 2*b*x + 2*a)/b^4
```

3.168.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{i\,\pi^3 + 6\,\pi^2a - 12i\,\pi a^2 - 8\,a^3}{4\left(2\,b^5x + i\,\pi b^4 + 2\,ab^4\right)} + \frac{x^2}{2\,b^2} + \frac{(-i\,\pi - 2\,a)x}{b^3} - \frac{3\left(\pi^2 - 4i\,\pi a - 4\,a^2\right)\log\left(i\,\pi + 2\,bx + 2\,a\right)}{4\,b^4}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
-1/4*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)/(2*b^5*x + I*pi*b^4 + 2*a*b
^4) + 1/2*x^2/b^2 + (-I*pi - 2*a)*x/b^3 - 3/4*(pi^2 - 4*I*pi*a - 4*a^2)*lo
g(I*pi + 2*b*x + 2*a)/b^4
```

3.168.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 490, normalized size of antiderivative = 6.53

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{x^2}{2\,b^2}$$
$$+\frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\right)\,\left(3\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2-12\,a\,\left(\right.\right.}{4\,b^4}$$
$$-\frac{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^3-8\,a^3-6\,a\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right.\right.}{4\,b\,\left(2\,a\,b^3+2\,b^4\,x-b^3\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\right.\right.\right.}$$
$$+\frac{x\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}{b^3}$$

input
```
int(x^3/acoth(tanh(a + b*x))^2,x)
```

output
```
x^2/(2*b^2) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))
- log(-2/(exp(2*a)*exp(2*b*x) - 1)))*(3*(2*a - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2
 - 12*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + lo
g(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 12*a^2))/(4*b^4) - ((2*a - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(
2*b*x) - 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 +
 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + lo
g(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(4*b*(2*a*b^3 + 2*b^4*x - b^3*(2
*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(
2*a)*exp(2*b*x) - 1)) + 2*b*x))) + (x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/b^3
```

3.169 $\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx$

3.169.1 Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{2x}{b^2} - \frac{x^2}{b\coth^{-1}(\tanh(a+bx))} + \frac{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}$$

output
```
2*x/b^2-x^2/b/arccoth(tanh(b*x+a))+2*(b*x-arccoth(tanh(b*x+a)))*ln(arccoth
(tanh(b*x+a)))/b^3
```

3.169.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{bx - \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2}{\coth^{-1}(\tanh(a+bx))} + 2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}$$

input
```
Integrate[x^2/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(b*x - (-(b*x) + ArcCoth[Tanh[a + b*x]])^2/ArcCoth[Tanh[a + b*x]] + 2*(b*x
 - ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^3
```

3.169.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2599, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2\int \frac{x}{\coth^{-1}(\tanh(a+bx))}dx}{b}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2589}$$

$$\frac{2\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2588}$$

$$\frac{2\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

$$\frac{2\left(\frac{(bx-\coth^{-1}(\tanh(a+bx)))\log(\coth^{-1}(\tanh(a+bx)))}{b^2}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[x^2/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
-(x^2/(b*ArcCoth[Tanh[a + b*x]])) + (2*(x/b + ((b*x - ArcCoth[Tanh[a + b*x
]])*Log[ArcCoth[Tanh[a + b*x]]])/b^2))/b
```

3.169.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.169.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 4626, normalized size of antiderivative = 92.52

method	result	size
risch	Expression too large to display	4626

input
```
int(x^2/arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
-4*I*x^2/b/(Pi*csgn(I*exp(2*b*x+2*a))^3-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp
(2*b*x+2*a))^2-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+
2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2
*a))+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-Pi*csgn(I/(exp(2*b*x+2
*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x
+2*a)+1))^3-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+4*I*ln(exp(b*x+a))+2*Pi)+2*x
/b^2+1/2*I/b^3*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(
I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3-2
*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+
2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x
+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*(ln(exp(b*x+a))-b*
x-a)+4*I*a+4*I*b*x+2*Pi)*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/
2*I/b^3*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3-2*Pi*csg
n(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2
*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a...
```

3.169.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.90

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx$$
$$= \frac{4\,b^2x^2 + 4\,abx + \pi^2 + 2i\,\pi(bx - 2\,a) - 4\,a^2 - 2\,(4\,abx - \pi^2 + 2i\,\pi(bx + 2\,a) + 4\,a^2)\log\left(i\,\pi + 2\,bx + 2\,a\right)}{2\,(2\,b^4x + i\,\pi b^3 + 2\,ab^3)}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/2*(4*b^2*x^2 + 4*a*b*x + pi^2 + 2*I*pi*(b*x - 2*a) - 4*a^2 - 2*(4*a*b*x
- pi^2 + 2*I*pi*(b*x + 2*a) + 4*a^2)*log(I*pi + 2*b*x + 2*a))/(2*b^4*x + I
*pi*b^3 + 2*a*b^3)
```

3.169.6 Sympy [F]

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{x^2}{\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**2/acoth(tanh(b*x+a))**2,x)
```

output
```
Integral(x**2/acoth(tanh(a + b*x))**2, x)
```

3.169.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.62

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{4\,b^2x^2+\pi^2+4i\,\pi a-4\,a^2-2\,(i\,\pi b-2\,ab)x}{2\,(2\,b^4x-i\,\pi b^3+2\,ab^3)} - \frac{(-i\,\pi+2\,a)\log\left(-i\,\pi+2\,bx+2\,a\right)}{b^3}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/2*(4*b^2*x^2 + pi^2 + 4*I*pi*a - 4*a^2 - 2*(I*pi*b - 2*a*b)*x)/(2*b^4*x
- I*pi*b^3 + 2*a*b^3) - (-I*pi + 2*a)*log(-I*pi + 2*b*x + 2*a)/b^3
```

3.169.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{\pi^2-4i\,\pi a-4\,a^2}{2\,(2\,b^4x+i\,\pi b^3+2\,ab^3)} + \frac{x}{b^2} - \frac{(i\,\pi+2\,a)\log\left(i\,\pi+2\,bx+2\,a\right)}{b^3}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
1/2*(pi^2 - 4*I*pi*a - 4*a^2)/(2*b^4*x + I*pi*b^3 + 2*a*b^3) + x/b^2 - (I*
pi + 2*a)*log(I*pi + 2*b*x + 2*a)/b^3
```

3.169.9 Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 302, normalized size of antiderivative = 6.04

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{x}{b^2} - \frac{\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)^2-4\,a\,\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b}{2\,b\,\left(2\,a\,b^2+2\,b^3\,x-b^2\,\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)\right)} + \frac{\ln\left(\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\right)\,\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}{b^3}$$

input
```
int(x^2/acoth(tanh(a + b*x))^2,x)
```

output
```
x/b^2 - ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + l
og(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1))
+ 2*b*x) + 4*a^2)/(2*b*(2*a*b^2 + 2*b^3*x - b^2*(2*a - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) +
2*b*x))) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - l
og(-2/(exp(2*a)*exp(2*b*x) - 1)))*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/b^3
```

3.170 $\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx$

3.170.1 Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x}{b\coth^{-1}(\tanh(a+bx))} + \frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}$$

output
```
-x/b/arccoth(tanh(b*x+a))+ln(arccoth(tanh(b*x+a)))/b^2
```

3.170.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{1-\frac{bx}{\coth^{-1}(\tanh(a+bx))}+\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}$$

input
```
Integrate[x/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
(1 - (b*x)/ArcCoth[Tanh[a + b*x]] + Log[ArcCoth[Tanh[a + b*x]]])/b^2
```

3.170.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2599, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \ 2599$$

$$\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}-\frac{x}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \ 2588$$

$$\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}-\frac{x}{b\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \ 14$$

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}-\frac{x}{b\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[x/ArcCoth[Tanh[a + b*x]]^2,x]
```

output
```
-(x/(b*ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/b^2
```

3.170.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.170.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

method	result
parallelrisch	$\frac{\ln(\operatorname{arccoth}(\tanh(bx+a)))\operatorname{arccoth}(\tanh(bx+a))-bx}{b^2\operatorname{arccoth}(\tanh(bx+a))}$
risch	$-\frac{}{b\left(\pi\operatorname{csgn}\left(ie^{2bx+2a}\right)^3-2\pi\operatorname{csgn}\left(ie^{bx+a}\right)\operatorname{csgn}\left(ie^{2bx+2a}\right)^2-\pi\operatorname{csgn}\left(ie^{2bx+2a}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2+\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\right.}$

input
```
int(x/arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
(ln(arccoth(tanh(b*x+a)))*arccoth(tanh(b*x+a))-b*x)/b^2/arccoth(tanh(b*x+a
))
```

3.170.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{i\pi + (i\pi + 2bx + 2a)\log(i\pi + 2bx + 2a) + 2a}{2b^3x + i\pi b^2 + 2ab^2}$$

input
```
integrate(x/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
(I*pi + (I*pi + 2*b*x + 2*a)*log(I*pi + 2*b*x + 2*a) + 2*a)/(2*b^3*x + I*p
i*b^2 + 2*a*b^2)
```

3.170.6 Sympy [A] (verification not implemented)

Time = 12.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \begin{cases} -\frac{x}{b\,\mathrm{acoth}\,(\tanh\,(a+bx))} + \frac{\log\,(\mathrm{acoth}\,(\tanh\,(a+bx)))}{b^2} & \text{for } b \neq 0 \\ \frac{x^2}{2\,\mathrm{acoth}^2\,(\tanh\,(a))} & \text{otherwise} \end{cases}$$

input
```
integrate(x/acoth(tanh(b*x+a))**2,x)
```

output
```
Piecewise((-x/(b*acoth(tanh(a + b*x))) + log(acoth(tanh(a + b*x)))/b**2, N
e(b, 0)), (x**2/(2*acoth(tanh(a))**2), True))
```

3.170.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{-i\,\pi + 2\,a}{2\,b^3x - i\,\pi b^2 + 2\,ab^2} + \frac{\log\,(-i\,\pi + 2\,bx + 2\,a)}{b^2}$$

input
```
integrate(x/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
(-I*pi + 2*a)/(2*b^3*x - I*pi*b^2 + 2*a*b^2) + log(-I*pi + 2*b*x + 2*a)/b^
2
```

3.170.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{-i\,\pi - 2\,a}{2\,b^3x + i\,\pi b^2 + 2\,ab^2} + \frac{\log\,(i\,\pi + 2\,bx + 2\,a)}{b^2}$$

input
```
integrate(x/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
-(-I*pi - 2*a)/(2*b^3*x + I*pi*b^2 + 2*a*b^2) + log(I*pi + 2*b*x + 2*a)/b^
2
```

3.170.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2} \, dx = \frac{\ln(\mathrm{acoth}(\tanh(a+b\,x)))}{b^2} - \frac{x}{b\,\mathrm{acoth}\,(\tanh\,(a+b\,x))}$$

input
```
int(x/acoth(tanh(a + b*x))^2,x)
```

output
```
log(acoth(tanh(a + b*x)))/b^2 - x/(b*acoth(tanh(a + b*x)))
```

3.171 $\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2} \, dx$

3.171.1 Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2} \, dx = -\frac{1}{b\coth^{-1}(\tanh(a+bx))}$$

output
```
-1/b/arccoth(tanh(b*x+a))
```

3.171.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2} \, dx = -\frac{1}{b\coth^{-1}(\tanh(a+bx))}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^(-2),x]
```

output
```
-(1/(b*ArcCoth[Tanh[a + b*x]]))
```

3.171.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}\,dx$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}d\coth^{-1}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{b\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^(-2),x]
```

output
```
-(1/(b*ArcCoth[Tanh[a + b*x]]))
```

3.171.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.171.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result
derivativedivides	$-\frac{1}{b\,\operatorname{arccoth}(\tanh(bx+a))}$
default	$-\frac{1}{b\,\operatorname{arccoth}(\tanh(bx+a))}$
parallelrisch	$-\frac{1}{b\,\operatorname{arccoth}(\tanh(bx+a))}$
risch	$-\frac{}{b\left(\pi\,\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{3}-2\pi\,\operatorname{csgn}\left(i\mathrm{e}^{bx+a}\right)\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{2}-\pi\,\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}+\pi\,\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right.\right.}$

input
```
int(1/arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/arccoth(tanh(b*x+a))
```

3.171.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{2}{2\,b^2x+i\,\pi b+2\,ab}$$

input
```
integrate(1/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
-2/(2*b^2*x + I*pi*b + 2*a*b)
```

3.171.6 Sympy [A] (verification not implemented)

Time = 12.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \begin{cases} -\frac{1}{b\,\operatorname{acoth}(\tanh(a+bx))} & \text{for } b \neq 0 \\ \frac{x}{\operatorname{acoth}^2(\tanh(a))} & \text{otherwise} \end{cases}$$

input
```
integrate(1/acoth(tanh(b*x+a))**2,x)
```

output
```
Piecewise((-1/(b*acoth(tanh(a + b*x))), Ne(b, 0)), (x/acoth(tanh(a))**2, T
rue))
```

3.171.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{4}{-2\,(i\,\pi+2\,bx+2\,a)b}$$

input
```
integrate(1/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
4/((-2*I*pi - 4*b*x - 4*a)*b)
```

3.171.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{2}{2\,b^2x+i\,\pi b+2\,ab}$$

input
```
integrate(1/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
-2/(2*b^2*x + I*pi*b + 2*a*b)
```

3.171.9 Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{b\,\mathrm{acoth}\left(\tanh\left(a+b\,x\right)\right)}$$

input
```
int(1/acoth(tanh(a + b*x))^2,x)
```

output
```
-1/(b*acoth(tanh(a + b*x)))
```

3.172 $\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2} dx$

3.172.1 Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2} dx = -\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))} + \frac{\log(x)}{(bx-\coth^{-1}(\tanh(a+bx)))^2} - \frac{\log(\coth^{-1}(\tanh(a+bx)))}{(bx-\coth^{-1}(\tanh(a+bx)))^2}$$

output
```
-1/(b*x-arccoth(tanh(b*x+a)))/arccoth(tanh(b*x+a))+ln(x)/(b*x-arccoth(tanh
(b*x+a)))^2-ln(arccoth(tanh(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^2
```

3.172.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2} dx = \frac{-bx+\coth^{-1}(\tanh(a+bx))(1+\log(bx)-\log(\coth^{-1}(\tanh(a+bx))))}{\coth^{-1}(\tanh(a+bx))(-bx+\coth^{-1}(\tanh(a+bx)))^2}$$

input
```
Integrate[1/(x*ArcCoth[Tanh[a + b*x]]^2),x]
```

output
```
(-(b*x) + ArcCoth[Tanh[a + b*x]]*(1 + Log[b*x] - Log[ArcCoth[Tanh[a + b*x]
]]))/(ArcCoth[Tanh[a + b*x]]*(-(b*x) + ArcCoth[Tanh[a + b*x]])^2)
```

3.172.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}dx$$

$$\Big\downarrow \text{2594}$$

$$-\frac{\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2591}$$

$$-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

$$-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2588}$$

$$-\frac{\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

$$-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}-\frac{\frac{\log(\coth^{-1}(\tanh(a+bx)))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[1/(x*ArcCoth[Tanh[a + b*x]]^2),x]
```

output
```
-(1/((b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]])) - (-(Log[x]/(
b*x - ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCot
h[Tanh[a + b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]])
```

3.172.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

3.172.4 Maple [F(-1)]

Timed out.

$$\int \frac{1}{x \operatorname{arccoth}(\tanh(bx+a))^2} dx$$

input
```
int(1/x/arccoth(tanh(b*x+a))^2,x)
```

output
```
int(1/x/arccoth(tanh(b*x+a))^2,x)
```

3.172.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}\,dx \\ = -\frac{4\left(-i\pi+(i\pi+2bx+2a)\log\left(i\pi+2bx+2a\right)+(-i\pi-2bx-2a)\log\left(x\right)-2a\right)}{8a^2bx-i\pi^3-2\pi^2(bx+3a)+8a^3+4i\pi(2abx+3a^2)}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
-4*(-I*pi + (I*pi + 2*b*x + 2*a)*log(I*pi + 2*b*x + 2*a) + (-I*pi - 2*b*x
- 2*a)*log(x) - 2*a)/(8*a^2*b*x - I*pi^3 - 2*pi^2*(b*x + 3*a) + 8*a^3 + 4*
I*pi*(2*a*b*x + 3*a^2))
```

3.172.6 Sympy [F]

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{1}{x\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x/acoth(tanh(b*x+a))**2,x)
```

output
```
Integral(1/(x*acoth(tanh(a + b*x))**2), x)
```

3.172.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{4\log\left(-i\pi+2bx+2a\right)}{\pi^2+4i\pi a-4a^2} - \frac{4\log\left(x\right)}{\pi^2+4i\pi a-4a^2} - \frac{4}{\pi^2+4i\pi a-4a^2-2\left(-i\pi b+2ab\right)x}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
4*log(-I*pi + 2*b*x + 2*a)/(pi^2 + 4*I*pi*a - 4*a^2) - 4*log(x)/(pi^2 + 4*
I*pi*a - 4*a^2) - 4/(pi^2 + 4*I*pi*a - 4*a^2 - 2*(-I*pi*b + 2*a*b)*x)
```

3.172.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{4\,\log\left(i\,\pi+2\,bx+2\,a\right)}{\pi^2-4i\,\pi a-4\,a^2}-\frac{4\,\log\left(x\right)}{\pi^2-4i\,\pi a-4\,a^2} + \frac{4}{2i\,\pi bx+4\,abx-\pi^2+4i\,\pi a+4\,a^2}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
4*log(I*pi + 2*b*x + 2*a)/(pi^2 - 4*I*pi*a - 4*a^2) - 4*log(x)/(pi^2 - 4*I
*pi*a - 4*a^2) + 4/(2*I*pi*b*x + 4*a*b*x - pi^2 + 4*I*pi*a + 4*a^2)
```

3.172.9 Mupad [B] (verification not implemented)

Time = 7.72 (sec) , antiderivative size = 421, normalized size of antiderivative = 6.01

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{4\,\ln\left(-\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-4\,\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+8\,b\,x+\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\,\mathrm{atan}\left(\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\,1{}\mathrm{i}-\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\,1{}\mathrm{i}+b}{\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b}\right.}{\left(\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(-\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\right)\,\left(\ln\left(-\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b}}\right.\right.}$$

input
```
int(1/(x*acoth(tanh(a + b*x))^2),x)
```

output
```
-(4*log(-1/(exp(2*a)*exp(2*b*x) - 1)) - 4*log((exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) - 1)) + 8*b*x + log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) - 1))*atan((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1
i - log(-2/(exp(2*a)*exp(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2
*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b
*x))*8i - atan((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1i
- log(-2/(exp(2*a)*exp(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2*b
*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x
))*(log(2) + log(-1/(exp(2*a)*exp(2*b*x) - 1)))*8i)/((log((exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-1/(exp(2*a)*exp(2*b*x) - 1)))*(log
(-1/(exp(2*a)*exp(2*b*x) - 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) - 1)) + 2*b*x)^2)
```

3.172. $\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2}\,dx$

3.173 $\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2} \, dx$

3.173.1 Optimal result

Integrand size = 13, antiderivative size = 102

$$\begin{aligned}\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2} \, dx = &-\frac{2b}{(bx-\coth^{-1}(\tanh(a+bx)))^2 \coth^{-1}(\tanh(a+bx))} \\ &+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))} \\ &+\frac{2b\log(x)}{(bx-\coth^{-1}(\tanh(a+bx)))^3} \\ &-\frac{2b\log\left(\coth^{-1}(\tanh(a+bx))\right)}{(bx-\coth^{-1}(\tanh(a+bx)))^3}\end{aligned}$$

output
```
-2*b/(b*x-arccoth(tanh(b*x+a)))^2/arccoth(tanh(b*x+a))+1/x/(b*x-arccoth(ta
nh(b*x+a)))/arccoth(tanh(b*x+a))+2*b*ln(x)/(b*x-arccoth(tanh(b*x+a)))^3-2*
b*ln(arccoth(tanh(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^3
```

3.173.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{-b^2x^2+\coth^{-1}(\tanh(a+bx))^2+2bx\coth^{-1}(\tanh(a+bx))\left(\log(x)-\log\left(\coth^{-1}(\tanh(a+bx))\right)\right)}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))}$$

input
```
Integrate[1/(x^2*ArcCoth[Tanh[a + b*x]]^2),x]
```

output
```
(-(b^2*x^2) + ArcCoth[Tanh[a + b*x]]^2 + 2*b*x*ArcCoth[Tanh[a + b*x]]*(Log
[x] - Log[ArcCoth[Tanh[a + b*x]]]))/(x*(b*x - ArcCoth[Tanh[a + b*x]])^3*Ar
cCoth[Tanh[a + b*x]])
```

3.173.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2602, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2}\,dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{2b\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}dx}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2594}$$

$$\frac{2b\left(-\frac{\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2591}$$

$$\frac{2b\left(-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\,(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

$$\frac{2b\left(-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\,(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2588}$$

$$\frac{2b\left(-\frac{\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\,(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{x\,(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}+\frac{2b\left(-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}-\frac{\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[1/(x^2*ArcCoth[Tanh[a + b*x]]^2),x]
```

output
```
1/(x*(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]) + (2*b*(-(1/((
b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]])) - (-(Log[x]/(b*x -
ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Tanh
[a + b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]])))/(b*x - ArcCoth[Tanh[a + b*x]
])
```

3.173. $\int \frac{1}{x^2\coth^{-1}(\tanh(a+bx))^2}\,dx$

3.173.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.173.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.35 (sec) , antiderivative size = 5357, normalized size of antiderivative = 52.52

output too large to display

input
```
int(1/x^2/arccoth(tanh(b*x+a))^2,x)
```

output
```
result too large to display
```

3.173.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{4\,(8\,abx - \pi^2 + 4i\,\pi(bx+a) + 4\,a^2 - 4\,(2\,b^2x^2 + i\,\pi bx + 2\,abx)\log\left(i\,\pi + 2\,bx + 2\,a\right) + 4\,(2\,b^2x^2 + i\,\pi bx}{16\,a^3bx^2 + \pi^4x + 16\,a^4x - 2i\,\pi^3(bx^2 + 4\,ax) - 12\,\pi^2(abx^2 + 2\,a^2x) + 8i\,\pi(3\,a^2bx^2 + 4\,a^3}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
-4*(8*a*b*x - pi^2 + 4*I*pi*(b*x + a) + 4*a^2 - 4*(2*b^2*x^2 + I*pi*b*x +
2*a*b*x)*log(I*pi + 2*b*x + 2*a) + 4*(2*b^2*x^2 + I*pi*b*x + 2*a*b*x)*log(
x))/(16*a^3*b*x^2 + pi^4*x + 16*a^4*x - 2*I*pi^3*(b*x^2 + 4*a*x) - 12*pi^2
*(a*b*x^2 + 2*a^2*x) + 8*I*pi*(3*a^2*b*x^2 + 4*a^3*x))
```

3.173.6 Sympy [F]

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{1}{x^2\,\mathrm{acoth}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**2/acoth(tanh(b*x+a))**2,x)
```

output
```
Integral(1/(x**2*acoth(tanh(a + b*x))**2), x)
```

3.173.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.32

$$\begin{aligned}\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2}\,dx &= -\frac{16\,b\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{-i\,\pi^3 + 6\,\pi^2a + 12i\,\pi a^2 - 8\,a^3} + \frac{16\,b\log\left(x\right)}{-i\,\pi^3 + 6\,\pi^2a + 12i\,\pi a^2 - 8\,a^3} \\ &\quad - \frac{4\,(i\,\pi - 4\,bx - 2\,a)}{2\,(\pi^2b + 4i\,\pi ab - 4\,a^2b)x^2 - (i\,\pi^3 - 6\,\pi^2a - 12i\,\pi a^2 + 8\,a^3)x}\end{aligned}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
-16*b*log(-I*pi + 2*b*x + 2*a)/(-I*pi^3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3)
+ 16*b*log(x)/(-I*pi^3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3) - 4*(I*pi - 4*b*x
 - 2*a)/(2*(pi^2*b + 4*I*pi*a*b - 4*a^2*b)*x^2 - (I*pi^3 - 6*pi^2*a - 12*I
*pi*a^2 + 8*a^3)*x)
```

3.173.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2} \, dx = \frac{16i \, b \log(i\pi + 2\,bx + 2\,a)}{\pi^3 - 6i\,\pi^2 a - 12\,\pi a^2 + 8i\,a^3} - \frac{16i\,b\log(x)}{\pi^3 - 6i\,\pi^2 a - 12\,\pi a^2 + 8i\,a^3} + \frac{8\,b}{2\,\pi^2 bx - 8i\,\pi abx - 8\,a^2 bx + i\,\pi^3 + 6\,\pi^2 a - 12i\,\pi a^2 - 8\,a^3} + \frac{4}{\pi^2 x - 4i\,\pi ax - 4\,a^2 x}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
16*I*b*log(I*pi + 2*b*x + 2*a)/(pi^3 - 6*I*pi^2*a - 12*pi*a^2 + 8*I*a^3) -
 16*I*b*log(x)/(pi^3 - 6*I*pi^2*a - 12*pi*a^2 + 8*I*a^3) + 8*b/(2*pi^2*b*x
 - 8*I*pi*a*b*x - 8*a^2*b*x + I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3) + 4
/(pi^2*x - 4*I*pi*a*x - 4*a^2*x)
```

3.173.9 Mupad [B] (verification not implemented)

Time = 6.68 (sec) , antiderivative size = 453, normalized size of antiderivative = 4.44

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2} \, dx = \frac{4\ln\left(-\frac{1}{e^{2a}\,e^{2bx}-1}\right)^2 - \ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)\left(8\ln\left(-\frac{1}{e^{2a}\,e^{2bx}-1}\right) + b\,x\,\mathrm{atan}\left(\frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)1i - \ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)1i + b\,x\,2i}{\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right) - \ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right) + 2\,b\,x}\right.\right.}{x\left(\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right) - \ln\left(-\frac{1}{e^{2a}\,e^{2bx}-1}\right)\right)\,(}$$

input
```
int(1/(x^2*acoth(tanh(a + b*x))^2),x)
```

output
```
(4*log(-1/(exp(2*a)*exp(2*b*x) - 1))^2 - log((exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) - 1))*(8*log(-1/(exp(2*a)*exp(2*b*x) - 1)) + b*x*atan((log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1i - log(-2/(exp(2*a)*ex
p(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))*32i) + 4*log((exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2 - 16*b^2*x^2 + b*x*log(-1/(e
xp(2*a)*exp(2*b*x) - 1))*atan((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) - 1))*1i - log(-2/(exp(2*a)*exp(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(e
xp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) - 1)) + 2*b*x))*32i)/(x*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
- 1)) - log(-1/(exp(2*a)*exp(2*b*x) - 1)))*(log(-1/(exp(2*a)*exp(2*b*x) -
1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3)
```

3.173. $\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2} \, dx$

3.174 $\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2}\,dx$

3.174.1 Optimal result

Integrand size = 13, antiderivative size = 143

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3 \coth^{-1}(\tanh(a+bx))} + \frac{3b}{2x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \coth^{-1}(\tanh(a+bx))} + \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))} + \frac{3b^2\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4} - \frac{3b^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4}$$

output
```
-3*b^2/(b*x-arccoth(tanh(b*x+a)))^3/arccoth(tanh(b*x+a))+3/2*b/x/(b*x-arcc
oth(tanh(b*x+a)))^2/arccoth(tanh(b*x+a))+1/2/x^2/(b*x-arccoth(tanh(b*x+a))
)/arccoth(tanh(b*x+a))+3*b^2*ln(x)/(b*x-arccoth(tanh(b*x+a)))^4-3*b^2*ln(a
rccoth(tanh(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^4
```

3.174.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2} \, dx = -\frac{2b^3x^3 - 6bx\coth^{-1}(\tanh(a+bx))^2 + \coth^{-1}(\tanh(a+bx))^3 - 3b^2x^2\coth^{-1}(\tanh(a+bx))\left(-1+2\log\right.}{2x^2\coth^{-1}(\tanh(a+bx))\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^4}$$

input
```
Integrate[1/(x^3*ArcCoth[Tanh[a + b*x]]^2),x]
```

output
```
-1/2*(2*b^3*x^3 - 6*b*x*ArcCoth[Tanh[a + b*x]]^2 + ArcCoth[Tanh[a + b*x]]^
3 - 3*b^2*x^2*ArcCoth[Tanh[a + b*x]]*(-1 + 2*Log[x] - 2*Log[ArcCoth[Tanh[a
 + b*x]]]))/(x^2*ArcCoth[Tanh[a + b*x]]*(-(b*x) + ArcCoth[Tanh[a + b*x]])^
4)
```

3.174.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2602, 2602, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2} \, dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{3b\int \frac{1}{x^2\coth^{-1}(\tanh(a+bx))^2}dx}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)} + \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2602}$$

$$\frac{3b\left(\frac{2b\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}dx}{bx-\coth^{-1}(\tanh(a+bx))} + \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}\right)}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)} + \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}$$

$$\Big\downarrow \text{2594}$$

$$\frac{3b\left(\frac{2b\left(-\frac{\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}\right)}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+$$
$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}$$

$\downarrow$ 2591

$$\frac{3b\left(\frac{2b\left(-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\text{co}}\right.}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$
$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}$$

$\downarrow$ 14

$$\frac{3b\left(\frac{2b\left(-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\text{co}}\right.}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$
$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}$$

$\downarrow$ 2588

$$\frac{3b\left(\frac{2b\left(-\frac{\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{}{x\left(bx-\coth^{-1}(\text{t}\right.}\right.}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$
$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}$$

$\downarrow$ 14

3.174. $\int \frac{1}{x^3\coth^{-1}(\tanh(a+bx))^2}\,dx$

$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}+$$

$$\frac{3b\left(\frac{1}{x(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}+\frac{2b\left(-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}-\frac{\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\cdots}{bx-\coth\cdots}}{bx-\coth^{-1}(\tanh(a+b\cdots}\right.}{bx-\coth^{-1}(\tanh(a+bx))}\right.}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

input
```
Int[1/(x^3*ArcCoth[Tanh[a + b*x]]^2),x]
```

output
```
1/(2*x^2*(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]) + (3*b*(1/
(x*(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]) + (2*b*(-(1/((b*
x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]])) - (-(Log[x]/(b*x - Ar
cCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Tanh[a
 + b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]])))/(b*x - ArcCoth[Tanh[a + b*x]])
))/(2*(b*x - ArcCoth[Tanh[a + b*x]]))
```

3.174.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.174.4 Maple [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)^2}dx$$

input
```
int(1/x^3/arccoth(tanh(b*x+a))^2,x)
```

output
```
int(1/x^3/arccoth(tanh(b*x+a))^2,x)
```

3.174.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.71

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{2\left(48\,ab^2x^2+24\,a^2bx+i\,\pi^3-6\,\pi^2(bx-a)-8\,a^3+12i\,\pi(2\,b^2x^2+2\,abx-a^2)-24\left(2\,b^3x^3+i\,\pi b^2x^2+2\right.}{32\,a^4bx^3+i\,\pi^5x^2+32\,a^5x^2+2\,\pi^4(bx^3+5\,ax^2)-8i\,\pi^3(2\,abx^3+5\,a^2x^2)-16\,\pi^2}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

output
```
2*(48*a*b^2*x^2 + 24*a^2*b*x + I*pi^3 - 6*pi^2*(b*x - a) - 8*a^3 + 12*I*pi
*(2*b^2*x^2 + 2*a*b*x - a^2) - 24*(2*b^3*x^3 + I*pi*b^2*x^2 + 2*a*b^2*x^2)
*log(I*pi + 2*b*x + 2*a) + 24*(2*b^3*x^3 + I*pi*b^2*x^2 + 2*a*b^2*x^2)*log
(x))/(32*a^4*b*x^3 + I*pi^5*x^2 + 32*a^5*x^2 + 2*pi^4*(b*x^3 + 5*a*x^2) -
8*I*pi^3*(2*a*b*x^3 + 5*a^2*x^2) - 16*pi^2*(3*a^2*b*x^3 + 5*a^3*x^2) + 16*
I*pi*(4*a^3*b*x^3 + 5*a^4*x^2))
```

3.174.6 Sympy [F]

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2}\,dx = \int \frac{1}{x^3 \operatorname{acoth}^2(\tanh(a+bx))}\,dx$$

input
```
integrate(1/x**3/acoth(tanh(b*x+a))**2,x)
```

output
```
Integral(1/(x**3*acoth(tanh(a + b*x))**2), x)
```

3.174.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{48\,b^2 \log(-i\,\pi + 2\,bx + 2\,a)}{\pi^4 + 8i\,\pi^3 a - 24\,\pi^2 a^2 - 32i\,\pi a^3 + 16\,a^4} + \frac{48\,b^2 \log(x)}{\pi^4 + 8i\,\pi^3 a - 24\,\pi^2 a^2 - 32i\,\pi a^3 + 16\,a^4} + \frac{2\,(24\,b^2 x^2 + \pi^2 + 4i\,\pi a - 4\,a^2 - 6\,(i\,\pi b - 2\,ab)x)}{2\,(i\,\pi^3 b - 6\,\pi^2 ab - 12i\,\pi a^2 b + 8\,a^3 b)x^3 + (\pi^4 + 8i\,\pi^3 a - 24\,\pi^2 a^2 - 32i\,\pi a^3 + 16\,a^4)x^2}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

output
```
-48*b^2*log(-I*pi + 2*b*x + 2*a)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*p
i*a^3 + 16*a^4) + 48*b^2*log(x)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi
*a^3 + 16*a^4) + 2*(24*b^2*x^2 + pi^2 + 4*I*pi*a - 4*a^2 - 6*(I*pi*b - 2*a
*b)*x)/(2*(I*pi^3*b - 6*pi^2*a*b - 12*I*pi*a^2*b + 8*a^3*b)*x^3 + (pi^4 +
8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi*a^3 + 16*a^4)*x^2)
```

3.174.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2} \, dx = -\frac{48 \, b^2 \log\left(i \, \pi + 2 \, bx + 2 \, a\right)}{\pi^4 - 8i \, \pi^3 a - 24 \, \pi^2 a^2 + 32i \, \pi a^3 + 16 \, a^4} + \frac{48 \, b^2 \log\left(x\right)}{\pi^4 - 8i \, \pi^3 a - 24 \, \pi^2 a^2 + 32i \, \pi a^3 + 16 \, a^4} + \frac{16 \, b^2}{-2i \, \pi^3 bx - 12 \, \pi^2 abx + 24i \, \pi a^2 bx + 16 \, a^3 bx + \pi^4 - 8i \, \pi^3 a - 24 \, \pi^2 a^2 + 32i \, \pi a^3 + 16 \, a^4} - \frac{4 \, \left(i \, \pi - 8 \, bx + 2 \, a\right)}{-2i \, \pi^3 x^2 - 12 \, \pi^2 a x^2 + 24i \, \pi a^2 x^2 + 16 \, a^3 x^2}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

output
```
-48*b^2*log(I*pi + 2*b*x + 2*a)/(pi^4 - 8*I*pi^3*a - 24*pi^2*a^2 + 32*I*pi
*a^3 + 16*a^4) + 48*b^2*log(x)/(pi^4 - 8*I*pi^3*a - 24*pi^2*a^2 + 32*I*pi*
a^3 + 16*a^4) + 16*b^2/(-2*I*pi^3*b*x - 12*pi^2*a*b*x + 24*I*pi*a^2*b*x +
16*a^3*b*x + pi^4 - 8*I*pi^3*a - 24*pi^2*a^2 + 32*I*pi*a^3 + 16*a^4) - 4*(
I*pi - 8*b*x + 2*a)/(-2*I*pi^3*x^2 - 12*pi^2*a*x^2 + 24*I*pi*a^2*x^2 + 16*
a^3*x^2)
```

3.174.9 Mupad [B] (verification not implemented)

Time = 7.85 (sec) , antiderivative size = 689, normalized size of antiderivative = 4.82

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2} \, dx = \frac{2\ln\left(-\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)^3-2\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)^3-6\,\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\,\ln\left(-\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)^2+6\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)^2\,\ln\left(-\frac{}{{\mathrm{e}}^{2}}\right.}{}$$

input
```
int(1/(x^3*acoth(tanh(a + b*x))^2),x)
```

output
```
(2*log(-1/(exp(2*a)*exp(2*b*x) - 1))^3 - 2*log((exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) - 1))^3 - 6*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 - 1))*log(-1/(exp(2*a)*exp(2*b*x) - 1))^2 + 6*log((exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) - 1))^2*log(-1/(exp(2*a)*exp(2*b*x) - 1)) - 32*b^3*x^3
 + 24*b*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2 + 24*b*x*
log(-1/(exp(2*a)*exp(2*b*x) - 1))^2 - 24*b^2*x^2*log((exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) - 1)) + 24*b^2*x^2*log(-1/(exp(2*a)*exp(2*b*x) - 1))
 - b^2*x^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*atan((log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1i - log(-2/(exp(2*a)*e
xp(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))*96i - 48*b*x*log(
(exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*log(-1/(exp(2*a)*exp(2*b*
x) - 1)) + b^2*x^2*log(-1/(exp(2*a)*exp(2*b*x) - 1))*atan((log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1i - log(-2/(exp(2*a)*exp(2*b*x) -
 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))*96i)/(x^2*(log((exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-1/(exp(2*a)*exp(2*b*x) - 1)))*(
log(-1/(exp(2*a)*exp(2*b*x) - 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) - 1)) + 2*b*x)^4)
```

3.174. $\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2} \, dx$

3.175 $\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3} dx$

3.175.1 Optimal result

Integrand size = 13, antiderivative size = 94

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3} dx = -\frac{x^m}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{mx^{-1+m}}{2b^2\coth^{-1}(\tanh(a+bx))} - \frac{mx^{-1+m}\,\text{Hypergeometric2F1}\left(1,-1+m,m,\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{2b^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

output
```
-1/2*x^m/b/arccoth(tanh(b*x+a))^2-1/2*m*x^(-1+m)/b^2/arccoth(tanh(b*x+a))-
1/2*m*x^(-1+m)*hypergeom([1, -1+m],[m],b*x/(b*x-arccoth(tanh(b*x+a))))/b^2
/(b*x-arccoth(tanh(b*x+a)))
```

3.175.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.54

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3} dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(3,1+m,2+m,-\frac{bx}{-bx+\coth^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3}$$

input
```
Integrate[x^m/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcCoth[Ta
nh[a + b*x]]))])/((1 + m)*(-(b*x) + ArcCoth[Tanh[a + b*x]])^3)
```

3.175.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2599, 2599, 2595}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx$$

$$\Big\downarrow \; 2599$$

$$\frac{m\int \frac{x^{m-1}}{\coth^{-1}(\tanh(a+bx))^2}dx}{2b}-\frac{x^m}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 2599$$

$$\frac{m\left(-\frac{(1-m)\int \frac{x^{m-2}}{\coth^{-1}(\tanh(a+bx))}dx}{b}-\frac{x^{m-1}}{b\coth^{-1}(\tanh(a+bx))}\right)}{2b}-\frac{x^m}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 2595$$

$$\frac{m\left(-\frac{x^{m-1}\,\text{Hypergeometric2F1}\left(1,m-1,m,\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\coth^{-1}(\tanh(a+bx))\right)}-\frac{x^{m-1}}{b\coth^{-1}(\tanh(a+bx))}\right)}{2b}-\frac{x^m}{2b\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Int[x^m/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*x^m/(b*ArcCoth[Tanh[a + b*x]]^2) + (m*(-(x^(-1 + m)/(b*ArcCoth[Tanh[a
 + b*x]])) - (x^(-1 + m)*Hypergeometric2F1[1, -1 + m, m, (b*x)/(b*x - ArcC
oth[Tanh[a + b*x]])])/(b*(b*x - ArcCoth[Tanh[a + b*x]]))))/(2*b)
```

3.175.3.1 Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] && !IntegerQ[n]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILt
Q[m, 0] && !IntegerQ[n]))
```

3.175.4 Maple [F]

$$\int \frac{x^m}{\operatorname{arccoth}(\tanh(bx+a))^3}dx$$

input
```
int(x^m/arccoth(tanh(b*x+a))^3,x)
```

output
```
int(x^m/arccoth(tanh(b*x+a))^3,x)
```

3.175.5 Fricas [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{x^m}{\operatorname{arcoth}(\tanh(bx+a))^3}\,dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
integral(x^m/arccoth(tanh(b*x + a))^3, x)
```

3.175.6 Sympy [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{x^m}{\operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**m/acoth(tanh(b*x+a))**3,x)
```

output
```
Integral(x**m/acoth(tanh(a + b*x))**3, x)
```

3.175.7 Maxima [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{x^m}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
integrate(x^m/arccoth(tanh(b*x + a))^3, x)
```

3.175.8 Giac [F]

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{x^m}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

input
```
integrate(x^m/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
integrate(x^m/arccoth(tanh(b*x + a))^3, x)
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{x^m}{\operatorname{acoth}\left(\tanh\left(a+b\,x\right)\right)^3}\,dx$$

input
```
int(x^m/acoth(tanh(a + b*x))^3,x)
```

output
```
int(x^m/acoth(tanh(a + b*x))^3, x)
```

3.176 $\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3} \, dx$

3.176.1 Optimal result

Integrand size = 13, antiderivative size = 92

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3} \, dx = \frac{3x^2}{b^3} + \frac{6x\left(bx - \coth^{-1}(\tanh(a+bx))\right)}{b^4} - \frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2\coth^{-1}(\tanh(a+bx))} + \frac{6\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^5}$$

output
```
3*x^2/b^3+6*x*(b*x-arccoth(tanh(b*x+a)))/b^4-1/2*x^4/b/arccoth(tanh(b*x+a)
)^2-2*x^3/b^2/arccoth(tanh(b*x+a))+6*(b*x-arccoth(tanh(b*x+a)))^2*ln(arcco
th(tanh(b*x+a)))/b^5
```

3.176.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3} \, dx = \frac{x^2}{2b^3} - \frac{3x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}{b^4} + \frac{4\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3}{b^5\coth^{-1}(\tanh(a+bx))} - \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^4}{2b^5\coth^{-1}(\tanh(a+bx))^2} + \frac{6\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2 \log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^5}$$

input
```
Integrate[x^4/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
x^2/(2*b^3) - (3*x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^4 + (4*(-(b*x) + A
rcCoth[Tanh[a + b*x]])^3)/(b^5*ArcCoth[Tanh[a + b*x]]) - (-(b*x) + ArcCoth
[Tanh[a + b*x]])^4/(2*b^5*ArcCoth[Tanh[a + b*x]]^2) + (6*(-(b*x) + ArcCoth
[Tanh[a + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^5
```

3.176.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2599, 2599, 2590, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{2\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}dx}{b}-\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2599}$$

$$\frac{2\left(\frac{3\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}dx}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}\right)}{b}-\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2590}$$

$$\frac{2\left(\frac{3\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{x}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}\right)}{b}-\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2589}$$

$$\frac{2\left(\frac{3\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}\right)}{b}-\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 2588$$

$$\frac{2\left(\frac{3\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}\right)}{b}-\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \; 14$$

$$\frac{2\left(\frac{3\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x}{b}\right)}{b}+\frac{x^2}{2b}\right)}{b}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}\right)}{b}-\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Int[x^4/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*x^4/(b*ArcCoth[Tanh[a + b*x]]^2) + (2*(-(x^3/(b*ArcCoth[Tanh[a + b*x]
])) + (3*(x^2/(2*b) + ((b*x - ArcCoth[Tanh[a + b*x]])*(x/b + ((b*x - ArcCo
th[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^2))/b))/b))/b
```

3.176. $\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}\,dx$

3.176.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2590
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Simp[(b*u - a*v)/a   Int[v^(n - 1)/u, x], x
] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
 1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.176.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.43 (sec) , antiderivative size = 29456, normalized size of antiderivative = 320.17

method	result	size
risch	Expression too large to display	29456

input
```
int(x^4/arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.176.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.88

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}\,dx$$
$$= \frac{16\,b^4x^4 - 64\,ab^3x^3 - 176\,a^2b^2x^2 + 32\,a^3bx + 7\,\pi^4 - 4i\,\pi^3(bx + 14\,a) + 112\,a^4 + 4\,\pi^2(11\,b^2x^2 - 6\,abx - 42}{}$$

input
```
integrate(x^4/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/8*(16*b^4*x^4 - 64*a*b^3*x^3 - 176*a^2*b^2*x^2 + 32*a^3*b*x + 7*pi^4 - 4
*I*pi^3*(b*x + 14*a) + 112*a^4 + 4*pi^2*(11*b^2*x^2 - 6*a*b*x - 42*a^2) -
16*I*pi*(2*b^3*x^3 + 11*a*b^2*x^2 - 3*a^2*b*x - 14*a^3) + 12*(16*a^2*b^2*x
^2 + 32*a^3*b*x + pi^4 - 4*I*pi^3*(b*x + 2*a) + 16*a^4 - 4*pi^2*(b^2*x^2 +
 6*a*b*x + 6*a^2) + 16*I*pi*(a*b^2*x^2 + 3*a^2*b*x + 2*a^3))*log(I*pi + 2*
b*x + 2*a))/(4*b^7*x^2 + 8*a*b^6*x - pi^2*b^5 + 4*a^2*b^5 + 4*I*pi*(b^6*x
+ a*b^5))
```

3.176.6 Sympy [F]

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{x^4}{\operatorname{acoth}^3(\tanh(a+bx))}\,dx$$

input
```
integrate(x**4/acoth(tanh(b*x+a))**3,x)
```

output
```
Integral(x**4/acoth(tanh(a + b*x))**3, x)
```

3.176.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.15

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}\,dx$$
$$= \frac{16\,b^4x^4 + 7\,\pi^4 + 56i\,\pi^3a - 168\,\pi^2a^2 - 224i\,\pi a^3 + 112\,a^4 - 32\,(-i\,\pi b^3 + 2\,ab^3)x^3 + 44\,(\pi^2b^2 + 4i\,\pi ab^2 - 4}{8\,(4\,b^7x^2 - \pi^2b^5 - 4i\,\pi ab^5 + 4\,a^2b^5 - 4\,(i\,\pi b^6 - 2\,ab^6)x)}$$
$$- \frac{3\,(\pi^2 + 4i\,\pi a - 4\,a^2)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{2\,b^5}$$

input
```
integrate(x^4/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
1/8*(16*b^4*x^4 + 7*pi^4 + 56*I*pi^3*a - 168*pi^2*a^2 - 224*I*pi*a^3 + 112
*a^4 - 32*(-I*pi*b^3 + 2*a*b^3)*x^3 + 44*(pi^2*b^2 + 4*I*pi*a*b^2 - 4*a^2*
b^2)*x^2 - 4*(-I*pi^3*b + 6*pi^2*a*b + 12*I*pi*a^2*b - 8*a^3*b)*x)/(4*b^7*
x^2 - pi^2*b^5 - 4*I*pi*a*b^5 + 4*a^2*b^5 - 4*(I*pi*b^6 - 2*a*b^6)*x) - 3/
2*(pi^2 + 4*I*pi*a - 4*a^2)*log(-I*pi + 2*b*x + 2*a)/b^5
```

3.176.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.77

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}\,dx =$$
$$-\frac{16\,\pi^3bx - 96i\,\pi^2abx - 192\,\pi a^2bx + 128i\,a^3bx + 7i\,\pi^4 + 56\,\pi^3a - 168i\,\pi^2a^2 - 224\,\pi a^3 + 112i\,a^4}{-32i\,b^7x^2 + 32\,\pi b^6x - 64i\,ab^6x + 8i\,\pi^2b^5 + 32\,\pi ab^5 - 32i\,a^2b^5}$$
$$+ \frac{x^2}{2\,b^3} - \frac{3\,(i\,\pi + 2\,a)x}{2\,b^4} - \frac{3\,(\pi^2 - 4i\,\pi a - 4\,a^2)\log\left(i\,\pi + 2\,bx + 2\,a\right)}{2\,b^5}$$

input
```
integrate(x^4/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
-(16*pi^3*b*x - 96*I*pi^2*a*b*x - 192*pi*a^2*b*x + 128*I*a^3*b*x + 7*I*pi^
4 + 56*pi^3*a - 168*I*pi^2*a^2 - 224*pi*a^3 + 112*I*a^4)/(-32*I*b^7*x^2 +
32*pi*b^6*x - 64*I*a*b^6*x + 8*I*pi^2*b^5 + 32*pi*a*b^5 - 32*I*a^2*b^5) +
1/2*x^2/b^3 - 3/2*(I*pi + 2*a)*x/b^4 - 3/2*(pi^2 - 4*I*pi*a - 4*a^2)*log(I
*pi + 2*b*x + 2*a)/b^5
```

3.176.9 Mupad [B] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 867, normalized size of antiderivative = 9.42

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \text{Too large to display}$$

input
```
int(x^4/acoth(tanh(a + b*x))^3,x)
```

output
```
((7*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-
2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) +
 2*b*x)^2 + 16*a^4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 32*a^3*(2*a
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a
)*exp(2*b*x) - 1)) + 2*b*x)))/(4*b) - x*(4*(2*a - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x
)^3 - 32*a^3 - 24*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 48*a^2*(2*a - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp
(2*b*x) - 1)) + 2*b*x)))/(2*b^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + x*(16
*a*b^5 - 8*b^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1
)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)) + 8*a^2*b^4 + 8*b^6*x^2 -
 8*a*b^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + l
og(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)) + x^2/(2*b^3) + (log(log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-2/(exp(2*a)*exp(2*b*x)
 - 1)))*(3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) +
 log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 12*a*(2*a - log((2*exp(...
```

3.177 $\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3} \, dx$

3.177.1 Optimal result

Integrand size = 13, antiderivative size = 71

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3} \, dx = \frac{3x}{b^3} - \frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\coth^{-1}(\tanh(a+bx))} + \frac{3\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^4}$$

output
```
3*x/b^3-1/2*x^3/b/arccoth(tanh(b*x+a))^2-3/2*x^2/b^2/arccoth(tanh(b*x+a))+
3*(b*x-arccoth(tanh(b*x+a)))*ln(arccoth(tanh(b*x+a)))/b^4
```

3.177.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3} \, dx = -\frac{b^3x^3 + 3b^2x^2\coth^{-1}(\tanh(a+bx)) + \coth^{-1}(\tanh(a+bx))^3\left(5+6\log\left(\coth^{-1}(\tanh(a+bx))\right)\right) - bx\,c}{2b^4\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Integrate[x^3/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*(b^3*x^3 + 3*b^2*x^2*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^
3*(5 + 6*Log[ArcCoth[Tanh[a + b*x]]]) - b*x*ArcCoth[Tanh[a + b*x]]^2*(11 +
 6*Log[ArcCoth[Tanh[a + b*x]]]))/(b^4*ArcCoth[Tanh[a + b*x]]^2)
```

3.177.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2599, 2599, 2589, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3}dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{3\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}dx}{2b}-\frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2599}$$

$$\frac{3\left(\frac{2\int \frac{x}{\coth^{-1}(\tanh(a+bx))}dx}{b}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}\right)}{2b}-\frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2589}$$

$$\frac{3\left(\frac{2\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}\right)}{2b}-\frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2588}$$

$$\frac{3\left(\frac{2\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}\right)}{2b}-\frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{3\left(\frac{2\left(\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x}{b}\right)}{b}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}\right)}{2b}-\frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Int[x^3/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*x^3/(b*ArcCoth[Tanh[a + b*x]]^2) + (3*(-(x^2/(b*ArcCoth[Tanh[a + b*x]
])) + (2*(x/b + ((b*x - ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]
])/b^2))/b))/(2*b)
```

3.177.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.177.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 4977, normalized size of antiderivative = 70.10

method	result	size
risch	Expression too large to display	4977

input
```
int(x^3/arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
-2*I*(3*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-3*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(
I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+6*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))
^3-6*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))^2+3*Pi*x^2*csgn(I*exp(b*x+a))^2*csg
n(I*exp(2*b*x+2*a))-6*Pi*x^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+3
*Pi*x^2*csgn(I*exp(2*b*x+2*a))^3-3*Pi*x^2*csgn(I*exp(2*b*x+2*a))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+3*Pi*x^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^3+12*I*x^2*ln(exp(b*x+a))+6*Pi*x^2+4*I*x^3*b)/b^2/(Pi*csgn(I*e
xp(2*b*x+2*a))^3-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-Pi*csgn(
I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(e
xp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+
2*a)+1))+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+Pi*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))^3-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3-2*Pi*csgn(I
/(exp(2*b*x+2*a)+1))^2+4*I*ln(exp(b*x+a))+2*Pi)^2+3/b^3*x+3/2*I/b^4*ln(Pi*
csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(e
xp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3-2*Pi*csgn(I/(exp(2*b*x
+2*a)+1))^2+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp
(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I...
```

3.177.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.75

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{16\,b^3x^3 + 32\,ab^2x^2 - 32\,a^2bx + 5i\,\pi^3 + 2\,\pi^2(4\,bx + 15\,a) - 40\,a^3 + 4i\,\pi(4\,b^2x^2 - 8\,abx - 15\,a^2) - 6\,(8\,ab^2x}{4\,(4\,b^6x^2 + 8\,ab^5x - \pi^2b^4 + 4\,a^2b}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/4*(16*b^3*x^3 + 32*a*b^2*x^2 - 32*a^2*b*x + 5*I*pi^3 + 2*pi^2*(4*b*x + 1
5*a) - 40*a^3 + 4*I*pi*(4*b^2*x^2 - 8*a*b*x - 15*a^2) - 6*(8*a*b^2*x^2 + 1
6*a^2*b*x - I*pi^3 - 2*pi^2*(2*b*x + 3*a) + 8*a^3 + 4*I*pi*(b^2*x^2 + 4*a*
b*x + 3*a^2))*log(I*pi + 2*b*x + 2*a))/(4*b^6*x^2 + 8*a*b^5*x - pi^2*b^4 +
 4*a^2*b^4 + 4*I*pi*(b^5*x + a*b^4))
```

3.177.6 Sympy [F]

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{x^3}{\operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(x**3/acoth(tanh(b*x+a))**3,x)
```

output
```
Integral(x**3/acoth(tanh(a + b*x))**3, x)
```

3.177.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.06

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{16\,b^3x^3 - 5i\,\pi^3 + 30\,\pi^2a + 60i\,\pi a^2 - 40\,a^3 - 16\,(i\,\pi b^2 - 2\,ab^2)x^2 + 8\,(\pi^2b + 4i\,\pi ab - 4\,a^2b)x}{4\,(4\,b^6x^2 - \pi^2b^4 - 4i\,\pi ab^4 + 4\,a^2b^4 - 4\,(i\,\pi b^5 - 2\,ab^5)x)} - \frac{3\,(-i\,\pi + 2\,a)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{2\,b^4}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
1/4*(16*b^3*x^3 - 5*I*pi^3 + 30*pi^2*a + 60*I*pi*a^2 - 40*a^3 - 16*(I*pi*b
^2 - 2*a*b^2)*x^2 + 8*(pi^2*b + 4*I*pi*a*b - 4*a^2*b)*x)/(4*b^6*x^2 - pi^2
*b^4 - 4*I*pi*a*b^4 + 4*a^2*b^4 - 4*(I*pi*b^5 - 2*a*b^5)*x) - 3/2*(-I*pi +
 2*a)*log(-I*pi + 2*b*x + 2*a)/b^4
```

3.177.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.73

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{12\,\pi^2 bx - 48i\,\pi abx - 48\,a^2 bx + 5i\,\pi^3 + 30\,\pi^2 a - 60i\,\pi a^2 - 40\,a^3}{4\,(4\,b^6x^2 + 4i\,\pi b^5x + 8\,ab^5x - \pi^2 b^4 + 4i\,\pi ab^4 + 4\,a^2b^4)} + \frac{x}{b^3} + \frac{3\,(-i\,\pi - 2\,a)\log\left(i\,\pi + 2\,bx + 2\,a\right)}{2\,b^4}$$

input
```
integrate(x^3/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
1/4*(12*pi^2*b*x - 48*I*pi*a*b*x - 48*a^2*b*x + 5*I*pi^3 + 30*pi^2*a - 60*
I*pi*a^2 - 40*a^3)/(4*b^6*x^2 + 4*I*pi*b^5*x + 8*a*b^5*x - pi^2*b^4 + 4*I*
pi*a*b^4 + 4*a^2*b^4) + x/b^3 + 3/2*(-I*pi - 2*a)*log(I*pi + 2*b*x + 2*a)/
b^4
```

3.177.9 Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 620, normalized size of antiderivative = 8.73

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{x}{b^3} - \frac{x\left(3\left(2\,a - \ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right) + 2\,b\,x\right)^2 - 12\,a\left(2\,a - \ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right.\right.}{b^3\left(2\,a - \ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right) + 2\,b\,x\right)^2 + x\left(8\,a\,b^4 - 4\,b^4\left(2\right.\right.} + \frac{\ln\left(\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right) - \ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)\right)\left(3\,\ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right) - 3\,\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right) + 6\,b\,x\right)}{2\,b^4}$$

input
```
int(x^3/acoth(tanh(a + b*x))^3,x)
```

output
```
x/b^3 - (x*(3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)
) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 12*a*(2*a - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x)
- 1)) + 2*b*x) + 12*a^2) - (5*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 8*a^3
- 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(
-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1))
+ 2*b*x)))/(4*b))/(b^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + x*(8*a*b^4 - 4
*b^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-
2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)) + 4*a^2*b^3 + 4*b^5*x^2 - 4*a*b^3*(
2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp
(2*a)*exp(2*b*x) - 1)) + 2*b*x)) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) - 1)) - log(-2/(exp(2*a)*exp(2*b*x) - 1)))*(3*log(-2/(exp(2
*a)*exp(2*b*x) - 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
- 1)) + 6*b*x))/(2*b^4)
```

3.177. $\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3} \, dx$

3.178 $\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}\,dx$

3.178.1 Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2\coth^{-1}(\tanh(a+bx))} + \frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}$$

output
```
-1/2*x^2/b/arccoth(tanh(b*x+a))^2-x/b^2/arccoth(tanh(b*x+a))+ln(arccoth(ta
nh(b*x+a)))/b^3
```

3.178.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{3-\frac{b^2x^2}{\coth^{-1}(\tanh(a+bx))^2}-\frac{2bx}{\coth^{-1}(\tanh(a+bx))}+2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{2b^3}$$

input
```
Integrate[x^2/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
(3 - (b^2*x^2)/ArcCoth[Tanh[a + b*x]]^2 - (2*b*x)/ArcCoth[Tanh[a + b*x]] +
 2*Log[ArcCoth[Tanh[a + b*x]]])/(2*b^3)
```

3.178.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2599, 2599, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}dx}{b}-\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{b}-\frac{x}{b\coth^{-1}(\tanh(a+bx))}}{b}-\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{b^2}-\frac{x}{b\coth^{-1}(\tanh(a+bx))}}{b}-\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\frac{\log(\coth^{-1}(\tanh(a+bx)))}{b^2}-\frac{x}{b\coth^{-1}(\tanh(a+bx))}}{b}-\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Int[x^2/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*x^2/(b*ArcCoth[Tanh[a + b*x]]^2) + (-(x/(b*ArcCoth[Tanh[a + b*x]])) +
 Log[ArcCoth[Tanh[a + b*x]]]/b^2)/b
```

3.178.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.178.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

method	result	size
parallelrisch	$\frac{-b^{2} x^{2}+2 \ln \left(\operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)\right) \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)^{2}-2 b x \,\operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)}{2 b^{3} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right)^{2}}$	54
risch	Expression too large to display	952

input
```
int(x^2/arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b^2*x^2+2*ln(arccoth(tanh(b*x+a)))*arccoth(tanh(b*x+a))^2-2*b*x*arcc
oth(tanh(b*x+a)))/b^3/arccoth(tanh(b*x+a))^2
```

3.178.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.62

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}\,dx$$
$$= \frac{16\,abx - 3\,\pi^2 + 4i\,\pi(2\,bx + 3\,a) + 12\,a^2 + 2\,(4\,b^2x^2 + 8\,abx - \pi^2 + 4i\,\pi(bx + a) + 4\,a^2)\log\left(i\,\pi + 2\,bx + 2\right.}{2\,(4\,b^5x^2 + 8\,ab^4x - \pi^2b^3 + 4\,a^2b^3 + 4i\,\pi(b^4x + ab^3))}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/2*(16*a*b*x - 3*pi^2 + 4*I*pi*(2*b*x + 3*a) + 12*a^2 + 2*(4*b^2*x^2 + 8*
a*b*x - pi^2 + 4*I*pi*(b*x + a) + 4*a^2)*log(I*pi + 2*b*x + 2*a))/(4*b^5*x
^2 + 8*a*b^4*x - pi^2*b^3 + 4*a^2*b^3 + 4*I*pi*(b^4*x + a*b^3))
```

3.178.6 Sympy [A] (verification not implemented)

Time = 24.64 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \begin{cases} -\frac{x^2}{2b\,\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)} - \frac{x}{b^2\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)} + \frac{\log\left(\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)\right)}{b^3} & \text{for } b \neq 0 \\ \frac{x^3}{3\operatorname{acoth}^3\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/acoth(tanh(b*x+a))**3,x)
```

output
```
Piecewise((-x**2/(2*b*acoth(tanh(a + b*x))**2) - x/(b**2*acoth(tanh(a + b*
x))) + log(acoth(tanh(a + b*x)))/b**3, Ne(b, 0)), (x**3/(3*acoth(tanh(a))*
*3), True))
```

3.178.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.04

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{3\,\pi^2+12i\,\pi a-12\,a^2-8\,(-i\,\pi b+2\,ab)x}{2\,(4\,b^5x^2-\pi^2b^3-4i\,\pi ab^3+4\,a^2b^3-4\,(i\,\pi b^4-2\,ab^4)x)} + \frac{\log\left(-i\,\pi+2\,bx+2\,a\right)}{b^3}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-1/2*(3*pi^2 + 12*I*pi*a - 12*a^2 - 8*(-I*pi*b + 2*a*b)*x)/(4*b^5*x^2 - pi
^2*b^3 - 4*I*pi*a*b^3 + 4*a^2*b^3 - 4*(I*pi*b^4 - 2*a*b^4)*x) + log(-I*pi
+ 2*b*x + 2*a)/b^3
```

3.178.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.96

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{8\,\pi bx-16i\,abx+3i\,\pi^2+12\,\pi a-12i\,a^2}{8i\,b^5x^2-8\,\pi b^4x+16i\,ab^4x-2i\,\pi^2b^3-8\,\pi ab^3+8i\,a^2b^3} + \frac{\log\left(i\,\pi+2\,bx+2\,a\right)}{b^3}$$

input
```
integrate(x^2/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
-(8*pi*b*x - 16*I*a*b*x + 3*I*pi^2 + 12*pi*a - 12*I*a^2)/(8*I*b^5*x^2 - 8*
pi*b^4*x + 16*I*a*b^4*x - 2*I*pi^2*b^3 - 8*pi*a*b^3 + 8*I*a^2*b^3) + log(I
*pi + 2*b*x + 2*a)/b^3
```

3.178.9 Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3} \, dx = \frac{\ln\left(\mathrm{acoth}(\tanh(a+b\,x))\right)}{b^3} - \frac{\frac{b^2\,x^2}{2} + b\,x\,\mathrm{acoth}(\tanh(a+b\,x))}{b^3\,\mathrm{acoth}\left(\tanh\left(a+b\,x\right)\right)^2}$$

input
```
int(x^2/acoth(tanh(a + b*x))^3,x)
```

output
```
log(acoth(tanh(a + b*x)))/b^3 - ((b^2*x^2)/2 + b*x*acoth(tanh(a + b*x)))/(
b^3*acoth(tanh(a + b*x))^2)
```

3.179 $\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3} \, dx$

3.179.1 Optimal result

Integrand size = 11, antiderivative size = 34

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3} \, dx = -\frac{x}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{1}{2b^2\coth^{-1}(\tanh(a+bx))}$$

output
```
-1/2*x/b/arccoth(tanh(b*x+a))^2-1/2/b^2/arccoth(tanh(b*x+a))
```

3.179.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3} \, dx = -\frac{bx+\coth^{-1}(\tanh(a+bx))}{2b^2\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Integrate[x/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*(b*x + ArcCoth[Tanh[a + b*x]])/(b^2*ArcCoth[Tanh[a + b*x]]^2)
```

3.179.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3} \, dx$$
$$\Big\downarrow \text{ 2599}$$
$$\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}dx}{2b} - \frac{x}{2b\coth^{-1}(\tanh(a+bx))^2}$$
$$\Big\downarrow \text{ 2588}$$
$$\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}d\coth^{-1}(\tanh(a+bx))}{2b^2} - \frac{x}{2b\coth^{-1}(\tanh(a+bx))^2}$$
$$\Big\downarrow \text{ 15}$$
$$-\frac{1}{2b^2\coth^{-1}(\tanh(a+bx))} - \frac{x}{2b\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Int[x/ArcCoth[Tanh[a + b*x]]^3,x]
```

output
```
-1/2*x/(b*ArcCoth[Tanh[a + b*x]]^2) - 1/(2*b^2*ArcCoth[Tanh[a + b*x]])
```

3.179.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.179.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

method	result
parallelrisch	$-\frac{bx+\operatorname{arccoth}(\tanh(bx+a))}{2b^2 \operatorname{arccoth}(\tanh(bx+a))^2}$
risch	$-\frac{2i\left(\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right) \operatorname{csgn}\left(i e^{2bx+2a}\right) \operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)-\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right) \operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2+2\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right.\right.}{b^2\left(\pi \operatorname{csgn}\left(i e^{2bx+2a}\right)^3-2\pi \operatorname{csgn}\left(i e^{bx+a}\right) \operatorname{csgn}\left(i e^{2bx+2a}\right)^2-\pi \operatorname{csgn}\left(i e^{2bx+2a}\right) \operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2+\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right) \right.}$

input
```
int(x/arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x+arccoth(tanh(b*x+a)))/b^2/arccoth(tanh(b*x+a))^2
```

3.179.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.76

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{-i\,\pi - 4\,bx - 2\,a}{4\,b^4x^2 + 8\,ab^3x - \pi^2b^2 + 4\,a^2b^2 + 4i\,\pi(b^3x + ab^2)}$$

input
```
integrate(x/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
(-I*pi - 4*b*x - 2*a)/(4*b^4*x^2 + 8*a*b^3*x - pi^2*b^2 + 4*a^2*b^2 + 4*I*
pi*(b^3*x + a*b^2))
```

3.179.6 Sympy [A] (verification not implemented)

Time = 24.81 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \begin{cases} -\frac{x}{2b\,\mathrm{acoth}^2\,(\tanh\,(a+bx))} - \frac{1}{2b^2\,\mathrm{acoth}\,(\tanh\,(a+bx))} & \text{for } b \neq 0 \\ \frac{x^2}{2\,\mathrm{acoth}^3\,(\tanh\,(a))} & \text{otherwise} \end{cases}$$

input
```
integrate(x/acoth(tanh(b*x+a))**3,x)
```

output
```
Piecewise((-x/(2*b*acoth(tanh(a + b*x))**2) - 1/(2*b**2*acoth(tanh(a + b*x
))), Ne(b, 0)), (x**2/(2*acoth(tanh(a))**3), True))
```

3.179.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.85

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{-i\,\pi + 4\,bx + 2\,a}{4\,b^4x^2 - \pi^2b^2 - 4i\,\pi ab^2 + 4\,a^2b^2 - 4\,(i\,\pi b^3 - 2\,ab^3)x}$$

input
```
integrate(x/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
-(-I*pi + 4*b*x + 2*a)/(4*b^4*x^2 - pi^2*b^2 - 4*I*pi*a*b^2 + 4*a^2*b^2 -
4*(I*pi*b^3 - 2*a*b^3)*x)
```

3.179.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{i\,\pi + 4\,bx + 2\,a}{4\,b^4x^2 + 4i\,\pi b^3x + 8\,ab^3x - \pi^2b^2 + 4i\,\pi ab^2 + 4\,a^2b^2}$$

input
```
integrate(x/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
-(I*pi + 4*b*x + 2*a)/(4*b^4*x^2 + 4*I*pi*b^3*x + 8*a*b^3*x - pi^2*b^2 + 4
*I*pi*a*b^2 + 4*a^2*b^2)
```

3.179.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{\operatorname{acoth}(\tanh(a+b\,x))+b\,x}{2\,b^2\operatorname{acoth}\left(\tanh\left(a+b\,x\right)\right)^2}$$

input
```
int(x/acoth(tanh(a + b*x))^3,x)
```

output
```
-(acoth(tanh(a + b*x)) + b*x)/(2*b^2*acoth(tanh(a + b*x))^2)
```

3.180 $\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3} \, dx$

3.180.1 Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3} \, dx = -\frac{1}{2b\coth^{-1}(\tanh(a+bx))^2}$$

output
```
-1/2/b/arccoth(tanh(b*x+a))^2
```

3.180.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3} \, dx = -\frac{1}{2b\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^(-3),x]
```

output
```
-1/2*1/(b*ArcCoth[Tanh[a + b*x]]^2)
```

3.180.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3} \, dx$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3}d\coth^{-1}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{1}{2b\coth^{-1}(\tanh(a+bx))^2}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^(-3),x]
```

output
```
-1/2*1/(b*ArcCoth[Tanh[a + b*x]]^2)
```

3.180.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.180.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{1}{2b\operatorname{arccoth}(\tanh(bx+a))^2}$
default	$-\frac{1}{2b\operatorname{arccoth}(\tanh(bx+a))^2}$
parallelrisch	$-\frac{1}{2b\operatorname{arccoth}(\tanh(bx+a))^2}$
risch	$\frac{}{b\left(\pi \operatorname{csgn}\left(i e^{2bx+2a}\right)^3-2\pi \operatorname{csgn}\left(i e^{bx+a}\right)\operatorname{csgn}\left(i e^{2bx+2a}\right)^2-\pi \operatorname{csgn}\left(i e^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2+\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\right.}$

input
```
int(1/arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/arccoth(tanh(b*x+a))^2
```

3.180.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{2}{4\,b^3x^2+8\,ab^2x-\pi^2b+4\,a^2b+4i\,\pi(b^2x+ab)}$$

input
```
integrate(1/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
-2/(4*b^3*x^2 + 8*a*b^2*x - pi^2*b + 4*a^2*b + 4*I*pi*(b^2*x + a*b))
```

3.180.6 Sympy [A] (verification not implemented)

Time = 24.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \begin{cases} -\frac{1}{2b\operatorname{acoth}^2(\tanh(a+bx))} & \text{for } b \neq 0 \\ \frac{x}{\operatorname{acoth}^3(\tanh(a))} & \text{otherwise} \end{cases}$$

input
```
integrate(1/acoth(tanh(b*x+a))**3,x)
```

output
```
Piecewise((-1/(2*b*acoth(tanh(a + b*x))**2), Ne(b, 0)), (x/acoth(tanh(a))*
*3, True))
```

3.180.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{2}{\left(\pi^2 - 4i\,\pi(bx+a) - 4\,(bx+a)^2\right)b}$$

input
```
integrate(1/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
2/((pi^2 - 4*I*pi*(b*x + a) - 4*(b*x + a)^2)*b)
```

3.180.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{2i}{4i\,b^3x^2 - 4\,\pi b^2x + 8i\,ab^2x - i\,\pi^2b - 4\,\pi ab + 4i\,a^2b}$$

input
```
integrate(1/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
-2*I/(4*I*b^3*x^2 - 4*pi*b^2*x + 8*I*a*b^2*x - I*pi^2*b - 4*pi*a*b + 4*I*a
^2*b)
```

3.180.9 Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2\,b\,\mathrm{acoth}\left(\tanh\left(a+b\,x\right)\right)^2}$$

input
```
int(1/acoth(tanh(a + b*x))^3,x)
```

output
```
-1/(2*b*acoth(tanh(a + b*x))^2)
```

3.181 $\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3} dx$

3.181.1 Optimal result

Integrand size = 13, antiderivative size = 97

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3} dx = -\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2} + \frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))} - \frac{\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3} + \frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3}$$

output
```
-1/2/(b*x-arccoth(tanh(b*x+a)))/arccoth(tanh(b*x+a))^2+1/(b*x-arccoth(tanh
(b*x+a)))^2/arccoth(tanh(b*x+a))-ln(x)/(b*x-arccoth(tanh(b*x+a)))^3+ln(arc
coth(tanh(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^3
```

3.181.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{b^2x^2-4bx\coth^{-1}(\tanh(a+bx))+\coth^{-1}(\tanh(a+bx))^2\left(3+2\log(bx)-2\log\left(\coth^{-1}(\tanh(a+bx))\right)\right)}{2\coth^{-1}(\tanh(a+bx))^2\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3}$$

input
```
Integrate[1/(x*ArcCoth[Tanh[a + b*x]]^3),x]
```

output
```
(b^2*x^2 - 4*b*x*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^2*(3 + 2*
Log[b*x] - 2*Log[ArcCoth[Tanh[a + b*x]]]))/(2*ArcCoth[Tanh[a + b*x]]^2*(-(
b*x) + ArcCoth[Tanh[a + b*x]])^3)
```

3.181.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2594, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^3}dx$$

$$\Big\downarrow \text{2594}$$

$$-\frac{\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2594}$$

$$-\frac{-\frac{\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2591}$$

$$-\frac{-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{14}$$

$$-\frac{-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2588}$$

$$-\frac{-\frac{\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{14}$$

$$-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}-\frac{-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}-\frac{\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[1/(x*ArcCoth[Tanh[a + b*x]]^3),x]
```

output
```
-1/2*1/((b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) - (-(1/((
b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]])) - (-(Log[x]/(b*x -
ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Tanh
[a + b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]]
)
```

3.181. $\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^3}\,dx$

3.181.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

3.181.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.35 (sec) , antiderivative size = 5655, normalized size of antiderivative = 58.30

output too large to display

input
```
int(1/x/arccoth(tanh(b*x+a))^3,x)
```

output
```
result too large to display
```

3.181.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.38

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^3}\,dx$$
$$= \frac{4\left(8\,abx - 3\,\pi^2 + 4i\,\pi(bx + 3\,a) + 12\,a^2 - 2\left(4\,b^2x^2 + 8\,abx - \pi^2 + 4i\,\pi(bx + a) + 4\,a^2\right)\log\left(i\,\pi + 2\,bx + 2\right.\right.}{32\,a^3b^2x^2 + 64\,a^4bx + i\,\pi^5 + 2\,\pi^4(2\,bx + 5\,a) + 32\,a^5 - 4i\,\pi^3(b^2x^2 + 8\,abx + 10\,a^2) - 8\,\pi^2(3\,ab^2x^2 +}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
4*(8*a*b*x - 3*pi^2 + 4*I*pi*(b*x + 3*a) + 12*a^2 - 2*(4*b^2*x^2 + 8*a*b*x
 - pi^2 + 4*I*pi*(b*x + a) + 4*a^2)*log(I*pi + 2*b*x + 2*a) + 2*(4*b^2*x^2
 + 8*a*b*x - pi^2 + 4*I*pi*(b*x + a) + 4*a^2)*log(x))/(32*a^3*b^2*x^2 + 64
*a^4*b*x + I*pi^5 + 2*pi^4*(2*b*x + 5*a) + 32*a^5 - 4*I*pi^3*(b^2*x^2 + 8*
a*b*x + 10*a^2) - 8*pi^2*(3*a*b^2*x^2 + 12*a^2*b*x + 10*a^3) + 16*I*pi*(3*
a^2*b^2*x^2 + 8*a^3*b*x + 5*a^4))
```

3.181.6 Sympy [F]

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3}\, dx = \int \frac{1}{x\,\mathrm{acoth}^3\left(\tanh\left(a+bx\right)\right)}\, dx$$

input
```
integrate(1/x/acoth(tanh(b*x+a))**3,x)
```

output
```
Integral(1/(x*acoth(tanh(a + b*x))**3), x)
```

3.181.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.76

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3}\, dx$$
$$= \frac{4\left(-3i\pi + 4bx + 6a\right)}{\pi^4 + 8i\pi^3 a - 24\pi^2 a^2 - 32i\pi a^3 + 16a^4 - 4\left(\pi^2 b^2 + 4i\pi a b^2 - 4a^2 b^2\right)x^2 - 4\left(-i\pi^3 b + 6\pi^2 ab + 12i\pi a^2 b - \right.}$$
$$+ \frac{8\log\left(-i\pi + 2bx + 2a\right)}{-i\pi^3 + 6\pi^2 a + 12i\pi a^2 - 8a^3} - \frac{8\log\left(x\right)}{-i\pi^3 + 6\pi^2 a + 12i\pi a^2 - 8a^3}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
4*(-3*I*pi + 4*b*x + 6*a)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi*a^3 +
 16*a^4 - 4*(pi^2*b^2 + 4*I*pi*a*b^2 - 4*a^2*b^2)*x^2 - 4*(-I*pi^3*b + 6*p
i^2*a*b + 12*I*pi*a^2*b - 8*a^3*b)*x) + 8*log(-I*pi + 2*b*x + 2*a)/(-I*pi^
3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3) - 8*log(x)/(-I*pi^3 + 6*pi^2*a + 12*I*
pi*a^2 - 8*a^3)
```

3.181.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.78

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3} dx$$
$$= \frac{4(-3i\pi - 4bx - 6a)}{4\pi^2b^2x^2 - 16i\pi ab^2x^2 - 16a^2b^2x^2 + 4i\pi^3bx + 24\pi^2abx - 48i\pi a^2bx - 32a^3bx - \pi^4 + 8i\pi^3a + 24\pi^2a^2 -}$$
$$- \frac{8i \log(i\pi + 2bx + 2a)}{\pi^3 - 6i\pi^2a - 12\pi a^2 + 8ia^3} + \frac{8i \log(x)}{\pi^3 - 6i\pi^2a - 12\pi a^2 + 8ia^3}$$

input
```
integrate(1/x/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
4*(-3*I*pi - 4*b*x - 6*a)/(4*pi^2*b^2*x^2 - 16*I*pi*a*b^2*x^2 - 16*a^2*b^2
*x^2 + 4*I*pi^3*b*x + 24*pi^2*a*b*x - 48*I*pi*a^2*b*x - 32*a^3*b*x - pi^4
+ 8*I*pi^3*a + 24*pi^2*a^2 - 32*I*pi*a^3 - 16*a^4) - 8*I*log(I*pi + 2*b*x
+ 2*a)/(pi^3 - 6*I*pi^2*a - 12*pi*a^2 + 8*I*a^3) + 8*I*log(x)/(pi^3 - 6*I*
pi^2*a - 12*pi*a^2 + 8*I*a^3)
```

3.181.9 Mupad [B] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 902, normalized size of antiderivative = 9.30

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3} dx = \text{Too large to display}$$

input
```
int(1/(x*acoth(tanh(a + b*x))^3),x)
```

output

```
- (16*atanh((16*(4*b*x - ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 8*a^3 - 6*a
*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(e
xp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b
*x))/((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(
-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2
*b*x) + 4*a^2))*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) -
 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2/16 - (a*(2*a - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b
*x) - 1)) + 2*b*x))/4 + a^2/4))/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3))/(log(-2/(ex
p(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 - 1)) + 2*b*x)^3 - (12/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) - (16*b*x)/((2*a - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(
2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 4*a^2))/(
(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/...
```

3.181. $\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3} \, dx$

3.182 $\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3} \, dx$

3.182.1 Optimal result

Integrand size = 13, antiderivative size = 131

$$\begin{aligned}\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3} \, dx = & -\frac{3b}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \coth^{-1}(\tanh(a+bx))^2} \\ & + \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))^2} \\ & + \frac{3b}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3 \coth^{-1}(\tanh(a+bx))} \\ & - \frac{3b\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4} \\ & + \frac{3b\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4}\end{aligned}$$

output
```
-3/2*b/(b*x-arccoth(tanh(b*x+a)))^2/arccoth(tanh(b*x+a))^2+1/x/(b*x-arccot
h(tanh(b*x+a)))/arccoth(tanh(b*x+a))^2+3*b/(b*x-arccoth(tanh(b*x+a)))^3/ar
ccoth(tanh(b*x+a))-3*b*ln(x)/(b*x-arccoth(tanh(b*x+a)))^4+3*b*ln(arccoth(t
anh(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^4
```

3.182.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3} dx = -\frac{b^3x^3 - 6b^2x^2 \coth^{-1}(\tanh(a+bx)) + 2\coth^{-1}(\tanh(a+bx))^3 + 3bx \coth^{-1}(\tanh(a+bx))^2 \left(1 + 2\log(\right.}{2x \coth^{-1}(\tanh(a+bx))^2 \left(-bx + \coth^{-1}(\tanh(a+bx))\right)^4}$$

input
```
Integrate[1/(x^2*ArcCoth[Tanh[a + b*x]]^3),x]
```

output
```
-1/2*(b^3*x^3 - 6*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 2*ArcCoth[Tanh[a + b*x]
]^3 + 3*b*x*ArcCoth[Tanh[a + b*x]]^2*(1 + 2*Log[x] - 2*Log[ArcCoth[Tanh[a
+ b*x]]]))/(x*ArcCoth[Tanh[a + b*x]]^2*(-(b*x) + ArcCoth[Tanh[a + b*x]])^4
)
```

3.182.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.38, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2602, 2594, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3} dx$$

$$\Big\downarrow \text{ 2602}$$

$$\frac{3b \int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3} dx}{bx - \coth^{-1}(\tanh(a+bx))} + \frac{1}{x\left(bx - \coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{ 2594}$$

$$\frac{3b \left(-\frac{\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2} dx}{bx - \coth^{-1}(\tanh(a+bx))} - \frac{1}{2\left(bx - \coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))^2}\right)}{bx - \coth^{-1}(\tanh(a+bx))} + \frac{1}{x\left(bx - \coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{ 2594}$$

$$\frac{3b\left(-\frac{-\frac{\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))^2}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$\big\downarrow$ 2591

$$\frac{3b\left(-\frac{-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))^2}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$\big\downarrow$ 14

$$\frac{3b\left(-\frac{-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))^2}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$\big\downarrow$ 2588

$$\frac{3b\left(-\frac{-\frac{\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))^2}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$\big\downarrow$ 14

3.182. $\int \frac{1}{x^2\coth^{-1}(\tanh(a+bx))^3}\,dx$

$$\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}+$$
$$\frac{3b\left(-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}-\frac{-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}-\frac{\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\cdots}{bx-\coth^{-1}\cdots}}{bx-\coth^{-1}(\tanh(a+bx\cdots}}{bx-\coth^{-1}(\tanh(a+bx))}\cdots}{bx-\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[1/(x^2*ArcCoth[Tanh[a + b*x]]^3),x]
```

output
```
1/(x*(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) + (3*b*(-1/2
*1/((b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) - (-(1/((b*x
- ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]])) - (-(Log[x]/(b*x - ArcC
oth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Tanh[a +
 b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]])))/
(b*x - ArcCoth[Tanh[a + b*x]])
```

3.182.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.182.4 Maple [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)^3} dx$$

input
```
int(1/x^2/arccoth(tanh(b*x+a))^3,x)
```

output
```
int(1/x^2/arccoth(tanh(b*x+a))^3,x)
```

3.182.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.60

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3} dx = -\frac{8\left(24\,ab^2x^2+36\,a^2bx-i\,\pi^3-3\,\pi^2(3\,bx+2\,a)+8\,a^3+12i\,\pi(b^2x^2+3\,abx+a^2)-6\left(4\,b^3x^3+8\,ab^2x^2-\right.\right.}{64\,a^4b^2x^3+128\,a^5bx^2-\pi^6x+64\,a^6x+4i\,\pi^5(bx^2+3\,ax)+4\,\pi^4(b^2x^3+10\,abx^2+15\,a^2x)}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
-8*(24*a*b^2*x^2 + 36*a^2*b*x - I*pi^3 - 3*pi^2*(3*b*x + 2*a) + 8*a^3 + 12
*I*pi*(b^2*x^2 + 3*a*b*x + a^2) - 6*(4*b^3*x^3 + 8*a*b^2*x^2 - pi^2*b*x +
4*a^2*b*x + 4*I*pi*(b^2*x^2 + a*b*x))*log(I*pi + 2*b*x + 2*a) + 6*(4*b^3*x
^3 + 8*a*b^2*x^2 - pi^2*b*x + 4*a^2*b*x + 4*I*pi*(b^2*x^2 + a*b*x))*log(x)
)/(64*a^4*b^2*x^3 + 128*a^5*b*x^2 - pi^6*x + 64*a^6*x + 4*I*pi^5*(b*x^2 +
3*a*x) + 4*pi^4*(b^2*x^3 + 10*a*b*x^2 + 15*a^2*x) - 32*I*pi^3*(a*b^2*x^3 +
 5*a^2*b*x^2 + 5*a^3*x) - 16*pi^2*(6*a^2*b^2*x^3 + 20*a^3*b*x^2 + 15*a^4*x
) + 64*I*pi*(2*a^3*b^2*x^3 + 5*a^4*b*x^2 + 3*a^5*x))
```

3.182.6 Sympy [F]

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{1}{x^2\,\mathrm{acoth}^3\,(\tanh\,(a+bx))}\,dx$$

input
```
integrate(1/x**2/acoth(tanh(b*x+a))**3,x)
```

output
```
Integral(1/(x**2*acoth(tanh(a + b*x))**3), x)
```

3.182.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.87

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3}\,dx$$
$$= \frac{48\,b\log\left(-i\,\pi+2\,bx+2\,a\right)}{\pi^4+8i\,\pi^3a-24\,\pi^2a^2-32i\,\pi a^3+16\,a^4} - \frac{48\,b\log\left(x\right)}{\pi^4+8i\,\pi^3a-24\,\pi^2a^2-32i\,\pi a^3+16\,a^4}$$
$$- \frac{8\left(12\,b^2x^2-\pi^2-4i\,\pi a+4\,a^2-9\left(i\,\pi b-2\,ab\right)x\right)}{4\left(i\,\pi^3b^2-6\,\pi^2ab^2-12i\,\pi a^2b^2+8\,a^3b^2\right)x^3+4\left(\pi^4b+8i\,\pi^3ab-24\,\pi^2a^2b-32i\,\pi a^3b+16\,a^4b\right)x^2-(i\,\pi^5}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
48*b*log(-I*pi + 2*b*x + 2*a)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi*a
^3 + 16*a^4) - 48*b*log(x)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi*a^3
+ 16*a^4) - 8*(12*b^2*x^2 - pi^2 - 4*I*pi*a + 4*a^2 - 9*(I*pi*b - 2*a*b)*x
)/(4*(I*pi^3*b^2 - 6*pi^2*a*b^2 - 12*I*pi*a^2*b^2 + 8*a^3*b^2)*x^3 + 4*(pi
^4*b + 8*I*pi^3*a*b - 24*pi^2*a^2*b - 32*I*pi*a^3*b + 16*a^4*b)*x^2 - (I*p
i^5 - 10*pi^4*a - 40*I*pi^3*a^2 + 80*pi^2*a^3 + 80*I*pi*a^4 - 32*a^5)*x)
```

3.182.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.97

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3}\,dx$$
$$= \frac{48\,b\log\left(i\,\pi+2\,bx+2\,a\right)}{\pi^4-8i\,\pi^3a-24\,\pi^2a^2+32i\,\pi a^3+16\,a^4} - \frac{48\,b\log\left(x\right)}{\pi^4-8i\,\pi^3a-24\,\pi^2a^2+32i\,\pi a^3+16\,a^4}$$
$$+ \frac{16\left(8\,b^2x+5i\,\pi b+10\,ab\right)}{8i\,\pi^3b^2x^2+48\,\pi^2ab^2x^2-96i\,\pi a^2b^2x^2-64\,a^3b^2x^2-8\,\pi^4bx+64i\,\pi^3abx+192\,\pi^2a^2bx-256i\,\pi a^3bx-1}$$
$$+ \frac{8}{i\,\pi^3x+6\,\pi^2ax-12i\,\pi a^2x-8\,a^3x}$$

input
```
integrate(1/x^2/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
48*b*log(I*pi + 2*b*x + 2*a)/(pi^4 - 8*I*pi^3*a - 24*pi^2*a^2 + 32*I*pi*a^
3 + 16*a^4) - 48*b*log(x)/(pi^4 - 8*I*pi^3*a - 24*pi^2*a^2 + 32*I*pi*a^3 +
 16*a^4) + 16*(8*b^2*x + 5*I*pi*b + 10*a*b)/(8*I*pi^3*b^2*x^2 + 48*pi^2*a*
b^2*x^2 - 96*I*pi*a^2*b^2*x^2 - 64*a^3*b^2*x^2 - 8*pi^4*b*x + 64*I*pi^3*a*
b*x + 192*pi^2*a^2*b*x - 256*I*pi*a^3*b*x - 128*a^4*b*x - 2*I*pi^5 - 20*pi
^4*a + 80*I*pi^3*a^2 + 160*pi^2*a^3 - 160*I*pi*a^4 - 64*a^5) + 8/(I*pi^3*x
 + 6*pi^2*a*x - 12*I*pi*a^2*x - 8*a^3*x)
```

3.182.9 Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 1074, normalized size of antiderivative = 8.20

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*acoth(tanh(a + b*x))^3),x)
```

output
```
(8/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) - 1)) + 2*b*x) - (72*b*x)/((2*a - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)
^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + l
og(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 4*a^2) + (96*b^2*x^2)/((log(-2
/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) - 1)) + 2*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2
*b*x) - 1)) + 2*b*x) + 4*a^2)))/(x*((2*a - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4
*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/
(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 4*a^2) + x^2*(8*a*b - 4*b*(2*a - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp
(2*b*x) - 1)) + 2*b*x)) + 4*b^2*x^3) + (96*b*atanh(((2*a - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)
) + 2*b*x)^4 + 24*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 16*a^4 - 8*a*(2
*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(
2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 32*a^3*(2*a - log((2*exp(2*a)*exp(2*...
```

3.182. $\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3}\,dx$

3.183 $\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3} \, dx$

3.183.1 Optimal result

Integrand size = 13, antiderivative size = 170

$$\begin{aligned}\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3} \, dx = &-\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3 \coth^{-1}(\tanh(a+bx))^2} \\ &+ \frac{2b}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \coth^{-1}(\tanh(a+bx))^2} \\ &+ \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))^2} \\ &+ \frac{6b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4 \coth^{-1}(\tanh(a+bx))} \\ &- \frac{6b^2 \log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^5} \\ &+ \frac{6b^2 \log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^5}\end{aligned}$$

output
```
-3*b^2/(b*x-arccoth(tanh(b*x+a)))^3/arccoth(tanh(b*x+a))^2+2*b/x/(b*x-arcc
oth(tanh(b*x+a)))^2/arccoth(tanh(b*x+a))^2+1/2/x^2/(b*x-arccoth(tanh(b*x+a
)))/arccoth(tanh(b*x+a))^2+6*b^2/(b*x-arccoth(tanh(b*x+a)))^4/arccoth(tanh
(b*x+a))-6*b^2*ln(x)/(b*x-arccoth(tanh(b*x+a)))^5+6*b^2*ln(arccoth(tanh(b*
x+a)))/(b*x-arccoth(tanh(b*x+a)))^5
```

3.183.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3} dx = \frac{-b^4x^4 + 8b^3x^3\coth^{-1}(\tanh(a+bx)) - 8bx\coth^{-1}(\tanh(a+bx))^3 + \coth^{-1}(\tanh(a+bx))^4 - 12b^2x^2 \cot}{2x^2\left(bx - \coth^{-1}(\tanh(a+bx))\right)^5 \coth^{-1}(\tanh(a+}$$

input
```
Integrate[1/(x^3*ArcCoth[Tanh[a + b*x]]^3),x]
```

output
```
(-(b^4*x^4) + 8*b^3*x^3*ArcCoth[Tanh[a + b*x]] - 8*b*x*ArcCoth[Tanh[a + b*
x]]^3 + ArcCoth[Tanh[a + b*x]]^4 - 12*b^2*x^2*ArcCoth[Tanh[a + b*x]]^2*(Lo
g[x] - Log[ArcCoth[Tanh[a + b*x]]]))/(2*x^2*(b*x - ArcCoth[Tanh[a + b*x]])
^5*ArcCoth[Tanh[a + b*x]]^2)
```

3.183.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {2602, 2602, 2594, 2594, 2591, 14, 2588, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3} dx$$

$$\Big\downarrow \text{2602}$$

$$\frac{2b\int \frac{1}{x^2\coth^{-1}(\tanh(a+bx))^3}dx}{bx-\coth^{-1}(\tanh(a+bx))} + \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2602}$$

$$\frac{2b\left(\frac{3b\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^3}dx}{bx-\coth^{-1}(\tanh(a+bx))} + \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}\right)}{bx-\coth^{-1}(\tanh(a+bx))} + \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$$\Big\downarrow \text{2594}$$

$$\frac{2b\left(\frac{3b\left(-\frac{\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}\right)}{bx-\coth^{-1}(\tanh(a+bx))}$$

$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$\downarrow$ 2594

$$\frac{2b\left(\frac{3b\left(-\frac{-\frac{\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}\right)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{\quad}{x(bx-}\right.}{bx-\coth^{-1}(\tanh(a+bx))}$$

$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$\downarrow$ 2591

$$\frac{2b\left(\frac{3b\left(-\frac{-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\int \frac{1}{x}dx}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\text{t}}\right.}{bx-\coth^{-1}(\tanh(a+bx))}\right.}{bx-\coth^{-1}(\tanh(a+bx))}$$

$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$\downarrow$ 14

3.183. $\int \frac{1}{x^3\coth^{-1}(\tanh(a+bx))^3}\,dx$

$$2b\Bigg(\frac{3b\left(-\frac{-\frac{\frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}dx}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(t}\right.}{bx-\coth^{-1}(\tanh(a+bx))}$$

$$\overline{\qquad bx-\coth^{-1}(\tanh(a+bx))}$$

$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$\Big\downarrow$ 2588

$$2b\Bigg(\frac{3b\left(-\frac{-\frac{\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}d\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{}{2\left(bx-\coth^{-1}(\tanh\right.}\right.}{bx-\coth^{-1}(\tanh(a+bx))}$$

$$\overline{\qquad bx-\coth^{-1}(\tanh(a+bx))}$$

$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}$$

$\Big\downarrow$ 14

$$\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}+$$

$$2b\Bigg(\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}+\frac{3b\left(-\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}-\frac{-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth}}{}\right.}{bx-\coth^{-1}(\tanh(a+}$$

$$\overline{\qquad bx-\coth^{-1}(\tanh(a+bx))}$$

input
```
Int[1/(x^3*ArcCoth[Tanh[a + b*x]]^3),x]
```

3.183. $\int \frac{1}{x^3\coth^{-1}(\tanh(a+bx))^3}\,dx$

output
```
1/(2*x^2*(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) + (2*b*(
1/(x*(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) + (3*b*(-1/2
*1/((b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) - (-(1/((b*x
- ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]])) - (-(Log[x]/(b*x - ArcC
oth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Tanh[a +
 b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]]))/(b*x - ArcCoth[Tanh[a + b*x]])))/
(b*x - ArcCoth[Tanh[a + b*x]])))/(b*x - ArcCoth[Tanh[a + b*x]])
```

3.183.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2591
```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Simp[b/(b*u - a*v)   Int[1/v, x], x] - Simp[a/(b*u - a*v)   Int[1
/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

rule 2594
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Simp[a*((n + 1)/((n +
 1)*(b*u - a*v)))   Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecew
iseLinearQ[u, v, x] && LtQ[n, -1]
```

rule 2602
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + S
imp[b*((m + n + 2)/((m + 1)*(b*u - a*v)))   Int[u^(m + 1)*v^n, x], x] /; Ne
Q[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m
, -1]
```

3.183. $\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3} \, dx$

3.183.4 Maple [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \operatorname{arccoth}(\tanh(bx+a))^3}dx$$

input
```
int(1/x^3/arccoth(tanh(b*x+a))^3,x)
```

output
```
int(1/x^3/arccoth(tanh(b*x+a))^3,x)
```

3.183.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.71

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{4\,(192\,ab^3x^3 + 288\,a^2b^2x^2 + 64\,a^3bx - \pi^4 - 8i\,\pi^3(bx-a) - 16\,a^4 - 24\,\pi^2(3\,b^2x^2 + 2\,abx - a^2) + 32i\,\pi(3\,b}{128\,a^5b^2x^4 + 256\,a^6bx^3 - i\,\pi^7x^2 + 128\,a^7x^2 - 2\,\pi^6(2\,bx^3 + 7\,ax^2) + 4i\,\pi^5(b^2x^4 + 12\,}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

output
```
4*(192*a*b^3*x^3 + 288*a^2*b^2*x^2 + 64*a^3*b*x - pi^4 - 8*I*pi^3*(b*x - a
) - 16*a^4 - 24*pi^2*(3*b^2*x^2 + 2*a*b*x - a^2) + 32*I*pi*(3*b^3*x^3 + 9*
a*b^2*x^2 + 3*a^2*b*x - a^3) - 48*(4*b^4*x^4 + 8*a*b^3*x^3 - pi^2*b^2*x^2
+ 4*a^2*b^2*x^2 + 4*I*pi*(b^3*x^3 + a*b^2*x^2))*log(I*pi + 2*b*x + 2*a) +
48*(4*b^4*x^4 + 8*a*b^3*x^3 - pi^2*b^2*x^2 + 4*a^2*b^2*x^2 + 4*I*pi*(b^3*x
^3 + a*b^2*x^2))*log(x))/(128*a^5*b^2*x^4 + 256*a^6*b*x^3 - I*pi^7*x^2 + 1
28*a^7*x^2 - 2*pi^6*(2*b*x^3 + 7*a*x^2) + 4*I*pi^5*(b^2*x^4 + 12*a*b*x^3 +
 21*a^2*x^2) + 40*pi^4*(a*b^2*x^4 + 6*a^2*b*x^3 + 7*a^3*x^2) - 80*I*pi^3*(
2*a^2*b^2*x^4 + 8*a^3*b*x^3 + 7*a^4*x^2) - 32*pi^2*(10*a^3*b^2*x^4 + 30*a^
4*b*x^3 + 21*a^5*x^2) + 64*I*pi*(5*a^4*b^2*x^4 + 12*a^5*b*x^3 + 7*a^6*x^2)
)
```

3.183.6 Sympy [F]

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3}\,dx = \int \frac{1}{x^3 \operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

input
```
integrate(1/x**3/acoth(tanh(b*x+a))**3,x)
```

output
```
Integral(1/(x**3*acoth(tanh(a + b*x))**3), x)
```

3.183.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.95

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{192\,b^2 \log\left(-i\,\pi + 2\,bx + 2\,a\right)}{i\,\pi^5 - 10\,\pi^4 a - 40i\,\pi^3 a^2 + 80\,\pi^2 a^3 + 80i\,\pi a^4 - 32\,a^5}$$
$$- \frac{192\,b^2 \log\left(x\right)}{i\,\pi^5 - 10\,\pi^4 a - 40i\,\pi^3 a^2 + 80\,\pi^2 a^3 + 80i\,\pi a^4 - 32\,a^5}$$
$$+ \frac{4\left(96\,b^3x^3 - i\,\pi^3 + 6\,\pi^2 a + 12i\,\pi a^2 - 8\,a^3 - 72\,(i\,\pi b\right.}{4\,(\pi^4b^2 + 8i\,\pi^3ab^2 - 24\,\pi^2a^2b^2 - 32i\,\pi a^3b^2 + 16\,a^4b^2)x^4 - 4\,(i\,\pi^5 b - 10\,\pi^4 ab - 40i\,\pi^3a^2b + 80\,\pi^2a^3b + 8}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

output
```
192*b^2*log(-I*pi + 2*b*x + 2*a)/(I*pi^5 - 10*pi^4*a - 40*I*pi^3*a^2 + 80*
pi^2*a^3 + 80*I*pi*a^4 - 32*a^5) - 192*b^2*log(x)/(I*pi^5 - 10*pi^4*a - 40
*I*pi^3*a^2 + 80*pi^2*a^3 + 80*I*pi*a^4 - 32*a^5) + 4*(96*b^3*x^3 - I*pi^3
 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3 - 72*(I*pi*b^2 - 2*a*b^2)*x^2 - 8*(pi^2*
b + 4*I*pi*a*b - 4*a^2*b)*x)/(4*(pi^4*b^2 + 8*I*pi^3*a*b^2 - 24*pi^2*a^2*b
^2 - 32*I*pi*a^3*b^2 + 16*a^4*b^2)*x^4 - 4*(I*pi^5*b - 10*pi^4*a*b - 40*I*
pi^3*a^2*b + 80*pi^2*a^3*b + 80*I*pi*a^4*b - 32*a^5*b)*x^3 - (pi^6 + 12*I*
pi^5*a - 60*pi^4*a^2 - 160*I*pi^3*a^3 + 240*pi^2*a^4 + 192*I*pi*a^5 - 64*a
^6)*x^2)
```

3.183.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.02

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{192i\,b^2\log(i\pi+2\,bx+2\,a)}{\pi^5-10i\,\pi^4a-40\,\pi^3a^2+80i\,\pi^2a^3+80\,\pi a^4-32i\,a^5} - \frac{192i\,b^2\log(x)}{\pi^5-10i\,\pi^4a-40\,\pi^3a^2+80i\,\pi^2a^3+80\,\pi a^4-32i\,a^5} - \frac{4\,(i\pi-12\,bx+2\,a)}{\pi^4x^2-8i\,\pi^3ax^2-24\,\pi^2a^2x^2+32i\,\pi a^3x^2+16\,a^4x^2} + \frac{16\,(12\,b^3x+\cdots}{4\,\pi^4b^2x^2-32i\,\pi^3ab^2x^2-96\,\pi^2a^2b^2x^2+128i\,\pi a^3b^2x^2+64\,a^4b^2x^2+4i\,\pi^5bx+40\,\pi^4abx-160i\,\pi^3a^2bx-\cdots}$$

input
```
integrate(1/x^3/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

output
```
192*I*b^2*log(I*pi + 2*b*x + 2*a)/(pi^5 - 10*I*pi^4*a - 40*pi^3*a^2 + 80*I
*pi^2*a^3 + 80*pi*a^4 - 32*I*a^5) - 192*I*b^2*log(x)/(pi^5 - 10*I*pi^4*a -
 40*pi^3*a^2 + 80*I*pi^2*a^3 + 80*pi*a^4 - 32*I*a^5) - 4*(I*pi - 12*b*x +
2*a)/(pi^4*x^2 - 8*I*pi^3*a*x^2 - 24*pi^2*a^2*x^2 + 32*I*pi*a^3*x^2 + 16*a
^4*x^2) + 16*(12*b^3*x + 7*I*pi*b^2 + 14*a*b^2)/(4*pi^4*b^2*x^2 - 32*I*pi^
3*a*b^2*x^2 - 96*pi^2*a^2*b^2*x^2 + 128*I*pi*a^3*b^2*x^2 + 64*a^4*b^2*x^2
+ 4*I*pi^5*b*x + 40*pi^4*a*b*x - 160*I*pi^3*a^2*b*x - 320*pi^2*a^3*b*x + 3
20*I*pi*a^4*b*x + 128*a^5*b*x - pi^6 + 12*I*pi^5*a + 60*pi^4*a^2 - 160*I*p
i^3*a^3 - 240*pi^2*a^4 + 192*I*pi*a^5 + 64*a^6)
```

3.183.9 Mupad [B] (verification not implemented)

Time = 10.02 (sec) , antiderivative size = 1251, normalized size of antiderivative = 7.36

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^3*acoth(tanh(a + b*x))^3),x)
```

output
```
(4/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) - 1)) + 2*b*x) + (32*b*x)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - (2
88*b^2*x^2)/((log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^
2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + lo
g(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 4*a^2)) + (384*b^3*x^3)/((log(-
2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) - 1)) + 2*b*x)^2*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*ex
p(2*b*x) - 1)) + 2*b*x) + 4*a^2)))/(x^3*(8*a*b - 4*b*(2*a - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1
)) + 2*b*x)) + x^2*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*
b*x) - 1)) + 2*b*x) + 4*a^2) + 4*b^2*x^4) - (384*b^2*atanh((4*b*x*((2*a -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*
exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(e...
```

3.183. $\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3} \, dx$

3.184 $\int x^m \coth^{-1}(\tanh(a+bx))^n\,dx$

3.184.1 Optimal result

Integrand size = 13, antiderivative size = 79

$$\int x^m \coth^{-1}(\tanh(a+bx))^n\,dx = \frac{x^m\left(\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)^{-m}\coth^{-1}(\tanh(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(-m,1+n,2+n,-\frac{\coth^{-1}(\tan}{bx-\coth^{-1}(t}\right.}{b(1+n)}$$

output
```
x^m*arccoth(tanh(b*x+a))^(1+n)*hypergeom([-m, 1+n],[2+n],-arccoth(tanh(b*x
+a))/(b*x-arccoth(tanh(b*x+a))))/b/(1+n)/((b*x/(b*x-arccoth(tanh(b*x+a))))
^m)
```

3.184.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int x^m \coth^{-1}(\tanh(a+bx))^n\,dx = \frac{x^{1+m}\coth^{-1}(\tanh(a+bx))^n\left(1+\frac{bx}{-bx+\coth^{-1}(\tanh(a+bx))}\right)^{-n}\text{Hypergeometric2F1}\left(1+m,-n,2+m,-\frac{}{-bx}\right.}{1+m}$$

input
```
Integrate[x^m*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
(x^(1 + m)*ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2F1[1 + m, -n, 2 + m, -(
(b*x)/(-(b*x) + ArcCoth[Tanh[a + b*x]]))])/((1 + m)*(1 + (b*x)/(-(b*x) + A
rcCoth[Tanh[a + b*x]]))^n)
```

3.184.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2604}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \coth^{-1}(\tanh(a+bx))^n \, dx$$

$$\Big\downarrow \; 2604$$

$$\frac{x^m \left(\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)^{-m} \coth^{-1}(\tanh(a+bx))^{n+1} \text{Hypergeometric2F1}\left(-m, n+1, n+2, -\frac{\coth^{-1}(\tanh(a+bx}{bx-\coth^{-1}(\tanh(a+}\right.}{b(n+1)}$$

input
```
Int[x^m*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
(x^m*ArcCoth[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, -(
ArcCoth[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/(b*(1 + n)*((b*x)
/(b*x - ArcCoth[Tanh[a + b*x]]))^m)
```

3.184.3.1 Defintions of rubi rules used

rule 2604
```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[u^m*(v^(n + 1)/(b*(n + 1)*(b*(u/(b*u - a*v)))^m))*Hype
rgeometric2F1[-m, n + 1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v,
 0]] /; PiecewiseLinearQ[u, v, x] && !IntegerQ[m] && !IntegerQ[n]
```

3.184.4 Maple [F]

$$\int x^m \operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)^n dx$$

input
```
int(x^m*arccoth(tanh(b*x+a))^n,x)
```

output
```
int(x^m*arccoth(tanh(b*x+a))^n,x)
```

3.184.5 Fricas [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))^n\,dx = \int x^m \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n\,dx$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral(x^m*arccoth(tanh(b*x + a))^n, x)
```

3.184.6 Sympy [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))^n\,dx = \int x^m \operatorname{acoth}^n\left(\tanh\left(a+bx\right)\right)dx$$

input
```
integrate(x**m*acoth(tanh(b*x+a))**n,x)
```

output
```
Integral(x**m*acoth(tanh(a + b*x))**n, x)
```

3.184.7 Maxima [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))^n\,dx = \int x^m \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n\,dx$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate(x^m*arccoth(tanh(b*x + a))^n, x)
```

3.184.8 Giac [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))^n\,dx = \int x^m \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n\,dx$$

input
```
integrate(x^m*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate(x^m*arccoth(tanh(b*x + a))^n, x)
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int x^m \coth^{-1}(\tanh(a+bx))^n\,dx = \int x^m \operatorname{acoth}(\tanh(a+b\,x))^n\,dx$$

input
```
int(x^m*acoth(tanh(a + b*x))^n,x)
```

output
```
int(x^m*acoth(tanh(a + b*x))^n, x)
```

3.185 $\int x^4 \coth^{-1}(\tanh(a+bx))^n \, dx$

3.185.1 Optimal result

Integrand size = 13, antiderivative size = 165

$$\int x^4 \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{x^4 \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3 \coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \coth^{-1}(\tanh(a+bx))^{3+n}}{b^3(1+n)(2+n)(3+n)} - \frac{24x \coth^{-1}(\tanh(a+bx))^{4+n}}{b^4(1+n)(2+n)(3+n)(4+n)} + \frac{24 \coth^{-1}(\tanh(a+bx))^{5+n}}{b^5(1+n)(2+n)(3+n)(4+n)(5+n)}$$

output
```
x^4*arccoth(tanh(b*x+a))^(1+n)/b/(1+n)-4*x^3*arccoth(tanh(b*x+a))^(2+n)/b^
2/(1+n)/(2+n)+12*x^2*arccoth(tanh(b*x+a))^(3+n)/b^3/(3+n)/(n^2+3*n+2)-24*x
*arccoth(tanh(b*x+a))^(4+n)/b^4/(n^2+5*n+4)/(n^2+5*n+6)+24*arccoth(tanh(b*
x+a))^(5+n)/b^5/(n^2+7*n+12)/(n^3+8*n^2+17*n+10)
```

3.185.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int x^4 \coth^{-1}(\tanh(a+bx))^n \, dx$$
$$= \frac{\coth^{-1}(\tanh(a+bx))^{1+n} \left(b^4(120+154n+71n^2+14n^3+n^4)\, x^4 - 4b^3(60+47n+12n^2+n^3)\, x^3 \coth^{-1}(\right.}{b^5(1+}$$

input
```
Integrate[x^4*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
(ArcCoth[Tanh[a + b*x]]^(1 + n)*(b^4*(120 + 154*n + 71*n^2 + 14*n^3 + n^4)
*x^4 - 4*b^3*(60 + 47*n + 12*n^2 + n^3)*x^3*ArcCoth[Tanh[a + b*x]] + 12*b^
2*(20 + 9*n + n^2)*x^2*ArcCoth[Tanh[a + b*x]]^2 - 24*b*(5 + n)*x*ArcCoth[T
anh[a + b*x]]^3 + 24*ArcCoth[Tanh[a + b*x]]^4))/(b^5*(1 + n)*(2 + n)*(3 +
n)*(4 + n)*(5 + n))
```

3.185.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2599, 2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \coth^{-1}(\tanh(a+bx))^n \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^4 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)} - \frac{4\int x^3 \coth^{-1}(\tanh(a+bx))^{n+1}dx}{b(n+1)}$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^4 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)} - \frac{4\left(\frac{x^3 \coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)} - \frac{3\int x^2 \coth^{-1}(\tanh(a+bx))^{n+2}dx}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^4\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{4\left(\frac{x^3\coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{3\left(\frac{x^2\coth^{-1}(\tanh(a+bx))^{n+3}}{b(n+3)}-\frac{2\int x\coth^{-1}(\tanh(a+bx))^{n+3}dx}{b(n+3)}\right)}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \; 2599$$

$$\frac{x^4\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{4\left(\frac{x^3\coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{3\left(\frac{x^2\coth^{-1}(\tanh(a+bx))^{n+3}}{b(n+3)}-\frac{2\left(\frac{x\coth^{-1}(\tanh(a+bx))^{n+4}}{b(n+4)}-\frac{\int\coth^{-1}(\tanh(a+bx))^{n+4}dx}{b(n+4)}\right)}{b(n+3)}\right)}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \; 2588$$

$$\frac{x^4\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{4\left(\frac{x^3\coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{3\left(\frac{x^2\coth^{-1}(\tanh(a+bx))^{n+3}}{b(n+3)}-\frac{2\left(\frac{x\coth^{-1}(\tanh(a+bx))^{n+4}}{b(n+4)}-\frac{\int\coth^{-1}(\tanh(a+bx))^{n+4}d\coth^{-1}(\tanh(a+bx))}{b^2(n+4)}\right)}{b(n+3)}\right)}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \; 15$$

$$\frac{x^4\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{4\left(\frac{x^3\coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{3\left(\frac{x^2\coth^{-1}(\tanh(a+bx))^{n+3}}{b(n+3)}-\frac{2\left(\frac{x\coth^{-1}(\tanh(a+bx))^{n+4}}{b(n+4)}-\frac{\coth^{-1}(\tanh(a+bx))^{n+5}}{b^2(n+4)(n+5)}\right)}{b(n+3)}\right)}{b(n+2)}\right)}{b(n+1)}$$

input
```
Int[x^4*ArcCoth[Tanh[a + b*x]]^n,x]
```

3.185. $\int x^4\coth^{-1}(\tanh(a+bx))^n\,dx$

output
```
(x^4*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (4*((x^3*ArcCoth[Tanh[a
 + b*x]]^(2 + n))/(b*(2 + n)) - (3*((x^2*ArcCoth[Tanh[a + b*x]]^(3 + n))/(
b*(3 + n)) - (2*((x*ArcCoth[Tanh[a + b*x]]^(4 + n))/(b*(4 + n)) - ArcCoth[
Tanh[a + b*x]]^(5 + n)/(b^2*(4 + n)*(5 + n))))/(b*(3 + n))))/(b*(2 + n)))
/(b*(1 + n))
```

3.185.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.185.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(165) = 330$.

Time = 16.41 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.55

method	result
parallelrisch	$-\frac{-24\,\mathrm{arccoth}(\tanh(bx+a))^n\,\mathrm{arccoth}(\tanh(bx+a))^5-120\,\mathrm{arccoth}(\tanh(bx+a))^n\,\mathrm{arccoth}(\tanh(bx+a))x^4b^4+240\,\mathrm{arccoth}(\mathrm{tan}}{}$
risch	Expression too large to display

input
```
int(x^4*arccoth(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
-(-24*arccoth(tanh(b*x+a))^n*arccoth(tanh(b*x+a))^5-120*arccoth(tanh(b*x+a
))^n*arccoth(tanh(b*x+a))*x^4*b^4+240*arccoth(tanh(b*x+a))^n*arccoth(tanh(
b*x+a))^2*x^3*b^3-240*arccoth(tanh(b*x+a))^n*arccoth(tanh(b*x+a))^3*x^2*b^
2+120*arccoth(tanh(b*x+a))^n*arccoth(tanh(b*x+a))^4*x*b+48*x^3*arccoth(tan
h(b*x+a))^2*arccoth(tanh(b*x+a))^n*b^3*n^2+188*x^3*arccoth(tanh(b*x+a))^2*
arccoth(tanh(b*x+a))^n*b^3*n-12*x^2*arccoth(tanh(b*x+a))^3*arccoth(tanh(b*
x+a))^n*b^2*n^2-108*x^2*arccoth(tanh(b*x+a))^3*arccoth(tanh(b*x+a))^n*b^2*
n+24*x*arccoth(tanh(b*x+a))^4*arccoth(tanh(b*x+a))^n*b*n-14*x^4*arccoth(ta
nh(b*x+a))*arccoth(tanh(b*x+a))^n*b^4*n^3-71*x^4*arccoth(tanh(b*x+a))*arcc
oth(tanh(b*x+a))^n*b^4*n^2+4*x^3*arccoth(tanh(b*x+a))^2*arccoth(tanh(b*x+a
))^n*b^3*n^3-154*x^4*arccoth(tanh(b*x+a))*arccoth(tanh(b*x+a))^n*b^4*n-x^4
*arccoth(tanh(b*x+a))*arccoth(tanh(b*x+a))^n*b^4*n^4)/b^5/(n^5+15*n^4+85*n
^3+225*n^2+274*n+120)
```

3.185.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 828, normalized size of antiderivative = 5.02

$$\int x^4 \coth^{-1}(\tanh(a+bx))^n\,dx =$$

$$-\frac{(96\,a^4bnx - 4\,(b^5n^4 + 10\,b^5n^3 + 35\,b^5n^2 + 50\,b^5n + 24\,b^5)x^5 - 3i\,\pi^5 + 6\,\pi^4(bnx - 5\,a) - 96\,a^5 - 4\,(ab^4n}{}$$

input
```
integrate(x^4*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
-1/4*((96*a^4*b*n*x - 4*(b^5*n^4 + 10*b^5*n^3 + 35*b^5*n^2 + 50*b^5*n + 24
*b^5)*x^5 - 3*I*pi^5 + 6*pi^4*(b*n*x - 5*a) - 96*a^5 - 4*(a*b^4*n^4 + 6*a*
b^4*n^3 + 11*a*b^4*n^2 + 6*a*b^4*n)*x^4 - 6*I*pi^3*(8*a*b*n*x - (b^2*n^2 +
 b^2*n)*x^2 - 20*a^2) + 16*(a^2*b^3*n^3 + 3*a^2*b^3*n^2 + 2*a^2*b^3*n)*x^3
 - 4*pi^2*(36*a^2*b*n*x + (b^3*n^3 + 3*b^3*n^2 + 2*b^3*n)*x^3 - 60*a^3 - 9
*(a*b^2*n^2 + a*b^2*n)*x^2) - 48*(a^3*b^2*n^2 + a^3*b^2*n)*x^2 + 2*I*pi*(9
6*a^3*b*n*x - (b^4*n^4 + 6*b^4*n^3 + 11*b^4*n^2 + 6*b^4*n)*x^4 - 120*a^4 +
 8*(a*b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 36*(a^2*b^2*n^2 + a^2*b^2*n
)*x^2))*cosh(n*log(1/2*I*pi + b*x + a)) + (96*a^4*b*n*x - 4*(b^5*n^4 + 10*
b^5*n^3 + 35*b^5*n^2 + 50*b^5*n + 24*b^5)*x^5 - 3*I*pi^5 + 6*pi^4*(b*n*x -
 5*a) - 96*a^5 - 4*(a*b^4*n^4 + 6*a*b^4*n^3 + 11*a*b^4*n^2 + 6*a*b^4*n)*x^
4 - 6*I*pi^3*(8*a*b*n*x - (b^2*n^2 + b^2*n)*x^2 - 20*a^2) + 16*(a^2*b^3*n^
3 + 3*a^2*b^3*n^2 + 2*a^2*b^3*n)*x^3 - 4*pi^2*(36*a^2*b*n*x + (b^3*n^3 + 3
*b^3*n^2 + 2*b^3*n)*x^3 - 60*a^3 - 9*(a*b^2*n^2 + a*b^2*n)*x^2) - 48*(a^3*
b^2*n^2 + a^3*b^2*n)*x^2 + 2*I*pi*(96*a^3*b*n*x - (b^4*n^4 + 6*b^4*n^3 + 1
1*b^4*n^2 + 6*b^4*n)*x^4 - 120*a^4 + 8*(a*b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*
n)*x^3 - 36*(a^2*b^2*n^2 + a^2*b^2*n)*x^2))*sinh(n*log(1/2*I*pi + b*x + a)
))/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

3.185.6 Sympy [F]

$$\int x^4 \coth^{-1}(\tanh(a+bx))^n\, dx = \text{Too large to display}$$

input
```
integrate(x**4*acoth(tanh(b*x+a))**n,x)
```

output
```
Piecewise((x**5*acoth(tanh(a))**n/5, Eq(b, 0)), (-x**4/(4*b*acoth(tanh(a +
 b*x))**4) - x**3/(3*b**2*acoth(tanh(a + b*x))**3) - x**2/(2*b**3*acoth(ta
nh(a + b*x))**2) - x/(b**4*acoth(tanh(a + b*x))) + log(acoth(tanh(a + b*x)
))/b**5, Eq(n, -5)), (Integral(x**4/acoth(tanh(a + b*x))**4, x), Eq(n, -4)
), (Integral(x**4/acoth(tanh(a + b*x))**3, x), Eq(n, -3)), (Integral(x**4/
acoth(tanh(a + b*x))**2, x), Eq(n, -2)), (Integral(x**4/acoth(tanh(a + b*x
)), x), Eq(n, -1)), (b**4*n**4*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*
x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5
*n + 120*b**5) + 14*b**4*n**3*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x
))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) + 71*b**4*n**2*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x)
)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n
 + 120*b**5) + 154*b**4*n*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**
n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
120*b**5) + 120*b**4*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b*
*5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b
**5) - 4*b**3*n**3*x**3*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b
**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*
b**5) - 48*b**3*n**2*x**3*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/
(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +...
```

3.185.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.30

$$\int x^4 \coth^{-1}(\tanh(a+bx))^n \, dx$$

$$= \frac{(4\,(n^4 + 10\,n^3 + 35\,n^2 + 50\,n + 24)b^5x^5 - 3i\,\pi^5 + 30\,\pi^4 a + 120i\,\pi^3 a^2 - 240\,\pi^2 a^3 - 240i\,\pi a^4 + 96\,a^5 - 2\,(\ldots}{\ldots}$$

input
```
integrate(x^4*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
(4*(n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 - 3*I*pi^5 + 30*pi^4*a + 12
0*I*pi^3*a^2 - 240*pi^2*a^3 - 240*I*pi*a^4 + 96*a^5 - 2*(I*pi*(n^4 + 6*n^3
 + 11*n^2 + 6*n)*b^4 - 2*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4)*x^4 + 4*(pi^2
*(n^3 + 3*n^2 + 2*n)*b^3 + 4*I*pi*(n^3 + 3*n^2 + 2*n)*a*b^3 - 4*(n^3 + 3*n
^2 + 2*n)*a^2*b^3)*x^3 - 6*(-I*pi^3*(n^2 + n)*b^2 + 6*pi^2*(n^2 + n)*a*b^2
 + 12*I*pi*(n^2 + n)*a^2*b^2 - 8*(n^2 + n)*a^3*b^2)*x^2 - 6*(pi^4*b*n + 8*
I*pi^3*a*b*n - 24*pi^2*a^2*b*n - 32*I*pi*a^3*b*n + 16*a^4*b*n)*x)*(cosh(-n
*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*b*x + 2*a)))/((2^(n + 2
)*n^5 + 15*2^(n + 2)*n^4 + 85*2^(n + 2)*n^3 + 225*2^(n + 2)*n^2 + 137*2^(n
 + 3)*n + 15*2^(n + 5))*b^5)
```

3.185.8 Giac [F]

$$\int x^4 \coth^{-1}(\tanh(a+bx))^n \, dx = \int x^4 \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n \, dx$$

input
```
integrate(x^4*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate(x^4*arccoth(tanh(b*x + a))^n, x)
```

3.185.9 Mupad [B] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 546, normalized size of antiderivative = 3.31

$$\int x^4 \coth^{-1}(\tanh(a+bx))^n\,dx = -\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2}-\frac{\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2}\right)^n \left(\frac{3\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^5}{4\,b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} - \frac{x^5\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120} + \frac{3\,n\,x\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^4}{2\,b^4\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} + \frac{n\,x^4\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)\,\left(n^3+6\,n^2+11\,n+6\right)}{2\,b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} + \frac{3\,n\,x^2\,(n+1)\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^3}{2\,b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} + \frac{n\,x^3\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2\,\left(n^2+3\,n+2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)$$

input
```
int(x^4*acoth(tanh(a + b*x))^n,x)
```

output
```
-(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 - log(-2/(exp(2
*a)*exp(2*b*x) - 1))/2)^n*((3*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^5)/(4*b^5*(274*n
+ 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (x^5*(50*n + 35*n^2 + 10*n^3 +
 n^4 + 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (3*n*x*(log(
-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) - 1)) + 2*b*x)^4)/(2*b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 +
 120)) + (n*x^4*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)*(11*n + 6*n^2 + n^3 + 6))/(2*b*
(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (3*n*x^2*(n + 1)*(log(-
2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) - 1)) + 2*b*x)^3)/(2*b^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 +
120)) + (n*x^3*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2*(3*n + n^2 + 2))/(b^2*(274*n +
 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
```

3.186 $\int x^3 \coth^{-1}(\tanh(a+bx))^n \, dx$

3.186.1 Optimal result

Integrand size = 13, antiderivative size = 121

$$\int x^3 \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{x^3 \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2 \coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x \coth^{-1}(\tanh(a+bx))^{3+n}}{b^3(1+n)(2+n)(3+n)} - \frac{6 \coth^{-1}(\tanh(a+bx))^{4+n}}{b^4(1+n)(2+n)(3+n)(4+n)}$$

output
```
x^3*arccoth(tanh(b*x+a))^(1+n)/b/(1+n)-3*x^2*arccoth(tanh(b*x+a))^(2+n)/b^
2/(1+n)/(2+n)+6*x*arccoth(tanh(b*x+a))^(3+n)/b^3/(3+n)/(n^2+3*n+2)-6*arcco
th(tanh(b*x+a))^(4+n)/b^4/(n^2+5*n+4)/(n^2+5*n+6)
```

3.186.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int x^3 \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{\coth^{-1}(\tanh(a+bx))^{1+n} \left(b^3(24+26n+9n^2+n^3)\,x^3 - 3b^2(12+7n+n^2)\,x^2 \coth^{-1}(\tanh(a+bx)) + 6b\right.}{b^4(1+n)(2+n)(3+n)(4+n)}$$

input
```
Integrate[x^3*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
(ArcCoth[Tanh[a + b*x]]^(1 + n)*(b^3*(24 + 26*n + 9*n^2 + n^3)*x^3 - 3*b^2
*(12 + 7*n + n^2)*x^2*ArcCoth[Tanh[a + b*x]] + 6*b*(4 + n)*x*ArcCoth[Tanh[
a + b*x]]^2 - 6*ArcCoth[Tanh[a + b*x]]^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4
 + n))
```

3.186.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2599, 2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(\tanh(a+bx))^n \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^3 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{3 \int x^2 \coth^{-1}(\tanh(a+bx))^{n+1}dx}{b(n+1)}$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^3 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{3 \left(\frac{x^2 \coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{2 \int x \coth^{-1}(\tanh(a+bx))^{n+2}dx}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \text{2599}$$

$$\frac{x^3 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{3 \left(\frac{x^2 \coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{2 \left(\frac{x \coth^{-1}(\tanh(a+bx))^{n+3}}{b(n+3)}-\frac{\int \coth^{-1}(\tanh(a+bx))^{n+3}dx}{b(n+3)}\right)}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \text{2588}$$

$$\frac{x^3 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{3 \left(\frac{x^2 \coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{2 \left(\frac{x \coth^{-1}(\tanh(a+bx))^{n+3}}{b(n+3)}-\frac{\int \coth^{-1}(\tanh(a+bx))^{n+3}d\coth^{-1}(\tanh(a+bx))}{b^2(n+3)}\right)}{b(n+2)}\right)}{b(n+1)}$$

$$\downarrow 15$$

$$\frac{x^3\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{3\left(\frac{x^2\coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{2\left(\frac{x\coth^{-1}(\tanh(a+bx))^{n+3}}{b(n+3)}-\frac{\coth^{-1}(\tanh(a+bx))^{n+4}}{b^2(n+3)(n+4)}\right)}{b(n+2)}\right)}{b(n+1)}$$

input
```
Int[x^3*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
(x^3*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (3*((x^2*ArcCoth[Tanh[a
 + b*x]]^(2 + n))/(b*(2 + n)) - (2*((x*ArcCoth[Tanh[a + b*x]]^(3 + n))/(b*
(3 + n)) - ArcCoth[Tanh[a + b*x]]^(4 + n)/(b^2*(3 + n)*(4 + n))))/(b*(2 +
n))))/(b*(1 + n))
```

3.186.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.186.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(121) = 242.

Time = 10.86 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.29

method	result
parallelrisch	$-\frac{-24 \operatorname{arccoth}(\tanh(bx+a))^{n} \operatorname{arccoth}(\tanh(bx+a))^{3} xb - x^{3} \operatorname{arccoth}(\tanh(bx+a)) \operatorname{arccoth}(\tanh(bx+a))^{n} b^{3} n^{3} - 9x^{3} \operatorname{arccoth}(\mathrm{ta}}{}$
risch	Expression too large to display

input
```
int(x^3*arccoth(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
-(-24*arccoth(tanh(b*x+a))^n*arccoth(tanh(b*x+a))^3*x*b-x^3*arccoth(tanh(b
*x+a))*arccoth(tanh(b*x+a))^n*b^3*n^3-9*x^3*arccoth(tanh(b*x+a))*arccoth(t
anh(b*x+a))^n*b^3*n^2-26*x^3*arccoth(tanh(b*x+a))*arccoth(tanh(b*x+a))^n*b
^3*n+3*x^2*arccoth(tanh(b*x+a))^2*arccoth(tanh(b*x+a))^n*b^2*n^2+21*x^2*ar
ccoth(tanh(b*x+a))^2*arccoth(tanh(b*x+a))^n*b^2*n-6*x*arccoth(tanh(b*x+a))
^3*arccoth(tanh(b*x+a))^n*b*n-24*arccoth(tanh(b*x+a))^n*arccoth(tanh(b*x+a
))*x^3*b^3+36*arccoth(tanh(b*x+a))^n*arccoth(tanh(b*x+a))^2*x^2*b^2+6*arcc
oth(tanh(b*x+a))^n*arccoth(tanh(b*x+a))^4)/(n^3+6*n^2+11*n+6)/(4+n)/b^4
```

3.186.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 498, normalized size of antiderivative = 4.12

$$\int x^{3} \coth^{-1}(\tanh(a+bx))^{n}\, dx$$
$$= \frac{(48\, a^{3}bnx + 8\,(b^{4}n^{3} + 6\, b^{4}n^{2} + 11\, b^{4}n + 6\, b^{4})x^{4} - 3\,\pi^{4} - 6i\,\pi^{3}(bnx - 4\,a) - 48\,a^{4} + 8\,(ab^{3}n^{3} + 3\,ab^{3}n^{2} + 2}{}$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
1/8*((48*a^3*b*n*x + 8*(b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 3*pi
^4 - 6*I*pi^3*(b*n*x - 4*a) - 48*a^4 + 8*(a*b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^
3*n)*x^3 - 6*pi^2*(6*a*b*n*x - (b^2*n^2 + b^2*n)*x^2 - 12*a^2) - 24*(a^2*b
^2*n^2 + a^2*b^2*n)*x^2 + 4*I*pi*(18*a^2*b*n*x + (b^3*n^3 + 3*b^3*n^2 + 2*
b^3*n)*x^3 - 24*a^3 - 6*(a*b^2*n^2 + a*b^2*n)*x^2))*cosh(n*log(1/2*I*pi +
b*x + a)) + (48*a^3*b*n*x + 8*(b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4
 - 3*pi^4 - 6*I*pi^3*(b*n*x - 4*a) - 48*a^4 + 8*(a*b^3*n^3 + 3*a*b^3*n^2 +
 2*a*b^3*n)*x^3 - 6*pi^2*(6*a*b*n*x - (b^2*n^2 + b^2*n)*x^2 - 12*a^2) - 24
*(a^2*b^2*n^2 + a^2*b^2*n)*x^2 + 4*I*pi*(18*a^2*b*n*x + (b^3*n^3 + 3*b^3*n
^2 + 2*b^3*n)*x^3 - 24*a^3 - 6*(a*b^2*n^2 + a*b^2*n)*x^2))*sinh(n*log(1/2*
I*pi + b*x + a)))/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

3.186.6 Sympy [F]

$$\int x^3 \coth^{-1}(\tanh(a+bx))^n\,dx = \text{Too large to display}$$

input
```
integrate(x**3*acoth(tanh(b*x+a))**n,x)
```

output
```
Piecewise((x**4*acoth(tanh(a))**n/4, Eq(b, 0)), (-x**3/(3*b*acoth(tanh(a +
 b*x))**3) - x**2/(2*b**2*acoth(tanh(a + b*x))**2) - x/(b**3*acoth(tanh(a
+ b*x))) + log(acoth(tanh(a + b*x)))/b**4, Eq(n, -4)), (Integral(x**3/acot
h(tanh(a + b*x))**3, x), Eq(n, -3)), (Integral(x**3/acoth(tanh(a + b*x))**
2, x), Eq(n, -2)), (Integral(x**3/acoth(tanh(a + b*x)), x), Eq(n, -1)), (b
**3*n**3*x**3*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10
*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*b**3*n**2*x**3*acoth(
tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4
*n**2 + 50*b**4*n + 24*b**4) + 26*b**3*n*x**3*acoth(tanh(a + b*x))*acoth(t
anh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24
*b**4) + 24*b**3*x**3*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**4*n
**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*b**2*n**2*x**
2*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**
3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 21*b**2*n*x**2*acoth(tanh(a + b*
x))**2*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 +
50*b**4*n + 24*b**4) - 36*b**2*x**2*acoth(tanh(a + b*x))**2*acoth(tanh(a +
 b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
+ 6*b*n*x*acoth(tanh(a + b*x))**3*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*
b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 24*b*x*acoth(tanh(a + b*
x))**3*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2...
```

3.186.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.12

$$\int x^3 \coth^{-1}(\tanh(a+bx))^n \, dx$$
$$= \frac{(8\,(n^3+6\,n^2+11\,n+6)b^4x^4 - 3\,\pi^4 - 24i\,\pi^3 a + 72\,\pi^2 a^2 + 96i\,\pi a^3 - 48\,a^4 - 4\,(i\,\pi(n^3+3\,n^2+2\,n)b^3 - 2}{}$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
(8*(n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 - 3*pi^4 - 24*I*pi^3*a + 72*pi^2*a^2 +
 96*I*pi*a^3 - 48*a^4 - 4*(I*pi*(n^3 + 3*n^2 + 2*n)*b^3 - 2*(n^3 + 3*n^2 +
 2*n)*a*b^3)*x^3 + 6*(pi^2*(n^2 + n)*b^2 + 4*I*pi*(n^2 + n)*a*b^2 - 4*(n^2
 + n)*a^2*b^2)*x^2 - 6*(-I*pi^3*b*n + 6*pi^2*a*b*n + 12*I*pi*a^2*b*n - 8*a
^3*b*n)*x)*(cosh(-n*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*b*x
+ 2*a)))/((2^(n + 3)*n^4 + 5*2^(n + 4)*n^3 + 35*2^(n + 3)*n^2 + 25*2^(n +
4)*n + 3*2^(n + 6))*b^4)
```

3.186.8 Giac [F]

$$\int x^3 \coth^{-1}(\tanh(a+bx))^n \, dx = \int x^3 \operatorname{arcoth}(\tanh(bx+a))^n \, dx$$

input
```
integrate(x^3*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate(x^3*arccoth(tanh(b*x + a))^n, x)
```

3.186.9 Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.45

$$\int x^3 \coth^{-1}(\tanh(a+bx))^n\,dx = -\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2}-\frac{\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2}\right)^n\left(\frac{3\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^4}{8\,b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)} - \frac{x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24} + \frac{3\,n\,x\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^3}{4\,b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)} + \frac{n\,x^3\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)\left(n^2+3\,n+2\right)}{2\,b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)} + \frac{3\,n\,x^2\,(n+1)\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2}{4\,b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)$$

input
```
int(x^3*acoth(tanh(a + b*x))^n,x)
```

output
```
-(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 - log(-2/(exp(2
*a)*exp(2*b*x) - 1))/2)^n*((3*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^4)/(8*b^4*(50*n +
 35*n^2 + 10*n^3 + n^4 + 24)) - (x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*
n^2 + 10*n^3 + n^4 + 24) + (3*n*x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3)/(4*b^3*(50
*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (n*x^3*(log(-2/(exp(2*a)*exp(2*b*x) -
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)*(3*n
 + n^2 + 2))/(2*b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (3*n*x^2*(n + 1)*
(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) - 1)) + 2*b*x)^2)/(4*b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)))
```

3.187 $\int x^2 \coth^{-1}(\tanh(a+bx))^n \, dx$

3.187.1 Optimal result

Integrand size = 13, antiderivative size = 82

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{x^2 \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2x \coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2 \coth^{-1}(\tanh(a+bx))^{3+n}}{b^3(1+n)(2+n)(3+n)}$$

output
```
x^2*arccoth(tanh(b*x+a))^(1+n)/b/(1+n)-2*x*arccoth(tanh(b*x+a))^(2+n)/b^2/
(1+n)/(2+n)+2*arccoth(tanh(b*x+a))^(3+n)/b^3/(3+n)/(n^2+3*n+2)
```

3.187.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{\coth^{-1}(\tanh(a+bx))^{1+n} \left(b^2(6+5n+n^2)\, x^2 - 2b(3+n)x \coth^{-1}(\tanh(a+bx)) + 2 \coth^{-1}(\tanh(a+bx\right.}{b^3(1+n)(2+n)(3+n)}$$

input
```
Integrate[x^2*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
(ArcCoth[Tanh[a + b*x]]^(1 + n)*(b^2*(6 + 5*n + n^2)*x^2 - 2*b*(3 + n)*x*A
rcCoth[Tanh[a + b*x]] + 2*ArcCoth[Tanh[a + b*x]]^2))/(b^3*(1 + n)*(2 + n)*
(3 + n))
```

3.187.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2599, 2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^2 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{2\int x \coth^{-1}(\tanh(a+bx))^{n+1}dx}{b(n+1)}$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^2 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{2\left(\frac{x \coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{\int \coth^{-1}(\tanh(a+bx))^{n+2}dx}{b(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \text{ 2588}$$

$$\frac{x^2 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{2\left(\frac{x \coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{\int \coth^{-1}(\tanh(a+bx))^{n+2}d\coth^{-1}(\tanh(a+bx))}{b^2(n+2)}\right)}{b(n+1)}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x^2 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{2\left(\frac{x \coth^{-1}(\tanh(a+bx))^{n+2}}{b(n+2)}-\frac{\coth^{-1}(\tanh(a+bx))^{n+3}}{b^2(n+2)(n+3)}\right)}{b(n+1)}$$

input
```
Int[x^2*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
(x^2*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (2*((x*ArcCoth[Tanh[a +
 b*x]]^(2 + n))/(b*(2 + n)) - ArcCoth[Tanh[a + b*x]]^(3 + n)/(b^2*(2 + n)*
(3 + n))))/(b*(1 + n))
```

3.187.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.187.4 Maple [A] (verified)

Time = 9.52 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.99

method	result
parallelrisch	$-\frac{-2\operatorname{arccoth}(\tanh(bx+a))^n\operatorname{arccoth}(\tanh(bx+a))^3-6\operatorname{arccoth}(\tanh(bx+a))^n x^2\operatorname{arccoth}(\tanh(bx+a))b^2+6\operatorname{arccoth}(\tanh(bx+}{}$
risch	Expression too large to display

input
```
int(x^2*arccoth(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
-(-2*arccoth(tanh(b*x+a))^n*arccoth(tanh(b*x+a))^3-6*arccoth(tanh(b*x+a))^
n*x^2*arccoth(tanh(b*x+a))*b^2+6*arccoth(tanh(b*x+a))^n*arccoth(tanh(b*x+a
))^2*x*b-x^2*arccoth(tanh(b*x+a))*arccoth(tanh(b*x+a))^n*b^2*n^2-5*x^2*arc
coth(tanh(b*x+a))*arccoth(tanh(b*x+a))^n*b^2*n+2*x*arccoth(tanh(b*x+a))^2*
arccoth(tanh(b*x+a))^n*b*n)/b^3/(n^3+6*n^2+11*n+6)
```

3.187.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.37

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n \, dx =$$
$$-\frac{(8\,a^2bnx - 4\,(b^3n^2 + 3\,b^3n + 2\,b^3)x^3 + i\,\pi^3 - 2\,\pi^2(bnx - 3\,a) - 8\,a^3 - 4\,(ab^2n^2 + ab^2n)x^2 + 2i\,\pi(4\,abnx$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
-1/4*((8*a^2*b*n*x - 4*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 + I*pi^3 - 2*pi^2*(
b*n*x - 3*a) - 8*a^3 - 4*(a*b^2*n^2 + a*b^2*n)*x^2 + 2*I*pi*(4*a*b*n*x - (
b^2*n^2 + b^2*n)*x^2 - 6*a^2))*cosh(n*log(1/2*I*pi + b*x + a)) + (8*a^2*b*
n*x - 4*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 + I*pi^3 - 2*pi^2*(b*n*x - 3*a) -
8*a^3 - 4*(a*b^2*n^2 + a*b^2*n)*x^2 + 2*I*pi*(4*a*b*n*x - (b^2*n^2 + b^2*n
)*x^2 - 6*a^2))*sinh(n*log(1/2*I*pi + b*x + a)))/(b^3*n^3 + 6*b^3*n^2 + 11
*b^3*n + 6*b^3)
```

3.187.6 Sympy [F]

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n \, dx$$
$$= \begin{cases} \frac{x^3 \operatorname{acoth}^n(\tanh(a))}{3} \\ -\frac{x^2}{2b \operatorname{acoth}^2(\tanh(a+bx))} - \frac{x}{b^2 \operatorname{acoth}(\tanh(a+bx))} + \frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b^3} \\ \int \frac{x^2}{\operatorname{acoth}^2(\tanh(a+bx))}\, dx \\ \int \frac{x^2}{\operatorname{acoth}(\tanh(a+bx))}\, dx \\ \frac{b^2n^2x^2 \operatorname{acoth}(\tanh(a+bx)) \operatorname{acoth}^n(\tanh(a+bx))}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{5b^2nx^2 \operatorname{acoth}(\tanh(a+bx)) \operatorname{acoth}^n(\tanh(a+bx))}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{6b^2x^2 \operatorname{acoth}(\tanh(a+bx)) \operatorname{acoth}^n}{b^3n^3+6b^3n^2+11b^3n+} \end{cases}$$

input
```
integrate(x**2*acoth(tanh(b*x+a))**n,x)
```

output
```
Piecewise((x**3*acoth(tanh(a))**n/3, Eq(b, 0)), (-x**2/(2*b*acoth(tanh(a +
 b*x))**2) - x/(b**2*acoth(tanh(a + b*x))) + log(acoth(tanh(a + b*x)))/b**
3, Eq(n, -3)), (Integral(x**2/acoth(tanh(a + b*x))**2, x), Eq(n, -2)), (In
tegral(x**2/acoth(tanh(a + b*x)), x), Eq(n, -1)), (b**2*n**2*x**2*acoth(ta
nh(a + b*x))*acoth(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n
+ 6*b**3) + 5*b**2*n*x**2*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b*
*3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 6*b**2*x**2*acoth(tanh(a + b
*x))*acoth(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3
) - 2*b*n*x*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**3*n**3 + 6
*b**3*n**2 + 11*b**3*n + 6*b**3) - 6*b*x*acoth(tanh(a + b*x))**2*acoth(tan
h(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*acoth(ta
nh(a + b*x))**3*acoth(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3
*n + 6*b**3), True))
```

3.187.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.02

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n \, dx$$
$$= \frac{(4\,(n^2+3\,n+2)b^3x^3 + i\,\pi^3 - 6\,\pi^2 a - 12i\,\pi a^2 + 8\,a^3 - 2\,(i\,\pi(n^2+n)b^2 - 2\,(n^2+n)ab^2)x^2 + 2\,(\pi^2 bn + 4a}{(2^{n+2}n^3 + 3\cdot 2^{n+3}n^2 + 11\cdot 2^{n+2}}$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
(4*(n^2 + 3*n + 2)*b^3*x^3 + I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3 - 2*(
I*pi*(n^2 + n)*b^2 - 2*(n^2 + n)*a*b^2)*x^2 + 2*(pi^2*b*n + 4*I*pi*a*b*n -
 4*a^2*b*n)*x)*(cosh(-n*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*
b*x + 2*a)))/((2^(n + 2)*n^3 + 3*2^(n + 3)*n^2 + 11*2^(n + 2)*n + 3*2^(n +
 3))*b^3)
```

3.187.8 Giac [F]

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n \, dx = \int x^2 \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n \, dx$$

input
```
integrate(x^2*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(tanh(b*x + a))^n, x)
```

3.187.9 Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.71

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n \, dx = -\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2}-\frac{\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2}\right)^n\left(\frac{\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^3}{4\,b^3\,\left(n^3+6\,n^2+11\,n+6\right)} - \frac{x^3\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}+\frac{n\,x\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2}{2\,b^2\,\left(n^3+6\,n^2+11\,n+6\right)} + \frac{n\,x^2\,\left(n+1\right)\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2\,b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)$$

input
```
int(x^2*acoth(tanh(a + b*x))^n,x)
```

output
```
-(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 - log(-2/(exp(2
*a)*exp(2*b*x) - 1))/2)^n*((log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3/(4*b^3*(11*n + 6*n
^2 + n^3 + 6)) - (x^3*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (n*x*(lo
g(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) - 1)) + 2*b*x)^2)/(2*b^2*(11*n + 6*n^2 + n^3 + 6)) + (n*x^2*(n +
1)*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b*(11*n + 6*n^2 + n^3 + 6)))
```

3.188 $\int x \coth^{-1}(\tanh(a+bx))^n \, dx$

3.188.1 Optimal result

Integrand size = 11, antiderivative size = 48

$$\int x \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{x \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}$$

output
```
x*arccoth(tanh(b*x+a))^(1+n)/b/(1+n)-arccoth(tanh(b*x+a))^(2+n)/b^2/(1+n)/
(2+n)
```

3.188.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int x \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{\left(b(2+n)x - \coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))^{1+n}}{b^2(1+n)(2+n)}$$

input
```
Integrate[x*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
((b*(2 + n)*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b
^2*(1 + n)*(2 + n))
```

3.188.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2599, 2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(\tanh(a+bx))^n \, dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)} - \frac{\int \coth^{-1}(\tanh(a+bx))^{n+1}dx}{b(n+1)}$$

$$\Big\downarrow \text{2588}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)} - \frac{\int \coth^{-1}(\tanh(a+bx))^{n+1}d\coth^{-1}(\tanh(a+bx))}{b^2(n+1)}$$

$$\Big\downarrow \text{15}$$

$$\frac{x \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)} - \frac{\coth^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}$$

input
```
Int[x*ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
(x*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - ArcCoth[Tanh[a + b*x]]^(2
 + n)/(b^2*(1 + n)*(2 + n))
```

3.188.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.188.4 Maple [A] (verified)

Time = 9.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.60

method	result
parallelrisch	$-\frac{\operatorname{arccoth}(\tanh(bx+a))^{n} \operatorname{arccoth}(\tanh(bx+a))^{2}-x \operatorname{arccoth}(\tanh(bx+a)) \operatorname{arccoth}(\tanh(bx+a))^{n} bn-2 \operatorname{arccoth}(\tanh(bx+a))^{n}}{b^{2}(1+n)(2+n)}$
risch	$\frac{\left(\frac{1}{2}\right)^{n}\left(2\ln\left(\mathrm{e}^{bx+a}\right)-\frac{i\pi\ \mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\left(-\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)+\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)\right)^{2}}{2}-\frac{i\pi\ \mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)\left(-\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)+\mathrm{csgn}\left(i\mathrm{e}\right.\right.}{}\right.}{2b(1+n)}$

input
```
int(x*arccoth(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
-(arccoth(tanh(b*x+a))^n*arccoth(tanh(b*x+a))^2-x*arccoth(tanh(b*x+a))*arc
coth(tanh(b*x+a))^n*b*n-2*arccoth(tanh(b*x+a))^n*x*arccoth(tanh(b*x+a))*b)
/b^2/(1+n)/(2+n)
```

3.188.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.71

$$\int x \coth^{-1}(\tanh(a+bx))^n\,dx = \frac{(4\,abnx+4\,(b^2n+b^2)x^2+\pi^2+2i\,\pi(bnx-2\,a)-4\,a^2)\cosh\left(n\log\left(\frac{1}{2}i\,\pi+bx+a\right)\right)+(4\,abnx+4\,(b^2n+}{4\,(b^2n^2+3\,b^2n+2\,b^2)}$$

input
```
integrate(x*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
1/4*((4*a*b*n*x + 4*(b^2*n + b^2)*x^2 + pi^2 + 2*I*pi*(b*n*x - 2*a) - 4*a^
2)*cosh(n*log(1/2*I*pi + b*x + a)) + (4*a*b*n*x + 4*(b^2*n + b^2)*x^2 + pi
^2 + 2*I*pi*(b*n*x - 2*a) - 4*a^2)*sinh(n*log(1/2*I*pi + b*x + a)))/(b^2*n
^2 + 3*b^2*n + 2*b^2)
```

3.188.6 Sympy [F]

$$\int x \coth^{-1}(\tanh(a+bx))^n \, dx$$
$$= \begin{cases} \frac{x^2 \operatorname{acoth}^n(\tanh(a))}{2} \\ -\frac{x}{b \operatorname{acoth}(\tanh(a+bx))} + \frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b^2} \\ \int \frac{x}{\operatorname{acoth}(\tanh(a+bx))} \, dx \\ \frac{bnx \operatorname{acoth}(\tanh(a+bx)) \operatorname{acoth}^n(\tanh(a+bx))}{b^2n^2+3b^2n+2b^2} + \frac{2bx \operatorname{acoth}(\tanh(a+bx)) \operatorname{acoth}^n(\tanh(a+bx))}{b^2n^2+3b^2n+2b^2} - \frac{\operatorname{acoth}^2(\tanh(a+bx)) \operatorname{acoth}^n(\tanh(a+b}{b^2n^2+3b^2n+2b^2} \end{cases}$$

input
```
integrate(x*acoth(tanh(b*x+a))**n,x)
```

output
```
Piecewise((x**2*acoth(tanh(a))**n/2, Eq(b, 0)), (-x/(b*acoth(tanh(a + b*x)
)) + log(acoth(tanh(a + b*x)))/b**2, Eq(n, -2)), (Integral(x/acoth(tanh(a
+ b*x)), x), Eq(n, -1)), (b*n*x*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))*
*n/(b**2*n**2 + 3*b**2*n + 2*b**2) + 2*b*x*acoth(tanh(a + b*x))*acoth(tanh
(a + b*x))**n/(b**2*n**2 + 3*b**2*n + 2*b**2) - acoth(tanh(a + b*x))**2*ac
oth(tanh(a + b*x))**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True))
```

3.188.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.12

$$\int x \coth^{-1}(\tanh(a+bx))^n \, dx$$
$$= \frac{(4\,b^2(n+1)x^2 + \pi^2 + 4i\,\pi a - 4\,a^2 - 2\,(i\,\pi bn - 2\,abn)x)(\cosh(-n\log(-i\,\pi + 2\,bx + 2\,a)) - \sinh(-n\log}{(2^{n+2}n^2 + 3\cdot 2^{n+2}n + 2^{n+3})b^2}$$

input
```
integrate(x*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
(4*b^2*(n + 1)*x^2 + pi^2 + 4*I*pi*a - 4*a^2 - 2*(I*pi*b*n - 2*a*b*n)*x)*(
cosh(-n*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*b*x + 2*a)))/((2
^(n + 2)*n^2 + 3*2^(n + 2)*n + 2^(n + 3))*b^2)
```

3.188.8 Giac [F]

$$\int x \coth^{-1}(\tanh(a+bx))^n \, dx = \int x \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n \, dx$$

input
```
integrate(x*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate(x*arccoth(tanh(b*x + a))^n, x)
```

3.188.9 Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 205, normalized size of antiderivative = 4.27

$$\int x \coth^{-1}(\tanh(a+bx))^n \, dx = -\left(\frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)}{2}-\frac{\ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)}{2}\right)^n\left(\frac{\left(\ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)+2\,b\,x\right)^2}{4\,b^2\,\left(n^2+3\,n+2\right)}-\frac{x^2\,\left(n+1\right)}{n^2+3\,n+2}+\frac{n\,x\,\left(\ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2\,b\,\left(n^2+3\,n+2\right)}\right)$$

input
```
int(x*acoth(tanh(a + b*x))^n,x)
```

output
```
-(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 - log(-2/(exp(2
*a)*exp(2*b*x) - 1))/2)^n*((log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2/(4*b^2*(3*n + n^2
+ 2)) - (x^2*(n + 1))/(3*n + n^2 + 2) + (n*x*(log(-2/(exp(2*a)*exp(2*b*x)
- 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(
2*b*(3*n + n^2 + 2)))
```

3.189 $\int \coth^{-1}(\tanh(a+bx))^n \, dx$

3.189.1 Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}$$

output
```
arccoth(tanh(b*x+a))^(1+n)/b/(1+n)
```

3.189.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

3.189.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(\tanh(a+bx))^n\,dx$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \coth^{-1}(\tanh(a+bx))^n d\coth^{-1}(\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^n,x]
```

output
```
ArcCoth[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

3.189.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.189.4 Maple [A] (verified)

Time = 9.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\operatorname{arccoth}(\tanh(bx+a))^{1+n}}{b(1+n)}$
default	$\frac{\operatorname{arccoth}(\tanh(bx+a))^{1+n}}{b(1+n)}$
parallelrisch	$\frac{\operatorname{arccoth}(\tanh(bx+a))^{n} \operatorname{arccoth}(\tanh(bx+a))}{b(1+n)}$
risch	$\frac{\left(\frac{1}{2}\right)^{n}\left(2\ln\left(\mathrm{e}^{bx+a}\right)-\frac{i\pi\ \mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\left(-\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)+\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)\right)^{2}}{2}-\frac{i\pi\ \mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)\left(-\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)+\cdots\right.}{2b(1+\cdots}$

input
```
int(arccoth(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
arccoth(tanh(b*x+a))^(1+n)/b/(1+n)
```

3.189.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.90

$$\int \coth^{-1}(\tanh(a+bx))^n\,dx$$
$$= \frac{(i\pi + 2\,bx + 2\,a)\cosh\left(n\log\left(\frac{1}{2}i\,\pi + bx + a\right)\right) + (i\,\pi + 2\,bx + 2\,a)\sinh\left(n\log\left(\frac{1}{2}i\,\pi + bx + a\right)\right)}{2\,(bn + b)}$$

input
```
integrate(arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

output
```
1/2*((I*pi + 2*b*x + 2*a)*cosh(n*log(1/2*I*pi + b*x + a)) + (I*pi + 2*b*x
+ 2*a)*sinh(n*log(1/2*I*pi + b*x + a)))/(b*n + b)
```

3.189.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \coth^{-1}(\tanh(a+bx))^n\,dx = \begin{cases} \frac{x}{\operatorname{acoth}(\tanh(a))} & \text{for } b=0 \wedge n=-1 \\ x\operatorname{acoth}^n(\tanh(a)) & \text{for } b=0 \\ \frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b} & \text{for } n=-1 \\ \frac{\operatorname{acoth}(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{bn+b} & \text{otherwise} \end{cases}$$

input
```
integrate(acoth(tanh(b*x+a))**n,x)
```

output
```
Piecewise((x/acoth(tanh(a)), Eq(b, 0) & Eq(n, -1)), (x*acoth(tanh(a))**n,
Eq(b, 0)), (log(acoth(tanh(a + b*x)))/b, Eq(n, -1)), (acoth(tanh(a + b*x))
*acoth(tanh(a + b*x))**n/(b*n + b), True))
```

3.189.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.25

$$\int \coth^{-1}(\tanh(a+bx))^n\,dx = \frac{(-i\pi+2bx+2a)(\cosh(-n\log(-i\pi+2bx+2a))-\sinh(-n\log(-i\pi+2bx+2a)))}{(2^{n+1}n+2^{n+1})b}$$

input
```
integrate(arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

output
```
(-I*pi + 2*b*x + 2*a)*(cosh(-n*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*
pi + 2*b*x + 2*a)))/((2^(n + 1)*n + 2^(n + 1))*b)
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \coth^{-1}(\tanh(a+bx))^n\,dx = \frac{\left(\frac{1}{2}\log\left(-e^{(2bx+2a)}\right)\right)^{n+1}}{b(n+1)}$$

input
```
integrate(arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

output
```
(1/2*log(-e^(2*b*x + 2*a)))^(n + 1)/(b*(n + 1))
```

3.189.9 Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 121, normalized size of antiderivative = 6.05

$$\int \coth^{-1}(\tanh(a+bx))^n\,dx = \left(\frac{1}{2}\right)^n\left(\frac{x}{n+1} - \frac{\frac{\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)}{2}-\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)}{2}+bx}{b\,(n+1)}\right)\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)-\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)\right)^n$$

input
```
int(acoth(tanh(a + b*x))^n,x)
```

output
```
(1/2)^n*(x/(n + 1) - (log(-2/(exp(2*a)*exp(2*b*x) - 1))/2 - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 + b*x)/(b*(n + 1)))*(log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-2/(exp(2*a)*exp(2*b*x)
 - 1)))^n
```

3.190 $\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x} dx$

3.190.1 Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x} dx = \frac{\coth^{-1}(\tanh(a+bx))^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(1+n)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

output
```
arccoth(tanh(b*x+a))^(1+n)*hypergeom([1, 1+n],[2+n],-arccoth(tanh(b*x+a))/
(b*x-arccoth(tanh(b*x+a))))/(1+n)/(b*x-arccoth(tanh(b*x+a)))
```

3.190.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x} dx = \frac{\coth^{-1}(\tanh(a+bx))^n \left(\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)^{-n} \text{Hypergeometric2F1}\left(-n,-n,1-n,1-\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)}{n}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^n/x,x]
```

output
```
(ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2F1[-n, -n, 1 - n, 1 - ArcCoth[Tan
h[a + b*x]]/(b*x)])/(n*(ArcCoth[Tanh[a + b*x]]/(b*x))^n)
```

3.190.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2595}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x} \, dx$$

$$\Big\downarrow \text{2595}$$

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(n+1)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^n/x,x]
```

output
```
(ArcCoth[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, -(ArcCo
th[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/((1 + n)*(b*x - ArcCot
h[Tanh[a + b*x]]))
```

3.190.3.1 Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] && !IntegerQ[n]
```

3.190.4 Maple [F]

$$\int \frac{\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)^{n}}{x}dx$$

input
```
int(arccoth(tanh(b*x+a))^n/x,x)
```

output
```
int(arccoth(tanh(b*x+a))^n/x,x)
```

3.190.5 Fricas [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x}\,dx = \int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n}{x}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x,x, algorithm="fricas")
```

output
```
integral(arccoth(tanh(b*x + a))^n/x, x)
```

3.190.6 Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x}\,dx = \int \frac{\operatorname{acoth}^n\left(\tanh\left(a+bx\right)\right)}{x}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))**n/x,x)
```

output
```
Integral(acoth(tanh(a + b*x))**n/x, x)
```

3.190.7 Maxima [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x}\,dx = \int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n}{x}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x,x, algorithm="maxima")
```

output
```
integrate(arccoth(tanh(b*x + a))^n/x, x)
```

3.190.8 Giac [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x}\,dx = \int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n}{x}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x,x, algorithm="giac")
```

output
```
integrate(arccoth(tanh(b*x + a))^n/x, x)
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x}\,dx = \int \frac{\operatorname{acoth}(\tanh(a+b\,x))^n}{x}\,dx$$

input
```
int(acoth(tanh(a + b*x))^n/x,x)
```

output
```
int(acoth(tanh(a + b*x))^n/x, x)
```

3.191 $\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2}\,dx$

3.191.1 Optimal result

Integrand size = 13, antiderivative size = 71

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2}\,dx = -\frac{\coth^{-1}(\tanh(a+bx))^n}{x} + \frac{b\coth^{-1}(\tanh(a+bx))^n\,\text{Hypergeometric2F1}\left(1,n,1+n,-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}$$

output
```
-arccoth(tanh(b*x+a))^n/x+b*arccoth(tanh(b*x+a))^n*hypergeom([1, n],[1+n],
-arccoth(tanh(b*x+a))/(b*x-arccoth(tanh(b*x+a))))/(b*x-arccoth(tanh(b*x+a)
))
```

3.191.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2}\,dx = \frac{\coth^{-1}(\tanh(a+bx))^n\left(\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)^{-n}\text{Hypergeometric2F1}\left(1-n,-n,2-n,1-\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)}{(-1+n)x}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^n/x^2,x]
```

output
```
(ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2F1[1 - n, -n, 2 - n, 1 - ArcCoth[
Tanh[a + b*x]]/(b*x)])/((-1 + n)*x*(ArcCoth[Tanh[a + b*x]]/(b*x))^n)
```

3.191.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2599, 2595}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2} \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$bn \int \frac{\coth^{-1}(\tanh(a+bx))^{n-1}}{x} dx - \frac{\coth^{-1}(\tanh(a+bx))^n}{x}$$

$$\Big\downarrow \text{ 2595}$$

$$\frac{b \coth^{-1}(\tanh(a+bx))^n \text{Hypergeometric2F1}\left(1,n,n+1,-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))} - \frac{\coth^{-1}(\tanh(a+bx))^n}{x}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^n/x^2,x]
```

output
```
-(ArcCoth[Tanh[a + b*x]]^n/x) + (b*ArcCoth[Tanh[a + b*x]]^n*Hypergeometric
2F1[1, n, 1 + n, -(ArcCoth[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))]
)/(b*x - ArcCoth[Tanh[a + b*x]])
```

3.191.3.1 Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.191.4 Maple [F]

$$\int \frac{\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{2}}dx$$

input
```
int(arccoth(tanh(b*x+a))^n/x^2,x)
```

output
```
int(arccoth(tanh(b*x+a))^n/x^2,x)
```

3.191.5 Fricas [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2}\,dx = \int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{2}}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x^2,x, algorithm="fricas")
```

output
```
integral(arccoth(tanh(b*x + a))^n/x^2, x)
```

3.191.6 Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2}\,dx = \int \frac{\operatorname{acoth}^n(\tanh(a+bx))}{x^2}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))**n/x**2,x)
```

output
```
Integral(acoth(tanh(a + b*x))**n/x**2, x)
```

3.191.7 Maxima [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2}\,dx = \int \frac{\operatorname{arcoth}(\tanh(bx+a))^n}{x^2}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x^2,x, algorithm="maxima")
```

output
```
integrate(arccoth(tanh(b*x + a))^n/x^2, x)
```

3.191.8 Giac [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2}\,dx = \int \frac{\operatorname{arcoth}(\tanh(bx+a))^n}{x^2}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x^2,x, algorithm="giac")
```

output
```
integrate(arccoth(tanh(b*x + a))^n/x^2, x)
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2} dx = \int \frac{\operatorname{acoth}(\tanh(a+b\,x))^n}{x^2} dx$$

input
```
int(acoth(tanh(a + b*x))^n/x^2,x)
```

output
```
int(acoth(tanh(a + b*x))^n/x^2, x)
```

3.192 $\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3} dx$

3.192.1 Optimal result

Integrand size = 13, antiderivative size = 101

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3} dx = -\frac{bn\coth^{-1}(\tanh(a+bx))^{-1+n}}{2x} - \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2} + \frac{b^2n\coth^{-1}(\tanh(a+bx))^{-1+n}\,\text{Hypergeometric2F1}\left(1,-1+n,n,-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

output
```
-1/2*b*n*arccoth(tanh(b*x+a))^(-1+n)/x-1/2*arccoth(tanh(b*x+a))^n/x^2+1/2*
b^2*n*arccoth(tanh(b*x+a))^(-1+n)*hypergeom([1, -1+n],[n],-arccoth(tanh(b*
x+a))/(b*x-arccoth(tanh(b*x+a))))/(b*x-arccoth(tanh(b*x+a)))
```

3.192.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.66

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3} dx = \frac{\coth^{-1}(\tanh(a+bx))^n\left(\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)^{-n}\text{Hypergeometric2F1}\left(2-n,-n,3-n,1-\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)}{(-2+n)x^2}$$

input
```
Integrate[ArcCoth[Tanh[a + b*x]]^n/x^3,x]
```

output
```
(ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2F1[2 - n, -n, 3 - n, 1 - ArcCoth[
Tanh[a + b*x]]/(b*x)])/((-2 + n)*x^2*(ArcCoth[Tanh[a + b*x]]/(b*x))^n)
```

3.192.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2599, 2599, 2595}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3} dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{2}bn\int \frac{\coth^{-1}(\tanh(a+bx))^{n-1}}{x^2}dx - \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2}$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{2}bn\left(-b(1-n)\int \frac{\coth^{-1}(\tanh(a+bx))^{n-2}}{x}dx - \frac{\coth^{-1}(\tanh(a+bx))^{n-1}}{x}\right) - \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2}$$

$$\Big\downarrow \text{2595}$$

$$\frac{1}{2}bn\left(\frac{b\coth^{-1}(\tanh(a+bx))^{n-1}\text{Hypergeometric2F1}\left(1,n-1,n,-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))} - \frac{\coth^{-1}(\tanh(a+bx)}{x}\right. \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2}$$

input
```
Int[ArcCoth[Tanh[a + b*x]]^n/x^3,x]
```

output
```
-1/2*ArcCoth[Tanh[a + b*x]]^n/x^2 + (b*n*(-(ArcCoth[Tanh[a + b*x]]^(-1 + n
)/x) + (b*ArcCoth[Tanh[a + b*x]]^(-1 + n)*Hypergeometric2F1[1, -1 + n, n,
-(ArcCoth[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/(b*x - ArcCoth[
Tanh[a + b*x]])))/2
```

3.192.3.1 Defintions of rubi rules used

rule 2595
```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] && !IntegerQ[n]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILt
Q[m, 0] && !IntegerQ[n]))
```

3.192.4 Maple [F]

$$\int \frac{\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{3}}dx$$

input
```
int(arccoth(tanh(b*x+a))^n/x^3,x)
```

output
```
int(arccoth(tanh(b*x+a))^n/x^3,x)
```

3.192.5 Fricas [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3}\,dx = \int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{3}}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x^3,x, algorithm="fricas")
```

output
```
integral(arccoth(tanh(b*x + a))^n/x^3, x)
```

3.192.6 Sympy [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3}\,dx = \int \frac{\operatorname{acoth}^n\left(\tanh\left(a+bx\right)\right)}{x^3}\,dx$$

input
```
integrate(acoth(tanh(b*x+a))**n/x**3,x)
```

output
```
Integral(acoth(tanh(a + b*x))**n/x**3, x)
```

3.192.7 Maxima [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3}\,dx = \int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n}{x^3}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x^3,x, algorithm="maxima")
```

output
```
integrate(arccoth(tanh(b*x + a))^n/x^3, x)
```

3.192.8 Giac [F]

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3}\,dx = \int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^n}{x^3}\,dx$$

input
```
integrate(arccoth(tanh(b*x+a))^n/x^3,x, algorithm="giac")
```

output
```
integrate(arccoth(tanh(b*x + a))^n/x^3, x)
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3}\,dx = \int \frac{\mathrm{acoth}(\tanh(a+b\,x))^n}{x^3}\,dx$$

input
```
int(acoth(tanh(a + b*x))^n/x^3,x)
```

output
```
int(acoth(tanh(a + b*x))^n/x^3, x)
```

3.193 $\int x^m \coth^{-1}(\tanh(a+bx))\,dx$

3.193.1 Optimal result

Integrand size = 11, antiderivative size = 37

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^{2+m}}{2+3m+m^2} + \frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}$$

output
```
-b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*arccoth(tanh(b*x+a))/(1+m)
```

3.193.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = x^m\left(\frac{bx^2}{2+m} + \frac{x(-bx+\coth^{-1}(\tanh(a+bx)))}{1+m}\right)$$

input
```
Integrate[x^m*ArcCoth[Tanh[a + b*x]],x]
```

output
```
x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/(1 + m))
```

3.193.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{b\int x^{m+1}dx}{m+1}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{(m+1)(m+2)}$$

input
```
Int[x^m*ArcCoth[Tanh[a + b*x]],x]
```

output
```
-((b*x^(2 + m))/((1 + m)*(2 + m))) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]])/(1
 + m)
```

3.193.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.193.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

method	result
parallelrisch	$-\frac{-x\,x^{m} \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right) m +b\,x^{2} x^{m}-2 \operatorname{arccoth}\left(\tanh \left(b x +a \right)\right) x\,x^{m}}{\left(1+m \right) \left(2+m \right)}$
risch	$\frac{x\,x^{m} \ln \left({\mathrm e}^{b x +a}\right)}{1+m}-\frac{x \left(4 b x +i \pi \,\operatorname{csgn}\left(\frac{i {\mathrm e}^{2 b x +2 a}}{{\mathrm e}^{2 b x +2 a}+1}\right)^{3} m +2 i \pi \,\operatorname{csgn}\left(\frac{i}{{\mathrm e}^{2 b x +2 a}+1}\right) \operatorname{csgn}\left(i {\mathrm e}^{2 b x +2 a}\right) \operatorname{csgn}\left(\frac{i {\mathrm e}^{2 b x +2 a}}{{\mathrm e}^{2 b x +2 a}+1}\right)-2 i \pi \,\operatorname{csgn}\left(i {\mathrm e}^{2}\right.}{}$

input
```
int(x^m*arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-(-x*x^m*arccoth(tanh(b*x+a))*m+b*x^2*x^m-2*arccoth(tanh(b*x+a))*x*x^m)/(1
+m)/(2+m)
```

3.193.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.19

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = \frac{(i\,\pi(m+2)x+2\,(bm+b)x^2+2\,(am+2\,a)x)\cosh\left(m\log\left(x\right)\right)+(i\,\pi(m+2)x+2\,(bm+b)x^2+2\,(am+}{2\,(m^2+3\,m+2)}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*((I*pi*(m + 2)*x + 2*(b*m + b)*x^2 + 2*(a*m + 2*a)*x)*cosh(m*log(x)) +
 (I*pi*(m + 2)*x + 2*(b*m + b)*x^2 + 2*(a*m + 2*a)*x)*sinh(m*log(x)))/(m^2
 + 3*m + 2)
```

3.193.6 Sympy [F]

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = \begin{cases} b\log(x) - \frac{\operatorname{acoth}(\tanh(a+bx))}{x} & \text{for } m=-2 \\ \int \frac{\operatorname{acoth}(\tanh(a+bx))}{x}\,dx & \text{for } m=-1 \\ -\frac{bx^2x^m}{m^2+3m+2} + \frac{mxx^m \operatorname{acoth}(\tanh(a+bx))}{m^2+3m+2} + \frac{2xx^m \operatorname{acoth}(\tanh(a+bx))}{m^2+3m+2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*acoth(tanh(b*x+a)),x)
```

output
```
Piecewise((b*log(x) - acoth(tanh(a + b*x))/x, Eq(m, -2)), (Integral(acoth(
tanh(a + b*x))/x, x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x*x**
m*acoth(tanh(a + b*x))/(m**2 + 3*m + 2) + 2*x*x**m*acoth(tanh(a + b*x))/(m
**2 + 3*m + 2), True))
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = -\frac{bx^2x^m}{(m+2)(m+1)} + \frac{x^{m+1}\operatorname{arcoth}(\tanh(bx+a))}{m+1}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-b*x^2*x^m/((m + 2)*(m + 1)) + x^(m + 1)*arccoth(tanh(b*x + a))/(m + 1)
```

3.193.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(37) = 74.

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = \frac{x^{m+1}\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{2\,(m+1)} - \frac{bx^{m+2}}{(m+2)(m+1)}$$

input
```
integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

output
```
1/2*x^(m + 1)*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((e^(
2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/(m + 1) - b*x^(m + 2)/((m +
2)*(m + 1))
```

3.193.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.59

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx$$
$$= \frac{2\,b\,x^m\,x^2\,(m+1)}{2\,m^2+6\,m+4} - \frac{x\,x^m\,(m+2)\,\left(\ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2\,m^2+6\,m+4}$$

input
```
int(x^m*acoth(tanh(a + b*x)),x)
```

output
```
(2*b*x^m*x^2*(m + 1))/(6*m + 2*m^2 + 4) - (x*x^m*(m + 2)*(log(-2/(exp(2*a)
*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))
 + 2*b*x))/(6*m + 2*m^2 + 4)
```

3.194 $\int x^2 \coth^{-1}(\coth(a+bx))\,dx$

3.194.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int x^2 \coth^{-1}(\coth(a+bx))\,dx = -\frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(\coth(a+bx))$$

output
```
-1/12*b*x^4+1/3*x^3*arccoth(coth(b*x+a))
```

3.194.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x^2 \coth^{-1}(\coth(a+bx))\,dx = -\frac{1}{12}x^3 \left(bx - 4\coth^{-1}(\coth(a+bx))\right)$$

input
```
Integrate[x^2*ArcCoth[Coth[a + b*x]],x]
```

output
```
-1/12*(x^3*(b*x - 4*ArcCoth[Coth[a + b*x]]))
```

3.194.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(\coth(a+bx))\,dx$$

$$\Big\downarrow \text{2599}$$

$$\frac{1}{3}x^3\coth^{-1}(\coth(a+bx)) - \frac{b\int x^3dx}{3}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{3}x^3\coth^{-1}(\coth(a+bx)) - \frac{bx^4}{12}$$

input
```
Int[x^2*ArcCoth[Coth[a + b*x]],x]
```

output
```
-1/12*(b*x^4) + (x^3*ArcCoth[Coth[a + b*x]])/3
```

3.194.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.194.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{b\,x^{4}}{12}+\frac{x^{3} \operatorname{arccoth}\left(\coth \left(b x +a \right)\right)}{3}$
parts	$-\frac{b\,x^{4}}{12}+\frac{x^{3} \operatorname{arccoth}\left(\coth \left(b x +a \right)\right)}{3}$
parallelrisch	$-\frac{x^{3} \left(b x -4 \operatorname{arccoth}\left(\frac{1}{\tanh \left(b x +a \right)}\right)\right)}{12}$
risch	$\frac{x^{3} \ln \left({\mathrm e}^{b x +a}\right)}{3}-\frac{b\,x^{4}}{12}-\frac{i \pi \,x^{3} \operatorname{csgn}\left(i {\mathrm e}^{2 b x +2 a}\right) \operatorname{csgn}\left(\frac{i}{{\mathrm e}^{2 b x +2 a}-1}\right) \operatorname{csgn}\left(\frac{i {\mathrm e}^{2 b x +2 a}}{{\mathrm e}^{2 b x +2 a}-1}\right)}{12}+\frac{i \pi \,x^{3} \operatorname{csgn}\left(\frac{i}{{\mathrm e}^{2 b x +2 a}-1}\right) \operatorname{csgn}\left(\frac{i {\mathrm e}^{2 b x}}{{\mathrm e}^{2 b x +}}\right.}{12}$

input
```
int(x^2*arccoth(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*b*x^4+1/3*x^3*arccoth(coth(b*x+a))
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x^2 \coth^{-1}(\coth(a+bx))\,dx = \frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

input
```
integrate(x^2*arccoth(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

3.194.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(19) = 38.

Time = 4.41 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.30

$$\int x^2 \coth^{-1}(\coth(a+bx))\,dx = \begin{cases} \frac{x^{3} \operatorname{acoth}{\left(\coth{\left(bx + \log{\left(- e^{- bx} \right)} \right)} \right)}}{3} & \text{for}\: a = \log{\left(- e^{- bx} \right)} \\ \frac{x^{3} \operatorname{acoth}{\left(\coth{\left(bx + \log{\left(e^{- bx} \right)} \right)} \right)}}{3} & \text{for}\: a = \log{\left(e^{- bx} \right)} \\ - \frac{bx^{4}}{12} + \frac{x^{3} \operatorname{acoth}{\left(\frac{1}{\tanh{\left(a + bx \right)}} \right)}}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*acoth(coth(b*x+a)),x)
```

output
```
Piecewise((x**3*acoth(coth(b*x + log(-exp(-b*x))))/3, Eq(a, log(-exp(-b*x)
))), (x**3*acoth(coth(b*x + log(exp(-b*x))))/3, Eq(a, log(exp(-b*x)))), (-
b*x**4/12 + x**3*acoth(1/tanh(a + b*x))/3, True))
```

3.194.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x^2 \coth^{-1}(\coth(a+bx))\,dx = \frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

input
```
integrate(x^2*arccoth(coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

3.194.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x^2 \coth^{-1}(\coth(a+bx))\,dx = \frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

input
```
integrate(x^2*arccoth(coth(b*x+a)),x, algorithm="giac")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

3.194.9 Mupad [B] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \coth^{-1}(\coth(a+bx))\,dx = \frac{x^3\,\mathrm{acoth}(\coth(a+b\,x))}{3} - \frac{b\,x^4}{12}$$

input
```
int(x^2*acoth(coth(a + b*x)),x)
```

output
```
(x^3*acoth(coth(a + b*x)))/3 - (b*x^4)/12
```

3.195 $\int x \coth^{-1}(\coth(a+bx))\,dx$

3.195.1 Optimal result

Integrand size = 9, antiderivative size = 23

$$\int x \coth^{-1}(\coth(a+bx))\,dx = -\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(\coth(a+bx))$$

output
```
-1/6*b*x^3+1/2*x^2*arccoth(coth(b*x+a))
```

3.195.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x \coth^{-1}(\coth(a+bx))\,dx = -\frac{1}{6}x^2\left(bx-3\coth^{-1}(\coth(a+bx))\right)$$

input
```
Integrate[x*ArcCoth[Coth[a + b*x]],x]
```

output
```
-1/6*(x^2*(b*x - 3*ArcCoth[Coth[a + b*x]]))
```

3.195.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6796, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(\coth(a+bx))\,dx$$

$$\Big\downarrow \text{6796}$$

$$\frac{1}{2}b\int -x^2dx+\frac{1}{2}x^2\coth^{-1}(\coth(a+bx))$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}x^2\coth^{-1}(\coth(a+bx))-\frac{bx^3}{6}$$

input
```
Int[x*ArcCoth[Coth[a + b*x]],x]
```

output
```
-1/6*(b*x^3) + (x^2*ArcCoth[Coth[a + b*x]])/2
```

3.195.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6796
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

3.195.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{b\,x^{3}}{6}+\frac{x^{2} \operatorname{arccoth}\left(\coth \left(b x +a \right)\right)}{2}$
parts	$-\frac{b\,x^{3}}{6}+\frac{x^{2} \operatorname{arccoth}\left(\coth \left(b x +a \right)\right)}{2}$
parallelrisch	$-\frac{x^{2} \left(b x -3 \,\operatorname{arccoth}\left(\frac{1}{\tanh \left(b x +a \right)}\right)\right)}{6}$
risch	$\frac{x^{2} \ln \left({\mathrm e}^{b x +a}\right)}{2}-\frac{b\,x^{3}}{6}+\frac{i \pi \,x^{2} \operatorname{csgn}\left(\frac{i}{{\mathrm e}^{2 b x +2 a}-1}\right) \operatorname{csgn}\left(\frac{i {\mathrm e}^{2 b x +2 a}}{{\mathrm e}^{2 b x +2 a}-1}\right)^{2}}{8}+\frac{i \pi \,x^{2} \operatorname{csgn}\left(i {\mathrm e}^{2 b x +2 a}\right) \operatorname{csgn}\left(\frac{i {\mathrm e}^{2 b x +2 a}}{{\mathrm e}^{2 b x +2 a}-1}\right)^{2}}{8}-\frac{i \pi \,x^{2}}{}$

input
```
int(x*arccoth(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*b*x^3+1/2*x^2*arccoth(coth(b*x+a))
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x \coth^{-1}(\coth(a+bx))\,dx=\frac{1}{3}x^3b+\frac{1}{2}x^2a$$

input
```
integrate(x*arccoth(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/3*x^3*b + 1/2*x^2*a
```

3.195.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(19) = 38$.

Time = 2.64 (sec) , antiderivative size = 180, normalized size of antiderivative = 7.83

$$\int x \coth^{-1}(\coth(a+bx))\,dx = \begin{cases} \frac{x^2 \operatorname{acoth}(\coth(a))}{2} & \text{for } b=0 \\ -\frac{x\log(-e^{-bx})\operatorname{acoth}(\coth(bx+\log(-e^{-bx})))}{b} - \frac{\log(-e^{-bx})^2\operatorname{acoth}(\coth(bx+\log(-e^{-bx})))}{2b^2} & \text{for } a=\log(-e^{-bx}) \\ -\frac{x\log(e^{-bx})\operatorname{acoth}(\coth(bx+\log(e^{-bx})))}{b} - \frac{\log(e^{-bx})^2\operatorname{acoth}(\coth(bx+\log(e^{-bx})))}{2b^2} & \text{for } a=\log(e^{-bx}) \\ \frac{x\operatorname{acoth}^2\left(\frac{1}{\tanh(a+bx)}\right)}{2b} - \frac{\operatorname{acoth}^3\left(\frac{1}{\tanh(a+bx)}\right)}{6b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acoth(coth(b*x+a)),x)
```

output
```
Piecewise((x**2*acoth(coth(a))/2, Eq(b, 0)), (-x*log(-exp(-b*x))*acoth(cot
h(b*x + log(-exp(-b*x))))/b - log(-exp(-b*x))**2*acoth(coth(b*x + log(-exp
(-b*x))))/(2*b**2), Eq(a, log(-exp(-b*x)))), (-x*log(exp(-b*x))*acoth(coth
(b*x + log(exp(-b*x))))/b - log(exp(-b*x))**2*acoth(coth(b*x + log(exp(-b*
x))))/(2*b**2), Eq(a, log(exp(-b*x)))), (x*acoth(1/tanh(a + b*x))**2/(2*b)
 - acoth(1/tanh(a + b*x))**3/(6*b**2), True))
```

3.195.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x \coth^{-1}(\coth(a+bx))\,dx = \frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

input
```
integrate(x*arccoth(coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*b*x^3 + 1/2*a*x^2
```

3.195.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x \coth^{-1}(\coth(a+bx))\,dx = \frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2$$

input
```
integrate(x*arccoth(coth(b*x+a)),x, algorithm="giac")
```

output
```
1/3*b*x^3 + 1/2*a*x^2
```

3.195.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x \coth^{-1}(\coth(a+bx))\,dx = \frac{x^2\,\mathrm{acoth}(\coth(a+b\,x))}{2} - \frac{b\,x^3}{6}$$

input
```
int(x*acoth(coth(a + b*x)),x)
```

output
```
(x^2*acoth(coth(a + b*x)))/2 - (b*x^3)/6
```

3.196 $\int \coth^{-1}(\coth(a+bx))\,dx$

3.196.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \coth^{-1}(\coth(a+bx))\,dx = \frac{\coth^{-1}(\coth(a+bx))^2}{2b}$$

output
```
1/2*arccoth(coth(b*x+a))^2/b
```

3.196.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \coth^{-1}(\coth(a+bx))\,dx = -\frac{bx^2}{2} + x\coth^{-1}(\coth(a+bx))$$

input
```
Integrate[ArcCoth[Coth[a + b*x]],x]
```

output
```
-1/2*(b*x^2) + x*ArcCoth[Coth[a + b*x]]
```

3.196.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(\coth(a+bx))\,dx$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \coth^{-1}(\coth(a+bx))d\coth^{-1}(\coth(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\coth^{-1}(\coth(a+bx))^2}{2b}$$

input
```
Int[ArcCoth[Coth[a + b*x]],x]
```

output
```
ArcCoth[Coth[a + b*x]]^2/(2*b)
```

3.196.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.196.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

method	result
parallelrisch	$-\frac{x\left(bx-2\,\mathrm{arccoth}\left(\frac{1}{\tanh(bx+a)}\right)\right)}{2}$
derivativedivides	$\frac{\mathrm{arctanh}(\coth(bx+a))\,\mathrm{arccoth}(\coth(bx+a))-\frac{\mathrm{arctanh}(\coth(bx+a))^{2}}{2}}{b}$
default	$\frac{\mathrm{arctanh}(\coth(bx+a))\,\mathrm{arccoth}(\coth(bx+a))-\frac{\mathrm{arctanh}(\coth(bx+a))^{2}}{2}}{b}$
parts	$x\ \mathrm{arccoth}\left(\coth\left(bx+a\right)\right)+\frac{-\frac{(bx+a)^{2}}{2}+(bx+a)a}{b}$
risch	$x\ln\left(\mathrm{e}^{bx+a}\right)-\frac{i\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)^{3}x}{4}+\frac{i\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)^{2}\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)x}{4}+\frac{i\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)^{2}\mathrm{csgn}}{4}$

input
```
int(arccoth(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(b*x-2*arccoth(1/tanh(b*x+a)))
```

3.196.5 Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \coth^{-1}(\coth(a+bx))\,dx = \frac{1}{2}x^2b + xa$$

input
```
integrate(arccoth(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*x^2*b + x*a
```

3.196.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(12) = 24.

Time = 1.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.88

$$\int \coth^{-1}(\coth(a+bx))\,dx = \begin{cases} x \operatorname{acoth}(\coth(a)) & \text{for } b=0 \\ x \operatorname{acoth}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right) & \text{for } a=\log\left(-e^{-bx}\right) \\ -\frac{\log(e^{-bx}) \operatorname{acoth}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right)}{b} & \text{for } a=\log\left(e^{-bx}\right) \\ \frac{\operatorname{acoth}^2\left(\frac{1}{\tanh(a+bx)}\right)}{2b} & \text{otherwise} \end{cases}$$

input
```
integrate(acoth(coth(b*x+a)),x)
```

output
```
Piecewise((x*acoth(coth(a)), Eq(b, 0)), (x*acoth(coth(b*x + log(-exp(-b*x)
))), Eq(a, log(-exp(-b*x)))), (-log(exp(-b*x))*acoth(coth(b*x + log(exp(-b
*x))))/b, Eq(a, log(exp(-b*x)))), (acoth(1/tanh(a + b*x))**2/(2*b), True))
```

3.196.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \coth^{-1}(\coth(a+bx))\,dx = \frac{1}{2}\,bx^2 + ax$$

input
```
integrate(arccoth(coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*b*x^2 + a*x
```

3.196.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \coth^{-1}(\coth(a+bx))\,dx = \frac{1}{2}\,bx^2 + ax$$

input
```
integrate(arccoth(coth(b*x+a)),x, algorithm="giac")
```

output
```
1/2*b*x^2 + a*x
```

3.196.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \coth^{-1}(\coth(a+bx))\,dx = x\,\mathrm{acoth}(\coth(a+b\,x)) - \frac{b\,x^2}{2}$$

input
```
int(acoth(coth(a + b*x)),x)
```

output
```
x*acoth(coth(a + b*x)) - (b*x^2)/2
```

3.197 $\int \frac{\coth^{-1}(\coth(a+bx))}{x} dx$

3.197.1 Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x} dx = bx - \left(bx - \coth^{-1}(\coth(a+bx))\right)\log(x)$$

output
```
b*x-(b*x-arccoth(coth(b*x+a)))*ln(x)
```

3.197.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x} dx = bx + \left(-bx + \coth^{-1}(\coth(a+bx))\right)\log(x)$$

input
```
Integrate[ArcCoth[Coth[a + b*x]]/x,x]
```

output
```
b*x + (-(b*x) + ArcCoth[Coth[a + b*x]])*Log[x]
```

3.197.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x} \, dx$$

$$\Big\downarrow \text{2589}$$

$$bx-\left(bx-\coth^{-1}(\coth(a+bx))\right)\int \frac{1}{x}dx$$

$$\Big\downarrow \text{14}$$

$$bx-\log(x)\left(bx-\coth^{-1}(\coth(a+bx))\right)$$

input
```
Int[ArcCoth[Coth[a + b*x]]/x,x]
```

output
```
b*x - (b*x - ArcCoth[Coth[a + b*x]])*Log[x]
```

3.197.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

3.197.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

method	result
default	$\ln(x)\,\mathrm{arccoth}\left(\coth\left(bx+a\right)\right)+b(-x\ln\left(x\right)+x)$
parts	$\ln(x)\,\mathrm{arccoth}\left(\coth\left(bx+a\right)\right)+b(-x\ln\left(x\right)+x)$
risch	$\ln\left(x\right)\ln\left(\mathrm{e}^{bx+a}\right)-b\ln\left(x\right)x+bx-\frac{i\pi\left(\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)^{2}\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)-2\,\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{2}+\mathrm{csgn}\left(i\mathrm{e}^{2bx+2}\right.\right.}{}$

input
```
int(arccoth(coth(b*x+a))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arccoth(coth(b*x+a))+b*(-x*ln(x)+x)
```

3.197.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x}\,dx = bx + a\log(x)$$

input
```
integrate(arccoth(coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
b*x + a*log(x)
```

3.197.6 Sympy [F]

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{acoth}(\coth(a+bx))}{x}\,dx$$

input
```
integrate(acoth(coth(b*x+a))/x,x)
```

output
```
Integral(acoth(coth(a + b*x))/x, x)
```

3.197.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x}\,dx = bx + a\log(x)$$

input
```
integrate(arccoth(coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
b*x + a*log(x)
```

3.197.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.43

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x}\,dx = bx + a\log(|x|)$$

input
```
integrate(arccoth(coth(b*x+a))/x,x, algorithm="giac")
```

output
```
b*x + a*log(abs(x))
```

3.197.9 Mupad [B] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.76

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x}\,dx = b\,x - \frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln(x)}{2} + \frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln(x)}{2} - b\,x\,\ln(x)$$

input
```
int(acoth(coth(a + b*x))/x,x)
```

output
```
b*x - (log(1/(exp(2*a)*exp(2*b*x) + 1))*log(x))/2 + (log((exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))*log(x))/2 - b*x*log(x)
```

3.198 $\int \frac{\coth^{-1}(\coth(a+bx))}{x^2} dx$

3.198.1 Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^2} dx = -\frac{\coth^{-1}(\coth(a+bx))}{x} + b\log(x)$$

output
```
-arccoth(coth(b*x+a))/x+b*ln(x)
```

3.198.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^2} dx = b - \frac{\coth^{-1}(\coth(a+bx))}{x} + b\log(x)$$

input
```
Integrate[ArcCoth[Coth[a + b*x]]/x^2,x]
```

output
```
b - ArcCoth[Coth[a + b*x]]/x + b*Log[x]
```

3.198.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^2} \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$b\int \frac{1}{x}dx-\frac{\coth^{-1}(\coth(a+bx))}{x}$$

$$\Big\downarrow \text{ 14}$$

$$b\log(x)-\frac{\coth^{-1}(\coth(a+bx))}{x}$$

input
```
Int[ArcCoth[Coth[a + b*x]]/x^2,x]
```

output
```
-(ArcCoth[Coth[a + b*x]]/x) + b*Log[x]
```

3.198.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.198.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

method	result
default	$-\frac{\operatorname{arccoth}(\coth(bx+a))}{x}+b\ln(-bx)$
parts	$-\frac{\operatorname{arccoth}(\coth(bx+a))}{x}+b\ln(-bx)$
parallelrisch	$\frac{b\ln(x)x-\operatorname{arccoth}\left(\frac{1}{\tanh(bx+a)}\right)}{x}$
risch	$-\frac{\ln(\mathrm{e}^{bx+a})}{x}+\frac{i\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)^{3}+i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{3}-i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)^{2}+i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)^{2}\mathrm{csgn}(}{}$

input
```
int(arccoth(coth(b*x+a))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccoth(coth(b*x+a))/x+b*ln(-b*x)
```

3.198.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^2}\,dx=\frac{bx\log(x)-a}{x}$$

input
```
integrate(arccoth(coth(b*x+a))/x^2,x, algorithm="fricas")
```

output
```
(b*x*log(x) - a)/x
```

3.198.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(14) = 28.

Time = 2.61 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.00

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^2}\,dx=\begin{cases}-\frac{\operatorname{acoth}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{x} & \text{for}\ a=\log\left(-e^{-bx}\right)\\ -\frac{\operatorname{acoth}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right)}{x} & \text{for}\ a=\log\left(e^{-bx}\right)\\ b\log(x)-\frac{\operatorname{acoth}\left(\frac{1}{\tanh(a+bx)}\right)}{x} & \text{otherwise}\end{cases}$$

input
```
integrate(acoth(coth(b*x+a))/x**2,x)
```

output
```
Piecewise((-acoth(coth(b*x + log(-exp(-b*x))))/x, Eq(a, log(-exp(-b*x)))),
 (-acoth(coth(b*x + log(exp(-b*x))))/x, Eq(a, log(exp(-b*x)))), (b*log(x)
- acoth(1/tanh(a + b*x))/x, True))
```

3.198.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^2}\,dx = b\log(x) - \frac{a}{x}$$

input
```
integrate(arccoth(coth(b*x+a))/x^2,x, algorithm="maxima")
```

output
```
b*log(x) - a/x
```

3.198.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^2}\,dx = b\log(|x|) - \frac{a}{x}$$

input
```
integrate(arccoth(coth(b*x+a))/x^2,x, algorithm="giac")
```

output
```
b*log(abs(x)) - a/x
```

3.198.9 Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^2}\,dx = b\,\ln(x) - \frac{\mathrm{acoth}(\coth(a+b\,x))}{x}$$

input
```
int(acoth(coth(a + b*x))/x^2,x)
```

output
```
b*log(x) - acoth(coth(a + b*x))/x
```

3.198. $\int \frac{\coth^{-1}(\coth(a+bx))}{x^2}\,dx$

3.199 $\int \frac{\coth^{-1}(\coth(a+bx))}{x^3} dx$

3.199.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^3} dx = -\frac{b}{2x} - \frac{\coth^{-1}(\coth(a+bx))}{2x^2}$$

output
```
-1/2*b/x-1/2*arccoth(coth(b*x+a))/x^2
```

3.199.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^3} dx = -\frac{bx + \coth^{-1}(\coth(a+bx))}{2x^2}$$

input
```
Integrate[ArcCoth[Coth[a + b*x]]/x^3,x]
```

output
```
-1/2*(b*x + ArcCoth[Coth[a + b*x]])/x^2
```

3.199.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^3} \, dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{2}b\int \frac{1}{x^2}dx-\frac{\coth^{-1}(\coth(a+bx))}{2x^2}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\coth^{-1}(\coth(a+bx))}{2x^2}-\frac{b}{2x}$$

input
```
Int[ArcCoth[Coth[a + b*x]]/x^3,x]
```

output
```
-1/2*b/x - ArcCoth[Coth[a + b*x]]/(2*x^2)
```

3.199.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.199.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{b}{2x}-\frac{\operatorname{arccoth}(\coth(bx+a))}{2x^2}$
parts	$-\frac{b}{2x}-\frac{\operatorname{arccoth}(\coth(bx+a))}{2x^2}$
parallelrisch	$\frac{-bx-\operatorname{arccoth}\left(\frac{1}{\tanh(bx+a)}\right)}{2x^2}$
risch	$-\frac{\ln(e^{bx+a})}{2x^2}-\frac{4bx+i\pi\operatorname{csgn}(ie^{2bx+2a})\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}-1}\right)^2-i\pi\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}-1}\right)^3-i\pi\operatorname{csgn}(ie^{2bx+2a})^3+i\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}-\cdots}\right.}{}$

input
```
int(arccoth(coth(b*x+a))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/x-1/2*arccoth(coth(b*x+a))/x^2
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^3}\,dx = -\frac{2\,bx+a}{2\,x^2}$$

input
```
integrate(arccoth(coth(b*x+a))/x^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b*x + a)/x^2
```

3.199.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(19) = 38.

Time = 4.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.48

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^3}\,dx = \begin{cases} -\frac{\operatorname{acoth}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{2x^2} & \text{for}\ a=\log\left(-e^{-bx}\right) \\ -\frac{\operatorname{acoth}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right)}{2x^2} & \text{for}\ a=\log\left(e^{-bx}\right) \\ -\frac{b}{2x}-\frac{\operatorname{acoth}\left(\frac{1}{\tanh(a+bx)}\right)}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(acoth(coth(b*x+a))/x**3,x)
```

output
```
Piecewise((-acoth(coth(b*x + log(-exp(-b*x))))/(2*x**2), Eq(a, log(-exp(-b
*x)))), (-acoth(coth(b*x + log(exp(-b*x))))/(2*x**2), Eq(a, log(exp(-b*x))
)), (-b/(2*x) - acoth(1/tanh(a + b*x))/(2*x**2), True))
```

3.199.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^3}\,dx = -\frac{2\,bx+a}{2\,x^2}$$

input
```
integrate(arccoth(coth(b*x+a))/x^3,x, algorithm="maxima")
```

output
```
-1/2*(2*b*x + a)/x^2
```

3.199.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^3}\,dx = -\frac{2\,bx+a}{2\,x^2}$$

input
```
integrate(arccoth(coth(b*x+a))/x^3,x, algorithm="giac")
```

output
```
-1/2*(2*b*x + a)/x^2
```

3.199.9 Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\coth^{-1}(\coth(a+bx))}{x^3}\,dx = -\frac{\mathrm{acoth}(\coth(a+b\,x))+b\,x}{2\,x^2}$$

input
```
int(acoth(coth(a + b*x))/x^3,x)
```

output
```
-(acoth(coth(a + b*x)) + b*x)/(2*x^2)
```

3.199. $\int \frac{\coth^{-1}(\coth(a+bx))}{x^3}\,dx$

3.200 $\int \coth^{-1}(\cosh(x))\,dx$

3.200.1 Optimal result

Integrand size = 3, antiderivative size = 27

$$\int \coth^{-1}(\cosh(x))\,dx = x\coth^{-1}(\cosh(x)) - 2x\text{arctanh}(e^x) - \text{PolyLog}\left(2,-e^x\right) + \text{PolyLog}\left(2,e^x\right)$$

output
```
x*arccoth(cosh(x))-2*x*arctanh(exp(x))-polylog(2,-exp(x))+polylog(2,exp(x)
)
```

3.200.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \coth^{-1}(\cosh(x))\,dx = x\coth^{-1}(\cosh(x)) + x(\log\left(1-e^x\right) - \log\left(1+e^x\right)) - \text{PolyLog}\left(2,-e^x\right) + \text{PolyLog}\left(2,e^x\right)$$

input
```
Integrate[ArcCoth[Cosh[x]],x]
```

output
```
x*ArcCoth[Cosh[x]] + x*(Log[1 - E^x] - Log[1 + E^x]) - PolyLog[2, -E^x] +
PolyLog[2, E^x]
```

3.200.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.333, Rules used = {6826, 25, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(\cosh(x))\,dx$$

$$\Big\downarrow \text{6826}$$

$$x\coth^{-1}(\cosh(x)) - \int -x\text{csch}(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int x\text{csch}(x)dx + x\coth^{-1}(\cosh(x))$$

$$\Big\downarrow \text{3042}$$

$$x\coth^{-1}(\cosh(x)) + \int ix\csc(ix)dx$$

$$\Big\downarrow \text{26}$$

$$x\coth^{-1}(\cosh(x)) + i\int x\csc(ix)dx$$

$$\Big\downarrow \text{4670}$$

$$x\coth^{-1}(\cosh(x)) + i\left(i\int \log(1-e^x)\,dx - i\int \log(1+e^x)\,dx + 2ix\text{arctanh}(e^x)\right)$$

$$\Big\downarrow \text{2715}$$

$$x\coth^{-1}(\cosh(x)) + i\left(i\int e^{-x}\log(1-e^x)\,de^x - i\int e^{-x}\log(1+e^x)\,de^x + 2ix\text{arctanh}(e^x)\right)$$

$$\Big\downarrow \text{2838}$$

$$x\coth^{-1}(\cosh(x)) + i(2ix\text{arctanh}(e^x) + i\,\text{PolyLog}(2,-e^x) - i\,\text{PolyLog}(2,e^x))$$

input
```
Int[ArcCoth[Cosh[x]],x]
```

output
```
x*ArcCoth[Cosh[x]] + I*((2*I)*x*ArcTanh[E^x] + I*PolyLog[2, -E^x] - I*Poly
Log[2, E^x])
```

3.200.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6826
```
Int[ArcCoth[u_], x_Symbol] :> Simp[x*ArcCoth[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

3.200.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

method	result
default	$x\ \mathrm{arccoth}\left(\cosh\left(x\right)\right)+x\ln\left(1-\mathrm{e}^{x}\right)+\mathrm{polylog}\left(2,\mathrm{e}^{x}\right)-x\ln\left(1+\mathrm{e}^{x}\right)-\mathrm{polylog}\left(2,-\mathrm{e}^{x}\right)$
parts	$x\ \mathrm{arccoth}\left(\cosh\left(x\right)\right)+x\ln\left(1-\mathrm{e}^{x}\right)+\mathrm{polylog}\left(2,\mathrm{e}^{x}\right)-x\ln\left(1+\mathrm{e}^{x}\right)-\mathrm{polylog}\left(2,-\mathrm{e}^{x}\right)$
risch	$-\frac{i\pi\ \mathrm{csgn}\left(i\left(\mathrm{e}^{x}-1\right)^{2}\right)\mathrm{csgn}\left(i\mathrm{e}^{-x}\left(\mathrm{e}^{x}-1\right)^{2}\right)^{2}x}{4}-\frac{i\pi\ \mathrm{csgn}\left(i\mathrm{e}^{-x}\right)\mathrm{csgn}\left(i\mathrm{e}^{-x}\left(\mathrm{e}^{x}-1\right)^{2}\right)^{2}x}{4}+\frac{i\pi\mathrm{csgn}\left(i\left(\mathrm{e}^{x}-1\right)^{2}\right)^{3}x}{4}-x\ln\left(\mathrm{e}^{x}-\right.$

input
```
int(arccoth(cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
x*arccoth(cosh(x))+x*ln(1-exp(x))+polylog(2,exp(x))-x*ln(1+exp(x))-polylog
(2,-exp(x))
```

3.200.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(22) = 44.

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int \coth^{-1}(\cosh(x))\,dx = \frac{1}{2}x\log\left(\frac{\cosh\left(x\right)+1}{\cosh\left(x\right)-1}\right)-x\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right) + x\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right) + \mathrm{Li}_2(\cosh\left(x\right)+\sinh\left(x\right))-\mathrm{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right))$$

input
```
integrate(arccoth(cosh(x)),x, algorithm="fricas")
```

output
```
1/2*x*log((cosh(x) + 1)/(cosh(x) - 1)) - x*log(cosh(x) + sinh(x) + 1) + x*
log(-cosh(x) - sinh(x) + 1) + dilog(cosh(x) + sinh(x)) - dilog(-cosh(x) -
sinh(x))
```

3.200.6 Sympy [F]

$$\int \coth^{-1}(\cosh(x))\,dx = \int \operatorname{acoth}(\cosh(x))\,dx$$

input
```
integrate(acoth(cosh(x)),x)
```

output
```
Integral(acoth(cosh(x)), x)
```

3.200.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \coth^{-1}(\cosh(x))\,dx = x\,\operatorname{arcoth}(\cosh(x)) - x\log(e^x + 1) + x\log(-e^x + 1) - \operatorname{Li}_2(-e^x) + \operatorname{Li}_2(e^x)$$

input
```
integrate(arccoth(cosh(x)),x, algorithm="maxima")
```

output
```
x*arccoth(cosh(x)) - x*log(e^x + 1) + x*log(-e^x + 1) - dilog(-e^x) + dilo
g(e^x)
```

3.200.8 Giac [F]

$$\int \coth^{-1}(\cosh(x))\,dx = \int \operatorname{arcoth}(\cosh(x))\,dx$$

input
```
integrate(arccoth(cosh(x)),x, algorithm="giac")
```

output
```
integrate(arccoth(cosh(x)), x)
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(\cosh(x))\,dx = \int \operatorname{acoth}(\cosh(x))\,dx$$

input
```
int(acoth(cosh(x)),x)
```

output
```
int(acoth(cosh(x)), x)
```

3.201 $\int x \coth^{-1}(\cosh(x))\,dx$

3.201.1 Optimal result

Integrand size = 5, antiderivative size = 51

$$\int x \coth^{-1}(\cosh(x))\,dx = \frac{1}{2}x^2\coth^{-1}(\cosh(x)) - x^2\text{arctanh}(e^x) - x\,\text{PolyLog}(2,-e^x) + x\,\text{PolyLog}(2,e^x) + \text{PolyLog}(3,-e^x) - \text{PolyLog}(3,e^x)$$

output
```
1/2*x^2*arccoth(cosh(x))-x^2*arctanh(exp(x))-x*polylog(2,-exp(x))+x*polylo
g(2,exp(x))+polylog(3,-exp(x))-polylog(3,exp(x))
```

3.201.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int x \coth^{-1}(\cosh(x))\,dx = \frac{1}{2}x^2\coth^{-1}(\cosh(x)) + \frac{1}{2}x^2\log(1-e^x) - \frac{1}{2}x^2\log(1+e^x) - x\,\text{PolyLog}(2,-e^x) + x\,\text{PolyLog}(2,e^x) + \text{PolyLog}(3,-e^x) - \text{PolyLog}(3,e^x)$$

input
```
Integrate[x*ArcCoth[Cosh[x]],x]
```

output
```
(x^2*ArcCoth[Cosh[x]])/2 + (x^2*Log[1 - E^x])/2 - (x^2*Log[1 + E^x])/2 - x
*PolyLog[2, -E^x] + x*PolyLog[2, E^x] + PolyLog[3, -E^x] - PolyLog[3, E^x]
```

3.201.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.600, Rules used = {6828, 25, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(\cosh(x))\,dx$$

$$\Big\downarrow \text{ 6828}$$

$$\frac{1}{2}x^2 \coth^{-1}(\cosh(x)) - \frac{1}{2}\int -x^2\text{csch}(x)dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\int x^2\text{csch}(x)dx + \frac{1}{2}x^2 \coth^{-1}(\cosh(x))$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}x^2 \coth^{-1}(\cosh(x)) + \frac{1}{2}\int ix^2 \csc(ix)dx$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{2}x^2 \coth^{-1}(\cosh(x)) + \frac{1}{2}i\int x^2 \csc(ix)dx$$

$$\Big\downarrow \text{ 4670}$$

$$\frac{1}{2}x^2 \coth^{-1}(\cosh(x)) + \frac{1}{2}i\left(2i\int x \log(1-e^x)\,dx - 2i\int x \log(1+e^x)\,dx + 2ix^2\text{arctanh}(e^x)\right)$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{1}{2}x^2 \coth^{-1}(\cosh(x)) + \frac{1}{2}i\left(-2i\left(\int \text{PolyLog}(2,-e^x)\,dx - x\,\text{PolyLog}(2,-e^x)\right) + 2i\left(\int \text{PolyLog}(2,e^x)\,dx - x\,\text{PolyLog}(2,e^x)\right) + 2ix^2\text{ar}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{2}x^2 \coth^{-1}(\cosh(x)) + \frac{1}{2}i\left(-2i\left(\int e^{-x}\,\text{PolyLog}(2,-e^x)\,de^x - x\,\text{PolyLog}(2,-e^x)\right) + 2i\left(\int e^{-x}\,\text{PolyLog}(2,e^x)\,de^x - x\,\text{PolyLog}(2,e^x)\right)$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{1}{2}x^2\coth^{-1}(\cosh(x))+$$
$$\frac{1}{2}i\left(2ix^2\text{arctanh}(e^x)-2i(\text{PolyLog}\left(3,-e^x\right)-x\,\text{PolyLog}\left(2,-e^x\right))+2i(\text{PolyLog}\left(3,e^x\right)-x\,\text{PolyLog}\left(2,e^x\right))\right)$$

input
```
Int[x*ArcCoth[Cosh[x]],x]
```

output
```
(x^2*ArcCoth[Cosh[x]])/2 + (I/2)*((2*I)*x^2*ArcTanh[E^x] - (2*I)*(-(x*Poly
Log[2, -E^x]) + PolyLog[3, -E^x]) + (2*I)*(-(x*PolyLog[2, E^x]) + PolyLog[
3, E^x]))
```

3.201.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6828
```
Int[((a_.) + ArcCoth[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCoth[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x],
x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
 &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
 + 1, x]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.201.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 379, normalized size of antiderivative = 7.43

method	result
risch	$-\frac{x^2 \ln(e^x-1)}{2} - x \operatorname{polylog}(2, -e^x) + \operatorname{polylog}(3, -e^x) - \frac{i\pi\left(\operatorname{csgn}(i(1+e^x))^2 \operatorname{csgn}\left(i(1+e^x)^2\right) - 2 \operatorname{csgn}(i(1+e^x))\operatorname{csgn}\right.}{}$

input
```
int(x*arccoth(cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2*ln(exp(x)-1)-x*polylog(2,-exp(x))+polylog(3,-exp(x))-1/8*I*Pi*(cs
gn(I*(1+exp(x)))^2*csgn(I*(1+exp(x))^2)-2*csgn(I*(1+exp(x)))*csgn(I*(1+exp
(x))^2)^2+csgn(I*(1+exp(x))^2)^3+csgn(I*(1+exp(x))^2)*csgn(I*exp(-x))*csgn
(I*exp(-x)*(1+exp(x))^2)-csgn(I*(1+exp(x))^2)*csgn(I*exp(-x)*(1+exp(x))^2)
^2-csgn(I*(exp(x)-1))^2*csgn(I*(exp(x)-1)^2)+2*csgn(I*(exp(x)-1))*csgn(I*(
exp(x)-1)^2)^2-csgn(I*(exp(x)-1)^2)^3-csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x))
*csgn(I*exp(-x)*(exp(x)-1)^2)+csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x)*(exp(x)-
1)^2)^2-csgn(I*exp(-x))*csgn(I*exp(-x)*(1+exp(x))^2)^2+csgn(I*exp(-x))*csg
n(I*exp(-x)*(exp(x)-1)^2)^2+csgn(I*exp(-x)*(1+exp(x))^2)^3-csgn(I*exp(-x)*
(exp(x)-1)^2)^3)*x^2+1/2*x^2*ln(1-exp(x))+x*polylog(2,exp(x))-polylog(3,ex
p(x))
```

3.201.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(42) = 84$.

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int x\coth^{-1}(\cosh(x))\,dx = \frac{1}{4}\,x^2\log\left(\frac{\cosh(x)+1}{\cosh(x)-1}\right) - \frac{1}{2}\,x^2\log(\cosh(x)+\sinh(x)+1) + \frac{1}{2}\,x^2\log(-\cosh(x)-\sinh(x)+1) + x\mathrm{Li}_2(\cosh(x)+\sinh(x)) - x\mathrm{Li}_2(-\cosh(x)-\sinh(x)) - \mathrm{polylog}(3,\cosh(x)+\sinh(x)) + \mathrm{polylog}(3,-\cosh(x)-\sinh(x))$$

input
```
integrate(x*arccoth(cosh(x)),x, algorithm="fricas")
```

output
```
1/4*x^2*log((cosh(x) + 1)/(cosh(x) - 1)) - 1/2*x^2*log(cosh(x) + sinh(x) +
 1) + 1/2*x^2*log(-cosh(x) - sinh(x) + 1) + x*dilog(cosh(x) + sinh(x)) - x
*dilog(-cosh(x) - sinh(x)) - polylog(3, cosh(x) + sinh(x)) + polylog(3, -c
osh(x) - sinh(x))
```

3.201.6 Sympy [F]

$$\int x \coth^{-1}(\cosh(x))\,dx = \int x \operatorname{acoth}(\cosh(x))\,dx$$

input
```
integrate(x*acoth(cosh(x)),x)
```

output
```
Integral(x*acoth(cosh(x)), x)
```

3.201.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int x \coth^{-1}(\cosh(x))\,dx = \frac{1}{2}x^2 \operatorname{arcoth}(\cosh(x)) - \frac{1}{2}x^2 \log(e^x+1) + \frac{1}{2}x^2 \log(-e^x+1) - x\mathrm{Li}_2(-e^x) + x\mathrm{Li}_2(e^x) + \mathrm{Li}_3(-e^x) - \mathrm{Li}_3(e^x)$$

input
```
integrate(x*arccoth(cosh(x)),x, algorithm="maxima")
```

output
```
1/2*x^2*arccoth(cosh(x)) - 1/2*x^2*log(e^x + 1) + 1/2*x^2*log(-e^x + 1) -
x*dilog(-e^x) + x*dilog(e^x) + polylog(3, -e^x) - polylog(3, e^x)
```

3.201.8 Giac [F]

$$\int x \coth^{-1}(\cosh(x))\,dx = \int x \operatorname{arcoth}(\cosh(x))\,dx$$

input
```
integrate(x*arccoth(cosh(x)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(cosh(x)), x)
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(\cosh(x))\,dx = \int x\,\mathrm{acoth}(\cosh(x))\,\,dx$$

input
```
int(x*acoth(cosh(x)),x)
```

output
```
int(x*acoth(cosh(x)), x)
```

3.202 $\int x^2 \coth^{-1}(\cosh(x))\,dx$

3.202.1 Optimal result

Integrand size = 7, antiderivative size = 77

$$\int x^2 \coth^{-1}(\cosh(x))\,dx = \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) - \frac{2}{3}x^3\text{arctanh}(e^x) - x^2\,\text{PolyLog}(2,-e^x) + x^2\,\text{PolyLog}(2,e^x) + 2x\,\text{PolyLog}(3,-e^x) - 2x\,\text{PolyLog}(3,e^x) - 2\,\text{PolyLog}(4,-e^x) + 2\,\text{PolyLog}(4,e^x)$$

output
```
1/3*x^3*arccoth(cosh(x))-2/3*x^3*arctanh(exp(x))-x^2*polylog(2,-exp(x))+x^
2*polylog(2,exp(x))+2*x*polylog(3,-exp(x))-2*x*polylog(3,exp(x))-2*polylog
(4,-exp(x))+2*polylog(4,exp(x))
```

3.202.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18

$$\int x^2 \coth^{-1}(\cosh(x))\,dx = \frac{1}{3}\big(x^3 \coth^{-1}(\cosh(x)) + x^3\log(1-e^x) - x^3\log(1+e^x) - 3x^2\,\text{PolyLog}(2,-e^x) + 3x^2\,\text{PolyLog}(2,e^x) + 6x\,\text{PolyLog}(3,-e^x) - 6x\,\text{PolyLog}(3,e^x) - 6\,\text{PolyLog}(4,-e^x) + 6\,\text{PolyLog}(4,e^x)\big)$$

input
```
Integrate[x^2*ArcCoth[Cosh[x]],x]
```

output
```
(x^3*ArcCoth[Cosh[x]] + x^3*Log[1 - E^x] - x^3*Log[1 + E^x] - 3*x^2*PolyLo
g[2, -E^x] + 3*x^2*PolyLog[2, E^x] + 6*x*PolyLog[3, -E^x] - 6*x*PolyLog[3,
 E^x] - 6*PolyLog[4, -E^x] + 6*PolyLog[4, E^x])/3
```

3.202.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.29, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {6828, 25, 3042, 26, 4670, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(\cosh(x))\,dx$$

$$\Big\downarrow \text{6828}$$

$$\frac{1}{3}x^3\coth^{-1}(\cosh(x))-\frac{1}{3}\int -x^3\text{csch}(x)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int x^3\text{csch}(x)dx+\frac{1}{3}x^3\coth^{-1}(\cosh(x))$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}x^3\coth^{-1}(\cosh(x))+\frac{1}{3}\int ix^3\csc(ix)dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{3}x^3\coth^{-1}(\cosh(x))+\frac{1}{3}i\int x^3\csc(ix)dx$$

$$\Big\downarrow \text{4670}$$

$$\frac{1}{3}x^3\coth^{-1}(\cosh(x))+\frac{1}{3}i\left(3i\int x^2\log\left(1-e^x\right)dx-3i\int x^2\log\left(1+e^x\right)dx+2ix^3\text{arctanh}(e^x)\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}x^3\coth^{-1}(\cosh(x))+$$
$$\frac{1}{3}i\left(-3i\left(2\int x\,\text{PolyLog}\left(2,-e^x\right)dx-x^2\,\text{PolyLog}\left(2,-e^x\right)\right)+3i\left(2\int x\,\text{PolyLog}\left(2,e^x\right)dx-x^2\,\text{PolyLog}\left(2,e^x\right)\right)\right)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3}x^3\coth^{-1}(\cosh(x))+ \frac{1}{3}i\left(-3i\left(2\left(x\,\text{PolyLog}\left(3,-e^x\right)-\int \text{PolyLog}\left(3,-e^x\right)dx\right)-x^2\,\text{PolyLog}\left(2,-e^x\right)\right)+3i\left(2\left(x\,\text{PolyLog}\left(3,e^x\right)-\int\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3}x^3\coth^{-1}(\cosh(x))+ \frac{1}{3}i\left(-3i\left(2\left(x\,\text{PolyLog}\left(3,-e^x\right)-\int e^{-x}\,\text{PolyLog}\left(3,-e^x\right)de^x\right)-x^2\,\text{PolyLog}\left(2,-e^x\right)\right)+3i\left(2\left(x\,\text{PolyLog}\left(3,e^x\right.\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{3}x^3\coth^{-1}(\cosh(x))+ \frac{1}{3}i(2ix^3\text{arctanh}(e^x)-3i(2(x\,\text{PolyLog}\left(3,-e^x\right)-\text{PolyLog}\left(4,-e^x\right))-x^2\,\text{PolyLog}\left(2,-e^x\right))+3i(2(x\,\text{PolyLog}\left(3,e\right.$$

input
```
Int[x^2*ArcCoth[Cosh[x]],x]
```

output
```
(x^3*ArcCoth[Cosh[x]])/3 + (I/3)*((2*I)*x^3*ArcTanh[E^x] - (3*I)*(-(x^2*Po
lyLog[2, -E^x]) + 2*(x*PolyLog[3, -E^x] - PolyLog[4, -E^x])) + (3*I)*(-(x^
2*PolyLog[2, E^x]) + 2*(x*PolyLog[3, E^x] - PolyLog[4, E^x])))
```

3.202.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6828
```
Int[((a_.) + ArcCoth[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCoth[u])/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x],
x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
 &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
 + 1, x]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.202. $\int x^2 \coth^{-1}(\cosh(x))\,dx$

3.202.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 401, normalized size of antiderivative = 5.21

method	result
risch	$-\frac{x^3 \ln(e^x-1)}{3} - x^2 \operatorname{polylog}(2, -e^x) + 2x \operatorname{polylog}(3, -e^x) - 2 \operatorname{polylog}(4, -e^x) - \frac{i\pi\left(\operatorname{csgn}(i(1+e^x))^2 \operatorname{csgn}\right.}{}$

input
```
int(x^2*arccoth(cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3*ln(exp(x)-1)-x^2*polylog(2,-exp(x))+2*x*polylog(3,-exp(x))-2*poly
log(4,-exp(x))-1/12*I*Pi*(csgn(I*(1+exp(x)))^2*csgn(I*(1+exp(x))^2)-2*csgn
(I*(1+exp(x)))*csgn(I*(1+exp(x))^2)^2+csgn(I*(1+exp(x))^2)^3+csgn(I*(1+exp
(x))^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(1+exp(x))^2)-csgn(I*(1+exp(x))^2)*
csgn(I*exp(-x)*(1+exp(x))^2)^2-csgn(I*(exp(x)-1))^2*csgn(I*(exp(x)-1)^2)+2
*csgn(I*(exp(x)-1))*csgn(I*(exp(x)-1)^2)^2-csgn(I*(exp(x)-1)^2)^3-csgn(I*(
exp(x)-1)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)+csgn(I*(exp(x)-1
)^2)*csgn(I*exp(-x)*(exp(x)-1)^2)^2-csgn(I*exp(-x))*csgn(I*exp(-x)*(1+exp(
x))^2)^2+csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)^2+csgn(I*exp(-x)*(1+
exp(x))^2)^3-csgn(I*exp(-x)*(exp(x)-1)^2)^3)*x^3+1/3*x^3*ln(1-exp(x))+x^2*
polylog(2,exp(x))-2*x*polylog(3,exp(x))+2*polylog(4,exp(x))
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.52

$$\begin{aligned}\int x^2 \coth^{-1}(\cosh(x))\,dx = {} & \frac{1}{6}\,x^3 \log\left(\frac{\cosh(x)+1}{\cosh(x)-1}\right) - \frac{1}{3}\,x^3 \log(\cosh(x)+\sinh(x)+1) \\ & + \frac{1}{3}\,x^3 \log(-\cosh(x)-\sinh(x)+1) + x^2 \mathrm{Li}_2(\cosh(x)+\sinh(x)) \\ & - x^2 \mathrm{Li}_2(-\cosh(x)-\sinh(x)) - 2\,x\mathrm{polylog}(3, \cosh(x)+\sinh(x)) \\ & + 2\,x\mathrm{polylog}(3, -\cosh(x)-\sinh(x)) \\ & + 2\,\mathrm{polylog}(4, \cosh(x)+\sinh(x)) \\ & - 2\,\mathrm{polylog}(4, -\cosh(x)-\sinh(x))\end{aligned}$$

input
```
integrate(x^2*arccoth(cosh(x)),x, algorithm="fricas")
```

output
```
1/6*x^3*log((cosh(x) + 1)/(cosh(x) - 1)) - 1/3*x^3*log(cosh(x) + sinh(x) +
 1) + 1/3*x^3*log(-cosh(x) - sinh(x) + 1) + x^2*dilog(cosh(x) + sinh(x)) -
 x^2*dilog(-cosh(x) - sinh(x)) - 2*x*polylog(3, cosh(x) + sinh(x)) + 2*x*p
olylog(3, -cosh(x) - sinh(x)) + 2*polylog(4, cosh(x) + sinh(x)) - 2*polylo
g(4, -cosh(x) - sinh(x))
```

3.202.6 Sympy [F]

$$\int x^2 \coth^{-1}(\cosh(x))\,dx = \int x^2 \operatorname{acoth}(\cosh(x))\,dx$$

input
```
integrate(x**2*acoth(cosh(x)),x)
```

output
```
Integral(x**2*acoth(cosh(x)), x)
```

3.202.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\begin{aligned}\int x^2 \coth^{-1}(\cosh(x))\,dx =& \frac{1}{3}\,x^3 \operatorname{arcoth}(\cosh(x)) - \frac{1}{3}\,x^3 \log(e^x+1) \\ &+ \frac{1}{3}\,x^3 \log(-e^x+1) - x^2\mathrm{Li}_2(-e^x) + x^2\mathrm{Li}_2(e^x) \\ &+ 2\,x\mathrm{Li}_3(-e^x) - 2\,x\mathrm{Li}_3(e^x) - 2\,\mathrm{Li}_4(-e^x) + 2\,\mathrm{Li}_4(e^x)\end{aligned}$$

input
```
integrate(x^2*arccoth(cosh(x)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(cosh(x)) - 1/3*x^3*log(e^x + 1) + 1/3*x^3*log(-e^x + 1) -
x^2*dilog(-e^x) + x^2*dilog(e^x) + 2*x*polylog(3, -e^x) - 2*x*polylog(3, e
^x) - 2*polylog(4, -e^x) + 2*polylog(4, e^x)
```

3.202.8 Giac [F]

$$\int x^2 \coth^{-1}(\cosh(x))\,dx = \int x^2 \operatorname{arcoth}(\cosh(x))\,dx$$

input
```
integrate(x^2*arccoth(cosh(x)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(cosh(x)), x)
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(\cosh(x))\,dx = \int x^2 \operatorname{acoth}(\cosh(x))\,dx$$

input
```
int(x^2*acoth(cosh(x)),x)
```

output
```
int(x^2*acoth(cosh(x)), x)
```

3.203 $\int x^2 \coth^{-1}(c+d\tanh(a+bx))\,dx$

3.203.1 Optimal result

Integrand size = 15, antiderivative size = 307

$$\begin{aligned}\int x^2\coth^{-1}(c+d\tanh(a+bx))\,dx &= \frac{1}{3}x^3\coth^{-1}(c+d\tanh(a+bx)) \\ &+\frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) \\ &-\frac{1}{6}x^3\log\left(1+\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right) \\ &+\frac{x^2\,\text{PolyLog}\left(2,-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b} \\ &-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b} \\ &-\frac{x\,\text{PolyLog}\left(3,-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b^2} \\ &+\frac{x\,\text{PolyLog}\left(3,-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b^2} \\ &+\frac{\text{PolyLog}\left(4,-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{8b^3} \\ &-\frac{\text{PolyLog}\left(4,-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arccoth(c+d*tanh(b*x+a))+1/6*x^3*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+
d))-1/6*x^3*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x^2*polylog(2,-(1-c-d
)*exp(2*b*x+2*a)/(1-c+d))/b-1/4*x^2*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c
-d))/b-1/4*x*polylog(3,-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/4*x*polylog(
3,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^2+1/8*polylog(4,-(1-c-d)*exp(2*b*x+2*
a)/(1-c+d))/b^3-1/8*polylog(4,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^3
```

3.203.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.86

$$\int x^2 \coth^{-1}(c+d\tanh(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(c+d\tanh(a+bx)) + \frac{4b^3x^3\log\left(1+\frac{(-1+c-d)e^{-2(a+bx)}}{-1+c+d}\right) - 4b^3x^3\log\left(1+\frac{(1+c-d)e^{-2(a+bx)}}{1+c+d}\right) - 6b^2x^2\operatorname{PolyLog}\left(2,\frac{(1-c+d)e^{-2(a+bx)}}{-1+c+d}\right)}{}$$

input
```
Integrate[x^2*ArcCoth[c + d*Tanh[a + b*x]],x]
```

output
```
(x^3*ArcCoth[c + d*Tanh[a + b*x]])/3 + (4*b^3*x^3*Log[1 + (-1 + c - d)/((-
1 + c + d)*E^(2*(a + b*x)))] - 4*b^3*x^3*Log[1 + (1 + c - d)/((1 + c + d)*
E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (1 - c + d)/((-1 + c + d)*E^(2*(a
 + b*x)))] + 6*b^2*x^2*PolyLog[2, (-1 - c + d)/((1 + c + d)*E^(2*(a + b*x)
))] - 6*b*x*PolyLog[3, (1 - c + d)/((-1 + c + d)*E^(2*(a + b*x)))] + 6*b*x
*PolyLog[3, (-1 - c + d)/((1 + c + d)*E^(2*(a + b*x)))] - 3*PolyLog[4, (1
- c + d)/((-1 + c + d)*E^(2*(a + b*x)))] + 3*PolyLog[4, (-1 - c + d)/((1 +
 c + d)*E^(2*(a + b*x)))])/(24*b^3)
```

3.203.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6798, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(d\tanh(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6798}$$

$$\frac{1}{3}b(-c-d+1)\int \frac{e^{2a+2bx}x^3}{-c+(-c-d+1)e^{2a+2bx}+d+1}dx-\frac{1}{3}b(c+d+1)\int \frac{e^{2a+2bx}x^3}{c+(c+d+1)e^{2a+2bx}-d+1}dx+\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}b(-c-d+1)\left(\frac{x^3\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{3\int x^2\log\left(\frac{e^{2a+2bx}(-c-d+1)}{-c+d+1}+1\right)dx}{2b(-c-d+1)}\right)-\frac{1}{3}b(c+d+1)\left(\frac{x^3\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{3\int x^2\log\left(\frac{e^{2a+2bx}(c+d+1)}{c-d+1}+1\right)dx}{2b(c+d+1)}\right)+\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}b(-c-d+1)\left(\frac{x^3\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}\right)}{2b(-c-d+1)}\right)-\frac{1}{3}b(c+d+1)\left(\frac{x^3\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}\right)}{2b(c+d+1)}\right)+\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{7163}$$

3.203. $\int x^2\coth^{-1}(c+d\tanh(a+bx))\,dx$

$$1)\left(\frac{x^3\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(-}{}\right.}{2b}\right.}{2b(-c-d+1)}\right.\frac{1}{3}b(-c-d+$$

$$1)\left(\frac{x^3\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(c+d+1)}{c-}\right.}{2b}\right.}{2b(c+d+1)}\right.\frac{1}{3}b(c+d+$$

$$\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+c)$$

$$\downarrow \text{2720}$$

$$1)\left(\frac{x^3\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2}{}\right.}{2b(-c-d+1)}\right.\frac{1}{3}b(-c-d+$$

$$1)\left(\frac{x^3\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyL}}{}\right.}{2b(c+d+1)}\right.\frac{1}{3}b(c+d+$$

$$\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+c)$$

$$\downarrow \text{7143}$$

3.203. $\int x^2\coth^{-1}(c+d\tanh(a+bx))\,dx$

$$1)\left(\frac{x^3\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\text{PolyLog}\left(4,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(-c-d\cdots}{-\cdots}\right.}{2b}\right.}{2b(-c-d+1)}\right.$$

$$\frac{1}{3}b(-c-d+$$

$$1)\left(\frac{x^3\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\text{PolyLog}\left(4,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a\cdots}}{c-d+1}\right.}{2b}\right.}{2b(c+d+1)}\right.$$

$$\frac{1}{3}b(c+d+$$

$$\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+c)$$

input
```
Int[x^2*ArcCoth[c + d*Tanh[a + b*x]],x]
```

output
```
(x^3*ArcCoth[c + d*Tanh[a + b*x]])/3 + (b*(1 - c - d)*((x^3*Log[1 + ((1 -
c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(2*b*(1 - c - d)) - (3*(-1/2*(x^2*Po
lyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/b + ((x*PolyLog[3,
 -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(2*b) - PolyLog[4, -(((1 -
 c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b^2))/b))/(2*b*(1 - c - d))))/3
- (b*(1 + c + d)*((x^3*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])
/(2*b*(1 + c + d)) - (3*(-1/2*(x^2*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*
x))/(1 + c - d))])/b + ((x*PolyLog[3, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 +
 c - d))])/(2*b) - PolyLog[4, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))
]/(4*b^2))/b))/(2*b*(1 + c + d))))/3
```

3.203. $\int x^2\coth^{-1}(c+d\tanh(a+bx))\,dx$

3.203.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6798
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + (Simp[b*((1 - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x))), x], x] - Simp[b*
((1 + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c -
 d + (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.203.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.91 (sec) , antiderivative size = 5257, normalized size of antiderivative = 17.12

method	result	size
risch	Expression too large to display	5257

input
```
int(x^2*arccoth(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.203.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(263) = 526$.

Time = 0.28 (sec) , antiderivative size = 899, normalized size of antiderivative = 2.93

$$\int x^2 \coth^{-1}(c + d\tanh(a + bx)) \, dx = \text{Too large to display}$$

input
```
integrate(x^2*arccoth(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(b^3*x^3*log(((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b
*x + a) + d*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt(-(c + d + 1)/(c - d + 1
))*(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt(-(c + d + 1)/(
c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt(-(c +
d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-sq
rt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2*
(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sq
rt(-(c + d + 1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c
 + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) -
a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c -
 d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x +
 a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d
 - 1))) + 6*b*x*polylog(3, sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) +
 sinh(b*x + a))) + 6*b*x*polylog(3, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(
b*x + a) + sinh(b*x + a))) - 6*b*x*polylog(3, sqrt(-(c + d - 1)/(c - d - 1
))*(cosh(b*x + a) + sinh(b*x + a))) - 6*b*x*polylog(3, -sqrt(-(c + d - 1)/
(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt(-
(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3 +
 a^3)*log(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))
+ 1) + (b^3*x^3 + a^3)*log(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a...
```

3.203.6 Sympy [F]

$$\int x^2 \coth^{-1}(c+d\tanh(a+bx))\,dx = \int x^2 \operatorname{acoth}\left(c+d\tanh\left(a+bx\right)\right)dx$$

input
```
integrate(x**2*acoth(c+d*tanh(b*x+a)),x)
```

output
```
Integral(x**2*acoth(c + d*tanh(a + b*x)), x)
```

3.203.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.92

$$\int x^2 \coth^{-1}(c + d\tanh(a + bx))\, dx = \frac{1}{3}\, x^3 \operatorname{arcoth}\left(d\tanh\left(bx + a\right) + c\right) - \frac{1}{18}\, bd\left(\frac{4\, b^3 x^3 \log\left(\frac{(c+d+1)e^{(2\, bx+2\, a)}}{c-d+1} + 1\right) + 6\, b^2 x^2 \mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2\, bx+2\, a)}}{c-d+1}\right) - 6\, bx \mathrm{Li}_3(-\frac{(c+d+1)e^{(2\, bx+2\, a)}}{c-d+1}) + 3\, \mathrm{I}}{b^4 d}\right.$$

input
```
integrate(x^2*arccoth(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(d*tanh(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log((c + d + 1
)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 6*b^2*x^2*dilog(-(c + d + 1)*e^(2*b*x
 + 2*a)/(c - d + 1)) - 6*b*x*polylog(3, -(c + d + 1)*e^(2*b*x + 2*a)/(c -
d + 1)) + 3*polylog(4, -(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d)
- (4*b^3*x^3*log((c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*
dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, -(c + d
 - 1)*e^(2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, -(c + d - 1)*e^(2*b*x +
2*a)/(c - d - 1)))/(b^4*d))
```

3.203.8 Giac [F]

$$\int x^2 \coth^{-1}(c + d\tanh(a + bx))\, dx = \int x^2 \operatorname{arcoth}\left(d\tanh\left(bx + a\right) + c\right)\, dx$$

input
```
integrate(x^2*arccoth(c+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(d*tanh(b*x + a) + c), x)
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(c + d\tanh(a + bx))\,dx = \int x^2\,\mathrm{acoth}(c + d\tanh(a + b\,x))\;dx$$

input
```
int(x^2*acoth(c + d*tanh(a + b*x)),x)
```

output
```
int(x^2*acoth(c + d*tanh(a + b*x)), x)
```

3.204 $\int x \coth^{-1}(c + d \tanh(a + bx)) \, dx$

3.204.1 Optimal result

Integrand size = 13, antiderivative size = 231

$$\begin{aligned}\int x \coth^{-1}(c + d \tanh(a + bx)) \, dx =& \frac{1}{2} x^2 \coth^{-1}(c + d \tanh(a + bx)) \\ &+ \frac{1}{4} x^2 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\ &- \frac{1}{4} x^2 \log\left(1 + \frac{(1 + c + d)e^{2a+2bx}}{1 + c - d}\right) \\ &+ \frac{x \operatorname{PolyLog}\left(2, -\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b} \\ &- \frac{x \operatorname{PolyLog}\left(2, -\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b} \\ &- \frac{\operatorname{PolyLog}\left(3, -\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{8b^2} \\ &+ \frac{\operatorname{PolyLog}\left(3, -\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{8b^2}\end{aligned}$$

output
```
1/2*x^2*arccoth(c+d*tanh(b*x+a))+1/4*x^2*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+
d))-1/4*x^2*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x*polylog(2,-(1-c-d)*
exp(2*b*x+2*a)/(1-c+d))/b-1/4*x*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))
/b-1/8*polylog(3,-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/8*polylog(3,-(1+c+
d)*exp(2*b*x+2*a)/(1+c-d))/b^2
```

3.204.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.86

$$\int x\coth^{-1}(c+d\tanh(a+bx))\,dx$$
$$=\frac{4b^2x^2\coth^{-1}(c+d\tanh(a+bx))+2b^2x^2\log\left(1+\frac{(-1+c-d)e^{-2(a+bx)}}{-1+c+d}\right)-2b^2x^2\log\left(1+\frac{(1+c-d)e^{-2(a+bx)}}{1+c+d}\right)-2}{}$$

input
```
Integrate[x*ArcCoth[c + d*Tanh[a + b*x]],x]
```

output
```
(4*b^2*x^2*ArcCoth[c + d*Tanh[a + b*x]] + 2*b^2*x^2*Log[1 + (-1 + c - d)/(
(-1 + c + d)*E^(2*(a + b*x)))] - 2*b^2*x^2*Log[1 + (1 + c - d)/((1 + c + d
)*E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, (1 - c + d)/((-1 + c + d)*E^(2*(a +
 b*x)))] + 2*b*x*PolyLog[2, (-1 - c + d)/((1 + c + d)*E^(2*(a + b*x)))] -
PolyLog[3, (1 - c + d)/((-1 + c + d)*E^(2*(a + b*x)))] + PolyLog[3, (-1 -
c + d)/((1 + c + d)*E^(2*(a + b*x)))])/(8*b^2)
```

3.204.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6798, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(d\tanh(a+bx)+c)\,dx$$
$$\Big\downarrow \text{6798}$$
$$\frac{1}{2}b(-c-d+1)\int\frac{e^{2a+2bx}x^2}{-c+(-c-d+1)e^{2a+2bx}+d+1}dx-\frac{1}{2}b(c+d+1)\int\frac{e^{2a+2bx}x^2}{c+(c+d+1)e^{2a+2bx}-d+1}dx+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+c)$$
$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}b(-c-d+1)\left(\frac{x^2\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{\int x\log\left(\frac{e^{2a+2bx}(-c-d+1)}{-c+d+1}+1\right)dx}{b(-c-d+1)}\right)-\frac{1}{2}b(c+d+1)\left(\frac{x^2\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{\int x\log\left(\frac{e^{2a+2bx}(c+d+1)}{c-d+1}+1\right)dx}{b(c+d+1)}\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{1}{2}b(-c-d+1)\left(\frac{x^2\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{\frac{\int \text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}\right)-\frac{1}{2}b(c+d+1)\left(\frac{x^2\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{\frac{\int \text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{2}b(-c-d+1)\left(\frac{x^2\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}\right)-\frac{1}{2}b(c+d+1)\left(\frac{x^2\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{ 7143}$$

3.204. $\int x\coth^{-1}(c+d\tanh(a+bx))\,dx$

$$\frac{1}{2}b(-c-d+1)\left(\frac{x^2\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{\frac{\operatorname{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}\right)-\frac{1}{2}b(c+d+1)\left(\frac{x^2\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{\frac{\operatorname{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+c)$$

input
```
Int[x*ArcCoth[c + d*Tanh[a + b*x]],x]
```

output
```
(x^2*ArcCoth[c + d*Tanh[a + b*x]])/2 + (b*(1 - c - d)*((x^2*Log[1 + ((1 -
c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(2*b*(1 - c - d)) - (-1/2*(x*PolyLog
[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/b + PolyLog[3, -(((1 -
c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b^2))/(b*(1 - c - d))))/2 - (b*(1
 + c + d)*((x^2*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(2*b*(
1 + c + d)) - (-1/2*(x*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c -
 d))])/b + PolyLog[3, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(4*b^2
))/(b*(1 + c + d))))/2
```

3.204.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6798
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + (Simp[b*((1 - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x))), x], x] - Simp[b*
((1 + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c -
 d + (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.204.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.48 (sec) , antiderivative size = 4953, normalized size of antiderivative = 21.44

method	result	size
risch	Expression too large to display	4953

input
```
int(x*arccoth(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^2*a^2*c/(1+c+d)*ln(c*exp(2*b*x+2*a)+d*exp(2*b*x+2*a)+exp(2*b*x+2*a)
+c-d+1)-1/4/b^2*a^2*d/(1+c+d)*ln(c*exp(2*b*x+2*a)+d*exp(2*b*x+2*a)+exp(2*b
*x+2*a)+c-d+1)+1/2/b^2*a*d/(1+c+d)*dilog((-exp(b*x+a)*c-exp(b*x+a)*d+(-(1+
c-d)*(1+c+d))^(1/2)-exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a*d/(1+c
+d)*dilog((exp(b*x+a)*c+exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/
(-(1+c-d)*(1+c+d))^(1/2))-1/4/b^2*c/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(-
1-c+d))*a^2-1/4/b*c/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*x-1
/4/b^2*c/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*a-1/4/b^2*d/(1
+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*a^2-1/4/b*d/(1+c+d)*polylog(2,
(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*x-1/4/b^2*d/(1+c+d)*polylog(2,(1+c+d)*exp
(2*b*x+2*a)/(-1-c+d))*a+1/2/b^2*a/(c+d-1)*dilog((-exp(b*x+a)*c-exp(b*x+a)*
d+(-(c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))+1/2/b^2*a
/(c+d-1)*dilog((exp(b*x+a)*c+exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)-exp(b*x
+a))/(-(c-d-1)*(c+d-1))^(1/2))+1/4*c/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(
1-c+d))*x^2-1/4/b^2*a^2/(c+d-1)*ln(c*exp(2*b*x+2*a)+d*exp(2*b*x+2*a)-exp(2
*b*x+2*a)+c-d-1)-1/8/b^2*c/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/(1-c+d
))-1/8/b^2*d/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/(1-c+d))-1/4/b^2/(c+
d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*a^2-1/4/b/(c+d-1)*polylog(2,(c+d
-1)*exp(2*b*x+2*a)/(1-c+d))*x-1/4/b^2/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+
2*a)/(1-c+d))*a+1/4*d/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*x^2-...
```

3.204.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(197) = 394$.

Time = 0.27 (sec) , antiderivative size = 745, normalized size of antiderivative = 3.23

$$\int x \coth^{-1}(c + d\tanh(a + bx))\, dx$$
$$= \frac{b^2x^2\log\left(\frac{(c+1)\cosh(bx+a)+d\sinh(bx+a)}{(c-1)\cosh(bx+a)+d\sinh(bx+a)}\right) - 2\,bx\mathrm{Li}_2\left(\sqrt{-\frac{c+d+1}{c-d+1}}(\cosh(bx+a)+\sinh(bx+a))\right) - 2\,bx\mathrm{Li}_2\left(-\sqrt{\cdots}\right.}{\cdots}$$

input
```
integrate(x*arccoth(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(b^2*x^2*log(((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b
*x + a) + d*sinh(b*x + a))) - 2*b*x*dilog(sqrt(-(c + d + 1)/(c - d + 1))*(
cosh(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt(-(c + d + 1)/(c - d +
1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt(-(c + d - 1)/(c -
d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt(-(c + d - 1)/
(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cosh
(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/
(c - d + 1))) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b
*x + a) - 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a^2*log(2*(c + d
 - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c
 + d - 1)/(c - d - 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d -
 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - (b^2*x
^2 - a^2)*log(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a
)) + 1) - (b^2*x^2 - a^2)*log(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x +
a) + sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(sqrt(-(c + d - 1)/(c - d -
1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt(-(c +
 d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3, s
qrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*polylog
(3, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*p
olylog(3, sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)...
```

3.204.6 Sympy [F]

$$\int x \coth^{-1}(c + d\tanh(a + bx))\,dx = \int x \operatorname{acoth}\left(c + d\tanh\left(a + bx\right)\right)dx$$

input
```
integrate(x*acoth(c+d*tanh(b*x+a)),x)
```

output
```
Integral(x*acoth(c + d*tanh(a + b*x)), x)
```

3.204.7 Maxima [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.93

$$\int x\coth^{-1}(c+d\tanh(a+bx))\,dx =$$
$$-\frac{1}{8}bd\left(\frac{2b^2x^2\log\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}+1\right)+2bx\mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)-\mathrm{Li}_3(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1})}{b^3d}-\frac{2b^2x^2\log}{}\right.$$
$$+\frac{1}{2}x^2\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+c\right)$$

input
```
integrate(x*arccoth(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*b*d*((2*b^2*x^2*log((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 2*
b*x*dilog(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - polylog(3, -(c + d +
 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^3*d) - (2*b^2*x^2*log((c + d - 1)*e^(
2*b*x + 2*a)/(c - d - 1) + 1) + 2*b*x*dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(
c - d - 1)) - polylog(3, -(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^3*d
)) + 1/2*x^2*arccoth(d*tanh(b*x + a) + c)
```

3.204.8 Giac [F]

$$\int x\coth^{-1}(c+d\tanh(a+bx))\,dx = \int x\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccoth(c+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(d*tanh(b*x + a) + c), x)
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int x\coth^{-1}(c+d\tanh(a+bx))\,dx=\int x\,\mathrm{acoth}(c+d\tanh(a+b\,x))\;dx$$

input
```
int(x*acoth(c + d*tanh(a + b*x)),x)
```

output
```
int(x*acoth(c + d*tanh(a + b*x)), x)
```

3.205 $\int \coth^{-1}(c+d\tanh(a+bx))\,dx$

3.205.1 Optimal result

Integrand size = 11, antiderivative size = 150

$$\int \coth^{-1}(c+d\tanh(a+bx))\,dx = x\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log\left(1+\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right) + \frac{\text{PolyLog}\left(2,-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b} - \frac{\text{PolyLog}\left(2,-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b}$$

output
```
x*arccoth(c+d*tanh(b*x+a))+1/2*x*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+d))-1/2*
x*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*polylog(2,-(1-c-d)*exp(2*b*x+2*
a)/(1-c+d))/b-1/4*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b
```

3.205.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \coth^{-1}(c+d\tanh(a+bx))\,dx = x\coth^{-1}(c+d\tanh(a+bx)) + \frac{2bx\left(\log\left(1+\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)-\log\left(1+\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)\right)+\operatorname{PolyLog}\left(2,-\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)-\operatorname{PolyLog}}{4b}$$

input
```
Integrate[ArcCoth[c + d*Tanh[a + b*x]],x]
```

output
```
x*ArcCoth[c + d*Tanh[a + b*x]] + (2*b*x*(Log[1 + ((-1 + c + d)*E^(2*(a + b
*x)))/(-1 + c - d)] - Log[1 + ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)])
+ PolyLog[2, -(((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d))] - PolyLog[2,
-(((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d))])/(4*b)
```

3.205.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.38, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6790, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d\tanh(a+bx)+c)\,dx$$

$$\Big\downarrow \text{ 6790}$$

$$b(-c-d+1)\int \frac{e^{2a+2bx}x}{-c+(-c-d+1)e^{2a+2bx}+d+1}dx - b(c+d+1)\int \frac{e^{2a+2bx}x}{c+(c+d+1)e^{2a+2bx}-d+1}dx + x\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{ 2620}$$

$$b(-c-d+1)\left(\frac{x\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)} - \frac{\int \log\left(\frac{e^{2a+2bx}(-c-d+1)}{-c+d+1}+1\right)dx}{2b(-c-d+1)}\right) - b(c+d+1)\left(\frac{x\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)} - \frac{\int \log\left(\frac{e^{2a+2bx}(c+d+1)}{c-d+1}+1\right)dx}{2b(c+d+1)}\right) + x\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2715}$$

$$b(-c-d+1)\left(\frac{x\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}-\frac{\int e^{-2a-2bx}\log\left(\frac{e^{2a+2bx}(-c-d+1)}{-c+d+1}+1\right)de^{2a+2bx}}{4b^2(-c-d+1)}\right)-b(c+d+1)\left(\frac{x\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}-\frac{\int e^{-2a-2bx}\log\left(\frac{e^{2a+2bx}(c+d+1)}{c-d+1}+1\right)de^{2a+2bx}}{4b^2(c+d+1)}\right)+x\coth^{-1}(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2838}$$

$$b(-c-d+1)\left(\frac{\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2(-c-d+1)}+\frac{x\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)}{2b(-c-d+1)}\right)-b(c+d+1)\left(\frac{\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2(c+d+1)}+\frac{x\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)}{2b(c+d+1)}\right)+x\coth^{-1}(d\tanh(a+bx)+c)$$

input
```
Int[ArcCoth[c + d*Tanh[a + b*x]],x]
```

output
```
x*ArcCoth[c + d*Tanh[a + b*x]] + b*(1 - c - d)*((x*Log[1 + ((1 - c - d)*E^
(2*a + 2*b*x))/(1 - c + d)])/(2*b*(1 - c - d)) + PolyLog[2, -(((1 - c - d)
*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b^2*(1 - c - d))) - b*(1 + c + d)*((x*L
og[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(2*b*(1 + c + d)) + Pol
yLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(4*b^2*(1 + c + d)))
```

3.205.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6790
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Tanh[a + b*x]], x] + (Simp[b*(1 - c - d)   Int[x*(E^(2*a + 2*b*
x)/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x))), x], x] - Simp[b*(1 + c + d)
  Int[x*(E^(2*a + 2*b*x)/(1 + c - d + (1 + c + d)*E^(2*a + 2*b*x))), x], x]
) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

3.205.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(138) = 276$.

Time = 2.75 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.32

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccoth}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}-\frac{d^2\left(-\frac{\frac{\operatorname{dilog}\left(\frac{-d\tanh(bx+a)}{-1-c-d}\right.}{2}}{}\right.}{}}{}$
default	$\frac{-\frac{\operatorname{arccoth}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}-\frac{d^2\left(-\frac{\frac{\operatorname{dilog}\left(\frac{-d\tanh(bx+a)}{-1-c-d}\right.}{2}}{}\right.}{}}{}$
risch	Expression too large to display

input
```
int(arccoth(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-1/2*arccoth(c+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)+1/2*arccoth(c+
d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)-d)-1/2*d^2*(1/d*(-1/2*dilog((-d*tanh(b*
x+a)-c-1)/(-1-c-d))-1/2*ln(-d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)-c-1)/(-1-c
-d))+1/2*dilog((-d*tanh(b*x+a)-c+1)/(1-c-d))+1/2*ln(-d*tanh(b*x+a)+d)*ln((
-d*tanh(b*x+a)-c+1)/(1-c-d)))-1/d*(-1/2*dilog((-d*tanh(b*x+a)-c-1)/(-1-c+d
))-1/2*ln(-d*tanh(b*x+a)-d)*ln((-d*tanh(b*x+a)-c-1)/(-1-c+d))+1/2*dilog((-
d*tanh(b*x+a)-c+1)/(1-c+d))+1/2*ln(-d*tanh(b*x+a)-d)*ln((-d*tanh(b*x+a)-c+
1)/(1-c+d)))))
```

3.205.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(128) = 256.

Time = 0.28 (sec) , antiderivative size = 551, normalized size of antiderivative = 3.67

$$\int \coth^{-1}(c+d\tanh(a+bx))\,dx$$
$$= \frac{bx\log\left(\frac{(c+1)\cosh(bx+a)+d\sinh(bx+a)}{(c-1)\cosh(bx+a)+d\sinh(bx+a)}\right)+a\log\left(2\left(c+d+1\right)\cosh\left(bx+a\right)+2\left(c+d+1\right)\sinh\left(bx+a\right)+2\left(c\right.\right.}{}$$

input
```
integrate(arccoth(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b*x*log(((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b*x +
 a) + d*sinh(b*x + a))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1
)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a*log(2*
(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sq
rt(-(c + d + 1)/(c - d + 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c +
 d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - a*
log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d -
 1)*sqrt(-(c + d - 1)/(c - d - 1))) - (b*x + a)*log(sqrt(-(c + d + 1)/(c -
 d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(-(c +
d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(s
qrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x
+ a)*log(-sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) +
 1) - dilog(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))
) - dilog(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)))
 + dilog(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) +
 dilog(-sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

3.205.6 Sympy [F]

$$\int \coth^{-1}(c+d\tanh(a+bx))\,dx=\int \operatorname{acoth}\left(c+d\tanh\left(a+bx\right)\right)dx$$

input
```
integrate(acoth(c+d*tanh(b*x+a)),x)
```

output
```
Integral(acoth(c + d*tanh(a + b*x)), x)
```

3.205.7 Maxima [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \coth^{-1}(c+d\tanh(a+bx))\,dx =$$
$$-\frac{1}{4}bd\left(\frac{2\,bx\log\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+\mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^2d}-\frac{2\,bx\log\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+\mathrm{Li}_2\left(-\right.}{b^2d}\right.$$
$$+x\,\mathrm{arcoth}\left(d\tanh\left(bx+a\right)+c\right)$$

input
```
integrate(arccoth(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/4*b*d*((2*b*x*log((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + dilog(
-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^2*d) - (2*b*x*log((c + d - 1
)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(c
 - d - 1)))/(b^2*d)) + x*arccoth(d*tanh(b*x + a) + c)
```

3.205.8 Giac [F]

$$\int \coth^{-1}(c+d\tanh(a+bx))\,dx = \int \mathrm{arcoth}\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arccoth(c+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(d*tanh(b*x + a) + c), x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(c+d\tanh(a+bx))\,dx = \int \mathrm{acoth}(c+d\tanh(a+b\,x))\,dx$$

input
```
int(acoth(c + d*tanh(a + b*x)),x)
```

output
```
int(acoth(c + d*tanh(a + b*x)), x)
```

3.206 $\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx$

3.206.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx=\text{Int}\left(\frac{\coth^{-1}(c+d\tanh(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccoth(c+d*tanh(b*x+a))/x,x)
```

3.206.2 Mathematica [N/A]

Not integrable

Time = 3.70 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx=\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[c + d*Tanh[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[c + d*Tanh[a + b*x]]/x, x]
```

3.206.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d\tanh(a+bx)+c)}{x}dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\coth^{-1}(d\tanh(a+bx)+c)}{x}dx$$

input
```
Int[ArcCoth[c + d*Tanh[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.206.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.206.4 Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(c+d\tanh\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(c+d*tanh(b*x+a))/x,x)
```

output
```
int(arccoth(c+d*tanh(b*x+a))/x,x)
```

3.206.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arccoth(d*tanh(b*x + a) + c)/x, x)
```

3.206.6 Sympy [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{acoth}\left(c+d\tanh\left(a+bx\right)\right)}{x}\,dx$$

input
```
integrate(acoth(c+d*tanh(b*x+a))/x,x)
```

output
```
Integral(acoth(c + d*tanh(a + b*x))/x, x)
```

3.206.7 Maxima [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arccoth(d*tanh(b*x + a) + c)/x, x)
```

3.206.8 Giac [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c + d\tanh(a + bx))}{x}\, dx = \int \frac{\operatorname{arcoth}\left(d\tanh\left(bx + a\right) + c\right)}{x}\, dx$$

input
```
integrate(arccoth(c+d*tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(d*tanh(b*x + a) + c)/x, x)
```

3.206.9 Mupad [N/A]

Not integrable

Time = 4.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c + d\tanh(a + bx))}{x}\, dx = \int \frac{\operatorname{acoth}(c + d\tanh(a + b\,x))}{x}\, dx$$

input
```
int(acoth(c + d*tanh(a + b*x))/x,x)
```

output
```
int(acoth(c + d*tanh(a + b*x))/x, x)
```

3.207 $\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx$

3.207.1 Optimal result

Integrand size = 16, antiderivative size = 155

$$\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{x^3\operatorname{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b} + \frac{3x^2\operatorname{PolyLog}\left(3,-\left((1+d)e^{2a+2bx}\right)\right)}{8b^2} - \frac{3x\operatorname{PolyLog}\left(4,-\left((1+d)e^{2a+2bx}\right)\right)}{8b^3} + \frac{3\operatorname{PolyLog}\left(5,-\left((1+d)e^{2a+2bx}\right)\right)}{16b^4}$$

output
```
1/20*b*x^5+1/4*x^4*arccoth(1+d+d*tanh(b*x+a))-1/8*x^4*ln(1+(1+d)*exp(2*b*x
+2*a))-1/4*x^3*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,-(1+d)
*exp(2*b*x+2*a))/b^2-3/8*x*polylog(4,-(1+d)*exp(2*b*x+2*a))/b^3+3/16*polyl
og(5,-(1+d)*exp(2*b*x+2*a))/b^4
```

3.207.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx$$
$$= \frac{4b^4x^4\coth^{-1}(1+d+d\tanh(a+bx)) - 2b^4x^4\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right) + 4b^3x^3\operatorname{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right) + 6b^2x^2}{16b^4}$$

input
```
Integrate[x^3*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

output
```
(4*b^4*x^4*ArcCoth[1 + d + d*Tanh[a + b*x]] - 2*b^4*x^4*Log[1 + 1/((1 + d)
*E^(2*(a + b*x)))] + 4*b^3*x^3*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))]
+ 6*b^2*x^2*PolyLog[3, -(1/((1 + d)*E^(2*(a + b*x))))] + 6*b*x*PolyLog[4,
-(1/((1 + d)*E^(2*(a + b*x))))] + 3*PolyLog[5, -(1/((1 + d)*E^(2*(a + b*x)
)))])/(16*b^4)
```

3.207.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6794, 2615, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(d\tanh(a+bx)+d+1)\,dx$$
$$\Big\downarrow \text{6794}$$
$$\frac{1}{4}b\int \frac{x^4}{e^{2a+2bx}(d+1)+1}dx + \frac{1}{4}x^4\coth^{-1}(d\tanh(a+bx)+d+1)$$
$$\Big\downarrow \text{2615}$$
$$\frac{1}{4}b\left(\frac{x^5}{5} - (d+1)\int \frac{e^{2a+2bx}x^4}{e^{2a+2bx}(d+1)+1}dx\right) + \frac{1}{4}x^4\coth^{-1}(d\tanh(a+bx)+d+1)$$
$$\Big\downarrow \text{2620}$$

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\int x^3\log\left(e^{2a+2bx}(d+1)+1\right)dx}{b(d+1)}\right)\right)+\frac{1}{4}x^4\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\downarrow$ 3011

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\left(\frac{3\int x^2\operatorname{PolyLog}\left(2,-\left((d+1)e^{2a+2bx}\right)\right)dx}{2b}-\frac{x^3\operatorname{PolyLog}\left(2,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}\right.}{b(d+1)}\right.\right.$$
$$\frac{1}{4}x^4\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\downarrow$ 7163

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\int x\operatorname{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)dx}{b}\right)}{2b}-\right.}{b(d+1)}\right.\right.$$
$$\frac{1}{4}x^4\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\downarrow$ 7163

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\int \operatorname{PolyLo}}{}}{b}\right.}{2b}\right.}{b(d+1)}\right.\right.$$
$$\frac{1}{4}x^4\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\downarrow$ 2720

3.207. $\int x^3\coth^{-1}(1+d+d\tanh(a+bx))\,dx$

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\int e^{-2a-}\cdots}{}}{}\right)}{2b}\cdots\right)}{b(d\cdots}\right.\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\big\downarrow$ 7143

$$\frac{1}{4}b\left(\frac{x^5}{5}-(d+1)\left(\frac{x^4\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\operatorname{PolyLog}\cdots}{}}{b}\right)}{2b}\cdots\right)}{b(d+1)}\right.\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d\tanh(a+bx)+d+1)$$

input
```
Int[x^3*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

output
```
(x^4*ArcCoth[1 + d + d*Tanh[a + b*x]])/4 + (b*(x^5/5 - (1 + d)*((x^4*Log[1
 + (1 + d)*E^(2*a + 2*b*x)])/(2*b*(1 + d)) - (2*(-1/2*(x^3*PolyLog[2, -((1
 + d)*E^(2*a + 2*b*x))])/b + (3*((x^2*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x)
)])/(2*b) - ((x*PolyLog[4, -((1 + d)*E^(2*a + 2*b*x))])/(2*b) - PolyLog[5,
 -((1 + d)*E^(2*a + 2*b*x))]/(4*b^2))/b))/(2*b)))/(b*(1 + d)))))/4
```

3.207. $\int x^3\coth^{-1}(1+d+d\tanh(a+bx))\,dx$

3.207.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6794
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.207.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.09 (sec) , antiderivative size = 1684, normalized size of antiderivative = 10.86

method	result	size
risch	Expression too large to display	1684

input
```
int(x^3*arccoth(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^3*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x*a^3+1/2/b^3*d*a^3/(1+d)*ln(1
+exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/
2))*x-3/8/b^4*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^4+3/8/b^2*d/(1+d)*polyl
og(3,-(1+d)*exp(2*b*x+2*a))*x^2-1/4/b^4*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x
+2*a))*a^3-3/8/b^3*d/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2*a))*x+1/2/b^3*a^3/
(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(-
d-1)^(1/2))*x+1/2/b^4*d*a^4/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^4*d*
a^4/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))+1/2/b^4*d*a^3/(1+d)*dilog(1+exp(b*
x+a)*(-d-1)^(1/2))+1/2/b^4*d*a^3/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))-1/
8/b^4*d*a^4/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)+1/20*b*x^5-1/8/b^4
*a^4/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)-1/4/b/(1+d)*polylog(2,-(1
+d)*exp(2*b*x+2*a))*x^3-3/8/b^4/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^4+3/8/b
^2/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x^2-1/4/b^4/(1+d)*polylog(2,-(1+
d)*exp(2*b*x+2*a))*a^3-3/8/b^3/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2*a))*x+3/
16/b^4*d/(1+d)*polylog(5,-(1+d)*exp(2*b*x+2*a))+1/2/b^4*a^4/(1+d)*ln(1+exp
(b*x+a)*(-d-1)^(1/2))+1/2/b^4*a^4/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))+1/2/
b^4*a^3/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^4*a^3/(1+d)*dilog(1-e
xp(b*x+a)*(-d-1)^(1/2))-1/8*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^4-1/8/(1+
d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^4+3/16/b^4/(1+d)*polylog(5,-(1+d)*exp(2*b*
x+2*a))-1/2/b^3/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x*a^3-1/4*x^4*ln(exp(b...
```

3.207.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(135) = 270.

Time = 0.27 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.90

$$\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx$$
$$= \frac{2\,b^5x^5 + 5\,b^4x^4\log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 20\,b^3x^3\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d-4}(\cosh(bx+a)+\sinh(bx+a))\right)}{}$$

input
```
integrate(x^3*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/40*(2*b^5*x^5 + 5*b^4*x^4*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(1/2*sqrt(-4*d - 4)
*(cosh(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-1/2*sqrt(-4*d - 4)*(
cosh(b*x + a) + sinh(b*x + a))) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d
 + 1)*sinh(b*x + a) + sqrt(-4*d - 4)) - 5*a^4*log(2*(d + 1)*cosh(b*x + a)
+ 2*(d + 1)*sinh(b*x + a) - sqrt(-4*d - 4)) + 60*b^2*x^2*polylog(3, 1/2*sq
rt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -1/2
*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, 1/2*
sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, -1/2*
sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(b^4*x^4 - a^4)*log(1/
2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^4*x^4 - a^4)*
log(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*polylog
(5, 1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -
1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

3.207.6 Sympy [F]

$$\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int x^3\,\mathrm{acoth}\,(d\tanh(a+bx)+d+1)\,dx$$

input
```
integrate(x**3*acoth(1+d+d*tanh(b*x+a)),x)
```

output
```
Integral(x**3*acoth(d*tanh(a + b*x) + d + 1), x)
```

3.207.7 Maxima [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

$$\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{1}{4}x^4 \operatorname{arcoth}(d\tanh(bx+a)+d+1) + \frac{1}{40}\left(\frac{2x^5}{d} - \frac{5\left(2b^4x^4\log\left((d+1)e^{(2bx+2a)}+1\right)+4b^3x^3\operatorname{Li}_2\left(-(d+1)e^{(2bx+2a)}\right)-6b^2x^2\operatorname{Li}_3(-(d+1)e^{(2\right.}{b^5d}\right.$$

input
```
integrate(x^3*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*x^4*arccoth(d*tanh(b*x + a) + d + 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*lo
g((d + 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog(-(d + 1)*e^(2*b*x + 2*a))
 - 6*b^2*x^2*polylog(3, -(d + 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, -(d +
 1)*e^(2*b*x + 2*a)) - 3*polylog(5, -(d + 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*
d
```

3.207.8 Giac [F]

$$\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int x^3 \operatorname{arcoth}(d\tanh(bx+a)+d+1)\,dx$$

input
```
integrate(x^3*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^3*arccoth(d*tanh(b*x + a) + d + 1), x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int x^3 \operatorname{acoth}(d+d\tanh(a+b\,x)+1)\,dx$$

input
```
int(x^3*acoth(d + d*tanh(a + b*x) + 1),x)
```

output
```
int(x^3*acoth(d + d*tanh(a + b*x) + 1), x)
```

3.208 $\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx$

3.208.1 Optimal result

Integrand size = 16, antiderivative size = 128

$$\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3 \log\left(1+(1+d)e^{2a+2bx}\right) - \frac{x^2 \operatorname{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b} + \frac{x \operatorname{PolyLog}\left(3,-\left((1+d)e^{2a+2bx}\right)\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4,-\left((1+d)e^{2a+2bx}\right)\right)}{8b^3}$$

output
```
1/12*b*x^4+1/3*x^3*arccoth(1+d+d*tanh(b*x+a))-1/6*x^3*ln(1+(1+d)*exp(2*b*x
+2*a))-1/4*x^2*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,-(1+d)*e
xp(2*b*x+2*a))/b^2-1/8*polylog(4,-(1+d)*exp(2*b*x+2*a))/b^3
```

3.208.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx$$
$$= \frac{8b^3x^3\coth^{-1}(1+d+d\tanh(a+bx)) - 4b^3x^3\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right) + 6b^2x^2\operatorname{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right) + 6bx\,\mathrm{P}}{24b^3}$$

input
```
Integrate[x^2*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCoth[1 + d + d*Tanh[a + b*x]] - 4*b^3*x^3*Log[1 + 1/((1 + d)
*E^(2*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))]
+ 6*b*x*PolyLog[3, -(1/((1 + d)*E^(2*(a + b*x))))] + 3*PolyLog[4, -(1/((1
+ d)*E^(2*(a + b*x))))])/(24*b^3)
```

3.208.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6794, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\coth^{-1}(d\tanh(a+bx)+d+1)\,dx$$
$$\Big\downarrow \text{6794}$$
$$\frac{1}{3}b\int \frac{x^3}{e^{2a+2bx}(d+1)+1}dx + \frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+d+1)$$
$$\Big\downarrow \text{2615}$$
$$\frac{1}{3}b\left(\frac{x^4}{4}-(d+1)\int \frac{e^{2a+2bx}x^3}{e^{2a+2bx}(d+1)+1}dx\right)+\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+d+1)$$
$$\Big\downarrow \text{2620}$$
$$\frac{1}{3}b\left(\frac{x^4}{4}-(d+1)\left(\frac{x^3\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{3\int x^2\log\left(e^{2a+2bx}(d+1)+1\right)dx}{2b(d+1)}\right)\right)+$$
$$\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\Big\downarrow$ 3011

$$\frac{1}{3}b\left(\frac{x^4}{4}-(d+1)\left(\frac{x^3\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-\left((d+1)e^{2a+2bx}\right)\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}\right)}{2b(d+1)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\Big\downarrow$ 7163

$$\frac{1}{3}b\left(\frac{x^4}{4}-(d+1)\left(\frac{x^3\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)dx}{2b}}{b}-\frac{x^2\,\text{Polyl}}{}\right.}{2b(d+1)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\Big\downarrow$ 2720

$$\frac{1}{3}b\left(\frac{x^4}{4}-(d+1)\left(\frac{x^3\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)de^{2a-}}{4b^2}}{b}\right.}{2b(d+1)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\Big\downarrow$ 7143

$$\frac{1}{3}b\left(\frac{x^4}{4}-(d+1)\left(\frac{x^3\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}-\frac{\text{PolyLog}\left(4,-\left((d+1)e^{2a+2bx}\right)\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}}{}\right.}{2b(d+1)}\right.\right.$$
$$\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+d+1)$$

input
```
Int[x^2*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

3.208. $\int x^2\coth^{-1}(1+d+d\tanh(a+bx))\,dx$

output
```
(x^3*ArcCoth[1 + d + d*Tanh[a + b*x]])/3 + (b*(x^4/4 - (1 + d)*((x^3*Log[1
 + (1 + d)*E^(2*a + 2*b*x)])/(2*b*(1 + d)) - (3*(-1/2*(x^2*PolyLog[2, -((1
 + d)*E^(2*a + 2*b*x))])/b + ((x*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(
2*b) - PolyLog[4, -((1 + d)*E^(2*a + 2*b*x))]/(4*b^2))/b))/(2*b*(1 + d))))
)/3
```

3.208.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6794
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.208.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.90 (sec) , antiderivative size = 1625, normalized size of antiderivative = 12.70

method	result	size
risch	Expression too large to display	1625

input
```
int(x^2*arccoth(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^3*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)+1/4/b^2*d/(1+d)*polylog(3,-(
1+d)*exp(2*b*x+2*a))*x-1/2/b^2*a^2/(1+d)*x*ln(1+exp(b*x+a)*(-d-1)^(1/2))-1
/2/b^2*a^2/(1+d)*x*ln(1-exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*a^3*d/(1+d)*ln(1+
exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*a^3*d/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))
-1/2/b^3*a^2*d/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*a^2*d/(1+d)*
dilog(1-exp(b*x+a)*(-d-1)^(1/2))+1/6/b^3*d*a^3/(1+d)*ln(d*exp(2*b*x+2*a)+e
xp(2*b*x+2*a)+1)+1/2/b^2/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^2*x-1/4/b*d/(1
+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^2+1/3/b^3*d/(1+d)*ln(1+(1+d)*exp(2*
b*x+2*a))*a^3+1/4/b^3*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a^2+1/12*b*
x^4-1/3*x^3*ln(exp(b*x+a))+1/2/b^2*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^2*
x-1/2/b^2*a^2*d/(1+d)*x*ln(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^2*a^2*d/(1+d)*
x*ln(1-exp(b*x+a)*(-d-1)^(1/2))-1/8/b^3/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2
*a))-1/6/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^3-1/6*d/(1+d)*ln(1+(1+d)*exp(2
*b*x+2*a))*x^3-1/8/b^3*d/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2*a))-1/2/b^3*a^
3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(-
d-1)^(1/2))-1/2/b^3*a^2/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*a^2
/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))+1/6/b^3*a^3/(1+d)*ln(d*exp(2*b*x+2
*a)+exp(2*b*x+2*a)+1)-1/4/b/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^2+1/3
/b^3/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^3+1/4/b^3/(1+d)*polylog(2,-(1+d)*e
xp(2*b*x+2*a))*a^2+1/4/b^2/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x-1/1...
```

3.208.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(111) = 222$.

Time = 0.26 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.98

$$\int x^2 \coth^{-1}(1 + d + d\tanh(a + bx))\, dx$$

$$= \frac{b^4x^4 + 2\,b^3x^3\log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d-4}(\cosh(bx+a) + \sinh(bx+a))\right) - 6}{}$$

input
```
integrate(x^2*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(b^4*x^4 + 2*b^3*x^3*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(d
*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d - 4)*(c
osh(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d - 4)*(cosh
(b*x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1
)*sinh(b*x + a) + sqrt(-4*d - 4)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*
(d + 1)*sinh(b*x + a) - sqrt(-4*d - 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d
- 4)*(cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d -
 4)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*d
 - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sq
rt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqrt
(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*d
 - 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

3.208.6 Sympy [F]

$$\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int x^2 \operatorname{acoth}\left(d\tanh\left(a+bx\right)+d+1\right)dx$$

input
```
integrate(x**2*acoth(1+d+d*tanh(b*x+a)),x)
```

output
```
Integral(x**2*acoth(d*tanh(a + b*x) + d + 1), x)
```

3.208.7 Maxima [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

$$\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{1}{3}x^3 \operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right) + \frac{1}{36}\left(\frac{3\,x^4}{d} - \frac{2\left(4\,b^3x^3\log\left((d+1)e^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-(d+1)e^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3(-(d+1)e^{(2\,b}\right.}{b^4d}\right.$$

input
```
integrate(x^2*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(d*tanh(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*lo
g((d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-(d + 1)*e^(2*b*x + 2*a))
 - 6*b*x*polylog(3, -(d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, -(d + 1)*e^(2
*b*x + 2*a)))/(b^4*d))*b*d
```

3.208.8 Giac [F]

$$\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int x^2 \operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(x^2*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(d*tanh(b*x + a) + d + 1), x)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int x^2 \operatorname{acoth}(d+d\tanh(a+b\,x)+1)\,dx$$

input
```
int(x^2*acoth(d + d*tanh(a + b*x) + 1),x)
```

output
```
int(x^2*acoth(d + d*tanh(a + b*x) + 1), x)
```

3.209 $\int x \coth^{-1}(1+d+d\tanh(a+bx))\,dx$

3.209.1 Optimal result

Integrand size = 14, antiderivative size = 101

$$\int x \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{4}x^2\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{x\,\text{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b} + \frac{\text{PolyLog}\left(3,-\left((1+d)e^{2a+2bx}\right)\right)}{8b^2}$$

output
```
1/6*b*x^3+1/2*x^2*arccoth(1+d+d*tanh(b*x+a))-1/4*x^2*ln(1+(1+d)*exp(2*b*x+
2*a))-1/4*x*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+1/8*polylog(3,-(1+d)*exp(2*
b*x+2*a))/b^2
```

3.209.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int x\coth^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{2b^2x^2\left(2\coth^{-1}(1+d+d\tanh(a+bx))-\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right)\right)+2bx\,\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right)+\text{PolyLog}}{8b^2}$$

input
```
Integrate[x*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcCoth[1 + d + d*Tanh[a + b*x]] - Log[1 + 1/((1 + d)*E^(2*(
a + b*x)))]) + 2*b*x*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))] + PolyLog[
3, -(1/((1 + d)*E^(2*(a + b*x))))])/(8*b^2)
```

3.209.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6794, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(d\tanh(a+bx)+d+1)\,dx$$

$$\Big\downarrow \text{6794}$$

$$\frac{1}{2}b\int \frac{x^2}{e^{2a+2bx}(d+1)+1}dx+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(d+1)\int \frac{e^{2a+2bx}x^2}{e^{2a+2bx}(d+1)+1}dx\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(d+1)\left(\frac{x^2\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\int x\log\left(e^{2a+2bx}(d+1)+1\right)dx}{b(d+1)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(d+1)\left(\frac{x^2\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\frac{\int \text{PolyLog}\left(2,-\left((d+1)e^{2a+2bx}\right)\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-\left((d+1)e^{2a+2bx}\right)\right)}{2b}}{b(d+1)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(d+1)\left(\frac{x^2\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\frac{\int e^{-2a-2bx}\operatorname{PolyLog}(2,-((d+1)e^{2a+2bx}))de^{2a+2bx}}{4b^2}-\frac{x\operatorname{PolyLog}(2,-((d+1)e^{2a}}{2b}}{b(d+1)}\right.\right.$$
$$\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+d+1)$$

$\downarrow$ 7143

$$\frac{1}{2}b\left(\frac{x^3}{3}-(d+1)\left(\frac{x^2\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\frac{\operatorname{PolyLog}(3,-((d+1)e^{2a+2bx}))}{4b^2}-\frac{x\operatorname{PolyLog}(2,-((d+1)e^{2a+2bx}))}{2b}}{b(d+1)}\right)\right)+$$
$$\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+d+1)$$

input
```
Int[x*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 + d + d*Tanh[a + b*x]])/2 + (b*(x^3/3 - (1 + d)*((x^2*Log[1
 + (1 + d)*E^(2*a + 2*b*x)])/(2*b*(1 + d)) - (-1/2*(x*PolyLog[2, -((1 + d)
*E^(2*a + 2*b*x))])/b + PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))]/(4*b^2))/(b
*(1 + d)))))/2
```

3.209.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6794
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.209.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.52 (sec) , antiderivative size = 1542, normalized size of antiderivative = 15.27

method	result	size
risch	Expression too large to display	1542

input
```
int(x*arccoth(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(I*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))^
3+I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2+2*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-I*Pi*csgn(I/(exp(2*
b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))+I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+
1)*d*exp(2*b*x+2*a))^2+I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))^2+I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(d*exp(2*b*x+2
*a)+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b
*x+2*a)+1))-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d)*csgn(
I/(exp(2*b*x+2*a)+1)*d*exp(2*b*x+2*a))-I*Pi*csgn(I*(d*exp(2*b*x+2*a)+exp(2
*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1
))^2-I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-I*Pi*csgn(I*exp(2*b*
x+2*a))^3-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+I*Pi*csgn(I*d)*
csgn(I/(exp(2*b*x+2*a)+1)*d*exp(2*b*x+2*a))^2-I*Pi*csgn(I/(exp(2*b*x+2*a)+
1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))^2-I*Pi*
csgn(I/(exp(2*b*x+2*a)+1)*d*exp(2*b*x+2*a))^3+2*ln(d))*x^2-1/2*x^2*ln(exp(
b*x+a))+1/6*b*x^3-1/2/b*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a*x+1/2/b*a*d/(
1+d)*x*ln(1+exp(b*x+a)*(-d-1)^(1/2))-1/4/b^2*a^2/(1+d)*ln(d*exp(2*b*x+2*a)
+exp(2*b*x+2*a)+1)-1/4/b^2/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^2-1/4/b/(1+d
)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x-1/4/b^2/(1+d)*polylog(2,-(1+d)*exp...
```

3.209.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(87) = 174$.

Time = 0.26 (sec) , antiderivative size = 322, normalized size of antiderivative = 3.19

$$\int x\coth^{-1}(1+d+d\tanh(a+bx))\,dx$$

$$= \frac{2\,b^3x^3+3\,b^2x^2\log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)-6\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d-4}(\cosh(bx+a)+\sinh(bx+a))\right)-6\cdots}{\cdots}$$

input
```
integrate(x*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*x^3 + 3*b^2*x^2*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(1/2*sqrt(-4*d - 4)*(cos
h(b*x + a) + sinh(b*x + a))) - 6*b*x*dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*x +
 a) + sinh(b*x + a))) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh
(b*x + a) + sqrt(-4*d - 4)) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1
)*sinh(b*x + a) - sqrt(-4*d - 4)) - 3*(b^2*x^2 - a^2)*log(1/2*sqrt(-4*d -
4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*sqrt(
-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*polylog(3, 1/2*sqrt(-4*
d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(3, -1/2*sqrt(-4*d - 4)
*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

3.209.6 Sympy [F]

$$\int x \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int x\,\mathrm{acoth}\,(d\tanh{(a+bx)}+d+1)\,dx$$

input
```
integrate(x*acoth(1+d+d*tanh(b*x+a)),x)
```

output
```
Integral(x*acoth(d*tanh(a + b*x) + d + 1), x)
```

3.209.7 Maxima [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int x \coth^{-1}(1+d+d\tanh(a+bx))\,dx$$
$$= \frac{1}{24}\left(\frac{4\,x^3}{d} - \frac{3\left(2\,b^2x^2\log\left((d+1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2(-(d+1)e^{(2\,bx+2\,a)})-\mathrm{Li}_3(-(d+1)e^{(2\,bx+2\,a)})\right)}{b^3d}\right)$$
$$+\frac{1}{2}\,x^2\,\mathrm{arcoth}\,(d\tanh{(bx+a)}+d+1)$$

input
```
integrate(x*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(4*x^3/d - 3*(2*b^2*x^2*log((d + 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g(-(d + 1)*e^(2*b*x + 2*a)) - polylog(3, -(d + 1)*e^(2*b*x + 2*a)))/(b^3*d
))*b*d + 1/2*x^2*arccoth(d*tanh(b*x + a) + d + 1)
```

3.209.8 Giac [F]

$$\int x \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int x \operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(x*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(d*tanh(b*x + a) + d + 1), x)
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \int x \operatorname{acoth}(d+d\tanh(a+b\,x)+1)\,dx$$

input
```
int(x*acoth(d + d*tanh(a + b*x) + 1),x)
```

output
```
int(x*acoth(d + d*tanh(a + b*x) + 1), x)
```

3.210 $\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx$

3.210.1 Optimal result

Integrand size = 12, antiderivative size = 69

$$\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{bx^2}{2}+x\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{\text{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b}$$

output
```
1/2*b*x^2+x*arccoth(1+d+d*tanh(b*x+a))-1/2*x*ln(1+(1+d)*exp(2*b*x+2*a))-1/
4*polylog(2,-(1+d)*exp(2*b*x+2*a))/b
```

3.210.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx = x\coth^{-1}(1+d+d\tanh(a+bx)) + \frac{-2bx\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right)+\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right)}{4b}$$

input
```
Integrate[ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

output
```
x*ArcCoth[1 + d + d*Tanh[a + b*x]] + (-2*b*x*Log[1 + 1/((1 + d)*E^(2*(a +
b*x)))] + PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))])/(4*b)
```

3.210.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6786, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d\tanh(a+bx)+d+1)\,dx$$

$$\Big\downarrow \text{6786}$$

$$b\int \frac{x}{e^{2a+2bx}(d+1)+1}dx+x\coth^{-1}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{2615}$$

$$b\left(\frac{x^2}{2}-(d+1)\int \frac{e^{2a+2bx}x}{e^{2a+2bx}(d+1)+1}dx\right)+x\coth^{-1}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{2620}$$

$$b\left(\frac{x^2}{2}-(d+1)\left(\frac{x\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\int \log\left(e^{2a+2bx}(d+1)+1\right)dx}{2b(d+1)}\right)\right)+ x\coth^{-1}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{2715}$$

$$b\left(\frac{x^2}{2}-(d+1)\left(\frac{x\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}-\frac{\int e^{-2a-2bx}\log\left(e^{2a+2bx}(d+1)+1\right)de^{2a+2bx}}{4b^2(d+1)}\right)\right)+ x\coth^{-1}(d\tanh(a+bx)+d+1)$$

$$\Big\downarrow \text{2838}$$

$$b\left(\frac{x^2}{2}-(d+1)\left(\frac{\text{PolyLog}\left(2,-\left((d+1)e^{2a+2bx}\right)\right)}{4b^2(d+1)}+\frac{x\log\left((d+1)e^{2a+2bx}+1\right)}{2b(d+1)}\right)\right)+ x\coth^{-1}(d\tanh(a+bx)+d+1)$$

input
```
Int[ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

output
```
x*ArcCoth[1 + d + d*Tanh[a + b*x]] + b*(x^2/2 - (1 + d)*((x*Log[1 + (1 + d
)*E^(2*a + 2*b*x)])/(2*b*(1 + d)) + PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))]
/(4*b^2*(1 + d))))
```

3.210.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6786
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Tanh[a + b*x]], x] + Simp[b   Int[x/(c - d + c*E^(2*a + 2*b*x))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

3.210.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(61) = 122$.

Time = 1.60 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.70

method	result
derivativedivides	$\frac{\frac{\operatorname{arccoth}(1+d+d\tanh(bx+a))d\ln(d+d\tanh(bx+a))}{2}-\frac{\operatorname{arccoth}(1+d+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{d^2\left(-\frac{\operatorname{dilog}\left(\frac{d\tanh(bx+a}{2}\right.}{2}\right.}{}}{}$
default	$\frac{\frac{\operatorname{arccoth}(1+d+d\tanh(bx+a))d\ln(d+d\tanh(bx+a))}{2}-\frac{\operatorname{arccoth}(1+d+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{d^2\left(-\frac{\operatorname{dilog}\left(\frac{d\tanh(bx+a}{2}\right.}{2}\right.}{}}{}$
risch	Expression too large to display

input
```
int(arccoth(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(1/2*arccoth(1+d+d*tanh(b*x+a))*d*ln(d+d*tanh(b*x+a))-1/2*arccoth(1+
d+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)+1/2*d^2*(1/d*(-1/2*dilog(1/2*d*tan
h(b*x+a)+1/2*d+1)-1/2*ln(d+d*tanh(b*x+a))*ln(1/2*d*tanh(b*x+a)+1/2*d+1)+1/
4*ln(d+d*tanh(b*x+a))^2)-1/d*(-1/2*dilog((-d*tanh(b*x+a)-d-2)/(-2*d-2))-1/
2*ln(-d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)-d-2)/(-2*d-2))+1/2*dilog(-1/2*(-
d*tanh(b*x+a)-d)/d)+1/2*ln(-d*tanh(b*x+a)+d)*ln(-1/2*(-d*tanh(b*x+a)-d)/d)
)))
```

3.210.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(60) = 120$.

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.45

$$\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx$$
$$= \frac{b^2x^2 + bx\log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) + a\log\left(2\,(d+1)\cosh\left(bx+a\right) + 2\,(d+1)\sinh\left(bx+a\right) + \sqrt{-4}\right.}{}$$

input
```
integrate(arccoth(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 + b*x*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b
*x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*si
nh(b*x + a) + sqrt(-4*d - 4)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*
sinh(b*x + a) - sqrt(-4*d - 4)) - (b*x + a)*log(1/2*sqrt(-4*d - 4)*(cosh(b
*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-1/2*sqrt(-4*d - 4)*(cosh(b*
x + a) + sinh(b*x + a)) + 1) - dilog(1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + s
inh(b*x + a))) - dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))
))/b
```

3.210.6 Sympy [F]

$$\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx=\int \operatorname{acoth}\left(d\tanh\left(a+bx\right)+d+1\right)dx$$

input
```
integrate(acoth(1+d+d*tanh(b*x+a)),x)
```

output
```
Integral(acoth(d*tanh(a + b*x) + d + 1), x)
```

3.210.7 Maxima [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx$$
$$=\frac{1}{4}\,bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left((d+1)e^{(2\,bx+2\,a)}+1\right)+\operatorname{Li}_2\left(-(d+1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right)$$
$$+x\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)$$

input
```
integrate(arccoth(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*b*d*(2*x^2/d - (2*b*x*log((d + 1)*e^(2*b*x + 2*a) + 1) + dilog(-(d + 1
)*e^(2*b*x + 2*a)))/(b^2*d)) + x*arccoth(d*tanh(b*x + a) + d + 1)
```

3.210.8 Giac [F]

$$\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx=\int \operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(arccoth(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(d*tanh(b*x + a) + d + 1), x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx=\int \operatorname{acoth}(d+d\tanh(a+b\,x)+1)\,dx$$

input
```
int(acoth(d + d*tanh(a + b*x) + 1),x)
```

output
```
int(acoth(d + d*tanh(a + b*x) + 1), x)
```

3.211 $\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$

3.211.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx=\operatorname{Int}\left(\frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccoth(1+d+d*tanh(b*x+a))/x,x)
```

3.211.2 Mathematica [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx=\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[1 + d + d*Tanh[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[1 + d + d*Tanh[a + b*x]]/x, x]
```

3.211.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d\tanh(a+bx)+d+1)}{x} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\coth^{-1}(d\tanh(a+bx)+d+1)}{x} dx$$

input
```
Int[ArcCoth[1 + d + d*Tanh[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.211.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.211.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(1+d+d\tanh\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(1+d+d*tanh(b*x+a))/x,x)
```

output
```
int(arccoth(1+d+d*tanh(b*x+a))/x,x)
```

3.211.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+d+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arccoth(d*tanh(b*x + a) + d + 1)/x, x)
```

3.211.6 Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{acoth}\left(d\tanh\left(a+bx\right)+d+1\right)}{x}\,dx$$

input
```
integrate(acoth(1+d+d*tanh(b*x+a))/x,x)
```

output
```
Integral(acoth(d*tanh(a + b*x) + d + 1)/x, x)
```

3.211.7 Maxima [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+d+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arccoth(d*tanh(b*x + a) + d + 1)/x, x)
```

3.211.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+d+d*tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(d*tanh(b*x + a) + d + 1)/x, x)
```

3.211.9 Mupad [N/A]

Not integrable

Time = 4.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{acoth}(d+d\tanh(a+b\,x)+1)}{x}\,dx$$

input
```
int(acoth(d + d*tanh(a + b*x) + 1)/x,x)
```

output
```
int(acoth(d + d*tanh(a + b*x) + 1)/x, x)
```

3.212 $\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx$

3.212.1 Optimal result

Integrand size = 19, antiderivative size = 168

$$\begin{aligned}\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = {} & \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx)) \\ & -\frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2bx}\right) \\ & -\frac{x^3\operatorname{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b} \\ & +\frac{3x^2\operatorname{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{8b^2} \\ & -\frac{3x\operatorname{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{8b^3} \\ & +\frac{3\operatorname{PolyLog}\left(5,-\left((1-d)e^{2a+2bx}\right)\right)}{16b^4}\end{aligned}$$

output

```
1/20*b*x^5+1/4*x^4*arccoth(1-d-d*tanh(b*x+a))-1/8*x^4*ln(1+(1-d)*exp(2*b*x
+2*a))-1/4*x^3*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,-(1-d)
*exp(2*b*x+2*a))/b^2-3/8*x*polylog(4,-(1-d)*exp(2*b*x+2*a))/b^3+3/16*polyl
og(5,-(1-d)*exp(2*b*x+2*a))/b^4
```

3.212.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.88

$$\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx$$
$$= \frac{4b^4x^4\coth^{-1}(1-d-d\tanh(a+bx)) - 2b^4x^4\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right) + 4b^3x^3\operatorname{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right) + 6b^2x^2\operatorname{P}}{16b^4}$$

input
```
Integrate[x^3*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

output
```
(4*b^4*x^4*ArcCoth[1 - d - d*Tanh[a + b*x]] - 2*b^4*x^4*Log[1 - 1/((-1 + d
)*E^(2*(a + b*x)))] + 4*b^3*x^3*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))] +
 6*b^2*x^2*PolyLog[3, 1/((-1 + d)*E^(2*(a + b*x)))] + 6*b*x*PolyLog[4, 1/(
(-1 + d)*E^(2*(a + b*x)))] + 3*PolyLog[5, 1/((-1 + d)*E^(2*(a + b*x)))])/(
16*b^4)
```

3.212.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {6794, 2615, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(d(-\tanh(a+bx))-d+1)\,dx$$
$$\Big\downarrow \text{6794}$$
$$\frac{1}{4}b\int \frac{x^4}{e^{2a+2bx}(1-d)+1}dx + \frac{1}{4}x^4\coth^{-1}(d(-\tanh(a+bx))-d+1)$$
$$\Big\downarrow \text{2615}$$
$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\int \frac{e^{2a+2bx}x^4}{e^{2a+2bx}(1-d)+1}dx\right) + \frac{1}{4}x^4\coth^{-1}(d(-\tanh(a+bx))-d+1)$$
$$\Big\downarrow \text{2620}$$

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\int x^3\log\left(e^{2a+2bx}(1-d)+1\right)dx}{b(1-d)}\right)\right)+\frac{1}{4}x^4\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$\downarrow$ 3011

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\left(\frac{3\int x^2\,\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)dx}{2b}-\frac{x^3\,\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}\right.}{b(1-d)}\right.\right.\frac{1}{4}x^4\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$\downarrow$ 7163

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\int x\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)dx}{b}\right)}{2b}-\right.}{b(1-d)}\right.\right.\frac{1}{4}x^4\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$\downarrow$ 7163

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\,\text{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\int \text{PolyLo}}{}}{b}\right.}{2b}\right.}{b(1-d)}\right.\right.\frac{1}{4}x^4\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$\downarrow$ 2720

3.212. $\int x^3\coth^{-1}(1-d-d\tanh(a+bx))\,dx$

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\int e^{-2a-}}{}}{}\right)}{2b}\right.}{b(1}\right.\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{1}{4}b\left(\frac{x^5}{5}-(1-d)\left(\frac{x^4\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\operatorname{PolyLog}}{}}{b}\right)}{2b}\right.}{b(1-d)}\right.\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

input
```
Int[x^3*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

output
```
(x^4*ArcCoth[1 - d - d*Tanh[a + b*x]])/4 + (b*(x^5/5 - (1 - d)*((x^4*Log[1
 + (1 - d)*E^(2*a + 2*b*x)])/(2*b*(1 - d)) - (2*(-1/2*(x^3*PolyLog[2, -((1
 - d)*E^(2*a + 2*b*x))])/b + (3*((x^2*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x)
)])/(2*b) - ((x*PolyLog[4, -((1 - d)*E^(2*a + 2*b*x))])/(2*b) - PolyLog[5,
 -((1 - d)*E^(2*a + 2*b*x))]/(4*b^2))/b))/(2*b)))/(b*(1 - d)))))/4
```

3.212. $\int x^3\coth^{-1}(1-d-d\tanh(a+bx))\,dx$

3.212.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6794
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.212.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.02 (sec) , antiderivative size = 1754, normalized size of antiderivative = 10.44

method	result	size
risch	Expression too large to display	1754

input
```
int(x^3*arccoth(1-d-d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^3/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x*a^3-1/4/b*d/(d-1)*polylog(2,(d-
1)*exp(2*b*x+2*a))*x^3-3/8/b^4*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^4+3/8/
b^2*d/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))*x^2-1/4/b^4*d/(d-1)*polylog(2,
(d-1)*exp(2*b*x+2*a))*a^3-3/8/b^3*d/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))*
x-1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x-1/2/b^3*a^3/(d-1)*ln(1+
exp(b*x+a)*(d-1)^(1/2))*x+1/2/b^4*d*a^4/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))
+1/2/b^4*d*a^4/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))+1/2/b^4*d*a^3/(d-1)*dilo
g(1-exp(b*x+a)*(d-1)^(1/2))-1/8/b^4*d*a^4/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*
b*x+2*a)-1)+1/20*b*x^5-1/8*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^4-1/2/b^4*
a^4/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))-1/2/b^4*a^3/(d-1)*dilog(1-exp(b*x+a
)*(d-1)^(1/2))-1/2/b^4*a^3/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))+3/16/b^4*
d/(d-1)*polylog(5,(d-1)*exp(2*b*x+2*a))+1/4/b/(d-1)*polylog(2,(d-1)*exp(2*
b*x+2*a))*x^3+3/8/b^4/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^4-3/8/b^2/(d-1)*p
olylog(3,(d-1)*exp(2*b*x+2*a))*x^2+1/4/b^4/(d-1)*polylog(2,(d-1)*exp(2*b*x
+2*a))*a^3+3/8/b^3/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))*x-1/2/b^4*a^4/(d-
1)*ln(1-exp(b*x+a)*(d-1)^(1/2))+1/8/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^4-3
/16/b^4/(d-1)*polylog(5,(d-1)*exp(2*b*x+2*a))+1/8/b^4*a^4/(d-1)*ln(d*exp(2
*b*x+2*a)-exp(2*b*x+2*a)-1)-1/4*x^4*ln(exp(b*x+a))+1/2/b^4*d*a^3/(d-1)*dil
og(1+exp(b*x+a)*(d-1)^(1/2))+1/2/b^3*d*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/
2))*x+1/2/b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x-1/2/b^3*d/(d-1...
```

3.212.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(135) = 270$.

Time = 0.27 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.52

$$\int x^3 \coth^{-1}(1 - d - d\tanh(a + bx))\, dx$$
$$= \frac{2\, b^5 x^5 - 5\, b^4 x^4 \log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right) - 20\, b^3 x^3 \mathrm{Li}_2\left(\sqrt{d-1}(\cosh(bx+a) + \sinh(bx+a))\right) - 20}{}$$

input
```
integrate(x^3*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/40*(2*b^5*x^5 - 5*b^4*x^4*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d -
2)*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(sqrt(d - 1)*(cosh(
b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-sqrt(d - 1)*(cosh(b*x + a)
+ sinh(b*x + a))) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x
 + a) + 2*sqrt(d - 1)) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sin
h(b*x + a) - 2*sqrt(d - 1)) + 60*b^2*x^2*polylog(3, sqrt(d - 1)*(cosh(b*x
+ a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -sqrt(d - 1)*(cosh(b*x + a)
 + sinh(b*x + a))) - 120*b*x*polylog(4, sqrt(d - 1)*(cosh(b*x + a) + sinh(
b*x + a))) - 120*b*x*polylog(4, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a
))) - 5*(b^4*x^4 - a^4)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) +
1) - 5*(b^4*x^4 - a^4)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) +
1) + 120*polylog(5, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) + 120*pol
ylog(5, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

3.212.6 Sympy [F]

$$\int x^3 \coth^{-1}(1 - d - d\tanh(a + bx))\, dx = -\int x^3 \operatorname{acoth}(d\tanh(a + bx) + d - 1)\, dx$$

input
```
integrate(x**3*acoth(1-d-d*tanh(b*x+a)),x)
```

output
```
-Integral(x**3*acoth(d*tanh(a + b*x) + d - 1), x)
```

3.212.7 Maxima [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.87

$$\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = -\frac{1}{4}\,x^4 \operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d-1\right) + \frac{1}{40}\left(\frac{2\,x^5}{d} - \frac{5\left(2\,b^4x^4\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+4\,b^3x^3\mathrm{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right)-6\,b^2x^2\mathrm{Li}_3((d-1)e^{(2\,bx}\right.}{b^5d}\right.$$

input
```
integrate(x^3*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/4*x^4*arccoth(d*tanh(b*x + a) + d - 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*l
og(-(d - 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog((d - 1)*e^(2*b*x + 2*a)
) - 6*b^2*x^2*polylog(3, (d - 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, (d -
1)*e^(2*b*x + 2*a)) - 3*polylog(5, (d - 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

3.212.8 Giac [F]

$$\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \int x^3 \operatorname{arcoth}\left(-d\tanh\left(bx+a\right)-d+1\right)\,dx$$

input
```
integrate(x^3*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^3*arccoth(-d*tanh(b*x + a) - d + 1), x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \int -x^3\,\mathrm{acoth}(d+d\tanh(a+b\,x)-1)\,dx$$

input
```
int(-x^3*acoth(d + d*tanh(a + b*x) - 1),x)
```

output
```
int(-x^3*acoth(d + d*tanh(a + b*x) - 1), x)
```

3.213 $\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx$

3.213.1 Optimal result

Integrand size = 19, antiderivative size = 139

$$\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(1-d-d\tanh(a+bx)) -\frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2bx}\right) -\frac{x^2\operatorname{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b} +\frac{x\operatorname{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{4b^2} -\frac{\operatorname{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{8b^3}$$

output
```
1/12*b*x^4+1/3*x^3*arccoth(1-d-d*tanh(b*x+a))-1/6*x^3*ln(1+(1-d)*exp(2*b*x
+2*a))-1/4*x^2*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,-(1-d)*e
xp(2*b*x+2*a))/b^2-1/8*polylog(4,-(1-d)*exp(2*b*x+2*a))/b^3
```

3.213.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx$$
$$= \frac{8b^3x^3\coth^{-1}(1-d-d\tanh(a+bx)) - 4b^3x^3\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right) + 6b^2x^2\operatorname{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right) + 6bx\operatorname{Pol}}{24b^3}$$

input
```
Integrate[x^2*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCoth[1 - d - d*Tanh[a + b*x]] - 4*b^3*x^3*Log[1 - 1/((-1 + d
)*E^(2*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))] +
 6*b*x*PolyLog[3, 1/((-1 + d)*E^(2*(a + b*x)))] + 3*PolyLog[4, 1/((-1 + d)
*E^(2*(a + b*x)))])/(24*b^3)
```

3.213.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6794, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\coth^{-1}(d(-\tanh(a+bx))-d+1)\,dx$$
$$\Big\downarrow \; 6794$$
$$\frac{1}{3}b\int \frac{x^3}{e^{2a+2bx}(1-d)+1}dx+\frac{1}{3}x^3\coth^{-1}(d(-\tanh(a+bx))-d+1)$$
$$\Big\downarrow \; 2615$$
$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\int \frac{e^{2a+2bx}x^3}{e^{2a+2bx}(1-d)+1}dx\right)+\frac{1}{3}x^3\coth^{-1}(d(-\tanh(a+bx))-d+1)$$
$$\Big\downarrow \; 2620$$
$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\left(\frac{x^3\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{3\int x^2\log\left(e^{2a+2bx}(1-d)+1\right)dx}{2b(1-d)}\right)\right)+$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \; 3011$$

$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\left(\frac{x^3\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}\right)}{2b(1-d)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \; 7163$$

$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\left(\frac{x^3\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)dx}{2b}}{b}-\frac{x^2\,\text{Poly}\cdots}{\cdots}\right.}{2b(1-d)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\left(\frac{x^3\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)de^{2a\cdots}}{4b^2}}{b}\cdots\right.}{2b(1-d)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \; 7143$$

$$\frac{1}{3}b\left(\frac{x^4}{4}-(1-d)\left(\frac{x^3\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{2b}-\frac{\text{PolyLog}\left(4,-\left((1-d)e^{2a+2bx}\right)\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\cdots}{\cdots}\right.}{2b(1-d)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

input
```
Int[x^2*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

3.213. $\int x^2\coth^{-1}(1-d-d\tanh(a+bx))\,dx$

output
```
(x^3*ArcCoth[1 - d - d*Tanh[a + b*x]])/3 + (b*(x^4/4 - (1 - d)*((x^3*Log[1
 + (1 - d)*E^(2*a + 2*b*x)])/(2*b*(1 - d)) - (3*(-1/2*(x^2*PolyLog[2, -((1
 - d)*E^(2*a + 2*b*x))])/b + ((x*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))])/(
2*b) - PolyLog[4, -((1 - d)*E^(2*a + 2*b*x))]/(4*b^2))/b))/(2*b*(1 - d))))
)/3
```

3.213.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6794
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.213.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.94 (sec) , antiderivative size = 1697, normalized size of antiderivative = 12.21

method	result	size
risch	Expression too large to display	1697

input
```
int(x^2*arccoth(1-d-d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2
*b*x+2*a)-exp(2*b*x+2*a)-1))^2-2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1)*d*exp(2*b*
x+2*a))^2+2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a
)-1))^2+I*Pi*csgn(I*d)*csgn(I/(exp(2*b*x+2*a)+1)*d*exp(2*b*x+2*a))^2-I*Pi*
csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(e
xp(2*b*x+2*a)+1))+I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)+1))^2-I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-I*Pi*csgn
(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^3-I*Pi*csgn(I*e
xp(2*b*x+2*a))^3+I*Pi*csgn(I/(exp(2*b*x+2*a)+1)*d*exp(2*b*x+2*a))^3+I*Pi*c
sgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*I*
Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-I*Pi*csgn(I*exp(2*b*x+2*a)/
(exp(2*b*x+2*a)+1))^3+I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(d*exp(2*b*x+
2*a)-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*
b*x+2*a)-1))-I*Pi*csgn(I*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))*csgn(I/(exp(
2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^2+I*Pi*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*d*exp(2*b*x+2*a))^2-I*
Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d)*csgn(I/(exp(2*b*x+2
*a)+1)*d*exp(2*b*x+2*a))+2*ln(d))*x^3+1/8/b^3/(d-1)*polylog(4,(d-1)*exp(2*
b*x+2*a))+1/6/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^3+1/12*b*x^4-1/3*x^3*ln(e
xp(b*x+a))+1/2/b^2*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^2*x-1/2/b^2*a^2...
```

3.213.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(112) = 224$.

Time = 0.28 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.58

$$\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx$$

$$= \frac{b^4x^4 - 2\,b^3x^3\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\sqrt{d-1}(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,b^2x^2 \cdots}{\cdots}$$

input
```
integrate(x^2*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(b^4*x^4 - 2*b^3*x^3*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)
*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d - 1)*(cosh(b*x
 + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d - 1)*(cosh(b*x + a) + si
nh(b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a
) + 2*sqrt(d - 1)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*
x + a) - 2*sqrt(d - 1)) + 12*b*x*polylog(3, sqrt(d - 1)*(cosh(b*x + a) + s
inh(b*x + a))) + 12*b*x*polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x
+ a))) - 2*(b^3*x^3 + a^3)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))
 + 1) - 2*(b^3*x^3 + a^3)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))
 + 1) - 12*polylog(4, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*po
lylog(4, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

3.213.6 Sympy [F]

$$\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = -\int x^2 \operatorname{acoth}\left(d\tanh\left(a+bx\right)+d-1\right)dx$$

input
```
integrate(x**2*acoth(1-d-d*tanh(b*x+a)),x)
```

output
```
-Integral(x**2*acoth(d*tanh(a + b*x) + d - 1), x)
```

3.213.7 Maxima [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = -\frac{1}{3}x^3 \operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d-1\right) + \frac{1}{36}\left(\frac{3x^4}{d} - \frac{2\left(4b^3x^3\log\left(-(d-1)e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left((d-1)e^{(2bx+2a)}\right)-6bx\mathrm{Li}_3((d-1)e^{(2bx+}\right.}{b^4d}\right.$$

input
```
integrate(x^2*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-1/3*x^3*arccoth(d*tanh(b*x + a) + d - 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*l
og(-(d - 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d - 1)*e^(2*b*x + 2*a)
) - 6*b*x*polylog(3, (d - 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d - 1)*e^(2*
b*x + 2*a)))/(b^4*d))*b*d
```

3.213.8 Giac [F]

$$\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \int x^2 \operatorname{arcoth}\left(-d\tanh\left(bx+a\right)-d+1\right)\,dx$$

input
```
integrate(x^2*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(-d*tanh(b*x + a) - d + 1), x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \int -x^2\,\mathrm{acoth}(d+d\tanh(a+b\,x)-1)\,dx$$

input
```
int(-x^2*acoth(d + d*tanh(a + b*x) - 1),x)
```

output
```
int(-x^2*acoth(d + d*tanh(a + b*x) - 1), x)
```

3.214 $\int x \coth^{-1}(1-d-d\tanh(a+bx))\,dx$

3.214.1 Optimal result

Integrand size = 17, antiderivative size = 110

$$\int x \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{4}x^2\log\left(1+(1-d)e^{2a+2bx}\right) - \frac{x\,\mathrm{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b} + \frac{\mathrm{PolyLog}\left(3,-\left((1-d)e^{2a+2bx}\right)\right)}{8b^2}$$

output
```
1/6*b*x^3+1/2*x^2*arccoth(1-d-d*tanh(b*x+a))-1/4*x^2*ln(1+(1-d)*exp(2*b*x+
2*a))-1/4*x*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+1/8*polylog(3,-(1-d)*exp(2*
b*x+2*a))/b^2
```

3.214.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int x\coth^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{2b^2x^2\left(2\coth^{-1}(1-d-d\tanh(a+bx))-\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right)\right)+2bx\,\mathrm{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right)+\mathrm{PolyLog}\left(\right.}{8b^2}$$

input
```
Integrate[x*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcCoth[1 - d - d*Tanh[a + b*x]] - Log[1 - 1/((-1 + d)*E^(2*
(a + b*x)))]) + 2*b*x*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))] + PolyLog[3
, 1/((-1 + d)*E^(2*(a + b*x)))])/(8*b^2)
```

3.214.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6794, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(d(-\tanh(a+bx))-d+1)\,dx$$

$$\Big\downarrow \text{6794}$$

$$\frac{1}{2}b\int \frac{x^2}{e^{2a+2bx}(1-d)+1}dx+\frac{1}{2}x^2\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(1-d)\int \frac{e^{2a+2bx}x^2}{e^{2a+2bx}(1-d)+1}dx\right)+\frac{1}{2}x^2\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(1-d)\left(\frac{x^2\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\int x\log\left(e^{2a+2bx}(1-d)+1\right)dx}{b(1-d)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(1-d)\left(\frac{x^2\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\frac{\int \text{PolyLog}(2,-((1-d)e^{2a+2bx}))dx}{2b}-\frac{x\,\text{PolyLog}(2,-((1-d)e^{2a+2bx}))}{2b}}{b(1-d)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(1-d)\left(\frac{x^2\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\frac{\int e^{-2a-2bx}\operatorname{PolyLog}(2,-((1-d)e^{2a+2bx}))de^{2a+2bx}}{4b^2}-\frac{x\operatorname{PolyLog}(2,-((1-d)e^{2a}}{2b}}{b(1-d)}\right.\right.$$
$$\frac{1}{2}x^2\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}b\left(\frac{x^3}{3}-(1-d)\left(\frac{x^2\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\frac{\operatorname{PolyLog}(3,-((1-d)e^{2a+2bx}))}{4b^2}-\frac{x\operatorname{PolyLog}(2,-((1-d)e^{2a+2bx}))}{2b}}{b(1-d)}\right)\right)+$$
$$\frac{1}{2}x^2\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

input
```
Int[x*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 - d - d*Tanh[a + b*x]])/2 + (b*(x^3/3 - (1 - d)*((x^2*Log[1
 + (1 - d)*E^(2*a + 2*b*x)])/(2*b*(1 - d)) - (-1/2*(x*PolyLog[2, -((1 - d)
*E^(2*a + 2*b*x))])/b + PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))]/(4*b^2))/(b
*(1 - d)))))/2
```

3.214.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6794
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.214.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.62 (sec) , antiderivative size = 1616, normalized size of antiderivative = 14.69

method	result	size
risch	Expression too large to display	1616

input
```
int(x*arccoth(1-d-d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*a/(d-1)*x*ln(1+exp(b*x+a)*(d-1)^(1/2))-1/2/b^2*a/(d-1)*dilog(1+exp(
b*x+a)*(d-1)^(1/2))+1/8/b^2*d/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))+1/4/b^
2/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^2+1/4/b/(d-1)*polylog(2,(d-1)*exp(2*b
*x+2*a))*x+1/4/b^2/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a-1/2/b^2*a^2/(d-
1)*ln(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1
/2))-1/2/b^2*a/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))+1/4/b^2*a^2/(d-1)*ln(
d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1)-1/4*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*
x^2+1/2/b^2*a^2*d/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b^2*a^2*d/(d-1)*l
n(1+exp(b*x+a)*(d-1)^(1/2))+1/2/b^2*a*d/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/
2))+1/2/b^2*a*d/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))+1/2/b/(d-1)*ln(1-(d-
1)*exp(2*b*x+2*a))*a*x-1/4/b^2*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^2-1/4/
b*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x-1/4/b^2*d/(d-1)*polylog(2,(d-1
)*exp(2*b*x+2*a))*a-1/4/b^2*a^2*d/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)
-1)-1/2/b*a/(d-1)*x*ln(1-exp(b*x+a)*(d-1)^(1/2))-1/2*x^2*ln(exp(b*x+a))+1/
6*b*x^3+1/2/b*a*d/(d-1)*x*ln(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b*a*d/(d-1)*x*l
n(1+exp(b*x+a)*(d-1)^(1/2))+1/4*x^2*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1)+
1/4/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^2-1/8/b^2/(d-1)*polylog(3,(d-1)*exp
(2*b*x+2*a))-1/2/b*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a*x-1/8*(-I*Pi*csgn(
I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*
x+2*a)-1))^2-2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1)*d*exp(2*b*x+2*a))^2+2*I*P...
```

3.214.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(89) = 178$.

Time = 0.26 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.77

$$\int x\coth^{-1}(1-d-d\tanh(a+bx))\,dx$$

$$= \frac{2\,b^3x^3 - 3\,b^2x^2\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,bx\mathrm{Li}_2\left(\sqrt{d-1}(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,bx\mathrm{Li}}{}$$

input
```
integrate(x*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*x^3 - 3*b^2*x^2*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d -
2)*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(sqrt(d - 1)*(cosh(b*x +
 a) + sinh(b*x + a))) - 6*b*x*dilog(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x
 + a))) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) + 2*
sqrt(d - 1)) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a)
 - 2*sqrt(d - 1)) - 3*(b^2*x^2 - a^2)*log(sqrt(d - 1)*(cosh(b*x + a) + sin
h(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-sqrt(d - 1)*(cosh(b*x + a) + sin
h(b*x + a)) + 1) + 6*polylog(3, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)
)) + 6*polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

3.214.6 Sympy [F]

$$\int x \coth^{-1}(1-d-d\tanh(a+bx))\,dx = -\int x\,\mathrm{acoth}\left(d\tanh\left(a+bx\right)+d-1\right)dx$$

input
```
integrate(x*acoth(1-d-d*tanh(b*x+a)),x)
```

output
```
-Integral(x*acoth(d*tanh(a + b*x) + d - 1), x)
```

3.214.7 Maxima [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

$$\int x \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left((d-1)e^{(2\,bx+2\,a)}\right)\right)}{b^3d}\right)b - \frac{1}{2}\,x^2\,\mathrm{arcoth}\left(d\tanh\left(bx+a\right)+d-1\right)$$

input
```
integrate(x*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(4*x^3/d - 3*(2*b^2*x^2*log(-(d - 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dil
og((d - 1)*e^(2*b*x + 2*a)) - polylog(3, (d - 1)*e^(2*b*x + 2*a)))/(b^3*d)
)*b*d - 1/2*x^2*arccoth(d*tanh(b*x + a) + d - 1)
```

3.214.8 Giac [F]

$$\int x \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \int x \operatorname{arcoth}\left(-d\tanh\left(bx+a\right)-d+1\right)\,dx$$

input
```
integrate(x*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(-d*tanh(b*x + a) - d + 1), x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \int -x\,\mathrm{acoth}(d+d\tanh(a+b\,x)-1)\,dx$$

input
```
int(-x*acoth(d + d*tanh(a + b*x) - 1),x)
```

output
```
int(-x*acoth(d + d*tanh(a + b*x) - 1), x)
```

3.215 $\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx$

3.215.1 Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{bx^2}{2}+x\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1-d)e^{2a+2bx}\right) - \frac{\operatorname{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b}$$

output
```
1/2*b*x^2+x*arccoth(1-d-d*tanh(b*x+a))-1/2*x*ln(1+(1-d)*exp(2*b*x+2*a))-1/
4*polylog(2,-(1-d)*exp(2*b*x+2*a))/b
```

3.215.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx = x\coth^{-1}(1-d-d\tanh(a+bx)) + \frac{-2bx\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right)+\operatorname{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right)}{4b}$$

input
```
Integrate[ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

output
```
x*ArcCoth[1 - d - d*Tanh[a + b*x]] + (-2*b*x*Log[1 - 1/((-1 + d)*E^(2*(a +
 b*x)))] + PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))])/(4*b)
```

3.215.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6786, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d(-\tanh(a+bx))-d+1)\,dx$$

$$\Big\downarrow \text{6786}$$

$$b\int \frac{x}{e^{2a+2bx}(1-d)+1}dx+x\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{2615}$$

$$b\left(\frac{x^2}{2}-(1-d)\int \frac{e^{2a+2bx}x}{e^{2a+2bx}(1-d)+1}dx\right)+x\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{2620}$$

$$b\left(\frac{x^2}{2}-(1-d)\left(\frac{x\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\int \log\left(e^{2a+2bx}(1-d)+1\right)dx}{2b(1-d)}\right)\right)+ x\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{2715}$$

$$b\left(\frac{x^2}{2}-(1-d)\left(\frac{x\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}-\frac{\int e^{-2a-2bx}\log\left(e^{2a+2bx}(1-d)+1\right)de^{2a+2bx}}{4b^2(1-d)}\right)\right)+ x\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

$$\Big\downarrow \text{2838}$$

$$b\left(\frac{x^2}{2}-(1-d)\left(\frac{\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b^2(1-d)}+\frac{x\log\left((1-d)e^{2a+2bx}+1\right)}{2b(1-d)}\right)\right)+ x\coth^{-1}(d(-\tanh(a+bx))-d+1)$$

input
```
Int[ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

output
```
x*ArcCoth[1 - d - d*Tanh[a + b*x]] + b*(x^2/2 - (1 - d)*((x*Log[1 + (1 - d
)*E^(2*a + 2*b*x)])/(2*b*(1 - d)) + PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))]
/(4*b^2*(1 - d))))
```

3.215.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6786
```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Tanh[a + b*x]], x] + Simp[b   Int[x/(c - d + c*E^(2*a + 2*b*x))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

3.215.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(68) = 136$.

Time = 1.63 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.57

method	result
derivativedivides	$-\frac{-\frac{\operatorname{arccoth}(1-d-d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}+\frac{\operatorname{arccoth}(1-d-d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}-\frac{d^2\left(-\frac{\operatorname{dilog}\left(-\frac{d\tanh}{}\right.}{}\right.}{}}{}$
default	$-\frac{-\frac{\operatorname{arccoth}(1-d-d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}+\frac{\operatorname{arccoth}(1-d-d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}-\frac{d^2\left(-\frac{\operatorname{dilog}\left(-\frac{d\tanh}{}\right.}{}\right.}{}}{}$
risch	Expression too large to display

input
```
int(arccoth(1-d-d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/b/d*(-1/2*arccoth(1-d-d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)-d)+1/2*arccoth
(1-d-d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)-1/2*d^2*(1/d*(-1/2*dilog(-1/2*d
*tanh(b*x+a)-1/2*d+1)-1/2*ln(-d*tanh(b*x+a)-d)*ln(-1/2*d*tanh(b*x+a)-1/2*d
+1)+1/4*ln(-d*tanh(b*x+a)-d)^2)-1/d*(-1/2*dilog((-d*tanh(b*x+a)-d+2)/(-2*d
+2))-1/2*ln(-d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)-d+2)/(-2*d+2))+1/2*dilog(
-1/2*(-d*tanh(b*x+a)-d)/d)+1/2*ln(-d*tanh(b*x+a)+d)*ln(-1/2*(-d*tanh(b*x+a
)-d)/d))))
```

3.215.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(63) = 126.

Time = 0.29 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.99

$$\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx$$
$$=\frac{b^2x^2-bx\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right)+a\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)+2\,\sqrt{d}\right.}{}$$

input
```
integrate(arccoth(1-d-d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 - b*x*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)*cosh(b
*x + a) + d*sinh(b*x + a))) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*si
nh(b*x + a) + 2*sqrt(d - 1)) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*s
inh(b*x + a) - 2*sqrt(d - 1)) - (b*x + a)*log(sqrt(d - 1)*(cosh(b*x + a) +
 sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*
x + a)) + 1) - dilog(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(
-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

3.215.6 Sympy [F]

$$\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx=-\int \operatorname{acoth}\left(d\tanh\left(a+bx\right)+d-1\right)dx$$

input
```
integrate(acoth(1-d-d*tanh(b*x+a)),x)
```

output
```
-Integral(acoth(d*tanh(a + b*x) + d - 1), x)
```

3.215.7 Maxima [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{4}\,bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right) - x\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d-1\right)$$

input
```
integrate(arccoth(1-d-d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*b*d*(2*x^2/d - (2*b*x*log(-(d - 1)*e^(2*b*x + 2*a) + 1) + dilog((d - 1
)*e^(2*b*x + 2*a)))/(b^2*d)) - x*arccoth(d*tanh(b*x + a) + d - 1)
```

3.215.8 Giac [F]

$$\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \int \operatorname{arcoth}\left(-d\tanh\left(bx+a\right)-d+1\right)\,dx$$

input
```
integrate(arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(-d*tanh(b*x + a) - d + 1), x)
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \int -\operatorname{acoth}(d+d\tanh(a+b\,x)-1)\,dx$$

input
```
int(-acoth(d + d*tanh(a + b*x) - 1),x)
```

output
```
int(-acoth(d + d*tanh(a + b*x) - 1), x)
```

3.216 $\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$

3.216.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx=\text{Int}\left(\frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccoth(1-d-d*tanh(b*x+a))/x,x)
```

3.216.2 Mathematica [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx=\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[1 - d - d*Tanh[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[1 - d - d*Tanh[a + b*x]]/x, x]
```

3.216.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d(-\tanh(a+bx))-d+1)}{x}\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\coth^{-1}(d(-\tanh(a+bx))-d+1)}{x}dx$$

input
```
Int[ArcCoth[1 - d - d*Tanh[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.216.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.216.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(1-d-d\tanh\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(1-d-d*tanh(b*x+a))/x,x)
```

output
```
int(arccoth(1-d-d*tanh(b*x+a))/x,x)
```

3.216.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(-d\tanh\left(bx+a\right)-d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-d-d*tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(-arccoth(d*tanh(b*x + a) + d - 1)/x, x)
```

3.216.6 Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx = -\int \frac{\operatorname{acoth}\left(d\tanh\left(a+bx\right)+d-1\right)}{x}\,dx$$

input
```
integrate(acoth(1-d-d*tanh(b*x+a))/x,x)
```

output
```
-Integral(acoth(d*tanh(a + b*x) + d - 1)/x, x)
```

3.216.7 Maxima [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(-d\tanh\left(bx+a\right)-d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-d-d*tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
-integrate(arccoth(d*tanh(b*x + a) + d - 1)/x, x)
```

3.216.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(-d\tanh\left(bx+a\right)-d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-d-d*tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(-d*tanh(b*x + a) - d + 1)/x, x)
```

3.216.9 Mupad [N/A]

Not integrable

Time = 4.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx = \int -\frac{\operatorname{acoth}(d+d\tanh(a+b\,x)-1)}{x}\,dx$$

input
```
int(-acoth(d + d*tanh(a + b*x) - 1)/x,x)
```

output
```
int(-acoth(d + d*tanh(a + b*x) - 1)/x, x)
```

3.217 $\int x^2 \coth^{-1}(c + d\coth(a + bx))\, dx$

3.217.1 Optimal result

Integrand size = 15, antiderivative size = 303

$$\begin{aligned}\int x^2 \coth^{-1}(c + d\coth(a + bx))\, dx = {} & \frac{1}{3}x^3 \coth^{-1}(c + d\coth(a + bx)) \\ & + \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\ & - \frac{1}{6}x^3 \log\left(1 - \frac{(1 + c + d)e^{2a+2bx}}{1 + c - d}\right) \\ & + \frac{x^2 \operatorname{PolyLog}\left(2, \frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b} \\ & - \frac{x^2 \operatorname{PolyLog}\left(2, \frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b} \\ & - \frac{x \operatorname{PolyLog}\left(3, \frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b^2} \\ & + \frac{x \operatorname{PolyLog}\left(3, \frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b^2} \\ & + \frac{\operatorname{PolyLog}\left(4, \frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{8b^3} \\ & - \frac{\operatorname{PolyLog}\left(4, \frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arccoth(c+d*coth(b*x+a))+1/6*x^3*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+
d))-1/6*x^3*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x^2*polylog(2,(1-c-d)
*exp(2*b*x+2*a)/(1-c+d))/b-1/4*x^2*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d
))/b-1/4*x*polylog(3,(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/4*x*polylog(3,(
1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^2+1/8*polylog(4,(1-c-d)*exp(2*b*x+2*a)/(1
-c+d))/b^3-1/8*polylog(4,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^3
```

3.217.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.87

$$\int x^2 \coth^{-1}(c+d\coth(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(c+d\coth(a+bx))$$
$$-\frac{-4b^3x^3\log\left(1+\frac{(1-c+d)e^{-2(a+bx)}}{-1+c+d}\right)+4b^3x^3\log\left(1+\frac{(-1-c+d)e^{-2(a+bx)}}{1+c+d}\right)+6b^2x^2\,\text{PolyLog}\left(2,\frac{(-1+c-d)e^{-2(a+b}}{-1+c+d}\right.}{}$$

input
```
Integrate[x^2*ArcCoth[c + d*Coth[a + b*x]],x]
```

output
```
(x^3*ArcCoth[c + d*Coth[a + b*x]])/3 - (-4*b^3*x^3*Log[1 + (1 - c + d)/((-
1 + c + d)*E^(2*(a + b*x)))] + 4*b^3*x^3*Log[1 + (-1 - c + d)/((1 + c + d)
*E^(2*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, (-1 + c - d)/((-1 + c + d)*E^(2*
(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (1 + c - d)/((1 + c + d)*E^(2*(a + b*x
)))] + 6*b*x*PolyLog[3, (-1 + c - d)/((-1 + c + d)*E^(2*(a + b*x)))] - 6*b
*x*PolyLog[3, (1 + c - d)/((1 + c + d)*E^(2*(a + b*x)))] + 3*PolyLog[4, (-
1 + c - d)/((-1 + c + d)*E^(2*(a + b*x)))] - 3*PolyLog[4, (1 + c - d)/((1
+ c + d)*E^(2*(a + b*x)))])/(24*b^3)
```

3.217.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6800, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\coth^{-1}(d\coth(a+bx)+c)\,dx$$

$$\Big\downarrow \ 6800$$

$$-\frac{1}{3}b(-c-d+1)\int \frac{e^{2a+2bx}x^3}{-c-(-c-d+1)e^{2a+2bx}+d+1}dx+\frac{1}{3}b(c+d+1)\int \frac{e^{2a+2bx}x^3}{c-(c+d+1)e^{2a+2bx}-d+1}dx+\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \ 2620$$

$$-\frac{1}{3}b(-c-d+1)\left(\frac{3\int x^2\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b(-c-d+1)}-\frac{x^3\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{3}b(c+d+1)\left(\frac{3\int x^2\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b(c+d+1)}-\frac{x^3\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \ 3011$$

$$-\frac{1}{3}b(-c-d+1)\left(\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}\right)}{2b(-c-d+1)}-\frac{x^3\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{3}b(c+d+1)\left(\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}\right)}{2b(c+d+1)}-\frac{x^3\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \ 7163$$

3.217. $\int x^2\coth^{-1}(c+d\coth(a+bx))\,dx$

$$-\frac{1}{3}b(-c-d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}\right)}{2b(-c-d+1)}-\frac{x^3\log\left(1-\frac{(-c-\cdots}{\cdots}\right.}{2b(-c-d\cdots}\right.$$

$$\frac{1}{3}b(c+d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}\right)}{2b(c+d+1)}-\frac{x^3\log\left(1-\frac{(c+d+1)e^{2\cdots}}{c-d+\cdots}\right.}{2b(c+d+1)}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow\ 2720$$

$$-\frac{1}{3}b(-c-d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}\right)}{2b(-c-d+1)}-\frac{x^3\,\text{l}\cdots}{\cdots}\right.$$

$$\frac{1}{3}b(c+d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}\right)}{2b(c+d+1)}-\frac{x^3\log\left(\cdots\right.}{2\cdots}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow\ 7143$$

3.217. $\int x^2\coth^{-1}(c+d\coth(a+bx))\,dx$

$$-\frac{1}{3}b(-c-d+1)\left(\frac{3\left(\frac{\frac{x\,\mathrm{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}-\frac{\mathrm{PolyLog}\left(4,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}}{b}-\frac{x^2\,\mathrm{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}\right)}{2b(-c-d+1)}-\frac{x^3\log\left(1-\frac{(-c-d+1}{-c+\ldots}\right.}{2b(-c-d+\ldots}\right.$$

$$\frac{1}{3}b(c+d+1)\left(\frac{3\left(\frac{\frac{x\,\mathrm{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}-\frac{\mathrm{PolyLog}\left(4,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}}{b}-\frac{x^2\,\mathrm{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}\right)}{2b(c+d+1)}-\frac{x^3\log\left(1-\frac{(c+d+1)e^{2a+2\ldots}}{c-d+1}\right.}{2b(c+d+1)}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+c)$$

input
```
Int[x^2*ArcCoth[c + d*Coth[a + b*x]],x]
```

output
```
(x^3*ArcCoth[c + d*Coth[a + b*x]])/3 - (b*(1 - c - d)*(-1/2*(x^3*Log[1 - (
(1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(b*(1 - c - d)) + (3*(-1/2*(x^2
*PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/b + ((x*PolyLog[3,
 ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(2*b) - PolyLog[4, ((1 - c -
d)*E^(2*a + 2*b*x))/(1 - c + d)]/(4*b^2))/b))/(2*b*(1 - c - d))))/3 + (b*(
1 + c + d)*(-1/2*(x^3*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/
(b*(1 + c + d)) + (3*(-1/2*(x^2*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(
1 + c - d)])/b + ((x*PolyLog[3, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]
)/(2*b) - PolyLog[4, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(4*b^2))/b
))/(2*b*(1 + c + d))))/3
```

3.217. $\int x^2 \coth^{-1}(c+d\coth(a+bx))\,dx$

3.217.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6800
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + (-Simp[b*((1 - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(
E^(2*a + 2*b*x)/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x))), x], x] + Simp[b
*((1 + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c
- d - (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.217.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.20 (sec) , antiderivative size = 5185, normalized size of antiderivative = 17.11

method	result	size
risch	Expression too large to display	5185

input
```
int(x^2*arccoth(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.217.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(259) = 518$.

Time = 0.29 (sec) , antiderivative size = 879, normalized size of antiderivative = 2.90

$$\int x^2 \coth^{-1}(c + d\coth(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate(x^2*arccoth(c+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(b^3*x^3*log((d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a
) + (c - 1)*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt((c + d + 1)/(c - d + 1)
)*(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt((c + d + 1)/(c
- d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt((c + d -
 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-sqrt(
(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2*(c +
 d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((
c + d + 1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d
+ 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) - a^3*lo
g(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1
)*sqrt((c + d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2
*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1)))
+ 6*b*x*polylog(3, sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x
 + a))) + 6*b*x*polylog(3, -sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) +
 sinh(b*x + a))) - 6*b*x*polylog(3, sqrt((c + d - 1)/(c - d - 1))*(cosh(b*
x + a) + sinh(b*x + a))) - 6*b*x*polylog(3, -sqrt((c + d - 1)/(c - d - 1))
*(cosh(b*x + a) + sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt((c + d + 1)/(
c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3 + a^3)*log(-sq
rt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^
3 + a^3)*log(sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + ...
```

3.217.6 Sympy [F]

$$\int x^2 \coth^{-1}(c + d\coth(a + bx))\,dx = \int x^2 \operatorname{acoth}\left(c + d\coth\left(a + bx\right)\right)dx$$

input
```
integrate(x**2*acoth(c+d*coth(b*x+a)),x)
```

output
```
Integral(x**2*acoth(c + d*coth(a + b*x)), x)
```

3.217.7 Maxima [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.91

$$\int x^2 \coth^{-1}(c+d\coth(a+bx))\,dx = \frac{1}{3}x^3 \operatorname{arcoth}\left(d\coth\left(bx+a\right)+c\right) - \frac{1}{18}bd\left(\frac{4b^3x^3\log\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}+1\right)+6b^2x^2\operatorname{Li}_2\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)-6bx\operatorname{Li}_3\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)+3\operatorname{Li}\right.}{b^4d}$$

input
```
integrate(x^2*arccoth(c+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(d*coth(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log(-(c + d +
1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 6*b^2*x^2*dilog((c + d + 1)*e^(2*b*x
 + 2*a)/(c - d + 1)) - 6*b*x*polylog(3, (c + d + 1)*e^(2*b*x + 2*a)/(c - d
 + 1)) + 3*polylog(4, (c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d) -
(4*b^3*x^3*log(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*d
ilog((c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, (c + d -
1)*e^(2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, (c + d - 1)*e^(2*b*x + 2*a)
/(c - d - 1)))/(b^4*d))
```

3.217.8 Giac [F]

$$\int x^2 \coth^{-1}(c+d\coth(a+bx))\,dx = \int x^2 \operatorname{arcoth}\left(d\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x^2*arccoth(c+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(d*coth(b*x + a) + c), x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(c + d\coth(a + bx))\, dx = \int x^2 \operatorname{acoth}(c + d\coth(a + b\, x))\ dx$$

input
```
int(x^2*acoth(c + d*coth(a + b*x)),x)
```

output
```
int(x^2*acoth(c + d*coth(a + b*x)), x)
```

3.218 $\int x \coth^{-1}(c + d\coth(a + bx))\,dx$

3.218.1 Optimal result

Integrand size = 13, antiderivative size = 229

$$\begin{aligned}\int x\coth^{-1}(c+d\coth(a+bx))\,dx &= \frac{1}{2}x^2\coth^{-1}(c+d\coth(a+bx)) \\ &\quad+\frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) \\ &\quad-\frac{1}{4}x^2\log\left(1-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right) \\ &\quad+\frac{x\,\text{PolyLog}\left(2,\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b} \\ &\quad-\frac{x\,\text{PolyLog}\left(2,\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b} \\ &\quad-\frac{\text{PolyLog}\left(3,\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{8b^2} \\ &\quad+\frac{\text{PolyLog}\left(3,\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{8b^2}\end{aligned}$$

output
```
1/2*x^2*arccoth(c+d*coth(b*x+a))+1/4*x^2*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+
d))-1/4*x^2*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x*polylog(2,(1-c-d)*e
xp(2*b*x+2*a)/(1-c+d))/b-1/4*x*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b
-1/8*polylog(3,(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/8*polylog(3,(1+c+d)*e
xp(2*b*x+2*a)/(1+c-d))/b^2
```

3.218.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.87

$$\int x\coth^{-1}(c+d\coth(a+bx))\,dx$$
$$=\frac{4b^2x^2\coth^{-1}(c+d\coth(a+bx))+2b^2x^2\log\left(1+\frac{(1-c+d)e^{-2(a+bx)}}{-1+c+d}\right)-2b^2x^2\log\left(1+\frac{(-1-c+d)e^{-2(a+bx)}}{1+c+d}\right)-2$$

input
```
Integrate[x*ArcCoth[c + d*Coth[a + b*x]],x]
```

output
```
(4*b^2*x^2*ArcCoth[c + d*Coth[a + b*x]] + 2*b^2*x^2*Log[1 + (1 - c + d)/((
-1 + c + d)*E^(2*(a + b*x)))] - 2*b^2*x^2*Log[1 + (-1 - c + d)/((1 + c + d
)*E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, (-1 + c - d)/((-1 + c + d)*E^(2*(a
+ b*x)))] + 2*b*x*PolyLog[2, (1 + c - d)/((1 + c + d)*E^(2*(a + b*x)))] -
PolyLog[3, (-1 + c - d)/((-1 + c + d)*E^(2*(a + b*x)))] + PolyLog[3, (1 +
c - d)/((1 + c + d)*E^(2*(a + b*x)))])/(8*b^2)
```

3.218.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6800, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(d\coth(a+bx)+c)\,dx$$
$$\Big\downarrow \text{6800}$$
$$-\frac{1}{2}b(-c-d+1)\int\frac{e^{2a+2bx}x^2}{-c-(-c-d+1)e^{2a+2bx}+d+1}dx+\frac{1}{2}b(c+d+1)\int\frac{e^{2a+2bx}x^2}{c-(c+d+1)e^{2a+2bx}-d+1}dx+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+c)$$
$$\Big\downarrow \text{2620}$$

$$-\frac{1}{2}b(-c-d+1)\left(\frac{\int x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{b(-c-d+1)}-\frac{x^2\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{2}b(c+d+1)\left(\frac{\int x\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{b(c+d+1)}-\frac{x^2\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$-\frac{1}{2}b(-c-d+1)\left(\frac{\frac{\int \text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}-\frac{x^2\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{2}b(c+d+1)\left(\frac{\frac{\int \text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}-\frac{x^2\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2720}$$

$$-\frac{1}{2}b(-c-d+1)\left(\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}-\frac{x^2\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{2}b(c+d+1)\left(\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}-\frac{x^2\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{7143}$$

3.218. $\int x\coth^{-1}(c+d\coth(a+bx))\,dx$

$$-\frac{1}{2}b(-c-d+1)\left(\frac{\frac{\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}-\frac{x\,\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b}}{b(-c-d+1)}-\frac{x^2\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+\frac{1}{2}b(c+d+1)\left(\frac{\frac{\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}-\frac{x\,\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b}}{b(c+d+1)}-\frac{x^2\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+c)$$

input
```
Int[x*ArcCoth[c + d*Coth[a + b*x]],x]
```

output
```
(x^2*ArcCoth[c + d*Coth[a + b*x]])/2 - (b*(1 - c - d)*(-1/2*(x^2*Log[1 - (
(1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(b*(1 - c - d)) + (-1/2*(x*Poly
Log[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/b + PolyLog[3, ((1 - c
- d)*E^(2*a + 2*b*x))/(1 - c + d)]/(4*b^2))/(b*(1 - c - d))))/2 + (b*(1 +
c + d)*(-1/2*(x^2*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(b*(
1 + c + d)) + (-1/2*(x*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d
)])/b + PolyLog[3, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(4*b^2))/(b*
(1 + c + d))))/2
```

3.218.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6800
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + (-Simp[b*((1 - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(
E^(2*a + 2*b*x)/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x))), x], x] + Simp[b
*((1 + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c
- d - (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.218.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.95 (sec) , antiderivative size = 4881, normalized size of antiderivative = 21.31

method	result	size
risch	Expression too large to display	4881

input
```
int(x*arccoth(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*a/(c+d-1)*dilog((exp(b*x+a)*c+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)
-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/4/b^2/(c+d-1)*ln(1-(c+d-1)*exp(2*b
*x+2*a)/(c-d-1))*a^2-1/4/b/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(c-d-1
))*x-1/4/b^2/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a-1/8/b^2*d
/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))-1/8/b^2*c/(c+d-1)*polyl
og(3,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))-1/4/b^2*a^2/(c+d-1)*ln(c*exp(2*b*x+2*
a)+d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-c+d+1)+1/4*c/(c+d-1)*ln(1-(c+d-1)*exp(2
*b*x+2*a)/(c-d-1))*x^2+1/4*d/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*
x^2+1/2/b^2*a/(c+d-1)*dilog((-exp(b*x+a)*c-exp(b*x+a)*d+((c-d-1)*(c+d-1))^
(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))+1/2/b^2*a/(1+c+d)*dilog((-exp(b
*x+a)*c-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))
^(1/2))+1/2/b^2*a/(1+c+d)*dilog((exp(b*x+a)*c+exp(b*x+a)*d+((1+c-d)*(1+c+d
))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))-1/4/b^2*a^2/(1+c+d)*ln(c*exp
(2*b*x+2*a)+d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-c+d-1)-1/4/b^2/(1+c+d)*ln(1-(1
+c+d)*exp(2*b*x+2*a)/(1+c-d))*a^2-1/4/b/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*
x+2*a)/(1+c-d))*x+1/8/b^2*c/(1+c+d)*polylog(3,(1+c+d)*exp(2*b*x+2*a)/(1+c-
d))+1/8/b^2*d/(1+c+d)*polylog(3,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))-1/4*d/(1+c
+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x^2-1/4*c/(1+c+d)*ln(1-(1+c+d)*ex
p(2*b*x+2*a)/(1+c-d))*x^2-1/4/b^2/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)
/(1+c-d))*a+1/8*I*Pi*(csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a...
```

3.218.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(195) = 390$.

Time = 0.30 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.18

$$\int x\coth^{-1}(c+d\coth(a+bx))\,dx$$
$$= \frac{b^2x^2\log\left(\frac{d\cosh(bx+a)+(c+1)\sinh(bx+a)}{d\cosh(bx+a)+(c-1)\sinh(bx+a)}\right) - 2\,bx\mathrm{Li}_2\left(\sqrt{\frac{c+d+1}{c-d+1}}(\cosh(bx+a)+\sinh(bx+a))\right) - 2\,bx\mathrm{Li}_2\left(-\sqrt{\frac{c+}{c-}}\right.}{}$$

input
```
integrate(x*arccoth(c+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(b^2*x^2*log((d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a
) + (c - 1)*sinh(b*x + a))) - 2*b*x*dilog(sqrt((c + d + 1)/(c - d + 1))*(c
osh(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt((c + d + 1)/(c - d + 1)
)*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt((c + d - 1)/(c - d -
 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt((c + d - 1)/(c -
 d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cosh(b*x
 + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d + 1)/(c -
d + 1))) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x +
a) - 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) + a^2*log(2*(c + d - 1)*
cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c + d -
1)/(c - d - 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sin
h(b*x + a) - 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) - (b^2*x^2 - a^2
)*log(sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
 (b^2*x^2 - a^2)*log(-sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(
b*x + a)) + 1) + (b^2*x^2 - a^2)*log(sqrt((c + d - 1)/(c - d - 1))*(cosh(b
*x + a) + sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt((c + d - 1)/(c -
 d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3, sqrt((c + d +
 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*polylog(3, -sqrt((c
+ d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, sqrt
((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(...
```

3.218.6 Sympy [F]

$$\int x \coth^{-1}(c + d \coth(a + bx)) \, dx = \int x \operatorname{acoth}(c + d \coth(a + bx)) \, dx$$

input
```
integrate(x*acoth(c+d*coth(b*x+a)),x)
```

output
```
Integral(x*acoth(c + d*coth(a + b*x)), x)
```

3.218.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.93

$$\int x\coth^{-1}(c+d\coth(a+bx))\,dx =$$
$$-\frac{1}{8}bd\left(\frac{2\,b^2x^2\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+2\,bx\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-\mathrm{Li}_3(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1})}{b^3d}-\frac{2\,b^2x^2\log(}{}\right.$$
$$+\frac{1}{2}x^2\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+c\right)$$

input
```
integrate(x*arccoth(c+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*b*d*((2*b^2*x^2*log(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 2
*b*x*dilog((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - polylog(3, (c + d +
1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^3*d) - (2*b^2*x^2*log(-(c + d - 1)*e^(
2*b*x + 2*a)/(c - d - 1) + 1) + 2*b*x*dilog((c + d - 1)*e^(2*b*x + 2*a)/(c
 - d - 1)) - polylog(3, (c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^3*d))
 + 1/2*x^2*arccoth(d*coth(b*x + a) + c)
```

3.218.8 Giac [F]

$$\int x\coth^{-1}(c+d\coth(a+bx))\,dx=\int x\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arccoth(c+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(d*coth(b*x + a) + c), x)
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(c + d \coth(a + bx)) \, dx = \int x \operatorname{acoth}(c + d \coth(a + b x)) \, dx$$

input
```
int(x*acoth(c + d*coth(a + b*x)),x)
```

output
```
int(x*acoth(c + d*coth(a + b*x)), x)
```

3.219 $\int \coth^{-1}(c+d\coth(a+bx))\,dx$

3.219.1 Optimal result

Integrand size = 11, antiderivative size = 150

$$\int \coth^{-1}(c+d\coth(a+bx))\,dx = x\coth^{-1}(c+d\coth(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log\left(1-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right) + \frac{\text{PolyLog}\left(2,\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)}{4b} - \frac{\text{PolyLog}\left(2,\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)}{4b}$$

output
```
x*arccoth(c+d*coth(b*x+a))+1/2*x*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))-1/2*
x*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*polylog(2,(1-c-d)*exp(2*b*x+2*a
)/(1-c+d))/b-1/4*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b
```

3.219.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \coth^{-1}(c+d\coth(a+bx))\,dx = x\coth^{-1}(c+d\coth(a+bx)) - \frac{-2bx\left(\log\left(1-\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)-\log\left(1-\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)\right)-\operatorname{PolyLog}\left(2,\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)+\operatorname{PolyLog}}{4b}$$

input
```
Integrate[ArcCoth[c + d*Coth[a + b*x]],x]
```

output
```
x*ArcCoth[c + d*Coth[a + b*x]] - (-2*b*x*(Log[1 - ((-1 + c + d)*E^(2*(a +
b*x)))/(-1 + c - d)] - Log[1 - ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)])
 - PolyLog[2, ((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d)] + PolyLog[2, ((
1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)])/(4*b)
```

3.219.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.38, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6792, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d\coth(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6792}$$

$$-b(-c-d+1)\int \frac{e^{2a+2bx}x}{-c-(-c-d+1)e^{2a+2bx}+d+1}dx+b(c+d+1)\int \frac{e^{2a+2bx}x}{c-(c+d+1)e^{2a+2bx}-d+1}dx+x\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$-b(-c-d+1)\left(\frac{\int \log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)dx}{2b(-c-d+1)}-\frac{x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+b(c+d+1)\left(\frac{\int \log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)dx}{2b(c+d+1)}-\frac{x\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+x\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2715}$$

$$-b(-c-d+1)\left(\frac{\int e^{-2a-2bx}\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)de^{2a+2bx}}{4b^2(-c-d+1)}-\frac{x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+b(c+d+1)\left(\frac{\int e^{-2a-2bx}\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)de^{2a+2bx}}{4b^2(c+d+1)}-\frac{x\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+x\coth^{-1}(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2838}$$

$$-b(-c-d+1)\left(-\frac{\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2(-c-d+1)}-\frac{x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{2b(-c-d+1)}\right)+b(c+d+1)\left(-\frac{\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2(c+d+1)}-\frac{x\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{2b(c+d+1)}\right)+x\coth^{-1}(d\coth(a+bx)+c)$$

input
```
Int[ArcCoth[c + d*Coth[a + b*x]],x]
```

output
```
x*ArcCoth[c + d*Coth[a + b*x]] - b*(1 - c - d)*(-1/2*(x*Log[1 - ((1 - c -
d)*E^(2*a + 2*b*x))/(1 - c + d)])/(b*(1 - c - d)) - PolyLog[2, ((1 - c - d
)*E^(2*a + 2*b*x))/(1 - c + d)]/(4*b^2*(1 - c - d))) + b*(1 + c + d)*(-1/2
*(x*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(b*(1 + c + d)) -
PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(4*b^2*(1 + c + d)))
```

3.219.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6792
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Coth[a + b*x]], x] + (-Simp[b*(1 - c - d)   Int[x*(E^(2*a + 2*b
*x)/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x))), x], x] + Simp[b*(1 + c + d)
   Int[x*(E^(2*a + 2*b*x)/(1 + c - d - (1 + c + d)*E^(2*a + 2*b*x))), x], x
]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

3.219.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(138) = 276$.

Time = 2.95 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.32

method	result
derivativedivides	$-\frac{\operatorname{arccoth}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{-d\coth(bx+a)-c}{1-c-d}\right.}{2}\right.}{}$
default	$-\frac{\operatorname{arccoth}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{-d\coth(bx+a)-c}{1-c-d}\right.}{2}\right.}{}$
risch	Expression too large to display

input
```
int(arccoth(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-1/2*arccoth(c+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)+1/2*arccoth(c+
d*coth(b*x+a))*d*ln(-d*coth(b*x+a)-d)-1/2*d^2*(1/d*(1/2*dilog((-d*coth(b*x
+a)-c+1)/(1-c-d))+1/2*ln(-d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)-c+1)/(1-c-d)
)-1/2*dilog((-d*coth(b*x+a)-c-1)/(-1-c-d))-1/2*ln(-d*coth(b*x+a)+d)*ln((-d
*coth(b*x+a)-c-1)/(-1-c-d)))-1/d*(-1/2*dilog((-d*coth(b*x+a)-c-1)/(-1-c+d)
)-1/2*ln(-d*coth(b*x+a)-d)*ln((-d*coth(b*x+a)-c-1)/(-1-c+d))+1/2*dilog((-d
*coth(b*x+a)-c+1)/(1-c+d))+1/2*ln(-d*coth(b*x+a)-d)*ln((-d*coth(b*x+a)-c+1
)/(1-c+d)))))
```

3.219.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(128) = 256$.

Time = 0.28 (sec) , antiderivative size = 539, normalized size of antiderivative = 3.59

$$\int \coth^{-1}(c + d\coth(a + bx))\,dx$$
$$= \frac{bx\log\left(\frac{d\cosh(bx+a)+(c+1)\sinh(bx+a)}{d\cosh(bx+a)+(c-1)\sinh(bx+a)}\right) + a\log\left(2\,(c+d+1)\cosh\left(bx+a\right) + 2\,(c+d+1)\sinh\left(bx+a\right) + 2\,(c}{}$$

input
```
integrate(arccoth(c+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b*x*log((d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a) +
(c - 1)*sinh(b*x + a))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1
)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) + a*log(2*(
c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqr
t((c + d + 1)/(c - d + 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d
 - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) - a*log
(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)
*sqrt((c + d - 1)/(c - d - 1))) - (b*x + a)*log(sqrt((c + d + 1)/(c - d +
1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt((c + d + 1)
/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(sqrt((c
 + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*lo
g(-sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - di
log(sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - dilog
(-sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(s
qrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(-sqr
t((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

3.219.6 Sympy [F]

$$\int \coth^{-1}(c + d\coth(a + bx))\,dx = \int \operatorname{acoth}\left(c + d\coth\left(a + bx\right)\right)dx$$

input
```
integrate(acoth(c+d*coth(b*x+a)),x)
```

output
```
Integral(acoth(c + d*coth(a + b*x)), x)
```

3.219.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \coth^{-1}(c+d\coth(a+bx))\,dx =$$
$$-\frac{1}{4}bd\left(\frac{2\,bx\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^2d}-\frac{2\,bx\log\left(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+\mathrm{Li}_2\left(\right.}{b^2d}\right.$$
$$+x\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+c\right)$$

input
```
integrate(arccoth(c+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
-1/4*b*d*((2*b*x*log(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + dilog
((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^2*d) - (2*b*x*log(-(c + d -
1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + dilog((c + d - 1)*e^(2*b*x + 2*a)/(c
 - d - 1)))/(b^2*d)) + x*arccoth(d*coth(b*x + a) + c)
```

3.219.8 Giac [F]

$$\int \coth^{-1}(c+d\coth(a+bx))\,dx = \int \mathrm{arcoth}\left(d\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arccoth(c+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(d*coth(b*x + a) + c), x)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(c+d\coth(a+bx))\,dx = \int \mathrm{acoth}(c+d\coth(a+b\,x))\,dx$$

input
```
int(acoth(c + d*coth(a + b*x)),x)
```

output
```
int(acoth(c + d*coth(a + b*x)), x)
```

3.220 $\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x} dx$

3.220.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x} dx = \text{Int}\left(\frac{\coth^{-1}(c+d\coth(a+bx))}{x}, x\right)$$

output
```
CannotIntegrate(arccoth(c+d*coth(b*x+a))/x,x)
```

3.220.2 Mathematica [N/A]

Not integrable

Time = 3.89 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x} dx = \int \frac{\coth^{-1}(c+d\coth(a+bx))}{x} dx$$

input
```
Integrate[ArcCoth[c + d*Coth[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[c + d*Coth[a + b*x]]/x, x]
```

3.220.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d\coth(a+bx)+c)}{x}\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{\coth^{-1}(d\coth(a+bx)+c)}{x}dx$$

input
```
Int[ArcCoth[c + d*Coth[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.220.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.220.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(c+d\coth\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(c+d*coth(b*x+a))/x,x)
```

output
```
int(arccoth(c+d*coth(b*x+a))/x,x)
```

3.220.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arccoth(d*coth(b*x + a) + c)/x, x)
```

3.220.6 Sympy [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{acoth}\left(c+d\coth\left(a+bx\right)\right)}{x}\,dx$$

input
```
integrate(acoth(c+d*coth(b*x+a))/x,x)
```

output
```
Integral(acoth(c + d*coth(a + b*x))/x, x)
```

3.220.7 Maxima [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arccoth(d*coth(b*x + a) + c)/x, x)
```

3.220.8 Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*coth(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(d*coth(b*x + a) + c)/x, x)
```

3.220.9 Mupad [N/A]

Not integrable

Time = 4.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\operatorname{acoth}(c+d\coth(a+b\,x))}{x}\,dx$$

input
```
int(acoth(c + d*coth(a + b*x))/x,x)
```

output
```
int(acoth(c + d*coth(a + b*x))/x, x)
```

3.221 $\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx$

3.221.1 Optimal result

Integrand size = 16, antiderivative size = 152

$$\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x^3\operatorname{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b} + \frac{3x^2\operatorname{PolyLog}\left(3,(1+d)e^{2a+2bx}\right)}{8b^2} - \frac{3x\operatorname{PolyLog}\left(4,(1+d)e^{2a+2bx}\right)}{8b^3} + \frac{3\operatorname{PolyLog}\left(5,(1+d)e^{2a+2bx}\right)}{16b^4}$$

output
```
1/20*b*x^5+1/4*x^4*arccoth(1+d+d*coth(b*x+a))-1/8*x^4*ln(1-(1+d)*exp(2*b*x
+2*a))-1/4*x^3*polylog(2,(1+d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,(1+d)*e
xp(2*b*x+2*a))/b^2-3/8*x*polylog(4,(1+d)*exp(2*b*x+2*a))/b^3+3/16*polylog(
5,(1+d)*exp(2*b*x+2*a))/b^4
```

3.221.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx$$
$$= \frac{4b^4x^4\coth^{-1}(1+d+d\coth(a+bx)) - 2b^4x^4\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right) + 4b^3x^3\operatorname{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right) + 6b^2x^2\operatorname{P}}{16b^4}$$

input
```
Integrate[x^3*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

output
```
(4*b^4*x^4*ArcCoth[1 + d + d*Coth[a + b*x]] - 2*b^4*x^4*Log[1 - 1/((1 + d)
*E^(2*(a + b*x)))] + 4*b^3*x^3*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))] + 6
*b^2*x^2*PolyLog[3, 1/((1 + d)*E^(2*(a + b*x)))] + 6*b*x*PolyLog[4, 1/((1
+ d)*E^(2*(a + b*x)))] + 3*PolyLog[5, 1/((1 + d)*E^(2*(a + b*x)))])/(16*b^
4)
```

3.221.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6796, 2615, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(d\coth(a+bx)+d+1)\,dx$$
$$\Big\downarrow \text{6796}$$
$$\frac{1}{4}b\int \frac{x^4}{1-(d+1)e^{2a+2bx}}dx + \frac{1}{4}x^4\coth^{-1}(d\coth(a+bx)+d+1)$$
$$\Big\downarrow \text{2615}$$
$$\frac{1}{4}b\left((d+1)\int \frac{e^{2a+2bx}x^4}{1-(d+1)e^{2a+2bx}}dx + \frac{x^5}{5}\right) + \frac{1}{4}x^4\coth^{-1}(d\coth(a+bx)+d+1)$$
$$\Big\downarrow \text{2620}$$

$$\frac{1}{4}b\left((d+1)\left(\frac{2\int x^3\log\left(1-(d+1)e^{2a+2bx}\right)dx}{b(d+1)}-\frac{x^4\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^5}{5}\right)+\frac{1}{4}x^4\coth^{-1}(d\coth(a+bx)+d+1)$$

$\downarrow$ 3011

$$\frac{1}{4}b\left((d+1)\left(\frac{2\left(\frac{3\int x^2\operatorname{PolyLog}(2,(d+1)e^{2a+2bx})dx}{2b}-\frac{x^3\operatorname{PolyLog}(2,(d+1)e^{2a+2bx})}{2b}\right)}{b(d+1)}-\frac{x^4\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^5}{5}\right)\ldots$$
$$\frac{1}{4}x^4\coth^{-1}(d\coth(a+bx)+d+1)$$

$\downarrow$ 7163

$$\frac{1}{4}b\left((d+1)\left(\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}(3,(d+1)e^{2a+2bx})}{2b}-\frac{\int x\operatorname{PolyLog}(3,(d+1)e^{2a+2bx})dx}{b}\right)}{2b}-\frac{x^3\operatorname{PolyLog}(2,(d+1)e^{2a+2bx})}{2b}\right)}{b(d+1)}-\frac{x^4\log\left(1-(d\ldots\right.}{2b(d\ldots}\right.\right.$$
$$\frac{1}{4}x^4\coth^{-1}(d\coth(a+bx)+d+1)$$

$\downarrow$ 7163

$$\frac{1}{4}b\left((d+1)\left(\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}(3,(d+1)e^{2a+2bx})}{2b}-\frac{\frac{x\operatorname{PolyLog}(4,(d+1)e^{2a+2bx})}{2b}-\frac{\int\operatorname{PolyLog}(4,(d+1)e^{2a+2bx})dx}{2b}}{b}\right)}{2b}-\frac{x^3\operatorname{PolyLog}(2,(d+1)e^{2a+2\ldots}}{2b}\right.}{b(d+1)}\right.\right.$$
$$\frac{1}{4}x^4\coth^{-1}(d\coth(a+bx)+d+1)$$

$\downarrow$ 2720

3.221. $\int x^3\coth^{-1}(1+d+d\coth(a+bx))\,dx$

$$\frac{1}{4}b\left((d+1)\left(\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\operatorname{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}\right)}{2b}-\frac{x^3\operatorname{PolyLo}\right)}{b(d+1)}\right.\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d\coth(a+bx)+d+1)$$

$\downarrow$ 7143

$$\frac{1}{4}b\left((d+1)\left(\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\operatorname{PolyLog}\left(5,(d+1)e^{2a+2bx}\right)}{4b^2}}{b}\right)}{2b}-\frac{x^3\operatorname{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{2b}\right)}{b(d+1)}\right.\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d\coth(a+bx)+d+1)$$

input
```
Int[x^3*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

output
```
(x^4*ArcCoth[1 + d + d*Coth[a + b*x]])/4 + (b*(x^5/5 + (1 + d)*(-1/2*(x^4*
Log[1 - (1 + d)*E^(2*a + 2*b*x)])/(b*(1 + d)) + (2*(-1/2*(x^3*PolyLog[2, (
1 + d)*E^(2*a + 2*b*x)])/b + (3*((x^2*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])
/(2*b) - ((x*PolyLog[4, (1 + d)*E^(2*a + 2*b*x)])/(2*b) - PolyLog[5, (1 +
d)*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b)))/(b*(1 + d)))))/4
```

3.221. $\int x^3\coth^{-1}(1+d+d\coth(a+bx))\,dx$

3.221.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6796
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.221. $\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.221.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.06 (sec) , antiderivative size = 1656, normalized size of antiderivative = 10.89

method	result	size
risch	Expression too large to display	1656

input
```
int(x^3*arccoth(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^4*d*a^4/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+3/16/b^4/(1+d)*polylog(5,
(1+d)*exp(2*b*x+2*a))-1/8/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^4+1/2/b^4*d*a
^4/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))+1/2/b^4*d*a^3/(1+d)*dilog(1-exp(b*x+
a)*(1+d)^(1/2))+1/2/b^4*d*a^3/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/4/b^
4*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a^3-3/8/b^3*d/(1+d)*polylog(4,(1
+d)*exp(2*b*x+2*a))*x+1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+1/2
/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x-1/4/b*d/(1+d)*polylog(2,(1+d
)*exp(2*b*x+2*a))*x^3-3/8/b^4*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^4+3/8/b
^2*d/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))*x^2-1/2/b^3*d/(1+d)*ln(1-(1+d)*
exp(2*b*x+2*a))*x*a^3+1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+1
/2/b^3*d*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x+1/20*b*x^5-1/2/b^3/(1+d)
*ln(1-(1+d)*exp(2*b*x+2*a))*x*a^3-1/8/b^4*a^4/(1+d)*ln(d*exp(2*b*x+2*a)+ex
p(2*b*x+2*a)-1)-1/4/b/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x^3-3/8/b^4/(1
+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^4+3/8/b^2/(1+d)*polylog(3,(1+d)*exp(2*b*x
+2*a))*x^2-1/4/b^4/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a^3-3/8/b^3/(1+d)
*polylog(4,(1+d)*exp(2*b*x+2*a))*x+3/16/b^4*d/(1+d)*polylog(5,(1+d)*exp(2*
b*x+2*a))+1/2/b^4*a^4/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^4*a^4/(1+d)
*ln(1+exp(b*x+a)*(1+d)^(1/2))+1/2/b^4*a^3/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(
1/2))+1/2/b^4*a^3/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/8*d/(1+d)*ln(1-(
1+d)*exp(2*b*x+2*a))*x^4-1/8/b^4*d*a^4/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*...
```

3.221.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(132) = 264$.

Time = 0.26 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.78

$$\int x^3 \coth^{-1}(1 + d + d\coth(a + bx))\,dx$$
$$= \frac{2\,b^5x^5 + 5\,b^4x^4\log\left(\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 20\,b^3x^3\mathrm{Li}_2\left(\sqrt{d+1}(\cosh(bx+a) + \sinh(bx+a))\right) - 20}{}$$

input
```
integrate(x^3*arccoth(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/40*(2*b^5*x^5 + 5*b^4*x^4*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(sqrt(d + 1)*(cosh(
b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-sqrt(d + 1)*(cosh(b*x + a)
+ sinh(b*x + a))) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x
 + a) + 2*sqrt(d + 1)) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sin
h(b*x + a) - 2*sqrt(d + 1)) + 60*b^2*x^2*polylog(3, sqrt(d + 1)*(cosh(b*x
+ a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -sqrt(d + 1)*(cosh(b*x + a)
 + sinh(b*x + a))) - 120*b*x*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh(
b*x + a))) - 120*b*x*polylog(4, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a
))) - 5*(b^4*x^4 - a^4)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) +
1) - 5*(b^4*x^4 - a^4)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) +
1) + 120*polylog(5, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) + 120*pol
ylog(5, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

3.221.6 Sympy [F]

$$\int x^3 \coth^{-1}(1 + d + d\coth(a + bx))\,dx = \int x^3 \operatorname{acoth}(d\coth(a + bx) + d + 1)\,dx$$

input
```
integrate(x**3*acoth(1+d+d*coth(b*x+a)),x)
```

output
```
Integral(x**3*acoth(d*coth(a + b*x) + d + 1), x)
```

3.221.7 Maxima [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.96

$$\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{1}{4}x^4 \operatorname{arcoth}(d\coth(bx+a)+d+1) + \frac{1}{40}\left(\frac{2x^5}{d} - \frac{5\left(2b^4x^4\log\left(-(d+1)e^{(2bx+2a)}+1\right)+4b^3x^3\mathrm{Li}_2\left((d+1)e^{(2bx+2a)}\right)-6b^2x^2\mathrm{Li}_3\left((d+1)e^{(2bx}\right.\right.}{b^5d}\right.$$

input
```
integrate(x^3*arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*x^4*arccoth(d*coth(b*x + a) + d + 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*lo
g(-(d + 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog((d + 1)*e^(2*b*x + 2*a))
 - 6*b^2*x^2*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, (d + 1
)*e^(2*b*x + 2*a)) - 3*polylog(5, (d + 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

3.221.8 Giac [F]

$$\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \int x^3 \operatorname{arcoth}(d\coth(bx+a)+d+1)\,dx$$

input
```
integrate(x^3*arccoth(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^3*arccoth(d*coth(b*x + a) + d + 1), x)
```

3.221.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \int x^3 \operatorname{acoth}(d+d\coth(a+b\,x)+1)\,dx$$

input
```
int(x^3*acoth(d + d*coth(a + b*x) + 1),x)
```

output
```
int(x^3*acoth(d + d*coth(a + b*x) + 1), x)
```

3.222 $\int x^2 \coth^{-1}(1+d+d\coth(a+bx))\,dx$

3.222.1 Optimal result

Integrand size = 16, antiderivative size = 126

$$\int x^2 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x^2\,\text{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b} + \frac{x\,\text{PolyLog}\left(3,(1+d)e^{2a+2bx}\right)}{4b^2} - \frac{\text{PolyLog}\left(4,(1+d)e^{2a+2bx}\right)}{8b^3}$$

output
```
1/12*b*x^4+1/3*x^3*arccoth(1+d+d*coth(b*x+a))-1/6*x^3*ln(1-(1+d)*exp(2*b*x
+2*a))-1/4*x^2*polylog(2,(1+d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,(1+d)*exp
(2*b*x+2*a))/b^2-1/8*polylog(4,(1+d)*exp(2*b*x+2*a))/b^3
```

3.222.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95

$$\int x^2 \coth^{-1}(1+d+d\coth(a+bx))\,dx$$
$$= \frac{8b^3x^3\coth^{-1}(1+d+d\coth(a+bx)) - 4b^3x^3\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right) + 6b^2x^2\operatorname{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right) + 6bx\operatorname{Pol}}{24b^3}$$

input
```
Integrate[x^2*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCoth[1 + d + d*Coth[a + b*x]] - 4*b^3*x^3*Log[1 - 1/((1 + d)
*E^(2*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))] + 6
*b*x*PolyLog[3, 1/((1 + d)*E^(2*(a + b*x)))] + 3*PolyLog[4, 1/((1 + d)*E^(
2*(a + b*x)))])/(24*b^3)
```

3.222.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6796, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\coth^{-1}(d\coth(a+bx)+d+1)\,dx$$
$$\Big\downarrow \text{6796}$$
$$\frac{1}{3}b\int\frac{x^3}{1-(d+1)e^{2a+2bx}}dx+\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+d+1)$$
$$\Big\downarrow \text{2615}$$
$$\frac{1}{3}b\left((d+1)\int\frac{e^{2a+2bx}x^3}{1-(d+1)e^{2a+2bx}}dx+\frac{x^4}{4}\right)+\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+d+1)$$
$$\Big\downarrow \text{2620}$$
$$\frac{1}{3}b\left((d+1)\left(\frac{3\int x^2\log\left(1-(d+1)e^{2a+2bx}\right)dx}{2b(d+1)}-\frac{x^3\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^4}{4}\right)+$$
$$\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}b\left((d+1)\left(\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{2b}\right)}{2b(d+1)}-\frac{x^3\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^4}{4}\right)+\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3}b\left((d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\int\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{2b}\right)}{2b(d+1)}-\frac{x^3\log\left(1-(d+1)e^{\right.}}{2b(d+1)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3}b\left((d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{2b}\right)}{2b(d+1)}-\frac{x^3\log\left(\right.}{}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{3}b\left((d+1)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{2b}-\frac{\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{2b}\right)}{2b(d+1)}-\frac{x^3\log\left(1-(d+1)e^{2a+\right.}}{2b(d+1)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+d+1)$$

input
```
Int[x^2*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

3.222. $\int x^2\coth^{-1}(1+d+d\coth(a+bx))\,dx$

output
```
(x^3*ArcCoth[1 + d + d*Coth[a + b*x]])/3 + (b*(x^4/4 + (1 + d)*(-1/2*(x^3*
Log[1 - (1 + d)*E^(2*a + 2*b*x)])/(b*(1 + d)) + (3*(-1/2*(x^2*PolyLog[2, (
1 + d)*E^(2*a + 2*b*x)])/b + ((x*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])/(2*b
) - PolyLog[4, (1 + d)*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b*(1 + d)))))/3
```

3.222.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6796
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.222.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.98 (sec) , antiderivative size = 1599, normalized size of antiderivative = 12.69

method	result	size
risch	Expression too large to display	1599

input
```
int(x^2*arccoth(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/12*b*x^4-1/3*x^3*ln(exp(b*x+a))-1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+
2*a))*x^2+1/3/b^3*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^3+1/4/b^3*d/(1+d)*p
olylog(2,(1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(1+d)*polylog(3,(1+d)*exp(2*b
*x+2*a))*x-1/2/b^2*a^2/(1+d)*x*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^2*a^2/(1
+d)*x*ln(1+exp(b*x+a)*(1+d)^(1/2))+1/6/b^3*d*a^3/(1+d)*ln(d*exp(2*b*x+2*a)
+exp(2*b*x+2*a)-1)-1/2/b^3*a^3*d/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^
3*a^3*d/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*a^2*d/(1+d)*dilog(1-exp
(b*x+a)*(1+d)^(1/2))-1/2/b^3*a^2*d/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))+1
/2/b^2/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^2*x-1/8/b^3/(1+d)*polylog(4,(1+d
)*exp(2*b*x+2*a))-1/6/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^3-1/4/b/(1+d)*pol
ylog(2,(1+d)*exp(2*b*x+2*a))*x^2+1/3/b^3/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*
a^3+1/4/b^3/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2/(1+d)*polylo
g(3,(1+d)*exp(2*b*x+2*a))*x-1/8/b^3*d/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a)
)-1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*a^3/(1+d)*ln(1+ex
p(b*x+a)*(1+d)^(1/2))-1/2/b^3*a^2/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))-1/
2/b^3*a^2/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/6*d/(1+d)*ln(1-(1+d)*exp
(2*b*x+2*a))*x^3+1/6/b^3*a^3/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)-1
/2/b^2*a^2*d/(1+d)*x*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^2*a^2*d/(1+d)*x*ln
(1+exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^2*
x-1/12*(-I*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3+I*Pi*csgn(I...
```

3.222.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(109) = 218$.

Time = 0.28 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.85

$$\int x^2 \coth^{-1}(1 + d + d\coth(a + bx))\,dx$$
$$= \frac{b^4x^4 + 2\,b^3x^3\log\left(\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\sqrt{d+1}(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,b^2x^2 \cdots}{\cdots}$$

input
```
integrate(x^2*arccoth(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(b^4*x^4 + 2*b^3*x^3*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d
*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d + 1)*(cosh(b*x
 + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d + 1)*(cosh(b*x + a) + si
nh(b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a
) + 2*sqrt(d + 1)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*
x + a) - 2*sqrt(d + 1)) + 12*b*x*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + s
inh(b*x + a))) + 12*b*x*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x
+ a))) - 2*(b^3*x^3 + a^3)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))
 + 1) - 2*(b^3*x^3 + a^3)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))
 + 1) - 12*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*po
lylog(4, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

3.222.6 Sympy [F]

$$\int x^2 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \int x^2 \operatorname{acoth}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

input
```
integrate(x**2*acoth(1+d+d*coth(b*x+a)),x)
```

output
```
Integral(x**2*acoth(d*coth(a + b*x) + d + 1), x)
```

3.222.7 Maxima [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

$$\int x^2 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{1}{3}x^3 \operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right) + \frac{1}{36}\left(\frac{3x^4}{d} - \frac{2\left(4b^3x^3\log\left(-(d+1)e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left((d+1)e^{(2bx+2a)}\right)-6bx\mathrm{Li}_3\left((d+1)e^{(2bx+\right.}\right.}{b^4d}\right.$$

input
```
integrate(x^2*arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(d*coth(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*lo
g(-(d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d + 1)*e^(2*b*x + 2*a))
 - 6*b*x*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d + 1)*e^(2*b
*x + 2*a)))/(b^4*d))*b*d
```

3.222.8 Giac [F]

$$\int x^2 \coth^{-1}(1 + d + d\coth(a + bx))\,dx = \int x^2 \operatorname{arcoth}\left(d\coth\left(bx + a\right) + d + 1\right)\,dx$$

input
```
integrate(x^2*arccoth(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(d*coth(b*x + a) + d + 1), x)
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(1 + d + d\coth(a + bx))\,dx = \int x^2 \operatorname{acoth}(d + d\coth(a + b\,x) + 1)\,dx$$

input
```
int(x^2*acoth(d + d*coth(a + b*x) + 1),x)
```

output
```
int(x^2*acoth(d + d*coth(a + b*x) + 1), x)
```

3.223 $\int x \coth^{-1}(1+d+d\coth(a+bx))\,dx$

3.223.1 Optimal result

Integrand size = 14, antiderivative size = 100

$$\int x \coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{4}x^2\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x\,\text{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b} + \frac{\text{PolyLog}\left(3,(1+d)e^{2a+2bx}\right)}{8b^2}$$

output
```
1/6*b*x^3+1/2*x^2*arccoth(1+d+d*coth(b*x+a))-1/4*x^2*ln(1-(1+d)*exp(2*b*x+
2*a))-1/4*x*polylog(2,(1+d)*exp(2*b*x+2*a))/b+1/8*polylog(3,(1+d)*exp(2*b*
x+2*a))/b^2
```

3.223.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int x \coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{2b^2x^2\left(2\coth^{-1}(1+d+d\coth(a+bx))-\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right)\right)+2bx\,\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right)+\text{PolyLog}\left(3\right.}{8b^2}$$

input
```
Integrate[x*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcCoth[1 + d + d*Coth[a + b*x]] - Log[1 - 1/((1 + d)*E^(2*(
a + b*x)))]) + 2*b*x*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))] + PolyLog[3,
1/((1 + d)*E^(2*(a + b*x)))])/(8*b^2)
```

3.223.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6796, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(d\coth(a+bx)+d+1)\,dx$$

$$\Big\downarrow \text{ 6796}$$

$$\frac{1}{2}b\int \frac{x^2}{1-(d+1)e^{2a+2bx}}dx+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{1}{2}b\left((d+1)\int \frac{e^{2a+2bx}x^2}{1-(d+1)e^{2a+2bx}}dx+\frac{x^3}{3}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{2}b\left((d+1)\left(\frac{\int x\log\left(1-(d+1)e^{2a+2bx}\right)dx}{b(d+1)}-\frac{x^2\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^3}{3}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{1}{2}b\left((d+1)\left(\frac{\frac{\int \text{PolyLog}(2,(d+1)e^{2a+2bx})dx}{2b}-\frac{x\,\text{PolyLog}(2,(d+1)e^{2a+2bx})}{2b}}{b(d+1)}-\frac{x^2\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^3}{3}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{2}b\left((d+1)\left(\frac{\frac{\int e^{-2a-2bx}\,\mathrm{PolyLog}(2,(d+1)e^{2a+2bx})de^{2a+2bx}}{4b^2}-\frac{x\,\mathrm{PolyLog}(2,(d+1)e^{2a+2bx})}{2b}}{b(d+1)}-\frac{x^2\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x}{}\right.$$
$$\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{1}{2}b\left((d+1)\left(\frac{\frac{\mathrm{PolyLog}(3,(d+1)e^{2a+2bx})}{4b^2}-\frac{x\,\mathrm{PolyLog}(2,(d+1)e^{2a+2bx})}{2b}}{b(d+1)}-\frac{x^2\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^3}{3}\right)+$$
$$\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+d+1)$$

input
```
Int[x*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 + d + d*Coth[a + b*x]])/2 + (b*(x^3/3 + (1 + d)*(-1/2*(x^2*
Log[1 - (1 + d)*E^(2*a + 2*b*x)])/(b*(1 + d)) + (-1/2*(x*PolyLog[2, (1 + d
)*E^(2*a + 2*b*x)])/b + PolyLog[3, (1 + d)*E^(2*a + 2*b*x)]/(4*b^2))/(b*(1
 + d)))))/2
```

3.223.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6796
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.223.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.64 (sec) , antiderivative size = 1518, normalized size of antiderivative = 15.18

method	result	size
risch	Expression too large to display	1518

input
```
int(x*arccoth(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a*x-1/4/b^2*d/(1+d)*ln(1-(1+d)*exp
(2*b*x+2*a))*a^2-1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x-1/4/b^2*d
/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a+1/2/b^2*a^2*d/(1+d)*ln(1-exp(b*x+
a)*(1+d)^(1/2))+1/2/b^2*a^2*d/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a
*d/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a*d/(1+d)*dilog(1+exp(b*x
+a)*(1+d)^(1/2))+1/2/b*a/(1+d)*x*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b*a/(1+d
)*x*ln(1+exp(b*x+a)*(1+d)^(1/2))-1/4/b^2*a^2*d/(1+d)*ln(d*exp(2*b*x+2*a)+e
xp(2*b*x+2*a)-1)+1/8/b^2/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))-1/4/(1+d)*l
n(1-(1+d)*exp(2*b*x+2*a))*x^2+1/2/b*a*d/(1+d)*x*ln(1-exp(b*x+a)*(1+d)^(1/2
))+1/2/b*a*d/(1+d)*x*ln(1+exp(b*x+a)*(1+d)^(1/2))-1/2/b*d/(1+d)*ln(1-(1+d)
*exp(2*b*x+2*a))*a*x-1/2*x^2*ln(exp(b*x+a))+1/6*b*x^3-1/4/b/(1+d)*polylog(
2,(1+d)*exp(2*b*x+2*a))*x-1/4/b^2/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a+
1/8/b^2*d/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))+1/2/b^2*a^2/(1+d)*ln(1-exp
(b*x+a)*(1+d)^(1/2))+1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))+1/2/b^
2*a/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a/(1+d)*dilog(1+exp(b*x+
a)*(1+d)^(1/2))-1/4*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^2-1/4/b^2*a^2/(1+
d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)-1/4/b^2/(1+d)*ln(1-(1+d)*exp(2*b*
x+2*a))*a^2+1/4*x^2*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)-1/8*(-I*Pi*csgn(
I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3+I*Pi*csgn(I/(exp(2*b*x+2*a)-1))*c
sgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+2*I*Pi*csgn(I*exp(b*x+a))*cs...
```

3.223.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(86) = 172$.

Time = 0.27 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.05

$$\int x \coth^{-1}(1 + d + d\coth(a + bx))\, dx$$
$$= \frac{2\,b^3x^3 + 3\,b^2x^2\log\left(\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,bx\mathrm{Li}_2\left(\sqrt{d+1}(\cosh(bx+a) + \sinh(bx+a))\right) - 6\,bx\mathrm{Li}}{}$$

input
```
integrate(x*arccoth(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*x^3 + 3*b^2*x^2*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(sqrt(d + 1)*(cosh(b*x +
 a) + sinh(b*x + a))) - 6*b*x*dilog(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x
 + a))) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) + 2*
sqrt(d + 1)) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a)
 - 2*sqrt(d + 1)) - 3*(b^2*x^2 - a^2)*log(sqrt(d + 1)*(cosh(b*x + a) + sin
h(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-sqrt(d + 1)*(cosh(b*x + a) + sin
h(b*x + a)) + 1) + 6*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)
)) + 6*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

3.223.6 Sympy [F]

$$\int x \coth^{-1}(1+d+d\coth(a+bx))\,dx = \int x\,\mathrm{acoth}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

input
```
integrate(x*acoth(1+d+d*coth(b*x+a)),x)
```

output
```
Integral(x*acoth(d*coth(a + b*x) + d + 1), x)
```

3.223.7 Maxima [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int x \coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left((d+1)e^{(2\,bx+2\,a)}\right)\right)}{b^3d}\right)b + \frac{1}{2}\,x^2\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)$$

input
```
integrate(x*arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(4*x^3/d - 3*(2*b^2*x^2*log(-(d + 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dil
og((d + 1)*e^(2*b*x + 2*a)) - polylog(3, (d + 1)*e^(2*b*x + 2*a)))/(b^3*d)
)*b*d + 1/2*x^2*arccoth(d*coth(b*x + a) + d + 1)
```

3.223.8 Giac [F]

$$\int x \coth^{-1}(1+d+d\coth(a+bx))\,dx = \int x \operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(x*arccoth(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(d*coth(b*x + a) + d + 1), x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(1+d+d\coth(a+bx))\,dx = \int x\,\operatorname{acoth}(d+d\coth(a+b\,x)+1)\,dx$$

input
```
int(x*acoth(d + d*coth(a + b*x) + 1),x)
```

output
```
int(x*acoth(d + d*coth(a + b*x) + 1), x)
```

3.224 $\int \coth^{-1}(1+d+d\coth(a+bx))\,dx$

3.224.1 Optimal result

Integrand size = 12, antiderivative size = 69

$$\int \coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{bx^2}{2}+x\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{\operatorname{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b}$$

output
```
1/2*b*x^2+x*arccoth(1+d+d*coth(b*x+a))-1/2*x*ln(1-(1+d)*exp(2*b*x+2*a))-1/
4*polylog(2,(1+d)*exp(2*b*x+2*a))/b
```

3.224.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \coth^{-1}(1+d+d\coth(a+bx))\,dx = x\coth^{-1}(1+d+d\coth(a+bx)) + \frac{-2bx\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right)+\operatorname{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right)}{4b}$$

input
```
Integrate[ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

output
```
x*ArcCoth[1 + d + d*Coth[a + b*x]] + (-2*b*x*Log[1 - 1/((1 + d)*E^(2*(a +
b*x)))] + PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))])/(4*b)
```

3.224.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6788, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d\coth(a+bx)+d+1)\,dx$$

$$\Big\downarrow \ 6788$$

$$b\int \frac{x}{1-(d+1)e^{2a+2bx}}dx+x\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \ 2615$$

$$b\left((d+1)\int \frac{e^{2a+2bx}x}{1-(d+1)e^{2a+2bx}}dx+\frac{x^2}{2}\right)+x\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \ 2620$$

$$b\left((d+1)\left(\frac{\int \log\left(1-(d+1)e^{2a+2bx}\right)dx}{2b(d+1)}-\frac{x\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^2}{2}\right)+x\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \ 2715$$

$$b\left((d+1)\left(\frac{\int e^{-2a-2bx}\log\left(1-(d+1)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2(d+1)}-\frac{x\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^2}{2}\right)+x\coth^{-1}(d\coth(a+bx)+d+1)$$

$$\Big\downarrow \ 2838$$

$$b\left((d+1)\left(-\frac{\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b^2(d+1)}-\frac{x\log\left(1-(d+1)e^{2a+2bx}\right)}{2b(d+1)}\right)+\frac{x^2}{2}\right)+x\coth^{-1}(d\coth(a+bx)+d+1)$$

input
```
Int[ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

output
```
x*ArcCoth[1 + d + d*Coth[a + b*x]] + b*(x^2/2 + (1 + d)*(-1/2*(x*Log[1 - (
1 + d)*E^(2*a + 2*b*x)])/(b*(1 + d)) - PolyLog[2, (1 + d)*E^(2*a + 2*b*x)]
/(4*b^2*(1 + d))))
```

3.224.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6788
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Coth[a + b*x]], x] + Simp[b   Int[x/(c - d - c*E^(2*a + 2*b*x))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

3.224.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(61) = 122$.

Time = 1.70 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.70

method	result
derivativedivides	$-\frac{\operatorname{arccoth}(1+d+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(1+d+d\coth(bx+a))d\ln(d+d\coth(bx+a))}{2}+\frac{d^2\left(\frac{\ln(d+d\coth(bx+a))}{4}\right.}{}$
default	$-\frac{\operatorname{arccoth}(1+d+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(1+d+d\coth(bx+a))d\ln(d+d\coth(bx+a))}{2}+\frac{d^2\left(\frac{\ln(d+d\coth(bx+a))}{4}\right.}{}$
risch	Expression too large to display

input
```
int(arccoth(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-1/2*arccoth(1+d+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)+1/2*arccoth(
1+d+d*coth(b*x+a))*d*ln(d+d*coth(b*x+a))+1/2*d^2*(1/d*(1/4*ln(d+d*coth(b*x
+a))^2-1/2*dilog(1/2*d*coth(b*x+a)+1/2*d+1)-1/2*ln(d+d*coth(b*x+a))*ln(1/2
*d*coth(b*x+a)+1/2*d+1))-1/d*(-1/2*dilog((-d*coth(b*x+a)-d-2)/(-2*d-2))-1/
2*ln(-d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)-d-2)/(-2*d-2))+1/2*dilog(-1/2*(-
d*coth(b*x+a)-d)/d)+1/2*ln(-d*coth(b*x+a)+d)*ln(-1/2*(-d*coth(b*x+a)-d)/d)
)))
```

3.224.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(60) = 120$.

Time = 0.27 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.28

$$\int \coth^{-1}(1+d+d\coth(a+bx))\,dx$$
$$= \frac{b^2x^2 + bx\log\left(\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) + a\log\left(2\,(d+1)\cosh\,(bx+a) + 2\,(d+1)\sinh\,(bx+a) + 2\,\sqrt{d}\right.}{}$$

input
```
integrate(arccoth(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 + b*x*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d*cosh(b
*x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*si
nh(b*x + a) + 2*sqrt(d + 1)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*s
inh(b*x + a) - 2*sqrt(d + 1)) - (b*x + a)*log(sqrt(d + 1)*(cosh(b*x + a) +
 sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*
x + a)) + 1) - dilog(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(
-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

3.224.6 Sympy [F]

$$\int \coth^{-1}(1+d+d\coth(a+bx))\,dx = \int \operatorname{acoth}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

input
```
integrate(acoth(1+d+d*coth(b*x+a)),x)
```

output
```
Integral(acoth(d*coth(a + b*x) + d + 1), x)
```

3.224.7 Maxima [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{1}{4}bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right) + x\operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)$$

input
```
integrate(arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*b*d*(2*x^2/d - (2*b*x*log(-(d + 1)*e^(2*b*x + 2*a) + 1) + dilog((d + 1
)*e^(2*b*x + 2*a)))/(b^2*d)) + x*arccoth(d*coth(b*x + a) + d + 1)
```

3.224.8 Giac [F]

$$\int \coth^{-1}(1+d+d\coth(a+bx))\,dx=\int \operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)\,dx$$

input
```
integrate(arccoth(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(d*coth(b*x + a) + d + 1), x)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(1+d+d\coth(a+bx))\,dx=\int \operatorname{acoth}(d+d\coth(a+b\,x)+1)\,dx$$

input
```
int(acoth(d + d*coth(a + b*x) + 1),x)
```

output
```
int(acoth(d + d*coth(a + b*x) + 1), x)
```

3.225 $\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x} dx$

3.225.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x} dx = \text{Int}\left(\frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}, x\right)$$

output
```
CannotIntegrate(arccoth(1+d+d*coth(b*x+a))/x,x)
```

3.225.2 Mathematica [N/A]

Not integrable

Time = 2.89 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x} dx = \int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x} dx$$

input
```
Integrate[ArcCoth[1 + d + d*Coth[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[1 + d + d*Coth[a + b*x]]/x, x]
```

3.225.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d\coth(a+bx)+d+1)}{x}dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\coth^{-1}(d\coth(a+bx)+d+1)}{x}dx$$

input
```
Int[ArcCoth[1 + d + d*Coth[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.225.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.225.4 Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}(1+d+d\coth(bx+a))}{x}dx$$

input
```
int(arccoth(1+d+d*coth(b*x+a))/x,x)
```

output
```
int(arccoth(1+d+d*coth(b*x+a))/x,x)
```

3.225.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1 + d + d\coth(a + bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\coth\left(bx + a\right) + d + 1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+d+d*coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arccoth(d*coth(b*x + a) + d + 1)/x, x)
```

3.225.6 Sympy [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\coth^{-1}(1 + d + d\coth(a + bx))}{x}\,dx = \int \frac{\operatorname{acoth}\left(d\coth\left(a + bx\right) + d + 1\right)}{x}\,dx$$

input
```
integrate(acoth(1+d+d*coth(b*x+a))/x,x)
```

output
```
Integral(acoth(d*coth(a + b*x) + d + 1)/x, x)
```

3.225.7 Maxima [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1 + d + d\coth(a + bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\coth\left(bx + a\right) + d + 1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+d+d*coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arccoth(d*coth(b*x + a) + d + 1)/x, x)
```

3.225.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+d+d*coth(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(d*coth(b*x + a) + d + 1)/x, x)
```

3.225.9 Mupad [N/A]

Not integrable

Time = 4.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx = \int \frac{\mathrm{acoth}(d+d\coth(a+b\,x)+1)}{x}\,dx$$

input
```
int(acoth(d + d*coth(a + b*x) + 1)/x,x)
```

output
```
int(acoth(d + d*coth(a + b*x) + 1)/x, x)
```

3.226 $\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx$

3.226.1 Optimal result

Integrand size = 19, antiderivative size = 165

$$\begin{aligned}\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx =& \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx))\\ &-\frac{1}{8}x^4\log\left(1-(1-d)e^{2a+2bx}\right)\\ &-\frac{x^3\operatorname{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}\\ &+\frac{3x^2\operatorname{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{8b^2}\\ &-\frac{3x\operatorname{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{8b^3}\\ &+\frac{3\operatorname{PolyLog}\left(5,(1-d)e^{2a+2bx}\right)}{16b^4}\end{aligned}$$

output
```
1/20*b*x^5+1/4*x^4*arccoth(1-d-d*coth(b*x+a))-1/8*x^4*ln(1-(1-d)*exp(2*b*x
+2*a))-1/4*x^3*polylog(2,(1-d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,(1-d)*e
xp(2*b*x+2*a))/b^2-3/8*x*polylog(4,(1-d)*exp(2*b*x+2*a))/b^3+3/16*polylog(
5,(1-d)*exp(2*b*x+2*a))/b^4
```

3.226.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92

$$\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx$$
$$= \frac{4b^4x^4\coth^{-1}(1-d-d\coth(a+bx)) - 2b^4x^4\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right) + 4b^3x^3\operatorname{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right) + 6b^2x^2}{16b^4}$$

input
```
Integrate[x^3*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

output
```
(4*b^4*x^4*ArcCoth[1 - d - d*Coth[a + b*x]] - 2*b^4*x^4*Log[1 + 1/((-1 + d
)*E^(2*(a + b*x)))] + 4*b^3*x^3*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))
] + 6*b^2*x^2*PolyLog[3, -(1/((-1 + d)*E^(2*(a + b*x))))] + 6*b*x*PolyLog[
4, -(1/((-1 + d)*E^(2*(a + b*x))))] + 3*PolyLog[5, -(1/((-1 + d)*E^(2*(a +
 b*x))))])/(16*b^4)
```

3.226.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6796, 2615, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}(d(-\coth(a+bx))-d+1)\,dx$$

$$\Big\downarrow \text{6796}$$

$$\frac{1}{4}b\int \frac{x^4}{1-(1-d)e^{2a+2bx}}dx + \frac{1}{4}x^4\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{4}b\left((1-d)\int \frac{e^{2a+2bx}x^4}{1-(1-d)e^{2a+2bx}}dx + \frac{x^5}{5}\right) + \frac{1}{4}x^4\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{4}b\left((1-d)\left(\frac{2\int x^3\log\left(1-(1-d)e^{2a+2bx}\right)dx}{b(1-d)}-\frac{x^4\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^5}{5}\right)+\frac{1}{4}x^4\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$\downarrow$ 3011

$$\frac{1}{4}b\left((1-d)\left(\frac{2\left(\frac{3\int x^2\,\text{PolyLog}(2,(1-d)e^{2a+2bx})dx}{2b}-\frac{x^3\,\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}\right)}{b(1-d)}-\frac{x^4\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^5}{5}\right)+\frac{1}{4}x^4\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$\downarrow$ 7163

$$\frac{1}{4}b\left((1-d)\left(\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\int x\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)dx}{b}\right)}{2b}-\frac{x^3\,\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}\right)}{b(1-d)}-\frac{x^4\log\left(1-(1\right.}{2b(1}\right.\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$\downarrow$ 7163

$$\frac{1}{4}b\left((1-d)\left(\frac{2\left(\frac{3\left(\frac{x^2\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\frac{x\,\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\int \text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)dx}{2b}}{b}\right)}{2b}-\frac{x^3\,\text{PolyLog}(2,(1-d)e^{2a+2}}{2b}\right.}{b(1-d)}\right.\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$\downarrow$ 2720

3.226. $\int x^3\coth^{-1}(1-d-d\coth(a+bx))\,dx$

$$\frac{1}{4}b\left((1-d)\left(\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\operatorname{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}\right)}{2b}-\frac{x^3\operatorname{PolyLo}}{}\right.}{b(1-d)}\right.\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$\downarrow$ 7143

$$\frac{1}{4}b\left((1-d)\left(\frac{2\left(\frac{3\left(\frac{x^2\operatorname{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\frac{x\operatorname{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\operatorname{PolyLog}\left(5,(1-d)e^{2a+2bx}\right)}{4b^2}}{b}\right)}{2b}-\frac{x^3\operatorname{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}\right.}{b(1-d)}\right.\right.$$

$$\frac{1}{4}x^4\coth^{-1}(d(-\coth(a+bx))-d+1)$$

input
```
Int[x^3*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

output
```
(x^4*ArcCoth[1 - d - d*Coth[a + b*x]])/4 + (b*(x^5/5 + (1 - d)*(-1/2*(x^4*
Log[1 - (1 - d)*E^(2*a + 2*b*x)])/(b*(1 - d)) + (2*(-1/2*(x^3*PolyLog[2, (
1 - d)*E^(2*a + 2*b*x)])/b + (3*((x^2*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])
/(2*b) - ((x*PolyLog[4, (1 - d)*E^(2*a + 2*b*x)])/(2*b) - PolyLog[5, (1 -
d)*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b)))/(b*(1 - d)))))/4
```

3.226. $\int x^3\coth^{-1}(1-d-d\coth(a+bx))\,dx$

3.226.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6796
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.226.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.02 (sec) , antiderivative size = 1782, normalized size of antiderivative = 10.80

method	result	size
risch	Expression too large to display	1782

input
```
int(x^3*arccoth(1-d-d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^4*d*a^3/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^4*a^3/(d-1)*dilo
g(1+exp(b*x+a)*(1-d)^(1/2))-1/8*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^4-1/2
/b^4*a^4/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^4*a^4/(d-1)*ln(1-exp(b*x
+a)*(1-d)^(1/2))+3/16/b^4*d/(d-1)*polylog(5,-(d-1)*exp(2*b*x+2*a))+1/4/b/(
d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^3+3/8/b^4/(d-1)*ln(1+(d-1)*exp(2*b
*x+2*a))*a^4-3/8/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x^2+1/4/b^4/(d
-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a^3-1/2/b^4*a^3/(d-1)*dilog(1-exp(b*x
+a)*(1-d)^(1/2))+3/8/b^3/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))*x-1/8/b^4*
d*a^4/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1)+1/2/b^4*d*a^3/(d-1)*dilo
g(1-exp(b*x+a)*(1-d)^(1/2))+1/2/b^3/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a^3
-1/4/b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^3-3/8/b^4*d/(d-1)*ln(1+(
d-1)*exp(2*b*x+2*a))*a^4+3/8/b^2*d/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*
x^2-1/4/b^4*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a^3-1/2/b^3*a^3/(d-1)
*ln(1+exp(b*x+a)*(1-d)^(1/2))*x-1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1
/2))*x-3/8/b^3*d/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))*x+1/2/b^4*d*a^4/(d
-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^4*d*a^4/(d-1)*ln(1-exp(b*x+a)*(1-d)
^(1/2))+1/20*b*x^5+1/8*x^4*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1)-3/16/b^4/
(d-1)*polylog(5,-(d-1)*exp(2*b*x+2*a))+1/8/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a)
)*x^4-1/2/b^3*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a^3+1/2/b^3*d*a^3/(d-1)
*ln(1+exp(b*x+a)*(1-d)^(1/2))*x+1/2/b^3*d*a^3/(d-1)*ln(1-exp(b*x+a)*(1-...
```

3.226.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(138) = 276$.

Time = 0.27 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.73

$$\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx$$
$$= \frac{2\,b^5x^5 - 5\,b^4x^4\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) - 20\,b^3x^3\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d+4}(\cosh(bx+a)+\sinh(bx+a))\right)}{}$$

input
```
integrate(x^3*arccoth(1-d-d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/40*(2*b^5*x^5 - 5*b^4*x^4*log((d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cos
h(b*x + a) + (d - 2)*sinh(b*x + a))) - 20*b^3*x^3*dilog(1/2*sqrt(-4*d + 4)
*(cosh(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-1/2*sqrt(-4*d + 4)*(
cosh(b*x + a) + sinh(b*x + a))) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d
 - 1)*sinh(b*x + a) + sqrt(-4*d + 4)) - 5*a^4*log(2*(d - 1)*cosh(b*x + a)
+ 2*(d - 1)*sinh(b*x + a) - sqrt(-4*d + 4)) + 60*b^2*x^2*polylog(3, 1/2*sq
rt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -1/2
*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, 1/2*
sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, -1/2*
sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(b^4*x^4 - a^4)*log(1/
2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^4*x^4 - a^4)*
log(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*polylog
(5, 1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -
1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

3.226.6 Sympy [F]

$$\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx = -\int x^3\,\mathrm{acoth}\,(d\coth\,(a+bx)+d-1)\,dx$$

input
```
integrate(x**3*acoth(1-d-d*coth(b*x+a)),x)
```

output
```
-Integral(x**3*acoth(d*coth(a + b*x) + d - 1), x)
```

3.226.7 Maxima [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.90

$$\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx = -\frac{1}{4}\,x^4\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+d-1\right) + \frac{1}{40}\left(\frac{2\,x^5}{d} - \frac{5\,\left(2\,b^4x^4\log\left((d-1)e^{(2\,bx+2\,a)}+1\right)+4\,b^3x^3\mathrm{Li}_2\left(-(d-1)e^{(2\,bx+2\,a)}\right)-6\,b^2x^2\mathrm{Li}_3(-(d-1)e^{(}}{b^5d}\right.$$

input
```
integrate(x^3*arccoth(1-d-d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
-1/4*x^4*arccoth(d*coth(b*x + a) + d - 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*l
og((d - 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog(-(d - 1)*e^(2*b*x + 2*a)
) - 6*b^2*x^2*polylog(3, -(d - 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, -(d
- 1)*e^(2*b*x + 2*a)) - 3*polylog(5, -(d - 1)*e^(2*b*x + 2*a)))/(b^5*d))*b
*d
```

3.226.8 Giac [F]

$$\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx = \int x^3\,\mathrm{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)\,dx$$

input
```
integrate(x^3*arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^3*arccoth(-d*coth(b*x + a) - d + 1), x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx = \int -x^3\,\mathrm{acoth}(d+d\coth(a+b\,x)-1)\,dx$$

input
```
int(-x^3*acoth(d + d*coth(a + b*x) - 1),x)
```

output
```
int(-x^3*acoth(d + d*coth(a + b*x) - 1), x)
```

3.227 $\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx$

3.227.1 Optimal result

Integrand size = 19, antiderivative size = 137

$$\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx = \frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x^2\,\mathrm{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b} + \frac{x\,\mathrm{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{4b^2} - \frac{\mathrm{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{8b^3}$$

output
```
1/12*b*x^4+1/3*x^3*arccoth(1-d-d*coth(b*x+a))-1/6*x^3*ln(1-(1-d)*exp(2*b*x
+2*a))-1/4*x^2*polylog(2,(1-d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,(1-d)*exp
(2*b*x+2*a))/b^2-1/8*polylog(4,(1-d)*exp(2*b*x+2*a))/b^3
```

3.227.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx$$
$$=\frac{8b^3x^3\coth^{-1}(1-d-d\coth(a+bx))-4b^3x^3\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right)+6b^2x^2\operatorname{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right)+6bx\operatorname{P}}{24b^3}$$

input
```
Integrate[x^2*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCoth[1 - d - d*Coth[a + b*x]] - 4*b^3*x^3*Log[1 + 1/((-1 + d
)*E^(2*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))
] + 6*b*x*PolyLog[3, -(1/((-1 + d)*E^(2*(a + b*x))))] + 3*PolyLog[4, -(1/(
(-1 + d)*E^(2*(a + b*x))))])/(24*b^3)
```

3.227.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6796, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(d(-\coth(a+bx))-d+1)\,dx$$

$$\Big\downarrow \text{ 6796}$$

$$\frac{1}{3}b\int \frac{x^3}{1-(1-d)e^{2a+2bx}}dx+\frac{1}{3}x^3\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{1}{3}b\left((1-d)\int \frac{e^{2a+2bx}x^3}{1-(1-d)e^{2a+2bx}}dx+\frac{x^4}{4}\right)+\frac{1}{3}x^3\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{3}b\left((1-d)\left(\frac{3\int x^2\log\left(1-(1-d)e^{2a+2bx}\right)dx}{2b(1-d)}-\frac{x^3\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^4}{4}\right)+$$
$$\frac{1}{3}x^3\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}b\left((1-d)\left(\frac{3\left(\frac{\int x\,\text{PolyLog}(2,(1-d)e^{2a+2bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}\right)}{2b(1-d)}-\frac{x^3\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^4}{4}\right)+\frac{1}{3}x^3\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3}b\left((1-d)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}\right)}{2b(1-d)}-\frac{x^3\log\left(1-(1-d)e\right.}{2b(1-d)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3}b\left((1-d)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}\right)}{2b(1-d)}-\frac{x^3\log\left(\right.}{}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{3}b\left((1-d)\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{2b}-\frac{\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}\right)}{2b(1-d)}-\frac{x^3\log\left(1-(1-d)e^{2a+}\right.}{2b(1-d)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d(-\coth(a+bx))-d+1)$$

input
```
Int[x^2*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

3.227. $\int x^2\coth^{-1}(1-d-d\coth(a+bx))\,dx$

output
```
(x^3*ArcCoth[1 - d - d*Coth[a + b*x]])/3 + (b*(x^4/4 + (1 - d)*(-1/2*(x^3*
Log[1 - (1 - d)*E^(2*a + 2*b*x)])/(b*(1 - d)) + (3*(-1/2*(x^2*PolyLog[2, (
1 - d)*E^(2*a + 2*b*x)])/b + ((x*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])/(2*b
) - PolyLog[4, (1 - d)*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b*(1 - d)))))/3
```

3.227.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6796
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.227.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.88 (sec) , antiderivative size = 1723, normalized size of antiderivative = 12.58

method	result	size
risch	Expression too large to display	1723

input
```
int(x^2*arccoth(1-d-d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x+1/2/b^3*a^3/(d-1)*ln(1+e
xp(b*x+a)*(1-d)^(1/2))+1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))+1/2/
b^3*a^2/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^3*a^2/(d-1)*dilog(1-ex
p(b*x+a)*(1-d)^(1/2))-1/6/b^3*a^3/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)
+1)-1/6*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^3-1/8/b^3*d/(d-1)*polylog(4,-
(d-1)*exp(2*b*x+2*a))+1/4/b/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^2-1/3
/b^3/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^3-1/4/b^3/(d-1)*polylog(2,-(d-1)*e
xp(2*b*x+2*a))*a^2-1/2/b^2*a^2*d/(d-1)*x*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/
b^2*a^2*d/(d-1)*x*ln(1-exp(b*x+a)*(1-d)^(1/2))+1/2/b^2*d/(d-1)*ln(1+(d-1)*
exp(2*b*x+2*a))*a^2*x+1/8/b^3/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))+1/6/(
d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^3+1/2/b^2*a^2/(d-1)*x*ln(1-exp(b*x+a)*(1
-d)^(1/2))+1/6/b^3*d*a^3/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1)-1/2/b
^3*a^3*d/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^3*a^3*d/(d-1)*ln(1-exp(b
*x+a)*(1-d)^(1/2))-1/2/b^3*a^2*d/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))-1/2
/b^3*a^2*d/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))-1/2/b^2/(d-1)*ln(1+(d-1)*
exp(2*b*x+2*a))*a^2*x-1/4/b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^2+1
/3/b^3*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^3+1/4/b^3*d/(d-1)*polylog(2,-(
d-1)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*
x+1/2/b^2*a^2/(d-1)*x*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/12*b*x^4-1/3*x^3*ln(e
xp(b*x+a))-1/12*(-I*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)-exp(...
```

3.227.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(114) = 228$.

Time = 0.27 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.78

$$\int x^2 \coth^{-1}(1 - d - d\coth(a + bx))\,dx$$

$$= \frac{b^4x^4 - 2\,b^3x^3\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d+4}(\cosh(bx+a)+\sinh(bx+a))\right) - \cdots}{\cdots}$$

input
```
integrate(x^2*arccoth(1-d-d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(b^4*x^4 - 2*b^3*x^3*log((d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(
b*x + a) + (d - 2)*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d + 4)*(c
osh(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d + 4)*(cosh
(b*x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1
)*sinh(b*x + a) + sqrt(-4*d + 4)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*
(d - 1)*sinh(b*x + a) - sqrt(-4*d + 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d
+ 4)*(cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d +
 4)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*d
 + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sq
rt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqrt
(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*d
 + 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

3.227.6 Sympy [F]

$$\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx = -\int x^2 \operatorname{acoth}\left(d\coth\left(a+bx\right)+d-1\right)dx$$

input
```
integrate(x**2*acoth(1-d-d*coth(b*x+a)),x)
```

output
```
-Integral(x**2*acoth(d*coth(a + b*x) + d - 1), x)
```

3.227.7 Maxima [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx = -\frac{1}{3}x^3\operatorname{arcoth}\left(d\coth\left(bx+a\right)+d-1\right) + \frac{1}{36}\left(\frac{3x^4}{d} - \frac{2\left(4b^3x^3\log\left((d-1)e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(-(d-1)e^{(2bx+2a)}\right)-6bx\mathrm{Li}_3(-(d-1)e^{(2b}\right.}{b^4d}\right.$$

input
```
integrate(x^2*arccoth(1-d-d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
-1/3*x^3*arccoth(d*coth(b*x + a) + d - 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*l
og((d - 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-(d - 1)*e^(2*b*x + 2*a)
) - 6*b*x*polylog(3, -(d - 1)*e^(2*b*x + 2*a)) + 3*polylog(4, -(d - 1)*e^(
2*b*x + 2*a)))/(b^4*d))*b*d
```

3.227.8 Giac [F]

$$\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx = \int x^2 \operatorname{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)\,dx$$

input
```
integrate(x^2*arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(-d*coth(b*x + a) - d + 1), x)
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx = \int -x^2\,\mathrm{acoth}(d+d\coth(a+b\,x)-1)\,dx$$

input
```
int(-x^2*acoth(d + d*coth(a + b*x) - 1),x)
```

output
```
int(-x^2*acoth(d + d*coth(a + b*x) - 1), x)
```

3.228 $\int x \coth^{-1}(1-d-d\coth(a+bx))\,dx$

3.228.1 Optimal result

Integrand size = 17, antiderivative size = 109

$$\int x \coth^{-1}(1-d-d\coth(a+bx))\,dx = \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\coth(a+bx)) -\frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right) -\frac{x\,\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b} +\frac{\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{8b^2}$$

output
```
1/6*b*x^3+1/2*x^2*arccoth(1-d-d*coth(b*x+a))-1/4*x^2*ln(1-(1-d)*exp(2*b*x+
2*a))-1/4*x*polylog(2,(1-d)*exp(2*b*x+2*a))/b+1/8*polylog(3,(1-d)*exp(2*b*
x+2*a))/b^2
```

3.228.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int x \coth^{-1}(1-d-d\coth(a+bx))\,dx = \frac{2b^2x^2\left(2\coth^{-1}(1-d-d\coth(a+bx))-\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right)\right)+2bx\,\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right)+\text{PolyLog}}{8b^2}$$

input
```
Integrate[x*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcCoth[1 - d - d*Coth[a + b*x]] - Log[1 + 1/((-1 + d)*E^(2*
(a + b*x)))]) + 2*b*x*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))] + PolyLo
g[3, -(1/((-1 + d)*E^(2*(a + b*x))))])/(8*b^2)
```

3.228.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6796, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(d(-\coth(a+bx))-d+1)\,dx$$

$$\Big\downarrow \; 6796$$

$$\frac{1}{2}b\int \frac{x^2}{1-(1-d)e^{2a+2bx}}dx+\frac{1}{2}x^2\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \; 2615$$

$$\frac{1}{2}b\left((1-d)\int \frac{e^{2a+2bx}x^2}{1-(1-d)e^{2a+2bx}}dx+\frac{x^3}{3}\right)+\frac{1}{2}x^2\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \; 2620$$

$$\frac{1}{2}b\left((1-d)\left(\frac{\int x\log\left(1-(1-d)e^{2a+2bx}\right)dx}{b(1-d)}-\frac{x^2\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^3}{3}\right)+\frac{1}{2}x^2\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \; 3011$$

$$\frac{1}{2}b\left((1-d)\left(\frac{\frac{\int \text{PolyLog}(2,(1-d)e^{2a+2bx})dx}{2b}-\frac{x\,\text{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}}{b(1-d)}-\frac{x^2\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^3}{3}\right)+\frac{1}{2}x^2\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{2}b\left((1-d)\left(\frac{\frac{\int e^{-2a-2bx}\operatorname{PolyLog}(2,(1-d)e^{2a+2bx})de^{2a+2bx}}{4b^2}-\frac{x\operatorname{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}}{b(1-d)}-\frac{x^2\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^3}{3}\right)+$$
$$\frac{1}{2}x^2\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{1}{2}b\left((1-d)\left(\frac{\frac{\operatorname{PolyLog}(3,(1-d)e^{2a+2bx})}{4b^2}-\frac{x\operatorname{PolyLog}(2,(1-d)e^{2a+2bx})}{2b}}{b(1-d)}-\frac{x^2\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^3}{3}\right)+$$
$$\frac{1}{2}x^2\coth^{-1}(d(-\coth(a+bx))-d+1)$$

input
```
Int[x*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 - d - d*Coth[a + b*x]])/2 + (b*(x^3/3 + (1 - d)*(-1/2*(x^2*
Log[1 - (1 - d)*E^(2*a + 2*b*x)])/(b*(1 - d)) + (-1/2*(x*PolyLog[2, (1 - d
)*E^(2*a + 2*b*x)])/b + PolyLog[3, (1 - d)*E^(2*a + 2*b*x)]/(4*b^2))/(b*(1
 - d)))))/2
```

3.228.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6796
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
 + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c
- d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.228.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.57 (sec) , antiderivative size = 1640, normalized size of antiderivative = 15.05

method	result	size
risch	Expression too large to display	1640

input
```
int(x*arccoth(1-d-d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*a*d/(d-1)*x*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b*a*d/(d-1)*x*ln(1-exp(
b*x+a)*(1-d)^(1/2))-1/2/b*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a*x-1/2*x^2*l
n(exp(b*x+a))+1/6*b*x^3+1/2/b^2*a^2*d/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1
/2/b^2*a^2*d/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))+1/2/b^2*a*d/(d-1)*dilog(1+
exp(b*x+a)*(1-d)^(1/2))+1/2/b^2*a*d/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))-
1/8/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))+1/4/(d-1)*ln(1+(d-1)*exp(2*
b*x+2*a))*x^2+1/8/b^2*d/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))+1/4/b^2/(d-
1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^2+1/4/b/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2
*a))*x+1/4/b^2/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a-1/2/b^2*a^2/(d-1)*
ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^2*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2)
)-1/2/b^2*a/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^2*a/(d-1)*dilog(1-
exp(b*x+a)*(1-d)^(1/2))+1/4/b^2*a^2/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*
a)+1)-1/4*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^2-1/4/b^2*d/(d-1)*ln(1+(d-1
)*exp(2*b*x+2*a))*a^2-1/4/b^2*a^2*d/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*
a)+1)-1/4/b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x-1/4/b^2*d/(d-1)*pol
ylog(2,-(d-1)*exp(2*b*x+2*a))*a+1/2/b/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a*x
-1/2/b*a/(d-1)*x*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b*a/(d-1)*x*ln(1-exp(b*x
+a)*(1-d)^(1/2))+1/4*x^2*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1)-1/8*(-I*Pi*
csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1))^3-I*Pi*csgn
(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)-exp(...
```

3.228.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(90) = 180$.

Time = 0.26 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.95

$$\int x\coth^{-1}(1-d-d\coth(a+bx))\,dx$$

$$= \frac{2\,b^3x^3 - 3\,b^2x^2\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) - 6\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d+4}(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right) - 6}{}$$

input
```
integrate(x*arccoth(1-d-d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*x^3 - 3*b^2*x^2*log((d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cos
h(b*x + a) + (d - 2)*sinh(b*x + a))) - 6*b*x*dilog(1/2*sqrt(-4*d + 4)*(cos
h(b*x + a) + sinh(b*x + a))) - 6*b*x*dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*x +
 a) + sinh(b*x + a))) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh
(b*x + a) + sqrt(-4*d + 4)) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1
)*sinh(b*x + a) - sqrt(-4*d + 4)) - 3*(b^2*x^2 - a^2)*log(1/2*sqrt(-4*d +
4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*sqrt(
-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*polylog(3, 1/2*sqrt(-4*
d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(3, -1/2*sqrt(-4*d + 4)
*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

3.228.6 Sympy [F]

$$\int x \coth^{-1}(1-d-d\coth(a+bx))\,dx = -\int x\,\mathrm{acoth}\,(d\coth(a+bx)+d-1)\,dx$$

input
```
integrate(x*acoth(1-d-d*coth(b*x+a)),x)
```

output
```
-Integral(x*acoth(d*coth(a + b*x) + d - 1), x)
```

3.228.7 Maxima [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x \coth^{-1}(1-d-d\coth(a+bx))\,dx = \frac{1}{24}\left(\frac{4x^3}{d} - \frac{3\left(2b^2x^2\log\left((d-1)e^{(2bx+2a)}+1\right)+2bx\mathrm{Li}_2\left(-(d-1)e^{(2bx+2a)}\right)-\mathrm{Li}_3\left(-(d-1)e^{(2bx+2a)}\right)\right)}{b^3d}\right) - \frac{1}{2}x^2\,\mathrm{arcoth}\,(d\coth(bx+a)+d-1)$$

input
```
integrate(x*arccoth(1-d-d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(4*x^3/d - 3*(2*b^2*x^2*log((d - 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g(-(d - 1)*e^(2*b*x + 2*a)) - polylog(3, -(d - 1)*e^(2*b*x + 2*a)))/(b^3*d
))*b*d - 1/2*x^2*arccoth(d*coth(b*x + a) + d - 1)
```

3.228.8 Giac [F]

$$\int x\coth^{-1}(1-d-d\coth(a+bx))\,dx=\int x\,\mathrm{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)\,dx$$

input
```
integrate(x*arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(-d*coth(b*x + a) - d + 1), x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int x\coth^{-1}(1-d-d\coth(a+bx))\,dx=\int -x\,\mathrm{acoth}(d+d\coth(a+b\,x)-1)\,dx$$

input
```
int(-x*acoth(d + d*coth(a + b*x) - 1),x)
```

output
```
int(-x*acoth(d + d*coth(a + b*x) - 1), x)
```

3.229 $\int \coth^{-1}(1-d-d\coth(a+bx))\,dx$

3.229.1 Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \coth^{-1}(1-d-d\coth(a+bx))\,dx = \frac{bx^2}{2} + x\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}$$

output
```
1/2*b*x^2+x*arccoth(1-d-d*coth(b*x+a))-1/2*x*ln(1-(1-d)*exp(2*b*x+2*a))-1/
4*polylog(2,(1-d)*exp(2*b*x+2*a))/b
```

3.229.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \coth^{-1}(1-d-d\coth(a+bx))\,dx = x\coth^{-1}(1-d-d\coth(a+bx)) + \frac{-2bx\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right) + \text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{4b}$$

input
```
Integrate[ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

output
```
x*ArcCoth[1 - d - d*Coth[a + b*x]] + (-2*b*x*Log[1 + 1/((-1 + d)*E^(2*(a +
 b*x)))] + PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))])/(4*b)
```

3.229.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.34, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6788, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d(-\coth(a+bx))-d+1)\,dx$$

$$\Big\downarrow \text{6788}$$

$$b\int \frac{x}{1-(1-d)e^{2a+2bx}}dx+x\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{2615}$$

$$b\left((1-d)\int \frac{e^{2a+2bx}x}{1-(1-d)e^{2a+2bx}}dx+\frac{x^2}{2}\right)+x\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{2620}$$

$$b\left((1-d)\left(\frac{\int \log\left(1-(1-d)e^{2a+2bx}\right)dx}{2b(1-d)}-\frac{x\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^2}{2}\right)+x\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{2715}$$

$$b\left((1-d)\left(\frac{\int e^{-2a-2bx}\log\left(1-(1-d)e^{2a+2bx}\right)de^{2a+2bx}}{4b^2(1-d)}-\frac{x\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^2}{2}\right)+x\coth^{-1}(d(-\coth(a+bx))-d+1)$$

$$\Big\downarrow \text{2838}$$

$$b\left((1-d)\left(-\frac{\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b^2(1-d)}-\frac{x\log\left(1-(1-d)e^{2a+2bx}\right)}{2b(1-d)}\right)+\frac{x^2}{2}\right)+x\coth^{-1}(d(-\coth(a+bx))-d+1)$$

input
```
Int[ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

output
```
x*ArcCoth[1 - d - d*Coth[a + b*x]] + b*(x^2/2 + (1 - d)*(-1/2*(x*Log[1 - (
1 - d)*E^(2*a + 2*b*x)])/(b*(1 - d)) - PolyLog[2, (1 - d)*E^(2*a + 2*b*x)]
/(4*b^2*(1 - d))))
```

3.229.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6788
```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Coth[a + b*x]], x] + Simp[b   Int[x/(c - d - c*E^(2*a + 2*b*x))
, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

3.229.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(68) = 136$.

Time = 1.66 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.57

method	result
derivativedivides	$-\frac{\frac{\operatorname{arccoth}(1-d-d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}-\frac{\operatorname{arccoth}(1-d-d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\ldots$
default	$-\frac{\frac{\operatorname{arccoth}(1-d-d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}-\frac{\operatorname{arccoth}(1-d-d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\ldots$
risch	Expression too large to display

input
```
int(arccoth(1-d-d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/b/d*(1/2*arccoth(1-d-d*coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)-1/2*arccoth(
1-d-d*coth(b*x+a))*d*ln(-d*coth(b*x+a)-d)-1/2*d^2*(1/d*(-1/2*dilog(-1/2*d*
coth(b*x+a)-1/2*d+1)-1/2*ln(-d*coth(b*x+a)-d)*ln(-1/2*d*coth(b*x+a)-1/2*d+
1)+1/4*ln(-d*coth(b*x+a)-d)^2)-1/d*(1/2*dilog(-1/2*(-d*coth(b*x+a)-d)/d)+1
/2*ln(-d*coth(b*x+a)+d)*ln(-1/2*(-d*coth(b*x+a)-d)/d)-1/2*dilog((-d*coth(b
*x+a)-d+2)/(-2*d+2))-1/2*ln(-d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)-d+2)/(-2*
d+2)))))
```

3.229.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(63) = 126$.

Time = 0.26 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.14

$$\int \coth^{-1}(1-d-d\coth(a+bx))\,dx$$
$$= \frac{b^2x^2 - bx\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) + a\log\left(2\,(d-1)\cosh\,(bx+a) + 2\,(d-1)\sinh\,(bx+a) + \sqrt{-4}\ldots\right.}{\ldots}$$

input
```
integrate(arccoth(1-d-d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 - b*x*log((d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b*x + a
) + (d - 2)*sinh(b*x + a))) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*si
nh(b*x + a) + sqrt(-4*d + 4)) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*
sinh(b*x + a) - sqrt(-4*d + 4)) - (b*x + a)*log(1/2*sqrt(-4*d + 4)*(cosh(b
*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-1/2*sqrt(-4*d + 4)*(cosh(b*
x + a) + sinh(b*x + a)) + 1) - dilog(1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + s
inh(b*x + a))) - dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))
))/b
```

3.229.6 Sympy [F]

$$\int \coth^{-1}(1-d-d\coth(a+bx))\,dx=-\int \operatorname{acoth}\left(d\coth\left(a+bx\right)+d-1\right)dx$$

input
```
integrate(acoth(1-d-d*coth(b*x+a)),x)
```

output
```
-Integral(acoth(d*coth(a + b*x) + d - 1), x)
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \coth^{-1}(1-d-d\coth(a+bx))\,dx$$
$$=\frac{1}{4}bd\left(\frac{2x^2}{d}-\frac{2bx\log\left((d-1)e^{(2bx+2a)}+1\right)+\mathrm{Li}_2\left(-(d-1)e^{(2bx+2a)}\right)}{b^2d}\right)$$
$$-x\operatorname{arcoth}\left(d\coth\left(bx+a\right)+d-1\right)$$

input
```
integrate(arccoth(1-d-d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*b*d*(2*x^2/d - (2*b*x*log((d - 1)*e^(2*b*x + 2*a) + 1) + dilog(-(d - 1
)*e^(2*b*x + 2*a)))/(b^2*d)) - x*arccoth(d*coth(b*x + a) + d - 1)
```

3.229.8 Giac [F]

$$\int \coth^{-1}(1-d-d\coth(a+bx))\,dx=\int \operatorname{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)\,dx$$

input
```
integrate(arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(-d*coth(b*x + a) - d + 1), x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(1-d-d\coth(a+bx))\,dx=\int -\mathrm{acoth}(d+d\coth(a+b\,x)-1)\,dx$$

input
```
int(-acoth(d + d*coth(a + b*x) - 1),x)
```

output
```
int(-acoth(d + d*coth(a + b*x) - 1), x)
```

3.230 $\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx$

3.230.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx = \mathrm{Int}\left(\frac{\coth^{-1}(1-d-d\coth(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccoth(1-d-d*coth(b*x+a))/x,x)
```

3.230.2 Mathematica [N/A]

Not integrable

Time = 2.91 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[1 - d - d*Coth[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[1 - d - d*Coth[a + b*x]]/x, x]
```

3.230.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d(-\coth(a+bx))-d+1)}{x} \, dx$$

$$\Big\downarrow 7299$$

$$\int \frac{\coth^{-1}(d(-\coth(a+bx))-d+1)}{x}dx$$

input
```
Int[ArcCoth[1 - d - d*Coth[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.230.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.230.4 Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(1-d-d\coth\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(1-d-d*coth(b*x+a))/x,x)
```

output
```
int(arccoth(1-d-d*coth(b*x+a))/x,x)
```

3.230.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-d-d*coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(-arccoth(d*coth(b*x + a) + d - 1)/x, x)
```

3.230.6 Sympy [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx = -\int \frac{\operatorname{acoth}\left(d\coth\left(a+bx\right)+d-1\right)}{x}\,dx$$

input
```
integrate(acoth(1-d-d*coth(b*x+a))/x,x)
```

output
```
-Integral(acoth(d*coth(a + b*x) + d - 1)/x, x)
```

3.230.7 Maxima [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-d-d*coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
-integrate(arccoth(d*coth(b*x + a) + d - 1)/x, x)
```

3.230.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-d-d*coth(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(-d*coth(b*x + a) - d + 1)/x, x)
```

3.230.9 Mupad [N/A]

Not integrable

Time = 4.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx = \int -\frac{\operatorname{acoth}(d+d\coth(a+b\,x)-1)}{x}\,dx$$

input
```
int(-acoth(d + d*coth(a + b*x) - 1)/x,x)
```

output
```
int(-acoth(d + d*coth(a + b*x) - 1)/x, x)
```

3.231 $\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx$

3.231.1 Optimal result

Integrand size = 15, antiderivative size = 302

$$\begin{aligned}\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx &= \frac{(e+fx)^4 \coth^{-1}(\tan(a+bx))}{4f} \\ &+ \frac{i(e+fx)^4 \arctan\left(e^{2i(a+bx)}\right)}{4f} \\ &- \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} \\ &+ \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b} \\ &+ \frac{3f(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2} \\ &- \frac{3f(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2} \\ &+ \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3} \\ &- \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3} \\ &- \frac{3f^3 \operatorname{PolyLog}\left(5,-ie^{2i(a+bx)}\right)}{16b^4} \\ &+ \frac{3f^3 \operatorname{PolyLog}\left(5,ie^{2i(a+bx)}\right)}{16b^4}\end{aligned}$$

output
```
1/4*(f*x+e)^4*arccoth(tan(b*x+a))/f+1/4*I*(f*x+e)^4*arctan(exp(2*I*(b*x+a)
))/f-1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^3*poly
log(2,I*exp(2*I*(b*x+a)))/b+3/8*f*(f*x+e)^2*polylog(3,-I*exp(2*I*(b*x+a)))
/b^2-3/8*f*(f*x+e)^2*polylog(3,I*exp(2*I*(b*x+a)))/b^2+3/8*I*f^2*(f*x+e)*p
olylog(4,-I*exp(2*I*(b*x+a)))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*I*(b
*x+a)))/b^3-3/16*f^3*polylog(5,-I*exp(2*I*(b*x+a)))/b^4+3/16*f^3*polylog(5
,I*exp(2*I*(b*x+a)))/b^4
```

3.231.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 654 vs. $2(302) = 604$.

Time = 0.20 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.17

$$\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx = \frac{1}{4}x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\coth^{-1}(\tan(a+bx))$$
$$+\frac{-8b^4e^3x\log\left(1-ie^{2i(a+bx)}\right)-12b^4e^2fx^2\log\left(1-ie^{2i(a+bx)}\right)-8b^4ef^2x^3\log\left(1-ie^{2i(a+bx)}\right)-2b^4f^3x^4\,\mathrm{lo}}{}$$

input
```
Integrate[(e + f*x)^3*ArcCoth[Tan[a + b*x]],x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcCoth[Tan[a + b*x]])/4 +
(-8*b^4*e^3*x*Log[1 - I*E^((2*I)*(a + b*x))] - 12*b^4*e^2*f*x^2*Log[1 - I*
E^((2*I)*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] - 2*
b^4*f^3*x^4*Log[1 - I*E^((2*I)*(a + b*x))] + 8*b^4*e^3*x*Log[1 + I*E^((2*I
)*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 8*b^4*e*
f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] + 2*b^4*f^3*x^4*Log[1 + I*E^((2*I)*
(a + b*x))] - (4*I)*b^3*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))] +
 (4*I)*b^3*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))] + 6*b^2*e^2*f*Pol
yLog[3, (-I)*E^((2*I)*(a + b*x))] + 12*b^2*e*f^2*x*PolyLog[3, (-I)*E^((2*I
)*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b^2
*e^2*f*PolyLog[3, I*E^((2*I)*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3, I*E^(
(2*I)*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^((2*I)*(a + b*x))] + (6*I
)*b*e*f^2*PolyLog[4, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b*f^3*x*PolyLog[4,
(-I)*E^((2*I)*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))
] - (6*I)*b*f^3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I
)*E^((2*I)*(a + b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

3.231.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6806, 3042, 4669, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx$$

$$\Big\downarrow \text{ 6806}$$

$$\frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f}-\frac{b\int (e+fx)^4\sec(2a+2bx)dx}{4f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f}-\frac{b\int (e+fx)^4\csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{4f}$$

$$\Big\downarrow \text{ 4669}$$

$$\frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f}-\frac{b\left(-\frac{2f\int (e+fx)^3\log\left(1-ie^{2i(a+bx)}\right)dx}{b}+\frac{2f\int (e+fx)^3\log\left(1+ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^4\arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{4f}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f}-\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\int (e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{2f\left(\frac{i(e+fx)^3\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{3if\int (e+fx)^2\text{PolyLog}\left(2\right.}{2b}\right.}{b}\right.}{4f}$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f}-\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\int (e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^2\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{2b}\right)}{b}-\frac{2f\left(\frac{i(e+fx)^3\text{PolyLo}}{2}\right.}{}\right.}{4f}$$

$\downarrow$ 7163

$$\frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f} -$$

$$b\left(\frac{2f\left(\frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\left(\frac{if\int\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)dx}{2b}-\frac{i(e+fx)\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}-\frac{i(e+fx)^2\operatorname{PolyLog}\left(3,-ie^{2i(a+b}\right.}{2b}\right.}{2b}\right.}{b}\right.$$

$\downarrow$ 2720

$$\frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f} -$$

$$b\left(\frac{2f\left(\frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\left(\frac{f\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}-\frac{i(e+fx)^2\,\mathrm{P}}{}\right.}{2b}\right.}{b}\right.$$

$\downarrow$ 7143

3.231. $\int (e+fx)^3\coth^{-1}(\tan(a+bx))\,dx$

$$\frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f} - \frac{b\left(-\frac{i(e+fx)^4\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{2f\left(\frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\left(\frac{f\operatorname{PolyLog}\left(5,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right.}{2b}\right.}{b}\right.}{}$$

input
```
Int[(e + f*x)^3*ArcCoth[Tan[a + b*x]],x]
```

output
```
((e + f*x)^4*ArcCoth[Tan[a + b*x]])/(4*f) - (b*(((-I)*(e + f*x)^4*ArcTan[E
^((2*I)*(a + b*x))])/b + (2*f*(((I/2)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)
*(a + b*x))])/b - (((3*I)/2)*f*(((-1/2*I)*(e + f*x)^2*PolyLog[3, (-I)*E^((
2*I)*(a + b*x))])/b + (I*f*(((-1/2*I)*(e + f*x)*PolyLog[4, (-I)*E^((2*I)*(
a + b*x))])/b + (f*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b))
/b - (2*f*(((I/2)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (((3*
I)/2)*f*(((-1/2*I)*(e + f*x)^2*PolyLog[3, I*E^((2*I)*(a + b*x))])/b + (I*f
*(((-1/2*I)*(e + f*x)*PolyLog[4, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[5,
 I*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b))/b))/(4*f)
```

3.231.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6806
```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Tan[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.231. $\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx$

3.231.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 24.71 (sec) , antiderivative size = 3640, normalized size of antiderivative = 12.05

method	result	size
risch	Expression too large to display	3640

input
```
int((f*x+e)^3*arccoth(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
3/2*f^2/b^2*e*a^2*ln(-I*exp(2*I*(b*x+a))+1)*x+1/4*I*f^3/b*polylog(2,I*exp(
2*I*(b*x+a)))*x^3+1/4*I*f^3/b^4*polylog(2,I*exp(2*I*(b*x+a)))*a^3-3/8*I*f^
3/b^3*polylog(4,I*exp(2*I*(b*x+a)))*x+3/2*f^2/b^3*a^3*e*ln(1-exp(I*(b*x+a)
)*(-1)^(3/4))-3/2*f/b^2*a^2*e^2*ln(1+exp(I*(b*x+a))*(-1)^(3/4))-3/16*f^3*p
olylog(5,-I*exp(2*I*(b*x+a)))/b^4+3/16*f^3*polylog(5,I*exp(2*I*(b*x+a)))/b
^4+3/4*f*e^2*ln(1+I*exp(2*I*(b*x+a)))*x^2+1/2*f^2*e*ln(1+I*exp(2*I*(b*x+a)
))*x^3+1/8*f^3/b^4*a^4*ln(-exp(2*I*(b*x+a))+I)-1/2*f^3/b^3*a^3*ln(1+exp(I*
(b*x+a))*(-1)^(3/4))*x-1/2*f^3/b^3*a^3*ln(1-exp(I*(b*x+a))*(-1)^(3/4))*x-f
^2/b^3*e*ln(1+I*exp(2*I*(b*x+a)))*a^3+1/2*f^3/b^3*ln(1+I*exp(2*I*(b*x+a)))
*x*a^3-1/2*f^2/b^3*a^3*e*ln(-exp(2*I*(b*x+a))+I)+3/4*f/b^2*a^2*e^2*ln(-exp
(2*I*(b*x+a))+I)-3/4*f^2/b^2*e*polylog(3,I*exp(2*I*(b*x+a)))*x-1/8/f*e^4*l
n(exp(2*I*(b*x+a))+I)-1/2*e^3*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))*x
-1/2*e^3*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))*x+1/8*(f*x+e)^4/f*ln(e
xp(2*I*(b*x+a))+I)+1/2*e^3*ln(1+exp(I*(b*x+a))*(-1)^(3/4))*x+1/2*e^3*ln(1-
exp(I*(b*x+a))*(-1)^(3/4))*x+1/8/f*e^4*ln(-exp(2*I*(b*x+a))+I)+1/8*f^3*ln(
1+I*exp(2*I*(b*x+a)))*x^4-3/4*I*f^2/b*e*polylog(2,-I*exp(2*I*(b*x+a)))*x^2
+3/4*I*f^2/b^3*e*polylog(2,-I*exp(2*I*(b*x+a)))*a^2-1/2*I*f^3/b^4*a^3*dilo
g(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I*f^3/b^4*a^3*dilog(((-I)^(1
/2)+exp(I*(b*x+a)))/(-I)^(1/2))+3/2*f/b*e^2*a*ln(((-I)^(1/2)-exp(I*(b*x+a)
))/(-I)^(1/2))*x+3/2*f/b*e^2*a*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2...
```

3.231.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1808 vs. 2(236) = 472.

Time = 0.32 (sec) , antiderivative size = 1808, normalized size of antiderivative = 5.99

$$\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*arccoth(tan(b*x+a)),x, algorithm="fricas")
```

output
```
-1/32*(3*f^3*polylog(5, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x +
 a)^2 + 1)) - 3*f^3*polylog(5, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/(ta
n(b*x + a)^2 + 1)) + 3*f^3*polylog(5, (-I*tan(b*x + a)^2 + 2*tan(b*x + a)
+ I)/(tan(b*x + a)^2 + 1)) - 3*f^3*polylog(5, (-I*tan(b*x + a)^2 - 2*tan(b
*x + a) + I)/(tan(b*x + a)^2 + 1)) + 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2
 - 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x
 + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f
^2*x^2 - 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(-((I + 1)*tan(b*x + a)^2 - 2*t
an(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + 4*(I*b^3*f^3*x^3 + 3*I*b^
3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(-(-(I - 1)*tan(b*x + a)^2
 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) + 4*(I*b^3*f^3*x^3 +
3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(-(-(I - 1)*tan(b*x
+ a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^4*f^3*x^
4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*
b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 + 2*tan(b
*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) - 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f
+ 4*a^3*b*e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a)
+ I - 1)/(tan(b*x + a)^2 + 1)) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*
b*e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)
/(tan(b*x + a)^2 + 1)) - 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f...
```

3.231.6 Sympy [F]

$$\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx = \int (e+fx)^3 \operatorname{acoth}(\tan(a+bx))\,dx$$

input
```
integrate((f*x+e)**3*acoth(tan(b*x+a)),x)
```

output
```
Integral((e + f*x)**3*acoth(tan(a + b*x)), x)
```

3.231.7 Maxima [F]

$$\int (e+fx)^3\coth^{-1}(\tan(a+bx))\,dx=\int (fx+e)^3\operatorname{arcoth}\left(\tan\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)^3*arccoth(tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/16*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a
)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*e
*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x +
 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*e*f^2*
x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^
3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*sin(4*b*x + 4*a)*sin(2*
b*x + 2*a) + (b*f^3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(2
*b*x + 2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)
 + 1), x)
```

3.231.8 Giac [F]

$$\int (e+fx)^3\coth^{-1}(\tan(a+bx))\,dx=\int (fx+e)^3\operatorname{arcoth}\left(\tan\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)^3*arccoth(tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*arccoth(tan(b*x + a)), x)
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3\coth^{-1}(\tan(a+bx))\,dx=\int \operatorname{acoth}(\tan(a+b\,x))\,\left(e+f\,x\right)^3\,dx$$

input
```
int(acoth(tan(a + b*x))*(e + f*x)^3,x)
```

output
```
int(acoth(tan(a + b*x))*(e + f*x)^3, x)
```

3.232 $\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx$

3.232.1 Optimal result

Integrand size = 15, antiderivative size = 234

$$\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx = \frac{(e+fx)^3 \coth^{-1}(\tan(a+bx))}{3f} + \frac{i(e+fx)^3 \arctan\left(e^{2i(a+bx)}\right)}{3f} - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b} + \frac{f(e+fx)\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2} - \frac{f(e+fx)\operatorname{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{4b^2} + \frac{if^2 \operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3} - \frac{if^2 \operatorname{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}$$

output
```
1/3*(f*x+e)^3*arccoth(tan(b*x+a))/f+1/3*I*(f*x+e)^3*arctan(exp(2*I*(b*x+a)
))/f-1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^2*poly
log(2,I*exp(2*I*(b*x+a)))/b+1/4*f*(f*x+e)*polylog(3,-I*exp(2*I*(b*x+a)))/b
^2-1/4*f*(f*x+e)*polylog(3,I*exp(2*I*(b*x+a)))/b^2+1/8*I*f^2*polylog(4,-I*
exp(2*I*(b*x+a)))/b^3-1/8*I*f^2*polylog(4,I*exp(2*I*(b*x+a)))/b^3
```

3.232.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.75

$$\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx = \frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\coth^{-1}(\tan(a+bx)) + \frac{-12b^3e^2x\log\left(1-ie^{2i(a+bx)}\right)-12b^3efx^2\log\left(1-ie^{2i(a+bx)}\right)-4b^3f^2x^3\log\left(1-ie^{2i(a+bx)}\right)+12b^3e^2x\log}{}$$

input
```
Integrate[(e + f*x)^2*ArcCoth[Tan[a + b*x]],x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCoth[Tan[a + b*x]])/3 + (-12*b^3*e^2*x*L
og[1 - I*E^((2*I)*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 - I*E^((2*I)*(a + b*x
))] - 4*b^3*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] + 12*b^3*e^2*x*Log[1 +
I*E^((2*I)*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 4
*b^3*f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] - (6*I)*b^2*(e + f*x)^2*PolyLo
g[2, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b^2*(e + f*x)^2*PolyLog[2, I*E^((2*
I)*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] + 6*b*f^2*x*
PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b*e*f*PolyLog[3, I*E^((2*I)*(a +
b*x))] - 6*b*f^2*x*PolyLog[3, I*E^((2*I)*(a + b*x))] + (3*I)*f^2*PolyLog[4
, (-I)*E^((2*I)*(a + b*x))] - (3*I)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])
/(24*b^3)
```

3.232.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6806, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx$$

$$\Big\downarrow \text{ 6806}$$

$$\frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f} - \frac{b\int (e+fx)^3\sec(2a+2bx)dx}{3f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f}-\frac{b\int(e+fx)^3\csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{3f}$$

$$\Big\downarrow \text{ 4669}$$

$$\frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f}-\frac{b\left(-\frac{3f\int(e+fx)^2\log\left(1-ie^{2i(a+bx)}\right)dx}{2b}+\frac{3f\int(e+fx)^2\log\left(1+ie^{2i(a+bx)}\right)dx}{2b}-\frac{i(e+fx)^3\arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{3f}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f}-\frac{b\left(\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\int(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{if\int(e+fx)\text{PolyLog}\left(2,ie^{2i\ldots}\right.}{b}\right.}{2b}\right.}{3f}$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f}-\frac{b\left(\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\left(\frac{if\int\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)dx}{2b}-\frac{i(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,ie^{2i(a\ldots}\right.}{2b}\right.}{}\right.}{3f}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f}-\frac{b\left(\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\left(\frac{f\int e^{-2i(a+bx)}\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{3f\left(\frac{i(e+f\ldots}{}\right.}{}\right.}{3\ldots}$$

$$\Big\downarrow \text{ 7143}$$

3.232. $\int(e+fx)^2\coth^{-1}(\tan(a+bx))\,dx$

$$\frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f} - \frac{b\left(-\frac{i(e+fx)^3\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\left(\frac{f\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{i(e+fx)\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b} - \cdots\right)}{3f}$$

input
```
Int[(e + f*x)^2*ArcCoth[Tan[a + b*x]],x]
```

output
```
((e + f*x)^3*ArcCoth[Tan[a + b*x]])/(3*f) - (b*(((-I)*(e + f*x)^3*ArcTan[E
^((2*I)*(a + b*x))])/b + (3*f*(((I/2)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)
*(a + b*x))])/b - (I*f*(((-1/2*I)*(e + f*x)*PolyLog[3, (-I)*E^((2*I)*(a +
b*x))])/b + (f*PolyLog[4, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/(2*b) -
 (3*f*(((I/2)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (I*f*(((-
1/2*I)*(e + f*x)*PolyLog[3, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[4, I*E^
((2*I)*(a + b*x))])/(4*b^2)))/b))/(2*b)))/(3*f)
```

3.232.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6806
```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Tan[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.232.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 20.18 (sec) , antiderivative size = 2719, normalized size of antiderivative = 11.62

method	result	size
risch	Expression too large to display	2719

input
```
int((f*x+e)^2*arccoth(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^2/b^3*ln(1+I*exp(2*I*(b*x+a)))*a^3+1/4*f^2/b^2*polylog(3,-I*exp(2*I
*(b*x+a)))*x+1/2*f*e*ln(1+I*exp(2*I*(b*x+a)))*x^2+1/2*f^2/b^3*a^3*ln(1+exp
(I*(b*x+a))*(-1)^(3/4))+1/2*f^2/b^3*a^3*ln(1-exp(I*(b*x+a))*(-1)^(3/4))-1/
6*f^2/b^3*a^3*ln(-exp(2*I*(b*x+a))+I)+1/4*f*e/b^2*polylog(3,-I*exp(2*I*(b*
x+a)))-1/8*I*f^2*polylog(4,I*exp(2*I*(b*x+a)))/b^3+1/8*I*f^2*polylog(4,-I*
exp(2*I*(b*x+a)))/b^3+1/12*I*Pi*(csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(
2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))-csgn(I/
(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a)
)+I)/(exp(2*I*(b*x+a))+1))-csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b
*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp
(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2-csgn(I*(exp(2*I*(b*x+a))-I))*csgn
(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+csgn(I*(exp(2*I*(b*x+a))+I
))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2-csgn(I*(exp(2*I*(b*
x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b
*x+a))+1))+csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2+csgn(I*
(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+csgn(I*(exp(2*I*(b*x+a))-I)/(
exp(2*I*(b*x+a))+1))*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))
-csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((1-I)*(exp(2*I*(b*
x+a))-I)/(exp(2*I*(b*x+a))+1))^2-csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x
+a))+1))^3+csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+I)...
```

3.232.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1282 vs. $2(180) = 360$.

Time = 0.29 (sec) , antiderivative size = 1282, normalized size of antiderivative = 5.48

$$\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*arccoth(tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/48*(3*I*f^2*polylog(4, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x
+ a)^2 + 1)) + 3*I*f^2*polylog(4, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/
(tan(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 + 2*tan(b*x
+ a) + I)/(tan(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 -
2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2*e*
f*x - I*b^2*e^2)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/
(tan(b*x + a)^2 + 1) + 1) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)
*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2
+ 1) + 1) - 6*(I*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-(-(I - 1)
*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(I
*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-(-(I - 1)*tan(b*x + a)^2
- 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - 4*(b^3*f^2*x^3 + 3*b
^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(((I +
1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 4*(3*a
*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*
x + a) + I - 1)/(tan(b*x + a)^2 + 1)) - 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3
*f^2)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a
)^2 + 1)) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3
*a^2*b*e*f + a^3*f^2)*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1
)/(tan(b*x + a)^2 + 1)) - 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x ...
```

3.232.6 Sympy [F]

$$\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\, dx = \int (e+fx)^2 \operatorname{acoth}(\tan(a+bx))\, dx$$

input
```
integrate((f*x+e)**2*acoth(tan(b*x+a)),x)
```

output
```
Integral((e + f*x)**2*acoth(tan(a + b*x)), x)
```

3.232.7 Maxima [F]

$$\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx = \int (fx+e)^2 \operatorname{arcoth}(\tan(bx+a))\,dx$$

input
```
integrate((f*x+e)^2*arccoth(tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*
x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x
)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2
) - integrate(2/3*((b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(4*b*x + 4*a)*
cos(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(2*b*x + 2*a))
/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

3.232.8 Giac [F]

$$\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx = \int (fx+e)^2 \operatorname{arcoth}(\tan(bx+a))\,dx$$

input
```
integrate((f*x+e)^2*arccoth(tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*arccoth(tan(b*x + a)), x)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx = \int \operatorname{acoth}(\tan(a+b\,x))\,(e+f\,x)^2\,dx$$

input
```
int(acoth(tan(a + b*x))*(e + f*x)^2,x)
```

output
```
int(acoth(tan(a + b*x))*(e + f*x)^2, x)
```

3.233 $\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx$

3.233.1 Optimal result

Integrand size = 13, antiderivative size = 162

$$\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx = \frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f} + \frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{2f} - \frac{i(e+fx)\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} + \frac{i(e+fx)\,\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b} + \frac{f\,\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2} - \frac{f\,\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2}$$

output
```
1/2*(f*x+e)^2*arccoth(tan(b*x+a))/f+1/2*I*(f*x+e)^2*arctan(exp(2*I*(b*x+a)
))/f-1/4*I*(f*x+e)*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)*polylog(
2,I*exp(2*I*(b*x+a)))/b+1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*pol
ylog(3,I*exp(2*I*(b*x+a)))/b^2
```

3.233.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.82

$$\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx = ex\coth^{-1}(\tan(a+bx))+\frac{1}{2}fx^2\coth^{-1}(\tan(a+bx)) - \frac{e\left((-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+\right.}{8b} + \frac{f(4ib^2x^2\arctan(\cos(2(a+bx))+i\sin(2(a+bx)))+2ibx\,\text{PolyLog}(2,i\cos(2(a+bx))-\sin(2(a+bx))}{}$$

input
```
Integrate[(e + f*x)*ArcCoth[Tan[a + b*x]],x]
```

output
```
e*x*ArcCoth[Tan[a + b*x]] + (f*x^2*ArcCoth[Tan[a + b*x]])/2 - (e*((-4*a +
Pi - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*x))] - Log[1 + I/E^((2*I)*(a + b*x)
)]) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*x]] + (2*I)*(PolyLog[2, (-I)/E^((2*
I)*(a + b*x))] - PolyLog[2, I/E^((2*I)*(a + b*x))])))/(8*b) + (f*((4*I)*b^
2*x^2*ArcTan[Cos[2*(a + b*x)] + I*Sin[2*(a + b*x)]] + (2*I)*b*x*PolyLog[2,
 I*Cos[2*(a + b*x)] - Sin[2*(a + b*x)]] - (2*I)*b*x*PolyLog[2, (-I)*Cos[2*
(a + b*x)] + Sin[2*(a + b*x)]] - PolyLog[3, I*Cos[2*(a + b*x)] - Sin[2*(a
+ b*x)]] + PolyLog[3, (-I)*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]]))/(8*b^2)
```

3.233.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6806, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx$$

$$\Big\downarrow \text{ 6806}$$

$$\frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f}-\frac{b\int (e+fx)^2\sec(2a+2bx)dx}{2f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f}-\frac{b\int (e+fx)^2\csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{2f}$$

$$\Big\downarrow \text{4669}$$

$$\frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f}-\frac{b\left(-\frac{f\int (e+fx)\log\left(1-ie^{2i(a+bx)}\right)dx}{b}+\frac{f\int (e+fx)\log\left(1+ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{2f}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f}-\frac{b\left(\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\int \text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{if\int \text{PolyLog}\left(2,ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{i(\ldots}{}$$

$$\Big\downarrow \text{2720}$$

$$\frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f}-\frac{b\left(\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{f\int e^{-2i(a+bx)}\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{f\int e^{-2i(a+bx)}\text{Po}\ldots}{}\right)}{b}\ldots}{2f}$$

$$\Big\downarrow \text{7143}$$

$$\frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f}-\frac{b\left(-\frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{f\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2}\right)}{b}-\frac{f\left(\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{f\text{Poly}\ldots}{}\right)}{b}\ldots}{2f}$$

input
```
Int[(e + f*x)*ArcCoth[Tan[a + b*x]],x]
```

output
```
((e + f*x)^2*ArcCoth[Tan[a + b*x]])/(2*f) - (b*(((-I)*(e + f*x)^2*ArcTan[E
^((2*I)*(a + b*x))])/b + (f*(((I/2)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a
+ b*x))])/b - (f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b - (f*((
(I/2)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (f*PolyLog[3, I*E^(
(2*I)*(a + b*x))])/(4*b^2)))/b))/(2*f)
```

3.233. $\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx$

3.233.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6806
```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Tan[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.233.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.83 (sec) , antiderivative size = 1818, normalized size of antiderivative = 11.22

method	result	size
risch	Expression too large to display	1818

input
```
int((f*x+e)*arccoth(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*e/b*a*ln(-exp(2*I*(b*x+a))+I)+1/2/b*f*a*ln(((-I)^(1/2)-exp(I*(b*x+a))
)/(-I)^(1/2))*x+1/2/b*f*a*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))*x-1/2
*I/b^2*f*a*dilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b^2*f*a*dil
og(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/2*f/b*ln(1+I*exp(2*I*(b*x+a))
)*a*x-1/4*I*f/b*polylog(2,-I*exp(2*I*(b*x+a)))*x-1/4*I*f/b^2*polylog(2,-I*
exp(2*I*(b*x+a)))*a-1/4*f/b^2*ln(-I*exp(2*I*(b*x+a))+1)*a^2+1/4/b^2*f*a^2*
ln(-exp(2*I*(b*x+a))+I)+1/4*f/b^2*ln(1+I*exp(2*I*(b*x+a)))*a^2+1/8*f*polyl
og(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylog(3,I*exp(2*I*(b*x+a)))/b^2+1/2*
e/b*ln(1+exp(I*(b*x+a))*(-1)^(3/4))*a+1/2*e/b*ln(1-exp(I*(b*x+a))*(-1)^(3/
4))*a-1/2*I*e/b*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I*e/b*dilog(1-exp(I
*(b*x+a))*(-1)^(3/4))-1/2/b^2*f*a^2*ln(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2/b^
2*f*a^2*ln(1-exp(I*(b*x+a))*(-1)^(3/4))+1/2/b^2*f*a^2*ln(((-I)^(1/2)-exp(I
*(b*x+a)))/(-I)^(1/2))+1/2/b^2*f*a^2*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(
1/2))-1/2*e/b*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))*a-1/2*e/b*ln(((-I
)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))*a+1/2*I*e/b*dilog(((-I)^(1/2)-exp(I*(b
*x+a)))/(-I)^(1/2))+1/2*I*e/b*dilog(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2)
)-1/4/b^2*f*a^2*ln(exp(2*I*(b*x+a))+I)-1/4*ln(exp(2*I*(b*x+a))-I)*x^2*f-1/
2*ln(exp(2*I*(b*x+a))-I)*e*x-1/2/b*f*a*ln(1+exp(I*(b*x+a))*(-1)^(3/4))*x-1
/2/b*f*a*ln(1-exp(I*(b*x+a))*(-1)^(3/4))*x+1/2*I/b^2*f*a*dilog(1+exp(I*(b*
x+a))*(-1)^(3/4))+1/2*I/b^2*f*a*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))-1/2*...
```

3.233.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 834 vs. 2(130) = 260.

Time = 0.29 (sec) , antiderivative size = 834, normalized size of antiderivative = 5.15

$$\int (e + fx) \coth^{-1}(\tan(a + bx)) \, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*arccoth(tan(b*x+a)),x, algorithm="fricas")
```

output
```
-1/16*(2*(-I*b*f*x - I*b*e)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a
) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(-I*b*f*x - I*b*e)*dilog(-((I + 1
)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(
I*b*f*x + I*b*e)*dilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)
/(tan(b*x + a)^2 + 1) + 1) + 2*(I*b*f*x + I*b*e)*dilog(-(-(I - 1)*tan(b*x
+ a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^2*f*x^2
+ 2*b^2*e*x + 2*a*b*e - a^2*f)*log(((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a
) - I + 1)/(tan(b*x + a)^2 + 1)) - 2*(2*a*b*e - a^2*f)*log(((I + 1)*tan(b*
x + a)^2 + 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 + 1)) + 2*(2*a*b*e -
a^2*f)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x +
a)^2 + 1)) - 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(((I + 1)*tan(
b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*f*x^2
+ 2*b^2*e*x + 2*a*b*e - a^2*f)*log((-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x +
a) + I + 1)/(tan(b*x + a)^2 + 1)) - 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a
^2*f)*log((-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)
^2 + 1)) - 2*(2*a*b*e - a^2*f)*log(((I - 1)*tan(b*x + a)^2 + 2*I*tan(b*x +
 a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*(2*a*b*e - a^2*f)*log(((I - 1)*tan(
b*x + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 4*(b^2*f*x^
2 + 2*b^2*e*x)*log((tan(b*x + a) + 1)/(tan(b*x + a) - 1)) - f*polylog(3, (
I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + f*polylo...
```

3.233.6 Sympy [F]

$$\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx=\int (e+fx)\,\mathrm{acoth}\,(\tan{(a+bx)})\,dx$$

input
```
integrate((f*x+e)*acoth(tan(b*x+a)),x)
```

output
```
Integral((e + f*x)*acoth(tan(a + b*x)), x)
```

3.233.7 Maxima [F]

$$\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx=\int (fx+e)\,\mathrm{arcoth}\,(\tan\,(bx+a))\,\,dx$$

input
```
integrate((f*x+e)*arccoth(tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*si
n(2*b*x + 2*a) + 2) - 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin
(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(((b*f*x^2 + 2*b*e*x)
*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)
^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

3.233.8 Giac [F]

$$\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx=\int (fx+e)\,\mathrm{arcoth}\,(\tan\,(bx+a))\,\,dx$$

input
```
integrate((f*x+e)*arccoth(tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*arccoth(tan(b*x + a)), x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx=\int \mathrm{acoth}(\tan(a+b\,x))\,\,(e+f\,x)\,\,dx$$

input
```
int(acoth(tan(a + b*x))*(e + f*x),x)
```

output
```
int(acoth(tan(a + b*x))*(e + f*x), x)
```

3.234 $\int \coth^{-1}(\tan(a+bx))\,dx$

3.234.1 Optimal result

Integrand size = 7, antiderivative size = 79

$$\int \coth^{-1}(\tan(a+bx))\,dx = x\coth^{-1}(\tan(a+bx)) + ix\arctan\left(e^{2i(a+bx)}\right) - \frac{i\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} + \frac{i\,\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}$$

output
```
x*arccoth(tan(b*x+a))+I*x*arctan(exp(2*I*(b*x+a)))-1/4*I*polylog(2,-I*exp(
2*I*(b*x+a)))/b+1/4*I*polylog(2,I*exp(2*I*(b*x+a)))/b
```

3.234.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.61

$$\int \coth^{-1}(\tan(a+bx))\,dx = x\coth^{-1}(\tan(a+bx)) - \frac{(-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+2a}{8b}$$

input
```
Integrate[ArcCoth[Tan[a + b*x]],x]
```

output
```
x*ArcCoth[Tan[a + b*x]] - ((-4*a + Pi - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*
x))] - Log[1 + I/E^((2*I)*(a + b*x))]) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*
x]] + (2*I)*(PolyLog[2, (-I)/E^((2*I)*(a + b*x))] - PolyLog[2, I/E^((2*I)*
(a + b*x))]))/(8*b)
```

3.234.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6802, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(\tan(a+bx))\,dx$$

$$\Big\downarrow \text{6802}$$

$$x\coth^{-1}(\tan(a+bx))-b\int x\sec(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$x\coth^{-1}(\tan(a+bx))-b\int x\csc\left(2a+2bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4669}$$

$$x\coth^{-1}(\tan(a+bx))-b\left(-\frac{\int \log\left(1-ie^{2i(a+bx)}\right)dx}{2b}+\frac{\int \log\left(1+ie^{2i(a+bx)}\right)dx}{2b}-\frac{ix\arctan\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\coth^{-1}(\tan(a+bx))-b\left(\frac{i\int e^{-2i(a+bx)}\log\left(1-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i\int e^{-2i(a+bx)}\log\left(1+ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{ix\arctan\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\coth^{-1}(\tan(a+bx))-b\left(-\frac{ix\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{i\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{i\,\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b^2}\right)$$

input
```
Int[ArcCoth[Tan[a + b*x]],x]
```

output
```
x*ArcCoth[Tan[a + b*x]] - b*(((-I)*x*ArcTan[E^((2*I)*(a + b*x))])/b + ((I/
4)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])/b^2 - ((I/4)*PolyLog[2, I*E^((2*I
)*(a + b*x))])/b^2)
```

3.234.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6802
```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcCoth[Tan[a + b
*x]], x] - Simp[b   Int[x*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

3.234.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(64) = 128$.

Time = 1.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.14

method	result
derivativedivides	$\frac{\arctan(\tan(bx+a))\,\mathrm{arccoth}(\tan(bx+a))+\frac{\arctan(\tan(bx+a))\ln\left(1+\frac{i(1+i\tan(bx+a))^2}{1+\tan(bx+a)^2}\right)}{2}-\frac{\arctan(\tan(bx+a))\ln\left(1-\frac{i(1+i\tan(b\ldots}{1+\tan(bx\ldots}\right.}{2}}{b}$
default	$\frac{\arctan(\tan(bx+a))\,\mathrm{arccoth}(\tan(bx+a))+\frac{\arctan(\tan(bx+a))\ln\left(1+\frac{i(1+i\tan(bx+a))^2}{1+\tan(bx+a)^2}\right)}{2}-\frac{\arctan(\tan(bx+a))\ln\left(1-\frac{i(1+i\tan(b\ldots}{1+\tan(bx\ldots}\right.}{2}}{b}$
risch	Expression too large to display

input
```
int(arccoth(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(arctan(tan(b*x+a))*arccoth(tan(b*x+a))+1/2*arctan(tan(b*x+a))*ln(1+I*
(1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2))-1/2*arctan(tan(b*x+a))*ln(1-I*(1+I*ta
n(b*x+a))^2/(1+tan(b*x+a)^2))-1/4*I*dilog(1+I*(1+I*tan(b*x+a))^2/(1+tan(b*
x+a)^2))+1/4*I*dilog(1-I*(1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2)))
```

3.234.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(57) = 114$.

Time = 0.28 (sec) , antiderivative size = 498, normalized size of antiderivative = 6.30

$$\int \coth^{-1}(\tan(a+bx))\,dx$$
$$= \frac{4\,bx\log\left(\frac{\tan(bx+a)+1}{\tan(bx+a)-1}\right) - 2\,(bx+a)\log\left(\frac{(i+1)\tan(bx+a)^2+2\tan(bx+a)-i+1}{\tan(bx+a)^2+1}\right) + 2\,a\log\left(\frac{(i+1)\tan(bx+a)^2+2i\tan(bx+a)-}{\tan(bx+a)^2+1}\right.}{}$$

input
```
integrate(arccoth(tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(4*b*x*log((tan(b*x + a) + 1)/(tan(b*x + a) - 1)) - 2*(b*x + a)*log(((
I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*
a*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2
+ 1)) - 2*a*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b
*x + a)^2 + 1)) + 2*(b*x + a)*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a)
 - I + 1)/(tan(b*x + a)^2 + 1)) - 2*(b*x + a)*log((-(I - 1)*tan(b*x + a)^2
 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*(b*x + a)*log((-(I -
1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*a*lo
g(((I - 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)
) - 2*a*log(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x +
 a)^2 + 1)) + I*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(
tan(b*x + a)^2 + 1) + 1) + I*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x +
a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*tan(b*x + a)^2
+ 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*t
an(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1))/b
```

3.234.6 Sympy [F]

$$\int \coth^{-1}(\tan(a+bx))\,dx = \int \operatorname{acoth}(\tan(a+bx))\,dx$$

input
```
integrate(acoth(tan(b*x+a)),x)
```

output
```
Integral(acoth(tan(a + b*x)), x)
```

3.234.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(57) = 114$.

Time = 0.34 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.30

$$\int \coth^{-1}(\tan(a+bx))\,dx = \frac{4\,(bx+a)\operatorname{arcoth}(\tan(bx+a)) + \left(\arctan\left(\frac{1}{2}\tan(bx+a)+\frac{1}{2},\frac{1}{2}\tan(bx+a)+\frac{1}{2}\right) - \arctan\left(\frac{1}{2}\tan(bx+\right.\right.}{}$$

input
```
integrate(arccoth(tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*(4*(b*x + a)*arccoth(tan(b*x + a)) + (arctan2(1/2*tan(b*x + a) + 1/2,
1/2*tan(b*x + a) + 1/2) - arctan2(1/2*tan(b*x + a) - 1/2, -1/2*tan(b*x + a
) + 1/2))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log(1/2*tan(b*x + a)^2 + tan
(b*x + a) + 1/2) + (b*x + a)*log(1/2*tan(b*x + a)^2 - tan(b*x + a) + 1/2)
- I*dilog((1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2) + I*dilog(-(1/2*I - 1/
2)*tan(b*x + a) + 1/2*I + 1/2) + I*dilog((1/2*I - 1/2)*tan(b*x + a) + 1/2*
I + 1/2) - I*dilog(-(1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2))/b
```

3.234.8 Giac [F]

$$\int \coth^{-1}(\tan(a+bx))\,dx = \int \operatorname{arcoth}\left(\tan\left(bx+a\right)\right)\,dx$$

input
```
integrate(arccoth(tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(tan(b*x + a)), x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(\tan(a+bx))\,dx = \int \operatorname{acoth}(\tan(a+b\,x))\,dx$$

input
```
int(acoth(tan(a + b*x)),x)
```

output
```
int(acoth(tan(a + b*x)), x)
```

3.235 $\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} dx$

3.235.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} dx = \text{Int}\left(\frac{\coth^{-1}(\tan(a+bx))}{e+fx}, x\right)$$

output
```
CannotIntegrate(arccoth(tan(b*x+a))/(f*x+e),x)
```

3.235.2 Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} dx = \int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} dx$$

input
```
Integrate[ArcCoth[Tan[a + b*x]]/(e + f*x),x]
```

output
```
Integrate[ArcCoth[Tan[a + b*x]]/(e + f*x), x]
```

3.235.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}dx$$

input
```
Int[ArcCoth[Tan[a + b*x]]/(e + f*x),x]
```

output
```
$Aborted
```

3.235.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.235.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(\tan\left(bx+a\right)\right)}{fx+e}dx$$

input
```
int(arccoth(tan(b*x+a))/(f*x+e),x)
```

output
```
int(arccoth(tan(b*x+a))/(f*x+e),x)
```

3.235.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{arcoth}(\tan(bx+a))}{fx+e}\,dx$$

input
```
integrate(arccoth(tan(b*x+a))/(f*x+e),x, algorithm="fricas")
```

output
```
integral(arccoth(tan(b*x + a))/(f*x + e), x)
```

3.235.6 Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{acoth}(\tan(a+bx))}{e+fx}\,dx$$

input
```
integrate(acoth(tan(b*x+a))/(f*x+e),x)
```

output
```
Integral(acoth(tan(a + b*x))/(e + f*x), x)
```

3.235.7 Maxima [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{arcoth}(\tan(bx+a))}{fx+e}\,dx$$

input
```
integrate(arccoth(tan(b*x+a))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(arccoth(tan(b*x + a))/(f*x + e), x)
```

3.235.8 Giac [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} \, dx = \int \frac{\operatorname{arcoth}(\tan(bx+a))}{fx+e} \, dx$$

input
```
integrate(arccoth(tan(b*x+a))/(f*x+e),x, algorithm="giac")
```

output
```
integrate(arccoth(tan(b*x + a))/(f*x + e), x)
```

3.235.9 Mupad [N/A]

Not integrable

Time = 4.42 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} \, dx = \int \frac{\operatorname{acoth}(\tan(a+b\,x))}{e+f\,x} \, dx$$

input
```
int(acoth(tan(a + b*x))/(e + f*x),x)
```

output
```
int(acoth(tan(a + b*x))/(e + f*x), x)
```

3.236 $\int x^2 \coth^{-1}(c + d\tan(a + bx))\,dx$

3.236.1 Optimal result

Integrand size = 15, antiderivative size = 395

$$\begin{aligned}\int x^2 \coth^{-1}(c + d\tan(a + bx))\,dx &= \frac{1}{3}x^3 \coth^{-1}(c + d\tan(a + bx)) \\ &+ \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) \\ &- \frac{1}{6}x^3 \log\left(1 + \frac{(1 + c - id)e^{2ia+2ibx}}{1 + c + id}\right) \\ &- \frac{ix^2 \operatorname{PolyLog}\left(2, -\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{4b} \\ &+ \frac{ix^2 \operatorname{PolyLog}\left(2, -\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{4b} \\ &+ \frac{x \operatorname{PolyLog}\left(3, -\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{4b^2} \\ &- \frac{x \operatorname{PolyLog}\left(3, -\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{4b^2} \\ &+ \frac{i \operatorname{PolyLog}\left(4, -\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{8b^3} \\ &- \frac{i \operatorname{PolyLog}\left(4, -\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arccoth(c+d*tan(b*x+a))+1/6*x^3*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/
(1-c-I*d))-1/6*x^3*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*x^2*
polylog(2,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*x^2*polylog(2,-
(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b+1/4*x*polylog(3,-(1-c+I*d)*exp(2
*I*a+2*I*b*x)/(1-c-I*d))/b^2-1/4*x*polylog(3,-(1+c-I*d)*exp(2*I*a+2*I*b*x)
/(1+c+I*d))/b^2+1/8*I*polylog(4,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b
^3-1/8*I*polylog(4,-(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b^3
```

3.236.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.88

$$\int x^2 \coth^{-1}(c + d\tan(a + bx))\,dx$$
$$= \frac{8b^3x^3\coth^{-1}(c + d\tan(a + bx)) + 4b^3x^3\log\left(1 + \frac{(-1+c+id)e^{-2i(a+bx)}}{-1+c-id}\right) - 4b^3x^3\log\left(1 + \frac{(1+c+id)e^{-2i(a+bx)}}{1+c-id}\right) + }{}$$

input
```
Integrate[x^2*ArcCoth[c + d*Tan[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCoth[c + d*Tan[a + b*x]] + 4*b^3*x^3*Log[1 + (-1 + c + I*d)/
((-1 + c - I*d)*E^((2*I)*(a + b*x)))] - 4*b^3*x^3*Log[1 + (1 + c + I*d)/((
1 + c - I*d)*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (1 - c - I*d
)/((-1 + c - I*d)*E^((2*I)*(a + b*x)))] - (6*I)*b^2*x^2*PolyLog[2, (-1 - c
 - I*d)/((1 + c - I*d)*E^((2*I)*(a + b*x)))] + 6*b*x*PolyLog[3, (1 - c - I
*d)/((-1 + c - I*d)*E^((2*I)*(a + b*x)))] - 6*b*x*PolyLog[3, (-1 - c - I*d
)/((1 + c - I*d)*E^((2*I)*(a + b*x)))] - (3*I)*PolyLog[4, (1 - c - I*d)/((
-1 + c - I*d)*E^((2*I)*(a + b*x)))] + (3*I)*PolyLog[4, (-1 - c - I*d)/((1
+ c - I*d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.236.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6822, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(d\tan(a+bx)+c)\,dx$$

$$\Big\downarrow \; 6822$$

$$-\frac{1}{3}b(ic+d+i)\int \frac{e^{2ia+2ibx}x^3}{c+(c-id+1)e^{2ia+2ibx}+id+1}dx+\frac{1}{3}b(-d+i(1-c))\int \frac{e^{2ia+2ibx}x^3}{-c+(-c+id+1)e^{2ia+2ibx}-id+1}dx+\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \; 2620$$

$$\frac{1}{3}b(-d+i(1-c))\left(\frac{x^3\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{3\int x^2\log\left(\frac{e^{2ia+2ibx}(-c+id+1)}{-c-id+1}+1\right)dx}{2b(-d+i(1-c))}\right)-\frac{1}{3}b(ic+d+i)\left(\frac{x^3\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{3\int x^2\log\left(\frac{e^{2ia+2ibx}(c-id+1)}{c+id+1}+1\right)dx}{2(bd+i(bc+b))}\right)+\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \; 3011$$

$$\frac{1}{3}b(-d+i(1-c))\left(\frac{x^3\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)dx}{b}\right)}{2b(-d+i(1-c))}\right)-\frac{1}{3}b(ic+d+i)\left(\frac{x^3\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)dx}{b}\right)}{2(bd+i(bc+b))}\right)+\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \; 7163$$

3.236. $\int x^2 \coth^{-1}(c+d\tan(a+bx))\,dx$

$$c)) \left(\frac{x^3 \log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))} - \frac{\frac{1}{3}b(-d+i(1-}{} \quad \frac{3\left(\frac{ix^2 \operatorname{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b} - \frac{i\left(\frac{i\int \operatorname{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)dx}{2b} - \frac{ix \operatorname{Po}}{}\right.}{b}\right.}{2b(-d+i(1-c))} \right.$$

$$i) \left(\frac{x^3 \log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))} - \frac{\frac{1}{3}b(ic+d+}{} \quad \frac{3\left(\frac{ix^2 \operatorname{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b} - \frac{i\left(\frac{i\int \operatorname{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)dx}{2b} - \frac{ix \operatorname{PolyLog}(}{}\right.}{b}\right.}{2(bd+i(bc+b))} \right.$$

$$\frac{1}{3}x^3 \coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \; 2720$$

3.236. $\int x^2 \coth^{-1}(c+d\tan(a+bx))\,dx$

$$\frac{1}{3}b(-d+i(1-$$

$$c))\left(\frac{x^3\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right.}{4b^2}\right.}{}\right.}{2b(-d+i(1-c))}\right.$$

$$\frac{1}{3}b(ic+d+$$

$$i)\left(\frac{x^3\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)de^{2ia}}{4b^2}\right.}{b}\right.}{2(bd+i(bc+b))}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{7143}$$

3.236. $\int x^2\coth^{-1}(c+d\tan(a+bx))\,dx$

$$\frac{1}{3}b(-d+i(1-c))\left(\frac{x^3\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}}{}\right.}{b}\right.}{2b(-d+i(1-c))}\right.$$

$$\frac{1}{3}b(ic+d+i)\left(\frac{x^3\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,-\right.}{}\right.}{b}\right.}{2(bd+i(bc+b))}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)+c)$$

input
```
Int[x^2*ArcCoth[c + d*Tan[a + b*x]],x]
```

output
```
(x^3*ArcCoth[c + d*Tan[a + b*x]])/3 + (b*(I*(1 - c) - d)*((x^3*Log[1 + ((1
 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d)])/(2*b*(I*(1 - c) - d))
 - (3*(((I/2)*x^2*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1
- c - I*d))])/b - (I*(((-1/2*I)*x*PolyLog[3, -(((1 - c + I*d)*E^((2*I)*a +
 (2*I)*b*x))/(1 - c - I*d))])/b + PolyLog[4, -(((1 - c + I*d)*E^((2*I)*a +
 (2*I)*b*x))/(1 - c - I*d))]/(4*b^2)))/b))/(2*b*(I*(1 - c) - d))))/3 - (b*
(I + I*c + d)*((x^3*Log[1 + ((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c
 + I*d)])/(2*(I*(b + b*c) + b*d)) - (3*(((I/2)*x^2*PolyLog[2, -(((1 + c -
I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b - (I*(((-1/2*I)*x*PolyLog
[3, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + PolyLog
[4, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))]/(4*b^2)))/b)
)/(2*(I*(b + b*c) + b*d))))/3
```

3.236. $\int x^2\coth^{-1}(c+d\tan(a+bx))\,dx$

3.236.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6822
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + (-Simp[I*b*((1 + c - I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)
*(E^(2*I*a + 2*I*b*x)/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x))), x
], x] + Simp[I*b*((1 - c + I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*
I*a + 2*I*b*x)/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x))), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.236.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 20.80 (sec) , antiderivative size = 6855, normalized size of antiderivative = 17.35

method	result	size
risch	Expression too large to display	6855

input
```
int(x^2*arccoth(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.236.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2164 vs. 2(279) = 558.

Time = 0.32 (sec) , antiderivative size = 2164, normalized size of antiderivative = 5.48

$$\int x^2 \coth^{-1}(c + d\tan(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate(x^2*arccoth(c+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/48*(8*b^3*x^3*log((d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) - 6
*I*b^2*x^2*dilog(2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d
 + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2
 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 6*I*b^2*x^2
*dilog(2*((-I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^
2 - 2*(c + 1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 +
 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 6*I*b^2*x^2*dilog(2
*((I*(c - 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c -
 1)*d - I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*
tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) - 6*I*b^2*x^2*dilog(2*((-I*(c -
 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d +
I*d^2 + 2*I*c - I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x
+ a)^2 + c^2 + d^2 - 2*c + 1) + 1) + 4*a^3*log(((I*(c + 1)*d + d^2)*tan(b*
x + a)^2 - c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) -
2*c - 1)/(tan(b*x + a)^2 + 1)) + 4*a^3*log(((I*(c + 1)*d - d^2)*tan(b*x +
a)^2 + c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c
+ 1)/(tan(b*x + a)^2 + 1)) - 4*a^3*log(((I*(c - 1)*d + d^2)*tan(b*x + a)^2
 - c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)
/(tan(b*x + a)^2 + 1)) - 4*a^3*log(((I*(c - 1)*d - d^2)*tan(b*x + a)^2 + c
^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) - 2*c + 1)/...
```

3.236.6 Sympy [F]

$$\int x^2 \coth^{-1}(c + d\tan(a + bx))\,dx = \int x^2 \operatorname{acoth}\left(c + d\tan\left(a + bx\right)\right)dx$$

input
```
integrate(x**2*acoth(c+d*tan(b*x+a)),x)
```

output
```
Integral(x**2*acoth(c + d*tan(a + b*x)), x)
```

3.236.7 Maxima [F]

$$\int x^2 \coth^{-1}(c + d\tan(a + bx))\,dx = \int x^2 \operatorname{arcoth}(d\tan(bx + a) + c)\,dx$$

input
```
integrate(x^2*arccoth(c+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*x^3*log((c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*
b*x + 2*a) + (c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2
 - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/12*x^3*log((c^2 + d^2 -
2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 -
2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*
x + 2*a) - 2*c + 1) - 4*b*d*integrate(-1/3*(2*(c^2 + d^2 - 1)*x^3*cos(2*b*
x + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^3*sin(2*b*x
+ 2*a)^2 + (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2
*a) - (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*
cos(2*b*x + 2*a) + (c^2 - d^2 - 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))
/(c^4 + d^4 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*
cos(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x
 + 2*a)^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 +
 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 +
2*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 - 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2
*b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*
a) + 4*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)
*d - 2*(c*d^3 + (c^3 - c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*si
n(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a
) + 1), x)
```

3.236.8 Giac [F]

$$\int x^2 \coth^{-1}(c + d\tan(a + bx))\,dx = \int x^2 \operatorname{arcoth}(d\tan(bx + a) + c)\,dx$$

input
```
integrate(x^2*arccoth(c+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(d*tan(b*x + a) + c), x)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(c+d\tan(a+bx))\,dx=\int x^2\,\mathrm{acoth}(c+d\tan(a+b\,x))\;dx$$

input
```
int(x^2*acoth(c + d*tan(a + b*x)),x)
```

output
```
int(x^2*acoth(c + d*tan(a + b*x)), x)
```

3.237 $\int x \coth^{-1}(c+d\tan(a+bx))\,dx$

3.237.1 Optimal result

Integrand size = 13, antiderivative size = 295

$$\begin{aligned}\int x\coth^{-1}(c+d\tan(a+bx))\,dx &= \frac{1}{2}x^2\coth^{-1}(c+d\tan(a+bx)) \\ &+ \frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) \\ &- \frac{1}{4}x^2\log\left(1+\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right) \\ &- \frac{ix\,\mathrm{PolyLog}\left(2,-\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{4b} \\ &+ \frac{ix\,\mathrm{PolyLog}\left(2,-\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{4b} \\ &+ \frac{\mathrm{PolyLog}\left(3,-\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{8b^2} \\ &- \frac{\mathrm{PolyLog}\left(3,-\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{8b^2}\end{aligned}$$

output
```
1/2*x^2*arccoth(c+d*tan(b*x+a))+1/4*x^2*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/
(1-c-I*d))-1/4*x^2*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*x*po
lylog(2,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*x*polylog(2,-(1+c
-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b+1/8*polylog(3,-(1-c+I*d)*exp(2*I*a+2
*I*b*x)/(1-c-I*d))/b^2-1/8*polylog(3,-(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*
d))/b^2
```

3.237.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.88

$$\int x\coth^{-1}(c+d\tan(a+bx))\,dx$$
$$=\frac{4b^2x^2\coth^{-1}(c+d\tan(a+bx))+2b^2x^2\log\left(1+\frac{(-1+c+id)e^{-2i(a+bx)}}{-1+c-id}\right)-2b^2x^2\log\left(1+\frac{(1+c+id)e^{-2i(a+bx)}}{1+c-id}\right)+$$

input
```
Integrate[x*ArcCoth[c + d*Tan[a + b*x]],x]
```

output
```
(4*b^2*x^2*ArcCoth[c + d*Tan[a + b*x]] + 2*b^2*x^2*Log[1 + (-1 + c + I*d)/
((-1 + c - I*d)*E^((2*I)*(a + b*x)))] - 2*b^2*x^2*Log[1 + (1 + c + I*d)/((
1 + c - I*d)*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (1 - c - I*d)/((
-1 + c - I*d)*E^((2*I)*(a + b*x)))] - (2*I)*b*x*PolyLog[2, (-1 - c - I*d)/
((1 + c - I*d)*E^((2*I)*(a + b*x)))] + PolyLog[3, (1 - c - I*d)/((-1 + c -
 I*d)*E^((2*I)*(a + b*x)))] - PolyLog[3, (-1 - c - I*d)/((1 + c - I*d)*E^(
(2*I)*(a + b*x)))])/(8*b^2)
```

3.237.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6822, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(d\tan(a+bx)+c)\,dx$$

$$\Big\downarrow \ 6822$$

$$-\frac{1}{2}b(ic+d+i)\int\frac{e^{2ia+2ibx}x^2}{c+(c-id+1)e^{2ia+2ibx}+id+1}dx+\frac{1}{2}b(-d+i(1-c))\int\frac{e^{2ia+2ibx}x^2}{-c+(-c+id+1)e^{2ia+2ibx}-id+1}dx+\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \ 2620$$

$$\frac{1}{2}b(-d+i(1-c))\left(\frac{x^2\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{\int x\log\left(\frac{e^{2ia+2ibx}(-c+id+1)}{-c-id+1}+1\right)dx}{b(-d+i(1-c))}\right)-\frac{1}{2}b(ic+d+i)\left(\frac{x^2\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{\int x\log\left(\frac{e^{2ia+2ibx}(c-id+1)}{c+id+1}+1\right)dx}{bd+i(bc+b)}\right)+\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \; 3011$$

$$\frac{1}{2}b(-d+i(1-c))\left(\frac{x^2\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{i\int\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)dx}{2b}}{b(-d+i(1-c))}\right)-\frac{1}{2}b(ic+d+i)\left(\frac{x^2\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{i\int\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)dx}{2b}}{bd+i(bc+b)}\right)+\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{2}b(-d+i(1-c))\left(\frac{x^2\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)de^{2ia+2ibx}}{4b^2}}{b(-d+i(1-c))}\right)-\frac{1}{2}b(ic+d+i)\left(\frac{x^2\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)de^{2ia+2ibx}}{4b^2}}{bd+i(bc+b)}\right)+\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \; 7143$$

3.237. $\int x\coth^{-1}(c+d\tan(a+bx))\,dx$

$$\frac{1}{2}b(-d+i(1-c))\left(\frac{x^2\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b}-\frac{\text{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2}}{b(-d+i(1-c))}\right)-\frac{1}{2}b(ic+d+i)\left(\frac{x^2\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2b}-\frac{\text{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2}}{bd+i(bc+b)}\right)+\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)+c)$$

input
```
Int[x*ArcCoth[c + d*Tan[a + b*x]],x]
```

output
```
(x^2*ArcCoth[c + d*Tan[a + b*x]])/2 + (b*(I*(1 - c) - d)*((x^2*Log[1 + ((1
 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d)])/(2*b*(I*(1 - c) - d))
 - (((I/2)*x*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c -
 I*d))])/b - PolyLog[3, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c -
 I*d))]/(4*b^2))/(b*(I*(1 - c) - d))))/2 - (b*(I + I*c + d)*((x^2*Log[1 +
((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d)])/(2*(I*(b + b*c) +
b*d)) - (((I/2)*x*PolyLog[2, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1
+ c + I*d))])/b - PolyLog[3, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1
+ c + I*d))]/(4*b^2))/(I*(b + b*c) + b*d)))/2
```

3.237.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6822
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + (-Simp[I*b*((1 + c - I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)
*(E^(2*I*a + 2*I*b*x)/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x))), x
], x] + Simp[I*b*((1 - c + I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*
I*a + 2*I*b*x)/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x))), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.237.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.92 (sec) , antiderivative size = 6481, normalized size of antiderivative = 21.97

method	result	size
risch	Expression too large to display	6481

input
```
int(x*arccoth(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.237.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1688 vs. 2(209) = 418.

Time = 0.33 (sec) , antiderivative size = 1688, normalized size of antiderivative = 5.72

$$\int x\coth^{-1}(c+d\tan(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate(x*arccoth(c+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/16*(4*b^2*x^2*log((d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) - 2
*I*b*x*dilog(2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d + (
I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 + d
^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 2*I*b*x*dilog(2
*((-I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^2 - 2*(c
 + 1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1
)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 2*I*b*x*dilog(2*((I*(c - 1)
*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c - 1)*d - I*d^
2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)
^2 + c^2 + d^2 - 2*c + 1) + 1) - 2*I*b*x*dilog(2*((-I*(c - 1)*d - d^2)*tan
(b*x + a)^2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c -
I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d
^2 - 2*c + 1) + 1) - 2*a^2*log(((I*(c + 1)*d + d^2)*tan(b*x + a)^2 - c^2 +
 I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/(tan(b*
x + a)^2 + 1)) - 2*a^2*log(((I*(c + 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(
c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c + 1)/(tan(b*x +
a)^2 + 1)) + 2*a^2*log(((I*(c - 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c -
1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/(tan(b*x + a)^2
 + 1)) + 2*a^2*log(((I*(c - 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c - 1)*d
 + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) - 2*c + 1)/(tan(b*x + a)^2 ...
```

3.237.6 Sympy [F]

$$\int x\coth^{-1}(c+d\tan(a+bx))\,dx = \int x\,\operatorname{acoth}\left(c+d\tan\left(a+bx\right)\right)dx$$

input
```
integrate(x*acoth(c+d*tan(b*x+a)),x)
```

output
```
Integral(x*acoth(c + d*tan(a + b*x)), x)
```

3.237.7 Maxima [F]

$$\int x \coth^{-1}(c + d\tan(a + bx))\,dx = \int x\,\mathrm{arcoth}\left(d\tan\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x*arccoth(c+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-2*b*d*integrate(-(2*(c^2 + d^2 - 1)*x^2*cos(2*b*x + 2*a)^2 + 2*c*d*x^2*si
n(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*sin(2*b*x + 2*a)^2 + (c^2 - d^2 - 1
)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) - (c^2 - d^2 - 1)*x^2
*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*x + 2*a) + (c^2 -
 d^2 - 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)
*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c
^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (c^4 + d^4 +
2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 -
1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 -
 1)*d^2 - 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(c*d^3 +
(c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 - d^4 - 2*c^2
 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)*d - 2*(c*d^3 + (c^3 - c)
*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin(4*b*x
 + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*log(
(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b*x + 2*a) +
(c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 + 2*c
+ 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/8*x^2*log((c^2 + d^2 - 2*c + 1)*cos(2
*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*sin(2
*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) - 2*c
 + 1)
```

3.237.8 Giac [F]

$$\int x \coth^{-1}(c + d\tan(a + bx))\,dx = \int x\,\mathrm{arcoth}\left(d\tan\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x*arccoth(c+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(d*tan(b*x + a) + c), x)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(c + d \tan(a + bx)) \, dx = \int x \operatorname{acoth}(c + d \tan(a + b x)) \, dx$$

input
```
int(x*acoth(c + d*tan(a + b*x)),x)
```

output
```
int(x*acoth(c + d*tan(a + b*x)), x)
```

3.238 $\int \coth^{-1}(c+d\tan(a+bx))\,dx$

3.238.1 Optimal result

Integrand size = 11, antiderivative size = 194

$$\begin{aligned}\int \coth^{-1}(c+d\tan(a+bx))\,dx = {} & x\coth^{-1}(c+d\tan(a+bx)) \\ & + \frac{1}{2}x\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) \\ & - \frac{1}{2}x\log\left(1+\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right) \\ & - \frac{i\,\mathrm{PolyLog}\left(2,-\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{4b} \\ & + \frac{i\,\mathrm{PolyLog}\left(2,-\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)}{4b}\end{aligned}$$

output
```
x*arccoth(c+d*tan(b*x+a))+1/2*x*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d
))-1/2*x*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*polylog(2,-(1-
c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*polylog(2,-(1+c-I*d)*exp(2*I*
a+2*I*b*x)/(1+c+I*d))/b
```

3.238.2 Mathematica [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.88

$$\int \coth^{-1}(c+d\tan(a+bx))\,dx = x\Bigg(\coth^{-1}(c+d\tan(a+bx)) + \frac{2a\log(1-c-d\tan(a+bx)) - i\log(1-i\tan(a+bx))\log\left(\frac{-1+c+d\tan(a+bx)}{-1+c-id}\right) + i\log(1+i\tan(a+bx)}{}$$

input
```
Integrate[ArcCoth[c + d*Tan[a + b*x]],x]
```

output
```
x*(ArcCoth[c + d*Tan[a + b*x]] + (2*a*Log[1 - c - d*Tan[a + b*x]] - I*Log[
1 - I*Tan[a + b*x]]*Log[(-1 + c + d*Tan[a + b*x])/(-1 + c - I*d)] + I*Log[
1 + I*Tan[a + b*x]]*Log[(-1 + c + d*Tan[a + b*x])/(-1 + c + I*d)] - 2*a*Lo
g[1 + c + d*Tan[a + b*x]] + I*Log[1 - I*Tan[a + b*x]]*Log[(1 + c + d*Tan[a
 + b*x])/(1 + c - I*d)] - I*Log[1 + I*Tan[a + b*x]]*Log[(1 + c + d*Tan[a +
 b*x])/(1 + c + I*d)] + I*PolyLog[2, -((d*(-I + Tan[a + b*x]))/(-1 + c + I
*d))] - I*PolyLog[2, -((d*(-I + Tan[a + b*x]))/(1 + c + I*d))] - I*PolyLog
[2, -((d*(I + Tan[a + b*x]))/(-1 + c - I*d))] + I*PolyLog[2, -((d*(I + Tan
[a + b*x]))/(1 + c - I*d))])/(4*a - (2*I)*Log[1 - I*Tan[a + b*x]] + (2*I)*
Log[1 + I*Tan[a + b*x]]))
```

3.238.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.48, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6814, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d\tan(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6814}$$

$$-b(ic+d+i)\int \frac{e^{2ia+2ibx}x}{c+(c-id+1)e^{2ia+2ibx}+id+1}dx + b(-d+i(1-c))\int \frac{e^{2ia+2ibx}x}{-c+(-c+id+1)e^{2ia+2ibx}-id+1}dx + x\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \; 2620$$

$$b(-d+i(1-c))\left(\frac{x\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{\int\log\left(\frac{e^{2ia+2ibx}(-c+id+1)}{-c-id+1}+1\right)dx}{2b(-d+i(1-c))}\right)-b(ic+d+i)\left(\frac{x\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{\int\log\left(\frac{e^{2ia+2ibx}(c-id+1)}{c+id+1}+1\right)dx}{2(bd+i(bc+b))}\right)+x\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \; 2715$$

$$b(-d+i(1-c))\left(\frac{i\int e^{-2ia-2ibx}\log\left(\frac{e^{2ia+2ibx}(-c+id+1)}{-c-id+1}+1\right)de^{2ia+2ibx}}{4b^2(-d+i(1-c))}+\frac{x\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}\right)-b(ic+d+i)\left(\frac{i\int e^{-2ia-2ibx}\log\left(\frac{e^{2ia+2ibx}(c-id+1)}{c+id+1}+1\right)de^{2ia+2ibx}}{4b(bd+i(bc+b))}+\frac{x\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}\right)+x\coth^{-1}(d\tan(a+bx)+c)$$

$$\Big\downarrow \; 2838$$

$$b(-d+i(1-c))\left(\frac{x\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{2b(-d+i(1-c))}-\frac{i\,\mathrm{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2(-d+i(1-c))}\right)-b(ic+d+i)\left(\frac{x\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{2(bd+i(bc+b))}-\frac{i\,\mathrm{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b(bd+i(bc+b))}\right)+x\coth^{-1}(d\tan(a+bx)+c)$$

input
```
Int[ArcCoth[c + d*Tan[a + b*x]],x]
```

output
```
x*ArcCoth[c + d*Tan[a + b*x]] + b*(I*(1 - c) - d)*((x*Log[1 + ((1 - c + I*
d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d)])/(2*b*(I*(1 - c) - d)) - ((I/4)
*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/(b^
2*(I*(1 - c) - d))) - b*(I + I*c + d)*((x*Log[1 + ((1 + c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 + c + I*d)])/(2*(I*(b + b*c) + b*d)) - ((I/4)*PolyLog[2
, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/(b*(I*(b + b*
c) + b*d)))
```

3.238. $\int \coth^{-1}(c+d\tan(a+bx))\,dx$

3.238.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6814
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Coth[c + d*Tan[a + b*x]], x] + (-Simp[I*b*(1 + c - I*d)   Int[x*(E^(2*I*a +
 2*I*b*x)/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x))), x], x] + Simp
[I*b*(1 - c + I*d)   Int[x*(E^(2*I*a + 2*I*b*x)/(1 - c - I*d + (1 - c + I*d
)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)
^2, 1]
```

3.238.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(164) = 328$.

Time = 2.64 (sec) , antiderivative size = 556, normalized size of antiderivative = 2.87

method	result
derivativedivides	$\frac{d\arctan(\tan(bx+a))\,\mathrm{arccoth}(c+d\tan(bx+a))+d^2\left(-\frac{\arctan\left(-\frac{c+d\tan(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)+c-1\right)}{2d}+\mathrm{arcta}\ldots\right.}{\ldots}$
default	$\frac{d\arctan(\tan(bx+a))\,\mathrm{arccoth}(c+d\tan(bx+a))+d^2\left(-\frac{\arctan\left(-\frac{c+d\tan(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)+c-1\right)}{2d}+\mathrm{arcta}\ldots\right.}{\ldots}$
risch	Expression too large to display

input
```
int(arccoth(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(d*arctan(tan(b*x+a))*arccoth(c+d*tan(b*x+a))+d^2*(-1/2*arctan(-(c+d
*tan(b*x+a))/d+c/d)/d*ln(d*((c+d*tan(b*x+a))/d-c/d)+c-1)+1/2*arctan(-(c+d*
tan(b*x+a))/d+c/d)/d*ln(d*((c+d*tan(b*x+a))/d-c/d)+c+1)+1/4*I*ln(d*((c+d*t
an(b*x+a))/d-c/d)+c-1)*(ln((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(I*d+c-1))-ln(
(I*d+d*((c+d*tan(b*x+a))/d-c/d))/(1-c+I*d)))/d+1/4*I*(dilog((I*d-d*((c+d*t
an(b*x+a))/d-c/d))/(I*d+c-1))-dilog((I*d+d*((c+d*tan(b*x+a))/d-c/d))/(1-c+
I*d)))/d-1/4*I*ln(d*((c+d*tan(b*x+a))/d-c/d)+c+1)*(ln((I*d-d*((c+d*tan(b*x
+a))/d-c/d))/(1+c+I*d))-ln((I*d+d*((c+d*tan(b*x+a))/d-c/d))/(I*d-c-1)))/d-
1/4*I*(dilog((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(1+c+I*d))-dilog((I*d+d*((c+
d*tan(b*x+a))/d-c/d))/(I*d-c-1)))/d))
```

3.238.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1184 vs. 2(136) = 272.

Time = 0.32 (sec) , antiderivative size = 1184, normalized size of antiderivative = 6.10

$$\int \coth^{-1}(c+d\tan(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate(arccoth(c+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(4*b*x*log((d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) - 2*(b*x
 + a)*log(-2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d + (I*
c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2
 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 2*(b*x + a)*log(-2*((
-I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^2 - 2*(c +
1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*t
an(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*log(-2*((I*(c - 1)*d -
 d^2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c - 1)*d - I*d^2 -
2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 +
 c^2 + d^2 - 2*c + 1)) + 2*(b*x + a)*log(-2*((-I*(c - 1)*d - d^2)*tan(b*x
+ a)^2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c - I)*ta
n(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 -
2*c + 1)) + 2*a*log(((I*(c + 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*
d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/(tan(b*x + a)^2 +
1)) + 2*a*log(((I*(c + 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c + 1)*d + (I
*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c + 1)/(tan(b*x + a)^2 + 1)) -
2*a*log(((I*(c - 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (I*c^2 +
 I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/(tan(b*x + a)^2 + 1)) - 2*a*lo
g(((I*(c - 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c - 1)*d + (I*c^2 + I*d^2
 - 2*I*c + I)*tan(b*x + a) - 2*c + 1)/(tan(b*x + a)^2 + 1)) - I*dilog(2...
```

3.238.6 Sympy [F]

$$\int \coth^{-1}(c+d\tan(a+bx))\,dx=\int \operatorname{acoth}\left(c+d\tan\left(a+bx\right)\right)dx$$

input
```
integrate(acoth(c+d*tan(b*x+a)),x)
```

output
```
Integral(acoth(c + d*tan(a + b*x)), x)
```

3.238.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(136) = 272$.

Time = 0.36 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.92

$$\int \coth^{-1}(c + d\tan(a + bx))\,dx$$
$$= \frac{4\,(bx + a)\,\mathrm{arcoth}\,(d\tan\,(bx + a) + c) + \left(\arctan\left(\frac{d^2\tan(bx+a)+(c+1)d}{c^2+d^2+2\,c+1}, \frac{(c+1)d\tan(bx+a)+c^2+2\,c+1}{c^2+d^2+2\,c+1}\right) - \arctan\left(\frac{d^2\,\mathrm{t}}{}\right.\right.}{}$$

input
```
integrate(arccoth(c+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*(4*(b*x + a)*arccoth(d*tan(b*x + a) + c) + (arctan2((d^2*tan(b*x + a)
+ (c + 1)*d)/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*tan(b*x + a) + c^2 + 2*c +
1)/(c^2 + d^2 + 2*c + 1)) - arctan2((d^2*tan(b*x + a) + (c - 1)*d)/(c^2 +
d^2 - 2*c + 1), ((c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d^2 - 2*c
+ 1)))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log((d^2*tan(b*x + a)^2 + 2*(c
+ 1)*d*tan(b*x + a) + c^2 + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + (b*x + a)*lo
g((d^2*tan(b*x + a)^2 + 2*(c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d
^2 - 2*c + 1)) - I*dilog(-(I*d*tan(b*x + a) - d)/(I*c + d + I)) + I*dilog(
-(I*d*tan(b*x + a) - d)/(I*c + d - I)) - I*dilog((I*d*tan(b*x + a) + d)/(-
I*c + d + I)) + I*dilog((I*d*tan(b*x + a) + d)/(-I*c + d - I)))/b
```

3.238.8 Giac [F]

$$\int \coth^{-1}(c + d\tan(a + bx))\,dx = \int \mathrm{arcoth}\,(d\tan\,(bx + a) + c)\;dx$$

input
```
integrate(arccoth(c+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(d*tan(b*x + a) + c), x)
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(c+d\tan(a+bx))\,dx = \int \mathrm{acoth}(c+d\tan(a+b\,x))\;dx$$

input
```
int(acoth(c + d*tan(a + b*x)),x)
```

output
```
int(acoth(c + d*tan(a + b*x)), x)
```

3.239 $\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx$

3.239.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx=\mathrm{Int}\left(\frac{\coth^{-1}(c+d\tan(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccoth(c+d*tan(b*x+a))/x,x)
```

3.239.2 Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx=\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[c + d*Tan[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[c + d*Tan[a + b*x]]/x, x]
```

3.239.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d\tan(a+bx)+c)}{x} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\coth^{-1}(d\tan(a+bx)+c)}{x}dx$$

input
```
Int[ArcCoth[c + d*Tan[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.239.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.239.4 Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(c+d\tan\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(c+d*tan(b*x+a))/x,x)
```

output
```
int(arccoth(c+d*tan(b*x+a))/x,x)
```

3.239.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\tan\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*tan(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arccoth(d*tan(b*x + a) + c)/x, x)
```

3.239.6 Sympy [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{acoth}\left(c+d\tan\left(a+bx\right)\right)}{x}\,dx$$

input
```
integrate(acoth(c+d*tan(b*x+a))/x,x)
```

output
```
Integral(acoth(c + d*tan(a + b*x))/x, x)
```

3.239.7 Maxima [N/A]

Not integrable

Time = 3.48 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\tan\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*tan(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arccoth(d*tan(b*x + a) + c)/x, x)
```

3.239.8 Giac [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\tan\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*tan(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(d*tan(b*x + a) + c)/x, x)
```

3.239.9 Mupad [N/A]

Not integrable

Time = 5.97 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx=\int \frac{\mathrm{acoth}(c+d\tan(a+b\,x))}{x}\,dx$$

input
```
int(acoth(c + d*tan(a + b*x))/x,x)
```

output
```
int(acoth(c + d*tan(a + b*x))/x, x)
```

3.240 $\int x^2 \coth^{-1}(1 - id + d\tan(a + bx))\,dx$

3.240.1 Optimal result

Integrand size = 20, antiderivative size = 170

$$\begin{aligned}\int x^2 \coth^{-1}(1 - id + d\tan(a + bx))\,dx &= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1 - id + d\tan(a + bx)) \\ &\quad - \frac{1}{6}x^3\log\left(1 + (1 - id)e^{2ia+2ibx}\right) \\ &\quad + \frac{ix^2\,\mathrm{PolyLog}\left(2, -\left((1 - id)e^{2ia+2ibx}\right)\right)}{4b} \\ &\quad - \frac{x\,\mathrm{PolyLog}\left(3, -\left((1 - id)e^{2ia+2ibx}\right)\right)}{4b^2} \\ &\quad - \frac{i\,\mathrm{PolyLog}\left(4, -\left((1 - id)e^{2ia+2ibx}\right)\right)}{8b^3}\end{aligned}$$

output
```
1/12*I*b*x^4+1/3*x^3*arccoth(1-I*d+d*tan(b*x+a))-1/6*x^3*ln(1+(1-I*d)*exp(
2*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*p
olylog(3,-(1-I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,-(1-I*d)*exp(2*I
*a+2*I*b*x))/b^3
```

3.240.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.91

$$\int x^2 \coth^{-1}(1-id+d\tan(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1-id+d\tan(a+bx)) - \frac{4b^3x^3\log\left(1+\frac{ie^{-2i(a+bx)}}{i+d}\right)+6ib^2x^2\operatorname{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{i+d}\right)+6bx\operatorname{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{i+d}\right)-3i\operatorname{PolyL}}{24b^3}$$

input
```
Integrate[x^2*ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 - I*d + d*Tan[a + b*x]])/3 - (4*b^3*x^3*Log[1 + I/((I + d)*
E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a
 + b*x)))] + 6*b*x*PolyLog[3, (-I)/((I + d)*E^((2*I)*(a + b*x)))] - (3*I)*
PolyLog[4, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.240.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6818, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\coth^{-1}(d\tan(a+bx)-id+1)\,dx$$

$$\Big\downarrow \text{ 6818}$$

$$\frac{1}{3}ib\int \frac{x^3}{e^{2ia+2ibx}(1-id)+1}dx+\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1-id)\int \frac{e^{2ia+2ibx}x^3}{e^{2ia+2ibx}(1-id)+1}dx\right)+\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1-id)\left(\frac{x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{3\int x^2\log\left(e^{2ia+2ibx}(1-id)+1\right)dx}{2b(d+i)}\right)\right)+\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1-id)\left(\frac{x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,-\left((1-id)e^{2ia+2\cdots}\right.\right.}{b}\right.}{2b(d+i)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1-id)\left(\frac{x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,-\left((1-id)e^{2ia+2\cdots}\right.\right.}{2b}\right.}{}\right.}{2b(d+i)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1-id)\left(\frac{x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,-\left((1-\cdots\right.\right.}{4b^2}\right.}{}\right.}{2b(d+i)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1-id)\left(\frac{x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,-\left((1-id)e^{2ia+2ibx}\right.\right.}{4b^2}\right.}{}\right.}{2b(d+i)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)-id+1)$$

3.240. $\int x^2\coth^{-1}(1-id+d\tan(a+bx))\,dx$

input
```
Int[x^2*ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 - I*d + d*Tan[a + b*x]])/3 + (I/3)*b*(x^4/4 - (1 - I*d)*((x
^3*Log[1 + (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I + d)) - (3*(((I/2)*
x^2*PolyLog[2, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))])/b - (I*(((-1/2*I)*x*
PolyLog[3, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))])/b + PolyLog[4, -((1 - I*
d)*E^((2*I)*a + (2*I)*b*x))]/(4*b^2)))/b))/(2*b*(I + d))))
```

3.240.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6818
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.240.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.31 (sec) , antiderivative size = 2273, normalized size of antiderivative = 13.37

method	result	size
risch	Expression too large to display	2273

input
```
int(x^2*arccoth(1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^2*x-1/2/b^2*d*a^2/(I+d)*l
n(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/2/b^2*d*a^2/(I+d)*ln(1+I*exp(I*
(b*x+a))*(-I*(I+d))^(1/2))*x+1/4*I/b*d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*
x+a)))*x^2+1/2*I/b^3*d*a^2/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2)
)+1/2*I/b^3*d*a^2/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2*I/b
^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/2*I/b^2*a^2/(I+d)
*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/2*I/b^2/(I+d)*ln(1-I*(I+d)*ex
p(2*I*(b*x+a)))*a^2*x-1/4*I/b^3*a^2*d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x
+a)))+1/12*I*b*x^4-1/6*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^3-1/2/b^3*
a^2/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/4/b/(I+d)*polylog(2
,I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*
x+a)))*a^2-1/2/b^3*a^2/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/
6*I/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^3-1/3*x^3*ln(exp(I*(b*x+a)))+1/
3/b^3*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/4/b^2*d/(I+d)*polylog(3
,I*(I+d)*exp(2*I*(b*x+a)))*x-1/2/b^3*d*a^3/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I
*(I+d))^(1/2))-1/2/b^3*d*a^3/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))
+1/6/b^3*a^3*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)+1/3*I/b^3
/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/4*I/b^2/(I+d)*polylog(3,I*(I+d
)*exp(2*I*(b*x+a)))*x-1/8*I/b^3*d/(I+d)*polylog(4,I*(I+d)*exp(2*I*(b*x+a))
)+1/6*I/b^3*a^3/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)-1/2*I...
```

3.240.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(118) = 236$.

Time = 0.28 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.02

$$\int x^2 \coth^{-1}(1 - id + d\tan(a + bx))\, dx$$
$$= \frac{i\, b^4x^4 + 2\, b^3x^3 \log\left(\frac{((d+i)e^{(2i\, bx+2i\, a)}+i)e^{(-2i\, bx-2i\, a)}}{d}\right) + 6i\, b^2x^2 \mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\, d - 4}e^{(i\, bx+i\, a)}\right) + 6i\, b^2x^2 \mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\, d\right.}{}$$

input
```
integrate(x^2*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(I*b^4*x^4 + 2*b^3*x^3*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*
b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) +
 6*I*b^2*x^2*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*l
og(1/2*(2*(d + I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(d + I)) + 2*a^3*lo
g(1/2*(2*(d + I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(d + I)) - 12*b*x*po
lylog(3, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2*sqr
t(4*I*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(4*I*d - 4)*
e^(I*b*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*x
 + I*a) + 1) - 12*I*polylog(4, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*I
*polylog(4, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

3.240.6 Sympy [F]

$$\int x^2 \coth^{-1}(1-id+d\tan(a+bx))\,dx=\int x^2\operatorname{acoth}\left(d\tan\left(a+bx\right)-id+1\right)dx$$

input
```
integrate(x**2*acoth(1-I*d+d*tan(b*x+a)),x)
```

output
```
Integral(x**2*acoth(d*tan(a + b*x) - I*d + 1), x)
```

3.240.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(118) = 236.

Time = 0.24 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.02

$$\int x^2 \coth^{-1}(1-id+d\tan(a+bx))\,dx$$
$$=\frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{arcoth}(d\tan(bx+a)-i\,d+1)}{b^2}-\frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(-4i\,(bx+a)^3+9i\,(bx+a\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(x^2*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*tan(b
*x + a) - I*d + 1)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*
x + a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^2
)*arctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) + cos
(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog
((I*d - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*
x + a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^
2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog
(3, (I*d - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d - 1)*e^(2*I*b*x +
 2*I*a)))/b^2)/b
```

3.240.8 Giac [F]

$$\int x^2 \coth^{-1}(1 - id + d\tan(a + bx))\,dx = \int x^2 \operatorname{arcoth}\left(d\tan\left(bx + a\right) - i\,d + 1\right)\,dx$$

input
```
integrate(x^2*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(d*tan(b*x + a) - I*d + 1), x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(1 - id + d\tan(a + bx))\,dx = \int x^2 \operatorname{acoth}(d\tan(a + b\,x) + 1 - d\,1\mathrm{i})\,dx$$

input
```
int(x^2*acoth(d*tan(a + b*x) - d*1i + 1),x)
```

output
```
int(x^2*acoth(d*tan(a + b*x) - d*1i + 1), x)
```

3.241 $\int x \coth^{-1}(1-id+d\tan(a+bx))\,dx$

3.241.1 Optimal result

Integrand size = 18, antiderivative size = 133

$$\int x \coth^{-1}(1-id+d\tan(a+bx))\,dx = \frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx)) -\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right) +\frac{ix\,\text{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b} -\frac{\text{PolyLog}\left(3,-\left((1-id)e^{2ia+2ibx}\right)\right)}{8b^2}$$

output
```
1/6*I*b*x^3+1/2*x^2*arccoth(1-I*d+d*tan(b*x+a))-1/4*x^2*ln(1+(1-I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylo
g(3,-(1-I*d)*exp(2*I*a+2*I*b*x))/b^2
```

3.241.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\int x \coth^{-1}(1-id+d\tan(a+bx))\,dx = \frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx)) - \frac{2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{i+d}\right)+2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{i+d}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{i+d}\right)}{8b^2}$$

input
```
Integrate[x*ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 - I*d + d*Tan[a + b*x]])/2 - (2*b^2*x^2*Log[1 + I/((I + d)*
E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a + b
*x)))] + PolyLog[3, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

3.241.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6818, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(d\tan(a+bx)-id+1)\,dx$$

$$\Big\downarrow \text{ 6818}$$

$$\frac{1}{2}ib\int \frac{x^2}{e^{2ia+2ibx}(1-id)+1}dx+\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1-id)\int \frac{e^{2ia+2ibx}x^2}{e^{2ia+2ibx}(1-id)+1}dx\right)+\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1-id)\left(\frac{x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{\int x\log\left(e^{2ia+2ibx}(1-id)+1\right)dx}{b(d+i)}\right)\right)+ \frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1-id)\left(\frac{x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)d}{2b}}{b(d+i)}\right.\right.$$
$$\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1-id)\left(\frac{x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{\frac{ix\,\text{PolyLog}(2,-((1-id)e^{2ia+2ibx}))}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}(2,-((1-id)e^{2}}{4b^2}}{b(d+i)}\right.\right.$$

$$\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)-id+1)$$

$\downarrow$ 7143

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1-id)\left(\frac{x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{\frac{ix\,\text{PolyLog}(2,-((1-id)e^{2ia+2ibx}))}{2b}-\frac{\text{PolyLog}(3,-((1-id)e^{2ia+2ibx}))}{4b^2}}{b(d+i)}\right)\right)$$

$$\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)-id+1)$$

input
```
Int[x*ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 - I*d + d*Tan[a + b*x]])/2 + (I/2)*b*(x^3/3 - (1 - I*d)*((x
^2*Log[1 + (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I + d)) - (((I/2)*x*P
olyLog[2, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 - I*d
)*E^((2*I)*a + (2*I)*b*x))]/(4*b^2))/(b*(I + d))))
```

3.241.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6818
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.241.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.98 (sec) , antiderivative size = 2183, normalized size of antiderivative = 16.41

method	result	size
risch	Expression too large to display	2183

input
```
int(x*arccoth(1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/6*I*b*x^3-1/2*I/b^2*a*d/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))
-1/2*I/b^2*a*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/2*I/b*a/
(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/2/b*a*d/(I+d)*ln(1+I*exp
(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/2/b*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)
))*a*x+1/2/b*a*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/2*I/b/(
I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a*x+1/4*I/b*d/(I+d)*polylog(2,I*(I+d)*
exp(2*I*(b*x+a)))*x+1/4*I/b^2*d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*
a+1/2*I/b*a/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/4*d/(I+d)*ln
(1-I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/2/b^2*a/(I+d)*dilog(1-I*exp(I*(b*x+a))*
(-I*(I+d))^(1/2))+1/2/b^2*a/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2
))-1/4/b/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x-1/4/b^2/(I+d)*polylog
(2,I*(I+d)*exp(2*I*(b*x+a)))*a-1/8/b^2*d/(I+d)*polylog(3,I*(I+d)*exp(2*I*(
b*x+a)))-1/8*I/b^2/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))-1/4*I/(I+d)*l
n(1-I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a)
)*(-I*(I+d))^(1/2))+1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(
1/2))-1/4/b^2*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^2-1/4/b^2*a^2*d/(I+
d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)+1/2*I/b^2*a^2/(I+d)*ln(1-I*
exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))
*(-I*(I+d))^(1/2))-1/4*I/b^2/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^2-1/4*
I/b^2*a^2/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)-1/2*x^2*ln(...
```

3.241.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(93) = 186$.

Time = 0.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.20

$$\int x \coth^{-1}(1 - id + d\tan(a + bx))\, dx$$
$$= \frac{2i\, b^3x^3 + 3\, b^2x^2 \log\left(\frac{((d+i)e^{(2i\, bx+2i\, a)}+i)e^{(-2i\, bx-2i\, a)}}{d}\right) + 2i\, a^3 + 6i\, bx\text{Li}_2\left(\frac{1}{2}\sqrt{4i\, d - 4}e^{(i\, bx+i\, a)}\right) + 6i\, bx\text{Li}_2\left(-\frac{1}{2}\right.}{}$$

input
```
integrate(x*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*I*b^3*x^3 + 3*b^2*x^2*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*
I*b*x - 2*I*a)/d) + 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x +
 I*a)) + 6*I*b*x*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 3*a^2*log(1
/2*(2*(d + I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(d + I)) - 3*a^2*log(1/
2*(2*(d + I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(d + I)) - 3*(b^2*x^2 -
a^2)*log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 3*(b^2*x^2 - a^2)*log(
-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 6*polylog(3, 1/2*sqrt(4*I*d -
4)*e^(I*b*x + I*a)) - 6*polylog(3, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/
b^2
```

3.241.6 Sympy [F]

$$\int x\coth^{-1}(1-id+d\tan(a+bx))\,dx=\int x\,\mathrm{acoth}\,(d\tan{(a+bx)}-id+1)\,dx$$

input
```
integrate(x*acoth(1-I*d+d*tan(b*x+a)),x)
```

output
```
Integral(x*acoth(d*tan(a + b*x) - I*d + 1), x)
```

3.241.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(93) = 186.

Time = 0.24 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.86

$$\int x\coth^{-1}(1-id+d\tan(a+bx))\,dx$$
$$=\frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\,\mathrm{arcoth}(d\tan(bx+a)-i\,d+1)}{b}-\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left((i\,d-1)e^{(2i\,bx+2i\,a)}\right)-6\left(-i\,(bx+a)^2+2i\,(bx+a\right.}{}}{}$$

input
```
integrate(x*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arccoth(d*tan(b*x + a) - I*d + 1)/b
 - (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((I*d - 1)*e^(2*I
*b*x + 2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(-d*cos(2*b*x
 + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1) + 3
*((b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1
)*sin(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*
polylog(3, (I*d - 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

3.241.8 Giac [F]

$$\int x \coth^{-1}(1 - id + d\tan(a + bx))\,dx = \int x\,\mathrm{arcoth}\left(d\tan\left(bx + a\right) - i\,d + 1\right)\,dx$$

input
```
integrate(x*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(d*tan(b*x + a) - I*d + 1), x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(1 - id + d\tan(a + bx))\,dx = \int x\,\mathrm{acoth}(d\tan(a + b\,x) + 1 - d\,1\mathrm{i})\,dx$$

input
```
int(x*acoth(d*tan(a + b*x) - d*1i + 1),x)
```

output
```
int(x*acoth(d*tan(a + b*x) - d*1i + 1), x)
```

3.242 $\int \coth^{-1}(1-id+d\tan(a+bx))\,dx$

3.242.1 Optimal result

Integrand size = 16, antiderivative size = 93

$$\int \coth^{-1}(1-id+d\tan(a+bx))\,dx = \frac{1}{2}ibx^2+x\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right) + \frac{i\,\mathrm{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b}$$

output
```
1/2*I*b*x^2+x*arccoth(1-I*d+d*tan(b*x+a))-1/2*x*ln(1+(1-I*d)*exp(2*I*a+2*I
*b*x))+1/4*I*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b
```

3.242.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 766 vs. 2(93) = 186.

Time = 4.35 (sec) , antiderivative size = 766, normalized size of antiderivative = 8.24

$$\int \coth^{-1}(1-id+d\tan(a+bx))\,dx = x\coth^{-1}(1-id+d\tan(a+bx)) + \frac{x\left(2ibx\log(2\cos(bx)(\cos(bx)-i\sin(bx)))-\log\left(\frac{\sec(bx)(\cos(a)-i\sin(a))((2i+d)\cos(a+bx)+id\sin(a+bx))}{2(i+d)}\right)\log(1-i\right.}{((2i+d)\cos(a+bx)+id\sin(a+bx))\left(\frac{i\log(1+i\tan(bx))\sec(bx)(d\cos(a)+i(2i+d)\sin(a))}{(2i+d)\cos(a+bx)+id\sin(a+bx)}+\frac{\log(}{}\right.}$$

input
```
Integrate[ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

output
```
x*ArcCoth[1 - I*d + d*Tan[a + b*x]] + (x*((2*I)*b*x*Log[2*Cos[b*x]*(Cos[b*
x] - I*Sin[b*x])] - Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((2*I + d)*Cos[a + b
*x] + I*d*Sin[a + b*x]))/(2*(I + d))]*Log[1 - I*Tan[b*x]] + Log[(Sec[b*x]*
((2 - I*d)*Cos[a + b*x] + d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Log[
1 + I*Tan[b*x]] - PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - PolyLog[2, (Sec
[b*x]*(d*Cos[a] + I*(2*I + d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*
(I + d))] + PolyLog[2, -1/2*((Cos[a] + I*Sin[a])*(d*Cos[a] + I*(2*I + d)*S
in[a])*(-I + Tan[b*x]))])*Sec[a + b*x]^2*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*
x] + Sin[b*x])*((2 - I*d)*Cos[a + b*x] + d*Sin[a + b*x]))/(((2*I + d)*Cos[
a + b*x] + I*d*Sin[a + b*x])*((I*Log[1 + I*Tan[b*x]]*Sec[b*x]*(d*Cos[a] +
I*(2*I + d)*Sin[a]))/((2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]) + (Log[1
- I*Tan[b*x]]*Sec[b*x]*((-I)*d*Cos[a] + (2*I + d)*Sin[a]))/((2*I + d)*Cos[
a + b*x] + I*d*Sin[a + b*x]) + 2*b*x*(1 - I*Tan[b*x]) + (Log[(Sec[b*x]*((2
 - I*d)*Cos[a + b*x] + d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*x
]^2)/(-I + Tan[b*x]) - (Log[1 + ((Cos[a] + I*Sin[a])*(d*Cos[a] + I*(2*I +
d)*Sin[a])*(-I + Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + Log[(Sec[b*x]
*(Cos[a] - I*Sin[a])*((2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]))/(2*(I +
d))]*(-I + Tan[b*x]) - (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((2*I + d)*Cos[a
 + b*x] + I*d*Sin[a + b*x]))/(2*(I + d))]*Sec[b*x]^2)/(I + Tan[b*x]))*(-I
+ Tan[a + b*x])*(2 - I*d + d*Tan[a + b*x]))
```

3.242.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6810, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d\tan(a+bx)-id+1)\,dx$$

$$\Big\downarrow \text{6810}$$

$$ib\int \frac{x}{e^{2ia+2ibx}(1-id)+1}dx+x\coth^{-1}(d\tan(a+bx)-id+1)$$

$$\Big\downarrow \text{2615}$$

$$ib\left(\frac{x^2}{2}-(1-id)\int \frac{e^{2ia+2ibx}x}{e^{2ia+2ibx}(1-id)+1}dx\right)+x\coth^{-1}(d\tan(a+bx)-id+1)$$

$\big\downarrow$ 2620

$$ib\left(\frac{x^2}{2}-(1-id)\left(\frac{x\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{\int \log\left(e^{2ia+2ibx}(1-id)+1\right)dx}{2b(d+i)}\right)\right)+x\coth^{-1}(d\tan(a+bx)-id+1)$$

$\big\downarrow$ 2715

$$ib\left(\frac{x^2}{2}-(1-id)\left(\frac{i\int e^{-2ia-2ibx}\log\left(e^{2ia+2ibx}(1-id)+1\right)de^{2ia+2ibx}}{4b^2(d+i)}+\frac{x\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+x\coth^{-1}(d\tan(a+bx)-id+1)$$

$\big\downarrow$ 2838

$$ib\left(\frac{x^2}{2}-(1-id)\left(\frac{x\log\left(1+(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}-\frac{i\,\mathrm{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b^2(d+i)}\right)\right)+x\coth^{-1}(d\tan(a+bx)-id+1)$$

input
```
Int[ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

output
```
x*ArcCoth[1 - I*d + d*Tan[a + b*x]] + I*b*(x^2/2 - (1 - I*d)*((x*Log[1 + (
1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I + d)) - ((I/4)*PolyLog[2, -((1
- I*d)*E^((2*I)*a + (2*I)*b*x))])/(b^2*(I + d))))
```

3.242.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6810
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Coth[c + d*Tan[a + b*x]], x] + Simp[I*b   Int[x/(c + I*d + c*E^(2*I*a + 2*I
*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, 1]
```

3.242.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(76) = 152.

Time = 2.32 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.30

method	result
derivativedivides	$-\frac{i \operatorname{arccoth}(1-id+d\tan(bx+a))d\ln(-id+d\tan(bx+a))}{2}+\frac{i \operatorname{arccoth}(1-id+d\tan(bx+a))d\ln(id+d\tan(bx+a))}{2}+\frac{d^2\left(-\frac{i\left(\frac{\ln(-id+d\tan\cdots}{4}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
default	$-\frac{i \operatorname{arccoth}(1-id+d\tan(bx+a))d\ln(-id+d\tan(bx+a))}{2}+\frac{i \operatorname{arccoth}(1-id+d\tan(bx+a))d\ln(id+d\tan(bx+a))}{2}+\frac{d^2\left(-\frac{i\left(\frac{\ln(-id+d\tan\cdots}{4}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
risch	Expression too large to display

input
```
int(arccoth(1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-1/2*I*arccoth(1-I*d+d*tan(b*x+a))*d*ln(-I*d+d*tan(b*x+a))+1/2*I*ar
ccoth(1-I*d+d*tan(b*x+a))*d*ln(I*d+d*tan(b*x+a))+1/2*d^2*(-I/d*(1/4*ln(-I*
d+d*tan(b*x+a))^2-1/2*dilog(1-1/2*I*d+1/2*d*tan(b*x+a))-1/2*ln(-I*d+d*tan(
b*x+a))*ln(1-1/2*I*d+1/2*d*tan(b*x+a)))+I/d*(-1/2*dilog(I*(I*d+d*tan(b*x+a
)-I*(2*I+2*d))/(2*I+2*d))-1/2*ln(I*d+d*tan(b*x+a))*ln(I*(I*d+d*tan(b*x+a)-
I*(2*I+2*d))/(2*I+2*d))+1/2*dilog(1/2*I*(-I*d+d*tan(b*x+a))/d)+1/2*ln(I*d+
d*tan(b*x+a))*ln(1/2*I*(-I*d+d*tan(b*x+a))/d))))
```

3.242.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(65) = 130$.

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.33

$$\int \coth^{-1}(1-id+d\tan(a+bx))\,dx$$
$$= \frac{i\,b^2x^2 + bx\log\left(\frac{((d+i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right) - i\,a^2 - (bx+a)\log\left(\frac{1}{2}\sqrt{4i\,d-4}e^{(i\,bx+i\,a)}+1\right) - (bx+a)\log \cdots}{\cdots}$$

input
```
integrate(arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(I*b^2*x^2 + b*x*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2
*I*a)/d) - I*a^2 - (b*x + a)*log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1)
- (b*x + a)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) + a*log(1/2*(2*(
d + I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(d + I)) + a*log(1/2*(2*(d + I
)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(d + I)) + I*dilog(1/2*sqrt(4*I*d -
 4)*e^(I*b*x + I*a)) + I*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b
```

3.242.6 Sympy [F]

$$\int \coth^{-1}(1-id+d\tan(a+bx))\,dx = \int \operatorname{acoth}\left(d\tan\left(a+bx\right)-id+1\right)dx$$

input
```
integrate(acoth(1-I*d+d*tan(b*x+a)),x)
```

output
```
Integral(acoth(d*tan(a + b*x) - I*d + 1), x)
```

3.242.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(65) = 130$.

Time = 0.29 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.82

$$\int \coth^{-1}(1 - id + d\tan(a + bx))\,dx =$$

$$-\frac{4\,(bx+a)d\left(\frac{\log(d\tan(bx+a)-i\,d+2)}{d} - \frac{\log(\tan(bx+a)-i)}{d}\right) - d\left(\frac{2i\left(\log(d\tan(bx+a)-i\,d+2)\log\left(-\frac{i\,d\tan(bx+a)+d+2i}{2\,(d+i)}+1\right)+\mathrm{Li}_2\left(\frac{i}{}\right.\right.}{d}\right.}{}$$

input
```
integrate(arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*(4*(b*x + a)*d*(log(d*tan(b*x + a) - I*d + 2)/d - log(tan(b*x + a) -
I)/d) - d*(2*I*(log(d*tan(b*x + a) - I*d + 2)*log(-1/2*(I*d*tan(b*x + a) +
 d + 2*I)/(d + I) + 1) + dilog(1/2*(I*d*tan(b*x + a) + d + 2*I)/(d + I)))/
d - (2*I*log(d*tan(b*x + a) - I*d + 2)*log(tan(b*x + a) - I) - I*log(tan(b
*x + a) - I)^2)/d + 2*I*(log(1/2*d*tan(b*x + a) - 1/2*I*d + 1)*log(tan(b*x
 + a) - I) + dilog(-1/2*d*tan(b*x + a) + 1/2*I*d))/d - 2*I*(log(tan(b*x +
a) - I)*log(-1/2*I*tan(b*x + a) + 1/2) + dilog(1/2*I*tan(b*x + a) + 1/2))/
d) - 8*(b*x + a)*arccoth(d*tan(b*x + a) - I*d + 1))/b
```

3.242.8 Giac [F]

$$\int \coth^{-1}(1 - id + d\tan(a + bx))\,dx = \int \operatorname{arcoth}\left(d\tan\left(bx + a\right) - i\,d + 1\right)\,dx$$

input
```
integrate(arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(d*tan(b*x + a) - I*d + 1), x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(1-id+d\tan(a+bx))\,dx = \int \mathrm{acoth}(d\tan(a+b\,x)+1-d\,1\mathrm{i})\,dx$$

input
```
int(acoth(d*tan(a + b*x) - d*1i + 1),x)
```

output
```
int(acoth(d*tan(a + b*x) - d*1i + 1), x)
```

3.243 $\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x} \, dx$

3.243.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x} \, dx = \text{Int}\left(\frac{\coth^{-1}(1-id+d\tan(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccoth(1-I*d+d*tan(b*x+a))/x,x)
```

3.243.2 Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x} \, dx = \int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x} \, dx$$

input
```
Integrate[ArcCoth[1 - I*d + d*Tan[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[1 - I*d + d*Tan[a + b*x]]/x, x]
```

3.243.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d\tan(a+bx)-id+1)}{x} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\coth^{-1}(d\tan(a+bx)-id+1)}{x}dx$$

input
```
Int[ArcCoth[1 - I*d + d*Tan[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.243.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.243.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arccoth}\left(1-id+d\tan\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(1-I*d+d*tan(b*x+a))/x,x)
```

output
```
int(arccoth(1-I*d+d*tan(b*x+a))/x,x)
```

3.243.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\tan\left(bx+a\right)-i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-I*d+d*tan(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d)
/x, x)
```

3.243.6 Sympy [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{acoth}\left(d\tan\left(a+bx\right)-id+1\right)}{x}\,dx$$

input
```
integrate(acoth(1-I*d+d*tan(b*x+a))/x,x)
```

output
```
Integral(acoth(d*tan(a + b*x) - I*d + 1)/x, x)
```

3.243.7 Maxima [N/A]

Not integrable

Time = 4.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 7.20

$$\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\tan\left(bx+a\right)-i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-I*d+d*tan(b*x+a))/x,x, algorithm="maxima")
```

output
```
-I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(d))*log(x) - 1/2*I*integrate(arctan2(-
d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2
*a) - 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1
)*sin(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1)/x, x
)
```

3.243.8 Giac [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(1 - id + d\tan(a + bx))}{x}\, dx = \int \frac{\operatorname{arcoth}\left(d\tan\left(bx + a\right) - i\,d + 1\right)}{x}\, dx$$

input
```
integrate(arccoth(1-I*d+d*tan(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(d*tan(b*x + a) - I*d + 1)/x, x)
```

3.243.9 Mupad [N/A]

Not integrable

Time = 5.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(1 - id + d\tan(a + bx))}{x}\, dx = \int \frac{\operatorname{acoth}(d\tan(a + b\,x) + 1 - d\,1\mathrm{i})}{x}\, dx$$

input
```
int(acoth(d*tan(a + b*x) - d*1i + 1)/x,x)
```

output
```
int(acoth(d*tan(a + b*x) - d*1i + 1)/x, x)
```

3.244 $\int x^2 \coth^{-1}(1+id-d\tan(a+bx))\,dx$

3.244.1 Optimal result

Integrand size = 21, antiderivative size = 171

$$\int x^2 \coth^{-1}(1+id-d\tan(a+bx))\,dx = \frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+id)e^{2ia+2ibx}\right) + \frac{ix^2\operatorname{PolyLog}\left(2,-\left((1+id)e^{2ia+2ibx}\right)\right)}{4b} - \frac{x\operatorname{PolyLog}\left(3,-\left((1+id)e^{2ia+2ibx}\right)\right)}{4b^2} - \frac{i\operatorname{PolyLog}\left(4,-\left((1+id)e^{2ia+2ibx}\right)\right)}{8b^3}$$

output
```
1/12*I*b*x^4+1/3*x^3*arccoth(1+I*d-d*tan(b*x+a))-1/6*x^3*ln(1+(1+I*d)*exp(
2*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*p
olylog(3,-(1+I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,-(1+I*d)*exp(2*I
*a+2*I*b*x))/b^3
```

3.244.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.91

$$\int x^2 \coth^{-1}(1+id-d\tan(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx)) - \frac{4b^3x^3\log\left(1-\frac{ie^{-2i(a+bx)}}{-i+d}\right)+6ib^2x^2\operatorname{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{-i+d}\right)+6bx\operatorname{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{-i+d}\right)-3i\operatorname{PolyLog}\left(\right.}{24b^3}$$

input
```
Integrate[x^2*ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 + I*d - d*Tan[a + b*x]])/3 - (4*b^3*x^3*Log[1 - I/((-I + d)
*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/((-I + d)*E^((2*I)*(a
+ b*x)))] + 6*b*x*PolyLog[3, I/((-I + d)*E^((2*I)*(a + b*x)))] - (3*I)*Pol
yLog[4, I/((-I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.244.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6818, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\coth^{-1}(d(-\tan(a+bx))+id+1)\,dx$$

$$\Big\downarrow \text{6818}$$

$$\frac{1}{3}ib\int \frac{x^3}{e^{2ia+2ibx}(id+1)+1}dx+\frac{1}{3}x^3\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\int \frac{e^{2ia+2ibx}x^3}{e^{2ia+2ibx}(id+1)+1}dx\right)+\frac{1}{3}x^3\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\left(\frac{x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{3\int x^2\log\left(e^{2ia+2ibx}(id+1)+1\right)dx}{2b(-d+i)}\right)\right)+\frac{1}{3}x^3\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\left(\frac{x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,-\left((id+1)e^{2ia+2\dots}\right.\right.}{b}\right.}{2b(-d+i)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\left(\frac{x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,-\left((id+1)e^{2ia+2\dots}\right.\right.}{2b}\right.}{\dots}\right.}{2b(-d+i)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\left(\frac{x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,-\left((id\dots\right.\right.}{4b^2}\right.}{\dots}\right.}{2b(-d+i\dots}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}-(1+id)\left(\frac{x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,-\left((id+1)e^{2ia+2ibx}\right.\right.}{4b^2}\right.}{\dots}\right.}{2b(-d+i)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d(-\tan(a+bx))+id+1)$$

3.244. $\int x^2\coth^{-1}(1+id-d\tan(a+bx))\,dx$

input
```
Int[x^2*ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 + I*d - d*Tan[a + b*x]])/3 + (I/3)*b*(x^4/4 - (1 + I*d)*((x
^3*Log[1 + (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I - d)) - (3*(((I/2)*
x^2*PolyLog[2, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))])/b - (I*(((-1/2*I)*x*
PolyLog[3, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))])/b + PolyLog[4, -((1 + I*
d)*E^((2*I)*a + (2*I)*b*x))]/(4*b^2)))/b))/(2*b*(I - d))))
```

3.244.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6818
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.244.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.41 (sec) , antiderivative size = 2383, normalized size of antiderivative = 13.94

method	result	size
risch	Expression too large to display	2383

input
```
int(x^2*arccoth(1+I*d-d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(
2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I
*(b*x+a))+1))-I*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^3+2*I*Pi*
csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-I*Pi*csgn((exp(2*I*(b*x+
a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+I*Pi*csgn(I*exp(2*I*(b
*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)
))^2-I*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*
(b*x+a))+1)*exp(2*I*(b*x+a)))-I*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(
b*x+a)))^3+I*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(ex
p(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-I*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp
(2*I*(b*x+a)))^2-I*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*cs
gn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+I*P
i*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1
))^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*
I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+I*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp
(2*I*(b*x+a)))^3-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))
)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))-I*Pi*csgn(I*exp(2*I*(b*x+a
)))^3-I*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+
a))+1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1
))^2-I*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+I*Pi*csgn((...
```

3.244.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(119) = 238.

Time = 0.27 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.01

$$\int x^2 \coth^{-1}(1 + id - d\tan(a + bx))\,dx$$
$$= \frac{i\,b^4x^4 - 2\,b^3x^3\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,d-4}e^{(i\,b\ldots}\right.}{}$$

input
```
integrate(x^2*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(I*b^4*x^4 - 2*b^3*x^3*log(d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x
+ 2*I*a) - I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) +
 6*I*b^2*x^2*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*
log(1/2*(2*(d - I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(d - I)) + 2*a^3*
log(1/2*(2*(d - I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(d - I)) - 12*b*x
*polylog(3, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2
*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*I*d
 - 4)*e^(I*b*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*I*d - 4)*e
^(I*b*x + I*a) + 1) - 12*I*polylog(4, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)
) - 12*I*polylog(4, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

3.244.6 Sympy [F]

$$\int x^2 \coth^{-1}(1+id-d\tan(a+bx))\,dx = \int x^2 \operatorname{acoth}\left(-d\tan\left(a+bx\right)+id+1\right)dx$$

input
```
integrate(x**2*acoth(1+I*d-d*tan(b*x+a)),x)
```

output
```
Integral(x**2*acoth(-d*tan(a + b*x) + I*d + 1), x)
```

3.244.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(119) = 238.

Time = 0.22 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.00

$$\int x^2 \coth^{-1}(1+id-d\tan(a+bx))\,dx =$$

$$-\frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{arcoth}(d\tan(bx+a)-i\,d-1)}{b^2}+\frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(-4i\,(bx+a)^3+9i\,(bx\right.}{}}{}$$

input
```
integrate(x^2*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*tan(
b*x + a) - I*d - 1)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b
*x + a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^
2)*arctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + co
s(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilo
g((-I*d - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(
b*x + a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a
)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polyl
og(3, (-I*d - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d - 1)*e^(2*I*b
*x + 2*I*a)))/b^2)/b
```

3.244.8 Giac [F]

$$\int x^2 \coth^{-1}(1 + id - d\tan(a + bx))\,dx = \int x^2 \operatorname{arcoth}\left(-d\tan\left(bx + a\right) + i\,d + 1\right)\,dx$$

input
```
integrate(x^2*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(-d*tan(b*x + a) + I*d + 1), x)
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(1 + id - d\tan(a + bx))\,dx = \int x^2 \operatorname{acoth}(1 - d\tan(a + b\,x) + d\,1\mathrm{i})\,dx$$

input
```
int(x^2*acoth(d*1i - d*tan(a + b*x) + 1),x)
```

output
```
int(x^2*acoth(d*1i - d*tan(a + b*x) + 1), x)
```

3.245 $\int x \coth^{-1}(1+id-d\tan(a+bx))\,dx$

3.245.1 Optimal result

Integrand size = 19, antiderivative size = 134

$$\begin{aligned}\int x \coth^{-1}(1+id-d\tan(a+bx))\,dx = &\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx))\\&-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)\\&+\frac{ix\operatorname{PolyLog}\left(2,-\left((1+id)e^{2ia+2ibx}\right)\right)}{4b}\\&-\frac{\operatorname{PolyLog}\left(3,-\left((1+id)e^{2ia+2ibx}\right)\right)}{8b^2}\end{aligned}$$

output
```
1/6*I*b*x^3+1/2*x^2*arccoth(1+I*d-d*tan(b*x+a))-1/4*x^2*ln(1+(1+I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylo
g(3,-(1+I*d)*exp(2*I*a+2*I*b*x))/b^2
```

3.245.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\begin{aligned}&\int x\coth^{-1}(1+id-d\tan(a+bx))\,dx\\&=\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx))\\&-\frac{2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{-i+d}\right)+2ibx\operatorname{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{-i+d}\right)+\operatorname{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{8b^2}\end{aligned}$$

input
```
Integrate[x*ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 + I*d - d*Tan[a + b*x]])/2 - (2*b^2*x^2*Log[1 - I/((-I + d)
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/((-I + d)*E^((2*I)*(a + b*
x)))] + PolyLog[3, I/((-I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

3.245.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6818, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(d(-\tan(a+bx))+id+1)\,dx$$

$$\Big\downarrow \; 6818$$

$$\frac{1}{2}ib\int \frac{x^2}{e^{2ia+2ibx}(id+1)+1}dx+\frac{1}{2}x^2\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \; 2615$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1+id)\int \frac{e^{2ia+2ibx}x^2}{e^{2ia+2ibx}(id+1)+1}dx\right)+\frac{1}{2}x^2\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \; 2620$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1+id)\left(\frac{x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{\int x\log\left(e^{2ia+2ibx}(id+1)+1\right)dx}{b(-d+i)}\right)\right)+$$
$$\frac{1}{2}x^2\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \; 3011$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1+id)\left(\frac{x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{\frac{ix\operatorname{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)}{2b}-\frac{i\int \operatorname{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)d}{2b}}{b(-d+i)}\right.\right.$$
$$\frac{1}{2}x^2\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1+id)\left(\frac{x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{\frac{ix\,\text{PolyLog}(2,-((id+1)e^{2ia+2ibx}))}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}(2,-((id+1)e^{2}}{4b^2}}{b(-d+i)}\right.\right.$$

$$\frac{1}{2}x^2\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}-(1+id)\left(\frac{x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{\frac{ix\,\text{PolyLog}(2,-((id+1)e^{2ia+2ibx}))}{2b}-\frac{\text{PolyLog}(3,-((id+1)e^{2ia+2ibx}))}{4b^2}}{b(-d+i)}\right)\right)$$

$$\frac{1}{2}x^2\coth^{-1}(d(-\tan(a+bx))+id+1)$$

input
```
Int[x*ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 + I*d - d*Tan[a + b*x]])/2 + (I/2)*b*(x^3/3 - (1 + I*d)*((x
^2*Log[1 + (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I - d)) - (((I/2)*x*P
olyLog[2, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 + I*d
)*E^((2*I)*a + (2*I)*b*x))]/(4*b^2))/(b*(I - d))))
```

3.245.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6818
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c + I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.245.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.16 (sec) , antiderivative size = 2285, normalized size of antiderivative = 17.05

method	result	size
risch	Expression too large to display	2285

input
```
int(x*arccoth(1+I*d-d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2
*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*
(b*x+a))+1))-I*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^3+2*I*Pi*c
sgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-I*Pi*csgn((exp(2*I*(b*x+a
))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+I*Pi*csgn(I*exp(2*I*(b*
x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a))
)^2-I*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(
b*x+a))+1)*exp(2*I*(b*x+a)))-I*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b
*x+a)))^3+I*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp
(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-I*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(
2*I*(b*x+a)))^2-I*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csg
n(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+I*Pi
*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1)
)^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I
*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+I*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(
2*I*(b*x+a)))^3-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a)))
*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))-I*Pi*csgn(I*exp(2*I*(b*x+a)
))^3-I*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a
))+1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1)
)^2-I*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+I*Pi*csgn((e...
```

3.245.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(94) = 188.

Time = 0.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.18

$$\int x \coth^{-1}(1 + id - d\tan(a + bx))\, dx$$
$$= \frac{2i\, b^3x^3 - 3\, b^2x^2 \log\left(\frac{de^{(2i\, bx+2i\, a)}}{(d-i)e^{(2i\, bx+2i\, a)}-i}\right) + 2i\, a^3 + 6i\, bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\, d-4}e^{(i\, bx+i\, a)}\right) + 6i\, bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\, d-\right.}{}$$

input
```
integrate(x*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*I*b^3*x^3 - 3*b^2*x^2*log(d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*
x + 2*I*a) - I)) + 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(-4*I*d - 4)*e^(I*b*x +
 I*a)) + 6*I*b*x*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 3*a^2*log(
1/2*(2*(d - I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(d - I)) - 3*a^2*log(
1/2*(2*(d - I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(d - I)) - 3*(b^2*x^2
 - a^2)*log(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 3*(b^2*x^2 - a^2)*
log(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 6*polylog(3, 1/2*sqrt(-4*
I*d - 4)*e^(I*b*x + I*a)) - 6*polylog(3, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x +
I*a)))/b^2
```

3.245.6 Sympy [F]

$$\int x \coth^{-1}(1+id-d\tan(a+bx))\,dx = \int x \operatorname{acoth}\left(-d\tan\left(a+bx\right)+id+1\right)dx$$

input
```
integrate(x*acoth(1+I*d-d*tan(b*x+a)),x)
```

output
```
Integral(x*acoth(-d*tan(a + b*x) + I*d + 1), x)
```

3.245.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(94) = 188$.

Time = 0.25 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.84

$$\int x \coth^{-1}(1+id-d\tan(a+bx))\,dx =$$

$$-\frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\operatorname{arcoth}(d\tan(bx+a)-i\,d-1)}{b}+\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\operatorname{Li}_2\left((-i\,d-1)e^{(2i\,bx+2i\,a)}\right)-6\left(-i\,(bx+a)^2+2i\,(\ldots}{\ldots}}{\ldots}$$

input
```
integrate(x*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arccoth(d*tan(b*x + a) - I*d - 1)/
b + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((-I*d - 1)*e^(2
*I*b*x + 2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(d*cos(2*b*
x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1) +
 3*((b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 +
 1)*sin(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) +
3*polylog(3, (-I*d - 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

3.245.8 Giac [F]

$$\int x \coth^{-1}(1 + id - d\tan(a + bx))\,dx = \int x\,\mathrm{arcoth}\left(-d\tan\left(bx + a\right) + i\,d + 1\right)\,dx$$

input
```
integrate(x*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(-d*tan(b*x + a) + I*d + 1), x)
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(1 + id - d\tan(a + bx))\,dx = \int x\,\mathrm{acoth}(1 - d\tan(a + b\,x) + d\,1\mathrm{i})\,dx$$

input
```
int(x*acoth(d*1i - d*tan(a + b*x) + 1),x)
```

output
```
int(x*acoth(d*1i - d*tan(a + b*x) + 1), x)
```

3.246 $\int \coth^{-1}(1+id-d\tan(a+bx))\,dx$

3.246.1 Optimal result

Integrand size = 17, antiderivative size = 94

$$\int \coth^{-1}(1+id-d\tan(a+bx))\,dx = \frac{1}{2}ibx^2+x\coth^{-1}(1+id-d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right) + \frac{i\operatorname{PolyLog}\left(2,-\left((1+id)e^{2ia+2ibx}\right)\right)}{4b}$$

output

```
1/2*I*b*x^2+x*arccoth(1+I*d-d*tan(b*x+a))-1/2*x*ln(1+(1+I*d)*exp(2*I*a+2*I
*b*x))+1/4*I*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b
```

3.246.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 723 vs. 2(94) = 188.

Time = 1.67 (sec) , antiderivative size = 723, normalized size of antiderivative = 7.69

$$\int \coth^{-1}(1+id-d\tan(a+bx))\,dx = x\coth^{-1}(1+id-d\tan(a+bx)) - \frac{x\left(-2ibx\log(2\cos(bx)(\cos(bx)-i\sin(bx)))+\log\left(\frac{\sec(bx)(\cos(a)-i\sin(a))((-2i+d)\cos(a+bx)+id\sin(a+bx))}{2(-i+d)}\right)\log(1\right.}{((-2i+d)\cos(a+bx)+id\sin(a+bx))\left(\frac{i\log(1-i\tan(bx))\sec(bx)(d\cos(a)+(2+id)\sin(a))}{(-2i+d)\cos(a+bx)+id\sin(a+bx)}+\frac{\log(1+i\tan(bx)}{(-2i+}\right.}$$

input
```
Integrate[ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

output
```
x*ArcCoth[1 + I*d - d*Tan[a + b*x]] - (x*((-2*I)*b*x*Log[2*Cos[b*x]*(Cos[b
*x] - I*Sin[b*x])] + Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-2*I + d)*Cos[a +
 b*x] + I*d*Sin[a + b*x]))/(2*(-I + d))]*Log[1 - I*Tan[b*x]] - Log[(Sec[b*
x]*((2 + I*d)*Cos[a + b*x] - d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*L
og[1 + I*Tan[b*x]] + PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] + PolyLog[2, (
Sec[b*x]*(d*Cos[a] + (2 + I*d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2
*(-I + d))] - PolyLog[2, ((Cos[a] + I*Sin[a])*(d*Cos[a] + (2 + I*d)*Sin[a]
)*(-I + Tan[b*x]))/2])*Sec[a + b*x]*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*x] +
Sin[b*x]))/(((-2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x])*((I*Log[1 - I*Tan
[b*x]]*Sec[b*x]*(d*Cos[a] + (2 + I*d)*Sin[a]))/((-2*I + d)*Cos[a + b*x] +
I*d*Sin[a + b*x]) + (Log[1 + I*Tan[b*x]]*Sec[b*x]*((-I)*d*Cos[a] + (-2*I +
 d)*Sin[a]))/((-2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]) - (Log[(Sec[b*x]
*((2 + I*d)*Cos[a + b*x] - d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec
[b*x]^2)/(-I + Tan[b*x]) + (Log[1 - ((Cos[a] + I*Sin[a])*(d*Cos[a] + (2 +
I*d)*Sin[a])*(-I + Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) - Log[1 - (Se
c[b*x]*(d*Cos[a] + (2 + I*d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*(
-I + d))]*(-I + Tan[b*x]) + (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-2*I + d)
*Cos[a + b*x] + I*d*Sin[a + b*x]))/(2*(-I + d))]*Sec[b*x]^2)/(I + Tan[b*x]
) + (2*I)*b*x*(I + Tan[b*x]))*(-I + Tan[a + b*x]))
```

3.246.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6810, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d(-\tan(a+bx))+id+1)\,dx$$

$$\Big\downarrow \text{6810}$$

$$ib\int \frac{x}{e^{2ia+2ibx}(id+1)+1}dx+x\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \text{2615}$$

$$ib\left(\frac{x^2}{2}-(1+id)\int \frac{e^{2ia+2ibx}x}{e^{2ia+2ibx}(id+1)+1}dx\right)+x\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \; 2620$$

$$ib\left(\frac{x^2}{2}-(1+id)\left(\frac{x\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{\int\log\left(e^{2ia+2ibx}(id+1)+1\right)dx}{2b(-d+i)}\right)\right)+ x\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \; 2715$$

$$ib\left(\frac{x^2}{2}-(1+id)\left(\frac{i\int e^{-2ia-2ibx}\log\left(e^{2ia+2ibx}(id+1)+1\right)de^{2ia+2ibx}}{4b^2(-d+i)}+\frac{x\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+ x\coth^{-1}(d(-\tan(a+bx))+id+1)$$

$$\Big\downarrow \; 2838$$

$$ib\left(\frac{x^2}{2}-(1+id)\left(\frac{x\log\left(1+(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}-\frac{i\,\text{PolyLog}\left(2,-\left((id+1)e^{2ia+2ibx}\right)\right)}{4b^2(-d+i)}\right)\right)+ x\coth^{-1}(d(-\tan(a+bx))+id+1)$$

input
```
Int[ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

output
```
x*ArcCoth[1 + I*d - d*Tan[a + b*x]] + I*b*(x^2/2 - (1 + I*d)*((x*Log[1 + (
1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(2*b*(I - d)) - ((I/4)*PolyLog[2, -((1
+ I*d)*E^((2*I)*a + (2*I)*b*x))])/(b^2*(I - d))))
```

3.246.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))    Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6810
```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Coth[c + d*Tan[a + b*x]], x] + Simp[I*b   Int[x/(c + I*d + c*E^(2*I*a + 2*I
*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, 1]
```

3.246.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(77) = 154.

Time = 2.50 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.40

method	result
derivativedivides	$-\frac{i \operatorname{arccoth}(1+id-d\tan(bx+a))d\ln(id-d\tan(bx+a))}{2}-\frac{i \operatorname{arccoth}(1+id-d\tan(bx+a))d\ln(-id-d\tan(bx+a))}{2}-\frac{d^2\left(-\frac{i\left(\frac{\ln(id-d\tan(}{4}\right.}{}\right.}{}$
default	$-\frac{i \operatorname{arccoth}(1+id-d\tan(bx+a))d\ln(id-d\tan(bx+a))}{2}-\frac{i \operatorname{arccoth}(1+id-d\tan(bx+a))d\ln(-id-d\tan(bx+a))}{2}-\frac{d^2\left(-\frac{i\left(\frac{\ln(id-d\tan(}{4}\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int(arccoth(1+I*d-d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/b/d*(1/2*I*arccoth(1+I*d-d*tan(b*x+a))*d*ln(I*d-d*tan(b*x+a))-1/2*I*arc
coth(1+I*d-d*tan(b*x+a))*d*ln(-I*d-d*tan(b*x+a))-1/2*d^2*(-I/d*(1/4*ln(I*d
-d*tan(b*x+a))^2-1/2*dilog(1+1/2*I*d-1/2*d*tan(b*x+a))-1/2*ln(I*d-d*tan(b*
x+a))*ln(1+1/2*I*d-1/2*d*tan(b*x+a)))+I/d*(1/2*dilog(-1/2*I*(I*d-d*tan(b*x
+a))/d)+1/2*ln(-I*d-d*tan(b*x+a))*ln(-1/2*I*(I*d-d*tan(b*x+a))/d)-1/2*dilo
g(I*(-I*d-d*tan(b*x+a)-I*(2*I-2*d))/(2*I-2*d))-1/2*ln(-I*d-d*tan(b*x+a))*l
n(I*(-I*d-d*tan(b*x+a)-I*(2*I-2*d))/(2*I-2*d)))))
```

3.246.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(66) = 132.

Time = 0.26 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.32

$$\int \coth^{-1}(1+id-d\tan(a+bx))\,dx$$
$$=\frac{i\,b^2x^2-bx\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right)-i\,a^2-(bx+a)\log\left(\frac{1}{2}\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}+1\right)-(bx+a)\log\left(-\frac{1}{2}\sqrt{-\cdots}\right.}{\cdots}$$

input
```
integrate(arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(I*b^2*x^2 - b*x*log(d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x + 2*I*a
) - I)) - I*a^2 - (b*x + a)*log(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1)
- (b*x + a)*log(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) + a*log(1/2*(2*
(d - I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(d - I)) + a*log(1/2*(2*(d -
 I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(d - I)) + I*dilog(1/2*sqrt(-4*I
*d - 4)*e^(I*b*x + I*a)) + I*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))
/b
```

3.246.6 Sympy [F]

$$\int \coth^{-1}(1+id-d\tan(a+bx))\,dx=\int \operatorname{acoth}\left(-d\tan\left(a+bx\right)+id+1\right)dx$$

input
```
integrate(acoth(1+I*d-d*tan(b*x+a)),x)
```

output
```
Integral(acoth(-d*tan(a + b*x) + I*d + 1), x)
```

3.246.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(66) = 132$.

Time = 0.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.77

$$\int \coth^{-1}(1+id-d\tan(a+bx))\,dx =$$

$$-\frac{4\,(bx+a)d\left(\frac{\log(d\tan(bx+a)-i\,d-2)}{d}-\frac{\log(\tan(bx+a)-i)}{d}\right)+d\left(-\frac{2i\left(\log(d\tan(bx+a)-i\,d-2)\log\left(-\frac{i\,d\tan(bx+a)+d-2i}{2\,(d-i)}+1\right)+\mathrm{Li}_2\right.}{d}\right.}{}$$

input
```
integrate(arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*(4*(b*x + a)*d*(log(d*tan(b*x + a) - I*d - 2)/d - log(tan(b*x + a) -
I)/d) + d*(-2*I*(log(d*tan(b*x + a) - I*d - 2)*log(-1/2*(I*d*tan(b*x + a)
+ d - 2*I)/(d - I) + 1) + dilog(1/2*(I*d*tan(b*x + a) + d - 2*I)/(d - I)))
/d + (2*I*log(d*tan(b*x + a) - I*d - 2)*log(tan(b*x + a) - I) - I*log(tan(
b*x + a) - I)^2)/d - 2*I*(log(-1/2*d*tan(b*x + a) + 1/2*I*d + 1)*log(tan(b
*x + a) - I) + dilog(1/2*d*tan(b*x + a) - 1/2*I*d))/d + 2*I*(log(tan(b*x +
 a) - I)*log(-1/2*I*tan(b*x + a) + 1/2) + dilog(1/2*I*tan(b*x + a) + 1/2))
/d) + 8*(b*x + a)*arccoth(d*tan(b*x + a) - I*d - 1))/b
```

3.246.8 Giac [F]

$$\int \coth^{-1}(1+id-d\tan(a+bx))\,dx = \int \operatorname{arcoth}\left(-d\tan\left(bx+a\right)+i\,d+1\right)\,dx$$

input
```
integrate(arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(-d*tan(b*x + a) + I*d + 1), x)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(1 + id - d\tan(a + bx))\, dx = \int \operatorname{acoth}(1 - d\tan(a + b\,x) + d\,1\mathrm{i})\; dx$$

input
```
int(acoth(d*1i - d*tan(a + b*x) + 1),x)
```

output
```
int(acoth(d*1i - d*tan(a + b*x) + 1), x)
```

3.247 $\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx$

3.247.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx = \text{Int}\left(\frac{\coth^{-1}(1+id-d\tan(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccoth(1+I*d-d*tan(b*x+a))/x,x)
```

3.247.2 Mathematica [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[1 + I*d - d*Tan[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[1 + I*d - d*Tan[a + b*x]]/x, x]
```

3.247.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d(-\tan(a+bx))+id+1)}{x} \, dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\coth^{-1}(d(-\tan(a+bx))+id+1)}{x}dx$$

input
```
Int[ArcCoth[1 + I*d - d*Tan[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.247.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.247.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arccoth}\left(1+id-d\tan\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(1+I*d-d*tan(b*x+a))/x,x)
```

output
```
int(arccoth(1+I*d-d*tan(b*x+a))/x,x)
```

3.247.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.71

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(-d\tan\left(bx+a\right)+i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+I*d-d*tan(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(-1/2*log(d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x + 2*I*a) - I))
/x, x)
```

3.247.6 Sympy [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx = \int \frac{\operatorname{acoth}\left(-d\tan\left(a+bx\right)+id+1\right)}{x}\,dx$$

input
```
integrate(acoth(1+I*d-d*tan(b*x+a))/x,x)
```

output
```
Integral(acoth(-d*tan(a + b*x) + I*d + 1)/x, x)
```

3.247.7 Maxima [N/A]

Not integrable

Time = 4.39 (sec) , antiderivative size = 141, normalized size of antiderivative = 6.71

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(-d\tan\left(bx+a\right)+i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+I*d-d*tan(b*x+a))/x,x, algorithm="maxima")
```

output
```
-I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(d))*log(x) + 1/2*I*integrate(arctan2(d
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*
a) + 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)
*sin(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1)/x, x)
```

3.247.8 Giac [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(-d\tan\left(bx+a\right)+i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+I*d-d*tan(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(-d*tan(b*x + a) + I*d + 1)/x, x)
```

3.247.9 Mupad [N/A]

Not integrable

Time = 4.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx=\int \frac{\operatorname{acoth}(1-d\tan(a+b\,x)+d\,1\mathrm{i})}{x}\,dx$$

input
```
int(acoth(d*1i - d*tan(a + b*x) + 1)/x,x)
```

output
```
int(acoth(d*1i - d*tan(a + b*x) + 1)/x, x)
```

3.248 $\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx$

3.248.1 Optimal result

Integrand size = 15, antiderivative size = 302

$$\begin{aligned}\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx &= \frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f} \\ &+ \frac{i(e+fx)^4 \arctan\left(e^{2i(a+bx)}\right)}{4f} \\ &- \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} \\ &+ \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b} \\ &+ \frac{3f(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2} \\ &- \frac{3f(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2} \\ &+ \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3} \\ &- \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3} \\ &- \frac{3f^3 \operatorname{PolyLog}\left(5,-ie^{2i(a+bx)}\right)}{16b^4} \\ &+ \frac{3f^3 \operatorname{PolyLog}\left(5,ie^{2i(a+bx)}\right)}{16b^4}\end{aligned}$$

output
```
1/4*(f*x+e)^4*arccoth(cot(b*x+a))/f+1/4*I*(f*x+e)^4*arctan(exp(2*I*(b*x+a)
))/f-1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^3*poly
log(2,I*exp(2*I*(b*x+a)))/b+3/8*f*(f*x+e)^2*polylog(3,-I*exp(2*I*(b*x+a)))
/b^2-3/8*f*(f*x+e)^2*polylog(3,I*exp(2*I*(b*x+a)))/b^2+3/8*I*f^2*(f*x+e)*p
olylog(4,-I*exp(2*I*(b*x+a)))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*I*(b
*x+a)))/b^3-3/16*f^3*polylog(5,-I*exp(2*I*(b*x+a)))/b^4+3/16*f^3*polylog(5
,I*exp(2*I*(b*x+a)))/b^4
```

3.248.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 654 vs. $2(302) = 604$.

Time = 0.17 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.17

$$\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx = \frac{1}{4}x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\coth^{-1}(\cot(a+bx)) + \frac{-8b^4e^3x\log\left(1-ie^{2i(a+bx)}\right)-12b^4e^2fx^2\log\left(1-ie^{2i(a+bx)}\right)-8b^4ef^2x^3\log\left(1-ie^{2i(a+bx)}\right)-2b^4f^3x^4\text{lo}}{}$$

input
```
Integrate[(e + f*x)^3*ArcCoth[Cot[a + b*x]],x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcCoth[Cot[a + b*x]])/4 +
(-8*b^4*e^3*x*Log[1 - I*E^((2*I)*(a + b*x))] - 12*b^4*e^2*f*x^2*Log[1 - I*
E^((2*I)*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] - 2*
b^4*f^3*x^4*Log[1 - I*E^((2*I)*(a + b*x))] + 8*b^4*e^3*x*Log[1 + I*E^((2*I
)*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 8*b^4*e*
f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] + 2*b^4*f^3*x^4*Log[1 + I*E^((2*I)*
(a + b*x))] - (4*I)*b^3*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))] +
 (4*I)*b^3*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))] + 6*b^2*e^2*f*Pol
yLog[3, (-I)*E^((2*I)*(a + b*x))] + 12*b^2*e*f^2*x*PolyLog[3, (-I)*E^((2*I
)*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b^2
*e^2*f*PolyLog[3, I*E^((2*I)*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3, I*E^(
(2*I)*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^((2*I)*(a + b*x))] + (6*I
)*b*e*f^2*PolyLog[4, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b*f^3*x*PolyLog[4,
(-I)*E^((2*I)*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))
] - (6*I)*b*f^3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I
)*E^((2*I)*(a + b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

3.248.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6808, 3042, 4669, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx$$

$$\Big\downarrow \text{6808}$$

$$\frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f}-\frac{b\int (e+fx)^4 \sec(2a+2bx)dx}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f}-\frac{b\int (e+fx)^4 \csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{4f}$$

$$\Big\downarrow \text{4669}$$

$$\frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f}-\frac{b\left(-\frac{2f\int (e+fx)^3 \log\left(1-ie^{2i(a+bx)}\right)dx}{b}+\frac{2f\int (e+fx)^3 \log\left(1+ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^4 \arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{4f}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f}-\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3 \text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\int (e+fx)^2 \text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{2f\left(\frac{i(e+fx)^3 \text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{3if\int (e+fx)^2 \text{PolyLog}\left(2\right.}{2b}\right.}{b}\right.}{4f}$$

$$\Big\downarrow \text{7163}$$

$$\frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f}-\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3 \text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\int (e+fx) \text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^2 \text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{2b}\right)}{b}-\frac{2f\left(\frac{i(e+fx)^3 \text{PolyLo}}{2}\right.}{}\right.}{4f}$$

$\downarrow$ 7163

$$\frac{(e+fx)^4\coth^{-1}(\cot(a+bx))}{4f}-\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\left(\frac{if\int\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)dx}{2b}-\frac{i(e+fx)\,\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}-\frac{i(e+fx)^2\,\text{PolyLog}\left(3,-ie^{2i(a+b\cdots}{2b}\cdots\right)}{2b}\cdots\right)}{b}\cdots\right)}{\cdots}$$

$\downarrow$ 2720

$$\frac{(e+fx)^4\coth^{-1}(\cot(a+bx))}{4f}-\frac{b\left(\frac{2f\left(\frac{i(e+fx)^3\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{3if\left(\frac{if\left(\frac{f\int e^{-2i(a+bx)}\,\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(e+fx)\,\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}-\frac{i(e+fx)^2\,\text{P}\cdots}{\cdots}\cdots\right)}{2b}\cdots\right)}{b}\cdots\right)}{\cdots}$$

$\downarrow$ 7143

3.248. $\int(e+fx)^3\coth^{-1}(\cot(a+bx))\,dx$

$$\frac{(e+fx)^4\coth^{-1}(\cot(a+bx))}{4f} - \frac{b\left(-\frac{i(e+fx)^4\arctan\left(e^{2i(a+bx)}\right)}{b} + \frac{2f\left(\frac{i(e+fx)^3\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b} - \frac{3if\left(\frac{if\left(\frac{f\operatorname{PolyLog}\left(5,-ie^{2i(a+bx)}\right)}{4b^2} - \frac{i(e+fx)\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right.}{2b}\right.}{b}\right.}{}$$

input
```
Int[(e + f*x)^3*ArcCoth[Cot[a + b*x]],x]
```

output
```
((e + f*x)^4*ArcCoth[Cot[a + b*x]])/(4*f) - (b*(((-I)*(e + f*x)^4*ArcTan[E
^((2*I)*(a + b*x))])/b + (2*f*(((I/2)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)
*(a + b*x))])/b - (((3*I)/2)*f*(((-1/2*I)*(e + f*x)^2*PolyLog[3, (-I)*E^((
2*I)*(a + b*x))])/b + (I*f*(((-1/2*I)*(e + f*x)*PolyLog[4, (-I)*E^((2*I)*(
a + b*x))])/b + (f*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b))
/b - (2*f*(((I/2)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (((3*
I)/2)*f*(((-1/2*I)*(e + f*x)^2*PolyLog[3, I*E^((2*I)*(a + b*x))])/b + (I*f
*(((-1/2*I)*(e + f*x)*PolyLog[4, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[5,
 I*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b))/b))/(4*f)
```

3.248.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6808
```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Cot[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.248. $\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx$

3.248.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 27.66 (sec) , antiderivative size = 3640, normalized size of antiderivative = 12.05

method	result	size
risch	Expression too large to display	3640

input
```
int((f*x+e)^3*arccoth(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
3/2*f^2/b^2*e*a^2*ln(-I*exp(2*I*(b*x+a))+1)*x+1/4*I*f^3/b*polylog(2,I*exp(
2*I*(b*x+a)))*x^3+1/4*I*f^3/b^4*polylog(2,I*exp(2*I*(b*x+a)))*a^3-3/8*I*f^
3/b^3*polylog(4,I*exp(2*I*(b*x+a)))*x+3/2*f^2/b^3*a^3*e*ln(1-exp(I*(b*x+a)
)*(-1)^(3/4))-3/2*f/b^2*a^2*e^2*ln(1+exp(I*(b*x+a))*(-1)^(3/4))-3/16*f^3*p
olylog(5,-I*exp(2*I*(b*x+a)))/b^4+3/16*f^3*polylog(5,I*exp(2*I*(b*x+a)))/b
^4+3/4*f*e^2*ln(1+I*exp(2*I*(b*x+a)))*x^2+1/2*f^2*e*ln(1+I*exp(2*I*(b*x+a)
))*x^3+1/8*f^3/b^4*a^4*ln(-exp(2*I*(b*x+a))+I)-1/2*f^3/b^3*a^3*ln(1+exp(I*
(b*x+a))*(-1)^(3/4))*x-1/2*f^3/b^3*a^3*ln(1-exp(I*(b*x+a))*(-1)^(3/4))*x-f
^2/b^3*e*ln(1+I*exp(2*I*(b*x+a)))*a^3+1/2*f^3/b^3*ln(1+I*exp(2*I*(b*x+a)))
*x*a^3-1/2*f^2/b^3*a^3*e*ln(-exp(2*I*(b*x+a))+I)+3/4*f/b^2*a^2*e^2*ln(-exp
(2*I*(b*x+a))+I)-3/4*f^2/b^2*e*polylog(3,I*exp(2*I*(b*x+a)))*x-1/8/f*e^4*l
n(exp(2*I*(b*x+a))+I)-1/2*e^3*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))*x
-1/2*e^3*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))*x+1/8*(f*x+e)^4/f*ln(e
xp(2*I*(b*x+a))+I)+1/2*e^3*ln(1+exp(I*(b*x+a))*(-1)^(3/4))*x+1/2*e^3*ln(1-
exp(I*(b*x+a))*(-1)^(3/4))*x+1/8/f*e^4*ln(-exp(2*I*(b*x+a))+I)+1/8*f^3*ln(
1+I*exp(2*I*(b*x+a)))*x^4-3/4*I*f^2/b*e*polylog(2,-I*exp(2*I*(b*x+a)))*x^2
+3/4*I*f^2/b^3*e*polylog(2,-I*exp(2*I*(b*x+a)))*a^2-1/2*I*f^3/b^4*a^3*dilo
g(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I*f^3/b^4*a^3*dilog(((-I)^(1
/2)+exp(I*(b*x+a)))/(-I)^(1/2))+3/2*f/b*e^2*a*ln(((-I)^(1/2)-exp(I*(b*x+a)
))/(-I)^(1/2))*x+3/2*f/b*e^2*a*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2...
```

3.248.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1566 vs. 2(236) = 472.

Time = 0.35 (sec) , antiderivative size = 1566, normalized size of antiderivative = 5.19

$$\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*arccoth(cot(b*x+a)),x, algorithm="fricas")
```

output
```
-1/32*(3*f^3*polylog(5, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*f^3*pol
ylog(5, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 3*f^3*polylog(5, -I*cos(2
*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*f^3*polylog(5, -I*cos(2*b*x + 2*a) - s
in(2*b*x + 2*a)) + 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f*x
 - I*b^3*e^3)*dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 4*(-I*b^3*f^3
*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(I*cos(2*b*x
+ 2*a) - sin(2*b*x + 2*a)) + 4*(I*b^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^
3*e^2*f*x + I*b^3*e^3)*dilog(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 4*(
I*b^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(-I*
cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 4*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 +
6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*log((cos(2*b*x + 2*a) + sin(2*b*x + 2*a) +
1)/(cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1)) - 2*(4*a*b^3*e^3 - 6*a^2*b^2
*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a
) + I) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(c
os(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x
^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3
*b*e*f^2 - a^4*f^3)*log(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) - 2*(b^
4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3
- 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(I*cos(2*b*x + 2*a) - sin(
2*b*x + 2*a) + 1) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 ...
```

3.248.6 Sympy [F]

$$\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\, dx = \int (e+fx)^3 \operatorname{acoth}(\cot(a+bx))\, dx$$

input
```
integrate((f*x+e)**3*acoth(cot(b*x+a)),x)
```

output
```
Integral((e + f*x)**3*acoth(cot(a + b*x)), x)
```

3.248.7 Maxima [F]

$$\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx = \int (fx+e)^3 \operatorname{arcoth}(\cot(bx+a))\,dx$$

input
```
integrate((f*x+e)^3*arccoth(cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/16*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a
)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*e
*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x +
 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*e*f^2*
x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^
3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*sin(4*b*x + 4*a)*sin(2*
b*x + 2*a) + (b*f^3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(2
*b*x + 2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)
 + 1), x)
```

3.248.8 Giac [F]

$$\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx = \int (fx+e)^3 \operatorname{arcoth}(\cot(bx+a))\,dx$$

input
```
integrate((f*x+e)^3*arccoth(cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*arccoth(cot(b*x + a)), x)
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx = \int \operatorname{acoth}(\cot(a+b\,x))\,(e+f\,x)^3\,dx$$

input
```
int(acoth(cot(a + b*x))*(e + f*x)^3,x)
```

output
```
int(acoth(cot(a + b*x))*(e + f*x)^3, x)
```

3.249 $\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx$

3.249.1 Optimal result

Integrand size = 15, antiderivative size = 234

$$\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx = \frac{(e+fx)^3 \coth^{-1}(\cot(a+bx))}{3f} + \frac{i(e+fx)^3 \arctan\left(e^{2i(a+bx)}\right)}{3f} - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b} + \frac{f(e+fx) \operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2} - \frac{f(e+fx) \operatorname{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{4b^2} + \frac{if^2 \operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3} - \frac{if^2 \operatorname{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}$$

output
```
1/3*(f*x+e)^3*arccoth(cot(b*x+a))/f+1/3*I*(f*x+e)^3*arctan(exp(2*I*(b*x+a)
))/f-1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^2*poly
log(2,I*exp(2*I*(b*x+a)))/b+1/4*f*(f*x+e)*polylog(3,-I*exp(2*I*(b*x+a)))/b
^2-1/4*f*(f*x+e)*polylog(3,I*exp(2*I*(b*x+a)))/b^2+1/8*I*f^2*polylog(4,-I*
exp(2*I*(b*x+a)))/b^3-1/8*I*f^2*polylog(4,I*exp(2*I*(b*x+a)))/b^3
```

3.249.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.75

$$\int (e+fx)^2\coth^{-1}(\cot(a+bx))\,dx = \frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\coth^{-1}(\cot(a+bx)) + \frac{-12b^3e^2x\log\left(1-ie^{2i(a+bx)}\right)-12b^3efx^2\log\left(1-ie^{2i(a+bx)}\right)-4b^3f^2x^3\log\left(1-ie^{2i(a+bx)}\right)+12b^3e^2x\log}{}$$

input
```
Integrate[(e + f*x)^2*ArcCoth[Cot[a + b*x]],x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCoth[Cot[a + b*x]])/3 + (-12*b^3*e^2*x*L
og[1 - I*E^((2*I)*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 - I*E^((2*I)*(a + b*x
))] - 4*b^3*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] + 12*b^3*e^2*x*Log[1 +
I*E^((2*I)*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 4
*b^3*f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] - (6*I)*b^2*(e + f*x)^2*PolyLo
g[2, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b^2*(e + f*x)^2*PolyLog[2, I*E^((2*
I)*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] + 6*b*f^2*x*
PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b*e*f*PolyLog[3, I*E^((2*I)*(a +
b*x))] - 6*b*f^2*x*PolyLog[3, I*E^((2*I)*(a + b*x))] + (3*I)*f^2*PolyLog[4
, (-I)*E^((2*I)*(a + b*x))] - (3*I)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])
/(24*b^3)
```

3.249.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6808, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2\coth^{-1}(\cot(a+bx))\,dx$$

$$\Big\downarrow \text{ 6808}$$

$$\frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f}-\frac{b\int (e+fx)^3\sec(2a+2bx)dx}{3f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f}-\frac{b\int(e+fx)^3\csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{3f}$$

$$\Big\downarrow \text{4669}$$

$$\frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f}-\frac{b\left(-\frac{3f\int(e+fx)^2\log\left(1-ie^{2i(a+bx)}\right)dx}{2b}+\frac{3f\int(e+fx)^2\log\left(1+ie^{2i(a+bx)}\right)dx}{2b}-\frac{i(e+fx)^3\arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{3f}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f}-\frac{b\left(\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\int(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{if\int(e+fx)\text{PolyLog}\left(2,ie^{2i\cdots}\right.}{b}\right.}{2b}\right.}{3f}$$

$$\Big\downarrow \text{7163}$$

$$\frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f}-\frac{b\left(\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\left(\frac{if\int\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)dx}{2b}-\frac{i(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,ie^{2i(a\cdots}\right.}{2b}\right.}{}\right.}{3f}$$

$$\Big\downarrow \text{2720}$$

$$\frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f}-\frac{b\left(\frac{3f\left(\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\left(\frac{f\int e^{-2i(a+bx)}\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{3f\left(\frac{i(e+f\cdots}{}\right.}{}\right.}{3\cdots}$$

$$\Big\downarrow \text{7143}$$

3.249. $\int(e+fx)^2\coth^{-1}(\cot(a+bx))\,dx$

$$\frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f} - \frac{b\left(-\frac{i(e+fx)^3\arctan\left(e^{2i(a+bx)}\right)}{b} + \frac{3f\left(\frac{i(e+fx)^2\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b} - \frac{if\left(\frac{f\operatorname{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{4b^2} - \frac{i(e+fx)\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b} - \right.}{3f}$$

input
```
Int[(e + f*x)^2*ArcCoth[Cot[a + b*x]],x]
```

output
```
((e + f*x)^3*ArcCoth[Cot[a + b*x]])/(3*f) - (b*(((-I)*(e + f*x)^3*ArcTan[E
^((2*I)*(a + b*x))])/b + (3*f*(((I/2)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)
*(a + b*x))])/b - (I*f*(((-1/2*I)*(e + f*x)*PolyLog[3, (-I)*E^((2*I)*(a +
b*x))])/b + (f*PolyLog[4, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b))/(2*b) -
 (3*f*(((I/2)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (I*f*(((-
1/2*I)*(e + f*x)*PolyLog[3, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[4, I*E^
((2*I)*(a + b*x))])/(4*b^2)))/b))/(2*b)))/(3*f)
```

3.249.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6808
```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Cot[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.249.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 22.22 (sec) , antiderivative size = 2719, normalized size of antiderivative = 11.62

method	result	size
risch	Expression too large to display	2719

input
```
int((f*x+e)^2*arccoth(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^2/b^3*ln(1+I*exp(2*I*(b*x+a)))*a^3+1/4*f^2/b^2*polylog(3,-I*exp(2*I
*(b*x+a)))*x+1/2*f*e*ln(1+I*exp(2*I*(b*x+a)))*x^2+1/2*f^2/b^3*a^3*ln(1+exp
(I*(b*x+a))*(-1)^(3/4))+1/2*f^2/b^3*a^3*ln(1-exp(I*(b*x+a))*(-1)^(3/4))-1/
6*f^2/b^3*a^3*ln(-exp(2*I*(b*x+a))+I)+1/4*f*e/b^2*polylog(3,-I*exp(2*I*(b*
x+a)))-1/8*I*f^2*polylog(4,I*exp(2*I*(b*x+a)))/b^3+1/8*I*f^2*polylog(4,-I*
exp(2*I*(b*x+a)))/b^3-1/12*I*Pi*(csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x
+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))+csgn((1-I)*
(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-csgn(I*(exp(2*I*(b*x+a))-I))*
csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))
-1))+csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*
x+a))-1))^2+csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn
(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))-csgn(I*(exp(2*I*(b*x+a))+I))
*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+csgn(I/(exp(2*I*(b*x+
a))-1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2-csgn(I/(exp(2*
I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-csgn(I*
(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^3-csgn(I*(exp(2*I*(b*x+a))-I)/(
exp(2*I*(b*x+a))-1))*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))
+csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*csgn((1+I)*(exp(2*I*(b*
x+a))-I)/(exp(2*I*(b*x+a))-1))^2+csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x
+a))-1))^3-csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)...
```

3.249.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1084 vs. 2(180) = 360.

Time = 0.32 (sec) , antiderivative size = 1084, normalized size of antiderivative = 4.63

$$\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*arccoth(cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/48*(-3*I*f^2*polylog(4, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*I*f^2
*polylog(4, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I*cos(2*b*x +
2*a) - sin(2*b*x + 2*a)) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)*
dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2
*e*f*x - I*b^2*e^2)*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 6*(I*b^
2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-I*cos(2*b*x + 2*a) + sin(2*b
*x + 2*a)) - 6*(I*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-I*cos(2*
b*x + 2*a) - sin(2*b*x + 2*a)) + 8*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^
2*x)*log((cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a) - sin
(2*b*x + 2*a) + 1)) + 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(cos(2*b*
x + 2*a) + I*sin(2*b*x + 2*a) + I) - 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^
2)*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - 4*(b^3*f^2*x^3 + 3*b^3
*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(I*cos(2*
b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^
3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(I*cos(2*b*x + 2*a) - si
n(2*b*x + 2*a) + 1) - 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b
^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)
 + 1) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2
*b*e*f + a^3*f^2)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) + 4*(...
```

3.249.6 Sympy [F]

$$\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx = \int (e+fx)^2 \operatorname{acoth}(\cot(a+bx))\,dx$$

input
```
integrate((f*x+e)**2*acoth(cot(b*x+a)),x)
```

output
```
Integral((e + f*x)**2*acoth(cot(a + b*x)), x)
```

3.249.7 Maxima [F]

$$\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx = \int (fx+e)^2 \operatorname{arcoth}(\cot(bx+a))\,dx$$

input
```
integrate((f*x+e)^2*arccoth(cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*
x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x
)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2
) - integrate(2/3*((b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(4*b*x + 4*a)*
cos(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(2*b*x + 2*a))
/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

3.249.8 Giac [F]

$$\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx = \int (fx+e)^2 \operatorname{arcoth}(\cot(bx+a))\,dx$$

input
```
integrate((f*x+e)^2*arccoth(cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*arccoth(cot(b*x + a)), x)
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx = \int \operatorname{acoth}(\cot(a+b\,x))\,(e+f\,x)^2\,dx$$

input
```
int(acoth(cot(a + b*x))*(e + f*x)^2,x)
```

output
```
int(acoth(cot(a + b*x))*(e + f*x)^2, x)
```

3.250 $\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx$

3.250.1 Optimal result

Integrand size = 13, antiderivative size = 162

$$\begin{aligned}\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx = {} & \frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f} \\ & + \frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{2f} \\ & - \frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} \\ & + \frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b} \\ & + \frac{f\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2} - \frac{f\operatorname{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2}\end{aligned}$$

output
```
1/2*(f*x+e)^2*arccoth(cot(b*x+a))/f+1/2*I*(f*x+e)^2*arctan(exp(2*I*(b*x+a)
))/f-1/4*I*(f*x+e)*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)*polylog(
2,I*exp(2*I*(b*x+a)))/b+1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*pol
ylog(3,I*exp(2*I*(b*x+a)))/b^2
```

3.250.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.82

$$\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx = ex\coth^{-1}(\cot(a+bx))+\frac{1}{2}fx^2\coth^{-1}(\cot(a+bx))$$
$$-\frac{e\left((-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+\right.}{8b}$$
$$+\frac{f(4ib^2x^2\arctan(\cos(2(a+bx))+i\sin(2(a+bx)))+2ibx\operatorname{PolyLog}(2,i\cos(2(a+bx))-\sin(2(a+bx))}{}$$

input
```
Integrate[(e + f*x)*ArcCoth[Cot[a + b*x]],x]
```

output
```
e*x*ArcCoth[Cot[a + b*x]] + (f*x^2*ArcCoth[Cot[a + b*x]])/2 - (e*((-4*a +
Pi - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*x))] - Log[1 + I/E^((2*I)*(a + b*x)
)]) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*x]] + (2*I)*(PolyLog[2, (-I)/E^((2*
I)*(a + b*x))] - PolyLog[2, I/E^((2*I)*(a + b*x))])))/(8*b) + (f*((4*I)*b^
2*x^2*ArcTan[Cos[2*(a + b*x)] + I*Sin[2*(a + b*x)]] + (2*I)*b*x*PolyLog[2,
 I*Cos[2*(a + b*x)] - Sin[2*(a + b*x)]] - (2*I)*b*x*PolyLog[2, (-I)*Cos[2*
(a + b*x)] + Sin[2*(a + b*x)]] - PolyLog[3, I*Cos[2*(a + b*x)] - Sin[2*(a
+ b*x)]] + PolyLog[3, (-I)*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]]))/(8*b^2)
```

3.250.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {6808, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx$$
$$\Big\downarrow \text{6808}$$
$$\frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f}-\frac{b\int (e+fx)^2\sec(2a+2bx)dx}{2f}$$
$$\Big\downarrow \text{3042}$$
$$\frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f}-\frac{b\int (e+fx)^2\csc\left(2a+2bx+\frac{\pi}{2}\right)dx}{2f}$$

$$\Big\downarrow \text{4669}$$

$$\frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f}-\frac{b\left(-\frac{f\int (e+fx)\log\left(1-ie^{2i(a+bx)}\right)dx}{b}+\frac{f\int (e+fx)\log\left(1+ie^{2i(a+bx)}\right)dx}{b}-\frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{b}\right)}{2f}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f}-\frac{b\left(\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{if\int \operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{if\int \operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{i(\ldots}{}\right.}{2f}$$

$$\Big\downarrow \text{2720}$$

$$\frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f}-\frac{b\left(\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{f\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{f\int e^{-2i(a+bx)}\operatorname{Po}\ldots}{}\right.}{b}\right.}{2f}$$

$$\Big\downarrow \text{7143}$$

$$\frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f}-\frac{b\left(-\frac{i(e+fx)^2\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{2b}-\frac{f\operatorname{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2}\right)}{b}-\frac{f\left(\frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{2b}-\frac{f\operatorname{Poly}\ldots}{}\right.}{b}\right.}{2f}$$

input
```
Int[(e + f*x)*ArcCoth[Cot[a + b*x]],x]
```

output
```
((e + f*x)^2*ArcCoth[Cot[a + b*x]])/(2*f) - (b*(((-I)*(e + f*x)^2*ArcTan[E
^((2*I)*(a + b*x))])/b + (f*(((I/2)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a
+ b*x))])/b - (f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2)))/b - (f*((
(I/2)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b - (f*PolyLog[3, I*E^(
(2*I)*(a + b*x))])/(4*b^2)))/b))/(2*f)
```

3.250. $\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx$

3.250.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6808
```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Cot[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.250.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.09 (sec) , antiderivative size = 1818, normalized size of antiderivative = 11.22

method	result	size
risch	Expression too large to display	1818

input
```
int((f*x+e)*arccoth(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
(1/4*f*x^2+1/2*e*x)*ln(exp(2*I*(b*x+a))+I)+1/2*e*ln(1+exp(I*(b*x+a))*(-1)^
(3/4))*x+1/2*e*ln(1-exp(I*(b*x+a))*(-1)^(3/4))*x-1/2*e/b*a*ln(-exp(2*I*(b*
x+a))+I)+1/2*e/b*ln(1+exp(I*(b*x+a))*(-1)^(3/4))*a+1/2*e/b*ln(1-exp(I*(b*x
+a))*(-1)^(3/4))*a-1/2*I*e/b*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I*e/b*
dilog(1-exp(I*(b*x+a))*(-1)^(3/4))+1/2*f/b*ln(1+I*exp(2*I*(b*x+a)))*a*x-1/
4*I*f/b*polylog(2,-I*exp(2*I*(b*x+a)))*x-1/4*I*f/b^2*polylog(2,-I*exp(2*I*
(b*x+a)))*a-1/2*e*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))*x-1/2*e*ln(((
-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))*x-1/2/b*f*a*ln(1+exp(I*(b*x+a))*(-1)
^(3/4))*x-1/2/b*f*a*ln(1-exp(I*(b*x+a))*(-1)^(3/4))*x+1/2*I/b^2*f*a*dilog(
1+exp(I*(b*x+a))*(-1)^(3/4))+1/2*I/b^2*f*a*dilog(1-exp(I*(b*x+a))*(-1)^(3/
4))+1/2/b*f*a*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))*x-1/4*f/b^2*ln(-I
*exp(2*I*(b*x+a))+1)*a^2+1/4/b^2*f*a^2*ln(-exp(2*I*(b*x+a))+I)-1/2*e/b*ln(
((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))*a-1/2*e/b*ln(((-I)^(1/2)+exp(I*(b*
x+a)))/(-I)^(1/2))*a+1/2*I*e/b*dilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2
))+1/2*I*e/b*dilog(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/2/b^2*f*a^2*l
n(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2/b^2*f*a^2*ln(((-I)^(1/2)+exp
(I*(b*x+a)))/(-I)^(1/2))+1/4*f/b^2*ln(1+I*exp(2*I*(b*x+a)))*a^2+1/8*f*poly
log(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylog(3,I*exp(2*I*(b*x+a)))/b^2-1/4
*I*Pi*(csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2
*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))+csgn((1-I)*(exp(2*I*(b*x+a))+I)/(e...
```

3.250.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 680 vs. 2(130) = 260.

Time = 0.32 (sec) , antiderivative size = 680, normalized size of antiderivative = 4.20

$$\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx =$$

$$-\frac{2\left(-i\,bfx-i\,be\right)\mathrm{Li}_2(i\,\cos\left(2\,bx+2\,a\right)+\sin\left(2\,bx+2\,a\right))+2\left(-i\,bfx-i\,be\right)\mathrm{Li}_2(i\,\cos\left(2\,bx+2\,a\right)-\sin\left(\right.}{}$$

input
```
integrate((f*x+e)*arccoth(cot(b*x+a)),x, algorithm="fricas")
```

output
```
-1/16*(2*(-I*b*f*x - I*b*e)*dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) +
 2*(-I*b*f*x - I*b*e)*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 2*(I*
b*f*x + I*b*e)*dilog(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 2*(I*b*f*x
+ I*b*e)*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 4*(b^2*f*x^2 + 2*
b^2*e*x)*log((cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a) -
 sin(2*b*x + 2*a) + 1)) - 2*(2*a*b*e - a^2*f)*log(cos(2*b*x + 2*a) + I*sin
(2*b*x + 2*a) + I) + 2*(2*a*b*e - a^2*f)*log(cos(2*b*x + 2*a) - I*sin(2*b*
x + 2*a) + I) + 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(I*cos(2*b*
x + 2*a) + sin(2*b*x + 2*a) + 1) - 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^
2*f)*log(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) + 2*(b^2*f*x^2 + 2*b^2
*e*x + 2*a*b*e - a^2*f)*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) -
2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(-I*cos(2*b*x + 2*a) - sin(
2*b*x + 2*a) + 1) - 2*(2*a*b*e - a^2*f)*log(-cos(2*b*x + 2*a) + I*sin(2*b*
x + 2*a) + I) + 2*(2*a*b*e - a^2*f)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x +
2*a) + I) - f*polylog(3, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + f*polylo
g(3, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - f*polylog(3, -I*cos(2*b*x +
2*a) + sin(2*b*x + 2*a)) + f*polylog(3, -I*cos(2*b*x + 2*a) - sin(2*b*x +
2*a)))/b^2
```

3.250.6 Sympy [F]

$$\int (e + fx) \coth^{-1}(\cot(a + bx))\, dx = \int (e + fx)\, \operatorname{acoth}\left(\cot\left(a + bx\right)\right) dx$$

input
```
integrate((f*x+e)*acoth(cot(b*x+a)),x)
```

output
```
Integral((e + f*x)*acoth(cot(a + b*x)), x)
```

3.250.7 Maxima [F]

$$\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx = \int (fx+e)\,\mathrm{arcoth}\left(\cot\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)*arccoth(cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*si
n(2*b*x + 2*a) + 2) - 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin
(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(((b*f*x^2 + 2*b*e*x)
*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)
^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

3.250.8 Giac [F]

$$\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx = \int (fx+e)\,\mathrm{arcoth}\left(\cot\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)*arccoth(cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*arccoth(cot(b*x + a)), x)
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx = \int \mathrm{acoth}(\cot(a+b\,x))\,\left(e+f\,x\right)\,dx$$

input
```
int(acoth(cot(a + b*x))*(e + f*x),x)
```

output
```
int(acoth(cot(a + b*x))*(e + f*x), x)
```

3.251 $\int \coth^{-1}(\cot(a+bx))\,dx$

3.251.1 Optimal result

Integrand size = 7, antiderivative size = 79

$$\int \coth^{-1}(\cot(a+bx))\,dx = x\coth^{-1}(\cot(a+bx)) + ix\arctan\left(e^{2i(a+bx)}\right) - \frac{i\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b} + \frac{i\,\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}$$

output
```
x*arccoth(cot(b*x+a))+I*x*arctan(exp(2*I*(b*x+a)))-1/4*I*polylog(2,-I*exp(
2*I*(b*x+a)))/b+1/4*I*polylog(2,I*exp(2*I*(b*x+a)))/b
```

3.251.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.61

$$\int \coth^{-1}(\cot(a+bx))\,dx = x\coth^{-1}(\cot(a+bx)) - \frac{(-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+2a}{8b}$$

input
```
Integrate[ArcCoth[Cot[a + b*x]],x]
```

output
```
x*ArcCoth[Cot[a + b*x]] - ((-4*a + Pi - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*
x))] - Log[1 + I/E^((2*I)*(a + b*x))]) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*
x]] + (2*I)*(PolyLog[2, (-I)/E^((2*I)*(a + b*x))] - PolyLog[2, I/E^((2*I)*
(a + b*x))]))/(8*b)
```

3.251.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6804, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(\cot(a+bx))\,dx$$

$$\Big\downarrow \text{6804}$$

$$x\coth^{-1}(\cot(a+bx))-b\int x\sec(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$x\coth^{-1}(\cot(a+bx))-b\int x\csc\left(2a+2bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4669}$$

$$x\coth^{-1}(\cot(a+bx))-b\left(-\frac{\int \log\left(1-ie^{2i(a+bx)}\right)dx}{2b}+\frac{\int \log\left(1+ie^{2i(a+bx)}\right)dx}{2b}-\frac{ix\arctan\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\coth^{-1}(\cot(a+bx))-b\left(\frac{i\int e^{-2i(a+bx)}\log\left(1-ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i\int e^{-2i(a+bx)}\log\left(1+ie^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{ix\arctan\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\coth^{-1}(\cot(a+bx))-b\left(-\frac{ix\arctan\left(e^{2i(a+bx)}\right)}{b}+\frac{i\,\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{i\,\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b^2}\right)$$

input
```
Int[ArcCoth[Cot[a + b*x]],x]
```

output
```
x*ArcCoth[Cot[a + b*x]] - b*(((-I)*x*ArcTan[E^((2*I)*(a + b*x))])/b + ((I/
4)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])/b^2 - ((I/4)*PolyLog[2, I*E^((2*I
)*(a + b*x))])/b^2)
```

3.251.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 6804
```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcCoth[Cot[a + b
*x]], x] - Simp[b   Int[x*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

3.251.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(64) = 128$.

Time = 1.43 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.38

method	result
derivativedivides	$\frac{-\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arccoth}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\ln\left(1+\frac{i(1+i\cot(bx+a))^2}{\cot(bx+a)^2+1}\right)}{2}+\frac{\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)}{}}{b}$
default	$\frac{-\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arccoth}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\ln\left(1+\frac{i(1+i\cot(bx+a))^2}{\cot(bx+a)^2+1}\right)}{2}+\frac{\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)}{}}{b}$
risch	Expression too large to display

input
```
int(arccoth(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-(1/2*Pi-arccot(cot(b*x+a)))*arccoth(cot(b*x+a))-1/2*(1/2*Pi-arccot(c
ot(b*x+a)))*ln(1+I*(1+I*cot(b*x+a))^2/(cot(b*x+a)^2+1))+1/2*(1/2*Pi-arccot
(cot(b*x+a)))*ln(1-I*(1+I*cot(b*x+a))^2/(cot(b*x+a)^2+1))+1/4*I*dilog(1+I*
(1+I*cot(b*x+a))^2/(cot(b*x+a)^2+1))-1/4*I*dilog(1-I*(1+I*cot(b*x+a))^2/(c
ot(b*x+a)^2+1)))
```

3.251.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(57) = 114$.

Time = 0.29 (sec) , antiderivative size = 388, normalized size of antiderivative = 4.91

$$\int \coth^{-1}(\cot(a+bx))\,dx$$
$$= \frac{4\,bx\log\left(\frac{\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a)+1}{\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a)+1}\right)+2\,a\log\left(\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right)+i\right)-2\,a\log\left(\cos\left(2\,bx+\right.\right.}{}$$

input
```
integrate(arccoth(cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(4*b*x*log((cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a)
 - sin(2*b*x + 2*a) + 1)) + 2*a*log(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)
+ I) - 2*a*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - 2*(b*x + a)*lo
g(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(b*x + a)*log(I*cos(2*b*x
 + 2*a) - sin(2*b*x + 2*a) + 1) - 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) + si
n(2*b*x + 2*a) + 1) + 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*
a) + 1) + 2*a*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 2*a*log(-c
os(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) + I*dilog(I*cos(2*b*x + 2*a) + s
in(2*b*x + 2*a)) + I*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - I*dilo
g(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - I*dilog(-I*cos(2*b*x + 2*a) -
sin(2*b*x + 2*a)))/b
```

3.251.6 Sympy [F]

$$\int \coth^{-1}(\cot(a+bx))\,dx = \int \operatorname{acoth}(\cot(a+bx))\,dx$$

input
```
integrate(acoth(cot(b*x+a)),x)
```

output
```
Integral(acoth(cot(a + b*x)), x)
```

3.251.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(57) = 114$.

Time = 0.33 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.33

$$\int \coth^{-1}(\cot(a+bx))\,dx$$
$$= \frac{4\,(bx+a)\,\operatorname{arcoth}\left(\frac{1}{\tan(bx+a)}\right) + \left(\arctan\left(\frac{1}{2}\,\tan(bx+a)+\frac{1}{2},\frac{1}{2}\,\tan(bx+a)+\frac{1}{2}\right) - \arctan\left(\frac{1}{2}\,\tan(bx+a)\right.\right.}{}$$

input
```
integrate(arccoth(cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*(4*(b*x + a)*arccoth(1/tan(b*x + a)) + (arctan2(1/2*tan(b*x + a) + 1/2
, 1/2*tan(b*x + a) + 1/2) - arctan2(1/2*tan(b*x + a) - 1/2, -1/2*tan(b*x +
 a) + 1/2))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log(1/2*tan(b*x + a)^2 + t
an(b*x + a) + 1/2) + (b*x + a)*log(1/2*tan(b*x + a)^2 - tan(b*x + a) + 1/2
) - I*dilog((1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2) + I*dilog(-(1/2*I -
1/2)*tan(b*x + a) + 1/2*I + 1/2) + I*dilog((1/2*I - 1/2)*tan(b*x + a) + 1/
2*I + 1/2) - I*dilog(-(1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2))/b
```

3.251.8 Giac [F]

$$\int \coth^{-1}(\cot(a+bx))\,dx = \int \operatorname{arcoth}\left(\cot\left(bx+a\right)\right)\,dx$$

input
```
integrate(arccoth(cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(cot(b*x + a)), x)
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(\cot(a+bx))\,dx = \int \operatorname{acoth}(\cot(a+b\,x))\,dx$$

input
```
int(acoth(cot(a + b*x)),x)
```

output
```
int(acoth(cot(a + b*x)), x)
```

3.252 $\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx$

3.252.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} \, dx = \text{Int}\left(\frac{\coth^{-1}(\cot(a+bx))}{e+fx},x\right)$$

output
```
CannotIntegrate(arccoth(cot(b*x+a))/(f*x+e),x)
```

3.252.2 Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} \, dx = \int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} \, dx$$

input
```
Integrate[ArcCoth[Cot[a + b*x]]/(e + f*x),x]
```

output
```
Integrate[ArcCoth[Cot[a + b*x]]/(e + f*x), x]
```

3.252.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}dx$$

input
```
Int[ArcCoth[Cot[a + b*x]]/(e + f*x),x]
```

output
```
$Aborted
```

3.252.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.252.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(\cot\left(bx+a\right)\right)}{fx+e}dx$$

input
```
int(arccoth(cot(b*x+a))/(f*x+e),x)
```

output
```
int(arccoth(cot(b*x+a))/(f*x+e),x)
```

3.252.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} \, dx = \int \frac{\operatorname{arcoth}(\cot(bx+a))}{fx+e} \, dx$$

input
```
integrate(arccoth(cot(b*x+a))/(f*x+e),x, algorithm="fricas")
```

output
```
integral(arccoth(cot(b*x + a))/(f*x + e), x)
```

3.252.6 Sympy [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} \, dx = \int \frac{\operatorname{acoth}(\cot(a+bx))}{e+fx} \, dx$$

input
```
integrate(acoth(cot(b*x+a))/(f*x+e),x)
```

output
```
Integral(acoth(cot(a + b*x))/(e + f*x), x)
```

3.252.7 Maxima [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} \, dx = \int \frac{\operatorname{arcoth}(\cot(bx+a))}{fx+e} \, dx$$

input
```
integrate(arccoth(cot(b*x+a))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(arccoth(cot(b*x + a))/(f*x + e), x)
```

3.252.8 Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{arcoth}\left(\cot\left(bx+a\right)\right)}{fx+e}\,dx$$

input
```
integrate(arccoth(cot(b*x+a))/(f*x+e),x, algorithm="giac")
```

output
```
integrate(arccoth(cot(b*x + a))/(f*x + e), x)
```

3.252.9 Mupad [N/A]

Not integrable

Time = 4.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{acoth}(\cot(a+b\,x))}{e+f\,x}\,dx$$

input
```
int(acoth(cot(a + b*x))/(e + f*x),x)
```

output
```
int(acoth(cot(a + b*x))/(e + f*x), x)
```

3.253 $\int x^2 \coth^{-1}(c + d\cot(a + bx))\,dx$

3.253.1 Optimal result

Integrand size = 15, antiderivative size = 391

$$\begin{aligned}\int x^2 \coth^{-1}(c+d\cot(a+bx))\,dx &= \frac{1}{3}x^3\coth^{-1}(c+d\cot(a+bx)) \\ &+ \frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) \\ &- \frac{1}{6}x^3\log\left(1-\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right) \\ &- \frac{ix^2\,\mathrm{PolyLog}\left(2,\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{4b} \\ &+ \frac{ix^2\,\mathrm{PolyLog}\left(2,\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{4b} \\ &+ \frac{x\,\mathrm{PolyLog}\left(3,\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{4b^2} \\ &- \frac{x\,\mathrm{PolyLog}\left(3,\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{4b^2} \\ &+ \frac{i\,\mathrm{PolyLog}\left(4,\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{8b^3} \\ &- \frac{i\,\mathrm{PolyLog}\left(4,\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arccoth(c+d*cot(b*x+a))+1/6*x^3*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/
(1-c+I*d))-1/6*x^3*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*x^2*
polylog(2,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*x^2*polylog(2,(1
+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b+1/4*x*polylog(3,(1-c-I*d)*exp(2*I*
a+2*I*b*x)/(1-c+I*d))/b^2-1/4*x*polylog(3,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+
c-I*d))/b^2+1/8*I*polylog(4,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b^3-1/
8*I*polylog(4,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b^3
```

3.253.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.87

$$\int x^2 \coth^{-1}(c + d\cot(a + bx))\,dx$$
$$= \frac{8b^3x^3 \coth^{-1}(c + d\cot(a + bx)) + 4b^3x^3 \log\left(1 + \frac{(1-c+id)e^{-2i(a+bx)}}{-1+c+id}\right) - 4b^3x^3 \log\left(1 + \frac{(-1-c+id)e^{-2i(a+bx)}}{1+c+id}\right) +$$

input
```
Integrate[x^2*ArcCoth[c + d*Cot[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcCoth[c + d*Cot[a + b*x]] + 4*b^3*x^3*Log[1 + (1 - c + I*d)/(
(-1 + c + I*d)*E^((2*I)*(a + b*x)))] - 4*b^3*x^3*Log[1 + (-1 - c + I*d)/((
1 + c + I*d)*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-1 + c - I*
d)/((-1 + c + I*d)*E^((2*I)*(a + b*x)))] - (6*I)*b^2*x^2*PolyLog[2, (1 + c
 - I*d)/((1 + c + I*d)*E^((2*I)*(a + b*x)))] + 6*b*x*PolyLog[3, (-1 + c -
I*d)/((-1 + c + I*d)*E^((2*I)*(a + b*x)))] - 6*b*x*PolyLog[3, (1 + c - I*d
)/((1 + c + I*d)*E^((2*I)*(a + b*x)))] - (3*I)*PolyLog[4, (-1 + c - I*d)/(
(-1 + c + I*d)*E^((2*I)*(a + b*x)))] + (3*I)*PolyLog[4, (1 + c - I*d)/((1
+ c + I*d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.253.3 Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6824, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(d\cot(a+bx)+c)\,dx$$

$$\Big\downarrow \text{ 6824}$$

$$-\frac{1}{3}b(-ic+d+i)\int \frac{e^{2ia+2ibx}x^3}{-c-(-c-id+1)e^{2ia+2ibx}+id+1}dx+\frac{1}{3}b(-d+i(c+1))\int \frac{e^{2ia+2ibx}x^3}{c-(c+id+1)e^{2ia+2ibx}-id+1}dx+\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{1}{3}b(-ic+d+i)\left(\frac{3\int x^2\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{2b(d+i(1-c))}-\frac{x^3\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+\frac{1}{3}b(-d+i(c+1))\left(\frac{3\int x^2\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{2(-bd+i(bc+b))}-\frac{x^3\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{1}{3}b(-ic+d+i)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{b}\right)}{2b(d+i(1-c))}-\frac{x^3\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+\frac{1}{3}b(-d+i(c+1))\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{b}\right)}{2(-bd+i(bc+b))}-\frac{x^3\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{ 7163}$$

3.253. $\int x^2 \coth^{-1}(c+d\cot(a+bx))\,dx$

$$-\frac{1}{3}b(-ic+d+$$

$$i)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}\right)}{b}\right)}{2b(d+i(1-c))}-\frac{x^3\log\right.$$

$$\frac{1}{3}b(-d+i(c+$$

$$1))\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}\right)}{b}\right)}{2(-bd+i(bc+b))}-\frac{x^3\log\left(1\right.}{2(-}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{2720}$$

3.253. $\int x^2\coth^{-1}(c+d\cot(a+bx))\,dx$

$$-\frac{1}{3}b(-ic+d+i)\left(\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\,\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}\right.}{b}\right.}{2b(d+i(1-c))}\right.$$

$$\frac{1}{3}b(-d+i(c+1))\left(\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\,\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}\right)}{b}\right.}{2(-bd+i(bc+b))}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{ 7143}$$

3.253. $\int x^2\coth^{-1}(c+d\cot(a+bx))\,dx$

$$-\frac{1}{3}b(-ic+d+$$

$$i)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}\right)}{b}\right)}{2b(d+i(1-c))}-\frac{x^3\log\left(1-\right.}{2b(}\right.$$

$$\frac{1}{3}b(-d+i(c+$$

$$1))\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}\right)}{b}\right)}{2(-bd+i(bc+b))}-\frac{x^3\log\left(1-\right.}{2(-bd-}\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+c)$$

input
```
Int[x^2*ArcCoth[c + d*Cot[a + b*x]],x]
```

output
```
(x^3*ArcCoth[c + d*Cot[a + b*x]])/3 - (b*(I - I*c + d)*(-1/2*(x^3*Log[1 -
((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(b*(I*(1 - c) + d)
) + (3*(((I/2)*x^2*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 -
 c + I*d)])/b - (I*(((-1/2*I)*x*PolyLog[3, ((1 - c - I*d)*E^((2*I)*a + (2*
I)*b*x))/(1 - c + I*d)])/b + PolyLog[4, ((1 - c - I*d)*E^((2*I)*a + (2*I)*
b*x))/(1 - c + I*d)]/(4*b^2)))/b))/(2*b*(I*(1 - c) + d))))/3 + (b*(I*(1 +
c) - d)*(-1/2*(x^3*Log[1 - ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c
- I*d)])/(I*(b + b*c) - b*d) + (3*(((I/2)*x^2*PolyLog[2, ((1 + c + I*d)*E^
((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b - (I*(((-1/2*I)*x*PolyLog[3, ((1
+ c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + PolyLog[4, ((1 + c
 + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)]/(4*b^2)))/b))/(2*(I*(b + b
*c) - b*d))))/3
```

3.253. $\int x^2 \coth^{-1}(c+d\cot(a+bx))\,dx$

3.253.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6824
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + (-Simp[I*b*((1 - c - I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)
*(E^(2*I*a + 2*I*b*x)/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x))), x
], x] + Simp[I*b*((1 + c + I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*
I*a + 2*I*b*x)/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x))), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.253.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 27.60 (sec) , antiderivative size = 6661, normalized size of antiderivative = 17.04

method	result	size
risch	Expression too large to display	6661

input
```
int(x^2*arccoth(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.253.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1798 vs. 2(275) = 550.

Time = 0.47 (sec) , antiderivative size = 1798, normalized size of antiderivative = 4.60

$$\int x^2 \coth^{-1}(c + d\cot(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate(x^2*arccoth(c+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/48*(8*b^3*x^3*log((d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d
*cos(2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 6*I*b^2*x^2*dilog(-(c
^2 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^
2 + 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^
2 + 2*c + 1) + 1) - 6*I*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d -
 d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c +
I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 6*I*b^2*x^2*di
log(-(c^2 + d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) +
 (-I*c^2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c
^2 + d^2 - 2*c + 1) + 1) + 6*I*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c -
 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2
*I*c + I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) + 4*a^3*l
og(1/2*c^2 + I*(c + 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x +
 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) - 4*a^
3*log(1/2*c^2 + I*(c - 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*
x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) + 4
*a^3*log(-1/2*c^2 + I*(c + 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(
2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2)
 - 4*a^3*log(-1/2*c^2 + I*(c - 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*
cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c...
```

3.253.6 Sympy [F]

$$\int x^2 \coth^{-1}(c + d\cot(a + bx))\,dx = \int x^2 \operatorname{acoth}\left(c + d\cot\left(a + bx\right)\right)dx$$

input
```
integrate(x**2*acoth(c+d*cot(b*x+a)),x)
```

output
```
Integral(x**2*acoth(c + d*cot(a + b*x)), x)
```

3.253.7 Maxima [F]

$$\int x^2 \coth^{-1}(c + d\cot(a + bx))\,dx = \int x^2 \operatorname{arcoth}\left(d\cot\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x^2*arccoth(c+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*x^3*log((c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*
b*x + 2*a) + (c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2
 - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/12*x^3*log((c^2 + d^2 -
2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 -
2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*
x + 2*a) - 2*c + 1) - 4*b*d*integrate(1/3*(2*(c^2 + d^2 - 1)*x^3*cos(2*b*x
 + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^3*sin(2*b*x +
 2*a)^2 - (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*
a) + (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*c
os(2*b*x + 2*a) - (c^2 - d^2 - 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/
(c^4 + d^4 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*c
os(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x
+ 2*a)^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 +
4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2
*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 - 2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*
b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a
) - 4*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*
d + 2*(c*d^3 + (c^3 - c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin
(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a)
 + 1), x)
```

3.253.8 Giac [F]

$$\int x^2 \coth^{-1}(c + d\cot(a + bx))\,dx = \int x^2 \operatorname{arcoth}\left(d\cot\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x^2*arccoth(c+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(d*cot(b*x + a) + c), x)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(c + d\cot(a + bx))\, dx = \int x^2\, \mathrm{acoth}(c + d\cot(a + b\, x))\ dx$$

input
```
int(x^2*acoth(c + d*cot(a + b*x)),x)
```

output
```
int(x^2*acoth(c + d*cot(a + b*x)), x)
```

3.254 $\int x \coth^{-1}(c+d\cot(a+bx))\,dx$

3.254.1 Optimal result

Integrand size = 13, antiderivative size = 293

$$\begin{aligned}\int x \coth^{-1}(c+d\cot(a+bx))\,dx =& \frac{1}{2}x^2\coth^{-1}(c+d\cot(a+bx)) \\ &+ \frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) \\ &- \frac{1}{4}x^2\log\left(1-\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right) \\ &- \frac{ix\,\text{PolyLog}\left(2,\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{4b} \\ &+ \frac{ix\,\text{PolyLog}\left(2,\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{4b} \\ &+ \frac{\text{PolyLog}\left(3,\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{8b^2} \\ &- \frac{\text{PolyLog}\left(3,\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{8b^2}\end{aligned}$$

output
```
1/2*x^2*arccoth(c+d*cot(b*x+a))+1/4*x^2*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/
(1-c+I*d))-1/4*x^2*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*x*po
lylog(2,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*x*polylog(2,(1+c+I
*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b+1/8*polylog(3,(1-c-I*d)*exp(2*I*a+2*I*
b*x)/(1-c+I*d))/b^2-1/8*polylog(3,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/
b^2
```

3.254.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.87

$$\int x\coth^{-1}(c+d\cot(a+bx))\,dx$$
$$=\frac{4b^2x^2\coth^{-1}(c+d\cot(a+bx))+2b^2x^2\log\left(1+\frac{(1-c+id)e^{-2i(a+bx)}}{-1+c+id}\right)-2b^2x^2\log\left(1+\frac{(-1-c+id)e^{-2i(a+bx)}}{1+c+id}\right)+}{}$$

input
```
Integrate[x*ArcCoth[c + d*Cot[a + b*x]],x]
```

output
```
(4*b^2*x^2*ArcCoth[c + d*Cot[a + b*x]] + 2*b^2*x^2*Log[1 + (1 - c + I*d)/(
(-1 + c + I*d)*E^((2*I)*(a + b*x)))] - 2*b^2*x^2*Log[1 + (-1 - c + I*d)/((
1 + c + I*d)*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-1 + c - I*d)/(
(-1 + c + I*d)*E^((2*I)*(a + b*x)))] - (2*I)*b*x*PolyLog[2, (1 + c - I*d)/
((1 + c + I*d)*E^((2*I)*(a + b*x)))] + PolyLog[3, (-1 + c - I*d)/((-1 + c
+ I*d)*E^((2*I)*(a + b*x)))] - PolyLog[3, (1 + c - I*d)/((1 + c + I*d)*E^(
(2*I)*(a + b*x)))])/(8*b^2)
```

3.254.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.34, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6824, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(d\cot(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6824}$$

$$-\frac{1}{2}b(-ic+d+i)\int \frac{e^{2ia+2ibx}x^2}{-c-(-c-id+1)e^{2ia+2ibx}+id+1}dx+\frac{1}{2}b(-d+i(c+1))\int \frac{e^{2ia+2ibx}x^2}{c-(c+id+1)e^{2ia+2ibx}-id+1}dx+\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$-\frac{1}{2}b(-ic+d+i)\left(\frac{\int x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{b(d+i(1-c))}-\frac{x^2\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+\frac{1}{2}b(-d+i(c+1))\left(\frac{\int x\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{-bd+i(bc+b)}-\frac{x^2\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$-\frac{1}{2}b(-ic+d+i)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{i\int\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{2b}}{b(d+i(1-c))}-\frac{x^2\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+\frac{1}{2}b(-d+i(c+1))\left(\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{i\int\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{2b}}{-bd+i(bc+b)}-\frac{x^2\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{2720}$$

$$-\frac{1}{2}b(-ic+d+i)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)de^{2ia+2ibx}}{4b^2}}{b(d+i(1-c))}-\frac{x^2\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+\frac{1}{2}b(-d+i(c+1))\left(\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)de^{2ia+2ibx}}{4b^2}}{-bd+i(bc+b)}-\frac{x^2\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{7143}$$

3.254. $\int x\coth^{-1}(c+d\cot(a+bx))\,dx$

$$-\frac{1}{2}b(-ic+d+i)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b}-\frac{\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2}}{b(d+i(1-c))}-\frac{x^2\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+\frac{1}{2}b(-d+i(c+1))\left(\frac{\frac{ix\,\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2b}-\frac{\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2}}{-bd+i(bc+b)}-\frac{x^2\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+c)$$

input
```
Int[x*ArcCoth[c + d*Cot[a + b*x]],x]
```

output
```
(x^2*ArcCoth[c + d*Cot[a + b*x]])/2 - (b*(I - I*c + d)*(-1/2*(x^2*Log[1 -
((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(b*(I*(1 - c) + d)
) + (((I/2)*x*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c +
I*d)])/b - PolyLog[3, ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d
)]/(4*b^2))/(b*(I*(1 - c) + d))))/2 + (b*(I*(1 + c) - d)*(-1/2*(x^2*Log[1
- ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/(I*(b + b*c) - b
*d) + (((I/2)*x*PolyLog[2, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c
- I*d)])/b - PolyLog[3, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I
*d)]/(4*b^2))/(I*(b + b*c) - b*d)))/2
```

3.254.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6824
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + (-Simp[I*b*((1 - c - I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)
*(E^(2*I*a + 2*I*b*x)/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x))), x
], x] + Simp[I*b*((1 + c + I*d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*
I*a + 2*I*b*x)/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x))), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.254.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.18 (sec) , antiderivative size = 6311, normalized size of antiderivative = 21.54

method	result	size
risch	Expression too large to display	6311

input
```
int(x*arccoth(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.254.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1462 vs. $2(207) = 414$.

Time = 0.43 (sec) , antiderivative size = 1462, normalized size of antiderivative = 4.99

$$\int x \coth^{-1}(c + d\cot(a + bx))\,dx = \text{Too large to display}$$

input
```
integrate(x*arccoth(c+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/16*(4*b^2*x^2*log((d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d
*cos(2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 2*I*b*x*dilog(-(c^2 +
 d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 +
2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 +
2*c + 1) + 1) - 2*I*b*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2
*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2
*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 2*I*b*x*dilog(-(c^2 +
d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2
*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2
*c + 1) + 1) + 2*I*b*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*
c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*
b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) - 2*a^2*log(1/2*c^2 + I*(
c + 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c
^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) + 2*a^2*log(1/2*c^2 +
I*(c - 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(
I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) - 2*a^2*log(-1/2*c^
2 + I*(c + 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1
/2*(I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) + 2*a^2*log(-1/
2*c^2 + I*(c - 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a)
 + 1/2*(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 2*(b^2...
```

3.254.6 Sympy [F]

$$\int x \coth^{-1}(c + d\cot(a + bx))\,dx = \int x\,\mathrm{acoth}\,(c + d\cot\,(a + bx))\,dx$$

input
```
integrate(x*acoth(c+d*cot(b*x+a)),x)
```

output
```
Integral(x*acoth(c + d*cot(a + b*x)), x)
```

3.254.7 Maxima [F]

$$\int x\coth^{-1}(c+d\cot(a+bx))\,dx=\int x\,\mathrm{arcoth}\,(d\cot(bx+a)+c)\;dx$$

input
```
integrate(x*arccoth(c+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-2*b*d*integrate((2*(c^2 + d^2 - 1)*x^2*cos(2*b*x + 2*a)^2 + 2*c*d*x^2*sin
(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*sin(2*b*x + 2*a)^2 - (c^2 - d^2 - 1)
*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) + (c^2 - d^2 - 1)*x^2*
cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*x + 2*a) - (c^2 -
d^2 - 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)*
d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c^
4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (c^4 + d^4 + 2
*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1
)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 -
1)*d^2 - 2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(c*d^3 + (
c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4 - d^4 - 2*c^2
+ 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*d + 2*(c*d^3 + (c^3 - c)*
d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin(4*b*x
+ 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*log((
c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b*x + 2*a) + (
c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 + 2*c +
 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/8*x^2*log((c^2 + d^2 - 2*c + 1)*cos(2*
b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*sin(2*
b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) - 2*c
+ 1)
```

3.254.8 Giac [F]

$$\int x\coth^{-1}(c+d\cot(a+bx))\,dx=\int x\,\mathrm{arcoth}\,(d\cot(bx+a)+c)\;dx$$

input
```
integrate(x*arccoth(c+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(d*cot(b*x + a) + c), x)
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int x\coth^{-1}(c+d\cot(a+bx))\,dx = \int x\,\mathrm{acoth}(c+d\cot(a+b\,x))\,dx$$

input
```
int(x*acoth(c + d*cot(a + b*x)),x)
```

output
```
int(x*acoth(c + d*cot(a + b*x)), x)
```

3.255 $\int \coth^{-1}(c+d\cot(a+bx))\,dx$

3.255.1 Optimal result

Integrand size = 11, antiderivative size = 194

$$\begin{aligned}\int \coth^{-1}(c+d\cot(a+bx))\,dx = {} & x\coth^{-1}(c+d\cot(a+bx)) \\ & + \frac{1}{2}x\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) \\ & - \frac{1}{2}x\log\left(1-\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right) \\ & - \frac{i\,\text{PolyLog}\left(2,\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{4b} \\ & + \frac{i\,\text{PolyLog}\left(2,\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)}{4b}\end{aligned}$$

output
```
x*arccoth(c+d*cot(b*x+a))+1/2*x*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d
))-1/2*x*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*polylog(2,(1-c
-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*polylog(2,(1+c+I*d)*exp(2*I*a+
2*I*b*x)/(1+c-I*d))/b
```

3.255.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 390 vs. 2(194) = 388.

Time = 0.52 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.01

$$\int \coth^{-1}(c+d\cot(a+bx))\,dx = x\Bigg(\coth^{-1}(c+d\cot(a+bx)) + \frac{2a\log(d+(-1+c)\tan(a+bx)) + i\log(1+i\tan(a+bx))\log\left(-\frac{i(d+(-1+c)\tan(a+bx))}{-1+c-id}\right) - i\log(1-i\tan\dots}{\dots}$$

input
```
Integrate[ArcCoth[c + d*Cot[a + b*x]],x]
```

output
```
x*(ArcCoth[c + d*Cot[a + b*x]] + (2*a*Log[d + (-1 + c)*Tan[a + b*x]] + I*L
og[1 + I*Tan[a + b*x]]*Log[((-I)*(d + (-1 + c)*Tan[a + b*x]))/(-1 + c - I*
d)] - I*Log[1 - I*Tan[a + b*x]]*Log[(I*(d + (-1 + c)*Tan[a + b*x]))/(-1 +
c + I*d)] - 2*a*Log[d + (1 + c)*Tan[a + b*x]] + I*Log[1 - I*Tan[a + b*x]]*
Log[(I*(d + (1 + c)*Tan[a + b*x]))/(1 + c + I*d)] - I*Log[1 + I*Tan[a + b*
x]]*Log[(d + (1 + c)*Tan[a + b*x])/(I*(1 + c) + d)] - I*PolyLog[2, ((-1 +
c)*(1 - I*Tan[a + b*x]))/(-1 + c + I*d)] + I*PolyLog[2, ((1 + c)*(1 - I*Ta
n[a + b*x]))/(1 + c + I*d)] + I*PolyLog[2, ((-1 + c)*(1 + I*Tan[a + b*x]))
/(-1 + c - I*d)] - I*PolyLog[2, ((1 + c)*(1 + I*Tan[a + b*x]))/(1 + c - I*
d)])/(4*a - (2*I)*Log[1 - I*Tan[a + b*x]] + (2*I)*Log[1 + I*Tan[a + b*x]])
)
```

3.255.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.46, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6816, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d\cot(a+bx)+c)\,dx$$

$$\Big\downarrow \text{6816}$$

$$-b(-ic+d+i)\int \frac{e^{2ia+2ibx}x}{-c-(-c-id+1)e^{2ia+2ibx}+id+1}dx+b(-d+i(c+1))\int \frac{e^{2ia+2ibx}x}{c-(c+id+1)e^{2ia+2ibx}-id+1}dx+x\coth^{-1}(d\cot(a+bx)+c)$$

$\big\downarrow$ 2620

$$-b(-ic+d+i)\left(\frac{\int \log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)dx}{2b(d+i(1-c))}-\frac{x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+b(-d+i(c+1))\left(\frac{\int \log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)dx}{2(-bd+i(bc+b))}-\frac{x\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+x\coth^{-1}(d\cot(a+bx)+c)$$

$\big\downarrow$ 2715

$$-b(-ic+d+i)\left(-\frac{i\int e^{-2ia-2ibx}\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)de^{2ia+2ibx}}{4b^2(d+i(1-c))}-\frac{x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+b(-d+i(c+1))\left(-\frac{i\int e^{-2ia-2ibx}\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)de^{2ia+2ibx}}{4b(-bd+i(bc+b))}-\frac{x\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+x\coth^{-1}(d\cot(a+bx)+c)$$

$\big\downarrow$ 2838

$$-b(-ic+d+i)\left(\frac{i\,\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2(d+i(1-c))}-\frac{x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{2b(d+i(1-c))}\right)+b(-d+i(c+1))\left(\frac{i\,\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b(-bd+i(bc+b))}-\frac{x\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{2(-bd+i(bc+b))}\right)+x\coth^{-1}(d\cot(a+bx)+c)$$

input
```
Int[ArcCoth[c + d*Cot[a + b*x]],x]
```

output
```
x*ArcCoth[c + d*Cot[a + b*x]] - b*(I - I*c + d)*(-1/2*(x*Log[1 - ((1 - c -
 I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(b*(I*(1 - c) + d)) + ((I/4
)*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(b^2*
(I*(1 - c) + d))) + b*(I*(1 + c) - d)*(-1/2*(x*Log[1 - ((1 + c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/(I*(b + b*c) - b*d) + ((I/4)*PolyLog[
2, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/(b*(I*(b + b*c)
 - b*d)))
```

3.255. $\int \coth^{-1}(c+d\cot(a+bx))\,dx$

3.255.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6816
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Coth[c + d*Cot[a + b*x]], x] + (-Simp[I*b*(1 - c - I*d)   Int[x*(E^(2*I*a +
 2*I*b*x)/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x))), x], x] + Simp
[I*b*(1 + c + I*d)   Int[x*(E^(2*I*a + 2*I*b*x)/(1 + c - I*d - (1 + c + I*d
)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - I*d)
^2, 1]
```

3.255.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(164) = 328$.

Time = 2.99 (sec) , antiderivative size = 564, normalized size of antiderivative = 2.91

method	result
derivativedivides	$\frac{-d\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arccoth}(c+d\cot(bx+a))-d^2\left(\frac{\arctan\left(-\frac{c+d\cot(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\cot(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2d}-\right.}{}$
default	$\frac{-d\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arccoth}(c+d\cot(bx+a))-d^2\left(\frac{\arctan\left(-\frac{c+d\cot(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\cot(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2d}-\right.}{}$
risch	Expression too large to display

input
```
int(arccoth(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-d*(1/2*Pi-arccot(cot(b*x+a)))*arccoth(c+d*cot(b*x+a))-d^2*(1/2*arc
tan(-(c+d*cot(b*x+a))/d+c/d)/d*ln(d*((c+d*cot(b*x+a))/d-c/d)+c+1)-1/2*arct
an(-(c+d*cot(b*x+a))/d+c/d)/d*ln(d*((c+d*cot(b*x+a))/d-c/d)+c-1)+1/4*I*ln(
d*((c+d*cot(b*x+a))/d-c/d)+c-1)*(ln((I*d-d*((c+d*cot(b*x+a))/d-c/d))/(I*d+
c-1))-ln((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(1-c+I*d)))/d+1/4*I*(dilog((I*d-
d*((c+d*cot(b*x+a))/d-c/d))/(I*d+c-1))-dilog((I*d+d*((c+d*cot(b*x+a))/d-c/
d))/(1-c+I*d)))/d-1/4*I*ln(d*((c+d*cot(b*x+a))/d-c/d)+c+1)*(ln((I*d-d*((c+
d*cot(b*x+a))/d-c/d))/(1+c+I*d))-ln((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(I*d-
c-1)))/d-1/4*I*(dilog((I*d-d*((c+d*cot(b*x+a))/d-c/d))/(1+c+I*d))-dilog((I
*d+d*((c+d*cot(b*x+a))/d-c/d))/(I*d-c-1)))/d))
```

3.255.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1098 vs. 2(136) = 272.

Time = 0.38 (sec) , antiderivative size = 1098, normalized size of antiderivative = 5.66

$$\int \coth^{-1}(c+d\cot(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate(arccoth(c+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(4*b*x*log((d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d*cos(
2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 2*a*log(1/2*c^2 + I*(c + 1
)*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 +
I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) - 2*a*log(1/2*c^2 + I*(c -
1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 +
 I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) + 2*a*log(-1/2*c^2 + I*(c
+ 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2
 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) - 2*a*log(-1/2*c^2 + I*(
c - 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c
^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 2*(b*x + a)*log((c^2
 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2
+ 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2
+ 2*c + 1)) - 2*(b*x + a)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*
c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*
b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*log((c^2 + d^2
- (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c
- 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c +
 1)) + 2*(b*x + a)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*
cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2
*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) + I*dilog(-(c^2 + d^2 - (c^2 + 2*...
```

3.255.6 Sympy [F]

$$\int \coth^{-1}(c+d\cot(a+bx))\,dx=\int \operatorname{acoth}\left(c+d\cot\left(a+bx\right)\right)dx$$

input
```
integrate(acoth(c+d*cot(b*x+a)),x)
```

output
```
Integral(acoth(c + d*cot(a + b*x)), x)
```

3.255.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(136) = 272$.

Time = 0.38 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.02

$$\int \coth^{-1}(c + d\cot(a + bx))\,dx$$
$$= \frac{4\,(bx+a)\operatorname{arcoth}\left(c+\frac{d}{\tan(bx+a)}\right)+\left(\arctan\left(\frac{(c+1)d+(c^2+2\,c+1)\tan(bx+a)}{c^2+d^2+2\,c+1},\frac{(c+1)d\tan(bx+a)+d^2}{c^2+d^2+2\,c+1}\right)-\arctan\left(\frac{(c-1)d-}{}\right.\right.}{}$$

input
```
integrate(arccoth(c+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*(4*(b*x + a)*arccoth(c + d/tan(b*x + a)) + (arctan2(((c + 1)*d + (c^2
+ 2*c + 1)*tan(b*x + a))/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*tan(b*x + a) +
d^2)/(c^2 + d^2 + 2*c + 1)) - arctan2(((c - 1)*d + (c^2 - 2*c + 1)*tan(b*x
 + a))/(c^2 + d^2 - 2*c + 1), ((c - 1)*d*tan(b*x + a) + d^2)/(c^2 + d^2 -
2*c + 1)))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log((2*(c + 1)*d*tan(b*x +
a) + (c^2 + 2*c + 1)*tan(b*x + a)^2 + d^2)/(c^2 + d^2 + 2*c + 1)) + (b*x +
 a)*log((2*(c - 1)*d*tan(b*x + a) + (c^2 - 2*c + 1)*tan(b*x + a)^2 + d^2)/
(c^2 + d^2 - 2*c + 1)) + I*dilog(-((c + 1)*tan(b*x + a) - I*c - I)/(I*c +
d + I)) - I*dilog(-((c - 1)*tan(b*x + a) - I*c + I)/(I*c + d - I)) + I*dil
og(-((c - 1)*tan(b*x + a) + I*c - I)/(-I*c + d + I)) - I*dilog(-((c + 1)*t
an(b*x + a) + I*c + I)/(-I*c + d - I)))/b
```

3.255.8 Giac [F]

$$\int \coth^{-1}(c + d\cot(a + bx))\,dx = \int \operatorname{arcoth}\left(d\cot\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(arccoth(c+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(d*cot(b*x + a) + c), x)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(c + d\cot(a + bx))\,dx = \int \operatorname{acoth}(c + d\cot(a + b\,x))\,dx$$

input
```
int(acoth(c + d*cot(a + b*x)),x)
```

output
```
int(acoth(c + d*cot(a + b*x)), x)
```

3.256 $\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx$

3.256.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx = \text{Int}\left(\frac{\coth^{-1}(c+d\cot(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccoth(c+d*cot(b*x+a))/x,x)
```

3.256.2 Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[c + d*Cot[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[c + d*Cot[a + b*x]]/x, x]
```

3.256.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d\cot(a+bx)+c)}{x} \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\coth^{-1}(d\cot(a+bx)+c)}{x}dx$$

input
```
Int[ArcCoth[c + d*Cot[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.256.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.256.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arccoth}\left(c+d\cot\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(c+d*cot(b*x+a))/x,x)
```

output
```
int(arccoth(c+d*cot(b*x+a))/x,x)
```

3.256.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\text{arcoth}\left(d\cot\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*cot(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arccoth(d*cot(b*x + a) + c)/x, x)
```

3.256.6 Sympy [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\text{acoth}\left(c+d\cot\left(a+bx\right)\right)}{x}\,dx$$

input
```
integrate(acoth(c+d*cot(b*x+a))/x,x)
```

output
```
Integral(acoth(c + d*cot(a + b*x))/x, x)
```

3.256.7 Maxima [N/A]

Not integrable

Time = 3.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\text{arcoth}\left(d\cot\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*cot(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arccoth(d*cot(b*x + a) + c)/x, x)
```

3.256.8 Giac [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\cot\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arccoth(c+d*cot(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(d*cot(b*x + a) + c)/x, x)
```

3.256.9 Mupad [N/A]

Not integrable

Time = 6.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx=\int \frac{\operatorname{acoth}(c+d\cot(a+b\,x))}{x}\,dx$$

input
```
int(acoth(c + d*cot(a + b*x))/x,x)
```

output
```
int(acoth(c + d*cot(a + b*x))/x, x)
```

3.257 $\int x^2 \coth^{-1}(1+id+d\cot(a+bx))\,dx$

3.257.1 Optimal result

Integrand size = 20, antiderivative size = 168

$$\int x^2 \coth^{-1}(1+id+d\cot(a+bx))\,dx = \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right) + \frac{ix^2\,\mathrm{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b} - \frac{x\,\mathrm{PolyLog}\left(3,(1+id)e^{2ia+2ibx}\right)}{4b^2} - \frac{i\,\mathrm{PolyLog}\left(4,(1+id)e^{2ia+2ibx}\right)}{8b^3}$$

output
```
1/12*I*b*x^4+1/3*x^3*arccoth(1+I*d+d*cot(b*x+a))-1/6*x^3*ln(1-(1+I*d)*exp(
2*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*po
lylog(3,(1+I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,(1+I*d)*exp(2*I*a+
2*I*b*x))/b^3
```

3.257.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.92

$$\int x^2 \coth^{-1}(1+id+d\cot(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx)) - \frac{4b^3x^3\log\left(1+\frac{ie^{-2i(a+bx)}}{-i+d}\right)+6ib^2x^2\operatorname{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)+6bx\operatorname{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)-3i\operatorname{PolyL}}{24b^3}$$

input
```
Integrate[x^2*ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 + I*d + d*Cot[a + b*x]])/3 - (4*b^3*x^3*Log[1 + I/((-I + d)
*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/((-I + d)*E^((2*I)*
(a + b*x)))] + 6*b*x*PolyLog[3, (-I)/((-I + d)*E^((2*I)*(a + b*x)))] - (3*
I)*PolyLog[4, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.257.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6820, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\coth^{-1}(d\cot(a+bx)+id+1)\,dx$$

$$\Big\downarrow \text{6820}$$

$$\frac{1}{3}ib\int \frac{x^3}{1-(id+1)e^{2ia+2ibx}}dx+\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\int \frac{e^{2ia+2ibx}x^3}{1-(id+1)e^{2ia+2ibx}}dx\right)+\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\left(\frac{3\int x^2\log\left(1-(id+1)e^{2ia+2ibx}\right)dx}{2b(-d+i)}-\frac{x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}(2,(id+1)e^{2ia+2ibx})}{2b}-\frac{i\int x\operatorname{PolyLog}(2,(id+1)e^{2ia+2ibx})dx}{b}\right)}{2b(-d+i)}-\frac{x^3\log\left(1-(1+id)e^{2ia+2ibx}\right.}{2b(-d+i)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}(2,(id+1)e^{2ia+2ibx})}{2b}-\frac{i\left(\frac{i\int \operatorname{PolyLog}\left(3,(id+1)e^{2ia+2ibx}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,(id+1)e^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2b(-d+i)}-\frac{x}{}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}(2,(id+1)e^{2ia+2ibx})}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,(id+1)e^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,(id+1)e^{2i\ldots}\right.}{2b}\right.}{b}\right.}{2b(-d+i)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1+id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}(2,(id+1)e^{2ia+2ibx})}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,(id+1)e^{2ia+2ibx}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,(id+1)e^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2b(-d+i)}-\frac{x^3\log}{}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+id+1)$$

3.257. $\int x^2\coth^{-1}(1+id+d\cot(a+bx))\,dx$

input
```
Int[x^2*ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 + I*d + d*Cot[a + b*x]])/3 + (I/3)*b*(x^4/4 + (1 + I*d)*(-1
/2*(x^3*Log[1 - (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I - d)) + (3*(((I/
2)*x^2*PolyLog[2, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/b - (I*(((-1/2*I)*x*
PolyLog[3, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/b + PolyLog[4, (1 + I*d)*E^
((2*I)*a + (2*I)*b*x)]/(4*b^2)))/b))/(2*b*(I - d))))
```

3.257.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6820
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.257.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.53 (sec) , antiderivative size = 2383, normalized size of antiderivative = 14.18

method	result	size
risch	Expression too large to display	2383

input
```
int(x^2*arccoth(1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/4/b^2*d/(I-d)*polylog(3,-I*(I-d)*exp(2*I*(b*x+a)))*x+1/2/b^3*d*a^3/(I-d)
*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/2/b^3*d*a^3/(I-d)*ln(1+I*exp(I*(
b*x+a))*(I*(I-d))^(1/2))-1/6/b^3*a^3*d/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I
*(b*x+a))*d-I)-1/3/b^3*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^3-1/4/b/(I
-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I-d)*polylog(2,-I*(I
-d)*exp(2*I*(b*x+a)))*a^2-1/2/b^3*a^2/(I-d)*dilog(1-I*exp(I*(b*x+a))*(I*(I
-d))^(1/2))-1/6*I/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x^3+1/6*d/(I-d)*ln(
1+I*(I-d)*exp(2*I*(b*x+a)))*x^3-1/2/b^3*a^2/(I-d)*dilog(1+I*exp(I*(b*x+a))
*(I*(I-d))^(1/2))+1/12*I*b*x^4+1/4*I/b^3*a^2*d/(I-d)*polylog(2,-I*(I-d)*ex
p(2*I*(b*x+a)))-1/3*x^3*ln(exp(I*(b*x+a)))+1/2/b^2*d*a^2/(I-d)*ln(1-I*exp(
I*(b*x+a))*(I*(I-d))^(1/2))*x+1/2/b^2*d*a^2/(I-d)*ln(1+I*exp(I*(b*x+a))*(I
*(I-d))^(1/2))*x-1/2/b^2*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^2*x-1/2*
I/b^3*d*a^2/(I-d)*dilog(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/2*I/b^3*d*a^
2/(I-d)*dilog(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/2*I/b^2*a^2/(I-d)*ln(1
-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))*x-1/2*I/b^2*a^2/(I-d)*ln(1+I*exp(I*(b*x
+a))*(I*(I-d))^(1/2))*x-1/4*I/b*d/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)
))*x^2+1/2*I/b^2/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^2*x+1/3*I/b^3/(I-d
)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^3-1/2*I/b^3*a^3/(I-d)*ln(1-I*exp(I*(b*x
+a))*(I*(I-d))^(1/2))-1/2*I/b^3*a^3/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^
(1/2))+1/6*I/b^3*a^3/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d-I)-...
```

3.257.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.07

$$\int x^2 \coth^{-1}(1 + id + d\cot(a + bx))\,dx$$
$$= \frac{2i\,b^4x^4 + 4\,b^3x^3\log\left(\frac{((d-i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(-(-i\,d-1)e^{(2i\,bx+2i\,a)}\right) - 2i\,a^4 + 4\,a^3\log\left(\cdots\right.}{\cdots}$$

input
```
integrate(x^2*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/24*(2*I*b^4*x^4 + 4*b^3*x^3*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*
I*b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(-(-I*d - 1)*e^(2*I*b*x + 2*I*a)) - 2
*I*a^4 + 4*a^3*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*b*x*poly
log(3, (I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((-I*d - 1)*e
^(2*I*b*x + 2*I*a) + 1) - 3*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b
^3
```

3.257.6 Sympy [F]

$$\int x^2 \coth^{-1}(1+id+d\cot(a+bx))\,dx = \int x^2 \operatorname{acoth}\left(d\cot\left(a+bx\right)+id+1\right)dx$$

input
```
integrate(x**2*acoth(1+I*d+d*cot(b*x+a)),x)
```

output
```
Integral(x**2*acoth(d*cot(a + b*x) + I*d + 1), x)
```

3.257.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(119) = 238$.

Time = 0.24 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.05

$$\int x^2 \coth^{-1}(1+id+d\cot(a+bx))\,dx = \frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{arcoth}(d\cot(bx+a)+i\,d+1)}{b^2} - \frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(4i\,(bx+a)^3-9i\,(bx+a)^2\right.}{}}{}$$

input
```
integrate(x^2*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*cot(b
*x + a) + I*d + 1)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*
x + a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2)
*arctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) - cos(2
*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog((
I*d + 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x
+ a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2
+ 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3
, (I*d + 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2
*I*a)))/b^2)/b
```

3.257.8 Giac [F]

$$\int x^2 \coth^{-1}(1+id+d\cot(a+bx))\,dx = \int x^2 \operatorname{arcoth}\left(d\cot\left(bx+a\right)+i\,d+1\right)\,dx$$

input
```
integrate(x^2*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(d*cot(b*x + a) + I*d + 1), x)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(1+id+d\cot(a+bx))\,dx = \int x^2 \operatorname{acoth}(d\cot(a+b\,x)+1+d\,1\mathrm{i})\,dx$$

input
```
int(x^2*acoth(d*1i + d*cot(a + b*x) + 1),x)
```

output
```
int(x^2*acoth(d*1i + d*cot(a + b*x) + 1), x)
```

3.258 $\int x \coth^{-1}(1+id+d\cot(a+bx))\,dx$

3.258.1 Optimal result

Integrand size = 18, antiderivative size = 132

$$\int x \coth^{-1}(1+id+d\cot(a+bx))\,dx = \frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx)) -\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right) +\frac{ix\,\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b} -\frac{\text{PolyLog}\left(3,(1+id)e^{2ia+2ibx}\right)}{8b^2}$$

output
```
1/6*I*b*x^3+1/2*x^2*arccoth(1+I*d+d*cot(b*x+a))-1/4*x^2*ln(1-(1+I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylog
(3,(1+I*d)*exp(2*I*a+2*I*b*x))/b^2
```

3.258.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.90

$$\int x \coth^{-1}(1+id+d\cot(a+bx))\,dx = \frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx)) -\frac{2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{-i+d}\right)+2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{8b^2}$$

input
```
Integrate[x*ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 + I*d + d*Cot[a + b*x]])/2 - (2*b^2*x^2*Log[1 + I/((-I + d)
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/((-I + d)*E^((2*I)*(a +
 b*x)))] + PolyLog[3, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

3.258.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6820, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(d\cot(a+bx)+id+1)\,dx$$

$$\Big\downarrow \text{6820}$$

$$\frac{1}{2}ib\int \frac{x^2}{1-(id+1)e^{2ia+2ibx}}dx+\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1+id)\int \frac{e^{2ia+2ibx}x^2}{1-(id+1)e^{2ia+2ibx}}dx\right)+\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1+id)\left(\frac{\int x\log\left(1-(id+1)e^{2ia+2ibx}\right)dx}{b(-d+i)}-\frac{x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1+id)\left(\frac{\frac{ix\,\text{PolyLog}(2,(id+1)e^{2ia+2ibx})}{2b}-\frac{i\int \text{PolyLog}(2,(id+1)e^{2ia+2ibx})dx}{2b}}{b(-d+i)}-\frac{x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1+id)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,(id+1)e^{2ia+2ibx}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,(id+1)e^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}}{b(-d+i)}-\frac{x^2\log\left(1-(1+id)\right.}{2b(-d+i}\right.\right.$$

$$\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \; 7143$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1+id)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,(id+1)e^{2ia+2ibx}\right)}{2b}-\frac{\text{PolyLog}\left(3,(id+1)e^{2ia+2ibx}\right)}{4b^2}}{b(-d+i)}-\frac{x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+$$

$$\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+id+1)$$

input
```
Int[x*ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 + I*d + d*Cot[a + b*x]])/2 + (I/2)*b*(x^3/3 + (1 + I*d)*(-1
/2*(x^2*Log[1 - (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I - d)) + (((I/2)*
x*PolyLog[2, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/b - PolyLog[3, (1 + I*d)*
E^((2*I)*a + (2*I)*b*x)]/(4*b^2))/(b*(I - d))))
```

3.258.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6820
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.258.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.09 (sec) , antiderivative size = 2285, normalized size of antiderivative = 17.31

method	result	size
risch	Expression too large to display	2285

input
```
int(x*arccoth(1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(I*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+
a))-1))^3+I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2*
I*(b*x+a)))^2-I*Pi*csgn(I*exp(2*I*(b*x+a)))^3+2*I*Pi*csgn(I*exp(I*(b*x+a))
)*csgn(I*exp(2*I*(b*x+a)))^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x
+a)))^3+I*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(I*d/(exp(2
*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-I*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(
2*I*(b*x+a)))^3+I*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(
1/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a))*d)^2+I*Pi*csgn(I*d)*csgn(I*d/(exp(
2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-I*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(
2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(
b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(
I/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+I*Pi*csgn(1/(exp(2*I*(b*x+a))-1
)*exp(2*I*(b*x+a))*d)^3-I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x
+a))-1))*csgn(I/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))-I*Pi*csgn(I*exp(I*(
b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-I*Pi*csgn(1/(exp(2*I*(b*x+a))-1)*exp(2
*I*(b*x+a))*d)^2-I*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I))*cs
gn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+I*P
i*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*c
sgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))+I*Pi*c
sgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a...
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.18

$$\int x \coth^{-1}(1 + id + d\cot(a + bx))\, dx$$
$$= \frac{4i\, b^3x^3 + 6\, b^2x^2 \log\left(\frac{((d-i)e^{(2i\, bx+2i\, a)}+i)e^{(-2i\, bx-2i\, a)}}{d}\right) + 4i\, a^3 + 6i\, bx\mathrm{Li}_2\left(-(-i\, d-1)e^{(2i\, bx+2i\, a)}\right) - 6\, a^2 \log\left(\frac{(d\cdots}{}\right.}{24\, b^2}$$

input
```
integrate(x*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/24*(4*I*b^3*x^3 + 6*b^2*x^2*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*
I*b*x - 2*I*a)/d) + 4*I*a^3 + 6*I*b*x*dilog(-(-I*d - 1)*e^(2*I*b*x + 2*I*a
)) - 6*a^2*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*(b^2*x^2 - a
^2)*log((-I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) - 3*polylog(3, (I*d + 1)*e^(2*
I*b*x + 2*I*a)))/b^2
```

3.258.6 Sympy [F]

$$\int x\coth^{-1}(1+id+d\cot(a+bx))\,dx=\int x\,\mathrm{acoth}\,(d\cot{(a+bx)}+id+1)\,dx$$

input
```
integrate(x*acoth(1+I*d+d*cot(b*x+a)),x)
```

output
```
Integral(x*acoth(d*cot(a + b*x) + I*d + 1), x)
```

3.258.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(94) = 188.

Time = 0.23 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.89

$$\int x\coth^{-1}(1+id+d\cot(a+bx))\,dx$$
$$=\frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\,\mathrm{arcoth}(d\cot(bx+a)+i\,d+1)}{b}-\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left((i\,d+1)e^{(2i\,bx+2i\,a)}\right)-6\left(i\,(bx+a)^2-2i\,(bx+a)a\right.}{}}{}$$

input
```
integrate(x*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arccoth(d*cot(b*x + a) + I*d + 1)/b
 - (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((I*d + 1)*e^(2*I
*b*x + 2*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(d*cos(2*b*x +
 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1) + 3*(
(b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*
sin(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*po
lylog(3, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

3.258.8 Giac [F]

$$\int x \coth^{-1}(1 + id + d \cot(a + bx)) \, dx = \int x \operatorname{arcoth}\left(d \cot\left(bx + a\right) + i\, d + 1\right) \, dx$$

input
```
integrate(x*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(d*cot(b*x + a) + I*d + 1), x)
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(1 + id + d \cot(a + bx)) \, dx = \int x \operatorname{acoth}(d \cot(a + b\, x) + 1 + d\, 1\mathrm{i}) \, dx$$

input
```
int(x*acoth(d*1i + d*cot(a + b*x) + 1),x)
```

output
```
int(x*acoth(d*1i + d*cot(a + b*x) + 1), x)
```

3.259 $\int \coth^{-1}(1+id+d\cot(a+bx))\,dx$

3.259.1 Optimal result

Integrand size = 16, antiderivative size = 93

$$\int \coth^{-1}(1+id+d\cot(a+bx))\,dx = \frac{1}{2}ibx^2 + x\coth^{-1}(1+id+d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right) + \frac{i\,\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}$$

output
```
1/2*I*b*x^2+x*arccoth(1+I*d+d*cot(b*x+a))-1/2*x*ln(1-(1+I*d)*exp(2*I*a+2*I
*b*x))+1/4*I*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b
```

3.259.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 709 vs. 2(93) = 186.

Time = 2.15 (sec) , antiderivative size = 709, normalized size of antiderivative = 7.62

$$\int \coth^{-1}(1+id+d\cot(a+bx))\,dx = x\coth^{-1}(1+id+d\cot(a+bx)) + \frac{x\csc^2(a+bx)\left(2bx\log(2\cos(bx)(\cos(bx)-i\sin(bx)))+i\right.}{(i+\cot(a+bx))(2+id+d\cot(a+bx))\left(2ibx+\log\left(1+\frac{1}{2}\sec(bx)((-2-id)\cos(a)+d\sin(a))(\cos(a\right.\right.}$$

input
```
Integrate[ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

output
```
x*ArcCoth[1 + I*d + d*Cot[a + b*x]] + (x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos[b
*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*(d*Cos[
a + b*x] + (2 + I*d)*Sin[a + b*x]))/(2*(-I + d))]*Log[1 - I*Tan[b*x]] - I*
Log[(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b
*x]))/2]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] -
I*PolyLog[2, (Sec[b*x]*((2 + I*d)*Cos[a] - d*Sin[a])*(Cos[a + b*x] + I*Sin
[a + b*x]))/2] + I*PolyLog[2, ((Cos[a] - I*Sin[a])*((-2 - I*d)*Cos[a] + d*
Sin[a])*(I + Tan[b*x]))/(2*(-I + d))])*(Cos[b*x] - I*Sin[b*x])*(Cos[b*x] +
 I*Sin[b*x]))/((I + Cot[a + b*x])*(2 + I*d + d*Cot[a + b*x])*((2*I)*b*x +
Log[1 + (Sec[b*x]*((-2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a +
 b*x]))/2] - Log[(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I
*d)*Sin[a + b*x]))/2] + ((-2*I + d)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] - Lo
g[1 + I*Tan[b*x]]))/(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x]) + (d*(Log[1
- I*Tan[b*x]] - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/((-I)*d*Cos[a + b*x] +
(-2*I + d)*Sin[a + b*x]) + 2*b*x*Tan[b*x] - I*Log[1 + (Sec[b*x]*((-2 - I*d
)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*Tan[b*x] + I*Log[
(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x])
)/2]*Tan[b*x] - I*Log[1 - I*Tan[b*x]]*Tan[b*x] + I*Log[1 + I*Tan[b*x]]*Tan
[b*x]))
```

3.259.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6812, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d\cot(a+bx)+id+1)\,dx$$

$$\Big\downarrow \text{ 6812}$$

$$ib\int \frac{x}{1-(id+1)e^{2ia+2ibx}}dx+x\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{ 2615}$$

$$ib\left(\frac{x^2}{2}+(1+id)\int \frac{e^{2ia+2ibx}x}{1-(id+1)e^{2ia+2ibx}}dx\right)+x\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2620}$$

$$ib\left(\frac{x^2}{2}+(1+id)\left(\frac{\int \log\left(1-(id+1)e^{2ia+2ibx}\right)dx}{2b(-d+i)}-\frac{x\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+ x\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2715}$$

$$ib\left(\frac{x^2}{2}+(1+id)\left(-\frac{i\int e^{-2ia-2ibx}\log\left(1-(id+1)e^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2(-d+i)}-\frac{x\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+ x\coth^{-1}(d\cot(a+bx)+id+1)$$

$$\Big\downarrow \text{2838}$$

$$ib\left(\frac{x^2}{2}+(1+id)\left(\frac{i\,\text{PolyLog}\left(2,(id+1)e^{2ia+2ibx}\right)}{4b^2(-d+i)}-\frac{x\log\left(1-(1+id)e^{2ia+2ibx}\right)}{2b(-d+i)}\right)\right)+ x\coth^{-1}(d\cot(a+bx)+id+1)$$

input
```
Int[ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

output
```
x*ArcCoth[1 + I*d + d*Cot[a + b*x]] + I*b*(x^2/2 + (1 + I*d)*(-1/2*(x*Log[
1 - (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I - d)) + ((I/4)*PolyLog[2, (1
 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(b^2*(I - d))))
```

3.259.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6812
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Coth[c + d*Cot[a + b*x]], x] + Simp[I*b   Int[x/(c - I*d - c*E^(2*I*a + 2*I
*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, 1]
```

3.259.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(76) = 152.

Time = 2.55 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.30

method	result
derivativedivides	$\frac{-\frac{i\,\mathrm{arccoth}(1+id+d\cot(bx+a))d\ln(id+d\cot(bx+a))}{2}+\frac{i\,\mathrm{arccoth}(1+id+d\cot(bx+a))d\ln(-id+d\cot(bx+a))}{2}+\frac{d^2\left(-\frac{i\left(-\frac{\mathrm{dilog}\left(1+\frac{id}{2}\right.}{}\right.}{}\right.}{}}{}$
default	$\frac{-\frac{i\,\mathrm{arccoth}(1+id+d\cot(bx+a))d\ln(id+d\cot(bx+a))}{2}+\frac{i\,\mathrm{arccoth}(1+id+d\cot(bx+a))d\ln(-id+d\cot(bx+a))}{2}+\frac{d^2\left(-\frac{i\left(-\frac{\mathrm{dilog}\left(1+\frac{id}{2}\right.}{}\right.}{}\right.}{}}{}$
risch	Expression too large to display

input
```
int(arccoth(1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-1/2*I*arccoth(1+I*d+d*cot(b*x+a))*d*ln(I*d+d*cot(b*x+a))+1/2*I*arc
coth(1+I*d+d*cot(b*x+a))*d*ln(-I*d+d*cot(b*x+a))+1/2*d^2*(-I/d*(-1/2*dilog
(1+1/2*I*d+1/2*d*cot(b*x+a))-1/2*ln(I*d+d*cot(b*x+a))*ln(1+1/2*I*d+1/2*d*c
ot(b*x+a))+1/4*ln(I*d+d*cot(b*x+a))^2)+I/d*(1/2*dilog(-1/2*I*(I*d+d*cot(b*
x+a))/d)+1/2*ln(-I*d+d*cot(b*x+a))*ln(-1/2*I*(I*d+d*cot(b*x+a))/d)-1/2*dil
og(I*(-I*d+d*cot(b*x+a)-I*(2*I-2*d))/(2*I-2*d))-1/2*ln(-I*d+d*cot(b*x+a))*
ln(I*(-I*d+d*cot(b*x+a)-I*(2*I-2*d))/(2*I-2*d)))))
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.30

$$\int \coth^{-1}(1+id+d\cot(a+bx))\,dx$$
$$= \frac{2i\,b^2x^2 + 2\,bx\log\left(\frac{((d-i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right) - 2i\,a^2 - 2\,(bx+a)\log\left((-i\,d-1)e^{(2i\,bx+2i\,a)}+1\right) + 2\,a\,\mathrm{lo}}{4\,b}$$

input
```
integrate(arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*I*b^2*x^2 + 2*b*x*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x
 - 2*I*a)/d) - 2*I*a^2 - 2*(b*x + a)*log((-I*d - 1)*e^(2*I*b*x + 2*I*a) +
1) + 2*a*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) + I*dilog(-(-I*d -
 1)*e^(2*I*b*x + 2*I*a)))/b
```

3.259.6 Sympy [F]

$$\int \coth^{-1}(1+id+d\cot(a+bx))\,dx = \int \operatorname{acoth}\left(d\cot\left(a+bx\right)+id+1\right)dx$$

input
```
integrate(acoth(1+I*d+d*cot(b*x+a)),x)
```

output
```
Integral(acoth(d*cot(a + b*x) + I*d + 1), x)
```

3.259.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(66) = 132$.

Time = 0.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.08

$$\int \coth^{-1}(1+id+d\cot(a+bx))\,dx =$$

$$-\frac{4(bx+a)d\left(\frac{\log((id+2)\tan(bx+a)+d)}{d}-\frac{\log(i\tan(bx+a)+1)}{d}\right)+d\left(-\frac{2i\left(\log((id+2)\tan(bx+a)+d)\log\left(\frac{(d-2i)\tan(bx+a)-id}{2id+2}+1\right)\right.}{d}\right.}{}$$

input
```
integrate(arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*(4*(b*x + a)*d*(log((I*d + 2)*tan(b*x + a) + d)/d - log(I*tan(b*x + a
) + 1)/d) + d*(-2*I*(log((I*d + 2)*tan(b*x + a) + d)*log(((d - 2*I)*tan(b*
x + a) - I*d)/(2*I*d + 2) + 1) + dilog(-((d - 2*I)*tan(b*x + a) - I*d)/(2*
I*d + 2)))/d - 2*I*(log(1/2*(d - 2*I)*tan(b*x + a) - 1/2*I*d)*log(I*tan(b*
x + a) + 1) + dilog(-1/2*(d - 2*I)*tan(b*x + a) + 1/2*I*d + 1))/d + (2*I*l
og((I*d + 2)*tan(b*x + a) + d)*log(I*tan(b*x + a) + 1) - I*log(I*tan(b*x +
 a) + 1)^2)/d + 2*I*(log(I*tan(b*x + a) + 1)*log(-1/2*I*tan(b*x + a) + 1/2
) + dilog(1/2*I*tan(b*x + a) + 1/2))/d) - 8*(b*x + a)*arccoth(I*d + d/tan(
b*x + a) + 1))/b
```

3.259.8 Giac [F]

$$\int \coth^{-1}(1+id+d\cot(a+bx))\,dx = \int \operatorname{arcoth}\left(d\cot\left(bx+a\right)+i\,d+1\right)\,dx$$

input
```
integrate(arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(d*cot(b*x + a) + I*d + 1), x)
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(1 + id + d\cot(a + bx))\, dx = \int \mathrm{acoth}(d\cot(a + b\,x) + 1 + d\,1\mathrm{i})\; dx$$

input
```
int(acoth(d*1i + d*cot(a + b*x) + 1),x)
```

output
```
int(acoth(d*1i + d*cot(a + b*x) + 1), x)
```

3.260 $\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx$

3.260.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx=\operatorname{Int}\left(\frac{\coth^{-1}(1+id+d\cot(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccoth(1+I*d+d*cot(b*x+a))/x,x)
```

3.260.2 Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx=\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[1 + I*d + d*Cot[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[1 + I*d + d*Cot[a + b*x]]/x, x]
```

3.260.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d\cot(a+bx)+id+1)}{x} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\coth^{-1}(d\cot(a+bx)+id+1)}{x}dx$$

input
```
Int[ArcCoth[1 + I*d + d*Cot[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.260.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.260.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arccoth}\left(1+id+d\cot\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(1+I*d+d*cot(b*x+a))/x,x)
```

output
```
int(arccoth(1+I*d+d*cot(b*x+a))/x,x)
```

3.260.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\cot\left(bx+a\right)+i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+I*d+d*cot(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d)
/x, x)
```

3.260.6 Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{acoth}\left(d\cot\left(a+bx\right)+id+1\right)}{x}\,dx$$

input
```
integrate(acoth(1+I*d+d*cot(b*x+a))/x,x)
```

output
```
Integral(acoth(d*cot(a + b*x) + I*d + 1)/x, x)
```

3.260.7 Maxima [N/A]

Not integrable

Time = 5.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 7.05

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx = \int \frac{\operatorname{arcoth}\left(d\cot\left(bx+a\right)+i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+I*d+d*cot(b*x+a))/x,x, algorithm="maxima")
```

output
```
-I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(d))*log(x) + 1/2*I*integrate(arctan2(d
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*
a) - 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)
*sin(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1)/x, x)
```

3.260.8 Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(d\cot\left(bx+a\right)+i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1+I*d+d*cot(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(d*cot(b*x + a) + I*d + 1)/x, x)
```

3.260.9 Mupad [N/A]

Not integrable

Time = 5.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx=\int \frac{\mathrm{acoth}(d\cot(a+b\,x)+1+d\,1\mathrm{i})}{x}\,dx$$

input
```
int(acoth(d*1i + d*cot(a + b*x) + 1)/x,x)
```

output
```
int(acoth(d*1i + d*cot(a + b*x) + 1)/x, x)
```

3.261 $\int x^2 \coth^{-1}(1 - id - d\cot(a + bx))\,dx$

3.261.1 Optimal result

Integrand size = 21, antiderivative size = 169

$$\begin{aligned}\int x^2 \coth^{-1}(1-id-d\cot(a+bx))\,dx =& \frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx)) \\ &-\frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+2ibx}\right) \\ &+\frac{ix^2\operatorname{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b} \\ &-\frac{x\operatorname{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{4b^2} \\ &-\frac{i\operatorname{PolyLog}\left(4,(1-id)e^{2ia+2ibx}\right)}{8b^3}\end{aligned}$$

output
```
1/12*I*b*x^4+1/3*x^3*arccoth(1-I*d-d*cot(b*x+a))-1/6*x^3*ln(1-(1-I*d)*exp(
2*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*po
lylog(3,(1-I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,(1-I*d)*exp(2*I*a+
2*I*b*x))/b^3
```

3.261.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.92

$$\int x^2 \coth^{-1}(1-id-d\cot(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx)) - \frac{4b^3x^3\log\left(1+\frac{e^{-2i(a+bx)}}{-1+id}\right)+6ib^2x^2\operatorname{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{i+d}\right)+6bx\operatorname{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{i+d}\right)-3i\operatorname{PolyLog}\left(\right.}{24b^3}$$

input
```
Integrate[x^2*ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 - I*d - d*Cot[a + b*x]])/3 - (4*b^3*x^3*Log[1 + 1/((-1 + I*
d)*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/((I + d)*E^((2*I)*(a
 + b*x)))] + 6*b*x*PolyLog[3, I/((I + d)*E^((2*I)*(a + b*x)))] - (3*I)*Pol
yLog[4, I/((I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.261.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6820, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\coth^{-1}(d(-\cot(a+bx))-id+1)\,dx$$

$$\Big\downarrow \text{6820}$$

$$\frac{1}{3}ib\int\frac{x^3}{1-(1-id)e^{2ia+2ibx}}dx+\frac{1}{3}x^3\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\int\frac{e^{2ia+2ibx}x^3}{1-(1-id)e^{2ia+2ibx}}dx\right)+\frac{1}{3}x^3\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\left(\frac{3\int x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)dx}{2b(d+i)}-\frac{x^3\log\left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+\frac{1}{3}x^3\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow 3011$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)dx}{b}\right)}{2b(d+i)}-\frac{x^3\log\left(1-(1-id)e^{2ia+2ibx}\right.}{2b(d+i)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow 7163$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2b(d+i)}-\frac{x}{}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow 2720$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,(1-id)e^{2i\ldots}\right.}{2b}\right.}{b}\right.}{2b(d+i)}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow 7143$$

$$\frac{1}{3}ib\left(\frac{x^4}{4}+(1-id)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,(1-id)e^{2ia+2ibx}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2b(d+i)}-\frac{x^3\log}{}\right.\right.$$

$$\frac{1}{3}x^3\coth^{-1}(d(-\cot(a+bx))-id+1)$$

3.261. $\int x^2\coth^{-1}(1-id-d\cot(a+bx))\,dx$

input
```
Int[x^2*ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

output
```
(x^3*ArcCoth[1 - I*d - d*Cot[a + b*x]])/3 + (I/3)*b*(x^4/4 + (1 - I*d)*(-1
/2*(x^3*Log[1 - (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I + d)) + (3*(((I/
2)*x^2*PolyLog[2, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/b - (I*(((-1/2*I)*x*
PolyLog[3, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/b + PolyLog[4, (1 - I*d)*E^
((2*I)*a + (2*I)*b*x)]/(4*b^2)))/b))/(2*b*(I + d))))
```

3.261.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6820
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.261.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.49 (sec) , antiderivative size = 2273, normalized size of antiderivative = 13.45

method	result	size
risch	Expression too large to display	2273

input
```
int(x^2*arccoth(1-I*d-d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^3*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)-1/2*I/b^2*a^2/(I+d)*ln
(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x+1/12*I*b*x^4-1/4/b/(I+d)*polylog(2,
-I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b
*x+a)))*a^2-1/2/b^3*a^2/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/
2/b^3*a^2/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/6*d/(I+d)*ln(1
+I*(I+d)*exp(2*I*(b*x+a)))*x^3-1/6*I/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*
x^3-1/3*x^3*ln(exp(I*(b*x+a)))+1/6/b^3*a^3*d/(I+d)*ln(I*exp(2*I*(b*x+a))+e
xp(2*I*(b*x+a))*d-I)-1/4/b^2*d/(I+d)*polylog(3,-I*(I+d)*exp(2*I*(b*x+a)))*
x+1/3/b^3*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/2/b^3*d*a^3/(I+d)*l
n(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/2/b^3*d*a^3/(I+d)*ln(1-I*exp(I*(b*
x+a))*(I*(I+d))^(1/2))-1/8*I/b^3*d/(I+d)*polylog(4,-I*(I+d)*exp(2*I*(b*x+a
)))-1/2*I/b^3*a^3/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/2*I/b^3*a
^3/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/6*I/b^3*a^3/(I+d)*ln(I*e
xp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)+1/3*I/b^3/(I+d)*ln(1+I*(I+d)*exp(2*I
*(b*x+a)))*a^3-1/4*I/b^2/(I+d)*polylog(3,-I*(I+d)*exp(2*I*(b*x+a)))*x-1/2/
b^2*d*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x-1/2/b^2*d*a^2/(I+
d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x+1/2/b^2*d/(I+d)*ln(1+I*(I+d)*e
xp(2*I*(b*x+a)))*a^2*x+1/2*I/b^3*d*a^2/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(
I+d))^(1/2))+1/2*I/b^3*d*a^2/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2
))-1/4*I/b^3*a^2*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))+1/2*I/b^2...
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.06

$$\int x^2 \coth^{-1}(1 - id - d\cot(a + bx))\,dx$$
$$= \frac{2i\,b^4x^4 - 4\,b^3x^3\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(-(i\,d-1)e^{(2i\,bx+2i\,a)}\right) - 2i\,a^4 + 4\,a^3\log\left(\frac{(d+i)e^{(2i\,bx+2i\,a)}}{d+i}\right.}{}$$

input
```
integrate(x^2*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/24*(2*I*b^4*x^4 - 4*b^3*x^3*log(d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*
x + 2*I*a) - I)) + 6*I*b^2*x^2*dilog(-(I*d - 1)*e^(2*I*b*x + 2*I*a)) - 2*I
*a^4 + 4*a^3*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) - 6*b*x*polylo
g(3, (-I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((I*d - 1)*e^(
2*I*b*x + 2*I*a) + 1) - 3*I*polylog(4, (-I*d + 1)*e^(2*I*b*x + 2*I*a)))/b^
3
```

3.261.6 Sympy [F]

$$\int x^2 \coth^{-1}(1 - id - d\cot(a + bx))\, dx = -\int x^2 \operatorname{acoth}\left(d\cot\left(a + bx\right) + id - 1\right) dx$$

input
```
integrate(x**2*acoth(1-I*d-d*cot(b*x+a)),x)
```

output
```
-Integral(x**2*acoth(d*cot(a + b*x) + I*d - 1), x)
```

3.261.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(120) = 240.

Time = 0.24 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.04

$$\int x^2 \coth^{-1}(1 - id - d\cot(a + bx))\, dx =$$
$$-\frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{arcoth}(d\cot(bx+a)+i\,d-1)}{b^2} + \frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(4i\,(bx+a)^3-9i\,(bx+\right.}{}}{}$$

input
```
integrate(x^2*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*cot(
b*x + a) + I*d - 1)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b
*x + a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2
)*arctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - co
s(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilo
g((-I*d + 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(
b*x + a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a
)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polyl
og(3, (-I*d + 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d + 1)*e^(2*I*b
*x + 2*I*a)))/b^2)/b
```

3.261.8 Giac [F]

$$\int x^2 \coth^{-1}(1 - id - d\cot(a + bx))\,dx = \int x^2 \operatorname{arcoth}\left(-d\cot\left(bx + a\right) - i\,d + 1\right)\,dx$$

input
```
integrate(x^2*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(-d*cot(b*x + a) - I*d + 1), x)
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(1 - id - d\cot(a + bx))\,dx = \int -x^2 \operatorname{acoth}(d\cot(a + b\,x) - 1 + d\,1\mathrm{i})\,dx$$

input
```
int(-x^2*acoth(d*1i + d*cot(a + b*x) - 1),x)
```

output
```
int(-x^2*acoth(d*1i + d*cot(a + b*x) - 1), x)
```

3.262 $\int x \coth^{-1}(1-id-d\cot(a+bx))\,dx$

3.262.1 Optimal result

Integrand size = 19, antiderivative size = 133

$$\int x \coth^{-1}(1-id-d\cot(a+bx))\,dx = \frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id-d\cot(a+bx)) - \frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right) + \frac{ix\,\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b} - \frac{\text{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{8b^2}$$

output
```
1/6*I*b*x^3+1/2*x^2*arccoth(1-I*d-d*cot(b*x+a))-1/4*x^2*ln(1-(1-I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylog
(3,(1-I*d)*exp(2*I*a+2*I*b*x))/b^2
```

3.262.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\int x \coth^{-1}(1-id-d\cot(a+bx))\,dx = \frac{1}{2}x^2\coth^{-1}(1-id-d\cot(a+bx)) - \frac{2b^2x^2\log\left(1+\frac{e^{-2i(a+bx)}}{-1+id}\right)+2ibx\,\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{i+d}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{i+d}\right)}{8b^2}$$

input
```
Integrate[x*ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 - I*d - d*Cot[a + b*x]])/2 - (2*b^2*x^2*Log[1 + 1/((-1 + I*
d)*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/((I + d)*E^((2*I)*(a + b
*x)))] + PolyLog[3, I/((I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

3.262.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6820, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}(d(-\cot(a+bx))-id+1)\,dx$$

$$\Big\downarrow \text{6820}$$

$$\frac{1}{2}ib\int \frac{x^2}{1-(1-id)e^{2ia+2ibx}}dx+\frac{1}{2}x^2\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1-id)\int \frac{e^{2ia+2ibx}x^2}{1-(1-id)e^{2ia+2ibx}}dx\right)+\frac{1}{2}x^2\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1-id)\left(\frac{\int x\log\left(1-(1-id)e^{2ia+2ibx}\right)dx}{b(d+i)}-\frac{x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1-id)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)dx}{2b}}{b(d+i)}-\frac{x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+\frac{1}{2}x^2\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1-id)\left(\frac{\frac{ix\operatorname{PolyLog}(2,(1-id)e^{2ia+2ibx})}{2b}-\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}(2,(1-id)e^{2ia+2ibx})de^{2ia+2ibx}}{4b^2}}{b(d+i)}-\frac{x^2\log\left(1-(1-id)\right.}{2b(d+i)}\right.\right.$$
$$\frac{1}{2}x^2\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}ib\left(\frac{x^3}{3}+(1-id)\left(\frac{\frac{ix\operatorname{PolyLog}(2,(1-id)e^{2ia+2ibx})}{2b}-\frac{\operatorname{PolyLog}(3,(1-id)e^{2ia+2ibx})}{4b^2}}{b(d+i)}-\frac{x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+$$
$$\frac{1}{2}x^2\coth^{-1}(d(-\cot(a+bx))-id+1)$$

input
```
Int[x*ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

output
```
(x^2*ArcCoth[1 - I*d - d*Cot[a + b*x]])/2 + (I/2)*b*(x^3/3 + (1 - I*d)*(-1
/2*(x^2*Log[1 - (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I + d)) + (((I/2)*
x*PolyLog[2, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/b - PolyLog[3, (1 - I*d)*
E^((2*I)*a + (2*I)*b*x)]/(4*b^2))/(b*(I + d))))
```

3.262.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6820
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^
(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
 EqQ[(c - I*d)^2, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.262.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.07 (sec) , antiderivative size = 2183, normalized size of antiderivative = 16.41

method	result	size
risch	Expression too large to display	2183

input
```
int(x*arccoth(1-I*d-d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/6*I*b*x^3-1/4*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x^2-1/4*I/(I+d)*ln(
1+I*(I+d)*exp(2*I*(b*x+a)))*x^2-1/8*I/b^2/(I+d)*polylog(3,-I*(I+d)*exp(2*I
*(b*x+a)))-1/4/b^2*a^2*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)
+1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/2/b^2*a^2*d/
(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/4/b^2*d/(I+d)*ln(1+I*(I+d)*
exp(2*I*(b*x+a)))*a^2-1/4*I/b^2*a^2/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b
*x+a))*d-I)+1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/2
*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/4*I/b^2/(I+d)*ln
(1+I*(I+d)*exp(2*I*(b*x+a)))*a^2+1/2/b^2*a/(I+d)*dilog(1-I*exp(I*(b*x+a))*
(I*(I+d))^(1/2))-1/4/b/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x-1/4/b^
2/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*a-1/8/b^2*d/(I+d)*polylog(3,-
I*(I+d)*exp(2*I*(b*x+a)))+1/2/b^2*a/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d
))^(1/2))-1/2*I/b^2*a*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/
2*I/b^2*a*d/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/2*I/b/(I+d)*
ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a*x+1/4*I/b*d/(I+d)*polylog(2,-I*(I+d)*exp(
2*I*(b*x+a)))*x+1/2/b*a*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x+1
/2/b*a*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x-1/2/b*d/(I+d)*ln(1
+I*(I+d)*exp(2*I*(b*x+a)))*a*x+1/4*I/b^2*d/(I+d)*polylog(2,-I*(I+d)*exp(2*
I*(b*x+a)))*a+1/2*I/b*a/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x+1/2
*I/b*a/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x-1/2*x^2*ln(exp(I*...
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.17

$$\int x\coth^{-1}(1-id-d\cot(a+bx))\,dx = \frac{4i\,b^3x^3-6\,b^2x^2\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right)+4i\,a^3+6i\,bx\text{Li}_2\left(-(i\,d-1)e^{(2i\,bx+2i\,a)}\right)-6\,a^2\log\left(\frac{(d+i)e^{(2i\,bx+2i\,a)}-i}{d+i}\right.}{24\,b^2}$$

input
```
integrate(x*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/24*(4*I*b^3*x^3 - 6*b^2*x^2*log(d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*
x + 2*I*a) - I)) + 4*I*a^3 + 6*I*b*x*dilog(-(I*d - 1)*e^(2*I*b*x + 2*I*a))
 - 6*a^2*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) - 6*(b^2*x^2 - a^2
)*log((I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) - 3*polylog(3, (-I*d + 1)*e^(2*I*
b*x + 2*I*a))/b^2
```

3.262.6 Sympy [F]

$$\int x \coth^{-1}(1 - id - d\cot(a + bx))\, dx = -\int x\,\text{acoth}\,(d\cot\,(a + bx) + id - 1)\, dx$$

input
```
integrate(x*acoth(1-I*d-d*cot(b*x+a)),x)
```

output
```
-Integral(x*acoth(d*cot(a + b*x) + I*d - 1), x)
```

3.262.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(95) = 190.

Time = 0.23 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.88

$$\int x \coth^{-1}(1 - id - d\cot(a + bx))\, dx =$$
$$-\frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\text{arcoth}(d\cot(bx+a)+i\,d-1)}{b}+\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\text{Li}_2\left((-i\,d+1)e^{(2i\,bx+2i\,a)}\right)-6\left(i\,(bx+a)^2-2i\,(bx\right.}{}}{}$$

input
```
integrate(x*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arccoth(d*cot(b*x + a) + I*d - 1)/
b + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((-I*d + 1)*e^(2
*I*b*x + 2*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(-d*cos(2*b*
x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1) +
 3*((b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 +
 1)*sin(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) +
3*polylog(3, (-I*d + 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

3.262.8 Giac [F]

$$\int x \coth^{-1}(1 - id - d\cot(a + bx))\,dx = \int x\,\mathrm{arcoth}\left(-d\cot\left(bx + a\right) - i\,d + 1\right)\,dx$$

input
```
integrate(x*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(-d*cot(b*x + a) - I*d + 1), x)
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(1 - id - d\cot(a + bx))\,dx = \int -x\,\mathrm{acoth}(d\cot(a + b\,x) - 1 + d\,1\mathrm{i})\,dx$$

input
```
int(-x*acoth(d*1i + d*cot(a + b*x) - 1),x)
```

output
```
int(-x*acoth(d*1i + d*cot(a + b*x) - 1), x)
```

3.263 $\int \coth^{-1}(1-id-d\cot(a+bx))\,dx$

3.263.1 Optimal result

Integrand size = 17, antiderivative size = 94

$$\int \coth^{-1}(1-id-d\cot(a+bx))\,dx = \frac{1}{2}ibx^2+x\coth^{-1}(1-id-d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right) + \frac{i\,\mathrm{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}$$

output
```
1/2*I*b*x^2+x*arccoth(1-I*d-d*cot(b*x+a))-1/2*x*ln(1-(1-I*d)*exp(2*I*a+2*I
*b*x))+1/4*I*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b
```

3.263.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 605 vs. 2(94) = 188.

Time = 1.84 (sec) , antiderivative size = 605, normalized size of antiderivative = 6.44

$$\int \coth^{-1}(1-id-d\cot(a+bx))\,dx = x\coth^{-1}(1-id-d\cot(a+bx)) + \frac{x\csc^2(a+bx)\left(2bx\log(2\cos(bx)(\cos(bx)-i\sin(bx)))+i\log\left(\frac{\sec(bx)(\cos(a)-i\sin(a))(d\cos(a+bx)+i(2i+d)\sin(a+b}{2(i+d)}\right.\right.}{(i+\cot(a+bx))(-2+id+d\cot(a+}$$

input
```
Integrate[ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

output
```
x*ArcCoth[1 - I*d - d*Cot[a + b*x]] + (x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos[b
*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*(d*Cos[
a + b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*(I + d))]*Log[1 - I*Tan[b*x]] - I
*Log[(I*Sec[b*x]*(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*Cos[a] -
(2*I)*Sin[a])]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*
x]] - I*PolyLog[2, (Sec[b*x]*((2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] +
 I*Sin[a + b*x]))/2] + I*PolyLog[2, ((Cos[a] - I*Sin[a])*((2 - I*d)*Cos[a]
 + d*Sin[a])*(I + Tan[b*x]))/(2*(I + d))])*(Cos[b*x] - I*Sin[b*x])*(Cos[b*
x] + I*Sin[b*x]))/((I + Cot[a + b*x])*(-2 + I*d + d*Cot[a + b*x])*(-((Log[
1 - I*Tan[b*x]]*Sec[b*x]*((2*I + d)*Cos[a] + I*d*Sin[a]))/(d*Cos[a + b*x]
+ I*(2*I + d)*Sin[a + b*x])) + (Log[1 + I*Tan[b*x]]*Sec[b*x]*((2*I + d)*Co
s[a] + I*d*Sin[a]))/(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]) + (Log[(I*
Sec[b*x]*(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Si
n[a])]*Sec[b*x]^2)/(1 + I*Tan[b*x]) - 2*b*x*(I + Tan[b*x]) + I*Log[1 - (Se
c[b*x]*((2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*(
I + Tan[b*x])))
```

3.263.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6812, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(d(-\cot(a+bx))-id+1)\,dx$$

$$\Big\downarrow \text{ 6812}$$

$$ib\int \frac{x}{1-(1-id)e^{2ia+2ibx}}dx+x\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{ 2615}$$

$$ib\left(\frac{x^2}{2}+(1-id)\int \frac{e^{2ia+2ibx}x}{1-(1-id)e^{2ia+2ibx}}dx\right)+x\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \text{ 2620}$$

$$ib\left(\frac{x^2}{2}+(1-id)\left(\frac{\int \log\left(1-(1-id)e^{2ia+2ibx}\right)dx}{2b(d+i)}-\frac{x\log\left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+$$
$$x\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \; 2715$$

$$ib\left(\frac{x^2}{2}+(1-id)\left(-\frac{i\int e^{-2ia-2ibx}\log\left(1-(1-id)e^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2(d+i)}-\frac{x\log\left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+$$
$$x\coth^{-1}(d(-\cot(a+bx))-id+1)$$

$$\Big\downarrow \; 2838$$

$$ib\left(\frac{x^2}{2}+(1-id)\left(\frac{i\,\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b^2(d+i)}-\frac{x\log\left(1-(1-id)e^{2ia+2ibx}\right)}{2b(d+i)}\right)\right)+$$
$$x\coth^{-1}(d(-\cot(a+bx))-id+1)$$

input
```
Int[ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

output
```
x*ArcCoth[1 - I*d - d*Cot[a + b*x]] + I*b*(x^2/2 + (1 - I*d)*(-1/2*(x*Log[
1 - (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(b*(I + d)) + ((I/4)*PolyLog[2, (1
 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(b^2*(I + d))))
```

3.263.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6812
```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Coth[c + d*Cot[a + b*x]], x] + Simp[I*b   Int[x/(c - I*d - c*E^(2*I*a + 2*I
*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, 1]
```

3.263.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(77) = 154$.

Time = 2.38 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.40

method	result
derivativedivides	$-\frac{-\frac{i \operatorname{arccoth}(1-id-d\cot(bx+a))d\ln(id-d\cot(bx+a))}{2}+\frac{i \operatorname{arccoth}(1-id-d\cot(bx+a))d\ln(-id-d\cot(bx+a))}{2}-\frac{d^2\left(-\frac{i\left(-\frac{\operatorname{dilog}\left(1-\right.}{}\right.}{}\right.}{}}{}$
default	$-\frac{-\frac{i \operatorname{arccoth}(1-id-d\cot(bx+a))d\ln(id-d\cot(bx+a))}{2}+\frac{i \operatorname{arccoth}(1-id-d\cot(bx+a))d\ln(-id-d\cot(bx+a))}{2}-\frac{d^2\left(-\frac{i\left(-\frac{\operatorname{dilog}\left(1-\right.}{}\right.}{}\right.}{}}{}$
risch	Expression too large to display

input
```
int(arccoth(1-I*d-d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/b/d*(-1/2*I*arccoth(1-I*d-d*cot(b*x+a))*d*ln(I*d-d*cot(b*x+a))+1/2*I*ar
ccoth(1-I*d-d*cot(b*x+a))*d*ln(-I*d-d*cot(b*x+a))-1/2*d^2*(-I/d*(-1/2*dilo
g(1-1/2*I*d-1/2*d*cot(b*x+a))-1/2*ln(-I*d-d*cot(b*x+a))*ln(1-1/2*I*d-1/2*d
*cot(b*x+a))+1/4*ln(-I*d-d*cot(b*x+a))^2)+I/d*(-1/2*dilog(I*(I*d-d*cot(b*x
+a)-I*(2*I+2*d))/(2*I+2*d))-1/2*ln(I*d-d*cot(b*x+a))*ln(I*(I*d-d*cot(b*x+a
)-I*(2*I+2*d))/(2*I+2*d))+1/2*dilog(1/2*I*(-I*d-d*cot(b*x+a))/d)+1/2*ln(I*
d-d*cot(b*x+a))*ln(1/2*I*(-I*d-d*cot(b*x+a))/d)))
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int \coth^{-1}(1 - id - d\cot(a + bx))\, dx$$
$$= \frac{2i\, b^2x^2 - 2\, bx \log\left(\frac{de^{(2i\, bx+2i\, a)}}{(d+i)e^{(2i\, bx+2i\, a)}-i}\right) - 2i\, a^2 - 2\,(bx + a)\log\left((i\, d - 1)e^{(2i\, bx+2i\, a)} + 1\right) + 2\, a\log\left(\frac{(d+i)e^{(2i\, bx+}}{d+i}\right.}{4\, b}$$

input
```
integrate(arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*I*b^2*x^2 - 2*b*x*log(d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x + 2
*I*a) - I)) - 2*I*a^2 - 2*(b*x + a)*log((I*d - 1)*e^(2*I*b*x + 2*I*a) + 1)
 + 2*a*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) + I*dilog(-(I*d - 1)
*e^(2*I*b*x + 2*I*a)))/b
```

3.263.6 Sympy [F]

$$\int \coth^{-1}(1 - id - d\cot(a + bx))\, dx = -\int \operatorname{acoth}\left(d\cot\left(a + bx\right) + id - 1\right) dx$$

input
```
integrate(acoth(1-I*d-d*cot(b*x+a)),x)
```

output
```
-Integral(acoth(d*cot(a + b*x) + I*d - 1), x)
```

3.263.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(67) = 134.

Time = 0.31 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.06

$$\int \coth^{-1}(1 - id - d\cot(a + bx))\, dx =$$
$$-\frac{4\,(bx + a)d\left(\frac{\log((i\, d-2)\tan(bx+a)+d)}{d} - \frac{\log(i\, \tan(bx+a)+1)}{d}\right) - d\left(\frac{2i\,\left(\log((i\, d-2)\tan(bx+a)+d)\log\left(\frac{(d+2i)\tan(bx+a)-i\, d}{2i\, d-2}+1\right)+\mathrm{I}\right.}{d}\right.}{}$$

input
```
integrate(arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*(4*(b*x + a)*d*(log((I*d - 2)*tan(b*x + a) + d)/d - log(I*tan(b*x + a
) + 1)/d) - d*(2*I*(log((I*d - 2)*tan(b*x + a) + d)*log(((d + 2*I)*tan(b*x
 + a) - I*d)/(2*I*d - 2) + 1) + dilog(-((d + 2*I)*tan(b*x + a) - I*d)/(2*I
*d - 2)))/d + 2*I*(log(-1/2*(d + 2*I)*tan(b*x + a) + 1/2*I*d)*log(I*tan(b*
x + a) + 1) + dilog(1/2*(d + 2*I)*tan(b*x + a) - 1/2*I*d + 1))/d - (2*I*lo
g((I*d - 2)*tan(b*x + a) + d)*log(I*tan(b*x + a) + 1) - I*log(I*tan(b*x +
a) + 1)^2)/d - 2*I*(log(I*tan(b*x + a) + 1)*log(-1/2*I*tan(b*x + a) + 1/2)
 + dilog(1/2*I*tan(b*x + a) + 1/2))/d) + 8*(b*x + a)*arccoth(I*d + d/tan(b
*x + a) - 1))/b
```

3.263.8 Giac [F]

$$\int \coth^{-1}(1-id-d\cot(a+bx))\,dx = \int \operatorname{arcoth}\left(-d\cot\left(bx+a\right)-i\,d+1\right)\,dx$$

input
```
integrate(arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(-d*cot(b*x + a) - I*d + 1), x)
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(1-id-d\cot(a+bx))\,dx = \int -\operatorname{acoth}(d\cot(a+b\,x)-1+d\,1\mathrm{i})\,dx$$

input
```
int(-acoth(d*1i + d*cot(a + b*x) - 1),x)
```

output
```
int(-acoth(d*1i + d*cot(a + b*x) - 1), x)
```

3.264 $\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx$

3.264.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx = \operatorname{Int}\left(\frac{\coth^{-1}(1-id-d\cot(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arccoth(1-I*d-d*cot(b*x+a))/x,x)
```

3.264.2 Mathematica [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx$$

input
```
Integrate[ArcCoth[1 - I*d - d*Cot[a + b*x]]/x,x]
```

output
```
Integrate[ArcCoth[1 - I*d - d*Cot[a + b*x]]/x, x]
```

3.264.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{-1}(d(-\cot(a+bx))-id+1)}{x} \, dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{\coth^{-1}(d(-\cot(a+bx))-id+1)}{x}dx$$

input
```
Int[ArcCoth[1 - I*d - d*Cot[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.264.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.264.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{arccoth}\left(1-id-d\cot\left(bx+a\right)\right)}{x}dx$$

input
```
int(arccoth(1-I*d-d*cot(b*x+a))/x,x)
```

output
```
int(arccoth(1-I*d-d*cot(b*x+a))/x,x)
```

3.264.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.71

$$\int \frac{\coth^{-1}(1 - id - d\cot(a + bx))}{x}\, dx = \int \frac{\operatorname{arcoth}\left(-d\cot\left(bx + a\right) - i\,d + 1\right)}{x}\, dx$$

input
```
integrate(arccoth(1-I*d-d*cot(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(-1/2*log(d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x + 2*I*a) - I))
/x, x)
```

3.264.6 Sympy [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\coth^{-1}(1 - id - d\cot(a + bx))}{x}\, dx = -\int \frac{\operatorname{acoth}\left(d\cot\left(a + bx\right) + id - 1\right)}{x}\, dx$$

input
```
integrate(acoth(1-I*d-d*cot(b*x+a))/x,x)
```

output
```
-Integral(acoth(d*cot(a + b*x) + I*d - 1)/x, x)
```

3.264.7 Maxima [N/A]

Not integrable

Time = 5.00 (sec) , antiderivative size = 144, normalized size of antiderivative = 6.86

$$\int \frac{\coth^{-1}(1 - id - d\cot(a + bx))}{x}\, dx = \int \frac{\operatorname{arcoth}\left(-d\cot\left(bx + a\right) - i\,d + 1\right)}{x}\, dx$$

input
```
integrate(arccoth(1-I*d-d*cot(b*x+a))/x,x, algorithm="maxima")
```

output
```
-I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(d))*log(x) - 1/2*I*integrate(arctan2(-
d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2
*a) + 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1
)*sin(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1)/x, x
)
```

3.264.8 Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx=\int \frac{\operatorname{arcoth}\left(-d\cot\left(bx+a\right)-i\,d+1\right)}{x}\,dx$$

input
```
integrate(arccoth(1-I*d-d*cot(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arccoth(-d*cot(b*x + a) - I*d + 1)/x, x)
```

3.264.9 Mupad [N/A]

Not integrable

Time = 5.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx=\int -\frac{\mathrm{acoth}(d\cot(a+b\,x)-1+d\,1\mathrm{i})}{x}\,dx$$

input
```
int(-acoth(d*1i + d*cot(a + b*x) - 1)/x,x)
```

output
```
int(-acoth(d*1i + d*cot(a + b*x) - 1)/x, x)
```

3.265 $\int \frac{(a+b\coth^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx$

3.265.1 Optimal result

Integrand size = 24, antiderivative size = 160

$$\int \frac{(a+b\coth^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx = ad\log(x)+\frac{ae\log^2(fx^m)}{2m}+\frac{bd\,\mathrm{PolyLog}\left(2,-\frac{x^{-n}}{c}\right)}{2n}+\frac{be\log(fx^m)\,\mathrm{PolyLog}\left(2,-\frac{x^{-n}}{c}\right)}{2n}-\frac{bd\,\mathrm{PolyLog}\left(2,\frac{x^{-n}}{c}\right)}{2n}-\frac{be\log(fx^m)\,\mathrm{PolyLog}\left(2,\frac{x^{-n}}{c}\right)}{2n}+\frac{bem\,\mathrm{PolyLog}\left(3,-\frac{x^{-n}}{c}\right)}{2n^2}-\frac{bem\,\mathrm{PolyLog}\left(3,\frac{x^{-n}}{c}\right)}{2n^2}$$

output
```
a*d*ln(x)+1/2*a*e*ln(f*x^m)^2/m+1/2*b*d*polylog(2,-1/c/(x^n))/n+1/2*b*e*ln
(f*x^m)*polylog(2,-1/c/(x^n))/n-1/2*b*d*polylog(2,1/c/(x^n))/n-1/2*b*e*ln(
f*x^m)*polylog(2,1/c/(x^n))/n+1/2*b*e*m*polylog(3,-1/c/(x^n))/n^2-1/2*b*e*
m*polylog(3,1/c/(x^n))/n^2
```

3.265.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.82

$$\int \frac{(a+b\coth^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx$$
$$= -\frac{bcemx^n\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2},\frac{3}{2};c^2x^{2n}\right)}{n^2}+\frac{bcx^n\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2};c^2x^{2n}\right)(d+e\log(fx^m))}{n}$$
$$-\frac{1}{2}\left(a+b\coth^{-1}(cx^n)-b\text{arctanh}(cx^n)\right)\log(x)\left(em\log(x)-2(d+e\log(fx^m))\right)$$

input
```
Integrate[((a + b*ArcCoth[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

output
```
-((b*c*e*m*x^n*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, c^2*
x^(2*n)])/n^2) + (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, c^2
*x^(2*n)]*(d + e*Log[f*x^m]))/n - ((a + b*ArcCoth[c*x^n] - b*ArcTanh[c*x^n
])*Log[x]*(e*m*Log[x] - 2*(d + e*Log[f*x^m])))/2
```

3.265.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\coth^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{d(a+b\coth^{-1}(cx^n))}{x}+\frac{e\log(fx^m)(a+b\coth^{-1}(cx^n))}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$ad\log(x)+\frac{ae\log^2(fx^m)}{2m}+\frac{bd\,\text{PolyLog}\left(2,-\frac{x^{-n}}{c}\right)}{2n}-\frac{bd\,\text{PolyLog}\left(2,\frac{x^{-n}}{c}\right)}{2n}+\frac{be\,\text{PolyLog}\left(2,-\frac{x^{-n}}{c}\right)\log(fx^m)}{2n}-\frac{be\,\text{PolyLog}\left(2,\frac{x^{-n}}{c}\right)\log(fx^m)}{2n}+\frac{bem\,\text{PolyLog}\left(3,-\frac{x^{-n}}{c}\right)}{2n^2}-\frac{bem\,\text{PolyLog}\left(3,\frac{x^{-n}}{c}\right)}{2n^2}$$

input
```
Int[((a + b*ArcCoth[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

output
```
a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) + (b*d*PolyLog[2, -(1/(c*x^n))])/(2*
n) + (b*e*Log[f*x^m]*PolyLog[2, -(1/(c*x^n))])/(2*n) - (b*d*PolyLog[2, 1/(
c*x^n)])/(2*n) - (b*e*Log[f*x^m]*PolyLog[2, 1/(c*x^n)])/(2*n) + (b*e*m*Pol
yLog[3, -(1/(c*x^n))])/(2*n^2) - (b*e*m*PolyLog[3, 1/(c*x^n)])/(2*n^2)
```

3.265.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.265.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 146.70 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.59

method	result
risch	$\frac{\left(-\frac{ie\pi\,\text{csgn}(if)\,\text{csgn}(ix^m)\,\text{csgn}(if\,x^m)}{4}+\frac{ie\pi\,\text{csgn}(if)\,\text{csgn}(if\,x^m)^2}{4}+\frac{ie\pi\,\text{csgn}(ix^m)\,\text{csgn}(if\,x^m)^2}{4}-\frac{ie\pi\,\text{csgn}(if\,x^m)^3}{4}+\frac{e\ln(f)}{2}+\frac{d}{2}\right)(-b\,\text{dilog}}{n}$

input
```
int((a+b*arccoth(c*x^n))*(d+e*ln(f*x^m))/x,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*I*e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/4*I*e*Pi*csgn(I*f)*csgn
(I*f*x^m)^2+1/4*I*e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/4*I*e*Pi*csgn(I*f*x^m
)^3+1/2*e*ln(f)+1/2*d)/n*(-b*dilog(c*x^n+1)+2*a*ln(x^n)-ln(c*x^n)*ln(c*x^n
-1)*b-dilog(c*x^n)*b)-1/2*e*b*m/n*ln(x)*polylog(2,-c*x^n)+1/2*b*e*m*polylo
g(3,-c*x^n)/n^2+1/2*e*b/n*dilog(c*x^n+1)*m*ln(x)-1/2*e*b/n*dilog(c*x^n+1)*
ln(x^m)+1/2*e*a/m*ln(x^m)^2+1/4*e*b*ln(c*x^n-1)*m*ln(x)^2-1/2*e*b*ln(x)*ln
(x^m)*ln(c*x^n-1)-1/4*e*b*m*ln(x)^2*ln(1-c*x^n)+1/2*e*b*m/n*ln(x)*polylog(
2,c*x^n)-1/2*b*e*m*polylog(3,c*x^n)/n^2+1/2*e*b*ln(1-c*x^n)*ln(x)*ln(x^m)+
1/2*e*b/n*ln(1-c*x^n)*ln(c*x^n)*m*ln(x)-1/2*e*b/n*ln(1-c*x^n)*ln(c*x^n)*ln
(x^m)+1/2*e*b/n*dilog(c*x^n)*m*ln(x)-1/2*e*b/n*dilog(c*x^n)*ln(x^m)
```

3.265.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(142) = 284$.

Time = 0.28 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.04

$$\int \frac{(a + b\coth^{-1}(cx^n))(d + e\log(fx^m))}{x}\,dx$$
$$= \frac{2\,aemn^2\log(x)^2 - 2\,bem\text{polylog}(3, c\cosh(n\log(x)) + c\sinh(n\log(x))) + 2\,bem\text{polylog}(3, -c\cosh(n\log}{}$$

input
```
integrate((a+b*arccoth(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="fricas")
```

output
```
1/4*(2*a*e*m*n^2*log(x)^2 - 2*b*e*m*polylog(3, c*cosh(n*log(x)) + c*sinh(n
*log(x))) + 2*b*e*m*polylog(3, -c*cosh(n*log(x)) - c*sinh(n*log(x))) + 2*(
b*e*m*n*log(x) + b*e*n*log(f) + b*d*n)*dilog(c*cosh(n*log(x)) + c*sinh(n*l
og(x))) - 2*(b*e*m*n*log(x) + b*e*n*log(f) + b*d*n)*dilog(-c*cosh(n*log(x)
) - c*sinh(n*log(x))) - (b*e*m*n^2*log(x)^2 + 2*(b*e*n^2*log(f) + b*d*n^2)
*log(x))*log(c*cosh(n*log(x)) + c*sinh(n*log(x)) + 1) + (b*e*m*n^2*log(x)^
2 + 2*(b*e*n^2*log(f) + b*d*n^2)*log(x))*log(-c*cosh(n*log(x)) - c*sinh(n*
log(x)) + 1) + 4*(a*e*n^2*log(f) + a*d*n^2)*log(x) + (b*e*m*n^2*log(x)^2 +
 2*(b*e*n^2*log(f) + b*d*n^2)*log(x))*log((c*cosh(n*log(x)) + c*sinh(n*log
(x)) + 1)/(c*cosh(n*log(x)) + c*sinh(n*log(x)) - 1)))/n^2
```

3.265.6 Sympy [F]

$$\int \frac{(a + b\coth^{-1}(cx^n))(d + e\log(fx^m))}{x}\,dx = \int \frac{(a + b\operatorname{acoth}(cx^n))(d + e\log(fx^m))}{x}\,dx$$

input
```
integrate((a+b*acoth(c*x**n))*(d+e*ln(f*x**m))/x,x)
```

output
```
Integral((a + b*acoth(c*x**n))*(d + e*log(f*x**m))/x, x)
```

3.265.7 Maxima [F]

$$\int \frac{(a + b\coth^{-1}(cx^n))(d + e\log(fx^m))}{x}\,dx = \int \frac{(b\operatorname{arcoth}(cx^n) + a)(e\log(fx^m) + d)}{x}\,dx$$

input
```
integrate((a+b*arccoth(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="maxima")
```

output
```
1/2*a*e*log(f*x^m)^2/m + a*d*log(x) - 1/4*(b*e*m*log(x)^2 - 2*b*e*log(x)*l
og(x^m) - 2*(e*log(f) + d)*b*log(x))*log(c*x^n + 1) + 1/4*(b*e*m*log(x)^2
- 2*b*e*log(x)*log(x^m) - 2*(e*log(f) + d)*b*log(x))*log(c*x^n - 1) + inte
grate(1/2*(2*b*c*e*n*x^n*log(x)*log(x^m) - (b*c*e*m*n*log(x)^2 - 2*(e*n*lo
g(f) + d*n)*b*c*log(x))*x^n)/(c^2*x*x^(2*n) - x), x)
```

3.265.8 Giac [F]

$$\int \frac{(a + b\coth^{-1}(cx^n))(d + e\log(fx^m))}{x}\,dx = \int \frac{(b\operatorname{arcoth}(cx^n) + a)(e\log(fx^m) + d)}{x}\,dx$$

input
```
integrate((a+b*arccoth(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="giac")
```

output
```
integrate((b*arccoth(c*x^n) + a)*(e*log(f*x^m) + d)/x, x)
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\coth^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx = \int \frac{(a+b\,\mathrm{acoth}(c\,x^n))\,(d+e\,\ln(f\,x^m))}{x}\,dx$$

input
```
int(((a + b*acoth(c*x^n))*(d + e*log(f*x^m)))/x,x)
```

output
```
int(((a + b*acoth(c*x^n))*(d + e*log(f*x^m)))/x, x)
```

3.266 $\int x^5 \left(a+b \coth^{-1}(cx)\right) \left(d+e \log \left(1-c^2 x^2\right)\right) dx$

3.266.1 Optimal result

Integrand size = 27, antiderivative size = 297

$$\int x^5 \left(a+b \coth^{-1}(cx)\right) \left(d+e \log \left(1-c^2 x^2\right)\right) dx = \frac{b(3d-e)x}{18c^5} - \frac{137bex}{180c^5} + \frac{b(3d-e)x^3}{54c^3} - \frac{47bex^3}{540c^3} + \frac{b(3d-e)x^5}{90c} - \frac{bex^5}{75c} - \frac{ex^2\left(a+b \coth^{-1}(cx)\right)}{6c^4} - \frac{ex^4\left(a+b \coth^{-1}(cx)\right)}{12c^2} - \frac{1}{18}ex^6\left(a+b \coth^{-1}(cx)\right) - \frac{b(3d-e)\text{arctanh}(cx)}{18c^6} + \frac{137be\text{arctanh}(cx)}{180c^6} + \frac{bex \log \left(1-c^2x^2\right)}{6c^5} + \frac{bex^3 \log \left(1-c^2x^2\right)}{18c^3} + \frac{bex^5 \log \left(1-c^2x^2\right)}{30c} - \frac{e\left(a+b \coth^{-1}(cx)\right) \log \left(1-c^2x^2\right)}{6c^6} + \frac{1}{6}x^6\left(a+b \coth^{-1}(cx)\right) \left(d+e \log \left(1-c^2x^2\right)\right)$$

output
```
1/36*b*(6*d-11*e)*x/c^5-23/45*b*e*x/c^5+1/108*b*(6*d-5*e)*x^3/c^3-8/135*b*
e*x^3/c^3+1/90*b*(3*d-e)*x^5/c-1/75*b*e*x^5/c-1/6*e*x^2*(a+b*arccoth(c*x))
/c^4-1/12*e*x^4*(a+b*arccoth(c*x))/c^2-1/18*e*x^6*(a+b*arccoth(c*x))-1/36*
b*(6*d-11*e)*arctanh(c*x)/c^6+23/45*b*e*arctanh(c*x)/c^6+1/6*b*e*x*ln(-c^2
*x^2+1)/c^5+1/18*b*e*x^3*ln(-c^2*x^2+1)/c^3+1/30*b*e*x^5*ln(-c^2*x^2+1)/c-
1/6*e*(a+b*arccoth(c*x))*ln(-c^2*x^2+1)/c^6+1/6*x^6*(a+b*arccoth(c*x))*(d+
e*ln(-c^2*x^2+1))
```

3.266.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.79

$$\int x^5\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{30bc(10d-49e)x-300ac^2ex^2+10bc^3(10d-19e)x^3-150ac^4ex^4+4bc^5(15d-11e)x^5+100ac^6(3d-e)}{}$$

input
```
Integrate[x^5*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(30*b*c*(10*d - 49*e)*x - 300*a*c^2*e*x^2 + 10*b*c^3*(10*d - 19*e)*x^3 - 1
50*a*c^4*e*x^4 + 4*b*c^5*(15*d - 11*e)*x^5 + 100*a*c^6*(3*d - e)*x^6 - 50*
b*c^2*x^2*(-6*c^4*d*x^4 + e*(6 + 3*c^2*x^2 + 2*c^4*x^4))*ArcCoth[c*x] + 15
*(10*b*d - 20*a*e - 49*b*e)*Log[1 - c*x] - 15*(10*b*d + 20*a*e - 49*b*e)*L
og[1 + c*x] + 20*e*(15*a*c^6*x^6 + b*c*x*(15 + 5*c^2*x^2 + 3*c^4*x^4) + 15
*b*(-1 + c^6*x^6)*ArcCoth[c*x])*Log[1 - c^2*x^2])/(1800*c^6)
```

3.266.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6646, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)\,dx$$

$$\Big\downarrow \text{6646}$$

$$-bc\int\left(\frac{(3d-e)x^6}{18\left(1-c^2x^2\right)}-\frac{ex^4}{12c^2\left(1-c^2x^2\right)}-\frac{ex^2}{6c^4\left(1-c^2x^2\right)}-\frac{e\left(c^4x^4+c^2x^2+1\right)\log\left(1-c^2x^2\right)}{6c^6}\right)dx-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{6c^4}+\frac{1}{6}x^6\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^4\left(a+b\coth^{-1}(cx)\right)}{12c^2}-\frac{e\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{6c^6}-\frac{1}{18}ex^6\left(a+b\coth^{-1}(cx)\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ex^2(a+b\coth^{-1}(cx))}{6c^4}+\frac{1}{6}x^6(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^4(a+b\coth^{-1}(cx))}{12c^2}-\frac{e\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{6c^6}-\frac{1}{18}ex^6(a+b\coth^{-1}(cx))-bc\left(\frac{(3d-e)\text{arctanh}(cx)}{18c^7}-\frac{137e\text{arctanh}(cx)}{180c^7}-\frac{x(3d-e)}{18c^6}+\frac{137ex}{180c^6}-\frac{x^3(3d-e)}{54c^4}+\frac{47ex^3}{540c^4}-\frac{x^5(3d-e)}{90c^2}+\frac{ex^5}{75c^2}-\right.$$

input
```
Int[x^5*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
-1/6*(e*x^2*(a + b*ArcCoth[c*x]))/c^4 - (e*x^4*(a + b*ArcCoth[c*x]))/(12*c
^2) - (e*x^6*(a + b*ArcCoth[c*x]))/18 - (e*(a + b*ArcCoth[c*x])*Log[1 - c^
2*x^2])/(6*c^6) + (x^6*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/6 -
b*c*(-1/18*((3*d - e)*x)/c^6 + (137*e*x)/(180*c^6) - ((3*d - e)*x^3)/(54*c
^4) + (47*e*x^3)/(540*c^4) - ((3*d - e)*x^5)/(90*c^2) + (e*x^5)/(75*c^2) +
 ((3*d - e)*ArcTanh[c*x])/(18*c^7) - (137*e*ArcTanh[c*x])/(180*c^7) - (e*x
*Log[1 - c^2*x^2])/(6*c^6) - (e*x^3*Log[1 - c^2*x^2])/(18*c^4) - (e*x^5*Lo
g[1 - c^2*x^2])/(30*c^2))
```

3.266.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6646
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Simp[(a + b*ArcCoth[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[
u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m
+ 1)/2, 0]
```

3.266.4 Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.06

method	result
parallelrisch	$\frac{150be\ln(-c^2x^2+1)\operatorname{arccoth}(cx)x^6c^6-150\operatorname{arccoth}(cx)bd-150\ln(-c^2x^2+1)ae-150\operatorname{arccoth}(cx)\ln(-c^2x^2+1)be-150ac^2ex^2}{}$
default	Expression too large to display
parts	Expression too large to display
risch	Expression too large to display

input
```
int(x^5*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/900*(150*b*e*ln(-c^2*x^2+1)*arccoth(c*x)*x^6*c^6-150*arccoth(c*x)*b*d-15
0*ln(-c^2*x^2+1)*a*e-150*arccoth(c*x)*ln(-c^2*x^2+1)*b*e-150*a*c^2*e*x^2+1
50*b*c*d*x-150*arccoth(c*x)*b*c^2*e*x^2+150*ln(-c^2*x^2+1)*b*c*e*x-735*b*x
*e*c+50*b*c^3*d*x^3-95*b*e*x^3*c^3+30*b*c^5*d*x^5-22*b*c^5*e*x^5-150*a*e+1
50*a*c^6*d*x^6-50*a*c^6*e*x^6-75*e*b*arccoth(c*x)*x^4*c^4+50*e*b*x^3*ln(-c
^2*x^2+1)*c^3+30*b*e*ln(-c^2*x^2+1)*x^5*c^5+150*a*e*ln(-c^2*x^2+1)*x^6*c^6
+150*b*arccoth(c*x)*x^6*c^6*d-50*b*arccoth(c*x)*x^6*c^6*e+735*arccoth(c*x)
*b*e-75*a*e*x^4*c^4)/c^6
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.83

$$\int x^5\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx = -\frac{150\,ac^4ex^4-100\,(3\,ac^6d-ac^6e)x^6+300\,ac^2ex^2-4\,(15\,bc^5d-11\,bc^5e)x^5-10\,(10\,bc^3d-19\,bc^3e)x^3-}{}$$

input
```
integrate(x^5*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="frica
s")
```

output
```
-1/1800*(150*a*c^4*e*x^4 - 100*(3*a*c^6*d - a*c^6*e)*x^6 + 300*a*c^2*e*x^2
 - 4*(15*b*c^5*d - 11*b*c^5*e)*x^5 - 10*(10*b*c^3*d - 19*b*c^3*e)*x^3 - 30
*(10*b*c*d - 49*b*c*e)*x - 20*(15*a*c^6*e*x^6 + 3*b*c^5*e*x^5 + 5*b*c^3*e*
x^3 + 15*b*c*e*x - 15*a*e)*log(-c^2*x^2 + 1) + 5*(15*b*c^4*e*x^4 - 10*(3*b
*c^6*d - b*c^6*e)*x^6 + 30*b*c^2*e*x^2 + 30*b*d - 147*b*e - 30*(b*c^6*e*x^
6 - b*e)*log(-c^2*x^2 + 1))*log((c*x + 1)/(c*x - 1)))/c^6
```

3.266.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.45 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.22

$$\int x^5\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx = \begin{cases} \frac{adx^6}{6}+\frac{aex^6\log\left(-c^2x^2+1\right)}{6}-\frac{aex^6}{18}-\frac{aex^4}{12c^2}-\frac{aex^2}{6c^4}-\frac{ae\log\left(-c^2x^2+1\right)}{6c^6}+\frac{bdx^6\operatorname{acoth}(cx)}{6}+\frac{bex^6\log\left(-c^2x^2+1\right)\operatorname{acoth}(cx)}{6}-\ldots \\ \frac{dx^6\left(a+\frac{i\pi b}{2}\right)}{6} \end{cases}$$

input
```
integrate(x**5*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**6/6 + a*e*x**6*log(-c**2*x**2 + 1)/6 - a*e*x**6/18 - a*e
*x**4/(12*c**2) - a*e*x**2/(6*c**4) - a*e*log(-c**2*x**2 + 1)/(6*c**6) + b
*d*x**6*acoth(c*x)/6 + b*e*x**6*log(-c**2*x**2 + 1)*acoth(c*x)/6 - b*e*x**
6*acoth(c*x)/18 + b*d*x**5/(30*c) + b*e*x**5*log(-c**2*x**2 + 1)/(30*c) -
11*b*e*x**5/(450*c) - b*e*x**4*acoth(c*x)/(12*c**2) + b*d*x**3/(18*c**3) +
 b*e*x**3*log(-c**2*x**2 + 1)/(18*c**3) - 19*b*e*x**3/(180*c**3) - b*e*x**
2*acoth(c*x)/(6*c**4) + b*d*x/(6*c**5) + b*e*x*log(-c**2*x**2 + 1)/(6*c**5
) - 49*b*e*x/(60*c**5) - b*d*acoth(c*x)/(6*c**6) - b*e*log(-c**2*x**2 + 1)
*acoth(c*x)/(6*c**6) + 49*b*e*acoth(c*x)/(60*c**6), Ne(c, 0)), (d*x**6*(a
+ I*pi*b/2)/6, True))
```

3.266.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.11

$$\int x^5\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx=\frac{1}{6}\,adx^6$$
$$+\frac{1}{36}\left(6\,x^6\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\,c^4x^6+3\,c^2x^4+6\,x^2}{c^6}+\frac{6\,\log\left(c^2x^2-1\right)}{c^8}\right)\right)be\,\mathrm{arcoth}\left(cx\right)$$
$$+\frac{1}{180}\left(30\,x^6\,\mathrm{arcoth}\left(cx\right)+c\left(\frac{2\,(3\,c^4x^5+5\,c^2x^3+15\,x)}{c^6}-\frac{15\,\log\left(cx+1\right)}{c^7}+\frac{15\,\log\left(cx-1\right)}{c^7}\right)\right)bd$$
$$+\frac{1}{36}\left(6\,x^6\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\,c^4x^6+3\,c^2x^4+6\,x^2}{c^6}+\frac{6\,\log\left(c^2x^2-1\right)}{c^8}\right)\right)ae$$
$$-\frac{(4\,(-15i\,\pi c^5+11\,c^5)x^5+10\,(-10i\,\pi c^3+19\,c^3)x^3+30\,(-10i\,\pi c+49\,c)x+5\,(30i\,\pi-12\,c^5x^5-20\,c^3x^3}{1800\,c^6}$$

input
```
integrate(x^5*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxim
a")
```

output
```
1/6*a*d*x^6 + 1/36*(6*x^6*log(-c^2*x^2 + 1) - c^2*((2*c^4*x^6 + 3*c^2*x^4
+ 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^8))*b*e*arccoth(c*x) + 1/180*(30*x^6*a
rccoth(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^
7 + 15*log(c*x - 1)/c^7))*b*d + 1/36*(6*x^6*log(-c^2*x^2 + 1) - c^2*((2*c^
4*x^6 + 3*c^2*x^4 + 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^8))*a*e - 1/1800*(4*
(-15*I*pi*c^5 + 11*c^5)*x^5 + 10*(-10*I*pi*c^3 + 19*c^3)*x^3 + 30*(-10*I*p
i*c + 49*c)*x + 5*(30*I*pi - 12*c^5*x^5 - 20*c^3*x^3 - 60*c*x - 147)*log(c
*x + 1) + 5*(-30*I*pi - 12*c^5*x^5 - 20*c^3*x^3 - 60*c*x + 147)*log(c*x -
1))*b*e/c^6
```

3.266.8 Giac [F(-2)]

Exception generated.

$$\int x^5 \left(a + b \coth^{-1}(cx)\right) \left(d + e \log\left(1 - c^2x^2\right)\right) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.266.9 Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.72

$$\int x^5\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\ln\left(1-c^2x^2\right)\left(\frac{a\,e\,x^6}{6}+\frac{b\,e\,x}{6\,c^5}+\frac{b\,e\,x^5}{30\,c}+\frac{b\,e\,x^3}{18\,c^3}\right)$$
$$-\ln\left(\frac{1}{c\,x}+1\right)\left(\ln\left(1-c^2x^2\right)\left(\frac{b\,e}{12\,c^6}-\frac{b\,e\,x^6}{12}\right)-\frac{b\,d\,x^6}{12}+\frac{b\,e\,x^6}{36}+\frac{b\,e\,x^4}{24\,c^2}+\frac{b\,e\,x^2}{12\,c^4}\right)$$
$$+\ln\left(1-\frac{1}{c\,x}\right)\left(\frac{\frac{b\,d\,x^7}{6}-\frac{b\,c^2\,d\,x^9}{6}}{2\,(c\,x^2+x)\,(c\,x-1)}+\frac{\frac{b\,e\,x^7}{36}+\frac{b\,e\,x^5}{12\,c^2}-\frac{b\,e\,x^3}{6\,c^4}+\frac{b\,c^2\,e\,x^9}{18}}{2\,(c\,x^2+x)\,(c\,x-1)}\right.$$
$$\left.+\frac{\ln\left(1-c^2x^2\right)\left(\frac{b\,e\,x^7}{6}-\frac{b\,c^2\,e\,x^9}{6}\right)}{2\,(c\,x^2+x)\,(c\,x-1)}-\frac{b\,e\,\ln\left(1-c^2x^2\right)\,(x-c^2x^3)}{12\,c^6\,(c\,x^2+x)\,(c\,x-1)}\right)$$
$$+x^4\left(\frac{a\,(3\,d-e)}{12\,c^2}-\frac{a\,d}{4\,c^2}\right)+x^3\left(\frac{b\,(15\,d-11\,e)}{270\,c^3}-\frac{7\,b\,e}{108\,c^3}\right)+x\left(\frac{\frac{b\,(15\,d-11\,e)}{90\,c^3}-\frac{7\,b\,e}{36\,c^3}}{c^2}-\frac{b\,e}{2\,c^5}\right)$$
$$+\frac{a\,x^6\,(3\,d-e)}{18}+\frac{x^2\left(\frac{a\,(3\,d-e)}{3\,c^2}-\frac{a\,d}{c^2}\right)}{2\,c^2}-\frac{\ln(c\,x-1)\,(20\,a\,e-10\,b\,d+49\,b\,e)}{120\,c^6}$$
$$-\frac{\ln(c\,x+1)\,(20\,a\,e+10\,b\,d-49\,b\,e)}{120\,c^6}+\frac{b\,x^5\,(15\,d-11\,e)}{450\,c}$$

input
```
int(x^5*(a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

output
```
log(1 - c^2*x^2)*((a*e*x^6)/6 + (b*e*x)/(6*c^5) + (b*e*x^5)/(30*c) + (b*e*
x^3)/(18*c^3)) - log(1/(c*x) + 1)*(log(1 - c^2*x^2)*((b*e)/(12*c^6) - (b*e
*x^6)/12) - (b*d*x^6)/12 + (b*e*x^6)/36 + (b*e*x^4)/(24*c^2) + (b*e*x^2)/(
12*c^4)) + log(1 - 1/(c*x))*(((b*d*x^7)/6 - (b*c^2*d*x^9)/6)/(2*(x + c*x^2
)*(c*x - 1)) + ((b*e*x^7)/36 + (b*e*x^5)/(12*c^2) - (b*e*x^3)/(6*c^4) + (b
*c^2*e*x^9)/18)/(2*(x + c*x^2)*(c*x - 1)) + (log(1 - c^2*x^2)*((b*e*x^7)/6
 - (b*c^2*e*x^9)/6))/(2*(x + c*x^2)*(c*x - 1)) - (b*e*log(1 - c^2*x^2)*(x
- c^2*x^3))/(12*c^6*(x + c*x^2)*(c*x - 1))) + x^4*((a*(3*d - e))/(12*c^2)
- (a*d)/(4*c^2)) + x^3*((b*(15*d - 11*e))/(270*c^3) - (7*b*e)/(108*c^3)) +
 x*(((b*(15*d - 11*e))/(90*c^3) - (7*b*e)/(36*c^3))/c^2 - (b*e)/(2*c^5)) +
 (a*x^6*(3*d - e))/18 + (x^2*((a*(3*d - e))/(3*c^2) - (a*d)/c^2))/(2*c^2)
- (log(c*x - 1)*(20*a*e - 10*b*d + 49*b*e))/(120*c^6) - (log(c*x + 1)*(20*
a*e + 10*b*d - 49*b*e))/(120*c^6) + (b*x^5*(15*d - 11*e))/(450*c)
```

3.267 $\int x^3 \left(a+b\coth^{-1}(cx)\right) \left(d+e\log\left(1-c^2x^2\right)\right) \, dx$

3.267.1 Optimal result

Integrand size = 27, antiderivative size = 225

$$\begin{aligned}
&\int x^3 \left(a+b\coth^{-1}(cx)\right) \left(d+e\log\left(1-c^2x^2\right)\right) \, dx \\
&= \frac{b(2d-3e)x}{8c^3} - \frac{2bex}{3c^3} + \frac{b(2d-e)x^3}{24c} - \frac{bex^3}{18c} - \frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2} \\
&\quad - \frac{1}{8}ex^4\left(a+b\coth^{-1}(cx)\right) - \frac{b(2d-3e)\text{arctanh}(cx)}{8c^4} \\
&\quad + \frac{2be\text{arctanh}(cx)}{3c^4} + \frac{bex\log\left(1-c^2x^2\right)}{4c^3} + \frac{bex^3\log\left(1-c^2x^2\right)}{12c} \\
&\quad - \frac{e\left(a+b\coth^{-1}(cx)\right)\log\left(1-c^2x^2\right)}{4c^4} + \frac{1}{4}x^4\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)
\end{aligned}$$

output
```
1/8*b*(2*d-3*e)*x/c^3-2/3*b*e*x/c^3+1/24*b*(2*d-e)*x^3/c-1/18*b*e*x^3/c-1/
4*e*x^2*(a+b*arccoth(c*x))/c^2-1/8*e*x^4*(a+b*arccoth(c*x))-1/8*b*(2*d-3*e
)*arctanh(c*x)/c^4+2/3*b*e*arctanh(c*x)/c^4+1/4*b*e*x*ln(-c^2*x^2+1)/c^3+1
/12*b*e*x^3*ln(-c^2*x^2+1)/c-1/4*e*(a+b*arccoth(c*x))*ln(-c^2*x^2+1)/c^4+1
/4*x^4*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))
```

3.267.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.85

$$\int x^3 \left(a+b\coth^{-1}(cx)\right) \left(d+e\log\left(1-c^2x^2\right)\right) dx$$
$$= \frac{6bc(6d-25e)x - 36ac^2ex^2 + 2bc^3(6d-7e)x^3 + 18ac^4(2d-e)x^4 - 18bc^2x^2(-2c^2dx^2+e(2+c^2x^2))\,\mathrm{coth}}{}$$

input
```
Integrate[x^3*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(6*b*c*(6*d - 25*e)*x - 36*a*c^2*e*x^2 + 2*b*c^3*(6*d - 7*e)*x^3 + 18*a*c^
4*(2*d - e)*x^4 - 18*b*c^2*x^2*(-2*c^2*d*x^2 + e*(2 + c^2*x^2))*ArcCoth[c*
x] + 3*(6*b*d - 12*a*e - 25*b*e)*Log[1 - c*x] - 3*(6*b*d + 12*a*e - 25*b*e
)*Log[1 + c*x] + 12*e*(3*a*c^4*x^4 + b*c*x*(3 + c^2*x^2) + 3*b*(-1 + c^4*x
^4)*ArcCoth[c*x])*Log[1 - c^2*x^2])/(144*c^4)
```

3.267.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {6646, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\coth^{-1}(cx)\right) \left(e\log\left(1-c^2x^2\right)+d\right) dx$$

$$\Big\downarrow \text{ 6646}$$

$$-bc\int \left(-\frac{\left(2e-c^2(2d-e)x^2\right)x^2}{8c^2\left(1-c^2x^2\right)}-\frac{e\left(c^2x^2+1\right)\log\left(1-c^2x^2\right)}{4c^4}\right)dx+\frac{1}{4}x^4\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2}-\frac{e\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{4c^4}-\frac{1}{8}ex^4\left(a+b\coth^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}x^4(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^2(a+b\coth^{-1}(cx))}{4c^2}-\frac{e\log\left(1-c^2x^2\right)(a+b\coth^{-1}(cx))}{4c^4}-\frac{1}{8}ex^4(a+b\coth^{-1}(cx))-bc\left(\frac{(2d-3e)\text{arctanh}(cx)}{8c^5}-\frac{2e\text{arctanh}(cx)}{3c^5}-\frac{x(2d-3e)}{8c^4}+\frac{2ex}{3c^4}-\frac{x^3(2d-e)}{24c^2}+\frac{ex^3}{18c^2}-\frac{ex^3\log\left(1-c^2x^2\right)}{12c^2}-\frac{ex\right.$$

input
```
Int[x^3*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
-1/4*(e*x^2*(a + b*ArcCoth[c*x]))/c^2 - (e*x^4*(a + b*ArcCoth[c*x]))/8 - (
e*(a + b*ArcCoth[c*x])*Log[1 - c^2*x^2])/(4*c^4) + (x^4*(a + b*ArcCoth[c*x
])*(d + e*Log[1 - c^2*x^2]))/4 - b*c*(-1/8*((2*d - 3*e)*x)/c^4 + (2*e*x)/(
3*c^4) - ((2*d - e)*x^3)/(24*c^2) + (e*x^3)/(18*c^2) + ((2*d - 3*e)*ArcTan
h[c*x])/(8*c^5) - (2*e*ArcTanh[c*x])/(3*c^5) - (e*x*Log[1 - c^2*x^2])/(4*c
^4) - (e*x^3*Log[1 - c^2*x^2])/(12*c^2))
```

3.267.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6646
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Simp[(a + b*ArcCoth[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[
u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m
+ 1)/2, 0]
```

3.267.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{18be\ln(-c^2x^2+1)\,\text{arccoth}(cx)x^4c^4-18\,\text{arccoth}(cx)bd-18\ln(-c^2x^2+1)ae-18\,\text{arccoth}(cx)\ln(-c^2x^2+1)be-18ac^2ex^2+18bc}{}$
default	Expression too large to display
parts	Expression too large to display
risch	Expression too large to display

input
```
int(x^3*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/72*(18*b*e*ln(-c^2*x^2+1)*arccoth(c*x)*x^4*c^4-18*arccoth(c*x)*b*d-18*ln
(-c^2*x^2+1)*a*e-18*arccoth(c*x)*ln(-c^2*x^2+1)*b*e-18*a*c^2*e*x^2+18*b*c*
d*x-18*arccoth(c*x)*b*c^2*e*x^2+18*ln(-c^2*x^2+1)*b*c*e*x-75*b*x*e*c+6*b*c
^3*d*x^3-7*b*e*x^3*c^3-18*a*e-9*e*b*arccoth(c*x)*x^4*c^4+6*e*b*x^3*ln(-c^2
*x^2+1)*c^3+18*a*e*ln(-c^2*x^2+1)*x^4*c^4+18*b*arccoth(c*x)*x^4*c^4*d+75*a
rccoth(c*x)*b*e+18*a*x^4*d*c^4-9*a*e*x^4*c^4)/c^4
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.87

$$\int x^3 \left(a + b \coth^{-1}(cx)\right) \left(d + e \log\left(1 - c^2x^2\right)\right) \, dx =$$
$$-\frac{36\, ac^2ex^2 - 18\,(2\,ac^4d - ac^4e)x^4 - 2\,(6\,bc^3d - 7\,bc^3e)x^3 - 6\,(6\,bcd - 25\,bce)x - 12\,(3\,ac^4ex^4 + bc^3ex^3 +}{}$$

input
```
integrate(x^3*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="frica
s")
```

output
```
-1/144*(36*a*c^2*e*x^2 - 18*(2*a*c^4*d - a*c^4*e)*x^4 - 2*(6*b*c^3*d - 7*b
*c^3*e)*x^3 - 6*(6*b*c*d - 25*b*c*e)*x - 12*(3*a*c^4*e*x^4 + b*c^3*e*x^3 +
 3*b*c*e*x - 3*a*e)*log(-c^2*x^2 + 1) + 3*(6*b*c^2*e*x^2 - 3*(2*b*c^4*d -
b*c^4*e)*x^4 + 6*b*d - 25*b*e - 6*(b*c^4*e*x^4 - b*e)*log(-c^2*x^2 + 1))*l
og((c*x + 1)/(c*x - 1)))/c^4
```

3.267.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.27

$$\int x^3 \left(a + b \coth^{-1}(cx)\right) \left(d + e \log\left(1 - c^2x^2\right)\right) \, dx$$
$$= \begin{cases} \frac{adx^4}{4} + \frac{aex^4 \log\left(-c^2x^2+1\right)}{4} - \frac{aex^4}{8} - \frac{aex^2}{4c^2} - \frac{ae \log\left(-c^2x^2+1\right)}{4c^4} + \frac{bdx^4 \operatorname{acoth}(cx)}{4} + \frac{bex^4 \log\left(-c^2x^2+1\right) \operatorname{acoth}(cx)}{4} - \frac{bex^4 \operatorname{acoth}}{8} \\ \frac{dx^4\left(a + \frac{i\pi b}{2}\right)}{4} \end{cases}$$

input
```
integrate(x**3*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**4/4 + a*e*x**4*log(-c**2*x**2 + 1)/4 - a*e*x**4/8 - a*e*
x**2/(4*c**2) - a*e*log(-c**2*x**2 + 1)/(4*c**4) + b*d*x**4*acoth(c*x)/4 +
 b*e*x**4*log(-c**2*x**2 + 1)*acoth(c*x)/4 - b*e*x**4*acoth(c*x)/8 + b*d*x
**3/(12*c) + b*e*x**3*log(-c**2*x**2 + 1)/(12*c) - 7*b*e*x**3/(72*c) - b*e
*x**2*acoth(c*x)/(4*c**2) + b*d*x/(4*c**3) + b*e*x*log(-c**2*x**2 + 1)/(4*
c**3) - 25*b*e*x/(24*c**3) - b*d*acoth(c*x)/(4*c**4) - b*e*log(-c**2*x**2
+ 1)*acoth(c*x)/(4*c**4) + 25*b*e*acoth(c*x)/(24*c**4), Ne(c, 0)), (d*x**4
*(a + I*pi*b/2)/4, True))
```

3.267.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.20

$$\begin{aligned}
&\int x^3\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx \\
&= \frac{1}{4}adx^4 + \frac{1}{8}\left(2x^4\log\left(-c^2x^2+1\right) - c^2\left(\frac{c^2x^4+2x^2}{c^4} + \frac{2\log\left(c^2x^2-1\right)}{c^6}\right)\right)be\,\mathrm{arcoth}\,(cx) \\
&\quad + \frac{1}{24}\left(6x^4\,\mathrm{arcoth}\,(cx) + c\left(\frac{2\left(c^2x^3+3x\right)}{c^4} - \frac{3\log\left(cx+1\right)}{c^5} + \frac{3\log\left(cx-1\right)}{c^5}\right)\right)bd \\
&\quad + \frac{1}{8}\left(2x^4\log\left(-c^2x^2+1\right) - c^2\left(\frac{c^2x^4+2x^2}{c^4} + \frac{2\log\left(c^2x^2-1\right)}{c^6}\right)\right)ae \\
&\quad - \frac{\left(2\left(-6i\pi c^3+7c^3\right)x^3 + 6\left(-6i\pi c+25c\right)x + 3\left(6i\pi-4c^3x^3-12cx-25\right)\log\left(cx+1\right) + 3\left(-6i\pi-4c\right.\right.}{144c^4}
\end{aligned}$$

input
```
integrate(x^3*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxim
a")
```

output
```
1/4*a*d*x^4 + 1/8*(2*x^4*log(-c^2*x^2 + 1) - c^2*((c^2*x^4 + 2*x^2)/c^4 +
2*log(c^2*x^2 - 1)/c^6))*b*e*arccoth(c*x) + 1/24*(6*x^4*arccoth(c*x) + c*(
2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d + 1/
8*(2*x^4*log(-c^2*x^2 + 1) - c^2*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 -
1)/c^6))*a*e - 1/144*(2*(-6*I*pi*c^3 + 7*c^3)*x^3 + 6*(-6*I*pi*c + 25*c)*x
 + 3*(6*I*pi - 4*c^3*x^3 - 12*c*x - 25)*log(c*x + 1) + 3*(-6*I*pi - 4*c^3*
x^3 - 12*c*x + 25)*log(c*x - 1))*b*e/c^4
```

3.267.8 Giac [F(-2)]

Exception generated.

$$\int x^3(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.267.9 Mupad [B] (verification not implemented)

Time = 5.42 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.84

$$\begin{aligned}
&\int x^3(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx \\
&= \ln\left(1-\frac{1}{c\,x}\right)\left(\frac{\frac{b\,e\,x^5}{8}-\frac{b\,e\,x^3}{4\,c^2}+\frac{b\,c^2\,e\,x^7}{8}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\frac{b\,d\,x^5}{4}-\frac{b\,c^2\,d\,x^7}{4}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}\right. \\
&\left.+\frac{\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e\,x^5}{4}-\frac{b\,c^2\,e\,x^7}{4}\right)}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{b\,e\,\ln\left(1-c^2\,x^2\right)\,\left(x-c^2\,x^3\right)}{8\,c^4\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}\right) \\
&+x\left(\frac{b\,\left(6\,d-7\,e\right)}{24\,c^3}-\frac{3\,b\,e}{4\,c^3}\right)+\ln\left(1-c^2\,x^2\right)\,\left(\frac{a\,e\,x^4}{4}+\frac{b\,e\,x}{4\,c^3}+\frac{b\,e\,x^3}{12\,c}\right) \\
&-\ln\left(\frac{1}{c\,x}+1\right)\left(\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e}{8\,c^4}-\frac{b\,e\,x^4}{8}\right)-\frac{b\,d\,x^4}{8}+\frac{b\,e\,x^4}{16}+\frac{b\,e\,x^2}{8\,c^2}\right) \\
&+x^2\left(\frac{a\,\left(2\,d-e\right)}{4\,c^2}-\frac{a\,d}{2\,c^2}\right)+\frac{a\,x^4\,\left(2\,d-e\right)}{8}-\frac{\ln\left(c\,x-1\right)\,\left(12\,a\,e-6\,b\,d+25\,b\,e\right)}{48\,c^4} \\
&-\frac{\ln\left(c\,x+1\right)\,\left(12\,a\,e+6\,b\,d-25\,b\,e\right)}{48\,c^4}+\frac{b\,x^3\,\left(6\,d-7\,e\right)}{72\,c}
\end{aligned}$$

input
```
int(x^3*(a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

output
```
log(1 - 1/(c*x))*(((b*e*x^5)/8 - (b*e*x^3)/(4*c^2) + (b*c^2*e*x^7)/8)/(2*(
x + c*x^2)*(c*x - 1)) + ((b*d*x^5)/4 - (b*c^2*d*x^7)/4)/(2*(x + c*x^2)*(c*
x - 1)) + (log(1 - c^2*x^2)*((b*e*x^5)/4 - (b*c^2*e*x^7)/4))/(2*(x + c*x^2
)*(c*x - 1)) - (b*e*log(1 - c^2*x^2)*(x - c^2*x^3))/(8*c^4*(x + c*x^2)*(c*
x - 1))) + x*((b*(6*d - 7*e))/(24*c^3) - (3*b*e)/(4*c^3)) + log(1 - c^2*x^
2)*((a*e*x^4)/4 + (b*e*x)/(4*c^3) + (b*e*x^3)/(12*c)) - log(1/(c*x) + 1)*(
log(1 - c^2*x^2)*((b*e)/(8*c^4) - (b*e*x^4)/8) - (b*d*x^4)/8 + (b*e*x^4)/1
6 + (b*e*x^2)/(8*c^2)) + x^2*((a*(2*d - e))/(4*c^2) - (a*d)/(2*c^2)) + (a*
x^4*(2*d - e))/8 - (log(c*x - 1)*(12*a*e - 6*b*d + 25*b*e))/(48*c^4) - (lo
g(c*x + 1)*(12*a*e + 6*b*d - 25*b*e))/(48*c^4) + (b*x^3*(6*d - 7*e))/(72*c
)
```

3.267. $\int x^3\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

3.268 $\int x(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)dx$

3.268.1 Optimal result

Integrand size = 25, antiderivative size = 140

$$\int x(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)dx$$
$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2(a+b\coth^{-1}(cx))-\frac{1}{2}ex^2(a+b\coth^{-1}(cx))$$
$$-\frac{b(d-e)\text{arctanh}(cx)}{2c^2}+\frac{b e\text{arctanh}(cx)}{c^2}+\frac{bex\log\left(1-c^2x^2\right)}{2c}$$
$$-\frac{e(1-c^2x^2)\left(a+b\coth^{-1}(cx)\right)\log\left(1-c^2x^2\right)}{2c^2}$$

output
```
1/2*b*(d-e)*x/c-b*e*x/c+1/2*d*x^2*(a+b*arccoth(c*x))-1/2*e*x^2*(a+b*arccot
h(c*x))-1/2*b*(d-e)*arctanh(c*x)/c^2+b*e*arctanh(c*x)/c^2+1/2*b*e*x*ln(-c^
2*x^2+1)/c-1/2*e*(-c^2*x^2+1)*(a+b*arccoth(c*x))*ln(-c^2*x^2+1)/c^2
```

3.268.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92

$$\int x(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)dx$$
$$=\frac{2bc(d-3e)x+2ac^2(d-e)x^2+2bc^2(d-e)x^2\coth^{-1}(cx)+(b(d-3e)-2ae)\log(1-cx)-(b(d-3e)+}{4c^2}$$

input
```
Integrate[x*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(2*b*c*(d - 3*e)*x + 2*a*c^2*(d - e)*x^2 + 2*b*c^2*(d - e)*x^2*ArcCoth[c*x
] + (b*(d - 3*e) - 2*a*e)*Log[1 - c*x] - (b*(d - 3*e) + 2*a*e)*Log[1 + c*x
] + 2*e*(c*x*(b + a*c*x) + b*(-1 + c^2*x^2)*ArcCoth[c*x])*Log[1 - c^2*x^2]
)/(4*c^2)
```

3.268.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6646, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)dx$$

$$\Big\downarrow \text{6646}$$

$$-bc\int\left(\frac{(d-e)x^2}{2\left(1-c^2x^2\right)}-\frac{e\log\left(1-c^2x^2\right)}{2c^2}\right)dx-\frac{e\left(1-c^2x^2\right)\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{2c^2}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e\left(1-c^2x^2\right)\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{2c^2}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)-bc\left(\frac{(d-e)\text{arctanh}(cx)}{2c^3}-\frac{e\,\text{arctanh}(cx)}{c^3}-\frac{x(d-e)}{2c^2}-\frac{ex\log\left(1-c^2x^2\right)}{2c^2}+\frac{ex}{c^2}\right)$$

input
```
Int[x*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(d*x^2*(a + b*ArcCoth[c*x]))/2 - (e*x^2*(a + b*ArcCoth[c*x]))/2 - (e*(1 -
c^2*x^2)*(a + b*ArcCoth[c*x])*Log[1 - c^2*x^2])/(2*c^2) - b*c*(-1/2*((d -
e)*x)/c^2 + (e*x)/c^2 + ((d - e)*ArcTanh[c*x])/(2*c^3) - (e*ArcTanh[c*x])/
c^3 - (e*x*Log[1 - c^2*x^2])/(2*c^2))
```

3.268.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6646
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Simp[(a + b*ArcCoth[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[
u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m
+ 1)/2, 0]
```

3.268.4 Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.24

method	result
parallelrisch	$\frac{\ln(-c^2x^2+1)\operatorname{arccoth}(cx)b\,c^2e\,x^2+\operatorname{arccoth}(cx)b\,c^2d\,x^2-\operatorname{arccoth}(cx)b\,c^2e\,x^2+\ln(-c^2x^2+1)a\,c^2e\,x^2+a\,c^2d\,x^2-a\,c^2e\,x^2+\ln(-c}{2c^2}$
default	Expression too large to display
parts	Expression too large to display
risch	Expression too large to display

input
```
int(x*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(ln(-c^2*x^2+1)*arccoth(c*x)*b*c^2*e*x^2+arccoth(c*x)*b*c^2*d*x^2-arcc
oth(c*x)*b*c^2*e*x^2+ln(-c^2*x^2+1)*a*c^2*e*x^2+a*c^2*d*x^2-a*c^2*e*x^2+ln
(-c^2*x^2+1)*b*c*e*x+b*c*d*x-3*b*x*e*c-arccoth(c*x)*ln(-c^2*x^2+1)*b*e-arc
coth(c*x)*b*d+3*arccoth(c*x)*b*e-ln(-c^2*x^2+1)*a*e)/c^2
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{2\,(ac^2d-ac^2e)x^2+2\,(bcd-3\,bce)x+2\,(ac^2ex^2+bcex-ae)\log\left(-c^2x^2+1\right)+((bc^2d-bc^2e)x^2-bd+3}{4\,c^2}$$

input
```
integrate(x*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas"
)
```

output
```
1/4*(2*(a*c^2*d - a*c^2*e)*x^2 + 2*(b*c*d - 3*b*c*e)*x + 2*(a*c^2*e*x^2 +
b*c*e*x - a*e)*log(-c^2*x^2 + 1) + ((b*c^2*d - b*c^2*e)*x^2 - b*d + 3*b*e
+ (b*c^2*e*x^2 - b*e)*log(-c^2*x^2 + 1))*log((c*x + 1)/(c*x - 1)))/c^2
```

3.268.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.49

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$

$$=\begin{cases}\frac{adx^2}{2}+\frac{aex^2\log\left(-c^2x^2+1\right)}{2}-\frac{aex^2}{2}-\frac{ae\log\left(-c^2x^2+1\right)}{2c^2}+\frac{bdx^2\operatorname{acoth}(cx)}{2}+\frac{bex^2\log\left(-c^2x^2+1\right)\operatorname{acoth}(cx)}{2}-\frac{bex^2\operatorname{acoth}(cx)}{2}+\\ \frac{dx^2\left(a+\frac{i\pi b}{2}\right)}{2}\end{cases}$$

input
```
integrate(x*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**2/2 + a*e*x**2*log(-c**2*x**2 + 1)/2 - a*e*x**2/2 - a*e*
log(-c**2*x**2 + 1)/(2*c**2) + b*d*x**2*acoth(c*x)/2 + b*e*x**2*log(-c**2*
x**2 + 1)*acoth(c*x)/2 - b*e*x**2*acoth(c*x)/2 + b*d*x/(2*c) + b*e*x*log(-
c**2*x**2 + 1)/(2*c) - 3*b*e*x/(2*c) - b*d*acoth(c*x)/(2*c**2) - b*e*log(-
c**2*x**2 + 1)*acoth(c*x)/(2*c**2) + 3*b*e*acoth(c*x)/(2*c**2), Ne(c, 0)),
 (d*x**2*(a + I*pi*b/2)/2, True))
```

3.268.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.22

$$\begin{aligned}\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx\\ &=\frac{1}{2}\,adx^2+\frac{1}{4}\left(2\,x^2\operatorname{arcoth}(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)bd\\ &\quad-\frac{\left(c^2x^2-\left(c^2x^2-1\right)\log\left(-c^2x^2+1\right)-1\right)be\operatorname{arcoth}(cx)}{2\,c^2}\\ &\quad-\frac{\left(c^2x^2-\left(c^2x^2-1\right)\log\left(-c^2x^2+1\right)-1\right)ae}{2\,c^2}\\ &\quad-\frac{\left(3\,cx-(cx+1)\log(cx+1)-(cx-1)\log(-cx+1)\right)be}{2\,c^2}\end{aligned}$$

input
```
integrate(x*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima"
)
```

output
```
1/2*a*d*x^2 + 1/4*(2*x^2*arccoth(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + lo
g(c*x - 1)/c^3))*b*d - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1)
*b*e*arccoth(c*x)/c^2 - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1
)*a*e/c^2 - 1/2*(3*c*x - (c*x + 1)*log(c*x + 1) - (c*x - 1)*log(-c*x + 1))
*b*e/c^2
```

3.268.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.72

$$\begin{aligned}\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx &= -\frac{1}{4}\,bex^2\log\left(-cx+1\right)^2-\frac{1}{4}\left(-i\,\pi bd+i\,\pi be-2\,ad+2\,ae\right)x^2\\&\quad+\frac{1}{4}\left(bex^2-\frac{be}{c^2}\right)\log\left(cx+1\right)^2\\&\quad-\frac{1}{4}\left(\left(-i\,\pi be-bd-2\,ae+be\right)x^2-\frac{2\,bex}{c}\right)\log\left(cx+1\right)-\frac{be\log\left(cx-1\right)^2}{4\,c^2}\\&\quad-\frac{1}{4}\left(\left(-i\,\pi be+bd-2\,ae-be\right)x^2-\frac{2\,bex}{c}-\frac{2\,be\log\left(cx-1\right)}{c^2}\right)\log\left(-cx+1\right)\\&\quad+\frac{\left(bd-3\,be\right)x}{2\,c}+\frac{\left(-i\,\pi be-bd-2\,ae+3\,be\right)\log\left(cx+1\right)}{4\,c^2}\\&\quad+\frac{\left(-i\,\pi be+bd-2\,ae-3\,be\right)\log\left(cx-1\right)}{4\,c^2}\end{aligned}$$

input
```
integrate(x*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

output
```
-1/4*b*e*x^2*log(-c*x + 1)^2 - 1/4*(-I*pi*b*d + I*pi*b*e - 2*a*d + 2*a*e)*
x^2 + 1/4*(b*e*x^2 - b*e/c^2)*log(c*x + 1)^2 - 1/4*((-I*pi*b*e - b*d - 2*a
*e + b*e)*x^2 - 2*b*e*x/c)*log(c*x + 1) - 1/4*b*e*log(c*x - 1)^2/c^2 - 1/4
*((-I*pi*b*e + b*d - 2*a*e - b*e)*x^2 - 2*b*e*x/c - 2*b*e*log(c*x - 1)/c^2
)*log(-c*x + 1) + 1/2*(b*d - 3*b*e)*x/c + 1/4*(-I*pi*b*e - b*d - 2*a*e + 3
*b*e)*log(c*x + 1)/c^2 + 1/4*(-I*pi*b*e + b*d - 2*a*e - 3*b*e)*log(c*x - 1
)/c^2
```

3.268.9 Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.35

$$\int x\big(a+b\coth^{-1}(cx)\big)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\ln\left(1-\frac{1}{c\,x}\right)\left(\frac{\frac{b\,d\,x^3}{2}-\frac{b\,c^2\,d\,x^5}{2}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{\frac{b\,e\,x^3}{2}-\frac{b\,c^2\,e\,x^5}{2}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e\,x^3}{2}-\frac{b\,c^2\,e\,x^5}{2}\right)}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{b\,e\,\ln\left(1-c^2\,x^2\right)\,\left(x-c^2\,x^3\right)}{4\,c^2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}\right)$$
$$+\ln\left(1-c^2\,x^2\right)\,\left(\frac{a\,e\,x^2}{2}+\frac{b\,e\,x}{2\,c}\right)$$
$$-\ln\left(\frac{1}{c\,x}+1\right)\,\left(\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e}{4\,c^2}-\frac{b\,e\,x^2}{4}\right)-\frac{b\,d\,x^2}{4}+\frac{b\,e\,x^2}{4}\right)+\frac{a\,x^2\,\left(d-e\right)}{2}$$
$$-\frac{\ln\left(c\,x+1\right)\,\left(2\,a\,e+b\,d-3\,b\,e\right)}{4\,c^2}-\frac{\ln\left(c\,x-1\right)\,\left(2\,a\,e-b\,d+3\,b\,e\right)}{4\,c^2}+\frac{b\,x\,\left(d-3\,e\right)}{2\,c}$$

input
```
int(x*(a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

output
```
log(1 - 1/(c*x))*(((b*d*x^3)/2 - (b*c^2*d*x^5)/2)/(2*(x + c*x^2)*(c*x - 1)
) - ((b*e*x^3)/2 - (b*c^2*e*x^5)/2)/(2*(x + c*x^2)*(c*x - 1)) + (log(1 - c
^2*x^2)*((b*e*x^3)/2 - (b*c^2*e*x^5)/2))/(2*(x + c*x^2)*(c*x - 1)) - (b*e*
log(1 - c^2*x^2)*(x - c^2*x^3))/(4*c^2*(x + c*x^2)*(c*x - 1))) + log(1 - c
^2*x^2)*((a*e*x^2)/2 + (b*e*x)/(2*c)) - log(1/(c*x) + 1)*(log(1 - c^2*x^2)
*((b*e)/(4*c^2) - (b*e*x^2)/4) - (b*d*x^2)/4 + (b*e*x^2)/4) + (a*x^2*(d -
e))/2 - (log(c*x + 1)*(2*a*e + b*d - 3*b*e))/(4*c^2) - (log(c*x - 1)*(2*a*
e - b*d + 3*b*e))/(4*c^2) + (b*x*(d - 3*e))/(2*c)
```

3.269 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x}\,dx$

3.269.1 Optimal result

Integrand size = 27, antiderivative size = 381

$$\begin{aligned}
&\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x}\,dx \\
&= -\frac{1}{2}be\log^2\left(1+\frac{1}{cx}\right)\log\left(-\frac{1}{cx}\right)+\frac{1}{2}be\log^2\left(1-\frac{1}{cx}\right)\log\left(\frac{1}{cx}\right)+ad\log(x) \\
&\quad - be\log\left(\frac{c+\frac{1}{x}}{c}\right)\operatorname{PolyLog}\left(2,\frac{c+\frac{1}{x}}{c}\right)+be\log\left(1-\frac{1}{cx}\right)\operatorname{PolyLog}\left(2,1-\frac{1}{cx}\right) \\
&\quad + \frac{1}{2}bd\operatorname{PolyLog}\left(2,-\frac{1}{cx}\right)+\frac{1}{2}be\log\left(-c^2x^2\right)\operatorname{PolyLog}\left(2,-\frac{1}{cx}\right) \\
&\quad - \frac{1}{2}be\left(\log\left(1-\frac{1}{cx}\right)+\log\left(1+\frac{1}{cx}\right)+\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)\right)\operatorname{PolyLog}\left(2,-\frac{1}{cx}\right) \\
&\quad - \frac{1}{2}bd\operatorname{PolyLog}\left(2,\frac{1}{cx}\right)-\frac{1}{2}be\log\left(-c^2x^2\right)\operatorname{PolyLog}\left(2,\frac{1}{cx}\right) \\
&\quad + \frac{1}{2}be\left(\log\left(1-\frac{1}{cx}\right)+\log\left(1+\frac{1}{cx}\right)+\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)\right)\operatorname{PolyLog}\left(2,\frac{1}{cx}\right) \\
&\quad - \frac{1}{2}ae\operatorname{PolyLog}\left(2,c^2x^2\right)+be\operatorname{PolyLog}\left(3,\frac{c+\frac{1}{x}}{c}\right) \\
&\quad - be\operatorname{PolyLog}\left(3,1-\frac{1}{cx}\right)+be\operatorname{PolyLog}\left(3,-\frac{1}{cx}\right)-be\operatorname{PolyLog}\left(3,\frac{1}{cx}\right)
\end{aligned}$$

output
```
-1/2*b*e*ln(1+1/c/x)^2*ln(-1/c/x)+1/2*b*e*ln(1-1/c/x)^2*ln(1/c/x)+a*d*ln(x
)-b*e*ln((c+1/x)/c)*polylog(2,(c+1/x)/c)+b*e*ln(1-1/c/x)*polylog(2,1-1/c/x
)+1/2*b*d*polylog(2,-1/c/x)+1/2*b*e*ln(-c^2*x^2)*polylog(2,-1/c/x)-1/2*b*e
*(ln(1-1/c/x)+ln(1+1/c/x)+ln(-c^2*x^2)-ln(-c^2*x^2+1))*polylog(2,-1/c/x)-1
/2*b*d*polylog(2,1/c/x)-1/2*b*e*ln(-c^2*x^2)*polylog(2,1/c/x)+1/2*b*e*(ln(
1-1/c/x)+ln(1+1/c/x)+ln(-c^2*x^2)-ln(-c^2*x^2+1))*polylog(2,1/c/x)-1/2*a*e
*polylog(2,c^2*x^2)+b*e*polylog(3,(c+1/x)/c)-b*e*polylog(3,1-1/c/x)+b*e*po
lylog(3,-1/c/x)-b*e*polylog(3,1/c/x)
```

3.269.2 Mathematica [F]

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx = \int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x,x]
```

output
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x, x]
```

3.269.3 Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.88, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6642, 6447, 6640, 2838, 6638, 2904, 2843, 27, 2881, 27, 2821, 6447, 6632, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{x}\,dx$$

$$\Big\downarrow \text{6642}$$

$$e\int \frac{\left(a+b\coth^{-1}(cx)\right)\log\left(1-c^2x^2\right)}{x}dx+d\int \frac{a+b\coth^{-1}(cx)}{x}dx$$

$$\Big\downarrow \text{6447}$$

$$e\int \frac{(a+b\coth^{-1}(cx))\log(1-c^2x^2)}{x}dx+$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\downarrow$ 6640

$$e\left(a\int \frac{\log(1-c^2x^2)}{x}dx+b\int \frac{\coth^{-1}(cx)\log(1-c^2x^2)}{x}dx\right)+$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\downarrow$ 2838

$$e\left(b\int \frac{\coth^{-1}(cx)\log(1-c^2x^2)}{x}dx-\frac{1}{2}a\,\text{PolyLog}(2,c^2x^2)\right)+$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\downarrow$ 6638

$$e\left(b\left(-\left(\left(\log(-c^2x^2)-\log(1-c^2x^2)+\log\left(1-\frac{1}{cx}\right)+\log\left(\frac{1}{cx}+1\right)\right)\int \frac{\coth^{-1}(cx)}{x}dx\right)+\int \frac{\coth^{-1}(cx)\log}{x}\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\downarrow$ 2904

$$e\left(b\left(-\left(\left(\log(-c^2x^2)-\log(1-c^2x^2)+\log\left(1-\frac{1}{cx}\right)+\log\left(\frac{1}{cx}+1\right)\right)\int \frac{\coth^{-1}(cx)}{x}dx\right)+\int \frac{\coth^{-1}(cx)\log}{x}\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\downarrow$ 2843

$$e\left(b\left(-\left(\left(\log(-c^2x^2)-\log(1-c^2x^2)+\log\left(1-\frac{1}{cx}\right)+\log\left(\frac{1}{cx}+1\right)\right)\int \frac{\coth^{-1}(cx)}{x}dx\right)+\int \frac{\coth^{-1}(cx)\log}{x}\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\downarrow$ 27

3.269. $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x}\,dx$

$$e\Bigg(b\Bigg(-\Bigg(\Bigg(\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)+\log\left(1-\frac{1}{cx}\right)+\log\left(\frac{1}{cx}+1\right)\Bigg)\int\frac{\coth^{-1}(cx)}{x}dx\Bigg)+\int\frac{\coth^{-1}(cx)\log}{x}$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\big\downarrow$ 2881

$$e\Bigg(b\Bigg(-\Bigg(\Bigg(\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)+\log\left(1-\frac{1}{cx}\right)+\log\left(\frac{1}{cx}+1\right)\Bigg)\int\frac{\coth^{-1}(cx)}{x}dx\Bigg)+\int\frac{\coth^{-1}(cx)\log}{x}$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\big\downarrow$ 27

$$e\Bigg(b\Bigg(-\Bigg(\Bigg(\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)+\log\left(1-\frac{1}{cx}\right)+\log\left(\frac{1}{cx}+1\right)\Bigg)\int\frac{\coth^{-1}(cx)}{x}dx\Bigg)+\int\frac{\coth^{-1}(cx)\log}{x}$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\big\downarrow$ 2821

$$e\Bigg(b\Bigg(-\Bigg(\Bigg(\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)+\log\left(1-\frac{1}{cx}\right)+\log\left(\frac{1}{cx}+1\right)\Bigg)\int\frac{\coth^{-1}(cx)}{x}dx\Bigg)+\int\frac{\coth^{-1}(cx)\log}{x}$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\big\downarrow$ 6447

$$e\Bigg(b\Bigg(\int\frac{\coth^{-1}(cx)\log\left(-c^2x^2\right)}{x}dx+\frac{1}{2}\Bigg(\log^2\left(1-\frac{1}{cx}\right)\log\left(\frac{1}{cx}\right)-2\Bigg(\int x\,\text{PolyLog}\left(2,1-\frac{1}{cx}\right)d\left(1-\frac{1}{cx}\right)-\text{F}$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\big\downarrow$ 6632

$$e\Bigg(b\Bigg(-\frac{1}{2}\int\frac{\log\left(1-\frac{1}{cx}\right)\log\left(-c^2x^2\right)}{x}dx+\frac{1}{2}\int\frac{\log\left(1+\frac{1}{cx}\right)\log\left(-c^2x^2\right)}{x}dx+\frac{1}{2}\Bigg(\log^2\left(1-\frac{1}{cx}\right)\log\left(\frac{1}{cx}\right)-2\Bigg(\int$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\big\downarrow$ 2821

3.269. $\int\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}dx$

$$e\left(b\left(\frac{1}{2}\left(\text{PolyLog}\left(2,-\frac{1}{cx}\right)\log\left(-c^2x^2\right)-2\int\frac{\text{PolyLog}\left(2,-\frac{1}{cx}\right)}{x}dx\right)+\frac{1}{2}\left(2\int\frac{\text{PolyLog}\left(2,\frac{1}{cx}\right)}{x}dx-\text{PolyLog}\left(2\right.\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$\Big\downarrow$ 7143

$$e\left(b\left(-\left(\left(\frac{1}{2}\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)\left(\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)+\log\left(1-\frac{1}{cx}\right)+\log\left(\frac{1}{cx}\right.\right.\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x,x]
```

output
```
d*(a*Log[x] + (b*PolyLog[2, -(1/(c*x))])/2 - (b*PolyLog[2, 1/(c*x)])/2) +
e*(-1/2*(a*PolyLog[2, c^2*x^2]) + b*(-((Log[1 - 1/(c*x)] + Log[1 + 1/(c*x)
] + Log[-(c^2*x^2)] - Log[1 - c^2*x^2])*(PolyLog[2, -(1/(c*x))]/2 - PolyLo
g[2, 1/(c*x)]/2)) + (Log[1 - 1/(c*x)]^2*Log[1/(c*x)] - 2*(-(Log[1 - 1/(c*x
)]*PolyLog[2, 1 - 1/(c*x)]) + PolyLog[3, 1 - 1/(c*x)]))/2 + (-(Log[1 + 1/(
c*x)]^2*Log[-(1/(c*x))]) + 2*(-(Log[1 + 1/(c*x)]*PolyLog[2, 1 + 1/(c*x)])
+ PolyLog[3, 1 + 1/(c*x)]))/2 + (Log[-(c^2*x^2)]*PolyLog[2, -(1/(c*x))] +
2*PolyLog[3, -(1/(c*x))])/2 + (-(Log[-(c^2*x^2)]*PolyLog[2, 1/(c*x)]) - 2*
PolyLog[3, 1/(c*x)])/2))
```

3.269.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.269. $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x}\,dx$

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 6447
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

rule 6632
```
Int[(ArcCoth[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)])/(x_), x_Symbol] :> Si
mp[1/2   Int[Log[d*x^m]*(Log[1 + 1/(c*x^n)]/x), x], x] - Simp[1/2   Int[Log
[d*x^m]*(Log[1 - 1/(c*x^n)]/x), x], x] /; FreeQ[{c, d, m, n}, x]
```

rule 6638
```
Int[(ArcCoth[(c_.)*(x_)]*Log[(f_.) + (g_.)*(x_)^2])/(x_), x_Symbol] :> Simp
[(Log[f + g*x^2] - Log[(-c^2)*x^2] - Log[1 - 1/(c*x)] - Log[1 + 1/(c*x)])
 Int[ArcCoth[c*x]/x, x], x] + (Int[Log[(-c^2)*x^2]*(ArcCoth[c*x]/x), x] + S
imp[1/2   Int[Log[1 + 1/(c*x)]^2/x, x], x] - Simp[1/2   Int[Log[1 - 1/(c*x)
]^2/x, x], x]) /; FreeQ[{c, f, g}, x] && EqQ[c^2*f + g, 0]
```

rule 6640
```
Int[(Log[(f_.) + (g_.)*(x_)^2]*(ArcCoth[(c_.)*(x_)]*(b_.) + (a_)))/(x_), x_
Symbol] :> Simp[a   Int[Log[f + g*x^2]/x, x], x] + Simp[b   Int[Log[f + g*x
^2]*(ArcCoth[c*x]/x), x], x] /; FreeQ[{a, b, c, f, g}, x]
```

3.269. $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x}\,dx$

rule 6642
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*(Log[(f_.) + (g_.)*(x_)^2]*(e_.) +
 (d_)))/(x_), x_Symbol] :> Simp[d   Int[(a + b*ArcCoth[c*x])/x, x], x] + Si
mp[e   Int[Log[f + g*x^2]*((a + b*ArcCoth[c*x])/x), x], x] /; FreeQ[{a, b,
c, d, e, f, g}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.269.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.72 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.55

method	result
risch	$-\frac{a\left(i e\pi\ \mathrm{csgn}(i(cx-1))\,\mathrm{csgn}(i(cx+1))\,\mathrm{csgn}(i(cx-1)(cx+1))-i e\pi\ \mathrm{csgn}(i(cx-1))\,\mathrm{csgn}(i(cx-1)(cx+1))^2-i e\pi\ \mathrm{csgn}(i(cx+1))\,\mathrm{csgn}(i(c\right.}{2}$

input
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(I*e*Pi*csgn(I*(c*x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))-I*e
*Pi*csgn(I*(c*x-1))*csgn(I*(c*x-1)*(c*x+1))^2-I*e*Pi*csgn(I*(c*x+1))*csgn(
I*(c*x-1)*(c*x+1))^2-I*e*Pi*csgn(I*(c*x-1)*(c*x+1))^3+2*I*e*Pi*csgn(I*(c*x
-1)*(c*x+1))^2-2*I*e*Pi-2*d)*ln(c*x)-1/4*(-I*Pi*b*e*csgn(I*(c*x-1))*csgn(I
*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))+I*Pi*b*e*csgn(I*(c*x-1))*csgn(I*(c*x-1)*
(c*x+1))^2+I*Pi*b*e*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+I*Pi*b*e*csg
n(I*(c*x-1)*(c*x+1))^3-2*I*Pi*b*e*csgn(I*(c*x-1)*(c*x+1))^2+2*I*e*Pi*b-4*a
*e+2*d*b)*(dilog(c*x)+ln(c*x-1)*ln(c*x))-1/2*ln(c*x-1)^2*ln(c*x)*b*e-ln(c*
x-1)*polylog(2,-c*x+1)*b*e+polylog(3,-c*x+1)*b*e-(-1/4*I*Pi*b*e*csgn(I*(c*
x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))+1/4*I*Pi*b*e*csgn(I*(c*x-1))
*csgn(I*(c*x-1)*(c*x+1))^2+1/4*I*Pi*b*e*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*
x+1))^2+1/4*I*Pi*b*e*csgn(I*(c*x-1)*(c*x+1))^3-1/2*I*Pi*b*e*csgn(I*(c*x-1)
*(c*x+1))^2+1/2*I*e*Pi*b+a*e+1/2*d*b)*dilog(c*x+1)+1/2*ln(-c*x)*ln(c*x+1)^
2*b*e+polylog(2,c*x+1)*ln(c*x+1)*b*e-polylog(3,c*x+1)*b*e
```

3.269.5 Fricas [F]

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx=\int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log\left(-c^2x^2+1\right)+d)}{x}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="fricas"
)
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
 1))/x, x)
```

3.269.6 Sympy [F]

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx = \int \frac{(a+b\,\mathrm{acoth}\,(cx))\,(d+e\log\left(-c^2x^2+1\right))}{x}\,dx$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x,x)
```

output
```
Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x, x)
```

3.269.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.44

$$\begin{aligned}
&\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx \\
&= i\,\pi ae\log(x) \\
&\quad-\frac{1}{2}\left(\log(cx-1)^2\log(cx)+2\,\mathrm{Li}_2(-cx+1)\log(cx-1)-2\,\mathrm{Li}_3(-cx+1)\right)be \\
&\quad+\frac{1}{2}\left(\log(cx+1)^2\log(-cx)+2\,\mathrm{Li}_2(cx+1)\log(cx+1)-2\,\mathrm{Li}_3(cx+1)\right)be \\
&\quad+ad\log(x)-\frac{1}{2}(i\,\pi be+bd-2\,ae)(\log(cx-1)\log(cx)+\mathrm{Li}_2(-cx+1)) \\
&\quad-\frac{1}{2}(-i\,\pi be-bd-2\,ae)(\log(cx+1)\log(-cx)+\mathrm{Li}_2(cx+1))
\end{aligned}$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="maxima"
)
```

output
```
I*pi*a*e*log(x) - 1/2*(log(c*x - 1)^2*log(c*x) + 2*dilog(-c*x + 1)*log(c*x
 - 1) - 2*polylog(3, -c*x + 1))*b*e + 1/2*(log(c*x + 1)^2*log(-c*x) + 2*di
log(c*x + 1)*log(c*x + 1) - 2*polylog(3, c*x + 1))*b*e + a*d*log(x) - 1/2*
(I*pi*b*e + b*d - 2*a*e)*(log(c*x - 1)*log(c*x) + dilog(-c*x + 1)) - 1/2*(
-I*pi*b*e - b*d - 2*a*e)*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))
```

3.269.8 Giac [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x}\,dx = \int \frac{(b\,\text{arcoth}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="giac")
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x, x)
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x}\,dx = \int \frac{(a+b\,\text{acoth}(c\,x))\,(d+e\,\ln(1-c^2\,x^2))}{x}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x, x)
```

3.270 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^3}\,dx$

3.270.1 Optimal result

Integrand size = 27, antiderivative size = 247

$$\begin{aligned}
&\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^3}\,dx \\
&= -\frac{1}{2}bc^2e\coth^{-1}(cx)^2-\frac{1}{2}bc^2e\mathrm{arctanh}(cx)^2-ac^2e\log(x) \\
&\quad+bc^2e\mathrm{arctanh}(cx)\log\left(\frac{2}{1-cx}\right)+\frac{1}{2}(a+b)c^2e\log(1-cx)+\frac{1}{2}(a-b)c^2e\log(1+cx) \\
&\quad-\frac{bc(d+e\log(1-c^2x^2))}{2x}-\frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{2x^2} \\
&\quad+\frac{1}{2}bc^2\mathrm{arctanh}(cx)\,(d+e\log(1-c^2x^2))-bc^2e\coth^{-1}(cx)\log\left(2-\frac{2}{1+cx}\right) \\
&\quad+\frac{1}{2}bc^2e\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx}\right)+\frac{1}{2}bc^2e\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)
\end{aligned}$$

output
```
-1/2*b*c^2*e*arccoth(c*x)^2-1/2*b*c^2*e*arctanh(c*x)^2-a*c^2*e*ln(x)+b*c^2
*e*arctanh(c*x)*ln(2/(-c*x+1))+1/2*(a+b)*c^2*e*ln(-c*x+1)+1/2*(a-b)*c^2*e*
ln(c*x+1)-1/2*b*c*(d+e*ln(-c^2*x^2+1))/x-1/2*(a+b*arccoth(c*x))*(d+e*ln(-c
^2*x^2+1))/x^2+1/2*b*c^2*arctanh(c*x)*(d+e*ln(-c^2*x^2+1))-b*c^2*e*arccoth
(c*x)*ln(2-2/(c*x+1))+1/2*b*c^2*e*polylog(2,1-2/(-c*x+1))+1/2*b*c^2*e*poly
log(2,-1+2/(c*x+1))
```

3.270.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.65

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^3}\,dx$$
$$=\frac{1}{2}\left(-\frac{ad}{x^2}-2ac^2e\log(x)+(a+b)c^2e\log(1-cx)+(a-b)c^2e\log(1+cx)-\frac{bd(2\coth^{-1}(cx)+cx(2+cx\log(1-cx)-cx\log(1+cx)))}{2x^2}-\frac{e(a+bcx+(b-bc^2x^2)\coth^{-1}(cx))\log(1-c^2x^2)}{x^2}-bc^2e\left(\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)\right)$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^3,x]
```

output
```
(-((a*d)/x^2) - 2*a*c^2*e*Log[x] + (a + b)*c^2*e*Log[1 - c*x] + (a - b)*c^
2*e*Log[1 + c*x] - (b*d*(2*ArcCoth[c*x] + c*x*(2 + c*x*Log[1 - c*x] - c*x*
Log[1 + c*x])))/(2*x^2) - (e*(a + b*c*x + (b - b*c^2*x^2)*ArcCoth[c*x])*Lo
g[1 - c^2*x^2])/x^2 - b*c^2*e*(PolyLog[2, -(1/(c*x))] - PolyLog[2, 1/(c*x)
]))/2
```

3.270.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6648, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{x^3}\,dx$$
$$\Big\downarrow \; 6648$$

$$2c^2e\int\left(\frac{bc^2x\text{arctanh}(cx)}{2(1-c^2x^2)}-\frac{a+bcx+b\coth^{-1}(cx)}{2x(1-c^2x^2)}\right)dx-\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{2x^2}+\frac{1}{2}bc^2\text{arctanh}(cx)(e\log(1-c^2x^2)+d)-\frac{bc(e\log(1-c^2x^2)+d)}{2x}$$

$$\Big\downarrow \text{ 2009}$$

$$2c^2e\left(\frac{1}{4}(a+b)\log(1-cx)+\frac{1}{4}(a-b)\log(cx+1)-\frac{1}{2}a\log(x)-\frac{1}{4}b\text{arctanh}(cx)^2+\frac{1}{2}b\text{arctanh}(cx)\log\left(\frac{2}{1-cx}\right)-\right.$$
$$-\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{2x^2}+\frac{1}{2}bc^2\text{arctanh}(cx)(e\log(1-c^2x^2)+d)-\frac{bc(e\log(1-c^2x^2)+d)}{2x}$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^3,x]
```

output
```
-1/2*(b*c*(d + e*Log[1 - c^2*x^2]))/x - ((a + b*ArcCoth[c*x])*(d + e*Log[1
 - c^2*x^2]))/(2*x^2) + (b*c^2*ArcTanh[c*x]*(d + e*Log[1 - c^2*x^2]))/2 +
2*c^2*e*(-1/4*(b*ArcCoth[c*x]^2) - (b*ArcTanh[c*x]^2)/4 - (a*Log[x])/2 + (
b*ArcTanh[c*x]*Log[2/(1 - c*x)])/2 + ((a + b)*Log[1 - c*x])/4 + ((a - b)*L
og[1 + c*x])/4 - (b*ArcCoth[c*x]*Log[2 - 2/(1 + c*x)])/2 + (b*PolyLog[2, 1
 - 2/(1 - c*x)])/4 + (b*PolyLog[2, -1 + 2/(1 + c*x)])/4)
```

3.270.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6648
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegran
d[x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Inte
gerQ[m] && NeQ[m, -1]
```

3.270.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arccoth}\,(cx))\,(d+e\ln{(-c^2x^2+1)})}{x^3}dx$$

input
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)
```

output
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)
```

3.270.5 Fricas [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log{(1-c^2x^2)})}{x^3}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log{(-c^2x^2+1)}+d)}{x^3}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="frica
s")
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
 1))/x^3, x)
```

3.270.6 Sympy [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log{(1-c^2x^2)})}{x^3}\,dx$$
$$=\int \frac{(a+b\,\mathrm{acoth}\,(cx))\,(d+e\log{(-c^2x^2+1)})}{x^3}\,dx$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**3,x)
```

output
```
Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**3, x)
```

3.270.7 Maxima [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^3}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="maxim
a")
```

output
```
1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arccoth(c*x)/x^2)*b*d +
 1/2*(c^2*(log(c^2*x^2 - 1) - log(x^2)) - log(-c^2*x^2 + 1)/x^2)*a*e - 1/4
*b*e*(log(c*x + 1)^2/x^2 - 2*integrate(-((c*x + 1)*log(c*x - 1)^2 - (I*pi
+ (I*pi*c + c)*x)*log(c*x + 1) - (-I*pi - I*pi*c*x)*log(c*x - 1))/(c*x^4 +
 x^3), x)) - 1/2*a*d/x^2
```

3.270.8 Giac [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^3}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="giac"
)
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^3, x)
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acoth}(c\,x))\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^3}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^3,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^3, x)
```

3.271 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^5}\,dx$

3.271.1 Optimal result

Integrand size = 27, antiderivative size = 339

$$\begin{aligned}
&\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^5}\,dx \\
&= \frac{ac^2e}{4x^2}+\frac{5bc^3e}{12x}+\frac{bc^2e\coth^{-1}(cx)}{4x^2}-\frac{1}{4}bc^4e\coth^{-1}(cx)^2-\frac{1}{4}bc^4e\operatorname{arctanh}(cx) \\
&\quad-\frac{1}{4}bc^4e\operatorname{arctanh}(cx)^2-\frac{1}{2}ac^4e\log(x)+\frac{1}{2}bc^4e\operatorname{arctanh}(cx)\log\left(\frac{2}{1-cx}\right) \\
&\quad+\frac{1}{12}(3a+4b)c^4e\log(1-cx)+\frac{1}{12}(3a-4b)c^4e\log(1+cx)-\frac{bc(d+e\log(1-c^2x^2))}{12x^3} \\
&\quad-\frac{bc^3(d+e\log(1-c^2x^2))}{4x}-\frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{4x^4} \\
&\quad+\frac{1}{4}bc^4\operatorname{arctanh}(cx)(d+e\log(1-c^2x^2))-\frac{1}{2}bc^4e\coth^{-1}(cx)\log\left(2-\frac{2}{1+cx}\right) \\
&\quad+\frac{1}{4}bc^4e\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)+\frac{1}{4}bc^4e\operatorname{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)
\end{aligned}$$

output
```
1/4*a*c^2*e/x^2+5/12*b*c^3*e/x+1/4*b*c^2*e*arccoth(c*x)/x^2-1/4*b*c^4*e*ar
ccoth(c*x)^2-1/4*b*c^4*e*arctanh(c*x)-1/4*b*c^4*e*arctanh(c*x)^2-1/2*a*c^4
*e*ln(x)+1/2*b*c^4*e*arctanh(c*x)*ln(2/(-c*x+1))+1/12*(3*a+4*b)*c^4*e*ln(-
c*x+1)+1/12*(3*a-4*b)*c^4*e*ln(c*x+1)-1/12*b*c*(d+e*ln(-c^2*x^2+1))/x^3-1/
4*b*c^3*(d+e*ln(-c^2*x^2+1))/x-1/4*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))
/x^4+1/4*b*c^4*arctanh(c*x)*(d+e*ln(-c^2*x^2+1))-1/2*b*c^4*e*arccoth(c*x)*
ln(2-2/(c*x+1))+1/4*b*c^4*e*polylog(2,1-2/(-c*x+1))+1/4*b*c^4*e*polylog(2,
-1+2/(c*x+1))
```

3.271.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^5}\,dx$$
$$= -\frac{ad}{4x^4}+\frac{ac^2e}{4x^2}+\frac{bc^3e}{6x}-\frac{1}{2}ac^4e\log(x)+\frac{1}{12}\left(3ac^4e+4bc^4e\right)\log(1-cx)$$
$$-\frac{1}{2}bc^4e\left(-\frac{\coth^{-1}(cx)}{2c^2x^2}+\frac{1}{2}\left(-\frac{1}{cx}-\frac{1}{2}\log(1-cx)+\frac{1}{2}\log(1+cx)\right)\right)$$
$$+bc^4d\left(-\frac{\coth^{-1}(cx)}{4c^4x^4}+\frac{1}{4}\left(-\frac{1}{3c^3x^3}-\frac{1}{cx}-\frac{1}{2}\log(1-cx)+\frac{1}{2}\log(1+cx)\right)\right)$$
$$+\frac{1}{12}\left(3ac^4e-4bc^4e\right)\log(1+cx)$$
$$+\frac{e\left(-3a-bcx-3bc^3x^3-3b\coth^{-1}(cx)+3bc^4x^4\coth^{-1}(cx)\right)\log\left(1-c^2x^2\right)}{12x^4}$$
$$-\frac{1}{4}bc^4e\left(\operatorname{PolyLog}\left(2,-\frac{1}{cx}\right)-\operatorname{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5,x]
```

output
```
-1/4*(a*d)/x^4 + (a*c^2*e)/(4*x^2) + (b*c^3*e)/(6*x) - (a*c^4*e*Log[x])/2
+ ((3*a*c^4*e + 4*b*c^4*e)*Log[1 - c*x])/12 - (b*c^4*e*(-1/2*ArcCoth[c*x]/
(c^2*x^2) + (-(1/(c*x)) - Log[1 - c*x]/2 + Log[1 + c*x]/2)/2))/2 + b*c^4*d
*(-1/4*ArcCoth[c*x]/(c^4*x^4) + (-1/3*1/(c^3*x^3) - 1/(c*x) - Log[1 - c*x]
/2 + Log[1 + c*x]/2)/4) + ((3*a*c^4*e - 4*b*c^4*e)*Log[1 + c*x])/12 + (e*(
-3*a - b*c*x - 3*b*c^3*x^3 - 3*b*ArcCoth[c*x] + 3*b*c^4*x^4*ArcCoth[c*x])*
Log[1 - c^2*x^2])/(12*x^4) - (b*c^4*e*(PolyLog[2, -(1/(c*x))] - PolyLog[2,
 1/(c*x)]))/4
```

3.271.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6648, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{x^5}\,dx$$

$$\Big\downarrow \text{ 6648}$$

$$2c^2e\int\left(\frac{bc^4x\text{arctanh}(cx)}{4(1-c^2x^2)}-\frac{3bc^3x^3+bcx+3a+3b\coth^{-1}(cx)}{12x^3(1-c^2x^2)}\right)dx-\frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{4x^4}+\frac{1}{4}bc^4\text{arctanh}(cx)\,(e\log(1-c^2x^2)+d)-\frac{bc(e\log(1-c^2x^2)+d)}{12x^3}-\frac{bc^3(e\log(1-c^2x^2)+d)}{4x}$$

$$\Big\downarrow \text{ 2009}$$

$$2c^2e\left(\frac{1}{24}c^2(3a+4b)\log(1-cx)+\frac{1}{24}c^2(3a-4b)\log(cx+1)-\frac{1}{4}ac^2\log(x)+\frac{a}{8x^2}-\frac{1}{8}bc^2\text{arctanh}(cx)^2-\frac{1}{8}bc^2\text{arc}\right.$$
$$-\frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{4x^4}+\frac{1}{4}bc^4\text{arctanh}(cx)\,(e\log(1-c^2x^2)+d)-\frac{bc(e\log(1-c^2x^2)+d)}{12x^3}-\frac{bc^3(e\log(1-c^2x^2)+d)}{4x}$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5,x]
```

output
```
-1/12*(b*c*(d + e*Log[1 - c^2*x^2]))/x^3 - (b*c^3*(d + e*Log[1 - c^2*x^2])
)/(4*x) - ((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/(4*x^4) + (b*c^4
*ArcTanh[c*x]*(d + e*Log[1 - c^2*x^2]))/4 + 2*c^2*e*(a/(8*x^2) + (5*b*c)/(
24*x) + (b*ArcCoth[c*x])/(8*x^2) - (b*c^2*ArcCoth[c*x]^2)/8 - (b*c^2*ArcTa
nh[c*x])/8 - (b*c^2*ArcTanh[c*x]^2)/8 - (a*c^2*Log[x])/4 + (b*c^2*ArcTanh[
c*x]*Log[2/(1 - c*x)])/4 + ((3*a + 4*b)*c^2*Log[1 - c*x])/24 + ((3*a - 4*b
)*c^2*Log[1 + c*x])/24 - (b*c^2*ArcCoth[c*x]*Log[2 - 2/(1 + c*x)])/4 + (b*
c^2*PolyLog[2, 1 - 2/(1 - c*x)])/8 + (b*c^2*PolyLog[2, -1 + 2/(1 + c*x)])/
8)
```

3.271. $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^5}\,dx$

3.271.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6648
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegran
d[x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Inte
gerQ[m] && NeQ[m, -1]
```

3.271.4 Maple [F]

$$\int \frac{(a + b \operatorname{arccoth}(cx)) (d + e \ln (-c^2x^2 + 1))}{x^5} dx$$

input
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^5,x)
```

output
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^5,x)
```

3.271.5 Fricas [F]

$$\int \frac{(a + b \coth^{-1}(cx)) (d + e \log (1 - c^2x^2))}{x^5} dx = \int \frac{(b \operatorname{arcoth}(cx) + a)(e \log (-c^2x^2 + 1) + d)}{x^5} dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="frica
s")
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
 1))/x^5, x)
```

3.271.6 Sympy [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^5}\,dx = \int \frac{(a+b\,\mathrm{acoth}\,(cx))\,(d+e\log(-c^2x^2+1))}{x^5}\,dx$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**5,x)
```

output
```
Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**5, x)
```

3.271.7 Maxima [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^5}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^5}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="maxim
a")
```

output
```
1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c
- 6*arccoth(c*x)/x^4)*b*d + 1/4*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/
x^2)*c^2 - log(-c^2*x^2 + 1)/x^4)*a*e - 1/8*b*e*(log(c*x + 1)^2/x^4 - 4*in
tegrate(-1/2*(2*(c*x + 1)*log(c*x - 1)^2 - (2*I*pi + (2*I*pi*c + c)*x)*log
(c*x + 1) + 2*(I*pi + I*pi*c*x)*log(c*x - 1))/(c*x^6 + x^5), x)) - 1/4*a*d
/x^4
```

3.271.8 Giac [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^5}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^5}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="giac"
)
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^5, x)
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^5}\,dx$$
$$=\int \frac{\left(a+b\,\mathrm{acoth}(c\,x)\right)\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^5}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^5,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^5, x)
```

3.272 $\int x^4\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

3.272.1 Optimal result

Integrand size = 27, antiderivative size = 315

$$\begin{aligned}
&\int x^4\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx\\
&=-\frac{2aex}{5c^4}-\frac{77bex^2}{300c^3}-\frac{2aex^3}{15c^2}-\frac{9bex^4}{200c}-\frac{2}{25}aex^5-\frac{2bex\coth^{-1}(cx)}{5c^4}\\
&\quad-\frac{2bex^3\coth^{-1}(cx)}{15c^2}-\frac{2}{25}bex^5\coth^{-1}(cx)+\frac{be\coth^{-1}(cx)^2}{5c^5}\\
&\quad-\frac{(4a+3b)e\log(1-cx)}{20c^5}+\frac{(4a-3b)e\log(1+cx)}{20c^5}-\frac{23be\log\left(1-c^2x^2\right)}{75c^5}\\
&\quad-\frac{be\log^2\left(1-c^2x^2\right)}{20c^5}+\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{10c^3}+\frac{bx^4\left(d+e\log\left(1-c^2x^2\right)\right)}{20c}\\
&\quad+\frac{1}{5}x^5\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)+\frac{b\log\left(1-c^2x^2\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{10c^5}
\end{aligned}$$

output
```
-2/5*a*e*x/c^4-77/300*b*e*x^2/c^3-2/15*a*e*x^3/c^2-9/200*b*e*x^4/c-2/25*a*
e*x^5-2/5*b*e*x*arccoth(c*x)/c^4-2/15*b*e*x^3*arccoth(c*x)/c^2-2/25*b*e*x^
5*arccoth(c*x)+1/5*b*e*arccoth(c*x)^2/c^5-1/20*(4*a+3*b)*e*ln(-c*x+1)/c^5+
1/20*(4*a-3*b)*e*ln(c*x+1)/c^5-23/75*b*e*ln(-c^2*x^2+1)/c^5-1/20*b*e*ln(-c
^2*x^2+1)^2/c^5+1/10*b*x^2*(d+e*ln(-c^2*x^2+1))/c^3+1/20*b*x^4*(d+e*ln(-c^
2*x^2+1))/c+1/5*x^5*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))+1/10*b*ln(-c^2
*x^2+1)*(d+e*ln(-c^2*x^2+1))/c^5
```

3.272.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.75

$$\int x^4\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{-240acex+2bc^2(30d-77e)x^2-80ac^3ex^3+3bc^4(10d-9e)x^4+24ac^5(5d-2e)x^5-8bcx(-15c^4dx^4+2}{}$$

input
```
Integrate[x^4*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(-240*a*c*e*x + 2*b*c^2*(30*d - 77*e)*x^2 - 80*a*c^3*e*x^3 + 3*b*c^4*(10*d
 - 9*e)*x^4 + 24*a*c^5*(5*d - 2*e)*x^5 - 8*b*c*x*(-15*c^4*d*x^4 + 2*e*(15
+ 5*c^2*x^2 + 3*c^4*x^4))*ArcCoth[c*x] + 120*b*e*ArcCoth[c*x]^2 + 2*(30*b*
d - 60*a*e - 137*b*e)*Log[1 - c*x] + 2*(30*b*d + 60*a*e - 137*b*e)*Log[1 +
 c*x] + 30*c^2*e*x^2*(4*a*c^3*x^3 + b*(2 + c^2*x^2) + 4*b*c^3*x^3*ArcCoth[
c*x])*Log[1 - c^2*x^2] + 30*b*e*Log[1 - c^2*x^2]^2)/(600*c^5)
```

3.272.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6648, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)\,dx$$

$$\Big\downarrow \text{6648}$$

$$2c^2e\int\left(\frac{4ac^3x^6+4bc^3\coth^{-1}(cx)x^6+bc^2x^5+2bx^3}{20c^3\left(1-c^2x^2\right)}+\frac{bx\log\left(1-c^2x^2\right)}{10c^5\left(1-c^2x^2\right)}\right)dx+\frac{1}{5}x^5\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{bx^4\left(e\log\left(1-c^2x^2\right)+d\right)}{20c}+\frac{b\log\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{10c^5}+\frac{bx^2\left(e\log\left(1-c^2x^2\right)+d\right)}{10c^3}$$

$$\Big\downarrow \text{2009}$$

$$2c^2e\left(\frac{a\text{arctanh}(cx)}{5c^7}-\frac{ax}{5c^6}-\frac{ax^3}{15c^4}-\frac{ax^5}{25c^2}+\frac{b\coth^{-1}(cx)^2}{10c^7}-\frac{bx\coth^{-1}(cx)}{5c^6}-\frac{77bx^2}{600c^5}-\frac{bx^3\coth^{-1}(cx)}{15c^4}-\frac{9bx^4}{400c^3}\right.$$

$$\frac{1}{5}x^5\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{bx^4\left(e\log\left(1-c^2x^2\right)+d\right)}{20c}+$$

$$\frac{b\log\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{10c^5}+\frac{bx^2\left(e\log\left(1-c^2x^2\right)+d\right)}{10c^3}$$

input
```
Int[x^4*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(b*x^2*(d + e*Log[1 - c^2*x^2]))/(10*c^3) + (b*x^4*(d + e*Log[1 - c^2*x^2]
))/(20*c) + (x^5*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/5 + (b*Log
[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(10*c^5) + 2*c^2*e*(-1/5*(a*x)/c^6
 - (77*b*x^2)/(600*c^5) - (a*x^3)/(15*c^4) - (9*b*x^4)/(400*c^3) - (a*x^5)
/(25*c^2) - (b*x*ArcCoth[c*x])/(5*c^6) - (b*x^3*ArcCoth[c*x])/(15*c^4) - (
b*x^5*ArcCoth[c*x])/(25*c^2) + (b*ArcCoth[c*x]^2)/(10*c^7) + (a*ArcTanh[c*
x])/(5*c^7) - (137*b*Log[1 - c^2*x^2])/(600*c^7) - (b*Log[1 - c^2*x^2]^2)/
(40*c^7))
```

3.272.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6648
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegran
d[x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Inte
gerQ[m] && NeQ[m, -1]
```

3.272.4 Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.94

method	result
parallelrisch	$\frac{60db-154be+60b\,c^2d\,x^2+30b\,c^4d\,x^4-240aecx-27be\,x^4c^4-154be\,x^2c^2-48a\,c^5e\,x^5+120a\,c^5d\,x^5+120b\,\mathrm{arccoth}(cx)x^5c^5d-48b}{}$
risch	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^4*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/600*(60*d*b-154*b*e+60*b*c^2*d*x^2+30*b*c^4*d*x^4-240*a*e*c*x-27*b*e*x^4
*c^4-154*b*e*x^2*c^2-48*a*c^5*e*x^5+120*a*c^5*d*x^5+120*b*arccoth(c*x)*x^5
*c^5*d-48*b*arccoth(c*x)*x^5*c^5*e+120*a*e*ln(-c^2*x^2+1)*x^5*c^5+60*x^2*l
n(-c^2*x^2+1)*b*c^2*e-240*e*b*x*arccoth(c*x)*c-80*e*b*arccoth(c*x)*x^3*c^3
+120*e*b*arccoth(c*x)^2+30*b*e*ln(-c^2*x^2+1)*x^4*c^4-80*a*e*x^3*c^3+120*b
*e*ln(-c^2*x^2+1)*arccoth(c*x)*x^5*c^5+30*e*b*ln(-c^2*x^2+1)^2+240*arccoth
(c*x)*a*e+60*ln(-c^2*x^2+1)*b*d-274*ln(-c^2*x^2+1)*b*e)/c^5
```

3.272.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.79

$$\int x^4\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx =$$
$$-\frac{80\,ac^3ex^3-24\,(5\,ac^5d-2\,ac^5e)x^5-3\,(10\,bc^4d-9\,bc^4e)x^4+240\,acex-30\,be\log\left(-c^2x^2+1\right)^2-30\,be}{}$$

input
```
integrate(x^4*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="frica
s")
```

output
```
-1/600*(80*a*c^3*e*x^3 - 24*(5*a*c^5*d - 2*a*c^5*e)*x^5 - 3*(10*b*c^4*d -
9*b*c^4*e)*x^4 + 240*a*c*e*x - 30*b*e*log(-c^2*x^2 + 1)^2 - 30*b*e*log((c*
x + 1)/(c*x - 1))^2 - 2*(30*b*c^2*d - 77*b*c^2*e)*x^2 - 2*(60*a*c^5*e*x^5
+ 15*b*c^4*e*x^4 + 30*b*c^2*e*x^2 + 30*b*d - 137*b*e)*log(-c^2*x^2 + 1) -
4*(15*b*c^5*e*x^5*log(-c^2*x^2 + 1) - 10*b*c^3*e*x^3 + 3*(5*b*c^5*d - 2*b*
c^5*e)*x^5 - 30*b*c*e*x + 30*a*e)*log((c*x + 1)/(c*x - 1)))/c^5
```

3.272.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.68 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.10

$$\int x^4\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\begin{cases}\frac{adx^5}{5}+\frac{aex^5\log\left(-c^2x^2+1\right)}{5}-\frac{2aex^5}{25}-\frac{2aex^3}{15c^2}-\frac{2aex}{5c^4}+\frac{2ae\operatorname{acoth}(cx)}{5c^5}+\frac{bdx^5\operatorname{acoth}(cx)}{5}+\frac{bex^5\log\left(-c^2x^2+1\right)\operatorname{acoth}(cx)}{5}-\frac{2b}{} \\ \frac{dx^5\left(a+\frac{i\pi b}{2}\right)}{5}\end{cases}$$

input
```
integrate(x**4*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**5/5 + a*e*x**5*log(-c**2*x**2 + 1)/5 - 2*a*e*x**5/25 - 2
*a*e*x**3/(15*c**2) - 2*a*e*x/(5*c**4) + 2*a*e*acoth(c*x)/(5*c**5) + b*d*x
**5*acoth(c*x)/5 + b*e*x**5*log(-c**2*x**2 + 1)*acoth(c*x)/5 - 2*b*e*x**5*
acoth(c*x)/25 + b*d*x**4/(20*c) + b*e*x**4*log(-c**2*x**2 + 1)/(20*c) - 9*
b*e*x**4/(200*c) - 2*b*e*x**3*acoth(c*x)/(15*c**2) + b*d*x**2/(10*c**3) +
b*e*x**2*log(-c**2*x**2 + 1)/(10*c**3) - 77*b*e*x**2/(300*c**3) - 2*b*e*x*
acoth(c*x)/(5*c**4) + b*d*log(-c**2*x**2 + 1)/(10*c**5) + b*e*log(-c**2*x*
*2 + 1)**2/(20*c**5) - 137*b*e*log(-c**2*x**2 + 1)/(300*c**5) + b*e*acoth(
c*x)**2/(5*c**5), Ne(c, 0)), (d*x**5*(a + I*pi*b/2)/5, True))
```

3.272.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.01

$$\int x^4\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx=\frac{1}{5}adx^5$$
$$+\frac{1}{75}\left(15x^5\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\left(3c^4x^5+5c^2x^3+15x\right)}{c^6}-\frac{15\log\left(cx+1\right)}{c^7}+\frac{15\log\left(cx-1\right)}{c^7}\right)\right)be\operatorname{ar}$$
$$+\frac{1}{20}\left(4x^5\operatorname{arcoth}(cx)+c\left(\frac{c^2x^4+2x^2}{c^4}+\frac{2\log\left(c^2x^2-1\right)}{c^6}\right)\right)bd$$
$$+\frac{1}{75}\left(15x^5\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\left(3c^4x^5+5c^2x^3+15x\right)}{c^6}-\frac{15\log\left(cx+1\right)}{c^7}+\frac{15\log\left(cx-1\right)}{c^7}\right)\right)ae$$
$$-\frac{\left(3\left(-10i\pi c^4+9c^4\right)x^4+2\left(-30i\pi c^2+77c^2\right)x^2+2\left(-30i\pi-15c^4x^4-30c^2x^2-60\log\left(cx-1\right)+13\right.}{600c^5}$$

input
```
integrate(x^4*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxim
a")
```

output
```
1/5*a*d*x^5 + 1/75*(15*x^5*log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^2*x
^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b*e*arccoth(c
*x) + 1/20*(4*x^5*arccoth(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2
- 1)/c^6))*b*d + 1/75*(15*x^5*log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^
2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*a*e - 1/60
0*(3*(-10*I*pi*c^4 + 9*c^4)*x^4 + 2*(-30*I*pi*c^2 + 77*c^2)*x^2 + 2*(-30*I
*pi - 15*c^4*x^4 - 30*c^2*x^2 - 60*log(c*x - 1) + 137)*log(c*x + 1) + 2*(-
30*I*pi - 15*c^4*x^4 - 30*c^2*x^2 + 137)*log(c*x - 1))*b*e/c^5
```

3.272.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.14

$$\int x^4\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$= -\frac{1}{10}bex^5\log\left(-cx+1\right)^2 - \frac{1}{50}(-5i\pi bd+2i\pi be-10\,ad+4\,ae)x^5 + \frac{(10\,bd-9\,be)x^4}{200\,c} + \frac{1}{10}\left(bex^5+\frac{be}{c^5}\right)\log\left(cx+1\right)^2 - \frac{(i\pi be+2\,ae)x^3}{15\,c^2} - \frac{1}{300}\left(6\,(-5i\pi be-5\,bd-10\,ae+2\,be)x^5 - \frac{15\,bex^4}{c} + \frac{20\,bex^3}{c^2} - \frac{30\,bex^2}{c^3} + \frac{60\,bex}{c^4}\right)\log\left(cx+1\right) - \frac{1}{300}\left(6\,(-5i\pi be+5\,bd-10\,ae-2\,be)x^5 - \frac{15\,bex^4}{c} - \frac{20\,bex^3}{c^2} - \frac{30\,bex^2}{c^3} - \frac{60\,bex}{c^4} - \frac{60\,be\log\left(cx-1\right)}{c^5}\right.$$
$$\left.+1\right) + \frac{(30\,bd-77\,be)x^2}{300\,c^3} - \frac{be\log\left(cx-1\right)^2}{10\,c^5} - \frac{(i\pi be+2\,ae)x}{5\,c^4} + \frac{(30i\pi be+30\,bd+60\,ae-137\,be)\log\left(cx+1\right)}{300\,c^5} + \frac{(-30i\pi be+30\,bd-60\,ae-137\,be)\log\left(cx-1\right)}{300\,c^5}$$

input
```
integrate(x^4*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac"
)
```

output
```
-1/10*b*e*x^5*log(-c*x + 1)^2 - 1/50*(-5*I*pi*b*d + 2*I*pi*b*e - 10*a*d +
4*a*e)*x^5 + 1/200*(10*b*d - 9*b*e)*x^4/c + 1/10*(b*e*x^5 + b*e/c^5)*log(c
*x + 1)^2 - 1/15*(I*pi*b*e + 2*a*e)*x^3/c^2 - 1/300*(6*(-5*I*pi*b*e - 5*b*
d - 10*a*e + 2*b*e)*x^5 - 15*b*e*x^4/c + 20*b*e*x^3/c^2 - 30*b*e*x^2/c^3 +
 60*b*e*x/c^4)*log(c*x + 1) - 1/300*(6*(-5*I*pi*b*e + 5*b*d - 10*a*e - 2*b
*e)*x^5 - 15*b*e*x^4/c - 20*b*e*x^3/c^2 - 30*b*e*x^2/c^3 - 60*b*e*x/c^4 -
60*b*e*log(c*x - 1)/c^5)*log(-c*x + 1) + 1/300*(30*b*d - 77*b*e)*x^2/c^3 -
 1/10*b*e*log(c*x - 1)^2/c^5 - 1/5*(I*pi*b*e + 2*a*e)*x/c^4 + 1/300*(30*I*
pi*b*e + 30*b*d + 60*a*e - 137*b*e)*log(c*x + 1)/c^5 + 1/300*(-30*I*pi*b*e
 + 30*b*d - 60*a*e - 137*b*e)*log(c*x - 1)/c^5
```

3.272.9 Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.58

$$
\begin{aligned}
&\int x^4\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx \\
&= \ln\left(\frac{1}{c\,x}+1\right)\left(\frac{b\,d\,x^5}{10}-\frac{\frac{2\,b\,e\,c^5\,x^5}{5}+\frac{2\,b\,e\,c^3\,x^3}{3}+2\,b\,e\,c\,x}{10\,c^5}+\frac{b\,e\,x^5\,\ln\left(1-c^2\,x^2\right)}{10}\right) \\
&\quad+\ln\left(1-\frac{1}{c\,x}\right)\left(\frac{\frac{b\,d\,x^6}{5}-\frac{b\,c^2\,d\,x^8}{5}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\frac{4\,b\,e\,x^6}{75}+\frac{4\,b\,e\,x^4}{15\,c^2}-\frac{2\,b\,e\,x^2}{5\,c^4}+\frac{2\,b\,c^2\,e\,x^8}{25}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}\right. \\
&\qquad\left.+\frac{\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e\,x^6}{5}-\frac{b\,c^2\,e\,x^8}{5}\right)}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{b\,e\,\ln\left(\frac{1}{c\,x}+1\right)}{10\,c^5}\right) \\
&\quad+x^3\left(\frac{a\,\left(5\,d-2\,e\right)}{15\,c^2}-\frac{a\,d}{3\,c^2}\right)+x^2\left(\frac{b\,\left(10\,d-9\,e\right)}{100\,c^3}-\frac{b\,e}{6\,c^3}\right)+\frac{x\,\left(\frac{a\,\left(5\,d-2\,e\right)}{5\,c^2}-\frac{a\,d}{c^2}\right)}{c^2} \\
&\quad+\frac{a\,x^5\,\left(5\,d-2\,e\right)}{25}+c^2\,\ln\left(1-c^2\,x^2\right)\,\left(\frac{a\,e\,x^5}{5\,c^2}+\frac{b\,e\,x^4}{20\,c^3}+\frac{b\,e\,x^2}{10\,c^5}\right) \\
&\quad-\frac{\ln\left(c\,x-1\right)\,\left(60\,a\,e-30\,b\,d+137\,b\,e\right)}{300\,c^5}+\frac{\ln\left(c\,x+1\right)\,\left(60\,a\,e+30\,b\,d-137\,b\,e\right)}{300\,c^5} \\
&\quad+\frac{b\,e\,{\ln\left(\frac{1}{c\,x}+1\right)}^2}{20\,c^5}+\frac{b\,e\,{\ln\left(1-\frac{1}{c\,x}\right)}^2}{20\,c^5}+\frac{b\,e\,{\ln\left(1-c^2\,x^2\right)}^2}{20\,c^5}+\frac{b\,x^4\,\left(10\,d-9\,e\right)}{200\,c}
\end{aligned}
$$

input
```
int(x^4*(a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

output
```
log(1/(c*x) + 1)*((b*d*x^5)/10 - (2*b*c*e*x + (2*b*c^3*e*x^3)/3 + (2*b*c^5
*e*x^5)/5)/(10*c^5) + (b*e*x^5*log(1 - c^2*x^2))/10) + log(1 - 1/(c*x))*((
(b*d*x^6)/5 - (b*c^2*d*x^8)/5)/(2*(x + c*x^2)*(c*x - 1)) + ((4*b*e*x^6)/75
 + (4*b*e*x^4)/(15*c^2) - (2*b*e*x^2)/(5*c^4) + (2*b*c^2*e*x^8)/25)/(2*(x
+ c*x^2)*(c*x - 1)) + (log(1 - c^2*x^2)*((b*e*x^6)/5 - (b*c^2*e*x^8)/5))/(
2*(x + c*x^2)*(c*x - 1)) - (b*e*log(1/(c*x) + 1))/(10*c^5)) + x^3*((a*(5*d
 - 2*e))/(15*c^2) - (a*d)/(3*c^2)) + x^2*((b*(10*d - 9*e))/(100*c^3) - (b*
e)/(6*c^3)) + (x*((a*(5*d - 2*e))/(5*c^2) - (a*d)/c^2))/c^2 + (a*x^5*(5*d
- 2*e))/25 + c^2*log(1 - c^2*x^2)*((a*e*x^5)/(5*c^2) + (b*e*x^4)/(20*c^3)
+ (b*e*x^2)/(10*c^5)) - (log(c*x - 1)*(60*a*e - 30*b*d + 137*b*e))/(300*c^
5) + (log(c*x + 1)*(60*a*e + 30*b*d - 137*b*e))/(300*c^5) + (b*e*log(1/(c*
x) + 1)^2)/(20*c^5) + (b*e*log(1 - 1/(c*x))^2)/(20*c^5) + (b*e*log(1 - c^2
*x^2)^2)/(20*c^5) + (b*x^4*(10*d - 9*e))/(200*c)
```

3.272. $\int x^4 \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

3.273 $\int x^2 \left(a+b \coth^{-1}(cx)\right) \left(d+e \log \left(1-c^2 x^2\right)\right) \, dx$

3.273.1 Optimal result

Integrand size = 27, antiderivative size = 247

$$\begin{aligned}
&\int x^2 \left(a+b \coth^{-1}(cx)\right) \left(d+e \log \left(1-c^2 x^2\right)\right) \, dx \\
&= -\frac{2aex}{3c^2} - \frac{5bex^2}{18c} - \frac{2}{9} aex^3 - \frac{2bex \coth^{-1}(cx)}{3c^2} - \frac{2}{9} bex^3 \coth^{-1}(cx) \\
&\quad + \frac{be \coth^{-1}(cx)^2}{3c^3} - \frac{(2a+b)e \log(1-cx)}{6c^3} + \frac{(2a-b)e \log(1+cx)}{6c^3} \\
&\quad - \frac{4be \log \left(1-c^2x^2\right)}{9c^3} - \frac{be \log^2 \left(1-c^2x^2\right)}{12c^3} + \frac{bx^2 (d+e \log \left(1-c^2x^2\right))}{6c} \\
&\quad + \frac{1}{3} x^3 \left(a+b \coth^{-1}(cx)\right) \left(d+e \log \left(1-c^2x^2\right)\right) + \frac{b \log \left(1-c^2x^2\right) (d+e \log \left(1-c^2x^2\right))}{6c^3}
\end{aligned}$$

output
```
-2/3*a*e*x/c^2-5/18*b*e*x^2/c-2/9*a*e*x^3-2/3*b*e*x*arccoth(c*x)/c^2-2/9*b
*e*x^3*arccoth(c*x)+1/3*b*e*arccoth(c*x)^2/c^3-1/6*(2*a+b)*e*ln(-c*x+1)/c^
3+1/6*(2*a-b)*e*ln(c*x+1)/c^3-4/9*b*e*ln(-c^2*x^2+1)/c^3-1/12*b*e*ln(-c^2*
x^2+1)^2/c^3+1/6*b*x^2*(d+e*ln(-c^2*x^2+1))/c+1/3*x^3*(a+b*arccoth(c*x))*(
d+e*ln(-c^2*x^2+1))+1/6*b*ln(-c^2*x^2+1)*(d+e*ln(-c^2*x^2+1))/c^3
```

3.273.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.74

$$\int x^2\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{-24acex+2bc^2(3d-5e)x^2+4ac^3(3d-2e)x^3+4bcx(3c^2dx^2-2e(3+c^2x^2))\coth^{-1}(cx)+12be\coth^{-1}(}{}$$

input
```
Integrate[x^2*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(-24*a*c*e*x + 2*b*c^2*(3*d - 5*e)*x^2 + 4*a*c^3*(3*d - 2*e)*x^3 + 4*b*c*x
*(3*c^2*d*x^2 - 2*e*(3 + c^2*x^2))*ArcCoth[c*x] + 12*b*e*ArcCoth[c*x]^2 +
2*(3*b*d - 6*a*e - 11*b*e)*Log[1 - c*x] + 2*(3*b*d + 6*a*e - 11*b*e)*Log[1
 + c*x] + 6*c^2*e*x^2*(b + 2*a*c*x + 2*b*c*x*ArcCoth[c*x])*Log[1 - c^2*x^2
] + 3*b*e*Log[1 - c^2*x^2]^2)/(36*c^3)
```

3.273.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.90, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6648, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)\,dx$$
$$\Big\downarrow \text{6648}$$
$$2c^2e\int\left(\frac{\left(2cx\coth^{-1}(cx)b+b+2acx\right)x^3}{6c\left(1-c^2x^2\right)}+\frac{b\log\left(1-c^2x^2\right)x}{6c^3\left(1-c^2x^2\right)}\right)dx+\frac{1}{3}x^3\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{bx^2\left(e\log\left(1-c^2x^2\right)+d\right)}{6c}+\frac{b\log\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{6c^3}$$
$$\Big\downarrow \text{2009}$$

$$2c^2e\left(\frac{a\operatorname{arctanh}(cx)}{3c^5}-\frac{ax}{3c^4}-\frac{ax^3}{9c^2}+\frac{b\coth^{-1}(cx)^2}{6c^5}-\frac{bx\coth^{-1}(cx)}{3c^4}-\frac{5bx^2}{36c^3}-\frac{bx^3\coth^{-1}(cx)}{9c^2}-\frac{b\log^2\left(1-c^2x^2\right)}{24c^5}\right.$$
$$\frac{1}{3}x^3\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{bx^2\left(e\log\left(1-c^2x^2\right)+d\right)}{6c}+\frac{b\log\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{6c^3}$$

input
```
Int[x^2*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
(b*x^2*(d + e*Log[1 - c^2*x^2]))/(6*c) + (x^3*(a + b*ArcCoth[c*x])*(d + e*
Log[1 - c^2*x^2]))/3 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(6*c^
3) + 2*c^2*e*(-1/3*(a*x)/c^4 - (5*b*x^2)/(36*c^3) - (a*x^3)/(9*c^2) - (b*x
*ArcCoth[c*x])/(3*c^4) - (b*x^3*ArcCoth[c*x])/(9*c^2) + (b*ArcCoth[c*x]^2)
/(6*c^5) + (a*ArcTanh[c*x])/(3*c^5) - (11*b*Log[1 - c^2*x^2])/(36*c^5) - (
b*Log[1 - c^2*x^2]^2)/(24*c^5))
```

3.273.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6648
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegran
d[x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Inte
gerQ[m] && NeQ[m, -1]
```

3.273.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.93

method	result
parallelrisch	$\frac{12\operatorname{arccoth}(cx)bd+6b\,c^2d\,x^2-24aecx-10be\,x^2c^2+12be\ln\left(-c^2x^2+1\right)\operatorname{arccoth}(cx)x^3c^3+12\ln(cx-1)bd-44\ln(cx-1)be+6x^2\ln(}{}$
risch	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^2*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/36*(12*arccoth(c*x)*b*d+6*b*c^2*d*x^2-24*a*e*c*x-10*b*e*x^2*c^2+12*b*e*l
n(-c^2*x^2+1)*arccoth(c*x)*x^3*c^3+12*ln(c*x-1)*b*d-44*ln(c*x-1)*b*e+6*x^2
*ln(-c^2*x^2+1)*b*c^2*e-24*e*b*x*arccoth(c*x)*c-8*e*b*arccoth(c*x)*x^3*c^3
+12*e*b*arccoth(c*x)^2+12*a*e*x^3*ln(-c^2*x^2+1)*c^3+12*b*arccoth(c*x)*x^3
*c^3*d-8*a*e*x^3*c^3-44*arccoth(c*x)*b*e+12*a*d*x^3*c^3+3*e*b*ln(-c^2*x^2+
1)^2+24*arccoth(c*x)*a*e)/c^3
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.80

$$\int x^2\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx =$$

$$-\frac{24\,acex-4\,(3\,ac^3d-2\,ac^3e)x^3-3\,be\log\left(-c^2x^2+1\right)^2-3\,be\log\left(\frac{cx+1}{cx-1}\right)^2-2\,(3\,bc^2d-5\,bc^2e)x^2-2\,(6$$

input
```
integrate(x^2*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="frica
s")
```

output
```
-1/36*(24*a*c*e*x - 4*(3*a*c^3*d - 2*a*c^3*e)*x^3 - 3*b*e*log(-c^2*x^2 + 1
)^2 - 3*b*e*log((c*x + 1)/(c*x - 1))^2 - 2*(3*b*c^2*d - 5*b*c^2*e)*x^2 - 2
*(6*a*c^3*e*x^3 + 3*b*c^2*e*x^2 + 3*b*d - 11*b*e)*log(-c^2*x^2 + 1) - 2*(3
*b*c^3*e*x^3*log(-c^2*x^2 + 1) - 6*b*c*e*x + (3*b*c^3*d - 2*b*c^3*e)*x^3 +
 6*a*e)*log((c*x + 1)/(c*x - 1)))/c^3
```

3.273.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.07

$$\int x^2\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$

$$=\begin{cases}\frac{adx^3}{3}+\frac{aex^3\log\left(-c^2x^2+1\right)}{3}-\frac{2aex^3}{9}-\frac{2aex}{3c^2}+\frac{2ae\operatorname{acoth}(cx)}{3c^3}+\frac{bdx^3\operatorname{acoth}(cx)}{3}+\frac{bex^3\log\left(-c^2x^2+1\right)\operatorname{acoth}(cx)}{3}-\frac{2bex^3\operatorname{acoth}(}{9}\\ \frac{dx^3\left(a+\frac{i\pi b}{2}\right)}{3}\end{cases}$$

input
```
integrate(x**2*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**3/3 + a*e*x**3*log(-c**2*x**2 + 1)/3 - 2*a*e*x**3/9 - 2*
a*e*x/(3*c**2) + 2*a*e*acoth(c*x)/(3*c**3) + b*d*x**3*acoth(c*x)/3 + b*e*x
**3*log(-c**2*x**2 + 1)*acoth(c*x)/3 - 2*b*e*x**3*acoth(c*x)/9 + b*d*x**2/
(6*c) + b*e*x**2*log(-c**2*x**2 + 1)/(6*c) - 5*b*e*x**2/(18*c) - 2*b*e*x*a
coth(c*x)/(3*c**2) + b*d*log(-c**2*x**2 + 1)/(6*c**3) + b*e*log(-c**2*x**2
 + 1)**2/(12*c**3) - 11*b*e*log(-c**2*x**2 + 1)/(18*c**3) + b*e*acoth(c*x)
**2/(3*c**3), Ne(c, 0)), (d*x**3*(a + I*pi*b/2)/3, True))
```

3.273.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.02

$$\int x^2\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx=\frac{1}{3}\,adx^3$$
$$+\frac{1}{9}\left(3\,x^3\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log\left(cx+1\right)}{c^5}+\frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)be\,\mathrm{arcoth}\left(cx\right)$$
$$+\frac{1}{6}\left(2\,x^3\,\mathrm{arcoth}\left(cx\right)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)bd$$
$$+\frac{1}{9}\left(3\,x^3\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log\left(cx+1\right)}{c^5}+\frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)ae$$
$$+\frac{((3i\,\pi c^2-5\,c^2)x^2+(3i\,\pi+3\,c^2x^2+6\,\log\left(cx-1\right)-11)\log\left(cx+1\right)+(3i\,\pi+3\,c^2x^2-11)\log\left(cx-1\right)}{18\,c^3}$$

input
```
integrate(x^2*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxim
a")
```

output
```
1/3*a*d*x^3 + 1/9*(3*x^3*log(-c^2*x^2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 -
3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*e*arccoth(c*x) + 1/6*(2*x^3*ar
ccoth(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d + 1/9*(3*x^3*log(-c^2
*x^2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x -
1)/c^5))*a*e + 1/18*((3*I*pi*c^2 - 5*c^2)*x^2 + (3*I*pi + 3*c^2*x^2 + 6*lo
g(c*x - 1) - 11)*log(c*x + 1) + (3*I*pi + 3*c^2*x^2 - 11)*log(c*x - 1))*b*
e/c^3
```

3.273.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.14

$$\int x^2\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$= -\frac{1}{6}\,bex^3\log\left(-cx+1\right)^2 - \frac{1}{18}\left(-3i\,\pi bd+2i\,\pi be-6\,ad+4\,ae\right)x^3 + \frac{1}{6}\left(bex^3+\frac{be}{c^3}\right)\log\left(cx+1\right)^2 + \frac{(3\,bd-5\,be)x^2}{18\,c} - \frac{1}{18}\left((-3i\,\pi be-3\,bd-6\,ae+2\,be)x^3-\frac{3\,bex^2}{c}+\frac{6\,bex}{c^2}\right)\log\left(cx+1\right) - \frac{1}{18}\left((-3i\,\pi be+3\,bd-6\,ae-2\,be)x^3-\frac{3\,bex^2}{c}-\frac{6\,bex}{c^2}-\frac{6\,be\log\left(cx-1\right)}{c^3}\right)\log\left(-cx+1\right) - \frac{be\log\left(cx-1\right)^2}{6\,c^3} - \frac{(i\,\pi be+2\,ae)x}{3\,c^2} + \frac{(3i\,\pi be+3\,bd+6\,ae-11\,be)\log\left(cx+1\right)}{18\,c^3} + \frac{(-3i\,\pi be+3\,bd-6\,ae-11\,be)\log\left(cx-1\right)}{18\,c^3}$$

input
```
integrate(x^2*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac"
)
```

output
```
-1/6*b*e*x^3*log(-c*x + 1)^2 - 1/18*(-3*I*pi*b*d + 2*I*pi*b*e - 6*a*d + 4*
a*e)*x^3 + 1/6*(b*e*x^3 + b*e/c^3)*log(c*x + 1)^2 + 1/18*(3*b*d - 5*b*e)*x
^2/c - 1/18*((-3*I*pi*b*e - 3*b*d - 6*a*e + 2*b*e)*x^3 - 3*b*e*x^2/c + 6*b
*e*x/c^2)*log(c*x + 1) - 1/18*((-3*I*pi*b*e + 3*b*d - 6*a*e - 2*b*e)*x^3 -
 3*b*e*x^2/c - 6*b*e*x/c^2 - 6*b*e*log(c*x - 1)/c^3)*log(-c*x + 1) - 1/6*b
*e*log(c*x - 1)^2/c^3 - 1/3*(I*pi*b*e + 2*a*e)*x/c^2 + 1/18*(3*I*pi*b*e +
3*b*d + 6*a*e - 11*b*e)*log(c*x + 1)/c^3 + 1/18*(-3*I*pi*b*e + 3*b*d - 6*a
*e - 11*b*e)*log(c*x - 1)/c^3
```

3.273.9 Mupad [B] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.68

$$\int x^2(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\ln\left(\frac{1}{c\,x}+1\right)\left(\frac{b\,d\,x^3}{6}-\frac{\frac{2\,b\,e\,c^3\,x^3}{3}+2\,b\,e\,c\,x}{6\,c^3}+\frac{b\,e\,x^3\,\ln\left(1-c^2\,x^2\right)}{6}\right)$$
$$+x\left(\frac{a\,(3\,d-2\,e)}{3\,c^2}-\frac{a\,d}{c^2}\right)+\ln\left(1-\frac{1}{c\,x}\right)\left(\frac{\frac{4\,b\,e\,x^4}{9}-\frac{2\,b\,e\,x^2}{3\,c^2}+\frac{2\,b\,c^2\,e\,x^6}{9}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}\right.$$
$$\left.+\frac{\frac{b\,d\,x^4}{3}-\frac{b\,c^2\,d\,x^6}{3}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e\,x^4}{3}-\frac{b\,c^2\,e\,x^6}{3}\right)}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{b\,e\,\ln\left(\frac{1}{c\,x}+1\right)}{6\,c^3}\right)$$
$$+\frac{a\,x^3\,(3\,d-2\,e)}{9}+c^2\,\ln\left(1-c^2\,x^2\right)\,\left(\frac{a\,e\,x^3}{3\,c^2}+\frac{b\,e\,x^2}{6\,c^3}\right)$$
$$-\frac{\ln\left(c\,x-1\right)\,\left(6\,a\,e-3\,b\,d+11\,b\,e\right)}{18\,c^3}+\frac{\ln\left(c\,x+1\right)\,\left(6\,a\,e+3\,b\,d-11\,b\,e\right)}{18\,c^3}$$
$$+\frac{b\,e\ln\left(\frac{1}{c\,x}+1\right)^2}{12\,c^3}+\frac{b\,e\ln\left(1-\frac{1}{c\,x}\right)^2}{12\,c^3}+\frac{b\,e\ln\left(1-c^2\,x^2\right)^2}{12\,c^3}+\frac{b\,x^2\,(3\,d-5\,e)}{18\,c}$$

input
```
int(x^2*(a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

output
```
log(1/(c*x) + 1)*((b*d*x^3)/6 - (2*b*c*e*x + (2*b*c^3*e*x^3)/3)/(6*c^3) +
(b*e*x^3*log(1 - c^2*x^2))/6) + x*((a*(3*d - 2*e))/(3*c^2) - (a*d)/c^2) +
log(1 - 1/(c*x))*(((4*b*e*x^4)/9 - (2*b*e*x^2)/(3*c^2) + (2*b*c^2*e*x^6)/9
)/(2*(x + c*x^2)*(c*x - 1)) + ((b*d*x^4)/3 - (b*c^2*d*x^6)/3)/(2*(x + c*x^
2)*(c*x - 1)) + (log(1 - c^2*x^2)*((b*e*x^4)/3 - (b*c^2*e*x^6)/3))/(2*(x +
 c*x^2)*(c*x - 1)) - (b*e*log(1/(c*x) + 1))/(6*c^3)) + (a*x^3*(3*d - 2*e))
/9 + c^2*log(1 - c^2*x^2)*((a*e*x^3)/(3*c^2) + (b*e*x^2)/(6*c^3)) - (log(c
*x - 1)*(6*a*e - 3*b*d + 11*b*e))/(18*c^3) + (log(c*x + 1)*(6*a*e + 3*b*d
- 11*b*e))/(18*c^3) + (b*e*log(1/(c*x) + 1)^2)/(12*c^3) + (b*e*log(1 - 1/(
c*x))^2)/(12*c^3) + (b*e*log(1 - c^2*x^2)^2)/(12*c^3) + (b*x^2*(3*d - 5*e)
)/(18*c)
```

3.274 $\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

3.274.1 Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx = -2aex-2bex\coth^{-1}(cx)+\frac{e\left(a+b\coth^{-1}(cx)\right)^2}{bc}-\frac{be\log\left(1-c^2x^2\right)}{c} + x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)+\frac{b\left(d+e\log\left(1-c^2x^2\right)\right)^2}{4ce}$$

output
```
-2*a*e*x-2*b*e*x*arccoth(c*x)+e*(a+b*arccoth(c*x))^2/b/c-b*e*ln(-c^2*x^2+1
)/c+x*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))+1/4*b*(d+e*ln(-c^2*x^2+1))^2
/c/e
```

3.274.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.38

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx = adx-2aex+bdx\coth^{-1}(cx)-2bex\coth^{-1}(cx)+\frac{be\coth^{-1}(cx)^2}{c} + \frac{2ae\text{arctanh}(cx)}{c}+\frac{bd\log\left(1-c^2x^2\right)}{2c}-\frac{be\log\left(1-c^2x^2\right)}{c} + aex\log\left(1-c^2x^2\right)+bex\coth^{-1}(cx)\log\left(1-c^2x^2\right)+\frac{be\log^2\left(1-c^2x^2\right)}{4c}$$

input
```
Integrate[(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
a*d*x - 2*a*e*x + b*d*x*ArcCoth[c*x] - 2*b*e*x*ArcCoth[c*x] + (b*e*ArcCoth
[c*x]^2)/c + (2*a*e*ArcTanh[c*x])/c + (b*d*Log[1 - c^2*x^2])/(2*c) - (b*e*
Log[1 - c^2*x^2])/c + a*e*x*Log[1 - c^2*x^2] + b*e*x*ArcCoth[c*x]*Log[1 -
c^2*x^2] + (b*e*Log[1 - c^2*x^2]^2)/(4*c)
```

3.274.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6636, 2925, 2837, 2738, 6543, 2009, 6511}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b \coth^{-1}(cx)\right) \left(e \log \left(1-c^2x^2\right)+d\right) \, dx$$

$$\Big\downarrow \text{6636}$$

$$2c^2e\int \frac{x^2\left(a+b \coth^{-1}(cx)\right)}{1-c^2x^2}dx-bc\int \frac{x\left(d+e \log \left(1-c^2x^2\right)\right)}{1-c^2x^2}dx+x\left(a+b \coth^{-1}(cx)\right) \left(e \log \left(1-c^2x^2\right)+d\right)$$

$$\Big\downarrow \text{2925}$$

$$2c^2e\int \frac{x^2\left(a+b \coth^{-1}(cx)\right)}{1-c^2x^2}dx-\frac{1}{2}bc\int \frac{d+e \log \left(1-c^2x^2\right)}{1-c^2x^2}dx^2+x\left(a+b \coth^{-1}(cx)\right) \left(e \log \left(1-c^2x^2\right)+d\right)$$

$$\Big\downarrow \text{2837}$$

$$2c^2e\int \frac{x^2\left(a+b \coth^{-1}(cx)\right)}{1-c^2x^2}dx+\frac{b\int \frac{d+e \log \left(1-c^2x^2\right)}{x^2}d\left(1-c^2x^2\right)}{2c}+x\left(a+b \coth^{-1}(cx)\right) \left(e \log \left(1-c^2x^2\right)+d\right)$$

$$\Big\downarrow \text{2738}$$

$$2c^2e\int \frac{x^2\left(a+b \coth^{-1}(cx)\right)}{1-c^2x^2}dx+x\left(a+b \coth^{-1}(cx)\right) \left(e \log \left(1-c^2x^2\right)+d\right)+\frac{b\left(e \log \left(1-c^2x^2\right)+d\right)^2}{4ce}$$

$$\Big\downarrow \text{6543}$$

$$2c^2e\left(\frac{\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx}{c^2}-\frac{\int \left(a+b\coth^{-1}(cx)\right)dx}{c^2}\right)+$$
$$x\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{b\left(e\log\left(1-c^2x^2\right)+d\right)^2}{4ce}$$

$$\Big\downarrow \text{2009}$$

$$2c^2e\left(\frac{\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx}{c^2}-\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{c^2}\right)+$$
$$x\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{b\left(e\log\left(1-c^2x^2\right)+d\right)^2}{4ce}$$

$$\Big\downarrow \text{6511}$$

$$x\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+$$
$$2c^2e\left(\frac{\left(a+b\coth^{-1}(cx)\right)^2}{2bc^3}-\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{c^2}\right)+\frac{b\left(e\log\left(1-c^2x^2\right)+d\right)^2}{4ce}$$

input
```
Int[(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

output
```
x*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]) + (b*(d + e*Log[1 - c^2*x^
2])^2)/(4*c*e) + 2*c^2*e*((a + b*ArcCoth[c*x])^2/(2*b*c^3) - (a*x + b*x*Ar
cCoth[c*x] + (b*Log[1 - c^2*x^2])/(2*c))/c^2)
```

3.274.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6543
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6636
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcCoth[c*x]), x]
 + (-Simp[b*c   Int[x*((d + e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp
[2*e*g   Int[x^2*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b
, c, d, e, f, g}, x]
```

3.274.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.36

method	result
parallelrisch	$\frac{4be\ln(-c^2x^2+1)x \operatorname{arccoth}(cx)c+4b \operatorname{arccoth}(cx)xcd-8ebx \operatorname{arccoth}(cx)c+4aex\ln(-c^2x^2+1)c+4acdx-8aecx+4eb\operatorname{arccoth}(cx}{4c}$
risch	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*b*e*ln(-c^2*x^2+1)*x*arccoth(c*x)*c+4*b*arccoth(c*x)*x*c*d-8*e*b*x*
arccoth(c*x)*c+4*a*e*x*ln(-c^2*x^2+1)*c+4*a*c*d*x-8*a*e*c*x+4*e*b*arccoth(
c*x)^2+e*b*ln(-c^2*x^2+1)^2+8*arccoth(c*x)*a*e+2*ln(-c^2*x^2+1)*b*d-4*ln(-
c^2*x^2+1)*b*e)/c
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.25

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\frac{be\log\left(-c^2x^2+1\right)^2+be\log\left(\frac{cx+1}{cx-1}\right)^2+4\left(acd-2\,ace\right)x+2\left(2\,acex+bd-2\,be\right)\log\left(-c^2x^2+1\right)+2\left(bcex\right.}{4\,c}$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas")
```

output
```
1/4*(b*e*log(-c^2*x^2 + 1)^2 + b*e*log((c*x + 1)/(c*x - 1))^2 + 4*(a*c*d -
 2*a*c*e)*x + 2*(2*a*c*e*x + b*d - 2*b*e)*log(-c^2*x^2 + 1) + 2*(b*c*e*x*l
og(-c^2*x^2 + 1) + 2*a*e + (b*c*d - 2*b*c*e)*x)*log((c*x + 1)/(c*x - 1)))/
c
```

3.274.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.49

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\begin{cases} adx+aex\log\left(-c^2x^2+1\right)-2aex+\frac{2ae\operatorname{acoth}(cx)}{c}+bdx\operatorname{acoth}(cx)+bex\log\left(-c^2x^2+1\right)\operatorname{acoth}(cx)-2b \\ dx\left(a+\frac{i\pi b}{2}\right) \end{cases}$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x + a*e*x*log(-c**2*x**2 + 1) - 2*a*e*x + 2*a*e*acoth(c*x)/
c + b*d*x*acoth(c*x) + b*e*x*log(-c**2*x**2 + 1)*acoth(c*x) - 2*b*e*x*acot
h(c*x) + b*d*log(-c**2*x**2 + 1)/(2*c) + b*e*log(-c**2*x**2 + 1)**2/(4*c)
- b*e*log(-c**2*x**2 + 1)/c + b*e*acoth(c*x)**2/c, Ne(c, 0)), (d*x*(a + I*
pi*b/2), True))
```

3.274.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.71

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=-\left(c^2\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)-x\log\left(-c^2x^2+1\right)\right)be\,\mathrm{arcoth}(cx)$$
$$-\left(c^2\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)-x\log\left(-c^2x^2+1\right)\right)ae$$
$$+adx+\frac{(2cx\,\mathrm{arcoth}(cx)+\log(-c^2x^2+1))bd}{2c}$$
$$+\frac{((i\pi+2\log(cx-1)-2)\log(cx+1)+(i\pi-2)\log(cx-1))be}{2c}$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima")
```

output
```
-(c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) - x*log(-c^2*x^2 + 1
))*b*e*arccoth(c*x) - (c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)
 - x*log(-c^2*x^2 + 1))*a*e + a*d*x + 1/2*(2*c*x*arccoth(c*x) + log(-c^2*x
^2 + 1))*b*d/c + 1/2*((I*pi + 2*log(c*x - 1) - 2)*log(c*x + 1) + (I*pi - 2
)*log(c*x - 1))*b*e/c
```

3.274.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.90

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=-\frac{1}{2}bex\log(-cx+1)^2-\frac{1}{2}(-i\pi be-bd-2ae+2be)x\log(cx+1)$$
$$+\frac{1}{2}\left(bex+\frac{be}{c}\right)\log(cx+1)^2-\frac{be\log(cx-1)^2}{2c}-\frac{1}{2}(-i\pi bd+2i\pi be-2ad+4ae)x$$
$$-\frac{1}{2}\left((-i\pi be+bd-2ae-2be)x-\frac{2be\log(cx-1)}{c}\right)\log(-cx+1)$$
$$+\frac{(i\pi be+bd+2ae-2be)\log(cx+1)}{2c}+\frac{(-i\pi be+bd-2ae-2be)\log(cx-1)}{2c}$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

output
```
-1/2*b*e*x*log(-c*x + 1)^2 - 1/2*(-I*pi*b*e - b*d - 2*a*e + 2*b*e)*x*log(c
*x + 1) + 1/2*(b*e*x + b*e/c)*log(c*x + 1)^2 - 1/2*b*e*log(c*x - 1)^2/c -
1/2*(-I*pi*b*d + 2*I*pi*b*e - 2*a*d + 4*a*e)*x - 1/2*((-I*pi*b*e + b*d - 2
*a*e - 2*b*e)*x - 2*b*e*log(c*x - 1)/c)*log(-c*x + 1) + 1/2*(I*pi*b*e + b*
d + 2*a*e - 2*b*e)*log(c*x + 1)/c + 1/2*(-I*pi*b*e + b*d - 2*a*e - 2*b*e)*
log(c*x - 1)/c
```

3.274.9 Mupad [B] (verification not implemented)

Time = 4.99 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.03

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$$
$$=\ln\left(\frac{1}{c\,x}+1\right)\left(\frac{b\,d\,x}{2}-b\,e\,x+\frac{b\,e\,x\,\ln\left(1-c^2\,x^2\right)}{2}\right)+\ln\left(1-\frac{1}{c\,x}\right)\left(\frac{b\,d\,x^2-b\,c^2\,d\,x^4}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{2\,b\,e\,x^2-2\,b\,c^2\,e\,x^4}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\ln\left(1-c^2\,x^2\right)\,\left(b\,e\,x^2-b\,c^2\,e\,x^4\right)}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{b\,e\,\ln\left(\frac{1}{c\,x}+1\right)}{2\,c}\right)+a\,x\,\left(d-2\,e\right)+\frac{\ln\left(c\,x+1\right)\,\left(2\,a\,e+b\,d-2\,b\,e\right)}{2\,c}-\frac{\ln\left(c\,x-1\right)\,\left(2\,a\,e-b\,d+2\,b\,e\right)}{2\,c}+\frac{b\,e\ln\left(\frac{1}{c\,x}+1\right)^2}{4\,c}+\frac{b\,e\ln\left(1-\frac{1}{c\,x}\right)^2}{4\,c}+\frac{b\,e\ln\left(1-c^2\,x^2\right)^2}{4\,c}+a\,e\,x\,\ln\left(1-c^2\,x^2\right)$$

input
```
int((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

output
```
log(1/(c*x) + 1)*((b*d*x)/2 - b*e*x + (b*e*x*log(1 - c^2*x^2))/2) + log(1
- 1/(c*x))*((b*d*x^2 - b*c^2*d*x^4)/(2*(x + c*x^2)*(c*x - 1)) - (2*b*e*x^2
 - 2*b*c^2*e*x^4)/(2*(x + c*x^2)*(c*x - 1)) + (log(1 - c^2*x^2)*(b*e*x^2 -
 b*c^2*e*x^4))/(2*(x + c*x^2)*(c*x - 1)) - (b*e*log(1/(c*x) + 1))/(2*c)) +
 a*x*(d - 2*e) + (log(c*x + 1)*(2*a*e + b*d - 2*b*e))/(2*c) - (log(c*x - 1
)*(2*a*e - b*d + 2*b*e))/(2*c) + (b*e*log(1/(c*x) + 1)^2)/(4*c) + (b*e*log
(1 - 1/(c*x))^2)/(4*c) + (b*e*log(1 - c^2*x^2)^2)/(4*c) + a*e*x*log(1 - c^
2*x^2)
```

3.275 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^2}\,dx$

3.275.1 Optimal result

Integrand size = 27, antiderivative size = 105

$$\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^2}\,dx = -\frac{ce\left(a+b\coth^{-1}(cx)\right)^2}{b}-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x} + \frac{1}{2}bc\left(d+e\log\left(1-c^2x^2\right)\right)\log\left(1-\frac{1}{1-c^2x^2}\right)-\frac{1}{2}bce\,\text{PolyLog}\left(2,\frac{1}{1-c^2x^2}\right)$$

output
```
-c*e*(a+b*arccoth(c*x))^2/b-(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x+1/2*
b*c*(d+e*ln(-c^2*x^2+1))*ln(1-1/(-c^2*x^2+1))-1/2*b*c*e*polylog(2,1/(-c^2*
x^2+1))
```

3.275.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 332 vs. 2(105) = 210.

Time = 0.11 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.16

$$\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^2}\,dx = -\frac{4ad+4bd\coth^{-1}(cx)+4bcex\coth^{-1}(cx)^2+8acex\,\text{arctanh}(cx)-4bcdx\log(x)-bcex\log^2\left(-\frac{1}{c}+x\right)-}{}$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^2,x]
```

output
```
-1/4*(4*a*d + 4*b*d*ArcCoth[c*x] + 4*b*c*e*x*ArcCoth[c*x]^2 + 8*a*c*e*x*Ar
cTanh[c*x] - 4*b*c*d*x*Log[x] - b*c*e*x*Log[-c^(-1) + x]^2 - b*c*e*x*Log[c
^(-1) + x]^2 - 2*b*c*e*x*Log[c^(-1) + x]*Log[(1 - c*x)/2] + 4*b*c*e*x*Log[
x]*Log[1 - c*x] - 2*b*c*e*x*Log[-c^(-1) + x]*Log[(1 + c*x)/2] + 4*b*c*e*x*
Log[x]*Log[1 + c*x] + 4*a*e*Log[1 - c^2*x^2] + 2*b*c*d*x*Log[1 - c^2*x^2]
+ 4*b*e*ArcCoth[c*x]*Log[1 - c^2*x^2] - 4*b*c*e*x*Log[x]*Log[1 - c^2*x^2]
+ 2*b*c*e*x*Log[-c^(-1) + x]*Log[1 - c^2*x^2] + 2*b*c*e*x*Log[c^(-1) + x]*
Log[1 - c^2*x^2] + 4*b*c*e*x*PolyLog[2, -(c*x)] + 4*b*c*e*x*PolyLog[2, c*x
] - 2*b*c*e*x*PolyLog[2, 1/2 - (c*x)/2] - 2*b*c*e*x*PolyLog[2, (1 + c*x)/2
])/x
```

3.275.3 Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6644, 2925, 2858, 27, 2779, 2838, 6511}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x^2}dx$$

$$\Big\downarrow \text{6644}$$

$$-2c^2e\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+bc\int \frac{d+e\log\left(1-c^2x^2\right)}{x\left(1-c^2x^2\right)}dx-\frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x}$$

$$\Big\downarrow \text{2925}$$

$$-2c^2e\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{d+e\log\left(1-c^2x^2\right)}{x^2\left(1-c^2x^2\right)}dx^2-\frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x}$$

$$\Big\downarrow \text{2858}$$

$$-2c^2e\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx-\frac{b\int \frac{d+e\log(1-c^2x^2)}{x^4}d(1-c^2x^2)}{2c}-\frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{x}$$

$$\Big\downarrow \text{27}$$

$$-2c^2e\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx-\frac{1}{2}bc\int \frac{d+e\log(1-c^2x^2)}{c^2x^4}d(1-c^2x^2)-\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{x}$$

$$\Big\downarrow \text{2779}$$

$$-2c^2e\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx-\frac{1}{2}bc\left(e\int \frac{\log\left(1-\frac{1}{x^2}\right)}{x^2}d(1-c^2x^2)-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)\right)-\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{x}$$

$$\Big\downarrow \text{2838}$$

$$-2c^2e\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx-\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{x}-\frac{1}{2}bc\left(e\,\text{PolyLog}\left(2,\frac{1}{x^2}\right)-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)\right)$$

$$\Big\downarrow \text{6511}$$

$$-\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{x}-\frac{ce(a+b\coth^{-1}(cx))^2}{b}-\frac{1}{2}bc\left(e\,\text{PolyLog}\left(2,\frac{1}{x^2}\right)-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)\right)$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^2,x]
```

output
```
-((c*e*(a + b*ArcCoth[c*x])^2)/b) - ((a + b*ArcCoth[c*x])*(d + e*Log[1 - c
^2*x^2]))/x - (b*c*(-(Log[1 - x^(-2)]*(d + e*Log[1 - c^2*x^2])) + e*PolyLo
g[2, x^(-2)]))/2
```

3.275. $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^2}\,dx$

3.275.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6644
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcCoth[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d +
e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m
+ 2)*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f
, g}, x] && ILtQ[m/2, 0]
```

3.275. $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^2}\,dx$

3.275.4 Maple [F]

$$\int \frac{(a + b\ \operatorname{arccoth}(cx))\,(d + e\ln(-c^2x^2 + 1))}{x^2}dx$$

input
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^2,x)
```

output
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^2,x)
```

3.275.5 Fricas [F]

$$\int \frac{\left(a + b\coth^{-1}(cx)\right)\left(d + e\log\left(1 - c^2x^2\right)\right)}{x^2}\,dx = \int \frac{(b\operatorname{arcoth}(cx) + a)(e\log(-c^2x^2 + 1) + d)}{x^2}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="frica
s")
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
 1))/x^2, x)
```

3.275.6 Sympy [F]

$$\begin{aligned}&\int \frac{\left(a + b\coth^{-1}(cx)\right)\left(d + e\log\left(1 - c^2x^2\right)\right)}{x^2}\,dx\\&= \int \frac{(a + b\operatorname{acoth}(cx))\,(d + e\log(-c^2x^2 + 1))}{x^2}\,dx\end{aligned}$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**2,x)
```

output
```
Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**2, x)
```

3.275.7 Maxima [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^2}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="maxim
a")
```

output
```
-1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arccoth(c*x)/x)*b*d - (c^2*(log(
c*x + 1)/c - log(c*x - 1)/c) + log(-c^2*x^2 + 1)/x)*a*e - 1/2*b*e*(log(c*x
 + 1)^2/x - integrate(-((c*x + 1)*log(c*x - 1)^2 - (I*pi + (I*pi*c + 2*c)*
x)*log(c*x + 1) - (-I*pi - I*pi*c*x)*log(c*x - 1))/(c*x^3 + x^2), x)) - a*
d/x
```

3.275.8 Giac [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^2}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="giac"
)
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^2, x)
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acoth}(c\,x))\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^2}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^2,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^2, x)
```

3.276 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^4}\,dx$

3.276.1 Optimal result

Integrand size = 27, antiderivative size = 197

$$\begin{aligned}\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^4}\,dx &= \frac{2c^2e(a+b\coth^{-1}(cx))}{3x}-\frac{c^3e(a+b\coth^{-1}(cx))^2}{3b}-bc^3e\log(x)+\frac{1}{3}bc^3e\log(1-c^2x^2)\\ &-\frac{bc(1-c^2x^2)(d+e\log(1-c^2x^2))}{6x^2}-\frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{3x^3}\\ &+\frac{1}{6}bc^3(d+e\log(1-c^2x^2))\log\left(1-\frac{1}{1-c^2x^2}\right)-\frac{1}{6}bc^3e\,\mathrm{PolyLog}\left(2,\frac{1}{1-c^2x^2}\right)\end{aligned}$$

output
```
2/3*c^2*e*(a+b*arccoth(c*x))/x-1/3*c^3*e*(a+b*arccoth(c*x))^2/b-b*c^3*e*ln
(x)+1/3*b*c^3*e*ln(-c^2*x^2+1)-1/6*b*c*(-c^2*x^2+1)*(d+e*ln(-c^2*x^2+1))/x
^2-1/3*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^3+1/6*b*c^3*(d+e*ln(-c^2*
x^2+1))*ln(1-1/(-c^2*x^2+1))-1/6*b*c^3*e*polylog(2,1/(-c^2*x^2+1))
```

3.276.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 457 vs. 2(197) = 394.

Time = 0.23 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.32

$$\int \frac{(a+b\coth^{-1}(cx))\left(d+e\log\left(1-c^2x^2\right)\right)}{x^4}\,dx$$
$$= \frac{1}{6}\Bigg(-\frac{2ad}{x^3}-\frac{bcd}{x^2}+\frac{4ac^2e}{x}-\frac{2bd\coth^{-1}(cx)}{x^3}+\frac{4bc^2e\coth^{-1}(cx)}{x}-2bc^3e\coth^{-1}(cx)^2$$
$$-4ac^3e\text{arctanh}(cx)-4bc^3e\log\left(\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}\right)+2bc^3d\log(x)-2bc^3e\log(x)$$
$$+\frac{1}{2}bc^3e\log^2\left(-\frac{1}{c}+x\right)+\frac{1}{2}bc^3e\log^2\left(\frac{1}{c}+x\right)+bc^3e\log\left(\frac{1}{c}+x\right)\log\left(\frac{1}{2}(1-cx)\right)$$
$$-2bc^3e\log(x)\log(1-cx)+bc^3e\log\left(-\frac{1}{c}+x\right)\log\left(\frac{1}{2}(1+cx)\right)$$
$$-2bc^3e\log(x)\log(1+cx)-bc^3d\log\left(1-c^2x^2\right)+bc^3e\log\left(1-c^2x^2\right)$$
$$-\frac{2ae\log\left(1-c^2x^2\right)}{x^3}-\frac{bce\log\left(1-c^2x^2\right)}{x^2}-\frac{2be\coth^{-1}(cx)\log\left(1-c^2x^2\right)}{x^3}$$
$$+2bc^3e\log(x)\log\left(1-c^2x^2\right)-bc^3e\log\left(-\frac{1}{c}+x\right)\log\left(1-c^2x^2\right)$$
$$-bc^3e\log\left(\frac{1}{c}+x\right)\log\left(1-c^2x^2\right)-2bc^3e\,\text{PolyLog}(2,-cx)-2bc^3e\,\text{PolyLog}(2,cx)$$
$$+bc^3e\,\text{PolyLog}\left(2,\frac{1}{2}-\frac{cx}{2}\right)+bc^3e\,\text{PolyLog}\left(2,\frac{1}{2}(1+cx)\right)\Bigg)$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^4,x]
```

output
```
((-2*a*d)/x^3 - (b*c*d)/x^2 + (4*a*c^2*e)/x - (2*b*d*ArcCoth[c*x])/x^3 + (
4*b*c^2*e*ArcCoth[c*x])/x - 2*b*c^3*e*ArcCoth[c*x]^2 - 4*a*c^3*e*ArcTanh[c
*x] - 4*b*c^3*e*Log[1/Sqrt[1 - 1/(c^2*x^2)]] + 2*b*c^3*d*Log[x] - 2*b*c^3*
e*Log[x] + (b*c^3*e*Log[-c^(-1) + x]^2)/2 + (b*c^3*e*Log[c^(-1) + x]^2)/2
+ b*c^3*e*Log[c^(-1) + x]*Log[(1 - c*x)/2] - 2*b*c^3*e*Log[x]*Log[1 - c*x]
 + b*c^3*e*Log[-c^(-1) + x]*Log[(1 + c*x)/2] - 2*b*c^3*e*Log[x]*Log[1 + c*
x] - b*c^3*d*Log[1 - c^2*x^2] + b*c^3*e*Log[1 - c^2*x^2] - (2*a*e*Log[1 -
c^2*x^2])/x^3 - (b*c*e*Log[1 - c^2*x^2])/x^2 - (2*b*e*ArcCoth[c*x]*Log[1 -
 c^2*x^2])/x^3 + 2*b*c^3*e*Log[x]*Log[1 - c^2*x^2] - b*c^3*e*Log[-c^(-1) +
 x]*Log[1 - c^2*x^2] - b*c^3*e*Log[c^(-1) + x]*Log[1 - c^2*x^2] - 2*b*c^3*
e*PolyLog[2, -(c*x)] - 2*b*c^3*e*PolyLog[2, c*x] + b*c^3*e*PolyLog[2, 1/2
- (c*x)/2] + b*c^3*e*PolyLog[2, (1 + c*x)/2])/6
```

3.276.3 Rubi [A] (warning: unable to verify)

Time = 1.54 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.90, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {6644, 2925, 2858, 27, 2789, 2751, 16, 2779, 2838, 6545, 6453, 243, 47, 14, 16, 6511}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{x^4}dx$$

$$\Big\downarrow \text{6644}$$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2\left(1-c^2x^2\right)}dx+\frac{1}{3}bc\int \frac{d+e\log\left(1-c^2x^2\right)}{x^3\left(1-c^2x^2\right)}dx-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \text{2925}$$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2\left(1-c^2x^2\right)}dx+\frac{1}{6}bc\int \frac{d+e\log\left(1-c^2x^2\right)}{x^4\left(1-c^2x^2\right)}dx^2-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \text{2858}$$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2\left(1-c^2x^2\right)}dx-\frac{b\int \frac{d+e\log\left(1-c^2x^2\right)}{x^6}d\left(1-c^2x^2\right)}{6c}-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2(1-c^2x^2)}dx-\frac{1}{6}bc^3\int \frac{d+e\log(1-c^2x^2)}{c^4x^6}d(1-c^2x^2)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log(1-c^2x^2)+d\right)}{3x^3}$$

$$\Big\downarrow \; 2789$$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2(1-c^2x^2)}dx-\frac{1}{6}bc^3\left(\int \frac{d+e\log(1-c^2x^2)}{c^2x^4}d(1-c^2x^2)+\int \frac{d+e\log(1-c^2x^2)}{c^4x^4}d(1-c^2x^2)\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log(1-c^2x^2)+d\right)}{3x^3}$$

$$\Big\downarrow \; 2751$$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2(1-c^2x^2)}dx-\frac{1}{6}bc^3\left(\int \frac{d+e\log(1-c^2x^2)}{c^2x^4}d(1-c^2x^2)-e\int \frac{1}{c^2x^2}d(1-c^2x^2)+\frac{(1-c^2x^2)\left(e\log(1-c^2x^2)+d\right)}{c^2x^2}\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log(1-c^2x^2)+d\right)}{3x^3}$$

$$\Big\downarrow \; 16$$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2(1-c^2x^2)}dx-\frac{1}{6}bc^3\left(\int \frac{d+e\log(1-c^2x^2)}{c^2x^4}d(1-c^2x^2)+\frac{(1-c^2x^2)\left(e\log(1-c^2x^2)+d\right)}{c^2x^2}+e\log(c^2x^2)\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log(1-c^2x^2)+d\right)}{3x^3}$$

$$\Big\downarrow \; 2779$$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2(1-c^2x^2)}dx-\frac{1}{6}bc^3\left(e\int \frac{\log\left(1-\frac{1}{x^2}\right)}{x^2}d(1-c^2x^2)+\frac{(1-c^2x^2)\left(e\log(1-c^2x^2)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log(1-c^2x^2)+d\right)+e\right.$$

$$-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log(1-c^2x^2)+d\right)}{3x^3}$$

$$\Big\downarrow \; 2838$$

3.276. $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^4}\,dx$

$$-\frac{2}{3}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^2(1-c^2x^2)}dx-\frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{3x^3}-$$
$$\frac{1}{6}bc^3\left(\frac{(1-c^2x^2)\,(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)+e\,\mathrm{PolyLog}\left(2,\frac{1}{x^2}\right.\right.$$

$$\Big\downarrow\ 6545$$

$$-\frac{2}{3}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\int \frac{a+b\coth^{-1}(cx)}{x^2}dx\right)-$$
$$\frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{3x^3}-$$
$$\frac{1}{6}bc^3\left(\frac{(1-c^2x^2)\,(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)+e\,\mathrm{PolyLog}\left(2,\frac{1}{x^2}\right.\right.$$

$$\Big\downarrow\ 6453$$

$$-\frac{2}{3}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+bc\int \frac{1}{x(1-c^2x^2)}dx-\frac{a+b\coth^{-1}(cx)}{x}\right)-$$
$$\frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{3x^3}-$$
$$\frac{1}{6}bc^3\left(\frac{(1-c^2x^2)\,(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)+e\,\mathrm{PolyLog}\left(2,\frac{1}{x^2}\right.\right.$$

$$\Big\downarrow\ 243$$

$$-\frac{2}{3}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)}dx^2-\frac{a+b\coth^{-1}(cx)}{x}\right)-$$
$$\frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{3x^3}-$$
$$\frac{1}{6}bc^3\left(\frac{(1-c^2x^2)\,(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)+e\,\mathrm{PolyLog}\left(2,\frac{1}{x^2}\right.\right.$$

$$\Big\downarrow\ 47$$

$$-\frac{2}{3}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\int \frac{1}{x^2}dx^2\right)-\frac{a+b\coth^{-1}(cx)}{x}\right)-$$
$$\frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{3x^3}-$$
$$\frac{1}{6}bc^3\left(\frac{(1-c^2x^2)\,(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)+e\,\mathrm{PolyLog}\left(2,\frac{1}{x^2}\right.\right.$$

$$\Big\downarrow\ 14$$

3.276. $\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^4}\,dx$

$$-\frac{2}{3}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\log\left(x^2\right)\right)-\frac{a+b\coth^{-1}(cx)}{x}\right)-\frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-\frac{1}{6}bc^3\left(\frac{(1-c^2x^2)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\,\text{PolyLog}\left(2,\frac{1}{x^2}\right)\right)$$

$$\Big\downarrow \ 16$$

$$-\frac{2}{3}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx-\frac{a+b\coth^{-1}(cx)}{x}+\frac{1}{2}bc(\log\left(x^2\right)-\log\left(1-c^2x^2\right))\right)-\frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-\frac{1}{6}bc^3\left(\frac{(1-c^2x^2)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\,\text{PolyLog}\left(2,\frac{1}{x^2}\right)\right)$$

$$\Big\downarrow \ 6511$$

$$-\frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-\frac{2}{3}c^2e\left(\frac{c(a+b\coth^{-1}(cx))^2}{2b}-\frac{a+b\coth^{-1}(cx)}{x}+\frac{1}{2}bc(\log\left(x^2\right)-\log\left(1-c^2x^2\right))\right)-\frac{1}{6}bc^3\left(\frac{(1-c^2x^2)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)+e\,\text{PolyLog}\left(2,\frac{1}{x^2}\right)\right)$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^4,x]
```

output
```
(-2*c^2*e*(-((a + b*ArcCoth[c*x])/x) + (c*(a + b*ArcCoth[c*x])^2)/(2*b) +
(b*c*(Log[x^2] - Log[1 - c^2*x^2]))/2))/3 - ((a + b*ArcCoth[c*x])*(d + e*L
og[1 - c^2*x^2]))/(3*x^3) - (b*c^3*(e*Log[c^2*x^2] + ((1 - c^2*x^2)*(d + e
*Log[1 - c^2*x^2]))/(c^2*x^2) - Log[1 - x^(-2)]*(d + e*Log[1 - c^2*x^2]) +
 e*PolyLog[2, x^(-2)]))/6
```

3.276.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

3.276. $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^4}\,dx$

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.276. $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^4}\,dx$

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6545
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6644
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcCoth[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d +
e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m
+ 2)*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f
, g}, x] && ILtQ[m/2, 0]
```

3.276.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arccoth}\,(cx))\,(d+e\ln\left(-c^2x^2+1\right))}{x^4}dx$$

input
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

output
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

3.276.5 Fricas [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^4}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^4}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="frica
s")
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
 1))/x^4, x)
```

3.276.6 Sympy [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{acoth}\,(cx))\,(d+e\log(-c^2x^2+1))}{x^4}\,dx$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**4,x)
```

output
```
Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**4, x)
```

3.276.7 Maxima [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^4}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log(-c^2x^2+1)+d)}{x^4}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="maxim
a")
```

output
```
-1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arccoth(c*x)/x^3
)*b*d - 1/3*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c^2 + log(-c^2*x^2 +
1)/x^3)*a*e - 1/6*b*e*(log(c*x + 1)^2/x^3 - 3*integrate(-1/3*(3*(c*x + 1)*
log(c*x - 1)^2 - (3*I*pi + (3*I*pi*c + 2*c)*x)*log(c*x + 1) + 3*(I*pi + I*
pi*c*x)*log(c*x - 1))/(c*x^5 + x^4), x)) - 1/3*a*d/x^3
```

3.276.8 Giac [F]

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^4}\,dx=\int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log\left(-c^2x^2+1\right)+d)}{x^4}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="giac"
)
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^4, x)
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^4}\,dx = \int \frac{\left(a+b\,\mathrm{acoth}(c\,x)\right)\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^4}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^4,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^4, x)
```

3.277 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^6}\,dx$

3.277.1 Optimal result

Integrand size = 27, antiderivative size = 256

$$\begin{aligned}
&\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^6}\,dx \\
&= \frac{7bc^3e}{60x^2}+\frac{2c^2e(a+b\coth^{-1}(cx))}{15x^3}+\frac{2c^4e(a+b\coth^{-1}(cx))}{5x}-\frac{c^5e(a+b\coth^{-1}(cx))^2}{5b} \\
&\quad -\frac{5}{6}bc^5e\log(x)+\frac{19}{60}bc^5e\log(1-c^2x^2)-\frac{bc(d+e\log(1-c^2x^2))}{20x^4} \\
&\quad -\frac{bc^3(1-c^2x^2)(d+e\log(1-c^2x^2))}{10x^2}-\frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{5x^5} \\
&\quad +\frac{1}{10}bc^5(d+e\log(1-c^2x^2))\log\left(1-\frac{1}{1-c^2x^2}\right)-\frac{1}{10}bc^5e\,\mathrm{PolyLog}\left(2,\frac{1}{1-c^2x^2}\right)
\end{aligned}$$

output
```
7/60*b*c^3*e/x^2+2/15*c^2*e*(a+b*arccoth(c*x))/x^3+2/5*c^4*e*(a+b*arccoth(
c*x))/x-1/5*c^5*e*(a+b*arccoth(c*x))^2/b-5/6*b*c^5*e*ln(x)+19/60*b*c^5*e*l
n(-c^2*x^2+1)-1/20*b*c*(d+e*ln(-c^2*x^2+1))/x^4-1/10*b*c^3*(-c^2*x^2+1)*(d
+e*ln(-c^2*x^2+1))/x^2-1/5*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^5+1/1
0*b*c^5*(d+e*ln(-c^2*x^2+1))*ln(1-1/(-c^2*x^2+1))-1/10*b*c^5*e*polylog(2,1
/(-c^2*x^2+1))
```

3.277.2 Mathematica [F]

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^6}\,dx = \int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^6}\,dx$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6,x]
```

output
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6, x]
```

3.277.3 Rubi [A] (warning: unable to verify)

Time = 2.54 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.14, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.926, Rules used = {6644, 2925, 2858, 27, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838, 6545, 6453, 243, 54, 2009, 6545, 6453, 243, 47, 14, 16, 6511}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{x^6}\,dx$$

$$\Big\downarrow \text{ 6644}$$

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4(1-c^2x^2)}dx+\frac{1}{5}bc\int \frac{d+e\log(1-c^2x^2)}{x^5(1-c^2x^2)}dx-\frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{5x^5}$$

$$\Big\downarrow \text{ 2925}$$

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4(1-c^2x^2)}dx+\frac{1}{10}bc\int \frac{d+e\log(1-c^2x^2)}{x^6(1-c^2x^2)}dx^2-\frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{5x^5}$$

$$\Big\downarrow \text{ 2858}$$

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4\left(1-c^2x^2\right)}dx-\frac{b\int \frac{d+e\log\left(1-c^2x^2\right)}{x^8}d(1-c^2x^2)}{10c}-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 27

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4\left(1-c^2x^2\right)}dx-\frac{1}{10}bc^5\int \frac{d+e\log\left(1-c^2x^2\right)}{c^6x^8}d(1-c^2x^2)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 2789

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4\left(1-c^2x^2\right)}dx-\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^6x^6}d(1-c^2x^2)+\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^6}d(1-c^2x^2)\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 2756

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4\left(1-c^2x^2\right)}dx-\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^6}d(1-c^2x^2)-\frac{1}{2}e\int \frac{1}{c^4x^6}d(1-c^2x^2)+\frac{e\log\left(1-c^2x^2\right)+d}{2c^4x^4}\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 54

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4\left(1-c^2x^2\right)}dx-\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^6}d(1-c^2x^2)-\frac{1}{2}e\int \left(\frac{1}{c^2x^2}+\frac{1}{x^2}+\frac{1}{c^4x^4}\right)d(1-c^2x^2)+\frac{e\log\left(1-c^2x^2\right)+d}{2c^4x^4}\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$\Big\downarrow$ 2009

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4\left(1-c^2x^2\right)}dx-\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^6}d(1-c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\left(c^2x^2\right)+\log\left(1-c^2x^2\right)\right)+\frac{e\log\left(1-c^2x^2\right)+d}{2c^4x^4}\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

3.277. $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}dx$

$$\Big\downarrow \text{2789}$$

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4\left(1-c^2x^2\right)}dx-$$
$$\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^2x^4}d(1-c^2x^2)+\int \frac{d+e\log\left(1-c^2x^2\right)}{c^4x^4}d(1-c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\left(c^2x^2\right)+\log\left(1\right.\right.\right.$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2751}$$

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4\left(1-c^2x^2\right)}dx-$$
$$\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^2x^4}d(1-c^2x^2)-e\int \frac{1}{c^2x^2}d(1-c^2x^2)+\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\frac{1}{2}e\left(\frac{1}{c^2x\right.\right.$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{16}$$

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4\left(1-c^2x^2\right)}dx-$$
$$\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(1-c^2x^2\right)}{c^2x^4}d(1-c^2x^2)+\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\left(c\right.\right.\right.$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2779}$$

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4\left(1-c^2x^2\right)}dx-$$
$$\frac{1}{10}bc^5\left(e\int \frac{\log\left(1-\frac{1}{x^2}\right)}{x^2}d(1-c^2x^2)+\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\right.$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2}{5}c^2e\int \frac{a+b\coth^{-1}(cx)}{x^4\left(1-c^2x^2\right)}dx-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\left(\right.\right.\right.$$

$$\Big\downarrow \text{6545}$$

3.277. $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx$

$$-\frac{2}{5}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{x^2(1-c^2x^2)}dx+\int \frac{a+b\coth^{-1}(cx)}{x^4}dx\right)-\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{5x^5}-\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log(\right.\right.$$

$\downarrow$ 6453

$$-\frac{2}{5}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{x^2(1-c^2x^2)}dx+\frac{1}{3}bc\int \frac{1}{x^3(1-c^2x^2)}dx-\frac{a+b\coth^{-1}(cx)}{3x^3}\right)-\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{5x^5}-\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log(\right.\right.$$

$\downarrow$ 243

$$-\frac{2}{5}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{x^2(1-c^2x^2)}dx+\frac{1}{6}bc\int \frac{1}{x^4(1-c^2x^2)}dx^2-\frac{a+b\coth^{-1}(cx)}{3x^3}\right)-\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{5x^5}-\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log(\right.\right.$$

$\downarrow$ 54

$$-\frac{2}{5}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{x^2(1-c^2x^2)}dx+\frac{1}{6}bc\int \left(-\frac{c^4}{c^2x^2-1}+\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{a+b\coth^{-1}(cx)}{3x^3}\right)-\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{5x^5}-\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log(\right.\right.$$

$\downarrow$ 2009

$$-\frac{2}{5}c^2e\left(c^2\int \frac{a+b\coth^{-1}(cx)}{x^2(1-c^2x^2)}dx-\frac{a+b\coth^{-1}(cx)}{3x^3}+\frac{1}{6}bc\left(c^2\log(x^2)-c^2\log(1-c^2x^2)-\frac{1}{x^2}\right)\right)-\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{5x^5}-\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log(\right.\right.$$

$\downarrow$ 6545

3.277. $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^6}\,dx$

$$-\frac{2}{5}c^2e\left(c^2\left(c^2\int\frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\int\frac{a+b\coth^{-1}(cx)}{x^2}dx\right)-\frac{a+b\coth^{-1}(cx)}{3x^3}+\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1\right.\right.\right.$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$$\Big\downarrow \text{6453}$$

$$-\frac{2}{5}c^2e\left(c^2\left(c^2\int\frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+bc\int\frac{1}{x\left(1-c^2x^2\right)}dx-\frac{a+b\coth^{-1}(cx)}{x}\right)-\frac{a+b\coth^{-1}(cx)}{3x^3}+\frac{1}{6}bc\left(c^2\log\right.\right.$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$$\Big\downarrow \text{243}$$

$$-\frac{2}{5}c^2e\left(c^2\left(c^2\int\frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int\frac{1}{x^2\left(1-c^2x^2\right)}dx^2-\frac{a+b\coth^{-1}(cx)}{x}\right)-\frac{a+b\coth^{-1}(cx)}{3x^3}+\frac{1}{6}bc\left(\right.\right.$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$$\Big\downarrow \text{47}$$

$$-\frac{2}{5}c^2e\left(c^2\left(c^2\int\frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int\frac{1}{1-c^2x^2}dx^2+\int\frac{1}{x^2}dx^2\right)-\frac{a+b\coth^{-1}(cx)}{x}\right)-\frac{a+b\coth^{-1}}{3x^3}\right.$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

$$\Big\downarrow \text{14}$$

$$-\frac{2}{5}c^2e\left(c^2\left(c^2\int\frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int\frac{1}{1-c^2x^2}dx^2+\log\left(x^2\right)\right)-\frac{a+b\coth^{-1}(cx)}{x}\right)-\frac{a+b\coth^{-1}(}{3x^3}\right.$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(\frac{\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)+e\log\left(c^2x^2\right)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log\right.\right.$$

3.277. $\int\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}dx$

$$\Big\downarrow 16$$

$$-\frac{2}{5}c^2e\left(c^2\left(c^2\int \frac{a+b\coth^{-1}(cx)}{1-c^2x^2}dx-\frac{a+b\coth^{-1}(cx)}{x}+\frac{1}{2}bc(\log(x^2)-\log(1-c^2x^2))\right)-\frac{a+b\coth^{-1}(cx)}{3x^3}\right.$$
$$\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log(\right.\right.$$

$$\Big\downarrow 6511$$

$$-\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{5x^5}-$$
$$\frac{2}{5}c^2e\left(c^2\left(\frac{c(a+b\coth^{-1}(cx))^2}{2b}-\frac{a+b\coth^{-1}(cx)}{x}+\frac{1}{2}bc(\log(x^2)-\log(1-c^2x^2))\right)-\frac{a+b\coth^{-1}(cx)}{3x^3}+\frac{1}{6}bc\right.$$
$$\frac{1}{10}bc^5\left(\frac{(1-c^2x^2)(e\log(1-c^2x^2)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(1-c^2x^2)+d)+e\log(c^2x^2)-\frac{1}{2}e\left(\frac{1}{c^2x^2}-\log(\right.\right.$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6,x]
```

output
```
-1/5*((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5 - (2*c^2*e*(-1/3*
(a + b*ArcCoth[c*x])/x^3 + c^2*(-((a + b*ArcCoth[c*x])/x) + (c*(a + b*ArcC
oth[c*x])^2)/(2*b) + (b*c*(Log[x^2] - Log[1 - c^2*x^2]))/2) + (b*c*(-x^(-2
) + c^2*Log[x^2] - c^2*Log[1 - c^2*x^2]))/6))/5 - (b*c^5*(e*Log[c^2*x^2] -
 (e*(1/(c^2*x^2) - Log[c^2*x^2] + Log[1 - c^2*x^2]))/2 + (d + e*Log[1 - c^
2*x^2])/(2*c^4*x^4) + ((1 - c^2*x^2)*(d + e*Log[1 - c^2*x^2]))/(c^2*x^2) -
 Log[1 - x^(-2)]*(d + e*Log[1 - c^2*x^2]) + e*PolyLog[2, x^(-2)]))/10
```

3.277.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.277. $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^6}\,dx$

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

3.277. $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^6}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 6453
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6511
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6545
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.277. $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^6}\,dx$

rule 6644
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcCoth[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d +
e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m
+ 2)*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f
, g}, x] && ILtQ[m/2, 0]
```

3.277.4 Maple [F]

$$\int \frac{(a + b\,\mathrm{arccoth}\,(cx))\,(d + e\ln\,(-c^2x^2 + 1))}{x^6}dx$$

input
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^6,x)
```

output
```
int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^6,x)
```

3.277.5 Fricas [F]

$$\int \frac{(a + b\coth^{-1}(cx))\,(d + e\log\,(1 - c^2x^2))}{x^6}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx) + a)(e\log\,(-c^2x^2 + 1) + d)}{x^6}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="frica
s")
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
 1))/x^6, x)
```

3.277.6 Sympy [F]

$$\int \frac{(a + b\coth^{-1}(cx))\,(d + e\log\,(1 - c^2x^2))}{x^6}\,dx = \int \frac{(a + b\,\mathrm{acoth}\,(cx))\,(d + e\log\,(-c^2x^2 + 1))}{x^6}\,dx$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**6,x)
```

output
```
Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**6, x)
```

3.277.7 Maxima [F]

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx=\int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log\left(-c^2x^2+1\right)+d)}{x^6}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="maxim
a")
```

output
```
-1/20*((2*c^4*log(c^2*x^2 - 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c +
 4*arccoth(c*x)/x^5)*b*d - 1/15*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1)
- 2*(3*c^2*x^2 + 1)/x^3)*c^2 + 3*log(-c^2*x^2 + 1)/x^5)*a*e - 1/10*b*e*(lo
g(c*x + 1)^2/x^5 - 5*integrate(-1/5*(5*(c*x + 1)*log(c*x - 1)^2 - (5*I*pi
+ (5*I*pi*c + 2*c)*x)*log(c*x + 1) + 5*(I*pi + I*pi*c*x)*log(c*x - 1))/(c*
x^7 + x^6), x)) - 1/5*a*d/x^5
```

3.277.8 Giac [F]

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx=\int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log\left(-c^2x^2+1\right)+d)}{x^6}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="giac"
)
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^6, x)
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx$$
$$=\int \frac{\left(a+b\,\mathrm{acoth}(c\,x)\right)\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^6}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^6,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^6, x)
```

3.278 $\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$

3.278.1 Optimal result

Integrand size = 22, antiderivative size = 512

$$\begin{aligned}
&\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx \\
&=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right) \\
&+\frac{be\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\mathrm{arctanh}(cx)}{2c^2}-\frac{be(c^2f+g)\,\mathrm{arctanh}(cx)\log\left(\frac{2}{1+cx}\right)}{c^2g} \\
&+\frac{be(c^2f+g)\,\mathrm{arctanh}(cx)\log\left(\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{\left(c\sqrt{-f}-\sqrt{g}\right)(1+cx)}\right)}{2c^2g} \\
&+\frac{be(c^2f+g)\,\mathrm{arctanh}(cx)\log\left(\frac{2c\left(\sqrt{-f}+\sqrt{g}x\right)}{\left(c\sqrt{-f}+\sqrt{g}\right)(1+cx)}\right)}{2c^2g}+\frac{bex\log\left(f+gx^2\right)}{2c} \\
&+\frac{e\left(f+gx^2\right)\left(a+b\coth^{-1}(cx)\right)\log\left(f+gx^2\right)}{2g}-\frac{be(c^2f+g)\,\mathrm{arctanh}(cx)\log\left(f+gx^2\right)}{2c^2g} \\
&+\frac{be(c^2f+g)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2c^2g}-\frac{be(c^2f+g)\,\mathrm{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{\left(c\sqrt{-f}-\sqrt{g}\right)(1+cx)}\right)}{4c^2g} \\
&-\frac{be(c^2f+g)\,\mathrm{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}+\sqrt{g}x\right)}{\left(c\sqrt{-f}+\sqrt{g}\right)(1+cx)}\right)}{4c^2g}
\end{aligned}$$

output
```
1/2*b*(d-e)*x/c-b*e*x/c+1/2*d*x^2*(a+b*arccoth(c*x))-1/2*e*x^2*(a+b*arccot
h(c*x))-1/2*b*(d-e)*arctanh(c*x)/c^2-b*e*(c^2*f+g)*arctanh(c*x)*ln(2/(c*x+
1))/c^2/g+1/2*b*e*x*ln(g*x^2+f)/c+1/2*e*(g*x^2+f)*(a+b*arccoth(c*x))*ln(g*
x^2+f)/g-1/2*b*e*(c^2*f+g)*arctanh(c*x)*ln(g*x^2+f)/c^2/g+1/2*b*e*(c^2*f+g
)*arctanh(c*x)*ln(2*c*((-f)^(1/2)-x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)-g^(1/2)
))/c^2/g+1/2*b*e*(c^2*f+g)*arctanh(c*x)*ln(2*c*((-f)^(1/2)+x*g^(1/2))/(c*x
+1)/(c*(-f)^(1/2)+g^(1/2)))/c^2/g+1/2*b*e*(c^2*f+g)*polylog(2,1-2/(c*x+1))
/c^2/g-1/4*b*e*(c^2*f+g)*polylog(2,1-2*c*((-f)^(1/2)-x*g^(1/2))/(c*x+1)/(c
*(-f)^(1/2)-g^(1/2)))/c^2/g-1/4*b*e*(c^2*f+g)*polylog(2,1-2*c*((-f)^(1/2)+
x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)+g^(1/2)))/c^2/g+b*e*arctan(x*g^(1/2)/f^(1
/2))*f^(1/2)/c/g^(1/2)
```

3.278.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.10 (sec) , antiderivative size = 1128, normalized size of antiderivative = 2.20

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$$

$$=\frac{2bcdgx-6bcegx+2ac^2dgx^2-2ac^2egx^2-2bdg\coth^{-1}(cx)+2beg\coth^{-1}(cx)+2bc^2dgx^2\coth^{-1}(cx)-\cdots}{\cdots}$$

input
```
Integrate[x*(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]),x]
```

output
```
(2*b*c*d*g*x - 6*b*c*e*g*x + 2*a*c^2*d*g*x^2 - 2*a*c^2*e*g*x^2 - 2*b*d*g*A
rcCoth[c*x] + 2*b*e*g*ArcCoth[c*x] + 2*b*c^2*d*g*x^2*ArcCoth[c*x] - 2*b*c^
2*e*g*x^2*ArcCoth[c*x] + 4*b*c*e*Sqrt[f]*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f
]] - (4*I)*b*c^2*e*f*ArcSin[Sqrt[g/(c^2*f + g)]]*ArcTanh[(c*f)/(Sqrt[-(c^2
*f*g)]*x)] - (4*I)*b*e*g*ArcSin[Sqrt[g/(c^2*f + g)]]*ArcTanh[(c*f)/(Sqrt[-
(c^2*f*g)]*x)] - 4*b*c^2*e*f*ArcCoth[c*x]*Log[1 - E^(-2*ArcCoth[c*x])] - 4
*b*e*g*ArcCoth[c*x]*Log[1 - E^(-2*ArcCoth[c*x])] + 2*b*c^2*e*f*ArcCoth[c*x
]*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*ArcCoth[c*x])*g - 2*Sqrt
[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] + 2*b*e*g*ArcCoth[c*x]*Log
[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*ArcCoth[c*x])*g - 2*Sqrt[-(c^
2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] - (2*I)*b*c^2*e*f*ArcSin[Sqrt[g
/(c^2*f + g)]]*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*ArcCoth[c*x
])*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] - (2*I)*b*e*g
*ArcSin[Sqrt[g/(c^2*f + g)]]*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^
(2*ArcCoth[c*x])*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))]
 + 2*b*c^2*e*f*ArcCoth[c*x]*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(
2*ArcCoth[c*x])*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))]
+ 2*b*e*g*ArcCoth[c*x]*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*Arc
Coth[c*x])*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] + (2*
I)*b*c^2*e*f*ArcSin[Sqrt[g/(c^2*f + g)]]*Log[(c^2*(-1 + E^(2*ArcCoth[c*...
```

3.278.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6646, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)dx$$

$$\Big\downarrow \text{6646}$$

$$-bc\int\left(\frac{(d-e)x^2}{2\left(1-c^2x^2\right)}+\frac{e\left(gx^2+f\right)\log\left(gx^2+f\right)}{2g(1-cx)(cx+1)}\right)dx+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)+\frac{e\left(f+gx^2\right)\log\left(f+gx^2\right)\left(a+b\coth^{-1}(cx)\right)}{2g}-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)+\frac{e\left(f+gx^2\right)\log\left(f+gx^2\right)\left(a+b\coth^{-1}(cx)\right)}{2g}-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)-bc\left(-\frac{e\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c^2\sqrt{g}}+\frac{(d-e)\text{arctanh}(cx)}{2c^3}+\frac{e\text{arctanh}(cx)\left(c^2f+g\right)\log\left(f+gx^2\right)}{2c^3g}+\frac{e\text{arctanh}(cx)\left(c^2f+g\right)}{c^3g}\right.$$

input
```
Int[x*(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]),x]
```

output
```
(d*x^2*(a + b*ArcCoth[c*x]))/2 - (e*x^2*(a + b*ArcCoth[c*x]))/2 + (e*(f +
g*x^2)*(a + b*ArcCoth[c*x])*Log[f + g*x^2])/(2*g) - b*c*(-1/2*((d - e)*x)/
c^2 + (e*x)/c^2 - (e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(c^2*Sqrt[g]) +
((d - e)*ArcTanh[c*x])/(2*c^3) + (e*(c^2*f + g)*ArcTanh[c*x]*Log[2/(1 + c*
x)])/(c^3*g) - (e*(c^2*f + g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x)
)/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/(2*c^3*g) - (e*(c^2*f + g)*ArcTanh[
c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])
/(2*c^3*g) - (e*x*Log[f + g*x^2])/(2*c^2) + (e*(c^2*f + g)*ArcTanh[c*x]*Lo
g[f + g*x^2])/(2*c^3*g) - (e*(c^2*f + g)*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c
^3*g) + (e*(c^2*f + g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqr
t[-f] - Sqrt[g])*(1 + c*x))])/(4*c^3*g) + (e*(c^2*f + g)*PolyLog[2, 1 - (2
*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(4*c^3*g))
```

3.278.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6646
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Simp[(a + b*ArcCoth[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[
u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m
+ 1)/2, 0]
```

3.278.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 951 vs. $2(448) = 896$.

Time = 6.95 (sec) , antiderivative size = 952, normalized size of antiderivative = 1.86

method	result
risch	$\frac{db\ln(cx+1)x^2}{4} - \frac{db\ln(cx+1)}{4c^2} + \frac{db\ln(cx-1)}{4c^2} - \frac{eb\ln(cx-1)\ln\left(\frac{c\sqrt{-fg}-g(cx-1)-g}{c\sqrt{-fg}-g}\right)f}{4g} - \frac{eb\ln(cx-1)\ln\left(\frac{c\sqrt{-fg}+g(cx-1)+g}{c\sqrt{-fg}+g}\right)f}{4g}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x*(a+b*arccoth(c*x))*(d+e*ln(g*x^2+f)),x,method=_RETURNVERBOSE)
```

output
```
1/4*d*b*ln(c*x+1)*x^2-1/4*d*b/c^2*ln(c*x+1)+1/4*d*b/c^2*ln(c*x-1)-1/4*e*b/
g*ln(c*x-1)*ln((c*(-f*g)^(1/2)-g*(c*x-1)-g)/(c*(-f*g)^(1/2)-g))*f-1/4*e*b/
g*ln(c*x-1)*ln((c*(-f*g)^(1/2)+g*(c*x-1)+g)/(c*(-f*g)^(1/2)+g))*f+1/4*e*b/
g*ln(c*x+1)*ln((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*f+1/4*e*b/
g*ln(c*x+1)*ln((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))*f+e/c*f*b/
(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/2*a*d*x^2-3/2*b*e*x/c+1/2/c*b*d*x-1/
4*b*d*x^2*ln(c*x-1)+1/4*b*e*x^2*ln(c*x-1)-1/2*a*e*x^2+(1/4*b*x^2*e*ln(c*x+
1)-1/4*e*(b*x^2*ln(c*x-1)*c^2-2*a*c^2*x^2-2*b*c*x+b*ln(c*x+1)-b*ln(c*x-1))
/c^2)*ln(g*x^2+f)+1/4/c^2*b*e*ln(c*x+1)+1/4*e*b/c^2*dilog((c*(-f*g)^(1/2)-
(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))+1/4*e*b/c^2*dilog((c*(-f*g)^(1/2)+(c*x+1)
*g-g)/(c*(-f*g)^(1/2)-g))-1/4*e*b/c^2*ln(c*x-1)-1/4*e*b/c^2*dilog((c*(-f*g
)^(1/2)-g*(c*x-1)-g)/(c*(-f*g)^(1/2)-g))-1/4*e*b/c^2*dilog((c*(-f*g)^(1/2)
+g*(c*x-1)+g)/(c*(-f*g)^(1/2)+g))-1/4*e*b/c^2*ln(c*x-1)*ln((c*(-f*g)^(1/2)
-g*(c*x-1)-g)/(c*(-f*g)^(1/2)-g))-1/4*e*b/c^2*ln(c*x-1)*ln((c*(-f*g)^(1/2)
+g*(c*x-1)+g)/(c*(-f*g)^(1/2)+g))+1/4*e*b/c^2*ln(c*x+1)*ln((c*(-f*g)^(1/2)
-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))+1/4*e*b/c^2*ln(c*x+1)*ln((c*(-f*g)^(1/2)
+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))+1/2*e*f/g*a*ln(g*x^2+f)+1/4*e*b/g*dilog(
(c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*f+1/4*e*b/g*dilog((c*(-f*
g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))*f-1/4*e*b/g*dilog((c*(-f*g)^(1/2
)-g*(c*x-1)-g)/(c*(-f*g)^(1/2)-g))*f-1/4*e*b/g*dilog((c*(-f*g)^(1/2)+g*...
```

3.278.5 Fricas [F]

$$\int x(a+b\coth^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\int (b\,\mathrm{arcoth}\,(cx)+a)\big(e\log\left(gx^2+f\right)+d\big)x\,dx$$

input
```
integrate(x*(a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

output
```
integral(b*d*x*arccoth(c*x) + a*d*x + (b*e*x*arccoth(c*x) + a*e*x)*log(g*x
^2 + f), x)
```

3.278.6 Sympy [F(-1)]

Timed out.

$$\int x(a+b\coth^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*acoth(c*x))*(d+e*ln(g*x**2+f)),x)
```

output
```
Timed out
```

3.278.7 Maxima [F]

$$\int x(a+b\coth^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\int (b\,\mathrm{arcoth}\,(cx)+a)\big(e\log\left(gx^2+f\right)+d\big)x\,dx$$

input
```
integrate(x*(a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

output
```
1/2*a*d*x^2 + 1/4*(2*x^2*arccoth(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + lo
g(c*x - 1)/c^3))*b*d - 1/4*(2*c^2*g*integrate(x^3*log(c*x + 1)/(c^2*g*x^2
+ c^2*f), x) - 2*c^2*g*integrate(x^3*log(c*x - 1)/(c^2*g*x^2 + c^2*f), x)
- 2*c*g*(-I*f*(log(I*g*x/sqrt(f*g) + 1) - log(-I*g*x/sqrt(f*g) + 1))/(sqrt
(f*g)*c^2*g) - 2*x/(c^2*g)) - 2*g*integrate(x*log(c*x + 1)/(c^2*g*x^2 + c^
2*f), x) + 2*g*integrate(x*log(c*x - 1)/(c^2*g*x^2 + c^2*f), x) - (2*c*x +
 (c^2*x^2 - 1)*log(c*x + 1) - (c^2*x^2 - 1)*log(c*x - 1))*log(g*x^2 + f)/c
^2)*b*e - 1/2*(g*x^2 - (g*x^2 + f)*log(g*x^2 + f) + f)*a*e/g
```

3.278.8 Giac [F]

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\int (b\,\mathrm{arcoth}\,(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)x\,dx$$

input
```
integrate(x*(a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)*x, x)
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\int x\left(a+b\,\mathrm{acoth}(c\,x)\right)\left(d+e\,\ln\left(g\,x^2+f\right)\right)\,dx$$

input
```
int(x*(a + b*acoth(c*x))*(d + e*log(f + g*x^2)),x)
```

output
```
int(x*(a + b*acoth(c*x))*(d + e*log(f + g*x^2)), x)
```

3.279 $\int (a+b\coth^{-1}(cx)) (d+e\log(f+gx^2)) \, dx$

3.279.1 Optimal result

Integrand size = 21, antiderivative size = 546

$$\begin{aligned}
&\int (a+b\coth^{-1}(cx)) (d+e\log(f+gx^2)) \, dx \\
&= -2aex-2bex\coth^{-1}(cx)+\frac{2ae\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-\frac{be\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right)}{\sqrt{g}} \\
&+\frac{be\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1+\frac{1}{cx}\right)}{\sqrt{g}}+\frac{be\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(1-cx)}{(ic\sqrt{f}-\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{g}} \\
&-\frac{be\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(1+cx)}{(ic\sqrt{f}+\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{g}}-\frac{be\log(1-c^2x^2)}{c} \\
&+x(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))+\frac{b\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)(d+e\log(f+gx^2))}{2c} \\
&+\frac{be\,\mathrm{PolyLog}\left(2,\frac{c^2(f+gx^2)}{c^2f+g}\right)}{2c}-\frac{ibe\sqrt{f}\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(1-cx)}{(ic\sqrt{f}-\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{g}} \\
&+\frac{ibe\sqrt{f}\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(1+cx)}{(ic\sqrt{f}+\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{g}}
\end{aligned}$$

output
```
-2*a*e*x-2*b*e*x*arccoth(c*x)-b*e*ln(-c^2*x^2+1)/c+x*(a+b*arccoth(c*x))*(d
+e*ln(g*x^2+f))+1/2*b*ln(g*(-c^2*x^2+1)/(c^2*f+g))*(d+e*ln(g*x^2+f))/c+1/2
*b*e*polylog(2,c^2*(g*x^2+f)/(c^2*f+g))/c+2*a*e*arctan(x*g^(1/2)/f^(1/2))*
f^(1/2)/g^(1/2)-b*e*arctan(x*g^(1/2)/f^(1/2))*ln(1-1/c/x)*f^(1/2)/g^(1/2)+
b*e*arctan(x*g^(1/2)/f^(1/2))*ln(1+1/c/x)*f^(1/2)/g^(1/2)+b*e*arctan(x*g^(
1/2)/f^(1/2))*ln(-2*(-c*x+1)*f^(1/2)*g^(1/2)/(I*c*f^(1/2)-g^(1/2))/(f^(1/2
)-I*x*g^(1/2)))*f^(1/2)/g^(1/2)-b*e*arctan(x*g^(1/2)/f^(1/2))*ln(2*(c*x+1)
*f^(1/2)*g^(1/2)/(I*c*f^(1/2)+g^(1/2))/(f^(1/2)-I*x*g^(1/2)))*f^(1/2)/g^(1
/2)-1/2*I*b*e*polylog(2,1+2*(-c*x+1)*f^(1/2)*g^(1/2)/(I*c*f^(1/2)-g^(1/2))
/(f^(1/2)-I*x*g^(1/2)))*f^(1/2)/g^(1/2)+1/2*I*b*e*polylog(2,1-2*(c*x+1)*f^
(1/2)*g^(1/2)/(I*c*f^(1/2)+g^(1/2))/(f^(1/2)-I*x*g^(1/2)))*f^(1/2)/g^(1/2)
```

3.279.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1287 vs. 2(546) = 1092.

Time = 2.39 (sec) , antiderivative size = 1287, normalized size of antiderivative = 2.36

$$\int \left(a + b \coth^{-1}(cx)\right) \left(d + e \log \left(f + gx^2\right)\right) \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]),x]
```

output
```
a*d*x - 2*a*e*x + b*d*x*ArcCoth[c*x] + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/S
qrt[f]])/Sqrt[g] + (b*d*Log[1 - c^2*x^2])/(2*c) + a*e*x*Log[f + g*x^2] + b
*e*(x*ArcCoth[c*x] + Log[1 - c^2*x^2]/(2*c))*Log[f + g*x^2] + (b*e*(-4*c*x
*ArcCoth[c*x] + 4*Log[1/(c*Sqrt[1 - 1/(c^2*x^2)]*x)] + (Sqrt[c^2*f*g]*((-2
*I)*ArcCos[(c^2*f - g)/(c^2*f + g)]*ArcTan[Sqrt[c^2*f*g]/(c*g*x)] + 4*ArcC
oth[c*x]*ArcTan[(c*g*x)/Sqrt[c^2*f*g]] - (ArcCos[(c^2*f - g)/(c^2*f + g)]
+ 2*ArcTan[Sqrt[c^2*f*g]/(c*g*x)])*Log[((2*I)*g*(I*c^2*f + Sqrt[c^2*f*g])*
(-1 + 1/(c*x)))/((c^2*f + g)*(g + (I*Sqrt[c^2*f*g])/(c*x)))] - (ArcCos[(c^
2*f - g)/(c^2*f + g)] - 2*ArcTan[Sqrt[c^2*f*g]/(c*g*x)])*Log[(2*g*(c^2*f +
 I*Sqrt[c^2*f*g])*(1 + 1/(c*x)))/((c^2*f + g)*(g + (I*Sqrt[c^2*f*g])/(c*x)
))] + (ArcCos[(c^2*f - g)/(c^2*f + g)] + 2*(ArcTan[Sqrt[c^2*f*g]/(c*g*x)]
+ ArcTan[(c*g*x)/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]*Sqrt[c^2*f*g])/(E^ArcCoth[c
*x]*Sqrt[c^2*f + g]*Sqrt[-(c^2*f) + g + (c^2*f + g)*Cosh[2*ArcCoth[c*x]]])
] + (ArcCos[(c^2*f - g)/(c^2*f + g)] - 2*(ArcTan[Sqrt[c^2*f*g]/(c*g*x)] +
ArcTan[(c*g*x)/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]*E^ArcCoth[c*x]*Sqrt[c^2*f*g])
/(Sqrt[c^2*f + g]*Sqrt[-(c^2*f) + g + (c^2*f + g)*Cosh[2*ArcCoth[c*x]]])]
+ I*(-PolyLog[2, ((-(c^2*f) + g + (2*I)*Sqrt[c^2*f*g])*(g - (I*Sqrt[c^2*f*
g])/(c*x)))/((c^2*f + g)*(g + (I*Sqrt[c^2*f*g])/(c*x)))] + PolyLog[2, ((c^
2*f - g + (2*I)*Sqrt[c^2*f*g])*(I*g + Sqrt[c^2*f*g]/(c*x)))/((c^2*f + g)*(
(-I)*g + Sqrt[c^2*f*g]/(c*x)))])))/g))/(2*c) - (b*e*g*(((-Log[-c^(-1) +...
```

3.279.3 Rubi [A] (verified)

Time = 2.21 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.44, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6636, 2925, 2841, 2840, 2838, 6543, 2009, 6537, 218, 6535, 2920, 27, 2005, 5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)dx$$

$$\Big\downarrow \text{6636}$$

$$-2eg\int \frac{x^2\left(a+b\coth^{-1}(cx)\right)}{gx^2+f}dx-bc\int \frac{x\left(d+e\log\left(gx^2+f\right)\right)}{1-c^2x^2}dx+x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \text{2925}$$

$$-2eg\int \frac{x^2\left(a+b\coth^{-1}(cx)\right)}{gx^2+f}dx-\frac{1}{2}bc\int \frac{d+e\log\left(gx^2+f\right)}{1-c^2x^2}dx^2+x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \text{2841}$$

$$-2eg\int \frac{x^2\left(a+b\coth^{-1}(cx)\right)}{gx^2+f}dx-\frac{1}{2}bc\left(\frac{eg\int \frac{\log\left(\frac{g\left(1-c^2x^2\right)}{fc^2+g}\right)}{gx^2+f}dx^2}{c^2}-\frac{\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}\right)+x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \text{2840}$$

$$-2eg\int \frac{x^2\left(a+b\coth^{-1}(cx)\right)}{gx^2+f}dx-\frac{1}{2}bc\left(\frac{e\int \frac{\log\left(1-\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)}{x^2}d\left(gx^2+f\right)}{c^2}-\frac{\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}\right)+x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$-2eg\int \frac{x^2\left(a+b\coth^{-1}(cx)\right)}{gx^2+f}dx+x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(-\frac{\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)}{c^2}\right)$$

$$\Big\downarrow \text{6543}$$

$$-2eg\left(\frac{\int \left(a+b\coth^{-1}(cx)\right)dx}{g}-\frac{f\int \frac{a+b\coth^{-1}(cx)}{gx^2+f}dx}{g}\right)+x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(-\frac{\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)}{c^2}\right)$$

$$\Big\downarrow \text{2009}$$

3.279. $\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\int\frac{a+b\coth^{-1}(cx)}{gx^2+f}dx}{g}\right)+$$
$$x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-$$
$$\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$\big\downarrow$ 6537

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\left(a\int\frac{1}{gx^2+f}dx+b\int\frac{\coth^{-1}(cx)}{gx^2+f}dx\right)}{g}\right)+$$
$$x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-$$
$$\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$\big\downarrow$ 218

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\left(b\int\frac{\coth^{-1}(cx)}{gx^2+f}dx+\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)}{g}\right)+$$
$$x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-$$
$$\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$\big\downarrow$ 6535

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\left(b\left(\frac{1}{2}\int\frac{\log\left(1+\frac{1}{cx}\right)}{gx^2+f}dx-\frac{1}{2}\int\frac{\log\left(1-\frac{1}{cx}\right)}{gx^2+f}dx\right)+\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)}{g}\right)+$$
$$x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-$$
$$\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$\big\downarrow$ 2920

3.279. $\int\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)dx$

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\left(b\left(\frac{1}{2}\left(\frac{\int\frac{c\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(c-\frac{1}{x}\right)x^2}dx}{c}-\frac{\log\left(1-\frac{1}{cx}\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)+\frac{1}{2}\left(\frac{\int\frac{c\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(c+\frac{1}{x}\right)\ldots}}{c}\right.\right.\right.}{g}\right.$$
$$x(a+b\coth^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$\downarrow$ 27

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\left(b\left(\frac{1}{2}\left(\frac{\int\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\left(c-\frac{1}{x}\right)x^2}dx}{\sqrt{f}\sqrt{g}}-\frac{\log\left(1-\frac{1}{cx}\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)+\frac{1}{2}\left(\frac{\int\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\left(c+\frac{1}{x}\right)x^2}\ldots}{\sqrt{f}\sqrt{g}}\right.\right.\right.}{g}\right.$$
$$x(a+b\coth^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$\downarrow$ 2005

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\left(b\left(\frac{1}{2}\left(\frac{\int\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x(cx-1)}dx}{\sqrt{f}\sqrt{g}}-\frac{\log\left(1-\frac{1}{cx}\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)+\frac{1}{2}\left(\frac{\int\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x(cx+1)}\ldots}{\sqrt{f}\sqrt{g}}\right.\right.\right.}{g}\right.$$
$$x(a+b\coth^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$\downarrow$ 5411

3.279. $\int\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\left(b\left(\frac{1}{2}\left(\frac{\int\left(\frac{c\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{cx-1}-\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x}\right)dx}{\sqrt{f}\sqrt{g}}-\frac{\log(1-\frac{1}{cx})\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)+\right.\right.}{}\right.$$

$$x(a+b\coth^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\operatorname{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

$\downarrow$ 2009

$$-2eg\left(\frac{ax+\frac{b\log(1-c^2x^2)}{2c}+bx\coth^{-1}(cx)}{g}-\frac{f\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}\left(-\frac{\log(1-\frac{1}{cx})\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{}{(-}\right.}{}\right.\right.\right.}{}\right.$$

$$x(a+b\coth^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(-\frac{\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\operatorname{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)}{c^2}\right)$$

input
```
Int[(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]),x]
```

3.279. $\int\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$

output
```
x*(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]) - (b*c*(-((Log[(g*(1 - c^2*x
^2))/(c^2*f + g)]*(d + e*Log[f + g*x^2]))/c^2) - (e*PolyLog[2, (c^2*(f + g
*x^2))/(c^2*f + g)])/c^2))/2 - 2*e*g*((a*x + b*x*ArcCoth[c*x] + (b*Log[1 -
 c^2*x^2])/(2*c))/g - (f*((a*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]
) + b*((-((ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 - 1/(c*x)])/(Sqrt[f]*Sqrt[g])
) + (-(ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)
]) + ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(1 - c*x))/((I*c*
Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))] - (I/2)*PolyLog[2, ((-I)*Sqrt
[g]*x)/Sqrt[f]] + (I/2)*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]] + (I/2)*PolyLog[
2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)] - (I/2)*PolyLog[2, 1 + (2*Sqrt
[f]*Sqrt[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])
/(Sqrt[f]*Sqrt[g]))/2 + ((ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 + 1/(c*x)])/(S
qrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] -
I*Sqrt[g]*x)] - ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(1 + c*
x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))] + (I/2)*PolyLog[2,
((-I)*Sqrt[g]*x)/Sqrt[f]] - (I/2)*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]] - (I/2
)*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)] + (I/2)*PolyLog[2, 1
 - (2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqr
t[g]*x))])/(Sqrt[f]*Sqrt[g]))/2)))/g)
```

3.279.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

rule 6535
```
Int[ArcCoth[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[1/2   Int
[Log[1 + 1/(c*x)]/(d + e*x^2), x], x] - Simp[1/2   Int[Log[1 - 1/(c*x)]/(d
+ e*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

rule 6537
```
Int[(ArcCoth[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
 Simp[a   Int[1/(d + e*x^2), x], x] + Simp[b   Int[ArcCoth[c*x]/(d + e*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.279. $\int (a + b \coth^{-1}(cx)) (d + e \log (f + gx^2)) \, dx$

rule 6543
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6636
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcCoth[c*x]), x]
 + (-Simp[b*c   Int[x*((d + e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp
[2*e*g   Int[x^2*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b
, c, d, e, f, g}, x]
```

3.279.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.48 (sec) , antiderivative size = 3508, normalized size of antiderivative = 6.42

method	result	size
risch	Expression too large to display	3508

input
```
int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f)),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*e*b/c*Pi*csgn(I*c^2)^3*ln(c*x+1)-1/4*I*e*b/c*Pi*csgn(I/c^2*(c^2*f+((
c*x+1)^2-2*c*x-1)*g))^3*ln(c*x+1)-1/4*I*e*b*Pi*csgn(I*(c^2*f+((c*x+1)^2-2*
c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))^2*x-1/4*I*e*b*Pi*csgn
(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))^3*ln(c*x+1)*x-1/4*I*e*b*Pi*csgn(I/c^
2)*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))^2*x+1/4*I*e*b*Pi*csgn(I*c^2)^
3*ln(c*x+1)*x-1/4*I*e*b/c*Pi*csgn(I/c^2)*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*
x-1)*g))^2-1/4*I*e*b/c*Pi*csgn(I*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*csgn(I/c^2
*(c^2*f+((c*x+1)^2-2*c*x-1)*g))^2-1/4*I*e*b/c*Pi*csgn(I/c^2)*csgn(I/c^2*(c
^2*f+((c*x-1)^2+2*c*x-1)*g))^2-1/4*I*e*b/c*Pi*csgn(I*(c^2*f+((c*x-1)^2+2*c
*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^2+1/4*I*e*b/c*Pi*csgn(
I*c^2)^3*ln(c*x-1)-1/4*I*e*b/c*Pi*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g)
)^3*ln(c*x-1)+1/4*I*e*b*Pi*csgn(I/c^2)*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-
1)*g))^2*x-1/4*I*e*b*Pi*csgn(I*c^2)^3*ln(c*x-1)*x+1/4*I*e*b*Pi*csgn(I*(c^2
*f+((c*x-1)^2+2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^2*x+1
/4*I*e*b*Pi*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^3*ln(c*x-1)*x+1/2*d*
b/c*ln(c*x+1)+1/2/c*ln(c*x-1)*b*d-1/2*b*d*ln(c*x-1)*x+b*e*x*ln(c*x-1)+a*e*
x*ln(g*x^2+f)+a*d*x-b*d/c-b*e/c*ln(c*x-1)+1/2*d*b*ln(c*x+1)*x-b*x*e*ln(c*x
+1)-1/4*I*e*b*Pi*csgn(I*c^2)*csgn(I*c)^2*ln(c*x-1)*x-1/4*I*e*b*Pi*csgn(I*(
c^2*f+((c*x-1)^2+2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^2*
ln(c*x-1)*x-1/4*I*e*b*Pi*csgn(I/c^2)*csgn(I*(c^2*f+((c*x-1)^2+2*c*x-1)*...
```

3.279.5 Fricas [F]

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx=\int (b\,\mathrm{arcoth}\,(cx)+a)\big(e\log\left(gx^2+f\right)+d\big)\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

output
```
integral(b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(g*x^2 + f),
 x)
```

3.279.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f)),x)
```

output
```
Timed out
```

3.279.7 Maxima [F]

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx=\int \left(b\,\mathrm{arcoth}\left(cx\right)+a\right)\left(e\log\left(gx^2+f\right)+d\right)\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

output
```
(2*g*(f*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g) - x/g) + x*log(g*x^2 + f))*a*e
 + a*d*x + 1/2*b*e*(((c*x + 1)*log(c*x + 1) - (c*x - 1)*log(c*x - 1))*log(
g*x^2 + f)/c - integrate(2*((c*g*x^2 + g*x)*log(c*x + 1) - (c*g*x^2 - g*x)
*log(c*x - 1))/(c*g*x^2 + c*f), x)) + 1/2*(2*c*x*arccoth(c*x) + log(-c^2*x
^2 + 1))*b*d/c
```

3.279.8 Giac [F]

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx=\int \left(b\,\mathrm{arcoth}\left(cx\right)+a\right)\left(e\log\left(gx^2+f\right)+d\right)\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d), x)
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\coth^{-1}(cx))\,(d+e\log(f+gx^2))\,dx = \int (a+b\,\mathrm{acoth}(c\,x))\,(d+e\,\ln(g\,x^2+f))\,dx$$

input
```
int((a + b*acoth(c*x))*(d + e*log(f + g*x^2)),x)
```

output
```
int((a + b*acoth(c*x))*(d + e*log(f + g*x^2)), x)
```

3.280 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x}\,dx$

3.280.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x}\,dx &= ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)\\ &\quad+\frac{1}{2}bd\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}bd\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\\ &\quad+\frac{1}{2}ae\,\text{PolyLog}\left(2,1+\frac{gx^2}{f}\right)+be\text{Int}\left(\frac{\coth^{-1}(cx)\log(f+gx^2)}{x},x\right)\end{aligned}$$

output
```
b*e*CannotIntegrate(arccoth(c*x)*ln(g*x^2+f)/x,x)+a*d*ln(x)+1/2*a*e*ln(-g*
x^2/f)*ln(g*x^2+f)+1/2*b*d*polylog(2,-1/c/x)-1/2*b*d*polylog(2,1/c/x)+1/2*
a*e*polylog(2,1+g*x^2/f)
```

3.280.2 Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\begin{aligned}&\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x}\,dx\\ &= \int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x}\,dx\end{aligned}$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

output
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x, x]
```

3.280.3 Rubi [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6642, 6447, 6640, 2904, 2841, 2752, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\coth^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}dx$$

$$\Big\downarrow \text{6642}$$

$$d\int \frac{a+b\coth^{-1}(cx)}{x}dx+e\int \frac{(a+b\coth^{-1}(cx))\log\left(gx^2+f\right)}{x}dx$$

$$\Big\downarrow \text{6447}$$

$$e\int \frac{(a+b\coth^{-1}(cx))\log\left(gx^2+f\right)}{x}dx+ d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{6640}$$

$$e\left(a\int \frac{\log\left(gx^2+f\right)}{x}dx+b\int \frac{\coth^{-1}(cx)\log\left(gx^2+f\right)}{x}dx\right)+ d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{2904}$$

$$e\left(\frac{1}{2}a\int \frac{\log\left(gx^2+f\right)}{x^2}dx^2+b\int \frac{\coth^{-1}(cx)\log\left(gx^2+f\right)}{x}dx\right)+ d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{2841}$$

$$e\left(\frac{1}{2}a\left(\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)-g\int\frac{\log\left(-\frac{gx^2}{f}\right)}{gx^2+f}dx^2\right)+b\int\frac{\coth^{-1}(cx)\log\left(gx^2+f\right)}{x}dx\right)+ d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{ 2752}$$

$$e\left(b\int\frac{\coth^{-1}(cx)\log\left(gx^2+f\right)}{x}dx+\frac{1}{2}a\left(\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)+\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)\right)\right)+ d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{ 7299}$$

$$e\left(b\int\frac{\coth^{-1}(cx)\log\left(gx^2+f\right)}{x}dx+\frac{1}{2}a\left(\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)+\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)\right)\right)+ d\left(a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

output
```
$Aborted
```

3.280.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 6447
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

rule 6640
```
Int[(Log[(f_.) + (g_.)*(x_)^2]*(ArcCoth[(c_.)*(x_)]*(b_.) + (a_)))/(x_), x_
Symbol] :> Simp[a   Int[Log[f + g*x^2]/x, x], x] + Simp[b   Int[Log[f + g*x
^2]*(ArcCoth[c*x]/x), x], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 6642
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*(Log[(f_.) + (g_.)*(x_)^2]*(e_.) +
 (d_)))/(x_), x_Symbol] :> Simp[d   Int[(a + b*ArcCoth[c*x])/x, x], x] + Si
mp[e   Int[Log[f + g*x^2]*((a + b*ArcCoth[c*x])/x), x], x] /; FreeQ[{a, b,
c, d, e, f, g}, x]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.280.4 Maple [N/A] (verified)

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \operatorname{arccoth}(cx))\,(d + e\ln(g\,x^2 + f))}{x}dx$$

input
```
int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

output
```
int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

3.280.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log\left(gx^2+f\right)+d)}{x}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="fricas")
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(g*x^2 + f)
)/x, x)
```

3.280.6 Sympy [N/A]

Not integrable

Time = 113.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx = \int \frac{\left(a+b\,\mathrm{acoth}\,(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f))/x,x)
```

output
```
Integral((a + b*acoth(c*x))*(d + e*log(f + g*x**2))/x, x)
```

3.280.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.88

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log\left(gx^2+f\right)+d)}{x}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="maxima")
```

output
```
a*d*log(x) + integrate(1/2*b*e*(log(1/(c*x) + 1) - log(-1/(c*x) + 1))*log(
g*x^2 + f)/x + 1/2*b*d*(log(1/(c*x) + 1) - log(-1/(c*x) + 1))/x + a*e*log(
g*x^2 + f)/x, x)
```

3.280.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(f+gx^2))}{x}\,dx$$
$$=\int \frac{(b\,\text{arcoth}\,(cx)+a)(e\log(gx^2+f)+d)}{x}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="giac")
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)/x, x)
```

3.280.9 Mupad [N/A]

Not integrable

Time = 5.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(f+gx^2))}{x}\,dx=\int \frac{(a+b\,\text{acoth}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)}{x}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x, x)
```

3.281 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x^2}\,dx$

3.281.1 Optimal result

Integrand size = 24, antiderivative size = 560

$$\begin{aligned}
\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x^2}\,dx
&= \frac{2ae\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right)}{\sqrt{f}} \\
&+ \frac{be\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1+\frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(1-cx)}{(ic\sqrt{f}-\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}} \\
&- \frac{be\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(1+cx)}{(ic\sqrt{f}+\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}} \\
&- \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x} + \frac{1}{2}bc\log\left(-\frac{gx^2}{f}\right)(d+e\log(f+gx^2)) \\
&- \frac{1}{2}bc\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)(d+e\log(f+gx^2)) - \frac{1}{2}bce\,\mathrm{PolyLog}\left(2,\frac{c^2(f+gx^2)}{c^2f+g}\right) \\
&+ \frac{1}{2}bce\,\mathrm{PolyLog}\left(2,1+\frac{gx^2}{f}\right) - \frac{ibe\sqrt{g}\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(1-cx)}{(ic\sqrt{f}-\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}} \\
&+ \frac{ibe\sqrt{g}\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(1+cx)}{(ic\sqrt{f}+\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}}
\end{aligned}$$

output
```
-(a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x+1/2*b*c*ln(-g*x^2/f)*(d+e*ln(g*x^2
+f))-1/2*b*c*ln(g*(-c^2*x^2+1)/(c^2*f+g))*(d+e*ln(g*x^2+f))-1/2*b*c*e*poly
log(2,c^2*(g*x^2+f)/(c^2*f+g))+1/2*b*c*e*polylog(2,1+g*x^2/f)+2*a*e*arctan
(x*g^(1/2)/f^(1/2))*g^(1/2)/f^(1/2)-b*e*arctan(x*g^(1/2)/f^(1/2))*ln(1-1/c
/x)*g^(1/2)/f^(1/2)+b*e*arctan(x*g^(1/2)/f^(1/2))*ln(1+1/c/x)*g^(1/2)/f^(1
/2)+b*e*arctan(x*g^(1/2)/f^(1/2))*ln(-2*(-c*x+1)*f^(1/2)*g^(1/2)/(I*c*f^(1
/2)-g^(1/2))/(f^(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(1/2)-b*e*arctan(x*g^(1/2)/f
^(1/2))*ln(2*(c*x+1)*f^(1/2)*g^(1/2)/(I*c*f^(1/2)+g^(1/2))/(f^(1/2)-I*x*g^
(1/2)))*g^(1/2)/f^(1/2)-1/2*I*b*e*polylog(2,1+2*(-c*x+1)*f^(1/2)*g^(1/2)/(
I*c*f^(1/2)-g^(1/2))/(f^(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(1/2)+1/2*I*b*e*poly
log(2,1-2*(c*x+1)*f^(1/2)*g^(1/2)/(I*c*f^(1/2)+g^(1/2))/(f^(1/2)-I*x*g^(1/
2)))*g^(1/2)/f^(1/2)
```

3.281.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1236 vs. 2(560) = 1120.

Time = 2.52 (sec) , antiderivative size = 1236, normalized size of antiderivative = 2.21

$$\int \frac{(a+b\coth^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

output
```
-((a*d)/x) - (b*d*ArcCoth[c*x])/x + b*c*d*Log[x] - (b*c*d*Log[1 - c^2*x^2]
)/2 + a*e*((2*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - Log[f + g*x^2
]/x) + (b*e*(-(((2*ArcCoth[c*x] + c*x*(-2*Log[x] + Log[1 - c^2*x^2]))*Log[
f + g*x^2])/x) - 2*c*(Log[x]*(Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + Log[1 + (I*
Sqrt[g]*x)/Sqrt[f]]) + PolyLog[2, ((-I)*Sqrt[g]*x)/Sqrt[f]] + PolyLog[2, (
I*Sqrt[g]*x)/Sqrt[f]]) + c*(Log[-c^(-1) + x]*Log[(c*(Sqrt[f] - I*Sqrt[g]*x
))/(c*Sqrt[f] - I*Sqrt[g])] + Log[c^(-1) + x]*Log[(c*(Sqrt[f] - I*Sqrt[g]*
x))/(c*Sqrt[f] + I*Sqrt[g])] + Log[-c^(-1) + x]*Log[(c*(Sqrt[f] + I*Sqrt[g
]*x))/(c*Sqrt[f] + I*Sqrt[g])] - (Log[-c^(-1) + x] + Log[c^(-1) + x] - Log
[1 - c^2*x^2])*Log[f + g*x^2] + Log[c^(-1) + x]*Log[1 - (Sqrt[g]*(1 + c*x)
)/(I*c*Sqrt[f] + Sqrt[g])] + PolyLog[2, (c*Sqrt[g]*(c^(-1) + x))/(I*c*Sqrt
[f] + Sqrt[g])] + PolyLog[2, (I*Sqrt[g]*(-1 + c*x))/(c*Sqrt[f] - I*Sqrt[g]
)] + PolyLog[2, ((-I)*Sqrt[g]*(-1 + c*x))/(c*Sqrt[f] + I*Sqrt[g])] + PolyL
og[2, (I*Sqrt[g]*(1 + c*x))/(c*Sqrt[f] + I*Sqrt[g])]) - (c*g*((2*I)*ArcCos
[(c^2*f - g)/(c^2*f + g)]*ArcTan[(c*f)/(Sqrt[c^2*f*g]*x)] - 4*ArcCoth[c*x]
*ArcTan[(c*g*x)/Sqrt[c^2*f*g]] + (ArcCos[(c^2*f - g)/(c^2*f + g)] + 2*ArcT
an[(c*f)/(Sqrt[c^2*f*g]*x)])*Log[(2*g*(c^2*f - I*Sqrt[c^2*f*g])*(-1 + c*x)
)/((c^2*f + g)*(I*Sqrt[c^2*f*g] + c*g*x))] + (ArcCos[(c^2*f - g)/(c^2*f +
g)] - 2*ArcTan[(c*f)/(Sqrt[c^2*f*g]*x)])*Log[(2*g*(c^2*f + I*Sqrt[c^2*f*g]
)*(1 + c*x))/((c^2*f + g)*(I*Sqrt[c^2*f*g] + c*g*x))] - (ArcCos[(c^2*f ...
```

3.281.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.39, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6644, 2925, 2863, 2009, 6537, 218, 6535, 2920, 27, 2005, 5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^2}dx$$

$$\Big\downarrow \text{6644}$$

$$2eg\int \frac{a+b\coth^{-1}(cx)}{gx^2+f}dx+bc\int \frac{d+e\log\left(gx^2+f\right)}{x\left(1-c^2x^2\right)}dx-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}$$

$$\Big\downarrow \text{2925}$$

$$2eg\int \frac{a+b\coth^{-1}(cx)}{gx^2+f}dx+\frac{1}{2}bc\int \frac{d+e\log\left(gx^2+f\right)}{x^2\left(1-c^2x^2\right)}dx^2-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}$$

$$\downarrow \text{2863}$$

$$2eg\int \frac{a+b\coth^{-1}(cx)}{gx^2+f}dx+\frac{1}{2}bc\int \left(\frac{d+e\log(gx^2+f)}{x^2}-\frac{c^2(d+e\log(gx^2+f))}{c^2x^2-1}\right)dx^2-\frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x}$$

$$\downarrow \text{2009}$$

$$2eg\int \frac{a+b\coth^{-1}(cx)}{gx^2+f}dx-\frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x}+\frac{1}{2}bc\left(-\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)(d+e\log(f+gx^2))-e\operatorname{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)(d+e\log(f+g$$

$$\downarrow \text{6537}$$

$$2eg\left(a\int \frac{1}{gx^2+f}dx+b\int \frac{\coth^{-1}(cx)}{gx^2+f}dx\right)-\frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x}+\frac{1}{2}bc\left(-\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)(d+e\log(f+gx^2))-e\operatorname{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)(d+e\log(f+g$$

$$\downarrow \text{218}$$

$$2eg\left(b\int \frac{\coth^{-1}(cx)}{gx^2+f}dx+\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)-\frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x}+\frac{1}{2}bc\left(-\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)(d+e\log(f+gx^2))-e\operatorname{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)(d+e\log(f+g$$

$$\downarrow \text{6535}$$

$$2eg\left(b\left(\frac{1}{2}\int \frac{\log\left(1+\frac{1}{cx}\right)}{gx^2+f}dx-\frac{1}{2}\int \frac{\log\left(1-\frac{1}{cx}\right)}{gx^2+f}dx\right)+\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)-\frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x}+\frac{1}{2}bc\left(-\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)(d+e\log(f+gx^2))-e\operatorname{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)(d+e\log(f+g$$

$$\downarrow \text{2920}$$

3.281. $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x^2}\,dx$

$$2eg\left(b\left(\frac{1}{2}\left(\frac{\int \frac{c\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(c-\frac{1}{x}\right)x^2}dx}{c}-\frac{\log\left(1-\frac{1}{cx}\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)+\frac{1}{2}\left(\frac{\int \frac{c\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(c+\frac{1}{x}\right)x^2}dx}{c}+\frac{\log\left(\frac{1}{cx}+1\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right.\right.\right.$$

$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$

$$\frac{1}{2}bc\left(-\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+gx^2\right)\right)\right.$$

$\downarrow$ 27

$$2eg\left(b\left(\frac{1}{2}\left(\frac{\int \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\left(c-\frac{1}{x}\right)x^2}dx}{\sqrt{f}\sqrt{g}}-\frac{\log\left(1-\frac{1}{cx}\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)+\frac{1}{2}\left(\frac{\int \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\left(c+\frac{1}{x}\right)x^2}dx}{\sqrt{f}\sqrt{g}}+\frac{\log\left(\frac{1}{cx}+1\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)\right.\right.$$

$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$

$$\frac{1}{2}bc\left(-\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+gx^2\right)\right)\right.$$

$\downarrow$ 2005

$$2eg\left(b\left(\frac{1}{2}\left(\frac{\int \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x(cx-1)}dx}{\sqrt{f}\sqrt{g}}-\frac{\log\left(1-\frac{1}{cx}\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)+\frac{1}{2}\left(\frac{\int \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x(cx+1)}dx}{\sqrt{f}\sqrt{g}}+\frac{\log\left(\frac{1}{cx}+1\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)\right.\right.$$

$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$

$$\frac{1}{2}bc\left(-\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+gx^2\right)\right)\right.$$

$\downarrow$ 5411

$$2eg\left(b\left(\frac{1}{2}\left(\frac{\int \left(\frac{c\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{cx-1}-\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x}\right)dx}{\sqrt{f}\sqrt{g}}-\frac{\log\left(1-\frac{1}{cx}\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)+\frac{1}{2}\left(\frac{\int \left(\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x}-\frac{c\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{cx+1}\right)dx}{\sqrt{f}\sqrt{g}}\right.\right.\right.$$

$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$

$$\frac{1}{2}bc\left(-\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+gx^2\right)\right)\right.$$

$\downarrow$ 2009

3.281. $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx$

$$2eg\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}\left(-\frac{\log\left(1-\frac{1}{cx}\right)\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(1-cx)}{(-\sqrt{g}+ic\sqrt{f})(\sqrt{f}-i\sqrt{g}x)}\right)-\text{arcta}}{}\right.\right.\right.$$

$$\frac{(a+b\coth^{-1}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$

$$\frac{1}{2}bc\left(-\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{fc^2+g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+g\right.\right.\right.$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

output
```
-(((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x) + (b*c*(Log[-((g*x^2)/f
)]*(d + e*Log[f + g*x^2]) - Log[(g*(1 - c^2*x^2))/(c^2*f + g)]*(d + e*Log[
f + g*x^2]) - e*PolyLog[2, (c^2*(f + g*x^2))/(c^2*f + g)] + e*PolyLog[2, 1
 + (g*x^2)/f]))/2 + 2*e*g*((a*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g
]) + b*((-((ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 - 1/(c*x)])/(Sqrt[f]*Sqrt[g]
)) + (-(ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x
)]) + ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(1 - c*x))/((I*c
*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))] - (I/2)*PolyLog[2, ((-I)*Sqr
t[g]*x)/Sqrt[f]] + (I/2)*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]] + (I/2)*PolyLog
[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)] - (I/2)*PolyLog[2, 1 + (2*Sqr
t[f]*Sqrt[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))]
)/(Sqrt[f]*Sqrt[g]))/2 + ((ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 + 1/(c*x)])/(
Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] -
 I*Sqrt[g]*x)] - ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(1 + c
*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))] + (I/2)*PolyLog[2,
 ((-I)*Sqrt[g]*x)/Sqrt[f]] - (I/2)*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]] - (I/
2)*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)] + (I/2)*PolyLog[2,
1 - (2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sq
rt[g]*x))])/(Sqrt[f]*Sqrt[g]))/2))
```

3.281.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

rule 6535
```
Int[ArcCoth[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[1/2   Int
[Log[1 + 1/(c*x)]/(d + e*x^2), x], x] - Simp[1/2   Int[Log[1 - 1/(c*x)]/(d
+ e*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

rule 6537
```
Int[(ArcCoth[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
 Simp[a   Int[1/(d + e*x^2), x], x] + Simp[b   Int[ArcCoth[c*x]/(d + e*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.281. $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x^2}\,dx$

rule 6644
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcCoth[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d +
e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m
+ 2)*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f
, g}, x] && ILtQ[m/2, 0]
```

3.281.4 Maple [F]

$$\int \frac{\left(a+b\ \mathrm{arccoth}\left(c x\right)\right)\left(d+e\ln\left(g\,x^{2}+f\right)\right)}{x^{2}}d x$$

input
```
int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

output
```
int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

3.281.5 Fricas [F]

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log\left(gx^2+f\right)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="fricas")
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(g*x^2 + f)
)/x^2, x)
```

3.281.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\coth^{-1}(cx))\,(d + e\log(f + gx^2))}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f))/x**2,x)
```

output
```
Timed out
```

3.281.7 Maxima [F]

$$\int \frac{(a + b\coth^{-1}(cx))\,(d + e\log(f + gx^2))}{x^2}\,dx = \int \frac{(b\,\operatorname{arcoth}(cx) + a)(e\log(gx^2 + f) + d)}{x^2}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arccoth(c*x)/x)*b*d + (2*g*arcta
n(g*x/sqrt(f*g))/sqrt(f*g) - log(g*x^2 + f)/x)*a*e + 1/2*b*e*integrate((lo
g(1/(c*x) + 1) - log(-1/(c*x) + 1))*log(g*x^2 + f)/x^2, x) - a*d/x
```

3.281.8 Giac [F]

$$\int \frac{(a + b\coth^{-1}(cx))\,(d + e\log(f + gx^2))}{x^2}\,dx = \int \frac{(b\,\operatorname{arcoth}(cx) + a)(e\log(gx^2 + f) + d)}{x^2}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="giac")
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)/x^2, x)
```

3.281.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx = \int \frac{\left(a+b\,\mathrm{acoth}(c\,x)\right)\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)}{x^2}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x^2,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x^2, x)
```

3.282 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x^3}\,dx$

3.282.1 Optimal result

Integrand size = 24, antiderivative size = 712

$$\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x^3}\,dx$$
$$= \frac{bce\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} + \frac{beg\coth^{-1}(cx)\log\left(\frac{2}{1+cx}\right)}{f}$$
$$+ bc^2e\operatorname{arctanh}(cx)\log\left(\frac{2}{1+cx}\right) - \frac{beg\coth^{-1}(cx)\log\left(\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})(1+cx)}\right)}{2f}$$
$$- \frac{1}{2}bc^2e\operatorname{arctanh}(cx)\log\left(\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})(1+cx)}\right)$$
$$- \frac{beg\coth^{-1}(cx)\log\left(\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+\sqrt{g})(1+cx)}\right)}{2f}$$
$$- \frac{1}{2}bc^2e\operatorname{arctanh}(cx)\log\left(\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+\sqrt{g})(1+cx)}\right) - \frac{aeg\log(f+gx^2)}{2f}$$
$$- \frac{bc(d+e\log(f+gx^2))}{2x} - \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{2x^2}$$
$$+ \frac{1}{2}bc^2\operatorname{arctanh}(cx)(d+e\log(f+gx^2)) + \frac{beg\operatorname{PolyLog}\left(2,-\frac{1}{cx}\right)}{2f}$$
$$- \frac{beg\operatorname{PolyLog}\left(2,\frac{1}{cx}\right)}{2f} - \frac{1}{2}bc^2e\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)$$
$$- \frac{beg\operatorname{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{2f} + \frac{1}{4}bc^2e\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})(1+cx)}\right)$$
$$+ \frac{beg\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})(1+cx)}\right)}{4f}$$
$$+ \frac{1}{4}bc^2e\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+\sqrt{g})(1+cx)}\right)$$
$$+ \frac{beg\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+\sqrt{g})(1+cx)}\right)}{4f}$$

output
```
a*e*g*ln(x)/f+b*e*g*arccoth(c*x)*ln(2/(c*x+1))/f+b*c^2*e*arctanh(c*x)*ln(2
/(c*x+1))-1/2*a*e*g*ln(g*x^2+f)/f-1/2*b*c*(d+e*ln(g*x^2+f))/x-1/2*(a+b*arc
coth(c*x))*(d+e*ln(g*x^2+f))/x^2+1/2*b*c^2*arctanh(c*x)*(d+e*ln(g*x^2+f))-
1/2*b*e*g*arccoth(c*x)*ln(2*c*((-f)^(1/2)-x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)
-g^(1/2)))/f-1/2*b*c^2*e*arctanh(c*x)*ln(2*c*((-f)^(1/2)-x*g^(1/2))/(c*x+1
)/(c*(-f)^(1/2)-g^(1/2)))-1/2*b*e*g*arccoth(c*x)*ln(2*c*((-f)^(1/2)+x*g^(1
/2))/(c*x+1)/(c*(-f)^(1/2)+g^(1/2)))/f-1/2*b*c^2*e*arctanh(c*x)*ln(2*c*((-
f)^(1/2)+x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)+g^(1/2)))+1/2*b*e*g*polylog(2,-1
/c/x)/f-1/2*b*e*g*polylog(2,1/c/x)/f-1/2*b*c^2*e*polylog(2,1-2/(c*x+1))-1/
2*b*e*g*polylog(2,1-2/(c*x+1))/f+1/4*b*c^2*e*polylog(2,1-2*c*((-f)^(1/2)-x
*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)-g^(1/2)))+1/4*b*e*g*polylog(2,1-2*c*((-f)^
(1/2)-x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)-g^(1/2)))/f+1/4*b*c^2*e*polylog(2,1
-2*c*((-f)^(1/2)+x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)+g^(1/2)))+1/4*b*e*g*poly
log(2,1-2*c*((-f)^(1/2)+x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)+g^(1/2)))/f+b*c*e
*arctan(x*g^(1/2)/f^(1/2))*g^(1/2)/f^(1/2)
```

3.282.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.96 (sec) , antiderivative size = 1193, normalized size of antiderivative = 1.68

$$\int \frac{(a + b\coth^{-1}(cx))\,(d + e\log(f + gx^2))}{x^3}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

output
```
(-2*a*d*f - 2*b*c*d*f*x - 2*b*d*f*ArcCoth[c*x] + 2*b*c^2*d*f*x^2*ArcCoth[c
*x] + 4*b*c*e*Sqrt[f]*Sqrt[g]*x^2*ArcTan[(Sqrt[g]*x)/Sqrt[f]] + (4*I)*b*c^
2*e*f*x^2*ArcSin[Sqrt[g/(c^2*f + g)]]*ArcTanh[(c*f)/(Sqrt[-(c^2*f*g)]*x)]
+ (4*I)*b*e*g*x^2*ArcSin[Sqrt[g/(c^2*f + g)]]*ArcTanh[(c*f)/(Sqrt[-(c^2*f*
g)]*x)] + 4*b*c^2*e*f*x^2*ArcCoth[c*x]*Log[1 - E^(-2*ArcCoth[c*x])] + 4*b*
e*g*x^2*ArcCoth[c*x]*Log[1 + E^(-2*ArcCoth[c*x])] - 2*b*c^2*e*f*x^2*ArcCot
h[c*x]*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*ArcCoth[c*x])*g - 2
*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] - 2*b*e*g*x^2*ArcCoth
[c*x]*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*ArcCoth[c*x])*g - 2*
Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] + (2*I)*b*c^2*e*f*x^2*
ArcSin[Sqrt[g/(c^2*f + g)]]*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(
2*ArcCoth[c*x])*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))]
+ (2*I)*b*e*g*x^2*ArcSin[Sqrt[g/(c^2*f + g)]]*Log[(c^2*(-1 + E^(2*ArcCoth[
c*x]))*f + g + E^(2*ArcCoth[c*x])*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*
x])*(c^2*f + g))] - 2*b*c^2*e*f*x^2*ArcCoth[c*x]*Log[(c^2*(-1 + E^(2*ArcCo
th[c*x]))*f + g + E^(2*ArcCoth[c*x])*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth
[c*x])*(c^2*f + g))] - 2*b*e*g*x^2*ArcCoth[c*x]*Log[(c^2*(-1 + E^(2*ArcCot
h[c*x]))*f + g + E^(2*ArcCoth[c*x])*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[
c*x])*(c^2*f + g))] - (2*I)*b*c^2*e*f*x^2*ArcSin[Sqrt[g/(c^2*f + g)]]*Log[
(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*ArcCoth[c*x])*g + 2*Sqrt[-(...
```

3.282.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6648, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}dx$$

$$\Big\downarrow \text{6648}$$

$$-2eg\int \left(\frac{bc^2x\text{arctanh}(cx)}{2\left(gx^2+f\right)}-\frac{a+bcx+b\coth^{-1}(cx)}{2x\left(gx^2+f\right)}\right)dx-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}+\frac{1}{2}bc^2\text{arctanh}(cx)\left(d+e\log\left(f+gx^2\right)\right)-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}$$

$$\Big\downarrow \text{2009}$$

$$-2eg\left(\frac{a\log\left(f+gx^2\right)}{4f}-\frac{a\log(x)}{2f}-\frac{bc\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}}+\frac{bc^2\text{arctanh}(cx)\log\left(\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(cx+1)(c\sqrt{-f}-\sqrt{g})}\right)}{4g}+\frac{bc^2\text{arctanh}(cx)}{\quad}\right.$$
$$\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}+\frac{1}{2}bc^2\text{arctanh}(cx)\left(d+e\log\left(f+gx^2\right)\right)-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}$$

input
```
Int[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

output
```
-1/2*(b*c*(d + e*Log[f + g*x^2]))/x - ((a + b*ArcCoth[c*x])*(d + e*Log[f +
 g*x^2]))/(2*x^2) + (b*c^2*ArcTanh[c*x]*(d + e*Log[f + g*x^2]))/2 - 2*e*g*
(-1/2*(b*c*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - (a*Log[x])/(2*
f) - (b*ArcCoth[c*x]*Log[2/(1 + c*x)])/(2*f) - (b*c^2*ArcTanh[c*x]*Log[2/(
1 + c*x)])/(2*g) + (b*ArcCoth[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sq
rt[-f] - Sqrt[g])*(1 + c*x))])/(4*f) + (b*c^2*ArcTanh[c*x]*Log[(2*c*(Sqrt[
-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/(4*g) + (b*ArcCoth[
c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])
/(4*f) + (b*c^2*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f]
 + Sqrt[g])*(1 + c*x))])/(4*g) + (a*Log[f + g*x^2])/(4*f) - (b*PolyLog[2,
-(1/(c*x))])/(4*f) + (b*PolyLog[2, 1/(c*x)])/(4*f) + (b*PolyLog[2, 1 - 2/(
1 + c*x)])/(4*f) + (b*c^2*PolyLog[2, 1 - 2/(1 + c*x)])/(4*g) - (b*PolyLog[
2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/(
8*f) - (b*c^2*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - S
qrt[g])*(1 + c*x))])/(8*g) - (b*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]*x)
)/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(8*f) - (b*c^2*PolyLog[2, 1 - (2*c*
(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(8*g))
```

3.282.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6648
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegran
d[x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Inte
gerQ[m] && NeQ[m, -1]
```

3.282.4 Maple [A] (verified)

Time = 9.69 (sec) , antiderivative size = 937, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{ad}{2x^2}-\frac{aeg\ln(g\,x^2+f)}{2f}+\frac{aeg\ln(x)}{f}+\left(-\frac{be\ln(cx+1)}{4x^2}-\frac{e(b\,x^2\ln(cx-1)c^2-b\,c^2\ln(cx+1)x^2+2bcx-b\ln(cx-1)+2a)}{4x^2}\right)\ln($

input
```
int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*d/x^2-1/2*a*e*g*ln(g*x^2+f)/f+a*e*g*ln(x)/f+(-1/4*b*e/x^2*ln(c*x+1)
-1/4*e*(b*x^2*ln(c*x-1)*c^2-b*c^2*ln(c*x+1)*x^2+2*b*c*x-b*ln(c*x-1)+2*a)/x
^2)*ln(g*x^2+f)-1/4*g*b*e/f*ln(c*x+1)*ln((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(
-f*g)^(1/2)+g))-1/4*g*b*e/f*ln(c*x+1)*ln((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(
-f*g)^(1/2)-g))+g*e*b*c/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/4*g*b*e/f*ln
(c*x-1)*ln((c*(-f*g)^(1/2)-g*(c*x-1)-g)/(c*(-f*g)^(1/2)-g))+1/4*g*b*e/f*ln
(c*x-1)*ln((c*(-f*g)^(1/2)+g*(c*x-1)+g)/(c*(-f*g)^(1/2)+g))-1/2*g*b*e/f*ln
(c*x-1)*ln(c*x)-1/2*d*b*c/x+1/4*d*b*c^2*ln(c*x+1)-1/4*d*b*ln(c*x+1)/x^2-1/
4*d*b*c^2*ln(c*x-1)+1/4*d*b*ln(c*x-1)/x^2-1/4*b*e*ln(c*x+1)*ln((c*(-f*g)^(
1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*c^2-1/4*b*e*ln(c*x+1)*ln((c*(-f*g)^(
1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))*c^2-1/4*g*b*e/f*dilog((c*(-f*g)^(1/2
)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))-1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)+(c*x+
1)*g-g)/(c*(-f*g)^(1/2)-g))+1/4*b*e*ln(c*x-1)*ln((c*(-f*g)^(1/2)-g*(c*x-1)
-g)/(c*(-f*g)^(1/2)-g))*c^2+1/4*b*e*ln(c*x-1)*ln((c*(-f*g)^(1/2)+g*(c*x-1)
+g)/(c*(-f*g)^(1/2)+g))*c^2+1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)-g*(c*x-1)-g)
/(c*(-f*g)^(1/2)-g))+1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)+g*(c*x-1)+g)/(c*(-f
*g)^(1/2)+g))-1/2*g*b*e/f*dilog(c*x)-1/2*g*b*e/f*dilog(c*x+1)-1/4*b*e*dilo
g((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*c^2-1/4*b*e*dilog((c*(-
f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))*c^2+1/4*b*e*dilog((c*(-f*g)^(1
/2)-g*(c*x-1)-g)/(c*(-f*g)^(1/2)-g))*c^2+1/4*b*e*dilog((c*(-f*g)^(1/2)+...
```

3.282.5 Fricas [F]

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx = \int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log\left(gx^2+f\right)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="fricas")
```

output
```
integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(g*x^2 + f)
)/x^3, x)
```

3.282.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f))/x**3,x)
```

output
```
Timed out
```

3.282.7 Maxima [F]

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx
=\int \frac{(b\,\mathrm{arcoth}\,(cx)+a)(e\log\left(gx^2+f\right)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="maxima")
```

output
```
1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arccoth(c*x)/x^2)*b*d -
 1/2*(g*(log(g*x^2 + f)/f - log(x^2)/f) + log(g*x^2 + f)/x^2)*a*e - 1/4*(2
*c^2*g*integrate(x^2*log(c*x + 1)/(g*x^3 + f*x), x) - 2*c^2*g*integrate(x^
2*log(c*x - 1)/(g*x^3 + f*x), x) + 2*I*c*g*(log(I*g*x/sqrt(f*g) + 1) - log
(-I*g*x/sqrt(f*g) + 1))/sqrt(f*g) - 2*g*integrate(log(c*x + 1)/(g*x^3 + f*
x), x) + 2*g*integrate(log(c*x - 1)/(g*x^3 + f*x), x) + (2*c*x - (c^2*x^2
- 1)*log(c*x + 1) + (c^2*x^2 - 1)*log(c*x - 1))*log(g*x^2 + f)/x^2)*b*e -
1/2*a*d/x^2
```

3.282.8 Giac [F]

$$\int \frac{(a + b\coth^{-1}(cx))\,(d + e\log(f + gx^2))}{x^3}\,dx = \int \frac{(b\,\text{arcoth}\,(cx) + a)(e\log(gx^2 + f) + d)}{x^3}\,dx$$

input
```
integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="giac")
```

output
```
integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)/x^3, x)
```

3.282.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\coth^{-1}(cx))\,(d + e\log(f + gx^2))}{x^3}\,dx = \int \frac{(a + b\,\text{acoth}(c\,x))\,(d + e\,\ln(g\,x^2 + f))}{x^3}\,dx$$

input
```
int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x^3,x)
```

output
```
int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x^3, x)
```

3.283 $\int \coth^{-1}(e^x)\,dx$

3.283.1 Optimal result

Integrand size = 4, antiderivative size = 25

$$\int \coth^{-1}(e^x)\,dx = \frac{\text{PolyLog}(2,-e^{-x})}{2} - \frac{\text{PolyLog}(2,e^{-x})}{2}$$

output
```
1/2*polylog(2,-1/exp(x))-1/2*polylog(2,exp(-x))
```

3.283.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(25) = 50.

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \coth^{-1}(e^x)\,dx = x\coth^{-1}(e^x) + \frac{1}{2}x\log(1-e^x) - \frac{1}{2}x\log(1+e^x) - \frac{\text{PolyLog}(2,-e^x)}{2} + \frac{\text{PolyLog}(2,e^x)}{2}$$

input
```
Integrate[ArcCoth[E^x],x]
```

output
```
x*ArcCoth[E^x] + (x*Log[1 - E^x])/2 - (x*Log[1 + E^x])/2 - PolyLog[2, -E^x
]/2 + PolyLog[2, E^x]/2
```

3.283.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2720, 6447}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}(e^x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int e^{-x}\coth^{-1}(e^x)\,de^x$$

$$\Big\downarrow \text{6447}$$

$$\frac{\text{PolyLog}(2,-e^{-x})}{2}-\frac{\text{PolyLog}(2,e^{-x})}{2}$$

input
```
Int[ArcCoth[E^x],x]
```

output
```
PolyLog[2, -E^(-x)]/2 - PolyLog[2, E^(-x)]/2
```

3.283.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6447
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

3.283.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
risch	$-\frac{x\ln(e^{x}-1)}{2}-\frac{\operatorname{dilog}(e^{x})}{2}-\frac{\operatorname{dilog}(1+e^{x})}{2}$	22
derivativedivides	$\ln(e^{x})\operatorname{arccoth}(e^{x})-\frac{\operatorname{dilog}(e^{x})}{2}-\frac{\operatorname{dilog}(1+e^{x})}{2}-\frac{\ln(e^{x})\ln(1+e^{x})}{2}$	31
default	$\ln(e^{x})\operatorname{arccoth}(e^{x})-\frac{\operatorname{dilog}(e^{x})}{2}-\frac{\operatorname{dilog}(1+e^{x})}{2}-\frac{\ln(e^{x})\ln(1+e^{x})}{2}$	31
parts	$x\operatorname{arccoth}(e^{x})+\frac{x\ln(1-e^{x})}{2}+\frac{\operatorname{polylog}(2,e^{x})}{2}-\frac{x\ln(1+e^{x})}{2}-\frac{\operatorname{polylog}(2,-e^{x})}{2}$	39

input
```
int(arccoth(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*ln(exp(x)-1)-1/2*dilog(exp(x))-1/2*dilog(1+exp(x))
```

3.283.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(17) = 34.

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.56

$$\int \coth^{-1}(e^x)\,dx = \frac{1}{2}x\log\left(\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right)-\frac{1}{2}x\log(\cosh(x)+\sinh(x)+1) + \frac{1}{2}x\log(-\cosh(x)-\sinh(x)+1) + \frac{1}{2}\operatorname{Li}_2(\cosh(x)+\sinh(x))-\frac{1}{2}\operatorname{Li}_2(-\cosh(x)-\sinh(x))$$

input
```
integrate(arccoth(exp(x)),x, algorithm="fricas")
```

output
```
1/2*x*log((cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/2*x*log(cos
h(x) + sinh(x) + 1) + 1/2*x*log(-cosh(x) - sinh(x) + 1) + 1/2*dilog(cosh(x
) + sinh(x)) - 1/2*dilog(-cosh(x) - sinh(x))
```

3.283.6 Sympy [F]

$$\int \coth^{-1}(e^x)\, dx = \int \operatorname{acoth}(e^x)\, dx$$

input
```
integrate(acoth(exp(x)),x)
```

output
```
Integral(acoth(exp(x)), x)
```

3.283.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(17) = 34.

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\begin{aligned}\int \coth^{-1}(e^x)\, dx =& -\frac{1}{2}x(\log(e^x+1)-\log(e^x-1)) + x\operatorname{arcoth}(e^x) \\ &+ \frac{1}{2}\log(-e^x)\log(e^x+1) - \frac{1}{2}x\log(e^x-1) \\ &+ \frac{1}{2}\operatorname{Li}_2(e^x+1) - \frac{1}{2}\operatorname{Li}_2(-e^x+1)\end{aligned}$$

input
```
integrate(arccoth(exp(x)),x, algorithm="maxima")
```

output
```
-1/2*x*(log(e^x + 1) - log(e^x - 1)) + x*arccoth(e^x) + 1/2*log(-e^x)*log(
e^x + 1) - 1/2*x*log(e^x - 1) + 1/2*dilog(e^x + 1) - 1/2*dilog(-e^x + 1)
```

3.283.8 Giac [F]

$$\int \coth^{-1}(e^x)\, dx = \int \operatorname{arcoth}(e^x)\, dx$$

input
```
integrate(arccoth(exp(x)),x, algorithm="giac")
```

output
```
integrate(arccoth(e^x), x)
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}(e^x)\, dx = \int \mathrm{acoth}(\mathrm{e}^x)\, dx$$

input
```
int(acoth(exp(x)),x)
```

output
```
int(acoth(exp(x)), x)
```

3.284 $\int x \coth^{-1}(e^x)\, dx$

3.284.1 Optimal result

Integrand size = 6, antiderivative size = 51

$$\int x \coth^{-1}(e^x)\, dx = \frac{1}{2}x \operatorname{PolyLog}(2,-e^{-x}) - \frac{1}{2}x \operatorname{PolyLog}(2,e^{-x}) + \frac{\operatorname{PolyLog}(3,-e^{-x})}{2} - \frac{\operatorname{PolyLog}(3,e^{-x})}{2}$$

output
```
1/2*x*polylog(2,-1/exp(x))-1/2*x*polylog(2,exp(-x))+1/2*polylog(3,-1/exp(x
))-1/2*polylog(3,exp(-x))
```

3.284.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

$$\int x \coth^{-1}(e^x)\, dx = \frac{1}{4}\left(2x^2 \coth^{-1}(e^x) + x^2 \log(1-e^x) - x^2 \log(1+e^x) - 2x \operatorname{PolyLog}(2,-e^x) + 2x \operatorname{PolyLog}(2,e^x) + 2 \operatorname{PolyLog}(3,-e^x) - 2 \operatorname{PolyLog}(3,e^x)\right)$$

input
```
Integrate[x*ArcCoth[E^x],x]
```

output
```
(2*x^2*ArcCoth[E^x] + x^2*Log[1 - E^x] - x^2*Log[1 + E^x] - 2*x*PolyLog[2,
 -E^x] + 2*x*PolyLog[2, E^x] + 2*PolyLog[3, -E^x] - 2*PolyLog[3, E^x])/4
```

3.284.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6768, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}(e^x) \, dx$$

$$\Big\downarrow \text{6768}$$

$$\frac{1}{2} \int x \log(1+e^{-x}) \, dx - \frac{1}{2} \int x \log(1-e^{-x}) \, dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2} \left(x \operatorname{PolyLog}(2,-e^{-x}) - \int \operatorname{PolyLog}(2,-e^{-x}) \, dx \right) + \frac{1}{2} \left(\int \operatorname{PolyLog}(2,e^{-x}) \, dx - x \operatorname{PolyLog}(2,e^{-x}) \right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2} \left(\int e^x \operatorname{PolyLog}(2,-e^{-x}) \, de^{-x} + x \operatorname{PolyLog}(2,-e^{-x}) \right) + \frac{1}{2} \left(-\int e^x \operatorname{PolyLog}(2,e^{-x}) \, de^{-x} - x \operatorname{PolyLog}(2,e^{-x}) \right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2} (x \operatorname{PolyLog}(2,-e^{-x}) + \operatorname{PolyLog}(3,-e^{-x})) + \frac{1}{2} (-x \operatorname{PolyLog}(2,e^{-x}) - \operatorname{PolyLog}(3,e^{-x}))$$

input
```
Int[x*ArcCoth[E^x],x]
```

output
```
(x*PolyLog[2, -E^(-x)] + PolyLog[3, -E^(-x)])/2 + (-(x*PolyLog[2, E^(-x)])
 - PolyLog[3, E^(-x)])/2
```

3.284.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6768
```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Simp[1/2   I
nt[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x]
&& IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.284.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{x^{2} \ln \left({\mathrm e}^{x}-1\right)}{4}-\frac{x \,\operatorname{polylog}\left(2,-{\mathrm e}^{x}\right)}{2}+\frac{\operatorname{polylog}\left(3,-{\mathrm e}^{x}\right)}{2}+\frac{x^{2} \ln \left(1-{\mathrm e}^{x}\right)}{4}+\frac{x \,\operatorname{polylog}\left(2,{\mathrm e}^{x}\right)}{2}-\frac{\operatorname{polylog}\left(3,{\mathrm e}^{x}\right)}{2}$
default	$\frac{x^{2} \operatorname{arccoth}\left({\mathrm e}^{x}\right)}{2}+\frac{x^{2} \ln \left(1-{\mathrm e}^{x}\right)}{4}+\frac{x \,\operatorname{polylog}\left(2,{\mathrm e}^{x}\right)}{2}-\frac{\operatorname{polylog}\left(3,{\mathrm e}^{x}\right)}{2}-\frac{x^{2} \ln \left(1+{\mathrm e}^{x}\right)}{4}-\frac{x \,\operatorname{polylog}\left(2,-{\mathrm e}^{x}\right)}{2}+\frac{\operatorname{polylog}\left(3,-{\mathrm e}^{x}\right)}{2}$
parts	$\frac{x^{2} \operatorname{arccoth}\left({\mathrm e}^{x}\right)}{2}+\frac{x^{2} \ln \left(1-{\mathrm e}^{x}\right)}{4}+\frac{x \,\operatorname{polylog}\left(2,{\mathrm e}^{x}\right)}{2}-\frac{\operatorname{polylog}\left(3,{\mathrm e}^{x}\right)}{2}-\frac{x^{2} \ln \left(1+{\mathrm e}^{x}\right)}{4}-\frac{x \,\operatorname{polylog}\left(2,-{\mathrm e}^{x}\right)}{2}+\frac{\operatorname{polylog}\left(3,-{\mathrm e}^{x}\right)}{2}$

input
```
int(x*arccoth(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2*ln(exp(x)-1)-1/2*x*polylog(2,-exp(x))+1/2*polylog(3,-exp(x))+1/4*
x^2*ln(1-exp(x))+1/2*x*polylog(2,exp(x))-1/2*polylog(3,exp(x))
```

3.284.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(37) = 74$.

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.84

$$\int x\coth^{-1}(e^x)\,dx = \frac{1}{4}x^2\log\left(\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right) - \frac{1}{4}x^2\log(\cosh(x)+\sinh(x)+1) + \frac{1}{4}x^2\log(-\cosh(x)-\sinh(x)+1) + \frac{1}{2}x\mathrm{Li}_2(\cosh(x)+\sinh(x)) - \frac{1}{2}x\mathrm{Li}_2(-\cosh(x)-\sinh(x)) - \frac{1}{2}\mathrm{polylog}(3,\cosh(x)+\sinh(x)) + \frac{1}{2}\mathrm{polylog}(3,-\cosh(x)-\sinh(x))$$

input
```
integrate(x*arccoth(exp(x)),x, algorithm="fricas")
```

output
```
1/4*x^2*log((cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/4*x^2*log
(cosh(x) + sinh(x) + 1) + 1/4*x^2*log(-cosh(x) - sinh(x) + 1) + 1/2*x*dilo
g(cosh(x) + sinh(x)) - 1/2*x*dilog(-cosh(x) - sinh(x)) - 1/2*polylog(3, co
sh(x) + sinh(x)) + 1/2*polylog(3, -cosh(x) - sinh(x))
```

3.284.6 Sympy [F]

$$\int x\coth^{-1}(e^x)\,dx = \int x\,\mathrm{acoth}(e^x)\,dx$$

input
```
integrate(x*acoth(exp(x)),x)
```

output
```
Integral(x*acoth(exp(x)), x)
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int x \coth^{-1}(e^x)\, dx = \frac{1}{2}x^2 \operatorname{arcoth}(e^x) - \frac{1}{4}x^2 \log(e^x + 1) + \frac{1}{4}x^2 \log(-e^x + 1) - \frac{1}{2}x\operatorname{Li}_2(-e^x) + \frac{1}{2}x\operatorname{Li}_2(e^x) + \frac{1}{2}\operatorname{Li}_3(-e^x) - \frac{1}{2}\operatorname{Li}_3(e^x)$$

input
```
integrate(x*arccoth(exp(x)),x, algorithm="maxima")
```

output
```
1/2*x^2*arccoth(e^x) - 1/4*x^2*log(e^x + 1) + 1/4*x^2*log(-e^x + 1) - 1/2*
x*dilog(-e^x) + 1/2*x*dilog(e^x) + 1/2*polylog(3, -e^x) - 1/2*polylog(3, e
^x)
```

3.284.8 Giac [F]

$$\int x \coth^{-1}(e^x)\, dx = \int x \operatorname{arcoth}(e^x)\, dx$$

input
```
integrate(x*arccoth(exp(x)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(e^x), x)
```

3.284.9 Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}(e^x)\, dx = \int x \operatorname{acoth}(e^x)\, dx$$

input
```
int(x*acoth(exp(x)),x)
```

output
```
int(x*acoth(exp(x)), x)
```

3.285 $\int x^2 \coth^{-1}(e^x)\,dx$

3.285.1 Optimal result

Integrand size = 8, antiderivative size = 70

$$\int x^2 \coth^{-1}(e^x)\,dx = \frac{1}{2}x^2 \operatorname{PolyLog}\left(2,-e^{-x}\right) - \frac{1}{2}x^2 \operatorname{PolyLog}\left(2,e^{-x}\right) + x \operatorname{PolyLog}\left(3,-e^{-x}\right) - x \operatorname{PolyLog}\left(3,e^{-x}\right) + \operatorname{PolyLog}\left(4,-e^{-x}\right) - \operatorname{PolyLog}\left(4,e^{-x}\right)$$

output
```
1/2*x^2*polylog(2,-1/exp(x))-1/2*x^2*polylog(2,exp(-x))+x*polylog(3,-1/exp
(x))-x*polylog(3,exp(-x))+polylog(4,-1/exp(x))-polylog(4,exp(-x))
```

3.285.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.33

$$\int x^2 \coth^{-1}(e^x)\,dx = \frac{1}{6}\left(2x^3 \coth^{-1}(e^x) + x^3 \log(1-e^x) - x^3 \log(1+e^x) - 3x^2 \operatorname{PolyLog}(2,-e^x) + 3x^2 \operatorname{PolyLog}(2,e^x) + 6x \operatorname{PolyLog}(3,-e^x) - 6x \operatorname{PolyLog}(3,e^x) - 6 \operatorname{PolyLog}(4,-e^x) + 6 \operatorname{PolyLog}(4,e^x)\right)$$

input
```
Integrate[x^2*ArcCoth[E^x],x]
```

output
```
(2*x^3*ArcCoth[E^x] + x^3*Log[1 - E^x] - x^3*Log[1 + E^x] - 3*x^2*PolyLog[
2, -E^x] + 3*x^2*PolyLog[2, E^x] + 6*x*PolyLog[3, -E^x] - 6*x*PolyLog[3, E
^x] - 6*PolyLog[4, -E^x] + 6*PolyLog[4, E^x])/6
```

3.285.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6768, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}(e^x) \, dx$$

$$\Big\downarrow \text{6768}$$

$$\frac{1}{2} \int x^2 \log \left(1+e^{-x}\right) dx-\frac{1}{2} \int x^2 \log \left(1-e^{-x}\right) dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2} \left(x^2 \text{PolyLog}\left(2,-e^{-x}\right)-2 \int x \text{PolyLog}\left(2,-e^{-x}\right) dx\right)+\frac{1}{2} \left(2 \int x \text{PolyLog}\left(2,e^{-x}\right) dx-x^2 \text{PolyLog}\left(2,e^{-x}\right)\right)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{2} \left(x^2 \text{PolyLog}\left(2,-e^{-x}\right)-2 \left(\int \text{PolyLog}\left(3,-e^{-x}\right) dx-x \text{PolyLog}\left(3,-e^{-x}\right)\right)\right)+\frac{1}{2} \left(2 \left(\int \text{PolyLog}\left(3,e^{-x}\right) dx-x \text{PolyLog}\left(3,e^{-x}\right)\right)-x^2 \text{PolyLog}\left(2,e^{-x}\right)\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2} \left(x^2 \text{PolyLog}\left(2,-e^{-x}\right)-2 \left(-\int e^x \text{PolyLog}\left(3,-e^{-x}\right) de^{-x}-x \text{PolyLog}\left(3,-e^{-x}\right)\right)\right)+\frac{1}{2} \left(2 \left(-\int e^x \text{PolyLog}\left(3,e^{-x}\right) de^{-x}-x \text{PolyLog}\left(3,e^{-x}\right)\right)-x^2 \text{PolyLog}\left(2,e^{-x}\right)\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2} (x^2 \text{PolyLog}\left(2,-e^{-x}\right)-2(-x \text{PolyLog}\left(3,-e^{-x}\right)-\text{PolyLog}\left(4,-e^{-x}\right)))+\frac{1}{2} (2(-x \text{PolyLog}\left(3,e^{-x}\right)-\text{PolyLog}\left(4,e^{-x}\right))-x^2 \text{PolyLog}\left(2,e^{-x}\right))$$

input
```
Int[x^2*ArcCoth[E^x],x]
```

output
```
(x^2*PolyLog[2, -E^(-x)] - 2*(-(x*PolyLog[3, -E^(-x)]) - PolyLog[4, -E^(-x
)]))/2 + (-(x^2*PolyLog[2, E^(-x)]) + 2*(-(x*PolyLog[3, E^(-x)]) - PolyLog
[4, E^(-x)]))/2
```

3.285.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6768
```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Simp[1/2   I
nt[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x]
&& IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.285.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{x^3 \ln(e^x-1)}{6} - \frac{x^2 \operatorname{polylog}(2,-e^x)}{2} + x \operatorname{polylog}(3,-e^x) - \operatorname{polylog}(4,-e^x) + \frac{x^3 \ln(1-e^x)}{6} + \frac{x^2 \operatorname{polylog}(2,e^x)}{2}$ -
default	$\frac{x^3 \operatorname{arccoth}(e^x)}{3} + \frac{x^3 \ln(1-e^x)}{6} + \frac{x^2 \operatorname{polylog}(2,e^x)}{2} - x \operatorname{polylog}(3,e^x) + \operatorname{polylog}(4,e^x) - \frac{x^3 \ln(1+e^x)}{6} - \frac{x^2 \operatorname{polylo}}{}$
parts	$\frac{x^3 \operatorname{arccoth}(e^x)}{3} + \frac{x^3 \ln(1-e^x)}{6} + \frac{x^2 \operatorname{polylog}(2,e^x)}{2} - x \operatorname{polylog}(3,e^x) + \operatorname{polylog}(4,e^x) - \frac{x^3 \ln(1+e^x)}{6} - \frac{x^2 \operatorname{polylo}}{}$

input
```
int(x^2*arccoth(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x^3*ln(exp(x)-1)-1/2*x^2*polylog(2,-exp(x))+x*polylog(3,-exp(x))-poly
log(4,-exp(x))+1/6*x^3*ln(1-exp(x))+1/2*x^2*polylog(2,exp(x))-x*polylog(3,
exp(x))+polylog(4,exp(x))
```

3.285.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(58) = 116.

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.70

$$
\begin{aligned}
\int x^2 \coth^{-1}(e^x)\, dx = {} & \frac{1}{6} x^3 \log\left(\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right) - \frac{1}{6} x^3 \log(\cosh(x)+\sinh(x)+1) \\
& + \frac{1}{6} x^3 \log(-\cosh(x)-\sinh(x)+1) \\
& + \frac{1}{2} x^2 \mathrm{Li}_2(\cosh(x)+\sinh(x)) - \frac{1}{2} x^2 \mathrm{Li}_2(-\cosh(x)-\sinh(x)) \\
& - x\operatorname{polylog}(3,\cosh(x)+\sinh(x)) + x\operatorname{polylog}(3,-\cosh(x)-\sinh(x)) \\
& + \operatorname{polylog}(4,\cosh(x)+\sinh(x)) - \operatorname{polylog}(4,-\cosh(x)-\sinh(x))
\end{aligned}
$$

input
```
integrate(x^2*arccoth(exp(x)),x, algorithm="fricas")
```

output
```
1/6*x^3*log((cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/6*x^3*log
(cosh(x) + sinh(x) + 1) + 1/6*x^3*log(-cosh(x) - sinh(x) + 1) + 1/2*x^2*di
log(cosh(x) + sinh(x)) - 1/2*x^2*dilog(-cosh(x) - sinh(x)) - x*polylog(3,
cosh(x) + sinh(x)) + x*polylog(3, -cosh(x) - sinh(x)) + polylog(4, cosh(x)
 + sinh(x)) - polylog(4, -cosh(x) - sinh(x))
```

3.285.6 Sympy [F]

$$\int x^2 \coth^{-1}(e^x)\,dx = \int x^2 \operatorname{acoth}(e^x)\,dx$$

input
```
integrate(x**2*acoth(exp(x)),x)
```

output
```
Integral(x**2*acoth(exp(x)), x)
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int x^2 \coth^{-1}(e^x)\,dx = \frac{1}{3}\,x^3 \operatorname{arcoth}(e^x) - \frac{1}{6}\,x^3 \log(e^x+1) + \frac{1}{6}\,x^3 \log(-e^x+1) - \frac{1}{2}\,x^2 \mathrm{Li}_2(-e^x) + \frac{1}{2}\,x^2 \mathrm{Li}_2(e^x) + x\mathrm{Li}_3(-e^x) - x\mathrm{Li}_3(e^x) - \mathrm{Li}_4(-e^x) + \mathrm{Li}_4(e^x)$$

input
```
integrate(x^2*arccoth(exp(x)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(e^x) - 1/6*x^3*log(e^x + 1) + 1/6*x^3*log(-e^x + 1) - 1/2*
x^2*dilog(-e^x) + 1/2*x^2*dilog(e^x) + x*polylog(3, -e^x) - x*polylog(3, e
^x) - polylog(4, -e^x) + polylog(4, e^x)
```

3.285.8 Giac [F]

$$\int x^2 \coth^{-1}(e^x)\,dx = \int x^2 \operatorname{arcoth}(e^x)\,dx$$

input
```
integrate(x^2*arccoth(exp(x)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(e^x), x)
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}(e^x)\,dx = \int x^2\,\mathrm{acoth}(\mathrm{e}^x)\,dx$$

input
```
int(x^2*acoth(exp(x)),x)
```

output
```
int(x^2*acoth(exp(x)), x)
```

3.286 $\int \coth^{-1}\left(e^{a+bx}\right) dx$

3.286.1 Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \coth^{-1}\left(e^{a+bx}\right) dx=\frac{\text{PolyLog}\left(2,-e^{-a-bx}\right)}{2b}-\frac{\text{PolyLog}\left(2,e^{-a-bx}\right)}{2b}$$

output
```
1/2*polylog(2,-exp(-b*x-a))/b-1/2*polylog(2,exp(-b*x-a))/b
```

3.286.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.66

$$\int \coth^{-1}\left(e^{a+bx}\right) dx =\frac{bx\left(2\coth^{-1}\left(e^{a+bx}\right)+\log\left(1-e^{a+bx}\right)-\log\left(1+e^{a+bx}\right)\right)-\text{PolyLog}\left(2,-e^{a+bx}\right)+\text{PolyLog}\left(2,e^{a+bx}\right)}{2b}$$

input
```
Integrate[ArcCoth[E^(a + b*x)],x]
```

output
```
(b*x*(2*ArcCoth[E^(a + b*x)] + Log[1 - E^(a + b*x)] - Log[1 + E^(a + b*x)]
) - PolyLog[2, -E^(a + b*x)] + PolyLog[2, E^(a + b*x)])/(2*b)
```

3.286.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 6447}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}\left(e^{a+bx}\right) dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int e^{-a-bx}\coth^{-1}\left(e^{a+bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \text{ 6447}$$

$$\frac{\frac{1}{2}\text{PolyLog}\left(2,-e^{-a-bx}\right)-\frac{1}{2}\text{PolyLog}\left(2,e^{-a-bx}\right)}{b}$$

input
```
Int[ArcCoth[E^(a + b*x)],x]
```

output
```
(PolyLog[2, -E^(-a - b*x)]/2 - PolyLog[2, E^(-a - b*x)]/2)/b
```

3.286.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6447
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

3.286.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{\operatorname{dilog}\left(e^{bx+a}+1\right)}{2b}-\frac{\ln\left(-1+e^{bx+a}\right)\ln\left(e^{bx+a}\right)}{2b}-\frac{\operatorname{dilog}\left(e^{bx+a}\right)}{2b}$
derivativedivides	$\frac{\ln\left(e^{bx+a}\right)\operatorname{arccoth}\left(e^{bx+a}\right)-\frac{\operatorname{dilog}\left(e^{bx+a}\right)}{2}-\frac{\operatorname{dilog}\left(e^{bx+a}+1\right)}{2}-\frac{\ln\left(e^{bx+a}\right)\ln\left(e^{bx+a}+1\right)}{2}}{b}$
default	$\frac{\ln\left(e^{bx+a}\right)\operatorname{arccoth}\left(e^{bx+a}\right)-\frac{\operatorname{dilog}\left(e^{bx+a}\right)}{2}-\frac{\operatorname{dilog}\left(e^{bx+a}+1\right)}{2}-\frac{\ln\left(e^{bx+a}\right)\ln\left(e^{bx+a}+1\right)}{2}}{b}$
parts	$x\ \operatorname{arccoth}\left(e^{bx+a}\right)+\frac{\frac{(bx+a)\ln\left(1-e^{bx+a}\right)}{2}+\frac{\operatorname{polylog}\left(2,e^{bx+a}\right)}{2}-\frac{(bx+a)\ln\left(e^{bx+a}+1\right)}{2}-\frac{\operatorname{polylog}\left(2,-e^{bx+a}\right)}{2}+a\ \arctan}{b}$

input
```
int(arccoth(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*dilog(exp(b*x+a)+1)-1/2/b*ln(-1+exp(b*x+a))*ln(exp(b*x+a))-1/2/b*di
log(exp(b*x+a))
```

3.286.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(33) = 66.

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.34

$$\int \coth^{-1}\left(e^{a+bx}\right)\,dx$$
$$=\frac{bx\log\left(\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right)-bx\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-a\log\left(\cosh\left(bx+a\right)+\sinh\left(\right.\right.}{}$$

input
```
integrate(arccoth(exp(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b*x*log((cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh(b*x
 + a) - 1)) - b*x*log(cosh(b*x + a) + sinh(b*x + a) + 1) - a*log(cosh(b*x
+ a) + sinh(b*x + a) - 1) + (b*x + a)*log(-cosh(b*x + a) - sinh(b*x + a) +
 1) + dilog(cosh(b*x + a) + sinh(b*x + a)) - dilog(-cosh(b*x + a) - sinh(b
*x + a)))/b
```

3.286.6 Sympy [F]

$$\int \coth^{-1}\left(e^{a+bx}\right)\,dx = \int \operatorname{acoth}\left(e^{a+bx}\right)dx$$

input
```
integrate(acoth(exp(b*x+a)),x)
```

output
```
Integral(acoth(exp(a + b*x)), x)
```

3.286.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(33) = 66$.

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.61

$$\int \coth^{-1}\left(e^{a+bx}\right)\,dx = \frac{(bx+a)\operatorname{arcoth}\left(e^{(bx+a)}\right)}{b} - \frac{(bx+a)\left(\log\left(e^{(bx+a)}+1\right)-\log\left(e^{(bx+a)}-1\right)\right)-\log\left(-e^{(bx+a)}\right)\log\left(e^{(bx+a)}+1\right)+(bx+a)\log\left(e^{(bx+a)}\right.}{2b}$$

input
```
integrate(arccoth(exp(b*x+a)),x, algorithm="maxima")
```

output
```
(b*x + a)*arccoth(e^(b*x + a))/b - 1/2*((b*x + a)*(log(e^(b*x + a) + 1) -
log(e^(b*x + a) - 1)) - log(-e^(b*x + a))*log(e^(b*x + a) + 1) + (b*x + a)
*log(e^(b*x + a) - 1) - dilog(e^(b*x + a) + 1) + dilog(-e^(b*x + a) + 1))/
b
```

3.286.8 Giac [F]

$$\int \coth^{-1}\left(e^{a+bx}\right)\,dx = \int \operatorname{arcoth}\left(e^{(bx+a)}\right)\,dx$$

input
```
integrate(arccoth(exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccoth(e^(b*x + a)), x)
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}\left(e^{a+bx}\right)\,dx = \int \operatorname{acoth}\left(e^{a+b\,x}\right)\,dx$$

input
```
int(acoth(exp(a + b*x)),x)
```

output
```
int(acoth(exp(a + b*x)), x)
```

3.287 $\int x \coth^{-1}\left(e^{a+bx}\right) dx$

3.287.1 Optimal result

Integrand size = 10, antiderivative size = 83

$$\int x \coth^{-1}\left(e^{a+bx}\right) dx = \frac{x \operatorname{PolyLog}\left(2,-e^{-a-bx}\right)}{2b} - \frac{x \operatorname{PolyLog}\left(2,e^{-a-bx}\right)}{2b} + \frac{\operatorname{PolyLog}\left(3,-e^{-a-bx}\right)}{2b^2} - \frac{\operatorname{PolyLog}\left(3,e^{-a-bx}\right)}{2b^2}$$

output
```
1/2*x*polylog(2,-exp(-b*x-a))/b-1/2*x*polylog(2,exp(-b*x-a))/b+1/2*polylog
(3,-exp(-b*x-a))/b^2-1/2*polylog(3,exp(-b*x-a))/b^2
```

3.287.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36

$$\int x \coth^{-1}\left(e^{a+bx}\right) dx = \frac{2b^2x^2 \coth^{-1}\left(e^{a+bx}\right) + b^2x^2 \log\left(1-e^{a+bx}\right) - b^2x^2 \log\left(1+e^{a+bx}\right) - 2bx \operatorname{PolyLog}\left(2,-e^{a+bx}\right) + 2bx \operatorname{PolyL}}{4b^2}$$

input
```
Integrate[x*ArcCoth[E^(a + b*x)],x]
```

output
```
(2*b^2*x^2*ArcCoth[E^(a + b*x)] + b^2*x^2*Log[1 - E^(a + b*x)] - b^2*x^2*L
og[1 + E^(a + b*x)] - 2*b*x*PolyLog[2, -E^(a + b*x)] + 2*b*x*PolyLog[2, E^
(a + b*x)] + 2*PolyLog[3, -E^(a + b*x)] - 2*PolyLog[3, E^(a + b*x)])/(4*b^
2)
```

3.287.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6768, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^{-1}\left(e^{a+bx}\right) dx$$

$$\Big\downarrow \text{6768}$$

$$\frac{1}{2}\int x \log\left(1+e^{-a-bx}\right) dx-\frac{1}{2}\int x \log\left(1-e^{-a-bx}\right) dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}\left(\frac{x \operatorname{PolyLog}\left(2,-e^{-a-bx}\right)}{b}-\frac{\int \operatorname{PolyLog}\left(2,-e^{-a-bx}\right) dx}{b}\right)+\frac{1}{2}\left(\frac{\int \operatorname{PolyLog}\left(2,e^{-a-bx}\right) dx}{b}-\frac{x \operatorname{PolyLog}\left(2,e^{-a-bx}\right)}{b}\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}\left(\frac{\int e^{a+bx} \operatorname{PolyLog}\left(2,-e^{-a-bx}\right) de^{-a-bx}}{b^2}+\frac{x \operatorname{PolyLog}\left(2,-e^{-a-bx}\right)}{b}\right)+\frac{1}{2}\left(-\frac{\int e^{a+bx} \operatorname{PolyLog}\left(2,e^{-a-bx}\right) de^{-a-bx}}{b^2}-\frac{x \operatorname{PolyLog}\left(2,e^{-a-bx}\right)}{b}\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}\left(\frac{\operatorname{PolyLog}\left(3,-e^{-a-bx}\right)}{b^2}+\frac{x \operatorname{PolyLog}\left(2,-e^{-a-bx}\right)}{b}\right)+\frac{1}{2}\left(-\frac{\operatorname{PolyLog}\left(3,e^{-a-bx}\right)}{b^2}-\frac{x \operatorname{PolyLog}\left(2,e^{-a-bx}\right)}{b}\right)$$

input
```
Int[x*ArcCoth[E^(a + b*x)],x]
```

output
```
((x*PolyLog[2, -E^(-a - b*x)])/b + PolyLog[3, -E^(-a - b*x)]/b^2)/2 + (-((
x*PolyLog[2, E^(-a - b*x)])/b) - PolyLog[3, E^(-a - b*x)]/b^2)/2
```

3.287.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6768
```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Simp[1/2   I
nt[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x]
&& IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.287.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(71) = 142$.

Time = 0.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.16

method	result
default	$\frac{x^2 \operatorname{arccoth}\left(e^{bx+a}\right)}{2}+\frac{-a^2 \operatorname{arctanh}\left(e^{bx+a}\right)+\frac{(bx+a)^2 \ln\left(1-e^{bx+a}\right)}{2}+(bx+a) \operatorname{polylog}\left(2,e^{bx+a}\right)-\operatorname{polylog}\left(3,e^{bx+a}\right)-\frac{(bx+a)^2 \ln\left(e^{bx+a}\right.}{2}}{}$
parts	$\frac{x^2 \operatorname{arccoth}\left(e^{bx+a}\right)}{2}+\frac{-a^2 \operatorname{arctanh}\left(e^{bx+a}\right)+\frac{(bx+a)^2 \ln\left(1-e^{bx+a}\right)}{2}+(bx+a) \operatorname{polylog}\left(2,e^{bx+a}\right)-\operatorname{polylog}\left(3,e^{bx+a}\right)-\frac{(bx+a)^2 \ln\left(e^{bx+a}\right.}{2}}{}$
risch	$-\frac{x^2 \ln\left(-1+e^{bx+a}\right)}{4}+\frac{x^2 \ln\left(1-e^{bx+a}\right)}{4}+\frac{\ln\left(1-e^{bx+a}\right) a x}{2b}+\frac{\ln\left(1-e^{bx+a}\right) a^2}{4b^2}+\frac{x \operatorname{polylog}\left(2,e^{bx+a}\right)}{2b}+\frac{a^2 \ln\left(-1+e^{bx+a}\right)}{4b^2}+$

input
```
int(x*arccoth(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arccoth(exp(b*x+a))+1/2/b^2*(-a^2*arctanh(exp(b*x+a))+1/2*(b*x+a)^
2*ln(1-exp(b*x+a))+(b*x+a)*polylog(2,exp(b*x+a))-polylog(3,exp(b*x+a))-1/2
*(b*x+a)^2*ln(exp(b*x+a)+1)-(b*x+a)*polylog(2,-exp(b*x+a))+polylog(3,-exp(
b*x+a))-a*(b*x+a)*ln(1-exp(b*x+a))+a*(b*x+a)*ln(exp(b*x+a)+1)-a*polylog(2,
exp(b*x+a))+a*polylog(2,-exp(b*x+a)))
```

3.287.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(69) = 138$.

Time = 0.25 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.39

$$\int x\coth^{-1}\left(e^{a+bx}\right)\,dx$$
$$= \frac{b^2x^2\log\left(\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right) - b^2x^2\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right) + 2\,bx\mathrm{Li}_2(\cosh\left(bx+a\right)+ \cdots}{\cdots}$$

input
```
integrate(x*arccoth(exp(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(b^2*x^2*log((cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh
(b*x + a) - 1)) - b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*b*x*d
ilog(cosh(b*x + a) + sinh(b*x + a)) - 2*b*x*dilog(-cosh(b*x + a) - sinh(b*
x + a)) + a^2*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (b^2*x^2 - a^2)*log
(-cosh(b*x + a) - sinh(b*x + a) + 1) - 2*polylog(3, cosh(b*x + a) + sinh(b
*x + a)) + 2*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^2
```

3.287.6 Sympy [F]

$$\int x\coth^{-1}\left(e^{a+bx}\right)\,dx = \int x\,\mathrm{acoth}\left(e^{a}e^{bx}\right)dx$$

input
```
integrate(x*acoth(exp(b*x+a)),x)
```

output
```
Integral(x*acoth(exp(a)*exp(b*x)), x)
```

3.287.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.30

$$\int x\coth^{-1}\left(e^{a+bx}\right)\,dx = \frac{1}{2}\,x^2\operatorname{arcoth}\left(e^{(bx+a)}\right) - \frac{1}{4}\,b\left(\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3\left(-e^{(bx+a)}\right)}{b^3} - \frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(e^{\cdots}\right.}{b^3}\right.$$

input
```
integrate(x*arccoth(exp(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*x^2*arccoth(e^(b*x + a)) - 1/4*b*((b^2*x^2*log(e^(b*x + a) + 1) + 2*b*
x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 - (b^2*x^2*log(-e^
(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
)
```

3.287.8 Giac [F]

$$\int x\coth^{-1}\left(e^{a+bx}\right)\,dx = \int x\operatorname{arcoth}\left(e^{(bx+a)}\right)\,dx$$

input
```
integrate(x*arccoth(exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(e^(b*x + a)), x)
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int x\coth^{-1}\left(e^{a+bx}\right)\,dx = \int x\,\mathrm{acoth}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

input
```
int(x*acoth(exp(a + b*x)),x)
```

output
```
int(x*acoth(exp(a + b*x)), x)
```

3.288 $\int x^2 \coth^{-1}\left(e^{a+bx}\right)\,dx$

3.288.1 Optimal result

Integrand size = 12, antiderivative size = 119

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right)\,dx = \frac{x^2\operatorname{PolyLog}\left(2,-e^{-a-bx}\right)}{2b} - \frac{x^2\operatorname{PolyLog}\left(2,e^{-a-bx}\right)}{2b} + \frac{x\operatorname{PolyLog}\left(3,-e^{-a-bx}\right)}{b^2} - \frac{x\operatorname{PolyLog}\left(3,e^{-a-bx}\right)}{b^2} + \frac{\operatorname{PolyLog}\left(4,-e^{-a-bx}\right)}{b^3} - \frac{\operatorname{PolyLog}\left(4,e^{-a-bx}\right)}{b^3}$$

output
```
1/2*x^2*polylog(2,-exp(-b*x-a))/b-1/2*x^2*polylog(2,exp(-b*x-a))/b+x*polyl
og(3,-exp(-b*x-a))/b^2-x*polylog(3,exp(-b*x-a))/b^2+polylog(4,-exp(-b*x-a)
)/b^3-polylog(4,exp(-b*x-a))/b^3
```

3.288.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.25

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right)\,dx = \frac{2b^3x^3\coth^{-1}\left(e^{a+bx}\right) + b^3x^3\log\left(1-e^{a+bx}\right) - b^3x^3\log\left(1+e^{a+bx}\right) - 3b^2x^2\operatorname{PolyLog}\left(2,-e^{a+bx}\right) + 3b^2x^2\operatorname{Po}}{}$$

input
```
Integrate[x^2*ArcCoth[E^(a + b*x)],x]
```

output
```
(2*b^3*x^3*ArcCoth[E^(a + b*x)] + b^3*x^3*Log[1 - E^(a + b*x)] - b^3*x^3*L
og[1 + E^(a + b*x)] - 3*b^2*x^2*PolyLog[2, -E^(a + b*x)] + 3*b^2*x^2*PolyL
og[2, E^(a + b*x)] + 6*b*x*PolyLog[3, -E^(a + b*x)] - 6*b*x*PolyLog[3, E^(
a + b*x)] - 6*PolyLog[4, -E^(a + b*x)] + 6*PolyLog[4, E^(a + b*x)])/(6*b^3
)
```

3.288.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6768, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right) dx$$

$$\Big\downarrow \text{6768}$$

$$\frac{1}{2}\int x^2 \log\left(1+e^{-a-bx}\right)dx-\frac{1}{2}\int x^2 \log\left(1-e^{-a-bx}\right)dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}\left(\frac{x^2 \operatorname{PolyLog}\left(2,-e^{-a-bx}\right)}{b}-\frac{2\int x \operatorname{PolyLog}\left(2,-e^{-a-bx}\right)dx}{b}\right)+\frac{1}{2}\left(\frac{2\int x \operatorname{PolyLog}\left(2,e^{-a-bx}\right)dx}{b}-\frac{x^2 \operatorname{PolyLog}\left(2,e^{-a-bx}\right)}{b}\right)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{2}\left(\frac{x^2 \operatorname{PolyLog}\left(2,-e^{-a-bx}\right)}{b}-\frac{2\left(\frac{\int \operatorname{PolyLog}\left(3,-e^{-a-bx}\right)dx}{b}-\frac{x \operatorname{PolyLog}\left(3,-e^{-a-bx}\right)}{b}\right)}{b}\right)+\frac{1}{2}\left(\frac{2\left(\frac{\int \operatorname{PolyLog}\left(3,e^{-a-bx}\right)dx}{b}-\frac{x \operatorname{PolyLog}\left(3,e^{-a-bx}\right)}{b}\right)}{b}-\frac{x^2 \operatorname{PolyLog}\left(2,e^{-a-bx}\right)}{b}\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}\left(\frac{x^2 \operatorname{PolyLog}\left(2,-e^{-a-bx}\right)}{b}-\frac{2\left(-\frac{\int e^{a+bx}\operatorname{PolyLog}(3,-e^{-a-bx})de^{-a-bx}}{b^2}-\frac{x\operatorname{PolyLog}(3,-e^{-a-bx})}{b}\right)}{b}\right)+\frac{1}{2}\left(\frac{2\left(-\frac{\int e^{a+bx}\operatorname{PolyLog}(3,e^{-a-bx})de^{-a-bx}}{b^2}-\frac{x\operatorname{PolyLog}(3,e^{-a-bx})}{b}\right)}{b}-\frac{x^2\operatorname{PolyLog}\left(2,e^{-a-bx}\right)}{b}\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}\left(\frac{x^2 \operatorname{PolyLog}\left(2,-e^{-a-bx}\right)}{b}-\frac{2\left(-\frac{\operatorname{PolyLog}(4,-e^{-a-bx})}{b^2}-\frac{x\operatorname{PolyLog}(3,-e^{-a-bx})}{b}\right)}{b}\right)+\frac{1}{2}\left(\frac{2\left(-\frac{\operatorname{PolyLog}(4,e^{-a-bx})}{b^2}-\frac{x\operatorname{PolyLog}(3,e^{-a-bx})}{b}\right)}{b}-\frac{x^2\operatorname{PolyLog}\left(2,e^{-a-bx}\right)}{b}\right)$$

input
```
Int[x^2*ArcCoth[E^(a + b*x)],x]
```

output
```
((x^2*PolyLog[2, -E^(-a - b*x)])/b - (2*(-((x*PolyLog[3, -E^(-a - b*x)])/b
) - PolyLog[4, -E^(-a - b*x)]/b^2))/b)/2 + (-((x^2*PolyLog[2, E^(-a - b*x)
])/b) + (2*(-((x*PolyLog[3, E^(-a - b*x)])/b) - PolyLog[4, E^(-a - b*x)]/b
^2))/b)/2
```

3.288.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6768
```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Simp[1/2   I
nt[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x]
&& IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.288.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(109) = 218$.

Time = 0.32 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.05

method	result
risch	$-\frac{x^3 \ln\left(-1+e^{bx+a}\right)}{6}+\frac{\ln\left(1-e^{bx+a}\right) x^3}{6}-\frac{\ln\left(1-e^{bx+a}\right) x\, a^2}{2 b^2}+\frac{x^2 \operatorname{polylog}\left(2, e^{bx+a}\right)}{2 b}-\frac{a^3 \ln\left(-1+e^{bx+a}\right)}{6 b^3}-\frac{a^3 \ln\left(1-e^{bx+a}\right)}{3 b^3}$
default	$\frac{x^3 \operatorname{arccoth}\left(e^{bx+a}\right)}{3}+\frac{\frac{(bx+a)^3 \ln\left(1-e^{bx+a}\right)}{2}+\frac{3(bx+a)^2 \operatorname{polylog}\left(2, e^{bx+a}\right)}{2}-3(bx+a) \operatorname{polylog}\left(3, e^{bx+a}\right)+3 \operatorname{polylog}\left(4, e^{bx+a}\right)-\frac{(bx+a)^3 \ln}{}}{}$
parts	$\frac{x^3 \operatorname{arccoth}\left(e^{bx+a}\right)}{3}+\frac{\frac{(bx+a)^3 \ln\left(1-e^{bx+a}\right)}{2}+\frac{3(bx+a)^2 \operatorname{polylog}\left(2, e^{bx+a}\right)}{2}-3(bx+a) \operatorname{polylog}\left(3, e^{bx+a}\right)+3 \operatorname{polylog}\left(4, e^{bx+a}\right)-\frac{(bx+a)^3 \ln}{}}{}$

input
```
int(x^2*arccoth(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x^3*ln(-1+exp(b*x+a))+1/6*ln(1-exp(b*x+a))*x^3-1/2/b^2*ln(1-exp(b*x+a
))*x*a^2+1/2*x^2*polylog(2,exp(b*x+a))/b-1/6/b^3*a^3*ln(-1+exp(b*x+a))-1/3
/b^3*a^3*ln(1-exp(b*x+a))-x*polylog(3,exp(b*x+a))/b^2-1/2/b^3*polylog(2,ex
p(b*x+a))*a^2-1/2/b^3*a^2*dilog(exp(b*x+a))+polylog(4,exp(b*x+a))/b^3-1/2*
x^2*polylog(2,-exp(b*x+a))/b-1/2/b^3*dilog(exp(b*x+a)+1)*a^2+x*polylog(3,-
exp(b*x+a))/b^2+1/2/b^3*polylog(2,-exp(b*x+a))*a^2-polylog(4,-exp(b*x+a))/
b^3
```

3.288.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(107) = 214$.

Time = 0.27 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.08

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right) dx$$
$$= \frac{b^3x^3 \log\left(\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right) - b^3x^3 \log\left(\cosh\left(bx+a\right) + \sinh\left(bx+a\right) + 1\right) + 3\,b^2x^2\mathrm{Li}_2(\cosh\left(bx+a\right) -$$

input
```
integrate(x^2*arccoth(exp(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(b^3*x^3*log((cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh
(b*x + a) - 1)) - b^3*x^3*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*b^2*x
^2*dilog(cosh(b*x + a) + sinh(b*x + a)) - 3*b^2*x^2*dilog(-cosh(b*x + a) -
 sinh(b*x + a)) - a^3*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 6*b*x*polyl
og(3, cosh(b*x + a) + sinh(b*x + a)) + 6*b*x*polylog(3, -cosh(b*x + a) - s
inh(b*x + a)) + (b^3*x^3 + a^3)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) +
6*polylog(4, cosh(b*x + a) + sinh(b*x + a)) - 6*polylog(4, -cosh(b*x + a)
- sinh(b*x + a)))/b^3
```

3.288.6 Sympy [F]

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right) dx = \int x^2 \operatorname{acoth}\left(e^a e^{bx}\right) dx$$

input
```
integrate(x**2*acoth(exp(b*x+a)),x)
```

output
```
Integral(x**2*acoth(exp(a)*exp(b*x)), x)
```

3.288.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.19

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right)\,dx = \frac{1}{3}x^3 \operatorname{arcoth}\left(e^{(bx+a)}\right) - \frac{1}{6}b\left(\frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3\left(-e^{(bx+a)}\right)+6\,\mathrm{Li}_4\left(-e^{(bx+a)}\right)}{b^4} - \frac{b^3x^3\log\left(-e^{(b\right.}\right.}{}$$

input
```
integrate(x^2*arccoth(exp(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(e^(b*x + a)) - 1/6*b*((b^3*x^3*log(e^(b*x + a) + 1) + 3*b^
2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4,
-e^(b*x + a)))/b^4 - (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b
*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4)
```

3.288.8 Giac [F]

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right)\,dx = \int x^2 \operatorname{arcoth}\left(e^{(bx+a)}\right)\,dx$$

input
```
integrate(x^2*arccoth(exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(e^(b*x + a)), x)
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right)\,dx = \int x^2 \operatorname{acoth}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

input
```
int(x^2*acoth(exp(a + b*x)),x)
```

output
```
int(x^2*acoth(exp(a + b*x)), x)
```

3.289 $\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$

3.289.1 Optimal result

Integrand size = 12, antiderivative size = 168

$$\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx=-\frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)}+\frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)\left(1+a+bf^{c+dx}\right)}\right)}{d\log(f)}+\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1+a+bf^{c+dx}}\right)}{2d\log(f)}-\frac{\operatorname{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(1-a)\left(1+a+bf^{c+dx}\right)}\right)}{2d\log(f)}$$

output
```
-arccoth(a+b*f^(d*x+c))*ln(2/(1+a+b*f^(d*x+c)))/d/ln(f)+arccoth(a+b*f^(d*x
+c))*ln(2*b*f^(d*x+c)/(1-a)/(1+a+b*f^(d*x+c)))/d/ln(f)+1/2*polylog(2,1-2/(
1+a+b*f^(d*x+c)))/d/ln(f)-1/2*polylog(2,1-2*b*f^(d*x+c)/(1-a)/(1+a+b*f^(d*
x+c)))/d/ln(f)
```

3.289.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.64

$$\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$$
$$= \frac{dx\log(f)\left(2\coth^{-1}\left(a+bf^{c+dx}\right)+\log\left(\frac{-1+a+bf^{c+dx}}{-1+a}\right)-\log\left(\frac{1+a+bf^{c+dx}}{1+a}\right)\right)+\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{-1+a}\right)-\operatorname{Pol}}{2d\log(f)}$$

input
```
Integrate[ArcCoth[a + b*f^(c + d*x)],x]
```

output
```
(d*x*Log[f]*(2*ArcCoth[a + b*f^(c + d*x)] + Log[(-1 + a + b*f^(c + d*x))/(
-1 + a)] - Log[(1 + a + b*f^(c + d*x))/(1 + a)]) + PolyLog[2, -((b*f^(c +
d*x))/(-1 + a))] - PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f])
```

3.289.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2720, 6662, 25, 27, 6473, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$$
$$\Big\downarrow \text{2720}$$
$$\frac{\int f^{-c-dx}\coth^{-1}\left(bf^{c+dx}+a\right)df^{c+dx}}{d\log(f)}$$
$$\Big\downarrow \text{6662}$$
$$\frac{\int f^{-c-dx}\coth^{-1}\left(bf^{c+dx}+a\right)d\left(bf^{c+dx}+a\right)}{bd\log(f)}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\int -f^{-c-dx}\coth^{-1}\left(bf^{c+dx}+a\right)d\left(bf^{c+dx}+a\right)}{bd\log(f)}$$
$$\Big\downarrow \text{27}$$

$$-\frac{\int -\frac{f^{-c-dx}\coth^{-1}(bf^{c+dx}+a)}{b}d(bf^{c+dx}+a)}{d\log(f)}$$

$\big\downarrow$ 6473

$$-\frac{-\int \frac{\log\left(\frac{2}{bf^{c+dx}+a+1}\right)}{1-f^{2c+2dx}}d(bf^{c+dx}+a)+\int \frac{\log\left(\frac{2bf^{c+dx}}{(1-a)\left(bf^{c+dx}+a+1\right)}\right)}{1-f^{2c+2dx}}d(bf^{c+dx}+a)+\log\left(\frac{2}{a+bf^{c+dx}+1}\right)\coth^{-1}\left(a+bf^{c+}\right.}{d\log(f)}$$

$\big\downarrow$ 2849

$$-\frac{-\int \frac{\log\left(\frac{2}{bf^{c+dx}+a+1}\right)}{1-\frac{2}{bf^{c+dx}+a+1}}d\frac{1}{bf^{c+dx}+a+1}+\int \frac{\log\left(\frac{2bf^{c+dx}}{(1-a)\left(bf^{c+dx}+a+1\right)}\right)}{1-f^{2c+2dx}}d(bf^{c+dx}+a)+\log\left(\frac{2}{a+bf^{c+dx}+1}\right)\coth^{-1}\left(a+bf^{c+dx}\right.}{d\log(f)}$$

$\big\downarrow$ 2752

$$-\frac{\int \frac{\log\left(\frac{2bf^{c+dx}}{(1-a)\left(bf^{c+dx}+a+1\right)}\right)}{1-f^{2c+2dx}}d(bf^{c+dx}+a)-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{bf^{c+dx}+a+1}\right)+\log\left(\frac{2}{a+bf^{c+dx}+1}\right)\coth^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}$$

$\big\downarrow$ 2897

$$-\frac{-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{bf^{c+dx}+a+1}\right)+\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(1-a)(bf^{c+dx}+a+1)}\right)+\log\left(\frac{2}{a+bf^{c+dx}+1}\right)\coth^{-1}\left(a+bf^{c+dx}\right.}{d\log(f)}$$

input
```
Int[ArcCoth[a + b*f^(c + d*x)],x]
```

output
```
-((ArcCoth[a + b*f^(c + d*x)]*Log[2/(1 + a + b*f^(c + d*x))] - ArcCoth[a +
 b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c + d*x)))] -
 PolyLog[2, 1 - 2/(1 + a + b*f^(c + d*x))]/2 + PolyLog[2, 1 - (2*b*f^(c +
d*x))/((1 - a)*(1 + a + b*f^(c + d*x)))]/2)/(d*Log[f]))
```

3.289. $\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$

3.289.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6473
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcCoth[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*ArcCoth
[c*x])*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(c/e)
Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Simp[b*(c/e)   Int[Log[2*c*((d
 + e*x)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x]) /; FreeQ[{a, b, c, d
, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

rule 6662
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.289.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\ln(-b\,f^{dx+c})\operatorname{arccoth}(a+b\,f^{dx+c})-\frac{\operatorname{dilog}\left(\frac{-b\,f^{dx+c}-a-1}{-1-a}\right)}{2}-\frac{\ln(-b\,f^{dx+c})\ln\left(\frac{-b\,f^{dx+c}-a-1}{-1-a}\right)}{2}+\frac{\operatorname{dilog}\left(\frac{1-a-b\,f^{dx+c}}{1-a}\right)}{2}+}{d\ln(f)}$
default	$\frac{\ln(-b\,f^{dx+c})\operatorname{arccoth}(a+b\,f^{dx+c})-\frac{\operatorname{dilog}\left(\frac{-b\,f^{dx+c}-a-1}{-1-a}\right)}{2}-\frac{\ln(-b\,f^{dx+c})\ln\left(\frac{-b\,f^{dx+c}-a-1}{-1-a}\right)}{2}+\frac{\operatorname{dilog}\left(\frac{1-a-b\,f^{dx+c}}{1-a}\right)}{2}+}{d\ln(f)}$
risch	$-\frac{x\ln(b\,f^{dx+c}+a-1)}{2}+\frac{\operatorname{dilog}\left(\frac{f^{dx}f^{c}b+a-1}{-1+a}\right)}{2\ln(f)d}+\frac{\ln\left(\frac{f^{dx}f^{c}b+a-1}{-1+a}\right)x}{2}+\frac{c\ln\left(\frac{f^{dx}f^{c}b+a-1}{-1+a}\right)}{2d}-\frac{c\ln(f^{dx}f^{c}b+a-1}{2d}$

input
```
int(arccoth(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/ln(f)*(ln(-b*f^(d*x+c))*arccoth(a+b*f^(d*x+c))-1/2*dilog((-b*f^(d*x+c)
-a-1)/(-1-a))-1/2*ln(-b*f^(d*x+c))*ln((-b*f^(d*x+c)-a-1)/(-1-a))+1/2*dilog
((1-a-b*f^(d*x+c))/(1-a))+1/2*ln(-b*f^(d*x+c))*ln((1-a-b*f^(d*x+c))/(1-a))
)
```

3.289.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.68

$$\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$$
$$=\frac{dx\log(f)\log\left(\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1}{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1}\right)+c\log\left(b\cosh\left((dx+c)\log(f)\right)+b\sinh\left((dx+c)\right.\right.}{}$$

input
```
integrate(arccoth(a+b*f^(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(d*x*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
 a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)) + c
*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log(f) -
 c*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)*log(f)
 - (d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f
)) + a + 1)/(a + 1)) + (d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b*
sinh((d*x + c)*log(f)) + a - 1)/(a - 1)) - dilog(-(b*cosh((d*x + c)*log(f)
) + b*sinh((d*x + c)*log(f)) + a + 1)/(a + 1) + 1) + dilog(-(b*cosh((d*x +
 c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1))/(d*log(f))
```

3.289.6 Sympy [F]

$$\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx = \int \operatorname{acoth}\left(a+bf^{c+dx}\right)dx$$

input
```
integrate(acoth(a+b*f**(d*x+c)),x)
```

output
```
Integral(acoth(a + b*f**(c + d*x)), x)
```

3.289.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.20

$$\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx = \frac{(dx+c)\operatorname{arcoth}\left(bf^{dx+c}+a\right)}{d} - \frac{(dx+c)b\left(\frac{\log(bf^{dx+c}+a+1)}{b}-\frac{\log(bf^{dx+c}+a-1)}{b}\right)\log(f) - b\left(\frac{\log(bf^{dx+c}+a+1)\log\left(-\frac{bf^{dx+c}+a+1}{a+1}+1\right)+\mathrm{Li}_2\left(\frac{bf^{dx+c}+a+1}{a+1}\right)}{b}\right.}{2\,d\log(f)}$$

input
```
integrate(arccoth(a+b*f^(d*x+c)),x, algorithm="maxima")
```

output
```
(d*x + c)*arccoth(b*f^(d*x + c) + a)/d - 1/2*((d*x + c)*b*(log(b*f^(d*x +
c) + a + 1)/b - log(b*f^(d*x + c) + a - 1)/b)*log(f) - b*((log(b*f^(d*x +
c) + a + 1)*log(-(b*f^(d*x + c) + a + 1)/(a + 1) + 1) + dilog((b*f^(d*x +
c) + a + 1)/(a + 1)))/b - (log(b*f^(d*x + c) + a - 1)*log(-(b*f^(d*x + c)
+ a - 1)/(a - 1) + 1) + dilog((b*f^(d*x + c) + a - 1)/(a - 1)))/b))/(d*log
(f))
```

3.289.8 Giac [F(-2)]

Exception generated.

$$\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccoth(a+b*f^(d*x+c)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,2,0,0,0]%%%}+%%%{2,[0,1,1,1,1,0]%%%}+%%%{-2,[0,1,1,0,0,0]%%%}+%%%{1,[0,
1,0,2,0,1]%%
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx = \int \operatorname{acoth}\left(a+b\,f^{c+d\,x}\right)\,dx$$

input
```
int(acoth(a + b*f^(c + d*x)),x)
```

output
```
int(acoth(a + b*f^(c + d*x)), x)
```

3.290 $\int x \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$

3.290.1 Optimal result

Integrand size = 14, antiderivative size = 216

$$\begin{aligned}\int x\coth^{-1}\left(a+bf^{c+dx}\right)\,dx &= \frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\left(1+\frac{bf^{c+dx}}{1+a}\right)\\ &\quad-\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}x^2\log\left(1+\frac{1}{a+bf^{c+dx}}\right)\\ &\quad+\frac{x\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{2d\log(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{1+a}\right)}{2d\log(f)}\\ &\quad-\frac{\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{2d^2\log^2(f)}+\frac{\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{1+a}\right)}{2d^2\log^2(f)}\end{aligned}$$

output
```
1/4*x^2*ln(1-b*f^(d*x+c)/(1-a))-1/4*x^2*ln(1+b*f^(d*x+c)/(1+a))-1/4*x^2*ln
(1-1/(a+b*f^(d*x+c)))+1/4*x^2*ln(1+1/(a+b*f^(d*x+c)))+1/2*x*polylog(2,b*f^
(d*x+c)/(1-a))/d/ln(f)-1/2*x*polylog(2,-b*f^(d*x+c)/(1+a))/d/ln(f)-1/2*pol
ylog(3,b*f^(d*x+c)/(1-a))/d^2/ln(f)^2+1/2*polylog(3,-b*f^(d*x+c)/(1+a))/d^
2/ln(f)^2
```

3.290.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.82

$$\int x\coth^{-1}\left(a+bf^{c+dx}\right)\,dx$$
$$=\frac{2d^2x^2\coth^{-1}\left(a+bf^{c+dx}\right)\log^2(f)+d^2x^2\log^2(f)\log\left(1+\frac{bf^{c+dx}}{-1+a}\right)-d^2x^2\log^2(f)\log\left(1+\frac{bf^{c+dx}}{1+a}\right)+2dx\,\mathrm{l}}{4}$$

input
```
Integrate[x*ArcCoth[a + b*f^(c + d*x)],x]
```

output
```
(2*d^2*x^2*ArcCoth[a + b*f^(c + d*x)]*Log[f]^2 + d^2*x^2*Log[f]^2*Log[1 +
(b*f^(c + d*x))/(-1 + a)] - d^2*x^2*Log[f]^2*Log[1 + (b*f^(c + d*x))/(1 +
a)] + 2*d*x*Log[f]*PolyLog[2, -((b*f^(c + d*x))/(-1 + a))] - 2*d*x*Log[f]*
PolyLog[2, -((b*f^(c + d*x))/(1 + a))] - 2*PolyLog[3, -((b*f^(c + d*x))/(-
1 + a))] + 2*PolyLog[3, -((b*f^(c + d*x))/(1 + a))])/(4*d^2*Log[f]^2)
```

3.290.3 Rubi [A] (verified)

Time = 2.70 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6768, 3031, 25, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^{-1}\left(a+bf^{c+dx}\right)\,dx$$
$$\Big\downarrow \text{6768}$$
$$\frac{1}{2}\int x\log\left(1+\frac{1}{bf^{c+dx}+a}\right)dx-\frac{1}{2}\int x\log\left(1-\frac{1}{bf^{c+dx}+a}\right)dx$$
$$\Big\downarrow \text{3031}$$
$$\frac{1}{2}\left(\frac{1}{2}\int -\frac{bdf^{c+dx}x^2\log(f)}{\left(-bf^{c+dx}-a+1\right)\left(bf^{c+dx}+a\right)}dx-\frac{1}{2}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}x^2\log\left(\frac{1}{a+bf^{c+dx}}+1\right)-\frac{1}{2}\int -\frac{bdf^{c+dx}x^2\log(f)}{\left(bf^{c+dx}+a\right)\left(bf^{c+dx}+a+1\right)}dx\right)$$
$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{bdf^{c+dx}x^2\log(f)}{(-bf^{c+dx}-a+1)(bf^{c+dx}+a)}dx-\frac{1}{2}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{bdf^{c+dx}x^2\log(f)}{(bf^{c+dx}+a)(bf^{c+dx}+a+1)}dx+\frac{1}{2}x^2\log\left(\frac{1}{a+bf^{c+dx}}+1\right)\right)$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(-\frac{1}{2}bd\log(f)\int \frac{f^{c+dx}x^2}{(-bf^{c+dx}-a+1)(bf^{c+dx}+a)}dx-\frac{1}{2}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}bd\log(f)\int \frac{f^{c+dx}x^2}{(bf^{c+dx}+a)(bf^{c+dx}+a+1)}dx+\frac{1}{2}x^2\log\left(\frac{1}{a+bf^{c+dx}}+1\right)\right)$$

$\Big\downarrow$ 7293

$$\frac{1}{2}\left(\frac{1}{2}bd\log(f)\int \left(\frac{x^2f^{c+dx}}{-bf^{c+dx}-a-1}+\frac{x^2f^{c+dx}}{bf^{c+dx}+a}\right)dx+\frac{1}{2}x^2\log\left(\frac{1}{a+bf^{c+dx}}+1\right)\right)+\frac{1}{2}\left(-\frac{1}{2}bd\log(f)\int \left(\frac{x^2f^{c+dx}}{-bf^{c+dx}-a+1}+\frac{x^2f^{c+dx}}{bf^{c+dx}+a}\right)dx-\frac{1}{2}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\right)$$

$\Big\downarrow$ 2009

$$\frac{1}{2}\left(-\frac{1}{2}bd\log(f)\left(\frac{2\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{bd^3\log^3(f)}-\frac{2\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a}\right)}{bd^3\log^3(f)}-\frac{2x\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{bd^2\log^2(f)}+\frac{2x\,\text{PolyLog}\left(2,-\right.}{bd^2\log^2(f}\right.$$
$$\frac{1}{2}\left(\frac{1}{2}bd\log(f)\left(-\frac{2\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a}\right)}{bd^3\log^3(f)}+\frac{2\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)}{bd^3\log^3(f)}+\frac{2x\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a}\right)}{bd^2\log^2(f)}-\frac{2x\,\text{PolyLog}\left(\right.}{bd^2\log}\right.\right.$$

input
```
Int[x*ArcCoth[a + b*f^(c + d*x)],x]
```

output
```
(-1/2*(x^2*Log[1 - (a + b*f^(c + d*x))^(-1)]) - (b*d*Log[f]*(-((x^2*Log[1
- (b*f^(c + d*x))/(1 - a)])/(b*d*Log[f])) + (x^2*Log[1 + (b*f^(c + d*x))/a
])/(b*d*Log[f]) - (2*x*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(b*d^2*Log[f]^
2) + (2*x*PolyLog[2, -((b*f^(c + d*x))/a)])/(b*d^2*Log[f]^2) + (2*PolyLog[
3, (b*f^(c + d*x))/(1 - a)])/(b*d^3*Log[f]^3) - (2*PolyLog[3, -((b*f^(c +
d*x))/a)])/(b*d^3*Log[f]^3)))/2)/2 + ((x^2*Log[1 + (a + b*f^(c + d*x))^(-1
)])/2 + (b*d*Log[f]*((x^2*Log[1 + (b*f^(c + d*x))/a])/(b*d*Log[f]) - (x^2*
Log[1 + (b*f^(c + d*x))/(1 + a)])/(b*d*Log[f]) + (2*x*PolyLog[2, -((b*f^(c
 + d*x))/a)])/(b*d^2*Log[f]^2) - (2*x*PolyLog[2, -((b*f^(c + d*x))/(1 + a)
)])/(b*d^2*Log[f]^2) - (2*PolyLog[3, -((b*f^(c + d*x))/a)])/(b*d^3*Log[f]^
3) + (2*PolyLog[3, -((b*f^(c + d*x))/(1 + a))])/(b*d^3*Log[f]^3)))/2)/2
```

3.290. $\int x\coth^{-1}\left(a+bf^{c+dx}\right)\,dx$

3.290.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3031
```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Simp[1/(b*(m + 1))   Int[SimplifyIntegrand[(a +
 b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunc
tionFreeQ[u, x] && NeQ[m, -1]
```

rule 6768
```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Simp[1/2   I
nt[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x]
&& IGtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.290.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. 2(200) = 400.

Time = 0.64 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.73

method	result
risch	$-\frac{x^{2} \ln \left(b\, f^{d x +c}+a -1\right)}{4}+\frac{x^{2} \ln \left(1+a +b\, f^{d x +c}\right)}{4}-\frac{\ln \left(1-\frac{b\, f^{d x} f^{c}}{-1-a}\right) x^{2}}{4}-\frac{\ln \left(1-\frac{b\, f^{d x} f^{c}}{-1-a}\right) c x}{2 d}-\frac{\ln \left(1-\frac{b\, f^{d x} f^{c}}{-1-a}\right) c^{2}}{4 d^{2}}-\frac{\operatorname{polylog}(}{2}$

input
```
int(x*arccoth(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2*ln(b*f^(d*x+c)+a-1)+1/4*x^2*ln(1+a+b*f^(d*x+c))-1/4*ln(1-b*f^(d*x
)*f^c/(-1-a))*x^2-1/2/d*ln(1-b*f^(d*x)*f^c/(-1-a))*c*x-1/4/d^2*ln(1-b*f^(d
*x)*f^c/(-1-a))*c^2-1/2/ln(f)/d*polylog(2,b*f^(d*x)*f^c/(-1-a))*x-1/2/ln(f
)/d^2*polylog(2,b*f^(d*x)*f^c/(-1-a))*c+1/2/ln(f)^2/d^2*polylog(3,b*f^(d*x
)*f^c/(-1-a))-1/4/d^2*c^2*ln(1+a+f^(d*x)*f^c*b)+1/2/ln(f)/d^2*c*dilog((1+a
+f^(d*x)*f^c*b)/(1+a))+1/2/d*c*ln((1+a+f^(d*x)*f^c*b)/(1+a))*x+1/2/d^2*c^2
*ln((1+a+f^(d*x)*f^c*b)/(1+a))+1/4*ln(1-b*f^(d*x)*f^c/(1-a))*x^2+1/2/d*ln(
1-b*f^(d*x)*f^c/(1-a))*c*x+1/4/d^2*ln(1-b*f^(d*x)*f^c/(1-a))*c^2+1/2/ln(f)
/d*polylog(2,b*f^(d*x)*f^c/(1-a))*x+1/2/ln(f)/d^2*polylog(2,b*f^(d*x)*f^c/
(1-a))*c-1/2/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^c/(1-a))+1/4/d^2*c^2*ln(f^(
d*x)*f^c*b+a-1)-1/2/ln(f)/d^2*c*dilog((f^(d*x)*f^c*b+a-1)/(-1+a))-1/2/d*c*
ln((f^(d*x)*f^c*b+a-1)/(-1+a))*x-1/2/d^2*c^2*ln((f^(d*x)*f^c*b+a-1)/(-1+a)
)
```

3.290.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(193) = 386$.

Time = 0.26 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.83

$$\int x \coth^{-1}\left(a + bf^{c+dx}\right) dx$$
$$= \frac{d^2x^2 \log(f)^2 \log\left(\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1}{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1}\right) - c^2 \log(b\cosh((dx+c)\log(f)) + b\sinh((dx+$$

input
```
integrate(x*arccoth(a+b*f^(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(d^2*x^2*log(f)^2*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log
(f)) + a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1
)) - c^2*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*
log(f)^2 + c^2*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a
 - 1)*log(f)^2 - 2*d*x*dilog(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)
*log(f)) + a + 1)/(a + 1) + 1)*log(f) + 2*d*x*dilog(-(b*cosh((d*x + c)*log
(f)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1)*log(f) - (d^2*x^2 -
c^2)*log(f)^2*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a
 + 1)/(a + 1)) + (d^2*x^2 - c^2)*log(f)^2*log((b*cosh((d*x + c)*log(f)) +
b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1)) + 2*polylog(3, -(b*cosh((d*x +
c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a + 1)) - 2*polylog(3, -(b*cosh((d
*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a - 1)))/(d^2*log(f)^2)
```

3.290.6 Sympy [F]

$$\int x \coth^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x\,\mathrm{acoth}\left(a+bf^{c+dx}\right)dx$$

input
```
integrate(x*acoth(a+b*f**(d*x+c)),x)
```

output
```
Integral(x*acoth(a + b*f**(c + d*x)), x)
```

3.290.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.90

$$\int x \coth^{-1}\left(a+bf^{c+dx}\right)\,dx=$$
$$-\frac{1}{4}bd\left(\frac{d^2x^2\log\left(\frac{bf^{dx}f^c}{a+1}+1\right)\log(f)^2+2\,dx\mathrm{Li}_2\left(-\frac{bf^{dx}f^c}{a+1}\right)\log(f)-2\,\mathrm{Li}_3(-\frac{bf^{dx}f^c}{a+1})}{bd^3\log(f)^3}-\frac{d^2x^2\log\left(\frac{bf^{dx}f^c}{a-1}+1\right.}{}\right.$$
$$+\frac{1}{2}x^2\,\mathrm{arcoth}\left(bf^{dx+c}+a\right)$$

input
```
integrate(x*arccoth(a+b*f^(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*b*d*((d^2*x^2*log(b*f^(d*x)*f^c/(a + 1) + 1)*log(f)^2 + 2*d*x*dilog(-
b*f^(d*x)*f^c/(a + 1))*log(f) - 2*polylog(3, -b*f^(d*x)*f^c/(a + 1)))/(b*d
^3*log(f)^3) - (d^2*x^2*log(b*f^(d*x)*f^c/(a - 1) + 1)*log(f)^2 + 2*d*x*di
log(-b*f^(d*x)*f^c/(a - 1))*log(f) - 2*polylog(3, -b*f^(d*x)*f^c/(a - 1)))
/(b*d^3*log(f)^3))*log(f) + 1/2*x^2*arccoth(b*f^(d*x + c) + a)
```

3.290.8 Giac [F]

$$\int x \coth^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x\,\mathrm{arcoth}\left(bf^{dx+c}+a\right)\,dx$$

input
```
integrate(x*arccoth(a+b*f^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x*arccoth(b*f^(d*x + c) + a), x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int x \coth^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x\,\mathrm{acoth}\left(a+b\,f^{c+d\,x}\right)\,dx$$

input
```
int(x*acoth(a + b*f^(c + d*x)),x)
```

output
```
int(x*acoth(a + b*f^(c + d*x)), x)
```

3.291 $\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$

3.291.1 Optimal result

Integrand size = 16, antiderivative size = 269

$$\begin{aligned}\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right)\,dx &= \frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log\left(1+\frac{bf^{c+dx}}{1+a}\right)\\ &-\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}x^3\log\left(1+\frac{1}{a+bf^{c+dx}}\right)\\ &+\frac{x^2\operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{2d\log(f)}-\frac{x^2\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{1+a}\right)}{2d\log(f)}\\ &-\frac{x\operatorname{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{d^2\log^2(f)}+\frac{x\operatorname{PolyLog}\left(3,-\frac{bf^{c+dx}}{1+a}\right)}{d^2\log^2(f)}\\ &+\frac{\operatorname{PolyLog}\left(4,\frac{bf^{c+dx}}{1-a}\right)}{d^3\log^3(f)}-\frac{\operatorname{PolyLog}\left(4,-\frac{bf^{c+dx}}{1+a}\right)}{d^3\log^3(f)}\end{aligned}$$

output

```
1/6*x^3*ln(1-b*f^(d*x+c)/(1-a))-1/6*x^3*ln(1+b*f^(d*x+c)/(1+a))-1/6*x^3*ln
(1-1/(a+b*f^(d*x+c)))+1/6*x^3*ln(1+1/(a+b*f^(d*x+c)))+1/2*x^2*polylog(2,b*
f^(d*x+c)/(1-a))/d/ln(f)-1/2*x^2*polylog(2,-b*f^(d*x+c)/(1+a))/d/ln(f)-x*p
olylog(3,b*f^(d*x+c)/(1-a))/d^2/ln(f)^2+x*polylog(3,-b*f^(d*x+c)/(1+a))/d^
2/ln(f)^2+polylog(4,b*f^(d*x+c)/(1-a))/d^3/ln(f)^3-polylog(4,-b*f^(d*x+c)/
(1+a))/d^3/ln(f)^3
```

3.291.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.87

$$\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right) dx$$
$$= \frac{2d^3x^3 \coth^{-1}\left(a+bf^{c+dx}\right)\log^3(f) + d^3x^3\log^3(f)\log\left(1+\frac{bf^{c+dx}}{-1+a}\right) - d^3x^3\log^3(f)\log\left(1+\frac{bf^{c+dx}}{1+a}\right) + 3d^2x^2}{}$$

input
```
Integrate[x^2*ArcCoth[a + b*f^(c + d*x)],x]
```

output
```
(2*d^3*x^3*ArcCoth[a + b*f^(c + d*x)]*Log[f]^3 + d^3*x^3*Log[f]^3*Log[1 +
(b*f^(c + d*x))/(-1 + a)] - d^3*x^3*Log[f]^3*Log[1 + (b*f^(c + d*x))/(1 +
a)] + 3*d^2*x^2*Log[f]^2*PolyLog[2, -((b*f^(c + d*x))/(-1 + a))] - 3*d^2*x
^2*Log[f]^2*PolyLog[2, -((b*f^(c + d*x))/(1 + a))] - 6*d*x*Log[f]*PolyLog[
3, -((b*f^(c + d*x))/(-1 + a))] + 6*d*x*Log[f]*PolyLog[3, -((b*f^(c + d*x)
)/(1 + a))] + 6*PolyLog[4, -((b*f^(c + d*x))/(-1 + a))] - 6*PolyLog[4, -((
b*f^(c + d*x))/(1 + a))])/(6*d^3*Log[f]^3)
```

3.291.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 557 vs. 2(269) = 538.

Time = 2.69 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6768, 3031, 25, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right) dx$$
$$\Big\downarrow \text{ 6768}$$
$$\frac{1}{2}\int x^2 \log\left(1+\frac{1}{bf^{c+dx}+a}\right)dx - \frac{1}{2}\int x^2 \log\left(1-\frac{1}{bf^{c+dx}+a}\right)dx$$
$$\Big\downarrow \text{ 3031}$$

$$\frac{1}{2}\left(\frac{1}{3}\int -\frac{bdf^{c+dx}x^3\log(f)}{(-bf^{c+dx}-a+1)(bf^{c+dx}+a)}dx-\frac{1}{3}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\right)+\frac{1}{2}\left(\frac{1}{3}x^3\log\left(\frac{1}{a+bf^{c+dx}}+1\right)-\frac{1}{3}\int -\frac{bdf^{c+dx}x^3\log(f)}{(bf^{c+dx}+a)(bf^{c+dx}+a+1)}dx\right)$$

$$\downarrow 25$$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{bdf^{c+dx}x^3\log(f)}{(-bf^{c+dx}-a+1)(bf^{c+dx}+a)}dx-\frac{1}{3}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\right)+\frac{1}{2}\left(\frac{1}{3}\int \frac{bdf^{c+dx}x^3\log(f)}{(bf^{c+dx}+a)(bf^{c+dx}+a+1)}dx+\frac{1}{3}x^3\log\left(\frac{1}{a+bf^{c+dx}}+1\right)\right)$$

$$\downarrow 27$$

$$\frac{1}{2}\left(-\frac{1}{3}bd\log(f)\int \frac{f^{c+dx}x^3}{(-bf^{c+dx}-a+1)(bf^{c+dx}+a)}dx-\frac{1}{3}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\right)+\frac{1}{2}\left(\frac{1}{3}bd\log(f)\int \frac{f^{c+dx}x^3}{(bf^{c+dx}+a)(bf^{c+dx}+a+1)}dx+\frac{1}{3}x^3\log\left(\frac{1}{a+bf^{c+dx}}+1\right)\right)$$

$$\downarrow 7293$$

$$\frac{1}{2}\left(\frac{1}{3}bd\log(f)\int\left(\frac{x^3f^{c+dx}}{-bf^{c+dx}-a-1}+\frac{x^3f^{c+dx}}{bf^{c+dx}+a}\right)dx+\frac{1}{3}x^3\log\left(\frac{1}{a+bf^{c+dx}}+1\right)\right)+\frac{1}{2}\left(-\frac{1}{3}bd\log(f)\int\left(\frac{x^3f^{c+dx}}{-bf^{c+dx}-a+1}+\frac{x^3f^{c+dx}}{bf^{c+dx}+a}\right)dx-\frac{1}{3}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\right)$$

$$\downarrow 2009$$

$$\frac{1}{2}\left(-\frac{1}{3}bd\log(f)\left(-\frac{6\,\text{PolyLog}\left(4,\frac{bf^{c+dx}}{1-a}\right)}{bd^4\log^4(f)}+\frac{6\,\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a}\right)}{bd^4\log^4(f)}+\frac{6x\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{bd^3\log^3(f)}-\frac{6x\,\text{PolyLog}\left(3,\right.}{bd^3\log^3}\right.\right.$$
$$\frac{1}{2}\left(\frac{1}{3}bd\log(f)\left(\frac{6\,\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a}\right)}{bd^4\log^4(f)}-\frac{6\,\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a+1}\right)}{bd^4\log^4(f)}-\frac{6x\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a}\right)}{bd^3\log^3(f)}+\frac{6x\,\text{PolyLog}\left(3,\right.}{bd^3\log^3}\right.\right.$$

input
```
Int[x^2*ArcCoth[a + b*f^(c + d*x)],x]
```

3.291. $\int x^2\coth^{-1}\left(a+bf^{c+dx}\right)\,dx$

output
```
(-1/3*(x^3*Log[1 - (a + b*f^(c + d*x))^(-1)]) - (b*d*Log[f]*(-((x^3*Log[1
- (b*f^(c + d*x))/(1 - a)])/(b*d*Log[f])) + (x^3*Log[1 + (b*f^(c + d*x))/a
])/(b*d*Log[f]) - (3*x^2*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(b*d^2*Log[f
]^2) + (3*x^2*PolyLog[2, -((b*f^(c + d*x))/a)])/(b*d^2*Log[f]^2) + (6*x*Po
lyLog[3, (b*f^(c + d*x))/(1 - a)])/(b*d^3*Log[f]^3) - (6*x*PolyLog[3, -((b
*f^(c + d*x))/a)])/(b*d^3*Log[f]^3) - (6*PolyLog[4, (b*f^(c + d*x))/(1 - a
)])/(b*d^4*Log[f]^4) + (6*PolyLog[4, -((b*f^(c + d*x))/a)])/(b*d^4*Log[f]^
4)))/3)/2 + ((x^3*Log[1 + (a + b*f^(c + d*x))^(-1)])/3 + (b*d*Log[f]*((x^3
*Log[1 + (b*f^(c + d*x))/a])/(b*d*Log[f]) - (x^3*Log[1 + (b*f^(c + d*x))/(
1 + a)])/(b*d*Log[f]) + (3*x^2*PolyLog[2, -((b*f^(c + d*x))/a)])/(b*d^2*Lo
g[f]^2) - (3*x^2*PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(b*d^2*Log[f]^2)
- (6*x*PolyLog[3, -((b*f^(c + d*x))/a)])/(b*d^3*Log[f]^3) + (6*x*PolyLog[3
, -((b*f^(c + d*x))/(1 + a))])/(b*d^3*Log[f]^3) + (6*PolyLog[4, -((b*f^(c
+ d*x))/a)])/(b*d^4*Log[f]^4) - (6*PolyLog[4, -((b*f^(c + d*x))/(1 + a))])
/(b*d^4*Log[f]^4)))/3)/2
```

3.291.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3031
```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Simp[1/(b*(m + 1))   Int[SimplifyIntegrand[(a +
 b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunc
tionFreeQ[u, x] && NeQ[m, -1]
```

rule 6768
```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Simp[1/2   I
nt[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x]
&& IGtQ[m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.291.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(257) = 514$.

Time = 1.04 (sec) , antiderivative size = 666, normalized size of antiderivative = 2.48

method	result
risch	$-\frac{x^3 \ln(b f^{dx+c}+a-1)}{6}+\frac{x^3 \ln(1+a+b f^{dx+c})}{6}-\frac{\ln\left(1-\frac{b f^{dx} f^c}{-1-a}\right)x^3}{6}+\frac{\ln\left(1-\frac{b f^{dx} f^c}{-1-a}\right)x c^2}{2d^2}+\frac{\ln\left(1-\frac{b f^{dx} f^c}{-1-a}\right)c^3}{3d^3}-$ polylog

input
```
int(x^2*arccoth(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x^3*ln(b*f^(d*x+c)+a-1)+1/6*x^3*ln(1+a+b*f^(d*x+c))-1/6*ln(1-b*f^(d*x
)*f^c/(-1-a))*x^3+1/2/d^2*ln(1-b*f^(d*x)*f^c/(-1-a))*x*c^2+1/3/d^3*ln(1-b*
f^(d*x)*f^c/(-1-a))*c^3-1/2/ln(f)/d*polylog(2,b*f^(d*x)*f^c/(-1-a))*x^2+1/
2/ln(f)/d^3*polylog(2,b*f^(d*x)*f^c/(-1-a))*c^2+1/ln(f)^2/d^2*polylog(3,b*
f^(d*x)*f^c/(-1-a))*x-1/ln(f)^3/d^3*polylog(4,b*f^(d*x)*f^c/(-1-a))+1/6/d^
3*c^3*ln(1+a+f^(d*x)*f^c*b)-1/2/ln(f)/d^3*c^2*dilog((1+a+f^(d*x)*f^c*b)/(1
+a))-1/2/d^2*c^2*ln((1+a+f^(d*x)*f^c*b)/(1+a))*x-1/2/d^3*c^3*ln((1+a+f^(d*
x)*f^c*b)/(1+a))+1/6*ln(1-b*f^(d*x)*f^c/(1-a))*x^3-1/2/d^2*ln(1-b*f^(d*x)*
f^c/(1-a))*x*c^2-1/3/d^3*ln(1-b*f^(d*x)*f^c/(1-a))*c^3+1/2/ln(f)/d*polylog
(2,b*f^(d*x)*f^c/(1-a))*x^2-1/2/ln(f)/d^3*polylog(2,b*f^(d*x)*f^c/(1-a))*c
^2-1/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^c/(1-a))*x+1/ln(f)^3/d^3*polylog(4,
b*f^(d*x)*f^c/(1-a))-1/6/d^3*c^3*ln(f^(d*x)*f^c*b+a-1)+1/2/ln(f)/d^3*c^2*d
ilog((f^(d*x)*f^c*b+a-1)/(-1+a))+1/2/d^2*c^2*ln((f^(d*x)*f^c*b+a-1)/(-1+a)
)*x+1/2/d^3*c^3*ln((f^(d*x)*f^c*b+a-1)/(-1+a))
```

3.291.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.78

$$\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$$
$$=\frac{d^3x^3\log(f)^3\log\left(\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1}{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1}\right)-3\,d^2x^2\mathrm{Li}_2\left(-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))\cdot}{a+1}\right.}{}$$

input
```
integrate(x^2*arccoth(a+b*f^(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(d^3*x^3*log(f)^3*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log
(f)) + a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1
)) - 3*d^2*x^2*dilog(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f))
 + a + 1)/(a + 1) + 1)*log(f)^2 + 3*d^2*x^2*dilog(-(b*cosh((d*x + c)*log(f
)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1)*log(f)^2 + c^3*log(b*c
osh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log(f)^3 - c^3*l
og(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)*log(f)^3 -
 (d^3*x^3 + c^3)*log(f)^3*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)
*log(f)) + a + 1)/(a + 1)) + (d^3*x^3 + c^3)*log(f)^3*log((b*cosh((d*x + c
)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1)) + 6*d*x*log(f)*poly
log(3, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a + 1)) - 6
*d*x*log(f)*polylog(3, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f
)))/(a - 1)) - 6*polylog(4, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*
log(f)))/(a + 1)) + 6*polylog(4, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x
+ c)*log(f)))/(a - 1)))/(d^3*log(f)^3)
```

3.291.6 Sympy [F]

$$\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x^2\operatorname{acoth}\left(a+bf^{c+dx}\right)dx$$

input
```
integrate(x**2*acoth(a+b*f**(d*x+c)),x)
```

output
```
Integral(x**2*acoth(a + b*f**(c + d*x)), x)
```

3.291.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.94

$$\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right)\,dx=\frac{1}{3}\,x^3\operatorname{arcoth}\left(bf^{dx+c}+a\right)$$
$$-\frac{1}{6}\,bd\left(\frac{d^3x^3\log\left(\frac{bf^{dx}f^c}{a+1}+1\right)\log\left(f\right)^3+3\,d^2x^2\mathrm{Li}_2\left(-\frac{bf^{dx}f^c}{a+1}\right)\log\left(f\right)^2-6\,dx\log\left(f\right)\mathrm{Li}_3\left(-\frac{bf^{dx}f^c}{a+1}\right)+6\,\mathrm{Li}_4\left(-\right.}{bd^4\log\left(f\right)^4}\right.$$

input
```
integrate(x^2*arccoth(a+b*f^(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccoth(b*f^(d*x + c) + a) - 1/6*b*d*((d^3*x^3*log(b*f^(d*x)*f^c/(
a + 1) + 1)*log(f)^3 + 3*d^2*x^2*dilog(-b*f^(d*x)*f^c/(a + 1))*log(f)^2 -
6*d*x*log(f)*polylog(3, -b*f^(d*x)*f^c/(a + 1)) + 6*polylog(4, -b*f^(d*x)*
f^c/(a + 1)))/(b*d^4*log(f)^4) - (d^3*x^3*log(b*f^(d*x)*f^c/(a - 1) + 1)*l
og(f)^3 + 3*d^2*x^2*dilog(-b*f^(d*x)*f^c/(a - 1))*log(f)^2 - 6*d*x*log(f)*
polylog(3, -b*f^(d*x)*f^c/(a - 1)) + 6*polylog(4, -b*f^(d*x)*f^c/(a - 1)))
/(b*d^4*log(f)^4))*log(f)
```

3.291.8 Giac [F]

$$\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x^2\operatorname{arcoth}\left(bf^{dx+c}+a\right)\,dx$$

input
```
integrate(x^2*arccoth(a+b*f^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^2*arccoth(b*f^(d*x + c) + a), x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right)\,dx=\int x^2\operatorname{acoth}\left(a+b\,f^{c+d\,x}\right)\,dx$$

input
```
int(x^2*acoth(a + b*f^(c + d*x)),x)
```

output
```
int(x^2*acoth(a + b*f^(c + d*x)), x)
```

3.292 $\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx$

3.292.1 Optimal result

Integrand size = 20, antiderivative size = 17

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx=-\frac{\log\left(1-2\coth^{-1}(x)\right)}{2ab}$$

output
```
-1/2*ln(1-2*arccoth(x))/a/b
```

3.292.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx=-\frac{\log\left(-1+2\coth^{-1}(x)\right)}{2ab}$$

input
```
Integrate[1/((a - a*x^2)*(b - 2*b*ArcCoth[x])),x]
```

output
```
-1/2*Log[-1 + 2*ArcCoth[x]]/(a*b)
```

3.292.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {6509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}dx$$

$$\Big\downarrow \; 6509$$

$$-\frac{\log\left(1-2\coth^{-1}(x)\right)}{2ab}$$

input
```
Int[1/((a - a*x^2)*(b - 2*b*ArcCoth[x])),x]
```

output
```
-1/2*Log[1 - 2*ArcCoth[x]]/(a*b)
```

3.292.3.1 Defintions of rubi rules used

rule 6509
```
Int[1/(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol
] :> Simp[Log[RemoveContent[a + b*ArcCoth[c*x], x]]/(b*c*d), x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

3.292.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
parallelrisch	$-\frac{\ln\left(\operatorname{arccoth}(x)-\frac{1}{2}\right)}{2ab}$	14
default	$-\frac{\ln\left(2b\ \operatorname{arccoth}(x)-b\right)}{2ab}$	19
risch	$-\frac{\ln\left(1-\ln(1+x)+\ln(x-1)\right)}{2ab}$	22

input
```
int(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(arccoth(x)-1/2)/a/b
```

3.292.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx = -\frac{\log\left(\log\left(\frac{x+1}{x-1}\right)-1\right)}{2\,ab}$$

input
```
integrate(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x, algorithm="fricas")
```

output
```
-1/2*log(log((x + 1)/(x - 1)) - 1)/(a*b)
```

3.292.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx = -\frac{\log\left(\operatorname{acoth}(x)-\frac{1}{2}\right)}{2ab}$$

input
```
integrate(1/(-a*x**2+a)/(b-2*b*acoth(x)),x)
```

output
```
-log(acoth(x) - 1/2)/(2*a*b)
```

3.292.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx = -\frac{\log\left(\log(x+1)-\log(x-1)-1\right)}{2\,ab}$$

input
```
integrate(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x, algorithm="maxima")
```

output
```
-1/2*log(log(x + 1) - log(x - 1) - 1)/(a*b)
```

3.292.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(15) = 30.

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.59

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx = -\frac{\log\left(\frac{1}{4}\,\pi^2(\mathrm{sgn}(x+1)\,\mathrm{sgn}(x-1)-1)^2+\left(\log\left(\frac{|x+1|}{|x-1|}\right)-1\right)^2\right)}{4\,ab}$$

input
```
integrate(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x, algorithm="giac")
```

output
```
-1/4*log(1/4*pi^2*(sgn(x + 1)*sgn(x - 1) - 1)^2 + (log(abs(x + 1)/abs(x -
1)) - 1)^2)/(a*b)
```

3.292.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx = -\frac{\ln\left(2\,\mathrm{acoth}(x)-1\right)}{2\,a\,b}$$

input
```
int(1/((a - a*x^2)*(b - 2*b*acoth(x))),x)
```

output
```
-log(2*acoth(x) - 1)/(2*a*b)
```

3.293 $\int x^3 \coth^{-1}(a+bx^4)\,dx$

3.293.1 Optimal result

Integrand size = 12, antiderivative size = 44

$$\int x^3 \coth^{-1}(a+bx^4)\,dx = \frac{(a+bx^4)\coth^{-1}(a+bx^4)}{4b} + \frac{\log\left(1-(a+bx^4)^2\right)}{8b}$$

output
```
1/4*(b*x^4+a)*arccoth(b*x^4+a)/b+1/8*ln(1-(b*x^4+a)^2)/b
```

3.293.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int x^3 \coth^{-1}(a+bx^4)\,dx = \frac{2(a+bx^4)\coth^{-1}(a+bx^4)+\log\left(1-(a+bx^4)^2\right)}{8b}$$

input
```
Integrate[x^3*ArcCoth[a + b*x^4],x]
```

output
```
(2*(a + b*x^4)*ArcCoth[a + b*x^4] + Log[1 - (a + b*x^4)^2])/(8*b)
```

3.293.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7266, 6654, 6437, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^{-1}\left(a+bx^4\right) dx$$

$$\Big\downarrow \text{ 7266}$$

$$\frac{1}{4}\int \coth^{-1}\left(bx^4+a\right) dx^4$$

$$\Big\downarrow \text{ 6654}$$

$$\frac{\int \coth^{-1}\left(bx^4+a\right) d\left(bx^4+a\right)}{4b}$$

$$\Big\downarrow \text{ 6437}$$

$$\frac{\left(a+bx^4\right)\coth^{-1}\left(a+bx^4\right)-\int \frac{bx^4+a}{1-x^8}d\left(bx^4+a\right)}{4b}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{\left(a+bx^4\right)\coth^{-1}\left(a+bx^4\right)+\frac{1}{2}\log\left(1-x^8\right)}{4b}$$

input
```
Int[x^3*ArcCoth[a + b*x^4],x]
```

output
```
((a + b*x^4)*ArcCoth[a + b*x^4] + Log[1 - x^8]/2)/(4*b)
```

3.293.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6654
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
}, x] && IGtQ[p, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.293.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{(b\,x^4+a)\operatorname{arccoth}(b\,x^4+a)+\frac{\ln\left((b\,x^4+a)^2-1\right)}{2}}{4b}$
default	$\frac{(b\,x^4+a)\operatorname{arccoth}(b\,x^4+a)+\frac{\ln\left((b\,x^4+a)^2-1\right)}{2}}{4b}$
parts	$\frac{x^4\operatorname{arccoth}(b\,x^4+a)}{4}+b\left(\frac{(1-a)\ln(b\,x^4+a-1)}{8b^2}+\frac{(1+a)\ln(b\,x^4+a+1)}{8b^2}\right)$
parallelrisch	$-\frac{-\operatorname{arccoth}(b\,x^4+a)x^4b^2-\operatorname{arccoth}(b\,x^4+a)ab-\ln(b\,x^4+a-1)b-b\operatorname{arccoth}(b\,x^4+a)}{4b^2}$
risch	$\frac{x^4\ln(b\,x^4+a+1)}{8}-\frac{x^4\ln(b\,x^4+a-1)}{8}+\frac{\ln(b\,x^4+a+1)a}{8b}-\frac{\ln(-b\,x^4-a+1)a}{8b}+\frac{\ln(b\,x^4+a+1)}{8b}+\frac{\ln(-b\,x^4-a+1}{8b}$

input
```
int(x^3*arccoth(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*((b*x^4+a)*arccoth(b*x^4+a)+1/2*ln((b*x^4+a)^2-1))
```

3.293.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int x^3 \coth^{-1}(a+bx^4)\,dx = \frac{bx^4 \log\left(\frac{bx^4+a+1}{bx^4+a-1}\right) + (a+1)\log(bx^4+a+1) - (a-1)\log(bx^4+a-1)}{8\,b}$$

input
```
integrate(x^3*arccoth(b*x^4+a),x, algorithm="fricas")
```

output
```
1/8*(b*x^4*log((b*x^4 + a + 1)/(b*x^4 + a - 1)) + (a + 1)*log(b*x^4 + a +
1) - (a - 1)*log(b*x^4 + a - 1))/b
```

3.293.6 Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

$$\int x^3 \coth^{-1}(a+bx^4)\,dx = \begin{cases} \frac{a\,\mathrm{acoth}(a+bx^4)}{4b} + \frac{x^4\,\mathrm{acoth}(a+bx^4)}{4} + \frac{\log(a+bx^4+1)}{4b} - \frac{\mathrm{acoth}(a+bx^4)}{4b} & \text{for } b \neq 0 \\ \frac{x^4\,\mathrm{acoth}(a)}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acoth(b*x**4+a),x)
```

output
```
Piecewise((a*acoth(a + b*x**4)/(4*b) + x**4*acoth(a + b*x**4)/4 + log(a +
b*x**4 + 1)/(4*b) - acoth(a + b*x**4)/(4*b), Ne(b, 0)), (x**4*acoth(a)/4,
True))
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int x^3 \coth^{-1}\left(a+bx^4\right)\,dx = \frac{2\left(bx^4+a\right)\operatorname{arcoth}\left(bx^4+a\right)+\log\left(-\left(bx^4+a\right)^2+1\right)}{8\,b}$$

input
```
integrate(x^3*arccoth(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*(2*(b*x^4 + a)*arccoth(b*x^4 + a) + log(-(b*x^4 + a)^2 + 1))/b
```

3.293.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(40) = 80.

Time = 0.28 (sec) , antiderivative size = 225, normalized size of antiderivative = 5.11

$$\int x^3 \coth^{-1}\left(a+bx^4\right)\,dx$$

$$=\frac{1}{8}\left((a+1)b-(a-1)b\right)\left(\frac{\log\left(\frac{|bx^4+a+1|}{|bx^4+a-1|}\right)}{b^2}-\frac{\log\left(\left|\frac{bx^4+a+1}{bx^4+a-1}-1\right|\right)}{b^2}+\frac{\log\left(-\frac{\frac{1}{a-\frac{\left(\frac{\left(bx^4+a+1\right)(a-1)}{bx^4+a-1}-a-1\right)b}{\frac{\left(bx^4+a+1\right)b}{bx^4+a-1}-b}}+1}{\frac{1}{a-\frac{\left(\frac{\left(bx^4+a+1\right)(a-1)}{bx^4+a-1}-a-1\right)b}{\frac{\left(bx^4+a+1\right)b}{bx^4+a-1}-b}}-1}\right)}{b^2\left(\frac{bx^4+a+1}{bx^4+a-1}-1\right)}\right)$$

input
```
integrate(x^3*arccoth(b*x^4+a),x, algorithm="giac")
```

output
```
1/8*((a + 1)*b - (a - 1)*b)*(log(abs(b*x^4 + a + 1)/abs(b*x^4 + a - 1))/b^
2 - log(abs((b*x^4 + a + 1)/(b*x^4 + a - 1) - 1))/b^2 + log(-(1/(a - ((b*x
^4 + a + 1)*(a - 1)/(b*x^4 + a - 1) - a - 1)*b/((b*x^4 + a + 1)*b/(b*x^4 +
 a - 1) - b)) + 1)/(1/(a - ((b*x^4 + a + 1)*(a - 1)/(b*x^4 + a - 1) - a -
1)*b/((b*x^4 + a + 1)*b/(b*x^4 + a - 1) - b)) - 1))/(b^2*((b*x^4 + a + 1)/
(b*x^4 + a - 1) - 1)))
```

3.293.9 Mupad [B] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.43

$$\int x^3 \coth^{-1}\left(a+bx^4\right)\,dx=\frac{x^4\,\ln\left(\frac{b\,x^4+a+1}{b\,x^4+a}\right)}{8}-\frac{x^4\,\ln\left(\frac{b\,x^4+a-1}{b\,x^4+a}\right)}{8}+\frac{\ln\left(b\,x^4+a-1\right)}{8\,b}+\frac{\ln\left(b\,x^4+a+1\right)}{8\,b}-\frac{a\,\ln\left(b\,x^4+a-1\right)}{8\,b}+\frac{a\,\ln\left(b\,x^4+a+1\right)}{8\,b}$$

input
```
int(x^3*acoth(a + b*x^4),x)
```

output
```
(x^4*log((a + b*x^4 + 1)/(a + b*x^4)))/8 - (x^4*log((a + b*x^4 - 1)/(a + b
*x^4)))/8 + log(a + b*x^4 - 1)/(8*b) + log(a + b*x^4 + 1)/(8*b) - (a*log(a
 + b*x^4 - 1))/(8*b) + (a*log(a + b*x^4 + 1))/(8*b)
```

3.294 $\int x^{-1+n} \coth^{-1}(a+bx^n)\,dx$

3.294.1 Optimal result

Integrand size = 14, antiderivative size = 47

$$\int x^{-1+n} \coth^{-1}(a+bx^n)\,dx = \frac{(a+bx^n)\coth^{-1}(a+bx^n)}{bn} + \frac{\log\left(1-(a+bx^n)^2\right)}{2bn}$$

output
```
(a+b*x^n)*arccoth(a+b*x^n)/b/n+1/2*ln(1-(a+b*x^n)^2)/b/n
```

3.294.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int x^{-1+n} \coth^{-1}(a+bx^n)\,dx = \frac{2(a+bx^n)\coth^{-1}(a+bx^n) + \log\left(1-(a+bx^n)^2\right)}{2bn}$$

input
```
Integrate[x^(-1 + n)*ArcCoth[a + b*x^n],x]
```

output
```
(2*(a + b*x^n)*ArcCoth[a + b*x^n] + Log[1 - (a + b*x^n)^2])/(2*b*n)
```

3.294.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7266, 6654, 6437, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}\coth^{-1}(a+bx^n)\,dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{\int \coth^{-1}(bx^n+a)\,dx^n}{n}$$

$$\Big\downarrow \text{6654}$$

$$\frac{\int \coth^{-1}(bx^n+a)\,d(bx^n+a)}{bn}$$

$$\Big\downarrow \text{6437}$$

$$\frac{(a+bx^n)\coth^{-1}(a+bx^n)-\int \frac{bx^n+a}{1-x^{2n}}d(bx^n+a)}{bn}$$

$$\Big\downarrow \text{240}$$

$$\frac{(a+bx^n)\coth^{-1}(a+bx^n)+\frac{1}{2}\log\left(1-x^{2n}\right)}{bn}$$

input
```
Int[x^(-1 + n)*ArcCoth[a + b*x^n],x]
```

output
```
((a + b*x^n)*ArcCoth[a + b*x^n] + Log[1 - x^(2*n)]/2)/(b*n)
```

3.294.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6654
```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
}, x] && IGtQ[p, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.294.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(45) = 90.

Time = 4.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.51

method	result	size
risch	$\frac{x^{n} \ln(a+b\,x^{n}+1)}{2n}-\frac{x^{n} \ln(-1+a+b\,x^{n})}{2n}+\frac{\ln(x^{n}+\frac{1+a}{b})a}{2bn}-\frac{\ln(x^{n}+\frac{-1+a}{b})a}{2bn}+\frac{\ln(x^{n}+\frac{1+a}{b})}{2bn}+\frac{\ln(x^{n}+\frac{-1+a}{b})}{2bn}$	118

input
```
int(x^(-1+n)*arccoth(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/2/n*x^n*ln(a+b*x^n+1)-1/2/n*x^n*ln(-1+a+b*x^n)+1/2/b/n*ln(x^n+(1+a)/b)*a
-1/2/b/n*ln(x^n+(-1+a)/b)*a+1/2/b/n*ln(x^n+(1+a)/b)+1/2/b/n*ln(x^n+(-1+a)/
b)
```

3.294.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(45) = 90.

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.30

$$\int x^{-1+n}\coth^{-1}(a+bx^n)\,dx$$
$$=\frac{(a+1)\log\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a+1\right)-(a-1)\log\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right.\right.\right.}{2\,bn}$$

input
```
integrate(x^(-1+n)*arccoth(a+b*x^n),x, algorithm="fricas")
```

output
```
1/2*((a + 1)*log(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a + 1) - (a - 1)*lo
g(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a - 1) + (b*cosh(n*log(x)) + b*sin
h(n*log(x)))*log((b*cosh(n*log(x)) + b*sinh(n*log(x)) + a + 1)/(b*cosh(n*l
og(x)) + b*sinh(n*log(x)) + a - 1)))/(b*n)
```

3.294.6 Sympy [F(-2)]

Exception generated.

$$\int x^{-1+n}\coth^{-1}(a+bx^n)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1+n)*acoth(a+b*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.294.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int x^{-1+n}\coth^{-1}(a+bx^n)\,dx = \frac{2\,(bx^n+a)\,\mathrm{arcoth}\,(bx^n+a)+\log\left(-(bx^n+a)^2+1\right)}{2\,bn}$$

input
```
integrate(x^(-1+n)*arccoth(a+b*x^n),x, algorithm="maxima")
```

output
```
1/2*(2*(b*x^n + a)*arccoth(b*x^n + a) + log(-(b*x^n + a)^2 + 1))/(b*n)
```

3.294.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(45) = 90.

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.53

$$\int x^{-1+n}\coth^{-1}(a+bx^n)\,dx = \frac{((a+1)b-(a-1)b)\left(\frac{\log\left(\frac{|bx^n+a+1|}{|bx^n+a-1|}\right)}{b^2}-\frac{\log\left(\left|\frac{bx^n+a+1}{bx^n+a-1}-1\right|\right)}{b^2}+\frac{\log\left(\frac{bx^n+a+1}{bx^n+a-1}\right)}{b^2\left(\frac{bx^n+a+1}{bx^n+a-1}-1\right)}\right)}{2\,n}$$

input
```
integrate(x^(-1+n)*arccoth(a+b*x^n),x, algorithm="giac")
```

output
```
1/2*((a + 1)*b - (a - 1)*b)*(log(abs(b*x^n + a + 1)/abs(b*x^n + a - 1))/b^
2 - log(abs((b*x^n + a + 1)/(b*x^n + a - 1) - 1))/b^2 + log((b*x^n + a + 1
)/(b*x^n + a - 1))/(b^2*((b*x^n + a + 1)/(b*x^n + a - 1) - 1)))/n
```

3.294.9 Mupad [B] (verification not implemented)

Time = 5.53 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int x^{-1+n}\coth^{-1}(a+bx^n)\,dx=\frac{\frac{\ln\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n-1\right)}{2}+a\,\mathrm{acoth}(a+b\,x^n)}{b\,n}+\frac{x^n\,\mathrm{acoth}(a+b\,x^n)}{n}$$

input
```
int(x^(n - 1)*acoth(a + b*x^n),x)
```

output
```
(log(a^2 + b^2*x^(2*n) + 2*a*b*x^n - 1)/2 + a*acoth(a + b*x^n))/(b*n) + (x
^n*acoth(a + b*x^n))/n
```

3.295 $\int e^{c(a+bx)} \coth^{-1}(\sinh(ac+bcx))\,dx$

3.295.1 Optimal result

Integrand size = 20, antiderivative size = 107

$$\int e^{c(a+bx)} \coth^{-1}(\sinh(ac+bcx))\,dx = \frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc} + \frac{(1-\sqrt{2})\log\left(3-2\sqrt{2}-e^{2c(a+bx)}\right)}{2bc} + \frac{(1+\sqrt{2})\log\left(3+2\sqrt{2}-e^{2c(a+bx)}\right)}{2bc}$$

output
```
exp(b*c*x+a*c)*arccoth(sinh(c*(b*x+a)))/b/c+1/2*ln(3-exp(2*c*(b*x+a))-2*2^
(1/2))*(1-2^(1/2))/b/c+1/2*ln(3-exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/
c
```

3.295.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.43

$$\int e^{c(a+bx)} \coth^{-1}(\sinh(ac+bcx))\,dx = \frac{-2e^{c(a+bx)}\coth^{-1}\left(\frac{1}{2}e^{-c(a+bx)}-\frac{1}{2}e^{c(a+bx)}\right)-2\sqrt{2}\text{arctanh}\left(\frac{-1+e^{c(a+bx)}}{\sqrt{2}}\right)+2\sqrt{2}\text{arctanh}\left(\frac{1+e^{c(a+bx)}}{\sqrt{2}}\right)+\log(1}{2bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCoth[Sinh[a*c + b*c*x]],x]
```

output
```
(-2*E^(c*(a + b*x))*ArcCoth[1/(2*E^(c*(a + b*x))) - E^(c*(a + b*x))/2] - 2
*Sqrt[2]*ArcTanh[(-1 + E^(c*(a + b*x)))/Sqrt[2]] + 2*Sqrt[2]*ArcTanh[(1 +
E^(c*(a + b*x)))/Sqrt[2]] + Log[1 - 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))]
 + Log[1 + 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))])/(2*b*c)
```

3.295.3 Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {7281, 6830, 2720, 27, 1576, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \coth^{-1}(\sinh(ac+bcx))\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc}\coth^{-1}(\sinh(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{6830}$$

$$\frac{e^{ac+bcx}\coth^{-1}(\sinh(ac+bcx))-\int \frac{e^{ac+bxc}\cosh(ac+bxc)}{1-\sinh^2(ac+bxc)}d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{2720}$$

$$\frac{e^{ac+bcx}\coth^{-1}(\sinh(ac+bcx))-\int -\frac{2e^{ac+bxc}\left(1+e^{2ac+2bxc}\right)}{1-6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{e^{ac+bxc}\left(1+e^{2ac+2bxc}\right)}{1-6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc}+e^{ac+bcx}\coth^{-1}(\sinh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{1576}$$

$$\frac{\int \frac{1+e^{2ac+2bxc}}{1-5e^{2ac+2bxc}}de^{2ac+2bxc}+e^{ac+bcx}\coth^{-1}(\sinh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{1141}$$

$$\frac{\int \left(-\frac{1+\sqrt{2}}{2\left(-ac-bxc+2\sqrt{2}+3\right)}-\frac{1-\sqrt{2}}{2\left(-ac-bxc-2\sqrt{2}+3\right)}\right)de^{2ac+2bxc}+e^{ac+bcx}\coth^{-1}(\sinh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}(1-\sqrt{2})\log\left(-ac-bcx-2\sqrt{2}+3\right)+\frac{1}{2}(1+\sqrt{2})\log\left(-ac-bcx+2\sqrt{2}+3\right)+e^{ac+bcx}\coth^{-1}(\sinh(ac+bcx))}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCoth[Sinh[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcCoth[Sinh[a*c + b*c*x]] + ((1 - Sqrt[2])*Log[3 - 2*Sqr
t[2] - a*c - b*c*x])/2 + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2] - a*c - b*c*x])/
2)/(b*c)
```

3.295.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.295. $\int e^{c(a+bx)}\coth^{-1}(\sinh(ac+bcx))\,dx$

rule 6830
```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcCoth[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.295.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.45 (sec) , antiderivative size = 794, normalized size of antiderivative = 7.42

method	result	size
risch	Expression too large to display	794

input
```
int(exp(c*(b*x+a))*arccoth(sinh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)+1/4*I/b/c*P
i*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c
*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+2*exp
(c*(b*x+a))-1))*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b
*x+a))+2*exp(c*(b*x+a))-1))*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x
+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^2*exp
(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*cs
gn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b
*x+a))+1))*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*
x+a))+2*exp(c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(2*c*(
b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*ex
p(c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(-ex
p(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(
-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b
*x+a))+2*exp(c*(b*x+a))+1))^2*exp(c*(b*x+a))-1/2/b/c*exp(c*(b*x+a))*ln(exp
(2*c*(b*x+a))-2*exp(c*(b*x+a))-1)+1/2/b/c*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^
2)*2^(1/2)-1/2/b/c*ln(exp(2*c*(b*x+a))-(2^(1/2)-1)^2)*2^(1/2)-2*a/b+1/2/b/
c*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^2)+1/2/b/c*ln(exp(2*c*(b*x+a))-(2^(1/2)-
1)^2)
```

3.295.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(90) = 180$.

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.18

$$\int e^{c(a+bx)} \coth^{-1}(\sinh(ac+bcx))\,dx$$
$$= \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(\frac{\sinh(bcx+ac)+1}{\sinh(bcx+ac)-1}\right)+\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh(bcx+ac)^2-4\left(3\sqrt{2}+4\right)\cosh(bcx+a}{\cosh(bcx+ac)^2+}\right.}{2bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(sinh(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log((sinh(b*c*x + a*c) + 1)/(
sinh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c)
^2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2)
+ 3)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*
x + a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/(
cosh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x +
 a*c)^2)))/(b*c)
```

3.295.6 Sympy [F]

$$\int e^{c(a+bx)} \coth^{-1}(\sinh(ac+bcx))\,dx = e^{ac}\int e^{bcx}\,\mathrm{acoth}\,(\sinh(ac+bcx))\,dx$$

input
```
integrate(exp(c*(b*x+a))*acoth(sinh(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*acoth(sinh(a*c + b*c*x)), x)
```

3.295.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(90) = 180.

Time = 0.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.72

$$\int e^{c(a+bx)}\coth^{-1}(\sinh(ac+bcx))\,dx = \frac{\operatorname{arcoth}\left(\sinh\left(bcx+ac\right)\right)e^{((bx+a)c)}}{bc} + \frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}+1}{\sqrt{2}+e^{(bcx+ac)}-1}\right)}{2\,bc} - \frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}-1}{\sqrt{2}+e^{(bcx+ac)}+1}\right)}{2\,bc} + \frac{\log\left(e^{(2\,bcx+2\,ac)}+2\,e^{(bcx+ac)}-1\right)}{2\,bc} + \frac{\log\left(e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}-1\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(sinh(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arccoth(sinh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(sqrt(
2) - e^(b*c*x + a*c) + 1)/(sqrt(2) + e^(b*c*x + a*c) - 1))/(b*c) - 1/2*sqr
t(2)*log(-(sqrt(2) - e^(b*c*x + a*c) - 1)/(sqrt(2) + e^(b*c*x + a*c) + 1))
/(b*c) + 1/2*log(e^(2*b*c*x + 2*a*c) + 2*e^(b*c*x + a*c) - 1)/(b*c) + 1/2*
log(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) - 1)/(b*c)
```

3.295.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.56

$$\int e^{c(a+bx)}\coth^{-1}(\sinh(ac+bcx))\,dx = \frac{e^{((bx+a)c)}\log\left(-\frac{\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}+1}{\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}-1}\right)}{2\,bc} + \frac{\sqrt{2}\log\left(\frac{\left|-4\,\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}{\left|4\,\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}\right)+\log\left(\left|e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right|\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(sinh(b*c*x+a*c)),x, algorithm="giac")
```

output
```
1/2*e^((b*x + a)*c)*log(-(2/(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 1)/(2/(
e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 1))/(b*c) + 1/2*(sqrt(2)*log(abs(-4*
sqrt(2) + 2*e^(2*b*c*x + 2*a*c) - 6)/abs(4*sqrt(2) + 2*e^(2*b*c*x + 2*a*c)
 - 6)) + log(abs(e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)))/(b*c)
```

3.295.9 Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.75

$$\int e^{c(a+bx)}\coth^{-1}(\sinh(ac+bcx))\,dx = \frac{\ln\left(6\sqrt{2}\,\mathrm{e}^{2\,c\,(a+b\,x)}-2\sqrt{2}-8\,\mathrm{e}^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}+1\right)}{2\,b\,c} - \frac{\mathrm{e}^{a\,c+b\,c\,x}\,\ln\left(1-\frac{1}{\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}-\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}\right)}{2\,b\,c} - \frac{\ln\left(2\sqrt{2}-8\,\mathrm{e}^{2\,c\,(a+b\,x)}-6\sqrt{2}\,\mathrm{e}^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}-1\right)}{2\,b\,c} + \frac{\ln\left(\frac{1}{\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}-\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}+1\right)\,\mathrm{e}^{a\,c+b\,c\,x}}{2\,b\,c}$$

input
```
int(exp(c*(a + b*x))*acoth(sinh(a*c + b*c*x)),x)
```

output
```
(log(6*2^(1/2)*exp(2*c*(a + b*x)) - 2*2^(1/2) - 8*exp(2*c*(a + b*x)))*(2^(
1/2) + 1))/(2*b*c) - (exp(a*c + b*c*x)*log(1 - 1/((exp(b*c*x)*exp(a*c))/2
- (exp(-b*c*x)*exp(-a*c))/2)))/(2*b*c) - (log(2*2^(1/2) - 8*exp(2*c*(a + b
*x)) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c) + (log(1/((exp
(b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2) + 1)*exp(a*c + b*c*x))/(2
*b*c)
```

3.296 $\int e^{c(a+bx)} \coth^{-1}(\cosh(ac+bcx))\,dx$

3.296.1 Optimal result

Integrand size = 20, antiderivative size = 49

$$\int e^{c(a+bx)} \coth^{-1}(\cosh(ac+bcx))\,dx = \frac{e^{ac+bcx}\coth^{-1}(\cosh(c(a+bx)))}{bc} + \frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

output
```
exp(b*c*x+a*c)*arccoth(cosh(c*(b*x+a)))/b/c+ln(1-exp(2*c*(b*x+a)))/b/c
```

3.296.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int e^{c(a+bx)} \coth^{-1}(\cosh(ac+bcx))\,dx = \frac{e^{c(a+bx)}\coth^{-1}\left(\frac{1}{2}e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)\right)+\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCoth[Cosh[a*c + b*c*x]],x]
```

output
```
(E^(c*(a + b*x))*ArcCoth[(1 + E^(2*c*(a + b*x)))/(2*E^(c*(a + b*x)))] + Lo
g[1 - E^(2*c*(a + b*x))])/(b*c)
```

3.296.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7281, 6830, 25, 2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \coth^{-1}(\cosh(ac+bcx)) \, dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc} \coth^{-1}(\cosh(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{6830}$$

$$\frac{e^{ac+bcx} \coth^{-1}(\cosh(ac+bcx)) - \int -e^{ac+bxc}\text{csch}(ac+bxc)d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int e^{ac+bxc}\text{csch}(ac+bxc)d(ac+bxc) + e^{ac+bcx} \coth^{-1}(\cosh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{2e^{ac+bxc}}{1-e^{2ac+2bxc}}de^{ac+bxc} + e^{ac+bcx} \coth^{-1}(\cosh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{ac+bcx} \coth^{-1}(\cosh(ac+bcx)) - 2\int \frac{e^{ac+bxc}}{1-e^{2ac+2bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \text{240}$$

$$\frac{\log\left(1-e^{2ac+2bcx}\right) + e^{ac+bcx} \coth^{-1}(\cosh(ac+bcx))}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCoth[Cosh[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcCoth[Cosh[a*c + b*c*x]] + Log[1 - E^(2*a*c + 2*b*c*x)]
)/(b*c)
```

3.296.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6830
```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcCoth[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.296.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 824, normalized size of antiderivative = 16.82

method	result	size
risch	Expression too large to display	824

input
```
int(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
1/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+1)+1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+
a)))*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-1
)^2)*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1))^2*csgn(I*(exp(
c*(b*x+a))+1)^2)*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)*
csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))-1/4*I/b/c*Pi
*csgn(I*exp(-c*(b*x+a)))*csgn(I*(exp(c*(b*x+a))+1)^2)*csgn(I*exp(-c*(b*x+a
))*(exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a)
)-1)^2)^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-
c*(b*x+a))*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*exp(
-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)^3*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(ex
p(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)
)^2*csgn(I*(exp(c*(b*x+a))-1)^2)*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c
*(b*x+a))*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*(exp(
c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))-1/2*I/b/c*Pi*
csgn(I*(exp(c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))-1
/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*
x+a))-1)^2)^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*e
xp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))-1/b/c*exp(c*(b*x+a))
*ln(exp(c*(b*x+a))-1)-2*a/b+1/b/c*ln(exp(2*c*(b*x+a))-1)
```

3.296.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.88

$$\int e^{c(a+bx)}\coth^{-1}(\cosh(ac+bcx))\,dx$$
$$=\frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(\frac{\cosh(bcx+ac)+1}{\cosh(bcx+ac)-1}\right)+2\log\left(\frac{2\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log((cosh(b*c*x + a*c) + 1)/(
cosh(b*c*x + a*c) - 1)) + 2*log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - s
inh(b*c*x + a*c))))/(b*c)
```

3.296.6 Sympy [F]

$$\int e^{c(a+bx)}\coth^{-1}(\cosh(ac+bcx))\,dx = e^{ac}\int e^{bcx}\operatorname{acoth}(\cosh(ac+bcx))\,dx$$

input
```
integrate(exp(c*(b*x+a))*acoth(cosh(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*acoth(cosh(a*c + b*c*x)), x)
```

3.296.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int e^{c(a+bx)}\coth^{-1}(\cosh(ac+bcx))\,dx = \frac{\operatorname{arcoth}(\cosh(bcx+ac))\,e^{((bx+a)c)}}{bc} + \frac{\log\left(e^{(bcx+ac)}+1\right)}{bc} + \frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arccoth(cosh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(b*c*x + a*c) + 1
)/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

3.296.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(47) = 94.

Time = 0.38 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.00

$$\int e^{c(a+bx)}\coth^{-1}(\cosh(ac+bcx))\,dx = \frac{e^{((bx+a)c)}\log\left(-\frac{\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}+1}{\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}-1}\right)}{2\,bc} + \frac{\log\left(\left|e^{(2\,bcx+2\,ac)}-1\right|\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x, algorithm="giac")
```

output
```
1/2*e^((b*x + a)*c)*log(-(2/(e^(b*c*x + a*c) + e^(-b*c*x - a*c)) + 1)/(2/(
e^(b*c*x + a*c) + e^(-b*c*x - a*c)) - 1))/(b*c) + log(abs(e^(2*b*c*x + 2*a
*c) - 1))/(b*c)
```

3.296.9 Mupad [B] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.43

$$\int e^{c(a+bx)} \coth^{-1}(\cosh(ac+bcx))\,dx = \frac{\ln\left(e^{2\,b\,c\,x}\,e^{2\,a\,c}-1\right)}{b\,c} - \frac{e^{a\,c+b\,c\,x}\,\ln\left(1-\frac{1}{\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}}\right)}{2\,b\,c} + \frac{\ln\left(\frac{1}{\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}}+1\right)\,e^{a\,c+b\,c\,x}}{2\,b\,c}$$

input
```
int(exp(c*(a + b*x))*acoth(cosh(a*c + b*c*x)),x)
```

output
```
log(exp(2*b*c*x)*exp(2*a*c) - 1)/(b*c) - (exp(a*c + b*c*x)*log(1 - 1/((exp
(b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2)))/(2*b*c) + (log(1/((exp(
b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2) + 1)*exp(a*c + b*c*x))/(2*
b*c)
```

3.297 $\int e^{c(a+bx)} \coth^{-1}(\tanh(ac+bcx)) \, dx$

3.297.1 Optimal result

Integrand size = 20, antiderivative size = 45

$$\int e^{c(a+bx)} \coth^{-1}(\tanh(ac+bcx)) \, dx = -\frac{e^{ac+bcx}}{bc} + \frac{e^{ac+bcx} \coth^{-1}(\tanh(c(a+bx)))}{bc}$$

output
```
-exp(b*c*x+a*c)/b/c+exp(b*c*x+a*c)*arccoth(tanh(c*(b*x+a)))/b/c
```

3.297.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int e^{c(a+bx)} \coth^{-1}(\tanh(ac+bcx)) \, dx = \frac{e^{c(a+bx)} \left(-1 + \coth^{-1}\left(\frac{-1+e^{2c(a+bx)}}{1+e^{2c(a+bx)}}\right)\right)}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCoth[Tanh[a*c + b*c*x]],x]
```

output
```
(E^(c*(a + b*x))*(-1 + ArcCoth[(-1 + E^(2*c*(a + b*x)))/(1 + E^(2*c*(a + b
*x)))]))/(b*c)
```

3.297.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7281, 6830, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \coth^{-1}(\tanh(ac+bcx))\,dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int e^{ac+bxc} \coth^{-1}(\tanh(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{ 6830}$$

$$\frac{e^{ac+bcx} \coth^{-1}(\tanh(ac+bcx)) - \int e^{ac+bxc}d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{e^{ac+bcx} \coth^{-1}(\tanh(ac+bcx)) - e^{ac+bcx}}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCoth[Tanh[a*c + b*c*x]],x]
```

output
```
(-E^(a*c + b*c*x) + E^(a*c + b*c*x)*ArcCoth[Tanh[a*c + b*c*x]])/(b*c)
```

3.297.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 6830
```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcCoth[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.297.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result
parallelrisch	$-\frac{-e^{c(bx+a)}\operatorname{arccoth}(\tanh(c(bx+a)))+e^{c(bx+a)}}{bc}$
risch	$\frac{e^{c(bx+a)}\ln\left(e^{c(bx+a)}\right)}{bc}-\frac{i\left(\pi\operatorname{csgn}\left(\frac{i}{e^{2c(bx+a)}+1}\right)\operatorname{csgn}\left(ie^{2c(bx+a)}\right)\operatorname{csgn}\left(\frac{ie^{2c(bx+a)}}{e^{2c(bx+a)}+1}\right)-\pi\operatorname{csgn}\left(\frac{i}{e^{2c(bx+a)}+1}\right)\operatorname{csgn}\left(\frac{ie^{2c}}{e^{2c(b}}\right.\right.}{}$

input
```
int(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
-(-exp(c*(b*x+a))*arccoth(tanh(c*(b*x+a)))+exp(c*(b*x+a)))/b/c
```

3.297.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int e^{c(a+bx)}\coth^{-1}(\tanh(ac+bcx))\,dx = \frac{(i\,\pi+2\,bcx+2\,ac-2)\cosh{(bcx+ac)}+(i\,\pi+2\,bcx+2\,ac-2)\sinh{(bcx+ac)}}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*((I*pi + 2*b*c*x + 2*a*c - 2)*cosh(b*c*x + a*c) + (I*pi + 2*b*c*x + 2*
a*c - 2)*sinh(b*c*x + a*c))/(b*c)
```

3.297.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int e^{c(a+bx)}\coth^{-1}(\tanh(ac+bcx))\,dx = \begin{cases} \frac{i\pi x}{2} & \text{for } b=0\wedge c=0 \\ xe^{ac}\operatorname{acoth}\left(\tanh\left(ac\right)\right) & \text{for } b=0 \\ \frac{i\pi x}{2} & \text{for } c=0 \\ \frac{e^{ac}e^{bcx}\operatorname{acoth}\left(\tanh\left(ac+bcx\right)\right)}{bc}-\frac{e^{ac}e^{bcx}}{bc} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(c*(b*x+a))*acoth(tanh(b*c*x+a*c)),x)
```

output
```
Piecewise((I*pi*x/2, Eq(b, 0) & Eq(c, 0)), (x*exp(a*c)*acoth(tanh(a*c)), E
q(b, 0)), (I*pi*x/2, Eq(c, 0)), (exp(a*c)*exp(b*c*x)*acoth(tanh(a*c + b*c*
x))/(b*c) - exp(a*c)*exp(b*c*x)/(b*c), True))
```

3.297.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int e^{c(a+bx)}\coth^{-1}(\tanh(ac+bcx))\,dx = \frac{\operatorname{arcoth}\left(\tanh\left(bcx+ac\right)\right)e^{((bx+a)c)}}{bc}-\frac{e^{(bcx+ac)}}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arccoth(tanh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - e^(b*c*x + a*c)/(b*c)
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int e^{c(a+bx)}\coth^{-1}(\tanh(ac+bcx))\,dx = \frac{\left(e^{(bcx)}\log\left(-e^{(2\,bcx+2\,ac)}\right)-2\,e^{(bcx)}\right)e^{(ac)}}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x, algorithm="giac")
```

output
```
1/2*(e^(b*c*x)*log(-e^(2*b*c*x + 2*a*c)) - 2*e^(b*c*x))*e^(a*c)/(b*c)
```

3.297.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.62

$$\int e^{c(a+bx)}\coth^{-1}(\tanh(ac+bcx))\,dx=\frac{\mathrm{e}^{a\,c+b\,c\,x}\,(\mathrm{acoth}(\tanh(a\,c+b\,c\,x))-1)}{b\,c}$$

input
```
int(exp(c*(a + b*x))*acoth(tanh(a*c + b*c*x)),x)
```

output
```
(exp(a*c + b*c*x)*(acoth(tanh(a*c + b*c*x)) - 1))/(b*c)
```

3.298 $\int e^{c(a+bx)} \coth^{-1}(\coth(ac+bcx))\,dx$

3.298.1 Optimal result

Integrand size = 20, antiderivative size = 45

$$\int e^{c(a+bx)} \coth^{-1}(\coth(ac+bcx))\,dx = -\frac{e^{ac+bcx}}{bc} + \frac{e^{ac+bcx}\coth^{-1}(\coth(c(a+bx)))}{bc}$$

output
```
-exp(b*c*x+a*c)/b/c+exp(b*c*x+a*c)*arccoth(coth(c*(b*x+a)))/b/c
```

3.298.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int e^{c(a+bx)} \coth^{-1}(\coth(ac+bcx))\,dx = \frac{e^{c(a+bx)}\left(-1+\coth^{-1}\left(\frac{1+e^{2c(a+bx)}}{-1+e^{2c(a+bx)}}\right)\right)}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCoth[Coth[a*c + b*c*x]],x]
```

output
```
(E^(c*(a + b*x))*(-1 + ArcCoth[(1 + E^(2*c*(a + b*x)))/(-1 + E^(2*c*(a + b
*x)))]))/(b*c)
```

3.298.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7281, 6830, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \coth^{-1}(\coth(ac+bcx)) \, dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int e^{ac+bxc} \coth^{-1}(\coth(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{ 6830}$$

$$\frac{e^{ac+bcx} \coth^{-1}(\coth(ac+bcx)) - \int e^{ac+bxc}d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{e^{ac+bcx} \coth^{-1}(\coth(ac+bcx)) - e^{ac+bcx}}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCoth[Coth[a*c + b*c*x]],x]
```

output
```
(-E^(a*c + b*c*x) + E^(a*c + b*c*x)*ArcCoth[Coth[a*c + b*c*x]])/(b*c)
```

3.298.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 6830
```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcCoth[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.298.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

method	result
parallelrisch	$\frac{e^{c(bx+a)}\left(\operatorname{arccoth}\left(\frac{1}{\tanh(c(bx+a))}\right)-1\right)}{bc}$
default	$\frac{e^{bcx+ac}(bcx+ac)-e^{bcx+ac}+e^{bcx+ac}(\operatorname{arccoth}(\coth(bcx+ac))-bcx-ac)}{bc}$
risch	$\frac{e^{c(bx+a)}\ln\left(e^{c(bx+a)}\right)}{bc}-\frac{i\left(\pi\,\mathrm{csgn}\left(ie^{c(bx+a)}\right)^{2}\mathrm{csgn}\left(ie^{2c(bx+a)}\right)-2\pi\,\mathrm{csgn}\left(ie^{c(bx+a)}\right)\mathrm{csgn}\left(ie^{2c(bx+a)}\right)^{2}+\pi\,\mathrm{csgn}\left(ie^{2c(bx+a)}\right)^{3}\right.}{}$

input
```
int(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
exp(c*(b*x+a))*(arccoth(1/tanh(c*(b*x+a)))-1)/b/c
```

3.298.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.56

$$\int e^{c(a+bx)}\coth^{-1}(\coth(ac+bcx))\,dx=\frac{(bcx+ac-1)e^{(bcx+ac)}}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
(b*c*x + a*c - 1)*e^(b*c*x + a*c)/(b*c)
```

3.298.6 Sympy [F]

$$\int e^{c(a+bx)}\coth^{-1}(\coth(ac+bcx))\,dx = e^{ac}\int e^{bcx}\operatorname{acoth}(\coth(ac+bcx))\,dx$$

input
```
integrate(exp(c*(b*x+a))*acoth(coth(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*acoth(coth(a*c + b*c*x)), x)
```

3.298.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int e^{c(a+bx)}\coth^{-1}(\coth(ac+bcx))\,dx = \frac{ae^{(bcx+ac)}}{b} + \frac{\left(bcxe^{(ac)} - e^{(ac)}\right)e^{(bcx)}}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
a*e^(b*c*x + a*c)/b + (b*c*x*e^(a*c) - e^(a*c))*e^(b*c*x)/(b*c)
```

3.298.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int e^{c(a+bx)}\coth^{-1}(\coth(ac+bcx))\,dx = \frac{(b^2c^2x + abc^2 - bc)e^{(bcx+ac)}}{b^2c^2}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x, algorithm="giac")
```

output
```
(b^2*c^2*x + a*b*c^2 - b*c)*e^(b*c*x + a*c)/(b^2*c^2)
```

3.298.9 Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.62

$$\int e^{c(a+bx)} \coth^{-1}(\coth(ac+bcx))\,dx = \frac{e^{a\,c+b\,c\,x}\,(\mathrm{acoth}(\coth(a\,c+b\,c\,x))-1)}{b\,c}$$

input
```
int(exp(c*(a + b*x))*acoth(coth(a*c + b*c*x)),x)
```

output
```
(exp(a*c + b*c*x)*(acoth(coth(a*c + b*c*x)) - 1))/(b*c)
```

3.299 $\int e^{c(a+bx)} \coth^{-1}(\mathbf{sech}(ac+bcx))\,dx$

3.299.1 Optimal result

Integrand size = 20, antiderivative size = 49

$$\int e^{c(a+bx)} \coth^{-1}(\operatorname{sech}(ac+bcx))\,dx=\frac{e^{ac+bcx}\coth^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

output
```
exp(b*c*x+a*c)*arccoth(sech(c*(b*x+a)))/b/c+ln(1-exp(2*c*(b*x+a)))/b/c
```

3.299.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20

$$\int e^{c(a+bx)} \coth^{-1}(\operatorname{sech}(ac+bcx))\,dx=\frac{e^{c(a+bx)}\coth^{-1}\left(\frac{2e^{c(a+bx)}}{1+e^{2c(a+bx)}}\right)+\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCoth[Sech[a*c + b*c*x]],x]
```

output
```
(E^(c*(a + b*x))*ArcCoth[(2*E^(c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))] + Lo
g[1 - E^(2*c*(a + b*x))])/(b*c)
```

3.299.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7281, 6830, 25, 2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\coth^{-1}(\operatorname{sech}(ac+bcx))\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc}\coth^{-1}(\operatorname{sech}(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{6830}$$

$$\frac{e^{ac+bcx}\coth^{-1}(\operatorname{sech}(ac+bcx))-\int -e^{ac+bxc}\operatorname{csch}(ac+bxc)d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int e^{ac+bxc}\operatorname{csch}(ac+bxc)d(ac+bxc)+e^{ac+bcx}\coth^{-1}(\operatorname{sech}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{2e^{ac+bxc}}{1-e^{2ac+2bxc}}de^{ac+bxc}+e^{ac+bcx}\coth^{-1}(\operatorname{sech}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{ac+bcx}\coth^{-1}(\operatorname{sech}(ac+bcx))-2\int \frac{e^{ac+bxc}}{1-e^{2ac+2bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \text{240}$$

$$\frac{\log\left(1-e^{2ac+2bcx}\right)+e^{ac+bcx}\coth^{-1}(\operatorname{sech}(ac+bcx))}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCoth[Sech[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcCoth[Sech[a*c + b*c*x]] + Log[1 - E^(2*a*c + 2*b*c*x)]
)/(b*c)
```

3.299.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6830
```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcCoth[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.299.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.35 (sec) , antiderivative size = 939, normalized size of antiderivative = 19.16

method	result	size
risch	Expression too large to display	939

input
```
int(exp(c*(b*x+a))*arccoth(sech(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1))^2*csgn(I*(exp(c*(b*x+a))-1)^2)*exp
(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a
))+1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*(exp(c*
(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(
exp(c*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))^3*exp(c*(b*x+a))-1/4*I/b/c*Pi*cs
gn(I/(exp(2*c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1
))^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)^3*exp(c*(b*x
+a))+1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2)^
2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a)
)+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)*csgn(I*(exp(c*(b*x+a))+1)^2/(e
xp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1
)^2)*csgn(I/(exp(2*c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2/(exp(2*c*(b*
x+a))+1))*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2/(exp(2*c
*(b*x+a))+1))^3*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)*c
sgn(I*(exp(c*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))-1/2*I/b/
c*Pi*csgn(I*(exp(c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+
a))-1/2*I/b/c*exp(c*(b*x+a))*Pi-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1))^2*
csgn(I*(exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*
x+a))-1)^2)*csgn(I/(exp(2*c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))-1)^2/(exp(
2*c*(b*x+a))+1))*exp(c*(b*x+a))-1/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-...
```

3.299.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.90

$$\int e^{c(a+bx)}\coth^{-1}(\operatorname{sech}(ac+bcx))\,dx = \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(-\frac{\cosh(bcx+ac)+1}{\cosh(bcx+ac)-1}\right)+2\log\left(\frac{2\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(sech(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log(-(cosh(b*c*x + a*c) + 1)/
(cosh(b*c*x + a*c) - 1)) + 2*log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) -
sinh(b*c*x + a*c))))/(b*c)
```

3.299.6 Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\coth^{-1}(\operatorname{sech}(ac+bcx))\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))*acoth(sech(b*c*x+a*c)),x)
```

output
```
Timed out
```

3.299.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int e^{c(a+bx)}\coth^{-1}(\operatorname{sech}(ac+bcx))\,dx = \frac{\operatorname{arcoth}\left(\operatorname{sech}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} + \frac{\log\left(e^{(bcx+ac)}+1\right)}{bc} + \frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(sech(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arccoth(sech(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(b*c*x + a*c) + 1
)/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

3.299.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(47) = 94.

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.00

$$\int e^{c(a+bx)}\coth^{-1}(\operatorname{sech}(ac+bcx))\,dx = \frac{\left(e^{(bcx)}\log\left(-\frac{e^{(2bcx+2ac)}}{e^{(2bcx+2ac)}-2e^{(bcx+ac)}+1} - \frac{2e^{(bcx+ac)}}{e^{(2bcx+2ac)}-2e^{(bcx+ac)}+1} - \frac{1}{e^{(2bcx+2ac)}-2e^{(bcx+ac)}+1}\right) + 2e^{(-ac)}\log\left(\left|e^{(2bcx-}\right.\right.}{2bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(sech(b*c*x+a*c)),x, algorithm="giac")
```

output
```
1/2*(e^(b*c*x)*log(-e^(2*b*c*x + 2*a*c)/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x
+ a*c) + 1) - 2*e^(b*c*x + a*c)/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) +
 1) - 1/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) + 1)) + 2*e^(-a*c)*log(ab
s(e^(2*b*c*x + 2*a*c) - 1)))*e^(a*c)/(b*c)
```

3.299.9 Mupad [B] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.27

$$\int e^{c(a+bx)}\coth^{-1}(\operatorname{sech}(ac+bcx))\,dx=\frac{\ln\left(\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}-1\right)}{b\,c}-\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,\ln\left(1-\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}-\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}\right)}{2\,b\,c}+\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,\ln\left(\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}+\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}+1\right)}{2\,b\,c}$$

input
```
int(acoth(1/cosh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

output
```
log(exp(2*b*c*x)*exp(2*a*c) - 1)/(b*c) - (exp(b*c*x)*exp(a*c)*log(1 - (exp
(-b*c*x)*exp(-a*c))/2 - (exp(b*c*x)*exp(a*c))/2))/(2*b*c) + (exp(b*c*x)*ex
p(a*c)*log((exp(b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2 + 1))/(2*b*
c)
```

3.300 $\int e^{c(a+bx)} \coth^{-1}(\mathbf{csch}(ac+bcx))\,dx$

3.300.1 Optimal result

Integrand size = 20, antiderivative size = 107

$$\int e^{c(a+bx)} \coth^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}-e^{2c(a+bx)}\right)}{2bc} + \frac{\left(1+\sqrt{2}\right)\log\left(3+2\sqrt{2}-e^{2c(a+bx)}\right)}{2bc}$$

output
```
exp(b*c*x+a*c)*arccoth(csch(c*(b*x+a)))/b/c+1/2*ln(3-exp(2*c*(b*x+a))-2*2^
(1/2))*(1-2^(1/2))/b/c+1/2*ln(3-exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/
c
```

3.300.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.40

$$\int e^{c(a+bx)} \coth^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{2e^{c(a+bx)}\coth^{-1}\left(\frac{2e^{c(a+bx)}}{-1+e^{2c(a+bx)}}\right) - 2\sqrt{2}\operatorname{arctanh}\left(\frac{-1+e^{c(a+bx)}}{\sqrt{2}}\right) + 2\sqrt{2}\operatorname{arctanh}\left(\frac{1+e^{c(a+bx)}}{\sqrt{2}}\right) + \log\left(1-2e^{c(a+bx)}\right.}{2bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcCoth[Csch[a*c + b*c*x]],x]
```

output
```
(2*E^(c*(a + b*x))*ArcCoth[(2*E^(c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))] -
 2*Sqrt[2]*ArcTanh[(-1 + E^(c*(a + b*x)))/Sqrt[2]] + 2*Sqrt[2]*ArcTanh[(1
+ E^(c*(a + b*x)))/Sqrt[2]] + Log[1 - 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x)
)] + Log[1 + 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))])/(2*b*c)
```

3.300.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {7281, 6830, 25, 2720, 27, 1576, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\coth^{-1}(\operatorname{csch}(ac+bcx))\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc}\coth^{-1}(\operatorname{csch}(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{6830}$$

$$\frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(ac+bcx))-\int -\frac{e^{ac+bxc}\coth(ac+bxc)\operatorname{csch}(ac+bxc)}{1-\operatorname{csch}^2(ac+bxc)}d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{e^{ac+bxc}\coth(ac+bxc)\operatorname{csch}(ac+bxc)}{1-\operatorname{csch}^2(ac+bxc)}d(ac+bxc)+e^{ac+bcx}\coth^{-1}(\operatorname{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{2e^{ac+bxc}\left(1+e^{2ac+2bxc}\right)}{1-6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc}+e^{ac+bcx}\coth^{-1}(\operatorname{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{e^{ac+bxc}\left(1+e^{2ac+2bxc}\right)}{1-6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc}+e^{ac+bcx}\coth^{-1}(\operatorname{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{1576}$$

$$\frac{\int \frac{1+e^{2ac+2bxc}}{1-5e^{2ac+2bxc}}de^{2ac+2bxc}+e^{ac+bcx}\coth^{-1}(\operatorname{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{1141}$$

$$\frac{\int \left(-\frac{1+\sqrt{2}}{2\left(-ac-bxc+2\sqrt{2}+3\right)}-\frac{1-\sqrt{2}}{2\left(-ac-bxc-2\sqrt{2}+3\right)}\right)de^{2ac+2bxc}+e^{ac+bcx}\coth^{-1}(\operatorname{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}(1-\sqrt{2})\log\left(-ac-bcx-2\sqrt{2}+3\right)+\frac{1}{2}(1+\sqrt{2})\log\left(-ac-bcx+2\sqrt{2}+3\right)+e^{ac+bcx}\coth^{-1}(\operatorname{csch}(ac+bcx))}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcCoth[Csch[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcCoth[Csch[a*c + b*c*x]] + ((1 - Sqrt[2])*Log[3 - 2*Sqr
t[2] - a*c - b*c*x])/2 + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2] - a*c - b*c*x])/
2)/(b*c)
```

3.300.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6830
```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Simp[(a + b*ArcCoth[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u,
x]/(1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x
] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; F
reeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.300.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.33 (sec) , antiderivative size = 920, normalized size of antiderivative = 8.60

method	result	size
risch	Expression too large to display	920

input
```
int(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)+1/4*I/b/c*P
i*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I*(-exp(2*c*(b*x+a))
+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*c
sgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I/(exp(2*c*(b*x+a))-1))
*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))*exp(c
*(b*x+a))-1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(-exp(2*c*(b*x+
a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4*I/b/c*P
i*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^3*ex
p(c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(e
xp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/2*I/b/c*exp(c*(b*x+a))*Pi+1/4*I/b/c
*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x
+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(2*c*(b
*x+a))+2*exp(c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I/(exp(2*c*(
b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*exp(c*(b*x+a))+1/4*I/b/c
*Pi*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))
-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*c
sgn(I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^3*exp(c*
(b*x+a))-1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-2*exp(c*(b*x+a))-1)+1/
2/b/c*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^2)*2^(1/2)-1/2/b/c*ln(exp(2*c*(b*x+a
))-(2^(1/2)-1)^2)*2^(1/2)-2*a/b+1/2/b/c*ln(exp(2*c*(b*x+a))-(1+2^(1/2))...
```

3.300.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(90) = 180.

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.19

$$\int e^{c(a+bx)}\coth^{-1}(\operatorname{csch}(ac+bcx))\,dx$$

$$= \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(-\frac{\sinh(bcx+ac)+1}{\sinh(bcx+ac)-1}\right)+\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh(bcx+ac)^2-4\left(3\sqrt{2}+4\right)\cosh(bcx\cdots}{\cosh(bcx+ac)^2\cdots}\right.}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log(-(sinh(b*c*x + a*c) + 1)/
(sinh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c
)^2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2)
 + 3)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c
*x + a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/
(cosh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x
+ a*c)^2)))/(b*c)
```

3.300.6 Sympy [F]

$$\int e^{c(a+bx)} \coth^{-1}(\operatorname{csch}(ac+bcx))\,dx = e^{ac} \int e^{bcx} \operatorname{acoth}\left(\operatorname{csch}\left(ac+bcx\right)\right) dx$$

input
```
integrate(exp(c*(b*x+a))*acoth(csch(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*acoth(csch(a*c + b*c*x)), x)
```

3.300.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(90) = 180$.

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.72

$$\int e^{c(a+bx)} \coth^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{\operatorname{arcoth}\left(\operatorname{csch}\left(bcx+ac\right)\right)e^{((bx+a)c)}}{bc} + \frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}+1}{\sqrt{2}+e^{(bcx+ac)}-1}\right)}{2\,bc} - \frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}-1}{\sqrt{2}+e^{(bcx+ac)}+1}\right)}{2\,bc} + \frac{\log\left(e^{(2\,bcx+2\,ac)}+2\,e^{(bcx+ac)}-1\right)}{2\,bc} + \frac{\log\left(e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}-1\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arccoth(csch(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(sqrt(
2) - e^(b*c*x + a*c) + 1)/(sqrt(2) + e^(b*c*x + a*c) - 1))/(b*c) - 1/2*sqr
t(2)*log(-(sqrt(2) - e^(b*c*x + a*c) - 1)/(sqrt(2) + e^(b*c*x + a*c) + 1))
/(b*c) + 1/2*log(e^(2*b*c*x + 2*a*c) + 2*e^(b*c*x + a*c) - 1)/(b*c) + 1/2*
log(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) - 1)/(b*c)
```

3.300.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.47

$$\int e^{c(a+bx)}\coth^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{e^{((bx+a)c)}\log\left(-\frac{e^{(bcx+ac)}-e^{(-bcx-ac)}+2}{e^{(bcx+ac)}-e^{(-bcx-ac)}-2}\right)}{2\,bc} + \frac{\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}{\left|4\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}\right)+\log\left(\left|e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right|\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x, algorithm="giac")
```

output
```
1/2*e^((b*x + a)*c)*log(-(e^(b*c*x + a*c) - e^(-b*c*x - a*c) + 2)/(e^(b*c*
x + a*c) - e^(-b*c*x - a*c) - 2))/(b*c) + 1/2*(sqrt(2)*log(abs(-4*sqrt(2)
+ 2*e^(2*b*c*x + 2*a*c) - 6)/abs(4*sqrt(2) + 2*e^(2*b*c*x + 2*a*c) - 6)) +
 log(abs(e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)))/(b*c)
```

3.300.9 Mupad [B] (verification not implemented)

Time = 4.47 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.67

$$\int e^{c(a+bx)}\coth^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{e^{a\,c+b\,c\,x}\,\ln\left(\frac{e^{b\,c\,x}\,e^{a\,c}}{2}-\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}+1\right)}{2\,b\,c} - \frac{e^{a\,c+b\,c\,x}\,\ln\left(\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}-\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+1\right)}{2\,b\,c} + \frac{\ln\left(6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}-2\,\sqrt{2}-8\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}+1\right)}{2\,b\,c} - \frac{\ln\left(2\,\sqrt{2}-8\,e^{2\,c\,(a+b\,x)}-6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}-1\right)}{2\,b\,c}$$

input
```
int(acoth(1/sinh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

output
```
(exp(a*c + b*c*x)*log((exp(b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2
+ 1))/(2*b*c) - (exp(a*c + b*c*x)*log((exp(-b*c*x)*exp(-a*c))/2 - (exp(b*c
*x)*exp(a*c))/2 + 1))/(2*b*c) + (log(6*2^(1/2)*exp(2*c*(a + b*x)) - 2*2^(1
/2) - 8*exp(2*c*(a + b*x)))*(2^(1/2) + 1))/(2*b*c) - (log(2*2^(1/2) - 8*ex
p(2*c*(a + b*x)) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c)
```

3.300. $\int e^{c(a+bx)} \coth^{-1}(\text{csch}(ac + bcx)) \, dx$

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
    return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):    #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
      return expnType(expn.args[0])    #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
  elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
  elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
  elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
   print ("Enter grade_antiderivative for sagemath")
   print("Enter grade_antiderivative, result=",result)
   print("Enter grade_antiderivative, optimal=",optimal)
   print("type(anti)=",type(result))
   print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
   if result.has(I):
      if optimal.has(I): #both result and optimal complex
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
         else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
      else: #result contains complex but optimal is not
         grade = "C"
         grade_annotation ="Result contains complex when optimal does not."
   else: # result do not contain complex, this assumes optimal do not as well
      if leaf_count_result <= 2*leaf_count_optimal:
         grade = "A"
         grade_annotation =" "
      else:
         grade = "B"
         grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
   grade = "C"
   grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```